Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/1.2.1.3/94-1.2.1.3-b

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**310**]. This is test number [94].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.68 (309)	0.32 (1)
Mathematica	97.10 (301)	2.90 (9)
Maple	92.26 (286)	7.74 (24)
Fricas	89.03 (276)	10.97 (34)
Giac	77.10 (239)	22.90 (71)
Reduce	76.13 (236)	23.87 (74)
Maxima	73.23 (227)	26.77 (83)
Mupad	69.03 (214)	30.97 (96)
Sympy	45.81 (142)	54.19 (168)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

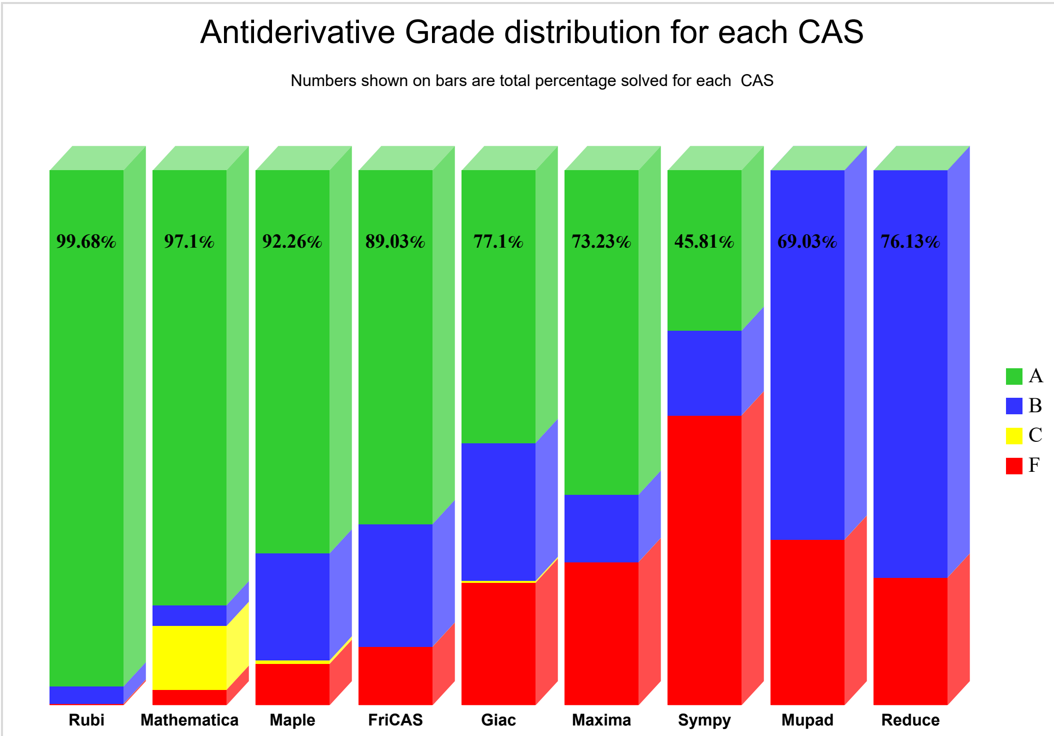
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

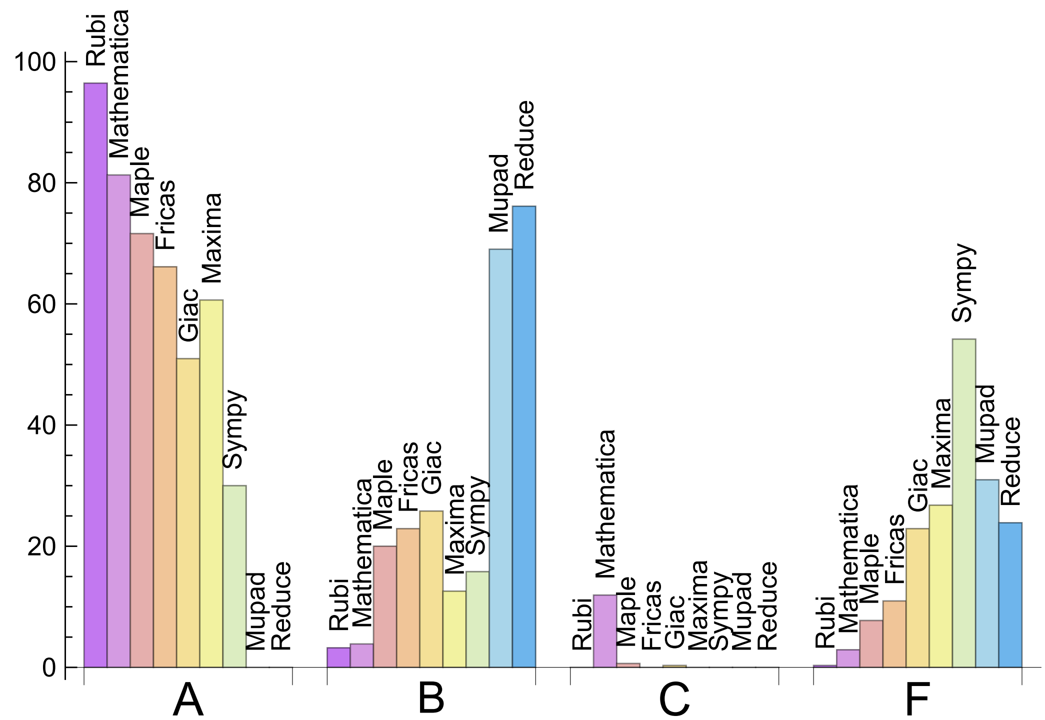
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.452	3.226	0.000	0.323
Mathematica	81.290	3.871	11.935	2.903
Maple	71.613	20.000	0.645	7.742
Fricas	66.129	22.903	0.000	10.968
Maxima	60.645	12.581	0.000	26.774
Giac	50.968	25.806	0.323	22.903
Sympy	30.000	15.806	0.000	54.194
Mupad	0.000	69.032	0.000	30.968
Reduce	0.000	76.129	0.000	23.871

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	9	100.00	0.00	0.00
Maple	24	100.00	0.00	0.00
Fricas	34	70.59	29.41	0.00
Giac	71	83.10	8.45	8.45
Reduce	74	100.00	0.00	0.00
Maxima	83	98.80	0.00	1.20
Mupad	96	0.00	100.00	0.00
Sympy	168	60.12	39.88	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.08
Giac	0.21
Rubi	0.37
Reduce	0.50
Maple	1.63
Fricas	1.90
Mathematica	3.70
Mupad	3.80
Sympy	5.08

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	197.83	1.20	163.00	1.03
Mathematica	207.10	1.19	150.00	1.04
Maxima	243.23	1.57	150.00	1.25
Maple	285.05	1.41	161.00	1.05
Giac	425.15	2.58	215.00	1.40
Reduce	579.07	3.18	258.50	1.95
Fricas	627.93	3.05	211.00	1.54
Sympy	796.22	3.87	174.00	1.60
Mupad	996.27	4.36	156.00	1.34

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

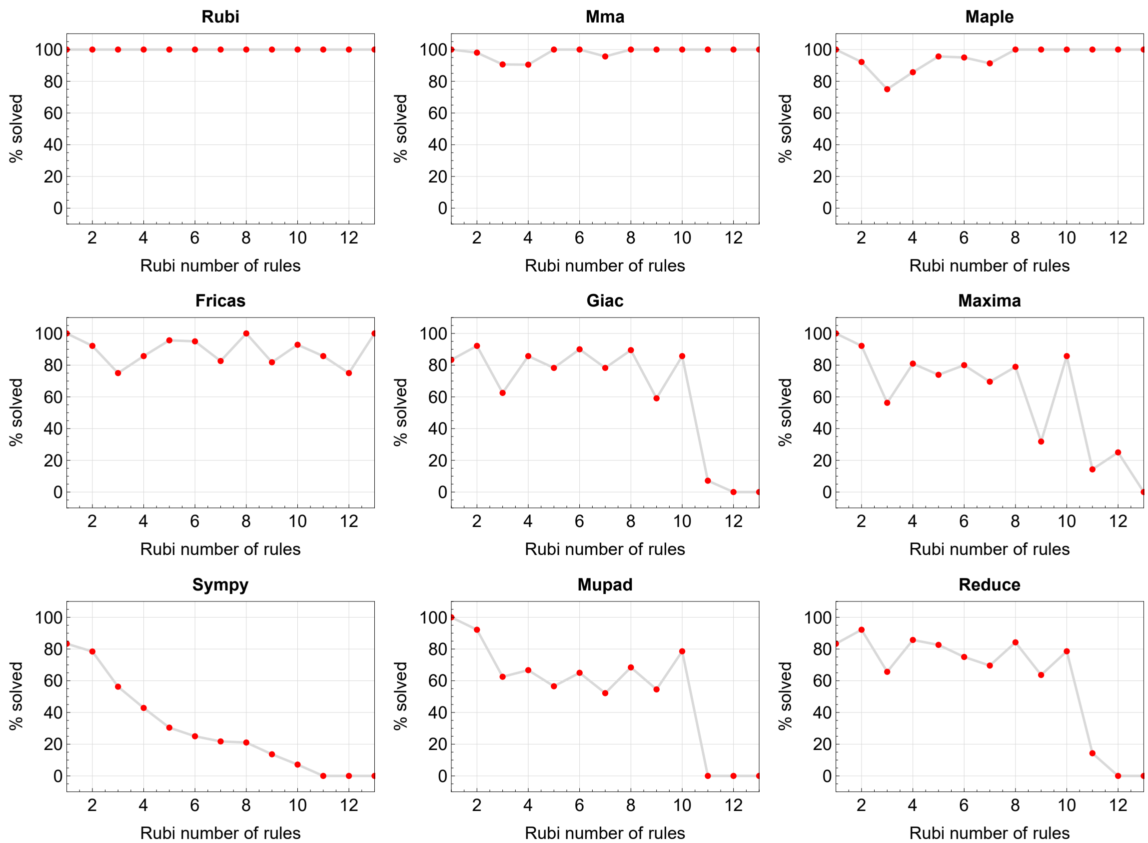


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

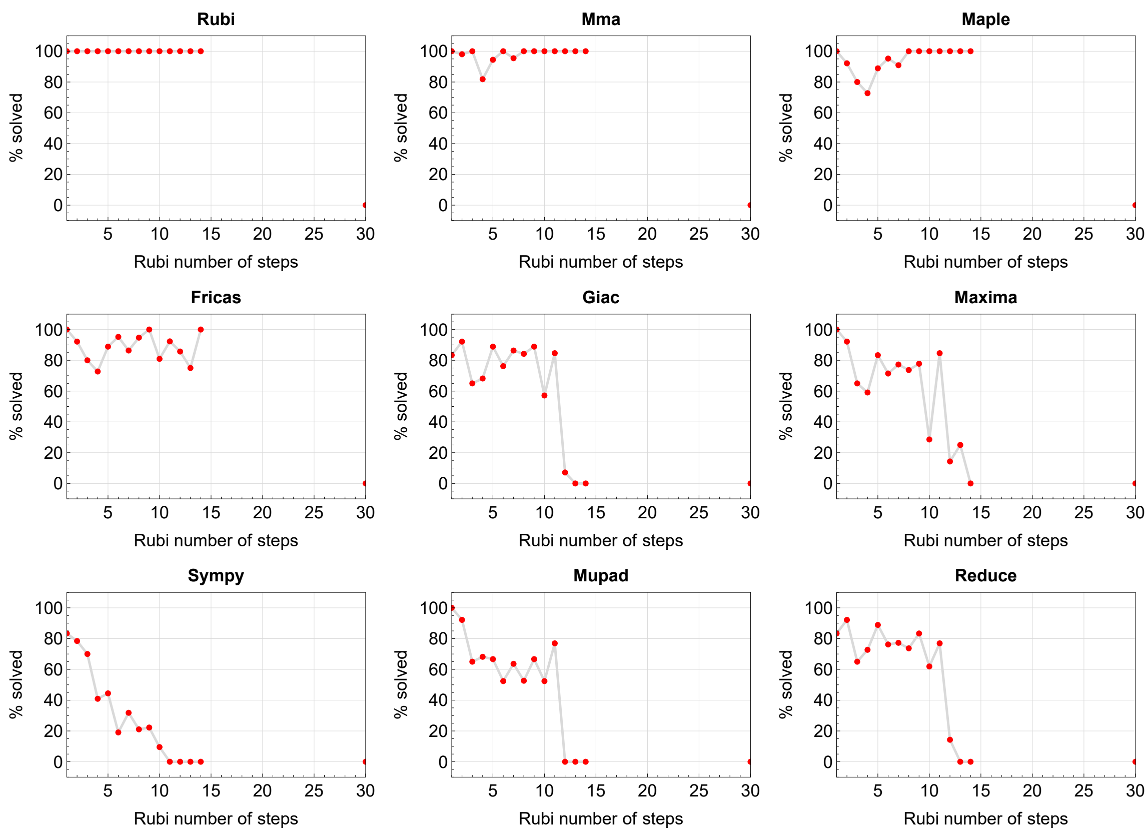


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

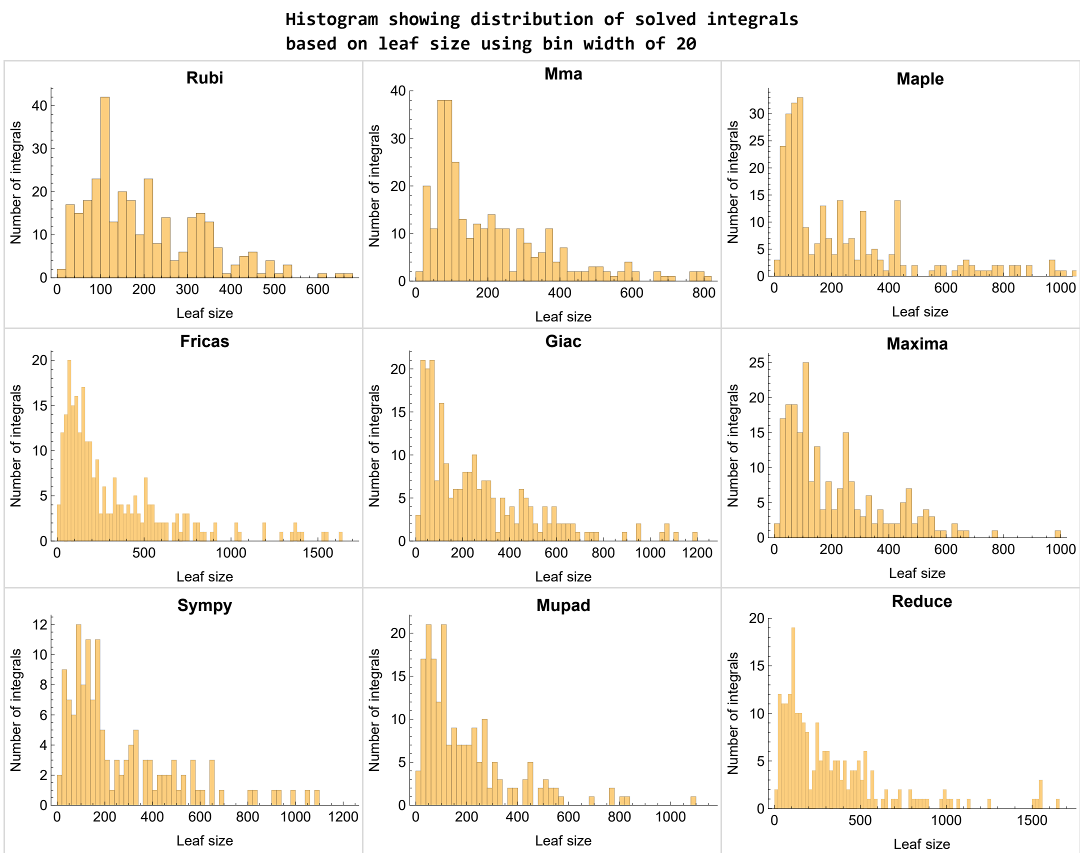


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

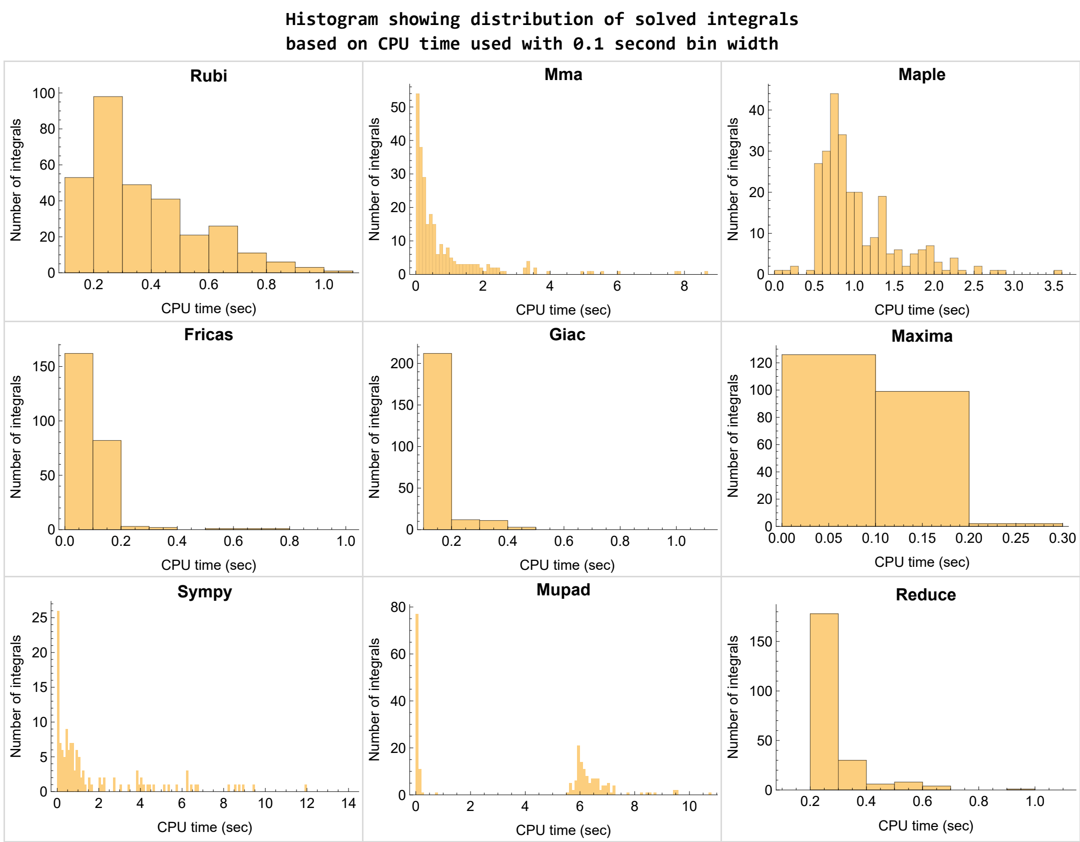


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

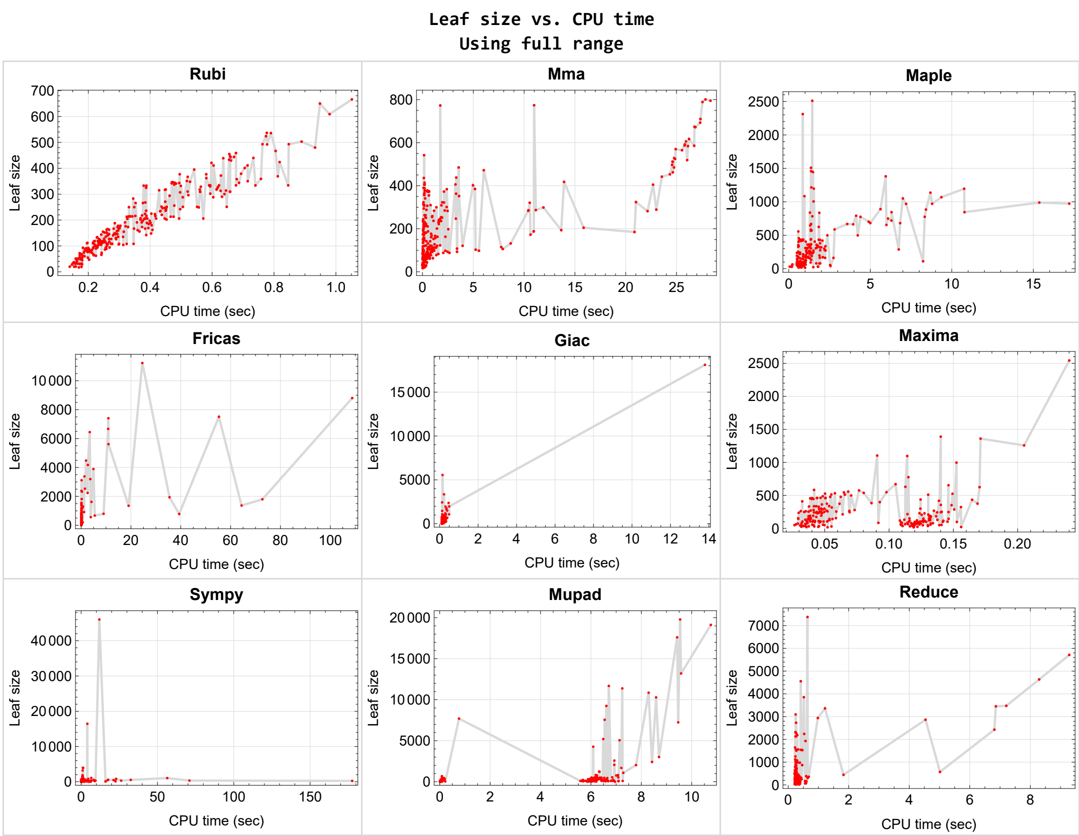


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{12, 13, 14\}$

Mathematica $\{7, 9, 11, 13, 14, 307\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

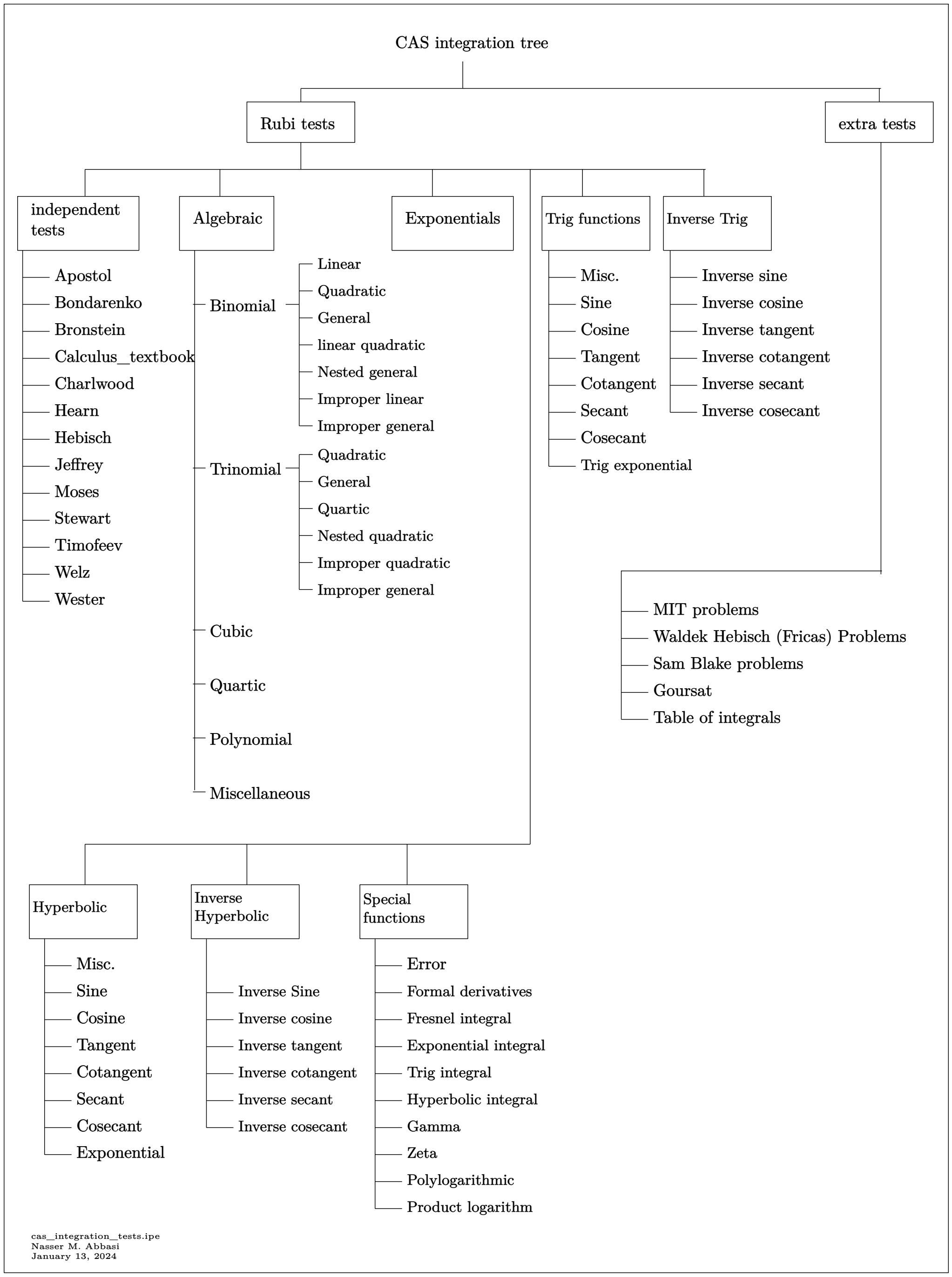
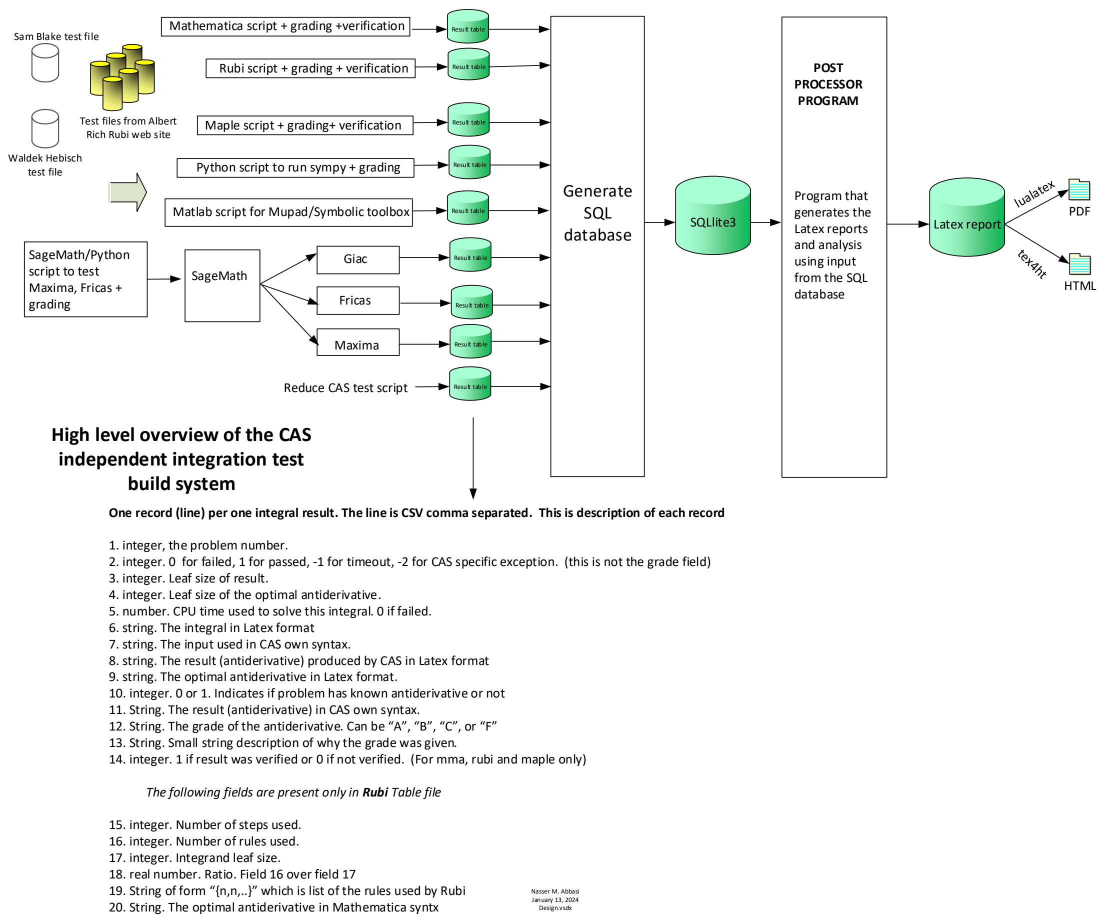


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310 }

B grade { 6, 7, 16, 17, 18, 25, 26, 139, 145, 147 }

C grade { }

F normal fail { 303 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 8, 9, 10, 11, 13, 14, 15, 19, 20, 21, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 291, 294, 296, 297, 298, 299, 300, 301, 307, 310 }

B grade { 6, 7, 16, 17, 18, 22, 23, 25, 26, 33, 185, 295 }

C grade { 4, 24, 139, 140, 142, 143, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293 }

F normal fail { 5, 12, 302, 303, 304, 305, 306, 308, 309 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 15, 19, 20, 21, 24, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 144, 145, 146, 148, 149, 150, 151, 152, 155, 156, 157, 158, 159, 162, 163, 164, 165, 166, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 186, 187, 188, 189, 190, 191, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 266, 272, 310 }

B grade { 3, 4, 16, 17, 18, 22, 23, 25, 26, 33, 46, 139, 143, 153, 154, 160, 161, 167, 168, 175, 176, 177, 183, 184, 185, 192, 193, 194, 262, 263, 264, 265, 267, 268, 269, 270, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297 }

C grade { 147, 259 }

F normal fail { 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 15, 20, 21, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 90, 91, 92, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 155, 156, 157, 158, 162, 163, 164, 165, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 186, 187, 188, 189, 190, 191, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 292, 293, 310 }

B grade { 3, 4, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 33, 46, 53, 54, 55, 56, 69, 70, 71, 72, 73, 74, 86, 87, 88, 89, 93, 94, 95, 96, 97, 101, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 142, 153, 175, 176, 177, 183, 184, 185, 192, 193, 194, 202, 247, 272, 282, 288, 289, 290, 291, 294, 295, 296, 297 }

C grade { }

F normal fail { 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309 }

F(-1) timedout fail { 102, 133, 138, 154, 159, 160, 161, 166, 167, 168 }

F(-2) exception fail { }

Maxima

A grade { 15, 20, 21, 24, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 146, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 167, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 186, 187, 188, 190, 191, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 220, 221, 222, 223, 224, 225, 226, 227, 228, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 297, 310 }

B grade { 16, 17, 18, 19, 22, 23, 25, 26, 27, 33, 46, 97, 102, 154, 161, 166, 168, 175, 176, 177, 183, 184, 185, 189, 192, 193, 194, 216, 217, 218, 219, 229, 230, 231, 232, 233, 234, 295, 296 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 147, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309 }

F(-1) timedout fail { }

F(-2) exception fail { 141 }

Giac

A grade { 19, 20, 21, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 141, 143, 144, 146, 147, 148, 149, 150, 151, 155, 156, 157, 158, 162, 163, 164, 165, 169, 170, 171, 172, 173, 178, 179, 180, 181, 182, 186, 187, 188, 189, 190, 191, 195, 196, 197, 198, 199, 200, 207, 208, 209, 210, 211, 212, 220, 221, 222, 223, 224, 225, 235, 236, 237, 238, 239, 244, 245, 246, 247, 248, 249, 252, 253, 254, 255, 256, 257, 258, 259 }

B grade { 16, 17, 18, 22, 23, 24, 25, 26, 27, 33, 34, 46, 56, 62, 72, 97, 105, 106, 111, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 145, 154, 161, 168, 176, 177, 183, 185, 192, 193, 194, 201, 202, 203, 204, 205, 206, 213, 214, 215, 216, 217, 218, 219, 226, 227, 228, 229, 230, 231, 232, 233, 234, 240, 241, 242, 243, 250, 251, 260, 261, 295, 296, 297 }

C grade { 140 }

F normal fail { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310 }

F(-1) timedout fail { 142, 153, 160, 167, 175, 184 }

F(-2) exception fail { 1, 152, 159, 166, 174, 290 }

Mupad

A grade { }

B grade { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 151, 158, 165, 172, 173, 181, 182, 189, 190, 191, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 295, 296, 297, 310 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 148, 149, 150, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 174, 175, 176, 177, 178, 179, 180, 183, 184, 185, 186, 187, 188, 192, 193, 194, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309 }

F(-2) exception fail { }

Sympy

A grade { 20, 28, 29, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 92, 98, 99, 100, 103, 104, 105, 106, 107, 108, 111, 112, 113, 114, 117, 118, 119, 120, 141, 146, 149, 150, 151, 158, 165, 169, 170, 171, 172, 173, 181, 182, 190, 195, 196, 197, 198, 199, 207, 208, 209, 210, 211, 220, 221, 222, 223, 224, 235, 236, 237, 238, 239, 248, 301 }

B grade { 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 30, 33, 34, 46, 62, 79, 80, 81, 82, 83, 87, 88, 89, 90, 91, 95, 96, 97, 109, 110, 115, 116, 121, 122, 148, 155, 156, 157, 162, 163, 164, 191, 247, 257, 258, 295, 296, 297 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 124, 125, 126, 127, 128, 139, 140, 142, 143, 144, 145, 147, 152, 153, 154, 159, 160, 161, 166, 167, 168, 174, 175, 176, 177, 178, 179, 180, 183, 184, 185, 186, 187, 188, 189, 192, 200, 201, 202, 203, 212, 213, 214, 240, 241, 242, 244, 245, 246, 249, 250, 252, 253, 254, 255, 256, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 286, 287, 289, 290, 291, 292, 293, 294, 298, 300 }

F(-1) timedout fail { 15, 59, 60, 61, 73, 74, 75, 76, 77, 78, 84, 85, 86, 93, 94, 101, 102, 123, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 193, 194, 204, 205, 206, 215, 216, 217, 218, 219, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 243, 251, 259, 260, 261, 283, 284, 285, 288, 299, 302, 303, 304, 305, 306, 307, 308, 309, 310 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 156, 157, 158, 160, 161, 163, 164, 167, 168, 171, 172, 173, 174, 175, 176, 177, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 255, 256, 257, 258, 259, 260, 261, 295, 296, 297 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 148, 155, 159, 162, 165, 166, 169, 170, 178, 179, 186, 194, 253, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	65	77	0	0	0	0	0	23	0
N.S.	1	1.08	1.28	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.173	0.798	0.000	0.000	0.000	0.000	0.000	0.263	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	101	102	0	0	0	0	0	54	0
N.S.	1	1.38	1.40	0.00	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.229	1.111	0.000	0.000	0.000	0.000	0.000	0.267	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	115	278	0	191	0	0	44	0
N.S.	1	1.00	1.28	3.09	0.00	2.12	0.00	0.00	0.49	0.00
time (sec)	N/A	0.225	7.748	2.204	0.000	0.079	0.000	0.000	2.291	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	131	282	275	0	191	0	0	42	0
N.S.	1	1.46	3.13	3.06	0.00	2.12	0.00	0.00	0.47	0.00
time (sec)	N/A	0.289	22.148	1.928	0.000	0.073	0.000	0.000	0.489	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	178	0	0	0	0	0	0	172	0
N.S.	1	1.45	0.00	0.00	0.00	0.00	0.00	0.00	1.40	0.00
time (sec)	N/A	0.304	0.000	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	250	324	0	0	0	0	0	192	0
N.S.	1	2.19	2.84	0.00	0.00	0.00	0.00	0.00	1.68	0.00
time (sec)	N/A	0.349	21.010	0.000	0.000	0.000	0.000	0.000	0.412	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	250	284	0	0	0	0	0	156	0
N.S.	1	2.23	2.54	0.00	0.00	0.00	0.00	0.00	1.39	0.00
time (sec)	N/A	0.331	10.406	0.000	0.000	0.000	0.000	0.000	0.258	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	163	185	0	0	0	0	0	175	0
N.S.	1	1.46	1.65	0.00	0.00	0.00	0.00	0.00	1.56	0.00
time (sec)	N/A	0.259	20.866	0.000	0.000	0.000	0.000	0.000	0.327	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	163	205	0	0	0	0	0	111	0
N.S.	1	1.46	1.83	0.00	0.00	0.00	0.00	0.00	0.99	0.00
time (sec)	N/A	0.267	15.855	0.000	0.000	0.000	0.000	0.000	0.319	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	152	173	0	0	0	0	0	53	0
N.S.	1	1.36	1.54	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.259	10.634	0.000	0.000	0.000	0.000	0.000	1.105	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	152	194	0	0	0	0	0	47	0
N.S.	1	1.33	1.70	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.252	13.654	0.000	0.000	0.000	0.000	0.000	0.332	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	244	332	0	0	0	0	0	0	77	0
N.S.	1	1.36	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.499	0.000	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	244	266	188	0	0	0	0	0	77	0
N.S.	1	1.09	0.77	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.377	10.946	0.000	0.000	0.000	0.000	0.000	0.310	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	244	266	188	0	0	0	0	0	106	0
N.S.	1	1.09	0.77	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.353	10.954	0.000	0.000	0.000	0.000	0.000	0.571	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	58	47	0	0	165	99
N.S.	1	1.00	1.00	0.97	1.87	1.52	0.00	0.00	5.32	3.19
time (sec)	N/A	0.172	0.182	1.785	0.109	0.106	0.000	0.000	0.298	6.796

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	249	138	157	238	238	245	158	131	129
N.S.	1	12.45	6.90	7.85	11.90	11.90	12.25	7.90	6.55	6.45
time (sec)	N/A	0.557	0.071	0.685	0.046	0.090	177.578	0.143	0.303	0.087

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	167	102	115	174	174	178	116	98	96
N.S.	1	8.35	5.10	5.75	8.70	8.70	8.90	5.80	4.90	4.80
time (sec)	N/A	0.384	0.050	0.714	0.043	0.076	3.801	0.110	0.351	7.259

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	94	66	74	110	110	112	131	65	63
N.S.	1	4.70	3.30	3.70	5.50	5.50	5.60	6.55	3.25	3.15
time (sec)	N/A	0.269	0.027	0.580	0.045	0.077	0.648	0.142	0.287	0.040

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	35	30	31	44	44	44	30	32	30
N.S.	1	1.94	1.67	1.72	2.44	2.44	2.44	1.67	1.78	1.67
time (sec)	N/A	0.152	0.012	0.569	0.032	0.068	0.145	0.142	0.271	0.026

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	15	18	17	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.79	0.95	0.89	0.89
time (sec)	N/A	0.178	0.013	0.787	0.052	0.072	0.262	0.120	0.258	6.955

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	34	34	33	33	33	32	32	34	34
N.S.	1	1.70	1.70	1.65	1.65	1.65	1.60	1.60	1.70	1.70
time (sec)	N/A	0.159	0.015	0.592	0.046	0.082	0.263	0.142	0.292	0.028

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	68	70	90	90	90	76	89	60
N.S.	1	1.00	3.40	3.50	4.50	4.50	4.50	3.80	4.45	3.00
time (sec)	N/A	0.159	0.025	0.655	0.046	0.076	1.216	0.128	0.284	7.086

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	104	108	143	143	144	118	153	93
N.S.	1	1.00	5.20	5.40	7.15	7.15	7.20	5.90	7.65	4.65
time (sec)	N/A	0.155	0.042	0.708	0.054	0.076	16.163	0.127	0.280	6.916

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	60	132	54	84	103	388	190	110	90
N.S.	1	1.25	2.75	1.12	1.75	2.15	8.08	3.96	2.29	1.88
time (sec)	N/A	0.188	0.537	0.628	0.092	0.090	4.043	0.167	0.266	7.100

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	142	117	135	146	146	163	150	149	138
N.S.	1	4.18	3.44	3.97	4.29	4.29	4.79	4.41	4.38	4.06
time (sec)	N/A	0.387	0.028	0.524	0.029	0.068	0.029	0.126	0.262	6.834

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	104	86	99	106	106	117	109	108	102
N.S.	1	3.06	2.53	2.91	3.12	3.12	3.44	3.21	3.18	3.00
time (sec)	N/A	0.313	0.021	0.517	0.039	0.084	0.025	0.118	0.249	6.034

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	66	57	63	66	66	73	68	67	66
N.S.	1	1.94	1.68	1.85	1.94	1.94	2.15	2.00	1.97	1.94
time (sec)	N/A	0.243	0.033	0.218	0.035	0.071	0.022	0.141	0.257	0.020

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	31	30	30	32	31	30	30
N.S.	1	1.00	0.88	0.97	0.94	0.94	1.00	0.97	0.94	0.94
time (sec)	N/A	0.179	0.007	0.059	0.035	0.061	0.020	0.131	0.241	0.023

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	32	26	29	38	29
N.S.	1	1.00	1.00	0.97	0.94	1.03	0.84	0.94	1.23	0.94
time (sec)	N/A	0.174	0.015	0.605	0.055	0.066	0.129	0.141	0.257	0.032

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	34	34	33	33	33	32	32	34	34
N.S.	1	1.70	1.70	1.65	1.65	1.65	1.60	1.60	1.70	1.70
time (sec)	N/A	0.156	0.011	0.576	0.035	0.086	0.248	0.128	0.223	0.001

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	33	45	45	46	33	44	45
N.S.	1	1.00	1.00	0.94	1.29	1.29	1.31	0.94	1.26	1.29
time (sec)	N/A	0.154	0.017	0.569	0.031	0.072	0.464	0.130	0.236	0.027

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	33	56	56	58	33	55	56
N.S.	1	1.00	1.00	0.94	1.60	1.60	1.66	0.94	1.57	1.60
time (sec)	N/A	0.156	0.018	0.582	0.040	0.084	0.584	0.120	0.223	0.031

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	233	238	237	237	287	268	275	231
N.S.	1	1.00	2.16	2.20	2.19	2.19	2.66	2.48	2.55	2.14
time (sec)	N/A	0.346	0.061	0.504	0.030	0.066	0.038	0.110	0.254	5.679

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	194	192	194	194	226	215	222	185
N.S.	1	1.00	1.80	1.78	1.80	1.80	2.09	1.99	2.06	1.71
time (sec)	N/A	0.324	0.053	0.503	0.047	0.091	0.032	0.132	0.311	5.671

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	150	148	148	148	173	164	169	141
N.S.	1	1.00	1.39	1.37	1.37	1.37	1.60	1.52	1.56	1.31
time (sec)	N/A	0.299	0.040	0.520	0.038	0.067	0.031	0.128	0.257	0.035

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	106	103	102	102	119	113	116	98
N.S.	1	1.00	0.98	0.95	0.94	0.94	1.10	1.05	1.07	0.91
time (sec)	N/A	0.277	0.028	0.510	0.040	0.099	0.029	0.117	0.241	5.621

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	55	56	56	66	62	63	55
N.S.	1	1.00	1.00	0.89	0.90	0.90	1.06	1.00	1.02	0.89
time (sec)	N/A	0.225	0.021	0.210	0.049	0.068	0.021	0.107	0.259	5.693

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	32	32	27	26	26	29	26	28	26
N.S.	1	1.03	1.03	0.87	0.84	0.84	0.94	0.84	0.90	0.84
time (sec)	N/A	0.165	0.002	0.184	0.032	0.073	0.018	0.137	0.233	0.024

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	94	97	98	82	100	118	100
N.S.	1	1.00	0.93	1.09	1.13	1.14	0.95	1.16	1.37	1.16
time (sec)	N/A	0.254	0.040	0.600	0.042	0.090	0.177	0.129	0.239	5.712

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	86	95	101	152	104	153	178	105
N.S.	1	1.00	0.97	1.07	1.13	1.71	1.17	1.72	2.00	1.18
time (sec)	N/A	0.259	0.058	0.598	0.036	0.071	0.320	0.149	0.250	5.582

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	88	99	111	168	117	98	189	111
N.S.	1	1.00	0.94	1.05	1.18	1.79	1.24	1.04	2.01	1.18
time (sec)	N/A	0.257	0.070	0.585	0.036	0.142	0.638	0.164	0.219	0.063

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	98	101	129	160	138	105	167	122
N.S.	1	1.00	0.97	1.00	1.28	1.58	1.37	1.04	1.65	1.21
time (sec)	N/A	0.261	0.047	0.570	0.038	0.079	1.140	0.139	0.244	5.782

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	87	96	132	132	150	145	112	128
N.S.	1	1.00	0.82	0.91	1.25	1.25	1.42	1.37	1.06	1.21
time (sec)	N/A	0.260	0.042	0.592	0.035	0.070	2.062	0.158	0.291	0.039

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	90	101	148	148	165	100	145	145
N.S.	1	1.00	0.83	0.94	1.37	1.37	1.53	0.93	1.34	1.34
time (sec)	N/A	0.262	0.040	0.596	0.039	0.088	3.465	0.127	0.231	0.044

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	87	95	153	153	173	98	156	150
N.S.	1	1.00	0.81	0.88	1.42	1.42	1.60	0.91	1.44	1.39
time (sec)	N/A	0.262	0.044	0.607	0.038	0.085	5.399	0.132	0.248	6.872

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	390	402	410	410	495	465	465	374
N.S.	1	1.00	1.89	1.95	1.99	1.99	2.40	2.26	2.26	1.82
time (sec)	N/A	0.572	0.109	0.596	0.033	0.068	0.059	0.129	0.224	5.974

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	314	327	332	332	398	377	377	298
N.S.	1	1.00	1.52	1.59	1.61	1.61	1.93	1.83	1.83	1.45
time (sec)	N/A	0.493	0.087	0.582	0.038	0.065	0.046	0.135	0.267	0.080

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	244	252	260	260	303	287	289	229
N.S.	1	1.00	1.18	1.22	1.26	1.26	1.47	1.39	1.40	1.11
time (sec)	N/A	0.447	0.065	0.577	0.035	0.072	0.041	0.151	0.237	0.058

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	174	177	184	184	211	200	201	168
N.S.	1	1.00	0.84	0.86	0.89	0.89	1.02	0.97	0.98	0.82
time (sec)	N/A	0.419	0.044	0.580	0.037	0.089	0.039	0.131	0.248	0.048

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	95	99	106	106	124	114	113	98
N.S.	1	1.00	0.90	0.93	1.00	1.00	1.17	1.08	1.07	0.92
time (sec)	N/A	0.298	0.044	0.506	0.052	0.064	0.026	0.142	0.258	0.030

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	60	51	50	50	58	50	51	50
N.S.	1	1.00	1.33	1.13	1.11	1.11	1.29	1.11	1.13	1.11
time (sec)	N/A	0.181	0.003	0.500	0.050	0.064	0.025	0.118	0.244	0.015

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	174	224	242	243	207	258	289	260
N.S.	1	1.00	1.03	1.33	1.43	1.44	1.22	1.53	1.71	1.54
time (sec)	N/A	0.394	0.097	0.701	0.042	0.070	0.331	0.123	0.260	0.039

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	175	233	249	354	246	328	399	311
N.S.	1	1.00	0.97	1.29	1.38	1.97	1.37	1.82	2.22	1.73
time (sec)	N/A	0.405	0.150	0.667	0.034	0.088	0.633	0.130	0.234	5.888

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	174	233	258	394	282	251	450	275
N.S.	1	1.00	0.94	1.26	1.39	2.13	1.52	1.36	2.43	1.49
time (sec)	N/A	0.395	0.154	0.657	0.052	0.081	1.517	0.110	0.257	6.072

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	232	233	270	411	294	251	455	268
N.S.	1	1.00	1.23	1.23	1.43	2.17	1.56	1.33	2.41	1.42
time (sec)	N/A	0.401	0.112	0.681	0.039	0.081	3.831	0.124	0.245	0.064

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	221	240	279	405	304	373	435	277
N.S.	1	1.00	1.17	1.27	1.48	2.14	1.61	1.97	2.30	1.47
time (sec)	N/A	0.389	0.109	0.696	0.041	0.080	9.430	0.132	0.236	6.078

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	212	234	298	365	326	252	368	243
N.S.	1	1.00	1.08	1.19	1.51	1.85	1.65	1.28	1.87	1.23
time (sec)	N/A	0.399	0.111	0.663	0.050	0.081	26.240	0.145	0.233	6.181

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	198	219	292	292	337	254	263	273
N.S.	1	1.00	0.97	1.07	1.43	1.43	1.65	1.25	1.29	1.34
time (sec)	N/A	0.381	0.092	0.671	0.046	0.079	70.948	0.142	0.247	6.258

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	202	233	317	317	0	258	326	299
N.S.	1	1.00	0.98	1.13	1.54	1.54	0.00	1.25	1.58	1.45
time (sec)	N/A	0.391	0.098	0.667	0.055	0.077	0.000	0.116	0.238	0.063

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	202	233	328	328	0	258	337	310
N.S.	1	1.00	0.98	1.13	1.59	1.59	0.00	1.25	1.64	1.50
time (sec)	N/A	0.384	0.092	0.649	0.045	0.083	0.000	0.132	0.237	6.314

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	202	233	339	339	0	258	348	321
N.S.	1	1.00	0.98	1.13	1.65	1.65	0.00	1.25	1.69	1.56
time (sec)	N/A	0.385	0.098	0.638	0.053	0.085	0.000	0.137	0.228	6.285

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	542	557	584	584	694	658	655	542
N.S.	1	1.00	1.62	1.67	1.75	1.75	2.08	1.97	1.96	1.62
time (sec)	N/A	0.846	0.157	0.585	0.042	0.074	0.059	0.124	0.224	6.843

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	436	455	478	478	564	536	532	438
N.S.	1	1.00	1.31	1.36	1.43	1.43	1.69	1.60	1.59	1.31
time (sec)	N/A	0.740	0.132	0.619	0.054	0.070	0.053	0.109	0.257	0.129

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	323	353	363	363	435	411	409	317
N.S.	1	1.00	0.97	1.06	1.09	1.09	1.30	1.23	1.22	0.95
time (sec)	N/A	0.677	0.097	0.581	0.048	0.068	0.044	0.128	0.235	0.099

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	238	251	262	262	306	287	286	227
N.S.	1	1.00	0.71	0.75	0.78	0.78	0.92	0.86	0.86	0.68
time (sec)	N/A	0.622	0.060	0.581	0.031	0.068	0.037	0.124	0.267	6.885

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	135	143	154	154	182	165	163	140
N.S.	1	1.00	0.91	0.97	1.04	1.04	1.23	1.11	1.10	0.95
time (sec)	N/A	0.403	0.050	0.514	0.029	0.070	0.033	0.131	0.235	0.040

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	55	85	74	73	73	85	73	74	73
N.S.	1	0.98	1.52	1.32	1.30	1.30	1.52	1.30	1.32	1.30
time (sec)	N/A	0.198	0.003	0.497	0.029	0.067	0.024	0.128	0.262	0.020

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	311	414	447	448	410	488	530	494
N.S.	1	1.00	1.07	1.43	1.54	1.54	1.41	1.68	1.83	1.70
time (sec)	N/A	0.632	0.167	0.743	0.041	0.082	0.496	0.125	0.223	6.350

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	405	433	456	621	454	563	689	826
N.S.	1	1.00	1.31	1.40	1.48	2.01	1.47	1.82	2.23	2.67
time (sec)	N/A	0.658	0.179	0.786	0.039	0.078	1.043	0.138	0.276	6.193

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	414	425	464	691	490	469	785	681
N.S.	1	1.00	1.38	1.42	1.55	2.30	1.63	1.56	2.62	2.27
time (sec)	N/A	0.640	0.203	0.711	0.047	0.083	2.720	0.135	0.239	0.086

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	294	431	478	732	530	464	833	548
N.S.	1	1.00	0.95	1.39	1.54	2.36	1.71	1.50	2.69	1.77
time (sec)	N/A	0.658	0.125	0.708	0.062	0.083	8.241	0.140	0.248	6.176

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	405	432	487	746	537	653	846	501
N.S.	1	1.00	1.29	1.38	1.55	2.38	1.71	2.08	2.69	1.60
time (sec)	N/A	0.645	0.211	0.655	0.048	0.086	32.514	0.151	0.265	6.141

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	388	433	499	730	0	458	808	494
N.S.	1	1.00	1.24	1.38	1.59	2.33	0.00	1.46	2.58	1.58
time (sec)	N/A	0.627	0.201	0.889	0.071	0.092	0.000	0.153	0.238	0.097

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	377	435	511	695	0	454	738	505
N.S.	1	1.00	1.18	1.36	1.60	2.17	0.00	1.42	2.31	1.58
time (sec)	N/A	0.616	0.218	0.868	0.066	0.100	0.000	0.177	0.271	6.414

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	366	426	527	624	0	459	637	448
N.S.	1	1.00	1.12	1.30	1.61	1.91	0.00	1.40	1.95	1.37
time (sec)	N/A	0.619	0.191	0.855	0.050	0.088	0.000	0.144	0.232	6.219

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	357	422	532	532	0	488	490	570
N.S.	1	1.00	1.08	1.28	1.61	1.61	0.00	1.48	1.48	1.73
time (sec)	N/A	0.594	0.178	0.852	0.058	0.093	0.000	0.119	0.227	0.092

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	359	425	546	546	0	488	577	513
N.S.	1	1.00	1.07	1.27	1.63	1.63	0.00	1.46	1.73	1.54
time (sec)	N/A	0.594	0.172	0.851	0.065	0.081	0.000	0.131	0.268	6.024

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	357	425	557	557	0	488	588	524
N.S.	1	1.00	1.07	1.27	1.67	1.67	0.00	1.46	1.76	1.57
time (sec)	N/A	0.590	0.175	0.876	0.068	0.085	0.000	0.127	0.230	6.242

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	217	232	230	550	908	257	337	249
N.S.	1	1.00	0.90	0.97	0.96	2.29	3.78	1.07	1.40	1.04
time (sec)	N/A	0.451	0.179	1.115	0.124	0.089	1.652	0.122	0.234	6.244

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	151	164	160	398	641	171	238	175
N.S.	1	1.00	0.90	0.98	0.96	2.38	3.84	1.02	1.43	1.05
time (sec)	N/A	0.348	0.139	1.066	0.132	0.126	1.084	0.108	0.271	0.114

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	99	97	100	235	425	102	138	114
N.S.	1	1.00	0.92	0.90	0.93	2.18	3.94	0.94	1.28	1.06
time (sec)	N/A	0.263	0.092	1.024	0.129	0.080	0.734	0.129	0.235	6.190

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	61	56	147	212	56	76	75
N.S.	1	1.00	1.02	0.95	0.88	2.30	3.31	0.88	1.19	1.17
time (sec)	N/A	0.218	0.053	0.877	0.132	0.087	0.386	0.129	0.257	0.060

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	31	98	124	31	34	32
N.S.	1	1.00	1.00	0.76	0.74	2.33	2.95	0.74	0.81	0.76
time (sec)	N/A	0.164	0.015	0.779	0.116	0.074	0.127	0.151	0.230	0.030

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	91	92	98	200	0	103	106	535
N.S.	1	1.00	0.83	0.84	0.90	1.83	0.00	0.94	0.97	4.91
time (sec)	N/A	0.274	0.064	1.337	0.153	0.640	0.000	0.137	0.224	7.087

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	148	157	216	562	0	237	434	810
N.S.	1	1.00	0.86	0.91	1.25	3.25	0.00	1.37	2.51	4.68
time (sec)	N/A	0.374	0.185	1.283	0.114	3.876	0.000	0.131	0.255	7.074

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	223	243	419	1350	0	399	1036	1680
N.S.	1	1.00	0.89	0.97	1.67	5.38	0.00	1.59	4.13	6.69
time (sec)	N/A	0.506	0.352	1.365	0.139	19.063	0.000	0.127	0.245	7.206

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	296	307	360	356	1190	1091	366	787	370
N.S.	1	1.00	1.03	1.21	1.20	4.01	3.67	1.23	2.65	1.25
time (sec)	N/A	0.523	0.164	1.145	0.140	0.103	6.674	0.129	0.230	0.190

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	231	263	268	849	836	273	577	276
N.S.	1	1.00	1.05	1.20	1.22	3.86	3.80	1.24	2.62	1.25
time (sec)	N/A	0.437	0.208	1.064	0.135	0.089	4.673	0.168	0.271	0.150

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	171	188	188	609	583	185	417	193
N.S.	1	1.00	1.06	1.17	1.17	3.78	3.62	1.15	2.59	1.20
time (sec)	N/A	0.359	0.126	1.029	0.132	0.085	2.729	0.132	0.239	6.114

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	118	119	127	130	384	382	128	255	203
N.S.	1	1.04	1.05	1.12	1.15	3.40	3.38	1.13	2.26	1.80
time (sec)	N/A	0.246	0.101	0.977	0.138	0.094	1.371	0.125	0.233	6.079

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	90	78	75	72	225	133	71	137	70
N.S.	1	1.29	1.11	1.07	1.03	3.21	1.90	1.01	1.96	1.00
time (sec)	N/A	0.200	0.071	0.791	0.123	0.085	0.493	0.140	0.298	0.068

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	49	48	140	90	47	67	44
N.S.	1	1.00	1.00	0.86	0.84	2.46	1.58	0.82	1.18	0.77
time (sec)	N/A	0.172	0.029	0.779	0.126	0.074	0.157	0.131	0.242	0.031

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	213	158	215	243	795	0	280	527	1086
N.S.	1	1.09	0.81	1.10	1.25	4.08	0.00	1.44	2.70	5.57
time (sec)	N/A	0.466	0.163	1.290	0.137	9.008	0.000	0.155	0.240	7.284

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	302	251	304	511	1940	0	518	1504	2029
N.S.	1	1.04	0.86	1.04	1.76	6.67	0.00	1.78	5.17	6.97
time (sec)	N/A	0.594	0.334	1.361	0.130	35.476	0.000	0.122	0.268	7.789

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	318	341	404	438	1403	1044	422	983	424
N.S.	1	1.04	1.11	1.32	1.43	4.58	3.41	1.38	3.21	1.39
time (sec)	N/A	0.614	0.258	1.044	0.125	0.110	56.413	0.156	0.250	0.181

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	236	263	309	352	1055	816	324	733	763
N.S.	1	1.09	1.21	1.42	1.62	4.86	3.76	1.49	3.38	3.52
time (sec)	N/A	0.387	0.200	1.017	0.149	0.101	22.880	0.130	0.278	7.005

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	127	186	215	248	752	468	239	524	265
N.S.	1	0.96	1.41	1.63	1.88	5.70	3.55	1.81	3.97	2.01
time (sec)	N/A	0.235	0.105	0.880	0.146	0.119	8.580	0.133	0.238	6.267

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	157	158	148	184	537	274	170	364	154
N.S.	1	1.03	1.04	0.97	1.21	3.53	1.80	1.12	2.39	1.01
time (sec)	N/A	0.293	0.105	0.867	0.137	0.095	3.994	0.131	0.237	6.646

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	119	101	95	114	355	180	98	229	100
N.S.	1	1.09	0.93	0.87	1.05	3.26	1.65	0.90	2.10	0.92
time (sec)	N/A	0.221	0.088	0.783	0.128	0.088	0.916	0.123	0.233	6.688

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	82	71	70	74	212	124	60	119	64
N.S.	1	1.09	0.95	0.93	0.99	2.83	1.65	0.80	1.59	0.85
time (sec)	N/A	0.188	0.051	0.846	0.121	0.082	0.252	0.151	0.254	6.683

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	350	263	435	525	1797	0	562	1251	2415
N.S.	1	1.14	0.86	1.42	1.71	5.85	0.00	1.83	4.07	7.87
time (sec)	N/A	0.715	0.297	1.353	0.149	72.752	0.000	0.140	0.238	8.425

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	480	378	567	997	0	0	884	3096	3015
N.S.	1	1.04	0.82	1.23	2.16	0.00	0.00	1.92	6.72	6.54
time (sec)	N/A	0.933	0.470	1.356	0.152	0.000	0.000	0.159	0.245	8.701

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	17	25	14	12	34	17
N.S.	1	1.00	1.00	0.76	1.00	1.47	0.82	0.71	2.00	1.00
time (sec)	N/A	0.170	0.017	0.998	0.119	0.072	0.060	0.127	0.255	0.027

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	20	21	21	26	24	21	19
N.S.	1	1.00	1.07	0.69	0.72	0.72	0.90	0.83	0.72	0.66
time (sec)	N/A	0.180	0.012	0.714	0.036	0.079	0.064	0.108	0.225	0.038

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	99	79	104	190	167	548	202	100
N.S.	1	1.00	0.85	0.68	0.90	1.64	1.44	4.72	1.74	0.86
time (sec)	N/A	0.268	0.121	1.928	0.046	0.076	0.934	0.138	0.221	0.051

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	96	78	104	143	167	310	149	100
N.S.	1	1.00	0.83	0.67	0.90	1.23	1.44	2.67	1.28	0.86
time (sec)	N/A	0.250	0.105	1.874	0.047	0.081	0.823	0.134	0.235	6.506

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	96	79	104	100	165	132	100	100
N.S.	1	1.00	0.84	0.69	0.91	0.88	1.45	1.16	0.88	0.88
time (sec)	N/A	0.241	0.100	1.684	0.049	0.076	0.838	0.126	0.260	0.041

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	97	83	112	110	150	137	102	111
N.S.	1	1.00	0.87	0.74	1.00	0.98	1.34	1.22	0.91	0.99
time (sec)	N/A	0.246	0.109	0.831	0.042	0.076	2.051	0.112	0.228	0.042

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	94	82	108	120	449	125	108	113
N.S.	1	1.00	0.84	0.73	0.96	1.07	4.01	1.12	0.96	1.01
time (sec)	N/A	0.245	0.115	0.836	0.052	0.078	0.382	0.125	0.230	6.416

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	95	80	109	133	653	117	120	100
N.S.	1	1.00	0.85	0.71	0.97	1.19	5.83	1.04	1.07	0.89
time (sec)	N/A	0.240	0.115	0.848	0.046	0.076	0.550	0.132	0.232	6.683

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	214	172	248	320	372	633	343	197
N.S.	1	1.00	0.98	0.79	1.14	1.47	1.71	2.90	1.57	0.90
time (sec)	N/A	0.374	0.213	2.096	0.050	0.082	1.000	0.133	0.263	0.062

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	213	170	248	247	371	280	259	197
N.S.	1	1.00	0.99	0.79	1.15	1.14	1.72	1.30	1.20	0.91
time (sec)	N/A	0.352	0.206	2.085	0.044	0.076	0.954	0.112	0.219	6.559

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	214	177	256	257	323	335	261	237
N.S.	1	1.00	1.00	0.83	1.20	1.20	1.51	1.57	1.22	1.11
time (sec)	N/A	0.334	0.252	1.031	0.047	0.081	6.498	0.132	0.231	0.040

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	214	180	254	269	301	320	268	249
N.S.	1	1.00	1.00	0.84	1.19	1.26	1.41	1.50	1.25	1.16
time (sec)	N/A	0.336	0.281	1.004	0.037	0.082	6.764	0.143	0.264	6.437

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	212	176	255	280	1426	317	279	251
N.S.	1	1.00	0.99	0.82	1.19	1.31	6.66	1.48	1.30	1.17
time (sec)	N/A	0.342	0.230	0.999	0.059	0.081	0.780	0.135	0.230	0.057

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	214	180	255	290	1855	311	290	258
N.S.	1	1.00	1.00	0.84	1.19	1.36	8.67	1.45	1.36	1.21
time (sec)	N/A	0.331	0.402	1.021	0.046	0.086	0.905	0.136	0.217	6.406

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	373	301	453	454	643	476	488	324
N.S.	1	1.00	1.07	0.86	1.30	1.30	1.85	1.37	1.40	0.93
time (sec)	N/A	0.529	0.496	2.177	0.040	0.076	1.198	0.159	0.217	0.083

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	373	298	461	463	561	621	490	394
N.S.	1	1.00	1.08	0.87	1.34	1.35	1.63	1.81	1.42	1.15
time (sec)	N/A	0.480	0.654	1.089	0.051	0.081	17.634	0.153	0.268	6.421

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	375	295	459	475	507	602	497	434
N.S.	1	1.00	1.08	0.85	1.33	1.37	1.47	1.74	1.44	1.25
time (sec)	N/A	0.473	0.440	1.096	0.050	0.083	17.607	0.173	0.229	6.305

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	373	304	461	487	476	599	508	455
N.S.	1	1.00	1.08	0.88	1.33	1.41	1.38	1.73	1.47	1.32
time (sec)	N/A	0.470	0.586	1.074	0.044	0.085	21.267	0.139	0.225	6.526

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	372	301	460	496	3218	594	519	452
N.S.	1	1.00	1.09	0.88	1.35	1.45	9.41	1.74	1.52	1.32
time (sec)	N/A	0.469	0.556	1.112	0.040	0.089	0.986	0.158	0.268	6.534

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	375	305	461	509	3952	589	530	454
N.S.	1	1.00	1.08	0.88	1.33	1.47	11.42	1.70	1.53	1.31
time (sec)	N/A	0.473	0.626	1.095	0.044	0.090	1.268	0.152	0.233	6.648

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	251	284	357	0	7410	0	680	899	11383
N.S.	1	1.06	1.20	1.51	0.00	31.27	0.00	2.87	3.79	48.03
time (sec)	N/A	0.647	1.139	1.704	0.000	10.943	0.000	0.218	0.249	7.244

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	213	259	271	0	4480	0	550	540	7560
N.S.	1	1.05	1.28	1.34	0.00	22.18	0.00	2.72	2.67	37.43
time (sec)	N/A	0.476	0.967	1.509	0.000	1.971	0.000	0.186	0.274	6.544

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	191	211	190	0	1538	0	316	258	4276
N.S.	1	1.07	1.18	1.06	0.00	8.59	0.00	1.77	1.44	23.89
time (sec)	N/A	0.373	0.504	1.411	0.000	0.132	0.000	0.171	0.233	6.090

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	160	188	133	0	2385	0	269	516	2065
N.S.	1	1.05	1.24	0.88	0.00	15.69	0.00	1.77	3.39	13.59
time (sec)	N/A	0.297	0.551	1.378	0.000	0.137	0.000	0.152	0.228	6.927

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	251	255	229	0	6448	0	941	983	10288
N.S.	1	1.27	1.29	1.16	0.00	32.73	0.00	4.78	4.99	52.22
time (sec)	N/A	0.454	0.943	1.444	0.000	3.452	0.000	0.241	0.315	8.585

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	318	300	345	0	11231	0	1751	2731	17610
N.S.	1	1.31	1.23	1.42	0.00	46.22	0.00	7.21	11.24	72.47
time (sec)	N/A	0.560	1.369	1.740	0.000	24.562	0.000	0.357	0.261	9.420

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	288	323	380	0	5611	0	755	1558	9253
N.S.	1	1.03	1.15	1.36	0.00	20.04	0.00	2.70	5.56	33.05
time (sec)	N/A	0.604	1.949	1.918	0.000	10.981	0.000	0.304	0.330	6.612

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	256	254	305	0	2327	0	546	870	5212
N.S.	1	1.08	1.07	1.28	0.00	9.78	0.00	2.29	3.66	21.90
time (sec)	N/A	0.478	1.838	1.655	0.000	0.328	0.000	0.267	0.463	6.488

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	235	285	244	0	3195	0	449	1528	5062
N.S.	1	1.04	1.27	1.08	0.00	14.20	0.00	2.00	6.79	22.50
time (sec)	N/A	0.433	2.184	1.510	0.000	3.679	0.000	0.203	0.468	7.135

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	310	309	336	0	7506	0	1182	2239	10862
N.S.	1	1.24	1.24	1.34	0.00	30.02	0.00	4.73	8.96	43.45
time (sec)	N/A	0.521	1.860	2.245	0.000	55.302	0.000	0.293	0.528	8.289

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	379	363	431	0	0	0	1970	3362	19787
N.S.	1	1.25	1.20	1.42	0.00	0.00	0.00	6.50	11.10	65.30
time (sec)	N/A	0.694	3.308	1.918	0.000	0.000	0.000	0.498	1.217	9.539

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	424	472	499	0	6669	0	1106	3452	11687
N.S.	1	1.07	1.19	1.26	0.00	16.84	0.00	2.79	8.72	29.51
time (sec)	N/A	0.818	6.033	2.369	0.000	10.858	0.000	0.324	6.861	6.710

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	401	402	420	0	3382	0	941	2428	7702
N.S.	1	1.08	1.08	1.13	0.00	9.09	0.00	2.53	6.53	20.70
time (sec)	N/A	0.705	4.970	2.066	0.000	1.345	0.000	0.306	6.814	0.764

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	411	352	337	0	4176	0	731	3475	7239
N.S.	1	1.17	1.01	0.96	0.00	11.93	0.00	2.09	9.93	20.68
time (sec)	N/A	0.714	3.597	1.798	0.000	2.712	0.000	0.273	7.207	9.463

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	440	438	433	0	8803	0	1633	4633	13200
N.S.	1	1.18	1.18	1.16	0.00	23.66	0.00	4.39	12.45	35.48
time (sec)	N/A	0.733	3.324	1.833	0.000	108.798	0.000	0.355	8.293	9.577

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	503	485	663	0	0	0	2380	5713	19125
N.S.	1	1.21	1.16	1.59	0.00	0.00	0.00	5.71	13.70	45.86
time (sec)	N/A	0.889	3.560	3.951	0.000	0.000	0.000	0.465	9.287	10.758

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	162	252	197	0	236	0	1495	113	310
N.S.	1	2.13	3.32	2.59	0.00	3.11	0.00	19.67	1.49	4.08
time (sec)	N/A	0.407	1.932	1.882	0.000	0.084	0.000	0.394	0.361	0.193

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	197	142	110	0	164	0	18107	59	206
N.S.	1	1.46	1.05	0.81	0.00	1.21	0.00	134.13	0.44	1.53
time (sec)	N/A	0.376	0.385	8.238	0.000	0.094	0.000	13.792	0.350	0.130

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	73	74	54	0	424	92	62	375	773
N.S.	1	1.11	1.12	0.82	0.00	6.42	1.39	0.94	5.68	11.71
time (sec)	N/A	0.243	0.245	2.536	0.000	0.103	3.873	0.115	0.376	6.238

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	467	91	287	0	1545	0	0	977	1244
N.S.	1	1.54	0.30	0.94	0.00	5.08	0.00	0.00	3.21	4.09
time (sec)	N/A	0.805	0.533	6.744	0.000	0.120	0.000	0.000	0.384	6.325

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	236	66	272	0	159	0	157	255	233
N.S.	1	1.52	0.43	1.75	0.00	1.03	0.00	1.01	1.65	1.50
time (sec)	N/A	0.450	0.221	2.254	0.000	0.078	0.000	0.265	0.400	0.071

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	67	22	41	0	22	0	63	42	38
N.S.	1	1.49	0.49	0.91	0.00	0.49	0.00	1.40	0.93	0.84
time (sec)	N/A	0.217	0.198	2.560	0.000	0.075	0.000	0.138	0.371	0.038

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	65	22	40	0	37	0	53	39	21
N.S.	1	2.41	0.81	1.48	0.00	1.37	0.00	1.96	1.44	0.78
time (sec)	N/A	0.218	0.113	0.915	0.000	0.076	0.000	0.149	0.365	6.214

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	47	24	22	21	14	22	21	19	26
N.S.	1	1.62	0.83	0.76	0.72	0.48	0.76	0.72	0.66	0.90
time (sec)	N/A	0.203	0.043	1.504	0.156	0.084	5.127	0.135	0.365	0.100

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	C	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	97	26	61	0	19	0	23	49	39
N.S.	1	2.16	0.58	1.36	0.00	0.42	0.00	0.51	1.09	0.87
time (sec)	N/A	0.227	0.295	1.036	0.000	0.078	0.000	0.134	0.391	0.218

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	247	241	266	301	580	493	303	24	0
N.S.	1	0.96	0.94	1.04	1.18	2.27	1.93	1.18	0.09	0.00
time (sec)	N/A	0.447	1.630	1.240	0.047	0.110	0.643	0.132	200.051	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	169	168	170	192	374	323	202	312	0
N.S.	1	0.99	0.98	0.99	1.12	2.19	1.89	1.18	1.82	0.00
time (sec)	N/A	0.301	0.900	1.066	0.039	0.100	0.580	0.127	0.430	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	111	104	96	118	232	175	114	165	0
N.S.	1	1.07	1.00	0.92	1.13	2.23	1.68	1.10	1.59	0.00
time (sec)	N/A	0.215	0.747	0.836	0.041	0.096	0.565	0.139	0.243	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	68	68	54	45	128	90	55	73	52
N.S.	1	1.01	1.01	0.81	0.67	1.91	1.34	0.82	1.09	0.78
time (sec)	N/A	0.178	0.454	0.792	0.031	0.101	0.403	0.132	0.246	6.580

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	156	153	234	213	784	0	0	2189	0
N.S.	1	1.05	1.03	1.57	1.43	5.26	0.00	0.00	14.69	0.00
time (sec)	N/A	0.365	1.083	1.306	0.064	39.351	0.000	0.000	0.301	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	155	157	441	265	1369	0	0	1018	0
N.S.	1	1.03	1.05	2.94	1.77	9.13	0.00	0.00	6.79	0.00
time (sec)	N/A	0.315	1.254	1.361	0.069	64.393	0.000	0.000	0.277	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	224	198	1445	548	0	0	617	1555	0
N.S.	1	1.17	1.04	7.57	2.87	0.00	0.00	3.23	8.14	0.00
time (sec)	N/A	0.383	2.543	1.484	0.098	0.000	0.000	0.200	0.255	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	271	322	314	374	804	921	424	24	0
N.S.	1	0.85	1.02	0.99	1.18	2.54	2.91	1.34	0.08	0.00
time (sec)	N/A	0.449	2.115	1.185	0.061	0.135	0.723	0.158	200.046	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	191	229	202	240	538	609	294	442	0
N.S.	1	0.89	1.07	0.94	1.12	2.51	2.85	1.37	2.07	0.00
time (sec)	N/A	0.319	1.579	1.037	0.045	0.116	0.780	0.185	1.827	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	133	142	148	155	332	335	178	245	0
N.S.	1	0.97	1.04	1.08	1.13	2.42	2.45	1.30	1.79	0.00
time (sec)	N/A	0.228	0.754	0.827	0.059	0.115	0.649	0.150	0.443	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	90	87	70	61	176	119	76	111	54
N.S.	1	1.03	1.00	0.80	0.70	2.02	1.37	0.87	1.28	0.62
time (sec)	N/A	0.191	0.517	0.783	0.028	0.096	0.425	0.153	0.305	6.013

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	251	240	367	384	0	0	0	24	0
N.S.	1	1.06	1.02	1.56	1.63	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.552	1.612	1.365	0.086	0.000	0.000	0.000	200.025	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	242	246	571	398	0	0	0	1076	0
N.S.	1	1.05	1.06	2.47	1.72	0.00	0.00	0.00	4.66	0.00
time (sec)	N/A	0.497	1.726	1.397	0.093	0.000	0.000	0.000	0.294	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	246	247	1005	776	0	0	666	2274	0
N.S.	1	1.03	1.04	4.22	3.26	0.00	0.00	2.80	9.55	0.00
time (sec)	N/A	0.440	3.282	1.519	0.115	0.000	0.000	0.190	0.263	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	294	406	362	446	1032	1486	547	24	0
N.S.	1	0.78	1.08	0.96	1.18	2.74	3.94	1.45	0.06	0.00
time (sec)	N/A	0.463	3.247	1.220	0.040	0.193	1.038	0.159	200.040	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	213	283	234	288	704	994	386	572	0
N.S.	1	0.83	1.10	0.91	1.12	2.74	3.87	1.50	2.23	0.00
time (sec)	N/A	0.324	2.465	1.111	0.048	0.143	0.767	0.184	5.016	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	155	177	185	192	434	561	238	325	0
N.S.	1	0.91	1.04	1.09	1.13	2.55	3.30	1.40	1.91	0.00
time (sec)	N/A	0.248	0.847	0.878	0.041	0.104	0.887	0.336	0.258	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	112	107	86	77	224	150	101	17	54
N.S.	1	1.05	1.00	0.80	0.72	2.09	1.40	0.94	0.16	0.50
time (sec)	N/A	0.197	0.490	0.797	0.036	0.094	0.482	0.233	200.039	6.161

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	369	373	585	654	0	0	0	24	0
N.S.	1	1.07	1.08	1.69	1.89	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.812	2.116	1.358	0.146	0.000	0.000	0.000	200.033	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	359	384	783	625	0	0	0	1929	0
N.S.	1	1.06	1.13	2.30	1.84	0.00	0.00	0.00	5.67	0.00
time (sec)	N/A	0.758	2.390	1.444	0.170	0.000	0.000	0.000	0.571	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	358	385	1198	1257	0	0	1067	2321	0
N.S.	1	1.05	1.13	3.51	3.69	0.00	0.00	3.13	6.81	0.00
time (sec)	N/A	0.680	5.167	1.536	0.205	0.000	0.000	0.494	0.240	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	309	232	254	336	516	374	278	24	0
N.S.	1	1.09	0.82	0.89	1.18	1.82	1.32	0.98	0.08	0.00
time (sec)	N/A	0.561	1.597	1.279	0.043	0.123	0.749	0.123	200.028	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	223	163	162	230	374	274	196	24	0
N.S.	1	1.11	0.81	0.81	1.14	1.86	1.36	0.98	0.12	0.00
time (sec)	N/A	0.415	1.472	1.211	0.048	0.126	0.605	0.118	200.030	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	145	110	103	144	226	184	123	186	0
N.S.	1	1.12	0.85	0.79	1.11	1.74	1.42	0.95	1.43	0.00
time (sec)	N/A	0.289	0.948	1.006	0.037	0.105	0.582	0.174	0.231	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	83	72	61	80	142	110	68	92	136
N.S.	1	1.15	1.00	0.85	1.11	1.97	1.53	0.94	1.28	1.89
time (sec)	N/A	0.199	0.587	0.807	0.032	0.119	0.562	0.162	0.218	6.460

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	37	29	92	71	39	38	36
N.S.	1	1.00	1.07	0.86	0.67	2.14	1.65	0.91	0.88	0.84
time (sec)	N/A	0.164	0.343	0.780	0.042	0.102	0.326	0.189	0.215	6.028

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	106	160	126	672	0	0	2126	0
N.S.	1	1.00	1.13	1.70	1.34	7.15	0.00	0.00	22.62	0.00
time (sec)	N/A	0.230	0.664	1.296	0.054	5.461	0.000	0.000	0.243	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	114	351	245	446	0	0	416	0
N.S.	1	1.00	1.10	3.38	2.36	4.29	0.00	0.00	4.00	0.00
time (sec)	N/A	0.241	0.677	1.339	0.069	0.249	0.000	0.000	0.268	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	191	167	668	538	914	0	583	1120	0
N.S.	1	1.08	0.94	3.77	3.04	5.16	0.00	3.29	6.33	0.00
time (sec)	N/A	0.357	1.245	1.367	0.080	1.178	0.000	0.132	0.233	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	288	286	1215	1097	1620	0	1076	2137	0
N.S.	1	1.12	1.11	4.71	4.25	6.28	0.00	4.17	8.28	0.00
time (sec)	N/A	0.505	10.435	1.385	0.114	4.160	0.000	0.136	0.263	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	252	237	260	336	692	0	300	24	0
N.S.	1	1.07	1.01	1.11	1.43	2.94	0.00	1.28	0.10	0.00
time (sec)	N/A	0.437	1.082	1.310	0.042	0.124	0.000	0.125	200.024	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	173	161	179	228	508	0	209	24	0
N.S.	1	1.11	1.03	1.15	1.46	3.26	0.00	1.34	0.15	0.00
time (sec)	N/A	0.329	0.848	1.190	0.044	0.115	0.000	0.122	200.034	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	117	109	124	143	312	0	130	303	0
N.S.	1	1.04	0.97	1.11	1.28	2.79	0.00	1.16	2.71	0.00
time (sec)	N/A	0.255	0.658	1.053	0.056	0.096	0.000	0.125	0.322	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	80	71	78	81	201	121	77	142	82
N.S.	1	1.14	1.01	1.11	1.16	2.87	1.73	1.10	2.03	1.17
time (sec)	N/A	0.196	0.457	0.799	0.037	0.105	4.162	0.131	0.343	6.103

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	26	31	35	46	23	47	24
N.S.	1	1.00	0.96	0.93	1.11	1.25	1.64	0.82	1.68	0.86
time (sec)	N/A	0.148	0.295	0.766	0.038	0.084	2.178	0.121	0.334	5.875

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	122	342	278	571	0	227	4555	0
N.S.	1	1.00	1.06	2.97	2.42	4.97	0.00	1.97	39.61	0.00
time (sec)	N/A	0.253	0.748	1.275	0.073	0.323	0.000	0.122	0.415	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	207	201	839	669	1297	0	0	1540	0
N.S.	1	1.09	1.06	4.42	3.52	6.83	0.00	0.00	8.11	0.00
time (sec)	N/A	0.369	1.420	1.299	0.105	0.745	0.000	0.000	0.421	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	307	774	1457	1390	2236	0	1048	2940	0
N.S.	1	1.05	2.65	4.99	4.76	7.66	0.00	3.59	10.07	0.00
time (sec)	N/A	0.497	10.986	1.375	0.140	2.602	0.000	0.149	0.979	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	329	301	405	514	1020	0	410	24	0
N.S.	1	1.10	1.01	1.35	1.72	3.41	0.00	1.37	0.08	0.00
time (sec)	N/A	0.564	2.378	1.595	0.055	0.146	0.000	0.131	200.031	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	248	236	310	375	724	0	299	704	0
N.S.	1	1.07	1.02	1.34	1.62	3.12	0.00	1.29	3.03	0.00
time (sec)	N/A	0.452	1.831	1.332	0.052	0.140	0.000	0.126	0.264	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	176	163	235	268	513	0	219	470	0
N.S.	1	1.08	1.00	1.44	1.64	3.15	0.00	1.34	2.88	0.00
time (sec)	N/A	0.320	2.253	1.147	0.051	0.144	0.000	0.124	0.259	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	98	107	169	127	0	110	282	106
N.S.	1	1.00	1.27	1.39	2.19	1.65	0.00	1.43	3.66	1.38
time (sec)	N/A	0.199	1.133	1.020	0.039	0.102	0.000	0.127	0.256	6.079

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	90	61	57	104	79	250	66	159	75
N.S.	1	1.17	0.79	0.74	1.35	1.03	3.25	0.86	2.06	0.97
time (sec)	N/A	0.196	0.715	0.790	0.033	0.120	8.729	0.124	0.248	6.133

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	43	40	48	62	146	37	96	41
N.S.	1	1.00	0.84	0.78	0.94	1.22	2.86	0.73	1.88	0.80
time (sec)	N/A	0.165	0.445	0.813	0.041	0.106	4.493	0.116	0.283	5.844

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	225	222	646	630	1196	0	1467	7379	0
N.S.	1	1.10	1.09	3.17	3.09	5.86	0.00	7.19	36.17	0.00
time (sec)	N/A	0.404	1.603	1.318	0.113	0.503	0.000	0.134	0.638	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	362	299	1509	1360	2534	0	1957	2864	0
N.S.	1	1.18	0.97	4.90	4.42	8.23	0.00	6.35	9.30	0.00
time (sec)	N/A	0.582	11.900	1.363	0.171	1.536	0.000	0.307	4.538	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	493	418	2511	2543	3894	0	3353	24	0
N.S.	1	1.13	0.96	5.76	5.83	8.93	0.00	7.69	0.06	0.00
time (sec)	N/A	0.848	13.932	1.440	0.240	4.938	0.000	0.228	200.049	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	141	76	55	90	70	131	64	110	55
N.S.	1	1.16	0.62	0.45	0.74	0.57	1.07	0.52	0.90	0.45
time (sec)	N/A	0.263	0.555	0.815	0.122	0.085	0.450	0.115	0.348	0.027

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	123	71	50	76	65	63	57	97	50
N.S.	1	1.23	0.71	0.50	0.76	0.65	0.63	0.57	0.97	0.50
time (sec)	N/A	0.237	0.460	0.782	0.132	0.087	1.033	0.113	0.291	0.021

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	92	66	45	62	60	95	54	84	45
N.S.	1	1.18	0.85	0.58	0.79	0.77	1.22	0.69	1.08	0.58
time (sec)	N/A	0.200	0.357	0.760	0.118	0.079	0.192	0.126	0.289	0.023

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	69	61	40	48	55	48	48	71	40
N.S.	1	1.23	1.09	0.71	0.86	0.98	0.86	0.86	1.27	0.71
time (sec)	N/A	0.173	0.210	0.660	0.118	0.081	0.651	0.101	0.291	0.020

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	51	54	35	36	50	61	44	58	33
N.S.	1	1.04	1.10	0.71	0.73	1.02	1.24	0.90	1.18	0.67
time (sec)	N/A	0.159	0.125	0.631	0.111	0.078	0.090	0.117	0.277	0.018

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	77	92	58	70	90	0	104	140	66
N.S.	1	1.07	1.28	0.81	0.97	1.25	0.00	1.44	1.94	0.92
time (sec)	N/A	0.205	0.843	0.958	0.119	0.090	0.000	0.143	0.399	0.108

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	95	77	76	109	0	285	172	80
N.S.	1	1.00	1.30	1.05	1.04	1.49	0.00	3.90	2.36	1.10
time (sec)	N/A	0.212	0.947	0.936	0.120	0.096	0.000	0.257	0.324	0.065

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	84	103	77	99	126	0	205	255	92
N.S.	1	1.06	1.30	0.97	1.25	1.59	0.00	2.59	3.23	1.16
time (sec)	N/A	0.212	0.930	0.983	0.127	0.086	0.000	0.138	0.265	0.066

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	87	78	70	115	104	0	232	191	106
N.S.	1	1.06	0.95	0.85	1.40	1.27	0.00	2.83	2.33	1.29
time (sec)	N/A	0.200	1.363	0.783	0.126	0.093	0.000	0.140	0.310	5.977

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	114	83	75	148	119	0	181	246	140
N.S.	1	1.10	0.80	0.72	1.42	1.14	0.00	1.74	2.37	1.35
time (sec)	N/A	0.234	1.562	0.796	0.127	0.101	0.000	0.116	0.312	0.069

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	141	88	80	186	134	0	322	301	178
N.S.	1	1.12	0.70	0.63	1.48	1.06	0.00	2.56	2.39	1.41
time (sec)	N/A	0.263	2.047	0.809	0.131	0.087	0.000	0.145	0.256	6.031

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	168	93	85	229	149	0	367	356	223
N.S.	1	1.14	0.63	0.57	1.55	1.01	0.00	2.48	2.41	1.51
time (sec)	N/A	0.298	2.472	0.815	0.128	0.135	0.000	0.138	0.270	5.934

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	167	86	65	102	80	162	72	136	65
N.S.	1	1.21	0.62	0.47	0.74	0.58	1.17	0.52	0.99	0.47
time (sec)	N/A	0.282	0.475	0.789	0.145	0.095	2.240	0.117	0.261	0.030

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	144	81	60	88	75	144	66	123	60
N.S.	1	1.24	0.70	0.52	0.76	0.65	1.24	0.57	1.06	0.52
time (sec)	N/A	0.242	0.560	0.747	0.120	0.087	1.544	0.116	0.252	0.026

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	117	76	55	74	70	129	62	110	55
N.S.	1	1.24	0.81	0.59	0.79	0.74	1.37	0.66	1.17	0.59
time (sec)	N/A	0.218	0.350	0.733	0.125	0.097	1.041	0.114	0.245	5.903

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	90	71	50	60	65	110	56	97	50
N.S.	1	1.25	0.99	0.69	0.83	0.90	1.53	0.78	1.35	0.69
time (sec)	N/A	0.195	0.441	0.630	0.124	0.087	0.703	0.113	0.262	5.842

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	72	66	45	48	60	97	53	84	45
N.S.	1	1.07	0.99	0.67	0.72	0.90	1.45	0.79	1.25	0.67
time (sec)	N/A	0.172	0.246	0.616	0.130	0.112	0.439	0.111	0.263	5.978

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	102	104	70	93	102	0	116	166	76
N.S.	1	1.11	1.13	0.76	1.01	1.11	0.00	1.26	1.80	0.83
time (sec)	N/A	0.232	0.727	0.934	0.123	0.100	0.000	0.142	0.252	0.073

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	107	111	87	99	121	0	475	198	108
N.S.	1	1.10	1.14	0.90	1.02	1.25	0.00	4.90	2.04	1.11
time (sec)	N/A	0.237	1.063	0.927	0.120	0.094	0.000	0.314	0.285	0.068

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	114	113	87	122	136	0	219	281	117
N.S.	1	1.15	1.14	0.88	1.23	1.37	0.00	2.21	2.84	1.18
time (sec)	N/A	0.256	1.074	0.921	0.131	0.108	0.000	0.147	0.290	0.073

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	116	113	87	150	151	0	265	370	133
N.S.	1	1.09	1.07	0.82	1.42	1.42	0.00	2.50	3.49	1.25
time (sec)	N/A	0.247	1.239	0.915	0.131	0.106	0.000	0.161	0.311	5.948

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	116	113	87	183	166	0	245	451	155
N.S.	1	1.09	1.07	0.82	1.73	1.57	0.00	2.31	4.25	1.46
time (sec)	N/A	0.244	1.356	0.928	0.132	0.102	0.000	0.257	0.336	5.976

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	119	88	80	209	134	0	318	301	179
N.S.	1	1.09	0.81	0.73	1.92	1.23	0.00	2.92	2.76	1.64
time (sec)	N/A	0.231	2.654	0.798	0.131	0.104	0.000	0.145	0.332	0.074

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	146	93	85	252	149	0	367	356	223
N.S.	1	1.11	0.71	0.65	1.92	1.14	0.00	2.80	2.72	1.70
time (sec)	N/A	0.267	3.399	0.810	0.141	0.098	0.000	0.148	0.265	5.995

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	173	98	90	300	164	0	408	411	272
N.S.	1	1.13	0.64	0.59	1.96	1.07	0.00	2.67	2.69	1.78
time (sec)	N/A	0.295	2.248	0.839	0.124	0.104	0.000	0.145	0.255	5.975

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	188	96	75	114	90	199	82	162	75
N.S.	1	1.22	0.62	0.49	0.74	0.58	1.29	0.53	1.05	0.49
time (sec)	N/A	0.286	0.477	0.815	0.116	0.100	8.927	0.110	0.248	0.030

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	165	91	70	100	85	180	76	149	70
N.S.	1	1.25	0.69	0.53	0.76	0.64	1.36	0.58	1.13	0.53
time (sec)	N/A	0.263	0.437	0.786	0.142	0.094	6.289	0.120	0.233	5.911

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	134	86	65	86	80	162	72	136	65
N.S.	1	1.22	0.78	0.59	0.78	0.73	1.47	0.65	1.24	0.59
time (sec)	N/A	0.229	0.370	0.756	0.110	0.121	4.377	0.116	0.273	0.026

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	111	81	60	72	75	143	63	123	60
N.S.	1	1.26	0.92	0.68	0.82	0.85	1.62	0.72	1.40	0.68
time (sec)	N/A	0.196	0.325	0.654	0.122	0.094	3.024	0.117	0.250	6.005

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	93	76	55	60	70	131	61	110	55
N.S.	1	1.12	0.92	0.66	0.72	0.84	1.58	0.73	1.33	0.66
time (sec)	N/A	0.183	0.233	0.624	0.120	0.099	2.238	0.112	0.269	5.907

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	127	114	80	116	112	0	125	192	86
N.S.	1	1.13	1.02	0.71	1.04	1.00	0.00	1.12	1.71	0.77
time (sec)	N/A	0.267	0.578	0.892	0.142	0.097	0.000	0.144	0.244	5.866

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	127	121	97	122	131	0	665	224	138
N.S.	1	1.09	1.03	0.83	1.04	1.12	0.00	5.68	1.91	1.18
time (sec)	N/A	0.274	0.809	0.962	0.122	0.111	0.000	0.398	0.363	0.080

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	141	121	97	145	146	0	230	307	147
N.S.	1	1.18	1.02	0.82	1.22	1.23	0.00	1.93	2.58	1.24
time (sec)	N/A	0.280	0.885	0.958	0.146	0.115	0.000	0.148	0.285	0.074

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	146	128	97	173	162	0	275	396	161
N.S.	1	1.14	1.00	0.76	1.35	1.27	0.00	2.15	3.09	1.26
time (sec)	N/A	0.294	1.182	0.948	0.132	0.103	0.000	0.148	0.293	0.075

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	148	121	97	206	176	0	440	477	180
N.S.	1	1.17	0.96	0.77	1.63	1.40	0.00	3.49	3.79	1.43
time (sec)	N/A	0.285	1.436	0.954	0.125	0.104	0.000	0.307	0.269	5.945

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	148	123	97	244	191	0	355	568	206
N.S.	1	1.11	0.92	0.73	1.83	1.44	0.00	2.67	4.27	1.55
time (sec)	N/A	0.287	1.737	0.974	0.140	0.105	0.000	0.153	0.271	5.939

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	148	121	97	287	206	0	389	647	238
N.S.	1	1.11	0.91	0.73	2.16	1.55	0.00	2.92	4.86	1.79
time (sec)	N/A	0.284	3.946	1.002	0.151	0.123	0.000	0.155	0.264	5.979

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	151	98	90	323	164	0	408	411	272
N.S.	1	1.11	0.72	0.66	2.38	1.21	0.00	3.00	3.02	2.00
time (sec)	N/A	0.260	5.557	0.876	0.156	0.093	0.000	0.148	0.258	5.998

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	178	103	95	376	179	0	457	466	326
N.S.	1	1.13	0.65	0.60	2.38	1.13	0.00	2.89	2.95	2.06
time (sec)	N/A	0.297	5.227	0.902	0.169	0.111	0.000	0.156	0.257	5.956

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	205	108	100	434	194	0	502	521	385
N.S.	1	1.14	0.60	0.56	2.41	1.08	0.00	2.79	2.89	2.14
time (sec)	N/A	0.326	3.397	0.960	0.165	0.101	0.000	0.154	0.277	0.100

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	128	66	45	78	60	97	53	84	45
N.S.	1	1.21	0.62	0.42	0.74	0.57	0.92	0.50	0.79	0.42
time (sec)	N/A	0.252	0.324	0.776	0.117	0.083	0.456	0.115	0.278	0.024

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	101	61	40	64	55	80	49	71	40
N.S.	1	1.20	0.73	0.48	0.76	0.65	0.95	0.58	0.85	0.48
time (sec)	N/A	0.221	0.278	0.753	0.115	0.109	0.256	0.114	0.242	0.020

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	74	56	35	50	50	63	42	58	35
N.S.	1	1.19	0.90	0.56	0.81	0.81	1.02	0.68	0.94	0.56
time (sec)	N/A	0.186	0.219	0.743	0.139	0.082	0.251	0.120	0.275	0.019

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	51	49	28	36	43	44	37	45	28
N.S.	1	1.28	1.22	0.70	0.90	1.08	1.10	0.92	1.12	0.70
time (sec)	N/A	0.167	0.171	0.638	0.118	0.089	0.119	0.114	0.328	0.018

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	44	25	24	40	29	34	34	25
N.S.	1	1.00	1.33	0.76	0.73	1.21	0.88	1.03	1.03	0.76
time (sec)	N/A	0.150	0.197	0.625	0.122	0.084	0.084	0.112	0.328	0.016

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	74	44	47	76	0	90	119	49
N.S.	1	1.00	1.42	0.85	0.90	1.46	0.00	1.73	2.29	0.94
time (sec)	N/A	0.180	0.506	0.867	0.120	0.093	0.000	0.132	0.297	5.939

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	68	50	53	74	0	125	83	53
N.S.	1	1.00	1.24	0.91	0.96	1.35	0.00	2.27	1.51	0.96
time (sec)	N/A	0.180	0.442	0.775	0.141	0.113	0.000	0.139	0.279	6.019

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	82	73	65	76	89	0	183	136	77
N.S.	1	1.06	0.95	0.84	0.99	1.16	0.00	2.38	1.77	1.00
time (sec)	N/A	0.204	0.567	0.772	0.121	0.088	0.000	0.139	0.248	0.063

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	109	78	70	104	104	0	232	191	106
N.S.	1	1.10	0.79	0.71	1.05	1.05	0.00	2.34	1.93	1.07
time (sec)	N/A	0.236	0.822	0.786	0.124	0.099	0.000	0.140	0.261	6.028

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	106	66	45	78	78	0	54	131	110
N.S.	1	1.19	0.74	0.51	0.88	0.88	0.00	0.61	1.47	1.24
time (sec)	N/A	0.220	0.395	0.803	0.134	0.096	0.000	0.116	0.270	5.855

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	79	61	40	64	73	0	47	118	105
N.S.	1	1.18	0.91	0.60	0.96	1.09	0.00	0.70	1.76	1.57
time (sec)	N/A	0.195	0.343	0.776	0.113	0.083	0.000	0.115	0.259	0.023

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	63	56	35	50	68	0	44	105	100
N.S.	1	1.05	0.93	0.58	0.83	1.13	0.00	0.73	1.75	1.67
time (sec)	N/A	0.181	0.295	0.749	0.110	0.091	0.000	0.113	0.297	5.979

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	51	51	30	36	62	99	39	92	88
N.S.	1	1.28	1.28	0.75	0.90	1.55	2.48	0.98	2.30	2.20
time (sec)	N/A	0.162	0.252	0.641	0.130	0.086	6.236	0.110	0.245	5.966

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	24	16	27	16	43	15
N.S.	1	1.00	1.00	0.85	1.20	0.80	1.35	0.80	2.15	0.75
time (sec)	N/A	0.140	0.179	0.602	0.045	0.096	6.232	0.112	0.230	0.025

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	66	48	58	83	0	84	242	106
N.S.	1	1.00	1.25	0.91	1.09	1.57	0.00	1.58	4.57	2.00
time (sec)	N/A	0.176	0.410	0.808	0.119	0.090	0.000	0.122	0.269	6.129

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	87	65	60	86	104	0	168	191	157
N.S.	1	1.16	0.87	0.80	1.15	1.39	0.00	2.24	2.55	2.09
time (sec)	N/A	0.203	0.716	0.784	0.114	0.089	0.000	0.131	0.293	0.074

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	114	83	65	128	119	0	199	246	181
N.S.	1	1.18	0.86	0.67	1.32	1.23	0.00	2.05	2.54	1.87
time (sec)	N/A	0.238	0.759	0.786	0.132	0.099	0.000	0.154	0.254	0.076

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	136	76	55	133	98	0	60	195	222
N.S.	1	1.17	0.66	0.47	1.15	0.84	0.00	0.52	1.68	1.91
time (sec)	N/A	0.257	0.522	0.852	0.112	0.095	0.000	0.116	0.285	5.978

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	111	71	50	119	93	0	55	24	217
N.S.	1	1.18	0.76	0.53	1.27	0.99	0.00	0.59	0.26	2.31
time (sec)	N/A	0.234	0.508	0.793	0.108	0.087	0.000	0.123	200.029	5.960

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	95	66	45	105	87	0	52	169	212
N.S.	1	1.09	0.76	0.52	1.21	1.00	0.00	0.60	1.94	2.44
time (sec)	N/A	0.217	0.484	0.766	0.116	0.099	0.000	0.114	0.556	0.030

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	81	61	40	91	81	0	48	156	200
N.S.	1	1.21	0.91	0.60	1.36	1.21	0.00	0.72	2.33	2.99
time (sec)	N/A	0.198	0.448	0.741	0.109	0.093	0.000	0.115	0.604	6.198

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	30	27	50	40	0	25	81	185
N.S.	1	1.04	0.65	0.59	1.09	0.87	0.00	0.54	1.76	4.02
time (sec)	N/A	0.175	0.360	0.715	0.026	0.078	0.000	0.112	0.567	0.028

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	48	25	22	36	35	122	23	68	161
N.S.	1	1.30	0.68	0.59	0.97	0.95	3.30	0.62	1.84	4.35
time (sec)	N/A	0.163	0.297	0.619	0.033	0.080	22.512	0.117	0.588	6.521

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	22	36	35	90	21	68	161
N.S.	1	1.00	0.68	0.59	0.97	0.95	2.43	0.57	1.84	4.35
time (sec)	N/A	0.154	0.249	0.677	0.033	0.122	15.974	0.112	0.558	0.024

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	78	63	75	81	103	0	93	384	218
N.S.	1	1.07	0.86	1.03	1.11	1.41	0.00	1.27	5.26	2.99
time (sec)	N/A	0.209	1.023	0.875	0.124	0.088	0.000	0.131	0.595	0.077

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	119	101	70	109	134	0	233	301	270
N.S.	1	1.25	1.06	0.74	1.15	1.41	0.00	2.45	3.17	2.84
time (sec)	N/A	0.238	0.993	0.816	0.126	0.092	0.000	0.135	0.645	6.625

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	146	93	75	151	149	0	208	356	301
N.S.	1	1.25	0.79	0.64	1.29	1.27	0.00	1.78	3.04	2.57
time (sec)	N/A	0.285	0.948	0.823	0.116	0.102	0.000	0.138	0.679	0.110

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	430	586	792	0	346	0	0	546	0
N.S.	1	0.99	1.35	1.83	0.00	0.80	0.00	0.00	1.26	0.00
time (sec)	N/A	0.661	26.739	4.131	0.000	0.107	0.000	0.000	3.295	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	351	519	668	0	271	0	0	363	0
N.S.	1	0.98	1.45	1.86	0.00	0.75	0.00	0.00	1.01	0.00
time (sec)	N/A	0.518	26.051	3.585	0.000	0.102	0.000	0.000	2.920	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	337	462	681	0	309	0	0	988	0
N.S.	1	0.97	1.34	1.97	0.00	0.89	0.00	0.00	2.86	0.00
time (sec)	N/A	0.478	24.636	6.829	0.000	0.103	0.000	0.000	9.099	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	421	525	775	0	515	0	0	0	0
N.S.	1	0.99	1.24	1.83	0.00	1.21	0.00	0.00	0.00	0.00
time (sec)	N/A	0.598	24.857	4.398	0.000	0.103	0.000	0.000	97.590	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	524	584	890	0	903	0	0	0	0
N.S.	1	0.99	1.10	1.68	0.00	1.71	0.00	0.00	0.00	0.00
time (sec)	N/A	0.774	26.128	5.625	0.000	0.160	0.000	0.000	76.550	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	603	609	801	1381	0	522	0	0	1039	0
N.S.	1	1.01	1.33	2.29	0.00	0.87	0.00	0.00	1.72	0.00
time (sec)	N/A	0.980	27.850	5.946	0.000	0.102	0.000	0.000	4.467	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	493	693	1050	0	417	0	0	788	0
N.S.	1	1.00	1.41	2.13	0.00	0.85	0.00	0.00	1.60	0.00
time (sec)	N/A	0.763	27.327	7.000	0.000	0.105	0.000	0.000	4.261	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	617	1137	0	506	0	0	0	0
N.S.	1	1.00	1.39	2.57	0.00	1.14	0.00	0.00	0.00	0.00
time (sec)	N/A	0.660	26.234	8.689	0.000	0.112	0.000	0.000	32.268	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	433	592	989	0	513	0	0	0	0
N.S.	1	1.00	1.37	2.29	0.00	1.19	0.00	0.00	0.00	0.00
time (sec)	N/A	0.655	25.819	15.365	0.000	0.127	0.000	0.000	140.258	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	537	672	973	0	885	0	0	0	0
N.S.	1	1.03	1.29	1.87	0.00	1.70	0.00	0.00	0.00	0.00
time (sec)	N/A	0.777	26.859	17.204	0.000	0.128	0.000	0.000	4.044	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	654	650	795	1069	0	1392	0	0	0	0
N.S.	1	0.99	1.22	1.63	0.00	2.13	0.00	0.00	0.00	0.00
time (sec)	N/A	0.948	28.342	9.361	0.000	0.211	0.000	0.000	12.316	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	377	512	682	0	270	0	0	464	0
N.S.	1	0.98	1.34	1.78	0.00	0.70	0.00	0.00	1.21	0.00
time (sec)	N/A	0.581	24.606	4.999	0.000	0.135	0.000	0.000	2.156	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	317	453	587	0	230	0	0	295	0
N.S.	1	0.98	1.39	1.81	0.00	0.71	0.00	0.00	0.91	0.00
time (sec)	N/A	0.448	24.351	2.809	0.000	0.100	0.000	0.000	1.437	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	268	405	498	0	182	0	0	90	0
N.S.	1	0.96	1.45	1.78	0.00	0.65	0.00	0.00	0.32	0.00
time (sec)	N/A	0.384	22.692	4.230	0.000	0.107	0.000	0.000	1.183	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	341	289	654	0	322	0	0	138	0
N.S.	1	1.01	0.85	1.93	0.00	0.95	0.00	0.00	0.41	0.00
time (sec)	N/A	0.484	23.013	5.990	0.000	0.093	0.000	0.000	2.975	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	439	485	776	0	545	0	0	186	0
N.S.	1	1.00	1.11	1.78	0.00	1.25	0.00	0.00	0.43	0.00
time (sec)	N/A	0.636	24.753	8.339	0.000	0.106	0.000	0.000	2.619	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	411	565	845	0	444	0	0	0	0
N.S.	1	1.01	1.40	2.09	0.00	1.10	0.00	0.00	0.00	0.00
time (sec)	N/A	0.605	25.545	10.778	0.000	0.101	0.000	0.000	4.418	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	343	497	703	0	328	0	0	0	0
N.S.	1	1.02	1.48	2.09	0.00	0.98	0.00	0.00	0.00	0.00
time (sec)	N/A	0.494	24.563	4.899	0.000	0.104	0.000	0.000	2.776	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	318	442	625	0	266	0	0	1442	0
N.S.	1	1.02	1.42	2.01	0.00	0.86	0.00	0.00	4.64	0.00
time (sec)	N/A	0.450	23.555	1.835	0.000	0.119	0.000	0.000	2.587	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	367	511	720	0	358	0	0	899	0
N.S.	1	1.05	1.46	2.05	0.00	1.02	0.00	0.00	2.56	0.00
time (sec)	N/A	0.521	24.634	6.297	0.000	0.111	0.000	0.000	1.814	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	459	570	884	0	804	0	0	1045	0
N.S.	1	1.02	1.26	1.96	0.00	1.78	0.00	0.00	2.31	0.00
time (sec)	N/A	0.677	24.920	8.415	0.000	0.117	0.000	0.000	30.910	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	492	710	969	0	752	0	0	0	0
N.S.	1	1.02	1.47	2.01	0.00	1.56	0.00	0.00	0.00	0.00
time (sec)	N/A	0.778	27.355	7.196	0.000	0.122	0.000	0.000	5.363	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	444	606	845	0	616	0	0	0	0
N.S.	1	1.02	1.40	1.95	0.00	1.42	0.00	0.00	0.00	0.00
time (sec)	N/A	0.671	25.935	6.303	0.000	0.104	0.000	0.000	6.632	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	446	528	750	0	459	0	0	0	0
N.S.	1	1.11	1.31	1.86	0.00	1.14	0.00	0.00	0.00	0.00
time (sec)	N/A	0.659	24.767	6.088	0.000	0.096	0.000	0.000	3.892	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	455	587	836	0	662	0	0	0	0
N.S.	1	1.04	1.34	1.91	0.00	1.51	0.00	0.00	0.00	0.00
time (sec)	N/A	0.656	25.790	1.871	0.000	0.106	0.000	0.000	3.511	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	536	674	971	0	828	0	0	352	0
N.S.	1	1.03	1.30	1.87	0.00	1.60	0.00	0.00	0.68	0.00
time (sec)	N/A	0.790	26.769	8.793	0.000	0.128	0.000	0.000	2.982	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	645	666	789	1195	0	1814	0	0	0	0
N.S.	1	1.03	1.22	1.85	0.00	2.81	0.00	0.00	0.00	0.00
time (sec)	N/A	1.051	27.555	10.758	0.000	0.235	0.000	0.000	70.346	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	135	288	161	0	191	0	0	42	0
N.S.	1	1.48	3.16	1.77	0.00	2.10	0.00	0.00	0.46	0.00
time (sec)	N/A	0.296	0.391	2.743	0.000	0.080	0.000	0.000	0.377	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	135	287	161	0	191	0	0	42	0
N.S.	1	1.48	3.15	1.77	0.00	2.10	0.00	0.00	0.46	0.00
time (sec)	N/A	0.312	11.153	2.227	0.000	0.080	0.000	0.000	0.392	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	106	164	0	191	0	0	44	0
N.S.	1	1.00	1.16	1.80	0.00	2.10	0.00	0.00	0.48	0.00
time (sec)	N/A	0.215	7.898	1.902	0.000	0.074	0.000	0.000	1.900	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	216	307	318	0	210	0	0	44	0
N.S.	1	1.58	2.24	2.32	0.00	1.53	0.00	0.00	0.32	0.00
time (sec)	N/A	0.388	0.468	1.741	0.000	0.085	0.000	0.000	0.347	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	216	321	318	0	210	0	0	44	0
N.S.	1	1.58	2.34	2.32	0.00	1.53	0.00	0.00	0.32	0.00
time (sec)	N/A	0.417	10.554	1.914	0.000	0.101	0.000	0.000	0.363	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	121	132	319	0	210	0	0	44	0
N.S.	1	1.01	1.10	2.66	0.00	1.75	0.00	0.00	0.37	0.00
time (sec)	N/A	0.251	8.678	1.993	0.000	0.081	0.000	0.000	1.753	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	773	2312	1104	3116	46038	5562	3854	2585
N.S.	1	1.00	2.08	6.22	2.97	8.38	123.76	14.95	10.36	6.95
time (sec)	N/A	0.629	1.751	0.857	0.091	0.181	11.935	0.150	0.516	6.929

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	355	1084	575	1373	16458	2435	1645	1229
N.S.	1	1.00	1.52	4.63	2.46	5.87	70.33	10.41	7.03	5.25
time (sec)	N/A	0.432	0.626	0.960	0.077	0.106	4.082	0.128	0.225	6.342

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	122	338	238	434	3958	768	489	446
N.S.	1	1.00	0.97	2.68	1.89	3.44	31.41	6.10	3.88	3.54
time (sec)	N/A	0.290	0.272	0.609	0.043	0.125	1.224	0.111	0.208	6.072

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	202	182	0	0	0	0	0	168	0
N.S.	1	0.95	0.86	0.00	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.453	0.359	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	365	344	310	0	0	0	0	0	849	0
N.S.	1	0.94	0.85	0.00	0.00	0.00	0.00	0.00	2.33	0.00
time (sec)	N/A	0.687	0.793	0.000	0.000	0.000	0.000	0.000	0.255	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	202	182	0	0	0	0	0	560	0
N.S.	1	0.95	0.86	0.00	0.00	0.00	0.00	0.00	2.64	0.00
time (sec)	N/A	0.369	0.289	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	119	122	0	0	0	134	0	894	0
N.S.	1	1.03	1.05	0.00	0.00	0.00	1.16	0.00	7.71	0.00
time (sec)	N/A	0.240	0.360	0.000	0.000	0.000	5.729	0.000	0.212	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.429	0.000	0.000	0.000	0.000	0.000	0.000	0.424	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	510	0	0	0	0	0	0	0	242	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	387	395	0	0	0	0	0	0	202	0
N.S.	1	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.542	0.000	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	0	0	0	0	0	0	162	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.345	0.000	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	0	0	0	0	0	0	122	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.495	0.000	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	253	0	0	0	0	0	82	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.387	0.289	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	0	0	0	0	0	0	52	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.388	0.000	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	0	0	0	0	0	0	111	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.379	0.000	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	58	47	0	0	165	98
N.S.	1	1.00	1.00	0.97	1.87	1.52	0.00	0.00	5.32	3.16
time (sec)	N/A	0.158	0.151	1.802	0.109	0.126	0.000	0.000	0.221	5.685

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [166] had the largest ratio of [.5000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.08	24	0.125
2	A	4	4	1.38	29	0.138
3	A	3	3	1.00	29	0.103
4	A	6	5	1.46	29	0.172
5	A	2	2	1.45	31	0.065
6	B	3	3	2.19	31	0.097
7	B	3	3	2.23	31	0.097
8	A	2	2	1.46	31	0.065
9	A	2	2	1.46	31	0.065
10	A	2	2	1.36	31	0.065
11	A	2	2	1.33	31	0.065
12	A	7	7	1.36	28	0.250
13	A	7	7	1.09	32	0.219
14	A	6	6	1.09	36	0.167
15	A	1	1	1.00	30	0.033
16	B	2	2	12.45	26	0.077
17	B	2	2	8.35	26	0.077
18	B	2	2	4.70	24	0.083
19	A	1	1	1.94	17	0.059
20	A	2	2	1.00	26	0.077
21	A	1	1	1.70	24	0.042

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	1	1	1.00	26	0.038
23	A	1	1	1.00	26	0.038
24	A	1	1	1.25	28	0.036
25	B	2	2	4.18	24	0.083
26	B	2	2	3.06	24	0.083
27	A	2	2	1.94	22	0.091
28	A	2	2	1.00	15	0.133
29	A	2	2	1.00	23	0.087
30	A	1	1	1.70	24	0.042
31	A	1	1	1.00	24	0.042
32	A	1	1	1.00	24	0.042
33	A	2	2	1.00	20	0.100
34	A	2	2	1.00	20	0.100
35	A	2	2	1.00	20	0.100
36	A	2	2	1.00	20	0.100
37	A	2	2	1.00	18	0.111
38	A	2	2	1.03	13	0.154
39	A	2	2	1.00	20	0.100
40	A	2	2	1.00	20	0.100
41	A	2	2	1.00	20	0.100
42	A	2	2	1.00	20	0.100
43	A	2	2	1.00	20	0.100
44	A	2	2	1.00	20	0.100
45	A	2	2	1.00	20	0.100
46	A	2	2	1.00	22	0.091
47	A	2	2	1.00	22	0.091
48	A	2	2	1.00	22	0.091
49	A	2	2	1.00	22	0.091
50	A	2	2	1.00	20	0.100
51	A	3	3	1.00	15	0.200
52	A	2	2	1.00	22	0.091
53	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	22	0.091
55	A	2	2	1.00	22	0.091
56	A	2	2	1.00	22	0.091
57	A	2	2	1.00	22	0.091
58	A	2	2	1.00	22	0.091
59	A	2	2	1.00	22	0.091
60	A	2	2	1.00	22	0.091
61	A	2	2	1.00	22	0.091
62	A	2	2	1.00	22	0.091
63	A	2	2	1.00	22	0.091
64	A	2	2	1.00	22	0.091
65	A	2	2	1.00	22	0.091
66	A	2	2	1.00	20	0.100
67	A	3	3	0.98	15	0.200
68	A	2	2	1.00	22	0.091
69	A	2	2	1.00	22	0.091
70	A	2	2	1.00	22	0.091
71	A	2	2	1.00	22	0.091
72	A	2	2	1.00	22	0.091
73	A	2	2	1.00	22	0.091
74	A	2	2	1.00	22	0.091
75	A	2	2	1.00	22	0.091
76	A	2	2	1.00	22	0.091
77	A	2	2	1.00	22	0.091
78	A	2	2	1.00	22	0.091
79	A	2	2	1.00	22	0.091
80	A	2	2	1.00	22	0.091
81	A	2	2	1.00	22	0.091
82	A	2	2	1.00	20	0.100
83	A	3	3	1.00	15	0.200
84	A	2	2	1.00	22	0.091
85	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.00	22	0.091
87	A	3	3	1.00	22	0.136
88	A	3	3	1.00	22	0.136
89	A	3	3	1.00	22	0.136
90	A	4	4	1.04	22	0.182
91	A	2	2	1.29	20	0.100
92	A	2	2	1.00	15	0.133
93	A	5	5	1.09	22	0.227
94	A	5	5	1.04	22	0.227
95	A	4	4	1.04	22	0.182
96	A	5	5	1.09	22	0.227
97	A	3	3	0.96	22	0.136
98	A	3	3	1.03	22	0.136
99	A	3	3	1.09	20	0.150
100	A	3	3	1.09	15	0.200
101	A	7	7	1.14	22	0.318
102	A	7	7	1.04	22	0.318
103	A	2	2	1.00	18	0.111
104	A	2	2	1.00	20	0.100
105	A	2	2	1.00	22	0.091
106	A	2	2	1.00	22	0.091
107	A	2	2	1.00	22	0.091
108	A	2	2	1.00	22	0.091
109	A	2	2	1.00	22	0.091
110	A	2	2	1.00	22	0.091
111	A	2	2	1.00	24	0.083
112	A	2	2	1.00	24	0.083
113	A	2	2	1.00	24	0.083
114	A	2	2	1.00	24	0.083
115	A	2	2	1.00	24	0.083
116	A	2	2	1.00	24	0.083
117	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.00	24	0.083
119	A	2	2	1.00	24	0.083
120	A	2	2	1.00	24	0.083
121	A	2	2	1.00	24	0.083
122	A	2	2	1.00	24	0.083
123	A	11	10	1.06	25	0.400
124	A	9	8	1.05	25	0.320
125	A	6	5	1.07	25	0.200
126	A	4	3	1.05	25	0.120
127	A	7	6	1.27	25	0.240
128	A	9	8	1.31	25	0.320
129	A	10	9	1.03	25	0.360
130	A	8	7	1.08	25	0.280
131	A	8	7	1.04	25	0.280
132	A	8	7	1.24	25	0.280
133	A	10	9	1.25	25	0.360
134	A	9	8	1.07	25	0.320
135	A	10	9	1.08	25	0.360
136	A	10	9	1.17	25	0.360
137	A	10	9	1.18	25	0.360
138	A	10	9	1.21	25	0.360
139	B	6	5	2.13	38	0.132
140	A	6	5	1.46	43	0.116
141	A	4	3	1.11	24	0.125
142	A	10	9	1.54	22	0.409
143	A	10	9	1.52	20	0.450
144	A	5	4	1.49	20	0.200
145	B	7	6	2.41	22	0.273
146	A	5	4	1.62	20	0.200
147	B	4	3	2.16	18	0.167
148	A	9	8	0.96	24	0.333
149	A	6	5	0.99	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	5	4	1.07	22	0.182
151	A	5	4	1.01	17	0.235
152	A	9	8	1.05	24	0.333
153	A	8	7	1.03	24	0.292
154	A	8	7	1.17	24	0.292
155	A	9	8	0.85	24	0.333
156	A	7	6	0.89	24	0.250
157	A	6	5	0.97	22	0.227
158	A	6	5	1.03	17	0.294
159	A	11	10	1.06	24	0.417
160	A	10	9	1.05	24	0.375
161	A	10	9	1.03	24	0.375
162	A	10	9	0.78	24	0.375
163	A	8	7	0.83	24	0.292
164	A	7	6	0.91	22	0.273
165	A	7	6	1.05	17	0.353
166	A	13	12	1.07	24	0.500
167	A	12	11	1.06	24	0.458
168	A	12	11	1.05	24	0.458
169	A	8	7	1.09	24	0.292
170	A	7	6	1.11	24	0.250
171	A	5	4	1.12	24	0.167
172	A	4	3	1.15	22	0.136
173	A	4	3	1.00	17	0.176
174	A	6	5	1.00	24	0.208
175	A	4	3	1.00	24	0.125
176	A	6	5	1.08	24	0.208
177	A	9	8	1.12	24	0.333
178	A	8	7	1.07	24	0.292
179	A	6	5	1.11	24	0.208
180	A	6	5	1.04	24	0.208
181	A	4	3	1.14	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	1	1	1.00	17	0.059
183	A	5	4	1.00	24	0.167
184	A	6	5	1.09	24	0.208
185	A	7	6	1.05	24	0.250
186	A	8	7	1.10	24	0.292
187	A	7	6	1.07	24	0.250
188	A	5	4	1.08	24	0.167
189	A	2	2	1.00	24	0.083
190	A	2	2	1.17	22	0.091
191	A	2	2	1.00	17	0.118
192	A	8	7	1.10	24	0.292
193	A	9	8	1.18	24	0.333
194	A	11	10	1.13	24	0.417
195	A	8	8	1.16	24	0.333
196	A	7	7	1.23	24	0.292
197	A	4	4	1.18	24	0.167
198	A	3	3	1.23	22	0.136
199	A	3	3	1.04	17	0.176
200	A	7	6	1.07	24	0.250
201	A	7	6	1.00	24	0.250
202	A	7	6	1.06	24	0.250
203	A	5	4	1.06	24	0.167
204	A	7	6	1.10	24	0.250
205	A	9	8	1.12	24	0.333
206	A	11	10	1.14	24	0.417
207	A	9	9	1.21	24	0.375
208	A	8	8	1.24	24	0.333
209	A	5	5	1.24	24	0.208
210	A	4	4	1.25	22	0.182
211	A	4	4	1.07	17	0.235
212	A	9	8	1.11	24	0.333
213	A	9	8	1.10	24	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	9	8	1.15	24	0.333
215	A	9	8	1.09	24	0.333
216	A	9	8	1.09	24	0.333
217	A	6	5	1.09	24	0.208
218	A	8	7	1.11	24	0.292
219	A	10	9	1.13	24	0.375
220	A	10	10	1.22	24	0.417
221	A	9	9	1.25	24	0.375
222	A	7	7	1.22	24	0.292
223	A	5	5	1.26	22	0.227
224	A	5	5	1.12	17	0.294
225	A	11	10	1.13	24	0.417
226	A	11	10	1.09	24	0.417
227	A	11	10	1.18	24	0.417
228	A	11	10	1.14	24	0.417
229	A	11	10	1.17	24	0.417
230	A	11	10	1.11	24	0.417
231	A	11	10	1.11	24	0.417
232	A	7	6	1.11	24	0.250
233	A	9	8	1.13	24	0.333
234	A	11	10	1.14	24	0.417
235	A	7	7	1.21	24	0.292
236	A	6	6	1.20	24	0.250
237	A	3	3	1.19	24	0.125
238	A	2	2	1.28	22	0.091
239	A	2	2	1.00	17	0.118
240	A	5	4	1.00	24	0.167
241	A	4	3	1.00	24	0.125
242	A	6	5	1.06	24	0.208
243	A	8	7	1.10	24	0.292
244	A	6	6	1.19	24	0.250
245	A	4	4	1.18	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	4	4	1.05	24	0.167
247	A	2	2	1.28	22	0.091
248	A	1	1	1.00	17	0.059
249	A	5	4	1.00	24	0.167
250	A	6	5	1.16	24	0.208
251	A	8	7	1.18	24	0.292
252	A	8	8	1.17	24	0.333
253	A	6	6	1.18	24	0.250
254	A	6	6	1.09	24	0.250
255	A	4	4	1.21	24	0.167
256	A	2	2	1.04	24	0.083
257	A	2	2	1.30	22	0.091
258	A	2	2	1.00	17	0.118
259	A	7	6	1.07	24	0.250
260	A	8	7	1.25	24	0.292
261	A	10	9	1.25	24	0.375
262	A	12	11	0.99	27	0.407
263	A	10	9	0.98	27	0.333
264	A	10	9	0.97	27	0.333
265	A	11	10	0.99	27	0.370
266	A	13	12	0.99	27	0.444
267	A	14	13	1.01	27	0.481
268	A	12	11	1.00	27	0.407
269	A	12	11	1.00	27	0.407
270	A	13	12	1.00	27	0.444
271	A	13	12	1.03	27	0.444
272	A	14	13	0.99	27	0.481
273	A	12	11	0.98	27	0.407
274	A	10	9	0.98	27	0.333
275	A	8	7	0.96	27	0.259
276	A	10	9	1.01	27	0.333
277	A	12	11	1.00	27	0.407

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	12	11	1.01	27	0.407
279	A	10	9	1.02	27	0.333
280	A	10	9	1.02	27	0.333
281	A	10	9	1.05	27	0.333
282	A	12	11	1.02	27	0.407
283	A	12	11	1.02	27	0.407
284	A	12	11	1.02	27	0.407
285	A	12	11	1.11	27	0.407
286	A	12	11	1.04	27	0.407
287	A	12	11	1.03	27	0.407
288	A	14	13	1.03	27	0.481
289	A	6	5	1.48	29	0.172
290	A	7	6	1.48	32	0.188
291	A	3	3	1.00	29	0.103
292	A	8	7	1.58	31	0.226
293	A	9	8	1.58	34	0.235
294	A	3	3	1.01	29	0.103
295	A	2	2	1.00	22	0.091
296	A	2	2	1.00	22	0.091
297	A	2	2	1.00	20	0.100
298	A	2	2	0.95	22	0.091
299	A	5	5	0.94	22	0.227
300	A	2	2	0.95	24	0.083
301	A	3	3	1.03	20	0.150
302	A	4	3	1.00	22	0.136
303	F	0	0	N/A	0.000	N/A
304	A	4	4	1.02	26	0.154
305	A	2	2	1.00	26	0.077
306	A	5	4	1.00	26	0.154
307	A	4	3	1.00	26	0.115
308	A	4	3	1.00	24	0.125
309	A	4	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	1	1	1.00	30	0.033

CHAPTER 3

Listing of integrals

3.1 $\int (c-acx)^p \sqrt{1-a^2x^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int (c-acx)^p \sqrt{1-a^2x^2} \, dx = -\frac{2\sqrt{2}(1-ax)^{3/2}(c-acx)^p \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}+p,\frac{5}{2}+p,\frac{1}{2}(1-ax)\right)}{a(3+2p)}$$

output
```
-2*2^(1/2)*(-a*x+1)^(3/2)*(-a*c*x+c)^p*hypergeom([-1/2, 3/2+p],[5/2+p],-1/
2*a*x+1/2)/a/(3+2*p)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.28

$$\int (c-acx)^p \sqrt{1-a^2x^2} \, dx = \frac{2^{1+p}(1-ax)^{-\frac{1}{2}-p}(1+ax)(c-acx)^p\sqrt{2-2a^2x^2} \text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{2}-p,\frac{5}{2},\frac{1}{2}(1+ax)\right)}{3a}$$

input
```
Integrate[(c - a*c*x)^p*Sqrt[1 - a^2*x^2],x]
```

output
```
(2^(1 + p)*(1 - a*x)^(-1/2 - p)*(1 + a*x)*(c - a*c*x)^p*Sqrt[2 - 2*a^2*x^2
]*Hypergeometric2F1[3/2, -1/2 - p, 5/2, (1 + a*x)/2])/(3*a)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {474, 456, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-a^2x^2}(c-acx)^p\,dx$$

$$\Big\downarrow \text{474}$$

$$(1-ax)^{-p}(c-acx)^p\int (1-ax)^p\sqrt{1-a^2x^2}dx$$

$$\Big\downarrow \text{456}$$

$$(1-ax)^{-p}(c-acx)^p\int (1-ax)^{p+\frac{1}{2}}\sqrt{ax+1}dx$$

$$\Big\downarrow \text{79}$$

$$\frac{2^{p+\frac{3}{2}}(ax+1)^{3/2}(1-ax)^{-p}(c-acx)^p\,\text{Hypergeometric2F1}\left(\frac{3}{2},-p-\frac{1}{2},\frac{5}{2},\frac{1}{2}(ax+1)\right)}{3a}$$

input
```
Int[(c - a*c*x)^p*Sqrt[1 - a^2*x^2],x]
```

output
```
(2^(3/2 + p)*(1 + a*x)^(3/2)*(c - a*c*x)^p*Hypergeometric2F1[3/2, -1/2 - p
, 5/2, (1 + a*x)/2])/(3*a*(1 - a*x)^p)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int (-acx+c)^p \sqrt{-a^2x^2+1}dx$$

input
```
int((-a*c*x+c)^p*(-a^2*x^2+1)^(1/2),x)
```

output
```
int((-a*c*x+c)^p*(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int (c-acx)^p\sqrt{1-a^2x^2}\,dx = \int \sqrt{-a^2x^2+1}(-acx+c)^p\,dx$$

input
```
integrate((-a*c*x+c)^p*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^p, x)
```

Sympy [F]

$$\int (c-acx)^p\sqrt{1-a^2x^2}\,dx=\int (-c(ax-1))^p\sqrt{-(ax-1)\,(ax+1)}\,dx$$

input
```
integrate((-a*c*x+c)**p*(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral((-c*(a*x - 1))**p*sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int (c-acx)^p\sqrt{1-a^2x^2}\,dx=\int \sqrt{-a^2x^2+1}(-acx+c)^p\,dx$$

input
```
integrate((-a*c*x+c)^p*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (c-acx)^p\sqrt{1-a^2x^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a*c*x+c)^p*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (c-acx)^p\sqrt{1-a^2x^2}\,dx=\int \sqrt{1-a^2\,x^2}\,(c-a\,c\,x)^p\,dx$$

input
```
int((1 - a^2*x^2)^(1/2)*(c - a*c*x)^p,x)
```

output
```
int((1 - a^2*x^2)^(1/2)*(c - a*c*x)^p, x)
```

Reduce [F]

$$\int (c-acx)^p\sqrt{1-a^2x^2}\,dx=\int (-acx+c)^p\,\sqrt{-a^2x^2+1}dx$$

input
```
int((-a*c*x+c)^p*(-a^2*x^2+1)^(1/2),x)
```

output
```
int(( - a*c*x + c)**p*sqrt( - a**2*x**2 + 1),x)
```

3.2 $\int \frac{(1+ax)(c-acx)^p}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 73

$$\int \frac{(1+ax)(c-acx)^p}{\sqrt{1-a^2x^2}}\,dx = -\frac{2\sqrt{2}(c-acx)^p\sqrt{1-a^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}+p,\frac{3}{2}+p,\frac{1}{2}(1-ax)\right)}{a(1+2p)\sqrt{1+ax}}$$

output
```
-2*2^(1/2)*(-a*c*x+c)^p*(-a^2*x^2+1)^(1/2)*hypergeom([-1/2, 1/2+p],[3/2+p]
,-1/2*a*x+1/2)/a/(1+2*p)/(a*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \frac{(1+ax)(c-acx)^p}{\sqrt{1-a^2x^2}}\,dx = \frac{2(1-ax)^{-p}(1+ax)(c-acx)^p\left(-(1-ax)^{1+p}+2^{\frac{1}{2}+p}\sqrt{1-ax}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2}-p,\frac{3}{2},\frac{1}{2}(1+\right.\right.}{(a+2ap)\sqrt{1-a^2x^2}}$$

input
```
Integrate[((1 + a*x)*(c - a*c*x)^p)/Sqrt[1 - a^2*x^2],x]
```

output
```
(2*(1 + a*x)*(c - a*c*x)^p*(-(1 - a*x)^(1 + p) + 2^(1/2 + p)*Sqrt[1 - a*x]
*Hypergeometric2F1[1/2, -1/2 - p, 3/2, (1 + a*x)/2]))/((a + 2*a*p)*(1 - a*
x)^p*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {672, 474, 456, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+1)(c-acx)^p}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \ 672$$

$$\frac{\int \frac{(c-acx)^p}{\sqrt{1-a^2x^2}}dx}{p+1}-\frac{\sqrt{1-a^2x^2}(c-acx)^p}{a(p+1)}$$

$$\Big\downarrow \ 474$$

$$\frac{(1-ax)^{-p}(c-acx)^p\int \frac{(1-ax)^p}{\sqrt{1-a^2x^2}}dx}{p+1}-\frac{\sqrt{1-a^2x^2}(c-acx)^p}{a(p+1)}$$

$$\Big\downarrow \ 456$$

$$\frac{(1-ax)^{-p}(c-acx)^p\int \frac{(1-ax)^{p-\frac{1}{2}}}{\sqrt{ax+1}}dx}{p+1}-\frac{\sqrt{1-a^2x^2}(c-acx)^p}{a(p+1)}$$

$$\Big\downarrow \ 79$$

$$\frac{2^{p+\frac{1}{2}}\sqrt{ax+1}(1-ax)^{-p}(c-acx)^p \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-p,\frac{3}{2},\frac{1}{2}(ax+1)\right)}{a(p+1)}-\frac{\sqrt{1-a^2x^2}(c-acx)^p}{a(p+1)}$$

input
```
Int[((1 + a*x)*(c - a*c*x)^p)/Sqrt[1 - a^2*x^2],x]
```

output
```
-(((c - a*c*x)^p*Sqrt[1 - a^2*x^2])/(a*(1 + p))) + (2^(1/2 + p)*Sqrt[1 + a
*x]*(c - a*c*x)^p*Hypergeometric2F1[1/2, 1/2 - p, 3/2, (1 + a*x)/2])/(a*(1
 + p)*(1 - a*x)^p)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

rule 672
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Simp[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2))   Int[(d + e*x
)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^
2 + a*e^2, 0] && NeQ[m + 2*p + 2, 0]
```

Maple [F]

$$\int \frac{(ax+1)\,(-acx+c)^p}{\sqrt{-a^2x^2+1}}dx$$

input
```
int((a*x+1)*(-a*c*x+c)^p/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((a*x+1)*(-a*c*x+c)^p/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{(1+ax)(c-acx)^p}{\sqrt{1-a^2x^2}}\,dx = \int \frac{(ax+1)(-acx+c)^p}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate((a*x+1)*(-a*c*x+c)^p/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*(-a*c*x + c)^p/(a*x - 1), x)
```

Sympy [F]

$$\int \frac{(1+ax)(c-acx)^p}{\sqrt{1-a^2x^2}}\,dx = \int \frac{(-c(ax-1))^p\,(ax+1)}{\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

input
```
integrate((a*x+1)*(-a*c*x+c)**p/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral((-c*(a*x - 1))**p*(a*x + 1)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{(1+ax)(c-acx)^p}{\sqrt{1-a^2x^2}}\,dx = \int \frac{(ax+1)(-acx+c)^p}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate((a*x+1)*(-a*c*x+c)^p/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*x + 1)*(-a*c*x + c)^p/sqrt(-a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{(1+ax)(c-acx)^p}{\sqrt{1-a^2x^2}}\,dx = \int \frac{(ax+1)(-acx+c)^p}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate((a*x+1)*(-a*c*x+c)^p/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*x + 1)*(-a*c*x + c)^p/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+ax)(c-acx)^p}{\sqrt{1-a^2x^2}}\,dx = \int \frac{(c-a\,c\,x)^p\,(a\,x+1)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(((c - a*c*x)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2),x)
```

output
```
int(((c - a*c*x)^p*(a*x + 1))/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(1+ax)(c-acx)^p}{\sqrt{1-a^2x^2}}\,dx = \int \frac{(-acx+c)^p}{\sqrt{-a^2x^2+1}}dx + \left(\int \frac{(-acx+c)^p\,x}{\sqrt{-a^2x^2+1}}dx\right)a$$

input
```
int((a*x+1)*(-a*c*x+c)^p/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(( - a*c*x + c)**p/sqrt( - a**2*x**2 + 1),x) + int((( - a*c*x + c)**p*x
)/sqrt( - a**2*x**2 + 1),x)*a
```

3.3 $\int \frac{\sqrt{1+cx}}{\sqrt{1-cx}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 90

$$\int \frac{\sqrt{1+cx}}{\sqrt{1-cx}\sqrt{d+ex}}\,dx=-\frac{2\sqrt{2}\sqrt{cd+e}\sqrt{\frac{c(d+ex)}{cd+e}}E\left(\arcsin\left(\frac{\sqrt{e}\sqrt{1-cx}}{\sqrt{cd+e}}\right)|\frac{cd+e}{2e}\right)}{c\sqrt{e}\sqrt{d+ex}}$$

output
```
-2*2^(1/2)*(c*d+e)^(1/2)*(c*(e*x+d)/(c*d+e))^(1/2)*EllipticE(e^(1/2)*(-c*x
+1)^(1/2)/(c*d+e)^(1/2),1/2*2^(1/2)*((c*d+e)/e)^(1/2))/c/e^(1/2)/(e*x+d)^(
1/2)
```

Mathematica [A] (verified)

Time = 7.75 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{1+cx}}{\sqrt{1-cx}\sqrt{d+ex}}\,dx=\frac{2\sqrt{d+ex}\left(\frac{1+cx}{\sqrt{1-cx}}-\frac{\sqrt{2}\sqrt{\frac{1+cx}{-1+cx}}E\left(\arcsin\left(\frac{\sqrt{2}}{\sqrt{1-cx}}\right)|\frac{cd+e}{2e}\right)}{\sqrt{\frac{c(d+ex)}{e(-1+cx)}}}\right)}{e\sqrt{1+cx}}$$

input
```
Integrate[Sqrt[1 + c*x]/(Sqrt[1 - c*x]*Sqrt[d + e*x]),x]
```

output
```
(2*Sqrt[d + e*x]*((1 + c*x)/Sqrt[1 - c*x] - (Sqrt[2]*Sqrt[(1 + c*x)/(-1 +
c*x)]*EllipticE[ArcSin[Sqrt[2]/Sqrt[1 - c*x]], (c*d + e)/(2*e)])/Sqrt[(c*(
d + e*x))/(e*(-1 + c*x))]))/(e*Sqrt[1 + c*x])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {124, 27, 123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx+1}}{\sqrt{1-cx}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 124}$$

$$\frac{\sqrt{2}\sqrt{\frac{c(d+ex)}{cd+e}}\int \frac{\sqrt{cx+1}}{\sqrt{2}\sqrt{1-cx}\sqrt{\frac{cd}{cd+e}+\frac{cex}{cd+e}}}dx}{\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{\frac{c(d+ex)}{cd+e}}\int \frac{\sqrt{cx+1}}{\sqrt{1-cx}\sqrt{\frac{cd}{cd+e}+\frac{cex}{cd+e}}}dx}{\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 123}$$

$$-\frac{2\sqrt{2}\sqrt{cd+e}\sqrt{\frac{c(d+ex)}{cd+e}}E\left(\arcsin\left(\frac{\sqrt{e}\sqrt{1-cx}}{\sqrt{cd+e}}\right)|\frac{cd+e}{2e}\right)}{c\sqrt{e}\sqrt{d+ex}}$$

input
```
Int[Sqrt[1 + c*x]/(Sqrt[1 - c*x]*Sqrt[d + e*x]),x]
```

output
```
(-2*Sqrt[2]*Sqrt[c*d + e]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticE[ArcSin[(
Sqrt[e]*Sqrt[1 - c*x])/Sqrt[c*d + e]], (c*d + e)/(2*e)])/(c*Sqrt[e]*Sqrt[d
 + e*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(77) = 154$.

Time = 2.20 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.09

method	result
default	$\frac{2\left(\operatorname{EllipticE}\left(\sqrt{\frac{c(ex+d)}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c^2d^2-2\operatorname{EllipticF}\left(\sqrt{\frac{c(ex+d)}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cde-\operatorname{EllipticE}\left(\sqrt{\frac{c(ex+d)}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)e^2+2\operatorname{EllipticF}\left(\sqrt{\frac{c(\cdots}{}}\right.\right.}{e^2c(e x^3c^2+c^2d x^2-ex-d)}$
elliptic	$\frac{\sqrt{-(ex+d)(c^2x^2-1)}\left(\frac{2\left(\frac{d}{e}-\frac{1}{c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{1}{c}}}\sqrt{\frac{x-\frac{1}{c}}{-\frac{d}{e}-\frac{1}{c}}}\sqrt{\frac{x+\frac{1}{c}}{-\frac{d}{e}+\frac{1}{c}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{1}{c}}},\sqrt{\frac{-\frac{d}{e}+\frac{1}{c}}{-\frac{d}{e}-\frac{1}{c}}}\right)}{\sqrt{-e x^3c^2-c^2d x^2+ex+d}}+\frac{2c\left(\frac{d}{e}-\frac{1}{c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{1}{c}}}\sqrt{\frac{x-\frac{1}{c}}{-\frac{d}{e}-\frac{1}{c}}}\sqrt{\frac{x+\frac{1}{c}}{-\frac{d}{e}+\frac{1}{c}}}\left(\cdots\right)}{\cdots}\right.}{\sqrt{cx+1}\sqrt{-cx+1}\sqrt{ex+d}}$

input
```
int((c*x+1)^(1/2)/(-c*x+1)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(EllipticE((c*(e*x+d)/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2-2*
EllipticF((c*(e*x+d)/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e-Ellipti
cE((c*(e*x+d)/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2+2*EllipticF((c*(
e*x+d)/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2)*(-e*(c*x+1)/(c*d-e))^(
1/2)*(-e*(c*x-1)/(c*d+e))^(1/2)*(c*(e*x+d)/(c*d-e))^(1/2)*(c*x+1)^(1/2)*(-
c*x+1)^(1/2)*(e*x+d)^(1/2)/e^2/c/(c^2*e*x^3+c^2*d*x^2-e*x-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(77) = 154$.

Time = 0.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.12

$$\int \frac{\sqrt{1+cx}}{\sqrt{1-cx}\sqrt{d+ex}}\,dx = \frac{2\left(3\sqrt{-c^2e}ce\text{weierstrassZeta}\left(\frac{4(c^2d^2+3e^2)}{3c^2e^2}, -\frac{8(c^2d^3-9de^2)}{27c^2e^3}, \text{weierstrassPInverse}\left(\frac{4(c^2d^2+3e^2)}{3c^2e^2}, -\frac{8(c^2d^3-9de^2)}{27c^2e^3}\right.\right.\right.}{3c^2e^2}$$

input
```
integrate((c*x+1)^(1/2)/(-c*x+1)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas"
)
```

output
```
2/3*(3*sqrt(-c^2*e)*c*e*weierstrassZeta(4/3*(c^2*d^2 + 3*e^2)/(c^2*e^2), -
8/27*(c^2*d^3 - 9*d*e^2)/(c^2*e^3), weierstrassPInverse(4/3*(c^2*d^2 + 3*e
^2)/(c^2*e^2), -8/27*(c^2*d^3 - 9*d*e^2)/(c^2*e^3), 1/3*(3*e*x + d)/e)) +
sqrt(-c^2*e)*(c*d - 3*e)*weierstrassPInverse(4/3*(c^2*d^2 + 3*e^2)/(c^2*e^
2), -8/27*(c^2*d^3 - 9*d*e^2)/(c^2*e^3), 1/3*(3*e*x + d)/e))/(c^2*e^2)
```

Sympy [F]

$$\int \frac{\sqrt{1+cx}}{\sqrt{1-cx}\sqrt{d+ex}}\,dx = \int \frac{\sqrt{cx+1}}{\sqrt{d+ex}\sqrt{-cx+1}}\,dx$$

input
```
integrate((c*x+1)**(1/2)/(-c*x+1)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Integral(sqrt(c*x + 1)/(sqrt(d + e*x)*sqrt(-c*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+cx}}{\sqrt{1-cx}\sqrt{d+ex}}\,dx = \int \frac{\sqrt{cx+1}}{\sqrt{-cx+1}\sqrt{ex+d}}\,dx$$

input
```
integrate((c*x+1)^(1/2)/(-c*x+1)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(c*x + 1)/(sqrt(-c*x + 1)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{\sqrt{1+cx}}{\sqrt{1-cx}\sqrt{d+ex}}\,dx = \int \frac{\sqrt{cx+1}}{\sqrt{-cx+1}\sqrt{ex+d}}\,dx$$

input
```
integrate((c*x+1)^(1/2)/(-c*x+1)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x + 1)/(sqrt(-c*x + 1)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+cx}}{\sqrt{1-cx}\sqrt{d+ex}}\,dx = \int \frac{\sqrt{c\,x+1}}{\sqrt{1-c\,x}\,\sqrt{d+e\,x}}\,dx$$

input
```
int((c*x + 1)^(1/2)/((1 - c*x)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
int((c*x + 1)^(1/2)/((1 - c*x)^(1/2)*(d + e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{1+cx}}{\sqrt{1-cx}\sqrt{d+ex}}\,dx = -\left(\int \frac{\sqrt{ex+d}\,\sqrt{cx+1}\,\sqrt{-cx+1}}{ce\,x^2+cdx-ex-d}dx\right)$$

input
```
int((c*x+1)^(1/2)/(-c*x+1)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
 - int((sqrt(d + e*x)*sqrt(c*x + 1)*sqrt( - c*x + 1))/(c*d*x + c*e*x**2 -
d - e*x),x)
```

3.4 $\int \frac{1+cx}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 90

$$\int \frac{1+cx}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx=-\frac{2\sqrt{2}\sqrt{cd+e}\sqrt{\frac{c(d+ex)}{cd+e}}E\left(\arcsin\left(\frac{\sqrt{e}\sqrt{1-cx}}{\sqrt{cd+e}}\right)|\frac{cd+e}{2e}\right)}{c\sqrt{e}\sqrt{d+ex}}$$

output
```
-2*2^(1/2)*(c*d+e)^(1/2)*(c*(e*x+d)/(c*d+e))^(1/2)*EllipticE(e^(1/2)*(-c*x
+1)^(1/2)/(c*d+e)^(1/2),1/2*2^(1/2)*((c*d+e)/e)^(1/2))/c/e^(1/2)/(e*x+d)^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.15 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.13

$$\int \frac{1+cx}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx = -\frac{2\left(e^2\sqrt{-\frac{cd+e}{c}}(1-c^2x^2)+ic(cd+e)\sqrt{\frac{e(-1+cx)}{c(d+ex)}}(d+ex)^{3/2}\sqrt{\frac{e+cex}{cd+cex}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}}\right)|\frac{cd-e}{cd+e}\right)-2i\right.}{ce^2\sqrt{-\frac{cd+e}{c}}\sqrt{d+ex}\sqrt{1-c^2x^2}}$$

input
```
Integrate[(1 + c*x)/(Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]),x]
```

output
```
(-2*(e^2*Sqrt[-((c*d + e)/c)]*(1 - c^2*x^2) + I*c*(c*d + e)*Sqrt[(e*(-1 +
c*x))/(c*(d + e*x))]*(d + e*x)^(3/2)*Sqrt[(e + c*e*x)/(c*d + c*e*x)]*Ellip
ticE[I*ArcSinh[Sqrt[-((c*d + e)/c)]/Sqrt[d + e*x]], (c*d - e)/(c*d + e)] -
 (2*I)*c*e*Sqrt[(e*(-1 + c*x))/(c*(d + e*x))]*(d + e*x)^(3/2)*Sqrt[(e + c*
e*x)/(c*d + c*e*x)]*EllipticF[I*ArcSinh[Sqrt[-((c*d + e)/c)]/Sqrt[d + e*x]
], (c*d - e)/(c*d + e)]))/(c*e^2*Sqrt[-((c*d + e)/c)]*Sqrt[d + e*x]*Sqrt[1
 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {600, 508, 327, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cx+1}{\sqrt{1-c^2x^2}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{600}$$

$$\frac{c\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{e}-\frac{(cd-e)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{e}$$

$$\Big\downarrow \text{508}$$

$$-\frac{(cd-e)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{e}-\frac{2\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{e\sqrt{\frac{c(d+ex)}{cd+e}}}$$

$$\Big\downarrow \text{327}$$

$$-\frac{(cd-e)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{e}-\frac{2\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{e\sqrt{\frac{c(d+ex)}{cd+e}}}$$

$$\Big\downarrow \text{511}$$

$$\frac{2(cd-e)\sqrt{\frac{c(d+ex)}{cd+e}}\int \frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{e\sqrt{\frac{c(d+ex)}{cd+e}}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2(cd-e)\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{e\sqrt{\frac{c(d+ex)}{cd+e}}}$$

input
```
Int[(1 + c*x)/(Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]),x]
```

output
```
(-2*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)
])/(e*Sqrt[(c*(d + e*x))/(c*d + e)]) + (2*(c*d - e)*Sqrt[(c*(d + e*x))/(c*
d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*e*Sq
rt[d + e*x])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(77) = 154$.

Time = 1.93 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.06

method	result
default	$\frac{2\left(\operatorname{EllipticE}\left(\sqrt{\frac{c(ex+d)}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c^2d^2-2\operatorname{EllipticF}\left(\sqrt{\frac{c(ex+d)}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cde-\operatorname{EllipticE}\left(\sqrt{\frac{c(ex+d)}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)e^2+2\operatorname{EllipticF}\left(\sqrt{\frac{c(}{}}\right.\right.}{e^2c(e\,x^3c^2+c^2d\,x^2-ex-d)}$
elliptic	$\frac{\sqrt{-(ex+d)(c^2x^2-1)}\left(\frac{2\left(\frac{d}{e}-\frac{1}{c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{1}{c}}}\sqrt{\frac{x-\frac{1}{c}}{-\frac{d}{e}-\frac{1}{c}}}\sqrt{\frac{x+\frac{1}{c}}{-\frac{d}{e}+\frac{1}{c}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{1}{c}}},\sqrt{\frac{-\frac{d}{e}+\frac{1}{c}}{-\frac{d}{e}-\frac{1}{c}}}\right)}{\sqrt{-e\,x^3c^2-c^2d\,x^2+ex+d}}+\frac{2c\left(\frac{d}{e}-\frac{1}{c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{1}{c}}}\sqrt{\frac{x-\frac{1}{c}}{-\frac{d}{e}-\frac{1}{c}}}\sqrt{\frac{x+\frac{1}{c}}{-\frac{d}{e}+\frac{1}{c}}}\left(\right.}{}\right.}{\sqrt{ex+d}\sqrt{-c^2x^2+1}}$

input
```
int((c*x+1)/(e*x+d)^(1/2)/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(EllipticE((c*(e*x+d)/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2-2*
EllipticF((c*(e*x+d)/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e-Ellipti
cE((c*(e*x+d)/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2+2*EllipticF((c*(
e*x+d)/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2)*(-e*(c*x+1)/(c*d-e))^(
1/2)*(-e*(c*x-1)/(c*d+e))^(1/2)*(c*(e*x+d)/(c*d-e))^(1/2)*(e*x+d)^(1/2)*(-
c^2*x^2+1)^(1/2)/e^2/c/(c^2*e*x^3+c^2*d*x^2-e*x-d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(77) = 154$.

Time = 0.07 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.12

$$\int \frac{1+cx}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx = \frac{2\left(3\sqrt{-c^2e}\,ce\,\text{weierstrassZeta}\left(\frac{4(c^2d^2+3e^2)}{3c^2e^2}, -\frac{8(c^2d^3-9de^2)}{27c^2e^3}, \text{weierstrassPInverse}\left(\frac{4(c^2d^2+3e^2)}{3c^2e^2}, -\frac{8(c^2d^3-9de^2)}{27c^2e^3}\right.\right.\right.}{3c^2e^2}$$

input
```
integrate((c*x+1)/(e*x+d)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(3*sqrt(-c^2*e)*c*e*weierstrassZeta(4/3*(c^2*d^2 + 3*e^2)/(c^2*e^2), -
8/27*(c^2*d^3 - 9*d*e^2)/(c^2*e^3), weierstrassPInverse(4/3*(c^2*d^2 + 3*e
^2)/(c^2*e^2), -8/27*(c^2*d^3 - 9*d*e^2)/(c^2*e^3), 1/3*(3*e*x + d)/e)) +
sqrt(-c^2*e)*(c*d - 3*e)*weierstrassPInverse(4/3*(c^2*d^2 + 3*e^2)/(c^2*e^
2), -8/27*(c^2*d^3 - 9*d*e^2)/(c^2*e^3), 1/3*(3*e*x + d)/e))/(c^2*e^2)
```

Sympy [F]

$$\int \frac{1+cx}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx = \int \frac{cx+1}{\sqrt{-(cx-1)(cx+1)}\sqrt{d+ex}}\,dx$$

input
```
integrate((c*x+1)/(e*x+d)**(1/2)/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral((c*x + 1)/(sqrt(-(c*x - 1)*(c*x + 1))*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1+cx}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx = \int \frac{cx+1}{\sqrt{-c^2x^2+1}\sqrt{ex+d}}\,dx$$

input
```
integrate((c*x+1)/(e*x+d)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x + 1)/(sqrt(-c^2*x^2 + 1)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{1+cx}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx = \int \frac{cx+1}{\sqrt{-c^2x^2+1}\sqrt{ex+d}}\,dx$$

input
```
integrate((c*x+1)/(e*x+d)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x + 1)/(sqrt(-c^2*x^2 + 1)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+cx}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx = \int \frac{c\,x+1}{\sqrt{1-c^2\,x^2}\,\sqrt{d+e\,x}}\,dx$$

input
```
int((c*x + 1)/((1 - c^2*x^2)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
int((c*x + 1)/((1 - c^2*x^2)^(1/2)*(d + e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1+cx}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\,dx = -\left(\int \frac{\sqrt{ex+d}\,\sqrt{-c^2x^2+1}}{ce\,x^2+cdx-ex-d}dx\right)$$

input
```
int((c*x+1)/(e*x+d)^(1/2)/(-c^2*x^2+1)^(1/2),x)
```

output
```
 - int((sqrt(d + e*x)*sqrt( - c**2*x**2 + 1))/(c*d*x + c*e*x**2 - d - e*x)
,x)
```

3.5 $\int (A+Bx)(c+dx)^{-3-2p} \left(A^2-B^2x^2\right)^p \, dx$

Optimal result

Integrand size = 31, antiderivative size = 123

$$\int (A+Bx)(c+dx)^{-3-2p} \left(A^2-B^2x^2\right)^p \, dx = -\frac{2^{1+p}\left(\frac{(Bc+Ad)(A+Bx)}{AB(c+dx)}\right)^{-1-p}(c+dx)^{-2(1+p)}\left(A^2-B^2x^2\right)^{1+p}\text{Hypergeometric2F1}\left(-1-p,1+p,2+p,\right.}{(Bc+Ad)(1+p)}$$

output
```
-2^(p+1)*((A*d+B*c)*(B*x+A)/A/B/(d*x+c))^(-1-p)*(-B^2*x^2+A^2)^(p+1)*hyper
geom([p+1, -1-p],[2+p],1/2*(-A*d+B*c)*(-B*x+A)/A/B/(d*x+c))/(A*d+B*c)/(p+1
)/((d*x+c)^(2*p+2))
```

Mathematica [F]

$$\int (A+Bx)(c+dx)^{-3-2p} \left(A^2-B^2x^2\right)^p \, dx = \int (A+Bx)(c+dx)^{-3-2p} \left(A^2-B^2x^2\right)^p \, dx$$

input
```
Integrate[(A + B*x)*(c + d*x)^(-3 - 2*p)*(A^2 - B^2*x^2)^p,x]
```

output
```
Integrate[(A + B*x)*(c + d*x)^(-3 - 2*p)*(A^2 - B^2*x^2)^p, x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {679, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)\left(A^2-B^2x^2\right)^p (c+dx)^{-2p-3}dx$$

$$\Big\downarrow \text{679}$$

$$\frac{AB\int (c+dx)^{-2(p+1)}\left(A^2-B^2x^2\right)^pdx}{Ad+Bc}-\frac{\left(A^2-B^2x^2\right)^{p+1}(c+dx)^{-2(p+1)}}{2(p+1)(Ad+Bc)}$$

$$\Big\downarrow \text{489}$$

$$\frac{AB(A+Bx)\left(A^2-B^2x^2\right)^p(c+dx)^{-2p-1}\left(-\frac{(A-Bx)(Bc-Ad)}{(A+Bx)(Ad+Bc)}\right)^{-p}\text{Hypergeometric2F1}\left(-2p-1,-p,-2p,\frac{2AB}{(Bc+Ad}\right.}{(2p+1)(Bc-Ad)(Ad+Bc)}$$
$$\frac{\left(A^2-B^2x^2\right)^{p+1}(c+dx)^{-2(p+1)}}{2(p+1)(Ad+Bc)}$$

input
```
Int[(A + B*x)*(c + d*x)^(-3 - 2*p)*(A^2 - B^2*x^2)^p,x]
```

output
```
-1/2*(A^2 - B^2*x^2)^(1 + p)/((B*c + A*d)*(1 + p)*(c + d*x)^(2*(1 + p))) +
 (A*B*(A + B*x)*(c + d*x)^(-1 - 2*p)*(A^2 - B^2*x^2)^p*Hypergeometric2F1[-
1 - 2*p, -p, -2*p, (2*A*B*(c + d*x))/((B*c + A*d)*(A + B*x))])/((B*c - A*d
)*(B*c + A*d)*(1 + 2*p)*(-(((B*c - A*d)*(A - B*x))/((B*c + A*d)*(A + B*x))
))^p)
```

Defintions of rubi rules used

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [F]

$$\int (Bx + A) (dx + c)^{-3-2p} \left(-B^2x^2 + A^2\right)^p dx$$

input
```
int((B*x+A)*(d*x+c)^(-3-2*p)*(-B^2*x^2+A^2)^p,x)
```

output
```
int((B*x+A)*(d*x+c)^(-3-2*p)*(-B^2*x^2+A^2)^p,x)
```

Fricas [F]

$$\int (A+Bx)(c+dx)^{-3-2p} \left(A^2-B^2x^2\right)^p dx = \int (Bx + A)\left(-B^2x^2 + A^2\right)^p (dx + c)^{-2p-3} dx$$

input
```
integrate((B*x+A)*(d*x+c)^(-3-2*p)*(-B^2*x^2+A^2)^p,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(-B^2*x^2 + A^2)^p*(d*x + c)^(-2*p - 3), x)
```

Sympy [F]

$$\int (A+Bx)(c+dx)^{-3-2p}\left(A^2-B^2x^2\right)^p\,dx$$
$$=\int\left(-(-A+Bx)\left(A+Bx\right)\right)^p\left(A+Bx\right)\left(c+dx\right)^{-2p-3}\,dx$$

input
```
integrate((B*x+A)*(d*x+c)**(-3-2*p)*(-B**2*x**2+A**2)**p,x)
```

output
```
Integral((-(-A + B*x)*(A + B*x))**p*(A + B*x)*(c + d*x)**(-2*p - 3), x)
```

Maxima [F]

$$\int (A+Bx)(c+dx)^{-3-2p}\left(A^2-B^2x^2\right)^p\,dx=\int (Bx+A)\left(-B^2x^2+A^2\right)^p(dx+c)^{-2\,p-3}\,dx$$

input
```
integrate((B*x+A)*(d*x+c)^(-3-2*p)*(-B^2*x^2+A^2)^p,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(-B^2*x^2 + A^2)^p*(d*x + c)^(-2*p - 3), x)
```

Giac [F]

$$\int (A+Bx)(c+dx)^{-3-2p}\left(A^2-B^2x^2\right)^p\,dx=\int (Bx+A)\left(-B^2x^2+A^2\right)^p(dx+c)^{-2\,p-3}\,dx$$

input
```
integrate((B*x+A)*(d*x+c)^(-3-2*p)*(-B^2*x^2+A^2)^p,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(-B^2*x^2 + A^2)^p*(d*x + c)^(-2*p - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)(c+dx)^{-3-2p}\left(A^2-B^2x^2\right)^p\,dx=\int \frac{(A+B\,x)\,\left(A^2-B^2\,x^2\right)^p}{(c+d\,x)^{2\,p+3}}\,dx$$

input
```
int(((A + B*x)*(A^2 - B^2*x^2)^p)/(c + d*x)^(2*p + 3),x)
```

output
```
int(((A + B*x)*(A^2 - B^2*x^2)^p)/(c + d*x)^(2*p + 3), x)
```

Reduce [F]

$$\begin{aligned}&\int (A+Bx)(c+dx)^{-3-2p}\left(A^2-B^2x^2\right)^p\,dx\\&=\left(\int \frac{(-b^2x^2+a^2)^p}{(dx+c)^{2p}c^3+3\,(dx+c)^{2p}c^2dx+3\,(dx+c)^{2p}c\,d^2x^2+(dx+c)^{2p}d^3x^3}dx\right)a\\&\quad+\left(\int \frac{(-b^2x^2+a^2)^p\,x}{(dx+c)^{2p}c^3+3\,(dx+c)^{2p}c^2dx+3\,(dx+c)^{2p}c\,d^2x^2+(dx+c)^{2p}d^3x^3}dx\right)b\end{aligned}$$

input
```
int((B*x+A)*(d*x+c)^(-3-2*p)*(-B^2*x^2+A^2)^p,x)
```

output
```
int((a**2 - b**2*x**2)**p/((c + d*x)**(2*p)*c**3 + 3*(c + d*x)**(2*p)*c**2
*d*x + 3*(c + d*x)**(2*p)*c*d**2*x**2 + (c + d*x)**(2*p)*d**3*x**3),x)*a +
 int(((a**2 - b**2*x**2)**p*x)/((c + d*x)**(2*p)*c**3 + 3*(c + d*x)**(2*p)
*c**2*d*x + 3*(c + d*x)**(2*p)*c*d**2*x**2 + (c + d*x)**(2*p)*d**3*x**3),x
)*b
```

3.6 $\int \frac{(A+Bx)\left(A^2-B^2x^2\right)^{2/3}}{(c+dx)^{13/3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 114

$$\int \frac{(A+Bx)\left(A^2-B^2x^2\right)^{2/3}}{(c+dx)^{13/3}}\,dx = -\frac{6\ 2^{2/3}\left(A^2-B^2x^2\right)^{5/3}\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{5}{3},\frac{8}{3},\frac{(Bc-Ad)(A-Bx)}{2AB(c+dx)}\right)}{5(Bc+Ad)\left(\frac{(Bc+Ad)(A+Bx)}{AB(c+dx)}\right)^{5/3}(c+dx)^{10/3}}$$

output
```
-6/5*2^(2/3)*(-B^2*x^2+A^2)^(5/3)*hypergeom([-5/3, 5/3],[8/3],1/2*(-A*d+B*
c)*(-B*x+A)/A/B/(d*x+c))/(A*d+B*c)/((A*d+B*c)*(B*x+A)/A/B/(d*x+c))^(5/3)/(
d*x+c)^(10/3)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 324 vs. 2(114) = 228.

Time = 21.01 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.84

$$\int \frac{(A+Bx)\left(A^2-B^2x^2\right)^{2/3}}{(c+dx)^{13/3}}\,dx = \frac{3\left(A^2-B^2x^2\right)\left(7(Bc-Ad)^3(Bc+Ad)^3-2B(Bc-Ad)^2(Bc+Ad)^2(7\right.}{}$$

input
```
Integrate[((A + B*x)*(A^2 - B^2*x^2)^(2/3))/(c + d*x)^(13/3),x]
```

output
```
(3*(A^2 - B^2*x^2)*(7*(B*c - A*d)^3*(B*c + A*d)^3 - 2*B*(B*c - A*d)^2*(B*c
 + A*d)^2*(7*B*c + 5*A*d)*(c + d*x) + B^2*(B*c - A*d)*(B*c + A*d)*(7*B^2*c
^2 + 10*A*B*c*d - 7*A^2*d^2)*(c + d*x)^2 + 40*A^3*B^3*d^3*(c + d*x)^3) - 4
0*A^3*B^3*d^2*(c + d*x)^3*(3*d*(A^2 - B^2*x^2) - (A^2*d + B^2*c*x)*(-(((-(
B^2*c^2) + A^2*d^2)*(A^2 - B^2*x^2))/(A^2*B^2*(c + d*x)^2)))^(1/3)*Hyperge
ometric2F1[1/3, 1/2, 3/2, (A^2*d + B^2*c*x)^2/(A^2*B^2*(c + d*x)^2)]))/(70
*d^2*(B*c - A*d)^2*(B*c + A*d)^3*(c + d*x)^(10/3)*(A^2 - B^2*x^2)^(1/3))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 250 vs. $2(114) = 228$.

Time = 0.35 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {679, 486, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(A^2-B^2x^2\right)^{2/3}}{(c+dx)^{13/3}}dx$$

$$\Big\downarrow \text{679}$$

$$\frac{AB\int \frac{\left(A^2-B^2x^2\right)^{2/3}}{(c+dx)^{10/3}}dx}{Ad+Bc}-\frac{3\left(A^2-B^2x^2\right)^{5/3}}{10(c+dx)^{10/3}(Ad+Bc)}$$

$$\Big\downarrow \text{486}$$

$$\frac{AB\left(\frac{4A^2B^2\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{A^2-B^2x^2}}dx}{7\left(B^2c^2-A^2d^2\right)}+\frac{3\left(A^2-B^2x^2\right)^{2/3}\left(A^2d+B^2cx\right)}{7(c+dx)^{7/3}\left(B^2c^2-A^2d^2\right)}\right)}{Ad+Bc}-\frac{3\left(A^2-B^2x^2\right)^{5/3}}{10(c+dx)^{10/3}(Ad+Bc)}$$

$$\Big\downarrow \text{489}$$

$$\frac{AB\left(\frac{12A^2B^2(A+Bx)\sqrt[3]{-\frac{(A-Bx)(Bc-Ad)}{(A+Bx)(Ad+Bc)}}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{2}{3},\frac{2AB(c+dx)}{(Bc+Ad)(A+Bx)}\right)}{7\sqrt[3]{A^2-B^2x^2}\sqrt[3]{c+dx}(Bc-Ad)(B^2c^2-A^2d^2)}+\frac{3(A^2-B^2x^2)^{2/3}(A^2d+B^2cx)}{7(c+dx)^{7/3}(B^2c^2-A^2d^2)}\right)}{Ad+Bc}$$
$$\frac{3(A^2-B^2x^2)^{5/3}}{10(c+dx)^{10/3}(Ad+Bc)}$$

input
```
Int[((A + B*x)*(A^2 - B^2*x^2)^(2/3))/(c + d*x)^(13/3),x]
```

output
```
(-3*(A^2 - B^2*x^2)^(5/3))/(10*(B*c + A*d)*(c + d*x)^(10/3)) + (A*B*((3*(A
^2*d + B^2*c*x)*(A^2 - B^2*x^2)^(2/3))/(7*(B^2*c^2 - A^2*d^2)*(c + d*x)^(7
/3)) + (12*A^2*B^2*(-(((B*c - A*d)*(A - B*x))/((B*c + A*d)*(A + B*x))))^(1
/3)*(A + B*x)*Hypergeometric2F1[-1/3, 1/3, 2/3, (2*A*B*(c + d*x))/((B*c +
A*d)*(A + B*x))])/(7*(B*c - A*d)*(B^2*c^2 - A^2*d^2)*(c + d*x)^(1/3)*(A^2
- B^2*x^2)^(1/3))))/(B*c + A*d)
```

Defintions of rubi rules used

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [F]

$$\int \frac{(Bx + A)\left(-B^2x^2 + A^2\right)^{\frac{2}{3}}}{(dx + c)^{\frac{13}{3}}}dx$$

input
```
int((B*x+A)*(-B^2*x^2+A^2)^(2/3)/(d*x+c)^(13/3),x)
```

output
```
int((B*x+A)*(-B^2*x^2+A^2)^(2/3)/(d*x+c)^(13/3),x)
```

Fricas [F]

$$\int \frac{(A + Bx)\left(A^2 - B^2x^2\right)^{2/3}}{(c + dx)^{13/3}}\,dx = \int \frac{\left(-B^2x^2 + A^2\right)^{\frac{2}{3}}(Bx + A)}{(dx + c)^{\frac{13}{3}}}\,dx$$

input
```
integrate((B*x+A)*(-B^2*x^2+A^2)^(2/3)/(d*x+c)^(13/3),x, algorithm="fricas
")
```

output
```
integral((-B^2*x^2 + A^2)^(2/3)*(B*x + A)*(d*x + c)^(2/3)/(d^5*x^5 + 5*c*d
^4*x^4 + 10*c^2*d^3*x^3 + 10*c^3*d^2*x^2 + 5*c^4*d*x + c^5), x)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(A^2-B^2x^2\right)^{2/3}}{(c+dx)^{13/3}}\,dx=\int \frac{(-(-A+Bx)\,(A+Bx))^{\frac{2}{3}}\,(A+Bx)}{(c+dx)^{\frac{13}{3}}}\,dx$$

input
```
integrate((B*x+A)*(-B**2*x**2+A**2)**(2/3)/(d*x+c)**(13/3),x)
```

output
```
Integral((-(-A + B*x)*(A + B*x))**(2/3)*(A + B*x)/(c + d*x)**(13/3), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\left(A^2-B^2x^2\right)^{2/3}}{(c+dx)^{13/3}}\,dx=\int \frac{(-B^2x^2+A^2)^{\frac{2}{3}}(Bx+A)}{(dx+c)^{\frac{13}{3}}}\,dx$$

input
```
integrate((B*x+A)*(-B^2*x^2+A^2)^(2/3)/(d*x+c)^(13/3),x, algorithm="maxima
")
```

output
```
integrate((-B^2*x^2 + A^2)^(2/3)*(B*x + A)/(d*x + c)^(13/3), x)
```

Giac [F]

$$\int \frac{(A+Bx)\left(A^2-B^2x^2\right)^{2/3}}{(c+dx)^{13/3}}\,dx=\int \frac{(-B^2x^2+A^2)^{\frac{2}{3}}(Bx+A)}{(dx+c)^{\frac{13}{3}}}\,dx$$

input
```
integrate((B*x+A)*(-B^2*x^2+A^2)^(2/3)/(d*x+c)^(13/3),x, algorithm="giac")
```

output
```
integrate((-B^2*x^2 + A^2)^(2/3)*(B*x + A)/(d*x + c)^(13/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(A^2-B^2x^2\right)^{2/3}}{(c+dx)^{13/3}}\,dx=\int \frac{(A+B\,x)\,\left(A^2-B^2\,x^2\right)^{2/3}}{\left(c+d\,x\right)^{13/3}}\,dx$$

input
```
int(((A + B*x)*(A^2 - B^2*x^2)^(2/3))/(c + d*x)^(13/3),x)
```

output
```
int(((A + B*x)*(A^2 - B^2*x^2)^(2/3))/(c + d*x)^(13/3), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\left(A^2-B^2x^2\right)^{2/3}}{(c+dx)^{13/3}}\,dx=\left(\int \frac{\left(-b^2x^2+a^2\right)^{\frac{2}{3}}}{(dx+c)^{\frac{1}{3}}\,c^4+4\,(dx+c)^{\frac{1}{3}}\,c^3dx+6\,(dx+c)^{\frac{1}{3}}\,c^2d^2x^2+4\,(dx+c)^{\frac{1}{3}}}\right.$$
$$+\left(\int \frac{\left(-b^2x^2+a^2\right)^{\frac{2}{3}}x}{(dx+c)^{\frac{1}{3}}\,c^4+4\,(dx+c)^{\frac{1}{3}}\,c^3dx+6\,(dx+c)^{\frac{1}{3}}\,c^2d^2x^2+4\,(dx+c)^{\frac{1}{3}}\,c\,d^3x^3+(dx+c)^{\frac{1}{3}}\,d^4x^4}dx\right)b$$

input
```
int((B*x+A)*(-B^2*x^2+A^2)^(2/3)/(d*x+c)^(13/3),x)
```

output
```
int((a**2 - b**2*x**2)**(2/3)/((c + d*x)**(1/3)*c**4 + 4*(c + d*x)**(1/3)*
c**3*d*x + 6*(c + d*x)**(1/3)*c**2*d**2*x**2 + 4*(c + d*x)**(1/3)*c*d**3*x
**3 + (c + d*x)**(1/3)*d**4*x**4),x)*a + int(((a**2 - b**2*x**2)**(2/3)*x)
/((c + d*x)**(1/3)*c**4 + 4*(c + d*x)**(1/3)*c**3*d*x + 6*(c + d*x)**(1/3)
*c**2*d**2*x**2 + 4*(c + d*x)**(1/3)*c*d**3*x**3 + (c + d*x)**(1/3)*d**4*x
**4),x)*b
```

3.7 $\int \frac{(A+Bx)\sqrt[3]{A^2-B^2x^2}}{(c+dx)^{11/3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 112

$$\int \frac{(A+Bx)\sqrt[3]{A^2-B^2x^2}}{(c+dx)^{11/3}}\,dx = -\frac{3(A^2-B^2x^2)^{4/3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{4}{3},\frac{7}{3},\frac{(Bc-Ad)(A-Bx)}{2AB(c+dx)}\right)}{2^{2/3}(Bc+Ad)\left(\frac{(Bc+Ad)(A+Bx)}{AB(c+dx)}\right)^{4/3}(c+dx)^{8/3}}$$

output
```
-3/2*(-B^2*x^2+A^2)^(4/3)*hypergeom([-4/3, 4/3],[7/3],1/2*(-A*d+B*c)*(-B*x
+A)/A/B/(d*x+c))*2^(1/3)/(A*d+B*c)/((A*d+B*c)*(B*x+A)/A/B/(d*x+c))^(4/3)/(
d*x+c)^(8/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 284 vs. 2(112) = 224.

Time = 10.41 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.54

$$\int \frac{(A+Bx)\sqrt[3]{A^2-B^2x^2}}{(c+dx)^{11/3}}\,dx = \frac{3\left(d(A^2-B^2x^2)\left(5A^4d^2+5B^4c^2x^2+8A^3Bd(c+dx)+8AB^3cx(c+dx)\right.\right.}{}$$

input
```
Integrate[((A + B*x)*(A^2 - B^2*x^2)^(1/3))/(c + d*x)^(11/3),x]
```

output
```
(3*(d*(A^2 - B^2*x^2)*(5*A^4*d^2 + 5*B^4*c^2*x^2 + 8*A^3*B*d*(c + d*x) + 8
*A*B^3*c*x*(c + d*x) - 5*A^2*B^2*(c^2 + d^2*x^2)) + 16*A^3*B^3*((d*(Sqrt[A
^2/B^2] - x))/(c + Sqrt[A^2/B^2]*d))^(1/3)*((d*(Sqrt[A^2/B^2] + x))/(-c +
Sqrt[A^2/B^2]*d))^(2/3)*(c + d*x)^3*Hypergeometric2F1[1/3, 2/3, 4/3, (2*Sq
rt[A^2/B^2]*(c + d*x))/((c - Sqrt[A^2/B^2]*d)*(Sqrt[A^2/B^2] - x))]))/(40*
d*(B*c - A*d)*(B*c + A*d)^2*(c + d*x)^(8/3)*(A^2 - B^2*x^2)^(2/3))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 250 vs. 2(112) = 224.

Time = 0.33 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.23, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {679, 486, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt[3]{A^2-B^2x^2}}{(c+dx)^{11/3}}dx$$

$$\Big\downarrow \text{679}$$

$$\frac{AB\int \frac{\sqrt[3]{A^2-B^2x^2}}{(c+dx)^{8/3}}dx}{Ad+Bc}-\frac{3\left(A^2-B^2x^2\right)^{4/3}}{8(c+dx)^{8/3}(Ad+Bc)}$$

$$\Big\downarrow \text{486}$$

$$\frac{AB\left(\frac{2A^2B^2\int \frac{1}{(c+dx)^{2/3}\left(A^2-B^2x^2\right)^{2/3}}dx}{5(B^2c^2-A^2d^2)}+\frac{3\sqrt[3]{A^2-B^2x^2}(A^2d+B^2cx)}{5(c+dx)^{5/3}(B^2c^2-A^2d^2)}\right)}{Ad+Bc}-\frac{3\left(A^2-B^2x^2\right)^{4/3}}{8(c+dx)^{8/3}(Ad+Bc)}$$

$$\Big\downarrow \text{489}$$

$$\frac{AB\left(\frac{3\sqrt[3]{A^2-B^2x^2}(A^2d+B^2cx)}{5(c+dx)^{5/3}(B^2c^2-A^2d^2)}-\frac{6A^2B^2(A+Bx)\sqrt[3]{c+dx}\left(-\frac{(A-Bx)(Bc-Ad)}{(A+Bx)(Ad+Bc)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{2AB(c+dx)}{(Bc+Ad)(A+Bx)}\right)}{5(A^2-B^2x^2)^{2/3}(Bc-Ad)(B^2c^2-A^2d^2)}\right)}{Ad+Bc}$$
$$\frac{3\left(A^2-B^2x^2\right)^{4/3}}{8(c+dx)^{8/3}(Ad+Bc)}$$

input
```
Int[((A + B*x)*(A^2 - B^2*x^2)^(1/3))/(c + d*x)^(11/3),x]
```

output
```
(-3*(A^2 - B^2*x^2)^(4/3))/(8*(B*c + A*d)*(c + d*x)^(8/3)) + (A*B*((3*(A^2
*d + B^2*c*x)*(A^2 - B^2*x^2)^(1/3))/(5*(B^2*c^2 - A^2*d^2)*(c + d*x)^(5/3
)) - (6*A^2*B^2*(-(((B*c - A*d)*(A - B*x))/((B*c + A*d)*(A + B*x))))^(2/3)
*(A + B*x)*(c + d*x)^(1/3)*Hypergeometric2F1[1/3, 2/3, 4/3, (2*A*B*(c + d*
x))/((B*c + A*d)*(A + B*x))])/(5*(B*c - A*d)*(B^2*c^2 - A^2*d^2)*(A^2 - B^
2*x^2)^(2/3))))/(B*c + A*d)
```

Defintions of rubi rules used

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 679

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [F]

$$\int \frac{(Bx+A)\left(-B^2x^2+A^2\right)^{\frac{1}{3}}}{(dx+c)^{\frac{11}{3}}}dx$$

input

```
int((B*x+A)*(-B^2*x^2+A^2)^(1/3)/(d*x+c)^(11/3),x)
```

output

```
int((B*x+A)*(-B^2*x^2+A^2)^(1/3)/(d*x+c)^(11/3),x)
```

Fricas [F]

$$\int \frac{(A+Bx)\sqrt[3]{A^2-B^2x^2}}{(c+dx)^{11/3}}\,dx=\int \frac{\left(-B^2x^2+A^2\right)^{\frac{1}{3}}(Bx+A)}{(dx+c)^{\frac{11}{3}}}\,dx$$

input

```
integrate((B*x+A)*(-B^2*x^2+A^2)^(1/3)/(d*x+c)^(11/3),x, algorithm="fricas
")
```

output

```
integral((-B^2*x^2 + A^2)^(1/3)*(B*x + A)*(d*x + c)^(1/3)/(d^4*x^4 + 4*c*d
^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt[3]{A^2-B^2x^2}}{(c+dx)^{11/3}}\,dx = \int \frac{\sqrt[3]{-(-A+Bx)\,(A+Bx)}(A+Bx)}{(c+dx)^{\frac{11}{3}}}\,dx$$

input
```
integrate((B*x+A)*(-B**2*x**2+A**2)**(1/3)/(d*x+c)**(11/3),x)
```

output
```
Integral((-(-A + B*x)*(A + B*x))**(1/3)*(A + B*x)/(c + d*x)**(11/3), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt[3]{A^2-B^2x^2}}{(c+dx)^{11/3}}\,dx = \int \frac{(-B^2x^2+A^2)^{\frac{1}{3}}(Bx+A)}{(dx+c)^{\frac{11}{3}}}\,dx$$

input
```
integrate((B*x+A)*(-B^2*x^2+A^2)^(1/3)/(d*x+c)^(11/3),x, algorithm="maxima
")
```

output
```
integrate((-B^2*x^2 + A^2)^(1/3)*(B*x + A)/(d*x + c)^(11/3), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt[3]{A^2-B^2x^2}}{(c+dx)^{11/3}}\,dx = \int \frac{(-B^2x^2+A^2)^{\frac{1}{3}}(Bx+A)}{(dx+c)^{\frac{11}{3}}}\,dx$$

input
```
integrate((B*x+A)*(-B^2*x^2+A^2)^(1/3)/(d*x+c)^(11/3),x, algorithm="giac")
```

output
```
integrate((-B^2*x^2 + A^2)^(1/3)*(B*x + A)/(d*x + c)^(11/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt[3]{A^2-B^2x^2}}{(c+dx)^{11/3}}\,dx=\int \frac{(A+B\,x)\,\left(A^2-B^2\,x^2\right)^{1/3}}{(c+d\,x)^{11/3}}\,dx$$

input
```
int(((A + B*x)*(A^2 - B^2*x^2)^(1/3))/(c + d*x)^(11/3),x)
```

output
```
int(((A + B*x)*(A^2 - B^2*x^2)^(1/3))/(c + d*x)^(11/3), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt[3]{A^2-B^2x^2}}{(c+dx)^{11/3}}\,dx=\left(\int \frac{\left(-b^2x^2+a^2\right)^{\frac{1}{3}}}{(dx+c)^{\frac{2}{3}}\,c^3+3\,(dx+c)^{\frac{2}{3}}\,c^2dx+3\,(dx+c)^{\frac{2}{3}}\,c\,d^2x^2+(dx+c)^{\frac{2}{3}}\,d^3x^3}$$
$$+\left(\int \frac{\left(-b^2x^2+a^2\right)^{\frac{1}{3}}x}{(dx+c)^{\frac{2}{3}}\,c^3+3\,(dx+c)^{\frac{2}{3}}\,c^2dx+3\,(dx+c)^{\frac{2}{3}}\,c\,d^2x^2+(dx+c)^{\frac{2}{3}}\,d^3x^3}dx\right)b$$

input
```
int((B*x+A)*(-B^2*x^2+A^2)^(1/3)/(d*x+c)^(11/3),x)
```

output
```
int((a**2 - b**2*x**2)**(1/3)/((c + d*x)**(2/3)*c**3 + 3*(c + d*x)**(2/3)*
c**2*d*x + 3*(c + d*x)**(2/3)*c*d**2*x**2 + (c + d*x)**(2/3)*d**3*x**3),x)
*a + int(((a**2 - b**2*x**2)**(1/3)*x)/((c + d*x)**(2/3)*c**3 + 3*(c + d*x
)**(2/3)*c**2*d*x + 3*(c + d*x)**(2/3)*c*d**2*x**2 + (c + d*x)**(2/3)*d**3
*x**3),x)*b
```

3.8 $\int \frac{A+Bx}{(c+dx)^{7/3}\sqrt[3]{A^2-B^2x^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 112

$$\int \frac{A+Bx}{(c+dx)^{7/3}\sqrt[3]{A^2-B^2x^2}}\,dx = -\frac{3\left(A^2-B^2x^2\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{(Bc-Ad)(A-Bx)}{2AB(c+dx)}\right)}{\sqrt[3]{2}(Bc+Ad)\left(\frac{(Bc+Ad)(A+Bx)}{AB(c+dx)}\right)^{2/3}(c+dx)^{4/3}}$$

output
```
-3/2*(-B^2*x^2+A^2)^(2/3)*hypergeom([-2/3, 2/3],[5/3],1/2*(-A*d+B*c)*(-B*x
+A)/A/B/(d*x+c))*2^(2/3)/(A*d+B*c)/((A*d+B*c)*(B*x+A)/A/B/(d*x+c))^(2/3)/(
d*x+c)^(4/3)
```

Mathematica [A] (verified)

Time = 20.87 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx}{(c+dx)^{7/3}\sqrt[3]{A^2-B^2x^2}}\,dx = \frac{3\left(-B^2c^2+A^2d^2\right)\left(A^2-B^2x^2\right)+4AB\left(A^2d+B^2cx\right)(c+dx)\sqrt[3]{-\frac{(-B^2c^2}{}}}{4(Bc-Ad)(Bc+Ad)^2(c+}$$

input
```
Integrate[(A + B*x)/((c + d*x)^(7/3)*(A^2 - B^2*x^2)^(1/3)),x]
```

output
```
(3*(-(B^2*c^2) + A^2*d^2)*(A^2 - B^2*x^2) + 4*A*B*(A^2*d + B^2*c*x)*(c + d
*x)*(-(((-(B^2*c^2) + A^2*d^2)*(A^2 - B^2*x^2))/(A^2*B^2*(c + d*x)^2)))^(1
/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (A^2*d + B^2*c*x)^2/(A^2*B^2*(c + d*x
)^2)])/(4*(B*c - A*d)*(B*c + A*d)^2*(c + d*x)^(4/3)*(A^2 - B^2*x^2)^(1/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {679, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt[3]{A^2-B^2x^2}(c+dx)^{7/3}}dx$$

$$\Big\downarrow \text{679}$$

$$\frac{AB\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{A^2-B^2x^2}}dx}{Ad+Bc}-\frac{3\left(A^2-B^2x^2\right)^{2/3}}{4(c+dx)^{4/3}(Ad+Bc)}$$

$$\Big\downarrow \text{489}$$

$$\frac{3AB(A+Bx)\sqrt[3]{-\frac{(A-Bx)(Bc-Ad)}{(A+Bx)(Ad+Bc)}}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{2}{3},\frac{2AB(c+dx)}{(Bc+Ad)(A+Bx)}\right)}{\sqrt[3]{A^2-B^2x^2}\sqrt[3]{c+dx}(Bc-Ad)(Ad+Bc)}-\frac{3\left(A^2-B^2x^2\right)^{2/3}}{4(c+dx)^{4/3}(Ad+Bc)}$$

input
```
Int[(A + B*x)/((c + d*x)^(7/3)*(A^2 - B^2*x^2)^(1/3)),x]
```

output
```
(-3*(A^2 - B^2*x^2)^(2/3))/(4*(B*c + A*d)*(c + d*x)^(4/3)) + (3*A*B*(-(((B
*c - A*d)*(A - B*x))/((B*c + A*d)*(A + B*x))))^(1/3)*(A + B*x)*Hypergeomet
ric2F1[-1/3, 1/3, 2/3, (2*A*B*(c + d*x))/((B*c + A*d)*(A + B*x))])/((B*c -
 A*d)*(B*c + A*d)*(c + d*x)^(1/3)*(A^2 - B^2*x^2)^(1/3))
```

Defintions of rubi rules used

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [F]

$$\int \frac{Bx + A}{(dx + c)^{\frac{7}{3}} \left(-B^2x^2 + A^2\right)^{\frac{1}{3}}}dx$$

input
```
int((B*x+A)/(d*x+c)^(7/3)/(-B^2*x^2+A^2)^(1/3),x)
```

output
```
int((B*x+A)/(d*x+c)^(7/3)/(-B^2*x^2+A^2)^(1/3),x)
```

Fricas [F]

$$\int \frac{A+Bx}{(c+dx)^{7/3}\sqrt[3]{A^2-B^2x^2}}\,dx=\int \frac{Bx+A}{(-B^2x^2+A^2)^{\frac{1}{3}}(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(7/3)/(-B^2*x^2+A^2)^(1/3),x, algorithm="fricas"
)
```

output
```
integral(-(-B^2*x^2 + A^2)^(2/3)*(d*x + c)^(2/3)/(B*d^3*x^4 - A*c^3 + (3*B
*c*d^2 - A*d^3)*x^3 + 3*(B*c^2*d - A*c*d^2)*x^2 + (B*c^3 - 3*A*c^2*d)*x),
x)
```

Sympy [F]

$$\int \frac{A+Bx}{(c+dx)^{7/3}\sqrt[3]{A^2-B^2x^2}}\,dx=\int \frac{A+Bx}{\sqrt[3]{-(-A+Bx)(A+Bx)}\,(c+dx)^{\frac{7}{3}}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)**(7/3)/(-B**2*x**2+A**2)**(1/3),x)
```

output
```
Integral((A + B*x)/((-(-A + B*x)*(A + B*x))**(1/3)*(c + d*x)**(7/3)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(c+dx)^{7/3}\sqrt[3]{A^2-B^2x^2}}\,dx=\int \frac{Bx+A}{(-B^2x^2+A^2)^{\frac{1}{3}}(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(7/3)/(-B^2*x^2+A^2)^(1/3),x, algorithm="maxima"
)
```

output
```
integrate((B*x + A)/((-B^2*x^2 + A^2)^(1/3)*(d*x + c)^(7/3)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(c+dx)^{7/3}\sqrt[3]{A^2-B^2x^2}}\,dx=\int \frac{Bx+A}{\left(-B^2x^2+A^2\right)^{\frac{1}{3}}(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(7/3)/(-B^2*x^2+A^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((B*x + A)/((-B^2*x^2 + A^2)^(1/3)*(d*x + c)^(7/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)^{7/3}\sqrt[3]{A^2-B^2x^2}}\,dx=\int \frac{A+B\,x}{\left(A^2-B^2\,x^2\right)^{1/3}\,\left(c+d\,x\right)^{7/3}}\,dx$$

input
```
int((A + B*x)/((A^2 - B^2*x^2)^(1/3)*(c + d*x)^(7/3)),x)
```

output
```
int((A + B*x)/((A^2 - B^2*x^2)^(1/3)*(c + d*x)^(7/3)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(c+dx)^{7/3}\sqrt[3]{A^2-B^2x^2}}\,dx=\left(\int \frac{x}{(dx+c)^{\frac{1}{3}}\left(-b^2x^2+a^2\right)^{\frac{1}{3}}c^2+2\,(dx+c)^{\frac{1}{3}}\left(-b^2x^2+a^2\right)^{\frac{1}{3}}cdx+(dx+\right.$$
$$+\left(\int \frac{1}{(dx+c)^{\frac{1}{3}}\left(-b^2x^2+a^2\right)^{\frac{1}{3}}c^2+2\,(dx+c)^{\frac{1}{3}}\left(-b^2x^2+a^2\right)^{\frac{1}{3}}cdx+(dx+c)^{\frac{1}{3}}\left(-b^2x^2+a^2\right)^{\frac{1}{3}}d^2x^2}dx\right)a$$

input
```
int((B*x+A)/(d*x+c)^(7/3)/(-B^2*x^2+A^2)^(1/3),x)
```

output
```
int(x/((c + d*x)**(1/3)*(a**2 - b**2*x**2)**(1/3)*c**2 + 2*(c + d*x)**(1/3
)*(a**2 - b**2*x**2)**(1/3)*c*d*x + (c + d*x)**(1/3)*(a**2 - b**2*x**2)**(
1/3)*d**2*x**2),x)*b + int(1/((c + d*x)**(1/3)*(a**2 - b**2*x**2)**(1/3)*c
**2 + 2*(c + d*x)**(1/3)*(a**2 - b**2*x**2)**(1/3)*c*d*x + (c + d*x)**(1/3
)*(a**2 - b**2*x**2)**(1/3)*d**2*x**2),x)*a
```

3.9 $\int \frac{A+Bx}{(c+dx)^{5/3}\left(A^2-B^2x^2\right)^{2/3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 112

$$\int \frac{A+Bx}{(c+dx)^{5/3}\left(A^2-B^2x^2\right)^{2/3}}\,dx = -\frac{3\sqrt[3]{2}\sqrt[3]{A^2-B^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{(Bc-Ad)(A-Bx)}{2AB(c+dx)}\right)}{(Bc+Ad)\sqrt[3]{\frac{(Bc+Ad)(A+Bx)}{AB(c+dx)}}(c+dx)^{2/3}}$$

output
```
-3*2^(1/3)*(-B^2*x^2+A^2)^(1/3)*hypergeom([-1/3, 1/3],[4/3],1/2*(-A*d+B*c)
*(-B*x+A)/A/B/(d*x+c))/(A*d+B*c)/((A*d+B*c)*(B*x+A)/A/B/(d*x+c))^(1/3)/(d*
x+c)^(2/3)
```

Mathematica [A] (warning: unable to verify)

Time = 15.85 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.83

$$\int \frac{A+Bx}{(c+dx)^{5/3}\left(A^2-B^2x^2\right)^{2/3}}\,dx = \frac{-3d(A^2-B^2x^2)+6AB\sqrt[3]{\frac{d\left(\sqrt{\frac{A^2}{B^2}}-x\right)}{c+\sqrt{\frac{A^2}{B^2}}d}}\left(\frac{d\left(\sqrt{\frac{A^2}{B^2}}+x\right)}{-c+\sqrt{\frac{A^2}{B^2}}d}\right)^{2/3}(c+dx)\,\mathrm{H}}{2d(Bc+Ad)(c+dx)^{2/3}\,(A^2-}$$

input
```
Integrate[(A + B*x)/((c + d*x)^(5/3)*(A^2 - B^2*x^2)^(2/3)),x]
```

output
```
(-3*d*(A^2 - B^2*x^2) + 6*A*B*((d*(Sqrt[A^2/B^2] - x))/(c + Sqrt[A^2/B^2]*
d))^(1/3)*((d*(Sqrt[A^2/B^2] + x))/(-c + Sqrt[A^2/B^2]*d))^(2/3)*(c + d*x)
*Hypergeometric2F1[1/3, 2/3, 4/3, (2*Sqrt[A^2/B^2]*(c + d*x))/((c - Sqrt[A
^2/B^2]*d)*(Sqrt[A^2/B^2] - x))])/(2*d*(B*c + A*d)*(c + d*x)^(2/3)*(A^2 -
B^2*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {679, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(A^2-B^2x^2)^{2/3}(c+dx)^{5/3}}dx$$

$$\Big\downarrow \ 679$$

$$\frac{AB\int \frac{1}{(c+dx)^{2/3}(A^2-B^2x^2)^{2/3}}dx}{Ad+Bc}-\frac{3\sqrt[3]{A^2-B^2x^2}}{2(c+dx)^{2/3}(Ad+Bc)}$$

$$\Big\downarrow \ 489$$

$$-\frac{3AB(A+Bx)\sqrt[3]{c+dx}\left(-\frac{(A-Bx)(Bc-Ad)}{(A+Bx)(Ad+Bc)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{2AB(c+dx)}{(Bc+Ad)(A+Bx)}\right)}{(A^2-B^2x^2)^{2/3}(Bc-Ad)(Ad+Bc)}-\frac{3\sqrt[3]{A^2-B^2x^2}}{2(c+dx)^{2/3}(Ad+Bc)}$$

input
```
Int[(A + B*x)/((c + d*x)^(5/3)*(A^2 - B^2*x^2)^(2/3)),x]
```

output
```
(-3*(A^2 - B^2*x^2)^(1/3))/(2*(B*c + A*d)*(c + d*x)^(2/3)) - (3*A*B*(-(((B
*c - A*d)*(A - B*x))/((B*c + A*d)*(A + B*x))))^(2/3)*(A + B*x)*(c + d*x)^(
1/3)*Hypergeometric2F1[1/3, 2/3, 4/3, (2*A*B*(c + d*x))/((B*c + A*d)*(A +
B*x))])/((B*c - A*d)*(B*c + A*d)*(A^2 - B^2*x^2)^(2/3))
```

Defintions of rubi rules used

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [F]

$$\int \frac{Bx + A}{(dx + c)^{\frac{5}{3}} (-B^2x^2 + A^2)^{\frac{2}{3}}} dx$$

input
```
int((B*x+A)/(d*x+c)^(5/3)/(-B^2*x^2+A^2)^(2/3),x)
```

output
```
int((B*x+A)/(d*x+c)^(5/3)/(-B^2*x^2+A^2)^(2/3),x)
```

Fricas [F]

$$\int \frac{A+Bx}{(c+dx)^{5/3}\left(A^2-B^2x^2\right)^{2/3}}\,dx=\int \frac{Bx+A}{\left(-B^2x^2+A^2\right)^{\frac{2}{3}}(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(5/3)/(-B^2*x^2+A^2)^(2/3),x, algorithm="fricas"
)
```

output
```
integral(-(-B^2*x^2 + A^2)^(1/3)*(d*x + c)^(1/3)/(B*d^2*x^3 - A*c^2 + (2*B
*c*d - A*d^2)*x^2 + (B*c^2 - 2*A*c*d)*x), x)
```

Sympy [F]

$$\int \frac{A+Bx}{(c+dx)^{5/3}\left(A^2-B^2x^2\right)^{2/3}}\,dx=\int \frac{A+Bx}{\left(-\left(-A+Bx\right)\left(A+Bx\right)\right)^{\frac{2}{3}}(c+dx)^{\frac{5}{3}}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)**(5/3)/(-B**2*x**2+A**2)**(2/3),x)
```

output
```
Integral((A + B*x)/((-(-A + B*x)*(A + B*x))**(2/3)*(c + d*x)**(5/3)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(c+dx)^{5/3}\left(A^2-B^2x^2\right)^{2/3}}\,dx=\int \frac{Bx+A}{\left(-B^2x^2+A^2\right)^{\frac{2}{3}}(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(5/3)/(-B^2*x^2+A^2)^(2/3),x, algorithm="maxima"
)
```

output
```
integrate((B*x + A)/((-B^2*x^2 + A^2)^(2/3)*(d*x + c)^(5/3)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(c+dx)^{5/3}\left(A^2-B^2x^2\right)^{2/3}}\,dx=\int \frac{Bx+A}{\left(-B^2x^2+A^2\right)^{\frac{2}{3}}(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(5/3)/(-B^2*x^2+A^2)^(2/3),x, algorithm="giac")
```

output
```
integrate((B*x + A)/((-B^2*x^2 + A^2)^(2/3)*(d*x + c)^(5/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)^{5/3}\left(A^2-B^2x^2\right)^{2/3}}\,dx=\int \frac{A+B\,x}{\left(A^2-B^2\,x^2\right)^{2/3}\,(c+d\,x)^{5/3}}\,dx$$

input
```
int((A + B*x)/((A^2 - B^2*x^2)^(2/3)*(c + d*x)^(5/3)),x)
```

output
```
int((A + B*x)/((A^2 - B^2*x^2)^(2/3)*(c + d*x)^(5/3)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(c+dx)^{5/3}\left(A^2-B^2x^2\right)^{2/3}}\,dx=\left(\int \frac{x}{(dx+c)^{\frac{2}{3}}\left(-b^2x^2+a^2\right)^{\frac{2}{3}}c+(dx+c)^{\frac{2}{3}}\left(-b^2x^2+a^2\right)^{\frac{2}{3}}dx}dx\right)b+\left(\int \frac{1}{(dx+c)^{\frac{2}{3}}\left(-b^2x^2+a^2\right)^{\frac{2}{3}}c+(dx+c)^{\frac{2}{3}}\left(-b^2x^2+a^2\right)^{\frac{2}{3}}dx}dx\right)a$$

input
```
int((B*x+A)/(d*x+c)^(5/3)/(-B^2*x^2+A^2)^(2/3),x)
```

output
```
int(x/((c + d*x)**(2/3)*(a**2 - b**2*x**2)**(2/3)*c + (c + d*x)**(2/3)*(a*
*2 - b**2*x**2)**(2/3)*d*x),x)*b + int(1/((c + d*x)**(2/3)*(a**2 - b**2*x**
*2)**(2/3)*c + (c + d*x)**(2/3)*(a**2 - b**2*x**2)**(2/3)*d*x),x)*a
```

3.10 $\int \frac{A+Bx}{\sqrt[3]{c+dx}\left(A^2-B^2x^2\right)^{4/3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 112

$$\int \frac{A+Bx}{\sqrt[3]{c+dx}\left(A^2-B^2x^2\right)^{4/3}}\,dx = \frac{3\sqrt[3]{\frac{(Bc+Ad)(A+Bx)}{AB(c+dx)}}(c+dx)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{2}{3},\frac{(Bc-A}{2AB}\right.}{\sqrt[3]{2}(Bc+Ad)\sqrt[3]{A^2-B^2x^2}}$$

output
```
3/2*((A*d+B*c)*(B*x+A)/A/B/(d*x+c))^(1/3)*(d*x+c)^(2/3)*hypergeom([-1/3, 1
/3],[2/3],1/2*(-A*d+B*c)*(-B*x+A)/A/B/(d*x+c))*2^(2/3)/(A*d+B*c)/(-B^2*x^2
+A^2)^(1/3)
```

Mathematica [A] (verified)

Time = 10.63 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.54

$$\int \frac{A+Bx}{\sqrt[3]{c+dx}\left(A^2-B^2x^2\right)^{4/3}}\,dx = \frac{\left(A^2-B^2x^2\right)^{2/3}\left(-3(Bc+Ad)(A+Bx)+\left(A^2d+B^2cx\right)\sqrt[3]{-\frac{\left(-B^2c^2+\right.}{A^2}}\right.}{2AB(Bc+Ad)(-A+Bx)}$$

input
```
Integrate[(A + B*x)/((c + d*x)^(1/3)*(A^2 - B^2*x^2)^(4/3)),x]
```

output
```
((A^2 - B^2*x^2)^(2/3)*(-3*(B*c + A*d)*(A + B*x) + (A^2*d + B^2*c*x)*(-(((
-(B^2*c^2) + A^2*d^2)*(A^2 - B^2*x^2))/(A^2*B^2*(c + d*x)^2)))^(1/3)*Hyper
geometric2F1[1/3, 1/2, 3/2, (A^2*d + B^2*c*x)^2/(A^2*B^2*(c + d*x)^2)]))/(
2*A*B*(B*c + A*d)*(-A + B*x)*(A + B*x)*(c + d*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.36, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {678, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\left(A^2-B^2x^2\right)^{4/3}\sqrt[3]{c+dx}}dx$$

$$\Big\downarrow \text{678}$$

$$\frac{3(A+Bx)}{2AB\sqrt[3]{A^2-B^2x^2}\sqrt[3]{c+dx}}-\frac{(Bc-Ad)\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{A^2-B^2x^2}}dx}{2AB}$$

$$\Big\downarrow \text{489}$$

$$\frac{3(A+Bx)}{2AB\sqrt[3]{A^2-B^2x^2}\sqrt[3]{c+dx}}-\frac{3(A+Bx)\sqrt[3]{-\frac{(A-Bx)(Bc-Ad)}{(A+Bx)(Ad+Bc)}}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{2}{3},\frac{2AB(c+dx)}{(Bc+Ad)(A+Bx)}\right)}{2AB\sqrt[3]{A^2-B^2x^2}\sqrt[3]{c+dx}}$$

input
```
Int[(A + B*x)/((c + d*x)^(1/3)*(A^2 - B^2*x^2)^(4/3)),x]
```

output
```
(3*(A + B*x))/(2*A*B*(c + d*x)^(1/3)*(A^2 - B^2*x^2)^(1/3)) - (3*(-(((B*c
- A*d)*(A - B*x))/((B*c + A*d)*(A + B*x))))^(1/3)*(A + B*x)*Hypergeometric
2F1[-1/3, 1/3, 2/3, (2*A*B*(c + d*x))/((B*c + A*d)*(A + B*x))])/(2*A*B*(c
+ d*x)^(1/3)*(A^2 - B^2*x^2)^(1/3))
```

Defintions of rubi rules used

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 678
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[m*((c*d*f + a*e*g)/(2*a*c*(p + 1)))   Int[(d + e*x)^(
m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && EqQ[S
implify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{Bx+A}{(dx+c)^{\frac{1}{3}}\left(-B^{2}x^{2}+A^{2}\right)^{\frac{4}{3}}}dx$$

input
```
int((B*x+A)/(d*x+c)^(1/3)/(-B^2*x^2+A^2)^(4/3),x)
```

output
```
int((B*x+A)/(d*x+c)^(1/3)/(-B^2*x^2+A^2)^(4/3),x)
```

Fricas [F]

$$\int \frac{A+Bx}{\sqrt[3]{c+dx}\,(A^2-B^2x^2)^{4/3}}\,dx=\int \frac{Bx+A}{\left(-B^{2}x^{2}+A^{2}\right)^{\frac{4}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(1/3)/(-B^2*x^2+A^2)^(4/3),x, algorithm="fricas"
)
```

output
```
integral((-B^2*x^2 + A^2)^(2/3)*(d*x + c)^(2/3)/(B^3*d*x^4 + A^3*c + (B^3*
c - A*B^2*d)*x^3 - (A*B^2*c + A^2*B*d)*x^2 - (A^2*B*c - A^3*d)*x), x)
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt[3]{c+dx}\,(A^2-B^2x^2)^{4/3}}\,dx=\int \frac{A+Bx}{(-(-A+Bx)\,(A+Bx))^{\frac{4}{3}}\,\sqrt[3]{c+dx}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)**(1/3)/(-B**2*x**2+A**2)**(4/3),x)
```

output
```
Integral((A + B*x)/((-(-A + B*x)*(A + B*x))**(4/3)*(c + d*x)**(1/3)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt[3]{c+dx}\,(A^2-B^2x^2)^{4/3}}\,dx=\int \frac{Bx+A}{(-B^2x^2+A^2)^{\frac{4}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(1/3)/(-B^2*x^2+A^2)^(4/3),x, algorithm="maxima"
)
```

output
```
integrate((B*x + A)/((-B^2*x^2 + A^2)^(4/3)*(d*x + c)^(1/3)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt[3]{c+dx}\,(A^2-B^2x^2)^{4/3}}\,dx=\int \frac{Bx+A}{(-B^2x^2+A^2)^{\frac{4}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(1/3)/(-B^2*x^2+A^2)^(4/3),x, algorithm="giac")
```

output
```
integrate((B*x + A)/((-B^2*x^2 + A^2)^(4/3)*(d*x + c)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt[3]{c+dx}\,(A^2-B^2x^2)^{4/3}}\,dx=\int \frac{A+B\,x}{(A^2-B^2\,x^2)^{4/3}\,(c+d\,x)^{1/3}}\,dx$$

input
```
int((A + B*x)/((A^2 - B^2*x^2)^(4/3)*(c + d*x)^(1/3)),x)
```

output
```
int((A + B*x)/((A^2 - B^2*x^2)^(4/3)*(c + d*x)^(1/3)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt[3]{c+dx}\,(A^2-B^2x^2)^{4/3}}\,dx=\int \frac{1}{(dx+c)^{\frac{1}{3}}\,(-b^2x^2+a^2)^{\frac{1}{3}}\,a-(dx+c)^{\frac{1}{3}}\,(-b^2x^2+a^2)^{\frac{1}{3}}\,bx}dx$$

input
```
int((B*x+A)/(d*x+c)^(1/3)/(-B^2*x^2+A^2)^(4/3),x)
```

output
```
int(1/((c + d*x)**(1/3)*(a**2 - b**2*x**2)**(1/3)*a - (c + d*x)**(1/3)*(a*
*2 - b**2*x**2)**(1/3)*b*x),x)
```

3.11 $\int \frac{(A+Bx)\sqrt[3]{c+dx}}{(A^2-B^2x^2)^{5/3}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 114

$$\int \frac{(A+Bx)\sqrt[3]{c+dx}}{(A^2-B^2x^2)^{5/3}}\,dx = \frac{3\left(\frac{(Bc+Ad)(A+Bx)}{AB(c+dx)}\right)^{2/3}(c+dx)^{4/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{1}{3},\frac{(Bc-Ad)(A-Bx)}{2AB(c+dx)}\right)}{2\,2^{2/3}(Bc+Ad)\,(A^2-B^2x^2)^{2/3}}$$

output
```
3/4*((A*d+B*c)*(B*x+A)/A/B/(d*x+c))^(2/3)*(d*x+c)^(4/3)*hypergeom([-2/3, 2
/3],[1/3],1/2*(-A*d+B*c)*(-B*x+A)/A/B/(d*x+c))*2^(1/3)/(A*d+B*c)/(-B^2*x^2
+A^2)^(2/3)
```

Mathematica [A] (warning: unable to verify)

Time = 13.65 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.70

$$\int \frac{(A+Bx)\sqrt[3]{c+dx}}{(A^2-B^2x^2)^{5/3}}\,dx = \frac{3\sqrt[3]{c+dx}\left(d(A+Bx)+(Bc-Ad)\sqrt[3]{\frac{d\left(\sqrt{\frac{A^2}{B^2}}-x\right)}{c+\sqrt{\frac{A^2}{B^2}}d}}\left(\frac{d\left(\sqrt{\frac{A^2}{B^2}}+x\right)}{-c+\sqrt{\frac{A^2}{B^2}}d}\right)^{2/3}\text{Hyperg}\right.}{4ABd\,(A^2-B^2x^2)^{2/3}}$$

input
```
Integrate[((A + B*x)*(c + d*x)^(1/3))/(A^2 - B^2*x^2)^(5/3),x]
```

output
```
(3*(c + d*x)^(1/3)*(d*(A + B*x) + (B*c - A*d)*((d*(Sqrt[A^2/B^2] - x))/(c
+ Sqrt[A^2/B^2]*d))^(1/3)*((d*(Sqrt[A^2/B^2] + x))/(-c + Sqrt[A^2/B^2]*d))
^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, (2*Sqrt[A^2/B^2]*(c + d*x))/((c -
Sqrt[A^2/B^2]*d)*(Sqrt[A^2/B^2] - x))]))/(4*A*B*d*(A^2 - B^2*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {678, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt[3]{c+dx}}{(A^2-B^2x^2)^{5/3}}dx$$

$$\Big\downarrow \text{ 678}$$

$$\frac{(Bc-Ad)\int \frac{1}{(c+dx)^{2/3}(A^2-B^2x^2)^{2/3}}dx}{4AB}+\frac{3(A+Bx)\sqrt[3]{c+dx}}{4AB\left(A^2-B^2x^2\right)^{2/3}}$$

$$\Big\downarrow \text{ 489}$$

$$\frac{3(A+Bx)\sqrt[3]{c+dx}}{4AB\left(A^2-B^2x^2\right)^{2/3}}-\frac{3(A+Bx)\sqrt[3]{c+dx}\left(-\frac{(A-Bx)(Bc-Ad)}{(A+Bx)(Ad+Bc)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{2AB(c+dx)}{(Bc+Ad)(A+Bx)}\right)}{4AB\left(A^2-B^2x^2\right)^{2/3}}$$

input
```
Int[((A + B*x)*(c + d*x)^(1/3))/(A^2 - B^2*x^2)^(5/3),x]
```

output
```
(3*(A + B*x)*(c + d*x)^(1/3))/(4*A*B*(A^2 - B^2*x^2)^(2/3)) - (3*(-(((B*c
- A*d)*(A - B*x))/((B*c + A*d)*(A + B*x))))^(2/3)*(A + B*x)*(c + d*x)^(1/3
)*Hypergeometric2F1[1/3, 2/3, 4/3, (2*A*B*(c + d*x))/((B*c + A*d)*(A + B*x
))])/(4*A*B*(A^2 - B^2*x^2)^(2/3))
```

Defintions of rubi rules used

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 678
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[m*((c*d*f + a*e*g)/(2*a*c*(p + 1)))   Int[(d + e*x)^(
m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && EqQ[S
implify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{(Bx+A)\,(dx+c)^{\frac{1}{3}}}{(-B^2x^2+A^2)^{\frac{5}{3}}}dx$$

input
```
int((B*x+A)*(d*x+c)^(1/3)/(-B^2*x^2+A^2)^(5/3),x)
```

output
```
int((B*x+A)*(d*x+c)^(1/3)/(-B^2*x^2+A^2)^(5/3),x)
```

Fricas [F]

$$\int \frac{(A+Bx)\sqrt[3]{c+dx}}{(A^2-B^2x^2)^{5/3}}\,dx=\int \frac{(Bx+A)(dx+c)^{\frac{1}{3}}}{(-B^2x^2+A^2)^{\frac{5}{3}}}\,dx$$

input
```
integrate((B*x+A)*(d*x+c)^(1/3)/(-B^2*x^2+A^2)^(5/3),x, algorithm="fricas"
)
```

output
```
integral((-B^2*x^2 + A^2)^(1/3)*(d*x + c)^(1/3)/(B^3*x^3 - A*B^2*x^2 - A^2
*B*x + A^3), x)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt[3]{c+dx}}{(A^2-B^2x^2)^{5/3}}\,dx = \int \frac{(A+Bx)\sqrt[3]{c+dx}}{(-(-A+Bx)(A+Bx))^{\frac{5}{3}}}\,dx$$

input
```
integrate((B*x+A)*(d*x+c)**(1/3)/(-B**2*x**2+A**2)**(5/3),x)
```

output
```
Integral((A + B*x)*(c + d*x)**(1/3)/(-(-A + B*x)*(A + B*x))**(5/3), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt[3]{c+dx}}{(A^2-B^2x^2)^{5/3}}\,dx = \int \frac{(Bx+A)(dx+c)^{\frac{1}{3}}}{(-B^2x^2+A^2)^{\frac{5}{3}}}\,dx$$

input
```
integrate((B*x+A)*(d*x+c)^(1/3)/(-B^2*x^2+A^2)^(5/3),x, algorithm="maxima"
)
```

output
```
integrate((B*x + A)*(d*x + c)^(1/3)/(-B^2*x^2 + A^2)^(5/3), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt[3]{c+dx}}{(A^2-B^2x^2)^{5/3}}\,dx = \int \frac{(Bx+A)(dx+c)^{\frac{1}{3}}}{(-B^2x^2+A^2)^{\frac{5}{3}}}\,dx$$

input
```
integrate((B*x+A)*(d*x+c)^(1/3)/(-B^2*x^2+A^2)^(5/3),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(d*x + c)^(1/3)/(-B^2*x^2 + A^2)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt[3]{c+dx}}{(A^2-B^2x^2)^{5/3}}\,dx = \int \frac{(A+B\,x)\,(c+d\,x)^{1/3}}{(A^2-B^2\,x^2)^{5/3}}\,dx$$

input
```
int(((A + B*x)*(c + d*x)^(1/3))/(A^2 - B^2*x^2)^(5/3),x)
```

output
```
int(((A + B*x)*(c + d*x)^(1/3))/(A^2 - B^2*x^2)^(5/3), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt[3]{c+dx}}{(A^2-B^2x^2)^{5/3}}\,dx = \int \frac{(dx+c)^{\frac{1}{3}}}{(-b^2x^2+a^2)^{\frac{2}{3}}\,a-(-b^2x^2+a^2)^{\frac{2}{3}}\,bx}dx$$

input
```
int((B*x+A)*(d*x+c)^(1/3)/(-B^2*x^2+A^2)^(5/3),x)
```

output
```
int((c + d*x)**(1/3)/((a**2 - b**2*x**2)**(2/3)*a - (a**2 - b**2*x**2)**(2
/3)*b*x),x)
```

3.12 $\int (1+x)(a+bx)^m \left(1-x^2\right)^{\frac{1}{2}(-5-m)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 244

$$\int (1+x)(a+bx)^m \left(1-x^2\right)^{\frac{1}{2}(-5-m)}\,dx = \frac{(1-x)^{\frac{1}{2}(-3-m)}(1+x)^{\frac{1}{2}(-1-m)}(a+bx)^{1+m}}{(a+b)(3+m)}$$
$$-\frac{b(a+2b+am)(a+bx)^{1+m}\left(1-x^2\right)^{\frac{1}{2}(-1-m)}}{(a-b)(a+b)^2(1+m)(3+m)}$$
$$+\frac{\left(2ab-b^2+a^2(2+m)\right)(1-x)\left(-\frac{(a+b)(1+x)}{(a-b)(1-x)}\right)^{\frac{3+m}{2}}(a+bx)^{2+m}\left(1-x^2\right)^{\frac{1}{2}(-3-m)}\text{Hypergeometric2F1}\left(2\right.}{(a-b)(a+b)^3(2+m)(3+m)}$$

output
```
(1-x)^(-3/2-1/2*m)*(1+x)^(-1/2-1/2*m)*(b*x+a)^(1+m)/(a+b)/(3+m)-b*(a*m+a+2
*b)*(b*x+a)^(1+m)*(-x^2+1)^(-1/2-1/2*m)/(a-b)/(a+b)^2/(1+m)/(3+m)+(2*a*b-b
^2+a^2*(2+m))*(1-x)*(-(a+b)*(1+x)/(a-b)/(1-x))^(3/2+1/2*m)*(b*x+a)^(2+m)*(
-x^2+1)^(-3/2-1/2*m)*hypergeom([2+m, 3/2+1/2*m],[3+m],2*(b*x+a)/(a-b)/(1-x
))/(a-b)/(a+b)^3/(2+m)/(3+m)
```

Mathematica [F]

$$\int (1+x)(a+bx)^m \left(1-x^2\right)^{\frac{1}{2}(-5-m)}\,dx = \int (1+x)(a+bx)^m \left(1-x^2\right)^{\frac{1}{2}(-5-m)}\,dx$$

input
```
Integrate[(1 + x)*(a + b*x)^m*(1 - x^2)^((-5 - m)/2),x]
```

output
```
Integrate[(1 + x)*(a + b*x)^m*(1 - x^2)^((-5 - m)/2), x]
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {689, 25, 27, 689, 25, 679, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)\left(1-x^2\right)^{\frac{1}{2}(-m-5)}(a+bx)^m\,dx$$

$$\Big\downarrow \; 689$$

$$\frac{\left(1-x^2\right)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+1)(a+b)}-\frac{\int -\left((a-b)(m-2x+1)(a+bx)^{m+1}\left(1-x^2\right)^{\frac{1}{2}(-m-5)}\right)dx}{(m+1)\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int (a-b)(m-2x+1)(a+bx)^{m+1}\left(1-x^2\right)^{\frac{1}{2}(-m-5)}dx}{(m+1)\left(a^2-b^2\right)}+\frac{\left(1-x^2\right)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+1)(a+b)}$$

$$\Big\downarrow \; 27$$

$$\frac{(a-b)\int (m-2x+1)(a+bx)^{m+1}\left(1-x^2\right)^{\frac{1}{2}(-m-5)}dx}{(m+1)\left(a^2-b^2\right)}+\frac{\left(1-x^2\right)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+1)(a+b)}$$

$$\Big\downarrow \; 689$$

$$\frac{(a-b)\left(-\frac{\int -(a+bx)^{m+2}((m+2)(ma+a+2b)+(2a+b+bm)x)(1-x^2)^{\frac{1}{2}(-m-5)}dx}{(m+2)(a^2-b^2)}-\frac{(1-x^2)^{\frac{1}{2}(-m-3)}(2a+bm+b)(a+bx)^{m+2}}{(m+2)(a^2-b^2)}\right)}{(m+1)(a^2-b^2)}+\frac{(1-x^2)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+1)(a+b)}$$

$\Big\downarrow$ 25

$$\frac{(a-b)\left(\frac{\int (a+bx)^{m+2}((m+2)(ma+a+2b)+(2a+b+bm)x)(1-x^2)^{\frac{1}{2}(-m-5)}dx}{(m+2)(a^2-b^2)}-\frac{(1-x^2)^{\frac{1}{2}(-m-3)}(2a+bm+b)(a+bx)^{m+2}}{(m+2)(a^2-b^2)}\right)}{(m+1)(a^2-b^2)}+\frac{(1-x^2)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+1)(a+b)}$$

$\Big\downarrow$ 679

$$\frac{(a-b)\left(\frac{\frac{(m+1)\left(a^2(m+2)+2ab-b^2\right)\int (a+bx)^{m+3}\left(1-x^2\right)^{\frac{1}{2}(-m-5)}dx}{a^2-b^2}+\frac{\left(1-x^2\right)^{\frac{1}{2}(-m-3)}\left(2a^2-ab(m+1)^2-2b^2(m+2)\right)(a+bx)^{m+3}}{(m+3)\left(a^2-b^2\right)}}{(m+2)(a^2-b^2)}-\frac{(1-x^2)^{\frac{1}{2}(-m}}{(}\right.}{(m+1)(a^2-b^2)}$$

$$\frac{(1-x^2)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+1)(a+b)}$$

$\Big\downarrow$ 489

$$\frac{(a-b)\left(\frac{\frac{\left(1-x^2\right)^{\frac{1}{2}(-m-3)}\left(2a^2-ab(m+1)^2-2b^2(m+2)\right)(a+bx)^{m+3}}{(m+3)\left(a^2-b^2\right)}-\frac{(m+1)(x+1)\left(1-x^2\right)^{\frac{1}{2}(-m-5)}\left(a^2(m+2)+2ab-b^2\right)\left(-\frac{(1-x)(a-b)}{(x+1)(a+b)}\right)^{\frac{m+5}{2}}(a+bx)}{(m+4)(a-b)\left(a^2-b^2\right)}}{(m+2)(a^2-b^2)}\right.}{(m+1)(a^2-b^2)}$$

$$\frac{(1-x^2)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+1)(a+b)}$$

input

```
Int[(1 + x)*(a + b*x)^m*(1 - x^2)^((-5 - m)/2),x]
```

output
```
((a + b*x)^(1 + m)*(1 - x^2)^((-3 - m)/2))/((a + b)*(1 + m)) + ((a - b)*(-
(((2*a + b + b*m)*(a + b*x)^(2 + m)*(1 - x^2)^((-3 - m)/2))/((a^2 - b^2)*(
2 + m))) + (((2*a^2 - a*b*(1 + m)^2 - 2*b^2*(2 + m))*(a + b*x)^(3 + m)*(1
- x^2)^((-3 - m)/2))/((a^2 - b^2)*(3 + m)) - ((1 + m)*(2*a*b - b^2 + a^2*(
2 + m))*(-(((a - b)*(1 - x))/((a + b)*(1 + x))))^((5 + m)/2)*(1 + x)*(a +
b*x)^(4 + m)*(1 - x^2)^((-5 - m)/2)*Hypergeometric2F1[4 + m, (5 + m)/2, 5
+ m, (2*(a + b*x))/((a + b)*(1 + x))])/((a - b)*(a^2 - b^2)*(4 + m)))/((a^
2 - b^2)*(2 + m))))/((a^2 - b^2)*(1 + m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 689
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && ILtQ[Sim
plify[m + 2*p + 3], 0] && NeQ[m, -1]
```

Maple [F]

$$\int (x+1)\,(bx+a)^m\left(-x^2+1\right)^{-\frac{5}{2}-\frac{m}{2}}\,dx$$

input
```
int((x+1)*(b*x+a)^m*(-x^2+1)^(-5/2-1/2*m),x)
```

output
```
int((x+1)*(b*x+a)^m*(-x^2+1)^(-5/2-1/2*m),x)
```

Fricas [F]

$$\int (1+x)(a+bx)^m\left(1-x^2\right)^{\frac{1}{2}(-5-m)}\,dx = \int (bx+a)^m\left(-x^2+1\right)^{-\frac{1}{2}m-\frac{5}{2}}(x+1)\,dx$$

input
```
integrate((1+x)*(b*x+a)^m*(-x^2+1)^(-5/2-1/2*m),x, algorithm="fricas")
```

output
```
integral((b*x + a)^m*(-x^2 + 1)^(-1/2*m - 5/2)*(x + 1), x)
```

Sympy [F]

$$\int (1+x)(a+bx)^m\left(1-x^2\right)^{\frac{1}{2}(-5-m)}\,dx = \int \left(-(x-1)\,(x+1)\right)^{-\frac{m}{2}-\frac{5}{2}}(a+bx)^m\,(x+1)\,dx$$

input
```
integrate((1+x)*(b*x+a)**m*(-x**2+1)**(-5/2-1/2*m),x)
```

output
```
Integral((-(x - 1)*(x + 1))**(-m/2 - 5/2)*(a + b*x)**m*(x + 1), x)
```

Maxima [F]

$$\int (1+x)(a+bx)^m \left(1-x^2\right)^{\frac{1}{2}(-5-m)} \, dx = \int (bx+a)^m \left(-x^2+1\right)^{-\frac{1}{2}m-\frac{5}{2}} (x+1) \, dx$$

input
```
integrate((1+x)*(b*x+a)^m*(-x^2+1)^(-5/2-1/2*m),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^m*(-x^2 + 1)^(-1/2*m - 5/2)*(x + 1), x)
```

Giac [F]

$$\int (1+x)(a+bx)^m \left(1-x^2\right)^{\frac{1}{2}(-5-m)} \, dx = \int (bx+a)^m \left(-x^2+1\right)^{-\frac{1}{2}m-\frac{5}{2}} (x+1) \, dx$$

input
```
integrate((1+x)*(b*x+a)^m*(-x^2+1)^(-5/2-1/2*m),x, algorithm="giac")
```

output
```
integrate((b*x + a)^m*(-x^2 + 1)^(-1/2*m - 5/2)*(x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+x)(a+bx)^m \left(1-x^2\right)^{\frac{1}{2}(-5-m)} \, dx = \int \frac{(x+1)\ (a+b\,x)^m}{\left(1-x^2\right)^{\frac{m}{2}+\frac{5}{2}}} \, dx$$

input
```
int(((x + 1)*(a + b*x)^m)/(1 - x^2)^(m/2 + 5/2),x)
```

output
```
int(((x + 1)*(a + b*x)^m)/(1 - x^2)^(m/2 + 5/2), x)
```

Reduce [F]

$$\int (1+x)(a+bx)^m \left(1-x^2\right)^{\frac{1}{2}(-5-m)}\,dx$$
$$=\int \frac{(bx+a)^m}{(-x^2+1)^{\frac{m}{2}+\frac{1}{2}}x^3-(-x^2+1)^{\frac{m}{2}+\frac{1}{2}}x^2-(-x^2+1)^{\frac{m}{2}+\frac{1}{2}}x+(-x^2+1)^{\frac{m}{2}+\frac{1}{2}}}dx$$

input
```
int((1+x)*(b*x+a)^m*(-x^2+1)^(-5/2-1/2*m),x)
```

output
```
int((a + b*x)**m/(( - x**2 + 1)**((m + 1)/2)*x**3 - ( - x**2 + 1)**((m + 1
)/2)*x**2 - ( - x**2 + 1)**((m + 1)/2)*x + ( - x**2 + 1)**((m + 1)/2)),x)
```

3.13 $\int \frac{(a+bx)^m \left(1-x^2\right)^{\frac{1}{2}(-3-m)}}{1-x} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 244

$$\int \frac{(a+bx)^m \left(1-x^2\right)^{\frac{1}{2}(-3-m)}}{1-x} \, dx = \frac{(1-x)^{\frac{1}{2}(-3-m)}(1+x)^{\frac{1}{2}(-1-m)}(a+bx)^{1+m}}{(a+b)(3+m)} - \frac{b(a+2b+am)(a+bx)^{1+m} \left(1-x^2\right)^{\frac{1}{2}(-1-m)}}{(a-b)(a+b)^2(1+m)(3+m)} + \frac{\left(2ab-b^2+a^2(2+m)\right)(1-x)\left(-\frac{(a+b)(1+x)}{(a-b)(1-x)}\right)^{\frac{3+m}{2}} (a+bx)^{2+m} \left(1-x^2\right)^{\frac{1}{2}(-3-m)} \text{Hypergeometric2F1}\left(2\right.}{(a-b)(a+b)^3(2+m)(3+m)}$$

output
```
(1-x)^(-3/2-1/2*m)*(1+x)^(-1/2-1/2*m)*(b*x+a)^(1+m)/(a+b)/(3+m)-b*(a*m+a+2
*b)*(b*x+a)^(1+m)*(-x^2+1)^(-1/2-1/2*m)/(a-b)/(a+b)^2/(1+m)/(3+m)+(2*a*b-b
^2+a^2*(2+m))*(1-x)*(-(a+b)*(1+x)/(a-b)/(1-x))^(3/2+1/2*m)*(b*x+a)^(2+m)*(
-x^2+1)^(-3/2-1/2*m)*hypergeom([2+m, 3/2+1/2*m],[3+m],2*(b*x+a)/(a-b)/(1-x
))/(a-b)/(a+b)^3/(2+m)/(3+m)
```

Mathematica [A] (warning: unable to verify)

Time = 10.95 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)^m \left(1-x^2\right)^{\frac{1}{2}(-3-m)}}{1-x}\,dx$$
$$= \frac{(a+bx)^{1+m}\left(1-x^2\right)^{-\frac{3}{2}-\frac{m}{2}}\left(4(1+x)-\frac{4(3b+a(2+m))(-1+x)(1+x)}{(a+b)(1+m)}+\frac{2^{\frac{3}{2}-\frac{m}{2}}\left(2ab-b^2+a^2(2+m)\right)\left(\frac{(a+b)(1+x)}{a+bx}\right)^{\frac{1}{2}(-1+m)}(\cdots}{(-\cdots}\right.}{4(a+b)(3+m)}$$

input
```
Integrate[((a + b*x)^m*(1 - x^2)^((-3 - m)/2))/(1 - x),x]
```

output
```
((a + b*x)^(1 + m)*(1 - x^2)^(-3/2 - m/2)*(4*(1 + x) - (4*(3*b + a*(2 + m)
)*(-1 + x)*(1 + x))/((a + b)*(1 + m)) + (2^(3/2 - m/2)*(2*a*b - b^2 + a^2*
(2 + m))*(((a + b)*(1 + x))/(a + b*x))^((-1 + m)/2)*(-1 + x^2)^2*Hypergeom
etric2F1[(1 - m)/2, (3 + m)/2, (3 - m)/2, -1/2*((a - b)*(-1 + x))/(a + b*x
)])/((-1 + m)*(a + b*x)^2)))/(4*(a + b)*(3 + m))
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {717, 144, 25, 172, 25, 27, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-x^2\right)^{\frac{1}{2}(-m-3)}(a+bx)^m}{1-x}\,dx$$
$$\Big\downarrow \; 717$$
$$\int (1-x)^{\frac{1}{2}(-m-3)-1}(x+1)^{\frac{1}{2}(-m-3)}(a+bx)^m dx$$
$$\Big\downarrow \; 144$$

$$\frac{(1-x)^{\frac{1}{2}(-m-3)}(x+1)^{\frac{1}{2}(-m-1)}(a+bx)^{m+1}}{(m+3)(a+b)}-\frac{\int -(1-x)^{\frac{1}{2}(-m-3)}(x+1)^{\frac{1}{2}(-m-3)}(a+bx)^m(xb+2b+a(m+2))dx}{(m+3)(a+b)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int (1-x)^{\frac{1}{2}(-m-3)}(x+1)^{\frac{1}{2}(-m-3)}(a+bx)^m(xb+2b+a(m+2))dx}{(m+3)(a+b)}+\frac{(x+1)^{\frac{1}{2}(-m-1)}(1-x)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+3)(a+b)}$$

$$\Big\downarrow \; 172$$

$$\frac{\frac{(1-x)^{\frac{1}{2}(-m-1)}(x+1)^{\frac{1}{2}(-m-1)}(a(m+2)+3b)(a+bx)^{m+1}}{(m+1)(a+b)}-\frac{\int -\left((m+1)\left((m+2)a^2+2ba-b^2\right)(1-x)^{\frac{1}{2}(-m-1)}(x+1)^{\frac{1}{2}(-m-3)}(a+bx)^m\right)dx}{(m+1)(a+b)}}{(m+3)(a+b)}+\frac{(x+1)^{\frac{1}{2}(-m-1)}(1-x)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+3)(a+b)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int (m+1)\left((m+2)a^2+2ba-b^2\right)(1-x)^{\frac{1}{2}(-m-1)}(x+1)^{\frac{1}{2}(-m-3)}(a+bx)^m dx}{(m+1)(a+b)}+\frac{(1-x)^{\frac{1}{2}(-m-1)}(x+1)^{\frac{1}{2}(-m-1)}(a(m+2)+3b)(a+bx)^{m+1}}{(m+1)(a+b)}}{(m+3)(a+b)}+\frac{(x+1)^{\frac{1}{2}(-m-1)}(1-x)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+3)(a+b)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(a^2(m+2)+2ab-b^2\right)\int (1-x)^{\frac{1}{2}(-m-1)}(x+1)^{\frac{1}{2}(-m-3)}(a+bx)^m dx}{a+b}+\frac{(1-x)^{\frac{1}{2}(-m-1)}(x+1)^{\frac{1}{2}(-m-1)}(a(m+2)+3b)(a+bx)^{m+1}}{(m+1)(a+b)}}{(m+3)(a+b)}+\frac{(x+1)^{\frac{1}{2}(-m-1)}(1-x)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+3)(a+b)}$$

$$\Big\downarrow \; 142$$

$$\frac{\frac{(1-x)^{\frac{1}{2}(-m-1)}(x+1)^{\frac{1}{2}(-m-1)}(a(m+2)+3b)(a+bx)^{m+1}}{(m+1)(a+b)}-\frac{2^{-\frac{m}{2}-\frac{1}{2}}(1-x)^{\frac{1-m}{2}}(x+1)^{\frac{1}{2}(-m-3)}\left(a^2(m+2)+2ab-b^2\right)\left(\frac{(x+1)(a+b)}{a+bx}\right)^{\frac{m+3}{2}}(a+bx}{(1-m)(a+b)^2}}{(m+3)(a+b)}$$

$$\frac{(x+1)^{\frac{1}{2}(-m-1)}(1-x)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+3)(a+b)}$$

input
```
Int[((a + b*x)^m*(1 - x^2)^((-3 - m)/2))/(1 - x),x]
```

output
```
((1 - x)^((-3 - m)/2)*(1 + x)^((-1 - m)/2)*(a + b*x)^(1 + m))/((a + b)*(3
+ m)) + (((3*b + a*(2 + m))*(1 - x)^((-1 - m)/2)*(1 + x)^((-1 - m)/2)*(a +
 b*x)^(1 + m))/((a + b)*(1 + m)) - (2^(-1/2 - m/2)*(2*a*b - b^2 + a^2*(2 +
 m))*(1 - x)^((1 - m)/2)*(1 + x)^((-3 - m)/2)*(((a + b)*(1 + x))/(a + b*x)
)^((3 + m)/2)*(a + b*x)^(1 + m)*Hypergeometric2F1[(1 - m)/2, (3 + m)/2, (3
 - m)/2, ((a - b)*(1 - x))/(2*(a + b*x))])/((a + b)^2*(1 - m)))/((a + b)*(
3 + m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 144
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[b*(a + b*x)^(m + 1)*(
c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x]
+ Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2))
- b*d*f*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && NeQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 717
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a
, 0] && GtQ[d, 0]
```

Maple [F]

$$\int \frac{(bx+a)^m (-x^2+1)^{-\frac{3}{2}-\frac{m}{2}}}{1-x} dx$$

input
```
int((b*x+a)^m*(-x^2+1)^(-3/2-1/2*m)/(1-x),x)
```

output
```
int((b*x+a)^m*(-x^2+1)^(-3/2-1/2*m)/(1-x),x)
```

Fricas [F]

$$\int \frac{(a+bx)^m (1-x^2)^{\frac{1}{2}(-3-m)}}{1-x} dx = \int -\frac{(bx+a)^m(-x^2+1)^{-\frac{1}{2}m-\frac{3}{2}}}{x-1} dx$$

input
```
integrate((b*x+a)^m*(-x^2+1)^(-3/2-1/2*m)/(1-x),x, algorithm="fricas")
```

output
```
integral(-(b*x + a)^m*(-x^2 + 1)^(-1/2*m - 3/2)/(x - 1), x)
```

Sympy [F]

$$\int \frac{(a+bx)^m (1-x^2)^{\frac{1}{2}(-3-m)}}{1-x}\,dx = -\int \frac{(1-x^2)^{-\frac{m}{2}-\frac{3}{2}} (a+bx)^m}{x-1}\,dx$$

input
```
integrate((b*x+a)**m*(-x**2+1)**(-3/2-1/2*m)/(1-x),x)
```

output
```
-Integral((1 - x**2)**(-m/2 - 3/2)*(a + b*x)**m/(x - 1), x)
```

Maxima [F]

$$\int \frac{(a+bx)^m (1-x^2)^{\frac{1}{2}(-3-m)}}{1-x}\,dx = \int -\frac{(bx+a)^m(-x^2+1)^{-\frac{1}{2}m-\frac{3}{2}}}{x-1}\,dx$$

input
```
integrate((b*x+a)^m*(-x^2+1)^(-3/2-1/2*m)/(1-x),x, algorithm="maxima")
```

output
```
-integrate((b*x + a)^m*(-x^2 + 1)^(-1/2*m - 3/2)/(x - 1), x)
```

Giac [F]

$$\int \frac{(a+bx)^m (1-x^2)^{\frac{1}{2}(-3-m)}}{1-x}\,dx = \int -\frac{(bx+a)^m(-x^2+1)^{-\frac{1}{2}m-\frac{3}{2}}}{x-1}\,dx$$

input
```
integrate((b*x+a)^m*(-x^2+1)^(-3/2-1/2*m)/(1-x),x, algorithm="giac")
```

output
```
integrate(-(b*x + a)^m*(-x^2 + 1)^(-1/2*m - 3/2)/(x - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m (1-x^2)^{\frac{1}{2}(-3-m)}}{1-x}\,dx = \int -\frac{(a+b\,x)^m}{(1-x^2)^{\frac{m}{2}+\frac{3}{2}}\,(x-1)}\,dx$$

input
```
int(-(a + b*x)^m/((1 - x^2)^(m/2 + 3/2)*(x - 1)),x)
```

output
```
int(-(a + b*x)^m/((1 - x^2)^(m/2 + 3/2)*(x - 1)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^m (1-x^2)^{\frac{1}{2}(-3-m)}}{1-x}\,dx$$
$$= \int \frac{(bx+a)^m}{(-x^2+1)^{\frac{m}{2}+\frac{1}{2}}\,x^3 - (-x^2+1)^{\frac{m}{2}+\frac{1}{2}}\,x^2 - (-x^2+1)^{\frac{m}{2}+\frac{1}{2}}\,x + (-x^2+1)^{\frac{m}{2}+\frac{1}{2}}}dx$$

input
```
int((b*x+a)^m*(-x^2+1)^(-3/2-1/2*m)/(1-x),x)
```

output
```
int((a + b*x)**m/(( - x**2 + 1)**((m + 1)/2)*x**3 - ( - x**2 + 1)**((m + 1
)/2)*x**2 - ( - x**2 + 1)**((m + 1)/2)*x + ( - x**2 + 1)**((m + 1)/2)),x)
```

3.14 $\int (1-x)^{\frac{1}{2}(-5-m)}(1+x)^{\frac{1}{2}(-3-m)}(a+bx)^m\,dx$

Optimal result

Integrand size = 36, antiderivative size = 244

$$\int (1-x)^{\frac{1}{2}(-5-m)}(1+x)^{\frac{1}{2}(-3-m)}(a+bx)^m\,dx=\frac{(1-x)^{\frac{1}{2}(-3-m)}(1+x)^{\frac{1}{2}(-1-m)}(a+bx)^{1+m}}{(a+b)(3+m)}$$
$$-\frac{b(a+2b+am)(a+bx)^{1+m}\left(1-x^2\right)^{\frac{1}{2}(-1-m)}}{(a-b)(a+b)^2(1+m)(3+m)}$$
$$+\frac{\left(2ab-b^2+a^2(2+m)\right)(1-x)\left(-\frac{(a+b)(1+x)}{(a-b)(1-x)}\right)^{\frac{3+m}{2}}(a+bx)^{2+m}\left(1-x^2\right)^{\frac{1}{2}(-3-m)}\text{Hypergeometric2F1}\left(2\right.}{(a-b)(a+b)^3(2+m)(3+m)}$$

output
```
(1-x)^(-3/2-1/2*m)*(1+x)^(-1/2-1/2*m)*(b*x+a)^(1+m)/(a+b)/(3+m)-b*(a*m+a+2
*b)*(b*x+a)^(1+m)*(-x^2+1)^(-1/2-1/2*m)/(a-b)/(a+b)^2/(1+m)/(3+m)+(2*a*b-b
^2+a^2*(2+m))*(1-x)*(-(a+b)*(1+x)/(a-b)/(1-x))^(3/2+1/2*m)*(b*x+a)^(2+m)*(
-x^2+1)^(-3/2-1/2*m)*hypergeom([2+m, 3/2+1/2*m],[3+m],2*(b*x+a)/(a-b)/(1-x
))/(a-b)/(a+b)^3/(2+m)/(3+m)
```

Mathematica [A] (warning: unable to verify)

Time = 10.95 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.77

$$\int (1-x)^{\frac{1}{2}(-5-m)}(1+x)^{\frac{1}{2}(-3-m)}(a+bx)^m\,dx$$
$$=\frac{(a+bx)^{1+m}\left(1-x^2\right)^{-\frac{3}{2}-\frac{m}{2}}\left(4(1+x)-\frac{4(3b+a(2+m))(-1+x)(1+x)}{(a+b)(1+m)}+\frac{2^{\frac{3}{2}-\frac{m}{2}}\left(2ab-b^2+a^2(2+m)\right)\left(\frac{(a+b)(1+x)}{a+bx}\right)^{\frac{1}{2}(-1+m)}(-\ldots}{(-\ldots}\right)}{4(a+b)(3+m)}$$

input
```
Integrate[(1 - x)^((-5 - m)/2)*(1 + x)^((-3 - m)/2)*(a + b*x)^m,x]
```

output
```
((a + b*x)^(1 + m)*(1 - x^2)^(-3/2 - m/2)*(4*(1 + x) - (4*(3*b + a*(2 + m)
)*(-1 + x)*(1 + x))/((a + b)*(1 + m)) + (2^(3/2 - m/2)*(2*a*b - b^2 + a^2*
(2 + m))*(((a + b)*(1 + x))/(a + b*x))^((-1 + m)/2)*(-1 + x^2)^2*Hypergeom
etric2F1[(1 - m)/2, (3 + m)/2, (3 - m)/2, -1/2*((a - b)*(-1 + x))/(a + b*x
)])/((-1 + m)*(a + b*x)^2)))/(4*(a + b)*(3 + m))
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {144, 25, 172, 25, 27, 142}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^{\frac{1}{2}(-m-5)}(x+1)^{\frac{1}{2}(-m-3)}(a+bx)^m\,dx$$
$$\Big\downarrow \; 144$$
$$\frac{(1-x)^{\frac{1}{2}(-m-3)}(x+1)^{\frac{1}{2}(-m-1)}(a+bx)^{m+1}}{(m+3)(a+b)}-\frac{\int -(1-x)^{\frac{1}{2}(-m-3)}(x+1)^{\frac{1}{2}(-m-3)}(a+bx)^m(xb+2b+a(m+2))dx}{(m+3)(a+b)}$$
$$\Big\downarrow \; 25$$

$$\frac{\int (1-x)^{\frac{1}{2}(-m-3)}(x+1)^{\frac{1}{2}(-m-3)}(a+bx)^m(xb+2b+a(m+2))dx}{(m+3)(a+b)}+\frac{(x+1)^{\frac{1}{2}(-m-1)}(1-x)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+3)(a+b)}$$

$\big\downarrow$ 172

$$\frac{\frac{(1-x)^{\frac{1}{2}(-m-1)}(x+1)^{\frac{1}{2}(-m-1)}(a(m+2)+3b)(a+bx)^{m+1}}{(m+1)(a+b)}-\frac{\int -\left((m+1)\left((m+2)a^2+2ba-b^2\right)(1-x)^{\frac{1}{2}(-m-1)}(x+1)^{\frac{1}{2}(-m-3)}(a+bx)^m\right)dx}{(m+1)(a+b)}}{(m+3)(a+b)}+\frac{(x+1)^{\frac{1}{2}(-m-1)}(1-x)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+3)(a+b)}$$

$\big\downarrow$ 25

$$\frac{\frac{\int (m+1)\left((m+2)a^2+2ba-b^2\right)(1-x)^{\frac{1}{2}(-m-1)}(x+1)^{\frac{1}{2}(-m-3)}(a+bx)^m dx}{(m+1)(a+b)}+\frac{(1-x)^{\frac{1}{2}(-m-1)}(x+1)^{\frac{1}{2}(-m-1)}(a(m+2)+3b)(a+bx)^{m+1}}{(m+1)(a+b)}}{(m+3)(a+b)}+\frac{(x+1)^{\frac{1}{2}(-m-1)}(1-x)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+3)(a+b)}$$

$\big\downarrow$ 27

$$\frac{\frac{\left(a^2(m+2)+2ab-b^2\right)\int (1-x)^{\frac{1}{2}(-m-1)}(x+1)^{\frac{1}{2}(-m-3)}(a+bx)^m dx}{a+b}+\frac{(1-x)^{\frac{1}{2}(-m-1)}(x+1)^{\frac{1}{2}(-m-1)}(a(m+2)+3b)(a+bx)^{m+1}}{(m+1)(a+b)}}{(m+3)(a+b)}+\frac{(x+1)^{\frac{1}{2}(-m-1)}(1-x)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+3)(a+b)}$$

$\big\downarrow$ 142

$$\frac{\frac{(1-x)^{\frac{1}{2}(-m-1)}(x+1)^{\frac{1}{2}(-m-1)}(a(m+2)+3b)(a+bx)^{m+1}}{(m+1)(a+b)}-\frac{2^{-\frac{m}{2}-\frac{1}{2}}(1-x)^{\frac{1-m}{2}}(x+1)^{\frac{1}{2}(-m-3)}\left(a^2(m+2)+2ab-b^2\right)\left(\frac{(x+1)(a+b)}{a+bx}\right)^{\frac{m+3}{2}}(a+bx}{(1-m)(a+b)^2}}{(m+3)(a+b)}$$
$$\frac{(x+1)^{\frac{1}{2}(-m-1)}(1-x)^{\frac{1}{2}(-m-3)}(a+bx)^{m+1}}{(m+3)(a+b)}$$

input
```
Int[(1 - x)^((-5 - m)/2)*(1 + x)^((-3 - m)/2)*(a + b*x)^m,x]
```

output
```
((1 - x)^((-3 - m)/2)*(1 + x)^((-1 - m)/2)*(a + b*x)^(1 + m))/((a + b)*(3
+ m)) + (((3*b + a*(2 + m))*(1 - x)^((-1 - m)/2)*(1 + x)^((-1 - m)/2)*(a +
 b*x)^(1 + m))/((a + b)*(1 + m)) - (2^(-1/2 - m/2)*(2*a*b - b^2 + a^2*(2 +
 m))*(1 - x)^((1 - m)/2)*(1 + x)^((-3 - m)/2)*(((a + b)*(1 + x))/(a + b*x)
)^((3 + m)/2)*(a + b*x)^(1 + m)*Hypergeometric2F1[(1 - m)/2, (3 + m)/2, (3
 - m)/2, ((a - b)*(1 - x))/(2*(a + b*x))])/((a + b)^2*(1 - m)))/((a + b)*(
3 + m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1))))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 144
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[b*(a + b*x)^(m + 1)*(
c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x]
+ Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2))
- b*d*f*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && NeQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (1-x)^{-\frac{5}{2}-\frac{m}{2}} (x+1)^{-\frac{3}{2}-\frac{m}{2}} (bx+a)^m \, dx$$

input
```
int((1-x)^(-5/2-1/2*m)*(x+1)^(-3/2-1/2*m)*(b*x+a)^m,x)
```

output
```
int((1-x)^(-5/2-1/2*m)*(x+1)^(-3/2-1/2*m)*(b*x+a)^m,x)
```

Fricas [F]

$$\int (1-x)^{\frac{1}{2}(-5-m)}(1+x)^{\frac{1}{2}(-3-m)}(a+bx)^m \, dx = \int (bx+a)^m (x+1)^{-\frac{1}{2}m-\frac{3}{2}}(-x+1)^{-\frac{1}{2}m-\frac{5}{2}} \, dx$$

input
```
integrate((1-x)^(-5/2-1/2*m)*(1+x)^(-3/2-1/2*m)*(b*x+a)^m,x, algorithm="fr
icas")
```

output
```
integral((b*x + a)^m*(x + 1)^(-1/2*m - 3/2)*(-x + 1)^(-1/2*m - 5/2), x)
```

Sympy [F]

$$\int (1-x)^{\frac{1}{2}(-5-m)}(1+x)^{\frac{1}{2}(-3-m)}(a+bx)^m\,dx = \int (1-x)^{-\frac{m}{2}-\frac{5}{2}}\,(a+bx)^m\,(x+1)^{-\frac{m}{2}-\frac{3}{2}}\,dx$$

input
```
integrate((1-x)**(-5/2-1/2*m)*(1+x)**(-3/2-1/2*m)*(b*x+a)**m,x)
```

output
```
Integral((1 - x)**(-m/2 - 5/2)*(a + b*x)**m*(x + 1)**(-m/2 - 3/2), x)
```

Maxima [F]

$$\int (1-x)^{\frac{1}{2}(-5-m)}(1+x)^{\frac{1}{2}(-3-m)}(a+bx)^m\,dx = \int (bx+a)^m(x+1)^{-\frac{1}{2}m-\frac{3}{2}}(-x+1)^{-\frac{1}{2}m-\frac{5}{2}}\,dx$$

input
```
integrate((1-x)^(-5/2-1/2*m)*(1+x)^(-3/2-1/2*m)*(b*x+a)^m,x, algorithm="ma
xima")
```

output
```
integrate((b*x + a)^m*(x + 1)^(-1/2*m - 3/2)*(-x + 1)^(-1/2*m - 5/2), x)
```

Giac [F]

$$\int (1-x)^{\frac{1}{2}(-5-m)}(1+x)^{\frac{1}{2}(-3-m)}(a+bx)^m\,dx = \int (bx+a)^m(x+1)^{-\frac{1}{2}m-\frac{3}{2}}(-x+1)^{-\frac{1}{2}m-\frac{5}{2}}\,dx$$

input
```
integrate((1-x)^(-5/2-1/2*m)*(1+x)^(-3/2-1/2*m)*(b*x+a)^m,x, algorithm="gi
ac")
```

output
```
integrate((b*x + a)^m*(x + 1)^(-1/2*m - 3/2)*(-x + 1)^(-1/2*m - 5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^{\frac{1}{2}(-5-m)}(1+x)^{\frac{1}{2}(-3-m)}(a+bx)^m\,dx = \int \frac{(a+b\,x)^m}{(1-x)^{\frac{m}{2}+\frac{5}{2}}\,(x+1)^{\frac{m}{2}+\frac{3}{2}}}\,dx$$

input
```
int((a + b*x)^m/((1 - x)^(m/2 + 5/2)*(x + 1)^(m/2 + 3/2)),x)
```

output
```
int((a + b*x)^m/((1 - x)^(m/2 + 5/2)*(x + 1)^(m/2 + 3/2)), x)
```

Reduce [F]

$$\int (1-x)^{\frac{1}{2}(-5-m)}(1+x)^{\frac{1}{2}(-3-m)}(a+bx)^m\,dx$$
$$= \int \frac{(bx+a)^m}{(x+1)^{\frac{m}{2}+\frac{1}{2}}\,(1-x)^{\frac{m}{2}+\frac{1}{2}}\,x^3 - (x+1)^{\frac{m}{2}+\frac{1}{2}}\,(1-x)^{\frac{m}{2}+\frac{1}{2}}\,x^2 - (x+1)^{\frac{m}{2}+\frac{1}{2}}\,(1-x)^{\frac{m}{2}+\frac{1}{2}}\,x + (x+1)^{\frac{m}{2}+\frac{1}{2}}\,(}$$

input
```
int((1-x)^(-5/2-1/2*m)*(1+x)^(-3/2-1/2*m)*(b*x+a)^m,x)
```

output
```
int((a + b*x)**m/((x + 1)**((m + 1)/2)*( - x + 1)**((m + 1)/2)*x**3 - (x +
 1)**((m + 1)/2)*( - x + 1)**((m + 1)/2)*x**2 - (x + 1)**((m + 1)/2)*( - x
 + 1)**((m + 1)/2)*x + (x + 1)**((m + 1)/2)*( - x + 1)**((m + 1)/2)),x)
```

3.15 $\int (ad-bcx)(c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 31

$$\int (ad-bcx)(c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx=-\frac{(c+dx)^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{2(1+p)}$$

output
```
-1/2*(b*x^2+a)^(p+1)/(p+1)/((d*x+c)^(2*p+2))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (ad-bcx)(c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx=-\frac{(c+dx)^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{2(1+p)}$$

input
```
Integrate[(a*d - b*c*x)*(c + d*x)^(-3 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-1/2*(a + b*x^2)^(1 + p)/((1 + p)*(c + d*x)^(2*(1 + p)))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {677}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^p (c+dx)^{-2p-3}(ad-bcx)\,dx$$

$$\Big\downarrow 677$$

$$-\frac{(a+bx^2)^{p+1} (c+dx)^{-2(p+1)}}{2(p+1)}$$

input
```
Int[(a*d - b*c*x)*(c + d*x)^(-3 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-1/2*(a + b*x^2)^(1 + p)/((1 + p)*(c + d*x)^(2*(1 + p)))
```

Defintions of rubi rules used

rule 677
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] &&
 EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*g, 0]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result
gosper	$-\frac{(b\,x^2+a)^{p+1}(dx+c)^{-2p-2}}{2(p+1)}$
orering	$-\frac{(dx+c)(b\,x^2+a)(dx+c)^{-3-2p}(b\,x^2+a)^{p}}{2(p+1)}$
parallelrisch	$-\frac{x^3(b\,x^2+a)^{p}(dx+c)^{-3-2p}b^2d^2+x^2(b\,x^2+a)^{p}(dx+c)^{-3-2p}b^2cd+x(b\,x^2+a)^{p}(dx+c)^{-3-2p}ab\,d^2+(b\,x^2+a)^{p}(dx+c)^{-3-2p}}{2(p+1)bd}$

input
```
int((-b*c*x+a*d)*(d*x+c)^(-3-2*p)*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(p+1)*(b*x^2+a)^(p+1)*(d*x+c)^(-2*p-2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int (ad-bcx)(c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx = -\frac{(bdx^3+bcx^2+adx+ac)(bx^2+a)^p(dx+c)^{-2\,p-3}}{2\,(p+1)}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)^(-3-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
-1/2*(b*d*x^3 + b*c*x^2 + a*d*x + a*c)*(b*x^2 + a)^p*(d*x + c)^(-2*p - 3)/
(p + 1)
```

Sympy [F(-1)]

Timed out.

$$\int (ad-bcx)(c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx = \text{Timed out}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)**(-3-2*p)*(b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int (ad-bcx)(c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx = -\frac{(bx^2+a)e^{(p\log(bx^2+a)-2\,p\log(dx+c))}}{2\,(d^2(p+1)x^2+2\,cd(p+1)x+c^2(p+1))}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)^(-3-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
-1/2*(b*x^2 + a)*e^(p*log(b*x^2 + a) - 2*p*log(d*x + c))/(d^2*(p + 1)*x^2
+ 2*c*d*(p + 1)*x + c^2*(p + 1))
```

Giac [F]

$$\int (ad-bcx)(c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx = \int -(bcx-ad)\left(bx^2+a\right)^p(dx+c)^{-2\,p-3}\,dx$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)^(-3-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate(-(b*c*x - a*d)*(b*x^2 + a)^p*(d*x + c)^(-2*p - 3), x)
```

Mupad [B] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.19

$$\int (ad-bcx)(c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx$$
$$= -\frac{\frac{a\,c\,(b\,x^2+a)^p}{2\,p+2}+\frac{a\,d\,x\,(b\,x^2+a)^p}{2\,p+2}+\frac{b\,c\,x^2\,(b\,x^2+a)^p}{2\,p+2}+\frac{b\,d\,x^3\,(b\,x^2+a)^p}{2\,p+2}}{(c+d\,x)^{2\,p+3}}$$

input
```
int(((a*d - b*c*x)*(a + b*x^2)^p)/(c + d*x)^(2*p + 3),x)
```

output
```
-((a*c*(a + b*x^2)^p)/(2*p + 2) + (a*d*x*(a + b*x^2)^p)/(2*p + 2) + (b*c*x
^2*(a + b*x^2)^p)/(2*p + 2) + (b*d*x^3*(a + b*x^2)^p)/(2*p + 2))/(c + d*x)
^(2*p + 3)
```

Reduce [F]

$$\int (ad-bcx)(c+dx)^{-3-2p}\left(a+bx^2\right)^p\,dx$$
$$=\left(\int \frac{\left(b\,x^2+a\right)^p}{\left(dx+c\right)^{2p}c^3+3\left(dx+c\right)^{2p}c^2dx+3\left(dx+c\right)^{2p}c\,d^2x^2+\left(dx+c\right)^{2p}d^3x^3}dx\right)ad$$
$$-\left(\int \frac{\left(b\,x^2+a\right)^p x}{\left(dx+c\right)^{2p}c^3+3\left(dx+c\right)^{2p}c^2dx+3\left(dx+c\right)^{2p}c\,d^2x^2+\left(dx+c\right)^{2p}d^3x^3}dx\right)bc$$

input
```
int((-b*c*x+a*d)*(d*x+c)^(-3-2*p)*(b*x^2+a)^p,x)
```

output
```
int((a + b*x**2)**p/((c + d*x)**(2*p)*c**3 + 3*(c + d*x)**(2*p)*c**2*d*x +
 3*(c + d*x)**(2*p)*c*d**2*x**2 + (c + d*x)**(2*p)*d**3*x**3),x)*a*d - int
(((a + b*x**2)**p*x)/((c + d*x)**(2*p)*c**3 + 3*(c + d*x)**(2*p)*c**2*d*x
+ 3*(c + d*x)**(2*p)*c*d**2*x**2 + (c + d*x)**(2*p)*d**3*x**3),x)*b*c
```

3.16 $\int \frac{(ad-bcx)(a+bx^2)^3}{(c+dx)^9}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 20

$$\int \frac{(ad-bcx)(a+bx^2)^3}{(c+dx)^9}\,dx = -\frac{(a+bx^2)^4}{8(c+dx)^8}$$

output
```
-1/8*(b*x^2+a)^4/(d*x+c)^8
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 138 vs. 2(20) = 40.

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 6.90

$$\int \frac{(ad-bcx)(a+bx^2)^3}{(c+dx)^9}\,dx = -\frac{a^4d^8 + 4a^3bd^8x^2 + 6a^2b^2d^8x^4 + 4ab^3d^8x^6 - b^4c(c^7 + 8c^6dx + 28c^5d^2x^2 + 56c^4d^3x^3 + 70c^3d^4x^4 + 56c^2d}{8d^8(c+dx)^8}$$

input
```
Integrate[((a*d - b*c*x)*(a + b*x^2)^3)/(c + d*x)^9,x]
```

output
```
-1/8*(a^4*d^8 + 4*a^3*b*d^8*x^2 + 6*a^2*b^2*d^8*x^4 + 4*a*b^3*d^8*x^6 - b^
4*c*(c^7 + 8*c^6*d*x + 28*c^5*d^2*x^2 + 56*c^4*d^3*x^3 + 70*c^3*d^4*x^4 +
56*c^2*d^5*x^5 + 28*c*d^6*x^6 + 8*d^7*x^7))/(d^8*(c + d*x)^8)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 249 vs. $2(20) = 40$.

Time = 0.56 (sec) , antiderivative size = 249, normalized size of antiderivative = 12.45, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{652, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (ad-bcx)}{(c+dx)^9} \, dx$$

$$\Big\downarrow \; 652$$

$$\int \left(\frac{b^2 (3a^2d^4 + 30abc^2d^2 + 35b^2c^4)}{d^7 (c+dx)^5} + \frac{b^3 (ad^2 + 7bc^2)}{d^7 (c+dx)^3} - \frac{3b^3c(3ad^2 + 7bc^2)}{d^7 (c+dx)^4} + \frac{5b^2c(-3ad^2 - 7bc^2)(ad^2 + bc^2)}{d^7 (c+dx)^6} \right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{b^2 (3a^2d^4 + 30abc^2d^2 + 35b^2c^4)}{4d^8 (c+dx)^4} - \frac{b^3 (ad^2 + 7bc^2)}{2d^8 (c+dx)^2} + \frac{b^3c(3ad^2 + 7bc^2)}{d^8 (c+dx)^3} + \frac{b^2c(ad^2 + bc^2)(3ad^2 + 7bc^2)}{d^8 (c+dx)^5} - \frac{b(ad^2 + bc^2)^2 (ad^2 + 7bc^2)}{2d^8 (c+dx)^6} + \frac{bc(ad^2 + bc^2)^3}{d^8 (c+dx)^7} - \frac{(ad^2 + bc^2)^4}{8d^8 (c+dx)^8} + \frac{b^4c}{d^8 (c+dx)}$$

input
```
Int[((a*d - b*c*x)*(a + b*x^2)^3)/(c + d*x)^9,x]
```

output
```
-1/8*(b*c^2 + a*d^2)^4/(d^8*(c + d*x)^8) + (b*c*(b*c^2 + a*d^2)^3)/(d^8*(c
 + d*x)^7) - (b*(b*c^2 + a*d^2)^2*(7*b*c^2 + a*d^2))/(2*d^8*(c + d*x)^6) +
 (b^2*c*(b*c^2 + a*d^2)*(7*b*c^2 + 3*a*d^2))/(d^8*(c + d*x)^5) - (b^2*(35*
b^2*c^4 + 30*a*b*c^2*d^2 + 3*a^2*d^4))/(4*d^8*(c + d*x)^4) + (b^3*c*(7*b*c
^2 + 3*a*d^2))/(d^8*(c + d*x)^3) - (b^3*(7*b*c^2 + a*d^2))/(2*d^8*(c + d*x
)^2) + (b^4*c)/(d^8*(c + d*x))
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(18) = 36$.

Time = 0.68 (sec) , antiderivative size = 157, normalized size of antiderivative = 7.85

method	result
risch	$\frac{\frac{c\,b^{4} x^{7}}{d}-\frac{b^{3} \left(a\,d^{2}-7 b\,c^{2}\right) x^{6}}{2 d^{2}}+\frac{7 c^{3} b^{4} x^{5}}{d^{3}}-\frac{b^{2} \left(3 a^{2} d^{4}-35 b^{2} c^{4}\right) x^{4}}{4 d^{4}}+\frac{7 c^{5} b^{4} x^{3}}{d^{5}}-\frac{b \left(a^{3} d^{6}-7 b^{3} c^{6}\right) x^{2}}{2 d^{6}}+\frac{c^{7} b^{4} x}{d^{7}}-\frac{a^{4} d^{8}-b^{4} c^{8}}{8 d^{8}}}{\left(d x+c\right)^{8}}$
gosper	$-\frac{-8 b^{4} c\,x^{7} d^{7}+4 a\,b^{3} d^{8} x^{6}-28 b^{4} c^{2} d^{6} x^{6}-56 b^{4} c^{3} x^{5} d^{5}+6 a^{2} b^{2} d^{8} x^{4}-70 b^{4} c^{4} d^{4} x^{4}-56 b^{4} c^{5} d^{3} x^{3}+4 a^{3} b\,d^{8} x^{2}-28 b^{4} c^{6} d^{2} x^{2}-8 b^{4} c^{7}}{8 \left(d x+c\right)^{8} d^{8}}$
norman	$\frac{\frac{c\,b^{4} x^{7}}{d}-\frac{\left(a\,b^{3} d^{2}-7 b^{4} c^{2}\right) x^{6}}{2 d^{2}}+\frac{7 c^{3} b^{4} x^{5}}{d^{3}}-\frac{\left(3 a^{2} b^{2} d^{4}-35 c^{4} b^{4}\right) x^{4}}{4 d^{4}}+\frac{7 c^{5} b^{4} x^{3}}{d^{5}}-\frac{\left(a^{3} b\,d^{6}-7 b^{4} c^{6}\right) x^{2}}{2 d^{6}}+\frac{c^{7} b^{4} x}{d^{7}}-\frac{a^{4} d^{8}-b^{4} c^{8}}{8 d^{8}}}{\left(d x+c\right)^{8}}$
parallelrisch	$\frac{8 b^{4} c\,x^{7} d^{7}-4 a\,b^{3} d^{8} x^{6}+28 b^{4} c^{2} d^{6} x^{6}+56 b^{4} c^{3} x^{5} d^{5}-6 a^{2} b^{2} d^{8} x^{4}+70 b^{4} c^{4} d^{4} x^{4}+56 b^{4} c^{5} d^{3} x^{3}-4 a^{3} b\,d^{8} x^{2}+28 b^{4} c^{6} d^{2} x^{2}+8 b^{4} c^{7} d x-}{8 d^{8} \left(d x+c\right)^{8}}$
orering	$-\frac{-8 b^{4} c\,x^{7} d^{7}+4 a\,b^{3} d^{8} x^{6}-28 b^{4} c^{2} d^{6} x^{6}-56 b^{4} c^{3} x^{5} d^{5}+6 a^{2} b^{2} d^{8} x^{4}-70 b^{4} c^{4} d^{4} x^{4}-56 b^{4} c^{5} d^{3} x^{3}+4 a^{3} b\,d^{8} x^{2}-28 b^{4} c^{6} d^{2} x^{2}-8 b^{4} c^{7}}{8 \left(d x+c\right)^{8} d^{8}}$
default	$-\frac{a^{4} d^{8}+4 a^{3} b\,c^{2} d^{6}+6 a^{2} b^{2} c^{4} d^{4}+4 a\,b^{3} c^{6} d^{2}+b^{4} c^{8}}{8 d^{8} \left(d x+c\right)^{8}}+\frac{b^{4} c}{d^{8} \left(d x+c\right)}-\frac{b^{3} \left(a\,d^{2}+7 b\,c^{2}\right)}{2 d^{8} \left(d x+c\right)^{2}}+\frac{b^{2} c \left(3 a^{2} d^{4}+10 b\,c^{2} d^{2} a+7 b^{2} c^{4}\right)}{d^{8} \left(d x+c\right)^{5}}-\frac{b(}{}$

input
```
int((-b*c*x+a*d)*(b*x^2+a)^3/(d*x+c)^9,x,method=_RETURNVERBOSE)
```

output
```
(c*b^4/d*x^7-1/2*b^3*(a*d^2-7*b*c^2)/d^2*x^6+7*c^3*b^4/d^3*x^5-1/4*b^2*(3*
a^2*d^4-35*b^2*c^4)/d^4*x^4+7*c^5*b^4/d^5*x^3-1/2*b*(a^3*d^6-7*b^3*c^6)/d^
6*x^2+c^7*b^4/d^7*x-1/8*(a^4*d^8-b^4*c^8)/d^8)/(d*x+c)^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(18) = 36.

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 11.90

$$\int \frac{(ad-bcx)\,(a+bx^2)^3}{(c+dx)^9}\,dx$$
$$= \frac{8\,b^4cd^7x^7+56\,b^4c^3d^5x^5+56\,b^4c^5d^3x^3+8\,b^4c^7dx+b^4c^8-a^4d^8+4\,(7\,b^4c^2d^6-ab^3d^8)x^6+2\,(35\,b^4c^4d^4-}{8\,(d^{16}x^8+8\,cd^{15}x^7+28\,c^2d^{14}x^6+56\,c^3d^{13}x^5+70\,c^4d^{12}x^4+56\,c^5d^{11}x^3+28\,c^6d^{10}x^2}$$

input
```
integrate((-b*c*x+a*d)*(b*x^2+a)^3/(d*x+c)^9,x, algorithm="fricas")
```

output
```
1/8*(8*b^4*c*d^7*x^7 + 56*b^4*c^3*d^5*x^5 + 56*b^4*c^5*d^3*x^3 + 8*b^4*c^7
*d*x + b^4*c^8 - a^4*d^8 + 4*(7*b^4*c^2*d^6 - a*b^3*d^8)*x^6 + 2*(35*b^4*c
^4*d^4 - 3*a^2*b^2*d^8)*x^4 + 4*(7*b^4*c^6*d^2 - a^3*b*d^8)*x^2)/(d^16*x^8
 + 8*c*d^15*x^7 + 28*c^2*d^14*x^6 + 56*c^3*d^13*x^5 + 70*c^4*d^12*x^4 + 56
*c^5*d^11*x^3 + 28*c^6*d^10*x^2 + 8*c^7*d^9*x + c^8*d^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(17) = 34.

Time = 177.58 (sec) , antiderivative size = 245, normalized size of antiderivative = 12.25

$$\int \frac{(ad-bcx)\,(a+bx^2)^3}{(c+dx)^9}\,dx =$$
$$-\frac{a^4d^8-b^4c^8-8b^4c^7dx-56b^4c^5d^3x^3-56b^4c^3d^5x^5-8b^4cd^7x^7+x^6\cdot(4ab^3d^8-28b^4c^2d^6)+x^4\cdot(6a^2b^2}{8c^8d^8+64c^7d^9x+224c^6d^{10}x^2+448c^5d^{11}x^3+560c^4d^{12}x^4+448c^3d^{13}x^5+224c^2d^{14}}$$

input
```
integrate((-b*c*x+a*d)*(b*x**2+a)**3/(d*x+c)**9,x)
```

output
```
-(a**4*d**8 - b**4*c**8 - 8*b**4*c**7*d*x - 56*b**4*c**5*d**3*x**3 - 56*b*
*4*c**3*d**5*x**5 - 8*b**4*c*d**7*x**7 + x**6*(4*a*b**3*d**8 - 28*b**4*c**
2*d**6) + x**4*(6*a**2*b**2*d**8 - 70*b**4*c**4*d**4) + x**2*(4*a**3*b*d**
8 - 28*b**4*c**6*d**2))/(8*c**8*d**8 + 64*c**7*d**9*x + 224*c**6*d**10*x**
2 + 448*c**5*d**11*x**3 + 560*c**4*d**12*x**4 + 448*c**3*d**13*x**5 + 224*
c**2*d**14*x**6 + 64*c*d**15*x**7 + 8*d**16*x**8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(18) = 36$.

Time = 0.05 (sec) , antiderivative size = 238, normalized size of antiderivative = 11.90

$$\int \frac{(ad-bcx)\left(a+bx^2\right)^3}{(c+dx)^9}\,dx = \frac{8\,b^4cd^7x^7+56\,b^4c^3d^5x^5+56\,b^4c^5d^3x^3+8\,b^4c^7dx+b^4c^8-a^4d^8+4\,(7\,b^4c^2d^6-ab^3d^8)x^6+2\,(35\,b^4c^4d^4-}{8\,(d^{16}x^8+8\,cd^{15}x^7+28\,c^2d^{14}x^6+56\,c^3d^{13}x^5+70\,c^4d^{12}x^4+56\,c^5d^{11}x^3+28\,c^6d^{10}x^2}$$

input
```
integrate((-b*c*x+a*d)*(b*x^2+a)^3/(d*x+c)^9,x, algorithm="maxima")
```

output
```
1/8*(8*b^4*c*d^7*x^7 + 56*b^4*c^3*d^5*x^5 + 56*b^4*c^5*d^3*x^3 + 8*b^4*c^7
*d*x + b^4*c^8 - a^4*d^8 + 4*(7*b^4*c^2*d^6 - a*b^3*d^8)*x^6 + 2*(35*b^4*c
^4*d^4 - 3*a^2*b^2*d^8)*x^4 + 4*(7*b^4*c^6*d^2 - a^3*b*d^8)*x^2)/(d^16*x^8
 + 8*c*d^15*x^7 + 28*c^2*d^14*x^6 + 56*c^3*d^13*x^5 + 70*c^4*d^12*x^4 + 56
*c^5*d^11*x^3 + 28*c^6*d^10*x^2 + 8*c^7*d^9*x + c^8*d^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(18) = 36$.

Time = 0.14 (sec) , antiderivative size = 158, normalized size of antiderivative = 7.90

$$\int \frac{(ad-bcx)\left(a+bx^2\right)^3}{(c+dx)^9}\,dx = \frac{8\,b^4cd^7x^7+28\,b^4c^2d^6x^6-4\,ab^3d^8x^6+56\,b^4c^3d^5x^5+70\,b^4c^4d^4x^4-6\,a^2b^2d^8x^4+56\,b^4c^5d^3x^3+28\,b^4c^6d^2x}{8\,(dx+c)^8d^8}$$

input
```
integrate((-b*c*x+a*d)*(b*x^2+a)^3/(d*x+c)^9,x, algorithm="giac")
```

output
```
1/8*(8*b^4*c*d^7*x^7 + 28*b^4*c^2*d^6*x^6 - 4*a*b^3*d^8*x^6 + 56*b^4*c^3*d
^5*x^5 + 70*b^4*c^4*d^4*x^4 - 6*a^2*b^2*d^8*x^4 + 56*b^4*c^5*d^3*x^3 + 28*
b^4*c^6*d^2*x^2 - 4*a^3*b*d^8*x^2 + 8*b^4*c^7*d*x + b^4*c^8 - a^4*d^8)/((d
*x + c)^8*d^8)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 6.45

$$\int \frac{(ad-bcx)(a+bx^2)^3}{(c+dx)^9}\,dx = -\frac{\frac{a^4}{8}+\frac{a^3\,b\,x^2}{2}+\frac{3\,a^2\,b^2\,x^4}{4}+\frac{a\,b^3\,x^6}{2}+\frac{b^4\,x^8}{8}}{c^8+8\,c^7\,d\,x+28\,c^6\,d^2\,x^2+56\,c^5\,d^3\,x^3+70\,c^4\,d^4\,x^4+56\,c^3\,d^5\,x^5+28\,c^2\,d^6\,x^6+8\,c\,d^7\,x^7+d^8\,x^8}$$

input
```
int(((a*d - b*c*x)*(a + b*x^2)^3)/(c + d*x)^9,x)
```

output
```
-(a^4/8 + (b^4*x^8)/8 + (a^3*b*x^2)/2 + (a*b^3*x^6)/2 + (3*a^2*b^2*x^4)/4)
/(c^8 + d^8*x^8 + 8*c*d^7*x^7 + 28*c^6*d^2*x^2 + 56*c^5*d^3*x^3 + 70*c^4*d
^4*x^4 + 56*c^3*d^5*x^5 + 28*c^2*d^6*x^6 + 8*c^7*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 6.55

$$\int \frac{(ad-bcx)(a+bx^2)^3}{(c+dx)^9}\,dx = \frac{-b^4x^8-4a\,b^3x^6-6a^2b^2x^4-4a^3b\,x^2-a^4}{8d^8x^8+64c\,d^7x^7+224c^2d^6x^6+448c^3d^5x^5+560c^4d^4x^4+448c^5d^3x^3+224c^6d^2x^2+64c^7dx+8c^8}$$

input
```
int((-b*c*x+a*d)*(b*x^2+a)^3/(d*x+c)^9,x)
```

output
```
( - a**4 - 4*a**3*b*x**2 - 6*a**2*b**2*x**4 - 4*a*b**3*x**6 - b**4*x**8)/(
8*(c**8 + 8*c**7*d*x + 28*c**6*d**2*x**2 + 56*c**5*d**3*x**3 + 70*c**4*d**
4*x**4 + 56*c**3*d**5*x**5 + 28*c**2*d**6*x**6 + 8*c*d**7*x**7 + d**8*x**8
))
```

3.17 $\int \frac{(ad-bcx)(a+bx^2)^2}{(c+dx)^7}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 20

$$\int \frac{(ad-bcx)(a+bx^2)^2}{(c+dx)^7}\,dx=-\frac{(a+bx^2)^3}{6(c+dx)^6}$$

output
```
-1/6*(b*x^2+a)^3/(d*x+c)^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 102 vs. 2(20) = 40.

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 5.10

$$\int \frac{(ad-bcx)(a+bx^2)^2}{(c+dx)^7}\,dx = -\frac{a^3d^6+3a^2bd^6x^2+3ab^2d^6x^4-b^3c(c^5+6c^4dx+15c^3d^2x^2+20c^2d^3x^3+15cd^4x^4+6d^5x^5)}{6d^6(c+dx)^6}$$

input
```
Integrate[((a*d - b*c*x)*(a + b*x^2)^2)/(c + d*x)^7,x]
```

output
```
-1/6*(a^3*d^6 + 3*a^2*b*d^6*x^2 + 3*a*b^2*d^6*x^4 - b^3*c*(c^5 + 6*c^4*d*x
 + 15*c^3*d^2*x^2 + 20*c^2*d^3*x^3 + 15*c*d^4*x^4 + 6*d^5*x^5))/(d^6*(c +
d*x)^6)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 167 vs. $2(20) = 40$.

Time = 0.38 (sec) , antiderivative size = 167, normalized size of antiderivative = 8.35, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (ad-bcx)}{(c+dx)^7} \, dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{b^2(ad^2+5bc^2)}{d^5(c+dx)^3} - \frac{2b^2c(3ad^2+5bc^2)}{d^5(c+dx)^4} + \frac{2b(ad^2+bc^2)(ad^2+5bc^2)}{d^5(c+dx)^5} - \frac{5bc(ad^2+bc^2)^2}{d^5(c+dx)^6} + \frac{(ad^2+bc^2)^3}{d^5(c+dx)^7} - \frac{}{d^5\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2(ad^2+5bc^2)}{2d^6(c+dx)^2} + \frac{2b^2c(3ad^2+5bc^2)}{3d^6(c+dx)^3} - \frac{b(ad^2+bc^2)(ad^2+5bc^2)}{2d^6(c+dx)^4} + \frac{bc(ad^2+bc^2)^2}{d^6(c+dx)^5} - \frac{(ad^2+bc^2)^3}{6d^6(c+dx)^6} + \frac{b^3c}{d^6(c+dx)}$$

input
```
Int[((a*d - b*c*x)*(a + b*x^2)^2)/(c + d*x)^7,x]
```

output
```
-1/6*(b*c^2 + a*d^2)^3/(d^6*(c + d*x)^6) + (b*c*(b*c^2 + a*d^2)^2)/(d^6*(c
 + d*x)^5) - (b*(b*c^2 + a*d^2)*(5*b*c^2 + a*d^2))/(2*d^6*(c + d*x)^4) + (
2*b^2*c*(5*b*c^2 + 3*a*d^2))/(3*d^6*(c + d*x)^3) - (b^2*(5*b*c^2 + a*d^2))
/(2*d^6*(c + d*x)^2) + (b^3*c)/(d^6*(c + d*x))
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(18) = 36$.

Time = 0.71 (sec) , antiderivative size = 115, normalized size of antiderivative = 5.75

method	result
risch	$\frac{\frac{c\,b^3x^5}{d}-\frac{b^2\left(a\,d^2-5b\,c^2\right)x^4}{2d^2}+\frac{10c^3b^3x^3}{3d^3}-\frac{b\left(a^2d^4-5b^2c^4\right)x^2}{2d^4}+\frac{c^5b^3x}{d^5}-\frac{a^3d^6-b^3c^6}{6d^6}}{(dx+c)^6}$
gosper	$-\frac{-6b^3c\,d^5x^5+3a\,b^2d^6x^4-15b^3c^2d^4x^4-20b^3c^3d^3x^3+3a^2b\,d^6x^2-15b^3c^4d^2x^2-6b^3c^5dx+a^3d^6-b^3c^6}{6(dx+c)^6d^6}$
norman	$\frac{\frac{c\,b^3x^5}{d}-\frac{\left(a\,b^2d^2-5b^3c^2\right)x^4}{2d^2}+\frac{10c^3b^3x^3}{3d^3}-\frac{\left(a^2b\,d^4-5b^3c^4\right)x^2}{2d^4}+\frac{c^5b^3x}{d^5}-\frac{a^3d^6-b^3c^6}{6d^6}}{(dx+c)^6}$
parallelrisch	$\frac{6b^3c\,d^5x^5-3a\,b^2d^6x^4+15b^3c^2d^4x^4+20b^3c^3d^3x^3-3a^2b\,d^6x^2+15b^3c^4d^2x^2+6b^3c^5dx-a^3d^6+b^3c^6}{6d^6(dx+c)^6}$
orering	$-\frac{-6b^3c\,d^5x^5+3a\,b^2d^6x^4-15b^3c^2d^4x^4-20b^3c^3d^3x^3+3a^2b\,d^6x^2-15b^3c^4d^2x^2-6b^3c^5dx+a^3d^6-b^3c^6}{6(dx+c)^6d^6}$
default	$\frac{b^3c}{d^6(dx+c)}-\frac{b^2(a\,d^2+5b\,c^2)}{2d^6(dx+c)^2}+\frac{bc(a^2d^4+2b\,c^2d^2a+b^2c^4)}{d^6(dx+c)^5}-\frac{a^3d^6+3a^2b\,c^2d^4+3a\,b^2c^4d^2+b^3c^6}{6d^6(dx+c)^6}+\frac{2b^2c(3a\,d^2+5b\,c^2)}{3d^6(dx+c)^3}-$

input
```
int((-b*c*x+a*d)*(b*x^2+a)^2/(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
(c*b^3/d*x^5-1/2*b^2*(a*d^2-5*b*c^2)/d^2*x^4+10/3*c^3*b^3/d^3*x^3-1/2*b*(a
^2*d^4-5*b^2*c^4)/d^4*x^2+c^5*b^3/d^5*x-1/6*(a^3*d^6-b^3*c^6)/d^6)/(d*x+c)
^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(18) = 36$.

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 8.70

$$\int \frac{(ad - bcx)(a + bx^2)^2}{(c + dx)^7} dx = \frac{6 b^3 c d^5 x^5 + 20 b^3 c^3 d^3 x^3 + 6 b^3 c^5 d x + b^3 c^6 - a^3 d^6 + 3 (5 b^3 c^2 d^4 - a b^2 d^6) x^4 + 3 (5 b^3 c^4 d^2 - a^2 b d^6) x^2}{6 (d^{12} x^6 + 6 c d^{11} x^5 + 15 c^2 d^{10} x^4 + 20 c^3 d^9 x^3 + 15 c^4 d^8 x^2 + 6 c^5 d^7 x + c^6 d^6)}$$

input
```
integrate((-b*c*x+a*d)*(b*x^2+a)^2/(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/6*(6*b^3*c*d^5*x^5 + 20*b^3*c^3*d^3*x^3 + 6*b^3*c^5*d*x + b^3*c^6 - a^3*
d^6 + 3*(5*b^3*c^2*d^4 - a*b^2*d^6)*x^4 + 3*(5*b^3*c^4*d^2 - a^2*b*d^6)*x^
2)/(d^12*x^6 + 6*c*d^11*x^5 + 15*c^2*d^10*x^4 + 20*c^3*d^9*x^3 + 15*c^4*d^
8*x^2 + 6*c^5*d^7*x + c^6*d^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(17) = 34$.

Time = 3.80 (sec) , antiderivative size = 178, normalized size of antiderivative = 8.90

$$\int \frac{(ad - bcx)(a + bx^2)^2}{(c + dx)^7} dx = -\frac{a^3 d^6 - b^3 c^6 - 6 b^3 c^5 d x - 20 b^3 c^3 d^3 x^3 - 6 b^3 c d^5 x^5 + x^4 \cdot (3 a b^2 d^6 - 15 b^3 c^2 d^4) + x^2 \cdot (3 a^2 b d^6 - 15 b^3 c^4 d^2)}{6 c^6 d^6 + 36 c^5 d^7 x + 90 c^4 d^8 x^2 + 120 c^3 d^9 x^3 + 90 c^2 d^{10} x^4 + 36 c d^{11} x^5 + 6 d^{12} x^6}$$

input
```
integrate((-b*c*x+a*d)*(b*x**2+a)**2/(d*x+c)**7,x)
```

output
```
-(a**3*d**6 - b**3*c**6 - 6*b**3*c**5*d*x - 20*b**3*c**3*d**3*x**3 - 6*b**
3*c*d**5*x**5 + x**4*(3*a*b**2*d**6 - 15*b**3*c**2*d**4) + x**2*(3*a**2*b*
d**6 - 15*b**3*c**4*d**2))/(6*c**6*d**6 + 36*c**5*d**7*x + 90*c**4*d**8*x*
*2 + 120*c**3*d**9*x**3 + 90*c**2*d**10*x**4 + 36*c*d**11*x**5 + 6*d**12*x
**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(18) = 36$.

Time = 0.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 8.70

$$\int \frac{(ad-bcx)\left(a+bx^2\right)^2}{(c+dx)^7}\,dx = \frac{6\,b^3cd^5x^5+20\,b^3c^3d^3x^3+6\,b^3c^5dx+b^3c^6-a^3d^6+3\,(5\,b^3c^2d^4-ab^2d^6)x^4+3\,(5\,b^3c^4d^2-a^2bd^6)x^2}{6\,(d^{12}x^6+6\,cd^{11}x^5+15\,c^2d^{10}x^4+20\,c^3d^9x^3+15\,c^4d^8x^2+6\,c^5d^7x+c^6d^6)}$$

input
```
integrate((-b*c*x+a*d)*(b*x^2+a)^2/(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/6*(6*b^3*c*d^5*x^5 + 20*b^3*c^3*d^3*x^3 + 6*b^3*c^5*d*x + b^3*c^6 - a^3*
d^6 + 3*(5*b^3*c^2*d^4 - a*b^2*d^6)*x^4 + 3*(5*b^3*c^4*d^2 - a^2*b*d^6)*x^
2)/(d^12*x^6 + 6*c*d^11*x^5 + 15*c^2*d^10*x^4 + 20*c^3*d^9*x^3 + 15*c^4*d^
8*x^2 + 6*c^5*d^7*x + c^6*d^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(18) = 36$.

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 5.80

$$\int \frac{(ad-bcx)\left(a+bx^2\right)^2}{(c+dx)^7}\,dx = \frac{6\,b^3cd^5x^5+15\,b^3c^2d^4x^4-3\,ab^2d^6x^4+20\,b^3c^3d^3x^3+15\,b^3c^4d^2x^2-3\,a^2bd^6x^2+6\,b^3c^5dx+b^3c^6-a^3d^6}{6\,(dx+c)^6d^6}$$

input
```
integrate((-b*c*x+a*d)*(b*x^2+a)^2/(d*x+c)^7,x, algorithm="giac")
```

output
```
1/6*(6*b^3*c*d^5*x^5 + 15*b^3*c^2*d^4*x^4 - 3*a*b^2*d^6*x^4 + 20*b^3*c^3*d
^3*x^3 + 15*b^3*c^4*d^2*x^2 - 3*a^2*b*d^6*x^2 + 6*b^3*c^5*d*x + b^3*c^6 -
a^3*d^6)/((d*x + c)^6*d^6)
```

Mupad [B] (verification not implemented)

Time = 7.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.80

$$\int \frac{(ad - bcx)\left(a + bx^2\right)^2}{(c + dx)^7}\,dx = -\frac{\frac{a^3}{6} + \frac{a^2\,b\,x^2}{2} + \frac{a\,b^2\,x^4}{2} + \frac{b^3\,x^6}{6}}{c^6 + 6\,c^5\,d\,x + 15\,c^4\,d^2\,x^2 + 20\,c^3\,d^3\,x^3 + 15\,c^2\,d^4\,x^4 + 6\,c\,d^5\,x^5 + d^6\,x^6}$$

input
```
int(((a*d - b*c*x)*(a + b*x^2)^2)/(c + d*x)^7,x)
```

output
```
-(a^3/6 + (b^3*x^6)/6 + (a^2*b*x^2)/2 + (a*b^2*x^4)/2)/(c^6 + d^6*x^6 + 6*
c*d^5*x^5 + 15*c^4*d^2*x^2 + 20*c^3*d^3*x^3 + 15*c^2*d^4*x^4 + 6*c^5*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 4.90

$$\int \frac{(ad - bcx)\left(a + bx^2\right)^2}{(c + dx)^7}\,dx = \frac{-b^3x^6 - 3a\,b^2x^4 - 3a^2b\,x^2 - a^3}{6d^6x^6 + 36c\,d^5x^5 + 90c^2d^4x^4 + 120c^3d^3x^3 + 90c^4d^2x^2 + 36c^5dx + 6c^6}$$

input
```
int((-b*c*x+a*d)*(b*x^2+a)^2/(d*x+c)^7,x)
```

output
```
( - a**3 - 3*a**2*b*x**2 - 3*a*b**2*x**4 - b**3*x**6)/(6*(c**6 + 6*c**5*d*
x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15*c**2*d**4*x**4 + 6*c*d**5*x
**5 + d**6*x**6))
```

3.18 $\int \frac{(ad-bcx)(a+bx^2)}{(c+dx)^5} dx$

Optimal result

Integrand size = 24, antiderivative size = 20

$$\int \frac{(ad-bcx)(a+bx^2)}{(c+dx)^5} dx = -\frac{(a+bx^2)^2}{4(c+dx)^4}$$

output
```
-1/4*(b*x^2+a)^2/(d*x+c)^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 66 vs. 2(20) = 40.

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.30

$$\int \frac{(ad-bcx)(a+bx^2)}{(c+dx)^5} dx = \frac{-a^2d^4-2abd^4x^2+b^2c(c^3+4c^2dx+6cd^2x^2+4d^3x^3)}{4d^4(c+dx)^4}$$

input
```
Integrate[((a*d - b*c*x)*(a + b*x^2))/(c + d*x)^5,x]
```

output
```
(-(a^2*d^4) - 2*a*b*d^4*x^2 + b^2*c*(c^3 + 4*c^2*d*x + 6*c*d^2*x^2 + 4*d^3
*x^3))/(4*d^4*(c + d*x)^4)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 94 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 4.70, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(ad-bcx)}{(c+dx)^5} \, dx$$

$$\Big\downarrow \ 652$$

$$\int \left(\frac{b(ad^2+3bc^2)}{d^3(c+dx)^3} - \frac{3bc(ad^2+bc^2)}{d^3(c+dx)^4} + \frac{(ad^2+bc^2)^2}{d^3(c+dx)^5} - \frac{b^2c}{d^3(c+dx)^2}\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{b(ad^2+3bc^2)}{2d^4(c+dx)^2} + \frac{bc(ad^2+bc^2)}{d^4(c+dx)^3} - \frac{(ad^2+bc^2)^2}{4d^4(c+dx)^4} + \frac{b^2c}{d^4(c+dx)}$$

input
```
Int[((a*d - b*c*x)*(a + b*x^2))/(c + d*x)^5,x]
```

output
```
-1/4*(b*c^2 + a*d^2)^2/(d^4*(c + d*x)^4) + (b*c*(b*c^2 + a*d^2))/(d^4*(c +
 d*x)^3) - (b*(3*b*c^2 + a*d^2))/(2*d^4*(c + d*x)^2) + (b^2*c)/(d^4*(c + d
*x))
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(18) = 36$.

Time = 0.58 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.70

method	result	size
risch	$\frac{\frac{c\,b^{2} x^{3}}{d}-\frac{b\left(a\,d^{2}-3 b\,c^{2}\right) x^{2}}{2 d^{2}}+\frac{c^{3} b^{2} x}{d^{3}}-\frac{a^{2} d^{4}-b^{2} c^{4}}{4 d^{4}}}{\left(d x +c \right)^{4}}$	74
gosper	$-\frac{-4 b^{2} c\,d^{3} x^{3}+2 a b\,d^{4} x^{2}-6 d^{2} c^{2} x^{2} b^{2}-4 b^{2} c^{3} d x +a^{2} d^{4}-b^{2} c^{4}}{4 \left(d x +c \right)^{4} d^{4}}$	75
parallelrisch	$\frac{4 b^{2} c\,d^{3} x^{3}-2 a b\,d^{4} x^{2}+6 d^{2} c^{2} x^{2} b^{2}+4 b^{2} c^{3} d x -a^{2} d^{4}+b^{2} c^{4}}{4 d^{4} \left(d x +c \right)^{4}}$	75
orering	$-\frac{-4 b^{2} c\,d^{3} x^{3}+2 a b\,d^{4} x^{2}-6 d^{2} c^{2} x^{2} b^{2}-4 b^{2} c^{3} d x +a^{2} d^{4}-b^{2} c^{4}}{4 \left(d x +c \right)^{4} d^{4}}$	75
norman	$\frac{\frac{c\,b^{2} x^{3}}{d}-\frac{\left(a b\,d^{2}-3 b^{2} c^{2}\right) x^{2}}{2 d^{2}}+\frac{c^{3} b^{2} x}{d^{3}}-\frac{a^{2} d^{4}-b^{2} c^{4}}{4 d^{4}}}{\left(d x +c \right)^{4}}$	76
default	$\frac{b^{2} c}{d^{4} \left(d x +c \right)}-\frac{b \left(a\,d^{2}+3 b\,c^{2}\right)}{2 d^{4} \left(d x +c \right)^{2}}+\frac{b c \left(a\,d^{2}+b\,c^{2}\right)}{d^{4} \left(d x +c \right)^{3}}-\frac{a^{2} d^{4}+2 b\,c^{2} d^{2} a +b^{2} c^{4}}{4 d^{4} \left(d x +c \right)^{4}}$	103

input
```
int((-b*c*x+a*d)*(b*x^2+a)/(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
(c/d*b^2*x^3-1/2*b*(a*d^2-3*b*c^2)/d^2*x^2+c^3*b^2/d^3*x-1/4*(a^2*d^4-b^2*
c^4)/d^4)/(d*x+c)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(18) = 36$.

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 5.50

$$\int \frac{(ad-bcx)\left(a+bx^2\right)}{(c+dx)^5}\,dx = \frac{4\,b^2cd^3x^3+4\,b^2c^3dx+b^2c^4-a^2d^4+2\,(3\,b^2c^2d^2-abd^4)x^2}{4\,(d^8x^4+4\,cd^7x^3+6\,c^2d^6x^2+4\,c^3d^5x+c^4d^4)}$$

input
```
integrate((-b*c*x+a*d)*(b*x^2+a)/(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/4*(4*b^2*c*d^3*x^3 + 4*b^2*c^3*d*x + b^2*c^4 - a^2*d^4 + 2*(3*b^2*c^2*d^
2 - a*b*d^4)*x^2)/(d^8*x^4 + 4*c*d^7*x^3 + 6*c^2*d^6*x^2 + 4*c^3*d^5*x + c
^4*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(17) = 34$.

Time = 0.65 (sec) , antiderivative size = 112, normalized size of antiderivative = 5.60

$$\int \frac{(ad-bcx)\,(a+bx^2)}{(c+dx)^5}\,dx$$
$$= -\frac{a^2d^4-b^2c^4-4b^2c^3dx-4b^2cd^3x^3+x^2\cdot(2abd^4-6b^2c^2d^2)}{4c^4d^4+16c^3d^5x+24c^2d^6x^2+16cd^7x^3+4d^8x^4}$$

input
```
integrate((-b*c*x+a*d)*(b*x**2+a)/(d*x+c)**5,x)
```

output
```
-(a**2*d**4 - b**2*c**4 - 4*b**2*c**3*d*x - 4*b**2*c*d**3*x**3 + x**2*(2*a
*b*d**4 - 6*b**2*c**2*d**2))/(4*c**4*d**4 + 16*c**3*d**5*x + 24*c**2*d**6*
x**2 + 16*c*d**7*x**3 + 4*d**8*x**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(18) = 36$.

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 5.50

$$\int \frac{(ad-bcx)\,(a+bx^2)}{(c+dx)^5}\,dx = \frac{4\,b^2cd^3x^3+4\,b^2c^3dx+b^2c^4-a^2d^4+2\,(3\,b^2c^2d^2-abd^4)x^2}{4\,(d^8x^4+4\,cd^7x^3+6\,c^2d^6x^2+4\,c^3d^5x+c^4d^4)}$$

input
```
integrate((-b*c*x+a*d)*(b*x^2+a)/(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/4*(4*b^2*c*d^3*x^3 + 4*b^2*c^3*d*x + b^2*c^4 - a^2*d^4 + 2*(3*b^2*c^2*d^
2 - a*b*d^4)*x^2)/(d^8*x^4 + 4*c*d^7*x^3 + 6*c^2*d^6*x^2 + 4*c^3*d^5*x + c
^4*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(18) = 36$.

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 6.55

$$\int \frac{(ad - bcx)(a + bx^2)}{(c + dx)^5} dx = \frac{\frac{4 b^2 c}{(dx+c)d} - \frac{6 b^2 c^2}{(dx+c)^2 d} + \frac{4 b^2 c^3}{(dx+c)^3 d} - \frac{b^2 c^4}{(dx+c)^4 d} - \frac{2 abd}{(dx+c)^2} + \frac{4 abcd}{(dx+c)^3} - \frac{2 abc^2 d}{(dx+c)^4} - \frac{a^2 d^3}{(dx+c)^4}}{4 d^3}$$

input
```
integrate((-b*c*x+a*d)*(b*x^2+a)/(d*x+c)^5,x, algorithm="giac")
```

output
```
1/4*(4*b^2*c/((d*x + c)*d) - 6*b^2*c^2/((d*x + c)^2*d) + 4*b^2*c^3/((d*x +
 c)^3*d) - b^2*c^4/((d*x + c)^4*d) - 2*a*b*d/(d*x + c)^2 + 4*a*b*c*d/(d*x
+ c)^3 - 2*a*b*c^2*d/(d*x + c)^4 - a^2*d^3/(d*x + c)^4)/d^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.15

$$\int \frac{(ad - bcx)(a + bx^2)}{(c + dx)^5} dx = -\frac{\frac{a^2}{4} + \frac{a\,b\,x^2}{2} + \frac{b^2\,x^4}{4}}{c^4 + 4\,c^3\,d\,x + 6\,c^2\,d^2\,x^2 + 4\,c\,d^3\,x^3 + d^4\,x^4}$$

input
```
int(((a*d - b*c*x)*(a + b*x^2))/(c + d*x)^5,x)
```

output
```
-(a^2/4 + (b^2*x^4)/4 + (a*b*x^2)/2)/(c^4 + d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*
d^2*x^2 + 4*c^3*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.25

$$\int \frac{(ad-bcx)(a+bx^2)}{(c+dx)^5}\,dx = \frac{-b^2x^4-2ab\,x^2-a^2}{4d^4x^4+16c\,d^3x^3+24c^2d^2x^2+16c^3dx+4c^4}$$

input
```
int((-b*c*x+a*d)*(b*x^2+a)/(d*x+c)^5,x)
```

output
```
( - a**2 - 2*a*b*x**2 - b**2*x**4)/(4*(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**
2 + 4*c*d**3*x**3 + d**4*x**4))
```

3.19 $\int \frac{ad-bcx}{(c+dx)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{ad-bcx}{(c+dx)^3} \, dx = -\frac{a+bx^2}{2(c+dx)^2}$$

output
```
-1/2*(b*x^2+a)/(d*x+c)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{ad-bcx}{(c+dx)^3} \, dx = \frac{-ad^2+bc(c+2dx)}{2d^2(c+dx)^2}$$

input
```
Integrate[(a*d - b*c*x)/(c + d*x)^3,x]
```

output
```
(-(a*d^2) + b*c*(c + 2*d*x))/(2*d^2*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ad-bcx}{(c+dx)^3}\,dx$$
$$\Big\downarrow \ 48$$
$$-\frac{(ad-bcx)^2}{2(c+dx)^2\,(ad^2+bc^2)}$$

input
```
Int[(a*d - b*c*x)/(c + d*x)^3,x]
```

output
```
-1/2*(a*d - b*c*x)^2/((b*c^2 + a*d^2)*(c + d*x)^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

method	result	size
gosper	$-\frac{-2bcdx+a\,d^{2}-b\,c^{2}}{2(dx+c)^{2}d^{2}}$	31
parallelrisch	$\frac{2bcdx-a\,d^{2}+b\,c^{2}}{2d^{2}(dx+c)^{2}}$	31
orering	$-\frac{-2bcdx+a\,d^{2}-b\,c^{2}}{2(dx+c)^{2}d^{2}}$	31
norman	$\frac{\frac{bcx}{d}-\frac{a\,d^{2}-b\,c^{2}}{2d^{2}}}{(dx+c)^{2}}$	34
risch	$\frac{\frac{bcx}{d}-\frac{a\,d^{2}-b\,c^{2}}{2d^{2}}}{(dx+c)^{2}}$	34
default	$\frac{bc}{d^{2}(dx+c)}-\frac{a\,d^{2}+b\,c^{2}}{2d^{2}(dx+c)^{2}}$	38

input
```
int((-b*c*x+a*d)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*b*c*d*x+a*d^2-b*c^2)/(d*x+c)^2/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(16) = 32$.

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{ad-bcx}{(c+dx)^3}\,dx = \frac{2\,bcdx+bc^2-ad^2}{2\left(d^4x^2+2\,cd^3x+c^2d^2\right)}$$

input
```
integrate((-b*c*x+a*d)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(2*b*c*d*x + b*c^2 - a*d^2)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{ad-bcx}{(c+dx)^3}\,dx = -\frac{ad^2-bc^2-2bcdx}{2c^2d^2+4cd^3x+2d^4x^2}$$

input
```
integrate((-b*c*x+a*d)/(d*x+c)**3,x)
```

output
```
-(a*d**2 - b*c**2 - 2*b*c*d*x)/(2*c**2*d**2 + 4*c*d**3*x + 2*d**4*x**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(16) = 32$.

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{ad-bcx}{(c+dx)^3}\,dx = \frac{2\,bcdx+bc^2-ad^2}{2\,(d^4x^2+2\,cd^3x+c^2d^2)}$$

input
```
integrate((-b*c*x+a*d)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/2*(2*b*c*d*x + b*c^2 - a*d^2)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{ad-bcx}{(c+dx)^3}\,dx = \frac{2\,bcdx+bc^2-ad^2}{2\,(dx+c)^2d^2}$$

input
```
integrate((-b*c*x+a*d)/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/2*(2*b*c*d*x + b*c^2 - a*d^2)/((d*x + c)^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{ad - bcx}{(c + dx)^3}\, dx = -\frac{\frac{b\,x^2}{2} + \frac{a}{2}}{c^2 + 2\,c\,d\,x + d^2\,x^2}$$

input
```
int((a*d - b*c*x)/(c + d*x)^3,x)
```

output
```
-(a/2 + (b*x^2)/2)/(c^2 + d^2*x^2 + 2*c*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \frac{ad - bcx}{(c + dx)^3}\, dx = \frac{-b\,x^2 - a}{2d^2x^2 + 4cdx + 2c^2}$$

input
```
int((-b*c*x+a*d)/(d*x+c)^3,x)
```

output
```
( - (a + b*x**2))/(2*(c**2 + 2*c*d*x + d**2*x**2))
```

3.20 $\int \frac{ad-bcx}{(c+dx)(a+bx^2)} dx$

Optimal result

Integrand size = 26, antiderivative size = 19

$$\int \frac{ad-bcx}{(c+dx)(a+bx^2)} dx = \log(c+dx) - \frac{1}{2}\log\left(a+bx^2\right)$$

output
```
ln(d*x+c)-1/2*ln(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{ad-bcx}{(c+dx)(a+bx^2)} dx = \log(c+dx) - \frac{1}{2}\log\left(a+bx^2\right)$$

input
```
Integrate[(a*d - b*c*x)/((c + d*x)*(a + b*x^2)),x]
```

output
```
Log[c + d*x] - Log[a + b*x^2]/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ad-bcx}{(a+bx^2)(c+dx)}dx$$

$$\Big\downarrow \text{657}$$

$$\int \left(\frac{d}{c+dx}-\frac{bx}{a+bx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(c+dx)-\frac{1}{2}\log\left(a+bx^2\right)$$

input
```
Int[(a*d - b*c*x)/((c + d*x)*(a + b*x^2)),x]
```

output
```
Log[c + d*x] - Log[a + b*x^2]/2
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$\ln\left(dx+c\right)-\frac{\ln\left(b\,x^{2}+a\right)}{2}$	18
norman	$\ln\left(dx+c\right)-\frac{\ln\left(b\,x^{2}+a\right)}{2}$	18
risch	$\ln\left(dx+c\right)-\frac{\ln\left(b\,x^{2}+a\right)}{2}$	18
parallelrisch	$\ln\left(dx+c\right)-\frac{\ln\left(b\,x^{2}+a\right)}{2}$	18

input
```
int((-b*c*x+a*d)/(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
ln(d*x+c)-1/2*ln(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{ad-bcx}{(c+dx)\left(a+bx^2\right)}\,dx=-\frac{1}{2}\log\left(bx^2+a\right)+\log\left(dx+c\right)$$

input
```
integrate((-b*c*x+a*d)/(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*log(b*x^2 + a) + log(d*x + c)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{ad-bcx}{(c+dx)\left(a+bx^2\right)}\,dx=-\frac{\log\left(\frac{a}{b}+x^2\right)}{2}+\log\left(\frac{c}{d}+x\right)$$

input
```
integrate((-b*c*x+a*d)/(d*x+c)/(b*x**2+a),x)
```

output
```
-log(a/b + x**2)/2 + log(c/d + x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{ad - bcx}{(c + dx)(a + bx^2)} dx = -\frac{1}{2} \log (bx^2 + a) + \log (dx + c)$$

input
```
integrate((-b*c*x+a*d)/(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*log(b*x^2 + a) + log(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{ad - bcx}{(c + dx)(a + bx^2)} dx = -\frac{1}{2} \log (bx^2 + a) + \log (|dx + c|)$$

input
```
integrate((-b*c*x+a*d)/(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*log(b*x^2 + a) + log(abs(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 6.96 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{ad - bcx}{(c + dx)(a + bx^2)} dx = \ln (c + d\,x) - \frac{\ln (b\,x^2 + a)}{2}$$

input
```
int((a*d - b*c*x)/((a + b*x^2)*(c + d*x)),x)
```

output
```
log(c + d*x) - log(a + b*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{ad - bcx}{(c + dx)(a + bx^2)}\,dx = -\frac{\log(b\,x^2 + a)}{2} + \log(dx + c)$$

input
```
int((-b*c*x+a*d)/(d*x+c)/(b*x^2+a),x)
```

output
```
( - log(a + b*x**2) + 2*log(c + d*x))/2
```

3.21 $\int \frac{(ad-bcx)(c+dx)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 20

$$\int \frac{(ad-bcx)(c+dx)}{(a+bx^2)^2}\,dx = \frac{(c+dx)^2}{2(a+bx^2)}$$

output
```
(d*x+c)^2/(2*b*x^2+2*a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{(ad-bcx)(c+dx)}{(a+bx^2)^2}\,dx = \frac{bc^2-ad^2+2bcdx}{2b(a+bx^2)}$$

input
```
Integrate[((a*d - b*c*x)*(c + d*x))/(a + b*x^2)^2,x]
```

output
```
(b*c^2 - a*d^2 + 2*b*c*d*x)/(2*b*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {673}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ad-bcx)}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \; 673$$

$$\frac{-ad^2+bc^2+2bcdx}{2b\,(a+bx^2)}$$

input
```
Int[((a*d - b*c*x)*(c + d*x))/(a + b*x^2)^2,x]
```

output
```
(b*c^2 - a*d^2 + 2*b*c*d*x)/(2*b*(a + b*x^2))
```

Defintions of rubi rules used

rule 673
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] /; FreeQ[{a, c, d, e, f, g, p}, x] && EqQ
[a*e*g - c*d*f*(2*p + 3), 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

method	result	size
gosper	$-\frac{-2bcdx+a\,d^2-b\,c^2}{2(b\,x^2+a)b}$	33
default	$\frac{cdx-\frac{a\,d^2-b\,c^2}{2b}}{b\,x^2+a}$	33
norman	$\frac{cdx-\frac{a\,d^2-b\,c^2}{2b}}{b\,x^2+a}$	33
risch	$\frac{cdx-\frac{a\,d^2-b\,c^2}{2b}}{b\,x^2+a}$	33
parallelrisch	$\frac{2bcdx-a\,d^2+b\,c^2}{2b(b\,x^2+a)}$	33
orering	$-\frac{-2bcdx+a\,d^2-b\,c^2}{2(b\,x^2+a)b}$	33

input
```
int((-b*c*x+a*d)*(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*b*c*d*x+a*d^2-b*c^2)/(b*x^2+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{(ad-bcx)(c+dx)}{(a+bx^2)^2}\,dx = \frac{2\,bcdx+bc^2-ad^2}{2\,(b^2x^2+ab)}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*(2*b*c*d*x + b*c^2 - a*d^2)/(b^2*x^2 + a*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{(ad - bcx)(c + dx)}{(a + bx^2)^2}\,dx = -\frac{ad^2 - bc^2 - 2bcdx}{2ab + 2b^2x^2}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)/(b*x**2+a)**2,x)
```

output
```
-(a*d**2 - b*c**2 - 2*b*c*d*x)/(2*a*b + 2*b**2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{(ad - bcx)(c + dx)}{(a + bx^2)^2}\,dx = \frac{2\,bcdx + bc^2 - ad^2}{2\,(b^2x^2 + ab)}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(2*b*c*d*x + b*c^2 - a*d^2)/(b^2*x^2 + a*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{(ad - bcx)(c + dx)}{(a + bx^2)^2}\,dx = \frac{2\,bcdx + bc^2 - ad^2}{2\,(bx^2 + a)b}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(2*b*c*d*x + b*c^2 - a*d^2)/((b*x^2 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{(ad-bcx)(c+dx)}{(a+bx^2)^2}\,dx = -\frac{\frac{a\,d^2-b\,c^2}{2\,b}-c\,d\,x}{b\,x^2+a}$$

input
```
int(((a*d - b*c*x)*(c + d*x))/(a + b*x^2)^2,x)
```

output
```
-((a*d^2 - b*c^2)/(2*b) - c*d*x)/(a + b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{(ad-bcx)(c+dx)}{(a+bx^2)^2}\,dx = \frac{x(a\,d^2x-b\,c^2x+2acd)}{2a\,(b\,x^2+a)}$$

input
```
int((-b*c*x+a*d)*(d*x+c)/(b*x^2+a)^2,x)
```

output
```
(x*(2*a*c*d + a*d**2*x - b*c**2*x))/(2*a*(a + b*x**2))
```

3.22 $\int \frac{(ad-bcx)(c+dx)^3}{\left(a+bx^2\right)^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 20

$$\int \frac{(ad-bcx)(c+dx)^3}{\left(a+bx^2\right)^3} \, dx=\frac{(c+dx)^4}{4\left(a+bx^2\right)^2}$$

output
```
1/4*(d*x+c)^4/(b*x^2+a)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 68 vs. 2(20) = 40.

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.40

$$\int \frac{(ad-bcx)(c+dx)^3}{\left(a+bx^2\right)^3} \, dx=\frac{-a^2d^4-2abd^4x^2+b^2c(c^3+4c^2dx+6cd^2x^2+4d^3x^3)}{4b^2\left(a+bx^2\right)^2}$$

input
```
Integrate[((a*d - b*c*x)*(c + d*x)^3)/(a + b*x^2)^3,x]
```

output
```
(-(a^2*d^4) - 2*a*b*d^4*x^2 + b^2*c*(c^3 + 4*c^2*d*x + 6*c*d^2*x^2 + 4*d^3
*x^3))/(4*b^2*(a + b*x^2)^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {677}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3(ad-bcx)}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{677}$$

$$\frac{(c+dx)^4}{4(a+bx^2)^2}$$

input
```
Int[((a*d - b*c*x)*(c + d*x)^3)/(a + b*x^2)^3,x]
```

output
```
(c + d*x)^4/(4*(a + b*x^2)^2)
```

Defintions of rubi rules used

rule 677
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] &&
 EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*g, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(18) = 36.

Time = 0.66 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.50

method	result	size
default	$\frac{c\,d^{3}x^{3}-\frac{d^{2}\left(a\,d^{2}-3b\,c^{2}\right)x^{2}}{2b}+c^{3}dx-\frac{a^{2}d^{4}-b^{2}c^{4}}{4b^{2}}}{\left(b\,x^{2}+a\right)^{2}}$	70
norman	$\frac{c\,d^{3}x^{3}-\frac{\left(a\,d^{4}-3b\,c^{2}d^{2}\right)x^{2}}{2b}+c^{3}dx-\frac{a^{2}d^{4}-b^{2}c^{4}}{4b^{2}}}{\left(b\,x^{2}+a\right)^{2}}$	70
risch	$\frac{c\,d^{3}x^{3}-\frac{d^{2}\left(a\,d^{2}-3b\,c^{2}\right)x^{2}}{2b}+c^{3}dx-\frac{a^{2}d^{4}-b^{2}c^{4}}{4b^{2}}}{\left(b\,x^{2}+a\right)^{2}}$	70
gosper	$-\frac{-4b^{2}c\,d^{3}x^{3}+2ab\,d^{4}x^{2}-6d^{2}c^{2}x^{2}b^{2}-4b^{2}c^{3}dx+a^{2}d^{4}-b^{2}c^{4}}{4\left(b\,x^{2}+a\right)^{2}b^{2}}$	77
parallelrisch	$\frac{4b^{2}c\,d^{3}x^{3}-2ab\,d^{4}x^{2}+6d^{2}c^{2}x^{2}b^{2}+4b^{2}c^{3}dx-a^{2}d^{4}+b^{2}c^{4}}{4b^{2}\left(b\,x^{2}+a\right)^{2}}$	77
orering	$-\frac{-4b^{2}c\,d^{3}x^{3}+2ab\,d^{4}x^{2}-6d^{2}c^{2}x^{2}b^{2}-4b^{2}c^{3}dx+a^{2}d^{4}-b^{2}c^{4}}{4\left(b\,x^{2}+a\right)^{2}b^{2}}$	77

input
```
int((-b*c*x+a*d)*(d*x+c)^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(c*d^3*x^3-1/2*d^2*(a*d^2-3*b*c^2)/b*x^2+c^3*d*x-1/4*(a^2*d^4-b^2*c^4)/b^2
)/(b*x^2+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(18) = 36$.

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.50

$$\int \frac{(ad-bcx)(c+dx)^3}{(a+bx^2)^3}\,dx = \frac{4\,b^2cd^3x^3+4\,b^2c^3dx+b^2c^4-a^2d^4+2\,(3\,b^2c^2d^2-abd^4)x^2}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)^3/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/4*(4*b^2*c*d^3*x^3 + 4*b^2*c^3*d*x + b^2*c^4 - a^2*d^4 + 2*(3*b^2*c^2*d^
2 - a*b*d^4)*x^2)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(15) = 30$.

Time = 1.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.50

$$\int \frac{(ad-bcx)(c+dx)^3}{(a+bx^2)^3}\,dx = -\frac{a^2d^4-b^2c^4-4b^2c^3dx-4b^2cd^3x^3+x^2\cdot(2abd^4-6b^2c^2d^2)}{4a^2b^2+8ab^3x^2+4b^4x^4}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)**3/(b*x**2+a)**3,x)
```

output
```
-(a**2*d**4 - b**2*c**4 - 4*b**2*c**3*d*x - 4*b**2*c*d**3*x**3 + x**2*(2*a
*b*d**4 - 6*b**2*c**2*d**2))/(4*a**2*b**2 + 8*a*b**3*x**2 + 4*b**4*x**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(18) = 36$.

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.50

$$\int \frac{(ad-bcx)(c+dx)^3}{(a+bx^2)^3}\,dx = \frac{4\,b^2cd^3x^3+4\,b^2c^3dx+b^2c^4-a^2d^4+2\,(3\,b^2c^2d^2-abd^4)x^2}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)^3/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/4*(4*b^2*c*d^3*x^3 + 4*b^2*c^3*d*x + b^2*c^4 - a^2*d^4 + 2*(3*b^2*c^2*d^
2 - a*b*d^4)*x^2)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(18) = 36$.

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.80

$$\int \frac{(ad-bcx)(c+dx)^3}{(a+bx^2)^3}\,dx = \frac{4\,b^2cd^3x^3+6\,b^2c^2d^2x^2-2\,abd^4x^2+4\,b^2c^3dx+b^2c^4-a^2d^4}{4\,(bx^2+a)^2b^2}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)^3/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/4*(4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 - 2*a*b*d^4*x^2 + 4*b^2*c^3*d*x +
 b^2*c^4 - a^2*d^4)/((b*x^2 + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 7.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.00

$$\int \frac{(ad-bcx)(c+dx)^3}{(a+bx^2)^3}\,dx = \frac{\frac{c^4}{4}+c^3\,d\,x+\frac{3\,c^2\,d^2\,x^2}{2}+c\,d^3\,x^3+\frac{d^4\,x^4}{4}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

input
```
int(((a*d - b*c*x)*(c + d*x)^3)/(a + b*x^2)^3,x)
```

output
```
(c^4/4 + (d^4*x^4)/4 + c*d^3*x^3 + (3*c^2*d^2*x^2)/2 + c^3*d*x)/(a^2 + b^2
*x^4 + 2*a*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.45

$$\int \frac{(ad-bcx)(c+dx)^3}{(a+bx^2)^3}\,dx = \frac{ab\,d^4x^4-3b^2c^2d^2x^4+4abc\,d^3x^3+4ab\,c^3dx-3a^2c^2d^2+ab\,c^4}{4ab\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int((-b*c*x+a*d)*(d*x+c)^3/(b*x^2+a)^3,x)
```

output
```
( - 3*a**2*c**2*d**2 + a*b*c**4 + 4*a*b*c**3*d*x + 4*a*b*c*d**3*x**3 + a*b
*d**4*x**4 - 3*b**2*c**2*d**2*x**4)/(4*a*b*(a**2 + 2*a*b*x**2 + b**2*x**4)
)
```

3.23 $\int \frac{(ad-bcx)(c+dx)^5}{(a+bx^2)^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 20

$$\int \frac{(ad-bcx)(c+dx)^5}{(a+bx^2)^4} dx = \frac{(c+dx)^6}{6(a+bx^2)^3}$$

output
```
1/6*(d*x+c)^6/(b*x^2+a)^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 104 vs. 2(20) = 40.

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 5.20

$$\int \frac{(ad-bcx)(c+dx)^5}{(a+bx^2)^4} dx = -\frac{a^3d^6+3a^2bd^6x^2+3ab^2d^6x^4-b^3c(c^5+6c^4dx+15c^3d^2x^2+20c^2d^3x^3+15cd^4x^4+6d^5x^5)}{6b^3(a+bx^2)^3}$$

input
```
Integrate[((a*d - b*c*x)*(c + d*x)^5)/(a + b*x^2)^4,x]
```

output
```
-1/6*(a^3*d^6 + 3*a^2*b*d^6*x^2 + 3*a*b^2*d^6*x^4 - b^3*c*(c^5 + 6*c^4*d*x
 + 15*c^3*d^2*x^2 + 20*c^2*d^3*x^3 + 15*c*d^4*x^4 + 6*d^5*x^5))/(b^3*(a +
b*x^2)^3)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{677\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^5(ad-bcx)}{(a+bx^2)^4} \, dx$$

$$\Big\downarrow \text{677}$$

$$\frac{(c+dx)^6}{6(a+bx^2)^3}$$

input
```
Int[((a*d - b*c*x)*(c + d*x)^5)/(a + b*x^2)^4,x]
```

output
```
(c + d*x)^6/(6*(a + b*x^2)^3)
```

Defintions of rubi rules used

rule 677
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] &&
 EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*g, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(18) = 36$.

Time = 0.71 (sec) , antiderivative size = 108, normalized size of antiderivative = 5.40

method	result	size
default	$\frac{c\,d^{5} x^{5}-\frac{d^{4} \left(a\,d^{2}-5 b\,c^{2}\right) x^{4}}{2 b}+\frac{10 d^{3} c^{3} x^{3}}{3}-\frac{d^{2} \left(a^{2} d^{4}-5 b^{2} c^{4}\right) x^{2}}{2 b^{2}}+c^{5} d x -\frac{a^{3} d^{6}-b^{3} c^{6}}{6 b^{3}}}{\left(b\,x^{2}+a \right)^{3}}$	108
norman	$\frac{c\,d^{5} x^{5}-\frac{\left(a\,d^{6}-5 b\,c^{2} d^{4}\right) x^{4}}{2 b}+\frac{10 d^{3} c^{3} x^{3}}{3}-\frac{\left(a^{2} d^{6}-5 c^{4} b^{2} d^{2}\right) x^{2}}{2 b^{2}}+c^{5} d x -\frac{a^{3} d^{6}-b^{3} c^{6}}{6 b^{3}}}{\left(b\,x^{2}+a \right)^{3}}$	108
risch	$\frac{c\,d^{5} x^{5}-\frac{d^{4} \left(a\,d^{2}-5 b\,c^{2}\right) x^{4}}{2 b}+\frac{10 d^{3} c^{3} x^{3}}{3}-\frac{d^{2} \left(a^{2} d^{4}-5 b^{2} c^{4}\right) x^{2}}{2 b^{2}}+c^{5} d x -\frac{a^{3} d^{6}-b^{3} c^{6}}{6 b^{3}}}{\left(b\,x^{2}+a \right)^{3}}$	108
gosper	$-\frac{-6 b^{3} c\,d^{5} x^{5}+3 a\,b^{2} d^{6} x^{4}-15 b^{3} c^{2} d^{4} x^{4}-20 b^{3} c^{3} d^{3} x^{3}+3 a^{2} b\,d^{6} x^{2}-15 b^{3} c^{4} d^{2} x^{2}-6 b^{3} c^{5} d x +a^{3} d^{6}-b^{3} c^{6}}{6 \left(b\,x^{2}+a \right)^{3} b^{3}}$	119
parallelrisch	$\frac{6 b^{3} c\,d^{5} x^{5}-3 a\,b^{2} d^{6} x^{4}+15 b^{3} c^{2} d^{4} x^{4}+20 b^{3} c^{3} d^{3} x^{3}-3 a^{2} b\,d^{6} x^{2}+15 b^{3} c^{4} d^{2} x^{2}+6 b^{3} c^{5} d x -a^{3} d^{6}+b^{3} c^{6}}{6 b^{3} \left(b\,x^{2}+a \right)^{3}}$	119
orering	$-\frac{-6 b^{3} c\,d^{5} x^{5}+3 a\,b^{2} d^{6} x^{4}-15 b^{3} c^{2} d^{4} x^{4}-20 b^{3} c^{3} d^{3} x^{3}+3 a^{2} b\,d^{6} x^{2}-15 b^{3} c^{4} d^{2} x^{2}-6 b^{3} c^{5} d x +a^{3} d^{6}-b^{3} c^{6}}{6 \left(b\,x^{2}+a \right)^{3} b^{3}}$	119

input
```
int((-b*c*x+a*d)*(d*x+c)^5/(b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(c*d^5*x^5-1/2*d^4*(a*d^2-5*b*c^2)/b*x^4+10/3*d^3*c^3*x^3-1/2*d^2*(a^2*d^4
-5*b^2*c^4)/b^2*x^2+c^5*d*x-1/6*(a^3*d^6-b^3*c^6)/b^3)/(b*x^2+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(18) = 36$.

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 7.15

$$\int \frac{(ad-bcx)(c+dx)^5}{(a+bx^2)^4}\,dx = \frac{6\,b^3cd^5x^5+20\,b^3c^3d^3x^3+6\,b^3c^5dx+b^3c^6-a^3d^6+3\,(5\,b^3c^2d^4-ab^2d^6)x^4+3\,(5\,b^3c^4d^2-a^2bd^6)x^2}{6\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)^5/(b*x^2+a)^4,x, algorithm="fricas")
```

output
```
1/6*(6*b^3*c*d^5*x^5 + 20*b^3*c^3*d^3*x^3 + 6*b^3*c^5*d*x + b^3*c^6 - a^3*
d^6 + 3*(5*b^3*c^2*d^4 - a*b^2*d^6)*x^4 + 3*(5*b^3*c^4*d^2 - a^2*b*d^6)*x^
2)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(15) = 30$.

Time = 16.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 7.20

$$\int \frac{(ad-bcx)(c+dx)^5}{(a+bx^2)^4}\,dx = -\frac{a^3d^6 - b^3c^6 - 6b^3c^5dx - 20b^3c^3d^3x^3 - 6b^3cd^5x^5 + x^4\cdot(3ab^2d^6 - 15b^3c^2d^4) + x^2\cdot(3a^2bd^6 - 15b^3c^4d^2)}{6a^3b^3 + 18a^2b^4x^2 + 18ab^5x^4 + 6b^6x^6}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)**5/(b*x**2+a)**4,x)
```

output
```
-(a**3*d**6 - b**3*c**6 - 6*b**3*c**5*d*x - 20*b**3*c**3*d**3*x**3 - 6*b**
3*c*d**5*x**5 + x**4*(3*a*b**2*d**6 - 15*b**3*c**2*d**4) + x**2*(3*a**2*b*
d**6 - 15*b**3*c**4*d**2))/(6*a**3*b**3 + 18*a**2*b**4*x**2 + 18*a*b**5*x*
*4 + 6*b**6*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(18) = 36$.

Time = 0.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 7.15

$$\int \frac{(ad-bcx)(c+dx)^5}{(a+bx^2)^4}\,dx = \frac{6\,b^3cd^5x^5 + 20\,b^3c^3d^3x^3 + 6\,b^3c^5dx + b^3c^6 - a^3d^6 + 3\,(5\,b^3c^2d^4 - ab^2d^6)x^4 + 3\,(5\,b^3c^4d^2 - a^2bd^6)x^2}{6\,(b^6x^6 + 3\,ab^5x^4 + 3\,a^2b^4x^2 + a^3b^3)}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)^5/(b*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/6*(6*b^3*c*d^5*x^5 + 20*b^3*c^3*d^3*x^3 + 6*b^3*c^5*d*x + b^3*c^6 - a^3*
d^6 + 3*(5*b^3*c^2*d^4 - a*b^2*d^6)*x^4 + 3*(5*b^3*c^4*d^2 - a^2*b*d^6)*x^
2)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(18) = 36$.

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 5.90

$$\int \frac{(ad-bcx)(c+dx)^5}{(a+bx^2)^4}\,dx = \frac{6\,b^3cd^5x^5+15\,b^3c^2d^4x^4-3\,ab^2d^6x^4+20\,b^3c^3d^3x^3+15\,b^3c^4d^2x^2-3\,a^2bd^6x^2+6\,b^3c^5dx+b^3c^6-a^3d^6}{6\,(bx^2+a)^3b^3}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)^5/(b*x^2+a)^4,x, algorithm="giac")
```

output
```
1/6*(6*b^3*c*d^5*x^5 + 15*b^3*c^2*d^4*x^4 - 3*a*b^2*d^6*x^4 + 20*b^3*c^3*d
^3*x^3 + 15*b^3*c^4*d^2*x^2 - 3*a^2*b*d^6*x^2 + 6*b^3*c^5*d*x + b^3*c^6 -
a^3*d^6)/((b*x^2 + a)^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 6.92 (sec) , antiderivative size = 93, normalized size of antiderivative = 4.65

$$\int \frac{(ad-bcx)(c+dx)^5}{(a+bx^2)^4}\,dx = \frac{\frac{c^6}{6}+c^5\,d\,x+\frac{5\,c^4\,d^2\,x^2}{2}+\frac{10\,c^3\,d^3\,x^3}{3}+\frac{5\,c^2\,d^4\,x^4}{2}+c\,d^5\,x^5+\frac{d^6\,x^6}{6}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}$$

input
```
int(((a*d - b*c*x)*(c + d*x)^5)/(a + b*x^2)^4,x)
```

output
```
(c^6/6 + (d^6*x^6)/6 + c*d^5*x^5 + (5*c^4*d^2*x^2)/2 + (10*c^3*d^3*x^3)/3
+ (5*c^2*d^4*x^4)/2 + c^5*d*x)/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 7.65

$$\int \frac{(ad-bcx)(c+dx)^5}{(a+bx^2)^4}\,dx$$
$$= \frac{a\,b^2d^6x^6-5b^3c^2d^4x^6+6a\,b^2c\,d^5x^5+20a\,b^2c^3d^3x^3-15a^2b\,c^2d^4x^2+15a\,b^2c^4d^2x^2+6a\,b^2c^5dx-5a^3c^2d^4}{6a\,b^2\,(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)}$$

input
```
int((-b*c*x+a*d)*(d*x+c)^5/(b*x^2+a)^4,x)
```

output
```
( - 5*a**3*c**2*d**4 - 15*a**2*b*c**2*d**4*x**2 + a*b**2*c**6 + 6*a*b**2*c
**5*d*x + 15*a*b**2*c**4*d**2*x**2 + 20*a*b**2*c**3*d**3*x**3 + 6*a*b**2*c
*d**5*x**5 + a*b**2*d**6*x**6 - 5*b**3*c**2*d**4*x**6)/(6*a*b**2*(a**3 + 3
*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.24 $\int (c+dx)(ad+bc(3+2p)x)\left(a+bx^2\right)^p\,dx$

Optimal result

Integrand size = 28, antiderivative size = 48

$$\int (c+dx)(ad+bc(3+2p)x)\left(a+bx^2\right)^p\,dx = \frac{\left(ad^2+bc^2(3+2p)+2bcd(1+p)x\right)\left(a+bx^2\right)^{1+p}}{2b(1+p)}$$

output
```
1/2*(a*d^2+b*c^2*(3+2*p)+2*b*c*d*(p+1)*x)*(b*x^2+a)^(p+1)/b/(p+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.75

$$\int (c+dx)(ad+bc(3+2p)x)\left(a+bx^2\right)^p\,dx = \frac{1}{6}\left(a+bx^2\right)^p\left(\frac{3\left(ad^2+bc^2(3+2p)\right)\left(a+bx^2\right)}{b(1+p)} + 6acdx\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) + 2bcd(3+2p)x^3\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[(c + d*x)*(a*d + b*c*(3 + 2*p)*x)*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*((3*(a*d^2 + b*c^2*(3 + 2*p))*(a + b*x^2))/(b*(1 + p)) + (6
*a*c*d*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
+ (2*b*c*d*(3 + 2*p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(1
 + (b*x^2)/a)^p))/6
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {673}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\left(a+bx^2\right)^p(ad+bc(2p+3)x)\,dx$$

$$\Big\downarrow \text{673}$$

$$\frac{(2p+3)\left(a+bx^2\right)^{p+1}\left(ad^2+bc^2(2p+3)+2bcd(p+1)x\right)}{2b\left(2p^2+5p+3\right)}$$

input
```
Int[(c + d*x)*(a*d + b*c*(3 + 2*p)*x)*(a + b*x^2)^p,x]
```

output
```
((3 + 2*p)*(a*d^2 + b*c^2*(3 + 2*p) + 2*b*c*d*(1 + p)*x)*(a + b*x^2)^(1 +
p))/(2*b*(3 + 5*p + 2*p^2))
```

Defintions of rubi rules used

rule 673
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] /; FreeQ[{a, c, d, e, f, g, p}, x] && EqQ
[a*e*g - c*d*f*(2*p + 3), 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

method	result
gosper	$\frac{(b\,x^2+a)^{p+1}(2bcdxp+2b\,c^2p+2bcdx+a\,d^2+3b\,c^2)}{2b(p+1)}$
orering	$\frac{(2bcdxp+2b\,c^2p+2bcdx+a\,d^2+3b\,c^2)(b\,x^2+a)(ad+bc(3+2p)x)(b\,x^2+a)^p}{2b(p+1)(2cbxp+3cbx+ad)}$
risch	$\frac{(2b^2cdp\,x^3+2b^2c^2p\,x^2+2b^2cd\,x^3+2abcdpx+ab\,d^2x^2+3b^2c^2x^2+2ab\,c^2p+2abcdx+a^2d^2+3ab\,c^2)(b\,x^2+a)^p}{2b(p+1)}$
norman	$adxc\,e^{p\ln(b\,x^2+a)}+bcd\,x^3e^{p\ln(b\,x^2+a)}+\frac{(2b\,c^2p+a\,d^2+3b\,c^2)x^2e^{p\ln(b\,x^2+a)}}{2+2p}+\frac{a(2b\,c^2p+a\,d^2+3b\,c^2)e^{p\ln(b\,x^2}}{2b(p+1)}$
parallelrisch	$\frac{2x^3(b\,x^2+a)^pb^2cdp+2x^3(b\,x^2+a)^pb^2cd+2x^2(b\,x^2+a)^pb^2c^2p+x^2(b\,x^2+a)^pab\,d^2+3x^2(b\,x^2+a)^pb^2c^2+2x(b\,x^2+a)^pabcdp+2}{2b(p+1)}$

input
```
int((d*x+c)*(a*d+b*c*(3+2*p)*x)*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2/b/(p+1)*(b*x^2+a)^(p+1)*(2*b*c*d*p*x+2*b*c^2*p+2*b*c*d*x+a*d^2+3*b*c^2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(46) = 92.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.15

$$\int (c+dx)(ad+bc(3+2p)x)\left(a+bx^2\right)^p\,dx$$
$$=\frac{(2\,abc^2p+3\,abc^2+a^2d^2+2\,(b^2cdp+b^2cd)x^3+(2\,b^2c^2p+3\,b^2c^2+abd^2)x^2+2\,(abcdp+abcd)x)(bx^2+}{2\,(bp+b)}$$

input
```
integrate((d*x+c)*(a*d+b*c*(3+2*p)*x)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*c^2*p + 3*a*b*c^2 + a^2*d^2 + 2*(b^2*c*d*p + b^2*c*d)*x^3 + (2*
b^2*c^2*p + 3*b^2*c^2 + a*b*d^2)*x^2 + 2*(a*b*c*d*p + a*b*c*d)*x)*(b*x^2 +
 a)^p/(b*p + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(42) = 84$.

Time = 4.04 (sec) , antiderivative size = 388, normalized size of antiderivative = 8.08

$$\int (c+dx)(ad+bc(3+2p)x)\left(a+bx^2\right)^p\,dx$$
$$= \begin{cases} d\left(cx+\frac{dx^2}{2}\right) \\ aa^pd\left(cx+\frac{dx^2}{2}\right) \\ \frac{ad^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b}+\frac{ad^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b}+\frac{c^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2}+\frac{c^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2}+cdx \\ \frac{a^2d^2(a+bx^2)^p}{2bp+2b}+\frac{2abc^2p(a+bx^2)^p}{2bp+2b}+\frac{3abc^2(a+bx^2)^p}{2bp+2b}+\frac{2abcdpx(a+bx^2)^p}{2bp+2b}+\frac{2abcdx(a+bx^2)^p}{2bp+2b}+\frac{abd^2x^2(a+bx^2)^p}{2bp+2b}+\frac{2b^2c^2px^2(a+\cdots}{2bp+2b} \end{cases}$$

input
```
integrate((d*x+c)*(a*d+b*c*(3+2*p)*x)*(b*x**2+a)**p,x)
```

output
```
Piecewise((d*(c*x + d*x**2/2), Eq(b, 0) & Eq(p, -1)), (a*a**p*d*(c*x + d*x
**2/2), Eq(b, 0)), (a*d**2*log(x - sqrt(-a/b))/(2*b) + a*d**2*log(x + sqrt
(-a/b))/(2*b) + c**2*log(x - sqrt(-a/b))/2 + c**2*log(x + sqrt(-a/b))/2 +
c*d*x, Eq(p, -1)), (a**2*d**2*(a + b*x**2)**p/(2*b*p + 2*b) + 2*a*b*c**2*p
*(a + b*x**2)**p/(2*b*p + 2*b) + 3*a*b*c**2*(a + b*x**2)**p/(2*b*p + 2*b)
+ 2*a*b*c*d*p*x*(a + b*x**2)**p/(2*b*p + 2*b) + 2*a*b*c*d*x*(a + b*x**2)**
p/(2*b*p + 2*b) + a*b*d**2*x**2*(a + b*x**2)**p/(2*b*p + 2*b) + 2*b**2*c**
2*p*x**2*(a + b*x**2)**p/(2*b*p + 2*b) + 3*b**2*c**2*x**2*(a + b*x**2)**p/
(2*b*p + 2*b) + 2*b**2*c*d*p*x**3*(a + b*x**2)**p/(2*b*p + 2*b) + 2*b**2*c
*d*x**3*(a + b*x**2)**p/(2*b*p + 2*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.75

$$\int (c+dx)(ad+bc(3+2p)x)\left(a+bx^2\right)^p\,dx$$
$$=\frac{(2\,b^2cd(p+1)x^3+2\,abcd(p+1)x+abc^2(2\,p+3)+a^2d^2+(b^2c^2(2\,p+3)+abd^2)x^2)(bx^2+a)^p}{2\,b(p+1)}$$

input
```
integrate((d*x+c)*(a*d+b*c*(3+2*p)*x)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
1/2*(2*b^2*c*d*(p + 1)*x^3 + 2*a*b*c*d*(p + 1)*x + a*b*c^2*(2*p + 3) + a^2
*d^2 + (b^2*c^2*(2*p + 3) + a*b*d^2)*x^2)*(b*x^2 + a)^p/(b*(p + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(46) = 92.

Time = 0.17 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.96

$$\int (c+dx)(ad+bc(3+2p)x)\left(a+bx^2\right)^p\,dx$$
$$=\frac{2\,(bx^2+a)^pb^2cdpx^3+2\,(bx^2+a)^pb^2c^2px^2+2\,(bx^2+a)^pb^2cdx^3+2\,(bx^2+a)^pabcdpx+3\,(bx^2+a)^pb^2c^2}{2\,(bp+}$$

input
```
integrate((d*x+c)*(a*d+b*c*(3+2*p)*x)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
1/2*(2*(b*x^2 + a)^p*b^2*c*d*p*x^3 + 2*(b*x^2 + a)^p*b^2*c^2*p*x^2 + 2*(b*
x^2 + a)^p*b^2*c*d*x^3 + 2*(b*x^2 + a)^p*a*b*c*d*p*x + 3*(b*x^2 + a)^p*b^2
*c^2*x^2 + (b*x^2 + a)^p*a*b*d^2*x^2 + 2*(b*x^2 + a)^p*a*b*c^2*p + 2*(b*x^
2 + a)^p*a*b*c*d*x + 3*(b*x^2 + a)^p*a*b*c^2 + (b*x^2 + a)^p*a^2*d^2)/(b*p
 + b)
```

Mupad [B] (verification not implemented)

Time = 7.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.88

$$\int (c+dx)(ad+bc(3+2p)x)\left(a+bx^2\right)^p\,dx = \left(b\,x^2+a\right)^p\left(\frac{a\,\left(a\,d^2+3\,b\,c^2+2\,b\,c^2\,p\right)}{2\,b\,\left(p+1\right)}+b\,c\,d\,x^3+\frac{x^2\,\left(3\,b^2\,c^2+2\,b^2\,c^2\,p+a\,b\,d^2\right)}{2\,b\,\left(p+1\right)}+a\,c\,d\,x\right)$$

input
```
int((a*d + b*c*x*(2*p + 3))*(a + b*x^2)^p*(c + d*x),x)
```

output
```
(a + b*x^2)^p*((a*(a*d^2 + 3*b*c^2 + 2*b*c^2*p))/(2*b*(p + 1)) + b*c*d*x^3
 + (x^2*(3*b^2*c^2 + 2*b^2*c^2*p + a*b*d^2))/(2*b*(p + 1)) + a*c*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.29

$$\int (c+dx)(ad+bc(3+2p)x)\left(a+bx^2\right)^p\,dx = \frac{\left(b\,x^2+a\right)^p\left(2b^2cdp\,x^3+2b^2c^2p\,x^2+2b^2cd\,x^3+2abcdpx+ab\,d^2x^2+3b^2c^2x^2+2ab\,c^2p+2abcdx+a^2d^2\right.}{2b\left(p+1\right)}$$

input
```
int((d*x+c)*(a*d+b*c*(3+2*p)*x)*(b*x^2+a)^p,x)
```

output
```
((a + b*x**2)**p*(a**2*d**2 + 2*a*b*c**2*p + 3*a*b*c**2 + 2*a*b*c*d*p*x +
2*a*b*c*d*x + a*b*d**2*x**2 + 2*b**2*c**2*p*x**2 + 3*b**2*c**2*x**2 + 2*b*
*2*c*d*p*x**3 + 2*b**2*c*d*x**3))/(2*b*(p + 1))
```

3.25 $\int (ad+9bcx)(c+dx)\left(a+bx^2\right)^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int (ad+9bcx)(c+dx)\left(a+bx^2\right)^3 \, dx = \frac{(9bc^2+ad^2+8bcdx)\left(a+bx^2\right)^4}{8b}$$

output
```
1/8*(8*b*c*d*x+a*d^2+9*b*c^2)*(b*x^2+a)^4/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 117 vs. 2(34) = 68.

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.44

$$\int (ad+9bcx)(c+dx)\left(a+bx^2\right)^3 \, dx = \frac{1}{8}x\left(4a^4d(2c+dx)+b^4cx^7(9c+8dx)+ab^3x^5\left(36c^2+32cdx+d^2x^2\right)+2a^2b^2x^3\left(27c^2+24cdx+2d^2x^2\right)+2a^3bx\left(18c^2+16cdx+3d^2x^2\right)\right)$$

input
```
Integrate[(a*d + 9*b*c*x)*(c + d*x)*(a + b*x^2)^3,x]
```

output
```
(x*(4*a^4*d*(2*c + d*x) + b^4*c*x^7*(9*c + 8*d*x) + a*b^3*x^5*(36*c^2 + 32
*c*d*x + d^2*x^2) + 2*a^2*b^2*x^3*(27*c^2 + 24*c*d*x + 2*d^2*x^2) + 2*a^3*
b*x*(18*c^2 + 16*c*d*x + 3*d^2*x^2)))/8
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 142 vs. $2(34) = 68$.

Time = 0.39 (sec) , antiderivative size = 142, normalized size of antiderivative = 4.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{652, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^3 (c+dx)(ad+9bcx)\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(a^4cd+a^3x(ad^2+9bc^2)+12a^3bcdx^2+30a^2b^2cdx^4+3a^2bx^3(ad^2+9bc^2)+b^3x^7(ad^2+9bc^2)+28ab^3cdx^6+\right.$$

$$\Big\downarrow \text{2009}$$

$$a^4cdx+\frac{1}{2}a^3x^2(ad^2+9bc^2)+4a^3bcdx^3+6a^2b^2cdx^5+\frac{3}{4}a^2bx^4(ad^2+9bc^2)+\frac{1}{8}b^3x^8(ad^2+9bc^2)+4ab^3cdx^7+\frac{1}{2}ab^2x^6(ad^2+9bc^2)+b^4cdx^9$$

input
```
Int[(a*d + 9*b*c*x)*(c + d*x)*(a + b*x^2)^3,x]
```

output
```
a^4*c*d*x + (a^3*(9*b*c^2 + a*d^2)*x^2)/2 + 4*a^3*b*c*d*x^3 + (3*a^2*b*(9*
b*c^2 + a*d^2)*x^4)/4 + 6*a^2*b^2*c*d*x^5 + (a*b^2*(9*b*c^2 + a*d^2)*x^6)/
2 + 4*a*b^3*c*d*x^7 + (b^3*(9*b*c^2 + a*d^2)*x^8)/8 + b^4*c*d*x^9
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(32) = 64$.

Time = 0.52 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.97

method	result
default	$b^4cd\,x^9+\frac{(a\,d^2+9b\,c^2)b^3x^8}{8}+4acd\,b^3x^7+\frac{(a\,d^2+9b\,c^2)a\,b^2x^6}{2}+6a^2cd\,b^2x^5+\frac{3(a\,d^2+9b\,c^2)a^2b\,x^4}{4}+4a^3$
norman	$b^4cd\,x^9+\left(\frac{1}{8}a\,b^3d^2+\frac{9}{8}b^4c^2\right)x^8+4acd\,b^3x^7+\left(\frac{1}{2}d^2a^2b^2+\frac{9}{2}a\,b^3c^2\right)x^6+6a^2cd\,b^2x^5+(\frac{3}{4}a^3d^2$
orering	$\frac{x(8d\,b^4c\,x^8+a\,b^3d^2x^7+9b^4c^2x^7+32acd\,b^3x^6+4a^2b^2d^2x^5+36a\,b^3c^2x^5+48a^2b^2cd\,x^4+6a^3b\,d^2x^3+54a^2b^2c^2x^3+32a^3dcb\,x^2+4}{8}$
gosper	$b^4cd\,x^9+\frac{1}{8}x^8a\,b^3d^2+\frac{9}{8}x^8b^4c^2+4acd\,b^3x^7+\frac{1}{2}x^6d^2a^2b^2+\frac{9}{2}x^6a\,b^3c^2+6a^2cd\,b^2x^5+\frac{3}{4}x^4a^3d$
risch	$b^4cd\,x^9+\frac{1}{8}x^8a\,b^3d^2+\frac{9}{8}x^8b^4c^2+4acd\,b^3x^7+\frac{1}{2}x^6d^2a^2b^2+\frac{9}{2}x^6a\,b^3c^2+6a^2cd\,b^2x^5+\frac{3}{4}x^4a^3d$
parallelrisch	$b^4cd\,x^9+\frac{1}{8}x^8a\,b^3d^2+\frac{9}{8}x^8b^4c^2+4acd\,b^3x^7+\frac{1}{2}x^6d^2a^2b^2+\frac{9}{2}x^6a\,b^3c^2+6a^2cd\,b^2x^5+\frac{3}{4}x^4a^3d$

input
```
int((9*b*c*x+a*d)*(d*x+c)*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
b^4*c*d*x^9+1/8*(a*d^2+9*b*c^2)*b^3*x^8+4*a*c*d*b^3*x^7+1/2*(a*d^2+9*b*c^2
)*a*b^2*x^6+6*a^2*c*d*b^2*x^5+3/4*(a*d^2+9*b*c^2)*a^2*b*x^4+4*a^3*d*c*b*x^
3+1/2*(a*d^2+9*b*c^2)*a^3*x^2+a^4*d*c*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(32) = 64$.

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 4.29

$$\int (ad+9bcx)(c+dx)\left(a+bx^2\right)^3\,dx = b^4cdx^9+4\,ab^3cdx^7+6\,a^2b^2cdx^5+4\,a^3bcdx^3+\frac{1}{8}\left(9\,b^4c^2+ab^3d^2\right)x^8+a^4cdx+\frac{1}{2}\left(9\,ab^3c^2+a^2b^2d^2\right)x^6+\frac{3}{4}\left(9\,a^2b^2c^2+a^3bd^2\right)x^4+\frac{1}{2}\left(9\,a^3bc^2+a^4d^2\right)x^2$$

input
```
integrate((9*b*c*x+a*d)*(d*x+c)*(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
b^4*c*d*x^9 + 4*a*b^3*c*d*x^7 + 6*a^2*b^2*c*d*x^5 + 4*a^3*b*c*d*x^3 + 1/8*
(9*b^4*c^2 + a*b^3*d^2)*x^8 + a^4*c*d*x + 1/2*(9*a*b^3*c^2 + a^2*b^2*d^2)*
x^6 + 3/4*(9*a^2*b^2*c^2 + a^3*b*d^2)*x^4 + 1/2*(9*a^3*b*c^2 + a^4*d^2)*x^
2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(31) = 62$.

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 4.79

$$\int (ad+9bcx)(c+dx)\left(a+bx^2\right)^3\,dx = a^4cdx+4a^3bcdx^3+6a^2b^2cdx^5+4ab^3cdx^7+b^4cdx^9+x^8\left(\frac{ab^3d^2}{8}+\frac{9b^4c^2}{8}\right)+x^6\left(\frac{a^2b^2d^2}{2}+\frac{9ab^3c^2}{2}\right)+x^4\cdot\left(\frac{3a^3bd^2}{4}+\frac{27a^2b^2c^2}{4}\right)+x^2\left(\frac{a^4d^2}{2}+\frac{9a^3bc^2}{2}\right)$$

input
```
integrate((9*b*c*x+a*d)*(d*x+c)*(b*x**2+a)**3,x)
```

output
```
a**4*c*d*x + 4*a**3*b*c*d*x**3 + 6*a**2*b**2*c*d*x**5 + 4*a*b**3*c*d*x**7
+ b**4*c*d*x**9 + x**8*(a*b**3*d**2/8 + 9*b**4*c**2/8) + x**6*(a**2*b**2*d
**2/2 + 9*a*b**3*c**2/2) + x**4*(3*a**3*b*d**2/4 + 27*a**2*b**2*c**2/4) +
x**2*(a**4*d**2/2 + 9*a**3*b*c**2/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(32) = 64$.

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 4.29

$$\int (ad+9bcx)(c+dx)\left(a+bx^2\right)^3\,dx = b^4cdx^9 + 4\,ab^3cdx^7 + 6\,a^2b^2cdx^5 + 4\,a^3bcdx^3 + \frac{1}{8}\left(9\,b^4c^2 + ab^3d^2\right)x^8 + a^4cdx + \frac{1}{2}\left(9\,ab^3c^2 + a^2b^2d^2\right)x^6 + \frac{3}{4}\left(9\,a^2b^2c^2 + a^3bd^2\right)x^4 + \frac{1}{2}\left(9\,a^3bc^2 + a^4d^2\right)x^2$$

input
```
integrate((9*b*c*x+a*d)*(d*x+c)*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
b^4*c*d*x^9 + 4*a*b^3*c*d*x^7 + 6*a^2*b^2*c*d*x^5 + 4*a^3*b*c*d*x^3 + 1/8*
(9*b^4*c^2 + a*b^3*d^2)*x^8 + a^4*c*d*x + 1/2*(9*a*b^3*c^2 + a^2*b^2*d^2)*
x^6 + 3/4*(9*a^2*b^2*c^2 + a^3*b*d^2)*x^4 + 1/2*(9*a^3*b*c^2 + a^4*d^2)*x^
2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(32) = 64$.

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 4.41

$$\int (ad+9bcx)(c+dx)\left(a+bx^2\right)^3\,dx = b^4cdx^9 + \frac{9}{8}\,b^4c^2x^8 + \frac{1}{8}\,ab^3d^2x^8 + 4\,ab^3cdx^7 + \frac{9}{2}\,ab^3c^2x^6 + \frac{1}{2}\,a^2b^2d^2x^6 + 6\,a^2b^2cdx^5 + \frac{27}{4}\,a^2b^2c^2x^4 + \frac{3}{4}\,a^3bd^2x^4 + 4\,a^3bcdx^3 + \frac{9}{2}\,a^3bc^2x^2 + \frac{1}{2}\,a^4d^2x^2 + a^4cdx$$

input
```
integrate((9*b*c*x+a*d)*(d*x+c)*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
b^4*c*d*x^9 + 9/8*b^4*c^2*x^8 + 1/8*a*b^3*d^2*x^8 + 4*a*b^3*c*d*x^7 + 9/2*
a*b^3*c^2*x^6 + 1/2*a^2*b^2*d^2*x^6 + 6*a^2*b^2*c*d*x^5 + 27/4*a^2*b^2*c^2
*x^4 + 3/4*a^3*b*d^2*x^4 + 4*a^3*b*c*d*x^3 + 9/2*a^3*b*c^2*x^2 + 1/2*a^4*d
^2*x^2 + a^4*c*d*x
```

Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 138, normalized size of antiderivative = 4.06

$$\int (ad+9bcx)(c+dx)\left(a+bx^2\right)^3\,dx = x^2\left(\frac{a^4\,d^2}{2}+\frac{9\,b\,a^3\,c^2}{2}\right)+x^8\left(\frac{9\,b^4\,c^2}{8}+\frac{a\,b^3\,d^2}{8}\right) + a^4\,c\,d\,x+b^4\,c\,d\,x^9+\frac{3\,a^2\,b\,x^4\,(9\,b\,c^2+a\,d^2)}{4} + \frac{a\,b^2\,x^6\,(9\,b\,c^2+a\,d^2)}{2}+4\,a^3\,b\,c\,d\,x^3 + 4\,a\,b^3\,c\,d\,x^7+6\,a^2\,b^2\,c\,d\,x^5$$

input
```
int((a*d + 9*b*c*x)*(a + b*x^2)^3*(c + d*x),x)
```

output
```
x^2*((a^4*d^2)/2 + (9*a^3*b*c^2)/2) + x^8*((9*b^4*c^2)/8 + (a*b^3*d^2)/8)
+ a^4*c*d*x + b^4*c*d*x^9 + (3*a^2*b*x^4*(a*d^2 + 9*b*c^2))/4 + (a*b^2*x^6
*(a*d^2 + 9*b*c^2))/2 + 4*a^3*b*c*d*x^3 + 4*a*b^3*c*d*x^7 + 6*a^2*b^2*c*d*
x^5
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 4.38

$$\int (ad+9bcx)(c+dx)\left(a+bx^2\right)^3\,dx = \frac{x(8b^4cd\,x^8+a\,b^3d^2x^7+9b^4c^2x^7+32a\,b^3cd\,x^6+4a^2b^2d^2x^5+36a\,b^3c^2x^5+48a^2b^2cd\,x^4+6a^3b\,d^2x^3+54a}{8}$$

input
```
int((9*b*c*x+a*d)*(d*x+c)*(b*x^2+a)^3,x)
```

output
```
(x*(8*a**4*c*d + 4*a**4*d**2*x + 36*a**3*b*c**2*x + 32*a**3*b*c*d*x**2 + 6
*a**3*b*d**2*x**3 + 54*a**2*b**2*c**2*x**3 + 48*a**2*b**2*c*d*x**4 + 4*a**
2*b**2*d**2*x**5 + 36*a*b**3*c**2*x**5 + 32*a*b**3*c*d*x**6 + a*b**3*d**2*
x**7 + 9*b**4*c**2*x**7 + 8*b**4*c*d*x**8))/8
```

3.26 $\int (ad+7bcx)(c+dx)\left(a+bx^2\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int (ad+7bcx)(c+dx)\left(a+bx^2\right)^2\,dx=\frac{(7bc^2+ad^2+6bcdx)\left(a+bx^2\right)^3}{6b}$$

output
```
1/6*(6*b*c*d*x+a*d^2+7*b*c^2)*(b*x^2+a)^3/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 86 vs. 2(34) = 68.

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.53

$$\int (ad+7bcx)(c+dx)\left(a+bx^2\right)^2\,dx=\frac{1}{6}x\left(3a^3d(2c+dx)+b^3cx^5(7c+6dx)+3a^2bx\left(7c^2+6cdx+d^2x^2\right)+ab^2x^3\left(21c^2+18cdx+d^2x^2\right)\right)$$

input
```
Integrate[(a*d + 7*b*c*x)*(c + d*x)*(a + b*x^2)^2,x]
```

output
```
(x*(3*a^3*d*(2*c + d*x) + b^3*c*x^5*(7*c + 6*d*x) + 3*a^2*b*x*(7*c^2 + 6*c
*d*x + d^2*x^2) + a*b^2*x^3*(21*c^2 + 18*c*d*x + d^2*x^2)))/6
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 104 vs. $2(34) = 68$.

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^2 (c+dx)(ad+7bcx)\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(a^3cd+a^2x\left(ad^2+7bc^2\right)+9a^2bcdx^2+b^2x^5\left(ad^2+7bc^2\right)+15ab^2cdx^4+2abx^3\left(ad^2+7bc^2\right)+7b^3cdx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^3cdx+\frac{1}{2}a^2x^2\left(ad^2+7bc^2\right)+3a^2bcdx^3+\frac{1}{6}b^2x^6\left(ad^2+7bc^2\right)+3ab^2cdx^5+\frac{1}{2}abx^4\left(ad^2+7bc^2\right)+b^3cdx^7$$

input
```
Int[(a*d + 7*b*c*x)*(c + d*x)*(a + b*x^2)^2,x]
```

output
```
a^3*c*d*x + (a^2*(7*b*c^2 + a*d^2)*x^2)/2 + 3*a^2*b*c*d*x^3 + (a*b*(7*b*c^
2 + a*d^2)*x^4)/2 + 3*a*b^2*c*d*x^5 + (b^2*(7*b*c^2 + a*d^2)*x^6)/6 + b^3*
c*d*x^7
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(32) = 64$.

Time = 0.52 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.91

method	result
default	$d\,x^7 c\,b^3+\frac{(a\,d^2+7b\,c^2)b^2x^6}{6}+3a\,b^2cd\,x^5+\frac{(a\,d^2+7b\,c^2)ab\,x^4}{2}+3a^2bcd\,x^3+\frac{(a\,d^2+7b\,c^2)a^2x^2}{2}+a^3cdx$
norman	$d\,x^7 c\,b^3+\left(\frac{1}{6}a\,b^2d^2+\frac{7}{6}b^3c^2\right)x^6+3a\,b^2cd\,x^5+\left(\frac{1}{2}a^2b\,d^2+\frac{7}{2}a\,c^2b^2\right)x^4+3a^2bcd\,x^3+\left(\frac{1}{2}a^3d^2\right.$
orering	$\frac{x(6b^3cd\,x^6+a\,b^2d^2x^5+7b^3c^2x^5+18cda\,b^2x^4+3d^2x^3a^2b+21a\,b^2c^2x^3+18x^2a^2bcd+3a^3d^2x+21a^2b\,c^2x+6a^3dc)}{6}$
gosper	$d\,x^7 c\,b^3+\frac{1}{6}a\,b^2d^2x^6+\frac{7}{6}b^3c^2x^6+3a\,b^2cd\,x^5+\frac{1}{2}a^2b\,d^2x^4+\frac{7}{2}a\,b^2c^2x^4+3a^2bcd\,x^3+\frac{1}{2}a^3d^2x^2$
risch	$d\,x^7 c\,b^3+\frac{1}{6}a\,b^2d^2x^6+\frac{7}{6}b^3c^2x^6+3a\,b^2cd\,x^5+\frac{1}{2}a^2b\,d^2x^4+\frac{7}{2}a\,b^2c^2x^4+3a^2bcd\,x^3+\frac{1}{2}a^3d^2x^2$
parallelrisch	$d\,x^7 c\,b^3+\frac{1}{6}a\,b^2d^2x^6+\frac{7}{6}b^3c^2x^6+3a\,b^2cd\,x^5+\frac{1}{2}a^2b\,d^2x^4+\frac{7}{2}a\,b^2c^2x^4+3a^2bcd\,x^3+\frac{1}{2}a^3d^2x^2$

input
```
int((7*b*c*x+a*d)*(d*x+c)*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d*x^7*c*b^3+1/6*(a*d^2+7*b*c^2)*b^2*x^6+3*a*b^2*c*d*x^5+1/2*(a*d^2+7*b*c^2
)*a*b*x^4+3*a^2*b*c*d*x^3+1/2*(a*d^2+7*b*c^2)*a^2*x^2+a^3*c*d*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(32) = 64$.

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.12

$$\int (ad+7bcx)(c+dx)\left(a+bx^2\right)^2\,dx = b^3cdx^7+3\,ab^2cdx^5+3\,a^2bcdx^3+\frac{1}{6}\left(7\,b^3c^2+ab^2d^2\right)x^6+a^3cdx+\frac{1}{2}\left(7\,ab^2c^2+a^2bd^2\right)x^4+\frac{1}{2}\left(7\,a^2bc^2+a^3d^2\right)x^2$$

input
```
integrate((7*b*c*x+a*d)*(d*x+c)*(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
b^3*c*d*x^7 + 3*a*b^2*c*d*x^5 + 3*a^2*b*c*d*x^3 + 1/6*(7*b^3*c^2 + a*b^2*d
^2)*x^6 + a^3*c*d*x + 1/2*(7*a*b^2*c^2 + a^2*b*d^2)*x^4 + 1/2*(7*a^2*b*c^2
 + a^3*d^2)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(31) = 62.

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.44

$$\int (ad+7bcx)(c+dx)\left(a+bx^2\right)^2\,dx = a^3cdx+3a^2bcdx^3+3ab^2cdx^5+b^3cdx^7+x^6\left(\frac{ab^2d^2}{6}+\frac{7b^3c^2}{6}\right)+x^4\left(\frac{a^2bd^2}{2}+\frac{7ab^2c^2}{2}\right)+x^2\left(\frac{a^3d^2}{2}+\frac{7a^2bc^2}{2}\right)$$

input
```
integrate((7*b*c*x+a*d)*(d*x+c)*(b*x**2+a)**2,x)
```

output
```
a**3*c*d*x + 3*a**2*b*c*d*x**3 + 3*a*b**2*c*d*x**5 + b**3*c*d*x**7 + x**6*
(a*b**2*d**2/6 + 7*b**3*c**2/6) + x**4*(a**2*b*d**2/2 + 7*a*b**2*c**2/2) +
 x**2*(a**3*d**2/2 + 7*a**2*b*c**2/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(32) = 64.

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.12

$$\int (ad+7bcx)(c+dx)\left(a+bx^2\right)^2\,dx = b^3cdx^7+3\,ab^2cdx^5+3\,a^2bcdx^3+\frac{1}{6}\left(7\,b^3c^2+ab^2d^2\right)x^6+a^3cdx+\frac{1}{2}\left(7\,ab^2c^2+a^2bd^2\right)x^4+\frac{1}{2}\left(7\,a^2bc^2+a^3d^2\right)x^2$$

input
```
integrate((7*b*c*x+a*d)*(d*x+c)*(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
b^3*c*d*x^7 + 3*a*b^2*c*d*x^5 + 3*a^2*b*c*d*x^3 + 1/6*(7*b^3*c^2 + a*b^2*d
^2)*x^6 + a^3*c*d*x + 1/2*(7*a*b^2*c^2 + a^2*b*d^2)*x^4 + 1/2*(7*a^2*b*c^2
 + a^3*d^2)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(32) = 64$.

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.21

$$\int (ad+7bcx)(c+dx)\left(a+bx^2\right)^2\,dx = b^3cdx^7 + \frac{7}{6}\,b^3c^2x^6 + \frac{1}{6}\,ab^2d^2x^6 + 3\,ab^2cdx^5 + \frac{7}{2}\,ab^2c^2x^4 + \frac{1}{2}\,a^2bd^2x^4 + 3\,a^2bcdx^3 + \frac{7}{2}\,a^2bc^2x^2 + \frac{1}{2}\,a^3d^2x^2 + a^3cdx$$

input
```
integrate((7*b*c*x+a*d)*(d*x+c)*(b*x^2+a)^2,x, algorithm="giac")
```

output
```
b^3*c*d*x^7 + 7/6*b^3*c^2*x^6 + 1/6*a*b^2*d^2*x^6 + 3*a*b^2*c*d*x^5 + 7/2*
a*b^2*c^2*x^4 + 1/2*a^2*b*d^2*x^4 + 3*a^2*b*c*d*x^3 + 7/2*a^2*b*c^2*x^2 +
1/2*a^3*d^2*x^2 + a^3*c*d*x
```

Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.00

$$\int (ad+7bcx)(c+dx)\left(a+bx^2\right)^2\,dx = x^2\left(\frac{a^3\,d^2}{2}+\frac{7\,b\,a^2\,c^2}{2}\right)+x^6\left(\frac{7\,b^3\,c^2}{6}+\frac{a\,b^2\,d^2}{6}\right) + a^3\,c\,d\,x + \frac{a\,b\,x^4\,(7\,b\,c^2+a\,d^2)}{2} + b^3\,c\,d\,x^7 + 3\,a^2\,b\,c\,d\,x^3 + 3\,a\,b^2\,c\,d\,x^5$$

input
```
int((a*d + 7*b*c*x)*(a + b*x^2)^2*(c + d*x),x)
```

output
```
x^2*((a^3*d^2)/2 + (7*a^2*b*c^2)/2) + x^6*((7*b^3*c^2)/6 + (a*b^2*d^2)/6)
+ a^3*c*d*x + (a*b*x^4*(a*d^2 + 7*b*c^2))/2 + b^3*c*d*x^7 + 3*a^2*b*c*d*x^
3 + 3*a*b^2*c*d*x^5
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.18

$$\int (ad + 7bcx)(c + dx)\left(a + bx^2\right)^2 \, dx$$
$$= \frac{x(6b^3cd\,x^6 + a\,b^2d^2x^5 + 7b^3c^2x^5 + 18a\,b^2cd\,x^4 + 3a^2b\,d^2x^3 + 21a\,b^2c^2x^3 + 18a^2bcd\,x^2 + 3a^3d^2x + 21a^2b\,c}{6}$$

input
```
int((7*b*c*x+a*d)*(d*x+c)*(b*x^2+a)^2,x)
```

output
```
(x*(6*a**3*c*d + 3*a**3*d**2*x + 21*a**2*b*c**2*x + 18*a**2*b*c*d*x**2 + 3
*a**2*b*d**2*x**3 + 21*a*b**2*c**2*x**3 + 18*a*b**2*c*d*x**4 + a*b**2*d**2
*x**5 + 7*b**3*c**2*x**5 + 6*b**3*c*d*x**6))/6
```

3.27 $\int (ad+5bcx)(c+dx)\left(a+bx^2\right) dx$

Optimal result

Integrand size = 22, antiderivative size = 34

$$\int (ad+5bcx)(c+dx)\left(a+bx^2\right) dx = \frac{(5bc^2+ad^2+4bcdx)\left(a+bx^2\right)^2}{4b}$$

output
```
1/4*(4*b*c*d*x+a*d^2+5*b*c^2)*(b*x^2+a)^2/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int (ad+5bcx)(c+dx)\left(a+bx^2\right) dx = \frac{1}{4}x\left(2a^2d(2c+dx)+b^2cx^3(5c+4dx)+abx\left(10c^2+8cdx+d^2x^2\right)\right)$$

input
```
Integrate[(a*d + 5*b*c*x)*(c + d*x)*(a + b*x^2),x]
```

output
```
(x*(2*a^2*d*(2*c + d*x) + b^2*c*x^3*(5*c + 4*d*x) + a*b*x*(10*c^2 + 8*c*d*
x + d^2*x^2)))/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(c+dx)(ad+5bcx)\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(a^2cd+bx^3\left(ad^2+5bc^2\right)+ax\left(ad^2+5bc^2\right)+6abcdx^2+5b^2cdx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2cdx+\frac{1}{4}bx^4\left(ad^2+5bc^2\right)+\frac{1}{2}ax^2\left(ad^2+5bc^2\right)+2abcdx^3+b^2cdx^5$$

input
```
Int[(a*d + 5*b*c*x)*(c + d*x)*(a + b*x^2),x]
```

output
```
a^2*c*d*x + (a*(5*b*c^2 + a*d^2)*x^2)/2 + 2*a*b*c*d*x^3 + (b*(5*b*c^2 + a*
d^2)*x^4)/4 + b^2*c*d*x^5
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

method	result	size
default	$b^{2} c d \,x^{5}+\frac{\left(a \,d^{2}+5 b \,c^{2}\right) b \,x^{4}}{4}+2 a b c d \,x^{3}+\frac{\left(a \,d^{2}+5 b \,c^{2}\right) a \,x^{2}}{2}+a^{2} c d x$	63
norman	$b^{2} c d \,x^{5}+\left(\frac{1}{4} a b \,d^{2}+\frac{5}{4} b^{2} c^{2}\right) x^{4}+2 a b c d \,x^{3}+\left(\frac{1}{2} a^{2} d^{2}+\frac{5}{2} a b \,c^{2}\right) x^{2}+a^{2} c d x$	67
orering	$\frac{x \left(4 b^{2} c d \,x^{4}+d^{2} x^{3} a b +5 b^{2} c^{2} x^{3}+8 a b c d \,x^{2}+2 a^{2} d^{2} x +10 a b \,c^{2} x +4 a^{2} c d \right)}{4}$	68
gosper	$b^{2} c d \,x^{5}+\frac{1}{4} a b \,d^{2} x^{4}+\frac{5}{4} b^{2} c^{2} x^{4}+2 a b c d \,x^{3}+\frac{1}{2} a^{2} d^{2} x^{2}+\frac{5}{2} a b \,c^{2} x^{2}+a^{2} c d x$	69
risch	$b^{2} c d \,x^{5}+\frac{1}{4} a b \,d^{2} x^{4}+\frac{5}{4} b^{2} c^{2} x^{4}+2 a b c d \,x^{3}+\frac{1}{2} a^{2} d^{2} x^{2}+\frac{5}{2} a b \,c^{2} x^{2}+a^{2} c d x$	69
parallelrisch	$b^{2} c d \,x^{5}+\frac{1}{4} a b \,d^{2} x^{4}+\frac{5}{4} b^{2} c^{2} x^{4}+2 a b c d \,x^{3}+\frac{1}{2} a^{2} d^{2} x^{2}+\frac{5}{2} a b \,c^{2} x^{2}+a^{2} c d x$	69

input
```
int((5*b*c*x+a*d)*(d*x+c)*(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
b^2*c*d*x^5+1/4*(a*d^2+5*b*c^2)*b*x^4+2*a*b*c*d*x^3+1/2*(a*d^2+5*b*c^2)*a*
x^2+a^2*c*d*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int (ad+5bcx)(c+dx)\left(a+bx^2\right)\,dx = b^2cdx^5+2\,abcdx^3+a^2cdx + \frac{1}{4}\left(5\,b^2c^2+abd^2\right)x^4+\frac{1}{2}\left(5\,abc^2+a^2d^2\right)x^2$$

input
```
integrate((5*b*c*x+a*d)*(d*x+c)*(b*x^2+a),x, algorithm="fricas")
```

output
```
b^2*c*d*x^5 + 2*a*b*c*d*x^3 + a^2*c*d*x + 1/4*(5*b^2*c^2 + a*b*d^2)*x^4 +
1/2*(5*a*b*c^2 + a^2*d^2)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(31) = 62$.

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.15

$$\int (ad+5bcx)(c+dx)\left(a+bx^2\right)\,dx = a^2cdx + 2abcdx^3 + b^2cdx^5 + x^4\left(\frac{abd^2}{4} + \frac{5b^2c^2}{4}\right) + x^2\left(\frac{a^2d^2}{2} + \frac{5abc^2}{2}\right)$$

input
```
integrate((5*b*c*x+a*d)*(d*x+c)*(b*x**2+a),x)
```

output
```
a**2*c*d*x + 2*a*b*c*d*x**3 + b**2*c*d*x**5 + x**4*(a*b*d**2/4 + 5*b**2*c*
*2/4) + x**2*(a**2*d**2/2 + 5*a*b*c**2/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int (ad+5bcx)(c+dx)\left(a+bx^2\right)\,dx = b^2cdx^5 + 2\,abcdx^3 + a^2cdx + \frac{1}{4}\left(5\,b^2c^2 + abd^2\right)x^4 + \frac{1}{2}\left(5\,abc^2 + a^2d^2\right)x^2$$

input
```
integrate((5*b*c*x+a*d)*(d*x+c)*(b*x^2+a),x, algorithm="maxima")
```

output
```
b^2*c*d*x^5 + 2*a*b*c*d*x^3 + a^2*c*d*x + 1/4*(5*b^2*c^2 + a*b*d^2)*x^4 +
1/2*(5*a*b*c^2 + a^2*d^2)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(32) = 64$.

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int (ad + 5bcx)(c + dx)\left(a + bx^2\right)\,dx = b^2cdx^5 + \frac{5}{4}\,b^2c^2x^4 + \frac{1}{4}\,abd^2x^4 + 2\,abcdx^3 + \frac{5}{2}\,abc^2x^2 + \frac{1}{2}\,a^2d^2x^2 + a^2cdx$$

input
```
integrate((5*b*c*x+a*d)*(d*x+c)*(b*x^2+a),x, algorithm="giac")
```

output
```
b^2*c*d*x^5 + 5/4*b^2*c^2*x^4 + 1/4*a*b*d^2*x^4 + 2*a*b*c*d*x^3 + 5/2*a*b*
c^2*x^2 + 1/2*a^2*d^2*x^2 + a^2*c*d*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int (ad + 5bcx)(c + dx)\left(a + bx^2\right)\,dx = x^2\left(\frac{a^2\,d^2}{2} + \frac{5\,b\,a\,c^2}{2}\right) + x^4\left(\frac{5\,b^2\,c^2}{4} + \frac{a\,b\,d^2}{4}\right) + a^2\,c\,d\,x + b^2\,c\,d\,x^5 + 2\,a\,b\,c\,d\,x^3$$

input
```
int((a*d + 5*b*c*x)*(a + b*x^2)*(c + d*x),x)
```

output
```
x^2*((a^2*d^2)/2 + (5*a*b*c^2)/2) + x^4*((5*b^2*c^2)/4 + (a*b*d^2)/4) + a^
2*c*d*x + b^2*c*d*x^5 + 2*a*b*c*d*x^3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int (ad+5bcx)(c+dx)\left(a+bx^2\right)\,dx = \frac{x(4b^2cd\,x^4+ab\,d^2x^3+5b^2c^2x^3+8abcd\,x^2+2a^2d^2x+10ab\,c^2x+4a^2cd)}{4}$$

input
```
int((5*b*c*x+a*d)*(d*x+c)*(b*x^2+a),x)
```

output
```
(x*(4*a**2*c*d + 2*a**2*d**2*x + 10*a*b*c**2*x + 8*a*b*c*d*x**2 + a*b*d**2
*x**3 + 5*b**2*c**2*x**3 + 4*b**2*c*d*x**4))/4
```

3.28 $\int (ad+3bcx)(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int (ad+3bcx)(c+dx)\,dx=\frac{(3bc^2+ad^2+2bcdx)\,(a+bx^2)}{2b}$$

output
```
1/2*(2*b*c*d*x+a*d^2+3*b*c^2)*(b*x^2+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int (ad+3bcx)(c+dx)\,dx=\frac{1}{2}x(ad(2c+dx)+bcx(3c+2dx))$$

input
```
Integrate[(a*d + 3*b*c*x)*(c + d*x),x]
```

output
```
(x*(a*d*(2*c + d*x) + b*c*x*(3*c + 2*d*x)))/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(ad+3bcx)\,dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(x\left(ad^2+3bc^2\right)+acd+3bcdx^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2\left(ad^2+3bc^2\right)+acdx+bcdx^3$$

input
```
Int[(a*d + 3*b*c*x)*(c + d*x),x]
```

output
```
a*c*d*x + ((3*b*c^2 + a*d^2)*x^2)/2 + b*c*d*x^3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result	size
default	$bcd\,x^3+\frac{(a\,d^2+3b\,c^2)\,x^2}{2}+adxc$	31
norman	$bcd\,x^3+\left(\frac{a\,d^2}{2}+\frac{3b\,c^2}{2}\right)x^2+adxc$	31
orering	$\frac{x(2bcd\,x^2+a\,d^2x+3c^2bx+2acd)}{2}$	31
gosper	$bcd\,x^3+\frac{1}{2}a\,d^2x^2+\frac{3}{2}b\,c^2x^2+adxc$	32
risch	$bcd\,x^3+\frac{1}{2}a\,d^2x^2+\frac{3}{2}b\,c^2x^2+adxc$	32
parallelrisch	$bcd\,x^3+\frac{1}{2}a\,d^2x^2+\frac{3}{2}b\,c^2x^2+adxc$	32

input
```
int((3*b*c*x+a*d)*(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
b*c*d*x^3+1/2*(a*d^2+3*b*c^2)*x^2+a*d*x*c
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int(ad+3bcx)(c+dx)\,dx=bcdx^3+acdx+\frac{1}{2}\left(3\,bc^2+ad^2\right)x^2$$

input
```
integrate((3*b*c*x+a*d)*(d*x+c),x, algorithm="fricas")
```

output
```
b*c*d*x^3 + a*c*d*x + 1/2*(3*b*c^2 + a*d^2)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (ad + 3bcx)(c + dx)\, dx = acdx + bcdx^3 + x^2 \left(\frac{ad^2}{2} + \frac{3bc^2}{2}\right)$$

input
```
integrate((3*b*c*x+a*d)*(d*x+c),x)
```

output
```
a*c*d*x + b*c*d*x**3 + x**2*(a*d**2/2 + 3*b*c**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int (ad + 3bcx)(c + dx)\, dx = bcdx^3 + acdx + \frac{1}{2}\left(3\, bc^2 + ad^2\right)x^2$$

input
```
integrate((3*b*c*x+a*d)*(d*x+c),x, algorithm="maxima")
```

output
```
b*c*d*x^3 + a*c*d*x + 1/2*(3*b*c^2 + a*d^2)*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int (ad + 3bcx)(c + dx)\, dx = bcdx^3 + \frac{3}{2}\, bc^2x^2 + \frac{1}{2}\, ad^2x^2 + acdx$$

input
```
integrate((3*b*c*x+a*d)*(d*x+c),x, algorithm="giac")
```

output
```
b*c*d*x^3 + 3/2*b*c^2*x^2 + 1/2*a*d^2*x^2 + a*c*d*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int (ad+3bcx)(c+dx)\,dx = b\,c\,d\,x^3+\left(\frac{3\,b\,c^2}{2}+\frac{a\,d^2}{2}\right)\,x^2+a\,c\,d\,x$$

input
```
int((a*d + 3*b*c*x)*(c + d*x),x)
```

output
```
x^2*((a*d^2)/2 + (3*b*c^2)/2) + b*c*d*x^3 + a*c*d*x
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int (ad+3bcx)(c+dx)\,dx = \frac{x(2bcd\,x^2+a\,d^2x+3b\,c^2x+2acd)}{2}$$

input
```
int((3*b*c*x+a*d)*(d*x+c),x)
```

output
```
(x*(2*a*c*d + a*d**2*x + 3*b*c**2*x + 2*b*c*d*x**2))/2
```

3.29 $\int \frac{(ad+bcx)(c+dx)}{a+bx^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 31

$$\int \frac{(ad+bcx)(c+dx)}{a+bx^2} \, dx = cdx + \frac{(bc^2+ad^2)\log(a+bx^2)}{2b}$$

output
```
c*d*x+1/2*(a*d^2+b*c^2)*ln(b*x^2+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(ad+bcx)(c+dx)}{a+bx^2} \, dx = cdx + \frac{(bc^2+ad^2)\log(a+bx^2)}{2b}$$

input
```
Integrate[((a*d + b*c*x)*(c + d*x))/(a + b*x^2),x]
```

output
```
c*d*x + ((b*c^2 + a*d^2)*Log[a + b*x^2])/(2*b)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ad+bcx)}{a+bx^2} \, dx$$

$$\Big\downarrow \text{657}$$

$$\int \left(\frac{x(ad^2+bc^2)}{a+bx^2}+cd\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ad^2+bc^2)\log(a+bx^2)}{2b}+cdx$$

input
```
Int[((a*d + b*c*x)*(c + d*x))/(a + b*x^2),x]
```

output
```
c*d*x + ((b*c^2 + a*d^2)*Log[a + b*x^2])/(2*b)
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$cdx+\frac{\left(a\,d^{2}+b\,c^{2}\right)\ln\left(b\,x^{2}+a\right)}{2b}$	30
norman	$cdx+\frac{\left(a\,d^{2}+b\,c^{2}\right)\ln\left(b\,x^{2}+a\right)}{2b}$	30
risch	$cdx+\frac{\ln\left(b\,x^{2}+a\right)a\,d^{2}}{2b}+\frac{\ln\left(b\,x^{2}+a\right)c^{2}}{2}$	36
parallelrisch	$\frac{\ln\left(b\,x^{2}+a\right)a\,d^{2}+\ln\left(b\,x^{2}+a\right)b\,c^{2}+2bcdx}{2b}$	39

input
```
int((b*c*x+a*d)*(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
c*d*x+1/2*(a*d^2+b*c^2)*ln(b*x^2+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(ad+bcx)(c+dx)}{a+bx^2}\,dx=\frac{2\,bcdx+(bc^2+ad^2)\log\left(bx^2+a\right)}{2\,b}$$

input
```
integrate((b*c*x+a*d)*(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(2*b*c*d*x + (b*c^2 + a*d^2)*log(b*x^2 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{(ad+bcx)(c+dx)}{a+bx^2}\,dx=cdx+\frac{(ad^2+bc^2)\log\left(a+bx^2\right)}{2b}$$

input
```
integrate((b*c*x+a*d)*(d*x+c)/(b*x**2+a),x)
```

output
```
c*d*x + (a*d**2 + b*c**2)*log(a + b*x**2)/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(ad+bcx)(c+dx)}{a+bx^2}\,dx = cdx + \frac{(bc^2+ad^2)\log(bx^2+a)}{2\,b}$$

input
```
integrate((b*c*x+a*d)*(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
c*d*x + 1/2*(b*c^2 + a*d^2)*log(b*x^2 + a)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(ad+bcx)(c+dx)}{a+bx^2}\,dx = cdx + \frac{(bc^2+ad^2)\log(bx^2+a)}{2\,b}$$

input
```
integrate((b*c*x+a*d)*(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
c*d*x + 1/2*(b*c^2 + a*d^2)*log(b*x^2 + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(ad+bcx)(c+dx)}{a+bx^2}\,dx = \frac{\ln(b\,x^2+a)\,\left(b\,c^2+a\,d^2\right)}{2\,b} + c\,d\,x$$

input
```
int(((a*d + b*c*x)*(c + d*x))/(a + b*x^2),x)
```

output
```
(log(a + b*x^2)*(a*d^2 + b*c^2))/(2*b) + c*d*x
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{(ad+bcx)(c+dx)}{a+bx^2}\,dx = \frac{\log(b\,x^2+a)\,a\,d^2+\log(b\,x^2+a)\,b\,c^2+2bcdx}{2b}$$

input
```
int((b*c*x+a*d)*(d*x+c)/(b*x^2+a),x)
```

output
```
(log(a + b*x**2)*a*d**2 + log(a + b*x**2)*b*c**2 + 2*b*c*d*x)/(2*b)
```

3.30 $\int \frac{(ad-bcx)(c+dx)}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 20

$$\int \frac{(ad-bcx)(c+dx)}{(a+bx^2)^2} dx = \frac{(c+dx)^2}{2(a+bx^2)}$$

output
```
(d*x+c)^2/(2*b*x^2+2*a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{(ad-bcx)(c+dx)}{(a+bx^2)^2} dx = \frac{bc^2-ad^2+2bcdx}{2b(a+bx^2)}$$

input
```
Integrate[((a*d - b*c*x)*(c + d*x))/(a + b*x^2)^2,x]
```

output
```
(b*c^2 - a*d^2 + 2*b*c*d*x)/(2*b*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {673}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ad-bcx)}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \; 673$$

$$\frac{-ad^2+bc^2+2bcdx}{2b\,(a+bx^2)}$$

input
```
Int[((a*d - b*c*x)*(c + d*x))/(a + b*x^2)^2,x]
```

output
```
(b*c^2 - a*d^2 + 2*b*c*d*x)/(2*b*(a + b*x^2))
```

Defintions of rubi rules used

rule 673
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] /; FreeQ[{a, c, d, e, f, g, p}, x] && EqQ
[a*e*g - c*d*f*(2*p + 3), 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

method	result	size
gosper	$-\frac{-2bcdx+a\,d^2-b\,c^2}{2(b\,x^2+a)b}$	33
default	$\frac{cdx-\frac{a\,d^2-b\,c^2}{2b}}{b\,x^2+a}$	33
norman	$\frac{cdx-\frac{a\,d^2-b\,c^2}{2b}}{b\,x^2+a}$	33
risch	$\frac{cdx-\frac{a\,d^2-b\,c^2}{2b}}{b\,x^2+a}$	33
parallelrisch	$\frac{2bcdx-a\,d^2+b\,c^2}{2b(b\,x^2+a)}$	33
orering	$-\frac{-2bcdx+a\,d^2-b\,c^2}{2(b\,x^2+a)b}$	33

input
```
int((-b*c*x+a*d)*(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*b*c*d*x+a*d^2-b*c^2)/(b*x^2+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{(ad-bcx)(c+dx)}{\left(a+bx^2\right)^2}\,dx=\frac{2\,bcdx+bc^2-ad^2}{2\,(b^2x^2+ab)}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*(2*b*c*d*x + b*c^2 - a*d^2)/(b^2*x^2 + a*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{(ad - bcx)(c + dx)}{(a + bx^2)^2} \, dx = -\frac{ad^2 - bc^2 - 2bcdx}{2ab + 2b^2x^2}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)/(b*x**2+a)**2,x)
```

output
```
-(a*d**2 - b*c**2 - 2*b*c*d*x)/(2*a*b + 2*b**2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{(ad - bcx)(c + dx)}{(a + bx^2)^2} \, dx = \frac{2\,bcdx + bc^2 - ad^2}{2\,(b^2x^2 + ab)}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(2*b*c*d*x + b*c^2 - a*d^2)/(b^2*x^2 + a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{(ad - bcx)(c + dx)}{(a + bx^2)^2} \, dx = \frac{2\,bcdx + bc^2 - ad^2}{2\,(bx^2 + a)b}$$

input
```
integrate((-b*c*x+a*d)*(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(2*b*c*d*x + b*c^2 - a*d^2)/((b*x^2 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{(ad-bcx)(c+dx)}{(a+bx^2)^2}\,dx = -\frac{\frac{a\,d^2-b\,c^2}{2\,b}-c\,d\,x}{b\,x^2+a}$$

input
```
int(((a*d - b*c*x)*(c + d*x))/(a + b*x^2)^2,x)
```

output
```
-((a*d^2 - b*c^2)/(2*b) - c*d*x)/(a + b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{(ad-bcx)(c+dx)}{(a+bx^2)^2}\,dx = \frac{x(a\,d^2x-b\,c^2x+2acd)}{2a\,(b\,x^2+a)}$$

input
```
int((-b*c*x+a*d)*(d*x+c)/(b*x^2+a)^2,x)
```

output
```
(x*(2*a*c*d + a*d**2*x - b*c**2*x))/(2*a*(a + b*x**2))
```

3.31 $\int \frac{(ad-3bcx)(c+dx)}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 35

$$\int \frac{(ad-3bcx)(c+dx)}{(a+bx^2)^3} \, dx = \frac{3bc^2-ad^2+4bcdx}{4b(a+bx^2)^2}$$

output
```
1/4*(4*b*c*d*x-a*d^2+3*b*c^2)/b/(b*x^2+a)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(ad-3bcx)(c+dx)}{(a+bx^2)^3} \, dx = \frac{3bc^2-ad^2+4bcdx}{4b(a+bx^2)^2}$$

input
```
Integrate[((a*d - 3*b*c*x)*(c + d*x))/(a + b*x^2)^3,x]
```

output
```
(3*b*c^2 - a*d^2 + 4*b*c*d*x)/(4*b*(a + b*x^2)^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {673}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ad-3bcx)}{(a+bx^2)^3} \, dx$$

$$\Big\downarrow \; 673$$

$$\frac{-ad^2+3bc^2+4bcdx}{4b\left(a+bx^2\right)^2}$$

input
```
Int[((a*d - 3*b*c*x)*(c + d*x))/(a + b*x^2)^3,x]
```

output
```
(3*b*c^2 - a*d^2 + 4*b*c*d*x)/(4*b*(a + b*x^2)^2)
```

Defintions of rubi rules used

rule 673
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] /; FreeQ[{a, c, d, e, f, g, p}, x] && EqQ
[a*e*g - c*d*f*(2*p + 3), 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{-4bcdx+a\,d^2-3b\,c^2}{4b(b\,x^2+a)^2}$	33
default	$\frac{cdx-\frac{a\,d^2-3b\,c^2}{4b}}{(b\,x^2+a)^2}$	33
risch	$\frac{cdx-\frac{a\,d^2-3b\,c^2}{4b}}{(b\,x^2+a)^2}$	33
orering	$-\frac{-4bcdx+a\,d^2-3b\,c^2}{4b(b\,x^2+a)^2}$	33
norman	$\frac{cdx+\frac{-ab\,d^2+3b^2c^2}{4b^2}}{(b\,x^2+a)^2}$	37
parallelrisch	$\frac{4b^2cxd-ab\,d^2+3b^2c^2}{4b^2(b\,x^2+a)^2}$	39

input
```
int((-3*b*c*x+a*d)*(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*(-4*b*c*d*x+a*d^2-3*b*c^2)/(b*x^2+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \frac{(ad-3bcx)(c+dx)}{(a+bx^2)^3}\,dx = \frac{4\,bcdx+3\,bc^2-ad^2}{4\,(b^3x^4+2\,ab^2x^2+a^2b)}$$

input
```
integrate((-3*b*c*x+a*d)*(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/4*(4*b*c*d*x + 3*b*c^2 - a*d^2)/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{(ad-3bcx)(c+dx)}{(a+bx^2)^3}\,dx = -\frac{ad^2-3bc^2-4bcdx}{4a^2b+8ab^2x^2+4b^3x^4}$$

input
```
integrate((-3*b*c*x+a*d)*(d*x+c)/(b*x**2+a)**3,x)
```

output
```
-(a*d**2 - 3*b*c**2 - 4*b*c*d*x)/(4*a**2*b + 8*a*b**2*x**2 + 4*b**3*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \frac{(ad-3bcx)(c+dx)}{(a+bx^2)^3}\,dx = \frac{4\,bcdx+3\,bc^2-ad^2}{4\,(b^3x^4+2\,ab^2x^2+a^2b)}$$

input
```
integrate((-3*b*c*x+a*d)*(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/4*(4*b*c*d*x + 3*b*c^2 - a*d^2)/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{(ad-3bcx)(c+dx)}{(a+bx^2)^3}\,dx = \frac{4\,bcdx+3\,bc^2-ad^2}{4\,(bx^2+a)^2b}$$

input
```
integrate((-3*b*c*x+a*d)*(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/4*(4*b*c*d*x + 3*b*c^2 - a*d^2)/((b*x^2 + a)^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \frac{(ad-3bcx)(c+dx)}{(a+bx^2)^3}\,dx = -\frac{\frac{a\,d^2-3\,b\,c^2}{4\,b}-c\,d\,x}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

input
```
int(((a*d - 3*b*c*x)*(c + d*x))/(a + b*x^2)^3,x)
```

output
```
-((a*d^2 - 3*b*c^2)/(4*b) - c*d*x)/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{(ad-3bcx)(c+dx)}{(a+bx^2)^3}\,dx = \frac{4bcdx-a\,d^2+3b\,c^2}{4b\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int((-3*b*c*x+a*d)*(d*x+c)/(b*x^2+a)^3,x)
```

output
```
( - a*d**2 + 3*b*c**2 + 4*b*c*d*x)/(4*b*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.32 $\int \frac{(ad-5bcx)(c+dx)}{(a+bx^2)^4} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 35

$$\int \frac{(ad-5bcx)(c+dx)}{(a+bx^2)^4} \, dx = \frac{5bc^2-ad^2+6bcdx}{6b(a+bx^2)^3}$$

output
```
1/6*(6*b*c*d*x-a*d^2+5*b*c^2)/b/(b*x^2+a)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(ad-5bcx)(c+dx)}{(a+bx^2)^4} \, dx = \frac{5bc^2-ad^2+6bcdx}{6b(a+bx^2)^3}$$

input
```
Integrate[((a*d - 5*b*c*x)*(c + d*x))/(a + b*x^2)^4,x]
```

output
```
(5*b*c^2 - a*d^2 + 6*b*c*d*x)/(6*b*(a + b*x^2)^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {673}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ad-5bcx)}{(a+bx^2)^4}\,dx$$

$$\Big\downarrow \; 673$$

$$\frac{-ad^2+5bc^2+6bcdx}{6b\left(a+bx^2\right)^3}$$

input
```
Int[((a*d - 5*b*c*x)*(c + d*x))/(a + b*x^2)^4,x]
```

output
```
(5*b*c^2 - a*d^2 + 6*b*c*d*x)/(6*b*(a + b*x^2)^3)
```

Defintions of rubi rules used

rule 673
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] /; FreeQ[{a, c, d, e, f, g, p}, x] && EqQ
[a*e*g - c*d*f*(2*p + 3), 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{-6bcdx+a\,d^2-5b\,c^2}{6b(b\,x^2+a)^3}$	33
default	$\frac{cdx-\frac{a\,d^2-5b\,c^2}{6b}}{(b\,x^2+a)^3}$	33
risch	$\frac{cdx-\frac{a\,d^2-5b\,c^2}{6b}}{(b\,x^2+a)^3}$	33
orering	$-\frac{-6bcdx+a\,d^2-5b\,c^2}{6b(b\,x^2+a)^3}$	33
norman	$\frac{cdx+\frac{-a\,b^2d^2+5b^3c^2}{6b^3}}{(b\,x^2+a)^3}$	39
parallelrisch	$\frac{6cdx\,b^3-a\,b^2d^2+5b^3c^2}{6b^3(b\,x^2+a)^3}$	41

input
```
int((-5*b*c*x+a*d)*(d*x+c)/(b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6/b*(-6*b*c*d*x+a*d^2-5*b*c^2)/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int \frac{(ad-5bcx)(c+dx)}{(a+bx^2)^4}\,dx = \frac{6\,bcdx+5\,bc^2-ad^2}{6\,(b^4x^6+3\,ab^3x^4+3\,a^2b^2x^2+a^3b)}$$

input
```
integrate((-5*b*c*x+a*d)*(d*x+c)/(b*x^2+a)^4,x, algorithm="fricas")
```

output
```
1/6*(6*b*c*d*x + 5*b*c^2 - a*d^2)/(b^4*x^6 + 3*a*b^3*x^4 + 3*a^2*b^2*x^2 +
 a^3*b)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

$$\int \frac{(ad-5bcx)(c+dx)}{(a+bx^2)^4}\,dx = -\frac{ad^2-5bc^2-6bcdx}{6a^3b+18a^2b^2x^2+18ab^3x^4+6b^4x^6}$$

input
```
integrate((-5*b*c*x+a*d)*(d*x+c)/(b*x**2+a)**4,x)
```

output
```
-(a*d**2 - 5*b*c**2 - 6*b*c*d*x)/(6*a**3*b + 18*a**2*b**2*x**2 + 18*a*b**3
*x**4 + 6*b**4*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int \frac{(ad-5bcx)(c+dx)}{(a+bx^2)^4}\,dx = \frac{6\,bcdx+5\,bc^2-ad^2}{6\,(b^4x^6+3\,ab^3x^4+3\,a^2b^2x^2+a^3b)}$$

input
```
integrate((-5*b*c*x+a*d)*(d*x+c)/(b*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/6*(6*b*c*d*x + 5*b*c^2 - a*d^2)/(b^4*x^6 + 3*a*b^3*x^4 + 3*a^2*b^2*x^2 +
 a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{(ad-5bcx)(c+dx)}{(a+bx^2)^4}\,dx = \frac{6\,bcdx+5\,bc^2-ad^2}{6\,(bx^2+a)^3b}$$

input
```
integrate((-5*b*c*x+a*d)*(d*x+c)/(b*x^2+a)^4,x, algorithm="giac")
```

output
```
1/6*(6*b*c*d*x + 5*b*c^2 - a*d^2)/((b*x^2 + a)^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int \frac{(ad-5bcx)(c+dx)}{(a+bx^2)^4}\,dx = -\frac{\frac{a\,d^2-5\,b\,c^2}{6\,b}-c\,d\,x}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}$$

input
```
int(((a*d - 5*b*c*x)*(c + d*x))/(a + b*x^2)^4,x)
```

output
```
-((a*d^2 - 5*b*c^2)/(6*b) - c*d*x)/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*
x^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int \frac{(ad-5bcx)(c+dx)}{(a+bx^2)^4}\,dx = \frac{6bcdx-a\,d^2+5b\,c^2}{6b\,(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)}$$

input
```
int((-5*b*c*x+a*d)*(d*x+c)/(b*x^2+a)^4,x)
```

output
```
( - a*d**2 + 5*b*c**2 + 6*b*c*d*x)/(6*b*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x
**4 + b**3*x**6))
```

3.33 $\int (A+Bx)(d+ex)^5 \left(a+cx^2\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int (A+Bx)(d+ex)^5 \left(a+cx^2\right) \, dx = -\frac{(Bd-Ae)\left(cd^2+ae^2\right)(d+ex)^6}{6e^4} + \frac{\left(3Bcd^2-2Acde+aBe^2\right)(d+ex)^7}{7e^4} - \frac{c(3Bd-Ae)(d+ex)^8}{8e^4} + \frac{Bc(d+ex)^9}{9e^4}$$

output

```
-1/6*(-A*e+B*d)*(a*e^2+c*d^2)*(e*x+d)^6/e^4+1/7*(-2*A*c*d*e+B*a*e^2+3*B*c*
d^2)*(e*x+d)^7/e^4-1/8*c*(-A*e+3*B*d)*(e*x+d)^8/e^4+1/9*B*c*(e*x+d)^9/e^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. $2(108) = 216$.

Time = 0.06 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.16

$$\int (A+Bx)(d+ex)^5\left(a+cx^2\right)\,dx = aAd^5x+\frac{1}{2}ad^4(Bd+5Ae)x^2+\frac{1}{3}d^3\left(Acd^2+5aBde+10aAe^2\right)x^3+\frac{1}{4}d^2\left(Bcd^3+5Acd^2e+10aBde^2+10aAe^3\right)x^4+de\left(Bcd^3+2Acd^2e+2aBde^2+aAe^3\right)x^5+\frac{1}{6}e^2\left(10Bcd^3+10Acd^2e+5aBde^2+aAe^3\right)x^6+\frac{1}{7}e^3\left(10Bcd^2+5Acde+aBe^2\right)x^7+\frac{1}{8}ce^4(5Bd+Ae)x^8+\frac{1}{9}Bce^5x^9$$

input
```
Integrate[(A + B*x)*(d + e*x)^5*(a + c*x^2),x]
```

output
```
a*A*d^5*x + (a*d^4*(B*d + 5*A*e)*x^2)/2 + (d^3*(A*c*d^2 + 5*a*B*d*e + 10*a
*A*e^2)*x^3)/3 + (d^2*(B*c*d^3 + 5*A*c*d^2*e + 10*a*B*d*e^2 + 10*a*A*e^3)*
x^4)/4 + d*e*(B*c*d^3 + 2*A*c*d^2*e + 2*a*B*d*e^2 + a*A*e^3)*x^5 + (e^2*(1
0*B*c*d^3 + 10*A*c*d^2*e + 5*a*B*d*e^2 + a*A*e^3)*x^6)/6 + (e^3*(10*B*c*d^
2 + 5*A*c*d*e + a*B*e^2)*x^7)/7 + (c*e^4*(5*B*d + A*e)*x^8)/8 + (B*c*e^5*x
^9)/9
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{652, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + cx^2)(A + Bx)(d + ex)^5 \, dx$$

$$\Big\downarrow \ 652$$

$$\int \left(\frac{(d+ex)^6 (aBe^2 - 2Acde + 3Bcd^2)}{e^3} + \frac{(d+ex)^5 (ae^2 + cd^2)(Ae - Bd)}{e^3} + \frac{c(d+ex)^7 (Ae - 3Bd)}{e^3} + \frac{Bc(d+}{e^3}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{(d+ex)^7 (aBe^2 - 2Acde + 3Bcd^2)}{7e^4} - \frac{(d+ex)^6 (ae^2 + cd^2)(Bd - Ae)}{6e^4} - \frac{c(d+ex)^8 (3Bd - Ae)}{8e^4} + \frac{Bc(d+ex)^9}{9e^4}$$

input
```
Int[(A + B*x)*(d + e*x)^5*(a + c*x^2),x]
```

output
```
-1/6*((B*d - A*e)*(c*d^2 + a*e^2)*(d + e*x)^6)/e^4 + ((3*B*c*d^2 - 2*A*c*d
*e + a*B*e^2)*(d + e*x)^7)/(7*e^4) - (c*(3*B*d - A*e)*(d + e*x)^8)/(8*e^4)
 + (B*c*(d + e*x)^9)/(9*e^4)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(100) = 200$.

Time = 0.50 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.20

method	result
norman	$\frac{B e^5 c x^9}{9}+\left(\frac{1}{8} A c e^5+\frac{5}{8} B c d e^4\right) x^8+\left(\frac{5}{7} A c d e^4+\frac{1}{7} B e^5 a+\frac{10}{7} B c d^2 e^3\right) x^7+\left(\frac{1}{6} A a e^5+\frac{5}{3} A c d^2\right.$
default	$\frac{B e^5 c x^9}{9}+\frac{(A e^5+5 B d e^4) c x^8}{8}+\frac{((5 A d e^4+10 B d^2 e^3) c+B e^5 a) x^7}{7}+\frac{((10 A d^2 e^3+10 B d^3 e^2) c+(A e^5+5 B d e^4) a) x^6}{6}$
gosper	$\frac{1}{9} B e^5 c x^9+\frac{1}{8} x^8 A c e^5+\frac{5}{8} x^8 B c d e^4+\frac{5}{7} x^7 A c d e^4+\frac{1}{7} x^7 B e^5 a+\frac{10}{7} x^7 B c d^2 e^3+\frac{1}{6} x^6 A a e^5+$
risch	$\frac{1}{9} B e^5 c x^9+\frac{1}{8} x^8 A c e^5+\frac{5}{8} x^8 B c d e^4+\frac{5}{7} x^7 A c d e^4+\frac{1}{7} x^7 B e^5 a+\frac{10}{7} x^7 B c d^2 e^3+\frac{1}{6} x^6 A a e^5+$
parallelrisch	$\frac{1}{9} B e^5 c x^9+\frac{1}{8} x^8 A c e^5+\frac{5}{8} x^8 B c d e^4+\frac{5}{7} x^7 A c d e^4+\frac{1}{7} x^7 B e^5 a+\frac{10}{7} x^7 B c d^2 e^3+\frac{1}{6} x^6 A a e^5+$
orering	$x(56 B e^5 c x^8+63 A c e^5 x^7+315 B c d e^4 x^7+360 A c d e^4 x^6+72 B a e^5 x^6+720 B c d^2 e^3 x^6+84 A a e^5 x^5+840 A c d^2 e^3 x^5+420 B a d e^4 x$

input

```
int((B*x+A)*(e*x+d)^5*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output

```
1/9*B*e^5*c*x^9+(1/8*A*c*e^5+5/8*B*c*d*e^4)*x^8+(5/7*A*c*d*e^4+1/7*B*e^5*a
+10/7*B*c*d^2*e^3)*x^7+(1/6*A*a*e^5+5/3*A*c*d^2*e^3+5/6*B*a*d*e^4+5/3*B*c*
d^3*e^2)*x^6+(A*a*d*e^4+2*A*c*d^3*e^2+2*B*a*d^2*e^3+B*c*d^4*e)*x^5+(5/2*A*
a*d^2*e^3+5/4*A*c*d^4*e+5/2*B*a*d^3*e^2+1/4*B*c*d^5)*x^4+(10/3*A*a*d^3*e^2
+1/3*A*d^5*c+5/3*B*a*d^4*e)*x^3+(5/2*A*a*d^4*e+1/2*B*a*d^5)*x^2+A*d^5*a*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(100) = 200$.

Time = 0.07 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.19

$$\begin{aligned}\int (A+Bx)(d+ex)^5\left(a+cx^2\right)\,dx &= \frac{1}{9}\,Bce^5x^9+\frac{1}{8}\left(5\,Bcde^4+Ace^5\right)x^8+Aad^5x \\ &\quad+\frac{1}{7}\left(10\,Bcd^2e^3+5\,Acde^4+Bae^5\right)x^7 \\ &\quad+\frac{1}{6}\left(10\,Bcd^3e^2+10\,Acd^2e^3+5\,Bade^4+Aae^5\right)x^6 \\ &\quad+\left(Bcd^4e+2\,Acd^3e^2+2\,Bad^2e^3+Aade^4\right)x^5 \\ &\quad+\frac{1}{4}\left(Bcd^5+5\,Acd^4e+10\,Bad^3e^2+10\,Aad^2e^3\right)x^4 \\ &\quad+\frac{1}{3}\left(Acd^5+5\,Bad^4e+10\,Aad^3e^2\right)x^3 \\ &\quad+\frac{1}{2}\left(Bad^5+5\,Aad^4e\right)x^2\end{aligned}$$

input
```
integrate((B*x+A)*(e*x+d)^5*(c*x^2+a),x, algorithm="fricas")
```

output
```
1/9*B*c*e^5*x^9 + 1/8*(5*B*c*d*e^4 + A*c*e^5)*x^8 + A*a*d^5*x + 1/7*(10*B*
c*d^2*e^3 + 5*A*c*d*e^4 + B*a*e^5)*x^7 + 1/6*(10*B*c*d^3*e^2 + 10*A*c*d^2*
e^3 + 5*B*a*d*e^4 + A*a*e^5)*x^6 + (B*c*d^4*e + 2*A*c*d^3*e^2 + 2*B*a*d^2*
e^3 + A*a*d*e^4)*x^5 + 1/4*(B*c*d^5 + 5*A*c*d^4*e + 10*B*a*d^3*e^2 + 10*A*
a*d^2*e^3)*x^4 + 1/3*(A*c*d^5 + 5*B*a*d^4*e + 10*A*a*d^3*e^2)*x^3 + 1/2*(B
*a*d^5 + 5*A*a*d^4*e)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(105) = 210$.

Time = 0.04 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.66

$$\int (A+Bx)(d+ex)^5\left(a+cx^2\right)\,dx = Aad^5x + \frac{Bce^5x^9}{9} + x^8\left(\frac{Ace^5}{8} + \frac{5Bcde^4}{8}\right) + x^7\cdot\left(\frac{5Acde^4}{7} + \frac{Bae^5}{7} + \frac{10Bcd^2e^3}{7}\right) + x^6\left(\frac{Aae^5}{6} + \frac{5Acd^2e^3}{3} + \frac{5Bade^4}{6} + \frac{5Bcd^3e^2}{3}\right) + x^5\left(Aade^4 + 2Acd^3e^2 + 2Bad^2e^3 + Bcd^4e\right) + x^4\cdot\left(\frac{5Aad^2e^3}{2} + \frac{5Acd^4e}{4} + \frac{5Bad^3e^2}{2} + \frac{Bcd^5}{4}\right) + x^3\cdot\left(\frac{10Aad^3e^2}{3} + \frac{Acd^5}{3} + \frac{5Bad^4e}{3}\right) + x^2\cdot\left(\frac{5Aad^4e}{2} + \frac{Bad^5}{2}\right)$$

input
```
integrate((B*x+A)*(e*x+d)**5*(c*x**2+a),x)
```

output
```
A*a*d**5*x + B*c*e**5*x**9/9 + x**8*(A*c*e**5/8 + 5*B*c*d*e**4/8) + x**7*(
5*A*c*d*e**4/7 + B*a*e**5/7 + 10*B*c*d**2*e**3/7) + x**6*(A*a*e**5/6 + 5*A
*c*d**2*e**3/3 + 5*B*a*d*e**4/6 + 5*B*c*d**3*e**2/3) + x**5*(A*a*d*e**4 +
2*A*c*d**3*e**2 + 2*B*a*d**2*e**3 + B*c*d**4*e) + x**4*(5*A*a*d**2*e**3/2
+ 5*A*c*d**4*e/4 + 5*B*a*d**3*e**2/2 + B*c*d**5/4) + x**3*(10*A*a*d**3*e**
2/3 + A*c*d**5/3 + 5*B*a*d**4*e/3) + x**2*(5*A*a*d**4*e/2 + B*a*d**5/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(100) = 200$.

Time = 0.03 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.19

$$\int (A+Bx)(d+ex)^5\left(a+cx^2\right)\,dx = \frac{1}{9}\,Bce^5x^9 + \frac{1}{8}\left(5\,Bcde^4 + Ace^5\right)x^8 + Aad^5x + \frac{1}{7}\left(10\,Bcd^2e^3 + 5\,Acde^4 + Bae^5\right)x^7 + \frac{1}{6}\left(10\,Bcd^3e^2 + 10\,Acd^2e^3 + 5\,Bade^4 + Aae^5\right)x^6 + \left(Bcd^4e + 2\,Acd^3e^2 + 2\,Bad^2e^3 + Aade^4\right)x^5 + \frac{1}{4}\left(Bcd^5 + 5\,Acd^4e + 10\,Bad^3e^2 + 10\,Aad^2e^3\right)x^4 + \frac{1}{3}\left(Acd^5 + 5\,Bad^4e + 10\,Aad^3e^2\right)x^3 + \frac{1}{2}\left(Bad^5 + 5\,Aad^4e\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)^5*(c*x^2+a),x, algorithm="maxima")
```

output
```
1/9*B*c*e^5*x^9 + 1/8*(5*B*c*d*e^4 + A*c*e^5)*x^8 + A*a*d^5*x + 1/7*(10*B*
c*d^2*e^3 + 5*A*c*d*e^4 + B*a*e^5)*x^7 + 1/6*(10*B*c*d^3*e^2 + 10*A*c*d^2*
e^3 + 5*B*a*d*e^4 + A*a*e^5)*x^6 + (B*c*d^4*e + 2*A*c*d^3*e^2 + 2*B*a*d^2*
e^3 + A*a*d*e^4)*x^5 + 1/4*(B*c*d^5 + 5*A*c*d^4*e + 10*B*a*d^3*e^2 + 10*A*
a*d^2*e^3)*x^4 + 1/3*(A*c*d^5 + 5*B*a*d^4*e + 10*A*a*d^3*e^2)*x^3 + 1/2*(B
*a*d^5 + 5*A*a*d^4*e)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(100) = 200$.

Time = 0.11 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.48

$$\int (A+Bx)(d+ex)^5\left(a+cx^2\right)\,dx = \frac{1}{9}\,Bce^5x^9+\frac{5}{8}\,Bcde^4x^8+\frac{1}{8}\,Ace^5x^8+\frac{10}{7}\,Bcd^2e^3x^7+\frac{5}{7}\,Acde^4x^7+\frac{1}{7}\,Bae^5x^7+\frac{5}{3}\,Bcd^3e^2x^6+\frac{5}{3}\,Acd^2e^3x^6+\frac{5}{6}\,Bade^4x^6+\frac{1}{6}\,Aae^5x^6+Bcd^4ex^5+2\,Acd^3e^2x^5+2\,Bad^2e^3x^5+Aade^4x^5+\frac{1}{4}\,Bcd^5x^4+\frac{5}{4}\,Acd^4ex^4+\frac{5}{2}\,Bad^3e^2x^4+\frac{5}{2}\,Aad^2e^3x^4+\frac{1}{3}\,Acd^5x^3+\frac{5}{3}\,Bad^4ex^3+\frac{10}{3}\,Aad^3e^2x^3+\frac{1}{2}\,Bad^5x^2+\frac{5}{2}\,Aad^4ex^2+Aad^5x$$

input
```
integrate((B*x+A)*(e*x+d)^5*(c*x^2+a),x, algorithm="giac")
```

output
```
1/9*B*c*e^5*x^9 + 5/8*B*c*d*e^4*x^8 + 1/8*A*c*e^5*x^8 + 10/7*B*c*d^2*e^3*x
^7 + 5/7*A*c*d*e^4*x^7 + 1/7*B*a*e^5*x^7 + 5/3*B*c*d^3*e^2*x^6 + 5/3*A*c*d
^2*e^3*x^6 + 5/6*B*a*d*e^4*x^6 + 1/6*A*a*e^5*x^6 + B*c*d^4*e*x^5 + 2*A*c*d
^3*e^2*x^5 + 2*B*a*d^2*e^3*x^5 + A*a*d*e^4*x^5 + 1/4*B*c*d^5*x^4 + 5/4*A*c
*d^4*e*x^4 + 5/2*B*a*d^3*e^2*x^4 + 5/2*A*a*d^2*e^3*x^4 + 1/3*A*c*d^5*x^3 +
 5/3*B*a*d^4*e*x^3 + 10/3*A*a*d^3*e^2*x^3 + 1/2*B*a*d^5*x^2 + 5/2*A*a*d^4*
e*x^2 + A*a*d^5*x
```

Mupad [B] (verification not implemented)

Time = 5.68 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.14

$$\int (A+Bx)(d+ex)^5\left(a+cx^2\right)\,dx = x^5\left(B\,c\,d^4\,e+2\,A\,c\,d^3\,e^2+2\,B\,a\,d^2\,e^3+A\,a\,d\,e^4\right) + x^3\left(\frac{A\,c\,d^5}{3}+\frac{5\,B\,a\,d^4\,e}{3}+\frac{10\,A\,a\,d^3\,e^2}{3}\right) + x^7\left(\frac{10\,B\,c\,d^2\,e^3}{7}+\frac{5\,A\,c\,d\,e^4}{7}+\frac{B\,a\,e^5}{7}\right) + x^4\left(\frac{B\,c\,d^5}{4}+\frac{5\,A\,c\,d^4\,e}{4}+\frac{5\,B\,a\,d^3\,e^2}{2}+\frac{5\,A\,a\,d^2\,e^3}{2}\right) + x^6\left(\frac{5\,B\,c\,d^3\,e^2}{3}+\frac{5\,A\,c\,d^2\,e^3}{3}+\frac{5\,B\,a\,d\,e^4}{6}+\frac{A\,a\,e^5}{6}\right) + A\,a\,d^5\,x + \frac{B\,c\,e^5\,x^9}{9} + \frac{a\,d^4\,x^2\,(5\,A\,e+B\,d)}{2} + \frac{c\,e^4\,x^8\,(A\,e+5\,B\,d)}{8}$$

input
```
int((a + c*x^2)*(A + B*x)*(d + e*x)^5,x)
```

output
```
x^5*(A*a*d*e^4 + B*c*d^4*e + 2*B*a*d^2*e^3 + 2*A*c*d^3*e^2) + x^3*((A*c*d^
5)/3 + (5*B*a*d^4*e)/3 + (10*A*a*d^3*e^2)/3) + x^7*((B*a*e^5)/7 + (5*A*c*d
*e^4)/7 + (10*B*c*d^2*e^3)/7) + x^4*((B*c*d^5)/4 + (5*A*c*d^4*e)/4 + (5*A*
a*d^2*e^3)/2 + (5*B*a*d^3*e^2)/2) + x^6*((A*a*e^5)/6 + (5*B*a*d*e^4)/6 + (
5*A*c*d^2*e^3)/3 + (5*B*c*d^3*e^2)/3) + A*a*d^5*x + (B*c*e^5*x^9)/9 + (a*d
^4*x^2*(5*A*e + B*d))/2 + (c*e^4*x^8*(A*e + 5*B*d))/8
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.55

$$\int (A+Bx)(d+ex)^5\left(a+cx^2\right)\,dx = \frac{x(56bc\,e^5x^8+63ac\,e^5x^7+315bcd\,e^4x^7+72ab\,e^5x^6+360acd\,e^4x^6+720bc\,d^2e^3x^6+84a^2e^5x^5+420abd\,e}{}$$

input
```
int((B*x+A)*(e*x+d)^5*(c*x^2+a),x)
```

output
```
(x*(504*a**2*d**5 + 1260*a**2*d**4*e*x + 1680*a**2*d**3*e**2*x**2 + 1260*a
**2*d**2*e**3*x**3 + 504*a**2*d*e**4*x**4 + 84*a**2*e**5*x**5 + 252*a*b*d*
*5*x + 840*a*b*d**4*e*x**2 + 1260*a*b*d**3*e**2*x**3 + 1008*a*b*d**2*e**3*
x**4 + 420*a*b*d*e**4*x**5 + 72*a*b*e**5*x**6 + 168*a*c*d**5*x**2 + 630*a*
c*d**4*e*x**3 + 1008*a*c*d**3*e**2*x**4 + 840*a*c*d**2*e**3*x**5 + 360*a*c
*d*e**4*x**6 + 63*a*c*e**5*x**7 + 126*b*c*d**5*x**3 + 504*b*c*d**4*e*x**4
+ 840*b*c*d**3*e**2*x**5 + 720*b*c*d**2*e**3*x**6 + 315*b*c*d*e**4*x**7 +
56*b*c*e**5*x**8))/504
```

3.34 $\int (A+Bx)(d+ex)^4 \left(a+cx^2\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int (A+Bx)(d+ex)^4 \left(a+cx^2\right) \, dx = -\frac{(Bd-Ae)\left(cd^2+ae^2\right)(d+ex)^5}{5e^4} + \frac{\left(3Bcd^2-2Acde+aBe^2\right)(d+ex)^6}{6e^4} - \frac{c(3Bd-Ae)(d+ex)^7}{7e^4} + \frac{Bc(d+ex)^8}{8e^4}$$

output
```
-1/5*(-A*e+B*d)*(a*e^2+c*d^2)*(e*x+d)^5/e^4+1/6*(-2*A*c*d*e+B*a*e^2+3*B*c*
d^2)*(e*x+d)^6/e^4-1/7*c*(-A*e+3*B*d)*(e*x+d)^7/e^4+1/8*B*c*(e*x+d)^8/e^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.80

$$\int (A+Bx)(d+ex)^4\left(a+cx^2\right)\,dx = aAd^4x+\frac{1}{2}ad^3(Bd+4Ae)x^2 +\frac{1}{3}d^2\left(Acd^2+4aBde+6aAe^2\right)x^3 +\frac{1}{4}d\left(Bcd^3+4Acd^2e+6aBde^2+4aAe^3\right)x^4 +\frac{1}{5}e\left(4Bcd^3+6Acd^2e+4aBde^2+aAe^3\right)x^5 +\frac{1}{6}e^2\left(6Bcd^2+4Acde+aBe^2\right)x^6 +\frac{1}{7}ce^3(4Bd+Ae)x^7+\frac{1}{8}Bce^4x^8$$

input
```
Integrate[(A + B*x)*(d + e*x)^4*(a + c*x^2),x]
```

output
```
a*A*d^4*x + (a*d^3*(B*d + 4*A*e)*x^2)/2 + (d^2*(A*c*d^2 + 4*a*B*d*e + 6*a*
A*e^2)*x^3)/3 + (d*(B*c*d^3 + 4*A*c*d^2*e + 6*a*B*d*e^2 + 4*a*A*e^3)*x^4)/
4 + (e*(4*B*c*d^3 + 6*A*c*d^2*e + 4*a*B*d*e^2 + a*A*e^3)*x^5)/5 + (e^2*(6*
B*c*d^2 + 4*A*c*d*e + a*B*e^2)*x^6)/6 + (c*e^3*(4*B*d + A*e)*x^7)/7 + (B*c
*e^4*x^8)/8
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(A+Bx)(d+ex)^4\,dx$$

$$\Big\downarrow \text{ 652}$$

$$\int \left(\frac{(d+ex)^5 \left(aBe^2-2Acde+3Bcd^2\right)}{e^3}+\frac{(d+ex)^4 \left(ae^2+cd^2\right)(Ae-Bd)}{e^3}+\frac{c(d+ex)^6(Ae-3Bd)}{e^3}+\frac{Bc(d+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^6 \left(aBe^2-2Acde+3Bcd^2\right)}{6e^4}-\frac{(d+ex)^5 \left(ae^2+cd^2\right)(Bd-Ae)}{5e^4}-\frac{c(d+ex)^7(3Bd-Ae)}{7e^4}+\frac{Bc(d+ex)^8}{8e^4}$$

input
```
Int[(A + B*x)*(d + e*x)^4*(a + c*x^2),x]
```

output
```
-1/5*((B*d - A*e)*(c*d^2 + a*e^2)*(d + e*x)^5)/e^4 + ((3*B*c*d^2 - 2*A*c*d
*e + a*B*e^2)*(d + e*x)^6)/(6*e^4) - (c*(3*B*d - A*e)*(d + e*x)^7)/(7*e^4)
 + (B*c*(d + e*x)^8)/(8*e^4)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.78

method	result
norman	$\frac{B e^4 c x^8}{8}+\left(\frac{1}{7} A c e^4+\frac{4}{7} B c d e^3\right) x^7+\left(\frac{2}{3} A c d e^3+\frac{1}{6} B e^4 a+B c d^2 e^2\right) x^6+\left(\frac{1}{5} A a e^4+\frac{6}{5} A c d^2 e^2\right.$
default	$\frac{B e^4 c x^8}{8}+\frac{\left(A e^4+4 B d e^3\right) c x^7}{7}+\frac{\left(\left(4 A d e^3+6 B d^2 e^2\right) c+B e^4 a\right) x^6}{6}+\frac{\left(\left(6 A d^2 e^2+4 B d^3 e\right) c+\left(A e^4+4 B d e^3\right) a\right) x^5}{5}+\frac{((}{}$
gosper	$\frac{1}{8} B e^4 c x^8+\frac{1}{7} x^7 A c e^4+\frac{4}{7} x^7 B c d e^3+\frac{2}{3} x^6 A c d e^3+\frac{1}{6} x^6 B e^4 a+x^6 B c d^2 e^2+\frac{1}{5} x^5 A a e^4+\frac{6}{5} x$
risch	$\frac{1}{8} B e^4 c x^8+\frac{1}{7} x^7 A c e^4+\frac{4}{7} x^7 B c d e^3+\frac{2}{3} x^6 A c d e^3+\frac{1}{6} x^6 B e^4 a+x^6 B c d^2 e^2+\frac{1}{5} x^5 A a e^4+\frac{6}{5} x$
parallelrisch	$\frac{1}{8} B e^4 c x^8+\frac{1}{7} x^7 A c e^4+\frac{4}{7} x^7 B c d e^3+\frac{2}{3} x^6 A c d e^3+\frac{1}{6} x^6 B e^4 a+x^6 B c d^2 e^2+\frac{1}{5} x^5 A a e^4+\frac{6}{5} x$
orering	$\frac{x\left(105 B e^4 c x^7+120 A c e^4 x^6+480 B c d e^3 x^6+560 A c d e^3 x^5+140 B a e^4 x^5+840 B c d^2 e^2 x^5+168 A a e^4 x^4+1008 A c d^2 e^2 x^4+672 B\right.}{}$

input
```
int((B*x+A)*(e*x+d)^4*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*B*e^4*c*x^8+(1/7*A*c*e^4+4/7*B*c*d*e^3)*x^7+(2/3*A*c*d*e^3+1/6*B*e^4*a
+B*c*d^2*e^2)*x^6+(1/5*A*a*e^4+6/5*A*c*d^2*e^2+4/5*B*a*d*e^3+4/5*B*c*d^3*e
)*x^5+(A*a*d*e^3+A*c*d^3*e+3/2*B*a*d^2*e^2+1/4*B*c*d^4)*x^4+(2*A*a*d^2*e^2
+1/3*A*c*d^4+4/3*B*a*d^3*e)*x^3+(2*A*a*d^3*e+1/2*B*a*d^4)*x^2+A*d^4*a*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.80

$$\begin{aligned}\int (A+Bx)(d+ex)^4\left(a+cx^2\right)\,dx &= \frac{1}{8}\,Bce^4x^8+\frac{1}{7}\left(4\,Bcde^3+Ace^4\right)x^7+Aad^4x\\&+\frac{1}{6}\left(6\,Bcd^2e^2+4\,Acde^3+Bae^4\right)x^6\\&+\frac{1}{5}\left(4\,Bcd^3e+6\,Acd^2e^2+4\,Bade^3+Aae^4\right)x^5\\&+\frac{1}{4}\left(Bcd^4+4\,Acd^3e+6\,Bad^2e^2+4\,Aade^3\right)x^4\\&+\frac{1}{3}\left(Acd^4+4\,Bad^3e+6\,Aad^2e^2\right)x^3\\&+\frac{1}{2}\left(Bad^4+4\,Aad^3e\right)x^2\end{aligned}$$

input
```
integrate((B*x+A)*(e*x+d)^4*(c*x^2+a),x, algorithm="fricas")
```

output
```
1/8*B*c*e^4*x^8 + 1/7*(4*B*c*d*e^3 + A*c*e^4)*x^7 + A*a*d^4*x + 1/6*(6*B*c
*d^2*e^2 + 4*A*c*d*e^3 + B*a*e^4)*x^6 + 1/5*(4*B*c*d^3*e + 6*A*c*d^2*e^2 +
 4*B*a*d*e^3 + A*a*e^4)*x^5 + 1/4*(B*c*d^4 + 4*A*c*d^3*e + 6*B*a*d^2*e^2 +
 4*A*a*d*e^3)*x^4 + 1/3*(A*c*d^4 + 4*B*a*d^3*e + 6*A*a*d^2*e^2)*x^3 + 1/2*
(B*a*d^4 + 4*A*a*d^3*e)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(102) = 204$.

Time = 0.03 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.09

$$\begin{aligned}\int (A+Bx)(d+ex)^4\left(a+cx^2\right)\,dx = &\ Aad^4x+\frac{Bce^4x^8}{8}+x^7\left(\frac{Ace^4}{7}+\frac{4Bcde^3}{7}\right)\\ &+x^6\cdot\left(\frac{2Acde^3}{3}+\frac{Bae^4}{6}+Bcd^2e^2\right)\\ &+x^5\left(\frac{Aae^4}{5}+\frac{6Acd^2e^2}{5}+\frac{4Bade^3}{5}+\frac{4Bcd^3e}{5}\right)\\ &+x^4\left(Aade^3+Acd^3e+\frac{3Bad^2e^2}{2}+\frac{Bcd^4}{4}\right)\\ &+x^3\cdot\left(2Aad^2e^2+\frac{Acd^4}{3}+\frac{4Bad^3e}{3}\right)\\ &+x^2\cdot\left(2Aad^3e+\frac{Bad^4}{2}\right)\end{aligned}$$

input
```
integrate((B*x+A)*(e*x+d)**4*(c*x**2+a),x)
```

output
```
A*a*d**4*x + B*c*e**4*x**8/8 + x**7*(A*c*e**4/7 + 4*B*c*d*e**3/7) + x**6*(
2*A*c*d*e**3/3 + B*a*e**4/6 + B*c*d**2*e**2) + x**5*(A*a*e**4/5 + 6*A*c*d*
*2*e**2/5 + 4*B*a*d*e**3/5 + 4*B*c*d**3*e/5) + x**4*(A*a*d*e**3 + A*c*d**3
*e + 3*B*a*d**2*e**2/2 + B*c*d**4/4) + x**3*(2*A*a*d**2*e**2 + A*c*d**4/3
+ 4*B*a*d**3*e/3) + x**2*(2*A*a*d**3*e + B*a*d**4/2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.80

$$\int (A+Bx)(d+ex)^4\left(a+cx^2\right)\,dx = \frac{1}{8}\,Bce^4x^8 + \frac{1}{7}\left(4\,Bcde^3 + Ace^4\right)x^7 + Aad^4x + \frac{1}{6}\left(6\,Bcd^2e^2 + 4\,Acde^3 + Bae^4\right)x^6 + \frac{1}{5}\left(4\,Bcd^3e + 6\,Acd^2e^2 + 4\,Bade^3 + Aae^4\right)x^5 + \frac{1}{4}\left(Bcd^4 + 4\,Acd^3e + 6\,Bad^2e^2 + 4\,Aade^3\right)x^4 + \frac{1}{3}\left(Acd^4 + 4\,Bad^3e + 6\,Aad^2e^2\right)x^3 + \frac{1}{2}\left(Bad^4 + 4\,Aad^3e\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)^4*(c*x^2+a),x, algorithm="maxima")
```

output
```
1/8*B*c*e^4*x^8 + 1/7*(4*B*c*d*e^3 + A*c*e^4)*x^7 + A*a*d^4*x + 1/6*(6*B*c
*d^2*e^2 + 4*A*c*d*e^3 + B*a*e^4)*x^6 + 1/5*(4*B*c*d^3*e + 6*A*c*d^2*e^2 +
 4*B*a*d*e^3 + A*a*e^4)*x^5 + 1/4*(B*c*d^4 + 4*A*c*d^3*e + 6*B*a*d^2*e^2 +
 4*A*a*d*e^3)*x^4 + 1/3*(A*c*d^4 + 4*B*a*d^3*e + 6*A*a*d^2*e^2)*x^3 + 1/2*
(B*a*d^4 + 4*A*a*d^3*e)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(100) = 200.

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.99

$$\int (A+Bx)(d+ex)^4\left(a+cx^2\right)\,dx = \frac{1}{8}\,Bce^4x^8 + \frac{4}{7}\,Bcde^3x^7 + \frac{1}{7}\,Ace^4x^7 + Bcd^2e^2x^6 + \frac{2}{3}\,Acde^3x^6 + \frac{1}{6}\,Bae^4x^6 + \frac{4}{5}\,Bcd^3ex^5 + \frac{6}{5}\,Acd^2e^2x^5 + \frac{4}{5}\,Bade^3x^5 + \frac{1}{5}\,Aae^4x^5 + \frac{1}{4}\,Bcd^4x^4 + Acd^3ex^4 + \frac{3}{2}\,Bad^2e^2x^4 + Aade^3x^4 + \frac{1}{3}\,Acd^4x^3 + \frac{4}{3}\,Bad^3ex^3 + 2\,Aad^2e^2x^3 + \frac{1}{2}\,Bad^4x^2 + 2\,Aad^3ex^2 + Aad^4x$$

input
```
integrate((B*x+A)*(e*x+d)^4*(c*x^2+a),x, algorithm="giac")
```

output
```
1/8*B*c*e^4*x^8 + 4/7*B*c*d*e^3*x^7 + 1/7*A*c*e^4*x^7 + B*c*d^2*e^2*x^6 +
2/3*A*c*d*e^3*x^6 + 1/6*B*a*e^4*x^6 + 4/5*B*c*d^3*e*x^5 + 6/5*A*c*d^2*e^2*
x^5 + 4/5*B*a*d*e^3*x^5 + 1/5*A*a*e^4*x^5 + 1/4*B*c*d^4*x^4 + A*c*d^3*e*x^
4 + 3/2*B*a*d^2*e^2*x^4 + A*a*d*e^3*x^4 + 1/3*A*c*d^4*x^3 + 4/3*B*a*d^3*e*
x^3 + 2*A*a*d^2*e^2*x^3 + 1/2*B*a*d^4*x^2 + 2*A*a*d^3*e*x^2 + A*a*d^4*x
```

Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.71

$$\int (A+Bx)(d+ex)^4\left(a+cx^2\right)\,dx = x^3\left(\frac{A\,c\,d^4}{3}+\frac{4\,B\,a\,d^3\,e}{3}+2\,A\,a\,d^2\,e^2\right) + x^6\left(B\,c\,d^2\,e^2+\frac{2\,A\,c\,d\,e^3}{3}+\frac{B\,a\,e^4}{6}\right) + x^4\left(\frac{B\,c\,d^4}{4}+A\,c\,d^3\,e+\frac{3\,B\,a\,d^2\,e^2}{2}+A\,a\,d\,e^3\right) + x^5\left(\frac{4\,B\,c\,d^3\,e}{5}+\frac{6\,A\,c\,d^2\,e^2}{5}+\frac{4\,B\,a\,d\,e^3}{5}+\frac{A\,a\,e^4}{5}\right) + A\,a\,d^4\,x+\frac{B\,c\,e^4\,x^8}{8} + \frac{a\,d^3\,x^2\,(4\,A\,e+B\,d)}{2}+\frac{c\,e^3\,x^7\,(A\,e+4\,B\,d)}{7}$$

input
```
int((a + c*x^2)*(A + B*x)*(d + e*x)^4,x)
```

output
```
x^3*((A*c*d^4)/3 + (4*B*a*d^3*e)/3 + 2*A*a*d^2*e^2) + x^6*((B*a*e^4)/6 + (
2*A*c*d*e^3)/3 + B*c*d^2*e^2) + x^4*((B*c*d^4)/4 + A*a*d*e^3 + A*c*d^3*e +
 (3*B*a*d^2*e^2)/2) + x^5*((A*a*e^4)/5 + (4*B*a*d*e^3)/5 + (4*B*c*d^3*e)/5
 + (6*A*c*d^2*e^2)/5) + A*a*d^4*x + (B*c*e^4*x^8)/8 + (a*d^3*x^2*(4*A*e +
B*d))/2 + (c*e^3*x^7*(A*e + 4*B*d))/7
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.06

$$\int (A+Bx)(d+ex)^4\left(a+cx^2\right)\,dx$$
$$= \frac{x(105bc\,e^4x^7+120ac\,e^4x^6+480bcd\,e^3x^6+140ab\,e^4x^5+560acd\,e^3x^5+840bc\,d^2e^2x^5+168a^2e^4x^4+672}{}$$

input
```
int((B*x+A)*(e*x+d)^4*(c*x^2+a),x)
```

output
```
(x*(840*a**2*d**4 + 1680*a**2*d**3*e*x + 1680*a**2*d**2*e**2*x**2 + 840*a*
*2*d*e**3*x**3 + 168*a**2*e**4*x**4 + 420*a*b*d**4*x + 1120*a*b*d**3*e*x**
2 + 1260*a*b*d**2*e**2*x**3 + 672*a*b*d*e**3*x**4 + 140*a*b*e**4*x**5 + 28
0*a*c*d**4*x**2 + 840*a*c*d**3*e*x**3 + 1008*a*c*d**2*e**2*x**4 + 560*a*c*
d*e**3*x**5 + 120*a*c*e**4*x**6 + 210*b*c*d**4*x**3 + 672*b*c*d**3*e*x**4
+ 840*b*c*d**2*e**2*x**5 + 480*b*c*d*e**3*x**6 + 105*b*c*e**4*x**7))/840
```

3.35 $\int (A+Bx)(d+ex)^3 \left(a+cx^2\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int (A+Bx)(d+ex)^3 \left(a+cx^2\right) \, dx = -\frac{(Bd-Ae)\left(cd^2+ae^2\right)(d+ex)^4}{4e^4} + \frac{\left(3Bcd^2-2Acde+aBe^2\right)(d+ex)^5}{5e^4} - \frac{c(3Bd-Ae)(d+ex)^6}{6e^4} + \frac{Bc(d+ex)^7}{7e^4}$$

output
```
-1/4*(-A*e+B*d)*(a*e^2+c*d^2)*(e*x+d)^4/e^4+1/5*(-2*A*c*d*e+B*a*e^2+3*B*c*
d^2)*(e*x+d)^5/e^4-1/6*c*(-A*e+3*B*d)*(e*x+d)^6/e^4+1/7*B*c*(e*x+d)^7/e^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.39

$$\int (A+Bx)(d+ex)^3\left(a+cx^2\right)\,dx = aAd^3x+\frac{1}{2}ad^2(Bd+3Ae)x^2 +\frac{1}{3}d\left(Acd^2+3aBde+3aAe^2\right)x^3 +\frac{1}{4}\left(Bcd^3+3Acd^2e+3aBde^2+aAe^3\right)x^4 +\frac{1}{5}e\left(3Bcd^2+3Acde+aBe^2\right)x^5 +\frac{1}{6}ce^2(3Bd+Ae)x^6+\frac{1}{7}Bce^3x^7$$

input
```
Integrate[(A + B*x)*(d + e*x)^3*(a + c*x^2),x]
```

output
```
a*A*d^3*x + (a*d^2*(B*d + 3*A*e)*x^2)/2 + (d*(A*c*d^2 + 3*a*B*d*e + 3*a*A*
e^2)*x^3)/3 + ((B*c*d^3 + 3*A*c*d^2*e + 3*a*B*d*e^2 + a*A*e^3)*x^4)/4 + (e
*(3*B*c*d^2 + 3*A*c*d*e + a*B*e^2)*x^5)/5 + (c*e^2*(3*B*d + A*e)*x^6)/6 +
(B*c*e^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(A+Bx)(d+ex)^3\,dx$$

$$\Big\downarrow \; 652$$

$$\int \left(\frac{(d+ex)^4\left(aBe^2-2Acde+3Bcd^2\right)}{e^3}+\frac{(d+ex)^3\left(ae^2+cd^2\right)(Ae-Bd)}{e^3}+\frac{c(d+ex)^5(Ae-3Bd)}{e^3}+\frac{Bc(d+}{e^3}\right.$$

$$\downarrow \text{2009}$$

$$\frac{(d+ex)^5 (aBe^2 - 2Acde + 3Bcd^2)}{5e^4} - \frac{(d+ex)^4 (ae^2+cd^2) (Bd-Ae)}{4e^4} - \frac{c(d+ex)^6 (3Bd-Ae)}{6e^4} + \frac{Bc(d+ex)^7}{7e^4}$$

input
```
Int[(A + B*x)*(d + e*x)^3*(a + c*x^2),x]
```

output
```
-1/4*((B*d - A*e)*(c*d^2 + a*e^2)*(d + e*x)^4)/e^4 + ((3*B*c*d^2 - 2*A*c*d
*e + a*B*e^2)*(d + e*x)^5)/(5*e^4) - (c*(3*B*d - A*e)*(d + e*x)^6)/(6*e^4)
 + (B*c*(d + e*x)^7)/(7*e^4)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.37

method	result
norman	$\frac{B\,e^3 c\,x^7}{7}+\left(\frac{1}{6}Ac\,e^3+\frac{1}{2}Bcd\,e^2\right)x^6+\left(\frac{3}{5}Acd\,e^2+\frac{1}{5}B\,e^3 a+\frac{3}{5}Bc\,d^2 e\right)x^5+\left(\frac{1}{4}Aa\,e^3+\frac{3}{4}Ac\,d^2 e\right.$
default	$\frac{B\,e^3 c\,x^7}{7}+\frac{(A\,e^3+3Bd\,e^2)c\,x^6}{6}+\frac{((3Ad\,e^2+3B\,d^2 e)c+B\,e^3 a)x^5}{5}+\frac{((3A\,d^2 e+B\,d^3)c+(A\,e^3+3Bd\,e^2)a)x^4}{4}+\frac{(Ac\,d^3}{}$
gosper	$\frac{1}{7}B\,e^3 c\,x^7+\frac{1}{6}x^6 Ac\,e^3+\frac{1}{2}x^6 Bcd\,e^2+\frac{3}{5}x^5 Acd\,e^2+\frac{1}{5}x^5 B\,e^3 a+\frac{3}{5}x^5 Bc\,d^2 e+\frac{1}{4}x^4 Aa\,e^3+\frac{3}{4}x$
risch	$\frac{1}{7}B\,e^3 c\,x^7+\frac{1}{6}x^6 Ac\,e^3+\frac{1}{2}x^6 Bcd\,e^2+\frac{3}{5}x^5 Acd\,e^2+\frac{1}{5}x^5 B\,e^3 a+\frac{3}{5}x^5 Bc\,d^2 e+\frac{1}{4}x^4 Aa\,e^3+\frac{3}{4}x$
parallelrisch	$\frac{1}{7}B\,e^3 c\,x^7+\frac{1}{6}x^6 Ac\,e^3+\frac{1}{2}x^6 Bcd\,e^2+\frac{3}{5}x^5 Acd\,e^2+\frac{1}{5}x^5 B\,e^3 a+\frac{3}{5}x^5 Bc\,d^2 e+\frac{1}{4}x^4 Aa\,e^3+\frac{3}{4}x$
orering	$\frac{x\left(60B\,e^3 c\,x^6+70Ac\,e^3 x^5+210Bcd\,e^2 x^5+252Acd\,e^2 x^4+84Ba\,e^3 x^4+252Bc\,d^2 e\,x^4+105Aa\,e^3 x^3+315Ac\,d^2 e\,x^3+315Bad\,e^2 x\right.}{420}$

input
```
int((B*x+A)*(e*x+d)^3*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/7*B*e^3*c*x^7+(1/6*A*c*e^3+1/2*B*c*d*e^2)*x^6+(3/5*A*c*d*e^2+1/5*B*e^3*a
+3/5*B*c*d^2*e)*x^5+(1/4*A*a*e^3+3/4*A*c*d^2*e+3/4*B*a*d*e^2+1/4*B*c*d^3)*
x^4+(A*a*d*e^2+1/3*A*c*d^3+B*a*d^2*e)*x^3+(3/2*A*a*d^2*e+1/2*B*a*d^3)*x^2+
A*d^3*a*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.37

$$\begin{aligned}\int (A+Bx)(d+ex)^3\left(a+cx^2\right)\,dx &= \frac{1}{7}\,Bce^3x^7+\frac{1}{6}\left(3\,Bcde^2+Ace^3\right)x^6+Aad^3x\\&\quad+\frac{1}{5}\left(3\,Bcd^2e+3\,Acde^2+Bae^3\right)x^5\\&\quad+\frac{1}{4}\left(Bcd^3+3\,Acd^2e+3\,Bade^2+Aae^3\right)x^4\\&\quad+\frac{1}{3}\left(Acd^3+3\,Bad^2e+3\,Aade^2\right)x^3\\&\quad+\frac{1}{2}\left(Bad^3+3\,Aad^2e\right)x^2\end{aligned}$$

input
```
integrate((B*x+A)*(e*x+d)^3*(c*x^2+a),x, algorithm="fricas")
```

output
```
1/7*B*c*e^3*x^7 + 1/6*(3*B*c*d*e^2 + A*c*e^3)*x^6 + A*a*d^3*x + 1/5*(3*B*c
*d^2*e + 3*A*c*d*e^2 + B*a*e^3)*x^5 + 1/4*(B*c*d^3 + 3*A*c*d^2*e + 3*B*a*d
*e^2 + A*a*e^3)*x^4 + 1/3*(A*c*d^3 + 3*B*a*d^2*e + 3*A*a*d*e^2)*x^3 + 1/2*
(B*a*d^3 + 3*A*a*d^2*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.60

$$\int (A+Bx)(d+ex)^3\left(a+cx^2\right)\,dx = Aad^3x+\frac{Bce^3x^7}{7}+x^6\left(\frac{Ace^3}{6}+\frac{Bcde^2}{2}\right)+x^5\cdot\left(\frac{3Acde^2}{5}+\frac{Bae^3}{5}+\frac{3Bcd^2e}{5}\right)+x^4\left(\frac{Aae^3}{4}+\frac{3Acd^2e}{4}+\frac{3Bade^2}{4}+\frac{Bcd^3}{4}\right)+x^3\left(Aade^2+\frac{Acd^3}{3}+Bad^2e\right)+x^2\cdot\left(\frac{3Aad^2e}{2}+\frac{Bad^3}{2}\right)$$

input
```
integrate((B*x+A)*(e*x+d)**3*(c*x**2+a),x)
```

output
```
A*a*d**3*x + B*c*e**3*x**7/7 + x**6*(A*c*e**3/6 + B*c*d*e**2/2) + x**5*(3*
A*c*d*e**2/5 + B*a*e**3/5 + 3*B*c*d**2*e/5) + x**4*(A*a*e**3/4 + 3*A*c*d**
2*e/4 + 3*B*a*d*e**2/4 + B*c*d**3/4) + x**3*(A*a*d*e**2 + A*c*d**3/3 + B*a
*d**2*e) + x**2*(3*A*a*d**2*e/2 + B*a*d**3/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.37

$$\int (A+Bx)(d+ex)^3\left(a+cx^2\right)\,dx = \frac{1}{7}\,Bce^3x^7+\frac{1}{6}\,\left(3\,Bcde^2+Ace^3\right)x^6+Aad^3x+\frac{1}{5}\,\left(3\,Bcd^2e+3\,Acde^2+Bae^3\right)x^5+\frac{1}{4}\,\left(Bcd^3+3\,Acd^2e+3\,Bade^2+Aae^3\right)x^4+\frac{1}{3}\,\left(Acd^3+3\,Bad^2e+3\,Aade^2\right)x^3+\frac{1}{2}\,\left(Bad^3+3\,Aad^2e\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)^3*(c*x^2+a),x, algorithm="maxima")
```

output
```
1/7*B*c*e^3*x^7 + 1/6*(3*B*c*d*e^2 + A*c*e^3)*x^6 + A*a*d^3*x + 1/5*(3*B*c
*d^2*e + 3*A*c*d*e^2 + B*a*e^3)*x^5 + 1/4*(B*c*d^3 + 3*A*c*d^2*e + 3*B*a*d
*e^2 + A*a*e^3)*x^4 + 1/3*(A*c*d^3 + 3*B*a*d^2*e + 3*A*a*d*e^2)*x^3 + 1/2*
(B*a*d^3 + 3*A*a*d^2*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.52

$$\int (A+Bx)(d+ex)^3\left(a+cx^2\right)\,dx = \frac{1}{7}Bce^3x^7+\frac{1}{2}Bcde^2x^6+\frac{1}{6}Ace^3x^6+\frac{3}{5}Bcd^2ex^5+\frac{3}{5}Acde^2x^5+\frac{1}{5}Bae^3x^5+\frac{1}{4}Bcd^3x^4+\frac{3}{4}Acd^2ex^4+\frac{3}{4}Bade^2x^4+\frac{1}{4}Aae^3x^4+\frac{1}{3}Acd^3x^3+Bad^2ex^3+Aade^2x^3+\frac{1}{2}Bad^3x^2+\frac{3}{2}Aad^2ex^2+Aad^3x$$

input
```
integrate((B*x+A)*(e*x+d)^3*(c*x^2+a),x, algorithm="giac")
```

output
```
1/7*B*c*e^3*x^7 + 1/2*B*c*d*e^2*x^6 + 1/6*A*c*e^3*x^6 + 3/5*B*c*d^2*e*x^5
+ 3/5*A*c*d*e^2*x^5 + 1/5*B*a*e^3*x^5 + 1/4*B*c*d^3*x^4 + 3/4*A*c*d^2*e*x^
4 + 3/4*B*a*d*e^2*x^4 + 1/4*A*a*e^3*x^4 + 1/3*A*c*d^3*x^3 + B*a*d^2*e*x^3
+ A*a*d*e^2*x^3 + 1/2*B*a*d^3*x^2 + 3/2*A*a*d^2*e*x^2 + A*a*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.31

$$\int (A+Bx)(d+ex)^3\left(a+cx^2\right)\,dx = x^4\left(\frac{B\,c\,d^3}{4}+\frac{3\,A\,c\,d^2\,e}{4}+\frac{3\,B\,a\,d\,e^2}{4}+\frac{A\,a\,e^3}{4}\right)+x^3\left(\frac{A\,c\,d^3}{3}+B\,a\,d^2\,e+A\,a\,d\,e^2\right)+x^5\left(\frac{3\,B\,c\,d^2\,e}{5}+\frac{3\,A\,c\,d\,e^2}{5}+\frac{B\,a\,e^3}{5}\right)+A\,a\,d^3\,x+\frac{B\,c\,e^3\,x^7}{7}+\frac{a\,d^2\,x^2\,(3\,A\,e+B\,d)}{2}+\frac{c\,e^2\,x^6\,(A\,e+3\,B\,d)}{6}$$

input
```
int((a + c*x^2)*(A + B*x)*(d + e*x)^3,x)
```

output
```
x^4*((A*a*e^3)/4 + (B*c*d^3)/4 + (3*B*a*d*e^2)/4 + (3*A*c*d^2*e)/4) + x^3*
((A*c*d^3)/3 + A*a*d*e^2 + B*a*d^2*e) + x^5*((B*a*e^3)/5 + (3*A*c*d*e^2)/5
 + (3*B*c*d^2*e)/5) + A*a*d^3*x + (B*c*e^3*x^7)/7 + (a*d^2*x^2*(3*A*e + B*
d))/2 + (c*e^2*x^6*(A*e + 3*B*d))/6
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.56

$$\int (A+Bx)(d+ex)^3\left(a+cx^2\right)\,dx$$
$$= \frac{x(60bc\,e^3x^6+70ac\,e^3x^5+210bcd\,e^2x^5+84ab\,e^3x^4+252acd\,e^2x^4+252bc\,d^2e\,x^4+105a^2e^3x^3+315abd\,e}{}$$

input
```
int((B*x+A)*(e*x+d)^3*(c*x^2+a),x)
```

output
```
(x*(420*a**2*d**3 + 630*a**2*d**2*e*x + 420*a**2*d*e**2*x**2 + 105*a**2*e*
*3*x**3 + 210*a*b*d**3*x + 420*a*b*d**2*e*x**2 + 315*a*b*d*e**2*x**3 + 84*
a*b*e**3*x**4 + 140*a*c*d**3*x**2 + 315*a*c*d**2*e*x**3 + 252*a*c*d*e**2*x
**4 + 70*a*c*e**3*x**5 + 105*b*c*d**3*x**3 + 252*b*c*d**2*e*x**4 + 210*b*c
*d*e**2*x**5 + 60*b*c*e**3*x**6))/420
```

3.36 $\int (A+Bx)(d+ex)^2 \left(a+cx^2\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int (A+Bx)(d+ex)^2 \left(a+cx^2\right) \, dx = -\frac{(Bd-Ae)\left(cd^2+ae^2\right)(d+ex)^3}{3e^4} + \frac{\left(3Bcd^2-2Acde+aBe^2\right)(d+ex)^4}{4e^4} - \frac{c(3Bd-Ae)(d+ex)^5}{5e^4} + \frac{Bc(d+ex)^6}{6e^4}$$

output
```
-1/3*(-A*e+B*d)*(a*e^2+c*d^2)*(e*x+d)^3/e^4+1/4*(-2*A*c*d*e+B*a*e^2+3*B*c*
d^2)*(e*x+d)^4/e^4-1/5*c*(-A*e+3*B*d)*(e*x+d)^5/e^4+1/6*B*c*(e*x+d)^6/e^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.98

$$\int (A+Bx)(d+ex)^2 \left(a+cx^2\right) \, dx = aAd^2x + \frac{1}{2}ad(Bd+2Ae)x^2 + \frac{1}{3}\left(Acd^2+2aBde+aAe^2\right)x^3 + \frac{1}{4}\left(Bcd^2+2Acde+aBe^2\right)x^4 + \frac{1}{5}ce(2Bd+Ae)x^5 + \frac{1}{6}Bce^2x^6$$

input
```
Integrate[(A + B*x)*(d + e*x)^2*(a + c*x^2),x]
```

output
```
a*A*d^2*x + (a*d*(B*d + 2*A*e)*x^2)/2 + ((A*c*d^2 + 2*a*B*d*e + a*A*e^2)*x
^3)/3 + ((B*c*d^2 + 2*A*c*d*e + a*B*e^2)*x^4)/4 + (c*e*(2*B*d + A*e)*x^5)/
5 + (B*c*e^2*x^6)/6
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)(A+Bx)(d+ex)^2\,dx$$

$$\Big\downarrow \ 652$$

$$\int \left(\frac{(d+ex)^3(aBe^2-2Acde+3Bcd^2)}{e^3}+\frac{(d+ex)^2(ae^2+cd^2)(Ae-Bd)}{e^3}+\frac{c(d+ex)^4(Ae-3Bd)}{e^3}+\frac{Bc(d+\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{(d+ex)^4(aBe^2-2Acde+3Bcd^2)}{4e^4}-\frac{(d+ex)^3(ae^2+cd^2)(Bd-Ae)}{3e^4}-\frac{c(d+ex)^5(3Bd-Ae)}{5e^4}+\frac{Bc(d+ex)^6}{6e^4}$$

input
```
Int[(A + B*x)*(d + e*x)^2*(a + c*x^2),x]
```

output
```
-1/3*((B*d - A*e)*(c*d^2 + a*e^2)*(d + e*x)^3)/e^4 + ((3*B*c*d^2 - 2*A*c*d
*e + a*B*e^2)*(d + e*x)^4)/(4*e^4) - (c*(3*B*d - A*e)*(d + e*x)^5)/(5*e^4)
 + (B*c*(d + e*x)^6)/(6*e^4)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95

method	result
default	$\frac{B e^2 c x^6}{6}+\frac{(A e^2+2Bde) c x^5}{5}+\frac{((2Ade+B d^2)c+Ba e^2)x^4}{4}+\frac{(Ac d^2+(A e^2+2Bde)a)x^3}{3}+\frac{(2Ade+B d^2)a x^2}{2}+A$
norman	$\frac{B e^2 c x^6}{6}+\left(\frac{1}{5}Ac e^2+\frac{2}{5}Bcde\right)x^5+\left(\frac{1}{2}Acde+\frac{1}{4}Ba e^2+\frac{1}{4}Bc d^2\right)x^4+\left(\frac{1}{3}Aa e^2+\frac{1}{3}Ac d^2+\frac{2}{3}B\right.$
gosper	$\frac{1}{6}B e^2 c x^6+\frac{1}{5}x^5 Ac e^2+\frac{2}{5}x^5 Bcde+\frac{1}{2}x^4 Acde+\frac{1}{4}x^4 Ba e^2+\frac{1}{4}x^4 Bc d^2+\frac{1}{3}x^3 Aa e^2+\frac{1}{3}x^3 Ac$
risch	$\frac{1}{6}B e^2 c x^6+\frac{1}{5}x^5 Ac e^2+\frac{2}{5}x^5 Bcde+\frac{1}{2}x^4 Acde+\frac{1}{4}x^4 Ba e^2+\frac{1}{4}x^4 Bc d^2+\frac{1}{3}x^3 Aa e^2+\frac{1}{3}x^3 Ac$
parallelrisch	$\frac{1}{6}B e^2 c x^6+\frac{1}{5}x^5 Ac e^2+\frac{2}{5}x^5 Bcde+\frac{1}{2}x^4 Acde+\frac{1}{4}x^4 Ba e^2+\frac{1}{4}x^4 Bc d^2+\frac{1}{3}x^3 Aa e^2+\frac{1}{3}x^3 Ac$
orering	$\frac{x(10B e^2 c x^5+12Ac e^2 x^4+24Bcde x^4+30Acde x^3+15Ba e^2 x^3+15Bc d^2 x^3+20Aa e^2 x^2+20Ac d^2 x^2+40Bade x^2+60Aadex+}{60}$

input
```
int((B*x+A)*(e*x+d)^2*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*B*e^2*c*x^6+1/5*(A*e^2+2*B*d*e)*c*x^5+1/4*((2*A*d*e+B*d^2)*c+B*a*e^2)*
x^4+1/3*(A*c*d^2+(A*e^2+2*B*d*e)*a)*x^3+1/2*(2*A*d*e+B*d^2)*a*x^2+A*a*d^2*
x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)\,dx=\frac{1}{6}\,Bce^2x^6+\frac{1}{5}\left(2\,Bcde+Ace^2\right)x^5+Aad^2x+\frac{1}{4}\left(Bcd^2+2\,Acde+Bae^2\right)x^4+\frac{1}{3}\left(Acd^2+2\,Bade+Aae^2\right)x^3+\frac{1}{2}\left(Bad^2+2\,Aade\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)^2*(c*x^2+a),x, algorithm="fricas")
```

output
```
1/6*B*c*e^2*x^6 + 1/5*(2*B*c*d*e + A*c*e^2)*x^5 + A*a*d^2*x + 1/4*(B*c*d^2
 + 2*A*c*d*e + B*a*e^2)*x^4 + 1/3*(A*c*d^2 + 2*B*a*d*e + A*a*e^2)*x^3 + 1/
2*(B*a*d^2 + 2*A*a*d*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)\,dx=Aad^2x+\frac{Bce^2x^6}{6}+x^5\left(\frac{Ace^2}{5}+\frac{2Bcde}{5}\right)+x^4\left(\frac{Acde}{2}+\frac{Bae^2}{4}+\frac{Bcd^2}{4}\right)+x^3\left(\frac{Aae^2}{3}+\frac{Acd^2}{3}+\frac{2Bade}{3}\right)+x^2\left(Aade+\frac{Bad^2}{2}\right)$$

input
```
integrate((B*x+A)*(e*x+d)**2*(c*x**2+a),x)
```

output
```
A*a*d**2*x + B*c*e**2*x**6/6 + x**5*(A*c*e**2/5 + 2*B*c*d*e/5) + x**4*(A*c
*d*e/2 + B*a*e**2/4 + B*c*d**2/4) + x**3*(A*a*e**2/3 + A*c*d**2/3 + 2*B*a*
d*e/3) + x**2*(A*a*d*e + B*a*d**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)\,dx = \frac{1}{6}\,Bce^2x^6 + \frac{1}{5}\left(2\,Bcde + Ace^2\right)x^5 + Aad^2x + \frac{1}{4}\left(Bcd^2 + 2\,Acde + Bae^2\right)x^4 + \frac{1}{3}\left(Acd^2 + 2\,Bade + Aae^2\right)x^3 + \frac{1}{2}\left(Bad^2 + 2\,Aade\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)^2*(c*x^2+a),x, algorithm="maxima")
```

output
```
1/6*B*c*e^2*x^6 + 1/5*(2*B*c*d*e + A*c*e^2)*x^5 + A*a*d^2*x + 1/4*(B*c*d^2
 + 2*A*c*d*e + B*a*e^2)*x^4 + 1/3*(A*c*d^2 + 2*B*a*d*e + A*a*e^2)*x^3 + 1/
2*(B*a*d^2 + 2*A*a*d*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)\,dx = \frac{1}{6}\,Bce^2x^6 + \frac{2}{5}\,Bcdex^5 + \frac{1}{5}\,Ace^2x^5 + \frac{1}{4}\,Bcd^2x^4 + \frac{1}{2}\,Acdex^4 + \frac{1}{4}\,Bae^2x^4 + \frac{1}{3}\,Acd^2x^3 + \frac{2}{3}\,Badex^3 + \frac{1}{3}\,Aae^2x^3 + \frac{1}{2}\,Bad^2x^2 + Aadex^2 + Aad^2x$$

input
```
integrate((B*x+A)*(e*x+d)^2*(c*x^2+a),x, algorithm="giac")
```

output
```
1/6*B*c*e^2*x^6 + 2/5*B*c*d*e*x^5 + 1/5*A*c*e^2*x^5 + 1/4*B*c*d^2*x^4 + 1/
2*A*c*d*e*x^4 + 1/4*B*a*e^2*x^4 + 1/3*A*c*d^2*x^3 + 2/3*B*a*d*e*x^3 + 1/3*
A*a*e^2*x^3 + 1/2*B*a*d^2*x^2 + A*a*d*e*x^2 + A*a*d^2*x
```

Mupad [B] (verification not implemented)

Time = 5.62 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.91

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)\,dx = x^3\left(\frac{A\,c\,d^2}{3}+\frac{2\,B\,a\,d\,e}{3}+\frac{A\,a\,e^2}{3}\right) + x^4\left(\frac{B\,c\,d^2}{4}+\frac{A\,c\,d\,e}{2}+\frac{B\,a\,e^2}{4}\right) + A\,a\,d^2\,x + \frac{a\,d\,x^2\,(2\,A\,e+B\,d)}{2} + \frac{c\,e\,x^5\,(A\,e+2\,B\,d)}{5} + \frac{B\,c\,e^2\,x^6}{6}$$

input
```
int((a + c*x^2)*(A + B*x)*(d + e*x)^2,x)
```

output
```
x^3*((A*a*e^2)/3 + (A*c*d^2)/3 + (2*B*a*d*e)/3) + x^4*((B*a*e^2)/4 + (B*c*
d^2)/4 + (A*c*d*e)/2) + A*a*d^2*x + (a*d*x^2*(2*A*e + B*d))/2 + (c*e*x^5*(
A*e + 2*B*d))/5 + (B*c*e^2*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)\,dx = \frac{x(10bc\,e^2x^5+12ac\,e^2x^4+24bcde\,x^4+15ab\,e^2x^3+30acde\,x^3+15bc\,d^2x^3+20a^2e^2x^2+40abde\,x^2+20a}{60}$$

input
```
int((B*x+A)*(e*x+d)^2*(c*x^2+a),x)
```

output
```
(x*(60*a**2*d**2 + 60*a**2*d*e*x + 20*a**2*e**2*x**2 + 30*a*b*d**2*x + 40*
a*b*d*e*x**2 + 15*a*b*e**2*x**3 + 20*a*c*d**2*x**2 + 30*a*c*d*e*x**3 + 12*
a*c*e**2*x**4 + 15*b*c*d**2*x**3 + 24*b*c*d*e*x**4 + 10*b*c*e**2*x**5))/60
```

3.37 $\int (A+Bx)(d+ex)\left(a+cx^2\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 62

$$\int (A+Bx)(d+ex)\left(a+cx^2\right) \, dx = aAdx+\frac{1}{2}a(Bd+Ae)x^2+\frac{1}{3}(Acd+aBe)x^3 +\frac{1}{4}c(Bd+Ae)x^4+\frac{1}{5}Bcex^5$$

output
```
a*A*d*x+1/2*a*(A*e+B*d)*x^2+1/3*(A*c*d+B*a*e)*x^3+1/4*c*(A*e+B*d)*x^4+1/5*
B*c*e*x^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int (A+Bx)(d+ex)\left(a+cx^2\right) \, dx = aAdx+\frac{1}{2}a(Bd+Ae)x^2+\frac{1}{3}(Acd+aBe)x^3 +\frac{1}{4}c(Bd+Ae)x^4+\frac{1}{5}Bcex^5$$

input
```
Integrate[(A + B*x)*(d + e*x)*(a + c*x^2),x]
```

output
```
a*A*d*x + (a*(B*d + A*e)*x^2)/2 + ((A*c*d + a*B*e)*x^3)/3 + (c*(B*d + A*e)
*x^4)/4 + (B*c*e*x^5)/5
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)(A+Bx)(d+ex)\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(x^2(aBe+Acd)+ax(Ae+Bd)+aAd+cx^3(Ae+Bd)+Bcex^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(aBe+Acd)+\frac{1}{2}ax^2(Ae+Bd)+aAdx+\frac{1}{4}cx^4(Ae+Bd)+\frac{1}{5}Bcex^5$$

input
```
Int[(A + B*x)*(d + e*x)*(a + c*x^2),x]
```

output
```
a*A*d*x + (a*(B*d + A*e)*x^2)/2 + ((A*c*d + a*B*e)*x^3)/3 + (c*(B*d + A*e)
*x^4)/4 + (B*c*e*x^5)/5
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result	size
default	$Aadx+\frac{a(Ae+Bd)x^2}{2}+\frac{(Acd+Bae)x^3}{3}+\frac{c(Ae+Bd)x^4}{4}+\frac{Bce\,x^5}{5}$	55
norman	$\frac{Bce\,x^5}{5}+\left(\frac{1}{4}Ace+\frac{1}{4}Bcd\right)x^4+\left(\frac{Acd}{3}+\frac{Bae}{3}\right)x^3+\left(\frac{1}{2}aAe+\frac{1}{2}Bad\right)x^2+Aadx$	60
orering	$\frac{x(12Bce\,x^4+15Ace\,x^3+15Bcd\,x^3+20Acd\,x^2+20Bae\,x^2+30Aaex+30Badx+60Aad)}{60}$	62
gosper	$\frac{1}{5}Bce\,x^5+\frac{1}{4}x^4Ace+\frac{1}{4}x^4Bcd+\frac{1}{3}x^3Acd+\frac{1}{3}x^3Bae+\frac{1}{2}x^2aAe+\frac{1}{2}x^2Bad+Aadx$	63
risch	$\frac{1}{5}Bce\,x^5+\frac{1}{4}x^4Ace+\frac{1}{4}x^4Bcd+\frac{1}{3}x^3Acd+\frac{1}{3}x^3Bae+\frac{1}{2}x^2aAe+\frac{1}{2}x^2Bad+Aadx$	63
parallelrisch	$\frac{1}{5}Bce\,x^5+\frac{1}{4}x^4Ace+\frac{1}{4}x^4Bcd+\frac{1}{3}x^3Acd+\frac{1}{3}x^3Bae+\frac{1}{2}x^2aAe+\frac{1}{2}x^2Bad+Aadx$	63

input
```
int((B*x+A)*(e*x+d)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
A*a*d*x+1/2*a*(A*e+B*d)*x^2+1/3*(A*c*d+B*a*e)*x^3+1/4*c*(A*e+B*d)*x^4+1/5*
B*c*e*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)\,dx=\frac{1}{5}\,Bcex^5+\frac{1}{4}\,(Bcd+Ace)x^4+Aadx+\frac{1}{3}\,(Acd+Bae)x^3+\frac{1}{2}\,(Bad+Aae)x^2$$

input
```
integrate((B*x+A)*(e*x+d)*(c*x^2+a),x, algorithm="fricas")
```

output
```
1/5*B*c*e*x^5 + 1/4*(B*c*d + A*c*e)*x^4 + A*a*d*x + 1/3*(A*c*d + B*a*e)*x^
3 + 1/2*(B*a*d + A*a*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)\,dx = Aadx + \frac{Bcex^5}{5} + x^4\left(\frac{Ace}{4} + \frac{Bcd}{4}\right) + x^3\left(\frac{Acd}{3} + \frac{Bae}{3}\right) + x^2\left(\frac{Aae}{2} + \frac{Bad}{2}\right)$$

input
```
integrate((B*x+A)*(e*x+d)*(c*x**2+a),x)
```

output
```
A*a*d*x + B*c*e*x**5/5 + x**4*(A*c*e/4 + B*c*d/4) + x**3*(A*c*d/3 + B*a*e/
3) + x**2*(A*a*e/2 + B*a*d/2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)\,dx = \frac{1}{5}\,Bcex^5 + \frac{1}{4}\,(Bcd+Ace)x^4 + Aadx + \frac{1}{3}\,(Acd+Bae)x^3 + \frac{1}{2}\,(Bad+Aae)x^2$$

input
```
integrate((B*x+A)*(e*x+d)*(c*x^2+a),x, algorithm="maxima")
```

output
```
1/5*B*c*e*x^5 + 1/4*(B*c*d + A*c*e)*x^4 + A*a*d*x + 1/3*(A*c*d + B*a*e)*x^
3 + 1/2*(B*a*d + A*a*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)\,dx = \frac{1}{5}Bcex^5+\frac{1}{4}Bcdx^4+\frac{1}{4}Acex^4+\frac{1}{3}Acdx^3+\frac{1}{3}Baex^3+\frac{1}{2}Badx^2+\frac{1}{2}Aaex^2+Aadx$$

input
```
integrate((B*x+A)*(e*x+d)*(c*x^2+a),x, algorithm="giac")
```

output
```
1/5*B*c*e*x^5 + 1/4*B*c*d*x^4 + 1/4*A*c*e*x^4 + 1/3*A*c*d*x^3 + 1/3*B*a*e*
x^3 + 1/2*B*a*d*x^2 + 1/2*A*a*e*x^2 + A*a*d*x
```

Mupad [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)\,dx = \frac{B\,c\,e\,x^5}{5}+\frac{c\,(A\,e+B\,d)\,x^4}{4}+\left(\frac{A\,c\,d}{3}+\frac{B\,a\,e}{3}\right)x^3+\frac{a\,(A\,e+B\,d)\,x^2}{2}+A\,a\,d\,x$$

input
```
int((a + c*x^2)*(A + B*x)*(d + e*x),x)
```

output
```
x^3*((A*c*d)/3 + (B*a*e)/3) + (a*x^2*(A*e + B*d))/2 + (c*x^4*(A*e + B*d))/
4 + (B*c*e*x^5)/5 + A*a*d*x
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)\,dx$$
$$=\frac{x(12bce\,x^4+15ace\,x^3+15bcd\,x^3+20abe\,x^2+20acd\,x^2+30a^2ex+30abdx+60a^2d)}{60}$$

input
```
int((B*x+A)*(e*x+d)*(c*x^2+a),x)
```

output
```
(x*(60*a**2*d + 30*a**2*e*x + 30*a*b*d*x + 20*a*b*e*x**2 + 20*a*c*d*x**2 +
 15*a*c*e*x**3 + 15*b*c*d*x**3 + 12*b*c*e*x**4))/60
```

3.38 $\int (A+Bx)\left(a+cx^2\right)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int (A+Bx)\left(a+cx^2\right)\,dx=aAx+\frac{1}{3}Acx^3+\frac{B(a+cx^2)^2}{4c}$$

output
```
a*A*x+1/3*A*c*x^3+1/4*B*(c*x^2+a)^2/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int (A+Bx)\left(a+cx^2\right)\,dx=aAx+\frac{1}{2}aBx^2+\frac{1}{3}Acx^3+\frac{1}{4}Bcx^4$$

input
```
Integrate[(A + B*x)*(a + c*x^2),x]
```

output
```
a*A*x + (a*B*x^2)/2 + (A*c*x^3)/3 + (B*c*x^4)/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {455, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)(A+Bx)\,dx$$

$$\Big\downarrow \text{455}$$

$$A\int (cx^2+a)\,dx+\frac{B(a+cx^2)^2}{4c}$$

$$\Big\downarrow \text{2009}$$

$$A\left(ax+\frac{cx^3}{3}\right)+\frac{B(a+cx^2)^2}{4c}$$

input
```
Int[(A + B*x)*(a + c*x^2),x]
```

output
```
(B*(a + c*x^2)^2)/(4*c) + A*(a*x + (c*x^3)/3)
```

Defintions of rubi rules used

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
gosper	$\frac{1}{4}Bc\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{2}Ba\,x^2+aAx$	27
default	$\frac{1}{4}Bc\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{2}Ba\,x^2+aAx$	27
norman	$\frac{1}{4}Bc\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{2}Ba\,x^2+aAx$	27
risch	$\frac{1}{4}Bc\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{2}Ba\,x^2+aAx$	27
parallelrisch	$\frac{1}{4}Bc\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{2}Ba\,x^2+aAx$	27
orering	$\frac{x(3Bc\,x^3+4Ac\,x^2+6Bax+12Aa)}{12}$	28

input
```
int((B*x+A)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*B*c*x^4+1/3*A*c*x^3+1/2*B*a*x^2+a*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (A+Bx)\left(a+cx^2\right)\,dx=\frac{1}{4}\,Bcx^4+\frac{1}{3}\,Acx^3+\frac{1}{2}\,Bax^2+Aax$$

input
```
integrate((B*x+A)*(c*x^2+a),x, algorithm="fricas")
```

output
```
1/4*B*c*x^4 + 1/3*A*c*x^3 + 1/2*B*a*x^2 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int (A+Bx)\left(a+cx^2\right)\,dx=Aax+\frac{Acx^3}{3}+\frac{Bax^2}{2}+\frac{Bcx^4}{4}$$

input
```
integrate((B*x+A)*(c*x**2+a),x)
```

output
```
A*a*x + A*c*x**3/3 + B*a*x**2/2 + B*c*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (A+Bx)\left(a+cx^2\right)\,dx=\frac{1}{4}\,Bcx^4+\frac{1}{3}\,Acx^3+\frac{1}{2}\,Bax^2+Aax$$

input
```
integrate((B*x+A)*(c*x^2+a),x, algorithm="maxima")
```

output
```
1/4*B*c*x^4 + 1/3*A*c*x^3 + 1/2*B*a*x^2 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (A+Bx)\left(a+cx^2\right)\,dx=\frac{1}{4}\,Bcx^4+\frac{1}{3}\,Acx^3+\frac{1}{2}\,Bax^2+Aax$$

input
```
integrate((B*x+A)*(c*x^2+a),x, algorithm="giac")
```

output
```
1/4*B*c*x^4 + 1/3*A*c*x^3 + 1/2*B*a*x^2 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (A+Bx)\left(a+cx^2\right)\,dx=\frac{B\,c\,x^4}{4}+\frac{A\,c\,x^3}{3}+\frac{B\,a\,x^2}{2}+A\,a\,x$$

input
```
int((a + c*x^2)*(A + B*x),x)
```

output
```
A*a*x + (B*a*x^2)/2 + (A*c*x^3)/3 + (B*c*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int (A+Bx)\left(a+cx^2\right)\,dx=\frac{x(3bc\,x^3+4ac\,x^2+6abx+12a^2)}{12}$$

input
```
int((B*x+A)*(c*x^2+a),x)
```

output
```
(x*(12*a**2 + 6*a*b*x + 4*a*c*x**2 + 3*b*c*x**3))/12
```

3.39 $\int \frac{(A+Bx)(a+cx^2)}{d+ex} dx$

Optimal result

Integrand size = 20, antiderivative size = 86

$$\int \frac{(A+Bx)\,(a+cx^2)}{d+ex}\,dx = \frac{(Bcd^2-Acde+aBe^2)\,x}{e^3}-\frac{c(Bd-Ae)x^2}{2e^2}+\frac{Bcx^3}{3e}-\frac{(Bd-Ae)\,(cd^2+ae^2)\log(d+ex)}{e^4}$$

output
```
(-A*c*d*e+B*a*e^2+B*c*d^2)*x/e^3-1/2*c*(-A*e+B*d)*x^2/e^2+1/3*B*c*x^3/e-(-
A*e+B*d)*(a*e^2+c*d^2)*ln(e*x+d)/e^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\,(a+cx^2)}{d+ex}\,dx = \frac{ex(6aBe^2+3Ace(-2d+ex)+Bc(6d^2-3dex+2e^2x^2))-6(Bd-Ae)\,(cd^2+ae^2)\log(d+ex)}{6e^4}$$

input
```
Integrate[((A + B*x)*(a + c*x^2))/(d + e*x),x]
```

output
```
(e*x*(6*a*B*e^2 + 3*A*c*e*(-2*d + e*x) + B*c*(6*d^2 - 3*d*e*x + 2*e^2*x^2)
) - 6*(B*d - A*e)*(c*d^2 + a*e^2)*Log[d + e*x])/(6*e^4)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(A+Bx)}{d+ex}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{(ae^2+cd^2)(Ae-Bd)}{e^3(d+ex)}+\frac{aBe^2-Acde+Bcd^2}{e^3}+\frac{cx(Ae-Bd)}{e^2}+\frac{Bcx^2}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(ae^2+cd^2)(Bd-Ae)\log(d+ex)}{e^4}+\frac{x(aBe^2-Acde+Bcd^2)}{e^3}-\frac{cx^2(Bd-Ae)}{2e^2}+\frac{Bcx^3}{3e}$$

input
```
Int[((A + B*x)*(a + c*x^2))/(d + e*x),x]
```

output
```
((B*c*d^2 - A*c*d*e + a*B*e^2)*x)/e^3 - (c*(B*d - A*e)*x^2)/(2*e^2) + (B*c
*x^3)/(3*e) - ((B*d - A*e)*(c*d^2 + a*e^2)*Log[d + e*x])/e^4
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

method	result
norman	$-\frac{(Acde-Ba\,e^2-Bc\,d^2)x}{e^3}+\frac{Bc\,x^3}{3e}+\frac{c(Ae-Bd)x^2}{2e^2}+\frac{(Aa\,e^3+Ac\,d^2e-Bad\,e^2-Bc\,d^3)\ln(ex+d)}{e^4}$
default	$-\frac{-\frac{1}{3}Bc\,x^3e^2-\frac{1}{2}Ac\,e^2x^2+\frac{1}{2}Bcde\,x^2+Acdex-Ba\,e^2x-Bc\,d^2x}{e^3}+\frac{(Aa\,e^3+Ac\,d^2e-Bad\,e^2-Bc\,d^3)\ln(ex+d)}{e^4}$
risch	$\frac{Bc\,x^3}{3e}+\frac{Ac\,x^2}{2e}-\frac{Bcd\,x^2}{2e^2}-\frac{Acdx}{e^2}+\frac{Bax}{e}+\frac{Bc\,d^2x}{e^3}+\frac{\ln(ex+d)Aa}{e}+\frac{\ln(ex+d)Ac\,d^2}{e^3}-\frac{\ln(ex+d)Bad}{e^2}-\frac{\ln(ex}{}$
parallelrisch	$\frac{2Bc\,x^3e^3+3A\,x^2c\,e^3-3B\,x^2cd\,e^2+6A\ln(ex+d)a\,e^3+6A\ln(ex+d)c\,d^2e-6Axcd\,e^2-6B\ln(ex+d)ad\,e^2-6B\ln(ex+d)c\,d^3+6B}{6e^4}$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-(A*c*d*e-B*a*e^2-B*c*d^2)/e^3*x+1/3*B*c*x^3/e+1/2*c/e^2*(A*e-B*d)*x^2+(A*
a*e^3+A*c*d^2*e-B*a*d*e^2-B*c*d^3)/e^4*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\,(a+cx^2)}{d+ex}\,dx$$
$$=\frac{2\,Bce^3x^3-3\,(Bcde^2-Ace^3)x^2+6\,(Bcd^2e-Acde^2+Bae^3)x-6\,(Bcd^3-Acd^2e+Bade^2-Aae^3)\log}{6\,e^4}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d),x, algorithm="fricas")
```

output
```
1/6*(2*B*c*e^3*x^3 - 3*(B*c*d*e^2 - A*c*e^3)*x^2 + 6*(B*c*d^2*e - A*c*d*e^
2 + B*a*e^3)*x - 6*(B*c*d^3 - A*c*d^2*e + B*a*d*e^2 - A*a*e^3)*log(e*x + d
))/e^4
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)(a+cx^2)}{d+ex}\,dx = \frac{Bcx^3}{3e} + x^2\left(\frac{Ac}{2e} - \frac{Bcd}{2e^2}\right) + x\left(-\frac{Acd}{e^2} + \frac{Ba}{e} + \frac{Bcd^2}{e^3}\right) - \frac{(-Ae+Bd)(ae^2+cd^2)\log(d+ex)}{e^4}$$

input
```
integrate((B*x+A)*(c*x**2+a)/(e*x+d),x)
```

output
```
B*c*x**3/(3*e) + x**2*(A*c/(2*e) - B*c*d/(2*e**2)) + x*(-A*c*d/e**2 + B*a/
e + B*c*d**2/e**3) - (-A*e + B*d)*(a*e**2 + c*d**2)*log(d + e*x)/e**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx)(a+cx^2)}{d+ex}\,dx = \frac{2\,Bce^2x^3 - 3\,(Bcde - Ace^2)x^2 + 6\,(Bcd^2 - Acde + Bae^2)x}{6\,e^3} - \frac{(Bcd^3 - Acd^2e + Bade^2 - Aae^3)\log(ex+d)}{e^4}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d),x, algorithm="maxima")
```

output
```
1/6*(2*B*c*e^2*x^3 - 3*(B*c*d*e - A*c*e^2)*x^2 + 6*(B*c*d^2 - A*c*d*e + B*
a*e^2)*x)/e^3 - (B*c*d^3 - A*c*d^2*e + B*a*d*e^2 - A*a*e^3)*log(e*x + d)/e
^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx)(a+cx^2)}{d+ex}\,dx = \frac{2\,Bce^2x^3-3\,Bcdex^2+3\,Ace^2x^2+6\,Bcd^2x-6\,Acdex+6\,Bae^2x}{6\,e^3} - \frac{(Bcd^3-Acd^2e+Bade^2-Aae^3)\log(|ex+d|)}{e^4}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d),x, algorithm="giac")
```

output
```
1/6*(2*B*c*e^2*x^3 - 3*B*c*d*e*x^2 + 3*A*c*e^2*x^2 + 6*B*c*d^2*x - 6*A*c*d
*e*x + 6*B*a*e^2*x)/e^3 - (B*c*d^3 - A*c*d^2*e + B*a*d*e^2 - A*a*e^3)*log(
abs(e*x + d))/e^4
```

Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx)(a+cx^2)}{d+ex}\,dx = x^2\left(\frac{A\,c}{2\,e}-\frac{B\,c\,d}{2\,e^2}\right)+x\left(\frac{B\,a}{e}-\frac{d\left(\frac{A\,c}{e}-\frac{B\,c\,d}{e^2}\right)}{e}\right) + \frac{\ln(d+e\,x)\,\left(-B\,c\,d^3+A\,c\,d^2\,e-B\,a\,d\,e^2+A\,a\,e^3\right)}{e^4} + \frac{B\,c\,x^3}{3\,e}$$

input
```
int(((a + c*x^2)*(A + B*x))/(d + e*x),x)
```

output
```
x^2*((A*c)/(2*e) - (B*c*d)/(2*e^2)) + x*((B*a)/e - (d*((A*c)/e - (B*c*d)/e
^2))/e) + (log(d + e*x)*(A*a*e^3 - B*c*d^3 - B*a*d*e^2 + A*c*d^2*e))/e^4 +
 (B*c*x^3)/(3*e)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx)(a+cx^2)}{d+ex}\,dx = \frac{6\log(ex+d)\,a^2e^3 - 6\log(ex+d)\,abd\,e^2 + 6\log(ex+d)\,ac\,d^2e - 6\log(ex+d)\,bc\,d^3 + 6ab\,e^3x - 6acd\,e^2}{6e^4}$$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d),x)
```

output
```
(6*log(d + e*x)*a**2*e**3 - 6*log(d + e*x)*a*b*d*e**2 + 6*log(d + e*x)*a*c
*d**2*e - 6*log(d + e*x)*b*c*d**3 + 6*a*b*e**3*x - 6*a*c*d*e**2*x + 3*a*c*
e**3*x**2 + 6*b*c*d**2*e*x - 3*b*c*d*e**2*x**2 + 2*b*c*e**3*x**3)/(6*e**4)
```

3.40 $\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^2}\,dx = -\frac{c(2Bd-Ae)x}{e^3}+\frac{Bcx^2}{2e^2}+\frac{(Bd-Ae)(cd^2+ae^2)}{e^4(d+ex)}+\frac{(3Bcd^2-2Acde+aBe^2)\log(d+ex)}{e^4}$$

output
```
-c*(-A*e+2*B*d)*x/e^3+1/2*B*c*x^2/e^2+(-A*e+B*d)*(a*e^2+c*d^2)/e^4/(e*x+d)
+(-2*A*c*d*e+B*a*e^2+3*B*c*d^2)*ln(e*x+d)/e^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^2}\,dx = \frac{2ce(-2Bd+Ae)x+Bce^2x^2+\frac{2(Bd-Ae)(cd^2+ae^2)}{d+ex}+2(3Bcd^2-2Acde+aBe^2)\log(d+ex)}{2e^4}$$

input
```
Integrate[((A + B*x)*(a + c*x^2))/(d + e*x)^2,x]
```

output
```
(2*c*e*(-2*B*d + A*e)*x + B*c*e^2*x^2 + (2*(B*d - A*e)*(c*d^2 + a*e^2))/(d
 + e*x) + 2*(3*B*c*d^2 - 2*A*c*d*e + a*B*e^2)*Log[d + e*x])/(2*e^4)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(A+Bx)}{(d+ex)^2}dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{aBe^2-2Acde+3Bcd^2}{e^3(d+ex)}+\frac{(ae^2+cd^2)(Ae-Bd)}{e^3(d+ex)^2}+\frac{c(Ae-2Bd)}{e^3}+\frac{Bcx}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ae^2+cd^2)(Bd-Ae)}{e^4(d+ex)}+\frac{\log(d+ex)(aBe^2-2Acde+3Bcd^2)}{e^4}-\frac{cx(2Bd-Ae)}{e^3}+\frac{Bcx^2}{2e^2}$$

input
```
Int[((A + B*x)*(a + c*x^2))/(d + e*x)^2,x]
```

output
```
-((c*(2*B*d - A*e)*x)/e^3) + (B*c*x^2)/(2*e^2) + ((B*d - A*e)*(c*d^2 + a*e
^2))/(e^4*(d + e*x)) + ((3*B*c*d^2 - 2*A*c*d*e + a*B*e^2)*Log[d + e*x])/e^
4
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

method	result
default	$\frac{c\left(\frac{1}{2}Be\,x^2+Aex-2Bdx\right)}{e^3}+\frac{\left(-2Acde+Ba\,e^2+3Bc\,d^2\right)\ln(ex+d)}{e^4}-\frac{Aa\,e^3+Ac\,d^2e-Bad\,e^2-Bc\,d^3}{e^4(ex+d)}$
norman	$\frac{-\frac{Aa\,e^3+2Ac\,d^2e-Bad\,e^2-3Bc\,d^3}{e^4}+\frac{Bc\,x^3}{2e}+\frac{c(2Ae-3Bd)x^2}{2e^2}}{ex+d}-\frac{\left(2Acde-Ba\,e^2-3Bc\,d^2\right)\ln(ex+d)}{e^4}$
risch	$\frac{Bc\,x^2}{2e^2}+\frac{cAx}{e^2}-\frac{2cBdx}{e^3}-\frac{Aa}{e(ex+d)}-\frac{Ac\,d^2}{e^3(ex+d)}+\frac{Bad}{e^2(ex+d)}+\frac{Bc\,d^3}{e^4(ex+d)}-\frac{2\ln(ex+d)Acd}{e^3}+\frac{\ln(ex+d)Ba}{e^2}+$
parallelrisch	$-\frac{-Bc\,x^3e^3+4A\ln(ex+d)xcd\,e^2-2A\,x^2c\,e^3-2B\ln(ex+d)xa\,e^3-6B\ln(ex+d)xc\,d^2e+3B\,x^2cd\,e^2+4A\ln(ex+d)c\,d^2e-2B\ln}{2e^4(ex+d)}$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c/e^3*(1/2*B*e*x^2+A*e*x-2*B*d*x)+(-2*A*c*d*e+B*a*e^2+3*B*c*d^2)*ln(e*x+d)
/e^4-(A*a*e^3+A*c*d^2*e-B*a*d*e^2-B*c*d^3)/e^4/(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.71

$$\int \frac{(A+Bx)\left(a+cx^2\right)}{(d+ex)^2}\,dx$$
$$=\frac{Bce^3x^3+2\,Bcd^3-2\,Acd^2e+2\,Bade^2-2\,Aae^3-(3\,Bcde^2-2\,Ace^3)x^2-2\,(2\,Bcd^2e-Acde^2)x+2\,(3}{2\,(e^5x+de^4)}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/2*(B*c*e^3*x^3 + 2*B*c*d^3 - 2*A*c*d^2*e + 2*B*a*d*e^2 - 2*A*a*e^3 - (3*
B*c*d*e^2 - 2*A*c*e^3)*x^2 - 2*(2*B*c*d^2*e - A*c*d*e^2)*x + 2*(3*B*c*d^3
- 2*A*c*d^2*e + B*a*d*e^2 + (3*B*c*d^2*e - 2*A*c*d*e^2 + B*a*e^3)*x)*log(e
*x + d))/(e^5*x + d*e^4)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^2}\,dx = \frac{Bcx^2}{2e^2} + x\left(\frac{Ac}{e^2} - \frac{2Bcd}{e^3}\right) + \frac{-Aae^3 - Acd^2e + Bade^2 + Bcd^3}{de^4 + e^5x} + \frac{(-2Acde + Bae^2 + 3Bcd^2)\log(d+ex)}{e^4}$$

input
```
integrate((B*x+A)*(c*x**2+a)/(e*x+d)**2,x)
```

output
```
B*c*x**2/(2*e**2) + x*(A*c/e**2 - 2*B*c*d/e**3) + (-A*a*e**3 - A*c*d**2*e
+ B*a*d*e**2 + B*c*d**3)/(d*e**4 + e**5*x) + (-2*A*c*d*e + B*a*e**2 + 3*B*
c*d**2)*log(d + e*x)/e**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^2}\,dx = \frac{Bcd^3 - Acd^2e + Bade^2 - Aae^3}{e^5x + de^4} + \frac{Bcex^2 - 2(2\,Bcd - Ace)x}{2\,e^3} + \frac{(3\,Bcd^2 - 2\,Acde + Bae^2)\log(ex+d)}{e^4}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
(B*c*d^3 - A*c*d^2*e + B*a*d*e^2 - A*a*e^3)/(e^5*x + d*e^4) + 1/2*(B*c*e*x
^2 - 2*(2*B*c*d - A*c*e)*x)/e^3 + (3*B*c*d^2 - 2*A*c*d*e + B*a*e^2)*log(e*
x + d)/e^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.72

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^2}\,dx = \frac{\left(Bc-\frac{2\,(3\,Bcde-Ace^2)}{(ex+d)e}\right)(ex+d)^2}{2\,e^4} - \frac{(3\,Bcd^2-2\,Acde+Bae^2)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^4} + \frac{\frac{Bcd^3e^2}{ex+d}-\frac{Acd^2e^3}{ex+d}+\frac{Bade^4}{ex+d}-\frac{Aae^5}{ex+d}}{e^6}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/2*(B*c - 2*(3*B*c*d*e - A*c*e^2)/((e*x + d)*e))*(e*x + d)^2/e^4 - (3*B*c
*d^2 - 2*A*c*d*e + B*a*e^2)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^4 + (
B*c*d^3*e^2/(e*x + d) - A*c*d^2*e^3/(e*x + d) + B*a*d*e^4/(e*x + d) - A*a*
e^5/(e*x + d))/e^6
```

Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^2}\,dx = x\left(\frac{A\,c}{e^2}-\frac{2\,B\,c\,d}{e^3}\right) - \frac{-B\,c\,d^3+A\,c\,d^2\,e-B\,a\,d\,e^2+A\,a\,e^3}{e\,\left(x\,e^4+d\,e^3\right)} + \frac{\ln\left(d+e\,x\right)\,\left(3\,B\,c\,d^2-2\,A\,c\,d\,e+B\,a\,e^2\right)}{e^4} + \frac{B\,c\,x^2}{2\,e^2}$$

input
```
int(((a + c*x^2)*(A + B*x))/(d + e*x)^2,x)
```

output
```
x*((A*c)/e^2 - (2*B*c*d)/e^3) - (A*a*e^3 - B*c*d^3 - B*a*d*e^2 + A*c*d^2*e
)/(e*(d*e^3 + e^4*x)) + (log(d + e*x)*(B*a*e^2 + 3*B*c*d^2 - 2*A*c*d*e))/e
^4 + (B*c*x^2)/(2*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.00

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^2}\,dx = \frac{2\log(ex+d)\,ab\,d^2e^2+2\log(ex+d)\,abd\,e^3x-4\log(ex+d)\,ac\,d^3e-4\log(ex+d)\,ac\,d^2e^2x+6\log(ex+}{2d}$$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^2,x)
```

output
```
(2*log(d + e*x)*a*b*d**2*e**2 + 2*log(d + e*x)*a*b*d*e**3*x - 4*log(d + e*
x)*a*c*d**3*e - 4*log(d + e*x)*a*c*d**2*e**2*x + 6*log(d + e*x)*b*c*d**4 +
 6*log(d + e*x)*b*c*d**3*e*x + 2*a**2*e**4*x - 2*a*b*d*e**3*x + 4*a*c*d**2
*e**2*x + 2*a*c*d*e**3*x**2 - 6*b*c*d**3*e*x - 3*b*c*d**2*e**2*x**2 + b*c*
d*e**3*x**3)/(2*d*e**4*(d + e*x))
```

3.41 $\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 94

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^3}\,dx = \frac{Bcx}{e^3} + \frac{(Bd-Ae)\,(cd^2+ae^2)}{2e^4(d+ex)^2} - \frac{3Bcd^2-2Acde+aBe^2}{e^4(d+ex)} - \frac{c(3Bd-Ae)\log(d+ex)}{e^4}$$

output
```
B*c*x/e^3+1/2*(-A*e+B*d)*(a*e^2+c*d^2)/e^4/(e*x+d)^2-(-2*A*c*d*e+B*a*e^2+3
*B*c*d^2)/e^4/(e*x+d)-c*(-A*e+3*B*d)*ln(e*x+d)/e^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^3}\,dx = \frac{2Bcex + \frac{(Bd-Ae)(cd^2+ae^2)}{(d+ex)^2} - \frac{2(3Bcd^2-2Acde+aBe^2)}{d+ex} + 2(-3Bcd+Ace)\log(d+ex)}{2e^4}$$

input
```
Integrate[((A + B*x)*(a + c*x^2))/(d + e*x)^3,x]
```

output
```
(2*B*c*e*x + ((B*d - A*e)*(c*d^2 + a*e^2))/(d + e*x)^2 - (2*(3*B*c*d^2 - 2
*A*c*d*e + a*B*e^2))/(d + e*x) + 2*(-3*B*c*d + A*c*e)*Log[d + e*x])/(2*e^4
)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(A+Bx)}{(d+ex)^3}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{aBe^2-2Acde+3Bcd^2}{e^3(d+ex)^2}+\frac{(ae^2+cd^2)(Ae-Bd)}{e^3(d+ex)^3}+\frac{c(Ae-3Bd)}{e^3(d+ex)}+\frac{Bc}{e^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ae^2+cd^2)(Bd-Ae)}{2e^4(d+ex)^2}-\frac{aBe^2-2Acde+3Bcd^2}{e^4(d+ex)}-\frac{c(3Bd-Ae)\log(d+ex)}{e^4}+\frac{Bcx}{e^3}$$

input
```
Int[((A + B*x)*(a + c*x^2))/(d + e*x)^3,x]
```

output
```
(B*c*x)/e^3 + ((B*d - A*e)*(c*d^2 + a*e^2))/(2*e^4*(d + e*x)^2) - (3*B*c*d
^2 - 2*A*c*d*e + a*B*e^2)/(e^4*(d + e*x)) - (c*(3*B*d - A*e)*Log[d + e*x])
/e^4
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

method	result
norman	$\frac{\frac{\left(2Acde-Ba\,e^{2}-6Bc\,d^{2}\right)x}{e^{3}}+\frac{Bc\,x^{3}}{e}-\frac{Aa\,e^{3}-3Ac\,d^{2}e+Bad\,e^{2}+9Bc\,d^{3}}{2e^{4}}}{(ex+d)^{2}}+\frac{c(Ae-3Bd)\ln(ex+d)}{e^{4}}$
default	$\frac{Bcx}{e^{3}}+\frac{c(Ae-3Bd)\ln(ex+d)}{e^{4}}-\frac{-2Acde+Ba\,e^{2}+3Bc\,d^{2}}{e^{4}(ex+d)}-\frac{Aa\,e^{3}+Ac\,d^{2}e-Bad\,e^{2}-Bc\,d^{3}}{2e^{4}(ex+d)^{2}}$
risch	$\frac{Bcx}{e^{3}}+\frac{(2Acde-Ba\,e^{2}-3Bc\,d^{2})x-\frac{Aa\,e^{3}-3Ac\,d^{2}e+Bad\,e^{2}+5Bc\,d^{3}}{2e}}{e^{3}(ex+d)^{2}}+\frac{c\ln(ex+d)A}{e^{3}}-\frac{3c\ln(ex+d)Bd}{e^{4}}$
parallelrisch	$\frac{2A\ln(ex+d)x^{2}c\,e^{3}-6B\ln(ex+d)x^{2}cd\,e^{2}+2Bc\,x^{3}e^{3}+4A\ln(ex+d)xcd\,e^{2}-12B\ln(ex+d)xc\,d^{2}e+2A\ln(ex+d)c\,d^{2}e+4Axcd\,e\cdots}{2e^{4}(ex+d)^{2}}$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
((2*A*c*d*e-B*a*e^2-6*B*c*d^2)/e^3*x+B*c*x^3/e-1/2*(A*a*e^3-3*A*c*d^2*e+B*
a*d*e^2+9*B*c*d^3)/e^4)/(e*x+d)^2+c/e^4*(A*e-3*B*d)*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.79

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^3}\,dx$$
$$=\frac{2\,Bce^3x^3+4\,Bcde^2x^2-5\,Bcd^3+3\,Acd^2e-Bade^2-Aae^3-2\,(2\,Bcd^2e-2\,Acde^2+Bae^3)x-2\,(3\,Bc\cdots}{2\,(e^6x^2+2\,de^5x+d^2e^4)}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*(2*B*c*e^3*x^3 + 4*B*c*d*e^2*x^2 - 5*B*c*d^3 + 3*A*c*d^2*e - B*a*d*e^2
 - A*a*e^3 - 2*(2*B*c*d^2*e - 2*A*c*d*e^2 + B*a*e^3)*x - 2*(3*B*c*d^3 - A*
c*d^2*e + (3*B*c*d*e^2 - A*c*e^3)*x^2 + 2*(3*B*c*d^2*e - A*c*d*e^2)*x)*log
(e*x + d))/(e^6*x^2 + 2*d*e^5*x + d^2*e^4)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^3}\,dx = \frac{Bcx}{e^3} - \frac{c(-Ae+3Bd)\log(d+ex)}{e^4} + \frac{-Aae^3+3Acd^2e-Bade^2-5Bcd^3+x(4Acde^2-2Bae^3-6Bcd^2e)}{2d^2e^4+4de^5x+2e^6x^2}$$

input
```
integrate((B*x+A)*(c*x**2+a)/(e*x+d)**3,x)
```

output
```
B*c*x/e**3 - c*(-A*e + 3*B*d)*log(d + e*x)/e**4 + (-A*a*e**3 + 3*A*c*d**2*
e - B*a*d*e**2 - 5*B*c*d**3 + x*(4*A*c*d*e**2 - 2*B*a*e**3 - 6*B*c*d**2*e)
)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^3}\,dx = -\frac{5\,Bcd^3-3\,Acd^2e+Bade^2+Aae^3+2\,(3\,Bcd^2e-2\,Acde^2+Bae^3)x}{2\,(e^6x^2+2\,de^5x+d^2e^4)} + \frac{Bcx}{e^3} - \frac{(3\,Bcd-Ace)\log(ex+d)}{e^4}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/2*(5*B*c*d^3 - 3*A*c*d^2*e + B*a*d*e^2 + A*a*e^3 + 2*(3*B*c*d^2*e - 2*A
*c*d*e^2 + B*a*e^3)*x)/(e^6*x^2 + 2*d*e^5*x + d^2*e^4) + B*c*x/e^3 - (3*B*
c*d - A*c*e)*log(e*x + d)/e^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^3}\,dx = \frac{Bcx}{e^3} - \frac{(3\,Bcd-Ace)\log(|ex+d|)}{e^4} - \frac{5\,Bcd^3-3\,Acd^2e+Bade^2+Aae^3+2\,(3\,Bcd^2e-2\,Acde^2+Bae^3)x}{2\,(ex+d)^2e^4}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^3,x, algorithm="giac")
```

output
```
B*c*x/e^3 - (3*B*c*d - A*c*e)*log(abs(e*x + d))/e^4 - 1/2*(5*B*c*d^3 - 3*A
*c*d^2*e + B*a*d*e^2 + A*a*e^3 + 2*(3*B*c*d^2*e - 2*A*c*d*e^2 + B*a*e^3)*x
)/((e*x + d)^2*e^4)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^3}\,dx = \frac{\ln(d+e\,x)\,(A\,c\,e-3\,B\,c\,d)}{e^4} - \frac{\frac{5\,B\,c\,d^3-3\,A\,c\,d^2\,e+B\,a\,d\,e^2+A\,a\,e^3}{2\,e}+x\,(3\,B\,c\,d^2-2\,A\,c\,d\,e+B\,a\,e^2)}{d^2\,e^3+2\,d\,e^4\,x+e^5\,x^2} + \frac{B\,c\,x}{e^3}$$

input
```
int(((a + c*x^2)*(A + B*x))/(d + e*x)^3,x)
```

output
```
(log(d + e*x)*(A*c*e - 3*B*c*d))/e^4 - ((A*a*e^3 + 5*B*c*d^3 + B*a*d*e^2 -
 3*A*c*d^2*e)/(2*e) + x*(B*a*e^2 + 3*B*c*d^2 - 2*A*c*d*e))/(d^2*e^3 + e^5*
x^2 + 2*d*e^4*x) + (B*c*x)/e^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.01

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{2\log(ex+d)\,ac\,d^3e + 4\log(ex+d)\,ac\,d^2e^2x + 2\log(ex+d)\,acd\,e^3x^2 - 6\log(ex+d)\,bc\,d^4 - 12\log(ex+}{2d\,e^4\,(e^2x}$$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^3,x)
```

output
```
(2*log(d + e*x)*a*c*d**3*e + 4*log(d + e*x)*a*c*d**2*e**2*x + 2*log(d + e*
x)*a*c*d*e**3*x**2 - 6*log(d + e*x)*b*c*d**4 - 12*log(d + e*x)*b*c*d**3*e*
x - 6*log(d + e*x)*b*c*d**2*e**2*x**2 - a**2*d*e**3 + a*b*e**4*x**2 + a*c*
d**3*e - 2*a*c*d*e**3*x**2 - 3*b*c*d**4 + 6*b*c*d**2*e**2*x**2 + 2*b*c*d*e
**3*x**3)/(2*d*e**4*(d**2 + 2*d*e*x + e**2*x**2))
```

3.42 $\int \frac{(A+Bx)\left(a+cx^2\right)}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int \frac{(A+Bx)\left(a+cx^2\right)}{(d+ex)^4}\,dx = \frac{(Bd-Ae)\left(cd^2+ae^2\right)}{3e^4(d+ex)^3} - \frac{3Bcd^2-2Acde+aBe^2}{2e^4(d+ex)^2} + \frac{c(3Bd-Ae)}{e^4(d+ex)} + \frac{Bc\log(d+ex)}{e^4}$$

output
```
1/3*(-A*e+B*d)*(a*e^2+c*d^2)/e^4/(e*x+d)^3-1/2*(-2*A*c*d*e+B*a*e^2+3*B*c*d
^2)/e^4/(e*x+d)^2+c*(-A*e+3*B*d)/e^4/(e*x+d)+B*c*ln(e*x+d)/e^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(a+cx^2\right)}{(d+ex)^4}\,dx = \frac{-2Ae(ae^2+c(d^2+3dex+3e^2x^2))+B(-ae^2(d+3ex)+cd(11d^2+27dex+18e^2x^2))+6Bc(d+ex)^3}{6e^4(d+ex)^3}$$

input
```
Integrate[((A + B*x)*(a + c*x^2))/(d + e*x)^4,x]
```

output
```
(-2*A*e*(a*e^2 + c*(d^2 + 3*d*e*x + 3*e^2*x^2)) + B*(-(a*e^2*(d + 3*e*x))
+ c*d*(11*d^2 + 27*d*e*x + 18*e^2*x^2)) + 6*B*c*(d + e*x)^3*Log[d + e*x])/
(6*e^4*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(A+Bx)}{(d+ex)^4}dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{aBe^2-2Acde+3Bcd^2}{e^3(d+ex)^3}+\frac{(ae^2+cd^2)(Ae-Bd)}{e^3(d+ex)^4}+\frac{c(Ae-3Bd)}{e^3(d+ex)^2}+\frac{Bc}{e^3(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aBe^2-2Acde+3Bcd^2}{2e^4(d+ex)^2}+\frac{(ae^2+cd^2)(Bd-Ae)}{3e^4(d+ex)^3}+\frac{c(3Bd-Ae)}{e^4(d+ex)}+\frac{Bc\log(d+ex)}{e^4}$$

input
```
Int[((A + B*x)*(a + c*x^2))/(d + e*x)^4,x]
```

output
```
((B*d - A*e)*(c*d^2 + a*e^2))/(3*e^4*(d + e*x)^3) - (3*B*c*d^2 - 2*A*c*d*e
 + a*B*e^2)/(2*e^4*(d + e*x)^2) + (c*(3*B*d - A*e))/(e^4*(d + e*x)) + (B*c
*Log[d + e*x])/e^4
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

method	result
risch	$\frac{-\frac{c(Ae-3Bd)x^2}{e^2}-\frac{\left(2Acde+Ba\,e^2-9Bc\,d^2\right)x}{2e^3}-\frac{2Aa\,e^3+2Ac\,d^2e+Bad\,e^2-11Bc\,d^3}{6e^4}}{(ex+d)^3}+\frac{Bc\ln(ex+d)}{e^4}$
norman	$\frac{-\frac{2Aa\,e^3+2Ac\,d^2e+Bad\,e^2-11Bc\,d^3}{6e^4}-\frac{(Ace-3Bcd)x^2}{e^2}-\frac{\left(2Acde+Ba\,e^2-9Bc\,d^2\right)x}{2e^3}}{(ex+d)^3}+\frac{Bc\ln(ex+d)}{e^4}$
default	$-\frac{Aa\,e^3+Ac\,d^2e-Bad\,e^2-Bc\,d^3}{3e^4(ex+d)^3}+\frac{Bc\ln(ex+d)}{e^4}-\frac{c(Ae-3Bd)}{e^4(ex+d)}-\frac{-2Acde+Ba\,e^2+3Bc\,d^2}{2e^4(ex+d)^2}$
parallelrisch	$-\frac{-6B\ln(ex+d)x^3c\,e^3-18B\ln(ex+d)x^2cd\,e^2+6A\,x^2c\,e^3-18B\ln(ex+d)xc\,d^2e-18B\,x^2cd\,e^2+6Axcd\,e^2-6B\ln(ex+d)c\,d^3}{6e^4(ex+d)^3}$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
(-c*(A*e-3*B*d)/e^2*x^2-1/2*(2*A*c*d*e+B*a*e^2-9*B*c*d^2)/e^3*x-1/6*(2*A*a
*e^3+2*A*c*d^2*e+B*a*d*e^2-11*B*c*d^3)/e^4)/(e*x+d)^3+B*c*ln(e*x+d)/e^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.58

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^4}\,dx$$
$$=\frac{11\,Bcd^3-2\,Acd^2e-Bade^2-2\,Aae^3+6\,(3\,Bcde^2-Ace^3)x^2+3\,(9\,Bcd^2e-2\,Acde^2-Bae^3)x+6\,(Bc}{6\,(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4)}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/6*(11*B*c*d^3 - 2*A*c*d^2*e - B*a*d*e^2 - 2*A*a*e^3 + 6*(3*B*c*d*e^2 - A
*c*e^3)*x^2 + 3*(9*B*c*d^2*e - 2*A*c*d*e^2 - B*a*e^3)*x + 6*(B*c*e^3*x^3 +
 3*B*c*d*e^2*x^2 + 3*B*c*d^2*e*x + B*c*d^3)*log(e*x + d))/(e^7*x^3 + 3*d*e
^6*x^2 + 3*d^2*e^5*x + d^3*e^4)
```

Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^4}\,dx = \frac{Bc\log(d+ex)}{e^4} + \frac{-2Aae^3-2Acd^2e-Bade^2+11Bcd^3+x^2(-6Ace^3+18Bcde^2)+x(-6Acde^2-3Bae^3+27Bcd^2e)}{6d^3e^4+18d^2e^5x+18de^6x^2+6e^7x^3}$$

input
```
integrate((B*x+A)*(c*x**2+a)/(e*x+d)**4,x)
```

output
```
B*c*log(d + e*x)/e**4 + (-2*A*a*e**3 - 2*A*c*d**2*e - B*a*d*e**2 + 11*B*c*
d**3 + x**2*(-6*A*c*e**3 + 18*B*c*d*e**2) + x*(-6*A*c*d*e**2 - 3*B*a*e**3
+ 27*B*c*d**2*e))/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*
x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^4}\,dx = \frac{11\,Bcd^3-2\,Acd^2e-Bade^2-2\,Aae^3+6\,(3\,Bcde^2-Ace^3)x^2+3\,(9\,Bcd^2e-2\,Acde^2-Bae^3)x}{6\,(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3e^4)} + \frac{Bc\log(ex+d)}{e^4}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
1/6*(11*B*c*d^3 - 2*A*c*d^2*e - B*a*d*e^2 - 2*A*a*e^3 + 6*(3*B*c*d*e^2 - A
*c*e^3)*x^2 + 3*(9*B*c*d^2*e - 2*A*c*d*e^2 - B*a*e^3)*x)/(e^7*x^3 + 3*d*e^
6*x^2 + 3*d^2*e^5*x + d^3*e^4) + B*c*log(e*x + d)/e^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^4}\,dx = \frac{Bc\log(|ex+d|)}{e^4} + \frac{6\,(3\,Bcde-Ace^2)x^2+3\,(9\,Bcd^2-2\,Acde-Bae^2)x+\frac{11\,Bcd^3-2\,Acd^2e-Bade^2-2\,Aae^3}{e}}{6\,(ex+d)^3e^3}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^4,x, algorithm="giac")
```

output
```
B*c*log(abs(e*x + d))/e^4 + 1/6*(6*(3*B*c*d*e - A*c*e^2)*x^2 + 3*(9*B*c*d^
2 - 2*A*c*d*e - B*a*e^2)*x + (11*B*c*d^3 - 2*A*c*d^2*e - B*a*d*e^2 - 2*A*a
*e^3)/e)/((e*x + d)^3*e^3)
```

Mupad [B] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^4}\,dx = \frac{B\,c\,\ln(d+e\,x)}{e^4} - \frac{\frac{-11\,B\,c\,d^3+2\,A\,c\,d^2\,e+B\,a\,d\,e^2+2\,A\,a\,e^3}{6\,e^4}+\frac{x\,(-9\,B\,c\,d^2+2\,A\,c\,d\,e+B\,a\,e^2)}{2\,e^3}+\frac{c\,x^2\,(A\,e-3\,B\,d)}{e^2}}{d^3+3\,d^2\,e\,x+3\,d\,e^2\,x^2+e^3\,x^3}$$

input
```
int(((a + c*x^2)*(A + B*x))/(d + e*x)^4,x)
```

output
```
(B*c*log(d + e*x))/e^4 - ((2*A*a*e^3 - 11*B*c*d^3 + B*a*d*e^2 + 2*A*c*d^2*
e)/(6*e^4) + (x*(B*a*e^2 - 9*B*c*d^2 + 2*A*c*d*e))/(2*e^3) + (c*x^2*(A*e -
 3*B*d))/e^2)/(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.65

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^4}\,dx$$
$$= \frac{6\log(ex+d)\,bc\,d^4+18\log(ex+d)\,bc\,d^3ex+18\log(ex+d)\,bc\,d^2e^2x^2+6\log(ex+d)\,bcd\,e^3x^3-2a^2d\,e^3}{6d\,e^4\,(e^3x^3+3d\,e^2x^2+3d^2ex+d^3)}$$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^4,x)
```

output
```
(6*log(d + e*x)*b*c*d**4 + 18*log(d + e*x)*b*c*d**3*e*x + 18*log(d + e*x)*
b*c*d**2*e**2*x**2 + 6*log(d + e*x)*b*c*d*e**3*x**3 - 2*a**2*d*e**3 - a*b*
d**2*e**2 - 3*a*b*d*e**3*x + 2*a*c*e**4*x**3 + 5*b*c*d**4 + 9*b*c*d**3*e*x
 - 6*b*c*d*e**3*x**3)/(6*d*e**4*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*
x**3))
```

3.43 $\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 106

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^5}\,dx = \frac{(Bd-Ae)(cd^2+ae^2)}{4e^4(d+ex)^4} - \frac{3Bcd^2-2Acde+aBe^2}{3e^4(d+ex)^3} + \frac{c(3Bd-Ae)}{2e^4(d+ex)^2} - \frac{Bc}{e^4(d+ex)}$$

output
```
1/4*(-A*e+B*d)*(a*e^2+c*d^2)/e^4/(e*x+d)^4-1/3*(-2*A*c*d*e+B*a*e^2+3*B*c*d
^2)/e^4/(e*x+d)^3+1/2*c*(-A*e+3*B*d)/e^4/(e*x+d)^2-B*c/e^4/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^5}\,dx = -\frac{3aAe^3+aBe^2(d+4ex)+Ace(d^2+4dex+6e^2x^2)+3Bc(d^3+4d^2ex+6de^2x^2+4e^3x^3)}{12e^4(d+ex)^4}$$

input
```
Integrate[((A + B*x)*(a + c*x^2))/(d + e*x)^5,x]
```

output
```
-1/12*(3*a*A*e^3 + a*B*e^2*(d + 4*e*x) + A*c*e*(d^2 + 4*d*e*x + 6*e^2*x^2)
 + 3*B*c*(d^3 + 4*d^2*e*x + 6*d*e^2*x^2 + 4*e^3*x^3))/(e^4*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(A+Bx)}{(d+ex)^5}dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{aBe^2-2Acde+3Bcd^2}{e^3(d+ex)^4}+\frac{(ae^2+cd^2)(Ae-Bd)}{e^3(d+ex)^5}+\frac{c(Ae-3Bd)}{e^3(d+ex)^3}+\frac{Bc}{e^3(d+ex)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aBe^2-2Acde+3Bcd^2}{3e^4(d+ex)^3}+\frac{(ae^2+cd^2)(Bd-Ae)}{4e^4(d+ex)^4}+\frac{c(3Bd-Ae)}{2e^4(d+ex)^2}-\frac{Bc}{e^4(d+ex)}$$

input
```
Int[((A + B*x)*(a + c*x^2))/(d + e*x)^5,x]
```

output
```
((B*d - A*e)*(c*d^2 + a*e^2))/(4*e^4*(d + e*x)^4) - (3*B*c*d^2 - 2*A*c*d*e
 + a*B*e^2)/(3*e^4*(d + e*x)^3) + (c*(3*B*d - A*e))/(2*e^4*(d + e*x)^2) -
(B*c)/(e^4*(d + e*x))
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{-\frac{Bc\,x^{3}}{e}-\frac{c(Ae+3Bd)x^{2}}{2e^{2}}-\frac{\left(Acde+Ba\,e^{2}+3Bc\,d^{2}\right)x}{3e^{3}}-\frac{3Aa\,e^{3}+Ac\,d^{2}e+Bad\,e^{2}+3Bc\,d^{3}}{12e^{4}}}{(ex+d)^{4}}$	96
norman	$\frac{-\frac{Bc\,x^{3}}{e}-\frac{(Ace+3Bcd)x^{2}}{2e^{2}}-\frac{\left(Acde+Ba\,e^{2}+3Bc\,d^{2}\right)x}{3e^{3}}-\frac{3Aa\,e^{3}+Ac\,d^{2}e+Bad\,e^{2}+3Bc\,d^{3}}{12e^{4}}}{(ex+d)^{4}}$	97
gosper	$-\frac{12Bc\,x^{3}e^{3}+6A\,x^{2}c\,e^{3}+18B\,x^{2}cd\,e^{2}+4Axcd\,e^{2}+4Bxa\,e^{3}+12Bxc\,d^{2}e+3Aa\,e^{3}+Ac\,d^{2}e+Bad\,e^{2}+3Bc\,d^{3}}{12(ex+d)^{4}e^{4}}$	99
parallelrisch	$-\frac{12Bc\,x^{3}e^{3}+6A\,x^{2}c\,e^{3}+18B\,x^{2}cd\,e^{2}+4Axcd\,e^{2}+4Bxa\,e^{3}+12Bxc\,d^{2}e+3Aa\,e^{3}+Ac\,d^{2}e+Bad\,e^{2}+3Bc\,d^{3}}{12(ex+d)^{4}e^{4}}$	99
orering	$-\frac{12Bc\,x^{3}e^{3}+6A\,x^{2}c\,e^{3}+18B\,x^{2}cd\,e^{2}+4Axcd\,e^{2}+4Bxa\,e^{3}+12Bxc\,d^{2}e+3Aa\,e^{3}+Ac\,d^{2}e+Bad\,e^{2}+3Bc\,d^{3}}{12(ex+d)^{4}e^{4}}$	99
default	$-\frac{-2Acde+Ba\,e^{2}+3Bc\,d^{2}}{3e^{4}(ex+d)^{3}}-\frac{Aa\,e^{3}+Ac\,d^{2}e-Bad\,e^{2}-Bc\,d^{3}}{4e^{4}(ex+d)^{4}}-\frac{Bc}{e^{4}(ex+d)}-\frac{c(Ae-3Bd)}{2e^{4}(ex+d)^{2}}$	110

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
(-B*c*x^3/e-1/2*c*(A*e+3*B*d)/e^2*x^2-1/3*(A*c*d*e+B*a*e^2+3*B*c*d^2)/e^3*
x-1/12*(3*A*a*e^3+A*c*d^2*e+B*a*d*e^2+3*B*c*d^3)/e^4)/(e*x+d)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)\left(a+cx^2\right)}{(d+ex)^5}\,dx = -\frac{12\,Bce^3x^3+3\,Bcd^3+Acd^2e+Bade^2+3\,Aae^3+6\,(3\,Bcde^2+Ace^3)x^2+4\,(3\,Bcd^2e+Acde^2+Bae}{12\,(e^8x^4+4\,de^7x^3+6\,d^2e^6x^2+4\,d^3e^5x+d^4e^4)}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^5,x, algorithm="fricas")
```

output
```
-1/12*(12*B*c*e^3*x^3 + 3*B*c*d^3 + A*c*d^2*e + B*a*d*e^2 + 3*A*a*e^3 + 6*
(3*B*c*d*e^2 + A*c*e^3)*x^2 + 4*(3*B*c*d^2*e + A*c*d*e^2 + B*a*e^3)*x)/(e^
8*x^4 + 4*d*e^7*x^3 + 6*d^2*e^6*x^2 + 4*d^3*e^5*x + d^4*e^4)
```

Sympy [A] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx)\left(a+cx^2\right)}{(d+ex)^5}\,dx = \frac{-3Aae^3-Acd^2e-Bade^2-3Bcd^3-12Bce^3x^3+x^2(-6Ace^3-18Bcde^2)+x(-4Acde^2-4Bae^3-12}{12d^4e^4+48d^3e^5x+72d^2e^6x^2+48de^7x^3+12e^8x^4}$$

input
```
integrate((B*x+A)*(c*x**2+a)/(e*x+d)**5,x)
```

output
```
(-3*A*a*e**3 - A*c*d**2*e - B*a*d*e**2 - 3*B*c*d**3 - 12*B*c*e**3*x**3 + x
**2*(-6*A*c*e**3 - 18*B*c*d*e**2) + x*(-4*A*c*d*e**2 - 4*B*a*e**3 - 12*B*c
*d**2*e))/(12*d**4*e**4 + 48*d**3*e**5*x + 72*d**2*e**6*x**2 + 48*d*e**7*x
**3 + 12*e**8*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^5}\,dx = -\frac{12\,Bce^3x^3+3\,Bcd^3+Acd^2e+Bade^2+3\,Aae^3+6\,(3\,Bcde^2+Ace^3)x^2+4\,(3\,Bcd^2e+Acde^2+Bae}{12\,(e^8x^4+4\,de^7x^3+6\,d^2e^6x^2+4\,d^3e^5x+d^4e^4)}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^5,x, algorithm="maxima")
```

output
```
-1/12*(12*B*c*e^3*x^3 + 3*B*c*d^3 + A*c*d^2*e + B*a*d*e^2 + 3*A*a*e^3 + 6*
(3*B*c*d*e^2 + A*c*e^3)*x^2 + 4*(3*B*c*d^2*e + A*c*d*e^2 + B*a*e^3)*x)/(e^
8*x^4 + 4*d*e^7*x^3 + 6*d^2*e^6*x^2 + 4*d^3*e^5*x + d^4*e^4)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^5}\,dx = -\frac{\frac{6\,Ac}{(ex+d)^2}-\frac{8\,Acd}{(ex+d)^3}+\frac{3\,Acd^2}{(ex+d)^4}+\frac{12\,Bc}{(ex+d)e}-\frac{18\,Bcd}{(ex+d)^2e}+\frac{12\,Bcd^2}{(ex+d)^3e}-\frac{3\,Bcd^3}{(ex+d)^4e}+\frac{4\,Bae}{(ex+d)^3}-\frac{3\,Bade}{(ex+d)^4}+\frac{3\,Aae^2}{(ex+d)^4}}{12\,e^3}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^5,x, algorithm="giac")
```

output
```
-1/12*(6*A*c/(e*x + d)^2 - 8*A*c*d/(e*x + d)^3 + 3*A*c*d^2/(e*x + d)^4 + 1
2*B*c/((e*x + d)*e) - 18*B*c*d/((e*x + d)^2*e) + 12*B*c*d^2/((e*x + d)^3*e
) - 3*B*c*d^3/((e*x + d)^4*e) + 4*B*a*e/(e*x + d)^3 - 3*B*a*d*e/(e*x + d)^
4 + 3*A*a*e^2/(e*x + d)^4)/e^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^5}\,dx$$
$$= -\frac{\frac{3\,B\,c\,d^3+A\,c\,d^2\,e+B\,a\,d\,e^2+3\,A\,a\,e^3}{12\,e^4}+\frac{x\,(3\,B\,c\,d^2+A\,c\,d\,e+B\,a\,e^2)}{3\,e^3}+\frac{B\,c\,x^3}{e}+\frac{c\,x^2\,(A\,e+3\,B\,d)}{2\,e^2}}{d^4+4\,d^3\,e\,x+6\,d^2\,e^2\,x^2+4\,d\,e^3\,x^3+e^4\,x^4}$$

input
```
int(((a + c*x^2)*(A + B*x))/(d + e*x)^5,x)
```

output
```
-((3*A*a*e^3 + 3*B*c*d^3 + B*a*d*e^2 + A*c*d^2*e)/(12*e^4) + (x*(B*a*e^2 +
 3*B*c*d^2 + A*c*d*e))/(3*e^3) + (B*c*x^3)/e + (c*x^2*(A*e + 3*B*d))/(2*e^
2))/(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^5}\,dx$$
$$= \frac{3bc\,e^3x^4-6acd\,e^2x^2-4abd\,e^2x-4ac\,d^2ex-3a^2d\,e^2-ab\,d^2e-ac\,d^3}{12d\,e^3\,(e^4x^4+4d\,e^3x^3+6d^2e^2x^2+4d^3ex+d^4)}$$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^5,x)
```

output
```
( - 3*a**2*d*e**2 - a*b*d**2*e - 4*a*b*d*e**2*x - a*c*d**3 - 4*a*c*d**2*e*
x - 6*a*c*d*e**2*x**2 + 3*b*c*e**3*x**4)/(12*d*e**3*(d**4 + 4*d**3*e*x + 6
*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))
```

3.44 $\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^6} dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^6} dx = \frac{(Bd-Ae)(cd^2+ae^2)}{5e^4(d+ex)^5} - \frac{3Bcd^2-2Acde+aBe^2}{4e^4(d+ex)^4} + \frac{c(3Bd-Ae)}{3e^4(d+ex)^3} - \frac{Bc}{2e^4(d+ex)^2}$$

output
```
1/5*(-A*e+B*d)*(a*e^2+c*d^2)/e^4/(e*x+d)^5-1/4*(-2*A*c*d*e+B*a*e^2+3*B*c*d
^2)/e^4/(e*x+d)^4+1/3*c*(-A*e+3*B*d)/e^4/(e*x+d)^3-1/2*B*c/e^4/(e*x+d)^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^6} dx = -\frac{2Ae(6ae^2+c(d^2+5dex+10e^2x^2))+3B(ae^2(d+5ex)+c(d^3+5d^2ex+10de^2x^2+10e^3x^3))}{60e^4(d+ex)^5}$$

input
```
Integrate[((A + B*x)*(a + c*x^2))/(d + e*x)^6,x]
```

output
```
-1/60*(2*A*e*(6*a*e^2 + c*(d^2 + 5*d*e*x + 10*e^2*x^2)) + 3*B*(a*e^2*(d +
5*e*x) + c*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3)))/(e^4*(d + e*x)^
5)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(A+Bx)}{(d+ex)^6} \, dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{aBe^2-2Acde+3Bcd^2}{e^3(d+ex)^5}+\frac{(ae^2+cd^2)(Ae-Bd)}{e^3(d+ex)^6}+\frac{c(Ae-3Bd)}{e^3(d+ex)^4}+\frac{Bc}{e^3(d+ex)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aBe^2-2Acde+3Bcd^2}{4e^4(d+ex)^4}+\frac{(ae^2+cd^2)(Bd-Ae)}{5e^4(d+ex)^5}+\frac{c(3Bd-Ae)}{3e^4(d+ex)^3}-\frac{Bc}{2e^4(d+ex)^2}$$

input
```
Int[((A + B*x)*(a + c*x^2))/(d + e*x)^6,x]
```

output
```
((B*d - A*e)*(c*d^2 + a*e^2))/(5*e^4*(d + e*x)^5) - (3*B*c*d^2 - 2*A*c*d*e
 + a*B*e^2)/(4*e^4*(d + e*x)^4) + (c*(3*B*d - A*e))/(3*e^4*(d + e*x)^3) -
(B*c)/(2*e^4*(d + e*x)^2)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{30Bc x^3 e^3+20A x^2 c e^3+30B x^2 cd e^2+10Axcd e^2+15Bxa e^3+15Bxc d^2 e+12Aa e^3+2Ac d^2 e+3Bad e^2+3Bc d^3}{60e^4(ex+d)^5}$	101
risch	$\frac{-\frac{Bc x^3}{2e}-\frac{c(2Ae+3Bd)x^2}{6e^2}-\frac{(2Acde+3Ba e^2+3Bc d^2)x}{12e^3}-\frac{12Aa e^3+2Ac d^2 e+3Bad e^2+3Bc d^3}{60e^4}}{(ex+d)^5}$	101
orering	$-\frac{30Bc x^3 e^3+20A x^2 c e^3+30B x^2 cd e^2+10Axcd e^2+15Bxa e^3+15Bxc d^2 e+12Aa e^3+2Ac d^2 e+3Bad e^2+3Bc d^3}{60e^4(ex+d)^5}$	101
parallelrisch	$-\frac{30Bc x^3 e^4+20Ac e^4 x^2+30Bcd e^3 x^2+10Acd e^3 x+15Ba e^4 x+15Bc d^2 e^2 x+12Aa e^4+2Ac d^2 e^2+3Bad e^3+3Bc d^3 e}{60e^5(ex+d)^5}$	106
default	$-\frac{c(Ae-3Bd)}{3e^4(ex+d)^3}-\frac{-2Acde+Ba e^2+3Bc d^2}{4e^4(ex+d)^4}-\frac{Aa e^3+Ac d^2 e-Bad e^2-Bc d^3}{5e^4(ex+d)^5}-\frac{Bc}{2e^4(ex+d)^2}$	110
norman	$\frac{-\frac{Bc x^3}{2e}-\frac{(2Ac e^2+3Bcde)x^2}{6e^3}-\frac{(2Acd e^2+3B e^3 a+3Bc d^2 e)x}{12e^4}-\frac{12Aa e^4+2Ac d^2 e^2+3Bad e^3+3Bc d^3 e}{60e^5}}{(ex+d)^5}$	111

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/60/e^4*(30*B*c*e^3*x^3+20*A*c*e^3*x^2+30*B*c*d*e^2*x^2+10*A*c*d*e^2*x+1
5*B*a*e^3*x+15*B*c*d^2*e*x+12*A*a*e^3+2*A*c*d^2*e+3*B*a*d*e^2+3*B*c*d^3)/(
e*x+d)^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^6}\,dx = -\frac{30\,Bce^3x^3+3\,Bcd^3+2\,Acd^2e+3\,Bade^2+12\,Aae^3+10\,(3\,Bcde^2+2\,Ace^3)x^2+5\,(3\,Bcd^2e+2\,Acde^2+3\,Bae^3)x}{60\,(e^9x^5+5\,de^8x^4+10\,d^2e^7x^3+10\,d^3e^6x^2+5\,d^4e^5x+d^5e^4)}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^6,x, algorithm="fricas")
```

output
```
-1/60*(30*B*c*e^3*x^3 + 3*B*c*d^3 + 2*A*c*d^2*e + 3*B*a*d*e^2 + 12*A*a*e^3
 + 10*(3*B*c*d*e^2 + 2*A*c*e^3)*x^2 + 5*(3*B*c*d^2*e + 2*A*c*d*e^2 + 3*B*a
*e^3)*x)/(e^9*x^5 + 5*d*e^8*x^4 + 10*d^2*e^7*x^3 + 10*d^3*e^6*x^2 + 5*d^4*
e^5*x + d^5*e^4)
```

Sympy [A] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.53

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^6}\,dx = \frac{-12Aae^3-2Acd^2e-3Bade^2-3Bcd^3-30Bce^3x^3+x^2(-20Ace^3-30Bcde^2)+x(-10Acde^2-15Bae^3-15Bcd^2e)}{60d^5e^4+300d^4e^5x+600d^3e^6x^2+600d^2e^7x^3+300de^8x^4+60e^9x^5}$$

input
```
integrate((B*x+A)*(c*x**2+a)/(e*x+d)**6,x)
```

output
```
(-12*A*a*e**3 - 2*A*c*d**2*e - 3*B*a*d*e**2 - 3*B*c*d**3 - 30*B*c*e**3*x**
3 + x**2*(-20*A*c*e**3 - 30*B*c*d*e**2) + x*(-10*A*c*d*e**2 - 15*B*a*e**3
- 15*B*c*d**2*e))/(60*d**5*e**4 + 300*d**4*e**5*x + 600*d**3*e**6*x**2 + 6
00*d**2*e**7*x**3 + 300*d*e**8*x**4 + 60*e**9*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^6}\,dx = -\frac{30\,Bce^3x^3+3\,Bcd^3+2\,Acd^2e+3\,Bade^2+12\,Aae^3+10\,(3\,Bcde^2+2\,Ace^3)x^2+5\,(3\,Bcd^2e+2\,Acde}{60\,(e^9x^5+5\,de^8x^4+10\,d^2e^7x^3+10\,d^3e^6x^2+5\,d^4e^5x+d^5e^4)}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^6,x, algorithm="maxima")
```

output
```
-1/60*(30*B*c*e^3*x^3 + 3*B*c*d^3 + 2*A*c*d^2*e + 3*B*a*d*e^2 + 12*A*a*e^3
 + 10*(3*B*c*d*e^2 + 2*A*c*e^3)*x^2 + 5*(3*B*c*d^2*e + 2*A*c*d*e^2 + 3*B*a
*e^3)*x)/(e^9*x^5 + 5*d*e^8*x^4 + 10*d^2*e^7*x^3 + 10*d^3*e^6*x^2 + 5*d^4*
e^5*x + d^5*e^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^6}\,dx = -\frac{30\,Bce^3x^3+30\,Bcde^2x^2+20\,Ace^3x^2+15\,Bcd^2ex+10\,Acde^2x+15\,Bae^3x+3\,Bcd^3+2\,Acd^2e+3\,B}{60\,(ex+d)^5e^4}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^6,x, algorithm="giac")
```

output
```
-1/60*(30*B*c*e^3*x^3 + 30*B*c*d*e^2*x^2 + 20*A*c*e^3*x^2 + 15*B*c*d^2*e*x
 + 10*A*c*d*e^2*x + 15*B*a*e^3*x + 3*B*c*d^3 + 2*A*c*d^2*e + 3*B*a*d*e^2 +
 12*A*a*e^3)/((e*x + d)^5*e^4)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)\left(a+cx^2\right)}{(d+ex)^6}\,dx = -\frac{\frac{3\,B\,c\,d^3+2\,A\,c\,d^2\,e+3\,B\,a\,d\,e^2+12\,A\,a\,e^3}{60\,e^4}+\frac{x\,\left(3\,B\,c\,d^2+2\,A\,c\,d\,e+3\,B\,a\,e^2\right)}{12\,e^3}+\frac{B\,c\,x^3}{2\,e}+\frac{c\,x^2\,\left(2\,A\,e+3\,B\,d\right)}{6\,e^2}}{d^5+5\,d^4\,e\,x+10\,d^3\,e^2\,x^2+10\,d^2\,e^3\,x^3+5\,d\,e^4\,x^4+e^5\,x^5}$$

input
```
int(((a + c*x^2)*(A + B*x))/(d + e*x)^6,x)
```

output
```
-((12*A*a*e^3 + 3*B*c*d^3 + 3*B*a*d*e^2 + 2*A*c*d^2*e)/(60*e^4) + (x*(3*B*
a*e^2 + 3*B*c*d^2 + 2*A*c*d*e))/(12*e^3) + (B*c*x^3)/(2*e) + (c*x^2*(2*A*e
 + 3*B*d))/(6*e^2))/(d^5 + e^5*x^5 + 5*d*e^4*x^4 + 10*d^3*e^2*x^2 + 10*d^2
*e^3*x^3 + 5*d^4*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)\left(a+cx^2\right)}{(d+ex)^6}\,dx = \frac{-30bc\,e^3x^3-20ac\,e^3x^2-30bcd\,e^2x^2-15ab\,e^3x-10acd\,e^2x-15bc\,d^2ex-12a^2e^3-3abd\,e^2-2ac\,d^2e-}{60e^4\left(e^5x^5+5d\,e^4x^4+10d^2e^3x^3+10d^3e^2x^2+5d^4ex+d^5\right)}$$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^6,x)
```

output
```
( - 12*a**2*e**3 - 3*a*b*d*e**2 - 15*a*b*e**3*x - 2*a*c*d**2*e - 10*a*c*d*
e**2*x - 20*a*c*e**3*x**2 - 3*b*c*d**3 - 15*b*c*d**2*e*x - 30*b*c*d*e**2*x
**2 - 30*b*c*e**3*x**3)/(60*e**4*(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 +
10*d**2*e**3*x**3 + 5*d*e**4*x**4 + e**5*x**5))
```

3.45 $\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^7} dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^7} dx = \frac{(Bd-Ae)(cd^2+ae^2)}{6e^4(d+ex)^6} - \frac{3Bcd^2-2Acde+aBe^2}{5e^4(d+ex)^5} + \frac{c(3Bd-Ae)}{4e^4(d+ex)^4} - \frac{Bc}{3e^4(d+ex)^3}$$

output
```
1/6*(-A*e+B*d)*(a*e^2+c*d^2)/e^4/(e*x+d)^6-1/5*(-2*A*c*d*e+B*a*e^2+3*B*c*d
^2)/e^4/(e*x+d)^5+1/4*c*(-A*e+3*B*d)/e^4/(e*x+d)^4-1/3*B*c/e^4/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^7} dx = -\frac{10aAe^3+2aBe^2(d+6ex)+Ace(d^2+6dex+15e^2x^2)+Bc(d^3+6d^2ex+15de^2x^2+20e^3x^3)}{60e^4(d+ex)^6}$$

input
```
Integrate[((A + B*x)*(a + c*x^2))/(d + e*x)^7,x]
```

output
```
-1/60*(10*a*A*e^3 + 2*a*B*e^2*(d + 6*e*x) + A*c*e*(d^2 + 6*d*e*x + 15*e^2*
x^2) + B*c*(d^3 + 6*d^2*e*x + 15*d*e^2*x^2 + 20*e^3*x^3))/(e^4*(d + e*x)^6
)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(A+Bx)}{(d+ex)^7}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{aBe^2-2Acde+3Bcd^2}{e^3(d+ex)^6}+\frac{(ae^2+cd^2)(Ae-Bd)}{e^3(d+ex)^7}+\frac{c(Ae-3Bd)}{e^3(d+ex)^5}+\frac{Bc}{e^3(d+ex)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aBe^2-2Acde+3Bcd^2}{5e^4(d+ex)^5}+\frac{(ae^2+cd^2)(Bd-Ae)}{6e^4(d+ex)^6}+\frac{c(3Bd-Ae)}{4e^4(d+ex)^4}-\frac{Bc}{3e^4(d+ex)^3}$$

input
```
Int[((A + B*x)*(a + c*x^2))/(d + e*x)^7,x]
```

output
```
((B*d - A*e)*(c*d^2 + a*e^2))/(6*e^4*(d + e*x)^6) - (3*B*c*d^2 - 2*A*c*d*e
 + a*B*e^2)/(5*e^4*(d + e*x)^5) + (c*(3*B*d - A*e))/(4*e^4*(d + e*x)^4) -
(B*c)/(3*e^4*(d + e*x)^3)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{-\frac{Bc\,x^{3}}{3e}-\frac{c(Ae+Bd)x^{2}}{4e^{2}}-\frac{\left(Acde+2Ba\,e^{2}+Bc\,d^{2}\right)x}{10e^{3}}-\frac{10Aa\,e^{3}+Ac\,d^{2}e+2Bad\,e^{2}+Bc\,d^{3}}{60e^{4}}}{(ex+d)^{6}}$	95
gosper	$-\frac{20Bc\,x^{3}e^{3}+15A\,x^{2}c\,e^{3}+15B\,x^{2}cd\,e^{2}+6Axcd\,e^{2}+12Bxa\,e^{3}+6Bxc\,d^{2}e+10Aa\,e^{3}+Ac\,d^{2}e+2Bad\,e^{2}+Bc\,d^{3}}{60e^{4}(ex+d)^{6}}$	99
orering	$-\frac{20Bc\,x^{3}e^{3}+15A\,x^{2}c\,e^{3}+15B\,x^{2}cd\,e^{2}+6Axcd\,e^{2}+12Bxa\,e^{3}+6Bxc\,d^{2}e+10Aa\,e^{3}+Ac\,d^{2}e+2Bad\,e^{2}+Bc\,d^{3}}{60e^{4}(ex+d)^{6}}$	99
parallelrisch	$-\frac{20Bc\,x^{3}e^{5}+15Ac\,e^{5}x^{2}+15Bcd\,e^{4}x^{2}+6Acd\,e^{4}x+12Ba\,e^{5}x+6Bc\,d^{2}e^{3}x+10Aa\,e^{5}+Ac\,d^{2}e^{3}+2Bad\,e^{4}+Bc\,d^{3}e^{2}}{60e^{6}(ex+d)^{6}}$	106
default	$-\frac{Bc}{3e^{4}(ex+d)^{3}}-\frac{c(Ae-3Bd)}{4e^{4}(ex+d)^{4}}-\frac{-2Acde+Ba\,e^{2}+3Bc\,d^{2}}{5e^{4}(ex+d)^{5}}-\frac{Aa\,e^{3}+Ac\,d^{2}e-Bad\,e^{2}-Bc\,d^{3}}{6e^{4}(ex+d)^{6}}$	110
norman	$\frac{-\frac{Bc\,x^{3}}{3e}-\frac{\left(Ac\,e^{3}+Bcd\,e^{2}\right)x^{2}}{4e^{4}}-\frac{\left(Acd\,e^{3}+2B\,e^{4}a+Bc\,d^{2}e^{2}\right)x}{10e^{5}}-\frac{10Aa\,e^{5}+Ac\,d^{2}e^{3}+2Bad\,e^{4}+Bc\,d^{3}e^{2}}{60e^{6}}}{(ex+d)^{6}}$	111

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*B*c*x^3/e-1/4*c/e^2*(A*e+B*d)*x^2-1/10/e^3*(A*c*d*e+2*B*a*e^2+B*c*d^
2)*x-1/60/e^4*(10*A*a*e^3+A*c*d^2*e+2*B*a*d*e^2+B*c*d^3))/(e*x+d)^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^7}\,dx = -\frac{20\,Bce^3x^3 + Bcd^3 + Acd^2e + 2\,Bade^2 + 10\,Aae^3 + 15\,(Bcde^2 + Ace^3)x^2 + 6\,(Bcd^2e + Acde^2 + 2\,Bae}{60\,(e^{10}x^6 + 6\,de^9x^5 + 15\,d^2e^8x^4 + 20\,d^3e^7x^3 + 15\,d^4e^6x^2 + 6\,d^5e^5x + d^6e^4)}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^7,x, algorithm="fricas")
```

output
```
-1/60*(20*B*c*e^3*x^3 + B*c*d^3 + A*c*d^2*e + 2*B*a*d*e^2 + 10*A*a*e^3 + 1
5*(B*c*d*e^2 + A*c*e^3)*x^2 + 6*(B*c*d^2*e + A*c*d*e^2 + 2*B*a*e^3)*x)/(e^
10*x^6 + 6*d*e^9*x^5 + 15*d^2*e^8*x^4 + 20*d^3*e^7*x^3 + 15*d^4*e^6*x^2 +
6*d^5*e^5*x + d^6*e^4)
```

Sympy [A] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.60

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^7}\,dx = \frac{-10Aae^3 - Acd^2e - 2Bade^2 - Bcd^3 - 20Bce^3x^3 + x^2(-15Ace^3 - 15Bcde^2) + x(-6Acde^2 - 12Bae^3 -}{60d^6e^4 + 360d^5e^5x + 900d^4e^6x^2 + 1200d^3e^7x^3 + 900d^2e^8x^4 + 360de^9x^5 + 60e^{10}x^6}$$

input
```
integrate((B*x+A)*(c*x**2+a)/(e*x+d)**7,x)
```

output
```
(-10*A*a*e**3 - A*c*d**2*e - 2*B*a*d*e**2 - B*c*d**3 - 20*B*c*e**3*x**3 +
x**2*(-15*A*c*e**3 - 15*B*c*d*e**2) + x*(-6*A*c*d*e**2 - 12*B*a*e**3 - 6*B
*c*d**2*e))/(60*d**6*e**4 + 360*d**5*e**5*x + 900*d**4*e**6*x**2 + 1200*d*
*3*e**7*x**3 + 900*d**2*e**8*x**4 + 360*d*e**9*x**5 + 60*e**10*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^7}\,dx = -\frac{20\,Bce^3x^3 + Bcd^3 + Acd^2e + 2\,Bade^2 + 10\,Aae^3 + 15\,(Bcde^2 + Ace^3)x^2 + 6\,(Bcd^2e + Acde^2 + 2\,Bae}{60\,(e^{10}x^6 + 6\,de^9x^5 + 15\,d^2e^8x^4 + 20\,d^3e^7x^3 + 15\,d^4e^6x^2 + 6\,d^5e^5x + d^6e^4)}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^7,x, algorithm="maxima")
```

output
```
-1/60*(20*B*c*e^3*x^3 + B*c*d^3 + A*c*d^2*e + 2*B*a*d*e^2 + 10*A*a*e^3 + 1
5*(B*c*d*e^2 + A*c*e^3)*x^2 + 6*(B*c*d^2*e + A*c*d*e^2 + 2*B*a*e^3)*x)/(e^
10*x^6 + 6*d*e^9*x^5 + 15*d^2*e^8*x^4 + 20*d^3*e^7*x^3 + 15*d^4*e^6*x^2 +
6*d^5*e^5*x + d^6*e^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^7}\,dx = -\frac{20\,Bce^3x^3 + 15\,Bcde^2x^2 + 15\,Ace^3x^2 + 6\,Bcd^2ex + 6\,Acde^2x + 12\,Bae^3x + Bcd^3 + Acd^2e + 2\,Bade^2}{60\,(ex+d)^6e^4}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^7,x, algorithm="giac")
```

output
```
-1/60*(20*B*c*e^3*x^3 + 15*B*c*d*e^2*x^2 + 15*A*c*e^3*x^2 + 6*B*c*d^2*e*x
+ 6*A*c*d*e^2*x + 12*B*a*e^3*x + B*c*d^3 + A*c*d^2*e + 2*B*a*d*e^2 + 10*A*
a*e^3)/((e*x + d)^6*e^4)
```

Mupad [B] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.39

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^7}\,dx = -\frac{\frac{B\,c\,d^3+A\,c\,d^2\,e+2\,B\,a\,d\,e^2+10\,A\,a\,e^3}{60\,e^4}+\frac{x\,\left(B\,c\,d^2+A\,c\,d\,e+2\,B\,a\,e^2\right)}{10\,e^3}+\frac{B\,c\,x^3}{3\,e}+\frac{c\,x^2\,\left(A\,e+B\,d\right)}{4\,e^2}}{d^6+6\,d^5\,e\,x+15\,d^4\,e^2\,x^2+20\,d^3\,e^3\,x^3+15\,d^2\,e^4\,x^4+6\,d\,e^5\,x^5+e^6\,x^6}$$

input
```
int(((a + c*x^2)*(A + B*x))/(d + e*x)^7,x)
```

output
```
-((10*A*a*e^3 + B*c*d^3 + 2*B*a*d*e^2 + A*c*d^2*e)/(60*e^4) + (x*(2*B*a*e^
2 + B*c*d^2 + A*c*d*e))/(10*e^3) + (B*c*x^3)/(3*e) + (c*x^2*(A*e + B*d))/(
4*e^2))/(d^6 + e^6*x^6 + 6*d*e^5*x^5 + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 + 1
5*d^2*e^4*x^4 + 6*d^5*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.44

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^7}\,dx = \frac{-20bc\,e^3x^3-15ac\,e^3x^2-15bcd\,e^2x^2-12ab\,e^3x-6acd\,e^2x-6bc\,d^2ex-10a^2e^3-2abd\,e^2-ac\,d^2e-bc}{60e^4\left(e^6x^6+6d\,e^5x^5+15d^2e^4x^4+20d^3e^3x^3+15d^4e^2x^2+6d^5ex+d^6\right)}$$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^7,x)
```

output
```
( - 10*a**2*e**3 - 2*a*b*d*e**2 - 12*a*b*e**3*x - a*c*d**2*e - 6*a*c*d*e**
2*x - 15*a*c*e**3*x**2 - b*c*d**3 - 6*b*c*d**2*e*x - 15*b*c*d*e**2*x**2 -
20*b*c*e**3*x**3)/(60*e**4*(d**6 + 6*d**5*e*x + 15*d**4*e**2*x**2 + 20*d**
3*e**3*x**3 + 15*d**2*e**4*x**4 + 6*d*e**5*x**5 + e**6*x**6))
```

3.46 $\int (A+Bx)(d+ex)^5 \left(a+cx^2\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 206

$$\int (A+Bx)(d+ex)^5 \left(a+cx^2\right)^2 \, dx = -\frac{(Bd-Ae)\left(cd^2+ae^2\right)^2(d+ex)^6}{6e^6} + \frac{\left(cd^2+ae^2\right)\left(5Bcd^2-4Acde+aBe^2\right)(d+ex)^7}{7e^6} - \frac{c\left(5Bcd^3-3Acd^2e+3aBde^2-aAe^3\right)(d+ex)^8}{4e^6} + \frac{2c\left(5Bcd^2-2Acde+aBe^2\right)(d+ex)^9}{9e^6} - \frac{c^2(5Bd-Ae)(d+ex)^{10}}{10e^6} + \frac{Bc^2(d+ex)^{11}}{11e^6}$$

output
```
-1/6*(-A*e+B*d)*(a*e^2+c*d^2)^2*(e*x+d)^6/e^6+1/7*(a*e^2+c*d^2)*(-4*A*c*d*
e+B*a*e^2+5*B*c*d^2)*(e*x+d)^7/e^6-1/4*c*(-A*a*e^3-3*A*c*d^2*e+3*B*a*d*e^2
+5*B*c*d^3)*(e*x+d)^8/e^6+2/9*c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)*(e*x+d)^9/e
^6-1/10*c^2*(-A*e+5*B*d)*(e*x+d)^10/e^6+1/11*B*c^2*(e*x+d)^11/e^6
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.89

$$\int (A+Bx)(d+ex)^5\left(a+cx^2\right)^2\,dx = a^2Ad^5x+\frac{1}{2}a^2d^4(Bd+5Ae)x^2 +\frac{1}{3}ad^3\left(2Acd^2+5aBde+10aAe^2\right)x^3 +\frac{1}{2}ad^2\left(Bcd^3+5Acd^2e+5aBde^2+5aAe^3\right)x^4 +\frac{1}{5}d\left(Ac^2d^4+10aBcd^3e+20aAcd^2e^2+10a^2Bde^3+5a^2Ae^4\right)x^5+\frac{1}{6}\left(Bc^2d^5+5Ac^2d^4e+20aBcd^3e^2+20aAcd^2e^3+5a^2Bde^4+a^2Ae^5\right)x^6+\frac{1}{7}e\left(5Bc^2d^4+10Ac^2d^3e+20aBcd^2e^2+10aAcde^3+a^2Be^4\right)x^7 +\frac{1}{4}ce^2\left(5Bcd^3+5Acd^2e+5aBde^2+aAe^3\right)x^8 +\frac{1}{9}ce^3\left(10Bcd^2+5Acde+2aBe^2\right)x^9 +\frac{1}{10}c^2e^4(5Bd+Ae)x^{10}+\frac{1}{11}Bc^2e^5x^{11}$$

input
```
Integrate[(A + B*x)*(d + e*x)^5*(a + c*x^2)^2,x]
```

output
```
a^2*A*d^5*x + (a^2*d^4*(B*d + 5*A*e)*x^2)/2 + (a*d^3*(2*A*c*d^2 + 5*a*B*d*
e + 10*a*A*e^2)*x^3)/3 + (a*d^2*(B*c*d^3 + 5*A*c*d^2*e + 5*a*B*d*e^2 + 5*a
*A*e^3)*x^4)/2 + (d*(A*c^2*d^4 + 10*a*B*c*d^3*e + 20*a*A*c*d^2*e^2 + 10*a^
2*B*d*e^3 + 5*a^2*A*e^4)*x^5)/5 + ((B*c^2*d^5 + 5*A*c^2*d^4*e + 20*a*B*c*d
^3*e^2 + 20*a*A*c*d^2*e^3 + 5*a^2*B*d*e^4 + a^2*A*e^5)*x^6)/6 + (e*(5*B*c^
2*d^4 + 10*A*c^2*d^3*e + 20*a*B*c*d^2*e^2 + 10*a*A*c*d*e^3 + a^2*B*e^4)*x^
7)/7 + (c*e^2*(5*B*c*d^3 + 5*A*c*d^2*e + 5*a*B*d*e^2 + a*A*e^3)*x^8)/4 + (
c*e^3*(10*B*c*d^2 + 5*A*c*d*e + 2*a*B*e^2)*x^9)/9 + (c^2*e^4*(5*B*d + A*e)
*x^10)/10 + (B*c^2*e^5*x^11)/11
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2 (A+Bx)(d+ex)^5 \, dx$$

$$\Big\downarrow \text{ 652}$$

$$\int \left(-\frac{2c(d+ex)^8 \left(-aBe^2+2Acde-5Bcd^2\right)}{e^5}+\frac{(d+ex)^6 \left(ae^2+cd^2\right) \left(aBe^2-4Acde+5Bcd^2\right)}{e^5}+\frac{(d+ex)^5 \left(a\right.}{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2c(d+ex)^9 \left(aBe^2-2Acde+5Bcd^2\right)}{9e^6}+\frac{(d+ex)^7 \left(ae^2+cd^2\right) \left(aBe^2-4Acde+5Bcd^2\right)}{7e^6}-\frac{(d+ex)^6 \left(ae^2+cd^2\right)^2 (Bd-Ae)}{6e^6}-\frac{c(d+ex)^8 \left(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3\right)}{4e^6}-\frac{c^2(d+ex)^{10}(5Bd-Ae)}{10e^6}+\frac{Bc^2(d+ex)^{11}}{11e^6}$$

input
```
Int[(A + B*x)*(d + e*x)^5*(a + c*x^2)^2,x]
```

output
```
-1/6*((B*d - A*e)*(c*d^2 + a*e^2)^2*(d + e*x)^6)/e^6 + ((c*d^2 + a*e^2)*(5
*B*c*d^2 - 4*A*c*d*e + a*B*e^2)*(d + e*x)^7)/(7*e^6) - (c*(5*B*c*d^3 - 3*A
*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*(d + e*x)^8)/(4*e^6) + (2*c*(5*B*c*d^2 -
 2*A*c*d*e + a*B*e^2)*(d + e*x)^9)/(9*e^6) - (c^2*(5*B*d - A*e)*(d + e*x)^
10)/(10*e^6) + (B*c^2*(d + e*x)^11)/(11*e^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(194) = 388.

Time = 0.60 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.95

method	result
default	$\frac{B e^5 c^2 x^{11}}{11}+\frac{(A e^5+5Bd e^4) c^2 x^{10}}{10}+\frac{((5Ad e^4+10B d^2 e^3) c^2+2B e^5 ac) x^9}{9}+\frac{((10A d^2 e^3+10B d^3 e^2) c^2+2(A e^5+5Bd e$
norman	$\frac{B e^5 c^2 x^{11}}{11}+\left(\frac{1}{10}A c^2 e^5+\frac{1}{2}B c^2 d e^4\right) x^{10}+\left(\frac{5}{9}A c^2 d e^4+\frac{2}{9}B e^5 ac+\frac{10}{9}B c^2 d^2 e^3\right) x^9+\left(\frac{1}{4}Aac e^5\right.$
gosper	$\frac{5}{7}x^7 B c^2 d^4 e+\frac{5}{6}x^6 A c^2 d^4 e+\frac{5}{6}x^6 B a^2 d e^4+x^5 A a^2 d e^4+2x^5 Bac d^4 e+\frac{5}{9}x^9 A c^2 d e^4+\frac{5}{2}x^4 A$
risch	$\frac{5}{7}x^7 B c^2 d^4 e+\frac{5}{6}x^6 A c^2 d^4 e+\frac{5}{6}x^6 B a^2 d e^4+x^5 A a^2 d e^4+2x^5 Bac d^4 e+\frac{5}{9}x^9 A c^2 d e^4+\frac{5}{2}x^4 A$
parallelrisch	$\frac{5}{7}x^7 B c^2 d^4 e+\frac{5}{6}x^6 A c^2 d^4 e+\frac{5}{6}x^6 B a^2 d e^4+x^5 A a^2 d e^4+2x^5 Bac d^4 e+\frac{5}{9}x^9 A c^2 d e^4+\frac{5}{2}x^4 A$
orering	$x(1260B e^5 c^2 x^{10}+1386A c^2 e^5 x^9+6930B c^2 d e^4 x^9+7700A c^2 d e^4 x^8+3080Bac e^5 x^8+15400B c^2 d^2 e^3 x^8+3465Aac e^5 x^7+173$

input
```
int((B*x+A)*(e*x+d)^5*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*B*e^5*c^2*x^11+1/10*(A*e^5+5*B*d*e^4)*c^2*x^10+1/9*((5*A*d*e^4+10*B*d
^2*e^3)*c^2+2*B*e^5*a*c)*x^9+1/8*((10*A*d^2*e^3+10*B*d^3*e^2)*c^2+2*(A*e^5
+5*B*d*e^4)*a*c)*x^8+1/7*((10*A*d^3*e^2+5*B*d^4*e)*c^2+2*(5*A*d*e^4+10*B*d
^2*e^3)*a*c+B*e^5*a^2)*x^7+1/6*((5*A*d^4*e+B*d^5)*c^2+2*(10*A*d^2*e^3+10*B
*d^3*e^2)*a*c+(A*e^5+5*B*d*e^4)*a^2)*x^6+1/5*(A*d^5*c^2+2*(10*A*d^3*e^2+5*
B*d^4*e)*a*c+(5*A*d*e^4+10*B*d^2*e^3)*a^2)*x^5+1/4*(2*(5*A*d^4*e+B*d^5)*a*
c+(10*A*d^2*e^3+10*B*d^3*e^2)*a^2)*x^4+1/3*(2*A*d^5*a*c+(10*A*d^3*e^2+5*B*
d^4*e)*a^2)*x^3+1/2*(5*A*d^4*e+B*d^5)*a^2*x^2+A*d^5*a^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(194) = 388$.

Time = 0.07 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.99

$$\int (A+Bx)(d+ex)^5 \left(a+cx^2\right)^2 \, dx = \frac{1}{11} Bc^2e^5x^{11} + \frac{1}{10} \left(5 Bc^2de^4 + Ac^2e^5\right)x^{10} + \frac{1}{9} \left(10 Bc^2d^2e^3 + 5 Ac^2de^4 + 2 Bace^5\right)x^9 + Aa^2d^5x + \frac{1}{4} \left(5 Bc^2d^3e^2 + 5 Ac^2d^2e^3 + 5 Bacde^4 + Aace^5\right)x^8 + \frac{1}{7} \left(5 Bc^2d^4e + 10 Ac^2d^3e^2 + 20 Bacd^2e^3 + 10 Aacde^4 + Ba^2e^5\right)x^7 + \frac{1}{6} \left(Bc^2d^5 + 5 Ac^2d^4e + 20 Bacd^3e^2 + 20 Aacd^2e^3 + 5 Ba^2de^4 + Aa^2e^5\right)x^6 + \frac{1}{5} \left(Ac^2d^5 + 10 Bacd^4e + 20 Aacd^3e^2 + 10 Ba^2d^2e^3 + 5 Aa^2de^4\right)x^5 + \frac{1}{2} \left(Bacd^5 + 5 Aacd^4e + 5 Ba^2d^3e^2 + 5 Aa^2d^2e^3\right)x^4 + \frac{1}{3} \left(2 Aacd^5 + 5 Ba^2d^4e + 10 Aa^2d^3e^2\right)x^3 + \frac{1}{2} \left(Ba^2d^5 + 5 Aa^2d^4e\right)x^2$$

input

```
integrate((B*x+A)*(e*x+d)^5*(c*x^2+a)^2,x, algorithm="fricas")
```

output

```
1/11*B*c^2*e^5*x^11 + 1/10*(5*B*c^2*d*e^4 + A*c^2*e^5)*x^10 + 1/9*(10*B*c^
2*d^2*e^3 + 5*A*c^2*d*e^4 + 2*B*a*c*e^5)*x^9 + A*a^2*d^5*x + 1/4*(5*B*c^2*
d^3*e^2 + 5*A*c^2*d^2*e^3 + 5*B*a*c*d*e^4 + A*a*c*e^5)*x^8 + 1/7*(5*B*c^2*
d^4*e + 10*A*c^2*d^3*e^2 + 20*B*a*c*d^2*e^3 + 10*A*a*c*d*e^4 + B*a^2*e^5)*
x^7 + 1/6*(B*c^2*d^5 + 5*A*c^2*d^4*e + 20*B*a*c*d^3*e^2 + 20*A*a*c*d^2*e^3
 + 5*B*a^2*d*e^4 + A*a^2*e^5)*x^6 + 1/5*(A*c^2*d^5 + 10*B*a*c*d^4*e + 20*A
*a*c*d^3*e^2 + 10*B*a^2*d^2*e^3 + 5*A*a^2*d*e^4)*x^5 + 1/2*(B*a*c*d^5 + 5*
A*a*c*d^4*e + 5*B*a^2*d^3*e^2 + 5*A*a^2*d^2*e^3)*x^4 + 1/3*(2*A*a*c*d^5 +
5*B*a^2*d^4*e + 10*A*a^2*d^3*e^2)*x^3 + 1/2*(B*a^2*d^5 + 5*A*a^2*d^4*e)*x^
2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(207) = 414$.

Time = 0.06 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.40

$$\int (A+Bx)(d+ex)^5\left(a+cx^2\right)^2\,dx = Aa^2d^5x + \frac{Bc^2e^5x^{11}}{11} + x^{10}\left(\frac{Ac^2e^5}{10} + \frac{Bc^2de^4}{2}\right) + x^9\cdot\left(\frac{5Ac^2de^4}{9} + \frac{2Bace^5}{9} + \frac{10Bc^2d^2e^3}{9}\right) + x^8\left(\frac{Aace^5}{4} + \frac{5Ac^2d^2e^3}{4} + \frac{5Bacde^4}{4} + \frac{5Bc^2d^3e^2}{4}\right) + x^7\cdot\left(\frac{10Aacde^4}{7} + \frac{10Ac^2d^3e^2}{7} + \frac{Ba^2e^5}{7} + \frac{20Bacd^2e^3}{7} + \frac{5Bc^2d^4e}{7}\right) + x^6\left(\frac{Aa^2e^5}{6} + \frac{10Aacd^2e^3}{3} + \frac{5Ac^2d^4e}{6} + \frac{5Ba^2de^4}{6} + \frac{10Bacd^3e^2}{3} + \frac{Bc^2d^5}{6}\right) + x^5\left(Aa^2de^4 + 4Aacd^3e^2 + \frac{Ac^2d^5}{5} + 2Ba^2d^2e^3 + 2Bacd^4e\right) + x^4\cdot\left(\frac{5Aa^2d^2e^3}{2} + \frac{5Aacd^4e}{2} + \frac{5Ba^2d^3e^2}{2} + \frac{Bacd^5}{2}\right) + x^3\cdot\left(\frac{10Aa^2d^3e^2}{3} + \frac{2Aacd^5}{3} + \frac{5Ba^2d^4e}{3}\right) + x^2\cdot\left(\frac{5Aa^2d^4e}{2} + \frac{Ba^2d^5}{2}\right)$$

input
```
integrate((B*x+A)*(e*x+d)**5*(c*x**2+a)**2,x)
```

output
```
A*a**2*d**5*x + B*c**2*e**5*x**11/11 + x**10*(A*c**2*e**5/10 + B*c**2*d*e*
*4/2) + x**9*(5*A*c**2*d*e**4/9 + 2*B*a*c*e**5/9 + 10*B*c**2*d**2*e**3/9)
+ x**8*(A*a*c*e**5/4 + 5*A*c**2*d**2*e**3/4 + 5*B*a*c*d*e**4/4 + 5*B*c**2*
d**3*e**2/4) + x**7*(10*A*a*c*d*e**4/7 + 10*A*c**2*d**3*e**2/7 + B*a**2*e*
*5/7 + 20*B*a*c*d**2*e**3/7 + 5*B*c**2*d**4*e/7) + x**6*(A*a**2*e**5/6 + 1
0*A*a*c*d**2*e**3/3 + 5*A*c**2*d**4*e/6 + 5*B*a**2*d*e**4/6 + 10*B*a*c*d**
3*e**2/3 + B*c**2*d**5/6) + x**5*(A*a**2*d*e**4 + 4*A*a*c*d**3*e**2 + A*c*
*2*d**5/5 + 2*B*a**2*d**2*e**3 + 2*B*a*c*d**4*e) + x**4*(5*A*a**2*d**2*e**
3/2 + 5*A*a*c*d**4*e/2 + 5*B*a**2*d**3*e**2/2 + B*a*c*d**5/2) + x**3*(10*A
*a**2*d**3*e**2/3 + 2*A*a*c*d**5/3 + 5*B*a**2*d**4*e/3) + x**2*(5*A*a**2*d
**4*e/2 + B*a**2*d**5/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(194) = 388$.

Time = 0.03 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.99

$$\begin{aligned}
&\int (A+Bx)(d+ex)^5\left(a+cx^2\right)^2\,dx \\
&= \frac{1}{11}Bc^2e^5x^{11} + \frac{1}{10}\left(5\,Bc^2de^4 + Ac^2e^5\right)x^{10} + \frac{1}{9}\left(10\,Bc^2d^2e^3 + 5\,Ac^2de^4 + 2\,Bace^5\right)x^9 \\
&\quad + Aa^2d^5x + \frac{1}{4}\left(5\,Bc^2d^3e^2 + 5\,Ac^2d^2e^3 + 5\,Bacde^4 + Aace^5\right)x^8 \\
&\quad + \frac{1}{7}\left(5\,Bc^2d^4e + 10\,Ac^2d^3e^2 + 20\,Bacd^2e^3 + 10\,Aacde^4 + Ba^2e^5\right)x^7 \\
&\quad + \frac{1}{6}\left(Bc^2d^5 + 5\,Ac^2d^4e + 20\,Bacd^3e^2 + 20\,Aacd^2e^3 + 5\,Ba^2de^4 + Aa^2e^5\right)x^6 \\
&\quad + \frac{1}{5}\left(Ac^2d^5 + 10\,Bacd^4e + 20\,Aacd^3e^2 + 10\,Ba^2d^2e^3 + 5\,Aa^2de^4\right)x^5 \\
&\quad + \frac{1}{2}\left(Bacd^5 + 5\,Aacd^4e + 5\,Ba^2d^3e^2 + 5\,Aa^2d^2e^3\right)x^4 \\
&\quad + \frac{1}{3}\left(2\,Aacd^5 + 5\,Ba^2d^4e + 10\,Aa^2d^3e^2\right)x^3 + \frac{1}{2}\left(Ba^2d^5 + 5\,Aa^2d^4e\right)x^2
\end{aligned}$$

input
```
integrate((B*x+A)*(e*x+d)^5*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/11*B*c^2*e^5*x^11 + 1/10*(5*B*c^2*d*e^4 + A*c^2*e^5)*x^10 + 1/9*(10*B*c^
2*d^2*e^3 + 5*A*c^2*d*e^4 + 2*B*a*c*e^5)*x^9 + A*a^2*d^5*x + 1/4*(5*B*c^2*
d^3*e^2 + 5*A*c^2*d^2*e^3 + 5*B*a*c*d*e^4 + A*a*c*e^5)*x^8 + 1/7*(5*B*c^2*
d^4*e + 10*A*c^2*d^3*e^2 + 20*B*a*c*d^2*e^3 + 10*A*a*c*d*e^4 + B*a^2*e^5)*
x^7 + 1/6*(B*c^2*d^5 + 5*A*c^2*d^4*e + 20*B*a*c*d^3*e^2 + 20*A*a*c*d^2*e^3
 + 5*B*a^2*d*e^4 + A*a^2*e^5)*x^6 + 1/5*(A*c^2*d^5 + 10*B*a*c*d^4*e + 20*A
*a*c*d^3*e^2 + 10*B*a^2*d^2*e^3 + 5*A*a^2*d*e^4)*x^5 + 1/2*(B*a*c*d^5 + 5*
A*a*c*d^4*e + 5*B*a^2*d^3*e^2 + 5*A*a^2*d^2*e^3)*x^4 + 1/3*(2*A*a*c*d^5 +
5*B*a^2*d^4*e + 10*A*a^2*d^3*e^2)*x^3 + 1/2*(B*a^2*d^5 + 5*A*a^2*d^4*e)*x^
2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(194) = 388$.

Time = 0.13 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.26

$$\begin{aligned}\int (A+Bx)(d+ex)^5\left(a+cx^2\right)^2\,dx &= \frac{1}{11}Bc^2e^5x^{11}+\frac{1}{2}Bc^2de^4x^{10}+\frac{1}{10}Ac^2e^5x^{10}\\&+\frac{10}{9}Bc^2d^2e^3x^9+\frac{5}{9}Ac^2de^4x^9+\frac{2}{9}Bace^5x^9\\&+\frac{5}{4}Bc^2d^3e^2x^8+\frac{5}{4}Ac^2d^2e^3x^8+\frac{5}{4}Bacde^4x^8\\&+\frac{1}{4}Aace^5x^8+\frac{5}{7}Bc^2d^4ex^7+\frac{10}{7}Ac^2d^3e^2x^7\\&+\frac{20}{7}Bacd^2e^3x^7+\frac{10}{7}Aacde^4x^7+\frac{1}{7}Ba^2e^5x^7\\&+\frac{1}{6}Bc^2d^5x^6+\frac{5}{6}Ac^2d^4ex^6+\frac{10}{3}Bacd^3e^2x^6\\&+\frac{10}{3}Aacd^2e^3x^6+\frac{5}{6}Ba^2de^4x^6+\frac{1}{6}Aa^2e^5x^6\\&+\frac{1}{5}Ac^2d^5x^5+2\,Bacd^4ex^5+4\,Aacd^3e^2x^5\\&+2\,Ba^2d^2e^3x^5+Aa^2de^4x^5+\frac{1}{2}Bacd^5x^4\\&+\frac{5}{2}Aacd^4ex^4+\frac{5}{2}Ba^2d^3e^2x^4+\frac{5}{2}Aa^2d^2e^3x^4\\&+\frac{2}{3}Aacd^5x^3+\frac{5}{3}Ba^2d^4ex^3+\frac{10}{3}Aa^2d^3e^2x^3\\&+\frac{1}{2}Ba^2d^5x^2+\frac{5}{2}Aa^2d^4ex^2+Aa^2d^5x\end{aligned}$$

input
```
integrate((B*x+A)*(e*x+d)^5*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/11*B*c^2*e^5*x^11 + 1/2*B*c^2*d*e^4*x^10 + 1/10*A*c^2*e^5*x^10 + 10/9*B*
c^2*d^2*e^3*x^9 + 5/9*A*c^2*d*e^4*x^9 + 2/9*B*a*c*e^5*x^9 + 5/4*B*c^2*d^3*
e^2*x^8 + 5/4*A*c^2*d^2*e^3*x^8 + 5/4*B*a*c*d*e^4*x^8 + 1/4*A*a*c*e^5*x^8
+ 5/7*B*c^2*d^4*e*x^7 + 10/7*A*c^2*d^3*e^2*x^7 + 20/7*B*a*c*d^2*e^3*x^7 +
10/7*A*a*c*d*e^4*x^7 + 1/7*B*a^2*e^5*x^7 + 1/6*B*c^2*d^5*x^6 + 5/6*A*c^2*d
^4*e*x^6 + 10/3*B*a*c*d^3*e^2*x^6 + 10/3*A*a*c*d^2*e^3*x^6 + 5/6*B*a^2*d*e
^4*x^6 + 1/6*A*a^2*e^5*x^6 + 1/5*A*c^2*d^5*x^5 + 2*B*a*c*d^4*e*x^5 + 4*A*a
*c*d^3*e^2*x^5 + 2*B*a^2*d^2*e^3*x^5 + A*a^2*d*e^4*x^5 + 1/2*B*a*c*d^5*x^4
 + 5/2*A*a*c*d^4*e*x^4 + 5/2*B*a^2*d^3*e^2*x^4 + 5/2*A*a^2*d^2*e^3*x^4 + 2
/3*A*a*c*d^5*x^3 + 5/3*B*a^2*d^4*e*x^3 + 10/3*A*a^2*d^3*e^2*x^3 + 1/2*B*a^
2*d^5*x^2 + 5/2*A*a^2*d^4*e*x^2 + A*a^2*d^5*x
```

Mupad [B] (verification not implemented)

Time = 5.97 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.82

$$\begin{aligned}
&\int (A+Bx)(d+ex)^5 \left(a+cx^2\right)^2 \, dx \\
&= x^5 \left(2\,B\,a^2\,d^2\,e^3+A\,a^2\,d\,e^4+2\,B\,a\,c\,d^4\,e+4\,A\,a\,c\,d^3\,e^2+\frac{A\,c^2\,d^5}{5}\right) \\
&\quad + x^7 \left(\frac{B\,a^2\,e^5}{7}+\frac{20\,B\,a\,c\,d^2\,e^3}{7}+\frac{10\,A\,a\,c\,d\,e^4}{7}+\frac{5\,B\,c^2\,d^4\,e}{7}+\frac{10\,A\,c^2\,d^3\,e^2}{7}\right) \\
&\quad + x^6 \left(\frac{5\,B\,a^2\,d\,e^4}{6}+\frac{A\,a^2\,e^5}{6}+\frac{10\,B\,a\,c\,d^3\,e^2}{3}+\frac{10\,A\,a\,c\,d^2\,e^3}{3}+\frac{B\,c^2\,d^5}{6}+\frac{5\,A\,c^2\,d^4\,e}{6}\right) \\
&\quad + \frac{a\,d^3\,x^3\,(2\,A\,c\,d^2+5\,B\,a\,d\,e+10\,A\,a\,e^2)}{3} + \frac{c\,e^3\,x^9\,(10\,B\,c\,d^2+5\,A\,c\,d\,e+2\,B\,a\,e^2)}{9} \\
&\quad + \frac{a^2\,d^4\,x^2\,(5\,A\,e+B\,d)}{2} + \frac{c^2\,e^4\,x^{10}\,(A\,e+5\,B\,d)}{10} + A\,a^2\,d^5\,x \\
&\quad + \frac{a\,d^2\,x^4\,(B\,c\,d^3+5\,A\,c\,d^2\,e+5\,B\,a\,d\,e^2+5\,A\,a\,e^3)}{2} \\
&\quad + \frac{c\,e^2\,x^8\,(5\,B\,c\,d^3+5\,A\,c\,d^2\,e+5\,B\,a\,d\,e^2+A\,a\,e^3)}{4} + \frac{B\,c^2\,e^5\,x^{11}}{11}
\end{aligned}$$

input
```
int((a + c*x^2)^2*(A + B*x)*(d + e*x)^5,x)
```

output
```
x^5*((A*c^2*d^5)/5 + A*a^2*d*e^4 + 2*B*a^2*d^2*e^3 + 2*B*a*c*d^4*e + 4*A*a
*c*d^3*e^2) + x^7*((B*a^2*e^5)/7 + (5*B*c^2*d^4*e)/7 + (10*A*c^2*d^3*e^2)/
7 + (10*A*a*c*d*e^4)/7 + (20*B*a*c*d^2*e^3)/7) + x^6*((A*a^2*e^5)/6 + (B*c
^2*d^5)/6 + (5*B*a^2*d*e^4)/6 + (5*A*c^2*d^4*e)/6 + (10*A*a*c*d^2*e^3)/3 +
 (10*B*a*c*d^3*e^2)/3) + (a*d^3*x^3*(10*A*a*e^2 + 2*A*c*d^2 + 5*B*a*d*e))/
3 + (c*e^3*x^9*(2*B*a*e^2 + 10*B*c*d^2 + 5*A*c*d*e))/9 + (a^2*d^4*x^2*(5*A
*e + B*d))/2 + (c^2*e^4*x^10*(A*e + 5*B*d))/10 + A*a^2*d^5*x + (a*d^2*x^4*
(5*A*a*e^3 + B*c*d^3 + 5*B*a*d*e^2 + 5*A*c*d^2*e))/2 + (c*e^2*x^8*(A*a*e^3
 + 5*B*c*d^3 + 5*B*a*d*e^2 + 5*A*c*d^2*e))/4 + (B*c^2*e^5*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.26

$$\int (A+Bx)(d+ex)^5\left(a+cx^2\right)^2\,dx = \frac{x(1260b\,c^2e^5x^{10}+1386a\,c^2e^5x^9+6930b\,c^2d\,e^4x^9+3080abc\,e^5x^8+7700a\,c^2d\,e^4x^8+15400b\,c^2d^2e^3x^8+3}{}$$

input
```
int((B*x+A)*(e*x+d)^5*(c*x^2+a)^2,x)
```

output
```
(x*(13860*a**3*d**5 + 34650*a**3*d**4*e*x + 46200*a**3*d**3*e**2*x**2 + 34
650*a**3*d**2*e**3*x**3 + 13860*a**3*d*e**4*x**4 + 2310*a**3*e**5*x**5 + 6
930*a**2*b*d**5*x + 23100*a**2*b*d**4*e*x**2 + 34650*a**2*b*d**3*e**2*x**3
 + 27720*a**2*b*d**2*e**3*x**4 + 11550*a**2*b*d*e**4*x**5 + 1980*a**2*b*e*
*5*x**6 + 9240*a**2*c*d**5*x**2 + 34650*a**2*c*d**4*e*x**3 + 55440*a**2*c*
d**3*e**2*x**4 + 46200*a**2*c*d**2*e**3*x**5 + 19800*a**2*c*d*e**4*x**6 +
3465*a**2*c*e**5*x**7 + 6930*a*b*c*d**5*x**3 + 27720*a*b*c*d**4*e*x**4 + 4
6200*a*b*c*d**3*e**2*x**5 + 39600*a*b*c*d**2*e**3*x**6 + 17325*a*b*c*d*e**
4*x**7 + 3080*a*b*c*e**5*x**8 + 2772*a*c**2*d**5*x**4 + 11550*a*c**2*d**4*
e*x**5 + 19800*a*c**2*d**3*e**2*x**6 + 17325*a*c**2*d**2*e**3*x**7 + 7700*
a*c**2*d*e**4*x**8 + 1386*a*c**2*e**5*x**9 + 2310*b*c**2*d**5*x**5 + 9900*
b*c**2*d**4*e*x**6 + 17325*b*c**2*d**3*e**2*x**7 + 15400*b*c**2*d**2*e**3*
x**8 + 6930*b*c**2*d*e**4*x**9 + 1260*b*c**2*e**5*x**10))/13860
```

3.47 $\int (A+Bx)(d+ex)^4 \left(a+cx^2\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 206

$$\begin{aligned}
&\int (A+Bx)(d+ex)^4 \left(a+cx^2\right)^2 \, dx \\
&= -\frac{(Bd-Ae)\left(cd^2+ae^2\right)^2(d+ex)^5}{5e^6} \\
&\quad+\frac{\left(cd^2+ae^2\right)\left(5Bcd^2-4Acde+aBe^2\right)(d+ex)^6}{6e^6} \\
&\quad-\frac{2c\left(5Bcd^3-3Acd^2e+3aBde^2-aAe^3\right)(d+ex)^7}{7e^6} \\
&\quad+\frac{c\left(5Bcd^2-2Acde+aBe^2\right)(d+ex)^8}{4e^6}-\frac{c^2(5Bd-Ae)(d+ex)^9}{9e^6}+\frac{Bc^2(d+ex)^{10}}{10e^6}
\end{aligned}$$

output

```
-1/5*(-A*e+B*d)*(a*e^2+c*d^2)^2*(e*x+d)^5/e^6+1/6*(a*e^2+c*d^2)*(-4*A*c*d*
e+B*a*e^2+5*B*c*d^2)*(e*x+d)^6/e^6-2/7*c*(-A*a*e^3-3*A*c*d^2*e+3*B*a*d*e^2
+5*B*c*d^3)*(e*x+d)^7/e^6+1/4*c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)*(e*x+d)^8/e
^6-1/9*c^2*(-A*e+5*B*d)*(e*x+d)^9/e^6+1/10*B*c^2*(e*x+d)^10/e^6
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.52

$$\int (A+Bx)(d+ex)^4 \left(a+cx^2\right)^2 \, dx = a^2Ad^4x+\frac{1}{2}a^2d^3(Bd+4Ae)x^2 +\frac{2}{3}ad^2\left(Acd^2+2aBde+3aAe^2\right)x^3 +\frac{1}{2}ad\left(Bcd^3+4Acd^2e+3aBde^2+2aAe^3\right)x^4 +\frac{1}{5}\left(Ac^2d^4+8aBcd^3e+12aAcd^2e^2+4a^2Bde^3 +a^2Ae^4\right)x^5+\frac{1}{6}\left(Bc^2d^4+4Ac^2d^3e+12aBcd^2e^2 +8aAcde^3+a^2Be^4\right)x^6 +\frac{2}{7}ce\left(2Bcd^3+3Acd^2e+4aBde^2+aAe^3\right)x^7 +\frac{1}{4}ce^2\left(3Bcd^2+2Acde+aBe^2\right)x^8 +\frac{1}{9}c^2e^3(4Bd+Ae)x^9+\frac{1}{10}Bc^2e^4x^{10}$$

input
```
Integrate[(A + B*x)*(d + e*x)^4*(a + c*x^2)^2,x]
```

output
```
a^2*A*d^4*x + (a^2*d^3*(B*d + 4*A*e)*x^2)/2 + (2*a*d^2*(A*c*d^2 + 2*a*B*d*
e + 3*a*A*e^2)*x^3)/3 + (a*d*(B*c*d^3 + 4*A*c*d^2*e + 3*a*B*d*e^2 + 2*a*A*
e^3)*x^4)/2 + ((A*c^2*d^4 + 8*a*B*c*d^3*e + 12*a*A*c*d^2*e^2 + 4*a^2*B*d*e
^3 + a^2*A*e^4)*x^5)/5 + ((B*c^2*d^4 + 4*A*c^2*d^3*e + 12*a*B*c*d^2*e^2 +
8*a*A*c*d*e^3 + a^2*B*e^4)*x^6)/6 + (2*c*e*(2*B*c*d^3 + 3*A*c*d^2*e + 4*a*
B*d*e^2 + a*A*e^3)*x^7)/7 + (c*e^2*(3*B*c*d^2 + 2*A*c*d*e + a*B*e^2)*x^8)/
4 + (c^2*e^3*(4*B*d + A*e)*x^9)/9 + (B*c^2*e^4*x^10)/10
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^2 (A+Bx)(d+ex)^4\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{2c(d+ex)^7\left(-aBe^2+2Acde-5Bcd^2\right)}{e^5}+\frac{(d+ex)^5\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^5}+\frac{(d+ex)^4\left(\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(d+ex)^8\left(aBe^2-2Acde+5Bcd^2\right)}{4e^6}+\frac{(d+ex)^6\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{6e^6}-\frac{(d+ex)^5\left(ae^2+cd^2\right)^2(Bd-Ae)}{5e^6}-\frac{2c(d+ex)^7\left(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3\right)}{7e^6}-\frac{c^2(d+ex)^9(5Bd-Ae)}{9e^6}+\frac{Bc^2(d+ex)^{10}}{10e^6}$$

input
```
Int[(A + B*x)*(d + e*x)^4*(a + c*x^2)^2,x]
```

output
```
-1/5*((B*d - A*e)*(c*d^2 + a*e^2)^2*(d + e*x)^5)/e^6 + ((c*d^2 + a*e^2)*(5
*B*c*d^2 - 4*A*c*d*e + a*B*e^2)*(d + e*x)^6)/(6*e^6) - (2*c*(5*B*c*d^3 - 3
*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*(d + e*x)^7)/(7*e^6) + (c*(5*B*c*d^2 -
 2*A*c*d*e + a*B*e^2)*(d + e*x)^8)/(4*e^6) - (c^2*(5*B*d - A*e)*(d + e*x)^
9)/(9*e^6) + (B*c^2*(d + e*x)^10)/(10*e^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.59

method	result
default	$\frac{B e^4 c^2 x^{10}}{10}+\frac{(A e^4+4Bd e^3) c^2 x^9}{9}+\frac{((4Ad e^3+6B d^2e^2)c^2+2B e^4ac) x^8}{8}+\frac{((6A d^2 e^2+4B d^3 e)c^2+2(A e^4+4Bd e^3)ac)}{7}$
norman	$\frac{B e^4 c^2 x^{10}}{10}+\left(\frac{1}{9}A c^2 e^4+\frac{4}{9}B c^2 d e^3\right)x^9+\left(\frac{1}{2}A c^2 d e^3+\frac{1}{4}B e^4 ac+\frac{3}{4}B c^2 d^2 e^2\right)x^8+\left(\frac{2}{7}Aac e^4+\right.$
gosper	$\frac{1}{2}x^4 Bac d^4+2x^3 A a^2 d^2 e^2+\frac{2}{3}x^3 A d^4 ac+2x^6 Bac d^2 e^2+\frac{3}{2}x^4 B a^2 d^2 e^2+\frac{12}{5}x^5 Aac d^2 e^2+\frac{4}{3}$
risch	$\frac{1}{2}x^4 Bac d^4+2x^3 A a^2 d^2 e^2+\frac{2}{3}x^3 A d^4 ac+2x^6 Bac d^2 e^2+\frac{3}{2}x^4 B a^2 d^2 e^2+\frac{12}{5}x^5 Aac d^2 e^2+\frac{4}{3}$
parallelrisch	$\frac{1}{2}x^4 Bac d^4+2x^3 A a^2 d^2 e^2+\frac{2}{3}x^3 A d^4 ac+2x^6 Bac d^2 e^2+\frac{3}{2}x^4 B a^2 d^2 e^2+\frac{12}{5}x^5 Aac d^2 e^2+\frac{4}{3}$
orering	$x(126B e^4 c^2 x^9+140A c^2 e^4 x^8+560B c^2 d e^3 x^8+630A c^2 d e^3 x^7+315Bac e^4 x^7+945B c^2 d^2 e^2 x^7+360Aac e^4 x^6+1080A c^2 d^2 e^2$

input
```
int((B*x+A)*(e*x+d)^4*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*B*e^4*c^2*x^10+1/9*(A*e^4+4*B*d*e^3)*c^2*x^9+1/8*((4*A*d*e^3+6*B*d^2*
e^2)*c^2+2*B*e^4*a*c)*x^8+1/7*((6*A*d^2*e^2+4*B*d^3*e)*c^2+2*(A*e^4+4*B*d*
e^3)*a*c)*x^7+1/6*((4*A*d^3*e+B*d^4)*c^2+2*(4*A*d*e^3+6*B*d^2*e^2)*a*c+B*e
^4*a^2)*x^6+1/5*(A*c^2*d^4+2*(6*A*d^2*e^2+4*B*d^3*e)*a*c+(A*e^4+4*B*d*e^3)
*a^2)*x^5+1/4*(2*(4*A*d^3*e+B*d^4)*a*c+(4*A*d*e^3+6*B*d^2*e^2)*a^2)*x^4+1/
3*(2*A*d^4*a*c+(6*A*d^2*e^2+4*B*d^3*e)*a^2)*x^3+1/2*(4*A*d^3*e+B*d^4)*a^2*
x^2+A*d^4*a^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.61

$$\int (A+Bx)(d+ex)^4\left(a+cx^2\right)^2\,dx = \frac{1}{10}Bc^2e^4x^{10}+\frac{1}{9}\left(4Bc^2de^3+Ac^2e^4\right)x^9+\frac{1}{4}\left(3Bc^2d^2e^2+2Ac^2de^3+Bace^4\right)x^8 + Aa^2d^4x+\frac{2}{7}\left(2Bc^2d^3e+3Ac^2d^2e^2+4Bacde^3+Aace^4\right)x^7 + \frac{1}{6}\left(Bc^2d^4+4Ac^2d^3e+12Bacd^2e^2+8Aacde^3+Ba^2e^4\right)x^6 + \frac{1}{5}\left(Ac^2d^4+8Bacd^3e+12Aacd^2e^2+4Ba^2de^3+Aa^2e^4\right)x^5 + \frac{1}{2}\left(Bacd^4+4Aacd^3e+3Ba^2d^2e^2+2Aa^2de^3\right)x^4 + \frac{2}{3}\left(Aacd^4+2Ba^2d^3e+3Aa^2d^2e^2\right)x^3+\frac{1}{2}\left(Ba^2d^4+4Aa^2d^3e\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)^4*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/10*B*c^2*e^4*x^10 + 1/9*(4*B*c^2*d*e^3 + A*c^2*e^4)*x^9 + 1/4*(3*B*c^2*d
^2*e^2 + 2*A*c^2*d*e^3 + B*a*c*e^4)*x^8 + A*a^2*d^4*x + 2/7*(2*B*c^2*d^3*e
 + 3*A*c^2*d^2*e^2 + 4*B*a*c*d*e^3 + A*a*c*e^4)*x^7 + 1/6*(B*c^2*d^4 + 4*A
*c^2*d^3*e + 12*B*a*c*d^2*e^2 + 8*A*a*c*d*e^3 + B*a^2*e^4)*x^6 + 1/5*(A*c^
2*d^4 + 8*B*a*c*d^3*e + 12*A*a*c*d^2*e^2 + 4*B*a^2*d*e^3 + A*a^2*e^4)*x^5
+ 1/2*(B*a*c*d^4 + 4*A*a*c*d^3*e + 3*B*a^2*d^2*e^2 + 2*A*a^2*d*e^3)*x^4 +
2/3*(A*a*c*d^4 + 2*B*a^2*d^3*e + 3*A*a^2*d^2*e^2)*x^3 + 1/2*(B*a^2*d^4 + 4
*A*a^2*d^3*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.93

$$\int (A+Bx)(d+ex)^4\left(a+cx^2\right)^2\,dx = Aa^2d^4x+\frac{Bc^2e^4x^{10}}{10}+x^9\left(\frac{Ac^2e^4}{9}+\frac{4Bc^2de^3}{9}\right)+x^8\left(\frac{Ac^2de^3}{2}+\frac{Bace^4}{4}+\frac{3Bc^2d^2e^2}{4}\right)+x^7\cdot\left(\frac{2Aace^4}{7}+\frac{6Ac^2d^2e^2}{7}+\frac{8Bacde^3}{7}+\frac{4Bc^2d^3e}{7}\right)+x^6\cdot\left(\frac{4Aacde^3}{3}+\frac{2Ac^2d^3e}{3}+\frac{Ba^2e^4}{6}+2Bacd^2e^2+\frac{Bc^2d^4}{6}\right)+x^5\left(\frac{Aa^2e^4}{5}+\frac{12Aacd^2e^2}{5}+\frac{Ac^2d^4}{5}+\frac{4Ba^2de^3}{5}+\frac{8Bacd^3e}{5}\right)+x^4\left(Aa^2de^3+2Aacd^3e+\frac{3Ba^2d^2e^2}{2}+\frac{Bacd^4}{2}\right)+x^3\cdot\left(2Aa^2d^2e^2+\frac{2Aacd^4}{3}+\frac{4Ba^2d^3e}{3}\right)+x^2\cdot\left(2Aa^2d^3e+\frac{Ba^2d^4}{2}\right)$$

input
```
integrate((B*x+A)*(e*x+d)**4*(c*x**2+a)**2,x)
```

output
```
A*a**2*d**4*x + B*c**2*e**4*x**10/10 + x**9*(A*c**2*e**4/9 + 4*B*c**2*d*e*
*3/9) + x**8*(A*c**2*d*e**3/2 + B*a*c*e**4/4 + 3*B*c**2*d**2*e**2/4) + x**
7*(2*A*a*c*e**4/7 + 6*A*c**2*d**2*e**2/7 + 8*B*a*c*d*e**3/7 + 4*B*c**2*d**
3*e/7) + x**6*(4*A*a*c*d*e**3/3 + 2*A*c**2*d**3*e/3 + B*a**2*e**4/6 + 2*B*
a*c*d**2*e**2 + B*c**2*d**4/6) + x**5*(A*a**2*e**4/5 + 12*A*a*c*d**2*e**2/
5 + A*c**2*d**4/5 + 4*B*a**2*d*e**3/5 + 8*B*a*c*d**3*e/5) + x**4*(A*a**2*d
*e**3 + 2*A*a*c*d**3*e + 3*B*a**2*d**2*e**2/2 + B*a*c*d**4/2) + x**3*(2*A*
a**2*d**2*e**2 + 2*A*a*c*d**4/3 + 4*B*a**2*d**3*e/3) + x**2*(2*A*a**2*d**3
*e + B*a**2*d**4/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.61

$$\int (A+Bx)(d+ex)^4\left(a+cx^2\right)^2\,dx$$
$$= \frac{1}{10}Bc^2e^4x^{10} + \frac{1}{9}\left(4Bc^2de^3 + Ac^2e^4\right)x^9 + \frac{1}{4}\left(3Bc^2d^2e^2 + 2Ac^2de^3 + Bace^4\right)x^8 + Aa^2d^4x + \frac{2}{7}\left(2Bc^2d^3e + 3Ac^2d^2e^2 + 4Bacde^3 + Aace^4\right)x^7 + \frac{1}{6}\left(Bc^2d^4 + 4Ac^2d^3e + 12Bacd^2e^2 + 8Aacde^3 + Ba^2e^4\right)x^6 + \frac{1}{5}\left(Ac^2d^4 + 8Bacd^3e + 12Aacd^2e^2 + 4Ba^2de^3 + Aa^2e^4\right)x^5 + \frac{1}{2}\left(Bacd^4 + 4Aacd^3e + 3Ba^2d^2e^2 + 2Aa^2de^3\right)x^4 + \frac{2}{3}\left(Aacd^4 + 2Ba^2d^3e + 3Aa^2d^2e^2\right)x^3 + \frac{1}{2}\left(Ba^2d^4 + 4Aa^2d^3e\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)^4*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/10*B*c^2*e^4*x^10 + 1/9*(4*B*c^2*d*e^3 + A*c^2*e^4)*x^9 + 1/4*(3*B*c^2*d
^2*e^2 + 2*A*c^2*d*e^3 + B*a*c*e^4)*x^8 + A*a^2*d^4*x + 2/7*(2*B*c^2*d^3*e
 + 3*A*c^2*d^2*e^2 + 4*B*a*c*d*e^3 + A*a*c*e^4)*x^7 + 1/6*(B*c^2*d^4 + 4*A
*c^2*d^3*e + 12*B*a*c*d^2*e^2 + 8*A*a*c*d*e^3 + B*a^2*e^4)*x^6 + 1/5*(A*c^
2*d^4 + 8*B*a*c*d^3*e + 12*A*a*c*d^2*e^2 + 4*B*a^2*d*e^3 + A*a^2*e^4)*x^5
+ 1/2*(B*a*c*d^4 + 4*A*a*c*d^3*e + 3*B*a^2*d^2*e^2 + 2*A*a^2*d*e^3)*x^4 +
2/3*(A*a*c*d^4 + 2*B*a^2*d^3*e + 3*A*a^2*d^2*e^2)*x^3 + 1/2*(B*a^2*d^4 + 4
*A*a^2*d^3*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.83

$$\int (A+Bx)(d+ex)^4 \left(a+cx^2\right)^2 \, dx = \frac{1}{10} Bc^2e^4x^{10} + \frac{4}{9} Bc^2de^3x^9 + \frac{1}{9} Ac^2e^4x^9 + \frac{3}{4} Bc^2d^2e^2x^8 + \frac{1}{2} Ac^2de^3x^8 + \frac{1}{4} Bace^4x^8 + \frac{4}{7} Bc^2d^3ex^7 + \frac{6}{7} Ac^2d^2e^2x^7 + \frac{8}{7} Bacde^3x^7 + \frac{2}{7} Aace^4x^7 + \frac{1}{6} Bc^2d^4x^6 + \frac{2}{3} Ac^2d^3ex^6 + 2 Bacd^2e^2x^6 + \frac{4}{3} Aacde^3x^6 + \frac{1}{6} Ba^2e^4x^6 + \frac{1}{5} Ac^2d^4x^5 + \frac{8}{5} Bacd^3ex^5 + \frac{12}{5} Aacd^2e^2x^5 + \frac{4}{5} Ba^2de^3x^5 + \frac{1}{5} Aa^2e^4x^5 + \frac{1}{2} Bacd^4x^4 + 2 Aacd^3ex^4 + \frac{3}{2} Ba^2d^2e^2x^4 + Aa^2de^3x^4 + \frac{2}{3} Aacd^4x^3 + \frac{4}{3} Ba^2d^3ex^3 + 2 Aa^2d^2e^2x^3 + \frac{1}{2} Ba^2d^4x^2 + 2 Aa^2d^3ex^2 + Aa^2d^4x$$

input
```
integrate((B*x+A)*(e*x+d)^4*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/10*B*c^2*e^4*x^10 + 4/9*B*c^2*d*e^3*x^9 + 1/9*A*c^2*e^4*x^9 + 3/4*B*c^2*
d^2*e^2*x^8 + 1/2*A*c^2*d*e^3*x^8 + 1/4*B*a*c*e^4*x^8 + 4/7*B*c^2*d^3*e*x^
7 + 6/7*A*c^2*d^2*e^2*x^7 + 8/7*B*a*c*d*e^3*x^7 + 2/7*A*a*c*e^4*x^7 + 1/6*
B*c^2*d^4*x^6 + 2/3*A*c^2*d^3*e*x^6 + 2*B*a*c*d^2*e^2*x^6 + 4/3*A*a*c*d*e^
3*x^6 + 1/6*B*a^2*e^4*x^6 + 1/5*A*c^2*d^4*x^5 + 8/5*B*a*c*d^3*e*x^5 + 12/5
*A*a*c*d^2*e^2*x^5 + 4/5*B*a^2*d*e^3*x^5 + 1/5*A*a^2*e^4*x^5 + 1/2*B*a*c*d
^4*x^4 + 2*A*a*c*d^3*e*x^4 + 3/2*B*a^2*d^2*e^2*x^4 + A*a^2*d*e^3*x^4 + 2/3
*A*a*c*d^4*x^3 + 4/3*B*a^2*d^3*e*x^3 + 2*A*a^2*d^2*e^2*x^3 + 1/2*B*a^2*d^4
*x^2 + 2*A*a^2*d^3*e*x^2 + A*a^2*d^4*x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.45

$$\int (A+Bx)(d+ex)^4 \left(a+cx^2\right)^2 \, dx$$
$$= x^5 \left(\frac{4\,B\,a^2\,d\,e^3}{5}+\frac{A\,a^2\,e^4}{5}+\frac{8\,B\,a\,c\,d^3\,e}{5}+\frac{12\,A\,a\,c\,d^2\,e^2}{5}+\frac{A\,c^2\,d^4}{5}\right)$$
$$+ x^6 \left(\frac{B\,a^2\,e^4}{6}+2\,B\,a\,c\,d^2\,e^2+\frac{4\,A\,a\,c\,d\,e^3}{3}+\frac{B\,c^2\,d^4}{6}+\frac{2\,A\,c^2\,d^3\,e}{3}\right)$$
$$+ \frac{2\,a\,d^2\,x^3\,(A\,c\,d^2+2\,B\,a\,d\,e+3\,A\,a\,e^2)}{3}+\frac{c\,e^2\,x^8\,(3\,B\,c\,d^2+2\,A\,c\,d\,e+B\,a\,e^2)}{4}$$
$$+ \frac{a^2\,d^3\,x^2\,(4\,A\,e+B\,d)}{2}+\frac{c^2\,e^3\,x^9\,(A\,e+4\,B\,d)}{9}$$
$$+ \frac{a\,d\,x^4\,(B\,c\,d^3+4\,A\,c\,d^2\,e+3\,B\,a\,d\,e^2+2\,A\,a\,e^3)}{2}$$
$$+ \frac{2\,c\,e\,x^7\,(2\,B\,c\,d^3+3\,A\,c\,d^2\,e+4\,B\,a\,d\,e^2+A\,a\,e^3)}{7}+A\,a^2\,d^4\,x+\frac{B\,c^2\,e^4\,x^{10}}{10}$$

input
```
int((a + c*x^2)^2*(A + B*x)*(d + e*x)^4,x)
```

output
```
x^5*((A*a^2*e^4)/5 + (A*c^2*d^4)/5 + (4*B*a^2*d*e^3)/5 + (8*B*a*c*d^3*e)/5
 + (12*A*a*c*d^2*e^2)/5) + x^6*((B*a^2*e^4)/6 + (B*c^2*d^4)/6 + (2*A*c^2*d
^3*e)/3 + (4*A*a*c*d*e^3)/3 + 2*B*a*c*d^2*e^2) + (2*a*d^2*x^3*(3*A*a*e^2 +
 A*c*d^2 + 2*B*a*d*e))/3 + (c*e^2*x^8*(B*a*e^2 + 3*B*c*d^2 + 2*A*c*d*e))/4
 + (a^2*d^3*x^2*(4*A*e + B*d))/2 + (c^2*e^3*x^9*(A*e + 4*B*d))/9 + (a*d*x^
4*(2*A*a*e^3 + B*c*d^3 + 3*B*a*d*e^2 + 4*A*c*d^2*e))/2 + (2*c*e*x^7*(A*a*e
^3 + 2*B*c*d^3 + 4*B*a*d*e^2 + 3*A*c*d^2*e))/7 + A*a^2*d^4*x + (B*c^2*e^4*
x^10)/10
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.83

$$\int (A+Bx)(d+ex)^4 \left(a+cx^2\right)^2 \, dx$$
$$= \frac{x(126b\,c^2e^4x^9+140a\,c^2e^4x^8+560b\,c^2d\,e^3x^8+315abc\,e^4x^7+630a\,c^2d\,e^3x^7+945b\,c^2d^2e^2x^7+360a^2c\,e^4x}{}$$

input
```
int((B*x+A)*(e*x+d)^4*(c*x^2+a)^2,x)
```

output
```
(x*(1260*a**3*d**4 + 2520*a**3*d**3*e*x + 2520*a**3*d**2*e**2*x**2 + 1260*
a**3*d*e**3*x**3 + 252*a**3*e**4*x**4 + 630*a**2*b*d**4*x + 1680*a**2*b*d*
*3*e*x**2 + 1890*a**2*b*d**2*e**2*x**3 + 1008*a**2*b*d*e**3*x**4 + 210*a**
2*b*e**4*x**5 + 840*a**2*c*d**4*x**2 + 2520*a**2*c*d**3*e*x**3 + 3024*a**2
*c*d**2*e**2*x**4 + 1680*a**2*c*d*e**3*x**5 + 360*a**2*c*e**4*x**6 + 630*a
*b*c*d**4*x**3 + 2016*a*b*c*d**3*e*x**4 + 2520*a*b*c*d**2*e**2*x**5 + 1440
*a*b*c*d*e**3*x**6 + 315*a*b*c*e**4*x**7 + 252*a*c**2*d**4*x**4 + 840*a*c*
*2*d**3*e*x**5 + 1080*a*c**2*d**2*e**2*x**6 + 630*a*c**2*d*e**3*x**7 + 140
*a*c**2*e**4*x**8 + 210*b*c**2*d**4*x**5 + 720*b*c**2*d**3*e*x**6 + 945*b*
c**2*d**2*e**2*x**7 + 560*b*c**2*d*e**3*x**8 + 126*b*c**2*e**4*x**9))/1260
```

3.48 $\int (A+Bx)(d+ex)^3 \left(a+cx^2\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 206

$$\int (A+Bx)(d+ex)^3 \left(a+cx^2\right)^2 \, dx = -\frac{(Bd-Ae)\left(cd^2+ae^2\right)^2(d+ex)^4}{4e^6} + \frac{\left(cd^2+ae^2\right)\left(5Bcd^2-4Acde+aBe^2\right)(d+ex)^5}{5e^6} - \frac{c\left(5Bcd^3-3Acd^2e+3aBde^2-aAe^3\right)(d+ex)^6}{3e^6} + \frac{2c\left(5Bcd^2-2Acde+aBe^2\right)(d+ex)^7}{7e^6} - \frac{c^2(5Bd-Ae)(d+ex)^8}{8e^6} + \frac{Bc^2(d+ex)^9}{9e^6}$$

output
```
-1/4*(-A*e+B*d)*(a*e^2+c*d^2)^2*(e*x+d)^4/e^6+1/5*(a*e^2+c*d^2)*(-4*A*c*d*
e+B*a*e^2+5*B*c*d^2)*(e*x+d)^5/e^6-1/3*c*(-A*a*e^3-3*A*c*d^2*e+3*B*a*d*e^2
+5*B*c*d^3)*(e*x+d)^6/e^6+2/7*c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)*(e*x+d)^7/e
^6-1/8*c^2*(-A*e+5*B*d)*(e*x+d)^8/e^6+1/9*B*c^2*(e*x+d)^9/e^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.18

$$\int (A+Bx)(d+ex)^3 \left(a+cx^2\right)^2 \, dx = a^2Ad^3x+\frac{1}{2}a^2d^2(Bd+3Ae)x^2 +\frac{1}{3}ad\left(2Acd^2+3aBde+3aAe^2\right)x^3 +\frac{1}{4}a\left(2Bcd^3+6Acd^2e+3aBde^2+aAe^3\right)x^4 +\frac{1}{5}\left(Ac^2d^3+6aBcd^2e+6aAcde^2+a^2Be^3\right)x^5 +\frac{1}{6}c\left(Bcd^3+3Acd^2e+6aBde^2+2aAe^3\right)x^6 +\frac{1}{7}ce\left(3Bcd^2+3Acde+2aBe^2\right)x^7 +\frac{1}{8}c^2e^2(3Bd+Ae)x^8+\frac{1}{9}Bc^2e^3x^9$$

input
```
Integrate[(A + B*x)*(d + e*x)^3*(a + c*x^2)^2,x]
```

output
```
a^2*A*d^3*x + (a^2*d^2*(B*d + 3*A*e)*x^2)/2 + (a*d*(2*A*c*d^2 + 3*a*B*d*e
+ 3*a*A*e^2)*x^3)/3 + (a*(2*B*c*d^3 + 6*A*c*d^2*e + 3*a*B*d*e^2 + a*A*e^3)
*x^4)/4 + ((A*c^2*d^3 + 6*a*B*c*d^2*e + 6*a*A*c*d*e^2 + a^2*B*e^3)*x^5)/5
+ (c*(B*c*d^3 + 3*A*c*d^2*e + 6*a*B*d*e^2 + 2*a*A*e^3)*x^6)/6 + (c*e*(3*B*
c*d^2 + 3*A*c*d*e + 2*a*B*e^2)*x^7)/7 + (c^2*e^2*(3*B*d + A*e)*x^8)/8 + (B
*c^2*e^3*x^9)/9
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2 (A+Bx)(d+ex)^3 \, dx$$

$$\downarrow 652$$

$$\int \left(-\frac{2c(d+ex)^6 \left(-aBe^2+2Acde-5Bcd^2\right)}{e^5}+\frac{(d+ex)^4 \left(ae^2+cd^2\right) \left(aBe^2-4Acde+5Bcd^2\right)}{e^5}+\frac{(d+ex)^3 (a$$

$$\downarrow 2009$$

$$\frac{2c(d+ex)^7 \left(aBe^2-2Acde+5Bcd^2\right)}{7e^6}+\frac{(d+ex)^5 \left(ae^2+cd^2\right) \left(aBe^2-4Acde+5Bcd^2\right)}{5e^6}-\frac{(d+ex)^4 \left(ae^2+cd^2\right)^2 (Bd-Ae)}{4e^6}-\frac{c(d+ex)^6 \left(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3\right)}{3e^6}-\frac{c^2(d+ex)^8(5Bd-Ae)}{8e^6}+\frac{Bc^2(d+ex)^9}{9e^6}$$

input
```
Int[(A + B*x)*(d + e*x)^3*(a + c*x^2)^2,x]
```

output
```
-1/4*((B*d - A*e)*(c*d^2 + a*e^2)^2*(d + e*x)^4)/e^6 + ((c*d^2 + a*e^2)*(5
*B*c*d^2 - 4*A*c*d*e + a*B*e^2)*(d + e*x)^5)/(5*e^6) - (c*(5*B*c*d^3 - 3*A
*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*(d + e*x)^6)/(3*e^6) + (2*c*(5*B*c*d^2 -
 2*A*c*d*e + a*B*e^2)*(d + e*x)^7)/(7*e^6) - (c^2*(5*B*d - A*e)*(d + e*x)^
8)/(8*e^6) + (B*c^2*(d + e*x)^9)/(9*e^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.22

method	result
default	$\frac{B e^3 c^2 x^9}{9} + \frac{(A e^3+3Bd e^2) c^2 x^8}{8} + \frac{((3Ad e^2+3B d^2 e) c^2+2B e^3 ac) x^7}{7} + \frac{((3A d^2 e+B d^3) c^2+2(A e^3+3Bd e^2) ac) x^6}{6} +$
norman	$\frac{B e^3 c^2 x^9}{9} + \left(\frac{1}{8}A c^2 e^3 + \frac{3}{8}B c^2 d e^2\right) x^8 + \left(\frac{3}{7}A c^2 d e^2 + \frac{2}{7}B e^3 ac + \frac{3}{7}B c^2 d^2 e\right) x^7 + \left(\frac{1}{3}Aac e^3 +\right.$
gosper	$\frac{1}{9}B e^3 c^2 x^9 + \frac{1}{8}x^8 A c^2 e^3 + \frac{3}{8}x^8 B c^2 d e^2 + \frac{3}{7}x^7 A c^2 d e^2 + \frac{2}{7}x^7 B e^3 ac + \frac{3}{7}x^7 B c^2 d^2 e + \frac{1}{3}x^6 Aac$
risch	$\frac{1}{9}B e^3 c^2 x^9 + \frac{1}{8}x^8 A c^2 e^3 + \frac{3}{8}x^8 B c^2 d e^2 + \frac{3}{7}x^7 A c^2 d e^2 + \frac{2}{7}x^7 B e^3 ac + \frac{3}{7}x^7 B c^2 d^2 e + \frac{1}{3}x^6 Aac$
parallelrisch	$\frac{1}{9}B e^3 c^2 x^9 + \frac{1}{8}x^8 A c^2 e^3 + \frac{3}{8}x^8 B c^2 d e^2 + \frac{3}{7}x^7 A c^2 d e^2 + \frac{2}{7}x^7 B e^3 ac + \frac{3}{7}x^7 B c^2 d^2 e + \frac{1}{3}x^6 Aac$
orering	$x(280B e^3 c^2 x^8+315A c^2 e^3 x^7+945B c^2 d e^2 x^7+1080A c^2 d e^2 x^6+720Bac e^3 x^6+1080B c^2 d^2 e x^6+840Aac e^3 x^5+1260A c^2 d^2 e$

input
```
int((B*x+A)*(e*x+d)^3*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*B*e^3*c^2*x^9+1/8*(A*e^3+3*B*d*e^2)*c^2*x^8+1/7*((3*A*d*e^2+3*B*d^2*e)
*c^2+2*B*e^3*a*c)*x^7+1/6*((3*A*d^2*e+B*d^3)*c^2+2*(A*e^3+3*B*d*e^2)*a*c)*
x^6+1/5*(A*c^2*d^3+2*(3*A*d*e^2+3*B*d^2*e)*a*c+B*e^3*a^2)*x^5+1/4*(2*(3*A*
d^2*e+B*d^3)*a*c+(A*e^3+3*B*d*e^2)*a^2)*x^4+1/3*(2*A*d^3*a*c+(3*A*d*e^2+3*
B*d^2*e)*a^2)*x^3+1/2*(3*A*d^2*e+B*d^3)*a^2*x^2+A*d^3*a^2*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.26

$$
\begin{aligned}
\int (A+Bx)(d+ex)^3\left(a+cx^2\right)^2\,dx = {} & \frac{1}{9}Bc^2e^3x^9 + \frac{1}{8}\left(3\,Bc^2de^2 + Ac^2e^3\right)x^8 \\
& + \frac{1}{7}\left(3\,Bc^2d^2e + 3\,Ac^2de^2 + 2\,Bace^3\right)x^7 + Aa^2d^3x \\
& + \frac{1}{6}\left(Bc^2d^3 + 3\,Ac^2d^2e + 6\,Bacde^2 + 2\,Aace^3\right)x^6 \\
& + \frac{1}{5}\left(Ac^2d^3 + 6\,Bacd^2e + 6\,Aacde^2 + Ba^2e^3\right)x^5 \\
& + \frac{1}{4}\left(2\,Bacd^3 + 6\,Aacd^2e + 3\,Ba^2de^2 + Aa^2e^3\right)x^4 \\
& + \frac{1}{3}\left(2\,Aacd^3 + 3\,Ba^2d^2e + 3\,Aa^2de^2\right)x^3 \\
& + \frac{1}{2}\left(Ba^2d^3 + 3\,Aa^2d^2e\right)x^2
\end{aligned}
$$

input
```
integrate((B*x+A)*(e*x+d)^3*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/9*B*c^2*e^3*x^9 + 1/8*(3*B*c^2*d*e^2 + A*c^2*e^3)*x^8 + 1/7*(3*B*c^2*d^2
*e + 3*A*c^2*d*e^2 + 2*B*a*c*e^3)*x^7 + A*a^2*d^3*x + 1/6*(B*c^2*d^3 + 3*A
*c^2*d^2*e + 6*B*a*c*d*e^2 + 2*A*a*c*e^3)*x^6 + 1/5*(A*c^2*d^3 + 6*B*a*c*d
^2*e + 6*A*a*c*d*e^2 + B*a^2*e^3)*x^5 + 1/4*(2*B*a*c*d^3 + 6*A*a*c*d^2*e +
 3*B*a^2*d*e^2 + A*a^2*e^3)*x^4 + 1/3*(2*A*a*c*d^3 + 3*B*a^2*d^2*e + 3*A*a
^2*d*e^2)*x^3 + 1/2*(B*a^2*d^3 + 3*A*a^2*d^2*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.47

$$\int (A+Bx)(d+ex)^3\left(a+cx^2\right)^2\,dx = Aa^2d^3x + \frac{Bc^2e^3x^9}{9} + x^8\left(\frac{Ac^2e^3}{8} + \frac{3Bc^2de^2}{8}\right) + x^7\cdot\left(\frac{3Ac^2de^2}{7} + \frac{2Bace^3}{7} + \frac{3Bc^2d^2e}{7}\right) + x^6\left(\frac{Aace^3}{3} + \frac{Ac^2d^2e}{2} + Bacde^2 + \frac{Bc^2d^3}{6}\right) + x^5\cdot\left(\frac{6Aacde^2}{5} + \frac{Ac^2d^3}{5} + \frac{Ba^2e^3}{5} + \frac{6Bacd^2e}{5}\right) + x^4\left(\frac{Aa^2e^3}{4} + \frac{3Aacd^2e}{2} + \frac{3Ba^2de^2}{4} + \frac{Bacd^3}{2}\right) + x^3\left(Aa^2de^2 + \frac{2Aacd^3}{3} + Ba^2d^2e\right) + x^2\cdot\left(\frac{3Aa^2d^2e}{2} + \frac{Ba^2d^3}{2}\right)$$

input
```
integrate((B*x+A)*(e*x+d)**3*(c*x**2+a)**2,x)
```

output
```
A*a**2*d**3*x + B*c**2*e**3*x**9/9 + x**8*(A*c**2*e**3/8 + 3*B*c**2*d*e**2
/8) + x**7*(3*A*c**2*d*e**2/7 + 2*B*a*c*e**3/7 + 3*B*c**2*d**2*e/7) + x**6
*(A*a*c*e**3/3 + A*c**2*d**2*e/2 + B*a*c*d*e**2 + B*c**2*d**3/6) + x**5*(6
*A*a*c*d*e**2/5 + A*c**2*d**3/5 + B*a**2*e**3/5 + 6*B*a*c*d**2*e/5) + x**4
*(A*a**2*e**3/4 + 3*A*a*c*d**2*e/2 + 3*B*a**2*d*e**2/4 + B*a*c*d**3/2) + x
**3*(A*a**2*d*e**2 + 2*A*a*c*d**3/3 + B*a**2*d**2*e) + x**2*(3*A*a**2*d**2
*e/2 + B*a**2*d**3/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.26

$$\begin{aligned}\int (A+Bx)(d+ex)^3\left(a+cx^2\right)^2\,dx &= \frac{1}{9}\,Bc^2e^3x^9 + \frac{1}{8}\left(3\,Bc^2de^2 + Ac^2e^3\right)x^8 \\ &+ \frac{1}{7}\left(3\,Bc^2d^2e + 3\,Ac^2de^2 + 2\,Bace^3\right)x^7 + Aa^2d^3x \\ &+ \frac{1}{6}\left(Bc^2d^3 + 3\,Ac^2d^2e + 6\,Bacde^2 + 2\,Aace^3\right)x^6 \\ &+ \frac{1}{5}\left(Ac^2d^3 + 6\,Bacd^2e + 6\,Aacde^2 + Ba^2e^3\right)x^5 \\ &+ \frac{1}{4}\left(2\,Bacd^3 + 6\,Aacd^2e + 3\,Ba^2de^2 + Aa^2e^3\right)x^4 \\ &+ \frac{1}{3}\left(2\,Aacd^3 + 3\,Ba^2d^2e + 3\,Aa^2de^2\right)x^3 \\ &+ \frac{1}{2}\left(Ba^2d^3 + 3\,Aa^2d^2e\right)x^2\end{aligned}$$

input
```
integrate((B*x+A)*(e*x+d)^3*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/9*B*c^2*e^3*x^9 + 1/8*(3*B*c^2*d*e^2 + A*c^2*e^3)*x^8 + 1/7*(3*B*c^2*d^2
*e + 3*A*c^2*d*e^2 + 2*B*a*c*e^3)*x^7 + A*a^2*d^3*x + 1/6*(B*c^2*d^3 + 3*A
*c^2*d^2*e + 6*B*a*c*d*e^2 + 2*A*a*c*e^3)*x^6 + 1/5*(A*c^2*d^3 + 6*B*a*c*d
^2*e + 6*A*a*c*d*e^2 + B*a^2*e^3)*x^5 + 1/4*(2*B*a*c*d^3 + 6*A*a*c*d^2*e +
 3*B*a^2*d*e^2 + A*a^2*e^3)*x^4 + 1/3*(2*A*a*c*d^3 + 3*B*a^2*d^2*e + 3*A*a
^2*d*e^2)*x^3 + 1/2*(B*a^2*d^3 + 3*A*a^2*d^2*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.39

$$\int (A+Bx)(d+ex)^3 \left(a+cx^2\right)^2 \,dx = \frac{1}{9}\,Bc^2e^3x^9 + \frac{3}{8}\,Bc^2de^2x^8 + \frac{1}{8}\,Ac^2e^3x^8 + \frac{3}{7}\,Bc^2d^2ex^7 + \frac{3}{7}\,Ac^2de^2x^7 + \frac{2}{7}\,Bace^3x^7 + \frac{1}{6}\,Bc^2d^3x^6 + \frac{1}{2}\,Ac^2d^2ex^6 + Bacde^2x^6 + \frac{1}{3}\,Aace^3x^6 + \frac{1}{5}\,Ac^2d^3x^5 + \frac{6}{5}\,Bacd^2ex^5 + \frac{6}{5}\,Aacde^2x^5 + \frac{1}{5}\,Ba^2e^3x^5 + \frac{1}{2}\,Bacd^3x^4 + \frac{3}{2}\,Aacd^2ex^4 + \frac{3}{4}\,Ba^2de^2x^4 + \frac{1}{4}\,Aa^2e^3x^4 + \frac{2}{3}\,Aacd^3x^3 + Ba^2d^2ex^3 + Aa^2de^2x^3 + \frac{1}{2}\,Ba^2d^3x^2 + \frac{3}{2}\,Aa^2d^2ex^2 + Aa^2d^3x$$

input
```
integrate((B*x+A)*(e*x+d)^3*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/9*B*c^2*e^3*x^9 + 3/8*B*c^2*d*e^2*x^8 + 1/8*A*c^2*e^3*x^8 + 3/7*B*c^2*d^
2*e*x^7 + 3/7*A*c^2*d*e^2*x^7 + 2/7*B*a*c*e^3*x^7 + 1/6*B*c^2*d^3*x^6 + 1/
2*A*c^2*d^2*e*x^6 + B*a*c*d*e^2*x^6 + 1/3*A*a*c*e^3*x^6 + 1/5*A*c^2*d^3*x^
5 + 6/5*B*a*c*d^2*e*x^5 + 6/5*A*a*c*d*e^2*x^5 + 1/5*B*a^2*e^3*x^5 + 1/2*B*
a*c*d^3*x^4 + 3/2*A*a*c*d^2*e*x^4 + 3/4*B*a^2*d*e^2*x^4 + 1/4*A*a^2*e^3*x^
4 + 2/3*A*a*c*d^3*x^3 + B*a^2*d^2*e*x^3 + A*a^2*d*e^2*x^3 + 1/2*B*a^2*d^3*
x^2 + 3/2*A*a^2*d^2*e*x^2 + A*a^2*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.11

$$\int (A+Bx)(d+ex)^3 \left(a+cx^2\right)^2 \, dx = x^5 \left(\frac{B\,a^2\,e^3}{5}+\frac{6\,B\,a\,c\,d^2\,e}{5}+\frac{6\,A\,a\,c\,d\,e^2}{5}+\frac{A\,c^2\,d^3}{5}\right) + \frac{a\,x^4\,(2\,B\,c\,d^3+6\,A\,c\,d^2\,e+3\,B\,a\,d\,e^2+A\,a\,e^3)}{4} + \frac{c\,x^6\,(B\,c\,d^3+3\,A\,c\,d^2\,e+6\,B\,a\,d\,e^2+2\,A\,a\,e^3)}{6} + \frac{a^2\,d^2\,x^2\,(3\,A\,e+B\,d)}{2} + \frac{c^2\,e^2\,x^8\,(A\,e+3\,B\,d)}{8} + A\,a^2\,d^3\,x + \frac{a\,d\,x^3\,(2\,A\,c\,d^2+3\,B\,a\,d\,e+3\,A\,a\,e^2)}{3} + \frac{c\,e\,x^7\,(3\,B\,c\,d^2+3\,A\,c\,d\,e+2\,B\,a\,e^2)}{7} + \frac{B\,c^2\,e^3\,x^9}{9}$$

input
```
int((a + c*x^2)^2*(A + B*x)*(d + e*x)^3,x)
```

output
```
x^5*((A*c^2*d^3)/5 + (B*a^2*e^3)/5 + (6*A*a*c*d*e^2)/5 + (6*B*a*c*d^2*e)/5
) + (a*x^4*(A*a*e^3 + 2*B*c*d^3 + 3*B*a*d*e^2 + 6*A*c*d^2*e))/4 + (c*x^6*(
2*A*a*e^3 + B*c*d^3 + 6*B*a*d*e^2 + 3*A*c*d^2*e))/6 + (a^2*d^2*x^2*(3*A*e
+ B*d))/2 + (c^2*e^2*x^8*(A*e + 3*B*d))/8 + A*a^2*d^3*x + (a*d*x^3*(3*A*a*
e^2 + 2*A*c*d^2 + 3*B*a*d*e))/3 + (c*e*x^7*(2*B*a*e^2 + 3*B*c*d^2 + 3*A*c*
d*e))/7 + (B*c^2*e^3*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.40

$$\int (A+Bx)(d+ex)^3 \left(a+cx^2\right)^2 \, dx = \frac{x(280b\,c^2e^3x^8+315a\,c^2e^3x^7+945b\,c^2d\,e^2x^7+720abc\,e^3x^6+1080a\,c^2d\,e^2x^6+1080b\,c^2d^2e\,x^6+840a^2c\,e}{}$$

input
```
int((B*x+A)*(e*x+d)^3*(c*x^2+a)^2,x)
```

output
```
(x*(2520*a**3*d**3 + 3780*a**3*d**2*e*x + 2520*a**3*d*e**2*x**2 + 630*a**3
*e**3*x**3 + 1260*a**2*b*d**3*x + 2520*a**2*b*d**2*e*x**2 + 1890*a**2*b*d*
e**2*x**3 + 504*a**2*b*e**3*x**4 + 1680*a**2*c*d**3*x**2 + 3780*a**2*c*d**
2*e*x**3 + 3024*a**2*c*d*e**2*x**4 + 840*a**2*c*e**3*x**5 + 1260*a*b*c*d**
3*x**3 + 3024*a*b*c*d**2*e*x**4 + 2520*a*b*c*d*e**2*x**5 + 720*a*b*c*e**3*
x**6 + 504*a*c**2*d**3*x**4 + 1260*a*c**2*d**2*e*x**5 + 1080*a*c**2*d*e**2
*x**6 + 315*a*c**2*e**3*x**7 + 420*b*c**2*d**3*x**5 + 1080*b*c**2*d**2*e*x
**6 + 945*b*c**2*d*e**2*x**7 + 280*b*c**2*e**3*x**8))/2520
```

3.49 $\int (A+Bx)(d+ex)^2 \left(a+cx^2\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 206

$$\begin{aligned}\int (A+Bx)(d+ex)^2 \left(a+cx^2\right)^2 \, dx &= -\frac{(Bd-Ae)\left(cd^2+ae^2\right)^2(d+ex)^3}{3e^6} \\ &+\frac{\left(cd^2+ae^2\right)\left(5Bcd^2-4Acde+aBe^2\right)(d+ex)^4}{4e^6} \\ &-\frac{2c(5Bcd^3-3Acd^2e+3aBde^2-aAe^3)(d+ex)^5}{5e^6} \\ &+\frac{c(5Bcd^2-2Acde+aBe^2)(d+ex)^6}{3e^6}-\frac{c^2(5Bd-Ae)(d+ex)^7}{7e^6}+\frac{Bc^2(d+ex)^8}{8e^6}\end{aligned}$$

output
```
-1/3*(-A*e+B*d)*(a*e^2+c*d^2)^2*(e*x+d)^3/e^6+1/4*(a*e^2+c*d^2)*(-4*A*c*d*
e+B*a*e^2+5*B*c*d^2)*(e*x+d)^4/e^6-2/5*c*(-A*a*e^3-3*A*c*d^2*e+3*B*a*d*e^2
+5*B*c*d^3)*(e*x+d)^5/e^6+1/3*c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)*(e*x+d)^6/e
^6-1/7*c^2*(-A*e+5*B*d)*(e*x+d)^7/e^6+1/8*B*c^2*(e*x+d)^8/e^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.84

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)^2\,dx = a^2Ad^2x+\frac{1}{2}a^2d(Bd+2Ae)x^2 +\frac{1}{3}a\left(2Acd^2+2aBde+aAe^2\right)x^3 +\frac{1}{4}a\left(2Bcd^2+4Acde+aBe^2\right)x^4 +\frac{1}{5}c\left(Acd^2+4aBde+2aAe^2\right)x^5 +\frac{1}{6}c\left(Bcd^2+2Acde+2aBe^2\right)x^6 +\frac{1}{7}c^2e(2Bd+Ae)x^7+\frac{1}{8}Bc^2e^2x^8$$

input
```
Integrate[(A + B*x)*(d + e*x)^2*(a + c*x^2)^2,x]
```

output
```
a^2*A*d^2*x + (a^2*d*(B*d + 2*A*e)*x^2)/2 + (a*(2*A*c*d^2 + 2*a*B*d*e + a*
A*e^2)*x^3)/3 + (a*(2*B*c*d^2 + 4*A*c*d*e + a*B*e^2)*x^4)/4 + (c*(A*c*d^2
+ 4*a*B*d*e + 2*a*A*e^2)*x^5)/5 + (c*(B*c*d^2 + 2*A*c*d*e + 2*a*B*e^2)*x^6
)/6 + (c^2*e*(2*B*d + A*e)*x^7)/7 + (B*c^2*e^2*x^8)/8
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2(A+Bx)(d+ex)^2\,dx$$

$$\Big\downarrow \; 652$$

$$\int \left(-\frac{2c(d+ex)^5\left(-aBe^2+2Acde-5Bcd^2\right)}{e^5}+\frac{(d+ex)^3\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^5}+\frac{(d+ex)^2\left(c\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(d+ex)^6\left(aBe^2-2Acde+5Bcd^2\right)}{3e^6}+\frac{(d+ex)^4\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{4e^6}-\frac{(d+ex)^3\left(ae^2+cd^2\right)^2(Bd-Ae)}{3e^6}-\frac{2c(d+ex)^5\left(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3\right)}{5e^6}-\frac{c^2(d+ex)^7(5Bd-Ae)}{7e^6}+\frac{Bc^2(d+ex)^8}{8e^6}$$

input
```
Int[(A + B*x)*(d + e*x)^2*(a + c*x^2)^2,x]
```

output
```
-1/3*((B*d - A*e)*(c*d^2 + a*e^2)^2*(d + e*x)^3)/e^6 + ((c*d^2 + a*e^2)*(5
*B*c*d^2 - 4*A*c*d*e + a*B*e^2)*(d + e*x)^4)/(4*e^6) - (2*c*(5*B*c*d^3 - 3
*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*(d + e*x)^5)/(5*e^6) + (c*(5*B*c*d^2 -
 2*A*c*d*e + a*B*e^2)*(d + e*x)^6)/(3*e^6) - (c^2*(5*B*d - A*e)*(d + e*x)^
7)/(7*e^6) + (B*c^2*(d + e*x)^8)/(8*e^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.86

method	result
default	$\frac{B\,e^{2}c^{2}x^{8}}{8}+\frac{(A\,e^{2}+2Bde)c^{2}x^{7}}{7}+\frac{((2Ade+B\,d^{2})c^{2}+2B\,e^{2}ac)x^{6}}{6}+\frac{(A\,c^{2}d^{2}+2(A\,e^{2}+2Bde)ac)x^{5}}{5}+\frac{(2(2Ade+B\,d^{2})}{4}$
norman	$\frac{B\,e^{2}c^{2}x^{8}}{8}+\left(\frac{1}{7}A\,c^{2}e^{2}+\frac{2}{7}B\,c^{2}de\right)x^{7}+\left(\frac{1}{3}A\,c^{2}de+\frac{1}{3}B\,e^{2}ac+\frac{1}{6}B\,c^{2}d^{2}\right)x^{6}+\left(\frac{2}{5}Aac\,e^{2}+\frac{1}{5}A\,c\right.$
gosper	$\frac{1}{8}B\,e^{2}c^{2}x^{8}+\frac{1}{7}x^{7}A\,c^{2}e^{2}+\frac{2}{7}x^{7}B\,c^{2}de+\frac{1}{3}x^{6}A\,c^{2}de+\frac{1}{3}x^{6}B\,e^{2}ac+\frac{1}{6}x^{6}B\,c^{2}d^{2}+\frac{2}{5}x^{5}Aac\,e^{2}+$
risch	$\frac{1}{8}B\,e^{2}c^{2}x^{8}+\frac{1}{7}x^{7}A\,c^{2}e^{2}+\frac{2}{7}x^{7}B\,c^{2}de+\frac{1}{3}x^{6}A\,c^{2}de+\frac{1}{3}x^{6}B\,e^{2}ac+\frac{1}{6}x^{6}B\,c^{2}d^{2}+\frac{2}{5}x^{5}Aac\,e^{2}+$
parallelrisch	$\frac{1}{8}B\,e^{2}c^{2}x^{8}+\frac{1}{7}x^{7}A\,c^{2}e^{2}+\frac{2}{7}x^{7}B\,c^{2}de+\frac{1}{3}x^{6}A\,c^{2}de+\frac{1}{3}x^{6}B\,e^{2}ac+\frac{1}{6}x^{6}B\,c^{2}d^{2}+\frac{2}{5}x^{5}Aac\,e^{2}+$
orering	$\frac{x(105B\,e^{2}c^{2}x^{7}+120A\,c^{2}e^{2}x^{6}+240B\,c^{2}de\,x^{6}+280A\,c^{2}de\,x^{5}+280Bac\,e^{2}x^{5}+140B\,c^{2}d^{2}x^{5}+336Aac\,e^{2}x^{4}+168A\,c^{2}d^{2}x^{4}+672}{}$

input
```
int((B*x+A)*(e*x+d)^2*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*B*e^2*c^2*x^8+1/7*(A*e^2+2*B*d*e)*c^2*x^7+1/6*((2*A*d*e+B*d^2)*c^2+2*B
*e^2*a*c)*x^6+1/5*(A*c^2*d^2+2*(A*e^2+2*B*d*e)*a*c)*x^5+1/4*(2*(2*A*d*e+B*
d^2)*a*c+B*e^2*a^2)*x^4+1/3*(2*A*d^2*a*c+(A*e^2+2*B*d*e)*a^2)*x^3+1/2*(2*A
*d*e+B*d^2)*a^2*x^2+A*a^2*d^2*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.89

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)^2\,dx=\frac{1}{8}\,Bc^2e^2x^8+\frac{1}{7}\left(2\,Bc^2de+Ac^2e^2\right)x^7+\frac{1}{6}\left(Bc^2d^2+2\,Ac^2de+2\,Bace^2\right)x^6+Aa^2d^2x+\frac{1}{5}\left(Ac^2d^2+4\,Bacde+2\,Aace^2\right)x^5+\frac{1}{4}\left(2\,Bacd^2+4\,Aacde+Ba^2e^2\right)x^4+\frac{1}{3}\left(2\,Aacd^2+2\,Ba^2de+Aa^2e^2\right)x^3+\frac{1}{2}\left(Ba^2d^2+2\,Aa^2de\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)^2*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*B*c^2*e^2*x^8 + 1/7*(2*B*c^2*d*e + A*c^2*e^2)*x^7 + 1/6*(B*c^2*d^2 + 2
*A*c^2*d*e + 2*B*a*c*e^2)*x^6 + A*a^2*d^2*x + 1/5*(A*c^2*d^2 + 4*B*a*c*d*e
 + 2*A*a*c*e^2)*x^5 + 1/4*(2*B*a*c*d^2 + 4*A*a*c*d*e + B*a^2*e^2)*x^4 + 1/
3*(2*A*a*c*d^2 + 2*B*a^2*d*e + A*a^2*e^2)*x^3 + 1/2*(B*a^2*d^2 + 2*A*a^2*d
*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.02

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)^2\,dx = Aa^2d^2x+\frac{Bc^2e^2x^8}{8}+x^7\left(\frac{Ac^2e^2}{7}+\frac{2Bc^2de}{7}\right)+x^6\left(\frac{Ac^2de}{3}+\frac{Bace^2}{3}+\frac{Bc^2d^2}{6}\right)+x^5\cdot\left(\frac{2Aace^2}{5}+\frac{Ac^2d^2}{5}+\frac{4Bacde}{5}\right)+x^4\left(Aacde+\frac{Ba^2e^2}{4}+\frac{Bacd^2}{2}\right)+x^3\left(\frac{Aa^2e^2}{3}+\frac{2Aacd^2}{3}+\frac{2Ba^2de}{3}\right)+x^2\left(Aa^2de+\frac{Ba^2d^2}{2}\right)$$

input
```
integrate((B*x+A)*(e*x+d)**2*(c*x**2+a)**2,x)
```

output
```
A*a**2*d**2*x + B*c**2*e**2*x**8/8 + x**7*(A*c**2*e**2/7 + 2*B*c**2*d*e/7)
 + x**6*(A*c**2*d*e/3 + B*a*c*e**2/3 + B*c**2*d**2/6) + x**5*(2*A*a*c*e**2
/5 + A*c**2*d**2/5 + 4*B*a*c*d*e/5) + x**4*(A*a*c*d*e + B*a**2*e**2/4 + B*
a*c*d**2/2) + x**3*(A*a**2*e**2/3 + 2*A*a*c*d**2/3 + 2*B*a**2*d*e/3) + x**
2*(A*a**2*d*e + B*a**2*d**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.89

$$\int (A+Bx)(d+ex)^2 \left(a+cx^2\right)^2 \, dx = \frac{1}{8} Bc^2e^2x^8 + \frac{1}{7} \left(2 Bc^2de + Ac^2e^2\right)x^7 + \frac{1}{6} \left(Bc^2d^2 + 2 Ac^2de + 2 Bace^2\right)x^6 + Aa^2d^2x + \frac{1}{5} \left(Ac^2d^2 + 4 Bacde + 2 Aace^2\right)x^5 + \frac{1}{4} \left(2 Bacd^2 + 4 Aacde + Ba^2e^2\right)x^4 + \frac{1}{3} \left(2 Aacd^2 + 2 Ba^2de + Aa^2e^2\right)x^3 + \frac{1}{2} \left(Ba^2d^2 + 2 Aa^2de\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)^2*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/8*B*c^2*e^2*x^8 + 1/7*(2*B*c^2*d*e + A*c^2*e^2)*x^7 + 1/6*(B*c^2*d^2 + 2
*A*c^2*d*e + 2*B*a*c*e^2)*x^6 + A*a^2*d^2*x + 1/5*(A*c^2*d^2 + 4*B*a*c*d*e
 + 2*A*a*c*e^2)*x^5 + 1/4*(2*B*a*c*d^2 + 4*A*a*c*d*e + B*a^2*e^2)*x^4 + 1/
3*(2*A*a*c*d^2 + 2*B*a^2*d*e + A*a^2*e^2)*x^3 + 1/2*(B*a^2*d^2 + 2*A*a^2*d
*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.97

$$\int (A+Bx)(d+ex)^2 \left(a+cx^2\right)^2 \, dx = \frac{1}{8} Bc^2e^2x^8 + \frac{2}{7} Bc^2dex^7 + \frac{1}{7} Ac^2e^2x^7 + \frac{1}{6} Bc^2d^2x^6 + \frac{1}{3} Ac^2dex^6 + \frac{1}{3} Bace^2x^6 + \frac{1}{5} Ac^2d^2x^5 + \frac{4}{5} Bacdex^5 + \frac{2}{5} Aace^2x^5 + \frac{1}{2} Bacd^2x^4 + Aacdex^4 + \frac{1}{4} Ba^2e^2x^4 + \frac{2}{3} Aacd^2x^3 + \frac{2}{3} Ba^2dex^3 + \frac{1}{3} Aa^2e^2x^3 + \frac{1}{2} Ba^2d^2x^2 + Aa^2dex^2 + Aa^2d^2x$$

input
```
integrate((B*x+A)*(e*x+d)^2*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/8*B*c^2*e^2*x^8 + 2/7*B*c^2*d*e*x^7 + 1/7*A*c^2*e^2*x^7 + 1/6*B*c^2*d^2*
x^6 + 1/3*A*c^2*d*e*x^6 + 1/3*B*a*c*e^2*x^6 + 1/5*A*c^2*d^2*x^5 + 4/5*B*a*
c*d*e*x^5 + 2/5*A*a*c*e^2*x^5 + 1/2*B*a*c*d^2*x^4 + A*a*c*d*e*x^4 + 1/4*B*
a^2*e^2*x^4 + 2/3*A*a*c*d^2*x^3 + 2/3*B*a^2*d*e*x^3 + 1/3*A*a^2*e^2*x^3 +
1/2*B*a^2*d^2*x^2 + A*a^2*d*e*x^2 + A*a^2*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.82

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)^2\,dx = x^3\left(\frac{2\,B\,a^2\,d\,e}{3}+\frac{A\,a^2\,e^2}{3}+\frac{2\,A\,c\,a\,d^2}{3}\right)+x^6\left(\frac{B\,c^2\,d^2}{6}+\frac{A\,c^2\,d\,e}{3}+\frac{B\,a\,c\,e^2}{3}\right)+\frac{c\,x^5\,\left(A\,c\,d^2+4\,B\,a\,d\,e+2\,A\,a\,e^2\right)}{5}+\frac{a\,x^4\,\left(2\,B\,c\,d^2+4\,A\,c\,d\,e+B\,a\,e^2\right)}{4}+A\,a^2\,d^2\,x+\frac{a^2\,d\,x^2\,\left(2\,A\,e+B\,d\right)}{2}+\frac{c^2\,e\,x^7\,\left(A\,e+2\,B\,d\right)}{7}+\frac{B\,c^2\,e^2\,x^8}{8}$$

input
```
int((a + c*x^2)^2*(A + B*x)*(d + e*x)^2,x)
```

output
```
x^3*((A*a^2*e^2)/3 + (2*A*a*c*d^2)/3 + (2*B*a^2*d*e)/3) + x^6*((B*c^2*d^2)
/6 + (B*a*c*e^2)/3 + (A*c^2*d*e)/3) + (c*x^5*(2*A*a*e^2 + A*c*d^2 + 4*B*a*
d*e))/5 + (a*x^4*(B*a*e^2 + 2*B*c*d^2 + 4*A*c*d*e))/4 + A*a^2*d^2*x + (a^2
*d*x^2*(2*A*e + B*d))/2 + (c^2*e*x^7*(A*e + 2*B*d))/7 + (B*c^2*e^2*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.98

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)^2\,dx$$
$$= \frac{x(105bc^2e^2x^7+120ac^2e^2x^6+240bc^2de\,x^6+280abce^2x^5+280ac^2de\,x^5+140bc^2d^2x^5+336a^2ce^2x^4+}{}$$

input
```
int((B*x+A)*(e*x+d)^2*(c*x^2+a)^2,x)
```

output
```
(x*(840*a**3*d**2 + 840*a**3*d*e*x + 280*a**3*e**2*x**2 + 420*a**2*b*d**2*
x + 560*a**2*b*d*e*x**2 + 210*a**2*b*e**2*x**3 + 560*a**2*c*d**2*x**2 + 84
0*a**2*c*d*e*x**3 + 336*a**2*c*e**2*x**4 + 420*a*b*c*d**2*x**3 + 672*a*b*c
*d*e*x**4 + 280*a*b*c*e**2*x**5 + 168*a*c**2*d**2*x**4 + 280*a*c**2*d*e*x*
*5 + 120*a*c**2*e**2*x**6 + 140*b*c**2*d**2*x**5 + 240*b*c**2*d*e*x**6 + 1
05*b*c**2*e**2*x**7))/840
```

3.50 $\int (A+Bx)(d+ex)\left(a+cx^2\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 106

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^2\,dx = a^2Adx+\frac{1}{2}a^2(Bd+Ae)x^2+\frac{1}{3}a(2Acd+aBe)x^3+\frac{1}{2}ac(Bd+Ae)x^4+\frac{1}{5}c(Acd+2aBe)x^5+\frac{1}{6}c^2(Bd+Ae)x^6+\frac{1}{7}Bc^2ex^7$$

output
```
a^2*A*d*x+1/2*a^2*(A*e+B*d)*x^2+1/3*a*(2*A*c*d+B*a*e)*x^3+1/2*a*c*(A*e+B*d
)*x^4+1/5*c*(A*c*d+2*B*a*e)*x^5+1/6*c^2*(A*e+B*d)*x^6+1/7*B*c^2*e*x^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^2\,dx = \frac{1}{210}x\left(35a^2(3A(2d+ex)+Bx(3d+2ex))+7acx^2(5A(4d+3ex)+3Bx(5d+4ex))+c^2x^4(7A(6d+5ex)+5Bx(7d+6ex))\right)$$

input
```
Integrate[(A + B*x)*(d + e*x)*(a + c*x^2)^2,x]
```

output
```
(x*(35*a^2*(3*A*(2*d + e*x) + B*x*(3*d + 2*e*x)) + 7*a*c*x^2*(5*A*(4*d + 3
*e*x) + 3*B*x*(5*d + 4*e*x)) + c^2*x^4*(7*A*(6*d + 5*e*x) + 5*B*x*(7*d + 6
*e*x))))/210
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2 (A+Bx)(d+ex)\,dx$$

$$\Big\downarrow \ 652$$

$$\int \left(a^2x(Ae+Bd)+a^2Ad+cx^4(2aBe+Acd)+2acx^3(Ae+Bd)+ax^2(aBe+2Acd)+c^2x^5(Ae+Bd)+Bc^2e$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}a^2x^2(Ae+Bd)+a^2Adx+\frac{1}{5}cx^5(2aBe+Acd)+\frac{1}{2}acx^4(Ae+Bd)+\frac{1}{3}ax^3(aBe+2Acd)+\frac{1}{6}c^2x^6(Ae+Bd)+\frac{1}{7}Bc^2ex^7$$

input
```
Int[(A + B*x)*(d + e*x)*(a + c*x^2)^2,x]
```

output
```
a^2*A*d*x + (a^2*(B*d + A*e)*x^2)/2 + (a*(2*A*c*d + a*B*e)*x^3)/3 + (a*c*(
B*d + A*e)*x^4)/2 + (c*(A*c*d + 2*a*B*e)*x^5)/5 + (c^2*(B*d + A*e)*x^6)/6
+ (B*c^2*e*x^7)/7
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

method	result
default	$\frac{B c^2 e x^7}{7}+\frac{c^2(Ae+Bd)x^6}{6}+\frac{(A c^2 d+2Beac)x^5}{5}+\frac{ac(Ae+Bd)x^4}{2}+\frac{(2Aacd+Be a^2)x^3}{3}+\frac{a^2(Ae+Bd)x^2}{2}+a^2Ad$
norman	$\frac{B c^2 e x^7}{7}+\left(\frac{1}{6}A c^2 e+\frac{1}{6}B c^2 d\right)x^6+\left(\frac{1}{5}A c^2 d+\frac{2}{5}Beac\right)x^5+\left(\frac{1}{2}Aace+\frac{1}{2}Bacd\right)x^4+\left(\frac{2}{3}Aacd\right.$
orering	$\frac{x(30Be c^2x^6+35A c^2e x^5+35B c^2d x^5+42A c^2d x^4+84Bace x^4+105Aace x^3+105Bacd x^3+140Aacd x^2+70B a^2e x^2+105A}{210}$
gosper	$\frac{1}{7}B c^2 e x^7+\frac{1}{6}x^6 A c^2 e+\frac{1}{6}x^6 B c^2 d+\frac{1}{5}x^5 A c^2 d+\frac{2}{5}x^5 Beac+\frac{1}{2}x^4 Aace+\frac{1}{2}x^4 Bacd+\frac{2}{3}x^3 Aa$
risch	$\frac{1}{7}B c^2 e x^7+\frac{1}{6}x^6 A c^2 e+\frac{1}{6}x^6 B c^2 d+\frac{1}{5}x^5 A c^2 d+\frac{2}{5}x^5 Beac+\frac{1}{2}x^4 Aace+\frac{1}{2}x^4 Bacd+\frac{2}{3}x^3 Aa$
parallelrisch	$\frac{1}{7}B c^2 e x^7+\frac{1}{6}x^6 A c^2 e+\frac{1}{6}x^6 B c^2 d+\frac{1}{5}x^5 A c^2 d+\frac{2}{5}x^5 Beac+\frac{1}{2}x^4 Aace+\frac{1}{2}x^4 Bacd+\frac{2}{3}x^3 Aa$

input
```
int((B*x+A)*(e*x+d)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*B*c^2*e*x^7+1/6*c^2*(A*e+B*d)*x^6+1/5*(A*c^2*d+2*B*a*c*e)*x^5+1/2*a*c*
(A*e+B*d)*x^4+1/3*(2*A*a*c*d+B*a^2*e)*x^3+1/2*a^2*(A*e+B*d)*x^2+a^2*A*d*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^2\,dx=\frac{1}{7}\,Bc^2ex^7+\frac{1}{6}\,\left(Bc^2d+Ac^2e\right)x^6+\frac{1}{5}\,\left(Ac^2d+2\,Bace\right)x^5+Aa^2dx+\frac{1}{2}\,(Bacd+Aace)x^4+\frac{1}{3}\,\left(2\,Aacd+Ba^2e\right)x^3+\frac{1}{2}\,\left(Ba^2d+Aa^2e\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/7*B*c^2*e*x^7 + 1/6*(B*c^2*d + A*c^2*e)*x^6 + 1/5*(A*c^2*d + 2*B*a*c*e)*
x^5 + A*a^2*d*x + 1/2*(B*a*c*d + A*a*c*e)*x^4 + 1/3*(2*A*a*c*d + B*a^2*e)*
x^3 + 1/2*(B*a^2*d + A*a^2*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.17

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^2\,dx=Aa^2dx+\frac{Bc^2ex^7}{7}+x^6\left(\frac{Ac^2e}{6}+\frac{Bc^2d}{6}\right)+x^5\left(\frac{Ac^2d}{5}+\frac{2Bace}{5}\right)+x^4\left(\frac{Aace}{2}+\frac{Bacd}{2}\right)+x^3\cdot\left(\frac{2Aacd}{3}+\frac{Ba^2e}{3}\right)+x^2\left(\frac{Aa^2e}{2}+\frac{Ba^2d}{2}\right)$$

input
```
integrate((B*x+A)*(e*x+d)*(c*x**2+a)**2,x)
```

output
```
A*a**2*d*x + B*c**2*e*x**7/7 + x**6*(A*c**2*e/6 + B*c**2*d/6) + x**5*(A*c*
*2*d/5 + 2*B*a*c*e/5) + x**4*(A*a*c*e/2 + B*a*c*d/2) + x**3*(2*A*a*c*d/3 +
 B*a**2*e/3) + x**2*(A*a**2*e/2 + B*a**2*d/2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^2\,dx = \frac{1}{7}Bc^2ex^7 + \frac{1}{6}\left(Bc^2d + Ac^2e\right)x^6 + \frac{1}{5}\left(Ac^2d + 2\,Bace\right)x^5 + Aa^2dx + \frac{1}{2}\left(Bacd + Aace\right)x^4 + \frac{1}{3}\left(2\,Aacd + Ba^2e\right)x^3 + \frac{1}{2}\left(Ba^2d + Aa^2e\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/7*B*c^2*e*x^7 + 1/6*(B*c^2*d + A*c^2*e)*x^6 + 1/5*(A*c^2*d + 2*B*a*c*e)*
x^5 + A*a^2*d*x + 1/2*(B*a*c*d + A*a*c*e)*x^4 + 1/3*(2*A*a*c*d + B*a^2*e)*
x^3 + 1/2*(B*a^2*d + A*a^2*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^2\,dx = \frac{1}{7}Bc^2ex^7 + \frac{1}{6}Bc^2dx^6 + \frac{1}{6}Ac^2ex^6 + \frac{1}{5}Ac^2dx^5 + \frac{2}{5}Bacex^5 + \frac{1}{2}Bacdx^4 + \frac{1}{2}Aacex^4 + \frac{2}{3}Aacdx^3 + \frac{1}{3}Ba^2ex^3 + \frac{1}{2}Ba^2dx^2 + \frac{1}{2}Aa^2ex^2 + Aa^2dx$$

input
```
integrate((B*x+A)*(e*x+d)*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/7*B*c^2*e*x^7 + 1/6*B*c^2*d*x^6 + 1/6*A*c^2*e*x^6 + 1/5*A*c^2*d*x^5 + 2/
5*B*a*c*e*x^5 + 1/2*B*a*c*d*x^4 + 1/2*A*a*c*e*x^4 + 2/3*A*a*c*d*x^3 + 1/3*
B*a^2*e*x^3 + 1/2*B*a^2*d*x^2 + 1/2*A*a^2*e*x^2 + A*a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^2\,dx = x^3\left(\frac{B\,e\,a^2}{3}+\frac{2\,A\,c\,d\,a}{3}\right)+x^5\left(\frac{A\,d\,c^2}{5}+\frac{2\,B\,a\,e\,c}{5}\right)+\frac{a^2\,x^2\,(A\,e+B\,d)}{2}+\frac{c^2\,x^6\,(A\,e+B\,d)}{6}+A\,a^2\,d\,x+\frac{a\,c\,x^4\,(A\,e+B\,d)}{2}+\frac{B\,c^2\,e\,x^7}{7}$$

input
```
int((a + c*x^2)^2*(A + B*x)*(d + e*x),x)
```

output
```
x^3*((B*a^2*e)/3 + (2*A*a*c*d)/3) + x^5*((A*c^2*d)/5 + (2*B*a*c*e)/5) + (a
^2*x^2*(A*e + B*d))/2 + (c^2*x^6*(A*e + B*d))/6 + A*a^2*d*x + (a*c*x^4*(A*
e + B*d))/2 + (B*c^2*e*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^2\,dx = \frac{x(30b\,c^2e\,x^6+35a\,c^2e\,x^5+35b\,c^2d\,x^5+84abce\,x^4+42a\,c^2d\,x^4+105a^2ce\,x^3+105abcd\,x^3+70a^2be\,x^2+}{210}$$

input
```
int((B*x+A)*(e*x+d)*(c*x^2+a)^2,x)
```

output
```
(x*(210*a**3*d + 105*a**3*e*x + 105*a**2*b*d*x + 70*a**2*b*e*x**2 + 140*a*
*2*c*d*x**2 + 105*a**2*c*e*x**3 + 105*a*b*c*d*x**3 + 84*a*b*c*e*x**4 + 42*
a*c**2*d*x**4 + 35*a*c**2*e*x**5 + 35*b*c**2*d*x**5 + 30*b*c**2*e*x**6))/2
10
```

3.51 $\int (A+Bx)\left(a+cx^2\right)^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int (A+Bx)\left(a+cx^2\right)^2\,dx=a^2Ax+\frac{2}{3}aAcx^3+\frac{1}{5}Ac^2x^5+\frac{B(a+cx^2)^3}{6c}$$

output
```
a^2*A*x+2/3*a*A*c*x^3+1/5*A*c^2*x^5+1/6*B*(c*x^2+a)^3/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int (A+Bx)\left(a+cx^2\right)^2\,dx=a^2Ax+\frac{1}{2}a^2Bx^2+\frac{2}{3}aAcx^3+\frac{1}{2}aBcx^4+\frac{1}{5}Ac^2x^5+\frac{1}{6}Bc^2x^6$$

input
```
Integrate[(A + B*x)*(a + c*x^2)^2,x]
```

output
```
a^2*A*x + (a^2*B*x^2)/2 + (2*a*A*c*x^3)/3 + (a*B*c*x^4)/2 + (A*c^2*x^5)/5
+ (B*c^2*x^6)/6
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {455, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^2 (A+Bx)\,dx$$

$$\Big\downarrow \text{455}$$

$$A\int (cx^2+a)^2\,dx+\frac{B(a+cx^2)^3}{6c}$$

$$\Big\downarrow \text{210}$$

$$A\int (c^2x^4+2acx^2+a^2)\,dx+\frac{B(a+cx^2)^3}{6c}$$

$$\Big\downarrow \text{2009}$$

$$A\left(a^2x+\frac{2}{3}acx^3+\frac{c^2x^5}{5}\right)+\frac{B(a+cx^2)^3}{6c}$$

input
```
Int[(A + B*x)*(a + c*x^2)^2,x]
```

output
```
(B*(a + c*x^2)^3)/(6*c) + A*(a^2*x + (2*a*c*x^3)/3 + (c^2*x^5)/5)
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

method	result	size
gosper	$\frac{1}{6}B\,c^2x^6+\frac{1}{5}A\,c^2x^5+\frac{1}{2}Bac\,x^4+\frac{2}{3}aAc\,x^3+\frac{1}{2}B\,a^2x^2+a^2Ax$	51
default	$\frac{1}{6}B\,c^2x^6+\frac{1}{5}A\,c^2x^5+\frac{1}{2}Bac\,x^4+\frac{2}{3}aAc\,x^3+\frac{1}{2}B\,a^2x^2+a^2Ax$	51
norman	$\frac{1}{6}B\,c^2x^6+\frac{1}{5}A\,c^2x^5+\frac{1}{2}Bac\,x^4+\frac{2}{3}aAc\,x^3+\frac{1}{2}B\,a^2x^2+a^2Ax$	51
risch	$\frac{1}{6}B\,c^2x^6+\frac{1}{5}A\,c^2x^5+\frac{1}{2}Bac\,x^4+\frac{2}{3}aAc\,x^3+\frac{1}{2}B\,a^2x^2+a^2Ax$	51
parallelrisch	$\frac{1}{6}B\,c^2x^6+\frac{1}{5}A\,c^2x^5+\frac{1}{2}Bac\,x^4+\frac{2}{3}aAc\,x^3+\frac{1}{2}B\,a^2x^2+a^2Ax$	51
orering	$\frac{x(5B\,c^2x^5+6x^4A\,c^2+15aBc\,x^3+20Aac\,x^2+15a^2Bx+30a^2A)}{30}$	52

input
```
int((B*x+A)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*B*c^2*x^6+1/5*A*c^2*x^5+1/2*B*a*c*x^4+2/3*a*A*c*x^3+1/2*B*a^2*x^2+a^2*
A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (A+Bx)\left(a+cx^2\right)^2\,dx=\frac{1}{6}\,Bc^2x^6+\frac{1}{5}\,Ac^2x^5+\frac{1}{2}\,Bacx^4+\frac{2}{3}\,Aacx^3+\frac{1}{2}\,Ba^2x^2+Aa^2x$$

input
```
integrate((B*x+A)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/6*B*c^2*x^6 + 1/5*A*c^2*x^5 + 1/2*B*a*c*x^4 + 2/3*A*a*c*x^3 + 1/2*B*a^2*
x^2 + A*a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int (A + Bx) \left(a + cx^2\right)^2 \, dx = Aa^2x + \frac{2Aacx^3}{3} + \frac{Ac^2x^5}{5} + \frac{Ba^2x^2}{2} + \frac{Bacx^4}{2} + \frac{Bc^2x^6}{6}$$

input
```
integrate((B*x+A)*(c*x**2+a)**2,x)
```

output
```
A*a**2*x + 2*A*a*c*x**3/3 + A*c**2*x**5/5 + B*a**2*x**2/2 + B*a*c*x**4/2 +
 B*c**2*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (A + Bx) \left(a + cx^2\right)^2 \, dx = \frac{1}{6} Bc^2x^6 + \frac{1}{5} Ac^2x^5 + \frac{1}{2} Bacx^4 + \frac{2}{3} Aacx^3 + \frac{1}{2} Ba^2x^2 + Aa^2x$$

input
```
integrate((B*x+A)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/6*B*c^2*x^6 + 1/5*A*c^2*x^5 + 1/2*B*a*c*x^4 + 2/3*A*a*c*x^3 + 1/2*B*a^2*
x^2 + A*a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (A + Bx) \left(a + cx^2\right)^2 \, dx = \frac{1}{6} Bc^2x^6 + \frac{1}{5} Ac^2x^5 + \frac{1}{2} Bacx^4 + \frac{2}{3} Aacx^3 + \frac{1}{2} Ba^2x^2 + Aa^2x$$

input
```
integrate((B*x+A)*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/6*B*c^2*x^6 + 1/5*A*c^2*x^5 + 1/2*B*a*c*x^4 + 2/3*A*a*c*x^3 + 1/2*B*a^2*
x^2 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (A+Bx)\left(a+cx^2\right)^2\,dx=\frac{B\,a^2\,x^2}{2}+A\,a^2\,x+\frac{B\,a\,c\,x^4}{2}+\frac{2\,A\,a\,c\,x^3}{3}+\frac{B\,c^2\,x^6}{6}+\frac{A\,c^2\,x^5}{5}$$

input
```
int((a + c*x^2)^2*(A + B*x),x)
```

output
```
(B*a^2*x^2)/2 + (A*c^2*x^5)/5 + (B*c^2*x^6)/6 + A*a^2*x + (2*A*a*c*x^3)/3
+ (B*a*c*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int (A+Bx)\left(a+cx^2\right)^2\,dx=\frac{x(5b\,c^2x^5+6a\,c^2x^4+15abc\,x^3+20a^2c\,x^2+15a^2bx+30a^3)}{30}$$

input
```
int((B*x+A)*(c*x^2+a)^2,x)
```

output
```
(x*(30*a**3 + 15*a**2*b*x + 20*a**2*c*x**2 + 15*a*b*c*x**3 + 6*a*c**2*x**4
 + 5*b*c**2*x**5))/30
```

3.52 $\int \frac{(A+Bx)(a+cx^2)^2}{d+ex} dx$

Optimal result

Integrand size = 22, antiderivative size = 169

$$\begin{aligned}\int \frac{(A+Bx)(a+cx^2)^2}{d+ex} dx = & \frac{\left(B(cd^2+ae^2)^2-Acde(cd^2+2ae^2)\right)x}{e^5} \\ & -\frac{c(Bd-Ae)(cd^2+2ae^2)x^2}{2e^4} \\ & +\frac{c(Bcd^2-Acde+2aBe^2)x^3}{3e^3}-\frac{c^2(Bd-Ae)x^4}{4e^2} \\ & +\frac{Bc^2x^5}{5e}-\frac{(Bd-Ae)(cd^2+ae^2)^2\log(d+ex)}{e^6}\end{aligned}$$

output
```
(B*(a*e^2+c*d^2)^2-A*c*d*e*(2*a*e^2+c*d^2))*x/e^5-1/2*c*(-A*e+B*d)*(2*a*e^
2+c*d^2)*x^2/e^4+1/3*c*(-A*c*d*e+2*B*a*e^2+B*c*d^2)*x^3/e^3-1/4*c^2*(-A*e+
B*d)*x^4/e^2+1/5*B*c^2*x^5/e-(-A*e+B*d)*(a*e^2+c*d^2)^2*ln(e*x+d)/e^6
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{d+ex}\,dx$$
$$=\frac{ex(5Ace(12ae^2(-2d+ex)+c(-12d^3+6d^2ex-4de^2x^2+3e^3x^3))+B(60a^2e^4+20ace^2(6d^2-3dex+2\ldots}{60e\ldots}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^2)/(d + e*x),x]
```

output
```
(e*x*(5*A*c*e*(12*a*e^2*(-2*d + e*x) + c*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^
2 + 3*e^3*x^3)) + B*(60*a^2*e^4 + 20*a*c*e^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2)
 + c^2*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x^3 + 12*e^4*x^4))
) - 60*(B*d - A*e)*(c*d^2 + a*e^2)^2*Log[d + e*x])/(60*e^6)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2(A+Bx)}{d+ex}\,dx$$
$$\Big\downarrow \ 652$$
$$\int \left(\frac{\left(ae^2+cd^2\right)^2(Ae-Bd)}{e^5(d+ex)}+\frac{B\left(ae^2+cd^2\right)^2-Acde\left(2ae^2+cd^2\right)}{e^5}+\frac{cx\left(2ae^2+cd^2\right)(Ae-Bd)}{e^4}-\frac{cx^2(-2aBe\ldots}{\ldots}\right.$$
$$\Big\downarrow \ 2009$$
$$-\frac{\left(ae^2+cd^2\right)^2(Bd-Ae)\log(d+ex)}{e^6}+\frac{x\left(B\left(ae^2+cd^2\right)^2-Acde\left(2ae^2+cd^2\right)\right)}{e^5}-\frac{cx^2\left(2ae^2+cd^2\right)(Bd-Ae)}{2e^4}+\frac{cx^3\left(2aBe^2-Acde+Bcd^2\right)}{3e^3}-\frac{c^2x^4(Bd-Ae)}{4e^2}+\frac{Bc^2x^5}{5e}$$

input
```
Int[((A + B*x)*(a + c*x^2)^2)/(d + e*x),x]
```

output
```
((B*(c*d^2 + a*e^2)^2 - A*c*d*e*(c*d^2 + 2*a*e^2))*x)/e^5 - (c*(B*d - A*e)
*(c*d^2 + 2*a*e^2)*x^2)/(2*e^4) + (c*(B*c*d^2 - A*c*d*e + 2*a*B*e^2)*x^3)/
(3*e^3) - (c^2*(B*d - A*e)*x^4)/(4*e^2) + (B*c^2*x^5)/(5*e) - ((B*d - A*e)
*(c*d^2 + a*e^2)^2*Log[d + e*x])/e^6
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.33

method	result
norman	$-\frac{\left(2Aacd\,e^{3}+A\,c^{2}d^{3}e-B\,e^{4}a^{2}-2Bac\,d^{2}e^{2}-B\,c^{2}d^{4}\right)x}{e^{5}}+\frac{B\,c^{2}x^{5}}{5e}+\frac{c\left(2Aa\,e^{3}+Ac\,d^{2}e-2Bad\,e^{2}-Bc\,d^{3}\right)x^{2}}{2e^{4}}-\frac{c(Acde-2 \ldots}{\ldots}$
default	$-\frac{-\frac{1}{5}B\,c^{2}x^{5}e^{4}-\frac{1}{4}A\,c^{2}e^{4}x^{4}+\frac{1}{4}B\,c^{2}d\,e^{3}x^{4}+\frac{1}{3}A\,c^{2}d\,e^{3}x^{3}-\frac{2}{3}Bac\,e^{4}x^{3}-\frac{1}{3}B\,c^{2}d^{2}e^{2}x^{3}-Aac\,e^{4}x^{2}-\frac{1}{2}A\,c^{2}d^{2}e^{2}x^{2}+Bacd\,e^{3}x^{2}+\ldots}{e^{5}}$
risch	$\frac{B\,c^{2}x^{5}}{5e}+\frac{A\,c^{2}x^{4}}{4e}-\frac{B\,c^{2}d\,x^{4}}{4e^{2}}-\frac{A\,c^{2}d\,x^{3}}{3e^{2}}+\frac{2Bac\,x^{3}}{3e}+\frac{B\,c^{2}d^{2}x^{3}}{3e^{3}}+\frac{Aac\,x^{2}}{e}+\frac{A\,c^{2}d^{2}x^{2}}{2e^{3}}-\frac{Bacd\,x^{2}}{e^{2}}-\frac{B\,c^{2}d^{3}x^{2}}{2e^{4}}\ldots$
parallelrisch	$\frac{12B\,x^{5}c^{2}e^{5}+15A\,x^{4}c^{2}e^{5}-15B\,x^{4}c^{2}d\,e^{4}-20A\,x^{3}c^{2}d\,e^{4}+40B\,x^{3}ac\,e^{5}+20B\,x^{3}c^{2}d^{2}e^{3}+60A\,x^{2}ac\,e^{5}+30A\,x^{2}c^{2}d^{2}e^{3}-60B\,x^{2}a\ldots}{\ldots}$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-(2*A*a*c*d*e^3+A*c^2*d^3*e-B*a^2*e^4-2*B*a*c*d^2*e^2-B*c^2*d^4)/e^5*x+1/5
*B*c^2*x^5/e+1/2*c/e^4*(2*A*a*e^3+A*c*d^2*e-2*B*a*d*e^2-B*c*d^3)*x^2-1/3*c
/e^3*(A*c*d*e-2*B*a*e^2-B*c*d^2)*x^3+1/4*c^2/e^2*(A*e-B*d)*x^4+(A*a^2*e^5+
2*A*a*c*d^2*e^3+A*c^2*d^4*e-B*a^2*d*e^4-2*B*a*c*d^3*e^2-B*c^2*d^5)/e^6*ln(
e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.44

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{d+ex}\,dx = \frac{12\,Bc^2e^5x^5-15\,(Bc^2de^4-Ac^2e^5)x^4+20\,(Bc^2d^2e^3-Ac^2de^4+2\,Bace^5)x^3-30\,(Bc^2d^3e^2-Ac^2d^2e^3+$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d),x, algorithm="fricas")
```

output
```
1/60*(12*B*c^2*e^5*x^5 - 15*(B*c^2*d*e^4 - A*c^2*e^5)*x^4 + 20*(B*c^2*d^2*
e^3 - A*c^2*d*e^4 + 2*B*a*c*e^5)*x^3 - 30*(B*c^2*d^3*e^2 - A*c^2*d^2*e^3 +
 2*B*a*c*d*e^4 - 2*A*a*c*e^5)*x^2 + 60*(B*c^2*d^4*e - A*c^2*d^3*e^2 + 2*B*
a*c*d^2*e^3 - 2*A*a*c*d*e^4 + B*a^2*e^5)*x - 60*(B*c^2*d^5 - A*c^2*d^4*e +
 2*B*a*c*d^3*e^2 - 2*A*a*c*d^2*e^3 + B*a^2*d*e^4 - A*a^2*e^5)*log(e*x + d)
)/e^6
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{d+ex}\,dx = \frac{Bc^2x^5}{5e}+x^4\left(\frac{Ac^2}{4e}-\frac{Bc^2d}{4e^2}\right)+x^3\left(-\frac{Ac^2d}{3e^2}+\frac{2Bac}{3e}+\frac{Bc^2d^2}{3e^3}\right)+x^2\left(\frac{Aac}{e}+\frac{Ac^2d^2}{2e^3}-\frac{Bacd}{e^2}-\frac{Bc^2d^3}{2e^4}\right)+x\left(-\frac{2Aacd}{e^2}-\frac{Ac^2d^3}{e^4}+\frac{Ba^2}{e}+\frac{2Bacd^2}{e^3}+\frac{Bc^2d^4}{e^5}\right)-\frac{(-Ae+Bd)\left(ae^2+cd^2\right)^2\log(d+ex)}{e^6}$$

input
```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d),x)
```

output
```
B*c**2*x**5/(5*e) + x**4*(A*c**2/(4*e) - B*c**2*d/(4*e**2)) + x**3*(-A*c**
2*d/(3*e**2) + 2*B*a*c/(3*e) + B*c**2*d**2/(3*e**3)) + x**2*(A*a*c/e + A*c
**2*d**2/(2*e**3) - B*a*c*d/e**2 - B*c**2*d**3/(2*e**4)) + x*(-2*A*a*c*d/e
**2 - A*c**2*d**3/e**4 + B*a**2/e + 2*B*a*c*d**2/e**3 + B*c**2*d**4/e**5)
- (-A*e + B*d)*(a*e**2 + c*d**2)**2*log(d + e*x)/e**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx)(a+cx^2)^2}{d+ex}\,dx$$
$$= \frac{12\,Bc^2e^4x^5 - 15\,(Bc^2de^3 - Ac^2e^4)x^4 + 20\,(Bc^2d^2e^2 - Ac^2de^3 + 2\,Bace^4)x^3 - 30\,(Bc^2d^3e - Ac^2d^2e^2 + 2}{60\,e^5}$$
$$- \frac{(Bc^2d^5 - Ac^2d^4e + 2\,Bacd^3e^2 - 2\,Aacd^2e^3 + Ba^2de^4 - Aa^2e^5)\log(ex+d)}{e^6}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d),x, algorithm="maxima")
```

output
```
1/60*(12*B*c^2*e^4*x^5 - 15*(B*c^2*d*e^3 - A*c^2*e^4)*x^4 + 20*(B*c^2*d^2*
e^2 - A*c^2*d*e^3 + 2*B*a*c*e^4)*x^3 - 30*(B*c^2*d^3*e - A*c^2*d^2*e^2 + 2
*B*a*c*d*e^3 - 2*A*a*c*e^4)*x^2 + 60*(B*c^2*d^4 - A*c^2*d^3*e + 2*B*a*c*d^
2*e^2 - 2*A*a*c*d*e^3 + B*a^2*e^4)*x)/e^5 - (B*c^2*d^5 - A*c^2*d^4*e + 2*B
*a*c*d^3*e^2 - 2*A*a*c*d^2*e^3 + B*a^2*d*e^4 - A*a^2*e^5)*log(e*x + d)/e^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.53

$$\int \frac{(A+Bx)(a+cx^2)^2}{d+ex}\,dx$$
$$= \frac{12\,Bc^2e^4x^5 - 15\,Bc^2de^3x^4 + 15\,Ac^2e^4x^4 + 20\,Bc^2d^2e^2x^3 - 20\,Ac^2de^3x^3 + 40\,Bace^4x^3 - 30\,Bc^2d^3ex^2 + 3}{}$$
$$- \frac{(Bc^2d^5 - Ac^2d^4e + 2\,Bacd^3e^2 - 2\,Aacd^2e^3 + Ba^2de^4 - Aa^2e^5)\log(|ex+d|)}{e^6}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d),x, algorithm="giac")
```

output
```
1/60*(12*B*c^2*e^4*x^5 - 15*B*c^2*d*e^3*x^4 + 15*A*c^2*e^4*x^4 + 20*B*c^2*
d^2*e^2*x^3 - 20*A*c^2*d*e^3*x^3 + 40*B*a*c*e^4*x^3 - 30*B*c^2*d^3*e*x^2 +
 30*A*c^2*d^2*e^2*x^2 - 60*B*a*c*d*e^3*x^2 + 60*A*a*c*e^4*x^2 + 60*B*c^2*d
^4*x - 60*A*c^2*d^3*e*x + 120*B*a*c*d^2*e^2*x - 120*A*a*c*d*e^3*x + 60*B*a
^2*e^4*x)/e^5 - (B*c^2*d^5 - A*c^2*d^4*e + 2*B*a*c*d^3*e^2 - 2*A*a*c*d^2*e
^3 + B*a^2*d*e^4 - A*a^2*e^5)*log(abs(e*x + d))/e^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.54

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{d+ex}\,dx$$
$$= x\left(\frac{B\,a^2}{e}-\frac{d\left(\frac{d\left(\frac{d\left(\frac{A\,c^2}{e}-\frac{B\,c^2\,d}{e^2}\right)}{e}-\frac{2\,B\,a\,c}{e}\right)}{e}+\frac{2\,A\,a\,c}{e}\right)}{e}\right)+x^4\left(\frac{A\,c^2}{4\,e}-\frac{B\,c^2\,d}{4\,e^2}\right)$$
$$-x^3\left(\frac{d\left(\frac{A\,c^2}{e}-\frac{B\,c^2\,d}{e^2}\right)}{3\,e}-\frac{2\,B\,a\,c}{3\,e}\right)+x^2\left(\frac{d\left(\frac{d\left(\frac{A\,c^2}{e}-\frac{B\,c^2\,d}{e^2}\right)}{e}-\frac{2\,B\,a\,c}{e}\right)}{2\,e}+\frac{A\,a\,c}{e}\right)$$
$$+\frac{\ln\left(d+e\,x\right)\left(-B\,a^2\,d\,e^4+A\,a^2\,e^5-2\,B\,a\,c\,d^3\,e^2+2\,A\,a\,c\,d^2\,e^3-B\,c^2\,d^5+A\,c^2\,d^4\,e\right)}{e^6}$$
$$+\frac{B\,c^2\,x^5}{5\,e}$$

input
```
int(((a + c*x^2)^2*(A + B*x))/(d + e*x),x)
```

output
```
x*((B*a^2)/e - (d*((d*((d*((A*c^2)/e - (B*c^2*d)/e^2))/e - (2*B*a*c)/e))/e
 + (2*A*a*c)/e))/e) + x^4*((A*c^2)/(4*e) - (B*c^2*d)/(4*e^2)) - x^3*((d*((
A*c^2)/e - (B*c^2*d)/e^2))/(3*e) - (2*B*a*c)/(3*e)) + x^2*((d*((d*((A*c^2)
/e - (B*c^2*d)/e^2))/e - (2*B*a*c)/e))/(2*e) + (A*a*c)/e) + (log(d + e*x)*
(A*a^2*e^5 - B*c^2*d^5 - B*a^2*d*e^4 + A*c^2*d^4*e + 2*A*a*c*d^2*e^3 - 2*B
*a*c*d^3*e^2))/e^6 + (B*c^2*x^5)/(5*e)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.71

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{d+ex}\,dx$$
$$= \frac{60\log(ex+d)\,a^3e^5 - 60\log(ex+d)\,a^2bd\,e^4 + 120\log(ex+d)\,a^2c\,d^2e^3 - 120\log(ex+d)\,abc\,d^3e^2 + 60\,\mathrm{lo}}{}$$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d),x)
```

output
```
(60*log(d + e*x)*a**3*e**5 - 60*log(d + e*x)*a**2*b*d*e**4 + 120*log(d + e
*x)*a**2*c*d**2*e**3 - 120*log(d + e*x)*a*b*c*d**3*e**2 + 60*log(d + e*x)*
a*c**2*d**4*e - 60*log(d + e*x)*b*c**2*d**5 + 60*a**2*b*e**5*x - 120*a**2*
c*d*e**4*x + 60*a**2*c*e**5*x**2 + 120*a*b*c*d**2*e**3*x - 60*a*b*c*d*e**4
*x**2 + 40*a*b*c*e**5*x**3 - 60*a*c**2*d**3*e**2*x + 30*a*c**2*d**2*e**3*x
**2 - 20*a*c**2*d*e**4*x**3 + 15*a*c**2*e**5*x**4 + 60*b*c**2*d**4*e*x - 3
0*b*c**2*d**3*e**2*x**2 + 20*b*c**2*d**2*e**3*x**3 - 15*b*c**2*d*e**4*x**4
 + 12*b*c**2*e**5*x**5)/(60*e**6)
```

3.53 $\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 180

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^2}\,dx = -\frac{c(4Bcd^3-3Acd^2e+4aBde^2-2aAe^3)\,x}{e^5} + \frac{c(3Bcd^2-2Acde+2aBe^2)\,x^2}{2e^4} - \frac{c^2(2Bd-Ae)x^3}{3e^3} + \frac{Bc^2x^4}{4e^2} + \frac{(Bd-Ae)\,(cd^2+ae^2)^2}{e^6(d+ex)} + \frac{(cd^2+ae^2)\,(5Bcd^2-4Acde+aBe^2)\log(d+ex)}{e^6}$$

output
```
-c*(-2*A*a*e^3-3*A*c*d^2*e+4*B*a*d*e^2+4*B*c*d^3)*x/e^5+1/2*c*(-2*A*c*d*e+
2*B*a*e^2+3*B*c*d^2)*x^2/e^4-1/3*c^2*(-A*e+2*B*d)*x^3/e^3+1/4*B*c^2*x^4/e^
2+(-A*e+B*d)*(a*e^2+c*d^2)^2/e^6/(e*x+d)+(a*e^2+c*d^2)*(-4*A*c*d*e+B*a*e^2
+5*B*c*d^2)*ln(e*x+d)/e^6
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^2}\,dx$$
$$= \frac{12ce(Ae(3cd^2+2ae^2)-4B(cd^3+ade^2))\,x+6ce^2(3Bcd^2-2Acde+2aBe^2)\,x^2+4c^2e^3(-2Bd+Ae)x^3}{12e^6}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^2,x]
```

output
```
(12*c*e*(A*e*(3*c*d^2 + 2*a*e^2) - 4*B*(c*d^3 + a*d*e^2))*x + 6*c*e^2*(3*B
*c*d^2 - 2*A*c*d*e + 2*a*B*e^2)*x^2 + 4*c^2*e^3*(-2*B*d + A*e)*x^3 + 3*B*c
^2*e^4*x^4 + (12*(B*d - A*e)*(c*d^2 + a*e^2)^2)/(d + e*x) + 12*(c*d^2 + a*
e^2)*(5*B*c*d^2 - 4*A*c*d*e + a*B*e^2)*Log[d + e*x])/(12*e^6)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2(A+Bx)}{(d+ex)^2}\,dx$$
$$\Big\downarrow \ 652$$
$$\int \left(\frac{(ae^2+cd^2)(aBe^2-4Acde+5Bcd^2)}{e^5(d+ex)}+\frac{(ae^2+cd^2)^2(Ae-Bd)}{e^5(d+ex)^2}-\frac{cx(-2aBe^2+2Acde-3Bcd^2)}{e^4}+\frac{c(2a\right.$$
$$\Big\downarrow \ 2009$$

$$\frac{(ae^2+cd^2)^2(Bd-Ae)}{e^6(d+ex)}+\frac{(ae^2+cd^2)\log(d+ex)(aBe^2-4Acde+5Bcd^2)}{e^6}+\frac{cx^2(2aBe^2-2Acde+3Bcd^2)}{2e^4}-\frac{cx(-2aAe^3+4aBde^2-3Acd^2e+4Bcd^3)}{e^5}-\frac{c^2x^3(2Bd-Ae)}{3e^3}+\frac{Bc^2x^4}{4e^2}$$

input
```
Int[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^2,x]
```

output
```
-((c*(4*B*c*d^3 - 3*A*c*d^2*e + 4*a*B*d*e^2 - 2*a*A*e^3)*x)/e^5) + (c*(3*B
*c*d^2 - 2*A*c*d*e + 2*a*B*e^2)*x^2)/(2*e^4) - (c^2*(2*B*d - A*e)*x^3)/(3*
e^3) + (B*c^2*x^4)/(4*e^2) + ((B*d - A*e)*(c*d^2 + a*e^2)^2)/(e^6*(d + e*x
)) + ((c*d^2 + a*e^2)*(5*B*c*d^2 - 4*A*c*d*e + a*B*e^2)*Log[d + e*x])/e^6
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.29

method	result
default	$\frac{c\left(\frac{1}{4}Bcx^4e^3+\frac{1}{3}Ax^3ce^3-\frac{2}{3}Bx^3cde^2-Ax^2cde^2+Bx^2ae^3+\frac{3}{2}Bx^2cd^2e+2Aae^3x+3Acd^2ex-4Bade^2x-4Bcd^3x\right)}{e^5}+(-4$
norman	$\frac{\frac{\left(Aa^2e^5+4Aacd^2e^3+4Ac^2d^4e-Ba^2de^4-6Bacd^3e^2-5Bc^2d^5\right)x}{de^5}+\frac{Bc^2x^5}{4e}-\frac{c\left(4Acde-6Bae^2-5Bcd^2\right)x^3}{6e^3}+\frac{c\left(4Aae^3+4Acd^2e-6Bad\right.}{2e^4}}{ex+d}$
risch	$\frac{Bc^2x^4}{4e^2}+\frac{c^2Ax^3}{3e^2}-\frac{2c^2Bx^3d}{3e^3}-\frac{c^2Ax^2d}{e^3}+\frac{cBx^2a}{e^2}+\frac{3c^2Bx^2d^2}{2e^4}+\frac{2cAax}{e^2}+\frac{3c^2Ad^2x}{e^4}-\frac{4cBadx}{e^3}-\frac{4c^2Bd^3x}{e^5}$
parallelrisch	$-\frac{-4Ax^4c^2e^5-60B\ln(ex+d)c^2d^5+48A\ln(ex+d)xc^2d^3e^2-60B\ln(ex+d)xc^2d^4e+48Aacd^2e^3-72Bacd^3e^2+5Bx^4c^2de^4+}{}$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c/e^5*(1/4*B*c*x^4*e^3+1/3*A*x^3*c*e^3-2/3*B*x^3*c*d*e^2-A*x^2*c*d*e^2+B*x
^2*a*e^3+3/2*B*x^2*c*d^2*e+2*A*a*e^3*x+3*A*c*d^2*e*x-4*B*a*d*e^2*x-4*B*c*d
^3*x)+(-4*A*a*c*d*e^3-4*A*c^2*d^3*e+B*a^2*e^4+6*B*a*c*d^2*e^2+5*B*c^2*d^4)
/e^6*ln(e*x+d)-(A*a^2*e^5+2*A*a*c*d^2*e^3+A*c^2*d^4*e-B*a^2*d*e^4-2*B*a*c*
d^3*e^2-B*c^2*d^5)/e^6/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(174) = 348$.

Time = 0.09 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.97

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^2}\,dx$$
$$= \frac{3\,Bc^2e^5x^5 + 12\,Bc^2d^5 - 12\,Ac^2d^4e + 24\,Bacd^3e^2 - 24\,Aacd^2e^3 + 12\,Ba^2de^4 - 12\,Aa^2e^5 - (5\,Bc^2de^4 - 4}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/12*(3*B*c^2*e^5*x^5 + 12*B*c^2*d^5 - 12*A*c^2*d^4*e + 24*B*a*c*d^3*e^2 -
 24*A*a*c*d^2*e^3 + 12*B*a^2*d*e^4 - 12*A*a^2*e^5 - (5*B*c^2*d*e^4 - 4*A*c
^2*e^5)*x^4 + 2*(5*B*c^2*d^2*e^3 - 4*A*c^2*d*e^4 + 6*B*a*c*e^5)*x^3 - 6*(5
*B*c^2*d^3*e^2 - 4*A*c^2*d^2*e^3 + 6*B*a*c*d*e^4 - 4*A*a*c*e^5)*x^2 - 12*(
4*B*c^2*d^4*e - 3*A*c^2*d^3*e^2 + 4*B*a*c*d^2*e^3 - 2*A*a*c*d*e^4)*x + 12*
(5*B*c^2*d^5 - 4*A*c^2*d^4*e + 6*B*a*c*d^3*e^2 - 4*A*a*c*d^2*e^3 + B*a^2*d
*e^4 + (5*B*c^2*d^4*e - 4*A*c^2*d^3*e^2 + 6*B*a*c*d^2*e^3 - 4*A*a*c*d*e^4
+ B*a^2*e^5)*x)*log(e*x + d))/(e^7*x + d*e^6)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.37

$$\begin{aligned}
&\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^2}\,dx \\
&= \frac{Bc^2x^4}{4e^2} + x^3\left(\frac{Ac^2}{3e^2} - \frac{2Bc^2d}{3e^3}\right) + x^2\left(-\frac{Ac^2d}{e^3} + \frac{Bac}{e^2} + \frac{3Bc^2d^2}{2e^4}\right) \\
&\quad + x\left(\frac{2Aac}{e^2} + \frac{3Ac^2d^2}{e^4} - \frac{4Bacd}{e^3} - \frac{4Bc^2d^3}{e^5}\right) \\
&\quad + \frac{-Aa^2e^5 - 2Aacd^2e^3 - Ac^2d^4e + Ba^2de^4 + 2Bacd^3e^2 + Bc^2d^5}{de^6 + e^7x} \\
&\quad + \frac{(ae^2 + cd^2)(-4Acde + Bae^2 + 5Bcd^2)\log(d+ex)}{e^6}
\end{aligned}$$

input
```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**2,x)
```

output
```
B*c**2*x**4/(4*e**2) + x**3*(A*c**2/(3*e**2) - 2*B*c**2*d/(3*e**3)) + x**2
*(-A*c**2*d/e**3 + B*a*c/e**2 + 3*B*c**2*d**2/(2*e**4)) + x*(2*A*a*c/e**2
+ 3*A*c**2*d**2/e**4 - 4*B*a*c*d/e**3 - 4*B*c**2*d**3/e**5) + (-A*a**2*e**
5 - 2*A*a*c*d**2*e**3 - A*c**2*d**4*e + B*a**2*d*e**4 + 2*B*a*c*d**3*e**2
+ B*c**2*d**5)/(d*e**6 + e**7*x) + (a*e**2 + c*d**2)*(-4*A*c*d*e + B*a*e**
2 + 5*B*c*d**2)*log(d + e*x)/e**6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.38

$$\begin{aligned}
&\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^2}\,dx \\
&= \frac{Bc^2d^5 - Ac^2d^4e + 2\,Bacd^3e^2 - 2\,Aacd^2e^3 + Ba^2de^4 - Aa^2e^5}{e^7x + de^6} \\
&\quad + \frac{3\,Bc^2e^3x^4 - 4\,(2\,Bc^2de^2 - Ac^2e^3)x^3 + 6\,(3\,Bc^2d^2e - 2\,Ac^2de^2 + 2\,Bace^3)x^2 - 12\,(4\,Bc^2d^3 - 3\,Ac^2d^2e}{12\,e^5} \\
&\quad + \frac{(5\,Bc^2d^4 - 4\,Ac^2d^3e + 6\,Bacd^2e^2 - 4\,Aacde^3 + Ba^2e^4)\log(ex+d)}{e^6}
\end{aligned}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^2,x, algorithm="maxima")
```

output
```
(B*c^2*d^5 - A*c^2*d^4*e + 2*B*a*c*d^3*e^2 - 2*A*a*c*d^2*e^3 + B*a^2*d*e^4
 - A*a^2*e^5)/(e^7*x + d*e^6) + 1/12*(3*B*c^2*e^3*x^4 - 4*(2*B*c^2*d*e^2 -
 A*c^2*e^3)*x^3 + 6*(3*B*c^2*d^2*e - 2*A*c^2*d*e^2 + 2*B*a*c*e^3)*x^2 - 12
*(4*B*c^2*d^3 - 3*A*c^2*d^2*e + 4*B*a*c*d*e^2 - 2*A*a*c*e^3)*x)/e^5 + (5*B
*c^2*d^4 - 4*A*c^2*d^3*e + 6*B*a*c*d^2*e^2 - 4*A*a*c*d*e^3 + B*a^2*e^4)*lo
g(e*x + d)/e^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.82

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^2} dx$$
$$= \frac{\left(3 Bc^2 - \frac{4(5 Bc^2de - Ac^2e^2)}{(ex+d)e} + \frac{12(5 Bc^2d^2e^2 - 2 Ac^2de^3 + Bace^4)}{(ex+d)^2e^2} - \frac{24(5 Bc^2d^3e^3 - 3 Ac^2d^2e^4 + 3 Bacde^5 - Aace^6)}{(ex+d)^3e^3}\right)(ex+d)^4}{12 e^6}$$
$$- \frac{(5 Bc^2d^4 - 4 Ac^2d^3e + 6 Bacd^2e^2 - 4 Aacde^3 + Ba^2e^4)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^6}$$
$$+ \frac{\frac{Bc^2d^5e^4}{ex+d} - \frac{Ac^2d^4e^5}{ex+d} + \frac{2 Bacd^3e^6}{ex+d} - \frac{2 Aacd^2e^7}{ex+d} + \frac{Ba^2de^8}{ex+d} - \frac{Aa^2e^9}{ex+d}}{e^{10}}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/12*(3*B*c^2 - 4*(5*B*c^2*d*e - A*c^2*e^2)/((e*x + d)*e) + 12*(5*B*c^2*d^
2*e^2 - 2*A*c^2*d*e^3 + B*a*c*e^4)/((e*x + d)^2*e^2) - 24*(5*B*c^2*d^3*e^3
 - 3*A*c^2*d^2*e^4 + 3*B*a*c*d*e^5 - A*a*c*e^6)/((e*x + d)^3*e^3))*(e*x +
d)^4/e^6 - (5*B*c^2*d^4 - 4*A*c^2*d^3*e + 6*B*a*c*d^2*e^2 - 4*A*a*c*d*e^3
+ B*a^2*e^4)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^6 + (B*c^2*d^5*e^4/(
e*x + d) - A*c^2*d^4*e^5/(e*x + d) + 2*B*a*c*d^3*e^6/(e*x + d) - 2*A*a*c*d
^2*e^7/(e*x + d) + B*a^2*d*e^8/(e*x + d) - A*a^2*e^9/(e*x + d))/e^10
```

Mupad [B] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.73

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^2}\,dx$$
$$= x^3\left(\frac{A\,c^2}{3\,e^2}-\frac{2\,B\,c^2\,d}{3\,e^3}\right)-x^2\left(\frac{d\left(\frac{A\,c^2}{e^2}-\frac{2\,B\,c^2\,d}{e^3}\right)}{e}-\frac{B\,a\,c}{e^2}+\frac{B\,c^2\,d^2}{2\,e^4}\right)$$
$$+x\left(\frac{2\,d\left(\frac{2\,d\left(\frac{A\,c^2}{e^2}-\frac{2\,B\,c^2\,d}{e^3}\right)}{e}-\frac{2\,B\,a\,c}{e^2}+\frac{B\,c^2\,d^2}{e^4}\right)}{e}-\frac{d^2\left(\frac{A\,c^2}{e^2}-\frac{2\,B\,c^2\,d}{e^3}\right)}{e^2}+\frac{2\,A\,a\,c}{e^2}\right)$$
$$+\frac{\ln(d+e\,x)\,\left(B\,a^2\,e^4+6\,B\,a\,c\,d^2\,e^2-4\,A\,a\,c\,d\,e^3+5\,B\,c^2\,d^4-4\,A\,c^2\,d^3\,e\right)}{e^6}$$
$$-\frac{-B\,a^2\,d\,e^4+A\,a^2\,e^5-2\,B\,a\,c\,d^3\,e^2+2\,A\,a\,c\,d^2\,e^3-B\,c^2\,d^5+A\,c^2\,d^4\,e}{e\,(x\,e^6+d\,e^5)}+\frac{B\,c^2\,x^4}{4\,e^2}$$

input
```
int(((a + c*x^2)^2*(A + B*x))/(d + e*x)^2,x)
```

output
```
x^3*((A*c^2)/(3*e^2) - (2*B*c^2*d)/(3*e^3)) - x^2*((d*((A*c^2)/e^2 - (2*B*
c^2*d)/e^3))/e - (B*a*c)/e^2 + (B*c^2*d^2)/(2*e^4)) + x*((2*d*((2*d*((A*c^
2)/e^2 - (2*B*c^2*d)/e^3))/e - (2*B*a*c)/e^2 + (B*c^2*d^2)/e^4))/e - (d^2*
((A*c^2)/e^2 - (2*B*c^2*d)/e^3))/e^2 + (2*A*a*c)/e^2) + (log(d + e*x)*(B*a
^2*e^4 + 5*B*c^2*d^4 - 4*A*c^2*d^3*e - 4*A*a*c*d*e^3 + 6*B*a*c*d^2*e^2))/e
^6 - (A*a^2*e^5 - B*c^2*d^5 - B*a^2*d*e^4 + A*c^2*d^4*e + 2*A*a*c*d^2*e^3
- 2*B*a*c*d^3*e^2)/(e*(d*e^5 + e^6*x)) + (B*c^2*x^4)/(4*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.22

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^2}\,dx$$
$$= \frac{12a^3e^6x-36abc\,d^2e^4x^2+12abcd\,e^5x^3+3b\,c^2d\,e^5x^5-5b\,c^2d^2e^4x^4+10b\,c^2d^3e^3x^3-60b\,c^2d^5ex-30b\,c^2d}{}$$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^2,x)
```

output
```
(12*log(d + e*x)*a**2*b*d**2*e**4 + 12*log(d + e*x)*a**2*b*d*e**5*x - 48*l
og(d + e*x)*a**2*c*d**3*e**3 - 48*log(d + e*x)*a**2*c*d**2*e**4*x + 72*log
(d + e*x)*a*b*c*d**4*e**2 + 72*log(d + e*x)*a*b*c*d**3*e**3*x - 48*log(d +
 e*x)*a*c**2*d**5*e - 48*log(d + e*x)*a*c**2*d**4*e**2*x + 60*log(d + e*x)
*b*c**2*d**6 + 60*log(d + e*x)*b*c**2*d**5*e*x + 12*a**3*e**6*x - 12*a**2*
b*d*e**5*x + 48*a**2*c*d**2*e**4*x + 24*a**2*c*d*e**5*x**2 - 72*a*b*c*d**3
*e**3*x - 36*a*b*c*d**2*e**4*x**2 + 12*a*b*c*d*e**5*x**3 + 48*a*c**2*d**4*
e**2*x + 24*a*c**2*d**3*e**3*x**2 - 8*a*c**2*d**2*e**4*x**3 + 4*a*c**2*d*e
**5*x**4 - 60*b*c**2*d**5*e*x - 30*b*c**2*d**4*e**2*x**2 + 10*b*c**2*d**3*
e**3*x**3 - 5*b*c**2*d**2*e**4*x**4 + 3*b*c**2*d*e**5*x**5)/(12*d*e**6*(d
+ e*x))
```

3.54 $\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 185

$$\begin{aligned}\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^3}\,dx &= \frac{c(6Bcd^2-3Acde+2aBe^2)\,x}{e^5}-\frac{c^2(3Bd-Ae)x^2}{2e^4}\\ &\quad+\frac{Bc^2x^3}{3e^3}+\frac{(Bd-Ae)\left(cd^2+ae^2\right)^2}{2e^6(d+ex)^2}\\ &\quad-\frac{\left(cd^2+ae^2\right)\left(5Bcd^2-4Acde+aBe^2\right)}{e^6(d+ex)}\\ &\quad-\frac{2c(5Bcd^3-3Acd^2e+3aBde^2-aAe^3)\log(d+ex)}{e^6}\end{aligned}$$

output
```
c*(-3*A*c*d*e+2*B*a*e^2+6*B*c*d^2)*x/e^5-1/2*c^2*(-A*e+3*B*d)*x^2/e^4+1/3*
B*c^2*x^3/e^3+1/2*(-A*e+B*d)*(a*e^2+c*d^2)^2/e^6/(e*x+d)^2-(a*e^2+c*d^2)*(
-4*A*c*d*e+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)-2*c*(-A*a*e^3-3*A*c*d^2*e+3*B*a*
d*e^2+5*B*c*d^3)*ln(e*x+d)/e^6
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^3}\,dx$$
$$= \frac{6ce(6Bcd^2-3Acde+2aBe^2)\,x+3c^2e^2(-3Bd+Ae)x^2+2Bc^2e^3x^3+\frac{3(Bd-Ae)\left(cd^2+ae^2\right)^2}{(d+ex)^2}-\frac{6(cd^2+ae^2)\left(5Bc\right.}{d}}{6e^6}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^3,x]
```

output
```
(6*c*e*(6*B*c*d^2 - 3*A*c*d*e + 2*a*B*e^2)*x + 3*c^2*e^2*(-3*B*d + A*e)*x^
2 + 2*B*c^2*e^3*x^3 + (3*(B*d - A*e)*(c*d^2 + a*e^2)^2)/(d + e*x)^2 - (6*(
c*d^2 + a*e^2)*(5*B*c*d^2 - 4*A*c*d*e + a*B*e^2))/(d + e*x) + 12*c*(-5*B*c
*d^3 + 3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*Log[d + e*x])/(6*e^6)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2(A+Bx)}{(d+ex)^3}\,dx$$
$$\Big\downarrow \; 652$$
$$\int \left(\frac{\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^5(d+ex)^2}+\frac{\left(ae^2+cd^2\right)^2(Ae-Bd)}{e^5(d+ex)^3}-\frac{c\left(-2aBe^2+3Acde-6Bcd^2\right)}{e^5}+\frac{2c(aA}{}\right.$$
$$\Big\downarrow \; 2009$$

$$-\frac{(ae^2+cd^2)\,(aBe^2-4Acde+5Bcd^2)}{e^6(d+ex)}+\frac{(ae^2+cd^2)^2\,(Bd-Ae)}{2e^6(d+ex)^2}+\frac{cx(2aBe^2-3Acde+6Bcd^2)}{e^5}-\frac{2c\log(d+ex)\,(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3)}{e^6}-\frac{c^2x^2(3Bd-Ae)}{2e^4}+\frac{Bc^2x^3}{3e^3}$$

input
```
Int[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^3,x]
```

output
```
(c*(6*B*c*d^2 - 3*A*c*d*e + 2*a*B*e^2)*x)/e^5 - (c^2*(3*B*d - A*e)*x^2)/(2
*e^4) + (B*c^2*x^3)/(3*e^3) + ((B*d - A*e)*(c*d^2 + a*e^2)^2)/(2*e^6*(d +
e*x)^2) - ((c*d^2 + a*e^2)*(5*B*c*d^2 - 4*A*c*d*e + a*B*e^2))/(e^6*(d + e*
x)) - (2*c*(5*B*c*d^3 - 3*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*Log[d + e*x])
/e^6
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.26

method	result
norman	$\frac{\frac{\left(4Aacd\,e^3+12A\,c^2d^3e-B\,e^4a^2-12Bac\,d^2e^2-20B\,c^2d^4\right)x}{e^5}-\frac{A\,a^2e^5-6Aac\,d^2e^3-18A\,c^2d^4e+B\,a^2d\,e^4+18Bac\,d^3e^2+30B\,c^2d^5}{2e^6}+\frac{B\,c^2x^5}{3e}}{(ex+d)^2}$
default	$-\frac{c\left(-\frac{1}{3}Bc\,x^3e^2-\frac{1}{2}Ac\,e^2x^2+\frac{3}{2}Bcde\,x^2+3Acdex-2Ba\,e^2x-6Bc\,d^2x\right)}{e^5}+\frac{2c\left(Aa\,e^3+3Ac\,d^2e-3Bad\,e^2-5Bc\,d^3\right)\ln(ex+d)}{e^6}$
risch	$\frac{B\,c^2x^3}{3e^3}+\frac{c^2A\,x^2}{2e^3}-\frac{3c^2Bd\,x^2}{2e^4}-\frac{3c^2Adx}{e^4}+\frac{2cBax}{e^3}+\frac{6c^2B\,d^2x}{e^5}+\frac{(4Aacd\,e^3+4A\,c^2d^3e-B\,e^4a^2-6Bac\,d^2e^2-5B\,c^2}{}$
parallelrisch	$\frac{3A\,x^4c^2e^5-60B\ln(ex+d)c^2d^5-6Bx\,a^2e^5+72A\ln(ex+d)x\,c^2d^3e^2-120B\ln(ex+d)x\,c^2d^4e+72Ax\,c^2d^3e^2+18Aac\,d^2e^3-54B}{}$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
((4*A*a*c*d*e^3+12*A*c^2*d^3*e-B*a^2*e^4-12*B*a*c*d^2*e^2-20*B*c^2*d^4)/e^
5*x-1/2*(A*a^2*e^5-6*A*a*c*d^2*e^3-18*A*c^2*d^4*e+B*a^2*d*e^4+18*B*a*c*d^3
*e^2+30*B*c^2*d^5)/e^6+1/3*B*c^2*x^5/e-2/3*c*(3*A*c*d*e-3*B*a*e^2-5*B*c*d^
2)/e^3*x^3+1/6*c^2*(3*A*e-5*B*d)/e^2*x^4)/(e*x+d)^2+2*c/e^6*(A*a*e^3+3*A*c
*d^2*e-3*B*a*d*e^2-5*B*c*d^3)*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(179) = 358$.

Time = 0.08 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.13

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^3}\,dx$$
$$= \frac{2\,Bc^2e^5x^5 - 27\,Bc^2d^5 + 21\,Ac^2d^4e - 30\,Bacd^3e^2 + 18\,Aacd^2e^3 - 3\,Ba^2de^4 - 3\,Aa^2e^5 - (5\,Bc^2de^4 - 3\,A}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/6*(2*B*c^2*e^5*x^5 - 27*B*c^2*d^5 + 21*A*c^2*d^4*e - 30*B*a*c*d^3*e^2 +
18*A*a*c*d^2*e^3 - 3*B*a^2*d*e^4 - 3*A*a^2*e^5 - (5*B*c^2*d*e^4 - 3*A*c^2*
e^5)*x^4 + 4*(5*B*c^2*d^2*e^3 - 3*A*c^2*d*e^4 + 3*B*a*c*e^5)*x^3 + 3*(21*B
*c^2*d^3*e^2 - 11*A*c^2*d^2*e^3 + 8*B*a*c*d*e^4)*x^2 + 6*(B*c^2*d^4*e + A*
c^2*d^3*e^2 - 4*B*a*c*d^2*e^3 + 4*A*a*c*d*e^4 - B*a^2*e^5)*x - 12*(5*B*c^2
*d^5 - 3*A*c^2*d^4*e + 3*B*a*c*d^3*e^2 - A*a*c*d^2*e^3 + (5*B*c^2*d^3*e^2
- 3*A*c^2*d^2*e^3 + 3*B*a*c*d*e^4 - A*a*c*e^5)*x^2 + 2*(5*B*c^2*d^4*e - 3*
A*c^2*d^3*e^2 + 3*B*a*c*d^2*e^3 - A*a*c*d*e^4)*x)*log(e*x + d))/(e^8*x^2 +
 2*d*e^7*x + d^2*e^6)
```

Sympy [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.52

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^3}\,dx$$
$$= \frac{Bc^2x^3}{3e^3} - \frac{2c(-Aae^3-3Acd^2e+3Bade^2+5Bcd^3)\log{(d+ex)}}{e^6} + x^2\left(\frac{Ac^2}{2e^3}-\frac{3Bc^2d}{2e^4}\right) + x\left(-\frac{3Ac^2d}{e^4}+\frac{2Bac}{e^3}+\frac{6Bc^2d^2}{e^5}\right) + \frac{-Aa^2e^5+6Aacd^2e^3+7Ac^2d^4e-Ba^2de^4-10Bacd^3e^2-9Bc^2d^5+x(8Aacde^4+8Ac^2d^3e^2-2Ba^2e^5}{2d^2e^6+4de^7x+2e^8x^2}$$

input
```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**3,x)
```

output
```
B*c**2*x**3/(3*e**3) - 2*c*(-A*a*e**3 - 3*A*c*d**2*e + 3*B*a*d*e**2 + 5*B*
c*d**3)*log(d + e*x)/e**6 + x**2*(A*c**2/(2*e**3) - 3*B*c**2*d/(2*e**4)) +
 x*(-3*A*c**2*d/e**4 + 2*B*a*c/e**3 + 6*B*c**2*d**2/e**5) + (-A*a**2*e**5
+ 6*A*a*c*d**2*e**3 + 7*A*c**2*d**4*e - B*a**2*d*e**4 - 10*B*a*c*d**3*e**2
 - 9*B*c**2*d**5 + x*(8*A*a*c*d*e**4 + 8*A*c**2*d**3*e**2 - 2*B*a**2*e**5
- 12*B*a*c*d**2*e**3 - 10*B*c**2*d**4*e))/(2*d**2*e**6 + 4*d*e**7*x + 2*e*
*8*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.39

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^3}\,dx = -\frac{9\,Bc^2d^5-7\,Ac^2d^4e+10\,Bacd^3e^2-6\,Aacd^2e^3+Ba^2de^4+Aa^2e^5+2\,(5\,Bc^2d^4e-4\,Ac^2d^3e^2+6\,Bacd}{2\,(e^8x^2+2\,de^7x+d^2e^6)} + \frac{2\,Bc^2e^2x^3-3\,(3\,Bc^2de-Ac^2e^2)x^2+6\,(6\,Bc^2d^2-3\,Ac^2de+2\,Bace^2)x}{6\,e^5} - \frac{2\,(5\,Bc^2d^3-3\,Ac^2d^2e+3\,Bacde^2-Aace^3)\log{(ex+d)}}{e^6}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/2*(9*B*c^2*d^5 - 7*A*c^2*d^4*e + 10*B*a*c*d^3*e^2 - 6*A*a*c*d^2*e^3 + B
*a^2*d*e^4 + A*a^2*e^5 + 2*(5*B*c^2*d^4*e - 4*A*c^2*d^3*e^2 + 6*B*a*c*d^2*
e^3 - 4*A*a*c*d*e^4 + B*a^2*e^5)*x)/(e^8*x^2 + 2*d*e^7*x + d^2*e^6) + 1/6*
(2*B*c^2*e^2*x^3 - 3*(3*B*c^2*d*e - A*c^2*e^2)*x^2 + 6*(6*B*c^2*d^2 - 3*A*
c^2*d*e + 2*B*a*c*e^2)*x)/e^5 - 2*(5*B*c^2*d^3 - 3*A*c^2*d^2*e + 3*B*a*c*d
*e^2 - A*a*c*e^3)*log(e*x + d)/e^6
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^3}\,dx = -\frac{2\left(5\,Bc^2d^3-3\,Ac^2d^2e+3\,Bacde^2-Aace^3\right)\log\left(|ex+d|\right)}{e^6}$$
$$-\frac{9\,Bc^2d^5-7\,Ac^2d^4e+10\,Bacd^3e^2-6\,Aacd^2e^3+Ba^2de^4+Aa^2e^5+2\left(5\,Bc^2d^4e-4\,Ac^2d^3e^2+6\,Bacc\right.}{2\left(ex+d\right)^2e^6}$$
$$+\frac{2\,Bc^2e^6x^3-9\,Bc^2de^5x^2+3\,Ac^2e^6x^2+36\,Bc^2d^2e^4x-18\,Ac^2de^5x+12\,Bace^6x}{6\,e^9}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^3,x, algorithm="giac")
```

output
```
-2*(5*B*c^2*d^3 - 3*A*c^2*d^2*e + 3*B*a*c*d*e^2 - A*a*c*e^3)*log(abs(e*x +
 d))/e^6 - 1/2*(9*B*c^2*d^5 - 7*A*c^2*d^4*e + 10*B*a*c*d^3*e^2 - 6*A*a*c*d
^2*e^3 + B*a^2*d*e^4 + A*a^2*e^5 + 2*(5*B*c^2*d^4*e - 4*A*c^2*d^3*e^2 + 6*
B*a*c*d^2*e^3 - 4*A*a*c*d*e^4 + B*a^2*e^5)*x)/((e*x + d)^2*e^6) + 1/6*(2*B
*c^2*e^6*x^3 - 9*B*c^2*d*e^5*x^2 + 3*A*c^2*e^6*x^2 + 36*B*c^2*d^2*e^4*x -
18*A*c^2*d*e^5*x + 12*B*a*c*e^6*x)/e^9
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.49

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^3}\,dx = x^2\left(\frac{A\,c^2}{2\,e^3}-\frac{3\,B\,c^2\,d}{2\,e^4}\right) - \frac{x\left(B\,a^2\,e^4+6\,B\,a\,c\,d^2\,e^2-4\,A\,a\,c\,d\,e^3+5\,B\,c^2\,d^4-4\,A\,c^2\,d^3\,e\right)+\frac{B\,a^2\,d\,e^4+A\,a^2\,e^5+10\,B\,a\,c\,d^3\,e^2-6\,A\,a\,c\,d^2\,e^3+9 \cdots}{2\,e}}{d^2\,e^5+2\,d\,e^6\,x+e^7\,x^2} - x\left(\frac{3\,d\left(\frac{A\,c^2}{e^3}-\frac{3\,B\,c^2\,d}{e^4}\right)}{e}-\frac{2\,B\,a\,c}{e^3}+\frac{3\,B\,c^2\,d^2}{e^5}\right) - \frac{\ln(d+e\,x)\left(10\,B\,c^2\,d^3-6\,A\,c^2\,d^2\,e+6\,B\,a\,c\,d\,e^2-2\,A\,a\,c\,e^3\right)}{e^6}+\frac{B\,c^2\,x^3}{3\,e^3}$$

input
```
int(((a + c*x^2)^2*(A + B*x))/(d + e*x)^3,x)
```

output
```
x^2*((A*c^2)/(2*e^3) - (3*B*c^2*d)/(2*e^4)) - (x*(B*a^2*e^4 + 5*B*c^2*d^4
- 4*A*c^2*d^3*e - 4*A*a*c*d*e^3 + 6*B*a*c*d^2*e^2) + (A*a^2*e^5 + 9*B*c^2*
d^5 + B*a^2*d*e^4 - 7*A*c^2*d^4*e - 6*A*a*c*d^2*e^3 + 10*B*a*c*d^3*e^2)/(2
*e))/(d^2*e^5 + e^7*x^2 + 2*d*e^6*x) - x*((3*d*((A*c^2)/e^3 - (3*B*c^2*d)/
e^4))/e - (2*B*a*c)/e^3 + (3*B*c^2*d^2)/e^5) - (log(d + e*x)*(10*B*c^2*d^3
 - 2*A*a*c*e^3 - 6*A*c^2*d^2*e + 6*B*a*c*d*e^2))/e^6 + (B*c^2*x^3)/(3*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.43

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^3}\,dx = \frac{36abc\,d^2e^4x^2+12abcd\,e^5x^3-3a^3d\,e^5-30b\,c^2d^6+2b\,c^2d\,e^5x^5-5b\,c^2d^2e^4x^4+20b\,c^2d^3e^3x^3+60b\,c^2d^4e^2 \cdots}{\cdots}$$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^3,x)
```

output
```
(12*log(d + e*x)*a**2*c*d**3*e**3 + 24*log(d + e*x)*a**2*c*d**2*e**4*x + 1
2*log(d + e*x)*a**2*c*d*e**5*x**2 - 36*log(d + e*x)*a*b*c*d**4*e**2 - 72*l
og(d + e*x)*a*b*c*d**3*e**3*x - 36*log(d + e*x)*a*b*c*d**2*e**4*x**2 + 36*
log(d + e*x)*a*c**2*d**5*e + 72*log(d + e*x)*a*c**2*d**4*e**2*x + 36*log(d
 + e*x)*a*c**2*d**3*e**3*x**2 - 60*log(d + e*x)*b*c**2*d**6 - 120*log(d +
e*x)*b*c**2*d**5*e*x - 60*log(d + e*x)*b*c**2*d**4*e**2*x**2 - 3*a**3*d*e*
*5 + 3*a**2*b*e**6*x**2 + 6*a**2*c*d**3*e**3 - 12*a**2*c*d*e**5*x**2 - 18*
a*b*c*d**4*e**2 + 36*a*b*c*d**2*e**4*x**2 + 12*a*b*c*d*e**5*x**3 + 18*a*c*
*2*d**5*e - 36*a*c**2*d**3*e**3*x**2 - 12*a*c**2*d**2*e**4*x**3 + 3*a*c**2
*d*e**5*x**4 - 30*b*c**2*d**6 + 60*b*c**2*d**4*e**2*x**2 + 20*b*c**2*d**3*
e**3*x**3 - 5*b*c**2*d**2*e**4*x**4 + 2*b*c**2*d*e**5*x**5)/(6*d*e**6*(d**
2 + 2*d*e*x + e**2*x**2))
```

3.55 $\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 189

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^4} dx = -\frac{c^2(4Bd-Ae)x}{e^5}+\frac{Bc^2x^2}{2e^4}+\frac{(Bd-Ae)(cd^2+ae^2)^2}{3e^6(d+ex)^3} - \frac{(cd^2+ae^2)(5Bcd^2-4Acde+aBe^2)}{2e^6(d+ex)^2} + \frac{2c(5Bcd^3-3Acd^2e+3aBde^2-aAe^3)}{e^6(d+ex)} + \frac{2c(5Bcd^2-2Acde+aBe^2)\log(d+ex)}{e^6}$$

output
```
-c^2*(-A*e+4*B*d)*x/e^5+1/2*B*c^2*x^2/e^4+1/3*(-A*e+B*d)*(a*e^2+c*d^2)^2/e
^6/(e*x+d)^3-1/2*(a*e^2+c*d^2)*(-4*A*c*d*e+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)^
2+2*c*(-A*a*e^3-3*A*c*d^2*e+3*B*a*d*e^2+5*B*c*d^3)/e^6/(e*x+d)+2*c*(-2*A*c
*d*e+B*a*e^2+5*B*c*d^2)*ln(e*x+d)/e^6
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^4}\,dx$$
$$= \frac{-2Ae(a^2e^4+2ace^2(d^2+3dex+3e^2x^2)+c^2(13d^4+27d^3ex+9d^2e^2x^2-9de^3x^3-3e^4x^4))+B(-a^2e^4(\cdots}{\cdots}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^4,x]
```

output
```
(-2*A*e*(a^2*e^4 + 2*a*c*e^2*(d^2 + 3*d*e*x + 3*e^2*x^2) + c^2*(13*d^4 + 2
7*d^3*e*x + 9*d^2*e^2*x^2 - 9*d*e^3*x^3 - 3*e^4*x^4)) + B*(-(a^2*e^4*(d +
3*e*x)) + 2*a*c*d*e^2*(11*d^2 + 27*d*e*x + 18*e^2*x^2) + c^2*(47*d^5 + 81*
d^4*e*x - 9*d^3*e^2*x^2 - 63*d^2*e^3*x^3 - 15*d*e^4*x^4 + 3*e^5*x^5)) + 12
*c*(5*B*c*d^2 - 2*A*c*d*e + a*B*e^2)*(d + e*x)^3*Log[d + e*x])/(6*e^6*(d +
 e*x)^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2(A+Bx)}{(d+ex)^4}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{2c\left(-aBe^2+2Acde-5Bcd^2\right)}{e^5(d+ex)}+\frac{\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^5(d+ex)^3}+\frac{\left(ae^2+cd^2\right)^2(Ae-Bd)}{e^5(d+ex)^4}+\frac{2c(\cdots}{\cdots}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(ae^2+cd^2)\,(aBe^2-4Acde+5Bcd^2)}{2e^6(d+ex)^2}+\frac{(ae^2+cd^2)^2\,(Bd-Ae)}{3e^6(d+ex)^3}+\frac{2c\log(d+ex)\,(aBe^2-2Acde+5Bcd^2)}{e^6}+\frac{2c(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3)}{e^6(d+ex)}-\frac{c^2x(4Bd-Ae)}{e^5}+\frac{Bc^2x^2}{2e^4}$$

input
```
Int[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^4,x]
```

output
```
-((c^2*(4*B*d - A*e)*x)/e^5) + (B*c^2*x^2)/(2*e^4) + ((B*d - A*e)*(c*d^2 +
 a*e^2)^2)/(3*e^6*(d + e*x)^3) - ((c*d^2 + a*e^2)*(5*B*c*d^2 - 4*A*c*d*e +
 a*B*e^2))/(2*e^6*(d + e*x)^2) + (2*c*(5*B*c*d^3 - 3*A*c*d^2*e + 3*a*B*d*e
^2 - a*A*e^3))/(e^6*(d + e*x)) + (2*c*(5*B*c*d^2 - 2*A*c*d*e + a*B*e^2)*Lo
g[d + e*x])/e^6
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.23

method	result
default	$\frac{c^2\left(\frac{1}{2}Bex^2+Aex-4Bdx\right)}{e^5}-\frac{Aa^2e^5+2Aacd^2e^3+Ac^2d^4e-Ba^2de^4-2Bacd^3e^2-Bc^2d^5}{3e^6(ex+d)^3}-\frac{2c\left(2Acde-Bae^2-5Bcd^2\right)\ln}{e^6}$
norman	$\frac{-\frac{2Aa^2e^5+4Aacd^2e^3+44Ac^2d^4e+Ba^2de^4-22Bacd^3e^2-110Bc^2d^5}{6e^6}-\frac{\left(2Aace^3+12Ac^2d^2e-6Bacde^2-30Bc^2d^3\right)x^2}{e^4}-\frac{(4Aacde^3+36}{}}{(ex+d)^3}$
risch	$\frac{Bc^2x^2}{2e^4}+\frac{c^2Ax}{e^4}-\frac{4c^2Bdx}{e^5}+\frac{\left(-2Aace^4-6Ac^2d^2e^2+6Bacde^3+10Bc^2d^3e\right)x^2+\left(-2Aacde^3-10Ac^2d^3e-\frac{1}{2}Be^4a^2+9\right.}{e^5(ex+}$
parallelrisch	$-\frac{-6Ax^4c^2e^5-60B\ln(ex+d)c^2d^5+3Bxa^2e^5+72A\ln(ex+d)xc^2d^3e^2-180B\ln(ex+d)xc^2d^4e+108Axc^2d^3e^2+4Aacd^2e^3-}{}$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
c^2/e^5*(1/2*B*e*x^2+A*e*x-4*B*d*x)-1/3*(A*a^2*e^5+2*A*a*c*d^2*e^3+A*c^2*d
^4*e-B*a^2*d*e^4-2*B*a*c*d^3*e^2-B*c^2*d^5)/e^6/(e*x+d)^3-2/e^6*c*(2*A*c*d
*e-B*a*e^2-5*B*c*d^2)*ln(e*x+d)-2*c/e^6*(A*a*e^3+3*A*c*d^2*e-3*B*a*d*e^2-5
*B*c*d^3)/(e*x+d)-1/2*(-4*A*a*c*d*e^3-4*A*c^2*d^3*e+B*a^2*e^4+6*B*a*c*d^2*
e^2+5*B*c^2*d^4)/e^6/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(183) = 366$.

Time = 0.08 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.17

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^4}\,dx = \frac{3\,Bc^2e^5x^5+47\,Bc^2d^5-26\,Ac^2d^4e+22\,Bacd^3e^2-4\,Aacd^2e^3-Ba^2de^4-2\,Aa^2e^5-3\,(5\,Bc^2de^4-2\,Ac}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/6*(3*B*c^2*e^5*x^5 + 47*B*c^2*d^5 - 26*A*c^2*d^4*e + 22*B*a*c*d^3*e^2 -
4*A*a*c*d^2*e^3 - B*a^2*d*e^4 - 2*A*a^2*e^5 - 3*(5*B*c^2*d*e^4 - 2*A*c^2*e
^5)*x^4 - 9*(7*B*c^2*d^2*e^3 - 2*A*c^2*d*e^4)*x^3 - 3*(3*B*c^2*d^3*e^2 + 6
*A*c^2*d^2*e^3 - 12*B*a*c*d*e^4 + 4*A*a*c*e^5)*x^2 + 3*(27*B*c^2*d^4*e - 1
8*A*c^2*d^3*e^2 + 18*B*a*c*d^2*e^3 - 4*A*a*c*d*e^4 - B*a^2*e^5)*x + 12*(5*
B*c^2*d^5 - 2*A*c^2*d^4*e + B*a*c*d^3*e^2 + (5*B*c^2*d^2*e^3 - 2*A*c^2*d*e
^4 + B*a*c*e^5)*x^3 + 3*(5*B*c^2*d^3*e^2 - 2*A*c^2*d^2*e^3 + B*a*c*d*e^4)*
x^2 + 3*(5*B*c^2*d^4*e - 2*A*c^2*d^3*e^2 + B*a*c*d^2*e^3)*x)*log(e*x + d))
/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*e^6)
```

Sympy [A] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.56

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^4}\,dx$$
$$= \frac{Bc^2x^2}{2e^4} + \frac{2c(-2Acde+Bae^2+5Bcd^2)\log(d+ex)}{e^6} + x\left(\frac{Ac^2}{e^4} - \frac{4Bc^2d}{e^5}\right) + \frac{-2Aa^2e^5 - 4Aacd^2e^3 - 26Ac^2d^4e - Ba^2de^4 + 22Bacd^3e^2 + 47Bc^2d^5 + x^2(-12Aace^5 - 36Ac^2d^2e^3 + \ldots}{6d^3e^6 + 18d^2e^7x + 18\ldots}$$

input
```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**4,x)
```

output
```
B*c**2*x**2/(2*e**4) + 2*c*(-2*A*c*d*e + B*a*e**2 + 5*B*c*d**2)*log(d + e*
x)/e**6 + x*(A*c**2/e**4 - 4*B*c**2*d/e**5) + (-2*A*a**2*e**5 - 4*A*a*c*d*
*2*e**3 - 26*A*c**2*d**4*e - B*a**2*d*e**4 + 22*B*a*c*d**3*e**2 + 47*B*c**
2*d**5 + x**2*(-12*A*a*c*e**5 - 36*A*c**2*d**2*e**3 + 36*B*a*c*d*e**4 + 60
*B*c**2*d**3*e**2) + x*(-12*A*a*c*d*e**4 - 60*A*c**2*d**3*e**2 - 3*B*a**2*
e**5 + 54*B*a*c*d**2*e**3 + 105*B*c**2*d**4*e))/(6*d**3*e**6 + 18*d**2*e**
7*x + 18*d*e**8*x**2 + 6*e**9*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^4}\,dx$$
$$= \frac{47\,Bc^2d^5 - 26\,Ac^2d^4e + 22\,Bacd^3e^2 - 4\,Aacd^2e^3 - Ba^2de^4 - 2\,Aa^2e^5 + 12\,(5\,Bc^2d^3e^2 - 3\,Ac^2d^2e^3 + 3\,B\ldots}{6\,(e^9x^3 + 3\,de^8x^2 + 3\,d^2e^7x\ldots} + \frac{Bc^2ex^2 - 2\,(4\,Bc^2d - Ac^2e)x}{2\,e^5} + \frac{2\,(5\,Bc^2d^2 - 2\,Ac^2de + Bace^2)\log(ex+d)}{e^6}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^4,x, algorithm="maxima")
```

output
```
1/6*(47*B*c^2*d^5 - 26*A*c^2*d^4*e + 22*B*a*c*d^3*e^2 - 4*A*a*c*d^2*e^3 -
B*a^2*d*e^4 - 2*A*a^2*e^5 + 12*(5*B*c^2*d^3*e^2 - 3*A*c^2*d^2*e^3 + 3*B*a*
c*d*e^4 - A*a*c*e^5)*x^2 + 3*(35*B*c^2*d^4*e - 20*A*c^2*d^3*e^2 + 18*B*a*c
*d^2*e^3 - 4*A*a*c*d*e^4 - B*a^2*e^5)*x)/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^
7*x + d^3*e^6) + 1/2*(B*c^2*e*x^2 - 2*(4*B*c^2*d - A*c^2*e)*x)/e^5 + 2*(5*
B*c^2*d^2 - 2*A*c^2*d*e + B*a*c*e^2)*log(e*x + d)/e^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.33

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^4}\,dx = \frac{2\,(5\,Bc^2d^2-2\,Ac^2de+Bace^2)\log(|ex+d|)}{e^6}+\frac{Bc^2e^4x^2-8\,Bc^2de^3x+2\,Ac^2e^4x}{2\,e^8} + \frac{47\,Bc^2d^5-26\,Ac^2d^4e+22\,Bacd^3e^2-4\,Aacd^2e^3-Ba^2de^4-2\,Aa^2e^5+12\,(5\,Bc^2d^3e^2-3\,Ac^2d^2e^3+\cdots}{6\,(ex+d)^3e^6}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^4,x, algorithm="giac")
```

output
```
2*(5*B*c^2*d^2 - 2*A*c^2*d*e + B*a*c*e^2)*log(abs(e*x + d))/e^6 + 1/2*(B*c
^2*e^4*x^2 - 8*B*c^2*d*e^3*x + 2*A*c^2*e^4*x)/e^8 + 1/6*(47*B*c^2*d^5 - 26
*A*c^2*d^4*e + 22*B*a*c*d^3*e^2 - 4*A*a*c*d^2*e^3 - B*a^2*d*e^4 - 2*A*a^2*
e^5 + 12*(5*B*c^2*d^3*e^2 - 3*A*c^2*d^2*e^3 + 3*B*a*c*d*e^4 - A*a*c*e^5)*x
^2 + 3*(35*B*c^2*d^4*e - 20*A*c^2*d^3*e^2 + 18*B*a*c*d^2*e^3 - 4*A*a*c*d*e
^4 - B*a^2*e^5)*x)/((e*x + d)^3*e^6)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^4}\,dx = x\left(\frac{A\,c^2}{e^4}-\frac{4\,B\,c^2\,d}{e^5}\right) - \frac{x\left(\frac{B\,a^2\,e^4}{2}-9\,B\,a\,c\,d^2\,e^2+2\,A\,a\,c\,d\,e^3-\frac{35\,B\,c^2\,d^4}{2}+10\,A\,c^2\,d^3\,e\right)+\frac{B\,a^2\,d\,e^4+2\,A\,a^2\,e^5-22\,B\,a\,c\,d^3\,e^2+4\,A\,a\,c\,d^2\,e^3\cdots}{6\,e}}{d^3\,e^5+3\,d^2\,e^6\,x+3\,d\,e^7\,x^2+\cdots} + \frac{\ln(d+e\,x)\,\left(10\,B\,c^2\,d^2-4\,A\,c^2\,d\,e+2\,B\,a\,c\,e^2\right)}{e^6}+\frac{B\,c^2\,x^2}{2\,e^4}$$

input
```
int(((a + c*x^2)^2*(A + B*x))/(d + e*x)^4,x)
```

output
```
x*((A*c^2)/e^4 - (4*B*c^2*d)/e^5) - (x*((B*a^2*e^4)/2 - (35*B*c^2*d^4)/2 +
 10*A*c^2*d^3*e + 2*A*a*c*d*e^3 - 9*B*a*c*d^2*e^2) + (2*A*a^2*e^5 - 47*B*c
^2*d^5 + B*a^2*d*e^4 + 26*A*c^2*d^4*e + 4*A*a*c*d^2*e^3 - 22*B*a*c*d^3*e^2
)/(6*e) + x^2*(2*A*a*c*e^4 - 10*B*c^2*d^3*e + 6*A*c^2*d^2*e^2 - 6*B*a*c*d*
e^3))/(d^3*e^5 + e^8*x^3 + 3*d^2*e^6*x + 3*d*e^7*x^2) + (log(d + e*x)*(10*
B*c^2*d^2 + 2*B*a*c*e^2 - 4*A*c^2*d*e))/e^6 + (B*c^2*x^2)/(2*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.41

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^4}\,dx$$
$$= \frac{-12abcd\,e^5x^3 - 2a^3d\,e^5 + 50b\,c^2d^6 + 3b\,c^2d\,e^5x^5 - 15b\,c^2d^2e^4x^4 - 60b\,c^2d^3e^3x^3 + 90b\,c^2d^5ex + 6a\,c^2d\,e^5x}{}$$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^4,x)
```

output
```
(12*log(d + e*x)*a*b*c*d**4*e**2 + 36*log(d + e*x)*a*b*c*d**3*e**3*x + 36*
log(d + e*x)*a*b*c*d**2*e**4*x**2 + 12*log(d + e*x)*a*b*c*d*e**5*x**3 - 24
*log(d + e*x)*a*c**2*d**5*e - 72*log(d + e*x)*a*c**2*d**4*e**2*x - 72*log(
d + e*x)*a*c**2*d**3*e**3*x**2 - 24*log(d + e*x)*a*c**2*d**2*e**4*x**3 + 6
0*log(d + e*x)*b*c**2*d**6 + 180*log(d + e*x)*b*c**2*d**5*e*x + 180*log(d
+ e*x)*b*c**2*d**4*e**2*x**2 + 60*log(d + e*x)*b*c**2*d**3*e**3*x**3 - 2*a
**3*d*e**5 - a**2*b*d**2*e**4 - 3*a**2*b*d*e**5*x + 4*a**2*c*e**6*x**3 + 1
0*a*b*c*d**4*e**2 + 18*a*b*c*d**3*e**3*x - 12*a*b*c*d*e**5*x**3 - 20*a*c**
2*d**5*e - 36*a*c**2*d**4*e**2*x + 24*a*c**2*d**2*e**4*x**3 + 6*a*c**2*d*e
**5*x**4 + 50*b*c**2*d**6 + 90*b*c**2*d**5*e*x - 60*b*c**2*d**3*e**3*x**3
- 15*b*c**2*d**2*e**4*x**4 + 3*b*c**2*d*e**5*x**5)/(6*d*e**6*(d**3 + 3*d**
2*e*x + 3*d*e**2*x**2 + e**3*x**3))
```

3.56 $\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 189

$$\begin{aligned}\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^5}\,dx = {} & \frac{Bc^2x}{e^5}+\frac{(Bd-Ae)\left(cd^2+ae^2\right)^2}{4e^6(d+ex)^4} \\ & -\frac{\left(cd^2+ae^2\right)\left(5Bcd^2-4Acde+aBe^2\right)}{3e^6(d+ex)^3} \\ & +\frac{c(5Bcd^3-3Acd^2e+3aBde^2-aAe^3)}{e^6(d+ex)^2} \\ & -\frac{2c(5Bcd^2-2Acde+aBe^2)}{e^6(d+ex)}-\frac{c^2(5Bd-Ae)\log(d+ex)}{e^6}\end{aligned}$$

output
```
B*c^2*x/e^5+1/4*(-A*e+B*d)*(a*e^2+c*d^2)^2/e^6/(e*x+d)^4-1/3*(a*e^2+c*d^2)
*(-4*A*c*d*e+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)^3+c*(-A*a*e^3-3*A*c*d^2*e+3*B*
a*d*e^2+5*B*c*d^3)/e^6/(e*x+d)^2-2*c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)/e^6/(e
*x+d)-c^2*(-A*e+5*B*d)*ln(e*x+d)/e^6
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^5}\,dx$$
$$= \frac{Ae(-3a^2e^4-2ace^2(d^2+4dex+6e^2x^2)+c^2d(25d^3+88d^2ex+108de^2x^2+48e^3x^3))-B(a^2e^4(d+4ex}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^5,x]
```

output
```
(A*e*(-3*a^2*e^4 - 2*a*c*e^2*(d^2 + 4*d*e*x + 6*e^2*x^2) + c^2*d*(25*d^3 +
 88*d^2*e*x + 108*d*e^2*x^2 + 48*e^3*x^3)) - B*(a^2*e^4*(d + 4*e*x) + 6*a*
c*e^2*(d^3 + 4*d^2*e*x + 6*d*e^2*x^2 + 4*e^3*x^3) + c^2*(77*d^5 + 248*d^4*
e*x + 252*d^3*e^2*x^2 + 48*d^2*e^3*x^3 - 48*d*e^4*x^4 - 12*e^5*x^5)) - 12*
c^2*(5*B*d - A*e)*(d + e*x)^4*Log[d + e*x])/(12*e^6*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2(A+Bx)}{(d+ex)^5}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{2c\left(-aBe^2+2Acde-5Bcd^2\right)}{e^5(d+ex)^2}+\frac{\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^5(d+ex)^4}+\frac{\left(ae^2+cd^2\right)^2(Ae-Bd)}{e^5(d+ex)^5}+\frac{2c(}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2c(aBe^2-2Acde+5Bcd^2)}{e^6(d+ex)}-\frac{(ae^2+cd^2)\,(aBe^2-4Acde+5Bcd^2)}{3e^6(d+ex)^3}+\frac{(ae^2+cd^2)^2\,(Bd-Ae)}{4e^6(d+ex)^4}+\frac{c(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3)}{e^6(d+ex)^2}-\frac{c^2(5Bd-Ae)\log(d+ex)}{e^6}+\frac{Bc^2x}{e^5}$$

input
```
Int[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^5,x]
```

output
```
(B*c^2*x)/e^5 + ((B*d - A*e)*(c*d^2 + a*e^2)^2)/(4*e^6*(d + e*x)^4) - ((c*
d^2 + a*e^2)*(5*B*c*d^2 - 4*A*c*d*e + a*B*e^2))/(3*e^6*(d + e*x)^3) + (c*(
5*B*c*d^3 - 3*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(e^6*(d + e*x)^2) - (2*c
*(5*B*c*d^2 - 2*A*c*d*e + a*B*e^2))/(e^6*(d + e*x)) - (c^2*(5*B*d - A*e)*L
og[d + e*x])/e^6
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.27

method	result
default	$\frac{B\,c^2x}{e^5}-\frac{-4Aacd\,e^3-4A\,c^2d^3e+B\,e^4a^2+6Bac\,d^2e^2+5B\,c^2d^4}{3e^6(ex+d)^3}-\frac{A\,a^2e^5+2Aac\,d^2e^3+A\,c^2d^4e-B\,a^2d\,e^4-2Bac\,d^3e^2-B\,c \cdots}{4e^6(ex+d)^4}$
norman	$\frac{\frac{B\,c^2x^5}{e}-\frac{3A\,a^2e^5+2Aac\,d^2e^3-25A\,c^2d^4e+B\,a^2d\,e^4+6Bac\,d^3e^2+125B\,c^2d^5}{12e^6}+\frac{2\left(2A\,c^2de-B\,e^2ac-10B\,c^2d^2\right)x^3}{e^3}-\frac{\left(Aac\,e^3-9A\,c^2d^2e+ \cdots\right.}{}}{(ex+d)^4}$
risch	$\frac{B\,c^2x}{e^5}+\frac{(4A\,c^2d\,e^3-2B\,e^4ac-10B\,c^2d^2e^2)x^3-ce\left(Aa\,e^3-9Ac\,d^2e+3Bad\,e^2+25Bc\,d^3\right)x^2+\left(-\frac{2}{3}Aacd\,e^3+\frac{22}{3}A\,c^2d^3e-\frac{1}{3}B \cdots\right.}{e^5(ex+d)^4}$
parallelrisch	$\frac{-60B\ln(ex+d)c^2d^5-4Bx\,a^2e^5+48A\ln(ex+d)x\,c^2d^3e^2-240B\ln(ex+d)x\,c^2d^4e+88Ax\,c^2d^3e^2-2Aac\,d^2e^3-6Bac\,d^3e^2+12A \cdots}{}$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
B*c^2*x/e^5-1/3*(-4*A*a*c*d*e^3-4*A*c^2*d^3*e+B*a^2*e^4+6*B*a*c*d^2*e^2+5*
B*c^2*d^4)/e^6/(e*x+d)^3-1/4*(A*a^2*e^5+2*A*a*c*d^2*e^3+A*c^2*d^4*e-B*a^2*
d*e^4-2*B*a*c*d^3*e^2-B*c^2*d^5)/e^6/(e*x+d)^4+c^2/e^6*(A*e-5*B*d)*ln(e*x+
d)+2/e^6*c*(2*A*c*d*e-B*a*e^2-5*B*c*d^2)/(e*x+d)-c/e^6*(A*a*e^3+3*A*c*d^2*
e-3*B*a*d*e^2-5*B*c*d^3)/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(185) = 370$.

Time = 0.08 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.14

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^5}\,dx$$
$$= \frac{12\,Bc^2e^5x^5 + 48\,Bc^2de^4x^4 - 77\,Bc^2d^5 + 25\,Ac^2d^4e - 6\,Bacd^3e^2 - 2\,Aacd^2e^3 - Ba^2de^4 - 3\,Aa^2e^5 - 24\,(}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^5,x, algorithm="fricas")
```

output
```
1/12*(12*B*c^2*e^5*x^5 + 48*B*c^2*d*e^4*x^4 - 77*B*c^2*d^5 + 25*A*c^2*d^4*
e - 6*B*a*c*d^3*e^2 - 2*A*a*c*d^2*e^3 - B*a^2*d*e^4 - 3*A*a^2*e^5 - 24*(2*
B*c^2*d^2*e^3 - 2*A*c^2*d*e^4 + B*a*c*e^5)*x^3 - 12*(21*B*c^2*d^3*e^2 - 9*
A*c^2*d^2*e^3 + 3*B*a*c*d*e^4 + A*a*c*e^5)*x^2 - 4*(62*B*c^2*d^4*e - 22*A*
c^2*d^3*e^2 + 6*B*a*c*d^2*e^3 + 2*A*a*c*d*e^4 + B*a^2*e^5)*x - 12*(5*B*c^2
*d^5 - A*c^2*d^4*e + (5*B*c^2*d*e^4 - A*c^2*e^5)*x^4 + 4*(5*B*c^2*d^2*e^3
- A*c^2*d*e^4)*x^3 + 6*(5*B*c^2*d^3*e^2 - A*c^2*d^2*e^3)*x^2 + 4*(5*B*c^2*
d^4*e - A*c^2*d^3*e^2)*x)*log(e*x + d))/(e^10*x^4 + 4*d*e^9*x^3 + 6*d^2*e^
8*x^2 + 4*d^3*e^7*x + d^4*e^6)
```

Sympy [A] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.61

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^5}\,dx = \frac{Bc^2x}{e^5} - \frac{c^2(-Ae+5Bd)\log(d+ex)}{e^6} + \frac{-3Aa^2e^5 - 2Aacd^2e^3 + 25Ac^2d^4e - Ba^2de^4 - 6Bacd^3e^2 - 77Bc^2d^5 + x^3\cdot(48Ac^2de^4 - 24Bace^5 - 12}{12d^4e^6}$$

input
```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**5,x)
```

output
```
B*c**2*x/e**5 - c**2*(-A*e + 5*B*d)*log(d + e*x)/e**6 + (-3*A*a**2*e**5 -
2*A*a*c*d**2*e**3 + 25*A*c**2*d**4*e - B*a**2*d*e**4 - 6*B*a*c*d**3*e**2 -
 77*B*c**2*d**5 + x**3*(48*A*c**2*d*e**4 - 24*B*a*c*e**5 - 120*B*c**2*d**2
*e**3) + x**2*(-12*A*a*c*e**5 + 108*A*c**2*d**2*e**3 - 36*B*a*c*d*e**4 - 3
00*B*c**2*d**3*e**2) + x*(-8*A*a*c*d*e**4 + 88*A*c**2*d**3*e**2 - 4*B*a**2
*e**5 - 24*B*a*c*d**2*e**3 - 260*B*c**2*d**4*e))/(12*d**4*e**6 + 48*d**3*e
**7*x + 72*d**2*e**8*x**2 + 48*d*e**9*x**3 + 12*e**10*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.48

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^5}\,dx = -\frac{77\,Bc^2d^5 - 25\,Ac^2d^4e + 6\,Bacd^3e^2 + 2\,Aacd^2e^3 + Ba^2de^4 + 3\,Aa^2e^5 + 24\,(5\,Bc^2d^2e^3 - 2\,Ac^2de^4 + Bc}{12\,(e^{10}x^4 +} + \frac{Bc^2x}{e^5} - \frac{(5\,Bc^2d - Ac^2e)\log(ex+d)}{e^6}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^5,x, algorithm="maxima")
```

output
```
-1/12*(77*B*c^2*d^5 - 25*A*c^2*d^4*e + 6*B*a*c*d^3*e^2 + 2*A*a*c*d^2*e^3 +
 B*a^2*d*e^4 + 3*A*a^2*e^5 + 24*(5*B*c^2*d^2*e^3 - 2*A*c^2*d*e^4 + B*a*c*e
^5)*x^3 + 12*(25*B*c^2*d^3*e^2 - 9*A*c^2*d^2*e^3 + 3*B*a*c*d*e^4 + A*a*c*e
^5)*x^2 + 4*(65*B*c^2*d^4*e - 22*A*c^2*d^3*e^2 + 6*B*a*c*d^2*e^3 + 2*A*a*c
*d*e^4 + B*a^2*e^5)*x)/(e^10*x^4 + 4*d*e^9*x^3 + 6*d^2*e^8*x^2 + 4*d^3*e^7
*x + d^4*e^6) + B*c^2*x/e^5 - (5*B*c^2*d - A*c^2*e)*log(e*x + d)/e^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(185) = 370$.

Time = 0.13 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.97

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^5}\,dx = \frac{(ex+d)Bc^2}{e^6} + \frac{(5\,Bc^2d - Ac^2e)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^6} - \frac{\frac{120\,Bc^2d^2e^{22}}{ex+d} - \frac{60\,Bc^2d^3e^{22}}{(ex+d)^2} + \frac{20\,Bc^2d^4e^{22}}{(ex+d)^3} - \frac{3\,Bc^2d^5e^{22}}{(ex+d)^4} - \frac{48\,Ac^2de^{23}}{ex+d} + \frac{36\,Ac^2d^2e^{23}}{(ex+d)^2} - \frac{16\,Ac^2d^3e^{23}}{(ex+d)^3} + \frac{3\,Ac^2d^4e^{23}}{(ex+d)^4} + \frac{24\,B\cdots}{ex\cdots}}{1\cdots}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^5,x, algorithm="giac")
```

output
```
(e*x + d)*B*c^2/e^6 + (5*B*c^2*d - A*c^2*e)*log(abs(e*x + d)/((e*x + d)^2*
abs(e)))/e^6 - 1/12*(120*B*c^2*d^2*e^22/(e*x + d) - 60*B*c^2*d^3*e^22/(e*x
 + d)^2 + 20*B*c^2*d^4*e^22/(e*x + d)^3 - 3*B*c^2*d^5*e^22/(e*x + d)^4 - 4
8*A*c^2*d*e^23/(e*x + d) + 36*A*c^2*d^2*e^23/(e*x + d)^2 - 16*A*c^2*d^3*e^
23/(e*x + d)^3 + 3*A*c^2*d^4*e^23/(e*x + d)^4 + 24*B*a*c*e^24/(e*x + d) -
36*B*a*c*d*e^24/(e*x + d)^2 + 24*B*a*c*d^2*e^24/(e*x + d)^3 - 6*B*a*c*d^3*
e^24/(e*x + d)^4 + 12*A*a*c*e^25/(e*x + d)^2 - 16*A*a*c*d*e^25/(e*x + d)^3
 + 6*A*a*c*d^2*e^25/(e*x + d)^4 + 4*B*a^2*e^26/(e*x + d)^3 - 3*B*a^2*d*e^2
6/(e*x + d)^4 + 3*A*a^2*e^27/(e*x + d)^4)/e^28
```

Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.47

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^5}\,dx = \frac{\ln\left(d+e\,x\right)\,\left(A\,c^2\,e-5\,B\,c^2\,d\right)}{e^6} - \frac{x^3\,\left(10\,B\,c^2\,d^2\,e^2-4\,A\,c^2\,d\,e^3+2\,B\,a\,c\,e^4\right)+x\,\left(\frac{B\,a^2\,e^4}{3}+2\,B\,a\,c\,d^2\,e^2+\frac{2\,A\,a\,c\,d\,e^3}{3}+\frac{65\,B\,c^2\,d^4}{3}-\frac{22\,A\,c^2\,d^3\,e}{3}\right.}{d^4\,e^5+4\,d^3\,e^6\,x+} + \frac{B\,c^2\,x}{e^5}$$

input
```
int(((a + c*x^2)^2*(A + B*x))/(d + e*x)^5,x)
```

output
```
(log(d + e*x)*(A*c^2*e - 5*B*c^2*d))/e^6 - (x^3*(2*B*a*c*e^4 - 4*A*c^2*d*e
^3 + 10*B*c^2*d^2*e^2) + x*((B*a^2*e^4)/3 + (65*B*c^2*d^4)/3 - (22*A*c^2*d
^3*e)/3 + (2*A*a*c*d*e^3)/3 + 2*B*a*c*d^2*e^2) + (3*A*a^2*e^5 + 77*B*c^2*d
^5 + B*a^2*d*e^4 - 25*A*c^2*d^4*e + 2*A*a*c*d^2*e^3 + 6*B*a*c*d^3*e^2)/(12
*e) + x^2*(A*a*c*e^4 + 25*B*c^2*d^3*e - 9*A*c^2*d^2*e^2 + 3*B*a*c*d*e^3))/
(d^4*e^5 + e^9*x^4 + 4*d^3*e^6*x + 4*d*e^8*x^3 + 6*d^2*e^7*x^2) + (B*c^2*x
)/e^5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.30

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^5}\,dx = \frac{-3a^3d\,e^5-65b\,c^2d^6+12b\,c^2d\,e^5x^5+60b\,c^2d^2e^4x^4-200b\,c^2d^5ex-180b\,c^2d^4e^2x^2-12a\,c^2d\,e^5x^4+36a\,c^2}{}$$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^5,x)
```

output
```
(12*log(d + e*x)*a*c**2*d**5*e + 48*log(d + e*x)*a*c**2*d**4*e**2*x + 72*l
og(d + e*x)*a*c**2*d**3*e**3*x**2 + 48*log(d + e*x)*a*c**2*d**2*e**4*x**3
+ 12*log(d + e*x)*a*c**2*d*e**5*x**4 - 60*log(d + e*x)*b*c**2*d**6 - 240*l
og(d + e*x)*b*c**2*d**5*e*x - 360*log(d + e*x)*b*c**2*d**4*e**2*x**2 - 240
*log(d + e*x)*b*c**2*d**3*e**3*x**3 - 60*log(d + e*x)*b*c**2*d**2*e**4*x**
4 - 3*a**3*d*e**5 - a**2*b*d**2*e**4 - 4*a**2*b*d*e**5*x - 2*a**2*c*d**3*e
**3 - 8*a**2*c*d**2*e**4*x - 12*a**2*c*d*e**5*x**2 + 6*a*b*c*e**6*x**4 + 1
3*a*c**2*d**5*e + 40*a*c**2*d**4*e**2*x + 36*a*c**2*d**3*e**3*x**2 - 12*a*
c**2*d*e**5*x**4 - 65*b*c**2*d**6 - 200*b*c**2*d**5*e*x - 180*b*c**2*d**4*
e**2*x**2 + 60*b*c**2*d**2*e**4*x**4 + 12*b*c**2*d*e**5*x**5)/(12*d*e**6*(
d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))
```

3.57 $\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^6} dx$

Optimal result

Integrand size = 22, antiderivative size = 197

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^6} dx = \frac{(Bd-Ae)(cd^2+ae^2)^2}{5e^6(d+ex)^5} - \frac{(cd^2+ae^2)(5Bcd^2-4Acde+aBe^2)}{4e^6(d+ex)^4} + \frac{2c(5Bcd^3-3Acd^2e+3aBde^2-aAe^3)}{3e^6(d+ex)^3} - \frac{c(5Bcd^2-2Acde+aBe^2)}{e^6(d+ex)^2} + \frac{c^2(5Bd-Ae)}{e^6(d+ex)} + \frac{Bc^2\log(d+ex)}{e^6}$$

output
```
1/5*(-A*e+B*d)*(a*e^2+c*d^2)^2/e^6/(e*x+d)^5-1/4*(a*e^2+c*d^2)*(-4*A*c*d*e
+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)^4+2/3*c*(-A*a*e^3-3*A*c*d^2*e+3*B*a*d*e^2+
5*B*c*d^3)/e^6/(e*x+d)^3-c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)^2+c^
2*(-A*e+5*B*d)/e^6/(e*x+d)+B*c^2*ln(e*x+d)/e^6
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^6}\,dx$$
$$= \frac{-4Ae(3a^2e^4+ace^2(d^2+5dex+10e^2x^2)+3c^2(d^4+5d^3ex+10d^2e^2x^2+10de^3x^3+5e^4x^4))+B(-3a^2e}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^6,x]
```

output
```
(-4*A*e*(3*a^2*e^4 + a*c*e^2*(d^2 + 5*d*e*x + 10*e^2*x^2) + 3*c^2*(d^4 + 5
*d^3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4)) + B*(-3*a^2*e^4*(d
+ 5*e*x) - 6*a*c*e^2*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3) + c^2*d
*(137*d^4 + 625*d^3*e*x + 1100*d^2*e^2*x^2 + 900*d*e^3*x^3 + 300*e^4*x^4))
 + 60*B*c^2*(d + e*x)^5*Log[d + e*x])/(60*e^6*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2(A+Bx)}{(d+ex)^6}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{2c\left(-aBe^2+2Acde-5Bcd^2\right)}{e^5(d+ex)^3}+\frac{\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^5(d+ex)^5}+\frac{\left(ae^2+cd^2\right)^2(Ae-Bd)}{e^5(d+ex)^6}+\frac{2c(\cdots}{\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c(aBe^2-2Acde+5Bcd^2)}{e^6(d+ex)^2}-\frac{(ae^2+cd^2)\left(aBe^2-4Acde+5Bcd^2\right)}{4e^6(d+ex)^4}+\frac{(ae^2+cd^2)^2(Bd-Ae)}{5e^6(d+ex)^5}+\frac{2c(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3)}{3e^6(d+ex)^3}+\frac{c^2(5Bd-Ae)}{e^6(d+ex)}+\frac{Bc^2\log(d+ex)}{e^6}$$

input
```
Int[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^6,x]
```

output
```
((B*d - A*e)*(c*d^2 + a*e^2)^2)/(5*e^6*(d + e*x)^5) - ((c*d^2 + a*e^2)*(5*
B*c*d^2 - 4*A*c*d*e + a*B*e^2))/(4*e^6*(d + e*x)^4) + (2*c*(5*B*c*d^3 - 3*
A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(3*e^6*(d + e*x)^3) - (c*(5*B*c*d^2 -
2*A*c*d*e + a*B*e^2))/(e^6*(d + e*x)^2) + (c^2*(5*B*d - A*e))/(e^6*(d + e*
x)) + (B*c^2*Log[d + e*x])/e^6
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.19

method	result
risch	$\frac{-\frac{c^2(Ae-5Bd)x^4}{e^2}-\frac{c\left(2Acde+Ba\,e^2-15Bc\,d^2\right)x^3}{e^3}-\frac{c\left(2Aa\,e^3+6Ac\,d^2e+3Bad\,e^2-55Bc\,d^3\right)x^2}{3e^4}-\frac{\left(4Aacd\,e^3+12A\,c^2d^3e+3B\,e^4a^2+6Bac\right.}{12e^5}}{(ex+d)^5}$
default	$-\frac{2c\left(Aa\,e^3+3Ac\,d^2e-3Bad\,e^2-5Bc\,d^3\right)}{3e^6(ex+d)^3}-\frac{-4Aacd\,e^3-4A\,c^2d^3e+B\,e^4a^2+6Bac\,d^2e^2+5B\,c^2d^4}{4e^6(ex+d)^4}+\frac{B\,c^2\ln(ex+d)}{e^6}-A$
norman	$\frac{-\frac{12A\,a^2e^5+4Aac\,d^2e^3+12A\,c^2d^4e+3B\,a^2d\,e^4+6Bac\,d^3e^2-137B\,c^2d^5}{60e^6}-\frac{\left(A\,c^2e-5B\,c^2d\right)x^4}{e^2}-\frac{\left(2A\,c^2de+B\,e^2ac-15B\,c^2d^2\right)x^3}{e^3}-\left(2Aa\right.}{(ex+d)^5}$
parallelrisch	$-\frac{60A\,x^4c^2e^5-60B\ln(ex+d)c^2d^5+15Bx\,a^2e^5-300B\ln(ex+d)x\,c^2d^4e+60Ax\,c^2d^3e^2+4Aac\,d^2e^3+6Bac\,d^3e^2-60B\ln(ex+}{}$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

output
```
(-c^2*(A*e-5*B*d)/e^2*x^4-c*(2*A*c*d*e+B*a*e^2-15*B*c*d^2)/e^3*x^3-1/3*c*(
2*A*a*e^3+6*A*c*d^2*e+3*B*a*d*e^2-55*B*c*d^3)/e^4*x^2-1/12*(4*A*a*c*d*e^3+
12*A*c^2*d^3*e+3*B*a^2*e^4+6*B*a*c*d^2*e^2-125*B*c^2*d^4)/e^5*x-1/60*(12*A
*a^2*e^5+4*A*a*c*d^2*e^3+12*A*c^2*d^4*e+3*B*a^2*d*e^4+6*B*a*c*d^3*e^2-137*
B*c^2*d^5)/e^6)/(e*x+d)^5+B*c^2*ln(e*x+d)/e^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.85

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^6}\,dx$$
$$= \frac{137\,Bc^2d^5 - 12\,Ac^2d^4e - 6\,Bacd^3e^2 - 4\,Aacd^2e^3 - 3\,Ba^2de^4 - 12\,Aa^2e^5 + 60\,(5\,Bc^2de^4 - Ac^2e^5)x^4 + 6}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^6,x, algorithm="fricas")
```

output
```
1/60*(137*B*c^2*d^5 - 12*A*c^2*d^4*e - 6*B*a*c*d^3*e^2 - 4*A*a*c*d^2*e^3 -
 3*B*a^2*d*e^4 - 12*A*a^2*e^5 + 60*(5*B*c^2*d*e^4 - A*c^2*e^5)*x^4 + 60*(1
5*B*c^2*d^2*e^3 - 2*A*c^2*d*e^4 - B*a*c*e^5)*x^3 + 20*(55*B*c^2*d^3*e^2 -
6*A*c^2*d^2*e^3 - 3*B*a*c*d*e^4 - 2*A*a*c*e^5)*x^2 + 5*(125*B*c^2*d^4*e -
12*A*c^2*d^3*e^2 - 6*B*a*c*d^2*e^3 - 4*A*a*c*d*e^4 - 3*B*a^2*e^5)*x + 60*(
B*c^2*e^5*x^5 + 5*B*c^2*d*e^4*x^4 + 10*B*c^2*d^2*e^3*x^3 + 10*B*c^2*d^3*e^
2*x^2 + 5*B*c^2*d^4*e*x + B*c^2*d^5)*log(e*x + d))/(e^11*x^5 + 5*d*e^10*x^
4 + 10*d^2*e^9*x^3 + 10*d^3*e^8*x^2 + 5*d^4*e^7*x + d^5*e^6)
```

Sympy [A] (verification not implemented)

Time = 26.24 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.65

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^6}\,dx = \frac{Bc^2\log{(d+ex)}}{e^6} + \frac{-12Aa^2e^5 - 4Aacd^2e^3 - 12Ac^2d^4e - 3Ba^2de^4 - 6Bacd^3e^2 + 137Bc^2d^5 + x^4(-60Ac^2e^5 + 300Bc^2de^4$$

input
```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**6,x)
```

output
```
B*c**2*log(d + e*x)/e**6 + (-12*A*a**2*e**5 - 4*A*a*c*d**2*e**3 - 12*A*c**
2*d**4*e - 3*B*a**2*d*e**4 - 6*B*a*c*d**3*e**2 + 137*B*c**2*d**5 + x**4*(-
60*A*c**2*e**5 + 300*B*c**2*d*e**4) + x**3*(-120*A*c**2*d*e**4 - 60*B*a*c*
e**5 + 900*B*c**2*d**2*e**3) + x**2*(-40*A*a*c*e**5 - 120*A*c**2*d**2*e**3
 - 60*B*a*c*d*e**4 + 1100*B*c**2*d**3*e**2) + x*(-20*A*a*c*d*e**4 - 60*A*c
**2*d**3*e**2 - 15*B*a**2*e**5 - 30*B*a*c*d**2*e**3 + 625*B*c**2*d**4*e))/
(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3
+ 300*d*e**10*x**4 + 60*e**11*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.51

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^6}\,dx = \frac{137\,Bc^2d^5 - 12\,Ac^2d^4e - 6\,Bacd^3e^2 - 4\,Aacd^2e^3 - 3\,Ba^2de^4 - 12\,Aa^2e^5 + 60\,(5\,Bc^2de^4 - Ac^2e^5)x^4 + 6}{60} + \frac{Bc^2\log{(ex+d)}}{e^6}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^6,x, algorithm="maxima")
```

output
```
1/60*(137*B*c^2*d^5 - 12*A*c^2*d^4*e - 6*B*a*c*d^3*e^2 - 4*A*a*c*d^2*e^3 -
 3*B*a^2*d*e^4 - 12*A*a^2*e^5 + 60*(5*B*c^2*d*e^4 - A*c^2*e^5)*x^4 + 60*(1
5*B*c^2*d^2*e^3 - 2*A*c^2*d*e^4 - B*a*c*e^5)*x^3 + 20*(55*B*c^2*d^3*e^2 -
6*A*c^2*d^2*e^3 - 3*B*a*c*d*e^4 - 2*A*a*c*e^5)*x^2 + 5*(125*B*c^2*d^4*e -
12*A*c^2*d^3*e^2 - 6*B*a*c*d^2*e^3 - 4*A*a*c*d*e^4 - 3*B*a^2*e^5)*x)/(e^11
*x^5 + 5*d*e^10*x^4 + 10*d^2*e^9*x^3 + 10*d^3*e^8*x^2 + 5*d^4*e^7*x + d^5*
e^6) + B*c^2*log(e*x + d)/e^6
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^6}\,dx = \frac{Bc^2\log(|ex+d|)}{e^6} + \frac{60\,(5\,Bc^2de^3 - Ac^2e^4)x^4 + 60\,(15\,Bc^2d^2e^2 - 2\,Ac^2de^3 - Bace^4)x^3 + 20\,(55\,Bc^2d^3e - 6\,Ac^2d^2e^2 - 3\,B}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^6,x, algorithm="giac")
```

output
```
B*c^2*log(abs(e*x + d))/e^6 + 1/60*(60*(5*B*c^2*d*e^3 - A*c^2*e^4)*x^4 + 6
0*(15*B*c^2*d^2*e^2 - 2*A*c^2*d*e^3 - B*a*c*e^4)*x^3 + 20*(55*B*c^2*d^3*e
- 6*A*c^2*d^2*e^2 - 3*B*a*c*d*e^3 - 2*A*a*c*e^4)*x^2 + 5*(125*B*c^2*d^4 -
12*A*c^2*d^3*e - 6*B*a*c*d^2*e^2 - 4*A*a*c*d*e^3 - 3*B*a^2*e^4)*x + (137*B
*c^2*d^5 - 12*A*c^2*d^4*e - 6*B*a*c*d^3*e^2 - 4*A*a*c*d^2*e^3 - 3*B*a^2*d*
e^4 - 12*A*a^2*e^5)/e)/((e*x + d)^5*e^5)
```

Mupad [B] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^6}\,dx = \frac{B\,c^2\,\ln(d+e\,x)}{e^6} - \frac{x^2\left(-\frac{55\,B\,c^2\,d^3\,e^2}{3} + 2\,A\,c^2\,d^2\,e^3 + B\,a\,c\,d\,e^4 + \frac{2\,A\,a\,c\,e^5}{3}\right) + x^3\left(-15\,B\,c^2\,d^2\,e^3 + 2\,A\,c^2\,d\,e^4 + B\,a\,c\,e^5\right) + x}{}$$

input
```
int(((a + c*x^2)^2*(A + B*x))/(d + e*x)^6,x)
```

output
```
(B*c^2*log(d + e*x))/e^6 - (x^2*((2*A*a*c*e^5)/3 + 2*A*c^2*d^2*e^3 - (55*B
*c^2*d^3*e^2)/3 + B*a*c*d*e^4) + x^3*(B*a*c*e^5 + 2*A*c^2*d*e^4 - 15*B*c^2
*d^2*e^3) + x^4*(A*c^2*e^5 - 5*B*c^2*d*e^4) + x*((B*a^2*e^5)/4 - (125*B*c^
2*d^4*e)/12 + A*c^2*d^3*e^2 + (A*a*c*d*e^4)/3 + (B*a*c*d^2*e^3)/2) + (A*a^
2*e^5)/5 - (137*B*c^2*d^5)/60 + (B*a^2*d*e^4)/20 + (A*c^2*d^4*e)/5 + (A*a*
c*d^2*e^3)/15 + (B*a*c*d^3*e^2)/10)/(e^6*(d + e*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.87

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^6}\,dx$$
$$= \frac{60\log(ex+d)\,b\,c^2d^6 + 300\log(ex+d)\,b\,c^2d^5ex + 600\log(ex+d)\,b\,c^2d^4e^2x^2 + 600\log(ex+d)\,b\,c^2d^3e^3x^3}{}$$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^6,x)
```

output
```
(60*log(d + e*x)*b*c**2*d**6 + 300*log(d + e*x)*b*c**2*d**5*e*x + 600*log(
d + e*x)*b*c**2*d**4*e**2*x**2 + 600*log(d + e*x)*b*c**2*d**3*e**3*x**3 +
300*log(d + e*x)*b*c**2*d**2*e**4*x**4 + 60*log(d + e*x)*b*c**2*d*e**5*x**
5 - 12*a**3*d*e**5 - 3*a**2*b*d**2*e**4 - 15*a**2*b*d*e**5*x - 4*a**2*c*d*
*3*e**3 - 20*a**2*c*d**2*e**4*x - 40*a**2*c*d*e**5*x**2 - 6*a*b*c*d**4*e**
2 - 30*a*b*c*d**3*e**3*x - 60*a*b*c*d**2*e**4*x**2 - 60*a*b*c*d*e**5*x**3
+ 12*a*c**2*e**6*x**5 + 77*b*c**2*d**6 + 325*b*c**2*d**5*e*x + 500*b*c**2*
d**4*e**2*x**2 + 300*b*c**2*d**3*e**3*x**3 - 60*b*c**2*d*e**5*x**5)/(60*d*
e**6*(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*e**3*x**3 + 5*d*e**4
*x**4 + e**5*x**5))
```

3.58 $\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^7} dx$

Optimal result

Integrand size = 22, antiderivative size = 204

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^7} dx = \frac{(Bd-Ae)(cd^2+ae^2)^2}{6e^6(d+ex)^6} - \frac{(cd^2+ae^2)(5Bcd^2-4Acde+aBe^2)}{5e^6(d+ex)^5} + \frac{c(5Bcd^3-3Acd^2e+3aBde^2-aAe^3)}{2e^6(d+ex)^4} - \frac{2c(5Bcd^2-2Acde+aBe^2)}{3e^6(d+ex)^3} + \frac{c^2(5Bd-Ae)}{2e^6(d+ex)^2} - \frac{Bc^2}{e^6(d+ex)}$$

output
```
1/6*(-A*e+B*d)*(a*e^2+c*d^2)^2/e^6/(e*x+d)^6-1/5*(a*e^2+c*d^2)*(-4*A*c*d*e
+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)^5+1/2*c*(-A*a*e^3-3*A*c*d^2*e+3*B*a*d*e^2+
5*B*c*d^3)/e^6/(e*x+d)^4-2/3*c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)^
3+1/2*c^2*(-A*e+5*B*d)/e^6/(e*x+d)^2-B*c^2/e^6/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^7}\,dx =$$
$$-\frac{Ae(5a^2e^4+ace^2(d^2+6dex+15e^2x^2)+c^2(d^4+6d^3ex+15d^2e^2x^2+20de^3x^3+15e^4x^4))+B(a^2e^4(d}{30}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^7,x]
```

output
```
-1/30*(A*e*(5*a^2*e^4 + a*c*e^2*(d^2 + 6*d*e*x + 15*e^2*x^2) + c^2*(d^4 +
6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4)) + B*(a^2*e^4*(d +
 6*e*x) + a*c*e^2*(d^3 + 6*d^2*e*x + 15*d*e^2*x^2 + 20*e^3*x^3) + 5*c^2*(d
^5 + 6*d^4*e*x + 15*d^3*e^2*x^2 + 20*d^2*e^3*x^3 + 15*d*e^4*x^4 + 6*e^5*x^
5)))/(e^6*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2(A+Bx)}{(d+ex)^7}\,dx$$
$$\Big\downarrow \text{652}$$
$$\int \left(-\frac{2c\left(-aBe^2+2Acde-5Bcd^2\right)}{e^5(d+ex)^4}+\frac{\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^5(d+ex)^6}+\frac{\left(ae^2+cd^2\right)^2(Ae-Bd)}{e^5(d+ex)^7}+\frac{2c(\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{2c(aBe^2-2Acde+5Bcd^2)}{3e^6(d+ex)^3}-\frac{(ae^2+cd^2)\,(aBe^2-4Acde+5Bcd^2)}{5e^6(d+ex)^5}+\frac{(ae^2+cd^2)^2\,(Bd-Ae)}{6e^6(d+ex)^6}+\frac{c(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3)}{2e^6(d+ex)^4}+\frac{c^2(5Bd-Ae)}{2e^6(d+ex)^2}-\frac{Bc^2}{e^6(d+ex)}$$

input
```
Int[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^7,x]
```

output
```
((B*d - A*e)*(c*d^2 + a*e^2)^2)/(6*e^6*(d + e*x)^6) - ((c*d^2 + a*e^2)*(5*
B*c*d^2 - 4*A*c*d*e + a*B*e^2))/(5*e^6*(d + e*x)^5) + (c*(5*B*c*d^3 - 3*A*
c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(2*e^6*(d + e*x)^4) - (2*c*(5*B*c*d^2 -
2*A*c*d*e + a*B*e^2))/(3*e^6*(d + e*x)^3) + (c^2*(5*B*d - A*e))/(2*e^6*(d
+ e*x)^2) - (B*c^2)/(e^6*(d + e*x))
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.07

method	result
risch	$\frac{-\frac{B c^2 x^5}{e}-\frac{c^2 (A e+5 B d) x^4}{2 e^2}-\frac{2 c \left(A c d e+B a e^2+5 B c d^2\right) x^3}{3 e^3}-\frac{c \left(A a e^3+A c d^2 e+B a d e^2+5 B c d^3\right) x^2}{2 e^4}-\frac{\left(A a c d e^3+A c^2 d^3 e+B e^4 a^2+B a c d^2 \cdots\right.}{5 e^5}}{(e x+d)^6}$
norman	$\frac{-\frac{B c^2 x^5}{e}-\frac{\left(A c^2 e+5 B c^2 d\right) x^4}{2 e^2}-\frac{2 \left(A c^2 d e+B e^2 a c+5 B c^2 d^2\right) x^3}{3 e^3}-\frac{\left(A a c e^3+A c^2 d^2 e+B a c d e^2+5 B c^2 d^3\right) x^2}{2 e^4}-\frac{\left(A a c d e^3+A c^2 d^3 e+B e^4 \cdots\right.}{5 \cdots}}{(e x+d)^6}$
default	$\frac{2 c \left(2 A c d e-B a e^2-5 B c d^2\right)}{3 e^6 (e x+d)^3}-\frac{c \left(A a e^3+3 A c d^2 e-3 B a d e^2-5 B c d^3\right)}{2 e^6 (e x+d)^4}-\frac{-4 A a c d e^3-4 A c^2 d^3 e+B e^4 a^2+6 B a c d^2 e^2+5 B c^2 \cdots}{5 e^6 (e x+d)^5}$
gosper	$-\frac{30 B x^5 c^2 e^5+15 A x^4 c^2 e^5+75 B x^4 c^2 d e^4+20 A x^3 c^2 d e^4+20 B x^3 a c e^5+100 B x^3 c^2 d^2 e^3+15 A x^2 a c e^5+15 A x^2 c^2 d^2 e^3+15 B \cdots}{\cdots}$
parallelrisch	$-\frac{30 B x^5 c^2 e^5+15 A x^4 c^2 e^5+75 B x^4 c^2 d e^4+20 A x^3 c^2 d e^4+20 B x^3 a c e^5+100 B x^3 c^2 d^2 e^3+15 A x^2 a c e^5+15 A x^2 c^2 d^2 e^3+15 B \cdots}{\cdots}$
orering	$-\frac{30 B x^5 c^2 e^5+15 A x^4 c^2 e^5+75 B x^4 c^2 d e^4+20 A x^3 c^2 d e^4+20 B x^3 a c e^5+100 B x^3 c^2 d^2 e^3+15 A x^2 a c e^5+15 A x^2 c^2 d^2 e^3+15 B \cdots}{\cdots}$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

output
```
(-B*c^2*x^5/e-1/2*c^2*(A*e+5*B*d)/e^2*x^4-2/3*c*(A*c*d*e+B*a*e^2+5*B*c*d^2
)/e^3*x^3-1/2*c*(A*a*e^3+A*c*d^2*e+B*a*d*e^2+5*B*c*d^3)/e^4*x^2-1/5*(A*a*c
*d*e^3+A*c^2*d^3*e+B*a^2*e^4+B*a*c*d^2*e^2+5*B*c^2*d^4)/e^5*x-1/30*(5*A*a^
2*e^5+A*a*c*d^2*e^3+A*c^2*d^4*e+B*a^2*d*e^4+B*a*c*d^3*e^2+5*B*c^2*d^5)/e^6
)/(e*x+d)^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^7}\,dx = -\frac{30\,Bc^2e^5x^5+5\,Bc^2d^5+Ac^2d^4e+Bacd^3e^2+Aacd^2e^3+Ba^2de^4+5\,Aa^2e^5+15\,(5\,Bc^2de^4+Ac^2e^5)x \cdots}{30\,(e^{12}x^6+6\,d \cdots}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^7,x, algorithm="fricas")
```

output
```
-1/30*(30*B*c^2*e^5*x^5 + 5*B*c^2*d^5 + A*c^2*d^4*e + B*a*c*d^3*e^2 + A*a*
c*d^2*e^3 + B*a^2*d*e^4 + 5*A*a^2*e^5 + 15*(5*B*c^2*d*e^4 + A*c^2*e^5)*x^4
 + 20*(5*B*c^2*d^2*e^3 + A*c^2*d*e^4 + B*a*c*e^5)*x^3 + 15*(5*B*c^2*d^3*e^
2 + A*c^2*d^2*e^3 + B*a*c*d*e^4 + A*a*c*e^5)*x^2 + 6*(5*B*c^2*d^4*e + A*c^
2*d^3*e^2 + B*a*c*d^2*e^3 + A*a*c*d*e^4 + B*a^2*e^5)*x)/(e^12*x^6 + 6*d*e^
11*x^5 + 15*d^2*e^10*x^4 + 20*d^3*e^9*x^3 + 15*d^4*e^8*x^2 + 6*d^5*e^7*x +
 d^6*e^6)
```

Sympy [A] (verification not implemented)

Time = 70.95 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.65

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^7}\,dx$$
$$= \frac{-5Aa^2e^5 - Aacd^2e^3 - Ac^2d^4e - Ba^2de^4 - Bacd^3e^2 - 5Bc^2d^5 - 30Bc^2e^5x^5 + x^4(-15Ac^2e^5 - 75Bc^2de^4 \ldots}{30d^6e^6 + \ldots}$$

input
```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**7,x)
```

output
```
(-5*A*a**2*e**5 - A*a*c*d**2*e**3 - A*c**2*d**4*e - B*a**2*d*e**4 - B*a*c*
d**3*e**2 - 5*B*c**2*d**5 - 30*B*c**2*e**5*x**5 + x**4*(-15*A*c**2*e**5 -
75*B*c**2*d*e**4) + x**3*(-20*A*c**2*d*e**4 - 20*B*a*c*e**5 - 100*B*c**2*d
**2*e**3) + x**2*(-15*A*a*c*e**5 - 15*A*c**2*d**2*e**3 - 15*B*a*c*d*e**4 -
 75*B*c**2*d**3*e**2) + x*(-6*A*a*c*d*e**4 - 6*A*c**2*d**3*e**2 - 6*B*a**2
*e**5 - 6*B*a*c*d**2*e**3 - 30*B*c**2*d**4*e))/(30*d**6*e**6 + 180*d**5*e*
*7*x + 450*d**4*e**8*x**2 + 600*d**3*e**9*x**3 + 450*d**2*e**10*x**4 + 180
*d*e**11*x**5 + 30*e**12*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^7}\,dx =$$
$$-\frac{30\,Bc^2e^5x^5 + 5\,Bc^2d^5 + Ac^2d^4e + Bacd^3e^2 + Aacd^2e^3 + Ba^2de^4 + 5\,Aa^2e^5 + 15\,(5\,Bc^2de^4 + Ac^2e^5)x \ldots}{30\,(e^{12}x^6 + 6\,d \ldots}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^7,x, algorithm="maxima")
```

output
```
-1/30*(30*B*c^2*e^5*x^5 + 5*B*c^2*d^5 + A*c^2*d^4*e + B*a*c*d^3*e^2 + A*a*
c*d^2*e^3 + B*a^2*d*e^4 + 5*A*a^2*e^5 + 15*(5*B*c^2*d*e^4 + A*c^2*e^5)*x^4
 + 20*(5*B*c^2*d^2*e^3 + A*c^2*d*e^4 + B*a*c*e^5)*x^3 + 15*(5*B*c^2*d^3*e^
2 + A*c^2*d^2*e^3 + B*a*c*d*e^4 + A*a*c*e^5)*x^2 + 6*(5*B*c^2*d^4*e + A*c^
2*d^3*e^2 + B*a*c*d^2*e^3 + A*a*c*d*e^4 + B*a^2*e^5)*x)/(e^12*x^6 + 6*d*e^
11*x^5 + 15*d^2*e^10*x^4 + 20*d^3*e^9*x^3 + 15*d^4*e^8*x^2 + 6*d^5*e^7*x +
 d^6*e^6)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^7}\,dx = -\frac{30\,Bc^2e^5x^5 + 75\,Bc^2de^4x^4 + 15\,Ac^2e^5x^4 + 100\,Bc^2d^2e^3x^3 + 20\,Ac^2de^4x^3 + 20\,Bace^5x^3 + 75\,Bc^2d^3e^2x}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^7,x, algorithm="giac")
```

output
```
-1/30*(30*B*c^2*e^5*x^5 + 75*B*c^2*d*e^4*x^4 + 15*A*c^2*e^5*x^4 + 100*B*c^
2*d^2*e^3*x^3 + 20*A*c^2*d*e^4*x^3 + 20*B*a*c*e^5*x^3 + 75*B*c^2*d^3*e^2*x
^2 + 15*A*c^2*d^2*e^3*x^2 + 15*B*a*c*d*e^4*x^2 + 15*A*a*c*e^5*x^2 + 30*B*c
^2*d^4*e*x + 6*A*c^2*d^3*e^2*x + 6*B*a*c*d^2*e^3*x + 6*A*a*c*d*e^4*x + 6*B
*a^2*e^5*x + 5*B*c^2*d^5 + A*c^2*d^4*e + B*a*c*d^3*e^2 + A*a*c*d^2*e^3 + B
*a^2*d*e^4 + 5*A*a^2*e^5)/((e*x + d)^6*e^6)
```

Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^7}\,dx =$$
$$-\frac{\frac{B\,a^2\,d\,e^4+5\,A\,a^2\,e^5+B\,a\,c\,d^3\,e^2+A\,a\,c\,d^2\,e^3+5\,B\,c^2\,d^5+A\,c^2\,d^4\,e}{30\,e^6}+\frac{x\,\left(B\,a^2\,e^4+B\,a\,c\,d^2\,e^2+A\,a\,c\,d\,e^3+5\,B\,c^2\,d^4+A\,c^2\,d^3\,e\right)}{5\,e^5}+\frac{2\,c\,x^3\,(5\,B}{}}{d^6+6\,d^5\,e\,x+15\,d^4\,e^2\,x^2+20\,d^3\,e^3\,x^3+15\,d^2\,e^4}$$

input
```
int(((a + c*x^2)^2*(A + B*x))/(d + e*x)^7,x)
```

output
```
-((5*A*a^2*e^5 + 5*B*c^2*d^5 + B*a^2*d*e^4 + A*c^2*d^4*e + A*a*c*d^2*e^3 +
 B*a*c*d^3*e^2)/(30*e^6) + (x*(B*a^2*e^4 + 5*B*c^2*d^4 + A*c^2*d^3*e + A*a
*c*d*e^3 + B*a*c*d^2*e^2))/(5*e^5) + (2*c*x^3*(B*a*e^2 + 5*B*c*d^2 + A*c*d
*e))/(3*e^3) + (c^2*x^4*(A*e + 5*B*d))/(2*e^2) + (c*x^2*(A*a*e^3 + 5*B*c*d
^3 + B*a*d*e^2 + A*c*d^2*e))/(2*e^4) + (B*c^2*x^5)/e)/(d^6 + e^6*x^6 + 6*d
*e^5*x^5 + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 + 15*d^2*e^4*x^4 + 6*d^5*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.29

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^7}\,dx$$
$$=\frac{5b\,c^2e^5x^6-15a\,c^2d\,e^4x^4-20abcd\,e^4x^3-20a\,c^2d^2e^3x^3-15a^2cd\,e^4x^2-15abc\,d^2e^3x^2-15a\,c^2d^3e^2x^2-6}{30d\,e^5\left(e^6x^6+6d\,e^5x^5+15d^2e^4x^4+20\right.}$$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^7,x)
```

output
```
( - 5*a**3*d*e**4 - a**2*b*d**2*e**3 - 6*a**2*b*d*e**4*x - a**2*c*d**3*e**
2 - 6*a**2*c*d**2*e**3*x - 15*a**2*c*d*e**4*x**2 - a*b*c*d**4*e - 6*a*b*c*
d**3*e**2*x - 15*a*b*c*d**2*e**3*x**2 - 20*a*b*c*d*e**4*x**3 - a*c**2*d**5
 - 6*a*c**2*d**4*e*x - 15*a*c**2*d**3*e**2*x**2 - 20*a*c**2*d**2*e**3*x**3
 - 15*a*c**2*d*e**4*x**4 + 5*b*c**2*e**5*x**6)/(30*d*e**5*(d**6 + 6*d**5*e
*x + 15*d**4*e**2*x**2 + 20*d**3*e**3*x**3 + 15*d**2*e**4*x**4 + 6*d*e**5*
x**5 + e**6*x**6))
```

3.59 $\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^8} dx$

Optimal result

Integrand size = 22, antiderivative size = 206

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^8} dx = \frac{(Bd-Ae)(cd^2+ae^2)^2}{7e^6(d+ex)^7} - \frac{(cd^2+ae^2)(5Bcd^2-4Acde+aBe^2)}{6e^6(d+ex)^6} + \frac{2c(5Bcd^3-3Acd^2e+3aBde^2-aAe^3)}{5e^6(d+ex)^5} - \frac{c(5Bcd^2-2Acde+aBe^2)}{2e^6(d+ex)^4} + \frac{c^2(5Bd-Ae)}{3e^6(d+ex)^3} - \frac{Bc^2}{2e^6(d+ex)^2}$$

output

```
1/7*(-A*e+B*d)*(a*e^2+c*d^2)^2/e^6/(e*x+d)^7-1/6*(a*e^2+c*d^2)*(-4*A*c*d*e
+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)^6+2/5*c*(-A*a*e^3-3*A*c*d^2*e+3*B*a*d*e^2+
5*B*c*d^3)/e^6/(e*x+d)^5-1/2*c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)^
4+1/3*c^2*(-A*e+5*B*d)/e^6/(e*x+d)^3-1/2*B*c^2/e^6/(e*x+d)^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^8}\,dx = -\frac{2Ae(15a^2e^4+2ace^2(d^2+7dex+21e^2x^2)+c^2(d^4+7d^3ex+21d^2e^2x^2+35de^3x^3+35e^4x^4))+B(5a^2}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^8,x]
```

output
```
-1/210*(2*A*e*(15*a^2*e^4 + 2*a*c*e^2*(d^2 + 7*d*e*x + 21*e^2*x^2) + c^2*(
d^4 + 7*d^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4)) + B*(5*a^2*
e^4*(d + 7*e*x) + 3*a*c*e^2*(d^3 + 7*d^2*e*x + 21*d*e^2*x^2 + 35*e^3*x^3)
+ 5*c^2*(d^5 + 7*d^4*e*x + 21*d^3*e^2*x^2 + 35*d^2*e^3*x^3 + 35*d*e^4*x^4
+ 21*e^5*x^5)))/(e^6*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2(A+Bx)}{(d+ex)^8}\,dx$$

$$\Big\downarrow \; 652$$

$$\int \left(-\frac{2c\left(-aBe^2+2Acde-5Bcd^2\right)}{e^5(d+ex)^5}+\frac{\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^5(d+ex)^7}+\frac{\left(ae^2+cd^2\right)^2(Ae-Bd)}{e^5(d+ex)^8}+\frac{2c(}{}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{c(aBe^2-2Acde+5Bcd^2)}{2e^6(d+ex)^4}-\frac{(ae^2+cd^2)\,(aBe^2-4Acde+5Bcd^2)}{6e^6(d+ex)^6}+\frac{(ae^2+cd^2)^2\,(Bd-Ae)}{7e^6(d+ex)^7}+\frac{2c(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3)}{5e^6(d+ex)^5}+\frac{c^2(5Bd-Ae)}{3e^6(d+ex)^3}-\frac{Bc^2}{2e^6(d+ex)^2}$$

input
```
Int[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^8,x]
```

output
```
((B*d - A*e)*(c*d^2 + a*e^2)^2)/(7*e^6*(d + e*x)^7) - ((c*d^2 + a*e^2)*(5*
B*c*d^2 - 4*A*c*d*e + a*B*e^2))/(6*e^6*(d + e*x)^6) + (2*c*(5*B*c*d^3 - 3*
A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(5*e^6*(d + e*x)^5) - (c*(5*B*c*d^2 -
2*A*c*d*e + a*B*e^2))/(2*e^6*(d + e*x)^4) + (c^2*(5*B*d - A*e))/(3*e^6*(d
+ e*x)^3) - (B*c^2)/(2*e^6*(d + e*x)^2)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.13

method	result
risch	$\frac{-\frac{B c^2 x^5}{2e}-\frac{c^2(2Ae+5Bd)x^4}{6e^2}-\frac{c\left(2Acde+3Ba e^2+5Bc d^2\right)x^3}{6e^3}-\frac{c\left(4Aa e^3+2Ac d^2 e+3Bad e^2+5Bc d^3\right)x^2}{10e^4}-\frac{\left(4Aacd e^3+2A c^2 d^3 e+5B e^4 a\right.}{30e^5}}{(ex+d)^7}$
default	$-\frac{c^2(Ae-5Bd)}{3e^6(ex+d)^3}+\frac{c(2Acde-Ba e^2-5Bc d^2)}{2e^6(ex+d)^4}-\frac{A a^2 e^5+2Aac d^2 e^3+A c^2 d^4 e-B a^2 d e^4-2Bac d^3 e^2-B c^2 d^5}{7e^6(ex+d)^7}-\frac{2c\left(Aa e^3\right.}{}$
gosper	$-\frac{105B x^5 c^2 e^5+70A x^4 c^2 e^5+175B x^4 c^2 d e^4+70A x^3 c^2 d e^4+105B x^3 ac e^5+175B x^3 c^2 d^2 e^3+84A x^2 ac e^5+42A x^2 c^2 d^2 e^3+6}{}$
orering	$-\frac{105B x^5 c^2 e^5+70A x^4 c^2 e^5+175B x^4 c^2 d e^4+70A x^3 c^2 d e^4+105B x^3 ac e^5+175B x^3 c^2 d^2 e^3+84A x^2 ac e^5+42A x^2 c^2 d^2 e^3+6}{}$
norman	$\frac{-\frac{B c^2 x^5}{2e}-\frac{\left(2A c^2 e^2+5B c^2 de\right)x^4}{6e^3}-\frac{\left(2A c^2 d e^2+3B e^3 ac+5B c^2 d^2 e\right)x^3}{6e^4}-\frac{\left(4Aac e^4+2A c^2 d^2 e^2+3Bacd e^3+5B c^2 d^3 e\right)x^2}{10e^5}-\frac{\left(4Aacd e^4+\right.}{}}{(ex+d)^7}$
parallelrisch	$-\frac{105B c^2 x^5 e^6+70A c^2 e^6 x^4+175B c^2 d e^5 x^4+70A c^2 d e^5 x^3+105Bac e^6 x^3+175B c^2 d^2 e^4 x^3+84Aac e^6 x^2+42A c^2 d^2 e^4 x^2+63}{}$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^8,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*B*c^2*x^5/e-1/6/e^2*c^2*(2*A*e+5*B*d)*x^4-1/6*c/e^3*(2*A*c*d*e+3*B*a
*e^2+5*B*c*d^2)*x^3-1/10*c/e^4*(4*A*a*e^3+2*A*c*d^2*e+3*B*a*d*e^2+5*B*c*d^
3)*x^2-1/30/e^5*(4*A*a*c*d*e^3+2*A*c^2*d^3*e+5*B*a^2*e^4+3*B*a*c*d^2*e^2+5
*B*c^2*d^4)*x-1/210/e^6*(30*A*a^2*e^5+4*A*a*c*d^2*e^3+2*A*c^2*d^4*e+5*B*a^
2*d*e^4+3*B*a*c*d^3*e^2+5*B*c^2*d^5))/(e*x+d)^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.54

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^8}\,dx =$$
$$-\frac{105\,Bc^2e^5x^5+5\,Bc^2d^5+2\,Ac^2d^4e+3\,Bacd^3e^2+4\,Aacd^2e^3+5\,Ba^2de^4+30\,Aa^2e^5+35\,(5\,Bc^2de^4+}{210\,(e^{13}x^7}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^8,x, algorithm="fricas")
```

output
```
-1/210*(105*B*c^2*e^5*x^5 + 5*B*c^2*d^5 + 2*A*c^2*d^4*e + 3*B*a*c*d^3*e^2
+ 4*A*a*c*d^2*e^3 + 5*B*a^2*d*e^4 + 30*A*a^2*e^5 + 35*(5*B*c^2*d*e^4 + 2*A
*c^2*e^5)*x^4 + 35*(5*B*c^2*d^2*e^3 + 2*A*c^2*d*e^4 + 3*B*a*c*e^5)*x^3 + 2
1*(5*B*c^2*d^3*e^2 + 2*A*c^2*d^2*e^3 + 3*B*a*c*d*e^4 + 4*A*a*c*e^5)*x^2 +
7*(5*B*c^2*d^4*e + 2*A*c^2*d^3*e^2 + 3*B*a*c*d^2*e^3 + 4*A*a*c*d*e^4 + 5*B
*a^2*e^5)*x)/(e^13*x^7 + 7*d*e^12*x^6 + 21*d^2*e^11*x^5 + 35*d^3*e^10*x^4
+ 35*d^4*e^9*x^3 + 21*d^5*e^8*x^2 + 7*d^6*e^7*x + d^7*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^8}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**8,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.54

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^8}\,dx = -\frac{105\,Bc^2e^5x^5+5\,Bc^2d^5+2\,Ac^2d^4e+3\,Bacd^3e^2+4\,Aacd^2e^3+5\,Ba^2de^4+30\,Aa^2e^5+35\,(5\,Bc^2de^4+}{210\,(e^{13}x^7}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^8,x, algorithm="maxima")
```

output
```
-1/210*(105*B*c^2*e^5*x^5 + 5*B*c^2*d^5 + 2*A*c^2*d^4*e + 3*B*a*c*d^3*e^2
+ 4*A*a*c*d^2*e^3 + 5*B*a^2*d*e^4 + 30*A*a^2*e^5 + 35*(5*B*c^2*d*e^4 + 2*A
*c^2*e^5)*x^4 + 35*(5*B*c^2*d^2*e^3 + 2*A*c^2*d*e^4 + 3*B*a*c*e^5)*x^3 + 2
1*(5*B*c^2*d^3*e^2 + 2*A*c^2*d^2*e^3 + 3*B*a*c*d*e^4 + 4*A*a*c*e^5)*x^2 +
7*(5*B*c^2*d^4*e + 2*A*c^2*d^3*e^2 + 3*B*a*c*d^2*e^3 + 4*A*a*c*d*e^4 + 5*B
*a^2*e^5)*x)/(e^13*x^7 + 7*d*e^12*x^6 + 21*d^2*e^11*x^5 + 35*d^3*e^10*x^4
+ 35*d^4*e^9*x^3 + 21*d^5*e^8*x^2 + 7*d^6*e^7*x + d^7*e^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^8}\,dx = -\frac{105\,Bc^2e^5x^5+175\,Bc^2de^4x^4+70\,Ac^2e^5x^4+175\,Bc^2d^2e^3x^3+70\,Ac^2de^4x^3+105\,Bace^5x^3+105\,Bc^2d}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^8,x, algorithm="giac")
```

output
```
-1/210*(105*B*c^2*e^5*x^5 + 175*B*c^2*d*e^4*x^4 + 70*A*c^2*e^5*x^4 + 175*B
*c^2*d^2*e^3*x^3 + 70*A*c^2*d*e^4*x^3 + 105*B*a*c*e^5*x^3 + 105*B*c^2*d^3*
e^2*x^2 + 42*A*c^2*d^2*e^3*x^2 + 63*B*a*c*d*e^4*x^2 + 84*A*a*c*e^5*x^2 + 3
5*B*c^2*d^4*e*x + 14*A*c^2*d^3*e^2*x + 21*B*a*c*d^2*e^3*x + 28*A*a*c*d*e^4
*x + 35*B*a^2*e^5*x + 5*B*c^2*d^5 + 2*A*c^2*d^4*e + 3*B*a*c*d^3*e^2 + 4*A*
a*c*d^2*e^3 + 5*B*a^2*d*e^4 + 30*A*a^2*e^5)/((e*x + d)^7*e^6)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.45

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^8}\,dx = -\frac{\frac{5\,B\,a^2\,d\,e^4+30\,A\,a^2\,e^5+3\,B\,a\,c\,d^3\,e^2+4\,A\,a\,c\,d^2\,e^3+5\,B\,c^2\,d^5+2\,A\,c^2\,d^4\,e}{210\,e^6}+\frac{x\,\left(5\,B\,a^2\,e^4+3\,B\,a\,c\,d^2\,e^2+4\,A\,a\,c\,d\,e^3+5\,B\,c^2\,d^4+2\,A\,c^2\,d^3\,e\right)}{30\,e^5}}{d^7+7\,d^6\,e\,x+21\,d^5\,e^2\,x^2+35\,d^4\,e^3\,x^3+35\,d^3\,e}$$

input
```
int(((a + c*x^2)^2*(A + B*x))/(d + e*x)^8,x)
```

output
```
-((30*A*a^2*e^5 + 5*B*c^2*d^5 + 5*B*a^2*d*e^4 + 2*A*c^2*d^4*e + 4*A*a*c*d^
2*e^3 + 3*B*a*c*d^3*e^2)/(210*e^6) + (x*(5*B*a^2*e^4 + 5*B*c^2*d^4 + 2*A*c
^2*d^3*e + 4*A*a*c*d*e^3 + 3*B*a*c*d^2*e^2))/(30*e^5) + (c*x^3*(3*B*a*e^2
+ 5*B*c*d^2 + 2*A*c*d*e))/(6*e^3) + (c^2*x^4*(2*A*e + 5*B*d))/(6*e^2) + (c
*x^2*(4*A*a*e^3 + 5*B*c*d^3 + 3*B*a*d*e^2 + 2*A*c*d^2*e))/(10*e^4) + (B*c^
2*x^5)/(2*e))/(d^7 + e^7*x^7 + 7*d*e^6*x^6 + 21*d^5*e^2*x^2 + 35*d^4*e^3*x
^3 + 35*d^3*e^4*x^4 + 21*d^2*e^5*x^5 + 7*d^6*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.58

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^8}\,dx$$
$$= \frac{-105b\,c^2e^5x^5 - 70a\,c^2e^5x^4 - 175b\,c^2d\,e^4x^4 - 105abc\,e^5x^3 - 70a\,c^2d\,e^4x^3 - 175b\,c^2d^2e^3x^3 - 84a^2c\,e^5x^2 -}{210e}$$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^8,x)
```

output
```
( - 30*a**3*e**5 - 5*a**2*b*d*e**4 - 35*a**2*b*e**5*x - 4*a**2*c*d**2*e**3
 - 28*a**2*c*d*e**4*x - 84*a**2*c*e**5*x**2 - 3*a*b*c*d**3*e**2 - 21*a*b*c
*d**2*e**3*x - 63*a*b*c*d*e**4*x**2 - 105*a*b*c*e**5*x**3 - 2*a*c**2*d**4*
e - 14*a*c**2*d**3*e**2*x - 42*a*c**2*d**2*e**3*x**2 - 70*a*c**2*d*e**4*x*
*3 - 70*a*c**2*e**5*x**4 - 5*b*c**2*d**5 - 35*b*c**2*d**4*e*x - 105*b*c**2
*d**3*e**2*x**2 - 175*b*c**2*d**2*e**3*x**3 - 175*b*c**2*d*e**4*x**4 - 105
*b*c**2*e**5*x**5)/(210*e**6*(d**7 + 7*d**6*e*x + 21*d**5*e**2*x**2 + 35*d
**4*e**3*x**3 + 35*d**3*e**4*x**4 + 21*d**2*e**5*x**5 + 7*d*e**6*x**6 + e*
*7*x**7))
```

3.60 $\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^9}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 206

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^9}\,dx = \frac{(Bd-Ae)(cd^2+ae^2)^2}{8e^6(d+ex)^8} - \frac{(cd^2+ae^2)(5Bcd^2-4Acde+aBe^2)}{7e^6(d+ex)^7} + \frac{c(5Bcd^3-3Acd^2e+3aBde^2-aAe^3)}{3e^6(d+ex)^6} - \frac{2c(5Bcd^2-2Acde+aBe^2)}{5e^6(d+ex)^5} + \frac{c^2(5Bd-Ae)}{4e^6(d+ex)^4} - \frac{Bc^2}{3e^6(d+ex)^3}$$

output
```
1/8*(-A*e+B*d)*(a*e^2+c*d^2)^2/e^6/(e*x+d)^8-1/7*(a*e^2+c*d^2)*(-4*A*c*d*e
+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)^7+1/3*c*(-A*a*e^3-3*A*c*d^2*e+3*B*a*d*e^2+
5*B*c*d^3)/e^6/(e*x+d)^6-2/5*c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)^
5+1/4*c^2*(-A*e+5*B*d)/e^6/(e*x+d)^4-1/3*B*c^2/e^6/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^9}\,dx =$$
$$-\frac{Ae(105a^2e^4+10ace^2(d^2+8dex+28e^2x^2)+3c^2(d^4+8d^3ex+28d^2e^2x^2+56de^3x^3+70e^4x^4))+B(1$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^9,x]
```

output
```
-1/840*(A*e*(105*a^2*e^4 + 10*a*c*e^2*(d^2 + 8*d*e*x + 28*e^2*x^2) + 3*c^2
*(d^4 + 8*d^3*e*x + 28*d^2*e^2*x^2 + 56*d*e^3*x^3 + 70*e^4*x^4)) + B*(15*a
^2*e^4*(d + 8*e*x) + 6*a*c*e^2*(d^3 + 8*d^2*e*x + 28*d*e^2*x^2 + 56*e^3*x^
3) + 5*c^2*(d^5 + 8*d^4*e*x + 28*d^3*e^2*x^2 + 56*d^2*e^3*x^3 + 70*d*e^4*x
^4 + 56*e^5*x^5)))/(e^6*(d + e*x)^8)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2(A+Bx)}{(d+ex)^9}\,dx$$
$$\Big\downarrow \; 652$$
$$\int \left(-\frac{2c\left(-aBe^2+2Acde-5Bcd^2\right)}{e^5(d+ex)^6}+\frac{\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^5(d+ex)^8}+\frac{\left(ae^2+cd^2\right)^2(Ae-Bd)}{e^5(d+ex)^9}+\frac{2c(c$$
$$\Big\downarrow \; 2009$$

$$-\frac{2c(aBe^2-2Acde+5Bcd^2)}{5e^6(d+ex)^5}-\frac{(ae^2+cd^2)\,(aBe^2-4Acde+5Bcd^2)}{7e^6(d+ex)^7}+\frac{(ae^2+cd^2)^2\,(Bd-Ae)}{8e^6(d+ex)^8}+\frac{c(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3)}{3e^6(d+ex)^6}+\frac{c^2(5Bd-Ae)}{4e^6(d+ex)^4}-\frac{Bc^2}{3e^6(d+ex)^3}$$

input
```
Int[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^9,x]
```

output
```
((B*d - A*e)*(c*d^2 + a*e^2)^2)/(8*e^6*(d + e*x)^8) - ((c*d^2 + a*e^2)*(5*
B*c*d^2 - 4*A*c*d*e + a*B*e^2))/(7*e^6*(d + e*x)^7) + (c*(5*B*c*d^3 - 3*A*
c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(3*e^6*(d + e*x)^6) - (2*c*(5*B*c*d^2 -
2*A*c*d*e + a*B*e^2))/(5*e^6*(d + e*x)^5) + (c^2*(5*B*d - A*e))/(4*e^6*(d
+ e*x)^4) - (B*c^2)/(3*e^6*(d + e*x)^3)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.13

method	result
risch	$\frac{-\frac{B c^2 x^5}{3e}-\frac{c^2(3Ae+5Bd)x^4}{12e^2}-\frac{c\left(3Acde+6Ba e^2+5Bc d^2\right)x^3}{15e^3}-\frac{c\left(10Aa e^3+3Ac d^2 e+6Bad e^2+5Bc d^3\right)x^2}{30e^4}-\frac{\left(10Aacd e^3+3A c^2 d^3 e+15B\right.}{10}}{(ex+d)^8}$
default	$-\frac{B c^2}{3e^6(ex+d)^3}-\frac{c^2(Ae-5Bd)}{4e^6(ex+d)^4}-\frac{-4Aacd e^3-4A c^2 d^3 e+B e^4 a^2+6Bac d^2 e^2+5B c^2 d^4}{7e^6(ex+d)^7}+\frac{2c\left(2Acde-Ba e^2-5Bc d^2\right)}{5e^6(ex+d)^5}-$
gosper	$-\frac{280B x^5 c^2 e^5+210A x^4 c^2 e^5+350B x^4 c^2 d e^4+168A x^3 c^2 d e^4+336B x^3 ac e^5+280B x^3 c^2 d^2 e^3+280A x^2 ac e^5+84A x^2 c^2 d^2 e^3}{}$
orering	$-\frac{280B x^5 c^2 e^5+210A x^4 c^2 e^5+350B x^4 c^2 d e^4+168A x^3 c^2 d e^4+336B x^3 ac e^5+280B x^3 c^2 d^2 e^3+280A x^2 ac e^5+84A x^2 c^2 d^2 e^3}{}$
parallelrisch	$-\frac{280B c^2 x^5 e^7+210A c^2 e^7 x^4+350B c^2 d e^6 x^4+168A c^2 d e^6 x^3+336Bac e^7 x^3+280B c^2 d^2 e^5 x^3+280Aac e^7 x^2+84A c^2 d^2 e^5 x^2}{}$
norman	$\frac{-\frac{B c^2 x^5}{3e}-\frac{\left(3A c^2 e^3+5B c^2 d e^2\right)x^4}{12e^4}-\frac{\left(3A c^2 d e^3+6B e^4 ac+5B c^2 d^2 e^2\right)x^3}{15e^5}-\frac{\left(10Aac e^5+3A c^2 d^2 e^3+6Bacd e^4+5B c^2 d^3 e^2\right)x^2}{30e^6}-\frac{\left(10Aa\right.}{}}{(ex+d)}$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^9,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*B*c^2*x^5/e-1/12*c^2/e^2*(3*A*e+5*B*d)*x^4-1/15*c/e^3*(3*A*c*d*e+6*B
*a*e^2+5*B*c*d^2)*x^3-1/30*c/e^4*(10*A*a*e^3+3*A*c*d^2*e+6*B*a*d*e^2+5*B*c
*d^3)*x^2-1/105/e^5*(10*A*a*c*d*e^3+3*A*c^2*d^3*e+15*B*a^2*e^4+6*B*a*c*d^2
*e^2+5*B*c^2*d^4)*x-1/840/e^6*(105*A*a^2*e^5+10*A*a*c*d^2*e^3+3*A*c^2*d^4*
e+15*B*a^2*d*e^4+6*B*a*c*d^3*e^2+5*B*c^2*d^5))/(e*x+d)^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.59

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^9}\,dx = -\frac{280\,Bc^2e^5x^5+5\,Bc^2d^5+3\,Ac^2d^4e+6\,Bacd^3e^2+10\,Aacd^2e^3+15\,Ba^2de^4+105\,Aa^2e^5+70\,(5\,Bc^2de}{840\,(e^{14}x^8+}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^9,x, algorithm="fricas")
```

output
```
-1/840*(280*B*c^2*e^5*x^5 + 5*B*c^2*d^5 + 3*A*c^2*d^4*e + 6*B*a*c*d^3*e^2
+ 10*A*a*c*d^2*e^3 + 15*B*a^2*d*e^4 + 105*A*a^2*e^5 + 70*(5*B*c^2*d*e^4 +
3*A*c^2*e^5)*x^4 + 56*(5*B*c^2*d^2*e^3 + 3*A*c^2*d*e^4 + 6*B*a*c*e^5)*x^3
+ 28*(5*B*c^2*d^3*e^2 + 3*A*c^2*d^2*e^3 + 6*B*a*c*d*e^4 + 10*A*a*c*e^5)*x^
2 + 8*(5*B*c^2*d^4*e + 3*A*c^2*d^3*e^2 + 6*B*a*c*d^2*e^3 + 10*A*a*c*d*e^4
+ 15*B*a^2*e^5)*x)/(e^14*x^8 + 8*d*e^13*x^7 + 28*d^2*e^12*x^6 + 56*d^3*e^1
1*x^5 + 70*d^4*e^10*x^4 + 56*d^5*e^9*x^3 + 28*d^6*e^8*x^2 + 8*d^7*e^7*x +
d^8*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^9}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**9,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.59

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^9}\,dx = -\frac{280\,Bc^2e^5x^5+5\,Bc^2d^5+3\,Ac^2d^4e+6\,Bacd^3e^2+10\,Aacd^2e^3+15\,Ba^2de^4+105\,Aa^2e^5+70\,(5\,Bc^2de}{840\,(e^{14}x^8+}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^9,x, algorithm="maxima")
```

output
```
-1/840*(280*B*c^2*e^5*x^5 + 5*B*c^2*d^5 + 3*A*c^2*d^4*e + 6*B*a*c*d^3*e^2
+ 10*A*a*c*d^2*e^3 + 15*B*a^2*d*e^4 + 105*A*a^2*e^5 + 70*(5*B*c^2*d*e^4 +
3*A*c^2*e^5)*x^4 + 56*(5*B*c^2*d^2*e^3 + 3*A*c^2*d*e^4 + 6*B*a*c*e^5)*x^3
+ 28*(5*B*c^2*d^3*e^2 + 3*A*c^2*d^2*e^3 + 6*B*a*c*d*e^4 + 10*A*a*c*e^5)*x^
2 + 8*(5*B*c^2*d^4*e + 3*A*c^2*d^3*e^2 + 6*B*a*c*d^2*e^3 + 10*A*a*c*d*e^4
+ 15*B*a^2*e^5)*x)/(e^14*x^8 + 8*d*e^13*x^7 + 28*d^2*e^12*x^6 + 56*d^3*e^1
1*x^5 + 70*d^4*e^10*x^4 + 56*d^5*e^9*x^3 + 28*d^6*e^8*x^2 + 8*d^7*e^7*x +
d^8*e^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^9}\,dx = -\frac{280\,Bc^2e^5x^5+350\,Bc^2de^4x^4+210\,Ac^2e^5x^4+280\,Bc^2d^2e^3x^3+168\,Ac^2de^4x^3+336\,Bace^5x^3+140\,B}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^9,x, algorithm="giac")
```

output
```
-1/840*(280*B*c^2*e^5*x^5 + 350*B*c^2*d*e^4*x^4 + 210*A*c^2*e^5*x^4 + 280*
B*c^2*d^2*e^3*x^3 + 168*A*c^2*d*e^4*x^3 + 336*B*a*c*e^5*x^3 + 140*B*c^2*d^
3*e^2*x^2 + 84*A*c^2*d^2*e^3*x^2 + 168*B*a*c*d*e^4*x^2 + 280*A*a*c*e^5*x^2
 + 40*B*c^2*d^4*e*x + 24*A*c^2*d^3*e^2*x + 48*B*a*c*d^2*e^3*x + 80*A*a*c*d
*e^4*x + 120*B*a^2*e^5*x + 5*B*c^2*d^5 + 3*A*c^2*d^4*e + 6*B*a*c*d^3*e^2 +
 10*A*a*c*d^2*e^3 + 15*B*a^2*d*e^4 + 105*A*a^2*e^5)/((e*x + d)^8*e^6)
```

Mupad [B] (verification not implemented)

Time = 6.31 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.50

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^9}\,dx = -\frac{\frac{15\,B\,a^2\,d\,e^4+105\,A\,a^2\,e^5+6\,B\,a\,c\,d^3\,e^2+10\,A\,a\,c\,d^2\,e^3+5\,B\,c^2\,d^5+3\,A\,c^2\,d^4\,e}{840\,e^6}+\frac{x\,\left(15\,B\,a^2\,e^4+6\,B\,a\,c\,d^2\,e^2+10\,A\,a\,c\,d\,e^3+5\,B\,c^2\,d^4+3\,A\,c^2}{105\,e^5}}{d^8+8\,d^7\,e\,x+28\,d^6\,e^2\,x^2+56\,d^5\,e^3\,x^3+70\,d^4\,e^4\,x^4}$$

input
```
int(((a + c*x^2)^2*(A + B*x))/(d + e*x)^9,x)
```

output
```
-((105*A*a^2*e^5 + 5*B*c^2*d^5 + 15*B*a^2*d*e^4 + 3*A*c^2*d^4*e + 10*A*a*c
*d^2*e^3 + 6*B*a*c*d^3*e^2)/(840*e^6) + (x*(15*B*a^2*e^4 + 5*B*c^2*d^4 + 3
*A*c^2*d^3*e + 10*A*a*c*d*e^3 + 6*B*a*c*d^2*e^2))/(105*e^5) + (c*x^3*(6*B*
a*e^2 + 5*B*c*d^2 + 3*A*c*d*e))/(15*e^3) + (c^2*x^4*(3*A*e + 5*B*d))/(12*e
^2) + (c*x^2*(10*A*a*e^3 + 5*B*c*d^3 + 6*B*a*d*e^2 + 3*A*c*d^2*e))/(30*e^4
) + (B*c^2*x^5)/(3*e))/(d^8 + e^8*x^8 + 8*d*e^7*x^7 + 28*d^6*e^2*x^2 + 56*
d^5*e^3*x^3 + 70*d^4*e^4*x^4 + 56*d^3*e^5*x^5 + 28*d^2*e^6*x^6 + 8*d^7*e*x
)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.64

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^9}\,dx$$
$$= \frac{-280b\,c^2e^5x^5 - 210a\,c^2e^5x^4 - 350b\,c^2d\,e^4x^4 - 336abc\,e^5x^3 - 168a\,c^2d\,e^4x^3 - 280b\,c^2d^2e^3x^3 - 280a^2c\,e^5x}{840e^6}$$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^9,x)
```

output
```
( - 105*a**3*e**5 - 15*a**2*b*d*e**4 - 120*a**2*b*e**5*x - 10*a**2*c*d**2*
e**3 - 80*a**2*c*d*e**4*x - 280*a**2*c*e**5*x**2 - 6*a*b*c*d**3*e**2 - 48*
a*b*c*d**2*e**3*x - 168*a*b*c*d*e**4*x**2 - 336*a*b*c*e**5*x**3 - 3*a*c**2
*d**4*e - 24*a*c**2*d**3*e**2*x - 84*a*c**2*d**2*e**3*x**2 - 168*a*c**2*d*
e**4*x**3 - 210*a*c**2*e**5*x**4 - 5*b*c**2*d**5 - 40*b*c**2*d**4*e*x - 14
0*b*c**2*d**3*e**2*x**2 - 280*b*c**2*d**2*e**3*x**3 - 350*b*c**2*d*e**4*x*
*4 - 280*b*c**2*e**5*x**5)/(840*e**6*(d**8 + 8*d**7*e*x + 28*d**6*e**2*x**
2 + 56*d**5*e**3*x**3 + 70*d**4*e**4*x**4 + 56*d**3*e**5*x**5 + 28*d**2*e*
*6*x**6 + 8*d*e**7*x**7 + e**8*x**8))
```

3.61 $\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{10}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 206

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{10}} \, dx = \frac{(Bd-Ae)\left(cd^2+ae^2\right)^2}{9e^6(d+ex)^9} - \frac{(cd^2+ae^2)\left(5Bcd^2-4Acde+aBe^2\right)}{8e^6(d+ex)^8} + \frac{2c(5Bcd^3-3Acd^2e+3aBde^2-aAe^3)}{7e^6(d+ex)^7} - \frac{c(5Bcd^2-2Acde+aBe^2)}{3e^6(d+ex)^6} + \frac{c^2(5Bd-Ae)}{5e^6(d+ex)^5} - \frac{Bc^2}{4e^6(d+ex)^4}$$

output
```
1/9*(-A*e+B*d)*(a*e^2+c*d^2)^2/e^6/(e*x+d)^9-1/8*(a*e^2+c*d^2)*(-4*A*c*d*e
+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)^8+2/7*c*(-A*a*e^3-3*A*c*d^2*e+3*B*a*d*e^2+
5*B*c*d^3)/e^6/(e*x+d)^7-1/3*c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)^
6+1/5*c^2*(-A*e+5*B*d)/e^6/(e*x+d)^5-1/4*B*c^2/e^6/(e*x+d)^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{10}}\,dx = -\frac{4Ae(70a^2e^4+5ace^2(d^2+9dex+36e^2x^2)+c^2(d^4+9d^3ex+36d^2e^2x^2+84de^3x^3+126e^4x^4))+5B(7}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^10,x]
```

output
```
-1/2520*(4*A*e*(70*a^2*e^4 + 5*a*c*e^2*(d^2 + 9*d*e*x + 36*e^2*x^2) + c^2*
(d^4 + 9*d^3*e*x + 36*d^2*e^2*x^2 + 84*d*e^3*x^3 + 126*e^4*x^4)) + 5*B*(7*
a^2*e^4*(d + 9*e*x) + 2*a*c*e^2*(d^3 + 9*d^2*e*x + 36*d*e^2*x^2 + 84*e^3*x
^3) + c^2*(d^5 + 9*d^4*e*x + 36*d^3*e^2*x^2 + 84*d^2*e^3*x^3 + 126*d*e^4*x
^4 + 126*e^5*x^5)))/(e^6*(d + e*x)^9)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2(A+Bx)}{(d+ex)^{10}}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{2c\left(-aBe^2+2Acde-5Bcd^2\right)}{e^5(d+ex)^7}+\frac{\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^5(d+ex)^9}+\frac{\left(ae^2+cd^2\right)^2(Ae-Bd)}{e^5(d+ex)^{10}}+\frac{2c(}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c(aBe^2-2Acde+5Bcd^2)}{3e^6(d+ex)^6}-\frac{(ae^2+cd^2)(aBe^2-4Acde+5Bcd^2)}{8e^6(d+ex)^8}+\frac{(ae^2+cd^2)^2(Bd-Ae)}{9e^6(d+ex)^9}+\frac{2c(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3)}{7e^6(d+ex)^7}+\frac{c^2(5Bd-Ae)}{5e^6(d+ex)^5}-\frac{Bc^2}{4e^6(d+ex)^4}$$

input
```
Int[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^10,x]
```

output
```
((B*d - A*e)*(c*d^2 + a*e^2)^2)/(9*e^6*(d + e*x)^9) - ((c*d^2 + a*e^2)*(5*
B*c*d^2 - 4*A*c*d*e + a*B*e^2))/(8*e^6*(d + e*x)^8) + (2*c*(5*B*c*d^3 - 3*
A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(7*e^6*(d + e*x)^7) - (c*(5*B*c*d^2 -
2*A*c*d*e + a*B*e^2))/(3*e^6*(d + e*x)^6) + (c^2*(5*B*d - A*e))/(5*e^6*(d
+ e*x)^5) - (B*c^2)/(4*e^6*(d + e*x)^4)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.13

method	result
risch	$\frac{-\frac{B c^2 x^5}{4e}-\frac{c^2(4Ae+5Bd)x^4}{20e^2}-\frac{c\left(4Acde+10Ba e^2+5Bc d^2\right)x^3}{30e^3}-\frac{c\left(20Aa e^3+4Ac d^2e+10Bad e^2+5Bc d^3\right)x^2}{70e^4}-\frac{\left(20Aacd e^3+4A c^2d^3e+35\right.}{}}{(ex+d)^9}$
default	$-\frac{B c^2}{4e^6(ex+d)^4}-\frac{A a^2e^5+2Aac d^2e^3+A c^2d^4e-B a^2d e^4-2Bac d^3e^2-B c^2d^5}{9e^6(ex+d)^9}-\frac{2c\left(Aa e^3+3Ac d^2e-3Bad e^2-5Bc d^3\right)}{7e^6(ex+d)^7}$
gosper	$-\frac{630B x^5c^2e^5+504A x^4c^2e^5+630B x^4c^2d e^4+336A x^3c^2d e^4+840B x^3ac e^5+420B x^3c^2d^2e^3+720A x^2ac e^5+144A x^2c^2d^2}{}$
orering	$-\frac{630B x^5c^2e^5+504A x^4c^2e^5+630B x^4c^2d e^4+336A x^3c^2d e^4+840B x^3ac e^5+420B x^3c^2d^2e^3+720A x^2ac e^5+144A x^2c^2d^2}{}$
parallelrisch	$-\frac{630B c^2x^5e^8+504A c^2e^8x^4+630B c^2d e^7x^4+336A c^2d e^7x^3+840Bac e^8x^3+420B c^2d^2e^6x^3+720Aac e^8x^2+144A c^2d^2e^6x}{}$
norman	$\frac{-\frac{B c^2x^5}{4e}-\frac{\left(4A c^2e^4+5B c^2d e^3\right)x^4}{20e^5}-\frac{\left(4A c^2d e^4+10B e^5ac+5B c^2d^2e^3\right)x^3}{30e^6}-\frac{\left(20Aac e^6+4A c^2d^2e^4+10Bacd e^5+5B c^2d^3e^3\right)x^2}{70e^7}-\frac{(20}{}}{(ex+}$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^10,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*B*c^2*x^5/e-1/20*c^2/e^2*(4*A*e+5*B*d)*x^4-1/30/e^3*c*(4*A*c*d*e+10*
B*a*e^2+5*B*c*d^2)*x^3-1/70/e^4*c*(20*A*a*e^3+4*A*c*d^2*e+10*B*a*d*e^2+5*B
*c*d^3)*x^2-1/280/e^5*(20*A*a*c*d*e^3+4*A*c^2*d^3*e+35*B*a^2*e^4+10*B*a*c*
d^2*e^2+5*B*c^2*d^4)*x-1/2520/e^6*(280*A*a^2*e^5+20*A*a*c*d^2*e^3+4*A*c^2*
d^4*e+35*B*a^2*d*e^4+10*B*a*c*d^3*e^2+5*B*c^2*d^5))/(e*x+d)^9
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.65

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{10}}\,dx = -\frac{630\,Bc^2e^5x^5+5\,Bc^2d^5+4\,Ac^2d^4e+10\,Bacd^3e^2+20\,Aacd^2e^3+35\,Ba^2de^4+280\,Aa^2e^5+126\,(5\,Bc^2}{2520\,(e^{15}x^9+9\,de}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^10,x, algorithm="fricas")
```

output
```
-1/2520*(630*B*c^2*e^5*x^5 + 5*B*c^2*d^5 + 4*A*c^2*d^4*e + 10*B*a*c*d^3*e^
2 + 20*A*a*c*d^2*e^3 + 35*B*a^2*d*e^4 + 280*A*a^2*e^5 + 126*(5*B*c^2*d*e^4
 + 4*A*c^2*e^5)*x^4 + 84*(5*B*c^2*d^2*e^3 + 4*A*c^2*d*e^4 + 10*B*a*c*e^5)*
x^3 + 36*(5*B*c^2*d^3*e^2 + 4*A*c^2*d^2*e^3 + 10*B*a*c*d*e^4 + 20*A*a*c*e^
5)*x^2 + 9*(5*B*c^2*d^4*e + 4*A*c^2*d^3*e^2 + 10*B*a*c*d^2*e^3 + 20*A*a*c*
d*e^4 + 35*B*a^2*e^5)*x)/(e^15*x^9 + 9*d*e^14*x^8 + 36*d^2*e^13*x^7 + 84*d
^3*e^12*x^6 + 126*d^4*e^11*x^5 + 126*d^5*e^10*x^4 + 84*d^6*e^9*x^3 + 36*d^
7*e^8*x^2 + 9*d^8*e^7*x + d^9*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{10}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**10,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.65

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{10}}\,dx = -\frac{630\,Bc^2e^5x^5 + 5\,Bc^2d^5 + 4\,Ac^2d^4e + 10\,Bacd^3e^2 + 20\,Aacd^2e^3 + 35\,Ba^2de^4 + 280\,Aa^2e^5 + 126\,(5\,Bc^2}{2520\,(e^{15}x^9 + 9\,de}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^10,x, algorithm="maxima")
```

output
```
-1/2520*(630*B*c^2*e^5*x^5 + 5*B*c^2*d^5 + 4*A*c^2*d^4*e + 10*B*a*c*d^3*e^
2 + 20*A*a*c*d^2*e^3 + 35*B*a^2*d*e^4 + 280*A*a^2*e^5 + 126*(5*B*c^2*d*e^4
 + 4*A*c^2*e^5)*x^4 + 84*(5*B*c^2*d^2*e^3 + 4*A*c^2*d*e^4 + 10*B*a*c*e^5)*
x^3 + 36*(5*B*c^2*d^3*e^2 + 4*A*c^2*d^2*e^3 + 10*B*a*c*d*e^4 + 20*A*a*c*e^
5)*x^2 + 9*(5*B*c^2*d^4*e + 4*A*c^2*d^3*e^2 + 10*B*a*c*d^2*e^3 + 20*A*a*c*
d*e^4 + 35*B*a^2*e^5)*x)/(e^15*x^9 + 9*d*e^14*x^8 + 36*d^2*e^13*x^7 + 84*d
^3*e^12*x^6 + 126*d^4*e^11*x^5 + 126*d^5*e^10*x^4 + 84*d^6*e^9*x^3 + 36*d^
7*e^8*x^2 + 9*d^8*e^7*x + d^9*e^6)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^{10}}\,dx = -\frac{630\,Bc^2e^5x^5+630\,Bc^2de^4x^4+504\,Ac^2e^5x^4+420\,Bc^2d^2e^3x^3+336\,Ac^2de^4x^3+840\,Bace^5x^3+180\,B\cdots}{\cdots}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^10,x, algorithm="giac")
```

output
```
-1/2520*(630*B*c^2*e^5*x^5 + 630*B*c^2*d*e^4*x^4 + 504*A*c^2*e^5*x^4 + 420
*B*c^2*d^2*e^3*x^3 + 336*A*c^2*d*e^4*x^3 + 840*B*a*c*e^5*x^3 + 180*B*c^2*d
^3*e^2*x^2 + 144*A*c^2*d^2*e^3*x^2 + 360*B*a*c*d*e^4*x^2 + 720*A*a*c*e^5*x
^2 + 45*B*c^2*d^4*e*x + 36*A*c^2*d^3*e^2*x + 90*B*a*c*d^2*e^3*x + 180*A*a*
c*d*e^4*x + 315*B*a^2*e^5*x + 5*B*c^2*d^5 + 4*A*c^2*d^4*e + 10*B*a*c*d^3*e
^2 + 20*A*a*c*d^2*e^3 + 35*B*a^2*d*e^4 + 280*A*a^2*e^5)/((e*x + d)^9*e^6)
```

Mupad [B] (verification not implemented)

Time = 6.28 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.56

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^{10}}\,dx = -\frac{\frac{35\,B\,a^2\,d\,e^4+280\,A\,a^2\,e^5+10\,B\,a\,c\,d^3\,e^2+20\,A\,a\,c\,d^2\,e^3+5\,B\,c^2\,d^5+4\,A\,c^2\,d^4\,e}{2520\,e^6}+\frac{x\,(35\,B\,a^2\,e^4+10\,B\,a\,c\,d^2\,e^2+20\,A\,a\,c\,d\,e^3+5\,B\,c^2\,d^4+4\,A\cdots}{280\,e^5}\cdots}{d^9+9\,d^8\,e\,x+36\,d^7\,e^2\,x^2+84\,d^6\,e^3\,x^3+126\,d^5\,e^4\,x^4+12\cdots}$$

input
```
int(((a + c*x^2)^2*(A + B*x))/(d + e*x)^10,x)
```

output
```
-((280*A*a^2*e^5 + 5*B*c^2*d^5 + 35*B*a^2*d*e^4 + 4*A*c^2*d^4*e + 20*A*a*c
*d^2*e^3 + 10*B*a*c*d^3*e^2)/(2520*e^6) + (x*(35*B*a^2*e^4 + 5*B*c^2*d^4 +
 4*A*c^2*d^3*e + 20*A*a*c*d*e^3 + 10*B*a*c*d^2*e^2))/(280*e^5) + (c*x^3*(1
0*B*a*e^2 + 5*B*c*d^2 + 4*A*c*d*e))/(30*e^3) + (c^2*x^4*(4*A*e + 5*B*d))/(
20*e^2) + (c*x^2*(20*A*a*e^3 + 5*B*c*d^3 + 10*B*a*d*e^2 + 4*A*c*d^2*e))/(7
0*e^4) + (B*c^2*x^5)/(4*e))/(d^9 + e^9*x^9 + 9*d*e^8*x^8 + 36*d^7*e^2*x^2
+ 84*d^6*e^3*x^3 + 126*d^5*e^4*x^4 + 126*d^4*e^5*x^5 + 84*d^3*e^6*x^6 + 36
*d^2*e^7*x^7 + 9*d^8*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.69

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{10}}\,dx = \frac{-630b\,c^2e^5x^5-504a\,c^2e^5x^4-630b\,c^2d\,e^4x^4-840abc\,e^5x^3-336a\,c^2d\,e^4x^3-420b\,c^2d^2e^3x^3-720a^2c\,e^5x}{2520e^6\,(e^9x^9}$$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^10,x)
```

output
```
( - 280*a**3*e**5 - 35*a**2*b*d*e**4 - 315*a**2*b*e**5*x - 20*a**2*c*d**2*
e**3 - 180*a**2*c*d*e**4*x - 720*a**2*c*e**5*x**2 - 10*a*b*c*d**3*e**2 - 9
0*a*b*c*d**2*e**3*x - 360*a*b*c*d*e**4*x**2 - 840*a*b*c*e**5*x**3 - 4*a*c*
*2*d**4*e - 36*a*c**2*d**3*e**2*x - 144*a*c**2*d**2*e**3*x**2 - 336*a*c**2
*d*e**4*x**3 - 504*a*c**2*e**5*x**4 - 5*b*c**2*d**5 - 45*b*c**2*d**4*e*x -
 180*b*c**2*d**3*e**2*x**2 - 420*b*c**2*d**2*e**3*x**3 - 630*b*c**2*d*e**4
*x**4 - 630*b*c**2*e**5*x**5)/(2520*e**6*(d**9 + 9*d**8*e*x + 36*d**7*e**2
*x**2 + 84*d**6*e**3*x**3 + 126*d**5*e**4*x**4 + 126*d**4*e**5*x**5 + 84*d
**3*e**6*x**6 + 36*d**2*e**7*x**7 + 9*d*e**8*x**8 + e**9*x**9))
```

3.62 $\int (A+Bx)(d+ex)^5 \left(a+cx^2\right)^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 334

$$
\begin{aligned}
\int (A+Bx)(d+ex)^5 \left(a+cx^2\right)^3 \, dx
&= -\frac{(Bd-Ae)\left(cd^2+ae^2\right)^3 (d+ex)^6}{6e^8} \\
&\quad + \frac{\left(cd^2+ae^2\right)^2 \left(7Bcd^2-6Acde+aBe^2\right)(d+ex)^7}{7e^8} \\
&\quad - \frac{3c\left(cd^2+ae^2\right)\left(7Bcd^3-5Acd^2e+3aBde^2-aAe^3\right)(d+ex)^8}{8e^8} \\
&\quad - \frac{c\left(4Acde\left(5cd^2+3ae^2\right)-B\left(35c^2d^4+30acd^2e^2+3a^2e^4\right)\right)(d+ex)^9}{9e^8} \\
&\quad - \frac{c^2\left(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3\right)(d+ex)^{10}}{10e^8} \\
&\quad + \frac{3c^2\left(7Bcd^2-2Acde+aBe^2\right)(d+ex)^{11}}{11e^8} \\
&\quad - \frac{c^3(7Bd-Ae)(d+ex)^{12}}{12e^8} + \frac{Bc^3(d+ex)^{13}}{13e^8}
\end{aligned}
$$

output
```
-1/6*(-A*e+B*d)*(a*e^2+c*d^2)^3*(e*x+d)^6/e^8+1/7*(a*e^2+c*d^2)^2*(-6*A*c*
d*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^7/e^8-3/8*c*(a*e^2+c*d^2)*(-A*a*e^3-5*A*c*d
^2*e+3*B*a*d*e^2+7*B*c*d^3)*(e*x+d)^8/e^8-1/9*c*(4*A*c*d*e*(3*a*e^2+5*c*d^
2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))*(e*x+d)^9/e^8-1/10*c^2*(-3*A*a
*e^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)*(e*x+d)^10/e^8+3/11*c^2*(-2*A*c
*d*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^11/e^8-1/12*c^3*(-A*e+7*B*d)*(e*x+d)^12/e^
8+1/13*B*c^3*(e*x+d)^13/e^8
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.62

$$\int (A+Bx)(d+ex)^5\left(a+cx^2\right)^3\,dx = a^3Ad^5x+\frac{1}{2}a^3d^4(Bd+5Ae)x^2 + \frac{1}{3}a^2d^3\left(3Acd^2+5aBde+10aAe^2\right)x^3 + \frac{1}{4}a^2d^2\left(3Bcd^3+15Acd^2e+10aBde^2 + 10aAe^3\right)x^4+\frac{1}{5}ad\left(5aBde\left(3cd^2+2ae^2\right) + A\left(3c^2d^4+30acd^2e^2+5a^2e^4\right)\right)x^5 + \frac{1}{6}a\left(Ae\left(15c^2d^4+30acd^2e^2+a^2e^4\right) + B\left(3c^2d^5+30acd^3e^2+5a^2de^4\right)\right)x^6 + \frac{1}{7}\left(aBe\left(15c^2d^4+30acd^2e^2+a^2e^4\right) + Acd\left(c^2d^4+30acd^2e^2+15a^2e^4\right)\right)x^7 + \frac{1}{8}c\left(Ae\left(5c^2d^4+30acd^2e^2+3a^2e^4\right) + B\left(c^2d^5+30acd^3e^2+15a^2de^4\right)\right)x^8 + \frac{1}{9}ce\left(5Acde\left(2cd^2+3ae^2\right) + B\left(5c^2d^4+30acd^2e^2+3a^2e^4\right)\right)x^9 + \frac{1}{10}c^2e^2\left(10Bcd^3+10Acd^2e+15aBde^2 + 3aAe^3\right)x^{10} + \frac{1}{11}c^2e^3\left(10Bcd^2+5Acde+3aBe^2\right)x^{11} + \frac{1}{12}c^3e^4(5Bd+Ae)x^{12}+\frac{1}{13}Bc^3e^5x^{13}$$

input
```
Integrate[(A + B*x)*(d + e*x)^5*(a + c*x^2)^3,x]
```

output
```
a^3*A*d^5*x + (a^3*d^4*(B*d + 5*A*e)*x^2)/2 + (a^2*d^3*(3*A*c*d^2 + 5*a*B*
d*e + 10*a*A*e^2)*x^3)/3 + (a^2*d^2*(3*B*c*d^3 + 15*A*c*d^2*e + 10*a*B*d*e
^2 + 10*a*A*e^3)*x^4)/4 + (a*d*(5*a*B*d*e*(3*c*d^2 + 2*a*e^2) + A*(3*c^2*d
^4 + 30*a*c*d^2*e^2 + 5*a^2*e^4))*x^5)/5 + (a*(A*e*(15*c^2*d^4 + 30*a*c*d^
2*e^2 + a^2*e^4) + B*(3*c^2*d^5 + 30*a*c*d^3*e^2 + 5*a^2*d*e^4))*x^6)/6 +
((a*B*e*(15*c^2*d^4 + 30*a*c*d^2*e^2 + a^2*e^4) + A*c*d*(c^2*d^4 + 30*a*c*
d^2*e^2 + 15*a^2*e^4))*x^7)/7 + (c*(A*e*(5*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^
2*e^4) + B*(c^2*d^5 + 30*a*c*d^3*e^2 + 15*a^2*d*e^4))*x^8)/8 + (c*e*(5*A*c
*d*e*(2*c*d^2 + 3*a*e^2) + B*(5*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4))*x^9
)/9 + (c^2*e^2*(10*B*c*d^3 + 10*A*c*d^2*e + 15*a*B*d*e^2 + 3*a*A*e^3)*x^10
)/10 + (c^2*e^3*(10*B*c*d^2 + 5*A*c*d*e + 3*a*B*e^2)*x^11)/11 + (c^3*e^4*(
5*B*d + A*e)*x^12)/12 + (B*c^3*e^5*x^13)/13
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3 (A+Bx)(d+ex)^5 \, dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{c(d+ex)^8 \left(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4\right)}{e^7}-\frac{3c^2(d+ex)^{10} \left(-aBe^2+2Acc\right.}{e^7}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c(d+ex)^9\left(4Acde\left(3ae^2+5cd^2\right)-B\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)\right)}{9e^8}+\frac{3c^2(d+ex)^{11}\left(aBe^2-2Acde+7Bcd^2\right)}{11e^8}-\frac{c^2(d+ex)^{10}\left(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3\right)}{10e^8}+\frac{(d+ex)^7\left(ae^2+cd^2\right)^2\left(aBe^2-6Acde+7Bcd^2\right)}{7e^8}-\frac{(d+ex)^6\left(ae^2+cd^2\right)^3(Bd-Ae)}{6e^8}-\frac{3c(d+ex)^8\left(ae^2+cd^2\right)\left(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3\right)}{8e^8}-\frac{c^3(d+ex)^{12}(7Bd-Ae)}{12e^8}+\frac{Bc^3(d+ex)^{13}}{13e^8}$$

input
```
Int[(A + B*x)*(d + e*x)^5*(a + c*x^2)^3,x]
```

output
```
-1/6*((B*d - A*e)*(c*d^2 + a*e^2)^3*(d + e*x)^6)/e^8 + ((c*d^2 + a*e^2)^2*
(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2)*(d + e*x)^7)/(7*e^8) - (3*c*(c*d^2 + a*e
^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*(d + e*x)^8)/(8*e^8)
 - (c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*
a^2*e^4))*(d + e*x)^9)/(9*e^8) - (c^2*(35*B*c*d^3 - 15*A*c*d^2*e + 15*a*B*
d*e^2 - 3*a*A*e^3)*(d + e*x)^10)/(10*e^8) + (3*c^2*(7*B*c*d^2 - 2*A*c*d*e
+ a*B*e^2)*(d + e*x)^11)/(11*e^8) - (c^3*(7*B*d - A*e)*(d + e*x)^12)/(12*e
^8) + (B*c^3*(d + e*x)^13)/(13*e^8)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.67

method	result
default	$\frac{B e^5 c^3 x^{13}}{13} + \frac{(A e^5+5B d e^4) c^3 x^{12}}{12} + \frac{((5A d e^4+10B d^2 e^3) c^3+3B e^5 a c^2) x^{11}}{11} + \frac{((10A d^2 e^3+10B d^3 e^2) c^3+3(A e^5+5B}{10}$
norman	$\frac{B e^5 c^3 x^{13}}{13} + \left(\frac{1}{12} A c^3 e^5 + \frac{5}{12} B c^3 d e^4\right) x^{12} + \left(\frac{5}{11} A c^3 d e^4 + \frac{3}{11} B e^5 a c^2 + \frac{10}{11} B c^3 d^2 e^3\right) x^{11} + \left(\frac{3}{10}\right.$
gosper	$\frac{5}{3} x^9 A a c^2 d e^4 + x^5 A a^3 d e^4 + \frac{3}{5} x^5 A d^5 a c^2 + 2 x^5 B a^3 d^2 e^3 + \frac{5}{2} x^4 A a^3 d^2 e^3 + \frac{15}{7} x^7 B a c^2 d^4 e +$
risch	$\frac{5}{3} x^9 A a c^2 d e^4 + x^5 A a^3 d e^4 + \frac{3}{5} x^5 A d^5 a c^2 + 2 x^5 B a^3 d^2 e^3 + \frac{5}{2} x^4 A a^3 d^2 e^3 + \frac{15}{7} x^7 B a c^2 d^4 e +$
parallelrisch	$\frac{5}{3} x^9 A a c^2 d e^4 + x^5 A a^3 d e^4 + \frac{3}{5} x^5 A d^5 a c^2 + 2 x^5 B a^3 d^2 e^3 + \frac{5}{2} x^4 A a^3 d^2 e^3 + \frac{15}{7} x^7 B a c^2 d^4 e +$
orering	$\frac{x(27720B e^5 c^3 x^{12}+30030A c^3 e^5 x^{11}+150150B c^3 d e^4 x^{11}+163800A c^3 d e^4 x^{10}+98280B a c^2 e^5 x^{10}+327600B c^3 d^2 e^3 x^{10}+108}{}$

input
```
int((B*x+A)*(e*x+d)^5*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*B*e^5*c^3*x^13+1/12*(A*e^5+5*B*d*e^4)*c^3*x^12+1/11*((5*A*d*e^4+10*B*
d^2*e^3)*c^3+3*B*e^5*a*c^2)*x^11+1/10*((10*A*d^2*e^3+10*B*d^3*e^2)*c^3+3*(
A*e^5+5*B*d*e^4)*a*c^2)*x^10+1/9*((10*A*d^3*e^2+5*B*d^4*e)*c^3+3*(5*A*d*e^
4+10*B*d^2*e^3)*a*c^2+3*B*e^5*a^2*c)*x^9+1/8*((5*A*d^4*e+B*d^5)*c^3+3*(10*
A*d^2*e^3+10*B*d^3*e^2)*a*c^2+3*(A*e^5+5*B*d*e^4)*a^2*c)*x^8+1/7*(A*d^5*c^
3+3*(10*A*d^3*e^2+5*B*d^4*e)*a*c^2+3*(5*A*d*e^4+10*B*d^2*e^3)*a^2*c+B*e^5*
a^3)*x^7+1/6*(3*(5*A*d^4*e+B*d^5)*a*c^2+3*(10*A*d^2*e^3+10*B*d^3*e^2)*a^2*
c+(A*e^5+5*B*d*e^4)*a^3)*x^6+1/5*(3*A*d^5*a*c^2+3*(10*A*d^3*e^2+5*B*d^4*e)
*a^2*c+(5*A*d*e^4+10*B*d^2*e^3)*a^3)*x^5+1/4*(3*(5*A*d^4*e+B*d^5)*a^2*c+(1
0*A*d^2*e^3+10*B*d^3*e^2)*a^3)*x^4+1/3*(3*A*d^5*a^2*c+(10*A*d^3*e^2+5*B*d^
4*e)*a^3)*x^3+1/2*(5*A*d^4*e+B*d^5)*a^3*x^2+A*d^5*a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.75

$$\int (A+Bx)(d+ex)^5 \left(a+cx^2\right)^3 \, dx$$
$$= \frac{1}{13} Bc^3e^5x^{13} + \frac{1}{12} \left(5 Bc^3de^4 + Ac^3e^5\right)x^{12} + \frac{1}{11} \left(10 Bc^3d^2e^3 + 5 Ac^3de^4 + 3 Bac^2e^5\right)x^{11} + \frac{1}{10} \left(10 Bc^3d^3e^2 + 10 Ac^3d^2e^3 + 15 Bac^2de^4 + 3 Aac^2e^5\right)x^{10} + Aa^3d^5x + \frac{1}{9} \left(5 Bc^3d^4e + 10 Ac^3d^3e^2 + 30 Bac^2d^2e^3 + 15 Aac^2de^4 + 3 Ba^2ce^5\right)x^9 + \frac{1}{8} \left(Bc^3d^5 + 5 Ac^3d^4e + 30 Bac^2d^3e^2 + 30 Aac^2d^2e^3 + 15 Ba^2cde^4 + 3 Aa^2ce^5\right)x^8 + \frac{1}{7} \left(Ac^3d^5 + 15 Bac^2d^4e + 30 Aac^2d^3e^2 + 30 Ba^2cd^2e^3 + 15 Aa^2cde^4 + Ba^3e^5\right)x^7 + \frac{1}{6} \left(3 Bac^2d^5 + 15 Aac^2d^4e + 30 Ba^2cd^3e^2 + 30 Aa^2cd^2e^3 + 5 Ba^3de^4 + Aa^3e^5\right)x^6 + \frac{1}{5} \left(3 Aac^2d^5 + 15 Ba^2cd^4e + 30 Aa^2cd^3e^2 + 10 Ba^3d^2e^3 + 5 Aa^3de^4\right)x^5 + \frac{1}{4} \left(3 Ba^2cd^5 + 15 Aa^2cd^4e + 10 Ba^3d^3e^2 + 10 Aa^3d^2e^3\right)x^4 + \frac{1}{3} \left(3 Aa^2cd^5 + 5 Ba^3d^4e + 10 Aa^3d^3e^2\right)x^3 + \frac{1}{2} \left(Ba^3d^5 + 5 Aa^3d^4e\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)^5*(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/13*B*c^3*e^5*x^13 + 1/12*(5*B*c^3*d*e^4 + A*c^3*e^5)*x^12 + 1/11*(10*B*c
^3*d^2*e^3 + 5*A*c^3*d*e^4 + 3*B*a*c^2*e^5)*x^11 + 1/10*(10*B*c^3*d^3*e^2
+ 10*A*c^3*d^2*e^3 + 15*B*a*c^2*d*e^4 + 3*A*a*c^2*e^5)*x^10 + A*a^3*d^5*x
+ 1/9*(5*B*c^3*d^4*e + 10*A*c^3*d^3*e^2 + 30*B*a*c^2*d^2*e^3 + 15*A*a*c^2*
d*e^4 + 3*B*a^2*c*e^5)*x^9 + 1/8*(B*c^3*d^5 + 5*A*c^3*d^4*e + 30*B*a*c^2*d
^3*e^2 + 30*A*a*c^2*d^2*e^3 + 15*B*a^2*c*d*e^4 + 3*A*a^2*c*e^5)*x^8 + 1/7*
(A*c^3*d^5 + 15*B*a*c^2*d^4*e + 30*A*a*c^2*d^3*e^2 + 30*B*a^2*c*d^2*e^3 +
15*A*a^2*c*d*e^4 + B*a^3*e^5)*x^7 + 1/6*(3*B*a*c^2*d^5 + 15*A*a*c^2*d^4*e
+ 30*B*a^2*c*d^3*e^2 + 30*A*a^2*c*d^2*e^3 + 5*B*a^3*d*e^4 + A*a^3*e^5)*x^6
 + 1/5*(3*A*a*c^2*d^5 + 15*B*a^2*c*d^4*e + 30*A*a^2*c*d^3*e^2 + 10*B*a^3*d
^2*e^3 + 5*A*a^3*d*e^4)*x^5 + 1/4*(3*B*a^2*c*d^5 + 15*A*a^2*c*d^4*e + 10*B
*a^3*d^3*e^2 + 10*A*a^3*d^2*e^3)*x^4 + 1/3*(3*A*a^2*c*d^5 + 5*B*a^3*d^4*e
+ 10*A*a^3*d^3*e^2)*x^3 + 1/2*(B*a^3*d^5 + 5*A*a^3*d^4*e)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(340) = 680$.

Time = 0.06 (sec) , antiderivative size = 694, normalized size of antiderivative = 2.08

$$\int (A+Bx)(d+ex)^5\left(a+cx^2\right)^3\,dx = Aa^3d^5x + \frac{Bc^3e^5x^{13}}{13} + x^{12}\left(\frac{Ac^3e^5}{12} + \frac{5Bc^3de^4}{12}\right) + x^{11}\cdot\left(\frac{5Ac^3de^4}{11} + \frac{3Bac^2e^5}{11} + \frac{10Bc^3d^2e^3}{11}\right) + x^{10}\cdot\left(\frac{3Aac^2e^5}{10} + Ac^3d^2e^3 + \frac{3Bac^2de^4}{2} + Bc^3d^3e^2\right) + x^9\cdot\left(\frac{5Aac^2de^4}{3} + \frac{10Ac^3d^3e^2}{9} + \frac{Ba^2ce^5}{3} + \frac{10Bac^2d^2e^3}{3} + \frac{5Bc^3d^4e}{9}\right) + x^8\cdot\left(\frac{3Aa^2ce^5}{8} + \frac{15Aac^2d^2e^3}{4} + \frac{5Ac^3d^4e}{8} + \frac{15Ba^2cde^4}{8} + \frac{15Bac^2d^3e^2}{4} + \frac{Bc^3d^5}{8}\right) + x^7\cdot\left(\frac{15Aa^2cde^4}{7} + \frac{30Aac^2d^3e^2}{7} + \frac{Ac^3d^5}{7} + \frac{Ba^3e^5}{7} + \frac{30Ba^2cd^2e^3}{7} + \frac{15Bac^2d^4e}{7}\right) + x^6\left(\frac{Aa^3e^5}{6} + 5Aa^2cd^2e^3 + \frac{5Aac^2d^4e}{2} + \frac{5Ba^3de^4}{6} + 5Ba^2cd^3e^2 + \frac{Bac^2d^5}{2}\right) + x^5\left(Aa^3de^4 + 6Aa^2cd^3e^2 + \frac{3Aac^2d^5}{5} + 2Ba^3d^2e^3 + 3Ba^2cd^4e\right) + x^4\cdot\left(\frac{5Aa^3d^2e^3}{2} + \frac{15Aa^2cd^4e}{4} + \frac{5Ba^3d^3e^2}{2} + \frac{3Ba^2cd^5}{4}\right) + x^3\cdot\left(\frac{10Aa^3d^3e^2}{3} + Aa^2cd^5 + \frac{5Ba^3d^4e}{3}\right) + x^2\cdot\left(\frac{5Aa^3d^4e}{2} + \frac{Ba^3d^5}{2}\right)$$

input
```
integrate((B*x+A)*(e*x+d)**5*(c*x**2+a)**3,x)
```

output
```
A*a**3*d**5*x + B*c**3*e**5*x**13/13 + x**12*(A*c**3*e**5/12 + 5*B*c**3*d*
e**4/12) + x**11*(5*A*c**3*d*e**4/11 + 3*B*a*c**2*e**5/11 + 10*B*c**3*d**2
*e**3/11) + x**10*(3*A*a*c**2*e**5/10 + A*c**3*d**2*e**3 + 3*B*a*c**2*d*e*
*4/2 + B*c**3*d**3*e**2) + x**9*(5*A*a*c**2*d*e**4/3 + 10*A*c**3*d**3*e**2
/9 + B*a**2*c*e**5/3 + 10*B*a*c**2*d**2*e**3/3 + 5*B*c**3*d**4*e/9) + x**8
*(3*A*a**2*c*e**5/8 + 15*A*a*c**2*d**2*e**3/4 + 5*A*c**3*d**4*e/8 + 15*B*a
**2*c*d*e**4/8 + 15*B*a*c**2*d**3*e**2/4 + B*c**3*d**5/8) + x**7*(15*A*a**
2*c*d*e**4/7 + 30*A*a*c**2*d**3*e**2/7 + A*c**3*d**5/7 + B*a**3*e**5/7 + 3
0*B*a**2*c*d**2*e**3/7 + 15*B*a*c**2*d**4*e/7) + x**6*(A*a**3*e**5/6 + 5*A
*a**2*c*d**2*e**3 + 5*A*a*c**2*d**4*e/2 + 5*B*a**3*d*e**4/6 + 5*B*a**2*c*d
**3*e**2 + B*a*c**2*d**5/2) + x**5*(A*a**3*d*e**4 + 6*A*a**2*c*d**3*e**2 +
 3*A*a*c**2*d**5/5 + 2*B*a**3*d**2*e**3 + 3*B*a**2*c*d**4*e) + x**4*(5*A*a
**3*d**2*e**3/2 + 15*A*a**2*c*d**4*e/4 + 5*B*a**3*d**3*e**2/2 + 3*B*a**2*c
*d**5/4) + x**3*(10*A*a**3*d**3*e**2/3 + A*a**2*c*d**5 + 5*B*a**3*d**4*e/3
) + x**2*(5*A*a**3*d**4*e/2 + B*a**3*d**5/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.75

$$\int (A+Bx)(d+ex)^5 \left(a+cx^2\right)^3 \, dx = \frac{1}{13} Bc^3e^5x^{13} + \frac{1}{12} \left(5 Bc^3de^4 + Ac^3e^5\right)x^{12} + \frac{1}{11} \left(10 Bc^3d^2e^3 + 5 Ac^3de^4 + 3 Bac^2e^5\right)x^{11} + \frac{1}{10} \left(10 Bc^3d^3e^2 + 10 Ac^3d^2e^3 + 15 Bac^2de^4 + 3 Aac^2e^5\right)x^{10} + Aa^3d^5x + \frac{1}{9} \left(5 Bc^3d^4e + 10 Ac^3d^3e^2 + 30 Bac^2d^2e^3 + 15 Aac^2de^4 + 3 Ba^2ce^5\right)x^9 + \frac{1}{8} \left(Bc^3d^5 + 5 Ac^3d^4e + 30 Bac^2d^3e^2 + 30 Aac^2d^2e^3 + 15 Ba^2cde^4 + 3 Aa^2ce^5\right)x^8 + \frac{1}{7} \left(Ac^3d^5 + 15 Bac^2d^4e + 30 Aac^2d^3e^2 + 30 Ba^2cd^2e^3 + 15 Aa^2cde^4 + Ba^3e^5\right)x^7 + \frac{1}{6} \left(3 Bac^2d^5 + 15 Aac^2d^4e + 30 Ba^2cd^3e^2 + 30 Aa^2cd^2e^3 + 5 Ba^3de^4 + Aa^3e^5\right)x^6 + \frac{1}{5} \left(3 Aac^2d^5 + 15 Ba^2cd^4e + 30 Aa^2cd^3e^2 + 10 Ba^3d^2e^3 + 5 Aa^3de^4\right)x^5 + \frac{1}{4} \left(3 Ba^2cd^5 + 15 Aa^2cd^4e + 10 Ba^3d^3e^2 + 10 Aa^3d^2e^3\right)x^4 + \frac{1}{3} \left(3 Aa^2cd^5 + 5 Ba^3d^4e + 10 Aa^3d^3e^2\right)x^3 + \frac{1}{2} \left(Ba^3d^5 + 5 Aa^3d^4e\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)^5*(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/13*B*c^3*e^5*x^13 + 1/12*(5*B*c^3*d*e^4 + A*c^3*e^5)*x^12 + 1/11*(10*B*c
^3*d^2*e^3 + 5*A*c^3*d*e^4 + 3*B*a*c^2*e^5)*x^11 + 1/10*(10*B*c^3*d^3*e^2
+ 10*A*c^3*d^2*e^3 + 15*B*a*c^2*d*e^4 + 3*A*a*c^2*e^5)*x^10 + A*a^3*d^5*x
+ 1/9*(5*B*c^3*d^4*e + 10*A*c^3*d^3*e^2 + 30*B*a*c^2*d^2*e^3 + 15*A*a*c^2*
d*e^4 + 3*B*a^2*c*e^5)*x^9 + 1/8*(B*c^3*d^5 + 5*A*c^3*d^4*e + 30*B*a*c^2*d
^3*e^2 + 30*A*a*c^2*d^2*e^3 + 15*B*a^2*c*d*e^4 + 3*A*a^2*c*e^5)*x^8 + 1/7*
(A*c^3*d^5 + 15*B*a*c^2*d^4*e + 30*A*a*c^2*d^3*e^2 + 30*B*a^2*c*d^2*e^3 +
15*A*a^2*c*d*e^4 + B*a^3*e^5)*x^7 + 1/6*(3*B*a*c^2*d^5 + 15*A*a*c^2*d^4*e
+ 30*B*a^2*c*d^3*e^2 + 30*A*a^2*c*d^2*e^3 + 5*B*a^3*d*e^4 + A*a^3*e^5)*x^6
 + 1/5*(3*A*a*c^2*d^5 + 15*B*a^2*c*d^4*e + 30*A*a^2*c*d^3*e^2 + 10*B*a^3*d
^2*e^3 + 5*A*a^3*d*e^4)*x^5 + 1/4*(3*B*a^2*c*d^5 + 15*A*a^2*c*d^4*e + 10*B
*a^3*d^3*e^2 + 10*A*a^3*d^2*e^3)*x^4 + 1/3*(3*A*a^2*c*d^5 + 5*B*a^3*d^4*e
+ 10*A*a^3*d^3*e^2)*x^3 + 1/2*(B*a^3*d^5 + 5*A*a^3*d^4*e)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. 2(318) = 636.

Time = 0.12 (sec) , antiderivative size = 658, normalized size of antiderivative = 1.97

$$\int (A+Bx)(d+ex)^5\left(a+cx^2\right)^3\,dx = \frac{1}{13}\,Bc^3e^5x^{13} + \frac{5}{12}\,Bc^3de^4x^{12} + \frac{1}{12}\,Ac^3e^5x^{12} + \frac{10}{11}\,Bc^3d^2e^3x^{11} + \frac{5}{11}\,Ac^3de^4x^{11} + \frac{3}{11}\,Bac^2e^5x^{11} + Bc^3d^3e^2x^{10} + Ac^3d^2e^3x^{10} + \frac{3}{2}\,Bac^2de^4x^{10} + \frac{3}{10}\,Aac^2e^5x^{10} + \frac{5}{9}\,Bc^3d^4ex^9 + \frac{10}{9}\,Ac^3d^3e^2x^9 + \frac{10}{3}\,Bac^2d^2e^3x^9 + \frac{5}{3}\,Aac^2de^4x^9 + \frac{1}{3}\,Ba^2ce^5x^9 + \frac{1}{8}\,Bc^3d^5x^8 + \frac{5}{8}\,Ac^3d^4ex^8 + \frac{15}{4}\,Bac^2d^3e^2x^8 + \frac{15}{4}\,Aac^2d^2e^3x^8 + \frac{15}{8}\,Ba^2cde^4x^8 + \frac{3}{8}\,Aa^2ce^5x^8 + \frac{1}{7}\,Ac^3d^5x^7 + \frac{15}{7}\,Bac^2d^4ex^7 + \frac{30}{7}\,Aac^2d^3e^2x^7 + \frac{30}{7}\,Ba^2cd^2e^3x^7 + \frac{15}{7}\,Aa^2cde^4x^7 + \frac{1}{7}\,Ba^3e^5x^7 + \frac{1}{2}\,Bac^2d^5x^6 + \frac{5}{2}\,Aac^2d^4ex^6 + 5\,Ba^2cd^3e^2x^6 + 5\,Aa^2cd^2e^3x^6 + \frac{5}{6}\,Ba^3de^4x^6 + \frac{1}{6}\,Aa^3e^5x^6 + \frac{3}{5}\,Aac^2d^5x^5 + 3\,Ba^2cd^4ex^5 + 6\,Aa^2cd^3e^2x^5 + 2\,Ba^3d^2e^3x^5 + Aa^3de^4x^5 + \frac{3}{4}\,Ba^2cd^5x^4 + \frac{15}{4}\,Aa^2cd^4ex^4 + \frac{5}{2}\,Ba^3d^3e^2x^4 + \frac{5}{2}\,Aa^3d^2e^3x^4 + Aa^2cd^5x^3 + \frac{5}{3}\,Ba^3d^4ex^3 + \frac{10}{3}\,Aa^3d^3e^2x^3 + \frac{1}{2}\,Ba^3d^5x^2 + \frac{5}{2}\,Aa^3d^4ex^2 + Aa^3d^5x$$

input
```
integrate((B*x+A)*(e*x+d)^5*(c*x^2+a)^3,x, algorithm="giac")
```

output

```
1/13*B*c^3*e^5*x^13 + 5/12*B*c^3*d*e^4*x^12 + 1/12*A*c^3*e^5*x^12 + 10/11*
B*c^3*d^2*e^3*x^11 + 5/11*A*c^3*d*e^4*x^11 + 3/11*B*a*c^2*e^5*x^11 + B*c^3
*d^3*e^2*x^10 + A*c^3*d^2*e^3*x^10 + 3/2*B*a*c^2*d*e^4*x^10 + 3/10*A*a*c^2
*e^5*x^10 + 5/9*B*c^3*d^4*e*x^9 + 10/9*A*c^3*d^3*e^2*x^9 + 10/3*B*a*c^2*d^
2*e^3*x^9 + 5/3*A*a*c^2*d*e^4*x^9 + 1/3*B*a^2*c*e^5*x^9 + 1/8*B*c^3*d^5*x^
8 + 5/8*A*c^3*d^4*e*x^8 + 15/4*B*a*c^2*d^3*e^2*x^8 + 15/4*A*a*c^2*d^2*e^3*
x^8 + 15/8*B*a^2*c*d*e^4*x^8 + 3/8*A*a^2*c*e^5*x^8 + 1/7*A*c^3*d^5*x^7 + 1
5/7*B*a*c^2*d^4*e*x^7 + 30/7*A*a*c^2*d^3*e^2*x^7 + 30/7*B*a^2*c*d^2*e^3*x^
7 + 15/7*A*a^2*c*d*e^4*x^7 + 1/7*B*a^3*e^5*x^7 + 1/2*B*a*c^2*d^5*x^6 + 5/2
*A*a*c^2*d^4*e*x^6 + 5*B*a^2*c*d^3*e^2*x^6 + 5*A*a^2*c*d^2*e^3*x^6 + 5/6*B
*a^3*d*e^4*x^6 + 1/6*A*a^3*e^5*x^6 + 3/5*A*a*c^2*d^5*x^5 + 3*B*a^2*c*d^4*e
*x^5 + 6*A*a^2*c*d^3*e^2*x^5 + 2*B*a^3*d^2*e^3*x^5 + A*a^3*d*e^4*x^5 + 3/4
*B*a^2*c*d^5*x^4 + 15/4*A*a^2*c*d^4*e*x^4 + 5/2*B*a^3*d^3*e^2*x^4 + 5/2*A*
a^3*d^2*e^3*x^4 + A*a^2*c*d^5*x^3 + 5/3*B*a^3*d^4*e*x^3 + 10/3*A*a^3*d^3*e
^2*x^3 + 1/2*B*a^3*d^5*x^2 + 5/2*A*a^3*d^4*e*x^2 + A*a^3*d^5*x
```

Mupad [B] (verification not implemented)

Time = 6.84 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.62

$$\int (A+Bx)(d+ex)^5 \left(a+cx^2\right)^3 \, dx = x^6 \left(\frac{5\,B\,a^3\,d\,e^4}{6}+\frac{A\,a^3\,e^5}{6}+5\,B\,a^2\,c\,d^3\,e^2+5\,A\,a^2\,c\,d^2\,e^3+\frac{B\,a\,c^2\,d^5}{2}+\frac{5\,A\,a\,c^2\,d^4\,e}{2}\right) + x^7 \left(\frac{B\,a^3\,e^5}{7}+\frac{30\,B\,a^2\,c\,d^2\,e^3}{7}+\frac{15\,A\,a^2\,c\,d\,e^4}{7}+\frac{15\,B\,a\,c^2\,d^4\,e}{7}+\frac{30\,A\,a\,c^2\,d^3\,e^2}{7}+\frac{A\,c^3\,d^5}{7}\right) + x^8 \left(\frac{15\,B\,a^2\,c\,d\,e^4}{8}+\frac{3\,A\,a^2\,c\,e^5}{8}+\frac{15\,B\,a\,c^2\,d^3\,e^2}{4}+\frac{15\,A\,a\,c^2\,d^2\,e^3}{4}+\frac{B\,c^3\,d^5}{8}+\frac{5\,A\,c^3\,d^4\,e}{8}\right) + x^5 \left(2\,B\,a^3\,d^2\,e^3+A\,a^3\,d\,e^4+3\,B\,a^2\,c\,d^4\,e+6\,A\,a^2\,c\,d^3\,e^2+\frac{3\,A\,a\,c^2\,d^5}{5}\right) + x^9 \left(\frac{B\,a^2\,c\,e^5}{3}+\frac{10\,B\,a\,c^2\,d^2\,e^3}{3}+\frac{5\,A\,a\,c^2\,d\,e^4}{3}+\frac{5\,B\,c^3\,d^4\,e}{9}+\frac{10\,A\,c^3\,d^3\,e^2}{9}\right) + \frac{c^2\,e^2\,x^{10}\,\left(10\,B\,c\,d^3+10\,A\,c\,d^2\,e+15\,B\,a\,d\,e^2+3\,A\,a\,e^3\right)}{10} + \frac{a^3\,d^4\,x^2\,\left(5\,A\,e+B\,d\right)}{2} + \frac{c^3\,e^4\,x^{12}\,\left(A\,e+5\,B\,d\right)}{12} + \frac{a^2\,d^3\,x^3\,\left(3\,A\,c\,d^2+5\,B\,a\,d\,e+10\,A\,a\,e^2\right)}{3} + \frac{c^2\,e^3\,x^{11}\,\left(10\,B\,c\,d^2+5\,A\,c\,d\,e+3\,B\,a\,e^2\right)}{11} + A\,a^3\,d^5\,x + \frac{a^2\,d^2\,x^4\,\left(3\,B\,c\,d^3+15\,A\,c\,d^2\,e+10\,B\,a\,d\,e^2+10\,A\,a\,e^3\right)}{4} + \frac{B\,c^3\,e^5\,x^{13}}{13}$$

input
```
int((a + c*x^2)^3*(A + B*x)*(d + e*x)^5,x)
```

output
```
x^6*((A*a^3*e^5)/6 + (B*a*c^2*d^5)/2 + (5*B*a^3*d*e^4)/6 + 5*A*a^2*c*d^2*e
^3 + 5*B*a^2*c*d^3*e^2 + (5*A*a*c^2*d^4*e)/2) + x^7*((A*c^3*d^5)/7 + (B*a^
3*e^5)/7 + (30*A*a*c^2*d^3*e^2)/7 + (30*B*a^2*c*d^2*e^3)/7 + (15*A*a^2*c*d
*e^4)/7 + (15*B*a*c^2*d^4*e)/7) + x^8*((B*c^3*d^5)/8 + (3*A*a^2*c*e^5)/8 +
 (5*A*c^3*d^4*e)/8 + (15*A*a*c^2*d^2*e^3)/4 + (15*B*a*c^2*d^3*e^2)/4 + (15
*B*a^2*c*d*e^4)/8) + x^5*((3*A*a*c^2*d^5)/5 + A*a^3*d*e^4 + 2*B*a^3*d^2*e^
3 + 6*A*a^2*c*d^3*e^2 + 3*B*a^2*c*d^4*e) + x^9*((B*a^2*c*e^5)/3 + (5*B*c^3
*d^4*e)/9 + (10*A*c^3*d^3*e^2)/9 + (10*B*a*c^2*d^2*e^3)/3 + (5*A*a*c^2*d*e
^4)/3) + (c^2*e^2*x^10*(3*A*a*e^3 + 10*B*c*d^3 + 15*B*a*d*e^2 + 10*A*c*d^2
*e))/10 + (a^3*d^4*x^2*(5*A*e + B*d))/2 + (c^3*e^4*x^12*(A*e + 5*B*d))/12
+ (a^2*d^3*x^3*(10*A*a*e^2 + 3*A*c*d^2 + 5*B*a*d*e))/3 + (c^2*e^3*x^11*(3*
B*a*e^2 + 10*B*c*d^2 + 5*A*c*d*e))/11 + A*a^3*d^5*x + (a^2*d^2*x^4*(10*A*a
*e^3 + 3*B*c*d^3 + 10*B*a*d*e^2 + 15*A*c*d^2*e))/4 + (B*c^3*e^5*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.96

$$\int (A+Bx)(d+ex)^5 \left(a+cx^2\right)^3 dx$$
$$= \frac{x(27720b\,c^3e^5x^{12} + 30030a\,c^3e^5x^{11} + 150150b\,c^3d\,e^4x^{11} + 98280ab\,c^2e^5x^{10} + 163800a\,c^3d\,e^4x^{10} + 327600}{}$$

input
```
int((B*x+A)*(e*x+d)^5*(c*x^2+a)^3,x)
```

output
```
(x*(360360*a**4*d**5 + 900900*a**4*d**4*e*x + 1201200*a**4*d**3*e**2*x**2
+ 900900*a**4*d**2*e**3*x**3 + 360360*a**4*d*e**4*x**4 + 60060*a**4*e**5*x
**5 + 180180*a**3*b*d**5*x + 600600*a**3*b*d**4*e*x**2 + 900900*a**3*b*d**
3*e**2*x**3 + 720720*a**3*b*d**2*e**3*x**4 + 300300*a**3*b*d*e**4*x**5 + 5
1480*a**3*b*e**5*x**6 + 360360*a**3*c*d**5*x**2 + 1351350*a**3*c*d**4*e*x*
*3 + 2162160*a**3*c*d**3*e**2*x**4 + 1801800*a**3*c*d**2*e**3*x**5 + 77220
0*a**3*c*d*e**4*x**6 + 135135*a**3*c*e**5*x**7 + 270270*a**2*b*c*d**5*x**3
 + 1081080*a**2*b*c*d**4*e*x**4 + 1801800*a**2*b*c*d**3*e**2*x**5 + 154440
0*a**2*b*c*d**2*e**3*x**6 + 675675*a**2*b*c*d*e**4*x**7 + 120120*a**2*b*c*
e**5*x**8 + 216216*a**2*c**2*d**5*x**4 + 900900*a**2*c**2*d**4*e*x**5 + 15
44400*a**2*c**2*d**3*e**2*x**6 + 1351350*a**2*c**2*d**2*e**3*x**7 + 600600
*a**2*c**2*d*e**4*x**8 + 108108*a**2*c**2*e**5*x**9 + 180180*a*b*c**2*d**5
*x**5 + 772200*a*b*c**2*d**4*e*x**6 + 1351350*a*b*c**2*d**3*e**2*x**7 + 12
01200*a*b*c**2*d**2*e**3*x**8 + 540540*a*b*c**2*d*e**4*x**9 + 98280*a*b*c*
*2*e**5*x**10 + 51480*a*c**3*d**5*x**6 + 225225*a*c**3*d**4*e*x**7 + 40040
0*a*c**3*d**3*e**2*x**8 + 360360*a*c**3*d**2*e**3*x**9 + 163800*a*c**3*d*e
**4*x**10 + 30030*a*c**3*e**5*x**11 + 45045*b*c**3*d**5*x**7 + 200200*b*c*
*3*d**4*e*x**8 + 360360*b*c**3*d**3*e**2*x**9 + 327600*b*c**3*d**2*e**3*x*
*10 + 150150*b*c**3*d*e**4*x**11 + 27720*b*c**3*e**5*x**12))/360360
```

3.63 $\int (A+Bx)(d+ex)^4 \left(a+cx^2\right)^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 334

$$\begin{aligned}
\int (A+Bx)(d+ex)^4 \left(a+cx^2\right)^3 \, dx
&= -\frac{(Bd-Ae)\left(cd^2+ae^2\right)^3 (d+ex)^5}{5e^8} \\
&\quad+\frac{\left(cd^2+ae^2\right)^2 \left(7Bcd^2-6Acde+aBe^2\right)(d+ex)^6}{6e^8} \\
&\quad-\frac{3c\left(cd^2+ae^2\right)\left(7Bcd^3-5Acd^2e+3aBde^2-aAe^3\right)(d+ex)^7}{7e^8} \\
&\quad-\frac{c\left(4Acde\left(5cd^2+3ae^2\right)-B\left(35c^2d^4+30acd^2e^2+3a^2e^4\right)\right)(d+ex)^8}{8e^8} \\
&\quad-\frac{c^2\left(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3\right)(d+ex)^9}{9e^8} \\
&\quad+\frac{3c^2\left(7Bcd^2-2Acde+aBe^2\right)(d+ex)^{10}}{10e^8} \\
&\quad-\frac{c^3(7Bd-Ae)(d+ex)^{11}}{11e^8}+\frac{Bc^3(d+ex)^{12}}{12e^8}
\end{aligned}$$

output
```
-1/5*(-A*e+B*d)*(a*e^2+c*d^2)^3*(e*x+d)^5/e^8+1/6*(a*e^2+c*d^2)^2*(-6*A*c*
d*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^6/e^8-3/7*c*(a*e^2+c*d^2)*(-A*a*e^3-5*A*c*d
^2*e+3*B*a*d*e^2+7*B*c*d^3)*(e*x+d)^7/e^8-1/8*c*(4*A*c*d*e*(3*a*e^2+5*c*d^
2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))*(e*x+d)^8/e^8-1/9*c^2*(-3*A*a*
e^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)*(e*x+d)^9/e^8+3/10*c^2*(-2*A*c*d
*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^10/e^8-1/11*c^3*(-A*e+7*B*d)*(e*x+d)^11/e^8+
1/12*B*c^3*(e*x+d)^12/e^8
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.31

$$\begin{aligned}\int (A+Bx)(d+ex)^4\left(a+cx^2\right)^3\,dx &= a^3Ad^4x+\frac{1}{2}a^3d^3(Bd+4Ae)x^2\\ &+\frac{1}{3}a^2d^2\left(3Acd^2+4aBde+6aAe^2\right)x^3\\ &+\frac{1}{4}a^2d\left(3Bcd^3+12Acd^2e+6aBde^2+4aAe^3\right)x^4\\ &+\frac{1}{5}a\left(4aBde\left(3cd^2+ae^2\right)\right.\\ &\left.\quad+A\left(3c^2d^4+18acd^2e^2+a^2e^4\right)\right)x^5\\ &+\frac{1}{6}a\left(12Acde\left(cd^2+ae^2\right)\right.\\ &\left.\quad+B\left(3c^2d^4+18acd^2e^2+a^2e^4\right)\right)x^6\\ &+\frac{1}{7}c\left(12aBde\left(cd^2+ae^2\right)\right.\\ &\left.\quad+A\left(c^2d^4+18acd^2e^2+3a^2e^4\right)\right)x^7\\ &+\frac{1}{8}c\left(4Acde\left(cd^2+3ae^2\right)\right.\\ &\left.\quad+B\left(c^2d^4+18acd^2e^2+3a^2e^4\right)\right)x^8\\ &+\frac{1}{9}c^2e\left(4Bcd^3+6Acd^2e+12aBde^2+3aAe^3\right)x^9\\ &+\frac{1}{10}c^2e^2\left(6Bcd^2+4Acde+3aBe^2\right)x^{10}\\ &+\frac{1}{11}c^3e^3(4Bd+Ae)x^{11}+\frac{1}{12}Bc^3e^4x^{12}\end{aligned}$$

input
```
Integrate[(A + B*x)*(d + e*x)^4*(a + c*x^2)^3,x]
```

output

```
a^3*A*d^4*x + (a^3*d^3*(B*d + 4*A*e)*x^2)/2 + (a^2*d^2*(3*A*c*d^2 + 4*a*B*
d*e + 6*a*A*e^2)*x^3)/3 + (a^2*d*(3*B*c*d^3 + 12*A*c*d^2*e + 6*a*B*d*e^2 +
 4*a*A*e^3)*x^4)/4 + (a*(4*a*B*d*e*(3*c*d^2 + a*e^2) + A*(3*c^2*d^4 + 18*a
*c*d^2*e^2 + a^2*e^4))*x^5)/5 + (a*(12*A*c*d*e*(c*d^2 + a*e^2) + B*(3*c^2*
d^4 + 18*a*c*d^2*e^2 + a^2*e^4))*x^6)/6 + (c*(12*a*B*d*e*(c*d^2 + a*e^2) +
 A*(c^2*d^4 + 18*a*c*d^2*e^2 + 3*a^2*e^4))*x^7)/7 + (c*(4*A*c*d*e*(c*d^2 +
 3*a*e^2) + B*(c^2*d^4 + 18*a*c*d^2*e^2 + 3*a^2*e^4))*x^8)/8 + (c^2*e*(4*B
*c*d^3 + 6*A*c*d^2*e + 12*a*B*d*e^2 + 3*a*A*e^3)*x^9)/9 + (c^2*e^2*(6*B*c*
d^2 + 4*A*c*d*e + 3*a*B*e^2)*x^10)/10 + (c^3*e^3*(4*B*d + A*e)*x^11)/11 +
(B*c^3*e^4*x^12)/12
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3 (A+Bx)(d+ex)^4 \, dx$$

$$\Big\downarrow \ 652$$

$$\int \left(-\frac{c(d+ex)^7 \left(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4\right)}{e^7}-\frac{3c^2(d+ex)^9 \left(-aBe^2+2Acde\right.}{e^7}\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{c(d+ex)^8 \left(4Acde\left(3ae^2+5cd^2\right)-B\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)\right)}{8e^8}+\frac{3c^2(d+ex)^{10} \left(aBe^2-2Acde+7Bcd^2\right)}{10e^8}-\frac{c^2(d+ex)^9 \left(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3\right)}{9e^8}+\frac{(d+ex)^6 \left(ae^2+cd^2\right)^2 \left(aBe^2-6Acde+7Bcd^2\right)}{6e^8}-\frac{(d+ex)^5 \left(ae^2+cd^2\right)^3 (Bd-Ae)}{5e^8}-\frac{3c(d+ex)^7 \left(ae^2+cd^2\right) \left(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3\right)}{7e^8}-\frac{c^3(d+ex)^{11}(7Bd-Ae)}{11e^8}+\frac{Bc^3(d+ex)^{12}}{12e^8}$$

input
```
Int[(A + B*x)*(d + e*x)^4*(a + c*x^2)^3,x]
```

output
```
-1/5*((B*d - A*e)*(c*d^2 + a*e^2)^3*(d + e*x)^5)/e^8 + ((c*d^2 + a*e^2)^2*
(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2)*(d + e*x)^6)/(6*e^8) - (3*c*(c*d^2 + a*e
^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*(d + e*x)^7)/(7*e^8)
 - (c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*
a^2*e^4))*(d + e*x)^8)/(8*e^8) - (c^2*(35*B*c*d^3 - 15*A*c*d^2*e + 15*a*B*
d*e^2 - 3*a*A*e^3)*(d + e*x)^9)/(9*e^8) + (3*c^2*(7*B*c*d^2 - 2*A*c*d*e +
a*B*e^2)*(d + e*x)^10)/(10*e^8) - (c^3*(7*B*d - A*e)*(d + e*x)^11)/(11*e^8
) + (B*c^3*(d + e*x)^12)/(12*e^8)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.36

method	result
default	$\frac{B\,e^4c^3x^{12}}{12}+\frac{(A\,e^4+4Bd\,e^3)c^3x^{11}}{11}+\frac{((4Ad\,e^3+6B\,d^2e^2)c^3+3B\,e^4a\,c^2)x^{10}}{10}+\frac{((6A\,d^2e^2+4B\,d^3e)c^3+3(A\,e^4+4Bd\,e^3)}{9}$
norman	$\frac{B\,e^4c^3x^{12}}{12}+\left(\frac{1}{11}A\,c^3e^4+\frac{4}{11}B\,c^3d\,e^3\right)x^{11}+\left(\frac{2}{5}A\,c^3d\,e^3+\frac{3}{10}B\,e^4a\,c^2+\frac{3}{5}B\,c^3d^2e^2\right)x^{10}+\left(\frac{1}{3}Aa\right.$
gosper	$\frac{3}{5}x^5A\,d^4a\,c^2+\frac{4}{5}x^5B\,a^3d\,e^3+\frac{4}{3}x^9Ba\,c^2d\,e^3+3x^6B\,a^2c\,d^2e^2+A\,d^4a^3x+\frac{1}{11}x^{11}A\,c^3e^4+\frac{1}{8}x^8$
risch	$\frac{3}{5}x^5A\,d^4a\,c^2+\frac{4}{5}x^5B\,a^3d\,e^3+\frac{4}{3}x^9Ba\,c^2d\,e^3+3x^6B\,a^2c\,d^2e^2+A\,d^4a^3x+\frac{1}{11}x^{11}A\,c^3e^4+\frac{1}{8}x^8$
parallelrisch	$\frac{3}{5}x^5A\,d^4a\,c^2+\frac{4}{5}x^5B\,a^3d\,e^3+\frac{4}{3}x^9Ba\,c^2d\,e^3+3x^6B\,a^2c\,d^2e^2+A\,d^4a^3x+\frac{1}{11}x^{11}A\,c^3e^4+\frac{1}{8}x^8$
orering	$x(2310B\,e^4c^3x^{11}+2520A\,c^3e^4x^{10}+10080B\,c^3d\,e^3x^{10}+11088A\,c^3d\,e^3x^9+8316Ba\,c^2e^4x^9+16632B\,c^3d^2e^2x^9+9240Aa\,c^2e^4x$

input
```
int((B*x+A)*(e*x+d)^4*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*B*e^4*c^3*x^12+1/11*(A*e^4+4*B*d*e^3)*c^3*x^11+1/10*((4*A*d*e^3+6*B*d
^2*e^2)*c^3+3*B*e^4*a*c^2)*x^10+1/9*((6*A*d^2*e^2+4*B*d^3*e)*c^3+3*(A*e^4+
4*B*d*e^3)*a*c^2)*x^9+1/8*((4*A*d^3*e+B*d^4)*c^3+3*(4*A*d*e^3+6*B*d^2*e^2)
*a*c^2+3*B*e^4*a^2*c)*x^8+1/7*(A*d^4*c^3+3*(6*A*d^2*e^2+4*B*d^3*e)*a*c^2+3
*(A*e^4+4*B*d*e^3)*a^2*c)*x^7+1/6*(3*(4*A*d^3*e+B*d^4)*a*c^2+3*(4*A*d*e^3+
6*B*d^2*e^2)*a^2*c+B*e^4*a^3)*x^6+1/5*(3*A*d^4*a*c^2+3*(6*A*d^2*e^2+4*B*d^
3*e)*a^2*c+(A*e^4+4*B*d*e^3)*a^3)*x^5+1/4*(3*(4*A*d^3*e+B*d^4)*a^2*c+(4*A*
d*e^3+6*B*d^2*e^2)*a^3)*x^4+1/3*(3*A*d^4*a^2*c+(6*A*d^2*e^2+4*B*d^3*e)*a^3
)*x^3+1/2*(4*A*d^3*e+B*d^4)*a^3*x^2+A*d^4*a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.43

$$\begin{aligned}
\int (A+Bx)(d+ex)^4\left(a+cx^2\right)^3\,dx
&= \frac{1}{12}Bc^3e^4x^{12}+\frac{1}{11}\left(4Bc^3de^3+Ac^3e^4\right)x^{11}+\frac{1}{10}\left(6Bc^3d^2e^2+4Ac^3de^3+3Bac^2e^4\right)x^{10}\\
&+\frac{1}{9}\left(4Bc^3d^3e+6Ac^3d^2e^2+12Bac^2de^3+3Aac^2e^4\right)x^9+Aa^3d^4x\\
&+\frac{1}{8}\left(Bc^3d^4+4Ac^3d^3e+18Bac^2d^2e^2+12Aac^2de^3+3Ba^2ce^4\right)x^8\\
&+\frac{1}{7}\left(Ac^3d^4+12Bac^2d^3e+18Aac^2d^2e^2+12Ba^2cde^3+3Aa^2ce^4\right)x^7\\
&+\frac{1}{6}\left(3Bac^2d^4+12Aac^2d^3e+18Ba^2cd^2e^2+12Aa^2cde^3+Ba^3e^4\right)x^6\\
&+\frac{1}{5}\left(3Aac^2d^4+12Ba^2cd^3e+18Aa^2cd^2e^2+4Ba^3de^3+Aa^3e^4\right)x^5\\
&+\frac{1}{4}\left(3Ba^2cd^4+12Aa^2cd^3e+6Ba^3d^2e^2+4Aa^3de^3\right)x^4\\
&+\frac{1}{3}\left(3Aa^2cd^4+4Ba^3d^3e+6Aa^3d^2e^2\right)x^3+\frac{1}{2}\left(Ba^3d^4+4Aa^3d^3e\right)x^2
\end{aligned}$$

input
```
integrate((B*x+A)*(e*x+d)^4*(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/12*B*c^3*e^4*x^12 + 1/11*(4*B*c^3*d*e^3 + A*c^3*e^4)*x^11 + 1/10*(6*B*c^
3*d^2*e^2 + 4*A*c^3*d*e^3 + 3*B*a*c^2*e^4)*x^10 + 1/9*(4*B*c^3*d^3*e + 6*A
*c^3*d^2*e^2 + 12*B*a*c^2*d*e^3 + 3*A*a*c^2*e^4)*x^9 + A*a^3*d^4*x + 1/8*(
B*c^3*d^4 + 4*A*c^3*d^3*e + 18*B*a*c^2*d^2*e^2 + 12*A*a*c^2*d*e^3 + 3*B*a^
2*c*e^4)*x^8 + 1/7*(A*c^3*d^4 + 12*B*a*c^2*d^3*e + 18*A*a*c^2*d^2*e^2 + 12
*B*a^2*c*d*e^3 + 3*A*a^2*c*e^4)*x^7 + 1/6*(3*B*a*c^2*d^4 + 12*A*a*c^2*d^3*
e + 18*B*a^2*c*d^2*e^2 + 12*A*a^2*c*d*e^3 + B*a^3*e^4)*x^6 + 1/5*(3*A*a*c^
2*d^4 + 12*B*a^2*c*d^3*e + 18*A*a^2*c*d^2*e^2 + 4*B*a^3*d*e^3 + A*a^3*e^4)
*x^5 + 1/4*(3*B*a^2*c*d^4 + 12*A*a^2*c*d^3*e + 6*B*a^3*d^2*e^2 + 4*A*a^3*d
*e^3)*x^4 + 1/3*(3*A*a^2*c*d^4 + 4*B*a^3*d^3*e + 6*A*a^3*d^2*e^2)*x^3 + 1/
2*(B*a^3*d^4 + 4*A*a^3*d^3*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.69

$$\int (A+Bx)(d+ex)^4\left(a+cx^2\right)^3\,dx = Aa^3d^4x+\frac{Bc^3e^4x^{12}}{12}+x^{11}\left(\frac{Ac^3e^4}{11}+\frac{4Bc^3de^3}{11}\right)+x^{10}\cdot\left(\frac{2Ac^3de^3}{5}+\frac{3Bac^2e^4}{10}+\frac{3Bc^3d^2e^2}{5}\right)+x^9\left(\frac{Aac^2e^4}{3}+\frac{2Ac^3d^2e^2}{3}+\frac{4Bac^2de^3}{3}+\frac{4Bc^3d^3e}{9}\right)+x^8\cdot\left(\frac{3Aac^2de^3}{2}+\frac{Ac^3d^3e}{2}+\frac{3Ba^2ce^4}{8}+\frac{9Bac^2d^2e^2}{4}+\frac{Bc^3d^4}{8}\right)+x^7\cdot\left(\frac{3Aa^2ce^4}{7}+\frac{18Aac^2d^2e^2}{7}+\frac{Ac^3d^4}{7}+\frac{12Ba^2cde^3}{7}+\frac{12Bac^2d^3e}{7}\right)+x^6\cdot\left(2Aa^2cde^3+2Aac^2d^3e+\frac{Ba^3e^4}{6}+3Ba^2cd^2e^2+\frac{Bac^2d^4}{2}\right)+x^5\left(\frac{Aa^3e^4}{5}+\frac{18Aa^2cd^2e^2}{5}+\frac{3Aac^2d^4}{5}+\frac{4Ba^3de^3}{5}+\frac{12Ba^2cd^3e}{5}\right)+x^4\left(Aa^3de^3+3Aa^2cd^3e+\frac{3Ba^3d^2e^2}{2}+\frac{3Ba^2cd^4}{4}\right)+x^3\cdot\left(2Aa^3d^2e^2+Aa^2cd^4+\frac{4Ba^3d^3e}{3}\right)+x^2\cdot\left(2Aa^3d^3e+\frac{Ba^3d^4}{2}\right)$$

input
```
integrate((B*x+A)*(e*x+d)**4*(c*x**2+a)**3,x)
```

output
```
A*a**3*d**4*x + B*c**3*e**4*x**12/12 + x**11*(A*c**3*e**4/11 + 4*B*c**3*d*
e**3/11) + x**10*(2*A*c**3*d*e**3/5 + 3*B*a*c**2*e**4/10 + 3*B*c**3*d**2*e
**2/5) + x**9*(A*a*c**2*e**4/3 + 2*A*c**3*d**2*e**2/3 + 4*B*a*c**2*d*e**3/
3 + 4*B*c**3*d**3*e/9) + x**8*(3*A*a*c**2*d*e**3/2 + A*c**3*d**3*e/2 + 3*B
*a**2*c*e**4/8 + 9*B*a*c**2*d**2*e**2/4 + B*c**3*d**4/8) + x**7*(3*A*a**2*
c*e**4/7 + 18*A*a*c**2*d**2*e**2/7 + A*c**3*d**4/7 + 12*B*a**2*c*d*e**3/7
+ 12*B*a*c**2*d**3*e/7) + x**6*(2*A*a**2*c*d*e**3 + 2*A*a*c**2*d**3*e + B*
a**3*e**4/6 + 3*B*a**2*c*d**2*e**2 + B*a*c**2*d**4/2) + x**5*(A*a**3*e**4/
5 + 18*A*a**2*c*d**2*e**2/5 + 3*A*a*c**2*d**4/5 + 4*B*a**3*d*e**3/5 + 12*B
*a**2*c*d**3*e/5) + x**4*(A*a**3*d*e**3 + 3*A*a**2*c*d**3*e + 3*B*a**3*d**
2*e**2/2 + 3*B*a**2*c*d**4/4) + x**3*(2*A*a**3*d**2*e**2 + A*a**2*c*d**4 +
 4*B*a**3*d**3*e/3) + x**2*(2*A*a**3*d**3*e + B*a**3*d**4/2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.43

$$\begin{aligned}
&\int (A+Bx)(d+ex)^4\left(a+cx^2\right)^3\,dx \\
&= \frac{1}{12}\,Bc^3e^4x^{12} + \frac{1}{11}\left(4\,Bc^3de^3 + Ac^3e^4\right)x^{11} + \frac{1}{10}\left(6\,Bc^3d^2e^2 + 4\,Ac^3de^3 + 3\,Bac^2e^4\right)x^{10} \\
&\quad + \frac{1}{9}\left(4\,Bc^3d^3e + 6\,Ac^3d^2e^2 + 12\,Bac^2de^3 + 3\,Aac^2e^4\right)x^9 + Aa^3d^4x \\
&\quad + \frac{1}{8}\left(Bc^3d^4 + 4\,Ac^3d^3e + 18\,Bac^2d^2e^2 + 12\,Aac^2de^3 + 3\,Ba^2ce^4\right)x^8 \\
&\quad + \frac{1}{7}\left(Ac^3d^4 + 12\,Bac^2d^3e + 18\,Aac^2d^2e^2 + 12\,Ba^2cde^3 + 3\,Aa^2ce^4\right)x^7 \\
&\quad + \frac{1}{6}\left(3\,Bac^2d^4 + 12\,Aac^2d^3e + 18\,Ba^2cd^2e^2 + 12\,Aa^2cde^3 + Ba^3e^4\right)x^6 \\
&\quad + \frac{1}{5}\left(3\,Aac^2d^4 + 12\,Ba^2cd^3e + 18\,Aa^2cd^2e^2 + 4\,Ba^3de^3 + Aa^3e^4\right)x^5 \\
&\quad + \frac{1}{4}\left(3\,Ba^2cd^4 + 12\,Aa^2cd^3e + 6\,Ba^3d^2e^2 + 4\,Aa^3de^3\right)x^4 \\
&\quad + \frac{1}{3}\left(3\,Aa^2cd^4 + 4\,Ba^3d^3e + 6\,Aa^3d^2e^2\right)x^3 + \frac{1}{2}\left(Ba^3d^4 + 4\,Aa^3d^3e\right)x^2
\end{aligned}$$

input
```
integrate((B*x+A)*(e*x+d)^4*(c*x^2+a)^3,x, algorithm="maxima")
```

output

```
1/12*B*c^3*e^4*x^12 + 1/11*(4*B*c^3*d*e^3 + A*c^3*e^4)*x^11 + 1/10*(6*B*c^
3*d^2*e^2 + 4*A*c^3*d*e^3 + 3*B*a*c^2*e^4)*x^10 + 1/9*(4*B*c^3*d^3*e + 6*A
*c^3*d^2*e^2 + 12*B*a*c^2*d*e^3 + 3*A*a*c^2*e^4)*x^9 + A*a^3*d^4*x + 1/8*(
B*c^3*d^4 + 4*A*c^3*d^3*e + 18*B*a*c^2*d^2*e^2 + 12*A*a*c^2*d*e^3 + 3*B*a^
2*c*e^4)*x^8 + 1/7*(A*c^3*d^4 + 12*B*a*c^2*d^3*e + 18*A*a*c^2*d^2*e^2 + 12
*B*a^2*c*d*e^3 + 3*A*a^2*c*e^4)*x^7 + 1/6*(3*B*a*c^2*d^4 + 12*A*a*c^2*d^3*
e + 18*B*a^2*c*d^2*e^2 + 12*A*a^2*c*d*e^3 + B*a^3*e^4)*x^6 + 1/5*(3*A*a*c^
2*d^4 + 12*B*a^2*c*d^3*e + 18*A*a^2*c*d^2*e^2 + 4*B*a^3*d*e^3 + A*a^3*e^4)
*x^5 + 1/4*(3*B*a^2*c*d^4 + 12*A*a^2*c*d^3*e + 6*B*a^3*d^2*e^2 + 4*A*a^3*d
*e^3)*x^4 + 1/3*(3*A*a^2*c*d^4 + 4*B*a^3*d^3*e + 6*A*a^3*d^2*e^2)*x^3 + 1/
2*(B*a^3*d^4 + 4*A*a^3*d^3*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.60

$$\int (A+Bx)(d+ex)^4\left(a+cx^2\right)^3\,dx = \frac{1}{12}\,Bc^3e^4x^{12} + \frac{4}{11}\,Bc^3de^3x^{11} + \frac{1}{11}\,Ac^3e^4x^{11} + \frac{3}{5}\,Bc^3d^2e^2x^{10} + \frac{2}{5}\,Ac^3de^3x^{10} + \frac{3}{10}\,Bac^2e^4x^{10} + \frac{4}{9}\,Bc^3d^3ex^9 + \frac{2}{3}\,Ac^3d^2e^2x^9 + \frac{4}{3}\,Bac^2de^3x^9 + \frac{1}{3}\,Aac^2e^4x^9 + \frac{1}{8}\,Bc^3d^4x^8 + \frac{1}{2}\,Ac^3d^3ex^8 + \frac{9}{4}\,Bac^2d^2e^2x^8 + \frac{3}{2}\,Aac^2de^3x^8 + \frac{3}{8}\,Ba^2ce^4x^8 + \frac{1}{7}\,Ac^3d^4x^7 + \frac{12}{7}\,Bac^2d^3ex^7 + \frac{18}{7}\,Aac^2d^2e^2x^7 + \frac{12}{7}\,Ba^2cde^3x^7 + \frac{3}{7}\,Aa^2ce^4x^7 + \frac{1}{2}\,Bac^2d^4x^6 + 2\,Aac^2d^3ex^6 + 3\,Ba^2cd^2e^2x^6 + 2\,Aa^2cde^3x^6 + \frac{1}{6}\,Ba^3e^4x^6 + \frac{3}{5}\,Aac^2d^4x^5 + \frac{12}{5}\,Ba^2cd^3ex^5 + \frac{18}{5}\,Aa^2cd^2e^2x^5 + \frac{4}{5}\,Ba^3de^3x^5 + \frac{1}{5}\,Aa^3e^4x^5 + \frac{3}{4}\,Ba^2cd^4x^4 + 3\,Aa^2cd^3ex^4 + \frac{3}{2}\,Ba^3d^2e^2x^4 + Aa^3de^3x^4 + Aa^2cd^4x^3 + \frac{4}{3}\,Ba^3d^3ex^3 + 2\,Aa^3d^2e^2x^3 + \frac{1}{2}\,Ba^3d^4x^2 + 2\,Aa^3d^3ex^2 + Aa^3d^4x$$

input
```
integrate((B*x+A)*(e*x+d)^4*(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/12*B*c^3*e^4*x^12 + 4/11*B*c^3*d*e^3*x^11 + 1/11*A*c^3*e^4*x^11 + 3/5*B*
c^3*d^2*e^2*x^10 + 2/5*A*c^3*d*e^3*x^10 + 3/10*B*a*c^2*e^4*x^10 + 4/9*B*c^
3*d^3*e*x^9 + 2/3*A*c^3*d^2*e^2*x^9 + 4/3*B*a*c^2*d*e^3*x^9 + 1/3*A*a*c^2*
e^4*x^9 + 1/8*B*c^3*d^4*x^8 + 1/2*A*c^3*d^3*e*x^8 + 9/4*B*a*c^2*d^2*e^2*x^
8 + 3/2*A*a*c^2*d*e^3*x^8 + 3/8*B*a^2*c*e^4*x^8 + 1/7*A*c^3*d^4*x^7 + 12/7
*B*a*c^2*d^3*e*x^7 + 18/7*A*a*c^2*d^2*e^2*x^7 + 12/7*B*a^2*c*d*e^3*x^7 + 3
/7*A*a^2*c*e^4*x^7 + 1/2*B*a*c^2*d^4*x^6 + 2*A*a*c^2*d^3*e*x^6 + 3*B*a^2*c
*d^2*e^2*x^6 + 2*A*a^2*c*d*e^3*x^6 + 1/6*B*a^3*e^4*x^6 + 3/5*A*a*c^2*d^4*x
^5 + 12/5*B*a^2*c*d^3*e*x^5 + 18/5*A*a^2*c*d^2*e^2*x^5 + 4/5*B*a^3*d*e^3*x
^5 + 1/5*A*a^3*e^4*x^5 + 3/4*B*a^2*c*d^4*x^4 + 3*A*a^2*c*d^3*e*x^4 + 3/2*B
*a^3*d^2*e^2*x^4 + A*a^3*d*e^3*x^4 + A*a^2*c*d^4*x^3 + 4/3*B*a^3*d^3*e*x^3
 + 2*A*a^3*d^2*e^2*x^3 + 1/2*B*a^3*d^4*x^2 + 2*A*a^3*d^3*e*x^2 + A*a^3*d^4
*x
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.31

$$
\begin{aligned}
\int (A+Bx)(d+ex)^4\left(a+cx^2\right)^3\,dx
&= x^5\left(\frac{4\,B\,a^3\,d\,e^3}{5}+\frac{A\,a^3\,e^4}{5}+\frac{12\,B\,a^2\,c\,d^3\,e}{5}+\frac{18\,A\,a^2\,c\,d^2\,e^2}{5}+\frac{3\,A\,a\,c^2\,d^4}{5}\right)\\
&\quad+x^8\left(\frac{3\,B\,a^2\,c\,e^4}{8}+\frac{9\,B\,a\,c^2\,d^2\,e^2}{4}+\frac{3\,A\,a\,c^2\,d\,e^3}{2}+\frac{B\,c^3\,d^4}{8}+\frac{A\,c^3\,d^3\,e}{2}\right)\\
&\quad+x^6\left(\frac{B\,a^3\,e^4}{6}+3\,B\,a^2\,c\,d^2\,e^2+2\,A\,a^2\,c\,d\,e^3+\frac{B\,a\,c^2\,d^4}{2}+2\,A\,a\,c^2\,d^3\,e\right)\\
&\quad+x^7\left(\frac{12\,B\,a^2\,c\,d\,e^3}{7}+\frac{3\,A\,a^2\,c\,e^4}{7}+\frac{12\,B\,a\,c^2\,d^3\,e}{7}+\frac{18\,A\,a\,c^2\,d^2\,e^2}{7}+\frac{A\,c^3\,d^4}{7}\right)\\
&\quad+\frac{a^3\,d^3\,x^2\,(4\,A\,e+B\,d)}{2}+\frac{c^3\,e^3\,x^{11}\,(A\,e+4\,B\,d)}{11}\\
&\quad+\frac{a^2\,d^2\,x^3\,(3\,A\,c\,d^2+4\,B\,a\,d\,e+6\,A\,a\,e^2)}{3}+\frac{c^2\,e^2\,x^{10}\,(6\,B\,c\,d^2+4\,A\,c\,d\,e+3\,B\,a\,e^2)}{10}\\
&\quad+A\,a^3\,d^4\,x+\frac{a^2\,d\,x^4\,(3\,B\,c\,d^3+12\,A\,c\,d^2\,e+6\,B\,a\,d\,e^2+4\,A\,a\,e^3)}{4}\\
&\quad+\frac{c^2\,e\,x^9\,(4\,B\,c\,d^3+6\,A\,c\,d^2\,e+12\,B\,a\,d\,e^2+3\,A\,a\,e^3)}{9}+\frac{B\,c^3\,e^4\,x^{12}}{12}
\end{aligned}
$$

input
```
int((a + c*x^2)^3*(A + B*x)*(d + e*x)^4,x)
```

output
```
x^5*((A*a^3*e^4)/5 + (3*A*a*c^2*d^4)/5 + (4*B*a^3*d*e^3)/5 + (18*A*a^2*c*d
^2*e^2)/5 + (12*B*a^2*c*d^3*e)/5) + x^8*((B*c^3*d^4)/8 + (3*B*a^2*c*e^4)/8
 + (A*c^3*d^3*e)/2 + (9*B*a*c^2*d^2*e^2)/4 + (3*A*a*c^2*d*e^3)/2) + x^6*((
B*a^3*e^4)/6 + (B*a*c^2*d^4)/2 + 3*B*a^2*c*d^2*e^2 + 2*A*a*c^2*d^3*e + 2*A
*a^2*c*d*e^3) + x^7*((A*c^3*d^4)/7 + (3*A*a^2*c*e^4)/7 + (18*A*a*c^2*d^2*e
^2)/7 + (12*B*a*c^2*d^3*e)/7 + (12*B*a^2*c*d*e^3)/7) + (a^3*d^3*x^2*(4*A*e
 + B*d))/2 + (c^3*e^3*x^11*(A*e + 4*B*d))/11 + (a^2*d^2*x^3*(6*A*a*e^2 + 3
*A*c*d^2 + 4*B*a*d*e))/3 + (c^2*e^2*x^10*(3*B*a*e^2 + 6*B*c*d^2 + 4*A*c*d*
e))/10 + A*a^3*d^4*x + (a^2*d*x^4*(4*A*a*e^3 + 3*B*c*d^3 + 6*B*a*d*e^2 + 1
2*A*c*d^2*e))/4 + (c^2*e*x^9*(3*A*a*e^3 + 4*B*c*d^3 + 12*B*a*d*e^2 + 6*A*c
*d^2*e))/9 + (B*c^3*e^4*x^12)/12
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.59

$$\int (A+Bx)(d+ex)^4 \left(a+cx^2\right)^3 \, dx$$
$$= \frac{x(2310b\,c^3e^4x^{11}+2520a\,c^3e^4x^{10}+10080b\,c^3d\,e^3x^{10}+8316ab\,c^2e^4x^9+11088a\,c^3d\,e^3x^9+16632b\,c^3d^2e^2x}{}$$

input
```
int((B*x+A)*(e*x+d)^4*(c*x^2+a)^3,x)
```

output
```
(x*(27720*a**4*d**4 + 55440*a**4*d**3*e*x + 55440*a**4*d**2*e**2*x**2 + 27
720*a**4*d*e**3*x**3 + 5544*a**4*e**4*x**4 + 13860*a**3*b*d**4*x + 36960*a
**3*b*d**3*e*x**2 + 41580*a**3*b*d**2*e**2*x**3 + 22176*a**3*b*d*e**3*x**4
 + 4620*a**3*b*e**4*x**5 + 27720*a**3*c*d**4*x**2 + 83160*a**3*c*d**3*e*x*
*3 + 99792*a**3*c*d**2*e**2*x**4 + 55440*a**3*c*d*e**3*x**5 + 11880*a**3*c
*e**4*x**6 + 20790*a**2*b*c*d**4*x**3 + 66528*a**2*b*c*d**3*e*x**4 + 83160
*a**2*b*c*d**2*e**2*x**5 + 47520*a**2*b*c*d*e**3*x**6 + 10395*a**2*b*c*e**
4*x**7 + 16632*a**2*c**2*d**4*x**4 + 55440*a**2*c**2*d**3*e*x**5 + 71280*a
**2*c**2*d**2*e**2*x**6 + 41580*a**2*c**2*d*e**3*x**7 + 9240*a**2*c**2*e**
4*x**8 + 13860*a*b*c**2*d**4*x**5 + 47520*a*b*c**2*d**3*e*x**6 + 62370*a*b
*c**2*d**2*e**2*x**7 + 36960*a*b*c**2*d*e**3*x**8 + 8316*a*b*c**2*e**4*x**
9 + 3960*a*c**3*d**4*x**6 + 13860*a*c**3*d**3*e*x**7 + 18480*a*c**3*d**2*e
**2*x**8 + 11088*a*c**3*d*e**3*x**9 + 2520*a*c**3*e**4*x**10 + 3465*b*c**3
*d**4*x**7 + 12320*b*c**3*d**3*e*x**8 + 16632*b*c**3*d**2*e**2*x**9 + 1008
0*b*c**3*d*e**3*x**10 + 2310*b*c**3*e**4*x**11))/27720
```

3.64 $\int (A+Bx)(d+ex)^3 \left(a+cx^2\right)^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 334

$$\begin{aligned}
&\int (A+Bx)(d+ex)^3 \left(a+cx^2\right)^3 \, dx \\
&= -\frac{(Bd-Ae)\left(cd^2+ae^2\right)^3(d+ex)^4}{4e^8} \\
&\quad + \frac{\left(cd^2+ae^2\right)^2\left(7Bcd^2-6Acde+aBe^2\right)(d+ex)^5}{5e^8} \\
&\quad - \frac{c\left(cd^2+ae^2\right)\left(7Bcd^3-5Acd^2e+3aBde^2-aAe^3\right)(d+ex)^6}{2e^8} \\
&\quad - \frac{c\left(4Acde\left(5cd^2+3ae^2\right)-B\left(35c^2d^4+30acd^2e^2+3a^2e^4\right)\right)(d+ex)^7}{7e^8} \\
&\quad - \frac{c^2\left(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3\right)(d+ex)^8}{8e^8} \\
&\quad + \frac{c^2\left(7Bcd^2-2Acde+aBe^2\right)(d+ex)^9}{3e^8} - \frac{c^3(7Bd-Ae)(d+ex)^{10}}{10e^8} + \frac{Bc^3(d+ex)^{11}}{11e^8}
\end{aligned}$$

output
```
-1/4*(-A*e+B*d)*(a*e^2+c*d^2)^3*(e*x+d)^4/e^8+1/5*(a*e^2+c*d^2)^2*(-6*A*c*
d*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^5/e^8-1/2*c*(a*e^2+c*d^2)*(-A*a*e^3-5*A*c*d
^2*e+3*B*a*d*e^2+7*B*c*d^3)*(e*x+d)^6/e^8-1/7*c*(4*A*c*d*e*(3*a*e^2+5*c*d^
2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))*(e*x+d)^7/e^8-1/8*c^2*(-3*A*a*
e^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)*(e*x+d)^8/e^8+1/3*c^2*(-2*A*c*d*
e+B*a*e^2+7*B*c*d^2)*(e*x+d)^9/e^8-1/10*c^3*(-A*e+7*B*d)*(e*x+d)^10/e^8+1/
11*B*c^3*(e*x+d)^11/e^8
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.97

$$\int (A+Bx)(d+ex)^3\left(a+cx^2\right)^3\,dx = a^3Ad^3x+\frac{1}{2}a^3d^2(Bd+3Ae)x^2+a^2d\left(Acd^2+aBde+aAe^2\right)x^3+\frac{1}{4}a^2\left(3Bcd^3+9Acd^2e+3aBde^2+aAe^3\right)x^4+\frac{1}{5}a\left(aBe\left(9cd^2+ae^2\right)+3Acd\left(cd^2+3ae^2\right)\right)x^5+\frac{1}{2}ac\left(Bcd^3+3Acd^2e+3aBde^2+aAe^3\right)x^6+\frac{1}{7}c\left(3aBe\left(3cd^2+ae^2\right)+Acd\left(cd^2+9ae^2\right)\right)x^7+\frac{1}{8}c^2\left(Bcd^3+3Acd^2e+9aBde^2+3aAe^3\right)x^8+\frac{1}{3}c^2e\left(Bcd^2+Acde+aBe^2\right)x^9+\frac{1}{10}c^3e^2(3Bd+Ae)x^{10}+\frac{1}{11}Bc^3e^3x^{11}$$

input
```
Integrate[(A + B*x)*(d + e*x)^3*(a + c*x^2)^3,x]
```

output
```
a^3*A*d^3*x + (a^3*d^2*(B*d + 3*A*e)*x^2)/2 + a^2*d*(A*c*d^2 + a*B*d*e + a
*A*e^2)*x^3 + (a^2*(3*B*c*d^3 + 9*A*c*d^2*e + 3*a*B*d*e^2 + a*A*e^3)*x^4)/
4 + (a*(a*B*e*(9*c*d^2 + a*e^2) + 3*A*c*d*(c*d^2 + 3*a*e^2))*x^5)/5 + (a*c
*(B*c*d^3 + 3*A*c*d^2*e + 3*a*B*d*e^2 + a*A*e^3)*x^6)/2 + (c*(3*a*B*e*(3*c
*d^2 + a*e^2) + A*c*d*(c*d^2 + 9*a*e^2))*x^7)/7 + (c^2*(B*c*d^3 + 3*A*c*d^
2*e + 9*a*B*d*e^2 + 3*a*A*e^3)*x^8)/8 + (c^2*e*(B*c*d^2 + A*c*d*e + a*B*e^
2)*x^9)/3 + (c^3*e^2*(3*B*d + A*e)*x^10)/10 + (B*c^3*e^3*x^11)/11
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3 (A+Bx)(d+ex)^3\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{c(d+ex)^6\left(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4\right)}{e^7}-\frac{3c^2(d+ex)^8\left(-aBe^2+2Acde\right.}{e^7}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c(d+ex)^7\left(4Acde\left(3ae^2+5cd^2\right)-B\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)\right)}{7e^8}+\frac{c^2(d+ex)^9\left(aBe^2-2Acde+7Bcd^2\right)}{3e^8}-\frac{c^2(d+ex)^8\left(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3\right)}{8e^8}+\frac{(d+ex)^5\left(ae^2+cd^2\right)^2\left(aBe^2-6Acde+7Bcd^2\right)}{5e^8}-\frac{(d+ex)^4\left(ae^2+cd^2\right)^3(Bd-Ae)}{4e^8}-\frac{c(d+ex)^6\left(ae^2+cd^2\right)\left(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3\right)}{2e^8}-\frac{c^3(d+ex)^{10}(7Bd-Ae)}{10e^8}+\frac{Bc^3(d+ex)^{11}}{11e^8}$$

input

```
Int[(A + B*x)*(d + e*x)^3*(a + c*x^2)^3,x]
```

output
```
-1/4*((B*d - A*e)*(c*d^2 + a*e^2)^3*(d + e*x)^4)/e^8 + ((c*d^2 + a*e^2)^2*
(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2)*(d + e*x)^5)/(5*e^8) - (c*(c*d^2 + a*e^2
)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*(d + e*x)^6)/(2*e^8) -
 (c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^
2*e^4))*(d + e*x)^7)/(7*e^8) - (c^2*(35*B*c*d^3 - 15*A*c*d^2*e + 15*a*B*d*
e^2 - 3*a*A*e^3)*(d + e*x)^8)/(8*e^8) + (c^2*(7*B*c*d^2 - 2*A*c*d*e + a*B*
e^2)*(d + e*x)^9)/(3*e^8) - (c^3*(7*B*d - A*e)*(d + e*x)^10)/(10*e^8) + (B
*c^3*(d + e*x)^11)/(11*e^8)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.06

method	result
default	$\frac{B\,e^3 c^3 x^{11}}{11}+\frac{(A\,e^3+3Bd\,e^2)c^3 x^{10}}{10}+\frac{((3Ad\,e^2+3B\,d^2 e)c^3+3B\,e^3 a\,c^2)x^9}{9}+\frac{((3A\,d^2 e+B\,d^3)c^3+3(A\,e^3+3Bd\,e^2)a\,c^2)x}{8}$
norman	$\frac{B\,e^3 c^3 x^{11}}{11}+\left(\frac{1}{10}A\,c^3 e^3+\frac{3}{10}B\,c^3 d\,e^2\right)x^{10}+\left(\frac{1}{3}A\,c^3 d\,e^2+\frac{1}{3}B\,e^3 a\,c^2+\frac{1}{3}B\,c^3 d^2 e\right)x^9+\left(\frac{3}{8}Aa\,c^2\right.$
gosper	$\frac{1}{3}x^9 A\,c^3 d\,e^2+\frac{1}{3}x^9 B\,e^3 a\,c^2+A\,a^3 d\,e^2 x^3+A\,a^2 c\,d^3 x^3+\frac{1}{4}x^4 A\,a^3 e^3+\frac{1}{2}x^2 a^3 B\,d^3+\frac{1}{2}x^6 A\,a^2$
risch	$\frac{1}{3}x^9 A\,c^3 d\,e^2+\frac{1}{3}x^9 B\,e^3 a\,c^2+A\,a^3 d\,e^2 x^3+A\,a^2 c\,d^3 x^3+\frac{1}{4}x^4 A\,a^3 e^3+\frac{1}{2}x^2 a^3 B\,d^3+\frac{1}{2}x^6 A\,a^2$
parallelrisch	$\frac{1}{3}x^9 A\,c^3 d\,e^2+\frac{1}{3}x^9 B\,e^3 a\,c^2+A\,a^3 d\,e^2 x^3+A\,a^2 c\,d^3 x^3+\frac{1}{4}x^4 A\,a^3 e^3+\frac{1}{2}x^2 a^3 B\,d^3+\frac{1}{2}x^6 A\,a^2$
orering	$x(840B\,e^3 c^3 x^{10}+924A\,c^3 e^3 x^9+2772B\,c^3 d\,e^2 x^9+3080A\,c^3 d\,e^2 x^8+3080Ba\,c^2 e^3 x^8+3080B\,c^3 d^2 e\,x^8+3465Aa\,c^2 e^3 x^7+3465$

input
```
int((B*x+A)*(e*x+d)^3*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/11*B*e^3*c^3*x^11+1/10*(A*e^3+3*B*d*e^2)*c^3*x^10+1/9*((3*A*d*e^2+3*B*d^
2*e)*c^3+3*B*e^3*a*c^2)*x^9+1/8*((3*A*d^2*e+B*d^3)*c^3+3*(A*e^3+3*B*d*e^2)
*a*c^2)*x^8+1/7*(A*c^3*d^3+3*(3*A*d*e^2+3*B*d^2*e)*a*c^2+3*B*e^3*a^2*c)*x^
7+1/6*(3*(3*A*d^2*e+B*d^3)*a*c^2+3*(A*e^3+3*B*d*e^2)*a^2*c)*x^6+1/5*(3*A*d
^3*a*c^2+3*(3*A*d*e^2+3*B*d^2*e)*a^2*c+B*e^3*a^3)*x^5+1/4*(3*(3*A*d^2*e+B*
d^3)*a^2*c+(A*e^3+3*B*d*e^2)*a^3)*x^4+1/3*(3*A*d^3*a^2*c+(3*A*d*e^2+3*B*d^
2*e)*a^3)*x^3+1/2*(3*A*d^2*e+B*d^3)*a^3*x^2+A*d^3*a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.09

$$\int (A+Bx)(d+ex)^3\left(a+cx^2\right)^3\,dx$$
$$= \frac{1}{11}Bc^3e^3x^{11} + \frac{1}{10}\left(3Bc^3de^2 + Ac^3e^3\right)x^{10} + \frac{1}{3}\left(Bc^3d^2e + Ac^3de^2 + Bac^2e^3\right)x^9 + \frac{1}{8}\left(Bc^3d^3 + 3Ac^3d^2e + 9Bac^2de^2 + 3Aac^2e^3\right)x^8 + Aa^3d^3x + \frac{1}{7}\left(Ac^3d^3 + 9Bac^2d^2e + 9Aac^2de^2 + 3Ba^2ce^3\right)x^7 + \frac{1}{2}\left(Bac^2d^3 + 3Aac^2d^2e + 3Ba^2cde^2 + Aa^2ce^3\right)x^6 + \frac{1}{5}\left(3Aac^2d^3 + 9Ba^2cd^2e + 9Aa^2cde^2 + Ba^3e^3\right)x^5 + \frac{1}{4}\left(3Ba^2cd^3 + 9Aa^2cd^2e + 3Ba^3de^2 + Aa^3e^3\right)x^4 + \left(Aa^2cd^3 + Ba^3d^2e + Aa^3de^2\right)x^3 + \frac{1}{2}\left(Ba^3d^3 + 3Aa^3d^2e\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)^3*(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/11*B*c^3*e^3*x^11 + 1/10*(3*B*c^3*d*e^2 + A*c^3*e^3)*x^10 + 1/3*(B*c^3*d
^2*e + A*c^3*d*e^2 + B*a*c^2*e^3)*x^9 + 1/8*(B*c^3*d^3 + 3*A*c^3*d^2*e + 9
*B*a*c^2*d*e^2 + 3*A*a*c^2*e^3)*x^8 + A*a^3*d^3*x + 1/7*(A*c^3*d^3 + 9*B*a
*c^2*d^2*e + 9*A*a*c^2*d*e^2 + 3*B*a^2*c*e^3)*x^7 + 1/2*(B*a*c^2*d^3 + 3*A
*a*c^2*d^2*e + 3*B*a^2*c*d*e^2 + A*a^2*c*e^3)*x^6 + 1/5*(3*A*a*c^2*d^3 + 9
*B*a^2*c*d^2*e + 9*A*a^2*c*d*e^2 + B*a^3*e^3)*x^5 + 1/4*(3*B*a^2*c*d^3 + 9
*A*a^2*c*d^2*e + 3*B*a^3*d*e^2 + A*a^3*e^3)*x^4 + (A*a^2*c*d^3 + B*a^3*d^2
*e + A*a^3*d*e^2)*x^3 + 1/2*(B*a^3*d^3 + 3*A*a^3*d^2*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.30

$$\int (A+Bx)(d+ex)^3\left(a+cx^2\right)^3\,dx = Aa^3d^3x + \frac{Bc^3e^3x^{11}}{11} + x^{10}\left(\frac{Ac^3e^3}{10} + \frac{3Bc^3de^2}{10}\right) + x^9\left(\frac{Ac^3de^2}{3} + \frac{Bac^2e^3}{3} + \frac{Bc^3d^2e}{3}\right) + x^8 \cdot \left(\frac{3Aac^2e^3}{8} + \frac{3Ac^3d^2e}{8} + \frac{9Bac^2de^2}{8} + \frac{Bc^3d^3}{8}\right) + x^7 \cdot \left(\frac{9Aac^2de^2}{7} + \frac{Ac^3d^3}{7} + \frac{3Ba^2ce^3}{7} + \frac{9Bac^2d^2e}{7}\right) + x^6\left(\frac{Aa^2ce^3}{2} + \frac{3Aac^2d^2e}{2} + \frac{3Ba^2cde^2}{2} + \frac{Bac^2d^3}{2}\right) + x^5 \cdot \left(\frac{9Aa^2cde^2}{5} + \frac{3Aac^2d^3}{5} + \frac{Ba^3e^3}{5} + \frac{9Ba^2cd^2e}{5}\right) + x^4\left(\frac{Aa^3e^3}{4} + \frac{9Aa^2cd^2e}{4} + \frac{3Ba^3de^2}{4} + \frac{3Ba^2cd^3}{4}\right) + x^3\left(Aa^3de^2 + Aa^2cd^3 + Ba^3d^2e\right) + x^2 \cdot \left(\frac{3Aa^3d^2e}{2} + \frac{Ba^3d^3}{2}\right)$$

input
```
integrate((B*x+A)*(e*x+d)**3*(c*x**2+a)**3,x)
```

output
```
A*a**3*d**3*x + B*c**3*e**3*x**11/11 + x**10*(A*c**3*e**3/10 + 3*B*c**3*d*
e**2/10) + x**9*(A*c**3*d*e**2/3 + B*a*c**2*e**3/3 + B*c**3*d**2*e/3) + x*
*8*(3*A*a*c**2*e**3/8 + 3*A*c**3*d**2*e/8 + 9*B*a*c**2*d*e**2/8 + B*c**3*d
**3/8) + x**7*(9*A*a*c**2*d*e**2/7 + A*c**3*d**3/7 + 3*B*a**2*c*e**3/7 + 9
*B*a*c**2*d**2*e/7) + x**6*(A*a**2*c*e**3/2 + 3*A*a*c**2*d**2*e/2 + 3*B*a*
*2*c*d*e**2/2 + B*a*c**2*d**3/2) + x**5*(9*A*a**2*c*d*e**2/5 + 3*A*a*c**2*
d**3/5 + B*a**3*e**3/5 + 9*B*a**2*c*d**2*e/5) + x**4*(A*a**3*e**3/4 + 9*A*
a**2*c*d**2*e/4 + 3*B*a**3*d*e**2/4 + 3*B*a**2*c*d**3/4) + x**3*(A*a**3*d*
e**2 + A*a**2*c*d**3 + B*a**3*d**2*e) + x**2*(3*A*a**3*d**2*e/2 + B*a**3*d
**3/2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.09

$$\int (A+Bx)(d+ex)^3\left(a+cx^2\right)^3\,dx = \frac{1}{11}Bc^3e^3x^{11}+\frac{1}{10}\left(3\,Bc^3de^2+Ac^3e^3\right)x^{10}+\frac{1}{3}\left(Bc^3d^2e+Ac^3de^2+Bac^2e^3\right)x^9 + \frac{1}{8}\left(Bc^3d^3+3\,Ac^3d^2e+9\,Bac^2de^2+3\,Aac^2e^3\right)x^8+Aa^3d^3x + \frac{1}{7}\left(Ac^3d^3+9\,Bac^2d^2e+9\,Aac^2de^2+3\,Ba^2ce^3\right)x^7 + \frac{1}{2}\left(Bac^2d^3+3\,Aac^2d^2e+3\,Ba^2cde^2+Aa^2ce^3\right)x^6 + \frac{1}{5}\left(3\,Aac^2d^3+9\,Ba^2cd^2e+9\,Aa^2cde^2+Ba^3e^3\right)x^5 + \frac{1}{4}\left(3\,Ba^2cd^3+9\,Aa^2cd^2e+3\,Ba^3de^2+Aa^3e^3\right)x^4 + \left(Aa^2cd^3+Ba^3d^2e+Aa^3de^2\right)x^3+\frac{1}{2}\left(Ba^3d^3+3\,Aa^3d^2e\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)^3*(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/11*B*c^3*e^3*x^11 + 1/10*(3*B*c^3*d*e^2 + A*c^3*e^3)*x^10 + 1/3*(B*c^3*d
^2*e + A*c^3*d*e^2 + B*a*c^2*e^3)*x^9 + 1/8*(B*c^3*d^3 + 3*A*c^3*d^2*e + 9
*B*a*c^2*d*e^2 + 3*A*a*c^2*e^3)*x^8 + A*a^3*d^3*x + 1/7*(A*c^3*d^3 + 9*B*a
*c^2*d^2*e + 9*A*a*c^2*d*e^2 + 3*B*a^2*c*e^3)*x^7 + 1/2*(B*a*c^2*d^3 + 3*A
*a*c^2*d^2*e + 3*B*a^2*c*d*e^2 + A*a^2*c*e^3)*x^6 + 1/5*(3*A*a*c^2*d^3 + 9
*B*a^2*c*d^2*e + 9*A*a^2*c*d*e^2 + B*a^3*e^3)*x^5 + 1/4*(3*B*a^2*c*d^3 + 9
*A*a^2*c*d^2*e + 3*B*a^3*d*e^2 + A*a^3*e^3)*x^4 + (A*a^2*c*d^3 + B*a^3*d^2
*e + A*a^3*d*e^2)*x^3 + 1/2*(B*a^3*d^3 + 3*A*a^3*d^2*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.23

$$\int (A+Bx)(d+ex)^3\left(a+cx^2\right)^3\,dx=\frac{1}{11}Bc^3e^3x^{11}+\frac{3}{10}Bc^3de^2x^{10}+\frac{1}{10}Ac^3e^3x^{10}+\frac{1}{3}Bc^3d^2ex^9+\frac{1}{3}Ac^3de^2x^9+\frac{1}{3}Bac^2e^3x^9+\frac{1}{8}Bc^3d^3x^8+\frac{3}{8}Ac^3d^2ex^8+\frac{9}{8}Bac^2de^2x^8+\frac{3}{8}Aac^2e^3x^8+\frac{1}{7}Ac^3d^3x^7+\frac{9}{7}Bac^2d^2ex^7+\frac{9}{7}Aac^2de^2x^7+\frac{3}{7}Ba^2ce^3x^7+\frac{1}{2}Bac^2d^3x^6+\frac{3}{2}Aac^2d^2ex^6+\frac{3}{2}Ba^2cde^2x^6+\frac{1}{2}Aa^2ce^3x^6+\frac{3}{5}Aac^2d^3x^5+\frac{9}{5}Ba^2cd^2ex^5+\frac{9}{5}Aa^2cde^2x^5+\frac{1}{5}Ba^3e^3x^5+\frac{3}{4}Ba^2cd^3x^4+\frac{9}{4}Aa^2cd^2ex^4+\frac{3}{4}Ba^3de^2x^4+\frac{1}{4}Aa^3e^3x^4+Aa^2cd^3x^3+Ba^3d^2ex^3+Aa^3de^2x^3+\frac{1}{2}Ba^3d^3x^2+\frac{3}{2}Aa^3d^2ex^2+Aa^3d^3x$$

input
```
integrate((B*x+A)*(e*x+d)^3*(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/11*B*c^3*e^3*x^11 + 3/10*B*c^3*d*e^2*x^10 + 1/10*A*c^3*e^3*x^10 + 1/3*B*
c^3*d^2*e*x^9 + 1/3*A*c^3*d*e^2*x^9 + 1/3*B*a*c^2*e^3*x^9 + 1/8*B*c^3*d^3*
x^8 + 3/8*A*c^3*d^2*e*x^8 + 9/8*B*a*c^2*d*e^2*x^8 + 3/8*A*a*c^2*e^3*x^8 +
1/7*A*c^3*d^3*x^7 + 9/7*B*a*c^2*d^2*e*x^7 + 9/7*A*a*c^2*d*e^2*x^7 + 3/7*B*
a^2*c*e^3*x^7 + 1/2*B*a*c^2*d^3*x^6 + 3/2*A*a*c^2*d^2*e*x^6 + 3/2*B*a^2*c*
d*e^2*x^6 + 1/2*A*a^2*c*e^3*x^6 + 3/5*A*a*c^2*d^3*x^5 + 9/5*B*a^2*c*d^2*e*
x^5 + 9/5*A*a^2*c*d*e^2*x^5 + 1/5*B*a^3*e^3*x^5 + 3/4*B*a^2*c*d^3*x^4 + 9/
4*A*a^2*c*d^2*e*x^4 + 3/4*B*a^3*d*e^2*x^4 + 1/4*A*a^3*e^3*x^4 + A*a^2*c*d^
3*x^3 + B*a^3*d^2*e*x^3 + A*a^3*d*e^2*x^3 + 1/2*B*a^3*d^3*x^2 + 3/2*A*a^3*
d^2*e*x^2 + A*a^3*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.95

$$\int (A+Bx)(d+ex)^3\left(a+cx^2\right)^3\,dx = x^5\left(\frac{B\,a^3\,e^3}{5}+\frac{9\,B\,a^2\,c\,d^2\,e}{5}+\frac{9\,A\,a^2\,c\,d\,e^2}{5}+\frac{3\,A\,a\,c^2\,d^3}{5}\right)+x^7\left(\frac{3\,B\,a^2\,c\,e^3}{7}+\frac{9\,B\,a\,c^2\,d^2\,e}{7}+\frac{9\,A\,a\,c^2\,d\,e^2}{7}+\frac{A\,c^3\,d^3}{7}\right)+\frac{a^2\,x^4\,(3\,B\,c\,d^3+9\,A\,c\,d^2\,e+3\,B\,a\,d\,e^2+A\,a\,e^3)}{4}+\frac{c^2\,x^8\,(B\,c\,d^3+3\,A\,c\,d^2\,e+9\,B\,a\,d\,e^2+3\,A\,a\,e^3)}{8}+a^2\,d\,x^3\left(A\,c\,d^2+B\,a\,d\,e+A\,a\,e^2\right)+\frac{c^2\,e\,x^9\,(B\,c\,d^2+A\,c\,d\,e+B\,a\,e^2)}{3}+\frac{a^3\,d^2\,x^2\,(3\,A\,e+B\,d)}{2}+\frac{c^3\,e^2\,x^{10}\,(A\,e+3\,B\,d)}{10}+\frac{a\,c\,x^6\,(B\,c\,d^3+3\,A\,c\,d^2\,e+3\,B\,a\,d\,e^2+A\,a\,e^3)}{2}+A\,a^3\,d^3\,x+\frac{B\,c^3\,e^3\,x^{11}}{11}$$

input
```
int((a + c*x^2)^3*(A + B*x)*(d + e*x)^3,x)
```

output
```
x^5*((B*a^3*e^3)/5 + (3*A*a*c^2*d^3)/5 + (9*A*a^2*c*d*e^2)/5 + (9*B*a^2*c*
d^2*e)/5) + x^7*((A*c^3*d^3)/7 + (3*B*a^2*c*e^3)/7 + (9*A*a*c^2*d*e^2)/7 +
 (9*B*a*c^2*d^2*e)/7) + (a^2*x^4*(A*a*e^3 + 3*B*c*d^3 + 3*B*a*d*e^2 + 9*A*
c*d^2*e))/4 + (c^2*x^8*(3*A*a*e^3 + B*c*d^3 + 9*B*a*d*e^2 + 3*A*c*d^2*e))/
8 + a^2*d*x^3*(A*a*e^2 + A*c*d^2 + B*a*d*e) + (c^2*e*x^9*(B*a*e^2 + B*c*d^
2 + A*c*d*e))/3 + (a^3*d^2*x^2*(3*A*e + B*d))/2 + (c^3*e^2*x^10*(A*e + 3*B
*d))/10 + (a*c*x^6*(A*a*e^3 + B*c*d^3 + 3*B*a*d*e^2 + 3*A*c*d^2*e))/2 + A*
a^3*d^3*x + (B*c^3*e^3*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.22

$$\int (A+Bx)(d+ex)^3 \left(a+cx^2\right)^3 \, dx$$
$$= \frac{x(840b\,c^3e^3x^{10}+924a\,c^3e^3x^9+2772b\,c^3d\,e^2x^9+3080ab\,c^2e^3x^8+3080a\,c^3d\,e^2x^8+3080b\,c^3d^2e\,x^8+3465}{}$$

input
```
int((B*x+A)*(e*x+d)^3*(c*x^2+a)^3,x)
```

output
```
(x*(9240*a**4*d**3 + 13860*a**4*d**2*e*x + 9240*a**4*d*e**2*x**2 + 2310*a*
*4*e**3*x**3 + 4620*a**3*b*d**3*x + 9240*a**3*b*d**2*e*x**2 + 6930*a**3*b*
d*e**2*x**3 + 1848*a**3*b*e**3*x**4 + 9240*a**3*c*d**3*x**2 + 20790*a**3*c
*d**2*e*x**3 + 16632*a**3*c*d*e**2*x**4 + 4620*a**3*c*e**3*x**5 + 6930*a**
2*b*c*d**3*x**3 + 16632*a**2*b*c*d**2*e*x**4 + 13860*a**2*b*c*d*e**2*x**5
+ 3960*a**2*b*c*e**3*x**6 + 5544*a**2*c**2*d**3*x**4 + 13860*a**2*c**2*d**
2*e*x**5 + 11880*a**2*c**2*d*e**2*x**6 + 3465*a**2*c**2*e**3*x**7 + 4620*a
*b*c**2*d**3*x**5 + 11880*a*b*c**2*d**2*e*x**6 + 10395*a*b*c**2*d*e**2*x**
7 + 3080*a*b*c**2*e**3*x**8 + 1320*a*c**3*d**3*x**6 + 3465*a*c**3*d**2*e*x
**7 + 3080*a*c**3*d*e**2*x**8 + 924*a*c**3*e**3*x**9 + 1155*b*c**3*d**3*x*
*7 + 3080*b*c**3*d**2*e*x**8 + 2772*b*c**3*d*e**2*x**9 + 840*b*c**3*e**3*x
**10))/9240
```

3.65 $\int (A+Bx)(d+ex)^2 \left(a+cx^2\right)^3 dx$

Optimal result

Integrand size = 22, antiderivative size = 334

$$\begin{aligned}
&\int (A+Bx)(d+ex)^2 \left(a+cx^2\right)^3 dx \\
&= -\frac{(Bd-Ae)\left(cd^2+ae^2\right)^3(d+ex)^3}{3e^8} \\
&\quad + \frac{\left(cd^2+ae^2\right)^2\left(7Bcd^2-6Acde+aBe^2\right)(d+ex)^4}{4e^8} \\
&\quad - \frac{3c\left(cd^2+ae^2\right)\left(7Bcd^3-5Acd^2e+3aBde^2-aAe^3\right)(d+ex)^5}{5e^8} \\
&\quad - \frac{c\left(4Acde\left(5cd^2+3ae^2\right)-B\left(35c^2d^4+30acd^2e^2+3a^2e^4\right)\right)(d+ex)^6}{6e^8} \\
&\quad - \frac{c^2\left(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3\right)(d+ex)^7}{7e^8} \\
&\quad + \frac{3c^2\left(7Bcd^2-2Acde+aBe^2\right)(d+ex)^8}{8e^8} - \frac{c^3(7Bd-Ae)(d+ex)^9}{9e^8} + \frac{Bc^3(d+ex)^{10}}{10e^8}
\end{aligned}$$

output

```
-1/3*(-A*e+B*d)*(a*e^2+c*d^2)^3*(e*x+d)^3/e^8+1/4*(a*e^2+c*d^2)^2*(-6*A*c*
d*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^4/e^8-3/5*c*(a*e^2+c*d^2)*(-A*a*e^3-5*A*c*d
^2*e+3*B*a*d*e^2+7*B*c*d^3)*(e*x+d)^5/e^8-1/6*c*(4*A*c*d*e*(3*a*e^2+5*c*d^
2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))*(e*x+d)^6/e^8-1/7*c^2*(-3*A*a*
e^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)*(e*x+d)^7/e^8+3/8*c^2*(-2*A*c*d*
e+B*a*e^2+7*B*c*d^2)*(e*x+d)^8/e^8-1/9*c^3*(-A*e+7*B*d)*(e*x+d)^9/e^8+1/10
*B*c^3*(e*x+d)^10/e^8
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.71

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)^3\,dx = a^3Ad^2x+\frac{1}{2}a^3d(Bd+2Ae)x^2 + \frac{1}{3}a^2\left(3Acd^2+2aBde+aAe^2\right)x^3 + \frac{1}{4}a^2\left(3Bcd^2+6Acde+aBe^2\right)x^4 + \frac{3}{5}ac\left(Acd^2+2aBde+aAe^2\right)x^5 + \frac{1}{2}ac\left(Bcd^2+2Acde+aBe^2\right)x^6 + \frac{1}{7}c^2\left(Acd^2+6aBde+3aAe^2\right)x^7 + \frac{1}{8}c^2\left(Bcd^2+2Acde+3aBe^2\right)x^8 + \frac{1}{9}c^3e(2Bd+Ae)x^9+\frac{1}{10}Bc^3e^2x^{10}$$

input
```
Integrate[(A + B*x)*(d + e*x)^2*(a + c*x^2)^3,x]
```

output
```
a^3*A*d^2*x + (a^3*d*(B*d + 2*A*e)*x^2)/2 + (a^2*(3*A*c*d^2 + 2*a*B*d*e +
a*A*e^2)*x^3)/3 + (a^2*(3*B*c*d^2 + 6*A*c*d*e + a*B*e^2)*x^4)/4 + (3*a*c*(
A*c*d^2 + 2*a*B*d*e + a*A*e^2)*x^5)/5 + (a*c*(B*c*d^2 + 2*A*c*d*e + a*B*e^
2)*x^6)/2 + (c^2*(A*c*d^2 + 6*a*B*d*e + 3*a*A*e^2)*x^7)/7 + (c^2*(B*c*d^2
+ 2*A*c*d*e + 3*a*B*e^2)*x^8)/8 + (c^3*e*(2*B*d + A*e)*x^9)/9 + (B*c^3*e^2
*x^10)/10
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3 (A+Bx)(d+ex)^2 \, dx$$

$$\Big\downarrow \; 652$$

$$\int \left(-\frac{c(d+ex)^5 \left(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4\right)}{e^7}-\frac{3c^2(d+ex)^7 \left(-aBe^2+2Acd\right.}{e^7}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{c(d+ex)^6 \left(4Acde\left(3ae^2+5cd^2\right)-B\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)\right)}{6e^8}+\frac{3c^2(d+ex)^8 \left(aBe^2-2Acde+7Bcd^2\right)}{8e^8}-\frac{c^2(d+ex)^7 \left(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3\right)}{7e^8}+\frac{(d+ex)^4 \left(ae^2+cd^2\right)^2 \left(aBe^2-6Acde+7Bcd^2\right)}{4e^8}-\frac{(d+ex)^3 \left(ae^2+cd^2\right)^3 (Bd-Ae)}{3e^8}-\frac{3c(d+ex)^5 \left(ae^2+cd^2\right) \left(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3\right)}{5e^8}-\frac{c^3(d+ex)^9(7Bd-Ae)}{9e^8}+\frac{Bc^3(d+ex)^{10}}{10e^8}$$

input
```
Int[(A + B*x)*(d + e*x)^2*(a + c*x^2)^3,x]
```

output
```
-1/3*((B*d - A*e)*(c*d^2 + a*e^2)^3*(d + e*x)^3)/e^8 + ((c*d^2 + a*e^2)^2*
(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2)*(d + e*x)^4)/(4*e^8) - (3*c*(c*d^2 + a*e
^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*(d + e*x)^5)/(5*e^8)
 - (c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*
a^2*e^4))*(d + e*x)^6)/(6*e^8) - (c^2*(35*B*c*d^3 - 15*A*c*d^2*e + 15*a*B*
d*e^2 - 3*a*A*e^3)*(d + e*x)^7)/(7*e^8) + (3*c^2*(7*B*c*d^2 - 2*A*c*d*e +
a*B*e^2)*(d + e*x)^8)/(8*e^8) - (c^3*(7*B*d - A*e)*(d + e*x)^9)/(9*e^8) +
(B*c^3*(d + e*x)^10)/(10*e^8)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.75

method	result
default	$\frac{B\,e^{2} c^{3} x^{10}}{10}+\frac{\left(A\,e^{2}+2 B d e \right) c^{3} x^{9}}{9}+\frac{\left(\left(2 A d e +B\,d^{2}\right) c^{3}+3 B\,e^{2} a\,c^{2}\right) x^{8}}{8}+\frac{\left(A\,c^{3} d^{2}+3 \left(A\,e^{2}+2 B d e \right) a\,c^{2}\right) x^{7}}{7}+\frac{(3(2Ade+B$
norman	$\frac{B\,e^{2} c^{3} x^{10}}{10}+\left(\frac{1}{9} A\,c^{3} e^{2}+\frac{2}{9} B\,c^{3} d e \right) x^{9}+\left(\frac{1}{4} A\,c^{3} d e +\frac{3}{8} B\,e^{2} a\,c^{2}+\frac{1}{8} B\,c^{3} d^{2}\right) x^{8}+\left(\frac{3}{7} A a\,c^{2} e^{2}+\frac{1}{7}\right.$
gosper	$\frac{1}{10} B\,e^{2} c^{3} x^{10}+\frac{1}{9} x^{9} A\,c^{3} e^{2}+\frac{2}{9} x^{9} B\,c^{3} d e +\frac{1}{4} x^{8} A\,c^{3} d e +\frac{3}{8} x^{8} B\,e^{2} a\,c^{2}+\frac{1}{8} x^{8} B\,c^{3} d^{2}+\frac{3}{7} x^{7} A a\,c^{2}$
risch	$\frac{1}{10} B\,e^{2} c^{3} x^{10}+\frac{1}{9} x^{9} A\,c^{3} e^{2}+\frac{2}{9} x^{9} B\,c^{3} d e +\frac{1}{4} x^{8} A\,c^{3} d e +\frac{3}{8} x^{8} B\,e^{2} a\,c^{2}+\frac{1}{8} x^{8} B\,c^{3} d^{2}+\frac{3}{7} x^{7} A a\,c^{2}$
parallelrisch	$\frac{1}{10} B\,e^{2} c^{3} x^{10}+\frac{1}{9} x^{9} A\,c^{3} e^{2}+\frac{2}{9} x^{9} B\,c^{3} d e +\frac{1}{4} x^{8} A\,c^{3} d e +\frac{3}{8} x^{8} B\,e^{2} a\,c^{2}+\frac{1}{8} x^{8} B\,c^{3} d^{2}+\frac{3}{7} x^{7} A a\,c^{2}$
orering	$\frac{x\left(252 B\,e^{2} c^{3} x^{9}+280 A\,c^{3} e^{2} x^{8}+560 B\,c^{3} d e\,x^{8}+630 A\,c^{3} d e\,x^{7}+945 B a\,c^{2} e^{2} x^{7}+315 B\,c^{3} d^{2} x^{7}+1080 A a\,c^{2} e^{2} x^{6}+360 A\,c^{3} d^{2} x^{6}+\right.}{}$

input
```
int((B*x+A)*(e*x+d)^2*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*B*e^2*c^3*x^10+1/9*(A*e^2+2*B*d*e)*c^3*x^9+1/8*((2*A*d*e+B*d^2)*c^3+3
*B*e^2*a*c^2)*x^8+1/7*(A*c^3*d^2+3*(A*e^2+2*B*d*e)*a*c^2)*x^7+1/6*(3*(2*A*
d*e+B*d^2)*a*c^2+3*B*e^2*a^2*c)*x^6+1/5*(3*A*d^2*a*c^2+3*(A*e^2+2*B*d*e)*a
^2*c)*x^5+1/4*(3*(2*A*d*e+B*d^2)*a^2*c+B*e^2*a^3)*x^4+1/3*(3*A*d^2*a^2*c+(
A*e^2+2*B*d*e)*a^3)*x^3+1/2*(2*A*d*e+B*d^2)*a^3*x^2+A*d^2*a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.78

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)^3\,dx=\frac{1}{10}Bc^3e^2x^{10}+\frac{1}{9}\left(2Bc^3de+Ac^3e^2\right)x^9+\frac{1}{8}\left(Bc^3d^2+2Ac^3de+3Bac^2e^2\right)x^8+\frac{1}{7}\left(Ac^3d^2+6Bac^2de+3Aac^2e^2\right)x^7+Aa^3d^2x+\frac{1}{2}\left(Bac^2d^2+2Aac^2de+Ba^2ce^2\right)x^6+\frac{3}{5}\left(Aac^2d^2+2Ba^2cde+Aa^2ce^2\right)x^5+\frac{1}{4}\left(3Ba^2cd^2+6Aa^2cde+Ba^3e^2\right)x^4+\frac{1}{3}\left(3Aa^2cd^2+2Ba^3de+Aa^3e^2\right)x^3+\frac{1}{2}\left(Ba^3d^2+2Aa^3de\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)^2*(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/10*B*c^3*e^2*x^10 + 1/9*(2*B*c^3*d*e + A*c^3*e^2)*x^9 + 1/8*(B*c^3*d^2 +
 2*A*c^3*d*e + 3*B*a*c^2*e^2)*x^8 + 1/7*(A*c^3*d^2 + 6*B*a*c^2*d*e + 3*A*a
*c^2*e^2)*x^7 + A*a^3*d^2*x + 1/2*(B*a*c^2*d^2 + 2*A*a*c^2*d*e + B*a^2*c*e
^2)*x^6 + 3/5*(A*a*c^2*d^2 + 2*B*a^2*c*d*e + A*a^2*c*e^2)*x^5 + 1/4*(3*B*a
^2*c*d^2 + 6*A*a^2*c*d*e + B*a^3*e^2)*x^4 + 1/3*(3*A*a^2*c*d^2 + 2*B*a^3*d
*e + A*a^3*e^2)*x^3 + 1/2*(B*a^3*d^2 + 2*A*a^3*d*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.92

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)^3\,dx = Aa^3d^2x + \frac{Bc^3e^2x^{10}}{10} + x^9\left(\frac{Ac^3e^2}{9} + \frac{2Bc^3de}{9}\right) + x^8\left(\frac{Ac^3de}{4} + \frac{3Bac^2e^2}{8} + \frac{Bc^3d^2}{8}\right) + x^7\cdot\left(\frac{3Aac^2e^2}{7} + \frac{Ac^3d^2}{7} + \frac{6Bac^2de}{7}\right) + x^6\left(Aac^2de + \frac{Ba^2ce^2}{2} + \frac{Bac^2d^2}{2}\right) + x^5\cdot\left(\frac{3Aa^2ce^2}{5} + \frac{3Aac^2d^2}{5} + \frac{6Ba^2cde}{5}\right) + x^4\cdot\left(\frac{3Aa^2cde}{2} + \frac{Ba^3e^2}{4} + \frac{3Ba^2cd^2}{4}\right) + x^3\left(\frac{Aa^3e^2}{3} + Aa^2cd^2 + \frac{2Ba^3de}{3}\right) + x^2\left(Aa^3de + \frac{Ba^3d^2}{2}\right)$$

input
```
integrate((B*x+A)*(e*x+d)**2*(c*x**2+a)**3,x)
```

output
```
A*a**3*d**2*x + B*c**3*e**2*x**10/10 + x**9*(A*c**3*e**2/9 + 2*B*c**3*d*e/
9) + x**8*(A*c**3*d*e/4 + 3*B*a*c**2*e**2/8 + B*c**3*d**2/8) + x**7*(3*A*a
*c**2*e**2/7 + A*c**3*d**2/7 + 6*B*a*c**2*d*e/7) + x**6*(A*a*c**2*d*e + B*
a**2*c*e**2/2 + B*a*c**2*d**2/2) + x**5*(3*A*a**2*c*e**2/5 + 3*A*a*c**2*d*
*2/5 + 6*B*a**2*c*d*e/5) + x**4*(3*A*a**2*c*d*e/2 + B*a**3*e**2/4 + 3*B*a*
*2*c*d**2/4) + x**3*(A*a**3*e**2/3 + A*a**2*c*d**2 + 2*B*a**3*d*e/3) + x**
2*(A*a**3*d*e + B*a**3*d**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.78

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)^3\,dx = \frac{1}{10}\,Bc^3e^2x^{10} + \frac{1}{9}\left(2\,Bc^3de + Ac^3e^2\right)x^9 + \frac{1}{8}\left(Bc^3d^2 + 2\,Ac^3de + 3\,Bac^2e^2\right)x^8 + \frac{1}{7}\left(Ac^3d^2 + 6\,Bac^2de + 3\,Aac^2e^2\right)x^7 + Aa^3d^2x + \frac{1}{2}\left(Bac^2d^2 + 2\,Aac^2de + Ba^2ce^2\right)x^6 + \frac{3}{5}\left(Aac^2d^2 + 2\,Ba^2cde + Aa^2ce^2\right)x^5 + \frac{1}{4}\left(3\,Ba^2cd^2 + 6\,Aa^2cde + Ba^3e^2\right)x^4 + \frac{1}{3}\left(3\,Aa^2cd^2 + 2\,Ba^3de + Aa^3e^2\right)x^3 + \frac{1}{2}\left(Ba^3d^2 + 2\,Aa^3de\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)^2*(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/10*B*c^3*e^2*x^10 + 1/9*(2*B*c^3*d*e + A*c^3*e^2)*x^9 + 1/8*(B*c^3*d^2 +
 2*A*c^3*d*e + 3*B*a*c^2*e^2)*x^8 + 1/7*(A*c^3*d^2 + 6*B*a*c^2*d*e + 3*A*a
*c^2*e^2)*x^7 + A*a^3*d^2*x + 1/2*(B*a*c^2*d^2 + 2*A*a*c^2*d*e + B*a^2*c*e
^2)*x^6 + 3/5*(A*a*c^2*d^2 + 2*B*a^2*c*d*e + A*a^2*c*e^2)*x^5 + 1/4*(3*B*a
^2*c*d^2 + 6*A*a^2*c*d*e + B*a^3*e^2)*x^4 + 1/3*(3*A*a^2*c*d^2 + 2*B*a^3*d
*e + A*a^3*e^2)*x^3 + 1/2*(B*a^3*d^2 + 2*A*a^3*d*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.86

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)^3\,dx = \frac{1}{10}Bc^3e^2x^{10}+\frac{2}{9}Bc^3dex^9+\frac{1}{9}Ac^3e^2x^9+\frac{1}{8}Bc^3d^2x^8+\frac{1}{4}Ac^3dex^8+\frac{3}{8}Bac^2e^2x^8+\frac{1}{7}Ac^3d^2x^7+\frac{6}{7}Bac^2dex^7+\frac{3}{7}Aac^2e^2x^7+\frac{1}{2}Bac^2d^2x^6+Aac^2dex^6+\frac{1}{2}Ba^2ce^2x^6+\frac{3}{5}Aac^2d^2x^5+\frac{6}{5}Ba^2cdex^5+\frac{3}{5}Aa^2ce^2x^5+\frac{3}{4}Ba^2cd^2x^4+\frac{3}{2}Aa^2cdex^4+\frac{1}{4}Ba^3e^2x^4+Aa^2cd^2x^3+\frac{2}{3}Ba^3dex^3+\frac{1}{3}Aa^3e^2x^3+\frac{1}{2}Ba^3d^2x^2+Aa^3dex^2+Aa^3d^2x$$

input
```
integrate((B*x+A)*(e*x+d)^2*(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/10*B*c^3*e^2*x^10 + 2/9*B*c^3*d*e*x^9 + 1/9*A*c^3*e^2*x^9 + 1/8*B*c^3*d^
2*x^8 + 1/4*A*c^3*d*e*x^8 + 3/8*B*a*c^2*e^2*x^8 + 1/7*A*c^3*d^2*x^7 + 6/7*
B*a*c^2*d*e*x^7 + 3/7*A*a*c^2*e^2*x^7 + 1/2*B*a*c^2*d^2*x^6 + A*a*c^2*d*e*
x^6 + 1/2*B*a^2*c*e^2*x^6 + 3/5*A*a*c^2*d^2*x^5 + 6/5*B*a^2*c*d*e*x^5 + 3/
5*A*a^2*c*e^2*x^5 + 3/4*B*a^2*c*d^2*x^4 + 3/2*A*a^2*c*d*e*x^4 + 1/4*B*a^3*
e^2*x^4 + A*a^2*c*d^2*x^3 + 2/3*B*a^3*d*e*x^3 + 1/3*A*a^3*e^2*x^3 + 1/2*B*
a^3*d^2*x^2 + A*a^3*d*e*x^2 + A*a^3*d^2*x
```

Mupad [B] (verification not implemented)

Time = 6.89 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.68

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)^3\,dx = x^3\left(\frac{2\,B\,a^3\,d\,e}{3}+\frac{A\,a^3\,e^2}{3}+A\,c\,a^2\,d^2\right)+x^8\left(\frac{B\,c^3\,d^2}{8}+\frac{A\,c^3\,d\,e}{4}+\frac{3\,B\,a\,c^2\,e^2}{8}\right)+\frac{c^2\,x^7\,(A\,c\,d^2+6\,B\,a\,d\,e+3\,A\,a\,e^2)}{7}+\frac{a^2\,x^4\,(3\,B\,c\,d^2+6\,A\,c\,d\,e+B\,a\,e^2)}{4}+A\,a^3\,d^2\,x+\frac{3\,a\,c\,x^5\,(A\,c\,d^2+2\,B\,a\,d\,e+A\,a\,e^2)}{5}+\frac{a\,c\,x^6\,(B\,c\,d^2+2\,A\,c\,d\,e+B\,a\,e^2)}{2}+\frac{a^3\,d\,x^2\,(2\,A\,e+B\,d)}{2}+\frac{c^3\,e\,x^9\,(A\,e+2\,B\,d)}{9}+\frac{B\,c^3\,e^2\,x^{10}}{10}$$

input
```
int((a + c*x^2)^3*(A + B*x)*(d + e*x)^2,x)
```

output
```
x^3*((A*a^3*e^2)/3 + (2*B*a^3*d*e)/3 + A*a^2*c*d^2) + x^8*((B*c^3*d^2)/8 +
 (A*c^3*d*e)/4 + (3*B*a*c^2*e^2)/8) + (c^2*x^7*(3*A*a*e^2 + A*c*d^2 + 6*B*
a*d*e))/7 + (a^2*x^4*(B*a*e^2 + 3*B*c*d^2 + 6*A*c*d*e))/4 + A*a^3*d^2*x +
(3*a*c*x^5*(A*a*e^2 + A*c*d^2 + 2*B*a*d*e))/5 + (a*c*x^6*(B*a*e^2 + B*c*d^
2 + 2*A*c*d*e))/2 + (a^3*d*x^2*(2*A*e + B*d))/2 + (c^3*e*x^9*(A*e + 2*B*d)
)/9 + (B*c^3*e^2*x^10)/10
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.86

$$\int (A+Bx)(d+ex)^2\left(a+cx^2\right)^3\,dx = \frac{x(252b\,c^3e^2x^9+280a\,c^3e^2x^8+560b\,c^3de\,x^8+945ab\,c^2e^2x^7+630a\,c^3de\,x^7+315b\,c^3d^2x^7+1080a^2c^2e^2x^6}{}$$

input
```
int((B*x+A)*(e*x+d)^2*(c*x^2+a)^3,x)
```

output
```
(x*(2520*a**4*d**2 + 2520*a**4*d*e*x + 840*a**4*e**2*x**2 + 1260*a**3*b*d*
*2*x + 1680*a**3*b*d*e*x**2 + 630*a**3*b*e**2*x**3 + 2520*a**3*c*d**2*x**2
 + 3780*a**3*c*d*e*x**3 + 1512*a**3*c*e**2*x**4 + 1890*a**2*b*c*d**2*x**3
+ 3024*a**2*b*c*d*e*x**4 + 1260*a**2*b*c*e**2*x**5 + 1512*a**2*c**2*d**2*x
**4 + 2520*a**2*c**2*d*e*x**5 + 1080*a**2*c**2*e**2*x**6 + 1260*a*b*c**2*d
**2*x**5 + 2160*a*b*c**2*d*e*x**6 + 945*a*b*c**2*e**2*x**7 + 360*a*c**3*d*
*2*x**6 + 630*a*c**3*d*e*x**7 + 280*a*c**3*e**2*x**8 + 315*b*c**3*d**2*x**
7 + 560*b*c**3*d*e*x**8 + 252*b*c**3*e**2*x**9))/2520
```

3.66 $\int (A+Bx)(d+ex)\left(a+cx^2\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 148

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^3\,dx = a^3Adx+\frac{1}{2}a^3(Bd+Ae)x^2+\frac{1}{3}a^2(3Acd+aBe)x^3 +\frac{3}{4}a^2c(Bd+Ae)x^4+\frac{3}{5}ac(Acd+aBe)x^5 +\frac{1}{2}ac^2(Bd+Ae)x^6+\frac{1}{7}c^2(Acd+3aBe)x^7 +\frac{1}{8}c^3(Bd+Ae)x^8+\frac{1}{9}Bc^3ex^9$$

output
```
a^3*A*d*x+1/2*a^3*(A*e+B*d)*x^2+1/3*a^2*(3*A*c*d+B*a*e)*x^3+3/4*a^2*c*(A*e
+B*d)*x^4+3/5*a*c*(A*c*d+B*a*e)*x^5+1/2*a*c^2*(A*e+B*d)*x^6+1/7*c^2*(A*c*d
+3*B*a*e)*x^7+1/8*c^3*(A*e+B*d)*x^8+1/9*B*c^3*e*x^9
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^3\,dx = \frac{1}{6}a^3x(3A(2d+ex)+Bx(3d+2ex)) + \frac{1}{20}a^2cx^3(5A(4d+3ex)+3Bx(5d+4ex)) + \frac{1}{70}ac^2x^5(7A(6d+5ex)+5Bx(7d+6ex)) + \frac{1}{504}c^3x^7(9A(8d+7ex)+7Bx(9d+8ex))$$

input
```
Integrate[(A + B*x)*(d + e*x)*(a + c*x^2)^3,x]
```

output
```
(a^3*x*(3*A*(2*d + e*x) + B*x*(3*d + 2*e*x)))/6 + (a^2*c*x^3*(5*A*(4*d + 3
*e*x) + 3*B*x*(5*d + 4*e*x)))/20 + (a*c^2*x^5*(7*A*(6*d + 5*e*x) + 5*B*x*(
7*d + 6*e*x)))/70 + (c^3*x^7*(9*A*(8*d + 7*e*x) + 7*B*x*(9*d + 8*e*x)))/50
4
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3(A+Bx)(d+ex)\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(a^3x(Ae+Bd)+a^3Ad+3a^2cx^3(Ae+Bd)+a^2x^2(aBe+3Acd)+c^2x^6(3aBe+Acd)+3ac^2x^5(Ae+Bd)+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^3x^2(Ae+Bd)+a^3Adx+\frac{3}{4}a^2cx^4(Ae+Bd)+\frac{1}{3}a^2x^3(aBe+3Acd)+\frac{1}{7}c^2x^7(3aBe+Acd)+\frac{1}{2}ac^2x^6(Ae+Bd)+\frac{3}{5}acx^5(aBe+Acd)+\frac{1}{8}c^3x^8(Ae+Bd)+\frac{1}{9}Bc^3ex^9$$

input
```
Int[(A + B*x)*(d + e*x)*(a + c*x^2)^3,x]
```

output
```
a^3*A*d*x + (a^3*(B*d + A*e)*x^2)/2 + (a^2*(3*A*c*d + a*B*e)*x^3)/3 + (3*a
^2*c*(B*d + A*e)*x^4)/4 + (3*a*c*(A*c*d + a*B*e)*x^5)/5 + (a*c^2*(B*d + A*
e)*x^6)/2 + (c^2*(A*c*d + 3*a*B*e)*x^7)/7 + (c^3*(B*d + A*e)*x^8)/8 + (B*c
^3*e*x^9)/9
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.97

method	result
default	$\frac{B\,c^3 e\,x^9}{9}+\frac{c^3(Ae+Bd)x^8}{8}+\frac{(A\,c^3 d+3Bea\,c^2)x^7}{7}+\frac{a\,c^2(Ae+Bd)x^6}{2}+\frac{(3Ada\,c^2+3Be\,a^2 c)x^5}{5}+\frac{3a^2 c(Ae+Bd)x^4}{4}+$
norman	$\frac{B\,c^3 e\,x^9}{9}+\left(\frac{1}{8}A\,c^3 e+\frac{1}{8}B\,c^3 d\right)x^8+\left(\frac{1}{7}A\,c^3 d+\frac{3}{7}Bea\,c^2\right)x^7+\left(\frac{1}{2}Aa\,c^2 e+\frac{1}{2}Ba\,c^2 d\right)x^6+\left(\frac{3}{5}A\right.$
gosper	$\frac{1}{9}B\,c^3 e\,x^9+\frac{1}{8}x^8 A\,c^3 e+\frac{1}{8}x^8 B\,c^3 d+\frac{1}{7}x^7 A\,c^3 d+\frac{3}{7}x^7 Bea\,c^2+\frac{1}{2}x^6 Aa\,c^2 e+\frac{1}{2}x^6 Ba\,c^2 d+\frac{3}{5}x$
risch	$\frac{1}{9}B\,c^3 e\,x^9+\frac{1}{8}x^8 A\,c^3 e+\frac{1}{8}x^8 B\,c^3 d+\frac{1}{7}x^7 A\,c^3 d+\frac{3}{7}x^7 Bea\,c^2+\frac{1}{2}x^6 Aa\,c^2 e+\frac{1}{2}x^6 Ba\,c^2 d+\frac{3}{5}x$
parallelrisch	$\frac{1}{9}B\,c^3 e\,x^9+\frac{1}{8}x^8 A\,c^3 e+\frac{1}{8}x^8 B\,c^3 d+\frac{1}{7}x^7 A\,c^3 d+\frac{3}{7}x^7 Bea\,c^2+\frac{1}{2}x^6 Aa\,c^2 e+\frac{1}{2}x^6 Ba\,c^2 d+\frac{3}{5}x$
orering	$\frac{x(280Be\,c^3 x^8+315A\,c^3 e\,x^7+315B\,c^3 d\,x^7+360A\,c^3 d\,x^6+1080Ba\,c^2 e\,x^6+1260Aa\,c^2 e\,x^5+1260Ba\,c^2 d\,x^5+1512Aa\,c^2 d\,x^4+15}{2520}$

input
```
int((B*x+A)*(e*x+d)*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*B*c^3*e*x^9+1/8*c^3*(A*e+B*d)*x^8+1/7*(A*c^3*d+3*B*a*c^2*e)*x^7+1/2*a*
c^2*(A*e+B*d)*x^6+1/5*(3*A*a*c^2*d+3*B*a^2*c*e)*x^5+3/4*a^2*c*(A*e+B*d)*x^
4+1/3*(3*A*a^2*c*d+B*a^3*e)*x^3+1/2*a^3*(A*e+B*d)*x^2+a^3*A*d*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.04

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^3\,dx = \frac{1}{9}Bc^3ex^9 + \frac{1}{8}\left(Bc^3d+Ac^3e\right)x^8 + \frac{1}{7}\left(Ac^3d+3\,Bac^2e\right)x^7 + \frac{1}{2}\left(Bac^2d+Aac^2e\right)x^6 + Aa^3dx + \frac{3}{5}\left(Aac^2d+Ba^2ce\right)x^5 + \frac{3}{4}\left(Ba^2cd+Aa^2ce\right)x^4 + \frac{1}{3}\left(3\,Aa^2cd+Ba^3e\right)x^3 + \frac{1}{2}\left(Ba^3d+Aa^3e\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)*(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/9*B*c^3*e*x^9 + 1/8*(B*c^3*d + A*c^3*e)*x^8 + 1/7*(A*c^3*d + 3*B*a*c^2*e
)*x^7 + 1/2*(B*a*c^2*d + A*a*c^2*e)*x^6 + A*a^3*d*x + 3/5*(A*a*c^2*d + B*a
^2*c*e)*x^5 + 3/4*(B*a^2*c*d + A*a^2*c*e)*x^4 + 1/3*(3*A*a^2*c*d + B*a^3*e
)*x^3 + 1/2*(B*a^3*d + A*a^3*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.23

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^3\,dx = Aa^3dx + \frac{Bc^3ex^9}{9} + x^8\left(\frac{Ac^3e}{8}+\frac{Bc^3d}{8}\right) + x^7\left(\frac{Ac^3d}{7}+\frac{3Bac^2e}{7}\right) + x^6\left(\frac{Aac^2e}{2}+\frac{Bac^2d}{2}\right) + x^5\cdot\left(\frac{3Aac^2d}{5}+\frac{3Ba^2ce}{5}\right) + x^4\cdot\left(\frac{3Aa^2ce}{4}+\frac{3Ba^2cd}{4}\right) + x^3\left(Aa^2cd+\frac{Ba^3e}{3}\right) + x^2\left(\frac{Aa^3e}{2}+\frac{Ba^3d}{2}\right)$$

input
```
integrate((B*x+A)*(e*x+d)*(c*x**2+a)**3,x)
```

output
```
A*a**3*d*x + B*c**3*e*x**9/9 + x**8*(A*c**3*e/8 + B*c**3*d/8) + x**7*(A*c*
*3*d/7 + 3*B*a*c**2*e/7) + x**6*(A*a*c**2*e/2 + B*a*c**2*d/2) + x**5*(3*A*
a*c**2*d/5 + 3*B*a**2*c*e/5) + x**4*(3*A*a**2*c*e/4 + 3*B*a**2*c*d/4) + x*
*3*(A*a**2*c*d + B*a**3*e/3) + x**2*(A*a**3*e/2 + B*a**3*d/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.04

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^3\,dx = \frac{1}{9}Bc^3ex^9 + \frac{1}{8}\left(Bc^3d+Ac^3e\right)x^8 + \frac{1}{7}\left(Ac^3d+3\,Bac^2e\right)x^7 + \frac{1}{2}\left(Bac^2d+Aac^2e\right)x^6 + Aa^3dx + \frac{3}{5}\left(Aac^2d+Ba^2ce\right)x^5 + \frac{3}{4}\left(Ba^2cd+Aa^2ce\right)x^4 + \frac{1}{3}\left(3\,Aa^2cd+Ba^3e\right)x^3 + \frac{1}{2}\left(Ba^3d+Aa^3e\right)x^2$$

input
```
integrate((B*x+A)*(e*x+d)*(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/9*B*c^3*e*x^9 + 1/8*(B*c^3*d + A*c^3*e)*x^8 + 1/7*(A*c^3*d + 3*B*a*c^2*e
)*x^7 + 1/2*(B*a*c^2*d + A*a*c^2*e)*x^6 + A*a^3*d*x + 3/5*(A*a*c^2*d + B*a
^2*c*e)*x^5 + 3/4*(B*a^2*c*d + A*a^2*c*e)*x^4 + 1/3*(3*A*a^2*c*d + B*a^3*e
)*x^3 + 1/2*(B*a^3*d + A*a^3*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^3\,dx = \frac{1}{9}Bc^3ex^9 + \frac{1}{8}Bc^3dx^8 + \frac{1}{8}Ac^3ex^8 + \frac{1}{7}Ac^3dx^7 + \frac{3}{7}Bac^2ex^7 + \frac{1}{2}Bac^2dx^6 + \frac{1}{2}Aac^2ex^6 + \frac{3}{5}Aac^2dx^5 + \frac{3}{5}Ba^2cex^5 + \frac{3}{4}Ba^2cdx^4 + \frac{3}{4}Aa^2cex^4 + Aa^2cdx^3 + \frac{1}{3}Ba^3ex^3 + \frac{1}{2}Ba^3dx^2 + \frac{1}{2}Aa^3ex^2 + Aa^3dx$$

input
```
integrate((B*x+A)*(e*x+d)*(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/9*B*c^3*e*x^9 + 1/8*B*c^3*d*x^8 + 1/8*A*c^3*e*x^8 + 1/7*A*c^3*d*x^7 + 3/
7*B*a*c^2*e*x^7 + 1/2*B*a*c^2*d*x^6 + 1/2*A*a*c^2*e*x^6 + 3/5*A*a*c^2*d*x^
5 + 3/5*B*a^2*c*e*x^5 + 3/4*B*a^2*c*d*x^4 + 3/4*A*a^2*c*e*x^4 + A*a^2*c*d*
x^3 + 1/3*B*a^3*e*x^3 + 1/2*B*a^3*d*x^2 + 1/2*A*a^3*e*x^2 + A*a^3*d*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^3\,dx = x^3\left(\frac{B\,e\,a^3}{3} + A\,c\,d\,a^2\right) + x^7\left(\frac{A\,d\,c^3}{7} + \frac{3\,B\,a\,e\,c^2}{7}\right) + x^5\left(\frac{3\,B\,e\,a^2\,c}{5} + \frac{3\,A\,d\,a\,c^2}{5}\right) + \frac{a^3\,x^2\,(A\,e+B\,d)}{2} + \frac{c^3\,x^8\,(A\,e+B\,d)}{8} + A\,a^3\,d\,x + \frac{B\,c^3\,e\,x^9}{9} + \frac{3\,a^2\,c\,x^4\,(A\,e+B\,d)}{4} + \frac{a\,c^2\,x^6\,(A\,e+B\,d)}{2}$$

input
```
int((a + c*x^2)^3*(A + B*x)*(d + e*x),x)
```

output
```
x^3*((B*a^3*e)/3 + A*a^2*c*d) + x^7*((A*c^3*d)/7 + (3*B*a*c^2*e)/7) + x^5*
((3*A*a*c^2*d)/5 + (3*B*a^2*c*e)/5) + (a^3*x^2*(A*e + B*d))/2 + (c^3*x^8*(
A*e + B*d))/8 + A*a^3*d*x + (B*c^3*e*x^9)/9 + (3*a^2*c*x^4*(A*e + B*d))/4
+ (a*c^2*x^6*(A*e + B*d))/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.10

$$\int (A+Bx)(d+ex)\left(a+cx^2\right)^3\,dx$$
$$= \frac{x(280b\,c^3e\,x^8+315a\,c^3e\,x^7+315b\,c^3d\,x^7+1080ab\,c^2e\,x^6+360a\,c^3d\,x^6+1260a^2c^2e\,x^5+1260ab\,c^2d\,x^5}{}$$

input
```
int((B*x+A)*(e*x+d)*(c*x^2+a)^3,x)
```

output
```
(x*(2520*a**4*d + 1260*a**4*e*x + 1260*a**3*b*d*x + 840*a**3*b*e*x**2 + 25
20*a**3*c*d*x**2 + 1890*a**3*c*e*x**3 + 1890*a**2*b*c*d*x**3 + 1512*a**2*b
*c*e*x**4 + 1512*a**2*c**2*d*x**4 + 1260*a**2*c**2*e*x**5 + 1260*a*b*c**2*
d*x**5 + 1080*a*b*c**2*e*x**6 + 360*a*c**3*d*x**6 + 315*a*c**3*e*x**7 + 31
5*b*c**3*d*x**7 + 280*b*c**3*e*x**8))/2520
```

3.67 $\int (A+Bx)\left(a+cx^2\right)^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int (A+Bx)\left(a+cx^2\right)^3\,dx=a^3Ax+a^2Acx^3+\frac{3}{5}aAc^2x^5+\frac{1}{7}Ac^3x^7+\frac{B(a+cx^2)^4}{8c}$$

output
```
a^3*A*x+a^2*A*c*x^3+3/5*a*A*c^2*x^5+1/7*A*c^3*x^7+1/8*B*(c*x^2+a)^4/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.52

$$\int (A+Bx)\left(a+cx^2\right)^3\,dx=a^3Ax+\frac{1}{2}a^3Bx^2+a^2Acx^3+\frac{3}{4}a^2Bcx^4+\frac{3}{5}aAc^2x^5+\frac{1}{2}aBc^2x^6+\frac{1}{7}Ac^3x^7+\frac{1}{8}Bc^3x^8$$

input
```
Integrate[(A + B*x)*(a + c*x^2)^3,x]
```

output
```
a^3*A*x + (a^3*B*x^2)/2 + a^2*A*c*x^3 + (3*a^2*B*c*x^4)/4 + (3*a*A*c^2*x^5
)/5 + (a*B*c^2*x^6)/2 + (A*c^3*x^7)/7 + (B*c^3*x^8)/8
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {455, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^3 (A+Bx) \, dx$$

$$\Big\downarrow \text{455}$$

$$A\int (cx^2+a)^3 dx + \frac{B(a+cx^2)^4}{8c}$$

$$\Big\downarrow \text{210}$$

$$A\int (c^3x^6+3ac^2x^4+3a^2cx^2+a^3) \, dx + \frac{B(a+cx^2)^4}{8c}$$

$$\Big\downarrow \text{2009}$$

$$A\left(a^3x+a^2cx^3+\frac{3}{5}ac^2x^5+\frac{c^3x^7}{7}\right)+\frac{B(a+cx^2)^4}{8c}$$

input
```
Int[(A + B*x)*(a + c*x^2)^3,x]
```

output
```
(B*(a + c*x^2)^4)/(8*c) + A*(a^3*x + a^2*c*x^3 + (3*a*c^2*x^5)/5 + (c^3*x^
7)/7)
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.32

method	result	siz
gosper	$\frac{1}{8}B\,c^{3}x^{8}+\frac{1}{7}A\,c^{3}x^{7}+\frac{1}{2}Ba\,c^{2}x^{6}+\frac{3}{5}aA\,c^{2}x^{5}+\frac{3}{4}B\,a^{2}c\,x^{4}+a^{2}Ac\,x^{3}+\frac{1}{2}B\,a^{3}x^{2}+a^{3}Ax$	74
default	$\frac{1}{8}B\,c^{3}x^{8}+\frac{1}{7}A\,c^{3}x^{7}+\frac{1}{2}Ba\,c^{2}x^{6}+\frac{3}{5}aA\,c^{2}x^{5}+\frac{3}{4}B\,a^{2}c\,x^{4}+a^{2}Ac\,x^{3}+\frac{1}{2}B\,a^{3}x^{2}+a^{3}Ax$	74
norman	$\frac{1}{8}B\,c^{3}x^{8}+\frac{1}{7}A\,c^{3}x^{7}+\frac{1}{2}Ba\,c^{2}x^{6}+\frac{3}{5}aA\,c^{2}x^{5}+\frac{3}{4}B\,a^{2}c\,x^{4}+a^{2}Ac\,x^{3}+\frac{1}{2}B\,a^{3}x^{2}+a^{3}Ax$	74
risch	$\frac{1}{8}B\,c^{3}x^{8}+\frac{1}{7}A\,c^{3}x^{7}+\frac{1}{2}Ba\,c^{2}x^{6}+\frac{3}{5}aA\,c^{2}x^{5}+\frac{3}{4}B\,a^{2}c\,x^{4}+a^{2}Ac\,x^{3}+\frac{1}{2}B\,a^{3}x^{2}+a^{3}Ax$	74
parallelrisch	$\frac{1}{8}B\,c^{3}x^{8}+\frac{1}{7}A\,c^{3}x^{7}+\frac{1}{2}Ba\,c^{2}x^{6}+\frac{3}{5}aA\,c^{2}x^{5}+\frac{3}{4}B\,a^{2}c\,x^{4}+a^{2}Ac\,x^{3}+\frac{1}{2}B\,a^{3}x^{2}+a^{3}Ax$	74
orering	$\frac{x\left(35B\,c^{3}x^{7}+40A\,c^{3}x^{6}+140Ba\,c^{2}x^{5}+168Aa\,c^{2}x^{4}+210B\,a^{2}c\,x^{3}+280a^{2}Ac\,x^{2}+140B\,a^{3}x+280a^{3}A\right)}{280}$	76

input
```
int((B*x+A)*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*B*c^3*x^8+1/7*A*c^3*x^7+1/2*B*a*c^2*x^6+3/5*a*A*c^2*x^5+3/4*B*a^2*c*x^
4+a^2*A*c*x^3+1/2*B*a^3*x^2+a^3*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int (A+Bx)\left(a+cx^2\right)^3\,dx=\frac{1}{8}\,Bc^3x^8+\frac{1}{7}\,Ac^3x^7+\frac{1}{2}\,Bac^2x^6+\frac{3}{5}\,Aac^2x^5+\frac{3}{4}\,Ba^2cx^4+Aa^2cx^3+\frac{1}{2}\,Ba^3x^2+Aa^3x$$

input
```
integrate((B*x+A)*(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/8*B*c^3*x^8 + 1/7*A*c^3*x^7 + 1/2*B*a*c^2*x^6 + 3/5*A*a*c^2*x^5 + 3/4*B*
a^2*c*x^4 + A*a^2*c*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.52

$$\int (A+Bx)\left(a+cx^2\right)^3\,dx = Aa^3x + Aa^2cx^3 + \frac{3Aac^2x^5}{5} + \frac{Ac^3x^7}{7} + \frac{Ba^3x^2}{2} + \frac{3Ba^2cx^4}{4} + \frac{Bac^2x^6}{2} + \frac{Bc^3x^8}{8}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3,x)
```

output
```
A*a**3*x + A*a**2*c*x**3 + 3*A*a*c**2*x**5/5 + A*c**3*x**7/7 + B*a**3*x**2
/2 + 3*B*a**2*c*x**4/4 + B*a*c**2*x**6/2 + B*c**3*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int (A+Bx)\left(a+cx^2\right)^3\,dx = \frac{1}{8}\,Bc^3x^8 + \frac{1}{7}\,Ac^3x^7 + \frac{1}{2}\,Bac^2x^6 + \frac{3}{5}\,Aac^2x^5 + \frac{3}{4}\,Ba^2cx^4 + Aa^2cx^3 + \frac{1}{2}\,Ba^3x^2 + Aa^3x$$

input
```
integrate((B*x+A)*(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*B*c^3*x^8 + 1/7*A*c^3*x^7 + 1/2*B*a*c^2*x^6 + 3/5*A*a*c^2*x^5 + 3/4*B*
a^2*c*x^4 + A*a^2*c*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int (A+Bx)\left(a+cx^2\right)^3\,dx=\frac{1}{8}Bc^3x^8+\frac{1}{7}Ac^3x^7+\frac{1}{2}Bac^2x^6+\frac{3}{5}Aac^2x^5+\frac{3}{4}Ba^2cx^4+Aa^2cx^3+\frac{1}{2}Ba^3x^2+Aa^3x$$

input
```
integrate((B*x+A)*(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*B*c^3*x^8 + 1/7*A*c^3*x^7 + 1/2*B*a*c^2*x^6 + 3/5*A*a*c^2*x^5 + 3/4*B*
a^2*c*x^4 + A*a^2*c*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int (A+Bx)\left(a+cx^2\right)^3\,dx=\frac{B\,a^3\,x^2}{2}+A\,a^3\,x+\frac{3\,B\,a^2\,c\,x^4}{4}+A\,a^2\,c\,x^3+\frac{B\,a\,c^2\,x^6}{2}+\frac{3\,A\,a\,c^2\,x^5}{5}+\frac{B\,c^3\,x^8}{8}+\frac{A\,c^3\,x^7}{7}$$

input
```
int((a + c*x^2)^3*(A + B*x),x)
```

output
```
(B*a^3*x^2)/2 + (A*c^3*x^7)/7 + (B*c^3*x^8)/8 + A*a^3*x + A*a^2*c*x^3 + (3
*A*a*c^2*x^5)/5 + (3*B*a^2*c*x^4)/4 + (B*a*c^2*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.32

$$\int (A+Bx)\left(a+cx^2\right)^3\,dx$$
$$=\frac{x(35b\,c^3x^7+40a\,c^3x^6+140ab\,c^2x^5+168a^2c^2x^4+210a^2bc\,x^3+280a^3c\,x^2+140a^3bx+280a^4)}{280}$$

input
```
int((B*x+A)*(c*x^2+a)^3,x)
```

output
```
(x*(280*a**4 + 140*a**3*b*x + 280*a**3*c*x**2 + 210*a**2*b*c*x**3 + 168*a*
*2*c**2*x**4 + 140*a*b*c**2*x**5 + 40*a*c**3*x**6 + 35*b*c**3*x**7))/280
```

3.68 $\int \frac{(A+Bx)(a+cx^2)^3}{d+ex} dx$

Optimal result

Integrand size = 22, antiderivative size = 290

$$\int \frac{(A+Bx)(a+cx^2)^3}{d+ex} dx = \frac{\left(B(cd^2+ae^2)^3 - Acde(c^2d^4+3acd^2e^2+3a^2e^4)\right)x}{e^7} - \frac{c(Bd-Ae)(c^2d^4+3acd^2e^2+3a^2e^4)x^2}{2e^6} - \frac{c(Acde(cd^2+3ae^2) - B(c^2d^4+3acd^2e^2+3a^2e^4))x^3}{3e^5} - \frac{c^2(Bd-Ae)(cd^2+3ae^2)x^4}{4e^4} + \frac{c^2(Bcd^2-Acde+3aBe^2)x^5}{5e^3} - \frac{c^3(Bd-Ae)x^6}{6e^2} + \frac{Bc^3x^7}{7e} - \frac{(Bd-Ae)(cd^2+ae^2)^3\log(d+ex)}{e^8}$$

output
```
(B*(a*e^2+c*d^2)^3-A*c*d*e*(3*a^2*e^4+3*a*c*d^2*e^2+c^2*d^4))*x/e^7-1/2*c*
(-A*e+B*d)*(3*a^2*e^4+3*a*c*d^2*e^2+c^2*d^4)*x^2/e^6-1/3*c*(A*c*d*e*(3*a*e
^2+c*d^2)-B*(3*a^2*e^4+3*a*c*d^2*e^2+c^2*d^4))*x^3/e^5-1/4*c^2*(-A*e+B*d)*
(3*a*e^2+c*d^2)*x^4/e^4+1/5*c^2*(-A*c*d*e+3*B*a*e^2+B*c*d^2)*x^5/e^3-1/6*c
^3*(-A*e+B*d)*x^6/e^2+1/7*B*c^3*x^7/e-(-A*e+B*d)*(a*e^2+c*d^2)^3*ln(e*x+d)
/e^8
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{d+ex}\,dx$$
$$= \frac{ex(7Ace(90a^2e^4(-2d+ex)+15ace^2(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+c^2(-60d^5+30d^4ex-20d^3e^2}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x),x]
```

output
```
(e*x*(7*A*c*e*(90*a^2*e^4*(-2*d + e*x) + 15*a*c*e^2*(-12*d^3 + 6*d^2*e*x -
 4*d*e^2*x^2 + 3*e^3*x^3) + c^2*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 1
5*d^2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5)) + B*(420*a^3*e^6 + 210*a^2*c*e
^4*(6*d^2 - 3*d*e*x + 2*e^2*x^2) + 21*a*c^2*e^2*(60*d^4 - 30*d^3*e*x + 20*
d^2*e^2*x^2 - 15*d*e^3*x^3 + 12*e^4*x^4) + c^3*(420*d^6 - 210*d^5*e*x + 14
0*d^4*e^2*x^2 - 105*d^3*e^3*x^3 + 84*d^2*e^4*x^4 - 70*d*e^5*x^5 + 60*e^6*x
^6))) - 420*(B*d - A*e)*(c*d^2 + a*e^2)^3*Log[d + e*x])/(420*e^8)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^3(A+Bx)}{d+ex}\,dx$$
$$\Big\downarrow \text{652}$$
$$\int \left(\frac{B\left(ae^2+cd^2\right)^3-Acde\left(3a^2e^4+3acd^2e^2+c^2d^4\right)}{e^7}+\frac{cx\left(3a^2e^4+3acd^2e^2+c^2d^4\right)(Ae-Bd)}{e^6}+\frac{cx^2\left(B\left(3a^2e^4\right.\right.}{}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{x\left(B\left(ae^2+cd^2\right)^3-Acde\left(3a^2e^4+3acd^2e^2+c^2d^4\right)\right)}{e^7}-\frac{cx^2\left(3a^2e^4+3acd^2e^2+c^2d^4\right)\left(Bd-Ae\right)}{2e^6}-\frac{cx^3\left(Acde\left(3ae^2+cd^2\right)-B\left(3a^2e^4+3acd^2e^2+c^2d^4\right)\right)}{3e^5}-\frac{c^2x^4\left(3ae^2+cd^2\right)\left(Bd-Ae\right)}{4e^4}+\frac{c^2x^5\left(3aBe^2-Acde+Bcd^2\right)}{5e^3}-\frac{\left(ae^2+cd^2\right)^3\left(Bd-Ae\right)\log(d+ex)}{e^8}-\frac{c^3x^6(Bd-Ae)}{6e^2}+\frac{Bc^3x^7}{7e}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x),x]
```

output
```
((B*(c*d^2 + a*e^2)^3 - A*c*d*e*(c^2*d^4 + 3*a*c*d^2*e^2 + 3*a^2*e^4))*x)/
e^7 - (c*(B*d - A*e)*(c^2*d^4 + 3*a*c*d^2*e^2 + 3*a^2*e^4)*x^2)/(2*e^6) -
(c*(A*c*d*e*(c*d^2 + 3*a*e^2) - B*(c^2*d^4 + 3*a*c*d^2*e^2 + 3*a^2*e^4))*x
^3)/(3*e^5) - (c^2*(B*d - A*e)*(c*d^2 + 3*a*e^2)*x^4)/(4*e^4) + (c^2*(B*c*
d^2 - A*c*d*e + 3*a*B*e^2)*x^5)/(5*e^3) - (c^3*(B*d - A*e)*x^6)/(6*e^2) +
(B*c^3*x^7)/(7*e) - ((B*d - A*e)*(c*d^2 + a*e^2)^3*Log[d + e*x])/e^8
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.43

method	result
norman	$-\frac{(3Aa^2cde^5+3Aac^2d^3e^3+Ac^3d^5e-Ba^3e^6-3Ba^2cd^2e^4-3Bac^2d^4e^2-Bc^3d^6)x}{e^7}+\frac{Bc^3x^7}{7e}+\frac{c(3Aa^2e^5+3Aacd^2e^3+$
default	$-\frac{3Aa^2cde^5x-3Bac^2d^4e^2x+\frac{3}{2}Bac^2d^3e^3x^2+\frac{3}{2}Ba^2cde^5x^2-Ba^2ce^6x^3-\frac{1}{3}Bc^3d^4e^2x^3-\frac{3}{2}Aa^2ce^6x^2+\frac{1}{6}Bc^3de^5x^6+\frac{1}{5}Ac$
risch	$\frac{3Aa^2cx^2}{2e}-\frac{Bc^3dx^6}{6e^2}-\frac{Ac^3dx^5}{5e^2}-\frac{Bc^3d^3x^4}{4e^4}-\frac{Aac^2dx^3}{e^2}+\frac{3Aac^2d^2x^2}{2e^3}-\frac{3Bac^2dx^4}{4e^2}+\frac{Bac^2d^2x^3}{e^3}+\frac{3\ln(ex$
parallelrisch	$\frac{70Ax^6c^3e^7+60Bx^7c^3e^7+420A\ln(ex+d)a^3e^7-420B\ln(ex+d)c^3d^7+420Bxa^3e^7+1260Bxac^2d^4e^3+105Ax^4c^3d^2e^5-105B$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-(3*A*a^2*c*d*e^5+3*A*a*c^2*d^3*e^3+A*c^3*d^5*e-B*a^3*e^6-3*B*a^2*c*d^2*e^
4-3*B*a*c^2*d^4*e^2-B*c^3*d^6)/e^7*x+1/7*B*c^3*x^7/e+1/2*c/e^6*(3*A*a^2*e^
5+3*A*a*c*d^2*e^3+A*c^2*d^4*e-3*B*a^2*d*e^4-3*B*a*c*d^3*e^2-B*c^2*d^5)*x^2
-1/3*c/e^5*(3*A*a*c*d*e^3+A*c^2*d^3*e-3*B*a^2*e^4-3*B*a*c*d^2*e^2-B*c^2*d^
4)*x^3+1/4*c^2/e^4*(3*A*a*e^3+A*c*d^2*e-3*B*a*d*e^2-B*c*d^3)*x^4-1/5*c^2/e
^3*(A*c*d*e-3*B*a*e^2-B*c*d^2)*x^5+1/6*c^3/e^2*(A*e-B*d)*x^6+(A*a^3*e^7+3*
A*a^2*c*d^2*e^5+3*A*a*c^2*d^4*e^3+A*c^3*d^6*e-B*a^3*d*e^6-3*B*a^2*c*d^3*e^
4-3*B*a*c^2*d^5*e^2-B*c^3*d^7)/e^8*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.54

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{d+ex}\,dx$$
$$=\frac{60\,Bc^3e^7x^7-70\,(Bc^3de^6-Ac^3e^7)x^6+84\,(Bc^3d^2e^5-Ac^3de^6+3\,Bac^2e^7)x^5-105\,(Bc^3d^3e^4-Ac^3d^2e^5}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d),x, algorithm="fricas")
```

output
```
1/420*(60*B*c^3*e^7*x^7 - 70*(B*c^3*d*e^6 - A*c^3*e^7)*x^6 + 84*(B*c^3*d^2
*e^5 - A*c^3*d*e^6 + 3*B*a*c^2*e^7)*x^5 - 105*(B*c^3*d^3*e^4 - A*c^3*d^2*e
^5 + 3*B*a*c^2*d*e^6 - 3*A*a*c^2*e^7)*x^4 + 140*(B*c^3*d^4*e^3 - A*c^3*d^3
*e^4 + 3*B*a*c^2*d^2*e^5 - 3*A*a*c^2*d*e^6 + 3*B*a^2*c*e^7)*x^3 - 210*(B*c
^3*d^5*e^2 - A*c^3*d^4*e^3 + 3*B*a*c^2*d^3*e^4 - 3*A*a*c^2*d^2*e^5 + 3*B*a
^2*c*d*e^6 - 3*A*a^2*c*e^7)*x^2 + 420*(B*c^3*d^6*e - A*c^3*d^5*e^2 + 3*B*a
*c^2*d^4*e^3 - 3*A*a*c^2*d^3*e^4 + 3*B*a^2*c*d^2*e^5 - 3*A*a^2*c*d*e^6 + B
*a^3*e^7)*x - 420*(B*c^3*d^7 - A*c^3*d^6*e + 3*B*a*c^2*d^5*e^2 - 3*A*a*c^2
*d^4*e^3 + 3*B*a^2*c*d^3*e^4 - 3*A*a^2*c*d^2*e^5 + B*a^3*d*e^6 - A*a^3*e^7
)*log(e*x + d))/e^8
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.41

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{d+ex}\,dx = \frac{Bc^3x^7}{7e} + x^6\left(\frac{Ac^3}{6e} - \frac{Bc^3d}{6e^2}\right) + x^5\left(-\frac{Ac^3d}{5e^2} + \frac{3Bac^2}{5e} + \frac{Bc^3d^2}{5e^3}\right) + x^4 \cdot \left(\frac{3Aac^2}{4e} + \frac{Ac^3d^2}{4e^3} - \frac{3Bac^2d}{4e^2} - \frac{Bc^3d^3}{4e^4}\right) + x^3\left(-\frac{Aac^2d}{e^2} - \frac{Ac^3d^3}{3e^4} + \frac{Ba^2c}{e} + \frac{Bac^2d^2}{e^3} + \frac{Bc^3d^4}{3e^5}\right) + x^2 \cdot \left(\frac{3Aa^2c}{2e} + \frac{3Aac^2d^2}{2e^3} + \frac{Ac^3d^4}{2e^5} - \frac{3Ba^2cd}{2e^2} - \frac{3Bac^2d^3}{2e^4} - \frac{Bc^3d^5}{2e^6}\right) + x\left(-\frac{3Aa^2cd}{e^2} - \frac{3Aac^2d^3}{e^4} - \frac{Ac^3d^5}{e^6} + \frac{Ba^3}{e} + \frac{3Ba^2cd^2}{e^3} + \frac{3Bac^2d^4}{e^5} + \frac{Bc^3d^6}{e^7}\right) - \frac{(-Ae+Bd)\,(ae^2+cd^2)^3\log(d+ex)}{e^8}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d),x)
```

output
```
B*c**3*x**7/(7*e) + x**6*(A*c**3/(6*e) - B*c**3*d/(6*e**2)) + x**5*(-A*c**
3*d/(5*e**2) + 3*B*a*c**2/(5*e) + B*c**3*d**2/(5*e**3)) + x**4*(3*A*a*c**2
/(4*e) + A*c**3*d**2/(4*e**3) - 3*B*a*c**2*d/(4*e**2) - B*c**3*d**3/(4*e**
4)) + x**3*(-A*a*c**2*d/e**2 - A*c**3*d**3/(3*e**4) + B*a**2*c/e + B*a*c**
2*d**2/e**3 + B*c**3*d**4/(3*e**5)) + x**2*(3*A*a**2*c/(2*e) + 3*A*a*c**2*
d**2/(2*e**3) + A*c**3*d**4/(2*e**5) - 3*B*a**2*c*d/(2*e**2) - 3*B*a*c**2*
d**3/(2*e**4) - B*c**3*d**5/(2*e**6)) + x*(-3*A*a**2*c*d/e**2 - 3*A*a*c**2
*d**3/e**4 - A*c**3*d**5/e**6 + B*a**3/e + 3*B*a**2*c*d**2/e**3 + 3*B*a*c*
*2*d**4/e**5 + B*c**3*d**6/e**7) - (-A*e + B*d)*(a*e**2 + c*d**2)**3*log(d
 + e*x)/e**8
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.54

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{d+ex}\,dx = \frac{60\,Bc^3e^6x^7 - 70\,(Bc^3de^5 - Ac^3e^6)x^6 + 84\,(Bc^3d^2e^4 - Ac^3de^5 + 3\,Bac^2e^6)x^5 - 105\,(Bc^3d^3e^3 - Ac^3d^2e^4}{} - \frac{(Bc^3d^7 - Ac^3d^6e + 3\,Bac^2d^5e^2 - 3\,Aac^2d^4e^3 + 3\,Ba^2cd^3e^4 - 3\,Aa^2cd^2e^5 + Ba^3de^6 - Aa^3e^7)\log(ex +}{e^8}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d),x, algorithm="maxima")
```

output
```
1/420*(60*B*c^3*e^6*x^7 - 70*(B*c^3*d*e^5 - A*c^3*e^6)*x^6 + 84*(B*c^3*d^2
*e^4 - A*c^3*d*e^5 + 3*B*a*c^2*e^6)*x^5 - 105*(B*c^3*d^3*e^3 - A*c^3*d^2*e
^4 + 3*B*a*c^2*d*e^5 - 3*A*a*c^2*e^6)*x^4 + 140*(B*c^3*d^4*e^2 - A*c^3*d^3
*e^3 + 3*B*a*c^2*d^2*e^4 - 3*A*a*c^2*d*e^5 + 3*B*a^2*c*e^6)*x^3 - 210*(B*c
^3*d^5*e - A*c^3*d^4*e^2 + 3*B*a*c^2*d^3*e^3 - 3*A*a*c^2*d^2*e^4 + 3*B*a^2
*c*d*e^5 - 3*A*a^2*c*e^6)*x^2 + 420*(B*c^3*d^6 - A*c^3*d^5*e + 3*B*a*c^2*d
^4*e^2 - 3*A*a*c^2*d^3*e^3 + 3*B*a^2*c*d^2*e^4 - 3*A*a^2*c*d*e^5 + B*a^3*e
^6)*x)/e^7 - (B*c^3*d^7 - A*c^3*d^6*e + 3*B*a*c^2*d^5*e^2 - 3*A*a*c^2*d^4*
e^3 + 3*B*a^2*c*d^3*e^4 - 3*A*a^2*c*d^2*e^5 + B*a^3*d*e^6 - A*a^3*e^7)*log
(e*x + d)/e^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.68

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{d+ex}\,dx$$
$$= \frac{60\,Bc^3e^6x^7 - 70\,Bc^3de^5x^6 + 70\,Ac^3e^6x^6 + 84\,Bc^3d^2e^4x^5 - 84\,Ac^3de^5x^5 + 252\,Bac^2e^6x^5 - 105\,Bc^3d^3e^3x}{}$$
$$- \frac{(Bc^3d^7 - Ac^3d^6e + 3\,Bac^2d^5e^2 - 3\,Aac^2d^4e^3 + 3\,Ba^2cd^3e^4 - 3\,Aa^2cd^2e^5 + Ba^3de^6 - Aa^3e^7)\log\left(|ex\right.}{e^8}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d),x, algorithm="giac")
```

output
```
1/420*(60*B*c^3*e^6*x^7 - 70*B*c^3*d*e^5*x^6 + 70*A*c^3*e^6*x^6 + 84*B*c^3
*d^2*e^4*x^5 - 84*A*c^3*d*e^5*x^5 + 252*B*a*c^2*e^6*x^5 - 105*B*c^3*d^3*e^
3*x^4 + 105*A*c^3*d^2*e^4*x^4 - 315*B*a*c^2*d*e^5*x^4 + 315*A*a*c^2*e^6*x^
4 + 140*B*c^3*d^4*e^2*x^3 - 140*A*c^3*d^3*e^3*x^3 + 420*B*a*c^2*d^2*e^4*x^
3 - 420*A*a*c^2*d*e^5*x^3 + 420*B*a^2*c*e^6*x^3 - 210*B*c^3*d^5*e*x^2 + 21
0*A*c^3*d^4*e^2*x^2 - 630*B*a*c^2*d^3*e^3*x^2 + 630*A*a*c^2*d^2*e^4*x^2 -
630*B*a^2*c*d*e^5*x^2 + 630*A*a^2*c*e^6*x^2 + 420*B*c^3*d^6*x - 420*A*c^3*
d^5*e*x + 1260*B*a*c^2*d^4*e^2*x - 1260*A*a*c^2*d^3*e^3*x + 1260*B*a^2*c*d
^2*e^4*x - 1260*A*a^2*c*d*e^5*x + 420*B*a^3*e^6*x)/e^7 - (B*c^3*d^7 - A*c^
3*d^6*e + 3*B*a*c^2*d^5*e^2 - 3*A*a*c^2*d^4*e^3 + 3*B*a^2*c*d^3*e^4 - 3*A*
a^2*c*d^2*e^5 + B*a^3*d*e^6 - A*a^3*e^7)*log(abs(e*x + d))/e^8
```

Mupad [B] (verification not implemented)

Time = 6.35 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.70

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{d+ex}\,dx$$

$$= x^2\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{A\,c^3}{e}-\frac{B\,c^3\,d}{e^2}\right)}{e}-\frac{3\,B\,a\,c^2}{e}\right)}{e}+\frac{3\,A\,a\,c^2}{e}\right)}{e}-\frac{3\,B\,a^2\,c}{e}\right)}{2\,e}+\frac{3\,A\,a^2\,c}{2\,e}\right)$$

$$+ x^4\left(\frac{d\left(\frac{d\left(\frac{A\,c^3}{e}-\frac{B\,c^3\,d}{e^2}\right)}{e}-\frac{3\,B\,a\,c^2}{e}\right)}{4\,e}+\frac{3\,A\,a\,c^2}{4\,e}\right)+x^6\left(\frac{A\,c^3}{6\,e}-\frac{B\,c^3\,d}{6\,e^2}\right)$$

$$- x^3\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{A\,c^3}{e}-\frac{B\,c^3\,d}{e^2}\right)}{e}-\frac{3\,B\,a\,c^2}{e}\right)}{e}+\frac{3\,A\,a\,c^2}{e}\right)}{3\,e}-\frac{B\,a^2\,c}{e}\right)$$

$$\left(\quad d\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{A\,c^3}{e}-\frac{B\,c^3\,d}{e^2}\right)}{e}-\frac{3\,B\,a\,c^2}{e}\right)}{e}+\frac{3\,A\,a\,c^2}{e}\right)}{e}-\frac{3\,B\,a^2\,c}{e}\right)}{e}+\frac{3\,A\,a^2\,c}{e}\right.\right.$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x),x)
```

output
```
x^2*((d*((d*((d*((d*((A*c^3)/e - (B*c^3*d)/e^2))/e - (3*B*a*c^2)/e))/e + (
3*A*a*c^2)/e))/e - (3*B*a^2*c)/e))/(2*e) + (3*A*a^2*c)/(2*e)) + x^4*((d*((
d*((A*c^3)/e - (B*c^3*d)/e^2))/e - (3*B*a*c^2)/e))/(4*e) + (3*A*a*c^2)/(4*
e)) + x^6*((A*c^3)/(6*e) - (B*c^3*d)/(6*e^2)) - x^3*((d*((d*((d*((A*c^3)/e
 - (B*c^3*d)/e^2))/e - (3*B*a*c^2)/e))/e + (3*A*a*c^2)/e))/(3*e) - (B*a^2*
c)/e) + x*((B*a^3)/e - (d*((d*((d*((d*((d*((A*c^3)/e - (B*c^3*d)/e^2))/e -
 (3*B*a*c^2)/e))/e + (3*A*a*c^2)/e))/e - (3*B*a^2*c)/e))/e + (3*A*a^2*c)/e
))/e) - x^5*((d*((A*c^3)/e - (B*c^3*d)/e^2))/(5*e) - (3*B*a*c^2)/(5*e)) +
(log(d + e*x)*(A*a^3*e^7 - B*c^3*d^7 - B*a^3*d*e^6 + A*c^3*d^6*e + 3*A*a*c
^2*d^4*e^3 + 3*A*a^2*c*d^2*e^5 - 3*B*a*c^2*d^5*e^2 - 3*B*a^2*c*d^3*e^4))/e
^8 + (B*c^3*x^7)/(7*e)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.83

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{d+ex}\,dx = \frac{420\log(ex+d)\,a^4e^7-420\log(ex+d)\,a^3bd\,e^6+1260\log(ex+d)\,a^3c\,d^2e^5+1260\log(ex+d)\,a^2c^2d^4e^3+}{}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d),x)
```

output
```
(420*log(d + e*x)*a**4*e**7 - 420*log(d + e*x)*a**3*b*d*e**6 + 1260*log(d
+ e*x)*a**3*c*d**2*e**5 - 1260*log(d + e*x)*a**2*b*c*d**3*e**4 + 1260*log(
d + e*x)*a**2*c**2*d**4*e**3 - 1260*log(d + e*x)*a*b*c**2*d**5*e**2 + 420*
log(d + e*x)*a*c**3*d**6*e - 420*log(d + e*x)*b*c**3*d**7 + 420*a**3*b*e**
7*x - 1260*a**3*c*d*e**6*x + 630*a**3*c*e**7*x**2 + 1260*a**2*b*c*d**2*e**
5*x - 630*a**2*b*c*d*e**6*x**2 + 420*a**2*b*c*e**7*x**3 - 1260*a**2*c**2*d
**3*e**4*x + 630*a**2*c**2*d**2*e**5*x**2 - 420*a**2*c**2*d*e**6*x**3 + 31
5*a**2*c**2*e**7*x**4 + 1260*a*b*c**2*d**4*e**3*x - 630*a*b*c**2*d**3*e**4
*x**2 + 420*a*b*c**2*d**2*e**5*x**3 - 315*a*b*c**2*d*e**6*x**4 + 252*a*b*c
**2*e**7*x**5 - 420*a*c**3*d**5*e**2*x + 210*a*c**3*d**4*e**3*x**2 - 140*a
*c**3*d**3*e**4*x**3 + 105*a*c**3*d**2*e**5*x**4 - 84*a*c**3*d*e**6*x**5 +
 70*a*c**3*e**7*x**6 + 420*b*c**3*d**6*e*x - 210*b*c**3*d**5*e**2*x**2 + 1
40*b*c**3*d**4*e**3*x**3 - 105*b*c**3*d**3*e**4*x**4 + 84*b*c**3*d**2*e**5
*x**5 - 70*b*c**3*d*e**6*x**6 + 60*b*c**3*e**7*x**7)/(420*e**8)
```

3.69 $\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 309

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^2} \, dx = -\frac{c\left(6Bd(cd^2+ae^2)^2-Ae(5c^2d^4+9acd^2e^2+3a^2e^4)\right)x}{e^7} - \frac{c(2Acde(2cd^2+3ae^2)-B(5c^2d^4+9acd^2e^2+3a^2e^4))\,x^2}{2e^6} + \frac{c^2(3Ae(cd^2+ae^2)-B(4cd^3+6ade^2))\,x^3}{3e^5} - \frac{c^2(2Acde-3B(cd^2+ae^2))\,x^4}{4e^4} - \frac{c^3(2Bd-Ae)x^5}{5e^3} + \frac{Bc^3x^6}{6e^2} + \frac{(Bd-Ae)\left(cd^2+ae^2\right)^3}{e^8(d+ex)} + \frac{\left(cd^2+ae^2\right)^2(7Bcd^2-6Acde+aBe^2)\log(d+ex)}{e^8}$$

output

```
-c*(6*B*d*(a*e^2+c*d^2)^2-A*e*(3*a^2*e^4+9*a*c*d^2*e^2+5*c^2*d^4))*x/e^7-1
/2*c*(2*A*c*d*e*(3*a*e^2+2*c*d^2)-B*(3*a^2*e^4+9*a*c*d^2*e^2+5*c^2*d^4))*x
^2/e^6+1/3*c^2*(3*A*e*(a*e^2+c*d^2)-B*(6*a*d*e^2+4*c*d^3))*x^3/e^5-1/4*c^2
*(2*A*c*d*e-3*B*(a*e^2+c*d^2))*x^4/e^4-1/5*c^3*(-A*e+2*B*d)*x^5/e^3+1/6*B*
c^3*x^6/e^2+(-A*e+B*d)*(a*e^2+c*d^2)^3/e^8/(e*x+d)+(a*e^2+c*d^2)^2*(-6*A*c
*d*e+B*a*e^2+7*B*c*d^2)*ln(e*x+d)/e^8
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.31

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^2}\,dx$$
$$= \frac{6Ae(-10a^3e^6+30a^2ce^4(-d^2+dex+e^2x^2)+10ac^2e^2(-3d^4+9d^3ex+6d^2e^2x^2-2de^3x^3+e^4x^4)+c^3(}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^2,x]
```

output
```
(6*A*e*(-10*a^3*e^6 + 30*a^2*c*e^4*(-d^2 + d*e*x + e^2*x^2) + 10*a*c^2*e^2
*(-3*d^4 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + c^3*(-10*d
^6 + 50*d^5*e*x + 30*d^4*e^2*x^2 - 10*d^3*e^3*x^3 + 5*d^2*e^4*x^4 - 3*d*e^
5*x^5 + 2*e^6*x^6)) + B*(60*a^3*d*e^6 + 90*a^2*c*e^4*(2*d^3 - 4*d^2*e*x -
3*d*e^2*x^2 + e^3*x^3) + 15*a*c^2*e^2*(12*d^5 - 48*d^4*e*x - 30*d^3*e^2*x^
2 + 10*d^2*e^3*x^3 - 5*d*e^4*x^4 + 3*e^5*x^5) + c^3*(60*d^7 - 360*d^6*e*x
- 210*d^5*e^2*x^2 + 70*d^4*e^3*x^3 - 35*d^3*e^4*x^4 + 21*d^2*e^5*x^5 - 14*
d*e^6*x^6 + 10*e^7*x^7)) + 60*(c*d^2 + a*e^2)^2*(7*B*c*d^2 - 6*A*c*d*e + a
*B*e^2)*(d + e*x)*Log[d + e*x])/(60*e^8*(d + e*x))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^3(A+Bx)}{(d+ex)^2}\,dx$$
$$\Big\downarrow \; 652$$
$$\int \left(\frac{c\left(Ae\left(3a^2e^4+9acd^2e^2+5c^2d^4\right)-6Bd\left(ae^2+cd^2\right)^2\right)}{e^7}-\frac{cx\left(-3a^2Be^4+6aAcde^3-9aBcd^2e^2+4Ac^2d^3e-\right.}{e^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{cx\left(6Bd\left(ae^2+cd^2\right)^2-Ae\left(3a^2e^4+9acd^2e^2+5c^2d^4\right)\right)}{e^7}-\frac{cx^2\left(2Acde\left(3ae^2+2cd^2\right)-B\left(3a^2e^4+9acd^2e^2+5c^2d^4\right)\right)}{2e^6}-\frac{c^2x^4\left(2Acde-3B\left(ae^2+cd^2\right)\right)}{4e^4}+\frac{c^2x^3\left(3Ae\left(ae^2+cd^2\right)-B\left(6ade^2+4cd^3\right)\right)}{3e^5}+\frac{\left(ae^2+cd^2\right)^3(Bd-Ae)}{e^8(d+ex)}+\frac{\left(ae^2+cd^2\right)^2\log(d+ex)\left(aBe^2-6Acde+7Bcd^2\right)}{e^8}-\frac{c^3x^5(2Bd-Ae)}{5e^3}+\frac{Bc^3x^6}{6e^2}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^2,x]
```

output
```
-((c*(6*B*d*(c*d^2 + a*e^2)^2 - A*e*(5*c^2*d^4 + 9*a*c*d^2*e^2 + 3*a^2*e^4
))*x)/e^7) - (c*(2*A*c*d*e*(2*c*d^2 + 3*a*e^2) - B*(5*c^2*d^4 + 9*a*c*d^2*
e^2 + 3*a^2*e^4))*x^2)/(2*e^6) + (c^2*(3*A*e*(c*d^2 + a*e^2) - B*(4*c*d^3
+ 6*a*d*e^2))*x^3)/(3*e^5) - (c^2*(2*A*c*d*e - 3*B*(c*d^2 + a*e^2))*x^4)/(
4*e^4) - (c^3*(2*B*d - A*e)*x^5)/(5*e^3) + (B*c^3*x^6)/(6*e^2) + ((B*d - A
*e)*(c*d^2 + a*e^2)^3)/(e^8*(d + e*x)) + ((c*d^2 + a*e^2)^2*(7*B*c*d^2 - 6
*A*c*d*e + a*B*e^2)*Log[d + e*x])/e^8
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.40

method	result
norman	$\frac{\left(A a^3 e^7+6 A a^2 c d^2 e^5+12 A a c^2 d^4 e^3+6 A c^3 d^6 e-B a^3 d e^6-9 B a^2 c d^3 e^4-15 B a c^2 d^5 e^2-7 B c^3 d^7\right) x}{d e^7}+\frac{B c^3 x^7}{6 e}-\frac{c\left(12 A a c d e^3+6 A c^2 d^3 e-\right.}{}$
default	$c\left(\frac{1}{6} B x^6 c^2 e^5+\frac{1}{5} A x^5 c^2 e^5-\frac{2}{5} B x^5 c^2 d e^4-\frac{1}{2} A x^4 c^2 d e^4+\frac{3}{4} B x^4 a c e^5+\frac{3}{4} B x^4 c^2 d^2 e^3+A x^3 a c e^5+A x^3 c^2 d^2 e^3-2 B x^3 a c d e^4-\right.$
risch	$\frac{c^3 A x^5}{5 e^2}-\frac{2 c^2 B x^3 a d}{e^3}-\frac{3 c^2 A x^2 a d}{e^3}+\frac{9 c^2 B x^2 a d^2}{2 e^4}+\frac{9 c^2 A a d^2 x}{e^4}-\frac{6 c B a^2 d x}{e^3}-\frac{12 c^2 B a d^3 x}{e^5}-\frac{3 A a^2 c d^2}{e^3(e x+d)}-\frac{3 A a}{e^5(e}$
parallelrisch	$-\frac{720 A \ln (e x+d) x a c^2 d^3 e^4-60 B \ln (e x+d) x a^3 e^7-540 B a^2 c d^3 e^4+360 A a^2 c d^2 e^5+720 A a c^2 d^4 e^3-12 A x^6 c^3 e^7-10 B x^7 c^3 e^7}{}$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
((A*a^3*e^7+6*A*a^2*c*d^2*e^5+12*A*a*c^2*d^4*e^3+6*A*c^3*d^6*e-B*a^3*d*e^6
-9*B*a^2*c*d^3*e^4-15*B*a*c^2*d^5*e^2-7*B*c^3*d^7)/d/e^7*x+1/6*B*c^3*x^7/e
-1/6*c*(12*A*a*c*d*e^3+6*A*c^2*d^3*e-9*B*a^2*e^4-15*B*a*c*d^2*e^2-7*B*c^2*
d^4)/e^5*x^3+1/2*c*(6*A*a^2*e^5+12*A*a*c*d^2*e^3+6*A*c^2*d^4*e-9*B*a^2*d*e
^4-15*B*a*c*d^3*e^2-7*B*c^2*d^5)/e^6*x^2-1/20*c^2*(6*A*c*d*e-15*B*a*e^2-7*
B*c*d^2)/e^3*x^5+1/12*c^2*(12*A*a*e^3+6*A*c*d^2*e-15*B*a*d*e^2-7*B*c*d^3)/
e^4*x^4+1/30*c^3*(6*A*e-7*B*d)/e^2*x^6)/(e*x+d)-(6*A*a^2*c*d*e^5+12*A*a*c^
2*d^3*e^3+6*A*c^3*d^5*e-B*a^3*e^6-9*B*a^2*c*d^2*e^4-15*B*a*c^2*d^4*e^2-7*B
*c^3*d^6)/e^8*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(299) = 598$.

Time = 0.08 (sec) , antiderivative size = 621, normalized size of antiderivative = 2.01

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^2}\,dx$$
$$=\frac{10\,Bc^3e^7x^7+60\,Bc^3d^7-60\,Ac^3d^6e+180\,Bac^2d^5e^2-180\,Aac^2d^4e^3+180\,Ba^2cd^3e^4-180\,Aa^2cd^2e^5+}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^2,x, algorithm="fricas")
```

output

```
1/60*(10*B*c^3*e^7*x^7 + 60*B*c^3*d^7 - 60*A*c^3*d^6*e + 180*B*a*c^2*d^5*e
^2 - 180*A*a*c^2*d^4*e^3 + 180*B*a^2*c*d^3*e^4 - 180*A*a^2*c*d^2*e^5 + 60*
B*a^3*d*e^6 - 60*A*a^3*e^7 - 2*(7*B*c^3*d*e^6 - 6*A*c^3*e^7)*x^6 + 3*(7*B*
c^3*d^2*e^5 - 6*A*c^3*d*e^6 + 15*B*a*c^2*e^7)*x^5 - 5*(7*B*c^3*d^3*e^4 - 6
*A*c^3*d^2*e^5 + 15*B*a*c^2*d*e^6 - 12*A*a*c^2*e^7)*x^4 + 10*(7*B*c^3*d^4*
e^3 - 6*A*c^3*d^3*e^4 + 15*B*a*c^2*d^2*e^5 - 12*A*a*c^2*d*e^6 + 9*B*a^2*c*
e^7)*x^3 - 30*(7*B*c^3*d^5*e^2 - 6*A*c^3*d^4*e^3 + 15*B*a*c^2*d^3*e^4 - 12
*A*a*c^2*d^2*e^5 + 9*B*a^2*c*d*e^6 - 6*A*a^2*c*e^7)*x^2 - 60*(6*B*c^3*d^6*
e - 5*A*c^3*d^5*e^2 + 12*B*a*c^2*d^4*e^3 - 9*A*a*c^2*d^3*e^4 + 6*B*a^2*c*d
^2*e^5 - 3*A*a^2*c*d*e^6)*x + 60*(7*B*c^3*d^7 - 6*A*c^3*d^6*e + 15*B*a*c^2
*d^5*e^2 - 12*A*a*c^2*d^4*e^3 + 9*B*a^2*c*d^3*e^4 - 6*A*a^2*c*d^2*e^5 + B*
a^3*d*e^6 + (7*B*c^3*d^6*e - 6*A*c^3*d^5*e^2 + 15*B*a*c^2*d^4*e^3 - 12*A*a
*c^2*d^3*e^4 + 9*B*a^2*c*d^2*e^5 - 6*A*a^2*c*d*e^6 + B*a^3*e^7)*x)*log(e*x
 + d))/(e^9*x + d*e^8)
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.47

$$\begin{aligned}\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^2}\,dx &= \frac{Bc^3x^6}{6e^2}+x^5\left(\frac{Ac^3}{5e^2}-\frac{2Bc^3d}{5e^3}\right)\\&\quad+x^4\left(-\frac{Ac^3d}{2e^3}+\frac{3Bac^2}{4e^2}+\frac{3Bc^3d^2}{4e^4}\right)+x^3\left(\frac{Aac^2}{e^2}+\frac{Ac^3d^2}{e^4}-\frac{2Bac^2d}{e^3}-\frac{4Bc^3d^3}{3e^5}\right)\\&\quad+x^2\left(-\frac{3Aac^2d}{e^3}-\frac{2Ac^3d^3}{e^5}+\frac{3Ba^2c}{2e^2}+\frac{9Bac^2d^2}{2e^4}+\frac{5Bc^3d^4}{2e^6}\right)\\&\quad+x\left(\frac{3Aa^2c}{e^2}+\frac{9Aac^2d^2}{e^4}+\frac{5Ac^3d^4}{e^6}-\frac{6Ba^2cd}{e^3}-\frac{12Bac^2d^3}{e^5}-\frac{6Bc^3d^5}{e^7}\right)\\&\quad+\frac{-Aa^3e^7-3Aa^2cd^2e^5-3Aac^2d^4e^3-Ac^3d^6e+Ba^3de^6+3Ba^2cd^3e^4+3Bac^2d^5e^2+Bc^3d^7}{de^8+e^9x}\\&\quad+\frac{\left(ae^2+cd^2\right)^2\left(-6Acde+Bae^2+7Bcd^2\right)\log\left(d+ex\right)}{e^8}\end{aligned}$$

input

```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**2,x)
```

output
```
B*c**3*x**6/(6*e**2) + x**5*(A*c**3/(5*e**2) - 2*B*c**3*d/(5*e**3)) + x**4
*(-A*c**3*d/(2*e**3) + 3*B*a*c**2/(4*e**2) + 3*B*c**3*d**2/(4*e**4)) + x**
3*(A*a*c**2/e**2 + A*c**3*d**2/e**4 - 2*B*a*c**2*d/e**3 - 4*B*c**3*d**3/(3
*e**5)) + x**2*(-3*A*a*c**2*d/e**3 - 2*A*c**3*d**3/e**5 + 3*B*a**2*c/(2*e*
*2) + 9*B*a*c**2*d**2/(2*e**4) + 5*B*c**3*d**4/(2*e**6)) + x*(3*A*a**2*c/e
**2 + 9*A*a*c**2*d**2/e**4 + 5*A*c**3*d**4/e**6 - 6*B*a**2*c*d/e**3 - 12*B
*a*c**2*d**3/e**5 - 6*B*c**3*d**5/e**7) + (-A*a**3*e**7 - 3*A*a**2*c*d**2*
e**5 - 3*A*a*c**2*d**4*e**3 - A*c**3*d**6*e + B*a**3*d*e**6 + 3*B*a**2*c*d
**3*e**4 + 3*B*a*c**2*d**5*e**2 + B*c**3*d**7)/(d*e**8 + e**9*x) + (a*e**2
 + c*d**2)**2*(-6*A*c*d*e + B*a*e**2 + 7*B*c*d**2)*log(d + e*x)/e**8
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.48

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^2}\,dx$$
$$= \frac{Bc^3d^7 - Ac^3d^6e + 3\,Bac^2d^5e^2 - 3\,Aac^2d^4e^3 + 3\,Ba^2cd^3e^4 - 3\,Aa^2cd^2e^5 + Ba^3de^6 - Aa^3e^7}{e^9x + de^8}$$
$$+ \frac{10\,Bc^3e^5x^6 - 12\,(2\,Bc^3de^4 - Ac^3e^5)x^5 + 15\,(3\,Bc^3d^2e^3 - 2\,Ac^3de^4 + 3\,Bac^2e^5)x^4 - 20\,(4\,Bc^3d^3e^2 - 3}{}$$
$$+ \frac{(7\,Bc^3d^6 - 6\,Ac^3d^5e + 15\,Bac^2d^4e^2 - 12\,Aac^2d^3e^3 + 9\,Ba^2cd^2e^4 - 6\,Aa^2cde^5 + Ba^3e^6)\log(ex+d)}{e^8}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^2,x, algorithm="maxima")
```

output
```
(B*c^3*d^7 - A*c^3*d^6*e + 3*B*a*c^2*d^5*e^2 - 3*A*a*c^2*d^4*e^3 + 3*B*a^2
*c*d^3*e^4 - 3*A*a^2*c*d^2*e^5 + B*a^3*d*e^6 - A*a^3*e^7)/(e^9*x + d*e^8)
 + 1/60*(10*B*c^3*e^5*x^6 - 12*(2*B*c^3*d*e^4 - A*c^3*e^5)*x^5 + 15*(3*B*c^
3*d^2*e^3 - 2*A*c^3*d*e^4 + 3*B*a*c^2*e^5)*x^4 - 20*(4*B*c^3*d^3*e^2 - 3*A
*c^3*d^2*e^3 + 6*B*a*c^2*d*e^4 - 3*A*a*c^2*e^5)*x^3 + 30*(5*B*c^3*d^4*e -
4*A*c^3*d^3*e^2 + 9*B*a*c^2*d^2*e^3 - 6*A*a*c^2*d*e^4 + 3*B*a^2*c*e^5)*x^2
 - 60*(6*B*c^3*d^5 - 5*A*c^3*d^4*e + 12*B*a*c^2*d^3*e^2 - 9*A*a*c^2*d^2*e^
3 + 6*B*a^2*c*d*e^4 - 3*A*a^2*c*e^5)*x)/e^7 + (7*B*c^3*d^6 - 6*A*c^3*d^5*e
 + 15*B*a*c^2*d^4*e^2 - 12*A*a*c^2*d^3*e^3 + 9*B*a^2*c*d^2*e^4 - 6*A*a^2*c
*d*e^5 + B*a^3*e^6)*log(e*x + d)/e^8
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.82

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^2}\,dx$$
$$= \frac{\left(10\,Bc^3 - \frac{12\left(7\,Bc^3de-Ac^3e^2\right)}{(ex+d)e} + \frac{45\left(7\,Bc^3d^2e^2-2\,Ac^3de^3+Bac^2e^4\right)}{(ex+d)^2e^2} - \frac{20\left(35\,Bc^3d^3e^3-15\,Ac^3d^2e^4+15\,Bac^2de^5-3\,Aac^2e^6\right)}{(ex+d)^3e^3} + 30\,(\cdots\right.}{}$$
$$- \frac{\left(7\,Bc^3d^6 - 6\,Ac^3d^5e + 15\,Bac^2d^4e^2 - 12\,Aac^2d^3e^3 + 9\,Ba^2cd^2e^4 - 6\,Aa^2cde^5 + Ba^3e^6\right)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^8}$$
$$+ \frac{\frac{Bc^3d^7e^6}{ex+d} - \frac{Ac^3d^6e^7}{ex+d} + \frac{3\,Bac^2d^5e^8}{ex+d} - \frac{3\,Aac^2d^4e^9}{ex+d} + \frac{3\,Ba^2cd^3e^{10}}{ex+d} - \frac{3\,Aa^2cd^2e^{11}}{ex+d} + \frac{Ba^3de^{12}}{ex+d} - \frac{Aa^3e^{13}}{ex+d}}{e^{14}}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/60*(10*B*c^3 - 12*(7*B*c^3*d*e - A*c^3*e^2)/((e*x + d)*e) + 45*(7*B*c^3*
d^2*e^2 - 2*A*c^3*d*e^3 + B*a*c^2*e^4)/((e*x + d)^2*e^2) - 20*(35*B*c^3*d^
3*e^3 - 15*A*c^3*d^2*e^4 + 15*B*a*c^2*d*e^5 - 3*A*a*c^2*e^6)/((e*x + d)^3*
e^3) + 30*(35*B*c^3*d^4*e^4 - 20*A*c^3*d^3*e^5 + 30*B*a*c^2*d^2*e^6 - 12*A
*a*c^2*d*e^7 + 3*B*a^2*c*e^8)/((e*x + d)^4*e^4) - 180*(7*B*c^3*d^5*e^5 - 5
*A*c^3*d^4*e^6 + 10*B*a*c^2*d^3*e^7 - 6*A*a*c^2*d^2*e^8 + 3*B*a^2*c*d*e^9
- A*a^2*c*e^10)/((e*x + d)^5*e^5))*(e*x + d)^6/e^8 - (7*B*c^3*d^6 - 6*A*c^
3*d^5*e + 15*B*a*c^2*d^4*e^2 - 12*A*a*c^2*d^3*e^3 + 9*B*a^2*c*d^2*e^4 - 6*
A*a^2*c*d*e^5 + B*a^3*e^6)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^8 + (B
*c^3*d^7*e^6/(e*x + d) - A*c^3*d^6*e^7/(e*x + d) + 3*B*a*c^2*d^5*e^8/(e*x
+ d) - 3*A*a*c^2*d^4*e^9/(e*x + d) + 3*B*a^2*c*d^3*e^10/(e*x + d) - 3*A*a^
2*c*d^2*e^11/(e*x + d) + B*a^3*d*e^12/(e*x + d) - A*a^3*e^13/(e*x + d))/e^
14
```

Mupad [B] (verification not implemented)

Time = 6.19 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.67

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^2,x)
```

output
```
x^3*((2*d*((2*d*((A*c^3)/e^2 - (2*B*c^3*d)/e^3))/e - (3*B*a*c^2)/e^2 + (B*
c^3*d^2)/e^4))/(3*e) - (d^2*((A*c^3)/e^2 - (2*B*c^3*d)/e^3))/(3*e^2) + (A*
a*c^2)/e^2) + x^5*((A*c^3)/(5*e^2) - (2*B*c^3*d)/(5*e^3)) - x^4*((d*((A*c^
3)/e^2 - (2*B*c^3*d)/e^3))/(2*e) - (3*B*a*c^2)/(4*e^2) + (B*c^3*d^2)/(4*e^
4)) - x*((2*d*((d^2*((2*d*((A*c^3)/e^2 - (2*B*c^3*d)/e^3))/e - (3*B*a*c^2)
/e^2 + (B*c^3*d^2)/e^4))/e^2 - (2*d*((2*d*((2*d*((A*c^3)/e^2 - (2*B*c^3*d)
/e^3))/e - (3*B*a*c^2)/e^2 + (B*c^3*d^2)/e^4))/e - (d^2*((A*c^3)/e^2 - (2*
B*c^3*d)/e^3))/e^2 + (3*A*a*c^2)/e^2))/e + (3*B*a^2*c)/e^2))/e + (d^2*((2*
d*((2*d*((A*c^3)/e^2 - (2*B*c^3*d)/e^3))/e - (3*B*a*c^2)/e^2 + (B*c^3*d^2)
/e^4))/e - (d^2*((A*c^3)/e^2 - (2*B*c^3*d)/e^3))/e^2 + (3*A*a*c^2)/e^2))/e
^2 - (3*A*a^2*c)/e^2) + x^2*((d^2*((2*d*((A*c^3)/e^2 - (2*B*c^3*d)/e^3))/e
 - (3*B*a*c^2)/e^2 + (B*c^3*d^2)/e^4))/(2*e^2) - (d*((2*d*((2*d*((A*c^3)/e
^2 - (2*B*c^3*d)/e^3))/e - (3*B*a*c^2)/e^2 + (B*c^3*d^2)/e^4))/e - (d^2*((
A*c^3)/e^2 - (2*B*c^3*d)/e^3))/e^2 + (3*A*a*c^2)/e^2))/e + (3*B*a^2*c)/(2*
e^2)) + (log(d + e*x)*(B*a^3*e^6 + 7*B*c^3*d^6 - 6*A*c^3*d^5*e - 12*A*a*c^
2*d^3*e^3 + 15*B*a*c^2*d^4*e^2 + 9*B*a^2*c*d^2*e^4 - 6*A*a^2*c*d*e^5))/e^8
 - (A*a^3*e^7 - B*c^3*d^7 - B*a^3*d*e^6 + A*c^3*d^6*e + 3*A*a*c^2*d^4*e^3
+ 3*A*a^2*c*d^2*e^5 - 3*B*a*c^2*d^5*e^2 - 3*B*a^2*c*d^3*e^4)/(e*(d*e^7 + e
^8*x)) + (B*c^3*x^6)/(6*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 689, normalized size of antiderivative = 2.23

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^2}\,dx$$
$$= \frac{-720\log(ex+d)\,a^2c^2d^4e^4x + 900\log(ex+d)\,ab\,c^2d^6e^2 - 360\log(ex+d)\,a\,c^3d^6e^2x + 420\log(ex+d)\,b\,c}{}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^2,x)
```

output

```
(60*log(d + e*x)*a**3*b*d**2*e**6 + 60*log(d + e*x)*a**3*b*d*e**7*x - 360*
log(d + e*x)*a**3*c*d**3*e**5 - 360*log(d + e*x)*a**3*c*d**2*e**6*x + 540*
log(d + e*x)*a**2*b*c*d**4*e**4 + 540*log(d + e*x)*a**2*b*c*d**3*e**5*x -
720*log(d + e*x)*a**2*c**2*d**5*e**3 - 720*log(d + e*x)*a**2*c**2*d**4*e**
4*x + 900*log(d + e*x)*a*b*c**2*d**6*e**2 + 900*log(d + e*x)*a*b*c**2*d**5
*e**3*x - 360*log(d + e*x)*a*c**3*d**7*e - 360*log(d + e*x)*a*c**3*d**6*e*
*2*x + 420*log(d + e*x)*b*c**3*d**8 + 420*log(d + e*x)*b*c**3*d**7*e*x + 6
0*a**4*e**8*x - 60*a**3*b*d*e**7*x + 360*a**3*c*d**2*e**6*x + 180*a**3*c*d
*e**7*x**2 - 540*a**2*b*c*d**3*e**5*x - 270*a**2*b*c*d**2*e**6*x**2 + 90*a
**2*b*c*d*e**7*x**3 + 720*a**2*c**2*d**4*e**4*x + 360*a**2*c**2*d**3*e**5*
x**2 - 120*a**2*c**2*d**2*e**6*x**3 + 60*a**2*c**2*d*e**7*x**4 - 900*a*b*c
**2*d**5*e**3*x - 450*a*b*c**2*d**4*e**4*x**2 + 150*a*b*c**2*d**3*e**5*x**
3 - 75*a*b*c**2*d**2*e**6*x**4 + 45*a*b*c**2*d*e**7*x**5 + 360*a*c**3*d**6
*e**2*x + 180*a*c**3*d**5*e**3*x**2 - 60*a*c**3*d**4*e**4*x**3 + 30*a*c**3
*d**3*e**5*x**4 - 18*a*c**3*d**2*e**6*x**5 + 12*a*c**3*d*e**7*x**6 - 420*b
*c**3*d**7*e*x - 210*b*c**3*d**6*e**2*x**2 + 70*b*c**3*d**5*e**3*x**3 - 35
*b*c**3*d**4*e**4*x**4 + 21*b*c**3*d**3*e**5*x**5 - 14*b*c**3*d**2*e**6*x*
*6 + 10*b*c**3*d*e**7*x**7)/(60*d*e**8*(d + e*x))
```

3.70 $\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 300

$$\begin{aligned}
&\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^3} \, dx \\
&= -\frac{c(Acde(10cd^2+9ae^2)-3B(5c^2d^4+6acd^2e^2+a^2e^4))\,x}{e^7} \\
&\quad -\frac{c^2(10Bcd^3-6Acd^2e+9aBde^2-3aAe^3)\,x^2}{2e^6} \\
&\quad +\frac{c^2(2Bcd^2-Acde+aBe^2)\,x^3}{e^5}-\frac{c^3(3Bd-Ae)x^4}{4e^4}+\frac{Bc^3x^5}{5e^3} \\
&\quad +\frac{(Bd-Ae)\left(cd^2+ae^2\right)^3}{2e^8(d+ex)^2}-\frac{\left(cd^2+ae^2\right)^2\left(7Bcd^2-6Acde+aBe^2\right)}{e^8(d+ex)} \\
&\quad -\frac{3c(cd^2+ae^2)\left(7Bcd^3-5Acd^2e+3aBde^2-aAe^3\right)\log(d+ex)}{e^8}
\end{aligned}$$

output
```
-c*(A*c*d*e*(9*a*e^2+10*c*d^2)-3*B*(a^2*e^4+6*a*c*d^2*e^2+5*c^2*d^4))*x/e^
7-1/2*c^2*(-3*A*a*e^3-6*A*c*d^2*e+9*B*a*d*e^2+10*B*c*d^3)*x^2/e^6+c^2*(-A*
c*d*e+B*a*e^2+2*B*c*d^2)*x^3/e^5-1/4*c^3*(-A*e+3*B*d)*x^4/e^4+1/5*B*c^3*x^
5/e^3+1/2*(-A*e+B*d)*(a*e^2+c*d^2)^3/e^8/(e*x+d)^2-(a*e^2+c*d^2)^2*(-6*A*c
*d*e+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)-3*c*(a*e^2+c*d^2)*(-A*a*e^3-5*A*c*d^2*
e+3*B*a*d*e^2+7*B*c*d^3)*ln(e*x+d)/e^8
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.38

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^3}\,dx$$
$$= \frac{5Ae(-2a^3e^6+6a^2cde^4(3d+4ex)+6ac^2e^2(7d^4+2d^3ex-11d^2e^2x^2-4de^3x^3+e^4x^4)+c^3(22d^6-16d^5}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^3,x]
```

output
```
(5*A*e*(-2*a^3*e^6 + 6*a^2*c*d*e^4*(3*d + 4*e*x) + 6*a*c^2*e^2*(7*d^4 + 2*
d^3*e*x - 11*d^2*e^2*x^2 - 4*d*e^3*x^3 + e^4*x^4) + c^3*(22*d^6 - 16*d^5*e
*x - 68*d^4*e^2*x^2 - 20*d^3*e^3*x^3 + 5*d^2*e^4*x^4 - 2*d*e^5*x^5 + e^6*x
^6)) + B*(-10*a^3*e^6*(d + 2*e*x) + 30*a^2*c*e^4*(-5*d^3 - 4*d^2*e*x + 4*d
*e^2*x^2 + 2*e^3*x^3) + 10*a*c^2*e^2*(-27*d^5 + 6*d^4*e*x + 63*d^3*e^2*x^2
 + 20*d^2*e^3*x^3 - 5*d*e^4*x^4 + 2*e^5*x^5) + c^3*(-130*d^7 + 160*d^6*e*x
 + 500*d^5*e^2*x^2 + 140*d^4*e^3*x^3 - 35*d^3*e^4*x^4 + 14*d^2*e^5*x^5 - 7
*d*e^6*x^6 + 4*e^7*x^7)) - 60*c*(c*d^2 + a*e^2)*(7*B*c*d^3 - 5*A*c*d^2*e +
 3*a*B*d*e^2 - a*A*e^3)*(d + e*x)^2*Log[d + e*x])/(20*e^8*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^3(A+Bx)}{(d+ex)^3}\,dx$$
$$\Big\downarrow \text{652}$$
$$\int \left(\frac{c\left(3B\left(a^2e^4+6acd^2e^2+5c^2d^4\right)-Acde\left(9ae^2+10cd^2\right)\right)}{e^7}-\frac{3c^2x^2\left(-aBe^2+Acde-2Bcd^2\right)}{e^5}+\frac{c^2x\left(3aAe^3-\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{cx\left(Acde\left(9ae^2+10cd^2\right)-3B\left(a^2e^4+6acd^2e^2+5c^2d^4\right)\right)}{e^7}+\frac{c^2x^3\left(aBe^2-Acde+2Bcd^2\right)}{e^5}-\frac{c^2x^2\left(-3aAe^3+9aBde^2-6Acd^2e+10Bcd^3\right)}{2e^6}-\frac{\left(ae^2+cd^2\right)^2\left(aBe^2-6Acde+7Bcd^2\right)}{e^8(d+ex)}+\frac{\left(ae^2+cd^2\right)^3(Bd-Ae)}{2e^8(d+ex)^2}-\frac{3c\left(ae^2+cd^2\right)\log(d+ex)\left(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3\right)}{e^8}-\frac{c^3x^4(3Bd-Ae)}{4e^4}+\frac{Bc^3x^5}{5e^3}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^3,x]
```

output
```
-((c*(A*c*d*e*(10*c*d^2 + 9*a*e^2) - 3*B*(5*c^2*d^4 + 6*a*c*d^2*e^2 + a^2*
e^4))*x)/e^7) - (c^2*(10*B*c*d^3 - 6*A*c*d^2*e + 9*a*B*d*e^2 - 3*a*A*e^3)*
x^2)/(2*e^6) + (c^2*(2*B*c*d^2 - A*c*d*e + a*B*e^2)*x^3)/e^5 - (c^3*(3*B*d
 - A*e)*x^4)/(4*e^4) + (B*c^3*x^5)/(5*e^3) + ((B*d - A*e)*(c*d^2 + a*e^2)^
3)/(2*e^8*(d + e*x)^2) - ((c*d^2 + a*e^2)^2*(7*B*c*d^2 - 6*A*c*d*e + a*B*e
^2))/(e^8*(d + e*x)) - (3*c*(c*d^2 + a*e^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a
*B*d*e^2 - a*A*e^3)*Log[d + e*x])/e^8
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.42

method	result
norman	$\frac{\frac{\left(6A\,a^{2}cd\,e^{5}+36Aa\,c^{2}d^{3}e^{3}+30A\,c^{3}d^{5}e-B\,a^{3}e^{6}-18B\,a^{2}c\,d^{2}e^{4}-60Ba\,c^{2}d^{4}e^{2}-42B\,c^{3}d^{6}\right)x}{e^{7}}-\frac{A\,a^{3}e^{7}-9A\,a^{2}c\,d^{2}e^{5}-54Aa\,c^{2}d^{4}e^{3}-45A\,c^{3}}{}}{}$
default	$-\frac{c\left(-\frac{1}{5}B\,c^{2}x^{5}e^{4}-\frac{1}{4}A\,c^{2}e^{4}x^{4}+\frac{3}{4}B\,c^{2}d\,e^{3}x^{4}+A\,c^{2}d\,e^{3}x^{3}-Bac\,e^{4}x^{3}-2B\,c^{2}d^{2}e^{2}x^{3}-\frac{3}{2}Aac\,e^{4}x^{2}-3A\,c^{2}d^{2}e^{2}x^{2}+\frac{9}{2}Bacd\,e^{3}x^{2}\right.}{e^{7}}$
risch	$\frac{B\,c^{3}x^{5}}{5e^{3}}+\frac{c^{3}A\,x^{4}}{4e^{3}}-\frac{3c^{3}Bd\,x^{4}}{4e^{4}}-\frac{c^{3}Ad\,x^{3}}{e^{4}}+\frac{c^{2}Ba\,x^{3}}{e^{3}}+\frac{2c^{3}B\,d^{2}x^{3}}{e^{5}}+\frac{3c^{2}Aa\,x^{2}}{2e^{3}}+\frac{3c^{3}A\,d^{2}x^{2}}{e^{5}}-\frac{9c^{2}Bad\,x^{2}}{2e^{4}}-$
parallelrisch	$\frac{720A\ln(ex+d)xa\,c^{2}d^{3}e^{4}-270B\,a^{2}c\,d^{3}e^{4}+90A\,a^{2}c\,d^{2}e^{5}+540Aa\,c^{2}d^{4}e^{3}+5A\,x^{6}c^{3}e^{7}+4B\,x^{7}c^{3}e^{7}-420B\ln(ex+d)c^{3}d^{7}-20Bx}{}$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
((6*A*a^2*c*d*e^5+36*A*a*c^2*d^3*e^3+30*A*c^3*d^5*e-B*a^3*e^6-18*B*a^2*c*d
^2*e^4-60*B*a*c^2*d^4*e^2-42*B*c^3*d^6)/e^7*x-1/2*(A*a^3*e^7-9*A*a^2*c*d^2
*e^5-54*A*a*c^2*d^4*e^3-45*A*c^3*d^6*e+B*a^3*d*e^6+27*B*a^2*c*d^3*e^4+90*B
*a*c^2*d^5*e^2+63*B*c^3*d^7)/e^8+1/5*B*c^3*x^7/e-c*(6*A*a*c*d*e^3+5*A*c^2*
d^3*e-3*B*a^2*e^4-10*B*a*c*d^2*e^2-7*B*c^2*d^4)/e^5*x^3-1/10*c^2*(5*A*c*d*
e-10*B*a*e^2-7*B*c*d^2)/e^3*x^5+1/4*c^2*(6*A*a*e^3+5*A*c*d^2*e-10*B*a*d*e^
2-7*B*c*d^3)/e^4*x^4+1/20*c^3*(5*A*e-7*B*d)/e^2*x^6)/(e*x+d)^2+3*c/e^8*(A*
a^2*e^5+6*A*a*c*d^2*e^3+5*A*c^2*d^4*e-3*B*a^2*d*e^4-10*B*a*c*d^3*e^2-7*B*c
^2*d^5)*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 691 vs. $2(292) = 584$.

Time = 0.08 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.30

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^3}\,dx = \frac{4\,Bc^3e^7x^7-130\,Bc^3d^7+110\,Ac^3d^6e-270\,Bac^2d^5e^2+210\,Aac^2d^4e^3-150\,Ba^2cd^3e^4+90\,Aa^2cd^2e^5-1}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/20*(4*B*c^3*e^7*x^7 - 130*B*c^3*d^7 + 110*A*c^3*d^6*e - 270*B*a*c^2*d^5*
e^2 + 210*A*a*c^2*d^4*e^3 - 150*B*a^2*c*d^3*e^4 + 90*A*a^2*c*d^2*e^5 - 10*
B*a^3*d*e^6 - 10*A*a^3*e^7 - (7*B*c^3*d*e^6 - 5*A*c^3*e^7)*x^6 + 2*(7*B*c^
3*d^2*e^5 - 5*A*c^3*d*e^6 + 10*B*a*c^2*e^7)*x^5 - 5*(7*B*c^3*d^3*e^4 - 5*A
*c^3*d^2*e^5 + 10*B*a*c^2*d*e^6 - 6*A*a*c^2*e^7)*x^4 + 20*(7*B*c^3*d^4*e^3
 - 5*A*c^3*d^3*e^4 + 10*B*a*c^2*d^2*e^5 - 6*A*a*c^2*d*e^6 + 3*B*a^2*c*e^7)
*x^3 + 10*(50*B*c^3*d^5*e^2 - 34*A*c^3*d^4*e^3 + 63*B*a*c^2*d^3*e^4 - 33*A
*a*c^2*d^2*e^5 + 12*B*a^2*c*d*e^6)*x^2 + 20*(8*B*c^3*d^6*e - 4*A*c^3*d^5*e
^2 + 3*B*a*c^2*d^4*e^3 + 3*A*a*c^2*d^3*e^4 - 6*B*a^2*c*d^2*e^5 + 6*A*a^2*c
*d*e^6 - B*a^3*e^7)*x - 60*(7*B*c^3*d^7 - 5*A*c^3*d^6*e + 10*B*a*c^2*d^5*e
^2 - 6*A*a*c^2*d^4*e^3 + 3*B*a^2*c*d^3*e^4 - A*a^2*c*d^2*e^5 + (7*B*c^3*d^
5*e^2 - 5*A*c^3*d^4*e^3 + 10*B*a*c^2*d^3*e^4 - 6*A*a*c^2*d^2*e^5 + 3*B*a^2
*c*d*e^6 - A*a^2*c*e^7)*x^2 + 2*(7*B*c^3*d^6*e - 5*A*c^3*d^5*e^2 + 10*B*a*
c^2*d^4*e^3 - 6*A*a*c^2*d^3*e^4 + 3*B*a^2*c*d^2*e^5 - A*a^2*c*d*e^6)*x)*lo
g(e*x + d))/(e^10*x^2 + 2*d*e^9*x + d^2*e^8)
```

Sympy [A] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.63

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^3}\,dx$$
$$= \frac{Bc^3x^5}{5e^3} - \frac{3c(ae^2+cd^2)\,(-Aae^3-5Acd^2e+3Bade^2+7Bcd^3)\log(d+ex)}{e^8}$$
$$+ x^4\left(\frac{Ac^3}{4e^3}-\frac{3Bc^3d}{4e^4}\right) + x^3\left(-\frac{Ac^3d}{e^4}+\frac{Bac^2}{e^3}+\frac{2Bc^3d^2}{e^5}\right)$$
$$+ x^2\cdot\left(\frac{3Aac^2}{2e^3}+\frac{3Ac^3d^2}{e^5}-\frac{9Bac^2d}{2e^4}-\frac{5Bc^3d^3}{e^6}\right)$$
$$+ x\left(-\frac{9Aac^2d}{e^4}-\frac{10Ac^3d^3}{e^6}+\frac{3Ba^2c}{e^3}+\frac{18Bac^2d^2}{e^5}+\frac{15Bc^3d^4}{e^7}\right)$$
$$+ \frac{-Aa^3e^7+9Aa^2cd^2e^5+21Aac^2d^4e^3+11Ac^3d^6e-Ba^3de^6-15Ba^2cd^3e^4-27Bac^2d^5e^2-13Bc^3d^7+}{2d^2e^8+4de^9x}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**3,x)
```

output
```
B*c**3*x**5/(5*e**3) - 3*c*(a*e**2 + c*d**2)*(-A*a*e**3 - 5*A*c*d**2*e + 3
*B*a*d*e**2 + 7*B*c*d**3)*log(d + e*x)/e**8 + x**4*(A*c**3/(4*e**3) - 3*B*
c**3*d/(4*e**4)) + x**3*(-A*c**3*d/e**4 + B*a*c**2/e**3 + 2*B*c**3*d**2/e*
*5) + x**2*(3*A*a*c**2/(2*e**3) + 3*A*c**3*d**2/e**5 - 9*B*a*c**2*d/(2*e**
4) - 5*B*c**3*d**3/e**6) + x*(-9*A*a*c**2*d/e**4 - 10*A*c**3*d**3/e**6 + 3
*B*a**2*c/e**3 + 18*B*a*c**2*d**2/e**5 + 15*B*c**3*d**4/e**7) + (-A*a**3*e
**7 + 9*A*a**2*c*d**2*e**5 + 21*A*a*c**2*d**4*e**3 + 11*A*c**3*d**6*e - B*
a**3*d*e**6 - 15*B*a**2*c*d**3*e**4 - 27*B*a*c**2*d**5*e**2 - 13*B*c**3*d*
*7 + x*(12*A*a**2*c*d*e**6 + 24*A*a*c**2*d**3*e**4 + 12*A*c**3*d**5*e**2 -
 2*B*a**3*e**7 - 18*B*a**2*c*d**2*e**5 - 30*B*a*c**2*d**4*e**3 - 14*B*c**3
*d**6*e))/(2*d**2*e**8 + 4*d*e**9*x + 2*e**10*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.55

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^3}\,dx = -\frac{13\,Bc^3d^7 - 11\,Ac^3d^6e + 27\,Bac^2d^5e^2 - 21\,Aac^2d^4e^3 + 15\,Ba^2cd^3e^4 - 9\,Aa^2cd^2e^5 + Ba^3de^6 + Aa^3e^7 + \cdots}{2\left(e^{10}x^2 + 2\,de^9x \cdots\right)} + \frac{4\,Bc^3e^4x^5 - 5\left(3\,Bc^3de^3 - Ac^3e^4\right)x^4 + 20\left(2\,Bc^3d^2e^2 - Ac^3de^3 + Bac^2e^4\right)x^3 - 10\left(10\,Bc^3d^3e - 6\,Ac^3d \cdots\right)}{20 \cdots} - \frac{3\left(7\,Bc^3d^5 - 5\,Ac^3d^4e + 10\,Bac^2d^3e^2 - 6\,Aac^2d^2e^3 + 3\,Ba^2cde^4 - Aa^2ce^5\right)\log\left(ex+d\right)}{e^8}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/2*(13*B*c^3*d^7 - 11*A*c^3*d^6*e + 27*B*a*c^2*d^5*e^2 - 21*A*a*c^2*d^4*
e^3 + 15*B*a^2*c*d^3*e^4 - 9*A*a^2*c*d^2*e^5 + B*a^3*d*e^6 + A*a^3*e^7 + 2
*(7*B*c^3*d^6*e - 6*A*c^3*d^5*e^2 + 15*B*a*c^2*d^4*e^3 - 12*A*a*c^2*d^3*e^
4 + 9*B*a^2*c*d^2*e^5 - 6*A*a^2*c*d*e^6 + B*a^3*e^7)*x)/(e^10*x^2 + 2*d*e^
9*x + d^2*e^8) + 1/20*(4*B*c^3*e^4*x^5 - 5*(3*B*c^3*d*e^3 - A*c^3*e^4)*x^4
 + 20*(2*B*c^3*d^2*e^2 - A*c^3*d*e^3 + B*a*c^2*e^4)*x^3 - 10*(10*B*c^3*d^3
*e - 6*A*c^3*d^2*e^2 + 9*B*a*c^2*d*e^3 - 3*A*a*c^2*e^4)*x^2 + 20*(15*B*c^3
*d^4 - 10*A*c^3*d^3*e + 18*B*a*c^2*d^2*e^2 - 9*A*a*c^2*d*e^3 + 3*B*a^2*c*e
^4)*x)/e^7 - 3*(7*B*c^3*d^5 - 5*A*c^3*d^4*e + 10*B*a*c^2*d^3*e^2 - 6*A*a*c
^2*d^2*e^3 + 3*B*a^2*c*d*e^4 - A*a^2*c*e^5)*log(e*x + d)/e^8
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.56

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^3}\,dx = -\frac{3\,(7\,Bc^3d^5-5\,Ac^3d^4e+10\,Bac^2d^3e^2-6\,Aac^2d^2e^3+3\,Ba^2cde^4-Aa^2ce^5)\log\left(|ex+d|\right)}{e^8} - \frac{13\,Bc^3d^7-11\,Ac^3d^6e+27\,Bac^2d^5e^2-21\,Aac^2d^4e^3+15\,Ba^2cd^3e^4-9\,Aa^2cd^2e^5+Ba^3de^6+Aa^3e^7+}{2\,(ex+d)^2} + \frac{4\,Bc^3e^{12}x^5-15\,Bc^3de^{11}x^4+5\,Ac^3e^{12}x^4+40\,Bc^3d^2e^{10}x^3-20\,Ac^3de^{11}x^3+20\,Bac^2e^{12}x^3-100\,Bc^3d^3}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^3,x, algorithm="giac")
```

output
```
-3*(7*B*c^3*d^5 - 5*A*c^3*d^4*e + 10*B*a*c^2*d^3*e^2 - 6*A*a*c^2*d^2*e^3 +
 3*B*a^2*c*d*e^4 - A*a^2*c*e^5)*log(abs(e*x + d))/e^8 - 1/2*(13*B*c^3*d^7
- 11*A*c^3*d^6*e + 27*B*a*c^2*d^5*e^2 - 21*A*a*c^2*d^4*e^3 + 15*B*a^2*c*d^
3*e^4 - 9*A*a^2*c*d^2*e^5 + B*a^3*d*e^6 + A*a^3*e^7 + 2*(7*B*c^3*d^6*e - 6
*A*c^3*d^5*e^2 + 15*B*a*c^2*d^4*e^3 - 12*A*a*c^2*d^3*e^4 + 9*B*a^2*c*d^2*e
^5 - 6*A*a^2*c*d*e^6 + B*a^3*e^7)*x)/((e*x + d)^2*e^8) + 1/20*(4*B*c^3*e^1
2*x^5 - 15*B*c^3*d*e^11*x^4 + 5*A*c^3*e^12*x^4 + 40*B*c^3*d^2*e^10*x^3 - 2
0*A*c^3*d*e^11*x^3 + 20*B*a*c^2*e^12*x^3 - 100*B*c^3*d^3*e^9*x^2 + 60*A*c^
3*d^2*e^10*x^2 - 90*B*a*c^2*d*e^11*x^2 + 30*A*a*c^2*e^12*x^2 + 300*B*c^3*d
^4*e^8*x - 200*A*c^3*d^3*e^9*x + 360*B*a*c^2*d^2*e^10*x - 180*A*a*c^2*d*e^
11*x + 60*B*a^2*c*e^12*x)/e^15
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.27

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^3}\,dx = x^4\left(\frac{A\,c^3}{4\,e^3}-\frac{3\,B\,c^3\,d}{4\,e^4}\right)$$

$$-\frac{\frac{B\,a^3\,d\,e^6+A\,a^3\,e^7+15\,B\,a^2\,c\,d^3\,e^4-9\,A\,a^2\,c\,d^2\,e^5+27\,B\,a\,c^2\,d^5\,e^2-21\,A\,a\,c^2\,d^4\,e^3+13\,B\,c^3\,d^7-11\,A\,c^3\,d^6\,e}{2\,e}+x\,(B\,a^3\,e^6+9\,B\,a^2\,c\,\cdots}{d^2\,e^7+2\,d\,e^8\,x+e^9\,x^2}$$

$$-x\left(\frac{3\,d\left(\frac{3\,d\left(\frac{3\,d\left(\frac{A\,c^3}{e^3}-\frac{3\,B\,c^3\,d}{e^4}\right)}{e}-\frac{3\,B\,a\,c^2}{e^3}+\frac{3\,B\,c^3\,d^2}{e^5}\right)}{e}-\frac{3\,d^2\left(\frac{A\,c^3}{e^3}-\frac{3\,B\,c^3\,d}{e^4}\right)}{e^2}+\frac{3\,A\,a\,c^2}{e^3}-\frac{B\,c^3\,d^3}{e^6}\right)}{e}\right.$$

$$\left.-\frac{3\,d^2\left(\frac{3\,d\left(\frac{A\,c^3}{e^3}-\frac{3\,B\,c^3\,d}{e^4}\right)}{e}-\frac{3\,B\,a\,c^2}{e^3}+\frac{3\,B\,c^3\,d^2}{e^5}\right)}{e^2}+\frac{d^3\left(\frac{A\,c^3}{e^3}-\frac{3\,B\,c^3\,d}{e^4}\right)}{e^3}-\frac{3\,B\,a^2\,c}{e^3}\right)$$

$$-x^3\left(\frac{d\left(\frac{A\,c^3}{e^3}-\frac{3\,B\,c^3\,d}{e^4}\right)}{e}-\frac{B\,a\,c^2}{e^3}+\frac{B\,c^3\,d^2}{e^5}\right)$$

$$+x^2\left(\frac{3\,d\left(\frac{3\,d\left(\frac{A\,c^3}{e^3}-\frac{3\,B\,c^3\,d}{e^4}\right)}{e}-\frac{3\,B\,a\,c^2}{e^3}+\frac{3\,B\,c^3\,d^2}{e^5}\right)}{2\,e}-\frac{3\,d^2\left(\frac{A\,c^3}{e^3}-\frac{3\,B\,c^3\,d}{e^4}\right)}{2\,e^2}+\frac{3\,A\,a\,c^2}{2\,e^3}\right.$$

$$\left.-\frac{B\,c^3\,d^3}{2\,e^6}\right)$$

$$-\frac{\ln(d+e\,x)\,\left(9\,B\,a^2\,c\,d\,e^4-3\,A\,a^2\,c\,e^5+30\,B\,a\,c^2\,d^3\,e^2-18\,A\,a\,c^2\,d^2\,e^3+21\,B\,c^3\,d^5-15\,A\,c^3\,d^4\,e\right)}{e^8}$$

$$+\frac{B\,c^3\,x^5}{5\,e^3}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^3,x)
```

output
```
x^4*((A*c^3)/(4*e^3) - (3*B*c^3*d)/(4*e^4)) - ((A*a^3*e^7 + 13*B*c^3*d^7 +
 B*a^3*d*e^6 - 11*A*c^3*d^6*e - 21*A*a*c^2*d^4*e^3 - 9*A*a^2*c*d^2*e^5 + 2
7*B*a*c^2*d^5*e^2 + 15*B*a^2*c*d^3*e^4)/(2*e) + x*(B*a^3*e^6 + 7*B*c^3*d^6
 - 6*A*c^3*d^5*e - 12*A*a*c^2*d^3*e^3 + 15*B*a*c^2*d^4*e^2 + 9*B*a^2*c*d^2
*e^4 - 6*A*a^2*c*d*e^5))/(d^2*e^7 + e^9*x^2 + 2*d*e^8*x) - x*((3*d*((3*d*(
(3*d*((A*c^3)/e^3 - (3*B*c^3*d)/e^4))/e - (3*B*a*c^2)/e^3 + (3*B*c^3*d^2)/
e^5))/e - (3*d^2*((A*c^3)/e^3 - (3*B*c^3*d)/e^4))/e^2 + (3*A*a*c^2)/e^3 -
(B*c^3*d^3)/e^6))/e - (3*d^2*((3*d*((A*c^3)/e^3 - (3*B*c^3*d)/e^4))/e - (3
*B*a*c^2)/e^3 + (3*B*c^3*d^2)/e^5))/e^2 + (d^3*((A*c^3)/e^3 - (3*B*c^3*d)/
e^4))/e^3 - (3*B*a^2*c)/e^3) - x^3*((d*((A*c^3)/e^3 - (3*B*c^3*d)/e^4))/e
- (B*a*c^2)/e^3 + (B*c^3*d^2)/e^5) + x^2*((3*d*((3*d*((A*c^3)/e^3 - (3*B*c
^3*d)/e^4))/e - (3*B*a*c^2)/e^3 + (3*B*c^3*d^2)/e^5))/(2*e) - (3*d^2*((A*c
^3)/e^3 - (3*B*c^3*d)/e^4))/(2*e^2) + (3*A*a*c^2)/(2*e^3) - (B*c^3*d^3)/(2
*e^6)) - (log(d + e*x)*(21*B*c^3*d^5 - 3*A*a^2*c*e^5 - 15*A*c^3*d^4*e - 18
*A*a*c^2*d^2*e^3 + 30*B*a*c^2*d^3*e^2 + 9*B*a^2*c*d*e^4))/e^8 + (B*c^3*x^5
)/(5*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 785, normalized size of antiderivative = 2.62

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^3,x)
```

output
```
(60*log(d + e*x)*a**3*c*d**3*e**5 + 120*log(d + e*x)*a**3*c*d**2*e**6*x +
60*log(d + e*x)*a**3*c*d*e**7*x**2 - 180*log(d + e*x)*a**2*b*c*d**4*e**4 -
 360*log(d + e*x)*a**2*b*c*d**3*e**5*x - 180*log(d + e*x)*a**2*b*c*d**2*e*
*6*x**2 + 360*log(d + e*x)*a**2*c**2*d**5*e**3 + 720*log(d + e*x)*a**2*c**
2*d**4*e**4*x + 360*log(d + e*x)*a**2*c**2*d**3*e**5*x**2 - 600*log(d + e*
x)*a*b*c**2*d**6*e**2 - 1200*log(d + e*x)*a*b*c**2*d**5*e**3*x - 600*log(d
 + e*x)*a*b*c**2*d**4*e**4*x**2 + 300*log(d + e*x)*a*c**3*d**7*e + 600*log
(d + e*x)*a*c**3*d**6*e**2*x + 300*log(d + e*x)*a*c**3*d**5*e**3*x**2 - 42
0*log(d + e*x)*b*c**3*d**8 - 840*log(d + e*x)*b*c**3*d**7*e*x - 420*log(d
+ e*x)*b*c**3*d**6*e**2*x**2 - 10*a**4*d*e**7 + 10*a**3*b*e**8*x**2 + 30*a
**3*c*d**3*e**5 - 60*a**3*c*d*e**7*x**2 - 90*a**2*b*c*d**4*e**4 + 180*a**2
*b*c*d**2*e**6*x**2 + 60*a**2*b*c*d*e**7*x**3 + 180*a**2*c**2*d**5*e**3 -
360*a**2*c**2*d**3*e**5*x**2 - 120*a**2*c**2*d**2*e**6*x**3 + 30*a**2*c**2
*d*e**7*x**4 - 300*a*b*c**2*d**6*e**2 + 600*a*b*c**2*d**4*e**4*x**2 + 200*
a*b*c**2*d**3*e**5*x**3 - 50*a*b*c**2*d**2*e**6*x**4 + 20*a*b*c**2*d*e**7*
x**5 + 150*a*c**3*d**7*e - 300*a*c**3*d**5*e**3*x**2 - 100*a*c**3*d**4*e**
4*x**3 + 25*a*c**3*d**3*e**5*x**4 - 10*a*c**3*d**2*e**6*x**5 + 5*a*c**3*d*
e**7*x**6 - 210*b*c**3*d**8 + 420*b*c**3*d**6*e**2*x**2 + 140*b*c**3*d**5*
e**3*x**3 - 35*b*c**3*d**4*e**4*x**4 + 14*b*c**3*d**3*e**5*x**5 - 7*b*c**3
*d**2*e**6*x**6 + 4*b*c**3*d*e**7*x**7)/(20*d*e**8*(d**2 + 2*d*e*x + e*...
```

3.71 $\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 310

$$\begin{aligned}
\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^4} dx
&= -\frac{c^2(20Bcd^3-10Acd^2e+12aBde^2-3aAe^3)x}{e^7} \\
&\quad+\frac{c^2(10Bcd^2-4Acde+3aBe^2)x^2}{2e^6}-\frac{c^3(4Bd-Ae)x^3}{3e^5}+\frac{Bc^3x^4}{4e^4} \\
&\quad+\frac{(Bd-Ae)(cd^2+ae^2)^3}{3e^8(d+ex)^3}-\frac{(cd^2+ae^2)^2(7Bcd^2-6Acde+aBe^2)}{2e^8(d+ex)^2} \\
&\quad+\frac{3c(cd^2+ae^2)(7Bcd^3-5Acd^2e+3aBde^2-aAe^3)}{e^8(d+ex)} \\
&\quad-\frac{c(4Acde(5cd^2+3ae^2)-B(35c^2d^4+30acd^2e^2+3a^2e^4))\log(d+ex)}{e^8}
\end{aligned}$$

output
```
-c^2*(-3*A*a*e^3-10*A*c*d^2*e+12*B*a*d*e^2+20*B*c*d^3)*x/e^7+1/2*c^2*(-4*A
*c*d*e+3*B*a*e^2+10*B*c*d^2)*x^2/e^6-1/3*c^3*(-A*e+4*B*d)*x^3/e^5+1/4*B*c^
3*x^4/e^4+1/3*(-A*e+B*d)*(a*e^2+c*d^2)^3/e^8/(e*x+d)^3-1/2*(a*e^2+c*d^2)^2
*(-6*A*c*d*e+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)^2+3*c*(a*e^2+c*d^2)*(-A*a*e^3-
5*A*c*d^2*e+3*B*a*d*e^2+7*B*c*d^3)/e^8/(e*x+d)-c*(4*A*c*d*e*(3*a*e^2+5*c*d
^2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))*ln(e*x+d)/e^8
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^4}\,dx$$
$$= \frac{12c^2e(Ae(10cd^2+3ae^2)-4B(5cd^3+3ade^2))\,x+6c^2e^2(10Bcd^2-4Acde+3aBe^2)\,x^2+4c^3e^3(-4Bd+$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^4,x]
```

output
```
(12*c^2*e*(A*e*(10*c*d^2 + 3*a*e^2) - 4*B*(5*c*d^3 + 3*a*d*e^2))*x + 6*c^2
*e^2*(10*B*c*d^2 - 4*A*c*d*e + 3*a*B*e^2)*x^2 + 4*c^3*e^3*(-4*B*d + A*e)*x
^3 + 3*B*c^3*e^4*x^4 + (4*(B*d - A*e)*(c*d^2 + a*e^2)^3)/(d + e*x)^3 - (6*
(c*d^2 + a*e^2)^2*(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2))/(d + e*x)^2 + (36*c*(
c*d^2 + a*e^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(d + e*x
) + 12*c*(-4*A*c*d*e*(5*c*d^2 + 3*a*e^2) + B*(35*c^2*d^4 + 30*a*c*d^2*e^2
+ 3*a^2*e^4))*Log[d + e*x])/(12*e^8)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^3(A+Bx)}{(d+ex)^4}\,dx$$
$$\Big\downarrow \text{652}$$
$$\int \left(-\frac{c\left(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4\right)}{e^7(d+ex)}-\frac{c^2x\left(-3aBe^2+4Acde-10Bcd^2\right)}{e^6}+\frac{c^2(}{}$$
$$\Big\downarrow \text{2009}$$

$$-\frac{c\log(d+ex)\left(4Acde\left(3ae^2+5cd^2\right)-B\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)\right)}{e^8}+\frac{c^2x^2\left(3aBe^2-4Acde+10Bcd^2\right)}{2e^6}-\frac{c^2x\left(-3aAe^3+12aBde^2-10Acd^2e+20Bcd^3\right)}{e^7}-\frac{\left(ae^2+cd^2\right)^2\left(aBe^2-6Acde+7Bcd^2\right)}{2e^8(d+ex)^2}+\frac{\left(ae^2+cd^2\right)^3(Bd-Ae)}{3e^8(d+ex)^3}+\frac{3c\left(ae^2+cd^2\right)\left(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3\right)}{e^8(d+ex)}-\frac{c^3x^3(4Bd-Ae)}{3e^5}+\frac{Bc^3x^4}{4e^4}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^4,x]
```

output
```
-((c^2*(20*B*c*d^3 - 10*A*c*d^2*e + 12*a*B*d*e^2 - 3*a*A*e^3)*x)/e^7) + (c
^2*(10*B*c*d^2 - 4*A*c*d*e + 3*a*B*e^2)*x^2)/(2*e^6) - (c^3*(4*B*d - A*e)*
x^3)/(3*e^5) + (B*c^3*x^4)/(4*e^4) + ((B*d - A*e)*(c*d^2 + a*e^2)^3)/(3*e^
8*(d + e*x)^3) - ((c*d^2 + a*e^2)^2*(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2))/(2*
e^8*(d + e*x)^2) + (3*c*(c*d^2 + a*e^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d
*e^2 - a*A*e^3))/(e^8*(d + e*x)) - (c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(
35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4))*Log[d + e*x])/e^8
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.39

method	result
norman	$-\frac{2A\,a^{3}e^{7}+6A\,a^{2}c\,d^{2}e^{5}+132Aa\,c^{2}d^{4}e^{3}+220A\,c^{3}d^{6}e+B\,a^{3}d\,e^{6}-33B\,a^{2}c\,d^{3}e^{4}-330Ba\,c^{2}d^{5}e^{2}-385B\,c^{3}d^{7}}{6e^{8}}-\frac{3\left(A\,a^{2}c\,e^{5}+12Aa\,c^{2}d^{2}e^{3}+2\right.}{}$
default	$\frac{c^{2}\left(\frac{1}{4}Bc\,x^{4}e^{3}+\frac{1}{3}A\,x^{3}c\,e^{3}-\frac{4}{3}B\,x^{3}cd\,e^{2}-2A\,x^{2}cd\,e^{2}+\frac{3}{2}B\,x^{2}a\,e^{3}+5B\,x^{2}c\,d^{2}e+3Aa\,e^{3}x+10Ac\,d^{2}ex-12Bad\,e^{2}x-20Bc\,d^{3}x\right)}{e^{7}}$
risch	$\frac{B\,c^{3}x^{4}}{4e^{4}}+\frac{c^{3}A\,x^{3}}{3e^{4}}-\frac{4c^{3}B\,x^{3}d}{3e^{5}}-\frac{2c^{3}A\,x^{2}d}{e^{5}}+\frac{3c^{2}B\,x^{2}a}{2e^{4}}+\frac{5c^{3}B\,x^{2}d^{2}}{e^{6}}+\frac{3c^{2}Aax}{e^{4}}+\frac{10c^{3}A\,d^{2}x}{e^{6}}-\frac{12c^{2}Badx}{e^{5}}-2$
parallelrisch	$-\frac{432A\ln(ex+d)xa\,c^{2}d^{3}e^{4}-66B\,a^{2}c\,d^{3}e^{4}+12A\,a^{2}c\,d^{2}e^{5}+264Aa\,c^{2}d^{4}e^{3}-4A\,x^{6}c^{3}e^{7}-3B\,x^{7}c^{3}e^{7}-420B\ln(ex+d)c^{3}d^{7}+6B}{}$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/6*(2*A*a^3*e^7+6*A*a^2*c*d^2*e^5+132*A*a*c^2*d^4*e^3+220*A*c^3*d^6*e+B
*a^3*d*e^6-33*B*a^2*c*d^3*e^4-330*B*a*c^2*d^5*e^2-385*B*c^3*d^7)/e^8-3*(A*
a^2*c*e^5+12*A*a*c^2*d^2*e^3+20*A*c^3*d^4*e-3*B*a^2*c*d*e^4-30*B*a*c^2*d^3
*e^2-35*B*c^3*d^5)/e^6*x^2-1/2*(6*A*a^2*c*d*e^5+108*A*a*c^2*d^3*e^3+180*A*
c^3*d^5*e+B*a^3*e^6-27*B*a^2*c*d^2*e^4-270*B*a*c^2*d^4*e^2-315*B*c^3*d^6)/
e^7*x+1/4*B*c^3*x^7/e-1/4*c^2*(4*A*c*d*e-6*B*a*e^2-7*B*c*d^2)/e^3*x^5+1/4*
c^2*(12*A*a*e^3+20*A*c*d^2*e-30*B*a*d*e^2-35*B*c*d^3)/e^4*x^4+1/12*c^3*(4*
A*e-7*B*d)/e^2*x^6)/(e*x+d)^3-1/e^8*c*(12*A*a*c*d*e^3+20*A*c^2*d^3*e-3*B*a
^2*e^4-30*B*a*c*d^2*e^2-35*B*c^2*d^4)*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 732 vs. $2(300) = 600$.

Time = 0.08 (sec) , antiderivative size = 732, normalized size of antiderivative = 2.36

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^4}\,dx$$
$$=\frac{3\,Bc^3e^7x^7+214\,Bc^3d^7-148\,Ac^3d^6e+282\,Bac^2d^5e^2-156\,Aac^2d^4e^3+66\,Ba^2cd^3e^4-12\,Aa^2cd^2e^5-2}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/12*(3*B*c^3*e^7*x^7 + 214*B*c^3*d^7 - 148*A*c^3*d^6*e + 282*B*a*c^2*d^5*
e^2 - 156*A*a*c^2*d^4*e^3 + 66*B*a^2*c*d^3*e^4 - 12*A*a^2*c*d^2*e^5 - 2*B*
a^3*d*e^6 - 4*A*a^3*e^7 - (7*B*c^3*d*e^6 - 4*A*c^3*e^7)*x^6 + 3*(7*B*c^3*d
^2*e^5 - 4*A*c^3*d*e^6 + 6*B*a*c^2*e^7)*x^5 - 3*(35*B*c^3*d^3*e^4 - 20*A*c
^3*d^2*e^5 + 30*B*a*c^2*d*e^6 - 12*A*a*c^2*e^7)*x^4 - 2*(278*B*c^3*d^4*e^3
 - 146*A*c^3*d^3*e^4 + 189*B*a*c^2*d^2*e^5 - 54*A*a*c^2*d*e^6)*x^3 - 6*(68
*B*c^3*d^5*e^2 - 26*A*c^3*d^4*e^3 + 9*B*a*c^2*d^3*e^4 + 18*A*a*c^2*d^2*e^5
 - 18*B*a^2*c*d*e^6 + 6*A*a^2*c*e^7)*x^2 + 6*(37*B*c^3*d^6*e - 34*A*c^3*d^
5*e^2 + 81*B*a*c^2*d^4*e^3 - 54*A*a*c^2*d^3*e^4 + 27*B*a^2*c*d^2*e^5 - 6*A
*a^2*c*d*e^6 - B*a^3*e^7)*x + 12*(35*B*c^3*d^7 - 20*A*c^3*d^6*e + 30*B*a*c
^2*d^5*e^2 - 12*A*a*c^2*d^4*e^3 + 3*B*a^2*c*d^3*e^4 + (35*B*c^3*d^4*e^3 -
20*A*c^3*d^3*e^4 + 30*B*a*c^2*d^2*e^5 - 12*A*a*c^2*d*e^6 + 3*B*a^2*c*e^7)*
x^3 + 3*(35*B*c^3*d^5*e^2 - 20*A*c^3*d^4*e^3 + 30*B*a*c^2*d^3*e^4 - 12*A*a
*c^2*d^2*e^5 + 3*B*a^2*c*d*e^6)*x^2 + 3*(35*B*c^3*d^6*e - 20*A*c^3*d^5*e^2
 + 30*B*a*c^2*d^4*e^3 - 12*A*a*c^2*d^3*e^4 + 3*B*a^2*c*d^2*e^5)*x)*log(e*x
 + d))/(e^11*x^3 + 3*d*e^10*x^2 + 3*d^2*e^9*x + d^3*e^8)
```

Sympy [A] (verification not implemented)

Time = 8.24 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.71

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^4}\,dx$$
$$= \frac{Bc^3x^4}{4e^4} + \frac{c(-12Aacde^3 - 20Ac^2d^3e + 3Ba^2e^4 + 30Bacd^2e^2 + 35Bc^2d^4)\log(d+ex)}{e^8} + x^3\left(\frac{Ac^3}{3e^4} - \frac{4Bc^3d}{3e^5}\right) + x^2\left(-\frac{2Ac^3d}{e^5} + \frac{3Bac^2}{2e^4} + \frac{5Bc^3d^2}{e^6}\right) + x\left(\frac{3Aac^2}{e^4} + \frac{10Ac^3d^2}{e^6} - \frac{12Bac^2d}{e^5} - \frac{20Bc^3d^3}{e^7}\right) + \frac{-2Aa^3e^7 - 6Aa^2cd^2e^5 - 78Aac^2d^4e^3 - 74Ac^3d^6e - Ba^3de^6 + 33Ba^2cd^3e^4 + 141Bac^2d^5e^2 + 107Bc^3d}{}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**4,x)
```

output
```
B*c**3*x**4/(4*e**4) + c*(-12*A*a*c*d*e**3 - 20*A*c**2*d**3*e + 3*B*a**2*e
**4 + 30*B*a*c*d**2*e**2 + 35*B*c**2*d**4)*log(d + e*x)/e**8 + x**3*(A*c**
3/(3*e**4) - 4*B*c**3*d/(3*e**5)) + x**2*(-2*A*c**3*d/e**5 + 3*B*a*c**2/(2
*e**4) + 5*B*c**3*d**2/e**6) + x*(3*A*a*c**2/e**4 + 10*A*c**3*d**2/e**6 -
12*B*a*c**2*d/e**5 - 20*B*c**3*d**3/e**7) + (-2*A*a**3*e**7 - 6*A*a**2*c*d
**2*e**5 - 78*A*a*c**2*d**4*e**3 - 74*A*c**3*d**6*e - B*a**3*d*e**6 + 33*B
*a**2*c*d**3*e**4 + 141*B*a*c**2*d**5*e**2 + 107*B*c**3*d**7 + x**2*(-18*A
*a**2*c*e**7 - 108*A*a*c**2*d**2*e**5 - 90*A*c**3*d**4*e**3 + 54*B*a**2*c*
d*e**6 + 180*B*a*c**2*d**3*e**4 + 126*B*c**3*d**5*e**2) + x*(-18*A*a**2*c*
d*e**6 - 180*A*a*c**2*d**3*e**4 - 162*A*c**3*d**5*e**2 - 3*B*a**3*e**7 + 8
1*B*a**2*c*d**2*e**5 + 315*B*a*c**2*d**4*e**3 + 231*B*c**3*d**6*e))/(6*d**
3*e**8 + 18*d**2*e**9*x + 18*d*e**10*x**2 + 6*e**11*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.54

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^4}\,dx$$
$$= \frac{107\,Bc^3d^7 - 74\,Ac^3d^6e + 141\,Bac^2d^5e^2 - 78\,Aac^2d^4e^3 + 33\,Ba^2cd^3e^4 - 6\,Aa^2cd^2e^5 - Ba^3de^6 - 2\,Aa^3e^7}{}$$
$$+ \frac{3\,Bc^3e^3x^4 - 4\,(4\,Bc^3de^2 - Ac^3e^3)x^3 + 6\,(10\,Bc^3d^2e - 4\,Ac^3de^2 + 3\,Bac^2e^3)x^2 - 12\,(20\,Bc^3d^3 - 10\,Ac}{12\,e^7}$$
$$+ \frac{(35\,Bc^3d^4 - 20\,Ac^3d^3e + 30\,Bac^2d^2e^2 - 12\,Aac^2de^3 + 3\,Ba^2ce^4)\log(ex+d)}{e^8}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^4,x, algorithm="maxima")
```

output
```
1/6*(107*B*c^3*d^7 - 74*A*c^3*d^6*e + 141*B*a*c^2*d^5*e^2 - 78*A*a*c^2*d^4
*e^3 + 33*B*a^2*c*d^3*e^4 - 6*A*a^2*c*d^2*e^5 - B*a^3*d*e^6 - 2*A*a^3*e^7
+ 18*(7*B*c^3*d^5*e^2 - 5*A*c^3*d^4*e^3 + 10*B*a*c^2*d^3*e^4 - 6*A*a*c^2*d
^2*e^5 + 3*B*a^2*c*d*e^6 - A*a^2*c*e^7)*x^2 + 3*(77*B*c^3*d^6*e - 54*A*c^3
*d^5*e^2 + 105*B*a*c^2*d^4*e^3 - 60*A*a*c^2*d^3*e^4 + 27*B*a^2*c*d^2*e^5 -
 6*A*a^2*c*d*e^6 - B*a^3*e^7)*x)/(e^11*x^3 + 3*d*e^10*x^2 + 3*d^2*e^9*x +
d^3*e^8) + 1/12*(3*B*c^3*e^3*x^4 - 4*(4*B*c^3*d*e^2 - A*c^3*e^3)*x^3 + 6*(
10*B*c^3*d^2*e - 4*A*c^3*d*e^2 + 3*B*a*c^2*e^3)*x^2 - 12*(20*B*c^3*d^3 - 1
0*A*c^3*d^2*e + 12*B*a*c^2*d*e^2 - 3*A*a*c^2*e^3)*x)/e^7 + (35*B*c^3*d^4 -
 20*A*c^3*d^3*e + 30*B*a*c^2*d^2*e^2 - 12*A*a*c^2*d*e^3 + 3*B*a^2*c*e^4)*l
og(e*x + d)/e^8
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.50

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^4}\,dx$$
$$= \frac{(35\,Bc^3d^4 - 20\,Ac^3d^3e + 30\,Bac^2d^2e^2 - 12\,Aac^2de^3 + 3\,Ba^2ce^4)\log\left(|ex+d|\right)}{e^8}$$
$$+ \frac{107\,Bc^3d^7 - 74\,Ac^3d^6e + 141\,Bac^2d^5e^2 - 78\,Aac^2d^4e^3 + 33\,Ba^2cd^3e^4 - 6\,Aa^2cd^2e^5 - Ba^3de^6 - 2\,Aa^3}{}$$
$$+ \frac{3\,Bc^3e^{12}x^4 - 16\,Bc^3de^{11}x^3 + 4\,Ac^3e^{12}x^3 + 60\,Bc^3d^2e^{10}x^2 - 24\,Ac^3de^{11}x^2 + 18\,Bac^2e^{12}x^2 - 240\,Bc^3d}{12\,e^{16}}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^4,x, algorithm="giac")
```

output
```
(35*B*c^3*d^4 - 20*A*c^3*d^3*e + 30*B*a*c^2*d^2*e^2 - 12*A*a*c^2*d*e^3 + 3
*B*a^2*c*e^4)*log(abs(e*x + d))/e^8 + 1/6*(107*B*c^3*d^7 - 74*A*c^3*d^6*e
+ 141*B*a*c^2*d^5*e^2 - 78*A*a*c^2*d^4*e^3 + 33*B*a^2*c*d^3*e^4 - 6*A*a^2*
c*d^2*e^5 - B*a^3*d*e^6 - 2*A*a^3*e^7 + 18*(7*B*c^3*d^5*e^2 - 5*A*c^3*d^4*
e^3 + 10*B*a*c^2*d^3*e^4 - 6*A*a*c^2*d^2*e^5 + 3*B*a^2*c*d*e^6 - A*a^2*c*e
^7)*x^2 + 3*(77*B*c^3*d^6*e - 54*A*c^3*d^5*e^2 + 105*B*a*c^2*d^4*e^3 - 60*
A*a*c^2*d^3*e^4 + 27*B*a^2*c*d^2*e^5 - 6*A*a^2*c*d*e^6 - B*a^3*e^7)*x)/((e
*x + d)^3*e^8) + 1/12*(3*B*c^3*e^12*x^4 - 16*B*c^3*d*e^11*x^3 + 4*A*c^3*e^
12*x^3 + 60*B*c^3*d^2*e^10*x^2 - 24*A*c^3*d*e^11*x^2 + 18*B*a*c^2*e^12*x^2
 - 240*B*c^3*d^3*e^9*x + 120*A*c^3*d^2*e^10*x - 144*B*a*c^2*d*e^11*x + 36*
A*a*c^2*e^12*x)/e^16
```

Mupad [B] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.77

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^4}\,dx = x^3\left(\frac{A\,c^3}{3\,e^4}-\frac{4\,B\,c^3\,d}{3\,e^5}\right) - \frac{\frac{B\,a^3\,d\,e^6+2\,A\,a^3\,e^7-33\,B\,a^2\,c\,d^3\,e^4+6\,A\,a^2\,c\,d^2\,e^5-141\,B\,a\,c^2\,d^5\,e^2+78\,A\,a\,c^2\,d^4\,e^3-107\,B\,c^3\,d^7+74\,A\,c^3\,d^6\,e}{6\,e}+x^2\left(-9\,B\,a^2\,c\,d\,e^5-\cdots\right.}{\cdots}$$

$$+x\left(\frac{4\,d\left(\frac{4\,d\left(\frac{A\,c^3}{e^4}-\frac{4\,B\,c^3\,d}{e^5}\right)}{e}-\frac{3\,B\,a\,c^2}{e^4}+\frac{6\,B\,c^3\,d^2}{e^6}\right)}{e}-\frac{6\,d^2\left(\frac{A\,c^3}{e^4}-\frac{4\,B\,c^3\,d}{e^5}\right)}{e^2}+\frac{3\,A\,a\,c^2}{e^4}-\frac{4\,B\,c^3\,d^3}{e^7}\right)-x^2\left(\frac{2\,d\left(\frac{A\,c^3}{e^4}-\frac{4\,B\,c^3\,d}{e^5}\right)}{e}-\frac{3\,B\,a\,c^2}{2\,e^4}+\frac{3\,B\,c^3\,d^2}{e^6}\right)$$

$$+\frac{\ln(d+e\,x)\,\left(3\,B\,a^2\,c\,e^4+30\,B\,a\,c^2\,d^2\,e^2-12\,A\,a\,c^2\,d\,e^3+35\,B\,c^3\,d^4-20\,A\,c^3\,d^3\,e\right)}{e^8}+\frac{B\,c^3\,x^4}{4\,e^4}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^4,x)
```

output
```
x^3*((A*c^3)/(3*e^4) - (4*B*c^3*d)/(3*e^5)) - ((2*A*a^3*e^7 - 107*B*c^3*d^
7 + B*a^3*d*e^6 + 74*A*c^3*d^6*e + 78*A*a*c^2*d^4*e^3 + 6*A*a^2*c*d^2*e^5
- 141*B*a*c^2*d^5*e^2 - 33*B*a^2*c*d^3*e^4)/(6*e) + x^2*(3*A*a^2*c*e^6 - 2
1*B*c^3*d^5*e + 15*A*c^3*d^4*e^2 + 18*A*a*c^2*d^2*e^4 - 30*B*a*c^2*d^3*e^3
 - 9*B*a^2*c*d*e^5) + x*((B*a^3*e^6)/2 - (77*B*c^3*d^6)/2 + 27*A*c^3*d^5*e
 + 30*A*a*c^2*d^3*e^3 - (105*B*a*c^2*d^4*e^2)/2 - (27*B*a^2*c*d^2*e^4)/2 +
 3*A*a^2*c*d*e^5))/(d^3*e^7 + e^10*x^3 + 3*d^2*e^8*x + 3*d*e^9*x^2) + x*((
4*d*((4*d*((A*c^3)/e^4 - (4*B*c^3*d)/e^5))/e - (3*B*a*c^2)/e^4 + (6*B*c^3*
d^2)/e^6))/e - (6*d^2*((A*c^3)/e^4 - (4*B*c^3*d)/e^5))/e^2 + (3*A*a*c^2)/e
^4 - (4*B*c^3*d^3)/e^7) - x^2*((2*d*((A*c^3)/e^4 - (4*B*c^3*d)/e^5))/e - (
3*B*a*c^2)/(2*e^4) + (3*B*c^3*d^2)/e^6) + (log(d + e*x)*(35*B*c^3*d^4 + 3*
B*a^2*c*e^4 - 20*A*c^3*d^3*e + 30*B*a*c^2*d^2*e^2 - 12*A*a*c^2*d*e^3))/e^8
 + (B*c^3*x^4)/(4*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 833, normalized size of antiderivative = 2.69

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^4} dx = \text{Too large to display}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^4,x)
```

output
```
(36*log(d + e*x)*a**2*b*c*d**4*e**4 + 108*log(d + e*x)*a**2*b*c*d**3*e**5*
x + 108*log(d + e*x)*a**2*b*c*d**2*e**6*x**2 + 36*log(d + e*x)*a**2*b*c*d*
e**7*x**3 - 144*log(d + e*x)*a**2*c**2*d**5*e**3 - 432*log(d + e*x)*a**2*c
**2*d**4*e**4*x - 432*log(d + e*x)*a**2*c**2*d**3*e**5*x**2 - 144*log(d +
e*x)*a**2*c**2*d**2*e**6*x**3 + 360*log(d + e*x)*a*b*c**2*d**6*e**2 + 1080
*log(d + e*x)*a*b*c**2*d**5*e**3*x + 1080*log(d + e*x)*a*b*c**2*d**4*e**4*
x**2 + 360*log(d + e*x)*a*b*c**2*d**3*e**5*x**3 - 240*log(d + e*x)*a*c**3*
d**7*e - 720*log(d + e*x)*a*c**3*d**6*e**2*x - 720*log(d + e*x)*a*c**3*d**
5*e**3*x**2 - 240*log(d + e*x)*a*c**3*d**4*e**4*x**3 + 420*log(d + e*x)*b*
c**3*d**8 + 1260*log(d + e*x)*b*c**3*d**7*e*x + 1260*log(d + e*x)*b*c**3*d
**6*e**2*x**2 + 420*log(d + e*x)*b*c**3*d**5*e**3*x**3 - 4*a**4*d*e**7 - 2
*a**3*b*d**2*e**6 - 6*a**3*b*d*e**7*x + 12*a**3*c*e**8*x**3 + 30*a**2*b*c*
d**4*e**4 + 54*a**2*b*c*d**3*e**5*x - 36*a**2*b*c*d*e**7*x**3 - 120*a**2*c
**2*d**5*e**3 - 216*a**2*c**2*d**4*e**4*x + 144*a**2*c**2*d**2*e**6*x**3 +
 36*a**2*c**2*d*e**7*x**4 + 300*a*b*c**2*d**6*e**2 + 540*a*b*c**2*d**5*e**
3*x - 360*a*b*c**2*d**3*e**5*x**3 - 90*a*b*c**2*d**2*e**6*x**4 + 18*a*b*c*
*2*d*e**7*x**5 - 200*a*c**3*d**7*e - 360*a*c**3*d**6*e**2*x + 240*a*c**3*d
**4*e**4*x**3 + 60*a*c**3*d**3*e**5*x**4 - 12*a*c**3*d**2*e**6*x**5 + 4*a*
c**3*d*e**7*x**6 + 350*b*c**3*d**8 + 630*b*c**3*d**7*e*x - 420*b*c**3*d**5
*e**3*x**3 - 105*b*c**3*d**4*e**4*x**4 + 21*b*c**3*d**3*e**5*x**5 - 7*b...
```

3.72 $\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 314

$$
\begin{aligned}
\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^5}\,dx = & -\frac{c^2(5Acde-3B(5cd^2+ae^2))\,x}{e^7}-\frac{c^3(5Bd-Ae)x^2}{2e^6} \\
& +\frac{Bc^3x^3}{3e^5}+\frac{(Bd-Ae)\left(cd^2+ae^2\right)^3}{4e^8(d+ex)^4} \\
& -\frac{\left(cd^2+ae^2\right)^2\left(7Bcd^2-6Acde+aBe^2\right)}{3e^8(d+ex)^3} \\
& +\frac{3c(cd^2+ae^2)\left(7Bcd^3-5Acd^2e+3aBde^2-aAe^3\right)}{2e^8(d+ex)^2} \\
& +\frac{c(4Acde(5cd^2+3ae^2)-B(35c^2d^4+30acd^2e^2+3a^2e^4))}{e^8(d+ex)} \\
& -\frac{c^2(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3)\log(d+ex)}{e^8}
\end{aligned}
$$

output

```
-c^2*(5*A*c*d*e-3*B*(a*e^2+5*c*d^2))*x/e^7-1/2*c^3*(-A*e+5*B*d)*x^2/e^6+1/
3*B*c^3*x^3/e^5+1/4*(-A*e+B*d)*(a*e^2+c*d^2)^3/e^8/(e*x+d)^4-1/3*(a*e^2+c*
d^2)^2*(-6*A*c*d*e+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)^3+3/2*c*(a*e^2+c*d^2)*(-
A*a*e^3-5*A*c*d^2*e+3*B*a*d*e^2+7*B*c*d^3)/e^8/(e*x+d)^2+c*(4*A*c*d*e*(3*a
*e^2+5*c*d^2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))/e^8/(e*x+d)-c^2*(-3
*A*a*e^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)*ln(e*x+d)/e^8
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.29

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^5}\,dx$$
$$= \frac{3Ae(-a^3e^6 - a^2ce^4(d^2+4dex+6e^2x^2) + ac^2de^2(25d^3+88d^2ex+108de^2x^2+48e^3x^3) + c^3(57d^6+168}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^5,x]
```

output
```
(3*A*e*(-(a^3*e^6) - a^2*c*e^4*(d^2 + 4*d*e*x + 6*e^2*x^2) + a*c^2*d*e^2*(
25*d^3 + 88*d^2*e*x + 108*d*e^2*x^2 + 48*e^3*x^3) + c^3*(57*d^6 + 168*d^5*
e*x + 132*d^4*e^2*x^2 - 32*d^3*e^3*x^3 - 68*d^2*e^4*x^4 - 12*d*e^5*x^5 + 2
*e^6*x^6)) - B*(a^3*e^6*(d + 4*e*x) + 9*a^2*c*e^4*(d^3 + 4*d^2*e*x + 6*d*e
^2*x^2 + 4*e^3*x^3) + 3*a*c^2*e^2*(77*d^5 + 248*d^4*e*x + 252*d^3*e^2*x^2
+ 48*d^2*e^3*x^3 - 48*d*e^4*x^4 - 12*e^5*x^5) + c^3*(319*d^7 + 856*d^6*e*x
 + 444*d^5*e^2*x^2 - 544*d^4*e^3*x^3 - 556*d^3*e^4*x^4 - 84*d^2*e^5*x^5 +
14*d*e^6*x^6 - 4*e^7*x^7)) + 12*c^2*(3*A*e*(5*c*d^2 + a*e^2) - 5*B*(7*c*d^
3 + 3*a*d*e^2))*(d + e*x)^4*Log[d + e*x])/(12*e^8*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^3(A+Bx)}{(d+ex)^5}\,dx$$
$$\Big\downarrow \text{652}$$
$$\int \left(-\frac{c(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4)}{e^7(d+ex)^2} - \frac{c^2(-3aBe^2+5Acde-15Bcd^2)}{e^7} + \frac{c^2(3\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(4Acde\left(3ae^2+5cd^2\right)-B\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)\right)}{e^8(d+ex)}-\frac{c^2x\left(5Acde-3B\left(ae^2+5cd^2\right)\right)}{e^7}-\frac{c^2\log(d+ex)\left(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3\right)}{e^8}-\frac{\left(ae^2+cd^2\right)^2\left(aBe^2-6Acde+7Bcd^2\right)}{3e^8(d+ex)^3}+\frac{\left(ae^2+cd^2\right)^3(Bd-Ae)}{4e^8(d+ex)^4}+\frac{3c\left(ae^2+cd^2\right)\left(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3\right)}{2e^8(d+ex)^2}-\frac{c^3x^2(5Bd-Ae)}{2e^6}+\frac{Bc^3x^3}{3e^5}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^5,x]
```

output
```
-((c^2*(5*A*c*d*e - 3*B*(5*c*d^2 + a*e^2))*x)/e^7) - (c^3*(5*B*d - A*e)*x^
2)/(2*e^6) + (B*c^3*x^3)/(3*e^5) + ((B*d - A*e)*(c*d^2 + a*e^2)^3)/(4*e^8*
(d + e*x)^4) - ((c*d^2 + a*e^2)^2*(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2))/(3*e^
8*(d + e*x)^3) + (3*c*(c*d^2 + a*e^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e
^2 - a*A*e^3))/(2*e^8*(d + e*x)^2) + (c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B
*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4)))/(e^8*(d + e*x)) - (c^2*(35*B*
c*d^3 - 15*A*c*d^2*e + 15*a*B*d*e^2 - 3*a*A*e^3)*Log[d + e*x])/e^8
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.38

method	result
default	$-\frac{c^2\left(-\frac{1}{3}Bc\,x^3e^2-\frac{1}{2}Ac\,e^2x^2+\frac{5}{2}Bcde\,x^2+5Acdex-3Ba\,e^2x-15Bc\,d^2x\right)}{e^7}-\frac{-6A\,a^2cd\,e^5-12Aa\,c^2d^3e^3-6A\,c^3d^5e+B\,a^3e^6}{3e^8(ex+d)}$
norman	$\frac{\frac{\left(12Aa\,c^2d\,e^3+60A\,c^3d^3e-3B\,e^4a^2c-60Ba\,c^2d^2e^2-140B\,c^3d^4\right)x^3}{e^5}-\frac{3A\,a^3e^7+3A\,a^2c\,d^2e^5-75Aa\,c^2d^4e^3-375A\,c^3d^6e+B\,a^3d\,e^6+9B\,a^2}{12e^8}}{}$
risch	$\frac{B\,c^3x^3}{3e^5}+\frac{c^3A\,x^2}{2e^5}-\frac{5c^3Bd\,x^2}{2e^6}-\frac{5c^3Adx}{e^6}+\frac{3c^2Bax}{e^5}+\frac{15c^3B\,d^2x}{e^7}+\frac{(12Aa\,c^2d\,e^5+20A\,c^3d^3e^3-3B\,a^2c\,e^6-30Ba\,c}{}$
parallelrisch	$\frac{144A\ln(ex+d)xa\,c^2d^3e^4-9B\,a^2c\,d^3e^4-3A\,a^2c\,d^2e^5+75Aa\,c^2d^4e^3+6A\,x^6c^3e^7+4B\,x^7c^3e^7-420B\ln(ex+d)c^3d^7-4Bx\,a^3e^7}{}$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-c^2/e^7*(-1/3*B*c*x^3*e^2-1/2*A*c*e^2*x^2+5/2*B*c*d*e*x^2+5*A*c*d*e*x-3*B
*a*e^2*x-15*B*c*d^2*x)-1/3*(-6*A*a^2*c*d*e^5-12*A*a*c^2*d^3*e^3-6*A*c^3*d^
5*e+B*a^3*e^6+9*B*a^2*c*d^2*e^4+15*B*a*c^2*d^4*e^2+7*B*c^3*d^6)/e^8/(e*x+d
)^3-1/4*(A*a^3*e^7+3*A*a^2*c*d^2*e^5+3*A*a*c^2*d^4*e^3+A*c^3*d^6*e-B*a^3*d
*e^6-3*B*a^2*c*d^3*e^4-3*B*a*c^2*d^5*e^2-B*c^3*d^7)/e^8/(e*x+d)^4+c^2/e^8*
(3*A*a*e^3+15*A*c*d^2*e-15*B*a*d*e^2-35*B*c*d^3)*ln(e*x+d)+1/e^8*c*(12*A*a
*c*d*e^3+20*A*c^2*d^3*e-3*B*a^2*e^4-30*B*a*c*d^2*e^2-35*B*c^2*d^4)/(e*x+d)
-3/2*c/e^8*(A*a^2*e^5+6*A*a*c*d^2*e^3+5*A*c^2*d^4*e-3*B*a^2*d*e^4-10*B*a*c
*d^3*e^2-7*B*c^2*d^5)/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. 2(304) = 608.

Time = 0.09 (sec) , antiderivative size = 746, normalized size of antiderivative = 2.38

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^5}\,dx$$
$$=\frac{4\,Bc^3e^7x^7-319\,Bc^3d^7+171\,Ac^3d^6e-231\,Bac^2d^5e^2+75\,Aac^2d^4e^3-9\,Ba^2cd^3e^4-3\,Aa^2cd^2e^5-Ba^3d}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^5,x, algorithm="fricas")
```

output
```
1/12*(4*B*c^3*e^7*x^7 - 319*B*c^3*d^7 + 171*A*c^3*d^6*e - 231*B*a*c^2*d^5*
e^2 + 75*A*a*c^2*d^4*e^3 - 9*B*a^2*c*d^3*e^4 - 3*A*a^2*c*d^2*e^5 - B*a^3*d
*e^6 - 3*A*a^3*e^7 - 2*(7*B*c^3*d*e^6 - 3*A*c^3*e^7)*x^6 + 12*(7*B*c^3*d^2
*e^5 - 3*A*c^3*d*e^6 + 3*B*a*c^2*e^7)*x^5 + 4*(139*B*c^3*d^3*e^4 - 51*A*c^
3*d^2*e^5 + 36*B*a*c^2*d*e^6)*x^4 + 4*(136*B*c^3*d^4*e^3 - 24*A*c^3*d^3*e^
4 - 36*B*a*c^2*d^2*e^5 + 36*A*a*c^2*d*e^6 - 9*B*a^2*c*e^7)*x^3 - 6*(74*B*c
^3*d^5*e^2 - 66*A*c^3*d^4*e^3 + 126*B*a*c^2*d^3*e^4 - 54*A*a*c^2*d^2*e^5 +
 9*B*a^2*c*d*e^6 + 3*A*a^2*c*e^7)*x^2 - 4*(214*B*c^3*d^6*e - 126*A*c^3*d^5
*e^2 + 186*B*a*c^2*d^4*e^3 - 66*A*a*c^2*d^3*e^4 + 9*B*a^2*c*d^2*e^5 + 3*A*
a^2*c*d*e^6 + B*a^3*e^7)*x - 12*(35*B*c^3*d^7 - 15*A*c^3*d^6*e + 15*B*a*c^
2*d^5*e^2 - 3*A*a*c^2*d^4*e^3 + (35*B*c^3*d^3*e^4 - 15*A*c^3*d^2*e^5 + 15*
B*a*c^2*d*e^6 - 3*A*a*c^2*e^7)*x^4 + 4*(35*B*c^3*d^4*e^3 - 15*A*c^3*d^3*e^
4 + 15*B*a*c^2*d^2*e^5 - 3*A*a*c^2*d*e^6)*x^3 + 6*(35*B*c^3*d^5*e^2 - 15*A
*c^3*d^4*e^3 + 15*B*a*c^2*d^3*e^4 - 3*A*a*c^2*d^2*e^5)*x^2 + 4*(35*B*c^3*d
^6*e - 15*A*c^3*d^5*e^2 + 15*B*a*c^2*d^4*e^3 - 3*A*a*c^2*d^3*e^4)*x)*log(e
*x + d))/(e^12*x^4 + 4*d*e^11*x^3 + 6*d^2*e^10*x^2 + 4*d^3*e^9*x + d^4*e^8
)
```

Sympy [A] (verification not implemented)

Time = 32.51 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.71

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^5}\,dx$$
$$= \frac{Bc^3x^3}{3e^5} - \frac{c^2(-3Aae^3-15Acd^2e+15Bade^2+35Bcd^3)\log(d+ex)}{e^8}$$
$$+ x^2\left(\frac{Ac^3}{2e^5} - \frac{5Bc^3d}{2e^6}\right) + x\left(-\frac{5Ac^3d}{e^6} + \frac{3Bac^2}{e^5} + \frac{15Bc^3d^2}{e^7}\right)$$
$$+ \frac{-3Aa^3e^7 - 3Aa^2cd^2e^5 + 75Aac^2d^4e^3 + 171Ac^3d^6e - Ba^3de^6 - 9Ba^2cd^3e^4 - 231Bac^2d^5e^2 - 319Bc^3d}{}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**5,x)
```

output
```
B*c**3*x**3/(3*e**5) - c**2*(-3*A*a*e**3 - 15*A*c*d**2*e + 15*B*a*d*e**2 +
 35*B*c*d**3)*log(d + e*x)/e**8 + x**2*(A*c**3/(2*e**5) - 5*B*c**3*d/(2*e*
*6)) + x*(-5*A*c**3*d/e**6 + 3*B*a*c**2/e**5 + 15*B*c**3*d**2/e**7) + (-3*
A*a**3*e**7 - 3*A*a**2*c*d**2*e**5 + 75*A*a*c**2*d**4*e**3 + 171*A*c**3*d*
*6*e - B*a**3*d*e**6 - 9*B*a**2*c*d**3*e**4 - 231*B*a*c**2*d**5*e**2 - 319
*B*c**3*d**7 + x**3*(144*A*a*c**2*d*e**6 + 240*A*c**3*d**3*e**4 - 36*B*a**
2*c*e**7 - 360*B*a*c**2*d**2*e**5 - 420*B*c**3*d**4*e**3) + x**2*(-18*A*a*
*2*c*e**7 + 324*A*a*c**2*d**2*e**5 + 630*A*c**3*d**4*e**3 - 54*B*a**2*c*d*
e**6 - 900*B*a*c**2*d**3*e**4 - 1134*B*c**3*d**5*e**2) + x*(-12*A*a**2*c*d
*e**6 + 264*A*a*c**2*d**3*e**4 + 564*A*c**3*d**5*e**2 - 4*B*a**3*e**7 - 36
*B*a**2*c*d**2*e**5 - 780*B*a*c**2*d**4*e**3 - 1036*B*c**3*d**6*e))/(12*d*
*4*e**8 + 48*d**3*e**9*x + 72*d**2*e**10*x**2 + 48*d*e**11*x**3 + 12*e**12
*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.55

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^5}\,dx = -\frac{319\,Bc^3d^7 - 171\,Ac^3d^6e + 231\,Bac^2d^5e^2 - 75\,Aac^2d^4e^3 + 9\,Ba^2cd^3e^4 + 3\,Aa^2cd^2e^5 + Ba^3de^6 + 3\,Aa^3}{} + \frac{2\,Bc^3e^2x^3 - 3\,(5\,Bc^3de - Ac^3e^2)x^2 + 6\,(15\,Bc^3d^2 - 5\,Ac^3de + 3\,Bac^2e^2)x}{6\,e^7} - \frac{(35\,Bc^3d^3 - 15\,Ac^3d^2e + 15\,Bac^2de^2 - 3\,Aac^2e^3)\log(ex+d)}{e^8}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^5,x, algorithm="maxima")
```

output
```
-1/12*(319*B*c^3*d^7 - 171*A*c^3*d^6*e + 231*B*a*c^2*d^5*e^2 - 75*A*a*c^2*
d^4*e^3 + 9*B*a^2*c*d^3*e^4 + 3*A*a^2*c*d^2*e^5 + B*a^3*d*e^6 + 3*A*a^3*e^
7 + 12*(35*B*c^3*d^4*e^3 - 20*A*c^3*d^3*e^4 + 30*B*a*c^2*d^2*e^5 - 12*A*a*
c^2*d*e^6 + 3*B*a^2*c*e^7)*x^3 + 18*(63*B*c^3*d^5*e^2 - 35*A*c^3*d^4*e^3 +
 50*B*a*c^2*d^3*e^4 - 18*A*a*c^2*d^2*e^5 + 3*B*a^2*c*d*e^6 + A*a^2*c*e^7)*
x^2 + 4*(259*B*c^3*d^6*e - 141*A*c^3*d^5*e^2 + 195*B*a*c^2*d^4*e^3 - 66*A*
a*c^2*d^3*e^4 + 9*B*a^2*c*d^2*e^5 + 3*A*a^2*c*d*e^6 + B*a^3*e^7)*x)/(e^12*
x^4 + 4*d*e^11*x^3 + 6*d^2*e^10*x^2 + 4*d^3*e^9*x + d^4*e^8) + 1/6*(2*B*c^
3*e^2*x^3 - 3*(5*B*c^3*d*e - A*c^3*e^2)*x^2 + 6*(15*B*c^3*d^2 - 5*A*c^3*d*
e + 3*B*a*c^2*e^2)*x)/e^7 - (35*B*c^3*d^3 - 15*A*c^3*d^2*e + 15*B*a*c^2*d*
e^2 - 3*A*a*c^2*e^3)*log(e*x + d)/e^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(304) = 608$.

Time = 0.15 (sec) , antiderivative size = 653, normalized size of antiderivative = 2.08

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^5}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^5,x, algorithm="giac")
```

output
```
1/6*(2*B*c^3 - 3*(7*B*c^3*d*e - A*c^3*e^2)/((e*x + d)*e) + 18*(7*B*c^3*d^2
*e^2 - 2*A*c^3*d*e^3 + B*a*c^2*e^4)/((e*x + d)^2*e^2))*(e*x + d)^3/e^8 + (
35*B*c^3*d^3 - 15*A*c^3*d^2*e + 15*B*a*c^2*d*e^2 - 3*A*a*c^2*e^3)*log(abs(
e*x + d)/((e*x + d)^2*abs(e)))/e^8 - 1/12*(420*B*c^3*d^4*e^36/(e*x + d) -
126*B*c^3*d^5*e^36/(e*x + d)^2 + 28*B*c^3*d^6*e^36/(e*x + d)^3 - 3*B*c^3*d
^7*e^36/(e*x + d)^4 - 240*A*c^3*d^3*e^37/(e*x + d) + 90*A*c^3*d^4*e^37/(e*
x + d)^2 - 24*A*c^3*d^5*e^37/(e*x + d)^3 + 3*A*c^3*d^6*e^37/(e*x + d)^4 +
360*B*a*c^2*d^2*e^38/(e*x + d) - 180*B*a*c^2*d^3*e^38/(e*x + d)^2 + 60*B*a
*c^2*d^4*e^38/(e*x + d)^3 - 9*B*a*c^2*d^5*e^38/(e*x + d)^4 - 144*A*a*c^2*d
*e^39/(e*x + d) + 108*A*a*c^2*d^2*e^39/(e*x + d)^2 - 48*A*a*c^2*d^3*e^39/(
e*x + d)^3 + 9*A*a*c^2*d^4*e^39/(e*x + d)^4 + 36*B*a^2*c*e^40/(e*x + d) -
54*B*a^2*c*d*e^40/(e*x + d)^2 + 36*B*a^2*c*d^2*e^40/(e*x + d)^3 - 9*B*a^2*
c*d^3*e^40/(e*x + d)^4 + 18*A*a^2*c*e^41/(e*x + d)^2 - 24*A*a^2*c*d*e^41/(
e*x + d)^3 + 9*A*a^2*c*d^2*e^41/(e*x + d)^4 + 4*B*a^3*e^42/(e*x + d)^3 - 3
*B*a^3*d*e^42/(e*x + d)^4 + 3*A*a^3*e^43/(e*x + d)^4)/e^44
```

Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.60

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^5}\,dx$$
$$= x^2\left(\frac{A\,c^3}{2\,e^5}-\frac{5\,B\,c^3\,d}{2\,e^6}\right)-x\left(\frac{5\,d\left(\frac{A\,c^3}{e^5}-\frac{5\,B\,c^3\,d}{e^6}\right)}{e}-\frac{3\,B\,a\,c^2}{e^5}+\frac{10\,B\,c^3\,d^2}{e^7}\right)$$
$$-\frac{\frac{B\,a^3\,d\,e^6+3\,A\,a^3\,e^7+9\,B\,a^2\,c\,d^3\,e^4+3\,A\,a^2\,c\,d^2\,e^5+231\,B\,a\,c^2\,d^5\,e^2-75\,A\,a\,c^2\,d^4\,e^3+319\,B\,c^3\,d^7-171\,A\,c^3\,d^6\,e}{12\,e}+x^2\left(\frac{9\,B\,a^2\,c\,d\,e^5}{2}+3\,A\right.}{}$$
$$-\frac{\ln(d+e\,x)\,\left(35\,B\,c^3\,d^3-15\,A\,c^3\,d^2\,e+15\,B\,a\,c^2\,d\,e^2-3\,A\,a\,c^2\,e^3\right)}{e^8}+\frac{B\,c^3\,x^3}{3\,e^5}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^5,x)
```

output
```
x^2*((A*c^3)/(2*e^5) - (5*B*c^3*d)/(2*e^6)) - x*((5*d*((A*c^3)/e^5 - (5*B*
c^3*d)/e^6))/e - (3*B*a*c^2)/e^5 + (10*B*c^3*d^2)/e^7) - ((3*A*a^3*e^7 + 3
19*B*c^3*d^7 + B*a^3*d*e^6 - 171*A*c^3*d^6*e - 75*A*a*c^2*d^4*e^3 + 3*A*a^
2*c*d^2*e^5 + 231*B*a*c^2*d^5*e^2 + 9*B*a^2*c*d^3*e^4)/(12*e) + x^2*((3*A*
a^2*c*e^6)/2 + (189*B*c^3*d^5*e)/2 - (105*A*c^3*d^4*e^2)/2 - 27*A*a*c^2*d^
2*e^4 + 75*B*a*c^2*d^3*e^3 + (9*B*a^2*c*d*e^5)/2) + x^3*(3*B*a^2*c*e^6 - 2
0*A*c^3*d^3*e^3 + 35*B*c^3*d^4*e^2 + 30*B*a*c^2*d^2*e^4 - 12*A*a*c^2*d*e^5
) + x*((B*a^3*e^6)/3 + (259*B*c^3*d^6)/3 - 47*A*c^3*d^5*e - 22*A*a*c^2*d^3
*e^3 + 65*B*a*c^2*d^4*e^2 + 3*B*a^2*c*d^2*e^4 + A*a^2*c*d*e^5))/(d^4*e^7 +
 e^11*x^4 + 4*d^3*e^8*x + 4*d*e^10*x^3 + 6*d^2*e^9*x^2) - (log(d + e*x)*(3
5*B*c^3*d^3 - 3*A*a*c^2*e^3 - 15*A*c^3*d^2*e + 15*B*a*c^2*d*e^2))/e^8 + (B
*c^3*x^3)/(3*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 846, normalized size of antiderivative = 2.69

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^5}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^5,x)
```

output

```
(36*log(d + e*x)*a**2*c**2*d**5*e**3 + 144*log(d + e*x)*a**2*c**2*d**4*e**
4*x + 216*log(d + e*x)*a**2*c**2*d**3*e**5*x**2 + 144*log(d + e*x)*a**2*c*
*2*d**2*e**6*x**3 + 36*log(d + e*x)*a**2*c**2*d*e**7*x**4 - 180*log(d + e*
x)*a*b*c**2*d**6*e**2 - 720*log(d + e*x)*a*b*c**2*d**5*e**3*x - 1080*log(d
 + e*x)*a*b*c**2*d**4*e**4*x**2 - 720*log(d + e*x)*a*b*c**2*d**3*e**5*x**3
 - 180*log(d + e*x)*a*b*c**2*d**2*e**6*x**4 + 180*log(d + e*x)*a*c**3*d**7
*e + 720*log(d + e*x)*a*c**3*d**6*e**2*x + 1080*log(d + e*x)*a*c**3*d**5*e
**3*x**2 + 720*log(d + e*x)*a*c**3*d**4*e**4*x**3 + 180*log(d + e*x)*a*c**
3*d**3*e**5*x**4 - 420*log(d + e*x)*b*c**3*d**8 - 1680*log(d + e*x)*b*c**3
*d**7*e*x - 2520*log(d + e*x)*b*c**3*d**6*e**2*x**2 - 1680*log(d + e*x)*b*
c**3*d**5*e**3*x**3 - 420*log(d + e*x)*b*c**3*d**4*e**4*x**4 - 3*a**4*d*e*
*7 - a**3*b*d**2*e**6 - 4*a**3*b*d*e**7*x - 3*a**3*c*d**3*e**5 - 12*a**3*c
*d**2*e**6*x - 18*a**3*c*d*e**7*x**2 + 9*a**2*b*c*e**8*x**4 + 39*a**2*c**2
*d**5*e**3 + 120*a**2*c**2*d**4*e**4*x + 108*a**2*c**2*d**3*e**5*x**2 - 36
*a**2*c**2*d*e**7*x**4 - 195*a*b*c**2*d**6*e**2 - 600*a*b*c**2*d**5*e**3*x
 - 540*a*b*c**2*d**4*e**4*x**2 + 180*a*b*c**2*d**2*e**6*x**4 + 36*a*b*c**2
*d*e**7*x**5 + 195*a*c**3*d**7*e + 600*a*c**3*d**6*e**2*x + 540*a*c**3*d**
5*e**3*x**2 - 180*a*c**3*d**3*e**5*x**4 - 36*a*c**3*d**2*e**6*x**5 + 6*a*c
**3*d*e**7*x**6 - 455*b*c**3*d**8 - 1400*b*c**3*d**7*e*x - 1260*b*c**3*d**
6*e**2*x**2 + 420*b*c**3*d**4*e**4*x**4 + 84*b*c**3*d**3*e**5*x**5 - 14...
```

3.73 $\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 313

$$\begin{aligned}\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^6}\,dx = &-\frac{c^3(6Bd-Ae)x}{e^7}+\frac{Bc^3x^2}{2e^6}+\frac{(Bd-Ae)(cd^2+ae^2)^3}{5e^8(d+ex)^5}\\ &-\frac{(cd^2+ae^2)^2(7Bcd^2-6Acde+aBe^2)}{4e^8(d+ex)^4}\\ &+\frac{c(cd^2+ae^2)(7Bcd^3-5Acd^2e+3aBde^2-aAe^3)}{e^8(d+ex)^3}\\ &+\frac{c(4Acde(5cd^2+3ae^2)-B(35c^2d^4+30acd^2e^2+3a^2e^4))}{2e^8(d+ex)^2}\\ &+\frac{c^2(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3)}{e^8(d+ex)}\\ &+\frac{3c^2(7Bcd^2-2Acde+aBe^2)\log(d+ex)}{e^8}\end{aligned}$$

output

```
-c^3*(-A*e+6*B*d)*x/e^7+1/2*B*c^3*x^2/e^6+1/5*(-A*e+B*d)*(a*e^2+c*d^2)^3/e
^8/(e*x+d)^5-1/4*(a*e^2+c*d^2)^2*(-6*A*c*d*e+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d
)^4+c*(a*e^2+c*d^2)*(-A*a*e^3-5*A*c*d^2*e+3*B*a*d*e^2+7*B*c*d^3)/e^8/(e*x+
d)^3+1/2*c*(4*A*c*d*e*(3*a*e^2+5*c*d^2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2
*d^4))/e^8/(e*x+d)^2+c^2*(-3*A*a*e^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)
/e^8/(e*x+d)+3*c^2*(-2*A*c*d*e+B*a*e^2+7*B*c*d^2)*ln(e*x+d)/e^8
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^6}\,dx$$
$$= \frac{-2Ae(2a^3e^6+a^2ce^4(d^2+5dex+10e^2x^2)+6ac^2e^2(d^4+5d^3ex+10d^2e^2x^2+10de^3x^3+5e^4x^4)+c^3(87d}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^6,x]
```

output
```
(-2*A*e*(2*a^3*e^6 + a^2*c*e^4*(d^2 + 5*d*e*x + 10*e^2*x^2) + 6*a*c^2*e^2*
(d^4 + 5*d^3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4) + c^3*(87*d^
6 + 375*d^5*e*x + 600*d^4*e^2*x^2 + 400*d^3*e^3*x^3 + 50*d^2*e^4*x^4 - 50*
d*e^5*x^5 - 10*e^6*x^6)) + B*(-(a^3*e^6*(d + 5*e*x)) - 3*a^2*c*e^4*(d^3 +
5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3) + a*c^2*d*e^2*(137*d^4 + 625*d^3*e*
x + 1100*d^2*e^2*x^2 + 900*d*e^3*x^3 + 300*e^4*x^4) + c^3*(459*d^7 + 1875*
d^6*e*x + 2700*d^5*e^2*x^2 + 1300*d^4*e^3*x^3 - 400*d^3*e^4*x^4 - 500*d^2*
e^5*x^5 - 70*d*e^6*x^6 + 10*e^7*x^7)) + 60*c^2*(7*B*c*d^2 - 2*A*c*d*e + a*
B*e^2)*(d + e*x)^5*Log[d + e*x])/(20*e^8*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^3(A+Bx)}{(d+ex)^6}\,dx$$
$$\Big\downarrow \; 652$$
$$\int \left(-\frac{c(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4)}{e^7(d+ex)^3}-\frac{3c^2\left(-aBe^2+2Acde-7Bcd^2\right)}{e^7(d+ex)}+\frac{c^2(3a}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(4Acde(3ae^2+5cd^2)-B(3a^2e^4+30acd^2e^2+35c^2d^4))}{2e^8(d+ex)^2}+\frac{3c^2\log(d+ex)\,(aBe^2-2Acde+7Bcd^2)}{e^8}+\frac{c^2(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3)}{e^8(d+ex)}-\frac{(ae^2+cd^2)^2\,(aBe^2-6Acde+7Bcd^2)}{4e^8(d+ex)^4}+\frac{(ae^2+cd^2)^3\,(Bd-Ae)}{5e^8(d+ex)^5}+\frac{c(ae^2+cd^2)\,(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3)}{e^8(d+ex)^3}-\frac{c^3x(6Bd-Ae)}{e^7}+\frac{Bc^3x^2}{2e^6}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^6,x]
```

output
```
-((c^3*(6*B*d - A*e)*x)/e^7) + (B*c^3*x^2)/(2*e^6) + ((B*d - A*e)*(c*d^2 +
 a*e^2)^3)/(5*e^8*(d + e*x)^5) - ((c*d^2 + a*e^2)^2*(7*B*c*d^2 - 6*A*c*d*e
 + a*B*e^2))/(4*e^8*(d + e*x)^4) + (c*(c*d^2 + a*e^2)*(7*B*c*d^3 - 5*A*c*d
^2*e + 3*a*B*d*e^2 - a*A*e^3))/(e^8*(d + e*x)^3) + (c*(4*A*c*d*e*(5*c*d^2
+ 3*a*e^2) - B*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4)))/(2*e^8*(d + e*x
)^2) + (c^2*(35*B*c*d^3 - 15*A*c*d^2*e + 15*a*B*d*e^2 - 3*a*A*e^3))/(e^8*(
d + e*x)) + (3*c^2*(7*B*c*d^2 - 2*A*c*d*e + a*B*e^2)*Log[d + e*x])/e^8
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.38

method	result
default	$\frac{c^3\left(\frac{1}{2}Be\,x^2+Aex-6Bdx\right)}{e^7}-\frac{c\left(A\,a^2e^5+6Aac\,d^2e^3+5A\,c^2d^4e-3B\,a^2d\,e^4-10Bac\,d^3e^2-7B\,c^2d^5\right)}{e^8(ex+d)^3}-\frac{-6A\,a^2cd\,e^5-12Aa\,c\cdots}{}$
norman	$\frac{-\frac{4A\,a^3e^7+2A\,a^2c\,d^2e^5+12Aa\,c^2d^4e^3+274A\,c^3d^6e+B\,a^3d\,e^6+3B\,a^2c\,d^3e^4-137Ba\,c^2d^5e^2-959B\,c^3d^7}{20e^8}-\frac{\left(3Aa\,c^2e^3+30A\,c^3d^2e-15Ba\,c\cdots\right.}{e^4}}{}$
risch	$\frac{B\,c^3x^2}{2e^6}+\frac{c^3Ax}{e^6}-\frac{6c^3Bdx}{e^7}+\frac{\left(-3Aa\,c^2e^6-15A\,c^3d^2e^4+15Ba\,c^2d\,e^5+35B\,c^3d^3e^3\right)x^4-\frac{e^2c\left(12Aacd\,e^3+100A\,c^2d^3e+3B\,e^4\cdots\right.}{2}}{}$
parallelrisch	$-\frac{3B\,a^2c\,d^3e^4+2A\,a^2c\,d^2e^5+12Aa\,c^2d^4e^3-20A\,x^6c^3e^7-10B\,x^7c^3e^7-420B\ln(ex+d)c^3d^7+5Bx\,a^3e^7+120A\ln(ex+d)x^5c^3\cdots}{}$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

output
```
c^3/e^7*(1/2*B*e*x^2+A*e*x-6*B*d*x)-c/e^8*(A*a^2*e^5+6*A*a*c*d^2*e^3+5*A*c
^2*d^4*e-3*B*a^2*d*e^4-10*B*a*c*d^3*e^2-7*B*c^2*d^5)/(e*x+d)^3-1/4*(-6*A*a
^2*c*d*e^5-12*A*a*c^2*d^3*e^3-6*A*c^3*d^5*e+B*a^3*e^6+9*B*a^2*c*d^2*e^4+15
*B*a*c^2*d^4*e^2+7*B*c^3*d^6)/e^8/(e*x+d)^4-3*c^2/e^8*(2*A*c*d*e-B*a*e^2-7
*B*c*d^2)*ln(e*x+d)-1/5*(A*a^3*e^7+3*A*a^2*c*d^2*e^5+3*A*a*c^2*d^4*e^3+A*c
^3*d^6*e-B*a^3*d*e^6-3*B*a^2*c*d^3*e^4-3*B*a*c^2*d^5*e^2-B*c^3*d^7)/e^8/(e
*x+d)^5-c^2/e^8*(3*A*a*e^3+15*A*c*d^2*e-15*B*a*d*e^2-35*B*c*d^3)/(e*x+d)+1
/2/e^8*c*(12*A*a*c*d*e^3+20*A*c^2*d^3*e-3*B*a^2*e^4-30*B*a*c*d^2*e^2-35*B*
c^2*d^4)/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. 2(305) = 610.

Time = 0.09 (sec) , antiderivative size = 730, normalized size of antiderivative = 2.33

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^6}\,dx$$
$$=\frac{10\,Bc^3e^7x^7+459\,Bc^3d^7-174\,Ac^3d^6e+137\,Bac^2d^5e^2-12\,Aac^2d^4e^3-3\,Ba^2cd^3e^4-2\,Aa^2cd^2e^5-Ba^3\cdots}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^6,x, algorithm="fricas")
```

output
```
1/20*(10*B*c^3*e^7*x^7 + 459*B*c^3*d^7 - 174*A*c^3*d^6*e + 137*B*a*c^2*d^5
*e^2 - 12*A*a*c^2*d^4*e^3 - 3*B*a^2*c*d^3*e^4 - 2*A*a^2*c*d^2*e^5 - B*a^3*
d*e^6 - 4*A*a^3*e^7 - 10*(7*B*c^3*d*e^6 - 2*A*c^3*e^7)*x^6 - 100*(5*B*c^3*
d^2*e^5 - A*c^3*d*e^6)*x^5 - 20*(20*B*c^3*d^3*e^4 + 5*A*c^3*d^2*e^5 - 15*B
*a*c^2*d*e^6 + 3*A*a*c^2*e^7)*x^4 + 10*(130*B*c^3*d^4*e^3 - 80*A*c^3*d^3*e
^4 + 90*B*a*c^2*d^2*e^5 - 12*A*a*c^2*d*e^6 - 3*B*a^2*c*e^7)*x^3 + 10*(270*
B*c^3*d^5*e^2 - 120*A*c^3*d^4*e^3 + 110*B*a*c^2*d^3*e^4 - 12*A*a*c^2*d^2*e
^5 - 3*B*a^2*c*d*e^6 - 2*A*a^2*c*e^7)*x^2 + 5*(375*B*c^3*d^6*e - 150*A*c^3
*d^5*e^2 + 125*B*a*c^2*d^4*e^3 - 12*A*a*c^2*d^3*e^4 - 3*B*a^2*c*d^2*e^5 -
2*A*a^2*c*d*e^6 - B*a^3*e^7)*x + 60*(7*B*c^3*d^7 - 2*A*c^3*d^6*e + B*a*c^2
*d^5*e^2 + (7*B*c^3*d^2*e^5 - 2*A*c^3*d*e^6 + B*a*c^2*e^7)*x^5 + 5*(7*B*c^
3*d^3*e^4 - 2*A*c^3*d^2*e^5 + B*a*c^2*d*e^6)*x^4 + 10*(7*B*c^3*d^4*e^3 - 2
*A*c^3*d^3*e^4 + B*a*c^2*d^2*e^5)*x^3 + 10*(7*B*c^3*d^5*e^2 - 2*A*c^3*d^4*
e^3 + B*a*c^2*d^3*e^4)*x^2 + 5*(7*B*c^3*d^6*e - 2*A*c^3*d^5*e^2 + B*a*c^2*
d^4*e^3)*x)*log(e*x + d))/(e^13*x^5 + 5*d*e^12*x^4 + 10*d^2*e^11*x^3 + 10*
d^3*e^10*x^2 + 5*d^4*e^9*x + d^5*e^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^6}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.59

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^6}\,dx = \frac{459\,Bc^3d^7 - 174\,Ac^3d^6e + 137\,Bac^2d^5e^2 - 12\,Aac^2d^4e^3 - 3\,Ba^2cd^3e^4 - 2\,Aa^2cd^2e^5 - Ba^3de^6 - 4\,Aa^3e^7}{} + \frac{Bc^3ex^2 - 2\,(6\,Bc^3d - Ac^3e)x}{2\,e^7} + \frac{3\,(7\,Bc^3d^2 - 2\,Ac^3de + Bac^2e^2)\log(ex+d)}{e^8}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^6,x, algorithm="maxima")
```

output
```
1/20*(459*B*c^3*d^7 - 174*A*c^3*d^6*e + 137*B*a*c^2*d^5*e^2 - 12*A*a*c^2*d
^4*e^3 - 3*B*a^2*c*d^3*e^4 - 2*A*a^2*c*d^2*e^5 - B*a^3*d*e^6 - 4*A*a^3*e^7
 + 20*(35*B*c^3*d^3*e^4 - 15*A*c^3*d^2*e^5 + 15*B*a*c^2*d*e^6 - 3*A*a*c^2*
e^7)*x^4 + 10*(245*B*c^3*d^4*e^3 - 100*A*c^3*d^3*e^4 + 90*B*a*c^2*d^2*e^5
- 12*A*a*c^2*d*e^6 - 3*B*a^2*c*e^7)*x^3 + 10*(329*B*c^3*d^5*e^2 - 130*A*c^
3*d^4*e^3 + 110*B*a*c^2*d^3*e^4 - 12*A*a*c^2*d^2*e^5 - 3*B*a^2*c*d*e^6 - 2
*A*a^2*c*e^7)*x^2 + 5*(399*B*c^3*d^6*e - 154*A*c^3*d^5*e^2 + 125*B*a*c^2*d
^4*e^3 - 12*A*a*c^2*d^3*e^4 - 3*B*a^2*c*d^2*e^5 - 2*A*a^2*c*d*e^6 - B*a^3*
e^7)*x)/(e^13*x^5 + 5*d*e^12*x^4 + 10*d^2*e^11*x^3 + 10*d^3*e^10*x^2 + 5*d
^4*e^9*x + d^5*e^8) + 1/2*(B*c^3*e*x^2 - 2*(6*B*c^3*d - A*c^3*e)*x)/e^7 +
3*(7*B*c^3*d^2 - 2*A*c^3*d*e + B*a*c^2*e^2)*log(e*x + d)/e^8
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.46

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^6}\,dx$$
$$= \frac{3\,(7\,Bc^3d^2 - 2\,Ac^3de + Bac^2e^2)\log(|ex+d|)}{e^8} + \frac{Bc^3e^6x^2 - 12\,Bc^3de^5x + 2\,Ac^3e^6x}{2\,e^{12}}$$
$$+ \frac{459\,Bc^3d^7 - 174\,Ac^3d^6e + 137\,Bac^2d^5e^2 - 12\,Aac^2d^4e^3 - 3\,Ba^2cd^3e^4 - 2\,Aa^2cd^2e^5 - Ba^3de^6 - 4\,Aa^3}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^6,x, algorithm="giac")
```

output
```
3*(7*B*c^3*d^2 - 2*A*c^3*d*e + B*a*c^2*e^2)*log(abs(e*x + d))/e^8 + 1/2*(B
*c^3*e^6*x^2 - 12*B*c^3*d*e^5*x + 2*A*c^3*e^6*x)/e^12 + 1/20*(459*B*c^3*d^
7 - 174*A*c^3*d^6*e + 137*B*a*c^2*d^5*e^2 - 12*A*a*c^2*d^4*e^3 - 3*B*a^2*c
*d^3*e^4 - 2*A*a^2*c*d^2*e^5 - B*a^3*d*e^6 - 4*A*a^3*e^7 + 20*(35*B*c^3*d^
3*e^4 - 15*A*c^3*d^2*e^5 + 15*B*a*c^2*d*e^6 - 3*A*a*c^2*e^7)*x^4 + 10*(245
*B*c^3*d^4*e^3 - 100*A*c^3*d^3*e^4 + 90*B*a*c^2*d^2*e^5 - 12*A*a*c^2*d*e^6
 - 3*B*a^2*c*e^7)*x^3 + 10*(329*B*c^3*d^5*e^2 - 130*A*c^3*d^4*e^3 + 110*B*
a*c^2*d^3*e^4 - 12*A*a*c^2*d^2*e^5 - 3*B*a^2*c*d*e^6 - 2*A*a^2*c*e^7)*x^2
+ 5*(399*B*c^3*d^6*e - 154*A*c^3*d^5*e^2 + 125*B*a*c^2*d^4*e^3 - 12*A*a*c^
2*d^3*e^4 - 3*B*a^2*c*d^2*e^5 - 2*A*a^2*c*d*e^6 - B*a^3*e^7)*x)/((e*x + d)
^5*e^8)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.58

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^6}\,dx = x\left(\frac{A\,c^3}{e^6}-\frac{6\,B\,c^3\,d}{e^7}\right) - \frac{\frac{B\,a^3\,d\,e^6+4\,A\,a^3\,e^7+3\,B\,a^2\,c\,d^3\,e^4+2\,A\,a^2\,c\,d^2\,e^5-137\,B\,a\,c^2\,d^5\,e^2+12\,A\,a\,c^2\,d^4\,e^3-459\,B\,c^3\,d^7+174\,A\,c^3\,d^6\,e}{20\,e}+x^2\left(\frac{3\,B\,a^2\,c\,d\,e^5}{2}+A\right.}{} + \frac{\ln(d+e\,x)\,\left(21\,B\,c^3\,d^2-6\,A\,c^3\,d\,e+3\,B\,a\,c^2\,e^2\right)}{e^8} + \frac{B\,c^3\,x^2}{2\,e^6}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^6,x)
```

output
```
x*((A*c^3)/e^6 - (6*B*c^3*d)/e^7) - ((4*A*a^3*e^7 - 459*B*c^3*d^7 + B*a^3*
d*e^6 + 174*A*c^3*d^6*e + 12*A*a*c^2*d^4*e^3 + 2*A*a^2*c*d^2*e^5 - 137*B*a
*c^2*d^5*e^2 + 3*B*a^2*c*d^3*e^4)/(20*e) + x^2*(A*a^2*c*e^6 - (329*B*c^3*d
^5*e)/2 + 65*A*c^3*d^4*e^2 + 6*A*a*c^2*d^2*e^4 - 55*B*a*c^2*d^3*e^3 + (3*B
*a^2*c*d*e^5)/2) + x^3*((3*B*a^2*c*e^6)/2 + 50*A*c^3*d^3*e^3 - (245*B*c^3*
d^4*e^2)/2 - 45*B*a*c^2*d^2*e^4 + 6*A*a*c^2*d*e^5) + x*((B*a^3*e^6)/4 - (3
99*B*c^3*d^6)/4 + (77*A*c^3*d^5*e)/2 + 3*A*a*c^2*d^3*e^3 - (125*B*a*c^2*d^
4*e^2)/4 + (3*B*a^2*c*d^2*e^4)/4 + (A*a^2*c*d*e^5)/2) + x^4*(3*A*a*c^2*e^6
 + 15*A*c^3*d^2*e^4 - 35*B*c^3*d^3*e^3 - 15*B*a*c^2*d*e^5))/(d^5*e^7 + e^1
2*x^5 + 5*d^4*e^8*x + 5*d*e^11*x^4 + 10*d^3*e^9*x^2 + 10*d^2*e^10*x^3) + (
log(d + e*x)*(21*B*c^3*d^2 - 6*A*c^3*d*e + 3*B*a*c^2*e^2))/e^8 + (B*c^3*x^
2)/(2*e^6)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 808, normalized size of antiderivative = 2.58

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^6} dx = \text{Too large to display}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^6,x)
```

output
```
(60*log(d + e*x)*a*b*c**2*d**6*e**2 + 300*log(d + e*x)*a*b*c**2*d**5*e**3*
x + 600*log(d + e*x)*a*b*c**2*d**4*e**4*x**2 + 600*log(d + e*x)*a*b*c**2*d
**3*e**5*x**3 + 300*log(d + e*x)*a*b*c**2*d**2*e**6*x**4 + 60*log(d + e*x)
*a*b*c**2*d*e**7*x**5 - 120*log(d + e*x)*a*c**3*d**7*e - 600*log(d + e*x)*
a*c**3*d**6*e**2*x - 1200*log(d + e*x)*a*c**3*d**5*e**3*x**2 - 1200*log(d
+ e*x)*a*c**3*d**4*e**4*x**3 - 600*log(d + e*x)*a*c**3*d**3*e**5*x**4 - 12
0*log(d + e*x)*a*c**3*d**2*e**6*x**5 + 420*log(d + e*x)*b*c**3*d**8 + 2100
*log(d + e*x)*b*c**3*d**7*e*x + 4200*log(d + e*x)*b*c**3*d**6*e**2*x**2 +
4200*log(d + e*x)*b*c**3*d**5*e**3*x**3 + 2100*log(d + e*x)*b*c**3*d**4*e*
*4*x**4 + 420*log(d + e*x)*b*c**3*d**3*e**5*x**5 - 4*a**4*d*e**7 - a**3*b*
d**2*e**6 - 5*a**3*b*d*e**7*x - 2*a**3*c*d**3*e**5 - 10*a**3*c*d**2*e**6*x
 - 20*a**3*c*d*e**7*x**2 - 3*a**2*b*c*d**4*e**4 - 15*a**2*b*c*d**3*e**5*x
- 30*a**2*b*c*d**2*e**6*x**2 - 30*a**2*b*c*d*e**7*x**3 + 12*a**2*c**2*e**8
*x**5 + 77*a*b*c**2*d**6*e**2 + 325*a*b*c**2*d**5*e**3*x + 500*a*b*c**2*d*
*4*e**4*x**2 + 300*a*b*c**2*d**3*e**5*x**3 - 60*a*b*c**2*d*e**7*x**5 - 154
*a*c**3*d**7*e - 650*a*c**3*d**6*e**2*x - 1000*a*c**3*d**5*e**3*x**2 - 600
*a*c**3*d**4*e**4*x**3 + 120*a*c**3*d**2*e**6*x**5 + 20*a*c**3*d*e**7*x**6
 + 539*b*c**3*d**8 + 2275*b*c**3*d**7*e*x + 3500*b*c**3*d**6*e**2*x**2 + 2
100*b*c**3*d**5*e**3*x**3 - 420*b*c**3*d**3*e**5*x**5 - 70*b*c**3*d**2*e**
6*x**6 + 10*b*c**3*d*e**7*x**7)/(20*d*e**8*(d**5 + 5*d**4*e*x + 10*d**3...
```

3.74 $\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 320

$$\begin{aligned}\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^7}\,dx &= \frac{Bc^3x}{e^7}+\frac{(Bd-Ae)\left(cd^2+ae^2\right)^3}{6e^8(d+ex)^6}\\
&\quad-\frac{\left(cd^2+ae^2\right)^2\left(7Bcd^2-6Acde+aBe^2\right)}{5e^8(d+ex)^5}\\
&\quad+\frac{3c\left(cd^2+ae^2\right)\left(7Bcd^3-5Acd^2e+3aBde^2-aAe^3\right)}{4e^8(d+ex)^4}\\
&\quad+\frac{c\left(4Acde\left(5cd^2+3ae^2\right)-B\left(35c^2d^4+30acd^2e^2+3a^2e^4\right)\right)}{3e^8(d+ex)^3}\\
&\quad+\frac{c^2\left(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3\right)}{2e^8(d+ex)^2}\\
&\quad-\frac{3c^2\left(7Bcd^2-2Acde+aBe^2\right)}{e^8(d+ex)}\\
&\quad-\frac{c^3(7Bd-Ae)\log(d+ex)}{e^8}\end{aligned}$$

output
```
B*c^3*x/e^7+1/6*(-A*e+B*d)*(a*e^2+c*d^2)^3/e^8/(e*x+d)^6-1/5*(a*e^2+c*d^2)
^2*(-6*A*c*d*e+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)^5+3/4*c*(a*e^2+c*d^2)*(-A*a*
e^3-5*A*c*d^2*e+3*B*a*d*e^2+7*B*c*d^3)/e^8/(e*x+d)^4+1/3*c*(4*A*c*d*e*(3*a
*e^2+5*c*d^2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))/e^8/(e*x+d)^3+1/2*c
^2*(-3*A*a*e^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)/e^8/(e*x+d)^2-3*c^2*(
-2*A*c*d*e+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)-c^3*(-A*e+7*B*d)*ln(e*x+d)/e^8
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^7}\,dx = -\frac{Ae(10a^3e^6+3a^2ce^4(d^2+6dex+15e^2x^2)+6ac^2e^2(d^4+6d^3ex+15d^2e^2x^2+20de^3x^3+15e^4x^4)-c^3d}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^7,x]
```

output
```
-1/60*(A*e*(10*a^3*e^6 + 3*a^2*c*e^4*(d^2 + 6*d*e*x + 15*e^2*x^2) + 6*a*c^
2*e^2*(d^4 + 6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4) - c^3
*d*(147*d^5 + 822*d^4*e*x + 1875*d^3*e^2*x^2 + 2200*d^2*e^3*x^3 + 1350*d*e
^4*x^4 + 360*e^5*x^5)) + B*(2*a^3*e^6*(d + 6*e*x) + 3*a^2*c*e^4*(d^3 + 6*d
^2*e*x + 15*d*e^2*x^2 + 20*e^3*x^3) + 30*a*c^2*e^2*(d^5 + 6*d^4*e*x + 15*d
^3*e^2*x^2 + 20*d^2*e^3*x^3 + 15*d*e^4*x^4 + 6*e^5*x^5) + c^3*(669*d^7 + 3
594*d^6*e*x + 7725*d^5*e^2*x^2 + 8200*d^4*e^3*x^3 + 4050*d^3*e^4*x^4 + 360
*d^2*e^5*x^5 - 360*d*e^6*x^6 - 60*e^7*x^7)) + 60*c^3*(7*B*d - A*e)*(d + e*
x)^6*Log[d + e*x])/(e^8*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3(A+Bx)}{(d+ex)^7}dx$$
$$\Big\downarrow \; 652$$
$$\int \left(-\frac{c(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4)}{e^7(d+ex)^4}-\frac{3c^2(-aBe^2+2Acde-7Bcd^2)}{e^7(d+ex)^2}+\frac{c^2(3a\right.$$
$$\Big\downarrow \; 2009$$
$$\frac{c(4Acde(3ae^2+5cd^2)-B(3a^2e^4+30acd^2e^2+35c^2d^4))}{3e^8(d+ex)^3}-\frac{3c^2(aBe^2-2Acde+7Bcd^2)}{e^8(d+ex)}+\frac{c^2(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3)}{2e^8(d+ex)^2}-\frac{(ae^2+cd^2)^2(aBe^2-6Acde+7Bcd^2)}{5e^8(d+ex)^5}+\frac{(ae^2+cd^2)^3(Bd-Ae)}{6e^8(d+ex)^6}+\frac{3c(ae^2+cd^2)(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3)}{4e^8(d+ex)^4}-\frac{c^3(7Bd-Ae)\log(d+ex)}{e^8}+\frac{Bc^3x}{e^7}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^7,x]
```

output
```
(B*c^3*x)/e^7 + ((B*d - A*e)*(c*d^2 + a*e^2)^3)/(6*e^8*(d + e*x)^6) - ((c*
d^2 + a*e^2)^2*(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2))/(5*e^8*(d + e*x)^5) + (3
*c*(c*d^2 + a*e^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(4*e
^8*(d + e*x)^4) + (c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(35*c^2*d^4 + 30*a
*c*d^2*e^2 + 3*a^2*e^4)))/(3*e^8*(d + e*x)^3) + (c^2*(35*B*c*d^3 - 15*A*c*
d^2*e + 15*a*B*d*e^2 - 3*a*A*e^3))/(2*e^8*(d + e*x)^2) - (3*c^2*(7*B*c*d^2
 - 2*A*c*d*e + a*B*e^2))/(e^8*(d + e*x)) - (c^3*(7*B*d - A*e)*Log[d + e*x]
)/e^8
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.36

method	result
risch	$\frac{B c^3 x}{e^7} + \frac{(6A c^3 d e^5 - 3Ba c^2 e^6 - 21B c^3 d^2 e^4) x^5 - \frac{c^2 e^3 (3Aa e^3 - 45Ac d^2 e + 15Bad e^2 + 175Bc d^3) x^4}{2} - \frac{e^2 c (6Aacd e^3 - 110A c^2 d^3 e +}{}}{}$
default	$\frac{B c^3 x}{e^7} + \frac{c(12Aacd e^3 + 20A c^2 d^3 e - 3B e^4 a^2 - 30Bac d^2 e^2 - 35B c^2 d^4)}{3e^8 (ex+d)^3} - \frac{3c(A a^2 e^5 + 6Aac d^2 e^3 + 5A c^2 d^4 e - 3B a^2 d e^4 - 10}{4e^8 (ex+d)^4}$
norman	$\frac{\frac{B c^3 x^7}{e} - \frac{10A a^3 e^7 + 3A a^2 c d^2 e^5 + 6Aa c^2 d^4 e^3 - 147A c^3 d^6 e + 2B a^3 d e^6 + 3B a^2 c d^3 e^4 + 30Ba c^2 d^5 e^2 + 1029B c^3 d^7}{60e^8} + \frac{3(2A c^3 de - B e^2 a c^2 -}{e^3}}{}$
parallelrisch	$\frac{-3B a^2 c d^3 e^4 - 3A a^2 c d^2 e^5 - 6Aa c^2 d^4 e^3 + 60B x^7 c^3 e^7 - 420B \ln(ex+d) c^3 d^7 - 12Bx a^3 e^7 + 360A \ln(ex+d) x^5 c^3 d e^6 - 2520B \ln}{}$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

output
```
B*c^3*x/e^7+((6*A*c^3*d*e^5-3*B*a*c^2*e^6-21*B*c^3*d^2*e^4)*x^5-1/2*c^2*e^
3*(3*A*a*e^3-45*A*c*d^2*e+15*B*a*d*e^2+175*B*c*d^3)*x^4-1/3*e^2*c*(6*A*a*c
*d*e^3-110*A*c^2*d^3*e+3*B*a^2*e^4+30*B*a*c*d^2*e^2+455*B*c^2*d^4)*x^3-1/4
*c*e*(3*A*a^2*e^5+6*A*a*c*d^2*e^3-125*A*c^2*d^4*e+3*B*a^2*d*e^4+30*B*a*c*d
^3*e^2+539*B*c^2*d^5)*x^2+(-3/10*A*a^2*c*d*e^5-3/5*A*a*c^2*d^3*e^3+137/10*
A*c^3*d^5*e-1/5*B*a^3*e^6-3/10*B*a^2*c*d^2*e^4-3*B*a*c^2*d^4*e^2-609/10*B*
c^3*d^6)*x-1/60/e*(10*A*a^3*e^7+3*A*a^2*c*d^2*e^5+6*A*a*c^2*d^4*e^3-147*A*
c^3*d^6*e+2*B*a^3*d*e^6+3*B*a^2*c*d^3*e^4+30*B*a*c^2*d^5*e^2+669*B*c^3*d^7
))/e^7/(e*x+d)^6+c^3/e^7*ln(e*x+d)*A-7*c^3/e^8*ln(e*x+d)*B*d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(310) = 620$.

Time = 0.10 (sec) , antiderivative size = 695, normalized size of antiderivative = 2.17

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^7}\,dx$$
$$= \frac{60\,Bc^3e^7x^7 + 360\,Bc^3de^6x^6 - 669\,Bc^3d^7 + 147\,Ac^3d^6e - 30\,Bac^2d^5e^2 - 6\,Aac^2d^4e^3 - 3\,Ba^2cd^3e^4 - 3\,Ac}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^7,x, algorithm="fricas")
```

output
```
1/60*(60*B*c^3*e^7*x^7 + 360*B*c^3*d*e^6*x^6 - 669*B*c^3*d^7 + 147*A*c^3*d
^6*e - 30*B*a*c^2*d^5*e^2 - 6*A*a*c^2*d^4*e^3 - 3*B*a^2*c*d^3*e^4 - 3*A*a^
2*c*d^2*e^5 - 2*B*a^3*d*e^6 - 10*A*a^3*e^7 - 180*(2*B*c^3*d^2*e^5 - 2*A*c^
3*d*e^6 + B*a*c^2*e^7)*x^5 - 90*(45*B*c^3*d^3*e^4 - 15*A*c^3*d^2*e^5 + 5*B
*a*c^2*d*e^6 + A*a*c^2*e^7)*x^4 - 20*(410*B*c^3*d^4*e^3 - 110*A*c^3*d^3*e^
4 + 30*B*a*c^2*d^2*e^5 + 6*A*a*c^2*d*e^6 + 3*B*a^2*c*e^7)*x^3 - 15*(515*B*
c^3*d^5*e^2 - 125*A*c^3*d^4*e^3 + 30*B*a*c^2*d^3*e^4 + 6*A*a*c^2*d^2*e^5 +
 3*B*a^2*c*d*e^6 + 3*A*a^2*c*e^7)*x^2 - 6*(599*B*c^3*d^6*e - 137*A*c^3*d^5
*e^2 + 30*B*a*c^2*d^4*e^3 + 6*A*a*c^2*d^3*e^4 + 3*B*a^2*c*d^2*e^5 + 3*A*a^
2*c*d*e^6 + 2*B*a^3*e^7)*x - 60*(7*B*c^3*d^7 - A*c^3*d^6*e + (7*B*c^3*d*e^
6 - A*c^3*e^7)*x^6 + 6*(7*B*c^3*d^2*e^5 - A*c^3*d*e^6)*x^5 + 15*(7*B*c^3*d
^3*e^4 - A*c^3*d^2*e^5)*x^4 + 20*(7*B*c^3*d^4*e^3 - A*c^3*d^3*e^4)*x^3 + 1
5*(7*B*c^3*d^5*e^2 - A*c^3*d^4*e^3)*x^2 + 6*(7*B*c^3*d^6*e - A*c^3*d^5*e^2
)*x)*log(e*x + d))/(e^14*x^6 + 6*d*e^13*x^5 + 15*d^2*e^12*x^4 + 20*d^3*e^1
1*x^3 + 15*d^4*e^10*x^2 + 6*d^5*e^9*x + d^6*e^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^7}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.60

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^7}\,dx =$$

$$-\frac{669\,Bc^3d^7 - 147\,Ac^3d^6e + 30\,Bac^2d^5e^2 + 6\,Aac^2d^4e^3 + 3\,Ba^2cd^3e^4 + 3\,Aa^2cd^2e^5 + 2\,Ba^3de^6 + 10\,Aa^3}{}$$

$$+\frac{Bc^3x}{e^7} - \frac{(7\,Bc^3d - Ac^3e)\log(ex+d)}{e^8}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^7,x, algorithm="maxima")
```

output
```
-1/60*(669*B*c^3*d^7 - 147*A*c^3*d^6*e + 30*B*a*c^2*d^5*e^2 + 6*A*a*c^2*d^
4*e^3 + 3*B*a^2*c*d^3*e^4 + 3*A*a^2*c*d^2*e^5 + 2*B*a^3*d*e^6 + 10*A*a^3*e
^7 + 180*(7*B*c^3*d^2*e^5 - 2*A*c^3*d*e^6 + B*a*c^2*e^7)*x^5 + 30*(175*B*c
^3*d^3*e^4 - 45*A*c^3*d^2*e^5 + 15*B*a*c^2*d*e^6 + 3*A*a*c^2*e^7)*x^4 + 20
*(455*B*c^3*d^4*e^3 - 110*A*c^3*d^3*e^4 + 30*B*a*c^2*d^2*e^5 + 6*A*a*c^2*d
*e^6 + 3*B*a^2*c*e^7)*x^3 + 15*(539*B*c^3*d^5*e^2 - 125*A*c^3*d^4*e^3 + 30
*B*a*c^2*d^3*e^4 + 6*A*a*c^2*d^2*e^5 + 3*B*a^2*c*d*e^6 + 3*A*a^2*c*e^7)*x^
2 + 6*(609*B*c^3*d^6*e - 137*A*c^3*d^5*e^2 + 30*B*a*c^2*d^4*e^3 + 6*A*a*c^
2*d^3*e^4 + 3*B*a^2*c*d^2*e^5 + 3*A*a^2*c*d*e^6 + 2*B*a^3*e^7)*x)/(e^14*x^
6 + 6*d*e^13*x^5 + 15*d^2*e^12*x^4 + 20*d^3*e^11*x^3 + 15*d^4*e^10*x^2 + 6
*d^5*e^9*x + d^6*e^8) + B*c^3*x/e^7 - (7*B*c^3*d - A*c^3*e)*log(e*x + d)/e
^8
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^7}\,dx = \frac{Bc^3x}{e^7} - \frac{(7\,Bc^3d - Ac^3e)\log\left(|ex+d|\right)}{e^8}$$
$$- \frac{669\,Bc^3d^7 - 147\,Ac^3d^6e + 30\,Bac^2d^5e^2 + 6\,Aac^2d^4e^3 + 3\,Ba^2cd^3e^4 + 3\,Aa^2cd^2e^5 + 2\,Ba^3de^6 + 10\,Aa^3 \cdots}{\cdots}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^7,x, algorithm="giac")
```

output
```
B*c^3*x/e^7 - (7*B*c^3*d - A*c^3*e)*log(abs(e*x + d))/e^8 - 1/60*(669*B*c^
3*d^7 - 147*A*c^3*d^6*e + 30*B*a*c^2*d^5*e^2 + 6*A*a*c^2*d^4*e^3 + 3*B*a^2
*c*d^3*e^4 + 3*A*a^2*c*d^2*e^5 + 2*B*a^3*d*e^6 + 10*A*a^3*e^7 + 180*(7*B*c
^3*d^2*e^5 - 2*A*c^3*d*e^6 + B*a*c^2*e^7)*x^5 + 30*(175*B*c^3*d^3*e^4 - 45
*A*c^3*d^2*e^5 + 15*B*a*c^2*d*e^6 + 3*A*a*c^2*e^7)*x^4 + 20*(455*B*c^3*d^4
*e^3 - 110*A*c^3*d^3*e^4 + 30*B*a*c^2*d^2*e^5 + 6*A*a*c^2*d*e^6 + 3*B*a^2*
c*e^7)*x^3 + 15*(539*B*c^3*d^5*e^2 - 125*A*c^3*d^4*e^3 + 30*B*a*c^2*d^3*e^
4 + 6*A*a*c^2*d^2*e^5 + 3*B*a^2*c*d*e^6 + 3*A*a^2*c*e^7)*x^2 + 6*(609*B*c^
3*d^6*e - 137*A*c^3*d^5*e^2 + 30*B*a*c^2*d^4*e^3 + 6*A*a*c^2*d^3*e^4 + 3*B
*a^2*c*d^2*e^5 + 3*A*a^2*c*d*e^6 + 2*B*a^3*e^7)*x)/((e*x + d)^6*e^8)
```

Mupad [B] (verification not implemented)

Time = 6.41 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.58

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^7}\,dx = \frac{\ln(d+e\,x)\,\left(A\,c^3\,e-7\,B\,c^3\,d\right)}{e^8} - \frac{\frac{2\,B\,a^3\,d\,e^6+10\,A\,a^3\,e^7+3\,B\,a^2\,c\,d^3\,e^4+3\,A\,a^2\,c\,d^2\,e^5+30\,B\,a\,c^2\,d^5\,e^2+6\,A\,a\,c^2\,d^4\,e^3+669\,B\,c^3\,d^7-147\,A\,c^3\,d^6\,e}{60\,e}+x^2\left(\frac{3\,B\,a^2\,c\,d\,e^5}{4}+\ldots\right.}{} + \frac{B\,c^3\,x}{e^7}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^7,x)
```

output
```
(log(d + e*x)*(A*c^3*e - 7*B*c^3*d))/e^8 - ((10*A*a^3*e^7 + 669*B*c^3*d^7
+ 2*B*a^3*d*e^6 - 147*A*c^3*d^6*e + 6*A*a*c^2*d^4*e^3 + 3*A*a^2*c*d^2*e^5
+ 30*B*a*c^2*d^5*e^2 + 3*B*a^2*c*d^3*e^4)/(60*e) + x^2*((3*A*a^2*c*e^6)/4
+ (539*B*c^3*d^5*e)/4 - (125*A*c^3*d^4*e^2)/4 + (3*A*a*c^2*d^2*e^4)/2 + (1
5*B*a*c^2*d^3*e^3)/2 + (3*B*a^2*c*d*e^5)/4) + x^3*(B*a^2*c*e^6 - (110*A*c^
3*d^3*e^3)/3 + (455*B*c^3*d^4*e^2)/3 + 10*B*a*c^2*d^2*e^4 + 2*A*a*c^2*d*e^
5) + x^5*(3*B*a*c^2*e^6 - 6*A*c^3*d*e^5 + 21*B*c^3*d^2*e^4) + x*((B*a^3*e^
6)/5 + (609*B*c^3*d^6)/10 - (137*A*c^3*d^5*e)/10 + (3*A*a*c^2*d^3*e^3)/5 +
 3*B*a*c^2*d^4*e^2 + (3*B*a^2*c*d^2*e^4)/10 + (3*A*a^2*c*d*e^5)/10) + x^4*
((3*A*a*c^2*e^6)/2 - (45*A*c^3*d^2*e^4)/2 + (175*B*c^3*d^3*e^3)/2 + (15*B*
a*c^2*d*e^5)/2))/(d^6*e^7 + e^13*x^6 + 6*d^5*e^8*x + 6*d*e^12*x^5 + 15*d^4
*e^9*x^2 + 20*d^3*e^10*x^3 + 15*d^2*e^11*x^4) + (B*c^3*x)/e^7
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 738, normalized size of antiderivative = 2.31

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^7}\,dx = \frac{-3a^3c\,d^3e^5-6a^2c^2d^5e^3+87a\,c^3d^7e-2a^3b\,d^2e^6+360\log(ex+d)\,a\,c^3d^6e^2x-2520\log(ex+d)\,b\,c^3d^7ex \ldots}{}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^7,x)
```

output

```
(60*log(d + e*x)*a*c**3*d**7*e + 360*log(d + e*x)*a*c**3*d**6*e**2*x + 900
*log(d + e*x)*a*c**3*d**5*e**3*x**2 + 1200*log(d + e*x)*a*c**3*d**4*e**4*x
**3 + 900*log(d + e*x)*a*c**3*d**3*e**5*x**4 + 360*log(d + e*x)*a*c**3*d**
2*e**6*x**5 + 60*log(d + e*x)*a*c**3*d*e**7*x**6 - 420*log(d + e*x)*b*c**3
*d**8 - 2520*log(d + e*x)*b*c**3*d**7*e*x - 6300*log(d + e*x)*b*c**3*d**6*
e**2*x**2 - 8400*log(d + e*x)*b*c**3*d**5*e**3*x**3 - 6300*log(d + e*x)*b*
c**3*d**4*e**4*x**4 - 2520*log(d + e*x)*b*c**3*d**3*e**5*x**5 - 420*log(d
+ e*x)*b*c**3*d**2*e**6*x**6 - 10*a**4*d*e**7 - 2*a**3*b*d**2*e**6 - 12*a*
*3*b*d*e**7*x - 3*a**3*c*d**3*e**5 - 18*a**3*c*d**2*e**6*x - 45*a**3*c*d*e
**7*x**2 - 3*a**2*b*c*d**4*e**4 - 18*a**2*b*c*d**3*e**5*x - 45*a**2*b*c*d*
*2*e**6*x**2 - 60*a**2*b*c*d*e**7*x**3 - 6*a**2*c**2*d**5*e**3 - 36*a**2*c
**2*d**4*e**4*x - 90*a**2*c**2*d**3*e**5*x**2 - 120*a**2*c**2*d**2*e**6*x*
*3 - 90*a**2*c**2*d*e**7*x**4 + 30*a*b*c**2*e**8*x**6 + 87*a*c**3*d**7*e +
 462*a*c**3*d**6*e**2*x + 975*a*c**3*d**5*e**3*x**2 + 1000*a*c**3*d**4*e**
4*x**3 + 450*a*c**3*d**3*e**5*x**4 - 60*a*c**3*d*e**7*x**6 - 609*b*c**3*d*
*8 - 3234*b*c**3*d**7*e*x - 6825*b*c**3*d**6*e**2*x**2 - 7000*b*c**3*d**5*
e**3*x**3 - 3150*b*c**3*d**4*e**4*x**4 + 420*b*c**3*d**2*e**6*x**6 + 60*b*
c**3*d*e**7*x**7)/(60*d*e**8*(d**6 + 6*d**5*e*x + 15*d**4*e**2*x**2 + 20*d
**3*e**3*x**3 + 15*d**2*e**4*x**4 + 6*d*e**5*x**5 + e**6*x**6))
```

3.75 $\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 327

$$\begin{aligned}\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^8}\,dx &= \frac{(Bd-Ae)\,(cd^2+ae^2)^3}{7e^8(d+ex)^7} \\ &- \frac{(cd^2+ae^2)^2\,(7Bcd^2-6Acde+aBe^2)}{6e^8(d+ex)^6} \\ &+ \frac{3c(cd^2+ae^2)\,(7Bcd^3-5Acd^2e+3aBde^2-aAe^3)}{5e^8(d+ex)^5} \\ &+ \frac{c(4Acde(5cd^2+3ae^2)-B(35c^2d^4+30acd^2e^2+3a^2e^4))}{4e^8(d+ex)^4} \\ &+ \frac{c^2(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3)}{3e^8(d+ex)^3} \\ &- \frac{3c^2(7Bcd^2-2Acde+aBe^2)}{2e^8(d+ex)^2} \\ &+ \frac{c^3(7Bd-Ae)}{e^8(d+ex)} + \frac{Bc^3\log(d+ex)}{e^8}\end{aligned}$$

output
```
1/7*(-A*e+B*d)*(a*e^2+c*d^2)^3/e^8/(e*x+d)^7-1/6*(a*e^2+c*d^2)^2*(-6*A*c*d
*e+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)^6+3/5*c*(a*e^2+c*d^2)*(-A*a*e^3-5*A*c*d^
2*e+3*B*a*d*e^2+7*B*c*d^3)/e^8/(e*x+d)^5+1/4*c*(4*A*c*d*e*(3*a*e^2+5*c*d^2
)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))/e^8/(e*x+d)^4+1/3*c^2*(-3*A*a*e
^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)/e^8/(e*x+d)^3-3/2*c^2*(-2*A*c*d*e
+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)^2+c^3*(-A*e+7*B*d)/e^8/(e*x+d)+B*c^3*ln(e*
x+d)/e^8
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^8}\,dx$$
$$= \frac{-12Ae(5a^3e^6+a^2ce^4(d^2+7dex+21e^2x^2)+ac^2e^2(d^4+7d^3ex+21d^2e^2x^2+35de^3x^3+35e^4x^4)+5c^3(d}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^8,x]
```

output
```
(-12*A*e*(5*a^3*e^6 + a^2*c*e^4*(d^2 + 7*d*e*x + 21*e^2*x^2) + a*c^2*e^2*(
d^4 + 7*d^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4) + 5*c^3*(d^6
 + 7*d^5*e*x + 21*d^4*e^2*x^2 + 35*d^3*e^3*x^3 + 35*d^2*e^4*x^4 + 21*d*e^5
*x^5 + 7*e^6*x^6)) + B*(-10*a^3*e^6*(d + 7*e*x) - 9*a^2*c*e^4*(d^3 + 7*d^2
*e*x + 21*d*e^2*x^2 + 35*e^3*x^3) - 30*a*c^2*e^2*(d^5 + 7*d^4*e*x + 21*d^3
*e^2*x^2 + 35*d^2*e^3*x^3 + 35*d*e^4*x^4 + 21*e^5*x^5) + c^3*d*(1089*d^6 +
 7203*d^5*e*x + 20139*d^4*e^2*x^2 + 30625*d^3*e^3*x^3 + 26950*d^2*e^4*x^4
+ 13230*d*e^5*x^5 + 2940*e^6*x^6)) + 420*B*c^3*(d + e*x)^7*Log[d + e*x])/(
420*e^8*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3 (A+Bx)}{(d+ex)^8} dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{c(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4)}{e^7(d+ex)^5} - \frac{3c^2(-aBe^2+2Acde-7Bcd^2)}{e^7(d+ex)^3} + \frac{c^2(3a\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(4Acde(3ae^2+5cd^2)-B(3a^2e^4+30acd^2e^2+35c^2d^4))}{4e^8(d+ex)^4} - \frac{3c^2(aBe^2-2Acde+7Bcd^2)}{2e^8(d+ex)^2} + \frac{c^2(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3)}{3e^8(d+ex)^3} - \frac{(ae^2+cd^2)^2(aBe^2-6Acde+7Bcd^2)}{6e^8(d+ex)^6} + \frac{(ae^2+cd^2)^3(Bd-Ae)}{7e^8(d+ex)^7} + \frac{3c(ae^2+cd^2)(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3)}{5e^8(d+ex)^5} + \frac{c^3(7Bd-Ae)}{e^8(d+ex)} + \frac{Bc^3\log(d+ex)}{e^8}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^8,x]
```

output
```
((B*d - A*e)*(c*d^2 + a*e^2)^3)/(7*e^8*(d + e*x)^7) - ((c*d^2 + a*e^2)^2*(
7*B*c*d^2 - 6*A*c*d*e + a*B*e^2))/(6*e^8*(d + e*x)^6) + (3*c*(c*d^2 + a*e^
2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(5*e^8*(d + e*x)^5)
+ (c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a
^2*e^4)))/(4*e^8*(d + e*x)^4) + (c^2*(35*B*c*d^3 - 15*A*c*d^2*e + 15*a*B*d
*e^2 - 3*a*A*e^3))/(3*e^8*(d + e*x)^3) - (3*c^2*(7*B*c*d^2 - 2*A*c*d*e + a
*B*e^2))/(2*e^8*(d + e*x)^2) + (c^3*(7*B*d - A*e))/(e^8*(d + e*x)) + (B*c^
3*Log[d + e*x])/e^8
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.30

method	result
risch	$\frac{-\frac{c^3(Ae-7Bd)x^6}{e^2}-\frac{3c^2\left(2Acde+Ba\,e^2-21Bc\,d^2\right)x^5}{2e^3}-\frac{c^2\left(6Aa\,e^3+30Ac\,d^2e+15Bad\,e^2-385Bc\,d^3\right)x^4}{6e^4}-\frac{c\left(12Aacd\,e^3+60A\,c^2d^3e+9B\,e^4}{12e}$
default	$-\frac{c^2\left(3Aa\,e^3+15Ac\,d^2e-15Bad\,e^2-35Bc\,d^3\right)}{3e^8(ex+d)^3}+\frac{c\left(12Aacd\,e^3+20A\,c^2d^3e-3B\,e^4a^2-30Bac\,d^2e^2-35B\,c^2d^4\right)}{4e^8(ex+d)^4}-\frac{A\,a^3e^7+}{}$
norman	$\frac{-\frac{60A\,a^3e^7+12A\,a^2c\,d^2e^5+12Aa\,c^2d^4e^3+60A\,c^3d^6e+10B\,a^3d\,e^6+9B\,a^2c\,d^3e^4+30Ba\,c^2d^5e^2-1089B\,c^3d^7}{420e^8}-\frac{\left(A\,c^3e-7B\,c^3d\right)x^6}{e^2}-\frac{3\left(2A\right.}{}}{}$
parallelrisch	$-\frac{9B\,a^2c\,d^3e^4+12A\,a^2c\,d^2e^5+12Aa\,c^2d^4e^3+420A\,x^6c^3e^7-420B\ln(ex+d)c^3d^7+70Bx\,a^3e^7-8820B\ln(ex+d)x^5c^3d^2e^5+21}{}$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^8,x,method=_RETURNVERBOSE)
```

output
```
(-c^3*(A*e-7*B*d)/e^2*x^6-3/2*c^2*(2*A*c*d*e+B*a*e^2-21*B*c*d^2)/e^3*x^5-1
/6*c^2*(6*A*a*e^3+30*A*c*d^2*e+15*B*a*d*e^2-385*B*c*d^3)/e^4*x^4-1/12*c*(1
2*A*a*c*d*e^3+60*A*c^2*d^3*e+9*B*a^2*e^4+30*B*a*c*d^2*e^2-875*B*c^2*d^4)/e
^5*x^3-1/20*c*(12*A*a^2*e^5+12*A*a*c*d^2*e^3+60*A*c^2*d^4*e+9*B*a^2*d*e^4+
30*B*a*c*d^3*e^2-959*B*c^2*d^5)/e^6*x^2-1/60*(12*A*a^2*c*d*e^5+12*A*a*c^2*
d^3*e^3+60*A*c^3*d^5*e+10*B*a^3*e^6+9*B*a^2*c*d^2*e^4+30*B*a*c^2*d^4*e^2-1
029*B*c^3*d^6)/e^7*x-1/420*(60*A*a^3*e^7+12*A*a^2*c*d^2*e^5+12*A*a*c^2*d^4
*e^3+60*A*c^3*d^6*e+10*B*a^3*d*e^6+9*B*a^2*c*d^3*e^4+30*B*a*c^2*d^5*e^2-10
89*B*c^3*d^7)/e^8)/(e*x+d)^7+B*c^3*ln(e*x+d)/e^8
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.91

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^8}\,dx$$
$$= \frac{1089\,Bc^3d^7 - 60\,Ac^3d^6e - 30\,Bac^2d^5e^2 - 12\,Aac^2d^4e^3 - 9\,Ba^2cd^3e^4 - 12\,Aa^2cd^2e^5 - 10\,Ba^3de^6 - 60\,Ac}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^8,x, algorithm="fricas")
```

output
```
1/420*(1089*B*c^3*d^7 - 60*A*c^3*d^6*e - 30*B*a*c^2*d^5*e^2 - 12*A*a*c^2*d
^4*e^3 - 9*B*a^2*c*d^3*e^4 - 12*A*a^2*c*d^2*e^5 - 10*B*a^3*d*e^6 - 60*A*a^
3*e^7 + 420*(7*B*c^3*d*e^6 - A*c^3*e^7)*x^6 + 630*(21*B*c^3*d^2*e^5 - 2*A*
c^3*d*e^6 - B*a*c^2*e^7)*x^5 + 70*(385*B*c^3*d^3*e^4 - 30*A*c^3*d^2*e^5 -
15*B*a*c^2*d*e^6 - 6*A*a*c^2*e^7)*x^4 + 35*(875*B*c^3*d^4*e^3 - 60*A*c^3*d
^3*e^4 - 30*B*a*c^2*d^2*e^5 - 12*A*a*c^2*d*e^6 - 9*B*a^2*c*e^7)*x^3 + 21*(
959*B*c^3*d^5*e^2 - 60*A*c^3*d^4*e^3 - 30*B*a*c^2*d^3*e^4 - 12*A*a*c^2*d^2
*e^5 - 9*B*a^2*c*d*e^6 - 12*A*a^2*c*e^7)*x^2 + 7*(1029*B*c^3*d^6*e - 60*A*
c^3*d^5*e^2 - 30*B*a*c^2*d^4*e^3 - 12*A*a*c^2*d^3*e^4 - 9*B*a^2*c*d^2*e^5
- 12*A*a^2*c*d*e^6 - 10*B*a^3*e^7)*x + 420*(B*c^3*e^7*x^7 + 7*B*c^3*d*e^6*
x^6 + 21*B*c^3*d^2*e^5*x^5 + 35*B*c^3*d^3*e^4*x^4 + 35*B*c^3*d^4*e^3*x^3 +
 21*B*c^3*d^5*e^2*x^2 + 7*B*c^3*d^6*e*x + B*c^3*d^7)*log(e*x + d))/(e^15*x
^7 + 7*d*e^14*x^6 + 21*d^2*e^13*x^5 + 35*d^3*e^12*x^4 + 35*d^4*e^11*x^3 +
21*d^5*e^10*x^2 + 7*d^6*e^9*x + d^7*e^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^8}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**8,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.61

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^8}\,dx = \frac{1089\,Bc^3d^7 - 60\,Ac^3d^6e - 30\,Bac^2d^5e^2 - 12\,Aac^2d^4e^3 - 9\,Ba^2cd^3e^4 - 12\,Aa^2cd^2e^5 - 10\,Ba^3de^6 - 60\,A}{} + \frac{Bc^3\log\left(ex+d\right)}{e^8}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^8,x, algorithm="maxima")
```

output
```
1/420*(1089*B*c^3*d^7 - 60*A*c^3*d^6*e - 30*B*a*c^2*d^5*e^2 - 12*A*a*c^2*d
^4*e^3 - 9*B*a^2*c*d^3*e^4 - 12*A*a^2*c*d^2*e^5 - 10*B*a^3*d*e^6 - 60*A*a^
3*e^7 + 420*(7*B*c^3*d*e^6 - A*c^3*e^7)*x^6 + 630*(21*B*c^3*d^2*e^5 - 2*A*
c^3*d*e^6 - B*a*c^2*e^7)*x^5 + 70*(385*B*c^3*d^3*e^4 - 30*A*c^3*d^2*e^5 -
15*B*a*c^2*d*e^6 - 6*A*a*c^2*e^7)*x^4 + 35*(875*B*c^3*d^4*e^3 - 60*A*c^3*d
^3*e^4 - 30*B*a*c^2*d^2*e^5 - 12*A*a*c^2*d*e^6 - 9*B*a^2*c*e^7)*x^3 + 21*(
959*B*c^3*d^5*e^2 - 60*A*c^3*d^4*e^3 - 30*B*a*c^2*d^3*e^4 - 12*A*a*c^2*d^2
*e^5 - 9*B*a^2*c*d*e^6 - 12*A*a^2*c*e^7)*x^2 + 7*(1029*B*c^3*d^6*e - 60*A*
c^3*d^5*e^2 - 30*B*a*c^2*d^4*e^3 - 12*A*a*c^2*d^3*e^4 - 9*B*a^2*c*d^2*e^5
- 12*A*a^2*c*d*e^6 - 10*B*a^3*e^7)*x)/(e^15*x^7 + 7*d*e^14*x^6 + 21*d^2*e^
13*x^5 + 35*d^3*e^12*x^4 + 35*d^4*e^11*x^3 + 21*d^5*e^10*x^2 + 7*d^6*e^9*x
 + d^7*e^8) + B*c^3*log(e*x + d)/e^8
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.40

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^8}\,dx = \frac{Bc^3\log\left(|ex+d|\right)}{e^8} + \frac{420\left(7\,Bc^3de^5 - Ac^3e^6\right)x^6 + 630\left(21\,Bc^3d^2e^4 - 2\,Ac^3de^5 - Bac^2e^6\right)x^5 + 70\left(385\,Bc^3d^3e^3 - 30\,Ac^3d^2e^4\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^8,x, algorithm="giac")
```

output
```
B*c^3*log(abs(e*x + d))/e^8 + 1/420*(420*(7*B*c^3*d*e^5 - A*c^3*e^6)*x^6 +
 630*(21*B*c^3*d^2*e^4 - 2*A*c^3*d*e^5 - B*a*c^2*e^6)*x^5 + 70*(385*B*c^3*
d^3*e^3 - 30*A*c^3*d^2*e^4 - 15*B*a*c^2*d*e^5 - 6*A*a*c^2*e^6)*x^4 + 35*(8
75*B*c^3*d^4*e^2 - 60*A*c^3*d^3*e^3 - 30*B*a*c^2*d^2*e^4 - 12*A*a*c^2*d*e^
5 - 9*B*a^2*c*e^6)*x^3 + 21*(959*B*c^3*d^5*e - 60*A*c^3*d^4*e^2 - 30*B*a*c
^2*d^3*e^3 - 12*A*a*c^2*d^2*e^4 - 9*B*a^2*c*d*e^5 - 12*A*a^2*c*e^6)*x^2 +
7*(1029*B*c^3*d^6 - 60*A*c^3*d^5*e - 30*B*a*c^2*d^4*e^2 - 12*A*a*c^2*d^3*e
^3 - 9*B*a^2*c*d^2*e^4 - 12*A*a^2*c*d*e^5 - 10*B*a^3*e^6)*x + (1089*B*c^3*
d^7 - 60*A*c^3*d^6*e - 30*B*a*c^2*d^5*e^2 - 12*A*a*c^2*d^4*e^3 - 9*B*a^2*c
*d^3*e^4 - 12*A*a^2*c*d^2*e^5 - 10*B*a^3*d*e^6 - 60*A*a^3*e^7)/e)/((e*x +
d)^7*e^7)
```

Mupad [B] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^8}\,dx = \frac{B\,c^3\,\ln\left(d+e\,x\right)}{e^8} - \frac{x^3\left(\frac{3\,B\,a^2\,c\,e^7}{4}+\frac{5\,B\,a\,c^2\,d^2\,e^5}{2}+A\,a\,c^2\,d\,e^6-\frac{875\,B\,c^3\,d^4\,e^3}{12}+5\,A\,c^3\,d^3\,e^4\right)+x^6\left(A\,c^3\,e^7-7\,B\,c^3\,d\,e^6\right)+x^2\left(9\right.}{}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^8,x)
```

output
```
(B*c^3*log(d + e*x))/e^8 - (x^3*((3*B*a^2*c*e^7)/4 + 5*A*c^3*d^3*e^4 - (87
5*B*c^3*d^4*e^3)/12 + (5*B*a*c^2*d^2*e^5)/2 + A*a*c^2*d*e^6) + x^6*(A*c^3*
e^7 - 7*B*c^3*d*e^6) + x^2*((3*A*a^2*c*e^7)/5 + 3*A*c^3*d^4*e^3 - (959*B*c
^3*d^5*e^2)/20 + (3*A*a*c^2*d^2*e^5)/5 + (3*B*a*c^2*d^3*e^4)/2 + (9*B*a^2*
c*d*e^6)/20) + x^5*((3*B*a*c^2*e^7)/2 + 3*A*c^3*d*e^6 - (63*B*c^3*d^2*e^5)
/2) + x*((B*a^3*e^7)/6 - (343*B*c^3*d^6*e)/20 + A*c^3*d^5*e^2 + (A*a*c^2*d
^3*e^4)/5 + (B*a*c^2*d^4*e^3)/2 + (3*B*a^2*c*d^2*e^5)/20 + (A*a^2*c*d*e^6)
/5) + x^4*(A*a*c^2*e^7 + 5*A*c^3*d^2*e^5 - (385*B*c^3*d^3*e^4)/6 + (5*B*a*
c^2*d*e^6)/2) + (A*a^3*e^7)/7 - (363*B*c^3*d^7)/140 + (B*a^3*d*e^6)/42 + (
A*c^3*d^6*e)/7 + (A*a*c^2*d^4*e^3)/35 + (A*a^2*c*d^2*e^5)/35 + (B*a*c^2*d^
5*e^2)/14 + (3*B*a^2*c*d^3*e^4)/140)/(e^8*(d + e*x)^7)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.95

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^8}\,dx$$
$$= \frac{60a\,c^3e^8x^7 - 12a^3c\,d^3e^5 - 12a^2c^2d^5e^3 + 420\log(ex+d)\,b\,c^3d\,e^7x^7 - 10a^3b\,d^2e^6 + 2940\log(ex+d)\,b\,c^3d^7}{}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^8,x)
```

output
```
(420*log(d + e*x)*b*c**3*d**8 + 2940*log(d + e*x)*b*c**3*d**7*e*x + 8820*l
og(d + e*x)*b*c**3*d**6*e**2*x**2 + 14700*log(d + e*x)*b*c**3*d**5*e**3*x*
*3 + 14700*log(d + e*x)*b*c**3*d**4*e**4*x**4 + 8820*log(d + e*x)*b*c**3*d
**3*e**5*x**5 + 2940*log(d + e*x)*b*c**3*d**2*e**6*x**6 + 420*log(d + e*x)
*b*c**3*d*e**7*x**7 - 60*a**4*d*e**7 - 10*a**3*b*d**2*e**6 - 70*a**3*b*d*e
**7*x - 12*a**3*c*d**3*e**5 - 84*a**3*c*d**2*e**6*x - 252*a**3*c*d*e**7*x*
*2 - 9*a**2*b*c*d**4*e**4 - 63*a**2*b*c*d**3*e**5*x - 189*a**2*b*c*d**2*e*
*6*x**2 - 315*a**2*b*c*d*e**7*x**3 - 12*a**2*c**2*d**5*e**3 - 84*a**2*c**2
*d**4*e**4*x - 252*a**2*c**2*d**3*e**5*x**2 - 420*a**2*c**2*d**2*e**6*x**3
 - 420*a**2*c**2*d*e**7*x**4 - 30*a*b*c**2*d**6*e**2 - 210*a*b*c**2*d**5*e
**3*x - 630*a*b*c**2*d**4*e**4*x**2 - 1050*a*b*c**2*d**3*e**5*x**3 - 1050*
a*b*c**2*d**2*e**6*x**4 - 630*a*b*c**2*d*e**7*x**5 + 60*a*c**3*e**8*x**7 +
 669*b*c**3*d**8 + 4263*b*c**3*d**7*e*x + 11319*b*c**3*d**6*e**2*x**2 + 15
925*b*c**3*d**5*e**3*x**3 + 12250*b*c**3*d**4*e**4*x**4 + 4410*b*c**3*d**3
*e**5*x**5 - 420*b*c**3*d*e**7*x**7)/(420*d*e**8*(d**7 + 7*d**6*e*x + 21*d
**5*e**2*x**2 + 35*d**4*e**3*x**3 + 35*d**3*e**4*x**4 + 21*d**2*e**5*x**5
+ 7*d*e**6*x**6 + e**7*x**7))
```

3.76 $\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^9} dx$

Optimal result

Integrand size = 22, antiderivative size = 330

$$\begin{aligned}\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^9} dx &= \frac{(Bd-Ae)(cd^2+ae^2)^3}{8e^8(d+ex)^8} \\ &- \frac{(cd^2+ae^2)^2(7Bcd^2-6Acde+aBe^2)}{7e^8(d+ex)^7} \\ &+ \frac{c(cd^2+ae^2)(7Bcd^3-5Acd^2e+3aBde^2-aAe^3)}{2e^8(d+ex)^6} \\ &+ \frac{c(4Acde(5cd^2+3ae^2)-B(35c^2d^4+30acd^2e^2+3a^2e^4))}{5e^8(d+ex)^5} \\ &+ \frac{c^2(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3)}{4e^8(d+ex)^4} \\ &- \frac{c^2(7Bcd^2-2Acde+aBe^2)}{e^8(d+ex)^3} \\ &+ \frac{c^3(7Bd-Ae)}{2e^8(d+ex)^2} - \frac{Bc^3}{e^8(d+ex)}\end{aligned}$$

output
```
1/8*(-A*e+B*d)*(a*e^2+c*d^2)^3/e^8/(e*x+d)^8-1/7*(a*e^2+c*d^2)^2*(-6*A*c*d
*e+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)^7+1/2*c*(a*e^2+c*d^2)*(-A*a*e^3-5*A*c*d^
2*e+3*B*a*d*e^2+7*B*c*d^3)/e^8/(e*x+d)^6+1/5*c*(4*A*c*d*e*(3*a*e^2+5*c*d^2
)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))/e^8/(e*x+d)^5+1/4*c^2*(-3*A*a*e
^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)/e^8/(e*x+d)^4-c^2*(-2*A*c*d*e+B*a
*e^2+7*B*c*d^2)/e^8/(e*x+d)^3+1/2*c^3*(-A*e+7*B*d)/e^8/(e*x+d)^2-B*c^3/e^8
/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^9}\,dx =$$
$$-\frac{Ae(35a^3e^6+5a^2ce^4(d^2+8dex+28e^2x^2)+3ac^2e^2(d^4+8d^3ex+28d^2e^2x^2+56de^3x^3+70e^4x^4)+5c^3$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^9,x]
```

output
```
-1/280*(A*e*(35*a^3*e^6 + 5*a^2*c*e^4*(d^2 + 8*d*e*x + 28*e^2*x^2) + 3*a*c
^2*e^2*(d^4 + 8*d^3*e*x + 28*d^2*e^2*x^2 + 56*d*e^3*x^3 + 70*e^4*x^4) + 5*
c^3*(d^6 + 8*d^5*e*x + 28*d^4*e^2*x^2 + 56*d^3*e^3*x^3 + 70*d^2*e^4*x^4 +
56*d*e^5*x^5 + 28*e^6*x^6)) + B*(5*a^3*e^6*(d + 8*e*x) + 3*a^2*c*e^4*(d^3
+ 8*d^2*e*x + 28*d*e^2*x^2 + 56*e^3*x^3) + 5*a*c^2*e^2*(d^5 + 8*d^4*e*x +
28*d^3*e^2*x^2 + 56*d^2*e^3*x^3 + 70*d*e^4*x^4 + 56*e^5*x^5) + 35*c^3*(d^7
 + 8*d^6*e*x + 28*d^5*e^2*x^2 + 56*d^4*e^3*x^3 + 70*d^3*e^4*x^4 + 56*d^2*e
^5*x^5 + 28*d*e^6*x^6 + 8*e^7*x^7)))/(e^8*(d + e*x)^8)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3 (A+Bx)}{(d+ex)^9} \, dx$$

$\big\downarrow$ 652

$$\int \left(-\frac{c(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4)}{e^7(d+ex)^6} - \frac{3c^2(-aBe^2+2Acde-7Bcd^2)}{e^7(d+ex)^4} + \frac{c^2(3a$$

$\big\downarrow$ 2009

$$\frac{c(4Acde(3ae^2+5cd^2)-B(3a^2e^4+30acd^2e^2+35c^2d^4))}{5e^8(d+ex)^5} - \frac{c^2(aBe^2-2Acde+7Bcd^2)}{e^8(d+ex)^3} + \frac{c^2(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3)}{4e^8(d+ex)^4} - \frac{(ae^2+cd^2)^2(aBe^2-6Acde+7Bcd^2)}{7e^8(d+ex)^7} + \frac{(ae^2+cd^2)^3(Bd-Ae)}{8e^8(d+ex)^8} + \frac{c(ae^2+cd^2)(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3)}{2e^8(d+ex)^6} + \frac{c^3(7Bd-Ae)}{2e^8(d+ex)^2} - \frac{Bc^3}{e^8(d+ex)}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^9,x]
```

output
```
((B*d - A*e)*(c*d^2 + a*e^2)^3)/(8*e^8*(d + e*x)^8) - ((c*d^2 + a*e^2)^2*(
7*B*c*d^2 - 6*A*c*d*e + a*B*e^2))/(7*e^8*(d + e*x)^7) + (c*(c*d^2 + a*e^2)
*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(2*e^8*(d + e*x)^6) +
(c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2
*e^4)))/(5*e^8*(d + e*x)^5) + (c^2*(35*B*c*d^3 - 15*A*c*d^2*e + 15*a*B*d*e
^2 - 3*a*A*e^3))/(4*e^8*(d + e*x)^4) - (c^2*(7*B*c*d^2 - 2*A*c*d*e + a*B*e
^2))/(e^8*(d + e*x)^3) + (c^3*(7*B*d - A*e))/(2*e^8*(d + e*x)^2) - (B*c^3)
/(e^8*(d + e*x))
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.28

method	result
risch	$\frac{-\frac{Bc^3x^7}{e}-\frac{c^3(Ae+7Bd)x^6}{2e^2}-\frac{c^2\left(Acde+Bae^2+7Bcd^2\right)x^5}{e^3}-\frac{c^2\left(3Aae^3+5Acd^2e+5Bade^2+35Bcd^3\right)x^4}{4e^4}-\frac{c\left(3Aacde^3+5Ac^2d^3e+3Be^4 \cdots\right.}{5e \cdots}}{\cdots}$
norman	$\frac{-\frac{Bc^3x^7}{e}-\frac{\left(Ac^3e+7Bc^3d\right)x^6}{2e^2}-\frac{\left(Ac^3de+Be^2ac^2+7Bc^3d^2\right)x^5}{e^3}-\frac{\left(3Aac^2e^3+5Ac^3d^2e+5Bac^2de^2+35Bc^3d^3\right)x^4}{4e^4}-\frac{\left(3Aac^2de^3+5A \cdots\right.}{\cdots}}{\cdots}$
default	$\frac{c^2\left(2Acde-Bae^2-7Bcd^2\right)}{e^8(ex+d)^3}-\frac{c^2\left(3Aae^3+15Acd^2e-15Bade^2-35Bcd^3\right)}{4e^8(ex+d)^4}-\frac{-6Aa^2cde^5-12Aac^2d^3e^3-6Ac^3d^5e+Ba^3 \cdots}{7e^8(ex+ \cdots}$
gosper	$-\frac{280Bx^7c^3e^7+140Ax^6c^3e^7+980Bx^6c^3de^6+280Ax^5c^3de^6+280Bx^5ac^2e^7+1960Bx^5c^3d^2e^5+210Ax^4ac^2e^7+350Ax^4c \cdots}{\cdots}$
parallelrisch	$-\frac{280Bx^7c^3e^7+140Ax^6c^3e^7+980Bx^6c^3de^6+280Ax^5c^3de^6+280Bx^5ac^2e^7+1960Bx^5c^3d^2e^5+210Ax^4ac^2e^7+350Ax^4c \cdots}{\cdots}$
orering	$-\frac{280Bx^7c^3e^7+140Ax^6c^3e^7+980Bx^6c^3de^6+280Ax^5c^3de^6+280Bx^5ac^2e^7+1960Bx^5c^3d^2e^5+210Ax^4ac^2e^7+350Ax^4c \cdots}{\cdots}$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^9,x,method=_RETURNVERBOSE)
```

output
```
(-B*c^3*x^7/e-1/2*c^3*(A*e+7*B*d)/e^2*x^6-c^2*(A*c*d*e+B*a*e^2+7*B*c*d^2)/
e^3*x^5-1/4*c^2*(3*A*a*e^3+5*A*c*d^2*e+5*B*a*d*e^2+35*B*c*d^3)/e^4*x^4-1/5
*c*(3*A*a*c*d*e^3+5*A*c^2*d^3*e+3*B*a^2*e^4+5*B*a*c*d^2*e^2+35*B*c^2*d^4)/
e^5*x^3-1/10*c*(5*A*a^2*e^5+3*A*a*c*d^2*e^3+5*A*c^2*d^4*e+3*B*a^2*d*e^4+5*
B*a*c*d^3*e^2+35*B*c^2*d^5)/e^6*x^2-1/35*(5*A*a^2*c*d*e^5+3*A*a*c^2*d^3*e^
3+5*A*c^3*d^5*e+5*B*a^3*e^6+3*B*a^2*c*d^2*e^4+5*B*a*c^2*d^4*e^2+35*B*c^3*d
^6)/e^7*x-1/280*(35*A*a^3*e^7+5*A*a^2*c*d^2*e^5+3*A*a*c^2*d^4*e^3+5*A*c^3*
d^6*e+5*B*a^3*d*e^6+3*B*a^2*c*d^3*e^4+5*B*a*c^2*d^5*e^2+35*B*c^3*d^7)/e^8)
/(e*x+d)^8
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.61

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^9}\,dx = -\frac{280\,Bc^3e^7x^7+35\,Bc^3d^7+5\,Ac^3d^6e+5\,Bac^2d^5e^2+3\,Aac^2d^4e^3+3\,Ba^2cd^3e^4+5\,Aa^2cd^2e^5+5\,Ba^3de \cdots}{\cdots}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^9,x, algorithm="fricas")
```

output
```
-1/280*(280*B*c^3*e^7*x^7 + 35*B*c^3*d^7 + 5*A*c^3*d^6*e + 5*B*a*c^2*d^5*e
^2 + 3*A*a*c^2*d^4*e^3 + 3*B*a^2*c*d^3*e^4 + 5*A*a^2*c*d^2*e^5 + 5*B*a^3*d
*e^6 + 35*A*a^3*e^7 + 140*(7*B*c^3*d*e^6 + A*c^3*e^7)*x^6 + 280*(7*B*c^3*d
^2*e^5 + A*c^3*d*e^6 + B*a*c^2*e^7)*x^5 + 70*(35*B*c^3*d^3*e^4 + 5*A*c^3*d
^2*e^5 + 5*B*a*c^2*d*e^6 + 3*A*a*c^2*e^7)*x^4 + 56*(35*B*c^3*d^4*e^3 + 5*A
*c^3*d^3*e^4 + 5*B*a*c^2*d^2*e^5 + 3*A*a*c^2*d*e^6 + 3*B*a^2*c*e^7)*x^3 +
28*(35*B*c^3*d^5*e^2 + 5*A*c^3*d^4*e^3 + 5*B*a*c^2*d^3*e^4 + 3*A*a*c^2*d^2
*e^5 + 3*B*a^2*c*d*e^6 + 5*A*a^2*c*e^7)*x^2 + 8*(35*B*c^3*d^6*e + 5*A*c^3*
d^5*e^2 + 5*B*a*c^2*d^4*e^3 + 3*A*a*c^2*d^3*e^4 + 3*B*a^2*c*d^2*e^5 + 5*A*
a^2*c*d*e^6 + 5*B*a^3*e^7)*x)/(e^16*x^8 + 8*d*e^15*x^7 + 28*d^2*e^14*x^6 +
 56*d^3*e^13*x^5 + 70*d^4*e^12*x^4 + 56*d^5*e^11*x^3 + 28*d^6*e^10*x^2 + 8
*d^7*e^9*x + d^8*e^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^9}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**9,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.61

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^9}\,dx = -\frac{280\,Bc^3e^7x^7 + 35\,Bc^3d^7 + 5\,Ac^3d^6e + 5\,Bac^2d^5e^2 + 3\,Aac^2d^4e^3 + 3\,Ba^2cd^3e^4 + 5\,Aa^2cd^2e^5 + 5\,Ba^3de}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^9,x, algorithm="maxima")
```

output
```
-1/280*(280*B*c^3*e^7*x^7 + 35*B*c^3*d^7 + 5*A*c^3*d^6*e + 5*B*a*c^2*d^5*e
^2 + 3*A*a*c^2*d^4*e^3 + 3*B*a^2*c*d^3*e^4 + 5*A*a^2*c*d^2*e^5 + 5*B*a^3*d
*e^6 + 35*A*a^3*e^7 + 140*(7*B*c^3*d*e^6 + A*c^3*e^7)*x^6 + 280*(7*B*c^3*d
^2*e^5 + A*c^3*d*e^6 + B*a*c^2*e^7)*x^5 + 70*(35*B*c^3*d^3*e^4 + 5*A*c^3*d
^2*e^5 + 5*B*a*c^2*d*e^6 + 3*A*a*c^2*e^7)*x^4 + 56*(35*B*c^3*d^4*e^3 + 5*A
*c^3*d^3*e^4 + 5*B*a*c^2*d^2*e^5 + 3*A*a*c^2*d*e^6 + 3*B*a^2*c*e^7)*x^3 +
28*(35*B*c^3*d^5*e^2 + 5*A*c^3*d^4*e^3 + 5*B*a*c^2*d^3*e^4 + 3*A*a*c^2*d^2
*e^5 + 3*B*a^2*c*d*e^6 + 5*A*a^2*c*e^7)*x^2 + 8*(35*B*c^3*d^6*e + 5*A*c^3*
d^5*e^2 + 5*B*a*c^2*d^4*e^3 + 3*A*a*c^2*d^3*e^4 + 3*B*a^2*c*d^2*e^5 + 5*A*
a^2*c*d*e^6 + 5*B*a^3*e^7)*x)/(e^16*x^8 + 8*d*e^15*x^7 + 28*d^2*e^14*x^6 +
 56*d^3*e^13*x^5 + 70*d^4*e^12*x^4 + 56*d^5*e^11*x^3 + 28*d^6*e^10*x^2 + 8
*d^7*e^9*x + d^8*e^8)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.48

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^9}\,dx = -\frac{280\,Bc^3e^7x^7+980\,Bc^3de^6x^6+140\,Ac^3e^7x^6+1960\,Bc^3d^2e^5x^5+280\,Ac^3de^6x^5+280\,Bac^2e^7x^5+2450}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^9,x, algorithm="giac")
```

output
```
-1/280*(280*B*c^3*e^7*x^7 + 980*B*c^3*d*e^6*x^6 + 140*A*c^3*e^7*x^6 + 1960
*B*c^3*d^2*e^5*x^5 + 280*A*c^3*d*e^6*x^5 + 280*B*a*c^2*e^7*x^5 + 2450*B*c^
3*d^3*e^4*x^4 + 350*A*c^3*d^2*e^5*x^4 + 350*B*a*c^2*d*e^6*x^4 + 210*A*a*c^
2*e^7*x^4 + 1960*B*c^3*d^4*e^3*x^3 + 280*A*c^3*d^3*e^4*x^3 + 280*B*a*c^2*d
^2*e^5*x^3 + 168*A*a*c^2*d*e^6*x^3 + 168*B*a^2*c*e^7*x^3 + 980*B*c^3*d^5*e
^2*x^2 + 140*A*c^3*d^4*e^3*x^2 + 140*B*a*c^2*d^3*e^4*x^2 + 84*A*a*c^2*d^2*
e^5*x^2 + 84*B*a^2*c*d*e^6*x^2 + 140*A*a^2*c*e^7*x^2 + 280*B*c^3*d^6*e*x +
 40*A*c^3*d^5*e^2*x + 40*B*a*c^2*d^4*e^3*x + 24*A*a*c^2*d^3*e^4*x + 24*B*a
^2*c*d^2*e^5*x + 40*A*a^2*c*d*e^6*x + 40*B*a^3*e^7*x + 35*B*c^3*d^7 + 5*A*
c^3*d^6*e + 5*B*a*c^2*d^5*e^2 + 3*A*a*c^2*d^4*e^3 + 3*B*a^2*c*d^3*e^4 + 5*
A*a^2*c*d^2*e^5 + 5*B*a^3*d*e^6 + 35*A*a^3*e^7)/((e*x + d)^8*e^8)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.73

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^9}\,dx = -\frac{5\,B\,a^3\,d\,e^6+40\,B\,a^3\,e^7\,x+35\,A\,a^3\,e^7+3\,B\,a^2\,c\,d^3\,e^4+24\,B\,a^2\,c\,d^2\,e^5\,x+5\,A\,a^2\,c\,d^2\,e^5+84\,B\,a^2\,c\,d\,e^{6}\cdots}{\cdots}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^9,x)
```

output
```
-(35*A*a^3*e^7 + 35*B*c^3*d^7 + 5*B*a^3*d*e^6 + 5*A*c^3*d^6*e + 40*B*a^3*e
^7*x + 140*A*c^3*e^7*x^6 + 280*B*c^3*e^7*x^7 + 280*B*c^3*d^6*e*x + 3*A*a*c
^2*d^4*e^3 + 5*A*a^2*c*d^2*e^5 + 5*B*a*c^2*d^5*e^2 + 3*B*a^2*c*d^3*e^4 + 1
40*A*a^2*c*e^7*x^2 + 210*A*a*c^2*e^7*x^4 + 168*B*a^2*c*e^7*x^3 + 280*B*a*c
^2*e^7*x^5 + 40*A*c^3*d^5*e^2*x + 280*A*c^3*d*e^6*x^5 + 980*B*c^3*d*e^6*x^
6 + 140*A*c^3*d^4*e^3*x^2 + 280*A*c^3*d^3*e^4*x^3 + 350*A*c^3*d^2*e^5*x^4
+ 980*B*c^3*d^5*e^2*x^2 + 1960*B*c^3*d^4*e^3*x^3 + 2450*B*c^3*d^3*e^4*x^4
+ 1960*B*c^3*d^2*e^5*x^5 + 84*A*a*c^2*d^2*e^5*x^2 + 140*B*a*c^2*d^3*e^4*x^
2 + 280*B*a*c^2*d^2*e^5*x^3 + 40*A*a^2*c*d*e^6*x + 24*A*a*c^2*d^3*e^4*x +
168*A*a*c^2*d*e^6*x^3 + 40*B*a*c^2*d^4*e^3*x + 24*B*a^2*c*d^2*e^5*x + 84*B
*a^2*c*d*e^6*x^2 + 350*B*a*c^2*d*e^6*x^4)/(280*d^8*e^8 + 280*e^16*x^8 + 22
40*d^7*e^9*x + 2240*d*e^15*x^7 + 7840*d^6*e^10*x^2 + 15680*d^5*e^11*x^3 +
19600*d^4*e^12*x^4 + 15680*d^3*e^13*x^5 + 7840*d^2*e^14*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.48

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^9}\,dx = \frac{35b\,c^3e^7x^8-140a\,c^3d\,e^6x^6-280ab\,c^2d\,e^6x^5-280a\,c^3d^2e^5x^5-210a^2c^2d\,e^6x^4-350ab\,c^2d^2e^5x^4-350a\,c^{\cdots}}{\cdots}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^9,x)
```

output

```
( - 35*a**4*d*e**6 - 5*a**3*b*d**2*e**5 - 40*a**3*b*d*e**6*x - 5*a**3*c*d*
*3*e**4 - 40*a**3*c*d**2*e**5*x - 140*a**3*c*d*e**6*x**2 - 3*a**2*b*c*d**4
*e**3 - 24*a**2*b*c*d**3*e**4*x - 84*a**2*b*c*d**2*e**5*x**2 - 168*a**2*b*
c*d*e**6*x**3 - 3*a**2*c**2*d**5*e**2 - 24*a**2*c**2*d**4*e**3*x - 84*a**2
*c**2*d**3*e**4*x**2 - 168*a**2*c**2*d**2*e**5*x**3 - 210*a**2*c**2*d*e**6
*x**4 - 5*a*b*c**2*d**6*e - 40*a*b*c**2*d**5*e**2*x - 140*a*b*c**2*d**4*e*
*3*x**2 - 280*a*b*c**2*d**3*e**4*x**3 - 350*a*b*c**2*d**2*e**5*x**4 - 280*
a*b*c**2*d*e**6*x**5 - 5*a*c**3*d**7 - 40*a*c**3*d**6*e*x - 140*a*c**3*d**
5*e**2*x**2 - 280*a*c**3*d**4*e**3*x**3 - 350*a*c**3*d**3*e**4*x**4 - 280*
a*c**3*d**2*e**5*x**5 - 140*a*c**3*d*e**6*x**6 + 35*b*c**3*e**7*x**8)/(280
*d*e**7*(d**8 + 8*d**7*e*x + 28*d**6*e**2*x**2 + 56*d**5*e**3*x**3 + 70*d*
*4*e**4*x**4 + 56*d**3*e**5*x**5 + 28*d**2*e**6*x**6 + 8*d*e**7*x**7 + e**
8*x**8))
```

3.77 $\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{10}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 334

$$\begin{aligned}\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{10}}\,dx &= \frac{(Bd-Ae)(cd^2+ae^2)^3}{9e^8(d+ex)^9} \\ &\quad - \frac{(cd^2+ae^2)^2(7Bcd^2-6Acde+aBe^2)}{8e^8(d+ex)^8} \\ &\quad + \frac{3c(cd^2+ae^2)(7Bcd^3-5Acd^2e+3aBde^2-aAe^3)}{7e^8(d+ex)^7} \\ &\quad + \frac{c(4Acde(5cd^2+3ae^2)-B(35c^2d^4+30acd^2e^2+3a^2e^4))}{6e^8(d+ex)^6} \\ &\quad + \frac{c^2(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3)}{5e^8(d+ex)^5} \\ &\quad - \frac{3c^2(7Bcd^2-2Acde+aBe^2)}{4e^8(d+ex)^4} \\ &\quad + \frac{c^3(7Bd-Ae)}{3e^8(d+ex)^3} - \frac{Bc^3}{2e^8(d+ex)^2}\end{aligned}$$

output
```
1/9*(-A*e+B*d)*(a*e^2+c*d^2)^3/e^8/(e*x+d)^9-1/8*(a*e^2+c*d^2)^2*(-6*A*c*d
*e+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)^8+3/7*c*(a*e^2+c*d^2)*(-A*a*e^3-5*A*c*d^
2*e+3*B*a*d*e^2+7*B*c*d^3)/e^8/(e*x+d)^7+1/6*c*(4*A*c*d*e*(3*a*e^2+5*c*d^2
)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))/e^8/(e*x+d)^6+1/5*c^2*(-3*A*a*e
^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)/e^8/(e*x+d)^5-3/4*c^2*(-2*A*c*d*e
+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)^4+1/3*c^3*(-A*e+7*B*d)/e^8/(e*x+d)^3-1/2*B
*c^3/e^8/(e*x+d)^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{10}}\,dx =$$
$$-\frac{2Ae(140a^3e^6+15a^2ce^4(d^2+9dex+36e^2x^2)+6ac^2e^2(d^4+9d^3ex+36d^2e^2x^2+84de^3x^3+126e^4x^4)\cdots}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^10,x]
```

output
```
-1/2520*(2*A*e*(140*a^3*e^6 + 15*a^2*c*e^4*(d^2 + 9*d*e*x + 36*e^2*x^2) +
6*a*c^2*e^2*(d^4 + 9*d^3*e*x + 36*d^2*e^2*x^2 + 84*d*e^3*x^3 + 126*e^4*x^4
) + 5*c^3*(d^6 + 9*d^5*e*x + 36*d^4*e^2*x^2 + 84*d^3*e^3*x^3 + 126*d^2*e^4
*x^4 + 126*d*e^5*x^5 + 84*e^6*x^6)) + 5*B*(7*a^3*e^6*(d + 9*e*x) + 3*a^2*c
*e^4*(d^3 + 9*d^2*e*x + 36*d*e^2*x^2 + 84*e^3*x^3) + 3*a*c^2*e^2*(d^5 + 9*
d^4*e*x + 36*d^3*e^2*x^2 + 84*d^2*e^3*x^3 + 126*d*e^4*x^4 + 126*e^5*x^5) +
 7*c^3*(d^7 + 9*d^6*e*x + 36*d^5*e^2*x^2 + 84*d^4*e^3*x^3 + 126*d^3*e^4*x^
4 + 126*d^2*e^5*x^5 + 84*d*e^6*x^6 + 36*e^7*x^7)))/(e^8*(d + e*x)^9)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3 (A+Bx)}{(d+ex)^{10}} \, dx$$

$$\Big\downarrow \text{ 652}$$

$$\int \left(-\frac{c(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4)}{e^7(d+ex)^7} - \frac{3c^2(-aBe^2+2Acde-7Bcd^2)}{e^7(d+ex)^5} + \frac{c^2(3a$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c(4Acde(3ae^2+5cd^2)-B(3a^2e^4+30acd^2e^2+35c^2d^4))}{6e^8(d+ex)^6} - \frac{3c^2(aBe^2-2Acde+7Bcd^2)}{4e^8(d+ex)^4} + \frac{c^2(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3)}{5e^8(d+ex)^5} - \frac{(ae^2+cd^2)^2(aBe^2-6Acde+7Bcd^2)}{8e^8(d+ex)^8} + \frac{(ae^2+cd^2)^3(Bd-Ae)}{9e^8(d+ex)^9} + \frac{3c(ae^2+cd^2)(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3)}{7e^8(d+ex)^7} + \frac{c^3(7Bd-Ae)}{3e^8(d+ex)^3} - \frac{Bc^3}{2e^8(d+ex)^2}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^10,x]
```

output
```
((B*d - A*e)*(c*d^2 + a*e^2)^3)/(9*e^8*(d + e*x)^9) - ((c*d^2 + a*e^2)^2*(
7*B*c*d^2 - 6*A*c*d*e + a*B*e^2))/(8*e^8*(d + e*x)^8) + (3*c*(c*d^2 + a*e^
2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(7*e^8*(d + e*x)^7)
+ (c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a
^2*e^4)))/(6*e^8*(d + e*x)^6) + (c^2*(35*B*c*d^3 - 15*A*c*d^2*e + 15*a*B*d
*e^2 - 3*a*A*e^3))/(5*e^8*(d + e*x)^5) - (3*c^2*(7*B*c*d^2 - 2*A*c*d*e + a
*B*e^2))/(4*e^8*(d + e*x)^4) + (c^3*(7*B*d - A*e))/(3*e^8*(d + e*x)^3) - (
B*c^3)/(2*e^8*(d + e*x)^2)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.27

method	result
risch	$\frac{-\frac{B c^3 x^7}{2e}-\frac{c^3(2Ae+7Bd)x^6}{6e^2}-\frac{c^2\left(2Acde+3Ba e^2+7Bc d^2\right)x^5}{4e^3}-\frac{c^2\left(12Aa e^3+10Ac d^2 e+15Bad e^2+35Bc d^3\right)x^4}{20e^4}-\frac{c\left(12Aacd e^3+10A c^2 d^3\right.}{}}{}$
default	$-\frac{c^3(Ae-7Bd)}{3e^8(ex+d)^3}+\frac{3c^2\left(2Acde-Ba e^2-7Bc d^2\right)}{4e^8(ex+d)^4}-\frac{A a^3 e^7+3A a^2 c d^2 e^5+3Aa c^2 d^4 e^3+A c^3 d^6 e-B a^3 d e^6-3B a^2 c d^3 e^4-3}{9e^8(ex+d)^9}$
norman	$\frac{-\frac{B c^3 x^7}{2e}-\frac{\left(2A c^3 e^2+7B c^3 de\right)x^6}{6e^3}-\frac{\left(2A c^3 d e^2+3B e^3 a c^2+7B c^3 d^2 e\right)x^5}{4e^4}-\frac{\left(12Aa c^2 e^4+10A c^3 d^2 e^2+15Ba c^2 d e^3+35B c^3 d^3 e\right)x^4}{20e^5}-\frac{(1}{}}{}$
gosper	$-\frac{1260B x^7 c^3 e^7+840A x^6 c^3 e^7+2940B x^6 c^3 d e^6+1260A x^5 c^3 d e^6+1890B x^5 a c^2 e^7+4410B x^5 c^3 d^2 e^5+1512A x^4 a c^2 e^7+126}{}$
orering	$-\frac{1260B x^7 c^3 e^7+840A x^6 c^3 e^7+2940B x^6 c^3 d e^6+1260A x^5 c^3 d e^6+1890B x^5 a c^2 e^7+4410B x^5 c^3 d^2 e^5+1512A x^4 a c^2 e^7+126}{}$
parallelrisch	$-\frac{1260B c^3 x^7 e^8+840A c^3 e^8 x^6+2940B c^3 d e^7 x^6+1260A c^3 d e^7 x^5+1890Ba c^2 e^8 x^5+4410B c^3 d^2 e^6 x^5+1512Aa c^2 e^8 x^4+1260}{}$

input

```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^10,x,method=_RETURNVERBOSE)
```

output

```
(-1/2*B*c^3*x^7/e-1/6*c^3/e^2*(2*A*e+7*B*d)*x^6-1/4/e^3*c^2*(2*A*c*d*e+3*B
*a*e^2+7*B*c*d^2)*x^5-1/20/e^4*c^2*(12*A*a*e^3+10*A*c*d^2*e+15*B*a*d*e^2+3
5*B*c*d^3)*x^4-1/30*c/e^5*(12*A*a*c*d*e^3+10*A*c^2*d^3*e+15*B*a^2*e^4+15*B
*a*c*d^2*e^2+35*B*c^2*d^4)*x^3-1/70*c/e^6*(30*A*a^2*e^5+12*A*a*c*d^2*e^3+1
0*A*c^2*d^4*e+15*B*a^2*d*e^4+15*B*a*c*d^3*e^2+35*B*c^2*d^5)*x^2-1/280/e^7*
(30*A*a^2*c*d*e^5+12*A*a*c^2*d^3*e^3+10*A*c^3*d^5*e+35*B*a^3*e^6+15*B*a^2*
c*d^2*e^4+15*B*a*c^2*d^4*e^2+35*B*c^3*d^6)*x-1/2520/e^8*(280*A*a^3*e^7+30*
A*a^2*c*d^2*e^5+12*A*a*c^2*d^4*e^3+10*A*c^3*d^6*e+35*B*a^3*d*e^6+15*B*a^2*
c*d^3*e^4+15*B*a*c^2*d^5*e^2+35*B*c^3*d^7))/(e*x+d)^9
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.63

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{10}}\,dx =$$
$$-\frac{1260\,Bc^3e^7x^7+35\,Bc^3d^7+10\,Ac^3d^6e+15\,Bac^2d^5e^2+12\,Aac^2d^4e^3+15\,Ba^2cd^3e^4+30\,Aa^2cd^2e^5+3}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^10,x, algorithm="fricas")
```

output
```
-1/2520*(1260*B*c^3*e^7*x^7 + 35*B*c^3*d^7 + 10*A*c^3*d^6*e + 15*B*a*c^2*d
^5*e^2 + 12*A*a*c^2*d^4*e^3 + 15*B*a^2*c*d^3*e^4 + 30*A*a^2*c*d^2*e^5 + 35
*B*a^3*d*e^6 + 280*A*a^3*e^7 + 420*(7*B*c^3*d*e^6 + 2*A*c^3*e^7)*x^6 + 630
*(7*B*c^3*d^2*e^5 + 2*A*c^3*d*e^6 + 3*B*a*c^2*e^7)*x^5 + 126*(35*B*c^3*d^3
*e^4 + 10*A*c^3*d^2*e^5 + 15*B*a*c^2*d*e^6 + 12*A*a*c^2*e^7)*x^4 + 84*(35*
B*c^3*d^4*e^3 + 10*A*c^3*d^3*e^4 + 15*B*a*c^2*d^2*e^5 + 12*A*a*c^2*d*e^6 +
 15*B*a^2*c*e^7)*x^3 + 36*(35*B*c^3*d^5*e^2 + 10*A*c^3*d^4*e^3 + 15*B*a*c^
2*d^3*e^4 + 12*A*a*c^2*d^2*e^5 + 15*B*a^2*c*d*e^6 + 30*A*a^2*c*e^7)*x^2 +
9*(35*B*c^3*d^6*e + 10*A*c^3*d^5*e^2 + 15*B*a*c^2*d^4*e^3 + 12*A*a*c^2*d^3
*e^4 + 15*B*a^2*c*d^2*e^5 + 30*A*a^2*c*d*e^6 + 35*B*a^3*e^7)*x)/(e^17*x^9
+ 9*d*e^16*x^8 + 36*d^2*e^15*x^7 + 84*d^3*e^14*x^6 + 126*d^4*e^13*x^5 + 12
6*d^5*e^12*x^4 + 84*d^6*e^11*x^3 + 36*d^7*e^10*x^2 + 9*d^8*e^9*x + d^9*e^8
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{10}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**10,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.63

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{10}}\,dx = -\frac{1260\,Bc^3e^7x^7 + 35\,Bc^3d^7 + 10\,Ac^3d^6e + 15\,Bac^2d^5e^2 + 12\,Aac^2d^4e^3 + 15\,Ba^2cd^3e^4 + 30\,Aa^2cd^2e^5 + 3}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^10,x, algorithm="maxima")
```

output
```
-1/2520*(1260*B*c^3*e^7*x^7 + 35*B*c^3*d^7 + 10*A*c^3*d^6*e + 15*B*a*c^2*d
^5*e^2 + 12*A*a*c^2*d^4*e^3 + 15*B*a^2*c*d^3*e^4 + 30*A*a^2*c*d^2*e^5 + 35
*B*a^3*d*e^6 + 280*A*a^3*e^7 + 420*(7*B*c^3*d*e^6 + 2*A*c^3*e^7)*x^6 + 630
*(7*B*c^3*d^2*e^5 + 2*A*c^3*d*e^6 + 3*B*a*c^2*e^7)*x^5 + 126*(35*B*c^3*d^3
*e^4 + 10*A*c^3*d^2*e^5 + 15*B*a*c^2*d*e^6 + 12*A*a*c^2*e^7)*x^4 + 84*(35*
B*c^3*d^4*e^3 + 10*A*c^3*d^3*e^4 + 15*B*a*c^2*d^2*e^5 + 12*A*a*c^2*d*e^6 +
 15*B*a^2*c*e^7)*x^3 + 36*(35*B*c^3*d^5*e^2 + 10*A*c^3*d^4*e^3 + 15*B*a*c^
2*d^3*e^4 + 12*A*a*c^2*d^2*e^5 + 15*B*a^2*c*d*e^6 + 30*A*a^2*c*e^7)*x^2 +
9*(35*B*c^3*d^6*e + 10*A*c^3*d^5*e^2 + 15*B*a*c^2*d^4*e^3 + 12*A*a*c^2*d^3
*e^4 + 15*B*a^2*c*d^2*e^5 + 30*A*a^2*c*d*e^6 + 35*B*a^3*e^7)*x)/(e^17*x^9
+ 9*d*e^16*x^8 + 36*d^2*e^15*x^7 + 84*d^3*e^14*x^6 + 126*d^4*e^13*x^5 + 12
6*d^5*e^12*x^4 + 84*d^6*e^11*x^3 + 36*d^7*e^10*x^2 + 9*d^8*e^9*x + d^9*e^8
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.46

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{10}}\,dx =$$
$$-\frac{1260\,Bc^3e^7x^7 + 2940\,Bc^3de^6x^6 + 840\,Ac^3e^7x^6 + 4410\,Bc^3d^2e^5x^5 + 1260\,Ac^3de^6x^5 + 1890\,Bac^2e^7x^5 +}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^10,x, algorithm="giac")
```

output
```
-1/2520*(1260*B*c^3*e^7*x^7 + 2940*B*c^3*d*e^6*x^6 + 840*A*c^3*e^7*x^6 + 4
410*B*c^3*d^2*e^5*x^5 + 1260*A*c^3*d*e^6*x^5 + 1890*B*a*c^2*e^7*x^5 + 4410
*B*c^3*d^3*e^4*x^4 + 1260*A*c^3*d^2*e^5*x^4 + 1890*B*a*c^2*d*e^6*x^4 + 151
2*A*a*c^2*e^7*x^4 + 2940*B*c^3*d^4*e^3*x^3 + 840*A*c^3*d^3*e^4*x^3 + 1260*
B*a*c^2*d^2*e^5*x^3 + 1008*A*a*c^2*d*e^6*x^3 + 1260*B*a^2*c*e^7*x^3 + 1260
*B*c^3*d^5*e^2*x^2 + 360*A*c^3*d^4*e^3*x^2 + 540*B*a*c^2*d^3*e^4*x^2 + 432
*A*a*c^2*d^2*e^5*x^2 + 540*B*a^2*c*d*e^6*x^2 + 1080*A*a^2*c*e^7*x^2 + 315*
B*c^3*d^6*e*x + 90*A*c^3*d^5*e^2*x + 135*B*a*c^2*d^4*e^3*x + 108*A*a*c^2*d
^3*e^4*x + 135*B*a^2*c*d^2*e^5*x + 270*A*a^2*c*d*e^6*x + 315*B*a^3*e^7*x +
 35*B*c^3*d^7 + 10*A*c^3*d^6*e + 15*B*a*c^2*d^5*e^2 + 12*A*a*c^2*d^4*e^3 +
 15*B*a^2*c*d^3*e^4 + 30*A*a^2*c*d^2*e^5 + 35*B*a^3*d*e^6 + 280*A*a^3*e^7)
/((e*x + d)^9*e^8)
```

Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.54

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{10}}\,dx =$$

$$-\frac{\frac{35\,B\,a^3\,d\,e^6+280\,A\,a^3\,e^7+15\,B\,a^2\,c\,d^3\,e^4+30\,A\,a^2\,c\,d^2\,e^5+15\,B\,a\,c^2\,d^5\,e^2+12\,A\,a\,c^2\,d^4\,e^3+35\,B\,c^3\,d^7+10\,A\,c^3\,d^6\,e}{2520\,e^8}+\frac{x\,\left(35\,B\,a^3\,e^6+15\,B\,a\right.}{}}{}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^10,x)
```

output
```
-((280*A*a^3*e^7 + 35*B*c^3*d^7 + 35*B*a^3*d*e^6 + 10*A*c^3*d^6*e + 12*A*a
*c^2*d^4*e^3 + 30*A*a^2*c*d^2*e^5 + 15*B*a*c^2*d^5*e^2 + 15*B*a^2*c*d^3*e^
4)/(2520*e^8) + (x*(35*B*a^3*e^6 + 35*B*c^3*d^6 + 10*A*c^3*d^5*e + 12*A*a*
c^2*d^3*e^3 + 15*B*a*c^2*d^4*e^2 + 15*B*a^2*c*d^2*e^4 + 30*A*a^2*c*d*e^5))
/(280*e^7) + (c^2*x^4*(12*A*a*e^3 + 35*B*c*d^3 + 15*B*a*d*e^2 + 10*A*c*d^2
*e))/(20*e^4) + (c*x^3*(15*B*a^2*e^4 + 35*B*c^2*d^4 + 10*A*c^2*d^3*e + 12*
A*a*c*d*e^3 + 15*B*a*c*d^2*e^2))/(30*e^5) + (c^3*x^6*(2*A*e + 7*B*d))/(6*e
^2) + (c^2*x^5*(3*B*a*e^2 + 7*B*c*d^2 + 2*A*c*d*e))/(4*e^3) + (c*x^2*(30*A
*a^2*e^5 + 35*B*c^2*d^5 + 15*B*a^2*d*e^4 + 10*A*c^2*d^4*e + 12*A*a*c*d^2*e
^3 + 15*B*a*c*d^3*e^2))/(70*e^6) + (B*c^3*x^7)/(2*e))/(d^9 + e^9*x^9 + 9*d
*e^8*x^8 + 36*d^7*e^2*x^2 + 84*d^6*e^3*x^3 + 126*d^5*e^4*x^4 + 126*d^4*e^5
*x^5 + 84*d^3*e^6*x^6 + 36*d^2*e^7*x^7 + 9*d^8*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.73

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{10}}\,dx$$
$$= \frac{-1260b\,c^3e^7x^7 - 840a\,c^3e^7x^6 - 2940b\,c^3d\,e^6x^6 - 1890ab\,c^2e^7x^5 - 1260a\,c^3d\,e^6x^5 - 4410b\,c^3d^2e^5x^5 - 151}{}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^10,x)
```

output
```
( - 280*a**4*e**7 - 35*a**3*b*d*e**6 - 315*a**3*b*e**7*x - 30*a**3*c*d**2*
e**5 - 270*a**3*c*d*e**6*x - 1080*a**3*c*e**7*x**2 - 15*a**2*b*c*d**3*e**4
 - 135*a**2*b*c*d**2*e**5*x - 540*a**2*b*c*d*e**6*x**2 - 1260*a**2*b*c*e**
7*x**3 - 12*a**2*c**2*d**4*e**3 - 108*a**2*c**2*d**3*e**4*x - 432*a**2*c**
2*d**2*e**5*x**2 - 1008*a**2*c**2*d*e**6*x**3 - 1512*a**2*c**2*e**7*x**4 -
 15*a*b*c**2*d**5*e**2 - 135*a*b*c**2*d**4*e**3*x - 540*a*b*c**2*d**3*e**4
*x**2 - 1260*a*b*c**2*d**2*e**5*x**3 - 1890*a*b*c**2*d*e**6*x**4 - 1890*a*
b*c**2*e**7*x**5 - 10*a*c**3*d**6*e - 90*a*c**3*d**5*e**2*x - 360*a*c**3*d
**4*e**3*x**2 - 840*a*c**3*d**3*e**4*x**3 - 1260*a*c**3*d**2*e**5*x**4 - 1
260*a*c**3*d*e**6*x**5 - 840*a*c**3*e**7*x**6 - 35*b*c**3*d**7 - 315*b*c**
3*d**6*e*x - 1260*b*c**3*d**5*e**2*x**2 - 2940*b*c**3*d**4*e**3*x**3 - 441
0*b*c**3*d**3*e**4*x**4 - 4410*b*c**3*d**2*e**5*x**5 - 2940*b*c**3*d*e**6*
x**6 - 1260*b*c**3*e**7*x**7)/(2520*e**8*(d**9 + 9*d**8*e*x + 36*d**7*e**2
*x**2 + 84*d**6*e**3*x**3 + 126*d**5*e**4*x**4 + 126*d**4*e**5*x**5 + 84*d
**3*e**6*x**6 + 36*d**2*e**7*x**7 + 9*d*e**8*x**8 + e**9*x**9))
```

3.78 $\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{11}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 334

$$
\begin{aligned}
\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{11}}\,dx &= \frac{(Bd-Ae)(cd^2+ae^2)^3}{10e^8(d+ex)^{10}} \\
&\quad - \frac{(cd^2+ae^2)^2(7Bcd^2-6Acde+aBe^2)}{9e^8(d+ex)^9} \\
&\quad + \frac{3c(cd^2+ae^2)(7Bcd^3-5Acd^2e+3aBde^2-aAe^3)}{8e^8(d+ex)^8} \\
&\quad + \frac{c(4Acde(5cd^2+3ae^2)-B(35c^2d^4+30acd^2e^2+3a^2e^4))}{7e^8(d+ex)^7} \\
&\quad + \frac{c^2(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3)}{6e^8(d+ex)^6} \\
&\quad - \frac{3c^2(7Bcd^2-2Acde+aBe^2)}{5e^8(d+ex)^5} \\
&\quad + \frac{c^3(7Bd-Ae)}{4e^8(d+ex)^4} - \frac{Bc^3}{3e^8(d+ex)^3}
\end{aligned}
$$

output
```
1/10*(-A*e+B*d)*(a*e^2+c*d^2)^3/e^8/(e*x+d)^10-1/9*(a*e^2+c*d^2)^2*(-6*A*c
*d*e+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)^9+3/8*c*(a*e^2+c*d^2)*(-A*a*e^3-5*A*c*
d^2*e+3*B*a*d*e^2+7*B*c*d^3)/e^8/(e*x+d)^8+1/7*c*(4*A*c*d*e*(3*a*e^2+5*c*d
^2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))/e^8/(e*x+d)^7+1/6*c^2*(-3*A*a
*e^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)/e^8/(e*x+d)^6-3/5*c^2*(-2*A*c*d
*e+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)^5+1/4*c^3*(-A*e+7*B*d)/e^8/(e*x+d)^4-1/3
*B*c^3/e^8/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{11}}\,dx =$$
$$-\frac{3Ae(84a^3e^6+7a^2ce^4(d^2+10dex+45e^2x^2)+2ac^2e^2(d^4+10d^3ex+45d^2e^2x^2+120de^3x^3+210e^4x^4)}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^11,x]
```

output
```
-1/2520*(3*A*e*(84*a^3*e^6 + 7*a^2*c*e^4*(d^2 + 10*d*e*x + 45*e^2*x^2) + 2
*a*c^2*e^2*(d^4 + 10*d^3*e*x + 45*d^2*e^2*x^2 + 120*d*e^3*x^3 + 210*e^4*x^
4) + c^3*(d^6 + 10*d^5*e*x + 45*d^4*e^2*x^2 + 120*d^3*e^3*x^3 + 210*d^2*e^
4*x^4 + 252*d*e^5*x^5 + 210*e^6*x^6)) + B*(28*a^3*e^6*(d + 10*e*x) + 9*a^2
*c*e^4*(d^3 + 10*d^2*e*x + 45*d*e^2*x^2 + 120*e^3*x^3) + 6*a*c^2*e^2*(d^5
+ 10*d^4*e*x + 45*d^3*e^2*x^2 + 120*d^2*e^3*x^3 + 210*d*e^4*x^4 + 252*e^5*
x^5) + 7*c^3*(d^7 + 10*d^6*e*x + 45*d^5*e^2*x^2 + 120*d^4*e^3*x^3 + 210*d^
3*e^4*x^4 + 252*d^2*e^5*x^5 + 210*d*e^6*x^6 + 120*e^7*x^7)))/(e^8*(d + e*x
)^10)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3 (A+Bx)}{(d+ex)^{11}} \, dx$$

$$\Big\downarrow \ 652$$

$$\int \left(-\frac{c(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4)}{e^7(d+ex)^8}-\frac{3c^2(-aBe^2+2Acde-7Bcd^2)}{e^7(d+ex)^6}+\frac{c^2(3a\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{c(4Acde(3ae^2+5cd^2)-B(3a^2e^4+30acd^2e^2+35c^2d^4))}{7e^8(d+ex)^7}-\frac{3c^2(aBe^2-2Acde+7Bcd^2)}{5e^8(d+ex)^5}+\frac{c^2(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3)}{6e^8(d+ex)^6}-\frac{(ae^2+cd^2)^2(aBe^2-6Acde+7Bcd^2)}{9e^8(d+ex)^9}+\frac{(ae^2+cd^2)^3(Bd-Ae)}{10e^8(d+ex)^{10}}+\frac{3c(ae^2+cd^2)(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3)}{8e^8(d+ex)^8}+\frac{c^3(7Bd-Ae)}{4e^8(d+ex)^4}-\frac{Bc^3}{3e^8(d+ex)^3}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^11,x]
```

output
```
((B*d - A*e)*(c*d^2 + a*e^2)^3)/(10*e^8*(d + e*x)^10) - ((c*d^2 + a*e^2)^2
*(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2))/(9*e^8*(d + e*x)^9) + (3*c*(c*d^2 + a*
e^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(8*e^8*(d + e*x)^8
) + (c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3
*a^2*e^4)))/(7*e^8*(d + e*x)^7) + (c^2*(35*B*c*d^3 - 15*A*c*d^2*e + 15*a*B
*d*e^2 - 3*a*A*e^3))/(6*e^8*(d + e*x)^6) - (3*c^2*(7*B*c*d^2 - 2*A*c*d*e +
 a*B*e^2))/(5*e^8*(d + e*x)^5) + (c^3*(7*B*d - A*e))/(4*e^8*(d + e*x)^4) -
 (B*c^3)/(3*e^8*(d + e*x)^3)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.27

method	result
risch	$\frac{-\frac{B c^3 x^7}{3e}-\frac{c^3(3Ae+7Bd)x^6}{12e^2}-\frac{c^2\left(3Acde+6Ba e^2+7Bc d^2\right)x^5}{10e^3}-\frac{c^2\left(6Aa e^3+3Ac d^2 e+6Bad e^2+7Bc d^3\right)x^4}{12e^4}-\frac{c\left(6Aacd e^3+3A c^2 d^3 e+9B\right.}{2}}{}$
default	$-\frac{B c^3}{3e^8(ex+d)^3}-\frac{c^3(Ae-7Bd)}{4e^8(ex+d)^4}-\frac{-6A a^2 cd e^5-12Aa c^2 d^3 e^3-6A c^3 d^5 e+B a^3 e^6+9B a^2 c d^2 e^4+15Ba c^2 d^4 e^2+7B c^3 d^6}{9e^8(ex+d)^9}-$
norman	$\frac{-\frac{B c^3 x^7}{3e}-\frac{\left(3A c^3 e^3+7B c^3 d e^2\right)x^6}{12e^4}-\frac{\left(3A c^3 d e^3+6B e^4 a c^2+7B c^3 d^2 e^2\right)x^5}{10e^5}-\frac{\left(6Aa c^2 e^5+3A c^3 d^2 e^3+6Ba c^2 d e^4+7B c^3 d^3 e^2\right)x^4}{12e^6}-\frac{(6A}{}}{}$
gosper	$-\frac{840B x^7 c^3 e^7+630A x^6 c^3 e^7+1470B x^6 c^3 d e^6+756A x^5 c^3 d e^6+1512B x^5 a c^2 e^7+1764B x^5 c^3 d^2 e^5+1260A x^4 a c^2 e^7+630A}{}$
orering	$-\frac{840B x^7 c^3 e^7+630A x^6 c^3 e^7+1470B x^6 c^3 d e^6+756A x^5 c^3 d e^6+1512B x^5 a c^2 e^7+1764B x^5 c^3 d^2 e^5+1260A x^4 a c^2 e^7+630A}{}$
parallelrisch	$-\frac{840B c^3 x^7 e^9+630A c^3 e^9 x^6+1470B c^3 d e^8 x^6+756A c^3 d e^8 x^5+1512Ba c^2 e^9 x^5+1764B c^3 d^2 e^7 x^5+1260Aa c^2 e^9 x^4+630A c}{}$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^11,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*B*c^3*x^7/e-1/12*c^3/e^2*(3*A*e+7*B*d)*x^6-1/10*c^2/e^3*(3*A*c*d*e+6
*B*a*e^2+7*B*c*d^2)*x^5-1/12*c^2/e^4*(6*A*a*e^3+3*A*c*d^2*e+6*B*a*d*e^2+7*
B*c*d^3)*x^4-1/21*c/e^5*(6*A*a*c*d*e^3+3*A*c^2*d^3*e+9*B*a^2*e^4+6*B*a*c*d
^2*e^2+7*B*c^2*d^4)*x^3-1/56*c/e^6*(21*A*a^2*e^5+6*A*a*c*d^2*e^3+3*A*c^2*d
^4*e+9*B*a^2*d*e^4+6*B*a*c*d^3*e^2+7*B*c^2*d^5)*x^2-1/252/e^7*(21*A*a^2*c*
d*e^5+6*A*a*c^2*d^3*e^3+3*A*c^3*d^5*e+28*B*a^3*e^6+9*B*a^2*c*d^2*e^4+6*B*a
*c^2*d^4*e^2+7*B*c^3*d^6)*x-1/2520/e^8*(252*A*a^3*e^7+21*A*a^2*c*d^2*e^5+6
*A*a*c^2*d^4*e^3+3*A*c^3*d^6*e+28*B*a^3*d*e^6+9*B*a^2*c*d^3*e^4+6*B*a*c^2*
d^5*e^2+7*B*c^3*d^7))/(e*x+d)^10
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.67

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{11}}\,dx = -\frac{840\,Bc^3e^7x^7+7\,Bc^3d^7+3\,Ac^3d^6e+6\,Bac^2d^5e^2+6\,Aac^2d^4e^3+9\,Ba^2cd^3e^4+21\,Aa^2cd^2e^5+28\,Ba^3d}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^11,x, algorithm="fricas")
```

output
```
-1/2520*(840*B*c^3*e^7*x^7 + 7*B*c^3*d^7 + 3*A*c^3*d^6*e + 6*B*a*c^2*d^5*e
^2 + 6*A*a*c^2*d^4*e^3 + 9*B*a^2*c*d^3*e^4 + 21*A*a^2*c*d^2*e^5 + 28*B*a^3
*d*e^6 + 252*A*a^3*e^7 + 210*(7*B*c^3*d*e^6 + 3*A*c^3*e^7)*x^6 + 252*(7*B*
c^3*d^2*e^5 + 3*A*c^3*d*e^6 + 6*B*a*c^2*e^7)*x^5 + 210*(7*B*c^3*d^3*e^4 +
3*A*c^3*d^2*e^5 + 6*B*a*c^2*d*e^6 + 6*A*a*c^2*e^7)*x^4 + 120*(7*B*c^3*d^4*
e^3 + 3*A*c^3*d^3*e^4 + 6*B*a*c^2*d^2*e^5 + 6*A*a*c^2*d*e^6 + 9*B*a^2*c*e^
7)*x^3 + 45*(7*B*c^3*d^5*e^2 + 3*A*c^3*d^4*e^3 + 6*B*a*c^2*d^3*e^4 + 6*A*a
*c^2*d^2*e^5 + 9*B*a^2*c*d*e^6 + 21*A*a^2*c*e^7)*x^2 + 10*(7*B*c^3*d^6*e +
 3*A*c^3*d^5*e^2 + 6*B*a*c^2*d^4*e^3 + 6*A*a*c^2*d^3*e^4 + 9*B*a^2*c*d^2*e
^5 + 21*A*a^2*c*d*e^6 + 28*B*a^3*e^7)*x)/(e^18*x^10 + 10*d*e^17*x^9 + 45*d
^2*e^16*x^8 + 120*d^3*e^15*x^7 + 210*d^4*e^14*x^6 + 252*d^5*e^13*x^5 + 210
*d^6*e^12*x^4 + 120*d^7*e^11*x^3 + 45*d^8*e^10*x^2 + 10*d^9*e^9*x + d^10*e
^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{11}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**11,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.67

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{11}}\,dx = -\frac{840\,Bc^3e^7x^7+7\,Bc^3d^7+3\,Ac^3d^6e+6\,Bac^2d^5e^2+6\,Aac^2d^4e^3+9\,Ba^2cd^3e^4+21\,Aa^2cd^2e^5+28\,Ba^3d}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^11,x, algorithm="maxima")
```

output
```
-1/2520*(840*B*c^3*e^7*x^7 + 7*B*c^3*d^7 + 3*A*c^3*d^6*e + 6*B*a*c^2*d^5*e
^2 + 6*A*a*c^2*d^4*e^3 + 9*B*a^2*c*d^3*e^4 + 21*A*a^2*c*d^2*e^5 + 28*B*a^3
*d*e^6 + 252*A*a^3*e^7 + 210*(7*B*c^3*d*e^6 + 3*A*c^3*e^7)*x^6 + 252*(7*B*
c^3*d^2*e^5 + 3*A*c^3*d*e^6 + 6*B*a*c^2*e^7)*x^5 + 210*(7*B*c^3*d^3*e^4 +
3*A*c^3*d^2*e^5 + 6*B*a*c^2*d*e^6 + 6*A*a*c^2*e^7)*x^4 + 120*(7*B*c^3*d^4*
e^3 + 3*A*c^3*d^3*e^4 + 6*B*a*c^2*d^2*e^5 + 6*A*a*c^2*d*e^6 + 9*B*a^2*c*e^
7)*x^3 + 45*(7*B*c^3*d^5*e^2 + 3*A*c^3*d^4*e^3 + 6*B*a*c^2*d^3*e^4 + 6*A*a
*c^2*d^2*e^5 + 9*B*a^2*c*d*e^6 + 21*A*a^2*c*e^7)*x^2 + 10*(7*B*c^3*d^6*e +
 3*A*c^3*d^5*e^2 + 6*B*a*c^2*d^4*e^3 + 6*A*a*c^2*d^3*e^4 + 9*B*a^2*c*d^2*e
^5 + 21*A*a^2*c*d*e^6 + 28*B*a^3*e^7)*x)/(e^18*x^10 + 10*d*e^17*x^9 + 45*d
^2*e^16*x^8 + 120*d^3*e^15*x^7 + 210*d^4*e^14*x^6 + 252*d^5*e^13*x^5 + 210
*d^6*e^12*x^4 + 120*d^7*e^11*x^3 + 45*d^8*e^10*x^2 + 10*d^9*e^9*x + d^10*e
^8)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.46

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{11}}\,dx = -\frac{840\,Bc^3e^7x^7+1470\,Bc^3de^6x^6+630\,Ac^3e^7x^6+1764\,Bc^3d^2e^5x^5+756\,Ac^3de^6x^5+1512\,Bac^2e^7x^5+1}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^11,x, algorithm="giac")
```

output
```
-1/2520*(840*B*c^3*e^7*x^7 + 1470*B*c^3*d*e^6*x^6 + 630*A*c^3*e^7*x^6 + 17
64*B*c^3*d^2*e^5*x^5 + 756*A*c^3*d*e^6*x^5 + 1512*B*a*c^2*e^7*x^5 + 1470*B
*c^3*d^3*e^4*x^4 + 630*A*c^3*d^2*e^5*x^4 + 1260*B*a*c^2*d*e^6*x^4 + 1260*A
*a*c^2*e^7*x^4 + 840*B*c^3*d^4*e^3*x^3 + 360*A*c^3*d^3*e^4*x^3 + 720*B*a*c
^2*d^2*e^5*x^3 + 720*A*a*c^2*d*e^6*x^3 + 1080*B*a^2*c*e^7*x^3 + 315*B*c^3*
d^5*e^2*x^2 + 135*A*c^3*d^4*e^3*x^2 + 270*B*a*c^2*d^3*e^4*x^2 + 270*A*a*c^
2*d^2*e^5*x^2 + 405*B*a^2*c*d*e^6*x^2 + 945*A*a^2*c*e^7*x^2 + 70*B*c^3*d^6
*e*x + 30*A*c^3*d^5*e^2*x + 60*B*a*c^2*d^4*e^3*x + 60*A*a*c^2*d^3*e^4*x +
90*B*a^2*c*d^2*e^5*x + 210*A*a^2*c*d*e^6*x + 280*B*a^3*e^7*x + 7*B*c^3*d^7
 + 3*A*c^3*d^6*e + 6*B*a*c^2*d^5*e^2 + 6*A*a*c^2*d^4*e^3 + 9*B*a^2*c*d^3*e
^4 + 21*A*a^2*c*d^2*e^5 + 28*B*a^3*d*e^6 + 252*A*a^3*e^7)/((e*x + d)^10*e^
8)
```

Mupad [B] (verification not implemented)

Time = 6.24 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.57

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{11}}\,dx =$$

$$-\frac{\frac{28\,B\,a^3\,d\,e^6+252\,A\,a^3\,e^7+9\,B\,a^2\,c\,d^3\,e^4+21\,A\,a^2\,c\,d^2\,e^5+6\,B\,a\,c^2\,d^5\,e^2+6\,A\,a\,c^2\,d^4\,e^3+7\,B\,c^3\,d^7+3\,A\,c^3\,d^6\,e}{2520\,e^8}+\frac{x\,\left(28\,B\,a^3\,e^6+9\,B\,a^2\,c\,d^2\,e\right.}{}}{}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^11,x)
```

output
```
-((252*A*a^3*e^7 + 7*B*c^3*d^7 + 28*B*a^3*d*e^6 + 3*A*c^3*d^6*e + 6*A*a*c^
2*d^4*e^3 + 21*A*a^2*c*d^2*e^5 + 6*B*a*c^2*d^5*e^2 + 9*B*a^2*c*d^3*e^4)/(2
520*e^8) + (x*(28*B*a^3*e^6 + 7*B*c^3*d^6 + 3*A*c^3*d^5*e + 6*A*a*c^2*d^3*
e^3 + 6*B*a*c^2*d^4*e^2 + 9*B*a^2*c*d^2*e^4 + 21*A*a^2*c*d*e^5))/(252*e^7)
 + (c^2*x^4*(6*A*a*e^3 + 7*B*c*d^3 + 6*B*a*d*e^2 + 3*A*c*d^2*e))/(12*e^4)
+ (c*x^3*(9*B*a^2*e^4 + 7*B*c^2*d^4 + 3*A*c^2*d^3*e + 6*A*a*c*d*e^3 + 6*B*
a*c*d^2*e^2))/(21*e^5) + (c^3*x^6*(3*A*e + 7*B*d))/(12*e^2) + (c^2*x^5*(6*
B*a*e^2 + 7*B*c*d^2 + 3*A*c*d*e))/(10*e^3) + (c*x^2*(21*A*a^2*e^5 + 7*B*c^
2*d^5 + 9*B*a^2*d*e^4 + 3*A*c^2*d^4*e + 6*A*a*c*d^2*e^3 + 6*B*a*c*d^3*e^2)
)/(56*e^6) + (B*c^3*x^7)/(3*e))/(d^10 + e^10*x^10 + 10*d*e^9*x^9 + 45*d^8*
e^2*x^2 + 120*d^7*e^3*x^3 + 210*d^6*e^4*x^4 + 252*d^5*e^5*x^5 + 210*d^4*e^
6*x^6 + 120*d^3*e^7*x^7 + 45*d^2*e^8*x^8 + 10*d^9*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.76

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{11}}\,dx$$
$$= \frac{-840b\,c^3e^7x^7-630a\,c^3e^7x^6-1470b\,c^3d\,e^6x^6-1512ab\,c^2e^7x^5-756a\,c^3d\,e^6x^5-1764b\,c^3d^2e^5x^5-1260a}{}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^11,x)
```

output
```
( - 252*a**4*e**7 - 28*a**3*b*d*e**6 - 280*a**3*b*e**7*x - 21*a**3*c*d**2*
e**5 - 210*a**3*c*d*e**6*x - 945*a**3*c*e**7*x**2 - 9*a**2*b*c*d**3*e**4 -
 90*a**2*b*c*d**2*e**5*x - 405*a**2*b*c*d*e**6*x**2 - 1080*a**2*b*c*e**7*x
**3 - 6*a**2*c**2*d**4*e**3 - 60*a**2*c**2*d**3*e**4*x - 270*a**2*c**2*d**
2*e**5*x**2 - 720*a**2*c**2*d*e**6*x**3 - 1260*a**2*c**2*e**7*x**4 - 6*a*b
*c**2*d**5*e**2 - 60*a*b*c**2*d**4*e**3*x - 270*a*b*c**2*d**3*e**4*x**2 -
720*a*b*c**2*d**2*e**5*x**3 - 1260*a*b*c**2*d*e**6*x**4 - 1512*a*b*c**2*e*
*7*x**5 - 3*a*c**3*d**6*e - 30*a*c**3*d**5*e**2*x - 135*a*c**3*d**4*e**3*x
**2 - 360*a*c**3*d**3*e**4*x**3 - 630*a*c**3*d**2*e**5*x**4 - 756*a*c**3*d
*e**6*x**5 - 630*a*c**3*e**7*x**6 - 7*b*c**3*d**7 - 70*b*c**3*d**6*e*x - 3
15*b*c**3*d**5*e**2*x**2 - 840*b*c**3*d**4*e**3*x**3 - 1470*b*c**3*d**3*e*
*4*x**4 - 1764*b*c**3*d**2*e**5*x**5 - 1470*b*c**3*d*e**6*x**6 - 840*b*c**
3*e**7*x**7)/(2520*e**8*(d**10 + 10*d**9*e*x + 45*d**8*e**2*x**2 + 120*d**
7*e**3*x**3 + 210*d**6*e**4*x**4 + 252*d**5*e**5*x**5 + 210*d**4*e**6*x**6
 + 120*d**3*e**7*x**7 + 45*d**2*e**8*x**8 + 10*d*e**9*x**9 + e**10*x**10))
```

3.79 $\int \frac{(A+Bx)(d+ex)^4}{a+cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 240

$$\begin{aligned}\int \frac{(A+Bx)(d+ex)^4}{a+cx^2}\,dx &= \frac{e(4Bcd^3+6Acd^2e-4aBde^2-aAe^3)\,x}{c^2} \\ &\quad+\frac{e^2(6Bcd^2+4Acde-aBe^2)\,x^2}{2c^2}+\frac{e^3(4Bd+Ae)x^3}{3c}+\frac{Be^4x^4}{4c} \\ &\quad-\frac{(4aBde(cd^2-ae^2)-A(c^2d^4-6acd^2e^2+a^2e^4))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}} \\ &\quad+\frac{(4Acde(cd^2-ae^2)+B(c^2d^4-6acd^2e^2+a^2e^4))\log(a+cx^2)}{2c^3}\end{aligned}$$

output

```
e*(-A*a*e^3+6*A*c*d^2*e-4*B*a*d*e^2+4*B*c*d^3)*x/c^2+1/2*e^2*(4*A*c*d*e-B*
a*e^2+6*B*c*d^2)*x^2/c^2+1/3*e^3*(A*e+4*B*d)*x^3/c+1/4*B*e^4*x^4/c-(4*a*B*
d*e*(-a*e^2+c*d^2)-A*(a^2*e^4-6*a*c*d^2*e^2+c^2*d^4))*arctan(c^(1/2)*x/a^(
1/2))/a^(1/2)/c^(5/2)+1/2*(4*A*c*d*e*(-a*e^2+c*d^2)+B*(a^2*e^4-6*a*c*d^2*e
^2+c^2*d^4))*ln(c*x^2+a)/c^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)(d+ex)^4}{a+cx^2}\,dx$$
$$= \frac{(4aBde(-cd^2+ae^2)+A(c^2d^4-6acd^2e^2+a^2e^4))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}}$$
$$+ \frac{cex(-12aAe^3-6aBe^2(8d+ex)+4Ace(18d^2+6dex+e^2x^2)+Bc(48d^3+36d^2ex+16de^2x^2+3e^3x}{12c^3}$$

input
```
Integrate[((A + B*x)*(d + e*x)^4)/(a + c*x^2),x]
```

output
```
((4*a*B*d*e*(-(c*d^2) + a*e^2) + A*(c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4))*Ar
cTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + (c*e*x*(-12*a*A*e^3 - 6*a*B
*e^2*(8*d + e*x) + 4*A*c*e*(18*d^2 + 6*d*e*x + e^2*x^2) + B*c*(48*d^3 + 36
*d^2*e*x + 16*d*e^2*x^2 + 3*e^3*x^3)) + 6*(4*A*c*d*e*(c*d^2 - a*e^2) + B*(
c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4))*Log[a + c*x^2])/(12*c^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^4}{a+cx^2}\,dx$$
$$\Big\downarrow 657$$
$$\int \left(\frac{x\left(B\left(a^2e^4-6acd^2e^2+c^2d^4\right)+4Acde\left(cd^2-ae^2\right)\right)+A\left(a^2e^4-6acd^2e^2+c^2d^4\right)-4aBde\left(cd^2-ae^2\right)}{c^2\left(a+cx^2\right)}+\frac{e^2x\right.$$
$$\Big\downarrow 2009$$

$$-\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(4aBde\left(cd^2-ae^2\right)-A\left(a^2e^4-6acd^2e^2+c^2d^4\right)\right)}{\sqrt{a}c^{5/2}}+\frac{\log\left(a+cx^2\right)\left(B\left(a^2e^4-6acd^2e^2+c^2d^4\right)+4Acde\left(cd^2-ae^2\right)\right)}{2c^3}+\frac{e^2x^2\left(-aBe^2+4Acde+6Bcd^2\right)}{2c^2}+\frac{ex\left(-aAe^3-4aBde^2+6Acd^2e+4Bcd^3\right)}{c^2}+\frac{e^3x^3(Ae+4Bd)}{3c}+\frac{Be^4x^4}{4c}$$

input
```
Int[((A + B*x)*(d + e*x)^4)/(a + c*x^2),x]
```

output
```
(e*(4*B*c*d^3 + 6*A*c*d^2*e - 4*a*B*d*e^2 - a*A*e^3)*x)/c^2 + (e^2*(6*B*c*
d^2 + 4*A*c*d*e - a*B*e^2)*x^2)/(2*c^2) + (e^3*(4*B*d + A*e)*x^3)/(3*c) +
(B*e^4*x^4)/(4*c) - ((4*a*B*d*e*(c*d^2 - a*e^2) - A*(c^2*d^4 - 6*a*c*d^2*e
^2 + a^2*e^4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + ((4*A*c*d*
e*(c*d^2 - a*e^2) + B*(c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4))*Log[a + c*x^2])
/(2*c^3)
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.97

method	result
default	$-\frac{e\left(-\frac{1}{4}Bcx^4e^3-\frac{1}{3}Ax^3ce^3-\frac{4}{3}Bx^3cde^2-2Ax^2cde^2+\frac{1}{2}Bx^2ae^3-3Bx^2cd^2e+Aae^3x-6Acd^2ex+4Bade^2x-4Bcd^3x\right)}{c^2}+\frac{(-4.}{}$
risch	Expression too large to display

input
```
int((B*x+A)*(e*x+d)^4/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-e/c^2*(-1/4*B*c*x^4*e^3-1/3*A*x^3*c*e^3-4/3*B*x^3*c*d*e^2-2*A*x^2*c*d*e^2
+1/2*B*x^2*a*e^3-3*B*x^2*c*d^2*e+A*a*e^3*x-6*A*c*d^2*e*x+4*B*a*d*e^2*x-4*B
*c*d^3*x)+1/c^2*(1/2*(-4*A*a*c*d*e^3+4*A*c^2*d^3*e+B*a^2*e^4-6*B*a*c*d^2*e
^2+B*c^2*d^4)/c*ln(c*x^2+a)+(A*a^2*e^4-6*A*a*c*d^2*e^2+A*c^2*d^4+4*B*a^2*d
*e^3-4*B*a*c*d^3*e)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.29

$$\int \frac{(A+Bx)(d+ex)^4}{a+cx^2}\,dx$$
$$= \left[\frac{3\,Bac^2e^4x^4 + 4\,(4\,Bac^2de^3 + Aac^2e^4)x^3 + 6\,(6\,Bac^2d^2e^2 + 4\,Aac^2de^3 - Ba^2ce^4)x^2 - 6\,(Ac^2d^4 - 4\,Bac}{}\right.$$

input
```
integrate((B*x+A)*(e*x+d)^4/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/12*(3*B*a*c^2*e^4*x^4 + 4*(4*B*a*c^2*d*e^3 + A*a*c^2*e^4)*x^3 + 6*(6*B*
a*c^2*d^2*e^2 + 4*A*a*c^2*d*e^3 - B*a^2*c*e^4)*x^2 - 6*(A*c^2*d^4 - 4*B*a*
c*d^3*e - 6*A*a*c*d^2*e^2 + 4*B*a^2*d*e^3 + A*a^2*e^4)*sqrt(-a*c)*log((c*x
^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 12*(4*B*a*c^2*d^3*e + 6*A*a*c^2*d^
2*e^2 - 4*B*a^2*c*d*e^3 - A*a^2*c*e^4)*x + 6*(B*a*c^2*d^4 + 4*A*a*c^2*d^3*
e - 6*B*a^2*c*d^2*e^2 - 4*A*a^2*c*d*e^3 + B*a^3*e^4)*log(c*x^2 + a))/(a*c^
3), 1/12*(3*B*a*c^2*e^4*x^4 + 4*(4*B*a*c^2*d*e^3 + A*a*c^2*e^4)*x^3 + 6*(6
*B*a*c^2*d^2*e^2 + 4*A*a*c^2*d*e^3 - B*a^2*c*e^4)*x^2 + 12*(A*c^2*d^4 - 4*
B*a*c*d^3*e - 6*A*a*c*d^2*e^2 + 4*B*a^2*d*e^3 + A*a^2*e^4)*sqrt(a*c)*arcta
n(sqrt(a*c)*x/a) + 12*(4*B*a*c^2*d^3*e + 6*A*a*c^2*d^2*e^2 - 4*B*a^2*c*d*e
^3 - A*a^2*c*e^4)*x + 6*(B*a*c^2*d^4 + 4*A*a*c^2*d^3*e - 6*B*a^2*c*d^2*e^2
 - 4*A*a^2*c*d*e^3 + B*a^3*e^4)*log(c*x^2 + a))/(a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 908 vs. $2(236) = 472$.

Time = 1.65 (sec) , antiderivative size = 908, normalized size of antiderivative = 3.78

$$\int \frac{(A+Bx)(d+ex)^4}{a+cx^2}\,dx = \frac{Be^4x^4}{4c} + x^3\left(\frac{Ae^4}{3c}+\frac{4Bde^3}{3c}\right) + x^2 \cdot \left(\frac{2Ade^3}{c}-\frac{Bae^4}{2c^2}+\frac{3Bd^2e^2}{c}\right) + x\left(-\frac{Aae^4}{c^2}+\frac{6Ad^2e^2}{c}-\frac{4Bade^3}{c^2}+\frac{4Bd^3e}{c}\right) + \left(\frac{-4Aacde^3+4Ac^2d^3e+Ba^2e^4-6Bacd^2e^2+Bc^2d^4}{2c^3} - \frac{\sqrt{-ac^7}(Aa^2e^4-6Aacd^2e^2+Ac^2d^4+4Ba^2de^3-4Bacd^3e)}{2ac^6}\right)\log\left(x+\frac{4Aa^2cde^3-4Aac^2d^3e-Ba^3e\cdots}{\cdots}\right) + \left(\frac{-4Aacde^3+4Ac^2d^3e+Ba^2e^4-6Bacd^2e^2+Bc^2d^4}{2c^3} + \frac{\sqrt{-ac^7}(Aa^2e^4-6Aacd^2e^2+Ac^2d^4+4Ba^2de^3-4Bacd^3e)}{2ac^6}\right)\log\left(x+\frac{4Aa^2cde^3-4Aac^2d^3e-Ba^3e\cdots}{\cdots}\right)$$

input
```
integrate((B*x+A)*(e*x+d)**4/(c*x**2+a),x)
```

output
```
B*e**4*x**4/(4*c) + x**3*(A*e**4/(3*c) + 4*B*d*e**3/(3*c)) + x**2*(2*A*d*e
**3/c - B*a*e**4/(2*c**2) + 3*B*d**2*e**2/c) + x*(-A*a*e**4/c**2 + 6*A*d**
2*e**2/c - 4*B*a*d*e**3/c**2 + 4*B*d**3*e/c) + ((-4*A*a*c*d*e**3 + 4*A*c**
2*d**3*e + B*a**2*e**4 - 6*B*a*c*d**2*e**2 + B*c**2*d**4)/(2*c**3) - sqrt(
-a*c**7)*(A*a**2*e**4 - 6*A*a*c*d**2*e**2 + A*c**2*d**4 + 4*B*a**2*d*e**3
- 4*B*a*c*d**3*e)/(2*a*c**6))*log(x + (4*A*a**2*c*d*e**3 - 4*A*a*c**2*d**3
*e - B*a**3*e**4 + 6*B*a**2*c*d**2*e**2 - B*a*c**2*d**4 + 2*a*c**3*((-4*A*
a*c*d*e**3 + 4*A*c**2*d**3*e + B*a**2*e**4 - 6*B*a*c*d**2*e**2 + B*c**2*d*
*4)/(2*c**3) - sqrt(-a*c**7)*(A*a**2*e**4 - 6*A*a*c*d**2*e**2 + A*c**2*d**
4 + 4*B*a**2*d*e**3 - 4*B*a*c*d**3*e)/(2*a*c**6)))/(A*a**2*c*e**4 - 6*A*a*
c**2*d**2*e**2 + A*c**3*d**4 + 4*B*a**2*c*d*e**3 - 4*B*a*c**2*d**3*e)) + (
(-4*A*a*c*d*e**3 + 4*A*c**2*d**3*e + B*a**2*e**4 - 6*B*a*c*d**2*e**2 + B*c
**2*d**4)/(2*c**3) + sqrt(-a*c**7)*(A*a**2*e**4 - 6*A*a*c*d**2*e**2 + A*c*
*2*d**4 + 4*B*a**2*d*e**3 - 4*B*a*c*d**3*e)/(2*a*c**6))*log(x + (4*A*a**2*
c*d*e**3 - 4*A*a*c**2*d**3*e - B*a**3*e**4 + 6*B*a**2*c*d**2*e**2 - B*a*c*
*2*d**4 + 2*a*c**3*((-4*A*a*c*d*e**3 + 4*A*c**2*d**3*e + B*a**2*e**4 - 6*B
*a*c*d**2*e**2 + B*c**2*d**4)/(2*c**3) + sqrt(-a*c**7)*(A*a**2*e**4 - 6*A*
a*c*d**2*e**2 + A*c**2*d**4 + 4*B*a**2*d*e**3 - 4*B*a*c*d**3*e)/(2*a*c**6)
))/(A*a**2*c*e**4 - 6*A*a*c**2*d**2*e**2 + A*c**3*d**4 + 4*B*a**2*c*d*e**3
 - 4*B*a*c**2*d**3*e))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.96

$$
\begin{aligned}
&\int \frac{(A+Bx)(d+ex)^4}{a+cx^2}\,dx \\
&= \frac{\left(Ac^2d^4-4\,Bacd^3e-6\,Aacd^2e^2+4\,Ba^2de^3+Aa^2e^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2} \\
&\quad+\frac{3\,Bce^4x^4+4\,(4\,Bcde^3+Ace^4)x^3+6\,(6\,Bcd^2e^2+4\,Acde^3-Bae^4)x^2+12\,(4\,Bcd^3e+6\,Acd^2e^2-4\,B}{12\,c^2} \\
&\quad+\frac{\left(Bc^2d^4+4\,Ac^2d^3e-6\,Bacd^2e^2-4\,Aacde^3+Ba^2e^4\right)\log\left(cx^2+a\right)}{2\,c^3}
\end{aligned}
$$

input
```
integrate((B*x+A)*(e*x+d)^4/(c*x^2+a),x, algorithm="maxima")
```

output
```
(A*c^2*d^4 - 4*B*a*c*d^3*e - 6*A*a*c*d^2*e^2 + 4*B*a^2*d*e^3 + A*a^2*e^4)*
arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/12*(3*B*c*e^4*x^4 + 4*(4*B*c*d*e
^3 + A*c*e^4)*x^3 + 6*(6*B*c*d^2*e^2 + 4*A*c*d*e^3 - B*a*e^4)*x^2 + 12*(4*
B*c*d^3*e + 6*A*c*d^2*e^2 - 4*B*a*d*e^3 - A*a*e^4)*x)/c^2 + 1/2*(B*c^2*d^4
 + 4*A*c^2*d^3*e - 6*B*a*c*d^2*e^2 - 4*A*a*c*d*e^3 + B*a^2*e^4)*log(c*x^2
+ a)/c^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)(d+ex)^4}{a+cx^2}\,dx$$
$$= \frac{(Ac^2d^4-4\,Bacd^3e-6\,Aacd^2e^2+4\,Ba^2de^3+Aa^2e^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2} + \frac{(Bc^2d^4+4\,Ac^2d^3e-6\,Bacd^2e^2-4\,Aacde^3+Ba^2e^4)\log\left(cx^2+a\right)}{2\,c^3} + \frac{3\,Bc^3e^4x^4+16\,Bc^3de^3x^3+4\,Ac^3e^4x^3+36\,Bc^3d^2e^2x^2+24\,Ac^3de^3x^2-6\,Bac^2e^4x^2+48\,Bc^3d^3ex+72}{12\,c^4}$$

input
```
integrate((B*x+A)*(e*x+d)^4/(c*x^2+a),x, algorithm="giac")
```

output
```
(A*c^2*d^4 - 4*B*a*c*d^3*e - 6*A*a*c*d^2*e^2 + 4*B*a^2*d*e^3 + A*a^2*e^4)*
arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/2*(B*c^2*d^4 + 4*A*c^2*d^3*e - 6
*B*a*c*d^2*e^2 - 4*A*a*c*d*e^3 + B*a^2*e^4)*log(c*x^2 + a)/c^3 + 1/12*(3*B
*c^3*e^4*x^4 + 16*B*c^3*d*e^3*x^3 + 4*A*c^3*e^4*x^3 + 36*B*c^3*d^2*e^2*x^2
 + 24*A*c^3*d*e^3*x^2 - 6*B*a*c^2*e^4*x^2 + 48*B*c^3*d^3*e*x + 72*A*c^3*d^
2*e^2*x - 48*B*a*c^2*d*e^3*x - 12*A*a*c^2*e^4*x)/c^4
```

Mupad [B] (verification not implemented)

Time = 6.24 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)(d+ex)^4}{a+cx^2}\,dx = \frac{x^3\,\left(A\,e^4+4\,B\,d\,e^3\right)}{3\,c} - x\,\left(\frac{a\,\left(A\,e^4+4\,B\,d\,e^3\right)}{c^2}-\frac{2\,d^2\,e\,\left(3\,A\,e+2\,B\,d\right)}{c}\right)-x^2\,\left(\frac{B\,a\,e^4}{2\,c^2}-\frac{d\,e^2\,\left(2\,A\,e+3\,B\,d\right)}{c}\right) + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(4\,B\,a^2\,d\,e^3+A\,a^2\,e^4-4\,B\,a\,c\,d^3\,e-6\,A\,a\,c\,d^2\,e^2+A\,c^2\,d^4\right)}{\sqrt{a}\,c^{5/2}} + \frac{\ln\left(c\,x^2+a\right)\,\left(4\,B\,a^3\,c^3\,e^4-24\,B\,a^2\,c^4\,d^2\,e^2-16\,A\,a^2\,c^4\,d\,e^3+4\,B\,a\,c^5\,d^4+16\,A\,a\,c^5\,d^3\,e\right)}{8\,a\,c^6} + \frac{B\,e^4\,x^4}{4\,c}$$

input
```
int(((A + B*x)*(d + e*x)^4)/(a + c*x^2),x)
```

output
```
(x^3*(A*e^4 + 4*B*d*e^3))/(3*c) - x*((a*(A*e^4 + 4*B*d*e^3))/c^2 - (2*d^2*
e*(3*A*e + 2*B*d))/c) - x^2*((B*a*e^4)/(2*c^2) - (d*e^2*(2*A*e + 3*B*d))/c
) + (atan((c^(1/2)*x)/a^(1/2))*(A*a^2*e^4 + A*c^2*d^4 + 4*B*a^2*d*e^3 - 4*
B*a*c*d^3*e - 6*A*a*c*d^2*e^2))/(a^(1/2)*c^(5/2)) + (log(a + c*x^2)*(4*B*a
*c^5*d^4 + 4*B*a^3*c^3*e^4 - 16*A*a^2*c^4*d*e^3 - 24*B*a^2*c^4*d^2*e^2 + 1
6*A*a*c^5*d^3*e))/(8*a*c^6) + (B*e^4*x^4)/(4*c)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.40

$$\int \frac{(A+Bx)(d+ex)^4}{a+cx^2}\,dx$$

$$= \frac{12\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2e^4+48\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)abd\,e^3-72\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)ac\,d^2e^2-48\sqrt{c}\,\sqrt{a}\,}{}$$

input
```
int((B*x+A)*(e*x+d)^4/(c*x^2+a),x)
```

output

```
(12*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*e**4 + 48*sqrt(c)*s
qrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*b*d*e**3 - 72*sqrt(c)*sqrt(a)*atan(
(c*x)/(sqrt(c)*sqrt(a)))*a*c*d**2*e**2 - 48*sqrt(c)*sqrt(a)*atan((c*x)/(sq
rt(c)*sqrt(a)))*b*c*d**3*e + 12*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a
)))*c**2*d**4 + 6*log(a + c*x**2)*a**2*b*e**4 - 24*log(a + c*x**2)*a**2*c*
d*e**3 - 36*log(a + c*x**2)*a*b*c*d**2*e**2 + 24*log(a + c*x**2)*a*c**2*d*
*3*e + 6*log(a + c*x**2)*b*c**2*d**4 - 12*a**2*c*e**4*x - 48*a*b*c*d*e**3*
x - 6*a*b*c*e**4*x**2 + 72*a*c**2*d**2*e**2*x + 24*a*c**2*d*e**3*x**2 + 4*
a*c**2*e**4*x**3 + 48*b*c**2*d**3*e*x + 36*b*c**2*d**2*e**2*x**2 + 16*b*c*
*2*d*e**3*x**3 + 3*b*c**2*e**4*x**4)/(12*c**3)
```

3.80 $\int \frac{(A+Bx)(d+ex)^3}{a+cx^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 167

$$\int \frac{(A+Bx)(d+ex)^3}{a+cx^2} dx = \frac{e(3Bcd^2+3Acde-aBe^2)\,x}{c^2} + \frac{e^2(3Bd+Ae)x^2}{2c} + \frac{Be^3x^3}{3c} + \frac{(Acd(cd^2-3ae^2)-aBe(3cd^2-ae^2))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}} + \frac{(Bcd^3+3Acd^2e-3aBde^2-aAe^3)\log(a+cx^2)}{2c^2}$$

output
```
e*(3*A*c*d*e-B*a*e^2+3*B*c*d^2)*x/c^2+1/2*e^2*(A*e+3*B*d)*x^2/c+1/3*B*e^3*
x^3/c+(A*c*d*(-3*a*e^2+c*d^2)-a*B*e*(-a*e^2+3*c*d^2))*arctan(c^(1/2)*x/a^(
1/2))/a^(1/2)/c^(5/2)+1/2*(-A*a*e^3+3*A*c*d^2*e-3*B*a*d*e^2+B*c*d^3)*ln(c*
x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)(d+ex)^3}{a+cx^2}\,dx = \frac{(Acd(cd^2-3ae^2)+aBe(-3cd^2+ae^2))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}} + \frac{ex(-6aBe^2+3Ace(6d+ex)+Bc(18d^2+9dex+2e^2x^2))+3(Bcd^3+3Acd^2e-3aBde^2-aAe^3)\log}{6c^2}$$

input
```
Integrate[((A + B*x)*(d + e*x)^3)/(a + c*x^2),x]
```

output
```
((A*c*d*(c*d^2 - 3*a*e^2) + a*B*e*(-3*c*d^2 + a*e^2))*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/(Sqrt[a]*c^(5/2)) + (e*x*(-6*a*B*e^2 + 3*A*c*e*(6*d + e*x) + B*c*
(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + 3*(B*c*d^3 + 3*A*c*d^2*e - 3*a*B*d*e^2 -
 a*A*e^3)*Log[a + c*x^2])/(6*c^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^3}{a+cx^2}\,dx$$

$$\Big\downarrow \; 657$$

$$\int \left(\frac{e(-aBe^2+3Acde+3Bcd^2)}{c^2} + \frac{cx(-aAe^3-3aBde^2+3Acd^2e+Bcd^3)+Acd(cd^2-3ae^2)-aBe(3cd^2-}{c^2(a+cx^2)}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd(cd^2-3ae^2)-aBe(3cd^2-ae^2)\right)}{\sqrt{a}c^{5/2}}+\frac{ex(-aBe^2+3Acde+3Bcd^2)}{c^2}+\frac{\log\left(a+cx^2\right)\left(-aAe^3-3aBde^2+3Acd^2e+Bcd^3\right)}{2c^2}+\frac{e^2x^2(Ae+3Bd)}{2c}+\frac{Be^3x^3}{3c}$$

input
```
Int[((A + B*x)*(d + e*x)^3)/(a + c*x^2),x]
```

output
```
(e*(3*B*c*d^2 + 3*A*c*d*e - a*B*e^2)*x)/c^2 + (e^2*(3*B*d + A*e)*x^2)/(2*c
) + (B*e^3*x^3)/(3*c) + ((A*c*d*(c*d^2 - 3*a*e^2) - a*B*e*(3*c*d^2 - a*e^2
))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + ((B*c*d^3 + 3*A*c*d^2*
e - 3*a*B*d*e^2 - a*A*e^3)*Log[a + c*x^2])/(2*c^2)
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.98

method	result
default	$\frac{e\left(\frac{1}{3}Bc\,x^3e^2+\frac{1}{2}Ac\,e^2x^2+\frac{3}{2}Bcde\,x^2+3Acdex-Ba\,e^2x+3Bc\,d^2x\right)}{c^2}+\frac{\frac{\left(-Aac\,e^3+3A\,c^2d^2e-3Bacd\,e^2+B\,c^2d^3\right)\ln\left(c\,x^2+a\right)}{2c}+\frac{(-3Aacd\,e^{}}{}}{c^2}$
risch	Expression too large to display

input
```
int((B*x+A)*(e*x+d)^3/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
e/c^2*(1/3*B*c*x^3*e^2+1/2*A*c*e^2*x^2+3/2*B*c*d*e*x^2+3*A*c*d*e*x-B*a*e^2
*x+3*B*c*d^2*x)+1/c^2*(1/2*(-A*a*c*e^3+3*A*c^2*d^2*e-3*B*a*c*d*e^2+B*c^2*d
^3)/c*ln(c*x^2+a)+(-3*A*a*c*d*e^2+A*c^2*d^3+B*a^2*e^3-3*B*a*c*d^2*e)/(a*c)
^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.38

$$\int \frac{(A+Bx)(d+ex)^3}{a+cx^2}\,dx$$
$$= \left[\frac{2\,Bac^2e^3x^3 + 3\,(3\,Bac^2de^2 + Aac^2e^3)x^2 - 3\,(Ac^2d^3 - 3\,Bacd^2e - 3\,Aacde^2 + Ba^2e^3)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{}}{cx}\right.}{}\right.$$

input
```
integrate((B*x+A)*(e*x+d)^3/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(2*B*a*c^2*e^3*x^3 + 3*(3*B*a*c^2*d*e^2 + A*a*c^2*e^3)*x^2 - 3*(A*c^2
*d^3 - 3*B*a*c*d^2*e - 3*A*a*c*d*e^2 + B*a^2*e^3)*sqrt(-a*c)*log((c*x^2 -
2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(3*B*a*c^2*d^2*e + 3*A*a*c^2*d*e^2 -
B*a^2*c*e^3)*x + 3*(B*a*c^2*d^3 + 3*A*a*c^2*d^2*e - 3*B*a^2*c*d*e^2 - A*a^
2*c*e^3)*log(c*x^2 + a))/(a*c^3), 1/6*(2*B*a*c^2*e^3*x^3 + 3*(3*B*a*c^2*d*
e^2 + A*a*c^2*e^3)*x^2 + 6*(A*c^2*d^3 - 3*B*a*c*d^2*e - 3*A*a*c*d*e^2 + B*
a^2*e^3)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 6*(3*B*a*c^2*d^2*e + 3*A*a*c^2*
d*e^2 - B*a^2*c*e^3)*x + 3*(B*a*c^2*d^3 + 3*A*a*c^2*d^2*e - 3*B*a^2*c*d*e^
2 - A*a^2*c*e^3)*log(c*x^2 + a))/(a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(170) = 340$.

Time = 1.08 (sec) , antiderivative size = 641, normalized size of antiderivative = 3.84

$$\int \frac{(A+Bx)(d+ex)^3}{a+cx^2}\,dx = \frac{Be^3x^3}{3c} + x^2\left(\frac{Ae^3}{2c} + \frac{3Bde^2}{2c}\right) + x\left(\frac{3Ade^2}{c} - \frac{Bae^3}{c^2} + \frac{3Bd^2e}{c}\right) + \left(-\frac{Aae^3 - 3Acd^2e + 3Bade^2 - Bcd^3}{2c^2} - \frac{\sqrt{-ac^5}(-3Aacde^2 + Ac^2d^3 + Ba^2e^3 - 3Bacd^2e)}{2ac^5}\right)\log\left(x + \frac{Aa^2e^3 - 3Aacd^2e + 3Ba^2de^2 - Bacd^3 -}{-3A}\right.$$
$$+ \left(-\frac{Aae^3 - 3Acd^2e + 3Bade^2 - Bcd^3}{2c^2} + \frac{\sqrt{-ac^5}(-3Aacde^2 + Ac^2d^3 + Ba^2e^3 - 3Bacd^2e)}{2ac^5}\right)\log\left(x + \frac{Aa^2e^3 - 3Aacd^2e + 3Ba^2de^2 - Bacd^3 +}{-3A}\right.$$

input
```
integrate((B*x+A)*(e*x+d)**3/(c*x**2+a),x)
```

output
```
B*e**3*x**3/(3*c) + x**2*(A*e**3/(2*c) + 3*B*d*e**2/(2*c)) + x*(3*A*d*e**2
/c - B*a*e**3/c**2 + 3*B*d**2*e/c) + (-(A*a*e**3 - 3*A*c*d**2*e + 3*B*a*d*
e**2 - B*c*d**3)/(2*c**2) - sqrt(-a*c**5)*(-3*A*a*c*d*e**2 + A*c**2*d**3 +
 B*a**2*e**3 - 3*B*a*c*d**2*e)/(2*a*c**5))*log(x + (A*a**2*e**3 - 3*A*a*c*
d**2*e + 3*B*a**2*d*e**2 - B*a*c*d**3 + 2*a*c**2*(-(A*a*e**3 - 3*A*c*d**2*
e + 3*B*a*d*e**2 - B*c*d**3)/(2*c**2) - sqrt(-a*c**5)*(-3*A*a*c*d*e**2 + A
*c**2*d**3 + B*a**2*e**3 - 3*B*a*c*d**2*e)/(2*a*c**5)))/(-3*A*a*c*d*e**2 +
 A*c**2*d**3 + B*a**2*e**3 - 3*B*a*c*d**2*e)) + (-(A*a*e**3 - 3*A*c*d**2*e
 + 3*B*a*d*e**2 - B*c*d**3)/(2*c**2) + sqrt(-a*c**5)*(-3*A*a*c*d*e**2 + A*
c**2*d**3 + B*a**2*e**3 - 3*B*a*c*d**2*e)/(2*a*c**5))*log(x + (A*a**2*e**3
 - 3*A*a*c*d**2*e + 3*B*a**2*d*e**2 - B*a*c*d**3 + 2*a*c**2*(-(A*a*e**3 -
3*A*c*d**2*e + 3*B*a*d*e**2 - B*c*d**3)/(2*c**2) + sqrt(-a*c**5)*(-3*A*a*c
*d*e**2 + A*c**2*d**3 + B*a**2*e**3 - 3*B*a*c*d**2*e)/(2*a*c**5)))/(-3*A*a
*c*d*e**2 + A*c**2*d**3 + B*a**2*e**3 - 3*B*a*c*d**2*e))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)(d+ex)^3}{a+cx^2}\,dx = \frac{(Bcd^3+3\,Acd^2e-3\,Bade^2-Aae^3)\log\left(cx^2+a\right)}{2\,c^2} + \frac{(Ac^2d^3-3\,Bacd^2e-3\,Aacde^2+Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2} + \frac{2\,Bce^3x^3+3\,(3\,Bcde^2+Ace^3)x^2+6\,(3\,Bcd^2e+3\,Acde^2-Bae^3)x}{6\,c^2}$$

input
```
integrate((B*x+A)*(e*x+d)^3/(c*x^2+a),x, algorithm="maxima")
```

output
```
1/2*(B*c*d^3 + 3*A*c*d^2*e - 3*B*a*d*e^2 - A*a*e^3)*log(c*x^2 + a)/c^2 + (
A*c^2*d^3 - 3*B*a*c*d^2*e - 3*A*a*c*d*e^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c
))/(sqrt(a*c)*c^2) + 1/6*(2*B*c*e^3*x^3 + 3*(3*B*c*d*e^2 + A*c*e^3)*x^2 +
6*(3*B*c*d^2*e + 3*A*c*d*e^2 - B*a*e^3)*x)/c^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)(d+ex)^3}{a+cx^2}\,dx = \frac{(Bcd^3+3\,Acd^2e-3\,Bade^2-Aae^3)\log\left(cx^2+a\right)}{2\,c^2} + \frac{(Ac^2d^3-3\,Bacd^2e-3\,Aacde^2+Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2} + \frac{2\,Bc^2e^3x^3+9\,Bc^2de^2x^2+3\,Ac^2e^3x^2+18\,Bc^2d^2ex+18\,Ac^2de^2x-6\,Bace^3x}{6\,c^3}$$

input
```
integrate((B*x+A)*(e*x+d)^3/(c*x^2+a),x, algorithm="giac")
```

output
```
1/2*(B*c*d^3 + 3*A*c*d^2*e - 3*B*a*d*e^2 - A*a*e^3)*log(c*x^2 + a)/c^2 + (
A*c^2*d^3 - 3*B*a*c*d^2*e - 3*A*a*c*d*e^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c
))/(sqrt(a*c)*c^2) + 1/6*(2*B*c^2*e^3*x^3 + 9*B*c^2*d*e^2*x^2 + 3*A*c^2*e^
3*x^2 + 18*B*c^2*d^2*e*x + 18*A*c^2*d*e^2*x - 6*B*a*c*e^3*x)/c^3
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)(d+ex)^3}{a+cx^2}\,dx = x\left(\frac{3\,d\,e\,(A\,e+B\,d)}{c}-\frac{B\,a\,e^3}{c^2}\right)+\frac{x^2\,(A\,e^3+3\,B\,d\,e^2)}{2\,c} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(B\,a^2\,e^3-3\,B\,a\,c\,d^2\,e-3\,A\,a\,c\,d\,e^2+A\,c^2\,d^3)}{\sqrt{a}\,c^{5/2}} + \frac{\ln\left(c\,x^2+a\right)\,(-12\,B\,a^2\,c^3\,d\,e^2-4\,A\,a^2\,c^3\,e^3+4\,B\,a\,c^4\,d^3+12\,A\,a\,c^4\,d^2\,e)}{8\,a\,c^5} + \frac{B\,e^3\,x^3}{3\,c}$$

input
```
int(((A + B*x)*(d + e*x)^3)/(a + c*x^2),x)
```

output
```
x*((3*d*e*(A*e + B*d))/c - (B*a*e^3)/c^2) + (x^2*(A*e^3 + 3*B*d*e^2))/(2*c
) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d^3 + B*a^2*e^3 - 3*A*a*c*d*e^2 - 3*
B*a*c*d^2*e))/(a^(1/2)*c^(5/2)) + (log(a + c*x^2)*(4*B*a*c^4*d^3 - 4*A*a^2
*c^3*e^3 - 12*B*a^2*c^3*d*e^2 + 12*A*a*c^4*d^2*e))/(8*a*c^5) + (B*e^3*x^3)
/(3*c)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx)(d+ex)^3}{a+cx^2}\,dx$$
$$= \frac{6\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)ab\,e^3 - 18\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)acd\,e^2 - 18\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)bc\,d^2e + 6\sqrt{c}\,\sqrt{a}\,ata}{}$$

input
```
int((B*x+A)*(e*x+d)^3/(c*x^2+a),x)
```

output
```
(6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*b*e**3 - 18*sqrt(c)*sqr
t(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c*d*e**2 - 18*sqrt(c)*sqrt(a)*atan((c
*x)/(sqrt(c)*sqrt(a)))*b*c*d**2*e + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*
sqrt(a)))*c**2*d**3 - 3*log(a + c*x**2)*a**2*c*e**3 - 9*log(a + c*x**2)*a*
b*c*d*e**2 + 9*log(a + c*x**2)*a*c**2*d**2*e + 3*log(a + c*x**2)*b*c**2*d*
*3 - 6*a*b*c*e**3*x + 18*a*c**2*d*e**2*x + 3*a*c**2*e**3*x**2 + 18*b*c**2*
d**2*e*x + 9*b*c**2*d*e**2*x**2 + 2*b*c**2*e**3*x**3)/(6*c**3)
```

3.81 $\int \frac{(A+Bx)(d+ex)^2}{a+cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 108

$$\int \frac{(A+Bx)(d+ex)^2}{a+cx^2}\,dx = \frac{e(2Bd+Ae)x}{c}+\frac{Be^2x^2}{2c}+\frac{(Acd^2-2aBde-aAe^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{(Bcd^2+2Acde-aBe^2)\log(a+cx^2)}{2c^2}$$

output
```
e*(A*e+2*B*d)*x/c+1/2*B*e^2*x^2/c+(-A*a*e^2+A*c*d^2-2*B*a*d*e)*arctan(c^(1
/2)*x/a^(1/2))/a^(1/2)/c^(3/2)+1/2*(2*A*c*d*e-B*a*e^2+B*c*d^2)*ln(c*x^2+a)
/c^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)(d+ex)^2}{a+cx^2}\,dx = \frac{cex(4Bd+2Ae+Bex)-\frac{2\sqrt{c}(-Acd^2+2aBde+aAe^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}}+(Bcd^2+2Acde-aBe^2)\log(a+cx^2)}{2c^2}$$

input
```
Integrate[((A + B*x)*(d + e*x)^2)/(a + c*x^2),x]
```

output
```
(c*e*x*(4*B*d + 2*A*e + B*e*x) - (2*Sqrt[c]*(-(A*c*d^2) + 2*a*B*d*e + a*A*
e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] + (B*c*d^2 + 2*A*c*d*e - a*B*e^2
)*Log[a + c*x^2])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^2}{a+cx^2} \, dx$$

$$\Big\downarrow \text{657}$$

$$\int \left(\frac{x\left(-aBe^2+2Acde+Bcd^2\right)-aAe^2-2aBde+Acd^2}{c\left(a+cx^2\right)}+\frac{e(Ae+2Bd)}{c}+\frac{Be^2x}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(-aAe^2-2aBde+Acd^2\right)}{\sqrt{a}c^{3/2}}+\frac{\log\left(a+cx^2\right)\left(-aBe^2+2Acde+Bcd^2\right)}{2c^2}+\frac{ex(Ae+2Bd)}{c}+\frac{Be^2x^2}{2c}$$

input
```
Int[((A + B*x)*(d + e*x)^2)/(a + c*x^2),x]
```

output
```
(e*(2*B*d + A*e)*x)/c + (B*e^2*x^2)/(2*c) + ((A*c*d^2 - 2*a*B*d*e - a*A*e^
2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + ((B*c*d^2 + 2*A*c*d*e
- a*B*e^2)*Log[a + c*x^2])/(2*c^2)
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

method	result
default	$\frac{e\left(\frac{1}{2}Be\,x^2+Aex+2Bdx\right)}{c}+\frac{\frac{\left(2Acde-Ba\,e^2+Bc\,d^2\right)\ln\left(c\,x^2+a\right)}{2c}+\frac{\left(-Aa\,e^2+Ac\,d^2-2Bade\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{c}$
risch	$\frac{B\,e^2x^2}{2c}+\frac{e^2Ax}{c}+\frac{2eBdx}{c}+\frac{\ln\left(-A\,a^2e^2+A\,d^2ac-2B\,a^2de-\sqrt{-ac(Aa\,e^2-Ac\,d^2+2Bade)^2}\,x\right)Ade}{c}-\frac{a\ln\left(-A\,a^2e^2+A\,d^2c\right.}{}$

input
```
int((B*x+A)*(e*x+d)^2/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
e/c*(1/2*B*e*x^2+A*e*x+2*B*d*x)+1/c*(1/2*(2*A*c*d*e-B*a*e^2+B*c*d^2)/c*ln(
c*x^2+a)+(-A*a*e^2+A*c*d^2-2*B*a*d*e)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.18

$$\int\frac{(A+Bx)(d+ex)^2}{a+cx^2}\,dx$$
$$=\left[\frac{Bace^2x^2+(Acd^2-2\,Bade-Aae^2)\sqrt{-ac}\log\left(\frac{cx^2+2\,\sqrt{-ac}x-a}{cx^2+a}\right)+2\,(2\,Bacde+Aace^2)x+(Bacd^2+2\,}{2\,ac^2}\right.$$

input
```
integrate((B*x+A)*(e*x+d)^2/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(B*a*c*e^2*x^2 + (A*c*d^2 - 2*B*a*d*e - A*a*e^2)*sqrt(-a*c)*log((c*x^
2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(2*B*a*c*d*e + A*a*c*e^2)*x + (B*
a*c*d^2 + 2*A*a*c*d*e - B*a^2*e^2)*log(c*x^2 + a))/(a*c^2), 1/2*(B*a*c*e^2
*x^2 + 2*(A*c*d^2 - 2*B*a*d*e - A*a*e^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) +
 2*(2*B*a*c*d*e + A*a*c*e^2)*x + (B*a*c*d^2 + 2*A*a*c*d*e - B*a^2*e^2)*log
(c*x^2 + a))/(a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(105) = 210$.

Time = 0.73 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.94

$$\int \frac{(A+Bx)(d+ex)^2}{a+cx^2}\,dx = \frac{Be^2x^2}{2c} + x\left(\frac{Ae^2}{c} + \frac{2Bde}{c}\right) + \left(-\frac{-2Acde+Bae^2-Bcd^2}{2c^2} - \frac{\sqrt{-ac^5}(Aae^2-Acd^2+2Bade)}{2ac^4}\right)\log\left(x + \frac{2Aacde - Ba^2e^2 + Bacd^2 - 2ac^2\left(-\frac{-2Acde+Bae^2-Bcd^2}{2c^2} - \cdots\right.}{Aace^2 - Ac^2d^2 + 2Bacde}\right. + \left(-\frac{-2Acde+Bae^2-Bcd^2}{2c^2} + \frac{\sqrt{-ac^5}(Aae^2-Acd^2+2Bade)}{2ac^4}\right)\log\left(x + \frac{2Aacde - Ba^2e^2 + Bacd^2 - 2ac^2\left(-\frac{-2Acde+Bae^2-Bcd^2}{2c^2} + \cdots\right.}{Aace^2 - Ac^2d^2 + 2Bacde}\right.$$

input
```
integrate((B*x+A)*(e*x+d)**2/(c*x**2+a),x)
```

output
```
B*e**2*x**2/(2*c) + x*(A*e**2/c + 2*B*d*e/c) + (-(-2*A*c*d*e + B*a*e**2 -
B*c*d**2)/(2*c**2) - sqrt(-a*c**5)*(A*a*e**2 - A*c*d**2 + 2*B*a*d*e)/(2*a*
c**4))*log(x + (2*A*a*c*d*e - B*a**2*e**2 + B*a*c*d**2 - 2*a*c**2*(-(-2*A*
c*d*e + B*a*e**2 - B*c*d**2)/(2*c**2) - sqrt(-a*c**5)*(A*a*e**2 - A*c*d**2
 + 2*B*a*d*e)/(2*a*c**4)))/(A*a*c*e**2 - A*c**2*d**2 + 2*B*a*c*d*e)) + (-(
-2*A*c*d*e + B*a*e**2 - B*c*d**2)/(2*c**2) + sqrt(-a*c**5)*(A*a*e**2 - A*c
*d**2 + 2*B*a*d*e)/(2*a*c**4))*log(x + (2*A*a*c*d*e - B*a**2*e**2 + B*a*c*
d**2 - 2*a*c**2*(-(-2*A*c*d*e + B*a*e**2 - B*c*d**2)/(2*c**2) + sqrt(-a*c*
*5)*(A*a*e**2 - A*c*d**2 + 2*B*a*d*e)/(2*a*c**4)))/(A*a*c*e**2 - A*c**2*d*
*2 + 2*B*a*c*d*e))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)(d+ex)^2}{a+cx^2}\,dx = \frac{(Acd^2-2\,Bade-Aae^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{Be^2x^2+2\,(2\,Bde+Ae^2)x}{2\,c} + \frac{(Bcd^2+2\,Acde-Bae^2)\log\left(cx^2+a\right)}{2\,c^2}$$

input
```
integrate((B*x+A)*(e*x+d)^2/(c*x^2+a),x, algorithm="maxima")
```

output
```
(A*c*d^2 - 2*B*a*d*e - A*a*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*
(B*e^2*x^2 + 2*(2*B*d*e + A*e^2)*x)/c + 1/2*(B*c*d^2 + 2*A*c*d*e - B*a*e^2
)*log(c*x^2 + a)/c^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)(d+ex)^2}{a+cx^2}\,dx = \frac{(Acd^2-2\,Bade-Aae^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{(Bcd^2+2\,Acde-Bae^2)\log\left(cx^2+a\right)}{2\,c^2} + \frac{Bce^2x^2+4\,Bcdex+2\,Ace^2x}{2\,c^2}$$

input
```
integrate((B*x+A)*(e*x+d)^2/(c*x^2+a),x, algorithm="giac")
```

output
```
(A*c*d^2 - 2*B*a*d*e - A*a*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*
(B*c*d^2 + 2*A*c*d*e - B*a*e^2)*log(c*x^2 + a)/c^2 + 1/2*(B*c*e^2*x^2 + 4*
B*c*d*e*x + 2*A*c*e^2*x)/c^2
```

Mupad [B] (verification not implemented)

Time = 6.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)(d+ex)^2}{a+cx^2}\,dx = \frac{x\,(A\,e^2+2\,B\,d\,e)}{c} + \frac{\ln\left(c\,x^2+a\right)\,\left(-4\,B\,a^2\,c^2\,e^2+4\,B\,a\,c^3\,d^2+8\,A\,a\,c^3\,d\,e\right)}{8\,a\,c^4} - \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(-A\,c\,d^2+2\,B\,a\,d\,e+A\,a\,e^2\right)}{\sqrt{a}\,c^{3/2}} + \frac{B\,e^2\,x^2}{2\,c}$$

input
```
int(((A + B*x)*(d + e*x)^2)/(a + c*x^2),x)
```

output
```
(x*(A*e^2 + 2*B*d*e))/c + (log(a + c*x^2)*(4*B*a*c^3*d^2 - 4*B*a^2*c^2*e^2
 + 8*A*a*c^3*d*e))/(8*a*c^4) - (atan((c^(1/2)*x)/a^(1/2))*(A*a*e^2 - A*c*d
^2 + 2*B*a*d*e))/(a^(1/2)*c^(3/2)) + (B*e^2*x^2)/(2*c)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx)(d+ex)^2}{a+cx^2}\,dx = \frac{-2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a\,e^2-4\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)bde+2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)c\,d^2-\log(c\,x^2+a)\,ab\,e^2}{2c^2}$$

input
```
int((B*x+A)*(e*x+d)^2/(c*x^2+a),x)
```

output
```
( - 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*e**2 - 4*sqrt(c)*sqr
t(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*b*d*e + 2*sqrt(c)*sqrt(a)*atan((c*x)/(s
qrt(c)*sqrt(a)))*c*d**2 - log(a + c*x**2)*a*b*e**2 + 2*log(a + c*x**2)*a*c
*d*e + log(a + c*x**2)*b*c*d**2 + 2*a*c*e**2*x + 4*b*c*d*e*x + b*c*e**2*x*
*2)/(2*c**2)
```

3.82 $\int \frac{(A+Bx)(d+ex)}{a+cx^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{(A+Bx)(d+ex)}{a+cx^2} dx = \frac{Bex}{c} + \frac{(Acd-aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}} + \frac{(Bd+Ae)\log(a+cx^2)}{2c}$$

output
```
B*e*x/c+(A*c*d-B*a*e)*arctan(c^(1/2)*x/a^(1/2))/a^(1/2)/c^(3/2)+1/2*(A*e+B
*d)*ln(c*x^2+a)/c
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)(d+ex)}{a+cx^2} dx = \frac{Bex}{c} - \frac{(-Acd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}} + \frac{(Bd+Ae)\log(a+cx^2)}{2c}$$

input
```
Integrate[((A + B*x)*(d + e*x))/(a + c*x^2),x]
```

output
```
(B*e*x)/c - ((-(A*c*d) + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3
/2)) + ((B*d + A*e)*Log[a + c*x^2])/(2*c)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)}{a+cx^2} \, dx$$

$$\Big\downarrow \text{657}$$

$$\int \left(\frac{-aBe+cx(Ae+Bd)+Acd}{c\left(a+cx^2\right)}+\frac{Be}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Acd-aBe)}{\sqrt{a}c^{3/2}}+\frac{\log\left(a+cx^2\right)(Ae+Bd)}{2c}+\frac{Bex}{c}$$

input
```
Int[((A + B*x)*(d + e*x))/(a + c*x^2),x]
```

output
```
(B*e*x)/c + ((A*c*d - a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)
) + ((B*d + A*e)*Log[a + c*x^2])/(2*c)
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

method	result
default	$\frac{Bex}{c}+\frac{\frac{(Ace+Bcd)\ln\left(c\,x^2+a\right)}{2c}+\frac{(Acd-Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{c}$
risch	$\frac{Bex}{c}+\frac{\ln\left(Aacd-Be\,a^2-\sqrt{-ac(Acd-Bae)^2}\,x\right)Ae}{2c}+\frac{\ln\left(Aacd-Be\,a^2-\sqrt{-ac(Acd-Bae)^2}\,x\right)Bd}{2c}+\frac{\ln\left(Aacd-Be\,a^2-\sqrt{-\right.}{}$

input
```
int((B*x+A)*(e*x+d)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
B*e*x/c+1/c*(1/2*(A*c*e+B*c*d)/c*ln(c*x^2+a)+(A*c*d-B*a*e)/(a*c)^(1/2)*arc
tan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.30

$$\int \frac{(A+Bx)(d+ex)}{a+cx^2}\,dx$$
$$=\left[\frac{2\,Bacex+(Acd-Bae)\sqrt{-ac}\log\left(\frac{cx^2+2\,\sqrt{-ac}x-a}{cx^2+a}\right)+(Bacd+Aace)\log\left(cx^2+a\right)}{2\,ac^2},\frac{2\,Bacex+2\,(Acd-\right.$$

input
```
integrate((B*x+A)*(e*x+d)/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*B*a*c*e*x + (A*c*d - B*a*e)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x
 - a)/(c*x^2 + a)) + (B*a*c*d + A*a*c*e)*log(c*x^2 + a))/(a*c^2), 1/2*(2*B
*a*c*e*x + 2*(A*c*d - B*a*e)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (B*a*c*d +
A*a*c*e)*log(c*x^2 + a))/(a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(60) = 120$.

Time = 0.39 (sec) , antiderivative size = 212, normalized size of antiderivative = 3.31

$$\int \frac{(A+Bx)(d+ex)}{a+cx^2}\,dx = \frac{Bex}{c} + \left(\frac{Ae+Bd}{2c} - \frac{\sqrt{-ac^3}(-Acd+Bae)}{2ac^3}\right)\log\left(x + \frac{Aae+Bad-2ac\left(\frac{Ae+Bd}{2c} - \frac{\sqrt{-ac^3}(-Acd+Bae)}{2ac^3}\right)}{-Acd+Bae}\right) + \left(\frac{Ae+Bd}{2c} + \frac{\sqrt{-ac^3}(-Acd+Bae)}{2ac^3}\right)\log\left(x + \frac{Aae+Bad-2ac\left(\frac{Ae+Bd}{2c} + \frac{\sqrt{-ac^3}(-Acd+Bae)}{2ac^3}\right)}{-Acd+Bae}\right)$$

input
```
integrate((B*x+A)*(e*x+d)/(c*x**2+a),x)
```

output
```
B*e*x/c + ((A*e + B*d)/(2*c) - sqrt(-a*c**3)*(-A*c*d + B*a*e)/(2*a*c**3))*
log(x + (A*a*e + B*a*d - 2*a*c*((A*e + B*d)/(2*c) - sqrt(-a*c**3)*(-A*c*d
+ B*a*e)/(2*a*c**3)))/(-A*c*d + B*a*e)) + ((A*e + B*d)/(2*c) + sqrt(-a*c**
3)*(-A*c*d + B*a*e)/(2*a*c**3))*log(x + (A*a*e + B*a*d - 2*a*c*((A*e + B*d
)/(2*c) + sqrt(-a*c**3)*(-A*c*d + B*a*e)/(2*a*c**3)))/(-A*c*d + B*a*e))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)(d+ex)}{a+cx^2}\,dx = \frac{Bex}{c} + \frac{(Bd+Ae)\log\left(cx^2+a\right)}{2\,c} + \frac{(Acd-Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c}$$

input
```
integrate((B*x+A)*(e*x+d)/(c*x^2+a),x, algorithm="maxima")
```

output
```
B*e*x/c + 1/2*(B*d + A*e)*log(c*x^2 + a)/c + (A*c*d - B*a*e)*arctan(c*x/sq
rt(a*c))/(sqrt(a*c)*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)(d+ex)}{a+cx^2}\,dx = \frac{Bex}{c} + \frac{(Bd+Ae)\log\left(cx^2+a\right)}{2\,c} + \frac{(Acd-Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c}$$

input
```
integrate((B*x+A)*(e*x+d)/(c*x^2+a),x, algorithm="giac")
```

output
```
B*e*x/c + 1/2*(B*d + A*e)*log(c*x^2 + a)/c + (A*c*d - B*a*e)*arctan(c*x/sq
rt(a*c))/(sqrt(a*c)*c)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)(d+ex)}{a+cx^2}\,dx = \frac{B\,e\,x}{c} + \frac{A\,e\,\ln\left(c\,x^2+a\right)}{2\,c} + \frac{B\,d\,\ln\left(c\,x^2+a\right)}{2\,c} + \frac{A\,d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}} - \frac{B\,\sqrt{a}\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}}$$

input
```
int(((A + B*x)*(d + e*x))/(a + c*x^2),x)
```

output
```
(B*e*x)/c + (A*e*log(a + c*x^2))/(2*c) + (B*d*log(a + c*x^2))/(2*c) + (A*d
*atan((c^(1/2)*x)/a^(1/2)))/(a^(1/2)*c^(1/2)) - (B*a^(1/2)*e*atan((c^(1/2)
*x)/a^(1/2)))/c^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx)(d+ex)}{a+cx^2}\,dx = \frac{-2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)be+2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)cd+\log(c\,x^2+a)\,ace+\log(c\,x^2+a)\,bcd+2bcex}{2c^2}$$

input
```
int((B*x+A)*(e*x+d)/(c*x^2+a),x)
```

output
```
( - 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*b*e + 2*sqrt(c)*sqrt(a
)*atan((c*x)/(sqrt(c)*sqrt(a)))*c*d + log(a + c*x**2)*a*c*e + log(a + c*x*
*2)*b*c*d + 2*b*c*e*x)/(2*c**2)
```

3.83 $\int \frac{A+Bx}{a+cx^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \frac{A+Bx}{a+cx^2} dx = \frac{A \arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} + \frac{B \log(a+cx^2)}{2c}$$

output
```
A*arctan(c^(1/2)*x/a^(1/2))/a^(1/2)/c^(1/2)+1/2*B*ln(c*x^2+a)/c
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{a+cx^2} dx = \frac{A \arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} + \frac{B \log(a+cx^2)}{2c}$$

input
```
Integrate[(A + B*x)/(a + c*x^2),x]
```

output
```
(A*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + (B*Log[a + c*x^2])/(2*
c)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{a+cx^2} \, dx$$

$$\Big\downarrow \text{452}$$

$$A\int \frac{1}{cx^2+a}dx+B\int \frac{x}{cx^2+a}dx$$

$$\Big\downarrow \text{218}$$

$$B\int \frac{x}{cx^2+a}dx+\frac{A\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

$$\Big\downarrow \text{240}$$

$$\frac{A\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}+\frac{B\log\left(a+cx^2\right)}{2c}$$

input
```
Int[(A + B*x)/(a + c*x^2),x]
```

output
```
(A*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + (B*Log[a + c*x^2])/(2*
c)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{B\ln\left(c\,x^{2}+a\right)}{2c}+\frac{A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$	32
risch	$\frac{\ln\left(-\sqrt{-ac}\,x+a\right)A\sqrt{-ac}}{2ac}+\frac{\ln\left(-\sqrt{-ac}\,x+a\right)B}{2c}-\frac{\ln\left(\sqrt{-ac}\,x+a\right)A\sqrt{-ac}}{2ac}+\frac{\ln\left(\sqrt{-ac}\,x+a\right)B}{2c}$	90

input
```
int((B*x+A)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*ln(c*x^2+a)/c+A/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.33

$$\int \frac{A+Bx}{a+cx^2}\,dx = \left[\frac{Ba\log\left(cx^2+a\right)-\sqrt{-ac}A\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right)}{2\,ac}, \frac{Ba\log\left(cx^2+a\right)+2\sqrt{ac}A\arctan\left(\frac{\sqrt{ac}x}{a}\right)}{2\,ac}\right]$$

input
```
integrate((B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(B*a*log(c*x^2 + a) - sqrt(-a*c)*A*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(
c*x^2 + a)))/(a*c), 1/2*(B*a*log(c*x^2 + a) + 2*sqrt(a*c)*A*arctan(sqrt(a*
c)*x/a))/(a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(37) = 74$.

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.95

$$\int \frac{A+Bx}{a+cx^2}\,dx = \left(-\frac{A\sqrt{-ac^3}}{2ac^2}+\frac{B}{2c}\right)\log\left(x+\frac{-Ba+2ac\left(-\frac{A\sqrt{-ac^3}}{2ac^2}+\frac{B}{2c}\right)}{Ac}\right) + \left(\frac{A\sqrt{-ac^3}}{2ac^2}+\frac{B}{2c}\right)\log\left(x+\frac{-Ba+2ac\left(\frac{A\sqrt{-ac^3}}{2ac^2}+\frac{B}{2c}\right)}{Ac}\right)$$

input
```
integrate((B*x+A)/(c*x**2+a),x)
```

output
```
(-A*sqrt(-a*c**3)/(2*a*c**2) + B/(2*c))*log(x + (-B*a + 2*a*c*(-A*sqrt(-a*
c**3)/(2*a*c**2) + B/(2*c)))/(A*c)) + (A*sqrt(-a*c**3)/(2*a*c**2) + B/(2*c
))*log(x + (-B*a + 2*a*c*(A*sqrt(-a*c**3)/(2*a*c**2) + B/(2*c)))/(A*c))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx}{a+cx^2}\,dx = \frac{A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} + \frac{B\log\left(cx^2+a\right)}{2\,c}$$

input
```
integrate((B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

output
```
A*arctan(c*x/sqrt(a*c))/sqrt(a*c) + 1/2*B*log(c*x^2 + a)/c
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx}{a+cx^2}\,dx = \frac{A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} + \frac{B\log\left(cx^2+a\right)}{2\,c}$$

input
```
integrate((B*x+A)/(c*x^2+a),x, algorithm="giac")
```

output
```
A*arctan(c*x/sqrt(a*c))/sqrt(a*c) + 1/2*B*log(c*x^2 + a)/c
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx}{a+cx^2}\,dx = \frac{B\,\ln\left(c\,x^2+a\right)}{2\,c} + \frac{A\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}$$

input
```
int((A + B*x)/(a + c*x^2),x)
```

output
```
(B*log(a + c*x^2))/(2*c) + (A*atan((c^(1/2)*x)/a^(1/2)))/(a^(1/2)*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{a+cx^2}\,dx = \frac{2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right) + \log(c\,x^2+a)\,b}{2c}$$

input
```
int((B*x+A)/(c*x^2+a),x)
```

output
```
(2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a))) + log(a + c*x**2)*b)/(2*c
)
```

3.84 $\int \frac{A+Bx}{(d+ex)(a+cx^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 109

$$\int \frac{A+Bx}{(d+ex)(a+cx^2)}\,dx=\frac{(Acd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}\,(cd^2+ae^2)}-\frac{(Bd-Ae)\log(d+ex)}{cd^2+ae^2}+\frac{(Bd-Ae)\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)}$$

output
```
(A*c*d+B*a*e)*arctan(c^(1/2)*x/a^(1/2))/a^(1/2)/c^(1/2)/(a*e^2+c*d^2)-(-A*
e+B*d)*ln(e*x+d)/(a*e^2+c*d^2)+(-A*e+B*d)*ln(c*x^2+a)/(2*a*e^2+2*c*d^2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx}{(d+ex)(a+cx^2)}\,dx=\frac{2(Acd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)-\sqrt{a}\sqrt{c}(Bd-Ae)\left(2\log(d+ex)-\log\left(a+cx^2\right)\right)}{2\sqrt{a}\sqrt{c}\,(cd^2+ae^2)}$$

input
```
Integrate[(A + B*x)/((d + e*x)*(a + c*x^2)),x]
```

output
```
(2*(A*c*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]] - Sqrt[a]*Sqrt[c]*(B*d - A*
e)*(2*Log[d + e*x] - Log[a + c*x^2]))/(2*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+cx^2)(d+ex)}\,dx$$

$$\Big\downarrow \text{657}$$

$$\int \left(\frac{aBe+cx(Bd-Ae)+Acd}{(a+cx^2)(ae^2+cd^2)}+\frac{e(Ae-Bd)}{(d+ex)(ae^2+cd^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+Acd)}{\sqrt{a}\sqrt{c}(ae^2+cd^2)}+\frac{\log(a+cx^2)(Bd-Ae)}{2(ae^2+cd^2)}-\frac{(Bd-Ae)\log(d+ex)}{ae^2+cd^2}$$

input
```
Int[(A + B*x)/((d + e*x)*(a + c*x^2)),x]
```

output
```
((A*c*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]*(c*d^2 + a*
e^2)) - ((B*d - A*e)*Log[d + e*x])/(c*d^2 + a*e^2) + ((B*d - A*e)*Log[a +
c*x^2])/(2*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

method	result
default	$\frac{(Ae-Bd)\ln(ex+d)}{a\,e^2+c\,d^2}+\frac{\frac{(-Ace+Bcd)\ln\left(c\,x^2+a\right)}{2c}+\frac{(Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{a\,e^2+c\,d^2}$
risch	$\frac{\ln(ex+d)Ae}{a\,e^2+c\,d^2}-\frac{\ln(ex+d)Bd}{a\,e^2+c\,d^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(a^2c\,e^2+a\,c^2d^2\right)\_Z^2+(2Aace-2Bacd)\_Z+c\,A^2+B^2a\right)}\_R\ln\left(\left((3ac\,e^2-c^2d^2)\_R\right.\right.\right.}{2}$

input
```
int((B*x+A)/(e*x+d)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(A*e-B*d)/(a*e^2+c*d^2)*ln(e*x+d)+1/(a*e^2+c*d^2)*(1/2*(-A*c*e+B*c*d)/c*ln
(c*x^2+a)+(A*c*d+B*a*e)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.83

$$\int \frac{A+Bx}{(d+ex)(a+cx^2)}\,dx$$
$$=\left[-\frac{(Acd+Bae)\sqrt{-ac}\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right)-(Bacd-Aace)\log\left(cx^2+a\right)+2\,(Bacd-Aace)\log\left(ex+\right.}{2\left(ac^2d^2+a^2ce^2\right)}\right.$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*((A*c*d + B*a*e)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2
+ a)) - (B*a*c*d - A*a*c*e)*log(c*x^2 + a) + 2*(B*a*c*d - A*a*c*e)*log(e*x
 + d))/(a*c^2*d^2 + a^2*c*e^2), 1/2*(2*(A*c*d + B*a*e)*sqrt(a*c)*arctan(sq
rt(a*c)*x/a) + (B*a*c*d - A*a*c*e)*log(c*x^2 + a) - 2*(B*a*c*d - A*a*c*e)*
log(e*x + d))/(a*c^2*d^2 + a^2*c*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x**2+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{(d+ex)\,(a+cx^2)}\,dx = \frac{(Bd-Ae)\log(cx^2+a)}{2\,(cd^2+ae^2)} - \frac{(Bd-Ae)\log(ex+d)}{cd^2+ae^2} + \frac{(Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(cd^2+ae^2)\sqrt{ac}}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="maxima")
```

output
```
1/2*(B*d - A*e)*log(c*x^2 + a)/(c*d^2 + a*e^2) - (B*d - A*e)*log(e*x + d)/
(c*d^2 + a*e^2) + (A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/((c*d^2 + a*e^2)*s
qrt(a*c))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{(d+ex)\,(a+cx^2)}\,dx = \frac{(Bd-Ae)\log{(cx^2+a)}}{2\,(cd^2+ae^2)} - \frac{(Bde-Ae^2)\log{(|ex+d|)}}{cd^2e+ae^3} + \frac{(Acd+Bae)\arctan{\left(\frac{cx}{\sqrt{ac}}\right)}}{(cd^2+ae^2)\sqrt{ac}}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="giac")
```

output
```
1/2*(B*d - A*e)*log(c*x^2 + a)/(c*d^2 + a*e^2) - (B*d*e - A*e^2)*log(abs(e
*x + d))/(c*d^2*e + a*e^3) + (A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/((c*d^2
 + a*e^2)*sqrt(a*c))
```

Mupad [B] (verification not implemented)

Time = 7.09 (sec) , antiderivative size = 535, normalized size of antiderivative = 4.91

$$\int \frac{A+Bx}{(d+ex)\,(a+cx^2)}\,dx = \frac{\ln{(d+e\,x)}\,\left(A\,e-B\,d\right)}{c\,d^2+a\,e^2} - \frac{\ln\left(B^2\,c\,e\,x-\frac{\left(c\,\left(a\,\left(\frac{A\,e}{2}-\frac{B\,d}{2}\right)+\frac{A\,d\,\sqrt{-a\,c}}{2}\right)+\frac{B\,a\,e\,\sqrt{-a\,c}}{2}\right)\,\left(x\,\left(3\,A\,c^2\,e^2-B\,c^2\,d\,e\right)-\frac{\left(c\,\left(a\,\left(\frac{A\,e}{2}-\frac{B\,d}{2}\right)+\frac{A\,d\,\sqrt{-a\,c}}{2}\right)+\frac{B\,a\,e\,\sqrt{-a\,c}}{2}\right)\,\left(x\,\left(\cdots\right.\right.}{a^2\,c\,e^2+a\,c^2\,d^2}\cdots\right.}{a^2\,c\,e^2+a\,c^2\,d^2}\cdots\right)}{a^2\,c\,e^2+a\,c^{\cdots}} - \frac{\ln\left(B^2\,c\,e\,x-\frac{\left(c\,\left(a\,\left(\frac{A\,e}{2}-\frac{B\,d}{2}\right)-\frac{A\,d\,\sqrt{-a\,c}}{2}\right)-\frac{B\,a\,e\,\sqrt{-a\,c}}{2}\right)\,\left(x\,\left(3\,A\,c^2\,e^2-B\,c^2\,d\,e\right)-\frac{\left(c\,\left(a\,\left(\frac{A\,e}{2}-\frac{B\,d}{2}\right)-\frac{A\,d\,\sqrt{-a\,c}}{2}\right)-\frac{B\,a\,e\,\sqrt{-a\,c}}{2}\right)\,\left(x\,\left(\cdots\right.\right.}{a^2\,c\,e^2+a\,c^2\,d^2}\cdots\right.}{a^2\,c\,e^2+a\,c^2\,d^2}\cdots\right)}{a^2\,c\,e^2+a\,c^{\cdots}}$$

input
```
int((A + B*x)/((a + c*x^2)*(d + e*x)),x)
```

output
```
(log(d + e*x)*(A*e - B*d))/(a*e^2 + c*d^2) - (log(B^2*c*e*x - ((c*(a*((A*e
)/2 - (B*d)/2) + (A*d*(-a*c)^(1/2))/2) + (B*a*e*(-a*c)^(1/2))/2)*(x*(3*A*c
^2*e^2 - B*c^2*d*e) - ((c*(a*((A*e)/2 - (B*d)/2) + (A*d*(-a*c)^(1/2))/2) +
 (B*a*e*(-a*c)^(1/2))/2)*(x*(6*a*c^2*e^3 - 2*c^3*d^2*e) + 8*a*c^2*d*e^2))/
(a*c^2*d^2 + a^2*c*e^2) - B*a*c*e^2 + A*c^2*d*e))/(a*c^2*d^2 + a^2*c*e^2)
+ A*B*c*e)*(c*(a*((A*e)/2 - (B*d)/2) + (A*d*(-a*c)^(1/2))/2) + (B*a*e*(-a*
c)^(1/2))/2))/(a*c^2*d^2 + a^2*c*e^2) - (log(B^2*c*e*x - ((c*(a*((A*e)/2 -
 (B*d)/2) - (A*d*(-a*c)^(1/2))/2) - (B*a*e*(-a*c)^(1/2))/2)*(x*(3*A*c^2*e^
2 - B*c^2*d*e) - ((c*(a*((A*e)/2 - (B*d)/2) - (A*d*(-a*c)^(1/2))/2) - (B*a
*e*(-a*c)^(1/2))/2)*(x*(6*a*c^2*e^3 - 2*c^3*d^2*e) + 8*a*c^2*d*e^2))/(a*c^
2*d^2 + a^2*c*e^2) - B*a*c*e^2 + A*c^2*d*e))/(a*c^2*d^2 + a^2*c*e^2) + A*B
*c*e)*(c*(a*((A*e)/2 - (B*d)/2) - (A*d*(-a*c)^(1/2))/2) - (B*a*e*(-a*c)^(1
/2))/2))/(a*c^2*d^2 + a^2*c*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{(d+ex)(a+cx^2)}\,dx$$
$$= \frac{2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)be + 2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)cd - \log(c\,x^2+a)\,ace + \log(c\,x^2+a)\,bcd + 2\log(ex+a}{2c\,(a\,e^2+c\,d^2)}$$

input
```
int((B*x+A)/(e*x+d)/(c*x^2+a),x)
```

output
```
(2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*b*e + 2*sqrt(c)*sqrt(a)*a
tan((c*x)/(sqrt(c)*sqrt(a)))*c*d - log(a + c*x**2)*a*c*e + log(a + c*x**2)
*b*c*d + 2*log(d + e*x)*a*c*e - 2*log(d + e*x)*b*c*d)/(2*c*(a*e**2 + c*d**
2))
```

3.85 $\int \frac{A+Bx}{(d+ex)^2(a+cx^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 173

$$\begin{aligned}\int \frac{A+Bx}{(d+ex)^2\,(a+cx^2)}\,dx &= \frac{Bd-Ae}{(cd^2+ae^2)\,(d+ex)} \\ &\quad+\frac{\sqrt{c}(Acd^2+2aBde-aAe^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\,(cd^2+ae^2)^2} \\ &\quad-\frac{(Bcd^2-2Acde-aBe^2)\log(d+ex)}{(cd^2+ae^2)^2} \\ &\quad+\frac{(Bcd^2-2Acde-aBe^2)\log\left(a+cx^2\right)}{2\,(cd^2+ae^2)^2}\end{aligned}$$

output
```
(-A*e+B*d)/(a*e^2+c*d^2)/(e*x+d)+c^(1/2)*(-A*a*e^2+A*c*d^2+2*B*a*d*e)*arct
an(c^(1/2)*x/a^(1/2))/a^(1/2)/(a*e^2+c*d^2)^2-(-2*A*c*d*e-B*a*e^2+B*c*d^2)
*ln(e*x+d)/(a*e^2+c*d^2)^2+1/2*(-2*A*c*d*e-B*a*e^2+B*c*d^2)*ln(c*x^2+a)/(a
*e^2+c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{(d+ex)^2(a+cx^2)}\,dx$$
$$= \frac{\frac{2(Bd-Ae)(cd^2+ae^2)}{d+ex} + \frac{2\sqrt{c}(Acd^2+2aBde-aAe^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}} + (-2Bcd^2+4Acde+2aBe^2)\log(d+ex) + (Bcd^2 -}{2(cd^2+ae^2)^2}$$

input
```
Integrate[(A + B*x)/((d + e*x)^2*(a + c*x^2)),x]
```

output
```
((2*(B*d - A*e)*(c*d^2 + a*e^2))/(d + e*x) + (2*Sqrt[c]*(A*c*d^2 + 2*a*B*d
*e - a*A*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] + (-2*B*c*d^2 + 4*A*c*d
*e + 2*a*B*e^2)*Log[d + e*x] + (B*c*d^2 - 2*A*c*d*e - a*B*e^2)*Log[a + c*x
^2])/(2*(c*d^2 + a*e^2)^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+cx^2)(d+ex)^2}\,dx$$
$$\Big\downarrow \text{657}$$
$$\int \left(\frac{c(x(-aBe^2-2Acde+Bcd^2)-aAe^2+2aBde+Acd^2)}{(a+cx^2)(ae^2+cd^2)^2} + \frac{e(Ae-Bd)}{(d+ex)^2(ae^2+cd^2)} + \frac{e(aBe^2+2Acde-Bc}{(d+ex)(ae^2+cd^2)^2}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(-aAe^2+2aBde+Acd^2\right)}{\sqrt{a}\left(ae^2+cd^2\right)^2}+\frac{\log\left(a+cx^2\right)\left(-aBe^2-2Acde+Bcd^2\right)}{2\left(ae^2+cd^2\right)^2}+\frac{Bd-Ae}{(d+ex)\left(ae^2+cd^2\right)}-\frac{\log(d+ex)\left(-aBe^2-2Acde+Bcd^2\right)}{\left(ae^2+cd^2\right)^2}$$

input
```
Int[(A + B*x)/((d + e*x)^2*(a + c*x^2)),x]
```

output
```
(B*d - A*e)/((c*d^2 + a*e^2)*(d + e*x)) + (Sqrt[c]*(A*c*d^2 + 2*a*B*d*e -
a*A*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^2) - ((B*c*
d^2 - 2*A*c*d*e - a*B*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^2 + ((B*c*d^2 - 2
*A*c*d*e - a*B*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91

method	result
default	$-\frac{Ae-Bd}{(a e^2+c d^2)(ex+d)}+\frac{(2Acde+Ba e^2-Bc d^2)\ln(ex+d)}{(a e^2+c d^2)^2}-\frac{c\left(\frac{(2Acde+Ba e^2-Bc d^2)\ln(c x^2+a)}{2c}+\frac{(Aa e^2-Ac d^2-2Bade)\arctan(}{\sqrt{ac}}\right.}{(a e^2+c d^2)^2}$
risch	Expression too large to display

input
```
int((B*x+A)/(e*x+d)^2/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-(A*e-B*d)/(a*e^2+c*d^2)/(e*x+d)+(2*A*c*d*e+B*a*e^2-B*c*d^2)/(a*e^2+c*d^2)
^2*ln(e*x+d)-c/(a*e^2+c*d^2)^2*(1/2*(2*A*c*d*e+B*a*e^2-B*c*d^2)/c*ln(c*x^2
+a)+(A*a*e^2-A*c*d^2-2*B*a*d*e)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.25

$$\int \frac{A+Bx}{(d+ex)^2\,(a+cx^2)}\,dx$$

$$= \left[\frac{2\,Bcd^3-2\,Acd^2e+2\,Bade^2-2\,Aae^3-(Acd^3+2\,Bad^2e-Aade^2+(Acd^2e+2\,Bade^2-Aae^3)x)\sqrt{-}}{}\right.$$

input
```
integrate((B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*B*c*d^3 - 2*A*c*d^2*e + 2*B*a*d*e^2 - 2*A*a*e^3 - (A*c*d^3 + 2*B*a
*d^2*e - A*a*d*e^2 + (A*c*d^2*e + 2*B*a*d*e^2 - A*a*e^3)*x)*sqrt(-c/a)*log
((c*x^2 - 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) + (B*c*d^3 - 2*A*c*d^2*e - B*
a*d*e^2 + (B*c*d^2*e - 2*A*c*d*e^2 - B*a*e^3)*x)*log(c*x^2 + a) - 2*(B*c*d
^3 - 2*A*c*d^2*e - B*a*d*e^2 + (B*c*d^2*e - 2*A*c*d*e^2 - B*a*e^3)*x)*log(
e*x + d))/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^
3 + a^2*e^5)*x), 1/2*(2*B*c*d^3 - 2*A*c*d^2*e + 2*B*a*d*e^2 - 2*A*a*e^3 +
2*(A*c*d^3 + 2*B*a*d^2*e - A*a*d*e^2 + (A*c*d^2*e + 2*B*a*d*e^2 - A*a*e^3)
*x)*sqrt(c/a)*arctan(x*sqrt(c/a)) + (B*c*d^3 - 2*A*c*d^2*e - B*a*d*e^2 + (
B*c*d^2*e - 2*A*c*d*e^2 - B*a*e^3)*x)*log(c*x^2 + a) - 2*(B*c*d^3 - 2*A*c*
d^2*e - B*a*d*e^2 + (B*c*d^2*e - 2*A*c*d*e^2 - B*a*e^3)*x)*log(e*x + d))/(
c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5
)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)^2\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)**2/(c*x**2+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.25

$$\begin{aligned}\int \frac{A+Bx}{(d+ex)^2\,(a+cx^2)}\,dx &= \frac{(Bcd^2-2\,Acde-Bae^2)\log\left(cx^2+a\right)}{2\,(c^2d^4+2\,acd^2e^2+a^2e^4)} \\ &\quad - \frac{(Bcd^2-2\,Acde-Bae^2)\log\left(ex+d\right)}{c^2d^4+2\,acd^2e^2+a^2e^4} \\ &\quad + \frac{(Ac^2d^2+2\,Bacde-Aace^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{ac}} \\ &\quad + \frac{Bd-Ae}{cd^3+ade^2+(cd^2e+ae^3)x}\end{aligned}$$

input
```
integrate((B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="maxima")
```

output
```
1/2*(B*c*d^2 - 2*A*c*d*e - B*a*e^2)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4) - (B*c*d^2 - 2*A*c*d*e - B*a*e^2)*log(e*x + d)/(c^2*d^4 + 2*a
*c*d^2*e^2 + a^2*e^4) + (A*c^2*d^2 + 2*B*a*c*d*e - A*a*c*e^2)*arctan(c*x/s
qrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) + (B*d - A*e)/(c
*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx}{(d+ex)^2(a+cx^2)}\,dx = \frac{(Bcd^2-2\,Acde-Bae^2)\log\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}{2\,(c^2d^4+2\,acd^2e^2+a^2e^4)} + \frac{\frac{Bde^2}{ex+d}-\frac{Ae^3}{ex+d}}{cd^2e^2+ae^4} + \frac{(Ac^2d^2e^2+2\,Bacde^3-Aace^4)\arctan\left(\frac{cd-\frac{cd^2}{ex+d}-\frac{ae^2}{ex+d}}{\sqrt{ace}}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{ac}e^2}$$

input
```
integrate((B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="giac")
```

output
```
1/2*(B*c*d^2 - 2*A*c*d*e - B*a*e^2)*log(c - 2*c*d/(e*x + d) + c*d^2/(e*x +
 d)^2 + a*e^2/(e*x + d)^2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + (B*d*e^2/
(e*x + d) - A*e^3/(e*x + d))/(c*d^2*e^2 + a*e^4) + (A*c^2*d^2*e^2 + 2*B*a*
c*d*e^3 - A*a*c*e^4)*arctan((c*d - c*d^2/(e*x + d) - a*e^2/(e*x + d))/(sqr
t(a*c)*e))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)*e^2)
```

Mupad [B] (verification not implemented)

Time = 7.07 (sec) , antiderivative size = 810, normalized size of antiderivative = 4.68

$$\int \frac{A+Bx}{(d+ex)^2(a+cx^2)}\,dx = \frac{\ln\left(3\,B\,a^3\,e^4+3\,B\,a\,c^2\,d^4+A\,c^3\,d^4\,x+A\,a^2\,e^4\,\sqrt{-a\,c}+A\,c^2\,d^4\,\sqrt{-a\,c}+14\,A\,d^2\,e^2\,(-a\,c)^{3/2}+A\,a^2\,c\,e^4\right.}{} - \frac{\ln(d+e\,x)\,\left(c\,(B\,d^2-2\,A\,d\,e)-B\,a\,e^2\right)}{a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4} - \frac{\ln\left(3\,B\,a^3\,e^4+8\,B\,d^3\,e\,(-a\,c)^{3/2}+3\,B\,a\,c^2\,d^4+A\,c^3\,d^4\,x-A\,a^2\,e^4\,\sqrt{-a\,c}-A\,c^2\,d^4\,\sqrt{-a\,c}+A\,a^2\,c\,e^4\right.}{} - \frac{A\,e-B\,d}{(c\,d^2+a\,e^2)\,(d+e\,x)}$$

input
```
int((A + B*x)/((a + c*x^2)*(d + e*x)^2),x)
```

output
```
(log(3*B*a^3*e^4 + 3*B*a*c^2*d^4 + A*c^3*d^4*x + A*a^2*e^4*(-a*c)^(1/2) +
A*c^2*d^4*(-a*c)^(1/2) + 14*A*d^2*e^2*(-a*c)^(3/2) + A*a^2*c*e^4*x - 8*B*a
^2*d*e^3*(-a*c)^(1/2) - 3*B*a^2*e^4*x*(-a*c)^(1/2) - 3*B*c^2*d^4*x*(-a*c)^
(1/2) - 10*B*a^2*c*d^2*e^2 + 8*A*d*e^3*x*(-a*c)^(3/2) - 8*A*a*c^2*d^3*e +
8*A*a^2*c*d*e^3 + 8*B*a*c*d^3*e*(-a*c)^(1/2) + 8*B*a*c^2*d^3*e*x - 8*B*a^2
*c*d*e^3*x + 8*A*c^2*d^3*e*x*(-a*c)^(1/2) - 14*A*a*c^2*d^2*e^2*x + 10*B*a*
c*d^2*e^2*x*(-a*c)^(1/2))*(c*((B*a*d^2)/2 + (A*d^2*(-a*c)^(1/2))/2 - A*a*d
*e) - e^2*((B*a^2)/2 + (A*a*(-a*c)^(1/2))/2) + B*a*d*e*(-a*c)^(1/2)))/(a^3
*e^4 + a*c^2*d^4 + 2*a^2*c*d^2*e^2) - (log(d + e*x)*(c*(B*d^2 - 2*A*d*e) -
 B*a*e^2))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) - (log(3*B*a^3*e^4 + 8*B*d^
3*e*(-a*c)^(3/2) + 3*B*a*c^2*d^4 + A*c^3*d^4*x - A*a^2*e^4*(-a*c)^(1/2) -
A*c^2*d^4*(-a*c)^(1/2) + A*a^2*c*e^4*x + 8*B*a^2*d*e^3*(-a*c)^(1/2) + 3*B*
a^2*e^4*x*(-a*c)^(1/2) + 3*B*c^2*d^4*x*(-a*c)^(1/2) + 10*B*d^2*e^2*x*(-a*c
)^(3/2) - 10*B*a^2*c*d^2*e^2 - 8*A*a*c^2*d^3*e + 8*A*a^2*c*d*e^3 + 8*B*a*c
^2*d^3*e*x - 8*B*a^2*c*d*e^3*x + 14*A*a*c*d^2*e^2*(-a*c)^(1/2) - 8*A*c^2*d
^3*e*x*(-a*c)^(1/2) - 14*A*a*c^2*d^2*e^2*x + 8*A*a*c*d*e^3*x*(-a*c)^(1/2))
*(e^2*((B*a^2)/2 - (A*a*(-a*c)^(1/2))/2) + c*((A*d^2*(-a*c)^(1/2))/2 - (B*
a*d^2)/2 + A*a*d*e) + B*a*d*e*(-a*c)^(1/2)))/(a^3*e^4 + a*c^2*d^4 + 2*a^2*
c*d^2*e^2) - (A*e - B*d)/((a*e^2 + c*d^2)*(d + e*x))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.51

$$\int \frac{A + Bx}{(d + ex)^2 (a + cx^2)} \, dx$$
$$= \frac{-2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right) a\,d^2e^2 - 2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right) ad\,e^3x + 4\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right) b\,d^3e + 4\sqrt{c}\,\sqrt{a}\,ata}{}$$

input
```
int((B*x+A)/(e*x+d)^2/(c*x^2+a),x)
```

output
```
( - 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*d**2*e**2 - 2*sqrt(c
)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*d*e**3*x + 4*sqrt(c)*sqrt(a)*ata
n((c*x)/(sqrt(c)*sqrt(a)))*b*d**3*e + 4*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c
)*sqrt(a)))*b*d**2*e**2*x + 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a))
)*c*d**4 + 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c*d**3*e*x - lo
g(a + c*x**2)*a*b*d**2*e**2 - log(a + c*x**2)*a*b*d*e**3*x - 2*log(a + c*x
**2)*a*c*d**3*e - 2*log(a + c*x**2)*a*c*d**2*e**2*x + log(a + c*x**2)*b*c*
d**4 + log(a + c*x**2)*b*c*d**3*e*x + 2*log(d + e*x)*a*b*d**2*e**2 + 2*log
(d + e*x)*a*b*d*e**3*x + 4*log(d + e*x)*a*c*d**3*e + 4*log(d + e*x)*a*c*d*
*2*e**2*x - 2*log(d + e*x)*b*c*d**4 - 2*log(d + e*x)*b*c*d**3*e*x + 2*a**2
*e**4*x - 2*a*b*d*e**3*x + 2*a*c*d**2*e**2*x - 2*b*c*d**3*e*x)/(2*d*(a**2*
d*e**4 + a**2*e**5*x + 2*a*c*d**3*e**2 + 2*a*c*d**2*e**3*x + c**2*d**5 + c
**2*d**4*e*x))
```

3.86 $\int \frac{A+Bx}{(d+ex)^3(a+cx^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 251

$$\begin{aligned}\int \frac{A+Bx}{(d+ex)^3(a+cx^2)}\,dx &= \frac{Bd-Ae}{2(cd^2+ae^2)(d+ex)^2}+\frac{Bcd^2-2Acde-aBe^2}{(cd^2+ae^2)^2(d+ex)}\\&\quad+\frac{\sqrt{c}(Acd(cd^2-3ae^2)+aBe(3cd^2-ae^2))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}(cd^2+ae^2)^3}\\&\quad-\frac{c(Bcd^3-3Acd^2e-3aBde^2+aAe^3)\log(d+ex)}{(cd^2+ae^2)^3}\\&\quad+\frac{c(Bcd^3-3Acd^2e-3aBde^2+aAe^3)\log(a+cx^2)}{2(cd^2+ae^2)^3}\end{aligned}$$

output
```
1/2*(-A*e+B*d)/(a*e^2+c*d^2)/(e*x+d)^2+(-2*A*c*d*e-B*a*e^2+B*c*d^2)/(a*e^2
+c*d^2)^2/(e*x+d)+c^(1/2)*(A*c*d*(-3*a*e^2+c*d^2)+a*B*e*(-a*e^2+3*c*d^2))*
arctan(c^(1/2)*x/a^(1/2))/a^(1/2)/(a*e^2+c*d^2)^3-c*(A*a*e^3-3*A*c*d^2*e-3
*B*a*d*e^2+B*c*d^3)*ln(e*x+d)/(a*e^2+c*d^2)^3+1/2*c*(A*a*e^3-3*A*c*d^2*e-3
*B*a*d*e^2+B*c*d^3)*ln(c*x^2+a)/(a*e^2+c*d^2)^3
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{(d+ex)^3(a+cx^2)}\,dx = \frac{\frac{(cd^2+ae^2)(B(-ae^2(d+2ex)+cd^2(3d+2ex))-Ae(ae^2+cd(5d+4ex)))}{(d+ex)^2}+\frac{2\sqrt{c}(Acd(cd^2-3ae^2)+aBe(3cd^2-ae^2))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}}-2c(Bc}{2(cd^2+ae}$$

input
```
Integrate[(A + B*x)/((d + e*x)^3*(a + c*x^2)),x]
```

output
```
(((c*d^2 + a*e^2)*(B*(-(a*e^2*(d + 2*e*x)) + c*d^2*(3*d + 2*e*x)) - A*e*(a
*e^2 + c*d*(5*d + 4*e*x))))/(d + e*x)^2 + (2*Sqrt[c]*(A*c*d*(c*d^2 - 3*a*e
^2) + a*B*e*(3*c*d^2 - a*e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] - 2*c*
(B*c*d^3 - 3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*Log[d + e*x] + c*(B*c*d^3
- 3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^
3)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+cx^2)(d+ex)^3}\,dx$$

$$\Big\downarrow \text{657}$$

$$\int \left(\frac{e(Ae-Bd)}{(d+ex)^3(ae^2+cd^2)}+\frac{e(aBe^2+2Acde-Bcd^2)}{(d+ex)^2(ae^2+cd^2)^2}+\frac{c(cx(aAe^3-3aBde^2-3Acd^2e+Bcd^3)+Acd(cd^2-}{(a+cx^2)(ae^2+cd^2)^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd(cd^2-3ae^2)+aBe(3cd^2-ae^2)\right)}{\sqrt{a}\,(ae^2+cd^2)^3}+\frac{Bd-Ae}{2(d+ex)^2\,(ae^2+cd^2)}+\frac{-aBe^2-2Acde+Bcd^2}{(d+ex)\,(ae^2+cd^2)^2}+\frac{c\log\left(a+cx^2\right)\left(aAe^3-3aBde^2-3Acd^2e+Bcd^3\right)}{2\,(ae^2+cd^2)^3}-\frac{c\log(d+ex)\left(aAe^3-3aBde^2-3Acd^2e+Bcd^3\right)}{(ae^2+cd^2)^3}$$

input
```
Int[(A + B*x)/((d + e*x)^3*(a + c*x^2)),x]
```

output
```
(B*d - A*e)/(2*(c*d^2 + a*e^2)*(d + e*x)^2) + (B*c*d^2 - 2*A*c*d*e - a*B*e
^2)/((c*d^2 + a*e^2)^2*(d + e*x)) + (Sqrt[c]*(A*c*d*(c*d^2 - 3*a*e^2) + a*
B*e*(3*c*d^2 - a*e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^
2)^3) - (c*(B*c*d^3 - 3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*Log[d + e*x])/(
c*d^2 + a*e^2)^3 + (c*(B*c*d^3 - 3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*Log[
a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.97

method	result
default	$-\frac{Ae-Bd}{2(a\,e^2+c\,d^2)(ex+d)^2}-\frac{2Acde+Ba\,e^2-Bc\,d^2}{(a\,e^2+c\,d^2)^2(ex+d)}-\frac{c(Aa\,e^3-3Ac\,d^2e-3Bad\,e^2+Bc\,d^3)\ln(ex+d)}{(a\,e^2+c\,d^2)^3}-\frac{c\left(\frac{(-Aac\,e^3+3A\,c^2d^2e+3}{}\right.}{}$
risch	Expression too large to display

input
```
int((B*x+A)/(e*x+d)^3/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(A*e-B*d)/(a*e^2+c*d^2)/(e*x+d)^2-(2*A*c*d*e+B*a*e^2-B*c*d^2)/(a*e^2+
c*d^2)^2/(e*x+d)-c*(A*a*e^3-3*A*c*d^2*e-3*B*a*d*e^2+B*c*d^3)*ln(e*x+d)/(a*
e^2+c*d^2)^3-c/(a*e^2+c*d^2)^3*(1/2*(-A*a*c*e^3+3*A*c^2*d^2*e+3*B*a*c*d*e^
2-B*c^2*d^3)/c*ln(c*x^2+a)+(3*A*a*c*d*e^2-A*c^2*d^3+B*a^2*e^3-3*B*a*c*d^2*
e)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(239) = 478$.

Time = 19.06 (sec) , antiderivative size = 1350, normalized size of antiderivative = 5.38

$$\int \frac{A+Bx}{(d+ex)^3\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(3*B*c^2*d^5 - 5*A*c^2*d^4*e + 2*B*a*c*d^3*e^2 - 6*A*a*c*d^2*e^3 - B*
a^2*d*e^4 - A*a^2*e^5 - (A*c^2*d^5 + 3*B*a*c*d^4*e - 3*A*a*c*d^3*e^2 - B*a
^2*d^2*e^3 + (A*c^2*d^3*e^2 + 3*B*a*c*d^2*e^3 - 3*A*a*c*d*e^4 - B*a^2*e^5)
*x^2 + 2*(A*c^2*d^4*e + 3*B*a*c*d^3*e^2 - 3*A*a*c*d^2*e^3 - B*a^2*d*e^4)*x
)*sqrt(-c/a)*log((c*x^2 - 2*a*x*sqrt(-c/a) - a)/(c*x^2 + a)) + 2*(B*c^2*d^
4*e - 2*A*c^2*d^3*e^2 - 2*A*a*c*d*e^4 - B*a^2*e^5)*x + (B*c^2*d^5 - 3*A*c^
2*d^4*e - 3*B*a*c*d^3*e^2 + A*a*c*d^2*e^3 + (B*c^2*d^3*e^2 - 3*A*c^2*d^2*e
^3 - 3*B*a*c*d*e^4 + A*a*c*e^5)*x^2 + 2*(B*c^2*d^4*e - 3*A*c^2*d^3*e^2 - 3
*B*a*c*d^2*e^3 + A*a*c*d*e^4)*x)*log(c*x^2 + a) - 2*(B*c^2*d^5 - 3*A*c^2*d
^4*e - 3*B*a*c*d^3*e^2 + A*a*c*d^2*e^3 + (B*c^2*d^3*e^2 - 3*A*c^2*d^2*e^3
- 3*B*a*c*d*e^4 + A*a*c*e^5)*x^2 + 2*(B*c^2*d^4*e - 3*A*c^2*d^3*e^2 - 3*B*
a*c*d^2*e^3 + A*a*c*d*e^4)*x)*log(e*x + d))/(c^3*d^8 + 3*a*c^2*d^6*e^2 + 3
*a^2*c*d^4*e^4 + a^3*d^2*e^6 + (c^3*d^6*e^2 + 3*a*c^2*d^4*e^4 + 3*a^2*c*d^
2*e^6 + a^3*e^8)*x^2 + 2*(c^3*d^7*e + 3*a*c^2*d^5*e^3 + 3*a^2*c*d^3*e^5 +
a^3*d*e^7)*x), 1/2*(3*B*c^2*d^5 - 5*A*c^2*d^4*e + 2*B*a*c*d^3*e^2 - 6*A*a*
c*d^2*e^3 - B*a^2*d*e^4 - A*a^2*e^5 + 2*(A*c^2*d^5 + 3*B*a*c*d^4*e - 3*A*a
*c*d^3*e^2 - B*a^2*d^2*e^3 + (A*c^2*d^3*e^2 + 3*B*a*c*d^2*e^3 - 3*A*a*c*d*
e^4 - B*a^2*e^5)*x^2 + 2*(A*c^2*d^4*e + 3*B*a*c*d^3*e^2 - 3*A*a*c*d^2*e^3
- B*a^2*d*e^4)*x)*sqrt(c/a)*arctan(x*sqrt(c/a)) + 2*(B*c^2*d^4*e - 2*A*c^2
*d^3*e^2 - 2*A*a*c*d*e^4 - B*a^2*e^5)*x + (B*c^2*d^5 - 3*A*c^2*d^4*e - ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)^3\,(a+cx^2)}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)**3/(c*x**2+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.67

$$\int \frac{A+Bx}{(d+ex)^3(a+cx^2)}\,dx = \frac{(Bc^2d^3-3\,Ac^2d^2e-3\,Bacde^2+Aace^3)\log(cx^2+a)}{2\,(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)} - \frac{(Bc^2d^3-3\,Ac^2d^2e-3\,Bacde^2+Aace^3)\log(ex+d)}{c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6} + \frac{(Ac^3d^3+3\,Bac^2d^2e-3\,Aac^2de^2-Ba^2ce^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)\sqrt{ac}} + \frac{3\,Bcd^3-5\,Acd^2e-Bade^2-Aae^3+2\,(Bcd^2e-2\,Acde^2-Bae^3)x}{2\,(c^2d^6+2\,acd^4e^2+a^2d^2e^4+(c^2d^4e^2+2\,acd^2e^4+a^2e^6)x^2+2\,(c^2d^5e+2\,acd^3e^3+a^2de^5)x)}$$

input
```
integrate((B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="maxima")
```

output
```
1/2*(B*c^2*d^3 - 3*A*c^2*d^2*e - 3*B*a*c*d*e^2 + A*a*c*e^3)*log(c*x^2 + a)
/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - (B*c^2*d^3 - 3*
A*c^2*d^2*e - 3*B*a*c*d*e^2 + A*a*c*e^3)*log(e*x + d)/(c^3*d^6 + 3*a*c^2*d
^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (A*c^3*d^3 + 3*B*a*c^2*d^2*e - 3*A*a
*c^2*d*e^2 - B*a^2*c*e^3)*arctan(c*x/sqrt(a*c))/((c^3*d^6 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*sqrt(a*c)) + 1/2*(3*B*c*d^3 - 5*A*c*d^2*e -
 B*a*d*e^2 - A*a*e^3 + 2*(B*c*d^2*e - 2*A*c*d*e^2 - B*a*e^3)*x)/(c^2*d^6 +
 2*a*c*d^4*e^2 + a^2*d^2*e^4 + (c^2*d^4*e^2 + 2*a*c*d^2*e^4 + a^2*e^6)*x^2
 + 2*(c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d*e^5)*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx}{(d+ex)^3(a+cx^2)}\,dx = \frac{(Bc^2d^3-3\,Ac^2d^2e-3\,Bacde^2+Aace^3)\log(cx^2+a)}{2\,(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)} - \frac{(Bc^2d^3e-3\,Ac^2d^2e^2-3\,Bacde^3+Aace^4)\log(|ex+d|)}{c^3d^6e+3\,ac^2d^4e^3+3\,a^2cd^2e^5+a^3e^7} + \frac{(Ac^3d^3+3\,Bac^2d^2e-3\,Aac^2de^2-Ba^2ce^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)\sqrt{ac}} + \frac{3\,Bc^2d^5-5\,Ac^2d^4e+2\,Bacd^3e^2-6\,Aacd^2e^3-Ba^2de^4-Aa^2e^5+2\,(Bc^2d^4e-2\,Ac^2d^3e^2-2\,Aacde^4}{2\,(cd^2+ae^2)^3(ex+d)^2}$$

input
```
integrate((B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="giac")
```

output
```
1/2*(B*c^2*d^3 - 3*A*c^2*d^2*e - 3*B*a*c*d*e^2 + A*a*c*e^3)*log(c*x^2 + a)
/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - (B*c^2*d^3*e -
3*A*c^2*d^2*e^2 - 3*B*a*c*d*e^3 + A*a*c*e^4)*log(abs(e*x + d))/(c^3*d^6*e
+ 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7) + (A*c^3*d^3 + 3*B*a*c^2*d^
2*e - 3*A*a*c^2*d*e^2 - B*a^2*c*e^3)*arctan(c*x/sqrt(a*c))/((c^3*d^6 + 3*a
*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*sqrt(a*c)) + 1/2*(3*B*c^2*d^5 -
5*A*c^2*d^4*e + 2*B*a*c*d^3*e^2 - 6*A*a*c*d^2*e^3 - B*a^2*d*e^4 - A*a^2*e^
5 + 2*(B*c^2*d^4*e - 2*A*c^2*d^3*e^2 - 2*A*a*c*d*e^4 - B*a^2*e^5)*x)/((c*d
^2 + a*e^2)^3*(e*x + d)^2)
```

Mupad [B] (verification not implemented)

Time = 7.21 (sec) , antiderivative size = 1680, normalized size of antiderivative = 6.69

$$\int \frac{A+Bx}{(d+ex)^3\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/((a + c*x^2)*(d + e*x)^3),x)
```

output
```
(log(B^2*a^7*e^10*(-a*c)^(1/2) - A^2*c^5*d^10*(-a*c)^(3/2) - 9*A^2*a^5*e^1
0*(-a*c)^(3/2) + 9*B^2*c^3*d^10*(-a*c)^(5/2) + A^2*a*c^7*d^10*x + B^2*a^7*
c*e^10*x + 6*A^2*a*d^4*e^6*(-a*c)^(7/2) + 6*B^2*a*d^6*e^4*(-a*c)^(7/2) - 1
06*A^2*c*d^6*e^4*(-a*c)^(7/2) + 27*B^2*c*d^8*e^2*(-a*c)^(7/2) + 9*A^2*a^6*
c^2*e^10*x + 9*B^2*a^2*c^6*d^10*x - 27*A^2*a^3*d^2*e^8*(-a*c)^(5/2) + 106*
B^2*a^3*d^4*e^6*(-a*c)^(5/2) - 77*B^2*a^5*d^2*e^8*(-a*c)^(3/2) + 77*A^2*c^
3*d^8*e^2*(-a*c)^(5/2) - 224*A*B*a*d^5*e^5*(-a*c)^(7/2) + 48*A*B*a^5*d*e^9
*(-a*c)^(3/2) - 64*A*B*c*d^7*e^3*(-a*c)^(7/2) - 48*A*B*c^3*d^9*e*(-a*c)^(5
/2) + 77*A^2*a^2*c^6*d^8*e^2*x + 106*A^2*a^3*c^5*d^6*e^4*x - 6*A^2*a^4*c^4
*d^4*e^6*x - 27*A^2*a^5*c^3*d^2*e^8*x - 27*B^2*a^3*c^5*d^8*e^2*x - 6*B^2*a
^4*c^4*d^6*e^4*x + 106*B^2*a^5*c^3*d^4*e^6*x + 77*B^2*a^6*c^2*d^2*e^8*x +
64*A*B*a^3*d^3*e^7*(-a*c)^(5/2) - 48*A*B*a^2*c^6*d^9*e*x - 48*A*B*a^6*c^2*
d*e^9*x + 64*A*B*a^3*c^5*d^7*e^3*x + 224*A*B*a^4*c^4*d^5*e^5*x + 64*A*B*a^
5*c^3*d^3*e^7*x)*(a^2*e^3*((A*c)/2 - (B*(-a*c)^(1/2))/2) - e^2*((3*B*a^2*c
*d)/2 + (3*A*a*c*d*(-a*c)^(1/2))/2) - a*e*((3*A*c^2*d^2)/2 - (3*B*c*d^2*(-
a*c)^(1/2))/2) + (B*a*c^2*d^3)/2 + (A*c^2*d^3*(-a*c)^(1/2))/2))/(a^4*e^6 +
 a*c^3*d^6 + 3*a^3*c*d^2*e^4 + 3*a^2*c^2*d^4*e^2) - (log(d + e*x)*(c^2*(B*
d^3 - 3*A*d^2*e) + a*c*(A*e^3 - 3*B*d*e^2)))/(a^3*e^6 + c^3*d^6 + 3*a*c^2*
d^4*e^2 + 3*a^2*c*d^2*e^4) - (log(9*A^2*a^5*e^10*(-a*c)^(3/2) + A^2*c^5*d^
10*(-a*c)^(3/2) - B^2*a^7*e^10*(-a*c)^(1/2) - 9*B^2*c^3*d^10*(-a*c)^(5/...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1036, normalized size of antiderivative = 4.13

$$\int \frac{A+Bx}{(d+ex)^3(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/(e*x+d)^3/(c*x^2+a),x)
```

output

```
( - 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*b*d**3*e**3 - 4*sqrt
(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*b*d**2*e**4*x - 2*sqrt(c)*sqrt
(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*b*d*e**5*x**2 - 6*sqrt(c)*sqrt(a)*atan
((c*x)/(sqrt(c)*sqrt(a)))*a*c*d**4*e**2 - 12*sqrt(c)*sqrt(a)*atan((c*x)/(s
qrt(c)*sqrt(a)))*a*c*d**3*e**3*x - 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*s
qrt(a)))*a*c*d**2*e**4*x**2 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a
)))*b*c*d**5*e + 12*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*b*c*d**4
*e**2*x + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*b*c*d**3*e**3*x*
*2 + 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**2*d**6 + 4*sqrt(c)
*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**2*d**5*e*x + 2*sqrt(c)*sqrt(a)*a
tan((c*x)/(sqrt(c)*sqrt(a)))*c**2*d**4*e**2*x**2 + log(a + c*x**2)*a**2*c*
d**3*e**3 + 2*log(a + c*x**2)*a**2*c*d**2*e**4*x + log(a + c*x**2)*a**2*c*
d*e**5*x**2 - 3*log(a + c*x**2)*a*b*c*d**4*e**2 - 6*log(a + c*x**2)*a*b*c*
d**3*e**3*x - 3*log(a + c*x**2)*a*b*c*d**2*e**4*x**2 - 3*log(a + c*x**2)*a
*c**2*d**5*e - 6*log(a + c*x**2)*a*c**2*d**4*e**2*x - 3*log(a + c*x**2)*a*
c**2*d**3*e**3*x**2 + log(a + c*x**2)*b*c**2*d**6 + 2*log(a + c*x**2)*b*c*
*2*d**5*e*x + log(a + c*x**2)*b*c**2*d**4*e**2*x**2 - 2*log(d + e*x)*a**2*
c*d**3*e**3 - 4*log(d + e*x)*a**2*c*d**2*e**4*x - 2*log(d + e*x)*a**2*c*d*
e**5*x**2 + 6*log(d + e*x)*a*b*c*d**4*e**2 + 12*log(d + e*x)*a*b*c*d**3*e*
*3*x + 6*log(d + e*x)*a*b*c*d**2*e**4*x**2 + 6*log(d + e*x)*a*c**2*d**5...
```

3.87 $\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 297

$$\begin{aligned}\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^2}\,dx &= -\frac{e^2(3Acd(2cd^2-5ae^2)-5aBe(6cd^2-ae^2))\,x}{2ac^3}-\frac{e^3(2Acd^2-5aBde-aAe^2)\,x^2}{ac^2}\\ &\quad -\frac{e^4(3Acd-5aBe)x^3}{6ac^2}-\frac{(d+ex)^4(a(Bd+Ae)-(Acd-aBe)x)}{2ac\,(a+cx^2)}\\ &\quad +\frac{(Acd(c^2d^4+10acd^2e^2-15a^2e^4)+5aBe(c^2d^4-6acd^2e^2+a^2e^4))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{7/2}}\\ &\quad +\frac{e^2(5Bcd^3+5Acd^2e-5aBde^2-aAe^3)\log(a+cx^2)}{c^3}\end{aligned}$$

output
```
-1/2*e^2*(3*A*c*d*(-5*a*e^2+2*c*d^2)-5*a*B*e*(-a*e^2+6*c*d^2))*x/a/c^3-e^3
*(-A*a*e^2+2*A*c*d^2-5*B*a*d*e)*x^2/a/c^2-1/6*e^4*(3*A*c*d-5*B*a*e)*x^3/a/
c^2-1/2*(e*x+d)^4*(a*(A*e+B*d)-(A*c*d-B*a*e)*x)/a/c/(c*x^2+a)+1/2*(A*c*d*(
-15*a^2*e^4+10*a*c*d^2*e^2+c^2*d^4)+5*a*B*e*(a^2*e^4-6*a*c*d^2*e^2+c^2*d^4
))*arctan(c^(1/2)*x/a^(1/2))/a^(3/2)/c^(7/2)+e^2*(-A*a*e^3+5*A*c*d^2*e-5*B
*a*d*e^2+5*B*c*d^3)*ln(c*x^2+a)/c^3
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^2}\,dx = \frac{e^3(10Bcd^2+5Acde-2aBe^2)\,x}{c^3}+\frac{e^4(5Bd+Ae)x^2}{2c^2}+\frac{Be^5x^3}{3c^2}$$
$$+\frac{Ac^3d^5x-a^3e^4(5Bd+Ae+Bex)+5a^2cde^2(2Bd(d+ex)+Ae(2d+ex))-ac^2d^3(5Ae(d+2ex)+B}{2ac^3\,(a+cx^2)}$$
$$+\frac{(Acd(c^2d^4+10acd^2e^2-15a^2e^4)+5aBe(c^2d^4-6acd^2e^2+a^2e^4))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{7/2}}$$
$$+\frac{e^2(5Bcd^3+5Acd^2e-5aBde^2-aAe^3)\log\left(a+cx^2\right)}{c^3}$$

input
```
Integrate[((A + B*x)*(d + e*x)^5)/(a + c*x^2)^2,x]
```

output
```
(e^3*(10*B*c*d^2 + 5*A*c*d*e - 2*a*B*e^2)*x)/c^3 + (e^4*(5*B*d + A*e)*x^2)
/(2*c^2) + (B*e^5*x^3)/(3*c^2) + (A*c^3*d^5*x - a^3*e^4*(5*B*d + A*e + B*e
*x) + 5*a^2*c*d*e^2*(2*B*d*(d + e*x) + A*e*(2*d + e*x)) - a*c^2*d^3*(5*A*e
*(d + 2*e*x) + B*d*(d + 5*e*x)))/(2*a*c^3*(a + c*x^2)) + ((A*c*d*(c^2*d^4
+ 10*a*c*d^2*e^2 - 15*a^2*e^4) + 5*a*B*e*(c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^
4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(7/2)) + (e^2*(5*B*c*d^3 + 5
*A*c*d^2*e - 5*a*B*d*e^2 - a*A*e^3)*Log[a + c*x^2])/c^3
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {684, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^2}\,dx$$
$$\Big\downarrow \; 684$$

$$\frac{\int \frac{(d+ex)^3(Acd^2+ae(5Bd+4Ae)-e(3Acd-5aBe)x)}{cx^2+a}dx}{2ac}-\frac{(d+ex)^4(a(Ae+Bd)-x(Acd-aBe))}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{657}$$

$$\frac{\int \left(-\frac{(3Acd-5aBe)x^2e^4}{c}-\frac{4(2Acd^2-5aBed-aAe^2)xe^3}{c}-\frac{(3Acd(2cd^2-5ae^2)-5aBe(6cd^2-ae^2))e^2}{c^2}+\frac{4ac(5Bcd^3+5Aced^2-5aBe^2d-aA}{}\right.}{2ac}$$

$$\frac{(d+ex)^4(a(Ae+Bd)-x(Acd-aBe))}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Acd(-15a^2e^4+10acd^2e^2+c^2d^4)+5aBe(a^2e^4-6acd^2e^2+c^2d^4))}{\sqrt{a}c^{5/2}}-\frac{e^2x(3Acd(2cd^2-5ae^2)-5aBe(6cd^2-ae^2))}{c^2}+\frac{2ae^2\log(a+c}{}}{2ac}$$

$$\frac{(d+ex)^4(a(Ae+Bd)-x(Acd-aBe))}{2ac\,(a+cx^2)}$$

input
```
Int[((A + B*x)*(d + e*x)^5)/(a + c*x^2)^2,x]
```

output
```
-1/2*((d + e*x)^4*(a*(B*d + A*e) - (A*c*d - a*B*e)*x))/(a*c*(a + c*x^2)) +
 (-((e^2*(3*A*c*d*(2*c*d^2 - 5*a*e^2) - 5*a*B*e*(6*c*d^2 - a*e^2))*x)/c^2)
 - (2*e^3*(2*A*c*d^2 - 5*a*B*d*e - a*A*e^2)*x^2)/c - (e^4*(3*A*c*d - 5*a*B
*e)*x^3)/(3*c) + ((A*c*d*(c^2*d^4 + 10*a*c*d^2*e^2 - 15*a^2*e^4) + 5*a*B*e
*(c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a
]*c^(5/2)) + (2*a*e^2*(5*B*c*d^3 + 5*A*c*d^2*e - 5*a*B*d*e^2 - a*A*e^3)*Lo
g[a + c*x^2])/c^2)/(2*a*c)
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.21

method	result
default	$\frac{e^3\left(\frac{1}{3}Bcx^3e^2+\frac{1}{2}Ace^2x^2+\frac{5}{2}Bcdex^2+5Acdex-2Bae^2x+10Bcd^2x\right)}{c^3} - \frac{-\frac{\left(5Aa^2cde^4-10Aac^2d^3e^2+Ad^5c^3-Be^5a^3+10Ba^2cd^2e^3-5\right.}{2a}}{}$
risch	Expression too large to display

input
```
int((B*x+A)*(e*x+d)^5/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
e^3/c^3*(1/3*B*c*x^3*e^2+1/2*A*c*e^2*x^2+5/2*B*c*d*e*x^2+5*A*c*d*e*x-2*B*a
*e^2*x+10*B*c*d^2*x)-1/c^3*((-1/2*(5*A*a^2*c*d*e^4-10*A*a*c^2*d^3*e^2+A*c^
3*d^5-B*a^3*e^5+10*B*a^2*c*d^2*e^3-5*B*a*c^2*d^4*e)/a*x+1/2*A*a^2*e^5-5*A*
a*c*d^2*e^3+5/2*A*c^2*d^4*e+5/2*B*a^2*d*e^4-5*B*a*c*d^3*e^2+1/2*B*c^2*d^5)
/(c*x^2+a)+1/2/a*(1/2*(4*A*a^2*c*e^5-20*A*a*c^2*d^2*e^3+20*B*a^2*c*d*e^4-2
0*B*a*c^2*d^3*e^2)/c*ln(c*x^2+a)+(15*A*a^2*c*d*e^4-10*A*a*c^2*d^3*e^2-A*c^
3*d^5-5*B*a^3*e^5+30*B*a^2*c*d^2*e^3-5*B*a*c^2*d^4*e)/(a*c)^(1/2)*arctan(c
*x/(a*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(281) = 562$.

Time = 0.10 (sec) , antiderivative size = 1190, normalized size of antiderivative = 4.01

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^5/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*B*a^2*c^3*e^5*x^5 - 6*B*a^2*c^3*d^5 - 30*A*a^2*c^3*d^4*e + 60*B*a
^3*c^2*d^3*e^2 + 60*A*a^3*c^2*d^2*e^3 - 30*B*a^4*c*d*e^4 - 6*A*a^4*c*e^5 +
 6*(5*B*a^2*c^3*d*e^4 + A*a^2*c^3*e^5)*x^4 + 20*(6*B*a^2*c^3*d^2*e^3 + 3*A
*a^2*c^3*d*e^4 - B*a^3*c^2*e^5)*x^3 + 6*(5*B*a^3*c^2*d*e^4 + A*a^3*c^2*e^5
)*x^2 - 3*(A*a*c^3*d^5 + 5*B*a^2*c^2*d^4*e + 10*A*a^2*c^2*d^3*e^2 - 30*B*a
^3*c*d^2*e^3 - 15*A*a^3*c*d*e^4 + 5*B*a^4*e^5 + (A*c^4*d^5 + 5*B*a*c^3*d^4
*e + 10*A*a*c^3*d^3*e^2 - 30*B*a^2*c^2*d^2*e^3 - 15*A*a^2*c^2*d*e^4 + 5*B*
a^3*c*e^5)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) +
 6*(A*a*c^4*d^5 - 5*B*a^2*c^3*d^4*e - 10*A*a^2*c^3*d^3*e^2 + 30*B*a^3*c^2*
d^2*e^3 + 15*A*a^3*c^2*d*e^4 - 5*B*a^4*c*e^5)*x + 12*(5*B*a^3*c^2*d^3*e^2
+ 5*A*a^3*c^2*d^2*e^3 - 5*B*a^4*c*d*e^4 - A*a^4*c*e^5 + (5*B*a^2*c^3*d^3*e
^2 + 5*A*a^2*c^3*d^2*e^3 - 5*B*a^3*c^2*d*e^4 - A*a^3*c^2*e^5)*x^2)*log(c*x
^2 + a))/(a^2*c^5*x^2 + a^3*c^4), 1/6*(2*B*a^2*c^3*e^5*x^5 - 3*B*a^2*c^3*d
^5 - 15*A*a^2*c^3*d^4*e + 30*B*a^3*c^2*d^3*e^2 + 30*A*a^3*c^2*d^2*e^3 - 15
*B*a^4*c*d*e^4 - 3*A*a^4*c*e^5 + 3*(5*B*a^2*c^3*d*e^4 + A*a^2*c^3*e^5)*x^4
 + 10*(6*B*a^2*c^3*d^2*e^3 + 3*A*a^2*c^3*d*e^4 - B*a^3*c^2*e^5)*x^3 + 3*(5
*B*a^3*c^2*d*e^4 + A*a^3*c^2*e^5)*x^2 + 3*(A*a*c^3*d^5 + 5*B*a^2*c^2*d^4*e
 + 10*A*a^2*c^2*d^3*e^2 - 30*B*a^3*c*d^2*e^3 - 15*A*a^3*c*d*e^4 + 5*B*a^4*
e^5 + (A*c^4*d^5 + 5*B*a*c^3*d^4*e + 10*A*a*c^3*d^3*e^2 - 30*B*a^2*c^2*d^2
*e^3 - 15*A*a^2*c^2*d*e^4 + 5*B*a^3*c*e^5)*x^2)*sqrt(a*c)*arctan(sqrt(a...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1091 vs. $2(294) = 588$.

Time = 6.67 (sec) , antiderivative size = 1091, normalized size of antiderivative = 3.67

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)**5/(c*x**2+a)**2,x)
```

output
```
B*e**5*x**3/(3*c**2) + x**2*(A*e**5/(2*c**2) + 5*B*d*e**4/(2*c**2)) + x*(5
*A*d*e**4/c**2 - 2*B*a*e**5/c**3 + 10*B*d**2*e**3/c**2) + (-e**2*(A*a*e**3
 - 5*A*c*d**2*e + 5*B*a*d*e**2 - 5*B*c*d**3)/c**3 - sqrt(-a**3*c**7)*(-15*
A*a**2*c*d*e**4 + 10*A*a*c**2*d**3*e**2 + A*c**3*d**5 + 5*B*a**3*e**5 - 30
*B*a**2*c*d**2*e**3 + 5*B*a*c**2*d**4*e)/(4*a**3*c**7))*log(x + (4*A*a**3*
e**5 - 20*A*a**2*c*d**2*e**3 + 20*B*a**3*d*e**4 - 20*B*a**2*c*d**3*e**2 +
4*a**2*c**3*(-e**2*(A*a*e**3 - 5*A*c*d**2*e + 5*B*a*d*e**2 - 5*B*c*d**3)/c
**3 - sqrt(-a**3*c**7)*(-15*A*a**2*c*d*e**4 + 10*A*a*c**2*d**3*e**2 + A*c*
*3*d**5 + 5*B*a**3*e**5 - 30*B*a**2*c*d**2*e**3 + 5*B*a*c**2*d**4*e)/(4*a*
*3*c**7)))/(-15*A*a**2*c*d*e**4 + 10*A*a*c**2*d**3*e**2 + A*c**3*d**5 + 5*
B*a**3*e**5 - 30*B*a**2*c*d**2*e**3 + 5*B*a*c**2*d**4*e)) + (-e**2*(A*a*e*
*3 - 5*A*c*d**2*e + 5*B*a*d*e**2 - 5*B*c*d**3)/c**3 + sqrt(-a**3*c**7)*(-1
5*A*a**2*c*d*e**4 + 10*A*a*c**2*d**3*e**2 + A*c**3*d**5 + 5*B*a**3*e**5 -
30*B*a**2*c*d**2*e**3 + 5*B*a*c**2*d**4*e)/(4*a**3*c**7))*log(x + (4*A*a**
3*e**5 - 20*A*a**2*c*d**2*e**3 + 20*B*a**3*d*e**4 - 20*B*a**2*c*d**3*e**2
+ 4*a**2*c**3*(-e**2*(A*a*e**3 - 5*A*c*d**2*e + 5*B*a*d*e**2 - 5*B*c*d**3)
/c**3 + sqrt(-a**3*c**7)*(-15*A*a**2*c*d*e**4 + 10*A*a*c**2*d**3*e**2 + A*
c**3*d**5 + 5*B*a**3*e**5 - 30*B*a**2*c*d**2*e**3 + 5*B*a*c**2*d**4*e)/(4*
a**3*c**7)))/(-15*A*a**2*c*d*e**4 + 10*A*a*c**2*d**3*e**2 + A*c**3*d**5 +
5*B*a**3*e**5 - 30*B*a**2*c*d**2*e**3 + 5*B*a*c**2*d**4*e)) + (-A*a**3*...
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.20

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^2}\,dx = -\frac{Bac^2d^5+5\,Aac^2d^4e-10\,Ba^2cd^3e^2-10\,Aa^2cd^2e^3+5\,Ba^3de^4+Aa^3e^5-(Ac^3d^5-5\,Bac^2d^4e-10\,A}{2\,(ac^4x^2+a^2c^3)}$$
$$+\frac{(5\,Bcd^3e^2+5\,Acd^2e^3-5\,Bade^4-Aae^5)\log(cx^2+a)}{c^3}$$
$$+\frac{2\,Bce^5x^3+3\,(5\,Bcde^4+Ace^5)x^2+6\,(10\,Bcd^2e^3+5\,Acde^4-2\,Bae^5)x}{6\,c^3}$$
$$+\frac{(Ac^3d^5+5\,Bac^2d^4e+10\,Aac^2d^3e^2-30\,Ba^2cd^2e^3-15\,Aa^2cde^4+5\,Ba^3e^5)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^3}$$

input
```
integrate((B*x+A)*(e*x+d)^5/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a*c^2*d^5 + 5*A*a*c^2*d^4*e - 10*B*a^2*c*d^3*e^2 - 10*A*a^2*c*d^2*
e^3 + 5*B*a^3*d*e^4 + A*a^3*e^5 - (A*c^3*d^5 - 5*B*a*c^2*d^4*e - 10*A*a*c^
2*d^3*e^2 + 10*B*a^2*c*d^2*e^3 + 5*A*a^2*c*d*e^4 - B*a^3*e^5)*x)/(a*c^4*x^
2 + a^2*c^3) + (5*B*c*d^3*e^2 + 5*A*c*d^2*e^3 - 5*B*a*d*e^4 - A*a*e^5)*log
(c*x^2 + a)/c^3 + 1/6*(2*B*c*e^5*x^3 + 3*(5*B*c*d*e^4 + A*c*e^5)*x^2 + 6*(
10*B*c*d^2*e^3 + 5*A*c*d*e^4 - 2*B*a*e^5)*x)/c^3 + 1/2*(A*c^3*d^5 + 5*B*a*
c^2*d^4*e + 10*A*a*c^2*d^3*e^2 - 30*B*a^2*c*d^2*e^3 - 15*A*a^2*c*d*e^4 + 5
*B*a^3*e^5)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^2}\,dx = \frac{(5\,Bcd^3e^2+5\,Acd^2e^3-5\,Bade^4-Aae^5)\log(cx^2+a)}{c^3}$$
$$+\frac{(Ac^3d^5+5\,Bac^2d^4e+10\,Aac^2d^3e^2-30\,Ba^2cd^2e^3-15\,Aa^2cde^4+5\,Ba^3e^5)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^3}$$
$$-\frac{Bac^2d^5+5\,Aac^2d^4e-10\,Ba^2cd^3e^2-10\,Aa^2cd^2e^3+5\,Ba^3de^4+Aa^3e^5-(Ac^3d^5-5\,Bac^2d^4e-10\,A}{2\,(cx^2+a)ac^3}$$
$$+\frac{2\,Bc^4e^5x^3+15\,Bc^4de^4x^2+3\,Ac^4e^5x^2+60\,Bc^4d^2e^3x+30\,Ac^4de^4x-12\,Bac^3e^5x}{6\,c^6}$$

input
```
integrate((B*x+A)*(e*x+d)^5/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
(5*B*c*d^3*e^2 + 5*A*c*d^2*e^3 - 5*B*a*d*e^4 - A*a*e^5)*log(c*x^2 + a)/c^3
 + 1/2*(A*c^3*d^5 + 5*B*a*c^2*d^4*e + 10*A*a*c^2*d^3*e^2 - 30*B*a^2*c*d^2*
e^3 - 15*A*a^2*c*d*e^4 + 5*B*a^3*e^5)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c
^3) - 1/2*(B*a*c^2*d^5 + 5*A*a*c^2*d^4*e - 10*B*a^2*c*d^3*e^2 - 10*A*a^2*c
*d^2*e^3 + 5*B*a^3*d*e^4 + A*a^3*e^5 - (A*c^3*d^5 - 5*B*a*c^2*d^4*e - 10*A
*a*c^2*d^3*e^2 + 10*B*a^2*c*d^2*e^3 + 5*A*a^2*c*d*e^4 - B*a^3*e^5)*x)/((c*
x^2 + a)*a*c^3) + 1/6*(2*B*c^4*e^5*x^3 + 15*B*c^4*d*e^4*x^2 + 3*A*c^4*e^5*
x^2 + 60*B*c^4*d^2*e^3*x + 30*A*c^4*d*e^4*x - 12*B*a*c^3*e^5*x)/c^6
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^2}\,dx = \frac{x^2\,(A\,e^5+5\,B\,d\,e^4)}{2\,c^2}$$

$$-\frac{\frac{A\,a^2\,e^5}{2}+\frac{B\,c^2\,d^5}{2}-\frac{x\,(-B\,a^3\,e^5+10\,B\,a^2\,c\,d^2\,e^3+5\,A\,a^2\,c\,d\,e^4-5\,B\,a\,c^2\,d^4\,e-10\,A\,a\,c^2\,d^3\,e^2+A\,c^3\,d^5)}{2\,a}+\frac{5\,B\,a^2\,d\,e^4}{2}+\frac{5\,A\,c^2\,d^4\,e}{2}-}{c^4\,x^2+a\,c^3}$$

$$-x\left(\frac{2\,B\,a\,e^5}{c^3}-\frac{5\,d\,e^3\,(A\,e+2\,B\,d)}{c^2}\right)$$

$$-\frac{\ln(c\,x^2+a)\,\left(160\,B\,a^4\,c^4\,d\,e^4+32\,A\,a^4\,c^4\,e^5-160\,B\,a^3\,c^5\,d^3\,e^2-160\,A\,a^3\,c^5\,d^2\,e^3\right)}{32\,a^3\,c^7}$$

$$+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(5\,B\,a^3\,e^5-30\,B\,a^2\,c\,d^2\,e^3-15\,A\,a^2\,c\,d\,e^4+5\,B\,a\,c^2\,d^4\,e+10\,A\,a\,c^2\,d^3\,e^2+A\,c^3\,d^5\right)}{2\,a^{3/2}\,c^{7/2}}$$

$$+\frac{B\,e^5\,x^3}{3\,c^2}$$

input
```
int(((A + B*x)*(d + e*x)^5)/(a + c*x^2)^2,x)
```

output
```
(x^2*(A*e^5 + 5*B*d*e^4))/(2*c^2) - ((A*a^2*e^5)/2 + (B*c^2*d^5)/2 - (x*(A
*c^3*d^5 - B*a^3*e^5 - 10*A*a*c^2*d^3*e^2 + 10*B*a^2*c*d^2*e^3 + 5*A*a^2*c
*d*e^4 - 5*B*a*c^2*d^4*e))/(2*a) + (5*B*a^2*d*e^4)/2 + (5*A*c^2*d^4*e)/2 -
 5*A*a*c*d^2*e^3 - 5*B*a*c*d^3*e^2)/(a*c^3 + c^4*x^2) - x*((2*B*a*e^5)/c^3
 - (5*d*e^3*(A*e + 2*B*d))/c^2) - (log(a + c*x^2)*(32*A*a^4*c^4*e^5 + 160*
B*a^4*c^4*d*e^4 - 160*A*a^3*c^5*d^2*e^3 - 160*B*a^3*c^5*d^3*e^2))/(32*a^3*
c^7) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^3*d^5 + 5*B*a^3*e^5 + 10*A*a*c^2*d^
3*e^2 - 30*B*a^2*c*d^2*e^3 - 15*A*a^2*c*d*e^4 + 5*B*a*c^2*d^4*e))/(2*a^(3/
2)*c^(7/2)) + (B*e^5*x^3)/(3*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 787, normalized size of antiderivative = 2.65

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^5/(c*x^2+a)^2,x)
```

output

```
(15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*b*e**5 - 45*sqrt(c)
*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*c*d*e**4 - 90*sqrt(c)*sqrt(a)*
atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*b*c*d**2*e**3 + 15*sqrt(c)*sqrt(a)*atan
((c*x)/(sqrt(c)*sqrt(a)))*a**2*b*c*e**5*x**2 + 30*sqrt(c)*sqrt(a)*atan((c*
x)/(sqrt(c)*sqrt(a)))*a**2*c**2*d**3*e**2 - 45*sqrt(c)*sqrt(a)*atan((c*x)/
(sqrt(c)*sqrt(a)))*a**2*c**2*d*e**4*x**2 + 15*sqrt(c)*sqrt(a)*atan((c*x)/(
sqrt(c)*sqrt(a)))*a*b*c**2*d**4*e - 90*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)
*sqrt(a)))*a*b*c**2*d**2*e**3*x**2 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)
*sqrt(a)))*a*c**3*d**5 + 30*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*
a*c**3*d**3*e**2*x**2 + 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*b
*c**3*d**4*e*x**2 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**4*d
**5*x**2 - 6*log(a + c*x**2)*a**4*c*e**5 - 30*log(a + c*x**2)*a**3*b*c*d*e
**4 + 30*log(a + c*x**2)*a**3*c**2*d**2*e**3 - 6*log(a + c*x**2)*a**3*c**2
*e**5*x**2 + 30*log(a + c*x**2)*a**2*b*c**2*d**3*e**2 - 30*log(a + c*x**2)
*a**2*b*c**2*d*e**4*x**2 + 30*log(a + c*x**2)*a**2*c**3*d**2*e**3*x**2 + 3
0*log(a + c*x**2)*a*b*c**3*d**3*e**2*x**2 - 15*a**3*b*c*e**5*x + 45*a**3*c
**2*d*e**4*x + 6*a**3*c**2*e**5*x**2 + 90*a**2*b*c**2*d**2*e**3*x + 30*a**
2*b*c**2*d*e**4*x**2 - 10*a**2*b*c**2*e**5*x**3 - 30*a**2*c**3*d**3*e**2*x
 - 30*a**2*c**3*d**2*e**3*x**2 + 30*a**2*c**3*d*e**4*x**3 + 3*a**2*c**3*e*
*5*x**4 - 15*a*b*c**3*d**4*e*x - 30*a*b*c**3*d**3*e**2*x**2 + 60*a*b*c*...
```

3.88 $\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 220

$$\begin{aligned}\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^2} \, dx &= -\frac{3e^2(Acd^2-4aBde-aAe^2)\,x}{2ac^2} - \frac{e^3(Acd-2aBe)x^2}{2ac^2} \\ &\quad - \frac{(d+ex)^3(a(Bd+Ae)-(Acd-aBe)x)}{2ac\,(a+cx^2)} \\ &\quad + \frac{(4aBde(cd^2-3ae^2)+A(c^2d^4+6acd^2e^2-3a^2e^4))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}} \\ &\quad + \frac{e^2(3Bcd^2+2Acde-aBe^2)\log(a+cx^2)}{c^3}\end{aligned}$$

output

```
-3/2*e^2*(-A*a*e^2+A*c*d^2-4*B*a*d*e)*x/a/c^2-1/2*e^3*(A*c*d-2*B*a*e)*x^2/
a/c^2-1/2*(e*x+d)^3*(a*(A*e+B*d)-(A*c*d-B*a*e)*x)/a/c/(c*x^2+a)+1/2*(4*a*B
*d*e*(-3*a*e^2+c*d^2)+A*(-3*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4))*arctan(c^(1/2)
*x/a^(1/2))/a^(3/2)/c^(5/2)+e^2*(2*A*c*d*e-B*a*e^2+3*B*c*d^2)*ln(c*x^2+a)/
c^3
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^2}\,dx$$
$$= \frac{2ce^3(4Bd+Ae)x+Bce^4x^2+\frac{-a^3Be^4+Ac^3d^4x+a^2ce^2(Ae(4d+ex)+2Bd(3d+2ex))-ac^2d^2(2Ae(2d+3ex)+Bd(d+4ex))}{a(a+cx^2)}+\frac{\sqrt{c}(4\cdots}{\cdots}}{2c^3}$$

input
```
Integrate[((A + B*x)*(d + e*x)^4)/(a + c*x^2)^2,x]
```

output
```
(2*c*e^3*(4*B*d + A*e)*x + B*c*e^4*x^2 + (-(a^3*B*e^4) + A*c^3*d^4*x + a^2
*c*e^2*(A*e*(4*d + e*x) + 2*B*d*(3*d + 2*e*x)) - a*c^2*d^2*(2*A*e*(2*d + 3
*e*x) + B*d*(d + 4*e*x)))/(a*(a + c*x^2)) + (Sqrt[c]*(4*a*B*d*e*(c*d^2 - 3
*a*e^2) + A*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4))*ArcTan[(Sqrt[c]*x)/Sqrt
[a]])/a^(3/2) + 2*e^2*(3*B*c*d^2 + 2*A*c*d*e - a*B*e^2)*Log[a + c*x^2])/(2
*c^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {684, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^2}\,dx$$
$$\Big\downarrow \; 684$$
$$\frac{\int \frac{(d+ex)^2(Acd^2+ae(4Bd+3Ae)-2e(Acd-2aBe)x)}{cx^2+a}dx}{2ac}-\frac{(d+ex)^3(a(Ae+Bd)-x(Acd-aBe))}{2ac\,(a+cx^2)}$$
$$\Big\downarrow \; 657$$

$$\frac{\int \left(-\frac{2(Acd-2aBe)xe^3}{c}-\frac{3(Acd^2-4aBed-aAe^2)e^2}{c}+\frac{4a(3Bcd^2+2Aced-aBe^2)xe^2+4aBd(cd^2-3ae^2)e+A(c^2d^4+6ace^2d^2-3a^2e^4)}{c(cx^2+a)}\right)dx}{2ac}$$
$$\frac{(d+ex)^3(a(Ae+Bd)-x(Acd-aBe))}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(A(-3a^2e^4+6acd^2e^2+c^2d^4)+4aBde(cd^2-3ae^2))}{\sqrt{a}c^{3/2}}+\frac{2ae^2\log(a+cx^2)(-aBe^2+2Acde+3Bcd^2)}{c^2}-\frac{3e^2x(-aAe^2-4aBde+Acd^2}{c}}{2ac}$$
$$\frac{(d+ex)^3(a(Ae+Bd)-x(Acd-aBe))}{2ac\,(a+cx^2)}$$

input
```
Int[((A + B*x)*(d + e*x)^4)/(a + c*x^2)^2,x]
```

output
```
-1/2*((d + e*x)^3*(a*(B*d + A*e) - (A*c*d - a*B*e)*x))/(a*c*(a + c*x^2)) +
 ((-3*e^2*(A*c*d^2 - 4*a*B*d*e - a*A*e^2)*x)/c - (e^3*(A*c*d - 2*a*B*e)*x^
2)/c + ((4*a*B*d*e*(c*d^2 - 3*a*e^2) + A*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*
e^4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (2*a*e^2*(3*B*c*d^2
 + 2*A*c*d*e - a*B*e^2)*Log[a + c*x^2])/c^2)/(2*a*c)
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.20

method	result
default	$\frac{e^{3}\left(\frac{1}{2} B e \,x^{2}+A e x +4 B d x \right)}{c^{2}}-\frac{\frac{-\frac{\left(A \,a^{2} e^{4}-6 A a c \,d^{2} e^{2}+A \,c^{2} d^{4}+4 B \,a^{2} d \,e^{3}-4 B a c \,d^{3} e \right) x}{2 a}-\frac{4 A a c d \,e^{3}-4 A \,c^{2} d^{3} e -B \,e^{4} a^{2}+6 B a c \,d^{2} e^{2}-B \,c^{2} d^{4}}{2 c}}{c \,x^{2}+a}+$
risch	Expression too large to display

input
```
int((B*x+A)*(e*x+d)^4/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
e^3/c^2*(1/2*B*e*x^2+A*e*x+4*B*d*x)-1/c^2*((-1/2*(A*a^2*e^4-6*A*a*c*d^2*e^
2+A*c^2*d^4+4*B*a^2*d*e^3-4*B*a*c*d^3*e)/a*x-1/2*(4*A*a*c*d*e^3-4*A*c^2*d^
3*e-B*a^2*e^4+6*B*a*c*d^2*e^2-B*c^2*d^4)/c)/(c*x^2+a)+1/2/a*(1/2*(-8*A*a*c
*d*e^3+4*B*a^2*e^4-12*B*a*c*d^2*e^2)/c*ln(c*x^2+a)+(3*A*a^2*e^4-6*A*a*c*d^
2*e^2-A*c^2*d^4+12*B*a^2*d*e^3-4*B*a*c*d^3*e)/(a*c)^(1/2)*arctan(c*x/(a*c)
^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(204) = 408.

Time = 0.09 (sec) , antiderivative size = 849, normalized size of antiderivative = 3.86

$$\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^2}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^4/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*B*a^2*c^2*e^4*x^4 + 2*B*a^3*c*e^4*x^2 - 2*B*a^2*c^2*d^4 - 8*A*a^2*
c^2*d^3*e + 12*B*a^3*c*d^2*e^2 + 8*A*a^3*c*d*e^3 - 2*B*a^4*e^4 + 4*(4*B*a^
2*c^2*d*e^3 + A*a^2*c^2*e^4)*x^3 + (A*a*c^2*d^4 + 4*B*a^2*c*d^3*e + 6*A*a^
2*c*d^2*e^2 - 12*B*a^3*d*e^3 - 3*A*a^3*e^4 + (A*c^3*d^4 + 4*B*a*c^2*d^3*e
+ 6*A*a*c^2*d^2*e^2 - 12*B*a^2*c*d*e^3 - 3*A*a^2*c*e^4)*x^2)*sqrt(-a*c)*lo
g((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(A*a*c^3*d^4 - 4*B*a^2*c^2
*d^3*e - 6*A*a^2*c^2*d^2*e^2 + 12*B*a^3*c*d*e^3 + 3*A*a^3*c*e^4)*x + 4*(3*
B*a^3*c*d^2*e^2 + 2*A*a^3*c*d*e^3 - B*a^4*e^4 + (3*B*a^2*c^2*d^2*e^2 + 2*A
*a^2*c^2*d*e^3 - B*a^3*c*e^4)*x^2)*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)
, 1/2*(B*a^2*c^2*e^4*x^4 + B*a^3*c*e^4*x^2 - B*a^2*c^2*d^4 - 4*A*a^2*c^2*d
^3*e + 6*B*a^3*c*d^2*e^2 + 4*A*a^3*c*d*e^3 - B*a^4*e^4 + 2*(4*B*a^2*c^2*d*
e^3 + A*a^2*c^2*e^4)*x^3 + (A*a*c^2*d^4 + 4*B*a^2*c*d^3*e + 6*A*a^2*c*d^2*
e^2 - 12*B*a^3*d*e^3 - 3*A*a^3*e^4 + (A*c^3*d^4 + 4*B*a*c^2*d^3*e + 6*A*a*
c^2*d^2*e^2 - 12*B*a^2*c*d*e^3 - 3*A*a^2*c*e^4)*x^2)*sqrt(a*c)*arctan(sqrt
(a*c)*x/a) + (A*a*c^3*d^4 - 4*B*a^2*c^2*d^3*e - 6*A*a^2*c^2*d^2*e^2 + 12*B
*a^3*c*d*e^3 + 3*A*a^3*c*e^4)*x + 2*(3*B*a^3*c*d^2*e^2 + 2*A*a^3*c*d*e^3 -
 B*a^4*e^4 + (3*B*a^2*c^2*d^2*e^2 + 2*A*a^2*c^2*d*e^3 - B*a^3*c*e^4)*x^2)*
log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 836 vs. $2(212) = 424$.

Time = 4.67 (sec) , antiderivative size = 836, normalized size of antiderivative = 3.80

$$\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^2}\,dx$$
$$= \frac{Be^4x^2}{2c^2} + x\left(\frac{Ae^4}{c^2} + \frac{4Bde^3}{c^2}\right) + \left(-\frac{e^2(-2Acde+Bae^2-3Bcd^2)}{c^3} - \frac{\sqrt{-a^3c^7}\cdot(3Aa^2e^4-6Aacd^2e^2-Ac^2d^4+12Ba^2de^3-4Bacd^3e)}{4a^3c^6}\right)\log\left(x+\frac{8Aa^2cde^3-4Ba^3e^4+1\right.$$
$$+\left(-\frac{e^2(-2Acde+Bae^2-3Bcd^2)}{c^3} + \frac{\sqrt{-a^3c^7}\cdot(3Aa^2e^4-6Aacd^2e^2-Ac^2d^4+12Ba^2de^3-4Bacd^3e)}{4a^3c^6}\right)\log\left(x+\frac{8Aa^2cde^3-4Ba^3e^4+1\right.$$
$$+\frac{4Aa^2cde^3-4Aac^2d^3e-Ba^3e^4+6Ba^2cd^2e^2-Bac^2d^4+x(Aa^2ce^4-6Aac^2d^2e^2+Ac^3d^4+4Ba^2cde^3}{2a^2c^3+2ac^4x^2}$$

input
```
integrate((B*x+A)*(e*x+d)**4/(c*x**2+a)**2,x)
```

output
```
B*e**4*x**2/(2*c**2) + x*(A*e**4/c**2 + 4*B*d*e**3/c**2) + (-e**2*(-2*A*c*
d*e + B*a*e**2 - 3*B*c*d**2)/c**3 - sqrt(-a**3*c**7)*(3*A*a**2*e**4 - 6*A*
a*c*d**2*e**2 - A*c**2*d**4 + 12*B*a**2*d*e**3 - 4*B*a*c*d**3*e)/(4*a**3*c
**6))*log(x + (8*A*a**2*c*d*e**3 - 4*B*a**3*e**4 + 12*B*a**2*c*d**2*e**2 -
 4*a**2*c**3*(-e**2*(-2*A*c*d*e + B*a*e**2 - 3*B*c*d**2)/c**3 - sqrt(-a**3
*c**7)*(3*A*a**2*e**4 - 6*A*a*c*d**2*e**2 - A*c**2*d**4 + 12*B*a**2*d*e**3
 - 4*B*a*c*d**3*e)/(4*a**3*c**6)))/(3*A*a**2*c*e**4 - 6*A*a*c**2*d**2*e**2
 - A*c**3*d**4 + 12*B*a**2*c*d*e**3 - 4*B*a*c**2*d**3*e)) + (-e**2*(-2*A*c
*d*e + B*a*e**2 - 3*B*c*d**2)/c**3 + sqrt(-a**3*c**7)*(3*A*a**2*e**4 - 6*A
*a*c*d**2*e**2 - A*c**2*d**4 + 12*B*a**2*d*e**3 - 4*B*a*c*d**3*e)/(4*a**3*
c**6))*log(x + (8*A*a**2*c*d*e**3 - 4*B*a**3*e**4 + 12*B*a**2*c*d**2*e**2
- 4*a**2*c**3*(-e**2*(-2*A*c*d*e + B*a*e**2 - 3*B*c*d**2)/c**3 + sqrt(-a**
3*c**7)*(3*A*a**2*e**4 - 6*A*a*c*d**2*e**2 - A*c**2*d**4 + 12*B*a**2*d*e**
3 - 4*B*a*c*d**3*e)/(4*a**3*c**6)))/(3*A*a**2*c*e**4 - 6*A*a*c**2*d**2*e**
2 - A*c**3*d**4 + 12*B*a**2*c*d*e**3 - 4*B*a*c**2*d**3*e)) + (4*A*a**2*c*d
*e**3 - 4*A*a*c**2*d**3*e - B*a**3*e**4 + 6*B*a**2*c*d**2*e**2 - B*a*c**2*
d**4 + x*(A*a**2*c*e**4 - 6*A*a*c**2*d**2*e**2 + A*c**3*d**4 + 4*B*a**2*c*
d*e**3 - 4*B*a*c**2*d**3*e))/(2*a**2*c**3 + 2*a*c**4*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx)(d+ex)^4}{\left(a+cx^2\right)^2}\,dx = -\frac{Bac^2d^4+4\,Aac^2d^3e-6\,Ba^2cd^2e^2-4\,Aa^2cde^3+Ba^3e^4-(Ac^3d^4-4\,Bac^2d^3e-6\,Aac^2d^2e^2+4\,Ba^2 \cdots}{2\,(ac^4x^2+a^2c^3)} + \frac{Be^4x^2+2\,(4\,Bde^3+Ae^4)x}{2\,c^2} + \frac{(3\,Bcd^2e^2+2\,Acde^3-Bae^4)\log\left(cx^2+a\right)}{c^3} + \frac{(Ac^2d^4+4\,Bacd^3e+6\,Aacd^2e^2-12\,Ba^2de^3-3\,Aa^2e^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2}$$

input
```
integrate((B*x+A)*(e*x+d)^4/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a*c^2*d^4 + 4*A*a*c^2*d^3*e - 6*B*a^2*c*d^2*e^2 - 4*A*a^2*c*d*e^3
+ B*a^3*e^4 - (A*c^3*d^4 - 4*B*a*c^2*d^3*e - 6*A*a*c^2*d^2*e^2 + 4*B*a^2*c
*d*e^3 + A*a^2*c*e^4)*x)/(a*c^4*x^2 + a^2*c^3) + 1/2*(B*e^4*x^2 + 2*(4*B*d
*e^3 + A*e^4)*x)/c^2 + (3*B*c*d^2*e^2 + 2*A*c*d*e^3 - B*a*e^4)*log(c*x^2 +
 a)/c^3 + 1/2*(A*c^2*d^4 + 4*B*a*c*d^3*e + 6*A*a*c*d^2*e^2 - 12*B*a^2*d*e^
3 - 3*A*a^2*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^2}\,dx = \frac{(3\,Bcd^2e^2+2\,Acde^3-Bae^4)\log(cx^2+a)}{c^3} + \frac{(Ac^2d^4+4\,Bacd^3e+6\,Aacd^2e^2-12\,Ba^2de^3-3\,Aa^2e^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2} + \frac{Bc^2e^4x^2+8\,Bc^2de^3x+2\,Ac^2e^4x}{2\,c^4} - \frac{Bac^2d^4+4\,Aac^2d^3e-6\,Ba^2cd^2e^2-4\,Aa^2cde^3+Ba^3e^4-(Ac^3d^4-4\,Bac^2d^3e-6\,Aac^2d^2e^2+4\,Ba^2c}{2\,(cx^2+a)ac^3}$$

input
```
integrate((B*x+A)*(e*x+d)^4/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
(3*B*c*d^2*e^2 + 2*A*c*d*e^3 - B*a*e^4)*log(c*x^2 + a)/c^3 + 1/2*(A*c^2*d^
4 + 4*B*a*c*d^3*e + 6*A*a*c*d^2*e^2 - 12*B*a^2*d*e^3 - 3*A*a^2*e^4)*arctan
(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2) + 1/2*(B*c^2*e^4*x^2 + 8*B*c^2*d*e^3*x +
 2*A*c^2*e^4*x)/c^4 - 1/2*(B*a*c^2*d^4 + 4*A*a*c^2*d^3*e - 6*B*a^2*c*d^2*e
^2 - 4*A*a^2*c*d*e^3 + B*a^3*e^4 - (A*c^3*d^4 - 4*B*a*c^2*d^3*e - 6*A*a*c^
2*d^2*e^2 + 4*B*a^2*c*d*e^3 + A*a^2*c*e^4)*x)/((c*x^2 + a)*a*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^2}\,dx = \frac{x\,(A\,e^4+4\,B\,d\,e^3)}{c^2} - \frac{\frac{B\,a^2\,e^4-6\,B\,a\,c\,d^2\,e^2-4\,A\,a\,c\,d\,e^3+B\,c^2\,d^4+4\,A\,c^2\,d^3\,e}{2\,c}-\frac{x\,(4\,B\,a^2\,d\,e^3+A\,a^2\,e^4-4\,B\,a\,c\,d^3\,e-6\,A\,a\,c\,d^2\,e^2+A\,c^2\,d^4)}{2\,a}}{c^3\,x^2+a\,c^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(-12\,B\,a^2\,d\,e^3-3\,A\,a^2\,e^4+4\,B\,a\,c\,d^3\,e+6\,A\,a\,c\,d^2\,e^2+A\,c^2\,d^4)}{2\,a^{3/2}\,c^{5/2}} + \frac{\ln(c\,x^2+a)\,(-32\,B\,a^4\,c^3\,e^4+96\,B\,a^3\,c^4\,d^2\,e^2+64\,A\,a^3\,c^4\,d\,e^3)}{32\,a^3\,c^6} + \frac{B\,e^4\,x^2}{2\,c^2}$$

input
```
int(((A + B*x)*(d + e*x)^4)/(a + c*x^2)^2,x)
```

output
```
(x*(A*e^4 + 4*B*d*e^3))/c^2 - ((B*a^2*e^4 + B*c^2*d^4 + 4*A*c^2*d^3*e - 4*
A*a*c*d*e^3 - 6*B*a*c*d^2*e^2)/(2*c) - (x*(A*a^2*e^4 + A*c^2*d^4 + 4*B*a^2
*d*e^3 - 4*B*a*c*d^3*e - 6*A*a*c*d^2*e^2))/(2*a))/(a*c^2 + c^3*x^2) + (ata
n((c^(1/2)*x)/a^(1/2))*(A*c^2*d^4 - 3*A*a^2*e^4 - 12*B*a^2*d*e^3 + 4*B*a*c
*d^3*e + 6*A*a*c*d^2*e^2))/(2*a^(3/2)*c^(5/2)) + (log(a + c*x^2)*(64*A*a^3
*c^4*d*e^3 - 32*B*a^4*c^3*e^4 + 96*B*a^3*c^4*d^2*e^2))/(32*a^3*c^6) + (B*e
^4*x^2)/(2*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.62

$$\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^2}\,dx = \frac{-12\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)abcd\,e^3x^2+4\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)abc\,d^3e+6\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a\,c^2d^2e^2x^2+4}{}$$

input
```
int((B*x+A)*(e*x+d)^4/(c*x^2+a)^2,x)
```

output
```
( - 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*e**4 - 12*sqrt(c)
*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*b*d*e**3 + 6*sqrt(c)*sqrt(a)*a
tan((c*x)/(sqrt(c)*sqrt(a)))*a**2*c*d**2*e**2 - 3*sqrt(c)*sqrt(a)*atan((c*
x)/(sqrt(c)*sqrt(a)))*a**2*c*e**4*x**2 + 4*sqrt(c)*sqrt(a)*atan((c*x)/(sqr
t(c)*sqrt(a)))*a*b*c*d**3*e - 12*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(
a)))*a*b*c*d*e**3*x**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c
**2*d**4 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c**2*d**2*e**
2*x**2 + 4*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*b*c**2*d**3*e*x**
2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**3*d**4*x**2 - 2*log(a
 + c*x**2)*a**3*b*e**4 + 4*log(a + c*x**2)*a**3*c*d*e**3 + 6*log(a + c*x**
2)*a**2*b*c*d**2*e**2 - 2*log(a + c*x**2)*a**2*b*c*e**4*x**2 + 4*log(a + c
*x**2)*a**2*c**2*d*e**3*x**2 + 6*log(a + c*x**2)*a*b*c**2*d**2*e**2*x**2 +
 3*a**3*c*e**4*x + 12*a**2*b*c*d*e**3*x + 2*a**2*b*c*e**4*x**2 - 6*a**2*c*
*2*d**2*e**2*x - 4*a**2*c**2*d*e**3*x**2 + 2*a**2*c**2*e**4*x**3 - 4*a*b*c
**2*d**3*e*x - 6*a*b*c**2*d**2*e**2*x**2 + 8*a*b*c**2*d*e**3*x**3 + a*b*c*
*2*e**4*x**4 + a*c**3*d**4*x + 4*a*c**3*d**3*e*x**2 + b*c**3*d**4*x**2)/(2
*a*c**3*(a + c*x**2))
```

3.89 $\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 161

$$\begin{aligned}\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^2} dx = &-\frac{e^2(Acd-3aBe)x}{2ac^2} \\ &-\frac{(d+ex)^2(a(Bd+Ae)-(Acd-aBe)x)}{2ac(a+cx^2)} \\ &+\frac{(3aBe(cd^2-ae^2)+Acd(cd^2+3ae^2))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}} \\ &+\frac{e^2(3Bd+Ae)\log(a+cx^2)}{2c^2}\end{aligned}$$

output
```
-1/2*e^2*(A*c*d-3*B*a*e)*x/a/c^2-1/2*(e*x+d)^2*(a*(A*e+B*d)-(A*c*d-B*a*e)*
x)/a/c/(c*x^2+a)+1/2*(3*a*B*e*(-a*e^2+c*d^2)+A*c*d*(3*a*e^2+c*d^2))*arctan
(c^(1/2)*x/a^(1/2))/a^(3/2)/c^(5/2)+1/2*e^2*(A*e+3*B*d)*ln(c*x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^2}\,dx$$
$$= \frac{2B\sqrt{c}e^3x+\frac{\sqrt{c}(Ac^2d^3x+a^2e^2(3Bd+Ae+Bex)-acd(3Ae(d+ex)+Bd(d+3ex)))}{a(a+cx^2)}+\frac{(3aBe(cd^2-ae^2)+Acd(cd^2+3ae^2))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a^{3/2}}}{2c^{5/2}}$$

input
```
Integrate[((A + B*x)*(d + e*x)^3)/(a + c*x^2)^2,x]
```

output
```
(2*B*Sqrt[c]*e^3*x + (Sqrt[c]*(A*c^2*d^3*x + a^2*e^2*(3*B*d + A*e + B*e*x)
 - a*c*d*(3*A*e*(d + e*x) + B*d*(d + 3*e*x))))/(a*(a + c*x^2)) + ((3*a*B*e
*(c*d^2 - a*e^2) + A*c*d*(c*d^2 + 3*a*e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a
^(3/2) + Sqrt[c]*e^2*(3*B*d + A*e)*Log[a + c*x^2])/(2*c^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {684, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^2}\,dx$$
$$\Big\downarrow \ 684$$
$$\frac{\int \frac{(d+ex)(Acd^2+ae(3Bd+2Ae)-e(Acd-3aBe)x)}{cx^2+a}dx}{2ac}-\frac{(d+ex)^2(a(Ae+Bd)-x(Acd-aBe))}{2ac\,(a+cx^2)}$$
$$\Big\downarrow \ 657$$
$$\frac{\int \left(\frac{2ac(3Bd+Ae)xe^2+3aB(cd^2-ae^2)e+Acd(cd^2+3ae^2)}{c(cx^2+a)}-e^2\left(Ad-\frac{3aBe}{c}\right)\right)dx}{2ac}-\frac{(d+ex)^2(a(Ae+Bd)-x(Acd-aBe))}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd(3ae^2+cd^2)+3aBe(cd^2-ae^2)\right)}{\sqrt{a}c^{3/2}}+\frac{ae^2\log(a+cx^2)(Ae+3Bd)}{c}-\left(e^2x\left(Ad-\frac{3aBe}{c}\right)\right)}{2ac}-\frac{(d+ex)^2(a(Ae+Bd)-x(Acd-aBe))}{2ac\,(a+cx^2)}$$

input
```
Int[((A + B*x)*(d + e*x)^3)/(a + c*x^2)^2,x]
```

output
```
-1/2*((d + e*x)^2*(a*(B*d + A*e) - (A*c*d - a*B*e)*x))/(a*c*(a + c*x^2)) +
 (-(e^2*(A*d - (3*a*B*e)/c)*x) + ((3*a*B*e*(c*d^2 - a*e^2) + A*c*d*(c*d^2
+ 3*a*e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (a*e^2*(3*B*d
 + A*e)*Log[a + c*x^2])/c)/(2*a*c)
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.17

method	result
default	$\frac{B e^3 x}{c^2}+\frac{\frac{-\frac{\left(3Aacd\,e^2-A\,c^2d^3-B\,e^3a^2+3Bac\,d^2e\right)x}{2a}+\frac{Aa\,e^3}{2}-\frac{3Ac\,d^2e}{2}+\frac{3Bad\,e^2}{2}-\frac{Bc\,d^3}{2}}{c\,x^2+a}+\frac{\left(2Aac\,e^3+6Bacd\,e^2\right)\ln\left(c\,x^2+a\right)}{2c}+\frac{\left(3Aacd\,e^2+\right.}{2a}}{c^2}$
risch	$\frac{B\,e^3x}{c^2}+\frac{-\frac{\left(3Aacd\,e^2-A\,c^2d^3-B\,e^3a^2+3Bac\,d^2e\right)x}{2a}+\frac{Aa\,e^3}{2}-\frac{3Ac\,d^2e}{2}+\frac{3Bad\,e^2}{2}-\frac{Bc\,d^3}{2}}{c^2\left(c\,x^2+a\right)}+\ln\Big(3A\,a^2cd\,e^2+A\,d^3a\,c^2-3B\,e^3a^3+3B\,c$

input
```
int((B*x+A)*(e*x+d)^3/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
B*e^3/c^2*x+1/c^2*((-1/2*(3*A*a*c*d*e^2-A*c^2*d^3-B*a^2*e^3+3*B*a*c*d^2*e)
/a*x+1/2*A*a*e^3-3/2*A*c*d^2*e+3/2*B*a*d*e^2-1/2*B*c*d^3)/(c*x^2+a)+1/2/a*
(1/2*(2*A*a*c*e^3+6*B*a*c*d*e^2)/c*ln(c*x^2+a)+(3*A*a*c*d*e^2+A*c^2*d^3-3*
B*a^2*e^3+3*B*a*c*d^2*e)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(145) = 290.

Time = 0.09 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.78

$$\int \frac{(A+Bx)(d+ex)^3}{\left(a+cx^2\right)^2}\,dx$$
$$=\left[\frac{4\,Ba^2c^2e^3x^3-2\,Ba^2c^2d^3-6\,Aa^2c^2d^2e+6\,Ba^3cde^2+2\,Aa^3ce^3+\left(Aac^2d^3+3\,Ba^2cd^2e+3\,Aa^2cde^2-\right.}{}\right.$$

input
```
integrate((B*x+A)*(e*x+d)^3/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*B*a^2*c^2*e^3*x^3 - 2*B*a^2*c^2*d^3 - 6*A*a^2*c^2*d^2*e + 6*B*a^3*
c*d*e^2 + 2*A*a^3*c*e^3 + (A*a*c^2*d^3 + 3*B*a^2*c*d^2*e + 3*A*a^2*c*d*e^2
 - 3*B*a^3*e^3 + (A*c^3*d^3 + 3*B*a*c^2*d^2*e + 3*A*a*c^2*d*e^2 - 3*B*a^2*
c*e^3)*x^2)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(
A*a*c^3*d^3 - 3*B*a^2*c^2*d^2*e - 3*A*a^2*c^2*d*e^2 + 3*B*a^3*c*e^3)*x + 2
*(3*B*a^3*c*d*e^2 + A*a^3*c*e^3 + (3*B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2)
*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3), 1/2*(2*B*a^2*c^2*e^3*x^3 - B*a^2
*c^2*d^3 - 3*A*a^2*c^2*d^2*e + 3*B*a^3*c*d*e^2 + A*a^3*c*e^3 + (A*a*c^2*d^
3 + 3*B*a^2*c*d^2*e + 3*A*a^2*c*d*e^2 - 3*B*a^3*e^3 + (A*c^3*d^3 + 3*B*a*c
^2*d^2*e + 3*A*a*c^2*d*e^2 - 3*B*a^2*c*e^3)*x^2)*sqrt(a*c)*arctan(sqrt(a*c
)*x/a) + (A*a*c^3*d^3 - 3*B*a^2*c^2*d^2*e - 3*A*a^2*c^2*d*e^2 + 3*B*a^3*c*
e^3)*x + (3*B*a^3*c*d*e^2 + A*a^3*c*e^3 + (3*B*a^2*c^2*d*e^2 + A*a^2*c^2*e
^3)*x^2)*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(148) = 296$.

Time = 2.73 (sec) , antiderivative size = 583, normalized size of antiderivative = 3.62

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^2}\,dx = \frac{Be^3x}{c^2} + \left(\frac{e^2(Ae+3Bd)}{2c^2} - \frac{\sqrt{-a^3c^5}(-3Aacde^2 - Ac^2d^3 + 3Ba^2e^3 - 3Bacd^2e)}{4a^3c^5}\right)\log\left(x + \frac{2Aa^2e^3 + 6Ba^2de^2 - 4a^2c^2\left(\frac{e^2(Ae+3Bd}{2c^2}\right.}{-3Aacde^2 - Ac^2d}\right. + \left(\frac{e^2(Ae+3Bd)}{2c^2} + \frac{\sqrt{-a^3c^5}(-3Aacde^2 - Ac^2d^3 + 3Ba^2e^3 - 3Bacd^2e)}{4a^3c^5}\right)\log\left(x + \frac{2Aa^2e^3 + 6Ba^2de^2 - 4a^2c^2\left(\frac{e^2(Ae+3Bd}{2c^2}\right.}{-3Aacde^2 - Ac^2d}\right. + \frac{Aa^2e^3 - 3Aacd^2e + 3Ba^2de^2 - Bacd^3 + x(-3Aacde^2 + Ac^2d^3 + Ba^2e^3 - 3Bacd^2e)}{2a^2c^2 + 2ac^3x^2}$$

input
```
integrate((B*x+A)*(e*x+d)**3/(c*x**2+a)**2,x)
```

output
```
B*e**3*x/c**2 + (e**2*(A*e + 3*B*d)/(2*c**2) - sqrt(-a**3*c**5)*(-3*A*a*c*
d*e**2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e)/(4*a**3*c**5))*log(
x + (2*A*a**2*e**3 + 6*B*a**2*d*e**2 - 4*a**2*c**2*(e**2*(A*e + 3*B*d)/(2*
c**2) - sqrt(-a**3*c**5)*(-3*A*a*c*d*e**2 - A*c**2*d**3 + 3*B*a**2*e**3 -
3*B*a*c*d**2*e)/(4*a**3*c**5)))/(-3*A*a*c*d*e**2 - A*c**2*d**3 + 3*B*a**2*
e**3 - 3*B*a*c*d**2*e)) + (e**2*(A*e + 3*B*d)/(2*c**2) + sqrt(-a**3*c**5)*
(-3*A*a*c*d*e**2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e)/(4*a**3*c
**5))*log(x + (2*A*a**2*e**3 + 6*B*a**2*d*e**2 - 4*a**2*c**2*(e**2*(A*e +
3*B*d)/(2*c**2) + sqrt(-a**3*c**5)*(-3*A*a*c*d*e**2 - A*c**2*d**3 + 3*B*a*
*2*e**3 - 3*B*a*c*d**2*e)/(4*a**3*c**5)))/(-3*A*a*c*d*e**2 - A*c**2*d**3 +
 3*B*a**2*e**3 - 3*B*a*c*d**2*e)) + (A*a**2*e**3 - 3*A*a*c*d**2*e + 3*B*a*
*2*d*e**2 - B*a*c*d**3 + x*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*a**2*e**3 -
3*B*a*c*d**2*e))/(2*a**2*c**2 + 2*a*c**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^2}\,dx = \frac{Be^3x}{c^2} - \frac{Bacd^3+3\,Aacd^2e-3\,Ba^2de^2-Aa^2e^3-(Ac^2d^3-3\,Bacd^2e-3\,Aacde^2+Ba^2e^3)x}{2\,(ac^3x^2+a^2c^2)} + \frac{(3\,Bde^2+Ae^3)\log(cx^2+a)}{2\,c^2} + \frac{(Ac^2d^3+3\,Bacd^2e+3\,Aacde^2-3\,Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2}$$

input
```
integrate((B*x+A)*(e*x+d)^3/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
B*e^3*x/c^2 - 1/2*(B*a*c*d^3 + 3*A*a*c*d^2*e - 3*B*a^2*d*e^2 - A*a^2*e^3 -
 (A*c^2*d^3 - 3*B*a*c*d^2*e - 3*A*a*c*d*e^2 + B*a^2*e^3)*x)/(a*c^3*x^2 + a
^2*c^2) + 1/2*(3*B*d*e^2 + A*e^3)*log(c*x^2 + a)/c^2 + 1/2*(A*c^2*d^3 + 3*
B*a*c*d^2*e + 3*A*a*c*d*e^2 - 3*B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c
)*a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^2}\,dx = \frac{Be^3x}{c^2} + \frac{(3\,Bde^2+Ae^3)\log\left(cx^2+a\right)}{2\,c^2} + \frac{(Ac^2d^3+3\,Bacd^2e+3\,Aacde^2-3\,Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2} - \frac{Bacd^3+3\,Aacd^2e-3\,Ba^2de^2-Aa^2e^3-(Ac^2d^3-3\,Bacd^2e-3\,Aacde^2+Ba^2e^3)x}{2\,(cx^2+a)ac^2}$$

input
```
integrate((B*x+A)*(e*x+d)^3/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
B*e^3*x/c^2 + 1/2*(3*B*d*e^2 + A*e^3)*log(c*x^2 + a)/c^2 + 1/2*(A*c^2*d^3
+ 3*B*a*c*d^2*e + 3*A*a*c*d*e^2 - 3*B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt
(a*c)*a*c^2) - 1/2*(B*a*c*d^3 + 3*A*a*c*d^2*e - 3*B*a^2*d*e^2 - A*a^2*e^3
- (A*c^2*d^3 - 3*B*a*c*d^2*e - 3*A*a*c*d*e^2 + B*a^2*e^3)*x)/((c*x^2 + a)*
a*c^2)
```

Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.20

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^2}\,dx = \frac{\frac{x\,\left(B\,a^2\,e^3-3\,B\,a\,c\,d^2\,e-3\,A\,a\,c\,d\,e^2+A\,c^2\,d^3\right)}{2\,a}+\frac{A\,a\,e^3}{2}-\frac{B\,c\,d^3}{2}+\frac{3\,B\,a\,d\,e^2}{2}-\frac{3\,A\,c\,d^2\,e}{2}}{c^3\,x^2+a\,c^2} + \frac{\ln\left(c\,x^2+a\right)\,\left(16\,A\,a^3\,c^3\,e^3+48\,B\,d\,a^3\,c^3\,e^2\right)}{32\,a^3\,c^5} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(-3\,B\,a^2\,e^3+3\,B\,a\,c\,d^2\,e+3\,A\,a\,c\,d\,e^2+A\,c^2\,d^3\right)}{2\,a^{3/2}\,c^{5/2}} + \frac{B\,e^3\,x}{c^2}$$

input
```
int(((A + B*x)*(d + e*x)^3)/(a + c*x^2)^2,x)
```

output
```
((x*(A*c^2*d^3 + B*a^2*e^3 - 3*A*a*c*d*e^2 - 3*B*a*c*d^2*e))/(2*a) + (A*a*
e^3)/2 - (B*c*d^3)/2 + (3*B*a*d*e^2)/2 - (3*A*c*d^2*e)/2)/(a*c^2 + c^3*x^2
) + (log(a + c*x^2)*(16*A*a^3*c^3*e^3 + 48*B*a^3*c^3*d*e^2))/(32*a^3*c^5)
+ (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d^3 - 3*B*a^2*e^3 + 3*A*a*c*d*e^2 + 3*
B*a*c*d^2*e))/(2*a^(3/2)*c^(5/2)) + (B*e^3*x)/c^2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.59

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^2}\,dx$$
$$= \frac{-3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2b\,e^3+3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2cd\,e^2+3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)abc\,d^2e-3\sqrt{c}\,\sqrt{a}\,a}{}$$

input
```
int((B*x+A)*(e*x+d)^3/(c*x^2+a)^2,x)
```

output
```
( - 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*b*e**3 + 3*sqrt(c
)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*c*d*e**2 + 3*sqrt(c)*sqrt(a)*
atan((c*x)/(sqrt(c)*sqrt(a)))*a*b*c*d**2*e - 3*sqrt(c)*sqrt(a)*atan((c*x)/
(sqrt(c)*sqrt(a)))*a*b*c*e**3*x**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*s
qrt(a)))*a*c**2*d**3 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c
**2*d*e**2*x**2 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*b*c**2*d
**2*e*x**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**3*d**3*x**2
+ log(a + c*x**2)*a**3*c*e**3 + 3*log(a + c*x**2)*a**2*b*c*d*e**2 + log(a
+ c*x**2)*a**2*c**2*e**3*x**2 + 3*log(a + c*x**2)*a*b*c**2*d*e**2*x**2 + 3
*a**2*b*c*e**3*x - 3*a**2*c**2*d*e**2*x - a**2*c**2*e**3*x**2 - 3*a*b*c**2
*d**2*e*x - 3*a*b*c**2*d*e**2*x**2 + 2*a*b*c**2*e**3*x**3 + a*c**3*d**3*x
+ 3*a*c**3*d**2*e*x**2 + b*c**3*d**3*x**2)/(2*a*c**3*(a + c*x**2))
```

3.90 $\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^2} dx = -\frac{(d+ex)(a(Bd+Ae)-(Acd-aBe)x)}{2ac(a+cx^2)} + \frac{(Acd^2+2aBde+aAe^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}} + \frac{Be^2\log(a+cx^2)}{2c^2}$$

output
```
-1/2*(e*x+d)*(a*(A*e+B*d)-(A*c*d-B*a*e)*x)/a/c/(c*x^2+a)+1/2*(A*a*e^2+A*c*
d^2+2*B*a*d*e)*arctan(c^(1/2)*x/a^(1/2))/a^(3/2)/c^(3/2)+1/2*B*e^2*ln(c*x^
2+a)/c^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^2}\,dx = \frac{\frac{a^2Be^2+Ac^2d^2x-ac(Ae(2d+ex)+Bd(d+2ex))}{a(a+cx^2)}+\frac{\sqrt{c}(Acd^2+2aBde+aAe^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a^{3/2}}+Be^2\log(a+cx^2)}{2c^2}$$

input
```
Integrate[((A + B*x)*(d + e*x)^2)/(a + c*x^2)^2,x]
```

output
```
((a^2*B*e^2 + A*c^2*d^2*x - a*c*(A*e*(2*d + e*x) + B*d*(d + 2*e*x)))/(a*(a
 + c*x^2)) + (Sqrt[c]*(A*c*d^2 + 2*a*B*d*e + a*A*e^2)*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/a^(3/2) + B*e^2*Log[a + c*x^2])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {684, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^2}\,dx$$

$$\Big\downarrow \text{684}$$

$$\frac{\int \frac{Acd^2+ae(2Bd+Ae)+2aBe^2x}{cx^2+a}dx}{2ac}-\frac{(d+ex)(a(Ae+Bd)-x(Acd-aBe))}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{452}$$

$$\frac{(ae(Ae+2Bd)+Acd^2)\int \frac{1}{cx^2+a}dx+2aBe^2\int \frac{x}{cx^2+a}dx}{2ac}-\frac{(d+ex)(a(Ae+Bd)-x(Acd-aBe))}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{2aBe^2\int\frac{x}{cx^2+a}dx+\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(ae(Ae+2Bd)+Acd^2)}{\sqrt{a}\sqrt{c}}}{2ac}-\frac{(d+ex)(a(Ae+Bd)-x(Acd-aBe))}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(ae(Ae+2Bd)+Acd^2)}{\sqrt{a}\sqrt{c}}+\frac{aBe^2\log(a+cx^2)}{c}}{2ac}-\frac{(d+ex)(a(Ae+Bd)-x(Acd-aBe))}{2ac\,(a+cx^2)}$$

input
```
Int[((A + B*x)*(d + e*x)^2)/(a + c*x^2)^2,x]
```

output
```
-1/2*((d + e*x)*(a*(B*d + A*e) - (A*c*d - a*B*e)*x))/(a*c*(a + c*x^2)) + (
((A*c*d^2 + a*e*(2*B*d + A*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[
c]) + (a*B*e^2*Log[a + c*x^2])/c)/(2*a*c)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

method	result
default	$\frac{-\frac{\left(A a\,e^{2}-A c\,d^{2}+2 B a d e \right) x}{2 a c}-\frac{2 A c d e -B a\,e^{2}+B c\,d^{2}}{2 c^{2}}}{c\,x^{2}+a}+\frac{\frac{B a\,e^{2} \ln \left(c\,x^{2}+a \right)}{c}+\frac{\left(A a\,e^{2}+A c\,d^{2}+2 B a d e \right) \arctan \left(\frac{c x}{\sqrt{a c}}\right)}{\sqrt{a c}}}{2 a c}$
risch	$\frac{-\frac{\left(A a\,e^{2}-A c\,d^{2}+2 B a d e \right) x}{2 a c}-\frac{2 A c d e -B a\,e^{2}+B c\,d^{2}}{2 c^{2}}}{c\,x^{2}+a}+\frac{\ln \left(A\,a^{2} e^{2}+A\,d^{2} a c+2 B\,a^{2} d e -\sqrt{-a c \left(A a\,e^{2}+A c\,d^{2}+2 B a d e \right)^{2}}\, x \right) B\,e^{2}}{2 c^{2}}+\frac{\ln \left(\right.}{}$

input
```
int((B*x+A)*(e*x+d)^2/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(A*a*e^2-A*c*d^2+2*B*a*d*e)/a/c*x-1/2*(2*A*c*d*e-B*a*e^2+B*c*d^2)/c^
2)/(c*x^2+a)+1/2/a/c*(B*a*e^2/c*ln(c*x^2+a)+(A*a*e^2+A*c*d^2+2*B*a*d*e)/(a
*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.40

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^2}\,dx$$

$$= \left[-\frac{2\,Ba^2cd^2+4\,Aa^2cde-2\,Ba^3e^2+(Aacd^2+2\,Ba^2de+Aa^2e^2+(Ac^2d^2+2\,Bacde+Aace^2)x^2)\sqrt{-a}}{4\,(a^2c^3x^2+}\right.$$

$$-\frac{Ba^2cd^2+2\,Aa^2cde-Ba^3e^2-(Aacd^2+2\,Ba^2de+Aa^2e^2+(Ac^2d^2+2\,Bacde+Aace^2)x^2)\sqrt{ac}\,\mathrm{arct}}{2\,(a^2c^3x^2+a^3c^2)}$$

input
```
integrate((B*x+A)*(e*x+d)^2/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*B*a^2*c*d^2 + 4*A*a^2*c*d*e - 2*B*a^3*e^2 + (A*a*c*d^2 + 2*B*a^2*
d*e + A*a^2*e^2 + (A*c^2*d^2 + 2*B*a*c*d*e + A*a*c*e^2)*x^2)*sqrt(-a*c)*lo
g((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(A*a*c^2*d^2 - 2*B*a^2*c*d
*e - A*a^2*c*e^2)*x - 2*(B*a^2*c*e^2*x^2 + B*a^3*e^2)*log(c*x^2 + a))/(a^2
*c^3*x^2 + a^3*c^2), -1/2*(B*a^2*c*d^2 + 2*A*a^2*c*d*e - B*a^3*e^2 - (A*a*
c*d^2 + 2*B*a^2*d*e + A*a^2*e^2 + (A*c^2*d^2 + 2*B*a*c*d*e + A*a*c*e^2)*x^
2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (A*a*c^2*d^2 - 2*B*a^2*c*d*e - A*a^2*
c*e^2)*x - (B*a^2*c*e^2*x^2 + B*a^3*e^2)*log(c*x^2 + a))/(a^2*c^3*x^2 + a^
3*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(105) = 210$.

Time = 1.37 (sec) , antiderivative size = 382, normalized size of antiderivative = 3.38

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^2}\,dx = \left(\frac{Be^2}{2c^2} - \frac{\sqrt{-a^3c^5}(Aae^2+Acd^2+2Bade)}{4a^3c^4}\right)\log\left(x + \frac{-2Ba^2e^2+4a^2c^2\left(\frac{Be^2}{2c^2} - \frac{\sqrt{-a^3c^5}(Aae^2+Acd^2+2Bade)}{4a^3c^4}\right)}{Aace^2+Ac^2d^2+2Bacde}\right) + \left(\frac{Be^2}{2c^2} + \frac{\sqrt{-a^3c^5}(Aae^2+Acd^2+2Bade)}{4a^3c^4}\right)\log\left(x + \frac{-2Ba^2e^2+4a^2c^2\left(\frac{Be^2}{2c^2} + \frac{\sqrt{-a^3c^5}(Aae^2+Acd^2+2Bade)}{4a^3c^4}\right)}{Aace^2+Ac^2d^2+2Bacde}\right) + \frac{-2Aacde+Ba^2e^2-Bacd^2+x(-Aace^2+Ac^2d^2-2Bacde)}{2a^2c^2+2ac^3x^2}$$

input
```
integrate((B*x+A)*(e*x+d)**2/(c*x**2+a)**2,x)
```

output
```
(B*e**2/(2*c**2) - sqrt(-a**3*c**5)*(A*a*e**2 + A*c*d**2 + 2*B*a*d*e)/(4*a
**3*c**4))*log(x + (-2*B*a**2*e**2 + 4*a**2*c**2*(B*e**2/(2*c**2) - sqrt(-
a**3*c**5)*(A*a*e**2 + A*c*d**2 + 2*B*a*d*e)/(4*a**3*c**4)))/(A*a*c*e**2 +
 A*c**2*d**2 + 2*B*a*c*d*e)) + (B*e**2/(2*c**2) + sqrt(-a**3*c**5)*(A*a*e*
*2 + A*c*d**2 + 2*B*a*d*e)/(4*a**3*c**4))*log(x + (-2*B*a**2*e**2 + 4*a**2
*c**2*(B*e**2/(2*c**2) + sqrt(-a**3*c**5)*(A*a*e**2 + A*c*d**2 + 2*B*a*d*e
)/(4*a**3*c**4)))/(A*a*c*e**2 + A*c**2*d**2 + 2*B*a*c*d*e)) + (-2*A*a*c*d*
e + B*a**2*e**2 - B*a*c*d**2 + x*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e)
)/(2*a**2*c**2 + 2*a*c**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^2}\,dx = \frac{Be^2\log(cx^2+a)}{2\,c^2} - \frac{Bacd^2+2\,Aacde-Ba^2e^2-(Ac^2d^2-2\,Bacde-Aace^2)x}{2\,(ac^3x^2+a^2c^2)} + \frac{(Acd^2+2\,Bade+Aae^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac}$$

input
```
integrate((B*x+A)*(e*x+d)^2/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*B*e^2*log(c*x^2 + a)/c^2 - 1/2*(B*a*c*d^2 + 2*A*a*c*d*e - B*a^2*e^2 -
(A*c^2*d^2 - 2*B*a*c*d*e - A*a*c*e^2)*x)/(a*c^3*x^2 + a^2*c^2) + 1/2*(A*c*
d^2 + 2*B*a*d*e + A*a*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^2}\,dx = \frac{Be^2\log(cx^2+a)}{2\,c^2} + \frac{(Acd^2+2\,Bade+Aae^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac} + \frac{(Acd^2-2\,Bade-Aae^2)x-\frac{Bacd^2+2\,Aacde-Ba^2e^2}{c}}{2\,(cx^2+a)ac}$$

input
```
integrate((B*x+A)*(e*x+d)^2/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*B*e^2*log(c*x^2 + a)/c^2 + 1/2*(A*c*d^2 + 2*B*a*d*e + A*a*e^2)*arctan(
c*x/sqrt(a*c))/(sqrt(a*c)*a*c) + 1/2*((A*c*d^2 - 2*B*a*d*e - A*a*e^2)*x -
(B*a*c*d^2 + 2*A*a*c*d*e - B*a^2*e^2)/c)/((c*x^2 + a)*a*c)
```

Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.80

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^2}\,dx = \frac{B\,a\,e^2}{2\,(c^3\,x^2+a\,c^2)} - \frac{B\,d^2}{2\,(c^2\,x^2+a\,c)} - \frac{A\,d\,e}{c^2\,x^2+a\,c} + \frac{A\,d^2\,x}{2\,(a^2+c\,a\,x^2)} - \frac{A\,e^2\,x}{2\,(c^2\,x^2+a\,c)} + \frac{B\,e^2\,\ln\left(c\,x^2+a\right)}{2\,c^2} + \frac{A\,d^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{c}} + \frac{A\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,c^{3/2}} - \frac{B\,d\,e\,x}{c^2\,x^2+a\,c} + \frac{B\,d\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{3/2}}$$

input
```
int(((A + B*x)*(d + e*x)^2)/(a + c*x^2)^2,x)
```

output
```
(B*a*e^2)/(2*(a*c^2 + c^3*x^2)) - (B*d^2)/(2*(a*c + c^2*x^2)) - (A*d*e)/(a
*c + c^2*x^2) + (A*d^2*x)/(2*(a^2 + a*c*x^2)) - (A*e^2*x)/(2*(a*c + c^2*x^
2)) + (B*e^2*log(a + c*x^2))/(2*c^2) + (A*d^2*atan((c^(1/2)*x)/a^(1/2)))/(
2*a^(3/2)*c^(1/2)) + (A*e^2*atan((c^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*c^(3/2))
 - (B*d*e*x)/(a*c + c^2*x^2) + (B*d*e*atan((c^(1/2)*x)/a^(1/2)))/(a^(1/2)*
c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.26

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^2}\,dx$$
$$= \frac{\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\sqrt{a}}\right)a^2e^2 + 2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\sqrt{a}}\right)abde + \sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\sqrt{a}}\right)ac\,d^2 + \sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\sqrt{a}}\right)}{}$$

input
```
int((B*x+A)*(e*x+d)^2/(c*x^2+a)^2,x)
```

output
```
(sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*e**2 + 2*sqrt(c)*sqrt(
a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*b*d*e + sqrt(c)*sqrt(a)*atan((c*x)/(sqr
t(c)*sqrt(a)))*a*c*d**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*
c*e**2*x**2 + 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*b*c*d*e*x**2
 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**2*d**2*x**2 + log(a +
c*x**2)*a**2*b*e**2 + log(a + c*x**2)*a*b*c*e**2*x**2 - a**2*c*e**2*x - 2*
a*b*c*d*e*x - a*b*c*e**2*x**2 + a*c**2*d**2*x + 2*a*c**2*d*e*x**2 + b*c**2
*d**2*x**2)/(2*a*c**2*(a + c*x**2))
```

3.91 $\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^2} \, dx = -\frac{(A+Bx)(ae-cdx)}{2ac(a+cx^2)} + \frac{(Acd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}$$

output
```
-1/2*(B*x+A)*(-c*d*x+a*e)/a/c/(c*x^2+a)+1/2*(A*c*d+B*a*e)*arctan(c^(1/2)*x
/a^(1/2))/a^(3/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^2} \, dx = \frac{-aBd-aAe+Acdx-aBex}{2ac(a+cx^2)} + \frac{(Acd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}$$

input
```
Integrate[((A + B*x)*(d + e*x))/(a + c*x^2)^2,x]
```

output
```
(-(a*B*d) - a*A*e + A*c*d*x - a*B*e*x)/(2*a*c*(a + c*x^2)) + ((A*c*d + a*B
*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {675, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^2}\,dx$$

$$\Big\downarrow \text{675}$$

$$\frac{(aBe+Acd)\int \frac{1}{cx^2+a}dx}{2ac}-\frac{Ae+Bd}{2c\,(a+cx^2)}+\frac{x(Acd-aBe)}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+Acd)}{2a^{3/2}c^{3/2}}-\frac{Ae+Bd}{2c\,(a+cx^2)}+\frac{x(Acd-aBe)}{2ac\,(a+cx^2)}$$

input
```
Int[((A + B*x)*(d + e*x))/(a + c*x^2)^2,x]
```

output
```
-1/2*(B*d + A*e)/(c*(a + c*x^2)) + ((A*c*d - a*B*e)*x)/(2*a*c*(a + c*x^2))
 + ((A*c*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\frac{(Acd-Bae)x}{2ac}-\frac{Ae+Bd}{2c}}{c\,x^{2}+a}+\frac{(Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2ac\sqrt{ac}}$	75
risch	$\frac{\frac{(Acd-Bae)x}{2ac}-\frac{Ae+Bd}{2c}}{c\,x^{2}+a}-\frac{\ln\left(cx+\sqrt{-ac}\right)Ad}{4\sqrt{-ac}\,a}-\frac{\ln\left(cx+\sqrt{-ac}\right)Be}{4\sqrt{-ac}\,c}+\frac{\ln\left(-cx+\sqrt{-ac}\right)Ad}{4\sqrt{-ac}\,a}+\frac{\ln\left(-cx+\sqrt{-ac}\right)Be}{4\sqrt{-ac}\,c}$	142

input
```
int((B*x+A)*(e*x+d)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(A*c*d-B*a*e)/a/c*x-1/2*(A*e+B*d)/c)/(c*x^2+a)+1/2*(A*c*d+B*a*e)/a/c/
(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.21

$$\int \frac{(A+Bx)(d+ex)}{\left(a+cx^2\right)^2}\,dx$$
$$=\left[-\frac{2\,Ba^2cd+2\,Aa^2ce+(Aacd+Ba^2e+(Ac^2d+Bace)x^2)\sqrt{-ac}\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right)-2\,(Aac^2d-Ba}{4\,(a^2c^3x^2+a^3c^2)}\right.$$
$$\left.-\frac{Ba^2cd+Aa^2ce-(Aacd+Ba^2e+(Ac^2d+Bace)x^2)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}x}{a}\right)-(Aac^2d-Ba^2ce)x}{2\,(a^2c^3x^2+a^3c^2)}\right]$$

input
```
integrate((B*x+A)*(e*x+d)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*B*a^2*c*d + 2*A*a^2*c*e + (A*a*c*d + B*a^2*e + (A*c^2*d + B*a*c*e
)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(A*a*c
^2*d - B*a^2*c*e)*x)/(a^2*c^3*x^2 + a^3*c^2), -1/2*(B*a^2*c*d + A*a^2*c*e
- (A*a*c*d + B*a^2*e + (A*c^2*d + B*a*c*e)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)
*x/a) - (A*a*c^2*d - B*a^2*c*e)*x)/(a^2*c^3*x^2 + a^3*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(65) = 130$.

Time = 0.49 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.90

$$\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^2}\,dx = -\frac{\sqrt{-\frac{1}{a^3c^3}}(Acd+Bae)\log\left(-a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3c^3}}(Acd+Bae)\log\left(a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4} + \frac{-Aae-Bad+x(Acd-Bae)}{2a^2c+2ac^2x^2}$$

input
```
integrate((B*x+A)*(e*x+d)/(c*x**2+a)**2,x)
```

output
```
-sqrt(-1/(a**3*c**3))*(A*c*d + B*a*e)*log(-a**2*c*sqrt(-1/(a**3*c**3)) + x
)/4 + sqrt(-1/(a**3*c**3))*(A*c*d + B*a*e)*log(a**2*c*sqrt(-1/(a**3*c**3))
 + x)/4 + (-A*a*e - B*a*d + x*(A*c*d - B*a*e))/(2*a**2*c + 2*a*c**2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^2}\,dx = -\frac{Bad+Aae-(Acd-Bae)x}{2\,(ac^2x^2+a^2c)} + \frac{(Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac}$$

input
```
integrate((B*x+A)*(e*x+d)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a*d + A*a*e - (A*c*d - B*a*e)*x)/(a*c^2*x^2 + a^2*c) + 1/2*(A*c*d
+ B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)(d+ex)}{\left(a+cx^2\right)^2}\,dx = \frac{(Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}ac} + \frac{Acdx-Baex-Bad-Aae}{2\left(cx^2+a\right)ac}$$

input
```
integrate((B*x+A)*(e*x+d)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c) + 1/2*(A*c*d*x -
 B*a*e*x - B*a*d - A*a*e)/((c*x^2 + a)*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)(d+ex)}{\left(a+cx^2\right)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(A\,c\,d+B\,a\,e\right)}{2\,a^{3/2}\,c^{3/2}} - \frac{\frac{A\,e+B\,d}{2\,c}-\frac{x\,\left(A\,c\,d-B\,a\,e\right)}{2\,a\,c}}{c\,x^2+a}$$

input
```
int(((A + B*x)*(d + e*x))/(a + c*x^2)^2,x)
```

output
```
(atan((c^(1/2)*x)/a^(1/2))*(A*c*d + B*a*e))/(2*a^(3/2)*c^(3/2)) - ((A*e +
B*d)/(2*c) - (x*(A*c*d - B*a*e))/(2*a*c))/(a + c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.96

$$\int \frac{(A+Bx)(d+ex)}{\left(a+cx^2\right)^2}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)abe+\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)acd+\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)bce\,x^2+\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)c}{2a\,c^2\left(c\,x^2+a\right)}$$

input
```
int((B*x+A)*(e*x+d)/(c*x^2+a)^2,x)
```

output
```
(sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*b*e + sqrt(c)*sqrt(a)*ata
n((c*x)/(sqrt(c)*sqrt(a)))*a*c*d + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqr
t(a)))*b*c*e*x**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**2*d*x
**2 - a*b*c*e*x + a*c**2*d*x + a*c**2*e*x**2 + b*c**2*d*x**2)/(2*a*c**2*(a
 + c*x**2))
```

3.92 $\int \frac{A+Bx}{(a+cx^2)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{A+Bx}{(a+cx^2)^2} dx = \frac{-aB+Acx}{2ac(a+cx^2)} + \frac{A\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}$$

output
```
1/2*(A*c*x-B*a)/a/c/(c*x^2+a)+1/2*A*arctan(c^(1/2)*x/a^(1/2))/a^(3/2)/c^(1
/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{(a+cx^2)^2} dx = \frac{-aB+Acx}{2ac(a+cx^2)} + \frac{A\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}$$

input
```
Integrate[(A + B*x)/(a + c*x^2)^2,x]
```

output
```
(-(a*B) + A*c*x)/(2*a*c*(a + c*x^2)) + (A*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*
a^(3/2)*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {454, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+cx^2)^2} dx$$

$$\Big\downarrow \ 454$$

$$\frac{A\int \frac{1}{cx^2+a}dx}{2a} - \frac{aB-Acx}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \ 218$$

$$\frac{A\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{aB-Acx}{2ac\,(a+cx^2)}$$

input
```
Int[(A + B*x)/(a + c*x^2)^2,x]
```

output
```
-1/2*(a*B - A*c*x)/(a*c*(a + c*x^2)) + (A*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*
a^(3/2)*Sqrt[c])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2Acx-2Ba}{4ac(c\,x^2+a)}+\frac{A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2a\sqrt{ac}}$	49
risch	$\frac{\frac{Ax}{2a}-\frac{B}{2c}}{c\,x^2+a}-\frac{A\ln\left(cx+\sqrt{-ac}\right)}{4\sqrt{-ac}\,a}+\frac{A\ln\left(-cx+\sqrt{-ac}\right)}{4\sqrt{-ac}\,a}$	73

input
```
int((B*x+A)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*A*c*x-2*B*a)/a/c/(c*x^2+a)+1/2*A/a/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.46

$$\int \frac{A+Bx}{(a+cx^2)^2}\,dx = \left[\frac{2\,Aacx-2\,Ba^2-(Acx^2+Aa)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)}{4\,(a^2c^2x^2+a^3c)}, \frac{Aacx-Ba^2+(Acx^2+Aa)\sqrt{ac}\arctan\left(\sqrt{\right.}\right.}{2\,(a^2c^2x^2+a^3c)}\right.$$

input
```
integrate((B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*A*a*c*x - 2*B*a^2 - (A*c*x^2 + A*a)*sqrt(-a*c)*log((c*x^2 - 2*sqrt
(-a*c)*x - a)/(c*x^2 + a)))/(a^2*c^2*x^2 + a^3*c), 1/2*(A*a*c*x - B*a^2 +
(A*c*x^2 + A*a)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(a^2*c^2*x^2 + a^3*c)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx}{(a+cx^2)^2}\,dx = A\left(-\frac{\sqrt{-\frac{1}{a^3c}}\log\left(-a^2\sqrt{-\frac{1}{a^3c}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3c}}\log\left(a^2\sqrt{-\frac{1}{a^3c}}+x\right)}{4}\right)+\frac{Acx-Ba}{2a^2c+2ac^2x^2}$$

input
```
integrate((B*x+A)/(c*x**2+a)**2,x)
```

output
```
A*(-sqrt(-1/(a**3*c))*log(-a**2*sqrt(-1/(a**3*c)) + x)/4 + sqrt(-1/(a**3*c
))*log(a**2*sqrt(-1/(a**3*c)) + x)/4) + (A*c*x - B*a)/(2*a**2*c + 2*a*c**2
*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{(a+cx^2)^2}\,dx = \frac{A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}a}+\frac{Acx-Ba}{2\left(ac^2x^2+a^2c\right)}$$

input
```
integrate((B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*A*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a) + 1/2*(A*c*x - B*a)/(a*c^2*x^2 +
 a^2*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx}{(a+cx^2)^2}\,dx = \frac{A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}a} + \frac{Acx-Ba}{2\,(cx^2+a)ac}$$

input
```
integrate((B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*A*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a) + 1/2*(A*c*x - B*a)/((c*x^2 + a)
*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx}{(a+cx^2)^2}\,dx = \frac{A\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{c}} - \frac{\frac{B}{2\,c}-\frac{A\,x}{2\,a}}{c\,x^2+a}$$

input
```
int((A + B*x)/(a + c*x^2)^2,x)
```

output
```
(A*atan((c^(1/2)*x)/a^(1/2)))/(2*a^(3/2)*c^(1/2)) - (B/(2*c) - (A*x)/(2*a)
)/(a + c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx}{(a+cx^2)^2}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a+\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)c\,x^2+acx+bc\,x^2}{2ac\,(c\,x^2+a)}$$

input
```
int((B*x+A)/(c*x^2+a)^2,x)
```

output
```
(sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a + sqrt(c)*sqrt(a)*atan((c
*x)/(sqrt(c)*sqrt(a)))*c*x**2 + a*c*x + b*c*x**2)/(2*a*c*(a + c*x**2))
```

3.93 $\int \frac{A+Bx}{(d+ex)\left(a+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 195

$$\int \frac{A+Bx}{(d+ex)\left(a+cx^2\right)^2}\,dx = -\frac{a(Bd-Ae)-(Acd+aBe)x}{2a\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\left(aBe(cd^2-ae^2)-Acd(cd^2+3ae^2)\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}\left(cd^2+ae^2\right)^2} - \frac{e^2(Bd-Ae)\log(d+ex)}{\left(cd^2+ae^2\right)^2} + \frac{e^2(Bd-Ae)\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^2}$$

output
```
-1/2*(a*(-A*e+B*d)-(A*c*d+B*a*e)*x)/a/(a*e^2+c*d^2)/(c*x^2+a)-1/2*(a*B*e*(
-a*e^2+c*d^2)-A*c*d*(3*a*e^2+c*d^2))*arctan(c^(1/2)*x/a^(1/2))/a^(3/2)/c^(
1/2)/(a*e^2+c*d^2)^2-e^2*(-A*e+B*d)*ln(e*x+d)/(a*e^2+c*d^2)^2+1/2*e^2*(-A*
e+B*d)*ln(c*x^2+a)/(a*e^2+c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{(d+ex)\left(a+cx^2\right)^2}\,dx = \frac{\frac{(cd^2+ae^2)(Acdx+a(-Bd+Ae+Bex))}{a(a+cx^2)} + \frac{(aBe(-cd^2+ae^2)+Acd(cd^2+3ae^2))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{c}} + 2e^2(-Bd+Ae)\log(d+ex)+e}{2\left(cd^2+ae^2\right)^2}$$

input
```
Integrate[(A + B*x)/((d + e*x)*(a + c*x^2)^2),x]
```

output
```
(((c*d^2 + a*e^2)*(A*c*d*x + a*(-(B*d) + A*e + B*e*x)))/(a*(a + c*x^2)) +
((a*B*e*(-(c*d^2) + a*e^2) + A*c*d*(c*d^2 + 3*a*e^2))*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/(a^(3/2)*Sqrt[c]) + 2*e^2*(-(B*d) + A*e)*Log[d + e*x] + e^2*(B*d
- A*e)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {686, 27, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\left(a+cx^2\right)^2(d+ex)}dx$$

$$\Big\downarrow \text{ 686}$$

$$-\frac{\int \frac{c\left(aBde-(Acd+aBe)xe-A\left(cd^2+2ae^2\right)\right)}{(d+ex)\left(cx^2+a\right)}dx}{2ac\left(ae^2+cd^2\right)}-\frac{a(Bd-Ae)-x(aBe+Acd)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int -\frac{Acd^2-aBed+2aAe^2+e(Acd+aBe)x}{(d+ex)\left(cx^2+a\right)}dx}{2a\left(ae^2+cd^2\right)}-\frac{a(Bd-Ae)-x(aBe+Acd)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Acd^2-aBed+2aAe^2+e(Acd+aBe)x}{(d+ex)(cx^2+a)}dx}{2a\left(ae^2+cd^2\right)}-\frac{a(Bd-Ae)-x(aBe+Acd)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{657}$$

$$\frac{\int \left(\frac{2a(Ae-Bd)e^3}{(cd^2+ae^2)(d+ex)}+\frac{2ac(Bd-Ae)xe^2-aB(cd^2-ae^2)e+Acd(cd^2+3ae^2)}{(cd^2+ae^2)(cx^2+a)}\right)dx}{2a\left(ae^2+cd^2\right)}-\frac{a(Bd-Ae)-x(aBe+Acd)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(aBe(cd^2-ae^2)-Acd(3ae^2+cd^2)\right)}{\sqrt{a}\sqrt{c}(ae^2+cd^2)}+\frac{ae^2\log(a+cx^2)(Bd-Ae)}{ae^2+cd^2}-\frac{2ae^2(Bd-Ae)\log(d+ex)}{ae^2+cd^2}}{2a\left(ae^2+cd^2\right)}-\frac{a(Bd-Ae)-x(aBe+Acd)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

input
```
Int[(A + B*x)/((d + e*x)*(a + c*x^2)^2),x]
```

output
```
-1/2*(a*(B*d - A*e) - (A*c*d + a*B*e)*x)/(a*(c*d^2 + a*e^2)*(a + c*x^2)) +
 (-(((a*B*e*(c*d^2 - a*e^2) - A*c*d*(c*d^2 + 3*a*e^2))*ArcTan[(Sqrt[c]*x)/
Sqrt[a]])/(Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2))) - (2*a*e^2*(B*d - A*e)*Log[d
+ e*x])/(c*d^2 + a*e^2) + (a*e^2*(B*d - A*e)*Log[a + c*x^2])/(c*d^2 + a*e^
2))/(2*a*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.10

method	result
default	$\frac{(Ae-Bd)e^2\ln(ex+d)}{(a\,e^2+c\,d^2)^2}+\frac{\frac{\frac{\left(Aacd\,e^2+A\,c^2d^3+B\,e^3a^2+Bac\,d^2e\right)x}{2a}+\frac{Aa\,e^3}{2}+\frac{Ac\,d^2e}{2}-\frac{Bad\,e^2}{2}-\frac{Bc\,d^3}{2}}{c\,x^2+a}+\frac{\frac{\left(-2Aac\,e^3+2Bacd\,e^2\right)\ln\left(c\,x^2+a\right)}{2c}+}{}}{(a\,e^2+c\,d^2)^2}$
risch	$\frac{\frac{(Acd+Bae)x}{2a\left(a\,e^2+c\,d^2\right)}+\frac{Ae-Bd}{2a\,e^2+2c\,d^2}}{c\,x^2+a}+\frac{e^3\ln(ex+d)A}{a^2e^4+2ac\,d^2e^2+c^2d^4}-\frac{e^2\ln(ex+d)Bd}{a^2e^4+2ac\,d^2e^2+c^2d^4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^5c\,e^4+2a^4c^2d^2e^2+a^3c^3d^4\right)\_Z^2\right.}\right.}{}$

input
```
int((B*x+A)/(e*x+d)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(A*e-B*d)*e^2/(a*e^2+c*d^2)^2*ln(e*x+d)+1/(a*e^2+c*d^2)^2*((1/2*(A*a*c*d*e
^2+A*c^2*d^3+B*a^2*e^3+B*a*c*d^2*e)/a*x+1/2*A*a*e^3+1/2*A*c*d^2*e-1/2*B*a*
d*e^2-1/2*B*c*d^3)/(c*x^2+a)+1/2/a*(1/2*(-2*A*a*c*e^3+2*B*a*c*d*e^2)/c*ln(
c*x^2+a)+(3*A*a*c*d*e^2+A*c^2*d^3+B*a^2*e^3-B*a*c*d^2*e)/(a*c)^(1/2)*arcta
n(c*x/(a*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(181) = 362$.

Time = 9.01 (sec) , antiderivative size = 795, normalized size of antiderivative = 4.08

$$\int \frac{A + Bx}{(d + ex)(a + cx^2)^2}\,dx$$
$$= \left[-\frac{2\,Ba^2c^2d^3 - 2\,Aa^2c^2d^2e + 2\,Ba^3cde^2 - 2\,Aa^3ce^3 + (Aac^2d^3 - Ba^2cd^2e + 3\,Aa^2cde^2 + Ba^3e^3 + (Ac^3}{} \right.$$
$$-\frac{Ba^2c^2d^3 - Aa^2c^2d^2e + Ba^3cde^2 - Aa^3ce^3 - (Aac^2d^3 - Ba^2cd^2e + 3\,Aa^2cde^2 + Ba^3e^3 + (Ac^3d^3 - B}{}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*B*a^2*c^2*d^3 - 2*A*a^2*c^2*d^2*e + 2*B*a^3*c*d*e^2 - 2*A*a^3*c*e
^3 + (A*a*c^2*d^3 - B*a^2*c*d^2*e + 3*A*a^2*c*d*e^2 + B*a^3*e^3 + (A*c^3*d
^3 - B*a*c^2*d^2*e + 3*A*a*c^2*d*e^2 + B*a^2*c*e^3)*x^2)*sqrt(-a*c)*log((c
*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(A*a*c^3*d^3 + B*a^2*c^2*d^2*e
 + A*a^2*c^2*d*e^2 + B*a^3*c*e^3)*x - 2*(B*a^3*c*d*e^2 - A*a^3*c*e^3 + (B*
a^2*c^2*d*e^2 - A*a^2*c^2*e^3)*x^2)*log(c*x^2 + a) + 4*(B*a^3*c*d*e^2 - A*
a^3*c*e^3 + (B*a^2*c^2*d*e^2 - A*a^2*c^2*e^3)*x^2)*log(e*x + d))/(a^3*c^3*
d^4 + 2*a^4*c^2*d^2*e^2 + a^5*c*e^4 + (a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2 + a
^4*c^2*e^4)*x^2), -1/2*(B*a^2*c^2*d^3 - A*a^2*c^2*d^2*e + B*a^3*c*d*e^2 -
A*a^3*c*e^3 - (A*a*c^2*d^3 - B*a^2*c*d^2*e + 3*A*a^2*c*d*e^2 + B*a^3*e^3 +
 (A*c^3*d^3 - B*a*c^2*d^2*e + 3*A*a*c^2*d*e^2 + B*a^2*c*e^3)*x^2)*sqrt(a*c
)*arctan(sqrt(a*c)*x/a) - (A*a*c^3*d^3 + B*a^2*c^2*d^2*e + A*a^2*c^2*d*e^2
 + B*a^3*c*e^3)*x - (B*a^3*c*d*e^2 - A*a^3*c*e^3 + (B*a^2*c^2*d*e^2 - A*a^
2*c^2*e^3)*x^2)*log(c*x^2 + a) + 2*(B*a^3*c*d*e^2 - A*a^3*c*e^3 + (B*a^2*c
^2*d*e^2 - A*a^2*c^2*e^3)*x^2)*log(e*x + d))/(a^3*c^3*d^4 + 2*a^4*c^2*d^2*
e^2 + a^5*c*e^4 + (a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2 + a^4*c^2*e^4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)\left(a+cx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx}{(d+ex)\left(a+cx^2\right)^2}\,dx = \frac{(Bde^2-Ae^3)\log\left(cx^2+a\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)} - \frac{(Bde^2-Ae^3)\log\left(ex+d\right)}{c^2d^4+2\,acd^2e^2+a^2e^4} + \frac{(Ac^2d^3-Bacd^2e+3\,Aacde^2+Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}} - \frac{Bad-Aae-(Acd+Bae)x}{2\left(a^2cd^2+a^3e^2+(ac^2d^2+a^2ce^2)x^2\right)}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*d*e^2 - A*e^3)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) -
 (B*d*e^2 - A*e^3)*log(e*x + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*
(A*c^2*d^3 - B*a*c*d^2*e + 3*A*a*c*d*e^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c)
)/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) - 1/2*(B*a*d - A*a*e
 - (A*c*d + B*a*e)*x)/(a^2*c*d^2 + a^3*e^2 + (a*c^2*d^2 + a^2*c*e^2)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx}{(d+ex)\left(a+cx^2\right)^2}\,dx = \frac{(Bde^2-Ae^3)\log\left(cx^2+a\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)} - \frac{(Bde^3-Ae^4)\log\left(|ex+d|\right)}{c^2d^4e+2\,acd^2e^3+a^2e^5} + \frac{(Ac^2d^3-Bacd^2e+3\,Aacde^2+Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}} - \frac{Bacd^3-Aacd^2e+Ba^2de^2-Aa^2e^3-(Ac^2d^3+Bacd^2e+Aacde^2+Ba^2e^3)x}{2\left(cd^2+ae^2\right)^2(cx^2+a)a}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(B*d*e^2 - A*e^3)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) -
 (B*d*e^3 - A*e^4)*log(abs(e*x + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)
 + 1/2*(A*c^2*d^3 - B*a*c*d^2*e + 3*A*a*c*d*e^2 + B*a^2*e^3)*arctan(c*x/sq
rt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) - 1/2*(B*a*c*
d^3 - A*a*c*d^2*e + B*a^2*d*e^2 - A*a^2*e^3 - (A*c^2*d^3 + B*a*c*d^2*e + A
*a*c*d*e^2 + B*a^2*e^3)*x)/((c*d^2 + a*e^2)^2*(c*x^2 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 7.28 (sec) , antiderivative size = 1086, normalized size of antiderivative = 5.57

$$\int \frac{A+Bx}{(d+ex)\left(a+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/((a + c*x^2)^2*(d + e*x)),x)
```

output
```
((A*e - B*d)/(2*(a*e^2 + c*d^2)) + (x*(A*c*d + B*a*e))/(2*a*(a*e^2 + c*d^2
)))/(a + c*x^2) - (log(A*c^3*d^5*(-a^3*c)^(1/2) - B*a^3*e^5*(-a^3*c)^(1/2)
 - 6*A*a^4*c*e^5 + B*a^4*c*e^5*x + 2*A*a^2*c^3*d^4*e + 12*A*a^3*c^2*d^2*e^
3 - 8*B*a^3*c^2*d^3*e^2 + 8*B*a^4*c*d*e^4 - A*a*c^4*d^5*x - 2*A*a^2*c^3*d^
3*e^2*x - 14*B*a^3*c^2*d^2*e^3*x + 2*A*a*c^2*d^3*e^2*(-a^3*c)^(1/2) + 14*B
*a^2*c*d^2*e^3*(-a^3*c)^(1/2) + 15*A*a^3*c^2*d*e^4*x + B*a^2*c^3*d^4*e*x -
 15*A*a^2*c*d*e^4*(-a^3*c)^(1/2) - B*a*c^2*d^4*e*(-a^3*c)^(1/2) - 6*A*a^2*
c*e^5*x*(-a^3*c)^(1/2) + 2*A*c^3*d^4*e*x*(-a^3*c)^(1/2) + 8*B*a^2*c*d*e^4*
x*(-a^3*c)^(1/2) + 12*A*a*c^2*d^2*e^3*x*(-a^3*c)^(1/2) - 8*B*a*c^2*d^3*e^2
*x*(-a^3*c)^(1/2))*(c*(a*((3*A*d*e^2*(-a^3*c)^(1/2))/4 - (B*d^2*e*(-a^3*c)
^(1/2))/4) + a^3*((A*e^3)/2 - (B*d*e^2)/2)) + (A*c^2*d^3*(-a^3*c)^(1/2))/4
 + (B*a^2*e^3*(-a^3*c)^(1/2))/4))/(a^5*c*e^4 + a^3*c^3*d^4 + 2*a^4*c^2*d^2
*e^2) + (log(A*c^3*d^5*(-a^3*c)^(1/2) - B*a^3*e^5*(-a^3*c)^(1/2) + 6*A*a^4
*c*e^5 - B*a^4*c*e^5*x - 2*A*a^2*c^3*d^4*e - 12*A*a^3*c^2*d^2*e^3 + 8*B*a^
3*c^2*d^3*e^2 - 8*B*a^4*c*d*e^4 + A*a*c^4*d^5*x + 2*A*a^2*c^3*d^3*e^2*x +
14*B*a^3*c^2*d^2*e^3*x + 2*A*a*c^2*d^3*e^2*(-a^3*c)^(1/2) + 14*B*a^2*c*d^2
*e^3*(-a^3*c)^(1/2) - 15*A*a^3*c^2*d*e^4*x - B*a^2*c^3*d^4*e*x - 15*A*a^2*
c*d*e^4*(-a^3*c)^(1/2) - B*a*c^2*d^4*e*(-a^3*c)^(1/2) - 6*A*a^2*c*e^5*x*(-
a^3*c)^(1/2) + 2*A*c^3*d^4*e*x*(-a^3*c)^(1/2) + 8*B*a^2*c*d*e^4*x*(-a^3*c)
^(1/2) + 12*A*a*c^2*d^2*e^3*x*(-a^3*c)^(1/2) - 8*B*a*c^2*d^3*e^2*x*(-a^...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.70

$$\int \frac{A+Bx}{(d+ex)(a+cx^2)^2}\,dx$$

$$= \frac{\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2b\,e^3 + 3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2cd\,e^2 - \sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)abc\,d^2e + \sqrt{c}\,\sqrt{a}\,atan\left(\right.}{}$$

input
```
int((B*x+A)/(e*x+d)/(c*x^2+a)^2,x)
```

output
```
(sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*b*e**3 + 3*sqrt(c)*sqr
t(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*c*d*e**2 - sqrt(c)*sqrt(a)*atan((c
*x)/(sqrt(c)*sqrt(a)))*a*b*c*d**2*e + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*
sqrt(a)))*a*b*c*e**3*x**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*
a*c**2*d**3 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c**2*d*e**
2*x**2 - sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*b*c**2*d**2*e*x**2
+ sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**3*d**3*x**2 - log(a + c
*x**2)*a**3*c*e**3 + log(a + c*x**2)*a**2*b*c*d*e**2 - log(a + c*x**2)*a**
2*c**2*e**3*x**2 + log(a + c*x**2)*a*b*c**2*d*e**2*x**2 + 2*log(d + e*x)*a
**3*c*e**3 - 2*log(d + e*x)*a**2*b*c*d*e**2 + 2*log(d + e*x)*a**2*c**2*e**
3*x**2 - 2*log(d + e*x)*a*b*c**2*d*e**2*x**2 + a**2*b*c*e**3*x + a**2*c**2
*d*e**2*x - a**2*c**2*e**3*x**2 + a*b*c**2*d**2*e*x + a*b*c**2*d*e**2*x**2
 + a*c**3*d**3*x - a*c**3*d**2*e*x**2 + b*c**3*d**3*x**2)/(2*a*c*(a**3*e**
4 + 2*a**2*c*d**2*e**2 + a**2*c*e**4*x**2 + a*c**2*d**4 + 2*a*c**2*d**2*e*
*2*x**2 + c**3*d**4*x**2))
```

3.94 $\int \frac{A+Bx}{(d+ex)^2(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 291

$$\begin{aligned}\int \frac{A+Bx}{(d+ex)^2(a+cx^2)^2}\,dx &= \frac{e^2(Bd-Ae)}{(cd^2+ae^2)^2(d+ex)} - \frac{a(Bcd^2-2Acde-aBe^2)-c(Acd^2+2aBde-aAe^2)x}{2a(cd^2+ae^2)^2(a+cx^2)} \\ &\quad - \frac{\sqrt{c}(2aBde(cd^2-3ae^2)-A(c^2d^4+6acd^2e^2-3a^2e^4))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}(cd^2+ae^2)^3} \\ &\quad - \frac{e^2(3Bcd^2-4Acde-aBe^2)\log(d+ex)}{(cd^2+ae^2)^3} \\ &\quad + \frac{e^2(3Bcd^2-4Acde-aBe^2)\log(a+cx^2)}{2(cd^2+ae^2)^3}\end{aligned}$$

output
```
e^2*(-A*e+B*d)/(a*e^2+c*d^2)^2/(e*x+d)-1/2*(a*(-2*A*c*d*e-B*a*e^2+B*c*d^2)
-c*(-A*a*e^2+A*c*d^2+2*B*a*d*e)*x)/a/(a*e^2+c*d^2)^2/(c*x^2+a)-1/2*c^(1/2)
*(2*a*B*d*e*(-3*a*e^2+c*d^2)-A*(-3*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4))*arctan(
c^(1/2)*x/a^(1/2))/a^(3/2)/(a*e^2+c*d^2)^3-e^2*(-4*A*c*d*e-B*a*e^2+3*B*c*d
^2)*ln(e*x+d)/(a*e^2+c*d^2)^3+1/2*e^2*(-4*A*c*d*e-B*a*e^2+3*B*c*d^2)*ln(c*
x^2+a)/(a*e^2+c*d^2)^3
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{(d+ex)^2\left(a+cx^2\right)^2}\,dx$$
$$=\frac{-\frac{2e^2(-Bd+Ae)(cd^2+ae^2)}{d+ex}+\frac{(cd^2+ae^2)(a^2Be^2+Ac^2d^2x-ac(Bd(d-2ex)+Ae(-2d+ex)))}{a(a+cx^2)}+\frac{\sqrt{c}(2aBde(-cd^2+3ae^2)+A(c^2d^4+6acd^2e^2}{a^{3/2}}}{2\,(cd}$$

input
```
Integrate[(A + B*x)/((d + e*x)^2*(a + c*x^2)^2),x]
```

output
```
((-2*e^2*(-(B*d) + A*e)*(c*d^2 + a*e^2))/(d + e*x) + ((c*d^2 + a*e^2)*(a^2
*B*e^2 + A*c^2*d^2*x - a*c*(B*d*(d - 2*e*x) + A*e*(-2*d + e*x))))/(a*(a +
c*x^2)) + (Sqrt[c]*(2*a*B*d*e*(-(c*d^2) + 3*a*e^2) + A*(c^2*d^4 + 6*a*c*d^
2*e^2 - 3*a^2*e^4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + 2*e^2*(-3*B*c*d
^2 + 4*A*c*d*e + a*B*e^2)*Log[d + e*x] - e^2*(-3*B*c*d^2 + 4*A*c*d*e + a*B
*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {686, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\left(a+cx^2\right)^2(d+ex)^2}\,dx$$

$$\Big\downarrow \; 686$$

$$-\frac{\int -\frac{c\left(Acd^2-2aBed+3aAe^2+2e(Acd+aBe)x\right)}{(d+ex)^2(cx^2+a)}dx}{2ac\left(ae^2+cd^2\right)}-\frac{a(Bd-Ae)-x(aBe+Acd)}{2a\left(a+cx^2\right)(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{c(Acd^2-2aBed+3aAe^2+2e(Acd+aBe)x)}{(d+ex)^2(cx^2+a)}dx}{2ac\left(ae^2+cd^2\right)}-\frac{a(Bd-Ae)-x(aBe+Acd)}{2a\left(a+cx^2\right)(d+ex)\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{Acd^2-2aBed+3aAe^2+2e(Acd+aBe)x}{(d+ex)^2(cx^2+a)}dx}{2a\left(ae^2+cd^2\right)}-\frac{a(Bd-Ae)-x(aBe+Acd)}{2a\left(a+cx^2\right)(d+ex)\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 657

$$\frac{\int \left(\frac{2a(-3Bcd^2+4Aced+aBe^2)e^3}{(cd^2+ae^2)^2(d+ex)}+\frac{(-Acd^2-4aBed+3aAe^2)e^2}{(cd^2+ae^2)(d+ex)^2}+\frac{c(2a(3Bcd^2-4Aced-aBe^2)xe^2-2aBd(cd^2-3ae^2)e+A(c^2d^4+6ace^2d^2\cdots}{(cd^2+ae^2)^2(cx^2+a)}\right.}{2a\left(ae^2+cd^2\right)}$$

$$\frac{a(Bd-Ae)-x(aBe+Acd)}{2a\left(a+cx^2\right)(d+ex)\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(2aBde(cd^2-3ae^2)-A(-3a^2e^4+6acd^2e^2+c^2d^4)\right)}{\sqrt{a}(ae^2+cd^2)^2}+\frac{ae^2\log(a+cx^2)(-aBe^2-4Acde+3Bcd^2)}{(ae^2+cd^2)^2}+\frac{e(-3aAe^2+4aBde+Ac\cdots}{(d+ex)(ae^2+cd^2)}}{2a\left(ae^2+cd^2\right)}$$

$$\frac{a(Bd-Ae)-x(aBe+Acd)}{2a\left(a+cx^2\right)(d+ex)\left(ae^2+cd^2\right)}$$

input

```
Int[(A + B*x)/((d + e*x)^2*(a + c*x^2)^2),x]
```

output

```
-1/2*(a*(B*d - A*e) - (A*c*d + a*B*e)*x)/(a*(c*d^2 + a*e^2)*(d + e*x)*(a +
 c*x^2)) + ((e*(A*c*d^2 + 4*a*B*d*e - 3*a*A*e^2))/((c*d^2 + a*e^2)*(d + e*
x)) - (Sqrt[c]*(2*a*B*d*e*(c*d^2 - 3*a*e^2) - A*(c^2*d^4 + 6*a*c*d^2*e^2 -
 3*a^2*e^4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^2) - (2
*a*e^2*(3*B*c*d^2 - 4*A*c*d*e - a*B*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^2 +
 (a*e^2*(3*B*c*d^2 - 4*A*c*d*e - a*B*e^2)*Log[a + c*x^2])/(c*d^2 + a*e^2)^
2)/(2*a*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.04

method	result
default	$\frac{e^{2}\left(4Acde+Ba\,e^{2}-3Bc\,d^{2}\right)\ln\left(ex+d\right)}{\left(a\,e^{2}+c\,d^{2}\right)^{3}}-\frac{\left(Ae-Bd\right)e^{2}}{\left(a\,e^{2}+c\,d^{2}\right)^{2}\left(ex+d\right)}-\frac{c\left(\frac{\frac{\left(A\,a^{2}e^{4}-A\,c^{2}d^{4}-2B\,a^{2}d\,e^{3}-2Bac\,d^{3}e\right)x}{2a}-\frac{2Aacd\,e^{3}+2A\,c^{2}d^{3}e+B}{2c}}{c\,x^{2}+a}\right.}{}$
risch	Expression too large to display

input
```
int((B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
e^2*(4*A*c*d*e+B*a*e^2-3*B*c*d^2)/(a*e^2+c*d^2)^3*ln(e*x+d)-(A*e-B*d)*e^2/
(a*e^2+c*d^2)^2/(e*x+d)-c/(a*e^2+c*d^2)^3*((1/2*(A*a^2*e^4-A*c^2*d^4-2*B*a
^2*d*e^3-2*B*a*c*d^3*e)/a*x-1/2*(2*A*a*c*d*e^3+2*A*c^2*d^3*e+B*a^2*e^4-B*c
^2*d^4)/c)/(c*x^2+a)+1/2/a*(1/2*(8*A*a*c*d*e^3+2*B*a^2*e^4-6*B*a*c*d^2*e^2
)/c*ln(c*x^2+a)+(3*A*a^2*e^4-6*A*a*c*d^2*e^2-A*c^2*d^4-6*B*a^2*d*e^3+2*B*a
*c*d^3*e)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 958 vs. $2(277) = 554$.

Time = 35.48 (sec) , antiderivative size = 1940, normalized size of antiderivative = 6.67

$$\int \frac{A+Bx}{(d+ex)^2\left(a+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*B*a*c^2*d^5 - 4*A*a*c^2*d^4*e - 4*B*a^2*c*d^3*e^2 - 6*B*a^3*d*e^4
 + 4*A*a^3*e^5 - 2*(A*c^3*d^4*e + 4*B*a*c^2*d^3*e^2 - 2*A*a*c^2*d^2*e^3 +
4*B*a^2*c*d*e^4 - 3*A*a^2*c*e^5)*x^2 + (A*a*c^2*d^5 - 2*B*a^2*c*d^4*e + 6*
A*a^2*c*d^3*e^2 + 6*B*a^3*d^2*e^3 - 3*A*a^3*d*e^4 + (A*c^3*d^4*e - 2*B*a*c
^2*d^3*e^2 + 6*A*a*c^2*d^2*e^3 + 6*B*a^2*c*d*e^4 - 3*A*a^2*c*e^5)*x^3 + (A
*c^3*d^5 - 2*B*a*c^2*d^4*e + 6*A*a*c^2*d^3*e^2 + 6*B*a^2*c*d^2*e^3 - 3*A*a
^2*c*d*e^4)*x^2 + (A*a*c^2*d^4*e - 2*B*a^2*c*d^3*e^2 + 6*A*a^2*c*d^2*e^3 +
 6*B*a^3*d*e^4 - 3*A*a^3*e^5)*x)*sqrt(-c/a)*log((c*x^2 - 2*a*x*sqrt(-c/a)
- a)/(c*x^2 + a)) - 2*(A*c^3*d^5 + B*a*c^2*d^4*e + 2*A*a*c^2*d^3*e^2 + 2*B
*a^2*c*d^2*e^3 + A*a^2*c*d*e^4 + B*a^3*e^5)*x - 2*(3*B*a^2*c*d^3*e^2 - 4*A
*a^2*c*d^2*e^3 - B*a^3*d*e^4 + (3*B*a*c^2*d^2*e^3 - 4*A*a*c^2*d*e^4 - B*a^
2*c*e^5)*x^3 + (3*B*a*c^2*d^3*e^2 - 4*A*a*c^2*d^2*e^3 - B*a^2*c*d*e^4)*x^2
 + (3*B*a^2*c*d^2*e^3 - 4*A*a^2*c*d*e^4 - B*a^3*e^5)*x)*log(c*x^2 + a) + 4
*(3*B*a^2*c*d^3*e^2 - 4*A*a^2*c*d^2*e^3 - B*a^3*d*e^4 + (3*B*a*c^2*d^2*e^3
 - 4*A*a*c^2*d*e^4 - B*a^2*c*e^5)*x^3 + (3*B*a*c^2*d^3*e^2 - 4*A*a*c^2*d^2
*e^3 - B*a^2*c*d*e^4)*x^2 + (3*B*a^2*c*d^2*e^3 - 4*A*a^2*c*d*e^4 - B*a^3*e
^5)*x)*log(e*x + d))/(a^2*c^3*d^7 + 3*a^3*c^2*d^5*e^2 + 3*a^4*c*d^3*e^4 +
a^5*d*e^6 + (a*c^4*d^6*e + 3*a^2*c^3*d^4*e^3 + 3*a^3*c^2*d^2*e^5 + a^4*c*e
^7)*x^3 + (a*c^4*d^7 + 3*a^2*c^3*d^5*e^2 + 3*a^3*c^2*d^3*e^4 + a^4*c*d*e^6
)*x^2 + (a^2*c^3*d^6*e + 3*a^3*c^2*d^4*e^3 + 3*a^4*c*d^2*e^5 + a^5*e^7)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)^2\,(a+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)**2/(c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.76

$$\int \frac{A+Bx}{(d+ex)^2\,(a+cx^2)^2}\,dx = \frac{(3\,Bcd^2e^2-4\,Acde^3-Bae^4)\log\left(cx^2+a\right)}{2\,(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)} - \frac{(3\,Bcd^2e^2-4\,Acde^3-Bae^4)\log\left(ex+d\right)}{c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6} + \frac{(Ac^3d^4-2\,Bac^2d^3e+6\,Aac^2d^2e^2+6\,Ba^2cde^3-3\,Aa^2ce^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,(ac^3d^6+3\,a^2c^2d^4e^2+3\,a^3cd^2e^4+a^4e^6)\sqrt{ac}} - \frac{Bacd^3-2\,Aacd^2e-3\,Ba^2de^2+2\,Aa^2e^3-(Ac^2d^2e+4\,Bacde^2-3\,Aace^3)x^2-(Ac^2d^3+Bacd \cdots}{2\,(a^2c^2d^5+2\,a^3cd^3e^2+a^4de^4+(ac^3d^4e+2\,a^2c^2d^2e^3+a^3ce^5)x^3+(ac^3d^5+2\,a^2c^2d^3e^2+a^3cde^4)x^2+ \cdots}$$

input
```
integrate((B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(3*B*c*d^2*e^2 - 4*A*c*d*e^3 - B*a*e^4)*log(c*x^2 + a)/(c^3*d^6 + 3*a*
c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - (3*B*c*d^2*e^2 - 4*A*c*d*e^3 -
B*a*e^4)*log(e*x + d)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e
^6) + 1/2*(A*c^3*d^4 - 2*B*a*c^2*d^3*e + 6*A*a*c^2*d^2*e^2 + 6*B*a^2*c*d*e
^3 - 3*A*a^2*c*e^4)*arctan(c*x/sqrt(a*c))/((a*c^3*d^6 + 3*a^2*c^2*d^4*e^2
+ 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(a*c)) - 1/2*(B*a*c*d^3 - 2*A*a*c*d^2*e -
 3*B*a^2*d*e^2 + 2*A*a^2*e^3 - (A*c^2*d^2*e + 4*B*a*c*d*e^2 - 3*A*a*c*e^3)
*x^2 - (A*c^2*d^3 + B*a*c*d^2*e + A*a*c*d*e^2 + B*a^2*e^3)*x)/(a^2*c^2*d^5
 + 2*a^3*c*d^3*e^2 + a^4*d*e^4 + (a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3 + a^3*c*
e^5)*x^3 + (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x^2 + (a^2*c^2*d^
4*e + 2*a^3*c*d^2*e^3 + a^4*e^5)*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.78

$$\int \frac{A+Bx}{(d+ex)^2\,(a+cx^2)^2}\,dx$$

$$= \frac{(3\,Bcd^2e^2-4\,Acde^3-Bae^4)\log\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}{2\,(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)} + \frac{\frac{Bde^6}{ex+d}-\frac{Ae^7}{ex+d}}{c^2d^4e^4+2\,acd^2e^6+a^2e^8} + \frac{(Ac^3d^4e^2-2\,Bac^2d^3e^3+6\,Aac^2d^2e^4+6\,Ba^2cde^5-3\,Aa^2ce^6)\arctan\left(\frac{cd-\frac{cd^2}{ex+d}-\frac{ae^2}{ex+d}}{\sqrt{ace}}\right)}{2\,(ac^3d^6+3\,a^2c^2d^4e^2+3\,a^3cd^2e^4+a^4e^6)\sqrt{ac}e^2} + \frac{\frac{Ac^3d^3e+3\,Bac^2d^2e^2-3\,Aac^2de^3-Ba^2ce^4}{cd^2+ae^2}-\frac{Ac^3d^4e^2+4\,Bac^2d^3e^3-6\,Aac^2d^2e^4-4\,Ba^2cde^5+Aa^2ce^6}{(cd^2+ae^2)(ex+d)e}}{2\,(cd^2+ae^2)^2a\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}$$

input
```
integrate((B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(3*B*c*d^2*e^2 - 4*A*c*d*e^3 - B*a*e^4)*log(c - 2*c*d/(e*x + d) + c*d^
2/(e*x + d)^2 + a*e^2/(e*x + d)^2)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^
2*e^4 + a^3*e^6) + (B*d*e^6/(e*x + d) - A*e^7/(e*x + d))/(c^2*d^4*e^4 + 2*
a*c*d^2*e^6 + a^2*e^8) + 1/2*(A*c^3*d^4*e^2 - 2*B*a*c^2*d^3*e^3 + 6*A*a*c^
2*d^2*e^4 + 6*B*a^2*c*d*e^5 - 3*A*a^2*c*e^6)*arctan((c*d - c*d^2/(e*x + d)
 - a*e^2/(e*x + d))/(sqrt(a*c)*e))/((a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3
*c*d^2*e^4 + a^4*e^6)*sqrt(a*c)*e^2) + 1/2*((A*c^3*d^3*e + 3*B*a*c^2*d^2*e
^2 - 3*A*a*c^2*d*e^3 - B*a^2*c*e^4)/(c*d^2 + a*e^2) - (A*c^3*d^4*e^2 + 4*B
*a*c^2*d^3*e^3 - 6*A*a*c^2*d^2*e^4 - 4*B*a^2*c*d*e^5 + A*a^2*c*e^6)/((c*d^
2 + a*e^2)*(e*x + d)*e))/((c*d^2 + a*e^2)^2*a*(c - 2*c*d/(e*x + d) + c*d^2
/(e*x + d)^2 + a*e^2/(e*x + d)^2))
```

Mupad [B] (verification not implemented)

Time = 7.79 (sec) , antiderivative size = 2029, normalized size of antiderivative = 6.97

$$\int \frac{A+Bx}{(d+ex)^2\,(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/((a + c*x^2)^2*(d + e*x)^2),x)
```

output
```
((x*(A*c*d + B*a*e))/(2*a*(a*e^2 + c*d^2)) - (2*A*a*e^3 + B*c*d^3 - 3*B*a*
d*e^2 - 2*A*c*d^2*e)/(2*(a*e^2 + c*d^2)^2) + (x^2*(A*c^2*d^2*e - 3*A*a*c*e
^3 + 4*B*a*c*d*e^2))/(2*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a*d + a*e
*x + c*d*x^2 + c*e*x^3) + (log(9*A^2*a^6*e^12*(-a^3*c)^(3/2) + A^2*c^6*d^1
2*(-a^3*c)^(3/2) - 36*B^2*a^10*e^12*(-a^3*c)^(1/2) - 558*A^2*a^2*d^2*e^10*
(-a^3*c)^(5/2) + 24*B^2*a^2*d^4*e^8*(-a^3*c)^(5/2) - 108*B^2*a^6*d^2*e^10*
(-a^3*c)^(3/2) - 612*A^2*c^2*d^6*e^6*(-a^3*c)^(5/2) - 308*B^2*c^2*d^8*e^4*
(-a^3*c)^(5/2) + 36*B^2*a^11*c*e^12*x + A^2*a^4*c^8*d^12*x + 9*A^2*a^10*c^
2*e^12*x + 276*A*B*a^2*d^3*e^9*(-a^3*c)^(5/2) + 808*A*B*c^2*d^7*e^5*(-a^3*
c)^(5/2) - 1119*A^2*a*c*d^4*e^8*(-a^3*c)^(5/2) - 424*B^2*a*c*d^6*e^6*(-a^3
*c)^(5/2) + 14*A^2*a^5*c^7*d^10*e^2*x + 55*A^2*a^6*c^6*d^8*e^4*x + 612*A^2
*a^7*c^5*d^6*e^6*x + 1119*A^2*a^8*c^4*d^4*e^8*x + 558*A^2*a^9*c^3*d^2*e^10
*x + 4*B^2*a^6*c^6*d^10*e^2*x + 308*B^2*a^7*c^5*d^8*e^4*x + 424*B^2*a^8*c^
4*d^6*e^6*x - 24*B^2*a^9*c^3*d^4*e^8*x - 108*B^2*a^10*c^2*d^2*e^10*x + 14*
A^2*a*c^5*d^10*e^2*(-a^3*c)^(3/2) + 252*A*B*a^6*d*e^11*(-a^3*c)^(3/2) + 55
*A^2*a^2*c^4*d^8*e^4*(-a^3*c)^(3/2) + 4*B^2*a^2*c^4*d^10*e^2*(-a^3*c)^(3/2
) - 4*A*B*a^5*c^7*d^11*e*x + 252*A*B*a^10*c^2*d*e^11*x + 1320*A*B*a*c*d^5*
e^7*(-a^3*c)^(5/2) - 4*A*B*a*c^5*d^11*e*(-a^3*c)^(3/2) - 20*A*B*a^6*c^6*d^
9*e^3*x - 808*A*B*a^7*c^5*d^7*e^5*x - 1320*A*B*a^8*c^4*d^5*e^7*x - 276*A*B
*a^9*c^3*d^3*e^9*x - 20*A*B*a^2*c^4*d^9*e^3*(-a^3*c)^(3/2))*(c*(a^3*((3...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1504, normalized size of antiderivative = 5.17

$$\int \frac{A+Bx}{(d+ex)^2\,(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x)
```

output
```
( - 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*d**2*e**4 - 3*sqr
t(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*d*e**5*x + 6*sqrt(c)*sqrt(
a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*b*d**3*e**3 + 6*sqrt(c)*sqrt(a)*atan
((c*x)/(sqrt(c)*sqrt(a)))*a**2*b*d**2*e**4*x + 6*sqrt(c)*sqrt(a)*atan((c*x
)/(sqrt(c)*sqrt(a)))*a**2*c*d**4*e**2 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt
(c)*sqrt(a)))*a**2*c*d**3*e**3*x - 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*s
qrt(a)))*a**2*c*d**2*e**4*x**2 - 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqr
t(a)))*a**2*c*d*e**5*x**3 - 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a))
)*a*b*c*d**5*e - 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*b*c*d**
4*e**2*x + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*b*c*d**3*e**3
*x**2 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*b*c*d**2*e**4*x*
*3 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c**2*d**6 + sqrt(c)*s
qrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c**2*d**5*e*x + 6*sqrt(c)*sqrt(a)*a
tan((c*x)/(sqrt(c)*sqrt(a)))*a*c**2*d**4*e**2*x**2 + 6*sqrt(c)*sqrt(a)*ata
n((c*x)/(sqrt(c)*sqrt(a)))*a*c**2*d**3*e**3*x**3 - 2*sqrt(c)*sqrt(a)*atan(
(c*x)/(sqrt(c)*sqrt(a)))*b*c**2*d**5*e*x**2 - 2*sqrt(c)*sqrt(a)*atan((c*x)
/(sqrt(c)*sqrt(a)))*b*c**2*d**4*e**2*x**3 + sqrt(c)*sqrt(a)*atan((c*x)/(sq
rt(c)*sqrt(a)))*c**3*d**6*x**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(
a)))*c**3*d**5*e*x**3 - log(a + c*x**2)*a**3*b*d**2*e**4 - log(a + c*x**2)
*a**3*b*d*e**5*x - 4*log(a + c*x**2)*a**3*c*d**3*e**3 - 4*log(a + c*x**...
```

3.95 $\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 306

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^3}\,dx = -\frac{e^2\left(\frac{5aBe(cd^2-3ae^2)}{c}+A(3cd^3+7ade^2)\right)x}{8a^2c^2} - \frac{(d+ex)^4(a(Bd+Ae)-(Acd-aBe)x)}{4ac\,(a+cx^2)^2} - \frac{(d+ex)^2\left(2ae(Acd^2+5aBde+2aAe^2)-(5aBe(cd^2-ae^2)+Acd(3cd^2+5ae^2))\,x\right)}{8a^2c^2\,(a+cx^2)} + \frac{(5aBe(c^2d^4+6acd^2e^2-3a^2e^4)+Acd(3c^2d^4+10acd^2e^2+15a^2e^4))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{7/2}} + \frac{e^4(5Bd+Ae)\log(a+cx^2)}{2c^3}$$

output
```
-1/8*e^2*(5*a*B*e*(-3*a*e^2+c*d^2)/c+A*(7*a*d*e^2+3*c*d^3))*x/a^2/c^2-1/4*
(e*x+d)^4*(a*(A*e+B*d)-(A*c*d-B*a*e)*x)/a/c/(c*x^2+a)^2-1/8*(e*x+d)^2*(2*a
*e*(2*A*a*e^2+A*c*d^2+5*B*a*d*e)-(5*a*B*e*(-a*e^2+c*d^2)+A*c*d*(5*a*e^2+3*
c*d^2))*x)/a^2/c^2/(c*x^2+a)+1/8*(5*a*B*e*(-3*a^2*e^4+6*a*c*d^2*e^2+c^2*d^
4)+A*c*d*(15*a^2*e^4+10*a*c*d^2*e^2+3*c^2*d^4))*arctan(c^(1/2)*x/a^(1/2))/
a^(5/2)/c^(7/2)+1/2*e^4*(A*e+5*B*d)*ln(c*x^2+a)/c^3
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^3}\,dx$$
$$= \frac{8B\sqrt{c}e^5x + \frac{2\sqrt{c}(Ac^3d^5x-a^3e^4(5Bd+Ae+Bex)+5a^2cde^2(2Bd(d+ex)+Ae(2d+ex))-ac^2d^3(5Ae(d+2ex)+Bd(d+5ex)))}{a(a+cx^2)^2} + \frac{\sqrt{c}(3Ac^3d^5}{}}{}$$

input
```
Integrate[((A + B*x)*(d + e*x)^5)/(a + c*x^2)^3,x]
```

output
```
(8*B*Sqrt[c]*e^5*x + (2*Sqrt[c]*(A*c^3*d^5*x - a^3*e^4*(5*B*d + A*e + B*e*
x) + 5*a^2*c*d*e^2*(2*B*d*(d + e*x) + A*e*(2*d + e*x)) - a*c^2*d^3*(5*A*e*
(d + 2*e*x) + B*d*(d + 5*e*x))))/(a*(a + c*x^2)^2) + (Sqrt[c]*(3*A*c^3*d^5
*x + 5*a*c^2*d^3*e*(B*d + 2*A*e)*x + a^3*e^4*(40*B*d + 8*A*e + 9*B*e*x) -
5*a^2*c*d*e^2*(2*B*d*(4*d + 5*e*x) + A*e*(8*d + 5*e*x))))/(a^2*(a + c*x^2)
) + ((5*a*B*e*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4) + A*c*d*(3*c^2*d^4 + 1
0*a*c*d^2*e^2 + 15*a^2*e^4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 4*Sqrt
[c]*e^4*(5*B*d + A*e)*Log[a + c*x^2])/(8*c^(7/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {684, 684, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^3}\,dx$$
$$\Big\downarrow \; 684$$
$$\frac{\int \frac{(d+ex)^3(3Acd^2+ae(5Bd+4Ae)-e(Acd-5aBe)x)}{(cx^2+a)^2}dx}{4ac} - \frac{(d+ex)^4(a(Ae+Bd)-x(Acd-aBe))}{4ac\,(a+cx^2)^2}$$
$$\Big\downarrow \; 684$$

$$\frac{\frac{\int \frac{(d+ex)\left(5aBde\left(cd^2+5ae^2\right)+A\left(3c^2d^4+7ace^2d^2+8a^2e^4\right)-e\left(5aBe\left(cd^2-3ae^2\right)+Acd\left(3cd^2+7ae^2\right)\right)x\right)}{cx^2+a}dx}{2ac}-\frac{(d+ex)^2\left(2ae\left(2aAe^2+5aBde+Acd^2\right)-x(A\ldots}{2ac(a+c\ldots}}{4ac}$$

$$\frac{(d+ex)^4(a(Ae+Bd)-x(Acd-aBe))}{4ac\left(a+cx^2\right)^2}$$

$\Big\downarrow$ 657

$$\frac{\frac{\int \left(\frac{8a^2c(5Bd+Ae)xe^4+5aB\left(c^2d^4+6ace^2d^2-3a^2e^4\right)e+Acd\left(3c^2d^4+10ace^2d^2+15a^2e^4\right)}{c\left(cx^2+a\right)}-e^2\left(3Acd^3+5aBed^2+7aAe^2d-\frac{15a^2Be^3}{c}\right)\right)dx}{2ac}-\frac{(d+ex)^2(2a\ldots}{}}{4ac}$$

$$\frac{(d+ex)^4(a(Ae+Bd)-x(Acd-aBe))}{4ac\left(a+cx^2\right)^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)+5aBe\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)\right)}{\sqrt{a}c^{3/2}}+\frac{4a^2e^4\log\left(a+cx^2\right)(Ae+5Bd)}{c}-e^2x\left(A\left(7ade^2+3cd^3\right)+\frac{5aBe\left(cd^2-3ae^2\ldots\right.}{c}\right.}{2ac}}{4ac}$$

$$\frac{(d+ex)^4(a(Ae+Bd)-x(Acd-aBe))}{4ac\left(a+cx^2\right)^2}$$

input

```
Int[((A + B*x)*(d + e*x)^5)/(a + c*x^2)^3,x]
```

output

```
-1/4*((d + e*x)^4*(a*(B*d + A*e) - (A*c*d - a*B*e)*x))/(a*c*(a + c*x^2)^2)
 + (-1/2*((d + e*x)^2*(2*a*e*(A*c*d^2 + 5*a*B*d*e + 2*a*A*e^2) - (5*a*B*e*
(c*d^2 - a*e^2) + A*c*d*(3*c*d^2 + 5*a*e^2))*x))/(a*c*(a + c*x^2)) + (-(e^
2*((5*a*B*e*(c*d^2 - 3*a*e^2))/c + A*(3*c*d^3 + 7*a*d*e^2))*x) + ((5*a*B*e
*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4) + A*c*d*(3*c^2*d^4 + 10*a*c*d^2*e^2
 + 15*a^2*e^4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (4*a^2*e^
4*(5*B*d + A*e)*Log[a + c*x^2])/c)/(2*a*c))/(4*a*c)
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.32

method	result
default	$\frac{B e^5 x}{c^3}+\frac{-\frac{c\left(25A a^2 c d e^4-10A a c^2 d^3 e^2-3A d^5 c^3-9B e^5 a^3+50B a^2 c d^2 e^3-5B a c^2 d^4 e\right)x^3}{8a^2}+\left(A a c e^5-5A c^2 d^2 e^3+5B a c d e^4-5B c^2 d^3 e^2\right)x^2-}{}$
risch	Expression too large to display

input
```
int((B*x+A)*(e*x+d)^5/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
B*e^5/c^3*x+1/c^3*((-1/8*c*(25*A*a^2*c*d*e^4-10*A*a*c^2*d^3*e^2-3*A*c^3*d^
5-9*B*a^3*e^5+50*B*a^2*c*d^2*e^3-5*B*a*c^2*d^4*e)/a^2*x^3+(A*a*c*e^5-5*A*c
^2*d^2*e^3+5*B*a*c*d*e^4-5*B*c^2*d^3*e^2)*x^2-1/8*(15*A*a^2*c*d*e^4+10*A*a
*c^2*d^3*e^2-5*A*c^3*d^5-7*B*a^3*e^5+30*B*a^2*c*d^2*e^3+5*B*a*c^2*d^4*e)/a
*x+3/4*A*a^2*e^5-5/2*A*a*c*d^2*e^3-5/4*A*c^2*d^4*e+15/4*B*a^2*d*e^4-5/2*B*
a*c*d^3*e^2-1/4*B*c^2*d^5)/(c*x^2+a)^2+1/8/a^2*(1/2*(8*A*a^2*c*e^5+40*B*a^
2*c*d*e^4)/c*ln(c*x^2+a)+(15*A*a^2*c*d*e^4+10*A*a*c^2*d^3*e^2+3*A*c^3*d^5-
15*B*a^3*e^5+30*B*a^2*c*d^2*e^3+5*B*a*c^2*d^4*e)/(a*c)^(1/2)*arctan(c*x/(a
*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 691 vs. $2(288) = 576$.

Time = 0.11 (sec) , antiderivative size = 1403, normalized size of antiderivative = 4.58

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^5/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*B*a^3*c^3*e^5*x^5 - 4*B*a^3*c^3*d^5 - 20*A*a^3*c^3*d^4*e - 40*B*
a^4*c^2*d^3*e^2 - 40*A*a^4*c^2*d^2*e^3 + 60*B*a^5*c*d*e^4 + 12*A*a^5*c*e^5
 + 2*(3*A*a*c^5*d^5 + 5*B*a^2*c^4*d^4*e + 10*A*a^2*c^4*d^3*e^2 - 50*B*a^3*
c^3*d^2*e^3 - 25*A*a^3*c^3*d*e^4 + 25*B*a^4*c^2*e^5)*x^3 - 16*(5*B*a^3*c^3
*d^3*e^2 + 5*A*a^3*c^3*d^2*e^3 - 5*B*a^4*c^2*d*e^4 - A*a^4*c^2*e^5)*x^2 +
(3*A*a^2*c^3*d^5 + 5*B*a^3*c^2*d^4*e + 10*A*a^3*c^2*d^3*e^2 + 30*B*a^4*c*d
^2*e^3 + 15*A*a^4*c*d*e^4 - 15*B*a^5*e^5 + (3*A*c^5*d^5 + 5*B*a*c^4*d^4*e
+ 10*A*a*c^4*d^3*e^2 + 30*B*a^2*c^3*d^2*e^3 + 15*A*a^2*c^3*d*e^4 - 15*B*a^
3*c^2*e^5)*x^4 + 2*(3*A*a*c^4*d^5 + 5*B*a^2*c^3*d^4*e + 10*A*a^2*c^3*d^3*e
^2 + 30*B*a^3*c^2*d^2*e^3 + 15*A*a^3*c^2*d*e^4 - 15*B*a^4*c*e^5)*x^2)*sqrt
(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 10*(A*a^2*c^4*d^5 -
 B*a^3*c^3*d^4*e - 2*A*a^3*c^3*d^3*e^2 - 6*B*a^4*c^2*d^2*e^3 - 3*A*a^4*c^2
*d*e^4 + 3*B*a^5*c*e^5)*x + 8*(5*B*a^5*c*d*e^4 + A*a^5*c*e^5 + (5*B*a^3*c^
3*d*e^4 + A*a^3*c^3*e^5)*x^4 + 2*(5*B*a^4*c^2*d*e^4 + A*a^4*c^2*e^5)*x^2)*
log(c*x^2 + a))/(a^3*c^6*x^4 + 2*a^4*c^5*x^2 + a^5*c^4), 1/8*(8*B*a^3*c^3*
e^5*x^5 - 2*B*a^3*c^3*d^5 - 10*A*a^3*c^3*d^4*e - 20*B*a^4*c^2*d^3*e^2 - 20
*A*a^4*c^2*d^2*e^3 + 30*B*a^5*c*d*e^4 + 6*A*a^5*c*e^5 + (3*A*a*c^5*d^5 + 5
*B*a^2*c^4*d^4*e + 10*A*a^2*c^4*d^3*e^2 - 50*B*a^3*c^3*d^2*e^3 - 25*A*a^3*
c^3*d*e^4 + 25*B*a^4*c^2*e^5)*x^3 - 8*(5*B*a^3*c^3*d^3*e^2 + 5*A*a^3*c^3*d
^2*e^3 - 5*B*a^4*c^2*d*e^4 - A*a^4*c^2*e^5)*x^2 + (3*A*a^2*c^3*d^5 + 5*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(299) = 598$.

Time = 56.41 (sec) , antiderivative size = 1044, normalized size of antiderivative = 3.41

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)**5/(c*x**2+a)**3,x)
```

output
```
B*e**5*x/c**3 + (e**4*(A*e + 5*B*d)/(2*c**3) - sqrt(-a**5*c**7)*(-15*A*a**
2*c*d*e**4 - 10*A*a*c**2*d**3*e**2 - 3*A*c**3*d**5 + 15*B*a**3*e**5 - 30*B
*a**2*c*d**2*e**3 - 5*B*a*c**2*d**4*e)/(16*a**5*c**7))*log(x + (8*A*a**3*e
**5 + 40*B*a**3*d*e**4 - 16*a**3*c**3*(e**4*(A*e + 5*B*d)/(2*c**3) - sqrt(
-a**5*c**7)*(-15*A*a**2*c*d*e**4 - 10*A*a*c**2*d**3*e**2 - 3*A*c**3*d**5 +
 15*B*a**3*e**5 - 30*B*a**2*c*d**2*e**3 - 5*B*a*c**2*d**4*e)/(16*a**5*c**7
)))/(-15*A*a**2*c*d*e**4 - 10*A*a*c**2*d**3*e**2 - 3*A*c**3*d**5 + 15*B*a*
*3*e**5 - 30*B*a**2*c*d**2*e**3 - 5*B*a*c**2*d**4*e)) + (e**4*(A*e + 5*B*d
)/(2*c**3) + sqrt(-a**5*c**7)*(-15*A*a**2*c*d*e**4 - 10*A*a*c**2*d**3*e**2
 - 3*A*c**3*d**5 + 15*B*a**3*e**5 - 30*B*a**2*c*d**2*e**3 - 5*B*a*c**2*d**
4*e)/(16*a**5*c**7))*log(x + (8*A*a**3*e**5 + 40*B*a**3*d*e**4 - 16*a**3*c
**3*(e**4*(A*e + 5*B*d)/(2*c**3) + sqrt(-a**5*c**7)*(-15*A*a**2*c*d*e**4 -
 10*A*a*c**2*d**3*e**2 - 3*A*c**3*d**5 + 15*B*a**3*e**5 - 30*B*a**2*c*d**2
*e**3 - 5*B*a*c**2*d**4*e)/(16*a**5*c**7)))/(-15*A*a**2*c*d*e**4 - 10*A*a*
c**2*d**3*e**2 - 3*A*c**3*d**5 + 15*B*a**3*e**5 - 30*B*a**2*c*d**2*e**3 -
5*B*a*c**2*d**4*e)) + (6*A*a**4*e**5 - 20*A*a**3*c*d**2*e**3 - 10*A*a**2*c
**2*d**4*e + 30*B*a**4*d*e**4 - 20*B*a**3*c*d**3*e**2 - 2*B*a**2*c**2*d**5
 + x**3*(-25*A*a**2*c**2*d*e**4 + 10*A*a*c**3*d**3*e**2 + 3*A*c**4*d**5 +
9*B*a**3*c*e**5 - 50*B*a**2*c**2*d**2*e**3 + 5*B*a*c**3*d**4*e) + x**2*(8*
A*a**3*c*e**5 - 40*A*a**2*c**2*d**2*e**3 + 40*B*a**3*c*d*e**4 - 40*B*a*...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^3}\,dx = \frac{Be^5x}{c^3}$$

$$- \frac{2\,Ba^2c^2d^5 + 10\,Aa^2c^2d^4e + 20\,Ba^3cd^3e^2 + 20\,Aa^3cd^2e^3 - 30\,Ba^4de^4 - 6\,Aa^4e^5 - (3\,Ac^4d^5 + 5\,Bac^3d^4}{}$$

$$+ \frac{(5\,Bde^4 + Ae^5)\log(cx^2+a)}{2\,c^3}$$

$$+ \frac{(3\,Ac^3d^5 + 5\,Bac^2d^4e + 10\,Aac^2d^3e^2 + 30\,Ba^2cd^2e^3 + 15\,Aa^2cde^4 - 15\,Ba^3e^5)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^3}$$

input
```
integrate((B*x+A)*(e*x+d)^5/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
B*e^5*x/c^3 - 1/8*(2*B*a^2*c^2*d^5 + 10*A*a^2*c^2*d^4*e + 20*B*a^3*c*d^3*e
^2 + 20*A*a^3*c*d^2*e^3 - 30*B*a^4*d*e^4 - 6*A*a^4*e^5 - (3*A*c^4*d^5 + 5*
B*a*c^3*d^4*e + 10*A*a*c^3*d^3*e^2 - 50*B*a^2*c^2*d^2*e^3 - 25*A*a^2*c^2*d
*e^4 + 9*B*a^3*c*e^5)*x^3 + 8*(5*B*a^2*c^2*d^3*e^2 + 5*A*a^2*c^2*d^2*e^3 -
 5*B*a^3*c*d*e^4 - A*a^3*c*e^5)*x^2 - (5*A*a*c^3*d^5 - 5*B*a^2*c^2*d^4*e -
 10*A*a^2*c^2*d^3*e^2 - 30*B*a^3*c*d^2*e^3 - 15*A*a^3*c*d*e^4 + 7*B*a^4*e^
5)*x)/(a^2*c^5*x^4 + 2*a^3*c^4*x^2 + a^4*c^3) + 1/2*(5*B*d*e^4 + A*e^5)*lo
g(c*x^2 + a)/c^3 + 1/8*(3*A*c^3*d^5 + 5*B*a*c^2*d^4*e + 10*A*a*c^2*d^3*e^2
 + 30*B*a^2*c*d^2*e^3 + 15*A*a^2*c*d*e^4 - 15*B*a^3*e^5)*arctan(c*x/sqrt(a
*c))/(sqrt(a*c)*a^2*c^3)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.38

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^3}\,dx = \frac{Be^5x}{c^3} + \frac{(5\,Bde^4 + Ae^5)\log(cx^2+a)}{2\,c^3} + \frac{(3\,Ac^3d^5 + 5\,Bac^2d^4e + 10\,Aac^2d^3e^2 + 30\,Ba^2cd^2e^3 + 15\,Aa^2cde^4 - 15\,Ba^3e^5)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^3} - \frac{2\,Ba^2c^2d^5 + 10\,Aa^2c^2d^4e + 20\,Ba^3cd^3e^2 + 20\,Aa^3cd^2e^3 - 30\,Ba^4de^4 - 6\,Aa^4e^5 - (3\,Ac^4d^5 + 5\,Bac^3d^4}{}$$

input
```
integrate((B*x+A)*(e*x+d)^5/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
B*e^5*x/c^3 + 1/2*(5*B*d*e^4 + A*e^5)*log(c*x^2 + a)/c^3 + 1/8*(3*A*c^3*d^
5 + 5*B*a*c^2*d^4*e + 10*A*a*c^2*d^3*e^2 + 30*B*a^2*c*d^2*e^3 + 15*A*a^2*c
*d*e^4 - 15*B*a^3*e^5)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c^3) - 1/8*(2*
B*a^2*c^2*d^5 + 10*A*a^2*c^2*d^4*e + 20*B*a^3*c*d^3*e^2 + 20*A*a^3*c*d^2*e
^3 - 30*B*a^4*d*e^4 - 6*A*a^4*e^5 - (3*A*c^4*d^5 + 5*B*a*c^3*d^4*e + 10*A*
a*c^3*d^3*e^2 - 50*B*a^2*c^2*d^2*e^3 - 25*A*a^2*c^2*d*e^4 + 9*B*a^3*c*e^5)
*x^3 + 8*(5*B*a^2*c^2*d^3*e^2 + 5*A*a^2*c^2*d^2*e^3 - 5*B*a^3*c*d*e^4 - A*
a^3*c*e^5)*x^2 - (5*A*a*c^3*d^5 - 5*B*a^2*c^2*d^4*e - 10*A*a^2*c^2*d^3*e^2
 - 30*B*a^3*c*d^2*e^3 - 15*A*a^3*c*d*e^4 + 7*B*a^4*e^5)*x)/((c*x^2 + a)^2*
a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.39

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^3}\,dx = \frac{\ln(c\,x^2+a)\,\left(256\,A\,a^5\,c^4\,e^5+1280\,B\,d\,a^5\,c^4\,e^4\right)}{512\,a^5\,c^7}$$
$$-\frac{\frac{B\,c^2\,d^5}{4}-\frac{3\,A\,a^2\,e^5}{4}-x^2\left(-5\,B\,c^2\,d^3\,e^2-5\,A\,c^2\,d^2\,e^3+5\,B\,a\,c\,d\,e^4+A\,a\,c\,e^5\right)-\frac{x^3\,(9\,B\,a^3\,c\,e^5-50\,B\,a^2\,c^2\,d^2\,e^3-2}{}}{}$$
$$+\frac{B\,e^5\,x}{c^3}$$
$$+\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(-15\,B\,a^3\,e^5+30\,B\,a^2\,c\,d^2\,e^3+15\,A\,a^2\,c\,d\,e^4+5\,B\,a\,c^2\,d^4\,e+10\,A\,a\,c^2\,d^3\,e^2+3\,A\,c^3\,d^5\right)}{8\,a^{5/2}\,c^{7/2}}$$

input
```
int(((A + B*x)*(d + e*x)^5)/(a + c*x^2)^3,x)
```

output
```
(log(a + c*x^2)*(256*A*a^5*c^4*e^5 + 1280*B*a^5*c^4*d*e^4))/(512*a^5*c^7)
- ((B*c^2*d^5)/4 - (3*A*a^2*e^5)/4 - x^2*(A*a*c*e^5 - 5*A*c^2*d^2*e^3 - 5*
B*c^2*d^3*e^2 + 5*B*a*c*d*e^4) - (x^3*(3*A*c^4*d^5 + 9*B*a^3*c*e^5 + 10*A*
a*c^3*d^3*e^2 - 25*A*a^2*c^2*d*e^4 - 50*B*a^2*c^2*d^2*e^3 + 5*B*a*c^3*d^4*
e))/(8*a^2) + (x*(10*A*a*c^2*d^3*e^2 - 7*B*a^3*e^5 - 5*A*c^3*d^5 + 30*B*a^
2*c*d^2*e^3 + 15*A*a^2*c*d*e^4 + 5*B*a*c^2*d^4*e))/(8*a) - (15*B*a^2*d*e^4
)/4 + (5*A*c^2*d^4*e)/4 + (5*A*a*c*d^2*e^3)/2 + (5*B*a*c*d^3*e^2)/2)/(a^2*
c^3 + c^5*x^4 + 2*a*c^4*x^2) + (B*e^5*x)/c^3 + (atan((c^(1/2)*x)/a^(1/2))*
(3*A*c^3*d^5 - 15*B*a^3*e^5 + 10*A*a*c^2*d^3*e^2 + 30*B*a^2*c*d^2*e^3 + 15
*A*a^2*c*d*e^4 + 5*B*a*c^2*d^4*e))/(8*a^(5/2)*c^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 983, normalized size of antiderivative = 3.21

$$\int \frac{(A+Bx)(d+ex)^5}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^5/(c*x^2+a)^3,x)
```

output

```
( - 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*b*e**5 + 15*sqrt
(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c*d*e**4 + 30*sqrt(c)*sqrt(
a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*b*c*d**2*e**3 - 30*sqrt(c)*sqrt(a)*a
tan((c*x)/(sqrt(c)*sqrt(a)))*a**3*b*c*e**5*x**2 + 10*sqrt(c)*sqrt(a)*atan(
(c*x)/(sqrt(c)*sqrt(a)))*a**3*c**2*d**3*e**2 + 30*sqrt(c)*sqrt(a)*atan((c*
x)/(sqrt(c)*sqrt(a)))*a**3*c**2*d*e**4*x**2 + 5*sqrt(c)*sqrt(a)*atan((c*x)
/(sqrt(c)*sqrt(a)))*a**2*b*c**2*d**4*e + 60*sqrt(c)*sqrt(a)*atan((c*x)/(sq
rt(c)*sqrt(a)))*a**2*b*c**2*d**2*e**3*x**2 - 15*sqrt(c)*sqrt(a)*atan((c*x)
/(sqrt(c)*sqrt(a)))*a**2*b*c**2*e**5*x**4 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(
sqrt(c)*sqrt(a)))*a**2*c**3*d**5 + 20*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*
sqrt(a)))*a**2*c**3*d**3*e**2*x**2 + 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c
)*sqrt(a)))*a**2*c**3*d*e**4*x**4 + 10*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)
*sqrt(a)))*a*b*c**3*d**4*e*x**2 + 30*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*s
qrt(a)))*a*b*c**3*d**2*e**3*x**4 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*s
qrt(a)))*a*c**4*d**5*x**2 + 10*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)
))*a*c**4*d**3*e**2*x**4 + 5*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))
*b*c**4*d**4*e*x**4 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**5
*d**5*x**4 + 4*log(a + c*x**2)*a**5*c*e**5 + 20*log(a + c*x**2)*a**4*b*c*d
*e**4 + 8*log(a + c*x**2)*a**4*c**2*e**5*x**2 + 40*log(a + c*x**2)*a**3*b*
c**2*d*e**4*x**2 + 4*log(a + c*x**2)*a**3*c**3*e**5*x**4 + 20*log(a + c...
```

3.96 $\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 217

$$\begin{aligned}\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^3}\,dx &= -\frac{(d+ex)^3(a(Bd+Ae)-(Acd-aBe)x)}{4ac\,(a+cx^2)^2}\\ &\quad-\frac{(d+ex)\,(ae(8aBde+3A(cd^2+ae^2))-(4aBe(cd^2-ae^2)+3Acd(cd^2+ae^2))\,x)}{8a^2c^2\,(a+cx^2)}\\ &\quad+\frac{\left(3A(cd^2+ae^2)^2+4aBde(cd^2+3ae^2)\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}+\frac{Be^4\log\left(a+cx^2\right)}{2c^3}\end{aligned}$$

output
```
-1/4*(e*x+d)^3*(a*(A*e+B*d)-(A*c*d-B*a*e)*x)/a/c/(c*x^2+a)^2-1/8*(e*x+d)*(
a*e*(8*B*a*d*e+3*A*(a*e^2+c*d^2))-(4*a*B*e*(-a*e^2+c*d^2)+3*A*c*d*(a*e^2+c
*d^2))*x)/a^2/c^2/(c*x^2+a)+1/8*(3*A*(a*e^2+c*d^2)^2+4*a*B*d*e*(3*a*e^2+c*
d^2))*arctan(c^(1/2)*x/a^(1/2))/a^(5/2)/c^(5/2)+1/2*B*e^4*ln(c*x^2+a)/c^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^3}\,dx$$
$$= \frac{\frac{-2a^3Be^4+2Ac^3d^4x+2a^2ce^2(Ae(4d+ex)+2Bd(3d+2ex))-2ac^2d^2(2Ae(2d+3ex)+Bd(d+4ex))}{a(a+cx^2)^2}+\frac{8a^3Be^4+3Ac^3d^4x+2ac^2d^2e(2Bd+3Ae)x-}{a^2(a+c}}{8c^3}$$

input
```
Integrate[((A + B*x)*(d + e*x)^4)/(a + c*x^2)^3,x]
```

output
```
((-2*a^3*B*e^4 + 2*A*c^3*d^4*x + 2*a^2*c*e^2*(A*e*(4*d + e*x) + 2*B*d*(3*d
 + 2*e*x)) - 2*a*c^2*d^2*(2*A*e*(2*d + 3*e*x) + B*d*(d + 4*e*x)))/(a*(a +
c*x^2)^2) + (8*a^3*B*e^4 + 3*A*c^3*d^4*x + 2*a*c^2*d^2*e*(2*B*d + 3*A*e)*x
 - a^2*c*e^2*(4*B*d*(6*d + 5*e*x) + A*e*(16*d + 5*e*x)))/(a^2*(a + c*x^2))
 + (Sqrt[c]*(3*A*(c*d^2 + a*e^2)^2 + 4*a*B*d*e*(c*d^2 + 3*a*e^2))*ArcTan[(
Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 4*B*e^4*Log[a + c*x^2])/(8*c^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {684, 684, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^3}\,dx$$

$$\Big\downarrow \text{684}$$

$$\frac{\int \frac{(d+ex)^2(3Acd^2+ae(4Bd+3Ae)+4aBe^2x)}{(cx^2+a)^2}dx}{4ac}-\frac{(d+ex)^3(a(Ae+Bd)-x(Acd-aBe))}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \text{684}$$

$$\frac{\frac{\int \frac{8a^2Bxe^4+4aBd\left(cd^2+3ae^2\right)e+3A\left(cd^2+ae^2\right)^2}{cx^2+a}dx}{2ac}-\frac{(d+ex)\left(x\left(4a^2Be^3-cd\left(ae(3Ae+4Bd)+3Acd^2\right)\right)+ae\left(3A\left(ae^2+cd^2\right)+8aBde\right)\right)}{2ac(a+cx^2)}}{4ac}-\frac{(d+ex)^3(a(Ae+Bd)-x(Acd-aBe))}{4ac\left(a+cx^2\right)^2}$$

$\Big\downarrow$ 452

$$\frac{\frac{8a^2Be^4\int \frac{x}{cx^2+a}dx+\left(3A\left(ae^2+cd^2\right)^2+4aBde\left(3ae^2+cd^2\right)\right)\int \frac{1}{cx^2+a}dx}{2ac}-\frac{(d+ex)\left(x\left(4a^2Be^3-cd\left(ae(3Ae+4Bd)+3Acd^2\right)\right)+ae\left(3A\left(ae^2+cd^2\right)+8aBde\right)\right)}{2ac(a+cx^2)}}{4ac}-\frac{(d+ex)^3(a(Ae+Bd)-x(Acd-aBe))}{4ac\left(a+cx^2\right)^2}$$

$\Big\downarrow$ 218

$$\frac{\frac{8a^2Be^4\int \frac{x}{cx^2+a}dx+\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(3A\left(ae^2+cd^2\right)^2+4aBde\left(3ae^2+cd^2\right)\right)}{\sqrt{a}\sqrt{c}}}{2ac}-\frac{(d+ex)\left(x\left(4a^2Be^3-cd\left(ae(3Ae+4Bd)+3Acd^2\right)\right)+ae\left(3A\left(ae^2+cd^2\right)+8aBde\right)\right)}{2ac(a+cx^2)}}{4ac}-\frac{(d+ex)^3(a(Ae+Bd)-x(Acd-aBe))}{4ac\left(a+cx^2\right)^2}$$

$\Big\downarrow$ 240

$$\frac{\frac{\frac{4a^2Be^4\log\left(a+cx^2\right)}{c}+\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(3A\left(ae^2+cd^2\right)^2+4aBde\left(3ae^2+cd^2\right)\right)}{\sqrt{a}\sqrt{c}}}{2ac}-\frac{(d+ex)\left(x\left(4a^2Be^3-cd\left(ae(3Ae+4Bd)+3Acd^2\right)\right)+ae\left(3A\left(ae^2+cd^2\right)+8aBde\right)\right)}{2ac(a+cx^2)}}{4ac}-\frac{(d+ex)^3(a(Ae+Bd)-x(Acd-aBe))}{4ac\left(a+cx^2\right)^2}$$

input
```
Int[((A + B*x)*(d + e*x)^4)/(a + c*x^2)^3,x]
```

output
```
-1/4*((d + e*x)^3*(a*(B*d + A*e) - (A*c*d - a*B*e)*x))/(a*c*(a + c*x^2)^2)
 + (-1/2*((d + e*x)*(a*e*(8*a*B*d*e + 3*A*(c*d^2 + a*e^2)) + (4*a^2*B*e^3
 - c*d*(3*A*c*d^2 + a*e*(4*B*d + 3*A*e)))*x))/(a*c*(a + c*x^2)) + (((3*A*(c
*d^2 + a*e^2)^2 + 4*a*B*d*e*(c*d^2 + 3*a*e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]]
)/(Sqrt[a]*Sqrt[c]) + (4*a^2*B*e^4*Log[a + c*x^2])/c)/(2*a*c))/(4*a*c)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.42

method	result
default	$\frac{-\frac{\left(5A\,a^2e^4-6Aac\,d^2e^2-3A\,c^2d^4+20B\,a^2d\,e^3-4Bac\,d^3e\right)x^3}{8a^2c}-\frac{e^2\left(2Acde-Ba\,e^2+3Bc\,d^2\right)x^2}{c^2}-\frac{\left(3A\,a^2e^4+6Aac\,d^2e^2-5A\,c^2d^4+12B\,a^2d\,e^3+4B}{8a\,c^2}}{\left(c\,x^2+a\right)^2}$
risch	$\frac{-\frac{\left(5A\,a^2e^4-6Aac\,d^2e^2-3A\,c^2d^4+20B\,a^2d\,e^3-4Bac\,d^3e\right)x^3}{8a^2c}-\frac{e^2\left(2Acde-Ba\,e^2+3Bc\,d^2\right)x^2}{c^2}-\frac{\left(3A\,a^2e^4+6Aac\,d^2e^2-5A\,c^2d^4+12B\,a^2d\,e^3+4B}{8a\,c^2}}{\left(c\,x^2+a\right)^2}$

input
```
int((B*x+A)*(e*x+d)^4/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8*(5*A*a^2*e^4-6*A*a*c*d^2*e^2-3*A*c^2*d^4+20*B*a^2*d*e^3-4*B*a*c*d^3*
e)/a^2/c*x^3-e^2*(2*A*c*d*e-B*a*e^2+3*B*c*d^2)/c^2*x^2-1/8*(3*A*a^2*e^4+6*
A*a*c*d^2*e^2-5*A*c^2*d^4+12*B*a^2*d*e^3+4*B*a*c*d^3*e)/a/c^2*x-1/4*(4*A*a
*c*d*e^3+4*A*c^2*d^3*e-3*B*a^2*e^4+6*B*a*c*d^2*e^2+B*c^2*d^4)/c^3)/(c*x^2+
a)^2+1/8/a^2/c^2*(4*B*e^4*a^2/c*ln(c*x^2+a)+(3*A*a^2*e^4+6*A*a*c*d^2*e^2+3
*A*c^2*d^4+12*B*a^2*d*e^3+4*B*a*c*d^3*e)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(201) = 402$.

Time = 0.10 (sec) , antiderivative size = 1055, normalized size of antiderivative = 4.86

$$\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^4/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*B*a^3*c^2*d^4 + 16*A*a^3*c^2*d^3*e + 24*B*a^4*c*d^2*e^2 + 16*A*a
^4*c*d*e^3 - 12*B*a^5*e^4 - 2*(3*A*a*c^4*d^4 + 4*B*a^2*c^3*d^3*e + 6*A*a^2
*c^3*d^2*e^2 - 20*B*a^3*c^2*d*e^3 - 5*A*a^3*c^2*e^4)*x^3 + 16*(3*B*a^3*c^2
*d^2*e^2 + 2*A*a^3*c^2*d*e^3 - B*a^4*c*e^4)*x^2 + (3*A*a^2*c^2*d^4 + 4*B*a
^3*c*d^3*e + 6*A*a^3*c*d^2*e^2 + 12*B*a^4*d*e^3 + 3*A*a^4*e^4 + (3*A*c^4*d
^4 + 4*B*a*c^3*d^3*e + 6*A*a*c^3*d^2*e^2 + 12*B*a^2*c^2*d*e^3 + 3*A*a^2*c^
2*e^4)*x^4 + 2*(3*A*a*c^3*d^4 + 4*B*a^2*c^2*d^3*e + 6*A*a^2*c^2*d^2*e^2 +
12*B*a^3*c*d*e^3 + 3*A*a^3*c*e^4)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c
)*x - a)/(c*x^2 + a)) - 2*(5*A*a^2*c^3*d^4 - 4*B*a^3*c^2*d^3*e - 6*A*a^3*c
^2*d^2*e^2 - 12*B*a^4*c*d*e^3 - 3*A*a^4*c*e^4)*x - 8*(B*a^3*c^2*e^4*x^4 +
2*B*a^4*c*e^4*x^2 + B*a^5*e^4)*log(c*x^2 + a))/(a^3*c^5*x^4 + 2*a^4*c^4*x^
2 + a^5*c^3), -1/8*(2*B*a^3*c^2*d^4 + 8*A*a^3*c^2*d^3*e + 12*B*a^4*c*d^2*e
^2 + 8*A*a^4*c*d*e^3 - 6*B*a^5*e^4 - (3*A*a*c^4*d^4 + 4*B*a^2*c^3*d^3*e +
6*A*a^2*c^3*d^2*e^2 - 20*B*a^3*c^2*d*e^3 - 5*A*a^3*c^2*e^4)*x^3 + 8*(3*B*a
^3*c^2*d^2*e^2 + 2*A*a^3*c^2*d*e^3 - B*a^4*c*e^4)*x^2 - (3*A*a^2*c^2*d^4 +
 4*B*a^3*c*d^3*e + 6*A*a^3*c*d^2*e^2 + 12*B*a^4*d*e^3 + 3*A*a^4*e^4 + (3*A
*c^4*d^4 + 4*B*a*c^3*d^3*e + 6*A*a*c^3*d^2*e^2 + 12*B*a^2*c^2*d*e^3 + 3*A*
a^2*c^2*e^4)*x^4 + 2*(3*A*a*c^3*d^4 + 4*B*a^2*c^2*d^3*e + 6*A*a^2*c^2*d^2*
e^2 + 12*B*a^3*c*d*e^3 + 3*A*a^3*c*e^4)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/
a) - (5*A*a^2*c^3*d^4 - 4*B*a^3*c^2*d^3*e - 6*A*a^3*c^2*d^2*e^2 - 12*B*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(202) = 404$.

Time = 22.88 (sec) , antiderivative size = 816, normalized size of antiderivative = 3.76

$$\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^3}\,dx = \left(\frac{Be^4}{2c^3} - \frac{\sqrt{-a^5c^7}\cdot(3Aa^2e^4+6Aacd^2e^2+3Ac^2d^4+12Ba^2de^3+4Bacd^3e)}{16a^5c^6}\right)\log\left(x+\frac{-8Ba^3e^4+16a^3c^3\left(\frac{Be^4}{2c^3}\right.}{3Aa^2ce^4+6Aa}\right. + \left(\frac{Be^4}{2c^3} + \frac{\sqrt{-a^5c^7}\cdot(3Aa^2e^4+6Aacd^2e^2+3Ac^2d^4+12Ba^2de^3+4Bacd^3e)}{16a^5c^6}\right)\log\left(x+\frac{-8Ba^3e^4+16a^3c^3\left(\frac{Be^4}{2c^3}\right.}{3Aa^2ce^4+6Aa}\right. + \frac{-8Aa^3cde^3-8Aa^2c^2d^3e+6Ba^4e^4-12Ba^3cd^2e^2-2Ba^2c^2d^4+x^3(-5Aa^2c^2e^4+6Aac^3d^2e^2+3Ac^4d}{}$$

input
```
integrate((B*x+A)*(e*x+d)**4/(c*x**2+a)**3,x)
```

output
```
(B*e**4/(2*c**3) - sqrt(-a**5*c**7)*(3*A*a**2*e**4 + 6*A*a*c*d**2*e**2 + 3
*A*c**2*d**4 + 12*B*a**2*d*e**3 + 4*B*a*c*d**3*e)/(16*a**5*c**6))*log(x +
(-8*B*a**3*e**4 + 16*a**3*c**3*(B*e**4/(2*c**3) - sqrt(-a**5*c**7)*(3*A*a*
*2*e**4 + 6*A*a*c*d**2*e**2 + 3*A*c**2*d**4 + 12*B*a**2*d*e**3 + 4*B*a*c*d
**3*e)/(16*a**5*c**6)))/(3*A*a**2*c*e**4 + 6*A*a*c**2*d**2*e**2 + 3*A*c**3
*d**4 + 12*B*a**2*c*d*e**3 + 4*B*a*c**2*d**3*e)) + (B*e**4/(2*c**3) + sqrt
(-a**5*c**7)*(3*A*a**2*e**4 + 6*A*a*c*d**2*e**2 + 3*A*c**2*d**4 + 12*B*a**
2*d*e**3 + 4*B*a*c*d**3*e)/(16*a**5*c**6))*log(x + (-8*B*a**3*e**4 + 16*a*
*3*c**3*(B*e**4/(2*c**3) + sqrt(-a**5*c**7)*(3*A*a**2*e**4 + 6*A*a*c*d**2*
e**2 + 3*A*c**2*d**4 + 12*B*a**2*d*e**3 + 4*B*a*c*d**3*e)/(16*a**5*c**6)))
/(3*A*a**2*c*e**4 + 6*A*a*c**2*d**2*e**2 + 3*A*c**3*d**4 + 12*B*a**2*c*d*e
**3 + 4*B*a*c**2*d**3*e)) + (-8*A*a**3*c*d*e**3 - 8*A*a**2*c**2*d**3*e + 6
*B*a**4*e**4 - 12*B*a**3*c*d**2*e**2 - 2*B*a**2*c**2*d**4 + x**3*(-5*A*a**
2*c**2*e**4 + 6*A*a*c**3*d**2*e**2 + 3*A*c**4*d**4 - 20*B*a**2*c**2*d*e**3
 + 4*B*a*c**3*d**3*e) + x**2*(-16*A*a**2*c**2*d*e**3 + 8*B*a**3*c*e**4 - 2
4*B*a**2*c**2*d**2*e**2) + x*(-3*A*a**3*c*e**4 - 6*A*a**2*c**2*d**2*e**2 +
 5*A*a*c**3*d**4 - 12*B*a**3*c*d*e**3 - 4*B*a**2*c**2*d**3*e))/(8*a**4*c**
3 + 16*a**3*c**4*x**2 + 8*a**2*c**5*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.62

$$\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^3}\,dx = \frac{Be^4\log\left(cx^2+a\right)}{2\,c^3}$$
$$- \frac{2\,Ba^2c^2d^4 + 8\,Aa^2c^2d^3e + 12\,Ba^3cd^2e^2 + 8\,Aa^3cde^3 - 6\,Ba^4e^4 - \left(3\,Ac^4d^4 + 4\,Bac^3d^3e + 6\,Aac^3d^2e^2 - \right.}{}$$
$$+ \frac{\left(3\,Ac^2d^4 + 4\,Bacd^3e + 6\,Aacd^2e^2 + 12\,Ba^2de^3 + 3\,Aa^2e^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^2}$$

input
```
integrate((B*x+A)*(e*x+d)^4/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/2*B*e^4*log(c*x^2 + a)/c^3 - 1/8*(2*B*a^2*c^2*d^4 + 8*A*a^2*c^2*d^3*e +
12*B*a^3*c*d^2*e^2 + 8*A*a^3*c*d*e^3 - 6*B*a^4*e^4 - (3*A*c^4*d^4 + 4*B*a*
c^3*d^3*e + 6*A*a*c^3*d^2*e^2 - 20*B*a^2*c^2*d*e^3 - 5*A*a^2*c^2*e^4)*x^3
+ 8*(3*B*a^2*c^2*d^2*e^2 + 2*A*a^2*c^2*d*e^3 - B*a^3*c*e^4)*x^2 - (5*A*a*c
^3*d^4 - 4*B*a^2*c^2*d^3*e - 6*A*a^2*c^2*d^2*e^2 - 12*B*a^3*c*d*e^3 - 3*A*
a^3*c*e^4)*x)/(a^2*c^5*x^4 + 2*a^3*c^4*x^2 + a^4*c^3) + 1/8*(3*A*c^2*d^4 +
 4*B*a*c*d^3*e + 6*A*a*c*d^2*e^2 + 12*B*a^2*d*e^3 + 3*A*a^2*e^4)*arctan(c*
x/sqrt(a*c))/(sqrt(a*c)*a^2*c^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.49

$$\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^3}\,dx = \frac{Be^4\log(cx^2+a)}{2\,c^3} + \frac{(3\,Ac^2d^4+4\,Bacd^3e+6\,Aacd^2e^2+12\,Ba^2de^3+3\,Aa^2e^4)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^2} + \frac{(3\,Ac^3d^4+4\,Bac^2d^3e+6\,Aac^2d^2e^2-20\,Ba^2cde^3-5\,Aa^2ce^4)x^3-8\,(3\,Ba^2cd^2e^2+2\,Aa^2cde^3-Ba^3e}{}$$

input
```
integrate((B*x+A)*(e*x+d)^4/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*B*e^4*log(c*x^2 + a)/c^3 + 1/8*(3*A*c^2*d^4 + 4*B*a*c*d^3*e + 6*A*a*c*
d^2*e^2 + 12*B*a^2*d*e^3 + 3*A*a^2*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a
^2*c^2) + 1/8*((3*A*c^3*d^4 + 4*B*a*c^2*d^3*e + 6*A*a*c^2*d^2*e^2 - 20*B*a
^2*c*d*e^3 - 5*A*a^2*c*e^4)*x^3 - 8*(3*B*a^2*c*d^2*e^2 + 2*A*a^2*c*d*e^3 -
 B*a^3*e^4)*x^2 + (5*A*a*c^2*d^4 - 4*B*a^2*c*d^3*e - 6*A*a^2*c*d^2*e^2 - 1
2*B*a^3*d*e^3 - 3*A*a^3*e^4)*x - 2*(B*a^2*c^2*d^4 + 4*A*a^2*c^2*d^3*e + 6*
B*a^3*c*d^2*e^2 + 4*A*a^3*c*d*e^3 - 3*B*a^4*e^4)/c)/((c*x^2 + a)^2*a^2*c^2
)
```

Mupad [B] (verification not implemented)

Time = 7.01 (sec) , antiderivative size = 763, normalized size of antiderivative = 3.52

$$\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^3}\,dx = \frac{5\,A\,d^4\,x}{8\,\left(a^3+2\,a^2\,c\,x^2+a\,c^2\,x^4\right)}-\frac{B\,d^4}{4\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}+\frac{3\,B\,a^2\,e^4}{4\,\left(a^2\,c^3+2\,a\,c^4\,x^2+c^5\,x^4\right)}-\frac{A\,d^3\,e}{a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4}-\frac{5\,A\,e^4\,x^3}{8\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}+\frac{B\,e^4\,\ln\left(c\,x^2+a\right)}{2\,c^3}-\frac{A\,a\,d\,e^3}{a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4}+\frac{3\,A\,c\,d^4\,x^3}{8\,\left(a^4+2\,a^3\,c\,x^2+a^2\,c^2\,x^4\right)}-\frac{3\,A\,a\,e^4\,x}{8\,\left(a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4\right)}+\frac{3\,A\,d^2\,e^2\,x^3}{4\,\left(a^3+2\,a^2\,c\,x^2+a\,c^2\,x^4\right)}-\frac{3\,A\,d^2\,e^2\,x}{4\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}-\frac{2\,A\,d\,e^3\,x^2}{a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4}-\frac{5\,B\,d\,e^3\,x^3}{2\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}-\frac{3\,B\,a\,d^2\,e^2}{2\,\left(a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4\right)}+\frac{B\,a\,e^4\,x^2}{a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4}-\frac{3\,B\,d^2\,e^2\,x^2}{a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4}+\frac{3\,A\,d^4\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{c}}+\frac{3\,A\,e^4\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,c^{5/2}}+\frac{B\,d^3\,e\,x^3}{2\,\left(a^3+2\,a^2\,c\,x^2+a\,c^2\,x^4\right)}-\frac{B\,d^3\,e\,x}{2\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}+\frac{3\,B\,d\,e^3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,c^{5/2}}+\frac{B\,d^3\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,c^{3/2}}+\frac{3\,A\,d^2\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{4\,a^{3/2}\,c^{3/2}}-\frac{3\,B\,a\,d\,e^3\,x}{2\,\left(a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4\right)}$$

input
```
int(((A + B*x)*(d + e*x)^4)/(a + c*x^2)^3,x)
```

output
```
(5*A*d^4*x)/(8*(a^3 + 2*a^2*c*x^2 + a*c^2*x^4)) - (B*d^4)/(4*(a^2*c + c^3*
x^4 + 2*a*c^2*x^2)) + (3*B*a^2*e^4)/(4*(a^2*c^3 + c^5*x^4 + 2*a*c^4*x^2))
- (A*d^3*e)/(a^2*c + c^3*x^4 + 2*a*c^2*x^2) - (5*A*e^4*x^3)/(8*(a^2*c + c^
3*x^4 + 2*a*c^2*x^2)) + (B*e^4*log(a + c*x^2))/(2*c^3) - (A*a*d*e^3)/(a^2*
c^2 + c^4*x^4 + 2*a*c^3*x^2) + (3*A*c*d^4*x^3)/(8*(a^4 + 2*a^3*c*x^2 + a^2
*c^2*x^4)) - (3*A*a*e^4*x)/(8*(a^2*c^2 + c^4*x^4 + 2*a*c^3*x^2)) + (3*A*d^
2*e^2*x^3)/(4*(a^3 + 2*a^2*c*x^2 + a*c^2*x^4)) - (3*A*d^2*e^2*x)/(4*(a^2*c
 + c^3*x^4 + 2*a*c^2*x^2)) - (2*A*d*e^3*x^2)/(a^2*c + c^3*x^4 + 2*a*c^2*x^
2) - (5*B*d*e^3*x^3)/(2*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) - (3*B*a*d^2*e^2)
/(2*(a^2*c^2 + c^4*x^4 + 2*a*c^3*x^2)) + (B*a*e^4*x^2)/(a^2*c^2 + c^4*x^4
+ 2*a*c^3*x^2) - (3*B*d^2*e^2*x^2)/(a^2*c + c^3*x^4 + 2*a*c^2*x^2) + (3*A*
d^4*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*c^(1/2)) + (3*A*e^4*atan((c^(1/2
)*x)/a^(1/2)))/(8*a^(1/2)*c^(5/2)) + (B*d^3*e*x^3)/(2*(a^3 + 2*a^2*c*x^2 +
 a*c^2*x^4)) - (B*d^3*e*x)/(2*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) + (3*B*d*e^
3*atan((c^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*c^(5/2)) + (B*d^3*e*atan((c^(1/2)*
x)/a^(1/2)))/(2*a^(3/2)*c^(3/2)) + (3*A*d^2*e^2*atan((c^(1/2)*x)/a^(1/2)))
/(4*a^(3/2)*c^(3/2)) - (3*B*a*d*e^3*x)/(2*(a^2*c^2 + c^4*x^4 + 2*a*c^3*x^2
))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 733, normalized size of antiderivative = 3.38

$$\int \frac{(A+Bx)(d+ex)^4}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^4/(c*x^2+a)^3,x)
```

output

```
(3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*e**4 + 12*sqrt(c)*sq
rt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*b*d*e**3 + 6*sqrt(c)*sqrt(a)*atan
((c*x)/(sqrt(c)*sqrt(a)))*a**3*c*d**2*e**2 + 6*sqrt(c)*sqrt(a)*atan((c*x)/
(sqrt(c)*sqrt(a)))*a**3*c*e**4*x**2 + 4*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c
)*sqrt(a)))*a**2*b*c*d**3*e + 24*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(
a)))*a**2*b*c*d*e**3*x**2 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a))
)*a**2*c**2*d**4 + 12*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*c
**2*d**2*e**2*x**2 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*
c**2*e**4*x**4 + 8*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*b*c**2*
d**3*e*x**2 + 12*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*b*c**2*d*
e**3*x**4 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c**3*d**4*x*
*2 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c**3*d**2*e**2*x**4
 + 4*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*b*c**3*d**3*e*x**4 + 3*
sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**4*d**4*x**4 + 4*log(a + c
*x**2)*a**4*b*e**4 + 8*log(a + c*x**2)*a**3*b*c*e**4*x**2 + 4*log(a + c*x*
*2)*a**2*b*c**2*e**4*x**4 + 2*a**4*b*e**4 - 3*a**4*c*e**4*x - 12*a**3*b*c*
d*e**3*x - 8*a**3*c**2*d**3*e - 6*a**3*c**2*d**2*e**2*x - 5*a**3*c**2*e**4
*x**3 - 2*a**2*b*c**2*d**4 - 4*a**2*b*c**2*d**3*e*x - 20*a**2*b*c**2*d*e**
3*x**3 - 4*a**2*b*c**2*e**4*x**4 + 5*a**2*c**3*d**4*x + 6*a**2*c**3*d**2*e
**2*x**3 + 8*a**2*c**3*d*e**3*x**4 + 4*a*b*c**3*d**3*e*x**3 + 12*a*b*c*...
```

3.97 $\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 132

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^3} dx = -\frac{(aB-Acx)(d+ex)^3}{4ac(a+cx^2)^2} - \frac{3(Acd+aBe)(2ade-(cd^2-ae^2)x)}{8a^2c^2(a+cx^2)} + \frac{3(Acd+aBe)(cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}$$

output
```
-1/4*(-A*c*x+B*a)*(e*x+d)^3/a/c/(c*x^2+a)^2-3/8*(A*c*d+B*a*e)*(2*a*d*e-(-a
*e^2+c*d^2)*x)/a^2/c^2/(c*x^2+a)+3/8*(A*c*d+B*a*e)*(a*e^2+c*d^2)*arctan(c^
(1/2)*x/a^(1/2))/a^(5/2)/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.41

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^3}\,dx = \frac{3Ac^2d^3x+3acde(Bd+Ae)x-a^2e^2(12Bd+4Ae+5Bex)}{8a^2c^2(a+cx^2)} + \frac{Ac^2d^3x+a^2e^2(3Bd+Ae+Bex)-acd(3Ae(d+ex)+Bd(d+3ex))}{4ac^2(a+cx^2)^2} + \frac{3(Acd+aBe)(cd^2+ae^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}$$

input
```
Integrate[((A + B*x)*(d + e*x)^3)/(a + c*x^2)^3,x]
```

output
```
(3*A*c^2*d^3*x + 3*a*c*d*e*(B*d + A*e)*x - a^2*e^2*(12*B*d + 4*A*e + 5*B*e
*x))/(8*a^2*c^2*(a + c*x^2)) + (A*c^2*d^3*x + a^2*e^2*(3*B*d + A*e + B*e*x
) - a*c*d*(3*A*e*(d + e*x) + B*d*(d + 3*e*x)))/(4*a*c^2*(a + c*x^2)^2) + (
3*(A*c*d + a*B*e)*(c*d^2 + a*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*
c^(5/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {678, 487, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^3}\,dx$$

$$\Big\downarrow \; 678$$

$$\frac{3(aBe+Acd)\int \frac{(d+ex)^2}{(cx^2+a)^2}dx}{4ac} - \frac{(d+ex)^3(aB-Acx)}{4ac(a+cx^2)^2}$$

$$\Big\downarrow \; 487$$

$$\frac{3(aBe+Acd)\left(\frac{(ae^2+cd^2)\int \frac{1}{cx^2+a}dx}{2ac}-\frac{(d+ex)(ae-cdx)}{2ac(a+cx^2)}\right)}{4ac}-\frac{(d+ex)^3(aB-Acx)}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{3(aBe+Acd)\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(ae^2+cd^2)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(ae-cdx)}{2ac(a+cx^2)}\right)}{4ac}-\frac{(d+ex)^3(aB-Acx)}{4ac\,(a+cx^2)^2}$$

input

```
Int[((A + B*x)*(d + e*x)^3)/(a + c*x^2)^3,x]
```

output

```
-1/4*((a*B - A*c*x)*(d + e*x)^3)/(a*c*(a + c*x^2)^2) + (3*(A*c*d + a*B*e)*
(-1/2*((a*e - c*d*x)*(d + e*x))/(a*c*(a + c*x^2)) + ((c*d^2 + a*e^2)*ArcTa
n[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))))/(4*a*c)
```

Defintions of rubi rules used

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 487

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[m*((c*d*f + a*e*g)/(2*a*c*(p + 1)))   Int[(d + e*x)^(
m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && EqQ[S
implify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.63

method	result
default	$\frac{\frac{\left(3Aacd\,e^{2}+3A\,c^{2}d^{3}-5B\,e^{3}a^{2}+3Bac\,d^{2}e\right)x^{3}}{8a^{2}c}-\frac{e^{2}(Ae+3Bd)x^{2}}{2c}-\frac{\left(3Aacd\,e^{2}-5A\,c^{2}d^{3}+3B\,e^{3}a^{2}+3Bac\,d^{2}e\right)x}{8a\,c^{2}}-\frac{Aa\,e^{3}+3Ac\,d^{2}e+3Bad\,e^{2}+Bc\,d^{3}}{4c^{2}}}{(c\,x^{2}+a)^{2}}$
risch	$\frac{\frac{\left(3Aacd\,e^{2}+3A\,c^{2}d^{3}-5B\,e^{3}a^{2}+3Bac\,d^{2}e\right)x^{3}}{8a^{2}c}-\frac{e^{2}(Ae+3Bd)x^{2}}{2c}-\frac{\left(3Aacd\,e^{2}-5A\,c^{2}d^{3}+3B\,e^{3}a^{2}+3Bac\,d^{2}e\right)x}{8a\,c^{2}}-\frac{Aa\,e^{3}+3Ac\,d^{2}e+3Bad\,e^{2}+Bc\,d^{3}}{4c^{2}}}{(c\,x^{2}+a)^{2}}$

input
```
int((B*x+A)*(e*x+d)^3/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*A*a*c*d*e^2+3*A*c^2*d^3-5*B*a^2*e^3+3*B*a*c*d^2*e)/a^2/c*x^3-1/2*e
^2*(A*e+3*B*d)*x^2/c-1/8*(3*A*a*c*d*e^2-5*A*c^2*d^3+3*B*a^2*e^3+3*B*a*c*d^
2*e)/a/c^2*x-1/4*(A*a*e^3+3*A*c*d^2*e+3*B*a*d*e^2+B*c*d^3)/c^2)/(c*x^2+a)^
2+3/8*(A*a*c*d*e^2+A*c^2*d^3+B*a^2*e^3+B*a*c*d^2*e)/a^2/c^2/(a*c)^(1/2)*ar
ctan(c*x/(a*c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(118) = 236.

Time = 0.12 (sec) , antiderivative size = 752, normalized size of antiderivative = 5.70

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^3}\,dx$$
$$= \left[-\frac{4\,Ba^3c^2d^3+12\,Aa^3c^2d^2e+12\,Ba^4cde^2+4\,Aa^4ce^3-2\,(3\,Aac^4d^3+3\,Ba^2c^3d^2e+3\,Aa^2c^3de^2-5\,Ba^3}{}\right.$$
$$-\frac{2\,Ba^3c^2d^3+6\,Aa^3c^2d^2e+6\,Ba^4cde^2+2\,Aa^4ce^3-(3\,Aac^4d^3+3\,Ba^2c^3d^2e+3\,Aa^2c^3de^2-5\,Ba^3c^2e^3}{}$$

input
```
integrate((B*x+A)*(e*x+d)^3/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*B*a^3*c^2*d^3 + 12*A*a^3*c^2*d^2*e + 12*B*a^4*c*d*e^2 + 4*A*a^4*
c*e^3 - 2*(3*A*a*c^4*d^3 + 3*B*a^2*c^3*d^2*e + 3*A*a^2*c^3*d*e^2 - 5*B*a^3
*c^2*e^3)*x^3 + 8*(3*B*a^3*c^2*d*e^2 + A*a^3*c^2*e^3)*x^2 + 3*(A*a^2*c^2*d
^3 + B*a^3*c*d^2*e + A*a^3*c*d*e^2 + B*a^4*e^3 + (A*c^4*d^3 + B*a*c^3*d^2*
e + A*a*c^3*d*e^2 + B*a^2*c^2*e^3)*x^4 + 2*(A*a*c^3*d^3 + B*a^2*c^2*d^2*e
+ A*a^2*c^2*d*e^2 + B*a^3*c*e^3)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)
*x - a)/(c*x^2 + a)) - 2*(5*A*a^2*c^3*d^3 - 3*B*a^3*c^2*d^2*e - 3*A*a^3*c^
2*d*e^2 - 3*B*a^4*c*e^3)*x)/(a^3*c^5*x^4 + 2*a^4*c^4*x^2 + a^5*c^3), -1/8*
(2*B*a^3*c^2*d^3 + 6*A*a^3*c^2*d^2*e + 6*B*a^4*c*d*e^2 + 2*A*a^4*c*e^3 - (
3*A*a*c^4*d^3 + 3*B*a^2*c^3*d^2*e + 3*A*a^2*c^3*d*e^2 - 5*B*a^3*c^2*e^3)*x
^3 + 4*(3*B*a^3*c^2*d*e^2 + A*a^3*c^2*e^3)*x^2 - 3*(A*a^2*c^2*d^3 + B*a^3*
c*d^2*e + A*a^3*c*d*e^2 + B*a^4*e^3 + (A*c^4*d^3 + B*a*c^3*d^2*e + A*a*c^3
*d*e^2 + B*a^2*c^2*e^3)*x^4 + 2*(A*a*c^3*d^3 + B*a^2*c^2*d^2*e + A*a^2*c^2
*d*e^2 + B*a^3*c*e^3)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (5*A*a^2*c^3*
d^3 - 3*B*a^3*c^2*d^2*e - 3*A*a^3*c^2*d*e^2 - 3*B*a^4*c*e^3)*x)/(a^3*c^5*x
^4 + 2*a^4*c^4*x^2 + a^5*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(129) = 258$.

Time = 8.58 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.55

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^3}\,dx$$
$$= -\frac{3\sqrt{-\frac{1}{a^5c^5}}(ae^2+cd^2)(Acd+Bae)\log\left(-\frac{3a^3c^2\sqrt{-\frac{1}{a^5c^5}}(ae^2+cd^2)(Acd+Bae)}{3Aacde^2+3Ac^2d^3+3Ba^2e^3+3Bacd^2e}+x\right)}{16}$$
$$+\frac{3\sqrt{-\frac{1}{a^5c^5}}(ae^2+cd^2)(Acd+Bae)\log\left(\frac{3a^3c^2\sqrt{-\frac{1}{a^5c^5}}(ae^2+cd^2)(Acd+Bae)}{3Aacde^2+3Ac^2d^3+3Ba^2e^3+3Bacd^2e}+x\right)}{16}$$
$$+\frac{-2Aa^3e^3-6Aa^2cd^2e-6Ba^3de^2-2Ba^2cd^3+x^3\cdot(3Aac^2de^2+3Ac^3d^3-5Ba^2ce^3+3Bac^2d^2e)+x^2}{8a^4c^2+16a^3c^3x^2+8a^2c^4}$$

input
```
integrate((B*x+A)*(e*x+d)**3/(c*x**2+a)**3,x)
```

output
```
-3*sqrt(-1/(a**5*c**5))*(a*e**2 + c*d**2)*(A*c*d + B*a*e)*log(-3*a**3*c**2
*sqrt(-1/(a**5*c**5))*(a*e**2 + c*d**2)*(A*c*d + B*a*e)/(3*A*a*c*d*e**2 +
3*A*c**2*d**3 + 3*B*a**2*e**3 + 3*B*a*c*d**2*e) + x)/16 + 3*sqrt(-1/(a**5*
c**5))*(a*e**2 + c*d**2)*(A*c*d + B*a*e)*log(3*a**3*c**2*sqrt(-1/(a**5*c**
5))*(a*e**2 + c*d**2)*(A*c*d + B*a*e)/(3*A*a*c*d*e**2 + 3*A*c**2*d**3 + 3*
B*a**2*e**3 + 3*B*a*c*d**2*e) + x)/16 + (-2*A*a**3*e**3 - 6*A*a**2*c*d**2*
e - 6*B*a**3*d*e**2 - 2*B*a**2*c*d**3 + x**3*(3*A*a*c**2*d*e**2 + 3*A*c**3
*d**3 - 5*B*a**2*c*e**3 + 3*B*a*c**2*d**2*e) + x**2*(-4*A*a**2*c*e**3 - 12
*B*a**2*c*d*e**2) + x*(-3*A*a**2*c*d*e**2 + 5*A*a*c**2*d**3 - 3*B*a**3*e**
3 - 3*B*a**2*c*d**2*e))/(8*a**4*c**2 + 16*a**3*c**3*x**2 + 8*a**2*c**4*x**
4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(118) = 236$.

Time = 0.15 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.88

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^3}\,dx = -\frac{2\,Ba^2cd^3 + 6\,Aa^2cd^2e + 6\,Ba^3de^2 + 2\,Aa^3e^3 - (3\,Ac^3d^3 + 3\,Bac^2d^2e + 3\,Aac^2de^2 - 5\,Ba^2ce^3)x^3 + 4\,(}{8\,(a^2c^4x^4 + 2\,a^3c^3x^2 + a^4c^2} + \frac{3\,(Ac^2d^3 + Bacd^2e + Aacde^2 + Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^2}$$

input
```
integrate((B*x+A)*(e*x+d)^3/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(2*B*a^2*c*d^3 + 6*A*a^2*c*d^2*e + 6*B*a^3*d*e^2 + 2*A*a^3*e^3 - (3*A
*c^3*d^3 + 3*B*a*c^2*d^2*e + 3*A*a*c^2*d*e^2 - 5*B*a^2*c*e^3)*x^3 + 4*(3*B
*a^2*c*d*e^2 + A*a^2*c*e^3)*x^2 - (5*A*a*c^2*d^3 - 3*B*a^2*c*d^2*e - 3*A*a
^2*c*d*e^2 - 3*B*a^3*e^3)*x)/(a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2) + 3/8
*(A*c^2*d^3 + B*a*c*d^2*e + A*a*c*d*e^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c))
/(sqrt(a*c)*a^2*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(118) = 236$.

Time = 0.13 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.81

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^3}\,dx = \frac{3\left(Ac^2d^3+Bacd^2e+Aacde^2+Ba^2e^3\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2c^2} + \frac{3\,Ac^3d^3x^3+3\,Bac^2d^2ex^3+3\,Aac^2de^2x^3-5\,Ba^2ce^3x^3-12\,Ba^2cde^2x^2-4\,Aa^2ce^3x^2+5\,Aac^2d^3x-3}{8\left(cx^2+a\right)^2a^2c^2}$$

input
```
integrate((B*x+A)*(e*x+d)^3/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
3/8*(A*c^2*d^3 + B*a*c*d^2*e + A*a*c*d*e^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*
c))/(sqrt(a*c)*a^2*c^2) + 1/8*(3*A*c^3*d^3*x^3 + 3*B*a*c^2*d^2*e*x^3 + 3*A
*a*c^2*d*e^2*x^3 - 5*B*a^2*c*e^3*x^3 - 12*B*a^2*c*d*e^2*x^2 - 4*A*a^2*c*e^
3*x^2 + 5*A*a*c^2*d^3*x - 3*B*a^2*c*d^2*e*x - 3*A*a^2*c*d*e^2*x - 3*B*a^3*
e^3*x - 2*B*a^2*c*d^3 - 6*A*a^2*c*d^2*e - 6*B*a^3*d*e^2 - 2*A*a^3*e^3)/((c
*x^2 + a)^2*a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 6.27 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.01

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^3}\,dx = \frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x\,\left(A\,c\,d+B\,a\,e\right)\,\left(c\,d^2+a\,e^2\right)}{\sqrt{a}\,\left(B\,a^2\,e^3+B\,a\,c\,d^2\,e+A\,a\,c\,d\,e^2+A\,c^2\,d^3\right)}\right)\,\left(A\,c\,d+B\,a\,e\right)\,\left(c\,d^2+a\,e^2\right)}{8\,a^{5/2}\,c^{5/2}} - \frac{\frac{B\,c\,d^3+3\,A\,c\,d^2\,e+3\,B\,a\,d\,e^2+A\,a\,e^3}{4\,c^2}+\frac{x^2\,\left(A\,e^3+3\,B\,d\,e^2\right)}{2\,c}+\frac{x\,\left(3\,B\,a^2\,e^3+3\,B\,a\,c\,d^2\,e+3\,A\,a\,c\,d\,e^2-5\,A\,c^2\,d^3\right)}{8\,a\,c^2}-\frac{x^3\,\left(-5\,B\,a^2\,e^3+3\,B\,a}{}}{a^2+2\,a\,c\,x^2+c^2\,x^4}$$

input
```
int(((A + B*x)*(d + e*x)^3)/(a + c*x^2)^3,x)
```

output
```
(3*atan((c^(1/2)*x*(A*c*d + B*a*e)*(a*e^2 + c*d^2))/(a^(1/2)*(A*c^2*d^3 +
B*a^2*e^3 + A*a*c*d*e^2 + B*a*c*d^2*e)))*(A*c*d + B*a*e)*(a*e^2 + c*d^2))/
(8*a^(5/2)*c^(5/2)) - ((A*a*e^3 + B*c*d^3 + 3*B*a*d*e^2 + 3*A*c*d^2*e)/(4*
c^2) + (x^2*(A*e^3 + 3*B*d*e^2))/(2*c) + (x*(3*B*a^2*e^3 - 5*A*c^2*d^3 + 3
*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/(8*a*c^2) - (x^3*(3*A*c^2*d^3 - 5*B*a^2*e^3
 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/(8*a^2*c))/(a^2 + c^2*x^4 + 2*a*c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.97

$$\int \frac{(A+Bx)(d+ex)^3}{(a+cx^2)^3}\,dx$$
$$= \frac{3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3b\,e^3+3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3cd\,e^2+3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2bc\,d^2e+6\sqrt{c}\,\sqrt{a}\,at}{}$$

input
```
int((B*x+A)*(e*x+d)^3/(c*x^2+a)^3,x)
```

output
```
(3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*b*e**3 + 3*sqrt(c)*s
qrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*c*d*e**2 + 3*sqrt(c)*sqrt(a)*ata
n((c*x)/(sqrt(c)*sqrt(a)))*a**2*b*c*d**2*e + 6*sqrt(c)*sqrt(a)*atan((c*x)/
(sqrt(c)*sqrt(a)))*a**2*b*c*e**3*x**2 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt
(c)*sqrt(a)))*a**2*c**2*d**3 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(
a)))*a**2*c**2*d*e**2*x**2 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)
))*a*b*c**2*d**2*e*x**2 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*
a*b*c**2*e**3*x**4 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c**
3*d**3*x**2 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c**3*d*e**
2*x**4 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*b*c**3*d**2*e*x**
4 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**4*d**3*x**4 - 3*a**
3*b*c*e**3*x - 6*a**3*c**2*d**2*e - 3*a**3*c**2*d*e**2*x - 2*a**2*b*c**2*d
**3 - 3*a**2*b*c**2*d**2*e*x - 5*a**2*b*c**2*e**3*x**3 + 5*a**2*c**3*d**3*
x + 3*a**2*c**3*d*e**2*x**3 + 2*a**2*c**3*e**3*x**4 + 3*a*b*c**3*d**2*e*x*
*3 + 6*a*b*c**3*d*e**2*x**4 + 3*a*c**4*d**3*x**3)/(8*a**2*c**3*(a**2 + 2*a
*c*x**2 + c**2*x**4))
```

3.98 $\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^3} dx = -\frac{(d+ex)(a(Bd+Ae)-(Acd-aBe)x)}{4ac(a+cx^2)^2} - \frac{2ae(Acd+aBe)-c(3Acd^2+2aBde+aAe^2)x}{8a^2c^2(a+cx^2)} + \frac{(3Acd^2+2aBde+aAe^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

output
```
-1/4*(e*x+d)*(a*(A*e+B*d)-(A*c*d-B*a*e)*x)/a/c/(c*x^2+a)^2-1/8*(2*a*e*(A*c
*d+B*a*e)-c*(A*a*e^2+3*A*c*d^2+2*B*a*d*e)*x)/a^2/c^2/(c*x^2+a)+1/8*(A*a*e^
2+3*A*c*d^2+2*B*a*d*e)*arctan(c^(1/2)*x/a^(1/2))/a^(5/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^3}\,dx$$
$$= \frac{\frac{\sqrt{a}(-4a^2Be^2+3Ac^2d^2x+ace(2Bd+Ae)x)}{a+cx^2}+\frac{2a^{3/2}(a^2Be^2+Ac^2d^2x-ac(Ae(2d+ex)+Bd(d+2ex)))}{(a+cx^2)^2}+\sqrt{c}(3Acd^2+2aBde+aAe}{8a^{5/2}c^2}$$

input
```
Integrate[((A + B*x)*(d + e*x)^2)/(a + c*x^2)^3,x]
```

output
```
((Sqrt[a]*(-4*a^2*B*e^2 + 3*A*c^2*d^2*x + a*c*e*(2*B*d + A*e)*x))/(a + c*x
^2) + (2*a^(3/2)*(a^2*B*e^2 + A*c^2*d^2*x - a*c*(A*e*(2*d + e*x) + B*d*(d
+ 2*e*x))))/(a + c*x^2)^2 + Sqrt[c]*(3*A*c*d^2 + 2*a*B*d*e + a*A*e^2)*ArcT
an[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {685, 675, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^3}\,dx$$
$$\Big\downarrow \text{685}$$
$$\frac{\int \frac{(d+ex)(3Acd+2aBe+Acex)}{(cx^2+a)^2}dx}{4ac}-\frac{(d+ex)^2(aB-Acx)}{4ac\,(a+cx^2)^2}$$
$$\Big\downarrow \text{675}$$
$$\frac{\frac{(aAe^2+2aBde+3Acd^2)\int \frac{1}{cx^2+a}dx}{2a}+\frac{x(-aAe^2+2aBde+3Acd^2)}{2a(a+cx^2)}-\frac{e(aBe+2Acd)}{c(a+cx^2)}}{4ac}-\frac{(d+ex)^2(aB-Acx)}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aAe^2+2aBde+3Acd^2)}{2a^{3/2}\sqrt{c}}+\frac{x(-aAe^2+2aBde+3Acd^2)}{2a(a+cx^2)}-\frac{e(aBe+2Acd)}{c(a+cx^2)}}{4ac}-\frac{(d+ex)^2(aB-Acx)}{4ac\,(a+cx^2)^2}$$

input
```
Int[((A + B*x)*(d + e*x)^2)/(a + c*x^2)^3,x]
```

output
```
-1/4*((a*B - A*c*x)*(d + e*x)^2)/(a*c*(a + c*x^2)^2) + (-((e*(2*A*c*d + a*
B*e))/(c*(a + c*x^2))) + ((3*A*c*d^2 + 2*a*B*d*e - a*A*e^2)*x)/(2*a*(a + c
*x^2)) + ((3*A*c*d^2 + 2*a*B*d*e + a*A*e^2)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(
2*a^(3/2)*Sqrt[c]))/(4*a*c)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 685
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^(m - 1)*(a + c*x^2)
^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m]
 || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

method	result
default	$\frac{\frac{\left(A a\,e^{2}+3 A c\,d^{2}+2 B a d e \right) x^{3}}{8 a^{2}}-\frac{B\,e^{2} x^{2}}{2 c}-\frac{\left(A a\,e^{2}-5 A c\,d^{2}+2 B a d e \right) x}{8 a c}-\frac{2 A c d e+B a\,e^{2}+B c\,d^{2}}{4 c^{2}}}{\left(c\,x^{2}+a \right)^{2}}+\frac{\left(A a\,e^{2}+3 A c\,d^{2}+2 B a d e \right) \arctan \left(\frac{c x}{\sqrt{a c}}\right)}{8 a^{2} c \sqrt{a c}}$
risch	$\frac{\frac{\left(A a\,e^{2}+3 A c\,d^{2}+2 B a d e \right) x^{3}}{8 a^{2}}-\frac{B\,e^{2} x^{2}}{2 c}-\frac{\left(A a\,e^{2}-5 A c\,d^{2}+2 B a d e \right) x}{8 a c}-\frac{2 A c d e+B a\,e^{2}+B c\,d^{2}}{4 c^{2}}}{\left(c\,x^{2}+a \right)^{2}}-\frac{\ln \left(c x+\sqrt{-a c}\right) A\,e^{2}}{16 \sqrt{-a c}\, c a}-\frac{3 \ln \left(c x+\sqrt{-a c}\right) A\,d^{2}}{16 \sqrt{-a c}\, a^{2}}$

input
```
int((B*x+A)*(e*x+d)^2/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(A*a*e^2+3*A*c*d^2+2*B*a*d*e)/a^2*x^3-1/2*B*e^2*x^2/c-1/8*(A*a*e^2-5*
A*c*d^2+2*B*a*d*e)/a/c*x-1/4*(2*A*c*d*e+B*a*e^2+B*c*d^2)/c^2)/(c*x^2+a)^2+
1/8*(A*a*e^2+3*A*c*d^2+2*B*a*d*e)/a^2/c/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.53

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^3}\,dx$$
$$= \left[-\frac{8\,Ba^3ce^2x^2+4\,Ba^3cd^2+8\,Aa^3cde+4\,Ba^4e^2-2\,(3\,Aac^3d^2+2\,Ba^2c^2de+Aa^2c^2e^2)x^3+(3\,Aa^2cd^2 \cdots}{\cdots}\right.$$
$$-\frac{4\,Ba^3ce^2x^2+2\,Ba^3cd^2+4\,Aa^3cde+2\,Ba^4e^2-(3\,Aac^3d^2+2\,Ba^2c^2de+Aa^2c^2e^2)x^3-(3\,Aa^2cd^2+2 \cdots}{\cdots}$$

input
```
integrate((B*x+A)*(e*x+d)^2/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(8*B*a^3*c*e^2*x^2 + 4*B*a^3*c*d^2 + 8*A*a^3*c*d*e + 4*B*a^4*e^2 -
2*(3*A*a*c^3*d^2 + 2*B*a^2*c^2*d*e + A*a^2*c^2*e^2)*x^3 + (3*A*a^2*c*d^2 +
 2*B*a^3*d*e + A*a^3*e^2 + (3*A*c^3*d^2 + 2*B*a*c^2*d*e + A*a*c^2*e^2)*x^4
 + 2*(3*A*a*c^2*d^2 + 2*B*a^2*c*d*e + A*a^2*c*e^2)*x^2)*sqrt(-a*c)*log((c*
x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(5*A*a^2*c^2*d^2 - 2*B*a^3*c*d*
e - A*a^3*c*e^2)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*c^2), -1/8*(4*B*a^3
*c*e^2*x^2 + 2*B*a^3*c*d^2 + 4*A*a^3*c*d*e + 2*B*a^4*e^2 - (3*A*a*c^3*d^2
+ 2*B*a^2*c^2*d*e + A*a^2*c^2*e^2)*x^3 - (3*A*a^2*c*d^2 + 2*B*a^3*d*e + A*
a^3*e^2 + (3*A*c^3*d^2 + 2*B*a*c^2*d*e + A*a*c^2*e^2)*x^4 + 2*(3*A*a*c^2*d
^2 + 2*B*a^2*c*d*e + A*a^2*c*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (
5*A*a^2*c^2*d^2 - 2*B*a^3*c*d*e - A*a^3*c*e^2)*x)/(a^3*c^4*x^4 + 2*a^4*c^3
*x^2 + a^5*c^2)]
```

Sympy [A] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.80

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5c^3}}(Aae^2+3Acd^2+2Bade)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5c^3}}(Aae^2+3Acd^2+2Bade)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16} + \frac{-4Aa^2cde-2Ba^3e^2-2Ba^2cd^2-4Ba^2ce^2x^2+x^3(Aac^2e^2+3Ac^3d^2+2Bac^2de)+x(-Aa^2ce^2+5A}{8a^4c^2+16a^3c^3x^2+8a^2c^4x^4}$$

input
```
integrate((B*x+A)*(e*x+d)**2/(c*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*c**3))*(A*a*e**2 + 3*A*c*d**2 + 2*B*a*d*e)*log(-a**3*c*sqrt
(-1/(a**5*c**3)) + x)/16 + sqrt(-1/(a**5*c**3))*(A*a*e**2 + 3*A*c*d**2 + 2
*B*a*d*e)*log(a**3*c*sqrt(-1/(a**5*c**3)) + x)/16 + (-4*A*a**2*c*d*e - 2*B
*a**3*e**2 - 2*B*a**2*c*d**2 - 4*B*a**2*c*e**2*x**2 + x**3*(A*a*c**2*e**2
+ 3*A*c**3*d**2 + 2*B*a*c**2*d*e) + x*(-A*a**2*c*e**2 + 5*A*a*c**2*d**2 -
2*B*a**2*c*d*e))/(8*a**4*c**2 + 16*a**3*c**3*x**2 + 8*a**2*c**4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^3}\,dx = -\frac{4\,Ba^2ce^2x^2+2\,Ba^2cd^2+4\,Aa^2cde+2\,Ba^3e^2-(3\,Ac^3d^2+2\,Bac^2de+Aac^2e^2)x^3-(5\,Aac^2d^2-2\,B}{8\,(a^2c^4x^4+2\,a^3c^3x^2+a^4c^2)} + \frac{(3\,Acd^2+2\,Bade+Aae^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c}$$

input
```
integrate((B*x+A)*(e*x+d)^2/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(4*B*a^2*c*e^2*x^2 + 2*B*a^2*c*d^2 + 4*A*a^2*c*d*e + 2*B*a^3*e^2 - (3
*A*c^3*d^2 + 2*B*a*c^2*d*e + A*a*c^2*e^2)*x^3 - (5*A*a*c^2*d^2 - 2*B*a^2*c
*d*e - A*a^2*c*e^2)*x)/(a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2) + 1/8*(3*A*
c*d^2 + 2*B*a*d*e + A*a*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^3}\,dx = \frac{(3\,Acd^2+2\,Bade+Aae^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c} + \frac{3\,Ac^3d^2x^3+2\,Bac^2dex^3+Aac^2e^2x^3-4\,Ba^2ce^2x^2+5\,Aac^2d^2x-2\,Ba^2cdex-Aa^2ce^2x-2\,Ba^2cd^2-}{8\,(cx^2+a)^2a^2c^2}$$

input
```
integrate((B*x+A)*(e*x+d)^2/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(3*A*c*d^2 + 2*B*a*d*e + A*a*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2
*c) + 1/8*(3*A*c^3*d^2*x^3 + 2*B*a*c^2*d*e*x^3 + A*a*c^2*e^2*x^3 - 4*B*a^2
*c*e^2*x^2 + 5*A*a*c^2*d^2*x - 2*B*a^2*c*d*e*x - A*a^2*c*e^2*x - 2*B*a^2*c
*d^2 - 4*A*a^2*c*d*e - 2*B*a^3*e^2)/((c*x^2 + a)^2*a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 6.65 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(3\,A\,c\,d^2+2\,B\,a\,d\,e+A\,a\,e^2\right)}{8\,a^{5/2}\,c^{3/2}} - \frac{\frac{B\,c\,d^2+2\,A\,c\,d\,e+B\,a\,e^2}{4\,c^2}-\frac{x^3\,\left(3\,A\,c\,d^2+2\,B\,a\,d\,e+A\,a\,e^2\right)}{8\,a^2}+\frac{x\,\left(-5\,A\,c\,d^2+2\,B\,a\,d\,e+A\,a\,e^2\right)}{8\,a\,c}+\frac{B\,e^2\,x^2}{2\,c}}{a^2+2\,a\,c\,x^2+c^2\,x^4}$$

input
```
int(((A + B*x)*(d + e*x)^2)/(a + c*x^2)^3,x)
```

output
```
(atan((c^(1/2)*x)/a^(1/2))*(A*a*e^2 + 3*A*c*d^2 + 2*B*a*d*e))/(8*a^(5/2)*c
^(3/2)) - ((B*a*e^2 + B*c*d^2 + 2*A*c*d*e)/(4*c^2) - (x^3*(A*a*e^2 + 3*A*c
*d^2 + 2*B*a*d*e))/(8*a^2) + (x*(A*a*e^2 - 5*A*c*d^2 + 2*B*a*d*e))/(8*a*c)
 + (B*e^2*x^2)/(2*c))/(a^2 + c^2*x^4 + 2*a*c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.39

$$\int \frac{(A+Bx)(d+ex)^2}{(a+cx^2)^3}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^3e^2+2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2bde+3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2c\,d^2+2\sqrt{c}\,\sqrt{a}\,atan\left(\right.}{}$$

input
```
int((B*x+A)*(e*x+d)^2/(c*x^2+a)^3,x)
```

output

```
(sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**3*e**2 + 2*sqrt(c)*sqrt(
a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*b*d*e + 3*sqrt(c)*sqrt(a)*atan((c*x)
/(sqrt(c)*sqrt(a)))*a**2*c*d**2 + 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sq
rt(a)))*a**2*c*e**2*x**2 + 4*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))
*a*b*c*d*e*x**2 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c**2*d
**2*x**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c**2*e**2*x**4
+ 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*b*c**2*d*e*x**4 + 3*sqrt
(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**3*d**2*x**4 - 4*a**3*c*d*e -
a**3*c*e**2*x - 2*a**2*b*c*d**2 - 2*a**2*b*c*d*e*x + 5*a**2*c**2*d**2*x +
a**2*c**2*e**2*x**3 + 2*a*b*c**2*d*e*x**3 + 2*a*b*c**2*e**2*x**4 + 3*a*c**
3*d**2*x**3)/(8*a**2*c**2*(a**2 + 2*a*c*x**2 + c**2*x**4))
```

3.99 $\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^3}\,dx = -\frac{(A+Bx)(ae-cdx)}{4ac\,(a+cx^2)^2} - \frac{2aBd-(3Acd+aBe)x}{8a^2c\,(a+cx^2)} + \frac{(3Acd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

output
```
-1/4*(B*x+A)*(-c*d*x+a*e)/a/c/(c*x^2+a)^2-1/8*(2*a*B*d-(3*A*c*d+B*a*e)*x)/
a^2/c/(c*x^2+a)+1/8*(3*A*c*d+B*a*e)*arctan(c^(1/2)*x/a^(1/2))/a^(5/2)/c^(3
/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^3}\,dx = \frac{3Ac^2dx^3-a^2(2Bd+2Ae+Bex)+acx(5Ad+Bex^2)}{8a^2c\,(a+cx^2)^2} + \frac{(3Acd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

input
```
Integrate[((A + B*x)*(d + e*x))/(a + c*x^2)^3,x]
```

output
```
(3*A*c^2*d*x^3 - a^2*(2*B*d + 2*A*e + B*e*x) + a*c*x*(5*A*d + B*e*x^2))/(8
*a^2*c*(a + c*x^2)^2) + ((3*A*c*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8
*a^(5/2)*c^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {675, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^3}dx$$

$$\Big\downarrow \text{ 675}$$

$$\frac{(aBe+3Acd)\int \frac{1}{(cx^2+a)^2}dx}{4ac}-\frac{Ae+Bd}{4c\left(a+cx^2\right)^2}+\frac{x(Acd-aBe)}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{(aBe+3Acd)\left(\frac{\int \frac{1}{cx^2+a}dx}{2a}+\frac{x}{2a(a+cx^2)}\right)}{4ac}-\frac{Ae+Bd}{4c\left(a+cx^2\right)^2}+\frac{x(Acd-aBe)}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a(a+cx^2)}\right)(aBe+3Acd)}{4ac}-\frac{Ae+Bd}{4c\left(a+cx^2\right)^2}+\frac{x(Acd-aBe)}{4ac\left(a+cx^2\right)^2}$$

input
```
Int[((A + B*x)*(d + e*x))/(a + c*x^2)^3,x]
```

output
```
-1/4*(B*d + A*e)/(c*(a + c*x^2)^2) + ((A*c*d - a*B*e)*x)/(4*a*c*(a + c*x^2
)^2) + ((3*A*c*d + a*B*e)*(x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a
]]/(2*a^(3/2)*Sqrt[c])))/(4*a*c)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.87

method	result
default	$\frac{\frac{(3Acd+Bae)x^3}{8a^2}+\frac{(5Acd-Bae)x}{8ac}-\frac{Ae+Bd}{4c}}{(cx^2+a)^2}+\frac{(3Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8a^2c\sqrt{ac}}$
risch	$\frac{\frac{(3Acd+Bae)x^3}{8a^2}+\frac{(5Acd-Bae)x}{8ac}-\frac{Ae+Bd}{4c}}{(cx^2+a)^2}-\frac{3\ln(cx+\sqrt{-ac})Ad}{16\sqrt{-ac}\,a^2}-\frac{\ln(cx+\sqrt{-ac})Be}{16\sqrt{-ac}\,ca}+\frac{3\ln(-cx+\sqrt{-ac})Ad}{16\sqrt{-ac}\,a^2}+\frac{\ln(-cx+\sqrt{-ac}}{16\sqrt{-ac}\,ca}$

input
```
int((B*x+A)*(e*x+d)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*A*c*d+B*a*e)/a^2*x^3+1/8*(5*A*c*d-B*a*e)/a/c*x-1/4*(A*e+B*d)/c)/(c
*x^2+a)^2+1/8*(3*A*c*d+B*a*e)/a^2/c/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.26

$$\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^3}\,dx$$
$$= \left[-\frac{4\,Ba^3cd+4\,Aa^3ce-2\,(3\,Aac^3d+Ba^2c^2e)x^3+(3\,Aa^2cd+Ba^3e+(3\,Ac^3d+Bac^2e)x^4+2\,(3\,Aac^2 \cdots}{16\,(a^3c^4x^4+2\,a^4c^3x^2+a^5c^2)}\right.$$
$$-\frac{2\,Ba^3cd+2\,Aa^3ce-(3\,Aac^3d+Ba^2c^2e)x^3-(3\,Aa^2cd+Ba^3e+(3\,Ac^3d+Bac^2e)x^4+2\,(3\,Aac^2d+ \cdots}{8\,(a^3c^4x^4+2\,a^4c^3x^2+a^5c^2)}$$

input
```
integrate((B*x+A)*(e*x+d)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*B*a^3*c*d + 4*A*a^3*c*e - 2*(3*A*a*c^3*d + B*a^2*c^2*e)*x^3 + (3
*A*a^2*c*d + B*a^3*e + (3*A*c^3*d + B*a*c^2*e)*x^4 + 2*(3*A*a*c^2*d + B*a^
2*c*e)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(
5*A*a^2*c^2*d - B*a^3*c*e)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*c^2), -1/
8*(2*B*a^3*c*d + 2*A*a^3*c*e - (3*A*a*c^3*d + B*a^2*c^2*e)*x^3 - (3*A*a^2*
c*d + B*a^3*e + (3*A*c^3*d + B*a*c^2*e)*x^4 + 2*(3*A*a*c^2*d + B*a^2*c*e)*
x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (5*A*a^2*c^2*d - B*a^3*c*e)*x)/(a^3
*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*c^2)]
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.65

$$\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^3}\,dx$$
$$= -\frac{\sqrt{-\frac{1}{a^5c^3}}\cdot(3Acd+Bae)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5c^3}}\cdot(3Acd+Bae)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16} + \frac{-2Aa^2e-2Ba^2d+x^3\cdot(3Ac^2d+Bace)+x(5Aacd-Ba^2e)}{8a^4c+16a^3c^2x^2+8a^2c^3x^4}$$

input
```
integrate((B*x+A)*(e*x+d)/(c*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*c**3))*(3*A*c*d + B*a*e)*log(-a**3*c*sqrt(-1/(a**5*c**3)) +
 x)/16 + sqrt(-1/(a**5*c**3))*(3*A*c*d + B*a*e)*log(a**3*c*sqrt(-1/(a**5*c
**3)) + x)/16 + (-2*A*a**2*e - 2*B*a**2*d + x**3*(3*A*c**2*d + B*a*c*e) +
x*(5*A*a*c*d - B*a**2*e))/(8*a**4*c + 16*a**3*c**2*x**2 + 8*a**2*c**3*x**4
)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^3}\,dx = -\frac{2\,Ba^2d+2\,Aa^2e-(3\,Ac^2d+Bace)x^3-(5\,Aacd-Ba^2e)x}{8\,(a^2c^3x^4+2\,a^3c^2x^2+a^4c)} + \frac{(3\,Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c}$$

input
```
integrate((B*x+A)*(e*x+d)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(2*B*a^2*d + 2*A*a^2*e - (3*A*c^2*d + B*a*c*e)*x^3 - (5*A*a*c*d - B*a
^2*e)*x)/(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4*c) + 1/8*(3*A*c*d + B*a*e)*arc
tan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^3}\,dx = \frac{(3\,Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c} + \frac{3\,Ac^2dx^3+Bacex^3+5\,Aacdx-Ba^2ex-2\,Ba^2d-2\,Aa^2e}{8\,(cx^2+a)^2a^2c}$$

input
```
integrate((B*x+A)*(e*x+d)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(3*A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8*(3*A*c
^2*d*x^3 + B*a*c*e*x^3 + 5*A*a*c*d*x - B*a^2*e*x - 2*B*a^2*d - 2*A*a^2*e)/
((c*x^2 + a)^2*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 6.69 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^3}\,dx = \frac{\frac{x^3\,(3\,A\,c\,d+B\,a\,e)}{8\,a^2}-\frac{A\,e+B\,d}{4\,c}+\frac{x\,(5\,A\,c\,d-B\,a\,e)}{8\,a\,c}}{a^2+2\,a\,c\,x^2+c^2\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(3\,A\,c\,d+B\,a\,e)}{8\,a^{5/2}\,c^{3/2}}$$

input
```
int(((A + B*x)*(d + e*x))/(a + c*x^2)^3,x)
```

output
```
((x^3*(3*A*c*d + B*a*e))/(8*a^2) - (A*e + B*d)/(4*c) + (x*(5*A*c*d - B*a*e
))/(8*a*c))/(a^2 + c^2*x^4 + 2*a*c*x^2) + (atan((c^(1/2)*x)/a^(1/2))*(3*A*
c*d + B*a*e))/(8*a^(5/2)*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.10

$$\int \frac{(A+Bx)(d+ex)}{(a+cx^2)^3}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2be+3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2cd+2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)abce\,x^2+6\sqrt{c}\,\sqrt{a}\,atan\left(\frac{}{\sqrt{}}\right.}{}$$

input
```
int((B*x+A)*(e*x+d)/(c*x^2+a)^3,x)
```

output
```
(sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*b*e + 3*sqrt(c)*sqrt(a
)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2*c*d + 2*sqrt(c)*sqrt(a)*atan((c*x)/(s
qrt(c)*sqrt(a)))*a*b*c*e*x**2 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt
(a)))*a*c**2*d*x**2 + sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*b*c**2
*e*x**4 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**3*d*x**4 - 2*
a**3*c*e - 2*a**2*b*c*d - a**2*b*c*e*x + 5*a**2*c**2*d*x + a*b*c**2*e*x**3
 + 3*a*c**3*d*x**3)/(8*a**2*c**2*(a**2 + 2*a*c*x**2 + c**2*x**4))
```

3.100 $\int \frac{A+Bx}{(a+cx^2)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{A+Bx}{(a+cx^2)^3}\,dx=\frac{-aB+Acx}{4ac\,(a+cx^2)^2}+\frac{3Ax}{8a^2\,(a+cx^2)}+\frac{3A\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}$$

output
```
1/4*(A*c*x-B*a)/a/c/(c*x^2+a)^2+3/8*A*x/a^2/(c*x^2+a)+3/8*A*arctan(c^(1/2)
*x/a^(1/2))/a^(5/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{(a+cx^2)^3}\,dx=\frac{\frac{\sqrt{a}(-2a^2B+5aAcx+3Ac^2x^3)}{(a+cx^2)^2}+3A\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c}$$

input
```
Integrate[(A + B*x)/(a + c*x^2)^3,x]
```

output
```
((Sqrt[a]*(-2*a^2*B + 5*a*A*c*x + 3*A*c^2*x^3))/(a + c*x^2)^2 + 3*A*Sqrt[c
]*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {454, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+cx^2)^3}\,dx$$

$$\Big\downarrow \text{454}$$

$$\frac{3A\int \frac{1}{(cx^2+a)^2}dx}{4a}-\frac{aB-Acx}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3A\left(\frac{\int \frac{1}{cx^2+a}dx}{2a}+\frac{x}{2a(a+cx^2)}\right)}{4a}-\frac{aB-Acx}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3A\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a(a+cx^2)}\right)}{4a}-\frac{aB-Acx}{4ac\,(a+cx^2)^2}$$

input
```
Int[(A + B*x)/(a + c*x^2)^3,x]
```

output
```
-1/4*(a*B - A*c*x)/(a*c*(a + c*x^2)^2) + (3*A*(x/(2*a*(a + c*x^2)) + ArcTa
n[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])))/(4*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{2Acx-2Ba}{8ac(c\,x^{2}+a)^{2}}+\frac{3A\left(\frac{x}{2a\left(c\,x^{2}+a\right)}+\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2a\sqrt{ac}}\right)}{4a}$	70
risch	$\frac{\frac{3Ac\,x^{3}}{8a^{2}}+\frac{5Ax}{8a}-\frac{B}{4c}}{(c\,x^{2}+a)^{2}}-\frac{3A\ln\left(cx+\sqrt{-ac}\right)}{16\sqrt{-ac}\,a^{2}}+\frac{3A\ln\left(-cx+\sqrt{-ac}\right)}{16\sqrt{-ac}\,a^{2}}$	83

input
```
int((B*x+A)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*A*c*x-2*B*a)/a/c/(c*x^2+a)^2+3/4*A/a*(1/2*x/a/(c*x^2+a)+1/2/a/(a*c)
^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.83

$$\int \frac{A+Bx}{\left(a+cx^2\right)^3}\,dx = \left[\frac{6\,Aac^2x^3+10\,Aa^2cx-4\,Ba^3-3\,(Ac^2x^4+2\,Aacx^2+Aa^2)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)}{16\,(a^3c^3x^4+2\,a^4c^2x^2+a^5c)}, \frac{3\,Aac^2x^3+5\,A\right.$$

input
```
integrate((B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(6*A*a*c^2*x^3 + 10*A*a^2*c*x - 4*B*a^3 - 3*(A*c^2*x^4 + 2*A*a*c*x^2
 + A*a^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a^3*c
^3*x^4 + 2*a^4*c^2*x^2 + a^5*c), 1/8*(3*A*a*c^2*x^3 + 5*A*a^2*c*x - 2*B*a^
3 + 3*(A*c^2*x^4 + 2*A*a*c*x^2 + A*a^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(
a^3*c^3*x^4 + 2*a^4*c^2*x^2 + a^5*c)]
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx}{\left(a+cx^2\right)^3}\,dx = A\left(-\frac{3\sqrt{-\frac{1}{a^5c}}\log\left(-a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{a^5c}}\log\left(a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16}\right) + \frac{5Aacx+3Ac^2x^3-2Ba^2}{8a^4c+16a^3c^2x^2+8a^2c^3x^4}$$

input
```
integrate((B*x+A)/(c*x**2+a)**3,x)
```

output
```
A*(-3*sqrt(-1/(a**5*c))*log(-a**3*sqrt(-1/(a**5*c)) + x)/16 + 3*sqrt(-1/(a
**5*c))*log(a**3*sqrt(-1/(a**5*c)) + x)/16) + (5*A*a*c*x + 3*A*c**2*x**3 -
 2*B*a**2)/(8*a**4*c + 16*a**3*c**2*x**2 + 8*a**2*c**3*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx}{(a+cx^2)^3}\,dx = \frac{3\,Ac^2x^3+5\,Aacx-2\,Ba^2}{8\,(a^2c^3x^4+2\,a^3c^2x^2+a^4c)}+\frac{3\,A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2}$$

input
```
integrate((B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(3*A*c^2*x^3 + 5*A*a*c*x - 2*B*a^2)/(a^2*c^3*x^4 + 2*a^3*c^2*x^2 + a^4
*c) + 3/8*A*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{(a+cx^2)^3}\,dx = \frac{3\,A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2}+\frac{3\,Ac^2x^3+5\,Aacx-2\,Ba^2}{8\,(cx^2+a)^2a^2c}$$

input
```
integrate((B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
3/8*A*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2) + 1/8*(3*A*c^2*x^3 + 5*A*a*c*x
 - 2*B*a^2)/((c*x^2 + a)^2*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 6.68 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx}{(a+cx^2)^3}\,dx = \frac{\frac{5\,A\,x}{8\,a}-\frac{B}{4\,c}+\frac{3\,A\,c\,x^3}{8\,a^2}}{a^2+2\,a\,c\,x^2+c^2\,x^4}+\frac{3\,A\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{c}}$$

input
```
int((A + B*x)/(a + c*x^2)^3,x)
```

output
```
((5*A*x)/(8*a) - B/(4*c) + (3*A*c*x^3)/(8*a^2))/(a^2 + c^2*x^4 + 2*a*c*x^2
) + (3*A*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx}{(a+cx^2)^3}\,dx = \frac{3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)a^2+6\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)ac\,x^2+3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{a}}\right)c^2x^4-2a^2b+5a^2cx+3a}{8a^2c\,(c^2x^4+2ac\,x^2+a^2)}$$

input
```
int((B*x+A)/(c*x^2+a)^3,x)
```

output
```
(3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**2 + 6*sqrt(c)*sqrt(a)*
atan((c*x)/(sqrt(c)*sqrt(a)))*a*c*x**2 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqr
t(c)*sqrt(a)))*c**2*x**4 - 2*a**2*b + 5*a**2*c*x + 3*a*c**2*x**3)/(8*a**2*
c*(a**2 + 2*a*c*x**2 + c**2*x**4))
```

3.101 $\int \frac{A+Bx}{(d+ex)\left(a+cx^2\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 307

$$\int \frac{A+Bx}{(d+ex)\left(a+cx^2\right)^3}\,dx = -\frac{a(Bd-Ae)-(Acd+aBe)x}{4a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} - \frac{4a^2e^2(Bd-Ae)+\left(aBe(cd^2-3ae^2)-Acd(3cd^2+7ae^2)\right)x}{8a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} - \frac{\left(aBe(c^2d^4+6acd^2e^2-3a^2e^4)-Acd(3c^2d^4+10acd^2e^2+15a^2e^4)\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}\left(cd^2+ae^2\right)^3} - \frac{e^4(Bd-Ae)\log(d+ex)}{\left(cd^2+ae^2\right)^3} + \frac{e^4(Bd-Ae)\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^3}$$

output
```
-1/4*(a*(-A*e+B*d)-(A*c*d+B*a*e)*x)/a/(a*e^2+c*d^2)/(c*x^2+a)^2-1/8*(4*a^2
*e^2*(-A*e+B*d)+(a*B*e*(-3*a*e^2+c*d^2)-A*c*d*(7*a*e^2+3*c*d^2))*x)/a^2/(a
*e^2+c*d^2)^2/(c*x^2+a)-1/8*(a*B*e*(-3*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)-A*c*
d*(15*a^2*e^4+10*a*c*d^2*e^2+3*c^2*d^4))*arctan(c^(1/2)*x/a^(1/2))/a^(5/2)
/c^(1/2)/(a*e^2+c*d^2)^3-e^4*(-A*e+B*d)*ln(e*x+d)/(a*e^2+c*d^2)^3+1/2*e^4*
(-A*e+B*d)*ln(c*x^2+a)/(a*e^2+c*d^2)^3
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{(d+ex)\left(a+cx^2\right)^3}\,dx$$
$$= \frac{\frac{2\left(cd^2+ae^2\right)^2(Acdx+a(-Bd+Ae+Bex))}{a\left(a+cx^2\right)^2} + \frac{\left(cd^2+ae^2\right)\left(3Ac^2d^3x+acde(-Bd+7Ae)x+a^2e^2(-4Bd+4Ae+3Bex)\right)}{a^2\left(a+cx^2\right)} + \frac{\left(aBe\left(-c^2d^4-6acd^2e^2\right.\right.}{}}{8\left(cd^2+\right.}$$

input
```
Integrate[(A + B*x)/((d + e*x)*(a + c*x^2)^3),x]
```

output
```
((2*(c*d^2 + a*e^2)^2*(A*c*d*x + a*(-(B*d) + A*e + B*e*x)))/(a*(a + c*x^2)
^2) + ((c*d^2 + a*e^2)*(3*A*c^2*d^3*x + a*c*d*e*(-(B*d) + 7*A*e)*x + a^2*e
^2*(-4*B*d + 4*A*e + 3*B*e*x)))/(a^2*(a + c*x^2)) + ((a*B*e*(-(c^2*d^4) -
6*a*c*d^2*e^2 + 3*a^2*e^4) + A*c*d*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^
4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[c]) + 8*e^4*(-(B*d) + A*e)*
Log[d + e*x] + 4*e^4*(B*d - A*e)*Log[a + c*x^2])/(8*(c*d^2 + a*e^2)^3)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {686, 25, 27, 686, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\left(a+cx^2\right)^3(d+ex)}\,dx$$
$$\Big\downarrow \ 686$$
$$-\frac{\int -\frac{c\left(3Acd^2-aBed+4aAe^2+3e(Acd+aBe)x\right)}{(d+ex)\left(cx^2+a\right)^2}dx}{4ac\left(ae^2+cd^2\right)} - \frac{a(Bd-Ae)-x(aBe+Acd)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$
$$\Big\downarrow \ 25$$

$$\frac{\int \frac{c(3Acd^2-aBed+4aAe^2+3e(Acd+aBe)x)}{(d+ex)(cx^2+a)^2}dx}{4ac\left(ae^2+cd^2\right)}-\frac{a(Bd-Ae)-x(aBe+Acd)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{3Acd^2-aBed+4aAe^2+3e(Acd+aBe)x}{(d+ex)(cx^2+a)^2}dx}{4a\left(ae^2+cd^2\right)}-\frac{a(Bd-Ae)-x(aBe+Acd)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 686

$$-\frac{\frac{\int \frac{c\left(aBde\left(cd^2+5ae^2\right)-A\left(3c^2d^4+7ace^2d^2+8a^2e^4\right)+e\left(aBe\left(cd^2-3ae^2\right)-Acd\left(3cd^2+7ae^2\right)\right)x\right)}{(d+ex)\left(cx^2+a\right)}dx}{2ac(ae^2+cd^2)}-\frac{4a^2e^2(Bd-Ae)+x\left(aBe(cd^2-3ae^2)-Acd(7ae^2\right.}{2a(a+cx^2)(ae^2+cd^2)}}{4a\left(ae^2+cd^2\right)}$$
$$\frac{a(Bd-Ae)-x(aBe+Acd)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{aBde\left(cd^2+5ae^2\right)-A\left(3c^2d^4+7ace^2d^2+8a^2e^4\right)+e\left(aBe\left(cd^2-3ae^2\right)-Acd\left(3cd^2+7ae^2\right)\right)x}{(d+ex)\left(cx^2+a\right)}dx}{2a(ae^2+cd^2)}-\frac{4a^2e^2(Bd-Ae)+x\left(aBe(cd^2-3ae^2)-Acd(7ae^2+3\right.}{2a(a+cx^2)(ae^2+cd^2)}}{4a\left(ae^2+cd^2\right)}$$
$$\frac{a(Bd-Ae)-x(aBe+Acd)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 657

$$-\frac{\frac{\int \left(\frac{-8a^2c(Bd-Ae)xe^4+aB\left(c^2d^4+6ace^2d^2-3a^2e^4\right)e-Acd\left(3c^2d^4+10ace^2d^2+15a^2e^4\right)}{\left(cd^2+ae^2\right)\left(cx^2+a\right)}-\frac{8a^2e^5(Ae-Bd)}{\left(cd^2+ae^2\right)(d+ex)}\right)dx}{2a(ae^2+cd^2)}-\frac{4a^2e^2(Bd-Ae)+x\left(aBe(cd^2-3ae\right.}{2a(a+cx^2)(ae^2-}}{4a\left(ae^2+cd^2\right)}$$
$$\frac{a(Bd-Ae)-x(aBe+Acd)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 2009

$$-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(aBe\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)-Acd\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)\right)}{\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)}-\frac{4a^2e^4\log\left(a+cx^2\right)(Bd-Ae)}{ae^2+cd^2}+\frac{8a^2e^4(Bd-Ae)\log(d+ex)}{ae^2+cd^2}}{2a(ae^2+cd^2)}-\frac{4a^2e^2(Bc}{}}{4a\left(ae^2+cd^2\right)}$$
$$\frac{a(Bd-Ae)-x(aBe+Acd)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

input
```
Int[(A + B*x)/((d + e*x)*(a + c*x^2)^3),x]
```

output
```
-1/4*(a*(B*d - A*e) - (A*c*d + a*B*e)*x)/(a*(c*d^2 + a*e^2)*(a + c*x^2)^2)
 + (-1/2*(4*a^2*e^2*(B*d - A*e) + (a*B*e*(c*d^2 - 3*a*e^2) - A*c*d*(3*c*d^
2 + 7*a*e^2))*x)/(a*(c*d^2 + a*e^2)*(a + c*x^2)) - (((a*B*e*(c^2*d^4 + 6*a
*c*d^2*e^2 - 3*a^2*e^4) - A*c*d*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4))
*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)) + (8*a^2*e
^4*(B*d - A*e)*Log[d + e*x])/(c*d^2 + a*e^2) - (4*a^2*e^4*(B*d - A*e)*Log[
a + c*x^2])/(c*d^2 + a*e^2))/(2*a*(c*d^2 + a*e^2)))/(4*a*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.42

method	result
default	$\frac{(Ae-Bd)e^4\ln(ex+d)}{(a\,e^2+c\,d^2)^3}+\frac{\frac{c\left(7A\,a^2cd\,e^4+10Aa\,c^2d^3e^2+3A\,d^5c^3+3B\,e^5a^3+2B\,a^2c\,d^2e^3-Ba\,c^2d^4e\right)x^3}{8a^2}+\left(\frac{1}{2}Aac\,e^5+\frac{1}{2}A\,c^2d^2e^3-\frac{1}{2}Bacd\,e^4-\right.}{}$
risch	Expression too large to display

input
```
int((B*x+A)/(e*x+d)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(A*e-B*d)*e^4/(a*e^2+c*d^2)^3*ln(e*x+d)+1/(a*e^2+c*d^2)^3*((1/8*c*(7*A*a^2
*c*d*e^4+10*A*a*c^2*d^3*e^2+3*A*c^3*d^5+3*B*a^3*e^5+2*B*a^2*c*d^2*e^3-B*a*
c^2*d^4*e)/a^2*x^3+(1/2*A*a*c*e^5+1/2*A*c^2*d^2*e^3-1/2*B*a*c*d*e^4-1/2*B*
c^2*d^3*e^2)*x^2+1/8*(9*A*a^2*c*d*e^4+14*A*a*c^2*d^3*e^2+5*A*c^3*d^5+5*B*a
^3*e^5+6*B*a^2*c*d^2*e^3+B*a*c^2*d^4*e)/a*x+3/4*A*a^2*e^5+A*a*c*d^2*e^3+1/
4*A*c^2*d^4*e-3/4*B*a^2*d*e^4-B*a*c*d^3*e^2-1/4*B*c^2*d^5)/(c*x^2+a)^2+1/8
/a^2*(1/2*(-8*A*a^2*c*e^5+8*B*a^2*c*d*e^4)/c*ln(c*x^2+a)+(15*A*a^2*c*d*e^4
+10*A*a*c^2*d^3*e^2+3*A*c^3*d^5+3*B*a^3*e^5-6*B*a^2*c*d^2*e^3-B*a*c^2*d^4*
e)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. 2(292) = 584.

Time = 72.75 (sec) , antiderivative size = 1797, normalized size of antiderivative = 5.85

$$\int \frac{A+Bx}{(d+ex)\,(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*B*a^3*c^3*d^5 - 4*A*a^3*c^3*d^4*e + 16*B*a^4*c^2*d^3*e^2 - 16*A*
a^4*c^2*d^2*e^3 + 12*B*a^5*c*d*e^4 - 12*A*a^5*c*e^5 - 2*(3*A*a*c^5*d^5 - B
*a^2*c^4*d^4*e + 10*A*a^2*c^4*d^3*e^2 + 2*B*a^3*c^3*d^2*e^3 + 7*A*a^3*c^3*
d*e^4 + 3*B*a^4*c^2*e^5)*x^3 + 8*(B*a^3*c^3*d^3*e^2 - A*a^3*c^3*d^2*e^3 +
B*a^4*c^2*d*e^4 - A*a^4*c^2*e^5)*x^2 + (3*A*a^2*c^3*d^5 - B*a^3*c^2*d^4*e
+ 10*A*a^3*c^2*d^3*e^2 - 6*B*a^4*c*d^2*e^3 + 15*A*a^4*c*d*e^4 + 3*B*a^5*e^
5 + (3*A*c^5*d^5 - B*a*c^4*d^4*e + 10*A*a*c^4*d^3*e^2 - 6*B*a^2*c^3*d^2*e^
3 + 15*A*a^2*c^3*d*e^4 + 3*B*a^3*c^2*e^5)*x^4 + 2*(3*A*a*c^4*d^5 - B*a^2*c
^3*d^4*e + 10*A*a^2*c^3*d^3*e^2 - 6*B*a^3*c^2*d^2*e^3 + 15*A*a^3*c^2*d*e^4
 + 3*B*a^4*c*e^5)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2
+ a)) - 2*(5*A*a^2*c^4*d^5 + B*a^3*c^3*d^4*e + 14*A*a^3*c^3*d^3*e^2 + 6*B*
a^4*c^2*d^2*e^3 + 9*A*a^4*c^2*d*e^4 + 5*B*a^5*c*e^5)*x - 8*(B*a^5*c*d*e^4
- A*a^5*c*e^5 + (B*a^3*c^3*d*e^4 - A*a^3*c^3*e^5)*x^4 + 2*(B*a^4*c^2*d*e^4
 - A*a^4*c^2*e^5)*x^2)*log(c*x^2 + a) + 16*(B*a^5*c*d*e^4 - A*a^5*c*e^5 +
(B*a^3*c^3*d*e^4 - A*a^3*c^3*e^5)*x^4 + 2*(B*a^4*c^2*d*e^4 - A*a^4*c^2*e^5
)*x^2)*log(e*x + d))/(a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4
+ a^8*c*e^6 + (a^3*c^6*d^6 + 3*a^4*c^5*d^4*e^2 + 3*a^5*c^4*d^2*e^4 + a^6*c
^3*e^6)*x^4 + 2*(a^4*c^5*d^6 + 3*a^5*c^4*d^4*e^2 + 3*a^6*c^3*d^2*e^4 + a^7
*c^2*e^6)*x^2), -1/8*(2*B*a^3*c^3*d^5 - 2*A*a^3*c^3*d^4*e + 8*B*a^4*c^2*d^
3*e^2 - 8*A*a^4*c^2*d^2*e^3 + 6*B*a^5*c*d*e^4 - 6*A*a^5*c*e^5 - (3*A*a*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)\left(a+cx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.71

$$\int \frac{A+Bx}{(d+ex)\left(a+cx^2\right)^3}\,dx$$
$$= \frac{(Bde^4 - Ae^5)\log(cx^2+a)}{2\,(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6)} - \frac{(Bde^4 - Ae^5)\log(ex+d)}{c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6}$$
$$+ \frac{(3\,Ac^3d^5 - Bac^2d^4e + 10\,Aac^2d^3e^2 - 6\,Ba^2cd^2e^3 + 15\,Aa^2cde^4 + 3\,Ba^3e^5)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,(a^2c^3d^6 + 3\,a^3c^2d^4e^2 + 3\,a^4cd^2e^4 + a^5e^6)\sqrt{ac}}$$
$$- \frac{2\,Ba^2cd^3 - 2\,Aa^2cd^2e + 6\,Ba^3de^2 - 6\,Aa^3e^3 - (3\,Ac^3d^3 - Bac^2d^2e + 7\,Aac^2de^2 + 3\,Ba^2ce^3)x^3 + 4\,(B}{8\,(a^4c^2d^4 + 2\,a^5cd^2e^2 + a^6e^4 + (a^2c^4d^4 + 2\,a^3c^3d^2e^2 + a^4c^2e^4)x^4 +}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/2*(B*d*e^4 - A*e^5)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*
d^2*e^4 + a^3*e^6) - (B*d*e^4 - A*e^5)*log(e*x + d)/(c^3*d^6 + 3*a*c^2*d^4
*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + 1/8*(3*A*c^3*d^5 - B*a*c^2*d^4*e + 10*
A*a*c^2*d^3*e^2 - 6*B*a^2*c*d^2*e^3 + 15*A*a^2*c*d*e^4 + 3*B*a^3*e^5)*arct
an(c*x/sqrt(a*c))/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^
5*e^6)*sqrt(a*c)) - 1/8*(2*B*a^2*c*d^3 - 2*A*a^2*c*d^2*e + 6*B*a^3*d*e^2 -
 6*A*a^3*e^3 - (3*A*c^3*d^3 - B*a*c^2*d^2*e + 7*A*a*c^2*d*e^2 + 3*B*a^2*c*
e^3)*x^3 + 4*(B*a^2*c*d*e^2 - A*a^2*c*e^3)*x^2 - (5*A*a*c^2*d^3 + B*a^2*c*
d^2*e + 9*A*a^2*c*d*e^2 + 5*B*a^3*e^3)*x)/(a^4*c^2*d^4 + 2*a^5*c*d^2*e^2 +
 a^6*e^4 + (a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2 + a^4*c^2*e^4)*x^4 + 2*(a^3*c^
3*d^4 + 2*a^4*c^2*d^2*e^2 + a^5*c*e^4)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.83

$$\int \frac{A+Bx}{(d+ex)(a+cx^2)^3}\,dx$$
$$= \frac{(Bde^4-Ae^5)\log(cx^2+a)}{2(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)} - \frac{(Bde^5-Ae^6)\log(|ex+d|)}{c^3d^6e+3\,ac^2d^4e^3+3\,a^2cd^2e^5+a^3e^7}$$
$$+ \frac{(3\,Ac^3d^5-Bac^2d^4e+10\,Aac^2d^3e^2-6\,Ba^2cd^2e^3+15\,Aa^2cde^4+3\,Ba^3e^5)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,(a^2c^3d^6+3\,a^3c^2d^4e^2+3\,a^4cd^2e^4+a^5e^6)\sqrt{ac}}$$
$$- \frac{2\,Ba^2c^2d^5-2\,Aa^2c^2d^4e+8\,Ba^3cd^3e^2-8\,Aa^3cd^2e^3+6\,Ba^4de^4-6\,Aa^4e^5-(3\,Ac^4d^5-Bac^3d^4e+10}{}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*(B*d*e^4 - A*e^5)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*
d^2*e^4 + a^3*e^6) - (B*d*e^5 - A*e^6)*log(abs(e*x + d))/(c^3*d^6*e + 3*a*
c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7) + 1/8*(3*A*c^3*d^5 - B*a*c^2*d^4*
e + 10*A*a*c^2*d^3*e^2 - 6*B*a^2*c*d^2*e^3 + 15*A*a^2*c*d*e^4 + 3*B*a^3*e^
5)*arctan(c*x/sqrt(a*c))/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e
^4 + a^5*e^6)*sqrt(a*c)) - 1/8*(2*B*a^2*c^2*d^5 - 2*A*a^2*c^2*d^4*e + 8*B*
a^3*c*d^3*e^2 - 8*A*a^3*c*d^2*e^3 + 6*B*a^4*d*e^4 - 6*A*a^4*e^5 - (3*A*c^4
*d^5 - B*a*c^3*d^4*e + 10*A*a*c^3*d^3*e^2 + 2*B*a^2*c^2*d^2*e^3 + 7*A*a^2*
c^2*d*e^4 + 3*B*a^3*c*e^5)*x^3 + 4*(B*a^2*c^2*d^3*e^2 - A*a^2*c^2*d^2*e^3
+ B*a^3*c*d*e^4 - A*a^3*c*e^5)*x^2 - (5*A*a*c^3*d^5 + B*a^2*c^2*d^4*e + 14
*A*a^2*c^2*d^3*e^2 + 6*B*a^3*c*d^2*e^3 + 9*A*a^3*c*d*e^4 + 5*B*a^4*e^5)*x)
/((c*d^2 + a*e^2)^3*(c*x^2 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 2415, normalized size of antiderivative = 7.87

$$\int \frac{A+Bx}{(d+ex)(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/((a + c*x^2)^3*(d + e*x)),x)
```

output
```
((3*A*a*e^3 - B*c*d^3 - 3*B*a*d*e^2 + A*c*d^2*e)/(4*(a^2*e^4 + c^2*d^4 + 2
*a*c*d^2*e^2)) + (x^2*(A*c*e^3 - B*c*d*e^2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c
*d^2*e^2)) + (x^3*(3*A*c^3*d^3 + 3*B*a^2*c*e^3 + 7*A*a*c^2*d*e^2 - B*a*c^2
*d^2*e))/(8*a^2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(5*A*c^2*d^3 + 5
*B*a^2*e^3 + 9*A*a*c*d*e^2 + B*a*c*d^2*e))/(8*a*(a^2*e^4 + c^2*d^4 + 2*a*c
*d^2*e^2)))/(a^2 + c^2*x^4 + 2*a*c*x^2) - (log(576*A^2*a^7*e^14*(-a^5*c)^(
3/2) + 9*A^2*c^7*d^14*(-a^5*c)^(3/2) - 9*B^2*a^13*e^14*(-a^5*c)^(1/2) + 55
8*B^2*a^7*d^2*e^12*(-a^5*c)^(3/2) + 9*B^2*a^15*c*e^14*x + 9*A^2*a^7*c^9*d^
14*x + 576*A^2*a^14*c^2*e^14*x - 1377*A^2*a*d^2*e^12*(-a^5*c)^(5/2) - 1119
*B^2*a*d^4*e^10*(-a^5*c)^(5/2) - 1326*A^2*c*d^4*e^10*(-a^5*c)^(5/2) - 612*
B^2*c*d^6*e^8*(-a^5*c)^(5/2) + 78*A^2*a^8*c^8*d^12*e^2*x + 319*A^2*a^9*c^7
*d^10*e^4*x + 740*A^2*a^10*c^6*d^8*e^6*x + 1015*A^2*a^11*c^5*d^6*e^8*x + 1
326*A^2*a^12*c^4*d^4*e^10*x + 1377*A^2*a^13*c^3*d^2*e^12*x + B^2*a^9*c^7*d
^12*e^2*x + 14*B^2*a^10*c^6*d^10*e^4*x + 55*B^2*a^11*c^5*d^8*e^6*x + 612*B
^2*a^12*c^4*d^6*e^8*x + 1119*B^2*a^13*c^3*d^4*e^10*x + 558*B^2*a^14*c^2*d^
2*e^12*x + 78*A^2*a*c^6*d^12*e^2*(-a^5*c)^(3/2) + 2244*A*B*a*d^3*e^11*(-a^
5*c)^(5/2) - 1062*A*B*a^7*d*e^13*(-a^5*c)^(3/2) + 1434*A*B*c*d^5*e^9*(-a^5
*c)^(5/2) + 319*A^2*a^2*c^5*d^10*e^4*(-a^5*c)^(3/2) + 740*A^2*a^3*c^4*d^8*
e^6*(-a^5*c)^(3/2) + 1015*A^2*a^4*c^3*d^6*e^8*(-a^5*c)^(3/2) + B^2*a^2*c^5
*d^12*e^2*(-a^5*c)^(3/2) + 14*B^2*a^3*c^4*d^10*e^4*(-a^5*c)^(3/2) + 55*...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1251, normalized size of antiderivative = 4.07

$$\int \frac{A+Bx}{(d+ex)\,(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/(e*x+d)/(c*x^2+a)^3,x)
```

output

```
(3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*b*e**5 + 15*sqrt(c)*
sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*c*d*e**4 - 6*sqrt(c)*sqrt(a)*at
an((c*x)/(sqrt(c)*sqrt(a)))*a**3*b*c*d**2*e**3 + 6*sqrt(c)*sqrt(a)*atan((c
*x)/(sqrt(c)*sqrt(a)))*a**3*b*c*e**5*x**2 + 10*sqrt(c)*sqrt(a)*atan((c*x)/
(sqrt(c)*sqrt(a)))*a**3*c**2*d**3*e**2 + 30*sqrt(c)*sqrt(a)*atan((c*x)/(sq
rt(c)*sqrt(a)))*a**3*c**2*d*e**4*x**2 - sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c
)*sqrt(a)))*a**2*b*c**2*d**4*e - 12*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sq
rt(a)))*a**2*b*c**2*d**2*e**3*x**2 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)
*sqrt(a)))*a**2*b*c**2*e**5*x**4 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*s
qrt(a)))*a**2*c**3*d**5 + 20*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))
*a**2*c**3*d**3*e**2*x**2 + 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)
))*a**2*c**3*d*e**4*x**4 - 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))
*a*b*c**3*d**4*e*x**2 - 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*
b*c**3*d**2*e**3*x**4 + 6*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*
c**4*d**5*x**2 + 10*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a*c**4*d
**3*e**2*x**4 - sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*b*c**4*d**4*
e*x**4 + 3*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*c**5*d**5*x**4 -
4*log(a + c*x**2)*a**5*c*e**5 + 4*log(a + c*x**2)*a**4*b*c*d*e**4 - 8*log(
a + c*x**2)*a**4*c**2*e**5*x**2 + 8*log(a + c*x**2)*a**3*b*c**2*d*e**4*x**
2 - 4*log(a + c*x**2)*a**3*c**3*e**5*x**4 + 4*log(a + c*x**2)*a**2*b*c*...
```

3.102 $\int \frac{A+Bx}{(d+ex)^2(a+cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 461

$$\begin{aligned}\int \frac{A+Bx}{(d+ex)^2(a+cx^2)^3}\,dx &= -\frac{e(2aBde(cd^2-11ae^2)-3A(c^2d^4+4acd^2e^2-5a^2e^4))}{8a^2(cd^2+ae^2)^3(d+ex)}\\ &- \frac{a(Bcd^2-2Acde-aBe^2)-c(Acd^2+2aBde-aAe^2)\,x}{4a(cd^2+ae^2)^2(a+cx^2)^2}\\ &- \frac{ae(3Acd^2+10aBde-7aAe^2)+(2aBe(cd^2-2ae^2)-3Acd(cd^2+3ae^2))\,x}{8a^2(cd^2+ae^2)^2(d+ex)(a+cx^2)}\\ &- \frac{\sqrt{c}(2aBde(c^2d^4+10acd^2e^2-15a^2e^4)-3A(c^3d^6+5ac^2d^4e^2+15a^2cd^2e^4-5a^3e^6))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}(cd^2+ae^2)^4}\\ &- \frac{e^4(5Bcd^2-6Acde-aBe^2)\log(d+ex)}{(cd^2+ae^2)^4}+\frac{e^4(5Bcd^2-6Acde-aBe^2)\log(a+cx^2)}{2(cd^2+ae^2)^4}\end{aligned}$$

output
```
-1/8*e*(2*a*B*d*e*(-11*a*e^2+c*d^2)-3*A*(-5*a^2*e^4+4*a*c*d^2*e^2+c^2*d^4)
)/a^2/(a*e^2+c*d^2)^3/(e*x+d)-1/4*(a*(-2*A*c*d*e-B*a*e^2+B*c*d^2)-c*(-A*a*
e^2+A*c*d^2+2*B*a*d*e)*x)/a/(a*e^2+c*d^2)^2/(c*x^2+a)^2-1/8*(a*e*(-7*A*a*e
^2+3*A*c*d^2+10*B*a*d*e)+(2*a*B*e*(-2*a*e^2+c*d^2)-3*A*c*d*(3*a*e^2+c*d^2)
)*x)/a^2/(a*e^2+c*d^2)^2/(e*x+d)/(c*x^2+a)-1/8*c^(1/2)*(2*a*B*d*e*(-15*a^2
*e^4+10*a*c*d^2*e^2+c^2*d^4)-3*A*(-5*a^3*e^6+15*a^2*c*d^2*e^4+5*a*c^2*d^4*
e^2+c^3*d^6))*arctan(c^(1/2)*x/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)^4-e^4*(-6*A*
c*d*e-B*a*e^2+5*B*c*d^2)*ln(e*x+d)/(a*e^2+c*d^2)^4+1/2*e^4*(-6*A*c*d*e-B*a
*e^2+5*B*c*d^2)*ln(c*x^2+a)/(a*e^2+c*d^2)^4
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx}{(d+ex)^2\,(a+cx^2)^3}\,dx$$

$$= \frac{-\frac{8e^4(-Bd+Ae)(cd^2+ae^2)}{d+ex} + \frac{(cd^2+ae^2)\left(4a^3Be^4+3Ac^3d^4x-2ac^2d^2e(Bd-6Ae)x+a^2ce^2(-2Bd(6d-7ex)+Ae(16d-7ex))\right)}{a^2(a+cx^2)} + \frac{2(cd^2+ae^2 \cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*x)/((d + e*x)^2*(a + c*x^2)^3),x]
```

output
```
((-8*e^4*(-(B*d) + A*e)*(c*d^2 + a*e^2))/(d + e*x) + ((c*d^2 + a*e^2)*(4*a
^3*B*e^4 + 3*A*c^3*d^4*x - 2*a*c^2*d^2*e*(B*d - 6*A*e)*x + a^2*c*e^2*(-2*B
*d*(6*d - 7*e*x) + A*e*(16*d - 7*e*x))))/(a^2*(a + c*x^2)) + (2*(c*d^2 + a
*e^2)^2*(a^2*B*e^2 + A*c^2*d^2*x - a*c*(B*d*(d - 2*e*x) + A*e*(-2*d + e*x)
)))/(a*(a + c*x^2)^2) + (Sqrt[c]*(2*a*B*d*e*(-(c^2*d^4) - 10*a*c*d^2*e^2 +
 15*a^2*e^4) + 3*A*(c^3*d^6 + 5*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a^3*e
^6))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 8*e^4*(-5*B*c*d^2 + 6*A*c*d*e
+ a*B*e^2)*Log[d + e*x] - 4*e^4*(-5*B*c*d^2 + 6*A*c*d*e + a*B*e^2)*Log[a +
 c*x^2])/(8*(c*d^2 + a*e^2)^4)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {686, 25, 27, 686, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+cx^2)^3 (d+ex)^2} dx$$

$$\Big\downarrow \; 686$$

$$-\frac{\int -\frac{c(3Acd^2-2aBed+5aAe^2+4e(Acd+aBe)x)}{(d+ex)^2(cx^2+a)^2}dx}{4ac(ae^2+cd^2)} - \frac{a(Bd-Ae)-x(aBe+Acd)}{4a(a+cx^2)^2(d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{c(3Acd^2-2aBed+5aAe^2+4e(Acd+aBe)x)}{(d+ex)^2(cx^2+a)^2}dx}{4ac(ae^2+cd^2)} - \frac{a(Bd-Ae)-x(aBe+Acd)}{4a(a+cx^2)^2(d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3Acd^2-2aBed+5aAe^2+4e(Acd+aBe)x}{(d+ex)^2(cx^2+a)^2}dx}{4a(ae^2+cd^2)} - \frac{a(Bd-Ae)-x(aBe+Acd)}{4a(a+cx^2)^2(d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \; 686$$

$$\frac{-\frac{\int \frac{c\left(2aBde\left(cd^2+7ae^2\right)-3A\left(c^2d^4+2ace^2d^2+5a^2e^4\right)+2e\left(2aBe\left(cd^2-2ae^2\right)-3Acd\left(cd^2+3ae^2\right)\right)x\right)}{(d+ex)^2\left(cx^2+a\right)}dx}{2ac(ae^2+cd^2)} - \frac{x\left(2aBe(cd^2-2ae^2)-3Acd(3ae^2+cd^2)\right)+ae}{2a(a+cx^2)(d+ex)(ae^2+}}{4a(ae^2+cd^2)}$$

$$\frac{a(Bd-Ae)-x(aBe+Acd)}{4a(a+cx^2)^2(d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{2aBde\left(cd^2+7ae^2\right)-3A\left(c^2d^4+2ace^2d^2+5a^2e^4\right)+2e\left(2aBe\left(cd^2-2ae^2\right)-3Acd\left(cd^2+3ae^2\right)\right)x}{(d+ex)^2\left(cx^2+a\right)}dx}{2a(ae^2+cd^2)} - \frac{x\left(2aBe(cd^2-2ae^2)-3Acd(3ae^2+cd^2)\right)+ae(-5}{2a(a+cx^2)(d+ex)(ae^2+cd}}{4a(ae^2+cd^2)}$$

$$\frac{a(Bd-Ae)-x(aBe+Acd)}{4a(a+cx^2)^2(d+ex)(ae^2+cd^2)}$$

$$\Big\downarrow \text{657}$$

$$-\frac{\frac{\int \left(-\frac{8a^2\left(-5Bcd^2+6Aced+aBe^2\right)e^5}{\left(cd^2+ae^2\right)^2(d+ex)}+\frac{\left(3A\left(c^2d^4+4ace^2d^2-5a^2e^4\right)-2aBde\left(cd^2-11ae^2\right)\right)e^2}{\left(cd^2+ae^2\right)(d+ex)^2}+\frac{c\left(-8a^2\left(5Bcd^2-6Aced-aBe^2\right)xe^4+2aBd\left(c^2d^4+10ace^2d\right.\right.}{\left(cd^2+ae^2\right)}\right.}{2a(ae^2+cd^2)}}{4a\left(ae^2+cd\right.}$$

$$\frac{a(Bd-Ae)-x(aBe+Acd)}{4a\left(a+cx^2\right)^2(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\frac{e\left(2aBde\left(cd^2-11ae^2\right)-3A\left(-5a^2e^4+4acd^2e^2+c^2d^4\right)\right)}{(d+ex)\left(ae^2+cd^2\right)}-\frac{4a^2e^4\log\left(a+cx^2\right)\left(-aBe^2-6Acde+5Bcd^2\right)}{\left(ae^2+cd^2\right)^2}+\frac{8a^2e^4\log(d+ex)\left(-aBe^2-6Acde+5Bcd^2\right)}{\left(ae^2+cd^2\right)^2}+\frac{\sqrt{c}\,\text{arct}}{}}{2a(ae^2+cd^2)}}{4}$$

$$\frac{a(Bd-Ae)-x(aBe+Acd)}{4a\left(a+cx^2\right)^2(d+ex)\left(ae^2+cd^2\right)}$$

input
```
Int[(A + B*x)/((d + e*x)^2*(a + c*x^2)^3),x]
```

output
```
-1/4*(a*(B*d - A*e) - (A*c*d + a*B*e)*x)/(a*(c*d^2 + a*e^2)*(d + e*x)*(a +
 c*x^2)^2) + (-1/2*(a*e*(A*c*d^2 + 6*a*B*d*e - 5*a*A*e^2) + (2*a*B*e*(c*d^
2 - 2*a*e^2) - 3*A*c*d*(c*d^2 + 3*a*e^2))*x)/(a*(c*d^2 + a*e^2)*(d + e*x)*
(a + c*x^2)) - ((e*(2*a*B*d*e*(c*d^2 - 11*a*e^2) - 3*A*(c^2*d^4 + 4*a*c*d^
2*e^2 - 5*a^2*e^4)))/((c*d^2 + a*e^2)*(d + e*x)) + (Sqrt[c]*(2*a*B*d*e*(c^
2*d^4 + 10*a*c*d^2*e^2 - 15*a^2*e^4) - 3*A*(c^3*d^6 + 5*a*c^2*d^4*e^2 + 15
*a^2*c*d^2*e^4 - 5*a^3*e^6))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2
+ a*e^2)^2) + (8*a^2*e^4*(5*B*c*d^2 - 6*A*c*d*e - a*B*e^2)*Log[d + e*x])/(
c*d^2 + a*e^2)^2 - (4*a^2*e^4*(5*B*c*d^2 - 6*A*c*d*e - a*B*e^2)*Log[a + c*
x^2])/(c*d^2 + a*e^2)^2)/(2*a*(c*d^2 + a*e^2)))/(4*a*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.23

method	result
default	$\frac{e^4(6Acde+Ba\,e^2-5Bc\,d^2)\ln(ex+d)}{(a\,e^2+c\,d^2)^4}-\frac{(Ae-Bd)e^4}{(a\,e^2+c\,d^2)^3(ex+d)}-\frac{c\left(\frac{c\left(7A\,a^3e^6-5A\,a^2c\,d^2e^4-15Aa\,c^2d^4e^2-3A\,c^3d^6-14B\,a^3d\,e^5-12B\right.}{8a^2}\right.}{}$
risch	Expression too large to display

input
```
int((B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
e^4*(6*A*c*d*e+B*a*e^2-5*B*c*d^2)/(a*e^2+c*d^2)^4*ln(e*x+d)-(A*e-B*d)*e^4/
(a*e^2+c*d^2)^3/(e*x+d)-c/(a*e^2+c*d^2)^4*((1/8*c*(7*A*a^3*e^6-5*A*a^2*c*d
^2*e^4-15*A*a*c^2*d^4*e^2-3*A*c^3*d^6-14*B*a^3*d*e^5-12*B*a^2*c*d^3*e^3+2*
B*a*c^2*d^5*e)/a^2*x^3+(-2*A*a*c*d*e^5-2*A*c^2*d^3*e^3-1/2*B*a^2*e^6+B*a*c
*d^2*e^4+3/2*B*c^2*d^4*e^2)*x^2+1/8*(9*A*a^3*e^6-3*A*a^2*c*d^2*e^4-17*A*a*
c^2*d^4*e^2-5*A*c^3*d^6-18*B*a^3*d*e^5-20*B*a^2*c*d^3*e^3-2*B*a*c^2*d^5*e)
/a*x-1/4*(10*A*a^2*c*d*e^5+12*A*a*c^2*d^3*e^3+2*A*c^3*d^5*e+3*B*a^3*e^6-3*
B*a^2*c*d^2*e^4-7*B*a*c^2*d^4*e^2-B*c^3*d^6)/c)/(c*x^2+a)^2+1/8/a^2*(1/2*(
48*A*a^2*c*d*e^5+8*B*a^3*e^6-40*B*a^2*c*d^2*e^4)/c*ln(c*x^2+a)+(15*A*a^3*e
^6-45*A*a^2*c*d^2*e^4-15*A*a*c^2*d^4*e^2-3*A*c^3*d^6-30*B*a^3*d*e^5+20*B*a
^2*c*d^3*e^3+2*B*a*c^2*d^5*e)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)^2\,(a+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)^2\,(a+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)**2/(c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. 2(444) = 888.

Time = 0.15 (sec) , antiderivative size = 997, normalized size of antiderivative = 2.16

$$\int \frac{A+Bx}{(d+ex)^2(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/2*(5*B*c*d^2*e^4 - 6*A*c*d*e^5 - B*a*e^6)*log(c*x^2 + a)/(c^4*d^8 + 4*a*
c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) - (5*B*c*d^2*
e^4 - 6*A*c*d*e^5 - B*a*e^6)*log(e*x + d)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a
^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + 1/8*(3*A*c^4*d^6 - 2*B*a*c^3
*d^5*e + 15*A*a*c^3*d^4*e^2 - 20*B*a^2*c^2*d^3*e^3 + 45*A*a^2*c^2*d^2*e^4
+ 30*B*a^3*c*d*e^5 - 15*A*a^3*c*e^6)*arctan(c*x/sqrt(a*c))/((a^2*c^4*d^8 +
 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(a
*c)) - 1/8*(2*B*a^2*c^2*d^5 - 4*A*a^2*c^2*d^4*e + 12*B*a^3*c*d^3*e^2 - 20*
A*a^3*c*d^2*e^3 - 14*B*a^4*d*e^4 + 8*A*a^4*e^5 - (3*A*c^4*d^4*e - 2*B*a*c^
3*d^3*e^2 + 12*A*a*c^3*d^2*e^3 + 22*B*a^2*c^2*d*e^4 - 15*A*a^2*c^2*e^5)*x^
4 - (3*A*c^4*d^5 - 2*B*a*c^3*d^4*e + 12*A*a*c^3*d^3*e^2 + 2*B*a^2*c^2*d^2*
e^3 + 9*A*a^2*c^2*d*e^4 + 4*B*a^3*c*e^5)*x^3 - (5*A*a*c^3*d^4*e - 10*B*a^2
*c^2*d^3*e^2 + 28*A*a^2*c^2*d^2*e^3 + 38*B*a^3*c*d*e^4 - 25*A*a^3*c*e^5)*x
^2 - (5*A*a*c^3*d^5 + 16*A*a^2*c^2*d^3*e^2 + 6*B*a^3*c*d^2*e^3 + 11*A*a^3*
c*d*e^4 + 6*B*a^4*e^5)*x)/(a^4*c^3*d^7 + 3*a^5*c^2*d^5*e^2 + 3*a^6*c*d^3*e
^4 + a^7*d*e^6 + (a^2*c^5*d^6*e + 3*a^3*c^4*d^4*e^3 + 3*a^4*c^3*d^2*e^5 +
a^5*c^2*e^7)*x^5 + (a^2*c^5*d^7 + 3*a^3*c^4*d^5*e^2 + 3*a^4*c^3*d^3*e^4 +
a^5*c^2*d*e^6)*x^4 + 2*(a^3*c^4*d^6*e + 3*a^4*c^3*d^4*e^3 + 3*a^5*c^2*d^2*
e^5 + a^6*c*e^7)*x^3 + 2*(a^3*c^4*d^7 + 3*a^4*c^3*d^5*e^2 + 3*a^5*c^2*d^3*
e^4 + a^6*c*d*e^6)*x^2 + (a^4*c^3*d^6*e + 3*a^5*c^2*d^4*e^3 + 3*a^6*c*d...
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 884, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx}{(d+ex)^2 (a+cx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*(5*B*c*d^2*e^4 - 6*A*c*d*e^5 - B*a*e^6)*log(c - 2*c*d/(e*x + d) + c*d^
2/(e*x + d)^2 + a*e^2/(e*x + d)^2)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*
d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + (B*d*e^10/(e*x + d) - A*e^11/(e*x +
 d))/(c^3*d^6*e^6 + 3*a*c^2*d^4*e^8 + 3*a^2*c*d^2*e^10 + a^3*e^12) + 1/8*(
3*A*c^4*d^6*e^2 - 2*B*a*c^3*d^5*e^3 + 15*A*a*c^3*d^4*e^4 - 20*B*a^2*c^2*d^
3*e^5 + 45*A*a^2*c^2*d^2*e^6 + 30*B*a^3*c*d*e^7 - 15*A*a^3*c*e^8)*arctan((
c*d - c*d^2/(e*x + d) - a*e^2/(e*x + d))/(sqrt(a*c)*e))/((a^2*c^4*d^8 + 4*
a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(a*c)
*e^2) + 1/8*(3*A*c^5*d^5*e - 2*B*a*c^4*d^4*e^2 + 14*A*a*c^4*d^3*e^3 + 32*B
*a^2*c^3*d^2*e^4 - 29*A*a^2*c^3*d*e^5 - 6*B*a^3*c^2*e^6 - (9*A*c^5*d^6*e^2
 - 6*B*a*c^4*d^5*e^3 + 41*A*a*c^4*d^4*e^4 + 116*B*a^2*c^3*d^3*e^5 - 121*A*
a^2*c^3*d^2*e^6 - 38*B*a^3*c^2*d*e^7 + 7*A*a^3*c^2*e^8)/((e*x + d)*e) + (9
*A*c^5*d^7*e^3 - 6*B*a*c^4*d^6*e^4 + 45*A*a*c^4*d^5*e^5 + 140*B*a^2*c^3*d^
4*e^6 - 145*A*a^2*c^3*d^3*e^7 - 22*B*a^3*c^2*d^2*e^8 - 21*A*a^3*c^2*d*e^9
- 8*B*a^4*c*e^10)/((e*x + d)^2*e^2) - (3*A*c^5*d^8*e^4 - 2*B*a*c^4*d^7*e^5
 + 18*A*a*c^4*d^6*e^6 + 58*B*a^2*c^3*d^5*e^7 - 60*A*a^2*c^3*d^4*e^8 + 26*B
*a^3*c^2*d^3*e^9 - 66*A*a^3*c^2*d^2*e^10 - 34*B*a^4*c*d*e^11 + 9*A*a^4*c*e
^12)/((e*x + d)^3*e^3))/((c*d^2 + a*e^2)^4*a^2*(c - 2*c*d/(e*x + d) + c*d^
2/(e*x + d)^2 + a*e^2/(e*x + d)^2)^2)
```

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 3015, normalized size of antiderivative = 6.54

$$\int \frac{A+Bx}{(d+ex)^2 (a+cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int((A + B*x)/((a + c*x^2)^3*(d + e*x)^2),x)
```

output
```
((x*(5*A*c^2*d^3 + 6*B*a^2*e^3 + 11*A*a*c*d*e^2))/(8*a*(a^2*e^4 + c^2*d^4
+ 2*a*c*d^2*e^2)) - (4*A*a^2*e^5 + B*c^2*d^5 - 7*B*a^2*d*e^4 - 2*A*c^2*d^4
*e - 10*A*a*c*d^2*e^3 + 6*B*a*c*d^3*e^2)/(4*(a*e^2 + c*d^2)*(a^2*e^4 + c^2
*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*A*c^3*d^3 + 4*B*a^2*c*e^3 + 9*A*a*c^2*d*e
^2 - 2*B*a*c^2*d^2*e))/(8*a^2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x^4*
(3*A*c^4*d^4*e - 15*A*a^2*c^2*e^5 + 12*A*a*c^3*d^2*e^3 - 2*B*a*c^3*d^3*e^2
 + 22*B*a^2*c^2*d*e^4))/(8*a^2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^
2*c*d^2*e^4)) + (x^2*(5*A*c^3*d^4*e - 25*A*a^2*c*e^5 + 28*A*a*c^2*d^2*e^3
- 10*B*a*c^2*d^3*e^2 + 38*B*a^2*c*d*e^4))/(8*a*(a*e^2 + c*d^2)*(a^2*e^4 +
c^2*d^4 + 2*a*c*d^2*e^2)))/(a^2*d + c^2*d*x^4 + c^2*e*x^5 + a^2*e*x + 2*a*
c*d*x^2 + 2*a*c*e*x^3) - (log(d + e*x)*(c*(5*B*d^2*e^4 - 6*A*d*e^5) - B*a*
e^6))/(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d
^4*e^4) + (log(576*B^2*a^14*e^16*(-a^5*c)^(1/2) - 9*A^2*c^8*d^16*(-a^5*c)^
(3/2) - 225*A^2*a^8*e^16*(-a^5*c)^(3/2) + 19836*A^2*a^2*d^2*e^14*(-a^5*c)^
(5/2) + 4056*B^2*a^2*d^4*e^12*(-a^5*c)^(5/2) + 3708*B^2*a^8*d^2*e^14*(-a^5
*c)^(3/2) + 23796*A^2*c^2*d^6*e^10*(-a^5*c)^(5/2) + 13840*B^2*c^2*d^8*e^8*
(-a^5*c)^(5/2) + 576*B^2*a^16*c*e^16*x + 9*A^2*a^7*c^10*d^16*x + 225*A^2*a
^15*c^2*e^16*x - 19236*A*B*a^2*d^3*e^13*(-a^5*c)^(5/2) - 33540*A*B*c^2*d^7
*e^9*(-a^5*c)^(5/2) + 40572*A^2*a*c*d^4*e^12*(-a^5*c)^(5/2) + 21820*B^2*a*
c*d^6*e^10*(-a^5*c)^(5/2) + 108*A^2*a^8*c^9*d^14*e^2*x + 684*A^2*a^9*c^...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 3096, normalized size of antiderivative = 6.72

$$\int \frac{A+Bx}{(d+ex)^2(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x)
```

output
```
( - 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**5*d**2*e**6 - 15*s
qrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**5*d*e**7*x + 30*sqrt(c)*sq
rt(a)*atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*b*d**3*e**5 + 30*sqrt(c)*sqrt(a)*
atan((c*x)/(sqrt(c)*sqrt(a)))*a**4*b*d**2*e**6*x + 45*sqrt(c)*sqrt(a)*atan
((c*x)/(sqrt(c)*sqrt(a)))*a**4*c*d**4*e**4 + 45*sqrt(c)*sqrt(a)*atan((c*x)
/(sqrt(c)*sqrt(a)))*a**4*c*d**3*e**5*x - 30*sqrt(c)*sqrt(a)*atan((c*x)/(sq
rt(c)*sqrt(a)))*a**4*c*d**2*e**6*x**2 - 30*sqrt(c)*sqrt(a)*atan((c*x)/(sqr
t(c)*sqrt(a)))*a**4*c*d*e**7*x**3 - 20*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)
*sqrt(a)))*a**3*b*c*d**5*e**3 - 20*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqr
t(a)))*a**3*b*c*d**4*e**4*x + 60*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(
a)))*a**3*b*c*d**3*e**5*x**2 + 60*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt
(a)))*a**3*b*c*d**2*e**6*x**3 + 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqr
t(a)))*a**3*c**2*d**6*e**2 + 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a
)))*a**3*c**2*d**5*e**3*x + 90*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a)
))*a**3*c**2*d**4*e**4*x**2 + 90*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(
a)))*a**3*c**2*d**3*e**5*x**3 - 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqr
t(a)))*a**3*c**2*d**2*e**6*x**4 - 15*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*s
qrt(a)))*a**3*c**2*d*e**7*x**5 - 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqr
t(a)))*a**2*b*c**2*d**7*e - 2*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt(a))
)*a**2*b*c**2*d**6*e**2*x - 40*sqrt(c)*sqrt(a)*atan((c*x)/(sqrt(c)*sqrt...
```

3.103 $\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 17

$$\int \frac{2+2x}{(-1+x)^3\,(1+x^2)}\,dx = -\frac{1}{(1-x)^2}+\frac{1}{-1+x}+\arctan(x)$$

output
```
-1/(1-x)^2+1/(-1+x)+arctan(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{2+2x}{(-1+x)^3\,(1+x^2)}\,dx = \frac{-2+x+(-1+x)^2\arctan(x)}{(-1+x)^2}$$

input
```
Integrate[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]
```

output
```
(-2 + x + (-1 + x)^2*ArcTan[x])/(-1 + x)^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+2}{(x-1)^3 (x^2+1)} \, dx$$

$$\Big\downarrow \text{657}$$

$$\int \left(\frac{1}{x^2+1} - \frac{1}{(x-1)^2} + \frac{2}{(x-1)^3}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\arctan(x) + \frac{1}{x-1} - \frac{1}{(1-x)^2}$$

input
```
Int[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]
```

output
```
-(1 - x)^(-2) + (-1 + x)^(-1) + ArcTan[x]
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{x-2}{(x-1)^2}+\arctan(x)$	13
default	$\arctan(x)-\frac{1}{(x-1)^2}+\frac{1}{x-1}$	16
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2-2i\ln(x-i)x+2i\ln(x+i)x+3+i\ln(x-i)-i\ln(x+i)-x^2}{2(x-1)^2}$	71

input
```
int((2+2*x)/(x-1)^3/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
(x-2)/(x-1)^2+arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx=\frac{(x^2-2\,x+1)\arctan(x)+x-2}{x^2-2\,x+1}$$

input
```
integrate((2+2*x)/(x-1)^3/(x^2+1),x, algorithm="fricas")
```

output
```
((x^2 - 2*x + 1)*arctan(x) + x - 2)/(x^2 - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx=\frac{x-2}{x^2-2x+1}+\operatorname{atan}(x)$$

input
```
integrate((2+2*x)/(x-1)**3/(x**2+1),x)
```

output
```
(x - 2)/(x**2 - 2*x + 1) + atan(x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{2+2x}{(-1+x)^3\,(1+x^2)}\,dx = \frac{x-2}{x^2-2\,x+1} + \arctan\left(x\right)$$

input
```
integrate((2+2*x)/(x-1)^3/(x^2+1),x, algorithm="maxima")
```

output
```
(x - 2)/(x^2 - 2*x + 1) + arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{2+2x}{(-1+x)^3\,(1+x^2)}\,dx = \frac{x-2}{\left(x-1\right)^2} + \arctan\left(x\right)$$

input
```
integrate((2+2*x)/(x-1)^3/(x^2+1),x, algorithm="giac")
```

output
```
(x - 2)/(x - 1)^2 + arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{2+2x}{(-1+x)^3\,(1+x^2)}\,dx = \mathrm{atan}(x) + \frac{x-2}{x^2-2\,x+1}$$

input
```
int((2*x + 2)/((x^2 + 1)*(x - 1)^3),x)
```

output
```
atan(x) + (x - 2)/(x^2 - 2*x + 1)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

$$\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx = \frac{2atan(x)\,x^2 - 4atan(x)\,x + 2atan(x) + x^2 - 3}{2x^2 - 4x + 2}$$

input
```
int((2+2*x)/(x-1)^3/(x^2+1),x)
```

output
```
(2*atan(x)*x**2 - 4*atan(x)*x + 2*atan(x) + x**2 - 3)/(2*(x**2 - 2*x + 1))
```

3.104 $\int \frac{-11+6x}{(-1+2x)(-1+x^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{-11+6x}{(-1+2x)\left(-1+x^2\right)}\,dx=\frac{16}{3}\log(1-2x)-\frac{5}{2}\log(1-x)-\frac{17}{6}\log(1+x)$$

output
```
16/3*ln(1-2*x)-5/2*ln(1-x)-17/6*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{-11+6x}{(-1+2x)\left(-1+x^2\right)}\,dx=-\frac{5}{2}\log(2-2x)+\frac{16}{3}\log(-1+2x)-\frac{17}{6}\log(2+2x)$$

input
```
Integrate[(-11 + 6*x)/((-1 + 2*x)*(-1 + x^2)),x]
```

output
```
(-5*Log[2 - 2*x])/2 + (16*Log[-1 + 2*x])/3 - (17*Log[2 + 2*x])/6
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{6x-11}{(2x-1)(x^2-1)}dx$$

$$\Big\downarrow \text{ 657}$$

$$\int \left(-\frac{17}{6(x+1)}+\frac{32}{3(2x-1)}-\frac{5}{2(x-1)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{16}{3}\log(1-2x)-\frac{5}{2}\log(1-x)-\frac{17}{6}\log(x+1)$$

input
```
Int[(-11 + 6*x)/((-1 + 2*x)*(-1 + x^2)),x]
```

output
```
(16*Log[1 - 2*x])/3 - (5*Log[1 - x])/2 - (17*Log[1 + x])/6
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{16\ln\left(x-\frac{1}{2}\right)}{3}-\frac{5\ln(x-1)}{2}-\frac{17\ln(x+1)}{6}$	20
default	$-\frac{17\ln(x+1)}{6}-\frac{5\ln(x-1)}{2}+\frac{16\ln(2x-1)}{3}$	22
norman	$-\frac{17\ln(x+1)}{6}-\frac{5\ln(x-1)}{2}+\frac{16\ln(2x-1)}{3}$	22
risch	$-\frac{17\ln(x+1)}{6}-\frac{5\ln(x-1)}{2}+\frac{16\ln(2x-1)}{3}$	22

input
```
int((-11+6*x)/(2*x-1)/(x^2-1),x,method=_RETURNVERBOSE)
```

output
```
16/3*ln(x-1/2)-5/2*ln(x-1)-17/6*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{-11+6x}{(-1+2x)(-1+x^2)}\,dx = \frac{16}{3}\log\left(2\,x-1\right)-\frac{17}{6}\log\left(x+1\right)-\frac{5}{2}\log\left(x-1\right)$$

input
```
integrate((-11+6*x)/(-1+2*x)/(x^2-1),x, algorithm="fricas")
```

output
```
16/3*log(2*x - 1) - 17/6*log(x + 1) - 5/2*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{-11+6x}{(-1+2x)(-1+x^2)}\,dx = -\frac{5\log\left(x-1\right)}{2}+\frac{16\log\left(x-\frac{1}{2}\right)}{3}-\frac{17\log\left(x+1\right)}{6}$$

input
```
integrate((-11+6*x)/(-1+2*x)/(x**2-1),x)
```

output
```
-5*log(x - 1)/2 + 16*log(x - 1/2)/3 - 17*log(x + 1)/6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{-11+6x}{(-1+2x)\left(-1+x^2\right)}\,dx = \frac{16}{3}\log\left(2\,x-1\right)-\frac{17}{6}\log\left(x+1\right)-\frac{5}{2}\log\left(x-1\right)$$

input
```
integrate((-11+6*x)/(-1+2*x)/(x^2-1),x, algorithm="maxima")
```

output
```
16/3*log(2*x - 1) - 17/6*log(x + 1) - 5/2*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{-11+6x}{(-1+2x)\left(-1+x^2\right)}\,dx = \frac{16}{3}\log\left(|2\,x-1|\right)-\frac{17}{6}\log\left(|x+1|\right)-\frac{5}{2}\log\left(|x-1|\right)$$

input
```
integrate((-11+6*x)/(-1+2*x)/(x^2-1),x, algorithm="giac")
```

output
```
16/3*log(abs(2*x - 1)) - 17/6*log(abs(x + 1)) - 5/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{-11+6x}{(-1+2x)\left(-1+x^2\right)}\,dx = \frac{16\,\ln\left(x-\frac{1}{2}\right)}{3}-\frac{17\,\ln\left(x+1\right)}{6}-\frac{5\,\ln\left(x-1\right)}{2}$$

input
```
int((6*x - 11)/((2*x - 1)*(x^2 - 1)),x)
```

output
```
(16*log(x - 1/2))/3 - (17*log(x + 1))/6 - (5*log(x - 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{-11+6x}{(-1+2x)\left(-1+x^2\right)}\,dx = \frac{16\log(2x-1)}{3} - \frac{5\log(x-1)}{2} - \frac{17\log(x+1)}{6}$$

input
```
int((-11+6*x)/(-1+2*x)/(x^2-1),x)
```

output
```
(32*log(2*x - 1) - 15*log(x - 1) - 17*log(x + 1))/6
```

3.105 $\int (A+Bx)(d+ex)^{3/2}\left(a+cx^2\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 116

$$\int (A+Bx)(d+ex)^{3/2}\left(a+cx^2\right)\,dx=-\frac{2(Bd-Ae)\left(cd^2+ae^2\right)(d+ex)^{5/2}}{5e^4}+\frac{2\left(3Bcd^2-2Acde+aBe^2\right)(d+ex)^{7/2}}{7e^4}-\frac{2c(3Bd-Ae)(d+ex)^{9/2}}{9e^4}+\frac{2Bc(d+ex)^{11/2}}{11e^4}$$

output
```
-2/5*(-A*e+B*d)*(a*e^2+c*d^2)*(e*x+d)^(5/2)/e^4+2/7*(-2*A*c*d*e+B*a*e^2+3*
B*c*d^2)*(e*x+d)^(7/2)/e^4-2/9*c*(-A*e+3*B*d)*(e*x+d)^(9/2)/e^4+2/11*B*c*(
e*x+d)^(11/2)/e^4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int (A+Bx)(d+ex)^{3/2}\left(a+cx^2\right)\,dx=\frac{2(d+ex)^{5/2}\left(11Ae\left(63ae^2+c\left(8d^2-20dex+35e^2x^2\right)\right)-3B\left(33ae^2(2d-5ex)+c\left(16d^3-40d^2\right.\right.\right.}{3465e^4}$$

input
```
Integrate[(A + B*x)*(d + e*x)^(3/2)*(a + c*x^2),x]
```

output
```
(2*(d + e*x)^(5/2)*(11*A*e*(63*a*e^2 + c*(8*d^2 - 20*d*e*x + 35*e^2*x^2))
- 3*B*(33*a*e^2*(2*d - 5*e*x) + c*(16*d^3 - 40*d^2*e*x + 70*d*e^2*x^2 - 10
5*e^3*x^3))))/(3465*e^4)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(A+Bx)(d+ex)^{3/2}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{(d+ex)^{5/2}\left(aBe^2-2Acde+3Bcd^2\right)}{e^3}+\frac{(d+ex)^{3/2}\left(ae^2+cd^2\right)(Ae-Bd)}{e^3}+\frac{c(d+ex)^{7/2}(Ae-3Bd)}{e^3}+\frac{B\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(d+ex)^{7/2}\left(aBe^2-2Acde+3Bcd^2\right)}{7e^4}-\frac{2(d+ex)^{5/2}\left(ae^2+cd^2\right)(Bd-Ae)}{5e^4}-\frac{2c(d+ex)^{9/2}(3Bd-Ae)}{9e^4}+\frac{2Bc(d+ex)^{11/2}}{11e^4}$$

input
```
Int[(A + B*x)*(d + e*x)^(3/2)*(a + c*x^2),x]
```

output
```
(-2*(B*d - A*e)*(c*d^2 + a*e^2)*(d + e*x)^(5/2))/(5*e^4) + (2*(3*B*c*d^2 -
 2*A*c*d*e + a*B*e^2)*(d + e*x)^(7/2))/(7*e^4) - (2*c*(3*B*d - A*e)*(d + e
*x)^(9/2))/(9*e^4) + (2*B*c*(d + e*x)^(11/2))/(11*e^4)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{2(ex+d)^{\frac{5}{2}}\left(\left(\frac{5\left(\frac{9Bx}{11}+A\right)x^2c}{9}+a\left(\frac{5Bx}{7}+A\right)\right)e^3-\frac{20\left(x\left(\frac{21Bx}{22}+A\right)c+\frac{9Ba}{10}\right)d\,e^2}{63}+\frac{8c\,d^2\left(\frac{15Bx}{11}+A\right)e}{63}-\frac{16Bc\,d^3}{231}\right)}{5e^4}$
gosper	$\frac{2(ex+d)^{\frac{5}{2}}(315Bc\,x^3e^3+385A\,x^2c\,e^3-210B\,x^2cd\,e^2-220Axcd\,e^2+495Bxa\,e^3+120Bxc\,d^2e+693Aa\,e^3+88Ac\,d^2e-19}{3465e^4}$
orering	$\frac{2(ex+d)^{\frac{5}{2}}(315Bc\,x^3e^3+385A\,x^2c\,e^3-210B\,x^2cd\,e^2-220Axcd\,e^2+495Bxa\,e^3+120Bxc\,d^2e+693Aa\,e^3+88Ac\,d^2e-19}{3465e^4}$
derivativedivides	$\frac{\frac{2Bc(ex+d)^{\frac{11}{2}}}{11}+\frac{2((Ae-Bd)c-2Bcd)(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(-2(Ae-Bd)cd+B\left(a\,e^2+c\,d^2\right)\right)(ex+d)^{\frac{7}{2}}}{7}+\frac{2(Ae-Bd)\left(a\,e^2+c\,d^2\right)(ex+d)^{\frac{5}{2}}}{5}}{e^4}$
default	$\frac{\frac{2Bc(ex+d)^{\frac{11}{2}}}{11}+\frac{2((Ae-Bd)c-2Bcd)(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(-2(Ae-Bd)cd+B\left(a\,e^2+c\,d^2\right)\right)(ex+d)^{\frac{7}{2}}}{7}+\frac{2(Ae-Bd)\left(a\,e^2+c\,d^2\right)(ex+d)^{\frac{5}{2}}}{5}}{e^4}$
trager	$2(315Bc\,e^5x^5+385Ac\,e^5x^4+420Bcd\,e^4x^4+550Acd\,e^4x^3+495B\,e^5a\,x^3+15Bc\,d^2e^3x^3+693Aa\,e^5x^2+33Ac\,d^2e^3x^2+79$
risch	$2(315Bc\,e^5x^5+385Ac\,e^5x^4+420Bcd\,e^4x^4+550Acd\,e^4x^3+495B\,e^5a\,x^3+15Bc\,d^2e^3x^3+693Aa\,e^5x^2+33Ac\,d^2e^3x^2+79$

input
```
int((B*x+A)*(e*x+d)^(3/2)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/5*(e*x+d)^(5/2)*((5/9*(9/11*B*x+A)*x^2*c+a*(5/7*B*x+A))*e^3-20/63*(x*(21
/22*B*x+A)*c+9/10*B*a)*d*e^2+8/63*c*d^2*(15/11*B*x+A)*e-16/231*B*c*d^3)/e^
4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.64

$$\int (A+Bx)(d+ex)^{3/2} \left(a+cx^2\right) dx = \frac{2 (315 Bce^5x^5 - 48 Bcd^5 + 88 Acd^4e - 198 Bad^3e^2 + 693 Aad^2e^3 + 35 (12 Bcde^4 + 11 Ace^5)x^4$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)*(c*x^2+a),x, algorithm="fricas")
```

output
```
2/3465*(315*B*c*e^5*x^5 - 48*B*c*d^5 + 88*A*c*d^4*e - 198*B*a*d^3*e^2 + 69
3*A*a*d^2*e^3 + 35*(12*B*c*d*e^4 + 11*A*c*e^5)*x^4 + 5*(3*B*c*d^2*e^3 + 11
0*A*c*d*e^4 + 99*B*a*e^5)*x^3 - 3*(6*B*c*d^3*e^2 - 11*A*c*d^2*e^3 - 264*B*
a*d*e^4 - 231*A*a*e^5)*x^2 + (24*B*c*d^4*e - 44*A*c*d^3*e^2 + 99*B*a*d^2*e
^3 + 1386*A*a*d*e^4)*x)*sqrt(e*x + d)/e^4
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.44

$$\int (A+Bx)(d+ex)^{3/2} \left(a+cx^2\right) dx = \begin{cases} \frac{2\left(\frac{Bc(d+ex)^{\frac{11}{2}}}{11e^3} + \frac{(d+ex)^{\frac{9}{2}}(Ace-3Bcd)}{9e^3} + \frac{(d+ex)^{\frac{7}{2}}\left(-2Acde+Bae^2+3Bcd^2\right)}{7e^3} + \frac{(d+ex)^{\frac{5}{2}}\left(Aae^3+Acd^2e-Bade^2-Bcd^3\right)}{5e^3}\right)}{e} & \text{for } e \neq \\ d^{\frac{3}{2}}\left(Aax + \frac{Acx^3}{3} + \frac{Bax^2}{2} + \frac{Bcx^4}{4}\right) & \text{otherwi} \end{cases}$$

input
```
integrate((B*x+A)*(e*x+d)**(3/2)*(c*x**2+a),x)
```

output
```
Piecewise((2*(B*c*(d + e*x)**(11/2)/(11*e**3) + (d + e*x)**(9/2)*(A*c*e -
3*B*c*d)/(9*e**3) + (d + e*x)**(7/2)*(-2*A*c*d*e + B*a*e**2 + 3*B*c*d**2)/
(7*e**3) + (d + e*x)**(5/2)*(A*a*e**3 + A*c*d**2*e - B*a*d*e**2 - B*c*d**3
)/(5*e**3))/e, Ne(e, 0)), (d**(3/2)*(A*a*x + A*c*x**3/3 + B*a*x**2/2 + B*c
*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int (A+Bx)(d+ex)^{3/2}\left(a+cx^2\right)\,dx = \frac{2\left(315\,(ex+d)^{\frac{11}{2}}Bc-385\,(3\,Bcd-Ace)(ex+d)^{\frac{9}{2}}+495\,(3\,Bcd^2-2\,Acde+Bae^2)(ex+d)^{\frac{7}{2}}\right.}{3465\,e^4}$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)*(c*x^2+a),x, algorithm="maxima")
```

output
```
2/3465*(315*(e*x + d)^(11/2)*B*c - 385*(3*B*c*d - A*c*e)*(e*x + d)^(9/2) +
 495*(3*B*c*d^2 - 2*A*c*d*e + B*a*e^2)*(e*x + d)^(7/2) - 693*(B*c*d^3 - A*
c*d^2*e + B*a*d*e^2 - A*a*e^3)*(e*x + d)^(5/2))/e^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. 2(100) = 200.

Time = 0.14 (sec) , antiderivative size = 548, normalized size of antiderivative = 4.72

$$\int (A+Bx)(d+ex)^{3/2}\left(a+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)*(c*x^2+a),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(e*x + d)*A*a*d^2 + 2310*((e*x + d)^(3/2) - 3*sqrt(e*x +
d)*d)*A*a*d + 1155*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*B*a*d^2/e + 231*(
3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*A*a + 231
*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*A*c*d^2
/e^2 + 462*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^
2)*B*a*d/e + 99*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(
3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*B*c*d^2/e^3 + 198*(5*(e*x + d)^(7/2) - 21
*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*A*c*d/
e^2 + 99*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^
2 - 35*sqrt(e*x + d)*d^3)*B*a/e + 22*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(
7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x
+ d)*d^4)*B*c*d/e^3 + 11*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378
*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*A*
c/e^2 + 5*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/
2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*
x + d)*d^5)*B*c/e^3)/e
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int (A+Bx)(d+ex)^{3/2}\left(a+cx^2\right)\,dx=\frac{(d+e\,x)^{7/2}\,(6\,B\,c\,d^2-4\,A\,c\,d\,e+2\,B\,a\,e^2)}{7\,e^4}+\frac{2\,B\,c\,(d+e\,x)^{11/2}}{11\,e^4}+\frac{2\,c\,(A\,e-3\,B\,d)\,(d+e\,x)^{9/2}}{9\,e^4}+\frac{2\,(c\,d^2+a\,e^2)\,(A\,e-B\,d)\,(d+e\,x)^{5/2}}{5\,e^4}$$

input
```
int((a + c*x^2)*(A + B*x)*(d + e*x)^(3/2),x)
```

output
```
((d + e*x)^(7/2)*(2*B*a*e^2 + 6*B*c*d^2 - 4*A*c*d*e))/(7*e^4) + (2*B*c*(d
+ e*x)^(11/2))/(11*e^4) + (2*c*(A*e - 3*B*d)*(d + e*x)^(9/2))/(9*e^4) + (2
*(a*e^2 + c*d^2)*(A*e - B*d)*(d + e*x)^(5/2))/(5*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.74

$$\int (A+Bx)(d+ex)^{3/2}\left(a+cx^2\right)\,dx = \frac{2\sqrt{ex+d}\,(315bc\,e^5x^5+385ac\,e^5x^4+420bcd\,e^4x^4+495ab\,e^5x^3+550acd\,e^4x^3+15bc\,d^2e^3x^3+$$

input
```
int((B*x+A)*(e*x+d)^(3/2)*(c*x^2+a),x)
```

output
```
(2*sqrt(d + e*x)*(693*a**2*d**2*e**3 + 1386*a**2*d*e**4*x + 693*a**2*e**5*
x**2 - 198*a*b*d**3*e**2 + 99*a*b*d**2*e**3*x + 792*a*b*d*e**4*x**2 + 495*
a*b*e**5*x**3 + 88*a*c*d**4*e - 44*a*c*d**3*e**2*x + 33*a*c*d**2*e**3*x**2
 + 550*a*c*d*e**4*x**3 + 385*a*c*e**5*x**4 - 48*b*c*d**5 + 24*b*c*d**4*e*x
 - 18*b*c*d**3*e**2*x**2 + 15*b*c*d**2*e**3*x**3 + 420*b*c*d*e**4*x**4 + 3
15*b*c*e**5*x**5))/(3465*e**4)
```

3.106 $\int (A+Bx)\sqrt{d+ex}(a+cx^2)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 116

$$\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)\,dx = -\frac{2(Bd-Ae)\left(cd^2+ae^2\right)(d+ex)^{3/2}}{3e^4} + \frac{2\left(3Bcd^2-2Acde+aBe^2\right)(d+ex)^{5/2}}{5e^4} - \frac{2c(3Bd-Ae)(d+ex)^{7/2}}{7e^4} + \frac{2Bc(d+ex)^{9/2}}{9e^4}$$

output

```
-2/3*(-A*e+B*d)*(a*e^2+c*d^2)*(e*x+d)^(3/2)/e^4+2/5*(-2*A*c*d*e+B*a*e^2+3*
B*c*d^2)*(e*x+d)^(5/2)/e^4-2/7*c*(-A*e+3*B*d)*(e*x+d)^(7/2)/e^4+2/9*B*c*(e
*x+d)^(9/2)/e^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)\,dx = \frac{2(d+ex)^{3/2}\left(105aAe^3+21aBe^2(-2d+3ex)+3Ace\left(8d^2-12dex+15e^2x^2\right)+Bc\left(-16d^3+24d^2ex-3\right.\right.}{315e^4}$$

input
```
Integrate[(A + B*x)*Sqrt[d + e*x]*(a + c*x^2),x]
```

output
```
(2*(d + e*x)^(3/2)*(105*a*A*e^3 + 21*a*B*e^2*(-2*d + 3*e*x) + 3*A*c*e*(8*d
^2 - 12*d*e*x + 15*e^2*x^2) + B*c*(-16*d^3 + 24*d^2*e*x - 30*d*e^2*x^2 + 3
5*e^3*x^3)))/(315*e^4)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(A+Bx)\sqrt{d+ex}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{(d+ex)^{3/2}\left(aBe^2-2Acde+3Bcd^2\right)}{e^3}+\frac{\sqrt{d+ex}\left(ae^2+cd^2\right)(Ae-Bd)}{e^3}+\frac{c(d+ex)^{5/2}(Ae-3Bd)}{e^3}+\frac{Bc(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(d+ex)^{5/2}\left(aBe^2-2Acde+3Bcd^2\right)}{5e^4}-\frac{2(d+ex)^{3/2}\left(ae^2+cd^2\right)(Bd-Ae)}{3e^4}-\frac{2c(d+ex)^{7/2}(3Bd-Ae)}{7e^4}+\frac{2Bc(d+ex)^{9/2}}{9e^4}$$

input
```
Int[(A + B*x)*Sqrt[d + e*x]*(a + c*x^2),x]
```

output
```
(-2*(B*d - A*e)*(c*d^2 + a*e^2)*(d + e*x)^(3/2))/(3*e^4) + (2*(3*B*c*d^2 -
 2*A*c*d*e + a*B*e^2)*(d + e*x)^(5/2))/(5*e^4) - (2*c*(3*B*d - A*e)*(d + e
*x)^(7/2))/(7*e^4) + (2*B*c*(d + e*x)^(9/2))/(9*e^4)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{2(ex+d)^{\frac{3}{2}}\left(\left(\left(\frac{3\left(\frac{7Bx}{9}+A\right)x^{2}c}{7}+a\left(\frac{3Bx}{5}+A\right)\right)e^{3}-\frac{12d\left(x\left(\frac{5Bx}{6}+A\right)c+\frac{7Ba}{6}\right)e^{2}}{35}+\frac{8c\,d^{2}(Bx+A)e}{35}-\frac{16Bc\,d^{3}}{105}\right)}{3e^{4}}$
gosper	$\frac{2(ex+d)^{\frac{3}{2}}(35Bc\,x^{3}e^{3}+45A\,x^{2}c\,e^{3}-30B\,x^{2}cd\,e^{2}-36Axcd\,e^{2}+63Bxa\,e^{3}+24Bxc\,d^{2}e+105Aa\,e^{3}+24Ac\,d^{2}e-42Bad\,e^{2}}{315e^{4}}$
orering	$\frac{2(ex+d)^{\frac{3}{2}}(35Bc\,x^{3}e^{3}+45A\,x^{2}c\,e^{3}-30B\,x^{2}cd\,e^{2}-36Axcd\,e^{2}+63Bxa\,e^{3}+24Bxc\,d^{2}e+105Aa\,e^{3}+24Ac\,d^{2}e-42Bad\,e^{2}}{315e^{4}}$
derivativedivides	$\frac{\frac{2Bc(ex+d)^{\frac{9}{2}}}{9}+\frac{2((Ae-Bd)c-2Bcd)(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(-2(Ae-Bd)cd+B\left(a\,e^{2}+c\,d^{2}\right)\right)(ex+d)^{\frac{5}{2}}}{5}+\frac{2(Ae-Bd)\left(a\,e^{2}+c\,d^{2}\right)(ex+d)^{\frac{3}{2}}}{3}}{e^{4}}$
default	$\frac{\frac{2Bc(ex+d)^{\frac{9}{2}}}{9}+\frac{2((Ae-Bd)c-2Bcd)(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(-2(Ae-Bd)cd+B\left(a\,e^{2}+c\,d^{2}\right)\right)(ex+d)^{\frac{5}{2}}}{5}+\frac{2(Ae-Bd)\left(a\,e^{2}+c\,d^{2}\right)(ex+d)^{\frac{3}{2}}}{3}}{e^{4}}$
trager	$\frac{2(35Bc\,e^{4}x^{4}+45Ac\,e^{4}x^{3}+5Bcd\,e^{3}x^{3}+9Acd\,e^{3}x^{2}+63Ba\,e^{4}x^{2}-6Bc\,d^{2}e^{2}x^{2}+105Aa\,e^{4}x-12Ac\,d^{2}e^{2}x+21Bad\,e^{3}x+8}{315e^{4}}$
risch	$\frac{2(35Bc\,e^{4}x^{4}+45Ac\,e^{4}x^{3}+5Bcd\,e^{3}x^{3}+9Acd\,e^{3}x^{2}+63Ba\,e^{4}x^{2}-6Bc\,d^{2}e^{2}x^{2}+105Aa\,e^{4}x-12Ac\,d^{2}e^{2}x+21Bad\,e^{3}x+8}{315e^{4}}$

input
```
int((B*x+A)*(e*x+d)^(1/2)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*(e*x+d)^(3/2)*((3/7*(7/9*B*x+A)*x^2*c+a*(3/5*B*x+A))*e^3-12/35*d*(x*(5
/6*B*x+A)*c+7/6*B*a)*e^2+8/35*c*d^2*(B*x+A)*e-16/105*B*c*d^3)/e^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.23

$$\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)\,dx$$
$$=\frac{2\,(35\,Bce^4x^4-16\,Bcd^4+24\,Acd^3e-42\,Bad^2e^2+105\,Aade^3+5\,(Bcde^3+9\,Ace^4)x^3-3\,(2\,Bcd^2e^2-3\ldots}{315\,e^4}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)*(c*x^2+a),x, algorithm="fricas")
```

output
```
2/315*(35*B*c*e^4*x^4 - 16*B*c*d^4 + 24*A*c*d^3*e - 42*B*a*d^2*e^2 + 105*A
*a*d*e^3 + 5*(B*c*d*e^3 + 9*A*c*e^4)*x^3 - 3*(2*B*c*d^2*e^2 - 3*A*c*d*e^3
- 21*B*a*e^4)*x^2 + (8*B*c*d^3*e - 12*A*c*d^2*e^2 + 21*B*a*d*e^3 + 105*A*a
*e^4)*x)*sqrt(e*x + d)/e^4
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.44

$$\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)\,dx$$
$$=\begin{cases}\frac{2\left(\frac{Bc(d+ex)^{\frac{9}{2}}}{9e^3}+\frac{(d+ex)^{\frac{7}{2}}(Ace-3Bcd)}{7e^3}+\frac{(d+ex)^{\frac{5}{2}}\left(-2Acde+Bae^2+3Bcd^2\right)}{5e^3}+\frac{(d+ex)^{\frac{3}{2}}\left(Aae^3+Acd^2e-Bade^2-Bcd^3\right)}{3e^3}\right)}{e} & \text{for } e\neq 0\\ \sqrt{d}\left(Aax+\frac{Acx^3}{3}+\frac{Bax^2}{2}+\frac{Bcx^4}{4}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((B*x+A)*(e*x+d)**(1/2)*(c*x**2+a),x)
```

output
```
Piecewise((2*(B*c*(d + e*x)**(9/2)/(9*e**3) + (d + e*x)**(7/2)*(A*c*e - 3*
B*c*d)/(7*e**3) + (d + e*x)**(5/2)*(-2*A*c*d*e + B*a*e**2 + 3*B*c*d**2)/(5
*e**3) + (d + e*x)**(3/2)*(A*a*e**3 + A*c*d**2*e - B*a*d*e**2 - B*c*d**3)/
(3*e**3))/e, Ne(e, 0)), (sqrt(d)*(A*a*x + A*c*x**3/3 + B*a*x**2/2 + B*c*x*
*4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)\,dx$$
$$=\frac{2\left(35\,(ex+d)^{\frac{9}{2}}Bc-45\,(3\,Bcd-Ace)(ex+d)^{\frac{7}{2}}+63\,(3\,Bcd^2-2\,Acde+Bae^2)(ex+d)^{\frac{5}{2}}-105\,(Bcd^3\right.}{315\,e^4}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)*(c*x^2+a),x, algorithm="maxima")
```

output
```
2/315*(35*(e*x + d)^(9/2)*B*c - 45*(3*B*c*d - A*c*e)*(e*x + d)^(7/2) + 63*
(3*B*c*d^2 - 2*A*c*d*e + B*a*e^2)*(e*x + d)^(5/2) - 105*(B*c*d^3 - A*c*d^2
*e + B*a*d*e^2 - A*a*e^3)*(e*x + d)^(3/2))/e^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(100) = 200.

Time = 0.13 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.67

$$\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)\,dx$$
$$=\frac{2\left(315\,\sqrt{ex+d}Aad+105\left((ex+d)^{\frac{3}{2}}-3\,\sqrt{ex+d}d\right)Aa+\frac{105\left((ex+d)^{\frac{3}{2}}-3\,\sqrt{ex+d}d\right)Bad}{e}+\frac{21\left(3\,(ex+d)^{\frac{5}{2}}-10\,(ex\right.}{}\right.}{}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)*(c*x^2+a),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(e*x + d)*A*a*d + 105*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)
*A*a + 105*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*B*a*d/e + 21*(3*(e*x + d)
^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*A*c*d/e^2 + 21*(3*(e
*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*B*a/e + 9*(5*
(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(
e*x + d)*d^3)*B*c*d/e^3 + 9*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35
*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*A*c/e^2 + (35*(e*x + d)^(9/2)
 - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d
^3 + 315*sqrt(e*x + d)*d^4)*B*c/e^3)/e
```

Mupad [B] (verification not implemented)

Time = 6.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)\,dx = \frac{(d+e\,x)^{5/2}\,\left(6\,B\,c\,d^2-4\,A\,c\,d\,e+2\,B\,a\,e^2\right)}{5\,e^4} + \frac{2\,B\,c\,(d+e\,x)^{9/2}}{9\,e^4} + \frac{2\,c\,(A\,e-3\,B\,d)\,\left(d+e\,x\right)^{7/2}}{7\,e^4} + \frac{2\,\left(c\,d^2+a\,e^2\right)\,\left(A\,e-B\,d\right)\,\left(d+e\,x\right)^{3/2}}{3\,e^4}$$

input
```
int((a + c*x^2)*(A + B*x)*(d + e*x)^(1/2),x)
```

output
```
((d + e*x)^(5/2)*(2*B*a*e^2 + 6*B*c*d^2 - 4*A*c*d*e))/(5*e^4) + (2*B*c*(d
+ e*x)^(9/2))/(9*e^4) + (2*c*(A*e - 3*B*d)*(d + e*x)^(7/2))/(7*e^4) + (2*(
a*e^2 + c*d^2)*(A*e - B*d)*(d + e*x)^(3/2))/(3*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.28

$$\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)\,dx = \frac{2\sqrt{ex+d}\,(35bc\,e^4x^4+45ac\,e^4x^3+5bcd\,e^3x^3+63ab\,e^4x^2+9acd\,e^3x^2-6bc\,d^2e^2x^2+105a^2e^4x+21abd}{315e^4}$$

input
```
int((B*x+A)*(e*x+d)^(1/2)*(c*x^2+a),x)
```

output
```
(2*sqrt(d + e*x)*(105*a**2*d*e**3 + 105*a**2*e**4*x - 42*a*b*d**2*e**2 + 2
1*a*b*d*e**3*x + 63*a*b*e**4*x**2 + 24*a*c*d**3*e - 12*a*c*d**2*e**2*x + 9
*a*c*d*e**3*x**2 + 45*a*c*e**4*x**3 - 16*b*c*d**4 + 8*b*c*d**3*e*x - 6*b*c
*d**2*e**2*x**2 + 5*b*c*d*e**3*x**3 + 35*b*c*e**4*x**4))/(315*e**4)
```

3.107 $\int \frac{(A+Bx)(a+cx^2)}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 114

$$\int \frac{(A+Bx)(a+cx^2)}{\sqrt{d+ex}}\,dx = -\frac{2(Bd-Ae)(cd^2+ae^2)\sqrt{d+ex}}{e^4} + \frac{2(3Bcd^2-2Acde+aBe^2)(d+ex)^{3/2}}{3e^4} - \frac{2c(3Bd-Ae)(d+ex)^{5/2}}{5e^4} + \frac{2Bc(d+ex)^{7/2}}{7e^4}$$

output
```
-2*(-A*e+B*d)*(a*e^2+c*d^2)*(e*x+d)^(1/2)/e^4+2/3*(-2*A*c*d*e+B*a*e^2+3*B*
c*d^2)*(e*x+d)^(3/2)/e^4-2/5*c*(-A*e+3*B*d)*(e*x+d)^(5/2)/e^4+2/7*B*c*(e*x
+d)^(7/2)/e^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)(a+cx^2)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}(105aAe^3+35aBe^2(-2d+ex)+7Ace(8d^2-4dex+3e^2x^2)-3Bc(16d^3-8d^2ex+6de^2x^2-}{105e^4}$$

input
```
Integrate[((A + B*x)*(a + c*x^2))/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(105*a*A*e^3 + 35*a*B*e^2*(-2*d + e*x) + 7*A*c*e*(8*d^2 -
 4*d*e*x + 3*e^2*x^2) - 3*B*c*(16*d^3 - 8*d^2*e*x + 6*d*e^2*x^2 - 5*e^3*x^
3)))/(105*e^4)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(A+Bx)}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{\sqrt{d+ex}(aBe^2-2Acde+3Bcd^2)}{e^3}+\frac{(ae^2+cd^2)(Ae-Bd)}{e^3\sqrt{d+ex}}+\frac{c(d+ex)^{3/2}(Ae-3Bd)}{e^3}+\frac{Bc(d+ex)^{5/2}}{e^3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(d+ex)^{3/2}(aBe^2-2Acde+3Bcd^2)}{3e^4}-\frac{2\sqrt{d+ex}(ae^2+cd^2)(Bd-Ae)}{e^4}-\frac{2c(d+ex)^{5/2}(3Bd-Ae)}{5e^4}+\frac{2Bc(d+ex)^{7/2}}{7e^4}$$

input
```
Int[((A + B*x)*(a + c*x^2))/Sqrt[d + e*x],x]
```

output
```
(-2*(B*d - A*e)*(c*d^2 + a*e^2)*Sqrt[d + e*x])/e^4 + (2*(3*B*c*d^2 - 2*A*c
*d*e + a*B*e^2)*(d + e*x)^(3/2))/(3*e^4) - (2*c*(3*B*d - A*e)*(d + e*x)^(5
/2))/(5*e^4) + (2*B*c*(d + e*x)^(7/2))/(7*e^4)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{2\sqrt{ex+d}\left(\left(\frac{x^2\left(\frac{5Bx}{7}+A\right)c}{5}+a\left(\frac{Bx}{3}+A\right)\right)e^3-\frac{4d\left(x\left(\frac{9Bx}{14}+A\right)c+\frac{5Ba}{2}\right)e^2}{15}+\frac{8c\,d^2\left(\frac{3Bx}{7}+A\right)e}{15}-\frac{16Bc\,d^3}{35}\right)}{e^4}$
gosper	$\frac{2\sqrt{ex+d}\,(15Bc\,x^3e^3+21A\,x^2c\,e^3-18B\,x^2cd\,e^2-28Axcd\,e^2+35Bxa\,e^3+24Bxc\,d^2e+105Aa\,e^3+56Ac\,d^2e-70Bad\,e^2-}{105e^4}$
trager	$\frac{2\sqrt{ex+d}\,(15Bc\,x^3e^3+21A\,x^2c\,e^3-18B\,x^2cd\,e^2-28Axcd\,e^2+35Bxa\,e^3+24Bxc\,d^2e+105Aa\,e^3+56Ac\,d^2e-70Bad\,e^2-}{105e^4}$
risch	$\frac{2\sqrt{ex+d}\,(15Bc\,x^3e^3+21A\,x^2c\,e^3-18B\,x^2cd\,e^2-28Axcd\,e^2+35Bxa\,e^3+24Bxc\,d^2e+105Aa\,e^3+56Ac\,d^2e-70Bad\,e^2-}{105e^4}$
orering	$\frac{2\sqrt{ex+d}\,(15Bc\,x^3e^3+21A\,x^2c\,e^3-18B\,x^2cd\,e^2-28Axcd\,e^2+35Bxa\,e^3+24Bxc\,d^2e+105Aa\,e^3+56Ac\,d^2e-70Bad\,e^2-}{105e^4}$
derivativedivides	$\frac{\frac{2Bc(ex+d)^{\frac{7}{2}}}{7}+\frac{2((Ae-Bd)c-2Bcd)(ex+d)^{\frac{5}{2}}}{5}+\frac{2\left(-2(Ae-Bd)cd+B\left(a\,e^2+c\,d^2\right)\right)(ex+d)^{\frac{3}{2}}}{3}+2(Ae-Bd)\left(a\,e^2+c\,d^2\right)\sqrt{ex+d}}{e^4}$
default	$\frac{\frac{2Bc(ex+d)^{\frac{7}{2}}}{7}+\frac{2((Ae-Bd)c-2Bcd)(ex+d)^{\frac{5}{2}}}{5}+\frac{2\left(-2(Ae-Bd)cd+B\left(a\,e^2+c\,d^2\right)\right)(ex+d)^{\frac{3}{2}}}{3}+2(Ae-Bd)\left(a\,e^2+c\,d^2\right)\sqrt{ex+d}}{e^4}$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(e*x+d)^(1/2)*((1/5*x^2*(5/7*B*x+A)*c+a*(1/3*B*x+A))*e^3-4/15*d*(x*(9/14
*B*x+A)*c+5/2*B*a)*e^2+8/15*c*d^2*(3/7*B*x+A)*e-16/35*B*c*d^3)/e^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)(a+cx^2)}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\left(15\,Bce^3x^3 - 48\,Bcd^3 + 56\,Acd^2e - 70\,Bade^2 + 105\,Aae^3 - 3\,(6\,Bcde^2 - 7\,Ace^3)x^2 + (24\,Bcd^2e - 28\right.}{105\,e^4}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*B*c*e^3*x^3 - 48*B*c*d^3 + 56*A*c*d^2*e - 70*B*a*d*e^2 + 105*A*a
*e^3 - 3*(6*B*c*d*e^2 - 7*A*c*e^3)*x^2 + (24*B*c*d^2*e - 28*A*c*d*e^2 + 35
*B*a*e^3)*x)*sqrt(e*x + d)/e^4
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.45

$$\int \frac{(A+Bx)(a+cx^2)}{\sqrt{d+ex}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{Bc(d+ex)^{\frac{7}{2}}}{7e^3} + \frac{(d+ex)^{\frac{5}{2}}(Ace-3Bcd)}{5e^3} + \frac{(d+ex)^{\frac{3}{2}}\left(-2Acde+Bae^2+3Bcd^2\right)}{3e^3} + \frac{\sqrt{d+ex}\left(Aae^3+Acd^2e-Bade^2-Bcd^3\right)}{e^3}\right)}{e} & \text{for } e \neq 0 \\ \frac{Aax+\frac{Acx^3}{3}+\frac{Bax^2}{2}+\frac{Bcx^4}{4}}{\sqrt{d}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)*(c*x**2+a)/(e*x+d)**(1/2),x)
```

output
```
Piecewise((2*(B*c*(d + e*x)**(7/2)/(7*e**3) + (d + e*x)**(5/2)*(A*c*e - 3*
B*c*d)/(5*e**3) + (d + e*x)**(3/2)*(-2*A*c*d*e + B*a*e**2 + 3*B*c*d**2)/(3
*e**3) + sqrt(d + e*x)*(A*a*e**3 + A*c*d**2*e - B*a*d*e**2 - B*c*d**3)/e**
3)/e, Ne(e, 0)), ((A*a*x + A*c*x**3/3 + B*a*x**2/2 + B*c*x**4/4)/sqrt(d),
True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)(a+cx^2)}{\sqrt{d+ex}}\,dx = \frac{2\left(15\,(ex+d)^{\frac{7}{2}}Bc - 21\,(3\,Bcd - Ace)(ex+d)^{\frac{5}{2}} + 35\,(3\,Bcd^2 - 2\,Acde + Bae^2)(ex+d)^{\frac{3}{2}} - 105\,(Bcd^3\right.}{105\,e^4}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
2/105*(15*(e*x + d)^(7/2)*B*c - 21*(3*B*c*d - A*c*e)*(e*x + d)^(5/2) + 35*
(3*B*c*d^2 - 2*A*c*d*e + B*a*e^2)*(e*x + d)^(3/2) - 105*(B*c*d^3 - A*c*d^2
*e + B*a*d*e^2 - A*a*e^3)*sqrt(e*x + d))/e^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx)(a+cx^2)}{\sqrt{d+ex}}\,dx = \frac{2\left(105\sqrt{ex+d}Aa + \frac{35\left((ex+d)^{\frac{3}{2}}-3\sqrt{ex+d}d\right)Ba}{e} + \frac{7\left(3\,(ex+d)^{\frac{5}{2}}-10\,(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}d^2\right)Ac}{e^2} + \frac{3\left(5\,(ex+d)^{\frac{7}{2}}-21\,(ex+d)^{\frac{5}{2}}\right.}{}\right.}{105\,e}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
2/105*(105*sqrt(e*x + d)*A*a + 35*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*B*
a/e + 7*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*
A*c/e^2 + 3*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)
*d^2 - 35*sqrt(e*x + d)*d^3)*B*c/e^3)/e
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\,(a+cx^2)}{\sqrt{d+ex}}\,dx = \frac{(d+e\,x)^{3/2}\,(6\,B\,c\,d^2-4\,A\,c\,d\,e+2\,B\,a\,e^2)}{3\,e^4} + \frac{2\,B\,c\,(d+e\,x)^{7/2}}{7\,e^4} + \frac{2\,c\,(A\,e-3\,B\,d)\,(d+e\,x)^{5/2}}{5\,e^4} + \frac{2\,(c\,d^2+a\,e^2)\,(A\,e-B\,d)\,\sqrt{d+e\,x}}{e^4}$$

input
```
int(((a + c*x^2)*(A + B*x))/(d + e*x)^(1/2),x)
```

output
```
((d + e*x)^(3/2)*(2*B*a*e^2 + 6*B*c*d^2 - 4*A*c*d*e))/(3*e^4) + (2*B*c*(d
+ e*x)^(7/2))/(7*e^4) + (2*c*(A*e - 3*B*d)*(d + e*x)^(5/2))/(5*e^4) + (2*(
a*e^2 + c*d^2)*(A*e - B*d)*(d + e*x)^(1/2))/e^4
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\,(a+cx^2)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{ex+d}\,(15bc\,e^3x^3+21ac\,e^3x^2-18bcd\,e^2x^2+35ab\,e^3x-28acd\,e^2x+24bc\,d^2ex+105a^2e^3-70abd\,e^2}{105e^4}$$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(d + e*x)*(105*a**2*e**3 - 70*a*b*d*e**2 + 35*a*b*e**3*x + 56*a*c*d
**2*e - 28*a*c*d*e**2*x + 21*a*c*e**3*x**2 - 48*b*c*d**3 + 24*b*c*d**2*e*x
 - 18*b*c*d*e**2*x**2 + 15*b*c*e**3*x**3))/(105*e**4)
```

3.108 $\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^{3/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{3/2}} \, dx = \frac{2(Bd-Ae)\,(cd^2+ae^2)}{e^4\sqrt{d+ex}} + \frac{2(3Bcd^2-2Acde+aBe^2)\sqrt{d+ex}}{e^4} - \frac{2c(3Bd-Ae)(d+ex)^{3/2}}{3e^4} + \frac{2Bc(d+ex)^{5/2}}{5e^4}$$

output
```
2*(-A*e+B*d)*(a*e^2+c*d^2)/e^4/(e*x+d)^(1/2)+2*(-2*A*c*d*e+B*a*e^2+3*B*c*d
^2)*(e*x+d)^(1/2)/e^4-2/3*c*(-A*e+3*B*d)*(e*x+d)^(3/2)/e^4+2/5*B*c*(e*x+d)
^(5/2)/e^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{3/2}} \, dx = \frac{-10Ae(3ae^2+c(8d^2+4dex-e^2x^2))+6B(5ae^2(2d+ex)+c(16d^3+8d^2ex}{15e^4\sqrt{d+ex}}$$

input
```
Integrate[((A + B*x)*(a + c*x^2))/(d + e*x)^(3/2),x]
```

output
```
(-10*A*e*(3*a*e^2 + c*(8*d^2 + 4*d*e*x - e^2*x^2)) + 6*B*(5*a*e^2*(2*d + e
*x) + c*(16*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3)))/(15*e^4*Sqrt[d + e*
x])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(A+Bx)}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{aBe^2-2Acde+3Bcd^2}{e^3\sqrt{d+ex}}+\frac{(ae^2+cd^2)(Ae-Bd)}{e^3(d+ex)^{3/2}}+\frac{c\sqrt{d+ex}(Ae-3Bd)}{e^3}+\frac{Bc(d+ex)^{3/2}}{e^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{d+ex}(aBe^2-2Acde+3Bcd^2)}{e^4}+\frac{2(ae^2+cd^2)(Bd-Ae)}{e^4\sqrt{d+ex}}-\frac{2c(d+ex)^{3/2}(3Bd-Ae)}{3e^4}+\frac{2Bc(d+ex)^{5/2}}{5e^4}$$

input
```
Int[((A + B*x)*(a + c*x^2))/(d + e*x)^(3/2),x]
```

output
```
(2*(B*d - A*e)*(c*d^2 + a*e^2))/(e^4*Sqrt[d + e*x]) + (2*(3*B*c*d^2 - 2*A*
c*d*e + a*B*e^2)*Sqrt[d + e*x])/e^4 - (2*c*(3*B*d - A*e)*(d + e*x)^(3/2))/
(3*e^4) + (2*B*c*(d + e*x)^(5/2))/(5*e^4)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{((6Bx^3+10Ax^2)c-30a(-Bx+A))e^3-40d\left(x\left(\frac{3Bx}{10}+A\right)c-\frac{3Ba}{2}\right)e^2-80cd^2\left(-\frac{3Bx}{5}+A\right)e+96Bcd^3}{15\sqrt{ex+d}\,e^4}$
gosper	$-\frac{2(-3Bcx^3e^3-5Ax^2ce^3+6Bx^2cde^2+20Axcde^2-15Bxae^3-24Bxcd^2e+15Aae^3+40Acd^2e-30Bade^2-48Bcd^3}{15\sqrt{ex+d}\,e^4}$
trager	$-\frac{2(-3Bcx^3e^3-5Ax^2ce^3+6Bx^2cde^2+20Axcde^2-15Bxae^3-24Bxcd^2e+15Aae^3+40Acd^2e-30Bade^2-48Bcd^3}{15\sqrt{ex+d}\,e^4}$
risch	$-\frac{2(-3e^2Bcx^2-5Ace^2x+9Bcdex+25Acde-15Bae^2-33Bcd^2)\sqrt{ex+d}}{15e^4}-\frac{2(Aae^3+Acd^2e-Bade^2-Bcd^3)}{e^4\sqrt{ex+d}}$
orering	$-\frac{2(-3Bcx^3e^3-5Ax^2ce^3+6Bx^2cde^2+20Axcde^2-15Bxae^3-24Bxcd^2e+15Aae^3+40Acd^2e-30Bade^2-48Bcd^3}{15\sqrt{ex+d}\,e^4}$
derivativedivides	$\frac{\frac{2Bc(ex+d)^{\frac{5}{2}}}{5}+\frac{2Ace(ex+d)^{\frac{3}{2}}}{3}-2Bcd(ex+d)^{\frac{3}{2}}-4Acde\sqrt{ex+d}+2Bae^2\sqrt{ex+d}+6Bcd^2\sqrt{ex+d}-\frac{2(Aae^3+Acd^2e-Bade^2}{\sqrt{ex+d}}}{e^4}$
default	$\frac{\frac{2Bc(ex+d)^{\frac{5}{2}}}{5}+\frac{2Ace(ex+d)^{\frac{3}{2}}}{3}-2Bcd(ex+d)^{\frac{3}{2}}-4Acde\sqrt{ex+d}+2Bae^2\sqrt{ex+d}+6Bcd^2\sqrt{ex+d}-\frac{2(Aae^3+Acd^2e-Bade^2}{\sqrt{ex+d}}}{e^4}$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(((6*B*x^3+10*A*x^2)*c-30*a*(-B*x+A))*e^3-40*d*(x*(3/10*B*x+A)*c-3/2*
B*a)*e^2-80*c*d^2*(-3/5*B*x+A)*e+96*B*c*d^3)/(e*x+d)^(1/2)/e^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+cx^2\right)}{(d+ex)^{3/2}}\,dx = \frac{2\left(3\,Bce^3x^3+48\,Bcd^3-40\,Acd^2e+30\,Bade^2-15\,Aae^3-\left(6\,Bcde^2-5\,Ace^3\right.\right.}{15\left(e^5x+de^4\right)}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*c*e^3*x^3 + 48*B*c*d^3 - 40*A*c*d^2*e + 30*B*a*d*e^2 - 15*A*a*e^
3 - (6*B*c*d*e^2 - 5*A*c*e^3)*x^2 + (24*B*c*d^2*e - 20*A*c*d*e^2 + 15*B*a*
e^3)*x)*sqrt(e*x + d)/(e^5*x + d*e^4)
```

Sympy [A] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)\left(a+cx^2\right)}{(d+ex)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{Bc(d+ex)^{\frac{5}{2}}}{5e^3}+\frac{(d+ex)^{\frac{3}{2}}(Ace-3Bcd)}{3e^3}+\frac{\sqrt{d+ex}\left(-2Acde+Bae^2+3Bcd^2\right)}{e^3}+\frac{(-Ae+Bd)\left(ae^2+cd^2\right)}{e^3\sqrt{d+ex}}\right)}{e} & \text{for } e \\ \frac{Aax+\frac{Acx^3}{3}+\frac{Bax^2}{2}+\frac{Bcx^4}{4}}{d^{\frac{3}{2}}} & \text{other} \end{cases}$$

input
```
integrate((B*x+A)*(c*x**2+a)/(e*x+d)**(3/2),x)
```

output
```
Piecewise((2*(B*c*(d + e*x)**(5/2)/(5*e**3) + (d + e*x)**(3/2)*(A*c*e - 3*
B*c*d)/(3*e**3) + sqrt(d + e*x)*(-2*A*c*d*e + B*a*e**2 + 3*B*c*d**2)/e**3
+ (-A*e + B*d)*(a*e**2 + c*d**2)/(e**3*sqrt(d + e*x)))/e, Ne(e, 0)), ((A*a
*x + A*c*x**3/3 + B*a*x**2/2 + B*c*x**4/4)/d**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{3\,(ex+d)^{\frac{5}{2}}Bc-5\,(3\,Bcd-Ace)(ex+d)^{\frac{3}{2}}+15\,(3\,Bcd^2-2\,Acde+Bae^2)\sqrt{ex+d}}{e^3}+\frac{15\,(Bcd^3-Acd^2e+B}{\sqrt{ex+d}e^3}\right.}{15\,e}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
2/15*((3*(e*x + d)^(5/2)*B*c - 5*(3*B*c*d - A*c*e)*(e*x + d)^(3/2) + 15*(3
*B*c*d^2 - 2*A*c*d*e + B*a*e^2)*sqrt(e*x + d))/e^3 + 15*(B*c*d^3 - A*c*d^2
*e + B*a*d*e^2 - A*a*e^3)/(sqrt(e*x + d)*e^3))/e
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^{3/2}}\,dx = \frac{2\,(Bcd^3-Acd^2e+Bade^2-Aae^3)}{\sqrt{ex+d}e^4}$$
$$+\frac{2\left(3\,(ex+d)^{\frac{5}{2}}Bce^{16}-15\,(ex+d)^{\frac{3}{2}}Bcde^{16}+45\,\sqrt{ex+d}Bcd^2e^{16}+5\,(ex+d)^{\frac{3}{2}}Ace^{17}-30\,\sqrt{ex+d}Acde\right.}{15\,e^{20}}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
2*(B*c*d^3 - A*c*d^2*e + B*a*d*e^2 - A*a*e^3)/(sqrt(e*x + d)*e^4) + 2/15*(
3*(e*x + d)^(5/2)*B*c*e^16 - 15*(e*x + d)^(3/2)*B*c*d*e^16 + 45*sqrt(e*x +
 d)*B*c*d^2*e^16 + 5*(e*x + d)^(3/2)*A*c*e^17 - 30*sqrt(e*x + d)*A*c*d*e^1
7 + 15*sqrt(e*x + d)*B*a*e^18)/e^20
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{3/2}}\,dx = \frac{\sqrt{d+e\,x}\,\left(6\,B\,c\,d^2-4\,A\,c\,d\,e+2\,B\,a\,e^2\right)}{e^4} - \frac{-2\,B\,c\,d^3+2\,A\,c\,d^2\,e-2\,B\,a\,d\,e^2+2\,A\,a\,e^3}{e^4\,\sqrt{d+e\,x}} + \frac{2\,B\,c\,{(d+e\,x)}^{5/2}}{5\,e^4} + \frac{2\,c\,\left(A\,e-3\,B\,d\right)\,{(d+e\,x)}^{3/2}}{3\,e^4}$$

input
```
int(((a + c*x^2)*(A + B*x))/(d + e*x)^(3/2),x)
```

output
```
((d + e*x)^(1/2)*(2*B*a*e^2 + 6*B*c*d^2 - 4*A*c*d*e))/e^4 - (2*A*a*e^3 - 2
*B*c*d^3 - 2*B*a*d*e^2 + 2*A*c*d^2*e)/(e^4*(d + e*x)^(1/2)) + (2*B*c*(d +
e*x)^(5/2))/(5*e^4) + (2*c*(A*e - 3*B*d)*(d + e*x)^(3/2))/(3*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{3/2}}\,dx = \frac{\frac{2}{5}bc\,e^3x^3+\frac{2}{3}ac\,e^3x^2-\frac{4}{5}bcd\,e^2x^2+2ab\,e^3x-\frac{8}{3}acd\,e^2x+\frac{16}{5}bc\,d^2ex-2a^2e^3+4}{\sqrt{ex+d}\,e^4}$$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^(3/2),x)
```

output
```
(2*( - 15*a**2*e**3 + 30*a*b*d*e**2 + 15*a*b*e**3*x - 40*a*c*d**2*e - 20*a
*c*d*e**2*x + 5*a*c*e**3*x**2 + 48*b*c*d**3 + 24*b*c*d**2*e*x - 6*b*c*d*e*
*2*x**2 + 3*b*c*e**3*x**3))/(15*sqrt(d + e*x)*e**4)
```

3.109 $\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{(A+Bx)\left(a+cx^2\right)}{(d+ex)^{5/2}} \, dx = \frac{2(Bd-Ae)\left(cd^2+ae^2\right)}{3e^4(d+ex)^{3/2}} - \frac{2(3Bcd^2-2Acde+aBe^2)}{e^4\sqrt{d+ex}} - \frac{2c(3Bd-Ae)\sqrt{d+ex}}{e^4} + \frac{2Bc(d+ex)^{3/2}}{3e^4}$$

output
```
2/3*(-A*e+B*d)*(a*e^2+c*d^2)/e^4/(e*x+d)^(3/2)-2*(-2*A*c*d*e+B*a*e^2+3*B*c
*d^2)/e^4/(e*x+d)^(1/2)-2*c*(-A*e+3*B*d)*(e*x+d)^(1/2)/e^4+2/3*B*c*(e*x+d)
^(3/2)/e^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(a+cx^2\right)}{(d+ex)^{5/2}} \, dx = -\frac{2(aAe^3+aBe^2(2d+3ex)-Ace(8d^2+12dex+3e^2x^2)+Bc(16d^3+24d^2ex+6de^2x^2-e^3x^3))}{3e^4(d+ex)^{3/2}}$$

input
```
Integrate[((A + B*x)*(a + c*x^2))/(d + e*x)^(5/2),x]
```

output
```
(-2*(a*A*e^3 + a*B*e^2*(2*d + 3*e*x) - A*c*e*(8*d^2 + 12*d*e*x + 3*e^2*x^2
) + B*c*(16*d^3 + 24*d^2*e*x + 6*d*e^2*x^2 - e^3*x^3)))/(3*e^4*(d + e*x)^(
3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(A+Bx)}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{aBe^2-2Acde+3Bcd^2}{e^3(d+ex)^{3/2}}+\frac{(ae^2+cd^2)(Ae-Bd)}{e^3(d+ex)^{5/2}}+\frac{c(Ae-3Bd)}{e^3\sqrt{d+ex}}+\frac{Bc\sqrt{d+ex}}{e^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(aBe^2-2Acde+3Bcd^2)}{e^4\sqrt{d+ex}}+\frac{2(ae^2+cd^2)(Bd-Ae)}{3e^4(d+ex)^{3/2}}-\frac{2c\sqrt{d+ex}(3Bd-Ae)}{e^4}+\frac{2Bc(d+ex)^{3/2}}{3e^4}$$

input
```
Int[((A + B*x)*(a + c*x^2))/(d + e*x)^(5/2),x]
```

output
```
(2*(B*d - A*e)*(c*d^2 + a*e^2))/(3*e^4*(d + e*x)^(3/2)) - (2*(3*B*c*d^2 -
2*A*c*d*e + a*B*e^2))/(e^4*Sqrt[d + e*x]) - (2*c*(3*B*d - A*e)*Sqrt[d + e*
x])/e^4 + (2*B*c*(d + e*x)^(3/2))/(3*e^4)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73

method	result	si
pseudoelliptic	$-\frac{2\left(\left(\left(-B x^3-3 A x^2\right) c+(3 B x+A) a\right) e^3-12 d\left(x\left(-\frac{B x}{2}+A\right) c-\frac{B a}{6}\right) e^2-8 c d^2(-3 B x+A) e+16 B c d^3\right)}{3(e x+d)^{\frac{3}{2}} e^4}$	82
risch	$\frac{2 c(B e x+3 A e-8 B d) \sqrt{e x+d}}{3 e^4}-\frac{2\left(-6 A x c d e^2+3 B x a e^3+9 B x c d^2 e+A a e^3-5 A c d^2 e+2 B a d e^2+8 B c d^3\right)}{3 e^4(e x+d)^{\frac{3}{2}}}$	9
gosper	$-\frac{2\left(-B c x^3 e^3-3 A x^2 c e^3+6 B x^2 c d e^2-12 A x c d e^2+3 B x a e^3+24 B x c d^2 e+A a e^3-8 A c d^2 e+2 B a d e^2+16 B c d^3\right)}{3(e x+d)^{\frac{3}{2}} e^4}$	1
trager	$-\frac{2\left(-B c x^3 e^3-3 A x^2 c e^3+6 B x^2 c d e^2-12 A x c d e^2+3 B x a e^3+24 B x c d^2 e+A a e^3-8 A c d^2 e+2 B a d e^2+16 B c d^3\right)}{3(e x+d)^{\frac{3}{2}} e^4}$	1
orering	$-\frac{2\left(-B c x^3 e^3-3 A x^2 c e^3+6 B x^2 c d e^2-12 A x c d e^2+3 B x a e^3+24 B x c d^2 e+A a e^3-8 A c d^2 e+2 B a d e^2+16 B c d^3\right)}{3(e x+d)^{\frac{3}{2}} e^4}$	1
derivativedivides	$\frac{\frac{2 B c(e x+d)^{\frac{3}{2}}}{3}+2 A c e \sqrt{e x+d}-6 B c d \sqrt{e x+d}-\frac{2\left(-2 A c d e+B a e^2+3 B c d^2\right)}{\sqrt{e x+d}}-\frac{2\left(A a e^3+A c d^2 e-B a d e^2-B c d^3\right)}{3(e x+d)^{\frac{3}{2}}}}{e^4}$	1
default	$\frac{\frac{2 B c(e x+d)^{\frac{3}{2}}}{3}+2 A c e \sqrt{e x+d}-6 B c d \sqrt{e x+d}-\frac{2\left(-2 A c d e+B a e^2+3 B c d^2\right)}{\sqrt{e x+d}}-\frac{2\left(A a e^3+A c d^2 e-B a d e^2-B c d^3\right)}{3(e x+d)^{\frac{3}{2}}}}{e^4}$	1

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(3/2)*(((-B*x^3-3*A*x^2)*c+(3*B*x+A)*a)*e^3-12*d*(x*(-1/2*B*x
+A)*c-1/6*B*a)*e^2-8*c*d^2*(-3*B*x+A)*e+16*B*c*d^3)/e^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{5/2}}\,dx = \frac{2\,(Bce^3x^3 - 16\,Bcd^3 + 8\,Acd^2e - 2\,Bade^2 - Aae^3 - 3\,(2\,Bcde^2 - Ace^3)x^2 - \cdots}{3\,(e^6x^2 + 2\,de^5x + d^2e^4)}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(B*c*e^3*x^3 - 16*B*c*d^3 + 8*A*c*d^2*e - 2*B*a*d*e^2 - A*a*e^3 - 3*(2
*B*c*d*e^2 - A*c*e^3)*x^2 - 3*(8*B*c*d^2*e - 4*A*c*d*e^2 + B*a*e^3)*x)*sqr
t(e*x + d)/(e^6*x^2 + 2*d*e^5*x + d^2*e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. 2(114) = 228.

Time = 0.38 (sec) , antiderivative size = 449, normalized size of antiderivative = 4.01

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{5/2}}\,dx = \begin{cases} -\frac{2Aae^3}{3de^4\sqrt{d+ex}+3e^5x\sqrt{d+ex}} + \frac{16Acd^2e}{3de^4\sqrt{d+ex}+3e^5x\sqrt{d+ex}} + \frac{24Acde^2x}{3de^4\sqrt{d+ex}+3e^5x\sqrt{d+ex}} + \cdots \\ \frac{Aax+\frac{Acx^3}{3}+\frac{Bax^2}{2}+\frac{Bcx^4}{4}}{d^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((B*x+A)*(c*x**2+a)/(e*x+d)**(5/2),x)
```

output
```
Piecewise((-2*A*a*e**3/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) +
 16*A*c*d**2*e/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) + 24*A*c*
d*e**2*x/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) + 6*A*c*e**3*x*
*2/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) - 4*B*a*d*e**2/(3*d*e
**4*sqrt(d + e*x) + 3*e**5*x*sqrt(d + e*x)) - 6*B*a*e**3*x/(3*d*e**4*sqrt(
d + e*x) + 3*e**5*x*sqrt(d + e*x)) - 32*B*c*d**3/(3*d*e**4*sqrt(d + e*x) +
 3*e**5*x*sqrt(d + e*x)) - 48*B*c*d**2*e*x/(3*d*e**4*sqrt(d + e*x) + 3*e**
5*x*sqrt(d + e*x)) - 12*B*c*d*e**2*x**2/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x
*sqrt(d + e*x)) + 2*B*c*e**3*x**3/(3*d*e**4*sqrt(d + e*x) + 3*e**5*x*sqrt(
d + e*x)), Ne(e, 0)), ((A*a*x + A*c*x**3/3 + B*a*x**2/2 + B*c*x**4/4)/d**(
5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{5/2}}\,dx = \frac{2\left(\frac{(ex+d)^{\frac{3}{2}}Bc-3\,(3\,Bcd-Ace)\sqrt{ex+d}}{e^3}+\frac{Bcd^3-Acd^2e+Bade^2-Aae^3-3\,(3\,Bcd^2-2\,Acde+Bae^2)\,(ex-}{(ex+d)^{\frac{3}{2}}e^3}\right.}{3\,e}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(((e*x + d)^(3/2)*B*c - 3*(3*B*c*d - A*c*e)*sqrt(e*x + d))/e^3 + (B*c*
d^3 - A*c*d^2*e + B*a*d*e^2 - A*a*e^3 - 3*(3*B*c*d^2 - 2*A*c*d*e + B*a*e^2
)*(e*x + d))/((e*x + d)^(3/2)*e^3))/e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{5/2}}\,dx = -\frac{2\,(9\,(ex+d)Bcd^2-Bcd^3-6\,(ex+d)Acde+Acd^2e+3\,(ex+d)Bae^2-Bade^2+Aae^3)}{3\,(ex+d)^{\frac{3}{2}}e^4} + \frac{2\left((ex+d)^{\frac{3}{2}}Bce^8-9\,\sqrt{ex+d}Bcde^8+3\,\sqrt{ex+d}Ace^9\right)}{3\,e^{12}}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(9*(e*x + d)*B*c*d^2 - B*c*d^3 - 6*(e*x + d)*A*c*d*e + A*c*d^2*e + 3*
(e*x + d)*B*a*e^2 - B*a*d*e^2 + A*a*e^3)/((e*x + d)^(3/2)*e^4) + 2/3*((e*x
 + d)^(3/2)*B*c*e^8 - 9*sqrt(e*x + d)*B*c*d*e^8 + 3*sqrt(e*x + d)*A*c*e^9)
/e^12
```

Mupad [B] (verification not implemented)

Time = 6.42 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{5/2}}\,dx = \frac{2\,B\,c\,(d+e\,x)^3-2\,A\,a\,e^3+2\,B\,c\,d^3+2\,B\,a\,d\,e^2-2\,A\,c\,d^2\,e-6\,B\,a\,e^2\,(d+e\,\dots}{3\,e^4\,\dots}$$

input
```
int(((a + c*x^2)*(A + B*x))/(d + e*x)^(5/2),x)
```

output
```
(2*B*c*(d + e*x)^3 - 2*A*a*e^3 + 2*B*c*d^3 + 2*B*a*d*e^2 - 2*A*c*d^2*e - 6
*B*a*e^2*(d + e*x) + 6*A*c*e*(d + e*x)^2 - 18*B*c*d*(d + e*x)^2 - 18*B*c*d
^2*(d + e*x) + 12*A*c*d*e*(d + e*x))/(3*e^4*(d + e*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{5/2}}\,dx = \frac{\frac{2}{3}bc\,e^3x^3+2ac\,e^3x^2-4bcd\,e^2x^2-2ab\,e^3x+8acd\,e^2x-16bc\,d^2ex-\frac{2}{3}a^2e^3-\dots}{\sqrt{ex+d}\,e^4\,(ex+d)}$$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^(5/2),x)
```

output
```
(2*( - a**2*e**3 - 2*a*b*d*e**2 - 3*a*b*e**3*x + 8*a*c*d**2*e + 12*a*c*d*e
**2*x + 3*a*c*e**3*x**2 - 16*b*c*d**3 - 24*b*c*d**2*e*x - 6*b*c*d*e**2*x**
2 + b*c*e**3*x**3))/(3*sqrt(d + e*x)*e**4*(d + e*x))
```

3.110 $\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{7/2}}\,dx = \frac{2(Bd-Ae)\,(cd^2+ae^2)}{5e^4(d+ex)^{5/2}} - \frac{2(3Bcd^2-2Acde+aBe^2)}{3e^4(d+ex)^{3/2}} + \frac{2c(3Bd-Ae)}{e^4\sqrt{d+ex}} + \frac{2Bc\sqrt{d+ex}}{e^4}$$

output
```
2/5*(-A*e+B*d)*(a*e^2+c*d^2)/e^4/(e*x+d)^(5/2)-2/3*(-2*A*c*d*e+B*a*e^2+3*B
*c*d^2)/e^4/(e*x+d)^(3/2)+2*c*(-A*e+3*B*d)/e^4/(e*x+d)^(1/2)+2*B*c*(e*x+d)
^(1/2)/e^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{7/2}}\,dx = -\frac{2(3aAe^3+aBe^2(2d+5ex)+Ace(8d^2+20dex+15e^2x^2)-3Bc(16d^3+40d^2ex+30de^2x^2+5e^3x^3))}{15e^4(d+ex)^{5/2}}$$

input
```
Integrate[((A + B*x)*(a + c*x^2))/(d + e*x)^(7/2),x]
```

output
```
(-2*(3*a*A*e^3 + a*B*e^2*(2*d + 5*e*x) + A*c*e*(8*d^2 + 20*d*e*x + 15*e^2*
x^2) - 3*B*c*(16*d^3 + 40*d^2*e*x + 30*d*e^2*x^2 + 5*e^3*x^3)))/(15*e^4*(d
 + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(A+Bx)}{(d+ex)^{7/2}}dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{aBe^2-2Acde+3Bcd^2}{e^3(d+ex)^{5/2}}+\frac{(ae^2+cd^2)(Ae-Bd)}{e^3(d+ex)^{7/2}}+\frac{c(Ae-3Bd)}{e^3(d+ex)^{3/2}}+\frac{Bc}{e^3\sqrt{d+ex}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(aBe^2-2Acde+3Bcd^2)}{3e^4(d+ex)^{3/2}}+\frac{2(ae^2+cd^2)(Bd-Ae)}{5e^4(d+ex)^{5/2}}+\frac{2c(3Bd-Ae)}{e^4\sqrt{d+ex}}+\frac{2Bc\sqrt{d+ex}}{e^4}$$

input
```
Int[((A + B*x)*(a + c*x^2))/(d + e*x)^(7/2),x]
```

output
```
(2*(B*d - A*e)*(c*d^2 + a*e^2))/(5*e^4*(d + e*x)^(5/2)) - (2*(3*B*c*d^2 -
2*A*c*d*e + a*B*e^2))/(3*e^4*(d + e*x)^(3/2)) + (2*c*(3*B*d - A*e))/(e^4*S
qrt[d + e*x]) + (2*B*c*Sqrt[d + e*x])/e^4
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{\left(-30x^2(-Bx+A)c-6\left(\frac{5Bx}{3}+A\right)a\right)e^3-40\left(x\left(-\frac{9Bx}{2}+A\right)c+\frac{Ba}{10}\right)d e^2-16c d^2(-15Bx+A)e+96Bc d^3}{15(ex+d)^{\frac{5}{2}}e^4}$
gosper	$-\frac{2(-15Bc x^3e^3+15A x^2c e^3-90B x^2cd e^2+20Axcd e^2+5Bxa e^3-120Bxc d^2e+3Aa e^3+8Ac d^2e+2Bad e^2-48Bc d^3}{15(ex+d)^{\frac{5}{2}}e^4}$
trager	$-\frac{2(-15Bc x^3e^3+15A x^2c e^3-90B x^2cd e^2+20Axcd e^2+5Bxa e^3-120Bxc d^2e+3Aa e^3+8Ac d^2e+2Bad e^2-48Bc d^3}{15(ex+d)^{\frac{5}{2}}e^4}$
orering	$-\frac{2(-15Bc x^3e^3+15A x^2c e^3-90B x^2cd e^2+20Axcd e^2+5Bxa e^3-120Bxc d^2e+3Aa e^3+8Ac d^2e+2Bad e^2-48Bc d^3}{15(ex+d)^{\frac{5}{2}}e^4}$
derivativedivides	$\frac{2Bc\sqrt{ex+d}-\frac{2\left(Aa e^3+Ac d^2e-Bad e^2-Bc d^3\right)}{5(ex+d)^{\frac{5}{2}}}-\frac{2c(Ae-3Bd)}{\sqrt{ex+d}}-\frac{2\left(-2Acde+Ba e^2+3Bc d^2\right)}{3(ex+d)^{\frac{3}{2}}}}{e^4}$
default	$\frac{2Bc\sqrt{ex+d}-\frac{2\left(Aa e^3+Ac d^2e-Bad e^2-Bc d^3\right)}{5(ex+d)^{\frac{5}{2}}}-\frac{2c(Ae-3Bd)}{\sqrt{ex+d}}-\frac{2\left(-2Acde+Ba e^2+3Bc d^2\right)}{3(ex+d)^{\frac{3}{2}}}}{e^4}$
risch	$\frac{2Bc\sqrt{ex+d}}{e^4}-\frac{2(15A x^2c e^3-45B x^2cd e^2+20Axcd e^2+5Bxa e^3-75Bxc d^2e+3Aa e^3+8Ac d^2e+2Bad e^2-33Bc d^3)}{15e^4\sqrt{ex+d}\,(e^2x^2+2dex+d^2)}$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*((-30*x^2*(-B*x+A)*c-6*(5/3*B*x+A)*a)*e^3-40*(x*(-9/2*B*x+A)*c+1/10*B
*a)*d*e^2-16*c*d^2*(-15*B*x+A)*e+96*B*c*d^3)/(e*x+d)^(5/2)/e^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{7/2}}\,dx = \frac{2\,(15\,Bce^3x^3+48\,Bcd^3-8\,Acd^2e-2\,Bade^2-3\,Aae^3+15\,(6\,Bcde^2-Ace^3)}{15\,(e^7x^3+3\,de^6x^2+3\,d^2e^5x+d^3}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
2/15*(15*B*c*e^3*x^3 + 48*B*c*d^3 - 8*A*c*d^2*e - 2*B*a*d*e^2 - 3*A*a*e^3
+ 15*(6*B*c*d*e^2 - A*c*e^3)*x^2 + 5*(24*B*c*d^2*e - 4*A*c*d*e^2 - B*a*e^3
)*x)*sqrt(e*x + d)/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. 2(116) = 232.

Time = 0.55 (sec) , antiderivative size = 653, normalized size of antiderivative = 5.83

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{7/2}}\,dx = \begin{cases} -\frac{6Aae^3}{15d^2e^4\sqrt{d+ex}+30de^5x\sqrt{d+ex}+15e^6x^2\sqrt{d+ex}} - \frac{16Acd^2e}{15d^2e^4\sqrt{d+ex}+30de^5x\sqrt{d+ex}+15e^6x^2\sqrt{d+ex}} - \\ \frac{Aax+\frac{Acx^3}{3}+\frac{Bax^2}{2}+\frac{Bcx^4}{4}}{d^{\frac{7}{2}}} \end{cases}$$

input
```
integrate((B*x+A)*(c*x**2+a)/(e*x+d)**(7/2),x)
```

output
```
Piecewise((-6*A*a*e**3/(15*d**2*e**4*sqrt(d + e*x) + 30*d*e**5*x*sqrt(d +
e*x) + 15*e**6*x**2*sqrt(d + e*x)) - 16*A*c*d**2*e/(15*d**2*e**4*sqrt(d +
e*x) + 30*d*e**5*x*sqrt(d + e*x) + 15*e**6*x**2*sqrt(d + e*x)) - 40*A*c*d*
e**2*x/(15*d**2*e**4*sqrt(d + e*x) + 30*d*e**5*x*sqrt(d + e*x) + 15*e**6*x
**2*sqrt(d + e*x)) - 30*A*c*e**3*x**2/(15*d**2*e**4*sqrt(d + e*x) + 30*d*e
**5*x*sqrt(d + e*x) + 15*e**6*x**2*sqrt(d + e*x)) - 4*B*a*d*e**2/(15*d**2*
e**4*sqrt(d + e*x) + 30*d*e**5*x*sqrt(d + e*x) + 15*e**6*x**2*sqrt(d + e*x
)) - 10*B*a*e**3*x/(15*d**2*e**4*sqrt(d + e*x) + 30*d*e**5*x*sqrt(d + e*x)
 + 15*e**6*x**2*sqrt(d + e*x)) + 96*B*c*d**3/(15*d**2*e**4*sqrt(d + e*x) +
 30*d*e**5*x*sqrt(d + e*x) + 15*e**6*x**2*sqrt(d + e*x)) + 240*B*c*d**2*e*
x/(15*d**2*e**4*sqrt(d + e*x) + 30*d*e**5*x*sqrt(d + e*x) + 15*e**6*x**2*s
qrt(d + e*x)) + 180*B*c*d*e**2*x**2/(15*d**2*e**4*sqrt(d + e*x) + 30*d*e**
5*x*sqrt(d + e*x) + 15*e**6*x**2*sqrt(d + e*x)) + 30*B*c*e**3*x**3/(15*d**
2*e**4*sqrt(d + e*x) + 30*d*e**5*x*sqrt(d + e*x) + 15*e**6*x**2*sqrt(d + e
*x)), Ne(e, 0)), ((A*a*x + A*c*x**3/3 + B*a*x**2/2 + B*c*x**4/4)/d**(7/2),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{7/2}}\,dx = \frac{2\left(\frac{15\sqrt{ex+d}Bc}{e^3} + \frac{3\,Bcd^3-3\,Acd^2e+3\,Bade^2-3\,Aae^3+15\,(3\,Bcd-Ace)(ex+d)^2-5\,(3\,Bcd^2-2\,Acde+B}{(ex+d)^{\frac{5}{2}}e^3}\right.}{15\,e}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
2/15*(15*sqrt(e*x + d)*B*c/e^3 + (3*B*c*d^3 - 3*A*c*d^2*e + 3*B*a*d*e^2 -
3*A*a*e^3 + 15*(3*B*c*d - A*c*e)*(e*x + d)^2 - 5*(3*B*c*d^2 - 2*A*c*d*e +
B*a*e^2)*(e*x + d))/((e*x + d)^(5/2)*e^3))/e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^{7/2}}\,dx = \frac{2\sqrt{ex+d}Bc}{e^4}$$
$$+\frac{2\left(45\,(ex+d)^2Bcd-15\,(ex+d)Bcd^2+3\,Bcd^3-15\,(ex+d)^2Ace+10\,(ex+d)Acde-3\,Acd^2e-5\,(ex\right.}{15\,(ex+d)^{\frac{5}{2}}e^4}$$

input
```
integrate((B*x+A)*(c*x^2+a)/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
2*sqrt(e*x + d)*B*c/e^4 + 2/15*(45*(e*x + d)^2*B*c*d - 15*(e*x + d)*B*c*d^
2 + 3*B*c*d^3 - 15*(e*x + d)^2*A*c*e + 10*(e*x + d)*A*c*d*e - 3*A*c*d^2*e
- 5*(e*x + d)*B*a*e^2 + 3*B*a*d*e^2 - 3*A*a*e^3)/((e*x + d)^(5/2)*e^4)
```

Mupad [B] (verification not implemented)

Time = 6.68 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)(a+cx^2)}{(d+ex)^{7/2}}\,dx =$$
$$-\frac{2\left(-48\,B\,c\,d^3-120\,B\,c\,d^2\,e\,x+8\,A\,c\,d^2\,e-90\,B\,c\,d\,e^2\,x^2+20\,A\,c\,d\,e^2\,x+2\,B\,a\,d\,e^2-15\,B\,c\,e^3\,x^3+15\right.}{15\,e^4\,(d+e\,x)^{5/2}}$$

input
```
int(((a + c*x^2)*(A + B*x))/(d + e*x)^(7/2),x)
```

output
```
-(2*(3*A*a*e^3 - 48*B*c*d^3 + 2*B*a*d*e^2 + 8*A*c*d^2*e + 5*B*a*e^3*x + 15
*A*c*e^3*x^2 - 15*B*c*e^3*x^3 - 90*B*c*d*e^2*x^2 + 20*A*c*d*e^2*x - 120*B*
c*d^2*e*x))/(15*e^4*(d + e*x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)\,(a+cx^2)}{(d+ex)^{7/2}}\,dx = \frac{2bc\,e^3x^3 - 2ac\,e^3x^2 + 12bcd\,e^2x^2 - \frac{2}{3}ab\,e^3x - \frac{8}{3}acd\,e^2x + 16bc\,d^2ex - \frac{2}{5}a^2e^3 -}{\sqrt{ex+d}\,e^4\,(e^2x^2+2dex+d^2)}$$

input
```
int((B*x+A)*(c*x^2+a)/(e*x+d)^(7/2),x)
```

output
```
(2*( - 3*a**2*e**3 - 2*a*b*d*e**2 - 5*a*b*e**3*x - 8*a*c*d**2*e - 20*a*c*d
*e**2*x - 15*a*c*e**3*x**2 + 48*b*c*d**3 + 120*b*c*d**2*e*x + 90*b*c*d*e**
2*x**2 + 15*b*c*e**3*x**3))/(15*sqrt(d + e*x)*e**4*(d**2 + 2*d*e*x + e**2*
x**2))
```

3.111 $\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 218

$$\begin{aligned}\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)^2\,dx &= -\frac{2(Bd-Ae)\left(cd^2+ae^2\right)^2(d+ex)^{3/2}}{3e^6}\\ &\quad+\frac{2\left(cd^2+ae^2\right)\left(5Bcd^2-4Acde+aBe^2\right)(d+ex)^{5/2}}{5e^6}\\ &\quad-\frac{4c\left(5Bcd^3-3Acd^2e+3aBde^2-aAe^3\right)(d+ex)^{7/2}}{7e^6}\\ &\quad+\frac{4c\left(5Bcd^2-2Acde+aBe^2\right)(d+ex)^{9/2}}{9e^6}\\ &\quad-\frac{2c^2(5Bd-Ae)(d+ex)^{11/2}}{11e^6}+\frac{2Bc^2(d+ex)^{13/2}}{13e^6}\end{aligned}$$

output

```
-2/3*(-A*e+B*d)*(a*e^2+c*d^2)^2*(e*x+d)^(3/2)/e^6+2/5*(a*e^2+c*d^2)*(-4*A*
c*d*e+B*a*e^2+5*B*c*d^2)*(e*x+d)^(5/2)/e^6-4/7*c*(-A*a*e^3-3*A*c*d^2*e+3*B
*a*d*e^2+5*B*c*d^3)*(e*x+d)^(7/2)/e^6+4/9*c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)
*(e*x+d)^(9/2)/e^6-2/11*c^2*(-A*e+5*B*d)*(e*x+d)^(11/2)/e^6+2/13*B*c^2*(e*
x+d)^(13/2)/e^6
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.98

$$\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)^2\,dx$$
$$= \frac{2(d+ex)^{3/2}\left(13Ae(1155a^2e^4+66ace^2(8d^2-12dex+15e^2x^2)+c^2(128d^4-192d^3ex+240d^2e^2x^2-280\right.}{}$$

input
```
Integrate[(A + B*x)*Sqrt[d + e*x]*(a + c*x^2)^2,x]
```

output
```
(2*(d + e*x)^(3/2)*(13*A*e*(1155*a^2*e^4 + 66*a*c*e^2*(8*d^2 - 12*d*e*x +
15*e^2*x^2) + c^2*(128*d^4 - 192*d^3*e*x + 240*d^2*e^2*x^2 - 280*d*e^3*x^3
 + 315*e^4*x^4)) + B*(3003*a^2*e^4*(-2*d + 3*e*x) + 286*a*c*e^2*(-16*d^3 +
 24*d^2*e*x - 30*d*e^2*x^2 + 35*e^3*x^3) - 5*c^2*(256*d^5 - 384*d^4*e*x +
480*d^3*e^2*x^2 - 560*d^2*e^3*x^3 + 630*d*e^4*x^4 - 693*e^5*x^5))))/(45045
*e^6)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2 (A+Bx)\sqrt{d+ex}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{2c(d+ex)^{7/2}\left(-aBe^2+2Acde-5Bcd^2\right)}{e^5}+\frac{(d+ex)^{3/2}\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^5}+\frac{\sqrt{d+ex}\cdots}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c(d+ex)^{9/2}\left(aBe^2-2Acde+5Bcd^2\right)}{9e^6}+$$
$$\frac{2(d+ex)^{5/2}\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{5e^6}-\frac{2(d+ex)^{3/2}\left(ae^2+cd^2\right)^2(Bd-Ae)}{3e^6}-$$
$$\frac{4c(d+ex)^{7/2}\left(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3\right)}{7e^6}-\frac{2c^2(d+ex)^{11/2}(5Bd-Ae)}{11e^6}+$$
$$\frac{2Bc^2(d+ex)^{13/2}}{13e^6}$$

input
```
Int[(A + B*x)*Sqrt[d + e*x]*(a + c*x^2)^2,x]
```

output
```
(-2*(B*d - A*e)*(c*d^2 + a*e^2)^2*(d + e*x)^(3/2))/(3*e^6) + (2*(c*d^2 + a
*e^2)*(5*B*c*d^2 - 4*A*c*d*e + a*B*e^2)*(d + e*x)^(5/2))/(5*e^6) - (4*c*(5
*B*c*d^3 - 3*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*(d + e*x)^(7/2))/(7*e^6) +
 (4*c*(5*B*c*d^2 - 2*A*c*d*e + a*B*e^2)*(d + e*x)^(9/2))/(9*e^6) - (2*c^2*
(5*B*d - A*e)*(d + e*x)^(11/2))/(11*e^6) + (2*B*c^2*(d + e*x)^(13/2))/(13*
e^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{2(ex+d)^{\frac{3}{2}}\left(\left(\left(\left(\frac{3}{13}Bx^5+\frac{3}{11}Ax^4\right)c^2+\frac{6\left(\frac{7Bx}{9}+A\right)x^2ac}{7}+a^2\left(\frac{3Bx}{5}+A\right)\right)e^5-\frac{24\left(\frac{35\left(\frac{45Bx}{52}+A\right)x^3c^2}{99}+ax\left(\frac{5Bx}{6}+A\right)c+\frac{7a^2B}{12}\right.}{35}\right.\right.}{3e^6}$
derivativedivides	$\frac{2Bc^2(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left((Ae-Bd)c^2-4Bc^2d\right)(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(-4(Ae-Bd)c^2d+B\left(2\left(ae^2+cd^2\right)c+4c^2d^2\right)\right)(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left((Ae-Bd)\left(2\right.\right.}{}$
default	$\frac{2Bc^2(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left((Ae-Bd)c^2-4Bc^2d\right)(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(-4(Ae-Bd)c^2d+B\left(2\left(ae^2+cd^2\right)c+4c^2d^2\right)\right)(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left((Ae-Bd)\left(2\right.\right.}{}$
gosper	$2(ex+d)^{\frac{3}{2}}\,(3465Bx^5c^2e^5+4095Ax^4c^2e^5-3150Bx^4c^2de^4-3640Ax^3c^2de^4+10010Bx^3ace^5+2800Bx^3c^2d^2e^3+1287$
orering	$2(ex+d)^{\frac{3}{2}}\,(3465Bx^5c^2e^5+4095Ax^4c^2e^5-3150Bx^4c^2de^4-3640Ax^3c^2de^4+10010Bx^3ace^5+2800Bx^3c^2d^2e^3+1287$
trager	$2(3465Bc^2e^6x^6+4095Ac^2e^6x^5+315Bc^2de^5x^5+455Ac^2de^5x^4+10010Bace^6x^4-350Bc^2d^2e^4x^4+12870Aace^6x^3-$
risch	$2(3465Bc^2e^6x^6+4095Ac^2e^6x^5+315Bc^2de^5x^5+455Ac^2de^5x^4+10010Bace^6x^4-350Bc^2d^2e^4x^4+12870Aace^6x^3-$

input
```
int((B*x+A)*(e*x+d)^(1/2)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(e*x+d)^(3/2)*(((3/13*B*x^5+3/11*A*x^4)*c^2+6/7*(7/9*B*x+A)*x^2*a*c+a^
2*(3/5*B*x+A))*e^5-24/35*(35/99*(45/52*B*x+A)*x^3*c^2+a*x*(5/6*B*x+A)*c+7/
12*a^2*B)*d*e^4+16/35*c*d^2*(5/11*x^2*(35/39*B*x+A)*c+a*(B*x+A))*e^3-64/38
5*c*d^3*(x*(25/26*B*x+A)*c+11/6*B*a)*e^2+128/1155*c^2*d^4*(15/13*B*x+A)*e-
256/3003*B*c^2*d^5)/e^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.47

$$\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)^2\,dx$$

$$=\frac{2\left(3465\,Bc^2e^6x^6-1280\,Bc^2d^6+1664\,Ac^2d^5e-4576\,Bacd^4e^2+6864\,Aacd^3e^3-6006\,Ba^2d^2e^4+15015\right.}{}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/45045*(3465*B*c^2*e^6*x^6 - 1280*B*c^2*d^6 + 1664*A*c^2*d^5*e - 4576*B*a
*c*d^4*e^2 + 6864*A*a*c*d^3*e^3 - 6006*B*a^2*d^2*e^4 + 15015*A*a^2*d*e^5 +
 315*(B*c^2*d*e^5 + 13*A*c^2*e^6)*x^5 - 35*(10*B*c^2*d^2*e^4 - 13*A*c^2*d*
e^5 - 286*B*a*c*e^6)*x^4 + 10*(40*B*c^2*d^3*e^3 - 52*A*c^2*d^2*e^4 + 143*B
*a*c*d*e^5 + 1287*A*a*c*e^6)*x^3 - 3*(160*B*c^2*d^4*e^2 - 208*A*c^2*d^3*e^
3 + 572*B*a*c*d^2*e^4 - 858*A*a*c*d*e^5 - 3003*B*a^2*e^6)*x^2 + (640*B*c^2
*d^5*e - 832*A*c^2*d^4*e^2 + 2288*B*a*c*d^3*e^3 - 3432*A*a*c*d^2*e^4 + 300
3*B*a^2*d*e^5 + 15015*A*a^2*e^6)*x)*sqrt(e*x + d)/e^6
```

Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.71

$$\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)^2\,dx$$
$$= \begin{cases} \frac{2\left(\frac{Bc^2(d+ex)^{\frac{13}{2}}}{13e^5}+\frac{(d+ex)^{\frac{11}{2}}\left(Ac^2e-5Bc^2d\right)}{11e^5}+\frac{(d+ex)^{\frac{9}{2}}\left(-4Ac^2de+2Bace^2+10Bc^2d^2\right)}{9e^5}+\frac{(d+ex)^{\frac{7}{2}}\cdot\left(2Aace^3+6Ac^2d^2e-6Bacde^2-10Bc^2d^3\right)}{7e^5}+\frac{(d+ex)^{\frac{5}{2}}}{}\right.}{e} \\ \sqrt{d}\left(Aa^2x+\frac{2Aacx^3}{3}+\frac{Ac^2x^5}{5}+\frac{Ba^2x^2}{2}+\frac{Bacx^4}{2}+\frac{Bc^2x^6}{6}\right) \end{cases}$$

input
```
integrate((B*x+A)*(e*x+d)**(1/2)*(c*x**2+a)**2,x)
```

output
```
Piecewise((2*(B*c**2*(d + e*x)**(13/2)/(13*e**5) + (d + e*x)**(11/2)*(A*c*
*2*e - 5*B*c**2*d)/(11*e**5) + (d + e*x)**(9/2)*(-4*A*c**2*d*e + 2*B*a*c*e
**2 + 10*B*c**2*d**2)/(9*e**5) + (d + e*x)**(7/2)*(2*A*a*c*e**3 + 6*A*c**2
*d**2*e - 6*B*a*c*d*e**2 - 10*B*c**2*d**3)/(7*e**5) + (d + e*x)**(5/2)*(-4
*A*a*c*d*e**3 - 4*A*c**2*d**3*e + B*a**2*e**4 + 6*B*a*c*d**2*e**2 + 5*B*c*
*2*d**4)/(5*e**5) + (d + e*x)**(3/2)*(A*a**2*e**5 + 2*A*a*c*d**2*e**3 + A*
c**2*d**4*e - B*a**2*d*e**4 - 2*B*a*c*d**3*e**2 - B*c**2*d**5)/(3*e**5))/e
, Ne(e, 0)), (sqrt(d)*(A*a**2*x + 2*A*a*c*x**3/3 + A*c**2*x**5/5 + B*a**2*
x**2/2 + B*a*c*x**4/2 + B*c**2*x**6/6), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.14

$$\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)^2\,dx$$
$$=\frac{2\left(3465\,(ex+d)^{\frac{13}{2}}Bc^2-4095\,(5\,Bc^2d-Ac^2e)(ex+d)^{\frac{11}{2}}+10010\,(5\,Bc^2d^2-2\,Ac^2de+Bace^2)(ex+d\right.}{}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/45045*(3465*(e*x + d)^(13/2)*B*c^2 - 4095*(5*B*c^2*d - A*c^2*e)*(e*x + d
)^(11/2) + 10010*(5*B*c^2*d^2 - 2*A*c^2*d*e + B*a*c*e^2)*(e*x + d)^(9/2) -
 12870*(5*B*c^2*d^3 - 3*A*c^2*d^2*e + 3*B*a*c*d*e^2 - A*a*c*e^3)*(e*x + d)
^(7/2) + 9009*(5*B*c^2*d^4 - 4*A*c^2*d^3*e + 6*B*a*c*d^2*e^2 - 4*A*a*c*d*e
^3 + B*a^2*e^4)*(e*x + d)^(5/2) - 15015*(B*c^2*d^5 - A*c^2*d^4*e + 2*B*a*c
*d^3*e^2 - 2*A*a*c*d^2*e^3 + B*a^2*d*e^4 - A*a^2*e^5)*(e*x + d)^(3/2))/e^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. 2(194) = 388.

Time = 0.13 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.90

$$\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(e*x + d)*A*a^2*d + 15015*((e*x + d)^(3/2) - 3*sqrt(e*x
 + d)*d)*A*a^2 + 15015*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*B*a^2*d/e + 6
006*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*A*a*
c*d/e^2 + 3003*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d
)*d^2)*B*a^2/e + 2574*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x
+ d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*B*a*c*d/e^3 + 2574*(5*(e*x + d)^(7/
2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)
*A*a*c/e^2 + 143*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x +
d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*A*c^2*d/e^
4 + 286*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*
d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*B*a*c/e^3 + 65*(63*
(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*
(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*B*
c^2*d/e^5 + 65*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d
)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sq
rt(e*x + d)*d^5)*A*c^2/e^4 + 15*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11
/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x +
d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*B*c^2/e^
5)/e
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.90

$$
\begin{aligned}
\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)^2\,dx
&= \frac{(d+e\,x)^{9/2}\,\left(20\,B\,c^2\,d^2-8\,A\,c^2\,d\,e+4\,B\,a\,c\,e^2\right)}{9\,e^6} \\
&\quad+\frac{4\,c\,(d+e\,x)^{7/2}\,\left(-5\,B\,c\,d^3+3\,A\,c\,d^2\,e-3\,B\,a\,d\,e^2+A\,a\,e^3\right)}{7\,e^6} \\
&\quad+\frac{2\,B\,c^2\,(d+e\,x)^{13/2}}{13\,e^6}+\frac{2\,\left(c\,d^2+a\,e^2\right)\,(d+e\,x)^{5/2}\,\left(5\,B\,c\,d^2-4\,A\,c\,d\,e+B\,a\,e^2\right)}{5\,e^6} \\
&\quad+\frac{2\,c^2\,(A\,e-5\,B\,d)\,(d+e\,x)^{11/2}}{11\,e^6}+\frac{2\,\left(c\,d^2+a\,e^2\right)^2\,(A\,e-B\,d)\,(d+e\,x)^{3/2}}{3\,e^6}
\end{aligned}
$$

input
```
int((a + c*x^2)^2*(A + B*x)*(d + e*x)^(1/2),x)
```

output
```
((d + e*x)^(9/2)*(20*B*c^2*d^2 + 4*B*a*c*e^2 - 8*A*c^2*d*e))/(9*e^6) + (4*
c*(d + e*x)^(7/2)*(A*a*e^3 - 5*B*c*d^3 - 3*B*a*d*e^2 + 3*A*c*d^2*e))/(7*e^
6) + (2*B*c^2*(d + e*x)^(13/2))/(13*e^6) + (2*(a*e^2 + c*d^2)*(d + e*x)^(5
/2)*(B*a*e^2 + 5*B*c*d^2 - 4*A*c*d*e))/(5*e^6) + (2*c^2*(A*e - 5*B*d)*(d +
 e*x)^(11/2))/(11*e^6) + (2*(a*e^2 + c*d^2)^2*(A*e - B*d)*(d + e*x)^(3/2))
/(3*e^6)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.57

$$\int (A+Bx)\sqrt{d+ex}\left(a+cx^2\right)^2\,dx$$
$$= \frac{2\sqrt{ex+d}\,(3465b\,c^2e^6x^6+4095a\,c^2e^6x^5+315b\,c^2d\,e^5x^5+10010abc\,e^6x^4+455a\,c^2d\,e^5x^4-350b\,c^2d^2e^4x^4}{}$$

input
```
int((B*x+A)*(e*x+d)^(1/2)*(c*x^2+a)^2,x)
```

output
```
(2*sqrt(d + e*x)*(15015*a**3*d*e**5 + 15015*a**3*e**6*x - 6006*a**2*b*d**2
*e**4 + 3003*a**2*b*d*e**5*x + 9009*a**2*b*e**6*x**2 + 6864*a**2*c*d**3*e*
*3 - 3432*a**2*c*d**2*e**4*x + 2574*a**2*c*d*e**5*x**2 + 12870*a**2*c*e**6
*x**3 - 4576*a*b*c*d**4*e**2 + 2288*a*b*c*d**3*e**3*x - 1716*a*b*c*d**2*e*
*4*x**2 + 1430*a*b*c*d*e**5*x**3 + 10010*a*b*c*e**6*x**4 + 1664*a*c**2*d**
5*e - 832*a*c**2*d**4*e**2*x + 624*a*c**2*d**3*e**3*x**2 - 520*a*c**2*d**2
*e**4*x**3 + 455*a*c**2*d*e**5*x**4 + 4095*a*c**2*e**6*x**5 - 1280*b*c**2*
d**6 + 640*b*c**2*d**5*e*x - 480*b*c**2*d**4*e**2*x**2 + 400*b*c**2*d**3*e
**3*x**3 - 350*b*c**2*d**2*e**4*x**4 + 315*b*c**2*d*e**5*x**5 + 3465*b*c**
2*e**6*x**6))/(45045*e**6)
```

3.112 $\int \frac{(A+Bx)(a+cx^2)^2}{\sqrt{d+ex}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 216

$$
\begin{aligned}
\int \frac{(A+Bx)\left(a+cx^2\right)^2}{\sqrt{d+ex}} \, dx = & -\frac{2(Bd-Ae)\left(cd^2+ae^2\right)^2\sqrt{d+ex}}{e^6} \\
& + \frac{2\left(cd^2+ae^2\right)\left(5Bcd^2-4Acde+aBe^2\right)(d+ex)^{3/2}}{3e^6} \\
& - \frac{4c\left(5Bcd^3-3Acd^2e+3aBde^2-aAe^3\right)(d+ex)^{5/2}}{5e^6} \\
& + \frac{4c\left(5Bcd^2-2Acde+aBe^2\right)(d+ex)^{7/2}}{7e^6} \\
& - \frac{2c^2(5Bd-Ae)(d+ex)^{9/2}}{9e^6} + \frac{2Bc^2(d+ex)^{11/2}}{11e^6}
\end{aligned}
$$

output

```
-2*(-A*e+B*d)*(a*e^2+c*d^2)^2*(e*x+d)^(1/2)/e^6+2/3*(a*e^2+c*d^2)*(-4*A*c*
d*e+B*a*e^2+5*B*c*d^2)*(e*x+d)^(3/2)/e^6-4/5*c*(-A*a*e^3-3*A*c*d^2*e+3*B*a
*d*e^2+5*B*c*d^3)*(e*x+d)^(5/2)/e^6+4/7*c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)*(
e*x+d)^(7/2)/e^6-2/9*c^2*(-A*e+5*B*d)*(e*x+d)^(9/2)/e^6+2/11*B*c^2*(e*x+d)
^(11/2)/e^6
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\sqrt{d+ex}(11Ae(315a^2e^4+42ace^2(8d^2-4dex+3e^2x^2)+c^2(128d^4-64d^3ex+48d^2e^2x^2-40de^3x^3+3}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^2)/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(11*A*e*(315*a^2*e^4 + 42*a*c*e^2*(8*d^2 - 4*d*e*x + 3*e^
2*x^2) + c^2*(128*d^4 - 64*d^3*e*x + 48*d^2*e^2*x^2 - 40*d*e^3*x^3 + 35*e^
4*x^4)) + B*(1155*a^2*e^4*(-2*d + e*x) + 198*a*c*e^2*(-16*d^3 + 8*d^2*e*x
- 6*d*e^2*x^2 + 5*e^3*x^3) - 5*c^2*(256*d^5 - 128*d^4*e*x + 96*d^3*e^2*x^2
 - 80*d^2*e^3*x^3 + 70*d*e^4*x^4 - 63*e^5*x^5))))/(3465*e^6)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2(A+Bx)}{\sqrt{d+ex}}\,dx$$
$$\Big\downarrow \; 652$$
$$\int \left(-\frac{2c(d+ex)^{5/2}\left(-aBe^2+2Acde-5Bcd^2\right)}{e^5}+\frac{\sqrt{d+ex}\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^5}+\frac{\left(ae^2+cd^2\right)}{e^5\sqrt{}}\right.$$
$$\Big\downarrow \; 2009$$

$$
\frac{4c(d+ex)^{7/2}\left(aBe^2-2Acde+5Bcd^2\right)}{7e^6}+
\frac{2(d+ex)^{3/2}\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{3e^6}-\frac{2\sqrt{d+ex}\left(ae^2+cd^2\right)^2\left(Bd-Ae\right)}{e^6}-
\frac{4c(d+ex)^{5/2}\left(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3\right)}{5e^6}-\frac{2c^2(d+ex)^{9/2}(5Bd-Ae)}{9e^6}+
\frac{2Bc^2(d+ex)^{11/2}}{11e^6}
$$

input

```
Int[((A + B*x)*(a + c*x^2)^2)/Sqrt[d + e*x],x]
```

output

```
(-2*(B*d - A*e)*(c*d^2 + a*e^2)^2*Sqrt[d + e*x])/e^6 + (2*(c*d^2 + a*e^2)*
(5*B*c*d^2 - 4*A*c*d*e + a*B*e^2)*(d + e*x)^(3/2))/(3*e^6) - (4*c*(5*B*c*d
^3 - 3*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*(d + e*x)^(5/2))/(5*e^6) + (4*c*
(5*B*c*d^2 - 2*A*c*d*e + a*B*e^2)*(d + e*x)^(7/2))/(7*e^6) - (2*c^2*(5*B*d
 - A*e)*(d + e*x)^(9/2))/(9*e^6) + (2*B*c^2*(d + e*x)^(11/2))/(11*e^6)
```

Defintions of rubi rules used

rule 652

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{2\sqrt{ex+d}\left(\left(\frac{\left(\frac{9Bx}{11}+A\right)x^4c^2}{9}+\frac{2x^2a\left(\frac{5Bx}{7}+A\right)c}{5}+a^2\left(\frac{Bx}{3}+A\right)\right)e^5-\frac{8\left(\frac{5\left(\frac{35Bx}{44}+A\right)x^3c^2}{21}+ax\left(\frac{9Bx}{14}+A\right)c+\frac{5a^2B}{4}\right)de^4}{15}+\frac{16}{}\cdots}{e^6}$
derivativedivides	$\frac{2Bc^2(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left((Ae-Bd)c^2-4Bc^2d\right)(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(-4(Ae-Bd)c^2d+B\left(2\left(ae^2+cd^2\right)c+4c^2d^2\right)\right)(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left((Ae-Bd)\left(2\left(\cdots\right.\right.\right.}{}$
default	$\frac{2Bc^2(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left((Ae-Bd)c^2-4Bc^2d\right)(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(-4(Ae-Bd)c^2d+B\left(2\left(ae^2+cd^2\right)c+4c^2d^2\right)\right)(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left((Ae-Bd)\left(2\left(\cdots\right.\right.\right.}{}$
gosper	$\frac{2\sqrt{ex+d}\,(315Bx^5c^2e^5+385Ax^4c^2e^5-350Bx^4c^2de^4-440Ax^3c^2de^4+990Bx^3ace^5+400Bx^3c^2d^2e^3+1386Ax^2ace^5\cdots}{}$
trager	$\frac{2\sqrt{ex+d}\,(315Bx^5c^2e^5+385Ax^4c^2e^5-350Bx^4c^2de^4-440Ax^3c^2de^4+990Bx^3ace^5+400Bx^3c^2d^2e^3+1386Ax^2ace^5\cdots}{}$
risch	$\frac{2\sqrt{ex+d}\,(315Bx^5c^2e^5+385Ax^4c^2e^5-350Bx^4c^2de^4-440Ax^3c^2de^4+990Bx^3ace^5+400Bx^3c^2d^2e^3+1386Ax^2ace^5\cdots}{}$
orering	$\frac{2\sqrt{ex+d}\,(315Bx^5c^2e^5+385Ax^4c^2e^5-350Bx^4c^2de^4-440Ax^3c^2de^4+990Bx^3ace^5+400Bx^3c^2d^2e^3+1386Ax^2ace^5\cdots}{}$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(e*x+d)^(1/2)*((1/9*(9/11*B*x+A)*x^4*c^2+2/5*x^2*a*(5/7*B*x+A)*c+a^2*(1/
3*B*x+A))*e^5-8/15*(5/21*(35/44*B*x+A)*x^3*c^2+a*x*(9/14*B*x+A)*c+5/4*a^2*
B)*d*e^4+16/15*c*d^2*(1/7*x^2*(25/33*B*x+A)*c+a*(3/7*B*x+A))*e^3-64/315*(x
*(15/22*B*x+A)*c+9/2*B*a)*c*d^3*e^2+128/315*(5/11*B*x+A)*c^2*d^4*e-256/693
*B*c^2*d^5)/e^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{\sqrt{d+ex}}\,dx$$
$$=\frac{2\,(315\,Bc^2e^5x^5-1280\,Bc^2d^5+1408\,Ac^2d^4e-3168\,Bacd^3e^2+3696\,Aacd^2e^3-2310\,Ba^2de^4+3465\,Aa^2\cdots}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*B*c^2*e^5*x^5 - 1280*B*c^2*d^5 + 1408*A*c^2*d^4*e - 3168*B*a*c
*d^3*e^2 + 3696*A*a*c*d^2*e^3 - 2310*B*a^2*d*e^4 + 3465*A*a^2*e^5 - 35*(10
*B*c^2*d*e^4 - 11*A*c^2*e^5)*x^4 + 10*(40*B*c^2*d^2*e^3 - 44*A*c^2*d*e^4 +
 99*B*a*c*e^5)*x^3 - 6*(80*B*c^2*d^3*e^2 - 88*A*c^2*d^2*e^3 + 198*B*a*c*d*
e^4 - 231*A*a*c*e^5)*x^2 + (640*B*c^2*d^4*e - 704*A*c^2*d^3*e^2 + 1584*B*a
*c*d^2*e^3 - 1848*A*a*c*d*e^4 + 1155*B*a^2*e^5)*x)*sqrt(e*x + d)/e^6
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.72

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{\sqrt{d+ex}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{Bc^2(d+ex)^{\frac{11}{2}}}{11e^5}+\frac{(d+ex)^{\frac{9}{2}}\left(Ac^2e-5Bc^2d\right)}{9e^5}+\frac{(d+ex)^{\frac{7}{2}}\left(-4Ac^2de+2Bace^2+10Bc^2d^2\right)}{7e^5}+\frac{(d+ex)^{\frac{5}{2}}\cdot\left(2Aace^3+6Ac^2d^2e-6Bacde^2-10Bc^2d^3\right)}{5e^5}+\frac{(d+ex)^{\frac{3}{2}}}{}\right.}{e} \\ \frac{Aa^2x+\frac{2Aacx^3}{3}+\frac{Ac^2x^5}{5}+\frac{Ba^2x^2}{2}+\frac{Bacx^4}{2}+\frac{Bc^2x^6}{6}}{\sqrt{d}} \end{cases}$$

input
```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**(1/2),x)
```

output
```
Piecewise((2*(B*c**2*(d + e*x)**(11/2)/(11*e**5) + (d + e*x)**(9/2)*(A*c**
2*e - 5*B*c**2*d)/(9*e**5) + (d + e*x)**(7/2)*(-4*A*c**2*d*e + 2*B*a*c*e**
2 + 10*B*c**2*d**2)/(7*e**5) + (d + e*x)**(5/2)*(2*A*a*c*e**3 + 6*A*c**2*d
**2*e - 6*B*a*c*d*e**2 - 10*B*c**2*d**3)/(5*e**5) + (d + e*x)**(3/2)*(-4*A
*a*c*d*e**3 - 4*A*c**2*d**3*e + B*a**2*e**4 + 6*B*a*c*d**2*e**2 + 5*B*c**2
*d**4)/(3*e**5) + sqrt(d + e*x)*(A*a**2*e**5 + 2*A*a*c*d**2*e**3 + A*c**2*
d**4*e - B*a**2*d*e**4 - 2*B*a*c*d**3*e**2 - B*c**2*d**5)/e**5)/e, Ne(e, 0
)), ((A*a**2*x + 2*A*a*c*x**3/3 + A*c**2*x**5/5 + B*a**2*x**2/2 + B*a*c*x*
*4/2 + B*c**2*x**6/6)/sqrt(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\left(315\,(ex+d)^{\frac{11}{2}}Bc^2 - 385\,(5\,Bc^2d - Ac^2e)(ex+d)^{\frac{9}{2}} + 990\,(5\,Bc^2d^2 - 2\,Ac^2de + Bace^2)(ex+d)^{\frac{7}{2}} - 1\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
2/3465*(315*(e*x + d)^(11/2)*B*c^2 - 385*(5*B*c^2*d - A*c^2*e)*(e*x + d)^(
9/2) + 990*(5*B*c^2*d^2 - 2*A*c^2*d*e + B*a*c*e^2)*(e*x + d)^(7/2) - 1386*
(5*B*c^2*d^3 - 3*A*c^2*d^2*e + 3*B*a*c*d*e^2 - A*a*c*e^3)*(e*x + d)^(5/2)
+ 1155*(5*B*c^2*d^4 - 4*A*c^2*d^3*e + 6*B*a*c*d^2*e^2 - 4*A*a*c*d*e^3 + B*
a^2*e^4)*(e*x + d)^(3/2) - 3465*(B*c^2*d^5 - A*c^2*d^4*e + 2*B*a*c*d^3*e^2
 - 2*A*a*c*d^2*e^3 + B*a^2*d*e^4 - A*a^2*e^5)*sqrt(e*x + d))/e^6
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\left(3465\,\sqrt{ex+d}Aa^2 + \frac{1155\left((ex+d)^{\frac{3}{2}}-3\,\sqrt{ex+d}d\right)Ba^2}{e} + \frac{462\left(3\,(ex+d)^{\frac{5}{2}}-10\,(ex+d)^{\frac{3}{2}}d+15\,\sqrt{ex+d}d^2\right)Aac}{e^2} + \frac{198\left(5\,(ex+d)^{\frac{7}{2}}\right.}{}\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(e*x + d)*A*a^2 + 1155*((e*x + d)^(3/2) - 3*sqrt(e*x + d)
*d)*B*a^2/e + 462*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x
+ d)*d^2)*A*a*c/e^2 + 198*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(
e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*B*a*c/e^3 + 11*(35*(e*x + d)^(9
/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2
)*d^3 + 315*sqrt(e*x + d)*d^4)*A*c^2/e^4 + 5*(63*(e*x + d)^(11/2) - 385*(e
*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 115
5*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*B*c^2/e^5)/e
```

Mupad [B] (verification not implemented)

Time = 6.56 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{\sqrt{d+ex}}\,dx = \frac{(d+e\,x)^{7/2}\,(20\,B\,c^2\,d^2-8\,A\,c^2\,d\,e+4\,B\,a\,c\,e^2)}{7\,e^6} + \frac{4\,c\,(d+e\,x)^{5/2}\,(-5\,B\,c\,d^3+3\,A\,c\,d^2\,e-3\,B\,a\,d\,e^2+A\,a\,e^3)}{5\,e^6} + \frac{2\,B\,c^2\,(d+e\,x)^{11/2}}{11\,e^6} + \frac{2\,(c\,d^2+a\,e^2)\,(d+e\,x)^{3/2}\,(5\,B\,c\,d^2-4\,A\,c\,d\,e+B\,a\,e^2)}{3\,e^6} + \frac{2\,c^2\,(A\,e-5\,B\,d)\,(d+e\,x)^{9/2}}{9\,e^6} + \frac{2\,(c\,d^2+a\,e^2)^2\,(A\,e-B\,d)\,\sqrt{d+e\,x}}{e^6}$$

input
```
int(((a + c*x^2)^2*(A + B*x))/(d + e*x)^(1/2),x)
```

output
```
((d + e*x)^(7/2)*(20*B*c^2*d^2 + 4*B*a*c*e^2 - 8*A*c^2*d*e))/(7*e^6) + (4*
c*(d + e*x)^(5/2)*(A*a*e^3 - 5*B*c*d^3 - 3*B*a*d*e^2 + 3*A*c*d^2*e))/(5*e^
6) + (2*B*c^2*(d + e*x)^(11/2))/(11*e^6) + (2*(a*e^2 + c*d^2)*(d + e*x)^(3
/2)*(B*a*e^2 + 5*B*c*d^2 - 4*A*c*d*e))/(3*e^6) + (2*c^2*(A*e - 5*B*d)*(d +
 e*x)^(9/2))/(9*e^6) + (2*(a*e^2 + c*d^2)^2*(A*e - B*d)*(d + e*x)^(1/2))/e
^6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.20

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\sqrt{ex+d}\,(315b\,c^2e^5x^5+385a\,c^2e^5x^4-350b\,c^2d\,e^4x^4+990abc\,e^5x^3-440a\,c^2d\,e^4x^3+400b\,c^2d^2e^3x^3+1}{}$$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(d + e*x)*(3465*a**3*e**5 - 2310*a**2*b*d*e**4 + 1155*a**2*b*e**5*x
 + 3696*a**2*c*d**2*e**3 - 1848*a**2*c*d*e**4*x + 1386*a**2*c*e**5*x**2 -
3168*a*b*c*d**3*e**2 + 1584*a*b*c*d**2*e**3*x - 1188*a*b*c*d*e**4*x**2 + 9
90*a*b*c*e**5*x**3 + 1408*a*c**2*d**4*e - 704*a*c**2*d**3*e**2*x + 528*a*c
**2*d**2*e**3*x**2 - 440*a*c**2*d*e**4*x**3 + 385*a*c**2*e**5*x**4 - 1280*
b*c**2*d**5 + 640*b*c**2*d**4*e*x - 480*b*c**2*d**3*e**2*x**2 + 400*b*c**2
*d**2*e**3*x**3 - 350*b*c**2*d*e**4*x**4 + 315*b*c**2*e**5*x**5))/(3465*e*
*6)
```

3.113 $\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 214

$$\begin{aligned}\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{3/2}}\,dx &= \frac{2(Bd-Ae)\left(cd^2+ae^2\right)^2}{e^6\sqrt{d+ex}}\\ &+\frac{2(cd^2+ae^2)\left(5Bcd^2-4Acde+aBe^2\right)\sqrt{d+ex}}{e^6}\\ &-\frac{4c(5Bcd^3-3Acd^2e+3aBde^2-aAe^3)\,(d+ex)^{3/2}}{3e^6}\\ &+\frac{4c(5Bcd^2-2Acde+aBe^2)\,(d+ex)^{5/2}}{5e^6}\\ &-\frac{2c^2(5Bd-Ae)(d+ex)^{7/2}}{7e^6}+\frac{2Bc^2(d+ex)^{9/2}}{9e^6}\end{aligned}$$

output
```
2*(-A*e+B*d)*(a*e^2+c*d^2)^2/e^6/(e*x+d)^(1/2)+2*(a*e^2+c*d^2)*(-4*A*c*d*e
+B*a*e^2+5*B*c*d^2)*(e*x+d)^(1/2)/e^6-4/3*c*(-A*a*e^3-3*A*c*d^2*e+3*B*a*d*
e^2+5*B*c*d^3)*(e*x+d)^(3/2)/e^6+4/5*c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)*(e*x
+d)^(5/2)/e^6-2/7*c^2*(-A*e+5*B*d)*(e*x+d)^(7/2)/e^6+2/9*B*c^2*(e*x+d)^(9/
2)/e^6
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{3/2}}\,dx = \frac{-6Ae(105a^2e^4+70ace^2(8d^2+4dex-e^2x^2)+3c^2(128d^4+64d^3ex-16d^2e^2$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^(3/2),x]
```

output
```
(-6*A*e*(105*a^2*e^4 + 70*a*c*e^2*(8*d^2 + 4*d*e*x - e^2*x^2) + 3*c^2*(128
*d^4 + 64*d^3*e*x - 16*d^2*e^2*x^2 + 8*d*e^3*x^3 - 5*e^4*x^4)) + 2*B*(315*
a^2*e^4*(2*d + e*x) + 126*a*c*e^2*(16*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*
x^3) + 5*c^2*(256*d^5 + 128*d^4*e*x - 32*d^3*e^2*x^2 + 16*d^2*e^3*x^3 - 10
*d*e^4*x^4 + 7*e^5*x^5)))/(315*e^6*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2(A+Bx)}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{2c(d+ex)^{3/2}\left(-aBe^2+2Acde-5Bcd^2\right)}{e^5}+\frac{\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^5\sqrt{d+ex}}+\frac{\left(ae^2+cd^2\right)^2(Ae-}{e^5(d+ex)^{3/2}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c(d+ex)^{5/2}\left(aBe^2-2Acde+5Bcd^2\right)}{5e^6}+\frac{2\sqrt{d+ex}(ae^2+cd^2)\left(aBe^2-4Acde+5Bcd^2\right)}{e^6}+\frac{2\left(ae^2+cd^2\right)^2(Bd-Ae)}{e^6\sqrt{d+ex}}-\frac{4c(d+ex)^{3/2}\left(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3\right)}{3e^6}-\frac{2c^2(d+ex)^{7/2}(5Bd-Ae)}{7e^6}+\frac{2Bc^2(d+ex)^{9/2}}{9e^6}$$

input
```
Int[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^(3/2),x]
```

output
```
(2*(B*d - A*e)*(c*d^2 + a*e^2)^2)/(e^6*Sqrt[d + e*x]) + (2*(c*d^2 + a*e^2)
*(5*B*c*d^2 - 4*A*c*d*e + a*B*e^2)*Sqrt[d + e*x])/e^6 - (4*c*(5*B*c*d^3 -
3*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*(d + e*x)^(3/2))/(3*e^6) + (4*c*(5*B*
c*d^2 - 2*A*c*d*e + a*B*e^2)*(d + e*x)^(5/2))/(5*e^6) - (2*c^2*(5*B*d - A*
e)*(d + e*x)^(7/2))/(7*e^6) + (2*B*c^2*(d + e*x)^(9/2))/(9*e^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{\left(\left(70B\,x^5+90A\,x^4\right)c^2+420\left(\frac{3Bx}{5}+A\right)x^2ac-630a^2\left(-Bx+A\right)\right)e^5-1680d\left(\left(\frac{5}{84}B\,x^4+\frac{3}{35}A\,x^3\right)c^2+ax\left(\frac{3Bx}{10}+A\right)c-\frac{3a^2}{4}\right.}{31}$
risch	$-\frac{2\left(-35e^4B\,c^2x^4-45e^4A\,c^2x^3+85d\,e^3B\,c^2x^3+117d\,e^3A\,c^2x^2-126e^4Bac\,x^2-165d^2e^2c^2B\,x^2-210e^4Aacx-261d^2e^2\right.}{315e}$
gosper	$-\frac{2\left(-35B\,x^5c^2e^5-45A\,x^4c^2e^5+50B\,x^4c^2d\,e^4+72A\,x^3c^2d\,e^4-126B\,x^3ac\,e^5-80B\,x^3c^2d^2e^3-210A\,x^2ac\,e^5-144A\,x^2\right.}{}$
trager	$-\frac{2\left(-35B\,x^5c^2e^5-45A\,x^4c^2e^5+50B\,x^4c^2d\,e^4+72A\,x^3c^2d\,e^4-126B\,x^3ac\,e^5-80B\,x^3c^2d^2e^3-210A\,x^2ac\,e^5-144A\,x^2\right.}{}$
orering	$-\frac{2\left(-35B\,x^5c^2e^5-45A\,x^4c^2e^5+50B\,x^4c^2d\,e^4+72A\,x^3c^2d\,e^4-126B\,x^3ac\,e^5-80B\,x^3c^2d^2e^3-210A\,x^2ac\,e^5-144A\,x^2\right.}{}$
derivativedivides	$\frac{\frac{2B\,c^2(ex+d)^{\frac{9}{2}}}{9}+\frac{2A\,c^2e(ex+d)^{\frac{7}{2}}}{7}-\frac{10B\,c^2d(ex+d)^{\frac{7}{2}}}{7}-\frac{8A\,c^2de(ex+d)^{\frac{5}{2}}}{5}+\frac{4Bac\,e^2(ex+d)^{\frac{5}{2}}}{5}+4B\,c^2d^2(ex+d)^{\frac{5}{2}}+\frac{4Aac\,e^3(ex+}{3}}{}$
default	$\frac{\frac{2B\,c^2(ex+d)^{\frac{9}{2}}}{9}+\frac{2A\,c^2e(ex+d)^{\frac{7}{2}}}{7}-\frac{10B\,c^2d(ex+d)^{\frac{7}{2}}}{7}-\frac{8A\,c^2de(ex+d)^{\frac{5}{2}}}{5}+\frac{4Bac\,e^2(ex+d)^{\frac{5}{2}}}{5}+4B\,c^2d^2(ex+d)^{\frac{5}{2}}+\frac{4Aac\,e^3(ex+}{3}}{}$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(((70*B*x^5+90*A*x^4)*c^2+420*(3/5*B*x+A)*x^2*a*c-630*a^2*(-B*x+A))*
e^5-1680*d*((5/84*B*x^4+3/35*A*x^3)*c^2+a*x*(3/10*B*x+A)*c-3/4*a^2*B)*e^4-
3360*c*d^2*(-3/35*(5/9*B*x+A)*x^2*c+a*(-3/5*B*x+A))*e^3-1152*c*d^3*(x*(5/1
8*B*x+A)*c-7/2*B*a)*e^2-2304*(-5/9*B*x+A)*c^2*d^4*e+2560*B*c^2*d^5)/(e*x+d
)^(1/2)/e^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.20

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{3/2}}\,dx = \frac{2\left(35\,Bc^2e^5x^5+1280\,Bc^2d^5-1152\,Ac^2d^4e+2016\,Bacd^3e^2-1680\,Aacd^2e^3\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*B*c^2*e^5*x^5 + 1280*B*c^2*d^5 - 1152*A*c^2*d^4*e + 2016*B*a*c*d
^3*e^2 - 1680*A*a*c*d^2*e^3 + 630*B*a^2*d*e^4 - 315*A*a^2*e^5 - 5*(10*B*c^
2*d*e^4 - 9*A*c^2*e^5)*x^4 + 2*(40*B*c^2*d^2*e^3 - 36*A*c^2*d*e^4 + 63*B*a
*c*e^5)*x^3 - 2*(80*B*c^2*d^3*e^2 - 72*A*c^2*d^2*e^3 + 126*B*a*c*d*e^4 - 1
05*A*a*c*e^5)*x^2 + (640*B*c^2*d^4*e - 576*A*c^2*d^3*e^2 + 1008*B*a*c*d^2*
e^3 - 840*A*a*c*d*e^4 + 315*B*a^2*e^5)*x)*sqrt(e*x + d)/(e^7*x + d*e^6)
```

Sympy [A] (verification not implemented)

Time = 6.50 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.51

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{Bc^2(d+ex)^{\frac{9}{2}}}{9e^5}+\frac{(d+ex)^{\frac{7}{2}}\left(Ac^2e-5Bc^2d\right)}{7e^5}+\frac{(d+ex)^{\frac{5}{2}}\left(-4Ac^2de+2Bace^2+10Bc^2d^2\right)}{5e^5}+\frac{(d+ex)^{\frac{3}{2}}\cdot\left(2Aace^3+6A\right.}{}\right.}{} \\ \frac{Aa^2x+\frac{2Aacx^3}{3}+\frac{Ac^2x^5}{5}+\frac{Ba^2x^2}{2}+\frac{Bacx^4}{2}+\frac{Bc^2x^6}{6}}{d^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**(3/2),x)
```

output
```
Piecewise((2*(B*c**2*(d + e*x)**(9/2)/(9*e**5) + (d + e*x)**(7/2)*(A*c**2*
e - 5*B*c**2*d)/(7*e**5) + (d + e*x)**(5/2)*(-4*A*c**2*d*e + 2*B*a*c*e**2
+ 10*B*c**2*d**2)/(5*e**5) + (d + e*x)**(3/2)*(2*A*a*c*e**3 + 6*A*c**2*d**
2*e - 6*B*a*c*d*e**2 - 10*B*c**2*d**3)/(3*e**5) + sqrt(d + e*x)*(-4*A*a*c*
d*e**3 - 4*A*c**2*d**3*e + B*a**2*e**4 + 6*B*a*c*d**2*e**2 + 5*B*c**2*d**4
)/e**5 + (-A*e + B*d)*(a*e**2 + c*d**2)**2/(e**5*sqrt(d + e*x)))/e, Ne(e,
0)), ((A*a**2*x + 2*A*a*c*x**3/3 + A*c**2*x**5/5 + B*a**2*x**2/2 + B*a*c*x
**4/2 + B*c**2*x**6/6)/d**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.20

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{35\,(ex+d)^{\frac{9}{2}}Bc^2-45\,(5\,Bc^2d-Ac^2e)(ex+d)^{\frac{7}{2}}+126\,(5\,Bc^2d^2-2\,Ac^2de+Bace^2)(ex+d)^{\frac{5}{2}}-210\,(5\,Bc^2}{}\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
2/315*((35*(e*x + d)^(9/2)*B*c^2 - 45*(5*B*c^2*d - A*c^2*e)*(e*x + d)^(7/2
) + 126*(5*B*c^2*d^2 - 2*A*c^2*d*e + B*a*c*e^2)*(e*x + d)^(5/2) - 210*(5*B
*c^2*d^3 - 3*A*c^2*d^2*e + 3*B*a*c*d*e^2 - A*a*c*e^3)*(e*x + d)^(3/2) + 31
5*(5*B*c^2*d^4 - 4*A*c^2*d^3*e + 6*B*a*c*d^2*e^2 - 4*A*a*c*d*e^3 + B*a^2*e
^4)*sqrt(e*x + d))/e^5 + 315*(B*c^2*d^5 - A*c^2*d^4*e + 2*B*a*c*d^3*e^2 -
2*A*a*c*d^2*e^3 + B*a^2*d*e^4 - A*a^2*e^5)/(sqrt(e*x + d)*e^5))/e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.57

$$\int \frac{(A+Bx)(a+cx^2)^2}{(d+ex)^{3/2}}\,dx = \frac{2\,(Bc^2d^5 - Ac^2d^4e + 2\,Bacd^3e^2 - 2\,Aacd^2e^3 + Ba^2de^4 - Aa^2e^5)}{\sqrt{ex+d}e^6} + \frac{2\left(35\,(ex+d)^{\frac{9}{2}}Bc^2e^{48} - 225\,(ex+d)^{\frac{7}{2}}Bc^2de^{48} + 630\,(ex+d)^{\frac{5}{2}}Bc^2d^2e^{48} - 1050\,(ex+d)^{\frac{3}{2}}Bc^2d^3e^{48} + 15\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
2*(B*c^2*d^5 - A*c^2*d^4*e + 2*B*a*c*d^3*e^2 - 2*A*a*c*d^2*e^3 + B*a^2*d*e
^4 - A*a^2*e^5)/(sqrt(e*x + d)*e^6) + 2/315*(35*(e*x + d)^(9/2)*B*c^2*e^48
 - 225*(e*x + d)^(7/2)*B*c^2*d*e^48 + 630*(e*x + d)^(5/2)*B*c^2*d^2*e^48 -
 1050*(e*x + d)^(3/2)*B*c^2*d^3*e^48 + 1575*sqrt(e*x + d)*B*c^2*d^4*e^48 +
 45*(e*x + d)^(7/2)*A*c^2*e^49 - 252*(e*x + d)^(5/2)*A*c^2*d*e^49 + 630*(e
*x + d)^(3/2)*A*c^2*d^2*e^49 - 1260*sqrt(e*x + d)*A*c^2*d^3*e^49 + 126*(e*
x + d)^(5/2)*B*a*c*e^50 - 630*(e*x + d)^(3/2)*B*a*c*d*e^50 + 1890*sqrt(e*x
 + d)*B*a*c*d^2*e^50 + 210*(e*x + d)^(3/2)*A*a*c*e^51 - 1260*sqrt(e*x + d)
*A*a*c*d*e^51 + 315*sqrt(e*x + d)*B*a^2*e^52)/e^54
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^{3/2}}\,dx = \frac{(d+e\,x)^{5/2}\,(20\,B\,c^2\,d^2-8\,A\,c^2\,d\,e+4\,B\,a\,c\,e^2)}{5\,e^6}$$
$$-\frac{-2\,B\,a^2\,d\,e^4+2\,A\,a^2\,e^5-4\,B\,a\,c\,d^3\,e^2+4\,A\,a\,c\,d^2\,e^3-2\,B\,c^2\,d^5+2\,A\,c^2\,d^4\,e}{e^6\,\sqrt{d+e\,x}}$$
$$+\frac{4\,c\,(d+e\,x)^{3/2}\,(-5\,B\,c\,d^3+3\,A\,c\,d^2\,e-3\,B\,a\,d\,e^2+A\,a\,e^3)}{3\,e^6}$$
$$+\frac{2\,B\,c^2\,(d+e\,x)^{9/2}}{9\,e^6}+\frac{2\,(c\,d^2+a\,e^2)\,\sqrt{d+e\,x}\,(5\,B\,c\,d^2-4\,A\,c\,d\,e+B\,a\,e^2)}{e^6}$$
$$+\frac{2\,c^2\,(A\,e-5\,B\,d)\,(d+e\,x)^{7/2}}{7\,e^6}$$

input
```
int(((a + c*x^2)^2*(A + B*x))/(d + e*x)^(3/2),x)
```

output
```
((d + e*x)^(5/2)*(20*B*c^2*d^2 + 4*B*a*c*e^2 - 8*A*c^2*d*e))/(5*e^6) - (2*
A*a^2*e^5 - 2*B*c^2*d^5 - 2*B*a^2*d*e^4 + 2*A*c^2*d^4*e + 4*A*a*c*d^2*e^3
- 4*B*a*c*d^3*e^2)/(e^6*(d + e*x)^(1/2)) + (4*c*(d + e*x)^(3/2)*(A*a*e^3 -
 5*B*c*d^3 - 3*B*a*d*e^2 + 3*A*c*d^2*e))/(3*e^6) + (2*B*c^2*(d + e*x)^(9/2
))/(9*e^6) + (2*(a*e^2 + c*d^2)*(d + e*x)^(1/2)*(B*a*e^2 + 5*B*c*d^2 - 4*A
*c*d*e))/e^6 + (2*c^2*(A*e - 5*B*d)*(d + e*x)^(7/2))/(7*e^6)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^{3/2}}\,dx = \frac{\frac{2}{9}b\,c^2e^5x^5+\frac{2}{7}a\,c^2e^5x^4-\frac{20}{63}b\,c^2d\,e^4x^4+\frac{4}{5}abc\,e^5x^3-\frac{16}{35}a\,c^2d\,e^4x^3+\frac{32}{63}b\,c^2d^2e^3x}{}$$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^(3/2),x)
```

output
```
(2*( - 315*a**3*e**5 + 630*a**2*b*d*e**4 + 315*a**2*b*e**5*x - 1680*a**2*c
*d**2*e**3 - 840*a**2*c*d*e**4*x + 210*a**2*c*e**5*x**2 + 2016*a*b*c*d**3*
e**2 + 1008*a*b*c*d**2*e**3*x - 252*a*b*c*d*e**4*x**2 + 126*a*b*c*e**5*x**
3 - 1152*a*c**2*d**4*e - 576*a*c**2*d**3*e**2*x + 144*a*c**2*d**2*e**3*x**
2 - 72*a*c**2*d*e**4*x**3 + 45*a*c**2*e**5*x**4 + 1280*b*c**2*d**5 + 640*b
*c**2*d**4*e*x - 160*b*c**2*d**3*e**2*x**2 + 80*b*c**2*d**2*e**3*x**3 - 50
*b*c**2*d*e**4*x**4 + 35*b*c**2*e**5*x**5))/(315*sqrt(d + e*x)*e**6)
```

3.114 $\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 214

$$\begin{aligned}\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{5/2}}\,dx = & \frac{2(Bd-Ae)\left(cd^2+ae^2\right)^2}{3e^6(d+ex)^{3/2}} \\ & -\frac{2\left(cd^2+ae^2\right)\left(5Bcd^2-4Acde+aBe^2\right)}{e^6\sqrt{d+ex}} \\ & -\frac{4c\left(5Bcd^3-3Acd^2e+3aBde^2-aAe^3\right)\sqrt{d+ex}}{e^6} \\ & +\frac{4c\left(5Bcd^2-2Acde+aBe^2\right)(d+ex)^{3/2}}{3e^6} \\ & -\frac{2c^2(5Bd-Ae)(d+ex)^{5/2}}{5e^6}+\frac{2Bc^2(d+ex)^{7/2}}{7e^6}\end{aligned}$$

output
```
2/3*(-A*e+B*d)*(a*e^2+c*d^2)^2/e^6/(e*x+d)^(3/2)-2*(a*e^2+c*d^2)*(-4*A*c*d
*e+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)^(1/2)-4*c*(-A*a*e^3-3*A*c*d^2*e+3*B*a*d*
e^2+5*B*c*d^3)*(e*x+d)^(1/2)/e^6+4/3*c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)*(e*x
+d)^(3/2)/e^6-2/5*c^2*(-A*e+5*B*d)*(e*x+d)^(5/2)/e^6+2/7*B*c^2*(e*x+d)^(7/
2)/e^6
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^{5/2}}\,dx = \frac{14Ae(-5a^2e^4+10ace^2(8d^2+12dex+3e^2x^2)+c^2(128d^4+192d^3ex+48d^2e$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^(5/2),x]
```

output
```
(14*A*e*(-5*a^2*e^4 + 10*a*c*e^2*(8*d^2 + 12*d*e*x + 3*e^2*x^2) + c^2*(128
*d^4 + 192*d^3*e*x + 48*d^2*e^2*x^2 - 8*d*e^3*x^3 + 3*e^4*x^4)) - 10*B*(7*
a^2*e^4*(2*d + 3*e*x) + 14*a*c*e^2*(16*d^3 + 24*d^2*e*x + 6*d*e^2*x^2 - e^
3*x^3) + c^2*(256*d^5 + 384*d^4*e*x + 96*d^3*e^2*x^2 - 16*d^2*e^3*x^3 + 6*
d*e^4*x^4 - 3*e^5*x^5)))/(105*e^6*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2\,(A+Bx)}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{2c\sqrt{d+ex}\,(-aBe^2+2Acde-5Bcd^2)}{e^5}+\frac{(ae^2+cd^2)\,(aBe^2-4Acde+5Bcd^2)}{e^5(d+ex)^{3/2}}+\frac{(ae^2+cd^2)^2\,(Ae-Bd}{e^5(d+ex)^{5/2}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c(d+ex)^{3/2}\left(aBe^2-2Acde+5Bcd^2\right)}{3e^6}-\frac{2\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^6\sqrt{d+ex}}+\frac{2\left(ae^2+cd^2\right)^2(Bd-Ae)}{3e^6(d+ex)^{3/2}}-\frac{4c\sqrt{d+ex}\left(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3\right)}{e^6}-\frac{2c^2(d+ex)^{5/2}(5Bd-Ae)}{5e^6}+\frac{2Bc^2(d+ex)^{7/2}}{7e^6}$$

input
```
Int[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^(5/2),x]
```

output
```
(2*(B*d - A*e)*(c*d^2 + a*e^2)^2)/(3*e^6*(d + e*x)^(3/2)) - (2*(c*d^2 + a*
e^2)*(5*B*c*d^2 - 4*A*c*d*e + a*B*e^2))/(e^6*Sqrt[d + e*x]) - (4*c*(5*B*c*
d^3 - 3*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*Sqrt[d + e*x])/e^6 + (4*c*(5*B*
c*d^2 - 2*A*c*d*e + a*B*e^2)*(d + e*x)^(3/2))/(3*e^6) - (2*c^2*(5*B*d - A*
e)*(d + e*x)^(5/2))/(5*e^6) + (2*B*c^2*(d + e*x)^(7/2))/(7*e^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{\left(\left(30B\,x^5+42A\,x^4\right)c^2+420x^2a\left(\frac{Bx}{3}+A\right)c-70a^2(3Bx+A)\right)e^5+1680d\left(\left(-\frac{1}{28}B\,x^4-\frac{1}{15}A\,x^3\right)c^2+ax\left(-\frac{Bx}{2}+A\right)c-\frac{a^2B}{12}\right.}{10}$
risch	$\frac{2c(15Bc\,x^3e^3+21A\,x^2c\,e^3-60B\,x^2cd\,e^2-98Axcd\,e^2+70Bxa\,e^3+185Bxc\,d^2e+210Aa\,e^3+511Ac\,d^2e-560Bad\,e^2-790}{105e^6}$
gosper	$-\frac{2(-15B\,x^5c^2e^5-21A\,x^4c^2e^5+30B\,x^4c^2d\,e^4+56A\,x^3c^2d\,e^4-70B\,x^3ac\,e^5-80B\,x^3c^2d^2e^3-210A\,x^2ac\,e^5-336A\,x^2c}{}$
trager	$-\frac{2(-15B\,x^5c^2e^5-21A\,x^4c^2e^5+30B\,x^4c^2d\,e^4+56A\,x^3c^2d\,e^4-70B\,x^3ac\,e^5-80B\,x^3c^2d^2e^3-210A\,x^2ac\,e^5-336A\,x^2c}{}$
orering	$-\frac{2(-15B\,x^5c^2e^5-21A\,x^4c^2e^5+30B\,x^4c^2d\,e^4+56A\,x^3c^2d\,e^4-70B\,x^3ac\,e^5-80B\,x^3c^2d^2e^3-210A\,x^2ac\,e^5-336A\,x^2c}{}$
derivativedivides	$\frac{\frac{2B\,c^2(ex+d)^{\frac{7}{2}}}{7}+\frac{2A\,c^2e(ex+d)^{\frac{5}{2}}}{5}-2B\,c^2d(ex+d)^{\frac{5}{2}}-\frac{8A\,c^2de(ex+d)^{\frac{3}{2}}}{3}+\frac{4Bac\,e^2(ex+d)^{\frac{3}{2}}}{3}+\frac{20B\,c^2d^2(ex+d)^{\frac{3}{2}}}{3}+4Aac\,e^3\sqrt{ex}}{}$
default	$\frac{\frac{2B\,c^2(ex+d)^{\frac{7}{2}}}{7}+\frac{2A\,c^2e(ex+d)^{\frac{5}{2}}}{5}-2B\,c^2d(ex+d)^{\frac{5}{2}}-\frac{8A\,c^2de(ex+d)^{\frac{3}{2}}}{3}+\frac{4Bac\,e^2(ex+d)^{\frac{3}{2}}}{3}+\frac{20B\,c^2d^2(ex+d)^{\frac{3}{2}}}{3}+4Aac\,e^3\sqrt{ex}}{}$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(((30*B*x^5+42*A*x^4)*c^2+420*x^2*a*(1/3*B*x+A)*c-70*a^2*(3*B*x+A))*
e^5+1680*d*((-1/28*B*x^4-1/15*A*x^3)*c^2+a*x*(-1/2*B*x+A)*c-1/12*a^2*B)*e^
4+1120*c*d^2*((1/7*B*x^3+3/5*A*x^2)*c+a*(-3*B*x+A))*e^3+2688*c*d^3*(x*(-5/
14*B*x+A)*c-5/6*B*a)*e^2+1792*c^2*d^4*(-15/7*B*x+A)*e-2560*B*c^2*d^5)/(e*x
+d)^(3/2)/e^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.26

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{5/2}}\,dx = \frac{2\left(15\,Bc^2e^5x^5-1280\,Bc^2d^5+896\,Ac^2d^4e-1120\,Bacd^3e^2+560\,Aacd^2e^3-7\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
2/105*(15*B*c^2*e^5*x^5 - 1280*B*c^2*d^5 + 896*A*c^2*d^4*e - 1120*B*a*c*d^
3*e^2 + 560*A*a*c*d^2*e^3 - 70*B*a^2*d*e^4 - 35*A*a^2*e^5 - 3*(10*B*c^2*d*
e^4 - 7*A*c^2*e^5)*x^4 + 2*(40*B*c^2*d^2*e^3 - 28*A*c^2*d*e^4 + 35*B*a*c*e
^5)*x^3 - 6*(80*B*c^2*d^3*e^2 - 56*A*c^2*d^2*e^3 + 70*B*a*c*d*e^4 - 35*A*a
*c*e^5)*x^2 - 3*(640*B*c^2*d^4*e - 448*A*c^2*d^3*e^2 + 560*B*a*c*d^2*e^3 -
 280*A*a*c*d*e^4 + 35*B*a^2*e^5)*x)*sqrt(e*x + d)/(e^8*x^2 + 2*d*e^7*x + d
^2*e^6)
```

Sympy [A] (verification not implemented)

Time = 6.76 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.41

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{Bc^2(d+ex)^{\frac{7}{2}}}{7e^5}+\frac{(d+ex)^{\frac{5}{2}}\left(Ac^2e-5Bc^2d\right)}{5e^5}+\frac{(d+ex)^{\frac{3}{2}}\left(-4Ac^2de+2Bace^2+10Bc^2d^2\right)}{3e^5}+\frac{\sqrt{d+ex}\left(2Aace^3+6Ac^2\right.}{}\right.}{e} \\ \frac{Aa^2x+\frac{2Aacx^3}{3}+\frac{Ac^2x^5}{5}+\frac{Ba^2x^2}{2}+\frac{Bacx^4}{2}+\frac{Bc^2x^6}{6}}{d^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**(5/2),x)
```

output
```
Piecewise((2*(B*c**2*(d + e*x)**(7/2)/(7*e**5) + (d + e*x)**(5/2)*(A*c**2*
e - 5*B*c**2*d)/(5*e**5) + (d + e*x)**(3/2)*(-4*A*c**2*d*e + 2*B*a*c*e**2
+ 10*B*c**2*d**2)/(3*e**5) + sqrt(d + e*x)*(2*A*a*c*e**3 + 6*A*c**2*d**2*e
 - 6*B*a*c*d*e**2 - 10*B*c**2*d**3)/e**5 - (a*e**2 + c*d**2)*(-4*A*c*d*e +
 B*a*e**2 + 5*B*c*d**2)/(e**5*sqrt(d + e*x)) + (-A*e + B*d)*(a*e**2 + c*d*
*2)**2/(3*e**5*(d + e*x)**(3/2)))/e, Ne(e, 0)), ((A*a**2*x + 2*A*a*c*x**3/
3 + A*c**2*x**5/5 + B*a**2*x**2/2 + B*a*c*x**4/2 + B*c**2*x**6/6)/d**(5/2)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{5/2}}\,dx = \frac{2\left(\frac{15\,(ex+d)^{\frac{7}{2}}Bc^2-21\,(5\,Bc^2d-Ac^2e)(ex+d)^{\frac{5}{2}}+70\,(5\,Bc^2d^2-2\,Ac^2de+Bace^2)(ex+d)^{\frac{3}{2}}-210\,(5\,Bc^2d}{e^5}\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
2/105*((15*(e*x + d)^(7/2)*B*c^2 - 21*(5*B*c^2*d - A*c^2*e)*(e*x + d)^(5/2
) + 70*(5*B*c^2*d^2 - 2*A*c^2*d*e + B*a*c*e^2)*(e*x + d)^(3/2) - 210*(5*B*
c^2*d^3 - 3*A*c^2*d^2*e + 3*B*a*c*d*e^2 - A*a*c*e^3)*sqrt(e*x + d))/e^5 +
35*(B*c^2*d^5 - A*c^2*d^4*e + 2*B*a*c*d^3*e^2 - 2*A*a*c*d^2*e^3 + B*a^2*d*
e^4 - A*a^2*e^5 - 3*(5*B*c^2*d^4 - 4*A*c^2*d^3*e + 6*B*a*c*d^2*e^2 - 4*A*a
*c*d*e^3 + B*a^2*e^4)*(e*x + d))/((e*x + d)^(3/2)*e^5))/e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.50

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{5/2}}\,dx =$$

$$-\frac{2\,(15\,(ex+d)Bc^2d^4-Bc^2d^5-12\,(ex+d)Ac^2d^3e+Ac^2d^4e+18\,(ex+d)Bacd^2e^2-2\,Bacd^3e^2-12\,(ex}{3\,(ex+d)^{\frac{3}{2}}e^6}$$

$$+\frac{2\left(15\,(ex+d)^{\frac{7}{2}}Bc^2e^{36}-105\,(ex+d)^{\frac{5}{2}}Bc^2de^{36}+350\,(ex+d)^{\frac{3}{2}}Bc^2d^2e^{36}-1050\,\sqrt{ex+d}Bc^2d^3e^{36}+21\,(\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(15*(e*x + d)*B*c^2*d^4 - B*c^2*d^5 - 12*(e*x + d)*A*c^2*d^3*e + A*c^
2*d^4*e + 18*(e*x + d)*B*a*c*d^2*e^2 - 2*B*a*c*d^3*e^2 - 12*(e*x + d)*A*a*
c*d*e^3 + 2*A*a*c*d^2*e^3 + 3*(e*x + d)*B*a^2*e^4 - B*a^2*d*e^4 + A*a^2*e^
5)/((e*x + d)^(3/2)*e^6) + 2/105*(15*(e*x + d)^(7/2)*B*c^2*e^36 - 105*(e*x
 + d)^(5/2)*B*c^2*d*e^36 + 350*(e*x + d)^(3/2)*B*c^2*d^2*e^36 - 1050*sqrt(
e*x + d)*B*c^2*d^3*e^36 + 21*(e*x + d)^(5/2)*A*c^2*e^37 - 140*(e*x + d)^(3
/2)*A*c^2*d*e^37 + 630*sqrt(e*x + d)*A*c^2*d^2*e^37 + 70*(e*x + d)^(3/2)*B
*a*c*e^38 - 630*sqrt(e*x + d)*B*a*c*d*e^38 + 210*sqrt(e*x + d)*A*a*c*e^39)
/e^42
```

Mupad [B] (verification not implemented)

Time = 6.44 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{5/2}}\,dx = \frac{(d+e\,x)^{3/2}\left(20\,B\,c^2\,d^2-8\,A\,c^2\,d\,e+4\,B\,a\,c\,e^2\right)}{3\,e^6}$$
$$-\frac{(d+e\,x)\,\left(2\,B\,a^2\,e^4+12\,B\,a\,c\,d^2\,e^2-8\,A\,a\,c\,d\,e^3+10\,B\,c^2\,d^4-8\,A\,c^2\,d^3\,e\right)+\frac{2\,A\,a^2\,e^5}{3}-\frac{2\,B\,c^2\,d^5}{3}-\frac{2\,B\,a^2\,d\,e}{3}}{e^6\,(d+e\,x)^{3/2}}$$
$$+\frac{4\,c\,\sqrt{d+e\,x}\,\left(-5\,B\,c\,d^3+3\,A\,c\,d^2\,e-3\,B\,a\,d\,e^2+A\,a\,e^3\right)}{e^6}$$
$$+\frac{2\,B\,c^2\,(d+e\,x)^{7/2}}{7\,e^6}+\frac{2\,c^2\,(A\,e-5\,B\,d)\,(d+e\,x)^{5/2}}{5\,e^6}$$

input
```
int(((a + c*x^2)^2*(A + B*x))/(d + e*x)^(5/2),x)
```

output
```
((d + e*x)^(3/2)*(20*B*c^2*d^2 + 4*B*a*c*e^2 - 8*A*c^2*d*e))/(3*e^6) - ((d
 + e*x)*(2*B*a^2*e^4 + 10*B*c^2*d^4 - 8*A*c^2*d^3*e - 8*A*a*c*d*e^3 + 12*B
*a*c*d^2*e^2) + (2*A*a^2*e^5)/3 - (2*B*c^2*d^5)/3 - (2*B*a^2*d*e^4)/3 + (2
*A*c^2*d^4*e)/3 + (4*A*a*c*d^2*e^3)/3 - (4*B*a*c*d^3*e^2)/3)/(e^6*(d + e*x
)^(3/2)) + (4*c*(d + e*x)^(1/2)*(A*a*e^3 - 5*B*c*d^3 - 3*B*a*d*e^2 + 3*A*c
*d^2*e))/e^6 + (2*B*c^2*(d + e*x)^(7/2))/(7*e^6) + (2*c^2*(A*e - 5*B*d)*(d
 + e*x)^(5/2))/(5*e^6)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{5/2}}\,dx = \frac{\frac{2}{7}b\,c^2e^5x^5+\frac{2}{5}a\,c^2e^5x^4-\frac{4}{7}b\,c^2d\,e^4x^4+\frac{4}{3}abc\,e^5x^3-\frac{16}{15}a\,c^2d\,e^4x^3+\frac{32}{21}b\,c^2d^2e^3x^3}{}$$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^(5/2),x)
```

output
```
(2*( - 35*a**3*e**5 - 70*a**2*b*d*e**4 - 105*a**2*b*e**5*x + 560*a**2*c*d*
*2*e**3 + 840*a**2*c*d*e**4*x + 210*a**2*c*e**5*x**2 - 1120*a*b*c*d**3*e**
2 - 1680*a*b*c*d**2*e**3*x - 420*a*b*c*d*e**4*x**2 + 70*a*b*c*e**5*x**3 +
896*a*c**2*d**4*e + 1344*a*c**2*d**3*e**2*x + 336*a*c**2*d**2*e**3*x**2 -
56*a*c**2*d*e**4*x**3 + 21*a*c**2*e**5*x**4 - 1280*b*c**2*d**5 - 1920*b*c*
*2*d**4*e*x - 480*b*c**2*d**3*e**2*x**2 + 80*b*c**2*d**2*e**3*x**3 - 30*b*
c**2*d*e**4*x**4 + 15*b*c**2*e**5*x**5))/(105*sqrt(d + e*x)*e**6*(d + e*x)
)
```

3.115 $\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 214

$$\begin{aligned}\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{7/2}}\,dx &= \frac{2(Bd-Ae)\left(cd^2+ae^2\right)^2}{5e^6(d+ex)^{5/2}}\\
&- \frac{2(cd^2+ae^2)\left(5Bcd^2-4Acde+aBe^2\right)}{3e^6(d+ex)^{3/2}}\\
&+ \frac{4c(5Bcd^3-3Acd^2e+3aBde^2-aAe^3)}{e^6\sqrt{d+ex}} + \frac{4c(5Bcd^2-2Acde+aBe^2)\sqrt{d+ex}}{e^6}\\
&- \frac{2c^2(5Bd-Ae)(d+ex)^{3/2}}{3e^6} + \frac{2Bc^2(d+ex)^{5/2}}{5e^6}\end{aligned}$$

output
```
2/5*(-A*e+B*d)*(a*e^2+c*d^2)^2/e^6/(e*x+d)^(5/2)-2/3*(a*e^2+c*d^2)*(-4*A*c
*d*e+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)^(3/2)+4*c*(-A*a*e^3-3*A*c*d^2*e+3*B*a*
d*e^2+5*B*c*d^3)/e^6/(e*x+d)^(1/2)+4*c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)*(e*x
+d)^(1/2)/e^6-2/3*c^2*(-A*e+5*B*d)*(e*x+d)^(3/2)/e^6+2/5*B*c^2*(e*x+d)^(5/
2)/e^6
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{7/2}}\,dx =$$
$$-\frac{2\left(3a^2Ae^5+a^2Be^4(2d+5ex)+2aAce^3\left(8d^2+20dex+15e^2x^2\right)-6aBce^2\left(16d^3+40d^2ex+30de^2x^2+5\right.\right.}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^(7/2),x]
```

output
```
(-2*(3*a^2*A*e^5 + a^2*B*e^4*(2*d + 5*e*x) + 2*a*A*c*e^3*(8*d^2 + 20*d*e*x
 + 15*e^2*x^2) - 6*a*B*c*e^2*(16*d^3 + 40*d^2*e*x + 30*d*e^2*x^2 + 5*e^3*x
^3) + A*c^2*e*(128*d^4 + 320*d^3*e*x + 240*d^2*e^2*x^2 + 40*d*e^3*x^3 - 5*
e^4*x^4) - B*c^2*(256*d^5 + 640*d^4*e*x + 480*d^3*e^2*x^2 + 80*d^2*e^3*x^3
 - 10*d*e^4*x^4 + 3*e^5*x^5)))/(15*e^6*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2(A+Bx)}{(d+ex)^{7/2}}\,dx$$
$$\Big\downarrow \ 652$$
$$\int \left(-\frac{2c\left(-aBe^2+2Acde-5Bcd^2\right)}{e^5\sqrt{d+ex}}+\frac{\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^5(d+ex)^{5/2}}+\frac{\left(ae^2+cd^2\right)^2(Ae-Bd)}{e^5(d+ex)^{7/2}}+\frac{2c(\cdots}{}\right.$$
$$\Big\downarrow \ 2009$$

$$\frac{4c\sqrt{d+ex}\left(aBe^2-2Acde+5Bcd^2\right)}{e^6}-\frac{2\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{3e^6(d+ex)^{3/2}}+\frac{2\left(ae^2+cd^2\right)^2(Bd-Ae)}{5e^6(d+ex)^{5/2}}+\frac{4c\left(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3\right)}{e^6\sqrt{d+ex}}-\frac{2c^2(d+ex)^{3/2}(5Bd-Ae)}{3e^6}+\frac{2Bc^2(d+ex)^{5/2}}{5e^6}$$

input
```
Int[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^(7/2),x]
```

output
```
(2*(B*d - A*e)*(c*d^2 + a*e^2)^2)/(5*e^6*(d + e*x)^(5/2)) - (2*(c*d^2 + a*
e^2)*(5*B*c*d^2 - 4*A*c*d*e + a*B*e^2))/(3*e^6*(d + e*x)^(3/2)) + (4*c*(5*
B*c*d^3 - 3*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(e^6*Sqrt[d + e*x]) + (4*c
*(5*B*c*d^2 - 2*A*c*d*e + a*B*e^2)*Sqrt[d + e*x])/e^6 - (2*c^2*(5*B*d - A*
e)*(d + e*x)^(3/2))/(3*e^6) + (2*B*c^2*(d + e*x)^(5/2))/(5*e^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{\left(\left(6B\,x^{5}+10A\,x^{4}\right)c^{2}-60a\,x^{2}(-Bx+A)c-6a^{2}\left(\frac{5Bx}{3}+A\right)\right)e^{5}-80d\left(x^{3}\left(\frac{Bx}{4}+A\right)c^{2}+ax\left(-\frac{9Bx}{2}+A\right)c+\frac{a^{2}B}{20}\right)e^{4}-32c\,d}{15(ex+d)}$
gosper	$-\frac{2\left(-3B\,x^{5}c^{2}e^{5}-5A\,x^{4}c^{2}e^{5}+10B\,x^{4}c^{2}d\,e^{4}+40A\,x^{3}c^{2}d\,e^{4}-30B\,x^{3}ac\,e^{5}-80B\,x^{3}c^{2}d^{2}e^{3}+30A\,x^{2}ac\,e^{5}+240A\,x^{2}c^{2}d^{2}\right.}{}$
derivativedivides	$\frac{\frac{2B\,c^{2}(ex+d)^{\frac{5}{2}}}{5}+\frac{2A\,c^{2}e(ex+d)^{\frac{3}{2}}}{3}-\frac{10B\,c^{2}d(ex+d)^{\frac{3}{2}}}{3}-8A\,c^{2}de\sqrt{ex+d}+4Bac\,e^{2}\sqrt{ex+d}+20B\,c^{2}d^{2}\sqrt{ex+d}-\frac{2\left(A\,a^{2}e^{5}+2Aa\right.}{}}{}$
default	$\frac{\frac{2B\,c^{2}(ex+d)^{\frac{5}{2}}}{5}+\frac{2A\,c^{2}e(ex+d)^{\frac{3}{2}}}{3}-\frac{10B\,c^{2}d(ex+d)^{\frac{3}{2}}}{3}-8A\,c^{2}de\sqrt{ex+d}+4Bac\,e^{2}\sqrt{ex+d}+20B\,c^{2}d^{2}\sqrt{ex+d}-\frac{2\left(A\,a^{2}e^{5}+2Aa\right.}{}}{}$
trager	$-\frac{2\left(-3B\,x^{5}c^{2}e^{5}-5A\,x^{4}c^{2}e^{5}+10B\,x^{4}c^{2}d\,e^{4}+40A\,x^{3}c^{2}d\,e^{4}-30B\,x^{3}ac\,e^{5}-80B\,x^{3}c^{2}d^{2}e^{3}+30A\,x^{2}ac\,e^{5}+240A\,x^{2}c^{2}d^{2}\right.}{}$
orering	$-\frac{2\left(-3B\,x^{5}c^{2}e^{5}-5A\,x^{4}c^{2}e^{5}+10B\,x^{4}c^{2}d\,e^{4}+40A\,x^{3}c^{2}d\,e^{4}-30B\,x^{3}ac\,e^{5}-80B\,x^{3}c^{2}d^{2}e^{3}+30A\,x^{2}ac\,e^{5}+240A\,x^{2}c^{2}d^{2}\right.}{}$
risch	$-\frac{2c\left(-3e^{2}Bc\,x^{2}-5Ac\,e^{2}x+19Bcdex+55Acde-30Ba\,e^{2}-128Bc\,d^{2}\right)\sqrt{ex+d}}{15e^{6}}-\frac{2\left(30A\,x^{2}ac\,e^{5}+90A\,x^{2}c^{2}d^{2}e^{3}-90B\right.}{}$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(((6*B*x^5+10*A*x^4)*c^2-60*a*x^2*(-B*x+A)*c-6*a^2*(5/3*B*x+A))*e^5-8
0*d*(x^3*(1/4*B*x+A)*c^2+a*x*(-9/2*B*x+A)*c+1/20*a^2*B)*e^4-32*c*d^2*((-5*
B*x^3+15*A*x^2)*c+a*(-15*B*x+A))*e^3-640*c*d^3*(x*(-3/2*B*x+A)*c-3/10*B*a)
*e^2-256*c^2*d^4*(-5*B*x+A)*e+512*B*c^2*d^5)/(e*x+d)^(5/2)/e^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.31

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{7/2}}\,dx = \frac{2\left(3\,Bc^2e^5x^5+256\,Bc^2d^5-128\,Ac^2d^4e+96\,Bacd^3e^2-16\,Aacd^2e^3-2\,Ba^2d\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*c^2*e^5*x^5 + 256*B*c^2*d^5 - 128*A*c^2*d^4*e + 96*B*a*c*d^3*e^2
 - 16*A*a*c*d^2*e^3 - 2*B*a^2*d*e^4 - 3*A*a^2*e^5 - 5*(2*B*c^2*d*e^4 - A*c
^2*e^5)*x^4 + 10*(8*B*c^2*d^2*e^3 - 4*A*c^2*d*e^4 + 3*B*a*c*e^5)*x^3 + 30*
(16*B*c^2*d^3*e^2 - 8*A*c^2*d^2*e^3 + 6*B*a*c*d*e^4 - A*a*c*e^5)*x^2 + 5*(
128*B*c^2*d^4*e - 64*A*c^2*d^3*e^2 + 48*B*a*c*d^2*e^3 - 8*A*a*c*d*e^4 - B*
a^2*e^5)*x)*sqrt(e*x + d)/(e^9*x^3 + 3*d*e^8*x^2 + 3*d^2*e^7*x + d^3*e^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1426 vs. $2(226) = 452$.

Time = 0.78 (sec) , antiderivative size = 1426, normalized size of antiderivative = 6.66

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**(7/2),x)
```

output
```
Piecewise((-6*A*a**2*e**5/(15*d**2*e**6*sqrt(d + e*x) + 30*d*e**7*x*sqrt(d
 + e*x) + 15*e**8*x**2*sqrt(d + e*x)) - 32*A*a*c*d**2*e**3/(15*d**2*e**6*s
qrt(d + e*x) + 30*d*e**7*x*sqrt(d + e*x) + 15*e**8*x**2*sqrt(d + e*x)) - 8
0*A*a*c*d*e**4*x/(15*d**2*e**6*sqrt(d + e*x) + 30*d*e**7*x*sqrt(d + e*x) +
 15*e**8*x**2*sqrt(d + e*x)) - 60*A*a*c*e**5*x**2/(15*d**2*e**6*sqrt(d + e
*x) + 30*d*e**7*x*sqrt(d + e*x) + 15*e**8*x**2*sqrt(d + e*x)) - 256*A*c**2
*d**4*e/(15*d**2*e**6*sqrt(d + e*x) + 30*d*e**7*x*sqrt(d + e*x) + 15*e**8*
x**2*sqrt(d + e*x)) - 640*A*c**2*d**3*e**2*x/(15*d**2*e**6*sqrt(d + e*x) +
 30*d*e**7*x*sqrt(d + e*x) + 15*e**8*x**2*sqrt(d + e*x)) - 480*A*c**2*d**2
*e**3*x**2/(15*d**2*e**6*sqrt(d + e*x) + 30*d*e**7*x*sqrt(d + e*x) + 15*e*
*8*x**2*sqrt(d + e*x)) - 80*A*c**2*d*e**4*x**3/(15*d**2*e**6*sqrt(d + e*x)
 + 30*d*e**7*x*sqrt(d + e*x) + 15*e**8*x**2*sqrt(d + e*x)) + 10*A*c**2*e**
5*x**4/(15*d**2*e**6*sqrt(d + e*x) + 30*d*e**7*x*sqrt(d + e*x) + 15*e**8*x
**2*sqrt(d + e*x)) - 4*B*a**2*d*e**4/(15*d**2*e**6*sqrt(d + e*x) + 30*d*e*
*7*x*sqrt(d + e*x) + 15*e**8*x**2*sqrt(d + e*x)) - 10*B*a**2*e**5*x/(15*d*
*2*e**6*sqrt(d + e*x) + 30*d*e**7*x*sqrt(d + e*x) + 15*e**8*x**2*sqrt(d +
e*x)) + 192*B*a*c*d**3*e**2/(15*d**2*e**6*sqrt(d + e*x) + 30*d*e**7*x*sqrt
(d + e*x) + 15*e**8*x**2*sqrt(d + e*x)) + 480*B*a*c*d**2*e**3*x/(15*d**2*e
**6*sqrt(d + e*x) + 30*d*e**7*x*sqrt(d + e*x) + 15*e**8*x**2*sqrt(d + e*x)
) + 360*B*a*c*d*e**4*x**2/(15*d**2*e**6*sqrt(d + e*x) + 30*d*e**7*x*sqr...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{7/2}}\,dx = \frac{2\left(\frac{3\,(ex+d)^{\frac{5}{2}}Bc^2-5\,(5\,Bc^2d-Ac^2e)(ex+d)^{\frac{3}{2}}+30\,(5\,Bc^2d^2-2\,Ac^2de+Bace^2)\sqrt{ex+d}}{e^5}+\frac{3\,Bc^2d^5-3\,A\ldots}{\ldots}\right.}{\ldots}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
2/15*((3*(e*x + d)^(5/2)*B*c^2 - 5*(5*B*c^2*d - A*c^2*e)*(e*x + d)^(3/2) +
 30*(5*B*c^2*d^2 - 2*A*c^2*d*e + B*a*c*e^2)*sqrt(e*x + d))/e^5 + (3*B*c^2*
d^5 - 3*A*c^2*d^4*e + 6*B*a*c*d^3*e^2 - 6*A*a*c*d^2*e^3 + 3*B*a^2*d*e^4 -
3*A*a^2*e^5 + 30*(5*B*c^2*d^3 - 3*A*c^2*d^2*e + 3*B*a*c*d*e^2 - A*a*c*e^3)
*(e*x + d)^2 - 5*(5*B*c^2*d^4 - 4*A*c^2*d^3*e + 6*B*a*c*d^2*e^2 - 4*A*a*c*
d*e^3 + B*a^2*e^4)*(e*x + d))/((e*x + d)^(5/2)*e^5))/e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.48

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{7/2}}\,dx = \frac{2\left(150\,(ex+d)^2Bc^2d^3-25\,(ex+d)Bc^2d^4+3\,Bc^2d^5-90\,(ex+d)^2Ac^2d^2e+\ldots\right.}{\ldots}$$
$$+\frac{2\left(3\,(ex+d)^{\frac{5}{2}}Bc^2e^{24}-25\,(ex+d)^{\frac{3}{2}}Bc^2de^{24}+150\,\sqrt{ex+d}Bc^2d^2e^{24}+5\,(ex+d)^{\frac{3}{2}}Ac^2e^{25}-60\,\sqrt{ex+d}\ldots\right.}{15\,e^{30}}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
2/15*(150*(e*x + d)^2*B*c^2*d^3 - 25*(e*x + d)*B*c^2*d^4 + 3*B*c^2*d^5 - 9
0*(e*x + d)^2*A*c^2*d^2*e + 20*(e*x + d)*A*c^2*d^3*e - 3*A*c^2*d^4*e + 90*
(e*x + d)^2*B*a*c*d*e^2 - 30*(e*x + d)*B*a*c*d^2*e^2 + 6*B*a*c*d^3*e^2 - 3
0*(e*x + d)^2*A*a*c*e^3 + 20*(e*x + d)*A*a*c*d*e^3 - 6*A*a*c*d^2*e^3 - 5*(
e*x + d)*B*a^2*e^4 + 3*B*a^2*d*e^4 - 3*A*a^2*e^5)/((e*x + d)^(5/2)*e^6) +
2/15*(3*(e*x + d)^(5/2)*B*c^2*e^24 - 25*(e*x + d)^(3/2)*B*c^2*d*e^24 + 150
*sqrt(e*x + d)*B*c^2*d^2*e^24 + 5*(e*x + d)^(3/2)*A*c^2*e^25 - 60*sqrt(e*x
 + d)*A*c^2*d*e^25 + 30*sqrt(e*x + d)*B*a*c*e^26)/e^30
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^{7/2}}\,dx = \frac{\sqrt{d+e\,x}\,(20\,B\,c^2\,d^2-8\,A\,c^2\,d\,e+4\,B\,a\,c\,e^2)}{e^6}$$
$$-\frac{(d+e\,x)\left(\frac{2\,B\,a^2\,e^4}{3}+4\,B\,a\,c\,d^2\,e^2-\frac{8\,A\,a\,c\,d\,e^3}{3}+\frac{10\,B\,c^2\,d^4}{3}-\frac{8\,A\,c^2\,d^3\,e}{3}\right)-(d+e\,x)^2\,(20\,B\,c^2\,d^3-12\,A\,c^2\,d^2\,e}{e^6\,(d+e\,x)^{5}}$$
$$+\frac{2\,B\,c^2\,(d+e\,x)^{5/2}}{5\,e^6}+\frac{2\,c^2\,(A\,e-5\,B\,d)\,(d+e\,x)^{3/2}}{3\,e^6}$$

input
```
int(((a + c*x^2)^2*(A + B*x))/(d + e*x)^(7/2),x)
```

output
```
((d + e*x)^(1/2)*(20*B*c^2*d^2 + 4*B*a*c*e^2 - 8*A*c^2*d*e))/e^6 - ((d + e
*x)*((2*B*a^2*e^4)/3 + (10*B*c^2*d^4)/3 - (8*A*c^2*d^3*e)/3 - (8*A*a*c*d*e
^3)/3 + 4*B*a*c*d^2*e^2) - (d + e*x)^2*(20*B*c^2*d^3 - 4*A*a*c*e^3 - 12*A*
c^2*d^2*e + 12*B*a*c*d*e^2) + (2*A*a^2*e^5)/5 - (2*B*c^2*d^5)/5 - (2*B*a^2
*d*e^4)/5 + (2*A*c^2*d^4*e)/5 + (4*A*a*c*d^2*e^3)/5 - (4*B*a*c*d^3*e^2)/5)
/(e^6*(d + e*x)^(5/2)) + (2*B*c^2*(d + e*x)^(5/2))/(5*e^6) + (2*c^2*(A*e -
 5*B*d)*(d + e*x)^(3/2))/(3*e^6)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^{7/2}}\,dx = \frac{\frac{2}{5}b\,c^2e^5x^5+\frac{2}{3}a\,c^2e^5x^4-\frac{4}{3}b\,c^2d\,e^4x^4+4abc\,e^5x^3-\frac{16}{3}a\,c^2d\,e^4x^3+\frac{32}{3}b\,c^2d^2e^3x^3}{}$$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^(7/2),x)
```

output
```
(2*( - 3*a**3*e**5 - 2*a**2*b*d*e**4 - 5*a**2*b*e**5*x - 16*a**2*c*d**2*e*
*3 - 40*a**2*c*d*e**4*x - 30*a**2*c*e**5*x**2 + 96*a*b*c*d**3*e**2 + 240*a
*b*c*d**2*e**3*x + 180*a*b*c*d*e**4*x**2 + 30*a*b*c*e**5*x**3 - 128*a*c**2
*d**4*e - 320*a*c**2*d**3*e**2*x - 240*a*c**2*d**2*e**3*x**2 - 40*a*c**2*d
*e**4*x**3 + 5*a*c**2*e**5*x**4 + 256*b*c**2*d**5 + 640*b*c**2*d**4*e*x +
480*b*c**2*d**3*e**2*x**2 + 80*b*c**2*d**2*e**3*x**3 - 10*b*c**2*d*e**4*x*
*4 + 3*b*c**2*e**5*x**5))/(15*sqrt(d + e*x)*e**6*(d**2 + 2*d*e*x + e**2*x*
*2))
```

3.116 $\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 214

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{9/2}}\,dx = \frac{2(Bd-Ae)\left(cd^2+ae^2\right)^2}{7e^6(d+ex)^{7/2}} - \frac{2\left(cd^2+ae^2\right)\left(5Bcd^2-4Acde+aBe^2\right)}{5e^6(d+ex)^{5/2}} + \frac{4c\left(5Bcd^3-3Acd^2e+3aBde^2-aAe^3\right)}{3e^6(d+ex)^{3/2}} - \frac{4c\left(5Bcd^2-2Acde+aBe^2\right)}{e^6\sqrt{d+ex}} - \frac{2c^2(5Bd-Ae)\sqrt{d+ex}}{e^6} + \frac{2Bc^2(d+ex)^{3/2}}{3e^6}$$

output
```
2/7*(-A*e+B*d)*(a*e^2+c*d^2)^2/e^6/(e*x+d)^(7/2)-2/5*(a*e^2+c*d^2)*(-4*A*c
*d*e+B*a*e^2+5*B*c*d^2)/e^6/(e*x+d)^(5/2)+4/3*c*(-A*a*e^3-3*A*c*d^2*e+3*B*
a*d*e^2+5*B*c*d^3)/e^6/(e*x+d)^(3/2)-4*c*(-2*A*c*d*e+B*a*e^2+5*B*c*d^2)/e^
6/(e*x+d)^(1/2)-2*c^2*(-A*e+5*B*d)*(e*x+d)^(1/2)/e^6+2/3*B*c^2*(e*x+d)^(3/
2)/e^6
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{9/2}}\,dx =$$
$$-\frac{2\left(Ae\left(15a^2e^4+2ace^2\left(8d^2+28dex+35e^2x^2\right)-3c^2\left(128d^4+448d^3ex+560d^2e^2x^2+280de^3x^3+35e^4x^4\right)\right)\right.}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^(9/2),x]
```

output
```
(-2*(A*e*(15*a^2*e^4 + 2*a*c*e^2*(8*d^2 + 28*d*e*x + 35*e^2*x^2) - 3*c^2*(
128*d^4 + 448*d^3*e*x + 560*d^2*e^2*x^2 + 280*d*e^3*x^3 + 35*e^4*x^4)) + B
*(3*a^2*e^4*(2*d + 7*e*x) + 6*a*c*e^2*(16*d^3 + 56*d^2*e*x + 70*d*e^2*x^2
+ 35*e^3*x^3) + 5*c^2*(256*d^5 + 896*d^4*e*x + 1120*d^3*e^2*x^2 + 560*d^2*
e^3*x^3 + 70*d*e^4*x^4 - 7*e^5*x^5))))/(105*e^6*(d + e*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2(A+Bx)}{(d+ex)^{9/2}}dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{2c\left(-aBe^2+2Acde-5Bcd^2\right)}{e^5(d+ex)^{3/2}}+\frac{\left(ae^2+cd^2\right)\left(aBe^2-4Acde+5Bcd^2\right)}{e^5(d+ex)^{7/2}}+\frac{\left(ae^2+cd^2\right)^2(Ae-Bd)}{e^5(d+ex)^{9/2}}+\frac{2c(}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4c(aBe^2-2Acde+5Bcd^2)}{e^6\sqrt{d+ex}}-\frac{2(ae^2+cd^2)(aBe^2-4Acde+5Bcd^2)}{5e^6(d+ex)^{5/2}}+\frac{2(ae^2+cd^2)^2(Bd-Ae)}{7e^6(d+ex)^{7/2}}+\frac{4c(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3)}{3e^6(d+ex)^{3/2}}-\frac{2c^2\sqrt{d+ex}(5Bd-Ae)}{e^6}+\frac{2Bc^2(d+ex)^{3/2}}{3e^6}$$

input
```
Int[((A + B*x)*(a + c*x^2)^2)/(d + e*x)^(9/2),x]
```

output
```
(2*(B*d - A*e)*(c*d^2 + a*e^2)^2)/(7*e^6*(d + e*x)^(7/2)) - (2*(c*d^2 + a*
e^2)*(5*B*c*d^2 - 4*A*c*d*e + a*B*e^2))/(5*e^6*(d + e*x)^(5/2)) + (4*c*(5*
B*c*d^3 - 3*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(3*e^6*(d + e*x)^(3/2)) -
(4*c*(5*B*c*d^2 - 2*A*c*d*e + a*B*e^2))/(e^6*Sqrt[d + e*x]) - (2*c^2*(5*B*
d - A*e)*Sqrt[d + e*x])/e^6 + (2*B*c^2*(d + e*x)^(3/2))/(3*e^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{\left(\left(70B\,x^5+210A\,x^4\right)c^2-140a\,x^2(3Bx+A)c-30a^2\left(\frac{7Bx}{5}+A\right)\right)e^5-112d\left(\left(\frac{25}{4}B\,x^4-15A\,x^3\right)c^2+ax\left(\frac{15Bx}{2}+A\right)c+\frac{3a^2B}{28}\right.}{105}$
derivativedivides	$\frac{\frac{2B\,c^2(ex+d)^{\frac{3}{2}}}{3}+2c^2eA\sqrt{ex+d}-10B\,c^2d\sqrt{ex+d}-\frac{2\left(-4Aacd\,e^3-4A\,c^2d^3e+B\,e^4a^2+6Bac\,d^2e^2+5B\,c^2d^4\right)}{5(ex+d)^{\frac{5}{2}}}+\frac{4c\left(2Acde-Ba\,e^2\right.}{\sqrt{ex+d}}}{e^6}$
default	$\frac{\frac{2B\,c^2(ex+d)^{\frac{3}{2}}}{3}+2c^2eA\sqrt{ex+d}-10B\,c^2d\sqrt{ex+d}-\frac{2\left(-4Aacd\,e^3-4A\,c^2d^3e+B\,e^4a^2+6Bac\,d^2e^2+5B\,c^2d^4\right)}{5(ex+d)^{\frac{5}{2}}}+\frac{4c\left(2Acde-Ba\,e^2\right.}{\sqrt{ex+d}}}{e^6}$
gosper	$-\frac{2\left(-35B\,x^5c^2e^5-105A\,x^4c^2e^5+350B\,x^4c^2d\,e^4-840A\,x^3c^2d\,e^4+210B\,x^3ac\,e^5+2800B\,x^3c^2d^2e^3+70A\,x^2ac\,e^5-168\right.}{}$
trager	$-\frac{2\left(-35B\,x^5c^2e^5-105A\,x^4c^2e^5+350B\,x^4c^2d\,e^4-840A\,x^3c^2d\,e^4+210B\,x^3ac\,e^5+2800B\,x^3c^2d^2e^3+70A\,x^2ac\,e^5-168\right.}{}$
orering	$-\frac{2\left(-35B\,x^5c^2e^5-105A\,x^4c^2e^5+350B\,x^4c^2d\,e^4-840A\,x^3c^2d\,e^4+210B\,x^3ac\,e^5+2800B\,x^3c^2d^2e^3+70A\,x^2ac\,e^5-168\right.}{}$
risch	$\frac{2c^2(Bex+3Ae-14Bd)\sqrt{ex+d}}{3e^6}-\frac{2\left(-420A\,x^3c^2d\,e^4+210B\,x^3ac\,e^5+1050B\,x^3c^2d^2e^3+70A\,x^2ac\,e^5-1050A\,x^2c^2d^2e\right.}{}$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(((70*B*x^5+210*A*x^4)*c^2-140*a*x^2*(3*B*x+A)*c-30*a^2*(7/5*B*x+A))
*e^5-112*d*((25/4*B*x^4-15*A*x^3)*c^2+a*x*(15/2*B*x+A)*c+3/28*a^2*B)*e^4-3
2*c*((175*B*x^3-105*A*x^2)*c+a*(21*B*x+A))*d^2*e^3+2688*c*(x*(-25/6*B*x+A)
*c-1/14*B*a)*d^3*e^2+768*c^2*d^4*(-35/3*B*x+A)*e-2560*B*c^2*d^5)/(e*x+d)^(
7/2)/e^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{9/2}}\,dx = \frac{2\left(35\,Bc^2e^5x^5-1280\,Bc^2d^5+384\,Ac^2d^4e-96\,Bacd^3e^2-16\,Aacd^2e^3-6\,Bc\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^(9/2),x, algorithm="fricas")
```

output
```
2/105*(35*B*c^2*e^5*x^5 - 1280*B*c^2*d^5 + 384*A*c^2*d^4*e - 96*B*a*c*d^3*
e^2 - 16*A*a*c*d^2*e^3 - 6*B*a^2*d*e^4 - 15*A*a^2*e^5 - 35*(10*B*c^2*d*e^4
 - 3*A*c^2*e^5)*x^4 - 70*(40*B*c^2*d^2*e^3 - 12*A*c^2*d*e^4 + 3*B*a*c*e^5)
*x^3 - 70*(80*B*c^2*d^3*e^2 - 24*A*c^2*d^2*e^3 + 6*B*a*c*d*e^4 + A*a*c*e^5
)*x^2 - 7*(640*B*c^2*d^4*e - 192*A*c^2*d^3*e^2 + 48*B*a*c*d^2*e^3 + 8*A*a*
c*d*e^4 + 3*B*a^2*e^5)*x)*sqrt(e*x + d)/(e^10*x^4 + 4*d*e^9*x^3 + 6*d^2*e^
8*x^2 + 4*d^3*e^7*x + d^4*e^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1855 vs. $2(226) = 452$.

Time = 0.91 (sec) , antiderivative size = 1855, normalized size of antiderivative = 8.67

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**(9/2),x)
```

output
```
Piecewise((-30*A*a**2*e**5/(105*d**3*e**6*sqrt(d + e*x) + 315*d**2*e**7*x*
sqrt(d + e*x) + 315*d*e**8*x**2*sqrt(d + e*x) + 105*e**9*x**3*sqrt(d + e*x
)) - 32*A*a*c*d**2*e**3/(105*d**3*e**6*sqrt(d + e*x) + 315*d**2*e**7*x*sqr
t(d + e*x) + 315*d*e**8*x**2*sqrt(d + e*x) + 105*e**9*x**3*sqrt(d + e*x))
- 112*A*a*c*d*e**4*x/(105*d**3*e**6*sqrt(d + e*x) + 315*d**2*e**7*x*sqrt(d
 + e*x) + 315*d*e**8*x**2*sqrt(d + e*x) + 105*e**9*x**3*sqrt(d + e*x)) - 1
40*A*a*c*e**5*x**2/(105*d**3*e**6*sqrt(d + e*x) + 315*d**2*e**7*x*sqrt(d +
 e*x) + 315*d*e**8*x**2*sqrt(d + e*x) + 105*e**9*x**3*sqrt(d + e*x)) + 768
*A*c**2*d**4*e/(105*d**3*e**6*sqrt(d + e*x) + 315*d**2*e**7*x*sqrt(d + e*x
) + 315*d*e**8*x**2*sqrt(d + e*x) + 105*e**9*x**3*sqrt(d + e*x)) + 2688*A*
c**2*d**3*e**2*x/(105*d**3*e**6*sqrt(d + e*x) + 315*d**2*e**7*x*sqrt(d + e
*x) + 315*d*e**8*x**2*sqrt(d + e*x) + 105*e**9*x**3*sqrt(d + e*x)) + 3360*
A*c**2*d**2*e**3*x**2/(105*d**3*e**6*sqrt(d + e*x) + 315*d**2*e**7*x*sqrt(
d + e*x) + 315*d*e**8*x**2*sqrt(d + e*x) + 105*e**9*x**3*sqrt(d + e*x)) +
1680*A*c**2*d*e**4*x**3/(105*d**3*e**6*sqrt(d + e*x) + 315*d**2*e**7*x*sqr
t(d + e*x) + 315*d*e**8*x**2*sqrt(d + e*x) + 105*e**9*x**3*sqrt(d + e*x))
+ 210*A*c**2*e**5*x**4/(105*d**3*e**6*sqrt(d + e*x) + 315*d**2*e**7*x*sqrt
(d + e*x) + 315*d*e**8*x**2*sqrt(d + e*x) + 105*e**9*x**3*sqrt(d + e*x)) -
 12*B*a**2*d*e**4/(105*d**3*e**6*sqrt(d + e*x) + 315*d**2*e**7*x*sqrt(d +
e*x) + 315*d*e**8*x**2*sqrt(d + e*x) + 105*e**9*x**3*sqrt(d + e*x)) - 4...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{9/2}}\,dx = \frac{2\left(\frac{35\left((ex+d)^{\frac{3}{2}}Bc^2-3\left(5Bc^2d-Ac^2e\right)\sqrt{ex+d}\right)}{e^5}+\frac{15Bc^2d^5-15Ac^2d^4e+30Bacd^3e^2-30Aacd^2e^3+1}{}\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^(9/2),x, algorithm="maxima")
```

output
```
2/105*(35*((e*x + d)^(3/2)*B*c^2 - 3*(5*B*c^2*d - A*c^2*e)*sqrt(e*x + d))/
e^5 + (15*B*c^2*d^5 - 15*A*c^2*d^4*e + 30*B*a*c*d^3*e^2 - 30*A*a*c*d^2*e^3
 + 15*B*a^2*d*e^4 - 15*A*a^2*e^5 - 210*(5*B*c^2*d^2 - 2*A*c^2*d*e + B*a*c*
e^2)*(e*x + d)^3 + 70*(5*B*c^2*d^3 - 3*A*c^2*d^2*e + 3*B*a*c*d*e^2 - A*a*c
*e^3)*(e*x + d)^2 - 21*(5*B*c^2*d^4 - 4*A*c^2*d^3*e + 6*B*a*c*d^2*e^2 - 4*
A*a*c*d*e^3 + B*a^2*e^4)*(e*x + d))/((e*x + d)^(7/2)*e^5))/e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.45

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{9/2}}\,dx =$$

$$-\frac{2\left(1050\,(ex+d)^3Bc^2d^2-350\,(ex+d)^2Bc^2d^3+105\,(ex+d)Bc^2d^4-15\,Bc^2d^5-420\,(ex+d)^3Ac^2de+\right.}{}$$

$$+\frac{2\left((ex+d)^{\frac{3}{2}}Bc^2e^{12}-15\,\sqrt{ex+d}Bc^2de^{12}+3\,\sqrt{ex+d}Ac^2e^{13}\right)}{3\,e^{18}}$$

input
```
integrate((B*x+A)*(c*x^2+a)^2/(e*x+d)^(9/2),x, algorithm="giac")
```

output
```
-2/105*(1050*(e*x + d)^3*B*c^2*d^2 - 350*(e*x + d)^2*B*c^2*d^3 + 105*(e*x
+ d)*B*c^2*d^4 - 15*B*c^2*d^5 - 420*(e*x + d)^3*A*c^2*d*e + 210*(e*x + d)^
2*A*c^2*d^2*e - 84*(e*x + d)*A*c^2*d^3*e + 15*A*c^2*d^4*e + 210*(e*x + d)^
3*B*a*c*e^2 - 210*(e*x + d)^2*B*a*c*d*e^2 + 126*(e*x + d)*B*a*c*d^2*e^2 -
30*B*a*c*d^3*e^2 + 70*(e*x + d)^2*A*a*c*e^3 - 84*(e*x + d)*A*a*c*d*e^3 + 3
0*A*a*c*d^2*e^3 + 21*(e*x + d)*B*a^2*e^4 - 15*B*a^2*d*e^4 + 15*A*a^2*e^5)/
((e*x + d)^(7/2)*e^6) + 2/3*((e*x + d)^(3/2)*B*c^2*e^12 - 15*sqrt(e*x + d)
*B*c^2*d*e^12 + 3*sqrt(e*x + d)*A*c^2*e^13)/e^18
```

Mupad [B] (verification not implemented)

Time = 6.41 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx)\left(a+cx^2\right)^2}{(d+ex)^{9/2}}\,dx =$$

$$-\frac{2\,(6\,B\,a^2\,d\,e^4+21\,B\,a^2\,e^5\,x+15\,A\,a^2\,e^5+96\,B\,a\,c\,d^3\,e^2+336\,B\,a\,c\,d^2\,e^3\,x+16\,A\,a\,c\,d^2\,e^3+420\,B\,a\,c\,d\,e}{}$$

input
```
int(((a + c*x^2)^2*(A + B*x))/(d + e*x)^(9/2),x)
```

output
```
-(2*(15*A*a^2*e^5 + 1280*B*c^2*d^5 + 6*B*a^2*d*e^4 - 384*A*c^2*d^4*e + 21*
B*a^2*e^5*x - 105*A*c^2*e^5*x^4 - 35*B*c^2*e^5*x^5 + 70*A*a*c*e^5*x^2 + 21
0*B*a*c*e^5*x^3 + 4480*B*c^2*d^4*e*x - 1344*A*c^2*d^3*e^2*x - 840*A*c^2*d*
e^4*x^3 + 350*B*c^2*d*e^4*x^4 - 1680*A*c^2*d^2*e^3*x^2 + 5600*B*c^2*d^3*e^
2*x^2 + 2800*B*c^2*d^2*e^3*x^3 + 16*A*a*c*d^2*e^3 + 96*B*a*c*d^3*e^2 + 56*
A*a*c*d*e^4*x + 336*B*a*c*d^2*e^3*x + 420*B*a*c*d*e^4*x^2))/(105*e^6*(d +
e*x)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx)\,(a+cx^2)^2}{(d+ex)^{9/2}}\,dx = \frac{\frac{2}{3}b\,c^2e^5x^5 + 2a\,c^2e^5x^4 - \frac{20}{3}b\,c^2d\,e^4x^4 - 4abc\,e^5x^3 + 16a\,c^2d\,e^4x^3 - \frac{160}{3}b\,c^2d^2e^3}{}$$

input
```
int((B*x+A)*(c*x^2+a)^2/(e*x+d)^(9/2),x)
```

output
```
(2*( - 15*a**3*e**5 - 6*a**2*b*d*e**4 - 21*a**2*b*e**5*x - 16*a**2*c*d**2*
e**3 - 56*a**2*c*d*e**4*x - 70*a**2*c*e**5*x**2 - 96*a*b*c*d**3*e**2 - 336
*a*b*c*d**2*e**3*x - 420*a*b*c*d*e**4*x**2 - 210*a*b*c*e**5*x**3 + 384*a*c
**2*d**4*e + 1344*a*c**2*d**3*e**2*x + 1680*a*c**2*d**2*e**3*x**2 + 840*a*
c**2*d*e**4*x**3 + 105*a*c**2*e**5*x**4 - 1280*b*c**2*d**5 - 4480*b*c**2*d
**4*e*x - 5600*b*c**2*d**3*e**2*x**2 - 2800*b*c**2*d**2*e**3*x**3 - 350*b*
c**2*d*e**4*x**4 + 35*b*c**2*e**5*x**5))/(105*sqrt(d + e*x)*e**6*(d**3 + 3
*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))
```

3.117 $\int \frac{(A+Bx)(a+cx^2)^3}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 348

$$\begin{aligned}
\int \frac{(A+Bx)\left(a+cx^2\right)^3}{\sqrt{d+ex}}\,dx
&= -\frac{2(Bd-Ae)\left(cd^2+ae^2\right)^3\sqrt{d+ex}}{e^8} \\
&\quad + \frac{2\left(cd^2+ae^2\right)^2\left(7Bcd^2-6Acde+aBe^2\right)(d+ex)^{3/2}}{3e^8} \\
&\quad - \frac{6c\left(cd^2+ae^2\right)\left(7Bcd^3-5Acd^2e+3aBde^2-aAe^3\right)(d+ex)^{5/2}}{5e^8} \\
&\quad - \frac{2c\left(4Acde\left(5cd^2+3ae^2\right)-B\left(35c^2d^4+30acd^2e^2+3a^2e^4\right)\right)(d+ex)^{7/2}}{7e^8} \\
&\quad - \frac{2c^2\left(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3\right)(d+ex)^{9/2}}{9e^8} \\
&\quad + \frac{6c^2\left(7Bcd^2-2Acde+aBe^2\right)(d+ex)^{11/2}}{11e^8} \\
&\quad - \frac{2c^3(7Bd-Ae)(d+ex)^{13/2}}{13e^8} + \frac{2Bc^3(d+ex)^{15/2}}{15e^8}
\end{aligned}$$

output
```
-2*(-A*e+B*d)*(a*e^2+c*d^2)^3*(e*x+d)^(1/2)/e^8+2/3*(a*e^2+c*d^2)^2*(-6*A*
c*d*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^(3/2)/e^8-6/5*c*(a*e^2+c*d^2)*(-A*a*e^3-5
*A*c*d^2*e+3*B*a*d*e^2+7*B*c*d^3)*(e*x+d)^(5/2)/e^8-2/7*c*(4*A*c*d*e*(3*a*
e^2+5*c*d^2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))*(e*x+d)^(7/2)/e^8-2/
9*c^2*(-3*A*a*e^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)*(e*x+d)^(9/2)/e^8+
6/11*c^2*(-2*A*c*d*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^(11/2)/e^8-2/13*c^3*(-A*e+
7*B*d)*(e*x+d)^(13/2)/e^8+2/15*B*c^3*(e*x+d)^(15/2)/e^8
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)(a+cx^2)^3}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\sqrt{d+ex}(3Ae(15015a^3e^6+3003a^2ce^4(8d^2-4dex+3e^2x^2)+143ac^2e^2(128d^4-64d^3ex+48d^2e^2x^2-4}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(3*A*e*(15015*a^3*e^6 + 3003*a^2*c*e^4*(8*d^2 - 4*d*e*x +
 3*e^2*x^2) + 143*a*c^2*e^2*(128*d^4 - 64*d^3*e*x + 48*d^2*e^2*x^2 - 40*d*
e^3*x^3 + 35*e^4*x^4) + 5*c^3*(1024*d^6 - 512*d^5*e*x + 384*d^4*e^2*x^2 -
320*d^3*e^3*x^3 + 280*d^2*e^4*x^4 - 252*d*e^5*x^5 + 231*e^6*x^6)) + B*(150
15*a^3*e^6*(-2*d + e*x) + 3861*a^2*c*e^4*(-16*d^3 + 8*d^2*e*x - 6*d*e^2*x^
2 + 5*e^3*x^3) + 195*a*c^2*e^2*(-256*d^5 + 128*d^4*e*x - 96*d^3*e^2*x^2 +
80*d^2*e^3*x^3 - 70*d*e^4*x^4 + 63*e^5*x^5) - 7*c^3*(2048*d^7 - 1024*d^6*e
*x + 768*d^5*e^2*x^2 - 640*d^4*e^3*x^3 + 560*d^3*e^4*x^4 - 504*d^2*e^5*x^5
 + 462*d*e^6*x^6 - 429*e^7*x^7))))/(45045*e^8)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^3 (A+Bx)}{\sqrt{d+ex}} \, dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{c(d+ex)^{5/2} \left(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4\right)}{e^7}-\frac{3c^2(d+ex)^{9/2} \left(-aBe^2+2A\right.}{e^7}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2c(d+ex)^{7/2} \left(4Acde\left(3ae^2+5cd^2\right)-B\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)\right)}{7e^8}+\frac{6c^2(d+ex)^{11/2} \left(aBe^2-2Acde+7Bcd^2\right)}{11e^8}-\frac{2c^2(d+ex)^{9/2} \left(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3\right)}{9e^8}+\frac{2(d+ex)^{3/2} \left(ae^2+cd^2\right)^2 \left(aBe^2-6Acde+7Bcd^2\right)}{3e^8}-\frac{2\sqrt{d+ex}\left(ae^2+cd^2\right)^3 (Bd-Ae)}{e^8}-\frac{6c(d+ex)^{5/2} \left(ae^2+cd^2\right) \left(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3\right)}{5e^8}-\frac{2c^3(d+ex)^{13/2}(7Bd-Ae)}{13e^8}+\frac{2Bc^3(d+ex)^{15/2}}{15e^8}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/Sqrt[d + e*x],x]
```

output
```
(-2*(B*d - A*e)*(c*d^2 + a*e^2)^3*Sqrt[d + e*x])/e^8 + (2*(c*d^2 + a*e^2)^
2*(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2)*(d + e*x)^(3/2))/(3*e^8) - (6*c*(c*d^2
 + a*e^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*(d + e*x)^(5/2
))/(5*e^8) - (2*c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(35*c^2*d^4 + 30*a*c*
d^2*e^2 + 3*a^2*e^4))*(d + e*x)^(7/2))/(7*e^8) - (2*c^2*(35*B*c*d^3 - 15*A
*c*d^2*e + 15*a*B*d*e^2 - 3*a*A*e^3)*(d + e*x)^(9/2))/(9*e^8) + (6*c^2*(7*
B*c*d^2 - 2*A*c*d*e + a*B*e^2)*(d + e*x)^(11/2))/(11*e^8) - (2*c^3*(7*B*d
- A*e)*(d + e*x)^(13/2))/(13*e^8) + (2*B*c^3*(d + e*x)^(15/2))/(15*e^8)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{2\left(\left(\left(\frac{1}{15} B\,x^{7}+\frac{1}{13} A\,x^{6}\right) c^{3}+\frac{\left(\frac{9 B x}{11}+A\right) x^{4} a\,c^{2}}{3}+\frac{3 x^{2} a^{2} \left(\frac{5 B x}{7}+A\right) c}{5}+a^{3} \left(\frac{B x}{3}+A\right)\right) e^{7}-\frac{4 d \left(\left(\frac{7}{78} B\,x^{6}+\frac{15}{143} A\,x^{5}\right) c^{3}+\frac{10\left(\frac{35 B}{44}\right.}{}\right.}{}}{}$
gosper	$\frac{2\sqrt{e x+d}\,\left(3003 B\,x^{7} c^{3} e^{7}+3465 A\,x^{6} c^{3} e^{7}-3234 B\,x^{6} c^{3} d\,e^{6}-3780 A\,x^{5} c^{3} d\,e^{6}+12285 B\,x^{5} a\,c^{2} e^{7}+3528 B\,x^{5} c^{3} d^{2} e^{5}+1501\right.}{}$
trager	$\frac{2\sqrt{e x+d}\,\left(3003 B\,x^{7} c^{3} e^{7}+3465 A\,x^{6} c^{3} e^{7}-3234 B\,x^{6} c^{3} d\,e^{6}-3780 A\,x^{5} c^{3} d\,e^{6}+12285 B\,x^{5} a\,c^{2} e^{7}+3528 B\,x^{5} c^{3} d^{2} e^{5}+1501\right.}{}$
risch	$\frac{2\sqrt{e x+d}\,\left(3003 B\,x^{7} c^{3} e^{7}+3465 A\,x^{6} c^{3} e^{7}-3234 B\,x^{6} c^{3} d\,e^{6}-3780 A\,x^{5} c^{3} d\,e^{6}+12285 B\,x^{5} a\,c^{2} e^{7}+3528 B\,x^{5} c^{3} d^{2} e^{5}+1501\right.}{}$
orering	$\frac{2\sqrt{e x+d}\,\left(3003 B\,x^{7} c^{3} e^{7}+3465 A\,x^{6} c^{3} e^{7}-3234 B\,x^{6} c^{3} d\,e^{6}-3780 A\,x^{5} c^{3} d\,e^{6}+12285 B\,x^{5} a\,c^{2} e^{7}+3528 B\,x^{5} c^{3} d^{2} e^{5}+1501\right.}{}$
derivativedivides	$\frac{\frac{2 B\,c^{3} (e x+d)^{\frac{15}{2}}}{15}+\frac{2\left(\left(A e-B d\right) c^{3}-6 B\,c^{3} d\right) (e x+d)^{\frac{13}{2}}}{13}+\frac{2\left(-6 (A e-B d) c^{3} d+B\left(\left(a\,e^{2}+c\,d^{2}\right) c^{2}+8 c^{3} d^{2}+c\left(2\left(a\,e^{2}+c\,d^{2}\right) c+4 c^{2} d^{2}\right)\right)\right.}{11}}{}$
default	$\frac{\frac{2 B\,c^{3} (e x+d)^{\frac{15}{2}}}{15}+\frac{2\left(\left(A e-B d\right) c^{3}-6 B\,c^{3} d\right) (e x+d)^{\frac{13}{2}}}{13}+\frac{2\left(-6 (A e-B d) c^{3} d+B\left(\left(a\,e^{2}+c\,d^{2}\right) c^{2}+8 c^{3} d^{2}+c\left(2\left(a\,e^{2}+c\,d^{2}\right) c+4 c^{2} d^{2}\right)\right)\right.}{11}}{}$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(((1/15*B*x^7+1/13*A*x^6)*c^3+1/3*(9/11*B*x+A)*x^4*a*c^2+3/5*x^2*a^2*(5/
7*B*x+A)*c+a^3*(1/3*B*x+A))*e^7-4/5*d*((7/78*B*x^6+15/143*A*x^5)*c^3+10/21
*(35/44*B*x+A)*x^3*a*c^2+a^2*x*(9/14*B*x+A)*c+5/6*B*a^3)*e^6+8/5*c*d^2*(25
/429*(21/25*B*x+A)*x^4*c^2+2/7*x^2*(25/33*B*x+A)*a*c+a^2*(3/7*B*x+A))*e^5-
64/105*c*d^3*(25/143*x^3*(49/60*B*x+A)*c^2+a*x*(15/22*B*x+A)*c+9/4*a^2*B)*
e^4+128/105*c^2*(15/143*(7/9*B*x+A)*x^2*c+a*(5/11*B*x+A))*d^4*e^3-512/3003
*c^2*d^5*(x*(7/10*B*x+A)*c+13/2*B*a)*e^2+1024/3003*c^3*d^6*(7/15*B*x+A)*e-
2048/6435*B*c^3*d^7)*(e*x+d)^(1/2)/e^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{\sqrt{d+ex}}\,dx = \frac{2\,(3003\,Bc^3e^7x^7 - 14336\,Bc^3d^7 + 15360\,Ac^3d^6e - 49920\,Bac^2d^5e^2 + 54912\,Aac^2d^4e^3 - 61776\,Ba^2cd^3e^4$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/45045*(3003*B*c^3*e^7*x^7 - 14336*B*c^3*d^7 + 15360*A*c^3*d^6*e - 49920*
B*a*c^2*d^5*e^2 + 54912*A*a*c^2*d^4*e^3 - 61776*B*a^2*c*d^3*e^4 + 72072*A*
a^2*c*d^2*e^5 - 30030*B*a^3*d*e^6 + 45045*A*a^3*e^7 - 231*(14*B*c^3*d*e^6
- 15*A*c^3*e^7)*x^6 + 63*(56*B*c^3*d^2*e^5 - 60*A*c^3*d*e^6 + 195*B*a*c^2*
e^7)*x^5 - 35*(112*B*c^3*d^3*e^4 - 120*A*c^3*d^2*e^5 + 390*B*a*c^2*d*e^6 -
 429*A*a*c^2*e^7)*x^4 + 5*(896*B*c^3*d^4*e^3 - 960*A*c^3*d^3*e^4 + 3120*B*
a*c^2*d^2*e^5 - 3432*A*a*c^2*d*e^6 + 3861*B*a^2*c*e^7)*x^3 - 3*(1792*B*c^3
*d^5*e^2 - 1920*A*c^3*d^4*e^3 + 6240*B*a*c^2*d^3*e^4 - 6864*A*a*c^2*d^2*e^
5 + 7722*B*a^2*c*d*e^6 - 9009*A*a^2*c*e^7)*x^2 + (7168*B*c^3*d^6*e - 7680*
A*c^3*d^5*e^2 + 24960*B*a*c^2*d^4*e^3 - 27456*A*a*c^2*d^3*e^4 + 30888*B*a^
2*c*d^2*e^5 - 36036*A*a^2*c*d*e^6 + 15015*B*a^3*e^7)*x)*sqrt(e*x + d)/e^8
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.85

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{\sqrt{d+ex}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{Bc^3(d+ex)^{\frac{15}{2}}}{15e^7}+\frac{(d+ex)^{\frac{13}{2}}\left(Ac^3e-7Bc^3d\right)}{13e^7}+\frac{(d+ex)^{\frac{11}{2}}\left(-6Ac^3de+3Bac^2e^2+21Bc^3d^2\right)}{11e^7}+\frac{(d+ex)^{\frac{9}{2}}\cdot\left(3Aac^2e^3+15Ac^3d^2e-15Bac^2de^2-35Bc^3d^3\right)}{9e^7}+\cdots \\ \frac{Aa^3x+Aa^2cx^3+\frac{3Aac^2x^5}{5}+\frac{Ac^3x^7}{7}+\frac{Ba^3x^2}{2}+\frac{3Ba^2cx^4}{4}+\frac{Bac^2x^6}{2}+\frac{Bc^3x^8}{8}}{\sqrt{d}} \end{cases}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**(1/2),x)
```

output
```
Piecewise((2*(B*c**3*(d + e*x)**(15/2)/(15*e**7) + (d + e*x)**(13/2)*(A*c*
*3*e - 7*B*c**3*d)/(13*e**7) + (d + e*x)**(11/2)*(-6*A*c**3*d*e + 3*B*a*c*
*2*e**2 + 21*B*c**3*d**2)/(11*e**7) + (d + e*x)**(9/2)*(3*A*a*c**2*e**3 +
15*A*c**3*d**2*e - 15*B*a*c**2*d*e**2 - 35*B*c**3*d**3)/(9*e**7) + (d + e*
x)**(7/2)*(-12*A*a*c**2*d*e**3 - 20*A*c**3*d**3*e + 3*B*a**2*c*e**4 + 30*B
*a*c**2*d**2*e**2 + 35*B*c**3*d**4)/(7*e**7) + (d + e*x)**(5/2)*(3*A*a**2*
c*e**5 + 18*A*a*c**2*d**2*e**3 + 15*A*c**3*d**4*e - 9*B*a**2*c*d*e**4 - 30
*B*a*c**2*d**3*e**2 - 21*B*c**3*d**5)/(5*e**7) + (d + e*x)**(3/2)*(-6*A*a*
*2*c*d*e**5 - 12*A*a*c**2*d**3*e**3 - 6*A*c**3*d**5*e + B*a**3*e**6 + 9*B*
a**2*c*d**2*e**4 + 15*B*a*c**2*d**4*e**2 + 7*B*c**3*d**6)/(3*e**7) + sqrt(
d + e*x)*(A*a**3*e**7 + 3*A*a**2*c*d**2*e**5 + 3*A*a*c**2*d**4*e**3 + A*c*
*3*d**6*e - B*a**3*d*e**6 - 3*B*a**2*c*d**3*e**4 - 3*B*a*c**2*d**5*e**2 -
B*c**3*d**7)/e**7)/e, Ne(e, 0)), ((A*a**3*x + A*a**2*c*x**3 + 3*A*a*c**2*x
**5/5 + A*c**3*x**7/7 + B*a**3*x**2/2 + 3*B*a**2*c*x**4/4 + B*a*c**2*x**6/
2 + B*c**3*x**8/8)/sqrt(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\left(3003\,(ex+d)^{\frac{15}{2}}Bc^3-3465\,(7\,Bc^3d-Ac^3e)(ex+d)^{\frac{13}{2}}+12285\,(7\,Bc^3d^2-2\,Ac^3de+Bac^2e^2)(ex+\cdots\right.}{\cdots}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
2/45045*(3003*(e*x + d)^(15/2)*B*c^3 - 3465*(7*B*c^3*d - A*c^3*e)*(e*x + d
)^(13/2) + 12285*(7*B*c^3*d^2 - 2*A*c^3*d*e + B*a*c^2*e^2)*(e*x + d)^(11/2
) - 5005*(35*B*c^3*d^3 - 15*A*c^3*d^2*e + 15*B*a*c^2*d*e^2 - 3*A*a*c^2*e^3
)*(e*x + d)^(9/2) + 6435*(35*B*c^3*d^4 - 20*A*c^3*d^3*e + 30*B*a*c^2*d^2*e
^2 - 12*A*a*c^2*d*e^3 + 3*B*a^2*c*e^4)*(e*x + d)^(7/2) - 27027*(7*B*c^3*d^
5 - 5*A*c^3*d^4*e + 10*B*a*c^2*d^3*e^2 - 6*A*a*c^2*d^2*e^3 + 3*B*a^2*c*d*e
^4 - A*a^2*c*e^5)*(e*x + d)^(5/2) + 15015*(7*B*c^3*d^6 - 6*A*c^3*d^5*e + 1
5*B*a*c^2*d^4*e^2 - 12*A*a*c^2*d^3*e^3 + 9*B*a^2*c*d^2*e^4 - 6*A*a^2*c*d*e
^5 + B*a^3*e^6)*(e*x + d)^(3/2) - 45045*(B*c^3*d^7 - A*c^3*d^6*e + 3*B*a*c
^2*d^5*e^2 - 3*A*a*c^2*d^4*e^3 + 3*B*a^2*c*d^3*e^4 - 3*A*a^2*c*d^2*e^5 + B
*a^3*d*e^6 - A*a^3*e^7)*sqrt(e*x + d))/e^8
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{\sqrt{d+ex}}\,dx = \frac{2\left(45045\sqrt{ex+d}Aa^3 + \frac{15015\left((ex+d)^{\frac{3}{2}}-3\sqrt{ex+d}d\right)Ba^3}{e} + \frac{9009\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}d^2\right)Aa^2c}{e^2} + \frac{3861\left(5(ex\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(e*x + d)*A*a^3 + 15015*((e*x + d)^(3/2) - 3*sqrt(e*x +
 d)*d)*B*a^3/e + 9009*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(
e*x + d)*d^2)*A*a^2*c/e^2 + 3861*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d
 + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*B*a^2*c/e^3 + 429*(35*(e
*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x
 + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*A*a*c^2/e^4 + 195*(63*(e*x + d)^(
11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(
5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*B*a*c^2/e^5 +
 15*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)
*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d
)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*A*c^3/e^6 + 7*(429*(e*x + d)^(15/2)
- 3465*(e*x + d)^(13/2)*d + 12285*(e*x + d)^(11/2)*d^2 - 25025*(e*x + d)^(
9/2)*d^3 + 32175*(e*x + d)^(7/2)*d^4 - 27027*(e*x + d)^(5/2)*d^5 + 15015*(
e*x + d)^(3/2)*d^6 - 6435*sqrt(e*x + d)*d^7)*B*c^3/e^7)/e
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{\sqrt{d+ex}}\,dx = \frac{(d+e\,x)^{7/2}\,\left(6\,B\,a^2\,c\,e^4+60\,B\,a\,c^2\,d^2\,e^2-24\,A\,a\,c^2\,d\,e^3+70\,B\,c^3\,d^4-40\,A\,c^3\,d^3\,e\right)}{7\,e^8} + \frac{(d+e\,x)^{11/2}\,\left(42\,B\,c^3\,d^2-12\,A\,c^3\,d\,e+6\,B\,a\,c^2\,e^2\right)}{11\,e^8} + \frac{2\,\left(c\,d^2+a\,e^2\right)^2\,(d+e\,x)^{3/2}\,\left(7\,B\,c\,d^2-6\,A\,c\,d\,e+B\,a\,e^2\right)}{3\,e^8} + \frac{2\,B\,c^3\,(d+e\,x)^{15/2}}{15\,e^8} + \frac{2\,c^2\,(d+e\,x)^{9/2}\,\left(-35\,B\,c\,d^3+15\,A\,c\,d^2\,e-15\,B\,a\,d\,e^2+3\,A\,a\,e^3\right)}{9\,e^8} + \frac{2\,c^3\,(A\,e-7\,B\,d)\,(d+e\,x)^{13/2}}{13\,e^8} + \frac{2\,\left(c\,d^2+a\,e^2\right)^3\,(A\,e-B\,d)\,\sqrt{d+e\,x}}{e^8} + \frac{6\,c\,\left(c\,d^2+a\,e^2\right)\,(d+e\,x)^{5/2}\,\left(-7\,B\,c\,d^3+5\,A\,c\,d^2\,e-3\,B\,a\,d\,e^2+A\,a\,e^3\right)}{5\,e^8}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^(1/2),x)
```

output
```
((d + e*x)^(7/2)*(70*B*c^3*d^4 + 6*B*a^2*c*e^4 - 40*A*c^3*d^3*e + 60*B*a*c
^2*d^2*e^2 - 24*A*a*c^2*d*e^3))/(7*e^8) + ((d + e*x)^(11/2)*(42*B*c^3*d^2
- 12*A*c^3*d*e + 6*B*a*c^2*e^2))/(11*e^8) + (2*(a*e^2 + c*d^2)^2*(d + e*x)
^(3/2)*(B*a*e^2 + 7*B*c*d^2 - 6*A*c*d*e))/(3*e^8) + (2*B*c^3*(d + e*x)^(15
/2))/(15*e^8) + (2*c^2*(d + e*x)^(9/2)*(3*A*a*e^3 - 35*B*c*d^3 - 15*B*a*d*
e^2 + 15*A*c*d^2*e))/(9*e^8) + (2*c^3*(A*e - 7*B*d)*(d + e*x)^(13/2))/(13*
e^8) + (2*(a*e^2 + c*d^2)^3*(A*e - B*d)*(d + e*x)^(1/2))/e^8 + (6*c*(a*e^2
 + c*d^2)*(d + e*x)^(5/2)*(A*a*e^3 - 7*B*c*d^3 - 3*B*a*d*e^2 + 5*A*c*d^2*e
))/(5*e^8)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.40

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\sqrt{ex+d}\,(3003b\,c^3e^7x^7 + 3465a\,c^3e^7x^6 - 3234b\,c^3d\,e^6x^6 + 12285ab\,c^2e^7x^5 - 3780a\,c^3d\,e^6x^5 + 3528b\,c^3d}{}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(d + e*x)*(45045*a**4*e**7 - 30030*a**3*b*d*e**6 + 15015*a**3*b*e**
7*x + 72072*a**3*c*d**2*e**5 - 36036*a**3*c*d*e**6*x + 27027*a**3*c*e**7*x
**2 - 61776*a**2*b*c*d**3*e**4 + 30888*a**2*b*c*d**2*e**5*x - 23166*a**2*b
*c*d*e**6*x**2 + 19305*a**2*b*c*e**7*x**3 + 54912*a**2*c**2*d**4*e**3 - 27
456*a**2*c**2*d**3*e**4*x + 20592*a**2*c**2*d**2*e**5*x**2 - 17160*a**2*c*
*2*d*e**6*x**3 + 15015*a**2*c**2*e**7*x**4 - 49920*a*b*c**2*d**5*e**2 + 24
960*a*b*c**2*d**4*e**3*x - 18720*a*b*c**2*d**3*e**4*x**2 + 15600*a*b*c**2*
d**2*e**5*x**3 - 13650*a*b*c**2*d*e**6*x**4 + 12285*a*b*c**2*e**7*x**5 + 1
5360*a*c**3*d**6*e - 7680*a*c**3*d**5*e**2*x + 5760*a*c**3*d**4*e**3*x**2
- 4800*a*c**3*d**3*e**4*x**3 + 4200*a*c**3*d**2*e**5*x**4 - 3780*a*c**3*d*
e**6*x**5 + 3465*a*c**3*e**7*x**6 - 14336*b*c**3*d**7 + 7168*b*c**3*d**6*e
*x - 5376*b*c**3*d**5*e**2*x**2 + 4480*b*c**3*d**4*e**3*x**3 - 3920*b*c**3
*d**3*e**4*x**4 + 3528*b*c**3*d**2*e**5*x**5 - 3234*b*c**3*d*e**6*x**6 + 3
003*b*c**3*e**7*x**7))/(45045*e**8)
```

3.118 $\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 344

$$\begin{aligned}\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{3/2}}\,dx &= \frac{2(Bd-Ae)\left(cd^2+ae^2\right)^3}{e^8\sqrt{d+ex}} \\ &+ \frac{2\left(cd^2+ae^2\right)^2\left(7Bcd^2-6Acde+aBe^2\right)\sqrt{d+ex}}{e^8} \\ &- \frac{2c\left(cd^2+ae^2\right)\left(7Bcd^3-5Acd^2e+3aBde^2-aAe^3\right)(d+ex)^{3/2}}{e^8} \\ &- \frac{2c\left(4Acde\left(5cd^2+3ae^2\right)-B\left(35c^2d^4+30acd^2e^2+3a^2e^4\right)\right)(d+ex)^{5/2}}{5e^8} \\ &- \frac{2c^2\left(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3\right)(d+ex)^{7/2}}{7e^8} \\ &+ \frac{2c^2\left(7Bcd^2-2Acde+aBe^2\right)(d+ex)^{9/2}}{3e^8} \\ &- \frac{2c^3(7Bd-Ae)(d+ex)^{11/2}}{11e^8} + \frac{2Bc^3(d+ex)^{13/2}}{13e^8}\end{aligned}$$

output
```
2*(-A*e+B*d)*(a*e^2+c*d^2)^3/e^8/(e*x+d)^(1/2)+2*(a*e^2+c*d^2)^2*(-6*A*c*d
*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^(1/2)/e^8-2*c*(a*e^2+c*d^2)*(-A*a*e^3-5*A*c*
d^2*e+3*B*a*d*e^2+7*B*c*d^3)*(e*x+d)^(3/2)/e^8-2/5*c*(4*A*c*d*e*(3*a*e^2+5
*c*d^2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))*(e*x+d)^(5/2)/e^8-2/7*c^2
*(-3*A*a*e^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)*(e*x+d)^(7/2)/e^8+2/3*c
^2*(-2*A*c*d*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^(9/2)/e^8-2/11*c^3*(-A*e+7*B*d)*
(e*x+d)^(11/2)/e^8+2/13*B*c^3*(e*x+d)^(13/2)/e^8
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{3/2}}\,dx = \frac{-26Ae(1155a^3e^6+1155a^2ce^4(8d^2+4dex-e^2x^2)+99ac^2e^2(128d^4+64d^3ex$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^(3/2),x]
```

output
```
(-26*A*e*(1155*a^3*e^6 + 1155*a^2*c*e^4*(8*d^2 + 4*d*e*x - e^2*x^2) + 99*a
*c^2*e^2*(128*d^4 + 64*d^3*e*x - 16*d^2*e^2*x^2 + 8*d*e^3*x^3 - 5*e^4*x^4)
 + 5*c^3*(1024*d^6 + 512*d^5*e*x - 128*d^4*e^2*x^2 + 64*d^3*e^3*x^3 - 40*d
^2*e^4*x^4 + 28*d*e^5*x^5 - 21*e^6*x^6)) + 2*B*(15015*a^3*e^6*(2*d + e*x)
+ 9009*a^2*c*e^4*(16*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3) + 715*a*c^2*
e^2*(256*d^5 + 128*d^4*e*x - 32*d^3*e^2*x^2 + 16*d^2*e^3*x^3 - 10*d*e^4*x^
4 + 7*e^5*x^5) + 35*c^3*(2048*d^7 + 1024*d^6*e*x - 256*d^5*e^2*x^2 + 128*d
^4*e^3*x^3 - 80*d^3*e^4*x^4 + 56*d^2*e^5*x^5 - 42*d*e^6*x^6 + 33*e^7*x^7))
)/(15015*e^8*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3 (A+Bx)}{(d+ex)^{3/2}} \, dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{c(d+ex)^{3/2} \left(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4\right)}{e^7} - \frac{3c^2(d+ex)^{7/2} \left(-aBe^2+2A\right.}{e^7}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2c(d+ex)^{5/2} \left(4Acde\left(3ae^2+5cd^2\right)-B\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)\right)}{5e^8}+\frac{2c^2(d+ex)^{9/2} \left(aBe^2-2Acde+7Bcd^2\right)}{3e^8}-\frac{2c^2(d+ex)^{7/2} \left(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3\right)}{7e^8}+\frac{2\sqrt{d+ex}\left(ae^2+cd^2\right)^2 \left(aBe^2-6Acde+7Bcd^2\right)}{e^8}+\frac{2\left(ae^2+cd^2\right)^3 (Bd-Ae)}{e^8\sqrt{d+ex}}-\frac{2c(d+ex)^{3/2} \left(ae^2+cd^2\right) \left(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3\right)}{e^8}-\frac{2c^3(d+ex)^{11/2}(7Bd-Ae)}{11e^8}+\frac{2Bc^3(d+ex)^{13/2}}{13e^8}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^(3/2),x]
```

output
```
(2*(B*d - A*e)*(c*d^2 + a*e^2)^3)/(e^8*Sqrt[d + e*x]) + (2*(c*d^2 + a*e^2)
^2*(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2)*Sqrt[d + e*x])/e^8 - (2*c*(c*d^2 + a*
e^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*(d + e*x)^(3/2))/e^
8 - (2*c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(35*c^2*d^4 + 30*a*c*d^2*e^2 +
 3*a^2*e^4))*(d + e*x)^(5/2))/(5*e^8) - (2*c^2*(35*B*c*d^3 - 15*A*c*d^2*e
+ 15*a*B*d*e^2 - 3*a*A*e^3)*(d + e*x)^(7/2))/(7*e^8) + (2*c^2*(7*B*c*d^2 -
 2*A*c*d*e + a*B*e^2)*(d + e*x)^(9/2))/(3*e^8) - (2*c^3*(7*B*d - A*e)*(d +
 e*x)^(11/2))/(11*e^8) + (2*B*c^3*(d + e*x)^(13/2))/(13*e^8)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(-\frac{1}{13}B\,x^{7}-\frac{1}{11}A\,x^{6}\right)c^{3}-\frac{3\left(\frac{7Bx}{9}+A\right)x^{4}a\,c^{2}}{7}-\left(\frac{3Bx}{5}+A\right)x^{2}a^{2}c+a^{3}(-Bx+A)\right)e^{7}+4d\left(\frac{x^{5}\left(\frac{21Bx}{26}+A\right)c^{3}}{33}+\frac{6x^{3}\left(2\right.}{}\right.\right.}{}$
risch	$-\frac{2(-1155e^{6}B\,c^{3}x^{6}-1365e^{6}A\,c^{3}x^{5}+2625d\,e^{5}B\,c^{3}x^{5}+3185d\,e^{5}A\,c^{3}x^{4}-5005e^{6}Ba\,c^{2}x^{4}-4585c^{3}d^{2}e^{4}B\,x^{4}-6435e^{6}Ac}{}$
gosper	$-\frac{2(-1155B\,x^{7}c^{3}e^{7}-1365A\,x^{6}c^{3}e^{7}+1470B\,x^{6}c^{3}d\,e^{6}+1820A\,x^{5}c^{3}d\,e^{6}-5005B\,x^{5}a\,c^{2}e^{7}-1960B\,x^{5}c^{3}d^{2}e^{5}-6435A\,x^{4}}{}$
trager	$-\frac{2(-1155B\,x^{7}c^{3}e^{7}-1365A\,x^{6}c^{3}e^{7}+1470B\,x^{6}c^{3}d\,e^{6}+1820A\,x^{5}c^{3}d\,e^{6}-5005B\,x^{5}a\,c^{2}e^{7}-1960B\,x^{5}c^{3}d^{2}e^{5}-6435A\,x^{4}}{}$
orering	$-\frac{2(-1155B\,x^{7}c^{3}e^{7}-1365A\,x^{6}c^{3}e^{7}+1470B\,x^{6}c^{3}d\,e^{6}+1820A\,x^{5}c^{3}d\,e^{6}-5005B\,x^{5}a\,c^{2}e^{7}-1960B\,x^{5}c^{3}d^{2}e^{5}-6435A\,x^{4}}{}$
derivativedivides	$\frac{12Ba\,c^{2}d^{2}e^{2}(ex+d)^{\frac{5}{2}}-14B\,c^{3}d^{5}(ex+d)^{\frac{3}{2}}-6B\,a^{2}cd\,e^{4}(ex+d)^{\frac{3}{2}}-20Ba\,c^{2}d^{3}e^{2}(ex+d)^{\frac{3}{2}}+12Aa\,c^{2}d^{2}e^{3}(ex+d)^{\frac{3}{2}}+18B}{}$
default	$\frac{12Ba\,c^{2}d^{2}e^{2}(ex+d)^{\frac{5}{2}}-14B\,c^{3}d^{5}(ex+d)^{\frac{3}{2}}-6B\,a^{2}cd\,e^{4}(ex+d)^{\frac{3}{2}}-20Ba\,c^{2}d^{3}e^{2}(ex+d)^{\frac{3}{2}}+12Aa\,c^{2}d^{2}e^{3}(ex+d)^{\frac{3}{2}}+18B}{}$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(e*x+d)^(1/2)*(((-1/13*B*x^7-1/11*A*x^6)*c^3-3/7*(7/9*B*x+A)*x^4*a*c^2-
(3/5*B*x+A)*x^2*a^2*c+a^3*(-B*x+A))*e^7+4*d*(1/33*x^5*(21/26*B*x+A)*c^3+6/
35*x^3*(25/36*B*x+A)*a*c^2+a^2*x*(3/10*B*x+A)*c-1/2*B*a^3)*e^6+8*c*(-5/231
*(49/65*B*x+A)*x^4*c^2-6/35*(5/9*B*x+A)*x^2*a*c+a^2*(-3/5*B*x+A))*d^2*e^5+
192/35*c*d^3*(5/99*x^3*(35/52*B*x+A)*c^2+a*x*(5/18*B*x+A)*c-7/4*a^2*B)*e^4
+384/35*c^2*d^4*(-5/99*x^2*(7/13*B*x+A)*c+a*(-5/9*B*x+A))*e^3+512/231*(x*(
7/26*B*x+A)*c-11/2*B*a)*c^2*d^5*e^2+1024/231*c^3*d^6*(-7/13*B*x+A)*e-2048/
429*B*c^3*d^7)/e^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.35

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^{3/2}}\,dx = \frac{2\,(1155\,Bc^3e^7x^7 + 71680\,Bc^3d^7 - 66560\,Ac^3d^6e + 183040\,Bac^2d^5e^2 - 16473$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
2/15015*(1155*B*c^3*e^7*x^7 + 71680*B*c^3*d^7 - 66560*A*c^3*d^6*e + 183040
*B*a*c^2*d^5*e^2 - 164736*A*a*c^2*d^4*e^3 + 144144*B*a^2*c*d^3*e^4 - 12012
0*A*a^2*c*d^2*e^5 + 30030*B*a^3*d*e^6 - 15015*A*a^3*e^7 - 105*(14*B*c^3*d*
e^6 - 13*A*c^3*e^7)*x^6 + 35*(56*B*c^3*d^2*e^5 - 52*A*c^3*d*e^6 + 143*B*a*
c^2*e^7)*x^5 - 5*(560*B*c^3*d^3*e^4 - 520*A*c^3*d^2*e^5 + 1430*B*a*c^2*d*e
^6 - 1287*A*a*c^2*e^7)*x^4 + (4480*B*c^3*d^4*e^3 - 4160*A*c^3*d^3*e^4 + 11
440*B*a*c^2*d^2*e^5 - 10296*A*a*c^2*d*e^6 + 9009*B*a^2*c*e^7)*x^3 - (8960*
B*c^3*d^5*e^2 - 8320*A*c^3*d^4*e^3 + 22880*B*a*c^2*d^3*e^4 - 20592*A*a*c^2
*d^2*e^5 + 18018*B*a^2*c*d*e^6 - 15015*A*a^2*c*e^7)*x^2 + (35840*B*c^3*d^6
*e - 33280*A*c^3*d^5*e^2 + 91520*B*a*c^2*d^4*e^3 - 82368*A*a*c^2*d^3*e^4 +
 72072*B*a^2*c*d^2*e^5 - 60060*A*a^2*c*d*e^6 + 15015*B*a^3*e^7)*x)*sqrt(e*
x + d)/(e^9*x + d*e^8)
```

Sympy [A] (verification not implemented)

Time = 17.63 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.63

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{Bc^3(d+ex)^{\frac{13}{2}}}{13e^7}+\frac{(d+ex)^{\frac{11}{2}}\left(Ac^3e-7Bc^3d\right)}{11e^7}+\frac{(d+ex)^{\frac{9}{2}}\left(-6Ac^3de+3Bac^2e^2+21Bc^3d^2\right)}{9e^7}+\frac{(d+ex)^{\frac{7}{2}}\cdot\left(3Aac^2e^3\right.}{} \right.}{} \\ \frac{Aa^3x+Aa^2cx^3+\frac{3Aac^2x^5}{5}+\frac{Ac^3x^7}{7}+\frac{Ba^3x^2}{2}+\frac{3Ba^2cx^4}{4}+\frac{Bac^2x^6}{2}+\frac{Bc^3x^8}{8}}{d^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**(3/2),x)
```

output
```
Piecewise((2*(B*c**3*(d + e*x)**(13/2)/(13*e**7) + (d + e*x)**(11/2)*(A*c*
*3*e - 7*B*c**3*d)/(11*e**7) + (d + e*x)**(9/2)*(-6*A*c**3*d*e + 3*B*a*c**
2*e**2 + 21*B*c**3*d**2)/(9*e**7) + (d + e*x)**(7/2)*(3*A*a*c**2*e**3 + 15
*A*c**3*d**2*e - 15*B*a*c**2*d*e**2 - 35*B*c**3*d**3)/(7*e**7) + (d + e*x)
**(5/2)*(-12*A*a*c**2*d*e**3 - 20*A*c**3*d**3*e + 3*B*a**2*c*e**4 + 30*B*a
*c**2*d**2*e**2 + 35*B*c**3*d**4)/(5*e**7) + (d + e*x)**(3/2)*(3*A*a**2*c*
e**5 + 18*A*a*c**2*d**2*e**3 + 15*A*c**3*d**4*e - 9*B*a**2*c*d*e**4 - 30*B
*a*c**2*d**3*e**2 - 21*B*c**3*d**5)/(3*e**7) + sqrt(d + e*x)*(-6*A*a**2*c*
d*e**5 - 12*A*a*c**2*d**3*e**3 - 6*A*c**3*d**5*e + B*a**3*e**6 + 9*B*a**2*
c*d**2*e**4 + 15*B*a*c**2*d**4*e**2 + 7*B*c**3*d**6)/e**7 + (-A*e + B*d)*(
a*e**2 + c*d**2)**3/(e**7*sqrt(d + e*x)))/e, Ne(e, 0)), ((A*a**3*x + A*a**
2*c*x**3 + 3*A*a*c**2*x**5/5 + A*c**3*x**7/7 + B*a**3*x**2/2 + 3*B*a**2*c*
x**4/4 + B*a*c**2*x**6/2 + B*c**3*x**8/8)/d**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{1155\,(ex+d)^{\frac{13}{2}}Bc^3-1365\,\left(7\,Bc^3d-Ac^3e\right)(ex+d)^{\frac{11}{2}}+5005\,\left(7\,Bc^3d^2-2\,Ac^3de+Bac^2e^2\right)(ex+d)^{\frac{9}{2}}-2}{}\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
2/15015*((1155*(e*x + d)^(13/2)*B*c^3 - 1365*(7*B*c^3*d - A*c^3*e)*(e*x +
d)^(11/2) + 5005*(7*B*c^3*d^2 - 2*A*c^3*d*e + B*a*c^2*e^2)*(e*x + d)^(9/2)
 - 2145*(35*B*c^3*d^3 - 15*A*c^3*d^2*e + 15*B*a*c^2*d*e^2 - 3*A*a*c^2*e^3)
*(e*x + d)^(7/2) + 3003*(35*B*c^3*d^4 - 20*A*c^3*d^3*e + 30*B*a*c^2*d^2*e^
2 - 12*A*a*c^2*d*e^3 + 3*B*a^2*c*e^4)*(e*x + d)^(5/2) - 15015*(7*B*c^3*d^5
 - 5*A*c^3*d^4*e + 10*B*a*c^2*d^3*e^2 - 6*A*a*c^2*d^2*e^3 + 3*B*a^2*c*d*e^
4 - A*a^2*c*e^5)*(e*x + d)^(3/2) + 15015*(7*B*c^3*d^6 - 6*A*c^3*d^5*e + 15
*B*a*c^2*d^4*e^2 - 12*A*a*c^2*d^3*e^3 + 9*B*a^2*c*d^2*e^4 - 6*A*a^2*c*d*e^
5 + B*a^3*e^6)*sqrt(e*x + d))/e^7 + 15015*(B*c^3*d^7 - A*c^3*d^6*e + 3*B*a
*c^2*d^5*e^2 - 3*A*a*c^2*d^4*e^3 + 3*B*a^2*c*d^3*e^4 - 3*A*a^2*c*d^2*e^5 +
 B*a^3*d*e^6 - A*a^3*e^7)/(sqrt(e*x + d)*e^7))/e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.81

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
2*(B*c^3*d^7 - A*c^3*d^6*e + 3*B*a*c^2*d^5*e^2 - 3*A*a*c^2*d^4*e^3 + 3*B*a
^2*c*d^3*e^4 - 3*A*a^2*c*d^2*e^5 + B*a^3*d*e^6 - A*a^3*e^7)/(sqrt(e*x + d)
*e^8) + 2/15015*(1155*(e*x + d)^(13/2)*B*c^3*e^96 - 9555*(e*x + d)^(11/2)*
B*c^3*d*e^96 + 35035*(e*x + d)^(9/2)*B*c^3*d^2*e^96 - 75075*(e*x + d)^(7/2
)*B*c^3*d^3*e^96 + 105105*(e*x + d)^(5/2)*B*c^3*d^4*e^96 - 105105*(e*x + d
)^(3/2)*B*c^3*d^5*e^96 + 105105*sqrt(e*x + d)*B*c^3*d^6*e^96 + 1365*(e*x +
 d)^(11/2)*A*c^3*e^97 - 10010*(e*x + d)^(9/2)*A*c^3*d*e^97 + 32175*(e*x +
d)^(7/2)*A*c^3*d^2*e^97 - 60060*(e*x + d)^(5/2)*A*c^3*d^3*e^97 + 75075*(e*
x + d)^(3/2)*A*c^3*d^4*e^97 - 90090*sqrt(e*x + d)*A*c^3*d^5*e^97 + 5005*(e
*x + d)^(9/2)*B*a*c^2*e^98 - 32175*(e*x + d)^(7/2)*B*a*c^2*d*e^98 + 90090*
(e*x + d)^(5/2)*B*a*c^2*d^2*e^98 - 150150*(e*x + d)^(3/2)*B*a*c^2*d^3*e^98
 + 225225*sqrt(e*x + d)*B*a*c^2*d^4*e^98 + 6435*(e*x + d)^(7/2)*A*a*c^2*e^
99 - 36036*(e*x + d)^(5/2)*A*a*c^2*d*e^99 + 90090*(e*x + d)^(3/2)*A*a*c^2*
d^2*e^99 - 180180*sqrt(e*x + d)*A*a*c^2*d^3*e^99 + 9009*(e*x + d)^(5/2)*B*
a^2*c*e^100 - 45045*(e*x + d)^(3/2)*B*a^2*c*d*e^100 + 135135*sqrt(e*x + d)
*B*a^2*c*d^2*e^100 + 15015*(e*x + d)^(3/2)*A*a^2*c*e^101 - 90090*sqrt(e*x
+ d)*A*a^2*c*d*e^101 + 15015*sqrt(e*x + d)*B*a^3*e^102)/e^104
```

Mupad [B] (verification not implemented)

Time = 6.42 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{3/2}}\,dx = \frac{(d+e\,x)^{5/2}\,\left(6\,B\,a^2\,c\,e^4+60\,B\,a\,c^2\,d^2\,e^2-24\,A\,a\,c^2\,d\,e^3+70\,B\,c^3\,d^4-40\,A\,c\right.}{5\,e^8}$$
$$-\frac{-2\,B\,a^3\,d\,e^6+2\,A\,a^3\,e^7-6\,B\,a^2\,c\,d^3\,e^4+6\,A\,a^2\,c\,d^2\,e^5-6\,B\,a\,c^2\,d^5\,e^2+6\,A\,a\,c^2\,d^4\,e^3-2\,B\,c^3\,d^7+2\,A\,c}{e^8\,\sqrt{d+e\,x}}$$
$$+\frac{(d+e\,x)^{9/2}\,\left(42\,B\,c^3\,d^2-12\,A\,c^3\,d\,e+6\,B\,a\,c^2\,e^2\right)}{9\,e^8}$$
$$+\frac{2\,\left(c\,d^2+a\,e^2\right)^2\,\sqrt{d+e\,x}\,\left(7\,B\,c\,d^2-6\,A\,c\,d\,e+B\,a\,e^2\right)}{e^8}+\frac{2\,B\,c^3\,(d+e\,x)^{13/2}}{13\,e^8}$$
$$+\frac{2\,c^2\,(d+e\,x)^{7/2}\,\left(-35\,B\,c\,d^3+15\,A\,c\,d^2\,e-15\,B\,a\,d\,e^2+3\,A\,a\,e^3\right)}{7\,e^8}$$
$$+\frac{2\,c^3\,(A\,e-7\,B\,d)\,\left(d+e\,x\right)^{11/2}}{11\,e^8}$$
$$+\frac{2\,c\,\left(c\,d^2+a\,e^2\right)\,\left(d+e\,x\right)^{3/2}\,\left(-7\,B\,c\,d^3+5\,A\,c\,d^2\,e-3\,B\,a\,d\,e^2+A\,a\,e^3\right)}{e^8}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^(3/2),x)
```

output
```
((d + e*x)^(5/2)*(70*B*c^3*d^4 + 6*B*a^2*c*e^4 - 40*A*c^3*d^3*e + 60*B*a*c
^2*d^2*e^2 - 24*A*a*c^2*d*e^3))/(5*e^8) - (2*A*a^3*e^7 - 2*B*c^3*d^7 - 2*B
*a^3*d*e^6 + 2*A*c^3*d^6*e + 6*A*a*c^2*d^4*e^3 + 6*A*a^2*c*d^2*e^5 - 6*B*a
*c^2*d^5*e^2 - 6*B*a^2*c*d^3*e^4)/(e^8*(d + e*x)^(1/2)) + ((d + e*x)^(9/2)
*(42*B*c^3*d^2 - 12*A*c^3*d*e + 6*B*a*c^2*e^2))/(9*e^8) + (2*(a*e^2 + c*d^
2)^2*(d + e*x)^(1/2)*(B*a*e^2 + 7*B*c*d^2 - 6*A*c*d*e))/e^8 + (2*B*c^3*(d
+ e*x)^(13/2))/(13*e^8) + (2*c^2*(d + e*x)^(7/2)*(3*A*a*e^3 - 35*B*c*d^3 -
 15*B*a*d*e^2 + 15*A*c*d^2*e))/(7*e^8) + (2*c^3*(A*e - 7*B*d)*(d + e*x)^(1
1/2))/(11*e^8) + (2*c*(a*e^2 + c*d^2)*(d + e*x)^(3/2)*(A*a*e^3 - 7*B*c*d^3
 - 3*B*a*d*e^2 + 5*A*c*d^2*e))/e^8
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^{3/2}}\,dx = \frac{\frac{4096}{429}b\,c^3d^7 - 8a^3cd\,e^6x + \frac{6}{5}a^2bc\,e^7x^3 + 4a^3bd\,e^6 - 16a^3c\,d^2e^5 - \frac{768}{35}a^2c^2d^4e^3 - \cdots}{\cdots}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^(3/2),x)
```

output
```
(2*( - 15015*a**4*e**7 + 30030*a**3*b*d*e**6 + 15015*a**3*b*e**7*x - 12012
0*a**3*c*d**2*e**5 - 60060*a**3*c*d*e**6*x + 15015*a**3*c*e**7*x**2 + 1441
44*a**2*b*c*d**3*e**4 + 72072*a**2*b*c*d**2*e**5*x - 18018*a**2*b*c*d*e**6
*x**2 + 9009*a**2*b*c*e**7*x**3 - 164736*a**2*c**2*d**4*e**3 - 82368*a**2*
c**2*d**3*e**4*x + 20592*a**2*c**2*d**2*e**5*x**2 - 10296*a**2*c**2*d*e**6
*x**3 + 6435*a**2*c**2*e**7*x**4 + 183040*a*b*c**2*d**5*e**2 + 91520*a*b*c
**2*d**4*e**3*x - 22880*a*b*c**2*d**3*e**4*x**2 + 11440*a*b*c**2*d**2*e**5
*x**3 - 7150*a*b*c**2*d*e**6*x**4 + 5005*a*b*c**2*e**7*x**5 - 66560*a*c**3
*d**6*e - 33280*a*c**3*d**5*e**2*x + 8320*a*c**3*d**4*e**3*x**2 - 4160*a*c
**3*d**3*e**4*x**3 + 2600*a*c**3*d**2*e**5*x**4 - 1820*a*c**3*d*e**6*x**5
+ 1365*a*c**3*e**7*x**6 + 71680*b*c**3*d**7 + 35840*b*c**3*d**6*e*x - 8960
*b*c**3*d**5*e**2*x**2 + 4480*b*c**3*d**4*e**3*x**3 - 2800*b*c**3*d**3*e**
4*x**4 + 1960*b*c**3*d**2*e**5*x**5 - 1470*b*c**3*d*e**6*x**6 + 1155*b*c**
3*e**7*x**7))/(15015*sqrt(d + e*x)*e**8)
```

3.119 $\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 346

$$
\begin{aligned}
\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{5/2}}\,dx &= \frac{2(Bd-Ae)\left(cd^2+ae^2\right)^3}{3e^8(d+ex)^{3/2}} \\
&- \frac{2\left(cd^2+ae^2\right)^2\left(7Bcd^2-6Acde+aBe^2\right)}{e^8\sqrt{d+ex}} \\
&- \frac{6c\left(cd^2+ae^2\right)\left(7Bcd^3-5Acd^2e+3aBde^2-aAe^3\right)\sqrt{d+ex}}{e^8} \\
&- \frac{2c\left(4Acde\left(5cd^2+3ae^2\right)-B\left(35c^2d^4+30acd^2e^2+3a^2e^4\right)\right)(d+ex)^{3/2}}{3e^8} \\
&- \frac{2c^2\left(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3\right)(d+ex)^{5/2}}{5e^8} \\
&+ \frac{6c^2\left(7Bcd^2-2Acde+aBe^2\right)(d+ex)^{7/2}}{7e^8} \\
&- \frac{2c^3(7Bd-Ae)(d+ex)^{9/2}}{9e^8} + \frac{2Bc^3(d+ex)^{11/2}}{11e^8}
\end{aligned}
$$

output
```
2/3*(-A*e+B*d)*(a*e^2+c*d^2)^3/e^8/(e*x+d)^(3/2)-2*(a*e^2+c*d^2)^2*(-6*A*c
*d*e+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)^(1/2)-6*c*(a*e^2+c*d^2)*(-A*a*e^3-5*A*
c*d^2*e+3*B*a*d*e^2+7*B*c*d^3)*(e*x+d)^(1/2)/e^8-2/3*c*(4*A*c*d*e*(3*a*e^2
+5*c*d^2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))*(e*x+d)^(3/2)/e^8-2/5*c
^2*(-3*A*a*e^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)*(e*x+d)^(5/2)/e^8+6/7
*c^2*(-2*A*c*d*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^(7/2)/e^8-2/9*c^3*(-A*e+7*B*d)
*(e*x+d)^(9/2)/e^8+2/11*B*c^3*(e*x+d)^(11/2)/e^8
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{5/2}}\,dx = \frac{22Ae(-105a^3e^6+315a^2ce^4(8d^2+12dex+3e^2x^2)+63ac^2e^2(128d^4+192d^3e$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^(5/2),x]
```

output
```
(22*A*e*(-105*a^3*e^6 + 315*a^2*c*e^4*(8*d^2 + 12*d*e*x + 3*e^2*x^2) + 63*
a*c^2*e^2*(128*d^4 + 192*d^3*e*x + 48*d^2*e^2*x^2 - 8*d*e^3*x^3 + 3*e^4*x^
4) + 5*c^3*(1024*d^6 + 1536*d^5*e*x + 384*d^4*e^2*x^2 - 64*d^3*e^3*x^3 + 2
4*d^2*e^4*x^4 - 12*d*e^5*x^5 + 7*e^6*x^6)) - 10*B*(231*a^3*e^6*(2*d + 3*e*
x) + 693*a^2*c*e^4*(16*d^3 + 24*d^2*e*x + 6*d*e^2*x^2 - e^3*x^3) + 99*a*c^
2*e^2*(256*d^5 + 384*d^4*e*x + 96*d^3*e^2*x^2 - 16*d^2*e^3*x^3 + 6*d*e^4*x
^4 - 3*e^5*x^5) + 7*c^3*(2048*d^7 + 3072*d^6*e*x + 768*d^5*e^2*x^2 - 128*d
^4*e^3*x^3 + 48*d^3*e^4*x^4 - 24*d^2*e^5*x^5 + 14*d*e^6*x^6 - 9*e^7*x^7)))
/(3465*e^8*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3 (A+Bx)}{(d+ex)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 652}$$

$$\int \left(-\frac{c\sqrt{d+ex}(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4)}{e^7}-\frac{3c^2(d+ex)^{5/2}(-aBe^2+2Acd}{e^7}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2c(d+ex)^{3/2}\left(4Acde\left(3ae^2+5cd^2\right)-B\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)\right)}{3e^8}+\frac{6c^2(d+ex)^{7/2}\left(aBe^2-2Acde+7Bcd^2\right)}{7e^8}-\frac{2c^2(d+ex)^{5/2}\left(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3\right)}{5e^8}-\frac{2\left(ae^2+cd^2\right)^2\left(aBe^2-6Acde+7Bcd^2\right)}{e^8\sqrt{d+ex}}+\frac{2\left(ae^2+cd^2\right)^3(Bd-Ae)}{3e^8(d+ex)^{3/2}}-\frac{6c\sqrt{d+ex}\left(ae^2+cd^2\right)\left(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3\right)}{e^8}-\frac{2c^3(d+ex)^{9/2}(7Bd-Ae)}{9e^8}+\frac{2Bc^3(d+ex)^{11/2}}{11e^8}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^(5/2),x]
```

output
```
(2*(B*d - A*e)*(c*d^2 + a*e^2)^3)/(3*e^8*(d + e*x)^(3/2)) - (2*(c*d^2 + a*
e^2)^2*(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2))/(e^8*Sqrt[d + e*x]) - (6*c*(c*d^
2 + a*e^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*Sqrt[d + e*x]
)/e^8 - (2*c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(35*c^2*d^4 + 30*a*c*d^2*e
^2 + 3*a^2*e^4))*(d + e*x)^(3/2))/(3*e^8) - (2*c^2*(35*B*c*d^3 - 15*A*c*d^
2*e + 15*a*B*d*e^2 - 3*a*A*e^3)*(d + e*x)^(5/2))/(5*e^8) + (6*c^2*(7*B*c*d
^2 - 2*A*c*d*e + a*B*e^2)*(d + e*x)^(7/2))/(7*e^8) - (2*c^3*(7*B*d - A*e)*
(d + e*x)^(9/2))/(9*e^8) + (2*B*c^3*(d + e*x)^(11/2))/(11*e^8)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$-\frac{2\left(\left(-\frac{\left(\frac{9Bx}{11}+A\right)x^6c^3}{3}-\frac{9x^4a\left(\frac{5Bx}{7}+A\right)c^2}{5}-9x^2a^2\left(\frac{Bx}{3}+A\right)c+a^3(3Bx+A)\right)e^7-36d\left(-\frac{\left(\frac{49Bx}{66}+A\right)x^5c^3}{63}-\frac{2\left(\frac{15Bx}{28}+\right.}{15}\right.}{}$
risch	$\frac{2c(315Bx^5c^2e^5+385Ax^4c^2e^5-1120Bx^4c^2de^4-1430Ax^3c^2de^4+1485Bx^3ace^5+2765Bx^3c^2d^2e^3+2079Ax^2ace^5+}{}$
gosper	$-\frac{2(-315Bx^7c^3e^7-385Ax^6c^3e^7+490Bx^6c^3de^6+660Ax^5c^3de^6-1485Bx^5ac^2e^7-840Bx^5c^3d^2e^5-2079Ax^4ac^2e^7}{}$
trager	$-\frac{2(-315Bx^7c^3e^7-385Ax^6c^3e^7+490Bx^6c^3de^6+660Ax^5c^3de^6-1485Bx^5ac^2e^7-840Bx^5c^3d^2e^5-2079Ax^4ac^2e^7}{}$
orering	$-\frac{2(-315Bx^7c^3e^7-385Ax^6c^3e^7+490Bx^6c^3de^6+660Ax^5c^3de^6-1485Bx^5ac^2e^7-840Bx^5c^3d^2e^5-2079Ax^4ac^2e^7}{}$
derivativedivides	$\frac{\frac{2Bc^3(ex+d)^{\frac{11}{2}}}{11}+\frac{2Ac^3e(ex+d)^{\frac{9}{2}}}{9}-\frac{14Bc^3d(ex+d)^{\frac{9}{2}}}{9}-\frac{12Ac^3de(ex+d)^{\frac{7}{2}}}{7}+\frac{6Bac^2e^2(ex+d)^{\frac{7}{2}}}{7}+6Bc^3d^2(ex+d)^{\frac{7}{2}}+\frac{6Aac^2e^3(}{5}}{}$
default	$\frac{\frac{2Bc^3(ex+d)^{\frac{11}{2}}}{11}+\frac{2Ac^3e(ex+d)^{\frac{9}{2}}}{9}-\frac{14Bc^3d(ex+d)^{\frac{9}{2}}}{9}-\frac{12Ac^3de(ex+d)^{\frac{7}{2}}}{7}+\frac{6Bac^2e^2(ex+d)^{\frac{7}{2}}}{7}+6Bc^3d^2(ex+d)^{\frac{7}{2}}+\frac{6Aac^2e^3(}{5}}{}$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(3/2)*((-1/3*(9/11*B*x+A)*x^6*c^3-9/5*x^4*a*(5/7*B*x+A)*c^2-9
*x^2*a^2*(1/3*B*x+A)*c+a^3*(3*B*x+A))*e^7-36*d*(-1/63*(49/66*B*x+A)*x^5*c^
3-2/15*(15/28*B*x+A)*x^3*a*c^2+a^2*x*(-1/2*B*x+A)*c-1/18*B*a^3)*e^6-24*c*(
1/21*(7/11*B*x+A)*x^4*c^2+6/5*(5/21*B*x+A)*x^2*a*c+a^2*(-3*B*x+A))*d^2*e^5
-576/5*c*d^3*(-5/189*(21/44*B*x+A)*x^3*c^2+a*x*(-5/14*B*x+A)*c-5/12*a^2*B)
*e^4-384/5*c^2*d^4*(5/21*(7/33*B*x+A)*x^2*c+a*(-15/7*B*x+A))*e^3-512/7*c^2
*d^5*(x*(-7/22*B*x+A)*c-3/2*B*a)*e^2-1024/21*c^3*d^6*(-21/11*B*x+A)*e+2048
/33*B*c^3*d^7)/e^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{5/2}}\,dx = \frac{2\,(315\,Bc^3e^7x^7 - 71680\,Bc^3d^7 + 56320\,Ac^3d^6e - 126720\,Bac^2d^5e^2 + 88704\,A$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
2/3465*(315*B*c^3*e^7*x^7 - 71680*B*c^3*d^7 + 56320*A*c^3*d^6*e - 126720*B
*a*c^2*d^5*e^2 + 88704*A*a*c^2*d^4*e^3 - 55440*B*a^2*c*d^3*e^4 + 27720*A*a
^2*c*d^2*e^5 - 2310*B*a^3*d*e^6 - 1155*A*a^3*e^7 - 35*(14*B*c^3*d*e^6 - 11
*A*c^3*e^7)*x^6 + 15*(56*B*c^3*d^2*e^5 - 44*A*c^3*d*e^6 + 99*B*a*c^2*e^7)*
x^5 - 3*(560*B*c^3*d^3*e^4 - 440*A*c^3*d^2*e^5 + 990*B*a*c^2*d*e^6 - 693*A
*a*c^2*e^7)*x^4 + (4480*B*c^3*d^4*e^3 - 3520*A*c^3*d^3*e^4 + 7920*B*a*c^2*
d^2*e^5 - 5544*A*a*c^2*d*e^6 + 3465*B*a^2*c*e^7)*x^3 - 3*(8960*B*c^3*d^5*e
^2 - 7040*A*c^3*d^4*e^3 + 15840*B*a*c^2*d^3*e^4 - 11088*A*a*c^2*d^2*e^5 +
6930*B*a^2*c*d*e^6 - 3465*A*a^2*c*e^7)*x^2 - 3*(35840*B*c^3*d^6*e - 28160*
A*c^3*d^5*e^2 + 63360*B*a*c^2*d^4*e^3 - 44352*A*a*c^2*d^3*e^4 + 27720*B*a^
2*c*d^2*e^5 - 13860*A*a^2*c*d*e^6 + 1155*B*a^3*e^7)*x)*sqrt(e*x + d)/(e^10
*x^2 + 2*d*e^9*x + d^2*e^8)
```

Sympy [A] (verification not implemented)

Time = 17.61 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.47

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{Bc^3(d+ex)^{\frac{11}{2}}}{11e^7}+\frac{(d+ex)^{\frac{9}{2}}\left(Ac^3e-7Bc^3d\right)}{9e^7}+\frac{(d+ex)^{\frac{7}{2}}\left(-6Ac^3de+3Bac^2e^2+21Bc^3d^2\right)}{7e^7}+\frac{(d+ex)^{\frac{5}{2}}\cdot\left(3Aac^2e^3\right.}{}\right.}{} \\ \frac{Aa^3x+Aa^2cx^3+\frac{3Aac^2x^5}{5}+\frac{Ac^3x^7}{7}+\frac{Ba^3x^2}{2}+\frac{3Ba^2cx^4}{4}+\frac{Bac^2x^6}{2}+\frac{Bc^3x^8}{8}}{d^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**(5/2),x)
```

output
```
Piecewise((2*(B*c**3*(d + e*x)**(11/2)/(11*e**7) + (d + e*x)**(9/2)*(A*c**
3*e - 7*B*c**3*d)/(9*e**7) + (d + e*x)**(7/2)*(-6*A*c**3*d*e + 3*B*a*c**2*
e**2 + 21*B*c**3*d**2)/(7*e**7) + (d + e*x)**(5/2)*(3*A*a*c**2*e**3 + 15*A
*c**3*d**2*e - 15*B*a*c**2*d*e**2 - 35*B*c**3*d**3)/(5*e**7) + (d + e*x)**
(3/2)*(-12*A*a*c**2*d*e**3 - 20*A*c**3*d**3*e + 3*B*a**2*c*e**4 + 30*B*a*c
**2*d**2*e**2 + 35*B*c**3*d**4)/(3*e**7) + sqrt(d + e*x)*(3*A*a**2*c*e**5
+ 18*A*a*c**2*d**2*e**3 + 15*A*c**3*d**4*e - 9*B*a**2*c*d*e**4 - 30*B*a*c*
*2*d**3*e**2 - 21*B*c**3*d**5)/e**7 - (a*e**2 + c*d**2)**2*(-6*A*c*d*e + B
*a*e**2 + 7*B*c*d**2)/(e**7*sqrt(d + e*x)) + (-A*e + B*d)*(a*e**2 + c*d**2
)**3/(3*e**7*(d + e*x)**(3/2)))/e, Ne(e, 0)), ((A*a**3*x + A*a**2*c*x**3 +
 3*A*a*c**2*x**5/5 + A*c**3*x**7/7 + B*a**3*x**2/2 + 3*B*a**2*c*x**4/4 + B
*a*c**2*x**6/2 + B*c**3*x**8/8)/d**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.33

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{5/2}}\,dx = \frac{2\left(\frac{315\,(ex+d)^{\frac{11}{2}}Bc^3-385\,\left(7\,Bc^3d-Ac^3e\right)(ex+d)^{\frac{9}{2}}+1485\,\left(7\,Bc^3d^2-2\,Ac^3de+Bac^2e^2\right)(ex+d)^{\frac{7}{2}}-693\,(}{}\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
2/3465*((315*(e*x + d)^(11/2)*B*c^3 - 385*(7*B*c^3*d - A*c^3*e)*(e*x + d)^
(9/2) + 1485*(7*B*c^3*d^2 - 2*A*c^3*d*e + B*a*c^2*e^2)*(e*x + d)^(7/2) - 6
93*(35*B*c^3*d^3 - 15*A*c^3*d^2*e + 15*B*a*c^2*d*e^2 - 3*A*a*c^2*e^3)*(e*x
 + d)^(5/2) + 1155*(35*B*c^3*d^4 - 20*A*c^3*d^3*e + 30*B*a*c^2*d^2*e^2 - 1
2*A*a*c^2*d*e^3 + 3*B*a^2*c*e^4)*(e*x + d)^(3/2) - 10395*(7*B*c^3*d^5 - 5*
A*c^3*d^4*e + 10*B*a*c^2*d^3*e^2 - 6*A*a*c^2*d^2*e^3 + 3*B*a^2*c*d*e^4 - A
*a^2*c*e^5)*sqrt(e*x + d))/e^7 + 1155*(B*c^3*d^7 - A*c^3*d^6*e + 3*B*a*c^2
*d^5*e^2 - 3*A*a*c^2*d^4*e^3 + 3*B*a^2*c*d^3*e^4 - 3*A*a^2*c*d^2*e^5 + B*a
^3*d*e^6 - A*a^3*e^7 - 3*(7*B*c^3*d^6 - 6*A*c^3*d^5*e + 15*B*a*c^2*d^4*e^2
 - 12*A*a*c^2*d^3*e^3 + 9*B*a^2*c*d^2*e^4 - 6*A*a^2*c*d*e^5 + B*a^3*e^6)*(
e*x + d))/((e*x + d)^(3/2)*e^7))/e
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.74

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{5/2}}\,dx =$$

$$-\frac{2\left(21\left(ex+d\right)Bc^3d^6-Bc^3d^7-18\left(ex+d\right)Ac^3d^5e+Ac^3d^6e+45\left(ex+d\right)Bac^2d^4e^2-3\,Bac^2d^5e^2-36\,(\ldots}{\ldots}$$

$$+\frac{2\left(315\left(ex+d\right)^{\frac{11}{2}}Bc^3e^{80}-2695\left(ex+d\right)^{\frac{9}{2}}Bc^3de^{80}+10395\left(ex+d\right)^{\frac{7}{2}}Bc^3d^2e^{80}-24255\left(ex+d\right)^{\frac{5}{2}}Bc^3d^3e^{\ldots}\right.}{\ldots}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(21*(e*x + d)*B*c^3*d^6 - B*c^3*d^7 - 18*(e*x + d)*A*c^3*d^5*e + A*c^
3*d^6*e + 45*(e*x + d)*B*a*c^2*d^4*e^2 - 3*B*a*c^2*d^5*e^2 - 36*(e*x + d)*
A*a*c^2*d^3*e^3 + 3*A*a*c^2*d^4*e^3 + 27*(e*x + d)*B*a^2*c*d^2*e^4 - 3*B*a
^2*c*d^3*e^4 - 18*(e*x + d)*A*a^2*c*d*e^5 + 3*A*a^2*c*d^2*e^5 + 3*(e*x + d
)*B*a^3*e^6 - B*a^3*d*e^6 + A*a^3*e^7)/((e*x + d)^(3/2)*e^8) + 2/3465*(315
*(e*x + d)^(11/2)*B*c^3*e^80 - 2695*(e*x + d)^(9/2)*B*c^3*d*e^80 + 10395*(
e*x + d)^(7/2)*B*c^3*d^2*e^80 - 24255*(e*x + d)^(5/2)*B*c^3*d^3*e^80 + 404
25*(e*x + d)^(3/2)*B*c^3*d^4*e^80 - 72765*sqrt(e*x + d)*B*c^3*d^5*e^80 + 3
85*(e*x + d)^(9/2)*A*c^3*e^81 - 2970*(e*x + d)^(7/2)*A*c^3*d*e^81 + 10395*
(e*x + d)^(5/2)*A*c^3*d^2*e^81 - 23100*(e*x + d)^(3/2)*A*c^3*d^3*e^81 + 51
975*sqrt(e*x + d)*A*c^3*d^4*e^81 + 1485*(e*x + d)^(7/2)*B*a*c^2*e^82 - 103
95*(e*x + d)^(5/2)*B*a*c^2*d*e^82 + 34650*(e*x + d)^(3/2)*B*a*c^2*d^2*e^82
 - 103950*sqrt(e*x + d)*B*a*c^2*d^3*e^82 + 2079*(e*x + d)^(5/2)*A*a*c^2*e^
83 - 13860*(e*x + d)^(3/2)*A*a*c^2*d*e^83 + 62370*sqrt(e*x + d)*A*a*c^2*d^
2*e^83 + 3465*(e*x + d)^(3/2)*B*a^2*c*e^84 - 31185*sqrt(e*x + d)*B*a^2*c*d
*e^84 + 10395*sqrt(e*x + d)*A*a^2*c*e^85)/e^88
```

Mupad [B] (verification not implemented)

Time = 6.30 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^{5/2}}\,dx = \frac{(d+e\,x)^{3/2}\,(6\,B\,a^2\,c\,e^4+60\,B\,a\,c^2\,d^2\,e^2-24\,A\,a\,c^2\,d\,e^3+70\,B\,c^3\,d^4-40\,A\,c}{3\,e^8}$$
$$+\frac{(d+e\,x)^{7/2}\,(42\,B\,c^3\,d^2-12\,A\,c^3\,d\,e+6\,B\,a\,c^2\,e^2)}{7\,e^8}$$
$$-\frac{(d+e\,x)\,\left(2\,B\,a^3\,e^6+18\,B\,a^2\,c\,d^2\,e^4-12\,A\,a^2\,c\,d\,e^5+30\,B\,a\,c^2\,d^4\,e^2-24\,A\,a\,c^2\,d^3\,e^3+14\,B\,c^3\,d^6-12\,A}{e^8\,(}$$
$$+\frac{2\,B\,c^3\,(d+e\,x)^{11/2}}{11\,e^8}+\frac{2\,c^2\,(d+e\,x)^{5/2}\,(-35\,B\,c\,d^3+15\,A\,c\,d^2\,e-15\,B\,a\,d\,e^2+3\,A\,a\,e^3)}{5\,e^8}$$
$$+\frac{2\,c^3\,(A\,e-7\,B\,d)\,(d+e\,x)^{9/2}}{9\,e^8}$$
$$+\frac{6\,c\,(c\,d^2+a\,e^2)\,\sqrt{d+e\,x}\,(-7\,B\,c\,d^3+5\,A\,c\,d^2\,e-3\,B\,a\,d\,e^2+A\,a\,e^3)}{e^8}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^(5/2),x)
```

output
```
((d + e*x)^(3/2)*(70*B*c^3*d^4 + 6*B*a^2*c*e^4 - 40*A*c^3*d^3*e + 60*B*a*c
^2*d^2*e^2 - 24*A*a*c^2*d*e^3))/(3*e^8) + ((d + e*x)^(7/2)*(42*B*c^3*d^2 -
 12*A*c^3*d*e + 6*B*a*c^2*e^2))/(7*e^8) - ((d + e*x)*(2*B*a^3*e^6 + 14*B*c
^3*d^6 - 12*A*c^3*d^5*e - 24*A*a*c^2*d^3*e^3 + 30*B*a*c^2*d^4*e^2 + 18*B*a
^2*c*d^2*e^4 - 12*A*a^2*c*d*e^5) + (2*A*a^3*e^7)/3 - (2*B*c^3*d^7)/3 - (2*
B*a^3*d*e^6)/3 + (2*A*c^3*d^6*e)/3 + 2*A*a*c^2*d^4*e^3 + 2*A*a^2*c*d^2*e^5
 - 2*B*a*c^2*d^5*e^2 - 2*B*a^2*c*d^3*e^4)/(e^8*(d + e*x)^(3/2)) + (2*B*c^3
*(d + e*x)^(11/2))/(11*e^8) + (2*c^2*(d + e*x)^(5/2)*(3*A*a*e^3 - 35*B*c*d
^3 - 15*B*a*d*e^2 + 15*A*c*d^2*e))/(5*e^8) + (2*c^3*(A*e - 7*B*d)*(d + e*x
)^(9/2))/(9*e^8) + (6*c*(a*e^2 + c*d^2)*(d + e*x)^(1/2)*(A*a*e^3 - 7*B*c*d
^3 - 3*B*a*d*e^2 + 5*A*c*d^2*e))/e^8
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.44

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{5/2}}\,dx = \frac{-\frac{4096}{99}bc^3d^7+24a^3cde^6x+2a^2bce^7x^3-\frac{4}{3}a^3bde^6+16a^3cd^2e^5+\frac{256}{5}a^2c^2d^4e^3}{}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^(5/2),x)
```

output
```
(2*( - 1155*a**4*e**7 - 2310*a**3*b*d*e**6 - 3465*a**3*b*e**7*x + 27720*a*
*3*c*d**2*e**5 + 41580*a**3*c*d*e**6*x + 10395*a**3*c*e**7*x**2 - 55440*a*
*2*b*c*d**3*e**4 - 83160*a**2*b*c*d**2*e**5*x - 20790*a**2*b*c*d*e**6*x**2
 + 3465*a**2*b*c*e**7*x**3 + 88704*a**2*c**2*d**4*e**3 + 133056*a**2*c**2*
d**3*e**4*x + 33264*a**2*c**2*d**2*e**5*x**2 - 5544*a**2*c**2*d*e**6*x**3
+ 2079*a**2*c**2*e**7*x**4 - 126720*a*b*c**2*d**5*e**2 - 190080*a*b*c**2*d
**4*e**3*x - 47520*a*b*c**2*d**3*e**4*x**2 + 7920*a*b*c**2*d**2*e**5*x**3
- 2970*a*b*c**2*d*e**6*x**4 + 1485*a*b*c**2*e**7*x**5 + 56320*a*c**3*d**6*
e + 84480*a*c**3*d**5*e**2*x + 21120*a*c**3*d**4*e**3*x**2 - 3520*a*c**3*d
**3*e**4*x**3 + 1320*a*c**3*d**2*e**5*x**4 - 660*a*c**3*d*e**6*x**5 + 385*
a*c**3*e**7*x**6 - 71680*b*c**3*d**7 - 107520*b*c**3*d**6*e*x - 26880*b*c*
*3*d**5*e**2*x**2 + 4480*b*c**3*d**4*e**3*x**3 - 1680*b*c**3*d**3*e**4*x**
4 + 840*b*c**3*d**2*e**5*x**5 - 490*b*c**3*d*e**6*x**6 + 315*b*c**3*e**7*x
**7))/(3465*sqrt(d + e*x)*e**8*(d + e*x))
```

3.120 $\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{7/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 346

$$\begin{aligned}
\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{7/2}} \, dx &= \frac{2(Bd-Ae)(cd^2+ae^2)^3}{5e^8(d+ex)^{5/2}} \\
&- \frac{2(cd^2+ae^2)^2(7Bcd^2-6Acde+aBe^2)}{3e^8(d+ex)^{3/2}} \\
&+ \frac{6c(cd^2+ae^2)(7Bcd^3-5Acd^2e+3aBde^2-aAe^3)}{e^8\sqrt{d+ex}} \\
&- \frac{2c(4Acde(5cd^2+3ae^2)-B(35c^2d^4+30acd^2e^2+3a^2e^4))\sqrt{d+ex}}{e^8} \\
&- \frac{2c^2(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3)(d+ex)^{3/2}}{3e^8} \\
&+ \frac{6c^2(7Bcd^2-2Acde+aBe^2)(d+ex)^{5/2}}{5e^8} \\
&- \frac{2c^3(7Bd-Ae)(d+ex)^{7/2}}{7e^8} + \frac{2Bc^3(d+ex)^{9/2}}{9e^8}
\end{aligned}$$

output
```
2/5*(-A*e+B*d)*(a*e^2+c*d^2)^3/e^8/(e*x+d)^(5/2)-2/3*(a*e^2+c*d^2)^2*(-6*A
*c*d*e+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)^(3/2)+6*c*(a*e^2+c*d^2)*(-A*a*e^3-5*
A*c*d^2*e+3*B*a*d*e^2+7*B*c*d^3)/e^8/(e*x+d)^(1/2)-2*c*(4*A*c*d*e*(3*a*e^2
+5*c*d^2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))*(e*x+d)^(1/2)/e^8-2/3*c
^2*(-3*A*a*e^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)*(e*x+d)^(3/2)/e^8+6/5
*c^2*(-2*A*c*d*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^(5/2)/e^8-2/7*c^3*(-A*e+7*B*d)
*(e*x+d)^(7/2)/e^8+2/9*B*c^3*(e*x+d)^(9/2)/e^8
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{7/2}}\,dx = \frac{2(-9Ae(7a^3e^6+7a^2ce^4(8d^2+20dex+15e^2x^2)+7ac^2e^2(128d^4+320d^3ex+$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^(7/2),x]
```

output
```
(2*(-9*A*e*(7*a^3*e^6 + 7*a^2*c*e^4*(8*d^2 + 20*d*e*x + 15*e^2*x^2) + 7*a*
c^2*e^2*(128*d^4 + 320*d^3*e*x + 240*d^2*e^2*x^2 + 40*d*e^3*x^3 - 5*e^4*x^
4) + c^3*(1024*d^6 + 2560*d^5*e*x + 1920*d^4*e^2*x^2 + 320*d^3*e^3*x^3 - 4
0*d^2*e^4*x^4 + 12*d*e^5*x^5 - 5*e^6*x^6)) + 7*B*(-3*a^3*e^6*(2*d + 5*e*x)
 + 27*a^2*c*e^4*(16*d^3 + 40*d^2*e*x + 30*d*e^2*x^2 + 5*e^3*x^3) + 9*a*c^2
*e^2*(256*d^5 + 640*d^4*e*x + 480*d^3*e^2*x^2 + 80*d^2*e^3*x^3 - 10*d*e^4*
x^4 + 3*e^5*x^5) + c^3*(2048*d^7 + 5120*d^6*e*x + 3840*d^5*e^2*x^2 + 640*d
^4*e^3*x^3 - 80*d^3*e^4*x^4 + 24*d^2*e^5*x^5 - 10*d*e^6*x^6 + 5*e^7*x^7)))
)/(315*e^8*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3 (A+Bx)}{(d+ex)^{7/2}} dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{c(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4)}{e^7\sqrt{d+ex}} - \frac{3c^2(d+ex)^{3/2}(-aBe^2+2Acde-7Bc\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2c\sqrt{d+ex}(4Acde(3ae^2+5cd^2)-B(3a^2e^4+30acd^2e^2+35c^2d^4))}{e^8} + \frac{6c^2(d+ex)^{5/2}(aBe^2-2Acde+7Bcd^2)}{5e^8} - \frac{2c^2(d+ex)^{3/2}(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3)}{3e^8} - \frac{2(ae^2+cd^2)^2(aBe^2-6Acde+7Bcd^2)}{3e^8(d+ex)^{3/2}} + \frac{2(ae^2+cd^2)^3(Bd-Ae)}{5e^8(d+ex)^{5/2}} + \frac{6c(ae^2+cd^2)(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3)}{e^8\sqrt{d+ex}} - \frac{2c^3(d+ex)^{7/2}(7Bd-Ae)}{7e^8} + \frac{2Bc^3(d+ex)^{9/2}}{9e^8}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^(7/2),x]
```

output
```
(2*(B*d - A*e)*(c*d^2 + a*e^2)^3)/(5*e^8*(d + e*x)^(5/2)) - (2*(c*d^2 + a*
e^2)^2*(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2))/(3*e^8*(d + e*x)^(3/2)) + (6*c*(
c*d^2 + a*e^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(e^8*Sqr
t[d + e*x]) - (2*c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(35*c^2*d^4 + 30*a*c
*d^2*e^2 + 3*a^2*e^4))*Sqrt[d + e*x])/e^8 - (2*c^2*(35*B*c*d^3 - 15*A*c*d^
2*e + 15*a*B*d*e^2 - 3*a*A*e^3)*(d + e*x)^(3/2))/(3*e^8) + (6*c^2*(7*B*c*d
^2 - 2*A*c*d*e + a*B*e^2)*(d + e*x)^(5/2))/(5*e^8) - (2*c^3*(7*B*d - A*e)*
(d + e*x)^(7/2))/(7*e^8) + (2*B*c^3*(d + e*x)^(9/2))/(9*e^8)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$-\frac{2\left(\left(-\frac{5\left(\frac{7Bx}{9}+A\right)x^{6}c^{3}}{7}-5\left(\frac{3Bx}{5}+A\right)x^{4}a\,c^{2}+15a^{2}x^{2}(-Bx+A)c+a^{3}\left(\frac{5Bx}{3}+A\right)\right)e^{7}+20\left(\left(\frac{1}{18}B\,x^{6}+\frac{3}{35}A\,x^{5}\right)c^{3}+2\left(\cdots\right.\right.}{\cdots}$
risch	$-\frac{2c\left(-35e^{4}B\,c^{2}x^{4}-45e^{4}A\,c^{2}x^{3}+175d\,e^{3}B\,c^{2}x^{3}+243d\,e^{3}A\,c^{2}x^{2}-189e^{4}Bac\,x^{2}-588d^{2}e^{2}c^{2}B\,x^{2}-315e^{4}Aacx-954d^{2}\cdots\right.}{3\cdots}$
gosper	$-\frac{2\left(-35B\,x^{7}c^{3}e^{7}-45A\,x^{6}c^{3}e^{7}+70B\,x^{6}c^{3}d\,e^{6}+108A\,x^{5}c^{3}d\,e^{6}-189B\,x^{5}a\,c^{2}e^{7}-168B\,x^{5}c^{3}d^{2}e^{5}-315A\,x^{4}a\,c^{2}e^{7}-360\cdots\right.}{\cdots}$
trager	$-\frac{2\left(-35B\,x^{7}c^{3}e^{7}-45A\,x^{6}c^{3}e^{7}+70B\,x^{6}c^{3}d\,e^{6}+108A\,x^{5}c^{3}d\,e^{6}-189B\,x^{5}a\,c^{2}e^{7}-168B\,x^{5}c^{3}d^{2}e^{5}-315A\,x^{4}a\,c^{2}e^{7}-360\cdots\right.}{\cdots}$
orering	$-\frac{2\left(-35B\,x^{7}c^{3}e^{7}-45A\,x^{6}c^{3}e^{7}+70B\,x^{6}c^{3}d\,e^{6}+108A\,x^{5}c^{3}d\,e^{6}-189B\,x^{5}a\,c^{2}e^{7}-168B\,x^{5}c^{3}d^{2}e^{5}-315A\,x^{4}a\,c^{2}e^{7}-360\cdots\right.}{\cdots}$
derivativedivides	$\frac{\frac{2B\,c^{3}(ex+d)^{\frac{9}{2}}}{9}+\frac{2A\,c^{3}e(ex+d)^{\frac{7}{2}}}{7}-2B\,c^{3}d(ex+d)^{\frac{7}{2}}-\frac{12A\,c^{3}de(ex+d)^{\frac{5}{2}}}{5}+\frac{6Ba\,c^{2}e^{2}(ex+d)^{\frac{5}{2}}}{5}+\frac{42B\,c^{3}d^{2}(ex+d)^{\frac{5}{2}}}{5}+2Aa\,c^{2}e^{3}(\cdots}{\cdots}$
default	$\frac{\frac{2B\,c^{3}(ex+d)^{\frac{9}{2}}}{9}+\frac{2A\,c^{3}e(ex+d)^{\frac{7}{2}}}{7}-2B\,c^{3}d(ex+d)^{\frac{7}{2}}-\frac{12A\,c^{3}de(ex+d)^{\frac{5}{2}}}{5}+\frac{6Ba\,c^{2}e^{2}(ex+d)^{\frac{5}{2}}}{5}+\frac{42B\,c^{3}d^{2}(ex+d)^{\frac{5}{2}}}{5}+2Aa\,c^{2}e^{3}(\cdots}{\cdots}$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(e*x+d)^(5/2)*((-5/7*(7/9*B*x+A)*x^6*c^3-5*(3/5*B*x+A)*x^4*a*c^2+15*a
^2*x^2*(-B*x+A)*c+a^3*(5/3*B*x+A))*e^7+20*((1/18*B*x^6+3/35*A*x^5)*c^3+2*(
1/4*B*x+A)*x^3*a*c^2+a^2*x*(-9/2*B*x+A)*c+1/30*B*a^3)*d*e^6+8*c*d^2*((-1/3
*B*x^5-5/7*A*x^4)*c^2+30*x^2*(-1/3*B*x+A)*a*c+a^2*(-15*B*x+A))*e^5+320*c*d
^3*(1/7*(7/36*B*x+A)*x^3*c^2+a*x*(-3/2*B*x+A)*c-3/20*a^2*B)*e^4+128*c^2*((
-5/9*B*x^3+15/7*A*x^2)*c+a*(-5*B*x+A))*d^4*e^3+2560/7*c^2*d^5*(x*(-7/6*B*x
+A)*c-7/10*B*a)*e^2+1024/7*c^3*d^6*(-35/9*B*x+A)*e-2048/9*B*c^3*d^7)/e^8
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.41

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{7/2}}\,dx = \frac{2\,(35\,Bc^3e^7x^7+14336\,Bc^3d^7-9216\,Ac^3d^6e+16128\,Bac^2d^5e^2-8064\,Aac^2d}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
2/315*(35*B*c^3*e^7*x^7 + 14336*B*c^3*d^7 - 9216*A*c^3*d^6*e + 16128*B*a*c
^2*d^5*e^2 - 8064*A*a*c^2*d^4*e^3 + 3024*B*a^2*c*d^3*e^4 - 504*A*a^2*c*d^2
*e^5 - 42*B*a^3*d*e^6 - 63*A*a^3*e^7 - 5*(14*B*c^3*d*e^6 - 9*A*c^3*e^7)*x^
6 + 3*(56*B*c^3*d^2*e^5 - 36*A*c^3*d*e^6 + 63*B*a*c^2*e^7)*x^5 - 5*(112*B*
c^3*d^3*e^4 - 72*A*c^3*d^2*e^5 + 126*B*a*c^2*d*e^6 - 63*A*a*c^2*e^7)*x^4 +
 5*(896*B*c^3*d^4*e^3 - 576*A*c^3*d^3*e^4 + 1008*B*a*c^2*d^2*e^5 - 504*A*a
*c^2*d*e^6 + 189*B*a^2*c*e^7)*x^3 + 15*(1792*B*c^3*d^5*e^2 - 1152*A*c^3*d^
4*e^3 + 2016*B*a*c^2*d^3*e^4 - 1008*A*a*c^2*d^2*e^5 + 378*B*a^2*c*d*e^6 -
63*A*a^2*c*e^7)*x^2 + 5*(7168*B*c^3*d^6*e - 4608*A*c^3*d^5*e^2 + 8064*B*a*
c^2*d^4*e^3 - 4032*A*a*c^2*d^3*e^4 + 1512*B*a^2*c*d^2*e^5 - 252*A*a^2*c*d*
e^6 - 21*B*a^3*e^7)*x)*sqrt(e*x + d)/(e^11*x^3 + 3*d*e^10*x^2 + 3*d^2*e^9*
x + d^3*e^8)
```

Sympy [A] (verification not implemented)

Time = 21.27 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.38

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^{7/2}}\,dx = \begin{cases} \frac{2\left(\frac{Bc^3(d+ex)^{\frac{9}{2}}}{9e^7}+\frac{3c\left(ae^2+cd^2\right)\left(-Aae^3-5Acd^2e+3Bade^2+7Bcd^3\right)}{e^7\sqrt{d+ex}}+\frac{(d+ex)^{\frac{7}{2}}\left(Ac^3e-7Bc^3d\right)}{7e^7}+\frac{(d+ex)^{\frac{5}{2}}\left(-6\right.}{}\right.}{} \\ \frac{Aa^3x+Aa^2cx^3+\frac{3Aac^2x^5}{5}+\frac{Ac^3x^7}{7}+\frac{Ba^3x^2}{2}+\frac{3Ba^2cx^4}{4}+\frac{Bac^2x^6}{2}+\frac{Bc^3x^8}{8}}{d^{\frac{7}{2}}} \end{cases}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**(7/2),x)
```

output
```
Piecewise((2*(B*c**3*(d + e*x)**(9/2)/(9*e**7) + 3*c*(a*e**2 + c*d**2)*(-A
*a*e**3 - 5*A*c*d**2*e + 3*B*a*d*e**2 + 7*B*c*d**3)/(e**7*sqrt(d + e*x)) +
 (d + e*x)**(7/2)*(A*c**3*e - 7*B*c**3*d)/(7*e**7) + (d + e*x)**(5/2)*(-6*
A*c**3*d*e + 3*B*a*c**2*e**2 + 21*B*c**3*d**2)/(5*e**7) + (d + e*x)**(3/2)
*(3*A*a*c**2*e**3 + 15*A*c**3*d**2*e - 15*B*a*c**2*d*e**2 - 35*B*c**3*d**3
)/(3*e**7) + sqrt(d + e*x)*(-12*A*a*c**2*d*e**3 - 20*A*c**3*d**3*e + 3*B*a
**2*c*e**4 + 30*B*a*c**2*d**2*e**2 + 35*B*c**3*d**4)/e**7 - (a*e**2 + c*d*
*2)**2*(-6*A*c*d*e + B*a*e**2 + 7*B*c*d**2)/(3*e**7*(d + e*x)**(3/2)) + (-
A*e + B*d)*(a*e**2 + c*d**2)**3/(5*e**7*(d + e*x)**(5/2)))/e, Ne(e, 0)), (
(A*a**3*x + A*a**2*c*x**3 + 3*A*a*c**2*x**5/5 + A*c**3*x**7/7 + B*a**3*x**
2/2 + 3*B*a**2*c*x**4/4 + B*a*c**2*x**6/2 + B*c**3*x**8/8)/d**(7/2), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.33

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^{7/2}}\,dx = \frac{2\left(\frac{35\,(ex+d)^{\frac{9}{2}}Bc^3-45\,(7\,Bc^3d-Ac^3e)\,(ex+d)^{\frac{7}{2}}+189\,(7\,Bc^3d^2-2\,Ac^3de+Bac^2e^2)\,(ex+d)^{\frac{5}{2}}-105\,(35\,B}{}\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
2/315*((35*(e*x + d)^(9/2)*B*c^3 - 45*(7*B*c^3*d - A*c^3*e)*(e*x + d)^(7/2
) + 189*(7*B*c^3*d^2 - 2*A*c^3*d*e + B*a*c^2*e^2)*(e*x + d)^(5/2) - 105*(3
5*B*c^3*d^3 - 15*A*c^3*d^2*e + 15*B*a*c^2*d*e^2 - 3*A*a*c^2*e^3)*(e*x + d)
^(3/2) + 315*(35*B*c^3*d^4 - 20*A*c^3*d^3*e + 30*B*a*c^2*d^2*e^2 - 12*A*a*
c^2*d*e^3 + 3*B*a^2*c*e^4)*sqrt(e*x + d))/e^7 + 21*(3*B*c^3*d^7 - 3*A*c^3*
d^6*e + 9*B*a*c^2*d^5*e^2 - 9*A*a*c^2*d^4*e^3 + 9*B*a^2*c*d^3*e^4 - 9*A*a^
2*c*d^2*e^5 + 3*B*a^3*d*e^6 - 3*A*a^3*e^7 + 45*(7*B*c^3*d^5 - 5*A*c^3*d^4*
e + 10*B*a*c^2*d^3*e^2 - 6*A*a*c^2*d^2*e^3 + 3*B*a^2*c*d*e^4 - A*a^2*c*e^5
)*(e*x + d)^2 - 5*(7*B*c^3*d^6 - 6*A*c^3*d^5*e + 15*B*a*c^2*d^4*e^2 - 12*A
*a*c^2*d^3*e^3 + 9*B*a^2*c*d^2*e^4 - 6*A*a^2*c*d*e^5 + B*a^3*e^6)*(e*x + d
))/((e*x + d)^(5/2)*e^7))/e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.73

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{7/2}}\,dx = \frac{2\left(315\,(ex+d)^2Bc^3d^5-35\,(ex+d)Bc^3d^6+3\,Bc^3d^7-225\,(ex+d)^2Ac^3d^4e\right.}{}$$

$$+\frac{2\left(35\,(ex+d)^{\frac{9}{2}}Bc^3e^{64}-315\,(ex+d)^{\frac{7}{2}}Bc^3de^{64}+1323\,(ex+d)^{\frac{5}{2}}Bc^3d^2e^{64}-3675\,(ex+d)^{\frac{3}{2}}Bc^3d^3e^{64}+1\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
2/15*(315*(e*x + d)^2*B*c^3*d^5 - 35*(e*x + d)*B*c^3*d^6 + 3*B*c^3*d^7 - 2
25*(e*x + d)^2*A*c^3*d^4*e + 30*(e*x + d)*A*c^3*d^5*e - 3*A*c^3*d^6*e + 45
0*(e*x + d)^2*B*a*c^2*d^3*e^2 - 75*(e*x + d)*B*a*c^2*d^4*e^2 + 9*B*a*c^2*d
^5*e^2 - 270*(e*x + d)^2*A*a*c^2*d^2*e^3 + 60*(e*x + d)*A*a*c^2*d^3*e^3 -
9*A*a*c^2*d^4*e^3 + 135*(e*x + d)^2*B*a^2*c*d*e^4 - 45*(e*x + d)*B*a^2*c*d
^2*e^4 + 9*B*a^2*c*d^3*e^4 - 45*(e*x + d)^2*A*a^2*c*e^5 + 30*(e*x + d)*A*a
^2*c*d*e^5 - 9*A*a^2*c*d^2*e^5 - 5*(e*x + d)*B*a^3*e^6 + 3*B*a^3*d*e^6 - 3
*A*a^3*e^7)/((e*x + d)^(5/2)*e^8) + 2/315*(35*(e*x + d)^(9/2)*B*c^3*e^64 -
 315*(e*x + d)^(7/2)*B*c^3*d*e^64 + 1323*(e*x + d)^(5/2)*B*c^3*d^2*e^64 -
3675*(e*x + d)^(3/2)*B*c^3*d^3*e^64 + 11025*sqrt(e*x + d)*B*c^3*d^4*e^64 +
 45*(e*x + d)^(7/2)*A*c^3*e^65 - 378*(e*x + d)^(5/2)*A*c^3*d*e^65 + 1575*(
e*x + d)^(3/2)*A*c^3*d^2*e^65 - 6300*sqrt(e*x + d)*A*c^3*d^3*e^65 + 189*(e
*x + d)^(5/2)*B*a*c^2*e^66 - 1575*(e*x + d)^(3/2)*B*a*c^2*d*e^66 + 9450*sq
rt(e*x + d)*B*a*c^2*d^2*e^66 + 315*(e*x + d)^(3/2)*A*a*c^2*e^67 - 3780*sqr
t(e*x + d)*A*a*c^2*d*e^67 + 945*sqrt(e*x + d)*B*a^2*c*e^68)/e^72
```

Mupad [B] (verification not implemented)

Time = 6.53 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.32

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^{7/2}}\,dx = \frac{\sqrt{d+e\,x}\,(6\,B\,a^2\,c\,e^4+60\,B\,a\,c^2\,d^2\,e^2-24\,A\,a\,c^2\,d\,e^3+70\,B\,c^3\,d^4-40\,A\,c^3\,d}{e^8}$$
$$-\frac{(d+e\,x)\,\left(\frac{2\,B\,a^3\,e^6}{3}+6\,B\,a^2\,c\,d^2\,e^4-4\,A\,a^2\,c\,d\,e^5+10\,B\,a\,c^2\,d^4\,e^2-8\,A\,a\,c^2\,d^3\,e^3+\frac{14\,B\,c^3\,d^6}{3}-4\,A\,c^3\,d^5\,e\right)}{}$$
$$+\frac{(d+e\,x)^{5/2}\,(42\,B\,c^3\,d^2-12\,A\,c^3\,d\,e+6\,B\,a\,c^2\,e^2)}{5\,e^8}+\frac{2\,B\,c^3\,(d+e\,x)^{9/2}}{9\,e^8}$$
$$+\frac{2\,c^2\,(d+e\,x)^{3/2}\,(-35\,B\,c\,d^3+15\,A\,c\,d^2\,e-15\,B\,a\,d\,e^2+3\,A\,a\,e^3)}{3\,e^8}$$
$$+\frac{2\,c^3\,(A\,e-7\,B\,d)\,(d+e\,x)^{7/2}}{7\,e^8}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^(7/2),x)
```

output
```
((d + e*x)^(1/2)*(70*B*c^3*d^4 + 6*B*a^2*c*e^4 - 40*A*c^3*d^3*e + 60*B*a*c
^2*d^2*e^2 - 24*A*a*c^2*d*e^3))/e^8 - ((d + e*x)*((2*B*a^3*e^6)/3 + (14*B*
c^3*d^6)/3 - 4*A*c^3*d^5*e - 8*A*a*c^2*d^3*e^3 + 10*B*a*c^2*d^4*e^2 + 6*B*
a^2*c*d^2*e^4 - 4*A*a^2*c*d*e^5) - (d + e*x)^2*(42*B*c^3*d^5 - 6*A*a^2*c*e
^5 - 30*A*c^3*d^4*e - 36*A*a*c^2*d^2*e^3 + 60*B*a*c^2*d^3*e^2 + 18*B*a^2*c
*d*e^4) + (2*A*a^3*e^7)/5 - (2*B*c^3*d^7)/5 - (2*B*a^3*d*e^6)/5 + (2*A*c^3
*d^6*e)/5 + (6*A*a*c^2*d^4*e^3)/5 + (6*A*a^2*c*d^2*e^5)/5 - (6*B*a*c^2*d^5
*e^2)/5 - (6*B*a^2*c*d^3*e^4)/5)/(e^8*(d + e*x)^(5/2)) + ((d + e*x)^(5/2)*
(42*B*c^3*d^2 - 12*A*c^3*d*e + 6*B*a*c^2*e^2))/(5*e^8) + (2*B*c^3*(d + e*x
)^(9/2))/(9*e^8) + (2*c^2*(d + e*x)^(3/2)*(3*A*a*e^3 - 35*B*c*d^3 - 15*B*a
*d*e^2 + 15*A*c*d^2*e))/(3*e^8) + (2*c^3*(A*e - 7*B*d)*(d + e*x)^(7/2))/(7
*e^8)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.47

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{7/2}}\,dx = \frac{\frac{4096}{45}bc^3d^7 - 8a^3cd\,e^6x + 6a^2bc\,e^7x^3 - \frac{4}{15}a^3bd\,e^6 - \frac{16}{5}a^3c\,d^2e^5 - \frac{256}{5}a^2c^2d^4e^3 - }{}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^(7/2),x)
```

output
```
(2*( - 63*a**4*e**7 - 42*a**3*b*d*e**6 - 105*a**3*b*e**7*x - 504*a**3*c*d*
*2*e**5 - 1260*a**3*c*d*e**6*x - 945*a**3*c*e**7*x**2 + 3024*a**2*b*c*d**3
*e**4 + 7560*a**2*b*c*d**2*e**5*x + 5670*a**2*b*c*d*e**6*x**2 + 945*a**2*b
*c*e**7*x**3 - 8064*a**2*c**2*d**4*e**3 - 20160*a**2*c**2*d**3*e**4*x - 15
120*a**2*c**2*d**2*e**5*x**2 - 2520*a**2*c**2*d*e**6*x**3 + 315*a**2*c**2*
e**7*x**4 + 16128*a*b*c**2*d**5*e**2 + 40320*a*b*c**2*d**4*e**3*x + 30240*
a*b*c**2*d**3*e**4*x**2 + 5040*a*b*c**2*d**2*e**5*x**3 - 630*a*b*c**2*d*e*
*6*x**4 + 189*a*b*c**2*e**7*x**5 - 9216*a*c**3*d**6*e - 23040*a*c**3*d**5*
e**2*x - 17280*a*c**3*d**4*e**3*x**2 - 2880*a*c**3*d**3*e**4*x**3 + 360*a*
c**3*d**2*e**5*x**4 - 108*a*c**3*d*e**6*x**5 + 45*a*c**3*e**7*x**6 + 14336
*b*c**3*d**7 + 35840*b*c**3*d**6*e*x + 26880*b*c**3*d**5*e**2*x**2 + 4480*
b*c**3*d**4*e**3*x**3 - 560*b*c**3*d**3*e**4*x**4 + 168*b*c**3*d**2*e**5*x
**5 - 70*b*c**3*d*e**6*x**6 + 35*b*c**3*e**7*x**7))/(315*sqrt(d + e*x)*e**
8*(d**2 + 2*d*e*x + e**2*x**2))
```

3.121 $\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 342

$$\begin{aligned}\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{9/2}}\,dx &= \frac{2(Bd-Ae)(cd^2+ae^2)^3}{7e^8(d+ex)^{7/2}} \\ &- \frac{2(cd^2+ae^2)^2(7Bcd^2-6Acde+aBe^2)}{5e^8(d+ex)^{5/2}} \\ &+ \frac{2c(cd^2+ae^2)(7Bcd^3-5Acd^2e+3aBde^2-aAe^3)}{e^8(d+ex)^{3/2}} \\ &+ \frac{2c(4Acde(5cd^2+3ae^2)-B(35c^2d^4+30acd^2e^2+3a^2e^4))}{e^8\sqrt{d+ex}} \\ &- \frac{2c^2(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3)\sqrt{d+ex}}{e^8} \\ &+ \frac{2c^2(7Bcd^2-2Acde+aBe^2)(d+ex)^{3/2}}{e^8} \\ &- \frac{2c^3(7Bd-Ae)(d+ex)^{5/2}}{5e^8} + \frac{2Bc^3(d+ex)^{7/2}}{7e^8}\end{aligned}$$

output
```
2/7*(-A*e+B*d)*(a*e^2+c*d^2)^3/e^8/(e*x+d)^(7/2)-2/5*(a*e^2+c*d^2)^2*(-6*A
*c*d*e+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)^(5/2)+2*c*(a*e^2+c*d^2)*(-A*a*e^3-5*
A*c*d^2*e+3*B*a*d*e^2+7*B*c*d^3)/e^8/(e*x+d)^(3/2)+2*c*(4*A*c*d*e*(3*a*e^2
+5*c*d^2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))/e^8/(e*x+d)^(1/2)-2*c^2
*(-3*A*a*e^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)*(e*x+d)^(1/2)/e^8+2*c^2
*(-2*A*c*d*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^(3/2)/e^8-2/5*c^3*(-A*e+7*B*d)*(e*
x+d)^(5/2)/e^8+2/7*B*c^3*(e*x+d)^(7/2)/e^8
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{9/2}}\,dx = \frac{2Ae(-5a^3e^6-a^2ce^4(8d^2+28dex+35e^2x^2)+3ac^2e^2(128d^4+448d^3ex+56}{}$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^(9/2),x]
```

output
```
(2*A*e*(-5*a^3*e^6 - a^2*c*e^4*(8*d^2 + 28*d*e*x + 35*e^2*x^2) + 3*a*c^2*e
^2*(128*d^4 + 448*d^3*e*x + 560*d^2*e^2*x^2 + 280*d*e^3*x^3 + 35*e^4*x^4)
+ c^3*(1024*d^6 + 3584*d^5*e*x + 4480*d^4*e^2*x^2 + 2240*d^3*e^3*x^3 + 280
*d^2*e^4*x^4 - 28*d*e^5*x^5 + 7*e^6*x^6)) - 2*B*(a^3*e^6*(2*d + 7*e*x) + 3
*a^2*c*e^4*(16*d^3 + 56*d^2*e*x + 70*d*e^2*x^2 + 35*e^3*x^3) + 5*a*c^2*e^2
*(256*d^5 + 896*d^4*e*x + 1120*d^3*e^2*x^2 + 560*d^2*e^3*x^3 + 70*d*e^4*x^
4 - 7*e^5*x^5) + c^3*(2048*d^7 + 7168*d^6*e*x + 8960*d^5*e^2*x^2 + 4480*d^
4*e^3*x^3 + 560*d^3*e^4*x^4 - 56*d^2*e^5*x^5 + 14*d*e^6*x^6 - 5*e^7*x^7)))
/(35*e^8*(d + e*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^3 (A+Bx)}{(d+ex)^{9/2}} \, dx$$

$$\Big\downarrow \text{ 652}$$

$$\int \left(-\frac{c\left(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4\right)}{e^7(d+ex)^{3/2}}-\frac{3c^2\sqrt{d+ex}\left(-aBe^2+2Acde-7Bcd^2\right)}{e^7}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2c\left(4Acde\left(3ae^2+5cd^2\right)-B\left(3a^2e^4+30acd^2e^2+35c^2d^4\right)\right)}{e^8\sqrt{d+ex}}+\frac{2c^2(d+ex)^{3/2}\left(aBe^2-2Acde+7Bcd^2\right)}{e^8}-\frac{2c^2\sqrt{d+ex}\left(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3\right)}{e^8}-\frac{2\left(ae^2+cd^2\right)^2\left(aBe^2-6Acde+7Bcd^2\right)}{5e^8(d+ex)^{5/2}}+\frac{2\left(ae^2+cd^2\right)^3(Bd-Ae)}{7e^8(d+ex)^{7/2}}+\frac{2c\left(ae^2+cd^2\right)\left(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3\right)}{e^8(d+ex)^{3/2}}-\frac{2c^3(d+ex)^{5/2}(7Bd-Ae)}{5e^8}+\frac{2Bc^3(d+ex)^{7/2}}{7e^8}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^(9/2),x]
```

output
```
(2*(B*d - A*e)*(c*d^2 + a*e^2)^3)/(7*e^8*(d + e*x)^(7/2)) - (2*(c*d^2 + a*
e^2)^2*(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2))/(5*e^8*(d + e*x)^(5/2)) + (2*c*(
c*d^2 + a*e^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(e^8*(d
+ e*x)^(3/2)) + (2*c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(35*c^2*d^4 + 30*a
*c*d^2*e^2 + 3*a^2*e^4)))/(e^8*Sqrt[d + e*x]) - (2*c^2*(35*B*c*d^3 - 15*A*
c*d^2*e + 15*a*B*d*e^2 - 3*a*A*e^3)*Sqrt[d + e*x])/e^8 + (2*c^2*(7*B*c*d^2
 - 2*A*c*d*e + a*B*e^2)*(d + e*x)^(3/2))/e^8 - (2*c^3*(7*B*d - A*e)*(d + e
*x)^(5/2))/(5*e^8) + (2*B*c^3*(d + e*x)^(7/2))/(7*e^8)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{\left(\left(10B\,x^{7}+14A\,x^{6}\right)c^{3}+210x^{4}a\left(\frac{Bx}{3}+A\right)c^{2}-70a^{2}x^{2}(3Bx+A)c-10a^{3}\left(\frac{7Bx}{5}+A\right)\right)e^{7}-56d\left(x^{5}\left(\frac{Bx}{2}+A\right)c^{3}-30\left(-\frac{5Bx}{12}\right.\right.}{}$
derivativedivides	$\frac{\frac{2B\,c^{3}(ex+d)^{\frac{7}{2}}}{7}+\frac{2A\,c^{3}e(ex+d)^{\frac{5}{2}}}{5}-\frac{14B\,c^{3}d(ex+d)^{\frac{5}{2}}}{5}-4A\,c^{3}de(ex+d)^{\frac{3}{2}}+2Ba\,c^{2}e^{2}(ex+d)^{\frac{3}{2}}+14B\,c^{3}d^{2}(ex+d)^{\frac{3}{2}}+6Aa\,c^{2}e}{}$
default	$\frac{\frac{2B\,c^{3}(ex+d)^{\frac{7}{2}}}{7}+\frac{2A\,c^{3}e(ex+d)^{\frac{5}{2}}}{5}-\frac{14B\,c^{3}d(ex+d)^{\frac{5}{2}}}{5}-4A\,c^{3}de(ex+d)^{\frac{3}{2}}+2Ba\,c^{2}e^{2}(ex+d)^{\frac{3}{2}}+14B\,c^{3}d^{2}(ex+d)^{\frac{3}{2}}+6Aa\,c^{2}e}{}$
risch	$\frac{2c^{2}\left(5Bc\,x^{3}e^{3}+7A\,x^{2}c\,e^{3}-34B\,x^{2}cd\,e^{2}-56Axcd\,e^{2}+35Bxa\,e^{3}+162Bxc\,d^{2}e+105Aa\,e^{3}+462Ac\,d^{2}e-490Bad\,e^{2}-1024\right.}{35e^{8}}$
gosper	$-\frac{2\left(-5B\,x^{7}c^{3}e^{7}-7A\,x^{6}c^{3}e^{7}+14B\,x^{6}c^{3}d\,e^{6}+28A\,x^{5}c^{3}d\,e^{6}-35B\,x^{5}a\,c^{2}e^{7}-56B\,x^{5}c^{3}d^{2}e^{5}-105A\,x^{4}a\,c^{2}e^{7}-280A\,x^{4}c\right.}{}$
trager	$-\frac{2\left(-5B\,x^{7}c^{3}e^{7}-7A\,x^{6}c^{3}e^{7}+14B\,x^{6}c^{3}d\,e^{6}+28A\,x^{5}c^{3}d\,e^{6}-35B\,x^{5}a\,c^{2}e^{7}-56B\,x^{5}c^{3}d^{2}e^{5}-105A\,x^{4}a\,c^{2}e^{7}-280A\,x^{4}c\right.}{}$
orering	$-\frac{2\left(-5B\,x^{7}c^{3}e^{7}-7A\,x^{6}c^{3}e^{7}+14B\,x^{6}c^{3}d\,e^{6}+28A\,x^{5}c^{3}d\,e^{6}-35B\,x^{5}a\,c^{2}e^{7}-56B\,x^{5}c^{3}d^{2}e^{5}-105A\,x^{4}a\,c^{2}e^{7}-280A\,x^{4}c\right.}{}$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(((10*B*x^7+14*A*x^6)*c^3+210*x^4*a*(1/3*B*x+A)*c^2-70*a^2*x^2*(3*B*x
+A)*c-10*a^3*(7/5*B*x+A))*e^7-56*d*(x^5*(1/2*B*x+A)*c^3-30*(-5/12*B*x+A)*x
^3*a*c^2+a^2*x*(15/2*B*x+A)*c+1/14*B*a^3)*e^6-16*c*d^2*((-7*B*x^5-35*A*x^4
)*c^2-210*x^2*(-5/3*B*x+A)*a*c+a^2*(21*B*x+A))*e^5+2688*c*d^3*(5/3*(-1/4*B
*x+A)*x^3*c^2+a*x*(-25/6*B*x+A)*c-1/28*a^2*B)*e^4+768*c^2*d^4*(35/3*x^2*(-
B*x+A)*c+a*(-35/3*B*x+A))*e^3+7168*c^2*d^5*(x*(-5/2*B*x+A)*c-5/14*B*a)*e^2
+2048*c^3*d^6*(-7*B*x+A)*e-4096*B*c^3*d^7)/(e*x+d)^(7/2)/e^8
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.45

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{9/2}}\,dx = \frac{2\left(5\,Bc^3e^7x^7 - 2048\,Bc^3d^7 + 1024\,Ac^3d^6e - 1280\,Bac^2d^5e^2 + 384\,Aac^2d^4e^3 -\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(9/2),x, algorithm="fricas")
```

output
```
2/35*(5*B*c^3*e^7*x^7 - 2048*B*c^3*d^7 + 1024*A*c^3*d^6*e - 1280*B*a*c^2*d
^5*e^2 + 384*A*a*c^2*d^4*e^3 - 48*B*a^2*c*d^3*e^4 - 8*A*a^2*c*d^2*e^5 - 2*
B*a^3*d*e^6 - 5*A*a^3*e^7 - 7*(2*B*c^3*d*e^6 - A*c^3*e^7)*x^6 + 7*(8*B*c^3
*d^2*e^5 - 4*A*c^3*d*e^6 + 5*B*a*c^2*e^7)*x^5 - 35*(16*B*c^3*d^3*e^4 - 8*A
*c^3*d^2*e^5 + 10*B*a*c^2*d*e^6 - 3*A*a*c^2*e^7)*x^4 - 35*(128*B*c^3*d^4*e
^3 - 64*A*c^3*d^3*e^4 + 80*B*a*c^2*d^2*e^5 - 24*A*a*c^2*d*e^6 + 3*B*a^2*c*
e^7)*x^3 - 35*(256*B*c^3*d^5*e^2 - 128*A*c^3*d^4*e^3 + 160*B*a*c^2*d^3*e^4
 - 48*A*a*c^2*d^2*e^5 + 6*B*a^2*c*d*e^6 + A*a^2*c*e^7)*x^2 - 7*(1024*B*c^3
*d^6*e - 512*A*c^3*d^5*e^2 + 640*B*a*c^2*d^4*e^3 - 192*A*a*c^2*d^3*e^4 + 2
4*B*a^2*c*d^2*e^5 + 4*A*a^2*c*d*e^6 + B*a^3*e^7)*x)*sqrt(e*x + d)/(e^12*x^
4 + 4*d*e^11*x^3 + 6*d^2*e^10*x^2 + 4*d^3*e^9*x + d^4*e^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3218 vs. $2(359) = 718$.

Time = 0.99 (sec) , antiderivative size = 3218, normalized size of antiderivative = 9.41

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**(9/2),x)
```

output
```
Piecewise((-10*A*a**3*e**7/(35*d**3*e**8*sqrt(d + e*x) + 105*d**2*e**9*x*s
qrt(d + e*x) + 105*d*e**10*x**2*sqrt(d + e*x) + 35*e**11*x**3*sqrt(d + e*x
)) - 16*A*a**2*c*d**2*e**5/(35*d**3*e**8*sqrt(d + e*x) + 105*d**2*e**9*x*s
qrt(d + e*x) + 105*d*e**10*x**2*sqrt(d + e*x) + 35*e**11*x**3*sqrt(d + e*x
)) - 56*A*a**2*c*d*e**6*x/(35*d**3*e**8*sqrt(d + e*x) + 105*d**2*e**9*x*sq
rt(d + e*x) + 105*d*e**10*x**2*sqrt(d + e*x) + 35*e**11*x**3*sqrt(d + e*x)
) - 70*A*a**2*c*e**7*x**2/(35*d**3*e**8*sqrt(d + e*x) + 105*d**2*e**9*x*sq
rt(d + e*x) + 105*d*e**10*x**2*sqrt(d + e*x) + 35*e**11*x**3*sqrt(d + e*x)
) + 768*A*a*c**2*d**4*e**3/(35*d**3*e**8*sqrt(d + e*x) + 105*d**2*e**9*x*s
qrt(d + e*x) + 105*d*e**10*x**2*sqrt(d + e*x) + 35*e**11*x**3*sqrt(d + e*x
)) + 2688*A*a*c**2*d**3*e**4*x/(35*d**3*e**8*sqrt(d + e*x) + 105*d**2*e**9
*x*sqrt(d + e*x) + 105*d*e**10*x**2*sqrt(d + e*x) + 35*e**11*x**3*sqrt(d +
 e*x)) + 3360*A*a*c**2*d**2*e**5*x**2/(35*d**3*e**8*sqrt(d + e*x) + 105*d*
*2*e**9*x*sqrt(d + e*x) + 105*d*e**10*x**2*sqrt(d + e*x) + 35*e**11*x**3*s
qrt(d + e*x)) + 1680*A*a*c**2*d*e**6*x**3/(35*d**3*e**8*sqrt(d + e*x) + 10
5*d**2*e**9*x*sqrt(d + e*x) + 105*d*e**10*x**2*sqrt(d + e*x) + 35*e**11*x*
*3*sqrt(d + e*x)) + 210*A*a*c**2*e**7*x**4/(35*d**3*e**8*sqrt(d + e*x) + 1
05*d**2*e**9*x*sqrt(d + e*x) + 105*d*e**10*x**2*sqrt(d + e*x) + 35*e**11*x
**3*sqrt(d + e*x)) + 2048*A*c**3*d**6*e/(35*d**3*e**8*sqrt(d + e*x) + 105*
d**2*e**9*x*sqrt(d + e*x) + 105*d*e**10*x**2*sqrt(d + e*x) + 35*e**11*x...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.35

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{9/2}}\,dx = \frac{2\left(\frac{5\,(ex+d)^{\frac{7}{2}}Bc^3-7\,(7\,Bc^3d-Ac^3e)\,(ex+d)^{\frac{5}{2}}+35\,(7\,Bc^3d^2-2\,Ac^3de+Bac^2e^2)\,(ex+d)^{\frac{3}{2}}-35\,(35\,Bc^3d^3}{e^7}\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(9/2),x, algorithm="maxima")
```

output
```
2/35*((5*(e*x + d)^(7/2)*B*c^3 - 7*(7*B*c^3*d - A*c^3*e)*(e*x + d)^(5/2) +
 35*(7*B*c^3*d^2 - 2*A*c^3*d*e + B*a*c^2*e^2)*(e*x + d)^(3/2) - 35*(35*B*c
^3*d^3 - 15*A*c^3*d^2*e + 15*B*a*c^2*d*e^2 - 3*A*a*c^2*e^3)*sqrt(e*x + d))
/e^7 + (5*B*c^3*d^7 - 5*A*c^3*d^6*e + 15*B*a*c^2*d^5*e^2 - 15*A*a*c^2*d^4*
e^3 + 15*B*a^2*c*d^3*e^4 - 15*A*a^2*c*d^2*e^5 + 5*B*a^3*d*e^6 - 5*A*a^3*e^
7 - 35*(35*B*c^3*d^4 - 20*A*c^3*d^3*e + 30*B*a*c^2*d^2*e^2 - 12*A*a*c^2*d*
e^3 + 3*B*a^2*c*e^4)*(e*x + d)^3 + 35*(7*B*c^3*d^5 - 5*A*c^3*d^4*e + 10*B*
a*c^2*d^3*e^2 - 6*A*a*c^2*d^2*e^3 + 3*B*a^2*c*d*e^4 - A*a^2*c*e^5)*(e*x +
d)^2 - 7*(7*B*c^3*d^6 - 6*A*c^3*d^5*e + 15*B*a*c^2*d^4*e^2 - 12*A*a*c^2*d^
3*e^3 + 9*B*a^2*c*d^2*e^4 - 6*A*a^2*c*d*e^5 + B*a^3*e^6)*(e*x + d))/((e*x
+ d)^(7/2)*e^7))/e
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.74

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{9/2}}\,dx =$$

$$-\frac{2\,(1225\,(ex+d)^3Bc^3d^4-245\,(ex+d)^2Bc^3d^5+49\,(ex+d)Bc^3d^6-5\,Bc^3d^7-700\,(ex+d)^3Ac^3d^3e+1}{}$$

$$+\frac{2\left(5\,(ex+d)^{\frac{7}{2}}Bc^3e^{48}-49\,(ex+d)^{\frac{5}{2}}Bc^3de^{48}+245\,(ex+d)^{\frac{3}{2}}Bc^3d^2e^{48}-1225\,\sqrt{ex+d}\,Bc^3d^3e^{48}+7\,(ex\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(9/2),x, algorithm="giac")
```

output
```
-2/35*(1225*(e*x + d)^3*B*c^3*d^4 - 245*(e*x + d)^2*B*c^3*d^5 + 49*(e*x +
d)*B*c^3*d^6 - 5*B*c^3*d^7 - 700*(e*x + d)^3*A*c^3*d^3*e + 175*(e*x + d)^2
*A*c^3*d^4*e - 42*(e*x + d)*A*c^3*d^5*e + 5*A*c^3*d^6*e + 1050*(e*x + d)^3
*B*a*c^2*d^2*e^2 - 350*(e*x + d)^2*B*a*c^2*d^3*e^2 + 105*(e*x + d)*B*a*c^2
*d^4*e^2 - 15*B*a*c^2*d^5*e^2 - 420*(e*x + d)^3*A*a*c^2*d*e^3 + 210*(e*x +
 d)^2*A*a*c^2*d^2*e^3 - 84*(e*x + d)*A*a*c^2*d^3*e^3 + 15*A*a*c^2*d^4*e^3
+ 105*(e*x + d)^3*B*a^2*c*e^4 - 105*(e*x + d)^2*B*a^2*c*d*e^4 + 63*(e*x +
d)*B*a^2*c*d^2*e^4 - 15*B*a^2*c*d^3*e^4 + 35*(e*x + d)^2*A*a^2*c*e^5 - 42*
(e*x + d)*A*a^2*c*d*e^5 + 15*A*a^2*c*d^2*e^5 + 7*(e*x + d)*B*a^3*e^6 - 5*B
*a^3*d*e^6 + 5*A*a^3*e^7)/((e*x + d)^(7/2)*e^8) + 2/35*(5*(e*x + d)^(7/2)*
B*c^3*e^48 - 49*(e*x + d)^(5/2)*B*c^3*d*e^48 + 245*(e*x + d)^(3/2)*B*c^3*d
^2*e^48 - 1225*sqrt(e*x + d)*B*c^3*d^3*e^48 + 7*(e*x + d)^(5/2)*A*c^3*e^49
 - 70*(e*x + d)^(3/2)*A*c^3*d*e^49 + 525*sqrt(e*x + d)*A*c^3*d^2*e^49 + 35
*(e*x + d)^(3/2)*B*a*c^2*e^50 - 525*sqrt(e*x + d)*B*a*c^2*d*e^50 + 105*sqr
t(e*x + d)*A*a*c^2*e^51)/e^56
```

Mupad [B] (verification not implemented)

Time = 6.53 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.32

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{9/2}}\,dx = \frac{(d+e\,x)^{3/2}\left(42\,B\,c^3\,d^2-12\,A\,c^3\,d\,e+6\,B\,a\,c^2\,e^2\right)}{3\,e^8}$$
$$-\frac{(d+e\,x)\left(\frac{2\,B\,a^3\,e^6}{5}+\frac{18\,B\,a^2\,c\,d^2\,e^4}{5}-\frac{12\,A\,a^2\,c\,d\,e^5}{5}+6\,B\,a\,c^2\,d^4\,e^2-\frac{24\,A\,a\,c^2\,d^3\,e^3}{5}+\frac{14\,B\,c^3\,d^6}{5}-\frac{12\,A\,c^3\,d^5\,e}{5}\right)+(d-}{}$$
$$+\frac{2\,B\,c^3\,(d+e\,x)^{7/2}}{7\,e^8}+\frac{2\,c^2\,\sqrt{d+e\,x}\left(-35\,B\,c\,d^3+15\,A\,c\,d^2\,e-15\,B\,a\,d\,e^2+3\,A\,a\,e^3\right)}{e^8}$$
$$+\frac{2\,c^3\,(A\,e-7\,B\,d)\,(d+e\,x)^{5/2}}{5\,e^8}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^(9/2),x)
```

output
```
((d + e*x)^(3/2)*(42*B*c^3*d^2 - 12*A*c^3*d*e + 6*B*a*c^2*e^2))/(3*e^8) -
((d + e*x)*((2*B*a^3*e^6)/5 + (14*B*c^3*d^6)/5 - (12*A*c^3*d^5*e)/5 - (24*
A*a*c^2*d^3*e^3)/5 + 6*B*a*c^2*d^4*e^2 + (18*B*a^2*c*d^2*e^4)/5 - (12*A*a^
2*c*d*e^5)/5) + (d + e*x)^3*(70*B*c^3*d^4 + 6*B*a^2*c*e^4 - 40*A*c^3*d^3*e
 + 60*B*a*c^2*d^2*e^2 - 24*A*a*c^2*d*e^3) - (d + e*x)^2*(14*B*c^3*d^5 - 2*
A*a^2*c*e^5 - 10*A*c^3*d^4*e - 12*A*a*c^2*d^2*e^3 + 20*B*a*c^2*d^3*e^2 + 6
*B*a^2*c*d*e^4) + (2*A*a^3*e^7)/7 - (2*B*c^3*d^7)/7 - (2*B*a^3*d*e^6)/7 +
(2*A*c^3*d^6*e)/7 + (6*A*a*c^2*d^4*e^3)/7 + (6*A*a^2*c*d^2*e^5)/7 - (6*B*a
*c^2*d^5*e^2)/7 - (6*B*a^2*c*d^3*e^4)/7)/(e^8*(d + e*x)^(7/2)) + (2*B*c^3*
(d + e*x)^(7/2))/(7*e^8) + (2*c^2*(d + e*x)^(1/2)*(3*A*a*e^3 - 35*B*c*d^3
- 15*B*a*d*e^2 + 15*A*c*d^2*e))/e^8 + (2*c^3*(A*e - 7*B*d)*(d + e*x)^(5/2)
)/(5*e^8)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.52

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{9/2}}\,dx = \frac{-\frac{4096}{35}b\,c^3d^7-\frac{8}{5}a^3cd\,e^6x-6a^2bc\,e^7x^3-\frac{4}{35}a^3bd\,e^6-\frac{16}{35}a^3c\,d^2e^5+\frac{768}{35}a^2c^2d^4e^3}{}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^(9/2),x)
```

output
```
(2*( - 5*a**4*e**7 - 2*a**3*b*d*e**6 - 7*a**3*b*e**7*x - 8*a**3*c*d**2*e**
5 - 28*a**3*c*d*e**6*x - 35*a**3*c*e**7*x**2 - 48*a**2*b*c*d**3*e**4 - 168
*a**2*b*c*d**2*e**5*x - 210*a**2*b*c*d*e**6*x**2 - 105*a**2*b*c*e**7*x**3
+ 384*a**2*c**2*d**4*e**3 + 1344*a**2*c**2*d**3*e**4*x + 1680*a**2*c**2*d*
*2*e**5*x**2 + 840*a**2*c**2*d*e**6*x**3 + 105*a**2*c**2*e**7*x**4 - 1280*
a*b*c**2*d**5*e**2 - 4480*a*b*c**2*d**4*e**3*x - 5600*a*b*c**2*d**3*e**4*x
**2 - 2800*a*b*c**2*d**2*e**5*x**3 - 350*a*b*c**2*d*e**6*x**4 + 35*a*b*c**
2*e**7*x**5 + 1024*a*c**3*d**6*e + 3584*a*c**3*d**5*e**2*x + 4480*a*c**3*d
**4*e**3*x**2 + 2240*a*c**3*d**3*e**4*x**3 + 280*a*c**3*d**2*e**5*x**4 - 2
8*a*c**3*d*e**6*x**5 + 7*a*c**3*e**7*x**6 - 2048*b*c**3*d**7 - 7168*b*c**3
*d**6*e*x - 8960*b*c**3*d**5*e**2*x**2 - 4480*b*c**3*d**4*e**3*x**3 - 560*
b*c**3*d**3*e**4*x**4 + 56*b*c**3*d**2*e**5*x**5 - 14*b*c**3*d*e**6*x**6 +
 5*b*c**3*e**7*x**7))/(35*sqrt(d + e*x)*e**8*(d**3 + 3*d**2*e*x + 3*d*e**2
*x**2 + e**3*x**3))
```

3.122 $\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 346

$$\begin{aligned}\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{11/2}}\,dx &= \frac{2(Bd-Ae)\left(cd^2+ae^2\right)^3}{9e^8(d+ex)^{9/2}}\\
&-\frac{2\left(cd^2+ae^2\right)^2\left(7Bcd^2-6Acde+aBe^2\right)}{7e^8(d+ex)^{7/2}}\\
&+\frac{6c\left(cd^2+ae^2\right)\left(7Bcd^3-5Acd^2e+3aBde^2-aAe^3\right)}{5e^8(d+ex)^{5/2}}\\
&+\frac{2c\left(4Acde\left(5cd^2+3ae^2\right)-B\left(35c^2d^4+30acd^2e^2+3a^2e^4\right)\right)}{3e^8(d+ex)^{3/2}}\\
&+\frac{2c^2\left(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3\right)}{e^8\sqrt{d+ex}}\\
&+\frac{6c^2\left(7Bcd^2-2Acde+aBe^2\right)\sqrt{d+ex}}{e^8}\\
&-\frac{2c^3(7Bd-Ae)(d+ex)^{3/2}}{3e^8}+\frac{2Bc^3(d+ex)^{5/2}}{5e^8}\end{aligned}$$

output
```
2/9*(-A*e+B*d)*(a*e^2+c*d^2)^3/e^8/(e*x+d)^(9/2)-2/7*(a*e^2+c*d^2)^2*(-6*A
*c*d*e+B*a*e^2+7*B*c*d^2)/e^8/(e*x+d)^(7/2)+6/5*c*(a*e^2+c*d^2)*(-A*a*e^3-
5*A*c*d^2*e+3*B*a*d*e^2+7*B*c*d^3)/e^8/(e*x+d)^(5/2)+2/3*c*(4*A*c*d*e*(3*a
*e^2+5*c*d^2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))/e^8/(e*x+d)^(3/2)+2
*c^2*(-3*A*a*e^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)/e^8/(e*x+d)^(1/2)+6
*c^2*(-2*A*c*d*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^(1/2)/e^8-2/3*c^3*(-A*e+7*B*d)
*(e*x+d)^(3/2)/e^8+2/5*B*c^3*(e*x+d)^(5/2)/e^8
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{11/2}}\,dx = \frac{-2Ae(35a^3e^6+3a^2ce^4(8d^2+36dex+63e^2x^2)+3ac^2e^2(128d^4+576d^3ex+$$

input
```
Integrate[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^(11/2),x]
```

output
```
(-2*A*e*(35*a^3*e^6 + 3*a^2*c*e^4*(8*d^2 + 36*d*e*x + 63*e^2*x^2) + 3*a*c^
2*e^2*(128*d^4 + 576*d^3*e*x + 1008*d^2*e^2*x^2 + 840*d*e^3*x^3 + 315*e^4*
x^4) + 5*c^3*(1024*d^6 + 4608*d^5*e*x + 8064*d^4*e^2*x^2 + 6720*d^3*e^3*x^
3 + 2520*d^2*e^4*x^4 + 252*d*e^5*x^5 - 21*e^6*x^6)) + 2*B*(-5*a^3*e^6*(2*d
 + 9*e*x) - 3*a^2*c*e^4*(16*d^3 + 72*d^2*e*x + 126*d*e^2*x^2 + 105*e^3*x^3
) + 15*a*c^2*e^2*(256*d^5 + 1152*d^4*e*x + 2016*d^3*e^2*x^2 + 1680*d^2*e^3
*x^3 + 630*d*e^4*x^4 + 63*e^5*x^5) + 7*c^3*(2048*d^7 + 9216*d^6*e*x + 1612
8*d^5*e^2*x^2 + 13440*d^4*e^3*x^3 + 5040*d^3*e^4*x^4 + 504*d^2*e^5*x^5 - 4
2*d*e^6*x^6 + 9*e^7*x^7)))/(315*e^8*(d + e*x)^(9/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3(A+Bx)}{(d+ex)^{11/2}}dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{c(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4)}{e^7(d+ex)^{5/2}}-\frac{3c^2(-aBe^2+2Acde-7Bcd^2)}{e^7\sqrt{d+ex}}+\frac{c^2(3a$$

$$\Big\downarrow \text{2009}$$

$$\frac{2c(4Acde(3ae^2+5cd^2)-B(3a^2e^4+30acd^2e^2+35c^2d^4))}{3e^8(d+ex)^{3/2}}+\frac{6c^2\sqrt{d+ex}(aBe^2-2Acde+7Bcd^2)}{e^8}+\frac{2c^2(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3)}{e^8\sqrt{d+ex}}-\frac{2(ae^2+cd^2)^2(aBe^2-6Acde+7Bcd^2)}{7e^8(d+ex)^{7/2}}+\frac{2(ae^2+cd^2)^3(Bd-Ae)}{9e^8(d+ex)^{9/2}}+\frac{6c(ae^2+cd^2)(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3)}{5e^8(d+ex)^{5/2}}-\frac{2c^3(d+ex)^{3/2}(7Bd-Ae)}{3e^8}+\frac{2Bc^3(d+ex)^{5/2}}{5e^8}$$

input
```
Int[((A + B*x)*(a + c*x^2)^3)/(d + e*x)^(11/2),x]
```

output
```
(2*(B*d - A*e)*(c*d^2 + a*e^2)^3)/(9*e^8*(d + e*x)^(9/2)) - (2*(c*d^2 + a*
e^2)^2*(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2))/(7*e^8*(d + e*x)^(7/2)) + (6*c*(
c*d^2 + a*e^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3))/(5*e^8*(
d + e*x)^(5/2)) + (2*c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2) - B*(35*c^2*d^4 + 30
*a*c*d^2*e^2 + 3*a^2*e^4)))/(3*e^8*(d + e*x)^(3/2)) + (2*c^2*(35*B*c*d^3 -
 15*A*c*d^2*e + 15*a*B*d*e^2 - 3*a*A*e^3))/(e^8*Sqrt[d + e*x]) + (6*c^2*(7
*B*c*d^2 - 2*A*c*d*e + a*B*e^2)*Sqrt[d + e*x])/e^8 - (2*c^3*(7*B*d - A*e)*
(d + e*x)^(3/2))/(3*e^8) + (2*B*c^3*(d + e*x)^(5/2))/(5*e^8)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{\left(\left(126B x^7+210A x^6\right)c^3-1890a x^4(-Bx+A)c^2-378\left(\frac{5Bx}{3}+A\right)x^2a^2c-70a^3\left(\frac{9Bx}{7}+A\right)\right)e^7-216d\left(\frac{35x^5\left(\frac{7Bx}{30}+A\right)c^3}{3}+\cdots\right.}{\cdots}$
derivativedivides	$\frac{\frac{2Bc^3(ex+d)^{\frac{5}{2}}}{5}+\frac{2Ac^3e(ex+d)^{\frac{3}{2}}}{3}-\frac{14Bc^3d(ex+d)^{\frac{3}{2}}}{3}-12Ac^3de\sqrt{ex+d}+6Bac^2e^2\sqrt{ex+d}+42Bc^3d^2\sqrt{ex+d}-\frac{2\left(Aa^3e^7+3\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{2Bc^3(ex+d)^{\frac{5}{2}}}{5}+\frac{2Ac^3e(ex+d)^{\frac{3}{2}}}{3}-\frac{14Bc^3d(ex+d)^{\frac{3}{2}}}{3}-12Ac^3de\sqrt{ex+d}+6Bac^2e^2\sqrt{ex+d}+42Bc^3d^2\sqrt{ex+d}-\frac{2\left(Aa^3e^7+3\cdots\right.}{\cdots}}{\cdots}$
gosper	$-\frac{2\left(-63Bx^7c^3e^7-105Ax^6c^3e^7+294Bx^6c^3de^6+1260Ax^5c^3de^6-945Bx^5ac^2e^7-3528Bx^5c^3d^2e^5+945Ax^4ac^2e^7\cdots\right.}{\cdots}$
trager	$-\frac{2\left(-63Bx^7c^3e^7-105Ax^6c^3e^7+294Bx^6c^3de^6+1260Ax^5c^3de^6-945Bx^5ac^2e^7-3528Bx^5c^3d^2e^5+945Ax^4ac^2e^7\cdots\right.}{\cdots}$
orering	$-\frac{2\left(-63Bx^7c^3e^7-105Ax^6c^3e^7+294Bx^6c^3de^6+1260Ax^5c^3de^6-945Bx^5ac^2e^7-3528Bx^5c^3d^2e^5+945Ax^4ac^2e^7\cdots\right.}{\cdots}$
risch	$-\frac{2c^2\left(-3e^2Bcx^2-5Ace^2x+29Bcdex+85Acde-45Bae^2-283Bcd^2\right)\sqrt{ex+d}}{15e^8}-\frac{2\left(945Ax^4ac^2e^7+4725Ax^4c^3d^2e^5\cdots\right.}{\cdots}$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(((126*B*x^7+210*A*x^6)*c^3-1890*a*x^4*(-B*x+A)*c^2-378*(5/3*B*x+A)*
x^2*a^2*c-70*a^3*(9/7*B*x+A))*e^7-216*d*(35/3*x^5*(7/30*B*x+A)*c^3+70/3*(-
15/4*B*x+A)*x^3*a*c^2+a^2*x*(7/2*B*x+A)*c+5/54*B*a^3)*e^6-48*c*d^2*((-147*
B*x^5+525*A*x^4)*c^2+126*(-25/3*B*x+A)*x^2*a*c+a^2*(9*B*x+A))*e^5-3456*c*d
^3*(175/9*(-21/20*B*x+A)*x^3*c^2+a*x*(-35/2*B*x+A)*c+1/36*a^2*B)*e^4-768*c
^2*((-245*B*x^3+105*A*x^2)*c+a*(-45*B*x+A))*d^4*e^3-46080*c^2*d^5*(x*(-49/
10*B*x+A)*c-1/6*B*a)*e^2-10240*c^3*(-63/5*B*x+A)*d^6*e+28672*B*c^3*d^7)/(e
*x+d)^(9/2)/e^8
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.47

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{11/2}}\,dx = \frac{2\,(63\,Bc^3e^7x^7 + 14336\,Bc^3d^7 - 5120\,Ac^3d^6e + 3840\,Bac^2d^5e^2 - 384\,Aac^2d^4e$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(11/2),x, algorithm="fricas")
```

output
```
2/315*(63*B*c^3*e^7*x^7 + 14336*B*c^3*d^7 - 5120*A*c^3*d^6*e + 3840*B*a*c^
2*d^5*e^2 - 384*A*a*c^2*d^4*e^3 - 48*B*a^2*c*d^3*e^4 - 24*A*a^2*c*d^2*e^5
- 10*B*a^3*d*e^6 - 35*A*a^3*e^7 - 21*(14*B*c^3*d*e^6 - 5*A*c^3*e^7)*x^6 +
63*(56*B*c^3*d^2*e^5 - 20*A*c^3*d*e^6 + 15*B*a*c^2*e^7)*x^5 + 315*(112*B*c
^3*d^3*e^4 - 40*A*c^3*d^2*e^5 + 30*B*a*c^2*d*e^6 - 3*A*a*c^2*e^7)*x^4 + 10
5*(896*B*c^3*d^4*e^3 - 320*A*c^3*d^3*e^4 + 240*B*a*c^2*d^2*e^5 - 24*A*a*c^
2*d*e^6 - 3*B*a^2*c*e^7)*x^3 + 63*(1792*B*c^3*d^5*e^2 - 640*A*c^3*d^4*e^3
+ 480*B*a*c^2*d^3*e^4 - 48*A*a*c^2*d^2*e^5 - 6*B*a^2*c*d*e^6 - 3*A*a^2*c*e
^7)*x^2 + 9*(7168*B*c^3*d^6*e - 2560*A*c^3*d^5*e^2 + 1920*B*a*c^2*d^4*e^3
- 192*A*a*c^2*d^3*e^4 - 24*B*a^2*c*d^2*e^5 - 12*A*a^2*c*d*e^6 - 5*B*a^3*e^
7)*x)*sqrt(e*x + d)/(e^13*x^5 + 5*d*e^12*x^4 + 10*d^2*e^11*x^3 + 10*d^3*e^
10*x^2 + 5*d^4*e^9*x + d^5*e^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3952 vs. $2(362) = 724$.

Time = 1.27 (sec) , antiderivative size = 3952, normalized size of antiderivative = 11.42

$$\int \frac{(A + Bx)(a + cx^2)^3}{(d + ex)^{11/2}}\, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**(11/2),x)
```

output
```
Piecewise((-70*A*a**3*e**7/(315*d**4*e**8*sqrt(d + e*x) + 1260*d**3*e**9*x
*sqrt(d + e*x) + 1890*d**2*e**10*x**2*sqrt(d + e*x) + 1260*d*e**11*x**3*sq
rt(d + e*x) + 315*e**12*x**4*sqrt(d + e*x)) - 48*A*a**2*c*d**2*e**5/(315*d
**4*e**8*sqrt(d + e*x) + 1260*d**3*e**9*x*sqrt(d + e*x) + 1890*d**2*e**10*
x**2*sqrt(d + e*x) + 1260*d*e**11*x**3*sqrt(d + e*x) + 315*e**12*x**4*sqrt
(d + e*x)) - 216*A*a**2*c*d*e**6*x/(315*d**4*e**8*sqrt(d + e*x) + 1260*d**
3*e**9*x*sqrt(d + e*x) + 1890*d**2*e**10*x**2*sqrt(d + e*x) + 1260*d*e**11
*x**3*sqrt(d + e*x) + 315*e**12*x**4*sqrt(d + e*x)) - 378*A*a**2*c*e**7*x*
*2/(315*d**4*e**8*sqrt(d + e*x) + 1260*d**3*e**9*x*sqrt(d + e*x) + 1890*d*
*2*e**10*x**2*sqrt(d + e*x) + 1260*d*e**11*x**3*sqrt(d + e*x) + 315*e**12*
x**4*sqrt(d + e*x)) - 768*A*a*c**2*d**4*e**3/(315*d**4*e**8*sqrt(d + e*x)
+ 1260*d**3*e**9*x*sqrt(d + e*x) + 1890*d**2*e**10*x**2*sqrt(d + e*x) + 12
60*d*e**11*x**3*sqrt(d + e*x) + 315*e**12*x**4*sqrt(d + e*x)) - 3456*A*a*c
**2*d**3*e**4*x/(315*d**4*e**8*sqrt(d + e*x) + 1260*d**3*e**9*x*sqrt(d + e
*x) + 1890*d**2*e**10*x**2*sqrt(d + e*x) + 1260*d*e**11*x**3*sqrt(d + e*x)
 + 315*e**12*x**4*sqrt(d + e*x)) - 6048*A*a*c**2*d**2*e**5*x**2/(315*d**4*
e**8*sqrt(d + e*x) + 1260*d**3*e**9*x*sqrt(d + e*x) + 1890*d**2*e**10*x**2
*sqrt(d + e*x) + 1260*d*e**11*x**3*sqrt(d + e*x) + 315*e**12*x**4*sqrt(d +
 e*x)) - 5040*A*a*c**2*d*e**6*x**3/(315*d**4*e**8*sqrt(d + e*x) + 1260*d**
3*e**9*x*sqrt(d + e*x) + 1890*d**2*e**10*x**2*sqrt(d + e*x) + 1260*d*e*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.33

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{11/2}}\,dx = \frac{2\left(\frac{21\left(3\,(ex+d)^{\frac{5}{2}}Bc^3-5\,(7\,Bc^3d-Ac^3e)(ex+d)^{\frac{3}{2}}+45\,(7\,Bc^3d^2-2\,Ac^3de+Bac^2e^2)\sqrt{ex+d}\right)}{e^7}+\frac{35\,Bc^3\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(11/2),x, algorithm="maxima")
```

output
```
2/315*(21*(3*(e*x + d)^(5/2)*B*c^3 - 5*(7*B*c^3*d - A*c^3*e)*(e*x + d)^(3/
2) + 45*(7*B*c^3*d^2 - 2*A*c^3*d*e + B*a*c^2*e^2)*sqrt(e*x + d))/e^7 + (35
*B*c^3*d^7 - 35*A*c^3*d^6*e + 105*B*a*c^2*d^5*e^2 - 105*A*a*c^2*d^4*e^3 +
105*B*a^2*c*d^3*e^4 - 105*A*a^2*c*d^2*e^5 + 35*B*a^3*d*e^6 - 35*A*a^3*e^7
+ 315*(35*B*c^3*d^3 - 15*A*c^3*d^2*e + 15*B*a*c^2*d*e^2 - 3*A*a*c^2*e^3)*(
e*x + d)^4 - 105*(35*B*c^3*d^4 - 20*A*c^3*d^3*e + 30*B*a*c^2*d^2*e^2 - 12*
A*a*c^2*d*e^3 + 3*B*a^2*c*e^4)*(e*x + d)^3 + 189*(7*B*c^3*d^5 - 5*A*c^3*d^
4*e + 10*B*a*c^2*d^3*e^2 - 6*A*a*c^2*d^2*e^3 + 3*B*a^2*c*d*e^4 - A*a^2*c*e
^5)*(e*x + d)^2 - 45*(7*B*c^3*d^6 - 6*A*c^3*d^5*e + 15*B*a*c^2*d^4*e^2 - 1
2*A*a*c^2*d^3*e^3 + 9*B*a^2*c*d^2*e^4 - 6*A*a^2*c*d*e^5 + B*a^3*e^6)*(e*x
+ d))/((e*x + d)^(9/2)*e^7))/e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.70

$$\int \frac{(A+Bx)\left(a+cx^2\right)^3}{(d+ex)^{11/2}}\,dx = \frac{2\left(11025\,(ex+d)^4Bc^3d^3-3675\,(ex+d)^3Bc^3d^4+1323\,(ex+d)^2Bc^3d^5-31\cdots\right.}{\cdots} + \frac{2\left(3\,(ex+d)^{\frac{5}{2}}Bc^3e^{32}-35\,(ex+d)^{\frac{3}{2}}Bc^3de^{32}+315\,\sqrt{ex+d}Bc^3d^2e^{32}+5\,(ex+d)^{\frac{3}{2}}Ac^3e^{33}-90\,\sqrt{ex+d}\cdots\right.}{15\,e^{40}}$$

input
```
integrate((B*x+A)*(c*x^2+a)^3/(e*x+d)^(11/2),x, algorithm="giac")
```

output
```
2/315*(11025*(e*x + d)^4*B*c^3*d^3 - 3675*(e*x + d)^3*B*c^3*d^4 + 1323*(e*
x + d)^2*B*c^3*d^5 - 315*(e*x + d)*B*c^3*d^6 + 35*B*c^3*d^7 - 4725*(e*x +
d)^4*A*c^3*d^2*e + 2100*(e*x + d)^3*A*c^3*d^3*e - 945*(e*x + d)^2*A*c^3*d^
4*e + 270*(e*x + d)*A*c^3*d^5*e - 35*A*c^3*d^6*e + 4725*(e*x + d)^4*B*a*c^
2*d*e^2 - 3150*(e*x + d)^3*B*a*c^2*d^2*e^2 + 1890*(e*x + d)^2*B*a*c^2*d^3*
e^2 - 675*(e*x + d)*B*a*c^2*d^4*e^2 + 105*B*a*c^2*d^5*e^2 - 945*(e*x + d)^
4*A*a*c^2*e^3 + 1260*(e*x + d)^3*A*a*c^2*d*e^3 - 1134*(e*x + d)^2*A*a*c^2*
d^2*e^3 + 540*(e*x + d)*A*a*c^2*d^3*e^3 - 105*A*a*c^2*d^4*e^3 - 315*(e*x +
 d)^3*B*a^2*c*e^4 + 567*(e*x + d)^2*B*a^2*c*d*e^4 - 405*(e*x + d)*B*a^2*c*
d^2*e^4 + 105*B*a^2*c*d^3*e^4 - 189*(e*x + d)^2*A*a^2*c*e^5 + 270*(e*x + d
)*A*a^2*c*d*e^5 - 105*A*a^2*c*d^2*e^5 - 45*(e*x + d)*B*a^3*e^6 + 35*B*a^3*
d*e^6 - 35*A*a^3*e^7)/((e*x + d)^(9/2)*e^8) + 2/15*(3*(e*x + d)^(5/2)*B*c^
3*e^32 - 35*(e*x + d)^(3/2)*B*c^3*d*e^32 + 315*sqrt(e*x + d)*B*c^3*d^2*e^3
2 + 5*(e*x + d)^(3/2)*A*c^3*e^33 - 90*sqrt(e*x + d)*A*c^3*d*e^33 + 45*sqrt
(e*x + d)*B*a*c^2*e^34)/e^40
```

Mupad [B] (verification not implemented)

Time = 6.65 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.31

$$\int \frac{(A+Bx)(a+cx^2)^3}{(d+ex)^{11/2}}\,dx = \frac{(d+e\,x)^4\,(70\,B\,c^3\,d^3-30\,A\,c^3\,d^2\,e+30\,B\,a\,c^2\,d\,e^2-6\,A\,a\,c^2\,e^3)-(d+e\,x)}{}$$
$$+\frac{\sqrt{d+e\,x}\,(42\,B\,c^3\,d^2-12\,A\,c^3\,d\,e+6\,B\,a\,c^2\,e^2)}{e^8}$$
$$+\frac{2\,B\,c^3\,(d+e\,x)^{5/2}}{5\,e^8}+\frac{2\,c^3\,(A\,e-7\,B\,d)\,(d+e\,x)^{3/2}}{3\,e^8}$$

input
```
int(((a + c*x^2)^3*(A + B*x))/(d + e*x)^(11/2),x)
```

output
```
((d + e*x)^4*(70*B*c^3*d^3 - 6*A*a*c^2*e^3 - 30*A*c^3*d^2*e + 30*B*a*c^2*d
*e^2) - (d + e*x)*((2*B*a^3*e^6)/7 + 2*B*c^3*d^6 - (12*A*c^3*d^5*e)/7 - (2
4*A*a*c^2*d^3*e^3)/7 + (30*B*a*c^2*d^4*e^2)/7 + (18*B*a^2*c*d^2*e^4)/7 - (
12*A*a^2*c*d*e^5)/7) - (d + e*x)^3*((70*B*c^3*d^4)/3 + 2*B*a^2*c*e^4 - (40
*A*c^3*d^3*e)/3 + 20*B*a*c^2*d^2*e^2 - 8*A*a*c^2*d*e^3) + (d + e*x)^2*((42
*B*c^3*d^5)/5 - (6*A*a^2*c*e^5)/5 - 6*A*c^3*d^4*e - (36*A*a*c^2*d^2*e^3)/5
 + 12*B*a*c^2*d^3*e^2 + (18*B*a^2*c*d*e^4)/5) - (2*A*a^3*e^7)/9 + (2*B*c^3
*d^7)/9 + (2*B*a^3*d*e^6)/9 - (2*A*c^3*d^6*e)/9 - (2*A*a*c^2*d^4*e^3)/3 -
(2*A*a^2*c*d^2*e^5)/3 + (2*B*a*c^2*d^5*e^2)/3 + (2*B*a^2*c*d^3*e^4)/3)/(e^
8*(d + e*x)^(9/2)) + ((d + e*x)^(1/2)*(42*B*c^3*d^2 - 12*A*c^3*d*e + 6*B*a
*c^2*e^2))/e^8 + (2*B*c^3*(d + e*x)^(5/2))/(5*e^8) + (2*c^3*(A*e - 7*B*d)*
(d + e*x)^(3/2))/(3*e^8)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.53

$$\int \frac{(A+Bx)\,(a+cx^2)^3}{(d+ex)^{11/2}}\,dx = \frac{\frac{4096}{45}b\,c^3d^7 - \frac{24}{35}a^3cd\,e^6x - 2a^2bc\,e^7x^3 - \frac{4}{63}a^3bd\,e^6 - \frac{16}{105}a^3c\,d^2e^5 - \frac{256}{105}a^2c^2d^4e^3}{}$$

input
```
int((B*x+A)*(c*x^2+a)^3/(e*x+d)^(11/2),x)
```

output
```
(2*( - 35*a**4*e**7 - 10*a**3*b*d*e**6 - 45*a**3*b*e**7*x - 24*a**3*c*d**2
*e**5 - 108*a**3*c*d*e**6*x - 189*a**3*c*e**7*x**2 - 48*a**2*b*c*d**3*e**4
 - 216*a**2*b*c*d**2*e**5*x - 378*a**2*b*c*d*e**6*x**2 - 315*a**2*b*c*e**7
*x**3 - 384*a**2*c**2*d**4*e**3 - 1728*a**2*c**2*d**3*e**4*x - 3024*a**2*c
**2*d**2*e**5*x**2 - 2520*a**2*c**2*d*e**6*x**3 - 945*a**2*c**2*e**7*x**4
+ 3840*a*b*c**2*d**5*e**2 + 17280*a*b*c**2*d**4*e**3*x + 30240*a*b*c**2*d*
*3*e**4*x**2 + 25200*a*b*c**2*d**2*e**5*x**3 + 9450*a*b*c**2*d*e**6*x**4 +
 945*a*b*c**2*e**7*x**5 - 5120*a*c**3*d**6*e - 23040*a*c**3*d**5*e**2*x -
40320*a*c**3*d**4*e**3*x**2 - 33600*a*c**3*d**3*e**4*x**3 - 12600*a*c**3*d
**2*e**5*x**4 - 1260*a*c**3*d*e**6*x**5 + 105*a*c**3*e**7*x**6 + 14336*b*c
**3*d**7 + 64512*b*c**3*d**6*e*x + 112896*b*c**3*d**5*e**2*x**2 + 94080*b*
c**3*d**4*e**3*x**3 + 35280*b*c**3*d**3*e**4*x**4 + 3528*b*c**3*d**2*e**5*
x**5 - 294*b*c**3*d*e**6*x**6 + 63*b*c**3*e**7*x**7))/(315*sqrt(d + e*x)*e
**8*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))
```

3.123 $\int \frac{(A+Bx)(d+ex)^{5/2}}{a-cx^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{a-cx^2} dx = -\frac{2(Bcd^2+2Acde+aBe^2)\sqrt{d+ex}}{c^2} - \frac{2(Bd+Ae)(d+ex)^{3/2}}{3c} - \frac{2B(d+ex)^{5/2}}{5c} + \frac{(\sqrt{a}B-A\sqrt{c})\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{9/4}} + \frac{(\sqrt{a}B+A\sqrt{c})\left(\sqrt{c}d+\sqrt{a}e\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}c^{9/4}}$$

output
```
-2*(2*A*c*d*e+B*a*e^2+B*c*d^2)*(e*x+d)^(1/2)/c^2-2/3*(A*e+B*d)*(e*x+d)^(3/
2)/c-2/5*B*(e*x+d)^(5/2)/c+(a^(1/2)*B-A*c^(1/2))*(c^(1/2)*d-a^(1/2)*e)^(5/
2)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(1/2)/c^(9
/4)+(a^(1/2)*B+A*c^(1/2))*(c^(1/2)*d+a^(1/2)*e)^(5/2)*arctanh(c^(1/4)*(e*x
+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(1/2)/c^(9/4)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.20

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{a-cx^2}\,dx = \frac{-2\sqrt{d+ex}(15aBe^2+5Ace(7d+ex)+Bc(23d^2+11dex+3e^2x^2)) + \frac{15(\sqrt{a}\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2),x]
```

output
```
(-2*Sqrt[d + e*x]*(15*a*B*e^2 + 5*A*c*e*(7*d + e*x) + B*c*(23*d^2 + 11*d*e
*x + 3*e^2*x^2)) + (15*(Sqrt[a]*B + A*Sqrt[c])*(Sqrt[c]*d + Sqrt[a]*e)^3*A
rcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a
]*e)])/(Sqrt[a]*Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]) - (15*(Sqrt[a]*B - A*Sqr
t[c])*(-(Sqrt[c]*d) + Sqrt[a]*e)^3*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e
]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/(Sqrt[a]*Sqrt[-(c*d) + Sqrt[a]*
Sqrt[c]*e]))/(15*c^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {653, 25, 653, 25, 27, 653, 25, 654, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{a-cx^2}dx$$

$$\Big\downarrow \text{653}$$

$$-\frac{\int -\frac{(d+ex)^{3/2}(Acd+aBe+c(Bd+Ae)x)}{a-cx^2}dx}{c}-\frac{2B(d+ex)^{5/2}}{5c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(d+ex)^{3/2}(Acd+aBe+c(Bd+Ae)x)}{a-cx^2}dx}{c}-\frac{2B(d+ex)^{5/2}}{5c}$$

$$\Big\downarrow \ 653$$

$$\frac{-\frac{\int -\frac{c\sqrt{d+ex}\left(Acd^2+2aBed+aAe^2+\left(Bcd^2+2Aced+aBe^2\right)x\right)}{a-cx^2}dx}{c}-\frac{2}{3}(d+ex)^{3/2}(Ae+Bd)}{c}-\frac{2B(d+ex)^{5/2}}{5c}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{c\sqrt{d+ex}\left(Acd^2+2aBed+aAe^2+\left(Bcd^2+2Aced+aBe^2\right)x\right)}{a-cx^2}dx}{c}-\frac{2}{3}(d+ex)^{3/2}(Ae+Bd)}{c}-\frac{2B(d+ex)^{5/2}}{5c}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{d+ex}(Acd^2+2aBed+aAe^2+(Bcd^2+2Aced+aBe^2)x)}{a-cx^2}dx-\frac{2}{3}(d+ex)^{3/2}(Ae+Bd)}{c}-\frac{2B(d+ex)^{5/2}}{5c}$$

$$\Big\downarrow \ 653$$

$$\frac{-\frac{\int -\frac{aBe\left(3cd^2+ae^2\right)+Acd\left(cd^2+3ae^2\right)+c\left(Bcd^3+3Aced^2+3aBe^2d+aAe^3\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{c}-\frac{2\sqrt{d+ex}(aBe^2+2Acde+Bcd^2)}{c}-\frac{2}{3}(d+ex)^{3/2}(Ae+Bd)}{c}$$
$$\frac{2B(d+ex)^{5/2}}{5c}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{aBe\left(3cd^2+ae^2\right)+Acd\left(cd^2+3ae^2\right)+c\left(Bcd^3+3Aced^2+3aBe^2d+aAe^3\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{c}-\frac{2\sqrt{d+ex}(aBe^2+2Acde+Bcd^2)}{c}-\frac{2}{3}(d+ex)^{3/2}(Ae+Bd)}{c}-$$
$$\frac{2B(d+ex)^{5/2}}{5c}$$

$$\Big\downarrow \ 654$$

$$\frac{\frac{2\int \frac{\left(cd^2-ae^2\right)\left(Bcd^2+2Aced+aBe^2\right)-c\left(Bcd^3+3Aced^2+3aBe^2d+aAe^3\right)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}-\frac{2\sqrt{d+ex}(aBe^2+2Acde+Bcd^2)}{c}-\frac{2}{3}(d+ex)^{3/2}(Ae}{c}$$
$$\frac{2B(d+ex)^{5/2}}{5c}$$

$$\Big\downarrow \ 1480$$

$$\frac{\frac{2\left(-\frac{\sqrt{c}\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{c}d-\sqrt{a}e\right)^3\int\frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}}-\frac{\sqrt{c}\left(\sqrt{a}B+A\sqrt{c}\right)\left(\sqrt{a}e+\sqrt{c}d\right)^3\int\frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}}\right)}{c}-\frac{2\sqrt{d+ex}(aB}{}}{c}$$

$$\frac{2B(d+ex)^{5/2}}{5c}$$

$\Big\downarrow$ 221

$$\frac{\frac{2\left(\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}\sqrt[4]{c}}+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\left(\sqrt{a}e+\sqrt{c}d\right)^{5/2}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}\sqrt[4]{c}}\right)}{c}-\frac{2\sqrt{d+ex}\left(aBe^2+2Acde+Bcd\right.}{c}}{c}$$

$$\frac{2B(d+ex)^{5/2}}{5c}$$

input
```
Int[((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2),x]
```

output
```
(-2*B*(d + e*x)^(5/2))/(5*c) + ((-2*(B*c*d^2 + 2*A*c*d*e + a*B*e^2)*Sqrt[d
 + e*x])/c - (2*(B*d + A*e)*(d + e*x)^(3/2))/3 + (2*(((Sqrt[a]*B - A*Sqrt[
c])*(Sqrt[c]*d - Sqrt[a]*e)^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqr
t[c]*d - Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4)) + ((Sqrt[a]*B + A*Sqrt[c])*(Sqrt
[c]*d + Sqrt[a]*e)^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d +
Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4))))/c)/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.51

method	result
pseudoelliptic	$-\frac{14\left(-\frac{9\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}\,c\left(\left(\left(-A\,d^2e-\frac{1}{3}B\,d^3\right)c-\frac{a\,e^2(Ae+3Bd)}{3}\right)\sqrt{ac\,e^2}+\left(\frac{A\,c^2d^3}{3}+ade(Ae+Bd)c+\frac{B\,e^3a^2}{3}\right)e\right)\arctan\left(\frac{\ldots}{\sqrt{(\ldots}}\right.}{14}\ldots\right.}{\ldots}$
risch	$-\frac{2(3e^2Bc\,x^2+5Ac\,e^2x+11Bcdex+35Acde+15Ba\,e^2+23Bc\,d^2)\sqrt{ex+d}}{15c^2}-\frac{2\left(\frac{\left(-3Aacd\,e^3-A\,c^2d^3e-B\,e^4a^2-3Bac\,d^2\ldots\right.}{\ldots}\right.}{\ldots}$
derivativedivides	$-\frac{2\left(\frac{Bc(ex+d)^{\frac{5}{2}}}{5}+\frac{Ace(ex+d)^{\frac{3}{2}}}{3}+\frac{Bcd(ex+d)^{\frac{3}{2}}}{3}+2Acde\sqrt{ex+d}+Ba\,e^2\sqrt{ex+d}+Bc\,d^2\sqrt{ex+d}\right)}{c^2}-\frac{2\left(\frac{\left(-3Aacd\,e^3-A\,c^2\ldots\right.}{\ldots}\right.}{\ldots}$
default	$-\frac{2\left(\frac{Bc(ex+d)^{\frac{5}{2}}}{5}+\frac{Ace(ex+d)^{\frac{3}{2}}}{3}+\frac{Bcd(ex+d)^{\frac{3}{2}}}{3}+2Acde\sqrt{ex+d}+Ba\,e^2\sqrt{ex+d}+Bc\,d^2\sqrt{ex+d}\right)}{c^2}+\frac{\left(3Aacd\,e^3+A\,c^2d^3e+\ldots\right.}{\ldots}$

input
```
int((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-14/3*(-9/14*((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*c*(((-A*d^2*e-1/3*B*d^3)*c-1/
3*a*e^2*(A*e+3*B*d))*(a*c*e^2)^(1/2)+(1/3*A*c^2*d^3+a*d*e*(A*e+B*d)*c+1/3*
B*e^3*a^2)*e)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))+(-9
/14*c*(((A*e+1/3*B*d)*d^2*c+1/3*a*e^2*(A*e+3*B*d))*(a*c*e^2)^(1/2)+(1/3*A*
c^2*d^3+a*d*e*(A*e+B*d)*c+1/3*B*e^3*a^2)*e)*arctanh(c*(e*x+d)^(1/2)/((c*d+
(a*c*e^2)^(1/2))*c)^(1/2))+((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*((23/35*B*d^2+e
*(11/35*B*x+A)*d+1/7*(3/5*B*x+A)*e^2*x)*c+3/7*B*a*e^2)*(e*x+d)^(1/2)*(a*c*
e^2)^(1/2))*((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))/((c*d+(a*c*e^2)^(1/2))*c)^(1
/2)/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7410 vs. 2(179) = 358.

Time = 10.94 (sec) , antiderivative size = 7410, normalized size of antiderivative = 31.27

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{a-cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{a-cx^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(e*x+d)**(5/2)/(-c*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{a-cx^2}\,dx = \int -\frac{(Bx+A)(ex+d)^{\frac{5}{2}}}{cx^2-a}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((B*x + A)*(e*x + d)^(5/2)/(c*x^2 - a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(179) = 358$.

Time = 0.22 (sec) , antiderivative size = 680, normalized size of antiderivative = 2.87

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{a-cx^2}\,dx = \frac{\left(\left(3\sqrt{ac}acd^2e+\sqrt{ac}a^2e^3\right)Ac^2e^2+\left(\sqrt{ac}acd^3+3\sqrt{ac}a^2de^2\right)Bc^2e^2-2\left(ac^3d^3 \cdots\right.\right.}{\cdots}$$

$$-\frac{\left(\left(3\sqrt{ac}acd^2e+\sqrt{ac}a^2e^3\right)Ac^2e^2+\left(\sqrt{ac}acd^3+3\sqrt{ac}a^2de^2\right)Bc^2e^2+2\left(ac^3d^3e-a^2c^2de^3\right)A|c||e|+\left(ac^3d \cdots\right.\right.}{\left(ac^4d+\cdots\right.}$$

$$-\frac{2\left(3(ex+d)^{\frac{5}{2}}Bc^4+5(ex+d)^{\frac{3}{2}}Bc^4d+15\sqrt{ex+d}Bc^4d^2+5(ex+d)^{\frac{3}{2}}Ac^4e+30\sqrt{ex+d}Ac^4de+15\cdots\right.}{15c^5}$$

input
```
integrate((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="giac")
```

output
```
((3*sqrt(a*c)*a*c*d^2*e + sqrt(a*c)*a^2*e^3)*A*c^2*e^2 + (sqrt(a*c)*a*c*d^
3 + 3*sqrt(a*c)*a^2*d*e^2)*B*c^2*e^2 - 2*(a*c^3*d^3*e - a^2*c^2*d*e^3)*A*a
bs(c)*abs(e) - (a*c^3*d^4 - a^3*c*e^4)*B*abs(c)*abs(e) - (sqrt(a*c)*c^4*d^
4*e + 3*sqrt(a*c)*a*c^3*d^2*e^3)*A - (3*sqrt(a*c)*a*c^3*d^3*e^2 + sqrt(a*c
)*a^2*c^2*d*e^4)*B)*arctan(sqrt(e*x + d)/sqrt(-(c^6*d + sqrt(c^12*d^2 - (c
^6*d^2 - a*c^5*e^2)*c^6))/c^6))/((a*c^4*d - sqrt(a*c)*a*c^3*e)*sqrt(-c^2*d
 - sqrt(a*c)*c*e)*abs(e)) - ((3*sqrt(a*c)*a*c*d^2*e + sqrt(a*c)*a^2*e^3)*A
*c^2*e^2 + (sqrt(a*c)*a*c*d^3 + 3*sqrt(a*c)*a^2*d*e^2)*B*c^2*e^2 + 2*(a*c^
3*d^3*e - a^2*c^2*d*e^3)*A*abs(c)*abs(e) + (a*c^3*d^4 - a^3*c*e^4)*B*abs(c
)*abs(e) - (sqrt(a*c)*c^4*d^4*e + 3*sqrt(a*c)*a*c^3*d^2*e^3)*A - (3*sqrt(a
*c)*a*c^3*d^3*e^2 + sqrt(a*c)*a^2*c^2*d*e^4)*B)*arctan(sqrt(e*x + d)/sqrt(
-(c^6*d - sqrt(c^12*d^2 - (c^6*d^2 - a*c^5*e^2)*c^6))/c^6))/((a*c^4*d + sq
rt(a*c)*a*c^3*e)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(e)) - 2/15*(3*(e*x + d)^
(5/2)*B*c^4 + 5*(e*x + d)^(3/2)*B*c^4*d + 15*sqrt(e*x + d)*B*c^4*d^2 + 5*(
e*x + d)^(3/2)*A*c^4*e + 30*sqrt(e*x + d)*A*c^4*d*e + 15*sqrt(e*x + d)*B*a
*c^3*e^2)/c^5
```

Mupad [B] (verification not implemented)

Time = 7.24 (sec) , antiderivative size = 11383, normalized size of antiderivative = 48.03

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{a-cx^2}\,dx = \text{Too large to display}$$

input
```
int(((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2),x)
```

output
```
- (2*d*((2*A*e - 2*B*d)/c + (4*B*d)/c) + (2*B*(a*e^2 - c*d^2))/c^2)*(d + e
*x)^(1/2) - ((2*A*e - 2*B*d)/(3*c) + (4*B*d)/(3*c))*(d + e*x)^(3/2) - atan
(((((8*(4*B*a^3*c^4*e^6 - 8*A*a*c^6*d^3*e^3 + 8*A*a^2*c^5*d*e^5 - 4*B*a*c^
6*d^4*e^2))/c^3 - 64*a*c^4*d*e^2*(d + e*x)^(1/2)*((B^2*a^2*c^7*d^5 + B^2*a
^3*e^5*(a^3*c^9)^(1/2) + A^2*a*c^8*d^5 + 10*A^2*a^2*c^7*d^3*e^2 + 10*B^2*a
^3*c^6*d^3*e^2 + A^2*a^2*c*e^5*(a^3*c^9)^(1/2) + 2*A*B*a^4*c^5*e^5 + 5*A^2
*c^3*d^4*e*(a^3*c^9)^(1/2) + 2*A*B*c^3*d^5*(a^3*c^9)^(1/2) + 5*A^2*a^3*c^6
*d*e^4 + 5*B^2*a^4*c^5*d*e^4 + 10*A*B*a^2*c^7*d^4*e + 5*B^2*a*c^2*d^4*e*(a
^3*c^9)^(1/2) + 10*A^2*a*c^2*d^2*e^3*(a^3*c^9)^(1/2) + 20*A*B*a^3*c^6*d^2*
e^3 + 10*B^2*a^2*c*d^2*e^3*(a^3*c^9)^(1/2) + 10*A*B*a^2*c*d*e^4*(a^3*c^9)^
(1/2) + 20*A*B*a*c^2*d^3*e^2*(a^3*c^9)^(1/2))/(4*a^2*c^9))^(1/2))*((B^2*a^
2*c^7*d^5 + B^2*a^3*e^5*(a^3*c^9)^(1/2) + A^2*a*c^8*d^5 + 10*A^2*a^2*c^7*d
^3*e^2 + 10*B^2*a^3*c^6*d^3*e^2 + A^2*a^2*c*e^5*(a^3*c^9)^(1/2) + 2*A*B*a^
4*c^5*e^5 + 5*A^2*c^3*d^4*e*(a^3*c^9)^(1/2) + 2*A*B*c^3*d^5*(a^3*c^9)^(1/2
) + 5*A^2*a^3*c^6*d*e^4 + 5*B^2*a^4*c^5*d*e^4 + 10*A*B*a^2*c^7*d^4*e + 5*B
^2*a*c^2*d^4*e*(a^3*c^9)^(1/2) + 10*A^2*a*c^2*d^2*e^3*(a^3*c^9)^(1/2) + 20
*A*B*a^3*c^6*d^2*e^3 + 10*B^2*a^2*c*d^2*e^3*(a^3*c^9)^(1/2) + 10*A*B*a^2*c
*d*e^4*(a^3*c^9)^(1/2) + 20*A*B*a*c^2*d^3*e^2*(a^3*c^9)^(1/2))/(4*a^2*c^9)
)^(1/2) + (16*(d + e*x)^(1/2)*(B^2*a^4*e^8 + A^2*c^4*d^6*e^2 + A^2*a^3*c*e
^8 + 15*A^2*a^2*c^2*d^2*e^6 + 15*B^2*a^2*c^2*d^4*e^4 + 15*A^2*a*c^3*d^4...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 899, normalized size of antiderivative = 3.79

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{a-cx^2}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a),x)
```

output
```
( - 30*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(
c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c*e**2 - 60*sqrt(a)*sqrt(sqrt(c)*sqrt
(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)
))*b*c*d*e - 30*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*
c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c**2*d**2 + 30*sqrt(c)*sqrt(sq
rt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a
)*e - c*d)))*a*b*e**2 + 60*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqr
t(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c*d*e + 30*sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(
c)*sqrt(a)*e - c*d)))*b*c*d**2 - 15*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*
log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*c*e**2 - 3
0*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*
d) + sqrt(c)*sqrt(d + e*x))*b*c*d*e - 15*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e +
c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c**2*d*
*2 + 15*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e +
 c*d) + sqrt(c)*sqrt(d + e*x))*a*c*e**2 + 30*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*
e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*b*c*d*
e + 15*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e +
c*d) + sqrt(c)*sqrt(d + e*x))*c**2*d**2 - 15*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*
e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*...
```

3.124 $\int \frac{(A+Bx)(d+ex)^{3/2}}{a-cx^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 202

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{a-cx^2} \, dx = -\frac{2(Bd+Ae)\sqrt{d+ex}}{c} - \frac{2B(d+ex)^{3/2}}{3c} + \frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{7/4}} + \frac{\left(\sqrt{a}B+A\sqrt{c}\right)\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}c^{7/4}}$$

output
```
-2*(A*e+B*d)*(e*x+d)^(1/2)/c-2/3*B*(e*x+d)^(3/2)/c+(a^(1/2)*B-A*c^(1/2))*(
c^(1/2)*d-a^(1/2)*e)^(3/2)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2
)*e)^(1/2))/a^(1/2)/c^(7/4)+(a^(1/2)*B+A*c^(1/2))*(c^(1/2)*d+a^(1/2)*e)^(3
/2)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(1/2)/c^(
7/4)
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{a-cx^2}\,dx =$$

$$-\frac{2c\sqrt{d+ex}(4Bd+3Ae+Bex)+\frac{3(\sqrt{a}B+A\sqrt{c})(\sqrt{c}d+\sqrt{a}e)\sqrt{-cd-\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}}+\frac{3(-\sqrt{a}B+A\sqrt{c})\cdot}{}}{3c^2}$$

input
```
Integrate[((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2),x]
```

output
```
-1/3*(2*c*Sqrt[d + e*x]*(4*B*d + 3*A*e + B*e*x) + (3*(Sqrt[a]*B + A*Sqrt[c
])*(Sqrt[c]*d + Sqrt[a]*e)*Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*ArcTan[(Sqrt[-
(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)])/Sqrt[a
] + (3*(-(Sqrt[a]*B) + A*Sqrt[c])*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)^2*ArcTan
[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)]
)/(Sqrt[a]*Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]))/c^2
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {653, 25, 653, 25, 27, 654, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{a-cx^2}dx$$

$$\Big\downarrow \text{653}$$

$$-\frac{\int -\frac{\sqrt{d+ex}(Acd+aBe+c(Bd+Ae)x)}{a-cx^2}dx}{c}-\frac{2B(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{d+ex}(Acd+aBe+c(Bd+Ae)x)}{a-cx^2}dx}{c}-\frac{2B(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \; 653$$

$$-\frac{-\frac{\int -\frac{c\left(Acd^2+2aBed+aAe^2+\left(Bcd^2+2Aced+aBe^2\right)x\right)}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{c}-2\sqrt{d+ex}(Ae+Bd)}{c}-\frac{2B(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{c\left(Acd^2+2aBed+aAe^2+\left(Bcd^2+2Aced+aBe^2\right)x\right)}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{c}-2\sqrt{d+ex}(Ae+Bd)}{c}-\frac{2B(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{Acd^2+2aBed+aAe^2+(Bcd^2+2Aced+aBe^2)x}{\sqrt{d+ex}(a-cx^2)}dx-2\sqrt{d+ex}(Ae+Bd)}{c}-\frac{2B(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \; 654$$

$$\frac{2\int \frac{(Bd+Ae)\left(cd^2-ae^2\right)-\left(Bcd^2+2Aced+aBe^2\right)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}-2\sqrt{d+ex}(Ae+Bd)}{c}-\frac{2B(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \; 1480$$

$$\frac{2\left(-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{c}d-\sqrt{a}e\right)^2\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}}-\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\left(\sqrt{a}e+\sqrt{c}d\right)^2\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}}\right)-2\sqrt{\cdots}}{c}-\frac{2B(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \; 221$$

$$\frac{2\left(\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}c^{3/4}}+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\left(\sqrt{a}e+\sqrt{c}d\right)^{3/2}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}c^{3/4}}\right)-2\sqrt{d+ex}(Ae\cdots}{c}-\frac{2B(d+ex)^{3/2}}{3c}$$

input
```
Int[((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2),x]
```

output
```
(-2*B*(d + e*x)^(3/2))/(3*c) + (-2*(B*d + A*e)*Sqrt[d + e*x] + 2*(((Sqrt[a
]*B - A*Sqrt[c])*(Sqrt[c]*d - Sqrt[a]*e)^(3/2)*ArcTanh[(c^(1/4)*Sqrt[d + e
*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(2*Sqrt[a]*c^(3/4)) + ((Sqrt[a]*B + A*S
qrt[c])*(Sqrt[c]*d + Sqrt[a]*e)^(3/2)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt
[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(3/4))))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.34

method	result
pseudoelliptic	$\frac{-\frac{2(Bex+3Ae+4Bd)\sqrt{ex+d}}{3}-\frac{\left(-Aac\,e^3-A\,c^2d^2e-2Bad\,e^2c+2A\sqrt{ac\,e^2}\,cde+B\sqrt{ac\,e^2}\,a\,e^2+B\sqrt{ac\,e^2}\,c\,d^2\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac}\ldots}}\right.}{\sqrt{ac\,e^2}\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}\ldots}{c}$
risch	$-\frac{2(Bex+3Ae+4Bd)\sqrt{ex+d}}{3c}-\frac{\left(-Aac\,e^3-A\,c^2d^2e-2Bad\,e^2c+2A\sqrt{ac\,e^2}\,cde+B\sqrt{ac\,e^2}\,a\,e^2+B\sqrt{ac\,e^2}\,c\,d^2\right)\operatorname{arctan}\ldots}{c\sqrt{ac\,e^2}\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}\ldots$
derivativedivides	$-\frac{2\left(\frac{B(ex+d)^{\frac{3}{2}}}{3}+Ae\sqrt{ex+d}+Bd\sqrt{ex+d}\right)}{c}+\frac{\left(Aac\,e^3+A\,c^2d^2e+2Bad\,e^2c+2A\sqrt{ac\,e^2}\,cde+B\sqrt{ac\,e^2}\,a\,e^2+B\sqrt{ac\,e^2}\ldots\right.}{c\sqrt{ac\,e^2}\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}\ldots$
default	$-\frac{2\left(\frac{B(ex+d)^{\frac{3}{2}}}{3}+Ae\sqrt{ex+d}+Bd\sqrt{ex+d}\right)}{c}-\frac{\left(-Aac\,e^3-A\,c^2d^2e-2Bad\,e^2c-2A\sqrt{ac\,e^2}\,cde-B\sqrt{ac\,e^2}\,a\,e^2-B\sqrt{ac\,e\ldots}\right.}{c\sqrt{ac\,e^2}\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}\ldots$

input
```
int((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-2/3*(B*e*x+3*A*e+4*B*d)*(e*x+d)^(1/2)-(-A*a*c*e^3-A*c^2*d^2*e-2*B*a*
d*e^2*c+2*A*(a*c*e^2)^(1/2)*c*d*e+B*(a*c*e^2)^(1/2)*a*e^2+B*(a*c*e^2)^(1/2
)*c*d^2)/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c*(e*x+d)
^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))+(A*a*c*e^3+A*c^2*d^2*e+2*B*a*d*e^
2*c+2*A*(a*c*e^2)^(1/2)*c*d*e+B*(a*c*e^2)^(1/2)*a*e^2+B*(a*c*e^2)^(1/2)*c*
d^2)/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/
2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4480 vs. $2(148) = 296$.

Time = 1.97 (sec) , antiderivative size = 4480, normalized size of antiderivative = 22.18

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{a-cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{a-cx^2}\,dx = -\int \frac{Ad\sqrt{d+ex}}{-a+cx^2}\,dx - \int \frac{Aex\sqrt{d+ex}}{-a+cx^2}\,dx - \int \frac{Bdx\sqrt{d+ex}}{-a+cx^2}\,dx - \int \frac{Bex^2\sqrt{d+ex}}{-a+cx^2}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)**(3/2)/(-c*x**2+a),x)
```

output
```
-Integral(A*d*sqrt(d + e*x)/(-a + c*x**2), x) - Integral(A*e*x*sqrt(d + e*
x)/(-a + c*x**2), x) - Integral(B*d*x*sqrt(d + e*x)/(-a + c*x**2), x) - In
tegral(B*e*x**2*sqrt(d + e*x)/(-a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{a-cx^2}\,dx = \int -\frac{(Bx+A)(ex+d)^{\frac{3}{2}}}{cx^2-a}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((B*x + A)*(e*x + d)^(3/2)/(c*x^2 - a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(148) = 296$.

Time = 0.19 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.72

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{a-cx^2}\,dx =$$

$$-\frac{\left(2\sqrt{ac}Bac^3d^2e^2-2\sqrt{ac}Aac^3de^3-\left(\sqrt{ac}acd^2+\sqrt{ac}a^2e^2\right)Bc^2e^2+\left(ac^3d^2e-a^2c^2e^3\right)A|c||e|+\left(ac^3d^3-\right.}{\left(ac^4d-\sqrt{ac}ac^3e\right)\sqrt{-c^2d-\sqrt{}}}$$

$$+\frac{\left(2\sqrt{ac}Bac^3d^2e^2-2\sqrt{ac}Aac^3de^3-\left(\sqrt{ac}acd^2+\sqrt{ac}a^2e^2\right)Bc^2e^2-\left(ac^3d^2e-a^2c^2e^3\right)A|c||e|-\left(ac^3d^3-\right.}{\left(ac^4d+\sqrt{ac}ac^3e\right)\sqrt{-c^2d+\sqrt{}}}$$

$$-\frac{2\left((ex+d)^{\frac{3}{2}}Bc^2+3\sqrt{ex+d}Bc^2d+3\sqrt{ex+d}Ac^2e\right)}{3\,c^3}$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="giac")
```

output
```
-(2*sqrt(a*c)*B*a*c^3*d^2*e^2 - 2*sqrt(a*c)*A*a*c^3*d*e^3 - (sqrt(a*c)*a*c
*d^2 + sqrt(a*c)*a^2*e^2)*B*c^2*e^2 + (a*c^3*d^2*e - a^2*c^2*e^3)*A*abs(c)
*abs(e) + (a*c^3*d^3 - a^2*c^2*d*e^2)*B*abs(c)*abs(e) + (sqrt(a*c)*c^4*d^3
*e + sqrt(a*c)*a*c^3*d*e^3)*A)*arctan(sqrt(e*x + d)/sqrt(-(c^4*d + sqrt(c^
8*d^2 - (c^4*d^2 - a*c^3*e^2)*c^4))/c^4))/((a*c^4*d - sqrt(a*c)*a*c^3*e)*s
qrt(-c^2*d - sqrt(a*c)*c*e)*abs(e)) + (2*sqrt(a*c)*B*a*c^3*d^2*e^2 - 2*sqr
t(a*c)*A*a*c^3*d*e^3 - (sqrt(a*c)*a*c*d^2 + sqrt(a*c)*a^2*e^2)*B*c^2*e^2 -
 (a*c^3*d^2*e - a^2*c^2*e^3)*A*abs(c)*abs(e) - (a*c^3*d^3 - a^2*c^2*d*e^2)
*B*abs(c)*abs(e) + (sqrt(a*c)*c^4*d^3*e + sqrt(a*c)*a*c^3*d*e^3)*A)*arctan
(sqrt(e*x + d)/sqrt(-(c^4*d - sqrt(c^8*d^2 - (c^4*d^2 - a*c^3*e^2)*c^4))/c
^4))/((a*c^4*d + sqrt(a*c)*a*c^3*e)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(e)) -
 2/3*((e*x + d)^(3/2)*B*c^2 + 3*sqrt(e*x + d)*B*c^2*d + 3*sqrt(e*x + d)*A*
c^2*e)/c^3
```

Mupad [B] (verification not implemented)

Time = 6.54 (sec) , antiderivative size = 7560, normalized size of antiderivative = 37.43

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{a-cx^2}\,dx = \text{Too large to display}$$

input
```
int(((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2),x)
```

output
```
- ((2*A*e - 2*B*d)/c + (4*B*d)/c)*(d + e*x)^(1/2) - atan(((((8*(4*A*a^2*c^
4*e^5 - 4*A*a*c^5*d^2*e^3 - 4*B*a*c^5*d^3*e^2 + 4*B*a^2*c^4*d*e^4))/c^2 -
64*a*c^4*d*e^2*(d + e*x)^(1/2)*((B^2*a^2*c^5*d^3 + B^2*a^2*e^3*(a^3*c^7)^(
1/2) + A^2*a*c^6*d^3 + 2*A*B*a^3*c^4*e^3 + 3*A^2*c^2*d^2*e*(a^3*c^7)^(1/2)
 + 2*A*B*c^2*d^3*(a^3*c^7)^(1/2) + 3*A^2*a^2*c^5*d*e^2 + 3*B^2*a^3*c^4*d*e
^2 + A^2*a*c*e^3*(a^3*c^7)^(1/2) + 3*B^2*a*c*d^2*e*(a^3*c^7)^(1/2) + 6*A*B
*a^2*c^5*d^2*e + 6*A*B*a*c*d*e^2*(a^3*c^7)^(1/2))/(4*a^2*c^7))^(1/2))*((B^
2*a^2*c^5*d^3 + B^2*a^2*e^3*(a^3*c^7)^(1/2) + A^2*a*c^6*d^3 + 2*A*B*a^3*c^
4*e^3 + 3*A^2*c^2*d^2*e*(a^3*c^7)^(1/2) + 2*A*B*c^2*d^3*(a^3*c^7)^(1/2) +
3*A^2*a^2*c^5*d*e^2 + 3*B^2*a^3*c^4*d*e^2 + A^2*a*c*e^3*(a^3*c^7)^(1/2) +
3*B^2*a*c*d^2*e*(a^3*c^7)^(1/2) + 6*A*B*a^2*c^5*d^2*e + 6*A*B*a*c*d*e^2*(a
^3*c^7)^(1/2))/(4*a^2*c^7))^(1/2) + (d + e*x)^(1/2)*(16*B^2*a^3*e^6 + 16*A
^2*c^3*d^4*e^2 + 16*A^2*a^2*c*e^6 + 96*A^2*a*c^2*d^2*e^4 + 16*B^2*a*c^2*d^
4*e^2 + 96*B^2*a^2*c*d^2*e^4 + 128*A*B*a^2*c*d*e^5 + 128*A*B*a*c^2*d^3*e^3
))*((B^2*a^2*c^5*d^3 + B^2*a^2*e^3*(a^3*c^7)^(1/2) + A^2*a*c^6*d^3 + 2*A*B
*a^3*c^4*e^3 + 3*A^2*c^2*d^2*e*(a^3*c^7)^(1/2) + 2*A*B*c^2*d^3*(a^3*c^7)^(
1/2) + 3*A^2*a^2*c^5*d*e^2 + 3*B^2*a^3*c^4*d*e^2 + A^2*a*c*e^3*(a^3*c^7)^(
1/2) + 3*B^2*a*c*d^2*e*(a^3*c^7)^(1/2) + 6*A*B*a^2*c^5*d^2*e + 6*A*B*a*c*d
*e^2*(a^3*c^7)^(1/2))/(4*a^2*c^7))^(1/2)*1i - (((8*(4*A*a^2*c^4*e^5 - 4*A*
a*c^5*d^2*e^3 - 4*B*a*c^5*d^3*e^2 + 4*B*a^2*c^4*d*e^4))/c^2 + 64*a*c^4*...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 540, normalized size of antiderivative = 2.67

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{a-cx^2}\,dx = \frac{-6\sqrt{a}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}}\right)be-6\sqrt{a}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\Big(}{}$$

input
```
int((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a),x)
```

output
```
( - 6*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*b*e - 6*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e -
 c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c*d
+ 6*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*
sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*e + 6*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c
*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*b*d -
3*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*
d) + sqrt(c)*sqrt(d + e*x))*b*e - 3*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*
log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c*d + 3*sqrt
(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt
(c)*sqrt(d + e*x))*b*e + 3*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(
sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c*d - 3*sqrt(c)*sqrt(sqr
t(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(
d + e*x))*a*e - 3*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c
)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*b*d + 3*sqrt(c)*sqrt(sqrt(c)*s
qrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))
*a*e + 3*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e
+ c*d) + sqrt(c)*sqrt(d + e*x))*b*d - 12*sqrt(d + e*x)*a*c*e - 16*sqrt(d +
 e*x)*b*c*d - 4*sqrt(d + e*x)*b*c*e*x)/(6*c**2)
```

3.125 $\int \frac{(A+Bx)\sqrt{d+ex}}{a-cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 179

$$\int \frac{(A+Bx)\sqrt{d+ex}}{a-cx^2}\,dx = -\frac{2B\sqrt{d+ex}}{c} + \frac{\left(\sqrt{a}B-A\sqrt{c}\right)\sqrt{\sqrt{c}d-\sqrt{a}e}\,\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{5/4}} + \frac{\left(\sqrt{a}B+A\sqrt{c}\right)\sqrt{\sqrt{c}d+\sqrt{a}e}\,\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}c^{5/4}}$$

output
```
-2*B*(e*x+d)^(1/2)/c+(a^(1/2)*B-A*c^(1/2))*(c^(1/2)*d-a^(1/2)*e)^(1/2)*arc
tanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(1/2)/c^(5/4)+(a
^(1/2)*B+A*c^(1/2))*(c^(1/2)*d+a^(1/2)*e)^(1/2)*arctanh(c^(1/4)*(e*x+d)^(1
/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(1/2)/c^(5/4)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\sqrt{d+ex}}{a-cx^2}\,dx = \frac{-2\sqrt{a}B\sqrt{c}\sqrt{d+ex}-\left(\sqrt{a}B+A\sqrt{c}\right)\sqrt{-cd-\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)+\left(-\sqrt{a}B+A\sqrt{c}\right)\cdots}{\sqrt{a}c^{3/2}}$$

input
```
Integrate[((A + B*x)*Sqrt[d + e*x])/(a - c*x^2),x]
```

output
```
(-2*Sqrt[a]*B*Sqrt[c]*Sqrt[d + e*x] - (Sqrt[a]*B + A*Sqrt[c])*Sqrt[-(c*d)
- Sqrt[a]*Sqrt[c]*e]*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x
])/(Sqrt[c]*d + Sqrt[a]*e)] + (-(Sqrt[a]*B) + A*Sqrt[c])*Sqrt[-(c*d) + Sqr
t[a]*Sqrt[c]*e]*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(S
qrt[c]*d - Sqrt[a]*e)])/(Sqrt[a]*c^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {653, 25, 654, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{d+ex}}{a-cx^2}\,dx$$

$$\Big\downarrow \text{653}$$

$$-\frac{\int -\frac{Acd+aBe+c(Bd+Ae)x}{\sqrt{d+ex}(a-cx^2)}dx}{c}-\frac{2B\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Acd+aBe+c(Bd+Ae)x}{\sqrt{d+ex}(a-cx^2)}dx}{c}-\frac{2B\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{654}$$

$$\frac{2\int \frac{B(cd^2-ae^2)-c(Bd+Ae)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}-\frac{2B\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{1480}$$

$$\frac{2\left(-\frac{\sqrt{c}(\sqrt{a}B-A\sqrt{c})(\sqrt{c}d-\sqrt{a}e)\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d-\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}}-\frac{\sqrt{c}(\sqrt{a}B+A\sqrt{c})(\sqrt{a}e+\sqrt{c}d)\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d+\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}}\right)}{c}-\frac{2B\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\left(\frac{(\sqrt{a}B-A\sqrt{c})\sqrt{\sqrt{c}d-\sqrt{a}e}\,\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}\sqrt[4]{c}}+\frac{(\sqrt{a}B+A\sqrt{c})\sqrt{\sqrt{a}e+\sqrt{c}d}\,\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}\sqrt[4]{c}}\right)}{c}-\frac{2B\sqrt{d+ex}}{c}$$

input
```
Int[((A + B*x)*Sqrt[d + e*x])/(a - c*x^2),x]
```

output
```
(-2*B*Sqrt[d + e*x])/c + (2*(((Sqrt[a]*B - A*Sqrt[c])*Sqrt[Sqrt[c]*d - Sqr
t[a]*e]*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(2*S
qrt[a]*c^(1/4)) + ((Sqrt[a]*B + A*Sqrt[c])*Sqrt[Sqrt[c]*d + Sqrt[a]*e]*Arc
Tanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(1
/4))))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
 - 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{2B\sqrt{ex+d}}{c}-\frac{\left(-Acde-Ba\,e^{2}+A\sqrt{ac\,e^{2}}\,e+B\sqrt{ac\,e^{2}}\,d\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{ac\,e^{2}}\,\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}+\frac{\left(Acde+Ba\,e^{2}+A\sqrt{ac\,e^{2}}\right.}{\sqrt{a}}$
risch	$-\frac{2B\sqrt{ex+d}}{c}-\frac{\left(-Acde-Ba\,e^{2}+A\sqrt{ac\,e^{2}}\,e+B\sqrt{ac\,e^{2}}\,d\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{ac\,e^{2}}\,\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}+\frac{\left(Acde+Ba\,e^{2}+A\sqrt{ac\,e^{2}}\right.}{\sqrt{a}}$
pseudoelliptic	$-\frac{2B\sqrt{ex+d}}{c}-\frac{\left(-Acde-Ba\,e^{2}+A\sqrt{ac\,e^{2}}\,e+B\sqrt{ac\,e^{2}}\,d\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{ac\,e^{2}}\,\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}+\frac{\left(Acde+Ba\,e^{2}+A\sqrt{ac\,e^{2}}\right.}{\sqrt{a}}$
default	$-\frac{2B\sqrt{ex+d}}{c}+\frac{\left(Acde+Ba\,e^{2}-A\sqrt{ac\,e^{2}}\,e-B\sqrt{ac\,e^{2}}\,d\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{ac\,e^{2}}\,\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}-\frac{\left(-Acde-Ba\,e^{2}-A\sqrt{ac\,e^{2}}\right.}{\sqrt{ac}}$

input
```
int((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2*B*(e*x+d)^(1/2)/c-(-A*c*d*e-B*a*e^2+A*(a*c*e^2)^(1/2)*e+B*(a*c*e^2)^(1/
2)*d)/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1
/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))+(A*c*d*e+B*a*e^2+A*(a*c*e^2)^(1/2)*e
+B*(a*c*e^2)^(1/2)*d)/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arct
anh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1538 vs. $2(129) = 258$.

Time = 0.13 (sec) , antiderivative size = 1538, normalized size of antiderivative = 8.59

$$\int \frac{(A+Bx)\sqrt{d+ex}}{a-cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*(c*sqrt((2*A*B*a*e + a*c^2*sqrt((4*A^2*B^2*c^2*d^2 + 4*(A*B^3*a*c + A
^3*B*c^2)*d*e + (B^4*a^2 + 2*A^2*B^2*a*c + A^4*c^2)*e^2)/(a*c^5)) + (B^2*a
 + A^2*c)*d)/(a*c^2))*log(-(2*(A*B^3*a*c - A^3*B*c^2)*d + (B^4*a^2 - A^4*c
^2)*e)*sqrt(e*x + d) + (2*A*B^2*a*c^2*d - A*a*c^4*sqrt((4*A^2*B^2*c^2*d^2
+ 4*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 + 2*A^2*B^2*a*c + A^4*c^2)*e^2)
/(a*c^5)) + (B^3*a^2*c + A^2*B*a*c^2)*e)*sqrt((2*A*B*a*e + a*c^2*sqrt((4*A
^2*B^2*c^2*d^2 + 4*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 + 2*A^2*B^2*a*c
+ A^4*c^2)*e^2)/(a*c^5)) + (B^2*a + A^2*c)*d)/(a*c^2))) - c*sqrt((2*A*B*a*
e + a*c^2*sqrt((4*A^2*B^2*c^2*d^2 + 4*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a
^2 + 2*A^2*B^2*a*c + A^4*c^2)*e^2)/(a*c^5)) + (B^2*a + A^2*c)*d)/(a*c^2))*
log(-(2*(A*B^3*a*c - A^3*B*c^2)*d + (B^4*a^2 - A^4*c^2)*e)*sqrt(e*x + d) -
 (2*A*B^2*a*c^2*d - A*a*c^4*sqrt((4*A^2*B^2*c^2*d^2 + 4*(A*B^3*a*c + A^3*B
*c^2)*d*e + (B^4*a^2 + 2*A^2*B^2*a*c + A^4*c^2)*e^2)/(a*c^5)) + (B^3*a^2*c
 + A^2*B*a*c^2)*e)*sqrt((2*A*B*a*e + a*c^2*sqrt((4*A^2*B^2*c^2*d^2 + 4*(A*
B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 + 2*A^2*B^2*a*c + A^4*c^2)*e^2)/(a*c^5
)) + (B^2*a + A^2*c)*d)/(a*c^2))) + c*sqrt((2*A*B*a*e - a*c^2*sqrt((4*A^2*
B^2*c^2*d^2 + 4*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 + 2*A^2*B^2*a*c + A
^4*c^2)*e^2)/(a*c^5)) + (B^2*a + A^2*c)*d)/(a*c^2))*log(-(2*(A*B^3*a*c - A
^3*B*c^2)*d + (B^4*a^2 - A^4*c^2)*e)*sqrt(e*x + d) + (2*A*B^2*a*c^2*d + A*
a*c^4*sqrt((4*A^2*B^2*c^2*d^2 + 4*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^...
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{a-cx^2}\,dx = -\int \frac{A\sqrt{d+ex}}{-a+cx^2}\,dx - \int \frac{Bx\sqrt{d+ex}}{-a+cx^2}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)**(1/2)/(-c*x**2+a),x)
```

output
```
-Integral(A*sqrt(d + e*x)/(-a + c*x**2), x) - Integral(B*x*sqrt(d + e*x)/(
-a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{a-cx^2}\,dx = \int -\frac{(Bx+A)\sqrt{ex+d}}{cx^2-a}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((B*x + A)*sqrt(e*x + d)/(c*x^2 - a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(129) = 258.

Time = 0.17 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.77

$$\int \frac{(A+Bx)\sqrt{d+ex}}{a-cx^2}\,dx = -\frac{2\sqrt{ex+d}B}{c} - \frac{\left(\sqrt{ac}Ac^3d^2e - \sqrt{ac}Aac^2e^3 + (ac^2d^2 - a^2ce^2)B|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{c^2d+\sqrt{c^4d^2-(c^2d^2-ace^2)c^2}}{c^2}}}\right)}{\left(ac^3d - \sqrt{ac}ac^2e\right)\sqrt{-c^2d - \sqrt{ac}ce}|e|} + \frac{\left(\sqrt{ac}Ac^3d^2e - \sqrt{ac}Aac^2e^3 - (ac^2d^2 - a^2ce^2)B|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{c^2d-\sqrt{c^4d^2-(c^2d^2-ace^2)c^2}}{c^2}}}\right)}{\left(ac^3d + \sqrt{ac}ac^2e\right)\sqrt{-c^2d + \sqrt{ac}ce}|e|}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="giac")
```

output
```
-2*sqrt(e*x + d)*B/c - (sqrt(a*c)*A*c^3*d^2*e - sqrt(a*c)*A*a*c^2*e^3 + (a
*c^2*d^2 - a^2*c*e^2)*B*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(c^2*d +
 sqrt(c^4*d^2 - (c^2*d^2 - a*c*e^2)*c^2))/c^2))/((a*c^3*d - sqrt(a*c)*a*c^
2*e)*sqrt(-c^2*d - sqrt(a*c)*c*e)*abs(e)) + (sqrt(a*c)*A*c^3*d^2*e - sqrt(
a*c)*A*a*c^2*e^3 - (a*c^2*d^2 - a^2*c*e^2)*B*abs(c)*abs(e))*arctan(sqrt(e*
x + d)/sqrt(-(c^2*d - sqrt(c^4*d^2 - (c^2*d^2 - a*c*e^2)*c^2))/c^2))/((a*c
^3*d + sqrt(a*c)*a*c^2*e)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 4276, normalized size of antiderivative = 23.89

$$\int \frac{(A+Bx)\sqrt{d+ex}}{a-cx^2}\,dx = \text{Too large to display}$$

input
```
int(((A + B*x)*(d + e*x)^(1/2))/(a - c*x^2),x)
```

output
```
- 2*atanh((32*A^2*a*c^2*e^4*(d + e*x)^(1/2)*((B^2*d)/(4*c^2) + (A*B*e)/(2*
c^2) + (A^2*d)/(4*a*c) + (A^2*e*(a^3*c^5)^(1/2))/(4*a^2*c^4) + (B^2*e*(a^3
*c^5)^(1/2))/(4*a*c^5) + (A*B*d*(a^3*c^5)^(1/2))/(2*a^2*c^4))^(1/2))/(16*A
^3*c^2*d^2*e^3 - 16*A^3*a*c*e^5 - 16*A*B^2*a^2*e^5 - (16*A^2*B*e^5*(a^3*c^
5)^(1/2))/c^2 - (16*B^3*a*e^5*(a^3*c^5)^(1/2))/c^3 + 32*A^2*B*c^2*d^3*e^2
+ (16*B^3*d^2*e^3*(a^3*c^5)^(1/2))/c^2 - 32*A^2*B*a*c*d*e^4 - (32*A*B^2*d*
e^4*(a^3*c^5)^(1/2))/c^2 + 16*A*B^2*a*c*d^2*e^3 + (32*A*B^2*d^3*e^2*(a^3*c
^5)^(1/2))/(a*c) + (16*A^2*B*d^2*e^3*(a^3*c^5)^(1/2))/(a*c)) + (32*B^2*a^2
*c*e^4*(d + e*x)^(1/2)*((B^2*d)/(4*c^2) + (A*B*e)/(2*c^2) + (A^2*d)/(4*a*c
) + (A^2*e*(a^3*c^5)^(1/2))/(4*a^2*c^4) + (B^2*e*(a^3*c^5)^(1/2))/(4*a*c^5
) + (A*B*d*(a^3*c^5)^(1/2))/(2*a^2*c^4))^(1/2))/(16*A^3*c^2*d^2*e^3 - 16*A
^3*a*c*e^5 - 16*A*B^2*a^2*e^5 - (16*A^2*B*e^5*(a^3*c^5)^(1/2))/c^2 - (16*B
^3*a*e^5*(a^3*c^5)^(1/2))/c^3 + 32*A^2*B*c^2*d^3*e^2 + (16*B^3*d^2*e^3*(a^
3*c^5)^(1/2))/c^2 - 32*A^2*B*a*c*d*e^4 - (32*A*B^2*d*e^4*(a^3*c^5)^(1/2))/
c^2 + 16*A*B^2*a*c*d^2*e^3 + (32*A*B^2*d^3*e^2*(a^3*c^5)^(1/2))/(a*c) + (1
6*A^2*B*d^2*e^3*(a^3*c^5)^(1/2))/(a*c)) + (32*A^2*d*e^3*(a^3*c^5)^(1/2)*(d
 + e*x)^(1/2)*((B^2*d)/(4*c^2) + (A*B*e)/(2*c^2) + (A^2*d)/(4*a*c) + (A^2*
e*(a^3*c^5)^(1/2))/(4*a^2*c^4) + (B^2*e*(a^3*c^5)^(1/2))/(4*a*c^5) + (A*B*
d*(a^3*c^5)^(1/2))/(2*a^2*c^4))^(1/2))/(16*A*B^2*a^3*e^5 + 16*A^3*a^2*c*e^
5 + (16*B^3*a^2*e^5*(a^3*c^5)^(1/2))/c^3 - 16*A^3*a*c^2*d^2*e^3 + (16*A...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.44

$$\int \frac{(A+Bx)\sqrt{d+ex}}{a-cx^2}\,dx$$

$$= \frac{-2\sqrt{a}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}}\right)c+2\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}}\right)b-\sqrt{a}\,\sqrt{\sqrt{c}}}{}$$

input
```
int((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a),x)
```

output
```
( - 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c + 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c
*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*b - sq
rt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) +
 sqrt(c)*sqrt(d + e*x))*c + sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt
(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c - sqrt(c)*sqrt(sqrt(c
)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d +
 e*x))*b + sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*
e + c*d) + sqrt(c)*sqrt(d + e*x))*b - 4*sqrt(d + e*x)*b*c)/(2*c**2)
```

3.126 $\int \frac{A+Bx}{\sqrt{d+ex}(a-cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 152

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)}\,dx=\frac{\left(B-\frac{A\sqrt{c}}{\sqrt{a}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{c^{3/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{c^{3/4}\sqrt{\sqrt{c}d+\sqrt{a}e}}$$

output
```
(B-A*c^(1/2)/a^(1/2))*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^
(1/2))/c^(3/4)/(c^(1/2)*d-a^(1/2)*e)^(1/2)+(B+A*c^(1/2)/a^(1/2))*arctanh(c
^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/c^(3/4)/(c^(1/2)*d+a^(1/
2)*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)}\,dx = \frac{\frac{(\sqrt{a}B+A\sqrt{c})\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{-cd-\sqrt{a}\sqrt{c}e}}+\frac{(\sqrt{a}B-A\sqrt{c})\arctan\left(\frac{\sqrt{-cd+\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-\sqrt{a}e}\right)}{\sqrt{-cd+\sqrt{a}\sqrt{c}e}}}{\sqrt{a}\sqrt{c}}$$

input
```
Integrate[(A + B*x)/(Sqrt[d + e*x]*(a - c*x^2)),x]
```

output
```
(((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d
+ e*x])/(Sqrt[c]*d + Sqrt[a]*e)])/Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e] + ((Sqr
t[a]*B - A*Sqrt[c])*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x]
)/(Sqrt[c]*d - Sqrt[a]*e)])/Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e])/(Sqrt[a]*Sqr
t[c])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {654, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a-cx^2)\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{654}$$

$$2\int \frac{Bd-Ae-B(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}$$

$$\Big\downarrow \text{1480}$$

$$2\left(-\frac{1}{2}\left(B-\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}-\frac{1}{2}\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right.}\right.$$

$$\downarrow 221$$

$$2\left(\frac{\left(B-\frac{A\sqrt{c}}{\sqrt{a}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2c^{3/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2c^{3/4}\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)$$

input
```
Int[(A + B*x)/(Sqrt[d + e*x]*(a - c*x^2)),x]
```

output
```
2*(((B - (A*Sqrt[c])/Sqrt[a])*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]
*d - Sqrt[a]*e]])/(2*c^(3/4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) + ((B + (A*Sqrt[
c])/Sqrt[a])*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])
/(2*c^(3/4)*Sqrt[Sqrt[c]*d + Sqrt[a]*e]))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$-\frac{-\frac{\left(Ace-B\sqrt{ac\,e^2}\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}\right)}{\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}-\frac{\left(Ace+B\sqrt{ac\,e^2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}\right)}{\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}}{\sqrt{ac\,e^2}}$	133
derivativedivides	$-2c\left(\frac{\left(-Ace+B\sqrt{ac\,e^2}\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}\right)}{2c\sqrt{ac\,e^2}\,\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}-\frac{\left(Ace+B\sqrt{ac\,e^2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}\right)}{2c\sqrt{ac\,e^2}\,\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}\right)$	148
default	$2c\left(\frac{\left(Ace-B\sqrt{ac\,e^2}\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}\right)}{2c\sqrt{ac\,e^2}\,\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}-\frac{\left(-Ace-B\sqrt{ac\,e^2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}\right)}{2c\sqrt{ac\,e^2}\,\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}\right)$	150

input
```
int((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*c*e^2)^(1/2)*(-(A*c*e-B*(a*c*e^2)^(1/2))/((-c*d+(a*c*e^2)^(1/2))*c)^
(1/2)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))-(A*c*e+B*(a
*c*e^2)^(1/2))/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)/((c
*d+(a*c*e^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2385 vs. 2(108) = 216.

Time = 0.14 (sec) , antiderivative size = 2385, normalized size of antiderivative = 15.69

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="fricas")
```

output
```
1/2*sqrt(-(2*A*B*a*e - (B^2*a + A^2*c)*d + (a*c^2*d^2 - a^2*c*e^2)*sqrt((4
*A^2*B^2*c^2*d^2 - 4*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 + 2*A^2*B^2*a*
c + A^4*c^2)*e^2)/(a*c^5*d^4 - 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4)))/(a*c^2*d
^2 - a^2*c*e^2))*log((2*(A*B^3*a*c - A^3*B*c^2)*d - (B^4*a^2 - A^4*c^2)*e)
*sqrt(e*x + d) + (2*A*B^2*a*c^2*d^2 - (B^3*a^2*c + 3*A^2*B*a*c^2)*d*e + (A
*B^2*a^2*c + A^3*a*c^2)*e^2 + (A*a*c^4*d^3 - B*a^2*c^3*d^2*e - A*a^2*c^3*d
*e^2 + B*a^3*c^2*e^3)*sqrt((4*A^2*B^2*c^2*d^2 - 4*(A*B^3*a*c + A^3*B*c^2)*
d*e + (B^4*a^2 + 2*A^2*B^2*a*c + A^4*c^2)*e^2)/(a*c^5*d^4 - 2*a^2*c^4*d^2*
e^2 + a^3*c^3*e^4)))*sqrt(-(2*A*B*a*e - (B^2*a + A^2*c)*d + (a*c^2*d^2 - a
^2*c*e^2)*sqrt((4*A^2*B^2*c^2*d^2 - 4*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a
^2 + 2*A^2*B^2*a*c + A^4*c^2)*e^2)/(a*c^5*d^4 - 2*a^2*c^4*d^2*e^2 + a^3*c^
3*e^4)))/(a*c^2*d^2 - a^2*c*e^2))) - 1/2*sqrt(-(2*A*B*a*e - (B^2*a + A^2*c
)*d + (a*c^2*d^2 - a^2*c*e^2)*sqrt((4*A^2*B^2*c^2*d^2 - 4*(A*B^3*a*c + A^3
*B*c^2)*d*e + (B^4*a^2 + 2*A^2*B^2*a*c + A^4*c^2)*e^2)/(a*c^5*d^4 - 2*a^2*
c^4*d^2*e^2 + a^3*c^3*e^4)))/(a*c^2*d^2 - a^2*c*e^2))*log((2*(A*B^3*a*c -
A^3*B*c^2)*d - (B^4*a^2 - A^4*c^2)*e)*sqrt(e*x + d) - (2*A*B^2*a*c^2*d^2 -
 (B^3*a^2*c + 3*A^2*B*a*c^2)*d*e + (A*B^2*a^2*c + A^3*a*c^2)*e^2 + (A*a*c^
4*d^3 - B*a^2*c^3*d^2*e - A*a^2*c^3*d*e^2 + B*a^3*c^2*e^3)*sqrt((4*A^2*B^2
*c^2*d^2 - 4*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 + 2*A^2*B^2*a*c + A^4*
c^2)*e^2)/(a*c^5*d^4 - 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4)))*sqrt(-(2*A*B*...
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)}\,dx = -\int \frac{A}{-a\sqrt{d+ex}+cx^2\sqrt{d+ex}}\,dx - \int \frac{Bx}{-a\sqrt{d+ex}+cx^2\sqrt{d+ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)**(1/2)/(-c*x**2+a),x)
```

output
```
-Integral(A/(-a*sqrt(d + e*x) + c*x**2*sqrt(d + e*x)), x) - Integral(B*x/(
-a*sqrt(d + e*x) + c*x**2*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)}\,dx = \int -\frac{Bx+A}{(cx^2-a)\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((B*x + A)/((c*x^2 - a)*sqrt(e*x + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(108) = 216$.

Time = 0.15 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.77

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)}\,dx =$$

$$-\frac{\left(Bacd|c||e|-Aace|c||e|+\sqrt{ac}Acde|c|-\sqrt{ac}Bae^2|c|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{cd+\sqrt{c^2d^2-(cd^2-ae^2)c}}{c}}}\right)}{\left(ac^2d-\sqrt{ac}ace\right)\sqrt{-c^2d-\sqrt{ac}ce}\,|e|}$$
$$-\frac{\left(Bacd|c||e|-Aace|c||e|-\sqrt{ac}Acde|c|+\sqrt{ac}Bae^2|c|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{cd-\sqrt{c^2d^2-(cd^2-ae^2)c}}{c}}}\right)}{\left(ac^2d+\sqrt{ac}ace\right)\sqrt{-c^2d+\sqrt{ac}ce}\,|e|}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="giac")
```

output
```
-(B*a*c*d*abs(c)*abs(e) - A*a*c*e*abs(c)*abs(e) + sqrt(a*c)*A*c*d*e*abs(c)
 - sqrt(a*c)*B*a*e^2*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(c*d + sqrt(c^2*d^
2 - (c*d^2 - a*e^2)*c))/c))/((a*c^2*d - sqrt(a*c)*a*c*e)*sqrt(-c^2*d - sqr
t(a*c)*c*e)*abs(e)) - (B*a*c*d*abs(c)*abs(e) - A*a*c*e*abs(c)*abs(e) - sqr
t(a*c)*A*c*d*e*abs(c) + sqrt(a*c)*B*a*e^2*abs(c))*arctan(sqrt(e*x + d)/sqr
t(-(c*d - sqrt(c^2*d^2 - (c*d^2 - a*e^2)*c))/c))/((a*c^2*d + sqrt(a*c)*a*c
*e)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.93 (sec) , antiderivative size = 2065, normalized size of antiderivative = 13.59

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/((a - c*x^2)*(d + e*x)^(1/2)),x)
```

output
```
atan((a^2*c^5*d^3*((B^2*a*e*(a^3*c^3)^(1/2) + A^2*c*e*(a^3*c^3)^(1/2) + A^
2*a*c^3*d + B^2*a^2*c^2*d - 2*A*B*a^2*c^2*e - 2*A*B*c*d*(a^3*c^3)^(1/2))/(
4*a^2*c^4*d^2 - 4*a^3*c^3*e^2))^(3/2)*(d + e*x)^(1/2)*8i + A^2*a^2*c^3*e^2
*((B^2*a*e*(a^3*c^3)^(1/2) + A^2*c*e*(a^3*c^3)^(1/2) + A^2*a*c^3*d + B^2*a
^2*c^2*d - 2*A*B*a^2*c^2*e - 2*A*B*c*d*(a^3*c^3)^(1/2))/(4*a^2*c^4*d^2 - 4
*a^3*c^3*e^2))^(1/2)*(d + e*x)^(1/2)*2i - B^2*a^2*c^3*d^2*((B^2*a*e*(a^3*c
^3)^(1/2) + A^2*c*e*(a^3*c^3)^(1/2) + A^2*a*c^3*d + B^2*a^2*c^2*d - 2*A*B*
a^2*c^2*e - 2*A*B*c*d*(a^3*c^3)^(1/2))/(4*a^2*c^4*d^2 - 4*a^3*c^3*e^2))^(1
/2)*(d + e*x)^(1/2)*2i + B^2*a^3*c^2*e^2*((B^2*a*e*(a^3*c^3)^(1/2) + A^2*c
*e*(a^3*c^3)^(1/2) + A^2*a*c^3*d + B^2*a^2*c^2*d - 2*A*B*a^2*c^2*e - 2*A*B
*c*d*(a^3*c^3)^(1/2))/(4*a^2*c^4*d^2 - 4*a^3*c^3*e^2))^(1/2)*(d + e*x)^(1/
2)*2i - A^2*a*c^4*d^2*((B^2*a*e*(a^3*c^3)^(1/2) + A^2*c*e*(a^3*c^3)^(1/2)
+ A^2*a*c^3*d + B^2*a^2*c^2*d - 2*A*B*a^2*c^2*e - 2*A*B*c*d*(a^3*c^3)^(1/2
))/(4*a^2*c^4*d^2 - 4*a^3*c^3*e^2))^(1/2)*(d + e*x)^(1/2)*2i - a^3*c^4*d*e
^2*((B^2*a*e*(a^3*c^3)^(1/2) + A^2*c*e*(a^3*c^3)^(1/2) + A^2*a*c^3*d + B^2
*a^2*c^2*d - 2*A*B*a^2*c^2*e - 2*A*B*c*d*(a^3*c^3)^(1/2))/(4*a^2*c^4*d^2 -
 4*a^3*c^3*e^2))^(3/2)*(d + e*x)^(1/2)*8i)/(A^3*c*e^2*(a^3*c^3)^(1/2) - B^
3*a^3*c*e^2 - 2*A^2*B*a*c^3*d^2 - B^3*a*d*e*(a^3*c^3)^(1/2) - A^2*B*a^2*c^
2*e^2 + A*B^2*a*e^2*(a^3*c^3)^(1/2) + 2*A*B^2*c*d^2*(a^3*c^3)^(1/2) + A^3*
a*c^3*d*e + 3*A*B^2*a^2*c^2*d*e - 3*A^2*B*c*d*e*(a^3*c^3)^(1/2)))*((B^2...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 516, normalized size of antiderivative = 3.39

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)}\,dx$$

$$= \frac{-2\sqrt{a}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}}\right)be+2\sqrt{a}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}}\right)cd+2\sqrt{c}\,\sqrt{}}{}$$

input
```
int((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a),x)
```

output
```
( - 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*b*e + 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e -
 c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c*d
+ 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*
sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*e - 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c
*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*b*d -
sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d)
 + sqrt(c)*sqrt(d + e*x))*b*e + sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(
 - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c*d + sqrt(a)*sq
rt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sq
rt(d + e*x))*b*e - sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*
sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c*d - sqrt(c)*sqrt(sqrt(c)*sqrt(
a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*
a*e + sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e
+ c*d) + sqrt(c)*sqrt(d + e*x))*b*d + sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e + c*d
)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*e - sqrt(c)
*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)
*sqrt(d + e*x))*b*d)/(2*c*(a*e**2 - c*d**2))
```

3.127 $\int \frac{A+Bx}{(d+ex)^{3/2}(a-cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{A+Bx}{(d+ex)^{3/2}(a-cx^2)}\,dx = -\frac{2(Bd-Ae)}{(cd^2-ae^2)\sqrt{d+ex}} + \frac{\left(\sqrt{a}B-A\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}} + \frac{\left(\sqrt{a}B+A\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}}$$

output

```
(2*A*e-2*B*d)/(-a*e^2+c*d^2)/(e*x+d)^(1/2)+(a^(1/2)*B-A*c^(1/2))*arctanh(c
^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(1/2)/c^(1/4)/(c^(1/2)
*d-a^(1/2)*e)^(3/2)+(a^(1/2)*B+A*c^(1/2))*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c
^(1/2)*d+a^(1/2)*e)^(1/2))/a^(1/2)/c^(1/4)/(c^(1/2)*d+a^(1/2)*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)}\,dx = \frac{-2Bd+2Ae}{\left(cd^2-ae^2\right)\sqrt{d+ex}} + \frac{\left(\sqrt{a}B+A\sqrt{c}\right)\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{-cd-\sqrt{a}\sqrt{c}e}} + \frac{\left(\sqrt{a}B-A\sqrt{c}\right)\arctan\left(\frac{\sqrt{-cd+\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-\sqrt{a}e}\right)}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{-cd+\sqrt{a}\sqrt{c}e}}$$

input
```
Integrate[(A + B*x)/((d + e*x)^(3/2)*(a - c*x^2)),x]
```

output
```
(-2*B*d + 2*A*e)/((c*d^2 - a*e^2)*Sqrt[d + e*x]) + ((Sqrt[a]*B + A*Sqrt[c]
)*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqr
t[a]*e)])/(Sqrt[a]*(Sqrt[c]*d + Sqrt[a]*e)*Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e
]) + ((Sqrt[a]*B - A*Sqrt[c])*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqr
t[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/(Sqrt[a]*(Sqrt[c]*d - Sqrt[a]*e)*Sqr
t[-(c*d) + Sqrt[a]*Sqrt[c]*e])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {655, 25, 654, 25, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\left(a-cx^2\right)(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{655}$$

$$-\frac{\int -\frac{Acd-aBe+c(Bd-Ae)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{cd^2-ae^2} - \frac{2(Bd-Ae)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{Acd-aBe+c(Bd-Ae)x}{\sqrt{d+ex}(a-cx^2)}dx}{cd^2-ae^2}-\frac{2(Bd-Ae)}{\sqrt{d+ex}\,(cd^2-ae^2)}$$

$$\Big\downarrow \; 654$$

$$\frac{2\int -\frac{2Acde-B(cd^2+ae^2)+c(Bd-Ae)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}-\frac{2(Bd-Ae)}{\sqrt{d+ex}\,(cd^2-ae^2)}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\int \frac{2Acde-B(cd^2+ae^2)+c(Bd-Ae)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}-\frac{2(Bd-Ae)}{\sqrt{d+ex}\,(cd^2-ae^2)}$$

$$\Big\downarrow \; 1480$$

$$\frac{2\left(-\frac{\sqrt{c}(\sqrt{a}B-A\sqrt{c})(\sqrt{a}e+\sqrt{c}d)\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d-\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}}-\frac{\sqrt{c}(\sqrt{a}B+A\sqrt{c})(\sqrt{c}d-\sqrt{a}e)\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d+\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}}\right)}{cd^2-ae^2}-\frac{2(Bd-Ae)}{\sqrt{d+ex}\,(cd^2-ae^2)}$$

$$\Big\downarrow \; 221$$

$$\frac{2\left(\frac{(\sqrt{a}B-A\sqrt{c})(\sqrt{a}e+\sqrt{c}d)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{(\sqrt{a}B+A\sqrt{c})(\sqrt{c}d-\sqrt{a}e)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}\sqrt[4]{c}\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{cd^2-ae^2}-\frac{2(Bd-Ae)}{\sqrt{d+ex}\,(cd^2-ae^2)}$$

input
```
Int[(A + B*x)/((d + e*x)^(3/2)*(a - c*x^2)),x]
```

output
```
(-2*(B*d - A*e))/((c*d^2 - a*e^2)*Sqrt[d + e*x]) + (2*(((Sqrt[a]*B - A*Sqr
t[c])*(Sqrt[c]*d + Sqrt[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]
*d - Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) + ((Sqrt
[a]*B + A*Sqrt[c])*(Sqrt[c]*d - Sqrt[a]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])
/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[a]
*e])))/(c*d^2 - a*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 655
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Simp[1/(c*d^2 + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g
- c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
&& FractionQ[m] && LtQ[m, -1]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.16

method	result
derivativedivides	$-\frac{2c\left(-\frac{\left(-Acde+Ba\,e^{2}+A\sqrt{ac\,e^{2}}\,e-B\sqrt{ac\,e^{2}}\,d\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2\sqrt{ac\,e^{2}}\,\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}+\frac{\left(Acde-Ba\,e^{2}+A\sqrt{ac\,e^{2}}\,e-B\sqrt{ac\,e^{2}}\,d\right)\arctan\left(\cdots\right.}{2\sqrt{ac\,e^{2}}\,\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)\cdots}}\cdots\right.}{a\,e^{2}-c\,d^{2}}\cdots$
default	$\frac{2c\left(-\frac{\left(Acde-Ba\,e^{2}-A\sqrt{ac\,e^{2}}\,e+B\sqrt{ac\,e^{2}}\,d\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2\sqrt{ac\,e^{2}}\,\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}+\frac{\left(-Acde+Ba\,e^{2}-A\sqrt{ac\,e^{2}}\,e+B\sqrt{ac\,e^{2}}\,d\right)\arctan\left(\cdots\right.}{2\sqrt{ac\,e^{2}}\,\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\cdots\right.}{a\,e^{2}-c\,d^{2}}\cdots$
pseudoelliptic	$-\frac{2\left(\frac{c\left(\left(Ae-Bd\right)\sqrt{ac\,e^{2}}+e\left(Acd-Bae\right)\right)\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}\,\sqrt{ex+d}\,\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2}+\left(\frac{c\left(\left(-Ae+Bd\right)\sqrt{ac\,e^{2}}+e\left(Acd-\cdots\right.\right.}{\cdots}\right.\right.}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}\,\sqrt{ac\,e^{2}}\,\sqrt{\left(cd+\sqrt{ac\,e^{2}}\cdots\right.}}\cdots$

input
```
int((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/(a*e^2-c*d^2)*c*(-1/2*(-A*c*d*e+B*a*e^2+A*(a*c*e^2)^(1/2)*e-B*(a*c*e^2)
^(1/2)*d)/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh(c*(e*x+d
)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+1/2*(A*c*d*e-B*a*e^2+A*(a*c*e^2)^
(1/2)*e-B*(a*c*e^2)^(1/2)*d)/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1
/2)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)))+2*(-A*e+B*d)
/(a*e^2-c*d^2)/(e*x+d)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6448 vs. $2(147) = 294$.

Time = 3.45 (sec) , antiderivative size = 6448, normalized size of antiderivative = 32.73

$$\int \frac{A+Bx}{(d+ex)^{3/2}\,(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}(a-cx^2)}\,dx = -\int \frac{A}{-ad\sqrt{d+ex}-aex\sqrt{d+ex}+cdx^2\sqrt{d+ex}+cex^3\sqrt{d+ex}}\,dx - \int \frac{Bx}{-ad\sqrt{d+ex}-aex\sqrt{d+ex}+cdx^2\sqrt{d+ex}+cex^3\sqrt{d+ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)**(3/2)/(-c*x**2+a),x)
```

output
```
-Integral(A/(-a*d*sqrt(d + e*x) - a*e*x*sqrt(d + e*x) + c*d*x**2*sqrt(d +
e*x) + c*e*x**3*sqrt(d + e*x)), x) - Integral(B*x/(-a*d*sqrt(d + e*x) - a*
e*x*sqrt(d + e*x) + c*d*x**2*sqrt(d + e*x) + c*e*x**3*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}(a-cx^2)}\,dx = \int -\frac{Bx+A}{(cx^2-a)(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((B*x + A)/((c*x^2 - a)*(e*x + d)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 941 vs. $2(147) = 294$.

Time = 0.24 (sec) , antiderivative size = 941, normalized size of antiderivative = 4.78

$$\int \frac{A+Bx}{(d+ex)^{3/2}(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a),x, algorithm="giac")
```

output
```
-2*(B*d - A*e)/((c*d^2 - a*e^2)*sqrt(e*x + d)) + ((c*d^2*e - a*e^3)^2*sqrt
(a*c)*B*a*d*abs(c) - (c*d^2*e - a*e^3)^2*sqrt(a*c)*A*a*e*abs(c) + 2*(a*c^2
*d^3*e - a^2*c*d*e^3)*A*abs(c*d^2*e - a*e^3)*abs(c) - (a*c^2*d^4 - a^3*e^4
)*B*abs(c*d^2*e - a*e^3)*abs(c) - (sqrt(a*c)*c^3*d^6*e - 2*sqrt(a*c)*a*c^2
*d^4*e^3 + sqrt(a*c)*a^2*c*d^2*e^5)*A*abs(c) + (sqrt(a*c)*a*c^2*d^5*e^2 -
2*sqrt(a*c)*a^2*c*d^3*e^4 + sqrt(a*c)*a^3*d*e^6)*B*abs(c))*arctan(sqrt(e*x
 + d)/sqrt(-(c^2*d^3 - a*c*d*e^2 + sqrt((c^2*d^3 - a*c*d*e^2)^2 - (c^2*d^4
 - 2*a*c*d^2*e^2 + a^2*e^4)*(c^2*d^2 - a*c*e^2)))/(c^2*d^2 - a*c*e^2)))/((
a*c^3*d^5 - 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4 - sqrt(a*c)*a*c^2*d^4*e + 2*sq
rt(a*c)*a^2*c*d^2*e^3 - sqrt(a*c)*a^3*e^5)*sqrt(-c^2*d - sqrt(a*c)*c*e)*ab
s(c*d^2*e - a*e^3)) - ((c*d^2*e - a*e^3)^2*sqrt(a*c)*B*a*d*abs(c) - (c*d^2
*e - a*e^3)^2*sqrt(a*c)*A*a*e*abs(c) - 2*(a*c^2*d^3*e - a^2*c*d*e^3)*A*abs
(c*d^2*e - a*e^3)*abs(c) + (a*c^2*d^4 - a^3*e^4)*B*abs(c*d^2*e - a*e^3)*ab
s(c) - (sqrt(a*c)*c^3*d^6*e - 2*sqrt(a*c)*a*c^2*d^4*e^3 + sqrt(a*c)*a^2*c*
d^2*e^5)*A*abs(c) + (sqrt(a*c)*a*c^2*d^5*e^2 - 2*sqrt(a*c)*a^2*c*d^3*e^4 +
 sqrt(a*c)*a^3*d*e^6)*B*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(c^2*d^3 - a*c*
d*e^2 - sqrt((c^2*d^3 - a*c*d*e^2)^2 - (c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)
*(c^2*d^2 - a*c*e^2)))/(c^2*d^2 - a*c*e^2)))/((a*c^3*d^5 - 2*a^2*c^2*d^3*e
^2 + a^3*c*d*e^4 + sqrt(a*c)*a*c^2*d^4*e - 2*sqrt(a*c)*a^2*c*d^2*e^3 + sqr
t(a*c)*a^3*e^5)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(c*d^2*e - a*e^3))
```

Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 10288, normalized size of antiderivative = 52.22

$$\int \frac{A+Bx}{(d+ex)^{3/2}(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/((a - c*x^2)*(d + e*x)^(3/2)),x)
```

output
```
atan((((-(B^2*a^2*c^2*d^3 + B^2*a^2*e^3*(a^3*c)^(1/2) + A^2*a*c^3*d^3 - 2*
A*B*c^2*d^3*(a^3*c)^(1/2) + 3*B^2*a^3*c*d*e^2 + A^2*a*c*e^3*(a^3*c)^(1/2)
+ 3*A^2*a^2*c^2*d*e^2 - 2*A*B*a^3*c*e^3 + 3*A^2*c^2*d^2*e*(a^3*c)^(1/2) -
6*A*B*a^2*c^2*d^2*e + 3*B^2*a*c*d^2*e*(a^3*c)^(1/2) - 6*A*B*a*c*d*e^2*(a^3
*c)^(1/2))/(4*(a^5*c*e^6 - a^2*c^4*d^6 + 3*a^3*c^3*d^4*e^2 - 3*a^4*c^2*d^2
*e^4)))^(1/2)*((d + e*x)^(1/2)*(-(B^2*a^2*c^2*d^3 + B^2*a^2*e^3*(a^3*c)^(1
/2) + A^2*a*c^3*d^3 - 2*A*B*c^2*d^3*(a^3*c)^(1/2) + 3*B^2*a^3*c*d*e^2 + A^
2*a*c*e^3*(a^3*c)^(1/2) + 3*A^2*a^2*c^2*d*e^2 - 2*A*B*a^3*c*e^3 + 3*A^2*c^
2*d^2*e*(a^3*c)^(1/2) - 6*A*B*a^2*c^2*d^2*e + 3*B^2*a*c*d^2*e*(a^3*c)^(1/2
) - 6*A*B*a*c*d*e^2*(a^3*c)^(1/2))/(4*(a^5*c*e^6 - a^2*c^4*d^6 + 3*a^3*c^3
*d^4*e^2 - 3*a^4*c^2*d^2*e^4)))^(1/2)*(64*a*c^9*d^11*e^2 - 64*a^6*c^4*d*e^
12 - 320*a^2*c^8*d^9*e^4 + 640*a^3*c^7*d^7*e^6 - 640*a^4*c^6*d^5*e^8 + 320
*a^5*c^5*d^3*e^10) - 32*B*a^6*c^3*e^12 + 64*A*a*c^8*d^9*e^3 + 64*A*a^5*c^4
*d*e^11 - 32*B*a*c^8*d^10*e^2 - 256*A*a^2*c^7*d^7*e^5 + 384*A*a^3*c^6*d^5*
e^7 - 256*A*a^4*c^5*d^3*e^9 + 96*B*a^2*c^7*d^8*e^4 - 64*B*a^3*c^6*d^6*e^6
- 64*B*a^4*c^5*d^4*e^8 + 96*B*a^5*c^4*d^2*e^10) + (d + e*x)^(1/2)*(16*A^2*
a^4*c^4*e^10 + 16*B^2*a^5*c^3*e^10 - 16*A^2*c^8*d^8*e^2 - 32*A^2*a^3*c^5*d
^2*e^8 + 32*B^2*a^2*c^6*d^6*e^4 - 32*B^2*a^4*c^4*d^2*e^8 + 32*A^2*a*c^7*d^
6*e^4 - 16*B^2*a*c^7*d^8*e^2 + 64*A*B*a*c^7*d^7*e^3 - 64*A*B*a^4*c^4*d*e^9
 - 192*A*B*a^2*c^6*d^5*e^5 + 192*A*B*a^3*c^5*d^3*e^7))*(-(B^2*a^2*c^2*d...
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 983, normalized size of antiderivative = 4.99

$$\int \frac{A+Bx}{(d+ex)^{3/2}(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a),x)
```

output
```
( - 2*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e
*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c*e**2 + 4*sqrt(a)*sqrt(
d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqr
t(sqrt(c)*sqrt(a)*e - c*d)))*b*c*d*e - 2*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c
)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e
- c*d)))*c**2*d**2 + 2*sqrt(c)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)
*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*b*e**2
- 4*sqrt(c)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x
)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c*d*e + 2*sqrt(c)*sqrt(d +
 e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(s
qrt(c)*sqrt(a)*e - c*d)))*b*c*d**2 - sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sq
rt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x
))*a*c*e**2 + 2*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( -
 sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*b*c*d*e - sqrt(a)*
sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e
+ c*d) + sqrt(c)*sqrt(d + e*x))*c**2*d**2 + sqrt(a)*sqrt(d + e*x)*sqrt(sqr
t(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d +
 e*x))*a*c*e**2 - 2*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*lo
g(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*b*c*d*e + sqrt(a)
*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e...
```

3.128 $\int \frac{A+Bx}{(d+ex)^{5/2}(a-cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 243

$$\int \frac{A+Bx}{(d+ex)^{5/2}(a-cx^2)}\,dx = -\frac{2(Bd-Ae)}{3(cd^2-ae^2)(d+ex)^{3/2}} - \frac{2(Bcd^2-2Acde+aBe^2)}{(cd^2-ae^2)^2\sqrt{d+ex}} + \frac{(\sqrt{a}B-A\sqrt{c})\sqrt[4]{c}\,\mathrm{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}} + \frac{(\sqrt{a}B+A\sqrt{c})\sqrt[4]{c}\,\mathrm{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}\left(\sqrt{c}d+\sqrt{a}e\right)^{5/2}}$$

output
```
1/3*(2*A*e-2*B*d)/(-a*e^2+c*d^2)/(e*x+d)^(3/2)-2*(-2*A*c*d*e+B*a*e^2+B*c*d
^2)/(-a*e^2+c*d^2)^2/(e*x+d)^(1/2)+(a^(1/2)*B-A*c^(1/2))*c^(1/4)*arctanh(c
^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(1/2)/(c^(1/2)*d-a^(1/
2)*e)^(5/2)+(a^(1/2)*B+A*c^(1/2))*c^(1/4)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c
^(1/2)*d+a^(1/2)*e)^(1/2))/a^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(5/2)
```

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx}{(d+ex)^{5/2}(a-cx^2)}\,dx = -\frac{2(aAe^3+aBe^2(2d+3ex)+Bcd^2(4d+3ex)-Acde(7d+6ex))}{3(cd^2-ae^2)^2(d+ex)^{3/2}} - \frac{(\sqrt{a}B+A\sqrt{c})\sqrt{-cd-\sqrt{a}\sqrt{c}e}\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}(\sqrt{c}d+\sqrt{a}e)^3} + \frac{(\sqrt{a}B\sqrt{c}-Ac)\arctan\left(\frac{\sqrt{-cd+\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-\sqrt{a}e}\right)}{\sqrt{a}(\sqrt{c}d-\sqrt{a}e)^2\sqrt{-cd+\sqrt{a}\sqrt{c}e}}$$

input
```
Integrate[(A + B*x)/((d + e*x)^(5/2)*(a - c*x^2)),x]
```

output
```
(-2*(a*A*e^3 + a*B*e^2*(2*d + 3*e*x) + B*c*d^2*(4*d + 3*e*x) - A*c*d*e*(7*
d + 6*e*x)))/(3*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)) - ((Sqrt[a]*B + A*Sqrt[
c])*Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]
*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)])/(Sqrt[a]*(Sqrt[c]*d + Sqrt[a]
*e)^3) + ((Sqrt[a]*B*Sqrt[c] - A*c)*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*
e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/(Sqrt[a]*(Sqrt[c]*d - Sqrt[a]*
e)^2*Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {655, 25, 655, 25, 27, 654, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a-cx^2)(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{655}$$

$$-\frac{\int -\frac{Acd-aBe+c(Bd-Ae)x}{(d+ex)^{3/2}(a-cx^2)}dx}{cd^2-ae^2}-\frac{2(Bd-Ae)}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{Acd-aBe+c(Bd-Ae)x}{(d+ex)^{3/2}(a-cx^2)}dx}{cd^2-ae^2}-\frac{2(Bd-Ae)}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

$\Big\downarrow$ 655

$$-\frac{-\frac{\int -\frac{c\left(Acd^2-2aBed+aAe^2+\left(Bcd^2-2Aced+aBe^2\right)x\right)}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{cd^2-ae^2}-\frac{2\left(aBe^2-2Acde+Bcd^2\right)}{\sqrt{d+ex}(cd^2-ae^2)}}{cd^2-ae^2}-\frac{2(Bd-Ae)}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{c\left(Acd^2-2aBed+aAe^2+\left(Bcd^2-2Aced+aBe^2\right)x\right)}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{cd^2-ae^2}-\frac{2\left(aBe^2-2Acde+Bcd^2\right)}{\sqrt{d+ex}(cd^2-ae^2)}}{cd^2-ae^2}-\frac{2(Bd-Ae)}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

$\Big\downarrow$ 27

$$\frac{\frac{c\int \frac{Acd^2-2aBed+aAe^2+\left(Bcd^2-2Aced+aBe^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{cd^2-ae^2}-\frac{2\left(aBe^2-2Acde+Bcd^2\right)}{\sqrt{d+ex}(cd^2-ae^2)}}{cd^2-ae^2}-\frac{2(Bd-Ae)}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

$\Big\downarrow$ 654

$$\frac{\frac{2c\int \frac{Bcd^3-3Aced^2+3aBe^2d-aAe^3-\left(Bcd^2-2Aced+aBe^2\right)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}-\frac{2\left(aBe^2-2Acde+Bcd^2\right)}{\sqrt{d+ex}(cd^2-ae^2)}}{cd^2-ae^2}-\frac{2(Bd-Ae)}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

$\Big\downarrow$ 1480

$$\frac{\frac{2c\left(-\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\left(\sqrt{c}d-\sqrt{a}e\right)^2\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}}-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{a}e+\sqrt{c}d\right)^2\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}}\right)}{cd^2-ae^2}-\frac{2\left(aBe^2-2Acde\right.}{\sqrt{d+ex}(cd^2-}}{cd^2-ae^2}$$

$$\frac{2(Bd-Ae)}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

$\Big\downarrow$ 221

$$\frac{\frac{2c\left(\frac{(\sqrt{a}B+A\sqrt{c})(\sqrt{c}d-\sqrt{a}e)^2\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}c^{3/4}\sqrt{\sqrt{a}e+\sqrt{c}d}}+\frac{(\sqrt{a}B-A\sqrt{c})(\sqrt{a}e+\sqrt{c}d)^2\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}c^{3/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{cd^2-ae^2}-\frac{2(aBe^2-2Acde+Bcd^2)}{\sqrt{d+ex}(cd^2-ae^2)}}{cd^2-ae^2}-\frac{2(Bd-Ae)}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

input
```
Int[(A + B*x)/((d + e*x)^(5/2)*(a - c*x^2)),x]
```

output
```
(-2*(B*d - A*e))/(3*(c*d^2 - a*e^2)*(d + e*x)^(3/2)) + ((-2*(B*c*d^2 - 2*A
*c*d*e + a*B*e^2))/((c*d^2 - a*e^2)*Sqrt[d + e*x]) + (2*c*(((Sqrt[a]*B - A
*Sqrt[c])*(Sqrt[c]*d + Sqrt[a]*e)^2*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[S
qrt[c]*d - Sqrt[a]*e]])/(2*Sqrt[a]*c^(3/4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) +
((Sqrt[a]*B + A*Sqrt[c])*(Sqrt[c]*d - Sqrt[a]*e)^2*ArcTanh[(c^(1/4)*Sqrt[d
 + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(3/4)*Sqrt[Sqrt[c]*d +
 Sqrt[a]*e])))/(c*d^2 - a*e^2))/(c*d^2 - a*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 655
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Simp[1/(c*d^2 + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g
- c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
&& FractionQ[m] && LtQ[m, -1]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.42

method	result
default	$-\frac{2(Ae-Bd)}{3(a e^2-c d^2)(ex+d)^{\frac{3}{2}}}-\frac{2(-2Acde+Ba e^2+Bc d^2)}{(a e^2-c d^2)^2\sqrt{ex+d}}-\frac{2c^2\left(\frac{(-Aac e^3-A c^2 d^2 e+2Bad e^2 c-2A\sqrt{ac e^2}\, cde+B\sqrt{ac e^2}}{2c\sqrt{ac e^2}\sqrt{(-cd}}\right.}{}$
derivativedivides	$\frac{-\frac{2Ae}{3}+\frac{2Bd}{3}}{(a e^2-c d^2)(ex+d)^{\frac{3}{2}}}+\frac{4Acde-2Ba e^2-2Bc d^2}{(a e^2-c d^2)^2\sqrt{ex+d}}-\frac{2c^2\left(\frac{(-Aac e^3-A c^2 d^2 e+2Bad e^2 c-2A\sqrt{ac e^2}\, cde+B\sqrt{ac e^2}\, a e^2+B}{2c\sqrt{ac e^2}\sqrt{(-cd+\sqrt{ac e}}}\right.}{}$
pseudoelliptic	$\frac{(ex+d)^{\frac{3}{2}} c\left(\left(\left((2Ade-B d^2)c-Ba e^2\right)\sqrt{ac e^2}+c\left(Ac d^2+ae(Ae-2Bd)\right)e\right)\sqrt{\left(cd+\sqrt{ac e^2}\right)c}\, \arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac e^2}\right)c}}\right)}{}$

input
```
int((B*x+A)/(e*x+d)^(5/2)/(-c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(A*e-B*d)/(a*e^2-c*d^2)/(e*x+d)^(3/2)-2/(a*e^2-c*d^2)^2*(-2*A*c*d*e+B
*a*e^2+B*c*d^2)/(e*x+d)^(1/2)-2*c^2/(a*e^2-c*d^2)^2*(1/2*(-A*a*c*e^3-A*c^2
*d^2*e+2*B*a*d*e^2*c-2*A*(a*c*e^2)^(1/2)*c*d*e+B*(a*c*e^2)^(1/2)*a*e^2+B*(
a*c*e^2)^(1/2)*c*d^2)/c/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*a
rctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))-1/2*(A*a*c*e^3+A*c
^2*d^2*e-2*B*a*d*e^2*c-2*A*(a*c*e^2)^(1/2)*c*d*e+B*(a*c*e^2)^(1/2)*a*e^2+B
*(a*c*e^2)^(1/2)*c*d^2)/c/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*
arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11231 vs. $2(189) = 378$.

Time = 24.56 (sec) , antiderivative size = 11231, normalized size of antiderivative = 46.22

$$\int \frac{A+Bx}{(d+ex)^{5/2}(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+Bx}{(d+ex)^{5/2}(a-cx^2)}\,dx =$$
$$-\int \frac{A}{-ad^2\sqrt{d+ex}-2adex\sqrt{d+ex}-ae^2x^2\sqrt{d+ex}+cd^2x^2\sqrt{d+ex}+2cdex^3\sqrt{d+ex}+ce^2x^4\sqrt{d+ex}}$$
$$-\int \frac{Bx}{-ad^2\sqrt{d+ex}-2adex\sqrt{d+ex}-ae^2x^2\sqrt{d+ex}+cd^2x^2\sqrt{d+ex}+2cdex^3\sqrt{d+ex}+ce^2x^4\sqrt{d+ex}}$$

input
```
integrate((B*x+A)/(e*x+d)**(5/2)/(-c*x**2+a),x)
```

output
```
-Integral(A/(-a*d**2*sqrt(d + e*x) - 2*a*d*e*x*sqrt(d + e*x) - a*e**2*x**2
*sqrt(d + e*x) + c*d**2*x**2*sqrt(d + e*x) + 2*c*d*e*x**3*sqrt(d + e*x) +
c*e**2*x**4*sqrt(d + e*x)), x) - Integral(B*x/(-a*d**2*sqrt(d + e*x) - 2*a
*d*e*x*sqrt(d + e*x) - a*e**2*x**2*sqrt(d + e*x) + c*d**2*x**2*sqrt(d + e*
x) + 2*c*d*e*x**3*sqrt(d + e*x) + c*e**2*x**4*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(d+ex)^{5/2}\left(a-cx^2\right)}\,dx=\int -\frac{Bx+A}{(cx^2-a)(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((B*x + A)/((c*x^2 - a)*(e*x + d)^(5/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1751 vs. $2(189) = 378$.

Time = 0.36 (sec) , antiderivative size = 1751, normalized size of antiderivative = 7.21

$$\int \frac{A+Bx}{(d+ex)^{5/2}\left(a-cx^2\right)}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^(5/2)/(-c*x^2+a),x, algorithm="giac")
```

output
```
-(2*(c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)^2*sqrt(a*c)*A*a*c*d*e*abs(c) - (
c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)^2*(sqrt(a*c)*a*c*d^2 + sqrt(a*c)*a^2*
e^2)*B*abs(c) - (3*a*c^4*d^6*e - 5*a^2*c^3*d^4*e^3 + a^3*c^2*d^2*e^5 + a^4
*c*e^7)*A*abs(c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)*abs(c) + (a*c^4*d^7 + a
^2*c^3*d^5*e^2 - 5*a^3*c^2*d^3*e^4 + 3*a^4*c*d*e^6)*B*abs(c^2*d^4*e - 2*a*
c*d^2*e^3 + a^2*e^5)*abs(c) + (sqrt(a*c)*c^6*d^11*e - 3*sqrt(a*c)*a*c^5*d^
9*e^3 + 2*sqrt(a*c)*a^2*c^4*d^7*e^5 + 2*sqrt(a*c)*a^3*c^3*d^5*e^7 - 3*sqrt
(a*c)*a^4*c^2*d^3*e^9 + sqrt(a*c)*a^5*c*d*e^11)*A*abs(c) - 2*(sqrt(a*c)*a*
c^5*d^10*e^2 - 4*sqrt(a*c)*a^2*c^4*d^8*e^4 + 6*sqrt(a*c)*a^3*c^3*d^6*e^6 -
 4*sqrt(a*c)*a^4*c^2*d^4*e^8 + sqrt(a*c)*a^5*c*d^2*e^10)*B*abs(c))*arctan(
sqrt(e*x + d)/sqrt(-(c^3*d^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4 + sqrt((c^3*d
^5 - 2*a*c^2*d^3*e^2 + a^2*c*d*e^4)^2 - (c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2
*c*d^2*e^4 - a^3*e^6)*(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)))/(c^3*d^4 -
 2*a*c^2*d^2*e^2 + a^2*c*e^4)))/((a*c^5*d^9 - 4*a^2*c^4*d^7*e^2 + 6*a^3*c^
3*d^5*e^4 - 4*a^4*c^2*d^3*e^6 + a^5*c*d*e^8 - sqrt(a*c)*a*c^4*d^8*e + 4*sq
rt(a*c)*a^2*c^3*d^6*e^3 - 6*sqrt(a*c)*a^3*c^2*d^4*e^5 + 4*sqrt(a*c)*a^4*c*
d^2*e^7 - sqrt(a*c)*a^5*e^9)*sqrt(-c^2*d - sqrt(a*c)*c*e)*abs(c^2*d^4*e -
2*a*c*d^2*e^3 + a^2*e^5)) + (2*(c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)^2*sqr
t(a*c)*A*a*c*d*e*abs(c) - (c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)^2*(sqrt(a*
c)*a*c*d^2 + sqrt(a*c)*a^2*e^2)*B*abs(c) + (3*a*c^4*d^6*e - 5*a^2*c^3*d...
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 17610, normalized size of antiderivative = 72.47

$$\int \frac{A+Bx}{(d+ex)^{5/2}\,(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/((a - c*x^2)*(d + e*x)^(5/2)),x)
```

output
```
- atan((((d + e*x)^(1/2)*(16*A^2*a^8*c^5*e^18 + 16*B^2*a^9*c^4*e^18 + 16*A
^2*c^13*d^16*e^2 - 320*A^2*a^2*c^11*d^12*e^6 + 1024*A^2*a^3*c^10*d^10*e^8
- 1440*A^2*a^4*c^9*d^8*e^10 + 1024*A^2*a^5*c^8*d^6*e^12 - 320*A^2*a^6*c^7*
d^4*e^14 - 320*B^2*a^3*c^10*d^12*e^6 + 1024*B^2*a^4*c^9*d^10*e^8 - 1440*B^
2*a^5*c^8*d^8*e^10 + 1024*B^2*a^6*c^7*d^6*e^12 - 320*B^2*a^7*c^6*d^4*e^14
+ 16*B^2*a*c^12*d^16*e^2 - 128*A*B*a*c^12*d^15*e^3 - 128*A*B*a^8*c^5*d*e^1
7 + 640*A*B*a^2*c^11*d^13*e^5 - 1152*A*B*a^3*c^10*d^11*e^7 + 640*A*B*a^4*c
^9*d^9*e^9 + 640*A*B*a^5*c^8*d^7*e^11 - 1152*A*B*a^6*c^7*d^5*e^13 + 640*A*
B*a^7*c^6*d^3*e^15) - (-(B^2*a^2*c^3*d^5 + B^2*a^3*e^5*(a^3*c)^(1/2) + A^2
*a*c^4*d^5 + 10*A^2*a^2*c^3*d^3*e^2 + 10*B^2*a^3*c^2*d^3*e^2 - 2*A*B*c^3*d
^5*(a^3*c)^(1/2) + 5*B^2*a^4*c*d*e^4 + 5*A^2*a^3*c^2*d*e^4 + A^2*a^2*c*e^5
*(a^3*c)^(1/2) - 2*A*B*a^4*c*e^5 + 5*A^2*c^3*d^4*e*(a^3*c)^(1/2) + 5*B^2*a
*c^2*d^4*e*(a^3*c)^(1/2) + 10*A^2*a*c^2*d^2*e^3*(a^3*c)^(1/2) - 10*A*B*a^2
*c^3*d^4*e + 10*B^2*a^2*c*d^2*e^3*(a^3*c)^(1/2) - 20*A*B*a^3*c^2*d^2*e^3 -
 10*A*B*a^2*c*d*e^4*(a^3*c)^(1/2) - 20*A*B*a*c^2*d^3*e^2*(a^3*c)^(1/2))/(4
*(a^7*e^10 - a^2*c^5*d^10 - 5*a^6*c*d^2*e^8 + 5*a^3*c^4*d^8*e^2 - 10*a^4*c
^3*d^6*e^4 + 10*a^5*c^2*d^4*e^6)))^(1/2)*((d + e*x)^(1/2)*(-(B^2*a^2*c^3*d
^5 + B^2*a^3*e^5*(a^3*c)^(1/2) + A^2*a*c^4*d^5 + 10*A^2*a^2*c^3*d^3*e^2 +
10*B^2*a^3*c^2*d^3*e^2 - 2*A*B*c^3*d^5*(a^3*c)^(1/2) + 5*B^2*a^4*c*d*e^4 +
 5*A^2*a^3*c^2*d*e^4 + A^2*a^2*c*e^5*(a^3*c)^(1/2) - 2*A*B*a^4*c*e^5 + ...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 2731, normalized size of antiderivative = 11.24

$$\int \frac{A+Bx}{(d+ex)^{5/2}\,(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/(e*x+d)^(5/2)/(-c*x^2+a),x)
```

output
```
( - 6*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e
*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*b*d*e**3 - 6*sqrt(a)*sqr
t(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*s
qrt(sqrt(c)*sqrt(a)*e - c*d)))*a*b*e**4*x + 18*sqrt(a)*sqrt(d + e*x)*sqrt(
sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt
(a)*e - c*d)))*a*c*d**2*e**2 + 18*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(
a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d))
)*a*c*d*e**3*x - 18*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*at
an((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*b*c*d**3*e -
 18*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x
)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*b*c*d**2*e**2*x + 6*sqrt(a)*
sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c**2*d**4 + 6*sqrt(a)*sqrt(d + e*x)*sqrt
(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqr
t(a)*e - c*d)))*c**2*d**3*e*x + 6*sqrt(c)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(
a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d))
)*a**2*d*e**3 + 6*sqrt(c)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan
((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*e**4*x -
18*sqrt(c)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)
*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*b*d**2*e**2 - 18*sqrt(c)...
```

3.129 $\int \frac{(A+Bx)(d+ex)^{5/2}}{\left(a-cx^2\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 280

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^2}\,dx = \frac{2Be^2\sqrt{d+ex}}{c^2} + \frac{\sqrt{d+ex}\left(a(Bcd^2+2Acde+aBe^2)+c(Acd^2+2aBde+aAe^2)\,x\right)}{2ac^2\,(a-cx^2)} - \frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\left(2Acd-5aBe+3\sqrt{a}A\sqrt{c}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}c^{9/4}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}\left(2Acd-5aBe-3\sqrt{a}A\sqrt{c}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{4a^{3/2}c^{9/4}}$$

output
```
2*B*e^2*(e*x+d)^(1/2)/c^2+1/2*(e*x+d)^(1/2)*(a*(2*A*c*d*e+B*a*e^2+B*c*d^2)
+c*(A*a*e^2+A*c*d^2+2*B*a*d*e)*x)/a/c^2/(-c*x^2+a)-1/4*(c^(1/2)*d-a^(1/2)*
e)^(3/2)*(2*A*c*d-5*B*a*e+3*a^(1/2)*A*c^(1/2)*e)*arctanh(c^(1/4)*(e*x+d)^(
1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(3/2)/c^(9/4)+1/4*(c^(1/2)*d+a^(1/2)*e
)^(3/2)*(2*A*c*d-5*B*a*e-3*a^(1/2)*A*c^(1/2)*e)*arctanh(c^(1/4)*(e*x+d)^(1
/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(3/2)/c^(9/4)
```

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^2}\,dx = \frac{-\frac{2\sqrt{a}\sqrt{d+ex}(5a^2Be^2+Ac^2d^2x+ac(Ae(2d+ex)+B(d^2+2dex-4e^2x^2)))}{-a+cx^2}+\frac{(\sqrt{c}d+\sqrt{a}e)^2(2Acd-5aB}{}}{}$$

input
```
Integrate[((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2)^2,x]
```

output
```
((-2*Sqrt[a]*Sqrt[d + e*x]*(5*a^2*B*e^2 + A*c^2*d^2*x + a*c*(A*e*(2*d + e*
x) + B*(d^2 + 2*d*e*x - 4*e^2*x^2))))/(-a + c*x^2) + ((Sqrt[c]*d + Sqrt[a]
*e)^2*(2*A*c*d - 5*a*B*e - 3*Sqrt[a]*A*Sqrt[c]*e)*ArcTan[(Sqrt[-(c*d) - Sq
rt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)])/Sqrt[-(c*d) - Sq
rt[a]*Sqrt[c]*e] - ((Sqrt[c]*d - Sqrt[a]*e)^2*(2*A*c*d - 5*a*B*e + 3*Sqrt[
a]*A*Sqrt[c]*e)*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(S
qrt[c]*d - Sqrt[a]*e)])/Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e])/(4*a^(3/2)*c^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {684, 27, 653, 25, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^2}dx$$

$$\Big\downarrow \text{684}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{2ac(a-cx^2)}-\frac{\int -\frac{\sqrt{d+ex}(2Acd^2-ae(5Bd+3Ae)-e(Acd+5aBe)x)}{2(a-cx^2)}dx}{2ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{d+ex}(2Acd^2-ae(5Bd+3Ae)-e(Acd+5aBe)x)}{a-cx^2}dx}{4ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{2ac(a-cx^2)}$$

$$\Big\downarrow \; 653$$

$$\frac{\frac{2e\sqrt{d+ex}(5aBe+Acd)}{c}-\frac{\int -\frac{2Acd\left(cd^2-2ae^2\right)-5aBe\left(cd^2+ae^2\right)+ce\left(Acd^2-10aBed-3aAe^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{c}}{4ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2Acd\left(cd^2-2ae^2\right)-5aBe\left(cd^2+ae^2\right)+ce\left(Acd^2-10aBed-3aAe^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{c}+\frac{2e\sqrt{d+ex}(5aBe+Acd)}{c}}{4ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \; 654$$

$$\frac{\frac{2\int -\frac{e\left((Acd+5aBe)\left(cd^2-ae^2\right)+c\left(Acd^2-10aBed-3aAe^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}+\frac{2e\sqrt{d+ex}(5aBe+Acd)}{c}}{4ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2e\sqrt{d+ex}(5aBe+Acd)}{c}-\frac{2\int \frac{e\left((Acd+5aBe)\left(cd^2-ae^2\right)+c\left(Acd^2-10aBed-3aAe^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}}{4ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2e\sqrt{d+ex}(5aBe+Acd)}{c}-\frac{2e\int \frac{(Acd+5aBe)\left(cd^2-ae^2\right)+c\left(Acd^2-10aBed-3aAe^2\right)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}}{4ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{2e\sqrt{d+ex}(5aBe+Acd)}{c}-\frac{2e\left(\frac{\sqrt{c}\left(\sqrt{a}e+\sqrt{c}d\right)^2\left(-3\sqrt{a}A\sqrt{c}e-5aBe+2Acd\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)^2\left(3\sqrt{a}A\sqrt{c}e-5aBe+2Acd\right)}{2\sqrt{a}e}\right.}{c}}{4ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{2ac\left(a-cx^2\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2e\sqrt{d+ex}(5aBe+Acd)}{c}-\frac{2e\left(\frac{(\sqrt{c}d-\sqrt{a}e)^{3/2}(3\sqrt{a}A\sqrt{c}e-5aBe+2Acd)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}\sqrt[4]{c}e}-\frac{(\sqrt{a}e+\sqrt{c}d)^{3/2}(-3\sqrt{a}A\sqrt{c}e-5aBe+2Acd)\text{arctan}}{2\sqrt{a}\sqrt[4]{c}e}\right.}{c}}{4ac}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{2ac\,(a-cx^2)}$$

input
```
Int[((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2)^2,x]
```

output
```
((d + e*x)^(3/2)*(a*(B*d + A*e) + (A*c*d + a*B*e)*x))/(2*a*c*(a - c*x^2))
+ ((2*e*(A*c*d + 5*a*B*e)*Sqrt[d + e*x])/c - (2*e*(((Sqrt[c]*d - Sqrt[a]*e
)^(3/2)*(2*A*c*d - 5*a*B*e + 3*Sqrt[a]*A*Sqrt[c]*e)*ArcTanh[(c^(1/4)*Sqrt[
d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4)*e) - ((Sqrt[c]*
d + Sqrt[a]*e)^(3/2)*(2*A*c*d - 5*a*B*e - 3*Sqrt[a]*A*Sqrt[c]*e)*ArcTanh[(
c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4)*e)
))/c)/(4*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{-ce\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}\left(\left(\frac{\left(Ac\,d^2-3\left(Ae+\frac{10Bd}{3}\right)ea\right)\sqrt{ac\,e^2}}{4}-\frac{A\,c^2d^3}{2}+ade\left(Ae+\frac{5Bd}{4}\right)c+\frac{5B\,e^3a^2}{4}\right)\left(-c\,x^2+a\right)\arctan\left(\frac{}{\sqrt{(-}}\right.\right.}{}$
derivativedivides	$2e^2\left(\frac{B\sqrt{ex+d}}{c^2}+\frac{\frac{c\left(Aa\,e^2+Ac\,d^2+2Bade\right)(ex+d)^{\frac{3}{2}}}{4ae}+\frac{\left(Aacd\,e^2-A\,c^2d^3+B\,e^3a^2-Bac\,d^2e\right)\sqrt{ex+d}}{4ae}}{-c(ex+d)^2+2cd(ex+d)+a\,e^2-c\,d^2}+\frac{c\left(-\frac{\left(4Aacd\,e^2-2A\,c\right.}{}\right.}{}\right.$
default	$2e^2\left(\frac{B\sqrt{ex+d}}{c^2}+\frac{\frac{c\left(Aa\,e^2+Ac\,d^2+2Bade\right)(ex+d)^{\frac{3}{2}}}{4ae}+\frac{\left(Aacd\,e^2-A\,c^2d^3+B\,e^3a^2-Bac\,d^2e\right)\sqrt{ex+d}}{4ae}}{-c(ex+d)^2+2cd(ex+d)+a\,e^2-c\,d^2}+\frac{c\left(-\frac{\left(4Aacd\,e^2-2A\,c\right.}{}\right.}{}\right.$
risch	$\frac{2B\,e^2\sqrt{ex+d}}{c^2}+\frac{2e^2\left(\frac{-\frac{c\left(Aa\,e^2+Ac\,d^2+2Bade\right)(ex+d)^{\frac{3}{2}}}{4ae}-\frac{\left(Aacd\,e^2-A\,c^2d^3+B\,e^3a^2-Bac\,d^2e\right)\sqrt{ex+d}}{4ae}}{c(ex+d)^2-2cd(ex+d)-a\,e^2+c\,d^2}+\frac{c\left(-\frac{\left(4Aacd\,e^2-\right.}{}\right.}{}\right.}{}$

input
```
int((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*
c)^(1/2)*(-c*e*((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*(1/4*(A*c*d^2-3*(A*e+10/3*B
*d)*e*a)*(a*c*e^2)^(1/2)-1/2*A*c^2*d^3+a*d*e*(A*e+5/4*B*d)*c+5/4*B*e^3*a^2
)*(-c*x^2+a)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))+((-c
*d+(a*c*e^2)^(1/2))*c)^(1/2)*(-c*e*(1/4*(-A*c*d^2+3*(A*e+10/3*B*d)*e*a)*(a
*c*e^2)^(1/2)-1/2*A*c^2*d^3+a*d*e*(A*e+5/4*B*d)*c+5/4*B*e^3*a^2)*(-c*x^2+a
)*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+((c*d+(a*c*e^2)
^(1/2))*c)^(1/2)*(e*x+d)^(1/2)*(a*c*e^2)^(1/2)*(1/2*A*c^2*d^2*x+a*((1/2*A*
x-2*B*x^2)*e^2+e*(B*x+A)*d+1/2*B*d^2)*c+5/2*B*e^2*a^2)))/c^2/(-c*x^2+a)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5611 vs. $2(223) = 446$.

Time = 10.98 (sec) , antiderivative size = 5611, normalized size of antiderivative = 20.04

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(e*x+d)**(5/2)/(-c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^2}\,dx = \int \frac{(Bx+A)(ex+d)^{\frac{5}{2}}}{(cx^2-a)^2}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x + d)^(5/2)/(c*x^2 - a)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. 2(223) = 446.

Time = 0.30 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.70

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

output
```
2*sqrt(e*x + d)*B*e^2/c^2 - 1/4*(10*sqrt(a*c)*B*a^3*d*e^4*abs(c) - (sqrt(a
*c)*c*d^2*e - 3*sqrt(a*c)*a*e^3)*A*a^2*e^2*abs(c) - (a*c^2*d^3*e - a^2*c*d
*e^3)*A*abs(a)*abs(c)*abs(e) - 5*(a^2*c*d^2*e^2 - a^3*e^4)*B*abs(a)*abs(c)
*abs(e) + 2*(sqrt(a*c)*a*c^2*d^4*e - 2*sqrt(a*c)*a^2*c*d^2*e^3)*A*abs(c) -
 5*(sqrt(a*c)*a^2*c*d^3*e^2 + sqrt(a*c)*a^3*d*e^4)*B*abs(c))*arctan(sqrt(e
*x + d)/sqrt(-(a*c^3*d + sqrt(a^2*c^6*d^2 - (a*c^3*d^2 - a^2*c^2*e^2)*a*c^
3))/(a*c^3)))/((a^2*c^3*d - sqrt(a*c)*a^2*c^2*e)*sqrt(-c^2*d - sqrt(a*c)*c
*e)*abs(a)*abs(e)) + 1/4*(10*B*a^3*c*d*e^4*abs(c) - (c^2*d^2*e - 3*a*c*e^3
)*A*a^2*e^2*abs(c) + (sqrt(a*c)*c^2*d^3*e - sqrt(a*c)*a*c*d*e^3)*A*abs(a)*
abs(c)*abs(e) + 5*(sqrt(a*c)*a*c*d^2*e^2 - sqrt(a*c)*a^2*e^4)*B*abs(a)*abs
(c)*abs(e) + 2*(a*c^3*d^4*e - 2*a^2*c^2*d^2*e^3)*A*abs(c) - 5*(a^2*c^2*d^3
*e^2 + a^3*c*d*e^4)*B*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a*c^3*d - sqrt(a
^2*c^6*d^2 - (a*c^3*d^2 - a^2*c^2*e^2)*a*c^3))/(a*c^3)))/((a^2*c^3*e + sqr
t(a*c)*a*c^3*d)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(a)*abs(e)) - 1/2*((e*x +
d)^(3/2)*A*c^2*d^2*e - sqrt(e*x + d)*A*c^2*d^3*e + 2*(e*x + d)^(3/2)*B*a*c
*d*e^2 - sqrt(e*x + d)*B*a*c*d^2*e^2 + (e*x + d)^(3/2)*A*a*c*e^3 + sqrt(e*
x + d)*A*a*c*d*e^3 + sqrt(e*x + d)*B*a^2*e^4)/(((e*x + d)^2*c - 2*(e*x + d
)*c*d + c*d^2 - a*e^2)*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 6.61 (sec) , antiderivative size = 9253, normalized size of antiderivative = 33.05

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^2} \, dx = \text{Too large to display}$$

input
```
int(((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2)^2,x)
```

output
```
atan(((((320*B*a^5*c^4*e^6 + 64*A*a^4*c^5*d*e^5 - 64*A*a^3*c^6*d^3*e^3 - 3
20*B*a^4*c^5*d^2*e^4)/(8*a^3*c^3) - 64*a*c^4*d*e^2*(d + e*x)^(1/2)*((4*A^2
*a^3*c^8*d^5 - 25*B^2*a^2*e^5*(a^9*c^9)^(1/2) - 15*A^2*a^4*c^7*d^3*e^2 + 2
5*B^2*a^5*c^6*d^3*e^2 + 30*A*B*a^6*c^5*e^5 + 5*A^2*c^2*d^2*e^3*(a^9*c^9)^(
1/2) + 15*A^2*a^5*c^6*d*e^4 + 75*B^2*a^6*c^5*d*e^4 - 9*A^2*a*c*e^5*(a^9*c^
9)^(1/2) + 30*A*B*c^2*d^3*e^2*(a^9*c^9)^(1/2) - 20*A*B*a^4*c^7*d^4*e - 75*
B^2*a*c*d^2*e^3*(a^9*c^9)^(1/2) + 30*A*B*a^5*c^6*d^2*e^3 - 70*A*B*a*c*d*e^
4*(a^9*c^9)^(1/2))/(64*a^6*c^9))^(1/2))*((4*A^2*a^3*c^8*d^5 - 25*B^2*a^2*e
^5*(a^9*c^9)^(1/2) - 15*A^2*a^4*c^7*d^3*e^2 + 25*B^2*a^5*c^6*d^3*e^2 + 30*
A*B*a^6*c^5*e^5 + 5*A^2*c^2*d^2*e^3*(a^9*c^9)^(1/2) + 15*A^2*a^5*c^6*d*e^4
 + 75*B^2*a^6*c^5*d*e^4 - 9*A^2*a*c*e^5*(a^9*c^9)^(1/2) + 30*A*B*c^2*d^3*e
^2*(a^9*c^9)^(1/2) - 20*A*B*a^4*c^7*d^4*e - 75*B^2*a*c*d^2*e^3*(a^9*c^9)^(
1/2) + 30*A*B*a^5*c^6*d^2*e^3 - 70*A*B*a*c*d*e^4*(a^9*c^9)^(1/2))/(64*a^6*
c^9))^(1/2) + ((d + e*x)^(1/2)*(25*B^2*a^4*e^8 + 4*A^2*c^4*d^6*e^2 + 9*A^2
*a^3*c*e^8 + 10*A^2*a^2*c^2*d^2*e^6 + 25*B^2*a^2*c^2*d^4*e^4 - 15*A^2*a*c^
3*d^4*e^4 + 150*B^2*a^3*c*d^2*e^6 + 100*A*B*a^3*c*d*e^7 - 20*A*B*a*c^3*d^5
*e^3))/(a^2*c))*((4*A^2*a^3*c^8*d^5 - 25*B^2*a^2*e^5*(a^9*c^9)^(1/2) - 15*
A^2*a^4*c^7*d^3*e^2 + 25*B^2*a^5*c^6*d^3*e^2 + 30*A*B*a^6*c^5*e^5 + 5*A^2*
c^2*d^2*e^3*(a^9*c^9)^(1/2) + 15*A^2*a^5*c^6*d*e^4 + 75*B^2*a^6*c^5*d*e^4
- 9*A^2*a*c*e^5*(a^9*c^9)^(1/2) + 30*A*B*c^2*d^3*e^2*(a^9*c^9)^(1/2) - ...
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1558, normalized size of antiderivative = 5.56

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^2,x)
```

output
```
(6*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*s
qrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c*e**2 + 10*sqrt(a)*sqrt(sqrt(c)*sqrt(
a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d))
)*a*b*c*d*e - 4*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*
c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**2*d**2 - 6*sqrt(a)*sqrt(s
qrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(
a)*e - c*d)))*a*c**2*e**2*x**2 - 10*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*
atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*b*c**2*d*e
*x**2 + 4*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sq
rt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c**3*d**2*x**2 - 10*sqrt(c)*sqrt(sqr
t(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)
*e - c*d)))*a**2*b*e**2 - 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sq
rt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c*d*e + 10*sq
rt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(s
qrt(c)*sqrt(a)*e - c*d)))*a*b*c*e**2*x**2 + 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)
*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*
a*c**2*d*e*x**2 + 3*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt
(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a**2*c*e**2 + 5*sqrt(a)*sqrt
(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*s
qrt(d + e*x))*a*b*c*d*e - 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( ...
```

3.130 $\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 238

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^2} dx = \frac{\sqrt{d+ex}(a(Bd+Ae)+(Acd+aBe)x)}{2ac(a-cx^2)} - \frac{\sqrt{\sqrt{c}d-\sqrt{a}e}(2Acd-3aBe+\sqrt{a}A\sqrt{c}e)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}c^{7/4}} + \frac{\sqrt{\sqrt{c}d+\sqrt{a}e}(2Acd-3aBe-\sqrt{a}A\sqrt{c}e)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{4a^{3/2}c^{7/4}}$$

output

```
1/2*(e*x+d)^(1/2)*(a*(A*e+B*d)+(A*c*d+B*a*e)*x)/a/c/(-c*x^2+a)-1/4*(c^(1/2
)*d-a^(1/2)*e)^(1/2)*(2*A*c*d-3*B*a*e+a^(1/2)*A*c^(1/2)*e)*arctanh(c^(1/4)
*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(3/2)/c^(7/4)+1/4*(c^(1/2)*d
+a^(1/2)*e)^(1/2)*(2*A*c*d-3*B*a*e-a^(1/2)*A*c^(1/2)*e)*arctanh(c^(1/4)*(e
*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(3/2)/c^(7/4)
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^2}\,dx = \frac{-\frac{2\sqrt{a}c\sqrt{d+ex}(aAe+Acdx+aB(d+ex))}{-a+cx^2}-\sqrt{-cd-\sqrt{a}\sqrt{c}e}\left(2Acd-3aBe-\sqrt{a}A\sqrt{ce}\right.}{}$$

input
```
Integrate[((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^2,x]
```

output
```
((-2*Sqrt[a]*c*Sqrt[d + e*x]*(a*A*e + A*c*d*x + a*B*(d + e*x)))/(-a + c*x^
2) - Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*(2*A*c*d - 3*a*B*e - Sqrt[a]*A*Sqrt[
c]*e)*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d +
 Sqrt[a]*e)] + Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*(2*A*c*d - 3*a*B*e + Sqrt[
a]*A*Sqrt[c]*e)*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(S
qrt[c]*d - Sqrt[a]*e)])/(4*a^(3/2)*c^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {684, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^2}\,dx$$

$$\Big\downarrow \text{684}$$

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{2ac\,(a-cx^2)}-\frac{\int -\frac{2Acd^2-3aBed-aAe^2+e(Acd-3aBe)x}{2\sqrt{d+ex}(a-cx^2)}dx}{2ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2Acd^2-3aBed-aAe^2+e(Acd-3aBe)x}{\sqrt{d+ex}(a-cx^2)}dx}{4ac}+\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{2ac\,(a-cx^2)}$$

$$\Big\downarrow \text{654}$$

$$\frac{\int -\frac{e(A(cd^2-ae^2)+(Acd-3aBe)(d+ex))}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}+\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{2ac\,(a-cx^2)}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{2ac\,(a-cx^2)}-\frac{\int \frac{e(A(cd^2-ae^2)+(Acd-3aBe)(d+ex))}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{2ac\,(a-cx^2)}-\frac{e\int \frac{A(cd^2-ae^2)+(Acd-3aBe)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}$$

$\Big\downarrow$ 1480

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{2ac\,(a-cx^2)}-$$
$$\frac{e\left(\frac{(\sqrt{a}e+\sqrt{c}d)(-\sqrt{a}A\sqrt{c}e-3aBe+2Acd)\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d+\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{(\sqrt{c}d-\sqrt{a}e)(\sqrt{a}A\sqrt{c}e-3aBe+2Acd)\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d-\sqrt{a}e}}{2\sqrt{a}e}\right.}{2ac}$$

$\Big\downarrow$ 221

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{2ac\,(a-cx^2)}-$$
$$\frac{e\left(\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}(\sqrt{a}A\sqrt{c}e-3aBe+2Acd)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}c^{3/4}e}-\frac{\sqrt{\sqrt{a}e+\sqrt{c}d}(-\sqrt{a}A\sqrt{c}e-3aBe+2Acd)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}c^{3/4}e}\right)}{2ac}$$

input
```
Int[((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^2,x]
```

output
```
(Sqrt[d + e*x]*(a*(B*d + A*e) + (A*c*d + a*B*e)*x))/(2*a*c*(a - c*x^2)) -
(e*((Sqrt[Sqrt[c]*d - Sqrt[a]*e]*(2*A*c*d - 3*a*B*e + Sqrt[a]*A*Sqrt[c]*e)
*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(2*Sqrt[a]*
c^(3/4)*e) - (Sqrt[Sqrt[c]*d + Sqrt[a]*e]*(2*A*c*d - 3*a*B*e - Sqrt[a]*A*S
qrt[c]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2
*Sqrt[a]*c^(3/4)*e)))/(2*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.28

method	result
pseudoelliptic	$\frac{-\frac{\left((Acd-3Bae)\sqrt{ac\,e^2}+c\left(-2Ac\,d^2+ae(Ae+3Bd)\right)\right)e\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}\,\left(-c\,x^2+a\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}\right)}{2}+\sqrt{\left(-cd+\sqrt{ac\right.}}}{2\sqrt{\left(cd+\right.}}$
derivativedivides	$2e^2\left(\frac{\frac{(Acd+Bae)(ex+d)^{\frac{3}{2}}}{4ace}+\frac{A\left(a\,e^2-c\,d^2\right)\sqrt{ex+d}}{4ace}}{-c(ex+d)^2+2cd(ex+d)+a\,e^2-c\,d^2}+\frac{-\frac{\left(Aac\,e^2-2A\,c^2d^2+3Bacde-A\sqrt{ac\,e^2}\,cd+3B\sqrt{ac\,e^2}\,ae\right)\operatorname{arctanh}\left(\right.}{2c\sqrt{ac\,e^2}\,\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}}{}\right.$
default	$2e^2\left(\frac{\frac{(Acd+Bae)(ex+d)^{\frac{3}{2}}}{4ace}+\frac{A\left(a\,e^2-c\,d^2\right)\sqrt{ex+d}}{4ace}}{-c(ex+d)^2+2cd(ex+d)+a\,e^2-c\,d^2}+\frac{-\frac{\left(Aac\,e^2-2A\,c^2d^2+3Bacde-A\sqrt{ac\,e^2}\,cd+3B\sqrt{ac\,e^2}\,ae\right)\operatorname{arctanh}\left(\right.}{2c\sqrt{ac\,e^2}\,\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}}{}\right.$

input
```
int((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)/(a*c*e^2)^(1/2)*(-1/2*((A*c*d-3*B*a*e)
*(a*c*e^2)^(1/2)+c*(-2*A*c*d^2+a*e*(A*e+3*B*d)))*e*((c*d+(a*c*e^2)^(1/2))*
c)^(1/2)*(-c*x^2+a)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2
))+((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*(-1/2*((-A*c*d+3*B*a*e)*(a*c*e^2)^(1/2
)+c*(-2*A*c*d^2+a*e*(A*e+3*B*d)))*e*(-c*x^2+a)*arctanh(c*(e*x+d)^(1/2)/((c
*d+(a*c*e^2)^(1/2))*c)^(1/2))+(c*x*A*d+a*(B*d+e*(B*x+A)))*((c*d+(a*c*e^2)^
(1/2))*c)^(1/2)*(e*x+d)^(1/2)*(a*c*e^2)^(1/2)))/((-c*d+(a*c*e^2)^(1/2))*c)
^(1/2)/a/c/(-c*x^2+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2327 vs. $2(183) = 366$.

Time = 0.33 (sec) , antiderivative size = 2327, normalized size of antiderivative = 9.78

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*((a*c^2*x^2 - a^2*c)*sqrt((4*A^2*c^2*d^3 - 12*A*B*a*c*d^2*e + 6*A*B*a^
2*e^3 + a^3*c^3*sqrt((36*A^2*B^2*c^2*d^2*e^4 - 12*(9*A*B^3*a*c + A^3*B*c^2
)*d*e^5 + (81*B^4*a^2 + 18*A^2*B^2*a*c + A^4*c^2)*e^6)/(a^3*c^7)) + 3*(3*B
^2*a^2 - A^2*a*c)*d*e^2)/(a^3*c^3))*log((24*A^3*B*c^3*d^3*e^2 - 4*(27*A^2*
B^2*a*c^2 + A^4*c^3)*d^2*e^3 + 6*(27*A*B^3*a^2*c + A^3*B*a*c^2)*d*e^4 - (8
1*B^4*a^3 - A^4*a*c^2)*e^5)*sqrt(e*x + d) + (6*A^2*B*a^2*c^3*d*e^3 - (9*A*
B^2*a^3*c^2 + A^3*a^2*c^3)*e^4 - (2*A*a^3*c^6*d - 3*B*a^4*c^5*e)*sqrt((36*
A^2*B^2*c^2*d^2*e^4 - 12*(9*A*B^3*a*c + A^3*B*c^2)*d*e^5 + (81*B^4*a^2 + 1
8*A^2*B^2*a*c + A^4*c^2)*e^6)/(a^3*c^7)))*sqrt((4*A^2*c^2*d^3 - 12*A*B*a*c
*d^2*e + 6*A*B*a^2*e^3 + a^3*c^3*sqrt((36*A^2*B^2*c^2*d^2*e^4 - 12*(9*A*B^
3*a*c + A^3*B*c^2)*d*e^5 + (81*B^4*a^2 + 18*A^2*B^2*a*c + A^4*c^2)*e^6)/(a
^3*c^7)) + 3*(3*B^2*a^2 - A^2*a*c)*d*e^2)/(a^3*c^3))) - (a*c^2*x^2 - a^2*c
)*sqrt((4*A^2*c^2*d^3 - 12*A*B*a*c*d^2*e + 6*A*B*a^2*e^3 + a^3*c^3*sqrt((3
6*A^2*B^2*c^2*d^2*e^4 - 12*(9*A*B^3*a*c + A^3*B*c^2)*d*e^5 + (81*B^4*a^2 +
 18*A^2*B^2*a*c + A^4*c^2)*e^6)/(a^3*c^7)) + 3*(3*B^2*a^2 - A^2*a*c)*d*e^2
)/(a^3*c^3))*log((24*A^3*B*c^3*d^3*e^2 - 4*(27*A^2*B^2*a*c^2 + A^4*c^3)*d^
2*e^3 + 6*(27*A*B^3*a^2*c + A^3*B*a*c^2)*d*e^4 - (81*B^4*a^3 - A^4*a*c^2)*
e^5)*sqrt(e*x + d) - (6*A^2*B*a^2*c^3*d*e^3 - (9*A*B^2*a^3*c^2 + A^3*a^2*c
^3)*e^4 - (2*A*a^3*c^6*d - 3*B*a^4*c^5*e)*sqrt((36*A^2*B^2*c^2*d^2*e^4 - 1
2*(9*A*B^3*a*c + A^3*B*c^2)*d*e^5 + (81*B^4*a^2 + 18*A^2*B^2*a*c + A^4*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(e*x+d)**(3/2)/(-c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^2}\,dx = \int \frac{(Bx+A)(ex+d)^{\frac{3}{2}}}{(cx^2-a)^2}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x + d)^(3/2)/(c*x^2 - a)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(183) = 366$.

Time = 0.27 (sec) , antiderivative size = 546, normalized size of antiderivative = 2.29

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^2}\,dx = \frac{\left(3\sqrt{ac}Ba^2c^3d^2e^2+\sqrt{ac}Aa^2c^3de^3-3\sqrt{ac}Ba^3c^2e^4+(ac^3d^2e-a^2c^2e^3)A|a||\right.}{4\left(a^2c^4d-\sqrt{ac}a^2c^3e\right.}$$

$$-\frac{\left(3Ba^2c^3d^2e^2+Aa^2c^3de^3-3Ba^3c^2e^4-\left(\sqrt{ac}c^2d^2e-\sqrt{ac}ace^3\right)A|a||c||e|-\left(2ac^4d^3e-a^2c^3de^3\right)A\right)\mathrm{arct}}{4\left(a^2c^3e+\sqrt{ac}ac^3d\right)\sqrt{-c^2d+\sqrt{ac}ce}|a||e|}$$

$$-\frac{(ex+d)^{\frac{3}{2}}Acde-\sqrt{ex+d}Acd^2e+(ex+d)^{\frac{3}{2}}Bae^2+\sqrt{ex+d}Aae^3}{2\left((ex+d)^2c-2(ex+d)cd+cd^2-ae^2\right)ac}$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/4*(3*sqrt(a*c)*B*a^2*c^3*d^2*e^2 + sqrt(a*c)*A*a^2*c^3*d*e^3 - 3*sqrt(a*
c)*B*a^3*c^2*e^4 + (a*c^3*d^2*e - a^2*c^2*e^3)*A*abs(a)*abs(c)*abs(e) - (2
*sqrt(a*c)*a*c^4*d^3*e - sqrt(a*c)*a^2*c^3*d*e^3)*A)*arctan(sqrt(e*x + d)/
sqrt(-(a*c^2*d + sqrt(a^2*c^4*d^2 - (a*c^2*d^2 - a^2*c*e^2)*a*c^2))/(a*c^2
)))/((a^2*c^4*d - sqrt(a*c)*a^2*c^3*e)*sqrt(-c^2*d - sqrt(a*c)*c*e)*abs(a)
*abs(e)) - 1/4*(3*B*a^2*c^3*d^2*e^2 + A*a^2*c^3*d*e^3 - 3*B*a^3*c^2*e^4 -
(sqrt(a*c)*c^2*d^2*e - sqrt(a*c)*a*c*e^3)*A*abs(a)*abs(c)*abs(e) - (2*a*c^
4*d^3*e - a^2*c^3*d*e^3)*A)*arctan(sqrt(e*x + d)/sqrt(-(a*c^2*d - sqrt(a^2
*c^4*d^2 - (a*c^2*d^2 - a^2*c*e^2)*a*c^2))/(a*c^2)))/((a^2*c^3*e + sqrt(a*
c)*a*c^3*d)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(a)*abs(e)) - 1/2*((e*x + d)^(
3/2)*A*c*d*e - sqrt(e*x + d)*A*c*d^2*e + (e*x + d)^(3/2)*B*a*e^2 + sqrt(e*
x + d)*A*a*e^3)/(((e*x + d)^2*c - 2*(e*x + d)*c*d + c*d^2 - a*e^2)*a*c)
```

Mupad [B] (verification not implemented)

Time = 6.49 (sec) , antiderivative size = 5212, normalized size of antiderivative = 21.90

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^2,x)
```

output
```
atan(((((64*A*a^4*c^4*e^5 - 64*A*a^3*c^5*d^2*e^3)/(8*a^3*c^2) - 64*a*c^4*d
*e^2*(d + e*x)^(1/2)*((4*A^2*a^3*c^6*d^3 + 9*B^2*a*e^3*(a^9*c^7)^(1/2) + A
^2*c*e^3*(a^9*c^7)^(1/2) + 6*A*B*a^5*c^4*e^3 - 3*A^2*a^4*c^5*d*e^2 + 9*B^2
*a^5*c^4*d*e^2 - 12*A*B*a^4*c^5*d^2*e - 6*A*B*c*d*e^2*(a^9*c^7)^(1/2))/(64
*a^6*c^7))^(1/2))*((4*A^2*a^3*c^6*d^3 + 9*B^2*a*e^3*(a^9*c^7)^(1/2) + A^2*
c*e^3*(a^9*c^7)^(1/2) + 6*A*B*a^5*c^4*e^3 - 3*A^2*a^4*c^5*d*e^2 + 9*B^2*a^
5*c^4*d*e^2 - 12*A*B*a^4*c^5*d^2*e - 6*A*B*c*d*e^2*(a^9*c^7)^(1/2))/(64*a^
6*c^7))^(1/2) + ((d + e*x)^(1/2)*(9*B^2*a^3*e^6 + 4*A^2*c^3*d^4*e^2 + A^2*
a^2*c*e^6 - 3*A^2*a*c^2*d^2*e^4 + 9*B^2*a^2*c*d^2*e^4 - 12*A*B*a*c^2*d^3*e
^3))/a^2)*((4*A^2*a^3*c^6*d^3 + 9*B^2*a*e^3*(a^9*c^7)^(1/2) + A^2*c*e^3*(a
^9*c^7)^(1/2) + 6*A*B*a^5*c^4*e^3 - 3*A^2*a^4*c^5*d*e^2 + 9*B^2*a^5*c^4*d*
e^2 - 12*A*B*a^4*c^5*d^2*e - 6*A*B*c*d*e^2*(a^9*c^7)^(1/2))/(64*a^6*c^7))^
(1/2)*1i - (((64*A*a^4*c^4*e^5 - 64*A*a^3*c^5*d^2*e^3)/(8*a^3*c^2) + 64*a*
c^4*d*e^2*(d + e*x)^(1/2)*((4*A^2*a^3*c^6*d^3 + 9*B^2*a*e^3*(a^9*c^7)^(1/2
) + A^2*c*e^3*(a^9*c^7)^(1/2) + 6*A*B*a^5*c^4*e^3 - 3*A^2*a^4*c^5*d*e^2 +
9*B^2*a^5*c^4*d*e^2 - 12*A*B*a^4*c^5*d^2*e - 6*A*B*c*d*e^2*(a^9*c^7)^(1/2)
)/(64*a^6*c^7))^(1/2))*((4*A^2*a^3*c^6*d^3 + 9*B^2*a*e^3*(a^9*c^7)^(1/2) +
 A^2*c*e^3*(a^9*c^7)^(1/2) + 6*A*B*a^5*c^4*e^3 - 3*A^2*a^4*c^5*d*e^2 + 9*B
^2*a^5*c^4*d*e^2 - 12*A*B*a^4*c^5*d^2*e - 6*A*B*c*d*e^2*(a^9*c^7)^(1/2))/(
64*a^6*c^7))^(1/2) - ((d + e*x)^(1/2)*(9*B^2*a^3*e^6 + 4*A^2*c^3*d^4*e^...
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 870, normalized size of antiderivative = 3.66

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^2,x)
```

output
```
(6*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*s
qrt(sqrt(c)*sqrt(a)*e - c*d)))*a*b*e - 4*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e -
c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c*d
 - 6*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)
*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*b*c*e*x**2 + 4*sqrt(a)*sqrt(sqrt(c)*sqrt(
a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d))
)*c**2*d*x**2 - 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x
)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*e + 2*sqrt(c)*sqrt(sqrt
(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*
e - c*d)))*a*c*e*x**2 + 3*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqr
t(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*b*e - 2*sqrt(a)*sqrt
(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*s
qrt(d + e*x))*a*c*d - 3*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(
sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*b*c*e*x**2 + 2*sqrt(a)*s
qrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c
)*sqrt(d + e*x))*c**2*d*x**2 - 3*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log
(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*b*e + 2*sqrt(a)*
sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*
sqrt(d + e*x))*a*c*d + 3*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sq
rt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*b*c*e*x**2 - 2*sqrt(a)*...
```

3.131 $\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 225

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^2}\,dx = \frac{(aB+Acx)\sqrt{d+ex}}{2ac(a-cx^2)} - \frac{\left(2Acd-aBe-\sqrt{a}A\sqrt{c}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}c^{5/4}\sqrt{\sqrt{c}d-\sqrt{a}e}} + \frac{\left(2Acd-aBe+\sqrt{a}A\sqrt{c}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{4a^{3/2}c^{5/4}\sqrt{\sqrt{c}d+\sqrt{a}e}}$$

output

```
1/2*(A*c*x+B*a)*(e*x+d)^(1/2)/a/c/(-c*x^2+a)-1/4*(2*A*c*d-B*a*e-a^(1/2)*A*
c^(1/2)*e)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(3
/2)/c^(5/4)/(c^(1/2)*d-a^(1/2)*e)^(1/2)+1/4*(2*A*c*d-B*a*e+a^(1/2)*A*c^(1/
2)*e)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(3/2)/c
^(5/4)/(c^(1/2)*d+a^(1/2)*e)^(1/2)
```

Mathematica [A] (verified)

Time = 2.18 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^2}\,dx$$
$$=\frac{-\frac{2\sqrt{a}\sqrt{c}(aB+Acx)\sqrt{d+ex}}{-a+cx^2}-\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}(2Acd-aBe+\sqrt{a}A\sqrt{c}e)\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{c}d+\sqrt{a}e}+\frac{\sqrt{-cd+\sqrt{a}\sqrt{c}e}(2Acd-aBe-\sqrt{a}A\cdots}{\sqrt{c}d\cdots}}{4a^{3/2}c^{3/2}}$$

input
```
Integrate[((A + B*x)*Sqrt[d + e*x])/(a - c*x^2)^2,x]
```

output
```
((-2*Sqrt[a]*Sqrt[c]*(a*B + A*c*x)*Sqrt[d + e*x])/(-a + c*x^2) - (Sqrt[-(c
*d) - Sqrt[a]*Sqrt[c]*e]*(2*A*c*d - a*B*e + Sqrt[a]*A*Sqrt[c]*e)*ArcTan[(S
qrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)])/(
Sqrt[c]*d + Sqrt[a]*e) + (Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*(2*A*c*d - a*B*
e - Sqrt[a]*A*Sqrt[c]*e)*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d +
 e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/(Sqrt[c]*d - Sqrt[a]*e))/(4*a^(3/2)*c^(3/
2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {685, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^2}\,dx$$

$$\Big\downarrow \text{685}$$

$$\frac{\sqrt{d+ex}(aB+Acx)}{2ac\,(a-cx^2)}-\frac{\int -\frac{2Acd-aBe+Acex}{2\sqrt{d+ex}(a-cx^2)}dx}{2ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2Acd-aBe+Acex}{\sqrt{d+ex}(a-cx^2)}dx}{4ac}+\frac{\sqrt{d+ex}(aB+Acx)}{2ac(a-cx^2)}$$

$\big\downarrow$ 654

$$\frac{\int -\frac{e(Acd-aBe+Ac(d+ex))}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}+\frac{\sqrt{d+ex}(aB+Acx)}{2ac(a-cx^2)}$$

$\big\downarrow$ 25

$$\frac{\sqrt{d+ex}(aB+Acx)}{2ac(a-cx^2)}-\frac{\int \frac{e(Acd-aBe+Ac(d+ex))}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}$$

$\big\downarrow$ 27

$$\frac{\sqrt{d+ex}(aB+Acx)}{2ac(a-cx^2)}-\frac{e\int \frac{Acd-aBe+Ac(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}$$

$\big\downarrow$ 1480

$$\frac{\sqrt{d+ex}(aB+Acx)}{2ac(a-cx^2)}-\frac{e\left(\frac{1}{2}\sqrt{c}\left(A\sqrt{c}-\frac{2Acd-aBe}{\sqrt{a}e}\right)\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d-\sqrt{a}e)}d\sqrt{d+ex}+\frac{1}{2}\sqrt{c}\left(\frac{2Acd-aBe}{\sqrt{a}e}+A\sqrt{c}\right)\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d+\sqrt{a}e)}d\sqrt{}\right.}{2ac}$$

$\big\downarrow$ 221

$$\frac{\sqrt{d+ex}(aB+Acx)}{2ac(a-cx^2)}-\frac{e\left(-\frac{\left(A\sqrt{c}-\frac{2Acd-aBe}{\sqrt{a}e}\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{a}e}}-\frac{\left(\frac{2Acd-aBe}{\sqrt{a}e}+A\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt[4]{c}\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2ac}$$

input
```
Int[((A + B*x)*Sqrt[d + e*x])/(a - c*x^2)^2,x]
```

output
```
((a*B + A*c*x)*Sqrt[d + e*x])/(2*a*c*(a - c*x^2)) - (e*(-1/2*((A*Sqrt[c] -
 (2*A*c*d - a*B*e)/(Sqrt[a]*e))*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[
c]*d - Sqrt[a]*e]])/(c^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) - ((A*Sqrt[c] +
(2*A*c*d - a*B*e)/(Sqrt[a]*e))*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c
]*d + Sqrt[a]*e]])/(2*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[a]*e])))/(2*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 685
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^(m - 1)*(a + c*x^2)
^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m]
 || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.08

method	result
derivativedivides	$2e^2\left(\frac{\frac{A(ex+d)^{\frac{3}{2}}}{4ae}-\frac{(Acd-Bae)\sqrt{ex+d}}{4aec}}{-c(ex+d)^2+2cd(ex+d)+a\,e^2-c\,d^2}+\frac{\frac{\left(2Acd-Bae-A\sqrt{ac\,e^2}\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}\right)}{2\sqrt{ac\,e^2}\,\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}-\frac{\left(-2Acd+Bae-A\sqrt{a\cdots}\right.}{2\sqrt{ac\cdots}}}{4ae}\right.$
default	$2e^2\left(\frac{\frac{A(ex+d)^{\frac{3}{2}}}{4ae}-\frac{(Acd-Bae)\sqrt{ex+d}}{4aec}}{-c(ex+d)^2+2cd(ex+d)+a\,e^2-c\,d^2}+\frac{\frac{\left(2Acd-Bae-A\sqrt{ac\,e^2}\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}\right)}{2\sqrt{ac\,e^2}\,\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}-\frac{\left(-2Acd+Bae-A\sqrt{a\cdots}\right.}{2\sqrt{ac\cdots}}}{4ae}\right.$
pseudoelliptic	$\frac{ce\left(Acd-\frac{Bae}{2}-\frac{A\sqrt{ac\,e^2}}{2}\right)\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}\,\left(-c\,x^2+a\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}\right)+\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}\,\left(c\left(Acd-\frac{B\cdots}{\cdots}\right.\right.}{2\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}\,\sqrt{ac\,e^2}\,\sqrt{\left(-cd+\sqrt{\cdots}\right.}}$

input
```
int((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*e^2*((1/4*A/a/e*(e*x+d)^(3/2)-1/4*(A*c*d-B*a*e)/a/e/c*(e*x+d)^(1/2))/(-c
*(e*x+d)^2+2*c*d*(e*x+d)+a*e^2-c*d^2)+1/4/a/e*(1/2*(2*A*c*d-B*a*e-A*(a*c*e
^2)^(1/2))/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c*(e*x+
d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))-1/2*(-2*A*c*d+B*a*e-A*(a*c*e^2)
^(1/2))/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^
(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3195 vs. $2(170) = 340$.

Time = 3.68 (sec) , antiderivative size = 3195, normalized size of antiderivative = 14.20

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(e*x+d)**(1/2)/(-c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^2}\,dx = \int \frac{(Bx+A)\sqrt{ex+d}}{(cx^2-a)^2}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*sqrt(e*x + d)/(c*x^2 - a)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(170) = 340$.

Time = 0.20 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.00

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^2}\,dx$$

$$= \frac{\left(2\,Aac^3d^2e - Ba^2c^2de^2 - Aa^2c^2e^3 - \sqrt{ac}Acde|a||c||e| + \sqrt{ac}Bae^2|a||c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{ac^2d+\sqrt{a^2c^4d^2-\left(ac^2d^2\right.}}{ac^2}}}\right.}{4\left(a^2c^2e - \sqrt{ac}ac^2d\right)\sqrt{-c^2d - \sqrt{ac}ce}|a||e|}$$

$$+ \frac{\left(2\,Aac^3d^2e - Ba^2c^2de^2 - Aa^2c^2e^3 + \sqrt{ac}Acde|a||c||e| - \sqrt{ac}Bae^2|a||c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{ac^2d-\sqrt{a^2c^4d^2-\left(ac\right.}}{ac^2}}}\right.}{4\left(a^2c^2e + \sqrt{ac}ac^2d\right)\sqrt{-c^2d + \sqrt{ac}ce}|a||e|}$$

$$- \frac{(ex+d)^{\frac{3}{2}}Ace - \sqrt{ex+d}Acde + \sqrt{ex+d}Bae^2}{2\left((ex+d)^2c - 2\,(ex+d)cd + cd^2 - ae^2\right)ac}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/4*(2*A*a*c^3*d^2*e - B*a^2*c^2*d*e^2 - A*a^2*c^2*e^3 - sqrt(a*c)*A*c*d*e
*abs(a)*abs(c)*abs(e) + sqrt(a*c)*B*a*e^2*abs(a)*abs(c)*abs(e))*arctan(sqr
t(e*x + d)/sqrt(-(a*c^2*d + sqrt(a^2*c^4*d^2 - (a*c^2*d^2 - a^2*c*e^2)*a*c
^2))/(a*c^2)))/((a^2*c^2*e - sqrt(a*c)*a*c^2*d)*sqrt(-c^2*d - sqrt(a*c)*c*
e)*abs(a)*abs(e)) + 1/4*(2*A*a*c^3*d^2*e - B*a^2*c^2*d*e^2 - A*a^2*c^2*e^3
 + sqrt(a*c)*A*c*d*e*abs(a)*abs(c)*abs(e) - sqrt(a*c)*B*a*e^2*abs(a)*abs(c
)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(a*c^2*d - sqrt(a^2*c^4*d^2 - (a*c^2*
d^2 - a^2*c*e^2)*a*c^2))/(a*c^2)))/((a^2*c^2*e + sqrt(a*c)*a*c^2*d)*sqrt(-
c^2*d + sqrt(a*c)*c*e)*abs(a)*abs(e)) - 1/2*((e*x + d)^(3/2)*A*c*e - sqrt(
e*x + d)*A*c*d*e + sqrt(e*x + d)*B*a*e^2)/(((e*x + d)^2*c - 2*(e*x + d)*c*
d + c*d^2 - a*e^2)*a*c)
```

Mupad [B] (verification not implemented)

Time = 7.14 (sec) , antiderivative size = 5062, normalized size of antiderivative = 22.50

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(((A + B*x)*(d + e*x)^(1/2))/(a - c*x^2)^2,x)
```

output
```
atan(((((64*B*a^4*c^2*e^4 - 64*A*a^3*c^3*d*e^3)/(8*a^3) - 64*a*c^4*d*e^2*(
d + e*x)^(1/2)*((4*A^2*a^3*c^5*d^3 + B^2*a*e^3*(a^9*c^5)^(1/2) + A^2*c*e^3
*(a^9*c^5)^(1/2) + 2*A*B*a^5*c^3*e^3 - 3*A^2*a^4*c^4*d*e^2 + B^2*a^5*c^3*d
*e^2 - 4*A*B*a^4*c^4*d^2*e - 2*A*B*c*d*e^2*(a^9*c^5)^(1/2))/(64*(a^6*c^6*d
^2 - a^7*c^5*e^2)))^(1/2))*((4*A^2*a^3*c^5*d^3 + B^2*a*e^3*(a^9*c^5)^(1/2)
 + A^2*c*e^3*(a^9*c^5)^(1/2) + 2*A*B*a^5*c^3*e^3 - 3*A^2*a^4*c^4*d*e^2 + B
^2*a^5*c^3*d*e^2 - 4*A*B*a^4*c^4*d^2*e - 2*A*B*c*d*e^2*(a^9*c^5)^(1/2))/(6
4*(a^6*c^6*d^2 - a^7*c^5*e^2)))^(1/2) + ((d + e*x)^(1/2)*(4*A^2*c^3*d^2*e^
2 + A^2*a*c^2*e^4 + B^2*a^2*c*e^4 - 4*A*B*a*c^2*d*e^3))/a^2)*((4*A^2*a^3*c
^5*d^3 + B^2*a*e^3*(a^9*c^5)^(1/2) + A^2*c*e^3*(a^9*c^5)^(1/2) + 2*A*B*a^5
*c^3*e^3 - 3*A^2*a^4*c^4*d*e^2 + B^2*a^5*c^3*d*e^2 - 4*A*B*a^4*c^4*d^2*e -
 2*A*B*c*d*e^2*(a^9*c^5)^(1/2))/(64*(a^6*c^6*d^2 - a^7*c^5*e^2)))^(1/2)*1i
 - (((64*B*a^4*c^2*e^4 - 64*A*a^3*c^3*d*e^3)/(8*a^3) + 64*a*c^4*d*e^2*(d +
 e*x)^(1/2)*((4*A^2*a^3*c^5*d^3 + B^2*a*e^3*(a^9*c^5)^(1/2) + A^2*c*e^3*(a
^9*c^5)^(1/2) + 2*A*B*a^5*c^3*e^3 - 3*A^2*a^4*c^4*d*e^2 + B^2*a^5*c^3*d*e^
2 - 4*A*B*a^4*c^4*d^2*e - 2*A*B*c*d*e^2*(a^9*c^5)^(1/2))/(64*(a^6*c^6*d^2
- a^7*c^5*e^2)))^(1/2))*((4*A^2*a^3*c^5*d^3 + B^2*a*e^3*(a^9*c^5)^(1/2) +
A^2*c*e^3*(a^9*c^5)^(1/2) + 2*A*B*a^5*c^3*e^3 - 3*A^2*a^4*c^4*d*e^2 + B^2*
a^5*c^3*d*e^2 - 4*A*B*a^4*c^4*d^2*e - 2*A*B*c*d*e^2*(a^9*c^5)^(1/2))/(64*(
a^6*c^6*d^2 - a^7*c^5*e^2)))^(1/2) - ((d + e*x)^(1/2)*(4*A^2*c^3*d^2*e^...
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 1528, normalized size of antiderivative = 6.79

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^2,x)
```

output
```
( - 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c*e**2 - 2*sqrt(a)*sqrt(sqrt(c)*sqr
t(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d
)))*a*b*c*d*e + 4*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x
)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**2*d**2 + 2*sqrt(a)*sqrt
(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqr
t(a)*e - c*d)))*a*c**2*e**2*x**2 + 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)
*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*b*c**2*d*
e*x**2 - 4*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(s
qrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c**3*d**2*x**2 - 2*sqrt(c)*sqrt(sqr
t(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)
*e - c*d)))*a**2*b*e**2 + 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sq
rt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c*d*e + 2*sqr
t(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sq
rt(c)*sqrt(a)*e - c*d)))*a*b*c*e**2*x**2 - 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*
e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a
*c**2*d*e*x**2 - sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)
*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a**2*c*e**2 - sqrt(a)*sqrt(sqrt
(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d
 + e*x))*a*b*c*d*e + 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqr...
```

3.132 $\int \frac{A+Bx}{\sqrt{d+ex}(a-cx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 250

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^2}\,dx = \frac{\sqrt{d+ex}(a(Bd-Ae)+(Acd-aBe)x)}{2a\,(cd^2-ae^2)\,(a-cx^2)} - \frac{\left(2Acd+aBe-3\sqrt{a}A\sqrt{c}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}} + \frac{\left(2Acd+aBe+3\sqrt{a}A\sqrt{c}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{4a^{3/2}c^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}}$$

output
```
1/2*(e*x+d)^(1/2)*(a*(-A*e+B*d)+(A*c*d-B*a*e)*x)/a/(-a*e^2+c*d^2)/(-c*x^2+
a)-1/4*(2*A*c*d+B*a*e-3*a^(1/2)*A*c^(1/2)*e)*arctanh(c^(1/4)*(e*x+d)^(1/2)
/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(3/2)/c^(3/4)/(c^(1/2)*d-a^(1/2)*e)^(3/2)+
1/4*(2*A*c*d+B*a*e+3*a^(1/2)*A*c^(1/2)*e)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c
^(1/2)*d+a^(1/2)*e)^(1/2))/a^(3/2)/c^(3/4)/(c^(1/2)*d+a^(1/2)*e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^2}\,dx$$
$$=\frac{\frac{2\sqrt{a}\sqrt{d+ex}(-Acdx+a(-Bd+Ae+Bex))}{(-cd^2+ae^2)(a-cx^2)}-\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}(2Acd+aBe+3\sqrt{a}A\sqrt{c}e)\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{c(\sqrt{c}d+\sqrt{a}e)^2}-\frac{(2Acd+aBe-3\sqrt{a}A\cdots}{\sqrt{c}(\sqrt{c}\cdots}}{4a^{3/2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[d + e*x]*(a - c*x^2)^2),x]
```

output
```
((2*Sqrt[a]*Sqrt[d + e*x]*(-(A*c*d*x) + a*(-(B*d) + A*e + B*e*x)))/((-(c*d
^2) + a*e^2)*(a - c*x^2)) - (Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*(2*A*c*d + a
*B*e + 3*Sqrt[a]*A*Sqrt[c]*e)*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqr
t[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)])/(c*(Sqrt[c]*d + Sqrt[a]*e)^2) - ((2*
A*c*d + a*B*e - 3*Sqrt[a]*A*Sqrt[c]*e)*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[
c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/(Sqrt[c]*(Sqrt[c]*d - Sqrt[
a]*e)*Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]))/(4*a^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {686, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a-cx^2)^2\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{ 686}$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{2a\,(a-cx^2)\,(cd^2-ae^2)}-\frac{\int -\frac{c(2Acd^2+aBed-3aAe^2+e(Acd-aBe)x)}{2\sqrt{d+ex}(a-cx^2)}dx}{2ac\,(cd^2-ae^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2Acd^2+aBed-3aAe^2+e(Acd-aBe)x}{\sqrt{d+ex}(a-cx^2)}dx}{4a\left(cd^2-ae^2\right)}+\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{2a\left(a-cx^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{654}$$

$$\frac{\int -\frac{e\left(Acd^2+2aBed-3aAe^2+(Acd-aBe)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a\left(cd^2-ae^2\right)}+\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{2a\left(a-cx^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{2a\left(a-cx^2\right)\left(cd^2-ae^2\right)}-\frac{\int \frac{e\left(Acd^2+2aBed-3aAe^2+(Acd-aBe)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{2a\left(a-cx^2\right)\left(cd^2-ae^2\right)}-\frac{e\int \frac{Acd^2+2aBed-3aAe^2+(Acd-aBe)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{2a\left(a-cx^2\right)\left(cd^2-ae^2\right)}-\frac{e\left(\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\left(3\sqrt{a}A\sqrt{c}e+aBe+2Acd\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\left(-3\sqrt{a}A\sqrt{c}e+aBe+2Acd\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}}{2\sqrt{a}e}\right.}{2a\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{2a\left(a-cx^2\right)\left(cd^2-ae^2\right)}-\frac{e\left(\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\left(-3\sqrt{a}A\sqrt{c}e+aBe+2Acd\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}c^{3/4}e\sqrt{\sqrt{c}d-\sqrt{a}e}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\left(3\sqrt{a}A\sqrt{c}e+aBe+2Acd\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}c^{3/4}e\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2a\left(cd^2-ae^2\right)}$$

input
```
Int[(A + B*x)/(Sqrt[d + e*x]*(a - c*x^2)^2),x]
```

output
```
(Sqrt[d + e*x]*(a*(B*d - A*e) + (A*c*d - a*B*e)*x))/(2*a*(c*d^2 - a*e^2)*(
a - c*x^2)) - (e*(((Sqrt[c]*d + Sqrt[a]*e)*(2*A*c*d + a*B*e - 3*Sqrt[a]*A*
Sqrt[c]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(
2*Sqrt[a]*c^(3/4)*e*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) - ((Sqrt[c]*d - Sqrt[a]*e
)*(2*A*c*d + a*B*e + 3*Sqrt[a]*A*Sqrt[c]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x]
)/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(3/4)*e*Sqrt[Sqrt[c]*d + Sqrt
[a]*e])))/(2*a*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.34

method	result
derivativedivides	$2e^2c^2\left(\frac{\frac{\left(Bae+A\sqrt{ac\,e^2}\right)\sqrt{ex+d}}{2c\left(cd+\sqrt{ac\,e^2}\right)\left(-ex+\frac{\sqrt{ac\,e^2}}{c}\right)}+\frac{\left(2Acd+Bae+3A\sqrt{ac\,e^2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}\right)}{2\left(cd+\sqrt{ac\,e^2}\right)\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}}{4eca\sqrt{ac\,e^2}}+-\frac{\left(Bae-A\sqrt{ac\,e^2}\right.}{2c\left(cd-\sqrt{ac\,e^2}\right)\left(-e\right.}\right.$
default	$2e^2c^2\left(\frac{\frac{\left(Bae+A\sqrt{ac\,e^2}\right)\sqrt{ex+d}}{2c\left(cd+\sqrt{ac\,e^2}\right)\left(-ex+\frac{\sqrt{ac\,e^2}}{c}\right)}+\frac{\left(2Acd+Bae+3A\sqrt{ac\,e^2}\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}\right)}{2\left(cd+\sqrt{ac\,e^2}\right)\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}}}{4eca\sqrt{ac\,e^2}}+-\frac{\left(Bae-A\sqrt{ac\,e^2}\right.}{2c\left(cd-\sqrt{ac\,e^2}\right)\left(-e\right.}\right.$
pseudoelliptic	$\frac{ce\left(\frac{\left(-c^2e^2x^2(Acd-Bae)\sqrt{ac\,e^2}+2A\,c^4d^2e^2x^2-2e^2a\left(\frac{3}{2}A\,e^2x^2-\frac{1}{2}Bde\,x^2+A\,d^2\right)c^3+a^2\left(3A\,e^4-Bd\,e^3\right)c^2+\left(ac\,e^2\right)^{\frac{3}{2}}Acd-\left(ac\right.\right.}{2}\right.}{}$

input
```
int((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*e^2*c^2*(1/4/e/c/a/(a*c*e^2)^(1/2)*(1/2/c/(c*d+(a*c*e^2)^(1/2))*(B*a*e+A
*(a*c*e^2)^(1/2))*(e*x+d)^(1/2)/(-e*x+(a*c*e^2)^(1/2)/c)+1/2*(2*A*c*d+B*a*
e+3*A*(a*c*e^2)^(1/2))/(c*d+(a*c*e^2)^(1/2))/((c*d+(a*c*e^2)^(1/2))*c)^(1/
2)*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)))+1/4/e/c/a/(a*
c*e^2)^(1/2)*(-1/2/c/(c*d-(a*c*e^2)^(1/2))*(B*a*e-A*(a*c*e^2)^(1/2))*(e*x+
d)^(1/2)/(-e*x-(a*c*e^2)^(1/2)/c)+1/2*(-2*A*c*d-B*a*e+3*A*(a*c*e^2)^(1/2))
/(-c*d+(a*c*e^2)^(1/2))/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^
(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7506 vs. $2(195) = 390$.

Time = 55.30 (sec) , antiderivative size = 7506, normalized size of antiderivative = 30.02

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)**(1/2)/(-c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^2}\,dx = \int \frac{Bx+A}{(cx^2-a)^2\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((c*x^2 - a)^2*sqrt(e*x + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1182 vs. $2(195) = 390$.

Time = 0.29 (sec) , antiderivative size = 1182, normalized size of antiderivative = 4.73

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/4*((a*c*d^2*e - a^2*e^3)^2*A*c*d*e*abs(c) - (a*c*d^2*e - a^2*e^3)^2*B*a
*e^2*abs(c) + (sqrt(a*c)*c^2*d^4*e - 4*sqrt(a*c)*a*c*d^2*e^3 + 3*sqrt(a*c)
*a^2*e^5)*A*abs(a*c*d^2*e - a^2*e^3)*abs(c) + 2*(sqrt(a*c)*a*c*d^3*e^2 - s
qrt(a*c)*a^2*d*e^4)*B*abs(a*c*d^2*e - a^2*e^3)*abs(c) - (2*a*c^4*d^7*e - 7
*a^2*c^3*d^5*e^3 + 8*a^3*c^2*d^3*e^5 - 3*a^4*c*d*e^7)*A*abs(c) - (a^2*c^3*
d^6*e^2 - 2*a^3*c^2*d^4*e^4 + a^4*c*d^2*e^6)*B*abs(c))*arctan(sqrt(e*x + d
)/sqrt(-(a*c^2*d^3 - a^2*c*d*e^2 + sqrt((a*c^2*d^3 - a^2*c*d*e^2)^2 - (a*c
^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)*(a*c^2*d^2 - a^2*c*e^2)))/(a*c^2*d^2 -
 a^2*c*e^2)))/((a^2*c^3*d^4*e - 2*a^3*c^2*d^2*e^3 + a^4*c*e^5 - sqrt(a*c)*
a*c^3*d^5 + 2*sqrt(a*c)*a^2*c^2*d^3*e^2 - sqrt(a*c)*a^3*c*d*e^4)*sqrt(-c^2
*d - sqrt(a*c)*c*e)*abs(a*c*d^2*e - a^2*e^3)) - 1/4*((a*c*d^2*e - a^2*e^3)
^2*sqrt(a*c)*A*c*d*e*abs(c) - (a*c*d^2*e - a^2*e^3)^2*sqrt(a*c)*B*a*e^2*ab
s(c) - (a*c^3*d^4*e - 4*a^2*c^2*d^2*e^3 + 3*a^3*c*e^5)*A*abs(a*c*d^2*e - a
^2*e^3)*abs(c) - 2*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*B*abs(a*c*d^2*e - a^2*e
^3)*abs(c) - (2*sqrt(a*c)*a*c^4*d^7*e - 7*sqrt(a*c)*a^2*c^3*d^5*e^3 + 8*sq
rt(a*c)*a^3*c^2*d^3*e^5 - 3*sqrt(a*c)*a^4*c*d*e^7)*A*abs(c) - (sqrt(a*c)*a
^2*c^3*d^6*e^2 - 2*sqrt(a*c)*a^3*c^2*d^4*e^4 + sqrt(a*c)*a^4*c*d^2*e^6)*B*
abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a*c^2*d^3 - a^2*c*d*e^2 - sqrt((a*c^2*
d^3 - a^2*c*d*e^2)^2 - (a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)*(a*c^2*d^2
- a^2*c*e^2)))/(a*c^2*d^2 - a^2*c*e^2)))/((a^2*c^4*d^5 - 2*a^3*c^3*d^3*...
```

Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 10862, normalized size of antiderivative = 43.45

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/((a - c*x^2)^2*(d + e*x)^(1/2)),x)
```

output
```
atan(((((192*A*a^5*c^3*e^7 - 128*B*a^5*c^3*d*e^6 + 64*A*a^3*c^5*d^4*e^3 -
256*A*a^4*c^4*d^2*e^5 + 128*B*a^4*c^4*d^3*e^4)/(8*(a^5*e^4 + a^3*c^2*d^4 -
 2*a^4*c*d^2*e^2)) + ((d + e*x)^(1/2)*((4*A^2*a^3*c^5*d^5 + B^2*a^2*e^5*(a
^9*c^3)^(1/2) - 15*A^2*a^4*c^4*d^3*e^2 + B^2*a^5*c^3*d^3*e^2 - 6*A*B*a^6*c
^2*e^5 - 5*A^2*c^2*d^2*e^3*(a^9*c^3)^(1/2) + 15*A^2*a^5*c^3*d*e^4 + 3*B^2*
a^6*c^2*d*e^4 + 9*A^2*a*c*e^5*(a^9*c^3)^(1/2) + 6*A*B*c^2*d^3*e^2*(a^9*c^3
)^(1/2) + 4*A*B*a^4*c^4*d^4*e + 3*B^2*a*c*d^2*e^3*(a^9*c^3)^(1/2) - 6*A*B*
a^5*c^3*d^2*e^3 - 14*A*B*a*c*d*e^4*(a^9*c^3)^(1/2))/(64*(a^6*c^6*d^6 - a^9
*c^3*e^6 - 3*a^7*c^5*d^4*e^2 + 3*a^8*c^4*d^2*e^4)))^(1/2)*(64*a^5*c^4*d*e^
6 + 64*a^3*c^6*d^5*e^2 - 128*a^4*c^5*d^3*e^4))/(a^4*e^4 + a^2*c^2*d^4 - 2*
a^3*c*d^2*e^2))*((4*A^2*a^3*c^5*d^5 + B^2*a^2*e^5*(a^9*c^3)^(1/2) - 15*A^2
*a^4*c^4*d^3*e^2 + B^2*a^5*c^3*d^3*e^2 - 6*A*B*a^6*c^2*e^5 - 5*A^2*c^2*d^2
*e^3*(a^9*c^3)^(1/2) + 15*A^2*a^5*c^3*d*e^4 + 3*B^2*a^6*c^2*d*e^4 + 9*A^2*
a*c*e^5*(a^9*c^3)^(1/2) + 6*A*B*c^2*d^3*e^2*(a^9*c^3)^(1/2) + 4*A*B*a^4*c^
4*d^4*e + 3*B^2*a*c*d^2*e^3*(a^9*c^3)^(1/2) - 6*A*B*a^5*c^3*d^2*e^3 - 14*A
*B*a*c*d*e^4*(a^9*c^3)^(1/2))/(64*(a^6*c^6*d^6 - a^9*c^3*e^6 - 3*a^7*c^5*d
^4*e^2 + 3*a^8*c^4*d^2*e^4)))^(1/2) - ((d + e*x)^(1/2)*(9*A^2*a^2*c^3*e^6
+ B^2*a^3*c^2*e^6 + 4*A^2*c^5*d^4*e^2 + B^2*a^2*c^3*d^2*e^4 - 11*A^2*a*c^4
*d^2*e^4 + 4*A*B*a*c^4*d^3*e^3 - 8*A*B*a^2*c^3*d*e^5))/(a^4*e^4 + a^2*c^2*
d^4 - 2*a^3*c*d^2*e^2))*((4*A^2*a^3*c^5*d^5 + B^2*a^2*e^5*(a^9*c^3)^(1/...
```

Reduce [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 2239, normalized size of antiderivative = 8.96

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^2,x)
```

output
```
( - 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*b*e**3 + 8*sqrt(a)*sqrt(sqrt(c)*sqr
t(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d
)))*a**2*c*d*e**2 - 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d +
 e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*b*c*d**2*e + 2*sqrt(a)
*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c
)*sqrt(a)*e - c*d)))*a*b*c*e**3*x**2 - 4*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e -
c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**
2*d**3 - 8*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(s
qrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**2*d*e**2*x**2 + 2*sqrt(a)*sqrt
(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqr
t(a)*e - c*d)))*b*c**2*d**2*e*x**2 + 4*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*
d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c**3*d*
*3*x**2 + 6*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(
sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*e**3 - 4*sqrt(c)*sqrt(sqrt(c)
*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e -
 c*d)))*a**2*b*d*e**2 - 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt
(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c*d**2*e - 6*sq
rt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(s
qrt(c)*sqrt(a)*e - c*d)))*a**2*c*e**3*x**2 + 4*sqrt(c)*sqrt(sqrt(c)*sqr...
```

3.133 $\int \frac{A+Bx}{(d+ex)^{3/2}(a-cx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 303

$$\begin{aligned}\int \frac{A+Bx}{(d+ex)^{3/2}(a-cx^2)^2}\,dx =& -\frac{e(Acd^2-6aBde+5aAe^2)}{2a(cd^2-ae^2)^2\sqrt{d+ex}} \\ &+\frac{a(Bd-Ae)+(Acd-aBe)x}{2a(cd^2-ae^2)\sqrt{d+ex}(a-cx^2)} \\ &-\frac{\left(2Acd+3aBe-5\sqrt{a}A\sqrt{c}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{4a^{3/2}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}} \\ &+\frac{\left(2Acd+3aBe+5\sqrt{a}A\sqrt{c}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{4a^{3/2}\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)^{5/2}}\end{aligned}$$

output

```
-1/2*e*(5*A*a*e^2+A*c*d^2-6*B*a*d*e)/a/(-a*e^2+c*d^2)^2/(e*x+d)^(1/2)+1/2*
(a*(-A*e+B*d)+(A*c*d-B*a*e)*x)/a/(-a*e^2+c*d^2)/(e*x+d)^(1/2)/(-c*x^2+a)-1
/4*(2*A*c*d+3*B*a*e-5*a^(1/2)*A*c^(1/2)*e)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(
c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(3/2)/c^(1/4)/(c^(1/2)*d-a^(1/2)*e)^(5/2)+1/
4*(2*A*c*d+3*B*a*e+5*a^(1/2)*A*c^(1/2)*e)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c
^(1/2)*d+a^(1/2)*e)^(1/2))/a^(3/2)/c^(1/4)/(c^(1/2)*d+a^(1/2)*e)^(5/2)
```

Mathematica [A] (verified)

Time = 3.31 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)^2}\,dx = \frac{-\frac{2\sqrt{a}\left(Ac^2d^2x(d+ex)+a^2e^2(5Bd-4Ae+Bex)+ac\left(Bd\left(d^2-dex-6e^2x^2\right)+Ae\left(-2d^2-dex+5e^2x^2\right)\right)\right)}{(cd^2-ae^2)^2\sqrt{d+ex}(-a+cx^2)} -$$

input
```
Integrate[(A + B*x)/((d + e*x)^(3/2)*(a - c*x^2)^2),x]
```

output
```
((-2*Sqrt[a]*(A*c^2*d^2*x*(d + e*x) + a^2*e^2*(5*B*d - 4*A*e + B*e*x) + a*
c*(B*d*(d^2 - d*e*x - 6*e^2*x^2) + A*e*(-2*d^2 - d*e*x + 5*e^2*x^2))))/((c
*d^2 - a*e^2)^2*Sqrt[d + e*x]*(-a + c*x^2)) + ((2*A*c*d + 3*a*B*e + 5*Sqrt
[a]*A*Sqrt[c]*e)*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(
Sqrt[c]*d + Sqrt[a]*e)])/((Sqrt[c]*d + Sqrt[a]*e)^2*Sqrt[-(c*d) - Sqrt[a]*
Sqrt[c]*e]) - ((2*A*c*d + 3*a*B*e - 5*Sqrt[a]*A*Sqrt[c]*e)*ArcTan[(Sqrt[-(
c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/((Sqrt[
c]*d - Sqrt[a]*e)^2*Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]))/(4*a^(3/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {686, 27, 655, 25, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\left(a-cx^2\right)^2(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{686}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)} - \frac{\int -\frac{c\left(2Acd^2+3aBed-5aAe^2+3e(Acd-aBe)x\right)}{2(d+ex)^{3/2}(a-cx^2)}dx}{2ac\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2Acd^2+3aBed-5aAe^2+3e(Acd-aBe)x}{(d+ex)^{3/2}(a-cx^2)}dx}{4a\left(cd^2-ae^2\right)}+\frac{x(Acd-aBe)+a(Bd-Ae)}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 655$$

$$\frac{-\frac{\int -\frac{2Acd\left(cd^2-4ae^2\right)+3aBe\left(cd^2+ae^2\right)+ce\left(Acd^2-6aBed+5aAe^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{cd^2-ae^2}-\frac{2e\left(5aAe^2-6aBde+Acd^2\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}}{4a\left(cd^2-ae^2\right)}+\frac{x(Acd-aBe)+a(Bd-Ae)}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2Acd\left(cd^2-4ae^2\right)+3aBe\left(cd^2+ae^2\right)+ce\left(Acd^2-6aBed+5aAe^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{cd^2-ae^2}-\frac{2e\left(5aAe^2-6aBde+Acd^2\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}}{4a\left(cd^2-ae^2\right)}+\frac{x(Acd-aBe)+a(Bd-Ae)}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 654$$

$$\frac{\frac{2\int -\frac{e\left(Acd\left(cd^2-13ae^2\right)+3aBe\left(3cd^2+ae^2\right)+c\left(Acd^2-6aBed+5aAe^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}-\frac{2e\left(5aAe^2-6aBde+Acd^2\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}}{4a\left(cd^2-ae^2\right)}+\frac{x(Acd-aBe)+a(Bd-Ae)}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{2\int \frac{e\left(Acd\left(cd^2-13ae^2\right)+3aBe\left(3cd^2+ae^2\right)+c\left(Acd^2-6aBed+5aAe^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}-\frac{2e\left(5aAe^2-6aBde+Acd^2\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}}{4a\left(cd^2-ae^2\right)}+\frac{x(Acd-aBe)+a(Bd-Ae)}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{2e\int \frac{Acd\left(cd^2-13ae^2\right)+3aBe\left(3cd^2+ae^2\right)+c\left(Acd^2-6aBed+5aAe^2\right)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{cd^2-ae^2}-\frac{2e\left(5aAe^2-6aBde+Acd^2\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}}{4a\left(cd^2-ae^2\right)}+\frac{x(Acd-aBe)+a(Bd-Ae)}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1480$$

$$-\frac{\frac{2e\left(\frac{\sqrt{c}(\sqrt{c}d-\sqrt{a}e)^2(5\sqrt{a}A\sqrt{c}e+3aBe+2Acd)\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d+\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{\sqrt{c}(\sqrt{a}e+\sqrt{c}d)^2(-5\sqrt{a}A\sqrt{c}e+3aBe+2Acd)\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d-\sqrt{a}e)}\cdots}{2\sqrt{a}e}\cdots\right.}{cd^2-ae^2}\cdots}{4a\left(cd^2-ae^2\right)}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{2e\left(\frac{(\sqrt{a}e+\sqrt{c}d)^2(-5\sqrt{a}A\sqrt{c}e+3aBe+2Acd)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}\sqrt[4]{c}e\sqrt{\sqrt{c}d-\sqrt{a}e}}-\frac{(\sqrt{c}d-\sqrt{a}e)^2(5\sqrt{a}A\sqrt{c}e+3aBe+2Acd)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}\sqrt[4]{c}e\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{cd^2-ae^2}-\frac{2e(5a\cdots}{\sqrt{\cdots}}}{4a\left(cd^2-ae^2\right)}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{2a\left(a-cx^2\right)\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

input
```
Int[(A + B*x)/((d + e*x)^(3/2)*(a - c*x^2)^2),x]
```

output
```
(a*(B*d - A*e) + (A*c*d - a*B*e)*x)/(2*a*(c*d^2 - a*e^2)*Sqrt[d + e*x]*(a
 - c*x^2)) + ((-2*e*(A*c*d^2 - 6*a*B*d*e + 5*a*A*e^2))/((c*d^2 - a*e^2)*Sqr
t[d + e*x]) - (2*e*(((Sqrt[c]*d + Sqrt[a]*e)^2*(2*A*c*d + 3*a*B*e - 5*Sqrt
[a]*A*Sqrt[c]*e)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*
e]])/(2*Sqrt[a]*c^(1/4)*e*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) - ((Sqrt[c]*d - Sqr
t[a]*e)^2*(2*A*c*d + 3*a*B*e + 5*Sqrt[a]*A*Sqrt[c]*e)*ArcTanh[(c^(1/4)*Sqr
t[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(1/4)*e*Sqrt[Sqrt[c
]*d + Sqrt[a]*e])))/(c*d^2 - a*e^2))/(4*a*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 655
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))
), x] + Simp[1/(c*d^2 + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d*f + a*e*g
- c*(e*f - d*g)*x, x]/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
&& FractionQ[m] && LtQ[m, -1]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.42

method	result
pseudoelliptic	$-\frac{2\left(ce\left(\frac{\left(A c\, d^{2}+5 e\left(A e-\frac{6 B d}{5}\right) a\right) \sqrt{a c\, e^{2}}}{8}-\frac{A\, c^{2} d^{3}}{4}+a d e\left(A e-\frac{3 B d}{8}\right) c-\frac{3 B\, e^{3} a^{2}}{8}\right) \sqrt{e x+d}\, \sqrt{\left(c d+\sqrt{a c\, e^{2}}\right) c}\, \left(-c\, x^{2}+a\right) \text{ar}}{}$
derivativedivides	$2 e^{2}\left(-\frac{A e-B d}{\left(a\, e^{2}-c\, d^{2}\right)^{2} \sqrt{e x+d}}-\frac{-\frac{c\left(A a\, e^{2}+A c\, d^{2}-2 B a d e\right) (e x+d)^{\frac{3}{2}}}{4 a e}+\frac{\left(3 A a c d\, e^{2}+A\, c^{2} d^{3}-B\, e^{3} a^{2}-3 B a c\, d^{2} e\right) \sqrt{e x+d}}{4 a e}}{-c (e x+d)^{2}+2 c d (e x+d)+a\, e^{2}-c\, d^{2}}+\frac{c}{}\right.$
default	$2 e^{2}\left(-\frac{A e-B d}{\left(a\, e^{2}-c\, d^{2}\right)^{2} \sqrt{e x+d}}-\frac{-\frac{c\left(A a\, e^{2}+A c\, d^{2}-2 B a d e\right) (e x+d)^{\frac{3}{2}}}{4 a e}+\frac{\left(3 A a c d\, e^{2}+A\, c^{2} d^{3}-B\, e^{3} a^{2}-3 B a c\, d^{2} e\right) \sqrt{e x+d}}{4 a e}}{-c (e x+d)^{2}+2 c d (e x+d)+a\, e^{2}-c\, d^{2}}+\frac{c}{}\right.$

input
```
int((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(c*e*(1/8*(A*c*d^2+5*e*(A*e-6/5*B*d)*a)*(a*c*e^2)^(1/2)-1/4*A*c^2*d^3+a
*d*e*(A*e-3/8*B*d)*c-3/8*B*e^3*a^2)*(e*x+d)^(1/2)*((c*d+(a*c*e^2)^(1/2))*c
)^(1/2)*(-c*x^2+a)*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)
)+(c*e*(1/8*(-A*c*d^2-5*e*(A*e-6/5*B*d)*a)*(a*c*e^2)^(1/2)-1/4*A*c^2*d^3+a
*d*e*(A*e-3/8*B*d)*c-3/8*B*e^3*a^2)*(e*x+d)^(1/2)*(-c*x^2+a)*arctanh(c*(e*
x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+(-1/4*A*d^2*x*(e*x+d)*c^2+1/2*
a*(-1/2*B*d^3+e*(1/2*B*x+A)*d^2+1/2*e^2*x*(6*B*x+A)*d-5/2*A*e^3*x^2)*c+(-5
/4*B*d+e*(-1/4*B*x+A))*e^2*a^2)*((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*(a*c*e^2)^
(1/2))*((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)/
(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)/(
a*e^2-c*d^2)^2/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)**(3/2)/(-c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)^2}\,dx=\int \frac{Bx+A}{\left(cx^2-a\right)^2(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((c*x^2 - a)^2*(e*x + d)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1970 vs. 2(244) = 488.

Time = 0.50 (sec) , antiderivative size = 1970, normalized size of antiderivative = 6.50

$$\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/4*(6*(a*c^2*d^4*e - 2*a^2*c*d^2*e^3 + a^3*e^5)^2*B*a*c*d*e^2*abs(c) - (a
*c^2*d^4*e - 2*a^2*c*d^2*e^3 + a^3*e^5)^2*(c^2*d^2*e + 5*a*c*e^3)*A*abs(c)
 - (sqrt(a*c)*c^4*d^7*e - 15*sqrt(a*c)*a*c^3*d^5*e^3 + 27*sqrt(a*c)*a^2*c^
2*d^3*e^5 - 13*sqrt(a*c)*a^3*c*d*e^7)*A*abs(a*c^2*d^4*e - 2*a^2*c*d^2*e^3
+ a^3*e^5)*abs(c) - 3*(3*sqrt(a*c)*a*c^3*d^6*e^2 - 5*sqrt(a*c)*a^2*c^2*d^4
*e^4 + sqrt(a*c)*a^3*c*d^2*e^6 + sqrt(a*c)*a^4*e^8)*B*abs(a*c^2*d^4*e - 2*
a^2*c*d^2*e^3 + a^3*e^5)*abs(c) + 2*(a*c^7*d^12*e - 8*a^2*c^6*d^10*e^3 + 2
2*a^3*c^5*d^8*e^5 - 28*a^4*c^4*d^6*e^7 + 17*a^5*c^3*d^4*e^9 - 4*a^6*c^2*d^
2*e^11)*A*abs(c) + 3*(a^2*c^6*d^11*e^2 - 3*a^3*c^5*d^9*e^4 + 2*a^4*c^4*d^7
*e^6 + 2*a^5*c^3*d^5*e^8 - 3*a^6*c^2*d^3*e^10 + a^7*c*d*e^12)*B*abs(c))*ar
ctan(sqrt(e*x + d)/sqrt(-(a*c^3*d^5 - 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4 + sq
rt((a*c^3*d^5 - 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)^2 - (a*c^3*d^6 - 3*a^2*c^
2*d^4*e^2 + 3*a^3*c*d^2*e^4 - a^4*e^6)*(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^
3*c*e^4)))/(a*c^3*d^4 - 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/((a^2*c^5*d^8*e -
 4*a^3*c^4*d^6*e^3 + 6*a^4*c^3*d^4*e^5 - 4*a^5*c^2*d^2*e^7 + a^6*c*e^9 - s
qrt(a*c)*a*c^5*d^9 + 4*sqrt(a*c)*a^2*c^4*d^7*e^2 - 6*sqrt(a*c)*a^3*c^3*d^5
*e^4 + 4*sqrt(a*c)*a^4*c^2*d^3*e^6 - sqrt(a*c)*a^5*c*d*e^8)*sqrt(-c^2*d -
sqrt(a*c)*c*e)*abs(a*c^2*d^4*e - 2*a^2*c*d^2*e^3 + a^3*e^5)) + 1/4*(6*(a*c
^2*d^4*e - 2*a^2*c*d^2*e^3 + a^3*e^5)^2*sqrt(a*c)*B*a*d*e^2*abs(c) - (a*c^
2*d^4*e - 2*a^2*c*d^2*e^3 + a^3*e^5)^2*(sqrt(a*c)*c*d^2*e + 5*sqrt(a*c)...
```

Mupad [B] (verification not implemented)

Time = 9.54 (sec) , antiderivative size = 19787, normalized size of antiderivative = 65.30

$$\int \frac{A+Bx}{(d+ex)^{3/2}(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/((a - c*x^2)^2*(d + e*x)^(3/2)),x)
```

output
```
(((d + e*x)*(B*a^2*e^4 - A*c^2*d^3*e - 11*A*a*c*d*e^3 + 11*B*a*c*d^2*e^2))
/(2*a*(a*e^2 - c*d^2)^2) - (2*(A*e^3 - B*d*e^2))/(a*e^2 - c*d^2) + (c*(d +
 e*x)^2*(5*A*a*e^3 - 6*B*a*d*e^2 + A*c*d^2*e))/(2*a*(a*e^2 - c*d^2)^2))/((
a*e^2 - c*d^2)*(d + e*x)^(1/2) - c*(d + e*x)^(5/2) + 2*c*d*(d + e*x)^(3/2)
) - atan((((d + e*x)^(1/2)*(800*A^2*a^12*c^4*e^20 + 288*B^2*a^13*c^3*e^20
+ 128*A^2*a^3*c^13*d^18*e^2 - 1760*A^2*a^4*c^12*d^16*e^4 + 10240*A^2*a^5*c
^11*d^14*e^6 - 30848*A^2*a^6*c^10*d^12*e^8 + 52480*A^2*a^7*c^9*d^10*e^10 -
 51008*A^2*a^8*c^8*d^8*e^12 + 25600*A^2*a^9*c^7*d^6*e^14 - 3200*A^2*a^10*c
^6*d^4*e^16 - 2432*A^2*a^11*c^5*d^2*e^18 + 288*B^2*a^5*c^11*d^16*e^4 - 576
0*B^2*a^7*c^9*d^12*e^8 + 18432*B^2*a^8*c^8*d^10*e^10 - 25920*B^2*a^9*c^7*d
^8*e^12 + 18432*B^2*a^10*c^6*d^6*e^14 - 5760*B^2*a^11*c^5*d^4*e^16 - 3456*
A*B*a^12*c^4*d*e^19 + 384*A*B*a^4*c^12*d^17*e^3 - 3840*A*B*a^5*c^11*d^15*e
^5 + 11520*A*B*a^6*c^10*d^13*e^7 - 9984*A*B*a^7*c^9*d^11*e^9 - 15360*A*B*a
^8*c^8*d^9*e^11 + 43776*A*B*a^9*c^7*d^7*e^13 - 42240*A*B*a^10*c^6*d^5*e^15
 + 19200*A*B*a^11*c^5*d^3*e^17) + (-(4*A^2*a^3*c^5*d^7 + 9*B^2*a^3*e^7*(a^
9*c)^(1/2) - 35*A^2*a^4*c^4*d^5*e^2 + 70*A^2*a^5*c^3*d^3*e^4 + 9*B^2*a^5*c
^3*d^5*e^2 + 90*B^2*a^6*c^2*d^3*e^4 - 35*A^2*c^3*d^4*e^3*(a^9*c)^(1/2) + 4
5*B^2*a^7*c*d*e^6 + 105*A^2*a^6*c^2*d*e^6 + 25*A^2*a^2*c*e^7*(a^9*c)^(1/2)
 - 30*A*B*a^7*c*e^7 + 30*A*B*c^3*d^5*e^2*(a^9*c)^(1/2) + 154*A^2*a*c^2*d^2
*e^5*(a^9*c)^(1/2) + 12*A*B*a^4*c^4*d^6*e + 45*B^2*a*c^2*d^4*e^3*(a^9*c...
```

Reduce [B] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 3362, normalized size of antiderivative = 11.10

$$\int \frac{A+Bx}{(d+ex)^{3/2}\,(a-cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^2,x)
```

output
```
( - 10*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d +
e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*c*e**4 + 18*sqrt(a)*
sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*b*c*d*e**3 - 18*sqrt(a)*sqrt(d + e*
x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt
(c)*sqrt(a)*e - c*d)))*a**2*c**2*d**2*e**2 + 10*sqrt(a)*sqrt(d + e*x)*sqrt
(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqr
t(a)*e - c*d)))*a**2*c**2*e**4*x**2 + 6*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)
*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e -
 c*d)))*a*b*c**2*d**3*e - 18*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e
- c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*b
*c**2*d*e**3*x**2 + 4*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*
atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**3*d**
4 + 18*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d +
e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**3*d**2*e**2*x**2 - 6
*sqrt(a)*sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c
)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*b*c**3*d**3*e*x**2 - 4*sqrt(a)*
sqrt(d + e*x)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c**4*d**4*x**2 + 6*sqrt(c)*sqrt(d + e*x)
*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqr...
```

3.134 $\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 396

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \frac{(d+ex)^{5/2}(a(Bd+Ae)+(Acd+aBe)x)}{4ac\,(a-cx^2)^2}$$
$$+\frac{\sqrt{d+ex}\left(ae(7Acd^2-14aBde-5aAe^2)+\left(2Acd(3cd^2-2ae^2)-7aBe(cd^2+ae^2)\right)x\right)}{16a^2c^2\,(a-cx^2)}$$
$$+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\left(7aBe(2\sqrt{c}d+3\sqrt{a}e)-A\left(12c^{3/2}d^2+18\sqrt{a}cde+5a\sqrt{c}e^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{11/4}}$$
$$-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}\left(7aBe(2\sqrt{c}d-3\sqrt{a}e)-A\left(12c^{3/2}d^2-18\sqrt{a}cde+5a\sqrt{c}e^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{32a^{5/2}c^{11/4}}$$

output
```
1/4*(e*x+d)^(5/2)*(a*(A*e+B*d)+(A*c*d+B*a*e)*x)/a/c/(-c*x^2+a)^2+1/16*(e*x
+d)^(1/2)*(a*e*(-5*A*a*e^2+7*A*c*d^2-14*B*a*d*e)+(2*A*c*d*(-2*a*e^2+3*c*d^
2)-7*a*B*e*(a*e^2+c*d^2))*x)/a^2/c^2/(-c*x^2+a)+1/32*(c^(1/2)*d-a^(1/2)*e)
^(3/2)*(7*a*B*e*(2*c^(1/2)*d+3*a^(1/2)*e)-A*(12*c^(3/2)*d^2+18*a^(1/2)*c*d
*e+5*a*c^(1/2)*e^2))*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(
1/2))/a^(5/2)/c^(11/4)-1/32*(c^(1/2)*d+a^(1/2)*e)^(3/2)*(7*a*B*e*(2*c^(1/2
)*d-3*a^(1/2)*e)-A*(12*c^(3/2)*d^2-18*a^(1/2)*c*d*e+5*a*c^(1/2)*e^2))*arct
anh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(5/2)/c^(11/4)
```

Mathematica [A] (verified)

Time = 6.03 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \frac{-\frac{2\sqrt{a}c\sqrt{d+ex}(6Ac^3d^3x^3+a^3e^2(5Ae+7B(2d+ex))-ac^2dx(7Bdex^2+A(10d^2+dex+8e^2x^2))-a^2c(Ae(1}{(a-cx^2)^2}}{}$$

input
```
Integrate[((A + B*x)*(d + e*x)^(7/2))/(a - c*x^2)^3,x]
```

output
```
((-2*Sqrt[a]*c*Sqrt[d + e*x]*(6*A*c^3*d^3*x^3 + a^3*e^2*(5*A*e + 7*B*(2*d
+ e*x)) - a*c^2*d*x*(7*B*d*e*x^2 + A*(10*d^2 + d*e*x + 8*e^2*x^2)) - a^2*c
*(A*e*(11*d^2 + 4*d*e*x + 9*e^2*x^2) + B*(4*d^3 + 5*d^2*e*x + 26*d*e^2*x^2
 + 11*e^3*x^3))))/(a - c*x^2)^2 - (Sqrt[c]*d + Sqrt[a]*e)*Sqrt[-(c*d) - Sq
rt[a]*Sqrt[c]*e]*(7*a*B*e*(-2*Sqrt[c]*d + 3*Sqrt[a]*e) + A*(12*c^(3/2)*d^2
 - 18*Sqrt[a]*c*d*e + 5*a*Sqrt[c]*e^2))*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt
[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)] - (Sqrt[c]*(Sqrt[c]*d - Sqr
t[a]*e)^2*(-7*a*B*e*(2*Sqrt[c]*d + 3*Sqrt[a]*e) + A*(12*c^(3/2)*d^2 + 18*S
qrt[a]*c*d*e + 5*a*Sqrt[c]*e^2))*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*
Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e])
/(32*a^(5/2)*c^3)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {684, 27, 684, 27, 654, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^3}\,dx$$

$$\Big\downarrow \ 684$$

$$\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\,(a-cx^2)^2} - \frac{\int -\frac{(d+ex)^{3/2}(6Acd^2-ae(7Bd+5Ae)+e(Acd-7aBe)x)}{2(a-cx^2)^2}dx}{4ac}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(d+ex)^{3/2}\left(6Acd^2-ae(7Bd+5Ae)+e(Acd-7aBe)x\right)}{\left(a-cx^2\right)^2}dx}{8ac}+\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \; 684$$

$$\frac{\frac{\sqrt{d+ex}\left(x\left(2Acd\left(3cd^2-2ae^2\right)-7aBe\left(ae^2+cd^2\right)\right)+ae\left(-5aAe^2-14aBde+7Acd^2\right)\right)}{2ac\left(a-cx^2\right)}-\frac{\int \frac{14aBde\left(cd^2-2ae^2\right)-A\left(12c^2d^4-19ace^2d^2+5a^2e^4\right)-e\left(2Acd \cdots\right.}{2\sqrt{d+ex}\left(a-cx^2\right)}}{2ac}}{8ac}$$

$$\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt{d+ex}\left(x\left(2Acd\left(3cd^2-2ae^2\right)-7aBe\left(ae^2+cd^2\right)\right)+ae\left(-5aAe^2-14aBde+7Acd^2\right)\right)}{2ac\left(a-cx^2\right)}-\frac{\int \frac{14aBde\left(cd^2-2ae^2\right)-A\left(12c^2d^4-19ace^2d^2+5a^2e^4\right)-e\left(2Acd \cdots\right.}{\sqrt{d+ex}\left(a-cx^2\right)}}{4ac}}{8ac}$$

$$\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \; 654$$

$$\frac{\frac{\sqrt{d+ex}\left(x\left(2Acd\left(3cd^2-2ae^2\right)-7aBe\left(ae^2+cd^2\right)\right)+ae\left(-5aAe^2-14aBde+7Acd^2\right)\right)}{2ac\left(a-cx^2\right)}-\frac{\int \frac{e\left(\left(cd^2-ae^2\right)\left(6Acd^2-7aBed-5aAe^2\right)+\left(2Acd\left(3cd^2-4ae^2\right) \cdots\right.\right.}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}}{2ac}}{8ac}$$

$$\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt{d+ex}\left(x\left(2Acd\left(3cd^2-2ae^2\right)-7aBe\left(ae^2+cd^2\right)\right)+ae\left(-5aAe^2-14aBde+7Acd^2\right)\right)}{2ac\left(a-cx^2\right)}-\frac{e\int \frac{\left(cd^2-ae^2\right)\left(6Acd^2-7aBed-5aAe^2\right)+\left(2Acd\left(3cd^2-4ae^2\right) \cdots\right.}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}}{2ac}}{8ac}$$

$$\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{\sqrt{d+ex}\left(x\left(2Acd\left(3cd^2-2ae^2\right)-7aBe\left(ae^2+cd^2\right)\right)+ae\left(-5aAe^2-14aBde+7Acd^2\right)\right)}{2ac\left(a-cx^2\right)}-\frac{e\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2\left(-21a^{3/2}Be^2+18\sqrt{a}Acde+5aA\sqrt{c}e^2-14aB\right.}{2\sqrt{a}}\right.}{}}{}$$

$$\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\sqrt{d+ex}\left(x\left(2Acd\left(3cd^2-2ae^2\right)-7aBe\left(ae^2+cd^2\right)\right)+ae\left(-5aAe^2-14aBde+7Acd^2\right)\right)}{2ac\left(a-cx^2\right)}-\frac{e\left(\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}\left(-21a^{3/2}Be^2+18\sqrt{a}Acde+5aA\sqrt{c}e^2-14aB\right.}{2\sqrt{a}c^{3/4}e}\right.}{}}{8ac}$$

$$\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

input
```
Int[((A + B*x)*(d + e*x)^(7/2))/(a - c*x^2)^3,x]
```

output
```
((d + e*x)^(5/2)*(a*(B*d + A*e) + (A*c*d + a*B*e)*x))/(4*a*c*(a - c*x^2)^2
) + ((Sqrt[d + e*x]*(a*e*(7*A*c*d^2 - 14*a*B*d*e - 5*a*A*e^2) + (2*A*c*d*(
3*c*d^2 - 2*a*e^2) - 7*a*B*e*(c*d^2 + a*e^2))*x))/(2*a*c*(a - c*x^2)) - (e
*(((Sqrt[c]*d - Sqrt[a]*e)^(3/2)*(12*A*c^(3/2)*d^2 - 14*a*B*Sqrt[c]*d*e +
18*Sqrt[a]*A*c*d*e - 21*a^(3/2)*B*e^2 + 5*a*A*Sqrt[c]*e^2)*ArcTanh[(c^(1/4
)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(2*Sqrt[a]*c^(3/4)*e) + ((S
qrt[c]*d + Sqrt[a]*e)^(3/2)*(7*a*B*e*(2*Sqrt[c]*d - 3*Sqrt[a]*e) - A*(12*c
^(3/2)*d^2 - 18*Sqrt[a]*c*d*e + 5*a*Sqrt[c]*e^2))*ArcTanh[(c^(1/4)*Sqrt[d
+ e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(3/4)*e)))/(2*a*c))/(8*
a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.26

method	result
pseudoelliptic	$-\frac{5\left(-\frac{e\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}\left(\frac{\left(-6A\,c^{2}d^{3}+8e\left(Ae+\frac{7Bd}{8}\right)dac-21B\,e^{3}a^{2}\right)\sqrt{ac\,e^{2}}}{5}+c\left(\frac{12A\,c^{2}d^{4}}{5}-\frac{19e\,d^{2}\left(Ae+\frac{14Bd}{19}\right)ac}{5}+a^{2}e^{3}\left(Ae+\right.\right.\right.}{2}\right.}{}$
default	$2e^{4}\left(\frac{\left(8Aacd\,e^{2}-6A\,c^{2}d^{3}+11B\,e^{3}a^{2}+7Bac\,d^{2}e\right)(ex+d)^{\frac{7}{2}}}{32a^{2}e^{3}c}+\frac{\left(9A\,a^{2}e^{4}-23Aac\,d^{2}e^{2}+18A\,c^{2}d^{4}-7B\,a^{2}d\,e^{3}-21Bac\,d^{3}e\right)(ex+d)^{\frac{5}{2}}}{32c\,a^{2}e^{3}}\right.$
derivativedivides	$-2e^{4}\left(-\frac{\left(8Aacd\,e^{2}-6A\,c^{2}d^{3}+11B\,e^{3}a^{2}+7Bac\,d^{2}e\right)(ex+d)^{\frac{7}{2}}}{32a^{2}e^{3}c}+\frac{\left(9A\,a^{2}e^{4}-23Aac\,d^{2}e^{2}+18A\,c^{2}d^{4}-7B\,a^{2}d\,e^{3}-21Bac\,d^{3}e\right)(e}{32c\,a^{2}e^{3}}\right.$

input
```
int((B*x+A)*(e*x+d)^(7/2)/(-c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-5/16/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)/(a*c*e^2)^(1/2)*(-1/2*e*((c*d+(a*c*e
^2)^(1/2))*c)^(1/2)*(1/5*(-6*A*c^2*d^3+8*e*(A*e+7/8*B*d)*d*a*c-21*B*e^3*a^
2)*(a*c*e^2)^(1/2)+c*(12/5*A*c^2*d^4-19/5*e*d^2*(A*e+14/19*B*d)*a*c+a^2*e^
3*(A*e+28/5*B*d)))*(-c*x^2+a)^2*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1
/2))*c)^(1/2))+((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*(-1/2*e*(1/5*(6*A*c^2*d^3-
8*e*(A*e+7/8*B*d)*d*a*c+21*B*e^3*a^2)*(a*c*e^2)^(1/2)+c*(12/5*A*c^2*d^4-19
/5*e*d^2*(A*e+14/19*B*d)*a*c+a^2*e^3*(A*e+28/5*B*d)))*(-c*x^2+a)^2*arctanh
(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+((c*d+(a*c*e^2)^(1/2))*c
)^(1/2)*(e*x+d)^(1/2)*(a*c*e^2)^(1/2)*(6/5*A*c^3*d^3*x^3-2*d*(A*d^2+1/10*e
*x*(7*B*x+A)*d+4/5*A*e^2*x^2)*x*a*c^2-11/5*(4/11*B*d^3+e*(5/11*B*x+A)*d^2+
4/11*e^2*(13/2*B*x+A)*x*d+9/11*e^3*x^2*(11/9*B*x+A))*a^2*c+e^2*a^3*(14/5*B
*d+e*(7/5*B*x+A)))))/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)/a^2/c^2/(-c*x^2+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6669 vs. $2(326) = 652$.

Time = 10.86 (sec) , antiderivative size = 6669, normalized size of antiderivative = 16.84

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(7/2)/(-c*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(e*x+d)**(7/2)/(-c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \int -\frac{(Bx+A)(ex+d)^{\frac{7}{2}}}{(cx^2-a)^3}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(7/2)/(-c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-integrate((B*x + A)*(e*x + d)^(7/2)/(c*x^2 - a)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1106 vs. $2(326) = 652$.

Time = 0.32 (sec) , antiderivative size = 1106, normalized size of antiderivative = 2.79

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(7/2)/(-c*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/32*(2*(3*a*c^2*d^3*e - 4*a^2*c*d*e^3)*A*e^2*abs(c) - 7*(a^2*c*d^2*e^2 -
 3*a^3*e^4)*B*e^2*abs(c) + (6*sqrt(a*c)*c^2*d^4*e - 11*sqrt(a*c)*a*c*d^2*e
^3 + 5*sqrt(a*c)*a^2*e^5)*A*abs(c)*abs(e) - 7*(sqrt(a*c)*a*c*d^3*e^2 - sqr
t(a*c)*a^2*d*e^4)*B*abs(c)*abs(e) - (12*c^3*d^5*e - 19*a*c^2*d^3*e^3 + 5*a
^2*c*d*e^5)*A*abs(c) + 14*(a*c^2*d^4*e^2 - 2*a^2*c*d^2*e^4)*B*abs(c))*arct
an(sqrt(e*x + d)/sqrt(-(a^2*c^3*d + sqrt(a^4*c^6*d^2 - (a^2*c^3*d^2 - a^3*
c^2*e^2)*a^2*c^3))/(a^2*c^3)))/((a^3*c^3*e - sqrt(a*c)*a^2*c^3*d)*sqrt(-c^
2*d - sqrt(a*c)*c*e)*abs(e)) - 1/32*(2*(3*a*c^2*d^3*e - 4*a^2*c*d*e^3)*A*e
^2*abs(c) - 7*(a^2*c*d^2*e^2 - 3*a^3*e^4)*B*e^2*abs(c) - (6*sqrt(a*c)*c^2*
d^4*e - 11*sqrt(a*c)*a*c*d^2*e^3 + 5*sqrt(a*c)*a^2*e^5)*A*abs(c)*abs(e) +
7*(sqrt(a*c)*a*c*d^3*e^2 - sqrt(a*c)*a^2*d*e^4)*B*abs(c)*abs(e) - (12*c^3*
d^5*e - 19*a*c^2*d^3*e^3 + 5*a^2*c*d*e^5)*A*abs(c) + 14*(a*c^2*d^4*e^2 - 2
*a^2*c*d^2*e^4)*B*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a^2*c^3*d - sqrt(a^4
*c^6*d^2 - (a^2*c^3*d^2 - a^3*c^2*e^2)*a^2*c^3))/(a^2*c^3)))/((a^3*c^3*e +
 sqrt(a*c)*a^2*c^3*d)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(e)) - 1/16*(6*(e*x
+ d)^(7/2)*A*c^3*d^3*e - 18*(e*x + d)^(5/2)*A*c^3*d^4*e + 18*(e*x + d)^(3/
2)*A*c^3*d^5*e - 6*sqrt(e*x + d)*A*c^3*d^6*e - 7*(e*x + d)^(7/2)*B*a*c^2*d
^2*e^2 + 21*(e*x + d)^(5/2)*B*a*c^2*d^3*e^2 - 21*(e*x + d)^(3/2)*B*a*c^2*d
^4*e^2 + 7*sqrt(e*x + d)*B*a*c^2*d^5*e^2 - 8*(e*x + d)^(7/2)*A*a*c^2*d*e^3
 + 23*(e*x + d)^(5/2)*A*a*c^2*d^2*e^3 - 32*(e*x + d)^(3/2)*A*a*c^2*d^3*...
```

Mupad [B] (verification not implemented)

Time = 6.71 (sec) , antiderivative size = 11687, normalized size of antiderivative = 29.51

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(((A + B*x)*(d + e*x)^(7/2))/(a - c*x^2)^3,x)
```

output
```
- atan(((((20480*A*a^7*c^6*e^7 + 28672*B*a^7*c^6*d*e^6 + 24576*A*a^5*c^8*d
^4*e^3 - 45056*A*a^6*c^7*d^2*e^5 - 28672*B*a^6*c^7*d^3*e^4)/(4096*a^6*c^5)
 - 64*a*c^4*d*e^2*(d + e*x)^(1/2)*((144*A^2*a^5*c^10*d^7 - 441*B^2*a^2*e^7
*(a^15*c^11)^(1/2) - 420*A^2*a^6*c^9*d^5*e^2 + 385*A^2*a^7*c^8*d^3*e^4 + 1
96*B^2*a^7*c^8*d^5*e^2 - 735*B^2*a^8*c^7*d^3*e^4 + 210*A*B*a^9*c^6*e^7 + 2
1*A^2*c^2*d^2*e^5*(a^15*c^11)^(1/2) - 105*A^2*a^8*c^7*d*e^6 + 735*B^2*a^9*
c^6*d*e^6 - 25*A^2*a*c*e^7*(a^15*c^11)^(1/2) - 210*A*B*c^2*d^3*e^4*(a^15*c
^11)^(1/2) - 336*A*B*a^6*c^9*d^6*e + 245*B^2*a*c*d^2*e^5*(a^15*c^11)^(1/2)
 + 1120*A*B*a^7*c^8*d^4*e^3 - 1050*A*B*a^8*c^7*d^2*e^5 + 266*A*B*a*c*d*e^6
*(a^15*c^11)^(1/2))/(4096*a^10*c^11))^(1/2))*((144*A^2*a^5*c^10*d^7 - 441*
B^2*a^2*e^7*(a^15*c^11)^(1/2) - 420*A^2*a^6*c^9*d^5*e^2 + 385*A^2*a^7*c^8*
d^3*e^4 + 196*B^2*a^7*c^8*d^5*e^2 - 735*B^2*a^8*c^7*d^3*e^4 + 210*A*B*a^9*
c^6*e^7 + 21*A^2*c^2*d^2*e^5*(a^15*c^11)^(1/2) - 105*A^2*a^8*c^7*d*e^6 + 7
35*B^2*a^9*c^6*d*e^6 - 25*A^2*a*c*e^7*(a^15*c^11)^(1/2) - 210*A*B*c^2*d^3*
e^4*(a^15*c^11)^(1/2) - 336*A*B*a^6*c^9*d^6*e + 245*B^2*a*c*d^2*e^5*(a^15*
c^11)^(1/2) + 1120*A*B*a^7*c^8*d^4*e^3 - 1050*A*B*a^8*c^7*d^2*e^5 + 266*A*
B*a*c*d*e^6*(a^15*c^11)^(1/2))/(4096*a^10*c^11))^(1/2) + ((d + e*x)^(1/2)*
(441*B^2*a^5*e^10 + 144*A^2*c^5*d^8*e^2 + 25*A^2*a^4*c*e^10 + 385*A^2*a^2*
c^3*d^4*e^6 - 126*A^2*a^3*c^2*d^2*e^8 + 196*B^2*a^2*c^3*d^6*e^4 - 735*B^2*
a^3*c^2*d^4*e^6 - 420*A^2*a*c^4*d^6*e^4 + 490*B^2*a^4*c*d^2*e^8 - 56*A*...
```

Reduce [B] (verification not implemented)

Time = 6.86 (sec) , antiderivative size = 3452, normalized size of antiderivative = 8.72

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^(7/2)/(-c*x^2+a)^3,x)
```

output
```
( - 42*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(
c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*b*e**3 + 26*sqrt(a)*sqrt(sqrt(c)*s
qrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c
*d)))*a**3*c*d*e**2 + 28*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(
d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*b*c*d**2*e + 84*
sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt
(sqrt(c)*sqrt(a)*e - c*d)))*a**2*b*c*e**3*x**2 - 24*sqrt(a)*sqrt(sqrt(c)*s
qrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c
*d)))*a**2*c**2*d**3 - 52*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt
(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c**2*d*e**2*x**
2 - 56*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(
c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*b*c**2*d**2*e*x**2 - 42*sqrt(a)*sqrt(
sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt
(a)*e - c*d)))*a*b*c**2*e**3*x**4 + 48*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*
d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**3*
d**3*x**2 + 26*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c
)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**3*d*e**2*x**4 + 28*sqrt(a)
*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c
)*sqrt(a)*e - c*d)))*b*c**3*d**2*e*x**4 - 24*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*
e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d))...
```

3.135 $\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 372

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^3} dx = \frac{(d+ex)^{3/2}(a(Bd+Ae)+(Acd+aBe)x)}{4ac\left(a-cx^2\right)^2} + \frac{\sqrt{d+ex}(ae(3Acd-5aBe)+c(6Acd^2-5aBde-3aAe^2)\,x)}{16a^2c^2\left(a-cx^2\right)} + \frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\left(5aBe\left(2\sqrt{c}d+\sqrt{a}e\right)-3A\left(4c^{3/2}d^2+2\sqrt{a}cde-a\sqrt{c}e^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{9/4}} - \frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\left(5aBe\left(2\sqrt{c}d-\sqrt{a}e\right)-3A\left(4c^{3/2}d^2-2\sqrt{a}cde-a\sqrt{c}e^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{32a^{5/2}c^{9/4}}$$

output
```
1/4*(e*x+d)^(3/2)*(a*(A*e+B*d)+(A*c*d+B*a*e)*x)/a/c/(-c*x^2+a)^2+1/16*(e*x
+d)^(1/2)*(a*e*(3*A*c*d-5*B*a*e)+c*(-3*A*a*e^2+6*A*c*d^2-5*B*a*d*e)*x)/a^2
/c^2/(-c*x^2+a)+1/32*(c^(1/2)*d-a^(1/2)*e)^(1/2)*(5*a*B*e*(2*c^(1/2)*d+a^(
1/2)*e)-3*A*(4*c^(3/2)*d^2+2*a^(1/2)*c*d*e-a*c^(1/2)*e^2))*arctanh(c^(1/4)
*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(5/2)/c^(9/4)-1/32*(c^(1/2)*
d+a^(1/2)*e)^(1/2)*(5*a*B*e*(2*c^(1/2)*d-a^(1/2)*e)-3*A*(4*c^(3/2)*d^2-2*a
^(1/2)*c*d*e-a*c^(1/2)*e^2))*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1
/2)*e)^(1/2))/a^(5/2)/c^(9/4)
```

Mathematica [A] (verified)

Time = 4.97 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^3}\,dx = \frac{-\frac{2\sqrt{a}\sqrt{c}\sqrt{d+ex}\left(5a^3Be^2+6Ac^3d^2x^3-ac^2x\left(5Bdex^2+A\left(10d^2+dex+3e^2x^2\right)\right)-a^2c\left(Ae(7d+ex)+B\left(4d^2\cdots\right.\right.\right.}{(a-cx^2)^2}}{\cdots}$$

input
```
Integrate[((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2)^3,x]
```

output
```
((-2*Sqrt[a]*Sqrt[c]*Sqrt[d + e*x]*(5*a^3*B*e^2 + 6*A*c^3*d^2*x^3 - a*c^2*
x*(5*B*d*e*x^2 + A*(10*d^2 + d*e*x + 3*e^2*x^2)) - a^2*c*(A*e*(7*d + e*x)
+ B*(4*d^2 + 3*d*e*x + 9*e^2*x^2))))/(a - c*x^2)^2 - Sqrt[-(c*d) - Sqrt[a]
*Sqrt[c]*e]*(5*a*B*e*(-2*Sqrt[c]*d + Sqrt[a]*e) + 3*A*(4*c^(3/2)*d^2 - 2*S
qrt[a]*c*d*e - a*Sqrt[c]*e^2))*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sq
rt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)] + Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*(
-5*a*B*e*(2*Sqrt[c]*d + Sqrt[a]*e) + 3*A*(4*c^(3/2)*d^2 + 2*Sqrt[a]*c*d*e
- a*Sqrt[c]*e^2))*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/
(Sqrt[c]*d - Sqrt[a]*e)])/(32*a^(5/2)*c^(5/2))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {684, 27, 685, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^3}\,dx$$

$$\Big\downarrow \; 684$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\,(a-cx^2)^2}-\frac{\int -\frac{\sqrt{d+ex}\left(6Acd^2-ae(5Bd+3Ae)+e(3Acd-5aBe)x\right)}{2(a-cx^2)^2}dx}{4ac}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{d+ex}(6Acd^2-ae(5Bd+3Ae)+e(3Acd-5aBe)x)}{(a-cx^2)^2}dx}{8ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{ 685}$$

$$\frac{\frac{\sqrt{d+ex}(cx(6Acd^2-ae(3Ae+5Bd))+ae(3Acd-5aBe))}{2ac(a-cx^2)}-\frac{\int -\frac{3Acd\left(4cd^2-3ae^2\right)-5aBe\left(2cd^2-ae^2\right)+ce\left(6Acd^2-ae(5Bd+3Ae)\right)x}{2\sqrt{d+ex}\left(a-cx^2\right)}dx}{2ac}}{8ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{3Acd\left(4cd^2-3ae^2\right)-5aBe\left(2cd^2-ae^2\right)+ce\left(6Acd^2-ae(5Bd+3Ae)\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{4ac}+\frac{\sqrt{d+ex}(cx(6Acd^2-ae(3Ae+5Bd))+ae(3Acd-5aBe))}{2ac(a-cx^2)}}{8ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{ 654}$$

$$\frac{\frac{\int -\frac{e\left((6Acd-5aBe)\left(cd^2-ae^2\right)+c\left(6Acd^2-ae(5Bd+3Ae)\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}+\frac{\sqrt{d+ex}(cx(6Acd^2-ae(3Ae+5Bd))+ae(3Acd-5aBe))}{2ac(a-cx^2)}}{8ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\sqrt{d+ex}(cx(6Acd^2-ae(3Ae+5Bd))+ae(3Acd-5aBe))}{2ac(a-cx^2)}-\frac{\int \frac{e\left((6Acd-5aBe)\left(cd^2-ae^2\right)+c\left(6Acd^2-ae(5Bd+3Ae)\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}}{8ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\sqrt{d+ex}(cx(6Acd^2-ae(3Ae+5Bd))+ae(3Acd-5aBe))}{2ac(a-cx^2)}-\frac{e\int \frac{(6Acd-5aBe)\left(cd^2-ae^2\right)+c\left(6Acd^2-ae(5Bd+3Ae)\right)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2ac}}{8ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$\Big\downarrow$ 1480

$$\frac{\frac{\sqrt{d+ex}(cx(6Acd^2-ae(3Ae+5Bd))+ae(3Acd-5aBe))}{2ac(a-cx^2)}-\frac{e\left(\frac{\sqrt{c}(\sqrt{c}d-\sqrt{a}e)\left(5aBe(\sqrt{a}e+2\sqrt{c}d)-3A\left(2\sqrt{a}cde-a\sqrt{c}e^2+4c^{3/2}d^2\right)\right)\int\frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d-}}{2\sqrt{a}e}\right.}{8ac}}{}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$\Big\downarrow$ 221

$$\frac{\frac{\sqrt{d+ex}(cx(6Acd^2-ae(3Ae+5Bd))+ae(3Acd-5aBe))}{2ac(a-cx^2)}-\frac{e\left(\frac{\sqrt{\sqrt{a}e+\sqrt{c}d}\left(5aBe(2\sqrt{c}d-\sqrt{a}e)-3A\left(-2\sqrt{a}cde-a\sqrt{c}e^2+4c^{3/2}d^2\right)\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+}}{\sqrt{\sqrt{a}e+\sqrt{}}}\right.}{2\sqrt{a}\sqrt[4]{c}e}\right.}{8ac}}{}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

input
```
Int[((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2)^3,x]
```

output
```
((d + e*x)^(3/2)*(a*(B*d + A*e) + (A*c*d + a*B*e)*x))/(4*a*c*(a - c*x^2)^2
) + ((Sqrt[d + e*x]*(a*e*(3*A*c*d - 5*a*B*e) + c*(6*A*c*d^2 - a*e*(5*B*d +
 3*A*e))*x))/(2*a*c*(a - c*x^2)) - (e*(-1/2*(Sqrt[Sqrt[c]*d - Sqrt[a]*e]*(
5*a*B*e*(2*Sqrt[c]*d + Sqrt[a]*e) - 3*A*(4*c^(3/2)*d^2 + 2*Sqrt[a]*c*d*e -
 a*Sqrt[c]*e^2))*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*
e]])/(Sqrt[a]*c^(1/4)*e) + (Sqrt[Sqrt[c]*d + Sqrt[a]*e]*(5*a*B*e*(2*Sqrt[c
]*d - Sqrt[a]*e) - 3*A*(4*c^(3/2)*d^2 - 2*Sqrt[a]*c*d*e - a*Sqrt[c]*e^2))*
ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c
^(1/4)*e)))/(2*a*c))/(8*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 685
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^(m - 1)*(a + c*x^2)
^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m]
 || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$-\frac{9ce\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}\left(\frac{\left(2Ac\,d^2-e\left(Ae+\frac{5Bd}{3}\right)a\right)\sqrt{ac\,e^2}}{3}-\frac{4A\,c^2d^3}{3}+ade\left(Ae+\frac{10Bd}{9}\right)c-\frac{5B\,e^3a^2}{9}\right)\left(-c\,x^2+a\right)^2\arctan\left(\frac{c\sqrt{}}{\sqrt{\left(-cd-\right.}}\right.}{32}$
default	$2e^4\left(\frac{\frac{\left(3Aa\,e^2-6Ac\,d^2+5Bade\right)(ex+d)^{\frac{7}{2}}}{32a^2e^3}-\frac{\left(8Aacd\,e^2-18A\,c^2d^3-9B\,e^3a^2+15Bac\,d^2e\right)(ex+d)^{\frac{5}{2}}}{32a^2e^3c}+\frac{\left(A\,a^2e^4+17Aac\,d^2e^2-18A\right.}{}}{\left(-c(ex+d)^2+2cd(ex+d)+\right.}\right.$
derivativedivides	$-2e^4\left(-\frac{\frac{\left(3Aa\,e^2-6Ac\,d^2+5Bade\right)(ex+d)^{\frac{7}{2}}}{32a^2e^3}-\frac{\left(8Aacd\,e^2-18A\,c^2d^3-9B\,e^3a^2+15Bac\,d^2e\right)(ex+d)^{\frac{5}{2}}}{32a^2e^3c}+\frac{\left(A\,a^2e^4+17Aac\,d^2e^2\right.}{}}{\left(-c(ex+d)^2+2cd(ex\right.}\right.$

input
```
int((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
7/16/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)/(a*c*e^2)^(1/2)*(-9/14*c*e*((c*d+(a*c
*e^2)^(1/2))*c)^(1/2)*(1/3*(2*A*c*d^2-e*(A*e+5/3*B*d)*a)*(a*c*e^2)^(1/2)-4
/3*A*c^2*d^3+a*d*e*(A*e+10/9*B*d)*c-5/9*B*e^3*a^2)*(-c*x^2+a)^2*arctan(c*(
e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))+((-c*d+(a*c*e^2)^(1/2))*c)^
(1/2)*(-9/14*c*e*(1/3*(-2*A*c*d^2+e*(A*e+5/3*B*d)*a)*(a*c*e^2)^(1/2)-4/3*A
*c^2*d^3+a*d*e*(A*e+10/9*B*d)*c-5/9*B*e^3*a^2)*(-c*x^2+a)^2*arctanh(c*(e*x
+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+((c*d+(a*c*e^2)^(1/2))*c)^(1/2)
*(e*x+d)^(1/2)*(a*c*e^2)^(1/2)*(-6/7*A*c^3*d^2*x^3+10/7*x*a*(A*d^2+1/10*e*
x*(5*B*x+A)*d+3/10*A*e^2*x^2)*c^2+(4/7*B*d^2+e*(3/7*B*x+A)*d+1/7*e^2*x*(9*
B*x+A))*a^2*c-5/7*B*e^2*a^3)))/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)/a^2/c^2/(-
c*x^2+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3382 vs. $2(302) = 604$.

Time = 1.35 (sec) , antiderivative size = 3382, normalized size of antiderivative = 9.09

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(e*x+d)**(5/2)/(-c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^3}\,dx = \int -\frac{(Bx+A)(ex+d)^{\frac{5}{2}}}{(cx^2-a)^3}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-integrate((B*x + A)*(e*x + d)^(5/2)/(c*x^2 - a)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 941 vs. $2(302) = 604$.

Time = 0.31 (sec) , antiderivative size = 941, normalized size of antiderivative = 2.53

$$\int \frac{(A + Bx)(d + ex)^{5/2}}{(a - cx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/32*(5*B*a^2*c*d*e^4*abs(c) - 3*(2*a*c^2*d^2*e - a^2*c*e^3)*A*e^2*abs(c)
- 6*(sqrt(a*c)*c^2*d^3*e - sqrt(a*c)*a*c*d*e^3)*A*abs(c)*abs(e) + 5*(sqrt(
a*c)*a*c*d^2*e^2 - sqrt(a*c)*a^2*e^4)*B*abs(c)*abs(e) + 3*(4*c^3*d^4*e - 3
*a*c^2*d^2*e^3)*A*abs(c) - 5*(2*a*c^2*d^3*e^2 - a^2*c*d*e^4)*B*abs(c))*arc
tan(sqrt(e*x + d)/sqrt(-(a^2*c^3*d + sqrt(a^4*c^6*d^2 - (a^2*c^3*d^2 - a^3
*c^2*e^2)*a^2*c^3))/(a^2*c^3)))/((a^3*c^3*e - sqrt(a*c)*a^2*c^3*d)*sqrt(-c
^2*d - sqrt(a*c)*c*e)*abs(e)) + 1/32*(5*B*a^2*c*d*e^4*abs(c) - 3*(2*a*c^2*
d^2*e - a^2*c*e^3)*A*e^2*abs(c) + 6*(sqrt(a*c)*c^2*d^3*e - sqrt(a*c)*a*c*d
*e^3)*A*abs(c)*abs(e) - 5*(sqrt(a*c)*a*c*d^2*e^2 - sqrt(a*c)*a^2*e^4)*B*ab
s(c)*abs(e) + 3*(4*c^3*d^4*e - 3*a*c^2*d^2*e^3)*A*abs(c) - 5*(2*a*c^2*d^3*
e^2 - a^2*c*d*e^4)*B*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a^2*c^3*d - sqrt(
a^4*c^6*d^2 - (a^2*c^3*d^2 - a^3*c^2*e^2)*a^2*c^3))/(a^2*c^3)))/((a^3*c^3*
e + sqrt(a*c)*a^2*c^3*d)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(e)) - 1/16*(6*(e
*x + d)^(7/2)*A*c^3*d^2*e - 18*(e*x + d)^(5/2)*A*c^3*d^3*e + 18*(e*x + d)^
(3/2)*A*c^3*d^4*e - 6*sqrt(e*x + d)*A*c^3*d^5*e - 5*(e*x + d)^(7/2)*B*a*c^
2*d*e^2 + 15*(e*x + d)^(5/2)*B*a*c^2*d^2*e^2 - 15*(e*x + d)^(3/2)*B*a*c^2*
d^3*e^2 + 5*sqrt(e*x + d)*B*a*c^2*d^4*e^2 - 3*(e*x + d)^(7/2)*A*a*c^2*e^3
+ 8*(e*x + d)^(5/2)*A*a*c^2*d*e^3 - 17*(e*x + d)^(3/2)*A*a*c^2*d^2*e^3 + 1
2*sqrt(e*x + d)*A*a*c^2*d^3*e^3 - 9*(e*x + d)^(5/2)*B*a^2*c*e^4 + 15*(e*x
+ d)^(3/2)*B*a^2*c*d*e^4 - 10*sqrt(e*x + d)*B*a^2*c*d^2*e^4 - (e*x + d)...
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 7702, normalized size of antiderivative = 20.70

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2)^3,x)
```

output
```
((e*(d + e*x)^(7/2)*(3*A*a*e^2 - 6*A*c*d^2 + 5*B*a*d*e))/(16*a^2) - ((d +
e*x)^(1/2)*(5*B*a^3*e^6 - 6*A*c^3*d^5*e + 12*A*a*c^2*d^3*e^3 + 5*B*a*c^2*d
^4*e^2 - 10*B*a^2*c*d^2*e^4 - 6*A*a^2*c*d*e^5))/(16*a^2*c^2) + ((d + e*x)^
(3/2)*(A*a^2*e^5 - 15*B*a^2*d*e^4 - 18*A*c^2*d^4*e + 17*A*a*c*d^2*e^3 + 15
*B*a*c*d^3*e^2))/(16*a^2*c) + (e*(d + e*x)^(5/2)*(18*A*c^2*d^3 + 9*B*a^2*e
^3 - 8*A*a*c*d*e^2 - 15*B*a*c*d^2*e))/(16*a^2*c))/(c^2*(d + e*x)^4 + a^2*e
^4 + c^2*d^4 + (6*c^2*d^2 - 2*a*c*e^2)*(d + e*x)^2 - (4*c^2*d^3 - 4*a*c*d*
e^2)*(d + e*x) - 4*c^2*d*(d + e*x)^3 - 2*a*c*d^2*e^2) - atan(((((20480*B*a
^7*c^4*e^6 - 24576*A*a^6*c^5*d*e^5 + 24576*A*a^5*c^6*d^3*e^3 - 20480*B*a^6
*c^5*d^2*e^4)/(4096*a^6*c^3) - 64*a*c^4*d*e^2*(d + e*x)^(1/2)*(-(25*B^2*a*
e^5*(a^15*c^9)^(1/2) - 144*A^2*a^5*c^8*d^5 + 9*A^2*c*e^5*(a^15*c^9)^(1/2)
+ 180*A^2*a^6*c^7*d^3*e^2 - 100*B^2*a^7*c^6*d^3*e^2 + 30*A*B*a^8*c^5*e^5 -
 45*A^2*a^7*c^6*d*e^4 + 75*B^2*a^8*c^5*d*e^4 + 240*A*B*a^6*c^7*d^4*e - 30*
A*B*c*d*e^4*(a^15*c^9)^(1/2) - 240*A*B*a^7*c^6*d^2*e^3)/(4096*a^10*c^9))^(
1/2))*(-(25*B^2*a*e^5*(a^15*c^9)^(1/2) - 144*A^2*a^5*c^8*d^5 + 9*A^2*c*e^5
*(a^15*c^9)^(1/2) + 180*A^2*a^6*c^7*d^3*e^2 - 100*B^2*a^7*c^6*d^3*e^2 + 30
*A*B*a^8*c^5*e^5 - 45*A^2*a^7*c^6*d*e^4 + 75*B^2*a^8*c^5*d*e^4 + 240*A*B*a
^6*c^7*d^4*e - 30*A*B*c*d*e^4*(a^15*c^9)^(1/2) - 240*A*B*a^7*c^6*d^2*e^3)/
(4096*a^10*c^9))^(1/2) + ((d + e*x)^(1/2)*(25*B^2*a^4*e^8 + 144*A^2*c^4*d^
6*e^2 + 9*A^2*a^3*c*e^8 + 45*A^2*a^2*c^2*d^2*e^6 + 100*B^2*a^2*c^2*d^4*...
```

Reduce [B] (verification not implemented)

Time = 6.81 (sec) , antiderivative size = 2428, normalized size of antiderivative = 6.53

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^3,x)
```

output

```
(6*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*s
qrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*c*e**2 + 20*sqrt(a)*sqrt(sqrt(c)*sqrt(
a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d))
)*a**2*b*c*d*e - 24*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e
*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c**2*d**2 - 12*sqrt(a
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(
c)*sqrt(a)*e - c*d)))*a**2*c**2*e**2*x**2 - 40*sqrt(a)*sqrt(sqrt(c)*sqrt(a
)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))
*a*b*c**2*d*e*x**2 + 48*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d
 + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**3*d**2*x**2 + 6*s
qrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(
sqrt(c)*sqrt(a)*e - c*d)))*a*c**3*e**2*x**4 + 20*sqrt(a)*sqrt(sqrt(c)*sqrt
(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)
))*b*c**3*d*e*x**4 - 24*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d
 + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c**4*d**2*x**4 + 10*sq
rt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(s
qrt(c)*sqrt(a)*e - c*d)))*a**3*b*e**2 - 12*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e
- c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**
3*c*d*e - 20*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/
(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*b*c*e**2*x**2 + 24*sqrt(c...
```

3.136 $\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 350

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^3} dx = \frac{\sqrt{d+ex}(a(Bd+Ae)+(Acd+aBe)x)}{4ac(a-cx^2)^2} - \frac{\sqrt{d+ex}(aAe-3(2Acd-aBe)x)}{16a^2c(a-cx^2)} + \frac{3\left(aBe\left(2\sqrt{c}d-\sqrt{a}e\right)-A\left(4c^{3/2}d^2-2\sqrt{a}cde-a\sqrt{c}e^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{7/4}\sqrt{\sqrt{c}d-\sqrt{a}e}} - \frac{3\left(aBe\left(2\sqrt{c}d+\sqrt{a}e\right)-A\left(4c^{3/2}d^2+2\sqrt{a}cde-a\sqrt{c}e^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{32a^{5/2}c^{7/4}\sqrt{\sqrt{c}d+\sqrt{a}e}}$$

output
```
1/4*(e*x+d)^(1/2)*(a*(A*e+B*d)+(A*c*d+B*a*e)*x)/a/c/(-c*x^2+a)^2-1/16*(e*x
+d)^(1/2)*(a*A*e-3*(2*A*c*d-B*a*e)*x)/a^2/c/(-c*x^2+a)+3/32*(a*B*e*(2*c^(1
/2)*d-a^(1/2)*e)-A*(4*c^(3/2)*d^2-2*a^(1/2)*c*d*e-a*c^(1/2)*e^2))*arctanh(
c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(5/2)/c^(7/4)/(c^(1/2
)*d-a^(1/2)*e)^(1/2)-3/32*(a*B*e*(2*c^(1/2)*d+a^(1/2)*e)-A*(4*c^(3/2)*d^2+
2*a^(1/2)*c*d*e-a*c^(1/2)*e^2))*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a
^(1/2)*e)^(1/2))/a^(5/2)/c^(7/4)/(c^(1/2)*d+a^(1/2)*e)^(1/2)
```

Mathematica [A] (verified)

Time = 3.60 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^3}\,dx = \frac{-\frac{2\sqrt{a}\sqrt{c}\sqrt{d+ex}(6Ac^2dx^3-a^2(4Bd+3Ae+Bex)-acx(10Ad+Aex+3Bex^2))}{(a-cx^2)^2} - \frac{3(aBe(2\sqrt{c}d+\sqrt{a}e)+}{}}{}$$

input
```
Integrate[((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^3,x]
```

output
```
((-2*Sqrt[a]*Sqrt[c]*Sqrt[d + e*x]*(6*A*c^2*d*x^3 - a^2*(4*B*d + 3*A*e + B
*e*x) - a*c*x*(10*A*d + A*e*x + 3*B*e*x^2)))/(a - c*x^2)^2 - (3*(a*B*e*(2*
Sqrt[c]*d + Sqrt[a]*e) + A*(-4*c^(3/2)*d^2 - 2*Sqrt[a]*c*d*e + a*Sqrt[c]*e
^2))*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d +
Sqrt[a]*e)])/Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e] - (3*(a*B*e*(-2*Sqrt[c]*d +
Sqrt[a]*e) + A*(4*c^(3/2)*d^2 - 2*Sqrt[a]*c*d*e - a*Sqrt[c]*e^2))*ArcTan[(
Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/
Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e])/(32*a^(5/2)*c^(3/2))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {684, 27, 686, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^3}\,dx$$

$$\Big\downarrow \ 684$$

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{4ac\,(a-cx^2)^2} - \frac{\int -\frac{6Acd^2-3aBed-aAe^2+e(5Acd-3aBe)x}{2\sqrt{d+ex}(a-cx^2)^2}dx}{4ac}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{6Acd^2-3aBed-aAe^2+e(5Acd-3aBe)x}{\sqrt{d+ex}(a-cx^2)^2}dx}{8ac}+\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{686}$$

$$\frac{-\frac{\int -\frac{3c\left(cd^2-ae^2\right)\left(4Acd^2-2aBed-aAe^2+e(2Acd-aBe)x\right)}{2\sqrt{d+ex}\left(a-cx^2\right)}dx}{2ac(cd^2-ae^2)}-\frac{\sqrt{d+ex}\left(aAe\left(cd^2-ae^2\right)-3x\left(cd^2-ae^2\right)(2Acd-aBe)\right)}{2a(a-cx^2)(cd^2-ae^2)}}{8ac}+\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{4Acd^2-2aBed-aAe^2+e(2Acd-aBe)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{4a}-\frac{\sqrt{d+ex}\left(aAe\left(cd^2-ae^2\right)-3x\left(cd^2-ae^2\right)(2Acd-aBe)\right)}{2a(a-cx^2)(cd^2-ae^2)}}{8ac}+\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{654}$$

$$\frac{\frac{3\int -\frac{e\left(2Acd^2-aBed-aAe^2+(2Acd-aBe)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}-\frac{\sqrt{d+ex}\left(aAe\left(cd^2-ae^2\right)-3x\left(cd^2-ae^2\right)(2Acd-aBe)\right)}{2a(a-cx^2)(cd^2-ae^2)}}{8ac}+\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{3\int \frac{e\left(2Acd^2-aBed-aAe^2+(2Acd-aBe)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}-\frac{\sqrt{d+ex}\left(aAe\left(cd^2-ae^2\right)-3x\left(cd^2-ae^2\right)(2Acd-aBe)\right)}{2a(a-cx^2)(cd^2-ae^2)}}{8ac}+\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{3e\int \frac{2Acd^2-aBed-aAe^2+(2Acd-aBe)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a}-\frac{\sqrt{d+ex}\left(aAe\left(cd^2-ae^2\right)-3x\left(cd^2-ae^2\right)(2Acd-aBe)\right)}{2a(a-cx^2)(cd^2-ae^2)}}{8ac}+\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{\frac{3e\left(\frac{\left(aBe(2\sqrt{c}d-\sqrt{a}e)-A\left(-2\sqrt{a}cde-a\sqrt{c}e^2+4c^{3/2}d^2\right)\right)\int\frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d-\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{\left(aBe(\sqrt{a}e+2\sqrt{c}d)-A\left(2\sqrt{a}cde-a\sqrt{c}e^2+4c^{3/2}d^2\right)\right)\int\frac{}{c(d+e}\cdots}{2\sqrt{a}e}\cdots\right.}{2a}\cdots}{8ac}$$

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{3e\left(\frac{\left(aBe(\sqrt{a}e+2\sqrt{c}d)-A\left(2\sqrt{a}cde-a\sqrt{c}e^2+4c^{3/2}d^2\right)\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}c^{3/4}e\sqrt{\sqrt{a}e+\sqrt{c}d}}-\frac{\left(aBe(2\sqrt{c}d-\sqrt{a}e)-A\left(-2\sqrt{a}cde-a\sqrt{c}e^2+4c^{3/2}d^2\right)\right)\text{arctanh}\left(\frac{\sqrt[4]{}}{\sqrt{\sqrt{}}}\cdots\right.}{2\sqrt{a}c^{3/4}e\sqrt{\sqrt{c}d-\sqrt{a}e}}\cdots\right.}{2a}\cdots}{8ac}$$

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{4ac\left(a-cx^2\right)^2}$$

input

```
Int[((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^3,x]
```

output

```
(Sqrt[d + e*x]*(a*(B*d + A*e) + (A*c*d + a*B*e)*x))/(4*a*c*(a - c*x^2)^2)
+ (-1/2*(Sqrt[d + e*x]*(a*A*e*(c*d^2 - a*e^2) - 3*(2*A*c*d - a*B*e)*(c*d^2
 - a*e^2)*x))/(a*(c*d^2 - a*e^2)*(a - c*x^2)) - (3*e*(-1/2*((a*B*e*(2*Sqrt
[c]*d - Sqrt[a]*e) - A*(4*c^(3/2)*d^2 - 2*Sqrt[a]*c*d*e - a*Sqrt[c]*e^2))*
ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(Sqrt[a]*c^(
3/4)*e*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) + ((a*B*e*(2*Sqrt[c]*d + Sqrt[a]*e) -
A*(4*c^(3/2)*d^2 + 2*Sqrt[a]*c*d*e - a*Sqrt[c]*e^2))*ArcTanh[(c^(1/4)*Sqrt
[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c^(3/4)*e*Sqrt[Sqrt[c]
*d + Sqrt[a]*e])))/(2*a))/(8*a*c)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{-\frac{3e\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}\left(\left(2Acd-Bae\right)\sqrt{ac\,e^2}+c\left(-4Ac\,d^2+ae(Ae+2Bd)\right)\right)\left(-c\,x^2+a\right)^2\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^2}\right)c}}\right)}{32}+\frac{3\sqrt{\left(-cd+\sqrt{}\right.}}{}}{}$
default	$2e^4\left(\frac{-\frac{3(2Acd-Bae)(ex+d)^{\frac{7}{2}}}{32a^2e^3}+\frac{\left(Aa\,e^2+18Ac\,d^2-9Bade\right)(ex+d)^{\frac{5}{2}}}{32a^2e^3}+\frac{\left(8Aacd\,e^2-18A\,c^2d^3+B\,e^3a^2+9Bac\,d^2e\right)(ex+d)^{\frac{3}{2}}}{32a^2e^3c}+\frac{3}{}}{\left(-c(ex+d)^2+2cd(ex+d)+a\,e^2-c\,d^2\right)^2}\right.$
derivativedivides	$-2e^4\left(-\frac{-\frac{3(2Acd-Bae)(ex+d)^{\frac{7}{2}}}{32a^2e^3}+\frac{\left(Aa\,e^2+18Ac\,d^2-9Bade\right)(ex+d)^{\frac{5}{2}}}{32a^2e^3}+\frac{\left(8Aacd\,e^2-18A\,c^2d^3+B\,e^3a^2+9Bac\,d^2e\right)(ex+d)^{}}{32a^2e^3c}}{\left(-c(ex+d)^2+2cd(ex+d)+a\,e^2-c\,d^2\right)^2}\right.$

input
```
int((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/16*(-1/2*e*((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*((2*A*c*d-B*a*e)*(a*c*e^2)^(1
/2)+c*(-4*A*c*d^2+a*e*(A*e+2*B*d)))*(-c*x^2+a)^2*arctan(c*(e*x+d)^(1/2)/((
-c*d+(a*c*e^2)^(1/2))*c)^(1/2))+((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*(-1/2*e*(
(-2*A*c*d+B*a*e)*(a*c*e^2)^(1/2)+c*(-4*A*c*d^2+a*e*(A*e+2*B*d)))*(-c*x^2+a
)^2*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+((c*d+(a*c*e^
2)^(1/2))*c)^(1/2)*(e*x+d)^(1/2)*(a*c*e^2)^(1/2)*(-2*A*c^2*d*x^3+10/3*x*(A
*d+1/10*e*x*(3*B*x+A))*a*c+(4/3*B*d+e*(1/3*B*x+A))*a^2)))/((c*d+(a*c*e^2)^
(1/2))*c)^(1/2)/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)/a^2/c/(-c
*x^2+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4176 vs. $2(280) = 560$.

Time = 2.71 (sec) , antiderivative size = 4176, normalized size of antiderivative = 11.93

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(e*x+d)**(3/2)/(-c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^3}\,dx = \int -\frac{(Bx+A)(ex+d)^{\frac{3}{2}}}{(cx^2-a)^3}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-integrate((B*x + A)*(e*x + d)^(3/2)/(c*x^2 - a)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(280) = 560$.

Time = 0.27 (sec) , antiderivative size = 731, normalized size of antiderivative = 2.09

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^3,x, algorithm="giac")
```

output
```
-3/32*(2*B*a*c^3*d^2*e^2 + 2*A*a*c^3*d*e^3 - B*a^2*c^2*e^4 - sqrt(a*c)*B*a
*c*d*e^2*abs(c)*abs(e) + (2*sqrt(a*c)*c^2*d^2*e - sqrt(a*c)*a*c*e^3)*A*abs
(c)*abs(e) - (4*c^4*d^3*e - a*c^3*d*e^3)*A)*arctan(sqrt(e*x + d)/sqrt(-(a^
2*c^2*d + sqrt(a^4*c^4*d^2 - (a^2*c^2*d^2 - a^3*c*e^2)*a^2*c^2))/(a^2*c^2)
))/((a^3*c^3*e - sqrt(a*c)*a^2*c^3*d)*sqrt(-c^2*d - sqrt(a*c)*c*e)*abs(e))
 - 3/32*(2*B*a*c^3*d^2*e^2 + 2*A*a*c^3*d*e^3 - B*a^2*c^2*e^4 + sqrt(a*c)*B
*a*c*d*e^2*abs(c)*abs(e) - (2*sqrt(a*c)*c^2*d^2*e - sqrt(a*c)*a*c*e^3)*A*a
bs(c)*abs(e) - (4*c^4*d^3*e - a*c^3*d*e^3)*A)*arctan(sqrt(e*x + d)/sqrt(-(
a^2*c^2*d - sqrt(a^4*c^4*d^2 - (a^2*c^2*d^2 - a^3*c*e^2)*a^2*c^2))/(a^2*c^
2)))/((a^3*c^3*e + sqrt(a*c)*a^2*c^3*d)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(e
)) - 1/16*(6*(e*x + d)^(7/2)*A*c^2*d*e - 18*(e*x + d)^(5/2)*A*c^2*d^2*e +
18*(e*x + d)^(3/2)*A*c^2*d^3*e - 6*sqrt(e*x + d)*A*c^2*d^4*e - 3*(e*x + d)
^(7/2)*B*a*c*e^2 + 9*(e*x + d)^(5/2)*B*a*c*d*e^2 - 9*(e*x + d)^(3/2)*B*a*c
*d^2*e^2 + 3*sqrt(e*x + d)*B*a*c*d^3*e^2 - (e*x + d)^(5/2)*A*a*c*e^3 - 8*(
e*x + d)^(3/2)*A*a*c*d*e^3 + 9*sqrt(e*x + d)*A*a*c*d^2*e^3 - (e*x + d)^(3/
2)*B*a^2*e^4 - 3*sqrt(e*x + d)*B*a^2*d*e^4 - 3*sqrt(e*x + d)*A*a^2*e^5)/((
(e*x + d)^2*c - 2*(e*x + d)*c*d + c*d^2 - a*e^2)^2*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 7239, normalized size of antiderivative = 20.68

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^3,x)
```

output
```
((3*(B*a*e^2 - 2*A*c*d*e)*(d + e*x)^(7/2))/(16*a^2) + ((d + e*x)^(5/2)*(A*
a*e^3 - 9*B*a*d*e^2 + 18*A*c*d^2*e))/(16*a^2) + (3*(d + e*x)^(1/2)*(A*a^2*
e^5 + B*a^2*d*e^4 + 2*A*c^2*d^4*e - 3*A*a*c*d^2*e^3 - B*a*c*d^3*e^2))/(16*
a^2*c) + ((d + e*x)^(3/2)*(B*a^2*e^4 - 18*A*c^2*d^3*e + 8*A*a*c*d*e^3 + 9*
B*a*c*d^2*e^2))/(16*a^2*c))/(c^2*(d + e*x)^4 + a^2*e^4 + c^2*d^4 + (6*c^2*
d^2 - 2*a*c*e^2)*(d + e*x)^2 - (4*c^2*d^3 - 4*a*c*d*e^2)*(d + e*x) - 4*c^2
*d*(d + e*x)^3 - 2*a*c*d^2*e^2) + atan(((((3*(4096*A*a^6*c^4*e^5 + 4096*B*
a^6*c^4*d*e^4 - 8192*A*a^5*c^5*d^2*e^3))/(4096*a^6*c^2) - 64*a*c^4*d*e^2*(
d + e*x)^(1/2)*(-(9*(B^2*a*e^5*(a^15*c^7)^(1/2) - 16*A^2*a^5*c^7*d^5 + A^2
*c*e^5*(a^15*c^7)^(1/2) + 20*A^2*a^6*c^6*d^3*e^2 - 4*B^2*a^7*c^5*d^3*e^2 +
 2*A*B*a^8*c^4*e^5 - 5*A^2*a^7*c^5*d*e^4 + 3*B^2*a^8*c^4*d*e^4 + 16*A*B*a^
6*c^6*d^4*e - 2*A*B*c*d*e^4*(a^15*c^7)^(1/2) - 16*A*B*a^7*c^5*d^2*e^3))/(4
096*(a^10*c^8*d^2 - a^11*c^7*e^2)))^(1/2))*(-(9*(B^2*a*e^5*(a^15*c^7)^(1/2
) - 16*A^2*a^5*c^7*d^5 + A^2*c*e^5*(a^15*c^7)^(1/2) + 20*A^2*a^6*c^6*d^3*e
^2 - 4*B^2*a^7*c^5*d^3*e^2 + 2*A*B*a^8*c^4*e^5 - 5*A^2*a^7*c^5*d*e^4 + 3*B
^2*a^8*c^4*d*e^4 + 16*A*B*a^6*c^6*d^4*e - 2*A*B*c*d*e^4*(a^15*c^7)^(1/2) -
 16*A*B*a^7*c^5*d^2*e^3))/(4096*(a^10*c^8*d^2 - a^11*c^7*e^2)))^(1/2) + ((
d + e*x)^(1/2)*(9*B^2*a^3*e^6 + 144*A^2*c^3*d^4*e^2 + 9*A^2*a^2*c*e^6 - 36
*A^2*a*c^2*d^2*e^4 + 36*B^2*a^2*c*d^2*e^4 - 144*A*B*a*c^2*d^3*e^3))/(64*a^
4))*(-(9*(B^2*a*e^5*(a^15*c^7)^(1/2) - 16*A^2*a^5*c^7*d^5 + A^2*c*e^5*(...
```

Reduce [B] (verification not implemented)

Time = 7.21 (sec) , antiderivative size = 3475, normalized size of antiderivative = 9.93

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^3,x)
```

output
```
(6*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*s
qrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*b*e**3 - 18*sqrt(a)*sqrt(sqrt(c)*sqrt(
a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d))
)*a**3*c*d*e**2 - 12*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d +
e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*b*c*d**2*e - 12*sqrt
(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqr
t(c)*sqrt(a)*e - c*d)))*a**2*b*c*e**3*x**2 + 24*sqrt(a)*sqrt(sqrt(c)*sqrt(
a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d))
)*a**2*c**2*d**3 + 36*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d +
 e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c**2*d*e**2*x**2 +
24*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*s
qrt(sqrt(c)*sqrt(a)*e - c*d)))*a*b*c**2*d**2*e*x**2 + 6*sqrt(a)*sqrt(sqrt(
c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e
 - c*d)))*a*b*c**2*e**3*x**4 - 48*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*at
an((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**3*d**3*
x**2 - 18*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sq
rt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*c**3*d*e**2*x**4 - 12*sqrt(a)*sqrt
(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqr
t(a)*e - c*d)))*b*c**3*d**2*e*x**4 + 24*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c
*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c**...
```

3.137 $\int \frac{(A+Bx)\sqrt{d+ex}}{\left(a-cx^2\right)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 372

$$\int \frac{(A+Bx)\sqrt{d+ex}}{\left(a-cx^2\right)^3} dx = \frac{(aB+Acx)\sqrt{d+ex}}{4ac\left(a-cx^2\right)^2} - \frac{\sqrt{d+ex}\left(ae(Acd-aBe)-c\left(6Acd^2-aBde-5aAe^2\right)x\right)}{16a^2c\left(cd^2-ae^2\right)\left(a-cx^2\right)} + \frac{\left(aBe\left(2\sqrt{c}d-3\sqrt{a}e\right)-A\left(12c^{3/2}d^2-18\sqrt{a}cde+5a\sqrt{c}e^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{3/2}} - \frac{\left(aBe\left(2\sqrt{c}d+3\sqrt{a}e\right)-A\left(12c^{3/2}d^2+18\sqrt{a}cde+5a\sqrt{c}e^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{32a^{5/2}c^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)^{3/2}}$$

output
```
1/4*(A*c*x+B*a)*(e*x+d)^(1/2)/a/c/(-c*x^2+a)^2-1/16*(e*x+d)^(1/2)*(a*e*(A*
c*d-B*a*e)-c*(-5*A*a*e^2+6*A*c*d^2-B*a*d*e)*x)/a^2/c/(-a*e^2+c*d^2)/(-c*x^
2+a)+1/32*(a*B*e*(2*c^(1/2)*d-3*a^(1/2)*e)-A*(12*c^(3/2)*d^2-18*a^(1/2)*c*
d*e+5*a*c^(1/2)*e^2))*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^
(1/2))/a^(5/2)/c^(5/4)/(c^(1/2)*d-a^(1/2)*e)^(3/2)-1/32*(a*B*e*(2*c^(1/2)*
d+3*a^(1/2)*e)-A*(12*c^(3/2)*d^2+18*a^(1/2)*c*d*e+5*a*c^(1/2)*e^2))*arctan
h(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(5/2)/c^(5/4)/(c^(1
/2)*d+a^(1/2)*e)^(3/2)
```

Mathematica [A] (verified)

Time = 3.32 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^3}\,dx$$

$$= \frac{\frac{2\sqrt{a}\sqrt{d+ex}\left(3a^3Be^2+6Ac^3d^2x^3+a^2c\left(Ae(d+9ex)+B\left(-4d^2+dex+e^2x^2\right)\right)-ac^2x\left(Bdex^2+A\left(10d^2+dex+5e^2x^2\right)\right)\right)}{(-cd^2+ae^2)(a-cx^2)^2}+\frac{\left(-aBe\left(2\sqrt{c}d+3\sqrt{a}e\right)+\right.}{}}{}$$

input
```
Integrate[((A + B*x)*Sqrt[d + e*x])/(a - c*x^2)^3,x]
```

output
```
((2*Sqrt[a]*Sqrt[d + e*x]*(3*a^3*B*e^2 + 6*A*c^3*d^2*x^3 + a^2*c*(A*e*(d +
 9*e*x) + B*(-4*d^2 + d*e*x + e^2*x^2)) - a*c^2*x*(B*d*e*x^2 + A*(10*d^2 +
 d*e*x + 5*e^2*x^2))))/((-(c*d^2) + a*e^2)*(a - c*x^2)^2) + ((-(a*B*e*(2*S
qrt[c]*d + 3*Sqrt[a]*e)) + A*(12*c^(3/2)*d^2 + 18*Sqrt[a]*c*d*e + 5*a*Sqrt
[c]*e^2))*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt[c]
*d + Sqrt[a]*e)])/((Sqrt[c]*d + Sqrt[a]*e)*Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e
]) - ((a*B*e*(-2*Sqrt[c]*d + 3*Sqrt[a]*e) + A*(12*c^(3/2)*d^2 - 18*Sqrt[a]
*c*d*e + 5*a*Sqrt[c]*e^2))*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d
 + e*x])/(Sqrt[c]*d - Sqrt[a]*e)])/((Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-(c*d) +
Sqrt[a]*Sqrt[c]*e]))/(32*a^(5/2)*c)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {685, 27, 686, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^3}\,dx$$

$$\Big\downarrow \; 685$$

$$\frac{\sqrt{d+ex}(aB+Acx)}{4ac\left(a-cx^2\right)^2}-\frac{\int -\frac{6Acd-aBe+5Acex}{2\sqrt{d+ex}(a-cx^2)^2}dx}{4ac}$$

$\big\downarrow$ 27

$$\frac{\int \frac{6Acd-aBe+5Acex}{\sqrt{d+ex}(a-cx^2)^2}dx}{8ac}+\frac{\sqrt{d+ex}(aB+Acx)}{4ac\left(a-cx^2\right)^2}$$

$\big\downarrow$ 686

$$\frac{-\frac{\int -\frac{c\left(Acd\left(12cd^2-13ae^2\right)-aBe\left(2cd^2-3ae^2\right)+ce\left(6Acd^2-aBed-5aAe^2\right)x\right)}{2\sqrt{d+ex}\left(a-cx^2\right)}dx}{2ac(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(Acd-aBe)-cx(-5aAe^2-aBde+6Acd^2))}{2a(a-cx^2)(cd^2-ae^2)}}{8ac}+\frac{\sqrt{d+ex}(aB+Acx)}{4ac\left(a-cx^2\right)^2}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{Acd\left(12cd^2-13ae^2\right)-aBe\left(2cd^2-3ae^2\right)+ce\left(6Acd^2-aBed-5aAe^2\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{4a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(Acd-aBe)-cx(-5aAe^2-aBde+6Acd^2))}{2a(a-cx^2)(cd^2-ae^2)}}{8ac}+\frac{\sqrt{d+ex}(aB+Acx)}{4ac\left(a-cx^2\right)^2}$$

$\big\downarrow$ 654

$$\frac{\frac{\int -\frac{e\left(2Acd\left(3cd^2-4ae^2\right)-aBe\left(cd^2-3ae^2\right)+c\left(6Acd^2-aBed-5aAe^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(Acd-aBe)-cx(-5aAe^2-aBde+6Acd^2))}{2a(a-cx^2)(cd^2-ae^2)}}{8ac}+\frac{\sqrt{d+ex}(aB+Acx)}{4ac\left(a-cx^2\right)^2}$$

$\big\downarrow$ 25

$$\frac{-\frac{\int \frac{e\left(2Acd\left(3cd^2-4ae^2\right)-aBe\left(cd^2-3ae^2\right)+c\left(6Acd^2-aBed-5aAe^2\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(Acd-aBe)-cx(-5aAe^2-aBde+6Acd^2))}{2a(a-cx^2)(cd^2-ae^2)}}{8ac}+\frac{\sqrt{d+ex}(aB+Acx)}{4ac\left(a-cx^2\right)^2}$$

$\big\downarrow$ 27

$$\frac{-\frac{e\int \frac{2Acd(3cd^2-4ae^2)-aBe(cd^2-3ae^2)+c(6Acd^2-aBed-5aAe^2)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(Acd-aBe)-cx(-5aAe^2-aBde+6Acd^2))}{2a(a-cx^2)(cd^2-ae^2)}}{8ac}+\frac{\sqrt{d+ex}(aB+Acx)}{4ac(a-cx^2)^2}$$

$\downarrow$ 1480

$$-\frac{e\left(\frac{\sqrt{c}(\sqrt{a}e+\sqrt{c}d)\left(aBe(2\sqrt{c}d-3\sqrt{a}e)-A\left(-18\sqrt{a}cde+5a\sqrt{c}e^2+12c^{3/2}d^2\right)\right)\int\frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d-\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{\sqrt{c}(\sqrt{c}d-\sqrt{a}e)\left(aBe(3\sqrt{a}e+2\sqrt{c}d)-A\left(18\right.\right.}{}\right.}{2a(cd^2-ae^2)}}{8ac}$$

$$\frac{\sqrt{d+ex}(aB+Acx)}{4ac(a-cx^2)^2}$$

$\downarrow$ 221

$$-\frac{e\left(\frac{(\sqrt{c}d-\sqrt{a}e)\left(aBe(3\sqrt{a}e+2\sqrt{c}d)-A\left(18\sqrt{a}cde+5a\sqrt{c}e^2+12c^{3/2}d^2\right)\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}\sqrt[4]{c}e\sqrt{\sqrt{a}e+\sqrt{c}d}}-\frac{(\sqrt{a}e+\sqrt{c}d)\left(aBe(2\sqrt{c}d-3\sqrt{a}e)-A\left(-18\sqrt{a}cde+5a\sqrt{c}e\right.\right.}{2\sqrt{a}\sqrt[4]{c}e\sqrt{\sqrt{c}d-}}\right.}{2a(cd^2-ae^2)}}{8ac}$$

$$\frac{\sqrt{d+ex}(aB+Acx)}{4ac(a-cx^2)^2}$$

input

```
Int[((A + B*x)*Sqrt[d + e*x])/(a - c*x^2)^3,x]
```

output

```
((a*B + A*c*x)*Sqrt[d + e*x])/(4*a*c*(a - c*x^2)^2) + (-1/2*(Sqrt[d + e*x]
*(a*e*(A*c*d - a*B*e) - c*(6*A*c*d^2 - a*B*d*e - 5*a*A*e^2)*x))/(a*(c*d^2
- a*e^2)*(a - c*x^2)) - (e*(-1/2*((Sqrt[c]*d + Sqrt[a]*e)*(a*B*e*(2*Sqrt[c
]*d - 3*Sqrt[a]*e) - A*(12*c^(3/2)*d^2 - 18*Sqrt[a]*c*d*e + 5*a*Sqrt[c]*e^
2))*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(Sqrt[a]
*c^(1/4)*e*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) + ((Sqrt[c]*d - Sqrt[a]*e)*(a*B*e*
(2*Sqrt[c]*d + 3*Sqrt[a]*e) - A*(12*c^(3/2)*d^2 + 18*Sqrt[a]*c*d*e + 5*a*S
qrt[c]*e^2))*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])
/(2*Sqrt[a]*c^(1/4)*e*Sqrt[Sqrt[c]*d + Sqrt[a]*e])))/(2*a*(c*d^2 - a*e^2))
)/(8*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 685
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^(m - 1)*(a + c*x^2)
^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m]
 || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{\frac{13ce\sqrt{\left(cd+\sqrt{ac\,e^2}\right)c}\left(\frac{\left(6Ac\,d^2-5e\left(Ae+\frac{Bd}{5}\right)a\right)\sqrt{ac\,e^2}}{13}-\frac{12A\,c^2d^3}{13}+ade\left(Ae+\frac{2Bd}{13}\right)c-\frac{3B\,e^3a^2}{13}\right)\left(-c\,x^2+a\right)^2\arctan\left(\frac{c\sqrt{e}}{\sqrt{\left(-cd+\right.}}\right.}{2}}{}$
default	$2e^4\left(\frac{-\frac{c\left(5Aa\,e^2-6Ac\,d^2+Bade\right)(ex+d)^{\frac{7}{2}}}{32a^2e^3\left(a\,e^2-c\,d^2\right)}+\frac{\left(14Aacd\,e^2-18A\,c^2d^3+B\,e^3a^2+3Bac\,d^2e\right)(ex+d)^{\frac{5}{2}}}{32a^2e^3\left(a\,e^2-c\,d^2\right)}+\frac{\left(9A\,a^2e^4-23Aac\,d^2e^2+18\right.}{32a}}{\left(-c(ex+d)^2+2cd(ex+d)+a\,e^2-c\right.}\right.$
derivativedivides	$-2e^4\left(-\frac{-\frac{c\left(5Aa\,e^2-6Ac\,d^2+Bade\right)(ex+d)^{\frac{7}{2}}}{32a^2e^3\left(a\,e^2-c\,d^2\right)}+\frac{\left(14Aacd\,e^2-18A\,c^2d^3+B\,e^3a^2+3Bac\,d^2e\right)(ex+d)^{\frac{5}{2}}}{32a^2e^3\left(a\,e^2-c\,d^2\right)}+\frac{\left(9A\,a^2e^4-23Aac\,d^2\right.}{}}{\left(-c(ex+d)^2+2cd(ex+d)+a\right.}\right.$

input
```
int((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(13/2*c*e*((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*(1/13*(6*A*c*d^2-5*e*(A*e+1
/5*B*d)*a)*(a*c*e^2)^(1/2)-12/13*A*c^2*d^3+a*d*e*(A*e+2/13*B*d)*c-3/13*B*e
^3*a^2)*(-c*x^2+a)^2*arctan(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/
2))+(13/2*c*(1/13*(-6*A*c*d^2+5*e*(A*e+1/5*B*d)*a)*(a*c*e^2)^(1/2)-12/13*A
*c^2*d^3+a*d*e*(A*e+2/13*B*d)*c-3/13*B*e^3*a^2)*e*(-c*x^2+a)^2*arctanh(c*(
e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+((c*d+(a*c*e^2)^(1/2))*c)^(1
/2)*(6*A*c^3*d^2*x^3-10*(A*d^2+1/10*e*x*(B*x+A)*d+1/2*A*e^2*x^2)*x*a*c^2+a
^2*(-4*B*d^2+e*(B*x+A)*d+9*e^2*(1/9*B*x+A)*x)*c+3*B*e^2*a^3)*(e*x+d)^(1/2)
*(a*c*e^2)^(1/2))*((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))/((c*d+(a*c*e^2)^(1/2))
*c)^(1/2)/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)/a^2/(a*e^2-c*d^
2)/c/(-c*x^2+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8803 vs. $2(302) = 604$.

Time = 108.80 (sec) , antiderivative size = 8803, normalized size of antiderivative = 23.66

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(e*x+d)**(1/2)/(-c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^3}\,dx = \int -\frac{(Bx+A)\sqrt{ex+d}}{(cx^2-a)^3}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-integrate((B*x + A)*sqrt(e*x + d)/(c*x^2 - a)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1633 vs. $2(302) = 604$.

Time = 0.36 (sec) , antiderivative size = 1633, normalized size of antiderivative = 4.39

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input

```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="giac")
```

output

```
1/32*((a^2*c^2*d^2*e - a^3*c*e^3)^2*B*a*d*e^2*abs(c) - (a^2*c^2*d^2*e - a^
3*c*e^3)^2*(6*c*d^2*e - 5*a*e^3)*A*abs(c) - 2*(3*sqrt(a*c)*a*c^3*d^5*e - 7
*sqrt(a*c)*a^2*c^2*d^3*e^3 + 4*sqrt(a*c)*a^3*c*d*e^5)*A*abs(a^2*c^2*d^2*e
- a^3*c*e^3)*abs(c) + (sqrt(a*c)*a^2*c^2*d^4*e^2 - 4*sqrt(a*c)*a^3*c*d^2*e
^4 + 3*sqrt(a*c)*a^4*e^6)*B*abs(a^2*c^2*d^2*e - a^3*c*e^3)*abs(c) + (12*a^
3*c^6*d^8*e - 37*a^4*c^5*d^6*e^3 + 38*a^5*c^4*d^4*e^5 - 13*a^6*c^3*d^2*e^7
)*A*abs(c) - (2*a^4*c^5*d^7*e^2 - 7*a^5*c^4*d^5*e^4 + 8*a^6*c^3*d^3*e^6 -
3*a^7*c^2*d*e^8)*B*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a^2*c^3*d^3 - a^3*c
^2*d*e^2 + sqrt((a^2*c^3*d^3 - a^3*c^2*d*e^2)^2 - (a^2*c^3*d^4 - 2*a^3*c^2
*d^2*e^2 + a^4*c*e^4)*(a^2*c^3*d^2 - a^3*c^2*e^2)))/(a^2*c^3*d^2 - a^3*c^2
*e^2)))/((a^4*c^4*d^4*e - 2*a^5*c^3*d^2*e^3 + a^6*c^2*e^5 - sqrt(a*c)*a^3*
c^4*d^5 + 2*sqrt(a*c)*a^4*c^3*d^3*e^2 - sqrt(a*c)*a^5*c^2*d*e^4)*sqrt(-c^2
*d - sqrt(a*c)*c*e)*abs(a^2*c^2*d^2*e - a^3*c*e^3)) + 1/32*((a^2*c^2*d^2*e
 - a^3*c*e^3)^2*B*a*d*e^2*abs(c) - (a^2*c^2*d^2*e - a^3*c*e^3)^2*(6*c*d^2*
e - 5*a*e^3)*A*abs(c) + 2*(3*sqrt(a*c)*a*c^3*d^5*e - 7*sqrt(a*c)*a^2*c^2*d
^3*e^3 + 4*sqrt(a*c)*a^3*c*d*e^5)*A*abs(a^2*c^2*d^2*e - a^3*c*e^3)*abs(c)
- (sqrt(a*c)*a^2*c^2*d^4*e^2 - 4*sqrt(a*c)*a^3*c*d^2*e^4 + 3*sqrt(a*c)*a^4
*e^6)*B*abs(a^2*c^2*d^2*e - a^3*c*e^3)*abs(c) + (12*a^3*c^6*d^8*e - 37*a^4
*c^5*d^6*e^3 + 38*a^5*c^4*d^4*e^5 - 13*a^6*c^3*d^2*e^7)*A*abs(c) - (2*a^4*
c^5*d^7*e^2 - 7*a^5*c^4*d^5*e^4 + 8*a^6*c^3*d^3*e^6 - 3*a^7*c^2*d*e^8)*...
```

Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 13200, normalized size of antiderivative = 35.48

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(((A + B*x)*(d + e*x)^(1/2))/(a - c*x^2)^3,x)
```

output
```
atan(((((12288*B*a^8*c^2*e^8 - 32768*A*a^7*c^3*d*e^7 - 24576*A*a^5*c^5*d^5
*e^3 + 57344*A*a^6*c^4*d^3*e^5 + 4096*B*a^6*c^4*d^4*e^4 - 16384*B*a^7*c^3*
d^2*e^6)/(4096*(a^8*e^4 + a^6*c^2*d^4 - 2*a^7*c*d^2*e^2)) - ((d + e*x)^(1/
2)*((144*A^2*a^5*c^7*d^7 - 9*B^2*a^2*e^7*(a^15*c^5)^(1/2) - 420*A^2*a^6*c^
6*d^5*e^2 + 385*A^2*a^7*c^5*d^3*e^4 + 4*B^2*a^7*c^5*d^5*e^2 - 15*B^2*a^8*c
^4*d^3*e^4 + 30*A*B*a^9*c^3*e^7 + 21*A^2*c^2*d^2*e^5*(a^15*c^5)^(1/2) - 10
5*A^2*a^8*c^4*d*e^6 + 15*B^2*a^9*c^3*d*e^6 - 25*A^2*a*c*e^7*(a^15*c^5)^(1/
2) - 30*A*B*c^2*d^3*e^4*(a^15*c^5)^(1/2) - 48*A*B*a^6*c^6*d^6*e + 5*B^2*a*
c*d^2*e^5*(a^15*c^5)^(1/2) + 160*A*B*a^7*c^5*d^4*e^3 - 150*A*B*a^8*c^4*d^2
*e^5 + 38*A*B*a*c*d*e^6*(a^15*c^5)^(1/2))/(4096*(a^10*c^8*d^6 - a^13*c^5*e
^6 - 3*a^11*c^7*d^4*e^2 + 3*a^12*c^6*d^2*e^4)))^(1/2)*(4096*a^7*c^4*d*e^6
+ 4096*a^5*c^6*d^5*e^2 - 8192*a^6*c^5*d^3*e^4))/(64*(a^6*e^4 + a^4*c^2*d^4
 - 2*a^5*c*d^2*e^2)))*((144*A^2*a^5*c^7*d^7 - 9*B^2*a^2*e^7*(a^15*c^5)^(1/
2) - 420*A^2*a^6*c^6*d^5*e^2 + 385*A^2*a^7*c^5*d^3*e^4 + 4*B^2*a^7*c^5*d^5
*e^2 - 15*B^2*a^8*c^4*d^3*e^4 + 30*A*B*a^9*c^3*e^7 + 21*A^2*c^2*d^2*e^5*(a
^15*c^5)^(1/2) - 105*A^2*a^8*c^4*d*e^6 + 15*B^2*a^9*c^3*d*e^6 - 25*A^2*a*c
*e^7*(a^15*c^5)^(1/2) - 30*A*B*c^2*d^3*e^4*(a^15*c^5)^(1/2) - 48*A*B*a^6*c
^6*d^6*e + 5*B^2*a*c*d^2*e^5*(a^15*c^5)^(1/2) + 160*A*B*a^7*c^5*d^4*e^3 -
150*A*B*a^8*c^4*d^2*e^5 + 38*A*B*a*c*d*e^6*(a^15*c^5)^(1/2))/(4096*(a^10*c
^8*d^6 - a^13*c^5*e^6 - 3*a^11*c^7*d^4*e^2 + 3*a^12*c^6*d^2*e^4)))^(1/2...
```

Reduce [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 4633, normalized size of antiderivative = 12.45

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^3,x)
```

output

```
( - 10*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(
c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**4*c*e**4 - 8*sqrt(a)*sqrt(sqrt(c)*sq
rt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*
d)))*a**3*b*c*d*e**3 + 38*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt
(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*c**2*d**2*e**2
+ 20*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)
*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*c**2*e**4*x**2 + 4*sqrt(a)*sqrt(sqrt
(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*
e - c*d)))*a**2*b*c**2*d**3*e + 16*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*a
tan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*b*c**2
*d*e**3*x**2 - 24*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x
)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c**3*d**4 - 76*sqrt(a)*
sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)
*sqrt(a)*e - c*d)))*a**2*c**3*d**2*e**2*x**2 - 10*sqrt(a)*sqrt(sqrt(c)*sqr
t(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d
)))*a**2*c**3*e**4*x**4 - 8*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sq
rt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*b*c**3*d**3*e*x*
*2 - 8*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(
c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*b*c**3*d*e**3*x**4 + 48*sqrt(a)*sqrt(
sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*s...
```

3.138 $\int \frac{A+Bx}{\sqrt{d+ex}(a-cx^2)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 417

$$\begin{aligned}\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^3}\,dx &= \frac{\sqrt{d+ex}(a(Bd-Ae)+(Acd-aBe)x)}{4a\,(cd^2-ae^2)\,(a-cx^2)^2} \\ &- \frac{\sqrt{d+ex}(ae(Acd^2+6aBde-7aAe^2)-(6Acd(cd^2-2ae^2)+aBe(cd^2+5ae^2))\,x)}{16a^2\,(cd^2-ae^2)^2\,(a-cx^2)} \\ &- \frac{\left(aBe\left(2\sqrt{c}d-5\sqrt{a}e\right)+3A\left(4c^{3/2}d^2-10\sqrt{a}cde+7a\sqrt{c}e^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{32a^{5/2}c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)^{5/2}} \\ &+ \frac{\left(aBe\left(2\sqrt{c}d+5\sqrt{a}e\right)+3A\left(4c^{3/2}d^2+10\sqrt{a}cde+7a\sqrt{c}e^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{32a^{5/2}c^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)^{5/2}}\end{aligned}$$

output
```
1/4*(e*x+d)^(1/2)*(a*(-A*e+B*d)+(A*c*d-B*a*e)*x)/a/(-a*e^2+c*d^2)/(-c*x^2+
a)^2-1/16*(e*x+d)^(1/2)*(a*e*(-7*A*a*e^2+A*c*d^2+6*B*a*d*e)-(6*A*c*d*(-2*a
*e^2+c*d^2)+a*B*e*(5*a*e^2+c*d^2))*x)/a^2/(-a*e^2+c*d^2)^2/(-c*x^2+a)-1/32
*(a*B*e*(2*c^(1/2)*d-5*a^(1/2)*e)+3*A*(4*c^(3/2)*d^2-10*a^(1/2)*c*d*e+7*a*
c^(1/2)*e^2))*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a
^(5/2)/c^(3/4)/(c^(1/2)*d-a^(1/2)*e)^(5/2)+1/32*(a*B*e*(2*c^(1/2)*d+5*a^(1
/2)*e)+3*A*(4*c^(3/2)*d^2+10*a^(1/2)*c*d*e+7*a*c^(1/2)*e^2))*arctanh(c^(1/
4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(5/2)/c^(3/4)/(c^(1/2)*d+a
^(1/2)*e)^(5/2)
```

Mathematica [A] (verified)

Time = 3.56 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^3}\,dx$$

$$= \frac{-\frac{2\sqrt{a}\sqrt{d+ex}(6Ac^3d^3x^3+a^3e^2(10Bd-11Ae-9Bex)-ac^2dx(-Bdex^2+A(10d^2+dex+12e^2x^2))+a^2c(Ae(5d^2+16dex+7e^2x^2)+B(-4d^3+3d^2}{(cd^2-ae^2)^2(a-cx^2)^2}}{}$$

input
```
Integrate[(A + B*x)/(Sqrt[d + e*x]*(a - c*x^2)^3),x]
```

output
```
((-2*Sqrt[a]*Sqrt[d + e*x]*(6*A*c^3*d^3*x^3 + a^3*e^2*(10*B*d - 11*A*e - 9
*B*e*x) - a*c^2*d*x*(-(B*d*e*x^2) + A*(10*d^2 + d*e*x + 12*e^2*x^2)) + a^2
*c*(A*e*(5*d^2 + 16*d*e*x + 7*e^2*x^2) + B*(-4*d^3 + 3*d^2*e*x - 6*d*e^2*x
^2 + 5*e^3*x^3))))/((c*d^2 - a*e^2)^2*(a - c*x^2)^2) - (Sqrt[-(c*d) - Sqrt
[a]*Sqrt[c]*e]*(a*B*e*(2*Sqrt[c]*d + 5*Sqrt[a]*e) + 3*A*(4*c^(3/2)*d^2 + 1
0*Sqrt[a]*c*d*e + 7*a*Sqrt[c]*e^2))*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*
e]*Sqrt[d + e*x])/(Sqrt[c]*d + Sqrt[a]*e)]])/(c*(Sqrt[c]*d + Sqrt[a]*e)^3)
 - ((a*B*e*(2*Sqrt[c]*d - 5*Sqrt[a]*e) + 3*A*(4*c^(3/2)*d^2 - 10*Sqrt[a]*c*
d*e + 7*a*Sqrt[c]*e^2))*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d +
e*x])/(Sqrt[c]*d - Sqrt[a]*e)]])/(Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)^2*Sqrt[-(
c*d) + Sqrt[a]*Sqrt[c]*e]))/(32*a^(5/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {686, 27, 686, 27, 654, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a-cx^2)^3\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \ 686$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{4a(a-cx^2)^2(cd^2-ae^2)}-\frac{\int -\frac{c(6Acd^2+aBed-7aAe^2+5e(Acd-aBe)x)}{2\sqrt{d+ex}(a-cx^2)^2}dx}{4ac(cd^2-ae^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{6Acd^2+aBed-7aAe^2+5e(Acd-aBe)x}{\sqrt{d+ex}(a-cx^2)^2}dx}{8a(cd^2-ae^2)}+\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{4a(a-cx^2)^2(cd^2-ae^2)}$$

$$\Big\downarrow \ 686$$

$$\frac{-\frac{\int -\frac{c\left(2aBde\left(cd^2-4ae^2\right)+3A\left(4c^2d^4-9ace^2d^2+7a^2e^4\right)+e\left(6Acd\left(cd^2-2ae^2\right)+aBe\left(cd^2+5ae^2\right)\right)x\right)}{2\sqrt{d+ex}\left(a-cx^2\right)}dx}{2ac(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(-7aAe^2+6aBde+Acd^2)-x(6A}{2a(a-cx^2)(cd^2}}{8a(cd^2-ae^2)}$$
$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{4a(a-cx^2)^2(cd^2-ae^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{2aBde\left(cd^2-4ae^2\right)+3A\left(4c^2d^4-9ace^2d^2+7a^2e^4\right)+e\left(6Acd\left(cd^2-2ae^2\right)+aBe\left(cd^2+5ae^2\right)\right)x}{\sqrt{d+ex}\left(a-cx^2\right)}dx}{4a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(-7aAe^2+6aBde+Acd^2)-x(6Acd(cd^2}{2a(a-cx^2)(cd^2-ae^2)}}{8a(cd^2-ae^2)}$$
$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{4a(a-cx^2)^2(cd^2-ae^2)}$$

$$\Big\downarrow \ 654$$

$$\frac{\frac{\int -\frac{e\left(aBde\left(cd^2-13ae^2\right)+3A\left(2c^2d^4-5ace^2d^2+7a^2e^4\right)+\left(6Acd\left(cd^2-2ae^2\right)+aBe\left(cd^2+5ae^2\right)\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}\left(ae\left(-7aAe^2+6aBde+Acd^2\right.\right.}{2a(a-c}}{8a\left(cd^2-ae^2\right)}$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{4a\left(a-cx^2\right)^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 25

$$\frac{-\frac{\int \frac{e\left(aBde\left(cd^2-13ae^2\right)+3A\left(2c^2d^4-5ace^2d^2+7a^2e^4\right)+\left(6Acd\left(cd^2-2ae^2\right)+aBe\left(cd^2+5ae^2\right)\right)(d+ex)\right)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}\left(ae\left(-7aAe^2+6aBde+Acd^2\right.\right.}{2a(a-}}{8a\left(cd^2-ae^2\right)}$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{4a\left(a-cx^2\right)^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 27

$$\frac{-\frac{e\int \frac{aBde\left(cd^2-13ae^2\right)+3A\left(2c^2d^4-5ace^2d^2+7a^2e^4\right)+\left(6Acd\left(cd^2-2ae^2\right)+aBe\left(cd^2+5ae^2\right)\right)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}\left(ae\left(-7aAe^2+6aBde+Acd^2\right)\right.}{2a(a-ca}}{8a\left(cd^2-ae^2\right)}$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{4a\left(a-cx^2\right)^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 1480

$$\frac{-\frac{e\left(\frac{(\sqrt{c}d-\sqrt{a}e)^2\left(3A\left(10\sqrt{a}cde+7a\sqrt{c}e^2+4c^{3/2}d^2\right)+aBe(5\sqrt{a}e+2\sqrt{c}d)\right)\int \frac{1}{c(d+ex)-\sqrt{c}(\sqrt{c}d+\sqrt{a}e)}d\sqrt{d+ex}}{2\sqrt{a}e}-\frac{(\sqrt{a}e+\sqrt{c}d)^2\left(3A\left(-10\sqrt{a}cde+7a\sqrt{c}e^2+4c^{3/2}\right.\right.}{}\right.}{2a(cd^2-ae^2)}}{8a\left(cd^2-\right.}$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{4a\left(a-cx^2\right)^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 221

$$\frac{-\frac{e\left(\frac{(\sqrt{a}e+\sqrt{c}d)^2\left(3A\left(-10\sqrt{a}cde+7a\sqrt{c}e^2+4c^{3/2}d^2\right)+aBe(2\sqrt{c}d-5\sqrt{a}e)\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}c^{3/4}e\sqrt{\sqrt{c}d-\sqrt{a}e}}-\frac{(\sqrt{c}d-\sqrt{a}e)^2\left(3A\left(10\sqrt{a}cde+7a\sqrt{c}e^2+4c^{3/2}d^2\right)+aI\right.}{2\sqrt{a}c^{3/4}e\sqrt{\sqrt{c}}}\right.}{2a(cd^2-ae^2)}}{8a\left(cd^2-ae^2\right)}$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{4a\left(a-cx^2\right)^2\left(cd^2-ae^2\right)}$$

input
```
Int[(A + B*x)/(Sqrt[d + e*x]*(a - c*x^2)^3),x]
```

output
```
(Sqrt[d + e*x]*(a*(B*d - A*e) + (A*c*d - a*B*e)*x))/(4*a*(c*d^2 - a*e^2)*(
a - c*x^2)^2) + (-1/2*(Sqrt[d + e*x]*(a*e*(A*c*d^2 + 6*a*B*d*e - 7*a*A*e^2
) - (6*A*c*d*(c*d^2 - 2*a*e^2) + a*B*e*(c*d^2 + 5*a*e^2))*x))/(a*(c*d^2 -
a*e^2)*(a - c*x^2)) - (e*(((Sqrt[c]*d + Sqrt[a]*e)^2*(a*B*e*(2*Sqrt[c]*d -
 5*Sqrt[a]*e) + 3*A*(4*c^(3/2)*d^2 - 10*Sqrt[a]*c*d*e + 7*a*Sqrt[c]*e^2))*
ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])/(2*Sqrt[a]*c
^(3/4)*e*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) - ((Sqrt[c]*d - Sqrt[a]*e)^2*(a*B*e*
(2*Sqrt[c]*d + 5*Sqrt[a]*e) + 3*A*(4*c^(3/2)*d^2 + 10*Sqrt[a]*c*d*e + 7*a*
Sqrt[c]*e^2))*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]]
)/(2*Sqrt[a]*c^(3/4)*e*Sqrt[Sqrt[c]*d + Sqrt[a]*e])))/(2*a*(c*d^2 - a*e^2)
))/(8*a*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 3.95 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.59

method	result
derivativedivides	$-2e^4c^3\left(\frac{\frac{\frac{\sqrt{ace^2}\left(6Acde+5Bae^2-9A\sqrt{ace^2}\,e-2B\sqrt{ace^2}\,d\right)(ex+d)^{\frac{3}{2}}}{4c^2e\left(ae^2+cd^2-2\sqrt{ace^2}\,d\right)}-\frac{\sqrt{ace^2}\left(6Acde+7Bae^2-11A\sqrt{ace^2}\,e-2B\sqrt{ace^2}\,d\right)}{4c^2e\left(cd-\sqrt{ace^2}\right)}}{\left(-ex-\frac{\sqrt{ace^2}}{c}\right)^2}}{16e^3\sqrt{ace^2}\,a}\right.$
default	$2e^4c^3\left(-\frac{\frac{\frac{\sqrt{ace^2}\left(6Acde+5Bae^2-9A\sqrt{ace^2}\,e-2B\sqrt{ace^2}\,d\right)(ex+d)^{\frac{3}{2}}}{4c^2e\left(ae^2+cd^2-2\sqrt{ace^2}\,d\right)}-\frac{\sqrt{ace^2}\left(6Acde+7Bae^2-11A\sqrt{ace^2}\,e-2B\sqrt{ace^2}\,d\right)}{4c^2e\left(cd-\sqrt{ace^2}\right)}}{\left(-ex-\frac{\sqrt{ace^2}}{c}\right)^2}}{16e^3\sqrt{ace^2}\,a}\right.$
pseudoelliptic	Expression too large to display

input
```
int((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*e^4*c^3*(1/16/e^3/(a*c*e^2)^(1/2)/a^2/c^2*((1/4*(a*c*e^2)^(1/2)/c^2/e*(
6*A*c*d*e+5*B*a*e^2-9*A*(a*c*e^2)^(1/2)*e-2*B*(a*c*e^2)^(1/2)*d)/(a*e^2+c*
d^2-2*(a*c*e^2)^(1/2)*d)*(e*x+d)^(3/2)-1/4*(a*c*e^2)^(1/2)/c^2/e*(6*A*c*d*
e+7*B*a*e^2-11*A*(a*c*e^2)^(1/2)*e-2*B*(a*c*e^2)^(1/2)*d)/(c*d-(a*c*e^2)^(
1/2))*(e*x+d)^(1/2))/(-e*x-(a*c*e^2)^(1/2)/c)^2-1/4*(-21*A*a*c*e^2-12*A*c^
2*d^2-2*B*a*c*d*e+30*A*(a*c*e^2)^(1/2)*c*d+5*B*(a*c*e^2)^(1/2)*a*e)/c/(-a*
e^2-c*d^2+2*(a*c*e^2)^(1/2)*d)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c*(
e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)))-1/16/e^3/(a*c*e^2)^(1/2)/a
^2/c^2*((-1/4*(a*c*e^2)^(1/2)/c^2/e*(6*A*c*d*e+5*B*a*e^2+9*A*(a*c*e^2)^(1/
2)*e+2*B*(a*c*e^2)^(1/2)*d)/(a*e^2+c*d^2+2*(a*c*e^2)^(1/2)*d)*(e*x+d)^(3/2
)+1/4*(a*c*e^2)^(1/2)/c^2/e*(6*A*c*d*e+7*B*a*e^2+11*A*(a*c*e^2)^(1/2)*e+2*
B*(a*c*e^2)^(1/2)*d)/(c*d+(a*c*e^2)^(1/2))*(e*x+d)^(1/2))/(-e*x+(a*c*e^2)^
(1/2)/c)^2+1/4*(21*A*a*c*e^2+12*A*c^2*d^2+2*B*a*c*d*e+30*A*(a*c*e^2)^(1/2)
*c*d+5*B*(a*c*e^2)^(1/2)*a*e)/c/(a*e^2+c*d^2+2*(a*c*e^2)^(1/2)*d)/((c*d+(a
*c*e^2)^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^
(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)**(1/2)/(-c*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^3}\,dx = \int -\frac{Bx+A}{(cx^2-a)^3\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-integrate((B*x + A)/((c*x^2 - a)^3*sqrt(e*x + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2380 vs. $2(347) = 694$.

Time = 0.46 (sec) , antiderivative size = 2380, normalized size of antiderivative = 5.71

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/32*(6*(a^2*c^2*d^4*e - 2*a^3*c*d^2*e^3 + a^4*e^5)^2*(c^2*d^3*e - 2*a*c*
d*e^3)*A*abs(c) + (a^2*c^2*d^4*e - 2*a^3*c*d^2*e^3 + a^4*e^5)^2*(a*c*d^2*e
^2 + 5*a^2*e^4)*B*abs(c) + 3*(2*sqrt(a*c)*a*c^4*d^8*e - 9*sqrt(a*c)*a^2*c^
3*d^6*e^3 + 19*sqrt(a*c)*a^3*c^2*d^4*e^5 - 19*sqrt(a*c)*a^4*c*d^2*e^7 + 7*
sqrt(a*c)*a^5*e^9)*A*abs(a^2*c^2*d^4*e - 2*a^3*c*d^2*e^3 + a^4*e^5)*abs(c)
 + (sqrt(a*c)*a^2*c^3*d^7*e^2 - 15*sqrt(a*c)*a^3*c^2*d^5*e^4 + 27*sqrt(a*c
)*a^4*c*d^3*e^6 - 13*sqrt(a*c)*a^5*d*e^8)*B*abs(a^2*c^2*d^4*e - 2*a^3*c*d^
2*e^3 + a^4*e^5)*abs(c) - 3*(4*a^3*c^7*d^13*e - 25*a^4*c^6*d^11*e^3 + 67*a
^5*c^5*d^9*e^5 - 98*a^6*c^4*d^7*e^7 + 82*a^7*c^3*d^5*e^9 - 37*a^8*c^2*d^3*
e^11 + 7*a^9*c*d*e^13)*A*abs(c) - 2*(a^4*c^6*d^12*e^2 - 8*a^5*c^5*d^10*e^4
 + 22*a^6*c^4*d^8*e^6 - 28*a^7*c^3*d^6*e^8 + 17*a^8*c^2*d^4*e^10 - 4*a^9*c
*d^2*e^12)*B*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(a^2*c^3*d^5 - 2*a^3*c^2*d
^3*e^2 + a^4*c*d*e^4 + sqrt((a^2*c^3*d^5 - 2*a^3*c^2*d^3*e^2 + a^4*c*d*e^4
)^2 - (a^2*c^3*d^6 - 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 - a^5*e^6)*(a^2*c
^3*d^4 - 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)))/(a^2*c^3*d^4 - 2*a^3*c^2*d^2*e^2
 + a^4*c*e^4)))/((a^4*c^5*d^8*e - 4*a^5*c^4*d^6*e^3 + 6*a^6*c^3*d^4*e^5 -
4*a^7*c^2*d^2*e^7 + a^8*c*e^9 - sqrt(a*c)*a^3*c^5*d^9 + 4*sqrt(a*c)*a^4*c^
4*d^7*e^2 - 6*sqrt(a*c)*a^5*c^3*d^5*e^4 + 4*sqrt(a*c)*a^6*c^2*d^3*e^6 - sq
rt(a*c)*a^7*c*d*e^8)*sqrt(-c^2*d - sqrt(a*c)*c*e)*abs(a^2*c^2*d^4*e - 2*a^
3*c*d^2*e^3 + a^4*e^5)) - 1/32*(6*(a^2*c^2*d^4*e - 2*a^3*c*d^2*e^3 + a^...
```

Mupad [B] (verification not implemented)

Time = 10.76 (sec) , antiderivative size = 19125, normalized size of antiderivative = 45.86

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/((a - c*x^2)^3*(d + e*x)^(1/2)),x)
```

output
```
- atan(((((86016*A*a^9*c^3*e^11 - 53248*B*a^9*c^3*d*e^10 + 24576*A*a^5*c^7
*d^8*e^3 - 110592*A*a^6*c^6*d^6*e^5 + 233472*A*a^7*c^5*d^4*e^7 - 233472*A*
a^8*c^4*d^2*e^9 + 4096*B*a^6*c^6*d^7*e^4 - 61440*B*a^7*c^5*d^5*e^6 + 11059
2*B*a^8*c^4*d^3*e^8)/(4096*(a^10*e^8 + a^6*c^4*d^8 - 4*a^9*c*d^2*e^6 - 4*a
^7*c^3*d^6*e^2 + 6*a^8*c^2*d^4*e^4)) - ((d + e*x)^(1/2)*((144*A^2*a^5*c^7*
d^9 - 25*B^2*a^3*e^9*(a^15*c^3)^(1/2) - 756*A^2*a^6*c^6*d^7*e^2 + 1701*A^2
*a^7*c^5*d^5*e^4 - 1890*A^2*a^8*c^4*d^3*e^6 + 4*B^2*a^7*c^5*d^7*e^2 - 35*B
^2*a^8*c^4*d^5*e^4 + 70*B^2*a^9*c^3*d^3*e^6 - 441*A^2*a^2*c*e^9*(a^15*c^3)
^(1/2) - 210*A*B*a^10*c^2*e^9 - 189*A^2*c^3*d^4*e^5*(a^15*c^3)^(1/2) + 945
*A^2*a^9*c^3*d*e^8 + 105*B^2*a^10*c^2*d*e^8 + 210*A*B*c^3*d^5*e^4*(a^15*c^
3)^(1/2) + 48*A*B*a^6*c^6*d^8*e + 486*A^2*a*c^2*d^2*e^7*(a^15*c^3)^(1/2) -
 336*A*B*a^7*c^5*d^6*e^3 + 630*A*B*a^8*c^4*d^4*e^5 - 420*A*B*a^9*c^3*d^2*e
^7 + 35*B^2*a*c^2*d^4*e^5*(a^15*c^3)^(1/2) - 154*B^2*a^2*c*d^2*e^7*(a^15*c
^3)^(1/2) + 666*A*B*a^2*c*d*e^8*(a^15*c^3)^(1/2) - 588*A*B*a*c^2*d^3*e^6*(
a^15*c^3)^(1/2))/(4096*(a^10*c^8*d^10 - a^15*c^3*e^10 - 5*a^11*c^7*d^8*e^2
 + 10*a^12*c^6*d^6*e^4 - 10*a^13*c^5*d^4*e^6 + 5*a^14*c^4*d^2*e^8)))^(1/2)
*(4096*a^9*c^4*d*e^10 + 4096*a^5*c^8*d^9*e^2 - 16384*a^6*c^7*d^7*e^4 + 245
76*a^7*c^6*d^5*e^6 - 16384*a^8*c^5*d^3*e^8))/(64*(a^8*e^8 + a^4*c^4*d^8 -
4*a^7*c*d^2*e^6 - 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((144*A^2*a^5*c
^7*d^9 - 25*B^2*a^3*e^9*(a^15*c^3)^(1/2) - 756*A^2*a^6*c^6*d^7*e^2 + 17...
```

Reduce [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 5713, normalized size of antiderivative = 13.70

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^3,x)
```

output
```
( - 10*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(
c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**4*b*e**5 + 66*sqrt(a)*sqrt(sqrt(c)*s
qrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c
*d)))*a**4*c*d*e**4 - 18*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(
d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*b*c*d**2*e**3 +
20*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*s
qrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*b*c*e**5*x**2 - 66*sqrt(a)*sqrt(sqrt(c
)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e
- c*d)))*a**3*c**2*d**3*e**2 - 132*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*a
tan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**3*c**2*d
*e**4*x**2 + 4*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c
)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*b*c**2*d**4*e + 36*sqrt(a)
*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c
)*sqrt(a)*e - c*d)))*a**2*b*c**2*d**2*e**3*x**2 - 10*sqrt(a)*sqrt(sqrt(c)*
sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e -
c*d)))*a**2*b*c**2*e**5*x**4 + 24*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*at
an((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c**3*d*
*5 + 132*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqr
t(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a**2*c**3*d**3*e**2*x**2 + 66*sqrt(a)
*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqr...
```

3.139 $\int \frac{A+Bx}{\sqrt{d+ex}(2ABd-A^2e-B^2ex^2)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 76

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(2ABd-A^2e-B^2ex^2)}\,dx=\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{B}\sqrt{2Bd-Ae}\sqrt{d+ex}}{2Bd-Ae+Bex}\right)}{\sqrt{B}e\sqrt{2Bd-Ae}}$$

output
```
2^(1/2)*arctanh(2^(1/2)*B^(1/2)*(-A*e+2*B*d)^(1/2)*(e*x+d)^(1/2)/(B*e*x-A*
e+2*B*d))/B^(1/2)/e/(-A*e+2*B*d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.32

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(2ABd-A^2e-B^2ex^2)}\,dx =\frac{-\frac{\left(-i\sqrt{A}\sqrt{e}+\sqrt{-2Bd+Ae}\right)\arctan\left(\frac{\sqrt{B}\sqrt{d+ex}}{\sqrt{-Bd-i\sqrt{A}\sqrt{e}\sqrt{-2Bd+Ae}}}\right)}{\sqrt{-Bd-i\sqrt{A}\sqrt{e}\sqrt{-2Bd+Ae}}}-\frac{\left(i\sqrt{A}\sqrt{e}+\sqrt{-2Bd+Ae}\right)\arctan\left(\frac{\sqrt{B}\sqrt{d+ex}}{\sqrt{-Bd+i\sqrt{A}\sqrt{e}\sqrt{-2Bd+Ae}}}\right)}{\sqrt{-Bd+i\sqrt{A}\sqrt{e}\sqrt{-2Bd+Ae}}}}{\sqrt{B}e\sqrt{-2Bd+Ae}}$$

input
```
Integrate[(A + B*x)/(Sqrt[d + e*x]*(2*A*B*d - A^2*e - B^2*e*x^2)),x]
```

output
```
(-((((-I)*Sqrt[A]*Sqrt[e] + Sqrt[-2*B*d + A*e])*ArcTan[(Sqrt[B]*Sqrt[d + e
*x])/Sqrt[-(B*d) - I*Sqrt[A]*Sqrt[e]*Sqrt[-2*B*d + A*e]]])/Sqrt[-(B*d) - I
*Sqrt[A]*Sqrt[e]*Sqrt[-2*B*d + A*e]]) - ((I*Sqrt[A]*Sqrt[e] + Sqrt[-2*B*d
+ A*e])*ArcTan[(Sqrt[B]*Sqrt[d + e*x])/Sqrt[-(B*d) + I*Sqrt[A]*Sqrt[e]*Sqr
t[-2*B*d + A*e]]])/Sqrt[-(B*d) + I*Sqrt[A]*Sqrt[e]*Sqrt[-2*B*d + A*e]])/(S
qrt[B]*e*Sqrt[-2*B*d + A*e])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 162 vs. $2(76) = 152$.

Time = 0.41 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {654, 1478, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(A^2(-e)+2ABd-B^2ex^2)}dx$$

$$\Big\downarrow \text{654}$$

$$2\int \frac{Bd-Ae-B(d+ex)}{e(d+ex)^2B^2-2de(d+ex)B^2+e(Bd-Ae)^2}d\sqrt{d+ex}$$

$$\Big\downarrow \text{1478}$$

$$2\left(-\frac{\int -\frac{\sqrt{2}\sqrt{2Bd-Ae}-2\sqrt{B}\sqrt{d+ex}}{\sqrt{B}\left(2d-\frac{Ae}{B}+ex-\frac{\sqrt{2}\sqrt{2Bd-Ae}\sqrt{d+ex}}{\sqrt{B}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{B}e\sqrt{2Bd-Ae}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2Bd-Ae}+\sqrt{2}\sqrt{B}\sqrt{d+ex}\right)}{\sqrt{B}\left(2d-\frac{Ae}{B}+ex+\frac{\sqrt{2}\sqrt{2Bd-Ae}\sqrt{d+ex}}{\sqrt{B}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{B}e\sqrt{2Bd-Ae}}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\int \frac{\sqrt{2}\sqrt{2Bd-Ae}-2\sqrt{B}\sqrt{d+ex}}{\sqrt{B}\left(2d-\frac{Ae}{B}+ex-\frac{\sqrt{2}\sqrt{2Bd-Ae}\sqrt{d+ex}}{\sqrt{B}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{B}e\sqrt{2Bd-Ae}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2Bd-Ae}+\sqrt{2}\sqrt{B}\sqrt{d+ex}\right)}{\sqrt{B}\left(2d-\frac{Ae}{B}+ex+\frac{\sqrt{2}\sqrt{2Bd-Ae}\sqrt{d+ex}}{\sqrt{B}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{B}e\sqrt{2Bd-Ae}}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{\int \frac{\sqrt{2}\sqrt{2Bd-Ae}-2\sqrt{B}\sqrt{d+ex}}{2d-\frac{Ae}{B}+ex-\frac{\sqrt{2}\sqrt{2Bd-Ae}\sqrt{d+ex}}{\sqrt{B}}}d\sqrt{d+ex}}{2\sqrt{2}Be\sqrt{2Bd-Ae}}+\frac{\int \frac{\sqrt{2Bd-Ae}+\sqrt{2}\sqrt{B}\sqrt{d+ex}}{2d-\frac{Ae}{B}+ex+\frac{\sqrt{2}\sqrt{2Bd-Ae}\sqrt{d+ex}}{\sqrt{B}}}d\sqrt{d+ex}}{2Be\sqrt{2Bd-Ae}}\right)$$

$\downarrow$ 1103

$$2\left(\frac{\log\left(\sqrt{2}\sqrt{B}\sqrt{d+ex}\sqrt{2Bd-Ae}-Ae+B(d+ex)+Bd\right)}{2\sqrt{2}\sqrt{B}e\sqrt{2Bd-Ae}}-\frac{\log\left(-\sqrt{2}\sqrt{B}\sqrt{d+ex}\sqrt{2Bd-Ae}-Ae+B(d\right.}{2\sqrt{2}\sqrt{B}e\sqrt{2Bd-Ae}}\right.$$

input
```
Int[(A + B*x)/(Sqrt[d + e*x]*(2*A*B*d - A^2*e - B^2*e*x^2)),x]
```

output
```
2*(-1/2*Log[B*d - A*e - Sqrt[2]*Sqrt[B]*Sqrt[2*B*d - A*e]*Sqrt[d + e*x] +
B*(d + e*x)]/(Sqrt[2]*Sqrt[B]*e*Sqrt[2*B*d - A*e]) + Log[B*d - A*e + Sqrt[
2]*Sqrt[B]*Sqrt[2*B*d - A*e]*Sqrt[d + e*x] + B*(d + e*x)]/(2*Sqrt[2]*Sqrt[
B]*e*Sqrt[2*B*d - A*e]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(62) = 124$.

Time = 1.88 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.59

method	result
pseudoelliptic	$-\frac{-\frac{\left(AeB+\sqrt{-e\,B^2A(Ae-2Bd)}\right)\operatorname{arctanh}\left(\frac{B\sqrt{ex+d}}{\sqrt{B^2d+\sqrt{-e\,B^2A(Ae-2Bd)}}}\right)}{\sqrt{B^2d+\sqrt{-e\,B^2A(Ae-2Bd)}}}+\frac{\left(AeB-\sqrt{-e\,B^2A(Ae-2Bd)}\right)\operatorname{arctanh}\left(\frac{B}{\sqrt{B^2d-\sqrt{}}}\right)}{\sqrt{B^2d-\sqrt{-e\,B^2A(Ae-2Bd)}}}}{e\sqrt{-e\,B^2A(Ae-2Bd)}}$
derivativedivides	$-\frac{2B^2\left(-\frac{\left(AeB+\sqrt{-e\,B^2A(Ae-2Bd)}\right)\operatorname{arctanh}\left(\frac{B\sqrt{ex+d}}{\sqrt{B^2d+\sqrt{-e\,B^2A(Ae-2Bd)}}}\right)}{2B^2\sqrt{-e\,B^2A(Ae-2Bd)}\sqrt{B^2d+\sqrt{-e\,B^2A(Ae-2Bd)}}}-\frac{\left(-AeB+\sqrt{-e\,B^2A(Ae-2Bd)}\right)\operatorname{arctanh}\left(\right.}{2B^2\sqrt{-e\,B^2A(Ae-2Bd)}\sqrt{B^2d-}}\right.}{e}$
default	$-\frac{2B^2\left(-\frac{\left(AeB+\sqrt{-e\,B^2A(Ae-2Bd)}\right)\operatorname{arctanh}\left(\frac{B\sqrt{ex+d}}{\sqrt{B^2d+\sqrt{-e\,B^2A(Ae-2Bd)}}}\right)}{2B^2\sqrt{-e\,B^2A(Ae-2Bd)}\sqrt{B^2d+\sqrt{-e\,B^2A(Ae-2Bd)}}}-\frac{\left(-AeB+\sqrt{-e\,B^2A(Ae-2Bd)}\right)\operatorname{arctanh}\left(\right.}{2B^2\sqrt{-e\,B^2A(Ae-2Bd)}\sqrt{B^2d-}}\right.}{e}$

input

```
int((B*x+A)/(e*x+d)^(1/2)/(-B^2*e*x^2-A^2*e+2*A*B*d),x,method=_RETURNVERBO
SE)
```

output

```
-1/e/(-e*B^2*A*(A*e-2*B*d))^(1/2)*(-(A*e*B+(-e*B^2*A*(A*e-2*B*d))^(1/2))/(
B^2*d+(-e*B^2*A*(A*e-2*B*d))^(1/2))^(1/2)*arctanh(B*(e*x+d)^(1/2)/(B^2*d+(
-e*B^2*A*(A*e-2*B*d))^(1/2))^(1/2))+(A*e*B-(-e*B^2*A*(A*e-2*B*d))^(1/2))*a
rctanh(B*(e*x+d)^(1/2)/(B^2*d-(-e*B^2*A*(A*e-2*B*d))^(1/2))^(1/2))/(B^2*d-
(-e*B^2*A*(A*e-2*B*d))^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.11

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(2ABd-A^2e-B^2ex^2)}\,dx = \left[\frac{\sqrt{2}\log\left(\frac{B^2e^2x^2+8\,B^2d^2-6\,ABde+A^2e^2+4\,(2\,B^2de-ABe^2)x+\frac{2\,\sqrt{2}\left(4\,B^3d^2-4\,AB^2de+A^2Be^2+\left(2\,B^3de-AB^2e^2\right)x\right)\sqrt{ex+d}}{\sqrt{2\,B^2d-ABe}}}{B^2ex^2-2\,ABd+A^2e}\right)}{2\,\sqrt{2\,B^2d-ABe}e}, -\frac{\sqrt{2}\sqrt{-\frac{1}{2\,B^2d-ABe}}\arctan\left(\frac{\sqrt{2}(Bex+2\,Bd-Ae)\sqrt{-\frac{1}{2\,B^2d-ABe}}}{2\,\sqrt{ex+d}}\right)}{e}\right]$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-B^2*e*x^2-A^2*e+2*A*B*d),x, algorithm="f
ricas")
```

output
```
[1/2*sqrt(2)*log((B^2*e^2*x^2 + 8*B^2*d^2 - 6*A*B*d*e + A^2*e^2 + 4*(2*B^2
*d*e - A*B*e^2)*x + 2*sqrt(2)*(4*B^3*d^2 - 4*A*B^2*d*e + A^2*B*e^2 + (2*B^
3*d*e - A*B^2*e^2)*x)*sqrt(e*x + d)/sqrt(2*B^2*d - A*B*e))/(B^2*e*x^2 - 2*
A*B*d + A^2*e))/(sqrt(2*B^2*d - A*B*e)*e), -sqrt(2)*sqrt(-1/(2*B^2*d - A*B
*e))*arctan(1/2*sqrt(2)*(B*e*x + 2*B*d - A*e)*sqrt(-1/(2*B^2*d - A*B*e))/s
qrt(e*x + d))/e]
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(2ABd-A^2e-B^2ex^2)}\,dx = -\int \frac{A}{A^2e\sqrt{d+ex}-2ABd\sqrt{d+ex}+B^2ex^2\sqrt{d+ex}}\,dx - \int \frac{Bx}{A^2e\sqrt{d+ex}-2ABd\sqrt{d+ex}+B^2ex^2\sqrt{d+ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)**(1/2)/(-B**2*e*x**2-A**2*e+2*A*B*d),x)
```

output
```
-Integral(A/(A**2*e*sqrt(d + e*x) - 2*A*B*d*sqrt(d + e*x) + B**2*e*x**2*sq
rt(d + e*x)), x) - Integral(B*x/(A**2*e*sqrt(d + e*x) - 2*A*B*d*sqrt(d + e
*x) + B**2*e*x**2*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(2ABd-A^2e-B^2ex^2)}\,dx = \int -\frac{Bx+A}{(B^2ex^2-2\,ABd+A^2e)\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-B^2*e*x^2-A^2*e+2*A*B*d),x, algorithm="m
axima")
```

output
```
-integrate((B*x + A)/((B^2*e*x^2 - 2*A*B*d + A^2*e)*sqrt(e*x + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1495 vs. $2(62) = 124$.

Time = 0.39 (sec) , antiderivative size = 1495, normalized size of antiderivative = 19.67

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(2ABd-A^2e-B^2ex^2)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-B^2*e*x^2-A^2*e+2*A*B*d),x, algorithm="g
iac")
```

output
```
(2*A*B^5*d^2*e^3 - A^2*B^4*d*e^4 - (4*A*B^3*d^2*e - 4*A^2*B^2*d*e^2 + A^3*
B*e^3 + (2*A*B^2*d*e - A^2*B*e^2)*sqrt(2*A*B*d*e - A^2*e^2))*B^2*e^2*sgn(e
) - (4*A*B^3*d^2*e - 4*A^2*B^2*d*e^2 + A^3*B*e^3 + (4*B^3*d^2 - 4*A*B^2*d*
e + A^2*B*e^2)*sqrt(2*A*B*d*e - A^2*e^2))*B^2*e^2 - (2*A*B^4*d^2*e^2 - 3*A
^2*B^3*d*e^3 + A^3*B^2*e^4 - (2*B^4*d^2*e - 3*A*B^3*d*e^2 + A^2*B^2*e^3)*s
qrt(2*A*B*d*e - A^2*e^2))*abs(B)*abs(e)*sgn(e) - (4*B^5*d^3*e - 8*A*B^4*d^
2*e^2 + 5*A^2*B^3*d*e^3 - A^3*B^2*e^4 + (2*B^4*d^2*e - 3*A*B^3*d*e^2 + A^2
*B^2*e^3)*sqrt(2*A*B*d*e - A^2*e^2))*abs(B)*abs(e) + (2*A*B^5*d^2*e^3 - A^
2*B^4*d*e^4 - sqrt(2*A*B*d*e - A^2*e^2)*A*B^4*d*e^3)*sgn(e) - (2*B^5*d^2*e
^2 - A*B^4*d*e^3)*sqrt(2*A*B*d*e - A^2*e^2))*arctan(sqrt(e*x + d)/sqrt(-(B
^2*d*e + sqrt(B^4*d^2*e^2 - (B^2*d^2*e - 2*A*B*d*e^2 + A^2*e^3)*B^2*e))/(B
^2*e)))/((2*sqrt(2)*sqrt(-2*B^2*d + A*B*e)*B^5*d^3*e^2 - 2*sqrt(2)*sqrt(-A
*B*e)*B^5*d^3*e^2 - 5*sqrt(2)*sqrt(-2*B^2*d + A*B*e)*A*B^4*d^2*e^3 + 5*sqr
t(2)*sqrt(-A*B*e)*A*B^4*d^2*e^3 + 4*sqrt(2)*sqrt(-2*B^2*d + A*B*e)*A^2*B^3
*d*e^4 - 4*sqrt(2)*sqrt(-A*B*e)*A^2*B^3*d*e^4 - sqrt(2)*sqrt(-2*B^2*d + A*
B*e)*A^3*B^2*e^5 + sqrt(2)*sqrt(-A*B*e)*A^3*B^2*e^5)*abs(B)*abs(e)) + (2*A
*B^5*d^2*e^3 - A^2*B^4*d*e^4 + (4*A*B^3*d^2*e - 4*A^2*B^2*d*e^2 + A^3*B*e^
3 - (2*A*B^2*d*e - A^2*B*e^2)*sqrt(2*A*B*d*e - A^2*e^2))*B^2*e^2*sgn(e) -
(4*A*B^3*d^2*e - 4*A^2*B^2*d*e^2 + A^3*B*e^3 + (4*B^3*d^2 - 4*A*B^2*d*e +
A^2*B*e^2)*sqrt(2*A*B*d*e - A^2*e^2))*B^2*e^2 + (2*A*B^4*d^2*e^2 - 3*A^...
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 310, normalized size of antiderivative = 4.08

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(2ABd-A^2e-B^2ex^2)}\,dx$$
$$=\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\left(A\,e^2\,\sqrt{A\,B\,e-2\,B^2\,d}-2\,B\,d\,e\,\sqrt{A\,B\,e-2\,B^2\,d}\right)\left(\left(\frac{\sqrt{2}\left(\frac{2\,B^4\,d-2\,A\,B^3\,e}{e^2}-\frac{4\,A^2\,B^4\,e^4-8\,A\,B^5\,d\,e^3+4\,B^6\,d^2\,e^2}{e^4\,\left(2\,B^2\,d-A\,B\,e\right)}\right)}{\left(A\,e-B\,d\right)\,\left(A\,e-2\,B\,d\right)}+\frac{4\,\sqrt{2}\,A\cdots}{e\,\left(2\,B^2\,d-A\,B\,e\right)\cdots}\cdots}{4\,A\,B^3\cdots}\cdots}{e\,\sqrt{A\,B\,e-2\,B^2\,d}\cdots}$$

input
```
int(-(A + B*x)/((d + e*x)^(1/2)*(A^2*e + B^2*e*x^2 - 2*A*B*d)),x)
```

output
```
(2^(1/2)*(atan(((A*e^2*(A*B*e - 2*B^2*d)^(1/2) - 2*B*d*e*(A*B*e - 2*B^2*d)
^(1/2))*(((2^(1/2)*((2*B^4*d - 2*A*B^3*e)/e^2 - (4*A^2*B^4*e^4 + 4*B^6*d^2
*e^2 - 8*A*B^5*d*e^3)/(e^4*(2*B^2*d - A*B*e))))/((A*e - B*d)*(A*e - 2*B*d)
) + (4*2^(1/2)*A*B^4)/(e*(2*B^2*d - A*B*e)*(A*e - B*d)))*(d + e*x)^(1/2) -
 (2^(1/2)*((2*B^4)/e^2 - (4*B^6*d)/(e^2*(2*B^2*d - A*B*e)))*(d + e*x)^(3/2
))/((A*e - B*d)*(A*e - 2*B*d))))/(4*A*B^3)) - atan((2^(1/2)*(A*B*e - 2*B^2
*d)^(1/2)*(d + e*x)^(1/2))/(2*(A*e - 2*B*d)))))/(e*(A*B*e - 2*B^2*d)^(1/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(2ABd-A^2e-B^2ex^2)}\,dx = -\frac{\sqrt{b}\,\sqrt{ae-2bd}\,\sqrt{2}\left(atan\left(\frac{2\sqrt{ex+d}\,b-\sqrt{e}\,\sqrt{b}\,\sqrt{a}\,\sqrt{2}}{\sqrt{b}\,\sqrt{ae-2bd}\,\sqrt{2}}\right)+atan\left(\frac{2\sqrt{ex+d}\,b+\sqrt{e}\,\sqrt{b}\,\sqrt{a}\,\sqrt{2}}{\sqrt{b}\,\sqrt{ae-2bd}\,\sqrt{2}}\right)\right)}{be\,(ae-2bd)}$$

input
```
int((B*x+A)/(e*x+d)^(1/2)/(-B^2*e*x^2-A^2*e+2*A*B*d),x)
```

output
```
( - sqrt(b)*sqrt(a*e - 2*b*d)*sqrt(2)*(atan((2*sqrt(d + e*x)*b - sqrt(e)*s
qrt(b)*sqrt(a)*sqrt(2))/(sqrt(b)*sqrt(a*e - 2*b*d)*sqrt(2))) + atan((2*sqr
t(d + e*x)*b + sqrt(e)*sqrt(b)*sqrt(a)*sqrt(2))/(sqrt(b)*sqrt(a*e - 2*b*d)
*sqrt(2)))))/(b*e*(a*e - 2*b*d))
```

3.140 $\int \frac{A+Bx}{\sqrt{\frac{A^2e-B^2e}{2AB}+ex(1+x^2)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 135

$$\int \frac{A+Bx}{\sqrt{\frac{A^2e-B^2e}{2AB}+ex(1+x^2)}}\,dx=\frac{\sqrt{2}\sqrt{A}\sqrt{B}\arctan\left(\frac{A}{B}+\frac{\sqrt{A}\sqrt{e\left(\frac{A}{B}-\frac{B}{A}+2x\right)}}{\sqrt{B}\sqrt{e}}\right)}{\sqrt{e}}-\frac{\sqrt{2}\sqrt{A}\sqrt{B}\arctan\left(\frac{A}{B}-\frac{\sqrt{A}\sqrt{\left(\frac{A}{B}-\frac{B}{A}\right)e+2ex}}{\sqrt{B}\sqrt{e}}\right)}{\sqrt{e}}$$

output
```
2^(1/2)*A^(1/2)*B^(1/2)*arctan(A/B+A^(1/2)*(e*(A/B-B/A+2*x))^(1/2)/B^(1/2)
/e^(1/2))/e^(1/2)-2^(1/2)*A^(1/2)*B^(1/2)*arctan(A/B-A^(1/2)*((A/B-B/A)*e+
2*e*x)^(1/2)/B^(1/2)/e^(1/2))/e^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx}{\sqrt{\frac{A^2e-B^2e}{2AB}+ex}\,(1+x^2)}\,dx = -\frac{i\sqrt{2}\sqrt{A}\sqrt{B}\sqrt{\frac{A}{B}-\frac{B}{A}+2x}\left(\operatorname{arctanh}\left(\frac{\sqrt{A}\sqrt{B}\sqrt{\frac{A}{B}-\frac{B}{A}+2x}}{A-iB}\right)-\operatorname{arctanh}\left(\frac{\sqrt{A}\sqrt{B}\sqrt{\frac{A}{B}-\frac{B}{A}+2x}}{A+iB}\right)\right)}{\sqrt{e\left(\frac{A}{B}-\frac{B}{A}+2x\right)}}$$

input
```
Integrate[(A + B*x)/(Sqrt[(A^2*e - B^2*e)/(2*A*B) + e*x]*(1 + x^2)),x]
```

output
```
((-I)*Sqrt[2]*Sqrt[A]*Sqrt[B]*Sqrt[A/B - B/A + 2*x]*(ArcTanh[(Sqrt[A]*Sqrt
[B]*Sqrt[A/B - B/A + 2*x])/(A - I*B)] - ArcTanh[(Sqrt[A]*Sqrt[B]*Sqrt[A/B
- B/A + 2*x])/(A + I*B)]))/Sqrt[e*(A/B - B/A + 2*x)]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.46, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {654, 27, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(x^2+1)\sqrt{\frac{A^2e-B^2e}{2AB}+ex}}dx$$

$$\Big\downarrow \text{654}$$

$$2\int \frac{2\left(\left(A^2+B^2\right)e+2AB\left(\frac{1}{2}\left(\frac{A}{B}-\frac{B}{A}\right)e+xe\right)\right)}{A\left(\frac{(A^2+B^2)^2e^2}{A^2B^2}-\frac{4(A^2-B^2)\left(\frac{1}{2}\left(\frac{A}{B}-\frac{B}{A}\right)e+xe\right)e}{AB}+4\left(\frac{1}{2}\left(\frac{A}{B}-\frac{B}{A}\right)e+xe\right)^2\right)}d\sqrt{\frac{1}{2}\left(\frac{A}{B}-\frac{B}{A}\right)e+xe}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{(A^2+B^2)e+2AB\left(\frac{1}{2}\left(\frac{A}{B}-\frac{B}{A}\right)e+xe\right)}{\frac{\left(A^2+B^2\right)^2e^2}{A^2B^2}-\frac{4\left(A^2-B^2\right)\left(\frac{1}{2}\left(\frac{A}{B}-\frac{B}{A}\right)e+xe\right)e}{AB}+4\left(\frac{1}{2}\left(\frac{A}{B}-\frac{B}{A}\right)e+xe\right)^2}d\sqrt{\frac{1}{2}\left(\frac{A}{B}-\frac{B}{A}\right)e+xe}}{A}$$

$$\Big\downarrow \text{1475}$$

$$\frac{4\left(\frac{1}{4}AB\int \frac{1}{\frac{1}{2}\left(\frac{A}{B}-\frac{B}{A}\right)e+\frac{(A^2+B^2)e}{2AB}+xe-\frac{\sqrt{2}\sqrt{A}\sqrt{\frac{1}{2}\left(\frac{A}{B}-\frac{B}{A}\right)e+xe}\sqrt{e}}{\sqrt{B}}}d\sqrt{\frac{1}{2}\left(\frac{A}{B}-\frac{B}{A}\right)e+xe}+\frac{1}{4}AB\int \frac{1}{\frac{1}{2}\left(\frac{A}{B}-\frac{B}{A}\right)e+\frac{(A^2+B^2)e}{2AB}+xe+\cdots}\right.}{A}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4\left(-\frac{1}{2}AB\int \frac{1}{-\frac{2Be}{A}-\frac{1}{2}\left(\frac{A}{B}-\frac{B}{A}\right)e-xe}d\left(2\sqrt{\frac{1}{2}\left(\frac{A}{B}-\frac{B}{A}\right)e+xe}-\frac{\sqrt{2}\sqrt{A}\sqrt{e}}{\sqrt{B}}\right)-\frac{1}{2}AB\int \frac{1}{-\frac{2Be}{A}-\frac{1}{2}\left(\frac{A}{B}-\frac{B}{A}\right)e-xe}d\left(\frac{\sqrt{2}\sqrt{A}\sqrt{e}}{\sqrt{B}}+\cdots\right.\right.}{A}$$

$$\Big\downarrow \text{217}$$

$$\frac{4\left(\frac{A^{3/2}\sqrt{B}\arctan\left(\frac{\sqrt{A}\left(2\sqrt{\frac{1}{2}e\left(\frac{A}{B}-\frac{B}{A}\right)+ex}-\frac{\sqrt{2}\sqrt{A}\sqrt{e}}{\sqrt{B}}\right)}{\sqrt{2}\sqrt{B}\sqrt{e}}\right)}{2\sqrt{2}\sqrt{e}}+\frac{A^{3/2}\sqrt{B}\arctan\left(\frac{\sqrt{A}\left(2\sqrt{\frac{1}{2}e\left(\frac{A}{B}-\frac{B}{A}\right)+ex}+\frac{\sqrt{2}\sqrt{A}\sqrt{e}}{\sqrt{B}}\right)}{\sqrt{2}\sqrt{B}\sqrt{e}}\right)}{2\sqrt{2}\sqrt{e}}\right)}{A}$$

input
```
Int[(A + B*x)/(Sqrt[(A^2*e - B^2*e)/(2*A*B) + e*x]*(1 + x^2)),x]
```

output
```
(4*((A^(3/2)*Sqrt[B]*ArcTan[(Sqrt[A]*(-((Sqrt[2]*Sqrt[A]*Sqrt[e])/Sqrt[B])
 + 2*Sqrt[((A/B - B/A)*e)/2 + e*x]))/(Sqrt[2]*Sqrt[B]*Sqrt[e])])/(2*Sqrt[2
]*Sqrt[e]) + (A^(3/2)*Sqrt[B]*ArcTan[(Sqrt[A]*((Sqrt[2]*Sqrt[A]*Sqrt[e])/S
qrt[B] + 2*Sqrt[((A/B - B/A)*e)/2 + e*x]))/(Sqrt[2]*Sqrt[B]*Sqrt[e])])/(2*
Sqrt[2]*Sqrt[e])))/A
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

Maple [A] (verified)

Time = 8.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{AB\sqrt{2}\left(\arctan\left(\frac{\sqrt{\frac{e\left(2xAB+A^2-B^2\right)}{AB}}\,AB-\sqrt{A^3Be}}{\sqrt{AeB}\,B}\right)+\arctan\left(\frac{\sqrt{\frac{e\left(2xAB+A^2-B^2\right)}{AB}}\,AB+\sqrt{A^3Be}}{\sqrt{AeB}\,B}\right)\right)}{\sqrt{AeB}}$
default	$2\sqrt{2}\,B^2A\left(\frac{\arctan\left(\frac{2AB\sqrt{2ex+\frac{e\left(A^2-B^2\right)}{AB}}+2\sqrt{A^3Be}}{2B\sqrt{AeB}}\right)}{2B\sqrt{AeB}}+\frac{\arctan\left(\frac{2AB\sqrt{2ex+\frac{e\left(A^2-B^2\right)}{AB}}-2\sqrt{A^3Be}}{2B\sqrt{AeB}}\right)}{2B\sqrt{AeB}}\right)$
derivativedivides	$4B^2A\left(\frac{\sqrt{2}\arctan\left(\frac{\left(2\sqrt{4ex+\frac{2A^2e-2B^2e}{AB}}\,AB+2\sqrt{2}\,\sqrt{A^3Be}\right)\sqrt{2}}{4B\sqrt{AeB}}\right)}{4B\sqrt{AeB}}+\frac{\sqrt{2}\arctan\left(\frac{\left(2\sqrt{4ex+\frac{2A^2e-2B^2e}{AB}}\,AB-2\sqrt{2}\,\sqrt{\ }\right.}{4B\sqrt{AeB}}\right.}{4B\sqrt{AeB}}\right.$

input
```
int(2*(B*x+A)/(2*(A^2*e-B^2*e)/A/B+4*e*x)^(1/2)/(x^2+1),x,method=_RETURNVE
RBOSE)
```

output
```
A*B*2^(1/2)*(arctan(((e*(2*A*B*x+A^2-B^2)/A/B)^(1/2)*A*B-(A^3*B*e)^(1/2))/
(A*e*B)^(1/2)/B)+arctan(((e*(2*A*B*x+A^2-B^2)/A/B)^(1/2)*A*B+(A^3*B*e)^(1/
2))/(A*e*B)^(1/2)/B))/(A*e*B)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx}{\sqrt{\frac{A^2e-B^2e}{2AB}+ex}\,(1+x^2)}\,dx$$

$$=\left[\frac{1}{2}\sqrt{2}\sqrt{-\frac{AB}{e}}\log\left(\frac{A^2x^2-4\,ABx-A^2+2\,B^2+2\,(Ax-B)\sqrt{-\frac{AB}{e}}\sqrt{\frac{2\,ABex+(A^2-B^2)e}{AB}}}{x^2+1}\right),\sqrt{2}\sqrt{\frac{AB}{e}}\,\text{ar}\right.$$

input
```
integrate(2*(B*x+A)/(2*(A^2*e-B^2*e)/A/B+4*e*x)^(1/2)/(x^2+1),x, algorithm
="fricas")
```

output
```
[1/2*sqrt(2)*sqrt(-A*B/e)*log((A^2*x^2 - 4*A*B*x - A^2 + 2*B^2 + 2*(A*x -
B)*sqrt(-A*B/e)*sqrt((2*A*B*e*x + (A^2 - B^2)*e)/(A*B)))/(x^2 + 1)), sqrt(
2)*sqrt(A*B/e)*arctan((A*x - B)*sqrt(A*B/e)*sqrt((2*A*B*e*x + (A^2 - B^2)*
e)/(A*B))/(2*A*B*x + A^2 - B^2))]
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{\frac{A^2e-B^2e}{2AB}+ex}\,(1+x^2)}\,dx = \sqrt{2}\left(\int \frac{A}{x^2\sqrt{\frac{Ae}{B}+2ex-\frac{Be}{A}}+\sqrt{\frac{Ae}{B}+2ex-\frac{Be}{A}}}\,dx + \int \frac{Bx}{x^2\sqrt{\frac{Ae}{B}+2ex-\frac{Be}{A}}+\sqrt{\frac{Ae}{B}+2ex-\frac{Be}{A}}}\,dx\right)$$

input
```
integrate(2*(B*x+A)/(2*(A**2*e-B**2*e)/A/B+4*e*x)**(1/2)/(x**2+1),x)
```

output
```
sqrt(2)*(Integral(A/(x**2*sqrt(A*e/B + 2*e*x - B*e/A) + sqrt(A*e/B + 2*e*x
 - B*e/A)), x) + Integral(B*x/(x**2*sqrt(A*e/B + 2*e*x - B*e/A) + sqrt(A*e
/B + 2*e*x - B*e/A)), x))
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{\frac{A^2e-B^2e}{2AB}+ex}\,(1+x^2)}\,dx = \int \frac{2\,(Bx+A)}{\sqrt{4\,ex+\frac{2\,(A^2e-B^2e)}{AB}}(x^2+1)}\,dx$$

input
```
integrate(2*(B*x+A)/(2*(A^2*e-B^2*e)/A/B+4*e*x)^(1/2)/(x^2+1),x, algorithm
="maxima")
```

output
```
2*integrate((B*x + A)/(sqrt(4*e*x + 2*(A^2*e - B^2*e)/(A*B))*(x^2 + 1)), x
)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 13.79 (sec) , antiderivative size = 18107, normalized size of antiderivative = 134.13

$$\int \frac{A+Bx}{\sqrt{\frac{A^2e-B^2e}{2AB}+ex}\,(1+x^2)}\,dx = \text{Too large to display}$$

input
```
integrate(2*(B*x+A)/(2*(A^2*e-B^2*e)/A/B+4*e*x)^(1/2)/(x^2+1),x, algorithm
="giac")
```

output
```
1/2*sqrt(2)*(6*A*B*e^2*abs(A)*abs(B)*cos(1/2*real_part(arccos(A^3*B*e/abs(
A^3*B*e + A*B^3*e) - A*B^3*e/abs(A^3*B*e + A*B^3*e))))^2*cosh(1/2*imag_par
t(arccos(A^3*B*e/abs(A^3*B*e + A*B^3*e) - A*B^3*e/abs(A^3*B*e + A*B^3*e)))
)^3*sin(1/2*real_part(arccos(A^3*B*e/abs(A^3*B*e + A*B^3*e) - A*B^3*e/abs(
A^3*B*e + A*B^3*e)))) - 2*A*B*e^2*abs(A)*abs(B)*cosh(1/2*imag_part(arccos(
A^3*B*e/abs(A^3*B*e + A*B^3*e) - A*B^3*e/abs(A^3*B*e + A*B^3*e))))^3*sin(1
/2*real_part(arccos(A^3*B*e/abs(A^3*B*e + A*B^3*e) - A*B^3*e/abs(A^3*B*e +
 A*B^3*e))))^3 - 18*A*B*e^2*abs(A)*abs(B)*cos(1/2*real_part(arccos(A^3*B*e
/abs(A^3*B*e + A*B^3*e) - A*B^3*e/abs(A^3*B*e + A*B^3*e))))^2*cosh(1/2*ima
g_part(arccos(A^3*B*e/abs(A^3*B*e + A*B^3*e) - A*B^3*e/abs(A^3*B*e + A*B^3
*e))))^2*sin(1/2*real_part(arccos(A^3*B*e/abs(A^3*B*e + A*B^3*e) - A*B^3*e
/abs(A^3*B*e + A*B^3*e))))*sinh(1/2*imag_part(arccos(A^3*B*e/abs(A^3*B*e +
 A*B^3*e) - A*B^3*e/abs(A^3*B*e + A*B^3*e)))) + 6*A*B*e^2*abs(A)*abs(B)*co
sh(1/2*imag_part(arccos(A^3*B*e/abs(A^3*B*e + A*B^3*e) - A*B^3*e/abs(A^3*B
*e + A*B^3*e))))^2*sin(1/2*real_part(arccos(A^3*B*e/abs(A^3*B*e + A*B^3*e)
 - A*B^3*e/abs(A^3*B*e + A*B^3*e))))^3*sinh(1/2*imag_part(arccos(A^3*B*e/a
bs(A^3*B*e + A*B^3*e) - A*B^3*e/abs(A^3*B*e + A*B^3*e)))) + 18*A*B*e^2*abs
(A)*abs(B)*cos(1/2*real_part(arccos(A^3*B*e/abs(A^3*B*e + A*B^3*e) - A*B^3
*e/abs(A^3*B*e + A*B^3*e))))^2*cosh(1/2*imag_part(arccos(A^3*B*e/abs(A^3*B
*e + A*B^3*e) - A*B^3*e/abs(A^3*B*e + A*B^3*e))))*sin(1/2*real_part(arc...
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx}{\sqrt{\frac{A^2e-B^2e}{2AB}+ex}\,(1+x^2)}\,dx = \frac{\sqrt{2}\,\sqrt{A}\,\sqrt{B}\left(\mathrm{atan}\left(\frac{A^{3/2}\left(4\,e\,x+\frac{2\,A^2\,e-2\,B^2\,e}{A\,B}\right)^{3/2}\sqrt{2\,B}}{8\,e^{3/2}\,(A^2+B^2)}-\frac{A^{5/2}\sqrt{8\,e\,x+\frac{2\,(2\,A^2\,e-2\,B^2\,e)}{A\,B}}}{4\,\sqrt{B}\,\sqrt{e}\,(A^2+B^2)}+\frac{3\,\sqrt{2}\,\sqrt{A}\,B^{3/2}\sqrt{4\,e\,x+\frac{2\,A^2\,e-2\,B^2\,e}{A\,B}}}{4\,\sqrt{e}\,(A^2+B^2)}\right.\right.}{\sqrt{e}}$$

input
```
int((2*A + 2*B*x)/((x^2 + 1)*(4*e*x + (2*A^2*e - 2*B^2*e)/(A*B))^(1/2)),x)
```

output
```
(2^(1/2)*A^(1/2)*B^(1/2)*(atan((A^(3/2)*(4*e*x + (2*A^2*e - 2*B^2*e)/(A*B)
)^(3/2)*(2*B)^(1/2))/(8*e^(3/2)*(A^2 + B^2)) - (A^(5/2)*(8*e*x + (2*(2*A^2
*e - 2*B^2*e))/(A*B))^(1/2))/(4*B^(1/2)*e^(1/2)*(A^2 + B^2)) + (3*2^(1/2)*
A^(1/2)*B^(3/2)*(4*e*x + (2*A^2*e - 2*B^2*e)/(A*B))^(1/2))/(4*e^(1/2)*(A^2
 + B^2))) + atan((2^(1/2)*A^(1/2)*(4*e*x + (2*A^2*e - 2*B^2*e)/(A*B))^(1/2
))/(4*B^(1/2)*e^(1/2)))))/e^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.44

$$\int \frac{A+Bx}{\sqrt{\frac{A^2e-B^2e}{2AB}+ex}\,(1+x^2)}\,dx = \frac{\sqrt{e}\,\sqrt{b}\,\sqrt{a}\,\sqrt{2}\left(atan\left(\frac{\sqrt{2abx+a^2-b^2}-a}{b}\right)+atan\left(\frac{\sqrt{2abx+a^2-b^2}+a}{b}\right)\right)}{e}$$

input
```
int(2*(B*x+A)/(2*(A^2*e-B^2*e)/A/B+4*e*x)^(1/2)/(x^2+1),x)
```

output
```
(sqrt(e)*sqrt(b)*sqrt(a)*sqrt(2)*(atan((sqrt(a**2 + 2*a*b*x - b**2) - a)/b
) + atan((sqrt(a**2 + 2*a*b*x - b**2) + a)/b)))/e
```

3.141 $\int \frac{A+Bx}{\sqrt{d+ex}(1-x^2)} dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1-x^2)}\,dx=-\frac{(A-B)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{\sqrt{d-e}}+\frac{(A+B)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d+e}}\right)}{\sqrt{d+e}}$$

output
```
-(A-B)*arctanh((e*x+d)^(1/2)/(d-e)^(1/2))/(d-e)^(1/2)+(A+B)*arctanh((e*x+d
)^(1/2)/(d+e)^(1/2))/(d+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1-x^2)}\,dx=-\frac{(A+B)\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{-d-e}}\right)}{\sqrt{-d-e}}+\frac{(A-B)\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{-d+e}}\right)}{\sqrt{-d+e}}$$

input
```
Integrate[(A + B*x)/(Sqrt[d + e*x]*(1 - x^2)),x]
```

output
```
-(((A + B)*ArcTan[Sqrt[d + e*x]/Sqrt[-d - e]])/Sqrt[-d - e]) + ((A - B)*Ar
cTan[Sqrt[d + e*x]/Sqrt[-d + e]])/Sqrt[-d + e]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {654, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(1-x^2)\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{654}$$

$$2\int \frac{Bd-Ae-B(d+ex)}{d^2-2(d+ex)d-e^2+(d+ex)^2}d\sqrt{d+ex}$$

$$\Big\downarrow \text{1480}$$

$$2\left(\frac{1}{2}(A-B)\int \frac{1}{xe+e}d\sqrt{d+ex}-\frac{1}{2}(A+B)\int \frac{1}{ex-e}d\sqrt{d+ex}\right)$$

$$\Big\downarrow \text{220}$$

$$2\left(\frac{(A+B)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d+e}}\right)}{2\sqrt{d+e}}-\frac{(A-B)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{2\sqrt{d-e}}\right)$$

input
```
Int[(A + B*x)/(Sqrt[d + e*x]*(1 - x^2)),x]
```

output
```
2*(-1/2*((A - B)*ArcTanh[Sqrt[d + e*x]/Sqrt[d - e]])/Sqrt[d - e] + ((A + B
)*ArcTanh[Sqrt[d + e*x]/Sqrt[d + e]])/(2*Sqrt[d + e]))
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$\frac{(A-B)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d+e}}\right)}{\sqrt{-d+e}}+\frac{(A+B)\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d+e}}\right)}{\sqrt{d+e}}$	54
derivativedivides	$-\frac{2\left(-\frac{A}{2}+\frac{B}{2}\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d+e}}\right)}{\sqrt{-d+e}}+\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d+e}}\right)}{\sqrt{d+e}}$	62
default	$-\frac{2\left(-\frac{A}{2}+\frac{B}{2}\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d+e}}\right)}{\sqrt{-d+e}}+\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d+e}}\right)}{\sqrt{d+e}}$	62

input
```
int((B*x+A)/(e*x+d)^(1/2)/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
(A-B)*arctan((e*x+d)^(1/2)/(-d+e)^(1/2))/(-d+e)^(1/2)+(A+B)*arctanh((e*x+d
)^(1/2)/(d+e)^(1/2))/(d+e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 424, normalized size of antiderivative = 6.42

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1-x^2)}\,dx$$
$$= \left[-\frac{((A-B)d+(A-B)e)\sqrt{d-e}\log\left(\frac{ex+2\sqrt{ex+d}\sqrt{d-e}+2d-e}{x+1}\right)-((A+B)d-(A+B)e)\sqrt{d+e}\log\left(\frac{ex+}{}\right.}{2\,(d^2-e^2)}\right.$$
$$-\frac{2\,((A+B)d-(A+B)e)\sqrt{-d-e}\arctan\left(\frac{\sqrt{-d-e}}{\sqrt{ex+d}}\right)+((A-B)d+(A-B)e)\sqrt{d-e}\log\left(\frac{ex+2\sqrt{ex+d}}{x+}\right.}{2\,(d^2-e^2)}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-x^2+1),x, algorithm="fricas")
```

output
```
[-1/2*(((A - B)*d + (A - B)*e)*sqrt(d - e)*log((e*x + 2*sqrt(e*x + d)*sqrt
(d - e) + 2*d - e)/(x + 1)) - ((A + B)*d - (A + B)*e)*sqrt(d + e)*log((e*x
 + 2*sqrt(e*x + d)*sqrt(d + e) + 2*d + e)/(x - 1)))/(d^2 - e^2), 1/2*(2*((
A - B)*d + (A - B)*e)*sqrt(-d + e)*arctan(sqrt(-d + e)/sqrt(e*x + d)) + ((
A + B)*d - (A + B)*e)*sqrt(d + e)*log((e*x + 2*sqrt(e*x + d)*sqrt(d + e) +
 2*d + e)/(x - 1)))/(d^2 - e^2), -1/2*(2*((A + B)*d - (A + B)*e)*sqrt(-d -
 e)*arctan(sqrt(-d - e)/sqrt(e*x + d)) + ((A - B)*d + (A - B)*e)*sqrt(d -
e)*log((e*x + 2*sqrt(e*x + d)*sqrt(d - e) + 2*d - e)/(x + 1)))/(d^2 - e^2)
, (((A - B)*d + (A - B)*e)*sqrt(-d + e)*arctan(sqrt(-d + e)/sqrt(e*x + d))
 - ((A + B)*d - (A + B)*e)*sqrt(-d - e)*arctan(sqrt(-d - e)/sqrt(e*x + d))
)/(d^2 - e^2)]
```

Sympy [A] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1-x^2)}\,dx = \begin{cases} \frac{2\left(\frac{e(A-B)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d+e}}\right)}{2\sqrt{-d+e}}-\frac{e(A+B)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d-e}}\right)}{2\sqrt{-d-e}}\right)}{e} & \text{for } e\neq 0 \\ \frac{A\left(-\frac{\log(x-1)}{2}+\frac{\log(x+1)}{2}\right)-\frac{B\log(1-x^2)}{2}}{\sqrt{d}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)/(e*x+d)**(1/2)/(-x**2+1),x)
```

output
```
Piecewise((2*(e*(A - B)*atan(sqrt(d + e*x)/sqrt(-d + e))/(2*sqrt(-d + e))
- e*(A + B)*atan(sqrt(d + e*x)/sqrt(-d - e))/(2*sqrt(-d - e)))/e, Ne(e, 0)
), ((A*(-log(x - 1)/2 + log(x + 1)/2) - B*log(1 - x**2)/2)/sqrt(d), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1-x^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-x^2+1),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d-4*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1-x^2)}\,dx = \frac{(A-B)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d+e}}\right)}{\sqrt{-d+e}} - \frac{(A+B)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d-e}}\right)}{\sqrt{-d-e}}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-x^2+1),x, algorithm="giac")
```

output
```
(A - B)*arctan(sqrt(e*x + d)/sqrt(-d + e))/sqrt(-d + e) - (A + B)*arctan(s
qrt(e*x + d)/sqrt(-d - e))/sqrt(-d - e)
```

Mupad [B] (verification not implemented)

Time = 6.24 (sec) , antiderivative size = 773, normalized size of antiderivative = 11.71

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1-x^2)}\,dx = \text{Too large to display}$$

input
```
int(-(A + B*x)/((x^2 - 1)*(d + e*x)^(1/2)),x)
```

output
```
- (atan(((((16*A^2*e^2 + 16*B^2*e^2)*(d + e*x)^(1/2) - ((A - B)*(32*B*d*e^
2 - 32*A*e^3 + (32*d*e^2*(A - B)*(d + e*x)^(1/2))/(d - e)^(1/2)))/(2*(d -
e)^(1/2)))*(A - B)*1i)/(2*(d - e)^(1/2)) + (((16*A^2*e^2 + 16*B^2*e^2)*(d
+ e*x)^(1/2) - ((A - B)*(32*A*e^3 - 32*B*d*e^2 + (32*d*e^2*(A - B)*(d + e*
x)^(1/2))/(d - e)^(1/2)))/(2*(d - e)^(1/2)))*(A - B)*1i)/(2*(d - e)^(1/2))
)/(16*B^3*e^2 - 16*A^2*B*e^2 + (((16*A^2*e^2 + 16*B^2*e^2)*(d + e*x)^(1/2)
 - ((A - B)*(32*B*d*e^2 - 32*A*e^3 + (32*d*e^2*(A - B)*(d + e*x)^(1/2))/(d
 - e)^(1/2)))/(2*(d - e)^(1/2)))*(A - B))/(2*(d - e)^(1/2)) - (((16*A^2*e^
2 + 16*B^2*e^2)*(d + e*x)^(1/2) - ((A - B)*(32*A*e^3 - 32*B*d*e^2 + (32*d*
e^2*(A - B)*(d + e*x)^(1/2))/(d - e)^(1/2)))/(2*(d - e)^(1/2)))*(A - B))/(
2*(d - e)^(1/2))))*(A - B)*1i)/(d - e)^(1/2) - (atan(((((16*A^2*e^2 + 16*B
^2*e^2)*(d + e*x)^(1/2) - ((A + B)*(32*B*d*e^2 - 32*A*e^3 + (32*d*e^2*(A +
 B)*(d + e*x)^(1/2))/(d + e)^(1/2)))/(2*(d + e)^(1/2)))*(A + B)*1i)/(2*(d
+ e)^(1/2)) + (((16*A^2*e^2 + 16*B^2*e^2)*(d + e*x)^(1/2) - ((A + B)*(32*A
*e^3 - 32*B*d*e^2 + (32*d*e^2*(A + B)*(d + e*x)^(1/2))/(d + e)^(1/2)))/(2*
(d + e)^(1/2)))*(A + B)*1i)/(2*(d + e)^(1/2)))/(16*B^3*e^2 - 16*A^2*B*e^2
+ (((16*A^2*e^2 + 16*B^2*e^2)*(d + e*x)^(1/2) - ((A + B)*(32*B*d*e^2 - 32*
A*e^3 + (32*d*e^2*(A + B)*(d + e*x)^(1/2))/(d + e)^(1/2)))/(2*(d + e)^(1/2
)))*(A + B))/(2*(d + e)^(1/2)) - (((16*A^2*e^2 + 16*B^2*e^2)*(d + e*x)^(1/
2) - ((A + B)*(32*A*e^3 - 32*B*d*e^2 + (32*d*e^2*(A + B)*(d + e*x)^(1/2...
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 375, normalized size of antiderivative = 5.68

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1-x^2)}\,dx$$
$$= \frac{\sqrt{d-e}\log\left(-\sqrt{d-e}+\sqrt{ex+d}\right)ad+\sqrt{d-e}\log\left(-\sqrt{d-e}+\sqrt{ex+d}\right)ae-\sqrt{d-e}\log\left(-\sqrt{d-e}+\sqrt{\right.}}{}$$

input
```
int((B*x+A)/(e*x+d)^(1/2)/(-x^2+1),x)
```

output
```
(sqrt(d - e)*log( - sqrt(d - e) + sqrt(d + e*x))*a*d + sqrt(d - e)*log( -
sqrt(d - e) + sqrt(d + e*x))*a*e - sqrt(d - e)*log( - sqrt(d - e) + sqrt(d
 + e*x))*b*d - sqrt(d - e)*log( - sqrt(d - e) + sqrt(d + e*x))*b*e - sqrt(
d - e)*log(sqrt(d - e) + sqrt(d + e*x))*a*d - sqrt(d - e)*log(sqrt(d - e)
+ sqrt(d + e*x))*a*e + sqrt(d - e)*log(sqrt(d - e) + sqrt(d + e*x))*b*d +
sqrt(d - e)*log(sqrt(d - e) + sqrt(d + e*x))*b*e - sqrt(d + e)*log(sqrt(d
+ e*x) - sqrt(d + e))*a*d + sqrt(d + e)*log(sqrt(d + e*x) - sqrt(d + e))*a
*e - sqrt(d + e)*log(sqrt(d + e*x) - sqrt(d + e))*b*d + sqrt(d + e)*log(sq
rt(d + e*x) - sqrt(d + e))*b*e + sqrt(d + e)*log(sqrt(d + e*x) + sqrt(d +
e))*a*d - sqrt(d + e)*log(sqrt(d + e*x) + sqrt(d + e))*a*e + sqrt(d + e)*l
og(sqrt(d + e*x) + sqrt(d + e))*b*d - sqrt(d + e)*log(sqrt(d + e*x) + sqrt
(d + e))*b*e)/(2*(d**2 - e**2))
```

3.142 $\int \frac{A+Bx}{\sqrt{d+ex}(1+x^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 304

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1+x^2)}\,dx = -\frac{\left(B-\frac{Bd-Ae}{\sqrt{d^2+e^2}}\right)\arctan\left(\frac{\sqrt{d+\sqrt{d^2+e^2}}-\sqrt{2}\sqrt{d+ex}}{\sqrt{-d+\sqrt{d^2+e^2}}}\right)}{\sqrt{2}\sqrt{-d+\sqrt{d^2+e^2}}} + \frac{\left(B-\frac{Bd-Ae}{\sqrt{d^2+e^2}}\right)\arctan\left(\frac{\sqrt{d+\sqrt{d^2+e^2}}+\sqrt{2}\sqrt{d+ex}}{\sqrt{-d+\sqrt{d^2+e^2}}}\right)}{\sqrt{2}\sqrt{-d+\sqrt{d^2+e^2}}} - \frac{\left(B+\frac{Bd-Ae}{\sqrt{d^2+e^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d+\sqrt{d^2+e^2}}\sqrt{d+ex}}{d+\sqrt{d^2+e^2}+ex}\right)}{\sqrt{2}\sqrt{d+\sqrt{d^2+e^2}}}$$

output
```
-1/2*(B-(-A*e+B*d)/(d^2+e^2)^(1/2))*arctan(((d+(d^2+e^2)^(1/2))^(1/2)-2^(1
/2)*(e*x+d)^(1/2))/(-d+(d^2+e^2)^(1/2))^(1/2))*2^(1/2)/(-d+(d^2+e^2)^(1/2)
)^(1/2)+1/2*(B-(-A*e+B*d)/(d^2+e^2)^(1/2))*arctan(((d+(d^2+e^2)^(1/2))^(1/
2)+2^(1/2)*(e*x+d)^(1/2))/(-d+(d^2+e^2)^(1/2))^(1/2))*2^(1/2)/(-d+(d^2+e^2
)^(1/2))^(1/2)-1/2*(B+(-A*e+B*d)/(d^2+e^2)^(1/2))*arctanh(2^(1/2)*(d+(d^2+
e^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/(d+(d^2+e^2)^(1/2)+e*x))*2^(1/2)/(d+(d^2+e
^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1+x^2)}\,dx = \frac{(-iA+B)\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{-d-ie}}\right)}{\sqrt{-d-ie}} + \frac{(iA+B)\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{-d+ie}}\right)}{\sqrt{-d+ie}}$$

input
```
Integrate[(A + B*x)/(Sqrt[d + e*x]*(1 + x^2)),x]
```

output
```
(((-I)*A + B)*ArcTan[Sqrt[d + e*x]/Sqrt[-d - I*e]])/Sqrt[-d - I*e] + ((I*A
 + B)*ArcTan[Sqrt[d + e*x]/Sqrt[-d + I*e]])/Sqrt[-d + I*e]
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.54, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {654, 25, 1483, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(x^2+1)\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 654}$$

$$2\int -\frac{Bd-Ae-B(d+ex)}{d^2-2(d+ex)d+e^2+(d+ex)^2}d\sqrt{d+ex}$$

$$\Big\downarrow \text{ 25}$$

$$-2\int \frac{Bd-Ae-B(d+ex)}{d^2-2(d+ex)d+e^2+(d+ex)^2}d\sqrt{d+ex}$$

$$\Big\downarrow \text{ 1483}$$

$$2\left(-\frac{\int \frac{\sqrt{2}(Bd-Ae)\sqrt{d+\sqrt{d^2+e^2}}-\left(Ae-B\left(d+\sqrt{d^2+e^2}\right)\right)\sqrt{d+ex}}{d+ex+\sqrt{d^2+e^2}+\sqrt{2}\sqrt{d+\sqrt{d^2+e^2}}\sqrt{d+ex}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{d^2+e^2}\sqrt{\sqrt{d^2+e^2}+d}}-\frac{\int \frac{\sqrt{2}\sqrt{d+\sqrt{d^2+e^2}}(Bd-Ae)+\left(Ae-B\left(d+\sqrt{d^2+e^2}\right)\right)\sqrt{d}}{d+ex+\sqrt{d^2+e^2}-\sqrt{2}\sqrt{d+\sqrt{d^2+e^2}}\sqrt{d+ex}}}{2\sqrt{2}\sqrt{d^2+e^2}\sqrt{\sqrt{d^2+e^2}+d}}\right.$$

$\downarrow$ 1142

$$2\left(-\frac{\frac{1}{2}\left(Ae-B\left(\sqrt{d^2+e^2}+d\right)\right)\int -\frac{\sqrt{2}\left(\sqrt{d+\sqrt{d^2+e^2}}-\sqrt{2}\sqrt{d+ex}\right)}{d+ex+\sqrt{d^2+e^2}-\sqrt{2}\sqrt{d+\sqrt{d^2+e^2}}\sqrt{d+ex}}d\sqrt{d+ex}-\frac{\sqrt{\sqrt{d^2+e^2}+d}\left(Ae-B\left(d-\sqrt{d^2+e^2}\right)\right)}{}}{2\sqrt{2}\sqrt{d^2+e^2}\sqrt{\sqrt{d^2+e^2}+d}}\right.$$

$\downarrow$ 25

$$2\left(-\frac{-\frac{\sqrt{\sqrt{d^2+e^2}+d}\left(Ae-B\left(d-\sqrt{d^2+e^2}\right)\right)\int \frac{1}{d+ex+\sqrt{d^2+e^2}-\sqrt{2}\sqrt{d+\sqrt{d^2+e^2}}\sqrt{d+ex}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{1}{2}\left(Ae-B\left(\sqrt{d^2+e^2}+d\right)\right)\int \frac{}{d+ex}}{2\sqrt{2}\sqrt{d^2+e^2}\sqrt{\sqrt{d^2+e^2}+d}}\right.$$

$\downarrow$ 27

$$2\left(-\frac{-\frac{\sqrt{\sqrt{d^2+e^2}+d}\left(Ae-B\left(d-\sqrt{d^2+e^2}\right)\right)\int \frac{1}{d+ex+\sqrt{d^2+e^2}-\sqrt{2}\sqrt{d+\sqrt{d^2+e^2}}\sqrt{d+ex}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\left(Ae-B\left(\sqrt{d^2+e^2}+d\right)\right)\int \frac{\sqrt{d+\sqrt{d^2+e}}}{d+ex+\sqrt{d^2+e^2}-\sqrt{2}}}{\sqrt{2}}}{2\sqrt{2}\sqrt{d^2+e^2}\sqrt{\sqrt{d^2+e^2}+d}}\right.$$

$\downarrow$ 1083

$$2\left(-\frac{\sqrt{2}\sqrt{\sqrt{d^2+e^2}+d}\left(Ae-B\left(d-\sqrt{d^2+e^2}\right)\right)\int \frac{1}{-d+2\left(d-\sqrt{d^2+e^2}\right)-ex}d\left(2\sqrt{d+ex}-\sqrt{2}\sqrt{d+\sqrt{d^2+e^2}}\right)-}{2\sqrt{2}\sqrt{d^2+e^2}\sqrt{\sqrt{d^2+e^2}+d}}\right.$$

$\downarrow$ 219

$$2\left(-\frac{\frac{\sqrt{\sqrt{d^2+e^2}+d}\left(Ae-B\left(d-\sqrt{d^2+e^2}\right)\right)\operatorname{arctanh}\left(\frac{2\sqrt{d+ex}-\sqrt{2}\sqrt{\sqrt{d^2+e^2}+d}}{\sqrt{2}\sqrt{d-\sqrt{d^2+e^2}}}\right)}{\sqrt{d-\sqrt{d^2+e^2}}}-\frac{\left(Ae-B\left(\sqrt{d^2+e^2}+d\right)\right)\int\frac{\sqrt{d+\sqrt{d^2+e^2}}-\sqrt{2}\sqrt{d+ex}}{d+ex+\sqrt{d^2+e^2}-\sqrt{2}\sqrt{d+\sqrt{d^2+e^2}}}}{\sqrt{2}}}{2\sqrt{2}\sqrt{d^2+e^2}\sqrt{\sqrt{d^2+e^2}+d}}\right.$$

$\downarrow$ 1103

$$2\left(-\frac{\frac{\sqrt{\sqrt{d^2+e^2}+d}\left(Ae-B\left(d-\sqrt{d^2+e^2}\right)\right)\operatorname{arctanh}\left(\frac{2\sqrt{d+ex}-\sqrt{2}\sqrt{\sqrt{d^2+e^2}+d}}{\sqrt{2}\sqrt{d-\sqrt{d^2+e^2}}}\right)}{\sqrt{d-\sqrt{d^2+e^2}}}+\frac{1}{2}\left(Ae-B\left(\sqrt{d^2+e^2}+d\right)\right)\log\left(-\sqrt{2}\sqrt{\sqrt{d^2}\right.}{2\sqrt{2}\sqrt{d^2+e^2}\sqrt{\sqrt{d^2+e^2}+d}}\right.$$

input
```
Int[(A + B*x)/(Sqrt[d + e*x]*(1 + x^2)),x]
```

output
```
2*(-1/2*((Sqrt[d + Sqrt[d^2 + e^2]]*(A*e - B*(d - Sqrt[d^2 + e^2]))*ArcTan
h[(-(Sqrt[2]*Sqrt[d + Sqrt[d^2 + e^2]]) + 2*Sqrt[d + e*x])/(Sqrt[2]*Sqrt[d
 - Sqrt[d^2 + e^2]])])/Sqrt[d - Sqrt[d^2 + e^2]] + ((A*e - B*(d + Sqrt[d^2
 + e^2]))*Log[d + Sqrt[d^2 + e^2] + e*x - Sqrt[2]*Sqrt[d + Sqrt[d^2 + e^2]
]*Sqrt[d + e*x]])/2)/(Sqrt[2]*Sqrt[d^2 + e^2]*Sqrt[d + Sqrt[d^2 + e^2]]) -
 ((Sqrt[d + Sqrt[d^2 + e^2]]*(A*e - B*(d - Sqrt[d^2 + e^2]))*ArcTanh[(Sqrt
[2]*Sqrt[d + Sqrt[d^2 + e^2]] + 2*Sqrt[d + e*x])/(Sqrt[2]*Sqrt[d - Sqrt[d^
2 + e^2]])])/Sqrt[d - Sqrt[d^2 + e^2]] - ((A*e - B*(d + Sqrt[d^2 + e^2]))*
Log[d + Sqrt[d^2 + e^2] + e*x + Sqrt[2]*Sqrt[d + Sqrt[d^2 + e^2]]*Sqrt[d +
 e*x]])/2)/(2*Sqrt[2]*Sqrt[d^2 + e^2]*Sqrt[d + Sqrt[d^2 + e^2]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 6.74 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{\frac{\sqrt{2\sqrt{d^2+e^2}-2d}\,\sqrt{2\sqrt{d^2+e^2}+2d}\,\left(\ln\left(d+ex-\sqrt{ex+d}\,\sqrt{2\sqrt{d^2+e^2}+2d}+\sqrt{d^2+e^2}\right)-\ln\left(ex+d+\sqrt{ex+d}\,\sqrt{2\sqrt{d^2+e^2}+2d}+\sqrt{d^2+e^2}\right)\right)}{4}}{\sqrt{d^2+e^2}\,\sqrt{2}}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((B*x+A)/(e*x+d)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/(d^2+e^2)^(1/2)*(1/4*(2*(d^2+e^2)^(1/2)-2*d)^(1/2)*(2*(d^2+e^2)^(1/2)+2*
d)^(1/2)*(ln(d+e*x-(e*x+d)^(1/2)*(2*(d^2+e^2)^(1/2)+2*d)^(1/2)+(d^2+e^2)^(
1/2))-ln(e*x+d+(e*x+d)^(1/2)*(2*(d^2+e^2)^(1/2)+2*d)^(1/2)+(d^2+e^2)^(1/2)
))*(A*d-A*(d^2+e^2)^(1/2)+B*e)+e*(arctan((2*(e*x+d)^(1/2)-(2*(d^2+e^2)^(1/
2)+2*d)^(1/2))/(2*(d^2+e^2)^(1/2)-2*d)^(1/2))+arctan((2*(e*x+d)^(1/2)+(2*(
d^2+e^2)^(1/2)+2*d)^(1/2))/(2*(d^2+e^2)^(1/2)-2*d)^(1/2)))*(A*e-B*d+B*(d^2
+e^2)^(1/2)))/(2*(d^2+e^2)^(1/2)-2*d)^(1/2)/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1545 vs. $2(249) = 498$.

Time = 0.12 (sec) , antiderivative size = 1545, normalized size of antiderivative = 5.08

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1+x^2)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-(2*A*B*e + (A^2 - B^2)*d + (d^2 + e^2)*sqrt(-(4*A^2*B^2*d^2 - 4
*(A^3*B - A*B^3)*d*e + (A^4 - 2*A^2*B^2 + B^4)*e^2)/(d^4 + 2*d^2*e^2 + e^4
)))/(d^2 + e^2))*log((2*(A^3*B + A*B^3)*d - (A^4 - B^4)*e)*sqrt(e*x + d) +
 (2*A*B^2*d^2 - (3*A^2*B - B^3)*d*e + (A^3 - A*B^2)*e^2 + (A*d^3 + B*d^2*e
 + A*d*e^2 + B*e^3)*sqrt(-(4*A^2*B^2*d^2 - 4*(A^3*B - A*B^3)*d*e + (A^4 -
2*A^2*B^2 + B^4)*e^2)/(d^4 + 2*d^2*e^2 + e^4)))*sqrt(-(2*A*B*e + (A^2 - B^
2)*d + (d^2 + e^2)*sqrt(-(4*A^2*B^2*d^2 - 4*(A^3*B - A*B^3)*d*e + (A^4 - 2
*A^2*B^2 + B^4)*e^2)/(d^4 + 2*d^2*e^2 + e^4)))/(d^2 + e^2))) + 1/2*sqrt(-(
2*A*B*e + (A^2 - B^2)*d + (d^2 + e^2)*sqrt(-(4*A^2*B^2*d^2 - 4*(A^3*B - A*
B^3)*d*e + (A^4 - 2*A^2*B^2 + B^4)*e^2)/(d^4 + 2*d^2*e^2 + e^4)))/(d^2 + e
^2))*log((2*(A^3*B + A*B^3)*d - (A^4 - B^4)*e)*sqrt(e*x + d) - (2*A*B^2*d^
2 - (3*A^2*B - B^3)*d*e + (A^3 - A*B^2)*e^2 + (A*d^3 + B*d^2*e + A*d*e^2 +
 B*e^3)*sqrt(-(4*A^2*B^2*d^2 - 4*(A^3*B - A*B^3)*d*e + (A^4 - 2*A^2*B^2 +
B^4)*e^2)/(d^4 + 2*d^2*e^2 + e^4)))*sqrt(-(2*A*B*e + (A^2 - B^2)*d + (d^2
+ e^2)*sqrt(-(4*A^2*B^2*d^2 - 4*(A^3*B - A*B^3)*d*e + (A^4 - 2*A^2*B^2 + B
^4)*e^2)/(d^4 + 2*d^2*e^2 + e^4)))/(d^2 + e^2))) - 1/2*sqrt(-(2*A*B*e + (A
^2 - B^2)*d - (d^2 + e^2)*sqrt(-(4*A^2*B^2*d^2 - 4*(A^3*B - A*B^3)*d*e + (
A^4 - 2*A^2*B^2 + B^4)*e^2)/(d^4 + 2*d^2*e^2 + e^4)))/(d^2 + e^2))*log((2*
(A^3*B + A*B^3)*d - (A^4 - B^4)*e)*sqrt(e*x + d) + (2*A*B^2*d^2 - (3*A^2*B
 - B^3)*d*e + (A^3 - A*B^2)*e^2 - (A*d^3 + B*d^2*e + A*d*e^2 + B*e^3)*s...
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1+x^2)}\,dx=\int \frac{A+Bx}{\sqrt{d+ex}\,(x^2+1)}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)**(1/2)/(x**2+1),x)
```

output
```
Integral((A + B*x)/(sqrt(d + e*x)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1+x^2)}\,dx = \int \frac{Bx+A}{\sqrt{ex+d}(x^2+1)}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(e*x + d)*(x^2 + 1)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1+x^2)}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 6.32 (sec) , antiderivative size = 1244, normalized size of antiderivative = 4.09

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1+x^2)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/((x^2 + 1)*(d + e*x)^(1/2)),x)
```

output
```
- atan((((32*B*d*e^2 - 32*A*e^3 + 64*d*e^2*(d + e*x)^(1/2)*((B^2*1i - A^2*
1i + 2*A*B)/(4*(d*1i - e)))^(1/2))*((B^2*1i - A^2*1i + 2*A*B)/(4*(d*1i - e
)))^(1/2) + (16*A^2*e^2 - 16*B^2*e^2)*(d + e*x)^(1/2))*((B^2*1i - A^2*1i +
 2*A*B)/(4*(d*1i - e)))^(1/2)*1i + ((32*A*e^3 - 32*B*d*e^2 + 64*d*e^2*(d +
 e*x)^(1/2)*((B^2*1i - A^2*1i + 2*A*B)/(4*(d*1i - e)))^(1/2))*((B^2*1i - A
^2*1i + 2*A*B)/(4*(d*1i - e)))^(1/2) + (16*A^2*e^2 - 16*B^2*e^2)*(d + e*x)
^(1/2))*((B^2*1i - A^2*1i + 2*A*B)/(4*(d*1i - e)))^(1/2)*1i)/(((32*A*e^3 -
 32*B*d*e^2 + 64*d*e^2*(d + e*x)^(1/2)*((B^2*1i - A^2*1i + 2*A*B)/(4*(d*1i
 - e)))^(1/2))*((B^2*1i - A^2*1i + 2*A*B)/(4*(d*1i - e)))^(1/2) + (16*A^2*
e^2 - 16*B^2*e^2)*(d + e*x)^(1/2))*((B^2*1i - A^2*1i + 2*A*B)/(4*(d*1i - e
)))^(1/2) - ((32*B*d*e^2 - 32*A*e^3 + 64*d*e^2*(d + e*x)^(1/2)*((B^2*1i -
A^2*1i + 2*A*B)/(4*(d*1i - e)))^(1/2))*((B^2*1i - A^2*1i + 2*A*B)/(4*(d*1i
 - e)))^(1/2) + (16*A^2*e^2 - 16*B^2*e^2)*(d + e*x)^(1/2))*((B^2*1i - A^2*
1i + 2*A*B)/(4*(d*1i - e)))^(1/2) + 16*B^3*e^2 + 16*A^2*B*e^2))*((B^2*1i -
 A^2*1i + 2*A*B)/(4*(d*1i - e)))^(1/2)*2i - atan((((16*A^2*e^2 - 16*B^2*e^
2)*(d + e*x)^(1/2) + ((B^2 - A^2 + A*B*2i)/(4*(d - e*1i)))^(1/2)*(32*B*d*e
^2 - 32*A*e^3 + 64*d*e^2*(d + e*x)^(1/2)*((B^2 - A^2 + A*B*2i)/(4*(d - e*1
i)))^(1/2)))*((B^2 - A^2 + A*B*2i)/(4*(d - e*1i)))^(1/2)*1i + ((16*A^2*e^2
 - 16*B^2*e^2)*(d + e*x)^(1/2) + ((B^2 - A^2 + A*B*2i)/(4*(d - e*1i)))^(1/
2)*(32*A*e^3 - 32*B*d*e^2 + 64*d*e^2*(d + e*x)^(1/2)*((B^2 - A^2 + A*B*...
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 977, normalized size of antiderivative = 3.21

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(1+x^2)}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/(e*x+d)^(1/2)/(x^2+1),x)
```

output

```
(sqrt(2)*( - 2*sqrt(sqrt(d**2 + e**2) - d)*sqrt(d**2 + e**2)*atan((sqrt(sq
rt(d**2 + e**2) + d)*sqrt(2) - 2*sqrt(d + e*x))/(sqrt(sqrt(d**2 + e**2) -
d)*sqrt(2)))*a*d - 2*sqrt(sqrt(d**2 + e**2) - d)*sqrt(d**2 + e**2)*atan((s
qrt(sqrt(d**2 + e**2) + d)*sqrt(2) - 2*sqrt(d + e*x))/(sqrt(sqrt(d**2 + e*
*2) - d)*sqrt(2)))*b*e - 2*sqrt(sqrt(d**2 + e**2) - d)*atan((sqrt(sqrt(d**
2 + e**2) + d)*sqrt(2) - 2*sqrt(d + e*x))/(sqrt(sqrt(d**2 + e**2) - d)*sqr
t(2)))*a*d**2 - 2*sqrt(sqrt(d**2 + e**2) - d)*atan((sqrt(sqrt(d**2 + e**2)
 + d)*sqrt(2) - 2*sqrt(d + e*x))/(sqrt(sqrt(d**2 + e**2) - d)*sqrt(2)))*a*
e**2 + 2*sqrt(sqrt(d**2 + e**2) - d)*sqrt(d**2 + e**2)*atan((sqrt(sqrt(d**
2 + e**2) + d)*sqrt(2) + 2*sqrt(d + e*x))/(sqrt(sqrt(d**2 + e**2) - d)*sqr
t(2)))*a*d + 2*sqrt(sqrt(d**2 + e**2) - d)*sqrt(d**2 + e**2)*atan((sqrt(sq
rt(d**2 + e**2) + d)*sqrt(2) + 2*sqrt(d + e*x))/(sqrt(sqrt(d**2 + e**2) -
d)*sqrt(2)))*b*e + 2*sqrt(sqrt(d**2 + e**2) - d)*atan((sqrt(sqrt(d**2 + e*
*2) + d)*sqrt(2) + 2*sqrt(d + e*x))/(sqrt(sqrt(d**2 + e**2) - d)*sqrt(2)))
*a*d**2 + 2*sqrt(sqrt(d**2 + e**2) - d)*atan((sqrt(sqrt(d**2 + e**2) + d)*
sqrt(2) + 2*sqrt(d + e*x))/(sqrt(sqrt(d**2 + e**2) - d)*sqrt(2)))*a*e**2 +
 sqrt(sqrt(d**2 + e**2) + d)*sqrt(d**2 + e**2)*log(sqrt(d**2 + e**2) - sqr
t(d + e*x)*sqrt(sqrt(d**2 + e**2) + d)*sqrt(2) + d + e*x)*a*d + sqrt(sqrt(
d**2 + e**2) + d)*sqrt(d**2 + e**2)*log(sqrt(d**2 + e**2) - sqrt(d + e*x)*
sqrt(sqrt(d**2 + e**2) + d)*sqrt(2) + d + e*x)*b*e - sqrt(sqrt(d**2 + e...
```

3.143 $\int \frac{(1-x)\sqrt{1+x}}{1+x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 155

$$\int \frac{(1-x)\sqrt{1+x}}{1+x^2}\,dx = -2\sqrt{1+x}-\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)+\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+x}}{1+\sqrt{2}+x}\right)}{\sqrt{1+\sqrt{2}}}$$

output
```
-2*(1+x)^(1/2)-(1+2^(1/2))^(1/2)*arctan(((2+2*2^(1/2))^(1/2)-2*(1+x)^(1/2)
)/(-2+2*2^(1/2))^(1/2))+(1+2^(1/2))^(1/2)*arctan(((2+2*2^(1/2))^(1/2)+2*(1
+x)^(1/2))/(-2+2*2^(1/2))^(1/2))+arctanh((2+2*2^(1/2))^(1/2)*(1+x)^(1/2)/(
1+2^(1/2)+x))/(1+2^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\int \frac{(1-x)\sqrt{1+x}}{1+x^2}\,dx = -2\sqrt{1+x}+\sqrt{2+2i}\arctan\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\sqrt{1+x}\right) + \sqrt{2-2i}\arctan\left(\sqrt{-\frac{1}{2}+\frac{i}{2}}\sqrt{1+x}\right)$$

input
```
Integrate[((1 - x)*Sqrt[1 + x])/(1 + x^2),x]
```

output
```
-2*Sqrt[1 + x] + Sqrt[2 + 2*I]*ArcTan[Sqrt[-1/2 - I/2]*Sqrt[1 + x]] + Sqrt
[2 - 2*I]*ArcTan[Sqrt[-1/2 + I/2]*Sqrt[1 + x]]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.52, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {653, 27, 484, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)\sqrt{x+1}}{x^2+1}dx$$

$$\Big\downarrow \text{ 653}$$

$$\int \frac{2}{\sqrt{x+1}\,(x^2+1)}dx-2\sqrt{x+1}$$

$$\Big\downarrow \text{ 27}$$

$$2\int \frac{1}{\sqrt{x+1}\,(x^2+1)}dx-2\sqrt{x+1}$$

$$\Big\downarrow \text{ 484}$$

$$4\int \frac{1}{(x+1)^2-2(x+1)+2}d\sqrt{x+1}-2\sqrt{x+1}$$

$\big\downarrow$ 1407

$$4\left(\frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{4\sqrt{1+\sqrt{2}}}+\frac{\int \frac{\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{4\sqrt{1+\sqrt{2}}}\right)-2\sqrt{x+1}$$

$\big\downarrow$ 1142

$$4\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}-\frac{1}{2}\int -\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{4\sqrt{1+\sqrt{2}}}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int}{}\right.$$

$$2\sqrt{x+1}$$

$\big\downarrow$ 25

$$4\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}+\frac{1}{2}\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}}{4\sqrt{1+\sqrt{2}}}+\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int}{}\right.$$

$$2\sqrt{x+1}$$

$\big\downarrow$ 1083

$$4\left(\frac{\frac{1}{2}\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}-\sqrt{2\left(1+\sqrt{2}\right)}\int \frac{1}{-x+2\left(1-\sqrt{2}\right)-1}d\left(2\sqrt{x+1}-\sqrt{2\left(1+\sqrt{2}\right)}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\frac{1}{2}\int}{}\right.$$

$$2\sqrt{x+1}$$

$\big\downarrow$ 217

$$4\left(\frac{\frac{1}{2}\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{x+1}+\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\frac{1}{2}\int \frac{2\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1}d\sqrt{\right.$$

$$2\sqrt{x+1}$$

$$\Big\downarrow \text{1103}$$

$$4\left(\frac{\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\arctan\left(\frac{2\sqrt{x+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{2}\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\arctan\left(\frac{2\sqrt{x+}\right.}{\sqrt{}}\right.$$

$$2\sqrt{x+1}$$

input
```
Int[((1 - x)*Sqrt[1 + x])/(1 + x^2),x]
```

output
```
-2*Sqrt[1 + x] + 4*((Sqrt[(1 + Sqrt[2])/(-1 + Sqrt[2])]*ArcTan[(-Sqrt[2*(1
 + Sqrt[2])] + 2*Sqrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]] - Log[1 + Sqrt[2] +
x - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]]/2)/(4*Sqrt[1 + Sqrt[2]]) + (Sqrt[(1
 + Sqrt[2])/(-1 + Sqrt[2])]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + x])
/Sqrt[2*(-1 + Sqrt[2])]] + Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqr
t[1 + x]]/2)/(4*Sqrt[1 + Sqrt[2]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 484
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[2*
d   Subst[Int[1/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(113) = 226.

Time = 2.25 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.75

method	result
derivativedivides	$-2\sqrt{x+1}+\frac{\left(\sqrt{2+2\sqrt{2}}\,\sqrt{2}-2\sqrt{2+2\sqrt{2}}\right)\ln\left(x+1-\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)}{4}+\frac{\left(2\sqrt{2}+\frac{\left(\sqrt{2+2\sqrt{2}}\,\sqrt{2}-2\sqrt{2+2\sqrt{2}}\right)}{2}\right.}{\sqrt{-}}$
default	$-2\sqrt{x+1}+\frac{\left(\sqrt{2+2\sqrt{2}}\,\sqrt{2}-2\sqrt{2+2\sqrt{2}}\right)\ln\left(x+1-\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)}{4}+\frac{\left(2\sqrt{2}+\frac{\left(\sqrt{2+2\sqrt{2}}\,\sqrt{2}-2\sqrt{2+2\sqrt{2}}\right)}{2}\right.}{\sqrt{-}}$
trager	$-2\sqrt{x+1}-\operatorname{RootOf}\left(\_Z^2+16\operatorname{RootOf}\left(512\_Z^4+32\_Z^2+1\right)^2+1\right)\ln\left(\frac{1024\operatorname{RootOf}}{}\right.$
risch	$-2\sqrt{x+1}-\frac{\ln\left(x+1+\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)\sqrt{2+2\sqrt{2}}\,\sqrt{2}}{4}+\frac{\ln\left(x+1+\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)\sqrt{2+2\sqrt{2}}}{2}+\frac{\text{ar}}{}$

input
```
int((1-x)*(x+1)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-2*(x+1)^(1/2)+1/4*((2+2*2^(1/2))^(1/2)*2^(1/2)-2*(2+2*2^(1/2))^(1/2))*ln(
x+1-(x+1)^(1/2)*(2+2*2^(1/2))^(1/2)+2^(1/2))+(2*2^(1/2)+1/2*((2+2*2^(1/2))
^(1/2)*2^(1/2)-2*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^
(1/2)*arctan((2*(x+1)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/4
*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2))*ln(x+1+(x+1)^(1/2)*(
2+2*2^(1/2))^(1/2)+2^(1/2))+(2*2^(1/2)-1/2*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2
*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan(((2
+2*2^(1/2))^(1/2)+2*(x+1)^(1/2))/(-2+2*2^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03

$$\int \frac{(1-x)\sqrt{1+x}}{1+x^2}\,dx = \sqrt{\sqrt{2}+1}\arctan\left(\left(\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}+\sqrt{2}\sqrt{x+1}\right)\sqrt{\sqrt{2}+1}\right) - \sqrt{\sqrt{2}+1}\arctan\left(\left(\left(\sqrt{2}+1\right)\sqrt{\sqrt{2}-1}-\sqrt{2}\sqrt{x+1}\right)\sqrt{\sqrt{2}+1}\right) + \frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(\sqrt{x+1}\left(\sqrt{2}+2\right)\sqrt{\sqrt{2}-1}+x+\sqrt{2}+1\right) - \frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(-\sqrt{x+1}\left(\sqrt{2}+2\right)\sqrt{\sqrt{2}-1}+x+\sqrt{2}+1\right) - 2\sqrt{x+1}$$

input
```
integrate((1-x)*(1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
sqrt(sqrt(2) + 1)*arctan(((sqrt(2) + 1)*sqrt(sqrt(2) - 1) + sqrt(2)*sqrt(x
 + 1))*sqrt(sqrt(2) + 1)) - sqrt(sqrt(2) + 1)*arctan(((sqrt(2) + 1)*sqrt(s
qrt(2) - 1) - sqrt(2)*sqrt(x + 1))*sqrt(sqrt(2) + 1)) + 1/2*sqrt(sqrt(2) -
 1)*log(sqrt(x + 1)*(sqrt(2) + 2)*sqrt(sqrt(2) - 1) + x + sqrt(2) + 1) - 1
/2*sqrt(sqrt(2) - 1)*log(-sqrt(x + 1)*(sqrt(2) + 2)*sqrt(sqrt(2) - 1) + x
+ sqrt(2) + 1) - 2*sqrt(x + 1)
```

Sympy [F]

$$\int \frac{(1-x)\sqrt{1+x}}{1+x^2}\,dx = -\int \left(-\frac{\sqrt{x+1}}{x^2+1}\right)dx - \int \frac{x\sqrt{x+1}}{x^2+1}\,dx$$

input
```
integrate((1-x)*(1+x)**(1/2)/(x**2+1),x)
```

output
```
-Integral(-sqrt(x + 1)/(x**2 + 1), x) - Integral(x*sqrt(x + 1)/(x**2 + 1),
 x)
```

Maxima [F]

$$\int \frac{(1-x)\sqrt{1+x}}{1+x^2}\,dx = \int -\frac{\sqrt{x+1}(x-1)}{x^2+1}\,dx$$

input
```
integrate((1-x)*(1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x + 1)*(x - 1)/(x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{(1-x)\sqrt{1+x}}{1+x^2}\,dx &= \sqrt{\sqrt{2}+1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)\\ &\quad+\sqrt{\sqrt{2}+1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)\\ &\quad+\frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right)\\ &\quad-\frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(-2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right)\\ &\quad-2\sqrt{x+1}\end{aligned}$$

input
```
integrate((1-x)*(1+x)^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
sqrt(sqrt(2) + 1)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(x
 + 1))/sqrt(-sqrt(2) + 2)) + sqrt(sqrt(2) + 1)*arctan(-1/2*2^(3/4)*(2^(1/4
)*sqrt(sqrt(2) + 2) - 2*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) + 1/2*sqrt(sqrt(2
) - 1)*log(2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) + x + sqrt(2) + 1) - 1/2*
sqrt(sqrt(2) - 1)*log(-2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) + x + sqrt(2)
 + 1) - 2*sqrt(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.50

$$\int \frac{(1-x)\sqrt{1+x}}{1+x^2}\,dx = \operatorname{atanh}\left(\frac{64\sqrt{2}\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{x+1}}{256\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64} - \frac{64\sqrt{2}\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{x+1}}{256\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64}\right)\left(2\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+2\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\right) - 2\sqrt{x+1} - \operatorname{atanh}\left(\frac{64\sqrt{2}\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{x+1}}{256\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64} + \frac{64\sqrt{2}\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{x+1}}{256\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64}\right)\left(2\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-2\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\right)$$

input
```
int(-((x - 1)*(x + 1)^(1/2))/(x^2 + 1),x)
```

output
```
atanh((64*2^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2))/(256*(2^(1/2)/4
 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) - 64) - (64*2^(1/2)*(2^(1/2)/4 - 1
/4)^(1/2)*(x + 1)^(1/2))/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^
(1/2) - 64))*(2*(- 2^(1/2)/4 - 1/4)^(1/2) + 2*(2^(1/2)/4 - 1/4)^(1/2)) - 2
*(x + 1)^(1/2) - atanh((64*2^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2)
)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) + 64) + (64*2^(1/
2)*(2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2))/(256*(2^(1/2)/4 - 1/4)^(1/2)*(-
2^(1/2)/4 - 1/4)^(1/2) + 64))*(2*(- 2^(1/2)/4 - 1/4)^(1/2) - 2*(2^(1/2)/4
- 1/4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.65

$$\int \frac{(1-x)\sqrt{1+x}}{1+x^2}\,dx = -\sqrt{\sqrt{2}-1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}-2\sqrt{x+1}}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right) - \sqrt{\sqrt{2}-1}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}-2\sqrt{x+1}}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right) + \sqrt{\sqrt{2}-1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}+2\sqrt{x+1}}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right) + \sqrt{\sqrt{2}-1}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}+2\sqrt{x+1}}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right) - \frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}\log\left(-\sqrt{x+1}\,\sqrt{\sqrt{2}+1}\,\sqrt{2}+\sqrt{2}+x+1\right)}{2} + \frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}\log\left(\sqrt{x+1}\,\sqrt{\sqrt{2}+1}\,\sqrt{2}+\sqrt{2}+x+1\right)}{2} + \frac{\sqrt{\sqrt{2}+1}\log\left(-\sqrt{x+1}\,\sqrt{\sqrt{2}+1}\,\sqrt{2}+\sqrt{2}+x+1\right)}{2} - \frac{\sqrt{\sqrt{2}+1}\log\left(\sqrt{x+1}\,\sqrt{\sqrt{2}+1}\,\sqrt{2}+\sqrt{2}+x+1\right)}{2} - 2\sqrt{x+1}$$

input
```
int((1-x)*(1+x)^(1/2)/(x^2+1),x)
```

output
```
( - 2*sqrt(sqrt(2) - 1)*sqrt(2)*atan((sqrt(sqrt(2) + 1)*sqrt(2) - 2*sqrt(x
 + 1))/(sqrt(sqrt(2) - 1)*sqrt(2))) - 2*sqrt(sqrt(2) - 1)*atan((sqrt(sqrt(
2) + 1)*sqrt(2) - 2*sqrt(x + 1))/(sqrt(sqrt(2) - 1)*sqrt(2))) + 2*sqrt(sqr
t(2) - 1)*sqrt(2)*atan((sqrt(sqrt(2) + 1)*sqrt(2) + 2*sqrt(x + 1))/(sqrt(s
qrt(2) - 1)*sqrt(2))) + 2*sqrt(sqrt(2) - 1)*atan((sqrt(sqrt(2) + 1)*sqrt(2
) + 2*sqrt(x + 1))/(sqrt(sqrt(2) - 1)*sqrt(2))) - sqrt(sqrt(2) + 1)*sqrt(2
)*log( - sqrt(x + 1)*sqrt(sqrt(2) + 1)*sqrt(2) + sqrt(2) + x + 1) + sqrt(s
qrt(2) + 1)*sqrt(2)*log(sqrt(x + 1)*sqrt(sqrt(2) + 1)*sqrt(2) + sqrt(2) +
x + 1) + sqrt(sqrt(2) + 1)*log( - sqrt(x + 1)*sqrt(sqrt(2) + 1)*sqrt(2) +
sqrt(2) + x + 1) - sqrt(sqrt(2) + 1)*log(sqrt(x + 1)*sqrt(sqrt(2) + 1)*sqr
t(2) + sqrt(2) + x + 1) - 4*sqrt(x + 1))/2
```

3.144 $\int \frac{3+x}{\sqrt{4+3x}(1+x^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{3+x}{\sqrt{4+3x}\,(1+x^2)}\,dx = -\sqrt{2}\arctan\left(3-\sqrt{2}\sqrt{4+3x}\right)+\sqrt{2}\arctan\left(3+\sqrt{8+6x}\right)$$

output
```
2^(1/2)*arctan(-3+2^(1/2)*(4+3*x)^(1/2))+2^(1/2)*arctan(3+(8+6*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

$$\int \frac{3+x}{\sqrt{4+3x}\,(1+x^2)}\,dx = \sqrt{2}\arctan\left(\frac{-1+3x}{\sqrt{8+6x}}\right)$$

input
```
Integrate[(3 + x)/(Sqrt[4 + 3*x]*(1 + x^2)),x]
```

output
```
Sqrt[2]*ArcTan[(-1 + 3*x)/Sqrt[8 + 6*x]]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.49, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {654, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+3}{\sqrt{3x+4}\,(x^2+1)}\,dx$$

$$\Big\downarrow \text{654}$$

$$2\int \frac{3x+9}{(3x+4)^2-8(3x+4)+25}d\sqrt{3x+4}$$

$$\Big\downarrow \text{1475}$$

$$2\left(\frac{1}{2}\int \frac{1}{3x-3\sqrt{2}\sqrt{3x+4}+9}d\sqrt{3x+4}+\frac{1}{2}\int \frac{1}{3x+3\sqrt{2}\sqrt{3x+4}+9}d\sqrt{3x+4}\right)$$

$$\Big\downarrow \text{1083}$$

$$2\left(-\int \frac{1}{-3x-6}d\left(2\sqrt{3x+4}-3\sqrt{2}\right)-\int \frac{1}{-3x-6}d\left(2\sqrt{3x+4}+3\sqrt{2}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{\arctan\left(\frac{2\sqrt{3x+4}-3\sqrt{2}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\arctan\left(\frac{2\sqrt{3x+4}+3\sqrt{2}}{\sqrt{2}}\right)}{\sqrt{2}}\right)$$

input
```
Int[(3 + x)/(Sqrt[4 + 3*x]*(1 + x^2)),x]
```

output
```
2*(ArcTan[(-3*Sqrt[2] + 2*Sqrt[4 + 3*x])/Sqrt[2]]/Sqrt[2] + ArcTan[(3*Sqrt
[2] + 2*Sqrt[4 + 3*x])/Sqrt[2]]/Sqrt[2])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$\left(\arctan\left(-3+\sqrt{2}\,\sqrt{3x+4}\right)+\arctan\left(\frac{\left(2\sqrt{3x+4}+3\sqrt{2}\right)\sqrt{2}}{2}\right)\right)\sqrt{2}$
derivativedivides	$\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{3x+4}-3\sqrt{2}\right)\sqrt{2}}{2}\right)+\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{3x+4}+3\sqrt{2}\right)\sqrt{2}}{2}\right)$
default	$\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{3x+4}-3\sqrt{2}\right)\sqrt{2}}{2}\right)+\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{3x+4}+3\sqrt{2}\right)\sqrt{2}}{2}\right)$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{9\operatorname{RootOf}\left(\_Z^2+2\right)x^2-12x\operatorname{RootOf}\left(\_Z^2+2\right)+12\sqrt{3x+4}\,x-7\operatorname{RootOf}\left(\_Z^2+2\right)-4\sqrt{3x+4}}{x^2+1}\right)}{2}$

input
```
int((3+x)/(3*x+4)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
(arctan(-3+2^(1/2)*(3*x+4)^(1/2))+arctan(1/2*(2*(3*x+4)^(1/2)+3*2^(1/2))*2
^(1/2)))*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

$$\int \frac{3+x}{\sqrt{4+3x}\,(1+x^2)}\,dx = \sqrt{2}\arctan\left(\frac{\sqrt{2}(3\,x-1)}{2\,\sqrt{3\,x+4}}\right)$$

input
```
integrate((3+x)/(4+3*x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
sqrt(2)*arctan(1/2*sqrt(2)*(3*x - 1)/sqrt(3*x + 4))
```

Sympy [F]

$$\int \frac{3+x}{\sqrt{4+3x}\,(1+x^2)}\,dx = \int \frac{x+3}{\sqrt{3x+4}\,(x^2+1)}\,dx$$

input
```
integrate((3+x)/(4+3*x)**(1/2)/(x**2+1),x)
```

output
```
Integral((x + 3)/(sqrt(3*x + 4)*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{3+x}{\sqrt{4+3x}\,(1+x^2)}\,dx = \int \frac{x+3}{(x^2+1)\sqrt{3\,x+4}}\,dx$$

input
```
integrate((3+x)/(4+3*x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate((x + 3)/((x^2 + 1)*sqrt(3*x + 4)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int \frac{3+x}{\sqrt{4+3x}\,(1+x^2)}\,dx = \sqrt{2}\arctan\left(\frac{1}{250}\cdot 25^{\frac{3}{4}}\sqrt{10}\left(3\cdot 25^{\frac{1}{4}}\sqrt{10}+10\,\sqrt{3\,x+4}\right)\right) + \sqrt{2}\arctan\left(-\frac{1}{250}\cdot 25^{\frac{3}{4}}\sqrt{10}\left(3\cdot 25^{\frac{1}{4}}\sqrt{10}-10\,\sqrt{3\,x+4}\right)\right)$$

input
```
integrate((3+x)/(4+3*x)^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(1/250*25^(3/4)*sqrt(10)*(3*25^(1/4)*sqrt(10) + 10*sqrt(3*x
+ 4))) + sqrt(2)*arctan(-1/250*25^(3/4)*sqrt(10)*(3*25^(1/4)*sqrt(10) - 10
*sqrt(3*x + 4)))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{3+x}{\sqrt{4+3x}\,(1+x^2)}\,dx = \sqrt{2}\left(\mathrm{atan}\left(\frac{\sqrt{2}\,(3\,x+4)^{3/2}}{10}-\frac{3\,\sqrt{6\,x+8}}{10}\right)+\mathrm{atan}\left(\frac{\sqrt{6\,x+8}}{2}\right)\right)$$

input
```
int((x + 3)/((3*x + 4)^(1/2)*(x^2 + 1)),x)
```

output
```
2^(1/2)*(atan((2^(1/2)*(3*x + 4)^(3/2))/10 - (3*(6*x + 8)^(1/2))/10) + ata
n((6*x + 8)^(1/2)/2))
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{3+x}{\sqrt{4+3x}\,(1+x^2)}\,dx = \sqrt{2}\left(atan\left(\frac{2\sqrt{3x+4}-3\sqrt{2}}{\sqrt{2}}\right)+atan\left(\frac{2\sqrt{3x+4}+3\sqrt{2}}{\sqrt{2}}\right)\right)$$

input
```
int((3+x)/(4+3*x)^(1/2)/(x^2+1),x)
```

output
```
sqrt(2)*(atan((2*sqrt(3*x + 4) - 3*sqrt(2))/sqrt(2)) + atan((2*sqrt(3*x +
4) + 3*sqrt(2))/sqrt(2)))
```

3.145 $\int \frac{1-3x}{\sqrt{4+3x}(1+x^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \frac{1-3x}{\sqrt{4+3x}\,(1+x^2)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{4+3x}}{3+x}\right)$$

output
```
2^(1/2)*arctanh(2^(1/2)*(4+3*x)^(1/2)/(3+x))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1-3x}{\sqrt{4+3x}\,(1+x^2)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{8+6x}}{3+x}\right)$$

input
```
Integrate[(1 - 3*x)/(Sqrt[4 + 3*x]*(1 + x^2)),x]
```

output
```
Sqrt[2]*ArcTanh[Sqrt[8 + 6*x]/(3 + x)]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. $2(27) = 54$.

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {654, 27, 1478, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-3x}{\sqrt{3x+4}\,(x^2+1)}\,dx$$

$$\Big\downarrow \text{ 654}$$

$$2\int \frac{3(1-3x)}{(3x+4)^2-8(3x+4)+25}d\sqrt{3x+4}$$

$$\Big\downarrow \text{ 27}$$

$$6\int \frac{1-3x}{(3x+4)^2-8(3x+4)+25}d\sqrt{3x+4}$$

$$\Big\downarrow \text{ 1478}$$

$$6\left(-\frac{\int -\frac{3\sqrt{2}-2\sqrt{3x+4}}{3x-3\sqrt{2}\sqrt{3x+4}+9}d\sqrt{3x+4}}{6\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{3x+4}+3\right)}{3x+3\sqrt{2}\sqrt{3x+4}+9}d\sqrt{3x+4}}{6\sqrt{2}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$6\left(\frac{\int \frac{3\sqrt{2}-2\sqrt{3x+4}}{3x-3\sqrt{2}\sqrt{3x+4}+9}d\sqrt{3x+4}}{6\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{3x+4}+3\right)}{3x+3\sqrt{2}\sqrt{3x+4}+9}d\sqrt{3x+4}}{6\sqrt{2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$6\left(\frac{\int \frac{3\sqrt{2}-2\sqrt{3x+4}}{3x-3\sqrt{2}\sqrt{3x+4}+9}d\sqrt{3x+4}}{6\sqrt{2}}+\frac{1}{6}\int \frac{\sqrt{2}\sqrt{3x+4}+3}{3x+3\sqrt{2}\sqrt{3x+4}+9}d\sqrt{3x+4}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$6\left(\frac{\log\left(3x+3\sqrt{2}\sqrt{3x+4}+9\right)}{6\sqrt{2}}-\frac{\log\left(3x-3\sqrt{2}\sqrt{3x+4}+9\right)}{6\sqrt{2}}\right)$$

input
```
Int[(1 - 3*x)/(Sqrt[4 + 3*x]*(1 + x^2)),x]
```

output
```
6*(-1/6*Log[9 + 3*x - 3*Sqrt[2]*Sqrt[4 + 3*x]]/Sqrt[2] + Log[9 + 3*x + 3*S
qrt[2]*Sqrt[4 + 3*x]]/(6*Sqrt[2]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

method	result
pseudoelliptic	$-\frac{\left(\ln\left(3+x-\sqrt{2}\,\sqrt{3x+4}\right)-\ln\left(3+x+\sqrt{2}\,\sqrt{3x+4}\right)\right)\sqrt{2}}{2}$
derivativedivides	$-\frac{\sqrt{2}\,\ln\left(3x+9-3\sqrt{2}\,\sqrt{3x+4}\right)}{2}+\frac{\sqrt{2}\,\ln\left(3x+9+3\sqrt{2}\,\sqrt{3x+4}\right)}{2}$
default	$-\frac{\sqrt{2}\,\ln\left(3x+9-3\sqrt{2}\,\sqrt{3x+4}\right)}{2}+\frac{\sqrt{2}\,\ln\left(3x+9+3\sqrt{2}\,\sqrt{3x+4}\right)}{2}$
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)x^2+12\,\mathrm{RootOf}\left(\_Z^2-2\right)x-4\sqrt{3x+4}\,x+17\,\mathrm{RootOf}\left(\_Z^2-2\right)-12\sqrt{3x+4}}{x^2+1}\right)}{2}$

input
```
int((1-3*x)/(3*x+4)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(3+x-2^(1/2)*(3*x+4)^(1/2))-ln(3+x+2^(1/2)*(3*x+4)^(1/2)))*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{1-3x}{\sqrt{4+3x}\,(1+x^2)}\,dx=\frac{1}{2}\,\sqrt{2}\log\left(\frac{2\,\sqrt{2}\sqrt{3\,x+4}(x+3)+x^2+12\,x+17}{x^2+1}\right)$$

input
```
integrate((1-3*x)/(4+3*x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log((2*sqrt(2)*sqrt(3*x + 4)*(x + 3) + x^2 + 12*x + 17)/(x^2 +
 1))
```

Sympy [F]

$$\int \frac{1-3x}{\sqrt{4+3x}\,(1+x^2)}\,dx = -\int \frac{3x}{x^2\sqrt{3x+4}+\sqrt{3x+4}}\,dx - \int \left(-\frac{1}{x^2\sqrt{3x+4}+\sqrt{3x+4}}\right)dx$$

input
```
integrate((1-3*x)/(4+3*x)**(1/2)/(x**2+1),x)
```

output
```
-Integral(3*x/(x**2*sqrt(3*x + 4) + sqrt(3*x + 4)), x) - Integral(-1/(x**2
*sqrt(3*x + 4) + sqrt(3*x + 4)), x)
```

Maxima [F]

$$\int \frac{1-3x}{\sqrt{4+3x}\,(1+x^2)}\,dx = \int -\frac{3\,x-1}{(x^2+1)\sqrt{3\,x+4}}\,dx$$

input
```
integrate((1-3*x)/(4+3*x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
-integrate((3*x - 1)/((x^2 + 1)*sqrt(3*x + 4)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(21) = 42$.

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\int \frac{1-3x}{\sqrt{4+3x}\,(1+x^2)}\,dx = \frac{1}{2}\sqrt{2}\log\left(\frac{3}{5}\cdot 25^{\frac{1}{4}}\sqrt{10}\sqrt{3\,x+4}+3\,x+9\right) - \frac{1}{2}\sqrt{2}\log\left(-\frac{3}{5}\cdot 25^{\frac{1}{4}}\sqrt{10}\sqrt{3\,x+4}+3\,x+9\right)$$

input
```
integrate((1-3*x)/(4+3*x)^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*log(3/5*25^(1/4)*sqrt(10)*sqrt(3*x + 4) + 3*x + 9) - 1/2*sqrt(
2)*log(-3/5*25^(1/4)*sqrt(10)*sqrt(3*x + 4) + 3*x + 9)
```

Mupad [B] (verification not implemented)

Time = 6.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1-3x}{\sqrt{4+3x}\,(1+x^2)}\,dx = \sqrt{2}\,\mathrm{atanh}\left(\frac{24\,\sqrt{6\,x+8}}{24\,x+72}\right)$$

input
```
int(-(3*x - 1)/((3*x + 4)^(1/2)*(x^2 + 1)),x)
```

output
```
2^(1/2)*atanh((24*(6*x + 8)^(1/2))/(24*x + 72))
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{1-3x}{\sqrt{4+3x}\,(1+x^2)}\,dx = \frac{\sqrt{2}\,\left(-\log\left(-3\sqrt{3x+4}\,\sqrt{2}+3x+9\right)+\log\left(3\sqrt{3x+4}\,\sqrt{2}+3x+9\right)\right)}{2}$$

input
```
int((1-3*x)/(4+3*x)^(1/2)/(x^2+1),x)
```

output
```
(sqrt(2)*( - log( - 3*sqrt(3*x + 4)*sqrt(2) + 3*x + 9) + log(3*sqrt(3*x +
4)*sqrt(2) + 3*x + 9)))/2
```

3.146 $\int \frac{2+x}{\sqrt{3+4x}(1+x^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{2+x}{\sqrt{3+4x}\,(1+x^2)}\,dx = -\arctan\left(2-\sqrt{3+4x}\right)+\arctan\left(2+\sqrt{3+4x}\right)$$

output
```
arctan(-2+(3+4*x)^(1/2))+arctan(2+(3+4*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{2+x}{\sqrt{3+4x}\,(1+x^2)}\,dx = \arctan\left(\frac{-\frac{5}{2}+\frac{1}{2}(3+4x)}{\sqrt{3+4x}}\right)$$

input
```
Integrate[(2 + x)/(Sqrt[3 + 4*x]*(1 + x^2)),x]
```

output
```
ArcTan[(-5/2 + (3 + 4*x)/2)/Sqrt[3 + 4*x]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {654, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{\sqrt{4x+3}\,(x^2+1)}dx$$

$$\Big\downarrow \; 654$$

$$2\int \frac{4x+8}{(4x+3)^2-6(4x+3)+25}d\sqrt{4x+3}$$

$$\Big\downarrow \; 1475$$

$$2\left(\frac{1}{2}\int \frac{1}{4x-4\sqrt{4x+3}+8}d\sqrt{4x+3}+\frac{1}{2}\int \frac{1}{4x+4\sqrt{4x+3}+8}d\sqrt{4x+3}\right)$$

$$\Big\downarrow \; 1083$$

$$2\left(-\int \frac{1}{-4x-7}d\left(2\sqrt{4x+3}-4\right)-\int \frac{1}{-4x-7}d\left(2\sqrt{4x+3}+4\right)\right)$$

$$\Big\downarrow \; 217$$

$$2\left(\frac{1}{2}\arctan\left(\frac{1}{2}\left(2\sqrt{4x+3}-4\right)\right)+\frac{1}{2}\arctan\left(\frac{1}{2}\left(2\sqrt{4x+3}+4\right)\right)\right)$$

input
```
Int[(2 + x)/(Sqrt[3 + 4*x]*(1 + x^2)),x]
```

output
```
2*(ArcTan[(-4 + 2*Sqrt[3 + 4*x])/2]/2 + ArcTan[(4 + 2*Sqrt[3 + 4*x])/2]/2)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	siz
derivativedivides	$\arctan\left(-2+\sqrt{3+4x}\right)+\arctan\left(2+\sqrt{3+4x}\right)$	22
default	$\arctan\left(-2+\sqrt{3+4x}\right)+\arctan\left(2+\sqrt{3+4x}\right)$	22
pseudoelliptic	$\arctan\left(-2+\sqrt{3+4x}\right)+\arctan\left(2+\sqrt{3+4x}\right)$	22
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^2+1\right)x^2-4\operatorname{RootOf}\left(\_Z^2+1\right)x-2\sqrt{3+4x}\,x-\operatorname{RootOf}\left(\_Z^2+1\right)+\sqrt{3+4x}}{x^2+1}\right)}{2}$	65

input
```
int((2+x)/(3+4*x)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
arctan(-2+(3+4*x)^(1/2))+arctan(2+(3+4*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.48

$$\int \frac{2+x}{\sqrt{3+4x}\,(1+x^2)}\,dx = \arctan\left(\frac{2\,x-1}{\sqrt{4\,x+3}}\right)$$

input
```
integrate((2+x)/(3+4*x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
arctan((2*x - 1)/sqrt(4*x + 3))
```

Sympy [A] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{2+x}{\sqrt{3+4x}\,(1+x^2)}\,dx = \operatorname{atan}\left(\sqrt{4x+3}-2\right) + \operatorname{atan}\left(\sqrt{4x+3}+2\right)$$

input
```
integrate((2+x)/(3+4*x)**(1/2)/(x**2+1),x)
```

output
```
atan(sqrt(4*x + 3) - 2) + atan(sqrt(4*x + 3) + 2)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{2+x}{\sqrt{3+4x}\,(1+x^2)}\,dx = \arctan\left(\sqrt{4\,x+3}+2\right) + \arctan\left(\sqrt{4\,x+3}-2\right)$$

input
```
integrate((2+x)/(3+4*x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
arctan(sqrt(4*x + 3) + 2) + arctan(sqrt(4*x + 3) - 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{2+x}{\sqrt{3+4x}\,(1+x^2)}\,dx = \arctan\left(\sqrt{4\,x+3}+2\right)+\arctan\left(\sqrt{4\,x+3}-2\right)$$

input
```
integrate((2+x)/(3+4*x)^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
arctan(sqrt(4*x + 3) + 2) + arctan(sqrt(4*x + 3) - 2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{2+x}{\sqrt{3+4x}\,(1+x^2)}\,dx = \mathrm{atan}\left(\frac{\sqrt{4\,x+3}}{2}\right)+\mathrm{atan}\left(\frac{(4\,x+2)\,\sqrt{4\,x+3}}{10}\right)$$

input
```
int((x + 2)/((4*x + 3)^(1/2)*(x^2 + 1)),x)
```

output
```
atan((4*x + 3)^(1/2)/2) + atan(((4*x + 2)*(4*x + 3)^(1/2))/10)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{2+x}{\sqrt{3+4x}\,(1+x^2)}\,dx = atan\left(\sqrt{4x+3}-2\right)+atan\left(\sqrt{4x+3}+2\right)$$

input
```
int((2+x)/(3+4*x)^(1/2)/(x^2+1),x)
```

output
```
atan(sqrt(4*x + 3) - 2) + atan(sqrt(4*x + 3) + 2)
```

3.147 $\int \frac{-2+x}{\sqrt{-3+x}(-8+x^2)} dx$

Optimal result

Integrand size = 18, antiderivative size = 45

$$\int \frac{-2+x}{\sqrt{-3+x}\,(-8+x^2)}\,dx = \frac{\arctan\left(\left(-1+\sqrt{2}\right)\sqrt{-3+x}\right)}{\sqrt{2}} + \frac{\arctan\left(\left(1+\sqrt{2}\right)\sqrt{-3+x}\right)}{\sqrt{2}}$$

output
```
1/2*arctan((2^(1/2)-1)*(-3+x)^(1/2))*2^(1/2)+1/2*arctan((1+2^(1/2))*(-3+x)
^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.58

$$\int \frac{-2+x}{\sqrt{-3+x}\,(-8+x^2)}\,dx = \frac{\arctan\left(\frac{-4+x}{2\sqrt{2}\sqrt{-3+x}}\right)}{\sqrt{2}}$$

input
```
Integrate[(-2 + x)/(Sqrt[-3 + x]*(-8 + x^2)),x]
```

output
```
ArcTan[(-4 + x)/(2*Sqrt[2]*Sqrt[-3 + x])]/Sqrt[2]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 97 vs. $2(45) = 90$.

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {654, 1477, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-2}{\sqrt{x-3}\,(x^2-8)}\,dx$$

$$\Big\downarrow \text{ 654}$$

$$2\int \frac{x-2}{(x-3)^2+6(x-3)+1}d\sqrt{x-3}$$

$$\Big\downarrow \text{ 1477}$$

$$2\left(\frac{1}{4}\left(2-\sqrt{2}\right)\int \frac{1}{x-2\sqrt{2}}d\sqrt{x-3}+\frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{x+2\sqrt{2}}d\sqrt{x-3}\right)$$

$$\Big\downarrow \text{ 216}$$

$$2\left(\frac{\left(2-\sqrt{2}\right)\arctan\left(\frac{\sqrt{x-3}}{\sqrt{3-2\sqrt{2}}}\right)}{4\sqrt{3-2\sqrt{2}}}+\frac{\left(2+\sqrt{2}\right)\arctan\left(\frac{\sqrt{x-3}}{\sqrt{3+2\sqrt{2}}}\right)}{4\sqrt{3+2\sqrt{2}}}\right)$$

input
```
Int[(-2 + x)/(Sqrt[-3 + x]*(-8 + x^2)),x]
```

output
```
2*(((2 - Sqrt[2])*ArcTan[Sqrt[-3 + x]/Sqrt[3 - 2*Sqrt[2]]])/(4*Sqrt[3 - 2*
Sqrt[2]]) + ((2 + Sqrt[2])*ArcTan[Sqrt[-3 + x]/Sqrt[3 + 2*Sqrt[2]]])/(4*Sq
rt[3 + 2*Sqrt[2]]))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1477
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && EqQ[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+2\right)x^2-16x\operatorname{RootOf}\left(\_Z^2+2\right)-8\sqrt{-3+x}\,x+40\operatorname{RootOf}\left(\_Z^2+2\right)+32\sqrt{-3+x}}{x^2-8}\right)}{4}$
derivativedivides	$\frac{\sqrt{2}\left(1+\sqrt{2}\right)\arctan\left(\frac{2\sqrt{-3+x}}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}+\frac{\left(\sqrt{2}-1\right)\sqrt{2}\arctan\left(\frac{2\sqrt{-3+x}}{-2+2\sqrt{2}}\right)}{-2+2\sqrt{2}}$
default	$\frac{\sqrt{2}\left(1+\sqrt{2}\right)\arctan\left(\frac{2\sqrt{-3+x}}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}+\frac{\left(\sqrt{2}-1\right)\sqrt{2}\arctan\left(\frac{2\sqrt{-3+x}}{-2+2\sqrt{2}}\right)}{-2+2\sqrt{2}}$

input
```
int((x-2)/(-3+x)^(1/2)/(x^2-8),x,method=_RETURNVERBOSE)
```

output
```
1/4*RootOf(_Z^2+2)*ln((RootOf(_Z^2+2)*x^2-16*x*RootOf(_Z^2+2)-8*(-3+x)^(1/
2)*x+40*RootOf(_Z^2+2)+32*(-3+x)^(1/2))/(x^2-8))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.42

$$\int \frac{-2+x}{\sqrt{-3+x}\,(-8+x^2)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}(x-4)}{4\sqrt{x-3}}\right)$$

input
```
integrate((-2+x)/(-3+x)^(1/2)/(x^2-8),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(1/4*sqrt(2)*(x - 4)/sqrt(x - 3))
```

Sympy [F]

$$\int \frac{-2+x}{\sqrt{-3+x}\,(-8+x^2)}\,dx = \int \frac{x-2}{\sqrt{x-3}\,(x^2-8)}\,dx$$

input
```
integrate((-2+x)/(-3+x)**(1/2)/(x**2-8),x)
```

output
```
Integral((x - 2)/(sqrt(x - 3)*(x**2 - 8)), x)
```

Maxima [F]

$$\int \frac{-2+x}{\sqrt{-3+x}\,(-8+x^2)}\,dx = \int \frac{x-2}{(x^2-8)\sqrt{x-3}}\,dx$$

input
```
integrate((-2+x)/(-3+x)^(1/2)/(x^2-8),x, algorithm="maxima")
```

output
```
integrate((x - 2)/((x^2 - 8)*sqrt(x - 3)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.51

$$\int \frac{-2+x}{\sqrt{-3+x}\,(-8+x^2)}\,dx = \frac{1}{4}\sqrt{2}\left(\pi + 2\arctan\left(\frac{\sqrt{2}(x-4)}{4\sqrt{x-3}}\right)\right)$$

input
```
integrate((-2+x)/(-3+x)^(1/2)/(x^2-8),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(pi + 2*arctan(1/4*sqrt(2)*(x - 4)/sqrt(x - 3)))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{-2+x}{\sqrt{-3+x}\,(-8+x^2)}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{x-3}}{4}\right) + \operatorname{atan}\left(\frac{7\sqrt{2}\sqrt{x-3}}{4} + \frac{\sqrt{2}\,(x-3)^{3/2}}{4}\right)\right)}{2}$$

input
```
int((x - 2)/((x^2 - 8)*(x - 3)^(1/2)),x)
```

output
```
(2^(1/2)*(atan((2^(1/2)*(x - 3)^(1/2))/4) + atan((7*2^(1/2)*(x - 3)^(1/2))
/4 + (2^(1/2)*(x - 3)^(3/2))/4)))/2
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int \frac{-2+x}{\sqrt{-3+x}\,(-8+x^2)}\,dx = \frac{\sqrt{2}\left(2atan\left(\frac{\sqrt{x-3}}{\sqrt{2}+1}\right) - \log\left(\sqrt{x-3}-\sqrt{2}\,i+i\right)i + \log\left(\sqrt{x-3}+\sqrt{2}\,i-i\right)i\right)}{4}$$

input
```
int((-2+x)/(-3+x)^(1/2)/(x^2-8),x)
```

output
```
(sqrt(2)*(2*atan(sqrt(x - 3)/(sqrt(2) + 1)) - log(sqrt(x - 3) - sqrt(2)*i
+ i)*i + log(sqrt(x - 3) + sqrt(2)*i - i)*i))/4
```

3.148 $\int (A+Bx)(c+dx)^3\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 256

$$\int (A+Bx)(c+dx)^3\sqrt{a+bx^2}\,dx = \frac{(2Abc(4bc^2-3ad^2)-aBd(6bc^2-ad^2))\,x\sqrt{a+bx^2}}{16b^2} + \frac{(Bc+2Ad)(c+dx)^2\left(a+bx^2\right)^{3/2}}{10b} + \frac{B(c+dx)^3\left(a+bx^2\right)^{3/2}}{6b} - \frac{(8(2ad^2(3Bc+Ad)-bc^2(Bc+12Ad))+3d(5aBd^2-2bc(Bc+7Ad))\,x)\left(a+bx^2\right)^{3/2}}{120b^2} + \frac{a(2Abc(4bc^2-3ad^2)-aBd(6bc^2-ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

output
```
1/16*(2*A*b*c*(-3*a*d^2+4*b*c^2)-a*B*d*(-a*d^2+6*b*c^2))*x*(b*x^2+a)^(1/2)
/b^2+1/10*(2*A*d+B*c)*(d*x+c)^2*(b*x^2+a)^(3/2)/b+1/6*B*(d*x+c)^3*(b*x^2+a
)^(3/2)/b-1/120*(16*a*d^2*(A*d+3*B*c)-8*b*c^2*(12*A*d+B*c)+3*d*(5*a*B*d^2-
2*b*c*(7*A*d+B*c))*x)*(b*x^2+a)^(3/2)/b^2+1/16*a*(2*A*b*c*(-3*a*d^2+4*b*c^
2)-a*B*d*(-a*d^2+6*b*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.94

$$\int (A+Bx)(c+dx)^3\sqrt{a+bx^2}\,dx$$
$$= \frac{\sqrt{b}\sqrt{a+bx^2}(-a^2d^2(96Bc+32Ad+15Bdx)+2ab(Ad(120c^2+45cdx+8d^2x^2)+B(40c^3+45c^2dx+2}{}$$

input
```
Integrate[(A + B*x)*(c + d*x)^3*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(-(a^2*d^2*(96*B*c + 32*A*d + 15*B*d*x)) + 2*a*b*
(A*d*(120*c^2 + 45*c*d*x + 8*d^2*x^2) + B*(40*c^3 + 45*c^2*d*x + 24*c*d^2*
x^2 + 5*d^3*x^3)) + 4*b^2*x*(3*A*(10*c^3 + 20*c^2*d*x + 15*c*d^2*x^2 + 4*d
^3*x^3) + B*x*(20*c^3 + 45*c^2*d*x + 36*c*d^2*x^2 + 10*d^3*x^3))) - 15*a*(
2*A*b*c*(4*b*c^2 - 3*a*d^2) + a*B*d*(-6*b*c^2 + a*d^2))*Log[-(Sqrt[b]*x) +
 Sqrt[a + b*x^2]])/(240*b^(5/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {687, 27, 687, 27, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(A+Bx)(c+dx)^3\,dx$$

$$\Big\downarrow \; 687$$

$$\frac{\int 3(c+dx)^2(2Abc-aBd+b(Bc+2Ad)x)\sqrt{bx^2+a}dx}{6b}+\frac{B\left(a+bx^2\right)^{3/2}(c+dx)^3}{6b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (c+dx)^2(2Abc-aBd+b(Bc+2Ad)x)\sqrt{bx^2+a}dx}{2b}+\frac{B\left(a+bx^2\right)^{3/2}(c+dx)^3}{6b}$$

$\downarrow$ 687

$$\frac{\frac{\int b(c+dx)\left(10Abc^2-7aBdc-4aAd^2-\left(5aBd^2-2bc(Bc+7Ad)\right)x\right)\sqrt{bx^2+a}dx}{5b}+\frac{1}{5}\left(a+bx^2\right)^{3/2}(c+dx)^2(2Ad+Bc)}{2b}+\frac{B\left(a+bx^2\right)^{3/2}(c+dx)^3}{6b}$$

$\downarrow$ 27

$$\frac{\frac{1}{5}\int (c+dx)\left(10Abc^2-7aBdc-4aAd^2-\left(5aBd^2-2bc(Bc+7Ad)\right)x\right)\sqrt{bx^2+a}dx+\frac{1}{5}\left(a+bx^2\right)^{3/2}(c+dx)^2(2}{2b}$$
$$\frac{B\left(a+bx^2\right)^{3/2}(c+dx)^3}{6b}$$

$\downarrow$ 676

$$\frac{\frac{1}{5}\left(\frac{5\left(2Abc\left(4bc^2-3ad^2\right)-aBd\left(6bc^2-ad^2\right)\right)\int\sqrt{bx^2+a}dx}{4b}-\frac{2\left(a+bx^2\right)^{3/2}\left(2ad^2(Ad+3Bc)-bc^2(12Ad+Bc)\right)}{3b}-\frac{dx\left(a+bx^2\right)^{3/2}\left(5aBd^2-2bc(7A\right.}{4b}}{2b}$$
$$\frac{B\left(a+bx^2\right)^{3/2}(c+dx)^3}{6b}$$

$\downarrow$ 211

$$\frac{\frac{1}{5}\left(\frac{5\left(2Abc\left(4bc^2-3ad^2\right)-aBd\left(6bc^2-ad^2\right)\right)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}-\frac{2\left(a+bx^2\right)^{3/2}\left(2ad^2(Ad+3Bc)-bc^2(12Ad+Bc)\right)}{3b}-\frac{dx\left(a+bx^2\right.}{}\right.}{2b}$$
$$\frac{B\left(a+bx^2\right)^{3/2}(c+dx)^3}{6b}$$

$\downarrow$ 224

$$\frac{\frac{1}{5}\left(\frac{5\left(2Abc\left(4bc^2-3ad^2\right)-aBd\left(6bc^2-ad^2\right)\right)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}-\frac{2\left(a+bx^2\right)^{3/2}\left(2ad^2(Ad+3Bc)-bc^2(12Ad+Bc)\right)}{3b}-d\right.}{2b}$$
$$\frac{B\left(a+bx^2\right)^{3/2}(c+dx)^3}{6b}$$

$\downarrow$ 219

$$\frac{\frac{1}{5}\left(\frac{5\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(2Abc\left(4bc^2-3ad^2\right)-aBd\left(6bc^2-ad^2\right)\right)}{4b}-\frac{2\left(a+bx^2\right)^{3/2}\left(2ad^2(Ad+3Bc)-bc^2(12Ad+Bc)\right)}{3b}-\frac{dx}{}\right.}{2b}$$

$$\frac{B\left(a+bx^2\right)^{3/2}(c+dx)^3}{6b}$$

input

```
Int[(A + B*x)*(c + d*x)^3*Sqrt[a + b*x^2],x]
```

output

```
(B*(c + d*x)^3*(a + b*x^2)^(3/2))/(6*b) + (((B*c + 2*A*d)*(c + d*x)^2*(a +
 b*x^2)^(3/2))/5 + ((-2*(2*a*d^2*(3*B*c + A*d) - b*c^2*(B*c + 12*A*d))*(a
+ b*x^2)^(3/2))/(3*b) - (d*(5*a*B*d^2 - 2*b*c*(B*c + 7*A*d))*x*(a + b*x^2)
^(3/2))/(4*b) + (5*(2*A*b*c*(4*b*c^2 - 3*a*d^2) - a*B*d*(6*b*c^2 - a*d^2))
*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt
[b]))/(4*b))/5)/(2*b)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.04

method	result
default	$A\,c^3\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+d^2(Ad+3Bc)\left(\frac{x^2(b\,x^2+a)^{\frac{3}{2}}}{5b}-\frac{2a(b\,x^2+a)^{\frac{3}{2}}}{15b^2}\right)+3cd(Ad+Bc$
risch	$-\frac{(-40B\,d^3b^2x^5-48A\,b^2d^3x^4-144B\,b^2c\,d^2x^4-180A\,b^2c\,d^2x^3-10Ba\,d^3b\,x^3-180B\,b^2c^2d\,x^3-16A\,d^3ab\,x^2-240A\,b^2c^2d\,x^2-48Ba}{2}$

input
```
int((B*x+A)*(d*x+c)^3*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*c^3*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+
d^2*(A*d+3*B*c)*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))+3*c
*d*(A*d+B*c)*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a
/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+1/3*c^2*(3*A*d+B*c)*(b*x^2+a)^(3/
2)/b+B*d^3*(1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-1/4
*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.27

$$\int (A + Bx)(c + dx)^3\sqrt{a + bx^2}\, dx$$

$$= \left[\frac{15\,(8\,Aab^2c^3 - 6\,Ba^2bc^2d - 6\,Aa^2bcd^2 + Ba^3d^3)\sqrt{b}\log\left(-2\,bx^2 - 2\,\sqrt{bx^2 + a}\sqrt{b}x - a\right) + 2\,(40\,Bb^3d^3x\right.$$

$$-\frac{15\,(8\,Aab^2c^3 - 6\,Ba^2bc^2d - 6\,Aa^2bcd^2 + Ba^3d^3)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - (40\,Bb^3d^3x^5 + 80\,Bab^2c^3 + 2$$

input
```
integrate((B*x+A)*(d*x+c)^3*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/480*(15*(8*A*a*b^2*c^3 - 6*B*a^2*b*c^2*d - 6*A*a^2*b*c*d^2 + B*a^3*d^3)
*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(40*B*b^3*d^3
*x^5 + 80*B*a*b^2*c^3 + 240*A*a*b^2*c^2*d - 96*B*a^2*b*c*d^2 - 32*A*a^2*b*
d^3 + 48*(3*B*b^3*c*d^2 + A*b^3*d^3)*x^4 + 10*(18*B*b^3*c^2*d + 18*A*b^3*c
*d^2 + B*a*b^2*d^3)*x^3 + 16*(5*B*b^3*c^3 + 15*A*b^3*c^2*d + 3*B*a*b^2*c*d
^2 + A*a*b^2*d^3)*x^2 + 15*(8*A*b^3*c^3 + 6*B*a*b^2*c^2*d + 6*A*a*b^2*c*d^
2 - B*a^2*b*d^3)*x)*sqrt(b*x^2 + a))/b^3, -1/240*(15*(8*A*a*b^2*c^3 - 6*B*
a^2*b*c^2*d - 6*A*a^2*b*c*d^2 + B*a^3*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt
(b*x^2 + a)) - (40*B*b^3*d^3*x^5 + 80*B*a*b^2*c^3 + 240*A*a*b^2*c^2*d - 96
*B*a^2*b*c*d^2 - 32*A*a^2*b*d^3 + 48*(3*B*b^3*c*d^2 + A*b^3*d^3)*x^4 + 10*
(18*B*b^3*c^2*d + 18*A*b^3*c*d^2 + B*a*b^2*d^3)*x^3 + 16*(5*B*b^3*c^3 + 15
*A*b^3*c^2*d + 3*B*a*b^2*c*d^2 + A*a*b^2*d^3)*x^2 + 15*(8*A*b^3*c^3 + 6*B*
a*b^2*c^2*d + 6*A*a*b^2*c*d^2 - B*a^2*b*d^3)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(241) = 482$.

Time = 0.64 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.93

$$\int (A+Bx)(c+dx)^3\sqrt{a+bx^2}\,dx$$

$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{Bd^3x^5}{6} + \frac{x^4(Abd^3+3Bbcd^2)}{5b} + \frac{x^3\cdot\left(3Abcd^2+\frac{Bad^3}{6}+3Bbc^2d\right)}{4b} + \frac{x^2\left(Aad^3+3Abc^2d+3Bacd^2+Bbc^3-\frac{4a\left(Abd^3+3Bbcd^2\right)}{5b}\right.}{3b}\right. \\ \sqrt{a}\left(Ac^3x + \frac{Bd^3x^5}{5} + \frac{x^4(Ad^3+3Bcd^2)}{4} + \frac{x^3\cdot(3Acd^2+3Bc^2d)}{3} + \frac{x^2\cdot(3Ac^2d+Bc^3)}{2}\right) \end{cases}$$

input
```
integrate((B*x+A)*(d*x+c)**3*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*d**3*x**5/6 + x**4*(A*b*d**3 + 3*B*b*c*d**2
)/(5*b) + x**3*(3*A*b*c*d**2 + B*a*d**3/6 + 3*B*b*c**2*d)/(4*b) + x**2*(A*
a*d**3 + 3*A*b*c**2*d + 3*B*a*c*d**2 + B*b*c**3 - 4*a*(A*b*d**3 + 3*B*b*c*
d**2)/(5*b))/(3*b) + x*(3*A*a*c*d**2 + A*b*c**3 + 3*B*a*c**2*d - 3*a*(3*A*
b*c*d**2 + B*a*d**3/6 + 3*B*b*c**2*d)/(4*b))/(2*b) + (3*A*a*c**2*d + B*a*c
**3 - 2*a*(A*a*d**3 + 3*A*b*c**2*d + 3*B*a*c*d**2 + B*b*c**3 - 4*a*(A*b*d*
*3 + 3*B*b*c*d**2)/(5*b))/(3*b))/b) + (A*a*c**3 - a*(3*A*a*c*d**2 + A*b*c*
*3 + 3*B*a*c**2*d - 3*a*(3*A*b*c*d**2 + B*a*d**3/6 + 3*B*b*c**2*d)/(4*b))/
(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0
)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(A*c**3*x + B*d**3
*x**5/5 + x**4*(A*d**3 + 3*B*c*d**2)/4 + x**3*(3*A*c*d**2 + 3*B*c**2*d)/3
+ x**2*(3*A*c**2*d + B*c**3)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.18

$$\int (A+Bx)(c+dx)^3\sqrt{a+bx^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Bd^3x^3}{6\,b} + \frac{1}{2}\sqrt{bx^2+a}Ac^3x - \frac{(bx^2+a)^{\frac{3}{2}}Bad^3x}{8\,b^2} + \frac{\sqrt{bx^2+a}Ba^2d^3x}{16\,b^2} + \frac{Aac^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} + \frac{Ba^3d^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} + \frac{(bx^2+a)^{\frac{3}{2}}Bc^3}{3\,b} + \frac{(bx^2+a)^{\frac{3}{2}}Ac^2d}{b} + \frac{(3\,Bcd^2+Ad^3)(bx^2+a)^{\frac{3}{2}}x^2}{5\,b} + \frac{3\,(Bc^2d+Acd^2)(bx^2+a)^{\frac{3}{2}}x}{4\,b} - \frac{3\,(Bc^2d+Acd^2)\sqrt{bx^2+a}ax}{8\,b} - \frac{3\,(Bc^2d+Acd^2)a^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} - \frac{2\,(3\,Bcd^2+Ad^3)(bx^2+a)^{\frac{3}{2}}a}{15\,b^2}$$

input
```
integrate((B*x+A)*(d*x+c)^3*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(3/2)*B*d^3*x^3/b + 1/2*sqrt(b*x^2 + a)*A*c^3*x - 1/8*(b*x
^2 + a)^(3/2)*B*a*d^3*x/b^2 + 1/16*sqrt(b*x^2 + a)*B*a^2*d^3*x/b^2 + 1/2*A
*a*c^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/16*B*a^3*d^3*arcsinh(b*x/sqrt(a*
b))/b^(5/2) + 1/3*(b*x^2 + a)^(3/2)*B*c^3/b + (b*x^2 + a)^(3/2)*A*c^2*d/b
+ 1/5*(3*B*c*d^2 + A*d^3)*(b*x^2 + a)^(3/2)*x^2/b + 3/4*(B*c^2*d + A*c*d^2
)*(b*x^2 + a)^(3/2)*x/b - 3/8*(B*c^2*d + A*c*d^2)*sqrt(b*x^2 + a)*a*x/b -
3/8*(B*c^2*d + A*c*d^2)*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/15*(3*B*c*d
^2 + A*d^3)*(b*x^2 + a)^(3/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.18

$$\int (A+Bx)(c+dx)^3\sqrt{a+bx^2}\,dx$$
$$=\frac{1}{240}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(5\,Bd^3x+\frac{6\,(3\,Bb^4cd^2+Ab^4d^3)}{b^4}\right)x+\frac{5\,(18\,Bb^4c^2d+18\,Ab^4cd^2+Bab^3d^3)}{b^4}\right)x\right.\right.\right.$$
$$-\frac{(8\,Aab^2c^3-6\,Ba^2bc^2d-6\,Aa^2bcd^2+Ba^3d^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^3*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/240*sqrt(b*x^2 + a)*((2*((4*(5*B*d^3*x + 6*(3*B*b^4*c*d^2 + A*b^4*d^3)/b
^4)*x + 5*(18*B*b^4*c^2*d + 18*A*b^4*c*d^2 + B*a*b^3*d^3)/b^4)*x + 8*(5*B*
b^4*c^3 + 15*A*b^4*c^2*d + 3*B*a*b^3*c*d^2 + A*a*b^3*d^3)/b^4)*x + 15*(8*A
*b^4*c^3 + 6*B*a*b^3*c^2*d + 6*A*a*b^3*c*d^2 - B*a^2*b^2*d^3)/b^4)*x + 16*
(5*B*a*b^3*c^3 + 15*A*a*b^3*c^2*d - 6*B*a^2*b^2*c*d^2 - 2*A*a^2*b^2*d^3)/b
^4) - 1/16*(8*A*a*b^2*c^3 - 6*B*a^2*b*c^2*d - 6*A*a^2*b*c*d^2 + B*a^3*d^3)
*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)(c+dx)^3\sqrt{a+bx^2}\,dx=\int \sqrt{b\,x^2+a}\,(A+B\,x)\,\left(c+d\,x\right)^3\,dx$$

input
```
int((a + b*x^2)^(1/2)*(A + B*x)*(c + d*x)^3,x)
```

output
```
int((a + b*x^2)^(1/2)*(A + B*x)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (A+Bx)(c+dx)^3\sqrt{a+bx^2}\,dx=\int (Bx+A)\,(dx+c)^3\,\sqrt{b\,x^2+a}dx$$

input
```
int((B*x+A)*(d*x+c)^3*(b*x^2+a)^(1/2),x)
```

output
```
int((B*x+A)*(d*x+c)^3*(b*x^2+a)^(1/2),x)
```

3.149 $\int (A+Bx)(c+dx)^2\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 171

$$\int (A+Bx)(c+dx)^2\sqrt{a+bx^2}\,dx = \frac{(4Abc^2-2aBcd-aAd^2)\,x\sqrt{a+bx^2}}{8b}+\frac{B(c+dx)^2\,(a+bx^2)^{3/2}}{5b} - \frac{(8(aBd^2-bc(Bc+5Ad))-3bd(2Bc+5Ad)x)\,(a+bx^2)^{3/2}}{60b^2} + \frac{a(4Abc^2-2aBcd-aAd^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output

```
1/8*(-A*a*d^2+4*A*b*c^2-2*B*a*c*d)*x*(b*x^2+a)^(1/2)/b+1/5*B*(d*x+c)^2*(b*
x^2+a)^(3/2)/b-1/60*(8*a*B*d^2-8*b*c*(5*A*d+B*c)-3*b*d*(5*A*d+2*B*c)*x)*(b
*x^2+a)^(3/2)/b^2+1/8*a*(-A*a*d^2+4*A*b*c^2-2*B*a*c*d)*arctanh(b^(1/2)*x/(
b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.98

$$\int (A+Bx)(c+dx)^2\sqrt{a+bx^2}\,dx$$
$$=\frac{\sqrt{a+bx^2}(-16a^2Bd^2+2b^2x(5A(6c^2+8cdx+3d^2x^2)+2Bx(10c^2+15cdx+6d^2x^2))+ab(5Ad(16c+3}{120b^2}$$

input
```
Integrate[(A + B*x)*(c + d*x)^2*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-16*a^2*B*d^2 + 2*b^2*x*(5*A*(6*c^2 + 8*c*d*x + 3*d^2*x^
2) + 2*B*x*(10*c^2 + 15*c*d*x + 6*d^2*x^2)) + a*b*(5*A*d*(16*c + 3*d*x) +
B*(40*c^2 + 30*c*d*x + 8*d^2*x^2))) + 15*a*Sqrt[b]*(-4*A*b*c^2 + 2*a*B*c*d
 + a*A*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(120*b^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {687, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(A+Bx)(c+dx)^2\,dx$$

$$\Big\downarrow \; 687$$

$$\frac{\int (c+dx)(5Abc-2aBd+b(2Bc+5Ad)x)\sqrt{bx^2+a}dx}{5b}+\frac{B\left(a+bx^2\right)^{3/2}(c+dx)^2}{5b}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{5}{4}\left(-aAd^2-2aBcd+4Abc^2\right)\int \sqrt{bx^2+a}dx-\frac{2\left(a+bx^2\right)^{3/2}\left(aBd^2-bc(5Ad+Bc)\right)}{3b}+\frac{1}{4}dx\left(a+bx^2\right)^{3/2}(5Ad+2Bc)}{5b}+\frac{B\left(a+bx^2\right)^{3/2}(c+dx)^2}{5b}$$

$$\Big\downarrow \textcolor{blue}{211}$$

$$\frac{\frac{5}{4}\left(-aAd^2-2aBcd+4Abc^2\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{2\left(a+bx^2\right)^{3/2}\left(aBd^2-bc(5Ad+Bc)\right)}{3b}+\frac{1}{4}dx\left(a+bx^2\right)^{3/2}}{5b}$$

$$\frac{B\left(a+bx^2\right)^{3/2}(c+dx)^2}{5b}$$

$$\Big\downarrow \textcolor{blue}{224}$$

$$\frac{\frac{5}{4}\left(-aAd^2-2aBcd+4Abc^2\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{2\left(a+bx^2\right)^{3/2}\left(aBd^2-bc(5Ad+Bc)\right)}{3b}+\frac{1}{4}dx\left(a+\right.}{5b}$$

$$\frac{B\left(a+bx^2\right)^{3/2}(c+dx)^2}{5b}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$\frac{\frac{5}{4}\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(-aAd^2-2aBcd+4Abc^2\right)-\frac{2\left(a+bx^2\right)^{3/2}\left(aBd^2-bc(5Ad+Bc)\right)}{3b}+\frac{1}{4}dx\left(a+bx\right.}{5b}$$

$$\frac{B\left(a+bx^2\right)^{3/2}(c+dx)^2}{5b}$$

input
```
Int[(A + B*x)*(c + d*x)^2*Sqrt[a + b*x^2],x]
```

output
```
(B*(c + d*x)^2*(a + b*x^2)^(3/2))/(5*b) + ((-2*(a*B*d^2 - b*c*(B*c + 5*A*d
))*(a + b*x^2)^(3/2))/(3*b) + (d*(2*B*c + 5*A*d)*x*(a + b*x^2)^(3/2))/4 +
(5*(4*A*b*c^2 - 2*a*B*c*d - a*A*d^2)*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4)/(5*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.99

method	result
default	$A\,c^{2}\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+d(Ad+2Bc)\left(\frac{x(b\,x^{2}+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)$
risch	$\frac{(24B\,d^{2}b^{2}x^{4}+30A\,b^{2}d^{2}x^{3}+60B\,b^{2}cd\,x^{3}+80A\,b^{2}cd\,x^{2}+8Bab\,d^{2}x^{2}+40c^{2}x^{2}B\,b^{2}+15Aa\,d^{2}xb+60A\,b^{2}c^{2}x+30Bacdxb+80Aabcd-16}{120b^{2}}$

input
```
int((B*x+A)*(d*x+c)^2*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*c^2*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+
d*(A*d+2*B*c)*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*
a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+1/3*c*(2*A*d+B*c)*(b*x^2+a)^(3/2
)/b+B*d^2*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.19

$$\int (A+Bx)(c+dx)^2\sqrt{a+bx^2}\,dx$$

$$=\left[-\frac{15\,(4\,Aabc^2-2\,Ba^2cd-Aa^2d^2)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(24\,Bb^2d^2x^4+40\,Babc^2}{}\right.$$

$$-\frac{15\,(4\,Aabc^2-2\,Ba^2cd-Aa^2d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(24\,Bb^2d^2x^4+40\,Babc^2+80\,Aabcd-16\,Ba}{}$$

input
```
integrate((B*x+A)*(d*x+c)^2*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/240*(15*(4*A*a*b*c^2 - 2*B*a^2*c*d - A*a^2*d^2)*sqrt(b)*log(-2*b*x^2 +
 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(24*B*b^2*d^2*x^4 + 40*B*a*b*c^2 + 8
0*A*a*b*c*d - 16*B*a^2*d^2 + 30*(2*B*b^2*c*d + A*b^2*d^2)*x^3 + 8*(5*B*b^2
*c^2 + 10*A*b^2*c*d + B*a*b*d^2)*x^2 + 15*(4*A*b^2*c^2 + 2*B*a*b*c*d + A*a
*b*d^2)*x)*sqrt(b*x^2 + a))/b^2, -1/120*(15*(4*A*a*b*c^2 - 2*B*a^2*c*d - A
*a^2*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (24*B*b^2*d^2*x^4
+ 40*B*a*b*c^2 + 80*A*a*b*c*d - 16*B*a^2*d^2 + 30*(2*B*b^2*c*d + A*b^2*d^2
)*x^3 + 8*(5*B*b^2*c^2 + 10*A*b^2*c*d + B*a*b*d^2)*x^2 + 15*(4*A*b^2*c^2 +
 2*B*a*b*c*d + A*a*b*d^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.89

$$\int (A+Bx)(c+dx)^2\sqrt{a+bx^2}\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{Bd^2x^4}{5}+\frac{x^3(Abd^2+2Bbcd)}{4b}+\frac{x^2\cdot\left(2Abcd+\frac{Bad^2}{5}+Bbc^2\right)}{3b}+\frac{x\left(Aad^2+Abc^2+2Bacd-\frac{3a\left(Abd^2+2Bbcd\right)}{4b}\right)}{2b}+\frac{2Aacd+B\right. \\ \sqrt{a}\left(Ac^2x+\frac{Bd^2x^4}{4}+\frac{x^3(Ad^2+2Bcd)}{3}+\frac{x^2\cdot(2Acd+Bc^2)}{2}\right) \end{cases}$$

input
```
integrate((B*x+A)*(d*x+c)**2*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*d**2*x**4/5 + x**3*(A*b*d**2 + 2*B*b*c*d)/(
4*b) + x**2*(2*A*b*c*d + B*a*d**2/5 + B*b*c**2)/(3*b) + x*(A*a*d**2 + A*b*
c**2 + 2*B*a*c*d - 3*a*(A*b*d**2 + 2*B*b*c*d)/(4*b))/(2*b) + (2*A*a*c*d +
B*a*c**2 - 2*a*(2*A*b*c*d + B*a*d**2/5 + B*b*c**2)/(3*b))/b) + (A*a*c**2 -
 a*(A*a*d**2 + A*b*c**2 + 2*B*a*c*d - 3*a*(A*b*d**2 + 2*B*b*c*d)/(4*b))/(2
*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0))
, (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(A*c**2*x + B*d**2*x
**4/4 + x**3*(A*d**2 + 2*B*c*d)/3 + x**2*(2*A*c*d + B*c**2)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.12

$$\int (A+Bx)(c+dx)^2\sqrt{a+bx^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Bd^2x^2}{5\,b} + \frac{1}{2}\,\sqrt{bx^2+a}Ac^2x + \frac{Aac^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}} + \frac{(bx^2+a)^{\frac{3}{2}}Bc^2}{3\,b} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Acd}{3\,b} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Bad^2}{15\,b^2} + \frac{(2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}x}{4\,b} - \frac{(2\,Bcd+Ad^2)\sqrt{bx^2+a}ax}{8\,b} - \frac{(2\,Bcd+Ad^2)a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^2*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a)^(3/2)*B*d^2*x^2/b + 1/2*sqrt(b*x^2 + a)*A*c^2*x + 1/2*A*a*
c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/3*(b*x^2 + a)^(3/2)*B*c^2/b + 2/3*(
b*x^2 + a)^(3/2)*A*c*d/b - 2/15*(b*x^2 + a)^(3/2)*B*a*d^2/b^2 + 1/4*(2*B*c
*d + A*d^2)*(b*x^2 + a)^(3/2)*x/b - 1/8*(2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*
a*x/b - 1/8*(2*B*c*d + A*d^2)*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.18

$$\int (A+Bx)(c+dx)^2\sqrt{a+bx^2}\,dx = \frac{1}{120}\,\sqrt{bx^2+a}\left(\left(2\left(3\left(4\,Bd^2x + \frac{5\,(2\,Bb^3cd+Ab^3d^2)}{b^3}\right)x + \frac{4\,(5\,Bb^3c^2+10\,Ab^3cd+Bab^2d^2)}{b^3}\right)x + \frac{15\,(\cdots}{} - \frac{(4\,Aabc^2-2\,Ba^2cd-Aa^2d^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^2*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/120*sqrt(b*x^2 + a)*((2*(3*(4*B*d^2*x + 5*(2*B*b^3*c*d + A*b^3*d^2)/b^3)
*x + 4*(5*B*b^3*c^2 + 10*A*b^3*c*d + B*a*b^2*d^2)/b^3)*x + 15*(4*A*b^3*c^2
 + 2*B*a*b^2*c*d + A*a*b^2*d^2)/b^3)*x + 8*(5*B*a*b^2*c^2 + 10*A*a*b^2*c*d
 - 2*B*a^2*b*d^2)/b^3) - 1/8*(4*A*a*b*c^2 - 2*B*a^2*c*d - A*a^2*d^2)*log(a
bs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)(c+dx)^2\sqrt{a+bx^2}\,dx=\int \sqrt{b\,x^2+a}\,(A+B\,x)\,\left(c+d\,x\right)^2\,dx$$

input
```
int((a + b*x^2)^(1/2)*(A + B*x)*(c + d*x)^2,x)
```

output
```
int((a + b*x^2)^(1/2)*(A + B*x)*(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.82

$$\int (A+Bx)(c+dx)^2\sqrt{a+bx^2}\,dx = \frac{80\sqrt{b\,x^2+a}\,a^2bcd+15\sqrt{b\,x^2+a}\,a^2b\,d^2x-16\sqrt{b\,x^2+a}\,a^2b\,d^2+60\sqrt{b\,x^2+a}\,a\,b^2c^2x+40\sqrt{b\,x^2+a}\,a\,b^2}{}$$

input
```
int((B*x+A)*(d*x+c)^2*(b*x^2+a)^(1/2),x)
```

output
```
(80*sqrt(a + b*x**2)*a**2*b*c*d + 15*sqrt(a + b*x**2)*a**2*b*d**2*x - 16*s
qrt(a + b*x**2)*a**2*b*d**2 + 60*sqrt(a + b*x**2)*a*b**2*c**2*x + 40*sqrt(
a + b*x**2)*a*b**2*c**2 + 80*sqrt(a + b*x**2)*a*b**2*c*d*x**2 + 30*sqrt(a
+ b*x**2)*a*b**2*c*d*x + 30*sqrt(a + b*x**2)*a*b**2*d**2*x**3 + 8*sqrt(a +
 b*x**2)*a*b**2*d**2*x**2 + 40*sqrt(a + b*x**2)*b**3*c**2*x**2 + 60*sqrt(a
 + b*x**2)*b**3*c*d*x**3 + 24*sqrt(a + b*x**2)*b**3*d**2*x**4 - 15*sqrt(b)
*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*d**2 + 60*sqrt(b)*log((s
qrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*c**2 - 30*sqrt(b)*log((sqrt(a
 + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*c*d)/(120*b**2)
```

3.150 $\int (A+Bx)(c+dx)\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int (A+Bx)(c+dx)\sqrt{a+bx^2}\,dx = \frac{(4Abc-aBd)x\sqrt{a+bx^2}}{8b} + \frac{(4(Bc+Ad)+3Bdx)\left(a+bx^2\right)^{3/2}}{12b} + \frac{a(4Abc-aBd)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output
```
1/8*(4*A*b*c-B*a*d)*x*(b*x^2+a)^(1/2)/b+1/12*(3*B*d*x+4*A*d+4*B*c)*(b*x^2+
a)^(3/2)/b+1/8*a*(4*A*b*c-B*a*d)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2
)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int (A+Bx)(c+dx)\sqrt{a+bx^2}\,dx = \frac{\sqrt{b}\sqrt{a+bx^2}(a(8Bc+8Ad+3Bdx)+2bx(Bx(4c+3dx)+A(6c+4dx)))+3a(-4Abc+aBd)\log\left(-\right.}{24b^{3/2}}$$

input
```
Integrate[(A + B*x)*(c + d*x)*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(a*(8*B*c + 8*A*d + 3*B*d*x) + 2*b*x*(B*x*(4*c +
3*d*x) + A*(6*c + 4*d*x))) + 3*a*(-4*A*b*c + a*B*d)*Log[-(Sqrt[b]*x) + Sqr
t[a + b*x^2]])/(24*b^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(A+Bx)(c+dx)\,dx$$

$$\Big\downarrow \text{676}$$

$$\frac{(4Abc-aBd)\int \sqrt{bx^2+a}dx}{4b}+\frac{\left(a+bx^2\right)^{3/2}(Ad+Bc)}{3b}+\frac{Bdx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{211}$$

$$\frac{(4Abc-aBd)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{\left(a+bx^2\right)^{3/2}(Ad+Bc)}{3b}+\frac{Bdx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{224}$$

$$\frac{(4Abc-aBd)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{\left(a+bx^2\right)^{3/2}(Ad+Bc)}{3b}+\frac{Bdx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(4Abc-aBd)}{4b}+\frac{\left(a+bx^2\right)^{3/2}(Ad+Bc)}{3b}+\frac{Bdx\left(a+bx^2\right)^{3/2}}{4b}$$

input
```
Int[(A + B*x)*(c + d*x)*Sqrt[a + b*x^2],x]
```

output
```
((B*c + A*d)*(a + b*x^2)^(3/2))/(3*b) + (B*d*x*(a + b*x^2)^(3/2))/(4*b) +
((4*A*b*c - a*B*d)*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(2*Sqrt[b])))/(4*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

method	result
risch	$\frac{(6Bbd\,x^3+8Abd\,x^2+8x^2Bbc+12Abcx+3Badx+8Aad+8aBc)\sqrt{b\,x^2+a}}{24b}+\frac{a(4Abc-Bad)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{3}{2}}}$
default	$Ac\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+\frac{(Ad+Bc)\left(b\,x^2+a\right)^{\frac{3}{2}}}{3b}+Bd\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2\right.}}{2\sqrt{b}}\right.}{4b}\right.$

input
```
int((B*x+A)*(d*x+c)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/b*(6*B*b*d*x^3+8*A*b*d*x^2+8*B*b*c*x^2+12*A*b*c*x+3*B*a*d*x+8*A*a*d+8
*B*a*c)*(b*x^2+a)^(1/2)+1/8*a*(4*A*b*c-B*a*d)/b^(3/2)*ln(b^(1/2)*x+(b*x^2+
a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.23

$$\int (A+Bx)(c+dx)\sqrt{a+bx^2}\,dx$$
$$=\left[-\frac{3\,(4\,Aabc-Ba^2d)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(6\,Bb^2dx^3+8\,Babc+8\,Aabd+8\,(Bb^2}{48\,b^2}\right.$$
$$-\frac{3\,(4\,Aabc-Ba^2d)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(6\,Bb^2dx^3+8\,Babc+8\,Aabd+8\,(Bb^2c+Ab^2d)x^2+3\,(4}{24\,b^2}$$

input
```
integrate((B*x+A)*(d*x+c)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(4*A*a*b*c - B*a^2*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(6*B*b^2*d*x^3 + 8*B*a*b*c + 8*A*a*b*d + 8*(B*b^2c + A*
b^2*d)*x^2 + 3*(4*A*b^2*c + B*a*b*d)*x)*sqrt(b*x^2 + a))/b^2, -1/24*(3*(4*
A*a*b*c - B*a^2*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*B*b^2*
d*x^3 + 8*B*a*b*c + 8*A*a*b*d + 8*(B*b^2*c + A*b^2*d)*x^2 + 3*(4*A*b^2*c +
 B*a*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.68

$$\int (A+Bx)(c+dx)\sqrt{a+bx^2}\,dx$$

$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{Bdx^3}{4}+\frac{x^2(Abd+Bbc)}{3b}+\frac{x\left(Abc+\frac{Bad}{4}\right)}{2b}+\frac{Aad+Bac-\frac{2a(Abd+Bbc)}{3b}}{b}\right)+\left(Aac-\frac{a\left(Abc+\frac{Bad}{4}\right)}{2b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\right.}{} \\ \frac{x\log(x)}{\sqrt{bx^2}}\end{cases}\right. \\ \sqrt{a}\left(Acx+\frac{Bdx^3}{3}+\frac{x^2(Ad+Bc)}{2}\right) \end{cases}$$

input
```
integrate((B*x+A)*(d*x+c)*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*d*x**3/4 + x**2*(A*b*d + B*b*c)/(3*b) + x*(
A*b*c + B*a*d/4)/(2*b) + (A*a*d + B*a*c - 2*a*(A*b*d + B*b*c)/(3*b))/b) +
(A*a*c - a*(A*b*c + B*a*d/4)/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x*
*2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0))
, (sqrt(a)*(A*c*x + B*d*x**3/3 + x**2*(A*d + B*c)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13

$$\int (A+Bx)(c+dx)\sqrt{a+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}Acx + \frac{(bx^2+a)^{\frac{3}{2}}Bdx}{4\,b} - \frac{\sqrt{bx^2+a}Badx}{8\,b} + \frac{Aac\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} - \frac{Ba^2d\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{(bx^2+a)^{\frac{3}{2}}Bc}{3\,b} + \frac{(bx^2+a)^{\frac{3}{2}}Ad}{3\,b}$$

input
```
integrate((B*x+A)*(d*x+c)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*A*c*x + 1/4*(b*x^2 + a)^(3/2)*B*d*x/b - 1/8*sqrt(b*x^2
 + a)*B*a*d*x/b + 1/2*A*a*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/8*B*a^2*d*a
rcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/3*(b*x^2 + a)^(3/2)*B*c/b + 1/3*(b*x^2 +
 a)^(3/2)*A*d/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

$$\int (A+Bx)(c+dx)\sqrt{a+bx^2}\,dx = \frac{1}{24}\sqrt{bx^2+a}\left(\left(2\left(3\,Bdx + \frac{4\,(Bb^2c+Ab^2d)}{b^2}\right)x + \frac{3\,(4\,Ab^2c+Babd)}{b^2}\right)x + \frac{8\,(Babc+Aabd)}{b^2}\right) - \frac{(4\,Aabc-Ba^2d)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a)*((2*(3*B*d*x + 4*(B*b^2*c + A*b^2*d)/b^2)*x + 3*(4*A*
b^2*c + B*a*b*d)/b^2)*x + 8*(B*a*b*c + A*a*b*d)/b^2) - 1/8*(4*A*a*b*c - B*
a^2*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)(c+dx)\sqrt{a+bx^2}\,dx=\int \sqrt{b\,x^2+a}\,(A+B\,x)\,\left(c+d\,x\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(A + B*x)*(c + d*x),x)
```

output
```
int((a + b*x^2)^(1/2)*(A + B*x)*(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.59

$$\int (A+Bx)(c+dx)\sqrt{a+bx^2}\,dx$$
$$=\frac{8\sqrt{b\,x^2+a}\,a^2d+12\sqrt{b\,x^2+a}\,abcx+8\sqrt{b\,x^2+a}\,abc+8\sqrt{b\,x^2+a}\,abd\,x^2+3\sqrt{b\,x^2+a}\,abdx+8\sqrt{b\,x^2+}}{24b}$$

input
```
int((B*x+A)*(d*x+c)*(b*x^2+a)^(1/2),x)
```

output
```
(8*sqrt(a + b*x**2)*a**2*d + 12*sqrt(a + b*x**2)*a*b*c*x + 8*sqrt(a + b*x*
*2)*a*b*c + 8*sqrt(a + b*x**2)*a*b*d*x**2 + 3*sqrt(a + b*x**2)*a*b*d*x + 8
*sqrt(a + b*x**2)*b**2*c*x**2 + 6*sqrt(a + b*x**2)*b**2*d*x**3 + 12*sqrt(b
)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c - 3*sqrt(b)*log((sqrt
(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*d)/(24*b)
```

3.151 $\int (A+Bx)\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B(a+bx^2)^{3/2}}{3b}+\frac{aA\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

output
```
1/2*A*x*(b*x^2+a)^(1/2)+1/3*B*(b*x^2+a)^(3/2)/b+1/2*a*A*arctanh(b^(1/2)*x/
(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \frac{\sqrt{a+bx^2}(2aB+3Abx+2bBx^2)}{6b}-\frac{aA\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

input
```
Integrate[(A + B*x)*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(2*a*B + 3*A*b*x + 2*b*B*x^2))/(6*b) - (a*A*Log[-(Sqrt[b]
*x) + Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(A+Bx)\,dx$$

$$\Big\downarrow \text{455}$$

$$A\int \sqrt{bx^2+a}dx+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{211}$$

$$A\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{224}$$

$$A\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{219}$$

$$A\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

input

```
Int[(A + B*x)*Sqrt[a + b*x^2],x]
```

output

```
(B*(a + b*x^2)^(3/2))/(3*b) + A*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

method	result	size
default	$A\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+\frac{B\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3b}$	54
risch	$\frac{\left(2Bb\,x^{2}+3Abx+2Ba\right)\sqrt{b\,x^{2}+a}}{6b}+\frac{Aa\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}$	56

input
```
int((B*x+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+1/3*
B*(b*x^2+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91

$$\int (A+Bx)\sqrt{a+bx^2}\,dx$$
$$=\left[\frac{3\,Aa\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,Bbx^2+3\,Abx+2\,Ba)\sqrt{bx^2+a}}{12\,b},\right.$$
$$\left.-\frac{3\,Aa\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Bbx^2+3\,Abx+2\,Ba)\sqrt{bx^2+a}}{6\,b}\right]$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*A*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
2*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2 + a))/b, -1/6*(3*A*a*sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2
+ a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int (A+Bx)\sqrt{a+bx^2}\,dx$$
$$=\begin{cases}\frac{Aa\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2}+\sqrt{a+bx^2}\left(\frac{Ax}{2}+\frac{Ba}{3b}+\frac{Bx^2}{3}\right) & \text{for } b\neq 0\\ \sqrt{a}\left(Ax+\frac{Bx^2}{2}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((B*x+A)*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((A*a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b),
 Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + sqrt(a + b*x**2)*(A*x/2 + B
*a/(3*b) + B*x**2/3), Ne(b, 0)), (sqrt(a)*(A*x + B*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

$$\int (A+Bx)\sqrt{a+bx^2}\,dx=\frac{1}{2}\sqrt{bx^2+a}Ax+\frac{Aa\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}+\frac{(bx^2+a)^{\frac{3}{2}}B}{3b}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*A*x + 1/2*A*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/3*(b*
x^2 + a)^(3/2)*B/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int (A+Bx)\sqrt{a+bx^2}\,dx=-\frac{Aa\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}+\frac{1}{6}\sqrt{bx^2+a}\left((2Bx+3A)x+\frac{2Ba}{b}\right)$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*A*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/6*sqrt(b*x^2 +
 a)*((2*B*x + 3*A)*x + 2*B*a/b)
```

Mupad [B] (verification not implemented)

Time = 6.58 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \frac{B\,{\left(b\,x^2+a\right)}^{3/2}}{3\,b}+\frac{A\,x\,\sqrt{b\,x^2+a}}{2}+\frac{A\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\,\sqrt{b}}$$

input
```
int((a + b*x^2)^(1/2)*(A + B*x),x)
```

output
```
(B*(a + b*x^2)^(3/2))/(3*b) + (A*x*(a + b*x^2)^(1/2))/2 + (A*a*log(b^(1/2)
*x + (a + b*x^2)^(1/2)))/(2*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \frac{3\sqrt{b\,x^2+a}\,abx+2\sqrt{b\,x^2+a}\,ab+2\sqrt{b\,x^2+a}\,b^2x^2+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2}{6b}$$

input
```
int((B*x+A)*(b*x^2+a)^(1/2),x)
```

output
```
(3*sqrt(a + b*x**2)*a*b*x + 2*sqrt(a + b*x**2)*a*b + 2*sqrt(a + b*x**2)*b*
*2*x**2 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2)/(6*b
)
```

3.152 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{c+dx}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{c+dx}\,dx=-\frac{(2(Bc-Ad)-Bdx)\sqrt{a+bx^2}}{2d^2}+\frac{(aBd^2+2bc(Bc-Ad))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}d^3}+\frac{(Bc-Ad)\sqrt{bc^2+ad^2}\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^3}$$

output

```
-1/2*(-B*d*x-2*A*d+2*B*c)*(b*x^2+a)^(1/2)/d^2+1/2*(a*B*d^2+2*b*c*(-A*d+B*c
))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d^3+(-A*d+B*c)*(a*d^2+b*c^2)
^(1/2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^3
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{c+dx}\,dx$$
$$= \frac{d(-2Bc+2Ad+Bdx)\sqrt{a+bx^2}-4(Bc-Ad)\sqrt{-bc^2-ad^2}\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)-\frac{(aBd^2+2bc(Bc-\ldots}{\ldots}}{2d^3}$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x^2])/(c + d*x),x]
```

output
```
(d*(-2*B*c + 2*A*d + B*d*x)*Sqrt[a + b*x^2] - 4*(B*c - A*d)*Sqrt[-(b*c^2)
- a*d^2]*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*
d^2]] - ((a*B*d^2 + 2*b*c*(B*c - A*d))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
)/Sqrt[b])/(2*d^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {682, 25, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{ 682}$$

$$\frac{\int -\frac{b(ad(Bc-2Ad)-(aBd^2+2bc(Bc-Ad))x)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}-\frac{\sqrt{a+bx^2}(2(Bc-Ad)-Bdx)}{2d^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{b(ad(Bc-2Ad)-(aBd^2+2bc(Bc-Ad))x)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}-\frac{\sqrt{a+bx^2}(2(Bc-Ad)-Bdx)}{2d^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{ad(Bc-2Ad)-(aBd^2+2bc(Bc-Ad))x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}-\frac{\sqrt{a+bx^2}(2(Bc-Ad)-Bdx)}{2d^2}$$

$$\Big\downarrow \text{719}$$

$$-\frac{\frac{2(ad^2+bc^2)(Bc-Ad)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{(aBd^2+2bc(Bc-Ad))\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}-\frac{\sqrt{a+bx^2}(2(Bc-Ad)-Bdx)}{2d^2}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{2(ad^2+bc^2)(Bc-Ad)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{(aBd^2+2bc(Bc-Ad))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}-\frac{\sqrt{a+bx^2}(2(Bc-Ad)-Bdx)}{2d^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{2(ad^2+bc^2)(Bc-Ad)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aBd^2+2bc(Bc-Ad))}{\sqrt{b}d}}{2d^2}-\frac{\sqrt{a+bx^2}(2(Bc-Ad)-Bdx)}{2d^2}$$

$$\Big\downarrow \text{488}$$

$$-\frac{-\frac{2(ad^2+bc^2)(Bc-Ad)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aBd^2+2bc(Bc-Ad))}{\sqrt{b}d}}{2d^2}-\frac{\sqrt{a+bx^2}(2(Bc-Ad)-Bdx)}{2d^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{2\sqrt{ad^2+bc^2}(Bc-Ad)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aBd^2+2bc(Bc-Ad))}{\sqrt{b}d}}{2d^2}-\frac{\sqrt{a+bx^2}(2(Bc-Ad)-Bdx)}{2d^2}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x^2])/(c + d*x),x]
```

output
```
-1/2*((2*(B*c - A*d) - B*d*x)*Sqrt[a + b*x^2])/d^2 - (-(((a*B*d^2 + 2*b*c*
(B*c - A*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d)) - (2*(B*c
- A*d)*Sqrt[b*c^2 + a*d^2]*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt
[a + b*x^2])])/d)/(2*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.57

method	result
risch	$\frac{(Bdx+2Ad-2Bc)\sqrt{b\,x^2+a}}{2d^2}-\frac{\frac{\left(2Abcd-aB\,d^2-2Bb\,c^2\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{d\sqrt{b}}+\frac{2\left(A\,d^3a+Ab\,c^2d-aBc\,d^2-bB\,c^3\right)\ln\left(\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\right.}{d}\right.}{d^2\sqrt{\frac{a\,d^2}{}}}}{2d^2}$
default	$\frac{B\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{d}+\frac{(Ad-Bc)\left(\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}-\frac{\sqrt{b}\,c\ln\left(\frac{-\frac{bc}{d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(\right.}{}}\right.}{d}\right.}{}$

input
```
int((B*x+A)*(b*x^2+a)^(1/2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*(B*d*x+2*A*d-2*B*c)*(b*x^2+a)^(1/2)/d^2-1/2/d^2*((2*A*b*c*d-B*a*d^2-2*
B*b*c^2)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+2*(A*a*d^3+A*b*c^2*d-B*a*
c*d^2-B*b*c^3)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c
/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2
+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [A] (verification not implemented)

Time = 39.35 (sec) , antiderivative size = 784, normalized size of antiderivative = 5.26

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/(d*x+c),x, algorithm="fricas")
```

output
```
[1/4*((2*B*b*c^2 - 2*A*b*c*d + B*a*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^
2 + a)*sqrt(b)*x - a) - 2*(B*b*c - A*b*d)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c
*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*
d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(B*b*d^
2*x - 2*B*b*c*d + 2*A*b*d^2)*sqrt(b*x^2 + a))/(b*d^3), -1/2*((2*B*b*c^2 -
2*A*b*c*d + B*a*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (B*b*c
- A*b*d)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b
^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a
))/(d^2*x^2 + 2*c*d*x + c^2)) - (B*b*d^2*x - 2*B*b*c*d + 2*A*b*d^2)*sqrt(b
*x^2 + a))/(b*d^3), 1/4*(4*(B*b*c - A*b*d)*sqrt(-b*c^2 - a*d^2)*arctan(sqr
t(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*
c^2 + a*b*d^2)*x^2)) + (2*B*b*c^2 - 2*A*b*c*d + B*a*d^2)*sqrt(b)*log(-2*b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(B*b*d^2*x - 2*B*b*c*d + 2*A*b*
d^2)*sqrt(b*x^2 + a))/(b*d^3), 1/2*(2*(B*b*c - A*b*d)*sqrt(-b*c^2 - a*d^2)
*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*
d^2 + (b^2*c^2 + a*b*d^2)*x^2)) - (2*B*b*c^2 - 2*A*b*c*d + B*a*d^2)*sqrt(-
b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (B*b*d^2*x - 2*B*b*c*d + 2*A*b*d^2
)*sqrt(b*x^2 + a))/(b*d^3)]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{c+dx}\,dx = \int \frac{(A+Bx)\sqrt{a+bx^2}}{c+dx}\,dx$$

input
```
integrate((B*x+A)*(b*x**2+a)**(1/2)/(d*x+c),x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x**2)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{c+dx}\,dx = \frac{\sqrt{bx^2+a}Bx}{2\,d} + \frac{B\sqrt{b}c^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^3} - \frac{A\sqrt{b}c \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^2} + \frac{Ba \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}d} - \frac{B\sqrt{a+\frac{bc^2}{d^2}}c \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d^2} + \frac{A\sqrt{a+\frac{bc^2}{d^2}} \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d} - \frac{\sqrt{bx^2+a}Bc}{d^2} + \frac{\sqrt{bx^2+a}A}{d}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*B*x/d + B*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^3 - A*s
qrt(b)*c*arcsinh(b*x/sqrt(a*b))/d^2 + 1/2*B*a*arcsinh(b*x/sqrt(a*b))/(sqrt
(b)*d) - B*sqrt(a + b*c^2/d^2)*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) -
a*d/(sqrt(a*b)*abs(d*x + c)))/d^2 + A*sqrt(a + b*c^2/d^2)*arcsinh(b*c*x/(s
qrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d - sqrt(b*x^2 + a)
*B*c/d^2 + sqrt(b*x^2 + a)*A/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{c+dx}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(A+B\,x)}{c+d\,x}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x))/(c + d*x),x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x))/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 2189, normalized size of antiderivative = 14.69

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{c+dx}\,dx=\text{Too large to display}$$

input
```
int((B*x+A)*(b*x^2+a)^(1/2)/(d*x+c),x)
```

output
```
( - 2*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*
sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*c*d + 2*sqrt(b)*sqrt(2*
sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c**2)
*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c*
*2)*c - a*d**2 - 2*b*c**2))*b*c**2 - 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2
)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sq
rt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**2*d**3 - 2*sqrt(2*s
qrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)
*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c*
*2))*a*b*c**2*d + 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*
c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 +
 b*c**2)*c - a*d**2 - 2*b*c**2))*a*b*c*d**2 + 2*sqrt(2*sqrt(b)*sqrt(a*d**2
 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/
sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*b**2*c**3 - s
qrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2)*sqrt(a*
d**2 + b*c**2)*log( - sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*
b*c**2) + sqrt(a + b*x**2)*d + sqrt(b)*d*x)*a*c*d + sqrt(b)*sqrt(2*sqrt(b)
*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2)*sqrt(a*d**2 + b*c**2)*log( -
 sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2) + sqrt(a +...
```

3.153 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 150

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^2} dx = \frac{(2Bc-Ad+Bdx)\sqrt{a+bx^2}}{d^2(c+dx)} - \frac{\sqrt{b}(2Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3} - \frac{(aBd^2+bc(2Bc-Ad))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^3\sqrt{bc^2+ad^2}}$$

output
```
(B*d*x-A*d+2*B*c)*(b*x^2+a)^(1/2)/d^2/(d*x+c)-b^(1/2)*(-A*d+2*B*c)*arctanh
(b^(1/2)*x/(b*x^2+a)^(1/2))/d^3-(a*B*d^2+b*c*(-A*d+2*B*c))*arctanh((-b*c*x
+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^3/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^2}\,dx$$
$$= \frac{\frac{d(2Bc-Ad+Bdx)\sqrt{a+bx^2}}{c+dx} - \frac{2(aBd^2+bc(2Bc-Ad))\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}} + \sqrt{b}(2Bc-Ad)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{d^3}$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x^2])/(c + d*x)^2,x]
```

output
```
((d*(2*B*c - A*d + B*d*x)*Sqrt[a + b*x^2])/(c + d*x) - (2*(a*B*d^2 + b*c*(
2*B*c - A*d))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2)
 - a*d^2]])/Sqrt[-(b*c^2) - a*d^2] + Sqrt[b]*(2*B*c - A*d)*Log[-(Sqrt[b]*x
) + Sqrt[a + b*x^2]])/d^3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {681, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{681}$$
$$\frac{\sqrt{a+bx^2}(-Ad+2Bc+Bdx)}{d^2(c+dx)} - \frac{\int -\frac{2(aBd-b(2Bc-Ad)x)}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{aBd-b(2Bc-Ad)x}{(c+dx)\sqrt{bx^2+a}}dx}{d^2} + \frac{\sqrt{a+bx^2}(-Ad+2Bc+Bdx)}{d^2(c+dx)}$$

$$\Big\downarrow \ 719$$

$$\frac{\frac{(aBd^2+bc(2Bc-Ad))\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{b(2Bc-Ad)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{d^2}+\frac{\sqrt{a+bx^2}(-Ad+2Bc+Bdx)}{d^2(c+dx)}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{(aBd^2+bc(2Bc-Ad))\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{b(2Bc-Ad)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{d^2}+\frac{\sqrt{a+bx^2}(-Ad+2Bc+Bdx)}{d^2(c+dx)}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{(aBd^2+bc(2Bc-Ad))\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2Bc-Ad)}{d}}{d^2}+\frac{\sqrt{a+bx^2}(-Ad+2Bc+Bdx)}{d^2(c+dx)}$$

$$\Big\downarrow \ 488$$

$$\frac{-\frac{(aBd^2+bc(2Bc-Ad))\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2Bc-Ad)}{d}}{d^2}+\frac{\sqrt{a+bx^2}(-Ad+2Bc+Bdx)}{d^2(c+dx)}$$

$$\Big\downarrow \ 219$$

$$\frac{-\frac{(aBd^2+bc(2Bc-Ad))\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\sqrt{ad^2+bc^2}}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2Bc-Ad)}{d}}{d^2}+\frac{\sqrt{a+bx^2}(-Ad+2Bc+Bdx)}{d^2(c+dx)}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x^2])/(c + d*x)^2,x]
```

output
```
((2*B*c - A*d + B*d*x)*Sqrt[a + b*x^2])/(d^2*(c + d*x)) + (-((Sqrt[b]*(2*B
*c - A*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d) - ((a*B*d^2 + b*c*(2*B*
c - A*d))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d
*Sqrt[b*c^2 + a*d^2]))/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(136) = 272$.

Time = 1.36 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.94

method	result
risch	$\frac{B\sqrt{b\,x^2+a}}{d^2} + \frac{\frac{\sqrt{b}\,(Ad-2Bc)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{d} + \frac{\left(2Abcd-aB\,d^2-3Bb\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x\right.}{}}}{x+\frac{c}{d}}\right.}{d^2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}}}{}$
default	$\frac{B\left(\sqrt{b(x+\frac{c}{d})^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}-\frac{\sqrt{b}\,c\ln\left(\frac{-\frac{bc}{d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}\right)}{d}-\frac{\left(a\,d^2+b\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-}{}\right.}{}\right.}{d^2}$

input
```
int((B*x+A)*(b*x^2+a)^(1/2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
B/d^2*(b*x^2+a)^(1/2)+1/d^2*(b^(1/2)*(A*d-2*B*c)/d*ln(b^(1/2)*x+(b*x^2+a)^
(1/2))+1/d^2*(2*A*b*c*d-B*a*d^2-3*B*b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2
*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^
2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+1/d^3*(A*a*d^3+A*b*c^
2*d-B*a*c*d^2-B*b*c^3)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*
(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(
1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(
b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(137) = 274$.

Time = 64.39 (sec) , antiderivative size = 1369, normalized size of antiderivative = 9.13

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
[-1/2*((2*B*b*c^4 - A*b*c^3*d + 2*B*a*c^2*d^2 - A*a*c*d^3 + (2*B*b*c^3*d -
 A*b*c^2*d^2 + 2*B*a*c*d^3 - A*a*d^4)*x)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x
^2 + a)*sqrt(b)*x - a) - (2*B*b*c^3 - A*b*c^2*d + B*a*c*d^2 + (2*B*b*c^2*d
 - A*b*c*d^2 + B*a*d^3)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2
- 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a
*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(2*B*b*c^3*d - A*b*c^2
*d^2 + 2*B*a*c*d^3 - A*a*d^4 + (B*b*c^2*d^2 + B*a*d^4)*x)*sqrt(b*x^2 + a))
/(b*c^3*d^3 + a*c*d^5 + (b*c^2*d^4 + a*d^6)*x), -1/2*(2*(2*B*b*c^3 - A*b*c
^2*d + B*a*c*d^2 + (2*B*b*c^2*d - A*b*c*d^2 + B*a*d^3)*x)*sqrt(-b*c^2 - a*
d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 +
a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + (2*B*b*c^4 - A*b*c^3*d + 2*B*a*c^2*d
^2 - A*a*c*d^3 + (2*B*b*c^3*d - A*b*c^2*d^2 + 2*B*a*c*d^3 - A*a*d^4)*x)*sq
rt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(2*B*b*c^3*d - A
*b*c^2*d^2 + 2*B*a*c*d^3 - A*a*d^4 + (B*b*c^2*d^2 + B*a*d^4)*x)*sqrt(b*x^2
 + a))/(b*c^3*d^3 + a*c*d^5 + (b*c^2*d^4 + a*d^6)*x), 1/2*(2*(2*B*b*c^4 -
A*b*c^3*d + 2*B*a*c^2*d^2 - A*a*c*d^3 + (2*B*b*c^3*d - A*b*c^2*d^2 + 2*B*a
*c*d^3 - A*a*d^4)*x)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*B*b*
c^3 - A*b*c^2*d + B*a*c*d^2 + (2*B*b*c^2*d - A*b*c*d^2 + B*a*d^3)*x)*sqrt(
b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d
^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2...
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^2}\,dx = \int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^2}\,dx$$

input
```
integrate((B*x+A)*(b*x**2+a)**(1/2)/(d*x+c)**2,x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x**2)/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.77

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^2}\,dx = \frac{\sqrt{bx^2+a}Bc}{d^3x+cd^2} - \frac{\sqrt{bx^2+a}A}{d^2x+cd} - \frac{2B\sqrt{b}c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^3} + \frac{A\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^2} + \frac{Bbc^2\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^4} - \frac{Abc\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^3} + \frac{B\sqrt{a+\frac{bc^2}{d^2}}\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d^2} + \frac{\sqrt{bx^2+a}B}{d^2}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a)*B*c/(d^3*x + c*d^2) - sqrt(b*x^2 + a)*A/(d^2*x + c*d) - 2*
B*sqrt(b)*c*arcsinh(b*x/sqrt(a*b))/d^3 + A*sqrt(b)*arcsinh(b*x/sqrt(a*b))/
d^2 + B*b*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(
d*x + c)))/(sqrt(a + b*c^2/d^2)*d^4) - A*b*c*arcsinh(b*c*x/(sqrt(a*b)*abs(
d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^3) + B*sq
rt(a + b*c^2/d^2)*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*
abs(d*x + c)))/d^2 + sqrt(b*x^2 + a)*B/d^2
```

Giac [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(A+B\,x)}{(c+d\,x)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x))/(c + d*x)^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1018, normalized size of antiderivative = 6.79

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(b*x^2+a)^(1/2)/(d*x+c)^2,x)
```

output

```
(2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a*b*c**2*d + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*s
qrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c*d**2*x - 2*sqrt(a*d**2 + b*c**2)
*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c*d**2 -
 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a*b*d**3*x - 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*s
qrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**3 - 4*sqrt(a*d**2 + b*c**2)*lo
g( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**2*d*x -
 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**2*d - 2*sqrt(a*d**2 + b*c**2)
*log(c + d*x)*a*b*c*d**2*x + 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c*d*
*2 + 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*d**3*x + 4*sqrt(a*d**2 + b*c
**2)*log(c + d*x)*b**2*c**3 + 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**2*c*
*2*d*x - 2*sqrt(a + b*x**2)*a**2*d**4 - 2*sqrt(a + b*x**2)*a*b*c**2*d**2 +
 4*sqrt(a + b*x**2)*a*b*c*d**3 + 2*sqrt(a + b*x**2)*a*b*d**4*x + 4*sqrt(a
+ b*x**2)*b**2*c**3*d + 2*sqrt(a + b*x**2)*b**2*c**2*d**2*x - sqrt(b)*log(
sqrt(a + b*x**2) - sqrt(b)*x)*a**2*c*d**3 - sqrt(b)*log(sqrt(a + b*x**2) -
 sqrt(b)*x)*a**2*d**4*x - sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*a*b*c*
*3*d - sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*a*b*c**2*d**2*x + 2*sqrt(
b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*a*b*c**2*d**2 + 2*sqrt(b)*log(sqrt(a
+ b*x**2) - sqrt(b)*x)*a*b*c*d**3*x + 2*sqrt(b)*log(sqrt(a + b*x**2) - ...
```

3.154 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 191

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^3}\,dx = \frac{(Bc-Ad)(ad-bcx)\sqrt{a+bx^2}}{2d\left(bc^2+ad^2\right)(c+dx)^2} - \frac{B\sqrt{a+bx^2}}{d^2(c+dx)} + \frac{\sqrt{b}B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3} + \frac{b(2bBc^3+ad^2(3Bc-Ad))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2d^3\left(bc^2+ad^2\right)^{3/2}}$$

output
```
1/2*(-A*d+B*c)*(-b*c*x+a*d)*(b*x^2+a)^(1/2)/d/(a*d^2+b*c^2)/(d*x+c)^2-B*(b
*x^2+a)^(1/2)/d^2/(d*x+c)+b^(1/2)*B*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^3
+1/2*b*(2*b*B*c^3+a*d^2*(-A*d+3*B*c))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(
1/2)/(b*x^2+a)^(1/2))/d^3/(a*d^2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^3}\,dx$$

$$= \frac{\frac{d\sqrt{a+bx^2}\left(-ad^2(Ad+B(c+2dx))+bc\left(Ad^2x-Bc(2c+3dx)\right)\right)}{(bc^2+ad^2)(c+dx)^2}-\frac{2b(2bBc^3+ad^2(3Bc-Ad))\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{(-bc^2-ad^2)^{3/2}}-2\sqrt{b}B\log\left(\cdots\right.}{2d^3}$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x^2])/(c + d*x)^3,x]
```

output
```
((d*Sqrt[a + b*x^2]*(-(a*d^2*(A*d + B*(c + 2*d*x))) + b*c*(A*d^2*x - B*c*(
2*c + 3*d*x))))/((b*c^2 + a*d^2)*(c + d*x)^2) - (2*b*(2*b*B*c^3 + a*d^2*(3
*B*c - A*d))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2)
- a*d^2]])/(-(b*c^2) - a*d^2)^(3/2) - 2*Sqrt[b]*B*Log[-(Sqrt[b]*x) + Sqrt[
a + b*x^2]])/(2*d^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {680, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{680}$$

$$-\frac{\int \frac{2b\left(ad(Bc-Ad)-2B\left(bc^2+ad^2\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{4d^2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(dx\left(2aBd^2+bc(3Bc-Ad)\right)+ad^2(Ad+Bc)+2bBc^3\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{ad(Bc-Ad)-2B\left(bc^2+ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(dx\left(2aBd^2+bc(3Bc-Ad)\right)+ad^2(Ad+Bc)+2bBc^3\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \textbf{719}$$

$$-\frac{b\left(\frac{\left(ad^2(3Bc-Ad)+2bBc^3\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{2B\left(ad^2+bc^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}\right)}{2d^2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(dx\left(2aBd^2+bc(3Bc-Ad)\right)+ad^2(Ad+Bc)+2bBc^3\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \textbf{224}$$

$$-\frac{b\left(\frac{\left(ad^2(3Bc-Ad)+2bBc^3\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{2B\left(ad^2+bc^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{2d^2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(dx\left(2aBd^2+bc(3Bc-Ad)\right)+ad^2(Ad+Bc)+2bBc^3\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{b\left(\frac{\left(ad^2(3Bc-Ad)+2bBc^3\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{2B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)}{\sqrt{b}d}\right)}{2d^2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(dx\left(2aBd^2+bc(3Bc-Ad)\right)+ad^2(Ad+Bc)+2bBc^3\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \textbf{488}$$

$$-\frac{b\left(-\frac{\left(ad^2(3Bc-Ad)+2bBc^3\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{2B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)}{\sqrt{b}d}\right)}{2d^2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(dx\left(2aBd^2+bc(3Bc-Ad)\right)+ad^2(Ad+Bc)+2bBc^3\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{b\left(-\frac{(ad^2(3Bc-Ad)+2bBc^3)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\sqrt{ad^2+bc^2}}-\frac{2B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad^2+bc^2)}{\sqrt{b}d}\right)}{2d^2\left(ad^2+bc^2\right)}-\frac{\sqrt{a+bx^2}\left(dx\left(2aBd^2+bc(3Bc-Ad)\right)+ad^2(Ad+Bc)+2bBc^3\right)}{2d^2(c+dx)^2\left(ad^2+bc^2\right)}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x^2])/(c + d*x)^3,x]
```

output
```
-1/2*((2*b*B*c^3 + a*d^2*(B*c + A*d) + d*(2*a*B*d^2 + b*c*(3*B*c - A*d))*x
)*Sqrt[a + b*x^2])/(d^2*(b*c^2 + a*d^2)*(c + d*x)^2) - (b*((-2*B*(b*c^2 +
a*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) - ((2*b*B*c^3 + a
*d^2*(3*B*c - A*d))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*
x^2])])/(d*Sqrt[b*c^2 + a*d^2])))/(2*d^2*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1444 vs. $2(171) = 342$.

Time = 1.48 (sec) , antiderivative size = 1445, normalized size of antiderivative = 7.57

method	result	size
default	Expression too large to display	1445

input
```
int((B*x+A)*(b*x^2+a)^(1/2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
B/d^3*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(3/2)-b*c*d/(a*d^2+b*c^2)*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b
*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*
b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d
^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1
/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+2*b/(
a*d^2+b*c^2)*d^2*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)
+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2
)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)
/d^2)^(1/2))))+(A*d-B*c)/d^4*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^
2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+1/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*
d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/
2)-b*c*d/(a*d^2+b*c^2)*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1
/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)
+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln(
(2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d
)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+2*b/(a*d^2+b*c^2)*
d^2*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2
)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+
b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^3}\,dx = \int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^3}\,dx$$

input
```
integrate((B*x+A)*(b*x**2+a)**(1/2)/(d*x+c)**3,x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x**2)/(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(172) = 344$.

Time = 0.10 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.87

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^3}\,dx = \frac{\sqrt{bx^2+a}Bbc^2}{2\left(bc^2d^3x+ad^5x+bc^3d^2+acd^4\right)} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}Bc}{2\left(bc^2d^2x^2+ad^4x^2+2\,bc^3dx+2\,acd^3x+bc^4+ac^2d^2\right)} - \frac{\sqrt{bx^2+a}Abc}{2\left(bc^2d^2x+ad^4x+bc^3d+acd^3\right)} - \frac{\sqrt{bx^2+a}Bbc}{2\left(bc^2d^2+ad^4\right)} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{2\left(bc^2dx^2+ad^3x^2+2\,bc^3x+2\,acd^2x+\frac{bc^4}{d}+ac^2d\right)} + \frac{\sqrt{bx^2+a}Ab}{2\left(bc^2d+ad^3\right)} - \frac{\sqrt{bx^2+a}B}{d^3x+cd^2} + \frac{B\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^3} + \frac{Bb^2c^3\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^6} - \frac{Ab^2c^2\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^5} - \frac{3\,Bbc\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\sqrt{a+\frac{bc^2}{d^2}}\,d^4} + \frac{Ab\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\sqrt{a+\frac{bc^2}{d^2}}\,d^3}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*B*b*c^2/(b*c^2*d^3*x + a*d^5*x + b*c^3*d^2 + a*c*d^4)
+ 1/2*(b*x^2 + a)^(3/2)*B*c/(b*c^2*d^2*x^2 + a*d^4*x^2 + 2*b*c^3*d*x + 2*a
*c*d^3*x + b*c^4 + a*c^2*d^2) - 1/2*sqrt(b*x^2 + a)*A*b*c/(b*c^2*d^2*x + a
*d^4*x + b*c^3*d + a*c*d^3) - 1/2*sqrt(b*x^2 + a)*B*b*c/(b*c^2*d^2 + a*d^4
) - 1/2*(b*x^2 + a)^(3/2)*A/(b*c^2*d*x^2 + a*d^3*x^2 + 2*b*c^3*x + 2*a*c*d
^2*x + b*c^4/d + a*c^2*d) + 1/2*sqrt(b*x^2 + a)*A*b/(b*c^2*d + a*d^3) - sq
rt(b*x^2 + a)*B/(d^3*x + c*d^2) + B*sqrt(b)*arcsinh(b*x/sqrt(a*b))/d^3 + 1
/2*B*b^2*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d
*x + c)))/((a + b*c^2/d^2)^(3/2)*d^6) - 1/2*A*b^2*c^2*arcsinh(b*c*x/(sqrt(
a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*
d^5) - 3/2*B*b*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*a
bs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^4) + 1/2*A*b*arcsinh(b*c*x/(sqrt(a*b)
*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(172) = 344$.

Time = 0.20 (sec) , antiderivative size = 617, normalized size of antiderivative = 3.23

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^3}\,dx$$

$$= -\frac{(2\,Bb^2c^3 + 3\,Babcd^2 - Aabd^3)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{(bc^2d^3+ad^5)\sqrt{-bc^2-ad^2}}$$

$$-\frac{B\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{d^3}$$

$$-\frac{4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Bb^2c^3d - 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab^2c^2d^2 + 3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Babcd^3 - \left(\sqrt{\right.}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-(2*B*b^2*c^3 + 3*B*a*b*c*d^2 - A*a*b*d^3)*arctan(-((sqrt(b)*x - sqrt(b*x^
2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b*c^2*d^3 + a*d^5)*sqrt(-b*
c^2 - a*d^2)) - B*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/d^3 - (4*
(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*b^2*c^3*d - 2*(sqrt(b)*x - sqrt(b*x^2 +
a))^3*A*b^2*c^2*d^2 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a*b*c*d^3 - (sqr
t(b)*x - sqrt(b*x^2 + a))^3*A*a*b*d^4 + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*
B*b^(5/2)*c^4 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*b^(5/2)*c^3*d + (sqrt(
b)*x - sqrt(b*x^2 + a))^2*B*a*b^(3/2)*c^2*d^2 + (sqrt(b)*x - sqrt(b*x^2 +
a))^2*A*a*b^(3/2)*c*d^3 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b)*
d^4 - 8*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a*b^2*c^3*d + 2*(sqrt(b)*x - sqrt(
b*x^2 + a))*A*a*b^2*c^2*d^2 - 5*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^2*b*c*d^
3 - (sqrt(b)*x - sqrt(b*x^2 + a))*A*a^2*b*d^4 + 3*B*a^2*b^(3/2)*c^2*d^2 -
A*a^2*b^(3/2)*c*d^3 + 2*B*a^3*sqrt(b)*d^4)/((b*c^2*d^3 + a*d^5)*((sqrt(b)*
x - sqrt(b*x^2 + a))^2*d + 2*(sqrt(b)*x - sqrt(b*x^2 + a))*sqrt(b)*c - a*d
)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^3}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(A+B\,x)}{(c+d\,x)^3}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x))/(c + d*x)^3,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x))/(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1555, normalized size of antiderivative = 8.14

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(b*x^2+a)^(1/2)/(d*x+c)^3,x)
```

output

```
(sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
b*c*x)*a**2*b*c**2*d**3 + 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqr
t(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c*d**4*x + sqrt(a*d**2 + b*c**2)*
log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*d**5*x**2
 - 3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*
d + b*c*x)*a*b**2*c**3*d**2 - 6*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)
*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**2*d**3*x - 3*sqrt(a*d**2 +
 b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*
c*d**4*x**2 - 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b
*c**2) - a*d + b*c*x)*b**3*c**5 - 4*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**4*d*x - 2*sqrt(a*d**2 +
b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**
3*d**2*x**2 - sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c**2*d**3 - 2*sqrt
(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c*d**4*x - sqrt(a*d**2 + b*c**2)*log
(c + d*x)*a**2*b*d**5*x**2 + 3*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c
**3*d**2 + 6*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**2*d**3*x + 3*sqr
t(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c*d**4*x**2 + 2*sqrt(a*d**2 + b*c**
2)*log(c + d*x)*b**3*c**5 + 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**3*c**4
*d*x + 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**3*c**3*d**2*x**2 - sqrt(a +
 b*x**2)*a**3*d**6 - sqrt(a + b*x**2)*a**2*b*c**2*d**4 + sqrt(a + b*x**...
```

3.155 $\int (A+Bx)(c+dx)^3 \left(a+bx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 317

$$\int (A+Bx)(c+dx)^3 \left(a+bx^2\right)^{3/2} \, dx = \frac{3a(8Abc(2bc^2-ad^2)-aBd(8bc^2-ad^2))\,x\sqrt{a+bx^2}}{128b^2} + \frac{(8Abc(2bc^2-ad^2)-aBd(8bc^2-ad^2))\,x(a+bx^2)^{3/2}}{64b^2} + \frac{(3Bc+8Ad)(c+dx)^2\left(a+bx^2\right)^{5/2}}{56b} + \frac{B(c+dx)^3\left(a+bx^2\right)^{5/2}}{8b} - \frac{(4(8ad^2(3Bc+Ad)-bc^2(3Bc+64Ad))+5d(7aBd^2-2bc(Bc+12Ad))\,x)\left(a+bx^2\right)^{5/2}}{560b^2} + \frac{3a^2(8Abc(2bc^2-ad^2)-aBd(8bc^2-ad^2))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

output

```
3/128*a*(8*A*b*c*(-a*d^2+2*b*c^2)-a*B*d*(-a*d^2+8*b*c^2))*x*(b*x^2+a)^(1/2
)/b^2+1/64*(8*A*b*c*(-a*d^2+2*b*c^2)-a*B*d*(-a*d^2+8*b*c^2))*x*(b*x^2+a)^(
3/2)/b^2+1/56*(8*A*d+3*B*c)*(d*x+c)^2*(b*x^2+a)^(5/2)/b+1/8*B*(d*x+c)^3*(b
*x^2+a)^(5/2)/b-1/560*(32*a*d^2*(A*d+3*B*c)-4*b*c^2*(64*A*d+3*B*c)+5*d*(7*
a*B*d^2-2*b*c*(12*A*d+B*c))*x)*(b*x^2+a)^(5/2)/b^2+3/128*a^2*(8*A*b*c*(-a*
d^2+2*b*c^2)-a*B*d*(-a*d^2+8*b*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^
(5/2)
```

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.02

$$\int (A+Bx)(c+dx)^3 \left(a+bx^2\right)^{3/2} dx = \frac{\sqrt{b}\sqrt{a+bx^2}(-a^3d^2(768Bc+256Ad+105Bdx)+16b^3x^3(2A(35c^3+84c^2dx+70cd^2x^2+2}{}$$

input
```
Integrate[(A + B*x)*(c + d*x)^3*(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(-(a^3*d^2*(768*B*c + 256*A*d + 105*B*d*x)) + 16*
b^3*x^3*(2*A*(35*c^3 + 84*c^2*d*x + 70*c*d^2*x^2 + 20*d^3*x^3) + B*x*(56*c
^3 + 140*c^2*d*x + 120*c*d^2*x^2 + 35*d^3*x^3)) + 2*a^2*b*(4*A*d*(336*c^2
+ 105*c*d*x + 16*d^2*x^2) + B*(448*c^3 + 420*c^2*d*x + 192*c*d^2*x^2 + 35*
d^3*x^3)) + 8*a*b^2*x*(2*A*(175*c^3 + 336*c^2*d*x + 245*c*d^2*x^2 + 64*d^3
*x^3) + B*x*(224*c^3 + 490*c^2*d*x + 384*c*d^2*x^2 + 105*d^3*x^3))) - 105*
a^2*(8*A*b*c*(2*b*c^2 - a*d^2) + a*B*d*(-8*b*c^2 + a*d^2))*Log[-(Sqrt[b]*x
) + Sqrt[a + b*x^2]])/(4480*b^(5/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {687, 687, 27, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} (A+Bx)(c+dx)^3 \, dx$$

$$\Big\downarrow \text{ 687}$$

$$\frac{\int (c+dx)^2(8Abc-3aBd+b(3Bc+8Ad)x) \left(bx^2+a\right)^{3/2} dx}{8b} + \frac{B\left(a+bx^2\right)^{5/2}(c+dx)^3}{8b}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{\frac{\int b(c+dx)\left(56Abc^2-27aBdc-16aAd^2-3\left(7aBd^2-2bc(Bc+12Ad)\right)x\right)\left(bx^2+a\right)^{3/2}dx}{7b}+\frac{1}{7}\left(a+bx^2\right)^{5/2}(c+dx)^2(8Ad+3Bc)}{8b}+\frac{B\left(a+bx^2\right)^{5/2}(c+dx)^3}{8b}$$

$\downarrow$ 27

$$\frac{\frac{1}{7}\int (c+dx)\left(56Abc^2-27aBdc-16aAd^2-3\left(7aBd^2-2bc(Bc+12Ad)\right)x\right)\left(bx^2+a\right)^{3/2}dx+\frac{1}{7}\left(a+bx^2\right)^{5/2}(c+\cdots}{8b}$$

$$\frac{B\left(a+bx^2\right)^{5/2}(c+dx)^3}{8b}$$

$\downarrow$ 676

$$\frac{\frac{1}{7}\left(\frac{7\left(8Abc\left(2bc^2-ad^2\right)-aBd\left(8bc^2-ad^2\right)\right)\int\left(bx^2+a\right)^{3/2}dx}{2b}-\frac{2\left(a+bx^2\right)^{5/2}\left(8ad^2(Ad+3Bc)-bc^2(64Ad+3Bc)\right)}{5b}-\frac{dx\left(a+bx^2\right)^{5/2}\left(7aBd^2-2bc\cdots\right.}{2b}\right.}{8b}$$

$$\frac{B\left(a+bx^2\right)^{5/2}(c+dx)^3}{8b}$$

$\downarrow$ 211

$$\frac{\frac{1}{7}\left(\frac{7\left(8Abc\left(2bc^2-ad^2\right)-aBd\left(8bc^2-ad^2\right)\right)\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}-\frac{2\left(a+bx^2\right)^{5/2}\left(8ad^2(Ad+3Bc)-bc^2(64Ad+3Bc)\right)}{5b}-\frac{dx(a+b\cdots}{\cdots}\right.}{8b}$$

$$\frac{B\left(a+bx^2\right)^{5/2}(c+dx)^3}{8b}$$

$\downarrow$ 211

$$\frac{\frac{1}{7}\left(\frac{7\left(8Abc\left(2bc^2-ad^2\right)-aBd\left(8bc^2-ad^2\right)\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}-\frac{2\left(a+bx^2\right)^{5/2}\left(8ad^2(Ad+3Bc)-bc^2(64A\cdots\right.}{5b}\right.}{8b}$$

$$\frac{B\left(a+bx^2\right)^{5/2}(c+dx)^3}{8b}$$

$\downarrow$ 224

$$\frac{\frac{1}{7}\left(\frac{7(8Abc(2bc^2-ad^2)-aBd(8bc^2-ad^2))\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{2b}-\frac{2(a+bx^2)^{5/2}(8ad^2(Ad+3Bc)-}{5b}\right.}{8b}$$

$$\frac{B(a+bx^2)^{5/2}(c+dx)^3}{8b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{7}\left(\frac{7\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)(8Abc(2bc^2-ad^2)-aBd(8bc^2-ad^2))}{2b}-\frac{2(a+bx^2)^{5/2}(8ad^2(Ad+3Bc)-b}{5b}\right.}{8b}$$

$$\frac{B(a+bx^2)^{5/2}(c+dx)^3}{8b}$$

input
```
Int[(A + B*x)*(c + d*x)^3*(a + b*x^2)^(3/2),x]
```

output
```
(B*(c + d*x)^3*(a + b*x^2)^(5/2))/(8*b) + (((3*B*c + 8*A*d)*(c + d*x)^2*(a
 + b*x^2)^(5/2))/7 + ((-2*(8*a*d^2*(3*B*c + A*d) - b*c^2*(3*B*c + 64*A*d))
*(a + b*x^2)^(5/2))/(5*b) - (d*(7*a*B*d^2 - 2*b*c*(B*c + 12*A*d))*x*(a + b
*x^2)^(5/2))/(2*b) + (7*(8*A*b*c*(2*b*c^2 - a*d^2) - a*B*d*(8*b*c^2 - a*d^
2))*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/(2*b))/7)/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.99

method	result
default	$A\,c^{3} \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b \,x^{2}+a}}{2}+\frac{a \ln \left(\sqrt{b}\, x +\sqrt{b \,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)+d^{2} \left(A d +3 B c \right) \left(\frac{x^{2} \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{7 b}-\frac{2 a \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{35 b^{2}}\right)+$
risch	$-\frac{\left(-560 B \,b^{3} d^{3} x^{7}-640 A \,b^{3} d^{3} x^{6}-1920 B \,b^{3} c \,d^{2} x^{6}-2240 A \,b^{3} c \,d^{2} x^{5}-840 B a \,b^{2} d^{3} x^{5}-2240 B \,b^{3} c^{2} d \,x^{5}-1024 A \,x^{4} a \,b^{2} d^{3}-2688 A \,b^{3} c^{}$

input
```
int((B*x+A)*(d*x+c)^3*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*c^3*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))))+d^2*(A*d+3*B*c)*(1/7*x^2*(b*x^2+a)^(5/2)/b-2
/35*a/b^2*(b*x^2+a)^(5/2))+3*c*d*(A*d+B*c)*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/
b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2)))))+1/5*c^2*(3*A*d+B*c)/b*(b*x^2+a)^(5/2)+B*d^3*(1/
8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b
*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x
^2+a)^(1/2))))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 804, normalized size of antiderivative = 2.54

$$\int (A+Bx)(c+dx)^3 \left(a+bx^2\right)^{3/2} \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(d*x+c)^3*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/8960*(105*(16*A*a^2*b^2*c^3 - 8*B*a^3*b*c^2*d - 8*A*a^3*b*c*d^2 + B*a^4
*d^3)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(560*B*b
^4*d^3*x^7 + 896*B*a^2*b^2*c^3 + 2688*A*a^2*b^2*c^2*d - 768*B*a^3*b*c*d^2
- 256*A*a^3*b*d^3 + 640*(3*B*b^4*c*d^2 + A*b^4*d^3)*x^6 + 280*(8*B*b^4*c^2
*d + 8*A*b^4*c*d^2 + 3*B*a*b^3*d^3)*x^5 + 128*(7*B*b^4*c^3 + 21*A*b^4*c^2*
d + 24*B*a*b^3*c*d^2 + 8*A*a*b^3*d^3)*x^4 + 70*(16*A*b^4*c^3 + 56*B*a*b^3*
c^2*d + 56*A*a*b^3*c*d^2 + B*a^2*b^2*d^3)*x^3 + 128*(14*B*a*b^3*c^3 + 42*A
*a*b^3*c^2*d + 3*B*a^2*b^2*c*d^2 + A*a^2*b^2*d^3)*x^2 + 35*(80*A*a*b^3*c^3
 + 24*B*a^2*b^2*c^2*d + 24*A*a^2*b^2*c*d^2 - 3*B*a^3*b*d^3)*x)*sqrt(b*x^2
+ a))/b^3, -1/4480*(105*(16*A*a^2*b^2*c^3 - 8*B*a^3*b*c^2*d - 8*A*a^3*b*c*
d^2 + B*a^4*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (560*B*b^4*
d^3*x^7 + 896*B*a^2*b^2*c^3 + 2688*A*a^2*b^2*c^2*d - 768*B*a^3*b*c*d^2 - 2
56*A*a^3*b*d^3 + 640*(3*B*b^4*c*d^2 + A*b^4*d^3)*x^6 + 280*(8*B*b^4*c^2*d
+ 8*A*b^4*c*d^2 + 3*B*a*b^3*d^3)*x^5 + 128*(7*B*b^4*c^3 + 21*A*b^4*c^2*d +
 24*B*a*b^3*c*d^2 + 8*A*a*b^3*d^3)*x^4 + 70*(16*A*b^4*c^3 + 56*B*a*b^3*c^2
*d + 56*A*a*b^3*c*d^2 + B*a^2*b^2*d^3)*x^3 + 128*(14*B*a*b^3*c^3 + 42*A*a*
b^3*c^2*d + 3*B*a^2*b^2*c*d^2 + A*a^2*b^2*d^3)*x^2 + 35*(80*A*a*b^3*c^3 +
24*B*a^2*b^2*c^2*d + 24*A*a^2*b^2*c*d^2 - 3*B*a^3*b*d^3)*x)*sqrt(b*x^2 + a
))/b^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(301) = 602$.

Time = 0.72 (sec) , antiderivative size = 921, normalized size of antiderivative = 2.91

$$\int (A+Bx)(c+dx)^3 \left(a+bx^2\right)^{3/2} \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(d*x+c)**3*(b*x**2+a)**(3/2),x)
```

output

```
Piecewise((sqrt(a + b*x**2)*(B*b*d**3*x**7/8 + x**6*(A*b**2*d**3 + 3*B*b**
2*c*d**2)/(7*b) + x**5*(3*A*b**2*c*d**2 + 9*B*a*b*d**3/8 + 3*B*b**2*c**2*d
)/(6*b) + x**4*(2*A*a*b*d**3 + 3*A*b**2*c**2*d + 6*B*a*b*c*d**2 + B*b**2*c
**3 - 6*a*(A*b**2*d**3 + 3*B*b**2*c*d**2)/(7*b))/(5*b) + x**3*(6*A*a*b*c*d
**2 + A*b**2*c**3 + B*a**2*d**3 + 6*B*a*b*c**2*d - 5*a*(3*A*b**2*c*d**2 +
9*B*a*b*d**3/8 + 3*B*b**2*c**2*d)/(6*b))/(4*b) + x**2*(A*a**2*d**3 + 6*A*a
*b*c**2*d + 3*B*a**2*c*d**2 + 2*B*a*b*c**3 - 4*a*(2*A*a*b*d**3 + 3*A*b**2*
c**2*d + 6*B*a*b*c*d**2 + B*b**2*c**3 - 6*a*(A*b**2*d**3 + 3*B*b**2*c*d**2
)/(7*b))/(5*b))/(3*b) + x*(3*A*a**2*c*d**2 + 2*A*a*b*c**3 + 3*B*a**2*c**2*
d - 3*a*(6*A*a*b*c*d**2 + A*b**2*c**3 + B*a**2*d**3 + 6*B*a*b*c**2*d - 5*a
*(3*A*b**2*c*d**2 + 9*B*a*b*d**3/8 + 3*B*b**2*c**2*d)/(6*b))/(4*b))/(2*b)
+ (3*A*a**2*c**2*d + B*a**2*c**3 - 2*a*(A*a**2*d**3 + 6*A*a*b*c**2*d + 3*B
*a**2*c*d**2 + 2*B*a*b*c**3 - 4*a*(2*A*a*b*d**3 + 3*A*b**2*c**2*d + 6*B*a*
b*c*d**2 + B*b**2*c**3 - 6*a*(A*b**2*d**3 + 3*B*b**2*c*d**2)/(7*b))/(5*b))
/(3*b))/b) + (A*a**2*c**3 - a*(3*A*a**2*c*d**2 + 2*A*a*b*c**3 + 3*B*a**2*c
**2*d - 3*a*(6*A*a*b*c*d**2 + A*b**2*c**3 + B*a**2*d**3 + 6*B*a*b*c**2*d -
 5*a*(3*A*b**2*c*d**2 + 9*B*a*b*d**3/8 + 3*B*b**2*c**2*d)/(6*b))/(4*b))/(2
*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0))
, (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(3/2)*(A*c**3*x + B*d**3*
x**5/5 + x**4*(A*d**3 + 3*B*c*d**2)/4 + x**3*(3*A*c*d**2 + 3*B*c**2*d)/...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.18

$$
\begin{aligned}
\int (A+Bx)(c+dx)^3\left(a+bx^2\right)^{3/2}\,dx &= \frac{(bx^2+a)^{\frac{5}{2}}Bd^3x^3}{8\,b} \\
&+ \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Ac^3x + \frac{3}{8}\sqrt{bx^2+a}Aac^3x - \frac{(bx^2+a)^{\frac{5}{2}}Bad^3x}{16\,b^2} \\
&+ \frac{(bx^2+a)^{\frac{3}{2}}Ba^2d^3x}{64\,b^2} + \frac{3\sqrt{bx^2+a}Ba^3d^3x}{128\,b^2} + \frac{3\,Aa^2c^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}} \\
&+ \frac{3\,Ba^4d^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}} + \frac{(bx^2+a)^{\frac{5}{2}}Bc^3}{5\,b} + \frac{3\,(bx^2+a)^{\frac{5}{2}}Ac^2d}{5\,b} \\
&+ \frac{(3\,Bcd^2+Ad^3)(bx^2+a)^{\frac{5}{2}}x^2}{7\,b} + \frac{(Bc^2d+Acd^2)(bx^2+a)^{\frac{5}{2}}x}{2\,b} \\
&- \frac{(Bc^2d+Acd^2)(bx^2+a)^{\frac{3}{2}}ax}{8\,b} - \frac{3\,(Bc^2d+Acd^2)\sqrt{bx^2+a}a^2x}{16\,b} \\
&- \frac{3\,(Bc^2d+Acd^2)a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} - \frac{2\,(3\,Bcd^2+Ad^3)(bx^2+a)^{\frac{5}{2}}a}{35\,b^2}
\end{aligned}
$$

input
```
integrate((B*x+A)*(d*x+c)^3*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(5/2)*B*d^3*x^3/b + 1/4*(b*x^2 + a)^(3/2)*A*c^3*x + 3/8*sq
rt(b*x^2 + a)*A*a*c^3*x - 1/16*(b*x^2 + a)^(5/2)*B*a*d^3*x/b^2 + 1/64*(b*x
^2 + a)^(3/2)*B*a^2*d^3*x/b^2 + 3/128*sqrt(b*x^2 + a)*B*a^3*d^3*x/b^2 + 3/
8*A*a^2*c^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 3/128*B*a^4*d^3*arcsinh(b*x/s
qrt(a*b))/b^(5/2) + 1/5*(b*x^2 + a)^(5/2)*B*c^3/b + 3/5*(b*x^2 + a)^(5/2)*
A*c^2*d/b + 1/7*(3*B*c*d^2 + A*d^3)*(b*x^2 + a)^(5/2)*x^2/b + 1/2*(B*c^2*d
 + A*c*d^2)*(b*x^2 + a)^(5/2)*x/b - 1/8*(B*c^2*d + A*c*d^2)*(b*x^2 + a)^(3
/2)*a*x/b - 3/16*(B*c^2*d + A*c*d^2)*sqrt(b*x^2 + a)*a^2*x/b - 3/16*(B*c^2
*d + A*c*d^2)*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/35*(3*B*c*d^2 + A*d^3
)*(b*x^2 + a)^(5/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.34

$$\int (A+Bx)(c+dx)^3 \left(a+bx^2\right)^{3/2} \, dx = \frac{1}{4480}\sqrt{bx^2+a}\left(\left(2\left(\left(\left(4\left(5\left(2\left(7\,Bbd^3x+\frac{8\,(3\,Bb^7cd^2+Ab^7d^3)}{b^6}\right)x+\frac{7\,(8\,Bb^7c^2d+8\,Ab^7\right.\right.\right.\right.\right.\right.\right.$$
$$-\frac{3\,(16\,Aa^2b^2c^3-8\,Ba^3bc^2d-8\,Aa^3bcd^2+Ba^4d^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^3*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/4480*sqrt(b*x^2 + a)*((2*((4*(5*(2*(7*B*b*d^3*x + 8*(3*B*b^7*c*d^2 + A*b
^7*d^3)/b^6)*x + 7*(8*B*b^7*c^2*d + 8*A*b^7*c*d^2 + 3*B*a*b^6*d^3)/b^6)*x
+ 16*(7*B*b^7*c^3 + 21*A*b^7*c^2*d + 24*B*a*b^6*c*d^2 + 8*A*a*b^6*d^3)/b^6
)*x + 35*(16*A*b^7*c^3 + 56*B*a*b^6*c^2*d + 56*A*a*b^6*c*d^2 + B*a^2*b^5*d
^3)/b^6)*x + 64*(14*B*a*b^6*c^3 + 42*A*a*b^6*c^2*d + 3*B*a^2*b^5*c*d^2 + A
*a^2*b^5*d^3)/b^6)*x + 35*(80*A*a*b^6*c^3 + 24*B*a^2*b^5*c^2*d + 24*A*a^2*
b^5*c*d^2 - 3*B*a^3*b^4*d^3)/b^6)*x + 128*(7*B*a^2*b^5*c^3 + 21*A*a^2*b^5*
c^2*d - 6*B*a^3*b^4*c*d^2 - 2*A*a^3*b^4*d^3)/b^6) - 3/128*(16*A*a^2*b^2*c^
3 - 8*B*a^3*b*c^2*d - 8*A*a^3*b*c*d^2 + B*a^4*d^3)*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)(c+dx)^3 \left(a+bx^2\right)^{3/2} \, dx = \int {\left(b\,x^2+a\right)}^{3/2}\,\left(A+B\,x\right)\,{\left(c+d\,x\right)}^3 \,d x$$

input
```
int((a + b*x^2)^(3/2)*(A + B*x)*(c + d*x)^3,x)
```

output
```
int((a + b*x^2)^(3/2)*(A + B*x)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (A+Bx)(c+dx)^3\left(a+bx^2\right)^{3/2}\,dx=\int (Bx+A)\,(dx+c)^3\left(b\,x^2+a\right)^{\frac{3}{2}}dx$$

input
```
int((B*x+A)*(d*x+c)^3*(b*x^2+a)^(3/2),x)
```

output
```
int((B*x+A)*(d*x+c)^3*(b*x^2+a)^(3/2),x)
```

3.156 $\int (A+Bx)(c+dx)^2 \left(a+bx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 214

$$\int (A+Bx)(c+dx)^2 \left(a+bx^2\right)^{3/2} \, dx = \frac{a(6Abc^2-2aBcd-aAd^2)\,x\sqrt{a+bx^2}}{16b} + \frac{(6Abc^2-2aBcd-aAd^2)\,x(a+bx^2)^{3/2}}{24b} + \frac{B(c+dx)^2\left(a+bx^2\right)^{5/2}}{7b} - \frac{(12(aBd^2-bc(Bc+7Ad))-5bd(2Bc+7Ad)x)\left(a+bx^2\right)^{5/2}}{210b^2} + \frac{a^2(6Abc^2-2aBcd-aAd^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output
```
1/16*a*(-A*a*d^2+6*A*b*c^2-2*B*a*c*d)*x*(b*x^2+a)^(1/2)/b+1/24*(-A*a*d^2+6
*A*b*c^2-2*B*a*c*d)*x*(b*x^2+a)^(3/2)/b+1/7*B*(d*x+c)^2*(b*x^2+a)^(5/2)/b-
1/210*(12*a*B*d^2-12*b*c*(7*A*d+B*c)-5*b*d*(7*A*d+2*B*c)*x)*(b*x^2+a)^(5/2
)/b^2+1/16*a^2*(-A*a*d^2+6*A*b*c^2-2*B*a*c*d)*arctanh(b^(1/2)*x/(b*x^2+a)^
(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.07

$$\int (A+Bx)(c+dx)^2 \left(a+bx^2\right)^{3/2} dx = \frac{\sqrt{a+bx^2}(-96a^3Bd^2+3a^2b(7Ad(32c+5dx)+2B(56c^2+35cdx+8d^2x^2))+4b^3x^3(7A(15$$

input
```
Integrate[(A + B*x)*(c + d*x)^2*(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[a + b*x^2]*(-96*a^3*B*d^2 + 3*a^2*b*(7*A*d*(32*c + 5*d*x) + 2*B*(56*
c^2 + 35*c*d*x + 8*d^2*x^2)) + 4*b^3*x^3*(7*A*(15*c^2 + 24*c*d*x + 10*d^2*
x^2) + 4*B*x*(21*c^2 + 35*c*d*x + 15*d^2*x^2)) + 2*a*b^2*x*(7*A*(75*c^2 +
96*c*d*x + 35*d^2*x^2) + 2*B*x*(168*c^2 + 245*c*d*x + 96*d^2*x^2))) + 105*
a^2*Sqrt[b]*(-6*A*b*c^2 + 2*a*B*c*d + a*A*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a +
 b*x^2]])/(1680*b^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {687, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} (A+Bx)(c+dx)^2 \, dx$$

$$\Big\downarrow \text{ 687}$$

$$\frac{\int (c+dx)(7Abc-2aBd+b(2Bc+7Ad)x)\left(bx^2+a\right)^{3/2} dx}{7b}+\frac{B\left(a+bx^2\right)^{5/2}(c+dx)^2}{7b}$$

$$\Big\downarrow \text{ 676}$$

$$\frac{\frac{7}{6}\left(-aAd^2-2aBcd+6Abc^2\right)\int \left(bx^2+a\right)^{3/2}dx-\frac{2\left(a+bx^2\right)^{5/2}\left(aBd^2-bc(7Ad+Bc)\right)}{5b}+\frac{1}{6}dx\left(a+bx^2\right)^{5/2}(7Ad+2Bc)}{7b}+$$

$$\frac{B\left(a+bx^2\right)^{5/2}(c+dx)^2}{7b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{7}{6}\left(-aAd^2-2aBcd+6Abc^2\right)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{2\left(a+bx^2\right)^{5/2}\left(aBd^2-bc(7Ad+Bc)\right)}{5b}+\frac{1}{6}dx\left(a+bx^2\right)}{7b}$$

$$\frac{B\left(a+bx^2\right)^{5/2}(c+dx)^2}{7b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{7}{6}\left(-aAd^2-2aBcd+6Abc^2\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{2\left(a+bx^2\right)^{5/2}\left(aBd^2-bc(7Ad+\right.}{5b}}{7b}$$

$$\frac{B\left(a+bx^2\right)^{5/2}(c+dx)^2}{7b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{7}{6}\left(-aAd^2-2aBcd+6Abc^2\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{2\left(a+bx^2\right)^{5/2}\left(aBd^2\right.}{5b}}{7b}$$

$$\frac{B\left(a+bx^2\right)^{5/2}(c+dx)^2}{7b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{7}{6}\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)\left(-aAd^2-2aBcd+6Abc^2\right)-\frac{2\left(a+bx^2\right)^{5/2}\left(aBd^2-\right.}{5b}}{7b}$$

$$\frac{B\left(a+bx^2\right)^{5/2}(c+dx)^2}{7b}$$

input
```
Int[(A + B*x)*(c + d*x)^2*(a + b*x^2)^(3/2),x]
```

output
```
(B*(c + d*x)^2*(a + b*x^2)^(5/2))/(7*b) + ((-2*(a*B*d^2 - b*c*(B*c + 7*A*d
))*(a + b*x^2)^(5/2))/(5*b) + (d*(2*B*c + 7*A*d)*x*(a + b*x^2)^(5/2))/6 +
(7*(6*A*b*c^2 - 2*a*B*c*d - a*A*d^2)*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*S
qrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))
/4))/6)/(7*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.94

method	result
default	$A c^2 \left(\frac{x(b x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+d(Ad+2Bc)\left(\frac{x(b x^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(b x^2+a)^{\frac{3}{2}}}{4}+\frac{3a}{}\right.}{}\right.$
risch	$\frac{(240B b^3 d^2 x^6+280A b^3 d^2 x^5+560B b^3 cd x^5+672A b^3 cd x^4+384Ba b^2 d^2 x^4+336B b^3 c^2 x^4+490Aa b^2 d^2 x^3+420A b^3 c^2 x^3+980Ba b^2}{}$

input
```
int((B*x+A)*(d*x+c)^2*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*c^2*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))))+d*(A*d+2*B*c)*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a
/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^
(1/2)*x+(b*x^2+a)^(1/2)))))+1/5*c*(2*A*d+B*c)/b*(b*x^2+a)^(5/2)+B*d^2*(1/7
*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 538, normalized size of antiderivative = 2.51

$$\int (A+Bx)(c+dx)^2 \left(a+bx^2\right)^{3/2} dx = \left[-\frac{105\left(6\,Aa^2bc^2-2\,Ba^3cd-Aa^3d^2\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(240\,Bb^3d}{} \right.$$

$$-\frac{105\left(6\,Aa^2bc^2-2\,Ba^3cd-Aa^3d^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(240\,Bb^3d^2x^6+336\,Ba^2bc^2+672\,Aa^2bcd-9\right.}{}$$

input
```
integrate((B*x+A)*(d*x+c)^2*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/3360*(105*(6*A*a^2*b*c^2 - 2*B*a^3*c*d - A*a^3*d^2)*sqrt(b)*log(-2*b*x
^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(240*B*b^3*d^2*x^6 + 336*B*a^2*b
*c^2 + 672*A*a^2*b*c*d - 96*B*a^3*d^2 + 280*(2*B*b^3*c*d + A*b^3*d^2)*x^5
+ 48*(7*B*b^3*c^2 + 14*A*b^3*c*d + 8*B*a*b^2*d^2)*x^4 + 70*(6*A*b^3*c^2 +
14*B*a*b^2*c*d + 7*A*a*b^2*d^2)*x^3 + 48*(14*B*a*b^2*c^2 + 28*A*a*b^2*c*d
+ B*a^2*b*d^2)*x^2 + 105*(10*A*a*b^2*c^2 + 2*B*a^2*b*c*d + A*a^2*b*d^2)*x)
*sqrt(b*x^2 + a))/b^2, -1/1680*(105*(6*A*a^2*b*c^2 - 2*B*a^3*c*d - A*a^3*d
^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (240*B*b^3*d^2*x^6 + 336
*B*a^2*b*c^2 + 672*A*a^2*b*c*d - 96*B*a^3*d^2 + 280*(2*B*b^3*c*d + A*b^3*d
^2)*x^5 + 48*(7*B*b^3*c^2 + 14*A*b^3*c*d + 8*B*a*b^2*d^2)*x^4 + 70*(6*A*b^
3*c^2 + 14*B*a*b^2*c*d + 7*A*a*b^2*d^2)*x^3 + 48*(14*B*a*b^2*c^2 + 28*A*a*
b^2*c*d + B*a^2*b*d^2)*x^2 + 105*(10*A*a*b^2*c^2 + 2*B*a^2*b*c*d + A*a^2*b
*d^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(209) = 418$.

Time = 0.78 (sec) , antiderivative size = 609, normalized size of antiderivative = 2.85

$$\int (A+Bx)(c+dx)^2 (a+bx^2)^{3/2}\, dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Bbd^2x^6}{7} + \frac{x^5(Ab^2d^2+2Bb^2cd)}{6b} + \frac{x^4\cdot\left(2Ab^2cd+\frac{8Babd^2}{7}+Bb^2c^2\right)}{5b} + \frac{x^3\cdot\left(2Aabd^2+Ab^2c^2+4Babcd-\frac{5a}{}\right.}{4b}\right. \\ a^{\frac{3}{2}}\left(Ac^2x + \frac{Bd^2x^4}{4} + \frac{x^3(Ad^2+2Bcd)}{3} + \frac{x^2\cdot(2Acd+Bc^2)}{2}\right) \end{cases}$$

input
```
integrate((B*x+A)*(d*x+c)**2*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*b*d**2*x**6/7 + x**5*(A*b**2*d**2 + 2*B*b**
2*c*d)/(6*b) + x**4*(2*A*b**2*c*d + 8*B*a*b*d**2/7 + B*b**2*c**2)/(5*b) +
x**3*(2*A*a*b*d**2 + A*b**2*c**2 + 4*B*a*b*c*d - 5*a*(A*b**2*d**2 + 2*B*b*
*2*c*d)/(6*b))/(4*b) + x**2*(4*A*a*b*c*d + B*a**2*d**2 + 2*B*a*b*c**2 - 4*
a*(2*A*b**2*c*d + 8*B*a*b*d**2/7 + B*b**2*c**2)/(5*b))/(3*b) + x*(A*a**2*d
**2 + 2*A*a*b*c**2 + 2*B*a**2*c*d - 3*a*(2*A*a*b*d**2 + A*b**2*c**2 + 4*B*
a*b*c*d - 5*a*(A*b**2*d**2 + 2*B*b**2*c*d)/(6*b))/(4*b))/(2*b) + (2*A*a**2
*c*d + B*a**2*c**2 - 2*a*(4*A*a*b*c*d + B*a**2*d**2 + 2*B*a*b*c**2 - 4*a*(
2*A*b**2*c*d + 8*B*a*b*d**2/7 + B*b**2*c**2)/(5*b))/(3*b))/b) + (A*a**2*c*
*2 - a*(A*a**2*d**2 + 2*A*a*b*c**2 + 2*B*a**2*c*d - 3*a*(2*A*a*b*d**2 + A*
b**2*c**2 + 4*B*a*b*c*d - 5*a*(A*b**2*d**2 + 2*B*b**2*c*d)/(6*b))/(4*b))/(
2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)
), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(3/2)*(A*c**2*x + B*d**2
*x**4/4 + x**3*(A*d**2 + 2*B*c*d)/3 + x**2*(2*A*c*d + B*c**2)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12

$$\int (A+Bx)(c+dx)^2\left(a+bx^2\right)^{3/2}\,dx = \frac{(bx^2+a)^{\frac{5}{2}}Bd^2x^2}{7\,b} + \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Ac^2x + \frac{3}{8}\sqrt{bx^2+a}Aac^2x + \frac{3\,Aa^2c^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}} + \frac{(bx^2+a)^{\frac{5}{2}}Bc^2}{5\,b} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Acd}{5\,b} - \frac{2\,(bx^2+a)^{\frac{5}{2}}Bad^2}{35\,b^2} + \frac{(2\,Bcd+Ad^2)(bx^2+a)^{\frac{5}{2}}x}{6\,b} - \frac{(2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}ax}{24\,b} - \frac{(2\,Bcd+Ad^2)\sqrt{bx^2+a}a^2x}{16\,b} - \frac{(2\,Bcd+Ad^2)a^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^2*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(5/2)*B*d^2*x^2/b + 1/4*(b*x^2 + a)^(3/2)*A*c^2*x + 3/8*sq
rt(b*x^2 + a)*A*a*c^2*x + 3/8*A*a^2*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1
/5*(b*x^2 + a)^(5/2)*B*c^2/b + 2/5*(b*x^2 + a)^(5/2)*A*c*d/b - 2/35*(b*x^2
 + a)^(5/2)*B*a*d^2/b^2 + 1/6*(2*B*c*d + A*d^2)*(b*x^2 + a)^(5/2)*x/b - 1/
24*(2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)*a*x/b - 1/16*(2*B*c*d + A*d^2)*sqrt
(b*x^2 + a)*a^2*x/b - 1/16*(2*B*c*d + A*d^2)*a^3*arcsinh(b*x/sqrt(a*b))/b^
(3/2)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.37

$$\int (A+Bx)(c+dx)^2\left(a+bx^2\right)^{3/2}\,dx=\frac{1}{1680}\sqrt{bx^2+a}\Bigg(\Bigg(2\Bigg(\Bigg(4\Bigg(5\Bigg(6\,Bbd^2x+\frac{7\,(2\,Bb^6cd+Ab^6d^2)}{b^5}\Bigg)x+\frac{6\,(7\,Bb^6c^2+14\,Ab^6cd+}{b^5}$$
$$-\frac{(6\,Aa^2bc^2-2\,Ba^3cd-Aa^3d^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^2*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/1680*sqrt(b*x^2 + a)*((2*((4*(5*(6*B*b*d^2*x + 7*(2*B*b^6*c*d + A*b^6*d^
2)/b^5)*x + 6*(7*B*b^6*c^2 + 14*A*b^6*c*d + 8*B*a*b^5*d^2)/b^5)*x + 35*(6*
A*b^6*c^2 + 14*B*a*b^5*c*d + 7*A*a*b^5*d^2)/b^5)*x + 24*(14*B*a*b^5*c^2 +
28*A*a*b^5*c*d + B*a^2*b^4*d^2)/b^5)*x + 105*(10*A*a*b^5*c^2 + 2*B*a^2*b^4
*c*d + A*a^2*b^4*d^2)/b^5)*x + 48*(7*B*a^2*b^4*c^2 + 14*A*a^2*b^4*c*d - 2*
B*a^3*b^3*d^2)/b^5) - 1/16*(6*A*a^2*b*c^2 - 2*B*a^3*c*d - A*a^3*d^2)*log(a
bs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)(c+dx)^2\left(a+bx^2\right)^{3/2}\,dx=\int \left(b\,x^2+a\right)^{3/2}(A+B\,x)\,\left(c+d\,x\right)^2dx$$

input
```
int((a + b*x^2)^(3/2)*(A + B*x)*(c + d*x)^2,x)
```

output
```
int((a + b*x^2)^(3/2)*(A + B*x)*(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.07

$$\int (A+Bx)(c+dx)^2\left(a+bx^2\right)^{3/2}\,dx=\frac{672\sqrt{b\,x^2+a}\,a^3bcd+105\sqrt{b\,x^2+a}\,a^3b\,d^2x-96\sqrt{b\,x^2+a}\,a^3b\,d^2+1050\sqrt{b\,x^2+a}\,a^2b^2c^2x+}{}$$

input
```
int((B*x+A)*(d*x+c)^2*(b*x^2+a)^(3/2),x)
```

output
```
(672*sqrt(a + b*x**2)*a**3*b*c*d + 105*sqrt(a + b*x**2)*a**3*b*d**2*x - 96
*sqrt(a + b*x**2)*a**3*b*d**2 + 1050*sqrt(a + b*x**2)*a**2*b**2*c**2*x + 3
36*sqrt(a + b*x**2)*a**2*b**2*c**2 + 1344*sqrt(a + b*x**2)*a**2*b**2*c*d*x
**2 + 210*sqrt(a + b*x**2)*a**2*b**2*c*d*x + 490*sqrt(a + b*x**2)*a**2*b**
2*d**2*x**3 + 48*sqrt(a + b*x**2)*a**2*b**2*d**2*x**2 + 420*sqrt(a + b*x**
2)*a*b**3*c**2*x**3 + 672*sqrt(a + b*x**2)*a*b**3*c**2*x**2 + 672*sqrt(a +
 b*x**2)*a*b**3*c*d*x**4 + 980*sqrt(a + b*x**2)*a*b**3*c*d*x**3 + 280*sqrt
(a + b*x**2)*a*b**3*d**2*x**5 + 384*sqrt(a + b*x**2)*a*b**3*d**2*x**4 + 33
6*sqrt(a + b*x**2)*b**4*c**2*x**4 + 560*sqrt(a + b*x**2)*b**4*c*d*x**5 + 2
40*sqrt(a + b*x**2)*b**4*d**2*x**6 - 105*sqrt(b)*log((sqrt(a + b*x**2) + s
qrt(b)*x)/sqrt(a))*a**4*d**2 + 630*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)
*x)/sqrt(a))*a**3*b*c**2 - 210*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/
sqrt(a))*a**3*b*c*d)/(1680*b**2)
```

3.157 $\int (A+Bx)(c+dx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 137

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{3/2}\,dx=\frac{a(6Abc-aBd)x\sqrt{a+bx^2}}{16b}+\frac{(6Abc-aBd)x(a+bx^2)^{3/2}}{24b}+\frac{(6(Bc+Ad)+5Bdx)\left(a+bx^2\right)^{5/2}}{30b}+\frac{a^2(6Abc-aBd)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output
```
1/16*a*(6*A*b*c-B*a*d)*x*(b*x^2+a)^(1/2)/b+1/24*(6*A*b*c-B*a*d)*x*(b*x^2+a
)^(3/2)/b+1/30*(5*B*d*x+6*A*d+6*B*c)*(b*x^2+a)^(5/2)/b+1/16*a^2*(6*A*b*c-B
*a*d)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{3/2}\,dx=\frac{\sqrt{b}\sqrt{a+bx^2}(3a^2(16Bc+16Ad+5Bdx)+4b^2x^3(3A(5c+4dx)+2Bx(6c+5dx))+2abx(}{240b^{3/2}}$$

input
```
Integrate[(A + B*x)*(c + d*x)*(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(3*a^2*(16*B*c + 16*A*d + 5*B*d*x) + 4*b^2*x^3*(3
*A*(5*c + 4*d*x) + 2*B*x*(6*c + 5*d*x)) + 2*a*b*x*(B*x*(48*c + 35*d*x) + A
*(75*c + 48*d*x))) + 15*a^2*(-6*A*b*c + a*B*d)*Log[-(Sqrt[b]*x) + Sqrt[a +
 b*x^2]])/(240*b^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2}(A+Bx)(c+dx)\,dx$$

$$\Big\downarrow \text{676}$$

$$\frac{(6Abc-aBd)\int \left(bx^2+a\right)^{3/2}dx}{6b}+\frac{\left(a+bx^2\right)^{5/2}(Ad+Bc)}{5b}+\frac{Bdx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{211}$$

$$\frac{(6Abc-aBd)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{\left(a+bx^2\right)^{5/2}(Ad+Bc)}{5b}+\frac{Bdx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{211}$$

$$\frac{(6Abc-aBd)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{\left(a+bx^2\right)^{5/2}(Ad+Bc)}{5b}+\frac{Bdx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{224}$$

$$\frac{(6Abc-aBd)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{\left(a+bx^2\right)^{5/2}(Ad+Bc)}{5b}+\frac{Bdx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)(6Abc-aBd)}{6b}+\frac{\left(a+bx^2\right)^{5/2}(Ad+Bc)}{5b}+\frac{Bdx\left(a+bx^2\right)^{5/2}}{6b}$$

input
```
Int[(A + B*x)*(c + d*x)*(a + b*x^2)^(3/2),x]
```

output
```
((B*c + A*d)*(a + b*x^2)^(5/2))/(5*b) + (B*d*x*(a + b*x^2)^(5/2))/(6*b) +
((6*A*b*c - a*B*d)*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2
+ (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/(6*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.08

method	result
risch	$\frac{(40B\,b^2 d\,x^5+48A\,b^2 d\,x^4+48x^4 B\,b^2 c+60A\,b^2 c\,x^3+70aBbd\,x^3+96Aabd\,x^2+96Babc\,x^2+150Aabcx+15B\,a^2 dx+48a^2 Ad+48B\,a^2 c)}{240b}$
default	$Ac\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{(Ad+Bc)(b\,x^2+a)^{\frac{5}{2}}}{5b}+Bd\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(b\,x^2+}{4}\right.}{}\right.$

input
```
int((B*x+A)*(d*x+c)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/240/b*(40*B*b^2*d*x^5+48*A*b^2*d*x^4+48*B*b^2*c*x^4+60*A*b^2*c*x^3+70*B*
a*b*d*x^3+96*A*a*b*d*x^2+96*B*a*b*c*x^2+150*A*a*b*c*x+15*B*a^2*d*x+48*A*a^
2*d+48*B*a^2*c)*(b*x^2+a)^(1/2)+1/16*a^2*(6*A*b*c-B*a*d)/b^(3/2)*ln(b^(1/2
)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.42

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{3/2}\,dx = \left[-\frac{15\left(6\,Aa^2bc - Ba^3d\right)\sqrt{b}\log\left(-2\,bx^2 + 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right) - 2\,(40\,Bb^3dx^5 + 48\,Ba^2bc \cdots}{\cdots}\right.$$

$$-\frac{15\left(6\,Aa^2bc - Ba^3d\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - (40\,Bb^3dx^5 + 48\,Ba^2bc + 48\,Aa^2bd + 48\,(Bb^3c + Ab^3d)x^4 \cdots}{240\,b^2}$$

input
```
integrate((B*x+A)*(d*x+c)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/480*(15*(6*A*a^2*b*c - B*a^3*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) - 2*(40*B*b^3*d*x^5 + 48*B*a^2*b*c + 48*A*a^2*b*d + 48*(
B*b^3*c + A*b^3*d)*x^4 + 10*(6*A*b^3*c + 7*B*a*b^2*d)*x^3 + 96*(B*a*b^2*c
+ A*a*b^2*d)*x^2 + 15*(10*A*a*b^2*c + B*a^2*b*d)*x)*sqrt(b*x^2 + a))/b^2,
-1/240*(15*(6*A*a^2*b*c - B*a^3*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
 a)) - (40*B*b^3*d*x^5 + 48*B*a^2*b*c + 48*A*a^2*b*d + 48*(B*b^3*c + A*b^3
*d)*x^4 + 10*(6*A*b^3*c + 7*B*a*b^2*d)*x^3 + 96*(B*a*b^2*c + A*a*b^2*d)*x^
2 + 15*(10*A*a*b^2*c + B*a^2*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(124) = 248$.

Time = 0.65 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.45

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{3/2}\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Bbdx^5}{6} + \frac{x^4\left(Ab^2d+Bb^2c\right)}{5b} + \frac{x^3\left(Ab^2c+\frac{7Babd}{6}\right)}{4b} + \frac{x^2\cdot\left(2Aabd+2Babc-\frac{4a\left(Ab^2d+Bb^2c\right)}{5b}\right)}{3b} + \frac{x\left(2Aa \cdots\right.}{\cdots}\right. \\ a^{\frac{3}{2}}\left(Acx + \frac{Bdx^3}{3} + \frac{x^2(Ad+Bc)}{2}\right) \end{cases}$$

input
```
integrate((B*x+A)*(d*x+c)*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*b*d*x**5/6 + x**4*(A*b**2*d + B*b**2*c)/(5*
b) + x**3*(A*b**2*c + 7*B*a*b*d/6)/(4*b) + x**2*(2*A*a*b*d + 2*B*a*b*c - 4
*a*(A*b**2*d + B*b**2*c)/(5*b))/(3*b) + x*(2*A*a*b*c + B*a**2*d - 3*a*(A*b
**2*c + 7*B*a*b*d/6)/(4*b))/(2*b) + (A*a**2*d + B*a**2*c - 2*a*(2*A*a*b*d
+ 2*B*a*b*c - 4*a*(A*b**2*d + B*b**2*c)/(5*b))/(3*b))/b) + (A*a**2*c - a*(
2*A*a*b*c + B*a**2*d - 3*a*(A*b**2*c + 7*B*a*b*d/6)/(4*b))/(2*b))*Piecewis
e((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/s
qrt(b*x**2), True)), Ne(b, 0)), (a**(3/2)*(A*c*x + B*d*x**3/3 + x**2*(A*d
+ B*c)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.13

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{3/2}\,dx = \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Acx + \frac{3}{8}\sqrt{bx^2+a}Aacx + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Bdx}{6\,b} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}Badx}{24\,b} - \frac{\sqrt{bx^2+a}Ba^2dx}{16\,b} + \frac{3\,Aa^2c\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}} - \frac{Ba^3d\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Bc}{5\,b} + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Ad}{5\,b}$$

input
```
integrate((B*x+A)*(d*x+c)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*A*c*x + 3/8*sqrt(b*x^2 + a)*A*a*c*x + 1/6*(b*x^2 + a
)^(5/2)*B*d*x/b - 1/24*(b*x^2 + a)^(3/2)*B*a*d*x/b - 1/16*sqrt(b*x^2 + a)*
B*a^2*d*x/b + 3/8*A*a^2*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/16*B*a^3*d*ar
csinh(b*x/sqrt(a*b))/b^(3/2) + 1/5*(b*x^2 + a)^(5/2)*B*c/b + 1/5*(b*x^2 +
a)^(5/2)*A*d/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.30

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{3/2}\,dx = \frac{1}{240}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(5\,Bbdx+\frac{6\,(Bb^5c+Ab^5d)}{b^4}\right)x+\frac{5\,(6\,Ab^5c+7\,Bab^4d)}{b^4}\right)x+\frac{48\,(B\right.\right.\right.$$
$$-\frac{(6\,Aa^2bc-Ba^3d)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/240*sqrt(b*x^2 + a)*((2*((4*(5*B*b*d*x + 6*(B*b^5*c + A*b^5*d)/b^4)*x +
5*(6*A*b^5*c + 7*B*a*b^4*d)/b^4)*x + 48*(B*a*b^4*c + A*a*b^4*d)/b^4)*x + 1
5*(10*A*a*b^4*c + B*a^2*b^3*d)/b^4)*x + 48*(B*a^2*b^3*c + A*a^2*b^3*d)/b^4
) - 1/16*(6*A*a^2*b*c - B*a^3*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^
(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{3/2}\,dx = \int \left(b\,x^2+a\right)^{3/2}\,(A+B\,x)\,(c+d\,x)\,dx$$

input
```
int((a + b*x^2)^(3/2)*(A + B*x)*(c + d*x),x)
```

output
```
int((a + b*x^2)^(3/2)*(A + B*x)*(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.79

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{3/2}\,dx = \frac{48\sqrt{b\,x^2+a}\,a^3d+150\sqrt{b\,x^2+a}\,a^2bcx+48\sqrt{b\,x^2+a}\,a^2bc+96\sqrt{b\,x^2+a}\,a^2bd\,x^2+15\sqrt{b\,x^2}}{}$$

input
```
int((B*x+A)*(d*x+c)*(b*x^2+a)^(3/2),x)
```

output
```
(48*sqrt(a + b*x**2)*a**3*d + 150*sqrt(a + b*x**2)*a**2*b*c*x + 48*sqrt(a
+ b*x**2)*a**2*b*c + 96*sqrt(a + b*x**2)*a**2*b*d*x**2 + 15*sqrt(a + b*x**
2)*a**2*b*d*x + 60*sqrt(a + b*x**2)*a*b**2*c*x**3 + 96*sqrt(a + b*x**2)*a*
b**2*c*x**2 + 48*sqrt(a + b*x**2)*a*b**2*d*x**4 + 70*sqrt(a + b*x**2)*a*b*
*2*d*x**3 + 48*sqrt(a + b*x**2)*b**3*c*x**4 + 40*sqrt(a + b*x**2)*b**3*d*x
**5 + 90*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*c - 15*s
qrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*d)/(240*b)
```

3.158 $\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{3}{8}aAx\sqrt{a+bx^2}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{3a^2A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

output
```
3/8*a*A*x*(b*x^2+a)^(1/2)+1/4*A*x*(b*x^2+a)^(3/2)+1/5*B*(b*x^2+a)^(5/2)/b+
3/8*a^2*A*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{\sqrt{a+bx^2}\left(8a^2B+2b^2x^3(5A+4Bx)+abx(25A+16Bx)\right)-15a^2A\sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+\right.}{40b}$$

input
```
Integrate[(A + B*x)*(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[a + b*x^2]*(8*a^2*B + 2*b^2*x^3*(5*A + 4*B*x) + a*b*x*(25*A + 16*B*x
)) - 15*a^2*A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(40*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} (A+Bx) \, dx$$

$$\Big\downarrow \text{455}$$

$$A \int \left(bx^2+a\right)^{3/2} dx + \frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{211}$$

$$A\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx + \frac{1}{4}x\left(a+bx^2\right)^{3/2}\right) + \frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{211}$$

$$A\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx + \frac{1}{2}x\sqrt{a+bx^2}\right) + \frac{1}{4}x\left(a+bx^2\right)^{3/2}\right) + \frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{224}$$

$$A\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}} + \frac{1}{2}x\sqrt{a+bx^2}\right) + \frac{1}{4}x\left(a+bx^2\right)^{3/2}\right) + \frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{219}$$

$$A\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}} + \frac{1}{2}x\sqrt{a+bx^2}\right) + \frac{1}{4}x\left(a+bx^2\right)^{3/2}\right) + \frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

input
```
Int[(A + B*x)*(a + b*x^2)^(3/2),x]
```

output
```
(B*(a + b*x^2)^(5/2))/(5*b) + A*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a
 + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

method	result	size
default	$A\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{B\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5b}$	70
risch	$\frac{\left(8x^{4}B\,b^{2}+10A\,x^{3}b^{2}+16Ba\,x^{2}b+25abAx+8a^{2}B\right)\sqrt{b\,x^{2}+a}}{40b}+\frac{3a^{2}A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8\sqrt{b}}$	80

input
```
int((B*x+A)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))))+1/5*B*(b*x^2+a)^(5/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.02

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \left[\frac{15\,Aa^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,Bb^2x^4+10\,Ab^2x^3+16\,Babx^2+25\,A}{80\,b}\right.$$
$$\left.-\frac{15\,Aa^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,Bb^2x^4+10\,Ab^2x^3+16\,Babx^2+25\,Aabx+8\,Ba^2)\sqrt{bx^2+a}}{40\,b}\right]$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/80*(15*A*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt(
b*x^2 + a))/b, -1/40*(15*A*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - (8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt
(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \begin{cases} \frac{3Aa^2\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{8}+\sqrt{a+bx^2}\cdot\left(\frac{5Aax}{8}+\frac{Abx^3}{4}+\frac{Ba^2}{5b}+\frac{2Bax^2}{5}+\frac{Bbx^4}{5}\right) \\ a^{\frac{3}{2}}\left(Ax+\frac{Bx^2}{2}\right) \end{cases}$$

input
```
integrate((B*x+A)*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((3*A*a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqr
t(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/8 + sqrt(a + b*x**2)*(5*A*
a*x/8 + A*b*x**3/4 + B*a**2/(5*b) + 2*B*a*x**2/5 + B*b*x**4/5), Ne(b, 0)),
 (a**(3/2)*(A*x + B*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Ax+\frac{3}{8}\sqrt{bx^2+a}Aax+\frac{3\,Aa^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{5\,b}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2 + a)*A*a*x + 3/8*A*a^2*arcsinh(
b*x/sqrt(a*b))/sqrt(b) + 1/5*(b*x^2 + a)^(5/2)*B/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=-\frac{3\,Aa^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}+\frac{1}{40}\,\sqrt{bx^2+a}\left(\frac{8\,Ba^2}{b}+(25\,Aa+2\,(8\,Ba+(4\,Bbx+5\,Ab)x)x)x\right)$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-3/8*A*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/40*sqrt(b*x^
2 + a)*(8*B*a^2/b + (25*A*a + 2*(8*B*a + (4*B*b*x + 5*A*b)*x)*x)*x)
```

Mupad [B] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.62

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{B\left(b\,x^2+a\right)^{5/2}}{5\,b}+\frac{A\,x\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

input
```
int((a + b*x^2)^(3/2)*(A + B*x),x)
```

output
```
(B*(a + b*x^2)^(5/2))/(5*b) + (A*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2]
, 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{25\sqrt{b\,x^2+a}\,a^2bx+8\sqrt{b\,x^2+a}\,a^2b+10\sqrt{b\,x^2+a}\,a\,b^2x^3+16\sqrt{b\,x^2+a}\,a\,b^2x^2+8\sqrt{b\,x^2+a}}{40b}$$

input
```
int((B*x+A)*(b*x^2+a)^(3/2),x)
```

output
```
(25*sqrt(a + b*x**2)*a**2*b*x + 8*sqrt(a + b*x**2)*a**2*b + 10*sqrt(a + b*
x**2)*a*b**2*x**3 + 16*sqrt(a + b*x**2)*a*b**2*x**2 + 8*sqrt(a + b*x**2)*b
**3*x**4 + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3)/(4
0*b)
```

3.159 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{c+dx}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 236

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{c+dx}\,dx =& \\ &-\frac{\left(8(Bc-Ad)\left(bc^2+ad^2\right)-d\left(3aBd^2+4bc(Bc-Ad)\right)x\right)\sqrt{a+bx^2}}{8d^4} \\ &-\frac{(4(Bc-Ad)-3Bdx)\left(a+bx^2\right)^{3/2}}{12d^2} \\ &+\frac{\left(3a^2Bd^4+8b^2c^3(Bc-Ad)+12abcd^2(Bc-Ad)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}d^5} \\ &+\frac{(Bc-Ad)\left(bc^2+ad^2\right)^{3/2}\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^5}\end{aligned}$$

output
```
-1/8*(8*(-A*d+B*c)*(a*d^2+b*c^2)-d*(3*a*B*d^2+4*b*c*(-A*d+B*c))*x)*(b*x^2+
a)^(1/2)/d^4-1/12*(-3*B*d*x-4*A*d+4*B*c)*(b*x^2+a)^(3/2)/d^2+1/8*(3*a^2*B*
d^4+8*b^2*c^3*(-A*d+B*c)+12*a*b*c*d^2*(-A*d+B*c))*arctanh(b^(1/2)*x/(b*x^2
+a)^(1/2))/b^(1/2)/d^5+(-A*d+B*c)*(a*d^2+b*c^2)^(3/2)*arctanh((-b*c*x+a*d)
/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^5
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{c+dx}\,dx = \frac{d\sqrt{a+bx^2}\left(ad^2(-32Bc+32Ad+15Bdx)+4Abd\left(6c^2-3cdx+2d^2x^2\right)+b\right.}{}$$

input
```
Integrate[((A + B*x)*(a + b*x^2)^(3/2))/(c + d*x),x]
```

output
```
(d*Sqrt[a + b*x^2]*(a*d^2*(-32*B*c + 32*A*d + 15*B*d*x) + 4*A*b*d*(6*c^2 -
 3*c*d*x + 2*d^2*x^2) + b*B*(-24*c^3 + 12*c^2*d*x - 8*c*d^2*x^2 + 6*d^3*x^
3)) + 48*(B*c - A*d)*(-(b*c^2) - a*d^2)^(3/2)*ArcTan[(Sqrt[b]*(c + d*x) -
d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] - (3*(3*a^2*B*d^4 + 8*b^2*c^3*(
B*c - A*d) + 12*a*b*c*d^2*(B*c - A*d))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
)/Sqrt[b])/(24*d^5)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {682, 25, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(A+Bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{682}$$

$$\frac{\int -\frac{b(ad(Bc-4Ad)-(3aBd^2+4bc(Bc-Ad))x)\sqrt{bx^2+a}}{c+dx}dx}{4bd^2}-\frac{\left(a+bx^2\right)^{3/2}(4(Bc-Ad)-3Bdx)}{12d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b(ad(Bc-4Ad)-(3aBd^2+4bc(Bc-Ad))x)\sqrt{bx^2+a}}{c+dx}dx}{4bd^2}-\frac{\left(a+bx^2\right)^{3/2}(4(Bc-Ad)-3Bdx)}{12d^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(ad(Bc-4Ad)-\left(3aBd^2+4bc(Bc-Ad)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4d^2}-\frac{\left(a+bx^2\right)^{3/2}(4(Bc-Ad)-3Bdx)}{12d^2}$$

$\downarrow$ 682

$$-\frac{\frac{\int \frac{b\left(ad\left(4b(Bc-Ad)c^2+ad^2(5Bc-8Ad)\right)-\left(3a^2Bd^4+12abc(Bc-Ad)d^2+8b^2c^3(Bc-Ad)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)(Bc-Ad)-dx\left(3aBd^2+4bc(Bc\right.\right.}{2d^2}}{4d^2}$$
$$\frac{\left(a+bx^2\right)^{3/2}(4(Bc-Ad)-3Bdx)}{12d^2}$$

$\downarrow$ 27

$$-\frac{\frac{\int \frac{ad\left(4b(Bc-Ad)c^2+ad^2(5Bc-8Ad)\right)-\left(3a^2Bd^4+12abc(Bc-Ad)d^2+8b^2c^3(Bc-Ad)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)(Bc-Ad)-dx\left(3aBd^2+4bc(Bc-A\right.\right.}{2d^2}}{4d^2}$$
$$\frac{\left(a+bx^2\right)^{3/2}(4(Bc-Ad)-3Bdx)}{12d^2}$$

$\downarrow$ 719

$$-\frac{\frac{\frac{8\left(ad^2+bc^2\right)^2(Bc-Ad)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(3a^2Bd^4+12abcd^2(Bc-Ad)+8b^2c^3(Bc-Ad)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)(Bc-Ad)-dx\left(3aB\right.\right.}{2d^2}}{4d^2}$$
$$\frac{\left(a+bx^2\right)^{3/2}(4(Bc-Ad)-3Bdx)}{12d^2}$$

$\downarrow$ 224

$$-\frac{\frac{\frac{8\left(ad^2+bc^2\right)^2(Bc-Ad)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(3a^2Bd^4+12abcd^2(Bc-Ad)+8b^2c^3(Bc-Ad)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)(Bc-Ad)\right.}{2d^2}}{4d^2}$$
$$\frac{\left(a+bx^2\right)^{3/2}(4(Bc-Ad)-3Bdx)}{12d^2}$$

$\downarrow$ 219

$$-\frac{\frac{\frac{8\left(ad^2+bc^2\right)^2(Bc-Ad)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2Bd^4+12abcd^2(Bc-Ad)+8b^2c^3(Bc-Ad)\right)}{\sqrt{b}d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)(Bc-Ad)\right.}{2d^2}}{4d^2}$$
$$\frac{\left(a+bx^2\right)^{3/2}(4(Bc-Ad)-3Bdx)}{12d^2}$$

$\downarrow$ 488

$$-\frac{\frac{-\frac{8\left(ad^2+bc^2\right)^2(Bc-Ad)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2Bd^4+12abcd^2(Bc-Ad)+8b^2c^3(Bc-Ad)\right)}{\sqrt{b}d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+\cdots\right.\right.}{\cdots}}{4d^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}(4(Bc-Ad)-3Bdx)}{12d^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2Bd^4+12abcd^2(Bc-Ad)+8b^2c^3(Bc-Ad)\right)}{\sqrt{b}d}-\frac{8\left(ad^2+bc^2\right)^{3/2}(Bc-Ad)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2\cdots\right.\right.}{\cdots}}{4d^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}(4(Bc-Ad)-3Bdx)}{12d^2}$$

input
```
Int[((A + B*x)*(a + b*x^2)^(3/2))/(c + d*x),x]
```

output
```
-1/12*((4*(B*c - A*d) - 3*B*d*x)*(a + b*x^2)^(3/2))/d^2 - (((8*(B*c - A*d)
*(b*c^2 + a*d^2) - d*(3*a*B*d^2 + 4*b*c*(B*c - A*d))*x)*Sqrt[a + b*x^2])/(
2*d^2) + (-(((3*a^2*B*d^4 + 8*b^2*c^3*(B*c - A*d) + 12*a*b*c*d^2*(B*c - A*
d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d)) - (8*(B*c - A*d)*(b
*c^2 + a*d^2)^(3/2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*
x^2])])/d)/(2*d^2))/(4*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.56

method	result
risch	$\frac{(6Bb\,d^3x^3+8Ab\,d^3x^2-8Bbc\,d^2x^2-12Abc\,d^2x+15Ba\,d^3x+12Bb\,c^2dx+32A\,d^3a+24Ab\,c^2d-32aBc\,d^2-24bB\,c^3)\sqrt{b\,x^2+a}}{24d^4}-\frac{(12A\cdots}{\cdots}$
default	$\frac{B\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{d}+\frac{(Ad-Bc)\left(\frac{\left(b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}\right)^{\frac{3}{2}}}{3}-\frac{bc\left(\frac{\left(2b\left(x+\frac{c}{d}\right)-2\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$

input
```
int((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*B*b*d^3*x^3+8*A*b*d^3*x^2-8*B*b*c*d^2*x^2-12*A*b*c*d^2*x+15*B*a*d^
3*x+12*B*b*c^2*d*x+32*A*a*d^3+24*A*b*c^2*d-32*B*a*c*d^2-24*B*b*c^3)*(b*x^2
+a)^(1/2)/d^4-1/8/d^4*((12*A*a*b*c*d^3+8*A*b^2*c^3*d-3*B*a^2*d^4-12*B*a*b*
c^2*d^2-8*B*b^2*c^4)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+8*(A*a^2*d^5+
2*A*a*b*c^2*d^3+A*b^2*c^4*d-B*a^2*c*d^4-2*B*a*b*c^3*d^2-B*b^2*c^5)/d^2/((a
*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b
*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x
+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{c+dx}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{c+dx}\,dx = \int \frac{(A+Bx)\left(a+bx^2\right)^{\frac{3}{2}}}{c+dx}\,dx$$

input
```
integrate((B*x+A)*(b*x**2+a)**(3/2)/(d*x+c),x)
```

output
```
Integral((A + B*x)*(a + b*x**2)**(3/2)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.63

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{c+dx}\,dx &= \frac{\sqrt{bx^2+a}\,Bbc^2x}{2\,d^3} - \frac{\sqrt{bx^2+a}\,Abcx}{2\,d^2} \\ &+ \frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx}{4\,d} + \frac{3\,\sqrt{bx^2+a}\,Bax}{8\,d} + \frac{Bb^{\frac{3}{2}}c^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^5} \\ &- \frac{Ab^{\frac{3}{2}}c^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^4} + \frac{3\,Ba\sqrt{b}c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,d^3} - \frac{3\,Aa\sqrt{b}c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,d^2} \\ &+ \frac{3\,Ba^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}d} - \frac{B\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}c\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d^2} \\ &+ \frac{A\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d} - \frac{\sqrt{bx^2+a}\,Bbc^3}{d^4} + \frac{\sqrt{bx^2+a}\,Abc^2}{d^3} \\ &- \frac{\left(bx^2+a\right)^{\frac{3}{2}}Bc}{3\,d^2} - \frac{\sqrt{bx^2+a}\,Bac}{d^2} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{3\,d} + \frac{\sqrt{bx^2+a}\,Aa}{d}\end{aligned}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*B*b*c^2*x/d^3 - 1/2*sqrt(b*x^2 + a)*A*b*c*x/d^2 + 1/4*
(b*x^2 + a)^(3/2)*B*x/d + 3/8*sqrt(b*x^2 + a)*B*a*x/d + B*b^(3/2)*c^4*arcs
inh(b*x/sqrt(a*b))/d^5 - A*b^(3/2)*c^3*arcsinh(b*x/sqrt(a*b))/d^4 + 3/2*B*
a*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^3 - 3/2*A*a*sqrt(b)*c*arcsinh(b*x/s
qrt(a*b))/d^2 + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d) - B*(a + b*c^
2/d^2)^(3/2)*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs
(d*x + c)))/d^2 + A*(a + b*c^2/d^2)^(3/2)*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x
 + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d - sqrt(b*x^2 + a)*B*b*c^3/d^4 + s
qrt(b*x^2 + a)*A*b*c^2/d^3 - 1/3*(b*x^2 + a)^(3/2)*B*c/d^2 - sqrt(b*x^2 +
a)*B*a*c/d^2 + 1/3*(b*x^2 + a)^(3/2)*A/d + sqrt(b*x^2 + a)*A*a/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{c+dx}\,dx = \int \frac{\left(b\,x^2+a\right)^{3/2}\,\left(A+B\,x\right)}{c+d\,x}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x))/(c + d*x),x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{c+dx}\,dx = \int \frac{(Bx+A)\left(b\,x^2+a\right)^{\frac{3}{2}}}{dx+c}dx$$

input
```
int((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c),x)
```

output
```
int((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c),x)
```

3.160 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 231

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{(c+dx)^2}\,dx &= \frac{(2(aBd^2+bc(4Bc-3Ad))-bd(4Bc-3Ad)x)\sqrt{a+bx^2}}{2d^4} \\ &+ \frac{(4Bc-3Ad+Bdx)\left(a+bx^2\right)^{3/2}}{3d^2(c+dx)} \\ &- \frac{\sqrt{b}(2aBcd^2+(4Bc-3Ad)\left(2bc^2+ad^2\right))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^5} \\ &- \frac{\sqrt{bc^2+ad^2}(aBd^2+bc(4Bc-3Ad))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^5}\end{aligned}$$

output
```
1/2*(2*a*B*d^2+2*b*c*(-3*A*d+4*B*c)-b*d*(-3*A*d+4*B*c)*x)*(b*x^2+a)^(1/2)/
d^4+1/3*(B*d*x-3*A*d+4*B*c)*(b*x^2+a)^(3/2)/d^2/(d*x+c)-1/2*b^(1/2)*(2*a*B
*c*d^2+(-3*A*d+4*B*c)*(a*d^2+2*b*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/
d^5-(a*d^2+b*c^2)^(1/2)*(a*B*d^2+b*c*(-3*A*d+4*B*c))*arctanh((-b*c*x+a*d)/
(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^5
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{(c+dx)^2}\,dx = \frac{\frac{d\sqrt{a+bx^2}\left(2ad^2(7Bc-3Ad+4Bdx)+3Abd\left(-6c^2-3cdx+d^2x^2\right)+2bB\left(12c^3+6c^2dx-2cd^2x^2+d^3x^3\right)\right)}{c+dx}+}{}$$

input
```
Integrate[((A + B*x)*(a + b*x^2)^(3/2))/(c + d*x)^2,x]
```

output
```
((d*Sqrt[a + b*x^2]*(2*a*d^2*(7*B*c - 3*A*d + 4*B*d*x) + 3*A*b*d*(-6*c^2 -
 3*c*d*x + d^2*x^2) + 2*b*B*(12*c^3 + 6*c^2*d*x - 2*c*d^2*x^2 + d^3*x^3)))
/(c + d*x) + 12*Sqrt[-(b*c^2) - a*d^2]*(a*B*d^2 + b*c*(4*B*c - 3*A*d))*Arc
Tan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] + 3*Sq
rt[b]*(2*b*c^2*(4*B*c - 3*A*d) - 3*a*d^2*(-2*B*c + A*d))*Log[-(Sqrt[b]*x)
+ Sqrt[a + b*x^2]])/(6*d^5)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {681, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(A+Bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{681}$$

$$\frac{\left(a+bx^2\right)^{3/2}(-3Ad+4Bc+Bdx)}{3d^2(c+dx)}-\frac{\int -\frac{2(aBd-b(4Bc-3Ad)x)\sqrt{bx^2+a}}{c+dx}dx}{2d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(aBd-b(4Bc-3Ad)x)\sqrt{bx^2+a}}{c+dx}dx}{d^2}+\frac{\left(a+bx^2\right)^{3/2}(-3Ad+4Bc+Bdx)}{3d^2(c+dx)}$$

$$\Big\downarrow \text{682}$$

$$\frac{\frac{\int \frac{b\left(ad\left(2aBd^2+bc(4Bc-3Ad)\right)-b\left(2aBcd^2+(4Bc-3Ad)\left(2bc^2+ad^2\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(2\left(aBd^2+bc(4Bc-3Ad)\right)-bdx(4Bc-3Ad)\right)}{2d^2}}{d^2}+\frac{\left(a+bx^2\right)^{3/2}(-3Ad+4Bc+Bdx)}{3d^2(c+dx)}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{ad\left(2aBd^2+bc(4Bc-3Ad)\right)-b\left(2aBcd^2+(4Bc-3Ad)\left(2bc^2+ad^2\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(aBd^2+bc(4Bc-3Ad)\right)-bdx(4Bc-3Ad)\right)}{2d^2}}{d^2}+\frac{\left(a+bx^2\right)^{3/2}(-3Ad+4Bc+Bdx)}{3d^2(c+dx)}$$

$\downarrow$ 719

$$\frac{\frac{\frac{2\left(ad^2+bc^2\right)\left(aBd^2+bc(4Bc-3Ad)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{b\left(\left(ad^2+2bc^2\right)(4Bc-3Ad)+2aBcd^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(aBd^2+bc(4Bc-3Ad)\right)-bdx \cdots\right.}{2d^2}}{d^2}+\frac{\left(a+bx^2\right)^{3/2}(-3Ad+4Bc+Bdx)}{3d^2(c+dx)}$$

$\downarrow$ 224

$$\frac{\frac{\frac{2\left(ad^2+bc^2\right)\left(aBd^2+bc(4Bc-3Ad)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{b\left(\left(ad^2+2bc^2\right)(4Bc-3Ad)+2aBcd^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(aBd^2+bc(4Bc-3A \cdots\right.\right.}{2d^2}}{d^2}+\frac{\left(a+bx^2\right)^{3/2}(-3Ad+4Bc+Bdx)}{3d^2(c+dx)}$$

$\downarrow$ 219

$$\frac{\frac{\frac{2\left(ad^2+bc^2\right)\left(aBd^2+bc(4Bc-3Ad)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(\left(ad^2+2bc^2\right)(4Bc-3Ad)+2aBcd^2\right)}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(aBd^2+bc(4Bc- \cdots\right.\right.}{2d^2}}{d^2}+\frac{\left(a+bx^2\right)^{3/2}(-3Ad+4Bc+Bdx)}{3d^2(c+dx)}$$

$\downarrow$ 488

$$\frac{\frac{-\frac{2\left(ad^2+bc^2\right)\left(aBd^2+bc(4Bc-3Ad)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(\left(ad^2+2bc^2\right)(4Bc-3Ad)+2aBcd^2\right)}{d}}{2d^2}+\frac{\sqrt{a+bx^2}(2(aB}{}}{d^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}(-3Ad+4Bc+Bdx)}{3d^2(c+dx)}$$

$\Big\downarrow$ 219

$$\frac{\frac{-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(\left(ad^2+2bc^2\right)(4Bc-3Ad)+2aBcd^2\right)}{d}-\frac{2\sqrt{ad^2+bc^2}\left(aBd^2+bc(4Bc-3Ad)\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}}{2d^2}+\frac{\sqrt{a+bx^2}(2(aB}{}}{d^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}(-3Ad+4Bc+Bdx)}{3d^2(c+dx)}$$

input
```
Int[((A + B*x)*(a + b*x^2)^(3/2))/(c + d*x)^2,x]
```

output
```
((4*B*c - 3*A*d + B*d*x)*(a + b*x^2)^(3/2))/(3*d^2*(c + d*x)) + (((2*(a*B*
d^2 + b*c*(4*B*c - 3*A*d)) - b*d*(4*B*c - 3*A*d)*x)*Sqrt[a + b*x^2])/(2*d^
2) + (-((Sqrt[b]*(2*a*B*c*d^2 + (4*B*c - 3*A*d)*(2*b*c^2 + a*d^2))*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d) - (2*Sqrt[b*c^2 + a*d^2]*(a*B*d^2 + b*c*
(4*B*c - 3*A*d))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2
])])/d)/(2*d^2))/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 682

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. 2(208) = 416.

Time = 1.40 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.47

method	result
risch	$-\frac{\left(-2Bb\,d^2x^2-3Ab\,d^2x+6Bbcdx+12Abcd-8aB\,d^2-18Bb\,c^2\right)\sqrt{b\,x^2+a}}{6d^4}+\frac{\frac{\sqrt{b}\left(3A\,d^3a+6Ab\,c^2d-6aBc\,d^2-8bB\,c^3\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{d}}{}$
default	Expression too large to display

input

```
int((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/6*(-2*B*b*d^2*x^2-3*A*b*d^2*x+6*B*b*c*d*x+12*A*b*c*d-8*B*a*d^2-18*B*b*c
^2)*(b*x^2+a)^(1/2)/d^4+1/2/d^4*(b^(1/2)*(3*A*a*d^3+6*A*b*c^2*d-6*B*a*c*d^
2-8*B*b*c^3)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+2/d^2*(4*A*a*b*c*d^3+4*A*b^2*
c^3*d-B*a^2*d^4-6*B*a*b*c^2*d^2-5*B*b^2*c^4)/((a*d^2+b*c^2)/d^2)^(1/2)*ln(
(2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d
)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+2*(A*a^2*d^5+2*A*a*
b*c^2*d^3+A*b^2*c^4*d-B*a^2*c*d^4-2*B*a*b*c^3*d^2-B*b^2*c^5)/d^3*(-1/(a*d^
2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)
-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b
*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d
^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{(c+dx)^2}\,dx = \text{Timed out}$$

input

```
integrate((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{(c+dx)^2}\,dx=\int \frac{(A+Bx)\left(a+bx^2\right)^{\frac{3}{2}}}{\left(c+dx\right)^2}\,dx$$

input
```
integrate((B*x+A)*(b*x**2+a)**(3/2)/(d*x+c)**2,x)
```

output
```
Integral((A + B*x)*(a + b*x**2)**(3/2)/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.72

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{(c+dx)^2}\,dx &= \frac{(bx^2+a)^{\frac{3}{2}}Bc}{d^3x+cd^2}-\frac{(bx^2+a)^{\frac{3}{2}}A}{d^2x+cd} \\ &\quad -\frac{2\sqrt{bx^2+a}Bbcx}{d^3}+\frac{3\sqrt{bx^2+a}Abx}{2\,d^2}-\frac{4\,Bb^{\frac{3}{2}}c^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^5} \\ &\quad +\frac{3\,Ab^{\frac{3}{2}}c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^4}-\frac{3\,Ba\sqrt{b}c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^3} \\ &\quad +\frac{3\,Aa\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,d^2}+\frac{3\,B\sqrt{a+\frac{bc^2}{d^2}}bc^2\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{d^4} \\ &\quad -\frac{3\,A\sqrt{a+\frac{bc^2}{d^2}}bc\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{d^3} \\ &\quad +\frac{B\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{d^2}+\frac{4\sqrt{bx^2+a}Bbc^2}{d^4} \\ &\quad -\frac{3\sqrt{bx^2+a}Abc}{d^3}+\frac{(bx^2+a)^{\frac{3}{2}}B}{3\,d^2}+\frac{\sqrt{bx^2+a}Ba}{d^2}\end{aligned}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(b*x^2 + a)^(3/2)*B*c/(d^3*x + c*d^2) - (b*x^2 + a)^(3/2)*A/(d^2*x + c*d)
- 2*sqrt(b*x^2 + a)*B*b*c*x/d^3 + 3/2*sqrt(b*x^2 + a)*A*b*x/d^2 - 4*B*b^(3
/2)*c^3*arcsinh(b*x/sqrt(a*b))/d^5 + 3*A*b^(3/2)*c^2*arcsinh(b*x/sqrt(a*b)
)/d^4 - 3*B*a*sqrt(b)*c*arcsinh(b*x/sqrt(a*b))/d^3 + 3/2*A*a*sqrt(b)*arcsi
nh(b*x/sqrt(a*b))/d^2 + 3*B*sqrt(a + b*c^2/d^2)*b*c^2*arcsinh(b*c*x/(sqrt(
a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^4 - 3*A*sqrt(a + b*c^
2/d^2)*b*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x
 + c)))/d^3 + B*(a + b*c^2/d^2)^(3/2)*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c
)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^2 + 4*sqrt(b*x^2 + a)*B*b*c^2/d^4 - 3
*sqrt(b*x^2 + a)*A*b*c/d^3 + 1/3*(b*x^2 + a)^(3/2)*B/d^2 + sqrt(b*x^2 + a)
*B*a/d^2
```

Giac [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{(c+dx)^2}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{(c+dx)^2}\,dx=\int \frac{\left(b\,x^2+a\right)^{3/2}\,\left(A+B\,x\right)}{\left(c+d\,x\right)^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x))/(c + d*x)^2,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1076, normalized size of antiderivative = 4.66

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c)^2,x)
```

output
```
(36*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a*b*c**2*d + 36*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)
*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c*d**2*x - 12*sqrt(a*d**2 + b*c*
*2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c*d**
2 - 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
 - a*d + b*c*x)*a*b*d**3*x - 48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x*
*2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**3 - 48*sqrt(a*d**2 + b*c*
*2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**2
*d*x - 36*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**2*d - 36*sqrt(a*d**2 +
 b*c**2)*log(c + d*x)*a*b*c*d**2*x + 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)
*a*b*c*d**2 + 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*d**3*x + 48*sqrt(a
*d**2 + b*c**2)*log(c + d*x)*b**2*c**3 + 48*sqrt(a*d**2 + b*c**2)*log(c +
d*x)*b**2*c**2*d*x - 12*sqrt(a + b*x**2)*a**2*d**4 - 36*sqrt(a + b*x**2)*a
*b*c**2*d**2 - 18*sqrt(a + b*x**2)*a*b*c*d**3*x + 28*sqrt(a + b*x**2)*a*b*
c*d**3 + 6*sqrt(a + b*x**2)*a*b*d**4*x**2 + 16*sqrt(a + b*x**2)*a*b*d**4*x
 + 48*sqrt(a + b*x**2)*b**2*c**3*d + 24*sqrt(a + b*x**2)*b**2*c**2*d**2*x
- 8*sqrt(a + b*x**2)*b**2*c*d**3*x**2 + 4*sqrt(a + b*x**2)*b**2*d**4*x**3
- 9*sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*a**2*c*d**3 - 9*sqrt(b)*log(
sqrt(a + b*x**2) - sqrt(b)*x)*a**2*d**4*x - 18*sqrt(b)*log(sqrt(a + b*x**2
) - sqrt(b)*x)*a*b*c**3*d - 18*sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x...
```

3.161 $\int \frac{(A+Bx)(a+bx^2)^{3/2}}{(c+dx)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 238

$$\begin{aligned}\int \frac{(A+Bx)(a+bx^2)^{3/2}}{(c+dx)^3} dx = & -\frac{3(aBd^2+2bc(2Bc-Ad)+bd(2Bc-Ad)x)\sqrt{a+bx^2}}{2d^4(c+dx)} \\ & +\frac{(2Bc-Ad+Bdx)(a+bx^2)^{3/2}}{2d^2(c+dx)^2} \\ & +\frac{3\sqrt{b}(aBd^2+2bc(2Bc-Ad))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^5} \\ & +\frac{3b(2bc^2(2Bc-Ad)+ad^2(3Bc-Ad))\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2d^5\sqrt{bc^2+ad^2}}\end{aligned}$$

output

```
-3/2*(a*B*d^2+2*b*c*(-A*d+2*B*c)+b*d*(-A*d+2*B*c)*x)*(b*x^2+a)^(1/2)/d^4/(
d*x+c)+1/2*(B*d*x-A*d+2*B*c)*(b*x^2+a)^(3/2)/d^2/(d*x+c)^2+3/2*b^(1/2)*(a*
B*d^2+2*b*c*(-A*d+2*B*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^5+3/2*b*(2*
b*c^2*(-A*d+2*B*c)+a*d^2*(-A*d+3*B*c))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^
(1/2)/(b*x^2+a)^(1/2))/d^5/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 3.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{(c+dx)^3}\,dx =$$

$$-\frac{\frac{d\sqrt{a+bx^2}(ad^2(Ad+B(c+2dx))+b(-Ad(6c^2+9cdx+2d^2x^2)+B(12c^3+18c^2dx+4cd^2x^2-d^3x^3)))}{(c+dx)^2}-\frac{6b(2bc^2(2Bc-Ad)+ad^2(3Bc-Ad))\operatorname{arcta}}{\sqrt{-bc^2-ad^2}}}{2d^5}$$

input
```
Integrate[((A + B*x)*(a + b*x^2)^(3/2))/(c + d*x)^3,x]
```

output
```
-1/2*((d*Sqrt[a + b*x^2]*(a*d^2*(A*d + B*(c + 2*d*x)) + b*(-(A*d*(6*c^2 +
9*c*d*x + 2*d^2*x^2)) + B*(12*c^3 + 18*c^2*d*x + 4*c*d^2*x^2 - d^3*x^3))))
/(c + d*x)^2 - (6*b*(2*b*c^2*(2*B*c - A*d) + a*d^2*(3*B*c - A*d))*ArcTan[(
Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/Sqrt[-(b*c
^2) - a*d^2] + 3*Sqrt[b]*(a*B*d^2 + 2*b*c*(2*B*c - A*d))*Log[-(Sqrt[b]*x)
+ Sqrt[a + b*x^2]])/d^5
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {681, 27, 681, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(A+Bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{ 681}$$

$$\frac{\left(a+bx^2\right)^{3/2}(-Ad+2Bc+Bdx)}{2d^2(c+dx)^2}-\frac{3\int -\frac{4(aBd-b(2Bc-Ad)x)\sqrt{bx^2+a}}{(c+dx)^2}dx}{8d^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\int \frac{(aBd-b(2Bc-Ad)x)\sqrt{bx^2+a}}{(c+dx)^2}dx}{2d^2}+\frac{\left(a+bx^2\right)^{3/2}(-Ad+2Bc+Bdx)}{2d^2(c+dx)^2}$$

$\Big\downarrow$ 681

$$\frac{3\left(-\frac{\int \frac{2b\left(ad(2Bc-Ad)-\left(aBd^2+2bc(2Bc-Ad)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}-\frac{\sqrt{a+bx^2}\left(aBd^2+bdx(2Bc-Ad)+2bc(2Bc-Ad)\right)}{d^2(c+dx)}\right)}{2d^2}+\frac{\left(a+bx^2\right)^{3/2}(-Ad+2Bc+Bdx)}{2d^2(c+dx)^2}$$

$\Big\downarrow$ 27

$$\frac{3\left(-\frac{b\int \frac{ad(2Bc-Ad)-\left(aBd^2+2bc(2Bc-Ad)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}-\frac{\sqrt{a+bx^2}\left(aBd^2+bdx(2Bc-Ad)+2bc(2Bc-Ad)\right)}{d^2(c+dx)}\right)}{2d^2}+\frac{\left(a+bx^2\right)^{3/2}(-Ad+2Bc+Bdx)}{2d^2(c+dx)^2}$$

$\Big\downarrow$ 719

$$\frac{3\left(-\frac{b\left(\frac{\left(ad^2(3Bc-Ad)+2bc^2(2Bc-Ad)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(aBd^2+2bc(2Bc-Ad)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}\right)}{d^2}-\frac{\sqrt{a+bx^2}\left(aBd^2+bdx(2Bc-Ad)+2bc(2Bc-\right.}{d^2(c+dx)}\right.}{2d^2}$$
$$\frac{\left(a+bx^2\right)^{3/2}(-Ad+2Bc+Bdx)}{2d^2(c+dx)^2}$$

$\Big\downarrow$ 224

$$\frac{3\left(-\frac{b\left(\frac{\left(ad^2(3Bc-Ad)+2bc^2(2Bc-Ad)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(aBd^2+2bc(2Bc-Ad)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{d^2}-\frac{\sqrt{a+bx^2}\left(aBd^2+bdx(2Bc-Ad)+\right.}{d^2(c+dx)}\right.}{2d^2}$$
$$\frac{\left(a+bx^2\right)^{3/2}(-Ad+2Bc+Bdx)}{2d^2(c+dx)^2}$$

$\Big\downarrow$ 219

$$\frac{3\left(-\frac{b\left(\frac{\left(ad^2(3Bc-Ad)+2bc^2(2Bc-Ad)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aBd^2+2bc(2Bc-Ad)\right)}{\sqrt{b}d}\right)}{d^2}-\frac{\sqrt{a+bx^2}\left(aBd^2+bdx(2Bc-Ad)+\right.}{d^2(c+dx)}}{2d^2}$$
$$\frac{\left(a+bx^2\right)^{3/2}(-Ad+2Bc+Bdx)}{2d^2(c+dx)^2}$$

$\big\downarrow$ 488

$$\frac{3\left(-\frac{b\left(-\frac{\left(ad^2(3Bc-Ad)+2bc^2(2Bc-Ad)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aBd^2+2bc(2Bc-Ad)\right)}{\sqrt{b}d}\right)}{d^2}-\frac{\sqrt{a+bx^2}\left(aBd^2+bc\right.}{}}{2d^2}$$
$$\frac{\left(a+bx^2\right)^{3/2}(-Ad+2Bc+Bdx)}{2d^2(c+dx)^2}$$

$\big\downarrow$ 219

$$\frac{3\left(-\frac{b\left(-\frac{\left(ad^2(3Bc-Ad)+2bc^2(2Bc-Ad)\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\sqrt{ad^2+bc^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aBd^2+2bc(2Bc-Ad)\right)}{\sqrt{b}d}\right)}{d^2}-\frac{\sqrt{a+bx^2}\left(aBd^2+bd\right.}{}}{2d^2}$$
$$\frac{\left(a+bx^2\right)^{3/2}(-Ad+2Bc+Bdx)}{2d^2(c+dx)^2}$$

input
```
Int[((A + B*x)*(a + b*x^2)^(3/2))/(c + d*x)^3,x]
```

output
```
((2*B*c - A*d + B*d*x)*(a + b*x^2)^(3/2))/(2*d^2*(c + d*x)^2) + (3*(-(((a*
B*d^2 + 2*b*c*(2*B*c - A*d) + b*d*(2*B*c - A*d)*x)*Sqrt[a + b*x^2])/(d^2*(
c + d*x))) - (b*(-(((a*B*d^2 + 2*b*c*(2*B*c - A*d))*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(Sqrt[b]*d)) - ((2*b*c^2*(2*B*c - A*d) + a*d^2*(3*B*c - A*
d))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[
b*c^2 + a*d^2])))/d^2))/(2*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. $2(214) = 428$.

Time = 1.52 (sec) , antiderivative size = 1005, normalized size of antiderivative = 4.22

method	result	size
risch	Expression too large to display	1005
default	Expression too large to display	2352

input
```
int((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*(B*d*x+2*A*d-6*B*c)*(b*x^2+a)^(1/2)/d^4-1/2/d^4*(2/d^3*(4*A*a*b*c*d^
3+4*A*b^2*c^3*d-B*a^2*d^4-6*B*a*b*c^2*d^2-5*B*b^2*c^4)*(-1/(a*d^2+b*c^2)*d
^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*
d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/
d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/
d^2)^(1/2))/(x+c/d)))-2*(A*a^2*d^5+2*A*a*b*c^2*d^3+A*b^2*c^4*d-B*a^2*c*d^4
-2*B*a*b*c^3*d^2-B*b^2*c^5)/d^4*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/
d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+3/2*b*c*d/(a*d^2+b*c^2)*(-1/
(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^
(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^
2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)
+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+1/2*b/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2
)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)
^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+3*
b^(1/2)*(2*A*b*c*d-B*a*d^2-4*B*b*c^2)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+4*b/
d^2*(A*a*d^3+3*A*b*c^2*d-3*B*a*c*d^2-5*B*b*c^3)/((a*d^2+b*c^2)/d^2)^(1/2)*
ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+
c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{(c+dx)^3}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{(c+dx)^3}\,dx=\int \frac{(A+Bx)\left(a+bx^2\right)^{\frac{3}{2}}}{\left(c+dx\right)^3}\,dx$$

input
```
integrate((B*x+A)*(b*x**2+a)**(3/2)/(d*x+c)**3,x)
```

output
```
Integral((A + B*x)*(a + b*x**2)**(3/2)/(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(215) = 430$.

Time = 0.12 (sec) , antiderivative size = 776, normalized size of antiderivative = 3.26

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{(c+dx)^3}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-3/2*sqrt(b*x^2 + a)*B*b^2*c^3/(b*c^2*d^4 + a*d^6) + 3/2*sqrt(b*x^2 + a)*B
*b^2*c^2*x/(b*c^2*d^3 + a*d^5) - 1/2*(b*x^2 + a)^(3/2)*B*b*c^2/(b*c^2*d^3*
x + a*d^5*x + b*c^3*d^2 + a*c*d^4) + 3/2*sqrt(b*x^2 + a)*A*b^2*c^2/(b*c^2*
d^3 + a*d^5) - 3/2*sqrt(b*x^2 + a)*A*b^2*c*x/(b*c^2*d^2 + a*d^4) + 1/2*(b*
x^2 + a)^(5/2)*B*c/(b*c^2*d^2*x^2 + a*d^4*x^2 + 2*b*c^3*d*x + 2*a*c*d^3*x
+ b*c^4 + a*c^2*d^2) + 1/2*(b*x^2 + a)^(3/2)*A*b*c/(b*c^2*d^2*x + a*d^4*x
+ b*c^3*d + a*c*d^3) - 1/2*(b*x^2 + a)^(3/2)*B*b*c/(b*c^2*d^2 + a*d^4) - 1
/2*(b*x^2 + a)^(5/2)*A/(b*c^2*d*x^2 + a*d^3*x^2 + 2*b*c^3*x + 2*a*c*d^2*x
+ b*c^4/d + a*c^2*d) + 1/2*(b*x^2 + a)^(3/2)*A*b/(b*c^2*d + a*d^3) - (b*x^
2 + a)^(3/2)*B/(d^3*x + c*d^2) + 3/2*sqrt(b*x^2 + a)*B*b*x/d^3 + 6*B*b^(3/
2)*c^2*arcsinh(b*x/sqrt(a*b))/d^5 - 3*A*b^(3/2)*c*arcsinh(b*x/sqrt(a*b))/d
^4 + 3/2*B*a*sqrt(b)*arcsinh(b*x/sqrt(a*b))/d^3 - 3/2*B*b^2*c^3*arcsinh(b*
c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c
^2/d^2)*d^6) + 3/2*A*b^2*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/
(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^5) - 9/2*B*sqrt(a + b*c^2
/d^2)*b*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x
+ c)))/d^4 + 3/2*A*sqrt(a + b*c^2/d^2)*b*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x
+ c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^3 - 9/2*sqrt(b*x^2 + a)*B*b*c/d^4
+ 3/2*sqrt(b*x^2 + a)*A*b/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(215) = 430$.

Time = 0.19 (sec) , antiderivative size = 666, normalized size of antiderivative = 2.80

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{(c+dx)^3}\,dx = \frac{1}{2}\sqrt{bx^2+a}\left(\frac{Bbx}{d^3} - \frac{2\left(3\,Bbcd^8 - Abd^9\right)}{d^{12}}\right) - \frac{3\left(4\,Bb^{\frac{3}{2}}c^2 - 2\,Ab^{\frac{3}{2}}cd + Ba\sqrt{b}d^2\right)\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{2\,d^5} - \frac{3\left(4\,Bb^2c^3 - 2\,Ab^2c^2d + 3\,Babcd^2 - Aabd^3\right)\arctan\left(-\frac{\left(\sqrt{b}x - \sqrt{bx^2+a}\right)d + \sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}\,d^5} - \frac{8\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^3 Bb^2c^3d - 6\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^3 Ab^2c^2d^2 + 3\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^3 Babcd^3 - \left(\sqrt{b}x\right.}{}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*(B*b*x/d^3 - 2*(3*B*b*c*d^8 - A*b*d^9)/d^12) - 3/2*(4*
B*b^(3/2)*c^2 - 2*A*b^(3/2)*c*d + B*a*sqrt(b)*d^2)*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/d^5 - 3*(4*B*b^2*c^3 - 2*A*b^2*c^2*d + 3*B*a*b*c*d^2 - A*a
*b*d^3)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2
- a*d^2))/(sqrt(-b*c^2 - a*d^2)*d^5) - (8*(sqrt(b)*x - sqrt(b*x^2 + a))^3*
B*b^2*c^3*d - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b^2*c^2*d^2 + 3*(sqrt(b)
*x - sqrt(b*x^2 + a))^3*B*a*b*c*d^3 - (sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a*
b*d^4 + 14*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*b^(5/2)*c^4 - 10*(sqrt(b)*x -
 sqrt(b*x^2 + a))^2*A*b^(5/2)*c^3*d - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*
a*b^(3/2)*c^2*d^2 + 5*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*b^(3/2)*c*d^3 -
2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b)*d^4 - 20*(sqrt(b)*x - sqrt
(b*x^2 + a))*B*a*b^2*c^3*d + 14*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b^2*c^2*
d^2 - 5*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^2*b*c*d^3 - (sqrt(b)*x - sqrt(b*
x^2 + a))*A*a^2*b*d^4 + 7*B*a^2*b^(3/2)*c^2*d^2 - 5*A*a^2*b^(3/2)*c*d^3 +
2*B*a^3*sqrt(b)*d^4)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*(sqrt(b)*x -
sqrt(b*x^2 + a))*sqrt(b)*c - a*d)^2*d^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{(c+dx)^3}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{3/2}\,\left(A+B\,x\right)}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x))/(c + d*x)^3,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x))/(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 2274, normalized size of antiderivative = 9.55

$$\int \frac{(A+Bx)\,(a+bx^2)^{3/2}}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(b*x^2+a)^(3/2)/(d*x+c)^3,x)
```

output
```
(6*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
+ b*c*x)*a**2*b*c**2*d**3 + 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*
sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c*d**4*x + 6*sqrt(a*d**2 + b*c
**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*d**5
*x**2 + 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2
) - a*d + b*c*x)*a*b**2*c**4*d + 24*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**3*d**2*x - 18*sqrt(a*d
**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*
b**2*c**3*d**2 + 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2
 + b*c**2) - a*d + b*c*x)*a*b**2*c**2*d**3*x**2 - 36*sqrt(a*d**2 + b*c**2)
*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**2*d**
3*x - 18*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
- a*d + b*c*x)*a*b**2*c*d**4*x**2 - 24*sqrt(a*d**2 + b*c**2)*log(sqrt(a +
b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**5 - 48*sqrt(a*d**2 +
b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**
4*d*x - 24*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2
) - a*d + b*c*x)*b**3*c**3*d**2*x**2 - 6*sqrt(a*d**2 + b*c**2)*log(c + d*x
)*a**2*b*c**2*d**3 - 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c*d**4*x
 - 6*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*d**5*x**2 - 12*sqrt(a*d**2
+ b*c**2)*log(c + d*x)*a*b**2*c**4*d - 24*sqrt(a*d**2 + b*c**2)*log(c +...
```

3.162 $\int (A+Bx)(c+dx)^3 \left(a+bx^2\right)^{5/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 377

$$\int (A+Bx)(c+dx)^3 \left(a+bx^2\right)^{5/2} dx = \frac{a^2(10Abc(8bc^2-3ad^2)-3aBd(10bc^2-ad^2))\,x\sqrt{a+bx^2}}{256b^2} + \frac{a(10Abc(8bc^2-3ad^2)-3aBd(10bc^2-ad^2))\,x(a+bx^2)^{3/2}}{384b^2} + \frac{(10Abc(8bc^2-3ad^2)-3aBd(10bc^2-ad^2))\,x(a+bx^2)^{5/2}}{480b^2} + \frac{(3Bc+10Ad)(c+dx)^2\left(a+bx^2\right)^{7/2}}{90b} + \frac{B(c+dx)^3\left(a+bx^2\right)^{7/2}}{10b} - \frac{(16(10ad^2(3Bc+Ad)-bc^2(3Bc+100Ad))+7d(27aBd^2-2bc(3Bc+55Ad))\,x)\left(a+bx^2\right)^{7/2}}{5040b^2} + \frac{a^3(10Abc(8bc^2-3ad^2)-3aBd(10bc^2-ad^2))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}$$

output
```
1/256*a^2*(10*A*b*c*(-3*a*d^2+8*b*c^2)-3*a*B*d*(-a*d^2+10*b*c^2))*x*(b*x^2
+a)^(1/2)/b^2+1/384*a*(10*A*b*c*(-3*a*d^2+8*b*c^2)-3*a*B*d*(-a*d^2+10*b*c^
2))*x*(b*x^2+a)^(3/2)/b^2+1/480*(10*A*b*c*(-3*a*d^2+8*b*c^2)-3*a*B*d*(-a*d
^2+10*b*c^2))*x*(b*x^2+a)^(5/2)/b^2+1/90*(10*A*d+3*B*c)*(d*x+c)^2*(b*x^2+a
)^(7/2)/b+1/10*B*(d*x+c)^3*(b*x^2+a)^(7/2)/b-1/5040*(160*a*d^2*(A*d+3*B*c)
-16*b*c^2*(100*A*d+3*B*c)+7*d*(27*a*B*d^2-2*b*c*(55*A*d+3*B*c))*x)*(b*x^2+
a)^(7/2)/b^2+1/256*a^3*(10*A*b*c*(-3*a*d^2+8*b*c^2)-3*a*B*d*(-a*d^2+10*b*c
^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 3.25 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.08

$$\int (A+Bx)(c+dx)^3 \left(a+bx^2\right)^{5/2} \, dx = \frac{\sqrt{b}\sqrt{a+bx^2}(-5a^4d^2(512Ad+3B(512c+63dx))+10a^3b(Ad(3456c^2+945cdx+128d^2x^2)}{}$$

input
```
Integrate[(A + B*x)*(c + d*x)^3*(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(-5*a^4*d^2*(512*A*d + 3*B*(512*c + 63*d*x)) + 10
*a^3*b*(A*d*(3456*c^2 + 945*c*d*x + 128*d^2*x^2) + 3*B*(384*c^3 + 315*c^2*
d*x + 128*c*d^2*x^2 + 21*d^3*x^3)) + 32*b^4*x^5*(5*A*(84*c^3 + 216*c^2*d*x
 + 189*c*d^2*x^2 + 56*d^3*x^3) + 3*B*x*(120*c^3 + 315*c^2*d*x + 280*c*d^2*
x^2 + 84*d^3*x^3)) + 12*a^2*b^2*x*(5*A*(924*c^3 + 1728*c^2*d*x + 1239*c*d^
2*x^2 + 320*d^3*x^3) + 3*B*x*(960*c^3 + 2065*c^2*d*x + 1600*c*d^2*x^2 + 43
4*d^3*x^3)) + 16*a*b^3*x^3*(5*A*(546*c^3 + 1296*c^2*d*x + 1071*c*d^2*x^2 +
 304*d^3*x^3) + 3*B*x*(720*c^3 + 1785*c^2*d*x + 1520*c*d^2*x^2 + 441*d^3*x
^3))) - 315*a^3*(10*A*b*c*(8*b*c^2 - 3*a*d^2) + 3*a*B*d*(-10*b*c^2 + a*d^2
))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(80640*b^(5/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.78, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {687, 687, 27, 676, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} (A+Bx)(c+dx)^3 \, dx$$

$$\Big\downarrow \text{687}$$

$$\frac{\int (c+dx)^2(10Abc-3aBd+b(3Bc+10Ad)x) \left(bx^2+a\right)^{5/2}dx}{10b}+\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^3}{10b}$$

$$\Big\downarrow \text{687}$$

$$\frac{\frac{\int b(c+dx)\left(90Abc^2-33aBdc-20aAd^2-\left(27aBd^2-2bc(3Bc+55Ad)\right)x\right)\left(bx^2+a\right)^{5/2}dx}{9b}+\frac{1}{9}\left(a+bx^2\right)^{7/2}(c+dx)^2(10Ad+3Bc)}{10b}+\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^3}{10b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{9}\int (c+dx)\left(90Abc^2-33aBdc-20aAd^2-\left(27aBd^2-2bc(3Bc+55Ad)\right)x\right)\left(bx^2+a\right)^{5/2}dx+\frac{1}{9}\left(a+bx^2\right)^{7/2}(c}{10b}$$
$$\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^3}{10b}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{1}{9}\left(\frac{9\left(10Abc\left(8bc^2-3ad^2\right)-3aBd\left(10bc^2-ad^2\right)\right)\int\left(bx^2+a\right)^{5/2}dx}{8b}-\frac{2\left(a+bx^2\right)^{7/2}\left(10ad^2(Ad+3Bc)-bc^2(100Ad+3Bc)\right)}{7b}-\frac{dx\left(a+bx^2\right)^{7/2}(27aB}{}\right.}{10b}$$
$$\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^3}{10b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{1}{9}\left(\frac{9\left(10Abc\left(8bc^2-3ad^2\right)-3aBd\left(10bc^2-ad^2\right)\right)\left(\frac{5}{6}a\int\left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}-\frac{2\left(a+bx^2\right)^{7/2}\left(10ad^2(Ad+3Bc)-bc^2(100Ad+3Bc)\right)}{7b}\right.}{10b}$$

$$\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^3}{10b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{1}{9}\left(\frac{9\left(10Abc\left(8bc^2-3ad^2\right)-3aBd\left(10bc^2-ad^2\right)\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}-\frac{2\left(a+bx^2\right)^{7/2}\left(10ad^2(Ad+3Bc)-b\right.}{7b}\right.}{10b}$$

$$\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^3}{10b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{1}{9}\left(\frac{9\left(10Abc\left(8bc^2-3ad^2\right)-3aBd\left(10bc^2-ad^2\right)\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}-\frac{2\left(a+bx^2\right)^{7/2}}{}\right.}{10b}$$

$$\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^3}{10b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{9}\left(\frac{9\left(10Abc\left(8bc^2-3ad^2\right)-3aBd\left(10bc^2-ad^2\right)\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}-2(\right.}{}$$

$$\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^3}{10b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{9}\left(\frac{9\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)\left(10Abc\left(8bc^2-3ad^2\right)-3aBd\left(10bc^2-ad^2\right)\right)}{8b}-2(a\right.}{}$$

$$\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^3}{10b}$$

input
```
Int[(A + B*x)*(c + d*x)^3*(a + b*x^2)^(5/2),x]
```

output
```
(B*(c + d*x)^3*(a + b*x^2)^(7/2))/(10*b) + (((3*B*c + 10*A*d)*(c + d*x)^2*
(a + b*x^2)^(7/2))/9 + ((-2*(10*a*d^2*(3*B*c + A*d) - b*c^2*(3*B*c + 100*A
*d))*(a + b*x^2)^(7/2))/(7*b) - (d*(27*a*B*d^2 - 2*b*c*(3*B*c + 55*A*d))*x
*(a + b*x^2)^(7/2))/(8*b) + (9*(10*A*b*c*(8*b*c^2 - 3*a*d^2) - 3*a*B*d*(10
*b*c^2 - a*d^2))*((x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(a + b*x^2)^(3/2))/4
+ (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(
2*Sqrt[b])))/4))/6))/(8*b))/9)/(10*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.96

method	result
default	$A\,c^3\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+d^2(Ad+3Bc)\left(\frac{x^2(b\,x^2+a)^{\frac{7}{2}}}{9b}-\right.$
risch	$-\frac{(-8064B\,b^4d^3x^9-8960A\,b^4d^3x^8-26880B\,b^4c\,d^2x^8-30240A\,b^4c\,d^2x^7-21168Ba\,b^3d^3x^7-30240B\,b^4c^2d\,x^7-24320Aa\,b^3d^3x^6-3}{}$

input
```
int((B*x+A)*(d*x+c)^3*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*c^3*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*
x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+d^2*(A*d+3*B*c
)*(1/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2))+3*c*d*(A*d+B*c)*(
1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2
+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a
)^(1/2))))))+1/7*c^2*(3*A*d+B*c)*(b*x^2+a)^(7/2)/b+B*d^3*(1/10*x^3*(b*x^2+
a)^(7/2)/b-3/10*a/b*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2
)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*
ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1032, normalized size of antiderivative = 2.74

$$\int (A+Bx)(c+dx)^3 \left(a+bx^2\right)^{5/2} \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(d*x+c)^3*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/161280*(315*(80*A*a^3*b^2*c^3 - 30*B*a^4*b*c^2*d - 30*A*a^4*b*c*d^2 + 3
*B*a^5*d^3)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8
064*B*b^5*d^3*x^9 + 11520*B*a^3*b^2*c^3 + 34560*A*a^3*b^2*c^2*d - 7680*B*a
^4*b*c*d^2 - 2560*A*a^4*b*d^3 + 8960*(3*B*b^5*c*d^2 + A*b^5*d^3)*x^8 + 302
4*(10*B*b^5*c^2*d + 10*A*b^5*c*d^2 + 7*B*a*b^4*d^3)*x^7 + 1280*(9*B*b^5*c^
3 + 27*A*b^5*c^2*d + 57*B*a*b^4*c*d^2 + 19*A*a*b^4*d^3)*x^6 + 168*(80*A*b^
5*c^3 + 510*B*a*b^4*c^2*d + 510*A*a*b^4*c*d^2 + 93*B*a^2*b^3*d^3)*x^5 + 38
40*(9*B*a*b^4*c^3 + 27*A*a*b^4*c^2*d + 15*B*a^2*b^3*c*d^2 + 5*A*a^2*b^3*d^
3)*x^4 + 210*(208*A*a*b^4*c^3 + 354*B*a^2*b^3*c^2*d + 354*A*a^2*b^3*c*d^2
+ 3*B*a^3*b^2*d^3)*x^3 + 1280*(27*B*a^2*b^3*c^3 + 81*A*a^2*b^3*c^2*d + 3*B
*a^3*b^2*c*d^2 + A*a^3*b^2*d^3)*x^2 + 315*(176*A*a^2*b^3*c^3 + 30*B*a^3*b^
2*c^2*d + 30*A*a^3*b^2*c*d^2 - 3*B*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/b^3, -1/
80640*(315*(80*A*a^3*b^2*c^3 - 30*B*a^4*b*c^2*d - 30*A*a^4*b*c*d^2 + 3*B*a
^5*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8064*B*b^5*d^3*x^9
+ 11520*B*a^3*b^2*c^3 + 34560*A*a^3*b^2*c^2*d - 7680*B*a^4*b*c*d^2 - 2560*
A*a^4*b*d^3 + 8960*(3*B*b^5*c*d^2 + A*b^5*d^3)*x^8 + 3024*(10*B*b^5*c^2*d
+ 10*A*b^5*c*d^2 + 7*B*a*b^4*d^3)*x^7 + 1280*(9*B*b^5*c^3 + 27*A*b^5*c^2*d
 + 57*B*a*b^4*c*d^2 + 19*A*a*b^4*d^3)*x^6 + 168*(80*A*b^5*c^3 + 510*B*a*b^
4*c^2*d + 510*A*a*b^4*c*d^2 + 93*B*a^2*b^3*d^3)*x^5 + 3840*(9*B*a*b^4*c^3
+ 27*A*a*b^4*c^2*d + 15*B*a^2*b^3*c*d^2 + 5*A*a^2*b^3*d^3)*x^4 + 210*(2...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1486 vs. $2(366) = 732$.

Time = 1.04 (sec) , antiderivative size = 1486, normalized size of antiderivative = 3.94

$$\int (A+Bx)(c+dx)^3\left(a+bx^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)*(d*x+c)**3*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*b**2*d**3*x**9/10 + x**8*(A*b**3*d**3 + 3*B
*b**3*c*d**2)/(9*b) + x**7*(3*A*b**3*c*d**2 + 21*B*a*b**2*d**3/10 + 3*B*b*
*3*c**2*d)/(8*b) + x**6*(3*A*a*b**2*d**3 + 3*A*b**3*c**2*d + 9*B*a*b**2*c*
d**2 + B*b**3*c**3 - 8*a*(A*b**3*d**3 + 3*B*b**3*c*d**2)/(9*b))/(7*b) + x*
*5*(9*A*a*b**2*c*d**2 + A*b**3*c**3 + 3*B*a**2*b*d**3 + 9*B*a*b**2*c**2*d
- 7*a*(3*A*b**3*c*d**2 + 21*B*a*b**2*d**3/10 + 3*B*b**3*c**2*d)/(8*b))/(6*
b) + x**4*(3*A*a**2*b*d**3 + 9*A*a*b**2*c**2*d + 9*B*a**2*b*c*d**2 + 3*B*a
*b**2*c**3 - 6*a*(3*A*a*b**2*d**3 + 3*A*b**3*c**2*d + 9*B*a*b**2*c*d**2 +
B*b**3*c**3 - 8*a*(A*b**3*d**3 + 3*B*b**3*c*d**2)/(9*b))/(7*b))/(5*b) + x*
*3*(9*A*a**2*b*c*d**2 + 3*A*a*b**2*c**3 + B*a**3*d**3 + 9*B*a**2*b*c**2*d
- 5*a*(9*A*a*b**2*c*d**2 + A*b**3*c**3 + 3*B*a**2*b*d**3 + 9*B*a*b**2*c**2
*d - 7*a*(3*A*b**3*c*d**2 + 21*B*a*b**2*d**3/10 + 3*B*b**3*c**2*d)/(8*b))/
(6*b))/(4*b) + x**2*(A*a**3*d**3 + 9*A*a**2*b*c**2*d + 3*B*a**3*c*d**2 + 3
*B*a**2*b*c**3 - 4*a*(3*A*a**2*b*d**3 + 9*A*a*b**2*c**2*d + 9*B*a**2*b*c*d
**2 + 3*B*a*b**2*c**3 - 6*a*(3*A*a*b**2*d**3 + 3*A*b**3*c**2*d + 9*B*a*b**
2*c*d**2 + B*b**3*c**3 - 8*a*(A*b**3*d**3 + 3*B*b**3*c*d**2)/(9*b))/(7*b))
/(5*b))/(3*b) + x*(3*A*a**3*c*d**2 + 3*A*a**2*b*c**3 + 3*B*a**3*c**2*d - 3
*a*(9*A*a**2*b*c*d**2 + 3*A*a*b**2*c**3 + B*a**3*d**3 + 9*B*a**2*b*c**2*d
- 5*a*(9*A*a*b**2*c*d**2 + A*b**3*c**3 + 3*B*a**2*b*d**3 + 9*B*a*b**2*c**2
*d - 7*a*(3*A*b**3*c*d**2 + 21*B*a*b**2*d**3/10 + 3*B*b**3*c**2*d)/(8*b...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.18

$$\int (A+Bx)(c+dx)^3 \left(a+bx^2\right)^{5/2} dx = \frac{(bx^2+a)^{\frac{7}{2}}Bd^3x^3}{10\,b} + \frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Ac^3x + \frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Aac^3x + \frac{5}{16}\sqrt{bx^2+a}Aa^2c^3x - \frac{3\left(bx^2+a\right)^{\frac{7}{2}}Bad^3x}{80\,b^2} + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Ba^2d^3x}{160\,b^2} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}Ba^3d^3x}{128\,b^2} + \frac{3\sqrt{bx^2+a}Ba^4d^3x}{256\,b^2} + \frac{5\,Aa^3c^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}} + \frac{3\,Ba^5d^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}} + \frac{\left(bx^2+a\right)^{\frac{7}{2}}Bc^3}{7\,b} + \frac{3\left(bx^2+a\right)^{\frac{7}{2}}Ac^2d}{7\,b} + \frac{(3\,Bcd^2+Ad^3)(bx^2+a)^{\frac{7}{2}}x^2}{9\,b} + \frac{3\,(Bc^2d+Acd^2)(bx^2+a)^{\frac{7}{2}}x}{8\,b} - \frac{(Bc^2d+Acd^2)(bx^2+a)^{\frac{5}{2}}ax}{16\,b} - \frac{5\,(Bc^2d+Acd^2)(bx^2+a)^{\frac{3}{2}}a^2x}{64\,b} - \frac{15\,(Bc^2d+Acd^2)\sqrt{bx^2+a}a^3x}{128\,b} - \frac{15\,(Bc^2d+Acd^2)a^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}} - \frac{2\,(3\,Bcd^2+Ad^3)(bx^2+a)^{\frac{7}{2}}a}{63\,b^2}$$

input
```
integrate((B*x+A)*(d*x+c)^3*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(7/2)*B*d^3*x^3/b + 1/6*(b*x^2 + a)^(5/2)*A*c^3*x + 5/24*
(b*x^2 + a)^(3/2)*A*a*c^3*x + 5/16*sqrt(b*x^2 + a)*A*a^2*c^3*x - 3/80*(b*x
^2 + a)^(7/2)*B*a*d^3*x/b^2 + 1/160*(b*x^2 + a)^(5/2)*B*a^2*d^3*x/b^2 + 1/
128*(b*x^2 + a)^(3/2)*B*a^3*d^3*x/b^2 + 3/256*sqrt(b*x^2 + a)*B*a^4*d^3*x/
b^2 + 5/16*A*a^3*c^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 3/256*B*a^5*d^3*arcs
inh(b*x/sqrt(a*b))/b^(5/2) + 1/7*(b*x^2 + a)^(7/2)*B*c^3/b + 3/7*(b*x^2 +
a)^(7/2)*A*c^2*d/b + 1/9*(3*B*c*d^2 + A*d^3)*(b*x^2 + a)^(7/2)*x^2/b + 3/8
*(B*c^2*d + A*c*d^2)*(b*x^2 + a)^(7/2)*x/b - 1/16*(B*c^2*d + A*c*d^2)*(b*x
^2 + a)^(5/2)*a*x/b - 5/64*(B*c^2*d + A*c*d^2)*(b*x^2 + a)^(3/2)*a^2*x/b -
 15/128*(B*c^2*d + A*c*d^2)*sqrt(b*x^2 + a)*a^3*x/b - 15/128*(B*c^2*d + A*
c*d^2)*a^4*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/63*(3*B*c*d^2 + A*d^3)*(b*x^
2 + a)^(7/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.45

$$\int (A+Bx)(c+dx)^3\left(a+bx^2\right)^{5/2}\,dx=\frac{1}{80640}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(\left(2\left(7\left(8\left(9\,Bb^2d^3x+\frac{10\,(3\,Bb^{10}cd^2+Ab^{10}d^3)}{b^8}\right)x+\frac{27\,(10\,Bb\right.\right.\right.\right.\right.\right.\right.$$
$$-\frac{(80\,Aa^3b^2c^3-30\,Ba^4bc^2d-30\,Aa^4bcd^2+3\,Ba^5d^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^3*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/80640*sqrt(b*x^2 + a)*((2*((4*((2*(7*(8*(9*B*b^2*d^3*x + 10*(3*B*b^10*c*
d^2 + A*b^10*d^3)/b^8)*x + 27*(10*B*b^10*c^2*d + 10*A*b^10*c*d^2 + 7*B*a*b
^9*d^3)/b^8)*x + 80*(9*B*b^10*c^3 + 27*A*b^10*c^2*d + 57*B*a*b^9*c*d^2 + 1
9*A*a*b^9*d^3)/b^8)*x + 21*(80*A*b^10*c^3 + 510*B*a*b^9*c^2*d + 510*A*a*b^
9*c*d^2 + 93*B*a^2*b^8*d^3)/b^8)*x + 480*(9*B*a*b^9*c^3 + 27*A*a*b^9*c^2*d
 + 15*B*a^2*b^8*c*d^2 + 5*A*a^2*b^8*d^3)/b^8)*x + 105*(208*A*a*b^9*c^3 + 3
54*B*a^2*b^8*c^2*d + 354*A*a^2*b^8*c*d^2 + 3*B*a^3*b^7*d^3)/b^8)*x + 640*(
27*B*a^2*b^8*c^3 + 81*A*a^2*b^8*c^2*d + 3*B*a^3*b^7*c*d^2 + A*a^3*b^7*d^3)
/b^8)*x + 315*(176*A*a^2*b^8*c^3 + 30*B*a^3*b^7*c^2*d + 30*A*a^3*b^7*c*d^2
 - 3*B*a^4*b^6*d^3)/b^8)*x + 1280*(9*B*a^3*b^7*c^3 + 27*A*a^3*b^7*c^2*d -
6*B*a^4*b^6*c*d^2 - 2*A*a^4*b^6*d^3)/b^8) - 1/256*(80*A*a^3*b^2*c^3 - 30*B
*a^4*b*c^2*d - 30*A*a^4*b*c*d^2 + 3*B*a^5*d^3)*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)(c+dx)^3\left(a+bx^2\right)^{5/2}\,dx=\int {\left(b\,x^2+a\right)}^{5/2}\,\left(A+B\,x\right)\,{\left(c+d\,x\right)}^3\,dx$$

input
```
int((a + b*x^2)^(5/2)*(A + B*x)*(c + d*x)^3,x)
```

output
```
int((a + b*x^2)^(5/2)*(A + B*x)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (A+Bx)(c+dx)^3 \left(a+bx^2\right)^{5/2} \, dx = \int (Bx+A)\,(dx+c)^3 \left(b\,x^2+a\right)^{\frac{5}{2}} dx$$

input
```
int((B*x+A)*(d*x+c)^3*(b*x^2+a)^(5/2),x)
```

output
```
int((B*x+A)*(d*x+c)^3*(b*x^2+a)^(5/2),x)
```

3.163 $\int (A+Bx)(c+dx)^2 \left(a+bx^2\right)^{5/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 257

$$\int (A+Bx)(c+dx)^2 \left(a+bx^2\right)^{5/2} \, dx = \frac{5a^2(8Abc^2-2aBcd-aAd^2)\,x\sqrt{a+bx^2}}{128b} + \frac{5a(8Abc^2-2aBcd-aAd^2)\,x(a+bx^2)^{3/2}}{192b} + \frac{(8Abc^2-2aBcd-aAd^2)\,x(a+bx^2)^{5/2}}{48b} + \frac{B(c+dx)^2\left(a+bx^2\right)^{7/2}}{9b} - \frac{(16(aBd^2-bc(Bc+9Ad))-7bd(2Bc+9Ad)x)\left(a+bx^2\right)^{7/2}}{504b^2} + \frac{5a^3(8Abc^2-2aBcd-aAd^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

output
```
5/128*a^2*(-A*a*d^2+8*A*b*c^2-2*B*a*c*d)*x*(b*x^2+a)^(1/2)/b+5/192*a*(-A*a
*d^2+8*A*b*c^2-2*B*a*c*d)*x*(b*x^2+a)^(3/2)/b+1/48*(-A*a*d^2+8*A*b*c^2-2*B
*a*c*d)*x*(b*x^2+a)^(5/2)/b+1/9*B*(d*x+c)^2*(b*x^2+a)^(7/2)/b-1/504*(16*a*
B*d^2-16*b*c*(9*A*d+B*c)-7*b*d*(9*A*d+2*B*c)*x)*(b*x^2+a)^(7/2)/b^2+5/128*
a^3*(-A*a*d^2+8*A*b*c^2-2*B*a*c*d)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3
/2)
```

Mathematica [A] (verified)

Time = 2.46 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.10

$$\int (A+Bx)(c+dx)^2 \left(a+bx^2\right)^{5/2} dx = \frac{\sqrt{a+bx^2}(-256a^4Bd^2+16b^4x^5(3A(28c^2+48cdx+21d^2x^2)+2Bx(36c^2+63cdx+28d^2x^2$$

input
```
Integrate[(A + B*x)*(c + d*x)^2*(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[a + b*x^2]*(-256*a^4*B*d^2 + 16*b^4*x^5*(3*A*(28*c^2 + 48*c*d*x + 21
*d^2*x^2) + 2*B*x*(36*c^2 + 63*c*d*x + 28*d^2*x^2)) + a^3*b*(9*A*d*(256*c
+ 35*d*x) + 2*B*(576*c^2 + 315*c*d*x + 64*d^2*x^2)) + 8*a*b^3*x^3*(2*B*x*(
216*c^2 + 357*c*d*x + 152*d^2*x^2) + A*(546*c^2 + 864*c*d*x + 357*d^2*x^2)
) + 6*a^2*b^2*x*(2*B*x*(288*c^2 + 413*c*d*x + 160*d^2*x^2) + A*(924*c^2 +
1152*c*d*x + 413*d^2*x^2))) + 315*a^3*Sqrt[b]*(-8*A*b*c^2 + 2*a*B*c*d + a*
A*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8064*b^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {687, 676, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2}(A+Bx)(c+dx)^2 dx$$

$$\Big\downarrow \text{687}$$

$$\frac{\int (c+dx)(9Abc-2aBd+b(2Bc+9Ad)x)\left(bx^2+a\right)^{5/2}dx}{9b}+\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^2}{9b}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{9}{8}\left(-aAd^2-2aBcd+8Abc^2\right)\int\left(bx^2+a\right)^{5/2}dx-\frac{2\left(a+bx^2\right)^{7/2}\left(aBd^2-bc(9Ad+Bc)\right)}{7b}+\frac{1}{8}dx\left(a+bx^2\right)^{7/2}(9Ad+2Bc)}{9b}+$$
$$\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^2}{9b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{9}{8}\left(-aAd^2-2aBcd+8Abc^2\right)\left(\frac{5}{6}a\int\left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)-\frac{2\left(a+bx^2\right)^{7/2}\left(aBd^2-bc(9Ad+Bc)\right)}{7b}+\frac{1}{8}dx\left(a+\right.}{9b}$$
$$\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^2}{9b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{9}{8}\left(-aAd^2-2aBcd+8Abc^2\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)-\frac{2\left(a+bx^2\right)^{7/2}\left(aBd^2-b\right.}{7b}}{9b}$$
$$\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^2}{9b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{9}{8}\left(-aAd^2-2aBcd+8Abc^2\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)-}{9b}$$
$$\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^2}{9b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{9}{8}\left(-aAd^2-2aBcd+8Abc^2\right)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right.}{9b}$$
$$\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^2}{9b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{9}{8}\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)\left(-aAd^2-2aBcd+8A\right.}{9b}$$

$$\frac{B\left(a+bx^2\right)^{7/2}(c+dx)^2}{9b}$$

input
```
Int[(A + B*x)*(c + d*x)^2*(a + b*x^2)^(5/2),x]
```

output
```
(B*(c + d*x)^2*(a + b*x^2)^(7/2))/(9*b) + ((-2*(a*B*d^2 - b*c*(B*c + 9*A*d
))*(a + b*x^2)^(7/2))/(7*b) + (d*(2*B*c + 9*A*d)*x*(a + b*x^2)^(7/2))/8 +
(9*(8*A*b*c^2 - 2*a*B*c*d - a*A*d^2)*((x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(
a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6))/8)/(9*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.91

method	result
default	$A\,c^{2}\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+d(Ad+2Bc)\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8b}-\right.$
risch	$\frac{(896B\,b^{4}d^{2}x^{8}+1008A\,b^{4}d^{2}x^{7}+2016B\,b^{4}cd\,x^{7}+2304A\,b^{4}cd\,x^{6}+2432Ba\,b^{3}d^{2}x^{6}+1152B\,b^{4}c^{2}x^{6}+2856Aa\,b^{3}d^{2}x^{5}+1344A\,b^{4}c^{2}x^{5}+5}{}$

input

```
int((B*x+A)*(d*x+c)^2*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
A*c^2*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*
x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+d*(A*d+2*B*c)*
(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^
2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+
a)^(1/2))))))+1/7*c*(2*A*d+B*c)*(b*x^2+a)^(7/2)/b+B*d^2*(1/9*x^2*(b*x^2+a)
^(7/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.74

$$\int (A+Bx)(c+dx)^2 \left(a+bx^2\right)^{5/2} dx = \left[-\frac{315\,(8\,Aa^3bc^2-2\,Ba^4cd-Aa^4d^2)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(896\,Bb^4d\right.$$

$$-\frac{315\,(8\,Aa^3bc^2-2\,Ba^4cd-Aa^4d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(896\,Bb^4d^2x^8+1008\,(2\,Bb^4cd+Ab^4d^2)x^7+}{}$$

input
```
integrate((B*x+A)*(d*x+c)^2*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/16128*(315*(8*A*a^3*b*c^2 - 2*B*a^4*c*d - A*a^4*d^2)*sqrt(b)*log(-2*b*
x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(896*B*b^4*d^2*x^8 + 1008*(2*B*
b^4*c*d + A*b^4*d^2)*x^7 + 1152*B*a^3*b*c^2 + 2304*A*a^3*b*c*d - 256*B*a^4
*d^2 + 128*(9*B*b^4*c^2 + 18*A*b^4*c*d + 19*B*a*b^3*d^2)*x^6 + 168*(8*A*b^
4*c^2 + 34*B*a*b^3*c*d + 17*A*a*b^3*d^2)*x^5 + 384*(9*B*a*b^3*c^2 + 18*A*a
*b^3*c*d + 5*B*a^2*b^2*d^2)*x^4 + 42*(104*A*a*b^3*c^2 + 118*B*a^2*b^2*c*d
 + 59*A*a^2*b^2*d^2)*x^3 + 128*(27*B*a^2*b^2*c^2 + 54*A*a^2*b^2*c*d + B*a^3
*b*d^2)*x^2 + 63*(88*A*a^2*b^2*c^2 + 10*B*a^3*b*c*d + 5*A*a^3*b*d^2)*x)*sq
rt(b*x^2 + a))/b^2, -1/8064*(315*(8*A*a^3*b*c^2 - 2*B*a^4*c*d - A*a^4*d^2)
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (896*B*b^4*d^2*x^8 + 1008*(
2*B*b^4*c*d + A*b^4*d^2)*x^7 + 1152*B*a^3*b*c^2 + 2304*A*a^3*b*c*d - 256*B
*a^4*d^2 + 128*(9*B*b^4*c^2 + 18*A*b^4*c*d + 19*B*a*b^3*d^2)*x^6 + 168*(8*
A*b^4*c^2 + 34*B*a*b^3*c*d + 17*A*a*b^3*d^2)*x^5 + 384*(9*B*a*b^3*c^2 + 18
*A*a*b^3*c*d + 5*B*a^2*b^2*d^2)*x^4 + 42*(104*A*a*b^3*c^2 + 118*B*a^2*b^2*
c*d + 59*A*a^2*b^2*d^2)*x^3 + 128*(27*B*a^2*b^2*c^2 + 54*A*a^2*b^2*c*d + B
*a^3*b*d^2)*x^2 + 63*(88*A*a^2*b^2*c^2 + 10*B*a^3*b*c*d + 5*A*a^3*b*d^2)*x
)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(257) = 514$.

Time = 0.77 (sec) , antiderivative size = 994, normalized size of antiderivative = 3.87

$$\int (A+Bx)(c+dx)^2 \left(a+bx^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(d*x+c)**2*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*b**2*d**2*x**8/9 + x**7*(A*b**3*d**2 + 2*B*
b**3*c*d)/(8*b) + x**6*(2*A*b**3*c*d + 19*B*a*b**2*d**2/9 + B*b**3*c**2)/(
7*b) + x**5*(3*A*a*b**2*d**2 + A*b**3*c**2 + 6*B*a*b**2*c*d - 7*a*(A*b**3*
d**2 + 2*B*b**3*c*d)/(8*b))/(6*b) + x**4*(6*A*a*b**2*c*d + 3*B*a**2*b*d**2
 + 3*B*a*b**2*c**2 - 6*a*(2*A*b**3*c*d + 19*B*a*b**2*d**2/9 + B*b**3*c**2)
/(7*b))/(5*b) + x**3*(3*A*a**2*b*d**2 + 3*A*a*b**2*c**2 + 6*B*a**2*b*c*d -
 5*a*(3*A*a*b**2*d**2 + A*b**3*c**2 + 6*B*a*b**2*c*d - 7*a*(A*b**3*d**2 +
2*B*b**3*c*d)/(8*b))/(6*b))/(4*b) + x**2*(6*A*a**2*b*c*d + B*a**3*d**2 + 3
*B*a**2*b*c**2 - 4*a*(6*A*a*b**2*c*d + 3*B*a**2*b*d**2 + 3*B*a*b**2*c**2 -
 6*a*(2*A*b**3*c*d + 19*B*a*b**2*d**2/9 + B*b**3*c**2)/(7*b))/(5*b))/(3*b)
 + x*(A*a**3*d**2 + 3*A*a**2*b*c**2 + 2*B*a**3*c*d - 3*a*(3*A*a**2*b*d**2
+ 3*A*a*b**2*c**2 + 6*B*a**2*b*c*d - 5*a*(3*A*a*b**2*d**2 + A*b**3*c**2 +
6*B*a*b**2*c*d - 7*a*(A*b**3*d**2 + 2*B*b**3*c*d)/(8*b))/(6*b))/(4*b))/(2*
b) + (2*A*a**3*c*d + B*a**3*c**2 - 2*a*(6*A*a**2*b*c*d + B*a**3*d**2 + 3*B
*a**2*b*c**2 - 4*a*(6*A*a*b**2*c*d + 3*B*a**2*b*d**2 + 3*B*a*b**2*c**2 - 6
*a*(2*A*b**3*c*d + 19*B*a*b**2*d**2/9 + B*b**3*c**2)/(7*b))/(5*b))/(3*b))/
b) + (A*a**3*c**2 - a*(A*a**3*d**2 + 3*A*a**2*b*c**2 + 2*B*a**3*c*d - 3*a*
(3*A*a**2*b*d**2 + 3*A*a*b**2*c**2 + 6*B*a**2*b*c*d - 5*a*(3*A*a*b**2*d**2
 + A*b**3*c**2 + 6*B*a*b**2*c*d - 7*a*(A*b**3*d**2 + 2*B*b**3*c*d)/(8*b))/
(6*b))/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.12

$$\int (A+Bx)(c+dx)^2 \left(a+bx^2\right)^{5/2} dx = \frac{(bx^2+a)^{\frac{7}{2}}Bd^2x^2}{9\,b} + \frac{1}{6}\,(bx^2+a)^{\frac{5}{2}}Ac^2x + \frac{5}{24}\,(bx^2+a)^{\frac{3}{2}}Aac^2x + \frac{5}{16}\,\sqrt{bx^2+a}Aa^2c^2x + \frac{5\,Aa^3c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}} + \frac{(bx^2+a)^{\frac{7}{2}}Bc^2}{7\,b} + \frac{2\,(bx^2+a)^{\frac{7}{2}}Acd}{7\,b} - \frac{2\,(bx^2+a)^{\frac{7}{2}}Bad^2}{63\,b^2} + \frac{(2\,Bcd+Ad^2)(bx^2+a)^{\frac{7}{2}}x}{8\,b} - \frac{(2\,Bcd+Ad^2)(bx^2+a)^{\frac{5}{2}}ax}{48\,b} - \frac{5\,(2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}a^2x}{192\,b} - \frac{5\,(2\,Bcd+Ad^2)\sqrt{bx^2+a}a^3x}{128\,b} - \frac{5\,(2\,Bcd+Ad^2)a^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^2*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/9*(b*x^2 + a)^(7/2)*B*d^2*x^2/b + 1/6*(b*x^2 + a)^(5/2)*A*c^2*x + 5/24*(
b*x^2 + a)^(3/2)*A*a*c^2*x + 5/16*sqrt(b*x^2 + a)*A*a^2*c^2*x + 5/16*A*a^3
*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/7*(b*x^2 + a)^(7/2)*B*c^2/b + 2/7*
(b*x^2 + a)^(7/2)*A*c*d/b - 2/63*(b*x^2 + a)^(7/2)*B*a*d^2/b^2 + 1/8*(2*B*
c*d + A*d^2)*(b*x^2 + a)^(7/2)*x/b - 1/48*(2*B*c*d + A*d^2)*(b*x^2 + a)^(5
/2)*a*x/b - 5/192*(2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)*a^2*x/b - 5/128*(2*B
*c*d + A*d^2)*sqrt(b*x^2 + a)*a^3*x/b - 5/128*(2*B*c*d + A*d^2)*a^4*arcsin
h(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.50

$$\int (A+Bx)(c+dx)^2 \left(a+bx^2\right)^{5/2} dx = \frac{1}{8064}\,\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(\left(2\left(7\left(8\,Bb^2d^2x + \frac{9\,(2\,Bb^9cd+Ab^9d^2)}{b^7}\right)x + \frac{8\,(9\,Bb^9c^2+18\,A}{b}\right.\right.\right.\right.\right.\right.\right. - \frac{5\,(8\,Aa^3bc^2 - 2\,Ba^4cd - Aa^4d^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^2*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/8064*sqrt(b*x^2 + a)*((2*((4*((2*(7*(8*B*b^2*d^2*x + 9*(2*B*b^9*c*d + A*
b^9*d^2)/b^7)*x + 8*(9*B*b^9*c^2 + 18*A*b^9*c*d + 19*B*a*b^8*d^2)/b^7)*x +
 21*(8*A*b^9*c^2 + 34*B*a*b^8*c*d + 17*A*a*b^8*d^2)/b^7)*x + 48*(9*B*a*b^8
*c^2 + 18*A*a*b^8*c*d + 5*B*a^2*b^7*d^2)/b^7)*x + 21*(104*A*a*b^8*c^2 + 11
8*B*a^2*b^7*c*d + 59*A*a^2*b^7*d^2)/b^7)*x + 64*(27*B*a^2*b^7*c^2 + 54*A*a
^2*b^7*c*d + B*a^3*b^6*d^2)/b^7)*x + 63*(88*A*a^2*b^7*c^2 + 10*B*a^3*b^6*c
*d + 5*A*a^3*b^6*d^2)/b^7)*x + 128*(9*B*a^3*b^6*c^2 + 18*A*a^3*b^6*c*d - 2
*B*a^4*b^5*d^2)/b^7) - 5/128*(8*A*a^3*b*c^2 - 2*B*a^4*c*d - A*a^4*d^2)*log
(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)(c+dx)^2 \left(a+bx^2\right)^{5/2} \, dx = \int \left(b\,x^2+a\right)^{5/2}\,\left(A+B\,x\right)\,\left(c+d\,x\right)^2 \,d x$$

input
```
int((a + b*x^2)^(5/2)*(A + B*x)*(c + d*x)^2,x)
```

output
```
int((a + b*x^2)^(5/2)*(A + B*x)*(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 5.02 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.23

$$\int (A+Bx)(c+dx)^2 \left(a+bx^2\right)^{5/2} \, dx = \frac{315\sqrt{b\,x^2+a}\,a^4 b\,d^2 x+5544\sqrt{b\,x^2+a}\,a^3 b^2 c^2 x+2478\sqrt{b\,x^2+a}\,a^3 b^2 d^2 x^3+128\sqrt{b\,x^2+a}\,a^3}{}$$

input
```
int((B*x+A)*(d*x+c)^2*(b*x^2+a)^(5/2),x)
```

output

```
(2304*sqrt(a + b*x**2)*a**4*b*c*d + 315*sqrt(a + b*x**2)*a**4*b*d**2*x - 2
56*sqrt(a + b*x**2)*a**4*b*d**2 + 5544*sqrt(a + b*x**2)*a**3*b**2*c**2*x +
 1152*sqrt(a + b*x**2)*a**3*b**2*c**2 + 6912*sqrt(a + b*x**2)*a**3*b**2*c*
d*x**2 + 630*sqrt(a + b*x**2)*a**3*b**2*c*d*x + 2478*sqrt(a + b*x**2)*a**3
*b**2*d**2*x**3 + 128*sqrt(a + b*x**2)*a**3*b**2*d**2*x**2 + 4368*sqrt(a +
 b*x**2)*a**2*b**3*c**2*x**3 + 3456*sqrt(a + b*x**2)*a**2*b**3*c**2*x**2 +
 6912*sqrt(a + b*x**2)*a**2*b**3*c*d*x**4 + 4956*sqrt(a + b*x**2)*a**2*b**
3*c*d*x**3 + 2856*sqrt(a + b*x**2)*a**2*b**3*d**2*x**5 + 1920*sqrt(a + b*x
**2)*a**2*b**3*d**2*x**4 + 1344*sqrt(a + b*x**2)*a*b**4*c**2*x**5 + 3456*s
qrt(a + b*x**2)*a*b**4*c**2*x**4 + 2304*sqrt(a + b*x**2)*a*b**4*c*d*x**6 +
 5712*sqrt(a + b*x**2)*a*b**4*c*d*x**5 + 1008*sqrt(a + b*x**2)*a*b**4*d**2
*x**7 + 2432*sqrt(a + b*x**2)*a*b**4*d**2*x**6 + 1152*sqrt(a + b*x**2)*b**
5*c**2*x**6 + 2016*sqrt(a + b*x**2)*b**5*c*d*x**7 + 896*sqrt(a + b*x**2)*b
**5*d**2*x**8 - 315*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*
*5*d**2 + 2520*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*
c**2 - 630*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*c*d)
/(8064*b**2)
```

3.164 $\int (A+Bx)(c+dx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 170

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{5/2}\,dx=\frac{5a^2(8Abc-aBd)x\sqrt{a+bx^2}}{128b}+\frac{5a(8Abc-aBd)x(a+bx^2)^{3/2}}{192b}+\frac{(8Abc-aBd)x(a+bx^2)^{5/2}}{48b}+\frac{(8(Bc+Ad)+7Bdx)\left(a+bx^2\right)^{7/2}}{56b}+\frac{5a^3(8Abc-aBd)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

output
```
5/128*a^2*(8*A*b*c-B*a*d)*x*(b*x^2+a)^(1/2)/b+5/192*a*(8*A*b*c-B*a*d)*x*(b
*x^2+a)^(3/2)/b+1/48*(8*A*b*c-B*a*d)*x*(b*x^2+a)^(5/2)/b+1/56*(7*B*d*x+8*A
*d+8*B*c)*(b*x^2+a)^(7/2)/b+5/128*a^3*(8*A*b*c-B*a*d)*arctanh(b^(1/2)*x/(b
*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{5/2}\,dx = \frac{\sqrt{b}\sqrt{a+bx^2}(3a^3(128Bc+128Ad+35Bdx)+16b^3x^5(4A(7c+6dx)+3Bx(8c+7dx))+8}{}$$

input
```
Integrate[(A + B*x)*(c + d*x)*(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(3*a^3*(128*B*c + 128*A*d + 35*B*d*x) + 16*b^3*x^
5*(4*A*(7*c + 6*d*x) + 3*B*x*(8*c + 7*d*x)) + 8*a*b^2*x^3*(2*A*(91*c + 72*
d*x) + B*x*(144*c + 119*d*x)) + 2*a^2*b*x*(12*A*(77*c + 48*d*x) + B*x*(576
*c + 413*d*x))) + 105*a^3*(-8*A*b*c + a*B*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b
*x^2]])/(2688*b^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {676, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2}(A+Bx)(c+dx)\,dx$$

$$\Big\downarrow \text{676}$$

$$\frac{(8Abc-aBd)\int \left(bx^2+a\right)^{5/2}dx}{8b}+\frac{\left(a+bx^2\right)^{7/2}(Ad+Bc)}{7b}+\frac{Bdx\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{211}$$

$$\frac{(8Abc-aBd)\left(\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{\left(a+bx^2\right)^{7/2}(Ad+Bc)}{7b}+\frac{Bdx\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \; 211$$

$$\frac{(8Abc-aBd)\left(\frac{5}{6}a\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{\left(a+bx^2\right)^{7/2}(Ad+Bc)}{7b}+\frac{Bdx\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \; 211$$

$$\frac{(8Abc-aBd)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{\left(a+bx^2\right)^{7/2}(Ad+Bc)}{7b}+\frac{Bdx\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \; 224$$

$$\frac{(8Abc-aBd)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{\left(a+bx^2\right)^{7/2}(Ad+Bc)}{7b}+\frac{Bdx\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)(8Abc-aBd)}{8b}+\frac{\left(a+bx^2\right)^{7/2}(Ad+Bc)}{7b}+\frac{Bdx\left(a+bx^2\right)^{7/2}}{8b}$$

input
```
Int[(A + B*x)*(c + d*x)*(a + b*x^2)^(5/2),x]
```

output
```
((B*c + A*d)*(a + b*x^2)^(7/2))/(7*b) + (B*d*x*(a + b*x^2)^(7/2))/(8*b) +
((8*A*b*c - a*B*d)*((x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(a + b*x^2)^(3/2))/
4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(2*Sqrt[b])))/4))/6))/(8*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.09

method	result
default	$Ac\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+\frac{(Ad+Bc)(b\,x^2+a)^{\frac{7}{2}}}{7b}+Bd\left(\frac{x(b\,x^2+\right.$
risch	$\frac{(336B\,b^3d\,x^7+384A\,b^3d\,x^6+384B\,b^3c\,x^6+448A\,b^3c\,x^5+952aB\,b^2d\,x^5+1152Aa\,b^2d\,x^4+1152Ba\,b^2c\,x^4+1456Aa\,b^2c\,x^3+826B\,a^2bd}{2688b}$

input
```
int((B*x+A)*(d*x+c)*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*c*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^
2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+1/7*(A*d+B*c)*(b
*x^2+a)^(7/2)/b+B*d*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2
)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*
ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.55

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{5/2}\,dx=\left[-\frac{105\left(8\,Aa^3bc-Ba^4d\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(336\,Bb^4dx^7+384\,(Bb\right.}{}\right.$$

$$-\frac{105\left(8\,Aa^3bc-Ba^4d\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(336\,Bb^4dx^7+384\,(Bb^4c+Ab^4d)x^6+384\,Ba^3bc+384\,A\right.}{}$$

input
```
integrate((B*x+A)*(d*x+c)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/5376*(105*(8*A*a^3*b*c - B*a^4*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2
+ a)*sqrt(b)*x - a) - 2*(336*B*b^4*d*x^7 + 384*(B*b^4*c + A*b^4*d)*x^6 + 3
84*B*a^3*b*c + 384*A*a^3*b*d + 56*(8*A*b^4*c + 17*B*a*b^3*d)*x^5 + 1152*(B
*a*b^3*c + A*a*b^3*d)*x^4 + 14*(104*A*a*b^3*c + 59*B*a^2*b^2*d)*x^3 + 1152
*(B*a^2*b^2*c + A*a^2*b^2*d)*x^2 + 21*(88*A*a^2*b^2*c + 5*B*a^3*b*d)*x)*sq
rt(b*x^2 + a))/b^2, -1/2688*(105*(8*A*a^3*b*c - B*a^4*d)*sqrt(-b)*arctan(s
qrt(-b)*x/sqrt(b*x^2 + a)) - (336*B*b^4*d*x^7 + 384*(B*b^4*c + A*b^4*d)*x^
6 + 384*B*a^3*b*c + 384*A*a^3*b*d + 56*(8*A*b^4*c + 17*B*a*b^3*d)*x^5 + 11
52*(B*a*b^3*c + A*a*b^3*d)*x^4 + 14*(104*A*a*b^3*c + 59*B*a^2*b^2*d)*x^3 +
 1152*(B*a^2*b^2*c + A*a^2*b^2*d)*x^2 + 21*(88*A*a^2*b^2*c + 5*B*a^3*b*d)*
x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(160) = 320$.

Time = 0.89 (sec) , antiderivative size = 561, normalized size of antiderivative = 3.30

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{5/2}\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Bb^2dx^7}{8}+\frac{x^6(Ab^3d+Bb^3c)}{7b}+\frac{x^5\left(Ab^3c+\frac{17Bab^2d}{8}\right)}{6b}+\frac{x^4\cdot\left(3Aab^2d+3Bab^2c-\frac{6a(Ab^3d+Bb^3c)}{7b}\right)}{5b}+ \cdots\right) \\ a^{\frac{5}{2}}\left(Acx+\frac{Bdx^3}{3}+\frac{x^2(Ad+Bc)}{2}\right) \end{cases}$$

input
```
integrate((B*x+A)*(d*x+c)*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*b**2*d*x**7/8 + x**6*(A*b**3*d + B*b**3*c)/
(7*b) + x**5*(A*b**3*c + 17*B*a*b**2*d/8)/(6*b) + x**4*(3*A*a*b**2*d + 3*B
*a*b**2*c - 6*a*(A*b**3*d + B*b**3*c)/(7*b))/(5*b) + x**3*(3*A*a*b**2*c +
3*B*a**2*b*d - 5*a*(A*b**3*c + 17*B*a*b**2*d/8)/(6*b))/(4*b) + x**2*(3*A*a
**2*b*d + 3*B*a**2*b*c - 4*a*(3*A*a*b**2*d + 3*B*a*b**2*c - 6*a*(A*b**3*d
+ B*b**3*c)/(7*b))/(5*b))/(3*b) + x*(3*A*a**2*b*c + B*a**3*d - 3*a*(3*A*a*
b**2*c + 3*B*a**2*b*d - 5*a*(A*b**3*c + 17*B*a*b**2*d/8)/(6*b))/(4*b))/(2*
b) + (A*a**3*d + B*a**3*c - 2*a*(3*A*a**2*b*d + 3*B*a**2*b*c - 4*a*(3*A*a*
b**2*d + 3*B*a*b**2*c - 6*a*(A*b**3*d + B*b**3*c)/(7*b))/(5*b))/(3*b))/b)
+ (A*a**3*c - a*(3*A*a**2*b*c + B*a**3*d - 3*a*(3*A*a*b**2*c + 3*B*a**2*b*
d - 5*a*(A*b**3*c + 17*B*a*b**2*d/8)/(6*b))/(4*b))/(2*b))*Piecewise((log(2
*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x*
*2), True)), Ne(b, 0)), (a**(5/2)*(A*c*x + B*d*x**3/3 + x**2*(A*d + B*c)/2
), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.13

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{5/2}\,dx = \frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Acx + \frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Aacx + \frac{5}{16}\sqrt{bx^2+a}Aa^2cx + \frac{\left(bx^2+a\right)^{\frac{7}{2}}Bdx}{8\,b} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}Badx}{48\,b} - \frac{5\left(bx^2+a\right)^{\frac{3}{2}}Ba^2dx}{192\,b} - \frac{5\sqrt{bx^2+a}Ba^3dx}{128\,b} + \frac{5\,Aa^3c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}} - \frac{5\,Ba^4d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}} + \frac{\left(bx^2+a\right)^{\frac{7}{2}}Bc}{7\,b} + \frac{\left(bx^2+a\right)^{\frac{7}{2}}Ad}{7\,b}$$

input
```
integrate((B*x+A)*(d*x+c)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*A*c*x + 5/24*(b*x^2 + a)^(3/2)*A*a*c*x + 5/16*sqrt(b
*x^2 + a)*A*a^2*c*x + 1/8*(b*x^2 + a)^(7/2)*B*d*x/b - 1/48*(b*x^2 + a)^(5/
2)*B*a*d*x/b - 5/192*(b*x^2 + a)^(3/2)*B*a^2*d*x/b - 5/128*sqrt(b*x^2 + a)
*B*a^3*d*x/b + 5/16*A*a^3*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 5/128*B*a^4*d
*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/7*(b*x^2 + a)^(7/2)*B*c/b + 1/7*(b*x^2
 + a)^(7/2)*A*d/b
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.40

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{5/2}\,dx = \frac{1}{2688}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(\left(6\left(7\,Bb^2dx+\frac{8\,(Bb^8c+Ab^8d)}{b^6}\right)x+\frac{7\,(8\,Ab^8c+17\,Bab^7d)}{b^6}\right)x\right.\right.\right.\right.\right. - \frac{5\,(8\,Aa^3bc-Ba^4d)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/2688*sqrt(b*x^2 + a)*((2*((4*((6*(7*B*b^2*d*x + 8*(B*b^8*c + A*b^8*d)/b^
6)*x + 7*(8*A*b^8*c + 17*B*a*b^7*d)/b^6)*x + 144*(B*a*b^7*c + A*a*b^7*d)/b
^6)*x + 7*(104*A*a*b^7*c + 59*B*a^2*b^6*d)/b^6)*x + 576*(B*a^2*b^6*c + A*a
^2*b^6*d)/b^6)*x + 21*(88*A*a^2*b^6*c + 5*B*a^3*b^5*d)/b^6)*x + 384*(B*a^3
*b^5*c + A*a^3*b^5*d)/b^6) - 5/128*(8*A*a^3*b*c - B*a^4*d)*log(abs(-sqrt(b
)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{5/2}\,dx = \int \left(b\,x^2+a\right)^{5/2}\,(A+B\,x)\,(c+d\,x)\,dx$$

input
```
int((a + b*x^2)^(5/2)*(A + B*x)*(c + d*x),x)
```

output
```
int((a + b*x^2)^(5/2)*(A + B*x)*(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.91

$$\int (A+Bx)(c+dx)\left(a+bx^2\right)^{5/2}\,dx = \frac{384\sqrt{b\,x^2+a}\,a^4d+1848\sqrt{b\,x^2+a}\,a^3bcx+384\sqrt{b\,x^2+a}\,a^3bc+1152\sqrt{b\,x^2+a}\,a^3bd\,x^2+10}{}$$

input
```
int((B*x+A)*(d*x+c)*(b*x^2+a)^(5/2),x)
```

output
```
(384*sqrt(a + b*x**2)*a**4*d + 1848*sqrt(a + b*x**2)*a**3*b*c*x + 384*sqrt
(a + b*x**2)*a**3*b*c + 1152*sqrt(a + b*x**2)*a**3*b*d*x**2 + 105*sqrt(a +
 b*x**2)*a**3*b*d*x + 1456*sqrt(a + b*x**2)*a**2*b**2*c*x**3 + 1152*sqrt(a
 + b*x**2)*a**2*b**2*c*x**2 + 1152*sqrt(a + b*x**2)*a**2*b**2*d*x**4 + 826
*sqrt(a + b*x**2)*a**2*b**2*d*x**3 + 448*sqrt(a + b*x**2)*a*b**3*c*x**5 +
1152*sqrt(a + b*x**2)*a*b**3*c*x**4 + 384*sqrt(a + b*x**2)*a*b**3*d*x**6 +
 952*sqrt(a + b*x**2)*a*b**3*d*x**5 + 384*sqrt(a + b*x**2)*b**4*c*x**6 + 3
36*sqrt(a + b*x**2)*b**4*d*x**7 + 840*sqrt(b)*log((sqrt(a + b*x**2) + sqrt
(b)*x)/sqrt(a))*a**4*c - 105*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sq
rt(a))*a**4*d)/(2688*b)
```

3.165 $\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 107

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}+\frac{5a^3A\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

output
```
5/16*a^2*A*x*(b*x^2+a)^(1/2)+5/24*a*A*x*(b*x^2+a)^(3/2)+1/6*A*x*(b*x^2+a)^
(5/2)+1/7*B*(b*x^2+a)^(7/2)/b+5/16*a^3*A*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2)
)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{\sqrt{a+bx^2}(48a^3B+8b^3x^5(7A+6Bx)+3a^2bx(77A+48Bx)+2ab^2x^3(91A+72Bx))-105}{336b}$$

input
```
Integrate[(A + B*x)*(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[a + b*x^2]*(48*a^3*B + 8*b^3*x^5*(7*A + 6*B*x) + 3*a^2*b*x*(77*A + 4
8*B*x) + 2*a*b^2*x^3*(91*A + 72*B*x)) - 105*a^3*A*Sqrt[b]*Log[-(Sqrt[b]*x)
 + Sqrt[a + b*x^2]])/(336*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {455, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{455}$$

$$A\int \left(bx^2+a\right)^{5/2}dx+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

$$\Big\downarrow \text{211}$$

$$A\left(\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

$$\Big\downarrow \text{211}$$

$$A\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

$$\Big\downarrow \text{211}$$

$$A\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

$$\Big\downarrow \text{224}$$

$$A\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

$$\Big\downarrow \text{ 219}$$

$$A\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

input
```
Int[(A + B*x)*(a + b*x^2)^(5/2),x]
```

output
```
(B*(a + b*x^2)^(7/2))/(7*b) + A*((x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(a + b
*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]])/(2*Sqrt[b])))/4))/6)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

method	result	si
default	$A\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+\frac{B\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{7b}$	86
risch	$\frac{\left(48B\,b^{3}x^{6}+56A\,b^{3}x^{5}+144Ba\,b^{2}x^{4}+182Aa\,x^{3}b^{2}+144B\,a^{2}b\,x^{2}+231A\,a^{2}bx+48B\,a^{3}\right)\sqrt{b\,x^{2}+a}}{336b}+\frac{5a^{3}A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16\sqrt{b}}$	10

input
```
int((B*x+A)*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+
a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+1/7*B*(b*x^2+a)^(7
/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.09

$$\int(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\left[\frac{105\,Aa^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(48\,Bb^3x^6+56\,Ab^3x^5+144\,Bab^2x^4+}{672\,b}\right.$$

$$-\frac{105\,Aa^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(48\,Bb^3x^6+56\,Ab^3x^5+144\,Bab^2x^4+182\,Aab^2x^3+144\,Ba^2bx^2+231}{336\,b}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/672*(105*A*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(48*B*b^3*x^6 + 56*A*b^3*x^5 + 144*B*a*b^2*x^4 + 182*A*a*b^2*x^3 + 144
*B*a^2*b*x^2 + 231*A*a^2*b*x + 48*B*a^3)*sqrt(b*x^2 + a))/b, -1/336*(105*A
*a^3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (48*B*b^3*x^6 + 56*A*b^
3*x^5 + 144*B*a*b^2*x^4 + 182*A*a*b^2*x^3 + 144*B*a^2*b*x^2 + 231*A*a^2*b*
x + 48*B*a^3)*sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.40

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\begin{cases}\frac{5Aa^3\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{16}+\sqrt{a+bx^2}\cdot\left(\frac{11Aa^2x}{16}+\frac{13Aabx^3}{24}+\frac{Ab^2x^5}{6}+\frac{Ba^3}{7b}+3\right. \\ a^{\frac{5}{2}}\left(Ax+\frac{Bx^2}{2}\right)\end{cases}$$

input
```
integrate((B*x+A)*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((5*A*a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqr
t(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/16 + sqrt(a + b*x**2)*(11*
A*a**2*x/16 + 13*A*a*b*x**3/24 + A*b**2*x**5/6 + B*a**3/(7*b) + 3*B*a**2*x
**2/7 + 3*B*a*b*x**4/7 + B*b**2*x**6/7), Ne(b, 0)), (a**(5/2)*(A*x + B*x**
2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Ax+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Aax+\frac{5}{16}\sqrt{bx^2+a}Aa^2x+\frac{5\,Aa^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}B}{7\,b}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*A*x + 5/24*(b*x^2 + a)^(3/2)*A*a*x + 5/16*sqrt(b*x^2
 + a)*A*a^2*x + 5/16*A*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/7*(b*x^2 + a
)^(7/2)*B/b
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx=-\frac{5\,Aa^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{336}\left(\frac{48\,Ba^3}{b}+\left(231\,Aa^2+2\left(72\,Ba^2+\left(91\,Aab+4\left(18\,Bab+\left(6\,Bb^2x+7\,Ab^2\right)x\right)x\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-5/16*A*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/336*(48*B*a
^3/b + (231*A*a^2 + 2*(72*B*a^2 + (91*A*a*b + 4*(18*B*a*b + (6*B*b^2*x + 7
*A*b^2)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 6.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{B\,{\left(b\,x^2+a\right)}^{7/2}}{7\,b}+\frac{A\,x\,{\left(b\,x^2+a\right)}^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{5/2}}$$

input
```
int((a + b*x^2)^(5/2)*(A + B*x),x)
```

output
```
(B*(a + b*x^2)^(7/2))/(7*b) + (A*x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2]
, 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(5/2)
```

Reduce [F]

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\int \left(Bx+A\right)\left(b\,x^2+a\right)^{\frac{5}{2}}dx$$

input
```
int((B*x+A)*(b*x^2+a)^(5/2),x)
```

output
```
int((B*x+A)*(b*x^2+a)^(5/2),x)
```

3.166 $\int \frac{(A+Bx)(a+bx^2)^{5/2}}{c+dx}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 346

$$\begin{aligned}\int \frac{(A+Bx)(a+bx^2)^{5/2}}{c+dx}\,dx &= -\frac{\left(16(Bc-Ad)(bc^2+ad^2)^2-d(5a^2Bd^4+8b^2c^3(Bc-Ad)+14abcd^2(Bc-Ad))x\right)\sqrt{a+bx^2}}{16d^6}\\
&\quad-\frac{(8(Bc-Ad)(bc^2+ad^2)-d(5aBd^2+6bc(Bc-Ad))x)(a+bx^2)^{3/2}}{24d^4}\\
&\quad-\frac{(6(Bc-Ad)-5Bdx)(a+bx^2)^{5/2}}{30d^2}\\
&\quad+\frac{(5a^3Bd^6+16b^3c^5(Bc-Ad)+40ab^2c^3d^2(Bc-Ad)+30a^2bcd^4(Bc-Ad))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}d^7}\\
&\quad+\frac{(Bc-Ad)(bc^2+ad^2)^{5/2}\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^7}\end{aligned}$$

output
```
-1/16*(16*(-A*d+B*c)*(a*d^2+b*c^2)^2-d*(5*a^2*B*d^4+8*b^2*c^3*(-A*d+B*c)+1
4*a*b*c*d^2*(-A*d+B*c))*x)*(b*x^2+a)^(1/2)/d^6-1/24*(8*(-A*d+B*c)*(a*d^2+b
*c^2)-d*(5*a*B*d^2+6*b*c*(-A*d+B*c))*x)*(b*x^2+a)^(3/2)/d^4-1/30*(-5*B*d*x
-6*A*d+6*B*c)*(b*x^2+a)^(5/2)/d^2+1/16*(5*a^3*B*d^6+16*b^3*c^5*(-A*d+B*c)+
40*a*b^2*c^3*d^2*(-A*d+B*c)+30*a^2*b*c*d^4*(-A*d+B*c))*arctanh(b^(1/2)*x/(
b*x^2+a)^(1/2))/b^(1/2)/d^7+(-A*d+B*c)*(a*d^2+b*c^2)^(5/2)*arctanh((-b*c*x
+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^7
```

Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{c+dx}\,dx = \frac{d\sqrt{a+bx^2}(a^2d^4(-368Bc+368Ad+165Bdx)-2abd^2(Ad(-280c^2+135c$$

input
```
Integrate[((A + B*x)*(a + b*x^2)^(5/2))/(c + d*x),x]
```

output
```
(d*Sqrt[a + b*x^2]*(a^2*d^4*(-368*B*c + 368*A*d + 165*B*d*x) - 2*a*b*d^2*(
A*d*(-280*c^2 + 135*c*d*x - 88*d^2*x^2) + B*(280*c^3 - 135*c^2*d*x + 88*c*
d^2*x^2 - 65*d^3*x^3)) - 4*b^2*(A*d*(-60*c^4 + 30*c^3*d*x - 20*c^2*d^2*x^2
 + 15*c*d^3*x^3 - 12*d^4*x^4) + B*(60*c^5 - 30*c^4*d*x + 20*c^3*d^2*x^2 -
15*c^2*d^3*x^3 + 12*c*d^4*x^4 - 10*d^5*x^5))) - 480*(B*c - A*d)*(-(b*c^2)
- a*d^2)^(5/2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2
) - a*d^2]] - (15*(5*a^3*B*d^6 + 16*b^3*c^5*(B*c - A*d) + 40*a*b^2*c^3*d^2
*(B*c - A*d) + 30*a^2*b*c*d^4*(B*c - A*d))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/Sqrt[b])/(240*d^7)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {682, 25, 27, 682, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}(A+Bx)}{c+dx}\,dx$$

$$\Big\downarrow \; 682$$

$$\frac{\int -\frac{b\left(ad(Bc-6Ad)-\left(5aBd^2+6bc(Bc-Ad)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{6bd^2}-\frac{\left(a+bx^2\right)^{5/2}(6(Bc-Ad)-5Bdx)}{30d^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{b\left(ad(Bc-6Ad)-\left(5aBd^2+6bc(Bc-Ad)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{6bd^2}-\frac{\left(a+bx^2\right)^{5/2}(6(Bc-Ad)-5Bdx)}{30d^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\left(ad(Bc-6Ad)-\left(5aBd^2+6bc(Bc-Ad)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{6d^2}-\frac{\left(a+bx^2\right)^{5/2}(6(Bc-Ad)-5Bdx)}{30d^2}$$

$$\Big\downarrow \; 682$$

$$-\frac{\frac{\int \frac{3b\left(ad\left(2b(Bc-Ad)c^2+ad^2(3Bc-8Ad)\right)-\left(5a^2Bd^4+14abc(Bc-Ad)d^2+8b^2c^3(Bc-Ad)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4bd^2}+\frac{\left(a+bx^2\right)^{3/2}\left(8\left(ad^2+bc^2\right)(Bc-Ad)-dx(5aB\cdots\right.}{4d^2}\cdots}{6d^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}(6(Bc-Ad)-5Bdx)}{30d^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{3\int \frac{\left(ad\left(2b(Bc-Ad)c^2+ad^2(3Bc-8Ad)\right)-\left(5a^2Bd^4+14abc(Bc-Ad)d^2+8b^2c^3(Bc-Ad)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4d^2}+\frac{\left(a+bx^2\right)^{3/2}\left(8\left(ad^2+bc^2\right)(Bc-Ad)-dx(5aB\cdots\right.}{4d^2}\cdots}{6d^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}(6(Bc-Ad)-5Bdx)}{30d^2}$$

$$\Big\downarrow \; 682$$

$$-\frac{\frac{3\left(\frac{\int \frac{b\left(ad\left(8b^2(Bc-Ad)c^4+18abd^2(Bc-Ad)c^2+a^2d^4(11Bc-16Ad)\right)-\left(5a^3Bd^6+30a^2bc(Bc-Ad)d^4+40ab^2c^3(Bc-Ad)d^2+16b^3c^5(Bc-Ad)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\sqrt{a+bx^2}\cdots\right.}{4d^2}}{6d^2\cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}(6(Bc-Ad)-5Bdx)}{30d^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{3\left(\frac{\int \frac{ad\left(8b^2(Bc-Ad)c^4+18abd^2(Bc-Ad)c^2+a^2d^4(11Bc-16Ad)\right)-\left(5a^3Bd^6+30a^2bc(Bc-Ad)d^4+40ab^2c^3(Bc-Ad)d^2+16b^3c^5(Bc-Ad)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(1\cdots\right.}{\cdots}\right)}{4d^2 \cdots}}{6d^2 \cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}(6(Bc-Ad)-5Bdx)}{30d^2}$$

$\downarrow$ 719

$$-\frac{3\left(\frac{\frac{16\left(ad^2+bc^2\right)^3(Bc-Ad)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(5a^3Bd^6+30a^2bcd^4(Bc-Ad)+40ab^2c^3d^2(Bc-Ad)+16b^3c^5(Bc-Ad)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(16\left(ad^2\cdots\right.\right.}{\cdots}\right)}{4d^2 \cdots}}{6d^2 \cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}(6(Bc-Ad)-5Bdx)}{30d^2}$$

$\downarrow$ 224

$$-\frac{3\left(\frac{\frac{16\left(ad^2+bc^2\right)^3(Bc-Ad)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(5a^3Bd^6+30a^2bcd^4(Bc-Ad)+40ab^2c^3d^2(Bc-Ad)+16b^3c^5(Bc-Ad)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\cdots}{\cdots}\right)}{4d^2 \cdots}}{6d \cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}(6(Bc-Ad)-5Bdx)}{30d^2}$$

$\downarrow$ 219

$$-\frac{3\left(\frac{\frac{16\left(ad^2+bc^2\right)^3(Bc-Ad)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(5a^3Bd^6+30a^2bcd^4(Bc-Ad)+40ab^2c^3d^2(Bc-Ad)+16b^3c^5(Bc-Ad)\right)}{\sqrt{b}d}}{2d^2}+\frac{\sqrt{a+bx^2}\cdots}{\cdots}\right)}{4d^2 \cdots}}{6d \cdots}$$

$$\frac{\left(a+bx^2\right)^{5/2}(6(Bc-Ad)-5Bdx)}{30d^2}$$

$\downarrow$ 488

$$-\frac{3\left(-\frac{\frac{16\left(ad^2+bc^2\right)^3(Bc-Ad)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(5a^3Bd^6+30a^2bcd^4(Bc-Ad)+40ab^2c^3d^2(Bc-Ad)+16b^3c^5(Bc-\right.}{\sqrt{b}d}}{2d^2}\right.}{4d^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}(6(Bc-Ad)-5Bdx)}{30d^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{3\left(\frac{\sqrt{a+bx^2}\left(16\left(ad^2+bc^2\right)^2(Bc-Ad)-dx\left(5a^2Bd^4+14abcd^2(Bc-Ad)+8b^2c^3(Bc-Ad)\right)\right)}{2d^2}+\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(5a^3Bd^6+30a^2bcd^4(Bc-Ad)+40ab^2c\right.}{\sqrt{b}d}}{}\right.}{4d^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}(6(Bc-Ad)-5Bdx)}{30d^2}$$

input
```
Int[((A + B*x)*(a + b*x^2)^(5/2))/(c + d*x),x]
```

output
```
-1/30*((6*(B*c - A*d) - 5*B*d*x)*(a + b*x^2)^(5/2))/d^2 - (((8*(B*c - A*d)
*(b*c^2 + a*d^2) - d*(5*a*B*d^2 + 6*b*c*(B*c - A*d))*x)*(a + b*x^2)^(3/2))
/(4*d^2) + (3*(((16*(B*c - A*d)*(b*c^2 + a*d^2)^2 - d*(5*a^2*B*d^4 + 8*b^2
*c^3*(B*c - A*d) + 14*a*b*c*d^2*(B*c - A*d))*x)*Sqrt[a + b*x^2])/(2*d^2) +
 (-(((5*a^3*B*d^6 + 16*b^3*c^5*(B*c - A*d) + 40*a*b^2*c^3*d^2*(B*c - A*d)
+ 30*a^2*b*c*d^4*(B*c - A*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[
b]*d)) - (16*(B*c - A*d)*(b*c^2 + a*d^2)^(5/2)*ArcTanh[(a*d - b*c*x)/(Sqrt
[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/d)/(2*d^2)))/(4*d^2))/(6*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.69

method	result
risch	$\frac{(40B b^2d^5x^5+48A b^2d^5x^4-48B b^2c d^4x^4-60A b^2c d^4x^3+130Bab d^5x^3+60B b^2c^2d^3x^3+176Aab d^5x^2+80A b^2c^2d^3x^2-176Babc d^4 \ldots}{}$
default	$\frac{B\left(\frac{x(b x^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(b x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{d}+\frac{(Ad-Bc)\left(\frac{\left(b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a d^2+b c^2}{d^2}\right)^{\frac{5}{2}}}{5} \ldots\right)}{}$

input
```
int((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/240*(40*B*b^2*d^5*x^5+48*A*b^2*d^5*x^4-48*B*b^2*c*d^4*x^4-60*A*b^2*c*d^4
*x^3+130*B*a*b*d^5*x^3+60*B*b^2*c^2*d^3*x^3+176*A*a*b*d^5*x^2+80*A*b^2*c^2
*d^3*x^2-176*B*a*b*c*d^4*x^2-80*B*b^2*c^3*d^2*x^2-270*A*a*b*c*d^4*x-120*A*
b^2*c^3*d^2*x+165*B*a^2*d^5*x+270*B*a*b*c^2*d^3*x+120*B*b^2*c^4*d*x+368*A*
a^2*d^5+560*A*a*b*c^2*d^3+240*A*b^2*c^4*d-368*B*a^2*c*d^4-560*B*a*b*c^3*d^
2-240*B*b^2*c^5)*(b*x^2+a)^(1/2)/d^6-1/16/d^6*((30*A*a^2*b*c*d^5+40*A*a*b^
2*c^3*d^3+16*A*b^3*c^5*d-5*B*a^3*d^6-30*B*a^2*b*c^2*d^4-40*B*a*b^2*c^4*d^2
-16*B*b^3*c^6)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+16*(A*a^3*d^7+3*A*a
^2*b*c^2*d^5+3*A*a*b^2*c^4*d^3+A*b^3*c^6*d-B*a^3*c*d^6-3*B*a^2*b*c^3*d^4-3
*B*a*b^2*c^5*d^2-B*b^3*c^7)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c
^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(
x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{c+dx}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{c+dx}\,dx=\int \frac{(A+Bx)\left(a+bx^2\right)^{\frac{5}{2}}}{c+dx}\,dx$$

input
```
integrate((B*x+A)*(b*x**2+a)**(5/2)/(d*x+c),x)
```

output
```
Integral((A + B*x)*(a + b*x**2)**(5/2)/(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(318) = 636$.

Time = 0.15 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.89

$$\int \frac{(A+Bx)\,(a+bx^2)^{5/2}}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*B*b^2*c^4*x/d^5 - 1/2*sqrt(b*x^2 + a)*A*b^2*c^3*x/d^4
+ 1/4*(b*x^2 + a)^(3/2)*B*b*c^2*x/d^3 + 7/8*sqrt(b*x^2 + a)*B*a*b*c^2*x/d^
3 - 1/4*(b*x^2 + a)^(3/2)*A*b*c*x/d^2 - 7/8*sqrt(b*x^2 + a)*A*a*b*c*x/d^2
+ 1/6*(b*x^2 + a)^(5/2)*B*x/d + 5/24*(b*x^2 + a)^(3/2)*B*a*x/d + 5/16*sqrt
(b*x^2 + a)*B*a^2*x/d + B*b^(5/2)*c^6*arcsinh(b*x/sqrt(a*b))/d^7 - A*b^(5/
2)*c^5*arcsinh(b*x/sqrt(a*b))/d^6 + 5/2*B*a*b^(3/2)*c^4*arcsinh(b*x/sqrt(a
*b))/d^5 - 5/2*A*a*b^(3/2)*c^3*arcsinh(b*x/sqrt(a*b))/d^4 + 15/8*B*a^2*sqr
t(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^3 - 15/8*A*a^2*sqrt(b)*c*arcsinh(b*x/sqr
t(a*b))/d^2 + 5/16*B*a^3*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d) - B*(a + b*c^2
/d^2)^(5/2)*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(
d*x + c)))/d^2 + A*(a + b*c^2/d^2)^(5/2)*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x
+ c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d - sqrt(b*x^2 + a)*B*b^2*c^5/d^6 +
sqrt(b*x^2 + a)*A*b^2*c^4/d^5 - 1/3*(b*x^2 + a)^(3/2)*B*b*c^3/d^4 - 2*sqrt
(b*x^2 + a)*B*a*b*c^3/d^4 + 1/3*(b*x^2 + a)^(3/2)*A*b*c^2/d^3 + 2*sqrt(b*x
^2 + a)*A*a*b*c^2/d^3 - 1/5*(b*x^2 + a)^(5/2)*B*c/d^2 - 1/3*(b*x^2 + a)^(3
/2)*B*a*c/d^2 - sqrt(b*x^2 + a)*B*a^2*c/d^2 + 1/5*(b*x^2 + a)^(5/2)*A/d +
1/3*(b*x^2 + a)^(3/2)*A*a/d + sqrt(b*x^2 + a)*A*a^2/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\,(a+bx^2)^{5/2}}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(a+bx^2)^{5/2}}{c+dx}\,dx = \int \frac{(b\,x^2+a)^{5/2}\,(A+B\,x)}{c+d\,x}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x))/(c + d*x),x)
```

output
```
int(((a + b*x^2)^(5/2)*(A + B*x))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(A+Bx)(a+bx^2)^{5/2}}{c+dx}\,dx = \int \frac{(Bx+A)\,(b\,x^2+a)^{\frac{5}{2}}}{dx+c}dx$$

input
```
int((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c),x)
```

output
```
int((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c),x)
```

3.167 $\int \frac{(A+Bx)(a+bx^2)^{5/2}}{(c+dx)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 340

$$\int \frac{(A+Bx)(a+bx^2)^{5/2}}{(c+dx)^2} dx = \frac{(8(bc^2+ad^2)(aBd^2+bc(6Bc-5Ad)) - bd(ad^2(22Bc-15Ad)+4bc^2(6B}{8d^6}$$
$$+ \frac{(4(aBd^2+bc(6Bc-5Ad)) - 3bd(6Bc-5Ad)x)(a+bx^2)^{3/2}}{12d^4}$$
$$+ \frac{(6Bc-5Ad+Bdx)(a+bx^2)^{5/2}}{5d^2(c+dx)}$$
$$- \frac{\sqrt{b}(8b^2c^4(6Bc-5Ad)+20abc^2d^2(4Bc-3Ad)+15a^2d^4(2Bc-Ad))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8d^7}$$
$$- \frac{(bc^2+ad^2)^{3/2}(aBd^2+bc(6Bc-5Ad))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^7}$$

output
```
1/8*(8*(a*d^2+b*c^2)*(a*B*d^2+b*c*(-5*A*d+6*B*c))-b*d*(a*d^2*(-15*A*d+22*B
*c)+4*b*c^2*(-5*A*d+6*B*c))*x)*(b*x^2+a)^(1/2)/d^6+1/12*(4*a*B*d^2+4*b*c*(
-5*A*d+6*B*c)-3*b*d*(-5*A*d+6*B*c)*x)*(b*x^2+a)^(3/2)/d^4+1/5*(B*d*x-5*A*d
+6*B*c)*(b*x^2+a)^(5/2)/d^2/(d*x+c)-1/8*b^(1/2)*(8*b^2*c^4*(-5*A*d+6*B*c)+
20*a*b*c^2*d^2*(-3*A*d+4*B*c)+15*a^2*d^4*(-A*d+2*B*c))*arctanh(b^(1/2)*x/(
b*x^2+a)^(1/2))/d^7-(a*d^2+b*c^2)^(3/2)*(a*B*d^2+b*c*(-5*A*d+6*B*c))*arcta
nh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^7
```

Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{(c+dx)^2}\,dx = \frac{d\sqrt{a+bx^2}\left(8a^2d^4(38Bc-15Ad+23Bdx)+abd^2\left(5Ad\left(-160c^2-85cdx+27d^2x^2\right)+2B\left(540c^3+285c^2dx-9\right.\right.\right.}{}$$

input
```
Integrate[((A + B*x)*(a + b*x^2)^(5/2))/(c + d*x)^2,x]
```

output
```
((d*Sqrt[a + b*x^2]*(8*a^2*d^4*(38*B*c - 15*A*d + 23*B*d*x) + a*b*d^2*(5*A
*d*(-160*c^2 - 85*c*d*x + 27*d^2*x^2) + 2*B*(540*c^3 + 285*c^2*d*x - 91*c*
d^2*x^2 + 44*d^3*x^3)) + 2*b^2*(5*A*d*(-60*c^4 - 30*c^3*d*x + 10*c^2*d^2*x
^2 - 5*c*d^3*x^3 + 3*d^4*x^4) + 6*B*(60*c^5 + 30*c^4*d*x - 10*c^3*d^2*x^2
+ 5*c^2*d^3*x^3 - 3*c*d^4*x^4 + 2*d^5*x^5))))/(c + d*x) - 240*(-(b*c^2) -
a*d^2)^(3/2)*(a*B*d^2 + b*c*(6*B*c - 5*A*d))*ArcTan[(Sqrt[b]*(c + d*x) - d
*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] + 15*Sqrt[b]*(8*b^2*c^4*(6*B*c -
 5*A*d) + 20*a*b*c^2*d^2*(4*B*c - 3*A*d) - 15*a^2*d^4*(-2*B*c + A*d))*Log[
-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(120*d^7)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {681, 27, 682, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}(A+Bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{681}$$

$$\frac{\left(a+bx^2\right)^{5/2}(-5Ad+6Bc+Bdx)}{5d^2(c+dx)}-\frac{\int -\frac{2(aBd-b(6Bc-5Ad)x)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{2d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(aBd-b(6Bc-5Ad)x)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{d^2}+\frac{\left(a+bx^2\right)^{5/2}(-5Ad+6Bc+Bdx)}{5d^2(c+dx)}$$

$\downarrow$ 682

$$\frac{\frac{\int \frac{b\left(ad\left(4aBd^2+bc(6Bc-5Ad)\right)-b\left(4b(6Bc-5Ad)c^2+ad^2(22Bc-15Ad)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4bd^2}+\frac{\left(a+bx^2\right)^{3/2}\left(4\left(aBd^2+bc(6Bc-5Ad)\right)-3bdx(6Bc-5Ad)\right)}{12d^2}}{d^2}+\frac{\left(a+bx^2\right)^{5/2}(-5Ad+6Bc+Bdx)}{5d^2(c+dx)}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\left(ad\left(4aBd^2+bc(6Bc-5Ad)\right)-b\left(4b(6Bc-5Ad)c^2+ad^2(22Bc-15Ad)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4d^2}+\frac{\left(a+bx^2\right)^{3/2}\left(4\left(aBd^2+bc(6Bc-5Ad)\right)-3bdx(6Bc-5Ad)\right)}{12d^2}}{d^2}+\frac{\left(a+bx^2\right)^{5/2}(-5Ad+6Bc+Bdx)}{5d^2(c+dx)}$$

$\downarrow$ 682

$$\frac{\frac{\frac{\int \frac{b\left(ad\left(8a^2Bd^4+abc(34Bc-25Ad)d^2+4b^2c^3(6Bc-5Ad)\right)-b\left(8b^2(6Bc-5Ad)c^4+20abd^2(4Bc-3Ad)c^2+15a^2d^4(2Bc-Ad)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)\left(aBd^2\cdots\right.\right.}{\cdots}}{4d^2}\cdots}{d^2}\cdots$$

$$\frac{\left(a+bx^2\right)^{5/2}(-5Ad+6Bc+Bdx)}{5d^2(c+dx)}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{ad\left(8a^2Bd^4+abc(34Bc-25Ad)d^2+4b^2c^3(6Bc-5Ad)\right)-b\left(8b^2(6Bc-5Ad)c^4+20abd^2(4Bc-3Ad)c^2+15a^2d^4(2Bc-Ad)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)\left(aBd^2+bc\cdots\right.\right.}{\cdots}}{4d^2}\cdots}{d^2}\cdots$$

$$\frac{\left(a+bx^2\right)^{5/2}(-5Ad+6Bc+Bdx)}{5d^2(c+dx)}$$

$\downarrow$ 719

$$\frac{\frac{\frac{8\left(ad^2+bc^2\right)^2\left(aBd^2+bc(6Bc-5Ad)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{b\left(15a^2d^4(2Bc-Ad)+20abc^2d^2(4Bc-3Ad)+8b^2c^4(6Bc-5Ad)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+b\cdots\right.\right.}{\cdots}}{4d^2}\cdots}{d^2}\cdots$$

$$\frac{\left(a+bx^2\right)^{5/2}(-5Ad+6Bc+Bdx)}{5d^2(c+dx)}$$

$\Big\downarrow$ 224

$$\frac{\frac{\frac{8\left(ad^2+bc^2\right)^2\left(aBd^2+bc(6Bc-5Ad)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{b\left(15a^2d^4(2Bc-Ad)+20abc^2d^2(4Bc-3Ad)+8b^2c^4(6Bc-5Ad)\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}+\frac{\sqrt{a+bx^2}}{}}{4d^2}}{d^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}(-5Ad+6Bc+Bdx)}{5d^2(c+dx)}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{8\left(ad^2+bc^2\right)^2\left(aBd^2+bc(6Bc-5Ad)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(15a^2d^4(2Bc-Ad)+20abc^2d^2(4Bc-3Ad)+8b^2c^4(6Bc-5Ad)\right)}{d}}{2d^2}+\frac{\sqrt{a+bx}}{}}{4d^2}}{d^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}(-5Ad+6Bc+Bdx)}{5d^2(c+dx)}$$

$\Big\downarrow$ 488

$$\frac{\frac{-\frac{8\left(ad^2+bc^2\right)^2\left(aBd^2+bc(6Bc-5Ad)\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(15a^2d^4(2Bc-Ad)+20abc^2d^2(4Bc-3Ad)+8b^2c^4(6Bc-\right.}{d}}{2d^2}}{4d^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}(-5Ad+6Bc+Bdx)}{5d^2(c+dx)}$$

$\Big\downarrow$ 219

$$\frac{\frac{-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(15a^2d^4(2Bc-Ad)+20abc^2d^2(4Bc-3Ad)+8b^2c^4(6Bc-5Ad)\right)}{d}-\frac{8\left(ad^2+bc^2\right)^{3/2}\left(aBd^2+bc(6Bc-5Ad)\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2}}\right.}{d}}{2d^2}}{4d^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}(-5Ad+6Bc+Bdx)}{5d^2(c+dx)}$$

input
```
Int[((A + B*x)*(a + b*x^2)^(5/2))/(c + d*x)^2,x]
```

output
```
((6*B*c - 5*A*d + B*d*x)*(a + b*x^2)^(5/2))/(5*d^2*(c + d*x)) + (((4*(a*B*
d^2 + b*c*(6*B*c - 5*A*d)) - 3*b*d*(6*B*c - 5*A*d)*x)*(a + b*x^2)^(3/2))/(
12*d^2) + (((8*(b*c^2 + a*d^2)*(a*B*d^2 + b*c*(6*B*c - 5*A*d)) - b*d*(a*d^
2*(22*B*c - 15*A*d) + 4*b*c^2*(6*B*c - 5*A*d))*x)*Sqrt[a + b*x^2])/(2*d^2)
 + (-((Sqrt[b]*(8*b^2*c^4*(6*B*c - 5*A*d) + 20*a*b*c^2*d^2*(4*B*c - 3*A*d)
 + 15*a^2*d^4*(2*B*c - A*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d) - (8
*(b*c^2 + a*d^2)^(3/2)*(a*B*d^2 + b*c*(6*B*c - 5*A*d))*ArcTanh[(a*d - b*c*
x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/d)/(2*d^2))/(4*d^2))/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. 2(313) = 626.

Time = 1.44 (sec) , antiderivative size = 783, normalized size of antiderivative = 2.30

method	result
risch	$-\frac{(-24B\,b^{2}x^{4}d^{4}-30A\,b^{2}d^{4}x^{3}+60B\,b^{2}c\,d^{3}x^{3}+80A\,b^{2}c\,d^{3}x^{2}-88Bab\,d^{4}x^{2}-120B\,b^{2}c^{2}d^{2}x^{2}-135Aab\,d^{4}x-180A\,b^{2}c^{2}d^{2}x+270Babc}{120d^{6}}$
default	Expression too large to display

input
```
int((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/120*(-24*B*b^2*d^4*x^4-30*A*b^2*d^4*x^3+60*B*b^2*c*d^3*x^3+80*A*b^2*c*d
^3*x^2-88*B*a*b*d^4*x^2-120*B*b^2*c^2*d^2*x^2-135*A*a*b*d^4*x-180*A*b^2*c^
2*d^2*x+270*B*a*b*c*d^3*x+240*B*b^2*c^3*d*x+560*A*a*b*c*d^3+480*A*b^2*c^3*
d-184*B*a^2*d^4-840*B*a*b*c^2*d^2-600*B*b^2*c^4)*(b*x^2+a)^(1/2)/d^6+1/8/d
^6*(b^(1/2)*(15*A*a^2*d^5+60*A*a*b*c^2*d^3+40*A*b^2*c^4*d-30*B*a^2*c*d^4-8
0*B*a*b*c^3*d^2-48*B*b^2*c^5)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+8/d^2*(6*A*a
^2*b*c*d^5+12*A*a*b^2*c^3*d^3+6*A*b^3*c^5*d-B*a^3*d^6-9*B*a^2*b*c^2*d^4-15
*B*a*b^2*c^4*d^2-7*B*b^3*c^6)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2
)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+
c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+8*(A*a^3*d^7+3*A*a^2*b*c^2*d^5+3*A
*a*b^2*c^4*d^3+A*b^3*c^6*d-B*a^3*c*d^6-3*B*a^2*b*c^3*d^4-3*B*a*b^2*c^5*d^2
-B*b^3*c^7)/d^3*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)
+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln
((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/
d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{(c+dx)^2}\,dx = \int \frac{(A+Bx)\left(a+bx^2\right)^{\frac{5}{2}}}{\left(c+dx\right)^2}\,dx$$

input
```
integrate((B*x+A)*(b*x**2+a)**(5/2)/(d*x+c)**2,x)
```

output
```
Integral((A + B*x)*(a + b*x**2)**(5/2)/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.84

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(b*x^2 + a)^(5/2)*B*c/(d^3*x + c*d^2) - (b*x^2 + a)^(5/2)*A/(d^2*x + c*d)
- 3*sqrt(b*x^2 + a)*B*b^2*c^3*x/d^5 + 5/2*sqrt(b*x^2 + a)*A*b^2*c^2*x/d^4
- 3/2*(b*x^2 + a)^(3/2)*B*b*c*x/d^3 - 11/4*sqrt(b*x^2 + a)*B*a*b*c*x/d^3 +
 5/4*(b*x^2 + a)^(3/2)*A*b*x/d^2 + 15/8*sqrt(b*x^2 + a)*A*a*b*x/d^2 - 6*B*
b^(5/2)*c^5*arcsinh(b*x/sqrt(a*b))/d^7 + 5*A*b^(5/2)*c^4*arcsinh(b*x/sqrt(
a*b))/d^6 - 10*B*a*b^(3/2)*c^3*arcsinh(b*x/sqrt(a*b))/d^5 + 15/2*A*a*b^(3/
2)*c^2*arcsinh(b*x/sqrt(a*b))/d^4 - 15/4*B*a^2*sqrt(b)*c*arcsinh(b*x/sqrt(
a*b))/d^3 + 15/8*A*a^2*sqrt(b)*arcsinh(b*x/sqrt(a*b))/d^2 + 5*B*(a + b*c^2
/d^2)^(3/2)*b*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*
abs(d*x + c)))/d^4 - 5*A*(a + b*c^2/d^2)^(3/2)*b*c*arcsinh(b*c*x/(sqrt(a*b
)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^3 + B*(a + b*c^2/d^2)^(5
/2)*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))
/d^2 + 6*sqrt(b*x^2 + a)*B*b^2*c^4/d^6 - 5*sqrt(b*x^2 + a)*A*b^2*c^3/d^5 +
 2*(b*x^2 + a)^(3/2)*B*b*c^2/d^4 + 7*sqrt(b*x^2 + a)*B*a*b*c^2/d^4 - 5/3*(
b*x^2 + a)^(3/2)*A*b*c/d^3 - 5*sqrt(b*x^2 + a)*A*a*b*c/d^3 + 1/5*(b*x^2 +
a)^(5/2)*B/d^2 + 1/3*(b*x^2 + a)^(3/2)*B*a/d^2 + sqrt(b*x^2 + a)*B*a^2/d^2
```

Giac [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{(c+dx)^2}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{5/2}\,\left(A+B\,x\right)}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x))/(c + d*x)^2,x)
```

output
```
int(((a + b*x^2)^(5/2)*(A + B*x))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 1929, normalized size of antiderivative = 5.67

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{(c+dx)^2}\,dx=\text{Too large to display}$$

input
```
int((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c)^2,x)
```

output

```
(1200*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
- a*d + b*c*x)*a**2*b*c**2*d**3 + 1200*sqrt(a*d**2 + b*c**2)*log( - sqrt(a
 + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c*d**4*x - 240*sqrt
(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*
c*x)*a**2*b*c*d**4 - 240*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqr
t(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*d**5*x + 1200*sqrt(a*d**2 + b*c**
2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**
4*d + 1200*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c
**2) - a*d + b*c*x)*a*b**2*c**3*d**2*x - 1680*sqrt(a*d**2 + b*c**2)*log( -
 sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**3*d**2 -
1680*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
 a*d + b*c*x)*a*b**2*c**2*d**3*x - 1440*sqrt(a*d**2 + b*c**2)*log( - sqrt(
a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**5 - 1440*sqrt(a*d
**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)
*b**3*c**4*d*x - 1200*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c**2*d**3
- 1200*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c*d**4*x + 240*sqrt(a*d**
2 + b*c**2)*log(c + d*x)*a**2*b*c*d**4 + 240*sqrt(a*d**2 + b*c**2)*log(c +
 d*x)*a**2*b*d**5*x - 1200*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**4*
d - 1200*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**3*d**2*x + 1680*sqrt
(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**3*d**2 + 1680*sqrt(a*d**2 + b*...
```

3.168 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 341

$$
\begin{aligned}
&\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{(c+dx)^3}\,dx = \\
&-\frac{5b(4(ad^2(3Bc-Ad)+b(6Bc^3-4Ac^2d))-d(3aBd^2+4bc(3Bc-2Ad))\,x)\sqrt{a+bx^2}}{8d^6} \\
&-\frac{5(3aBd^2+4bc(3Bc-2Ad)+bd(3Bc-2Ad)x)\left(a+bx^2\right)^{3/2}}{12d^4(c+dx)} \\
&+\frac{(3Bc-2Ad+Bdx)\left(a+bx^2\right)^{5/2}}{4d^2(c+dx)^2} \\
&+\frac{5\sqrt{b}(3a^2Bd^4+8b^2c^3(3Bc-2Ad)+12abcd^2(2Bc-Ad))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8d^7} \\
&+\frac{5b\sqrt{bc^2+ad^2}(2bc^2(3Bc-2Ad)+ad^2(3Bc-Ad))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2d^7}
\end{aligned}
$$

output
```
-5/8*b*(4*a*d^2*(-A*d+3*B*c)+4*b*(-4*A*c^2*d+6*B*c^3)-d*(3*a*B*d^2+4*b*c*(
-2*A*d+3*B*c))*x)*(b*x^2+a)^(1/2)/d^6-5/12*(3*a*B*d^2+4*b*c*(-2*A*d+3*B*c)
+b*d*(-2*A*d+3*B*c)*x)*(b*x^2+a)^(3/2)/d^4/(d*x+c)+1/4*(B*d*x-2*A*d+3*B*c)
*(b*x^2+a)^(5/2)/d^2/(d*x+c)^2+5/8*b^(1/2)*(3*a^2*B*d^4+8*b^2*c^3*(-2*A*d+
3*B*c)+12*a*b*c*d^2*(-A*d+2*B*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^7+5
/2*b*(a*d^2+b*c^2)^(1/2)*(2*b*c^2*(-2*A*d+3*B*c)+a*d^2*(-A*d+3*B*c))*arcta
nh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^7
```

Mathematica [A] (verified)

Time = 5.17 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{(c+dx)^3}\,dx =$$

$$-\frac{\frac{d\sqrt{a+bx^2}\left(12a^2d^4(Ad+B(c+2dx))+abd^2\left(-4Ad\left(35c^2+55cdx+14d^2x^2\right)+3B\left(100c^3+155c^2dx+38cd^2x^2-9d^3x^3\right)\right)+2b^2\left(-2Ad\left(60c^4+90c^3dx+2\right.\right.\right.}{(c+dx)^2}}{}$$

input
```
Integrate[((A + B*x)*(a + b*x^2)^(5/2))/(c + d*x)^3,x]
```

output
```
-1/24*((d*Sqrt[a + b*x^2]*(12*a^2*d^4*(A*d + B*(c + 2*d*x)) + a*b*d^2*(-4*
A*d*(35*c^2 + 55*c*d*x + 14*d^2*x^2) + 3*B*(100*c^3 + 155*c^2*d*x + 38*c*d
^2*x^2 - 9*d^3*x^3)) + 2*b^2*(-2*A*d*(60*c^4 + 90*c^3*d*x + 20*c^2*d^2*x^2
 - 5*c*d^3*x^3 + 2*d^4*x^4) + 3*B*(60*c^5 + 90*c^4*d*x + 20*c^3*d^2*x^2 -
5*c^2*d^3*x^3 + 2*c*d^4*x^4 - d^5*x^5))))/(c + d*x)^2 + 120*b*Sqrt[-(b*c^2
) - a*d^2]*(2*b*c^2*(3*B*c - 2*A*d) + a*d^2*(3*B*c - A*d))*ArcTan[(Sqrt[b]
*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] + 15*Sqrt[b]*(3*a^
2*B*d^4 + 8*b^2*c^3*(3*B*c - 2*A*d) + 12*a*b*c*d^2*(2*B*c - A*d))*Log[-(Sq
rt[b]*x) + Sqrt[a + b*x^2]])/d^7
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {681, 27, 681, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}(A+Bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{681}$$

$$\frac{\left(a+bx^2\right)^{5/2}(-2Ad+3Bc+Bdx)}{4d^2(c+dx)^2}-\frac{5\int -\frac{4(aBd-b(3Bc-2Ad)x)\left(bx^2+a\right)^{3/2}}{(c+dx)^2}dx}{16d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\int \frac{(aBd-b(3Bc-2Ad)x)\left(bx^2+a\right)^{3/2}}{(c+dx)^2}dx}{4d^2}+\frac{\left(a+bx^2\right)^{5/2}(-2Ad+3Bc+Bdx)}{4d^2(c+dx)^2}$$

$$\Big\downarrow \text{681}$$

$$\frac{5\left(-\frac{\int \frac{2b\left(ad(3Bc-2Ad)-\left(3aBd^2+4bc(3Bc-2Ad)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{2d^2}-\frac{\left(a+bx^2\right)^{3/2}\left(3aBd^2+bdx(3Bc-2Ad)+4bc(3Bc-2Ad)\right)}{3d^2(c+dx)}\right)}{4d^2}+\frac{\left(a+bx^2\right)^{5/2}(-2Ad+3Bc+Bdx)}{4d^2(c+dx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\left(-\frac{b\int \frac{\left(ad(3Bc-2Ad)-\left(3aBd^2+4bc(3Bc-2Ad)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{d^2}-\frac{\left(a+bx^2\right)^{3/2}\left(3aBd^2+bdx(3Bc-2Ad)+4bc(3Bc-2Ad)\right)}{3d^2(c+dx)}\right)}{4d^2}+\frac{\left(a+bx^2\right)^{5/2}(-2Ad+3Bc+Bdx)}{4d^2(c+dx)^2}$$

$$\Big\downarrow \text{682}$$

$$5\left(-\frac{b\left(\frac{\int \frac{b\left(ad\left(4b(3Bc-2Ad)c^2+ad^2(9Bc-4Ad)\right)-\left(3a^2Bd^4+12abc(2Bc-Ad)d^2+8b^2c^3(3Bc-2Ad)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(4\left(ad^2(3Bc-Ad)+b\left(6Bc^3-4Ac^2d\right)\right)\right.}{2d^2}\right.}{d^2}\right.$$

$$\overline{\qquad\qquad 4d^2 \qquad\qquad}$$

$$\frac{\left(a+bx^2\right)^{5/2}(-2Ad+3Bc+Bdx)}{4d^2(c+dx)^2}$$

$\downarrow$ 27

$$5\left(-\frac{b\left(\frac{\int \frac{ad\left(4b(3Bc-2Ad)c^2+ad^2(9Bc-4Ad)\right)-\left(3a^2Bd^4+12abc(2Bc-Ad)d^2+8b^2c^3(3Bc-2Ad)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(4\left(ad^2(3Bc-Ad)+b\left(6Bc^3-4Ac^2d\right)\right)-dx\right.}{2d^2}\right.}{d^2}\right.$$

$$\overline{\qquad\qquad 4d^2 \qquad\qquad}$$

$$\frac{\left(a+bx^2\right)^{5/2}(-2Ad+3Bc+Bdx)}{4d^2(c+dx)^2}$$

$\downarrow$ 719

$$5\left(-\frac{b\left(\frac{\frac{4\left(ad^2+bc^2\right)\left(ad^2(3Bc-Ad)+2bc^2(3Bc-2Ad)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(3a^2Bd^4+12abcd^2(2Bc-Ad)+8b^2c^3(3Bc-2Ad)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(4\right.}{}\right.}{d^2}\right.$$

$$\overline{\qquad\qquad 4d^2 \qquad\qquad}$$

$$\frac{\left(a+bx^2\right)^{5/2}(-2Ad+3Bc+Bdx)}{4d^2(c+dx)^2}$$

$\downarrow$ 224

$$\frac{5\left(-\frac{b\left(\frac{\frac{4\left(ad^2+bc^2\right)\left(ad^2(3Bc-Ad)+2bc^2(3Bc-2Ad)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(3a^2Bd^4+12abcd^2(2Bc-Ad)+8b^2c^3(3Bc-2Ad)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}+\cdots\right)}{d^2}\cdots\right)}{4d^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}(-2Ad+3Bc+Bdx)}{4d^2(c+dx)^2}$$

$\downarrow$ 219

$$\frac{5\left(-\frac{b\left(\frac{\frac{4\left(ad^2+bc^2\right)\left(ad^2(3Bc-Ad)+2bc^2(3Bc-2Ad)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2Bd^4+12abcd^2(2Bc-Ad)+8b^2c^3(3Bc-2Ad)\right)}{\sqrt{b}d}}{2d^2}+\cdots\right)}{d^2}\cdots\right)}{4d^2}$$

$$\frac{\left(a+bx^2\right)^{5/2}(-2Ad+3Bc+Bdx)}{4d^2(c+dx)^2}$$

$\downarrow$ 488

$$5\left(-\frac{b\left(\frac{-\frac{4\left(ad^2+bc^2\right)\left(ad^2(3Bc-Ad)+2bc^2(3Bc-2Ad)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2Bd^4+12abcd^2(2Bc-Ad)+8b^2c^3(3\cdots\right.}{\sqrt{b}d}}{2d^2}\cdots\right)}{d^2}\cdots\right)$$

$$\frac{\left(a+bx^2\right)^{5/2}(-2Ad+3Bc+Bdx)}{4d^2(c+dx)^2}$$

$\downarrow$ 219

$$\frac{5\left(-\frac{b\left(\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2Bd^4+12abcd^2(2Bc-Ad)+8b^2c^3(3Bc-2Ad)\right)}{\sqrt{b}d}-\frac{4\sqrt{ad^2+bc^2}\left(ad^2(3Bc-Ad)+2bc^2(3Bc-2Ad)\right)\text{arctanh}\left(\frac{ad-b}{\sqrt{a+bx^2}\sqrt{c}}\right.}{d}}{2d^2}\right.}{d^2}\right.}{4}$$

$$\frac{\left(a+bx^2\right)^{5/2}(-2Ad+3Bc+Bdx)}{4d^2(c+dx)^2}$$

input
```
Int[((A + B*x)*(a + b*x^2)^(5/2))/(c + d*x)^3,x]
```

output
```
((3*B*c - 2*A*d + B*d*x)*(a + b*x^2)^(5/2))/(4*d^2*(c + d*x)^2) + (5*(-1/3
*((3*a*B*d^2 + 4*b*c*(3*B*c - 2*A*d) + b*d*(3*B*c - 2*A*d)*x)*(a + b*x^2)^
(3/2))/(d^2*(c + d*x)) - (b*(((4*(a*d^2*(3*B*c - A*d) + b*(6*B*c^3 - 4*A*c
^2*d)) - d*(3*a*B*d^2 + 4*b*c*(3*B*c - 2*A*d))*x)*Sqrt[a + b*x^2])/(2*d^2)
 + (-(((3*a^2*B*d^4 + 8*b^2*c^3*(3*B*c - 2*A*d) + 12*a*b*c*d^2*(2*B*c - A*
d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d)) - (4*Sqrt[b*c^2 + a
*d^2]*(2*b*c^2*(3*B*c - 2*A*d) + a*d^2*(3*B*c - A*d))*ArcTanh[(a*d - b*c*x
)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/d)/(2*d^2)))/d^2))/(4*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1197 vs. $2(312) = 624$.

Time = 1.54 (sec) , antiderivative size = 1198, normalized size of antiderivative = 3.51

method	result	size
risch	Expression too large to display	1198
default	Expression too large to display	3562

input
```
int((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/24*b*(6*B*b*d^3*x^3+8*A*b*d^3*x^2-24*B*b*c*d^2*x^2-36*A*b*c*d^2*x+27*B*a
*d^3*x+72*B*b*c^2*d*x+56*A*a*d^3+144*A*b*c^2*d-168*B*a*c*d^2-240*B*b*c^3)*
(b*x^2+a)^(1/2)/d^6-1/8/d^6*(8/d^3*(6*A*a^2*b*c*d^5+12*A*a*b^2*c^3*d^3+6*A
*b^3*c^5*d-B*a^3*d^6-9*B*a^2*b*c^2*d^4-15*B*a*b^2*c^4*d^2-7*B*b^3*c^6)*(-1
/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)
^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d
^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d
)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-8*(A*a^3*d^7+3*A*a^2*b*c^2*d^5+3*A*a
*b^2*c^4*d^3+A*b^3*c^6*d-B*a^3*c*d^6-3*B*a^2*b*c^3*d^4-3*B*a*b^2*c^5*d^2-B
*b^3*c^7)/d^4*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/
d)+(a*d^2+b*c^2)/d^2)^(1/2)+3/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/
(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2
+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+
2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2
)^(1/2))/(x+c/d)))+1/2*b/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2
*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^
2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+5*b^(1/2)*(12*A*a*b*
c*d^3+16*A*b^2*c^3*d-3*B*a^2*d^4-24*B*a*b*c^2*d^2-24*B*b^2*c^4)/d*ln(b^(1/
2)*x+(b*x^2+a)^(1/2))+24*b/d^2*(A*a^2*d^5+6*A*a*b*c^2*d^3+5*A*b^2*c^4*d-3*
B*a^2*c*d^4-10*B*a*b*c^3*d^2-7*B*b^2*c^5)/((a*d^2+b*c^2)/d^2)^(1/2)*ln(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx)\,(a+bx^2)^{5/2}}{(c+dx)^3}\,dx = \int \frac{(A+Bx)\,(a+bx^2)^{\frac{5}{2}}}{(c+dx)^3}\,dx$$

input
```
integrate((B*x+A)*(b*x**2+a)**(5/2)/(d*x+c)**3,x)
```

output
```
Integral((A + B*x)*(a + b*x**2)**(5/2)/(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1257 vs. 2(313) = 626.

Time = 0.21 (sec) , antiderivative size = 1257, normalized size of antiderivative = 3.69

$$\int \frac{(A+Bx)\,(a+bx^2)^{5/2}}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-15/4*B*b^4*c^6*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*c^2*d^7 + a*sqrt(b)*d^9) +
 15/4*A*b^4*c^5*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*c^2*d^6 + a*sqrt(b)*d^8) -
 15/4*B*a*b^3*c^4*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*c^2*d^5 + a*sqrt(b)*d^7)
 + 15/4*sqrt(b*x^2 + a)*B*b^3*c^4*x/(b*c^2*d^5 + a*d^7) + 15/4*A*a*b^3*c^3
*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*c^2*d^4 + a*sqrt(b)*d^6) - 15/4*sqrt(b*x^
2 + a)*A*b^3*c^3*x/(b*c^2*d^4 + a*d^6) - 5/2*(b*x^2 + a)^(3/2)*B*b^2*c^3/(
b*c^2*d^4 + a*d^6) + 5/2*(b*x^2 + a)^(3/2)*B*b^2*c^2*x/(b*c^2*d^3 + a*d^5)
 + 15/4*sqrt(b*x^2 + a)*B*a*b^2*c^2*x/(b*c^2*d^3 + a*d^5) - 3/2*(b*x^2 + a
)^(5/2)*B*b*c^2/(b*c^2*d^3*x + a*d^5*x + b*c^3*d^2 + a*c*d^4) + 5/2*(b*x^2
 + a)^(3/2)*A*b^2*c^2/(b*c^2*d^3 + a*d^5) - 5/2*(b*x^2 + a)^(3/2)*A*b^2*c*
x/(b*c^2*d^2 + a*d^4) - 15/4*sqrt(b*x^2 + a)*A*a*b^2*c*x/(b*c^2*d^2 + a*d^
4) + 1/2*(b*x^2 + a)^(7/2)*B*c/(b*c^2*d^2*x^2 + a*d^4*x^2 + 2*b*c^3*d*x +
2*a*c*d^3*x + b*c^4 + a*c^2*d^2) + 3/2*(b*x^2 + a)^(5/2)*A*b*c/(b*c^2*d^2*
x + a*d^4*x + b*c^3*d + a*c*d^3) - 1/2*(b*x^2 + a)^(5/2)*B*b*c/(b*c^2*d^2
+ a*d^4) - 1/2*(b*x^2 + a)^(7/2)*A/(b*c^2*d*x^2 + a*d^3*x^2 + 2*b*c^3*x +
2*a*c*d^2*x + b*c^4/d + a*c^2*d) + 1/2*(b*x^2 + a)^(5/2)*A*b/(b*c^2*d + a*
d^3) - (b*x^2 + a)^(5/2)*B/(d^3*x + c*d^2) + 15/4*sqrt(b*x^2 + a)*B*b^2*c^
2*x/d^5 - 5/4*sqrt(b*x^2 + a)*A*b^2*c*x/d^4 + 5/4*(b*x^2 + a)^(3/2)*B*b*x/
d^3 + 15/8*sqrt(b*x^2 + a)*B*a*b*x/d^3 + 75/4*B*b^(5/2)*c^4*arcsinh(b*x/sq
rt(a*b))/d^7 - 55/4*A*b^(5/2)*c^3*arcsinh(b*x/sqrt(a*b))/d^6 + 15*B*a*b...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1067 vs. 2(313) = 626.

Time = 0.49 (sec) , antiderivative size = 1067, normalized size of antiderivative = 3.13

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a)*((2*(3*B*b^2*x/d^3 - 4*(3*B*b^4*c*d^21 - A*b^4*d^22)/
(b^2*d^25))*x + 9*(8*B*b^4*c^2*d^20 - 4*A*b^4*c*d^21 + 3*B*a*b^3*d^22)/(b^
2*d^25))*x - 8*(30*B*b^4*c^3*d^19 - 18*A*b^4*c^2*d^20 + 21*B*a*b^3*c*d^21
- 7*A*a*b^3*d^22)/(b^2*d^25)) - 5/8*(24*B*b^(5/2)*c^4 - 16*A*b^(5/2)*c^3*d
 + 24*B*a*b^(3/2)*c^2*d^2 - 12*A*a*b^(3/2)*c*d^3 + 3*B*a^2*sqrt(b)*d^4)*lo
g(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/d^7 - 5*(6*B*b^3*c^5 - 4*A*b^3*c^4*d
+ 9*B*a*b^2*c^3*d^2 - 5*A*a*b^2*c^2*d^3 + 3*B*a^2*b*c*d^4 - A*a^2*b*d^5)*a
rctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))
/(sqrt(-b*c^2 - a*d^2)*d^7) - (12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*b^3*c^
5*d - 10*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b^3*c^4*d^2 + 15*(sqrt(b)*x - s
qrt(b*x^2 + a))^3*B*a*b^2*c^3*d^3 - 11*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a
*b^2*c^2*d^4 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a^2*b*c*d^5 - (sqrt(b)*
x - sqrt(b*x^2 + a))^3*A*a^2*b*d^6 + 22*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*
b^(7/2)*c^6 - 18*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*b^(7/2)*c^5*d + 15*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*B*a*b^(5/2)*c^4*d^2 - 9*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*A*a*b^(5/2)*c^3*d^3 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*b^
(3/2)*c^2*d^4 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(3/2)*c*d^5 - 2*
(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*sqrt(b)*d^6 - 32*(sqrt(b)*x - sqrt(b
*x^2 + a))*B*a*b^3*c^5*d + 26*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b^3*c^4*d^
2 - 37*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^2*b^2*c^3*d^3 + 25*(sqrt(b)*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{(c+dx)^3}\,dx=\int \frac{\left(b\,x^2+a\right)^{5/2}\,\left(A+B\,x\right)}{\left(c+d\,x\right)^3}\,dx$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x))/(c + d*x)^3,x)
```

output
```
int(((a + b*x^2)^(5/2)*(A + B*x))/(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2321, normalized size of antiderivative = 6.81

$$\int \frac{(A+Bx)(a+bx^2)^{5/2}}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(b*x^2+a)^(5/2)/(d*x+c)^3,x)
```

output
```
(120*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*
d + b*c*x)*a**2*b*c**2*d**3 + 240*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**
2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c*d**4*x + 120*sqrt(a*d**2
+ b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b
*d**5*x**2 + 480*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 +
b*c**2) - a*d + b*c*x)*a*b**2*c**4*d + 960*sqrt(a*d**2 + b*c**2)*log(sqrt(
a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**3*d**2*x - 360*
sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b
*c*x)*a*b**2*c**3*d**2 + 480*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sq
rt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**2*d**3*x**2 - 720*sqrt(a*d**2
 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**
2*c**2*d**3*x - 360*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2
 + b*c**2) - a*d + b*c*x)*a*b**2*c*d**4*x**2 - 720*sqrt(a*d**2 + b*c**2)*l
og(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**5 - 1440*
sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b
*c*x)*b**3*c**4*d*x - 720*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(
a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**3*d**2*x**2 - 120*sqrt(a*d**2 + b*
c**2)*log(c + d*x)*a**2*b*c**2*d**3 - 240*sqrt(a*d**2 + b*c**2)*log(c + d*
x)*a**2*b*c*d**4*x - 120*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*d**5*x*
*2 - 480*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**4*d - 960*sqrt(a*...
```

3.169 $\int \frac{(A+Bx)(c+dx)^4}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 284

$$\int \frac{(A+Bx)(c+dx)^4}{\sqrt{a+bx^2}}\,dx = -\frac{(16aBd^2-bc(12Bc+35Ad))\,(c+dx)^2\sqrt{a+bx^2}}{60b^2} + \frac{(4Bc+5Ad)(c+dx)^3\sqrt{a+bx^2}}{20b} + \frac{B(c+dx)^4\sqrt{a+bx^2}}{5b} + \frac{(4(16a^2Bd^4-16abcd^2(7Bc+5Ad)+b^2c^3(12Bc+95Ad))-bd(ad^2(116Bc+45Ad)-2bc^2(12Bc+6}{120b^3} - \frac{(4aBcd(4bc^2-3ad^2)-A(8b^2c^4-24abc^2d^2+3a^2d^4))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
-1/60*(16*a*B*d^2-b*c*(35*A*d+12*B*c))*(d*x+c)^2*(b*x^2+a)^(1/2)/b^2+1/20*
(5*A*d+4*B*c)*(d*x+c)^3*(b*x^2+a)^(1/2)/b+1/5*B*(d*x+c)^4*(b*x^2+a)^(1/2)/
b+1/120*(64*a^2*B*d^4-64*a*b*c*d^2*(5*A*d+7*B*c)+4*b^2*c^3*(95*A*d+12*B*c)
-b*d*(a*d^2*(45*A*d+116*B*c)-2*b*c^2*(65*A*d+12*B*c))*x)*(b*x^2+a)^(1/2)/b
^3-1/8*(4*a*B*c*d*(-3*a*d^2+4*b*c^2)-A*(3*a^2*d^4-24*a*b*c^2*d^2+8*b^2*c^4
))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)(c+dx)^4}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(64a^2Bd^4 - abd^2(5Ad(64c+9dx)+4B(120c^2+45cdx+8d^2x^2)) + 2b^2(5Ad(48c^3+36c^2dx+$$

input
```
Integrate[((A + B*x)*(c + d*x)^4)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(64*a^2*B*d^4 - a*b*d^2*(5*A*d*(64*c + 9*d*x) + 4*B*(120*
c^2 + 45*c*d*x + 8*d^2*x^2)) + 2*b^2*(5*A*d*(48*c^3 + 36*c^2*d*x + 16*c*d^
2*x^2 + 3*d^3*x^3) + 12*B*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x
^3 + d^4*x^4))) - 15*Sqrt[b]*(4*a*B*c*d*(-4*b*c^2 + 3*a*d^2) + A*(8*b^2*c^
4 - 24*a*b*c^2*d^2 + 3*a^2*d^4))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(120
*b^3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {687, 687, 27, 687, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(c+dx)^4}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{687}$$

$$\frac{\int \frac{(c+dx)^3(5Abc-4aBd+b(4Bc+5Ad)x)}{\sqrt{bx^2+a}}dx}{5b} + \frac{B\sqrt{a+bx^2}(c+dx)^4}{5b}$$

$$\Big\downarrow \text{687}$$

$$\frac{\frac{\int \frac{b(c+dx)^2\left(20Abc^2-28aBdc-15aAd^2-\left(16aBd^2-bc(12Bc+35Ad)\right)x\right)}{\sqrt{bx^2+a}}dx}{4b}+\frac{1}{4}\sqrt{a+bx^2}(c+dx)^3(5Ad+4Bc)}{5b}+\frac{B\sqrt{a+bx^2}(c+dx)^4}{5b}$$

$\downarrow$ 27

$$\frac{\frac{1}{4}\int \frac{(c+dx)^2(20Abc^2-28aBdc-15aAd^2-(16aBd^2-bc(12Bc+35Ad))x)}{\sqrt{bx^2+a}}dx+\frac{1}{4}\sqrt{a+bx^2}(c+dx)^3(5Ad+4Bc)}{5b}+\frac{B\sqrt{a+bx^2}(c+dx)^4}{5b}$$

$\downarrow$ 687

$$\frac{\frac{1}{4}\left(\frac{\int \frac{(c+dx)\left(5Abc\left(12bc^2-23ad^2\right)-4aBd\left(27bc^2-8ad^2\right)-b\left(ad^2(116Bc+45Ad)-2bc^2(12Bc+65Ad)\right)x\right)}{\sqrt{bx^2+a}}dx}{3b}-\frac{\sqrt{a+bx^2}(c+dx)^2(16aBd^2-bc(35Ad+12B}{3b}\right.}{5b}$$

$$\frac{B\sqrt{a+bx^2}(c+dx)^4}{5b}$$

$\downarrow$ 676

$$\frac{1}{4}\left(\frac{-\frac{15}{2}\left(4aBcd(4bc^2-3ad^2)-A\left(3a^2d^4-24abc^2d^2+8b^2c^4\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{2\sqrt{a+bx^2}\left(16a^2Bd^4-16abcd^2(5Ad+7Bc)+b^2c^3(95Ad+12Bc)\right)}{b}-\frac{1}{2}dx\sqrt{a+}}{3b}\right.$$

5

$$\frac{B\sqrt{a+bx^2}(c+dx)^4}{5b}$$

$\downarrow$ 224

$$\frac{1}{4}\left(\frac{-\frac{15}{2}\left(4aBcd(4bc^2-3ad^2)-A\left(3a^2d^4-24abc^2d^2+8b^2c^4\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{2\sqrt{a+bx^2}\left(16a^2Bd^4-16abcd^2(5Ad+7Bc)+b^2c^3(95Ad+12Bc)\right)}{b}-}{3b}\right.$$

$$\frac{B\sqrt{a+bx^2}(c+dx)^4}{5b}$$

$\downarrow$ 219

$$\frac{1}{4}\left(\frac{-\frac{15\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(4aBcd\left(4bc^2-3ad^2\right)-A\left(3a^2d^4-24abc^2d^2+8b^2c^4\right)\right)}{2\sqrt{b}}+\frac{2\sqrt{a+bx^2}\left(16a^2Bd^4-16abcd^2(5Ad+7Bc)+b^2c^3(95Ad+12Bc)\right)}{b}-\frac{1}{2}dx}{3b}\right.$$

$$\frac{B\sqrt{a+bx^2}(c+dx)^4}{5b}$$

input
```
Int[((A + B*x)*(c + d*x)^4)/Sqrt[a + b*x^2],x]
```

output
```
(B*(c + d*x)^4*Sqrt[a + b*x^2])/(5*b) + (((4*B*c + 5*A*d)*(c + d*x)^3*Sqrt
[a + b*x^2])/4 + (-1/3*((16*a*B*d^2 - b*c*(12*B*c + 35*A*d))*(c + d*x)^2*S
qrt[a + b*x^2])/b + ((2*(16*a^2*B*d^4 - 16*a*b*c*d^2*(7*B*c + 5*A*d) + b^2
*c^3*(12*B*c + 95*A*d))*Sqrt[a + b*x^2])/b - (d*(a*d^2*(116*B*c + 45*A*d)
- 2*b*c^2*(12*B*c + 65*A*d))*x*Sqrt[a + b*x^2])/2 - (15*(4*a*B*c*d*(4*b*c^
2 - 3*a*d^2) - A*(8*b^2*c^4 - 24*a*b*c^2*d^2 + 3*a^2*d^4))*ArcTanh[(Sqrt[b
]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))/(3*b))/4)/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{(-24B\,b^2x^4d^4-30A\,b^2d^4x^3-120B\,b^2c\,d^3x^3-160A\,b^2c\,d^3x^2+32Bab\,d^4x^2-240B\,b^2c^2d^2x^2+45Aab\,d^4x-360A\,b^2c^2d^2x+180Bab}{120b^3}$
default	$\frac{A\,c^4\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+d^3(Ad+4Bc)\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+2c\,d^2(2Ad+$

input
```
int((B*x+A)*(d*x+c)^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(-24*B*b^2*d^4*x^4-30*A*b^2*d^4*x^3-120*B*b^2*c*d^3*x^3-160*A*b^2*c
*d^3*x^2+32*B*a*b*d^4*x^2-240*B*b^2*c^2*d^2*x^2+45*A*a*b*d^4*x-360*A*b^2*c
^2*d^2*x+180*B*a*b*c*d^3*x-240*B*b^2*c^3*d*x+320*A*a*b*c*d^3-480*A*b^2*c^3
*d-64*B*a^2*d^4+480*B*a*b*c^2*d^2-120*B*b^2*c^4)*(b*x^2+a)^(1/2)/b^3+1/8/b
^(5/2)*(3*A*a^2*d^4-24*A*a*b*c^2*d^2+8*A*b^2*c^4+12*B*a^2*c*d^3-16*B*a*b*c
^3*d)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.82

$$\int \frac{(A+Bx)(c+dx)^4}{\sqrt{a+bx^2}}\,dx$$

$$= \left[\frac{15\,(8\,Ab^2c^4 - 16\,Babc^3d - 24\,Aabc^2d^2 + 12\,Ba^2cd^3 + 3\,Aa^2d^4)\sqrt{b}\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right)}{}\right.$$

$$-\frac{15\,(8\,Ab^2c^4 - 16\,Babc^3d - 24\,Aabc^2d^2 + 12\,Ba^2cd^3 + 3\,Aa^2d^4)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - (24\,Bb^2d^4x^4 +}{}$$

input
```
integrate((B*x+A)*(d*x+c)^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/240*(15*(8*A*b^2*c^4 - 16*B*a*b*c^3*d - 24*A*a*b*c^2*d^2 + 12*B*a^2*c*d
^3 + 3*A*a^2*d^4)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(24*B*b^2*d^4*x^4 + 120*B*b^2*c^4 + 480*A*b^2*c^3*d - 480*B*a*b*c^2*d^
2 - 320*A*a*b*c*d^3 + 64*B*a^2*d^4 + 30*(4*B*b^2*c*d^3 + A*b^2*d^4)*x^3 +
16*(15*B*b^2*c^2*d^2 + 10*A*b^2*c*d^3 - 2*B*a*b*d^4)*x^2 + 15*(16*B*b^2*c^
3*d + 24*A*b^2*c^2*d^2 - 12*B*a*b*c*d^3 - 3*A*a*b*d^4)*x)*sqrt(b*x^2 + a))
/b^3, -1/120*(15*(8*A*b^2*c^4 - 16*B*a*b*c^3*d - 24*A*a*b*c^2*d^2 + 12*B*a
^2*c*d^3 + 3*A*a^2*d^4)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (24*
B*b^2*d^4*x^4 + 120*B*b^2*c^4 + 480*A*b^2*c^3*d - 480*B*a*b*c^2*d^2 - 320*
A*a*b*c*d^3 + 64*B*a^2*d^4 + 30*(4*B*b^2*c*d^3 + A*b^2*d^4)*x^3 + 16*(15*B
*b^2*c^2*d^2 + 10*A*b^2*c*d^3 - 2*B*a*b*d^4)*x^2 + 15*(16*B*b^2*c^3*d + 24
*A*b^2*c^2*d^2 - 12*B*a*b*c*d^3 - 3*A*a*b*d^4)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.32

$$\int \frac{(A+Bx)(c+dx)^4}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{Bd^4x^4}{5b}+\frac{x^3(Ad^4+4Bcd^3)}{4b}+\frac{x^2\cdot\left(4Acd^3-\frac{4Bad^4}{5b}+6Bc^2d^2\right)}{3b}+\frac{x\left(6Ac^2d^2+4Bc^3d-\frac{3a\left(Ad^4+4Bcd^3\right)}{4b}\right)}{2b}+\frac{4Ac^3d+Bc^4}{}\right. \\ \frac{Ac^4x+\frac{Bd^4x^6}{6}+\frac{x^5\left(Ad^4+4Bcd^3\right)}{5}+\frac{x^4\cdot\left(4Acd^3+6Bc^2d^2\right)}{4}+\frac{x^3\cdot\left(6Ac^2d^2+4Bc^3d\right)}{3}+\frac{x^2\cdot\left(4Ac^3d+Bc^4\right)}{2}}{\sqrt{a}} \end{cases}$$

input
```
integrate((B*x+A)*(d*x+c)**4/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*d**4*x**4/(5*b) + x**3*(A*d**4 + 4*B*c*d**3
)/(4*b) + x**2*(4*A*c*d**3 - 4*B*a*d**4/(5*b) + 6*B*c**2*d**2)/(3*b) + x*(
6*A*c**2*d**2 + 4*B*c**3*d - 3*a*(A*d**4 + 4*B*c*d**3)/(4*b))/(2*b) + (4*A
*c**3*d + B*c**4 - 2*a*(4*A*c*d**3 - 4*B*a*d**4/(5*b) + 6*B*c**2*d**2)/(3*
b))/b) + (A*c**4 - a*(6*A*c**2*d**2 + 4*B*c**3*d - 3*a*(A*d**4 + 4*B*c*d**
3)/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b
), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), ((A*c**4*x + B*d*
*4*x**6/6 + x**5*(A*d**4 + 4*B*c*d**3)/5 + x**4*(4*A*c*d**3 + 6*B*c**2*d**
2)/4 + x**3*(6*A*c**2*d**2 + 4*B*c**3*d)/3 + x**2*(4*A*c**3*d + B*c**4)/2)
/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)(c+dx)^4}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bd^4x^4}{5\,b} - \frac{4\,\sqrt{bx^2+a}Bad^4x^2}{15\,b^2} + \frac{Ac^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{\sqrt{bx^2+a}Bc^4}{b} + \frac{4\,\sqrt{bx^2+a}Ac^3d}{b} + \frac{8\,\sqrt{bx^2+a}Ba^2d^4}{15\,b^3} + \frac{(4\,Bcd^3+Ad^4)\sqrt{bx^2+a}x^3}{4\,b} + \frac{2\,(3\,Bc^2d^2+2\,Acd^3)\sqrt{bx^2+a}x^2}{3\,b} - \frac{3\,(4\,Bcd^3+Ad^4)\sqrt{bx^2+a}ax}{8\,b^2} + \frac{(2\,Bc^3d+3\,Ac^2d^2)\sqrt{bx^2+a}x}{b} + \frac{3\,(4\,Bcd^3+Ad^4)a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{(2\,Bc^3d+3\,Ac^2d^2)a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{4\,(3\,Bc^2d^2+2\,Acd^3)\sqrt{bx^2+a}a}{3\,b^2}$$

input
```
integrate((B*x+A)*(d*x+c)^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(b*x^2 + a)*B*d^4*x^4/b - 4/15*sqrt(b*x^2 + a)*B*a*d^4*x^2/b^2 + A
*c^4*arcsinh(b*x/sqrt(a*b))/sqrt(b) + sqrt(b*x^2 + a)*B*c^4/b + 4*sqrt(b*x
^2 + a)*A*c^3*d/b + 8/15*sqrt(b*x^2 + a)*B*a^2*d^4/b^3 + 1/4*(4*B*c*d^3 +
A*d^4)*sqrt(b*x^2 + a)*x^3/b + 2/3*(3*B*c^2*d^2 + 2*A*c*d^3)*sqrt(b*x^2 +
a)*x^2/b - 3/8*(4*B*c*d^3 + A*d^4)*sqrt(b*x^2 + a)*a*x/b^2 + (2*B*c^3*d +
3*A*c^2*d^2)*sqrt(b*x^2 + a)*x/b + 3/8*(4*B*c*d^3 + A*d^4)*a^2*arcsinh(b*x
/sqrt(a*b))/b^(5/2) - (2*B*c^3*d + 3*A*c^2*d^2)*a*arcsinh(b*x/sqrt(a*b))/b
^(3/2) - 4/3*(3*B*c^2*d^2 + 2*A*c*d^3)*sqrt(b*x^2 + a)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)(c+dx)^4}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{1}{120}\sqrt{bx^2+a}\left(\left(2\left(3\left(\frac{4\,Bd^4x}{b}+\frac{5\,(4\,Bb^4cd^3+Ab^4d^4)}{b^5}\right)x+\frac{8\,(15\,Bb^4c^2d^2+10\,Ab^4cd^3-2\,Bab^3d^4)}{b^5}\right)x\right.\right.$$
$$-\frac{(8\,Ab^2c^4-16\,Babc^3d-24\,Aabc^2d^2+12\,Ba^2cd^3+3\,Aa^2d^4)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/120*sqrt(b*x^2 + a)*((2*(3*(4*B*d^4*x/b + 5*(4*B*b^4*c*d^3 + A*b^4*d^4)/
b^5)*x + 8*(15*B*b^4*c^2*d^2 + 10*A*b^4*c*d^3 - 2*B*a*b^3*d^4)/b^5)*x + 15
*(16*B*b^4*c^3*d + 24*A*b^4*c^2*d^2 - 12*B*a*b^3*c*d^3 - 3*A*a*b^3*d^4)/b^
5)*x + 8*(15*B*b^4*c^4 + 60*A*b^4*c^3*d - 60*B*a*b^3*c^2*d^2 - 40*A*a*b^3*
c*d^3 + 8*B*a^2*b^2*d^4)/b^5) - 1/8*(8*A*b^2*c^4 - 16*B*a*b*c^3*d - 24*A*a
*b*c^2*d^2 + 12*B*a^2*c*d^3 + 3*A*a^2*d^4)*log(abs(-sqrt(b)*x + sqrt(b*x^2
 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(c+dx)^4}{\sqrt{a+bx^2}}\,dx=\int \frac{(A+B\,x)\,\left(c+d\,x\right)^4}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((A + B*x)*(c + d*x)^4)/(a + b*x^2)^(1/2),x)
```

output
```
int(((A + B*x)*(c + d*x)^4)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)(c+dx)^4}{\sqrt{a+bx^2}}\,dx = \int \frac{(Bx+A)\,(dx+c)^4}{\sqrt{b\,x^2+a}}dx$$

input
```
int((B*x+A)*(d*x+c)^4/(b*x^2+a)^(1/2),x)
```

output
```
int((B*x+A)*(d*x+c)^4/(b*x^2+a)^(1/2),x)
```

3.170 $\int \frac{(A+Bx)(c+dx)^3}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 201

$$\int \frac{(A+Bx)(c+dx)^3}{\sqrt{a+bx^2}}\,dx = \frac{(3Bc+4Ad)(c+dx)^2\sqrt{a+bx^2}}{12b}+\frac{B(c+dx)^3\sqrt{a+bx^2}}{4b} - \frac{(4(4ad^2(3Bc+Ad)-bc^2(3Bc+16Ad))+d(9aBd^2-2bc(3Bc+10Ad))x)\sqrt{a+bx^2}}{24b^2} + \frac{(4Abc(2bc^2-3ad^2)-3aBd(4bc^2-ad^2))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
1/12*(4*A*d+3*B*c)*(d*x+c)^2*(b*x^2+a)^(1/2)/b+1/4*B*(d*x+c)^3*(b*x^2+a)^(
1/2)/b-1/24*(16*a*d^2*(A*d+3*B*c)-4*b*c^2*(16*A*d+3*B*c)+d*(9*a*B*d^2-2*b*
c*(10*A*d+3*B*c))*x)*(b*x^2+a)^(1/2)/b^2+1/8*(4*A*b*c*(-3*a*d^2+2*b*c^2)-3
*a*B*d*(-a*d^2+4*b*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)(c+dx)^3}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-ad^2(48Bc+16Ad+9Bdx)+4Abd(18c^2+9cdx+2d^2x^2)+6bB(4c^3+6c^2dx+4cd^2x^2+d}{24b^2}$$
$$-\frac{(4Abc(2bc^2-3ad^2)+3aBd(-4bc^2+ad^2))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

input
```
Integrate[((A + B*x)*(c + d*x)^3)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-(a*d^2*(48*B*c + 16*A*d + 9*B*d*x)) + 4*A*b*d*(18*c^2 +
 9*c*d*x + 2*d^2*x^2) + 6*b*B*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))
)/(24*b^2) - ((4*A*b*c*(2*b*c^2 - 3*a*d^2) + 3*a*B*d*(-4*b*c^2 + a*d^2))*L
og[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {687, 687, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(c+dx)^3}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{ 687}$$
$$\frac{\int \frac{(c+dx)^2(4Abc-3aBd+b(3Bc+4Ad)x)}{\sqrt{bx^2+a}}dx}{4b}+\frac{B\sqrt{a+bx^2}(c+dx)^3}{4b}$$
$$\Big\downarrow \text{ 687}$$

$$\frac{\frac{\int \frac{b(c+dx)\left(12Abc^2-15aBdc-8aAd^2-\left(9aBd^2-2bc(3Bc+10Ad)\right)x\right)}{\sqrt{bx^2+a}}dx}{3b}+\frac{1}{3}\sqrt{a+bx^2}(c+dx)^2(4Ad+3Bc)}{4b}+\frac{B\sqrt{a+bx^2}(c+dx)^3}{4b}$$

$\big\downarrow$ 27

$$\frac{\frac{1}{3}\int \frac{(c+dx)(12Abc^2-15aBdc-8aAd^2-(9aBd^2-2bc(3Bc+10Ad))x)}{\sqrt{bx^2+a}}dx+\frac{1}{3}\sqrt{a+bx^2}(c+dx)^2(4Ad+3Bc)}{4b}+\frac{B\sqrt{a+bx^2}(c+dx)^3}{4b}$$

$\big\downarrow$ 676

$$\frac{\frac{1}{3}\left(\frac{3\left(4Abc(2bc^2-3ad^2)-3aBd(4bc^2-ad^2)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{2\sqrt{a+bx^2}\left(4ad^2(Ad+3Bc)-bc^2(16Ad+3Bc)\right)}{b}-\frac{dx\sqrt{a+bx^2}\left(9aBd^2-2bc(10Ad+\right.}{2b}\right.}{4b}$$
$$\frac{B\sqrt{a+bx^2}(c+dx)^3}{4b}$$

$\big\downarrow$ 224

$$\frac{\frac{1}{3}\left(\frac{3\left(4Abc(2bc^2-3ad^2)-3aBd(4bc^2-ad^2)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{2\sqrt{a+bx^2}\left(4ad^2(Ad+3Bc)-bc^2(16Ad+3Bc)\right)}{b}-\frac{dx\sqrt{a+bx^2}\left(9aBd^2-2b\right.}{2b}\right.}{4b}$$
$$\frac{B\sqrt{a+bx^2}(c+dx)^3}{4b}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{3}\left(\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(4Abc(2bc^2-3ad^2)-3aBd(4bc^2-ad^2)\right)}{2b^{3/2}}-\frac{2\sqrt{a+bx^2}\left(4ad^2(Ad+3Bc)-bc^2(16Ad+3Bc)\right)}{b}-\frac{dx\sqrt{a+bx^2}\left(9aBd^2-2b\right.}{2b}\right.}{4b}$$
$$\frac{B\sqrt{a+bx^2}(c+dx)^3}{4b}$$

input
```
Int[((A + B*x)*(c + d*x)^3)/Sqrt[a + b*x^2],x]
```

output
```
(B*(c + d*x)^3*Sqrt[a + b*x^2])/(4*b) + (((3*B*c + 4*A*d)*(c + d*x)^2*Sqrt
[a + b*x^2])/3 + ((-2*(4*a*d^2*(3*B*c + A*d) - b*c^2*(3*B*c + 16*A*d))*Sqr
t[a + b*x^2])/b - (d*(9*a*B*d^2 - 2*b*c*(3*B*c + 10*A*d))*x*Sqrt[a + b*x^2
])/(2*b) + (3*(4*A*b*c*(2*b*c^2 - 3*a*d^2) - 3*a*B*d*(4*b*c^2 - a*d^2))*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/3)/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\left(-6Bb\,d^{3}x^{3}-8Ab\,d^{3}x^{2}-24Bbc\,d^{2}x^{2}-36Abc\,d^{2}x+9Ba\,d^{3}x-36Bb\,c^{2}dx+16A\,d^{3}a-72Ab\,c^{2}d+48aBc\,d^{2}-24bB\,c^{3}\right)\sqrt{b\,x^{2}+a}}{24b^{2}}-$
default	$\frac{A\,c^{3}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+d^{2}(Ad+3Bc)\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)+3cd(Ad+Bc)\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{\right.}\right.}{}\right.$

input
```
int((B*x+A)*(d*x+c)^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-6*B*b*d^3*x^3-8*A*b*d^3*x^2-24*B*b*c*d^2*x^2-36*A*b*c*d^2*x+9*B*a*
d^3*x-36*B*b*c^2*d*x+16*A*a*d^3-72*A*b*c^2*d+48*B*a*c*d^2-24*B*b*c^3)*(b*x
^2+a)^(1/2)/b^2-1/8*(12*A*a*b*c*d^2-8*A*b^2*c^3-3*B*a^2*d^3+12*B*a*b*c^2*d
)/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.86

$$\int \frac{(A+Bx)(c+dx)^3}{\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{3\left(8\,Ab^2c^3-12\,Babc^2d-12\,Aabcd^2+3\,Ba^2d^3\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(6\,Bb^2d^3x^3\right.}{}\right.$$
$$-\frac{3\left(8\,Ab^2c^3-12\,Babc^2d-12\,Aabcd^2+3\,Ba^2d^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(6\,Bb^2d^3x^3+24\,Bb^2c^3+72\right.}{2}$$

input
```
integrate((B*x+A)*(d*x+c)^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(8*A*b^2*c^3 - 12*B*a*b*c^2*d - 12*A*a*b*c*d^2 + 3*B*a^2*d^3)*sqr
t(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*B*b^2*d^3*x^3
+ 24*B*b^2*c^3 + 72*A*b^2*c^2*d - 48*B*a*b*c*d^2 - 16*A*a*b*d^3 + 8*(3*B*b
^2*c*d^2 + A*b^2*d^3)*x^2 + 9*(4*B*b^2*c^2*d + 4*A*b^2*c*d^2 - B*a*b*d^3)*
x)*sqrt(b*x^2 + a))/b^3, -1/24*(3*(8*A*b^2*c^3 - 12*B*a*b*c^2*d - 12*A*a*b
*c*d^2 + 3*B*a^2*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*B*b
^2*d^3*x^3 + 24*B*b^2*c^3 + 72*A*b^2*c^2*d - 48*B*a*b*c*d^2 - 16*A*a*b*d^3
 + 8*(3*B*b^2*c*d^2 + A*b^2*d^3)*x^2 + 9*(4*B*b^2*c^2*d + 4*A*b^2*c*d^2 -
B*a*b*d^3)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx)(c+dx)^3}{\sqrt{a+bx^2}}\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Bd^3x^3}{4b} + \frac{x^2(Ad^3+3Bcd^2)}{3b} + \frac{x\left(3Acd^2 - \frac{3Bad^3}{4b} + 3Bc^2d\right)}{2b} + \frac{3Ac^2d + Bc^3 - \frac{2a\left(Ad^3+3Bcd^2\right)}{3b}}{b}\right) + \left(Ac^3 - \frac{a\left(3Acd^2 - \right.}{} \right. \\ \frac{Ac^3x + \frac{Bd^3x^5}{5} + \frac{x^4\left(Ad^3+3Bcd^2\right)}{4} + \frac{x^3\cdot\left(3Acd^2+3Bc^2d\right)}{3} + \frac{x^2\cdot\left(3Ac^2d+Bc^3\right)}{2}}{\sqrt{a}} \end{cases}$$

input
```
integrate((B*x+A)*(d*x+c)**3/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*d**3*x**3/(4*b) + x**2*(A*d**3 + 3*B*c*d**2
)/(3*b) + x*(3*A*c*d**2 - 3*B*a*d**3/(4*b) + 3*B*c**2*d)/(2*b) + (3*A*c**2
*d + B*c**3 - 2*a*(A*d**3 + 3*B*c*d**2)/(3*b))/b) + (A*c**3 - a*(3*A*c*d**
2 - 3*B*a*d**3/(4*b) + 3*B*c**2*d)/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a
+ b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(
b, 0)), ((A*c**3*x + B*d**3*x**5/5 + x**4*(A*d**3 + 3*B*c*d**2)/4 + x**3*(
3*A*c*d**2 + 3*B*c**2*d)/3 + x**2*(3*A*c**2*d + B*c**3)/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)(c+dx)^3}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bd^3x^3}{4\,b} - \frac{3\,\sqrt{bx^2+a}Bad^3x}{8\,b^2} + \frac{Ac^3 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{3\,Ba^2d^3 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} + \frac{\sqrt{bx^2+a}Bc^3}{b} + \frac{3\,\sqrt{bx^2+a}Ac^2d}{b} + \frac{(3\,Bcd^2+Ad^3)\sqrt{bx^2+a}x^2}{3\,b} + \frac{3\,(Bc^2d+Acd^2)\sqrt{bx^2+a}x}{2\,b} - \frac{3\,(Bc^2d+Acd^2)a \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} - \frac{2\,(3\,Bcd^2+Ad^3)\sqrt{bx^2+a}a}{3\,b^2}$$

input

```
integrate((B*x+A)*(d*x+c)^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output

```
1/4*sqrt(b*x^2 + a)*B*d^3*x^3/b - 3/8*sqrt(b*x^2 + a)*B*a*d^3*x/b^2 + A*c^
3*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 3/8*B*a^2*d^3*arcsinh(b*x/sqrt(a*b))/b^
(5/2) + sqrt(b*x^2 + a)*B*c^3/b + 3*sqrt(b*x^2 + a)*A*c^2*d/b + 1/3*(3*B*c
*d^2 + A*d^3)*sqrt(b*x^2 + a)*x^2/b + 3/2*(B*c^2*d + A*c*d^2)*sqrt(b*x^2 +
 a)*x/b - 3/2*(B*c^2*d + A*c*d^2)*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/3*(
3*B*c*d^2 + A*d^3)*sqrt(b*x^2 + a)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)(c+dx)^3}{\sqrt{a+bx^2}}\,dx = \frac{1}{24}\,\sqrt{bx^2+a}\left(\left(2\left(\frac{3\,Bd^3x}{b} + \frac{4\,(3\,Bb^3cd^2+Ab^3d^3)}{b^4}\right)x + \frac{9\,(4\,Bb^3c^2d+4\,Ab^3cd^2-Bab^2d^3)}{b^4}\right)x + \frac{8\,(3\,B\right. - \frac{(8\,Ab^2c^3-12\,Babc^2d-12\,Aabcd^2+3\,Ba^2d^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input

```
integrate((B*x+A)*(d*x+c)^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a)*((2*(3*B*d^3*x/b + 4*(3*B*b^3*c*d^2 + A*b^3*d^3)/b^4)
*x + 9*(4*B*b^3*c^2*d + 4*A*b^3*c*d^2 - B*a*b^2*d^3)/b^4)*x + 8*(3*B*b^3*c
^3 + 9*A*b^3*c^2*d - 6*B*a*b^2*c*d^2 - 2*A*a*b^2*d^3)/b^4) - 1/8*(8*A*b^2*
c^3 - 12*B*a*b*c^2*d - 12*A*a*b*c*d^2 + 3*B*a^2*d^3)*log(abs(-sqrt(b)*x +
sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(c+dx)^3}{\sqrt{a+bx^2}}\,dx=\int \frac{(A+B\,x)\,{(c+d\,x)}^3}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((A + B*x)*(c + d*x)^3)/(a + b*x^2)^(1/2),x)
```

output
```
int(((A + B*x)*(c + d*x)^3)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)(c+dx)^3}{\sqrt{a+bx^2}}\,dx=\int \frac{(Bx+A)\,(dx+c)^3}{\sqrt{b\,x^2+a}}dx$$

input
```
int((B*x+A)*(d*x+c)^3/(b*x^2+a)^(1/2),x)
```

output
```
int((B*x+A)*(d*x+c)^3/(b*x^2+a)^(1/2),x)
```

3.171 $\int \frac{(A+Bx)(c+dx)^2}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \frac{(A+Bx)(c+dx)^2}{\sqrt{a+bx^2}}\,dx = \frac{B(c+dx)^2\sqrt{a+bx^2}}{3b} - \frac{(4(aBd^2-bc(Bc+3Ad))-bd(2Bc+3Ad)x)\sqrt{a+bx^2}}{6b^2} + \frac{(2Abc^2-2aBcd-aAd^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
1/3*B*(d*x+c)^2*(b*x^2+a)^(1/2)/b-1/6*(4*a*B*d^2-4*b*c*(3*A*d+B*c)-b*d*(3*
A*d+2*B*c)*x)*(b*x^2+a)^(1/2)/b^2+1/2*(-A*a*d^2+2*A*b*c^2-2*B*a*c*d)*arcta
nh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)(c+dx)^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-4aBd^2+3Abd(4c+dx)+2bB(3c^2+3cdx+d^2x^2))+3\sqrt{b}(-2Abc^2+2aBcd+aAd^2)\log\left(\right.}{6b^2}$$

input
```
Integrate[((A + B*x)*(c + d*x)^2)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-4*a*B*d^2 + 3*A*b*d*(4*c + d*x) + 2*b*B*(3*c^2 + 3*c*d*
x + d^2*x^2)) + 3*Sqrt[b]*(-2*A*b*c^2 + 2*a*B*c*d + a*A*d^2)*Log[-(Sqrt[b]
*x) + Sqrt[a + b*x^2]])/(6*b^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {687, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(c+dx)^2}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{687}$$

$$\frac{\int \frac{(c+dx)(3Abc-2aBd+b(2Bc+3Ad)x)}{\sqrt{bx^2+a}}dx}{3b}+\frac{B\sqrt{a+bx^2}(c+dx)^2}{3b}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{3}{2}\left(-aAd^2-2aBcd+2Abc^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{2\sqrt{a+bx^2}(aBd^2-bc(3Ad+Bc))}{b}+\frac{1}{2}dx\sqrt{a+bx^2}(3Ad+2Bc)}{3b}+\frac{B\sqrt{a+bx^2}(c+dx)^2}{3b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3}{2}\left(-aAd^2-2aBcd+2Abc^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{2\sqrt{a+bx^2}(aBd^2-bc(3Ad+Bc))}{b}+\frac{1}{2}dx\sqrt{a+bx^2}(3Ad+2Bc)}{3b}+\frac{B\sqrt{a+bx^2}(c+dx)^2}{3b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-aAd^2-2aBcd+2Abc^2)}{2\sqrt{b}}-\frac{2\sqrt{a+bx^2}(aBd^2-bc(3Ad+Bc))}{b}+\frac{1}{2}dx\sqrt{a+bx^2}(3Ad+2Bc)}{3b}+\frac{B\sqrt{a+bx^2}(c+dx)^2}{3b}$$

input
```
Int[((A + B*x)*(c + d*x)^2)/Sqrt[a + b*x^2],x]
```

output
```
(B*(c + d*x)^2*Sqrt[a + b*x^2])/(3*b) + ((-2*(a*B*d^2 - b*c*(B*c + 3*A*d))
*Sqrt[a + b*x^2])/b + (d*(2*B*c + 3*A*d)*x*Sqrt[a + b*x^2])/2 + (3*(2*A*b*
c^2 - 2*a*B*c*d - a*A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b
]))/(3*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] && !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

method	result
risch	$\frac{(2Bb\,d^2x^2+3Ab\,d^2x+6Bbcdx+12Abcd-4aB\,d^2+6Bb\,c^2)\sqrt{b\,x^2+a}}{6b^2}-\frac{(Aa\,d^2-2Ab\,c^2+2Bacd)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}$
default	$\frac{A\,c^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+d(Ad+2Bc)\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)+\frac{c(2Ad+Bc)\sqrt{b\,x^2+a}}{b}+B\,d^2\Big(\frac{x^2}{}$

input
```
int((B*x+A)*(d*x+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*B*b*d^2*x^2+3*A*b*d^2*x+6*B*b*c*d*x+12*A*b*c*d-4*B*a*d^2+6*B*b*c^2)
*(b*x^2+a)^(1/2)/b^2-1/2/b^(3/2)*(A*a*d^2-2*A*b*c^2+2*B*a*c*d)*ln(b^(1/2)*
x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.74

$$\int \frac{(A+Bx)(c+dx)^2}{\sqrt{a+bx^2}}\,dx$$
$$=\left[-\frac{3\,(2\,Abc^2-2\,Bacd-Aad^2)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(2\,Bbd^2x^2+6\,Bbc^2+12\,Abc}{12\,b^2}\right.$$
$$-\frac{3\,(2\,Abc^2-2\,Bacd-Aad^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Bbd^2x^2+6\,Bbc^2+12\,Abcd-4\,Bad^2+3\,(2\,B}{6\,b^2}$$

input
```
integrate((B*x+A)*(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(2*A*b*c^2 - 2*B*a*c*d - A*a*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(
b*x^2 + a)*sqrt(b)*x - a) - 2*(2*B*b*d^2*x^2 + 6*B*b*c^2 + 12*A*b*c*d - 4*
B*a*d^2 + 3*(2*B*b*c*d + A*b*d^2)*x)*sqrt(b*x^2 + a))/b^2, -1/6*(3*(2*A*b*
c^2 - 2*B*a*c*d - A*a*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (
2*B*b*d^2*x^2 + 6*B*b*c^2 + 12*A*b*c*d - 4*B*a*d^2 + 3*(2*B*b*c*d + A*b*d^
2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx)(c+dx)^2}{\sqrt{a+bx^2}}\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Bd^2x^2}{3b}+\frac{x(Ad^2+2Bcd)}{2b}+\frac{2Acd-\frac{2Bad^2}{3b}+Bc^2}{b}\right)+\left(Ac^2-\frac{a(Ad^2+2Bcd)}{2b}\right)\left(\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for} \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{oth}\end{cases}\right.\right. \\ \frac{Ac^2x+\frac{Bd^2x^4}{4}+\frac{x^3\left(Ad^2+2Bcd\right)}{3}+\frac{x^2\cdot\left(2Acd+Bc^2\right)}{2}}{\sqrt{a}} \end{cases}$$

input
```
integrate((B*x+A)*(d*x+c)**2/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*d**2*x**2/(3*b) + x*(A*d**2 + 2*B*c*d)/(2*b
) + (2*A*c*d - 2*B*a*d**2/(3*b) + B*c**2)/b) + (A*c**2 - a*(A*d**2 + 2*B*c
*d)/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(
a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), ((A*c**2*x + B*d**2*x**
4/4 + x**3*(A*d**2 + 2*B*c*d)/3 + x**2*(2*A*c*d + B*c**2)/2)/sqrt(a), True
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)(c+dx)^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bd^2x^2}{3\,b} + \frac{Ac^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{\sqrt{bx^2+a}Bc^2}{b} + \frac{2\,\sqrt{bx^2+a}Acd}{b} - \frac{2\,\sqrt{bx^2+a}Bad^2}{3\,b^2} + \frac{(2\,Bcd+Ad^2)\sqrt{bx^2+a}x}{2\,b} - \frac{(2\,Bcd+Ad^2)a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a)*B*d^2*x^2/b + A*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + s
qrt(b*x^2 + a)*B*c^2/b + 2*sqrt(b*x^2 + a)*A*c*d/b - 2/3*sqrt(b*x^2 + a)*B
*a*d^2/b^2 + 1/2*(2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*x/b - 1/2*(2*B*c*d + A*
d^2)*a*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)(c+dx)^2}{\sqrt{a+bx^2}}\,dx = \frac{1}{6}\sqrt{bx^2+a}\left(\left(\frac{2\,Bd^2x}{b} + \frac{3\,(2\,Bb^2cd+Ab^2d^2)}{b^3}\right)x + \frac{2\,(3\,Bb^2c^2+6\,Ab^2cd-2\,Babd^2)}{b^3}\right) - \frac{(2\,Abc^2-2\,Bacd-Aad^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(b*x^2 + a)*((2*B*d^2*x/b + 3*(2*B*b^2*c*d + A*b^2*d^2)/b^3)*x + 2
*(3*B*b^2*c^2 + 6*A*b^2*c*d - 2*B*a*b*d^2)/b^3) - 1/2*(2*A*b*c^2 - 2*B*a*c
*d - A*a*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(c+dx)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{(A+B\,x)\,\left(c+d\,x\right)^2}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((A + B*x)*(c + d*x)^2)/(a + b*x^2)^(1/2),x)
```

output
```
int(((A + B*x)*(c + d*x)^2)/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx)(c+dx)^2}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{12\sqrt{b\,x^2+a}\,abcd+3\sqrt{b\,x^2+a}\,ab\,d^2x-4\sqrt{b\,x^2+a}\,ab\,d^2+6\sqrt{b\,x^2+a}\,b^2c^2+6\sqrt{b\,x^2+a}\,b^2cdx+2\sqrt{b\,x}}{6b}$$

input
```
int((B*x+A)*(d*x+c)^2/(b*x^2+a)^(1/2),x)
```

output
```
(12*sqrt(a + b*x**2)*a*b*c*d + 3*sqrt(a + b*x**2)*a*b*d**2*x - 4*sqrt(a +
b*x**2)*a*b*d**2 + 6*sqrt(a + b*x**2)*b**2*c**2 + 6*sqrt(a + b*x**2)*b**2*
c*d*x + 2*sqrt(a + b*x**2)*b**2*d**2*x**2 - 3*sqrt(b)*log((sqrt(a + b*x**2
) + sqrt(b)*x)/sqrt(a))*a**2*d**2 + 6*sqrt(b)*log((sqrt(a + b*x**2) + sqrt
(b)*x)/sqrt(a))*a*b*c**2 - 6*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sq
rt(a))*a*b*c*d)/(6*b**2)
```

3.172 $\int \frac{(A+Bx)(c+dx)}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{(A+Bx)(c+dx)}{\sqrt{a+bx^2}}\,dx = \frac{(2(Bc+Ad)+Bdx)\sqrt{a+bx^2}}{2b} + \frac{(2Abc-aBd)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
1/2*(B*d*x+2*A*d+2*B*c)*(b*x^2+a)^(1/2)/b+1/2*(2*A*b*c-B*a*d)*arctanh(b^(1
/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)(c+dx)}{\sqrt{a+bx^2}}\,dx = \frac{(2Bc+2Ad+Bdx)\sqrt{a+bx^2}}{2b} + \frac{(-2Abc+aBd)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

input
```
Integrate[((A + B*x)*(c + d*x))/Sqrt[a + b*x^2],x]
```

output
```
((2*B*c + 2*A*d + B*d*x)*Sqrt[a + b*x^2])/(2*b) + ((-2*A*b*c + a*B*d)*Log[
-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(c+dx)}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \; 676$$

$$\frac{(2Abc-aBd)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{\sqrt{a+bx^2}(Ad+Bc)}{b}+\frac{Bdx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \; 224$$

$$\frac{(2Abc-aBd)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{\sqrt{a+bx^2}(Ad+Bc)}{b}+\frac{Bdx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \; 219$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2Abc-aBd)}{2b^{3/2}}+\frac{\sqrt{a+bx^2}(Ad+Bc)}{b}+\frac{Bdx\sqrt{a+bx^2}}{2b}$$

input
```
Int[((A + B*x)*(c + d*x))/Sqrt[a + b*x^2],x]
```

output
```
((B*c + A*d)*Sqrt[a + b*x^2])/b + (B*d*x*Sqrt[a + b*x^2])/(2*b) + ((2*A*b*
c - a*B*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{(Bdx+2Ad+2Bc)\sqrt{b\,x^{2}+a}}{2b}+\frac{(2Abc-Bad)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}$	61
default	$\frac{Ac\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{(Ad+Bc)\sqrt{b\,x^{2}+a}}{b}+Bd\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)$	85

input
```
int((B*x+A)*(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(B*d*x+2*A*d+2*B*c)*(b*x^2+a)^(1/2)/b+1/2*(2*A*b*c-B*a*d)/b^(3/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.97

$$\int \frac{(A+Bx)(c+dx)}{\sqrt{a+bx^2}}\,dx = \left[-\frac{(2\,Abc-Bad)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(Bbdx+2\,Bbc+2\,Abd)\sqrt{bx^2+a}}{4\,b^2},\ -\frac{(2\,Abc-Bad)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Bbdx+2\,Bbc+2\,Abd)\sqrt{bx^2+a}}{2\,b^2}\right]$$

input

```
integrate((B*x+A)*(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output

```
[-1/4*((2*A*b*c - B*a*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a) - 2*(B*b*d*x + 2*B*b*c + 2*A*b*d)*sqrt(b*x^2 + a))/b^2, -1/2*((2*A*
b*c - B*a*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (B*b*d*x + 2*B*
b*c + 2*A*b*d)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.53

$$\int \frac{(A+Bx)(c+dx)}{\sqrt{a+bx^2}}\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Bdx}{2b}+\frac{Ad+Bc}{b}\right)+\left(Ac-\frac{Bad}{2b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right) & \text{for}\ b\neq 0\\ \frac{Acx+\frac{Bdx^3}{3}+\frac{x^2(Ad+Bc)}{2}}{\sqrt{a}} & \text{otherwise}\end{cases}$$

input

```
integrate((B*x+A)*(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*d*x/(2*b) + (A*d + B*c)/b) + (A*c - B*a*d/(
2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)
), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), ((A*c*x + B*d*x**3/3 + x**2*
(A*d + B*c)/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)(c+dx)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bdx}{2\,b} + \frac{Ac\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{Bad\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{\sqrt{bx^2+a}Bc}{b} + \frac{\sqrt{bx^2+a}Ad}{b}$$

input
```
integrate((B*x+A)*(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*B*d*x/b + A*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*B*a
*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + sqrt(b*x^2 + a)*B*c/b + sqrt(b*x^2 + a
)*A*d/b
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)(c+dx)}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\sqrt{bx^2+a}\left(\frac{Bdx}{b} + \frac{2\,(Bbc+Abd)}{b^2}\right) - \frac{(2\,Abc-Bad)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*(B*d*x/b + 2*(B*b*c + A*b*d)/b^2) - 1/2*(2*A*b*c - B*a
*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 6.46 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.89

$$\int \frac{(A+Bx)(c+dx)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \frac{6\,A\,c\,x+3\,A\,d\,x^2+3\,B\,c\,x^2+2\,B\,d\,x^3}{6\,\sqrt{a}} & \text{if } b=0 \\ \frac{A\,d\,\sqrt{b\,x^2+a}}{b}+\frac{B\,c\,\sqrt{b\,x^2+a}}{b}+\frac{A\,c\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{B\,a\,d\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}+\frac{B\,d\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if } b\neq 0 \end{cases}$$

input
```
int(((A + B*x)*(c + d*x))/(a + b*x^2)^(1/2),x)
```

output
```
piecewise(b == 0, (6*A*c*x + 3*A*d*x^2 + 3*B*c*x^2 + 2*B*d*x^3)/(6*a^(1/2)
), b ~= 0, (A*d*(a + b*x^2)^(1/2))/b + (B*c*(a + b*x^2)^(1/2))/b + (A*c*lo
g(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (B*a*d*log(2*b^(1/2)*x + 2*(a
+ b*x^2)^(1/2)))/(2*b^(3/2)) + (B*d*x*(a + b*x^2)^(1/2))/(2*b))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx)(c+dx)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{2\sqrt{b\,x^2+a}\,ad+2\sqrt{b\,x^2+a}\,bc+\sqrt{b\,x^2+a}\,bdx+2\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)ac-\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)ad}{2b}$$

input
```
int((B*x+A)*(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**2)*a*d + 2*sqrt(a + b*x**2)*b*c + sqrt(a + b*x**2)*b*d*x
+ 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c - sqrt(b)*log(
(sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*d)/(2*b)
```

3.173 $\int \frac{A+Bx}{\sqrt{a+bx^2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{A+Bx}{\sqrt{a+bx^2}} dx = \frac{B\sqrt{a+bx^2}}{b} + \frac{A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

output
```
B*(b*x^2+a)^(1/2)/b+A*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx}{\sqrt{a+bx^2}} dx = \frac{B\sqrt{a+bx^2}}{b} - \frac{A\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

input
```
Integrate[(A + B*x)/Sqrt[a + b*x^2],x]
```

output
```
(B*Sqrt[a + b*x^2])/b - (A*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \ 455$$

$$A\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{B\sqrt{a+bx^2}}{b}$$

$$\Big\downarrow \ 224$$

$$A\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{B\sqrt{a+bx^2}}{b}$$

$$\Big\downarrow \ 219$$

$$\frac{A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

input
```
Int[(A + B*x)/Sqrt[a + b*x^2],x]
```

output
```
(B*Sqrt[a + b*x^2])/b + (A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{B\sqrt{b\,x^2+a}}{b}$	37
risch	$\frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{B\sqrt{b\,x^2+a}}{b}$	37

input
```
int((B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+B*(b*x^2+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.14

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \left[\frac{A\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,\sqrt{bx^2+a}B}{2\,b}, -\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}B}{b}\right]$$

input
```
integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(A*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b
*x^2 + a)*B)/b, -(A*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt(b*x
^2 + a)*B)/b]
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \begin{cases} A\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right) + \frac{B\sqrt{a+bx^2}}{b} & \text{for}\ b\neq 0 \\ \frac{Ax+\frac{Bx^2}{2}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((A*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), N
e(a, 0)), (x*log(x)/sqrt(b*x**2), True)) + B*sqrt(a + b*x**2)/b, Ne(b, 0))
, ((A*x + B*x**2/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{\sqrt{bx^2+a}B}{b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
A*arcsinh(b*x/sqrt(a*b))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = -\frac{A\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}+\frac{\sqrt{bx^2+a}B}{b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-A*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \frac{B\sqrt{b\,x^2+a}}{b}+\frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

input
```
int((A + B*x)/(a + b*x^2)^(1/2),x)
```

output
```
(B*(a + b*x^2)^(1/2))/b + (A*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a}\,b+\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a}{b}$$

input
```
int((B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*b + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*
a)/b
```

3.174 $\int \frac{A+Bx}{(c+dx)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{A+Bx}{(c+dx)\sqrt{a+bx^2}}\,dx=\frac{B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}+\frac{(Bc-Ad)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d\sqrt{bc^2+ad^2}}$$

output
```
B*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d+(-A*d+B*c)*arctanh((-b*c*x+
a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx}{(c+dx)\sqrt{a+bx^2}}\,dx=\frac{\frac{2(Bc-Ad)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}}-\frac{B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}}{d}$$

input
```
Integrate[(A + B*x)/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
((2*(B*c - A*d)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^
2) - a*d^2]])/Sqrt[-(b*c^2) - a*d^2] - (B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/Sqrt[b])/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+bx^2}(c+dx)}dx$$

$$\Big\downarrow \text{719}$$

$$\frac{B\int \frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{(Bc-Ad)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}$$

$$\Big\downarrow \text{224}$$

$$\frac{B\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{(Bc-Ad)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}-\frac{(Bc-Ad)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}$$

$$\Big\downarrow \text{488}$$

$$\frac{(Bc-Ad)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}$$

$$\Big\downarrow \text{219}$$

$$\frac{(Bc-Ad)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\sqrt{ad^2+bc^2}}+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}$$

input
```
Int[(A + B*x)/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) + ((B*c - A*d)*ArcTan
h[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*
d^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.70

method	result	size
default	$\frac{B \ln\left(\sqrt{b}\, x+\sqrt{b\,x^{2}+a}\right)}{d \sqrt{b}}-\frac{\left(A d -B c \right) \ln\left(\frac{\frac{2 a \,d^{2}+2 b \,c^{2}}{d^{2}}-\frac{2 b c \left(x +\frac{c}{d}\right)}{d}+2 \sqrt{\frac{a \,d^{2}+b \,c^{2}}{d^{2}}}\, \sqrt{b \left(x +\frac{c}{d}\right)^{2}-\frac{2 b c \left(x +\frac{c}{d}\right)}{d}+\frac{a \,d^{2}+b \,c^{2}}{d^{2}}}}{x +\frac{c}{d}}\right)}{d^{2} \sqrt{\frac{a \,d^{2}+b \,c^{2}}{d^{2}}}}$	160

input
```
int((B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-(A*d-B*c)/d^2/((a*d^2+b*c^2)/d^2
)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2
)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))
```

Fricas [A] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 672, normalized size of antiderivative = 7.15

$$\int \frac{A+Bx}{(c+dx)\sqrt{a+bx^2}}\,dx$$

$$= \left[\frac{(Bbc^2+Bad^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-(Bbc-Abd)\sqrt{bc^2+ad^2}\log\left(\frac{2\,abcdx-abc^2-2\,a^2d^2\cdots}{\cdots}\right.}{2\,(b^2c^2d+abd^3)}\right.$$

$$-\frac{2\,(Bbc^2+Bad^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(Bbc-Abd)\sqrt{bc^2+ad^2}\log\left(\frac{2\,abcdx-abc^2-2\,a^2d^2-(2\,b^2c^2+abd^2)x^2\cdots}{d^2x^2+2\,cdx\cdots}\right.}{2\,(b^2c^2d+abd^3)}$$

input
```
integrate((B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((B*b*c^2 + B*a*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)
*x - a) - (B*b*c - A*b*d)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 -
 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*
d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)))/(b^2*c^2*d + a*b*d^3), -1/
2*(2*(B*b*c^2 + B*a*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (B*
b*c - A*b*d)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 -
(2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2
 + a))/(d^2*x^2 + 2*c*d*x + c^2)))/(b^2*c^2*d + a*b*d^3), 1/2*(2*(B*b*c -
A*b*d)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt
(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + (B*b*c^2 + B*
a*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(b^2*c^2*d
 + a*b*d^3), ((B*b*c - A*b*d)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*
d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2
)*x^2)) - (B*b*c^2 + B*a*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))
/(b^2*c^2*d + a*b*d^3)]
```

Sympy [F]

$$\int \frac{A+Bx}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{A+Bx}{\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx}{(c+dx)\sqrt{a+bx^2}}\,dx = \frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d} - \frac{Bc\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^2} + \frac{A\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d}$$

input
```
integrate((B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
B*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d) - B*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*
x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^2) + A*arcs
inh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a
 + b*c^2/d^2)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{A+B\,x}{\sqrt{b\,x^2+a}\,(c+d\,x)}\,dx$$

input
```
int((A + B*x)/((a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((A + B*x)/((a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2126, normalized size of antiderivative = 22.62

$$\int \frac{A+Bx}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*
sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*c*d + 2*sqrt(b)*sqrt(2*
sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c**2)
*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c*
*2)*c - a*d**2 - 2*b*c**2))*b*c**2 - 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2
)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sq
rt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**2*d**3 - 2*sqrt(2*s
qrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)
*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c*
*2))*a*b*c**2*d + 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*
c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 +
 b*c**2)*c - a*d**2 - 2*b*c**2))*a*b*c*d**2 + 2*sqrt(2*sqrt(b)*sqrt(a*d**2
 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/
sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*b**2*c**3 - s
qrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2)*sqrt(a*
d**2 + b*c**2)*log( - sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*
b*c**2) + sqrt(a + b*x**2)*d + sqrt(b)*d*x)*a*c*d + sqrt(b)*sqrt(2*sqrt(b)
*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2)*sqrt(a*d**2 + b*c**2)*log( -
 sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c + a*d**2 + 2*b*c**2) + sqrt(a +...
```

3.175 $\int \frac{A+Bx}{(c+dx)^2\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{A+Bx}{(c+dx)^2\sqrt{a+bx^2}}\,dx=\frac{(Bc-Ad)\sqrt{a+bx^2}}{(bc^2+ad^2)\,(c+dx)}-\frac{(Abc+aBd)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{(bc^2+ad^2)^{3/2}}$$

output
```
(-A*d+B*c)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)/(d*x+c)-(A*b*c+B*a*d)*arctanh((-b
*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx}{(c+dx)^2\sqrt{a+bx^2}}\,dx =\frac{(Bc-Ad)\sqrt{a+bx^2}}{(bc^2+ad^2)\,(c+dx)}+\frac{2(Abc+aBd)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{(-bc^2-ad^2)^{3/2}}$$

input
```
Integrate[(A + B*x)/((c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
((B*c - A*d)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)) + (2*(A*b*c + a*
B*d)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]
])/(-(b*c^2) - a*d^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+bx^2}(c+dx)^2}\,dx$$

$$\Big\downarrow \text{679}$$

$$\frac{(aBd+Abc)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}+\frac{\sqrt{a+bx^2}(Bc-Ad)}{(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{488}$$

$$\frac{\sqrt{a+bx^2}(Bc-Ad)}{(c+dx)\left(ad^2+bc^2\right)}-\frac{(aBd+Abc)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a+bx^2}(Bc-Ad)}{(c+dx)\left(ad^2+bc^2\right)}-\frac{(aBd+Abc)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\left(ad^2+bc^2\right)^{3/2}}$$

input
```
Int[(A + B*x)/((c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
((B*c - A*d)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)) - ((A*b*c + a*B*
d)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 +
a*d^2)^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(96) = 192$.

Time = 1.34 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.38

method	result
default	$-\frac{B\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}{x+\frac{c}{d}}\right)}{d^2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}}+\frac{(Ad-Bc)\left(-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d}{}}}{\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)}\right.}{}$

input
```
int((B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-B/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2
*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)
^(1/2))/(x+c/d))+(A*d-B*c)/d^3*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-
2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2
)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)
^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(97) = 194$.

Time = 0.25 (sec) , antiderivative size = 446, normalized size of antiderivative = 4.29

$$\int \frac{A+Bx}{(c+dx)^2\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{(Abc^2+Bacd+(Abcd+Bad^2)x)\sqrt{bc^2+ad^2}\log\left(\frac{2\,abcdx-abc^2-2\,a^2d^2-(2\,b^2c^2+abd^2)x^2-2\,\sqrt{bc^2+ad^2}(bcx-ad)\sqrt{bx^2+a}}{d^2x^2+2\,cdx+c^2}\right)}{2\,(b^2c^5+2\,abc^3d^2+a^2cd^4+(b^2c^4d+2\,abc^2d^3+a^2d^5)}\right.$$
$$-\frac{(Abc^2+Bacd+(Abcd+Bad^2)x)\sqrt{-bc^2-ad^2}\arctan\left(\frac{\sqrt{-bc^2-ad^2}(bcx-ad)\sqrt{bx^2+a}}{abc^2+a^2d^2+(b^2c^2+abd^2)x^2}\right)-(Bbc^3-Abc^2d+}{b^2c^5+2\,abc^3d^2+a^2cd^4+(b^2c^4d+2\,abc^2d^3+a^2d^5)x}$$

input
```
integrate((B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((A*b*c^2 + B*a*c*d + (A*b*c*d + B*a*d^2)*x)*sqrt(b*c^2 + a*d^2)*log(
(2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*
c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) + 2
*(B*b*c^3 - A*b*c^2*d + B*a*c*d^2 - A*a*d^3)*sqrt(b*x^2 + a))/(b^2*c^5 + 2
*a*b*c^3*d^2 + a^2*c*d^4 + (b^2*c^4*d + 2*a*b*c^2*d^3 + a^2*d^5)*x), -((A*
b*c^2 + B*a*c*d + (A*b*c*d + B*a*d^2)*x)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(
-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^
2 + a*b*d^2)*x^2)) - (B*b*c^3 - A*b*c^2*d + B*a*c*d^2 - A*a*d^3)*sqrt(b*x^
2 + a))/(b^2*c^5 + 2*a*b*c^3*d^2 + a^2*c*d^4 + (b^2*c^4*d + 2*a*b*c^2*d^3
+ a^2*d^5)*x)]
```

Sympy [F]

$$\int \frac{A+Bx}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{A+Bx}{\sqrt{a+bx^2}\,(c+dx)^2}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)**2/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a + b*x**2)*(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(97) = 194$.

Time = 0.07 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.36

$$\int \frac{A+Bx}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bc}{bc^2dx+ad^3x+bc^3+acd^2} - \frac{\sqrt{bx^2+a}A}{bc^2x+ad^2x+\frac{bc^3}{d}+acd} - \frac{Bbc^2\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^4} + \frac{Abc\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^3} + \frac{B\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^2}$$

input
```
integrate((B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a)*B*c/(b*c^2*d*x + a*d^3*x + b*c^3 + a*c*d^2) - sqrt(b*x^2 +
 a)*A/(b*c^2*x + a*d^2*x + b*c^3/d + a*c*d) - B*b*c^2*arcsinh(b*c*x/(sqrt(
a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*
d^4) + A*b*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d
*x + c)))/((a + b*c^2/d^2)^(3/2)*d^3) + B*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x
 + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^2)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{A+B\,x}{\sqrt{b\,x^2+a}\,(c+d\,x)^2}\,dx$$

input
```
int((A + B*x)/((a + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int((A + B*x)/((a + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 416, normalized size of antiderivative = 4.00

$$\int \frac{A+Bx}{(c+dx)^2\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a\,d^2+b\,c^2}\log\left(\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}-ad+bcx\right)ab\,c^2+\sqrt{a\,d^2+b\,c^2}\log\left(\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}-ac\right.}{}$$

input
```
int((B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
b*c*x)*a*b*c**2 + sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 +
 b*c**2) - a*d + b*c*x)*a*b*c*d*x + sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c*d + sqrt(a*d**2 + b*c**2)*
log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*d**2*x - sqr
t(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**2 - sqrt(a*d**2 + b*c**2)*log(c + d
*x)*a*b*c*d*x - sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c*d - sqrt(a*d**2 +
 b*c**2)*log(c + d*x)*a*b*d**2*x - sqrt(a + b*x**2)*a**2*d**3 - sqrt(a + b
*x**2)*a*b*c**2*d + sqrt(a + b*x**2)*a*b*c*d**2 + sqrt(a + b*x**2)*b**2*c*
*3)/(a**2*c*d**4 + a**2*d**5*x + 2*a*b*c**3*d**2 + 2*a*b*c**2*d**3*x + b**
2*c**5 + b**2*c**4*d*x)
```

3.176 $\int \frac{A+Bx}{(c+dx)^3\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 177

$$\int \frac{A+Bx}{(c+dx)^3\sqrt{a+bx^2}}\,dx = \frac{(Bc-Ad)\sqrt{a+bx^2}}{2(bc^2+ad^2)(c+dx)^2} - \frac{(2aBd^2-bc(Bc-3Ad))\sqrt{a+bx^2}}{2(bc^2+ad^2)^2(c+dx)} - \frac{b(2Abc^2+3aBcd-aAd^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2(bc^2+ad^2)^{5/2}}$$

output
```
1/2*(-A*d+B*c)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)/(d*x+c)^2-1/2*(2*a*B*d^2-b*c*
(-3*A*d+B*c))*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^2/(d*x+c)-1/2*b*(-A*a*d^2+2*A*
b*c^2+3*B*a*c*d)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))
/(a*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{(c+dx)^3\sqrt{a+bx^2}}\,dx$$
$$=\frac{\sqrt{a+bx^2}(-ad^2(Ad+B(c+2dx))+bc(Bc(2c+dx)-Ad(4c+3dx)))}{2\left(bc^2+ad^2\right)^2(c+dx)^2} - \frac{b(2Abc^2+3aBcd-aAd^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\left(-bc^2-ad^2\right)^{5/2}}$$

input
```
Integrate[(A + B*x)/((c + d*x)^3*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-(a*d^2*(A*d + B*(c + 2*d*x))) + b*c*(B*c*(2*c + d*x) -
A*d*(4*c + 3*d*x))))/(2*(b*c^2 + a*d^2)^2*(c + d*x)^2) - (b*(2*A*b*c^2 + 3
*a*B*c*d - a*A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(
b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {688, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+bx^2}(c+dx)^3}\,dx$$
$$\Big\downarrow \text{ 688}$$
$$\frac{\sqrt{a+bx^2}(Bc-Ad)}{2(c+dx)^2\left(ad^2+bc^2\right)}-\frac{\int -\frac{2(Abc+aBd)+b(Bc-Ad)x}{(c+dx)^2\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}$$
$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2(Abc+aBd)+b(Bc-Ad)x}{(c+dx)^2\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}(Bc-Ad)}{2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 679}$$

$$\frac{\frac{b\left(-aAd^2+3aBcd+2Abc^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(2aBd^2-bc(Bc-3Ad)\right)}{(c+dx)\left(ad^2+bc^2\right)}}{2\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}(Bc-Ad)}{2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{-\frac{b\left(-aAd^2+3aBcd+2Abc^2\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(2aBd^2-bc(Bc-3Ad)\right)}{(c+dx)\left(ad^2+bc^2\right)}}{2\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}(Bc-Ad)}{2(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{-\frac{b\left(-aAd^2+3aBcd+2Abc^2\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}\left(2aBd^2-bc(Bc-3Ad)\right)}{(c+dx)\left(ad^2+bc^2\right)}}{2\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}(Bc-Ad)}{2(c+dx)^2\left(ad^2+bc^2\right)}$$

input

```
Int[(A + B*x)/((c + d*x)^3*Sqrt[a + b*x^2]),x]
```

output

```
((B*c - A*d)*Sqrt[a + b*x^2])/(2*(b*c^2 + a*d^2)*(c + d*x)^2) + (-((((2*a*B
*d^2 - b*c*(B*c - 3*A*d))*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x))) -
(b*(2*A*b*c^2 + 3*a*B*c*d - a*A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a
*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(3/2))/(2*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(161) = 322$.

Time = 1.37 (sec) , antiderivative size = 668, normalized size of antiderivative = 3.77

method	result
default	$\frac{B\left(-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}{\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)}-\frac{bcd\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}{x+\frac{c}{d}}\right)}{\left(a\,d^2+b\,c^2\right)\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}}\right)}{d^3}+$

input
```
int((B*x+A)/(d*x+c)^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B/d^3*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2
+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c
/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+(A*d-B*c)/d^4*(-1/2/(a*d^2+
b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)
+3/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/
d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)
^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)
*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+1/2*b/(a
*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*
(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(162) = 324$.

Time = 1.18 (sec) , antiderivative size = 914, normalized size of antiderivative = 5.16

$$\int \frac{A + Bx}{(c + dx)^3\sqrt{a + bx^2}}\, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*A*b^2*c^4 + 3*B*a*b*c^3*d - A*a*b*c^2*d^2 + (2*A*b^2*c^2*d^2 + 3
*B*a*b*c*d^3 - A*a*b*d^4)*x^2 + 2*(2*A*b^2*c^3*d + 3*B*a*b*c^2*d^2 - A*a*b
*c*d^3)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2
*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 +
 a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(2*B*b^2*c^5 - 4*A*b^2*c^4*d + B*a*b*c
^3*d^2 - 5*A*a*b*c^2*d^3 - B*a^2*c*d^4 - A*a^2*d^5 + (B*b^2*c^4*d - 3*A*b^
2*c^3*d^2 - B*a*b*c^2*d^3 - 3*A*a*b*c*d^4 - 2*B*a^2*d^5)*x)*sqrt(b*x^2 + a
))/(b^3*c^8 + 3*a*b^2*c^6*d^2 + 3*a^2*b*c^4*d^4 + a^3*c^2*d^6 + (b^3*c^6*d
^2 + 3*a*b^2*c^4*d^4 + 3*a^2*b*c^2*d^6 + a^3*d^8)*x^2 + 2*(b^3*c^7*d + 3*a
*b^2*c^5*d^3 + 3*a^2*b*c^3*d^5 + a^3*c*d^7)*x), -1/2*((2*A*b^2*c^4 + 3*B*a
*b*c^3*d - A*a*b*c^2*d^2 + (2*A*b^2*c^2*d^2 + 3*B*a*b*c*d^3 - A*a*b*d^4)*x
^2 + 2*(2*A*b^2*c^3*d + 3*B*a*b*c^2*d^2 - A*a*b*c*d^3)*x)*sqrt(-b*c^2 - a*
d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 +
a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) - (2*B*b^2*c^5 - 4*A*b^2*c^4*d + B*a*b
*c^3*d^2 - 5*A*a*b*c^2*d^3 - B*a^2*c*d^4 - A*a^2*d^5 + (B*b^2*c^4*d - 3*A*
b^2*c^3*d^2 - B*a*b*c^2*d^3 - 3*A*a*b*c*d^4 - 2*B*a^2*d^5)*x)*sqrt(b*x^2 +
 a))/(b^3*c^8 + 3*a*b^2*c^6*d^2 + 3*a^2*b*c^4*d^4 + a^3*c^2*d^6 + (b^3*c^6
*d^2 + 3*a*b^2*c^4*d^4 + 3*a^2*b*c^2*d^6 + a^3*d^8)*x^2 + 2*(b^3*c^7*d + 3
*a*b^2*c^5*d^3 + 3*a^2*b*c^3*d^5 + a^3*c*d^7)*x)]
```

Sympy [F]

$$\int \frac{A+Bx}{(c+dx)^3\sqrt{a+bx^2}}\,dx=\int \frac{A+Bx}{\sqrt{a+bx^2}\,(c+dx)^3}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)**3/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a + b*x**2)*(c + d*x)**3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(162) = 324$.

Time = 0.08 (sec) , antiderivative size = 538, normalized size of antiderivative = 3.04

$$\int \frac{A+Bx}{(c+dx)^3\sqrt{a+bx^2}}\,dx = \frac{3\sqrt{bx^2+a}Bbc^2}{2\left(b^2c^4dx+2\,abc^2d^3x+a^2d^5x+b^2c^5+2\,abc^3d^2+a^2cd^4\right)} - \frac{3\sqrt{bx^2+a}Abc}{2\left(b^2c^4x+2\,abc^2d^2x+a^2d^4x+\frac{b^2c^5}{d}+2\,abc^3d+a^2cd^3\right)} + \frac{\sqrt{bx^2+a}Bc}{2\left(bc^2d^2x^2+ad^4x^2+2\,bc^3dx+2\,acd^3x+bc^4+ac^2d^2\right)} - \frac{\sqrt{bx^2+a}A}{2\left(bc^2dx^2+ad^3x^2+2\,bc^3x+2\,acd^2x+\frac{bc^4}{d}+ac^2d\right)} - \frac{\sqrt{bx^2+a}B}{bc^2dx+ad^3x+bc^3+acd^2} - \frac{3\,Bb^2c^3\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\left(a+\frac{bc^2}{d^2}\right)^{\frac{5}{2}}d^6} + \frac{3\,Ab^2c^2\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\left(a+\frac{bc^2}{d^2}\right)^{\frac{5}{2}}d^5} + \frac{3\,Bbc\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^4} - \frac{Ab\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{2\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^3}$$

input
```
integrate((B*x+A)/(d*x+c)^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
3/2*sqrt(b*x^2 + a)*B*b*c^2/(b^2*c^4*d*x + 2*a*b*c^2*d^3*x + a^2*d^5*x + b
^2*c^5 + 2*a*b*c^3*d^2 + a^2*c*d^4) - 3/2*sqrt(b*x^2 + a)*A*b*c/(b^2*c^4*x
 + 2*a*b*c^2*d^2*x + a^2*d^4*x + b^2*c^5/d + 2*a*b*c^3*d + a^2*c*d^3) + 1/
2*sqrt(b*x^2 + a)*B*c/(b*c^2*d^2*x^2 + a*d^4*x^2 + 2*b*c^3*d*x + 2*a*c*d^3
*x + b*c^4 + a*c^2*d^2) - 1/2*sqrt(b*x^2 + a)*A/(b*c^2*d*x^2 + a*d^3*x^2 +
 2*b*c^3*x + 2*a*c*d^2*x + b*c^4/d + a*c^2*d) - sqrt(b*x^2 + a)*B/(b*c^2*d
*x + a*d^3*x + b*c^3 + a*c*d^2) - 3/2*B*b^2*c^3*arcsinh(b*c*x/(sqrt(a*b)*a
bs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(5/2)*d^6) +
 3/2*A*b^2*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs
(d*x + c)))/((a + b*c^2/d^2)^(5/2)*d^5) + 3/2*B*b*c*arcsinh(b*c*x/(sqrt(a*
b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^
4) - 1/2*A*b*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d
*x + c)))/((a + b*c^2/d^2)^(3/2)*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(162) = 324$.

Time = 0.13 (sec) , antiderivative size = 583, normalized size of antiderivative = 3.29

$$\int \frac{A+Bx}{(c+dx)^3\sqrt{a+bx^2}}\,dx = -\frac{(2\,Ab^2c^2+3\,Babcd-Aabd^2)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{(b^2c^4+2\,abc^2d^2+a^2d^4)\sqrt{-bc^2-ad^2}} - \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab^2c^2d^2+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Babcd^3-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Aabd^4-2\left(\sqrt{b}\right.}{}$$

input
```
integrate((B*x+A)/(d*x+c)^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-(2*A*b^2*c^2 + 3*B*a*b*c*d - A*a*b*d^2)*arctan(((sqrt(b)*x - sqrt(b*x^2 +
 a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*
d^4)*sqrt(-b*c^2 - a*d^2)) - (2*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b^2*c^2*
d^2 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a*b*c*d^3 - (sqrt(b)*x - sqrt(b*
x^2 + a))^3*A*a*b*d^4 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*b^(5/2)*c^4 +
6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*b^(5/2)*c^3*d + 5*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*B*a*b^(3/2)*c^2*d^2 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*b^
(3/2)*c*d^3 - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b)*d^4 + 4*(sqr
t(b)*x - sqrt(b*x^2 + a))*B*a*b^2*c^3*d - 10*(sqrt(b)*x - sqrt(b*x^2 + a))
*A*a*b^2*c^2*d^2 - 5*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^2*b*c*d^3 - (sqrt(b
)*x - sqrt(b*x^2 + a))*A*a^2*b*d^4 - B*a^2*b^(3/2)*c^2*d^2 + 3*A*a^2*b^(3/
2)*c*d^3 + 2*B*a^3*sqrt(b)*d^4)/((b^2*c^4*d + 2*a*b*c^2*d^3 + a^2*d^5)*((s
qrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*(sqrt(b)*x - sqrt(b*x^2 + a))*sqrt(b)*
c - a*d)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)^3\sqrt{a+bx^2}}\,dx=\int \frac{A+B\,x}{\sqrt{b\,x^2+a}\,(c+d\,x)^3}\,dx$$

input
```
int((A + B*x)/((a + b*x^2)^(1/2)*(c + d*x)^3),x)
```

output
```
int((A + B*x)/((a + b*x^2)^(1/2)*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1120, normalized size of antiderivative = 6.33

$$\int \frac{A+Bx}{(c+dx)^3\sqrt{a+bx^2}}\,dx=\text{Too large to display}$$

input
```
int((B*x+A)/(d*x+c)^3/(b*x^2+a)^(1/2),x)
```

output

```
(sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**2*b*c**2*d**2 + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**
2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c*d**3*x + sqrt(a*d**2 + b*
c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*
d**4*x**2 - 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 +
b*c**2) - a*d + b*c*x)*a*b**2*c**4 - 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a
 + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**3*d*x - 3*sqrt(a
*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*
x)*a*b**2*c**3*d - 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*
d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**2*d**2*x**2 - 6*sqrt(a*d**2 + b*c*
*2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c*
*2*d**2*x - 3*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 +
b*c**2) - a*d + b*c*x)*a*b**2*c*d**3*x**2 - sqrt(a*d**2 + b*c**2)*log(c +
d*x)*a**2*b*c**2*d**2 - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c*d**3
*x - sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*d**4*x**2 + 2*sqrt(a*d**2 +
 b*c**2)*log(c + d*x)*a*b**2*c**4 + 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a
*b**2*c**3*d*x + 3*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**3*d + 2*sq
rt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**2*d**2*x**2 + 6*sqrt(a*d**2 + b
*c**2)*log(c + d*x)*a*b**2*c**2*d**2*x + 3*sqrt(a*d**2 + b*c**2)*log(c + d
*x)*a*b**2*c*d**3*x**2 - sqrt(a + b*x**2)*a**3*d**5 - 5*sqrt(a + b*x**2...
```

3.177 $\int \frac{A+Bx}{(c+dx)^4\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 258

$$\begin{aligned}
\int \frac{A+Bx}{(c+dx)^4\sqrt{a+bx^2}}\,dx &= \frac{(Bc-Ad)\sqrt{a+bx^2}}{3(bc^2+ad^2)(c+dx)^3}-\frac{(3aBd^2-bc(2Bc-5Ad))\sqrt{a+bx^2}}{6(bc^2+ad^2)^2(c+dx)^2}\\
&\quad+\frac{b(bc^2(2Bc-11Ad)-ad^2(13Bc-4Ad))\sqrt{a+bx^2}}{6(bc^2+ad^2)^3(c+dx)}\\
&\quad-\frac{b(Abc(2bc^2-3ad^2)+aBd(4bc^2-ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2(bc^2+ad^2)^{7/2}}
\end{aligned}$$

output
```
1/3*(-A*d+B*c)*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)/(d*x+c)^3-1/6*(3*a*B*d^2-b*c*
(-5*A*d+2*B*c))*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^2/(d*x+c)^2+1/6*b*(b*c^2*(-1
1*A*d+2*B*c)-a*d^2*(-4*A*d+13*B*c))*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^3/(d*x+c
)-1/2*b*(A*b*c*(-3*a*d^2+2*b*c^2)+a*B*d*(-a*d^2+4*b*c^2))*arctanh((-b*c*x+
a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(7/2)
```

Mathematica [A] (verified)

Time = 10.43 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx}{(c+dx)^4\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}\left(2(Bc-Ad)\left(bc^2+ad^2\right)^2+\left(bc^2+ad^2\right)\left(-3aBd^2+bc(2Bc-5Ad)\right)(c+dx)+b(b\right.}{}$$

input
```
Integrate[(A + B*x)/((c + d*x)^4*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2]*(2*(B*c - A*d)*(b*c^2 + a*d^2)^2 + (b
*c^2 + a*d^2)*(-3*a*B*d^2 + b*c*(2*B*c - 5*A*d))*(c + d*x) + b*(b*c^2*(2*B
*c - 11*A*d) + a*d^2*(-13*B*c + 4*A*d))*(c + d*x)^2) + 3*b*(A*b*c*(2*b*c^2
 - 3*a*d^2) + a*B*d*(4*b*c^2 - a*d^2))*(c + d*x)^3*Log[c + d*x] - 3*b*(A*b
*c*(2*b*c^2 - 3*a*d^2) + a*B*d*(4*b*c^2 - a*d^2))*(c + d*x)^3*Log[a*d - b*
c*x + Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2]])/(6*(b*c^2 + a*d^2)^(7/2)*(c +
d*x)^3)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {688, 25, 688, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+bx^2}(c+dx)^4}dx$$

$$\Big\downarrow \text{ 688}$$

$$\frac{\sqrt{a+bx^2}(Bc-Ad)}{3(c+dx)^3\left(ad^2+bc^2\right)}-\frac{\int -\frac{3(Abc+aBd)+2b(Bc-Ad)x}{(c+dx)^3\sqrt{bx^2+a}}dx}{3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3(Abc+aBd)+2b(Bc-Ad)x}{(c+dx)^3\sqrt{bx^2+a}}dx}{3\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}(Bc-Ad)}{3(c+dx)^3\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 688

$$\frac{-\frac{\int -\frac{b\left(2\left(3Abc^2+5aBdc-2aAd^2\right)-\left(3aBd^2-bc(2Bc-5Ad)\right)x\right)}{(c+dx)^2\sqrt{bx^2+a}}dx}{2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(3aBd^2-bc(2Bc-5Ad))}{2(c+dx)^2(ad^2+bc^2)}}{3\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}(Bc-Ad)}{3(c+dx)^3\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{b\left(2\left(3Abc^2+5aBdc-2aAd^2\right)-\left(3aBd^2-bc(2Bc-5Ad)\right)x\right)}{(c+dx)^2\sqrt{bx^2+a}}dx}{2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(3aBd^2-bc(2Bc-5Ad))}{2(c+dx)^2(ad^2+bc^2)}}{3\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}(Bc-Ad)}{3(c+dx)^3\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{b\int \frac{2\left(3Abc^2+5aBdc-2aAd^2\right)-\left(3aBd^2-bc(2Bc-5Ad)\right)x}{(c+dx)^2\sqrt{bx^2+a}}dx}{2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(3aBd^2-bc(2Bc-5Ad))}{2(c+dx)^2(ad^2+bc^2)}}{3\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}(Bc-Ad)}{3(c+dx)^3\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 679

$$\frac{\frac{b\left(\frac{3\left(Abc\left(2bc^2-3ad^2\right)+aBd\left(4bc^2-ad^2\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}+\frac{\sqrt{a+bx^2}\left(bc^2(2Bc-11Ad)-ad^2(13Bc-4Ad)\right)}{(c+dx)\left(ad^2+bc^2\right)}\right)}{2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(3aBd^2-bc(2Bc-5Ad))}{2(c+dx)^2(ad^2+bc^2)}}{3\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}(Bc-Ad)}{3(c+dx)^3\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 488

$$\frac{\frac{b\left(\frac{\sqrt{a+bx^2}\left(bc^2(2Bc-11Ad)-ad^2(13Bc-4Ad)\right)}{(c+dx)\left(ad^2+bc^2\right)}-\frac{3\left(Abc\left(2bc^2-3ad^2\right)+aBd\left(4bc^2-ad^2\right)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}\right)}{2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(3aBd^2-bc(2Bc-5Ad))}{2(c+dx)^2(ad^2+bc^2)}}{3\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}(Bc-Ad)}{3(c+dx)^3\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{b\left(\frac{\sqrt{a+bx^2}\left(bc^2(2Bc-11Ad)-ad^2(13Bc-4Ad)\right)}{(c+dx)\left(ad^2+bc^2\right)}-\frac{3\left(Abc\left(2bc^2-3ad^2\right)+aBd\left(4bc^2-ad^2\right)\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\left(ad^2+bc^2\right)^{3/2}}\right)}{2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(3aBd^2-bc(2B}{2(c+dx)^2(ad^2+b}}{3\left(ad^2+bc^2\right)}$$

$$\frac{\sqrt{a+bx^2}(Bc-Ad)}{3(c+dx)^3\left(ad^2+bc^2\right)}$$

input
```
Int[(A + B*x)/((c + d*x)^4*Sqrt[a + b*x^2]),x]
```

output
```
((B*c - A*d)*Sqrt[a + b*x^2])/(3*(b*c^2 + a*d^2)*(c + d*x)^3) + (-1/2*((3*
a*B*d^2 - b*c*(2*B*c - 5*A*d))*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)
^2) + (b*(((b*c^2*(2*B*c - 11*A*d) - a*d^2*(13*B*c - 4*A*d))*Sqrt[a + b*x^
2])/((b*c^2 + a*d^2)*(c + d*x)) - (3*(A*b*c*(2*b*c^2 - 3*a*d^2) + a*B*d*(4
*b*c^2 - a*d^2))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2
]))])/(b*c^2 + a*d^2)^(3/2)))/(2*(b*c^2 + a*d^2)))/(3*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1214 vs. $2(238) = 476$.

Time = 1.38 (sec) , antiderivative size = 1215, normalized size of antiderivative = 4.71

method	result	size
default	Expression too large to display	1215

input
```
int((B*x+A)/(d*x+c)^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B/d^4*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(1/2)+3/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*
(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/
((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^
2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))
/(x+c/d)))+1/2*b/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+
b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/
d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+(A*d-B*c)/d^5*(-1/3/(a*d^2+b
*c^2)*d^2/(x+c/d)^3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+
5/3*b*c*d/(a*d^2+b*c^2)*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2*(b*(x+c/d)^2-2*b
*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+3/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b
*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*
c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/
d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+
b*c^2)/d^2)^(1/2))/(x+c/d)))+1/2*b/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(
1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(
b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-2/3*b/(a*d
^2+b*c^2)*d^2*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(
a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((
2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(239) = 478$.

Time = 4.16 (sec) , antiderivative size = 1620, normalized size of antiderivative = 6.28

$$\int \frac{A+Bx}{(c+dx)^4\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(2*A*b^3*c^6 + 4*B*a*b^2*c^5*d - 3*A*a*b^2*c^4*d^2 - B*a^2*b*c^3
*d^3 + (2*A*b^3*c^3*d^3 + 4*B*a*b^2*c^2*d^4 - 3*A*a*b^2*c*d^5 - B*a^2*b*d^
6)*x^3 + 3*(2*A*b^3*c^4*d^2 + 4*B*a*b^2*c^3*d^3 - 3*A*a*b^2*c^2*d^4 - B*a^
2*b*c*d^5)*x^2 + 3*(2*A*b^3*c^5*d + 4*B*a*b^2*c^4*d^2 - 3*A*a*b^2*c^3*d^3
- B*a^2*b*c^2*d^4)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a
^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*s
qrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(6*B*b^3*c^7 - 18*A*b^3*c^6
*d - 4*B*a*b^2*c^5*d^2 - 23*A*a*b^2*c^4*d^3 - 11*B*a^2*b*c^3*d^4 - 7*A*a^2
*b*c^2*d^5 - B*a^3*c*d^6 - 2*A*a^3*d^7 + (2*B*b^3*c^5*d^2 - 11*A*b^3*c^4*d
^3 - 11*B*a*b^2*c^3*d^4 - 7*A*a*b^2*c^2*d^5 - 13*B*a^2*b*c*d^6 + 4*A*a^2*b
*d^7)*x^2 + 3*(2*B*b^3*c^6*d - 9*A*b^3*c^5*d^2 - 7*B*a*b^2*c^4*d^3 - 8*A*a
*b^2*c^3*d^4 - 10*B*a^2*b*c^2*d^5 + A*a^2*b*c*d^6 - B*a^3*d^7)*x)*sqrt(b*x
^2 + a))/(b^4*c^11 + 4*a*b^3*c^9*d^2 + 6*a^2*b^2*c^7*d^4 + 4*a^3*b*c^5*d^6
 + a^4*c^3*d^8 + (b^4*c^8*d^3 + 4*a*b^3*c^6*d^5 + 6*a^2*b^2*c^4*d^7 + 4*a^
3*b*c^2*d^9 + a^4*d^11)*x^3 + 3*(b^4*c^9*d^2 + 4*a*b^3*c^7*d^4 + 6*a^2*b^2
*c^5*d^6 + 4*a^3*b*c^3*d^8 + a^4*c*d^10)*x^2 + 3*(b^4*c^10*d + 4*a*b^3*c^8
*d^3 + 6*a^2*b^2*c^6*d^5 + 4*a^3*b*c^4*d^7 + a^4*c^2*d^9)*x), -1/6*(3*(2*A
*b^3*c^6 + 4*B*a*b^2*c^5*d - 3*A*a*b^2*c^4*d^2 - B*a^2*b*c^3*d^3 + (2*A*b^
3*c^3*d^3 + 4*B*a*b^2*c^2*d^4 - 3*A*a*b^2*c*d^5 - B*a^2*b*d^6)*x^3 + 3*(2*
A*b^3*c^4*d^2 + 4*B*a*b^2*c^3*d^3 - 3*A*a*b^2*c^2*d^4 - B*a^2*b*c*d^5)*...
```

Sympy [F]

$$\int \frac{A+Bx}{(c+dx)^4\sqrt{a+bx^2}}\,dx=\int \frac{A+Bx}{\sqrt{a+bx^2}\,(c+dx)^4}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)**4/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a + b*x**2)*(c + d*x)**4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1097 vs. $2(239) = 478$.

Time = 0.11 (sec) , antiderivative size = 1097, normalized size of antiderivative = 4.25

$$\int \frac{A+Bx}{(c+dx)^4\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
5/2*sqrt(b*x^2 + a)*B*b^2*c^3/(b^3*c^6*d*x + 3*a*b^2*c^4*d^3*x + 3*a^2*b*c
^2*d^5*x + a^3*d^7*x + b^3*c^7 + 3*a*b^2*c^5*d^2 + 3*a^2*b*c^3*d^4 + a^3*c
*d^6) - 5/2*sqrt(b*x^2 + a)*A*b^2*c^2/(b^3*c^6*x + 3*a*b^2*c^4*d^2*x + 3*a
^2*b*c^2*d^4*x + a^3*d^6*x + b^3*c^7/d + 3*a*b^2*c^5*d + 3*a^2*b*c^3*d^3 +
 a^3*c*d^5) + 5/6*sqrt(b*x^2 + a)*B*b*c^2/(b^2*c^4*d^2*x^2 + 2*a*b*c^2*d^4
*x^2 + a^2*d^6*x^2 + 2*b^2*c^5*d*x + 4*a*b*c^3*d^3*x + 2*a^2*c*d^5*x + b^2
*c^6 + 2*a*b*c^4*d^2 + a^2*c^2*d^4) - 5/6*sqrt(b*x^2 + a)*A*b*c/(b^2*c^4*d
*x^2 + 2*a*b*c^2*d^3*x^2 + a^2*d^5*x^2 + 2*b^2*c^5*x + 4*a*b*c^3*d^2*x + 2
*a^2*c*d^4*x + b^2*c^6/d + 2*a*b*c^4*d + a^2*c^2*d^3) - 13/6*sqrt(b*x^2 +
a)*B*b*c/(b^2*c^4*d*x + 2*a*b*c^2*d^3*x + a^2*d^5*x + b^2*c^5 + 2*a*b*c^3*
d^2 + a^2*c*d^4) + 2/3*sqrt(b*x^2 + a)*A*b/(b^2*c^4*x + 2*a*b*c^2*d^2*x +
a^2*d^4*x + b^2*c^5/d + 2*a*b*c^3*d + a^2*c*d^3) + 1/3*sqrt(b*x^2 + a)*B*c
/(b*c^2*d^3*x^3 + a*d^5*x^3 + 3*b*c^3*d^2*x^2 + 3*a*c*d^4*x^2 + 3*b*c^4*d*
x + 3*a*c^2*d^3*x + b*c^5 + a*c^3*d^2) - 1/3*sqrt(b*x^2 + a)*A/(b*c^2*d^2*
x^3 + a*d^4*x^3 + 3*b*c^3*d*x^2 + 3*a*c*d^3*x^2 + 3*b*c^4*x + 3*a*c^2*d^2*
x + b*c^5/d + a*c^3*d) - 1/2*sqrt(b*x^2 + a)*B/(b*c^2*d^2*x^2 + a*d^4*x^2
+ 2*b*c^3*d*x + 2*a*c*d^3*x + b*c^4 + a*c^2*d^2) - 5/2*B*b^3*c^4*arcsinh(b
*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/
d^2)^(7/2)*d^8) + 5/2*A*b^3*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a
*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(7/2)*d^7) + 3*B*b^2*c^2*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(239) = 478$.

Time = 0.14 (sec) , antiderivative size = 1076, normalized size of antiderivative = 4.17

$$\int \frac{A+Bx}{(c+dx)^4\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
(2*A*b^3*c^3 + 4*B*a*b^2*c^2*d - 3*A*a*b^2*c*d^2 - B*a^2*b*d^3)*arctan(-((
sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^3*c^
6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)*sqrt(-b*c^2 - a*d^2)) - 1
/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*b^3*c^3*d^3 + 12*(sqrt(b)*x - sqrt
(b*x^2 + a))^5*B*a*b^2*c^2*d^4 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a*b^2
*c*d^5 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*a^2*b*d^6 + 30*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*A*b^(7/2)*c^4*d^2 + 60*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B
*a*b^(5/2)*c^3*d^3 - 45*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*b^(5/2)*c^2*d^
4 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2)*c*d^5 - 8*(sqrt(b)*x
- sqrt(b*x^2 + a))^3*B*b^4*c^6 + 44*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b^4*
c^5*d + 64*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a*b^3*c^4*d^2 - 82*(sqrt(b)*x
 - sqrt(b*x^2 + a))^3*A*a*b^3*c^3*d^3 - 78*(sqrt(b)*x - sqrt(b*x^2 + a))^3
*B*a^2*b^2*c^2*d^4 + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^2*b^2*c*d^5 +
24*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*b^(7/2)*c^5*d - 102*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*A*a*b^(7/2)*c^4*d^2 - 102*(sqrt(b)*x - sqrt(b*x^2 + a))^2
*B*a^2*b^(5/2)*c^3*d^3 + 36*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(5/2)*
c^2*d^4 + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2)*c*d^5 - 12*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(3/2)*d^6 - 12*(sqrt(b)*x - sqrt(b*x^2
 + a))*B*a^2*b^3*c^4*d^2 + 60*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^2*b^3*c^3*
d^3 + 66*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^3*b^2*c^2*d^4 - 15*(sqrt(b)*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)^4\sqrt{a+bx^2}}\,dx=\int \frac{A+B\,x}{\sqrt{b\,x^2+a}\,(c+d\,x)^4}\,dx$$

input
```
int((A + B*x)/((a + b*x^2)^(1/2)*(c + d*x)^4),x)
```

output
```
int((A + B*x)/((a + b*x^2)^(1/2)*(c + d*x)^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 2137, normalized size of antiderivative = 8.28

$$\int \frac{A+Bx}{(c+dx)^4\sqrt{a+bx^2}}\,dx=\text{Too large to display}$$

input
```
int((B*x+A)/(d*x+c)^4/(b*x^2+a)^(1/2),x)
```

output

```
(9*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a**2*b**2*c**4*d**2 + 27*sqrt(a*d**2 + b*c**2)*log( - sqrt(a +
 b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**3*d**3*x + 3*sq
rt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
b*c*x)*a**2*b**2*c**3*d**3 + 27*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x*
*2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**2*d**4*x**2 + 9*sqrt
(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*
c*x)*a**2*b**2*c**2*d**4*x + 9*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**
2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*d**5*x**3 + 9*sqrt(a*d
**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)
*a**2*b**2*c*d**5*x**2 + 3*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*s
qrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*d**6*x**3 - 6*sqrt(a*d**2 +
b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**
3*c**6 - 18*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*
c**2) - a*d + b*c*x)*a*b**3*c**5*d*x - 12*sqrt(a*d**2 + b*c**2)*log( - sqr
t(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**5*d - 18*sqrt
(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*
c*x)*a*b**3*c**4*d**2*x**2 - 36*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x*
*2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**4*d**2*x - 6*sqrt(a*d**
2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x...
```

3.178 $\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 235

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{3/2}}\,dx = -\frac{(c+dx)^3(a(Bc+Ad)-(Abc-aBd)x)}{ab\sqrt{a+bx^2}} - \frac{d(3Abc-4aBd)(c+dx)^2\sqrt{a+bx^2}}{3ab^2} - \frac{d(4(3Abc(bc^2-4ad^2)-4aBd(4bc^2-ad^2))+bd(6Abc^2-20aBcd-9aAd^2)\,x)\sqrt{a+bx^2}}{6ab^3} - \frac{d(3ad^2(4Bc+Ad)-4bc^2(2Bc+3Ad))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

output
```
-(d*x+c)^3*(a*(A*d+B*c)-(A*b*c-B*a*d)*x)/a/b/(b*x^2+a)^(1/2)-1/3*d*(3*A*b*
c-4*B*a*d)*(d*x+c)^2*(b*x^2+a)^(1/2)/a/b^2-1/6*d*(12*A*b*c*(-4*a*d^2+b*c^2
)-16*a*B*d*(-a*d^2+4*b*c^2)+b*d*(-9*A*a*d^2+6*A*b*c^2-20*B*a*c*d)*x)*(b*x^
2+a)^(1/2)/a/b^3-1/2*d*(3*a*d^2*(A*d+4*B*c)-4*b*c^2*(3*A*d+2*B*c))*arctanh
(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{3/2}}\,dx = \frac{-16a^3Bd^4+6Ab^3c^4x+a^2bd^2(3Ad(16c+3dx)+4B(18c^2+9cdx-2d^2x^2))+}{}$$

input
```
Integrate[((A + B*x)*(c + d*x)^4)/(a + b*x^2)^(3/2),x]
```

output
```
(-16*a^3*B*d^4 + 6*A*b^3*c^4*x + a^2*b*d^2*(3*A*d*(16*c + 3*d*x) + 4*B*(18
*c^2 + 9*c*d*x - 2*d^2*x^2)) + a*b^2*(3*A*d*(-8*c^3 - 12*c^2*d*x + 8*c*d^2
*x^2 + d^3*x^3) + B*(-6*c^4 - 24*c^3*d*x + 36*c^2*d^2*x^2 + 12*c*d^3*x^3 +
 2*d^4*x^4)) + 3*a*Sqrt[b]*d*(3*a*d^2*(4*B*c + A*d) - 4*b*c^2*(2*B*c + 3*A
*d))*Sqrt[a + b*x^2]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(6*a*b^3*Sqrt[a
+ b*x^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {684, 27, 687, 25, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 684}$$

$$\frac{\int \frac{d(c+dx)^2(a(4Bc+3Ad)-(3Abc-4aBd)x)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^3(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\int \frac{(c+dx)^2(a(4Bc+3Ad)-(3Abc-4aBd)x)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^3(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{d\left(\frac{\int -\frac{(c+dx)\left(a\left(8aBd^2-3bc(4Bc+5Ad)\right)+b\left(6Abc^2-20aBdc-9aAd^2\right)x\right)}{\sqrt{bx^2+a}}dx}{3b}-\frac{\sqrt{a+bx^2}(c+dx)^2(3Abc-4aBd)}{3b}\right)}{ab}-\frac{(c+dx)^3(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\left(-\frac{\int \frac{(c+dx)\left(a\left(8aBd^2-3bc(4Bc+5Ad)\right)+b\left(6Abc^2-20aBdc-9aAd^2\right)x\right)}{\sqrt{bx^2+a}}dx}{3b}-\frac{\sqrt{a+bx^2}(c+dx)^2(3Abc-4aBd)}{3b}\right)}{ab}-\frac{(c+dx)^3(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{676}$$

$$\frac{d\left(-\frac{\frac{3}{2}a\left(3ad^2(Ad+4Bc)-4bc^2(3Ad+2Bc)\right)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{2\sqrt{a+bx^2}\left(3Abc\left(bc^2-4ad^2\right)-4aBd\left(4bc^2-ad^2\right)\right)}{b}+\frac{1}{2}dx\sqrt{a+bx^2}\left(-9aAd^2-20aBcd+6A\right.}{3b}\right.}{ab}$$

$$\frac{(c+dx)^3(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{d\left(-\frac{\frac{3}{2}a\left(3ad^2(Ad+4Bc)-4bc^2(3Ad+2Bc)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{2\sqrt{a+bx^2}\left(3Abc\left(bc^2-4ad^2\right)-4aBd\left(4bc^2-ad^2\right)\right)}{b}+\frac{1}{2}dx\sqrt{a+bx^2}\left(-9aAd^2-20a\right.}{3b}\right.}{ab}$$

$$\frac{(c+dx)^3(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(-\frac{\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3ad^2(Ad+4Bc)-4bc^2(3Ad+2Bc)\right)}{2\sqrt{b}}+\frac{2\sqrt{a+bx^2}\left(3Abc\left(bc^2-4ad^2\right)-4aBd\left(4bc^2-ad^2\right)\right)}{b}+\frac{1}{2}dx\sqrt{a+bx^2}\left(-9aAd^2-20aBcd\right.}{3b}\right.}{ab}$$

$$\frac{(c+dx)^3(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$

input
```
Int[((A + B*x)*(c + d*x)^4)/(a + b*x^2)^(3/2),x]
```

output
```
-(((c + d*x)^3*(a*(B*c + A*d) - (A*b*c - a*B*d)*x))/(a*b*Sqrt[a + b*x^2]))
 + (d*(-1/3*((3*A*b*c - 4*a*B*d)*(c + d*x)^2*Sqrt[a + b*x^2])/b - ((2*(3*A
*b*c*(b*c^2 - 4*a*d^2) - 4*a*B*d*(4*b*c^2 - a*d^2))*Sqrt[a + b*x^2])/b + (
d*(6*A*b*c^2 - 20*a*B*c*d - 9*a*A*d^2)*x*Sqrt[a + b*x^2])/2 + (3*a*(3*a*d^
2*(4*B*c + A*d) - 4*b*c^2*(2*B*c + 3*A*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(2*Sqrt[b]))/(3*b)))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.11

method	result
risch	$\frac{d^2(2Bbd^2x^2+3Abd^2x+12Bbcdx+24Abcd-10aBd^2+36Bbc^2)\sqrt{bx^2+a}}{6b^3} - \frac{bd(3Ad^3a-12Abc^2d+12aBcd^2-8bBc^3)\left(-\frac{x}{b\sqrt{bx^2+}}\right.}{}$
default	$\frac{Ac^4x}{\sqrt{bx^2+a}\,a} + d^3(Ad+4Bc)\left(\frac{x^3}{2b\sqrt{bx^2+a}} - \frac{3a\left(-\frac{x}{b\sqrt{bx^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right) + 2c\,d^2(2Ad+3Bc)\left(\frac{}{b\sqrt{}}\right.$

input
```
int((B*x+A)*(d*x+c)^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*d^2*(2*B*b*d^2*x^2+3*A*b*d^2*x+12*B*b*c*d*x+24*A*b*c*d-10*B*a*d^2+36*B
*b*c^2)*(b*x^2+a)^(1/2)/b^3-1/2/b^2*(b*d*(3*A*a*d^3-12*A*b*c^2*d+12*B*a*c*
d^2-8*B*b*c^3)*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2
)))-(8*A*a*b*c*d^3-8*A*b^2*c^3*d-2*B*a^2*d^4+12*B*a*b*c^2*d^2-2*B*b^2*c^4)
/b/(b*x^2+a)^(1/2)+A*a*d^4*x/(b*x^2+a)^(1/2)-2*A*b^2*c^4*x/a/(b*x^2+a)^(1/
2)+4*B*a*c*d^3*x/(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 692, normalized size of antiderivative = 2.94

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{3\,(8\,Ba^2bc^3d+12\,Aa^2bc^2d^2-12\,Ba^3cd^3-3\,Aa^3d^4+(8\,Bab^2c^3d+12\,Aab\right.$$

$$-\frac{3\,(8\,Ba^2bc^3d+12\,Aa^2bc^2d^2-12\,Ba^3cd^3-3\,Aa^3d^4+(8\,Bab^2c^3d+12\,Aab^2c^2d^2-12\,Ba^2bcd^3-3\,Aa^2bd}{}$$

input
```
integrate((B*x+A)*(d*x+c)^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(8*B*a^2*b*c^3*d + 12*A*a^2*b*c^2*d^2 - 12*B*a^3*c*d^3 - 3*A*a^3
*d^4 + (8*B*a*b^2*c^3*d + 12*A*a*b^2*c^2*d^2 - 12*B*a^2*b*c*d^3 - 3*A*a^2*
b*d^4)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(2
*B*a*b^2*d^4*x^4 - 6*B*a*b^2*c^4 - 24*A*a*b^2*c^3*d + 72*B*a^2*b*c^2*d^2 +
 48*A*a^2*b*c*d^3 - 16*B*a^3*d^4 + 3*(4*B*a*b^2*c*d^3 + A*a*b^2*d^4)*x^3 +
 4*(9*B*a*b^2*c^2*d^2 + 6*A*a*b^2*c*d^3 - 2*B*a^2*b*d^4)*x^2 + 3*(2*A*b^3*
c^4 - 8*B*a*b^2*c^3*d - 12*A*a*b^2*c^2*d^2 + 12*B*a^2*b*c*d^3 + 3*A*a^2*b*
d^4)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3), -1/6*(3*(8*B*a^2*b*c^3*d +
 12*A*a^2*b*c^2*d^2 - 12*B*a^3*c*d^3 - 3*A*a^3*d^4 + (8*B*a*b^2*c^3*d + 12
*A*a*b^2*c^2*d^2 - 12*B*a^2*b*c*d^3 - 3*A*a^2*b*d^4)*x^2)*sqrt(-b)*arctan(
sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*a*b^2*d^4*x^4 - 6*B*a*b^2*c^4 - 24*A*a*
b^2*c^3*d + 72*B*a^2*b*c^2*d^2 + 48*A*a^2*b*c*d^3 - 16*B*a^3*d^4 + 3*(4*B*
a*b^2*c*d^3 + A*a*b^2*d^4)*x^3 + 4*(9*B*a*b^2*c^2*d^2 + 6*A*a*b^2*c*d^3 -
2*B*a^2*b*d^4)*x^2 + 3*(2*A*b^3*c^4 - 8*B*a*b^2*c^3*d - 12*A*a*b^2*c^2*d^2
 + 12*B*a^2*b*c*d^3 + 3*A*a^2*b*d^4)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*
b^3)]
```

Sympy [F]

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{3/2}}\,dx = \int \frac{(A+Bx)\,(c+dx)^4}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(d*x+c)**4/(b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x)*(c + d*x)**4/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.43

$$
\begin{aligned}
\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{3/2}}\,dx &= \frac{Bd^4x^4}{3\sqrt{bx^2+a}b} - \frac{4\,Bad^4x^2}{3\sqrt{bx^2+a}b^2} \\
&+ \frac{Ac^4x}{\sqrt{bx^2+a}a} - \frac{Bc^4}{\sqrt{bx^2+a}b} - \frac{4\,Ac^3d}{\sqrt{bx^2+a}b} - \frac{8\,Ba^2d^4}{3\sqrt{bx^2+a}b^3} \\
&+ \frac{(4\,Bcd^3+Ad^4)x^3}{2\sqrt{bx^2+a}b} + \frac{2\,(3\,Bc^2d^2+2\,Acd^3)x^2}{\sqrt{bx^2+a}b} + \frac{3\,(4\,Bcd^3+Ad^4)ax}{2\sqrt{bx^2+a}b^2} \\
&- \frac{2\,(2\,Bc^3d+3\,Ac^2d^2)x}{\sqrt{bx^2+a}b} - \frac{3\,(4\,Bcd^3+Ad^4)a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} \\
&+ \frac{2\,(2\,Bc^3d+3\,Ac^2d^2)\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{4\,(3\,Bc^2d^2+2\,Acd^3)a}{\sqrt{bx^2+a}b^2}
\end{aligned}
$$

input
```
integrate((B*x+A)*(d*x+c)^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/3*B*d^4*x^4/(sqrt(b*x^2 + a)*b) - 4/3*B*a*d^4*x^2/(sqrt(b*x^2 + a)*b^2)
+ A*c^4*x/(sqrt(b*x^2 + a)*a) - B*c^4/(sqrt(b*x^2 + a)*b) - 4*A*c^3*d/(sqr
t(b*x^2 + a)*b) - 8/3*B*a^2*d^4/(sqrt(b*x^2 + a)*b^3) + 1/2*(4*B*c*d^3 + A
*d^4)*x^3/(sqrt(b*x^2 + a)*b) + 2*(3*B*c^2*d^2 + 2*A*c*d^3)*x^2/(sqrt(b*x^
2 + a)*b) + 3/2*(4*B*c*d^3 + A*d^4)*a*x/(sqrt(b*x^2 + a)*b^2) - 2*(2*B*c^3
*d + 3*A*c^2*d^2)*x/(sqrt(b*x^2 + a)*b) - 3/2*(4*B*c*d^3 + A*d^4)*a*arcsin
h(b*x/sqrt(a*b))/b^(5/2) + 2*(2*B*c^3*d + 3*A*c^2*d^2)*arcsinh(b*x/sqrt(a*
b))/b^(3/2) + 4*(3*B*c^2*d^2 + 2*A*c*d^3)*a/(sqrt(b*x^2 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\left(\left(\frac{2\,Bd^4x}{b}+\frac{3\,(4\,Bab^4cd^3+Aab^4d^4)}{ab^5}\right)x+\frac{4\,(9\,Bab^4c^2d^2+6\,Aab^4cd^3-2\,Ba^2b^3d^4)}{ab^5}\right)x+\frac{3\,(2\,Ab^5c^4\cdots}{\cdots}\right.\right.}{\cdots}$$
$$-\frac{(8\,Bbc^3d+12\,Abc^2d^2-12\,Bacd^3-3\,Aad^4)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/6*((((2*B*d^4*x/b + 3*(4*B*a*b^4*c*d^3 + A*a*b^4*d^4)/(a*b^5))*x + 4*(9*
B*a*b^4*c^2*d^2 + 6*A*a*b^4*c*d^3 - 2*B*a^2*b^3*d^4)/(a*b^5))*x + 3*(2*A*b
^5*c^4 - 8*B*a*b^4*c^3*d - 12*A*a*b^4*c^2*d^2 + 12*B*a^2*b^3*c*d^3 + 3*A*a
^2*b^3*d^4)/(a*b^5))*x - 2*(3*B*a*b^4*c^4 + 12*A*a*b^4*c^3*d - 36*B*a^2*b^
3*c^2*d^2 - 24*A*a^2*b^3*c*d^3 + 8*B*a^3*b^2*d^4)/(a*b^5))/sqrt(b*x^2 + a)
 - 1/2*(8*B*b*c^3*d + 12*A*b*c^2*d^2 - 12*B*a*c*d^3 - 3*A*a*d^4)*log(abs(-
sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{3/2}}\,dx = \int \frac{(A+B\,x)\,(c+d\,x)^4}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((A + B*x)*(c + d*x)^4)/(a + b*x^2)^(3/2),x)
```

output
```
int(((A + B*x)*(c + d*x)^4)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{3/2}}\,dx = \int \frac{(Bx+A)\,(dx+c)^4}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((B*x+A)*(d*x+c)^4/(b*x^2+a)^(3/2),x)
```

output
```
int((B*x+A)*(d*x+c)^4/(b*x^2+a)^(3/2),x)
```

3.179 $\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{3/2}}\,dx = -\frac{(c+dx)^2(a(Bc+Ad)-(Abc-aBd)x)}{ab\sqrt{a+bx^2}} - \frac{d(4(Abc^2-3aBcd-aAd^2)+d(2Abc-3aBd)x)\sqrt{a+bx^2}}{2ab^2} - \frac{3d(aBd^2-2bc(Bc+Ad))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

output

```
-(d*x+c)^2*(a*(A*d+B*c)-(A*b*c-B*a*d)*x)/a/b/(b*x^2+a)^(1/2)-1/2*d*(-4*A*a
*d^2+4*A*b*c^2-12*B*a*c*d+d*(2*A*b*c-3*B*a*d)*x)*(b*x^2+a)^(1/2)/a/b^2-3/2
*d*(a*B*d^2-2*b*c*(A*d+B*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{3/2}}\,dx = \frac{2Ab^2c^3x+a^2d^2(4Ad+3B(4c+dx))+ab(2Ad(-3c^2-3cdx+d^2x^2)+B(-2}{2ab^2\sqrt{a+bx^2}}$$
$$+\frac{3d(aBd^2-2bc(Bc+Ad))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{5/2}}$$

input
```
Integrate[((A + B*x)*(c + d*x)^3)/(a + b*x^2)^(3/2),x]
```

output
```
(2*A*b^2*c^3*x + a^2*d^2*(4*A*d + 3*B*(4*c + d*x)) + a*b*(2*A*d*(-3*c^2 -
3*c*d*x + d^2*x^2) + B*(-2*c^3 - 6*c^2*d*x + 6*c*d^2*x^2 + d^3*x^3)))/(2*a
*b^2*Sqrt[a + b*x^2]) + (3*d*(a*B*d^2 - 2*b*c*(B*c + A*d))*Log[-(Sqrt[b]*x
) + Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {684, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{3/2}}\,dx$$
$$\Big\downarrow \; 684$$
$$\frac{\int \frac{d(c+dx)(a(3Bc+2Ad)-(2Abc-3aBd)x)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$
$$\Big\downarrow \; 27$$
$$\frac{d\int \frac{(c+dx)(a(3Bc+2Ad)-(2Abc-3aBd)x)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$
$$\Big\downarrow \; 676$$

$$\frac{d\left(-\frac{3a(aBd^2-2bc(Ad+Bc))\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{2\sqrt{a+bx^2}(-aAd^2-3aBcd+Abc^2)}{b}-\frac{dx\sqrt{a+bx^2}(2Abc-3aBd)}{2b}\right)}{ab}-\frac{(c+dx)^2(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 224$$

$$\frac{d\left(-\frac{3a(aBd^2-2bc(Ad+Bc))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{2\sqrt{a+bx^2}(-aAd^2-3aBcd+Abc^2)}{b}-\frac{dx\sqrt{a+bx^2}(2Abc-3aBd)}{2b}\right)}{ab}-\frac{(c+dx)^2(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{d\left(-\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aBd^2-2bc(Ad+Bc))}{2b^{3/2}}-\frac{2\sqrt{a+bx^2}(-aAd^2-3aBcd+Abc^2)}{b}-\frac{dx\sqrt{a+bx^2}(2Abc-3aBd)}{2b}\right)}{ab}-\frac{(c+dx)^2(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$

input
```
Int[((A + B*x)*(c + d*x)^3)/(a + b*x^2)^(3/2),x]
```

output
```
-(((c + d*x)^2*(a*(B*c + A*d) - (A*b*c - a*B*d)*x))/(a*b*Sqrt[a + b*x^2]))
 + (d*((-2*(A*b*c^2 - 3*a*B*c*d - a*A*d^2)*Sqrt[a + b*x^2])/b - (d*(2*A*b*
c - 3*a*B*d)*x*Sqrt[a + b*x^2])/(2*b) - (3*a*(a*B*d^2 - 2*b*c*(B*c + A*d))
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.15

method	result
risch	$\frac{d^{2}\left(Bdx+2Ad+6Bc\right)\sqrt{b\,x^{2}+a}}{2b^{2}}+\frac{3bd\left(2Abcd-aB\,d^{2}+2Bb\,c^{2}\right)\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)+\frac{2A\,d^{3}a-6Ab\,c^{2}d+6aBc\,d^{2}-2bB\,c}{\sqrt{b\,x^{2}+a}}}{2b^{2}}$
default	$\frac{A\,c^{3}x}{\sqrt{b\,x^{2}+a}\,a}+d^{2}\left(Ad+3Bc\right)\left(\frac{x^{2}}{b\sqrt{b\,x^{2}+a}}+\frac{2a}{b^{2}\sqrt{b\,x^{2}+a}}\right)+3cd\left(Ad+Bc\right)\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right.$

input
```
int((B*x+A)*(d*x+c)^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*d^2*(B*d*x+2*A*d+6*B*c)*(b*x^2+a)^(1/2)/b^2+1/2/b^2*(3*b*d*(2*A*b*c*d-
B*a*d^2+2*B*b*c^2)*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^
(1/2)))+2*(A*a*d^3-3*A*b*c^2*d+3*B*a*c*d^2-B*b*c^3)/(b*x^2+a)^(1/2)+2*A*b^
2*c^3*x/a/(b*x^2+a)^(1/2)-a*B*d^3*x/(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 508, normalized size of antiderivative = 3.26

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{3\,(2\,Ba^2bc^2d+2\,Aa^2bcd^2-Ba^3d^3+(2\,Bab^2c^2d+2\,Aab^2cd^2-Ba^2bd^3)x^2)}{}\right.$$

$$-\frac{3\,(2\,Ba^2bc^2d+2\,Aa^2bcd^2-Ba^3d^3+(2\,Bab^2c^2d+2\,Aab^2cd^2-Ba^2bd^3)x^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Ba}{}$$

input
```
integrate((B*x+A)*(d*x+c)^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(3*(2*B*a^2*b*c^2*d + 2*A*a^2*b*c*d^2 - B*a^3*d^3 + (2*B*a*b^2*c^2*d
 + 2*A*a*b^2*c*d^2 - B*a^2*b*d^3)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2
 + a)*sqrt(b)*x - a) - 2*(B*a*b^2*d^3*x^3 - 2*B*a*b^2*c^3 - 6*A*a*b^2*c^2*
d + 12*B*a^2*b*c*d^2 + 4*A*a^2*b*d^3 + 2*(3*B*a*b^2*c*d^2 + A*a*b^2*d^3)*x
^2 + (2*A*b^3*c^3 - 6*B*a*b^2*c^2*d - 6*A*a*b^2*c*d^2 + 3*B*a^2*b*d^3)*x)*
sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3), -1/2*(3*(2*B*a^2*b*c^2*d + 2*A*a^2
*b*c*d^2 - B*a^3*d^3 + (2*B*a*b^2*c^2*d + 2*A*a*b^2*c*d^2 - B*a^2*b*d^3)*x
^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (B*a*b^2*d^3*x^3 - 2*B*a
*b^2*c^3 - 6*A*a*b^2*c^2*d + 12*B*a^2*b*c*d^2 + 4*A*a^2*b*d^3 + 2*(3*B*a*b
^2*c*d^2 + A*a*b^2*d^3)*x^2 + (2*A*b^3*c^3 - 6*B*a*b^2*c^2*d - 6*A*a*b^2*c
*d^2 + 3*B*a^2*b*d^3)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [F]

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(A+Bx)\,(c+dx)^3}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(d*x+c)**3/(b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x)*(c + d*x)**3/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.46

$$\begin{aligned}\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{3/2}}\,dx &= \frac{Bd^3x^3}{2\sqrt{bx^2+a}b} + \frac{Ac^3x}{\sqrt{bx^2+a}a} + \frac{3\,Bad^3x}{2\sqrt{bx^2+a}b^2} \\ &\quad - \frac{3\,Bad^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} - \frac{Bc^3}{\sqrt{bx^2+a}b} - \frac{3\,Ac^2d}{\sqrt{bx^2+a}b} + \frac{(3\,Bcd^2+Ad^3)x^2}{\sqrt{bx^2+a}b} \\ &\quad - \frac{3\,(Bc^2d+Acd^2)x}{\sqrt{bx^2+a}b} + \frac{3\,(Bc^2d+Acd^2)\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{2\,(3\,Bcd^2+Ad^3)a}{\sqrt{bx^2+a}b^2}\end{aligned}$$

input
```
integrate((B*x+A)*(d*x+c)^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*B*d^3*x^3/(sqrt(b*x^2 + a)*b) + A*c^3*x/(sqrt(b*x^2 + a)*a) + 3/2*B*a*
d^3*x/(sqrt(b*x^2 + a)*b^2) - 3/2*B*a*d^3*arcsinh(b*x/sqrt(a*b))/b^(5/2) -
 B*c^3/(sqrt(b*x^2 + a)*b) - 3*A*c^2*d/(sqrt(b*x^2 + a)*b) + (3*B*c*d^2 +
A*d^3)*x^2/(sqrt(b*x^2 + a)*b) - 3*(B*c^2*d + A*c*d^2)*x/(sqrt(b*x^2 + a)*
b) + 3*(B*c^2*d + A*c*d^2)*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 2*(3*B*c*d^2 +
 A*d^3)*a/(sqrt(b*x^2 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{Bd^3x}{b} + \frac{2\,(3\,Bab^3cd^2+Aab^3d^3)}{ab^4}\right)x + \frac{2\,Ab^4c^3-6\,Bab^3c^2d-6\,Aab^3cd^2+3\,Ba^2b^2d^3}{ab^4}\right)x - \frac{2\,(Bab^3c}{}}{2\sqrt{bx^2+a}} - \frac{3\,(2\,Bbc^2d + 2\,Abcd^2 - Bad^3)\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(((B*d^3*x/b + 2*(3*B*a*b^3*c*d^2 + A*a*b^3*d^3)/(a*b^4))*x + (2*A*b^4
*c^3 - 6*B*a*b^3*c^2*d - 6*A*a*b^3*c*d^2 + 3*B*a^2*b^2*d^3)/(a*b^4))*x - 2
*(B*a*b^3*c^3 + 3*A*a*b^3*c^2*d - 6*B*a^2*b^2*c*d^2 - 2*A*a^2*b^2*d^3)/(a*
b^4))/sqrt(b*x^2 + a) - 3/2*(2*B*b*c^2*d + 2*A*b*c*d^2 - B*a*d^3)*log(abs(
-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(A+B\,x)\,(c+d\,x)^3}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((A + B*x)*(c + d*x)^3)/(a + b*x^2)^(3/2),x)
```

output
```
int(((A + B*x)*(c + d*x)^3)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(Bx+A)\,(dx+c)^3}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((B*x+A)*(d*x+c)^3/(b*x^2+a)^(3/2),x)
```

output
```
int((B*x+A)*(d*x+c)^3/(b*x^2+a)^(3/2),x)
```

3.180 $\int \frac{(A+Bx)(c+dx)^2}{\left(a+bx^2\right)^{3/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{(A+Bx)(c+dx)^2}{\left(a+bx^2\right)^{3/2}} \, dx = -\frac{(c+dx)(a(Bc+Ad)-(Abc-aBd)x)}{ab\sqrt{a+bx^2}} - \frac{d(Abc-2aBd)\sqrt{a+bx^2}}{ab^2} + \frac{d(2Bc+Ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
-(d*x+c)*(a*(A*d+B*c)-(A*b*c-B*a*d)*x)/a/b/(b*x^2+a)^(1/2)-d*(A*b*c-2*B*a*
d)*(b*x^2+a)^(1/2)/a/b^2+d*(A*d+2*B*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/
b^(3/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)(c+dx)^2}{\left(a+bx^2\right)^{3/2}} \, dx = \frac{2a^2Bd^2+Ab^2c^2x-ab(Ad(2c+dx)+B(c^2+2cdx-d^2x^2))}{ab^2\sqrt{a+bx^2}} - \frac{d(2Bc+Ad)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[((A + B*x)*(c + d*x)^2)/(a + b*x^2)^(3/2),x]
```

output
```
(2*a^2*B*d^2 + A*b^2*c^2*x - a*b*(A*d*(2*c + d*x) + B*(c^2 + 2*c*d*x - d^2
*x^2)))/(a*b^2*Sqrt[a + b*x^2]) - (d*(2*B*c + A*d)*Log[-(Sqrt[b]*x) + Sqrt
[a + b*x^2]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {684, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(c+dx)^2}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \ 684$$

$$\frac{\int \frac{d(a(2Bc+Ad)-(Abc-2aBd)x)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{d\int \frac{a(2Bc+Ad)-(Abc-2aBd)x}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 455$$

$$\frac{d\left(a(Ad+2Bc)\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(Abc-2aBd)}{b}\right)}{ab}-\frac{(c+dx)(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 224$$

$$\frac{d\left(a(Ad+2Bc)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}(Abc-2aBd)}{b}\right)}{ab}-\frac{(c+dx)(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 219$$

$$\frac{d\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Ad+2Bc)}{\sqrt{b}}-\frac{\sqrt{a+bx^2}(Abc-2aBd)}{b}\right)}{ab}-\frac{(c+dx)(a(Ad+Bc)-x(Abc-aBd))}{ab\sqrt{a+bx^2}}$$

input
```
Int[((A + B*x)*(c + d*x)^2)/(a + b*x^2)^(3/2),x]
```

output
```
-(((c + d*x)*(a*(B*c + A*d) - (A*b*c - a*B*d)*x))/(a*b*Sqrt[a + b*x^2])) +
 (d*(-(((A*b*c - 2*a*B*d)*Sqrt[a + b*x^2])/b) + (a*(2*B*c + A*d)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

method	result
risch	$\frac{d^2 B\sqrt{b\,x^2+a}}{b^2}+\frac{bd(Ad+2Bc)\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)-\frac{2Abcd-aB\,d^2+Bb\,c^2}{b\sqrt{b\,x^2+a}}+\frac{Ab\,c^2x}{a\sqrt{b\,x^2+a}}}{b}$
default	$\frac{A\,c^2x}{\sqrt{b\,x^2+a}\,a}+d(Ad+2Bc)\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)-\frac{c(2Ad+Bc)}{b\sqrt{b\,x^2+a}}+B\,d^2\left(\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2a}{b^2\sqrt{b\,x^2+}}\right.$

input
```
int((B*x+A)*(d*x+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
d^2*B/b^2*(b*x^2+a)^(1/2)+1/b*(b*d*(A*d+2*B*c)*(-x/b/(b*x^2+a)^(1/2)+1/b^(
3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))-(2*A*b*c*d-B*a*d^2+B*b*c^2)/b/(b*x^2+a
)^(1/2)+A*b*c^2*x/a/(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.79

$$\int \frac{(A+Bx)(c+dx)^2}{(a+bx^2)^{3/2}}\,dx = \left[\frac{(2\,Ba^2cd+Aa^2d^2+(2\,Babcd+Aabd^2)x^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x\right.}{}\right.$$

$$-\frac{(2\,Ba^2cd+Aa^2d^2+(2\,Babcd+Aabd^2)x^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Babd^2x^2-Babc^2-2\,Aabcd+2\,Bc}{ab^3x^2+a^2b^2}$$

input
```
integrate((B*x+A)*(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((2*B*a^2*c*d + A*a^2*d^2 + (2*B*a*b*c*d + A*a*b*d^2)*x^2)*sqrt(b)*lo
g(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(B*a*b*d^2*x^2 - B*a*b*c
^2 - 2*A*a*b*c*d + 2*B*a^2*d^2 + (A*b^2*c^2 - 2*B*a*b*c*d - A*a*b*d^2)*x)*
sqrt(b*x^2 + a))/(a*b^3*x^2 + a^2*b^2), -((2*B*a^2*c*d + A*a^2*d^2 + (2*B*
a*b*c*d + A*a*b*d^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (B
*a*b*d^2*x^2 - B*a*b*c^2 - 2*A*a*b*c*d + 2*B*a^2*d^2 + (A*b^2*c^2 - 2*B*a*
b*c*d - A*a*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [F]

$$\int \frac{(A+Bx)(c+dx)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(A+Bx)\,(c+dx)^2}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(d*x+c)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x)*(c + d*x)**2/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx)(c+dx)^2}{(a+bx^2)^{3/2}}\,dx = \frac{Bd^2x^2}{\sqrt{bx^2+a}b} + \frac{Ac^2x}{\sqrt{bx^2+a}a} - \frac{Bc^2}{\sqrt{bx^2+a}b} - \frac{2\,Acd}{\sqrt{bx^2+a}b} + \frac{2\,Bad^2}{\sqrt{bx^2+a}b^2} - \frac{(2\,Bcd+Ad^2)x}{\sqrt{bx^2+a}b} + \frac{(2\,Bcd+Ad^2)\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
B*d^2*x^2/(sqrt(b*x^2 + a)*b) + A*c^2*x/(sqrt(b*x^2 + a)*a) - B*c^2/(sqrt(
b*x^2 + a)*b) - 2*A*c*d/(sqrt(b*x^2 + a)*b) + 2*B*a*d^2/(sqrt(b*x^2 + a)*b
^2) - (2*B*c*d + A*d^2)*x/(sqrt(b*x^2 + a)*b) + (2*B*c*d + A*d^2)*arcsinh(
b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx)(c+dx)^2}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\frac{Bd^2x}{b}+\frac{Ab^3c^2-2\,Bab^2cd-Aab^2d^2}{ab^3}\right)x-\frac{Bab^2c^2+2\,Aab^2cd-2\,Ba^2bd^2}{ab^3}}{\sqrt{bx^2+a}} - \frac{(2\,Bcd+Ad^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((B*d^2*x/b + (A*b^3*c^2 - 2*B*a*b^2*c*d - A*a*b^2*d^2)/(a*b^3))*x - (B*a*
b^2*c^2 + 2*A*a*b^2*c*d - 2*B*a^2*b*d^2)/(a*b^3))/sqrt(b*x^2 + a) - (2*B*c
*d + A*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(c+dx)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(A+B\,x)\,(c+d\,x)^2}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((A + B*x)*(c + d*x)^2)/(a + b*x^2)^(3/2),x)
```

output
```
int(((A + B*x)*(c + d*x)^2)/(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.71

$$\int \frac{(A+Bx)(c+dx)^2}{(a+bx^2)^{3/2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,abcd-\sqrt{b\,x^2+a}\,ab\,d^2x+2\sqrt{b\,x^2+a}\,ab\,d^2+\sqrt{b\,x^2+a}\,b^2c^2x-}{}$$

input
```
int((B*x+A)*(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a*b*c*d - sqrt(a + b*x**2)*a*b*d**2*x + 2*sqrt(a +
b*x**2)*a*b*d**2 + sqrt(a + b*x**2)*b**2*c**2*x - sqrt(a + b*x**2)*b**2*c*
*2 - 2*sqrt(a + b*x**2)*b**2*c*d*x + sqrt(a + b*x**2)*b**2*d**2*x**2 + sqr
t(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*d**2 + 2*sqrt(b)*log
((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c*d + sqrt(b)*log((sqrt(a + b
*x**2) + sqrt(b)*x)/sqrt(a))*a*b*d**2*x**2 + 2*sqrt(b)*log((sqrt(a + b*x**
2) + sqrt(b)*x)/sqrt(a))*b**2*c*d*x**2 - sqrt(b)*a**2*d**2 + sqrt(b)*a*b*c
**2 - 2*sqrt(b)*a*b*c*d - sqrt(b)*a*b*d**2*x**2 + sqrt(b)*b**2*c**2*x**2 -
 2*sqrt(b)*b**2*c*d*x**2)/(b**2*(a + b*x**2))
```

3.181 $\int \frac{(A+Bx)(c+dx)}{\left(a+bx^2\right)^{3/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{(A+Bx)(c+dx)}{\left(a+bx^2\right)^{3/2}} \, dx = -\frac{a(Bc+Ad)-(Abc-aBd)x}{ab\sqrt{a+bx^2}}+\frac{Bd\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
-(a*(A*d+B*c)-(A*b*c-B*a*d)*x)/a/b/(b*x^2+a)^(1/2)+B*d*arctanh(b^(1/2)*x/(
b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)(c+dx)}{\left(a+bx^2\right)^{3/2}} \, dx = \frac{-aBc-aAd+Abcx-aBdx}{ab\sqrt{a+bx^2}}-\frac{Bd\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[((A + B*x)*(c + d*x))/(a + b*x^2)^(3/2),x]
```

output
```
(-(a*B*c) - a*A*d + A*b*c*x - a*B*d*x)/(a*b*Sqrt[a + b*x^2]) - (B*d*Log[-(
Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {675, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 675}$$

$$\frac{Bd\int \frac{1}{\sqrt{bx^2+a}}dx}{b}-\frac{Ad+Bc}{b\sqrt{a+bx^2}}+\frac{x(Abc-aBd)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{Bd\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}-\frac{Ad+Bc}{b\sqrt{a+bx^2}}+\frac{x(Abc-aBd)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{Ad+Bc}{b\sqrt{a+bx^2}}+\frac{x(Abc-aBd)}{ab\sqrt{a+bx^2}}+\frac{Bd\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

input
```
Int[((A + B*x)*(c + d*x))/(a + b*x^2)^(3/2),x]
```

output
```
-((B*c + A*d)/(b*Sqrt[a + b*x^2])) + ((A*b*c - a*B*d)*x)/(a*b*Sqrt[a + b*x
^2]) + (B*d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{Acx}{\sqrt{b\,x^{2}+a}\,a}-\frac{Ad+Bc}{b\sqrt{b\,x^{2}+a}}+Bd\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)$	78

input
```
int((B*x+A)*(d*x+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*c/(b*x^2+a)^(1/2)/a*x-(A*d+B*c)/b/(b*x^2+a)^(1/2)+B*d*(-x/b/(b*x^2+a)^(1
/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.87

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{3/2}}\,dx = \left[\frac{(Babdx^2+Ba^2d)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(Babc+Aabd-\right.}{2\,(ab^3x^2+a^2b^2)}$$

$$\left.-\frac{(Babdx^2+Ba^2d)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(Babc+Aabd-(Ab^2c-Babd)x)\sqrt{bx^2+a}}{ab^3x^2+a^2b^2}\right]$$

input
```
integrate((B*x+A)*(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((B*a*b*d*x^2 + B*a^2*d)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a) - 2*(B*a*b*c + A*a*b*d - (A*b^2*c - B*a*b*d)*x)*sqrt(b*x^2 + a
))/(a*b^3*x^2 + a^2*b^2), -((B*a*b*d*x^2 + B*a^2*d)*sqrt(-b)*arctan(sqrt(-
b)*x/sqrt(b*x^2 + a)) + (B*a*b*c + A*a*b*d - (A*b^2*c - B*a*b*d)*x)*sqrt(b
*x^2 + a))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [A] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.73

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{3/2}}\,dx = Ad\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{Acx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

$$+Bc\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+Bd\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((B*x+A)*(d*x+c)/(b*x**2+a)**(3/2),x)
```

output
```
A*d*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), Tru
e)) + A*c*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*c*Piecewise((-1/(b*sqrt(a +
b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) + B*d*(asinh(sqrt(b)*x/sqr
t(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{3/2}}\,dx = \frac{Acx}{\sqrt{bx^2+a}a} - \frac{Bdx}{\sqrt{bx^2+a}b} + \frac{Bd\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{Bc}{\sqrt{bx^2+a}b} - \frac{Ad}{\sqrt{bx^2+a}b}$$

input
```
integrate((B*x+A)*(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
A*c*x/(sqrt(b*x^2 + a)*a) - B*d*x/(sqrt(b*x^2 + a)*b) + B*d*arcsinh(b*x/sq
rt(a*b))/b^(3/2) - B*c/(sqrt(b*x^2 + a)*b) - A*d/(sqrt(b*x^2 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{3/2}}\,dx = -\frac{Bd\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}} + \frac{\frac{(Ab^2c-Babd)x}{ab^2} - \frac{Babc+Aabd}{ab^2}}{\sqrt{bx^2+a}}$$

input
```
integrate((B*x+A)*(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-B*d*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + ((A*b^2*c - B*a*b*d)
*x/(a*b^2) - (B*a*b*c + A*a*b*d)/(a*b^2))/sqrt(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 6.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{3/2}}\,dx = \frac{B\,d\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}} - \frac{B\,c}{b\,\sqrt{b\,x^2+a}} - \frac{\frac{A\,d}{b}-\frac{A\,c\,x}{a}}{\sqrt{b\,x^2+a}} - \frac{B\,d\,x}{b\,\sqrt{b\,x^2+a}}$$

input
```
int(((A + B*x)*(c + d*x))/(a + b*x^2)^(3/2),x)
```

output
```
(B*d*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - (B*c)/(b*(a + b*x^2)^(1
/2)) - ((A*d)/b - (A*c*x)/a)/(a + b*x^2)^(1/2) - (B*d*x)/(b*(a + b*x^2)^(1
/2))
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.03

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{3/2}}\,dx = \frac{-\sqrt{b\,x^2+a}\,ad+\sqrt{b\,x^2+a}\,bcx-\sqrt{b\,x^2+a}\,bc-\sqrt{b\,x^2+a}\,bdx+\sqrt{b}\log\Big(\frac{\sqrt{b\,x^2}}{}}{b\,(b\,x}$$

input
```
int((B*x+A)*(d*x+c)/(b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2)*a*d + sqrt(a + b*x**2)*b*c*x - sqrt(a + b*x**2)*b*c -
 sqrt(a + b*x**2)*b*d*x + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a*d + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b*d*x**2 + s
qrt(b)*a*c - sqrt(b)*a*d + sqrt(b)*b*c*x**2 - sqrt(b)*b*d*x**2)/(b*(a + b*
x**2))
```

3.182 $\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 28

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}} \, dx = -\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

output
```
-(-A*b*x+B*a)/a/b/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}} \, dx = \frac{-aB+Abx}{ab\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x)/(a + b*x^2)^(3/2),x]
```

output
```
(-(a*B) + A*b*x)/(a*b*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 453}$$

$$-\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x)/(a + b*x^2)^(3/2),x]
```

output
```
-((a*B - A*b*x)/(a*b*Sqrt[a + b*x^2]))
```

Defintions of rubi rules used

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{Abx-Ba}{\sqrt{b\,x^{2}+a}\,ab}$	26
trager	$\frac{Abx-Ba}{\sqrt{b\,x^{2}+a}\,ab}$	26
orering	$\frac{Abx-Ba}{\sqrt{b\,x^{2}+a}\,ab}$	26
default	$\frac{Ax}{\sqrt{b\,x^{2}+a}\,a}-\frac{B}{b\sqrt{b\,x^{2}+a}}$	32

input
```
int((B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(A*b*x-B*a)/(b*x^2+a)^(1/2)/a/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{(Abx-Ba)\sqrt{bx^2+a}}{ab^2x^2+a^2b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
(A*b*x - B*a)*sqrt(b*x^2 + a)/(a*b^2*x^2 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}} + B\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*Piecewise((-1/(b*sqrt(a + b*x**2)),
Ne(b, 0)), (x**2/(2*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx=\frac{Ax}{\sqrt{bx^2+a}a}-\frac{B}{\sqrt{bx^2+a}b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
A*x/(sqrt(b*x^2 + a)*a) - B/(sqrt(b*x^2 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx=\frac{\frac{Ax}{a}-\frac{B}{b}}{\sqrt{bx^2+a}}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(A*x/a - B/b)/sqrt(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx=-\frac{\frac{B}{b}-\frac{A\,x}{a}}{\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x)/(a + b*x^2)^(3/2),x)
```

output
```
-(B/b - (A*x)/a)/(a + b*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,bx-\sqrt{b\,x^2+a}\,b+\sqrt{b}\,a+\sqrt{b}\,b\,x^2}{b\,(b\,x^2+a)}$$

input
```
int((B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*b*x - sqrt(a + b*x**2)*b + sqrt(b)*a + sqrt(b)*b*x**2)/(
b*(a + b*x**2))
```

3.183 $\int \frac{A+Bx}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{A+Bx}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=-\frac{a(Bc-Ad)-(Abc+aBd)x}{a\left(bc^2+ad^2\right)\sqrt{a+bx^2}}+\frac{d(Bc-Ad)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{\left(bc^2+ad^2\right)^{3/2}}$$

output
```
-(a*(-A*d+B*c)-(A*b*c+B*a*d)*x)/a/(a*d^2+b*c^2)/(b*x^2+a)^(1/2)+d*(-A*d+B*
c)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)
^(3/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\frac{Abcx+a(-Bc+Ad+Bdx)}{a\left(bc^2+ad^2\right)\sqrt{a+bx^2}}-\frac{2d(Bc-Ad)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\left(-bc^2-ad^2\right)^{3/2}}$$

input
```
Integrate[(A + B*x)/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
(A*b*c*x + a*(-(B*c) + A*d + B*d*x))/(a*(b*c^2 + a*d^2)*Sqrt[a + b*x^2]) -
 (2*d*(B*c - A*d)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*
c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {686, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}(c+dx)}dx$$

$$\Big\downarrow \text{686}$$

$$-\frac{\int \frac{abd(Bc-Ad)}{(c+dx)\sqrt{bx^2+a}}dx}{ab(ad^2+bc^2)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{a\sqrt{a+bx^2}(ad^2+bc^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d(Bc-Ad)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{a(Bc-Ad)-x(aBd+Abc)}{a\sqrt{a+bx^2}(ad^2+bc^2)}$$

$$\Big\downarrow \text{488}$$

$$\frac{d(Bc-Ad)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}-\frac{a(Bc-Ad)-x(aBd+Abc)}{a\sqrt{a+bx^2}(ad^2+bc^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{d(Bc-Ad)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{(ad^2+bc^2)^{3/2}}-\frac{a(Bc-Ad)-x(aBd+Abc)}{a\sqrt{a+bx^2}(ad^2+bc^2)}$$

input
```
Int[(A + B*x)/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
-((a*(B*c - A*d) - (A*b*c + a*B*d)*x)/(a*(b*c^2 + a*d^2)*Sqrt[a + b*x^2]))
 + (d*(B*c - A*d)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^
2])])/(b*c^2 + a*d^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(107) = 214$.

Time = 1.28 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.97

method	result
default	$\frac{Bx}{d\sqrt{b\,x^2+a}\,a}+\frac{(Ad-Bc)\left(\frac{d^2}{\left(a\,d^2+b\,c^2\right)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}+\frac{2bcd\left(2b\left(x+\frac{c}{d}\right)-\frac{2bc}{d}\right)}{\left(a\,d^2+b\,c^2\right)\left(\frac{4b\left(a\,d^2+b\,c^2\right)}{d^2}-\frac{4b^2c^2}{d^2}\right)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\right.}{d}}}\right.}{d^2}$

input
```
int((B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
B/d/(b*x^2+a)^(1/2)/a*x+(A*d-B*c)/d^2*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*
b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-
2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d
)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*l
n((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c
/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(108) = 216$.

Time = 0.32 (sec) , antiderivative size = 571, normalized size of antiderivative = 4.97

$$\int \frac{A+Bx}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = \left[-\frac{\left(Ba^2cd-Aa^2d^2+\left(Babcd-Aabd^2\right)x^2\right)\sqrt{bc^2+ad^2}\log\left(\frac{2\,abcdx-abc^2-2\,a^2d^2\cdots}{\cdots}\right.}{2\cdots}\right.$$

input
```
integrate((B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*((B*a^2*c*d - A*a^2*d^2 + (B*a*b*c*d - A*a*b*d^2)*x^2)*sqrt(b*c^2 +
a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2
- 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x
+ c^2)) + 2*(B*a*b*c^3 - A*a*b*c^2*d + B*a^2*c*d^2 - A*a^2*d^3 - (A*b^2*c^
3 + B*a*b*c^2*d + A*a*b*c*d^2 + B*a^2*d^3)*x)*sqrt(b*x^2 + a))/(a^2*b^2*c^
4 + 2*a^3*b*c^2*d^2 + a^4*d^4 + (a*b^3*c^4 + 2*a^2*b^2*c^2*d^2 + a^3*b*d^4
)*x^2), ((B*a^2*c*d - A*a^2*d^2 + (B*a*b*c*d - A*a*b*d^2)*x^2)*sqrt(-b*c^2
 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c
^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) - (B*a*b*c^3 - A*a*b*c^2*d + B*a^
2*c*d^2 - A*a^2*d^3 - (A*b^2*c^3 + B*a*b*c^2*d + A*a*b*c*d^2 + B*a^2*d^3)*
x)*sqrt(b*x^2 + a))/(a^2*b^2*c^4 + 2*a^3*b*c^2*d^2 + a^4*d^4 + (a*b^3*c^4
+ 2*a^2*b^2*c^2*d^2 + a^3*b*d^4)*x^2)]
```

Sympy [F]

$$\int \frac{A+Bx}{(c+dx)\,(a+bx^2)^{3/2}}\,dx=\int \frac{A+Bx}{(a+bx^2)^{\frac{3}{2}}\,(c+dx)}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)/(b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x)/((a + b*x**2)**(3/2)*(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(108) = 216$.

Time = 0.07 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.42

$$\begin{aligned}\int \frac{A+Bx}{(c+dx)\,(a+bx^2)^{3/2}}\,dx &= -\frac{Bbc^2x}{\sqrt{bx^2+a}\,abc^2d+\sqrt{bx^2+a}\,a^2d^3}\\ &+\frac{Abcx}{\sqrt{bx^2+a}\,abc^2+\sqrt{bx^2+a}\,a^2d^2}-\frac{Bc}{\sqrt{bx^2+a}\,bc^2+\sqrt{bx^2+a}\,ad^2}\\ &+\frac{A}{\frac{\sqrt{bx^2+a}\,bc^2}{d}+\sqrt{bx^2+a}\,ad}+\frac{Bx}{\sqrt{bx^2+a}\,ad}\\ &-\frac{Bc\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^2}+\frac{A\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d}\end{aligned}$$

input
```
integrate((B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-B*b*c^2*x/(sqrt(b*x^2 + a)*a*b*c^2*d + sqrt(b*x^2 + a)*a^2*d^3) + A*b*c*x
/(sqrt(b*x^2 + a)*a*b*c^2 + sqrt(b*x^2 + a)*a^2*d^2) - B*c/(sqrt(b*x^2 + a
)*b*c^2 + sqrt(b*x^2 + a)*a*d^2) + A/(sqrt(b*x^2 + a)*b*c^2/d + sqrt(b*x^2
 + a)*a*d) + B*x/(sqrt(b*x^2 + a)*a*d) - B*c*arcsinh(b*c*x/(sqrt(a*b)*abs(
d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^2) + A*
arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a
 + b*c^2/d^2)^(3/2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(108) = 216$.

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.97

$$\int \frac{A+Bx}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = \frac{\frac{(Ab^2c^3+Babc^2d+Aabcd^2+Ba^2d^3)x}{ab^2c^4+2\,a^2bc^2d^2+a^3d^4} - \frac{Babc^3-Aabc^2d+Ba^2cd^2-Aa^2d^3}{ab^2c^4+2\,a^2bc^2d^2+a^3d^4}}{\sqrt{bx^2+a}} + \frac{2\,(Bcd-Ad^2)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{(bc^2+ad^2)\sqrt{-bc^2-ad^2}}$$

input
```
integrate((B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((A*b^2*c^3 + B*a*b*c^2*d + A*a*b*c*d^2 + B*a^2*d^3)*x/(a*b^2*c^4 + 2*a^2*
b*c^2*d^2 + a^3*d^4) - (B*a*b*c^3 - A*a*b*c^2*d + B*a^2*c*d^2 - A*a^2*d^3)
/(a*b^2*c^4 + 2*a^2*b*c^2*d^2 + a^3*d^4))/sqrt(b*x^2 + a) + 2*(B*c*d - A*d
^2)*arctan(((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d
^2))/((b*c^2 + a*d^2)*sqrt(-b*c^2 - a*d^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)\,(a+bx^2)^{3/2}}\,dx = \int \frac{A+B\,x}{(b\,x^2+a)^{3/2}\,(c+d\,x)}\,dx$$

input
```
int((A + B*x)/((a + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((A + B*x)/((a + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 4555, normalized size of antiderivative = 39.61

$$\int \frac{A+Bx}{(c+dx)\,(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x)
```

output
```
( - 2*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*
sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**2*c*d + 2*sqrt(b)*sqrt
(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c*
*2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b
*c**2)*c - a*d**2 - 2*b*c**2))*a*b*c**2 - 2*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*
d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c**2)*atan((sqrt(a +
 b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2
- 2*b*c**2))*a*b*c*d*x**2 + 2*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)
*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + s
qrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*b
**2*c**2*x**2 - 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c*
*2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b
*c**2)*c - a*d**2 - 2*b*c**2))*a**3*d**3 - 2*sqrt(2*sqrt(b)*sqrt(a*d**2 +
b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqr
t(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**2*b*c**2*d +
2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a
 + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**
2 - 2*b*c**2))*a**2*b*c*d**2 - 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c -
a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(...
```

3.184 $\int \frac{A+Bx}{(c+dx)^2(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 190

$$\int \frac{A+Bx}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = \frac{Bc-Ad}{(bc^2+ad^2)(c+dx)\sqrt{a+bx^2}} + \frac{a(aBd^2-bc(2Bc-3Ad))+b(Abc^2+3aBcd-2aAd^2)\,x}{a(bc^2+ad^2)^2\sqrt{a+bx^2}} - \frac{d(aBd^2-bc(2Bc-3Ad))\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{(bc^2+ad^2)^{5/2}}$$

output

```
(-A*d+B*c)/(a*d^2+b*c^2)/(d*x+c)/(b*x^2+a)^(1/2)+(a*(a*B*d^2-b*c*(-3*A*d+2
*B*c))+b*(-2*A*a*d^2+A*b*c^2+3*B*a*c*d)*x)/a/(a*d^2+b*c^2)^2/(b*x^2+a)^(1/
2)-d*(a*B*d^2-b*c*(-3*A*d+2*B*c))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)
/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx}{(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx=\frac{Ab^2c^2x(c+dx)+a^2d^2(2Bc-Ad+Bdx)+ab(Ad(2c^2+cdx-2d^2x^2)+B}{a\left(bc^2+ad^2\right)^2(c+dx)\sqrt{a+bx^2}}$$
$$-\frac{2d(aBd^2+bc(-2Bc+3Ad))\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\left(-bc^2-ad^2\right)^{5/2}}$$

input
```
Integrate[(A + B*x)/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
(A*b^2*c^2*x*(c + d*x) + a^2*d^2*(2*B*c - A*d + B*d*x) + a*b*(A*d*(2*c^2 +
 c*d*x - 2*d^2*x^2) + B*c*(-c^2 + c*d*x + 3*d^2*x^2)))/(a*(b*c^2 + a*d^2)^
2*(c + d*x)*Sqrt[a + b*x^2]) - (2*d*(a*B*d^2 + b*c*(-2*B*c + 3*A*d))*ArcTa
n[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^
2) - a*d^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {686, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}(c+dx)^2}dx$$
$$\Big\downarrow \text{686}$$
$$-\frac{\int \frac{bd(2a(Bc-Ad)-(Abc+aBd)x)}{(c+dx)^2\sqrt{bx^2+a}}dx}{ab\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{a\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}$$
$$\Big\downarrow \text{27}$$
$$-\frac{d\int \frac{2a(Bc-Ad)-(Abc+aBd)x}{(c+dx)^2\sqrt{bx^2+a}}dx}{a\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{a\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \textbf{679}$$

$$-\frac{d\left(-\frac{a\left(aBd^2-bc(2Bc-3Ad)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(-2aAd^2+3aBcd+Abc^2\right)}{(c+dx)(ad^2+bc^2)}\right)}{a\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{a\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \textbf{488}$$

$$-\frac{d\left(\frac{a\left(aBd^2-bc(2Bc-3Ad)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(-2aAd^2+3aBcd+Abc^2\right)}{(c+dx)(ad^2+bc^2)}\right)}{a\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{a\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{d\left(\frac{a\left(aBd^2-bc(2Bc-3Ad)\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{(ad^2+bc^2)^{3/2}}-\frac{\sqrt{a+bx^2}\left(-2aAd^2+3aBcd+Abc^2\right)}{(c+dx)(ad^2+bc^2)}\right)}{a\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{a\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}$$

input

```
Int[(A + B*x)/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output

```
-((a*(B*c - A*d) - (A*b*c + a*B*d)*x)/(a*(b*c^2 + a*d^2)*(c + d*x)*Sqrt[a
+ b*x^2])) - (d*(-(((A*b*c^2 + 3*a*B*c*d - 2*a*A*d^2)*Sqrt[a + b*x^2])/((b
*c^2 + a*d^2)*(c + d*x))) + (a*(a*B*d^2 - b*c*(2*B*c - 3*A*d))*ArcTanh[(a*
d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(3/2)))
/(a*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 838 vs. $2(180) = 360$.

Time = 1.30 (sec) , antiderivative size = 839, normalized size of antiderivative = 4.42

method	result
default	$\frac{B\left(\frac{d^{2}}{\left(a\,d^{2}+b\,c^{2}\right)\sqrt{b\left(x+\frac{c}{d}\right)^{2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}}+\frac{2bcd\left(2b\left(x+\frac{c}{d}\right)-\frac{2bc}{d}\right)}{\left(a\,d^{2}+b\,c^{2}\right)\left(\frac{4b\left(a\,d^{2}+b\,c^{2}\right)}{d^{2}}-\frac{4b^{2}c^{2}}{d^{2}}\right)\sqrt{b\left(x+\frac{c}{d}\right)^{2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}}-\frac{d^{2}\ln\left(\cdots\right.}{\cdots}\right.}{d^{2}}$

input

```
int((B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
B/d^2*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)
^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-
4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*
d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(
x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c
^2)/d^2)^(1/2))/(x+c/d)))+(A*d-B*c)/d^3*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)/(b*(
x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+3*b*c*d/(a*d^2+b*c^2)*(1
/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2
*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^
2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^
2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2
*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)
^(1/2))/(x+c/d)))-4*b/(a*d^2+b*c^2)*d^2*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+
b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^
(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(181) = 362$.

Time = 0.75 (sec) , antiderivative size = 1297, normalized size of antiderivative = 6.83

$$\int \frac{A+Bx}{(c+dx)^2 (a+bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*((2*B*a^2*b*c^3*d - 3*A*a^2*b*c^2*d^2 - B*a^3*c*d^3 + (2*B*a*b^2*c^2
*d^2 - 3*A*a*b^2*c*d^3 - B*a^2*b*d^4)*x^3 + (2*B*a*b^2*c^3*d - 3*A*a*b^2*c
^2*d^2 - B*a^2*b*c*d^3)*x^2 + (2*B*a^2*b*c^2*d^2 - 3*A*a^2*b*c*d^3 - B*a^3
*d^4)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b
^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a
))/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(B*a*b^2*c^5 - 2*A*a*b^2*c^4*d - B*a^2*b
*c^3*d^2 - A*a^2*b*c^2*d^3 - 2*B*a^3*c*d^4 + A*a^3*d^5 - (A*b^3*c^4*d + 3*
B*a*b^2*c^3*d^2 - A*a*b^2*c^2*d^3 + 3*B*a^2*b*c*d^4 - 2*A*a^2*b*d^5)*x^2 -
 (A*b^3*c^5 + B*a*b^2*c^4*d + 2*A*a*b^2*c^3*d^2 + 2*B*a^2*b*c^2*d^3 + A*a^
2*b*c*d^4 + B*a^3*d^5)*x)*sqrt(b*x^2 + a))/(a^2*b^3*c^7 + 3*a^3*b^2*c^5*d^
2 + 3*a^4*b*c^3*d^4 + a^5*c*d^6 + (a*b^4*c^6*d + 3*a^2*b^3*c^4*d^3 + 3*a^3
*b^2*c^2*d^5 + a^4*b*d^7)*x^3 + (a*b^4*c^7 + 3*a^2*b^3*c^5*d^2 + 3*a^3*b^2
*c^3*d^4 + a^4*b*c*d^6)*x^2 + (a^2*b^3*c^6*d + 3*a^3*b^2*c^4*d^3 + 3*a^4*b
*c^2*d^5 + a^5*d^7)*x), ((2*B*a^2*b*c^3*d - 3*A*a^2*b*c^2*d^2 - B*a^3*c*d^
3 + (2*B*a*b^2*c^2*d^2 - 3*A*a*b^2*c*d^3 - B*a^2*b*d^4)*x^3 + (2*B*a*b^2*c
^3*d - 3*A*a*b^2*c^2*d^2 - B*a^2*b*c*d^3)*x^2 + (2*B*a^2*b*c^2*d^2 - 3*A*a
^2*b*c*d^3 - B*a^3*d^4)*x)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2
)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x
^2)) - (B*a*b^2*c^5 - 2*A*a*b^2*c^4*d - B*a^2*b*c^3*d^2 - A*a^2*b*c^2*d^3
- 2*B*a^3*c*d^4 + A*a^3*d^5 - (A*b^3*c^4*d + 3*B*a*b^2*c^3*d^2 - A*a*b^...
```

Sympy [F]

$$\int \frac{A+Bx}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx = \int \frac{A+Bx}{(a+bx^2)^{\frac{3}{2}}\,(c+dx)^2}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x)/((a + b*x**2)**(3/2)*(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(181) = 362$.

Time = 0.11 (sec) , antiderivative size = 669, normalized size of antiderivative = 3.52

$$\int \frac{A+Bx}{(c+dx)^2(a+bx^2)^{3/2}}\,dx =$$
$$-\frac{3\,Bb^2c^3x}{\sqrt{bx^2+a}ab^2c^4d+2\,\sqrt{bx^2+a}a^2bc^2d^3+\sqrt{bx^2+a}a^3d^5}+\frac{3\,Ab^2c^2x}{\sqrt{bx^2+a}ab^2c^4+2\,\sqrt{bx^2+a}a^2bc^2d^2+\sqrt{bx^2+a}a^3d^4}-\frac{3\,Bbc^2}{\sqrt{bx^2+a}b^2c^4+2\,\sqrt{bx^2+a}abc^2d^2+\sqrt{bx^2+a}a^2d^4}+\frac{3\,Bbcx}{\sqrt{bx^2+a}abc^2d+\sqrt{bx^2+a}a^2d^3}+\frac{3\,Abc}{\frac{\sqrt{bx^2+a}b^2c^4}{d}+2\,\sqrt{bx^2+a}abc^2d+\sqrt{bx^2+a}a^2d^3}-\frac{2\,Abx}{\sqrt{bx^2+a}abc^2+\sqrt{bx^2+a}a^2d^2}+\frac{Bc}{\sqrt{bx^2+a}bc^2dx+\sqrt{bx^2+a}ad^3x+\sqrt{bx^2+a}bc^3+\sqrt{bx^2+a}acd^2}-\frac{A}{\sqrt{bx^2+a}bc^2x+\sqrt{bx^2+a}ad^2x+\frac{\sqrt{bx^2+a}bc^3}{d}+\sqrt{bx^2+a}acd}+\frac{B}{\sqrt{bx^2+a}bc^2+\sqrt{bx^2+a}ad^2}-\frac{3\,Bbc^2\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{5}{2}}d^4}+\frac{3\,Abc\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{5}{2}}d^3}+\frac{B\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^2}$$

input
```
integrate((B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-3*B*b^2*c^3*x/(sqrt(b*x^2 + a)*a*b^2*c^4*d + 2*sqrt(b*x^2 + a)*a^2*b*c^2*
d^3 + sqrt(b*x^2 + a)*a^3*d^5) + 3*A*b^2*c^2*x/(sqrt(b*x^2 + a)*a*b^2*c^4
+ 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^2 + sqrt(b*x^2 + a)*a^3*d^4) - 3*B*b*c^2/(
sqrt(b*x^2 + a)*b^2*c^4 + 2*sqrt(b*x^2 + a)*a*b*c^2*d^2 + sqrt(b*x^2 + a)*
a^2*d^4) + 3*B*b*c*x/(sqrt(b*x^2 + a)*a*b*c^2*d + sqrt(b*x^2 + a)*a^2*d^3)
 + 3*A*b*c/(sqrt(b*x^2 + a)*b^2*c^4/d + 2*sqrt(b*x^2 + a)*a*b*c^2*d + sqrt
(b*x^2 + a)*a^2*d^3) - 2*A*b*x/(sqrt(b*x^2 + a)*a*b*c^2 + sqrt(b*x^2 + a)*
a^2*d^2) + B*c/(sqrt(b*x^2 + a)*b*c^2*d*x + sqrt(b*x^2 + a)*a*d^3*x + sqrt
(b*x^2 + a)*b*c^3 + sqrt(b*x^2 + a)*a*c*d^2) - A/(sqrt(b*x^2 + a)*b*c^2*x
+ sqrt(b*x^2 + a)*a*d^2*x + sqrt(b*x^2 + a)*b*c^3/d + sqrt(b*x^2 + a)*a*c*
d) + B/(sqrt(b*x^2 + a)*b*c^2 + sqrt(b*x^2 + a)*a*d^2) - 3*B*b*c^2*arcsinh
(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^
2/d^2)^(5/2)*d^4) + 3*A*b*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(
sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(5/2)*d^3) + B*arcsinh(b*c*x/(sq
rt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/
2)*d^2)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx=\int \frac{A+B\,x}{{\left(b\,x^2+a\right)}^{3/2}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((A + B*x)/((a + b*x^2)^(3/2)*(c + d*x)^2),x)
```

output
```
int((A + B*x)/((a + b*x^2)^(3/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 1540, normalized size of antiderivative = 8.11

$$\int \frac{A+Bx}{(c+dx)^2 (a+bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int((B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

output
```
(3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
+ b*c*x)*a**2*b*c**2*d**2 + 3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*s
qrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c*d**3*x + sqrt(a*d**2 + b*c**2
)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c*d**3
+ sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
 b*c*x)*a**2*b*d**4*x - 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(
a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**3*d + 3*sqrt(a*d**2 + b*c**2)*lo
g(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**2*d**2*x
**2 - 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
 a*d + b*c*x)*a*b**2*c**2*d**2*x + 3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*
x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c*d**3*x**3 + sqrt(a*d**
2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*
*2*c*d**3*x**2 + sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 +
b*c**2) - a*d + b*c*x)*a*b**2*d**4*x**3 - 2*sqrt(a*d**2 + b*c**2)*log(sqrt
(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**3*d*x**2 - 2*sqr
t(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*
x)*b**3*c**2*d**2*x**3 - 3*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c**2*
d**2 - 3*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c*d**3*x - sqrt(a*d**2
+ b*c**2)*log(c + d*x)*a**2*b*c*d**3 - sqrt(a*d**2 + b*c**2)*log(c + d*x)*
a**2*b*d**4*x + 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**3*d - 3*...
```

3.185 $\int \frac{A+Bx}{(c+dx)^3\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 292

$$\int \frac{A+Bx}{(c+dx)^3\left(a+bx^2\right)^{3/2}}\,dx=\frac{Bc-Ad}{2\left(bc^2+ad^2\right)(c+dx)^2\sqrt{a+bx^2}}-\frac{2aBd^2-bc(3Bc-5Ad)}{2\left(bc^2+ad^2\right)^2(c+dx)\sqrt{a+bx^2}}-\frac{b\left(3a\left(2bc^2(Bc-2Ad)-ad^2(3Bc-Ad)\right)-\left(Abc\left(2bc^2-13ad^2\right)+aBd\left(11bc^2-4ad^2\right)\right)x\right)}{2a\left(bc^2+ad^2\right)^3\sqrt{a+bx^2}}+\frac{3bd\left(2bc^2(Bc-2Ad)-ad^2(3Bc-Ad)\right)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2\left(bc^2+ad^2\right)^{7/2}}$$

output
```
1/2*(-A*d+B*c)/(a*d^2+b*c^2)/(d*x+c)^2/(b*x^2+a)^(1/2)-1/2*(2*a*B*d^2-b*c*
(-5*A*d+3*B*c))/(a*d^2+b*c^2)^2/(d*x+c)/(b*x^2+a)^(1/2)-1/2*b*(3*a*(2*b*c^
2*(-2*A*d+B*c)-a*d^2*(-A*d+3*B*c))-(A*b*c*(-13*a*d^2+2*b*c^2)+a*B*d*(-4*a*
d^2+11*b*c^2))*x)/a/(a*d^2+b*c^2)^3/(b*x^2+a)^(1/2)+3/2*b*d*(2*b*c^2*(-2*A
*d+B*c)-a*d^2*(-A*d+3*B*c))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^
2+a)^(1/2))/(a*d^2+b*c^2)^(7/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 774 vs. $2(292) = 584$.

Time = 10.99 (sec) , antiderivative size = 774, normalized size of antiderivative = 2.65

$$\int \frac{A+Bx}{(c+dx)^3\,(a+bx^2)^{3/2}}\,dx =$$

$$-\frac{\sqrt{-bc^2-ad^2}\left(4b^3c^2x\left(\sqrt{b}x-\sqrt{a+bx^2}\right)\left(Bcx(2c^2+9cdx+6d^2x^2)+A(c^3-4c^2dx-18cd^2x^2-12d^3x^3\right.\right.}{}$$

input
```
Integrate[(A + B*x)/((c + d*x)^3*(a + b*x^2)^(3/2)),x]
```

output
```
-1/2*(Sqrt[-(b*c^2) - a*d^2]*(4*b^3*c^2*x*(Sqrt[b]*x - Sqrt[a + b*x^2])*(B
*c*x*(2*c^2 + 9*c*d*x + 6*d^2*x^2) + A*(c^3 - 4*c^2*d*x - 18*c*d^2*x^2 - 1
2*d^3*x^3)) + a*b^2*(Sqrt[b]*B*c*x*(6*c^4 + 49*c^3*d*x + 14*c^2*d^2*x^2 -
54*c*d^3*x^3 - 36*d^4*x^4) + 3*A*d*Sqrt[a + b*x^2]*(2*c^4 + 15*c^3*d*x + 8
*c^2*d^2*x^2 - 6*c*d^3*x^3 - 4*d^4*x^4) + B*c*Sqrt[a + b*x^2]*(-2*c^4 - 31
*c^3*d*x - 2*c^2*d^2*x^2 + 54*c*d^3*x^3 + 36*d^4*x^4) + A*Sqrt[b]*(2*c^5 -
 14*c^4*d*x - 81*c^3*d^2*x^2 - 48*c^2*d^3*x^3 + 18*c*d^4*x^4 + 12*d^5*x^5)
) - a^3*d^3*(d*Sqrt[a + b*x^2]*(A*d + B*(c + 2*d*x)) + Sqrt[b]*(-3*A*d^2*x
 + B*(4*c^2 + 5*c*d*x - 2*d^2*x^2))) + a^2*b*d*(d*Sqrt[a + b*x^2]*(3*B*c*(
4*c^2 + 9*c*d*x + 7*d^2*x^2) - A*d*(10*c^2 + 11*c*d*x + 7*d^2*x^2)) + Sqrt
[b]*(B*c*(11*c^3 - 14*c^2*d*x - 54*c*d^2*x^2 - 39*d^3*x^3) + A*d*(-13*c^3
+ 4*c^2*d*x + 20*c*d^2*x^2 + 13*d^3*x^3)))) + 6*b*d*(-2*b*c^2*(B*c - 2*A*d
) + a*d^2*(3*B*c - A*d))*(c + d*x)^2*(a^2 + 5*a*b*x^2 + 4*b^2*x^4 - 3*a*Sq
rt[b]*x*Sqrt[a + b*x^2] - 4*b^(3/2)*x^3*Sqrt[a + b*x^2])*ArcTan[(-(Sqrt[b]
*(c + d*x)) + d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/((-(b*c^2) - a*d
^2)^(7/2)*(c + d*x)^2*Sqrt[a + b*x^2]*(a*(-3*Sqrt[b]*x + Sqrt[a + b*x^2])
+ 4*b*x^2*(-(Sqrt[b]*x) + Sqrt[a + b*x^2])))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {686, 27, 688, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}(c+dx)^3}dx$$

$$\Big\downarrow \text{686}$$

$$-\frac{\int \frac{bd(3a(Bc-Ad)-2(Abc+aBd)x)}{(c+dx)^3\sqrt{bx^2+a}}dx}{ab\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{3a(Bc-Ad)-2(Abc+aBd)x}{(c+dx)^3\sqrt{bx^2+a}}dx}{a\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{688}$$

$$-\frac{d\left(-\frac{\int \frac{2a\left(2aBd^2-bc(3Bc-5Ad)\right)+b\left(2Abc^2+5aBdc-3aAd^2\right)x}{(c+dx)^2\sqrt{bx^2+a}}dx}{2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(-3aAd^2+5aBcd+2Abc^2)}{2(c+dx)^2(ad^2+bc^2)}\right)}{a\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{679}$$

$$-\frac{d\left(-\frac{\frac{\sqrt{a+bx^2}\left(Abc\left(2bc^2-13ad^2\right)+aBd\left(11bc^2-4ad^2\right)\right)}{(c+dx)\left(ad^2+bc^2\right)}-\frac{3ab\left(2bc^2(Bc-2Ad)-ad^2(3Bc-Ad)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}}{2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(-3aAd^2+5aBcd+2A}{2(c+dx)^2(ad^2+bc^2)}\right.}{a\left(ad^2+bc^2\right)}$$
$$\frac{a(Bc-Ad)-x(aBd+Abc)}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{488}$$

$$-\frac{d\left(-\frac{\frac{3ab\left(2bc^2(Bc-2Ad)-ad^2(3Bc-Ad)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}+\frac{\sqrt{a+bx^2}\left(Abc\left(2bc^2-13ad^2\right)+aBd\left(11bc^2-4ad^2\right)\right)}{(c+dx)\left(ad^2+bc^2\right)}}{2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(-3aAd}{2(c+dx)^2}\right.}{a\left(ad^2+bc^2\right)}$$

$$\frac{a(Bc-Ad)-x(aBd+Abc)}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$\downarrow$ 219

$$-\frac{d\left(-\frac{\frac{3ab\left(2bc^2(Bc-2Ad)-ad^2(3Bc-Ad)\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\left(ad^2+bc^2\right)^{3/2}}+\frac{\sqrt{a+bx^2}\left(Abc\left(2bc^2-13ad^2\right)+aBd\left(11bc^2-4ad^2\right)\right)}{(c+dx)\left(ad^2+bc^2\right)}}{2(ad^2+bc^2)}-\frac{\sqrt{a+bx^2}(-3aAd^2}{2(c+dx)^2(}\right.}{a\left(ad^2+bc^2\right)}$$

$$\frac{a(Bc-Ad)-x(aBd+Abc)}{a\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}$$

input
```
Int[(A + B*x)/((c + d*x)^3*(a + b*x^2)^(3/2)),x]
```

output
```
-((a*(B*c - A*d) - (A*b*c + a*B*d)*x)/(a*(b*c^2 + a*d^2)*(c + d*x)^2*Sqrt[
a + b*x^2])) - (d*(-1/2*((2*A*b*c^2 + 5*a*B*c*d - 3*a*A*d^2)*Sqrt[a + b*x^
2])/((b*c^2 + a*d^2)*(c + d*x)^2) - (((A*b*c*(2*b*c^2 - 13*a*d^2) + a*B*d*
(11*b*c^2 - 4*a*d^2))*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)) + (3*a*
b*(2*b*c^2*(B*c - 2*A*d) - a*d^2*(3*B*c - A*d))*ArcTanh[(a*d - b*c*x)/(Sqr
t[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(3/2))/(2*(b*c^2 + a*d
^2))))/(a*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1456 vs. $2(272) = 544$.

Time = 1.38 (sec) , antiderivative size = 1457, normalized size of antiderivative = 4.99

method	result	size
default	Expression too large to display	1457

input
```
int((B*x+A)/(d*x+c)^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
B/d^3*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(1/2)+3*b*c*d/(a*d^2+b*c^2)*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*
b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-
2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d
)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*l
n((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c
/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-4*b/(a*d^2+b*c^2
)*d^2*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/
d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))+(A*d-B*c)/d^4*(-1/2/(a*d^2+
b*c^2)*d^2/(x+c/d)^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)
+5/2*b*c*d/(a*d^2+b*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)/(b*(x+c/d)^2-2*b*c/
d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+3*b*c*d/(a*d^2+b*c^2)*(1/(a*d^2+b*c^2)*
d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b
*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/
d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+
b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)
/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)
))-4*b/(a*d^2+b*c^2)*d^2*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^
2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-3/2*b/(a
*d^2+b*c^2)*d^2*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. $2(273) = 546$.

Time = 2.60 (sec) , antiderivative size = 2236, normalized size of antiderivative = 7.66

$$\int \frac{A+Bx}{(c+dx)^3 (a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((B*x+A)/(d*x+c)^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output

```
[1/4*(3*(2*B*a^2*b^2*c^5*d - 4*A*a^2*b^2*c^4*d^2 - 3*B*a^3*b*c^3*d^3 + A*a
^3*b*c^2*d^4 + (2*B*a*b^3*c^3*d^3 - 4*A*a*b^3*c^2*d^4 - 3*B*a^2*b^2*c*d^5
+ A*a^2*b^2*d^6)*x^4 + 2*(2*B*a*b^3*c^4*d^2 - 4*A*a*b^3*c^3*d^3 - 3*B*a^2*
b^2*c^2*d^4 + A*a^2*b^2*c*d^5)*x^3 + (2*B*a*b^3*c^5*d - 4*A*a*b^3*c^4*d^2
- B*a^2*b^2*c^3*d^3 - 3*A*a^2*b^2*c^2*d^4 - 3*B*a^3*b*c*d^5 + A*a^3*b*d^6)
*x^2 + 2*(2*B*a^2*b^2*c^4*d^2 - 4*A*a^2*b^2*c^3*d^3 - 3*B*a^3*b*c^2*d^4 +
A*a^3*b*c*d^5)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d
^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(
b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(2*B*a*b^3*c^7 - 6*A*a*b^3*c^6*
d - 10*B*a^2*b^2*c^5*d^2 + 4*A*a^2*b^2*c^4*d^3 - 11*B*a^3*b*c^3*d^4 + 11*A
*a^3*b*c^2*d^5 + B*a^4*c*d^6 + A*a^4*d^7 - (2*A*b^4*c^5*d^2 + 11*B*a*b^3*c
^4*d^3 - 11*A*a*b^3*c^3*d^4 + 7*B*a^2*b^2*c^2*d^5 - 13*A*a^2*b^2*c*d^6 - 4
*B*a^3*b*d^7)*x^3 - (4*A*b^4*c^6*d + 16*B*a*b^3*c^5*d^2 - 10*A*a*b^3*c^4*d
^3 + 17*B*a^2*b^2*c^3*d^4 - 17*A*a^2*b^2*c^2*d^5 + B*a^3*b*c*d^6 - 3*A*a^3
*b*d^7)*x^2 - (2*A*b^4*c^7 + 2*B*a*b^3*c^6*d + 8*A*a*b^3*c^5*d^2 + 17*B*a^
2*b^2*c^4*d^3 - 5*A*a^2*b^2*c^3*d^4 + 13*B*a^3*b*c^2*d^5 - 11*A*a^3*b*c*d^
6 - 2*B*a^4*d^7)*x)*sqrt(b*x^2 + a))/(a^2*b^4*c^10 + 4*a^3*b^3*c^8*d^2 + 6
*a^4*b^2*c^6*d^4 + 4*a^5*b*c^4*d^6 + a^6*c^2*d^8 + (a*b^5*c^8*d^2 + 4*a^2*
b^4*c^6*d^4 + 6*a^3*b^3*c^4*d^6 + 4*a^4*b^2*c^2*d^8 + a^5*b*d^10)*x^4 + 2*
(a*b^5*c^9*d + 4*a^2*b^4*c^7*d^3 + 6*a^3*b^3*c^5*d^5 + 4*a^4*b^2*c^3*d^...
```

Sympy [F]

$$\int \frac{A+Bx}{(c+dx)^3 (a+bx^2)^{3/2}}\,dx = \int \frac{A+Bx}{(a+bx^2)^{\frac{3}{2}} (c+dx)^3}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)**3/(b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x)/((a + b*x**2)**(3/2)*(c + d*x)**3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1390 vs. $2(273) = 546$.

Time = 0.14 (sec) , antiderivative size = 1390, normalized size of antiderivative = 4.76

$$\int \frac{A+Bx}{(c+dx)^3 (a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-15/2*B*b^3*c^4*x/(sqrt(b*x^2 + a)*a*b^3*c^6*d + 3*sqrt(b*x^2 + a)*a^2*b^2
*c^4*d^3 + 3*sqrt(b*x^2 + a)*a^3*b*c^2*d^5 + sqrt(b*x^2 + a)*a^4*d^7) + 15
/2*A*b^3*c^3*x/(sqrt(b*x^2 + a)*a*b^3*c^6 + 3*sqrt(b*x^2 + a)*a^2*b^2*c^4*
d^2 + 3*sqrt(b*x^2 + a)*a^3*b*c^2*d^4 + sqrt(b*x^2 + a)*a^4*d^6) - 15/2*B*
b^2*c^3/(sqrt(b*x^2 + a)*b^3*c^6 + 3*sqrt(b*x^2 + a)*a*b^2*c^4*d^2 + 3*sqr
t(b*x^2 + a)*a^2*b*c^2*d^4 + sqrt(b*x^2 + a)*a^3*d^6) + 19/2*B*b^2*c^2*x/(
sqrt(b*x^2 + a)*a*b^2*c^4*d + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^3 + sqrt(b*x^2
 + a)*a^3*d^5) + 15/2*A*b^2*c^2/(sqrt(b*x^2 + a)*b^3*c^6/d + 3*sqrt(b*x^2
+ a)*a*b^2*c^4*d + 3*sqrt(b*x^2 + a)*a^2*b*c^2*d^3 + sqrt(b*x^2 + a)*a^3*d
^5) - 13/2*A*b^2*c*x/(sqrt(b*x^2 + a)*a*b^2*c^4 + 2*sqrt(b*x^2 + a)*a^2*b*
c^2*d^2 + sqrt(b*x^2 + a)*a^3*d^4) + 5/2*B*b*c^2/(sqrt(b*x^2 + a)*b^2*c^4*
d*x + 2*sqrt(b*x^2 + a)*a*b*c^2*d^3*x + sqrt(b*x^2 + a)*a^2*d^5*x + sqrt(b
*x^2 + a)*b^2*c^5 + 2*sqrt(b*x^2 + a)*a*b*c^3*d^2 + sqrt(b*x^2 + a)*a^2*c*
d^4) - 5/2*A*b*c/(sqrt(b*x^2 + a)*b^2*c^4*x + 2*sqrt(b*x^2 + a)*a*b*c^2*d^
2*x + sqrt(b*x^2 + a)*a^2*d^4*x + sqrt(b*x^2 + a)*b^2*c^5/d + 2*sqrt(b*x^2
 + a)*a*b*c^3*d + sqrt(b*x^2 + a)*a^2*c*d^3) + 9/2*B*b*c/(sqrt(b*x^2 + a)*
b^2*c^4 + 2*sqrt(b*x^2 + a)*a*b*c^2*d^2 + sqrt(b*x^2 + a)*a^2*d^4) - 2*B*b
*x/(sqrt(b*x^2 + a)*a*b*c^2*d + sqrt(b*x^2 + a)*a^2*d^3) - 3/2*A*b/(sqrt(b
*x^2 + a)*b^2*c^4/d + 2*sqrt(b*x^2 + a)*a*b*c^2*d + sqrt(b*x^2 + a)*a^2*d^
3) + 1/2*B*c/(sqrt(b*x^2 + a)*b*c^2*d^2*x^2 + sqrt(b*x^2 + a)*a*d^4*x^2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. $2(273) = 546$.

Time = 0.15 (sec) , antiderivative size = 1048, normalized size of antiderivative = 3.59

$$\int \frac{A+Bx}{(c+dx)^3(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((A*b^6*c^9 + 3*B*a*b^5*c^8*d + 8*B*a^2*b^4*c^6*d^3 - 6*A*a^2*b^4*c^5*d^4
+ 6*B*a^3*b^3*c^4*d^5 - 8*A*a^3*b^3*c^3*d^6 - 3*A*a^4*b^2*c*d^8 - B*a^5*b*
d^9)*x/(a*b^6*c^12 + 6*a^2*b^5*c^10*d^2 + 15*a^3*b^4*c^8*d^4 + 20*a^4*b^3*
c^6*d^6 + 15*a^5*b^2*c^4*d^8 + 6*a^6*b*c^2*d^10 + a^7*d^12) - (B*a*b^5*c^9
 - 3*A*a*b^5*c^8*d - 8*A*a^2*b^4*c^6*d^3 - 6*B*a^3*b^3*c^5*d^4 - 6*A*a^3*b
^3*c^4*d^5 - 8*B*a^4*b^2*c^3*d^6 - 3*B*a^5*b*c*d^8 + A*a^5*b*d^9)/(a*b^6*c
^12 + 6*a^2*b^5*c^10*d^2 + 15*a^3*b^4*c^8*d^4 + 20*a^4*b^3*c^6*d^6 + 15*a^
5*b^2*c^4*d^8 + 6*a^6*b*c^2*d^10 + a^7*d^12))/sqrt(b*x^2 + a) + 3*(2*B*b^2
*c^3*d - 4*A*b^2*c^2*d^2 - 3*B*a*b*c*d^3 + A*a*b*d^4)*arctan(((sqrt(b)*x -
 sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^3*c^6 + 3*a*b^2
*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)*sqrt(-b*c^2 - a*d^2)) + (4*(sqrt(b)*
x - sqrt(b*x^2 + a))^3*B*b^2*c^3*d^2 - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A
*b^2*c^2*d^3 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a*b*c*d^4 + (sqrt(b)*x
- sqrt(b*x^2 + a))^3*A*a*b*d^5 + 10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*b^(5
/2)*c^4*d - 14*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*b^(5/2)*c^3*d^2 - 9*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*B*a*b^(3/2)*c^2*d^3 + 7*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*A*a*b^(3/2)*c*d^4 + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(
b)*d^5 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a*b^2*c^3*d^2 + 22*(sqrt(b)*x
- sqrt(b*x^2 + a))*A*a*b^2*c^2*d^3 + 5*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^2
*b*c*d^4 + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a^2*b*d^5 + 5*B*a^2*b^(3/2)*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)^3 (a+bx^2)^{3/2}}\,dx = \int \frac{A+B\,x}{(b\,x^2+a)^{3/2}\,(c+d\,x)^3}\,dx$$

input
```
int((A + B*x)/((a + b*x^2)^(3/2)*(c + d*x)^3),x)
```

output
```
int((A + B*x)/((a + b*x^2)^(3/2)*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 2940, normalized size of antiderivative = 10.07

$$\int \frac{A+Bx}{(c+dx)^3\left(a+bx^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((B*x+A)/(d*x+c)^3/(b*x^2+a)^(3/2),x)
```

output
```
(3*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a**3*b*c**2*d**4 + 6*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b*c*d**5*x + 3*sqrt(a*d**2
+ b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**
3*b*d**6*x**2 - 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d*
*2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**4*d**2 - 24*sqrt(a*d**2 + b*c**2)
*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*
*3*d**3*x - 9*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 +
b*c**2) - a*d + b*c*x)*a**2*b**2*c**3*d**3 - 9*sqrt(a*d**2 + b*c**2)*log(
- sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**2*d**
4*x**2 - 18*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*
c**2) - a*d + b*c*x)*a**2*b**2*c**2*d**4*x + 6*sqrt(a*d**2 + b*c**2)*log(
- sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*d**5*x
**3 - 9*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2
) - a*d + b*c*x)*a**2*b**2*c*d**5*x**2 + 3*sqrt(a*d**2 + b*c**2)*log( - sq
rt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*d**6*x**4 +
6*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*
d + b*c*x)*a*b**3*c**5*d - 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2
)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**4*d**2*x**2 + 12*sqrt(a*d
**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c...
```

3.186 $\int \frac{(A+Bx)(c+dx)^5}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 299

$$\int \frac{(A+Bx)(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = -\frac{(c+dx)^4(a(Bc+Ad)-(Abc-aBd)x)}{3ab\,(a+bx^2)^{3/2}} - \frac{(c+dx)^2\left(2a^2d^2(5Bc+2Ad)-\left(5aBd(bc^2-ad^2)+2Abc(bc^2+3ad^2)\right)x\right)}{3a^2b^2\sqrt{a+bx^2}} - \frac{d\left(4\left(5aBcd(bc^2-4ad^2)+2A(b^2c^4+4abc^2d^2-2a^2d^4)\right)+d\left(5aBd(2bc^2-3ad^2)+2Abc(2bc^2+7ad^2)\right)x\right)}{6a^2b^3} - \frac{5d^3(aBd^2-2bc(2Bc+Ad))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}$$

output

```
-1/3*(d*x+c)^4*(a*(A*d+B*c)-(A*b*c-B*a*d)*x)/a/b/(b*x^2+a)^(3/2)-1/3*(d*x+
c)^2*(2*a^2*d^2*(2*A*d+5*B*c)-(5*a*B*d*(-a*d^2+b*c^2)+2*A*b*c*(3*a*d^2+b*c
^2))*x)/a^2/b^2/(b*x^2+a)^(1/2)-1/6*d*(20*a*B*c*d*(-4*a*d^2+b*c^2)+8*A*(-2
*a^2*d^4+4*a*b*c^2*d^2+b^2*c^4)+d*(5*a*B*d*(-3*a*d^2+2*b*c^2)+2*A*b*c*(7*a
*d^2+2*b*c^2))*x)*(b*x^2+a)^(1/2)/a^2/b^3-5/2*d^3*(a*B*d^2-2*b*c*(A*d+2*B*
c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 2.38 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = \frac{4Ab^4c^5x^3+a^4d^4(80Bc+16Ad+15Bdx)+2ab^3c^3x(3Ac^2+5Bcdx^2+10Ad^2}{}$$
$$+\frac{5d^3(aBd^2-2bc(2Bc+Ad))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{7/2}}$$

input
```
Integrate[((A + B*x)*(c + d*x)^5)/(a + b*x^2)^(5/2),x]
```

output
```
(4*A*b^4*c^5*x^3 + a^4*d^4*(80*B*c + 16*A*d + 15*B*d*x) + 2*a*b^3*c^3*x*(3
*A*c^2 + 5*B*c*d*x^2 + 10*A*d^2*x^2) - 2*a^3*b*d^2*(A*d*(20*c^2 + 15*c*d*x
 - 12*d^2*x^2) + 10*B*(2*c^3 + 3*c^2*d*x - 6*c*d^2*x^2 - d^3*x^3)) - a^2*b
^2*(2*A*d*(5*c^4 + 30*c^2*d^2*x^2 + 20*c*d^3*x^3 - 3*d^4*x^4) + B*(2*c^5 +
 60*c^3*d^2*x^2 + 80*c^2*d^3*x^3 - 30*c*d^4*x^4 - 3*d^5*x^5)))/(6*a^2*b^3*
(a + b*x^2)^(3/2)) + (5*d^3*(a*B*d^2 - 2*b*c*(2*B*c + A*d))*Log[-(Sqrt[b]*
x) + Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {684, 684, 25, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(c+dx)^5}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 684$$

$$\frac{\int \frac{(c+dx)^3(2Abc^2+ad(5Bc+4Ad)-d(2Abc-5aBd)x)}{(bx^2+a)^{3/2}}dx}{3ab}-\frac{(c+dx)^4(a(Ad+Bc)-x(Abc-aBd))}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 684$$

$$\frac{\frac{\int -\frac{d(c+dx)\left(ad\left(2Abc^2-25aBdc-8aAd^2\right)+\left(5aBd\left(2bc^2-3ad^2\right)+2Abc\left(2bc^2+7ad^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2\left(2a^2d^2(2Ad+5Bc)-x\left(2Abc\left(3ad^2+bc^2\right)+5aBd(\right.\right.}{ab\sqrt{a+bx^2}}}{3ab}$$

$$\frac{(c+dx)^4(a(Ad+Bc)-x(Abc-aBd))}{3ab\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 25

$$\frac{-\frac{\int \frac{d(c+dx)\left(ad\left(2Abc^2-25aBdc-8aAd^2\right)+\left(5aBd\left(2bc^2-3ad^2\right)+2Abc\left(2bc^2+7ad^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2\left(2a^2d^2(2Ad+5Bc)-x\left(2Abc\left(3ad^2+bc^2\right)+5aBd\right.\right.}{ab\sqrt{a+bx^2}}}{3ab}$$

$$\frac{(c+dx)^4(a(Ad+Bc)-x(Abc-aBd))}{3ab\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{d\int \frac{(c+dx)\left(ad\left(2Abc^2-25aBdc-8aAd^2\right)+\left(5aBd\left(2bc^2-3ad^2\right)+2Abc\left(2bc^2+7ad^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2\left(2a^2d^2(2Ad+5Bc)-x\left(2Abc\left(3ad^2+bc^2\right)+5aBd\right.\right.}{ab\sqrt{a+bx^2}}}{3ab}$$

$$\frac{(c+dx)^4(a(Ad+Bc)-x(Abc-aBd))}{3ab\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 676

$$\frac{-\frac{d\left(\frac{15a^2d^2\left(aBd^2-2bc(Ad+2Bc)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{2\sqrt{a+bx^2}\left(2A\left(-2a^2d^4+4abc^2d^2+b^2c^4\right)+5aBcd\left(bc^2-4ad^2\right)\right)}{b}+\frac{dx\sqrt{a+bx^2}\left(2Abc\left(7ad^2+2bc^2\right)+5aBd\left(2\right.\right.}{2b}\right.}{ab}}{3ab}$$

$$\frac{(c+dx)^4(a(Ad+Bc)-x(Abc-aBd))}{3ab\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 224

$$\frac{-\frac{d\left(\frac{15a^2d^2\left(aBd^2-2bc(Ad+2Bc)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{2\sqrt{a+bx^2}\left(2A\left(-2a^2d^4+4abc^2d^2+b^2c^4\right)+5aBcd\left(bc^2-4ad^2\right)\right)}{b}+\frac{dx\sqrt{a+bx^2}\left(2Abc\left(7ad^2+2bc^2\right)\right.}{2b}\right.}{ab}}{3ab}$$

$$\frac{(c+dx)^4(a(Ad+Bc)-x(Abc-aBd))}{3ab\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 219

$$-\frac{-\frac{d\left(\frac{15a^2d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aBd^2-2bc(Ad+2Bc)\right)}{2b^{3/2}}+\frac{2\sqrt{a+bx^2}\left(2A\left(-2a^2d^4+4abc^2d^2+b^2c^4\right)+5aBcd\left(bc^2-4ad^2\right)\right)}{b}+\frac{dx\sqrt{a+bx^2}\left(2Abc\left(7ad^2+2bc^2\right)}{2b}}{ab}}{3ab}$$

$$\frac{(c+dx)^4(a(Ad+Bc)-x(Abc-aBd))}{3ab\left(a+bx^2\right)^{3/2}}$$

input
```
Int[((A + B*x)*(c + d*x)^5)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*((c + d*x)^4*(a*(B*c + A*d) - (A*b*c - a*B*d)*x))/(a*b*(a + b*x^2)^(3
/2)) + (-(((c + d*x)^2*(2*a^2*d^2*(5*B*c + 2*A*d) - (5*a*B*d*(b*c^2 - a*d^
2) + 2*A*b*c*(b*c^2 + 3*a*d^2))*x))/(a*b*Sqrt[a + b*x^2])) - (d*((2*(5*a*B
*c*d*(b*c^2 - 4*a*d^2) + 2*A*(b^2*c^4 + 4*a*b*c^2*d^2 - 2*a^2*d^4))*Sqrt[a
 + b*x^2])/b + (d*(5*a*B*d*(2*b*c^2 - 3*a*d^2) + 2*A*b*c*(2*b*c^2 + 7*a*d^
2))*x*Sqrt[a + b*x^2])/(2*b) + (15*a^2*d^2*(a*B*d^2 - 2*b*c*(2*B*c + A*d))
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(a*b))/(3*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.35

method	result
default	$A\,c^5\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+d^4(Ad+5Bc)\left(\frac{x^4}{b(b\,x^2+a)^{\frac{3}{2}}}-\frac{4a\left(-\frac{x^2}{b\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^2\left(b\,x^2+a\right)^{\frac{3}{2}}}\right)}{b}\right)+5$
risch	Expression too large to display

input
```
int((B*x+A)*(d*x+c)^5/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*c^5*(1/3*x/a/(b*x^2+a)^(3/2)+2/3/a^2/(b*x^2+a)^(1/2)*x)+d^4*(A*d+5*B*c)*
(x^4/b/(b*x^2+a)^(3/2)-4*a/b*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(
3/2)))+5*c*d^3*(A*d+2*B*c)*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)
^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+10*c^2*d^2*(A*d+B*c)*(-x^
2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2))+5*d*c^3*(2*A*d+B*c)*(-1/2*x
/b/(b*x^2+a)^(3/2)+1/2*a/b*(1/3*x/a/(b*x^2+a)^(3/2)+2/3/a^2/(b*x^2+a)^(1/2
)*x))-1/3*c^4*(5*A*d+B*c)/b/(b*x^2+a)^(3/2)+B*d^5*(1/2*x^5/b/(b*x^2+a)^(3/
2)-5/2*a/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)
*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1020, normalized size of antiderivative = 3.41

$$\int \frac{(A+Bx)(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(d*x+c)^5/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(15*(4*B*a^4*b*c^2*d^3 + 2*A*a^4*b*c*d^4 - B*a^5*d^5 + (4*B*a^2*b^3
*c^2*d^3 + 2*A*a^2*b^3*c*d^4 - B*a^3*b^2*d^5)*x^4 + 2*(4*B*a^3*b^2*c^2*d^3
 + 2*A*a^3*b^2*c*d^4 - B*a^4*b*d^5)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x
^2 + a)*sqrt(b)*x - a) - 2*(3*B*a^2*b^3*d^5*x^5 - 2*B*a^2*b^3*c^5 - 10*A*a
^2*b^3*c^4*d - 40*B*a^3*b^2*c^3*d^2 - 40*A*a^3*b^2*c^2*d^3 + 80*B*a^4*b*c*
d^4 + 16*A*a^4*b*d^5 + 6*(5*B*a^2*b^3*c*d^4 + A*a^2*b^3*d^5)*x^4 + 2*(2*A*
b^5*c^5 + 5*B*a*b^4*c^4*d + 10*A*a*b^4*c^3*d^2 - 40*B*a^2*b^3*c^2*d^3 - 20
*A*a^2*b^3*c*d^4 + 10*B*a^3*b^2*d^5)*x^3 - 12*(5*B*a^2*b^3*c^3*d^2 + 5*A*a
^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 - 2*A*a^3*b^2*d^5)*x^2 + 3*(2*A*a*b^4*
c^5 - 20*B*a^3*b^2*c^2*d^3 - 10*A*a^3*b^2*c*d^4 + 5*B*a^4*b*d^5)*x)*sqrt(b
*x^2 + a))/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4), -1/6*(15*(4*B*a^4*b*c^
2*d^3 + 2*A*a^4*b*c*d^4 - B*a^5*d^5 + (4*B*a^2*b^3*c^2*d^3 + 2*A*a^2*b^3*c
*d^4 - B*a^3*b^2*d^5)*x^4 + 2*(4*B*a^3*b^2*c^2*d^3 + 2*A*a^3*b^2*c*d^4 - B
*a^4*b*d^5)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*B*a^2*b^
3*d^5*x^5 - 2*B*a^2*b^3*c^5 - 10*A*a^2*b^3*c^4*d - 40*B*a^3*b^2*c^3*d^2 -
40*A*a^3*b^2*c^2*d^3 + 80*B*a^4*b*c*d^4 + 16*A*a^4*b*d^5 + 6*(5*B*a^2*b^3*
c*d^4 + A*a^2*b^3*d^5)*x^4 + 2*(2*A*b^5*c^5 + 5*B*a*b^4*c^4*d + 10*A*a*b^4
*c^3*d^2 - 40*B*a^2*b^3*c^2*d^3 - 20*A*a^2*b^3*c*d^4 + 10*B*a^3*b^2*d^5)*x
^3 - 12*(5*B*a^2*b^3*c^3*d^2 + 5*A*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 -
2*A*a^3*b^2*d^5)*x^2 + 3*(2*A*a*b^4*c^5 - 20*B*a^3*b^2*c^2*d^3 - 10*A*a...
```

Sympy [F]

$$\int \frac{(A+Bx)(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = \int \frac{(A+Bx)\,(c+dx)^5}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(d*x+c)**5/(b*x**2+a)**(5/2),x)
```

output
```
Integral((A + B*x)*(c + d*x)**5/(a + b*x**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.72

$$\int \frac{(A+Bx)(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = \frac{Bd^5x^5}{2\,(bx^2+a)^{\frac{3}{2}}b} + \frac{5\,Bad^5x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{6\,b} + \frac{2\,Ac^5x}{3\,\sqrt{bx^2+a}a^2} + \frac{Ac^5x}{3\,(bx^2+a)^{\frac{3}{2}}a} + \frac{5\,Bad^5x}{6\,\sqrt{bx^2+a}b^3} - \frac{5\,Bad^5\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}} - \frac{Bc^5}{3\,(bx^2+a)^{\frac{3}{2}}b} - \frac{5\,Ac^4d}{3\,(bx^2+a)^{\frac{3}{2}}b} - \frac{5}{3}\,(2\,Bc^2d^3+Acd^4)x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) + \frac{(5\,Bcd^4+Ad^5)x^4}{(bx^2+a)^{\frac{3}{2}}b} + \frac{4\,(5\,Bcd^4+Ad^5)ax^2}{(bx^2+a)^{\frac{3}{2}}b^2} - \frac{10\,(Bc^3d^2+Ac^2d^3)x^2}{(bx^2+a)^{\frac{3}{2}}b} - \frac{5\,(2\,Bc^2d^3+Acd^4)x}{3\,\sqrt{bx^2+a}b^2} - \frac{5\,(Bc^4d+2\,Ac^3d^2)x}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{5\,(Bc^4d+2\,Ac^3d^2)x}{3\,\sqrt{bx^2+a}ab} + \frac{5\,(2\,Bc^2d^3+Acd^4)\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}} + \frac{8\,(5\,Bcd^4+Ad^5)a^2}{3\,(bx^2+a)^{\frac{3}{2}}b^3} - \frac{20\,(Bc^3d^2+Ac^2d^3)a}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

input

```
integrate((B*x+A)*(d*x+c)^5/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/2*B*d^5*x^5/((b*x^2 + a)^(3/2)*b) + 5/6*B*a*d^5*x*(3*x^2/((b*x^2 + a)^(3
/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b + 2/3*A*c^5*x/(sqrt(b*x^2 + a)*a^2
) + 1/3*A*c^5*x/((b*x^2 + a)^(3/2)*a) + 5/6*B*a*d^5*x/(sqrt(b*x^2 + a)*b^3
) - 5/2*B*a*d^5*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 1/3*B*c^5/((b*x^2 + a)^(3
/2)*b) - 5/3*A*c^4*d/((b*x^2 + a)^(3/2)*b) - 5/3*(2*B*c^2*d^3 + A*c*d^4)*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) + (5*B*c*d^4
+ A*d^5)*x^4/((b*x^2 + a)^(3/2)*b) + 4*(5*B*c*d^4 + A*d^5)*a*x^2/((b*x^2 +
 a)^(3/2)*b^2) - 10*(B*c^3*d^2 + A*c^2*d^3)*x^2/((b*x^2 + a)^(3/2)*b) - 5/
3*(2*B*c^2*d^3 + A*c*d^4)*x/(sqrt(b*x^2 + a)*b^2) - 5/3*(B*c^4*d + 2*A*c^3
*d^2)*x/((b*x^2 + a)^(3/2)*b) + 5/3*(B*c^4*d + 2*A*c^3*d^2)*x/(sqrt(b*x^2
+ a)*a*b) + 5*(2*B*c^2*d^3 + A*c*d^4)*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 8/3
*(5*B*c*d^4 + A*d^5)*a^2/((b*x^2 + a)^(3/2)*b^3) - 20/3*(B*c^3*d^2 + A*c^2
*d^3)*a/((b*x^2 + a)^(3/2)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx)(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(\left(3\left(\frac{Bd^5x}{b}+\frac{2\,(5\,Ba^2b^5cd^4+Aa^2b^5d^5)}{a^2b^6}\right)x+\frac{2\,(2\,Ab^7c^5+5\,Bab^6c^4d+10\,Aab^6c^3d^2-40\,Ba^2b^5c^2d^3}{a^2b^6}\right.\right.\right.}{} - \frac{5\,(4\,Bbc^2d^3+2\,Abcd^4-Bad^5)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^5/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/6*((((3*(B*d^5*x/b + 2*(5*B*a^2*b^5*c*d^4 + A*a^2*b^5*d^5)/(a^2*b^6))*x
+ 2*(2*A*b^7*c^5 + 5*B*a*b^6*c^4*d + 10*A*a*b^6*c^3*d^2 - 40*B*a^2*b^5*c^2
*d^3 - 20*A*a^2*b^5*c*d^4 + 10*B*a^3*b^4*d^5)/(a^2*b^6))*x - 12*(5*B*a^2*b
^5*c^3*d^2 + 5*A*a^2*b^5*c^2*d^3 - 10*B*a^3*b^4*c*d^4 - 2*A*a^3*b^4*d^5)/(
a^2*b^6))*x + 3*(2*A*a*b^6*c^5 - 20*B*a^3*b^4*c^2*d^3 - 10*A*a^3*b^4*c*d^4
 + 5*B*a^4*b^3*d^5)/(a^2*b^6))*x - 2*(B*a^2*b^5*c^5 + 5*A*a^2*b^5*c^4*d +
20*B*a^3*b^4*c^3*d^2 + 20*A*a^3*b^4*c^2*d^3 - 40*B*a^4*b^3*c*d^4 - 8*A*a^4
*b^3*d^5)/(a^2*b^6))/(b*x^2 + a)^(3/2) - 5/2*(4*B*b*c^2*d^3 + 2*A*b*c*d^4
- B*a*d^5)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = \int \frac{(A+B\,x)\,\left(c+d\,x\right)^5}{\left(b\,x^2+a\right)^{5/2}}\,dx$$

input
```
int(((A + B*x)*(c + d*x)^5)/(a + b*x^2)^(5/2),x)
```

output
```
int(((A + B*x)*(c + d*x)^5)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)(c+dx)^5}{(a+bx^2)^{5/2}}\,dx = \int \frac{(Bx+A)\,(dx+c)^5}{(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((B*x+A)*(d*x+c)^5/(b*x^2+a)^(5/2),x)
```

output
```
int((B*x+A)*(d*x+c)^5/(b*x^2+a)^(5/2),x)
```

3.187 $\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 232

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{5/2}}\,dx = -\frac{(c+dx)^3(a(Bc+Ad)-(Abc-aBd)x)}{3ab\,(a+bx^2)^{3/2}} - \frac{(c+dx)\,(ad(Abc^2+8aBcd+3aAd^2)-2(2aBd(bc^2-ad^2)+Abc(bc^2+2ad^2))\,x)}{3a^2b^2\sqrt{a+bx^2}} - \frac{d(4aBd(bc^2-2ad^2)+Abc(2bc^2+5ad^2))\,\sqrt{a+bx^2}}{3a^2b^3} + \frac{d^3(4Bc+Ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

output
```
-1/3*(d*x+c)^3*(a*(A*d+B*c)-(A*b*c-B*a*d)*x)/a/b/(b*x^2+a)^(3/2)-1/3*(d*x+
c)*(a*d*(3*A*a*d^2+A*b*c^2+8*B*a*c*d)-2*(2*a*B*d*(-a*d^2+b*c^2)+A*b*c*(2*a
*d^2+b*c^2))*x)/a^2/b^2/(b*x^2+a)^(1/2)-1/3*d*(4*a*B*d*(-2*a*d^2+b*c^2)+A*
b*c*(5*a*d^2+2*b*c^2))*(b*x^2+a)^(1/2)/a^2/b^3+d^3*(A*d+4*B*c)*arctanh(b^(
1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{5/2}}\,dx = \frac{8a^4Bd^4+2Ab^4c^4x^3-a^3bd^2(Ad(8c+3dx)+12B(c^2+cdx-d^2x^2))+ab^3c^2x}{}$$

input
```
Integrate[((A + B*x)*(c + d*x)^4)/(a + b*x^2)^(5/2),x]
```

output
```
(8*a^4*B*d^4 + 2*A*b^4*c^4*x^3 - a^3*b*d^2*(A*d*(8*c + 3*d*x) + 12*B*(c^2
+ c*d*x - d^2*x^2)) + a*b^3*c^2*x*(4*B*c*d*x^2 + 3*A*(c^2 + 2*d^2*x^2)) -
a^2*b^2*(4*A*d*(c^3 + 3*c*d^2*x^2 + d^3*x^3) + B*(c^4 + 18*c^2*d^2*x^2 + 1
6*c*d^3*x^3 - 3*d^4*x^4)) - 3*a^2*Sqrt[b]*d^3*(4*B*c + A*d)*(a + b*x^2)^(3
/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(3*a^2*b^3*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {684, 684, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{684}$$

$$\frac{\int \frac{(c+dx)^2(2Abc^2+ad(4Bc+3Ad)-d(Abc-4aBd)x)}{(bx^2+a)^{3/2}}dx}{3ab}-\frac{(c+dx)^3(a(Ad+Bc)-x(Abc-aBd))}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{684}$$

$$\frac{\frac{\int \frac{d\left(3a^2d^2(4Bc+Ad)-\left(4aBd\left(bc^2-2ad^2\right)+Abc\left(2bc^2+5ad^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)\left(ad\left(3aAd^2+8aBcd+Abc^2\right)-2x\left(Abc\left(2ad^2+bc^2\right)+2aBd\left(bc^2-ad^2\right)\right)\right)}{ab\sqrt{a+bx^2}}}{3ab} -$$

$$\frac{(c+dx)^3(a(Ad+Bc)-x(Abc-aBd))}{3ab\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{d\int \frac{3a^2d^2(4Bc+Ad)-\left(4aBd\left(bc^2-2ad^2\right)+Abc\left(2bc^2+5ad^2\right)\right)x}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)\left(ad\left(3aAd^2+8aBcd+Abc^2\right)-2x\left(Abc\left(2ad^2+bc^2\right)+2aBd\left(bc^2-ad^2\right)\right)\right)}{ab\sqrt{a+bx^2}}}{3ab} -$$

$$\frac{(c+dx)^3(a(Ad+Bc)-x(Abc-aBd))}{3ab\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 455

$$\frac{\frac{d\left(3a^2d^2(Ad+4Bc)\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(Abc\left(5ad^2+2bc^2\right)+4aBd\left(bc^2-2ad^2\right)\right)}{b}\right)}{ab}-\frac{(c+dx)\left(ad\left(3aAd^2+8aBcd+Abc^2\right)-2x\left(Abc\left(2ad^2+bc^2\right)+2\right.\right.}{ab\sqrt{a+bx^2}}}{3ab}$$

$$\frac{(c+dx)^3(a(Ad+Bc)-x(Abc-aBd))}{3ab\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 224

$$\frac{\frac{d\left(3a^2d^2(Ad+4Bc)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}\left(Abc\left(5ad^2+2bc^2\right)+4aBd\left(bc^2-2ad^2\right)\right)}{b}\right)}{ab}-\frac{(c+dx)\left(ad\left(3aAd^2+8aBcd+Abc^2\right)-2x\left(Abc\left(2ad^2\right.\right.\right.}{ab\sqrt{a+bx^2}}}{3ab}$$

$$\frac{(c+dx)^3(a(Ad+Bc)-x(Abc-aBd))}{3ab\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{d\left(\frac{3a^2d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Ad+4Bc)}{\sqrt{b}}-\frac{\sqrt{a+bx^2}\left(Abc\left(5ad^2+2bc^2\right)+4aBd\left(bc^2-2ad^2\right)\right)}{b}\right)}{ab}-\frac{(c+dx)\left(ad\left(3aAd^2+8aBcd+Abc^2\right)-2x\left(Abc\left(2ad^2+\right.\right.\right.}{ab\sqrt{a+bx^2}}}{3ab}$$

$$\frac{(c+dx)^3(a(Ad+Bc)-x(Abc-aBd))}{3ab\left(a+bx^2\right)^{3/2}}$$

input

```
Int[((A + B*x)*(c + d*x)^4)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*((c + d*x)^3*(a*(B*c + A*d) - (A*b*c - a*B*d)*x))/(a*b*(a + b*x^2)^(3
/2)) + (-(((c + d*x)*(a*d*(A*b*c^2 + 8*a*B*c*d + 3*a*A*d^2) - 2*(2*a*B*d*(
b*c^2 - a*d^2) + A*b*c*(b*c^2 + 2*a*d^2))*x))/(a*b*Sqrt[a + b*x^2])) + (d*
(-(((4*a*B*d*(b*c^2 - 2*a*d^2) + A*b*c*(2*b*c^2 + 5*a*d^2))*Sqrt[a + b*x^2
])/b) + (3*a^2*d^2*(4*B*c + A*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqr
t[b]))/(a*b))/(3*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.34

method	result
default	$A\,c^4\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+d^3(Ad+4Bc)\left(-\frac{x^3}{3b(b\,x^2+a)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}\right)+2c$
risch	$\frac{B\,d^4\sqrt{b\,x^2+a}}{b^3}+\frac{\frac{d^3(Ad+4Bc)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{\left(4A\sqrt{-ab}\,abc\,d^3-4A\sqrt{-ab}\,b^2c^3d+A\,a^2b\,d^4-6Aa\,b^2c^2d^2+A\,b^3c^4-B\sqrt{-ab}\,a^2d^4+6B\sqrt{-}\right.}{}}{}$

input
```
int((B*x+A)*(d*x+c)^4/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*c^4*(1/3*x/a/(b*x^2+a)^(3/2)+2/3/a^2/(b*x^2+a)^(1/2)*x)+d^3*(A*d+4*B*c)*
(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)
*x+(b*x^2+a)^(1/2))))+2*c*d^2*(2*A*d+3*B*c)*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/
b^2/(b*x^2+a)^(3/2))+2*c^2*d*(3*A*d+2*B*c)*(-1/2*x/b/(b*x^2+a)^(3/2)+1/2*a
/b*(1/3*x/a/(b*x^2+a)^(3/2)+2/3/a^2/(b*x^2+a)^(1/2)*x))-1/3*c^3*(4*A*d+B*c
)/b/(b*x^2+a)^(3/2)+B*d^4*(x^4/b/(b*x^2+a)^(3/2)-4*a/b*(-x^2/b/(b*x^2+a)^(
3/2)-2/3*a/b^2/(b*x^2+a)^(3/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 724, normalized size of antiderivative = 3.12

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{5/2}}\,dx = \left[\frac{3\left(4\,Ba^4cd^3+Aa^4d^4+\left(4\,Ba^2b^2cd^3+Aa^2b^2d^4\right)x^4+2\left(4\,Ba^3bcd^3+Aa^3bd^4\right)x\right.}{}\right.$$

$$-\frac{3\left(4\,Ba^4cd^3+Aa^4d^4+\left(4\,Ba^2b^2cd^3+Aa^2b^2d^4\right)x^4+2\left(4\,Ba^3bcd^3+Aa^3bd^4\right)x^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-}{}$$

input
```
integrate((B*x+A)*(d*x+c)^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(4*B*a^4*c*d^3 + A*a^4*d^4 + (4*B*a^2*b^2*c*d^3 + A*a^2*b^2*d^4)*x
^4 + 2*(4*B*a^3*b*c*d^3 + A*a^3*b*d^4)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(b)*x - a) + 2*(3*B*a^2*b^2*d^4*x^4 - B*a^2*b^2*c^4 - 4*A*a
^2*b^2*c^3*d - 12*B*a^3*b*c^2*d^2 - 8*A*a^3*b*c*d^3 + 8*B*a^4*d^4 + 2*(A*b
^4*c^4 + 2*B*a*b^3*c^3*d + 3*A*a*b^3*c^2*d^2 - 8*B*a^2*b^2*c*d^3 - 2*A*a^2
*b^2*d^4)*x^3 - 6*(3*B*a^2*b^2*c^2*d^2 + 2*A*a^2*b^2*c*d^3 - 2*B*a^3*b*d^4
)*x^2 + 3*(A*a*b^3*c^4 - 4*B*a^3*b*c*d^3 - A*a^3*b*d^4)*x)*sqrt(b*x^2 + a)
)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3), -1/3*(3*(4*B*a^4*c*d^3 + A*a^4*
d^4 + (4*B*a^2*b^2*c*d^3 + A*a^2*b^2*d^4)*x^4 + 2*(4*B*a^3*b*c*d^3 + A*a^3
*b*d^4)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*B*a^2*b^2*d^
4*x^4 - B*a^2*b^2*c^4 - 4*A*a^2*b^2*c^3*d - 12*B*a^3*b*c^2*d^2 - 8*A*a^3*b
*c*d^3 + 8*B*a^4*d^4 + 2*(A*b^4*c^4 + 2*B*a*b^3*c^3*d + 3*A*a*b^3*c^2*d^2
- 8*B*a^2*b^2*c*d^3 - 2*A*a^2*b^2*d^4)*x^3 - 6*(3*B*a^2*b^2*c^2*d^2 + 2*A*
a^2*b^2*c*d^3 - 2*B*a^3*b*d^4)*x^2 + 3*(A*a*b^3*c^4 - 4*B*a^3*b*c*d^3 - A*
a^3*b*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3)]
```

Sympy [F]

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{5/2}}\,dx = \int \frac{(A+Bx)\,(c+dx)^4}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(d*x+c)**4/(b*x**2+a)**(5/2),x)
```

output
```
Integral((A + B*x)*(c + d*x)**4/(a + b*x**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.62

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{5/2}}\,dx = \frac{Bd^4x^4}{(bx^2+a)^{\frac{3}{2}}b} + \frac{4\,Bad^4x^2}{(bx^2+a)^{\frac{3}{2}}b^2} + \frac{2\,Ac^4x}{3\,\sqrt{bx^2+a}a^2} + \frac{Ac^4x}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{1}{3}\,(4\,Bcd^3+Ad^4)x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) - \frac{Bc^4}{3\,(bx^2+a)^{\frac{3}{2}}b} - \frac{4\,Ac^3d}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{8\,Ba^2d^4}{3\,(bx^2+a)^{\frac{3}{2}}b^3} - \frac{2\,(3\,Bc^2d^2+2\,Acd^3)x^2}{(bx^2+a)^{\frac{3}{2}}b} - \frac{(4\,Bcd^3+Ad^4)x}{3\,\sqrt{bx^2+a}b^2} - \frac{2\,(2\,Bc^3d+3\,Ac^2d^2)x}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,(2\,Bc^3d+3\,Ac^2d^2)x}{3\,\sqrt{bx^2+a}ab} + \frac{(4\,Bcd^3+Ad^4)\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}} - \frac{4\,(3\,Bc^2d^2+2\,Acd^3)a}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

input
```
integrate((B*x+A)*(d*x+c)^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
B*d^4*x^4/((b*x^2 + a)^(3/2)*b) + 4*B*a*d^4*x^2/((b*x^2 + a)^(3/2)*b^2) +
2/3*A*c^4*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*c^4*x/((b*x^2 + a)^(3/2)*a) - 1/
3*(4*B*c*d^3 + A*d^4)*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3
/2)*b^2)) - 1/3*B*c^4/((b*x^2 + a)^(3/2)*b) - 4/3*A*c^3*d/((b*x^2 + a)^(3/
2)*b) + 8/3*B*a^2*d^4/((b*x^2 + a)^(3/2)*b^3) - 2*(3*B*c^2*d^2 + 2*A*c*d^3
)*x^2/((b*x^2 + a)^(3/2)*b) - 1/3*(4*B*c*d^3 + A*d^4)*x/(sqrt(b*x^2 + a)*b
^2) - 2/3*(2*B*c^3*d + 3*A*c^2*d^2)*x/((b*x^2 + a)^(3/2)*b) + 2/3*(2*B*c^3
*d + 3*A*c^2*d^2)*x/(sqrt(b*x^2 + a)*a*b) + (4*B*c*d^3 + A*d^4)*arcsinh(b*
x/sqrt(a*b))/b^(5/2) - 4/3*(3*B*c^2*d^2 + 2*A*c*d^3)*a/((b*x^2 + a)^(3/2)*
b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.29

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(\left(\frac{3\,Bd^4x}{b} + \frac{2\,(Ab^6c^4+2\,Bab^5c^3d+3\,Aab^5c^2d^2-8\,Ba^2b^4cd^3-2\,Aa^2b^4d^4)}{a^2b^5}\right)x - \frac{6\,(3\,Ba^2b^4c^2d^2+2\,A}{a^2}\right.\right.}{}$$

$$-\frac{(4\,Bcd^3+Ad^4)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*((((3*B*d^4*x/b + 2*(A*b^6*c^4 + 2*B*a*b^5*c^3*d + 3*A*a*b^5*c^2*d^2 -
 8*B*a^2*b^4*c*d^3 - 2*A*a^2*b^4*d^4)/(a^2*b^5))*x - 6*(3*B*a^2*b^4*c^2*d^
2 + 2*A*a^2*b^4*c*d^3 - 2*B*a^3*b^3*d^4)/(a^2*b^5))*x + 3*(A*a*b^5*c^4 - 4
*B*a^3*b^3*c*d^3 - A*a^3*b^3*d^4)/(a^2*b^5))*x - (B*a^2*b^4*c^4 + 4*A*a^2*
b^4*c^3*d + 12*B*a^3*b^3*c^2*d^2 + 8*A*a^3*b^3*c*d^3 - 8*B*a^4*b^2*d^4)/(a
^2*b^5))/(b*x^2 + a)^(3/2) - (4*B*c*d^3 + A*d^4)*log(abs(-sqrt(b)*x + sqrt
(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{5/2}}\,dx = \int \frac{(A+B\,x)\,\,(c+d\,x)^4}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((A + B*x)*(c + d*x)^4)/(a + b*x^2)^(5/2),x)
```

output
```
int(((A + B*x)*(c + d*x)^4)/(a + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 704, normalized size of antiderivative = 3.03

$$\int \frac{(A+Bx)(c+dx)^4}{(a+bx^2)^{5/2}}\,dx = \frac{-\sqrt{b\,x^2+a}\,a\,b^3c^4+8\sqrt{b\,x^2+a}\,a^3b\,d^4+2\sqrt{b\,x^2+a}\,b^4c^4x^3+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a+}}{\sqrt{a}}\right.}{}$$

input
```
int((B*x+A)*(d*x+c)^4/(b*x^2+a)^(5/2),x)
```

output
```
( - 8*sqrt(a + b*x**2)*a**3*b*c*d**3 - 3*sqrt(a + b*x**2)*a**3*b*d**4*x +
8*sqrt(a + b*x**2)*a**3*b*d**4 - 4*sqrt(a + b*x**2)*a**2*b**2*c**3*d - 12*
sqrt(a + b*x**2)*a**2*b**2*c**2*d**2 - 12*sqrt(a + b*x**2)*a**2*b**2*c*d**
3*x**2 - 12*sqrt(a + b*x**2)*a**2*b**2*c*d**3*x - 4*sqrt(a + b*x**2)*a**2*
b**2*d**4*x**3 + 12*sqrt(a + b*x**2)*a**2*b**2*d**4*x**2 + 3*sqrt(a + b*x*
*2)*a*b**3*c**4*x - sqrt(a + b*x**2)*a*b**3*c**4 + 6*sqrt(a + b*x**2)*a*b*
*3*c**2*d**2*x**3 - 18*sqrt(a + b*x**2)*a*b**3*c**2*d**2*x**2 - 16*sqrt(a
+ b*x**2)*a*b**3*c*d**3*x**3 + 3*sqrt(a + b*x**2)*a*b**3*d**4*x**4 + 2*sqr
t(a + b*x**2)*b**4*c**4*x**3 + 4*sqrt(a + b*x**2)*b**4*c**3*d*x**3 + 3*sqr
t(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*d**4 + 12*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*c*d**3 + 6*sqrt(b)*log((s
qrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*d**4*x**2 + 24*sqrt(b)*log((s
qrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c*d**3*x**2 + 3*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*d**4*x**4 + 12*sqrt(b)
*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**3*c*d**3*x**4 + 6*sqrt(b
)*a**3*b*c**2*d**2 - 2*sqrt(b)*a**2*b**2*c**4 + 4*sqrt(b)*a**2*b**2*c**3*d
 + 12*sqrt(b)*a**2*b**2*c**2*d**2*x**2 - 4*sqrt(b)*a*b**3*c**4*x**2 + 8*sq
rt(b)*a*b**3*c**3*d*x**2 + 6*sqrt(b)*a*b**3*c**2*d**2*x**4 - 2*sqrt(b)*b**
4*c**4*x**4 + 4*sqrt(b)*b**4*c**3*d*x**4)/(3*a*b**3*(a**2 + 2*a*b*x**2 + b
**2*x**4))
```

3.188 $\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\begin{aligned}\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{5/2}}\,dx &= -\frac{(c+dx)^2(a(Bc+Ad)-(Abc-aBd)x)}{3ab\,(a+bx^2)^{3/2}} \\ &\quad - \frac{2ad(Abc^2+3aBcd+aAd^2)-(3aBd(bc^2-ad^2)+2Abc(bc^2+ad^2))\,x}{3a^2b^2\sqrt{a+bx^2}} \\ &\quad + \frac{Bd^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}\end{aligned}$$

output
```
-1/3*(d*x+c)^2*(a*(A*d+B*c)-(A*b*c-B*a*d)*x)/a/b/(b*x^2+a)^(3/2)-1/3*(2*a*
d*(A*a*d^2+A*b*c^2+3*B*a*c*d)-(3*a*B*d*(-a*d^2+b*c^2)+2*A*b*c*(a*d^2+b*c^2
))*x)/a^2/b^2/(b*x^2+a)^(1/2)+B*d^3*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(
5/2)
```

Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{5/2}}\,dx = \frac{2Ab^3c^3x^3 - a^3d^2(6Bc+2Ad+3Bdx) + 3ab^2cx(Bcdx^2 + A(c^2+d^2x^2)) - a^2b}{3a^2b^2\left(a+bx^2\right)^{3/2}}$$
$$- \frac{Bd^3\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{5/2}}$$

input
```
Integrate[((A + B*x)*(c + d*x)^3)/(a + b*x^2)^(5/2),x]
```

output
```
(2*A*b^3*c^3*x^3 - a^3*d^2*(6*B*c + 2*A*d + 3*B*d*x) + 3*a*b^2*c*x*(B*c*d*
x^2 + A*(c^2 + d^2*x^2)) - a^2*b*(3*A*d*(c^2 + d^2*x^2) + B*(c^3 + 9*c*d^2
*x^2 + 4*d^3*x^3)))/(3*a^2*b^2*(a + b*x^2)^(3/2)) - (B*d^3*Log[-(Sqrt[b]*x
) + Sqrt[a + b*x^2]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {684, 675, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 684$$

$$\frac{\int \frac{(c+dx)\left(2Abc^2+ad(3Bc+2Ad)+3aBd^2x\right)}{(bx^2+a)^{3/2}}dx}{3ab} - \frac{(c+dx)^2(a(Ad+Bc)-x(Abc-aBd))}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 675$$

$$\frac{\frac{3aBd^3\int \frac{1}{\sqrt{bx^2+a}}dx}{b}+\frac{x\left(\frac{2Abc^3}{a}-\frac{3aBd^3}{b}+cd(2Ad+3Bc)\right)}{\sqrt{a+bx^2}}-\frac{2d(aAd^2+3aBcd+Abc^2)}{b\sqrt{a+bx^2}}}{3ab}-\frac{(c+dx)^2(a(Ad+Bc)-x(Abc-aBd))}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3aBd^3\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}+\frac{x\left(\frac{2Abc^3}{a}-\frac{3aBd^3}{b}+cd(2Ad+3Bc)\right)}{\sqrt{a+bx^2}}-\frac{2d(aAd^2+3aBcd+Abc^2)}{b\sqrt{a+bx^2}}}{3ab}-\frac{(c+dx)^2(a(Ad+Bc)-x(Abc-aBd))}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{x\left(\frac{2Abc^3}{a}-\frac{3aBd^3}{b}+cd(2Ad+3Bc)\right)}{\sqrt{a+bx^2}}-\frac{2d(aAd^2+3aBcd+Abc^2)}{b\sqrt{a+bx^2}}+\frac{3aBd^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}}{3ab}-\frac{(c+dx)^2(a(Ad+Bc)-x(Abc-aBd))}{3ab\,(a+bx^2)^{3/2}}$$

input
```
Int[((A + B*x)*(c + d*x)^3)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*((c + d*x)^2*(a*(B*c + A*d) - (A*b*c - a*B*d)*x))/(a*b*(a + b*x^2)^(3
/2)) + ((-2*d*(A*b*c^2 + 3*a*B*c*d + a*A*d^2))/(b*Sqrt[a + b*x^2]) + (((2*
A*b*c^3)/a - (3*a*B*d^3)/b + c*d*(3*B*c + 2*A*d))*x)/Sqrt[a + b*x^2] + (3*
a*B*d^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2))/(3*a*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.44

method	result
default	$A c^3\left(\frac{x}{3a(b x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b x^2+a}}\right)+d^2(Ad+3Bc)\left(-\frac{x^2}{b(b x^2+a)^{\frac{3}{2}}}-\frac{2a}{3b^2(b x^2+a)^{\frac{3}{2}}}\right)+3cd(Ad+Bc)\Bigg($

input
```
int((B*x+A)*(d*x+c)^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*c^3*(1/3*x/a/(b*x^2+a)^(3/2)+2/3/a^2/(b*x^2+a)^(1/2)*x)+d^2*(A*d+3*B*c)*
(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2))+3*c*d*(A*d+B*c)*(-1/2*x
/b/(b*x^2+a)^(3/2)+1/2*a/b*(1/3*x/a/(b*x^2+a)^(3/2)+2/3/a^2/(b*x^2+a)^(1/2
)*x))-1/3*c^2*(3*A*d+B*c)/b/(b*x^2+a)^(3/2)+B*d^3*(-1/3*x^3/b/(b*x^2+a)^(3
/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.15

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{5/2}}\,dx = \left[\frac{3\left(Ba^2b^2d^3x^4+2\,Ba^3bd^3x^2+Ba^4d^3\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{}\right.$$

$$-\frac{3\left(Ba^2b^2d^3x^4+2\,Ba^3bd^3x^2+Ba^4d^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(Ba^2b^2c^3+3\,Aa^2b^2c^2d+6\,Ba^3bcd^2+2\,A\right.}{3\,(a^2}$$

input
```
integrate((B*x+A)*(d*x+c)^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(B*a^2*b^2*d^3*x^4 + 2*B*a^3*b*d^3*x^2 + B*a^4*d^3)*sqrt(b)*log(-2
*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(B*a^2*b^2*c^3 + 3*A*a^2*b^2
*c^2*d + 6*B*a^3*b*c*d^2 + 2*A*a^3*b*d^3 - (2*A*b^4*c^3 + 3*B*a*b^3*c^2*d
+ 3*A*a*b^3*c*d^2 - 4*B*a^2*b^2*d^3)*x^3 + 3*(3*B*a^2*b^2*c*d^2 + A*a^2*b^
2*d^3)*x^2 - 3*(A*a*b^3*c^3 - B*a^3*b*d^3)*x)*sqrt(b*x^2 + a))/(a^2*b^5*x^
4 + 2*a^3*b^4*x^2 + a^4*b^3), -1/3*(3*(B*a^2*b^2*d^3*x^4 + 2*B*a^3*b*d^3*x
^2 + B*a^4*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (B*a^2*b^2*c
^3 + 3*A*a^2*b^2*c^2*d + 6*B*a^3*b*c*d^2 + 2*A*a^3*b*d^3 - (2*A*b^4*c^3 +
3*B*a*b^3*c^2*d + 3*A*a*b^3*c*d^2 - 4*B*a^2*b^2*d^3)*x^3 + 3*(3*B*a^2*b^2*
c*d^2 + A*a^2*b^2*d^3)*x^2 - 3*(A*a*b^3*c^3 - B*a^3*b*d^3)*x)*sqrt(b*x^2 +
 a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3)]
```

Sympy [F]

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{5/2}}\,dx = \int \frac{(A+Bx)\,(c+dx)^3}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(d*x+c)**3/(b*x**2+a)**(5/2),x)
```

output
```
Integral((A + B*x)*(c + d*x)**3/(a + b*x**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.64

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3}\,Bd^3x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) + \frac{2\,Ac^3x}{3\,\sqrt{bx^2+a}a^2}+\frac{Ac^3x}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{Bd^3x}{3\,\sqrt{bx^2+a}b^2}+\frac{Bd^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}} - \frac{Bc^3}{3\,(bx^2+a)^{\frac{3}{2}}b}-\frac{Ac^2d}{(bx^2+a)^{\frac{3}{2}}b}-\frac{(3\,Bcd^2+Ad^3)x^2}{(bx^2+a)^{\frac{3}{2}}b} - \frac{(Bc^2d+Acd^2)x}{(bx^2+a)^{\frac{3}{2}}b}+\frac{(Bc^2d+Acd^2)x}{\sqrt{bx^2+a}ab}-\frac{2\,(3\,Bcd^2+Ad^3)a}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

input
```
integrate((B*x+A)*(d*x+c)^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*B*d^3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) +
 2/3*A*c^3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*c^3*x/((b*x^2 + a)^(3/2)*a) - 1
/3*B*d^3*x/(sqrt(b*x^2 + a)*b^2) + B*d^3*arcsinh(b*x/sqrt(a*b))/b^(5/2) -
1/3*B*c^3/((b*x^2 + a)^(3/2)*b) - A*c^2*d/((b*x^2 + a)^(3/2)*b) - (3*B*c*d
^2 + A*d^3)*x^2/((b*x^2 + a)^(3/2)*b) - (B*c^2*d + A*c*d^2)*x/((b*x^2 + a)
^(3/2)*b) + (B*c^2*d + A*c*d^2)*x/(sqrt(b*x^2 + a)*a*b) - 2/3*(3*B*c*d^2 +
 A*d^3)*a/((b*x^2 + a)^(3/2)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{5/2}}\,dx = -\frac{Bd^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}} + \frac{\left(x\left(\frac{(2\,Ab^5c^3+3\,Bab^4c^2d+3\,Aab^4cd^2-4\,Ba^2b^3d^3)x}{a^2b^4}-\frac{3\,(3\,Ba^2b^3cd^2+Aa^2b^3d^3)}{a^2b^4}\right)+\frac{3\,(Aab^4c^3-Ba^3b^2d^3)}{a^2b^4}\right)x-\frac{Ba^2b^3c^3+3\,Aa^2b^3c^2d+}{a^2}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-B*d^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) + 1/3*((x*((2*A*b^5*
c^3 + 3*B*a*b^4*c^2*d + 3*A*a*b^4*c*d^2 - 4*B*a^2*b^3*d^3)*x/(a^2*b^4) - 3
*(3*B*a^2*b^3*c*d^2 + A*a^2*b^3*d^3)/(a^2*b^4)) + 3*(A*a*b^4*c^3 - B*a^3*b
^2*d^3)/(a^2*b^4))*x - (B*a^2*b^3*c^3 + 3*A*a^2*b^3*c^2*d + 6*B*a^3*b^2*c*
d^2 + 2*A*a^3*b^2*d^3)/(a^2*b^4))/(b*x^2 + a)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{5/2}}\,dx = \int \frac{(A+B\,x)\,(c+d\,x)^3}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((A + B*x)*(c + d*x)^3)/(a + b*x^2)^(5/2),x)
```

output
```
int(((A + B*x)*(c + d*x)^3)/(a + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.88

$$\int \frac{(A+Bx)(c+dx)^3}{(a+bx^2)^{5/2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^3d^3-3\sqrt{b\,x^2+a}\,a^2b\,c^2d-6\sqrt{b\,x^2+a}\,a^2bc\,d^2-3\sqrt{b\,x^2+a}\,a^2b}{}$$

input
```
int((B*x+A)*(d*x+c)^3/(b*x^2+a)^(5/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**3*d**3 - 3*sqrt(a + b*x**2)*a**2*b*c**2*d - 6*sq
rt(a + b*x**2)*a**2*b*c*d**2 - 3*sqrt(a + b*x**2)*a**2*b*d**3*x**2 - 3*sqr
t(a + b*x**2)*a**2*b*d**3*x + 3*sqrt(a + b*x**2)*a*b**2*c**3*x - sqrt(a +
b*x**2)*a*b**2*c**3 + 3*sqrt(a + b*x**2)*a*b**2*c*d**2*x**3 - 9*sqrt(a + b
*x**2)*a*b**2*c*d**2*x**2 - 4*sqrt(a + b*x**2)*a*b**2*d**3*x**3 + 2*sqrt(a
 + b*x**2)*b**3*c**3*x**3 + 3*sqrt(a + b*x**2)*b**3*c**2*d*x**3 + 3*sqrt(b
)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*d**3 + 6*sqrt(b)*log((s
qrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d**3*x**2 + 3*sqrt(b)*log((sq
rt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*d**3*x**4 + 3*sqrt(b)*a**3*c*d
**2 - 2*sqrt(b)*a**2*b*c**3 + 3*sqrt(b)*a**2*b*c**2*d + 6*sqrt(b)*a**2*b*c
*d**2*x**2 - 4*sqrt(b)*a*b**2*c**3*x**2 + 6*sqrt(b)*a*b**2*c**2*d*x**2 + 3
*sqrt(b)*a*b**2*c*d**2*x**4 - 2*sqrt(b)*b**3*c**3*x**4 + 3*sqrt(b)*b**3*c*
*2*d*x**4)/(3*a*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.189 $\int \frac{(A+Bx)(c+dx)^2}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{(A+Bx)(c+dx)^2}{\left(a+bx^2\right)^{5/2}}\,dx=-\frac{(aB-Abx)(c+dx)^2}{3ab\left(a+bx^2\right)^{3/2}}-\frac{2(Abc+aBd)(ad-bcx)}{3a^2b^2\sqrt{a+bx^2}}$$

output
```
-1/3*(-A*b*x+B*a)*(d*x+c)^2/a/b/(b*x^2+a)^(3/2)-2/3*(A*b*c+B*a*d)*(-b*c*x+
a*d)/a^2/b^2/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)(c+dx)^2}{\left(a+bx^2\right)^{5/2}}\,dx=\frac{-2a^3Bd^2+2Ab^3c^2x^3+ab^2x(3Ac^2+2Bcdx^2+Ad^2x^2)-a^2b(2Acd+B(c^2+}{3a^2b^2\left(a+bx^2\right)^{3/2}}$$

input
```
Integrate[((A + B*x)*(c + d*x)^2)/(a + b*x^2)^(5/2),x]
```

output
```
(-2*a^3*B*d^2 + 2*A*b^3*c^2*x^3 + a*b^2*x*(3*A*c^2 + 2*B*c*d*x^2 + A*d^2*x
^2) - a^2*b*(2*A*c*d + B*(c^2 + 3*d^2*x^2)))/(3*a^2*b^2*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {678, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(c+dx)^2}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{678}$$

$$\frac{2(aBd+Abc)\int \frac{c+dx}{(bx^2+a)^{3/2}}dx}{3ab}-\frac{(c+dx)^2(aB-Abx)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{453}$$

$$-\frac{2(ad-bcx)(aBd+Abc)}{3a^2b^2\sqrt{a+bx^2}}-\frac{(c+dx)^2(aB-Abx)}{3ab\,(a+bx^2)^{3/2}}$$

input
```
Int[((A + B*x)*(c + d*x)^2)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*((a*B - A*b*x)*(c + d*x)^2)/(a*b*(a + b*x^2)^(3/2)) - (2*(A*b*c + a*B
*d)*(a*d - b*c*x))/(3*a^2*b^2*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 678
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[m*((c*d*f + a*e*g)/(2*a*c*(p + 1)))   Int[(d + e*x)^(
m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && EqQ[S
implify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

method	result
gosper	$-\frac{-A a \,b^{2} d^{2} x^{3}-2 A \,b^{3} c^{2} x^{3}-2 B a \,b^{2} c d \,x^{3}+3 B \,a^{2} b \,d^{2} x^{2}-3 A a \,b^{2} c^{2} x +2 A \,a^{2} b c d +2 B \,a^{3} d^{2}+B \,a^{2} b \,c^{2}}{3 \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{2} b^{2}}$
trager	$-\frac{-A a \,b^{2} d^{2} x^{3}-2 A \,b^{3} c^{2} x^{3}-2 B a \,b^{2} c d \,x^{3}+3 B \,a^{2} b \,d^{2} x^{2}-3 A a \,b^{2} c^{2} x +2 A \,a^{2} b c d +2 B \,a^{3} d^{2}+B \,a^{2} b \,c^{2}}{3 \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{2} b^{2}}$
orering	$-\frac{-A a \,b^{2} d^{2} x^{3}-2 A \,b^{3} c^{2} x^{3}-2 B a \,b^{2} c d \,x^{3}+3 B \,a^{2} b \,d^{2} x^{2}-3 A a \,b^{2} c^{2} x +2 A \,a^{2} b c d +2 B \,a^{3} d^{2}+B \,a^{2} b \,c^{2}}{3 \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{2} b^{2}}$
default	$A \,c^{2} \left(\frac{x}{3 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b \,x^{2}+a}}\right)+d \left(A d +2 B c \right) \left(-\frac{x}{2 b \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{a \left(\frac{x}{3 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b \,x^{2}+a}}\right)}{2 b}\right)-\frac{c(}{3b(}$

input
```
int((B*x+A)*(d*x+c)^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-A*a*b^2*d^2*x^3-2*A*b^3*c^2*x^3-2*B*a*b^2*c*d*x^3+3*B*a^2*b*d^2*x^2
-3*A*a*b^2*c^2*x+2*A*a^2*b*c*d+2*B*a^3*d^2+B*a^2*b*c^2)/(b*x^2+a)^(3/2)/a^
2/b^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.65

$$\int \frac{(A+Bx)(c+dx)^2}{(a+bx^2)^{5/2}}\,dx =$$

$$-\frac{\left(3\,Ba^2bd^2x^2-3\,Aab^2c^2x+Ba^2bc^2+2\,Aa^2bcd+2\,Ba^3d^2-\left(2\,Ab^3c^2+2\,Bab^2cd+Aab^2d^2\right)x^3\right)\sqrt{bx^2+a}}{3\,\left(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2\right)}$$

input
```
integrate((B*x+A)*(d*x+c)^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(3*B*a^2*b*d^2*x^2 - 3*A*a*b^2*c^2*x + B*a^2*b*c^2 + 2*A*a^2*b*c*d +
2*B*a^3*d^2 - (2*A*b^3*c^2 + 2*B*a*b^2*c*d + A*a*b^2*d^2)*x^3)*sqrt(b*x^2
+ a)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [F]

$$\int \frac{(A+Bx)(c+dx)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(A+Bx)\,(c+dx)^2}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(d*x+c)**2/(b*x**2+a)**(5/2),x)
```

output
```
Integral((A + B*x)*(c + d*x)**2/(a + b*x**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(69) = 138.

Time = 0.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.19

$$\int \frac{(A+Bx)(c+dx)^2}{(a+bx^2)^{5/2}}\,dx = -\frac{Bd^2x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,Ac^2x}{3\,\sqrt{bx^2+a}a^2} + \frac{Ac^2x}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{Bc^2}{3\,(bx^2+a)^{\frac{3}{2}}b} - \frac{2\,Acd}{3\,(bx^2+a)^{\frac{3}{2}}b} - \frac{2\,Bad^2}{3\,(bx^2+a)^{\frac{3}{2}}b^2} - \frac{(2\,Bcd+Ad^2)x}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{(2\,Bcd+Ad^2)x}{3\,\sqrt{bx^2+a}ab}$$

input
```
integrate((B*x+A)*(d*x+c)^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-B*d^2*x^2/((b*x^2 + a)^(3/2)*b) + 2/3*A*c^2*x/(sqrt(b*x^2 + a)*a^2) + 1/3
*A*c^2*x/((b*x^2 + a)^(3/2)*a) - 1/3*B*c^2/((b*x^2 + a)^(3/2)*b) - 2/3*A*c
*d/((b*x^2 + a)^(3/2)*b) - 2/3*B*a*d^2/((b*x^2 + a)^(3/2)*b^2) - 1/3*(2*B*
c*d + A*d^2)*x/((b*x^2 + a)^(3/2)*b) + 1/3*(2*B*c*d + A*d^2)*x/(sqrt(b*x^2
 + a)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx)(c+dx)^2}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{3\,Ac^2}{a} - \left(\frac{3\,Bd^2}{b} - \frac{(2\,Ab^3c^2+2\,Bab^2cd+Aab^2d^2)x}{a^2b^2}\right)x\right)x - \frac{Ba^2bc^2+2\,Aa^2bcd+2\,Ba^3d^2}{a^2b^2}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*((3*A*c^2/a - (3*B*d^2/b - (2*A*b^3*c^2 + 2*B*a*b^2*c*d + A*a*b^2*d^2)
*x/(a^2*b^2))*x)*x - (B*a^2*b*c^2 + 2*A*a^2*b*c*d + 2*B*a^3*d^2)/(a^2*b^2)
)/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.38

$$\int \frac{(A+Bx)(c+dx)^2}{(a+bx^2)^{5/2}}\,dx = -\frac{2\,B\,a^3\,d^2+B\,a^2\,b\,c^2+2\,A\,a^2\,b\,c\,d+3\,B\,a^2\,b\,d^2\,x^2-3\,A\,a\,b^2\,c^2\,x-2\,B\,a\,b^2\,c\,d\,x^3-A\,a\,b^2\,d^2\,x^3-2\,A\,b^3\,c^2}{3\,a^2\,b^2\,(b\,x^2+a)^{3/2}}$$

input
```
int(((A + B*x)*(c + d*x)^2)/(a + b*x^2)^(5/2),x)
```

output
```
-(2*B*a^3*d^2 + B*a^2*b*c^2 - 2*A*b^3*c^2*x^3 - A*a*b^2*d^2*x^3 + 3*B*a^2*
b*d^2*x^2 + 2*A*a^2*b*c*d - 3*A*a*b^2*c^2*x - 2*B*a*b^2*c*d*x^3)/(3*a^2*b^
2*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.66

$$\int \frac{(A+Bx)(c+dx)^2}{(a+bx^2)^{5/2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^2bcd - 2\sqrt{b\,x^2+a}\,a^2b\,d^2 + 3\sqrt{b\,x^2+a}\,a\,b^2c^2x - \sqrt{b\,x^2+a}\,a\,b^2c}{}$$

input
```
int((B*x+A)*(d*x+c)^2/(b*x^2+a)^(5/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**2*b*c*d - 2*sqrt(a + b*x**2)*a**2*b*d**2 + 3*sqr
t(a + b*x**2)*a*b**2*c**2*x - sqrt(a + b*x**2)*a*b**2*c**2 + sqrt(a + b*x*
*2)*a*b**2*d**2*x**3 - 3*sqrt(a + b*x**2)*a*b**2*d**2*x**2 + 2*sqrt(a + b*
x**2)*b**3*c**2*x**3 + 2*sqrt(a + b*x**2)*b**3*c*d*x**3 + sqrt(b)*a**3*d**
2 - 2*sqrt(b)*a**2*b*c**2 + 2*sqrt(b)*a**2*b*c*d + 2*sqrt(b)*a**2*b*d**2*x
**2 - 4*sqrt(b)*a*b**2*c**2*x**2 + 4*sqrt(b)*a*b**2*c*d*x**2 + sqrt(b)*a*b
**2*d**2*x**4 - 2*sqrt(b)*b**3*c**2*x**4 + 2*sqrt(b)*b**3*c*d*x**4)/(3*a*b
**2*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.190 $\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{5/2}} \, dx = -\frac{a(Bc+Ad)-(Abc-aBd)x}{3ab\,(a+bx^2)^{3/2}}+\frac{(2Abc+aBd)x}{3a^2b\sqrt{a+bx^2}}$$

output
```
-1/3*(a*(A*d+B*c)-(A*b*c-B*a*d)*x)/a/b/(b*x^2+a)^(3/2)+1/3*(2*A*b*c+B*a*d)
*x/a^2/b/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{5/2}} \, dx = \frac{-a^2Bc-a^2Ad+3aAbcx+2Ab^2cx^3+abBdx^3}{3a^2b\,(a+bx^2)^{3/2}}$$

input
```
Integrate[((A + B*x)*(c + d*x))/(a + b*x^2)^(5/2),x]
```

output
```
(-(a^2*B*c) - a^2*A*d + 3*a*A*b*c*x + 2*A*b^2*c*x^3 + a*b*B*d*x^3)/(3*a^2*
b*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {675, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{675}$$

$$\frac{(aBd+2Abc)\int \frac{1}{(bx^2+a)^{3/2}}dx}{3ab} - \frac{Ad+Bc}{3b(a+bx^2)^{3/2}} + \frac{x(Abc-aBd)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{x(aBd+2Abc)}{3a^2b\sqrt{a+bx^2}} - \frac{Ad+Bc}{3b(a+bx^2)^{3/2}} + \frac{x(Abc-aBd)}{3ab(a+bx^2)^{3/2}}$$

input
```
Int[((A + B*x)*(c + d*x))/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(B*c + A*d)/(b*(a + b*x^2)^(3/2)) + ((A*b*c - a*B*d)*x)/(3*a*b*(a + b
*x^2)^(3/2)) + ((2*A*b*c + a*B*d)*x)/(3*a^2*b*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

method	result	si
gosper	$-\frac{-2A\,b^{2}c\,x^{3}-aBbd\,x^{3}-3Aabcx+a^{2}Ad+B\,a^{2}c}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b}$	57
trager	$-\frac{-2A\,b^{2}c\,x^{3}-aBbd\,x^{3}-3Aabcx+a^{2}Ad+B\,a^{2}c}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b}$	57
orering	$-\frac{-2A\,b^{2}c\,x^{3}-aBbd\,x^{3}-3Aabcx+a^{2}Ad+B\,a^{2}c}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b}$	57
default	$Ac\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)-\frac{Ad+Bc}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+Bd\left(-\frac{x}{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)}{2b}\right)$	11

input
```
int((B*x+A)*(d*x+c)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-2*A*b^2*c*x^3-B*a*b*d*x^3-3*A*a*b*c*x+A*a^2*d+B*a^2*c)/(b*x^2+a)^(3
/2)/a^2/b
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{5/2}}\,dx = \frac{(3\,Aabcx - Ba^2c - Aa^2d + (2\,Ab^2c + Babd)x^3)\sqrt{bx^2+a}}{3\,(a^2b^3x^4 + 2\,a^3b^2x^2 + a^4b)}$$

input
```
integrate((B*x+A)*(d*x+c)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(3*A*a*b*c*x - B*a^2*c - A*a^2*d + (2*A*b^2*c + B*a*b*d)*x^3)*sqrt(b*x
^2 + a)/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)
```

Sympy [A] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.25

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{5/2}}\,dx = Ac\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + Ad\left(\begin{cases}-\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right) + Bc\left(\begin{cases}-\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right) + \frac{Bdx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((B*x+A)*(d*x+c)/(b*x**2+a)**(5/2),x)
```

output
```
A*c*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x
**2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt
(1 + b*x**2/a))) + A*d*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2
*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True)) + B*c*Piecewise(
(-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne(b, 0)), (x
**2/(2*a**(5/2)), True)) + B*d*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**
(3/2)*b*x**2*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{5/2}}\,dx = \frac{2\,Acx}{3\,\sqrt{bx^2+a}a^2} + \frac{Acx}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{Bdx}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{Bdx}{3\,\sqrt{bx^2+a}ab} - \frac{Bc}{3\,(bx^2+a)^{\frac{3}{2}}b} - \frac{Ad}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate((B*x+A)*(d*x+c)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*A*c*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*c*x/((b*x^2 + a)^(3/2)*a) - 1/3*B*
d*x/((b*x^2 + a)^(3/2)*b) + 1/3*B*d*x/(sqrt(b*x^2 + a)*a*b) - 1/3*B*c/((b*
x^2 + a)^(3/2)*b) - 1/3*A*d/((b*x^2 + a)^(3/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{3\,Ac}{a}+\frac{(2\,Ab^2c+Babd)x^2}{a^2b}\right)x-\frac{Ba^2c+Aa^2d}{a^2b}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(d*x+c)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*((3*A*c/a + (2*A*b^2*c + B*a*b*d)*x^2/(a^2*b))*x - (B*a^2*c + A*a^2*d)
/(a^2*b))/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{5/2}}\,dx = \frac{\frac{a\,(B\,b\,d\,x^3+3\,A\,b\,c\,x)}{3\,(b\,x^2+a)^{3/2}}+\frac{2\,A\,b^2\,c\,x^3}{3\,(b\,x^2+a)^{3/2}}}{a^2\,b}-\frac{A\,d+B\,c}{3\,b\,(b\,x^2+a)^{3/2}}$$

input
```
int(((A + B*x)*(c + d*x))/(a + b*x^2)^(5/2),x)
```

output
```
((a*(B*b*d*x^3 + 3*A*b*c*x))/(3*(a + b*x^2)^(3/2)) + (2*A*b^2*c*x^3)/(3*(a
 + b*x^2)^(3/2)))/(a^2*b) - (A*d + B*c)/(3*b*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.06

$$\int \frac{(A+Bx)(c+dx)}{(a+bx^2)^{5/2}}\,dx = \frac{-\sqrt{b\,x^2+a}\,a^2d+3\sqrt{b\,x^2+a}\,abcx-\sqrt{b\,x^2+a}\,abc+2\sqrt{b\,x^2+a}\,b^2c\,x^3+\sqrt{b\,x}}{3ab\,(b^2x^4}$$

input
```
int((B*x+A)*(d*x+c)/(b*x^2+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**2)*a**2*d + 3*sqrt(a + b*x**2)*a*b*c*x - sqrt(a + b*x**2
)*a*b*c + 2*sqrt(a + b*x**2)*b**2*c*x**3 + sqrt(a + b*x**2)*b**2*d*x**3 -
2*sqrt(b)*a**2*c + sqrt(b)*a**2*d - 4*sqrt(b)*a*b*c*x**2 + 2*sqrt(b)*a*b*d
*x**2 - 2*sqrt(b)*b**2*c*x**4 + sqrt(b)*b**2*d*x**4)/(3*a*b*(a**2 + 2*a*b*
x**2 + b**2*x**4))
```

3.191 $\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx=\frac{-aB+Abx}{3ab\,(a+bx^2)^{3/2}}+\frac{2Ax}{3a^2\sqrt{a+bx^2}}$$

output
```
1/3*(A*b*x-B*a)/a/b/(b*x^2+a)^(3/2)+2/3*A*x/a^2/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx=\frac{-a^2B+3aAbx+2Ab^2x^3}{3a^2b\,(a+bx^2)^{3/2}}$$

input
```
Integrate[(A + B*x)/(a + b*x^2)^(5/2),x]
```

output
```
(-(a^2*B) + 3*a*A*b*x + 2*A*b^2*x^3)/(3*a^2*b*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {454, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{454}$$

$$\frac{2A\int \frac{1}{(bx^2+a)^{3/2}}dx}{3a}-\frac{aB-Abx}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}-\frac{aB-Abx}{3ab\,(a+bx^2)^{3/2}}$$

input
```
Int[(A + B*x)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(a*B - A*b*x)/(a*b*(a + b*x^2)^(3/2)) + (2*A*x)/(3*a^2*Sqrt[a + b*x^2
])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{2A\,x^{3}b^{2}+3abAx-a^{2}B}{3(b\,x^{2}+a)^{\frac{3}{2}}\,a^{2}b}$	40
trager	$\frac{2A\,x^{3}b^{2}+3abAx-a^{2}B}{3(b\,x^{2}+a)^{\frac{3}{2}}\,a^{2}b}$	40
orering	$\frac{2A\,x^{3}b^{2}+3abAx-a^{2}B}{3(b\,x^{2}+a)^{\frac{3}{2}}\,a^{2}b}$	40
default	$A\left(\frac{x}{3a(b\,x^{2}+a)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)-\frac{B}{3b(b\,x^{2}+a)^{\frac{3}{2}}}$	50

input
```
int((B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*A*b^2*x^3+3*A*a*b*x-B*a^2)/(b*x^2+a)^(3/2)/a^2/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{(2\,Ab^2x^3+3\,Aabx-Ba^2)\sqrt{bx^2+a}}{3\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*A*b^2*x^3 + 3*A*a*b*x - B*a^2)*sqrt(b*x^2 + a)/(a^2*b^3*x^4 + 2*a^3
*b^2*x^2 + a^4*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(44) = 88$.

Time = 4.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.86

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\begin{cases}-\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**
2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1
 + b*x**2/a))) + B*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqr
t(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{2\,Ax}{3\,\sqrt{bx^2+a}a^2} + \frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{B}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1/3*B/((b*
x^2 + a)^(3/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{2\,Abx^2}{a^2}+\frac{3\,A}{a}\right)x-\frac{B}{b}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*((2*A*b*x^2/a^2 + 3*A/a)*x - B/b)/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{2\,A\,b\,x\,(b\,x^2+a)-B\,a^2+A\,a\,b\,x}{3\,a^2\,b\,(b\,x^2+a)^{3/2}}$$

input
```
int((A + B*x)/(a + b*x^2)^(5/2),x)
```

output
```
(2*A*b*x*(a + b*x^2) - B*a^2 + A*a*b*x)/(3*a^2*b*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{3\sqrt{b\,x^2+a}\,abx-\sqrt{b\,x^2+a}\,ab+2\sqrt{b\,x^2+a}\,b^2x^3-2\sqrt{b}\,a^2-4\sqrt{b}\,ab\,x^2-2\sqrt{b}\,b^2x^4}{3ab\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int((B*x+A)/(b*x^2+a)^(5/2),x)
```

output
```
(3*sqrt(a + b*x**2)*a*b*x - sqrt(a + b*x**2)*a*b + 2*sqrt(a + b*x**2)*b**2
*x**3 - 2*sqrt(b)*a**2 - 4*sqrt(b)*a*b*x**2 - 2*sqrt(b)*b**2*x**4)/(3*a*b*
(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.192 $\int \frac{A+Bx}{(c+dx)(a+bx^2)^{5/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 204

$$\int \frac{A+Bx}{(c+dx)(a+bx^2)^{5/2}} dx = -\frac{a(Bc-Ad)-(Abc+aBd)x}{3a(bc^2+ad^2)(a+bx^2)^{3/2}} - \frac{3a^2d^2(Bc-Ad)+(aBd(bc^2-2ad^2)-Abc(2bc^2+5ad^2))x}{3a^2(bc^2+ad^2)^2\sqrt{a+bx^2}} + \frac{d^3(Bc-Ad)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{(bc^2+ad^2)^{5/2}}$$

output
```
-1/3*(a*(-A*d+B*c)-(A*b*c+B*a*d)*x)/a/(a*d^2+b*c^2)/(b*x^2+a)^(3/2)-1/3*(3
*a^2*d^2*(-A*d+B*c)+(a*B*d*(-2*a*d^2+b*c^2)-A*b*c*(5*a*d^2+2*b*c^2))*x)/a^
2/(a*d^2+b*c^2)^2/(b*x^2+a)^(1/2)+d^3*(-A*d+B*c)*arctanh((-b*c*x+a*d)/(a*d
^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx}{(c+dx)\left(a+bx^2\right)^{5/2}}\,dx = \frac{2Ab^3c^3x^3+a^3d^2(-4Bc+4Ad+3Bdx)+ab^2cx\left(3Ac^2-Bcdx^2+5Ad^2x^2\right)-}{3a^2\left(bc^2+ad^2\right)^2\left(a+bx\right.}$$
$$+\frac{2d^3(Bc-Ad)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\left(-bc^2-ad^2\right)^{5/2}}$$

input
```
Integrate[(A + B*x)/((c + d*x)*(a + b*x^2)^(5/2)),x]
```

output
```
(2*A*b^3*c^3*x^3 + a^3*d^2*(-4*B*c + 4*A*d + 3*B*d*x) + a*b^2*c*x*(3*A*c^2
 - B*c*d*x^2 + 5*A*d^2*x^2) + a^2*b*(A*d*(c^2 + 6*c*d*x + 3*d^2*x^2) - B*(
c^3 + 3*c*d^2*x^2 - 2*d^3*x^3)))/(3*a^2*(b*c^2 + a*d^2)^2*(a + b*x^2)^(3/2
)) + (2*d^3*(B*c - A*d)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqr
t[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {686, 25, 27, 686, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{5/2}(c+dx)}dx$$

$$\Big\downarrow \text{686}$$

$$-\frac{\int -\frac{b\left(2Abc^2-aBdc+3aAd^2+2d(Abc+aBd)x\right)}{(c+dx)\left(bx^2+a\right)^{3/2}}dx}{3ab\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b(2Abc^2-aBdc+3aAd^2+2d(Abc+aBd)x)}{(c+dx)(bx^2+a)^{3/2}}dx}{3ab\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{2Abc^2-aBdc+3aAd^2+2d(Abc+aBd)x}{(c+dx)(bx^2+a)^{3/2}}dx}{3a\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 686

$$\frac{-\frac{\int \frac{3a^2bd^3(Bc-Ad)}{(c+dx)\sqrt{bx^2+a}}dx}{ab(ad^2+bc^2)}-\frac{3a^2d^2(Bc-Ad)+x\left(aBd\left(bc^2-2ad^2\right)-Abc\left(5ad^2+2bc^2\right)\right)}{a\sqrt{a+bx^2}(ad^2+bc^2)}}{3a\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 27

$$\frac{-\frac{3ad^3(Bc-Ad)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{3a^2d^2(Bc-Ad)+x\left(aBd\left(bc^2-2ad^2\right)-Abc\left(5ad^2+2bc^2\right)\right)}{a\sqrt{a+bx^2}(ad^2+bc^2)}}{3a\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 488

$$\frac{\frac{3ad^3(Bc-Ad)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}-\frac{3a^2d^2(Bc-Ad)+x\left(aBd\left(bc^2-2ad^2\right)-Abc\left(5ad^2+2bc^2\right)\right)}{a\sqrt{a+bx^2}(ad^2+bc^2)}}{3a\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 219

$$\frac{\frac{3ad^3(Bc-Ad)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{(ad^2+bc^2)^{3/2}}-\frac{3a^2d^2(Bc-Ad)+x\left(aBd\left(bc^2-2ad^2\right)-Abc\left(5ad^2+2bc^2\right)\right)}{a\sqrt{a+bx^2}(ad^2+bc^2)}}{3a\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}\left(ad^2+bc^2\right)}$$

input
```
Int[(A + B*x)/((c + d*x)*(a + b*x^2)^(5/2)),x]
```

output
```
-1/3*(a*(B*c - A*d) - (A*b*c + a*B*d)*x)/(a*(b*c^2 + a*d^2)*(a + b*x^2)^(3
/2)) + (-((3*a^2*d^2*(B*c - A*d) + (a*B*d*(b*c^2 - 2*a*d^2) - A*b*c*(2*b*c
^2 + 5*a*d^2))*x)/(a*(b*c^2 + a*d^2)*Sqrt[a + b*x^2])) + (3*a*d^3*(B*c - A
*d)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 +
 a*d^2)^(3/2))/(3*a*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(190) = 380$.

Time = 1.32 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.17

method	result
default	$\frac{B\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)}{d}+\frac{(Ad-Bc)\left(\frac{d^{2}}{3\left(a\,d^{2}+b\,c^{2}\right)\left(b\left(x+\frac{c}{d}\right)^{2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^{2}+b\,c^{2}}{d^{2}}\right)^{\frac{3}{2}}}+\frac{bcd\left(\frac{4\cdots}{\left(\frac{4b\left(a\,d^{2}+b\,c^{2}\right)}{d^{2}}-\frac{4b^{2}c^{2}}{d^{2}}\right)\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$

input

```
int((B*x+A)/(d*x+c)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
B/d*(1/3*x/a/(b*x^2+a)^(3/2)+2/3/a^2/(b*x^2+a)^(1/2)*x)+(A*d-B*c)/d^2*(1/3
/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+b
*c*d/(a*d^2+b*c^2)*(2/3*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2
*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+16/3*b/(4*
b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)^2*(2*b*(x+c/d)-2*b*c/d)/(b*(x+c/d)^2-2*
b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))+1/(a*d^2+b*c^2)*d^2*(1/(a*d^2+b*c^
2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^
2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x
+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d
^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c
^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c
/d))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(192) = 384$.

Time = 0.50 (sec) , antiderivative size = 1196, normalized size of antiderivative = 5.86

$$\int \frac{A+Bx}{(c+dx)(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(B*a^4*c*d^3 - A*a^4*d^4 + (B*a^2*b^2*c*d^3 - A*a^2*b^2*d^4)*x^4
+ 2*(B*a^3*b*c*d^3 - A*a^3*b*d^4)*x^2)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*
x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2
)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(B*a^2*b^2
*c^5 - A*a^2*b^2*c^4*d + 5*B*a^3*b*c^3*d^2 - 5*A*a^3*b*c^2*d^3 + 4*B*a^4*c
*d^4 - 4*A*a^4*d^5 - (2*A*b^4*c^5 - B*a*b^3*c^4*d + 7*A*a*b^3*c^3*d^2 + B*
a^2*b^2*c^2*d^3 + 5*A*a^2*b^2*c*d^4 + 2*B*a^3*b*d^5)*x^3 + 3*(B*a^2*b^2*c^
3*d^2 - A*a^2*b^2*c^2*d^3 + B*a^3*b*c*d^4 - A*a^3*b*d^5)*x^2 - 3*(A*a*b^3*
c^5 + 3*A*a^2*b^2*c^3*d^2 + B*a^3*b*c^2*d^3 + 2*A*a^3*b*c*d^4 + B*a^4*d^5)
*x)*sqrt(b*x^2 + a))/(a^4*b^3*c^6 + 3*a^5*b^2*c^4*d^2 + 3*a^6*b*c^2*d^4 +
a^7*d^6 + (a^2*b^5*c^6 + 3*a^3*b^4*c^4*d^2 + 3*a^4*b^3*c^2*d^4 + a^5*b^2*d
^6)*x^4 + 2*(a^3*b^4*c^6 + 3*a^4*b^3*c^4*d^2 + 3*a^5*b^2*c^2*d^4 + a^6*b*d
^6)*x^2), 1/3*(3*(B*a^4*c*d^3 - A*a^4*d^4 + (B*a^2*b^2*c*d^3 - A*a^2*b^2*d
^4)*x^4 + 2*(B*a^3*b*c*d^3 - A*a^3*b*d^4)*x^2)*sqrt(-b*c^2 - a*d^2)*arctan
(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (
b^2*c^2 + a*b*d^2)*x^2)) - (B*a^2*b^2*c^5 - A*a^2*b^2*c^4*d + 5*B*a^3*b*c^
3*d^2 - 5*A*a^3*b*c^2*d^3 + 4*B*a^4*c*d^4 - 4*A*a^4*d^5 - (2*A*b^4*c^5 - B
*a*b^3*c^4*d + 7*A*a*b^3*c^3*d^2 + B*a^2*b^2*c^2*d^3 + 5*A*a^2*b^2*c*d^4 +
 2*B*a^3*b*d^5)*x^3 + 3*(B*a^2*b^2*c^3*d^2 - A*a^2*b^2*c^2*d^3 + B*a^3*b*c
*d^4 - A*a^3*b*d^5)*x^2 - 3*(A*a*b^3*c^5 + 3*A*a^2*b^2*c^3*d^2 + B*a^3*...
```

Sympy [F]

$$\int \frac{A+Bx}{(c+dx)\,(a+bx^2)^{5/2}}\,dx = \int \frac{A+Bx}{(a+bx^2)^{\frac{5}{2}}\,(c+dx)}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)/(b*x**2+a)**(5/2),x)
```

output
```
Integral((A + B*x)/((a + b*x**2)**(5/2)*(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(192) = 384$.

Time = 0.11 (sec) , antiderivative size = 630, normalized size of antiderivative = 3.09

$$\int \frac{A+Bx}{(c+dx)\left(a+bx^2\right)^{5/2}}\,dx =$$

$$-\frac{Bbc^2x}{\frac{\sqrt{bx^2+a}ab^2c^4}{d}+2\sqrt{bx^2+a}a^2bc^2d+\sqrt{bx^2+a}a^3d^3}-\frac{Bbc^2x}{3\left((bx^2+a)^{\frac{3}{2}}abc^2d+(bx^2+a)^{\frac{3}{2}}a^2d^3\right)}-\frac{2\,Bbc^2x}{3\left(\sqrt{bx^2+a}a^2bc^2d+\sqrt{bx^2+a}a^3d^3\right)}+\frac{Abcx}{3\left((bx^2+a)^{\frac{3}{2}}abc^2+(bx^2+a)^{\frac{3}{2}}a^2d^2\right)}+\frac{Abcx}{2\sqrt{bx^2+a}a^2bc^2+\frac{\sqrt{bx^2+a}ab^2c^4}{d^2}+\sqrt{bx^2+a}a^3d^2}+\frac{2\,Abcx}{3\left(\sqrt{bx^2+a}a^2bc^2+\sqrt{bx^2+a}a^3d^2\right)}-\frac{Bc}{3\left((bx^2+a)^{\frac{3}{2}}bc^2+(bx^2+a)^{\frac{3}{2}}ad^2\right)}-\frac{Bc}{2\sqrt{bx^2+a}abc^2+\frac{\sqrt{bx^2+a}b^2c^4}{d^2}+\sqrt{bx^2+a}a^2d^2}+\frac{A}{\frac{\sqrt{bx^2+a}b^2c^4}{d^3}+\frac{2\sqrt{bx^2+a}abc^2}{d}+\sqrt{bx^2+a}a^2d}+\frac{A}{3\left(\frac{(bx^2+a)^{\frac{3}{2}}bc^2}{d}+(bx^2+a)^{\frac{3}{2}}ad\right)}+\frac{2\,Bx}{3\sqrt{bx^2+a}a^2d}+\frac{Bx}{3\,(bx^2+a)^{\frac{3}{2}}ad}-\frac{Bc\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{5}{2}}d^2}+\frac{A\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{5}{2}}d}$$

input
```
integrate((B*x+A)/(d*x+c)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-B*b*c^2*x/(sqrt(b*x^2 + a)*a*b^2*c^4/d + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d +
sqrt(b*x^2 + a)*a^3*d^3) - 1/3*B*b*c^2*x/((b*x^2 + a)^(3/2)*a*b*c^2*d + (b
*x^2 + a)^(3/2)*a^2*d^3) - 2/3*B*b*c^2*x/(sqrt(b*x^2 + a)*a^2*b*c^2*d + sq
rt(b*x^2 + a)*a^3*d^3) + 1/3*A*b*c*x/((b*x^2 + a)^(3/2)*a*b*c^2 + (b*x^2 +
 a)^(3/2)*a^2*d^2) + A*b*c*x/(2*sqrt(b*x^2 + a)*a^2*b*c^2 + sqrt(b*x^2 + a
)*a*b^2*c^4/d^2 + sqrt(b*x^2 + a)*a^3*d^2) + 2/3*A*b*c*x/(sqrt(b*x^2 + a)*
a^2*b*c^2 + sqrt(b*x^2 + a)*a^3*d^2) - 1/3*B*c/((b*x^2 + a)^(3/2)*b*c^2 +
(b*x^2 + a)^(3/2)*a*d^2) - B*c/(2*sqrt(b*x^2 + a)*a*b*c^2 + sqrt(b*x^2 + a
)*b^2*c^4/d^2 + sqrt(b*x^2 + a)*a^2*d^2) + A/(sqrt(b*x^2 + a)*b^2*c^4/d^3
+ 2*sqrt(b*x^2 + a)*a*b*c^2/d + sqrt(b*x^2 + a)*a^2*d) + 1/3*A/((b*x^2 + a
)^(3/2)*b*c^2/d + (b*x^2 + a)^(3/2)*a*d) + 2/3*B*x/(sqrt(b*x^2 + a)*a^2*d)
 + 1/3*B*x/((b*x^2 + a)^(3/2)*a*d) - B*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x
+ c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(5/2)*d^2) + A*arcs
inh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b
*c^2/d^2)^(5/2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1467 vs. $2(192) = 384$.

Time = 0.13 (sec) , antiderivative size = 1467, normalized size of antiderivative = 7.19

$$\int \frac{A + Bx}{(c + dx)\,(a + bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
2*(B*c*d^3 - A*d^4)*arctan(((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/s
qrt(-b*c^2 - a*d^2))/((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*sqrt(-b*c^2 - a*
d^2)) + 1/3*((((2*A*b^10*c^15 - B*a*b^9*c^14*d + 17*A*a*b^9*c^13*d^2 - 4*B
*a^2*b^8*c^12*d^3 + 60*A*a^2*b^8*c^11*d^4 - 3*B*a^3*b^7*c^10*d^5 + 115*A*a
^3*b^7*c^9*d^6 + 10*B*a^4*b^6*c^8*d^7 + 130*A*a^4*b^6*c^7*d^8 + 25*B*a^5*b
^5*c^6*d^9 + 87*A*a^5*b^5*c^5*d^10 + 24*B*a^6*b^4*c^4*d^11 + 32*A*a^6*b^4*
c^3*d^12 + 11*B*a^7*b^3*c^2*d^13 + 5*A*a^7*b^3*c*d^14 + 2*B*a^8*b^2*d^15)*
x/(a^2*b^9*c^16 + 8*a^3*b^8*c^14*d^2 + 28*a^4*b^7*c^12*d^4 + 56*a^5*b^6*c^
10*d^6 + 70*a^6*b^5*c^8*d^8 + 56*a^7*b^4*c^6*d^10 + 28*a^8*b^3*c^4*d^12 +
8*a^9*b^2*c^2*d^14 + a^10*b*d^16) - 3*(B*a^2*b^8*c^13*d^2 - A*a^2*b^8*c^12
*d^3 + 6*B*a^3*b^7*c^11*d^4 - 6*A*a^3*b^7*c^10*d^5 + 15*B*a^4*b^6*c^9*d^6
- 15*A*a^4*b^6*c^8*d^7 + 20*B*a^5*b^5*c^7*d^8 - 20*A*a^5*b^5*c^6*d^9 + 15*
B*a^6*b^4*c^5*d^10 - 15*A*a^6*b^4*c^4*d^11 + 6*B*a^7*b^3*c^3*d^12 - 6*A*a^
7*b^3*c^2*d^13 + B*a^8*b^2*c*d^14 - A*a^8*b^2*d^15)/(a^2*b^9*c^16 + 8*a^3*
b^8*c^14*d^2 + 28*a^4*b^7*c^12*d^4 + 56*a^5*b^6*c^10*d^6 + 70*a^6*b^5*c^8*
d^8 + 56*a^7*b^4*c^6*d^10 + 28*a^8*b^3*c^4*d^12 + 8*a^9*b^2*c^2*d^14 + a^1
0*b*d^16))*x + 3*(A*a*b^9*c^15 + 8*A*a^2*b^8*c^13*d^2 + B*a^3*b^7*c^12*d^3
 + 27*A*a^3*b^7*c^11*d^4 + 6*B*a^4*b^6*c^10*d^5 + 50*A*a^4*b^6*c^9*d^6 + 1
5*B*a^5*b^5*c^8*d^7 + 55*A*a^5*b^5*c^7*d^8 + 20*B*a^6*b^4*c^6*d^9 + 36*A*a
^6*b^4*c^5*d^10 + 15*B*a^7*b^3*c^4*d^11 + 13*A*a^7*b^3*c^3*d^12 + 6*B*a...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)\,(a+bx^2)^{5/2}}\,dx = \int \frac{A+B\,x}{(b\,x^2+a)^{5/2}\,(c+d\,x)}\,dx$$

input
```
int((A + B*x)/((a + b*x^2)^(5/2)*(c + d*x)),x)
```

output
```
int((A + B*x)/((a + b*x^2)^(5/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 7379, normalized size of antiderivative = 36.17

$$\int \frac{A+Bx}{(c+dx)\,(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/(d*x+c)/(b*x^2+a)^(5/2),x)
```

output
```
( - 6*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*
sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**3*c*d**2 + 6*sqrt(b)*s
qrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b
*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2
+ b*c**2)*c - a*d**2 - 2*b*c**2))*a**2*b*c**2*d - 12*sqrt(b)*sqrt(2*sqrt(b
)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c**2)*atan(
(sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c
- a*d**2 - 2*b*c**2))*a**2*b*c*d**2*x**2 + 12*sqrt(b)*sqrt(2*sqrt(b)*sqrt(
a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c**2)*atan((sqrt(a
 + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**
2 - 2*b*c**2))*a*b**2*c**2*d*x**2 - 6*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 +
 b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**
2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*
c**2))*a*b**2*c*d**2*x**4 + 6*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)
*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + s
qrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*b
**3*c**2*d*x**4 - 6*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*
c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 +
 b*c**2)*c - a*d**2 - 2*b*c**2))*a**4*d**4 - 6*sqrt(2*sqrt(b)*sqrt(a*d*...
```

3.193 $\int \frac{A+Bx}{(c+dx)^2\left(a+bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 308

$$\begin{aligned}\int \frac{A+Bx}{(c+dx)^2\left(a+bx^2\right)^{5/2}}\,dx &= \frac{Bc-Ad}{(bc^2+ad^2)\,(c+dx)\left(a+bx^2\right)^{3/2}}\\ &+\frac{a(aBd^2-bc(4Bc-5Ad))+b(Abc^2+5aBcd-4aAd^2)\,x}{3a\left(bc^2+ad^2\right)^2\left(a+bx^2\right)^{3/2}}\\ &+\frac{3a^2d^2(aBd^2-bc(4Bc-5Ad))-b(aBcd(2bc^2-13ad^2)-A(2b^2c^4+9abc^2d^2-8a^2d^4))\,x}{3a^2\left(bc^2+ad^2\right)^3\sqrt{a+bx^2}}\\ &-\frac{d^3(aBd^2-bc(4Bc-5Ad))\,\mathrm{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{\left(bc^2+ad^2\right)^{7/2}}\end{aligned}$$

output
```
(-A*d+B*c)/(a*d^2+b*c^2)/(d*x+c)/(b*x^2+a)^(3/2)+1/3*(a*(a*B*d^2-b*c*(-5*A
*d+4*B*c))+b*(-4*A*a*d^2+A*b*c^2+5*B*a*c*d)*x)/a/(a*d^2+b*c^2)^2/(b*x^2+a)
^(3/2)+1/3*(3*a^2*d^2*(a*B*d^2-b*c*(-5*A*d+4*B*c))-b*(a*B*c*d*(-13*a*d^2+2
*b*c^2)-A*(-8*a^2*d^4+9*a*b*c^2*d^2+2*b^2*c^4))*x)/a^2/(a*d^2+b*c^2)^3/(b*
x^2+a)^(1/2)-d^3*(a*B*d^2-b*c*(-5*A*d+4*B*c))*arctanh((-b*c*x+a*d)/(a*d^2+
b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2)^(7/2)
```

Mathematica [A] (verified)

Time = 11.90 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{(c+dx)^2\left(a+bx^2\right)^{5/2}}\,dx = \frac{\frac{d\left(aBcd(-2bc^2+13ad^2)+A(2b^2c^4+9abc^2d^2-8a^2d^4)\right)\sqrt{a+bx^2}}{a(bc^2+ad^2)^2}+\frac{Abcx+a(-Bc+Ad+Bdx)}{(a+bx^2)^{3/2}}+\frac{2Ab^2c\cdots}{\cdots}}{3a\cdots}$$

input
```
Integrate[(A + B*x)/((c + d*x)^2*(a + b*x^2)^(5/2)),x]
```

output
```
((d*(a*B*c*d*(-2*b*c^2 + 13*a*d^2) + A*(2*b^2*c^4 + 9*a*b*c^2*d^2 - 8*a^2*
d^4))*Sqrt[a + b*x^2])/(a*(b*c^2 + a*d^2)^2) + (A*b*c*x + a*(-(B*c) + A*d
+ B*d*x))/(a + b*x^2)^(3/2) + (2*A*b^2*c^3*x + a^2*d^2*(-5*B*c + 4*A*d + 3
*B*d*x) - a*b*c*d*(2*B*c*x + A*(c - 7*d*x)))/(a*(b*c^2 + a*d^2)*Sqrt[a + b
*x^2]) - (3*a*d^3*(a*B*d^2 + b*c*(-4*B*c + 5*A*d))*(c + d*x)*ArcTanh[(a*d
- b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(5/2))/(3
*a*(b*c^2 + a*d^2)*(c + d*x))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {686, 25, 27, 686, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{5/2}(c+dx)^2}dx$$

$$\Big\downarrow \text{686}$$

$$-\frac{\int -\frac{b\left(2\left(Abc^2-aBdc+2aAd^2\right)+3d(Abc+aBd)x\right)}{(c+dx)^2(bx^2+a)^{3/2}}dx}{3ab\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b(2(Abc^2-aBdc+2aAd^2)+3d(Abc+aBd)x)}{(c+dx)^2(bx^2+a)^{3/2}}dx}{3ab\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}(c+dx)\left(ad^2+bc^2\right)}$$

$\big\downarrow$ 27

$$\frac{\int \frac{2\left(Abc^2-aBdc+2aAd^2\right)+3d(Abc+aBd)x}{(c+dx)^2(bx^2+a)^{3/2}}dx}{3a\left(ad^2+bc^2\right)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}(c+dx)\left(ad^2+bc^2\right)}$$

$\big\downarrow$ 686

$$\frac{-\frac{\int \frac{bd\left(2ad\left(Abc^2+5aBdc-4aAd^2\right)+\left(aBd\left(2bc^2-3ad^2\right)-Abc\left(2bc^2+7ad^2\right)\right)x\right)}{(c+dx)^2\sqrt{bx^2+a}}dx}{ab(ad^2+bc^2)}-\frac{x\left(aBd(2bc^2-3ad^2)-Abc(7ad^2+2bc^2)\right)+ad\left(-4aAd^2+5aBcd+A\cdots\right.}{a\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)}}{3a\left(ad^2+bc^2\right)}$$

$$\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}(c+dx)\left(ad^2+bc^2\right)}$$

$\big\downarrow$ 27

$$\frac{-\frac{d\int \frac{2ad\left(Abc^2+5aBdc-4aAd^2\right)+\left(aBd\left(2bc^2-3ad^2\right)-Abc\left(2bc^2+7ad^2\right)\right)x}{(c+dx)^2\sqrt{bx^2+a}}dx}{a(ad^2+bc^2)}-\frac{x\left(aBd(2bc^2-3ad^2)-Abc(7ad^2+2bc^2)\right)+ad\left(-4aAd^2+5aBcd+Abc\cdots\right.}{a\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)}}{3a\left(ad^2+bc^2\right)}$$

$$\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}(c+dx)\left(ad^2+bc^2\right)}$$

$\big\downarrow$ 679

$$\frac{-\frac{d\left(\frac{\sqrt{a+bx^2}\left(aBcd\left(2bc^2-13ad^2\right)-A\left(-8a^2d^4+9abc^2d^2+2b^2c^4\right)\right)}{(c+dx)\left(ad^2+bc^2\right)}-\frac{3a^2d^2\left(aBd^2-bc(4Bc-5Ad)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}\right)}{a(ad^2+bc^2)}-\frac{x\left(aBd(2bc^2-3ad^2)-Abc(\cdots\right.}{a\sqrt{a+\cdots}}}{3a\left(ad^2+bc^2\right)}$$

$$\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}(c+dx)\left(ad^2+bc^2\right)}$$

$\big\downarrow$ 488

$$\frac{-\frac{d\left(\frac{3a^2d^2\left(aBd^2-bc(4Bc-5Ad)\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}+\frac{\sqrt{a+bx^2}\left(aBcd\left(2bc^2-13ad^2\right)-A\left(-8a^2d^4+9abc^2d^2+2b^2c^4\right)\right)}{(c+dx)\left(ad^2+bc^2\right)}\right)}{a(ad^2+bc^2)}-\frac{x\left(aBd(2bc^2-\cdots\right.}{\cdots}}{3a\left(ad^2+bc^2\right)}$$

$$\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}(c+dx)\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 219

$$-\frac{-\frac{d\left(\frac{3a^2d^2\left(aBd^2-bc(4Bc-5Ad)\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\left(ad^2+bc^2\right)^{3/2}}+\frac{\sqrt{a+bx^2}\left(aBcd\left(2bc^2-13ad^2\right)-A\left(-8a^2d^4+9abc^2d^2+2b^2c^4\right)\right)}{(c+dx)\left(ad^2+bc^2\right)}\right)}{a(ad^2+bc^2)}-\frac{x(aBd(2bc^2-3}{}}{3a\left(ad^2+bc^2\right)}$$

$$\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}(c+dx)\left(ad^2+bc^2\right)}$$

input
```
Int[(A + B*x)/((c + d*x)^2*(a + b*x^2)^(5/2)),x]
```

output
```
-1/3*(a*(B*c - A*d) - (A*b*c + a*B*d)*x)/(a*(b*c^2 + a*d^2)*(c + d*x)*(a +
 b*x^2)^(3/2)) + (-((a*d*(A*b*c^2 + 5*a*B*c*d - 4*a*A*d^2) + (a*B*d*(2*b*c
^2 - 3*a*d^2) - A*b*c*(2*b*c^2 + 7*a*d^2))*x)/(a*(b*c^2 + a*d^2)*(c + d*x)
*Sqrt[a + b*x^2])) - (d*(((a*B*c*d*(2*b*c^2 - 13*a*d^2) - A*(2*b^2*c^4 + 9
*a*b*c^2*d^2 - 8*a^2*d^4))*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)) +
(3*a^2*d^2*(a*B*d^2 - b*c*(4*B*c - 5*A*d))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c
^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(3/2)))/(a*(b*c^2 + a*d^2))
)/(3*a*(b*c^2 + a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 686

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1508 vs. $2(292) = 584$.

Time = 1.36 (sec) , antiderivative size = 1509, normalized size of antiderivative = 4.90

method	result	size
default	Expression too large to display	1509

input

```
int((B*x+A)/(d*x+c)^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
B/d^2*(1/3/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^
2)^(3/2)+b*c*d/(a*d^2+b*c^2)*(2/3*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)
/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+
16/3*b/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)^2*(2*b*(x+c/d)-2*b*c/d)/(b*(x
+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))+1/(a*d^2+b*c^2)*d^2*(1/(
a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b
*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/
d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)
*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*(
(a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(
1/2))/(x+c/d))))+(A*d-B*c)/d^3*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)/(b*(x+c/d)^2-
2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+5*b*c*d/(a*d^2+b*c^2)*(1/3/(a*d^2
+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+b*c*d/(a
*d^2+b*c^2)*(2/3*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^
2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+16/3*b/(4*b*(a*d^
2+b*c^2)/d^2-4*b^2*c^2/d^2)^2*(2*b*(x+c/d)-2*b*c/d)/(b*(x+c/d)^2-2*b*c/d*(
x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))+1/(a*d^2+b*c^2)*d^2*(1/(a*d^2+b*c^2)*d^2/
(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2
)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2
-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1254 vs. $2(293) = 586$.

Time = 1.54 (sec) , antiderivative size = 2534, normalized size of antiderivative = 8.23

$$\int \frac{A+Bx}{(c+dx)^2\,(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(4*B*a^4*b*c^3*d^3 - 5*A*a^4*b*c^2*d^4 - B*a^5*c*d^5 + (4*B*a^2*b
^3*c^2*d^4 - 5*A*a^2*b^3*c*d^5 - B*a^3*b^2*d^6)*x^5 + (4*B*a^2*b^3*c^3*d^3
 - 5*A*a^2*b^3*c^2*d^4 - B*a^3*b^2*c*d^5)*x^4 + 2*(4*B*a^3*b^2*c^2*d^4 - 5
*A*a^3*b^2*c*d^5 - B*a^4*b*d^6)*x^3 + 2*(4*B*a^3*b^2*c^3*d^3 - 5*A*a^3*b^2
*c^2*d^4 - B*a^4*b*c*d^5)*x^2 + (4*B*a^4*b*c^2*d^4 - 5*A*a^4*b*c*d^5 - B*a
^5*d^6)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2
*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 +
 a))/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(B*a^2*b^3*c^7 - 2*A*a^2*b^3*c^6*d + 1
0*B*a^3*b^2*c^5*d^2 - 16*A*a^3*b^2*c^4*d^3 + 2*B*a^4*b*c^3*d^4 - 11*A*a^4*
b*c^2*d^5 - 7*B*a^5*c*d^6 + 3*A*a^5*d^7 - (2*A*b^5*c^6*d - 2*B*a*b^4*c^5*d
^2 + 11*A*a*b^4*c^4*d^3 + 11*B*a^2*b^3*c^3*d^4 + A*a^2*b^3*c^2*d^5 + 13*B*
a^3*b^2*c*d^6 - 8*A*a^3*b^2*d^7)*x^4 - (2*A*b^5*c^7 - 2*B*a*b^4*c^6*d + 11
*A*a*b^4*c^5*d^2 - B*a^2*b^3*c^4*d^3 + 16*A*a^2*b^3*c^3*d^4 + 4*B*a^3*b^2*
c^2*d^5 + 7*A*a^3*b^2*c*d^6 + 3*B*a^4*b*d^7)*x^3 - 3*(A*a*b^4*c^6*d - 3*B*
a^2*b^3*c^5*d^2 + 8*A*a^2*b^3*c^4*d^3 + 4*B*a^3*b^2*c^3*d^4 + 3*A*a^3*b^2*
c^2*d^5 + 7*B*a^4*b*c*d^6 - 4*A*a^4*b*d^7)*x^2 - (3*A*a*b^4*c^7 - B*a^2*b^
3*c^6*d + 14*A*a^2*b^3*c^5*d^2 + 2*B*a^3*b^2*c^4*d^3 + 19*A*a^3*b^2*c^3*d^
4 + 7*B*a^4*b*c^2*d^5 + 8*A*a^4*b*c*d^6 + 4*B*a^5*d^7)*x)*sqrt(b*x^2 + a))
/(a^4*b^4*c^9 + 4*a^5*b^3*c^7*d^2 + 6*a^6*b^2*c^5*d^4 + 4*a^7*b*c^3*d^6 +
a^8*c*d^8 + (a^2*b^6*c^8*d + 4*a^3*b^5*c^6*d^3 + 6*a^4*b^4*c^4*d^5 + 4*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)^2\left(a+bx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(d*x+c)**2/(b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. 2(293) = 586.

Time = 0.17 (sec) , antiderivative size = 1360, normalized size of antiderivative = 4.42

$$\int \frac{A+Bx}{(c+dx)^2 (a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-5*B*b^2*c^3*x/(sqrt(b*x^2 + a)*a*b^3*c^6/d + 3*sqrt(b*x^2 + a)*a^2*b^2*c^
4*d + 3*sqrt(b*x^2 + a)*a^3*b*c^2*d^3 + sqrt(b*x^2 + a)*a^4*d^5) - 5/3*B*b
^2*c^3*x/((b*x^2 + a)^(3/2)*a*b^2*c^4*d + 2*(b*x^2 + a)^(3/2)*a^2*b*c^2*d^
3 + (b*x^2 + a)^(3/2)*a^3*d^5) - 10/3*B*b^2*c^3*x/(sqrt(b*x^2 + a)*a^2*b^2
*c^4*d + 2*sqrt(b*x^2 + a)*a^3*b*c^2*d^3 + sqrt(b*x^2 + a)*a^4*d^5) + 5/3*
A*b^2*c^2*x/((b*x^2 + a)^(3/2)*a*b^2*c^4 + 2*(b*x^2 + a)^(3/2)*a^2*b*c^2*d
^2 + (b*x^2 + a)^(3/2)*a^3*d^4) + 5*A*b^2*c^2*x/(3*sqrt(b*x^2 + a)*a^2*b^2
*c^4 + sqrt(b*x^2 + a)*a*b^3*c^6/d^2 + 3*sqrt(b*x^2 + a)*a^3*b*c^2*d^2 + s
qrt(b*x^2 + a)*a^4*d^4) + 10/3*A*b^2*c^2*x/(sqrt(b*x^2 + a)*a^2*b^2*c^4 +
2*sqrt(b*x^2 + a)*a^3*b*c^2*d^2 + sqrt(b*x^2 + a)*a^4*d^4) - 5/3*B*b*c^2/(
(b*x^2 + a)^(3/2)*b^2*c^4 + 2*(b*x^2 + a)^(3/2)*a*b*c^2*d^2 + (b*x^2 + a)^
(3/2)*a^2*d^4) - 5*B*b*c^2/(3*sqrt(b*x^2 + a)*a*b^2*c^4 + sqrt(b*x^2 + a)*
b^3*c^6/d^2 + 3*sqrt(b*x^2 + a)*a^2*b*c^2*d^2 + sqrt(b*x^2 + a)*a^3*d^4) +
 B*b*c*x/(sqrt(b*x^2 + a)*a*b^2*c^4/d + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d + sq
rt(b*x^2 + a)*a^3*d^3) + 5/3*B*b*c*x/((b*x^2 + a)^(3/2)*a*b*c^2*d + (b*x^2
 + a)^(3/2)*a^2*d^3) + 10/3*B*b*c*x/(sqrt(b*x^2 + a)*a^2*b*c^2*d + sqrt(b*
x^2 + a)*a^3*d^3) + 5*A*b*c/(sqrt(b*x^2 + a)*b^3*c^6/d^3 + 3*sqrt(b*x^2 +
a)*a*b^2*c^4/d + 3*sqrt(b*x^2 + a)*a^2*b*c^2*d + sqrt(b*x^2 + a)*a^3*d^3)
+ 5/3*A*b*c/((b*x^2 + a)^(3/2)*b^2*c^4/d + 2*(b*x^2 + a)^(3/2)*a*b*c^2*d +
 (b*x^2 + a)^(3/2)*a^2*d^3) - 4/3*A*b*x/((b*x^2 + a)^(3/2)*a*b*c^2 + (b...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1957 vs. $2(293) = 586$.

Time = 0.31 (sec) , antiderivative size = 1957, normalized size of antiderivative = 6.35

$$\int \frac{A+Bx}{(c+dx)^2\,(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*((12*B*a^2*b^(3/2)*c^2*d^6*log(abs(-b*c*d + sqrt(b*c^2 + a*d^2)*sqrt(
b)*abs(d))) - 15*A*a^2*b^(3/2)*c*d^7*log(abs(-b*c*d + sqrt(b*c^2 + a*d^2)*
sqrt(b)*abs(d))) - 3*B*a^3*sqrt(b)*d^8*log(abs(-b*c*d + sqrt(b*c^2 + a*d^2
)*sqrt(b)*abs(d))) + 2*sqrt(b*c^2 + a*d^2)*A*b^3*c^4*d^2*abs(d) - 2*sqrt(b
*c^2 + a*d^2)*B*a*b^2*c^3*d^3*abs(d) + 9*sqrt(b*c^2 + a*d^2)*A*a*b^2*c^2*d
^4*abs(d) + 13*sqrt(b*c^2 + a*d^2)*B*a^2*b*c*d^5*abs(d) - 8*sqrt(b*c^2 + a
*d^2)*A*a^2*b*d^6*abs(d))*sgn(1/(d*x + c))*sgn(d)/(sqrt(b*c^2 + a*d^2)*a^2
*b^(7/2)*c^6*abs(d) + 3*sqrt(b*c^2 + a*d^2)*a^3*b^(5/2)*c^4*d^2*abs(d) + 3
*sqrt(b*c^2 + a*d^2)*a^4*b^(3/2)*c^2*d^4*abs(d) + sqrt(b*c^2 + a*d^2)*a^5*
sqrt(b)*d^6*abs(d)) - ((2*A*b^5*c^4*d^13*sgn(1/(d*x + c))*sgn(d) - 2*B*a*b
^4*c^3*d^14*sgn(1/(d*x + c))*sgn(d) + 9*A*a*b^4*c^2*d^15*sgn(1/(d*x + c))*
sgn(d) + 13*B*a^2*b^3*c*d^16*sgn(1/(d*x + c))*sgn(d) - 8*A*a^2*b^3*d^17*sg
n(1/(d*x + c))*sgn(d))/(a^2*b^4*c^6*d^11*sgn(1/(d*x + c))^2*sgn(d)^2 + 3*a
^3*b^3*c^4*d^13*sgn(1/(d*x + c))^2*sgn(d)^2 + 3*a^4*b^2*c^2*d^15*sgn(1/(d*
x + c))^2*sgn(d)^2 + a^5*b*d^17*sgn(1/(d*x + c))^2*sgn(d)^2) - (3*(2*A*b^5
*c^5*d^14*sgn(1/(d*x + c))*sgn(d) - 2*B*a*b^4*c^4*d^15*sgn(1/(d*x + c))*sg
n(d) + 9*A*a*b^4*c^3*d^16*sgn(1/(d*x + c))*sgn(d) + 17*B*a^2*b^3*c^2*d^17*
sgn(1/(d*x + c))*sgn(d) - 13*A*a^2*b^3*c*d^18*sgn(1/(d*x + c))*sgn(d) - B*
a^3*b^2*d^19*sgn(1/(d*x + c))*sgn(d))/(a^2*b^4*c^6*d^11*sgn(1/(d*x + c))^2
*sgn(d)^2 + 3*a^3*b^3*c^4*d^13*sgn(1/(d*x + c))^2*sgn(d)^2 + 3*a^4*b^2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)^2\left(a+bx^2\right)^{5/2}}\,dx=\int \frac{A+B\,x}{\left(b\,x^2+a\right)^{5/2}\,\left(c+d\,x\right)^2}\,dx$$

input
```
int((A + B*x)/((a + b*x^2)^(5/2)*(c + d*x)^2),x)
```

output
```
int((A + B*x)/((a + b*x^2)^(5/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 2864, normalized size of antiderivative = 9.30

$$\int \frac{A+Bx}{(c+dx)^2\left(a+bx^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((B*x+A)/(d*x+c)^2/(b*x^2+a)^(5/2),x)
```

output
```
(15*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**4*b*c**2*d**4 + 15*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)
*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**4*b*c*d**5*x + 3*sqrt(a*d**2 + b*
c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**4*b*c*d
**5 + 3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
 a*d + b*c*x)*a**4*b*d**6*x - 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2
)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**2*c**3*d**3 + 30*sqrt(a*d**
2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3
*b**2*c**2*d**4*x**2 - 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(
a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**2*c**2*d**4*x + 30*sqrt(a*d**2 + b
*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**2
*c*d**5*x**3 + 6*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 +
b*c**2) - a*d + b*c*x)*a**3*b**2*c*d**5*x**2 + 6*sqrt(a*d**2 + b*c**2)*log
(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b**2*d**6*x**3
 - 24*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a**2*b**3*c**3*d**3*x**2 + 15*sqrt(a*d**2 + b*c**2)*log(sqrt(a
 + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**3*c**2*d**4*x**4 -
 24*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**2*b**3*c**2*d**4*x**3 + 15*sqrt(a*d**2 + b*c**2)*log(sqrt(a +
 b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**3*c*d**5*x**5 + 3...
```

3.194 $\int \frac{A+Bx}{(c+dx)^3\left(a+bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 436

$$\int \frac{A+Bx}{(c+dx)^3\left(a+bx^2\right)^{5/2}}\,dx=\frac{Bc-Ad}{2\left(bc^2+ad^2\right)(c+dx)^2\left(a+bx^2\right)^{3/2}}$$
$$-\frac{2aBd^2-bc(5Bc-7Ad)}{2\left(bc^2+ad^2\right)^2(c+dx)\left(a+bx^2\right)^{3/2}}$$
$$-\frac{b(5a(2bc^2(2Bc-3Ad)-ad^2(3Bc-Ad))-(Abc(2bc^2-33ad^2)+aBd(27bc^2-8ad^2))\,x)}{6a\left(bc^2+ad^2\right)^3\left(a+bx^2\right)^{3/2}}$$
$$-\frac{b(15a^2d^2(2bc^2(2Bc-3Ad)-ad^2(3Bc-Ad))-(Abc(4b^2c^4+28abc^2d^2-81a^2d^4)-aBd(6b^2c^4-83abc}{6a^2\left(bc^2+ad^2\right)^4\sqrt{a+bx^2}}$$
$$-\frac{5bd^3(ad^2(3Bc-Ad)-b(4Bc^3-6Ac^2d))\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{2\left(bc^2+ad^2\right)^{9/2}}$$

output
```
1/2*(-A*d+B*c)/(a*d^2+b*c^2)/(d*x+c)^2/(b*x^2+a)^(3/2)-1/2*(2*a*B*d^2-b*c*
(-7*A*d+5*B*c))/(a*d^2+b*c^2)^2/(d*x+c)/(b*x^2+a)^(3/2)-1/6*b*(5*a*(2*b*c^
2*(-3*A*d+2*B*c)-a*d^2*(-A*d+3*B*c))-(A*b*c*(-33*a*d^2+2*b*c^2)+a*B*d*(-8*
a*d^2+27*b*c^2))*x)/a/(a*d^2+b*c^2)^3/(b*x^2+a)^(3/2)-1/6*b*(15*a^2*d^2*(2
*b*c^2*(-3*A*d+2*B*c)-a*d^2*(-A*d+3*B*c))-(A*b*c*(-81*a^2*d^4+28*a*b*c^2*d
^2+4*b^2*c^4)-a*B*d*(16*a^2*d^4-83*a*b*c^2*d^2+6*b^2*c^4))*x)/a^2/(a*d^2+b
*c^2)^4/(b*x^2+a)^(1/2)-5/2*b*d^3*(a*d^2*(-A*d+3*B*c)-b*(-6*A*c^2*d+4*B*c^
3))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a*d^2+b*c^2
)^(9/2)
```

Mathematica [A] (verified)

Time = 13.93 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx}{(c+dx)^3\,(a+bx^2)^{5/2}}\,dx = \frac{1}{6}\left(\frac{\sqrt{a+bx^2}\left(-\frac{3d^4(-Bc+Ad)(bc^2+ad^2)}{(c+dx)^2}-\frac{3d^4(2aBd^2+bc(-9Bc+11Ad))}{c+dx}+\frac{2b(bc^2+ad^2)(\ldots}{\ldots}\right.}{\ldots}\right.$$
$$-\frac{15bd^3(2bc^2(2Bc-3Ad)+ad^2(-3Bc+Ad))\log(c+dx)}{(bc^2+ad^2)^{9/2}}$$
$$\left.+\frac{15bd^3(2bc^2(2Bc-3Ad)+ad^2(-3Bc+Ad))\log\left(ad-bcx+\sqrt{bc^2+ad^2}\sqrt{a+bx^2}\right)}{(bc^2+ad^2)^{9/2}}\right)$$

input
```
Integrate[(A + B*x)/((c + d*x)^3*(a + b*x^2)^(5/2)),x]
```

output
```
((Sqrt[a + b*x^2]*((-3*d^4*(-(B*c) + A*d)*(b*c^2 + a*d^2))/(c + d*x)^2 - (
3*d^4*(2*a*B*d^2 + b*c*(-9*B*c + 11*A*d)))/(c + d*x) + (2*b*(b*c^2 + a*d^2
)*(A*b^2*c^3*x - a^2*d^2*(-3*B*c + A*d + B*d*x) + a*b*c*(-(B*c*(c - 3*d*x)
) + 3*A*d*(c - d*x))))/(a*(a + b*x^2)^2) + (2*b*(2*A*b^3*c^5*x + a*b^2*c^3
*d*(-3*B*c + 14*A*d)*x + a^3*d^4*(18*B*c - 6*A*d - 5*B*d*x) + 2*a^2*b*c*d^
2*(3*A*d*(5*c - 4*d*x) + B*c*(-9*c + 14*d*x))))/(a^2*(a + b*x^2))))/(b*c^2
 + a*d^2)^4 - (15*b*d^3*(2*b*c^2*(2*B*c - 3*A*d) + a*d^2*(-3*B*c + A*d))*L
og[c + d*x])/(b*c^2 + a*d^2)^(9/2) + (15*b*d^3*(2*b*c^2*(2*B*c - 3*A*d) +
a*d^2*(-3*B*c + A*d))*Log[a*d - b*c*x + Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2
]])/(b*c^2 + a*d^2)^(9/2))/6
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {686, 25, 27, 686, 27, 688, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}(c+dx)^3}dx$$

$$\Big\downarrow \ 686$$

$$-\frac{\int -\frac{b(2Abc^2-3aBdc+5aAd^2+4d(Abc+aBd)x)}{(c+dx)^3(bx^2+a)^{3/2}}dx}{3ab(ad^2+bc^2)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{3a(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{b(2Abc^2-3aBdc+5aAd^2+4d(Abc+aBd)x)}{(c+dx)^3(bx^2+a)^{3/2}}dx}{3ab(ad^2+bc^2)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{3a(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2Abc^2-3aBdc+5aAd^2+4d(Abc+aBd)x}{(c+dx)^3(bx^2+a)^{3/2}}dx}{3a(ad^2+bc^2)}-\frac{a(Bc-Ad)-x(aBd+Abc)}{3a(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}$$

$$\Big\downarrow \ 686$$

$$\frac{-\frac{\int \frac{bd\left(3ad\left(2Abc^2+7aBdc-5aAd^2\right)+2\left(aBd\left(3bc^2-4ad^2\right)-Abc\left(2bc^2+9ad^2\right)\right)x\right)}{(c+dx)^3\sqrt{bx^2+a}}dx}{ab(ad^2+bc^2)}-\frac{x(aBd(3bc^2-4ad^2)-Abc(9ad^2+2bc^2))+ad(-5aAd^2+7aBcd\cdots}{a\sqrt{a+bx^2}(c+dx)^2(ad^2+bc^2)}}{3a(ad^2+bc^2)}$$
$$\frac{a(Bc-Ad)-x(aBd+Abc)}{3a(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{d\int \frac{3ad\left(2Abc^2+7aBdc-5aAd^2\right)+2\left(aBd\left(3bc^2-4ad^2\right)-Abc\left(2bc^2+9ad^2\right)\right)x}{(c+dx)^3\sqrt{bx^2+a}}dx}{a(ad^2+bc^2)}-\frac{x(aBd(3bc^2-4ad^2)-Abc(9ad^2+2bc^2))+ad(-5aAd^2+7aBcd+2\cdots}{a\sqrt{a+bx^2}(c+dx)^2(ad^2+bc^2)}}{3a(ad^2+bc^2)}$$
$$\frac{a(Bc-Ad)-x(aBd+Abc)}{3a(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}$$

$$\Big\downarrow \; 688$$

$$-\frac{-\frac{d\left(\frac{\sqrt{a+bx^2}\left(aBcd\left(6bc^2-29ad^2\right)-A\left(-15a^2d^4+24abc^2d^2+4b^2c^4\right)\right)}{2(c+dx)^2\left(ad^2+bc^2\right)}-\frac{\int -\frac{2ad\left(Abc\left(2bc^2-33ad^2\right)+aBd\left(27bc^2-8ad^2\right)\right)+b\left(aBcd\left(6bc^2-29ad^2\right)-A\left(4b^2c^4+\cdots\right.\right.}{(c+dx)^2\sqrt{bx^2+a}}\cdots}{2\left(ad^2+bc^2\right)}\cdots\right.}{a(ad^2+bc^2)}\cdots}{3a\left(ad^2+bc^2\right)}\cdots$$

$$\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{d\left(\frac{\int \frac{2ad\left(Abc\left(2bc^2-33ad^2\right)+aBd\left(27bc^2-8ad^2\right)\right)+b\left(aBcd\left(6bc^2-29ad^2\right)-A\left(4b^2c^4+24abd^2c^2-15a^2d^4\right)\right)x}{(c+dx)^2\sqrt{bx^2+a}}dx}{2\left(ad^2+bc^2\right)}+\frac{\sqrt{a+bx^2}\left(aBcd\left(6bc^2-29ad^2\right)-A\left(-15a^2\cdots\right.\right.}{2(c+dx)^2\left(ad^2+bc^2\cdots\right.}\cdots\right.}{a(ad^2+bc^2)}\cdots}{3a\left(ad^2+bc^2\right)}\cdots$$

$$\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 679$$

$$-\frac{-\frac{d\left(\frac{\frac{15a^2bd^2\left(2bc^2(2Bc-3Ad)-ad^2(3Bc-Ad)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(Abc\left(-81a^2d^4+28abc^2d^2+4b^2c^4\right)-aBd\left(16a^2d^4-83abc^2d^2+6b^2c^4\right)\right)}{(c+dx)\left(ad^2+bc^2\right)}}{2\left(ad^2+bc^2\right)}+\frac{\sqrt{a\cdots}}{\cdots}\right.}{a(ad^2+bc^2)}\cdots}{3a\left(ad^2+b\cdots\right.}\cdots$$

$$\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 488$$

$$-\frac{-\frac{d\left(\frac{-\frac{15a^2bd^2\left(2bc^2(2Bc-3Ad)-ad^2(3Bc-Ad)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(Abc\left(-81a^2d^4+28abc^2d^2+4b^2c^4\right)-aBd\left(16a^2d^4-83abc^2d^2+\cdots\right.\right.}{(c+dx)\left(ad^2+bc^2\right)}\cdots}{2\left(ad^2+bc^2\right)}\cdots\right.}{a(ad^2+bc^2)}\cdots}{3a\left(a\cdots\right.}\cdots$$

$$\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$-\frac{d\left(\frac{-\frac{15a^2bd^2\left(2bc^2(2Bc-3Ad)-ad^2(3Bc-Ad)\right)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}\left(Abc\left(-81a^2d^4+28abc^2d^2+4b^2c^4\right)-aBd\left(16a^2d^4-83abc^2d^2+\cdots\right.\right.}{(c+dx)\left(ad^2+bc^2\right)}}{2\left(ad^2+bc^2\right)}\cdots\right)}{a(ad^2+bc^2)} \cdots}{3a\,(ad\cdots}$$

$$\frac{a(Bc-Ad)-x(aBd+Abc)}{3a\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2+bc^2\right)}$$

input
```
Int[(A + B*x)/((c + d*x)^3*(a + b*x^2)^(5/2)),x]
```

output
```
-1/3*(a*(B*c - A*d) - (A*b*c + a*B*d)*x)/(a*(b*c^2 + a*d^2)*(c + d*x)^2*(a
 + b*x^2)^(3/2)) + (-((a*d*(2*A*b*c^2 + 7*a*B*c*d - 5*a*A*d^2) + (a*B*d*(3
*b*c^2 - 4*a*d^2) - A*b*c*(2*b*c^2 + 9*a*d^2))*x)/(a*(b*c^2 + a*d^2)*(c +
d*x)^2*Sqrt[a + b*x^2])) - (d*(((a*B*c*d*(6*b*c^2 - 29*a*d^2) - A*(4*b^2*c
^4 + 24*a*b*c^2*d^2 - 15*a^2*d^4))*Sqrt[a + b*x^2])/(2*(b*c^2 + a*d^2)*(c
+ d*x)^2) + (-(((A*b*c*(4*b^2*c^4 + 28*a*b*c^2*d^2 - 81*a^2*d^4) - a*B*d*(
6*b^2*c^4 - 83*a*b*c^2*d^2 + 16*a^2*d^4))*Sqrt[a + b*x^2])/((b*c^2 + a*d^2
)*(c + d*x))) - (15*a^2*b*d^2*(2*b*c^2*(2*B*c - 3*A*d) - a*d^2*(3*B*c - A*
d))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 +
 a*d^2)^(3/2))/(2*(b*c^2 + a*d^2))))/(a*(b*c^2 + a*d^2)))/(3*a*(b*c^2 + a*
d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2510 vs. $2(412) = 824$.

Time = 1.44 (sec) , antiderivative size = 2511, normalized size of antiderivative = 5.76

method	result	size
default	Expression too large to display	2511

input
```
int((B*x+A)/(d*x+c)^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
B/d^3*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(3/2)+5*b*c*d/(a*d^2+b*c^2)*(1/3/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-
2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+b*c*d/(a*d^2+b*c^2)*(2/3*(2*b*(x+
c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(
x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+16/3*b/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^
2)^2*(2*b*(x+c/d)-2*b*c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)
^(1/2))+1/(a*d^2+b*c^2)*d^2*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c
/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(
4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b
*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^
2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*
c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))-4*b/(a*d^2+b*c^2)*d^2*(2/
3*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2
-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+16/3*b/(4*b*(a*d^2+b*c^2)/d^2-4*
b^2*c^2/d^2)^2*(2*b*(x+c/d)-2*b*c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b
*c^2)/d^2)^(1/2)))+(A*d-B*c)/d^4*(-1/2/(a*d^2+b*c^2)*d^2/(x+c/d)^2/(b*(x+c
/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+7/2*b*c*d/(a*d^2+b*c^2)*(-1
/(a*d^2+b*c^2)*d^2/(x+c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)
^(3/2)+5*b*c*d/(a*d^2+b*c^2)*(1/3/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(
x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)+b*c*d/(a*d^2+b*c^2)*(2/3*(2*b*(x+c/d)-2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1934 vs. $2(413) = 826$.

Time = 4.94 (sec) , antiderivative size = 3894, normalized size of antiderivative = 8.93

$$\int \frac{A+Bx}{(c+dx)^3 (a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)^3 (a+bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(d*x+c)**3/(b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2543 vs. $2(413) = 826$.

Time $= 0.24$ (sec) , antiderivative size $= 2543$, normalized size of antiderivative $= 5.83$

$$\int \frac{A+Bx}{(c+dx)^3 (a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-35/2*B*b^3*c^4*x/(sqrt(b*x^2 + a)*a*b^4*c^8/d + 4*sqrt(b*x^2 + a)*a^2*b^3
*c^6*d + 6*sqrt(b*x^2 + a)*a^3*b^2*c^4*d^3 + 4*sqrt(b*x^2 + a)*a^4*b*c^2*d
^5 + sqrt(b*x^2 + a)*a^5*d^7) - 35/6*B*b^3*c^4*x/((b*x^2 + a)^(3/2)*a*b^3*
c^6*d + 3*(b*x^2 + a)^(3/2)*a^2*b^2*c^4*d^3 + 3*(b*x^2 + a)^(3/2)*a^3*b*c^
2*d^5 + (b*x^2 + a)^(3/2)*a^4*d^7) - 35/3*B*b^3*c^4*x/(sqrt(b*x^2 + a)*a^2
*b^3*c^6*d + 3*sqrt(b*x^2 + a)*a^3*b^2*c^4*d^3 + 3*sqrt(b*x^2 + a)*a^4*b*c
^2*d^5 + sqrt(b*x^2 + a)*a^5*d^7) + 35/6*A*b^3*c^3*x/((b*x^2 + a)^(3/2)*a*
b^3*c^6 + 3*(b*x^2 + a)^(3/2)*a^2*b^2*c^4*d^2 + 3*(b*x^2 + a)^(3/2)*a^3*b*
c^2*d^4 + (b*x^2 + a)^(3/2)*a^4*d^6) + 35/2*A*b^3*c^3*x/(4*sqrt(b*x^2 + a)
*a^2*b^3*c^6 + sqrt(b*x^2 + a)*a*b^4*c^8/d^2 + 6*sqrt(b*x^2 + a)*a^3*b^2*c
^4*d^2 + 4*sqrt(b*x^2 + a)*a^4*b*c^2*d^4 + sqrt(b*x^2 + a)*a^5*d^6) + 35/3
*A*b^3*c^3*x/(sqrt(b*x^2 + a)*a^2*b^3*c^6 + 3*sqrt(b*x^2 + a)*a^3*b^2*c^4*
d^2 + 3*sqrt(b*x^2 + a)*a^4*b*c^2*d^4 + sqrt(b*x^2 + a)*a^5*d^6) - 35/6*B*
b^2*c^3/((b*x^2 + a)^(3/2)*b^3*c^6 + 3*(b*x^2 + a)^(3/2)*a*b^2*c^4*d^2 + 3
*(b*x^2 + a)^(3/2)*a^2*b*c^2*d^4 + (b*x^2 + a)^(3/2)*a^3*d^6) - 35/2*B*b^2
*c^3/(4*sqrt(b*x^2 + a)*a*b^3*c^6 + sqrt(b*x^2 + a)*b^4*c^8/d^2 + 6*sqrt(b
*x^2 + a)*a^2*b^2*c^4*d^2 + 4*sqrt(b*x^2 + a)*a^3*b*c^2*d^4 + sqrt(b*x^2 +
 a)*a^4*d^6) + 15/2*B*b^2*c^2*x/(sqrt(b*x^2 + a)*a*b^3*c^6/d + 3*sqrt(b*x^
2 + a)*a^2*b^2*c^4*d + 3*sqrt(b*x^2 + a)*a^3*b*c^2*d^3 + sqrt(b*x^2 + a)*a
^4*d^5) + 43/6*B*b^2*c^2*x/((b*x^2 + a)^(3/2)*a*b^2*c^4*d + 2*(b*x^2 + ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3353 vs. $2(413) = 826$.

Time = 0.23 (sec) , antiderivative size = 3353, normalized size of antiderivative = 7.69

$$\int \frac{A + Bx}{(c + dx)^3 (a + bx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
5*(4*B*b^2*c^3*d^3 - 6*A*b^2*c^2*d^4 - 3*B*a*b*c*d^5 + A*a*b*d^6)*arctan((
(sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b^4*c
^8 + 4*a*b^3*c^6*d^2 + 6*a^2*b^2*c^4*d^4 + 4*a^3*b*c^2*d^6 + a^4*d^8)*sqrt
(-b*c^2 - a*d^2)) + 1/3*((((2*A*b^18*c^29 - 3*B*a*b^17*c^28*d + 38*A*a*b^1
7*c^27*d^2 - 8*B*a^2*b^16*c^26*d^3 + 276*A*a^2*b^16*c^25*d^4 + 133*B*a^3*b
^15*c^24*d^5 + 1076*A*a^3*b^15*c^23*d^6 + 1128*B*a^4*b^14*c^22*d^7 + 2486*
A*a^4*b^14*c^21*d^8 + 4345*B*a^5*b^13*c^20*d^9 + 3234*A*a^5*b^13*c^19*d^10
 + 10384*B*a^6*b^12*c^18*d^11 + 1056*A*a^6*b^12*c^17*d^12 + 16929*B*a^7*b^
11*c^16*d^13 - 4488*A*a^7*b^11*c^15*d^14 + 19536*B*a^8*b^10*c^14*d^15 - 10
098*A*a^8*b^10*c^13*d^16 + 16071*B*a^9*b^9*c^12*d^17 - 11638*A*a^9*b^9*c^1
1*d^18 + 9240*B*a^10*b^8*c^10*d^19 - 8668*A*a^10*b^8*c^9*d^20 + 3487*B*a^1
1*b^7*c^8*d^21 - 4332*A*a^11*b^7*c^7*d^22 + 712*B*a^12*b^6*c^6*d^23 - 1414
*A*a^12*b^6*c^5*d^24 + 3*B*a^13*b^5*c^4*d^25 - 274*A*a^13*b^5*c^3*d^26 - 3
2*B*a^14*b^4*c^2*d^27 - 24*A*a^14*b^4*c*d^28 - 5*B*a^15*b^3*d^29)*x/(a^2*b
^17*c^32 + 16*a^3*b^16*c^30*d^2 + 120*a^4*b^15*c^28*d^4 + 560*a^5*b^14*c^2
6*d^6 + 1820*a^6*b^13*c^24*d^8 + 4368*a^7*b^12*c^22*d^10 + 8008*a^8*b^11*c
^20*d^12 + 11440*a^9*b^10*c^18*d^14 + 12870*a^10*b^9*c^16*d^16 + 11440*a^1
1*b^8*c^14*d^18 + 8008*a^12*b^7*c^12*d^20 + 4368*a^13*b^6*c^10*d^22 + 1820
*a^14*b^5*c^8*d^24 + 560*a^15*b^4*c^6*d^26 + 120*a^16*b^3*c^4*d^28 + 16*a^
17*b^2*c^2*d^30 + a^18*b*d^32) - 6*(3*B*a^2*b^16*c^27*d^2 - 5*A*a^2*b^1...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(c+dx)^3 \left(a+bx^2\right)^{5/2}}\,dx = \int \frac{A+B\,x}{{\left(b\,x^2+a\right)}^{5/2}\,{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((A + B*x)/((a + b*x^2)^(5/2)*(c + d*x)^3),x)
```

output
```
int((A + B*x)/((a + b*x^2)^(5/2)*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(c+dx)^3(a+bx^2)^{5/2}}\,dx=\int \frac{Bx+A}{(dx+c)^3(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((B*x+A)/(d*x+c)^3/(b*x^2+a)^(5/2),x)
```

output
```
int((B*x+A)/(d*x+c)^3/(b*x^2+a)^(5/2),x)
```

3.195 $\int (5-x)(3+2x)^4\sqrt{2+3x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 122

$$\begin{aligned}\int (5-x)(3+2x)^4\sqrt{2+3x^2}\,dx &= \frac{2341}{18}x\sqrt{2+3x^2}+\frac{923}{315}(3+2x)^2\left(2+3x^2\right)^{3/2}+\frac{29}{63}(3+2x)^3\left(2+3x^2\right)^{3/2}\\&\quad-\frac{1}{21}(3+2x)^4\left(2+3x^2\right)^{3/2}+\frac{2}{405}(13781+4599x)\left(2+3x^2\right)^{3/2}+\frac{2341\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}\end{aligned}$$

output
```
2341/18*x*(3*x^2+2)^(1/2)+923/315*(3+2*x)^2*(3*x^2+2)^(3/2)+29/63*(3+2*x)^
3*(3*x^2+2)^(3/2)-1/21*(3+2*x)^4*(3*x^2+2)^(3/2)+2/405*(13781+4599*x)*(3*x
^2+2)^(3/2)+2341/27*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.62

$$\begin{aligned}\int (5-x)(3+2x)^4\sqrt{2+3x^2}\,dx &=\\&-\frac{\sqrt{2+3x^2}\left(-1167988-1558935x-1956174x^2-1222200x^3-297648x^4+15120x^5+12960x^6\right)}{5670}\\&-\frac{2341\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{9\sqrt{3}}\end{aligned}$$

input
```
Integrate[(5 - x)*(3 + 2*x)^4*Sqrt[2 + 3*x^2],x]
```

output
```
-1/5670*(Sqrt[2 + 3*x^2]*(-1167988 - 1558935*x - 1956174*x^2 - 1222200*x^3
 - 297648*x^4 + 15120*x^5 + 12960*x^6)) - (2341*Log[-(Sqrt[3]*x) + Sqrt[2
+ 3*x^2]])/(9*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {687, 687, 27, 687, 27, 676, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-x)(2x+3)^4\sqrt{3x^2+2}\,dx$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{21}\int (2x+3)^3(174x+331)\sqrt{3x^2+2}dx-\frac{1}{21}(2x+3)^4\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{21}\left(\frac{1}{18}\int 18(2x+3)^2(923x+877)\sqrt{3x^2+2}dx+\frac{29}{3}\left(3x^2+2\right)^{3/2}(2x+3)^3\right)-\frac{1}{21}(2x+3)^4\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{21}\left(\int (2x+3)^2(923x+877)\sqrt{3x^2+2}dx+\frac{29}{3}\left(3x^2+2\right)^{3/2}(2x+3)^3\right)-\frac{1}{21}(2x+3)^4\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{21}\left(\frac{1}{15}\int 7(2x+3)(6132x+4583)\sqrt{3x^2+2}dx+\frac{29}{3}\left(3x^2+2\right)^{3/2}(2x+3)^3+\frac{923}{15}\left(3x^2+2\right)^{3/2}(2x+3)^2\right)-\frac{1}{21}(2x+3)^4\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{21}\left(\frac{7}{15}\int (2x+3)(6132x+4583)\sqrt{3x^2+2}dx+\frac{29}{3}\left(3x^2+2\right)^{3/2}(2x+3)^3+\frac{923}{15}\left(3x^2+2\right)^{3/2}(2x+3)^2\right)-\frac{1}{21}(2x+3)^4\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \textbf{676}$$

$$\frac{1}{21}\left(\frac{7}{15}\left(11705\int \sqrt{3x^2+2}dx+1022x\left(3x^2+2\right)^{3/2}+\frac{27562}{9}\left(3x^2+2\right)^{3/2}\right)+\frac{29}{3}\left(3x^2+2\right)^{3/2}(2x+3)^3+\frac{923}{15}\right.$$
$$\frac{1}{21}(2x+3)^4\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \textbf{211}$$

$$\frac{1}{21}\left(\frac{7}{15}\left(11705\left(\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\sqrt{3x^2+2}x\right)+1022x\left(3x^2+2\right)^{3/2}+\frac{27562}{9}\left(3x^2+2\right)^{3/2}\right)+\frac{29}{3}\left(3x^2+2\right)^{3/}\right.$$
$$\frac{1}{21}(2x+3)^4\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \textbf{222}$$

$$\frac{1}{21}\left(\frac{7}{15}\left(11705\left(\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3x^2+2}x\right)+1022x\left(3x^2+2\right)^{3/2}+\frac{27562}{9}\left(3x^2+2\right)^{3/2}\right)+\frac{29}{3}\left(3x^2+2\right)\right.$$
$$\frac{1}{21}(2x+3)^4\left(3x^2+2\right)^{3/2}$$

input
```
Int[(5 - x)*(3 + 2*x)^4*Sqrt[2 + 3*x^2],x]
```

output
```
-1/21*((3 + 2*x)^4*(2 + 3*x^2)^(3/2)) + ((923*(3 + 2*x)^2*(2 + 3*x^2)^(3/2
))/15 + (29*(3 + 2*x)^3*(2 + 3*x^2)^(3/2))/3 + (7*((27562*(2 + 3*x^2)^(3/2
))/9 + 1022*x*(2 + 3*x^2)^(3/2) + 11705*((x*Sqrt[2 + 3*x^2])/2 + ArcSinh[S
qrt[3/2]*x]/Sqrt[3])))/15)/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.45

method	result
risch	$-\frac{(12960x^6+15120x^5-297648x^4-1222200x^3-1956174x^2-1558935x-1167988)\sqrt{3x^2+2}}{5670}+\frac{2341\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{27}$
trager	$\left(-\frac{16}{7}x^6-\frac{8}{3}x^5+\frac{5512}{105}x^4+\frac{1940}{9}x^3+\frac{326029}{945}x^2+\frac{4949}{18}x+\frac{583994}{2835}\right)\sqrt{3x^2+2}+\frac{2341\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\right.}{}$
default	$\frac{2341x\sqrt{3x^2+2}}{18}+\frac{2341\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{27}+\frac{291997(3x^2+2)^{\frac{3}{2}}}{2835}+\frac{652x(3x^2+2)^{\frac{3}{2}}}{9}+\frac{5672x^2(3x^2+2)^{\frac{3}{2}}}{315}-\frac{8x^3(3x^2+2)^{\frac{3}{2}}}{9}-$
meijerg	$-\frac{135\sqrt{3}\left(-\sqrt{6}\,\sqrt{\pi}\,x\sqrt{\frac{3x^2}{2}+1}-2\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{2\sqrt{\pi}}+\frac{32\sqrt{3}\left(\frac{\sqrt{6}\,\sqrt{\pi}\,x\left(-90x^4-15x^2+15\right)\sqrt{\frac{3x^2}{2}+1}}{120}-\frac{\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{4}\right)}{27\sqrt{\pi}}$

input
```
int((5-x)*(2*x+3)^4*(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5670*(12960*x^6+15120*x^5-297648*x^4-1222200*x^3-1956174*x^2-1558935*x-
1167988)*(3*x^2+2)^(1/2)+2341/27*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.57

$$\int(5-x)(3+2x)^4\sqrt{2+3x^2}\,dx = -\frac{1}{5670}\left(12960\,x^6+15120\,x^5-297648\,x^4-1222200\,x^3-1956174\,x^2-1558935\,x-1167988\right)\sqrt{3\,x^2+} + \frac{2341}{54}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3+2*x)^4*(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/5670*(12960*x^6 + 15120*x^5 - 297648*x^4 - 1222200*x^3 - 1956174*x^2 -
1558935*x - 1167988)*sqrt(3*x^2 + 2) + 2341/54*sqrt(3)*log(-sqrt(3)*sqrt(3
*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int (5-x)(3+2x)^4\sqrt{2+3x^2}\,dx = -\frac{16x^6\sqrt{3x^2+2}}{7} - \frac{8x^5\sqrt{3x^2+2}}{3} + \frac{5512x^4\sqrt{3x^2+2}}{105} + \frac{1940x^3\sqrt{3x^2+2}}{9} + \frac{326029x^2\sqrt{3x^2+2}}{945} + \frac{4949x\sqrt{3x^2+2}}{18} + \frac{583994\sqrt{3x^2+2}}{2835} + \frac{2341\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27}$$

input
```
integrate((5-x)*(3+2*x)**4*(3*x**2+2)**(1/2),x)
```

output
```
-16*x**6*sqrt(3*x**2 + 2)/7 - 8*x**5*sqrt(3*x**2 + 2)/3 + 5512*x**4*sqrt(3
*x**2 + 2)/105 + 1940*x**3*sqrt(3*x**2 + 2)/9 + 326029*x**2*sqrt(3*x**2 +
2)/945 + 4949*x*sqrt(3*x**2 + 2)/18 + 583994*sqrt(3*x**2 + 2)/2835 + 2341*
sqrt(3)*asinh(sqrt(6)*x/2)/27
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

$$\int (5-x)(3+2x)^4\sqrt{2+3x^2}\,dx = -\frac{16}{21}\left(3x^2+2\right)^{\frac{3}{2}}x^4 - \frac{8}{9}\left(3x^2+2\right)^{\frac{3}{2}}x^3 + \frac{5672}{315}\left(3x^2+2\right)^{\frac{3}{2}}x^2 + \frac{652}{9}\left(3x^2+2\right)^{\frac{3}{2}}x + \frac{291997}{2835}\left(3x^2+2\right)^{\frac{3}{2}} + \frac{2341}{18}\sqrt{3x^2+2}x + \frac{2341}{27}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)$$

input
```
integrate((5-x)*(3+2*x)^4*(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
-16/21*(3*x^2 + 2)^(3/2)*x^4 - 8/9*(3*x^2 + 2)^(3/2)*x^3 + 5672/315*(3*x^2
 + 2)^(3/2)*x^2 + 652/9*(3*x^2 + 2)^(3/2)*x + 291997/2835*(3*x^2 + 2)^(3/2
) + 2341/18*sqrt(3*x^2 + 2)*x + 2341/27*sqrt(3)*arcsinh(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

$$\int (5-x)(3+2x)^4\sqrt{2+3x^2}\,dx =$$
$$-\frac{1}{5670}\left(3\left(2\left(12\left(6\left(5\left(6\,x+7\right)x-689\right)x-16975\right)x-326029\right)x-519645\right)x-1167988\right)\sqrt{3\,x^2+2}$$
$$-\frac{2341}{27}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

input
```
integrate((5-x)*(3+2*x)^4*(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-1/5670*(3*(2*(12*(6*(5*(6*x + 7)*x - 689)*x - 16975)*x - 326029)*x - 5196
45)*x - 1167988)*sqrt(3*x^2 + 2) - 2341/27*sqrt(3)*log(-sqrt(3)*x + sqrt(3
*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.45

$$\int (5-x)(3+2x)^4\sqrt{2+3x^2}\,dx$$
$$=\frac{2341\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{27}$$
$$+\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(-\frac{48\,x^6}{7}-8\,x^5+\frac{5512\,x^4}{35}+\frac{1940\,x^3}{3}+\frac{326029\,x^2}{315}+\frac{4949\,x}{6}+\frac{583994}{945}\right)}{3}$$

input
```
int(-(2*x + 3)^4*(3*x^2 + 2)^(1/2)*(x - 5),x)
```

output
```
(2341*3^(1/2)*asinh((6^(1/2)*x)/2))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((4949
*x)/6 + (326029*x^2)/315 + (1940*x^3)/3 + (5512*x^4)/35 - 8*x^5 - (48*x^6)
/7 + 583994/945))/3
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.90

$$\int (5-x)(3+2x)^4\sqrt{2+3x^2}\,dx = -\frac{16\sqrt{3x^2+2}\,x^6}{7} - \frac{8\sqrt{3x^2+2}\,x^5}{3} + \frac{5512\sqrt{3x^2+2}\,x^4}{105} + \frac{1940\sqrt{3x^2+2}\,x^3}{9} + \frac{326029\sqrt{3x^2+2}\,x^2}{945} + \frac{4949\sqrt{3x^2+2}\,x}{18} + \frac{583994\sqrt{3x^2+2}}{2835} + \frac{2341\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{27}$$

input
```
int((5-x)*(3+2*x)^4*(3*x^2+2)^(1/2),x)
```

output
```
( - 12960*sqrt(3*x**2 + 2)*x**6 - 15120*sqrt(3*x**2 + 2)*x**5 + 297648*sqr
t(3*x**2 + 2)*x**4 + 1222200*sqrt(3*x**2 + 2)*x**3 + 1956174*sqrt(3*x**2 +
 2)*x**2 + 1558935*sqrt(3*x**2 + 2)*x + 1167988*sqrt(3*x**2 + 2) + 491610*
sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/5670
```

3.196 $\int (5-x)(3+2x)^3\sqrt{2+3x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int (5-x)(3+2x)^3\sqrt{2+3x^2}\,dx = \frac{511}{9}x\sqrt{2+3x^2}+\frac{17}{30}(3+2x)^2\left(2+3x^2\right)^{3/2} - \frac{1}{18}(3+2x)^3\left(2+3x^2\right)^{3/2} + \frac{7}{270}(898+267x)\left(2+3x^2\right)^{3/2}+\frac{1022\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}$$

output
```
511/9*x*(3*x^2+2)^(1/2)+17/30*(3+2*x)^2*(3*x^2+2)^(3/2)-1/18*(3+2*x)^3*(3*
x^2+2)^(3/2)+7/270*(898+267*x)*(3*x^2+2)^(3/2)+1022/27*arcsinh(1/2*x*6^(1/
2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.71

$$\int (5-x)(3+2x)^3\sqrt{2+3x^2}\,dx = -\frac{1}{270}\sqrt{2+3x^2}\left(-14516-21120x-21918x^2-8445x^3 - 216x^4+360x^5\right)-\frac{1022\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{9\sqrt{3}}$$

input
```
Integrate[(5 - x)*(3 + 2*x)^3*Sqrt[2 + 3*x^2],x]
```

output
```
-1/270*(Sqrt[2 + 3*x^2]*(-14516 - 21120*x - 21918*x^2 - 8445*x^3 - 216*x^4
 + 360*x^5)) - (1022*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(9*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {687, 27, 687, 27, 676, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-x)(2x+3)^3\sqrt{3x^2+2}\,dx$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{18}\int 3(2x+3)^2(51x+94)\sqrt{3x^2+2}dx-\frac{1}{18}(2x+3)^3\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int (2x+3)^2(51x+94)\sqrt{3x^2+2}dx-\frac{1}{18}(2x+3)^3\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{6}\left(\frac{1}{15}\int 42(2x+3)(89x+91)\sqrt{3x^2+2}dx+\frac{17}{5}\left(3x^2+2\right)^{3/2}(2x+3)^2\right)-\frac{1}{18}(2x+3)^3\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{14}{5}\int (2x+3)(89x+91)\sqrt{3x^2+2}dx+\frac{17}{5}\left(3x^2+2\right)^{3/2}(2x+3)^2\right)-\frac{1}{18}(2x+3)^3\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{676}$$

$$\frac{1}{6}\left(\frac{14}{5}\left(\frac{730}{3}\int \sqrt{3x^2+2}dx+\frac{89}{6}x\left(3x^2+2\right)^{3/2}+\frac{449}{9}\left(3x^2+2\right)^{3/2}\right)+\frac{17}{5}\left(3x^2+2\right)^{3/2}(2x+3)^2\right)-\frac{1}{18}(2x+3)^3\left(3x^2+2\right)^{3/2}$$

$\Big\downarrow$ 211

$$\frac{1}{6}\left(\frac{14}{5}\left(\frac{730}{3}\left(\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{89}{6}x\left(3x^2+2\right)^{3/2}+\frac{449}{9}\left(3x^2+2\right)^{3/2}\right)+\frac{17}{5}\left(3x^2+2\right)^{3/2}(2x+\right.$$
$$\frac{1}{18}(2x+3)^3\left(3x^2+2\right)^{3/2}$$

$\Big\downarrow$ 222

$$\frac{1}{6}\left(\frac{14}{5}\left(\frac{730}{3}\left(\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{89}{6}x\left(3x^2+2\right)^{3/2}+\frac{449}{9}\left(3x^2+2\right)^{3/2}\right)+\frac{17}{5}\left(3x^2+2\right)^{3/2}(2x\right.$$
$$\frac{1}{18}(2x+3)^3\left(3x^2+2\right)^{3/2}$$

input
```
Int[(5 - x)*(3 + 2*x)^3*Sqrt[2 + 3*x^2],x]
```

output
```
-1/18*((3 + 2*x)^3*(2 + 3*x^2)^(3/2)) + ((17*(3 + 2*x)^2*(2 + 3*x^2)^(3/2)
)/5 + (14*((449*(2 + 3*x^2)^(3/2))/9 + (89*x*(2 + 3*x^2)^(3/2))/6 + (730*(
(x*Sqrt[2 + 3*x^2])/2 + ArcSinh[Sqrt[3/2]*x]/Sqrt[3]))/3))/5)/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

method	result
risch	$-\frac{(360x^5-216x^4-8445x^3-21918x^2-21120x-14516)\sqrt{3x^2+2}}{270}+\frac{1022\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{27}$
trager	$\left(-\frac{4}{3}x^5+\frac{4}{5}x^4+\frac{563}{18}x^3+\frac{3653}{45}x^2+\frac{704}{9}x+\frac{7258}{135}\right)\sqrt{3x^2+2}-\frac{1022\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2-3\right)\right.}{27}$
default	$\frac{511x\sqrt{3x^2+2}}{9}+\frac{1022\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{27}+\frac{3629(3x^2+2)^{\frac{3}{2}}}{135}+\frac{193x(3x^2+2)^{\frac{3}{2}}}{18}+\frac{4x^2(3x^2+2)^{\frac{3}{2}}}{15}-\frac{4x^3(3x^2+2)^{\frac{3}{2}}}{9}$
meijerg	$-\frac{45\sqrt{3}\left(-\sqrt{6}\sqrt{\pi}\,x\sqrt{\frac{3x^2}{2}+1}-2\sqrt{\pi}\operatorname{arcsinh}\left(\frac{x\sqrt{2}\sqrt{3}}{2}\right)\right)}{2\sqrt{\pi}}-\frac{4\sqrt{2}\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}\left(-\frac{9x^2}{2}+2\right)}{15}\right)}{9\sqrt{\pi}}-\frac{14\sqrt{3}\left(-\frac{\sqrt{6}\sqrt{\pi}\,x(9\right.}{}$

input
```
int((5-x)*(2*x+3)^3*(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/270*(360*x^5-216*x^4-8445*x^3-21918*x^2-21120*x-14516)*(3*x^2+2)^(1/2)+
1022/27*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.65

$$\int (5-x)(3+2x)^3\sqrt{2+3x^2}\,dx = -\frac{1}{270}\left(360\,x^5-216\,x^4-8445\,x^3-21918\,x^2-21120\,x-14516\right)\sqrt{3\,x^2+2} + \frac{511}{27}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3+2*x)^3*(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/270*(360*x^5 - 216*x^4 - 8445*x^3 - 21918*x^2 - 21120*x - 14516)*sqrt(3
*x^2 + 2) + 511/27*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.63

$$\int (5-x)(3+2x)^3\sqrt{2+3x^2}\,dx = \sqrt{3x^2+2}\left(-\frac{4x^5}{3}+\frac{4x^4}{5}+\frac{563x^3}{18}+\frac{3653x^2}{45}+\frac{704x}{9}+\frac{7258}{135}\right)+\frac{1022\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27}$$

input
```
integrate((5-x)*(3+2*x)**3*(3*x**2+2)**(1/2),x)
```

output
```
sqrt(3*x**2 + 2)*(-4*x**5/3 + 4*x**4/5 + 563*x**3/18 + 3653*x**2/45 + 704*
x/9 + 7258/135) + 1022*sqrt(3)*asinh(sqrt(6)*x/2)/27
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int (5-x)(3+2x)^3\sqrt{2+3x^2}\,dx = -\frac{4}{9}\left(3x^2+2\right)^{\frac{3}{2}}x^3 + \frac{4}{15}\left(3x^2+2\right)^{\frac{3}{2}}x^2 + \frac{193}{18}\left(3x^2+2\right)^{\frac{3}{2}}x + \frac{3629}{135}\left(3x^2+2\right)^{\frac{3}{2}} + \frac{511}{9}\sqrt{3x^2+2}x + \frac{1022}{27}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)$$

input
```
integrate((5-x)*(3+2*x)^3*(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
-4/9*(3*x^2 + 2)^(3/2)*x^3 + 4/15*(3*x^2 + 2)^(3/2)*x^2 + 193/18*(3*x^2 +
2)^(3/2)*x + 3629/135*(3*x^2 + 2)^(3/2) + 511/9*sqrt(3*x^2 + 2)*x + 1022/2
7*sqrt(3)*arcsinh(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.57

$$\int (5-x)(3+2x)^3\sqrt{2+3x^2}\,dx = -\frac{1}{270}\left(3\left(\left(\left(24\left(5x-3\right)x-2815\right)x-7306\right)x-7040\right)x-14516\right)\sqrt{3x^2+2} - \frac{1022}{27}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)$$

input
```
integrate((5-x)*(3+2*x)^3*(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-1/270*(3*((((24*(5*x - 3)*x - 2815)*x - 7306)*x - 7040)*x - 14516)*sqrt(3*
x^2 + 2) - 1022/27*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

$$\int (5-x)(3+2x)^3\sqrt{2+3x^2}\,dx$$
$$=\frac{1022\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{27}+\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(-4\,x^5+\frac{12\,x^4}{5}+\frac{563\,x^3}{6}+\frac{3653\,x^2}{15}+\frac{704\,x}{3}+\frac{7258}{45}\right)}{3}$$

input
```
int(-(2*x + 3)^3*(3*x^2 + 2)^(1/2)*(x - 5),x)
```

output
```
(1022*3^(1/2)*asinh((6^(1/2)*x)/2))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((704*
x)/3 + (3653*x^2)/15 + (563*x^3)/6 + (12*x^4)/5 - 4*x^5 + 7258/45))/3
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int (5-x)(3+2x)^3\sqrt{2+3x^2}\,dx=-\frac{4\sqrt{3x^2+2}\,x^5}{3}+\frac{4\sqrt{3x^2+2}\,x^4}{5}+\frac{563\sqrt{3x^2+2}\,x^3}{18}+\frac{3653\sqrt{3x^2+2}\,x^2}{45}+\frac{704\sqrt{3x^2+2}\,x}{9}+\frac{7258\sqrt{3x^2+2}}{135}+\frac{1022\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{27}$$

input
```
int((5-x)*(3+2*x)^3*(3*x^2+2)^(1/2),x)
```

output
```
( - 360*sqrt(3*x**2 + 2)*x**5 + 216*sqrt(3*x**2 + 2)*x**4 + 8445*sqrt(3*x*
*2 + 2)*x**3 + 21918*sqrt(3*x**2 + 2)*x**2 + 21120*sqrt(3*x**2 + 2)*x + 14
516*sqrt(3*x**2 + 2) + 10220*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sq
rt(2)))/270
```

3.197 $\int (5-x)(3+2x)^2\sqrt{2+3x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 78

$$\int (5-x)(3+2x)^2\sqrt{2+3x^2}\,dx=\frac{131}{6}x\sqrt{2+3x^2}-\frac{1}{15}(3+2x)^2\left(2+3x^2\right)^{3/2}+\frac{2}{135}(431+99x)\left(2+3x^2\right)^{3/2}+\frac{131\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
131/6*x*(3*x^2+2)^(1/2)-1/15*(3+2*x)^2*(3*x^2+2)^(3/2)+2/135*(431+99*x)*(3
*x^2+2)^(3/2)+131/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int (5-x)(3+2x)^2\sqrt{2+3x^2}\,dx=-\frac{1}{270}\sqrt{2+3x^2}\left(-3124-6255x-4542x^2-540x^3+216x^4\right)-\frac{131\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[(5 - x)*(3 + 2*x)^2*Sqrt[2 + 3*x^2],x]
```

output
```
-1/270*(Sqrt[2 + 3*x^2]*(-3124 - 6255*x - 4542*x^2 - 540*x^3 + 216*x^4)) -
 (131*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {687, 676, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-x)(2x+3)^2\sqrt{3x^2+2}\,dx$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{15}\int (2x+3)(132x+233)\sqrt{3x^2+2}dx-\frac{1}{15}(2x+3)^2\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{676}$$

$$\frac{1}{15}\left(655\int \sqrt{3x^2+2}dx+22x\left(3x^2+2\right)^{3/2}+\frac{862}{9}\left(3x^2+2\right)^{3/2}\right)-\frac{1}{15}(2x+3)^2\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{15}\left(655\left(\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\sqrt{3x^2+2}x\right)+22x\left(3x^2+2\right)^{3/2}+\frac{862}{9}\left(3x^2+2\right)^{3/2}\right)-\frac{1}{15}(2x+3)^2\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{15}\left(655\left(\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3x^2+2}x\right)+22x\left(3x^2+2\right)^{3/2}+\frac{862}{9}\left(3x^2+2\right)^{3/2}\right)-\frac{1}{15}(2x+3)^2\left(3x^2+2\right)^{3/2}$$

input
```
Int[(5 - x)*(3 + 2*x)^2*Sqrt[2 + 3*x^2],x]
```

output
```
-1/15*((3 + 2*x)^2*(2 + 3*x^2)^(3/2)) + ((862*(2 + 3*x^2)^(3/2))/9 + 22*x*
(2 + 3*x^2)^(3/2) + 655*((x*Sqrt[2 + 3*x^2])/2 + ArcSinh[Sqrt[3/2]*x]/Sqrt
[3]))/15
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{(216x^4-540x^3-4542x^2-6255x-3124)\sqrt{3x^2+2}}{270}+\frac{131\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}$
trager	$\left(-\frac{4}{5}x^4+2x^3+\frac{757}{45}x^2+\frac{139}{6}x+\frac{1562}{135}\right)\sqrt{3x^2+2}-\frac{131\,\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right.}{9}$
default	$\frac{131x\sqrt{3x^2+2}}{6}+\frac{131\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}+\frac{781(3x^2+2)^{\frac{3}{2}}}{135}+\frac{2x(3x^2+2)^{\frac{3}{2}}}{3}-\frac{4x^2(3x^2+2)^{\frac{3}{2}}}{15}$
meijerg	$-\frac{15\sqrt{3}\left(-\sqrt{6}\,\sqrt{\pi}\,x\sqrt{\frac{3x^2}{2}+1}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{2\sqrt{\pi}}-\frac{8\sqrt{3}\left(-\frac{\sqrt{6}\,\sqrt{\pi}\,x\left(9x^2+3\right)\sqrt{\frac{3x^2}{2}+1}}{12}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{2}\right)}{9\sqrt{\pi}}-\frac{17\sqrt{2}}{}$

input
```
int((5-x)*(2*x+3)^2*(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/270*(216*x^4-540*x^3-4542*x^2-6255*x-3124)*(3*x^2+2)^(1/2)+131/9*arcsin
h(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77

$$\int (5-x)(3+2x)^2\sqrt{2+3x^2}\,dx$$
$$=-\frac{1}{270}\left(216\,x^4-540\,x^3-4542\,x^2-6255\,x-3124\right)\sqrt{3\,x^2+2}$$
$$+\frac{131}{18}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3+2*x)^2*(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/270*(216*x^4 - 540*x^3 - 4542*x^2 - 6255*x - 3124)*sqrt(3*x^2 + 2) + 13
1/18*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.22

$$\int (5-x)(3+2x)^2\sqrt{2+3x^2}\,dx = -\frac{4x^4\sqrt{3x^2+2}}{5} + 2x^3\sqrt{3x^2+2} + \frac{757x^2\sqrt{3x^2+2}}{45} + \frac{139x\sqrt{3x^2+2}}{6} + \frac{1562\sqrt{3x^2+2}}{135} + \frac{131\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

input
```
integrate((5-x)*(3+2*x)**2*(3*x**2+2)**(1/2),x)
```

output
```
-4*x**4*sqrt(3*x**2 + 2)/5 + 2*x**3*sqrt(3*x**2 + 2) + 757*x**2*sqrt(3*x**
2 + 2)/45 + 139*x*sqrt(3*x**2 + 2)/6 + 1562*sqrt(3*x**2 + 2)/135 + 131*sqr
t(3)*asinh(sqrt(6)*x/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int (5-x)(3+2x)^2\sqrt{2+3x^2}\,dx = -\frac{4}{15}\left(3x^2+2\right)^{\frac{3}{2}}x^2 + \frac{2}{3}\left(3x^2+2\right)^{\frac{3}{2}}x + \frac{781}{135}\left(3x^2+2\right)^{\frac{3}{2}} + \frac{131}{6}\sqrt{3x^2+2}x + \frac{131}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)$$

input
```
integrate((5-x)*(3+2*x)^2*(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
-4/15*(3*x^2 + 2)^(3/2)*x^2 + 2/3*(3*x^2 + 2)^(3/2)*x + 781/135*(3*x^2 + 2
)^(3/2) + 131/6*sqrt(3*x^2 + 2)*x + 131/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.69

$$\int (5-x)(3+2x)^2\sqrt{2+3x^2}\,dx$$
$$= -\frac{1}{270}\,(3\,(2\,(18\,(2\,x-5)x-757)x-2085)x-3124)\sqrt{3\,x^2+2}$$
$$-\frac{131}{9}\,\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

input
```
integrate((5-x)*(3+2*x)^2*(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-1/270*(3*(2*(18*(2*x - 5)*x - 757)*x - 2085)*x - 3124)*sqrt(3*x^2 + 2) -
131/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.58

$$\int (5-x)(3+2x)^2\sqrt{2+3x^2}\,dx = \frac{131\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}$$
$$+\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(-\frac{12\,x^4}{5}+6\,x^3+\frac{757\,x^2}{15}+\frac{139\,x}{2}+\frac{1562}{45}\right)}{3}$$

input
```
int(-(2*x + 3)^2*(3*x^2 + 2)^(1/2)*(x - 5),x)
```

output
```
(131*3^(1/2)*asinh((6^(1/2)*x)/2))/9 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((139*x)
/2 + (757*x^2)/15 + 6*x^3 - (12*x^4)/5 + 1562/45))/3
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int (5-x)(3+2x)^2\sqrt{2+3x^2}\,dx = -\frac{4\sqrt{3x^2+2}\,x^4}{5}+2\sqrt{3x^2+2}\,x^3 +\frac{757\sqrt{3x^2+2}\,x^2}{45}+\frac{139\sqrt{3x^2+2}\,x}{6} +\frac{1562\sqrt{3x^2+2}}{135}+\frac{131\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{9}$$

input
```
int((5-x)*(3+2*x)^2*(3*x^2+2)^(1/2),x)
```

output
```
( - 216*sqrt(3*x**2 + 2)*x**4 + 540*sqrt(3*x**2 + 2)*x**3 + 4542*sqrt(3*x*
*2 + 2)*x**2 + 6255*sqrt(3*x**2 + 2)*x + 3124*sqrt(3*x**2 + 2) + 3930*sqrt
(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/270
```

3.198 $\int (5-x)(3+2x)\sqrt{2+3x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 56

$$\int (5-x)(3+2x)\sqrt{2+3x^2}\,dx = \frac{23}{3}x\sqrt{2+3x^2}+\frac{1}{18}(14-3x)\left(2+3x^2\right)^{3/2}+\frac{46\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
23/3*x*(3*x^2+2)^(1/2)+1/18*(14-3*x)*(3*x^2+2)^(3/2)+46/9*arcsinh(1/2*x*6^
(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int (5-x)(3+2x)\sqrt{2+3x^2}\,dx = -\frac{1}{18}\sqrt{2+3x^2}\left(-28-132x-42x^2+9x^3\right)-\frac{46\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[(5 - x)*(3 + 2*x)*Sqrt[2 + 3*x^2],x]
```

output
```
-1/18*(Sqrt[2 + 3*x^2]*(-28 - 132*x - 42*x^2 + 9*x^3)) - (46*Log[-(Sqrt[3]
*x) + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {676, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-x)(2x+3)\sqrt{3x^2+2}\,dx$$

$$\Big\downarrow \text{676}$$

$$\frac{46}{3}\int \sqrt{3x^2+2}dx-\frac{1}{6}x\left(3x^2+2\right)^{3/2}+\frac{7}{9}\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{46}{3}\left(\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\sqrt{3x^2+2}x\right)-\frac{1}{6}x\left(3x^2+2\right)^{3/2}+\frac{7}{9}\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{46}{3}\left(\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3x^2+2}x\right)-\frac{1}{6}x\left(3x^2+2\right)^{3/2}+\frac{7}{9}\left(3x^2+2\right)^{3/2}$$

input
```
Int[(5 - x)*(3 + 2*x)*Sqrt[2 + 3*x^2],x]
```

output
```
(7*(2 + 3*x^2)^(3/2))/9 - (x*(2 + 3*x^2)^(3/2))/6 + (46*((x*Sqrt[2 + 3*x^2
])/2 + ArcSinh[Sqrt[3/2]*x]/Sqrt[3]))/3
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] && !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{(9x^3-42x^2-132x-28)\sqrt{3x^2+2}}{18}+\frac{46\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}$
default	$\frac{23x\sqrt{3x^2+2}}{3}+\frac{46\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}+\frac{7(3x^2+2)^{\frac{3}{2}}}{9}-\frac{x(3x^2+2)^{\frac{3}{2}}}{6}$
trager	$\left(-\frac{1}{2}x^3+\frac{7}{3}x^2+\frac{22}{3}x+\frac{14}{9}\right)\sqrt{3x^2+2}-\frac{46\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
meijerg	$-\frac{5\sqrt{3}\left(-\sqrt{6}\,\sqrt{\pi}\,x\sqrt{\frac{3x^2}{2}+1}-2\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{2\sqrt{\pi}}-\frac{7\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(3x^2+2)\sqrt{\frac{3x^2}{2}+1}}{3}\right)}{6\sqrt{\pi}}+\frac{2\sqrt{3}\left(-\frac{\sqrt{6}\,\sqrt{\pi}\,x(9x^2+3)\sqrt{3}}{12}\right.}{9}$

input
```
int((5-x)*(2*x+3)*(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/18*(9*x^3-42*x^2-132*x-28)*(3*x^2+2)^(1/2)+46/9*arcsinh(1/2*6^(1/2)*x)*
3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int (5-x)(3+2x)\sqrt{2+3x^2}\,dx = -\frac{1}{18}\left(9\,x^3-42\,x^2-132\,x-28\right)\sqrt{3\,x^2+2} + \frac{23}{9}\,\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3+2*x)*(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/18*(9*x^3 - 42*x^2 - 132*x - 28)*sqrt(3*x^2 + 2) + 23/9*sqrt(3)*log(-sq
rt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (5-x)(3+2x)\sqrt{2+3x^2}\,dx = \sqrt{3x^2+2}\left(-\frac{x^3}{2}+\frac{7x^2}{3}+\frac{22x}{3}+\frac{14}{9}\right)+\frac{46\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

input
```
integrate((5-x)*(3+2*x)*(3*x**2+2)**(1/2),x)
```

output
```
sqrt(3*x**2 + 2)*(-x**3/2 + 7*x**2/3 + 22*x/3 + 14/9) + 46*sqrt(3)*asinh(s
qrt(6)*x/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (5-x)(3+2x)\sqrt{2+3x^2}\,dx = -\frac{1}{6}\left(3\,x^2+2\right)^{\frac{3}{2}}x+\frac{7}{9}\left(3\,x^2+2\right)^{\frac{3}{2}} + \frac{23}{3}\sqrt{3\,x^2+2}x+\frac{46}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)$$

input
```
integrate((5-x)*(3+2*x)*(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(3*x^2 + 2)^(3/2)*x + 7/9*(3*x^2 + 2)^(3/2) + 23/3*sqrt(3*x^2 + 2)*x
+ 46/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (5-x)(3+2x)\sqrt{2+3x^2}\,dx = -\frac{1}{18}\left(3\left((3\,x-14)x-44\right)x-28\right)\sqrt{3\,x^2+2} - \frac{46}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

input
```
integrate((5-x)*(3+2*x)*(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-1/18*(3*((3*x - 14)*x - 44)*x - 28)*sqrt(3*x^2 + 2) - 46/9*sqrt(3)*log(-s
qrt(3)*x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.71

$$\int (5-x)(3+2x)\sqrt{2+3x^2}\,dx = \frac{46\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(-\frac{3\,x^3}{2}+7\,x^2+22\,x+\frac{14}{3}\right)}{3}$$

input
```
int(-(2*x + 3)*(3*x^2 + 2)^(1/2)*(x - 5),x)
```

output
```
(46*3^(1/2)*asinh((6^(1/2)*x)/2))/9 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(22*x + 7
*x^2 - (3*x^3)/2 + 14/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27

$$\int (5-x)(3+2x)\sqrt{2+3x^2}\,dx = -\frac{\sqrt{3x^2+2}\,x^3}{2} + \frac{7\sqrt{3x^2+2}\,x^2}{3} + \frac{22\sqrt{3x^2+2}\,x}{3} + \frac{14\sqrt{3x^2+2}}{9} + \frac{46\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{9}$$

input
```
int((5-x)*(3+2*x)*(3*x^2+2)^(1/2),x)
```

output
```
( - 9*sqrt(3*x**2 + 2)*x**3 + 42*sqrt(3*x**2 + 2)*x**2 + 132*sqrt(3*x**2 +
 2)*x + 28*sqrt(3*x**2 + 2) + 92*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x
)/sqrt(2)))/18
```

3.199 $\int (5-x)\sqrt{2+3x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int (5-x)\sqrt{2+3x^2}\,dx = \frac{5}{2}x\sqrt{2+3x^2}-\frac{1}{9}\left(2+3x^2\right)^{3/2}+\frac{5\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

output
```
5/2*x*(3*x^2+2)^(1/2)-1/9*(3*x^2+2)^(3/2)+5/3*arcsinh(1/2*x*6^(1/2))*3^(1/
2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int (5-x)\sqrt{2+3x^2}\,dx = -\frac{1}{18}\sqrt{2+3x^2}\left(4-45x+6x^2\right)-\frac{5\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[(5 - x)*Sqrt[2 + 3*x^2],x]
```

output
```
-1/18*(Sqrt[2 + 3*x^2]*(4 - 45*x + 6*x^2)) - (5*Log[-(Sqrt[3]*x) + Sqrt[2
+ 3*x^2]])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {455, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-x)\sqrt{3x^2+2}\,dx$$

$$\Big\downarrow \text{455}$$

$$5\int \sqrt{3x^2+2}dx-\frac{1}{9}\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$5\left(\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\sqrt{3x^2+2}x\right)-\frac{1}{9}\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{222}$$

$$5\left(\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3x^2+2}x\right)-\frac{1}{9}\left(3x^2+2\right)^{3/2}$$

input
```
Int[(5 - x)*Sqrt[2 + 3*x^2],x]
```

output
```
-1/9*(2 + 3*x^2)^(3/2) + 5*((x*Sqrt[2 + 3*x^2])/2 + ArcSinh[Sqrt[3/2]*x]/S
qrt[3])
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
risch	$-\frac{(6x^2-45x+4)\sqrt{3x^2+2}}{18}+\frac{5\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{3}$	35
default	$\frac{5x\sqrt{3x^2+2}}{2}-\frac{(3x^2+2)^{\frac{3}{2}}}{9}+\frac{5\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{3}$	37
trager	$\left(-\frac{1}{3}x^2+\frac{5}{2}x-\frac{2}{9}\right)\sqrt{3x^2+2}-\frac{5\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{3}$	52
meijerg	$-\frac{5\sqrt{3}\left(-\sqrt{6}\,\sqrt{\pi}\,x\sqrt{\frac{3x^2}{2}+1}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{6\sqrt{\pi}}+\frac{\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(3x^2+2)\sqrt{\frac{3x^2}{2}+1}}{3}\right)}{6\sqrt{\pi}}$	79

input
```
int((5-x)*(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/18*(6*x^2-45*x+4)*(3*x^2+2)^(1/2)+5/3*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int (5-x)\sqrt{2+3x^2}\,dx = -\frac{1}{18}\left(6\,x^2-45\,x+4\right)\sqrt{3\,x^2+2} + \frac{5}{6}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/18*(6*x^2 - 45*x + 4)*sqrt(3*x^2 + 2) + 5/6*sqrt(3)*log(-sqrt(3)*sqrt(3
*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int (5-x)\sqrt{2+3x^2}\,dx = -\frac{x^2\sqrt{3x^2+2}}{3} + \frac{5x\sqrt{3x^2+2}}{2} - \frac{2\sqrt{3x^2+2}}{9} + \frac{5\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{3}$$

input
```
integrate((5-x)*(3*x**2+2)**(1/2),x)
```

output
```
-x**2*sqrt(3*x**2 + 2)/3 + 5*x*sqrt(3*x**2 + 2)/2 - 2*sqrt(3*x**2 + 2)/9 +
 5*sqrt(3)*asinh(sqrt(6)*x/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int (5-x)\sqrt{2+3x^2}\,dx = -\frac{1}{9}\left(3\,x^2+2\right)^{\frac{3}{2}} + \frac{5}{2}\sqrt{3\,x^2+2}x + \frac{5}{3}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/9*(3*x^2 + 2)^(3/2) + 5/2*sqrt(3*x^2 + 2)*x + 5/3*sqrt(3)*arcsinh(1/2*s
qrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int (5-x)\sqrt{2+3x^2}\,dx = -\frac{1}{18}\left(3\left(2\,x-15\right)x+4\right)\sqrt{3\,x^2+2} - \frac{5}{3}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-1/18*(3*(2*x - 15)*x + 4)*sqrt(3*x^2 + 2) - 5/3*sqrt(3)*log(-sqrt(3)*x +
sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int (5-x)\sqrt{2+3x^2}\,dx = \frac{5\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{3} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(x^2-\frac{15\,x}{2}+\frac{2}{3}\right)}{3}$$

input
```
int(-(3*x^2 + 2)^(1/2)*(x - 5),x)
```

output
```
(5*3^(1/2)*asinh((6^(1/2)*x)/2))/3 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(x^2 - (15
*x)/2 + 2/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int (5-x)\sqrt{2+3x^2}\,dx = -\frac{\sqrt{3x^2+2}\,x^2}{3} + \frac{5\sqrt{3x^2+2}\,x}{2} - \frac{2\sqrt{3x^2+2}}{9} + \frac{5\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{3}$$

input
```
int((5-x)*(3*x^2+2)^(1/2),x)
```

output
```
( - 6*sqrt(3*x**2 + 2)*x**2 + 45*sqrt(3*x**2 + 2)*x - 4*sqrt(3*x**2 + 2) +
 30*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/18
```

3.200 $\int \frac{(5-x)\sqrt{2+3x^2}}{3+2x} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{(5-x)\sqrt{2+3x^2}}{3+2x} \, dx = \frac{1}{4}(13-x)\sqrt{2+3x^2} - \frac{121\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{8\sqrt{3}} - \frac{13}{8}\sqrt{35}\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)$$

output
```
1/4*(13-x)*(3*x^2+2)^(1/2)-121/24*arcsinh(1/2*x*6^(1/2))*3^(1/2)-13/8*35^(
1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int \frac{(5-x)\sqrt{2+3x^2}}{3+2x} \, dx = \frac{1}{24}\left(-6(-13+x)\sqrt{2+3x^2} + 78\sqrt{35}\text{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right) + 121\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)\right)$$

input
```
Integrate[((5 - x)*Sqrt[2 + 3*x^2])/(3 + 2*x),x]
```

output
```
(-6*(-13 + x)*Sqrt[2 + 3*x^2] + 78*Sqrt[35]*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]
*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]] + 121*Sqrt[3]*Log[-(Sqrt[3]*x) + Sqrt[2
+ 3*x^2]])/24
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {682, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\sqrt{3x^2+2}}{2x+3}dx$$

$$\Big\downarrow \text{682}$$

$$\frac{1}{24}\int \frac{6(46-121x)}{(2x+3)\sqrt{3x^2+2}}dx+\frac{1}{4}\sqrt{3x^2+2}(13-x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{46-121x}{(2x+3)\sqrt{3x^2+2}}dx+\frac{1}{4}\sqrt{3x^2+2}(13-x)$$

$$\Big\downarrow \text{719}$$

$$\frac{1}{4}\left(\frac{455}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{121}{2}\int \frac{1}{\sqrt{3x^2+2}}dx\right)+\frac{1}{4}\sqrt{3x^2+2}(13-x)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{4}\left(\frac{455}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{121\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)+\frac{1}{4}\sqrt{3x^2+2}(13-x)$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{4}\left(-\frac{455}{2}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{121\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)+\frac{1}{4}\sqrt{3x^2+2}(13-x)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(-\frac{121\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{13}{2}\sqrt{35}\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)\right)+\frac{1}{4}\sqrt{3x^2+2}(13-x)$$

input
```
Int[((5 - x)*Sqrt[2 + 3*x^2])/(3 + 2*x),x]
```

output
```
((13 - x)*Sqrt[2 + 3*x^2])/4 + ((-121*ArcSinh[Sqrt[3/2]*x])/(2*Sqrt[3]) -
(13*Sqrt[35]*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/2)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{(-13+x)\sqrt{3x^2+2}}{4}-\frac{121\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{24}-\frac{13\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{8}$
default	$-\frac{x\sqrt{3x^2+2}}{4}-\frac{121\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{24}+\frac{13\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}{8}-\frac{13\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{8}$
trager	$\left(\frac{13}{4}-\frac{x}{4}\right)\sqrt{3x^2+2}+\frac{121\,\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{24}-\frac{13\,\operatorname{RootOf}\left(\_Z^2-35\right)\ln\left(-\frac{9\,\mathrm{R}\cdots}{\cdots}\right.}{\cdots}$

input
```
int((5-x)*(3*x^2+2)^(1/2)/(2*x+3),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-13+x)*(3*x^2+2)^(1/2)-121/24*arcsinh(1/2*6^(1/2)*x)*3^(1/2)-13/8*35
^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \frac{(5-x)\sqrt{2+3x^2}}{3+2x}\,dx = -\frac{1}{4}\sqrt{3\,x^2+2}(x-13)+\frac{121}{48}\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right) +\frac{13}{16}\sqrt{35}\log\left(-\frac{\sqrt{35}\sqrt{3\,x^2+2}(9\,x-4)+93\,x^2-36\,x+43}{4\,x^2+12\,x+9}\right)$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x),x, algorithm="fricas")
```

output
```
-1/4*sqrt(3*x^2 + 2)*(x - 13) + 121/48*sqrt(3)*log(sqrt(3)*sqrt(3*x^2 + 2)
*x - 3*x^2 - 1) + 13/16*sqrt(35)*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4)
+ 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9))
```

Sympy [F]

$$\int \frac{(5-x)\sqrt{2+3x^2}}{3+2x}\,dx = -\int \left(-\frac{5\sqrt{3x^2+2}}{2x+3}\right)dx - \int \frac{x\sqrt{3x^2+2}}{2x+3}\,dx$$

input
```
integrate((5-x)*(3*x**2+2)**(1/2)/(3+2*x),x)
```

output
```
-Integral(-5*sqrt(3*x**2 + 2)/(2*x + 3), x) - Integral(x*sqrt(3*x**2 + 2)/
(2*x + 3), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{(5-x)\sqrt{2+3x^2}}{3+2x}\,dx = -\frac{1}{4}\sqrt{3\,x^2+2}x-\frac{121}{24}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) +\frac{13}{8}\sqrt{35}\,\mathrm{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|}-\frac{2\sqrt{6}}{3\,|2\,x+3|}\right)+\frac{13}{4}\sqrt{3\,x^2+2}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x),x, algorithm="maxima")
```

output
```
-1/4*sqrt(3*x^2 + 2)*x - 121/24*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 13/8*sqrt
(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3)) + 13/4
*sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.44

$$\int \frac{(5-x)\sqrt{2+3x^2}}{3+2x}\,dx = -\frac{1}{4}\sqrt{3\,x^2+2}(x-13) + \frac{121}{24}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{13}{8}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3\,x^2+2}}\right)$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x),x, algorithm="giac")
```

output
```
-1/4*sqrt(3*x^2 + 2)*(x - 13) + 121/24*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2
 + 2)) + 13/8*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sq
rt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2)))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{(5-x)\sqrt{2+3x^2}}{3+2x}\,dx = \frac{\sqrt{35}\left(910\,\ln\left(x+\frac{3}{2}\right)-910\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)\right)}{560} - \frac{121\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{24} - \frac{\sqrt{3}\left(\frac{3\,x}{4}-\frac{39}{4}\right)\sqrt{x^2+\frac{2}{3}}}{3}$$

input
```
int(-((3*x^2 + 2)^(1/2)*(x - 5))/(2*x + 3),x)
```

output
```
(35^(1/2)*(910*log(x + 3/2) - 910*log(x - (3^(1/2)*35^(1/2)*(x^2 + 2/3)^(1
/2))/9 - 4/9)))/560 - (121*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/24 - (3^(
1/2)*((3*x)/4 - 39/4)*(x^2 + 2/3)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.94

$$\int \frac{(5-x)\sqrt{2+3x^2}}{3+2x}\,dx = \frac{13\sqrt{35}\,atan\left(\frac{2\sqrt{3x^2+2}\,i+2\sqrt{3}\,ix}{\sqrt{35}-3\sqrt{3}}\right)i}{8} - \frac{\sqrt{3x^2+2}\,x}{4} + \frac{13\sqrt{3x^2+2}}{4} + \frac{13\sqrt{35}\log\left(4\sqrt{3x^2+2}\,\sqrt{3}\,x+3\sqrt{105}+12x^2-27\right)}{16} - \frac{13\sqrt{35}\log\left(\frac{2\sqrt{3x^2+2}+\sqrt{35}+2\sqrt{3}\,x+3\sqrt{3}}{\sqrt{2}}\right)}{8} - \frac{121\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{24}$$

input
```
int((5-x)*(3*x^2+2)^(1/2)/(3+2*x),x)
```

output
```
(78*sqrt(35)*atan((2*sqrt(3*x**2 + 2)*i + 2*sqrt(3)*i*x)/(sqrt(35) - 3*sqr
t(3)))*i - 12*sqrt(3*x**2 + 2)*x + 156*sqrt(3*x**2 + 2) + 39*sqrt(35)*log(
4*sqrt(3*x**2 + 2)*sqrt(3)*x + 3*sqrt(105) + 12*x**2 - 27) - 78*sqrt(35)*l
og((2*sqrt(3*x**2 + 2) + sqrt(35) + 2*sqrt(3)*x + 3*sqrt(3))/sqrt(2)) - 24
2*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/48
```

3.201 $\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 73

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^2}\,dx = -\frac{(8+x)\sqrt{2+3x^2}}{2(3+2x)} + 2\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right) + \frac{19\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{\sqrt{35}}$$

output
```
-1/2*(8+x)*(3*x^2+2)^(1/2)/(3+2*x)+2*arcsinh(1/2*x*6^(1/2))*3^(1/2)+19/35*
35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.30

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^2}\,dx = -\frac{(8+x)\sqrt{2+3x^2}}{6+4x} - \frac{38\text{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{\sqrt{35}} - 2\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)$$

input
```
Integrate[((5 - x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^2,x]
```

output
```
-(((8 + x)*Sqrt[2 + 3*x^2])/(6 + 4*x)) - (38*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3
]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]])/Sqrt[35] - 2*Sqrt[3]*Log[-(Sqrt[3]*x)
+ Sqrt[2 + 3*x^2]]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {681, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\sqrt{3x^2+2}}{(2x+3)^2}dx$$

$$\Big\downarrow \text{ 681}$$

$$-\frac{1}{8}\int \frac{8(1-12x)}{(2x+3)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}(x+8)}{2(2x+3)}$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{1-12x}{(2x+3)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}(x+8)}{2(2x+3)}$$

$$\Big\downarrow \text{ 719}$$

$$6\int \frac{1}{\sqrt{3x^2+2}}dx-19\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}(x+8)}{2(2x+3)}$$

$$\Big\downarrow \text{ 222}$$

$$-19\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx+2\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-\frac{\sqrt{3x^2+2}(x+8)}{2(2x+3)}$$

$$\Big\downarrow \text{ 488}$$

$$19\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}+2\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-\frac{\sqrt{3x^2+2}(x+8)}{2(2x+3)}$$

$$\Big\downarrow \text{ 219}$$

$$2\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)+\frac{19\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{\sqrt{35}}-\frac{\sqrt{3x^2+2}(x+8)}{2(2x+3)}$$

input
```
Int[((5 - x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^2,x]
```

output
```
-1/2*((8 + x)*Sqrt[2 + 3*x^2])/(3 + 2*x) + 2*Sqrt[3]*ArcSinh[Sqrt[3/2]*x]
+ (19*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/Sqrt[35]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{3x^3+24x^2+2x+16}{2(2x+3)\sqrt{3x^2+2}}+2\,\text{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}+\frac{19\sqrt{35}\,\text{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{35}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{70\left(x+\frac{3}{2}\right)}-\frac{19\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}{35}+2\,\text{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}+\frac{19\sqrt{35}\,\text{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x}}\right)}{35}$
trager	$-\frac{(8+x)\sqrt{3x^2+2}}{2(2x+3)}-\frac{19\,\text{RootOf}\left(\_Z^2-35\right)\ln\left(\frac{9\,\text{RootOf}\left(\_Z^2-35\right)x-4\,\text{RootOf}\left(\_Z^2-35\right)+35\sqrt{3x^2+2}}{2x+3}\right)}{35}+2\,\text{RootOf}\left(\_Z\right.$

input
```
int((5-x)*(3*x^2+2)^(1/2)/(2*x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(3*x^3+24*x^2+2*x+16)/(2*x+3)/(3*x^2+2)^(1/2)+2*arcsinh(1/2*6^(1/2)*x
)*3^(1/2)+19/35*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-
19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.49

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^2}\,dx$$
$$=\frac{70\sqrt{3}(2x+3)\log\left(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)+19\sqrt{35}(2x+3)\log\left(\frac{\sqrt{35}\sqrt{3x^2+2}(9x-4)-93x^2+36x-43}{4x^2+12x+9}\right)-}{70\,(2x+3)}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^2,x, algorithm="fricas")
```

output
```
1/70*(70*sqrt(3)*(2*x + 3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + 1
9*sqrt(35)*(2*x + 3)*log((sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) - 93*x^2 + 36
*x - 43)/(4*x^2 + 12*x + 9)) - 35*sqrt(3*x^2 + 2)*(x + 8))/(2*x + 3)
```

Sympy [F]

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^2}\,dx = -\int \left(-\frac{5\sqrt{3x^2+2}}{4x^2+12x+9}\right)dx - \int \frac{x\sqrt{3x^2+2}}{4x^2+12x+9}\,dx$$

input
```
integrate((5-x)*(3*x**2+2)**(1/2)/(3+2*x)**2,x)
```

output
```
-Integral(-5*sqrt(3*x**2 + 2)/(4*x**2 + 12*x + 9), x) - Integral(x*sqrt(3*
x**2 + 2)/(4*x**2 + 12*x + 9), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^2}\,dx = {} & 2\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) \\ & - \frac{19}{35}\sqrt{35}\operatorname{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|} - \frac{2\sqrt{6}}{3\,|2\,x+3|}\right) \\ & - \frac{1}{4}\sqrt{3\,x^2+2} - \frac{13\sqrt{3\,x^2+2}}{4\,(2\,x+3)}\end{aligned}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^2,x, algorithm="maxima")
```

output
```
2*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 19/35*sqrt(35)*arcsinh(3/2*sqrt(6)*x/ab
s(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3)) - 1/4*sqrt(3*x^2 + 2) - 13/4*sqrt(3
*x^2 + 2)/(2*x + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(59) = 118$.

Time = 0.26 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.90

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^2}\,dx$$
$$= \frac{19}{35}\sqrt{35}\log\left(\sqrt{35}\left(\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)-9\right)\operatorname{sgn}\left(\frac{1}{2x+3}\right)$$
$$-2\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{2\sqrt{35}}{2x+3}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)}\right)\operatorname{sgn}\left(\frac{1}{2x+3}\right)$$
$$-\frac{13}{8}\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}\operatorname{sgn}\left(\frac{1}{2x+3}\right)$$
$$+\frac{3\left(3\left(\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)\operatorname{sgn}\left(\frac{1}{2x+3}\right)-\sqrt{35}\operatorname{sgn}\left(\frac{1}{2x+3}\right)\right)}{4\left(\left(\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)^2-3\right)}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^2,x, algorithm="giac")
```

output
```
19/35*sqrt(35)*log(sqrt(35)*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sq
rt(35)/(2*x + 3)) - 9)*sgn(1/(2*x + 3)) - 2*sqrt(3)*log(1/2*abs(-2*sqrt(3)
 + 2*sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + 2*sqrt(35)/(2*x + 3))/(sqr
t(3) + sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3)))*sgn
(1/(2*x + 3)) - 13/8*sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3)*sgn(1/(2*x +
 3)) + 3/4*(3*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x +
3))*sgn(1/(2*x + 3)) - sqrt(35)*sgn(1/(2*x + 3)))/((sqrt(-18/(2*x + 3) + 3
5/(2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3))^2 - 3)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^2}\,dx = 2\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right) - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{4} - \frac{19\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{35} + \frac{19\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{35} - \frac{13\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{8\left(x+\frac{3}{2}\right)}$$

input
```
int(-((3*x^2 + 2)^(1/2)*(x - 5))/(2*x + 3)^2,x)
```

output
```
2*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2) - (3^(1/2)*(x^2 + 2/3)^(1/2))/4 - (
19*35^(1/2)*log(x + 3/2))/35 + (19*35^(1/2)*log(x - (3^(1/2)*35^(1/2)*(x^2
 + 2/3)^(1/2))/9 - 4/9))/35 - (13*3^(1/2)*(x^2 + 2/3)^(1/2))/(8*(x + 3/2))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.36

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^2}\,dx = \frac{-35\sqrt{3x^2+2}\,x - 280\sqrt{3x^2+2} + 76\sqrt{35}\log\left(-\sqrt{3x^2+2}\,\sqrt{35}+9x-4\right)x + 114\sqrt{35}\log\left(-\sqrt{3x^2+2}\,\sqrt{\right.}$$

input
```
int((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^2,x)
```

output
```
( - 35*sqrt(3*x**2 + 2)*x - 280*sqrt(3*x**2 + 2) + 76*sqrt(35)*log( - sqrt
(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x + 114*sqrt(35)*log( - sqrt(3*x**2 + 2)*
sqrt(35) + 9*x - 4) - 76*sqrt(35)*log(2*x + 3)*x - 114*sqrt(35)*log(2*x +
3) - 140*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x - 210*sqrt(3)*log(sqr
t(3*x**2 + 2) - sqrt(3)*x) + 140*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)
*x + 210*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x))/(70*(2*x + 3))
```

3.202 $\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^3}\,dx = \frac{(53+187x)\sqrt{2+3x^2}}{140(3+2x)^2} - \frac{1}{8}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right) - \frac{471\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{280\sqrt{35}}$$

output
```
1/140*(53+187*x)*(3*x^2+2)^(1/2)/(3+2*x)^2-1/8*arcsinh(1/2*x*6^(1/2))*3^(1
/2)-471/9800*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.30

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^3}\,dx = \frac{(53+187x)\sqrt{2+3x^2}}{140(3+2x)^2} + \frac{471\text{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{140\sqrt{35}} + \frac{1}{8}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)$$

input
```
Integrate[((5 - x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^3,x]
```

output
```
((53 + 187*x)*Sqrt[2 + 3*x^2])/(140*(3 + 2*x)^2) + (471*ArcTanh[(3*Sqrt[3]
 + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]])/(140*Sqrt[35]) + (Sqrt[3]*L
og[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/8
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {680, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\sqrt{3x^2+2}}{(2x+3)^3}dx$$

$$\Big\downarrow \text{680}$$

$$\frac{(187x+53)\sqrt{3x^2+2}}{140(2x+3)^2}-\frac{1}{560}\int -\frac{12(26-35x)}{(2x+3)\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{140}\int \frac{26-35x}{(2x+3)\sqrt{3x^2+2}}dx+\frac{\sqrt{3x^2+2}(187x+53)}{140(2x+3)^2}$$

$$\Big\downarrow \text{719}$$

$$\frac{3}{140}\left(\frac{157}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{35}{2}\int \frac{1}{\sqrt{3x^2+2}}dx\right)+\frac{\sqrt{3x^2+2}(187x+53)}{140(2x+3)^2}$$

$$\Big\downarrow \text{222}$$

$$\frac{3}{140}\left(\frac{157}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{35\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)+\frac{\sqrt{3x^2+2}(187x+53)}{140(2x+3)^2}$$

$$\Big\downarrow \text{488}$$

$$\frac{3}{140}\left(-\frac{157}{2}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{35\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)+\frac{\sqrt{3x^2+2}(187x+53)}{140(2x+3)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{140}\left(-\frac{35\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{157\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{2\sqrt{35}}\right)+\frac{\sqrt{3x^2+2}(187x+53)}{140(2x+3)^2}$$

input
```
Int[((5 - x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^3,x]
```

output
```
((53 + 187*x)*Sqrt[2 + 3*x^2])/(140*(3 + 2*x)^2) + (3*((-35*ArcSinh[Sqrt[3
/2]*x])/(2*Sqrt[3]) - (157*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/
(2*Sqrt[35])))/140
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

method	result
risch	$\frac{561x^3+159x^2+374x+106}{140(2x+3)^2\sqrt{3x^2+2}}-\frac{\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{8}-\frac{471\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{9800}$
trager	$\frac{(53+187x)\sqrt{3x^2+2}}{140(2x+3)^2}-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{8}+\frac{471\operatorname{RootOf}\left(\_Z^2-35\right)\ln\left(\frac{9\operatorname{RootOf}\left(\_Z^2\right.}{}\right.}{980}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{280\left(x+\frac{3}{2}\right)^2}-\frac{47\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{4900\left(x+\frac{3}{2}\right)}+\frac{471\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}{9800}-\frac{471\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x}}\right.}{9800}$

input
```
int((5-x)*(3*x^2+2)^(1/2)/(2*x+3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/140*(561*x^3+159*x^2+374*x+106)/(2*x+3)^2/(3*x^2+2)^(1/2)-1/8*arcsinh(1/
2*6^(1/2)*x)*3^(1/2)-471/9800*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(
x+3/2)^2-36*x-19)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(61) = 122$.

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.59

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^3}\,dx$$
$$= \frac{1225\sqrt{3}(4x^2+12x+9)\log\left(\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)+471\sqrt{35}(4x^2+12x+9)\log\left(-\frac{\sqrt{35}\sqrt{3x^2+2}(9}{4x}\right.}{19600\,(4x^2+12x+9)}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^3,x, algorithm="fricas")
```

output
```
1/19600*(1225*sqrt(3)*(4*x^2 + 12*x + 9)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3
*x^2 - 1) + 471*sqrt(35)*(4*x^2 + 12*x + 9)*log(-(sqrt(35)*sqrt(3*x^2 + 2)
*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9)) + 140*sqrt(3*x^2 + 2)
*(187*x + 53))/(4*x^2 + 12*x + 9)
```

Sympy [F]

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^3}\,dx = -\int \left(-\frac{5\sqrt{3x^2+2}}{8x^3+36x^2+54x+27}\right)dx - \int \frac{x\sqrt{3x^2+2}}{8x^3+36x^2+54x+27}\,dx$$

input
```
integrate((5-x)*(3*x**2+2)**(1/2)/(3+2*x)**3,x)
```

output
```
-Integral(-5*sqrt(3*x**2 + 2)/(8*x**3 + 36*x**2 + 54*x + 27), x) - Integra
l(x*sqrt(3*x**2 + 2)/(8*x**3 + 36*x**2 + 54*x + 27), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.25

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^3}\,dx = -\frac{1}{8}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{471}{9800}\sqrt{35}\operatorname{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|}-\frac{2\sqrt{6}}{3\,|2\,x+3|}\right) + \frac{39}{280}\sqrt{3\,x^2+2} - \frac{13\,(3\,x^2+2)^{\frac{3}{2}}}{70\,(4\,x^2+12\,x+9)} - \frac{47\sqrt{3\,x^2+2}}{280\,(2\,x+3)}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^3,x, algorithm="maxima")
```

output
```
-1/8*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 471/9800*sqrt(35)*arcsinh(3/2*sqrt(6
)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3)) + 39/280*sqrt(3*x^2 + 2) - 13
/70*(3*x^2 + 2)^(3/2)/(4*x^2 + 12*x + 9) - 47/280*sqrt(3*x^2 + 2)/(2*x + 3
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(61) = 122.

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.59

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^3}\,dx = \frac{1}{8}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{471}{9800}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3\,x^2+2}}\right) + \frac{3048\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^3+4301\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2-7368\sqrt{3}x+1496\sqrt{3}+7368\sqrt{3\,x^2+2}}{560\left(\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2+3\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)-2\right)^2}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^3,x, algorithm="giac")
```

output
```
1/8*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 471/9800*sqrt(35)*log(-abs
(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - s
qrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/560*(3048*(sqrt(3)*x - sqrt(
3*x^2 + 2))^3 + 4301*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 - 7368*sqrt(3
)*x + 1496*sqrt(3) + 7368*sqrt(3*x^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^
2 + 3*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^2
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^3}\,dx = \frac{471\,\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{9800} - \frac{\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{8} - \frac{471\,\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{35}\,\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{9800} + \frac{187\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{560\,\left(x+\frac{3}{2}\right)} - \frac{13\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{32\,\left(x^2+3\,x+\frac{9}{4}\right)}$$

input
```
int(-((3*x^2 + 2)^(1/2)*(x - 5))/(2*x + 3)^3,x)
```

output
```
(471*35^(1/2)*log(x + 3/2))/9800 - (3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/
8 - (471*35^(1/2)*log(x - (3^(1/2)*35^(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/9
800 + (187*3^(1/2)*(x^2 + 2/3)^(1/2))/(560*(x + 3/2)) - (13*3^(1/2)*(x^2 +
 2/3)^(1/2))/(32*(3*x + x^2 + 9/4))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.23

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^3}\,dx = \frac{26180\sqrt{3x^2+2}\,x+7420\sqrt{3x^2+2}+3768\sqrt{35}\log\left(\sqrt{3x^2+2}\,\sqrt{35}+9x-4\right)x^2+11304\sqrt{35}\log\left(\sqrt{3x^2+\right.}{}$$

input
```
int((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^3,x)
```

output
```
(26180*sqrt(3*x**2 + 2)*x + 7420*sqrt(3*x**2 + 2) + 3768*sqrt(35)*log(sqrt
(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**2 + 11304*sqrt(35)*log(sqrt(3*x**2 + 2
)*sqrt(35) + 9*x - 4)*x + 8478*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*
x - 4) - 3768*sqrt(35)*log(2*x + 3)*x**2 - 11304*sqrt(35)*log(2*x + 3)*x -
 8478*sqrt(35)*log(2*x + 3) + 4900*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*
x)*x**2 + 14700*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x + 11025*sqrt(3
)*log(sqrt(3*x**2 + 2) - sqrt(3)*x) - 4900*sqrt(3)*log(sqrt(3*x**2 + 2) +
sqrt(3)*x)*x**2 - 14700*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x - 1102
5*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x))/(19600*(4*x**2 + 12*x + 9))
```

3.203 $\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^4} dx = -\frac{41(4-9x)\sqrt{2+3x^2}}{2450(3+2x)^2} - \frac{13(2+3x^2)^{3/2}}{105(3+2x)^3} - \frac{123\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{1225\sqrt{35}}$$

output
```
-41/2450*(4-9*x)*(3*x^2+2)^(1/2)/(3+2*x)^2-13/105*(3*x^2+2)^(3/2)/(3+2*x)^
3-123/42875*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^4} dx = -\frac{\sqrt{2+3x^2}(3296-2337x+516x^2)}{7350(3+2x)^3} + \frac{246\operatorname{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{1225\sqrt{35}}$$

input
```
Integrate[((5 - x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^4,x]
```

output
```
-1/7350*(Sqrt[2 + 3*x^2]*(3296 - 2337*x + 516*x^2))/(3 + 2*x)^3 + (246*Arc
Tanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]])/(1225*Sqrt[3
5])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {679, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\sqrt{3x^2+2}}{(2x+3)^4}dx$$

$$\Big\downarrow \text{679}$$

$$\frac{41}{35}\int \frac{\sqrt{3x^2+2}}{(2x+3)^3}dx-\frac{13\left(3x^2+2\right)^{3/2}}{105(2x+3)^3}$$

$$\Big\downarrow \text{486}$$

$$\frac{41}{35}\left(\frac{3}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{(4-9x)\sqrt{3x^2+2}}{70(2x+3)^2}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{105(2x+3)^3}$$

$$\Big\downarrow \text{488}$$

$$\frac{41}{35}\left(-\frac{3}{35}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{105(2x+3)^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{41}{35}\left(-\frac{3\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{105(2x+3)^3}$$

input
```
Int[((5 - x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^4,x]
```

output
```
(-13*(2 + 3*x^2)^(3/2))/(105*(3 + 2*x)^3) + (41*(-1/70*((4 - 9*x)*Sqrt[2 +
 3*x^2])/(3 + 2*x)^2 - (3*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(
35*Sqrt[35])))/35
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{1548x^4-7011x^3+10920x^2-4674x+6592}{7350(2x+3)^3\sqrt{3x^2+2}}-\frac{123\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{42875}$
trager	$-\frac{(516x^2-2337x+3296)\sqrt{3x^2+2}}{7350(2x+3)^3}-\frac{123\,\mathrm{RootOf}\left(\_Z^2-35\right)\ln\left(-\frac{9\,\mathrm{RootOf}\left(\_Z^2-35\right)x-4\,\mathrm{RootOf}\left(\_Z^2-35\right)-35\sqrt{3x^2+2}}{2x+3}\right)}{42875}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{840\left(x+\frac{3}{2}\right)^3}-\frac{41\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{4900\left(x+\frac{3}{2}\right)^2}-\frac{369\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{85750\left(x+\frac{3}{2}\right)}+\frac{123\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}{42875}-\frac{123\sqrt{}}{}$

input
```
int((5-x)*(3*x^2+2)^(1/2)/(2*x+3)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/7350*(1548*x^4-7011*x^3+10920*x^2-4674*x+6592)/(2*x+3)^3/(3*x^2+2)^(1/2
)-123/42875*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^4}\,dx = \frac{369\sqrt{35}(8x^3+36x^2+54x+27)\log\left(-\frac{\sqrt{35}\sqrt{3x^2+2}(9x-4)+93x^2-36x+43}{4x^2+12x+9}\right)-35(516x^2-2337x+3296)\sqrt{}}{257250(8x^3+36x^2+54x+27)}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^4,x, algorithm="fricas")
```

output
```
1/257250*(369*sqrt(35)*(8*x^3 + 36*x^2 + 54*x + 27)*log(-(sqrt(35)*sqrt(3*
x^2 + 2)*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9)) - 35*(516*x^2
 - 2337*x + 3296)*sqrt(3*x^2 + 2))/(8*x^3 + 36*x^2 + 54*x + 27)
```

Sympy [F]

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^4}\,dx = -\int \left(-\frac{5\sqrt{3x^2+2}}{16x^4+96x^3+216x^2+216x+81}\right)dx - \int \frac{x\sqrt{3x^2+2}}{16x^4+96x^3+216x^2+216x+81}\,dx$$

input
```
integrate((5-x)*(3*x**2+2)**(1/2)/(3+2*x)**4,x)
```

output
```
-Integral(-5*sqrt(3*x**2 + 2)/(16*x**4 + 96*x**3 + 216*x**2 + 216*x + 81),
 x) - Integral(x*sqrt(3*x**2 + 2)/(16*x**4 + 96*x**3 + 216*x**2 + 216*x +
81), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.40

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^4}\,dx = \frac{123}{42875}\sqrt{35}\,\text{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|}-\frac{2\sqrt{6}}{3\,|2\,x+3|}\right) + \frac{123}{4900}\sqrt{3\,x^2+2} - \frac{13\,(3\,x^2+2)^{\frac{3}{2}}}{105\,(8\,x^3+36\,x^2+54\,x+27)} - \frac{41\,(3\,x^2+2)^{\frac{3}{2}}}{1225\,(4\,x^2+12\,x+9)} - \frac{369\,\sqrt{3\,x^2+2}}{4900\,(2\,x+3)}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^4,x, algorithm="maxima")
```

output
```
123/42875*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*
x + 3)) + 123/4900*sqrt(3*x^2 + 2) - 13/105*(3*x^2 + 2)^(3/2)/(8*x^3 + 36*
x^2 + 54*x + 27) - 41/1225*(3*x^2 + 2)^(3/2)/(4*x^2 + 12*x + 9) - 369/4900
*sqrt(3*x^2 + 2)/(2*x + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(67) = 134$.

Time = 0.14 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.83

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^4}\,dx = \frac{123}{42875}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3x^2+2}}\right) - \frac{\sqrt{3}\left(1553\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^5+30\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^4+3870\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^3-25740\right.}{9800\left(\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2+3\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+\right.\right.}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^4,x, algorithm="giac")
```

output
```
123/42875*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqrt(3
*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) - 1/9
800*sqrt(3)*(1553*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^5 + 30*(sqrt(3)*x
- sqrt(3*x^2 + 2))^4 + 3870*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 - 2574
0*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 - 20*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2
)) - 1376)/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 3*sqrt(3)*(sqrt(3)*x - sqrt(
3*x^2 + 2)) - 2)^3
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.29

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^4}\,dx = \frac{123\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{42875} - \frac{123\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{42875} - \frac{43\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{4900\left(x+\frac{3}{2}\right)} + \frac{37\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{560\left(x^2+3x+\frac{9}{4}\right)} - \frac{13\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{96\left(x^3+\frac{9x^2}{2}+\frac{27x}{4}+\frac{27}{8}\right)}$$

input
```
int(-((3*x^2 + 2)^(1/2)*(x - 5))/(2*x + 3)^4,x)
```

output
```
(123*35^(1/2)*log(x + 3/2))/42875 - (123*35^(1/2)*log(x - (3^(1/2)*35^(1/2
)*(x^2 + 2/3)^(1/2))/9 - 4/9))/42875 - (43*3^(1/2)*(x^2 + 2/3)^(1/2))/(490
0*(x + 3/2)) + (37*3^(1/2)*(x^2 + 2/3)^(1/2))/(560*(3*x + x^2 + 9/4)) - (1
3*3^(1/2)*(x^2 + 2/3)^(1/2))/(96*((27*x)/4 + (9*x^2)/2 + x^3 + 27/8))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.33

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^4}\,dx$$
$$= \frac{-18060\sqrt{3x^2+2}\,x^2 + 81795\sqrt{3x^2+2}\,x - 115360\sqrt{3x^2+2} + 5904\sqrt{35}\log\left(\sqrt{3x^2+2}\,\sqrt{35}+9x-4\right)x^3}{}$$

input
```
int((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^4,x)
```

output
```
( - 18060*sqrt(3*x**2 + 2)*x**2 + 81795*sqrt(3*x**2 + 2)*x - 115360*sqrt(3
*x**2 + 2) + 5904*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**3 +
 26568*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**2 + 39852*sqrt
(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x + 19926*sqrt(35)*log(sqrt(
3*x**2 + 2)*sqrt(35) + 9*x - 4) - 5904*sqrt(35)*log(2*x + 3)*x**3 - 26568*
sqrt(35)*log(2*x + 3)*x**2 - 39852*sqrt(35)*log(2*x + 3)*x - 19926*sqrt(35
)*log(2*x + 3))/(257250*(8*x**3 + 36*x**2 + 54*x + 27))
```

3.204 $\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^5}\,dx = -\frac{(297-257x)\sqrt{2+3x^2}}{840(3+2x)^4} - \frac{3\sqrt{2+3x^2}}{3920(3+2x)^2} - \frac{739\sqrt{2+3x^2}}{137200(3+2x)} - \frac{198\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{8575\sqrt{35}}$$

output
```
-1/840*(297-257*x)*(3*x^2+2)^(1/2)/(3+2*x)^4-3/3920*(3*x^2+2)^(1/2)/(3+2*x
)^2-739*(3*x^2+2)^(1/2)/(411600+274400*x)-198/300125*35^(1/2)*arctanh(1/35
*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^5}\,dx = -\frac{\sqrt{2+3x^2}(26028-304x+10134x^2+2217x^3)}{51450(3+2x)^4} + \frac{396\operatorname{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{8575\sqrt{35}}$$

input
```
Integrate[((5 - x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^5,x]
```

output
```
-1/51450*(Sqrt[2 + 3*x^2]*(26028 - 304*x + 10134*x^2 + 2217*x^3))/(3 + 2*x
)^4 + (396*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]]
)/(8575*Sqrt[35])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {688, 25, 679, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\sqrt{3x^2+2}}{(2x+3)^5}dx$$

$$\Big\downarrow \text{688}$$

$$-\frac{1}{140}\int -\frac{(164-39x)\sqrt{3x^2+2}}{(2x+3)^4}dx-\frac{13\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{140}\int \frac{(164-39x)\sqrt{3x^2+2}}{(2x+3)^4}dx-\frac{13\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}$$

$$\Big\downarrow \text{679}$$

$$\frac{1}{140}\left(\frac{264}{7}\int \frac{\sqrt{3x^2+2}}{(2x+3)^3}dx-\frac{89\left(3x^2+2\right)^{3/2}}{21(2x+3)^3}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}$$

$$\Big\downarrow \text{486}$$

$$\frac{1}{140}\left(\frac{264}{7}\left(\frac{3}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{(4-9x)\sqrt{3x^2+2}}{70(2x+3)^2}\right)-\frac{89\left(3x^2+2\right)^{3/2}}{21(2x+3)^3}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{140}\left(\frac{264}{7}\left(-\frac{3}{35}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{89\left(3x^2+2\right)^{3/2}}{21(2x+3)^3}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}$$

$\Big\downarrow$ 219

$$\frac{1}{140}\left(\frac{264}{7}\left(-\frac{3\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{89\left(3x^2+2\right)^{3/2}}{21(2x+3)^3}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}$$

input
```
Int[((5 - x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^5,x]
```

output
```
(-13*(2 + 3*x^2)^(3/2))/(140*(3 + 2*x)^4) + ((-89*(2 + 3*x^2)^(3/2))/(21*(
3 + 2*x)^3) + (264*(-1/70*((4 - 9*x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^2 - (3*Arc
Tanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(35*Sqrt[35])))/7)/140
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{6651x^5+30402x^4+3522x^3+98352x^2-608x+52056}{51450(2x+3)^4\sqrt{3x^2+2}}-\frac{198\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{300125}$
trager	$-\frac{(2217x^3+10134x^2-304x+26028)\sqrt{3x^2+2}}{51450(2x+3)^4}-\frac{198\,\mathrm{RootOf}\left(\_Z^2-35\right)\ln\left(-\frac{9\,\mathrm{RootOf}\left(\_Z^2-35\right)x-4\,\mathrm{RootOf}\left(\_Z^2-35\right)-35\sqrt{3\cdots}}{2x+3}\cdots\right)}{300125}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{2240\left(x+\frac{3}{2}\right)^4}-\frac{89\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{23520\left(x+\frac{3}{2}\right)^3}-\frac{33\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{17150\left(x+\frac{3}{2}\right)^2}-\frac{297\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{300125\left(x+\frac{3}{2}\right)}+\frac{198\sqrt{1\cdots}}{\cdots}$

input
```
int((5-x)*(3*x^2+2)^(1/2)/(2*x+3)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/51450*(6651*x^5+30402*x^4+3522*x^3+98352*x^2-608*x+52056)/(2*x+3)^4/(3*
x^2+2)^(1/2)-198/300125*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)
^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.14

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^5}\,dx$$
$$= \frac{594\sqrt{35}(16x^4+96x^3+216x^2+216x+81)\log\left(-\frac{\sqrt{35}\sqrt{3x^2+2}(9x-4)+93x^2-36x+43}{4x^2+12x+9}\right)-35\,(2217x^3+101\ldots}{1800750\,(16x^4+96x^3+216x^2+216x+81)}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^5,x, algorithm="fricas")
```

output
```
1/1800750*(594*sqrt(35)*(16*x^4 + 96*x^3 + 216*x^2 + 216*x + 81)*log(-(sqr
t(35)*sqrt(3*x^2 + 2)*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9))
- 35*(2217*x^3 + 10134*x^2 - 304*x + 26028)*sqrt(3*x^2 + 2))/(16*x^4 + 96*
x^3 + 216*x^2 + 216*x + 81)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^5}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(1/2)/(3+2*x)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.42

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^5}\,dx = \frac{198}{300125}\sqrt{35}\,\mathrm{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|}-\frac{2\sqrt{6}}{3\,|2\,x+3|}\right) + \frac{99}{17150}\sqrt{3\,x^2+2} - \frac{13\,(3\,x^2+2)^{\frac{3}{2}}}{140\,(16\,x^4+96\,x^3+216\,x^2+216\,x+81)} - \frac{89\,(3\,x^2+2)^{\frac{3}{2}}}{2940\,(8\,x^3+36\,x^2+54\,x+27)} - \frac{66\,(3\,x^2+2)^{\frac{3}{2}}}{8575\,(4\,x^2+12\,x+9)} - \frac{297\,\sqrt{3\,x^2+2}}{17150\,(2\,x+3)}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^5,x, algorithm="maxima")
```

output
```
198/300125*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2
*x + 3)) + 99/17150*sqrt(3*x^2 + 2) - 13/140*(3*x^2 + 2)^(3/2)/(16*x^4 + 9
6*x^3 + 216*x^2 + 216*x + 81) - 89/2940*(3*x^2 + 2)^(3/2)/(8*x^3 + 36*x^2
+ 54*x + 27) - 66/8575*(3*x^2 + 2)^(3/2)/(4*x^2 + 12*x + 9) - 297/17150*sq
rt(3*x^2 + 2)/(2*x + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(85) = 170$.

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.74

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^5}\,dx$$
$$= \frac{1}{9604000}\sqrt{35}\left(739\sqrt{35}\sqrt{3}+6336\log\left(\sqrt{35}\sqrt{3}-9\right)\right)\operatorname{sgn}\left(\frac{1}{2x+3}\right) - \frac{198}{300125}\sqrt{35}\log\left(\sqrt{35}\left(\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)-9\right)\operatorname{sgn}\left(\frac{1}{2x+3}\right) - \frac{1}{823200}\left(\frac{35\left(\frac{7\left(\frac{1365\operatorname{sgn}\left(\frac{1}{2x+3}\right)}{2x+3}-257\operatorname{sgn}\left(\frac{1}{2x+3}\right)\right)}{2x+3}+9\operatorname{sgn}\left(\frac{1}{2x+3}\right)\right)}{2x+3}+2217\operatorname{sgn}\left(\frac{1}{2x+3}\right)\right)\sqrt{-\frac{18}{2x+3}+\frac{}{(}}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^5,x, algorithm="giac")
```

output
```
1/9604000*sqrt(35)*(739*sqrt(35)*sqrt(3) + 6336*log(sqrt(35)*sqrt(3) - 9))
*sgn(1/(2*x + 3)) - 198/300125*sqrt(35)*log(sqrt(35)*(sqrt(-18/(2*x + 3) +
 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3)) - 9)*sgn(1/(2*x + 3)) - 1/82320
0*(35*(7*(1365*sgn(1/(2*x + 3))/(2*x + 3) - 257*sgn(1/(2*x + 3)))/(2*x + 3
) + 9*sgn(1/(2*x + 3)))/(2*x + 3) + 2217*sgn(1/(2*x + 3)))*sqrt(-18/(2*x +
 3) + 35/(2*x + 3)^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.35

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^5}\,dx = \frac{198\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{300125} - \frac{198\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{300125} - \frac{13\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{256\left(x^4+6x^3+\frac{27x^2}{2}+\frac{27x}{2}+\frac{81}{16}\right)} - \frac{739\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{274400\left(x+\frac{3}{2}\right)} - \frac{3\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{15680\left(x^2+3x+\frac{9}{4}\right)} + \frac{257\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{13440\left(x^3+\frac{9x^2}{2}+\frac{27x}{4}+\frac{27}{8}\right)}$$

input
```
int(-((3*x^2 + 2)^(1/2)*(x - 5))/(2*x + 3)^5,x)
```

output
```
(198*35^(1/2)*log(x + 3/2))/300125 - (198*35^(1/2)*log(x - (3^(1/2)*35^(1/
2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/300125 - (13*3^(1/2)*(x^2 + 2/3)^(1/2))/(2
56*((27*x)/2 + (27*x^2)/2 + 6*x^3 + x^4 + 81/16)) - (739*3^(1/2)*(x^2 + 2/
3)^(1/2))/(274400*(x + 3/2)) - (3*3^(1/2)*(x^2 + 2/3)^(1/2))/(15680*(3*x +
 x^2 + 9/4)) + (257*3^(1/2)*(x^2 + 2/3)^(1/2))/(13440*((27*x)/4 + (9*x^2)/
2 + x^3 + 27/8))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.37

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^5}\,dx = \frac{-77595\sqrt{3x^2+2}\,x^3-354690\sqrt{3x^2+2}\,x^2+10640\sqrt{3x^2+2}\,x-910980\sqrt{3x^2+2}+19008\sqrt{35}\log\left(\sqrt{3x\right.}}{}$$

input
```
int((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^5,x)
```

output

```
( - 77595*sqrt(3*x**2 + 2)*x**3 - 354690*sqrt(3*x**2 + 2)*x**2 + 10640*sqr
t(3*x**2 + 2)*x - 910980*sqrt(3*x**2 + 2) + 19008*sqrt(35)*log(sqrt(3*x**2
 + 2)*sqrt(35) + 9*x - 4)*x**4 + 114048*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt
(35) + 9*x - 4)*x**3 + 256608*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x
 - 4)*x**2 + 256608*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x +
96228*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4) - 19008*sqrt(35)*l
og(2*x + 3)*x**4 - 114048*sqrt(35)*log(2*x + 3)*x**3 - 256608*sqrt(35)*log
(2*x + 3)*x**2 - 256608*sqrt(35)*log(2*x + 3)*x - 96228*sqrt(35)*log(2*x +
 3))/(1800750*(16*x**4 + 96*x**3 + 216*x**2 + 216*x + 81))
```

3.205 $\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^6}\,dx = -\frac{(118-73x)\sqrt{2+3x^2}}{350(3+2x)^5}+\frac{\sqrt{2+3x^2}}{875(3+2x)^3}-\frac{12\sqrt{2+3x^2}}{6125(3+2x)^2}-\frac{366\sqrt{2+3x^2}}{214375(3+2x)}-\frac{1017\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{214375\sqrt{35}}$$

output
```
-1/350*(118-73*x)*(3*x^2+2)^(1/2)/(3+2*x)^5+1/875*(3*x^2+2)^(1/2)/(3+2*x)^
3-12/6125*(3*x^2+2)^(1/2)/(3+2*x)^2-366*(3*x^2+2)^(1/2)/(643125+428750*x)-
1017/7503125*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^6}\,dx = \frac{-\frac{35\sqrt{2+3x^2}(222112+108167x+186392x^2+76992x^3+11712x^4)}{(3+2x)^5}+4068\sqrt{35}\operatorname{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{15006250}$$

input
```
Integrate[((5 - x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^6,x]
```

output
```
((-35*Sqrt[2 + 3*x^2]*(222112 + 108167*x + 186392*x^2 + 76992*x^3 + 11712*
x^4))/(3 + 2*x)^5 + 4068*Sqrt[35]*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqr
t[2 + 3*x^2])/Sqrt[35]])/15006250
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {688, 25, 688, 27, 679, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\sqrt{3x^2+2}}{(2x+3)^6}dx$$

$$\Big\downarrow \text{688}$$

$$-\frac{1}{175}\int -\frac{(205-78x)\sqrt{3x^2+2}}{(2x+3)^5}dx-\frac{13\left(3x^2+2\right)^{3/2}}{175(2x+3)^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{175}\int \frac{(205-78x)\sqrt{3x^2+2}}{(2x+3)^5}dx-\frac{13\left(3x^2+2\right)^{3/2}}{175(2x+3)^5}$$

$$\Big\downarrow \text{688}$$

$$\frac{1}{175}\left(-\frac{1}{140}\int -\frac{84(73-23x)\sqrt{3x^2+2}}{(2x+3)^4}dx-\frac{23\left(3x^2+2\right)^{3/2}}{5(2x+3)^4}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{175(2x+3)^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{175}\left(\frac{3}{5}\int \frac{(73-23x)\sqrt{3x^2+2}}{(2x+3)^4}dx-\frac{23\left(3x^2+2\right)^{3/2}}{5(2x+3)^4}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{175(2x+3)^5}$$

$$\Big\downarrow \text{679}$$

$$\frac{1}{175}\left(\frac{3}{5}\left(\frac{113}{7}\int \frac{\sqrt{3x^2+2}}{(2x+3)^3}dx-\frac{43\left(3x^2+2\right)^{3/2}}{21(2x+3)^3}\right)-\frac{23\left(3x^2+2\right)^{3/2}}{5(2x+3)^4}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{175(2x+3)^5}$$

$$\Big\downarrow \text{486}$$

$$\frac{1}{175}\left(\frac{3}{5}\left(\frac{113}{7}\left(\frac{3}{35}\int\frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{(4-9x)\sqrt{3x^2+2}}{70(2x+3)^2}\right)-\frac{43\left(3x^2+2\right)^{3/2}}{21(2x+3)^3}\right)-\frac{23\left(3x^2+2\right)^{3/2}}{5(2x+3)^4}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{175(2x+3)^5}$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{175}\left(\frac{3}{5}\left(\frac{113}{7}\left(-\frac{3}{35}\int\frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{43\left(3x^2+2\right)^{3/2}}{21(2x+3)^3}\right)-\frac{23\left(3x^2+2\right)^{3/2}}{5(2x+3)^4}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{175(2x+3)^5}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{175}\left(\frac{3}{5}\left(\frac{113}{7}\left(-\frac{3\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{43\left(3x^2+2\right)^{3/2}}{21(2x+3)^3}\right)-\frac{23\left(3x^2+2\right)^{3/2}}{5(2x+3)^4}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{175(2x+3)^5}$$

input
```
Int[((5 - x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^6,x]
```

output
```
(-13*(2 + 3*x^2)^(3/2))/(175*(3 + 2*x)^5) + ((-23*(2 + 3*x^2)^(3/2))/(5*(3
 + 2*x)^4) + (3*((-43*(2 + 3*x^2)^(3/2))/(21*(3 + 2*x)^3) + (113*(-1/70*((
4 - 9*x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^2 - (3*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqr
t[2 + 3*x^2])])/(35*Sqrt[35])))/7))/5)/175
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{35136x^6+230976x^5+582600x^4+478485x^3+1039120x^2+216334x+444224}{428750(2x+3)^5\sqrt{3x^2+2}}-\frac{1017\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{7503125}$
trager	$-\frac{(11712x^4+76992x^3+186392x^2+108167x+222112)\sqrt{3x^2+2}}{428750(2x+3)^5}+\frac{1017\,\mathrm{RootOf}\left(\_Z^2-35\right)\ln\left(\frac{9\,\mathrm{RootOf}\left(\_Z^2-35\right)x-4\,\mathrm{RootOf}\left(\right.}{2x+3}\right.}{7503125}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{5600\left(x+\frac{3}{2}\right)^5}-\frac{23\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{14000\left(x+\frac{3}{2}\right)^4}-\frac{43\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{49000\left(x+\frac{3}{2}\right)^3}-\frac{339\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{857500\left(x+\frac{3}{2}\right)^2}-\frac{3051(}{1}$

input
```
int((5-x)*(3*x^2+2)^(1/2)/(2*x+3)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/428750*(35136*x^6+230976*x^5+582600*x^4+478485*x^3+1039120*x^2+216334*x
+444224)/(2*x+3)^5/(3*x^2+2)^(1/2)-1017/7503125*35^(1/2)*arctanh(2/35*(4-9
*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^6}\,dx$$
$$=\frac{1017\sqrt{35}(32x^5+240x^4+720x^3+1080x^2+810x+243)\log\left(-\frac{\sqrt{35}\sqrt{3x^2+2}(9x-4)+93x^2-36x+43}{4x^2+12x+9}\right)-35\,(1}{15006250\,(32x^5+240x^4+720x^3+1080x^2+81}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^6,x, algorithm="fricas")
```

output
```
1/15006250*(1017*sqrt(35)*(32*x^5 + 240*x^4 + 720*x^3 + 1080*x^2 + 810*x +
 243)*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^
2 + 12*x + 9)) - 35*(11712*x^4 + 76992*x^3 + 186392*x^2 + 108167*x + 22211
2)*sqrt(3*x^2 + 2))/(32*x^5 + 240*x^4 + 720*x^3 + 1080*x^2 + 810*x + 243)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^6}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(1/2)/(3+2*x)**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.48

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^6}\,dx = \frac{1017}{7503125}\sqrt{35}\,\mathrm{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|}-\frac{2\sqrt{6}}{3\,|2\,x+3|}\right) + \frac{1017}{857500}\sqrt{3\,x^2+2} - \frac{13\,(3\,x^2+2)^{\frac{3}{2}}}{175\,(32\,x^5+240\,x^4+720\,x^3+1080\,x^2+810\,x+243)} - \frac{23\,(3\,x^2+2)^{\frac{3}{2}}}{875\,(16\,x^4+96\,x^3+216\,x^2+216\,x+81)} - \frac{43\,(3\,x^2+2)^{\frac{3}{2}}}{6125\,(8\,x^3+36\,x^2+54\,x+27)} - \frac{339\,(3\,x^2+2)^{\frac{3}{2}}}{214375\,(4\,x^2+12\,x+9)} - \frac{3051\sqrt{3\,x^2+2}}{857500\,(2\,x+3)}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^6,x, algorithm="maxima")
```

output
```
1017/7503125*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs
(2*x + 3)) + 1017/857500*sqrt(3*x^2 + 2) - 13/175*(3*x^2 + 2)^(3/2)/(32*x^
5 + 240*x^4 + 720*x^3 + 1080*x^2 + 810*x + 243) - 23/875*(3*x^2 + 2)^(3/2)
/(16*x^4 + 96*x^3 + 216*x^2 + 216*x + 81) - 43/6125*(3*x^2 + 2)^(3/2)/(8*x
^3 + 36*x^2 + 54*x + 27) - 339/214375*(3*x^2 + 2)^(3/2)/(4*x^2 + 12*x + 9)
 - 3051/857500*sqrt(3*x^2 + 2)/(2*x + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(103) = 206$.

Time = 0.14 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.56

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^6}\,dx = \frac{1017}{7503125}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3x^2+2}}\right) - \frac{3\sqrt{3}\left(904\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^9+36612\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^8+254217\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^7-\cdots\right.}{\cdots}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^6,x, algorithm="giac")
```

output
```
1017/7503125*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqr
t(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) -
3/1715000*sqrt(3)*(904*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^9 + 36612*(sq
rt(3)*x - sqrt(3*x^2 + 2))^8 + 254217*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)
)^7 - 142464*(sqrt(3)*x - sqrt(3*x^2 + 2))^6 - 338184*sqrt(3)*(sqrt(3)*x -
 sqrt(3*x^2 + 2))^5 - 4315808*(sqrt(3)*x - sqrt(3*x^2 + 2))^4 + 1676892*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 - 1737184*(sqrt(3)*x - sqrt(3*x^2 +
2))^2 + 219776*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 31232)/((sqrt(3)*x
- sqrt(3*x^2 + 2))^2 + 3*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^5
```

Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.41

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^6}\,dx = \frac{1017\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{7503125} - \frac{1017\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{7503125} + \frac{73\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{11200\left(x^4+6x^3+\frac{27x^2}{2}+\frac{27x}{2}+\frac{81}{16}\right)} - \frac{13\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{640\left(x^5+\frac{15x^4}{2}+\frac{45x^3}{2}+\frac{135x^2}{4}+\frac{405x}{16}+\frac{243}{32}\right)} - \frac{183\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{214375\left(x+\frac{3}{2}\right)} - \frac{3\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{6125\left(x^2+3x+\frac{9}{4}\right)} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{7000\left(x^3+\frac{9x^2}{2}+\frac{27x}{4}+\frac{27}{8}\right)}$$

input
```
int(-((3*x^2 + 2)^(1/2)*(x - 5))/(2*x + 3)^6,x)
```

output
```
(1017*35^(1/2)*log(x + 3/2))/7503125 - (1017*35^(1/2)*log(x - (3^(1/2)*35^
(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/7503125 + (73*3^(1/2)*(x^2 + 2/3)^(1/2)
)/(11200*((27*x)/2 + (27*x^2)/2 + 6*x^3 + x^4 + 81/16)) - (13*3^(1/2)*(x^2
 + 2/3)^(1/2))/(640*((405*x)/16 + (135*x^2)/4 + (45*x^3)/2 + (15*x^4)/2 +
x^5 + 243/32)) - (183*3^(1/2)*(x^2 + 2/3)^(1/2))/(214375*(x + 3/2)) - (3*3
^(1/2)*(x^2 + 2/3)^(1/2))/(6125*(3*x + x^2 + 9/4)) + (3^(1/2)*(x^2 + 2/3)^
(1/2))/(7000*((27*x)/4 + (9*x^2)/2 + x^3 + 27/8))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.39

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^6}\,dx$$

$$= \frac{-409920\sqrt{3x^2+2}\,x^4 - 2694720\sqrt{3x^2+2}\,x^3 - 6523720\sqrt{3x^2+2}\,x^2 - 3785845\sqrt{3x^2+2}\,x - 7773920\sqrt{}}{}$$

input
```
int((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^6,x)
```

output
```
( - 409920*sqrt(3*x**2 + 2)*x**4 - 2694720*sqrt(3*x**2 + 2)*x**3 - 6523720
*sqrt(3*x**2 + 2)*x**2 - 3785845*sqrt(3*x**2 + 2)*x - 7773920*sqrt(3*x**2
+ 2) + 65088*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**5 + 4881
60*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**4 + 1464480*sqrt(3
5)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**3 + 2196720*sqrt(35)*log(sq
rt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**2 + 1647540*sqrt(35)*log(sqrt(3*x**2
 + 2)*sqrt(35) + 9*x - 4)*x + 494262*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35
) + 9*x - 4) - 65088*sqrt(35)*log(2*x + 3)*x**5 - 488160*sqrt(35)*log(2*x
+ 3)*x**4 - 1464480*sqrt(35)*log(2*x + 3)*x**3 - 2196720*sqrt(35)*log(2*x
+ 3)*x**2 - 1647540*sqrt(35)*log(2*x + 3)*x - 494262*sqrt(35)*log(2*x + 3)
)/(15006250*(32*x**5 + 240*x**4 + 720*x**3 + 1080*x**2 + 810*x + 243))
```

3.206 $\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^7}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 148

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^7}\,dx = -\frac{(647-327x)\sqrt{2+3x^2}}{2100(3+2x)^6}+\frac{127\sqrt{2+3x^2}}{98000(3+2x)^4} -\frac{479\sqrt{2+3x^2}}{490000(3+2x)^3}-\frac{2727\sqrt{2+3x^2}}{3430000(3+2x)^2} -\frac{53511\sqrt{2+3x^2}}{120050000(3+2x)}-\frac{6102\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{7503125\sqrt{35}}$$

output
```
-1/2100*(647-327*x)*(3*x^2+2)^(1/2)/(3+2*x)^6+127/98000*(3*x^2+2)^(1/2)/(3
+2*x)^4-479/490000*(3*x^2+2)^(1/2)/(3+2*x)^3-2727/3430000*(3*x^2+2)^(1/2)/
(3+2*x)^2-53511*(3*x^2+2)^(1/2)/(360150000+240100000*x)-6102/262609375*35^
(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 2.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.63

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^7}\,dx = \frac{-\frac{35\sqrt{2+3x^2}(22308548+18651300x+30753930x^2+18236055x^3+5388660x^4+642132x^5)}{(3+2x)^6}+73224\sqrt{35}\text{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{1575656250}$$

input
```
Integrate[((5 - x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^7,x]
```

output
```
((-35*Sqrt[2 + 3*x^2]*(22308548 + 18651300*x + 30753930*x^2 + 18236055*x^3
 + 5388660*x^4 + 642132*x^5))/(3 + 2*x)^6 + 73224*Sqrt[35]*ArcTanh[(3*Sqrt
[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]])/1575656250
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {688, 27, 688, 27, 688, 27, 679, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\sqrt{3x^2+2}}{(2x+3)^7}dx$$

$$\Big\downarrow \; 688$$

$$-\frac{1}{210}\int -\frac{3(82-39x)\sqrt{3x^2+2}}{(2x+3)^6}dx-\frac{13(3x^2+2)^{3/2}}{210(2x+3)^6}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{70}\int \frac{(82-39x)\sqrt{3x^2+2}}{(2x+3)^6}dx-\frac{13(3x^2+2)^{3/2}}{210(2x+3)^6}$$

$$\Big\downarrow \; 688$$

$$\frac{1}{70}\left(-\frac{1}{175}\int -\frac{6(485-281x)\sqrt{3x^2+2}}{(2x+3)^5}dx-\frac{281\left(3x^2+2\right)^{3/2}}{175(2x+3)^5}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{210(2x+3)^6}$$

$\downarrow$ 27

$$\frac{1}{70}\left(\frac{6}{175}\int \frac{(485-281x)\sqrt{3x^2+2}}{(2x+3)^5}dx-\frac{281\left(3x^2+2\right)^{3/2}}{175(2x+3)^5}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{210(2x+3)^6}$$

$\downarrow$ 688

$$\frac{1}{70}\left(\frac{6}{175}\left(-\frac{1}{140}\int -\frac{7(1852-777x)\sqrt{3x^2+2}}{(2x+3)^4}dx-\frac{259\left(3x^2+2\right)^{3/2}}{20(2x+3)^4}\right)-\frac{281\left(3x^2+2\right)^{3/2}}{175(2x+3)^5}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{210(2x+3)^6}$$

$\downarrow$ 27

$$\frac{1}{70}\left(\frac{6}{175}\left(\frac{1}{20}\int \frac{(1852-777x)\sqrt{3x^2+2}}{(2x+3)^4}dx-\frac{259\left(3x^2+2\right)^{3/2}}{20(2x+3)^4}\right)-\frac{281\left(3x^2+2\right)^{3/2}}{175(2x+3)^5}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{210(2x+3)^6}$$

$\downarrow$ 679

$$\frac{1}{70}\left(\frac{6}{175}\left(\frac{1}{20}\left(\frac{2712}{7}\int \frac{\sqrt{3x^2+2}}{(2x+3)^3}dx-\frac{1207\left(3x^2+2\right)^{3/2}}{21(2x+3)^3}\right)-\frac{259\left(3x^2+2\right)^{3/2}}{20(2x+3)^4}\right)-\frac{281\left(3x^2+2\right)^{3/2}}{175(2x+3)^5}\right)-\frac{13\left(3x^2+2\right)^{3/2}}{210(2x+3)^6}$$

$\downarrow$ 486

$$\frac{1}{70}\left(\frac{6}{175}\left(\frac{1}{20}\left(\frac{2712}{7}\left(\frac{3}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{(4-9x)\sqrt{3x^2+2}}{70(2x+3)^2}\right)-\frac{1207\left(3x^2+2\right)^{3/2}}{21(2x+3)^3}\right)-\frac{259\left(3x^2+2\right)^{3}}{20(2x+3)^4}\right.\right.$$
$$\frac{13\left(3x^2+2\right)^{3/2}}{210(2x+3)^6}$$

$\downarrow$ 488

$$\frac{1}{70}\left(\frac{6}{175}\left(\frac{1}{20}\left(\frac{2712}{7}\left(-\frac{3}{35}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{1207\left(3x^2+2\right)^{3/2}}{21(2x+3)^3}\right)-\frac{259(3x^2}{20(2x}\right.\right.$$
$$\frac{13\left(3x^2+2\right)^{3/2}}{210(2x+3)^6}$$

$$\big\downarrow \text{219}$$

$$\frac{1}{70}\left(\frac{6}{175}\left(\frac{1}{20}\left(\frac{2712}{7}\left(-\frac{3\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{1207(3x^2+2)^{3/2}}{21(2x+3)^3}\right)-\frac{259(3x^2+2)^{3/}}{20(2x+3)^4}\right.\right.$$
$$\frac{13(3x^2+2)^{3/2}}{210(2x+3)^6}$$

input
```
Int[((5 - x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^7,x]
```

output
```
(-13*(2 + 3*x^2)^(3/2))/(210*(3 + 2*x)^6) + ((-281*(2 + 3*x^2)^(3/2))/(175
*(3 + 2*x)^5) + (6*((-259*(2 + 3*x^2)^(3/2))/(20*(3 + 2*x)^4) + ((-1207*(2
 + 3*x^2)^(3/2))/(21*(3 + 2*x)^3) + (2712*(-1/70*((4 - 9*x)*Sqrt[2 + 3*x^2
])/(3 + 2*x)^2 - (3*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(35*Sqr
t[35])))/7)/20))/175)/70
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.57

method	result
risch	$-\frac{1926396x^7+16165980x^6+55992429x^5+103039110x^4+92426010x^3+128433504x^2+37302600x+44617096}{45018750(2x+3)^6\sqrt{3x^2+2}}-\frac{6102\sqrt{35}\ \mathrm{arctanh}}{}$
trager	$-\frac{(642132x^5+5388660x^4+18236055x^3+30753930x^2+18651300x+22308548)\sqrt{3x^2+2}}{45018750(2x+3)^6}-\frac{6102\,\mathrm{RootOf}\left(\_Z^2-35\right)\ln\left(-\frac{9\,\mathrm{RootO}}{}\right.}{}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{13440\left(x+\frac{3}{2}\right)^6}-\frac{281\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{392000\left(x+\frac{3}{2}\right)^5}-\frac{111\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{280000\left(x+\frac{3}{2}\right)^4}-\frac{1207\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{6860000\left(x+\frac{3}{2}\right)^3}-101$

input
```
int((5-x)*(3*x^2+2)^(1/2)/(2*x+3)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/45018750*(1926396*x^7+16165980*x^6+55992429*x^5+103039110*x^4+92426010*
x^3+128433504*x^2+37302600*x+44617096)/(2*x+3)^6/(3*x^2+2)^(1/2)-6102/2626
09375*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^7}\,dx$$
$$= \frac{18306\sqrt{35}(64\,x^6+576\,x^5+2160\,x^4+4320\,x^3+4860\,x^2+2916\,x+729)\log\left(-\frac{\sqrt{35}\sqrt{3\,x^2+2}(9\,x-4)+93\,x^2-3}{4\,x^2+12\,x+9}\right.}{1575656250\,(64\,x^6+576\,x^5+2160\,x}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^7,x, algorithm="fricas")
```

output
```
1/1575656250*(18306*sqrt(35)*(64*x^6 + 576*x^5 + 2160*x^4 + 4320*x^3 + 486
0*x^2 + 2916*x + 729)*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) + 93*x^2 -
36*x + 43)/(4*x^2 + 12*x + 9)) - 35*(642132*x^5 + 5388660*x^4 + 18236055*x
^3 + 30753930*x^2 + 18651300*x + 22308548)*sqrt(3*x^2 + 2))/(64*x^6 + 576*
x^5 + 2160*x^4 + 4320*x^3 + 4860*x^2 + 2916*x + 729)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^7}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(1/2)/(3+2*x)**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.55

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^7}\,dx$$
$$= \frac{6102}{262609375}\sqrt{35}\,\operatorname{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|}-\frac{2\sqrt{6}}{3\,|2\,x+3|}\right)+\frac{3051}{15006250}\sqrt{3\,x^2+2}$$
$$-\frac{13\,(3\,x^2+2)^{\frac{3}{2}}}{210\,(64\,x^6+576\,x^5+2160\,x^4+4320\,x^3+4860\,x^2+2916\,x+729)}$$
$$-\frac{281\,(3\,x^2+2)^{\frac{3}{2}}}{12250\,(32\,x^5+240\,x^4+720\,x^3+1080\,x^2+810\,x+243)}$$
$$-\frac{111\,(3\,x^2+2)^{\frac{3}{2}}}{17500\,(16\,x^4+96\,x^3+216\,x^2+216\,x+81)}-\frac{1207\,(3\,x^2+2)^{\frac{3}{2}}}{857500\,(8\,x^3+36\,x^2+54\,x+27)}$$
$$-\frac{2034\,(3\,x^2+2)^{\frac{3}{2}}}{7503125\,(4\,x^2+12\,x+9)}-\frac{9153\sqrt{3\,x^2+2}}{15006250\,(2\,x+3)}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^7,x, algorithm="maxima")
```

output
```
6102/262609375*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/a
bs(2*x + 3)) + 3051/15006250*sqrt(3*x^2 + 2) - 13/210*(3*x^2 + 2)^(3/2)/(6
4*x^6 + 576*x^5 + 2160*x^4 + 4320*x^3 + 4860*x^2 + 2916*x + 729) - 281/122
50*(3*x^2 + 2)^(3/2)/(32*x^5 + 240*x^4 + 720*x^3 + 1080*x^2 + 810*x + 243)
 - 111/17500*(3*x^2 + 2)^(3/2)/(16*x^4 + 96*x^3 + 216*x^2 + 216*x + 81) -
1207/857500*(3*x^2 + 2)^(3/2)/(8*x^3 + 36*x^2 + 54*x + 27) - 2034/7503125*
(3*x^2 + 2)^(3/2)/(4*x^2 + 12*x + 9) - 9153/15006250*sqrt(3*x^2 + 2)/(2*x
+ 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(121) = 242$.

Time = 0.14 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.48

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^7}\,dx$$
$$= \frac{6102}{262609375}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3x^2+2}}\right)$$
$$-\frac{3\sqrt{3}\Big(21696\sqrt{3}\big(\sqrt{3}x-\sqrt{3x^2+2}\big)^{11}+1073952\big(\sqrt{3}x-\sqrt{3x^2+2}\big)^{10}+6978880\sqrt{3}\big(\sqrt{3}x-\sqrt{3x^2+}$$

input
```
integrate((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^7,x, algorithm="giac")
```

output
```
6102/262609375*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*s
qrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2)))
- 3/240100000*sqrt(3)*(21696*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^11 + 10
73952*(sqrt(3)*x - sqrt(3*x^2 + 2))^10 + 6978880*sqrt(3)*(sqrt(3)*x - sqrt
(3*x^2 + 2))^9 + 87678735*(sqrt(3)*x - sqrt(3*x^2 + 2))^8 - 66333990*sqrt(
3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^7 - 258582989*(sqrt(3)*x - sqrt(3*x^2 + 2
))^6 - 426764436*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^5 + 755892540*(sqrt
(3)*x - sqrt(3*x^2 + 2))^4 - 355133440*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2
))^3 + 207134880*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 - 19853952*sqrt(3)*(sqrt(
3)*x - sqrt(3*x^2 + 2)) + 2283136)/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 3*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^6
```

Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.51

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^7}\,dx = \frac{6102\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{262609375} - \frac{6102\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{262609375} + \frac{127\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1568000\left(x^4+6x^3+\frac{27x^2}{2}+\frac{27x}{2}+\frac{81}{16}\right)} + \frac{109\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{44800\left(x^5+\frac{15x^4}{2}+\frac{45x^3}{2}+\frac{135x^2}{4}+\frac{405x}{16}+\frac{243}{32}\right)} - \frac{53511\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{240100000\left(x+\frac{3}{2}\right)} - \frac{13\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1536\left(x^6+9x^5+\frac{135x^4}{4}+\frac{135x^3}{2}+\frac{1215x^2}{16}+\frac{729x}{16}+\frac{729}{64}\right)} - \frac{2727\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{13720000\left(x^2+3x+\frac{9}{4}\right)} - \frac{479\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{3920000\left(x^3+\frac{9x^2}{2}+\frac{27x}{4}+\frac{27}{8}\right)}$$

input
```
int(-((3*x^2 + 2)^(1/2)*(x - 5))/(2*x + 3)^7,x)
```

output
```
(6102*35^(1/2)*log(x + 3/2))/262609375 - (6102*35^(1/2)*log(x - (3^(1/2)*3
5^(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/262609375 + (127*3^(1/2)*(x^2 + 2/3)^
(1/2))/(1568000*((27*x)/2 + (27*x^2)/2 + 6*x^3 + x^4 + 81/16)) + (109*3^(1
/2)*(x^2 + 2/3)^(1/2))/(44800*((405*x)/16 + (135*x^2)/4 + (45*x^3)/2 + (15
*x^4)/2 + x^5 + 243/32)) - (53511*3^(1/2)*(x^2 + 2/3)^(1/2))/(240100000*(x
 + 3/2)) - (13*3^(1/2)*(x^2 + 2/3)^(1/2))/(1536*((729*x)/16 + (1215*x^2)/1
6 + (135*x^3)/2 + (135*x^4)/4 + 9*x^5 + x^6 + 729/64)) - (2727*3^(1/2)*(x^
2 + 2/3)^(1/2))/(13720000*(3*x + x^2 + 9/4)) - (479*3^(1/2)*(x^2 + 2/3)^(1
/2))/(3920000*((27*x)/4 + (9*x^2)/2 + x^3 + 27/8))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.41

$$\int \frac{(5-x)\sqrt{2+3x^2}}{(3+2x)^7}\,dx$$
$$= \frac{-22474620\sqrt{3x^2+2}\,x^5-188603100\sqrt{3x^2+2}\,x^4-638261925\sqrt{3x^2+2}\,x^3-1076387550\sqrt{3x^2+2}\,x^2 -}{}$$

input
```
int((5-x)*(3*x^2+2)^(1/2)/(3+2*x)^7,x)
```

output
```
( - 22474620*sqrt(3*x**2 + 2)*x**5 - 188603100*sqrt(3*x**2 + 2)*x**4 - 638
261925*sqrt(3*x**2 + 2)*x**3 - 1076387550*sqrt(3*x**2 + 2)*x**2 - 65279550
0*sqrt(3*x**2 + 2)*x - 780799180*sqrt(3*x**2 + 2) + 2343168*sqrt(35)*log(s
qrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**6 + 21088512*sqrt(35)*log(sqrt(3*x*
*2 + 2)*sqrt(35) + 9*x - 4)*x**5 + 79081920*sqrt(35)*log(sqrt(3*x**2 + 2)*
sqrt(35) + 9*x - 4)*x**4 + 158163840*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35
) + 9*x - 4)*x**3 + 177934320*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x
 - 4)*x**2 + 106760592*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x
 + 26690148*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4) - 2343168*sq
rt(35)*log(2*x + 3)*x**6 - 21088512*sqrt(35)*log(2*x + 3)*x**5 - 79081920*
sqrt(35)*log(2*x + 3)*x**4 - 158163840*sqrt(35)*log(2*x + 3)*x**3 - 177934
320*sqrt(35)*log(2*x + 3)*x**2 - 106760592*sqrt(35)*log(2*x + 3)*x - 26690
148*sqrt(35)*log(2*x + 3))/(1575656250*(64*x**6 + 576*x**5 + 2160*x**4 + 4
320*x**3 + 4860*x**2 + 2916*x + 729))
```

3.207 $\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 138

$$\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{3/2} \, dx = \frac{2777}{12} x\sqrt{2+3x^2} + \frac{2777}{36} x\left(2+3x^2\right)^{3/2} + \frac{4421(3+2x)^2 \left(2+3x^2\right)^{5/2}}{2268} + \frac{13}{36}(3+2x)^3 \left(2+3x^2\right)^{5/2} - \frac{1}{27}(3+2x)^4 \left(2+3x^2\right)^{5/2} + \frac{(661583+226755x)\left(2+3x^2\right)^{5/2}}{17010} + \frac{2777\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{6\sqrt{3}}$$

output
```
2777/12*x*(3*x^2+2)^(1/2)+2777/36*x*(3*x^2+2)^(3/2)+4421/2268*(3+2*x)^2*(3
*x^2+2)^(5/2)+13/36*(3+2*x)^3*(3*x^2+2)^(5/2)-1/27*(3+2*x)^4*(3*x^2+2)^(5/
2)+1/17010*(661583+226755*x)*(3*x^2+2)^(5/2)+2777/18*arcsinh(1/2*x*6^(1/2)
)*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.62

$$\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{3/2} \, dx =$$

$$-\frac{\sqrt{2+3x^2}(-8598544-19683405x-27537072x^2-27468315x^3-24490404x^4-14492520x^5-3676320}{34020}$$

$$-\frac{2777 \log \left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{6\sqrt{3}}$$

input
```
Integrate[(5 - x)*(3 + 2*x)^4*(2 + 3*x^2)^(3/2),x]
```

output
```
-1/34020*(Sqrt[2 + 3*x^2]*(-8598544 - 19683405*x - 27537072*x^2 - 27468315
*x^3 - 24490404*x^4 - 14492520*x^5 - 3676320*x^6 + 204120*x^7 + 181440*x^8
)) - (2777*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(6*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {687, 687, 27, 687, 27, 676, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-x)(2x+3)^4 \left(3x^2+2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{27} \int (2x+3)^3(234x+421) \left(3x^2+2\right)^{3/2} dx - \frac{1}{27}(2x+3)^4 \left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{27}\left(\frac{1}{24}\int 6(2x+3)^2(4421x+4584) \left(3x^2+2\right)^{3/2} dx + \frac{39}{4}\left(3x^2+2\right)^{5/2}(2x+3)^3\right) - \frac{1}{27}(2x+3)^4 \left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{27}\left(\frac{1}{4}\int (2x+3)^2(4421x+4584)\left(3x^2+2\right)^{3/2}dx+\frac{39}{4}\left(3x^2+2\right)^{5/2}(2x+3)^3\right)-\frac{1}{27}(2x+3)^4\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{27}\left(\frac{1}{4}\left(\frac{1}{21}\int 2(2x+3)(136053x+126712)\left(3x^2+2\right)^{3/2}dx+\frac{4421}{21}(2x+3)^2\left(3x^2+2\right)^{5/2}\right)+\frac{39}{4}\left(3x^2+2\right)^{5/2}(2\right.$$
$$\frac{1}{27}(2x+3)^4\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{27}\left(\frac{1}{4}\left(\frac{2}{21}\int (2x+3)(136053x+126712)\left(3x^2+2\right)^{3/2}dx+\frac{4421}{21}(2x+3)^2\left(3x^2+2\right)^{5/2}\right)+\frac{39}{4}\left(3x^2+2\right)^{5/2}(2x\right.$$
$$\frac{1}{27}(2x+3)^4\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{676}$$

$$\frac{1}{27}\left(\frac{1}{4}\left(\frac{2}{21}\left(349902\int \left(3x^2+2\right)^{3/2}dx+15117x\left(3x^2+2\right)^{5/2}+\frac{661583}{15}\left(3x^2+2\right)^{5/2}\right)+\frac{4421}{21}(2x+3)^2\left(3x^2+2\right.\right.\right.$$
$$\frac{1}{27}(2x+3)^4\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{27}\left(\frac{1}{4}\left(\frac{2}{21}\left(349902\left(\frac{3}{2}\int \sqrt{3x^2+2}dx+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+15117x\left(3x^2+2\right)^{5/2}+\frac{661583}{15}\left(3x^2+2\right)^{5/2}\right)+\frac{44}{2}\right.\right.$$
$$\frac{1}{27}(2x+3)^4\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{27}\left(\frac{1}{4}\left(\frac{2}{21}\left(349902\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+15117x\left(3x^2+2\right)^{5/2}+\frac{661583}{15}\right.\right.\right.$$
$$\frac{1}{27}(2x+3)^4\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{27}\left(\frac{1}{4}\left(\frac{2}{21}\left(349902\left(\frac{3}{2}\left(\frac{\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+15117x\left(3x^2+2\right)^{5/2}+\frac{66158}{15}\right.\right.\right.$$
$$\frac{1}{27}(2x+3)^4\left(3x^2+2\right)^{5/2}$$

input
```
Int[(5 - x)*(3 + 2*x)^4*(2 + 3*x^2)^(3/2),x]
```

output
```
-1/27*((3 + 2*x)^4*(2 + 3*x^2)^(5/2)) + ((39*(3 + 2*x)^3*(2 + 3*x^2)^(5/2)
)/4 + ((4421*(3 + 2*x)^2*(2 + 3*x^2)^(5/2))/21 + (2*((661583*(2 + 3*x^2)^(
5/2))/15 + 15117*x*(2 + 3*x^2)^(5/2) + 349902*((x*(2 + 3*x^2)^(3/2))/4 + (
3*((x*Sqrt[2 + 3*x^2])/2 + ArcSinh[Sqrt[3/2]*x]/Sqrt[3]))/2)))/21)/4)/27
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.47

method	result
risch	$-\frac{(181440x^8+204120x^7-3676320x^6-14492520x^5-24490404x^4-27468315x^3-27537072x^2-19683405x-8598544)\sqrt{3x^2+2}}{34020}+$
trager	$\left(-\frac{16}{3}x^8-6x^7+\frac{6808}{63}x^6+426x^5+\frac{226763}{315}x^4+\frac{9689}{12}x^3+\frac{2294756}{2835}x^2+\frac{6943}{12}x+\frac{2149636}{8505}\right)\sqrt{3x^2+2}-$
default	$\frac{2777x(3x^2+2)^{\frac{3}{2}}}{36}+\frac{2777x\sqrt{3x^2+2}}{12}+\frac{2777\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{18}+\frac{537409(3x^2+2)^{\frac{5}{2}}}{8505}+\frac{434x(3x^2+2)^{\frac{5}{2}}}{9}+\frac{7256x^2(3x^2+2)^{\frac{5}{2}}}{567}$
meijerg	$\frac{405\sqrt{3}\left(\frac{4\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\left(\frac{3x^2}{8}+\frac{5}{8}\right)\sqrt{\frac{3x^2}{2}+1}}{3}+\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{2\sqrt{\pi}}-\frac{32\sqrt{3}\left(-\frac{\sqrt{6}\,\sqrt{\pi}\,x\left(-270x^6-270x^4-15x^2+15\right)\sqrt{\frac{3x^2}{2}+1}}{480}+\frac{\sqrt{\pi}\,\mathrm{ar}}{}\right.}{9\sqrt{\pi}}$

input
```
int((5-x)*(2*x+3)^4*(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/34020*(181440*x^8+204120*x^7-3676320*x^6-14492520*x^5-24490404*x^4-2746
8315*x^3-27537072*x^2-19683405*x-8598544)*(3*x^2+2)^(1/2)+2777/18*arcsinh(
1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.58

$$\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{3/2} \, dx =$$
$$-\frac{1}{34020} \left(181440\,x^{8} + 204120\,x^{7} - 3676320\,x^{6} - 14492520\,x^{5} - 24490404\,x^{4} - 27468315\,x^{3} - 27537072\,x^{2}\right.$$
$$+ \frac{2777}{36}\,\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^{2}+2}x - 3\,x^{2} - 1\right)$$

input
```
integrate((5-x)*(3+2*x)^4*(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/34020*(181440*x^8 + 204120*x^7 - 3676320*x^6 - 14492520*x^5 - 24490404*
x^4 - 27468315*x^3 - 27537072*x^2 - 19683405*x - 8598544)*sqrt(3*x^2 + 2)
+ 2777/36*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.17

$$\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{3/2} \, dx = -\frac{16x^{8}\sqrt{3x^{2}+2}}{3} - 6x^{7}\sqrt{3x^{2}+2} + \frac{6808x^{6}\sqrt{3x^{2}+2}}{63} + 426x^{5}\sqrt{3x^{2}+2} + \frac{226763x^{4}\sqrt{3x^{2}+2}}{315} + \frac{9689x^{3}\sqrt{3x^{2}+2}}{12} + \frac{2294756x^{2}\sqrt{3x^{2}+2}}{2835} + \frac{6943x\sqrt{3x^{2}+2}}{12} + \frac{2149636\sqrt{3x^{2}+2}}{8505} + \frac{2777\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{18}$$

input
```
integrate((5-x)*(3+2*x)**4*(3*x**2+2)**(3/2),x)
```

output
```
-16*x**8*sqrt(3*x**2 + 2)/3 - 6*x**7*sqrt(3*x**2 + 2) + 6808*x**6*sqrt(3*x
**2 + 2)/63 + 426*x**5*sqrt(3*x**2 + 2) + 226763*x**4*sqrt(3*x**2 + 2)/315
 + 9689*x**3*sqrt(3*x**2 + 2)/12 + 2294756*x**2*sqrt(3*x**2 + 2)/2835 + 69
43*x*sqrt(3*x**2 + 2)/12 + 2149636*sqrt(3*x**2 + 2)/8505 + 2777*sqrt(3)*as
inh(sqrt(6)*x/2)/18
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.74

$$\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{3/2} \, dx = -\frac{16}{27} \left(3 x^2+2\right)^{\frac{5}{2}} x^4 - \frac{2}{3} \left(3 x^2+2\right)^{\frac{5}{2}} x^3 + \frac{7256}{567} \left(3 x^2+2\right)^{\frac{5}{2}} x^2 + \frac{434}{9} \left(3 x^2+2\right)^{\frac{5}{2}} x + \frac{537409}{8505} \left(3 x^2+2\right)^{\frac{5}{2}} + \frac{2777}{36} \left(3 x^2+2\right)^{\frac{3}{2}} x + \frac{2777}{12} \sqrt{3 x^2+2} x + \frac{2777}{18} \sqrt{3} \operatorname{arsinh}\left(\frac{1}{2} \sqrt{6} x\right)$$

input
```
integrate((5-x)*(3+2*x)^4*(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
-16/27*(3*x^2 + 2)^(5/2)*x^4 - 2/3*(3*x^2 + 2)^(5/2)*x^3 + 7256/567*(3*x^2
 + 2)^(5/2)*x^2 + 434/9*(3*x^2 + 2)^(5/2)*x + 537409/8505*(3*x^2 + 2)^(5/2
) + 2777/36*(3*x^2 + 2)^(3/2)*x + 2777/12*sqrt(3*x^2 + 2)*x + 2777/18*sqrt
(3)*arcsinh(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.52

$$\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{3/2} \, dx = -\frac{1}{34020} (3 ((9 (4 (10 ((21 (8 x+9) x-3404) x-13419) x-226763) x-1017345) x-9179024) x-656113 - \frac{2777}{18} \sqrt{3} \log\left(-\sqrt{3} x+\sqrt{3 x^2+2}\right)$$

input
```
integrate((5-x)*(3+2*x)^4*(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
-1/34020*(3*((9*(4*(10*((21*(8*x + 9)*x - 3404)*x - 13419)*x - 226763)*x -
 1017345)*x - 9179024)*x - 6561135)*x - 8598544)*sqrt(3*x^2 + 2) - 2777/18
*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.47

$$\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{3/2} \, dx = \frac{2777\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{18} + \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(-16\,x^8-18\,x^7+\frac{6808\,x^6}{21}+1278\,x^5+\frac{226763\,x^4}{105}+\frac{9689\,x^3}{4}+\frac{2294756\,x^2}{945}+\frac{6943\,x}{4}+\frac{2149636}{2835}\right)}{3}$$

input
```
int(-(2*x + 3)^4*(3*x^2 + 2)^(3/2)*(x - 5),x)
```

output
```
(2777*3^(1/2)*asinh((6^(1/2)*x)/2))/18 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((6943
*x)/4 + (2294756*x^2)/945 + (9689*x^3)/4 + (226763*x^4)/105 + 1278*x^5 + (
6808*x^6)/21 - 18*x^7 - 16*x^8 + 2149636/2835))/3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{3/2} \, dx = -\frac{16\sqrt{3x^2+2}\,x^8}{3} - 6\sqrt{3x^2+2}\,x^7 + \frac{6808\sqrt{3x^2+2}\,x^6}{63} + 426\sqrt{3x^2+2}\,x^5 + \frac{226763\sqrt{3x^2+2}\,x^4}{315} + \frac{9689\sqrt{3x^2+2}\,x^3}{12} + \frac{2294756\sqrt{3x^2+2}\,x^2}{2835} + \frac{6943\sqrt{3x^2+2}\,x}{12} + \frac{2149636\sqrt{3x^2+2}}{8505} + \frac{2777\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{18}$$

input
```
int((5-x)*(3+2*x)^4*(3*x^2+2)^(3/2),x)
```

output
```
( - 181440*sqrt(3*x**2 + 2)*x**8 - 204120*sqrt(3*x**2 + 2)*x**7 + 3676320*
sqrt(3*x**2 + 2)*x**6 + 14492520*sqrt(3*x**2 + 2)*x**5 + 24490404*sqrt(3*x
**2 + 2)*x**4 + 27468315*sqrt(3*x**2 + 2)*x**3 + 27537072*sqrt(3*x**2 + 2)
*x**2 + 19683405*sqrt(3*x**2 + 2)*x + 8598544*sqrt(3*x**2 + 2) + 5248530*s
qrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/34020
```

3.208 $\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{3/2} \, dx = \frac{1087}{12}x\sqrt{2+3x^2} + \frac{1087}{36}x\left(2+3x^2\right)^{3/2} + \frac{71}{168}(3+2x)^2\left(2+3x^2\right)^{5/2} - \frac{1}{24}(3+2x)^3\left(2+3x^2\right)^{5/2} + \frac{(16973+5405x)\left(2+3x^2\right)^{5/2}}{1260} + \frac{1087\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{6\sqrt{3}}$$

output
```
1087/12*x*(3*x^2+2)^(1/2)+1087/36*x*(3*x^2+2)^(3/2)+71/168*(3+2*x)^2*(3*x^
2+2)^(5/2)-1/24*(3+2*x)^3*(3*x^2+2)^(5/2)+1/1260*(16973+5405*x)*(3*x^2+2)^
(5/2)+1087/18*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{3/2} \, dx = -\frac{\sqrt{2+3x^2}(-81392-226065x-245136x^2-219975x^3-186012x^4-75600x^5-2160x^6+3780x^7)}{1260} - \frac{1087\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{6\sqrt{3}}$$

input
```
Integrate[(5 - x)*(3 + 2*x)^3*(2 + 3*x^2)^(3/2),x]
```

output
```
-1/1260*(Sqrt[2 + 3*x^2]*(-81392 - 226065*x - 245136*x^2 - 219975*x^3 - 18
6012*x^4 - 75600*x^5 - 2160*x^6 + 3780*x^7)) - (1087*Log[-(Sqrt[3]*x) + Sq
rt[2 + 3*x^2]])/(6*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {687, 27, 687, 27, 676, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-x)(2x+3)^3\left(3x^2+2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 687}$$

$$\frac{1}{24}\int 3(2x+3)^2(71x+124)\left(3x^2+2\right)^{3/2}dx-\frac{1}{24}(2x+3)^3\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8}\int (2x+3)^2(71x+124)\left(3x^2+2\right)^{3/2}dx-\frac{1}{24}(2x+3)^3\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{1}{8}\left(\frac{1}{21}\int 2(2x+3)(3243x+3622)\left(3x^2+2\right)^{3/2}dx+\frac{71}{21}(2x+3)^2\left(3x^2+2\right)^{5/2}\right)-\frac{1}{24}(2x+3)^3\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8}\left(\frac{2}{21}\int (2x+3)(3243x+3622)\left(3x^2+2\right)^{3/2}dx+\frac{71}{21}(2x+3)^2\left(3x^2+2\right)^{5/2}\right)-\frac{1}{24}(2x+3)^3\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{ 676}$$

$$\frac{1}{8}\left(\frac{2}{21}\left(\frac{30436}{3}\int \left(3x^2+2\right)^{3/2}dx+\frac{1081}{3}x\left(3x^2+2\right)^{5/2}+\frac{16973}{15}\left(3x^2+2\right)^{5/2}\right)+\frac{71}{21}(2x+3)^2\left(3x^2+2\right)^{5/2}\right)-\frac{1}{24}(2x+3)^3\left(3x^2+2\right)^{5/2}$$

$\downarrow$ 211

$$\frac{1}{8}\left(\frac{2}{21}\left(\frac{30436}{3}\left(\frac{3}{2}\int \sqrt{3x^2+2}dx+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1081}{3}x\left(3x^2+2\right)^{5/2}+\frac{16973}{15}\left(3x^2+2\right)^{5/2}\right)+\frac{71}{21}(2x+3\right.$$
$$\frac{1}{24}(2x+3)^3\left(3x^2+2\right)^{5/2}$$

$\downarrow$ 211

$$\frac{1}{8}\left(\frac{2}{21}\left(\frac{30436}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1081}{3}x\left(3x^2+2\right)^{5/2}+\frac{16973}{15}\left(3x^2+2\right.\right.\right.$$
$$\frac{1}{24}(2x+3)^3\left(3x^2+2\right)^{5/2}$$

$\downarrow$ 222

$$\frac{1}{8}\left(\frac{2}{21}\left(\frac{30436}{3}\left(\frac{3}{2}\left(\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1081}{3}x\left(3x^2+2\right)^{5/2}+\frac{16973}{15}\left(3x^2\right.\right.\right.$$
$$\frac{1}{24}(2x+3)^3\left(3x^2+2\right)^{5/2}$$

input
```
Int[(5 - x)*(3 + 2*x)^3*(2 + 3*x^2)^(3/2),x]
```

output
```
-1/24*((3 + 2*x)^3*(2 + 3*x^2)^(5/2)) + ((71*(3 + 2*x)^2*(2 + 3*x^2)^(5/2)
)/21 + (2*((16973*(2 + 3*x^2)^(5/2))/15 + (1081*x*(2 + 3*x^2)^(5/2))/3 + (
30436*((x*(2 + 3*x^2)^(3/2))/4 + (3*((x*Sqrt[2 + 3*x^2])/2 + ArcSinh[Sqrt[
3/2]*x]/Sqrt[3]))/2))/3))/21)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

method	result
risch	$-\frac{(3780x^7-2160x^6-75600x^5-186012x^4-219975x^3-245136x^2-226065x-81392)\sqrt{3x^2+2}}{1260}+\frac{1087\,\text{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{18}$
trager	$\left(-3x^7+\frac{12}{7}x^6+60x^5+\frac{5167}{35}x^4+\frac{2095}{12}x^3+\frac{20428}{105}x^2+\frac{2153}{12}x+\frac{20348}{315}\right)\sqrt{3x^2+2}-\frac{1087\,\text{RootOf}\left(\_Z^2-\right.}{}$
default	$\frac{1087x(3x^2+2)^{\frac{3}{2}}}{36}+\frac{1087x\sqrt{3x^2+2}}{12}+\frac{1087\,\text{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{18}+\frac{5087(3x^2+2)^{\frac{5}{2}}}{315}+\frac{64x(3x^2+2)^{\frac{5}{2}}}{9}+\frac{4x^2(3x^2+2)^{\frac{5}{2}}}{21}-\frac{x^3}{}$
meijerg	$\frac{135\sqrt{3}\left(\frac{4\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\left(\frac{3x^2}{8}+\frac{5}{8}\right)\sqrt{\frac{3x^2}{2}+1}}{3}+\sqrt{\pi}\,\text{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{2\sqrt{\pi}}+\frac{4\sqrt{2}\left(\frac{16\sqrt{\pi}}{105}-\frac{2\sqrt{\pi}\left(-\frac{135}{2}x^6-72x^4-6x^2+8\right)\sqrt{\frac{3x^2}{2}+1}}{105}\right)}{3\sqrt{\pi}}+-\frac{42}{}$

input
```
int((5-x)*(2*x+3)^3*(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1260*(3780*x^7-2160*x^6-75600*x^5-186012*x^4-219975*x^3-245136*x^2-2260
65*x-81392)*(3*x^2+2)^(1/2)+1087/18*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

$$\int (5-x)(3+2x)^3\left(2+3x^2\right)^{3/2}\,dx = -\frac{1}{1260}\left(3780\,x^7-2160\,x^6-75600\,x^5-186012\,x^4-219975\,x^3-245136\,x^2-226065\,x-81392\right)\sqrt{3\,x^2+}$$
$$+\frac{1087}{36}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3+2*x)^3*(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/1260*(3780*x^7 - 2160*x^6 - 75600*x^5 - 186012*x^4 - 219975*x^3 - 24513
6*x^2 - 226065*x - 81392)*sqrt(3*x^2 + 2) + 1087/36*sqrt(3)*log(-sqrt(3)*s
qrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.24

$$\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{3/2} dx = -3x^7\sqrt{3x^2+2} + \frac{12x^6\sqrt{3x^2+2}}{7} + 60x^5\sqrt{3x^2+2} + \frac{5167x^4\sqrt{3x^2+2}}{35} + \frac{2095x^3\sqrt{3x^2+2}}{12} + \frac{20428x^2\sqrt{3x^2+2}}{105} + \frac{2153x\sqrt{3x^2+2}}{12} + \frac{20348\sqrt{3x^2+2}}{315} + \frac{1087\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{18}$$

input
```
integrate((5-x)*(3+2*x)**3*(3*x**2+2)**(3/2),x)
```

output
```
-3*x**7*sqrt(3*x**2 + 2) + 12*x**6*sqrt(3*x**2 + 2)/7 + 60*x**5*sqrt(3*x**
2 + 2) + 5167*x**4*sqrt(3*x**2 + 2)/35 + 2095*x**3*sqrt(3*x**2 + 2)/12 + 2
0428*x**2*sqrt(3*x**2 + 2)/105 + 2153*x*sqrt(3*x**2 + 2)/12 + 20348*sqrt(3
*x**2 + 2)/315 + 1087*sqrt(3)*asinh(sqrt(6)*x/2)/18
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

$$\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{3/2} dx = -\frac{1}{3}\left(3x^2+2\right)^{\frac{5}{2}}x^3 + \frac{4}{21}\left(3x^2+2\right)^{\frac{5}{2}}x^2 + \frac{64}{9}\left(3x^2+2\right)^{\frac{5}{2}}x + \frac{5087}{315}\left(3x^2+2\right)^{\frac{5}{2}} + \frac{1087}{36}\left(3x^2+2\right)^{\frac{3}{2}}x + \frac{1087}{12}\sqrt{3x^2+2}x + \frac{1087}{18}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)$$

input
```
integrate((5-x)*(3+2*x)^3*(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*(3*x^2 + 2)^(5/2)*x^3 + 4/21*(3*x^2 + 2)^(5/2)*x^2 + 64/9*(3*x^2 + 2)
^(5/2)*x + 5087/315*(3*x^2 + 2)^(5/2) + 1087/36*(3*x^2 + 2)^(3/2)*x + 1087
/12*sqrt(3*x^2 + 2)*x + 1087/18*sqrt(3)*arcsinh(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.57

$$\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{3/2} \, dx = -\frac{1}{1260}\left(3\left(\left(\left(12\left(15\left((7x-4)x-140\right)x-5167\right)x-73325\right)x-81712\right)x-75355\right)x-81392\right)\sqrt{3x^2+2} - \frac{1087}{18}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)$$

input
```
integrate((5-x)*(3+2*x)^3*(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
-1/1260*(3*(((12*(15*((7*x - 4)*x - 140)*x - 5167)*x - 73325)*x - 81712)*x
 - 75355)*x - 81392)*sqrt(3*x^2 + 2) - 1087/18*sqrt(3)*log(-sqrt(3)*x + sq
rt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{3/2} \, dx = \frac{1087\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{18} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(-9x^7+\frac{36x^6}{7}+180x^5+\frac{15501x^4}{35}+\frac{2095x^3}{4}+\frac{20428x^2}{35}+\frac{2153x}{4}+\frac{20348}{105}\right)}{3}$$

input
```
int(-(2*x + 3)^3*(3*x^2 + 2)^(3/2)*(x - 5),x)
```

output
```
(1087*3^(1/2)*asinh((6^(1/2)*x)/2))/18 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((2153
*x)/4 + (20428*x^2)/35 + (2095*x^3)/4 + (15501*x^4)/35 + 180*x^5 + (36*x^6
)/7 - 9*x^7 + 20348/105))/3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

$$\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{3/2} \, dx = -3\sqrt{3x^2+2}\, x^7 + \frac{12\sqrt{3x^2+2}\, x^6}{7} + 60\sqrt{3x^2+2}\, x^5 + \frac{5167\sqrt{3x^2+2}\, x^4}{35} + \frac{2095\sqrt{3x^2+2}\, x^3}{12} + \frac{20428\sqrt{3x^2+2}\, x^2}{105} + \frac{2153\sqrt{3x^2+2}\, x}{12} + \frac{20348\sqrt{3x^2+2}}{315} + \frac{1087\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{18}$$

input
```
int((5-x)*(3+2*x)^3*(3*x^2+2)^(3/2),x)
```

output
```
( - 3780*sqrt(3*x**2 + 2)*x**7 + 2160*sqrt(3*x**2 + 2)*x**6 + 75600*sqrt(3
*x**2 + 2)*x**5 + 186012*sqrt(3*x**2 + 2)*x**4 + 219975*sqrt(3*x**2 + 2)*x
**3 + 245136*sqrt(3*x**2 + 2)*x**2 + 226065*sqrt(3*x**2 + 2)*x + 81392*sqr
t(3*x**2 + 2) + 76090*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))
/1260
```

3.209 $\int (5-x)(3+2x)^2 \left(2+3x^2\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int (5-x)(3+2x)^2 \left(2+3x^2\right)^{3/2} dx = \frac{397}{12}x\sqrt{2+3x^2}+\frac{397}{36}x\left(2+3x^2\right)^{3/2} - \frac{1}{21}(3+2x)^2 \left(2+3x^2\right)^{5/2}+\frac{2}{315}(611+160x) \left(2+3x^2\right)^{5/2}+\frac{397\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{6\sqrt{3}}$$

output
```
397/12*x*(3*x^2+2)^(1/2)+397/36*x*(3*x^2+2)^(3/2)-1/21*(3+2*x)^2*(3*x^2+2)
^(5/2)+2/315*(611+160*x)*(3*x^2+2)^(5/2)+397/18*arcsinh(1/2*x*6^(1/2))*3^(
1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

$$\int (5-x)(3+2x)^2 \left(2+3x^2\right)^{3/2} dx = -\frac{\sqrt{2+3x^2}(-17392-71715x-51216x^2-48405x^3-36252x^4-5040x^5+2160x^6)}{1260} - \frac{397\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{6\sqrt{3}}$$

input
```
Integrate[(5 - x)*(3 + 2*x)^2*(2 + 3*x^2)^(3/2),x]
```

output
```
-1/1260*(Sqrt[2 + 3*x^2]*(-17392 - 71715*x - 51216*x^2 - 48405*x^3 - 36252
*x^4 - 5040*x^5 + 2160*x^6)) - (397*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(
6*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {687, 676, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-x)(2x+3)^2 \left(3x^2+2\right)^{3/2} dx$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{21}\int (2x+3)(192x+323)\left(3x^2+2\right)^{3/2}dx-\frac{1}{21}(2x+3)^2\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{676}$$

$$\frac{1}{21}\left(\frac{2779}{3}\int \left(3x^2+2\right)^{3/2}dx+\frac{64}{3}x\left(3x^2+2\right)^{5/2}+\frac{1222}{15}\left(3x^2+2\right)^{5/2}\right)-\frac{1}{21}(2x+3)^2\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{21}\left(\frac{2779}{3}\left(\frac{3}{2}\int \sqrt{3x^2+2}dx+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{64}{3}x\left(3x^2+2\right)^{5/2}+\frac{1222}{15}\left(3x^2+2\right)^{5/2}\right)-\frac{1}{21}(2x+3)^2\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{21}\left(\frac{2779}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{64}{3}x\left(3x^2+2\right)^{5/2}+\frac{1222}{15}\left(3x^2+2\right)^{5/2}\right)-\frac{1}{21}(2x+3)^2\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \; 222$$

$$\frac{1}{21}\left(\frac{2779}{3}\left(\frac{3}{2}\left(\frac{\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{64}{3}x\left(3x^2+2\right)^{5/2}+\frac{1222}{15}\left(3x^2+2\right)^{5/2}\right)-\frac{1}{21}(2x+3)^2\left(3x^2+2\right)^{5/2}$$

input
```
Int[(5 - x)*(3 + 2*x)^2*(2 + 3*x^2)^(3/2),x]
```

output
```
-1/21*((3 + 2*x)^2*(2 + 3*x^2)^(5/2)) + ((1222*(2 + 3*x^2)^(5/2))/15 + (64
*x*(2 + 3*x^2)^(5/2))/3 + (2779*((x*(2 + 3*x^2)^(3/2))/4 + (3*((x*Sqrt[2 +
 3*x^2])/2 + ArcSinh[Sqrt[3/2]*x]/Sqrt[3]))/2))/3)/21
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{(2160x^6-5040x^5-36252x^4-48405x^3-51216x^2-71715x-17392)\sqrt{3x^2+2}}{1260}+\frac{397\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{18}$
trager	$\left(-\frac{12}{7}x^6+4x^5+\frac{1007}{35}x^4+\frac{461}{12}x^3+\frac{4268}{105}x^2+\frac{683}{12}x+\frac{4348}{315}\right)\sqrt{3x^2+2}-\frac{397\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootC}\right.}{18}$
default	$\frac{397x(3x^2+2)^{\frac{3}{2}}}{36}+\frac{397x\sqrt{3x^2+2}}{12}+\frac{397\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{18}+\frac{1087(3x^2+2)^{\frac{5}{2}}}{315}+\frac{4x(3x^2+2)^{\frac{5}{2}}}{9}-\frac{4x^2(3x^2+2)^{\frac{5}{2}}}{21}$
meijerg	$\frac{45\sqrt{3}\left(\frac{4\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\left(\frac{3x^2}{8}+\frac{5}{8}\right)\sqrt{\frac{3x^2}{2}+1}}{3}+\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{2\sqrt{\pi}}+\frac{8\sqrt{3}\left(\frac{\sqrt{6}\,\sqrt{\pi}\,x\left(18x^4+21x^2+3\right)\sqrt{\frac{3x^2}{2}+1}}{36}-\frac{\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{6}\right.}{3\sqrt{\pi}}$

input
```
int((5-x)*(2*x+3)^2*(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1260*(2160*x^6-5040*x^5-36252*x^4-48405*x^3-51216*x^2-71715*x-17392)*(3
*x^2+2)^(1/2)+397/18*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int (5-x)(3+2x)^2\left(2+3x^2\right)^{3/2}\,dx = -\frac{1}{1260}\left(2160\,x^6-5040\,x^5-36252\,x^4-48405\,x^3-51216\,x^2-71715\,x-17392\right)\sqrt{3\,x^2+2}+\frac{397}{36}\,\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3+2*x)^2*(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/1260*(2160*x^6 - 5040*x^5 - 36252*x^4 - 48405*x^3 - 51216*x^2 - 71715*x
 - 17392)*sqrt(3*x^2 + 2) + 397/36*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x
- 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.37

$$\int (5-x)(3+2x)^2\left(2+3x^2\right)^{3/2}\,dx=-\frac{12x^6\sqrt{3x^2+2}}{7}+4x^5\sqrt{3x^2+2}+\frac{1007x^4\sqrt{3x^2+2}}{35}+\frac{461x^3\sqrt{3x^2+2}}{12}+\frac{4268x^2\sqrt{3x^2+2}}{105}+\frac{683x\sqrt{3x^2+2}}{12}+\frac{4348\sqrt{3x^2+2}}{315}+\frac{397\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{18}$$

input
```
integrate((5-x)*(3+2*x)**2*(3*x**2+2)**(3/2),x)
```

output
```
-12*x**6*sqrt(3*x**2 + 2)/7 + 4*x**5*sqrt(3*x**2 + 2) + 1007*x**4*sqrt(3*x
**2 + 2)/35 + 461*x**3*sqrt(3*x**2 + 2)/12 + 4268*x**2*sqrt(3*x**2 + 2)/10
5 + 683*x*sqrt(3*x**2 + 2)/12 + 4348*sqrt(3*x**2 + 2)/315 + 397*sqrt(3)*as
inh(sqrt(6)*x/2)/18
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.79

$$\int (5-x)(3+2x)^2\left(2+3x^2\right)^{3/2}\,dx=-\frac{4}{21}\left(3\,x^2+2\right)^{\frac{5}{2}}x^2+\frac{4}{9}\left(3\,x^2+2\right)^{\frac{5}{2}}x+\frac{1087}{315}\left(3\,x^2+2\right)^{\frac{5}{2}}+\frac{397}{36}\left(3\,x^2+2\right)^{\frac{3}{2}}x+\frac{397}{12}\sqrt{3\,x^2+2}x+\frac{397}{18}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)$$

input
```
integrate((5-x)*(3+2*x)^2*(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
-4/21*(3*x^2 + 2)^(5/2)*x^2 + 4/9*(3*x^2 + 2)^(5/2)*x + 1087/315*(3*x^2 +
2)^(5/2) + 397/36*(3*x^2 + 2)^(3/2)*x + 397/12*sqrt(3*x^2 + 2)*x + 397/18*
sqrt(3)*arcsinh(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.66

$$\int (5-x)(3+2x)^2 \left(2+3x^2\right)^{3/2} \, dx = -\frac{1}{1260} \left(3 \left(\left(\left(12 \left(20 \left(3 x-7\right)x-1007\right)x-16135\right)x-17072\right)x-23905\right)x-17392\right)\sqrt{3 x^2+2} - \frac{397}{18} \sqrt{3} \log \left(-\sqrt{3}x+\sqrt{3 x^2+2}\right)$$

input
```
integrate((5-x)*(3+2*x)^2*(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
-1/1260*(3*(((12*(20*(3*x - 7)*x - 1007)*x - 16135)*x - 17072)*x - 23905)*
x - 17392)*sqrt(3*x^2 + 2) - 397/18*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 +
2))
```

Mupad [B] (verification not implemented)

Time = 5.90 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

$$\int (5-x)(3+2x)^2 \left(2+3x^2\right)^{3/2} \, dx = \frac{397 \sqrt{3} \operatorname{asinh}\left(\frac{\sqrt{6}\, x}{2}\right)}{18} + \frac{\sqrt{3} \sqrt{x^2+\frac{2}{3}} \left(-\frac{36 x^6}{7}+12 x^5+\frac{3021 x^4}{35}+\frac{461 x^3}{4}+\frac{4268 x^2}{35}+\frac{683 x}{4}+\frac{4348}{105}\right)}{3}$$

input
```
int(-(2*x + 3)^2*(3*x^2 + 2)^(3/2)*(x - 5),x)
```

output
```
(397*3^(1/2)*asinh((6^(1/2)*x)/2))/18 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((683*x
)/4 + (4268*x^2)/35 + (461*x^3)/4 + (3021*x^4)/35 + 12*x^5 - (36*x^6)/7 +
4348/105))/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.17

$$\int (5-x)(3+2x)^2 \left(2+3x^2\right)^{3/2} dx = -\frac{12\sqrt{3x^2+2}\,x^6}{7} + 4\sqrt{3x^2+2}\,x^5 + \frac{1007\sqrt{3x^2+2}\,x^4}{35} + \frac{461\sqrt{3x^2+2}\,x^3}{12} + \frac{4268\sqrt{3x^2+2}\,x^2}{105} + \frac{683\sqrt{3x^2+2}\,x}{12} + \frac{4348\sqrt{3x^2+2}}{315} + \frac{397\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{18}$$

input
```
int((5-x)*(3+2*x)^2*(3*x^2+2)^(3/2),x)
```

output
```
( - 2160*sqrt(3*x**2 + 2)*x**6 + 5040*sqrt(3*x**2 + 2)*x**5 + 36252*sqrt(3
*x**2 + 2)*x**4 + 48405*sqrt(3*x**2 + 2)*x**3 + 51216*sqrt(3*x**2 + 2)*x**
2 + 71715*sqrt(3*x**2 + 2)*x + 17392*sqrt(3*x**2 + 2) + 27790*sqrt(3)*log(
(sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/1260
```

3.210 $\int (5-x)(3+2x) \left(2+3x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int (5-x)(3+2x) \left(2+3x^2\right)^{3/2} \, dx = \frac{137}{12} x\sqrt{2+3x^2} + \frac{137}{36} x \left(2+3x^2\right)^{3/2} + \frac{1}{45}(21-5x) \left(2+3x^2\right)^{5/2} + \frac{137 \text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{6\sqrt{3}}$$

output
```
137/12*x*(3*x^2+2)^(1/2)+137/36*x*(3*x^2+2)^(3/2)+1/45*(21-5*x)*(3*x^2+2)^
(5/2)+137/18*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int (5-x)(3+2x) \left(2+3x^2\right)^{3/2} \, dx = -\frac{1}{60}\sqrt{2+3x^2}\left(-112-1115x-336x^2-605x^3-252x^4+60x^5\right) - \frac{137 \log \left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{6\sqrt{3}}$$

input
```
Integrate[(5 - x)*(3 + 2*x)*(2 + 3*x^2)^(3/2),x]
```

output
```
-1/60*(Sqrt[2 + 3*x^2]*(-112 - 1115*x - 336*x^2 - 605*x^3 - 252*x^4 + 60*x
^5)) - (137*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(6*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {676, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-x)(2x+3)\left(3x^2+2\right)^{3/2}dx$$

$$\Big\downarrow \text{676}$$

$$\frac{137}{9}\int \left(3x^2+2\right)^{3/2}dx-\frac{1}{9}x\left(3x^2+2\right)^{5/2}+\frac{7}{15}\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{137}{9}\left(\frac{3}{2}\int \sqrt{3x^2+2}dx+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)-\frac{1}{9}x\left(3x^2+2\right)^{5/2}+\frac{7}{15}\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{137}{9}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)-\frac{1}{9}x\left(3x^2+2\right)^{5/2}+\frac{7}{15}\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{137}{9}\left(\frac{3}{2}\left(\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)-\frac{1}{9}x\left(3x^2+2\right)^{5/2}+\frac{7}{15}\left(3x^2+2\right)^{5/2}$$

input
```
Int[(5 - x)*(3 + 2*x)*(2 + 3*x^2)^(3/2),x]
```

output
```
(7*(2 + 3*x^2)^(5/2))/15 - (x*(2 + 3*x^2)^(5/2))/9 + (137*((x*(2 + 3*x^2)^
(3/2))/4 + (3*((x*Sqrt[2 + 3*x^2])/2 + ArcSinh[Sqrt[3/2]*x]/Sqrt[3]))/2))/
9
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\left(60 x^{5}-252 x^{4}-605 x^{3}-336 x^{2}-1115 x -112\right) \sqrt{3 x^{2}+2}}{60}+\frac{137 \,\operatorname{arcsinh}\left(\frac{\sqrt{6}\, x}{2}\right) \sqrt{3}}{18}$
default	$\frac{137 x \left(3 x^{2}+2\right)^{\frac{3}{2}}}{36}+\frac{137 x \sqrt{3 x^{2}+2}}{12}+\frac{137 \,\operatorname{arcsinh}\left(\frac{\sqrt{6}\, x}{2}\right) \sqrt{3}}{18}+\frac{7 \left(3 x^{2}+2\right)^{\frac{5}{2}}}{15}-\frac{x \left(3 x^{2}+2\right)^{\frac{5}{2}}}{9}$
trager	$\left(-x^{5}+\frac{21}{5} x^{4}+\frac{121}{12} x^{3}+\frac{28}{5} x^{2}+\frac{223}{12} x +\frac{28}{15}\right) \sqrt{3 x^{2}+2}-\frac{137 \operatorname{RootOf}\left(\_Z^{2}-3\right) \ln\left(-\operatorname{RootOf}\left(\_Z^{2}-3\right) \sqrt{3 x^{2}\ldots}\right.}{18}$
meijerg	$\frac{15 \sqrt{3}\, \left(\frac{4 \sqrt{\pi}\, x \sqrt{2}\, \sqrt{3}\, \left(\frac{3 x^{2}}{8}+\frac{5}{8}\right) \sqrt{\frac{3 x^{2}}{2}+1}}{3}+\sqrt{\pi}\, \operatorname{arcsinh}\left(\frac{x \sqrt{2}\, \sqrt{3}}{2}\right)\right)}{2 \sqrt{\pi}}+\frac{7 \sqrt{2}\, \left(-\frac{8 \sqrt{\pi}}{15}+\frac{4 \sqrt{\pi}\, \left(\frac{9}{2} x^{4}+6 x^{2}+2\right) \sqrt{\frac{3 x^{2}}{2}+1}}{15}\right)}{2 \sqrt{\pi}}-\frac{2 \sqrt{3}\, \left(\frac{\sqrt{6}\, \sqrt{\ldots}}{\ldots}\right.}{\ldots}$

input
```
int((5-x)*(2*x+3)*(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*(60*x^5-252*x^4-605*x^3-336*x^2-1115*x-112)*(3*x^2+2)^(1/2)+137/18*a
rcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int (5-x)(3+2x)\left(2+3x^2\right)^{3/2}\,dx = -\frac{1}{60}\left(60\,x^5-252\,x^4-605\,x^3-336\,x^2-1115\,x-112\right)\sqrt{3\,x^2+2} + \frac{137}{36}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3+2*x)*(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/60*(60*x^5 - 252*x^4 - 605*x^3 - 336*x^2 - 1115*x - 112)*sqrt(3*x^2 + 2
) + 137/36*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.53

$$\int (5-x)(3+2x)\left(2+3x^2\right)^{3/2}\,dx = -x^5\sqrt{3x^2+2}+\frac{21x^4\sqrt{3x^2+2}}{5}+\frac{121x^3\sqrt{3x^2+2}}{12} + \frac{28x^2\sqrt{3x^2+2}}{5}+\frac{223x\sqrt{3x^2+2}}{12}+\frac{28\sqrt{3x^2+2}}{15}+\frac{137\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{18}$$

input
```
integrate((5-x)*(3+2*x)*(3*x**2+2)**(3/2),x)
```

output
```
-x**5*sqrt(3*x**2 + 2) + 21*x**4*sqrt(3*x**2 + 2)/5 + 121*x**3*sqrt(3*x**2
 + 2)/12 + 28*x**2*sqrt(3*x**2 + 2)/5 + 223*x*sqrt(3*x**2 + 2)/12 + 28*sqr
t(3*x**2 + 2)/15 + 137*sqrt(3)*asinh(sqrt(6)*x/2)/18
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int (5-x)(3+2x)\left(2+3x^2\right)^{3/2}\,dx = -\frac{1}{9}\left(3\,x^2+2\right)^{\frac{5}{2}}x+\frac{7}{15}\left(3\,x^2+2\right)^{\frac{5}{2}} + \frac{137}{36}\left(3\,x^2+2\right)^{\frac{3}{2}}x+\frac{137}{12}\sqrt{3\,x^2+2}x+\frac{137}{18}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)$$

input
```
integrate((5-x)*(3+2*x)*(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
-1/9*(3*x^2 + 2)^(5/2)*x + 7/15*(3*x^2 + 2)^(5/2) + 137/36*(3*x^2 + 2)^(3/
2)*x + 137/12*sqrt(3*x^2 + 2)*x + 137/18*sqrt(3)*arcsinh(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int (5-x)(3+2x)\left(2+3x^2\right)^{3/2}\,dx = -\frac{1}{60}\left(\left(\left(\left(12\left(5\,x-21\right)x-605\right)x-336\right)x-1115\right)x-112\right)\sqrt{3\,x^2+2} - \frac{137}{18}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

input
```
integrate((5-x)*(3+2*x)*(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
-1/60*(((((12*(5*x - 21)*x - 605)*x - 336)*x - 1115)*x - 112)*sqrt(3*x^2 +
2) - 137/18*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int (5-x)(3+2x)\left(2+3x^2\right)^{3/2}\,dx=\frac{137\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{18}+\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(-3\,x^5+\frac{63\,x^4}{5}+\frac{121\,x^3}{4}+\frac{84\,x^2}{5}+\frac{223\,x}{4}+\frac{28}{5}\right)}{3}$$

input
```
int(-(2*x + 3)*(3*x^2 + 2)^(3/2)*(x - 5),x)
```

output
```
(137*3^(1/2)*asinh((6^(1/2)*x)/2))/18 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((223*x
)/4 + (84*x^2)/5 + (121*x^3)/4 + (63*x^4)/5 - 3*x^5 + 28/5))/3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.35

$$\int (5-x)(3+2x)\left(2+3x^2\right)^{3/2}\,dx=-\sqrt{3x^2+2}\,x^5+\frac{21\sqrt{3x^2+2}\,x^4}{5}+\frac{121\sqrt{3x^2+2}\,x^3}{12}+\frac{28\sqrt{3x^2+2}\,x^2}{5}+\frac{223\sqrt{3x^2+2}\,x}{12}+\frac{28\sqrt{3x^2+2}}{15}+\frac{137\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{18}$$

input
```
int((5-x)*(3+2*x)*(3*x^2+2)^(3/2),x)
```

output
```
( - 180*sqrt(3*x**2 + 2)*x**5 + 756*sqrt(3*x**2 + 2)*x**4 + 1815*sqrt(3*x*
*2 + 2)*x**3 + 1008*sqrt(3*x**2 + 2)*x**2 + 3345*sqrt(3*x**2 + 2)*x + 336*
sqrt(3*x**2 + 2) + 1370*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)
))/180
```

3.211 $\int (5-x) \left(2+3x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int (5-x) \left(2+3x^2\right)^{3/2} \, dx = \frac{15}{4}x\sqrt{2+3x^2} + \frac{5}{4}x\left(2+3x^2\right)^{3/2} - \frac{1}{15}\left(2+3x^2\right)^{5/2} + \frac{5}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)$$

output
```
15/4*x*(3*x^2+2)^(1/2)+5/4*x*(3*x^2+2)^(3/2)-1/15*(3*x^2+2)^(5/2)+5/2*arcs
inh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int (5-x) \left(2+3x^2\right)^{3/2} \, dx = -\frac{1}{60}\sqrt{2+3x^2}\left(16-375x+48x^2-225x^3+36x^4\right) - \frac{5}{2}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)$$

input
```
Integrate[(5 - x)*(2 + 3*x^2)^(3/2),x]
```

output
```
-1/60*(Sqrt[2 + 3*x^2]*(16 - 375*x + 48*x^2 - 225*x^3 + 36*x^4)) - (5*Sqrt
[3]*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {455, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-x) \left(3x^2+2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{455}$$

$$5 \int \left(3x^2+2\right)^{3/2} \, dx - \frac{1}{15}\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$5\left(\frac{3}{2}\int \sqrt{3x^2+2}dx + \frac{1}{4}x\left(3x^2+2\right)^{3/2}\right) - \frac{1}{15}\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$5\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{3x^2+2}}dx + \frac{1}{2}\sqrt{3x^2+2}x\right) + \frac{1}{4}x\left(3x^2+2\right)^{3/2}\right) - \frac{1}{15}\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{222}$$

$$5\left(\frac{3}{2}\left(\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}} + \frac{1}{2}\sqrt{3x^2+2}x\right) + \frac{1}{4}x\left(3x^2+2\right)^{3/2}\right) - \frac{1}{15}\left(3x^2+2\right)^{5/2}$$

input
```
Int[(5 - x)*(2 + 3*x^2)^(3/2),x]
```

output
```
-1/15*(2 + 3*x^2)^(5/2) + 5*((x*(2 + 3*x^2)^(3/2))/4 + (3*((x*Sqrt[2 + 3*x
^2])/2 + ArcSinh[Sqrt[3/2]*x]/Sqrt[3]))/2)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{\left(36x^{4}-225x^{3}+48x^{2}-375x+16\right)\sqrt{3x^{2}+2}}{60}+\frac{5\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{2}$	45
default	$\frac{15x\sqrt{3x^{2}+2}}{4}+\frac{5x\left(3x^{2}+2\right)^{\frac{3}{2}}}{4}-\frac{\left(3x^{2}+2\right)^{\frac{5}{2}}}{15}+\frac{5\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{2}$	49
trager	$\left(-\frac{3}{5}x^{4}+\frac{15}{4}x^{3}-\frac{4}{5}x^{2}+\frac{25}{4}x-\frac{4}{15}\right)\sqrt{3x^{2}+2}+\frac{5\,\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}-3\right)\sqrt{3x^{2}+2}+3x\right)}{2}$	61
meijerg	$\frac{5\sqrt{3}\left(\frac{4\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\,\left(\frac{3x^{2}}{8}+\frac{5}{8}\right)\sqrt{\frac{3x^{2}}{2}+1}}{3}+\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{2\sqrt{\pi}}-\frac{\sqrt{2}\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,\left(\frac{9}{2}x^{4}+6x^{2}+2\right)\sqrt{\frac{3x^{2}}{2}+1}}{15}\right)}{2\sqrt{\pi}}$	93

input
```
int((5-x)*(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*(36*x^4-225*x^3+48*x^2-375*x+16)*(3*x^2+2)^(1/2)+5/2*arcsinh(1/2*6^(
1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int (5-x)\left(2+3x^2\right)^{3/2}\,dx = -\frac{1}{60}\left(36\,x^4-225\,x^3+48\,x^2-375\,x+16\right)\sqrt{3\,x^2+2} + \frac{5}{4}\,\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/60*(36*x^4 - 225*x^3 + 48*x^2 - 375*x + 16)*sqrt(3*x^2 + 2) + 5/4*sqrt(
3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.45

$$\int (5-x)\left(2+3x^2\right)^{3/2}\,dx = -\frac{3x^4\sqrt{3x^2+2}}{5} + \frac{15x^3\sqrt{3x^2+2}}{4} - \frac{4x^2\sqrt{3x^2+2}}{5} + \frac{25x\sqrt{3x^2+2}}{4} - \frac{4\sqrt{3x^2+2}}{15} + \frac{5\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{2}$$

input
```
integrate((5-x)*(3*x**2+2)**(3/2),x)
```

output
```
-3*x**4*sqrt(3*x**2 + 2)/5 + 15*x**3*sqrt(3*x**2 + 2)/4 - 4*x**2*sqrt(3*x*
*2 + 2)/5 + 25*x*sqrt(3*x**2 + 2)/4 - 4*sqrt(3*x**2 + 2)/15 + 5*sqrt(3)*as
inh(sqrt(6)*x/2)/2
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int (5-x)\left(2+3x^2\right)^{3/2}\,dx = -\frac{1}{15}\left(3\,x^2+2\right)^{\frac{5}{2}} + \frac{5}{4}\left(3\,x^2+2\right)^{\frac{3}{2}}x + \frac{15}{4}\sqrt{3\,x^2+2}x + \frac{5}{2}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
-1/15*(3*x^2 + 2)^(5/2) + 5/4*(3*x^2 + 2)^(3/2)*x + 15/4*sqrt(3*x^2 + 2)*x
 + 5/2*sqrt(3)*arcsinh(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int (5-x)\left(2+3x^2\right)^{3/2}\,dx = -\frac{1}{60}\left(3\left(\left(3\left(4\,x-25\right)x+16\right)x-125\right)x+16\right)\sqrt{3\,x^2+2} - \frac{5}{2}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
-1/60*(3*((3*(4*x - 25)*x + 16)*x - 125)*x + 16)*sqrt(3*x^2 + 2) - 5/2*sqr
t(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

$$\int (5-x) \left(2+3x^2\right)^{3/2} \, dx = \frac{5\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{2} - \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{9\,x^4}{5}-\frac{45\,x^3}{4}+\frac{12\,x^2}{5}-\frac{75\,x}{4}+\frac{4}{5}\right)}{3}$$

input
```
int(-(3*x^2 + 2)^(3/2)*(x - 5),x)
```

output
```
(5*3^(1/2)*asinh((6^(1/2)*x)/2))/2 - (3^(1/2)*(x^2 + 2/3)^(1/2)*((12*x^2)/
5 - (75*x)/4 - (45*x^3)/4 + (9*x^4)/5 + 4/5))/3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.25

$$\int (5-x) \left(2+3x^2\right)^{3/2} \, dx = -\frac{3\sqrt{3x^2+2}\,x^4}{5} + \frac{15\sqrt{3x^2+2}\,x^3}{4} - \frac{4\sqrt{3x^2+2}\,x^2}{5} + \frac{25\sqrt{3x^2+2}\,x}{4} - \frac{4\sqrt{3x^2+2}}{15} + \frac{5\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{2}$$

input
```
int((5-x)*(3*x^2+2)^(3/2),x)
```

output
```
( - 36*sqrt(3*x**2 + 2)*x**4 + 225*sqrt(3*x**2 + 2)*x**3 - 48*sqrt(3*x**2
+ 2)*x**2 + 375*sqrt(3*x**2 + 2)*x - 16*sqrt(3*x**2 + 2) + 150*sqrt(3)*log
((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/60
```

3.212 $\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{3+2x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{3+2x}\,dx=\frac{1}{16}(455-123x)\sqrt{2+3x^2}+\frac{1}{24}(26-3x)\left(2+3x^2\right)^{3/2}-\frac{1529}{32}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-\frac{455}{32}\sqrt{35}\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)$$

output
```
1/16*(455-123*x)*(3*x^2+2)^(1/2)+1/24*(26-3*x)*(3*x^2+2)^(3/2)-1529/32*arc
sinh(1/2*x*6^(1/2))*3^(1/2)-455/32*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/
(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{3+2x}\,dx=\frac{1}{96}\left(-2\sqrt{2+3x^2}\left(-1469+381x-156x^2+18x^3\right)+2730\sqrt{35}\text{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)+4587\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)\right)$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x),x]
```

output
```
(-2*Sqrt[2 + 3*x^2]*(-1469 + 381*x - 156*x^2 + 18*x^3) + 2730*Sqrt[35]*Arc
Tanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]] + 4587*Sqrt[3
]*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/96
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {682, 27, 682, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{3/2}}{2x+3}dx$$

$$\Big\downarrow \text{682}$$

$$\frac{1}{48}\int \frac{12(43-123x)\sqrt{3x^2+2}}{2x+3}dx+\frac{1}{24}(26-3x)\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{(43-123x)\sqrt{3x^2+2}}{2x+3}dx+\frac{1}{24}(26-3x)\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{682}$$

$$\frac{1}{4}\left(\frac{1}{24}\int \frac{6(1082-4587x)}{(2x+3)\sqrt{3x^2+2}}dx+\frac{1}{4}\sqrt{3x^2+2}(455-123x)\right)+\frac{1}{24}(26-3x)\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{1}{4}\int \frac{1082-4587x}{(2x+3)\sqrt{3x^2+2}}dx+\frac{1}{4}\sqrt{3x^2+2}(455-123x)\right)+\frac{1}{24}(26-3x)\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{719}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(\frac{15925}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{4587}{2}\int \frac{1}{\sqrt{3x^2+2}}dx\right)+\frac{1}{4}\sqrt{3x^2+2}(455-123x)\right)+\frac{1}{24}(26-3x)\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(\frac{15925}{2}\int\frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{1529}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\right)+\frac{1}{4}\sqrt{3x^2+2}(455-123x)\right)+\frac{1}{24}(26-3x)\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(-\frac{15925}{2}\int\frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{1529}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\right)+\frac{1}{4}\sqrt{3x^2+2}(455-123x)\right)+\frac{1}{24}(26-3x)\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(-\frac{1529}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-\frac{455}{2}\sqrt{35}\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)\right)+\frac{1}{4}\sqrt{3x^2+2}(455-123x)\right)+\frac{1}{24}(26-3x)\left(3x^2+2\right)^{3/2}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x),x]
```

output
```
((26 - 3*x)*(2 + 3*x^2)^(3/2))/24 + (((455 - 123*x)*Sqrt[2 + 3*x^2])/4 + (
(-1529*Sqrt[3]*ArcSinh[Sqrt[3/2]*x])/2 - (455*Sqrt[35]*ArcTanh[(4 - 9*x)/(
Sqrt[35]*Sqrt[2 + 3*x^2])])/2)/4)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\left(18x^3-156x^2+381x-1469\right)\sqrt{3x^2+2}}{48}-\frac{1529\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{32}-\frac{455\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{32}$
trager	$\left(-\frac{3}{8}x^3+\frac{13}{4}x^2-\frac{127}{16}x+\frac{1469}{48}\right)\sqrt{3x^2+2}+\frac{455\,\operatorname{RootOf}\left(\_Z^2-35\right)\ln\left(\frac{9\,\operatorname{RootOf}\left(\_Z^2-35\right)x-4\,\operatorname{RootOf}\left(\_Z^2-35\right)}{2x+3}\right.}{32}$
default	$-\frac{x\left(3x^2+2\right)^{\frac{3}{2}}}{8}-\frac{3x\sqrt{3x^2+2}}{8}-\frac{1529\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{32}+\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{12}-\frac{117x\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}{16}+\frac{455\sqrt{1}}{}$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(2*x+3),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(18*x^3-156*x^2+381*x-1469)*(3*x^2+2)^(1/2)-1529/32*arcsinh(1/2*6^(1
/2)*x)*3^(1/2)-455/32*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2
-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.11

$$\int \frac{(5-x)(2+3x^2)^{3/2}}{3+2x}\,dx = -\frac{1}{48}\left(18\,x^3-156\,x^2+381\,x-1469\right)\sqrt{3\,x^2+2} + \frac{1529}{64}\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right) + \frac{455}{64}\sqrt{35}\log\left(-\frac{\sqrt{35}\sqrt{3\,x^2+2}(9\,x-4)+93\,x^2-36\,x+43}{4\,x^2+12\,x+9}\right)$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x),x, algorithm="fricas")
```

output
```
-1/48*(18*x^3 - 156*x^2 + 381*x - 1469)*sqrt(3*x^2 + 2) + 1529/64*sqrt(3)*
log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + 455/64*sqrt(35)*log(-(sqrt(35
)*sqrt(3*x^2 + 2)*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9))
```

Sympy [F]

$$\int \frac{(5-x)(2+3x^2)^{3/2}}{3+2x}\,dx = -\int\left(-\frac{10\sqrt{3x^2+2}}{2x+3}\right)dx - \int\frac{2x\sqrt{3x^2+2}}{2x+3}\,dx - \int\left(-\frac{15x^2\sqrt{3x^2+2}}{2x+3}\right)dx - \int\frac{3x^3\sqrt{3x^2+2}}{2x+3}\,dx$$

input
```
integrate((5-x)*(3*x**2+2)**(3/2)/(3+2*x),x)
```

output
```
-Integral(-10*sqrt(3*x**2 + 2)/(2*x + 3), x) - Integral(2*x*sqrt(3*x**2 +
2)/(2*x + 3), x) - Integral(-15*x**2*sqrt(3*x**2 + 2)/(2*x + 3), x) - Inte
gral(3*x**3*sqrt(3*x**2 + 2)/(2*x + 3), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{3+2x}\,dx = -\frac{1}{8}\left(3\,x^2+2\right)^{\frac{3}{2}}x+\frac{13}{12}\left(3\,x^2+2\right)^{\frac{3}{2}} - \frac{123}{16}\sqrt{3\,x^2+2}x-\frac{1529}{32}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{455}{32}\sqrt{35}\,\mathrm{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|}-\frac{2\sqrt{6}}{3\,|2\,x+3|}\right)+\frac{455}{16}\sqrt{3\,x^2+2}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x),x, algorithm="maxima")
```

output
```
-1/8*(3*x^2 + 2)^(3/2)*x + 13/12*(3*x^2 + 2)^(3/2) - 123/16*sqrt(3*x^2 + 2
)*x - 1529/32*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 455/32*sqrt(35)*arcsinh(3/2
*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3)) + 455/16*sqrt(3*x^2 +
2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.26

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{3+2x}\,dx = -\frac{1}{48}\left(3\left(2\left(3\,x-26\right)x+127\right)x-1469\right)\sqrt{3\,x^2+2} + \frac{1529}{32}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{455}{32}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3\,x^2+2}}\right)$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x),x, algorithm="giac")
```

output
```
-1/48*(3*(2*(3*x - 26)*x + 127)*x - 1469)*sqrt(3*x^2 + 2) + 1529/32*sqrt(3
)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 455/32*sqrt(35)*log(-abs(-2*sqrt(3)*
x - sqrt(35) - 3*sqrt(3) + 2*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*
sqrt(3) - 2*sqrt(3*x^2 + 2)))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{3+2x}\,dx = \frac{\sqrt{35}\left(31850\,\ln\left(x+\frac{3}{2}\right)-31850\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)\right)}{2240} - \frac{1529\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{32} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{9\,x^3}{8}-\frac{39\,x^2}{4}+\frac{381\,x}{16}-\frac{1469}{16}\right)}{3}$$

input
```
int(-((3*x^2 + 2)^(3/2)*(x - 5))/(2*x + 3),x)
```

output
```
(35^(1/2)*(31850*log(x + 3/2) - 31850*log(x - (3^(1/2)*35^(1/2)*(x^2 + 2/3
)^(1/2))/9 - 4/9)))/2240 - (1529*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/32
- (3^(1/2)*(x^2 + 2/3)^(1/2)*((381*x)/16 - (39*x^2)/4 + (9*x^3)/8 - 1469/1
6))/3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.80

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{3+2x}\,dx = \frac{455\sqrt{35}\,atan\left(\frac{2\sqrt{3x^2+2}\,i+2\sqrt{3}\,ix}{\sqrt{35}-3\sqrt{3}}\right)i}{32} - \frac{3\sqrt{3x^2+2}\,x^3}{8} + \frac{13\sqrt{3x^2+2}\,x^2}{4} - \frac{127\sqrt{3x^2+2}\,x}{16} + \frac{1469\sqrt{3x^2+2}}{48} + \frac{455\sqrt{35}\log\left(4\sqrt{3x^2+2}\,\sqrt{3}\,x+3\sqrt{105}+12x^2-27\right)}{64} - \frac{455\sqrt{35}\log\left(\frac{2\sqrt{3x^2+2}+\sqrt{35}+2\sqrt{3}\,x+3\sqrt{3}}{\sqrt{2}}\right)}{32} - \frac{1529\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{32}$$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(3+2*x),x)
```

output
```
(2730*sqrt(35)*atan((2*sqrt(3*x**2 + 2)*i + 2*sqrt(3)*i*x)/(sqrt(35) - 3*s
qrt(3)))*i - 72*sqrt(3*x**2 + 2)*x**3 + 624*sqrt(3*x**2 + 2)*x**2 - 1524*s
qrt(3*x**2 + 2)*x + 5876*sqrt(3*x**2 + 2) + 1365*sqrt(35)*log(4*sqrt(3*x**
2 + 2)*sqrt(3)*x + 3*sqrt(105) + 12*x**2 - 27) - 2730*sqrt(35)*log((2*sqrt
(3*x**2 + 2) + sqrt(35) + 2*sqrt(3)*x + 3*sqrt(3))/sqrt(2)) - 9174*sqrt(3)
*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/192
```

3.213 $\int \frac{(5-x)(2+3x^2)^{3/2}}{(3+2x)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{(5-x)(2+3x^2)^{3/2}}{(3+2x)^2}\,dx = -\frac{1}{8}(193-63x)\sqrt{2+3x^2}-\frac{(21+x)(2+3x^2)^{3/2}}{6(3+2x)} + \frac{663}{16}\sqrt{3}\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)+\frac{193}{16}\sqrt{35}\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)$$

output

```
-1/8*(193-63*x)*(3*x^2+2)^(1/2)-(21+x)*(3*x^2+2)^(3/2)/(18+12*x)+663/16*ar
csinh(1/2*x*6^(1/2))*3^(1/2)+193/16*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)
/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14

$$\int \frac{(5-x)(2+3x^2)^{3/2}}{(3+2x)^2}\,dx = \frac{1}{48}\left(-\frac{2\sqrt{2+3x^2}(1905+599x-126x^2+12x^3)}{3+2x} - 1158\sqrt{35}\operatorname{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right) - 1989\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)\right)$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^2,x]
```

output
```
((-2*Sqrt[2 + 3*x^2]*(1905 + 599*x - 126*x^2 + 12*x^3))/(3 + 2*x) - 1158*S
qrt[35]*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]] -
1989*Sqrt[3]*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/48
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {681, 27, 682, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x) \left(3 x^2+2\right)^{3/2}}{(2 x+3)^2} \, dx$$

$$\Big\downarrow \text{ 681}$$

$$-\frac{1}{8} \int \frac{4(2-63 x) \sqrt{3 x^2+2}}{2 x+3}dx-\frac{(x+21) \left(3 x^2+2\right)^{3/2}}{6(2 x+3)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{2} \int \frac{(2-63 x) \sqrt{3 x^2+2}}{2 x+3}dx-\frac{(x+21) \left(3 x^2+2\right)^{3/2}}{6(2 x+3)}$$

$$\Big\downarrow \text{ 682}$$

$$\frac{1}{2} \left(-\frac{1}{24} \int \frac{6(394-1989 x)}{(2 x+3) \sqrt{3 x^2+2}}dx-\frac{1}{4} \sqrt{3 x^2+2}(193-63 x)\right)-\frac{(x+21) \left(3 x^2+2\right)^{3/2}}{6(2 x+3)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2} \left(-\frac{1}{4} \int \frac{394-1989 x}{(2 x+3) \sqrt{3 x^2+2}}dx-\frac{1}{4} \sqrt{3 x^2+2}(193-63 x)\right)-\frac{(x+21) \left(3 x^2+2\right)^{3/2}}{6(2 x+3)}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1989}{2}\int \frac{1}{\sqrt{3x^2+2}}dx-\frac{6755}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx\right)-\frac{1}{4}(193-63x)\sqrt{3x^2+2}\right)-\frac{(x+21)\left(3x^2+2\right)^{3/2}}{6(2x+3)}$$

$\Big\downarrow$ 222

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{663}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-\frac{6755}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx\right)-\frac{1}{4}(193-63x)\sqrt{3x^2+2}\right)-\frac{(x+21)\left(3x^2+2\right)^{3/2}}{6(2x+3)}$$

$\Big\downarrow$ 488

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{6755}{2}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}+\frac{663}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\right)-\frac{1}{4}(193-63x)\sqrt{3x^2+2}\right)-\frac{(x+21)\left(3x^2+2\right)^{3/2}}{6(2x+3)}$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{663}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)+\frac{193}{2}\sqrt{35}\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)\right)-\frac{1}{4}(193-63x)\sqrt{3x^2+2}\right)-\frac{(x+21)\left(3x^2+2\right)^{3/2}}{6(2x+3)}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^2,x]
```

output
```
-1/6*((21 + x)*(2 + 3*x^2)^(3/2))/(3 + 2*x) + (-1/4*((193 - 63*x)*Sqrt[2 +
 3*x^2]) + ((663*Sqrt[3]*ArcSinh[Sqrt[3/2]*x])/2 + (193*Sqrt[35]*ArcTanh[(
4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/2)/4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{36x^5-378x^4+1821x^3+5463x^2+1198x+3810}{24(2x+3)\sqrt{3x^2+2}}+\frac{663\,\text{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{16}+\frac{193\sqrt{35}\,\text{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{16}$
trager	$-\frac{(12x^3-126x^2+599x+1905)\sqrt{3x^2+2}}{24(2x+3)}+\frac{\text{RootOf}\left(\_Z^2-1303715\right)\ln\left(\frac{-9\,\text{RootOf}\left(\_Z^2-1303715\right)x+6755\sqrt{3x^2+2}+4\,\text{RootOf}\left(\_\right.}{2x+3}\right.}{16}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{70\left(x+\frac{3}{2}\right)}-\frac{193\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{210}+\frac{63x\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}{8}+\frac{663\,\text{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{16}-\frac{193\sqrt{12(x}}{}$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(2*x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(36*x^5-378*x^4+1821*x^3+5463*x^2+1198*x+3810)/(2*x+3)/(3*x^2+2)^(1/
2)+663/16*arcsinh(1/2*6^(1/2)*x)*3^(1/2)+193/16*35^(1/2)*arctanh(2/35*(4-9
*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.25

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^2}\,dx=\frac{1989\sqrt{3}(2\,x+3)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+579\sqrt{35}(2\,x+3)\log\left(\right.}{96\,(2\,x}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^2,x, algorithm="fricas")
```

output
```
1/96*(1989*sqrt(3)*(2*x + 3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
 579*sqrt(35)*(2*x + 3)*log((sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) - 93*x^2 +
 36*x - 43)/(4*x^2 + 12*x + 9)) - 4*(12*x^3 - 126*x^2 + 599*x + 1905)*sqrt
(3*x^2 + 2))/(2*x + 3)
```

Sympy [F]

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^2}\,dx = -\int \left(-\frac{10\sqrt{3x^2+2}}{4x^2+12x+9}\right)dx - \int \frac{2x\sqrt{3x^2+2}}{4x^2+12x+9}\,dx - \int \left(-\frac{15x^2\sqrt{3x^2+2}}{4x^2+12x+9}\right)dx - \int \frac{3x^3\sqrt{3x^2+2}}{4x^2+12x+9}\,dx$$

input
```
integrate((5-x)*(3*x**2+2)**(3/2)/(3+2*x)**2,x)
```

output
```
-Integral(-10*sqrt(3*x**2 + 2)/(4*x**2 + 12*x + 9), x) - Integral(2*x*sqrt
(3*x**2 + 2)/(4*x**2 + 12*x + 9), x) - Integral(-15*x**2*sqrt(3*x**2 + 2)/
(4*x**2 + 12*x + 9), x) - Integral(3*x**3*sqrt(3*x**2 + 2)/(4*x**2 + 12*x
+ 9), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^2}\,dx = -\frac{1}{12}\left(3x^2+2\right)^{\frac{3}{2}} + \frac{63}{8}\sqrt{3x^2+2}x + \frac{663}{16}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) - \frac{193}{16}\sqrt{35}\operatorname{arsinh}\left(\frac{3\sqrt{6}x}{2|2x+3|} - \frac{2\sqrt{6}}{3|2x+3|}\right) - \frac{193}{8}\sqrt{3x^2+2} - \frac{13\left(3x^2+2\right)^{\frac{3}{2}}}{4(2x+3)}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^2,x, algorithm="maxima")
```

output
```
-1/12*(3*x^2 + 2)^(3/2) + 63/8*sqrt(3*x^2 + 2)*x + 663/16*sqrt(3)*arcsinh(
1/2*sqrt(6)*x) - 193/16*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*
sqrt(6)/abs(2*x + 3)) - 193/8*sqrt(3*x^2 + 2) - 13/4*(3*x^2 + 2)^(3/2)/(2*
x + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(75) = 150$.

Time = 0.31 (sec) , antiderivative size = 475, normalized size of antiderivative = 4.90

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^2}\,dx = \frac{193}{16}\sqrt{35}\log\left(\sqrt{35}\left(\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)-9\right)\operatorname{sgn}\left(\frac{1}{2x+3}\right) - \frac{663}{16}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{2\sqrt{35}}{2x+3}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)}\right)\operatorname{sgn}\left(\frac{1}{2x+3}\right) - \frac{455}{32}\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}\operatorname{sgn}\left(\frac{1}{2x+3}\right) + \frac{3\left(704\left(\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)^5\operatorname{sgn}\left(\frac{1}{2x+3}\right)-323\sqrt{35}\left(\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)^4\operatorname{sgn}\left(\frac{1}{2x\ldots}\right.}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^2,x, algorithm="giac")
```

output
```
193/16*sqrt(35)*log(sqrt(35)*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + s
qrt(35)/(2*x + 3)) - 9)*sgn(1/(2*x + 3)) - 663/16*sqrt(3)*log(1/2*abs(-2*s
qrt(3) + 2*sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + 2*sqrt(35)/(2*x + 3)
)/(sqrt(3) + sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3)
))*sgn(1/(2*x + 3)) - 455/32*sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3)*sgn(
1/(2*x + 3)) + 3/8*(704*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(3
5)/(2*x + 3))^5*sgn(1/(2*x + 3)) - 323*sqrt(35)*(sqrt(-18/(2*x + 3) + 35/(
2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3))^4*sgn(1/(2*x + 3)) - 1944*(sqrt(-18/
(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3))^3*sgn(1/(2*x + 3)) +
 1158*sqrt(35)*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x +
 3))^2*sgn(1/(2*x + 3)) + 1872*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) +
 sqrt(35)/(2*x + 3))*sgn(1/(2*x + 3)) - 1263*sqrt(35)*sgn(1/(2*x + 3)))/((
sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3))^2 - 3)^3
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

$$\int \frac{(5-x)\,(2+3x^2)^{3/2}}{(3+2x)^2}\,dx = \frac{663\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{16} - \frac{815\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{48} - \frac{193\,\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{16} + \frac{193\,\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{35}\,\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{16} - \frac{\sqrt{3}\,x^2\,\sqrt{x^2+\frac{2}{3}}}{4} - \frac{455\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{32\,\left(x+\frac{3}{2}\right)} + 3\,\sqrt{3}\,x\,\sqrt{x^2+\frac{2}{3}}$$

input
```
int(-((3*x^2 + 2)^(3/2)*(x - 5))/(2*x + 3)^2,x)
```

output
```
(663*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/16 - (815*3^(1/2)*(x^2 + 2/3)^(
1/2))/48 - (193*35^(1/2)*log(x + 3/2))/16 + (193*35^(1/2)*log(x - (3^(1/2)
*35^(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/16 - (3^(1/2)*x^2*(x^2 + 2/3)^(1/2)
)/4 - (455*3^(1/2)*(x^2 + 2/3)^(1/2))/(32*(x + 3/2)) + 3*3^(1/2)*x*(x^2 +
2/3)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.04

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^2}\,dx = \frac{-48\sqrt{3x^2+2}\,x^3+504\sqrt{3x^2+2}\,x^2-2396\sqrt{3x^2+2}\,x-7620\sqrt{3x^2+2}+231}{}$$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^2,x)
```

output
```
( - 48*sqrt(3*x**2 + 2)*x**3 + 504*sqrt(3*x**2 + 2)*x**2 - 2396*sqrt(3*x**
2 + 2)*x - 7620*sqrt(3*x**2 + 2) + 2316*sqrt(35)*log( - sqrt(3*x**2 + 2)*s
qrt(35) + 9*x - 4)*x + 3474*sqrt(35)*log( - sqrt(3*x**2 + 2)*sqrt(35) + 9*
x - 4) - 2316*sqrt(35)*log(2*x + 3)*x - 3474*sqrt(35)*log(2*x + 3) - 3978*
sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x - 5967*sqrt(3)*log(sqrt(3*x**2
 + 2) - sqrt(3)*x) + 3978*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x + 59
67*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x))/(96*(2*x + 3))
```

3.214 $\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^3}\,dx=\frac{9}{140}(127-38x)\sqrt{2+3x^2}+\frac{(404+421x)\left(2+3x^2\right)^{3/2}}{140(3+2x)^2}-\frac{111}{8}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-\frac{1143\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{8\sqrt{35}}$$

output
```
9/140*(127-38*x)*(3*x^2+2)^(1/2)+1/140*(404+421*x)*(3*x^2+2)^(3/2)/(3+2*x)
^2-111/8*arcsinh(1/2*x*6^(1/2))*3^(1/2)-1143/280*35^(1/2)*arctanh(1/35*(4-
9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.14

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^3}\,dx=-\frac{\sqrt{2+3x^2}\left(-317-328x-48x^2+3x^3\right)}{4(3+2x)^2}+\frac{1143\text{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{4\sqrt{35}}+\frac{111}{8}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^3,x]
```

output
```
-1/4*(Sqrt[2 + 3*x^2]*(-317 - 328*x - 48*x^2 + 3*x^3))/(3 + 2*x)^2 + (1143
*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]])/(4*Sqrt[
35]) + (111*Sqrt[3]*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/8
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {681, 27, 681, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x) \left(3 x^2+2\right)^{3/2}}{(2 x+3)^3} \, dx$$

$$\Big\downarrow \text{ 681}$$

$$-\frac{3}{32} \int \frac{16(1-12 x) \sqrt{3 x^2+2}}{(2 x+3)^2}dx-\frac{(x+8) \left(3 x^2+2\right)^{3/2}}{4 (2 x+3)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{2} \int \frac{(1-12 x) \sqrt{3 x^2+2}}{(2 x+3)^2}dx-\frac{(x+8) \left(3 x^2+2\right)^{3/2}}{4 (2 x+3)^2}$$

$$\Big\downarrow \text{ 681}$$

$$-\frac{3}{2} \left(-\frac{1}{8} \int \frac{12 (8-37 x)}{(2 x+3) \sqrt{3 x^2+2}}dx-\frac{\sqrt{3 x^2+2} (12 x+37)}{2 (2 x+3)}\right)-\frac{(x+8) \left(3 x^2+2\right)^{3/2}}{4 (2 x+3)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{2} \left(-\frac{3}{2} \int \frac{8-37 x}{(2 x+3) \sqrt{3 x^2+2}}dx-\frac{\sqrt{3 x^2+2} (12 x+37)}{2 (2 x+3)}\right)-\frac{(x+8) \left(3 x^2+2\right)^{3/2}}{4 (2 x+3)^2}$$

$$\Big\downarrow \text{ 719}$$

$$-\frac{3}{2}\left(-\frac{3}{2}\left(\frac{127}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{37}{2}\int \frac{1}{\sqrt{3x^2+2}}dx\right)-\frac{\sqrt{3x^2+2}(12x+37)}{2(2x+3)}\right)-\frac{(x+8)\left(3x^2+2\right)^{3/2}}{4(2x+3)^2}$$

$$\Big\downarrow \; 222$$

$$-\frac{3}{2}\left(-\frac{3}{2}\left(\frac{127}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{37\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)-\frac{\sqrt{3x^2+2}(12x+37)}{2(2x+3)}\right)-\frac{(x+8)\left(3x^2+2\right)^{3/2}}{4(2x+3)^2}$$

$$\Big\downarrow \; 488$$

$$-\frac{3}{2}\left(-\frac{3}{2}\left(-\frac{127}{2}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{37\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)-\frac{\sqrt{3x^2+2}(12x+37)}{2(2x+3)}\right)-\frac{(x+8)\left(3x^2+2\right)^{3/2}}{4(2x+3)^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{3}{2}\left(-\frac{3}{2}\left(-\frac{37\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{127\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{2\sqrt{35}}\right)-\frac{\sqrt{3x^2+2}(12x+37)}{2(2x+3)}\right)-\frac{(x+8)\left(3x^2+2\right)^{3/2}}{4(2x+3)^2}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^3,x]
```

output
```
-1/4*((8 + x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^2 - (3*(-1/2*((37 + 12*x)*Sqrt[
2 + 3*x^2])/(3 + 2*x) - (3*((-37*ArcSinh[Sqrt[3/2]*x])/(2*Sqrt[3]) - (127*
ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(2*Sqrt[35])))/2))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{9x^5-144x^4-978x^3-1047x^2-656x-634}{4(2x+3)^2\sqrt{3x^2+2}}-\frac{111\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{8}-\frac{1143\sqrt{35}\,\mathrm{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{280}$
trager	$-\frac{(3x^3-48x^2-328x-317)\sqrt{3x^2+2}}{4(2x+3)^2}-\frac{1143\,\mathrm{RootOf}\left(\_Z^2-35\right)\ln\left(-\frac{9\,\mathrm{RootOf}\left(\_Z^2-35\right)x-4\,\mathrm{RootOf}\left(\_Z^2-35\right)-35\sqrt{3x^2+2}}{2x+3}\right)}{280}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{280\left(x+\frac{3}{2}\right)^2}+\frac{187\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{4900\left(x+\frac{3}{2}\right)}+\frac{381\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{1225}-\frac{171x\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}{70}-\frac{111\ a}{}$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(2*x+3)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(9*x^5-144*x^4-978*x^3-1047*x^2-656*x-634)/(2*x+3)^2/(3*x^2+2)^(1/2)-
111/8*arcsinh(1/2*6^(1/2)*x)*3^(1/2)-1143/280*35^(1/2)*arctanh(2/35*(4-9*x
)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.37

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^3}\,dx=\frac{3885\,\sqrt{3}(4\,x^2+12\,x+9)\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+1143\,\sqrt{35}(4\,x^2+}{560}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^3,x, algorithm="fricas")
```

output
```
1/560*(3885*sqrt(3)*(4*x^2 + 12*x + 9)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x
^2 - 1) + 1143*sqrt(35)*(4*x^2 + 12*x + 9)*log(-(sqrt(35)*sqrt(3*x^2 + 2)*
(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9)) - 140*(3*x^3 - 48*x^2
- 328*x - 317)*sqrt(3*x^2 + 2))/(4*x^2 + 12*x + 9)
```

Sympy [F]

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^3}\,dx = -\int \left(-\frac{10\sqrt{3x^2+2}}{8x^3+36x^2+54x+27}\right)dx - \int \frac{2x\sqrt{3x^2+2}}{8x^3+36x^2+54x+27}\,dx - \int \left(-\frac{15x^2\sqrt{3x^2+2}}{8x^3+36x^2+54x+27}\right)dx - \int \frac{3x^3\sqrt{3x^2+2}}{8x^3+36x^2+54x+27}\,dx$$

input
```
integrate((5-x)*(3*x**2+2)**(3/2)/(3+2*x)**3,x)
```

output
```
-Integral(-10*sqrt(3*x**2 + 2)/(8*x**3 + 36*x**2 + 54*x + 27), x) - Integr
al(2*x*sqrt(3*x**2 + 2)/(8*x**3 + 36*x**2 + 54*x + 27), x) - Integral(-15*
x**2*sqrt(3*x**2 + 2)/(8*x**3 + 36*x**2 + 54*x + 27), x) - Integral(3*x**3
*sqrt(3*x**2 + 2)/(8*x**3 + 36*x**2 + 54*x + 27), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.23

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^3}\,dx = \frac{39}{280}\left(3x^2+2\right)^{\frac{3}{2}} - \frac{13\left(3x^2+2\right)^{\frac{5}{2}}}{70\left(4x^2+12x+9\right)} - \frac{171}{70}\sqrt{3x^2+2}x - \frac{111}{8}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{1143}{280}\sqrt{35}\operatorname{arsinh}\left(\frac{3\sqrt{6}x}{2\left|2x+3\right|} - \frac{2\sqrt{6}}{3\left|2x+3\right|}\right) + \frac{1143}{140}\sqrt{3x^2+2} + \frac{187\left(3x^2+2\right)^{\frac{3}{2}}}{280\left(2x+3\right)}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^3,x, algorithm="maxima")
```

output
```
39/280*(3*x^2 + 2)^(3/2) - 13/70*(3*x^2 + 2)^(5/2)/(4*x^2 + 12*x + 9) - 17
1/70*sqrt(3*x^2 + 2)*x - 111/8*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 1143/280*s
qrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3)) + 1
143/140*sqrt(3*x^2 + 2) + 187/280*(3*x^2 + 2)^(3/2)/(2*x + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(77) = 154$.

Time = 0.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.21

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^3}\,dx = -\frac{3}{16}\sqrt{3\,x^2+2}(x-19)+\frac{111}{8}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{1143}{280}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3\,x^2+2}}\right) + \frac{5\left(1452\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^3+3013\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2-6528\sqrt{3}x+1048\sqrt{3}+6528\sqrt{3\,x^2+2}\right)}{64\left(\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2+3\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)-2\right)^2}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^3,x, algorithm="giac")
```

output
```
-3/16*sqrt(3*x^2 + 2)*(x - 19) + 111/8*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2
 + 2)) + 1143/280*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) +
2*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2)
)) + 5/64*(1452*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 + 3013*sqrt(3)*(sqrt(3)*x
- sqrt(3*x^2 + 2))^2 - 6528*sqrt(3)*x + 1048*sqrt(3) + 6528*sqrt(3*x^2 + 2
))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 3*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 +
2)) - 2)^2
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.18

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^3}\,dx = \frac{1143\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{280}+\frac{57\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{16} - \frac{111\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{8}-\frac{1143\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{280} + \frac{655\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{64\left(x+\frac{3}{2}\right)}-\frac{455\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{128\left(x^2+3\,x+\frac{9}{4}\right)}-\frac{3\sqrt{3}\,x\sqrt{x^2+\frac{2}{3}}}{16}$$

input
```
int(-((3*x^2 + 2)^(3/2)*(x - 5))/(2*x + 3)^3,x)
```

output
```
(1143*35^(1/2)*log(x + 3/2))/280 + (57*3^(1/2)*(x^2 + 2/3)^(1/2))/16 - (11
1*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/8 - (1143*35^(1/2)*log(x - (3^(1/2
)*35^(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/280 + (655*3^(1/2)*(x^2 + 2/3)^(1/
2))/(64*(x + 3/2)) - (455*3^(1/2)*(x^2 + 2/3)^(1/2))/(128*(3*x + x^2 + 9/4
)) - (3*3^(1/2)*x*(x^2 + 2/3)^(1/2))/16
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.84

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^3}\,dx = \frac{-420\sqrt{3x^2+2}\,x^3+6720\sqrt{3x^2+2}\,x^2+45920\sqrt{3x^2+2}\,x+44380\sqrt{3x^2+2}\,+}{}$$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^3,x)
```

output
```
( - 420*sqrt(3*x**2 + 2)*x**3 + 6720*sqrt(3*x**2 + 2)*x**2 + 45920*sqrt(3*
x**2 + 2)*x + 44380*sqrt(3*x**2 + 2) + 9144*sqrt(35)*log(sqrt(3*x**2 + 2)*
sqrt(35) + 9*x - 4)*x**2 + 27432*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) +
9*x - 4)*x + 20574*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4) - 914
4*sqrt(35)*log(2*x + 3)*x**2 - 27432*sqrt(35)*log(2*x + 3)*x - 20574*sqrt(
35)*log(2*x + 3) + 15540*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x**2 +
46620*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x + 34965*sqrt(3)*log(sqrt
(3*x**2 + 2) - sqrt(3)*x) - 15540*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x
)*x**2 - 46620*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x - 34965*sqrt(3)
*log(sqrt(3*x**2 + 2) + sqrt(3)*x))/(560*(4*x**2 + 12*x + 9))
```

3.215 $\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 106

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^4}\,dx = -\frac{3(385+111x)\sqrt{2+3x^2}}{280(3+2x)} + \frac{(229+456x)\left(2+3x^2\right)^{3/2}}{420(3+2x)^3} + \frac{33}{16}\sqrt{3}\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right) + \frac{11727\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{560\sqrt{35}}$$

output
```
-3*(385+111*x)*(3*x^2+2)^(1/2)/(840+560*x)+1/420*(229+456*x)*(3*x^2+2)^(3/
2)/(3+2*x)^3+33/16*arcsinh(1/2*x*6^(1/2))*3^(1/2)+11727/19600*35^(1/2)*arc
tanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^4}\,dx = -\frac{\sqrt{2+3x^2}\left(30269+48747x+24474x^2+1260x^3\right)}{840(3+2x)^3} - \frac{11727\operatorname{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{280\sqrt{35}} - \frac{33}{16}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^4,x]
```

output
```
-1/840*(Sqrt[2 + 3*x^2]*(30269 + 48747*x + 24474*x^2 + 1260*x^3))/(3 + 2*x
)^3 - (11727*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35
]])/(280*Sqrt[35]) - (33*Sqrt[3]*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/16
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {680, 27, 681, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{3/2}}{(2x+3)^4}dx$$

$$\Big\downarrow \text{680}$$

$$\frac{(456x+229)\left(3x^2+2\right)^{3/2}}{420(2x+3)^3}-\frac{1}{560}\int -\frac{12(52-111x)\sqrt{3x^2+2}}{(2x+3)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{140}\int \frac{(52-111x)\sqrt{3x^2+2}}{(2x+3)^2}dx+\frac{(456x+229)\left(3x^2+2\right)^{3/2}}{420(2x+3)^3}$$

$$\Big\downarrow \text{681}$$

$$\frac{3}{140}\left(-\frac{1}{8}\int \frac{12(74-385x)}{(2x+3)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}(111x+385)}{2(2x+3)}\right)+\frac{(456x+229)\left(3x^2+2\right)^{3/2}}{420(2x+3)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{140}\left(-\frac{3}{2}\int \frac{74-385x}{(2x+3)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}(111x+385)}{2(2x+3)}\right)+\frac{(456x+229)\left(3x^2+2\right)^{3/2}}{420(2x+3)^3}$$

$$\Big\downarrow \text{719}$$

$$\frac{3}{140}\left(-\frac{3}{2}\left(\frac{1303}{2}\int\frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{385}{2}\int\frac{1}{\sqrt{3x^2+2}}dx\right)-\frac{\sqrt{3x^2+2}(111x+385)}{2(2x+3)}\right)+\frac{(456x+229)\left(3x^2+2\right)^{3/2}}{420(2x+3)^3}$$

$$\Big\downarrow \text{222}$$

$$\frac{3}{140}\left(-\frac{3}{2}\left(\frac{1303}{2}\int\frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{385\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)-\frac{\sqrt{3x^2+2}(111x+385)}{2(2x+3)}\right)+\frac{(456x+229)\left(3x^2+2\right)^{3/2}}{420(2x+3)^3}$$

$$\Big\downarrow \text{488}$$

$$\frac{3}{140}\left(-\frac{3}{2}\left(-\frac{1303}{2}\int\frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{385\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)-\frac{\sqrt{3x^2+2}(111x+385)}{2(2x+3)}\right)+\frac{(456x+229)\left(3x^2+2\right)^{3/2}}{420(2x+3)^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{140}\left(-\frac{3}{2}\left(-\frac{385\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{1303\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{2\sqrt{35}}\right)-\frac{\sqrt{3x^2+2}(111x+385)}{2(2x+3)}\right)+\frac{(456x+229)\left(3x^2+2\right)^{3/2}}{420(2x+3)^3}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^4,x]
```

output
```
((229 + 456*x)*(2 + 3*x^2)^(3/2))/(420*(3 + 2*x)^3) + (3*(-1/2*((385 + 111
*x)*Sqrt[2 + 3*x^2])/(3 + 2*x) - (3*((-385*ArcSinh[Sqrt[3/2]*x])/(2*Sqrt[3
]) - (1303*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(2*Sqrt[35])))/2
))/140
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{3780x^5+73422x^4+148761x^3+139755x^2+97494x+60538}{840(2x+3)^3\sqrt{3x^2+2}}+\frac{33\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{16}+\frac{11727\sqrt{35}\,\mathrm{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x}}\right.}{19600}$
trager	$-\frac{(1260x^3+24474x^2+48747x+30269)\sqrt{3x^2+2}}{840(2x+3)^3}-\frac{33\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{16}+\frac{11727\,\mathrm{Root}}{}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{840\left(x+\frac{3}{2}\right)^3}-\frac{\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{2450\left(x+\frac{3}{2}\right)^2}-\frac{446\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{42875\left(x+\frac{3}{2}\right)}-\frac{3909\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{85750}+\frac{3933x}{}$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(2*x+3)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/840*(3780*x^5+73422*x^4+148761*x^3+139755*x^2+97494*x+60538)/(2*x+3)^3/
(3*x^2+2)^(1/2)+33/16*arcsinh(1/2*6^(1/2)*x)*3^(1/2)+11727/19600*35^(1/2)*
arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.42

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^4}\,dx = \frac{121275\,\sqrt{3}(8\,x^3+36\,x^2+54\,x+27)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+3518}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^4,x, algorithm="fricas")
```

output
```
1/117600*(121275*sqrt(3)*(8*x^3 + 36*x^2 + 54*x + 27)*log(-sqrt(3)*sqrt(3*
x^2 + 2)*x - 3*x^2 - 1) + 35181*sqrt(35)*(8*x^3 + 36*x^2 + 54*x + 27)*log(
(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) - 93*x^2 + 36*x - 43)/(4*x^2 + 12*x +
9)) - 140*(1260*x^3 + 24474*x^2 + 48747*x + 30269)*sqrt(3*x^2 + 2))/(8*x^3
 + 36*x^2 + 54*x + 27)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^4}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(3/2)/(3+2*x)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.42

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^4}\,dx = \frac{3}{2450}\left(3\,x^2+2\right)^{\frac{3}{2}} - \frac{13\left(3\,x^2+2\right)^{\frac{5}{2}}}{105\left(8\,x^3+36\,x^2+54\,x+27\right)} - \frac{2\left(3\,x^2+2\right)^{\frac{5}{2}}}{1225\left(4\,x^2+12\,x+9\right)} + \frac{3933}{9800}\sqrt{3\,x^2+2}x + \frac{33}{16}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) - \frac{11727}{19600}\sqrt{35}\,\mathrm{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|} - \frac{2\sqrt{6}}{3\,|2\,x+3|}\right) - \frac{11727}{9800}\sqrt{3\,x^2+2} - \frac{223\left(3\,x^2+2\right)^{\frac{3}{2}}}{1225\left(2\,x+3\right)}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^4,x, algorithm="maxima")
```

output
```
3/2450*(3*x^2 + 2)^(3/2) - 13/105*(3*x^2 + 2)^(5/2)/(8*x^3 + 36*x^2 + 54*x
 + 27) - 2/1225*(3*x^2 + 2)^(5/2)/(4*x^2 + 12*x + 9) + 3933/9800*sqrt(3*x^
2 + 2)*x + 33/16*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 11727/19600*sqrt(35)*arc
sinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3)) - 11727/9800*s
qrt(3*x^2 + 2) - 223/1225*(3*x^2 + 2)^(3/2)/(2*x + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(84) = 168$.

Time = 0.16 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.50

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^4}\,dx = -\frac{33}{16}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$
$$-\frac{11727}{19600}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3\,x^2+2}}\right)-\frac{3}{16}\sqrt{3\,x^2+2}$$
$$-\frac{\sqrt{3}\left(14792\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^5+189285\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^4+141030\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^3-5\cdots\right.}{1120\left(\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2+3\sqrt{3}\left(\sqrt{3}x-\sqrt{3}\cdots\right.\right.}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^4,x, algorithm="giac")
```

output
```
-33/16*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 11727/19600*sqrt(35)*lo
g(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqrt(3*x^2 + 2))/(2*sqrt(3)
*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) - 3/16*sqrt(3*x^2 + 2) - 1
/1120*sqrt(3)*(14792*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^5 + 189285*(sqr
t(3)*x - sqrt(3*x^2 + 2))^4 + 141030*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))
^3 - 561630*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 166480*sqrt(3)*(sqrt(3)*x -
sqrt(3*x^2 + 2)) - 50144)/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 3*sqrt(3)*(sq
rt(3)*x - sqrt(3*x^2 + 2)) - 2)^3
```

Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.25

$$\int \frac{(5-x)\,(2+3x^2)^{3/2}}{(3+2x)^4}\,dx = \frac{33\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{16} - \frac{3\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{16} - \frac{11727\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{19600} + \frac{11727\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{19600} - \frac{1567\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{560\,\left(x+\frac{3}{2}\right)} + \frac{77\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{32\,\left(x^2+3\,x+\frac{9}{4}\right)} - \frac{455\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{384\,\left(x^3+\frac{9\,x^2}{2}+\frac{27\,x}{4}+\frac{27}{8}\right)}$$

input
```
int(-((3*x^2 + 2)^(3/2)*(x - 5))/(2*x + 3)^4,x)
```

output
```
(33*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/16 - (3*3^(1/2)*(x^2 + 2/3)^(1/2
))/16 - (11727*35^(1/2)*log(x + 3/2))/19600 + (11727*35^(1/2)*log(x - (3^(
1/2)*35^(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/19600 - (1567*3^(1/2)*(x^2 + 2/
3)^(1/2))/(560*(x + 3/2)) + (77*3^(1/2)*(x^2 + 2/3)^(1/2))/(32*(3*x + x^2
+ 9/4)) - (455*3^(1/2)*(x^2 + 2/3)^(1/2))/(384*((27*x)/4 + (9*x^2)/2 + x^3
 + 27/8))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.49

$$\int \frac{(5-x)\,(2+3x^2)^{3/2}}{(3+2x)^4}\,dx = \frac{-176400\sqrt{3x^2+2}\,x^3 - 3426360\sqrt{3x^2+2}\,x^2 - 6824580\sqrt{3x^2+2}\,x - 423766}{}$$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^4,x)
```

output
```
( - 176400*sqrt(3*x**2 + 2)*x**3 - 3426360*sqrt(3*x**2 + 2)*x**2 - 6824580
*sqrt(3*x**2 + 2)*x - 4237660*sqrt(3*x**2 + 2) + 562896*sqrt(35)*log( - sq
rt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**3 + 2533032*sqrt(35)*log( - sqrt(3*x
**2 + 2)*sqrt(35) + 9*x - 4)*x**2 + 3799548*sqrt(35)*log( - sqrt(3*x**2 +
2)*sqrt(35) + 9*x - 4)*x + 1899774*sqrt(35)*log( - sqrt(3*x**2 + 2)*sqrt(3
5) + 9*x - 4) - 562896*sqrt(35)*log(2*x + 3)*x**3 - 2533032*sqrt(35)*log(2
*x + 3)*x**2 - 3799548*sqrt(35)*log(2*x + 3)*x - 1899774*sqrt(35)*log(2*x
+ 3) - 970200*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x**3 - 4365900*sqr
t(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x**2 - 6548850*sqrt(3)*log(sqrt(3*x
**2 + 2) - sqrt(3)*x)*x - 3274425*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x
) + 970200*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x**3 + 4365900*sqrt(3
)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x**2 + 6548850*sqrt(3)*log(sqrt(3*x**2
 + 2) + sqrt(3)*x)*x + 3274425*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x))/
(117600*(8*x**3 + 36*x**2 + 54*x + 27))
```

3.216 $\int \frac{(5-x)(2+3x^2)^{3/2}}{(3+2x)^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 106

$$\int \frac{(5-x)(2+3x^2)^{3/2}}{(3+2x)^5}\,dx = \frac{3(2943+4097x)\sqrt{2+3x^2}}{19600(3+2x)^2} + \frac{(54+491x)(2+3x^2)^{3/2}}{840(3+2x)^4} - \frac{3}{32}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right) - \frac{39663\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{39200\sqrt{35}}$$

output
```
3/19600*(2943+4097*x)*(3*x^2+2)^(1/2)/(3+2*x)^2+1/840*(54+491*x)*(3*x^2+2)
^(3/2)/(3+2*x)^4-3/32*arcsinh(1/2*x*6^(1/2))*3^(1/2)-39663/1372000*35^(1/2
)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07

$$\int \frac{(5-x)(2+3x^2)^{3/2}}{(3+2x)^5}\,dx = \frac{\sqrt{2+3x^2}(245943+718441x+559764x^2+250602x^3)}{58800(3+2x)^4} + \frac{39663\text{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{19600\sqrt{35}} + \frac{3}{32}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^5,x]
```

output
```
(Sqrt[2 + 3*x^2]*(245943 + 718441*x + 559764*x^2 + 250602*x^3))/(58800*(3
+ 2*x)^4) + (39663*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/S
qrt[35]])/(19600*Sqrt[35]) + (3*Sqrt[3]*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]
])/32
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {680, 27, 680, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{3/2}}{(2x+3)^5}dx$$

$$\Big\downarrow \text{ 680}$$

$$\frac{(491x+54)\left(3x^2+2\right)^{3/2}}{840(2x+3)^4}-\frac{\int -\frac{24(39-35x)\sqrt{3x^2+2}}{(2x+3)^3}dx}{1120}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{140}\int \frac{(39-35x)\sqrt{3x^2+2}}{(2x+3)^3}dx+\frac{(491x+54)\left(3x^2+2\right)^{3/2}}{840(2x+3)^4}$$

$$\Big\downarrow \text{ 680}$$

$$\frac{3}{140}\left(\frac{(4097x+2943)\sqrt{3x^2+2}}{140(2x+3)^2}-\frac{1}{560}\int -\frac{12(366-1225x)}{(2x+3)\sqrt{3x^2+2}}dx\right)+\frac{(491x+54)\left(3x^2+2\right)^{3/2}}{840(2x+3)^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{140}\left(\frac{3}{140}\int \frac{366-1225x}{(2x+3)\sqrt{3x^2+2}}dx+\frac{\sqrt{3x^2+2}(4097x+2943)}{140(2x+3)^2}\right)+\frac{(491x+54)\left(3x^2+2\right)^{3/2}}{840(2x+3)^4}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{3}{140}\left(\frac{3}{140}\left(\frac{4407}{2}\int\frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{1225}{2}\int\frac{1}{\sqrt{3x^2+2}}dx\right)+\frac{\sqrt{3x^2+2}(4097x+2943)}{140(2x+3)^2}\right)+\frac{(491x+54)\left(3x^2+2\right)^{3/2}}{840(2x+3)^4}$$

$$\Big\downarrow \text{222}$$

$$\frac{3}{140}\left(\frac{3}{140}\left(\frac{4407}{2}\int\frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{1225\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)+\frac{\sqrt{3x^2+2}(4097x+2943)}{140(2x+3)^2}\right)+\frac{(491x+54)\left(3x^2+2\right)^{3/2}}{840(2x+3)^4}$$

$$\Big\downarrow \text{488}$$

$$\frac{3}{140}\left(\frac{3}{140}\left(-\frac{4407}{2}\int\frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{1225\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)+\frac{\sqrt{3x^2+2}(4097x+2943)}{140(2x+3)^2}\right)+\frac{(491x+54)\left(3x^2+2\right)^{3/2}}{840(2x+3)^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{140}\left(\frac{3}{140}\left(-\frac{1225\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{4407\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{2\sqrt{35}}\right)+\frac{\sqrt{3x^2+2}(4097x+2943)}{140(2x+3)^2}\right)+\frac{(491x+54)\left(3x^2+2\right)^{3/2}}{840(2x+3)^4}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^5,x]
```

output
```
((54 + 491*x)*(2 + 3*x^2)^(3/2))/(840*(3 + 2*x)^4) + (3*(((2943 + 4097*x)*
Sqrt[2 + 3*x^2])/(140*(3 + 2*x)^2) + (3*((-1225*ArcSinh[Sqrt[3/2]*x])/(2*S
qrt[3]) - (4407*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(2*Sqrt[35]
)))/140))/140
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

method	result
risch	$\frac{751806x^5+1679292x^4+2656527x^3+1857357x^2+1436882x+491886}{58800(2x+3)^4\sqrt{3x^2+2}} - \frac{3\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{32} - \frac{39663\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x}{35\sqrt{12\left(x+\frac{3}{2}\right.}}\right.}{1372000}$
trager	$\frac{(250602x^3+559764x^2+718441x+245943)\sqrt{3x^2+2}}{58800(2x+3)^4} + \frac{3\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{32} - \frac{39663\,\mathrm{Ro}}{}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{2240\left(x+\frac{3}{2}\right)^4} - \frac{211\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{117600\left(x+\frac{3}{2}\right)^3} - \frac{999\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{686000\left(x+\frac{3}{2}\right)^2} - \frac{5779\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{12005000\left(x+\frac{3}{2}\right)} + \frac{132}{}$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(2*x+3)^5,x,method=_RETURNVERBOSE)
```

output
```
1/58800*(751806*x^5+1679292*x^4+2656527*x^3+1857357*x^2+1436882*x+491886)/
(2*x+3)^4/(3*x^2+2)^(1/2)-3/32*arcsinh(1/2*6^(1/2)*x)*3^(1/2)-39663/137200
0*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.57

$$\int \frac{(5-x)\,(2+3x^2)^{3/2}}{(3+2x)^5}\,dx = \frac{385875\,\sqrt{3}(16\,x^4+96\,x^3+216\,x^2+216\,x+81)\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-\right.}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^5,x, algorithm="fricas")
```

output
```
1/8232000*(385875*sqrt(3)*(16*x^4 + 96*x^3 + 216*x^2 + 216*x + 81)*log(sqr
t(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + 118989*sqrt(35)*(16*x^4 + 96*x^3 + 2
16*x^2 + 216*x + 81)*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) + 93*x^2 - 3
6*x + 43)/(4*x^2 + 12*x + 9)) + 140*(250602*x^3 + 559764*x^2 + 718441*x +
245943)*sqrt(3*x^2 + 2))/(16*x^4 + 96*x^3 + 216*x^2 + 216*x + 81)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^5}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(3/2)/(3+2*x)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(84) = 168.

Time = 0.13 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.73

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^5}\,dx = \frac{2997}{686000}\left(3\,x^2+2\right)^{\frac{3}{2}} - \frac{13\left(3\,x^2+2\right)^{\frac{5}{2}}}{140\left(16\,x^4+96\,x^3+216\,x^2+216\,x+81\right)} - \frac{211\left(3\,x^2+2\right)^{\frac{5}{2}}}{14700\left(8\,x^3+36\,x^2+54\,x+27\right)} - \frac{999\left(3\,x^2+2\right)^{\frac{5}{2}}}{171500\left(4\,x^2+12\,x+9\right)} - \frac{7227}{686000}\sqrt{3\,x^2+2}x - \frac{3}{32}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{39663}{1372000}\sqrt{35}\operatorname{arsinh}\left(\frac{3\sqrt{6}x}{2\left|2\,x+3\right|} - \frac{2\sqrt{6}}{3\left|2\,x+3\right|}\right) + \frac{39663}{686000}\sqrt{3\,x^2+2} - \frac{5779\left(3\,x^2+2\right)^{\frac{3}{2}}}{686000\left(2\,x+3\right)}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^5,x, algorithm="maxima")
```

output
```
2997/686000*(3*x^2 + 2)^(3/2) - 13/140*(3*x^2 + 2)^(5/2)/(16*x^4 + 96*x^3
+ 216*x^2 + 216*x + 81) - 211/14700*(3*x^2 + 2)^(5/2)/(8*x^3 + 36*x^2 + 54
*x + 27) - 999/171500*(3*x^2 + 2)^(5/2)/(4*x^2 + 12*x + 9) - 7227/686000*s
qrt(3*x^2 + 2)*x - 3/32*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 39663/1372000*sqr
t(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3)) + 396
63/686000*sqrt(3*x^2 + 2) - 5779/686000*(3*x^2 + 2)^(3/2)/(2*x + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(84) = 168$.

Time = 0.26 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.31

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^5}\,dx =$$

$$-\frac{39663}{1372000}\sqrt{35}\log\left(\sqrt{35}\left(\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)-9\right)\operatorname{sgn}\left(\frac{1}{2x+3}\right)$$

$$+\frac{3}{32}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{2\sqrt{35}}{2x+3}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)}\right)\operatorname{sgn}\left(\frac{1}{2x+3}\right)$$

$$-\frac{1}{470400}\left(\frac{35\left(\frac{35\left(\frac{1365\operatorname{sgn}\left(\frac{1}{2x+3}\right)}{2x+3}-1193\operatorname{sgn}\left(\frac{1}{2x+3}\right)\right)}{2x+3}+16227\operatorname{sgn}\left(\frac{1}{2x+3}\right)\right)}{2x+3}-125301\operatorname{sgn}\left(\frac{1}{2x+3}\right)\right)\sqrt{-\frac{18}{2x+}}$$

input

```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^5,x, algorithm="giac")
```

output

```
-39663/1372000*sqrt(35)*log(sqrt(35)*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2
+ 3) + sqrt(35)/(2*x + 3)) - 9)*sgn(1/(2*x + 3)) + 3/32*sqrt(3)*log(1/2*ab
s(-2*sqrt(3) + 2*sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + 2*sqrt(35)/(2*
x + 3))/(sqrt(3) + sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*
x + 3)))*sgn(1/(2*x + 3)) - 1/470400*(35*(35*(1365*sgn(1/(2*x + 3))/(2*x +
 3) - 1193*sgn(1/(2*x + 3)))/(2*x + 3) + 16227*sgn(1/(2*x + 3)))/(2*x + 3)
 - 125301*sgn(1/(2*x + 3)))*sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.46

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^5}\,dx = \frac{39663\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{1372000} - \frac{3\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{32} - \frac{39663\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{1372000} - \frac{455\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1024\left(x^4+6x^3+\frac{27x^2}{2}+\frac{27x}{2}+\frac{81}{16}\right)} + \frac{41767\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{156800\left(x+\frac{3}{2}\right)} - \frac{5409\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{8960\left(x^2+3x+\frac{9}{4}\right)} + \frac{1193\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1536\left(x^3+\frac{9x^2}{2}+\frac{27x}{4}+\frac{27}{8}\right)}$$

input
```
int(-((3*x^2 + 2)^(3/2)*(x - 5))/(2*x + 3)^5,x)
```

output
```
(39663*35^(1/2)*log(x + 3/2))/1372000 - (3*3^(1/2)*asinh((2^(1/2)*3^(1/2)*
x)/2))/32 - (39663*35^(1/2)*log(x - (3^(1/2)*35^(1/2)*(x^2 + 2/3)^(1/2))/9
 - 4/9))/1372000 - (455*3^(1/2)*(x^2 + 2/3)^(1/2))/(1024*((27*x)/2 + (27*x
^2)/2 + 6*x^3 + x^4 + 81/16)) + (41767*3^(1/2)*(x^2 + 2/3)^(1/2))/(156800*
(x + 3/2)) - (5409*3^(1/2)*(x^2 + 2/3)^(1/2))/(8960*(3*x + x^2 + 9/4)) + (
1193*3^(1/2)*(x^2 + 2/3)^(1/2))/(1536*((27*x)/4 + (9*x^2)/2 + x^3 + 27/8))
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 451, normalized size of antiderivative = 4.25

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^5}\,dx = \frac{35084280\sqrt{3x^2+2}\,x^3+78366960\sqrt{3x^2+2}\,x^2+100581740\sqrt{3x^2+2}\,x+344}{}$$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^5,x)
```

output

```
(35084280*sqrt(3*x**2 + 2)*x**3 + 78366960*sqrt(3*x**2 + 2)*x**2 + 1005817
40*sqrt(3*x**2 + 2)*x + 34432020*sqrt(3*x**2 + 2) + 3807648*sqrt(35)*log(s
qrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**4 + 22845888*sqrt(35)*log(sqrt(3*x*
*2 + 2)*sqrt(35) + 9*x - 4)*x**3 + 51403248*sqrt(35)*log(sqrt(3*x**2 + 2)*
sqrt(35) + 9*x - 4)*x**2 + 51403248*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35)
 + 9*x - 4)*x + 19276218*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)
 - 3807648*sqrt(35)*log(2*x + 3)*x**4 - 22845888*sqrt(35)*log(2*x + 3)*x**
3 - 51403248*sqrt(35)*log(2*x + 3)*x**2 - 51403248*sqrt(35)*log(2*x + 3)*x
 - 19276218*sqrt(35)*log(2*x + 3) + 6174000*sqrt(3)*log(sqrt(3*x**2 + 2) -
 sqrt(3)*x)*x**4 + 37044000*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x**3
 + 83349000*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x**2 + 83349000*sqrt
(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x + 31255875*sqrt(3)*log(sqrt(3*x**2
 + 2) - sqrt(3)*x) - 6174000*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x**
4 - 37044000*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x**3 - 83349000*sqr
t(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x**2 - 83349000*sqrt(3)*log(sqrt(3*
x**2 + 2) + sqrt(3)*x)*x - 31255875*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)
*x))/(8232000*(16*x**4 + 96*x**3 + 216*x**2 + 216*x + 81))
```

3.217 $\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^6}\,dx=-\frac{369(4-9x)\sqrt{2+3x^2}}{171500(3+2x)^2}-\frac{41(4-9x)\left(2+3x^2\right)^{3/2}}{4900(3+2x)^4}-\frac{13\left(2+3x^2\right)^{5/2}}{175(3+2x)^5}-\frac{1107\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{85750\sqrt{35}}$$

output
```
-369/171500*(4-9*x)*(3*x^2+2)^(1/2)/(3+2*x)^2-41/4900*(4-9*x)*(3*x^2+2)^(3
/2)/(3+2*x)^4-13/175*(3*x^2+2)^(5/2)/(3+2*x)^5-1107/3001250*35^(1/2)*arcta
nh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 2.65 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^6}\,dx=\frac{-\frac{35\sqrt{2+3x^2}\left(125252-64493x+26682x^2-189543x^3+10602x^4\right)}{(3+2x)^5}+4428\sqrt{35}\text{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-}{\sqrt{3}}\right.}{6002500}$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^6,x]
```

output
```
((-35*Sqrt[2 + 3*x^2]*(125252 - 64493*x + 26682*x^2 - 189543*x^3 + 10602*x
^4))/(3 + 2*x)^5 + 4428*Sqrt[35]*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt
[2 + 3*x^2])/Sqrt[35]])/6002500
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {679, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{3/2}}{(2x+3)^6}dx$$

$$\Big\downarrow \text{679}$$

$$\frac{41}{35}\int \frac{\left(3x^2+2\right)^{3/2}}{(2x+3)^5}dx-\frac{13\left(3x^2+2\right)^{5/2}}{175(2x+3)^5}$$

$$\Big\downarrow \text{486}$$

$$\frac{41}{35}\left(\frac{9}{70}\int \frac{\sqrt{3x^2+2}}{(2x+3)^3}dx-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{13\left(3x^2+2\right)^{5/2}}{175(2x+3)^5}$$

$$\Big\downarrow \text{486}$$

$$\frac{41}{35}\left(\frac{9}{70}\left(\frac{3}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{(4-9x)\sqrt{3x^2+2}}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{13\left(3x^2+2\right)^{5/2}}{175(2x+3)^5}$$

$$\Big\downarrow \text{488}$$

$$\frac{41}{35}\left(\frac{9}{70}\left(-\frac{3}{35}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{13\left(3x^2+2\right)^{5/2}}{175(2x+3)^5}$$

$$\Big\downarrow \text{219}$$

$$\frac{41}{35}\left(\frac{9}{70}\left(-\frac{3\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{13\left(3x^2+2\right)^{5/2}}{175(2x+3)^5}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^6,x]
```

output
```
(-13*(2 + 3*x^2)^(5/2))/(175*(3 + 2*x)^5) + (41*(-1/140*((4 - 9*x)*(2 + 3*
x^2)^(3/2))/(3 + 2*x)^4 + (9*(-1/70*((4 - 9*x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^
2 - (3*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(35*Sqrt[35])))/70))
/35
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{31806x^6-568629x^5+101250x^4-572565x^3+429120x^2-128986x+250504}{171500(2x+3)^5\sqrt{3x^2+2}}-\frac{1107\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{3001250}$
trager	$-\frac{(10602x^4-189543x^3+26682x^2-64493x+125252)\sqrt{3x^2+2}}{171500(2x+3)^5}-\frac{1107\operatorname{RootOf}\left(\_Z^2-35\right)\ln\left(-\frac{9\operatorname{RootOf}\left(\_Z^2-35\right)x-4\operatorname{RootOf}(}{2x+3}\right.}{3001250}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{5600\left(x+\frac{3}{2}\right)^5}-\frac{41\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{39200\left(x+\frac{3}{2}\right)^4}-\frac{369\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{686000\left(x+\frac{3}{2}\right)^3}-\frac{3813\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{12005000\left(x+\frac{3}{2}\right)^2}-\frac{4317}{}$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(2*x+3)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/171500*(31806*x^6-568629*x^5+101250*x^4-572565*x^3+429120*x^2-128986*x+
250504)/(2*x+3)^5/(3*x^2+2)^(1/2)-1107/3001250*35^(1/2)*arctanh(2/35*(4-9*
x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^6}\,dx=\frac{1107\sqrt{35}(32x^5+240x^4+720x^3+1080x^2+810x+243)\log\left(-\frac{\sqrt{35}\sqrt{3x^2+2}(}{4}\right.}{6002500\,(32x^5+240x}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^6,x, algorithm="fricas")
```

output
```
1/6002500*(1107*sqrt(35)*(32*x^5 + 240*x^4 + 720*x^3 + 1080*x^2 + 810*x +
243)*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2
 + 12*x + 9)) - 35*(10602*x^4 - 189543*x^3 + 26682*x^2 - 64493*x + 125252)
*sqrt(3*x^2 + 2))/(32*x^5 + 240*x^4 + 720*x^3 + 1080*x^2 + 810*x + 243)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)(2+3x^2)^{3/2}}{(3+2x)^6}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(3/2)/(3+2*x)**6,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(90) = 180$.

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.92

$$\begin{aligned}\int \frac{(5-x)(2+3x^2)^{3/2}}{(3+2x)^6}\,dx &= \frac{11439}{12005000}\left(3x^2+2\right)^{\frac{3}{2}} \\ &- \frac{13\left(3x^2+2\right)^{\frac{5}{2}}}{175\left(32x^5+240x^4+720x^3+1080x^2+810x+243\right)} \\ &- \frac{41\left(3x^2+2\right)^{\frac{5}{2}}}{2450\left(16x^4+96x^3+216x^2+216x+81\right)} \\ &- \frac{369\left(3x^2+2\right)^{\frac{5}{2}}}{85750\left(8x^3+36x^2+54x+27\right)} - \frac{3813\left(3x^2+2\right)^{\frac{5}{2}}}{3001250\left(4x^2+12x+9\right)} \\ &+ \frac{9963}{6002500}\sqrt{3x^2+2}x + \frac{1107}{3001250}\sqrt{35}\,\text{arsinh}\left(\frac{3\sqrt{6}x}{2|2x+3|} - \frac{2\sqrt{6}}{3|2x+3|}\right) \\ &+ \frac{1107}{1500625}\sqrt{3x^2+2} - \frac{43173\left(3x^2+2\right)^{\frac{3}{2}}}{12005000(2x+3)}\end{aligned}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^6,x, algorithm="maxima")
```

output
```
11439/12005000*(3*x^2 + 2)^(3/2) - 13/175*(3*x^2 + 2)^(5/2)/(32*x^5 + 240*
x^4 + 720*x^3 + 1080*x^2 + 810*x + 243) - 41/2450*(3*x^2 + 2)^(5/2)/(16*x^
4 + 96*x^3 + 216*x^2 + 216*x + 81) - 369/85750*(3*x^2 + 2)^(5/2)/(8*x^3 +
36*x^2 + 54*x + 27) - 3813/3001250*(3*x^2 + 2)^(5/2)/(4*x^2 + 12*x + 9) +
9963/6002500*sqrt(3*x^2 + 2)*x + 1107/3001250*sqrt(35)*arcsinh(3/2*sqrt(6)
*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3)) + 1107/1500625*sqrt(3*x^2 + 2)
 - 43173/12005000*(3*x^2 + 2)^(3/2)/(2*x + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(90) = 180$.

Time = 0.15 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.92

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^6}\,dx = \frac{1107}{3001250}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3x^2+2}}\right)$$
$$-\frac{9\left(89686\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^9+138886\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^8+1224478\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^7+2451\right.}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^6,x, algorithm="giac")
```

output
```
1107/3001250*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqr
t(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) -
9/2744000*(89686*(sqrt(3)*x - sqrt(3*x^2 + 2))^9 + 138886*sqrt(3)*(sqrt(3)
*x - sqrt(3*x^2 + 2))^8 + 1224478*(sqrt(3)*x - sqrt(3*x^2 + 2))^7 + 245133
*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^6 - 1224531*(sqrt(3)*x - sqrt(3*x^2
 + 2))^5 - 4374874*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^4 + 4855928*(sqrt
(3)*x - sqrt(3*x^2 + 2))^3 - 1339152*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))
^2 - 586816*sqrt(3)*x - 37696*sqrt(3) + 586816*sqrt(3*x^2 + 2))/((sqrt(3)*
x - sqrt(3*x^2 + 2))^2 + 3*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^5
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.64

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^6}\,dx = \frac{1107\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{3001250} - \frac{1107\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{3001250} + \frac{731\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{2560\left(x^4+6x^3+\frac{27x^2}{2}+\frac{27x}{2}+\frac{81}{16}\right)} - \frac{91\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{512\left(x^5+\frac{15x^4}{2}+\frac{45x^3}{2}+\frac{135x^2}{4}+\frac{405x}{16}+\frac{243}{32}\right)} - \frac{5301\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{2744000\left(x+\frac{3}{2}\right)} + \frac{7233\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{156800\left(x^2+3x+\frac{9}{4}\right)} - \frac{8349\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{44800\left(x^3+\frac{9x^2}{2}+\frac{27x}{4}+\frac{27}{8}\right)}$$

input
```
int(-((3*x^2 + 2)^(3/2)*(x - 5))/(2*x + 3)^6,x)
```

output
```
(1107*35^(1/2)*log(x + 3/2))/3001250 - (1107*35^(1/2)*log(x - (3^(1/2)*35^
(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/3001250 + (731*3^(1/2)*(x^2 + 2/3)^(1/2
))/(2560*((27*x)/2 + (27*x^2)/2 + 6*x^3 + x^4 + 81/16)) - (91*3^(1/2)*(x^2
 + 2/3)^(1/2))/(512*((405*x)/16 + (135*x^2)/4 + (45*x^3)/2 + (15*x^4)/2 +
x^5 + 243/32)) - (5301*3^(1/2)*(x^2 + 2/3)^(1/2))/(2744000*(x + 3/2)) + (7
233*3^(1/2)*(x^2 + 2/3)^(1/2))/(156800*(3*x + x^2 + 9/4)) - (8349*3^(1/2)*
(x^2 + 2/3)^(1/2))/(44800*((27*x)/4 + (9*x^2)/2 + x^3 + 27/8))
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.76

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^6}\,dx = \frac{-371070\sqrt{3x^2+2}\,x^4+6634005\sqrt{3x^2+2}\,x^3-933870\sqrt{3x^2+2}\,x^2+225725}{}$$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^6,x)
```

output
```
( - 371070*sqrt(3*x**2 + 2)*x**4 + 6634005*sqrt(3*x**2 + 2)*x**3 - 933870*
sqrt(3*x**2 + 2)*x**2 + 2257255*sqrt(3*x**2 + 2)*x - 4383820*sqrt(3*x**2 +
 2) + 70848*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**5 + 53136
0*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**4 + 1594080*sqrt(35
)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**3 + 2391120*sqrt(35)*log(sqr
t(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**2 + 1793340*sqrt(35)*log(sqrt(3*x**2
+ 2)*sqrt(35) + 9*x - 4)*x + 538002*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35)
 + 9*x - 4) - 70848*sqrt(35)*log(2*x + 3)*x**5 - 531360*sqrt(35)*log(2*x +
 3)*x**4 - 1594080*sqrt(35)*log(2*x + 3)*x**3 - 2391120*sqrt(35)*log(2*x +
 3)*x**2 - 1793340*sqrt(35)*log(2*x + 3)*x - 538002*sqrt(35)*log(2*x + 3))
/(6002500*(32*x**5 + 240*x**4 + 720*x**3 + 1080*x**2 + 810*x + 243))
```

3.218 $\int \frac{(5-x)(2+3x^2)^{3/2}}{(3+2x)^7} dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(5-x)(2+3x^2)^{3/2}}{(3+2x)^7} dx = -\frac{3(67-2x)\sqrt{2+3x^2}}{3500(3+2x)^4} + \frac{9\sqrt{2+3x^2}}{7000(3+2x)^2} - \frac{9\sqrt{2+3x^2}}{35000(3+2x)} - \frac{(296-561x)(2+3x^2)^{3/2}}{2100(3+2x)^6} - \frac{27\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{8750\sqrt{35}}$$

output
```
-3/3500*(67-2*x)*(3*x^2+2)^(1/2)/(3+2*x)^4+9/7000*(3*x^2+2)^(1/2)/(3+2*x)^
2-9*(3*x^2+2)^(1/2)/(105000+70000*x)-1/2100*(296-561*x)*(3*x^2+2)^(3/2)/(3
+2*x)^6-27/306250*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 3.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.71

$$\int \frac{(5-x)(2+3x^2)^{3/2}}{(3+2x)^7} dx = -\frac{\sqrt{2+3x^2}(39748+3675x+33180x^2-39195x^3+2160x^4+432x^5)}{52500(3+2x)^6} + \frac{27\operatorname{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{4375\sqrt{35}}$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^7,x]
```

output
```
-1/52500*(Sqrt[2 + 3*x^2]*(39748 + 3675*x + 33180*x^2 - 39195*x^3 + 2160*x
^4 + 432*x^5))/(3 + 2*x)^6 + (27*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt
[2 + 3*x^2])/Sqrt[35]])/(4375*Sqrt[35])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {688, 27, 679, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{3/2}}{(2x+3)^7}dx$$

$$\Big\downarrow \text{688}$$

$$-\frac{1}{210}\int -\frac{3(82-13x)\left(3x^2+2\right)^{3/2}}{(2x+3)^6}dx-\frac{13\left(3x^2+2\right)^{5/2}}{210(2x+3)^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{70}\int \frac{(82-13x)\left(3x^2+2\right)^{3/2}}{(2x+3)^6}dx-\frac{13\left(3x^2+2\right)^{5/2}}{210(2x+3)^6}$$

$$\Big\downarrow \text{679}$$

$$\frac{1}{70}\left(\frac{98}{5}\int \frac{\left(3x^2+2\right)^{3/2}}{(2x+3)^5}dx-\frac{29\left(3x^2+2\right)^{5/2}}{25(2x+3)^5}\right)-\frac{13\left(3x^2+2\right)^{5/2}}{210(2x+3)^6}$$

$$\Big\downarrow \text{486}$$

$$\frac{1}{70}\left(\frac{98}{5}\left(\frac{9}{70}\int \frac{\sqrt{3x^2+2}}{(2x+3)^3}dx-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{29\left(3x^2+2\right)^{5/2}}{25(2x+3)^5}\right)-\frac{13\left(3x^2+2\right)^{5/2}}{210(2x+3)^6}$$

$$\Big\downarrow \text{486}$$

$$\frac{1}{70}\left(\frac{98}{5}\left(\frac{9}{70}\left(\frac{3}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{(4-9x)\sqrt{3x^2+2}}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{29\left(3x^2+2\right)^{5/2}}{25(2x+3)^5}\right)-\frac{13\left(3x^2+2\right)^{5/2}}{210(2x+3)^6}$$

$$\Big\downarrow \; 488$$

$$\frac{1}{70}\left(\frac{98}{5}\left(\frac{9}{70}\left(-\frac{3}{35}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{29\left(3x^2+2\right)^{5/2}}{25(2x+3)^5}\right)-\frac{13\left(3x^2+2\right)^{5/2}}{210(2x+3)^6}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{70}\left(\frac{98}{5}\left(\frac{9}{70}\left(-\frac{3\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{29\left(3x^2+2\right)^{5/2}}{25(2x+3)^5}\right)-\frac{13\left(3x^2+2\right)^{5/2}}{210(2x+3)^6}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^7,x]
```

output
```
(-13*(2 + 3*x^2)^(5/2))/(210*(3 + 2*x)^6) + ((-29*(2 + 3*x^2)^(5/2))/(25*(
3 + 2*x)^5) + (98*(-1/140*((4 - 9*x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^4 + (9*(
-1/70*((4 - 9*x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^2 - (3*ArcTanh[(4 - 9*x)/(Sqrt
[35]*Sqrt[2 + 3*x^2])])/(35*Sqrt[35])))/70))/5)/70
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.65

method	result
risch	$-\frac{1296x^7+6480x^6-116721x^5+103860x^4-67365x^3+185604x^2+7350x+79496}{52500(2x+3)^6\sqrt{3x^2+2}}-\frac{27\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{306250}$
trager	$-\frac{(432x^5+2160x^4-39195x^3+33180x^2+3675x+39748)\sqrt{3x^2+2}}{52500(2x+3)^6}-\frac{27\,\mathrm{RootOf}\left(\_Z^2-35\right)\ln\left(-\frac{9\,\mathrm{RootOf}\left(\_Z^2-35\right)x-4\,\mathrm{RootOf}}{2x+3}\right.}{306250}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{13440\left(x+\frac{3}{2}\right)^6}-\frac{29\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{56000\left(x+\frac{3}{2}\right)^5}-\frac{\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{4000\left(x+\frac{3}{2}\right)^4}-\frac{9\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{70000\left(x+\frac{3}{2}\right)^3}-\frac{93\left(3\left(x+\frac{3}{2}\right.\right.}{122500}$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(2*x+3)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/52500*(1296*x^7+6480*x^6-116721*x^5+103860*x^4-67365*x^3+185604*x^2+735
0*x+79496)/(2*x+3)^6/(3*x^2+2)^(1/2)-27/306250*35^(1/2)*arctanh(2/35*(4-9*
x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^7}\,dx=\frac{81\sqrt{35}(64\,x^6+576\,x^5+2160\,x^4+4320\,x^3+4860\,x^2+2916\,x+729)\log\left(-\right.}{1837500\,(64\,x^6+576\,}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^7,x, algorithm="fricas")
```

output
```
1/1837500*(81*sqrt(35)*(64*x^6 + 576*x^5 + 2160*x^4 + 4320*x^3 + 4860*x^2
+ 2916*x + 729)*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) + 93*x^2 - 36*x +
 43)/(4*x^2 + 12*x + 9)) - 35*(432*x^5 + 2160*x^4 - 39195*x^3 + 33180*x^2
+ 3675*x + 39748)*sqrt(3*x^2 + 2))/(64*x^6 + 576*x^5 + 2160*x^4 + 4320*x^3
 + 4860*x^2 + 2916*x + 729)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^7}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(3/2)/(3+2*x)**7,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(108) = 216.

Time = 0.14 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.92

$$\begin{aligned}\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^7}\,dx &= \frac{279}{1225000}\left(3\,x^2+2\right)^{\frac{3}{2}} \\ &- \frac{13\left(3\,x^2+2\right)^{\frac{5}{2}}}{210\left(64\,x^6+576\,x^5+2160\,x^4+4320\,x^3+4860\,x^2+2916\,x+729\right)} \\ &- \frac{29\left(3\,x^2+2\right)^{\frac{5}{2}}}{1750\left(32\,x^5+240\,x^4+720\,x^3+1080\,x^2+810\,x+243\right)} \\ &- \frac{\left(3\,x^2+2\right)^{\frac{5}{2}}}{250\left(16\,x^4+96\,x^3+216\,x^2+216\,x+81\right)} - \frac{9\left(3\,x^2+2\right)^{\frac{5}{2}}}{8750\left(8\,x^3+36\,x^2+54\,x+27\right)} \\ &- \frac{93\left(3\,x^2+2\right)^{\frac{5}{2}}}{306250\left(4\,x^2+12\,x+9\right)} + \frac{243}{612500}\sqrt{3\,x^2+2}x \\ &+ \frac{27}{306250}\sqrt{35}\,\operatorname{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|} - \frac{2\sqrt{6}}{3\,|2\,x+3|}\right) \\ &+ \frac{27}{153125}\sqrt{3\,x^2+2} - \frac{1053\left(3\,x^2+2\right)^{\frac{3}{2}}}{1225000\left(2\,x+3\right)}\end{aligned}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^7,x, algorithm="maxima")
```

output
```
279/1225000*(3*x^2 + 2)^(3/2) - 13/210*(3*x^2 + 2)^(5/2)/(64*x^6 + 576*x^5
 + 2160*x^4 + 4320*x^3 + 4860*x^2 + 2916*x + 729) - 29/1750*(3*x^2 + 2)^(5
/2)/(32*x^5 + 240*x^4 + 720*x^3 + 1080*x^2 + 810*x + 243) - 1/250*(3*x^2 +
 2)^(5/2)/(16*x^4 + 96*x^3 + 216*x^2 + 216*x + 81) - 9/8750*(3*x^2 + 2)^(5
/2)/(8*x^3 + 36*x^2 + 54*x + 27) - 93/306250*(3*x^2 + 2)^(5/2)/(4*x^2 + 12
*x + 9) + 243/612500*sqrt(3*x^2 + 2)*x + 27/306250*sqrt(35)*arcsinh(3/2*sq
rt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3)) + 27/153125*sqrt(3*x^2 +
2) - 1053/1225000*(3*x^2 + 2)^(3/2)/(2*x + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(108) = 216$.

Time = 0.15 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.80

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^7}\,dx = \frac{27}{306250}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3x^2+2}}\right)$$

$$-\frac{3\sqrt{3}\left(96\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^{11}+17877\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^{10}-4120\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^{9}+2586\right.}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^7,x, algorithm="giac")
```

output
```
27/306250*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqrt(3
*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) - 3/2
80000*sqrt(3)*(96*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^11 + 17877*(sqrt(3
)*x - sqrt(3*x^2 + 2))^10 - 4120*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^9 +
 25860*(sqrt(3)*x - sqrt(3*x^2 + 2))^8 - 225240*sqrt(3)*(sqrt(3)*x - sqrt(
3*x^2 + 2))^7 - 173964*(sqrt(3)*x - sqrt(3*x^2 + 2))^6 - 648336*sqrt(3)*(s
qrt(3)*x - sqrt(3*x^2 + 2))^5 + 641040*(sqrt(3)*x - sqrt(3*x^2 + 2))^4 - 3
09440*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 - 135120*(sqrt(3)*x - sqrt(3
*x^2 + 2))^2 - 10752*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) + 1536)/((sqrt(
3)*x - sqrt(3*x^2 + 2))^2 + 3*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^6
```

Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.70

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^7}\,dx = \frac{27\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{306250} - \frac{27\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{306250} - \frac{5977\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{89600\left(x^4+6x^3+\frac{27x^2}{2}+\frac{27x}{2}+\frac{81}{16}\right)} + \frac{577\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{5120\left(x^5+\frac{15x^4}{2}+\frac{45x^3}{2}+\frac{135x^2}{4}+\frac{405x}{16}+\frac{243}{32}\right)} - \frac{9\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{70000\left(x+\frac{3}{2}\right)} - \frac{455\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{6144\left(x^6+9x^5+\frac{135x^4}{4}+\frac{135x^3}{2}+\frac{1215x^2}{16}+\frac{729x}{16}+\frac{729}{64}\right)} + \frac{9\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{28000\left(x^2+3x+\frac{9}{4}\right)} + \frac{2829\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{224000\left(x^3+\frac{9x^2}{2}+\frac{27x}{4}+\frac{27}{8}\right)}$$

input
```
int(-((3*x^2 + 2)^(3/2)*(x - 5))/(2*x + 3)^7,x)
```

output
```
(27*35^(1/2)*log(x + 3/2))/306250 - (27*35^(1/2)*log(x - (3^(1/2)*35^(1/2)
*(x^2 + 2/3)^(1/2))/9 - 4/9))/306250 - (5977*3^(1/2)*(x^2 + 2/3)^(1/2))/(8
9600*((27*x)/2 + (27*x^2)/2 + 6*x^3 + x^4 + 81/16)) + (577*3^(1/2)*(x^2 +
2/3)^(1/2))/(5120*((405*x)/16 + (135*x^2)/4 + (45*x^3)/2 + (15*x^4)/2 + x^
5 + 243/32)) - (9*3^(1/2)*(x^2 + 2/3)^(1/2))/(70000*(x + 3/2)) - (455*3^(1
/2)*(x^2 + 2/3)^(1/2))/(6144*((729*x)/16 + (1215*x^2)/16 + (135*x^3)/2 + (
135*x^4)/4 + 9*x^5 + x^6 + 729/64)) + (9*3^(1/2)*(x^2 + 2/3)^(1/2))/(28000
*(3*x + x^2 + 9/4)) + (2829*3^(1/2)*(x^2 + 2/3)^(1/2))/(224000*((27*x)/4 +
 (9*x^2)/2 + x^3 + 27/8))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.72

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^7}\,dx = \frac{-15120\sqrt{3x^2+2}\,x^5-75600\sqrt{3x^2+2}\,x^4+1371825\sqrt{3x^2+2}\,x^3-1161300\sqrt{}}{}$$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^7,x)
```

output
```
( - 15120*sqrt(3*x**2 + 2)*x**5 - 75600*sqrt(3*x**2 + 2)*x**4 + 1371825*sq
rt(3*x**2 + 2)*x**3 - 1161300*sqrt(3*x**2 + 2)*x**2 - 128625*sqrt(3*x**2 +
 2)*x - 1391180*sqrt(3*x**2 + 2) + 10368*sqrt(35)*log(sqrt(3*x**2 + 2)*sqr
t(35) + 9*x - 4)*x**6 + 93312*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x
 - 4)*x**5 + 349920*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**4
 + 699840*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**3 + 787320*
sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**2 + 472392*sqrt(35)*l
og(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x + 118098*sqrt(35)*log(sqrt(3*x**
2 + 2)*sqrt(35) + 9*x - 4) - 10368*sqrt(35)*log(2*x + 3)*x**6 - 93312*sqrt
(35)*log(2*x + 3)*x**5 - 349920*sqrt(35)*log(2*x + 3)*x**4 - 699840*sqrt(3
5)*log(2*x + 3)*x**3 - 787320*sqrt(35)*log(2*x + 3)*x**2 - 472392*sqrt(35)
*log(2*x + 3)*x - 118098*sqrt(35)*log(2*x + 3))/(1837500*(64*x**6 + 576*x*
*5 + 2160*x**4 + 4320*x**3 + 4860*x**2 + 2916*x + 729))
```

3.219 $\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 153

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^8}\,dx=\frac{7569\sqrt{2+3x^2}}{6860000(3+2x)^3}+\frac{3897\sqrt{2+3x^2}}{48020000(3+2x)^2}-\frac{212679\sqrt{2+3x^2}}{1680700000(3+2x)}-\frac{3(22557+7898x)\sqrt{2+3x^2}}{1372000(3+2x)^5}-\frac{(471-596x)\left(2+3x^2\right)^{3/2}}{2940(3+2x)^7}-\frac{72603\,\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{105043750\sqrt{35}}$$

output
```
7569/6860000*(3*x^2+2)^(1/2)/(3+2*x)^3+3897/48020000*(3*x^2+2)^(1/2)/(3+2*
x)^2-212679*(3*x^2+2)^(1/2)/(5042100000+3361400000*x)-3/1372000*(22557+789
8*x)*(3*x^2+2)^(1/2)/(3+2*x)^5-1/2940*(471-596*x)*(3*x^2+2)^(3/2)/(3+2*x)^
7-72603/3676531250*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.64

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^8}\,dx = \frac{-\frac{35\sqrt{2+3x^2}(471103116+256388969x+740031210x^2-98810025x^3+148868010x^4+44301924x^5+5104296}{(3+2x)^7}}{22059187500}$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^8,x]
```

output
```
((-35*Sqrt[2 + 3*x^2]*(471103116 + 256388969*x + 740031210*x^2 - 98810025*
x^3 + 148868010*x^4 + 44301924*x^5 + 5104296*x^6))/(3 + 2*x)^7 + 871236*Sq
rt[35]*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]])/22
059187500
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {688, 25, 688, 27, 679, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{3/2}}{(2x+3)^8}dx$$

$$\Big\downarrow \text{688}$$

$$-\frac{1}{245}\int -\frac{(287-78x)\left(3x^2+2\right)^{3/2}}{(2x+3)^7}dx-\frac{13\left(3x^2+2\right)^{5/2}}{245(2x+3)^7}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{245}\int \frac{(287-78x)\left(3x^2+2\right)^{3/2}}{(2x+3)^7}dx-\frac{13\left(3x^2+2\right)^{5/2}}{245(2x+3)^7}$$

$$\Big\downarrow \text{688}$$

$$\frac{1}{245}\left(-\frac{1}{210}\int -\frac{6(2271-404x)\left(3x^2+2\right)^{3/2}}{(2x+3)^6}dx-\frac{404\left(3x^2+2\right)^{5/2}}{105(2x+3)^6}\right)-\frac{13\left(3x^2+2\right)^{5/2}}{245(2x+3)^7}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{245}\left(\frac{1}{35}\int \frac{(2271-404x)\left(3x^2+2\right)^{3/2}}{(2x+3)^6}dx-\frac{404\left(3x^2+2\right)^{5/2}}{105(2x+3)^6}\right)-\frac{13\left(3x^2+2\right)^{5/2}}{245(2x+3)^7}$$

$$\Big\downarrow \; 679$$

$$\frac{1}{245}\left(\frac{1}{35}\left(\frac{2689}{5}\int \frac{\left(3x^2+2\right)^{3/2}}{(2x+3)^5}dx-\frac{822\left(3x^2+2\right)^{5/2}}{25(2x+3)^5}\right)-\frac{404\left(3x^2+2\right)^{5/2}}{105(2x+3)^6}\right)-\frac{13\left(3x^2+2\right)^{5/2}}{245(2x+3)^7}$$

$$\Big\downarrow \; 486$$

$$\frac{1}{245}\left(\frac{1}{35}\left(\frac{2689}{5}\left(\frac{9}{70}\int \frac{\sqrt{3x^2+2}}{(2x+3)^3}dx-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{822\left(3x^2+2\right)^{5/2}}{25(2x+3)^5}\right)-\frac{404\left(3x^2+2\right)^{5/2}}{105(2x+3)^6}\right)-\frac{13\left(3x^2+2\right)^{5/2}}{245(2x+3)^7}$$

$$\Big\downarrow \; 486$$

$$\frac{1}{245}\left(\frac{1}{35}\left(\frac{2689}{5}\left(\frac{9}{70}\left(\frac{3}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{(4-9x)\sqrt{3x^2+2}}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{822(3x^2 +}{25(2x +}\right.\right.$$
$$\frac{13\left(3x^2+2\right)^{5/2}}{245(2x+3)^7}$$

$$\Big\downarrow \; 488$$

$$\frac{1}{245}\left(\frac{1}{35}\left(\frac{2689}{5}\left(\frac{9}{70}\left(-\frac{3}{35}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{822(}{25}\right.\right.$$
$$\frac{13\left(3x^2+2\right)^{5/2}}{245(2x+3)^7}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{245}\left(\frac{1}{35}\left(\frac{2689}{5}\left(\frac{9}{70}\left(-\frac{3\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{822\left(3x^2+\right.}{25(2x+}\right.\right.$$

$$\frac{13\left(3x^2+2\right)^{5/2}}{245(2x+3)^7}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^8,x]
```

output
```
(-13*(2 + 3*x^2)^(5/2))/(245*(3 + 2*x)^7) + ((-404*(2 + 3*x^2)^(5/2))/(105
*(3 + 2*x)^6) + ((-822*(2 + 3*x^2)^(5/2))/(25*(3 + 2*x)^5) + (2689*(-1/140
*((4 - 9*x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^4 + (9*(-1/70*((4 - 9*x)*Sqrt[2 +
 3*x^2])/(3 + 2*x)^2 - (3*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(
35*Sqrt[35])))/70))/5)/35)/245
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{15312888x^8+132905772x^7+456812622x^6-207826227x^5+2517829650x^4+571546857x^3+2893371768x^2+512777938x+9422062}{630262500(2x+3)^7\sqrt{3x^2+2}}$
trager	$-\frac{(5104296x^6+44301924x^5+148868010x^4-98810025x^3+740031210x^2+256388969x+471103116)\sqrt{3x^2+2}}{630262500(2x+3)^7}+\frac{72603\,\mathrm{RootOf}\left(\_\right.}{}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{31360\left(x+\frac{3}{2}\right)^7}-\frac{101\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{411600\left(x+\frac{3}{2}\right)^6}-\frac{411\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{3430000\left(x+\frac{3}{2}\right)^5}-\frac{2689\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{48020000\left(x+\frac{3}{2}\right)^4}-\frac{242}{}$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(2*x+3)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/630262500*(15312888*x^8+132905772*x^7+456812622*x^6-207826227*x^5+25178
29650*x^4+571546857*x^3+2893371768*x^2+512777938*x+942206232)/(2*x+3)^7/(3
*x^2+2)^(1/2)-72603/3676531250*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*
(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.07

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^8}\,dx = \frac{217809\sqrt{35}(128\,x^7 + 1344\,x^6 + 6048\,x^5 + 15120\,x^4 + 22680\,x^3 + 20412\,x^2 +$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^8,x, algorithm="fricas")
```

output
```
1/22059187500*(217809*sqrt(35)*(128*x^7 + 1344*x^6 + 6048*x^5 + 15120*x^4
+ 22680*x^3 + 20412*x^2 + 10206*x + 2187)*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(
9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9)) - 35*(5104296*x^6 + 443
01924*x^5 + 148868010*x^4 - 98810025*x^3 + 740031210*x^2 + 256388969*x + 4
71103116)*sqrt(3*x^2 + 2))/(128*x^7 + 1344*x^6 + 6048*x^5 + 15120*x^4 + 22
680*x^3 + 20412*x^2 + 10206*x + 2187)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^8}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(3/2)/(3+2*x)**8,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(126) = 252$.

Time = 0.12 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.96

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^8}\,dx = \frac{750231}{14706125000}\left(3\,x^2+2\right)^{\frac{3}{2}} - \frac{13\left(3\,x^2+2\right)^{\frac{5}{2}}}{245\left(128\,x^7+1344\,x^6+6048\,x^5+15120\,x^4+22680\,x^3+20412\,x^2+10206\,x+2187\right)} - \frac{404\left(3\,x^2+2\right)^{\frac{5}{2}}}{25725\left(64\,x^6+576\,x^5+2160\,x^4+4320\,x^3+4860\,x^2+2916\,x+729\right)} - \frac{822\left(3\,x^2+2\right)^{\frac{5}{2}}}{214375\left(32\,x^5+240\,x^4+720\,x^3+1080\,x^2+810\,x+243\right)} - \frac{2689\left(3\,x^2+2\right)^{\frac{5}{2}}}{3001250\left(16\,x^4+96\,x^3+216\,x^2+216\,x+81\right)} - \frac{24201\left(3\,x^2+2\right)^{\frac{5}{2}}}{105043750\left(8\,x^3+36\,x^2+54\,x+27\right)} - \frac{250077\left(3\,x^2+2\right)^{\frac{5}{2}}}{3676531250\left(4\,x^2+12\,x+9\right)} + \frac{653427}{7353062500}\sqrt{3\,x^2+2}x + \frac{72603}{3676531250}\sqrt{35}\,\mathrm{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|} - \frac{2\sqrt{6}}{3\,|2\,x+3|}\right) + \frac{72603}{1838265625}\sqrt{3\,x^2+2} - \frac{2831517\left(3\,x^2+2\right)^{\frac{3}{2}}}{14706125000\,(2\,x+3)}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^8,x, algorithm="maxima")
```

output
```
750231/14706125000*(3*x^2 + 2)^(3/2) - 13/245*(3*x^2 + 2)^(5/2)/(128*x^7 +
 1344*x^6 + 6048*x^5 + 15120*x^4 + 22680*x^3 + 20412*x^2 + 10206*x + 2187)
 - 404/25725*(3*x^2 + 2)^(5/2)/(64*x^6 + 576*x^5 + 2160*x^4 + 4320*x^3 + 4
860*x^2 + 2916*x + 729) - 822/214375*(3*x^2 + 2)^(5/2)/(32*x^5 + 240*x^4 +
 720*x^3 + 1080*x^2 + 810*x + 243) - 2689/3001250*(3*x^2 + 2)^(5/2)/(16*x^
4 + 96*x^3 + 216*x^2 + 216*x + 81) - 24201/105043750*(3*x^2 + 2)^(5/2)/(8*
x^3 + 36*x^2 + 54*x + 27) - 250077/3676531250*(3*x^2 + 2)^(5/2)/(4*x^2 + 1
2*x + 9) + 653427/7353062500*sqrt(3*x^2 + 2)*x + 72603/3676531250*sqrt(35)
*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3)) + 72603/18
38265625*sqrt(3*x^2 + 2) - 2831517/14706125000*(3*x^2 + 2)^(3/2)/(2*x + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(126) = 252.

Time = 0.15 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.67

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^8}\,dx = \frac{72603}{3676531250}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3x^2+2}}\right)$$
$$-\frac{9\left(258144\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^{13}+5033808\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^{12}+225898166\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^{11}\right.}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^8,x, algorithm="giac")
```

output
```
72603/3676531250*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2
*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))
) - 9/3361400000*(258144*(sqrt(3)*x - sqrt(3*x^2 + 2))^13 + 5033808*sqrt(3
)*(sqrt(3)*x - sqrt(3*x^2 + 2))^12 + 225898166*(sqrt(3)*x - sqrt(3*x^2 + 2
))^11 + 26360013*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^10 + 555459995*(sqr
t(3)*x - sqrt(3*x^2 + 2))^9 - 2679767547*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 +
 2))^8 - 4252091247*(sqrt(3)*x - sqrt(3*x^2 + 2))^7 - 6029804778*sqrt(3)*(
sqrt(3)*x - sqrt(3*x^2 + 2))^6 + 11677158028*(sqrt(3)*x - sqrt(3*x^2 + 2))
^5 - 7324195080*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^4 + 2245361152*(sqrt
(3)*x - sqrt(3*x^2 + 2))^3 - 675266496*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2
))^2 + 174039168*sqrt(3)*x - 6049536*sqrt(3) - 174039168*sqrt(3*x^2 + 2))/
((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 3*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))
 - 2)^7
```

Mupad [B] (verification not implemented)

Time = 5.97 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.78

$$\int \frac{(5-x)\left(2+3x^2\right)^{3/2}}{(3+2x)^8}\,dx = \frac{72603\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{3676531250} - \frac{72603\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{3676531250} + \frac{92453\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{21952000\left(x^4+6x^3+\frac{27x^2}{2}+\frac{27x}{2}+\frac{81}{16}\right)} - \frac{507\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{19600\left(x^5+\frac{15x^4}{2}+\frac{45x^3}{2}+\frac{135x^2}{4}+\frac{405x}{16}+\frac{243}{32}\right)} - \frac{212679\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{3361400000\left(x+\frac{3}{2}\right)} + \frac{125\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{2688\left(x^6+9x^5+\frac{135x^4}{4}+\frac{135x^3}{2}+\frac{1215x^2}{16}+\frac{729x}{16}+\frac{729}{64}\right)} + \frac{3897\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{192080000\left(x^2+3x+\frac{9}{4}\right)} - \frac{65\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{2048\left(x^7+\frac{21x^6}{2}+\frac{189x^5}{4}+\frac{945x^4}{8}+\frac{2835x^3}{16}+\frac{5103x^2}{32}+\frac{5103x}{64}+\frac{2187}{128}\right)} + \frac{7569\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{54880000\left(x^3+\frac{9x^2}{2}+\frac{27x}{4}+\frac{27}{8}\right)}$$

input

```
int(-((3*x^2 + 2)^(3/2)*(x - 5))/(2*x + 3)^8,x)
```

output
```
(72603*35^(1/2)*log(x + 3/2))/3676531250 - (72603*35^(1/2)*log(x - (3^(1/2
)*35^(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/3676531250 + (92453*3^(1/2)*(x^2 +
 2/3)^(1/2))/(21952000*((27*x)/2 + (27*x^2)/2 + 6*x^3 + x^4 + 81/16)) - (5
07*3^(1/2)*(x^2 + 2/3)^(1/2))/(19600*((405*x)/16 + (135*x^2)/4 + (45*x^3)/
2 + (15*x^4)/2 + x^5 + 243/32)) - (212679*3^(1/2)*(x^2 + 2/3)^(1/2))/(3361
400000*(x + 3/2)) + (125*3^(1/2)*(x^2 + 2/3)^(1/2))/(2688*((729*x)/16 + (1
215*x^2)/16 + (135*x^3)/2 + (135*x^4)/4 + 9*x^5 + x^6 + 729/64)) + (3897*3
^(1/2)*(x^2 + 2/3)^(1/2))/(192080000*(3*x + x^2 + 9/4)) - (65*3^(1/2)*(x^2
 + 2/3)^(1/2))/(2048*((5103*x)/64 + (5103*x^2)/32 + (2835*x^3)/16 + (945*x
^4)/8 + (189*x^5)/4 + (21*x^6)/2 + x^7 + 2187/128)) + (7569*3^(1/2)*(x^2 +
 2/3)^(1/2))/(54880000*((27*x)/4 + (9*x^2)/2 + x^3 + 27/8))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.69

$$\int \frac{(5-x)\,(2+3x^2)^{3/2}}{(3+2x)^8}\,dx = \frac{-178650360\sqrt{3x^2+2}\,x^6 - 1550567340\sqrt{3x^2+2}\,x^5 - 5210380350\sqrt{3x^2+2}\,x}{}$$

input
```
int((5-x)*(3*x^2+2)^(3/2)/(3+2*x)^8,x)
```

output
```
( - 178650360*sqrt(3*x**2 + 2)*x**6 - 1550567340*sqrt(3*x**2 + 2)*x**5 - 5
210380350*sqrt(3*x**2 + 2)*x**4 + 3458350875*sqrt(3*x**2 + 2)*x**3 - 25901
092350*sqrt(3*x**2 + 2)*x**2 - 8973613915*sqrt(3*x**2 + 2)*x - 16488609060
*sqrt(3*x**2 + 2) + 55759104*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x
- 4)*x**7 + 585470592*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x*
*6 + 2634617664*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**5 + 6
586544160*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**4 + 9879816
240*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**3 + 8891834616*sq
rt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**2 + 4445917308*sqrt(35)
*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x + 952696566*sqrt(35)*log(sqrt(
3*x**2 + 2)*sqrt(35) + 9*x - 4) - 55759104*sqrt(35)*log(2*x + 3)*x**7 - 58
5470592*sqrt(35)*log(2*x + 3)*x**6 - 2634617664*sqrt(35)*log(2*x + 3)*x**5
 - 6586544160*sqrt(35)*log(2*x + 3)*x**4 - 9879816240*sqrt(35)*log(2*x + 3
)*x**3 - 8891834616*sqrt(35)*log(2*x + 3)*x**2 - 4445917308*sqrt(35)*log(2
*x + 3)*x - 952696566*sqrt(35)*log(2*x + 3))/(22059187500*(128*x**7 + 1344
*x**6 + 6048*x**5 + 15120*x**4 + 22680*x**3 + 20412*x**2 + 10206*x + 2187)
)
```

3.220 $\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{5/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 154

$$\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{5/2} \, dx = \frac{4991}{12}x\sqrt{2+3x^2} + \frac{4991}{36}x\left(2+3x^2\right)^{3/2} + \frac{4991}{90}x\left(2+3x^2\right)^{5/2} + \frac{6433(3+2x)^2 \left(2+3x^2\right)^{7/2}}{4455} + \frac{49}{165}(3+2x)^3 \left(2+3x^2\right)^{7/2} - \frac{1}{33}(3+2x)^4 \left(2+3x^2\right)^{7/2} + \frac{2(181243+62244x)\left(2+3x^2\right)^{7/2}}{13365} + \frac{4991\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{6\sqrt{3}}$$

output
```
4991/12*x*(3*x^2+2)^(1/2)+4991/36*x*(3*x^2+2)^(3/2)+4991/90*x*(3*x^2+2)^(5
/2)+6433/4455*(3+2*x)^2*(3*x^2+2)^(7/2)+49/165*(3+2*x)^3*(3*x^2+2)^(7/2)-1
/33*(3+2*x)^4*(3*x^2+2)^(7/2)+2/13365*(181243+62244*x)*(3*x^2+2)^(7/2)+499
1/18*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.62

$$\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{5/2} dx =$$
$$-\frac{\sqrt{2+3x^2}(-19537120-64370295x-92160240x^2-127123425x^3-150762600x^4-129966606x^5-936}{53460}$$
$$-\frac{4991 \log \left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{6\sqrt{3}}$$

input
```
Integrate[(5 - x)*(3 + 2*x)^4*(2 + 3*x^2)^(5/2),x]
```

output
```
-1/53460*(Sqrt[2 + 3*x^2]*(-19537120 - 64370295*x - 92160240*x^2 - 1271234
25*x^3 - 150762600*x^4 - 129966606*x^5 - 93646260*x^6 - 50615928*x^7 - 129
21120*x^8 + 769824*x^9 + 699840*x^10)) - (4991*Log[-(Sqrt[3]*x) + Sqrt[2 +
 3*x^2]])/(6*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {687, 27, 687, 27, 687, 676, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-x)(2x+3)^4 \left(3x^2+2\right)^{5/2} dx$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{33}\int 7(2x+3)^3(42x+73)\left(3x^2+2\right)^{5/2}dx-\frac{1}{33}(2x+3)^4\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{33}\int (2x+3)^3(42x+73)\left(3x^2+2\right)^{5/2}dx-\frac{1}{33}(2x+3)^4\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \text{687}$$

$$\frac{7}{33}\left(\frac{1}{30}\int 6(2x+3)^2(919x+1011)\left(3x^2+2\right)^{5/2}dx+\frac{7}{5}(2x+3)^3\left(3x^2+2\right)^{7/2}\right)-\frac{1}{33}(2x+3)^4\left(3x^2+2\right)^{7/2}$$

$\Big\downarrow$ 27

$$\frac{7}{33}\left(\frac{1}{5}\int (2x+3)^2(919x+1011)\left(3x^2+2\right)^{5/2}dx+\frac{7}{5}(2x+3)^3\left(3x^2+2\right)^{7/2}\right)-\frac{1}{33}(2x+3)^4\left(3x^2+2\right)^{7/2}$$

$\Big\downarrow$ 687

$$\frac{7}{33}\left(\frac{1}{5}\left(\frac{1}{27}\int (2x+3)(71136x+74539)\left(3x^2+2\right)^{5/2}dx+\frac{919}{27}(2x+3)^2\left(3x^2+2\right)^{7/2}\right)+\frac{7}{5}(2x+3)^3\left(3x^2+2\right)^{7\ldots}\right.$$
$$\frac{1}{33}(2x+3)^4\left(3x^2+2\right)^{7/2}$$

$\Big\downarrow$ 676

$$\frac{7}{33}\left(\frac{1}{5}\left(\frac{1}{27}\left(211761\int \left(3x^2+2\right)^{5/2}dx+5928x\left(3x^2+2\right)^{7/2}+\frac{362486}{21}\left(3x^2+2\right)^{7/2}\right)+\frac{919}{27}(2x+3)^2\left(3x^2+2\right)^{7\ldots}\right.\right.$$
$$\frac{1}{33}(2x+3)^4\left(3x^2+2\right)^{7/2}$$

$\Big\downarrow$ 211

$$\frac{7}{33}\left(\frac{1}{5}\left(\frac{1}{27}\left(211761\left(\frac{5}{3}\int \left(3x^2+2\right)^{3/2}dx+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)+5928x\left(3x^2+2\right)^{7/2}+\frac{362486}{21}\left(3x^2+2\right)^{7/2}\right)+\ldots\right.\right.$$
$$\frac{1}{33}(2x+3)^4\left(3x^2+2\right)^{7/2}$$

$\Big\downarrow$ 211

$$\frac{7}{33}\left(\frac{1}{5}\left(\frac{1}{27}\left(211761\left(\frac{5}{3}\left(\frac{3}{2}\int \sqrt{3x^2+2}dx+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)+5928x\left(3x^2+2\right)^{7/2}+\frac{3624\ldots}{21}\right.\right.\right.$$
$$\frac{1}{33}(2x+3)^4\left(3x^2+2\right)^{7/2}$$

$\Big\downarrow$ 211

$$\frac{7}{33}\left(\frac{1}{5}\left(\frac{1}{27}\left(211761\left(\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)+5928x(\ldots\right.\right.\right.$$
$$\frac{1}{33}(2x+3)^4\left(3x^2+2\right)^{7/2}$$

$\Big\downarrow$ 222

$$\frac{7}{33}\left(\frac{1}{5}\left(\frac{1}{27}\left(211761\left(\frac{5}{3}\left(\frac{3}{2}\left(\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)+5928\right.\right.\right.$$
$$\frac{1}{33}(2x+3)^4\left(3x^2+2\right)^{7/2}$$

input
```
Int[(5 - x)*(3 + 2*x)^4*(2 + 3*x^2)^(5/2),x]
```

output
```
-1/33*((3 + 2*x)^4*(2 + 3*x^2)^(7/2)) + (7*((7*(3 + 2*x)^3*(2 + 3*x^2)^(7/
2))/5 + ((919*(3 + 2*x)^2*(2 + 3*x^2)^(7/2))/27 + ((362486*(2 + 3*x^2)^(7/
2))/21 + 5928*x*(2 + 3*x^2)^(7/2) + 211761*((x*(2 + 3*x^2)^(5/2))/6 + (5*(
(x*(2 + 3*x^2)^(3/2))/4 + (3*((x*Sqrt[2 + 3*x^2])/2 + ArcSinh[Sqrt[3/2]*x]
/Sqrt[3]))/2))/3))/27)/5))/33
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.49

method	result
risch	$-\frac{(699840x^{10}+769824x^9-12921120x^8-50615928x^7-93646260x^6-129966606x^5-150762600x^4-127123425x^3-92160240x^2-64}{53460}$
trager	$\left(-\frac{144}{11}x^{10}-\frac{72}{5}x^9+\frac{7976}{33}x^8+\frac{4734}{5}x^7+\frac{173419}{99}x^6+\frac{24311}{10}x^5+\frac{279190}{99}x^4+\frac{28535}{12}x^3+\frac{1536004}{891}x^2+\frac{1444}{12}\right.$
default	$\frac{4991x(3x^2+2)^{\frac{5}{2}}}{90}+\frac{4991x(3x^2+2)^{\frac{3}{2}}}{36}+\frac{4991x\sqrt{3x^2+2}}{12}+\frac{4991\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{18}+\frac{122107(3x^2+2)^{\frac{7}{2}}}{2673}+\frac{542x(3x^2+2)^{\frac{7}{2}}}{15}$
meijerg	$-\frac{2025\sqrt{3}\left(-\frac{8\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\left(\frac{3}{8}x^4+\frac{13}{16}x^2+\frac{11}{16}\right)\sqrt{\frac{3x^2}{2}+1}}{15}-\frac{\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{3}\right)}{2\sqrt{\pi}}+\frac{160\sqrt{3}\left(\frac{\sqrt{6}\,\sqrt{\pi}\,x\left(-648x^8-1134x^6-558x^4-15x^2+\right.}{2400}\right.}{9\sqrt{\pi}}$

input
```
int((5-x)*(2*x+3)^4*(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/53460*(699840*x^10+769824*x^9-12921120*x^8-50615928*x^7-93646260*x^6-12
9966606*x^5-150762600*x^4-127123425*x^3-92160240*x^2-64370295*x-19537120)*
(3*x^2+2)^(1/2)+4991/18*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.58

$$\int (5-x)(3+2x)^4\left(2+3x^2\right)^{5/2}\,dx =$$

$$-\frac{1}{53460}\left(699840\,x^{10}+769824\,x^9-12921120\,x^8-50615928\,x^7-93646260\,x^6-129966606\,x^5-150762\right.$$

$$+\frac{4991}{36}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3+2*x)^4*(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
-1/53460*(699840*x^10 + 769824*x^9 - 12921120*x^8 - 50615928*x^7 - 9364626
0*x^6 - 129966606*x^5 - 150762600*x^4 - 127123425*x^3 - 92160240*x^2 - 643
70295*x - 19537120)*sqrt(3*x^2 + 2) + 4991/36*sqrt(3)*log(-sqrt(3)*sqrt(3*
x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.29

$$\int (5-x)(3+2x)^4\left(2+3x^2\right)^{5/2}\,dx = -\frac{144x^{10}\sqrt{3x^2+2}}{11} - \frac{72x^9\sqrt{3x^2+2}}{5} + \frac{7976x^8\sqrt{3x^2+2}}{33} + \frac{4734x^7\sqrt{3x^2+2}}{5} + \frac{173419x^6\sqrt{3x^2+2}}{99} + \frac{24311x^5\sqrt{3x^2+2}}{10} + \frac{279190x^4\sqrt{3x^2+2}}{99} + \frac{28535x^3\sqrt{3x^2+2}}{12} + \frac{1536004x^2\sqrt{3x^2+2}}{891} + \frac{14449x\sqrt{3x^2+2}}{12} + \frac{976856\sqrt{3x^2+2}}{2673} + \frac{4991\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{18}$$

input
```
integrate((5-x)*(3+2*x)**4*(3*x**2+2)**(5/2),x)
```

output
```
-144*x**10*sqrt(3*x**2 + 2)/11 - 72*x**9*sqrt(3*x**2 + 2)/5 + 7976*x**8*sq
rt(3*x**2 + 2)/33 + 4734*x**7*sqrt(3*x**2 + 2)/5 + 173419*x**6*sqrt(3*x**2
 + 2)/99 + 24311*x**5*sqrt(3*x**2 + 2)/10 + 279190*x**4*sqrt(3*x**2 + 2)/9
9 + 28535*x**3*sqrt(3*x**2 + 2)/12 + 1536004*x**2*sqrt(3*x**2 + 2)/891 + 1
4449*x*sqrt(3*x**2 + 2)/12 + 976856*sqrt(3*x**2 + 2)/2673 + 4991*sqrt(3)*a
sinh(sqrt(6)*x/2)/18
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.74

$$\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{5/2} dx = -\frac{16}{33}\left(3x^2+2\right)^{\frac{7}{2}}x^4 - \frac{8}{15}\left(3x^2+2\right)^{\frac{7}{2}}x^3 + \frac{8840}{891}\left(3x^2+2\right)^{\frac{7}{2}}x^2 + \frac{542}{15}\left(3x^2+2\right)^{\frac{7}{2}}x + \frac{122107}{2673}\left(3x^2+2\right)^{\frac{7}{2}} + \frac{4991}{90}\left(3x^2+2\right)^{\frac{5}{2}}x + \frac{4991}{36}\left(3x^2+2\right)^{\frac{3}{2}}x + \frac{4991}{12}\sqrt{3x^2+2}x + \frac{4991}{18}\sqrt{3}\,\text{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)$$

input
```
integrate((5-x)*(3+2*x)^4*(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
-16/33*(3*x^2 + 2)^(7/2)*x^4 - 8/15*(3*x^2 + 2)^(7/2)*x^3 + 8840/891*(3*x^
2 + 2)^(7/2)*x^2 + 542/15*(3*x^2 + 2)^(7/2)*x + 122107/2673*(3*x^2 + 2)^(7
/2) + 4991/90*(3*x^2 + 2)^(5/2)*x + 4991/36*(3*x^2 + 2)^(3/2)*x + 4991/12*
sqrt(3*x^2 + 2)*x + 4991/18*sqrt(3)*arcsinh(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.53

$$\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{5/2} dx = -\frac{1}{53460}(3((9(2((2(6(4(27(10x+11)x-4985)x-78111)x-867095)x-2406789)x-2791900)x-4 - \frac{4991}{18}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)$$

input
```
integrate((5-x)*(3+2*x)^4*(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
-1/53460*(3*((9*(2*((2*(6*(4*(27*(10*x + 11)*x - 4985)*x - 78111)*x - 8670
95)*x - 2406789)*x - 2791900)*x - 4708275)*x - 30720080)*x - 21456765)*x -
 19537120)*sqrt(3*x^2 + 2) - 4991/18*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 +
 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.49

$$\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{5/2} \, dx = \frac{4991\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{18} + \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(-\frac{432\,x^{10}}{11}-\frac{216\,x^9}{5}+\frac{7976\,x^8}{11}+\frac{14202\,x^7}{5}+\frac{173419\,x^6}{33}+\frac{72933\,x^5}{10}+\frac{279190\,x^4}{33}+\frac{28535\,x^3}{4}+\frac{1536004\,x^2}{297}+\right.}{3}$$

input
```
int(-(2*x + 3)^4*(3*x^2 + 2)^(5/2)*(x - 5),x)
```

output
```
(4991*3^(1/2)*asinh((6^(1/2)*x)/2))/18 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((1444
9*x)/4 + (1536004*x^2)/297 + (28535*x^3)/4 + (279190*x^4)/33 + (72933*x^5)
/10 + (173419*x^6)/33 + (14202*x^7)/5 + (7976*x^8)/11 - (216*x^9)/5 - (432
*x^10)/11 + 976856/891))/3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05

$$\int (5-x)(3+2x)^4 \left(2+3x^2\right)^{5/2} \, dx = -\frac{144\sqrt{3x^2+2}\,x^{10}}{11} - \frac{72\sqrt{3x^2+2}\,x^9}{5} + \frac{7976\sqrt{3x^2+2}\,x^8}{33} + \frac{4734\sqrt{3x^2+2}\,x^7}{5} + \frac{173419\sqrt{3x^2+2}\,x^6}{99} + \frac{24311\sqrt{3x^2+2}\,x^5}{10} + \frac{279190\sqrt{3x^2+2}\,x^4}{99} + \frac{28535\sqrt{3x^2+2}\,x^3}{12} + \frac{1536004\sqrt{3x^2+2}\,x^2}{891} + \frac{14449\sqrt{3x^2+2}\,x}{12} + \frac{976856\sqrt{3x^2+2}}{2673} + \frac{4991\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{18}$$

input
```
int((5-x)*(3+2*x)^4*(3*x^2+2)^(5/2),x)
```

output
```
( - 699840*sqrt(3*x**2 + 2)*x**10 - 769824*sqrt(3*x**2 + 2)*x**9 + 1292112
0*sqrt(3*x**2 + 2)*x**8 + 50615928*sqrt(3*x**2 + 2)*x**7 + 93646260*sqrt(3
*x**2 + 2)*x**6 + 129966606*sqrt(3*x**2 + 2)*x**5 + 150762600*sqrt(3*x**2
+ 2)*x**4 + 127123425*sqrt(3*x**2 + 2)*x**3 + 92160240*sqrt(3*x**2 + 2)*x*
*2 + 64370295*sqrt(3*x**2 + 2)*x + 19537120*sqrt(3*x**2 + 2) + 14823270*sq
rt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/53460
```

3.221 $\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{5/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{5/2} \, dx = \frac{3731}{24} x\sqrt{2+3x^2} + \frac{3731}{72} x\left(2+3x^2\right)^{3/2} + \frac{3731}{180} x\left(2+3x^2\right)^{5/2} + \frac{91}{270}(3+2x)^2 \left(2+3x^2\right)^{7/2} - \frac{1}{30}(3+2x)^3 \left(2+3x^2\right)^{7/2} + \frac{(15244+4977x)\left(2+3x^2\right)^{7/2}}{1620} + \frac{3731\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{12\sqrt{3}}$$

output
```
3731/24*x*(3*x^2+2)^(1/2)+3731/72*x*(3*x^2+2)^(3/2)+3731/180*x*(3*x^2+2)^(
5/2)+91/270*(3+2*x)^2*(3*x^2+2)^(7/2)-1/30*(3+2*x)^3*(3*x^2+2)^(7/2)+1/162
0*(15244+4977*x)*(3*x^2+2)^(7/2)+3731/36*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.69

$$\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{5/2} \, dx = -\frac{\sqrt{2+3x^2}(-299200-1245915x-1350240x^2-1922805x^3-2036880x^4-1503522x^5-1035720x^6-4}{3240} - \frac{3731\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{12\sqrt{3}}$$

input
```
Integrate[(5 - x)*(3 + 2*x)^3*(2 + 3*x^2)^(5/2),x]
```

output
```
-1/3240*(Sqrt[2 + 3*x^2]*(-299200 - 1245915*x - 1350240*x^2 - 1922805*x^3
- 2036880*x^4 - 1503522*x^5 - 1035720*x^6 - 418446*x^7 - 12960*x^8 + 23328
*x^9)) - (3731*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(12*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {687, 27, 687, 27, 676, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-x)(2x+3)^3\left(3x^2+2\right)^{5/2}dx$$

$$\Big\downarrow \ 687$$

$$\frac{1}{30}\int 21(2x+3)^2(13x+22)\left(3x^2+2\right)^{5/2}dx-\frac{1}{30}(2x+3)^3\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \ 27$$

$$\frac{7}{10}\int (2x+3)^2(13x+22)\left(3x^2+2\right)^{5/2}dx-\frac{1}{30}(2x+3)^3\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \ 687$$

$$\frac{7}{10}\left(\frac{1}{27}\int 2(2x+3)(711x+839)\left(3x^2+2\right)^{5/2}dx+\frac{13}{27}(2x+3)^2\left(3x^2+2\right)^{7/2}\right)-\frac{1}{30}(2x+3)^3\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \ 27$$

$$\frac{7}{10}\left(\frac{2}{27}\int (2x+3)(711x+839)\left(3x^2+2\right)^{5/2}dx+\frac{13}{27}(2x+3)^2\left(3x^2+2\right)^{7/2}\right)-\frac{1}{30}(2x+3)^3\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \ 676$$

$$\frac{7}{10}\left(\frac{2}{27}\left(\frac{4797}{2}\int \left(3x^2+2\right)^{5/2}dx+\frac{237}{4}x\left(3x^2+2\right)^{7/2}+\frac{3811}{21}\left(3x^2+2\right)^{7/2}\right)+\frac{13}{27}(2x+3)^2\left(3x^2+2\right)^{7/2}\right)-\frac{1}{30}(2x+3)^3\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \; 211$$

$$\frac{7}{10}\left(\frac{2}{27}\left(\frac{4797}{2}\left(\frac{5}{3}\int \left(3x^2+2\right)^{3/2}dx+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)+\frac{237}{4}x\left(3x^2+2\right)^{7/2}+\frac{3811}{21}\left(3x^2+2\right)^{7/2}\right)+\frac{13}{27}(2x+\right.$$
$$\frac{1}{30}(2x+3)^3\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \; 211$$

$$\frac{7}{10}\left(\frac{2}{27}\left(\frac{4797}{2}\left(\frac{5}{3}\left(\frac{3}{2}\int \sqrt{3x^2+2}dx+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)+\frac{237}{4}x\left(3x^2+2\right)^{7/2}+\frac{3811}{21}\left(3x^2\right.\right.\right.$$
$$\frac{1}{30}(2x+3)^3\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \; 211$$

$$\frac{7}{10}\left(\frac{2}{27}\left(\frac{4797}{2}\left(\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)+\frac{237}{4}x\left(3x^2+\right.\right.\right.$$
$$\frac{1}{30}(2x+3)^3\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \; 222$$

$$\frac{7}{10}\left(\frac{2}{27}\left(\frac{4797}{2}\left(\frac{5}{3}\left(\frac{3}{2}\left(\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)+\frac{237}{4}x\left(3x^2\right.\right.\right.$$
$$\frac{1}{30}(2x+3)^3\left(3x^2+2\right)^{7/2}$$

input
```
Int[(5 - x)*(3 + 2*x)^3*(2 + 3*x^2)^(5/2),x]
```

output
```
-1/30*((3 + 2*x)^3*(2 + 3*x^2)^(7/2)) + (7*((13*(3 + 2*x)^2*(2 + 3*x^2)^(7
/2))/27 + (2*((3811*(2 + 3*x^2)^(7/2))/21 + (237*x*(2 + 3*x^2)^(7/2))/4 +
(4797*((x*(2 + 3*x^2)^(5/2))/6 + (5*((x*(2 + 3*x^2)^(3/2))/4 + (3*((x*Sqrt
[2 + 3*x^2])/2 + ArcSinh[Sqrt[3/2]*x]/Sqrt[3]))/2))/3))/2))/27))/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

method	result
risch	$-\frac{(23328x^9-12960x^8-418446x^7-1035720x^6-1503522x^5-2036880x^4-1922805x^3-1350240x^2-1245915x-299200)\sqrt{3x^2+2}}{3240}+$
trager	$\left(-\frac{36}{5}x^9+4x^8+\frac{2583}{20}x^7+\frac{959}{3}x^6+\frac{9281}{20}x^5+\frac{1886}{3}x^4+\frac{14243}{24}x^3+\frac{11252}{27}x^2+\frac{9229}{24}x+\frac{7480}{81}\right)\sqrt{3x^2+}$
default	$\frac{3731x(3x^2+2)^{\frac{5}{2}}}{180}+\frac{3731x(3x^2+2)^{\frac{3}{2}}}{72}+\frac{3731x\sqrt{3x^2+2}}{24}+\frac{3731\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{36}+\frac{935(3x^2+2)^{\frac{7}{2}}}{81}+\frac{319x(3x^2+2)^{\frac{7}{2}}}{60}+$
meijerg	$-\frac{675\sqrt{3}\left(-\frac{8\sqrt{\pi}\,x\sqrt{2}\sqrt{3}\left(\frac{3}{8}x^4+\frac{13}{16}x^2+\frac{11}{16}\right)\sqrt{\frac{3x^2}{2}+1}}{15}-\frac{\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\sqrt{3}}{2}\right)}{3}\right)}{2\sqrt{\pi}}-\frac{20\sqrt{2}\left(-\frac{32\sqrt{\pi}}{945}+\frac{4\sqrt{\pi}\left(-\frac{567}{4}x^8-\frac{513}{2}x^6-135x^4-6x\right.}{945}\right.}{3\sqrt{\pi}}$

input
```
int((5-x)*(2*x+3)^3*(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3240*(23328*x^9-12960*x^8-418446*x^7-1035720*x^6-1503522*x^5-2036880*x^
4-1922805*x^3-1350240*x^2-1245915*x-299200)*(3*x^2+2)^(1/2)+3731/36*arcsin
h(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.64

$$\int (5-x)(3+2x)^3\left(2+3x^2\right)^{5/2}\,dx =$$
$$-\frac{1}{3240}\left(23328\,x^9-12960\,x^8-418446\,x^7-1035720\,x^6-1503522\,x^5-2036880\,x^4-1922805\,x^3-13502\right.$$
$$+\frac{3731}{72}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3+2*x)^3*(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3240*(23328*x^9 - 12960*x^8 - 418446*x^7 - 1035720*x^6 - 1503522*x^5 -
2036880*x^4 - 1922805*x^3 - 1350240*x^2 - 1245915*x - 299200)*sqrt(3*x^2 +
 2) + 3731/72*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 6.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.36

$$\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{5/2} \, dx = -\frac{36x^9\sqrt{3x^2+2}}{5} + 4x^8\sqrt{3x^2+2} + \frac{2583x^7\sqrt{3x^2+2}}{20} + \frac{959x^6\sqrt{3x^2+2}}{3} + \frac{9281x^5\sqrt{3x^2+2}}{20} + \frac{1886x^4\sqrt{3x^2+2}}{3} + \frac{14243x^3\sqrt{3x^2+2}}{24} + \frac{11252x^2\sqrt{3x^2+2}}{27} + \frac{9229x\sqrt{3x^2+2}}{24} + \frac{7480\sqrt{3x^2+2}}{81} + \frac{3731\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{36}$$

input
```
integrate((5-x)*(3+2*x)**3*(3*x**2+2)**(5/2),x)
```

output
```
-36*x**9*sqrt(3*x**2 + 2)/5 + 4*x**8*sqrt(3*x**2 + 2) + 2583*x**7*sqrt(3*x
**2 + 2)/20 + 959*x**6*sqrt(3*x**2 + 2)/3 + 9281*x**5*sqrt(3*x**2 + 2)/20
+ 1886*x**4*sqrt(3*x**2 + 2)/3 + 14243*x**3*sqrt(3*x**2 + 2)/24 + 11252*x*
*2*sqrt(3*x**2 + 2)/27 + 9229*x*sqrt(3*x**2 + 2)/24 + 7480*sqrt(3*x**2 + 2
)/81 + 3731*sqrt(3)*asinh(sqrt(6)*x/2)/36
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{5/2} \, dx = -\frac{4}{15}\left(3x^2+2\right)^{\frac{7}{2}}x^3 + \frac{4}{27}\left(3x^2+2\right)^{\frac{7}{2}}x^2 + \frac{319}{60}\left(3x^2+2\right)^{\frac{7}{2}}x + \frac{935}{81}\left(3x^2+2\right)^{\frac{7}{2}} + \frac{3731}{180}\left(3x^2+2\right)^{\frac{5}{2}}x + \frac{3731}{72}\left(3x^2+2\right)^{\frac{3}{2}}x + \frac{3731}{24}\sqrt{3x^2+2}x + \frac{3731}{36}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)$$

input
```
integrate((5-x)*(3+2*x)^3*(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
-4/15*(3*x^2 + 2)^(7/2)*x^3 + 4/27*(3*x^2 + 2)^(7/2)*x^2 + 319/60*(3*x^2 +
 2)^(7/2)*x + 935/81*(3*x^2 + 2)^(7/2) + 3731/180*(3*x^2 + 2)^(5/2)*x + 37
31/72*(3*x^2 + 2)^(3/2)*x + 3731/24*sqrt(3*x^2 + 2)*x + 3731/36*sqrt(3)*ar
csinh(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

$$\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{5/2} \, dx = -\frac{1}{3240} (3 ((9 (2 (((3 (16 (9 x - 5)x - 2583)x - 19180)x - 27843)x - 37720)x - 71215)x - 450080)x - 41 - \frac{3731}{36} \sqrt{3} \log \left(-\sqrt{3}x + \sqrt{3 x^2 + 2}\right)$$

input
```
integrate((5-x)*(3+2*x)^3*(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
-1/3240*(3*((9*(2*(((3*(16*(9*x - 5)*x - 2583)*x - 19180)*x - 27843)*x - 3
7720)*x - 71215)*x - 450080)*x - 415305)*x - 299200)*sqrt(3*x^2 + 2) - 373
1/36*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

$$\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{5/2} \, dx = \frac{3731 \sqrt{3} \operatorname{asinh}\left(\frac{\sqrt{6} x}{2}\right)}{36} + \frac{\sqrt{3} \sqrt{x^2+\frac{2}{3}} \left(-\frac{108 x^9}{5} + 12 x^8 + \frac{7749 x^7}{20} + 959 x^6 + \frac{27843 x^5}{20} + 1886 x^4 + \frac{14243 x^3}{8} + \frac{11252 x^2}{9} + \frac{9229 x}{8} + \frac{7480}{27}\right)}{3}$$

input
```
int(-(2*x + 3)^3*(3*x^2 + 2)^(5/2)*(x - 5),x)
```

output
```
(3731*3^(1/2)*asinh((6^(1/2)*x)/2))/36 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((9229
*x)/8 + (11252*x^2)/9 + (14243*x^3)/8 + 1886*x^4 + (27843*x^5)/20 + 959*x^
6 + (7749*x^7)/20 + 12*x^8 - (108*x^9)/5 + 7480/27))/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.13

$$\int (5-x)(3+2x)^3 \left(2+3x^2\right)^{5/2} \, dx = -\frac{36\sqrt{3x^2+2}\,x^9}{5} + 4\sqrt{3x^2+2}\,x^8 + \frac{2583\sqrt{3x^2+2}\,x^7}{20} + \frac{959\sqrt{3x^2+2}\,x^6}{3} + \frac{9281\sqrt{3x^2+2}\,x^5}{20} + \frac{1886\sqrt{3x^2+2}\,x^4}{3} + \frac{14243\sqrt{3x^2+2}\,x^3}{24} + \frac{11252\sqrt{3x^2+2}\,x^2}{27} + \frac{9229\sqrt{3x^2+2}\,x}{24} + \frac{7480\sqrt{3x^2+2}}{81} + \frac{3731\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{36}$$

input
```
int((5-x)*(3+2*x)^3*(3*x^2+2)^(5/2),x)
```

output
```
( - 23328*sqrt(3*x**2 + 2)*x**9 + 12960*sqrt(3*x**2 + 2)*x**8 + 418446*sqr
t(3*x**2 + 2)*x**7 + 1035720*sqrt(3*x**2 + 2)*x**6 + 1503522*sqrt(3*x**2 +
 2)*x**5 + 2036880*sqrt(3*x**2 + 2)*x**4 + 1922805*sqrt(3*x**2 + 2)*x**3 +
 1350240*sqrt(3*x**2 + 2)*x**2 + 1245915*sqrt(3*x**2 + 2)*x + 299200*sqrt(
3*x**2 + 2) + 335790*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/
3240
```

3.222 $\int (5-x)(3+2x)^2 \left(2+3x^2\right)^{5/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 110

$$\int (5-x)(3+2x)^2 \left(2+3x^2\right)^{5/2} dx = \frac{665}{12}x\sqrt{2+3x^2} + \frac{665}{36}x\left(2+3x^2\right)^{3/2} + \frac{133}{18}x\left(2+3x^2\right)^{5/2} - \frac{1}{27}(3+2x)^2 \left(2+3x^2\right)^{7/2} + \frac{1}{81}(226+63x) \left(2+3x^2\right)^{7/2} + \frac{665\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{6\sqrt{3}}$$

output
```
665/12*x*(3*x^2+2)^(1/2)+665/36*x*(3*x^2+2)^(3/2)+133/18*x*(3*x^2+2)^(5/2)
-1/27*(3+2*x)^2*(3*x^2+2)^(7/2)+1/81*(226+63*x)*(3*x^2+2)^(7/2)+665/18*arc
sinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int (5-x)(3+2x)^2 \left(2+3x^2\right)^{5/2} dx = -\frac{1}{324}\sqrt{2+3x^2}\left(-6368-40365x-28272x^2-50571x^3-41256x^4 - 27378x^5-18900x^6-2916x^7+1296x^8\right) - \frac{665\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{6\sqrt{3}}$$

input
```
Integrate[(5 - x)*(3 + 2*x)^2*(2 + 3*x^2)^(5/2),x]
```

output
```
-1/324*(Sqrt[2 + 3*x^2]*(-6368 - 40365*x - 28272*x^2 - 50571*x^3 - 41256*x
^4 - 27378*x^5 - 18900*x^6 - 2916*x^7 + 1296*x^8)) - (665*Log[-(Sqrt[3]*x)
 + Sqrt[2 + 3*x^2]])/(6*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {687, 27, 676, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-x)(2x+3)^2 \left(3x^2+2\right)^{5/2} \, dx$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{27} \int 7(2x+3)(36x+59) \left(3x^2+2\right)^{5/2} dx - \frac{1}{27}(2x+3)^2 \left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{27} \int (2x+3)(36x+59) \left(3x^2+2\right)^{5/2} dx - \frac{1}{27}(2x+3)^2 \left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \text{676}$$

$$\frac{7}{27} \left(171 \int \left(3x^2+2\right)^{5/2} dx + 3x \left(3x^2+2\right)^{7/2} + \frac{226}{21} \left(3x^2+2\right)^{7/2}\right) - \frac{1}{27}(2x+3)^2 \left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{27} \left(171 \left(\frac{5}{3} \int \left(3x^2+2\right)^{3/2} dx + \frac{1}{6} x \left(3x^2+2\right)^{5/2}\right) + 3x \left(3x^2+2\right)^{7/2} + \frac{226}{21} \left(3x^2+2\right)^{7/2}\right) - \frac{1}{27}(2x+3)^2 \left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{27}\left(171\left(\frac{5}{3}\left(\frac{3}{2}\int \sqrt{3x^2+2}dx+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)+3x\left(3x^2+2\right)^{7/2}+\frac{226}{21}\left(3x^2+2\right)^{7/2}\right)-\frac{1}{27}(2x+3)^2\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \ 211$$

$$\frac{7}{27}\left(171\left(\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)+3x\left(3x^2+2\right)^{7/2}+\frac{2}{\cdot}\right.$$
$$\frac{1}{27}(2x+3)^2\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \ 222$$

$$\frac{7}{27}\left(171\left(\frac{5}{3}\left(\frac{3}{2}\left(\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)+3x\left(3x^2+2\right)^{7/2}\right.$$
$$\frac{1}{27}(2x+3)^2\left(3x^2+2\right)^{7/2}$$

input
```
Int[(5 - x)*(3 + 2*x)^2*(2 + 3*x^2)^(5/2),x]
```

output
```
-1/27*((3 + 2*x)^2*(2 + 3*x^2)^(7/2)) + (7*((226*(2 + 3*x^2)^(7/2))/21 + 3
*x*(2 + 3*x^2)^(7/2) + 171*((x*(2 + 3*x^2)^(5/2))/6 + (5*((x*(2 + 3*x^2)^(
3/2))/4 + (3*((x*Sqrt[2 + 3*x^2])/2 + ArcSinh[Sqrt[3/2]*x]/Sqrt[3]))/2))/3
)))/27
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{(1296x^8-2916x^7-18900x^6-27378x^5-41256x^4-50571x^3-28272x^2-40365x-6368)\sqrt{3x^2+2}}{324}+\frac{665\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{18}$
trager	$\left(-4x^8+9x^7+\frac{175}{3}x^6+\frac{169}{2}x^5+\frac{382}{3}x^4+\frac{1873}{12}x^3+\frac{2356}{27}x^2+\frac{1495}{12}x+\frac{1592}{81}\right)\sqrt{3x^2+2}-\frac{665\,\mathrm{RootOf}(\cdots}{\cdots}$
default	$\frac{133x(3x^2+2)^{\frac{5}{2}}}{18}+\frac{665x(3x^2+2)^{\frac{3}{2}}}{36}+\frac{665x\sqrt{3x^2+2}}{12}+\frac{665\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{18}+\frac{199(3x^2+2)^{\frac{7}{2}}}{81}+\frac{x(3x^2+2)^{\frac{7}{2}}}{3}-\frac{4x^2(3x^2\cdots}{27}$
meijerg	$-\frac{225\sqrt{3}\left(-\frac{8\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\left(\frac{3}{8}x^4+\frac{13}{16}x^2+\frac{11}{16}\right)\sqrt{\frac{3x^2}{2}+1}}{15}-\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{3}\right)}{2\sqrt{\pi}}-\frac{40\sqrt{3}\left(-\frac{\sqrt{6}\,\sqrt{\pi}\,x\left(162x^6+306x^4+177x^2+15\right)\sqrt{\frac{3x^2}{2}\cdots}}{720}\cdots\right.}{3\sqrt{\pi}}$

input
```
int((5-x)*(2*x+3)^2*(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/324*(1296*x^8-2916*x^7-18900*x^6-27378*x^5-41256*x^4-50571*x^3-28272*x^
2-40365*x-6368)*(3*x^2+2)^(1/2)+665/18*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

$$\int (5-x)(3+2x)^2 \left(2+3x^2\right)^{5/2} \, dx = -\frac{1}{324} \left(1296 \, x^{8} - 2916 \, x^{7} - 18900 \, x^{6} - 27378 \, x^{5} - 41256 \, x^{4} - 50571 \, x^{3} - 28272 \, x^{2} - 40365 \, x - 6368\right) \sqrt{3 \, x^{2} + 2} + \frac{665}{36} \, \sqrt{3} \log \left(-\sqrt{3} \sqrt{3 \, x^{2} + 2} x - 3 \, x^{2} - 1\right)$$

input
```
integrate((5-x)*(3+2*x)^2*(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
-1/324*(1296*x^8 - 2916*x^7 - 18900*x^6 - 27378*x^5 - 41256*x^4 - 50571*x^
3 - 28272*x^2 - 40365*x - 6368)*sqrt(3*x^2 + 2) + 665/36*sqrt(3)*log(-sqrt
(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.47

$$\int (5-x)(3+2x)^2 \left(2+3x^2\right)^{5/2} \, dx = -4x^{8}\sqrt{3x^{2} + 2} + 9x^{7}\sqrt{3x^{2} + 2} + \frac{175x^{6}\sqrt{3x^{2} + 2}}{3} + \frac{169x^{5}\sqrt{3x^{2} + 2}}{2} + \frac{382x^{4}\sqrt{3x^{2} + 2}}{3} + \frac{1873x^{3}\sqrt{3x^{2} + 2}}{12} + \frac{2356x^{2}\sqrt{3x^{2} + 2}}{27} + \frac{1495x\sqrt{3x^{2} + 2}}{12} + \frac{1592\sqrt{3x^{2} + 2}}{81} + \frac{665\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{18}$$

input
```
integrate((5-x)*(3+2*x)**2*(3*x**2+2)**(5/2),x)
```

output
```
-4*x**8*sqrt(3*x**2 + 2) + 9*x**7*sqrt(3*x**2 + 2) + 175*x**6*sqrt(3*x**2
+ 2)/3 + 169*x**5*sqrt(3*x**2 + 2)/2 + 382*x**4*sqrt(3*x**2 + 2)/3 + 1873*
x**3*sqrt(3*x**2 + 2)/12 + 2356*x**2*sqrt(3*x**2 + 2)/27 + 1495*x*sqrt(3*x
**2 + 2)/12 + 1592*sqrt(3*x**2 + 2)/81 + 665*sqrt(3)*asinh(sqrt(6)*x/2)/18
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int (5-x)(3+2x)^2 \left(2+3x^2\right)^{5/2} \, dx = -\frac{4}{27} \left(3 x^2+2\right)^{\frac{7}{2}} x^2 + \frac{1}{3} \left(3 x^2+2\right)^{\frac{7}{2}} x + \frac{199}{81} \left(3 x^2+2\right)^{\frac{7}{2}} + \frac{133}{18} \left(3 x^2+2\right)^{\frac{5}{2}} x + \frac{665}{36} \left(3 x^2+2\right)^{\frac{3}{2}} x + \frac{665}{12} \sqrt{3 x^2+2} x + \frac{665}{18} \sqrt{3} \operatorname{arsinh}\left(\frac{1}{2} \sqrt{6} x\right)$$

input
```
integrate((5-x)*(3+2*x)^2*(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
-4/27*(3*x^2 + 2)^(7/2)*x^2 + 1/3*(3*x^2 + 2)^(7/2)*x + 199/81*(3*x^2 + 2)
^(7/2) + 133/18*(3*x^2 + 2)^(5/2)*x + 665/36*(3*x^2 + 2)^(3/2)*x + 665/12*
sqrt(3*x^2 + 2)*x + 665/18*sqrt(3)*arcsinh(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.65

$$\int (5-x)(3+2x)^2 \left(2+3x^2\right)^{5/2} \, dx = -\frac{1}{324} \left(3 \left(\left(9 \left(2 \left(\left(2 \left(3 \left(4 x-9\right) x-175\right) x-507\right) x-764\right) x-1873\right) x-9424\right) x-13455\right) x-6368\right) \sqrt{3 x^2+2} - \frac{665}{18} \sqrt{3} \log\left(-\sqrt{3} x+\sqrt{3 x^2+2}\right)$$

input
```
integrate((5-x)*(3+2*x)^2*(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
-1/324*(3*((9*(2*((2*(3*(4*x - 9)*x - 175)*x - 507)*x - 764)*x - 1873)*x -
 9424)*x - 13455)*x - 6368)*sqrt(3*x^2 + 2) - 665/18*sqrt(3)*log(-sqrt(3)*
x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.59

$$\int (5-x)(3+2x)^2 \left(2+3x^2\right)^{5/2} \, dx = \frac{665\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{18} + \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(-12\,x^8+27\,x^7+175\,x^6+\frac{507\,x^5}{2}+382\,x^4+\frac{1873\,x^3}{4}+\frac{2356\,x^2}{9}+\frac{1495\,x}{4}+\frac{1592}{27}\right)}{3}$$

input
```
int(-(2*x + 3)^2*(3*x^2 + 2)^(5/2)*(x - 5),x)
```

output
```
(665*3^(1/2)*asinh((6^(1/2)*x)/2))/18 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((1495*
x)/4 + (2356*x^2)/9 + (1873*x^3)/4 + 382*x^4 + (507*x^5)/2 + 175*x^6 + 27*
x^7 - 12*x^8 + 1592/27))/3
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.24

$$\int (5-x)(3+2x)^2 \left(2+3x^2\right)^{5/2} \, dx = -4\sqrt{3x^2+2}\,x^8+9\sqrt{3x^2+2}\,x^7 + \frac{175\sqrt{3x^2+2}\,x^6}{3} + \frac{169\sqrt{3x^2+2}\,x^5}{2} + \frac{382\sqrt{3x^2+2}\,x^4}{3} + \frac{1873\sqrt{3x^2+2}\,x^3}{12} + \frac{2356\sqrt{3x^2+2}\,x^2}{27} + \frac{1495\sqrt{3x^2+2}\,x}{12} + \frac{1592\sqrt{3x^2+2}}{81} + \frac{665\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{18}$$

input
```
int((5-x)*(3+2*x)^2*(3*x^2+2)^(5/2),x)
```

output
```
( - 1296*sqrt(3*x**2 + 2)*x**8 + 2916*sqrt(3*x**2 + 2)*x**7 + 18900*sqrt(3
*x**2 + 2)*x**6 + 27378*sqrt(3*x**2 + 2)*x**5 + 41256*sqrt(3*x**2 + 2)*x**
4 + 50571*sqrt(3*x**2 + 2)*x**3 + 28272*sqrt(3*x**2 + 2)*x**2 + 40365*sqrt
(3*x**2 + 2)*x + 6368*sqrt(3*x**2 + 2) + 11970*sqrt(3)*log((sqrt(3*x**2 +
2) + sqrt(3)*x)/sqrt(2)))/324
```

3.223 $\int (5-x)(3+2x)\left(2+3x^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int (5-x)(3+2x)\left(2+3x^2\right)^{5/2}\,dx=\frac{455}{24}x\sqrt{2+3x^2}+\frac{455}{72}x\left(2+3x^2\right)^{3/2}+\frac{91}{36}x\left(2+3x^2\right)^{5/2}+\frac{1}{12}(4-x)\left(2+3x^2\right)^{7/2}+\frac{455\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{12\sqrt{3}}$$

output
```
455/24*x*(3*x^2+2)^(1/2)+455/72*x*(3*x^2+2)^(3/2)+91/36*x*(3*x^2+2)^(5/2)+
1/12*(4-x)*(3*x^2+2)^(7/2)+455/36*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int (5-x)(3+2x)\left(2+3x^2\right)^{5/2}\,dx=-\frac{1}{24}\sqrt{2+3x^2}\left(-64-985x-288x^2-1111x^3-432x^4-438x^5-216x^6+54x^7\right)-\frac{455\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{12\sqrt{3}}$$

input
```
Integrate[(5 - x)*(3 + 2*x)*(2 + 3*x^2)^(5/2),x]
```

output
```
-1/24*(Sqrt[2 + 3*x^2]*(-64 - 985*x - 288*x^2 - 1111*x^3 - 432*x^4 - 438*x
^5 - 216*x^6 + 54*x^7)) - (455*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(12*Sq
rt[3])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {676, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-x)(2x+3)\left(3x^2+2\right)^{5/2}dx$$

$$\Big\downarrow \text{676}$$

$$\frac{91}{6}\int \left(3x^2+2\right)^{5/2}dx-\frac{1}{12}x\left(3x^2+2\right)^{7/2}+\frac{1}{3}\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{91}{6}\left(\frac{5}{3}\int \left(3x^2+2\right)^{3/2}dx+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)-\frac{1}{12}x\left(3x^2+2\right)^{7/2}+\frac{1}{3}\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{91}{6}\left(\frac{5}{3}\left(\frac{3}{2}\int \sqrt{3x^2+2}dx+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)-\frac{1}{12}x\left(3x^2+2\right)^{7/2}+\frac{1}{3}\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{91}{6}\left(\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)-\frac{1}{12}x\left(3x^2+2\right)^{7/2}+\frac{1}{3}\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{91}{6}\left(\frac{5}{3}\left(\frac{3}{2}\left(\frac{\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)-\frac{1}{12}x\left(3x^2+2\right)^{7/2}+\frac{1}{3}\left(3x^2+2\right)^{7/2}$$

input
```
Int[(5 - x)*(3 + 2*x)*(2 + 3*x^2)^(5/2),x]
```

output
```
(2 + 3*x^2)^(7/2)/3 - (x*(2 + 3*x^2)^(7/2))/12 + (91*((x*(2 + 3*x^2)^(5/2)
)/6 + (5*((x*(2 + 3*x^2)^(3/2))/4 + (3*((x*Sqrt[2 + 3*x^2])/2 + ArcSinh[Sq
rt[3/2]*x]/Sqrt[3]))/2))/3))/6
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{(54x^7-216x^6-438x^5-432x^4-1111x^3-288x^2-985x-64)\sqrt{3x^2+2}}{24}+\frac{455\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{36}$
default	$\frac{91x(3x^2+2)^{\frac{5}{2}}}{36}+\frac{455x(3x^2+2)^{\frac{3}{2}}}{72}+\frac{455x\sqrt{3x^2+2}}{24}+\frac{455\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{36}+\frac{(3x^2+2)^{\frac{7}{2}}}{3}-\frac{x(3x^2+2)^{\frac{7}{2}}}{12}$
trager	$\left(-\frac{9}{4}x^7+9x^6+\frac{73}{4}x^5+18x^4+\frac{1111}{24}x^3+12x^2+\frac{985}{24}x+\frac{8}{3}\right)\sqrt{3x^2+2}-\frac{455\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{Roo}\right.}{36}$
meijerg	$-\frac{75\sqrt{3}\left(-\frac{8\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\left(\frac{3}{8}x^4+\frac{13}{16}x^2+\frac{11}{16}\right)\sqrt{\frac{3x^2}{2}+1}}{15}-\frac{\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{3}\right)}{2\sqrt{\pi}}-\frac{35\sqrt{2}\left(\frac{16\sqrt{\pi}}{105}-\frac{8\sqrt{\pi}\left(\frac{27}{4}x^6+\frac{27}{2}x^4+9x^2+2\right)\sqrt{\frac{3x^2}{2}+1}}{105}\right.}{2\sqrt{\pi}}$

input
```
int((5-x)*(2*x+3)*(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(54*x^7-216*x^6-438*x^5-432*x^4-1111*x^3-288*x^2-985*x-64)*(3*x^2+2)
^(1/2)+455/36*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

$$\int (5-x)(3+2x)\left(2+3x^2\right)^{5/2}\,dx = -\frac{1}{24}\left(54\,x^7-216\,x^6-438\,x^5-432\,x^4-1111\,x^3-288\,x^2-985\,x-64\right)\sqrt{3\,x^2+2} + \frac{455}{72}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3+2*x)*(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
-1/24*(54*x^7 - 216*x^6 - 438*x^5 - 432*x^4 - 1111*x^3 - 288*x^2 - 985*x -
 64)*sqrt(3*x^2 + 2) + 455/72*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x
^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 3.02 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.62

$$\int (5-x)(3+2x)\left(2+3x^2\right)^{5/2}\,dx = -\frac{9x^7\sqrt{3x^2+2}}{4}+9x^6\sqrt{3x^2+2}+\frac{73x^5\sqrt{3x^2+2}}{4}+18x^4\sqrt{3x^2+2}+\frac{1111x^3\sqrt{3x^2+2}}{24}+12x^2\sqrt{3x^2+2}+\frac{985x\sqrt{3x^2+2}}{24}+\frac{8\sqrt{3x^2+2}}{3}+\frac{455\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{36}$$

input
```
integrate((5-x)*(3+2*x)*(3*x**2+2)**(5/2),x)
```

output
```
-9*x**7*sqrt(3*x**2 + 2)/4 + 9*x**6*sqrt(3*x**2 + 2) + 73*x**5*sqrt(3*x**2
 + 2)/4 + 18*x**4*sqrt(3*x**2 + 2) + 1111*x**3*sqrt(3*x**2 + 2)/24 + 12*x*
*2*sqrt(3*x**2 + 2) + 985*x*sqrt(3*x**2 + 2)/24 + 8*sqrt(3*x**2 + 2)/3 + 4
55*sqrt(3)*asinh(sqrt(6)*x/2)/36
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int (5-x)(3+2x)\left(2+3x^2\right)^{5/2}\,dx = -\frac{1}{12}\left(3x^2+2\right)^{\frac{7}{2}}x+\frac{1}{3}\left(3x^2+2\right)^{\frac{7}{2}}+\frac{91}{36}\left(3x^2+2\right)^{\frac{5}{2}}x+\frac{455}{72}\left(3x^2+2\right)^{\frac{3}{2}}x+\frac{455}{24}\sqrt{3x^2+2}x+\frac{455}{36}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)$$

input
```
integrate((5-x)*(3+2*x)*(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
-1/12*(3*x^2 + 2)^(7/2)*x + 1/3*(3*x^2 + 2)^(7/2) + 91/36*(3*x^2 + 2)^(5/2
)*x + 455/72*(3*x^2 + 2)^(3/2)*x + 455/24*sqrt(3*x^2 + 2)*x + 455/36*sqrt(
3)*arcsinh(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int (5-x)(3+2x)\left(2+3x^2\right)^{5/2}\,dx = -\frac{1}{24}\left((((6\,((9\,(x-4)x-73)x-72)x-1111)x-288)x-985)x-64\right)\sqrt{3\,x^2+2} - \frac{455}{36}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

input
```
integrate((5-x)*(3+2*x)*(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
-1/24*((((6*((9*(x - 4)*x - 73)*x - 72)*x - 1111)*x - 288)*x - 985)*x - 64
)*sqrt(3*x^2 + 2) - 455/36*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.68

$$\int (5-x)(3+2x)\left(2+3x^2\right)^{5/2}\,dx = \frac{455\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{36} + \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(-\frac{27\,x^7}{4}+27\,x^6+\frac{219\,x^5}{4}+54\,x^4+\frac{1111\,x^3}{8}+36\,x^2+\frac{985\,x}{8}+8\right)}{3}$$

input
```
int(-(2*x + 3)*(3*x^2 + 2)^(5/2)*(x - 5),x)
```

output
```
(455*3^(1/2)*asinh((6^(1/2)*x)/2))/36 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((985*x
)/8 + 36*x^2 + (1111*x^3)/8 + 54*x^4 + (219*x^5)/4 + 27*x^6 - (27*x^7)/4 +
 8))/3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.40

$$\int (5-x)(3+2x)\left(2+3x^2\right)^{5/2}\,dx = -\frac{9\sqrt{3x^2+2}\,x^7}{4}+9\sqrt{3x^2+2}\,x^6+\frac{73\sqrt{3x^2+2}\,x^5}{4}+18\sqrt{3x^2+2}\,x^4+\frac{1111\sqrt{3x^2+2}\,x^3}{24}+12\sqrt{3x^2+2}\,x^2+\frac{985\sqrt{3x^2+2}\,x}{24}+\frac{8\sqrt{3x^2+2}}{3}+\frac{455\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{36}$$

input
```
int((5-x)*(3+2*x)*(3*x^2+2)^(5/2),x)
```

output
```
( - 162*sqrt(3*x**2 + 2)*x**7 + 648*sqrt(3*x**2 + 2)*x**6 + 1314*sqrt(3*x*
*2 + 2)*x**5 + 1296*sqrt(3*x**2 + 2)*x**4 + 3333*sqrt(3*x**2 + 2)*x**3 + 8
64*sqrt(3*x**2 + 2)*x**2 + 2955*sqrt(3*x**2 + 2)*x + 192*sqrt(3*x**2 + 2)
+ 910*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/72
```

3.224 $\int (5-x)\left(2+3x^2\right)^{5/2} dx$

Optimal result

Integrand size = 17, antiderivative size = 83

$$\int (5-x)\left(2+3x^2\right)^{5/2} dx = \frac{25}{4}x\sqrt{2+3x^2} + \frac{25}{12}x\left(2+3x^2\right)^{3/2} + \frac{5}{6}x\left(2+3x^2\right)^{5/2} - \frac{1}{21}\left(2+3x^2\right)^{7/2} + \frac{25\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}$$

output
```
25/4*x*(3*x^2+2)^(1/2)+25/12*x*(3*x^2+2)^(3/2)+5/6*x*(3*x^2+2)^(5/2)-1/21*
(3*x^2+2)^(7/2)+25/6*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

$$\int (5-x)\left(2+3x^2\right)^{5/2} dx = -\frac{1}{84}\sqrt{2+3x^2}\left(32-1155x+144x^2-1365x^3+216x^4-630x^5+108x^6\right) - \frac{25\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{2\sqrt{3}}$$

input
```
Integrate[(5 - x)*(2 + 3*x^2)^(5/2),x]
```

output
```
-1/84*(Sqrt[2 + 3*x^2]*(32 - 1155*x + 144*x^2 - 1365*x^3 + 216*x^4 - 630*x
^5 + 108*x^6)) - (25*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(2*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {455, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-x)\left(3x^2+2\right)^{5/2}dx$$

$$\Big\downarrow \text{455}$$

$$5\int \left(3x^2+2\right)^{5/2}dx-\frac{1}{21}\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \text{211}$$

$$5\left(\frac{5}{3}\int \left(3x^2+2\right)^{3/2}dx+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)-\frac{1}{21}\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \text{211}$$

$$5\left(\frac{5}{3}\left(\frac{3}{2}\int \sqrt{3x^2+2}dx+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)-\frac{1}{21}\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \text{211}$$

$$5\left(\frac{5}{3}\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)-\frac{1}{21}\left(3x^2+2\right)^{7/2}$$

$$\Big\downarrow \text{222}$$

$$5\left(\frac{5}{3}\left(\frac{3}{2}\left(\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3x^2+2}x\right)+\frac{1}{4}x\left(3x^2+2\right)^{3/2}\right)+\frac{1}{6}x\left(3x^2+2\right)^{5/2}\right)-\frac{1}{21}\left(3x^2+2\right)^{7/2}$$

input
```
Int[(5 - x)*(2 + 3*x^2)^(5/2),x]
```

output
```
-1/21*(2 + 3*x^2)^(7/2) + 5*((x*(2 + 3*x^2)^(5/2))/6 + (5*((x*(2 + 3*x^2)^
(3/2))/4 + (3*((x*Sqrt[2 + 3*x^2])/2 + ArcSinh[Sqrt[3/2]*x]/Sqrt[3]))/2))/
3)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{\left(108 x^{6}-630 x^{5}+216 x^{4}-1365 x^{3}+144 x^{2}-1155 x +32\right) \sqrt{3 x^{2}+2}}{84}+\frac{25 \,\operatorname{arcsinh}\left(\frac{\sqrt{6}\, x}{2}\right) \sqrt{3}}{6}$
default	$\frac{25 x \sqrt{3 x^{2}+2}}{4}+\frac{25 x \left(3 x^{2}+2\right)^{\frac{3}{2}}}{12}+\frac{5 x \left(3 x^{2}+2\right)^{\frac{5}{2}}}{6}-\frac{\left(3 x^{2}+2\right)^{\frac{7}{2}}}{21}+\frac{25 \,\operatorname{arcsinh}\left(\frac{\sqrt{6}\, x}{2}\right) \sqrt{3}}{6}$
trager	$\left(-\frac{9}{7} x^{6}+\frac{15}{2} x^{5}-\frac{18}{7} x^{4}+\frac{65}{4} x^{3}-\frac{12}{7} x^{2}+\frac{55}{4} x -\frac{8}{21}\right) \sqrt{3 x^{2}+2}+\frac{25 \operatorname{RootOf}\left(\_Z^{2}-3\right) \ln\left(\operatorname{RootOf}\left(\_Z^{2}-3\right)\right.}{6}$
meijerg	$-\frac{25 \sqrt{3}\, \left(-\frac{8 \sqrt{\pi}\, x \sqrt{2}\, \sqrt{3}\, \left(\frac{3}{8} x^{4}+\frac{13}{16} x^{2}+\frac{11}{16}\right) \sqrt{\frac{3 x^{2}}{2}+1}}{15}-\frac{\sqrt{\pi}\, \operatorname{arcsinh}\left(\frac{x \sqrt{2}\, \sqrt{3}}{2}\right)}{3}\right)}{2 \sqrt{\pi}}+\frac{5 \sqrt{2}\, \left(\frac{16 \sqrt{\pi}}{105}-\frac{8 \sqrt{\pi}\, \left(\frac{27}{4} x^{6}+\frac{27}{2} x^{4}+9 x^{2}+2\right) \sqrt{\frac{3 x^{2}}{2}+1}}{105}\right.}{2 \sqrt{\pi}}$

input
```
int((5-x)*(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/84*(108*x^6-630*x^5+216*x^4-1365*x^3+144*x^2-1155*x+32)*(3*x^2+2)^(1/2)
+25/6*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int (5-x)\left(2+3x^2\right)^{5/2}\,dx = -\frac{1}{84}\left(108\,x^6-630\,x^5+216\,x^4-1365\,x^3+144\,x^2-1155\,x+32\right)\sqrt{3\,x^2+2} + \frac{25}{12}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
-1/84*(108*x^6 - 630*x^5 + 216*x^4 - 1365*x^3 + 144*x^2 - 1155*x + 32)*sqr
t(3*x^2 + 2) + 25/12*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.58

$$\int (5-x)\left(2+3x^2\right)^{5/2}\,dx = -\frac{9x^6\sqrt{3x^2+2}}{7}+\frac{15x^5\sqrt{3x^2+2}}{2}-\frac{18x^4\sqrt{3x^2+2}}{7} + \frac{65x^3\sqrt{3x^2+2}}{4}-\frac{12x^2\sqrt{3x^2+2}}{7}+\frac{55x\sqrt{3x^2+2}}{4}-\frac{8\sqrt{3x^2+2}}{21}+\frac{25\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

input
```
integrate((5-x)*(3*x**2+2)**(5/2),x)
```

output
```
-9*x**6*sqrt(3*x**2 + 2)/7 + 15*x**5*sqrt(3*x**2 + 2)/2 - 18*x**4*sqrt(3*x
**2 + 2)/7 + 65*x**3*sqrt(3*x**2 + 2)/4 - 12*x**2*sqrt(3*x**2 + 2)/7 + 55*
x*sqrt(3*x**2 + 2)/4 - 8*sqrt(3*x**2 + 2)/21 + 25*sqrt(3)*asinh(sqrt(6)*x/
2)/6
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int (5-x)\left(2+3x^2\right)^{5/2}\,dx = -\frac{1}{21}\left(3\,x^2+2\right)^{\frac{7}{2}}+\frac{5}{6}\left(3\,x^2+2\right)^{\frac{5}{2}}x+\frac{25}{12}\left(3\,x^2+2\right)^{\frac{3}{2}}x+\frac{25}{4}\,\sqrt{3\,x^2+2}x+\frac{25}{6}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right)$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
-1/21*(3*x^2 + 2)^(7/2) + 5/6*(3*x^2 + 2)^(5/2)*x + 25/12*(3*x^2 + 2)^(3/2
)*x + 25/4*sqrt(3*x^2 + 2)*x + 25/6*sqrt(3)*arcsinh(1/2*sqrt(6)*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int (5-x)\left(2+3x^2\right)^{5/2}\,dx = -\frac{1}{84}\left(3\left(\left(\left(6\left(\left(6\,x-35\right)x+12\right)x-455\right)x+48\right)x-385\right)x+32\right)\sqrt{3\,x^2+2}-\frac{25}{6}\,\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
-1/84*(3*(((6*((6*x - 35)*x + 12)*x - 455)*x + 48)*x - 385)*x + 32)*sqrt(3
*x^2 + 2) - 25/6*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

$$\int (5-x) \left(2+3x^2\right)^{5/2} \, dx = \frac{25 \sqrt{3} \operatorname{asinh}\left(\frac{\sqrt{6}\, x}{2}\right)}{6} - \frac{\sqrt{3} \sqrt{x^2+\frac{2}{3}} \left(\frac{27\, x^6}{7} - \frac{45\, x^5}{2} + \frac{54\, x^4}{7} - \frac{195\, x^3}{4} + \frac{36\, x^2}{7} - \frac{165\, x}{4} + \frac{8}{7}\right)}{3}$$

input
```
int(-(3*x^2 + 2)^(5/2)*(x - 5),x)
```

output
```
(25*3^(1/2)*asinh((6^(1/2)*x)/2))/6 - (3^(1/2)*(x^2 + 2/3)^(1/2)*((36*x^2)
/7 - (165*x)/4 - (195*x^3)/4 + (54*x^4)/7 - (45*x^5)/2 + (27*x^6)/7 + 8/7)
)/3
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int (5-x) \left(2+3x^2\right)^{5/2} \, dx = -\frac{9\sqrt{3x^2+2}\, x^6}{7} + \frac{15\sqrt{3x^2+2}\, x^5}{2} - \frac{18\sqrt{3x^2+2}\, x^4}{7} + \frac{65\sqrt{3x^2+2}\, x^3}{4} - \frac{12\sqrt{3x^2+2}\, x^2}{7} + \frac{55\sqrt{3x^2+2}\, x}{4} - \frac{8\sqrt{3x^2+2}}{21} + \frac{25\sqrt{3} \log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\, x}{\sqrt{2}}\right)}{6}$$

input
```
int((5-x)*(3*x^2+2)^(5/2),x)
```

output
```
( - 108*sqrt(3*x**2 + 2)*x**6 + 630*sqrt(3*x**2 + 2)*x**5 - 216*sqrt(3*x**
2 + 2)*x**4 + 1365*sqrt(3*x**2 + 2)*x**3 - 144*sqrt(3*x**2 + 2)*x**2 + 115
5*sqrt(3*x**2 + 2)*x - 32*sqrt(3*x**2 + 2) + 350*sqrt(3)*log((sqrt(3*x**2
+ 2) + sqrt(3)*x)/sqrt(2)))/84
```

3.225 $\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{3+2x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{3+2x}\,dx=\frac{7}{64}(2275-691x)\sqrt{2+3x^2}+\frac{7}{96}(130-53x)\left(2+3x^2\right)^{3/2}+\frac{1}{60}(39-5x)\left(2+3x^2\right)^{5/2}-\frac{162673\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{128\sqrt{3}}-\frac{15925}{128}\sqrt{35}\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)$$

output
```
7/64*(2275-691*x)*(3*x^2+2)^(1/2)+7/96*(130-53*x)*(3*x^2+2)^(3/2)+1/60*(39
-5*x)*(3*x^2+2)^(5/2)-162673/384*arcsinh(1/2*x*6^(1/2))*3^(1/2)-15925/128*
35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{3+2x}\,dx = \frac{-2\sqrt{2+3x^2}\left(-259571+80295x-34788x^2+12090x^3-5616x^4+720x^5\right)+}{19}$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x),x]
```

output
```
(-2*Sqrt[2 + 3*x^2]*(-259571 + 80295*x - 34788*x^2 + 12090*x^3 - 5616*x^4
+ 720*x^5) + 477750*Sqrt[35]*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 +
 3*x^2])/Sqrt[35]] + 813365*Sqrt[3]*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/1
920
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {682, 27, 682, 27, 682, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{5/2}}{2x+3}dx$$

$$\Big\downarrow \text{682}$$

$$\frac{1}{72}\int \frac{42(18-53x)\left(3x^2+2\right)^{3/2}}{2x+3}dx+\frac{1}{60}(39-5x)\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{12}\int \frac{(18-53x)\left(3x^2+2\right)^{3/2}}{2x+3}dx+\frac{1}{60}(39-5x)\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{682}$$

$$\frac{7}{12}\left(\frac{1}{48}\int \frac{36(101-691x)\sqrt{3x^2+2}}{2x+3}dx+\frac{1}{8}(130-53x)\left(3x^2+2\right)^{3/2}\right)+\frac{1}{60}(39-5x)\left(3x^2+2\right)^{5/2}$$

$\downarrow$ 27

$$\frac{7}{12}\left(\frac{3}{4}\int \frac{(101-691x)\sqrt{3x^2+2}}{2x+3}dx+\frac{1}{8}(130-53x)\left(3x^2+2\right)^{3/2}\right)+\frac{1}{60}(39-5x)\left(3x^2+2\right)^{5/2}$$

$\downarrow$ 682

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{24}\int \frac{6(4954-23239x)}{(2x+3)\sqrt{3x^2+2}}dx+\frac{1}{4}\sqrt{3x^2+2}(2275-691x)\right)+\frac{1}{8}(130-53x)\left(3x^2+2\right)^{3/2}\right)+\frac{1}{60}(39-5x)\left(3x^2+2\right)^{5/2}$$

$\downarrow$ 27

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{4}\int \frac{4954-23239x}{(2x+3)\sqrt{3x^2+2}}dx+\frac{1}{4}\sqrt{3x^2+2}(2275-691x)\right)+\frac{1}{8}(130-53x)\left(3x^2+2\right)^{3/2}\right)+\frac{1}{60}(39-5x)\left(3x^2+2\right)^{5/2}$$

$\downarrow$ 719

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{4}\left(\frac{79625}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{23239}{2}\int \frac{1}{\sqrt{3x^2+2}}dx\right)+\frac{1}{4}\sqrt{3x^2+2}(2275-691x)\right)+\frac{1}{8}(130-5\right.$$
$$\frac{1}{60}(39-5x)\left(3x^2+2\right)^{5/2}$$

$\downarrow$ 222

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{4}\left(\frac{79625}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{23239\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)+\frac{1}{4}\sqrt{3x^2+2}(2275-691x)\right)+\frac{1}{8}(130-\right.$$
$$\frac{1}{60}(39-5x)\left(3x^2+2\right)^{5/2}$$

$\downarrow$ 488

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{4}\left(-\frac{79625}{2}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{23239\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)+\frac{1}{4}\sqrt{3x^2+2}(2275-691x)\right)+\frac{1}{8}(1\right.$$
$$\frac{1}{60}(39-5x)\left(3x^2+2\right)^{5/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{7}{12}\left(\frac{3}{4}\left(\frac{1}{4}\left(-\frac{23239\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{2275}{2}\sqrt{35}\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)\right)+\frac{1}{4}\sqrt{3x^2+2}(2275-691x)\right)+\frac{1}{8}(\right.$$
$$\frac{1}{60}(39-5x)\left(3x^2+2\right)^{5/2}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x),x]
```

output
```
((39 - 5*x)*(2 + 3*x^2)^(5/2))/60 + (7*(((130 - 53*x)*(2 + 3*x^2)^(3/2))/8
 + (3*(((2275 - 691*x)*Sqrt[2 + 3*x^2])/4 + ((-23239*ArcSinh[Sqrt[3/2]*x])
/(2*Sqrt[3]) - (2275*Sqrt[35]*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])
])/2)/4))/4))/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{(720x^5-5616x^4+12090x^3-34788x^2+80295x-259571)\sqrt{3x^2+2}}{960}-\frac{162673\,\text{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{384}-\frac{15925\sqrt{35}\,\text{arctanh}\left(\frac{2}{35\sqrt{12(}}\right.}{128}$
trager	$\left(-\frac{3}{4}x^5+\frac{117}{20}x^4-\frac{403}{32}x^3+\frac{2899}{80}x^2-\frac{5353}{64}x+\frac{259571}{960}\right)\sqrt{3x^2+2}-\frac{162673\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(\text{RootOf}\left(\_Z\right.\right.}{384}$
default	$-\frac{x(3x^2+2)^{\frac{5}{2}}}{12}-\frac{5x(3x^2+2)^{\frac{3}{2}}}{24}-\frac{5x\sqrt{3x^2+2}}{8}-\frac{162673\,\text{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{384}+\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{20}-\frac{117x\left(3\left(x+\frac{3}{2}\right)^2-\right.}{32}$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(2*x+3),x,method=_RETURNVERBOSE)
```

output
```
-1/960*(720*x^5-5616*x^4+12090*x^3-34788*x^2+80295*x-259571)*(3*x^2+2)^(1/
2)-162673/384*arcsinh(1/2*6^(1/2)*x)*3^(1/2)-15925/128*35^(1/2)*arctanh(2/
35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{3+2x}\,dx = -\frac{1}{960}\left(720\,x^5-5616\,x^4+12090\,x^3-34788\,x^2+80295\,x-259571\right)\sqrt{3\,x^2+2} + \frac{162673}{768}\,\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right) + \frac{15925}{256}\,\sqrt{35}\log\left(-\frac{\sqrt{35}\sqrt{3\,x^2+2}(9\,x-4)+93\,x^2-36\,x+43}{4\,x^2+12\,x+9}\right)$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x),x, algorithm="fricas")
```

output
```
-1/960*(720*x^5 - 5616*x^4 + 12090*x^3 - 34788*x^2 + 80295*x - 259571)*sqr
t(3*x^2 + 2) + 162673/768*sqrt(3)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 -
1) + 15925/256*sqrt(35)*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) + 93*x^2
- 36*x + 43)/(4*x^2 + 12*x + 9))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{3+2x}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(5/2)/(3+2*x),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{3+2x}\,dx = -\frac{1}{12}\left(3\,x^2+2\right)^{\frac{5}{2}}x + \frac{13}{20}\left(3\,x^2+2\right)^{\frac{5}{2}} - \frac{371}{96}\left(3\,x^2+2\right)^{\frac{3}{2}}x + \frac{455}{48}\left(3\,x^2+2\right)^{\frac{3}{2}} - \frac{4837}{64}\sqrt{3\,x^2+2}x - \frac{162673}{384}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{15925}{128}\sqrt{35}\,\mathrm{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|} - \frac{2\sqrt{6}}{3\,|2\,x+3|}\right) + \frac{15925}{64}\sqrt{3\,x^2+2}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x),x, algorithm="maxima")
```

output
```
-1/12*(3*x^2 + 2)^(5/2)*x + 13/20*(3*x^2 + 2)^(5/2) - 371/96*(3*x^2 + 2)^(
3/2)*x + 455/48*(3*x^2 + 2)^(3/2) - 4837/64*sqrt(3*x^2 + 2)*x - 162673/384
*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 15925/128*sqrt(35)*arcsinh(3/2*sqrt(6)*x
/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3)) + 15925/64*sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.12

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{3+2x}\,dx = -\frac{1}{960}\left(3\left(2\left(\left(24\left(5\,x-39\right)x+2015\right)x-5798\right)x+26765\right)x-259571\right)\sqrt{3\,x^2+2} + \frac{162673}{384}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{15925}{128}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3\,x^2+2}}\right)$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x),x, algorithm="giac")
```

output
```
-1/960*(3*(2*((24*(5*x - 39)*x + 2015)*x - 5798)*x + 26765)*x - 259571)*sq
rt(3*x^2 + 2) + 162673/384*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 159
25/128*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqrt(3*x^
2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2)))
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{3+2x}\,dx = \frac{\sqrt{35}\left(1114750\,\ln\left(x+\frac{3}{2}\right)-1114750\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{35}\,\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)\right)}{8960} - \frac{162673\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{384} - \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{9\,x^5}{4}-\frac{351\,x^4}{20}+\frac{1209\,x^3}{32}-\frac{8697\,x^2}{80}+\frac{16059\,x}{64}-\frac{259571}{320}\right)}{3}$$

input
```
int(-((3*x^2 + 2)^(5/2)*(x - 5))/(2*x + 3),x)
```

output
```
(35^(1/2)*(1114750*log(x + 3/2) - 1114750*log(x - (3^(1/2)*35^(1/2)*(x^2 +
 2/3)^(1/2))/9 - 4/9)))/8960 - (162673*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2
))/384 - (3^(1/2)*(x^2 + 2/3)^(1/2)*((16059*x)/64 - (8697*x^2)/80 + (1209*
x^3)/32 - (351*x^4)/20 + (9*x^5)/4 - 259571/320))/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.71

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{3+2x}\,dx = \frac{15925\sqrt{35}\,atan\left(\frac{2\sqrt{3x^2+2}\,i+2\sqrt{3}\,ix}{\sqrt{35}-3\sqrt{3}}\right)i}{128} - \frac{3\sqrt{3x^2+2}\,x^5}{4} + \frac{117\sqrt{3x^2+2}\,x^4}{20} - \frac{403\sqrt{3x^2+2}\,x^3}{32} + \frac{2899\sqrt{3x^2+2}\,x^2}{80} - \frac{5353\sqrt{3x^2+2}\,x}{64} + \frac{259571\sqrt{3x^2+2}}{960} + \frac{15925\sqrt{35}\log\left(4\sqrt{3x^2+2}\,\sqrt{3}\,x+3\sqrt{105}+12x^2-27\right)}{256} - \frac{15925\sqrt{35}\log\left(\frac{2\sqrt{3x^2+2}+\sqrt{35}+2\sqrt{3}\,x+3\sqrt{3}}{\sqrt{2}}\right)}{128} - \frac{162673\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{384}$$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(3+2*x),x)
```

output
```
(477750*sqrt(35)*atan((2*sqrt(3*x**2 + 2)*i + 2*sqrt(3)*i*x)/(sqrt(35) - 3
*sqrt(3)))*i - 2880*sqrt(3*x**2 + 2)*x**5 + 22464*sqrt(3*x**2 + 2)*x**4 -
48360*sqrt(3*x**2 + 2)*x**3 + 139152*sqrt(3*x**2 + 2)*x**2 - 321180*sqrt(3
*x**2 + 2)*x + 1038284*sqrt(3*x**2 + 2) + 238875*sqrt(35)*log(4*sqrt(3*x**
2 + 2)*sqrt(3)*x + 3*sqrt(105) + 12*x**2 - 27) - 477750*sqrt(35)*log((2*sq
rt(3*x**2 + 2) + sqrt(35) + 2*sqrt(3)*x + 3*sqrt(3))/sqrt(2)) - 1626730*sq
rt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/3840
```

3.226 $\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^2}\,dx = -\frac{7}{16}(775-243x)\sqrt{2+3x^2}-\frac{1}{24}(310-153x)\left(2+3x^2\right)^{3/2}-\frac{(34+x)\left(2+3x^2\right)^{5/2}}{10(3+2x)} + \frac{18543}{32}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)+\frac{5425}{32}\sqrt{35}\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)$$

output
```
-7/16*(775-243*x)*(3*x^2+2)^(1/2)-1/24*(310-153*x)*(3*x^2+2)^(3/2)-(34+x)*
(3*x^2+2)^(5/2)/(30+20*x)+18543/32*arcsinh(1/2*x*6^(1/2))*3^(1/2)+5425/32*
35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^2}\,dx = \frac{1}{480}\left(-\frac{2\sqrt{2+3x^2}(265989+89521x-19458x^2+5118x^3-1836x^4+216x^5)}{3+2x}\right.$$
$$-162750\sqrt{35}\text{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)$$
$$\left.-278145\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)\right)$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^2,x]
```

output
```
((-2*Sqrt[2 + 3*x^2]*(265989 + 89521*x - 19458*x^2 + 5118*x^3 - 1836*x^4 +
 216*x^5))/(3 + 2*x) - 162750*Sqrt[35]*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x -
2*Sqrt[2 + 3*x^2])/Sqrt[35]] - 278145*Sqrt[3]*Log[-(Sqrt[3]*x) + Sqrt[2 +
3*x^2]])/480
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {681, 27, 682, 27, 682, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{5/2}}{(2x+3)^2}\,dx$$

$$\Big\downarrow \text{681}$$

$$-\frac{1}{8}\int \frac{8(1-51x)\left(3x^2+2\right)^{3/2}}{2x+3}dx-\frac{(x+34)\left(3x^2+2\right)^{5/2}}{10(2x+3)}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{(1-51x)\left(3x^2+2\right)^{3/2}}{2x+3}dx-\frac{(x+34)\left(3x^2+2\right)^{5/2}}{10(2x+3)}$$

$\Big\downarrow$ 682

$$-\frac{1}{48}\int \frac{84(23-243x)\sqrt{3x^2+2}}{2x+3}dx-\frac{(x+34)\left(3x^2+2\right)^{5/2}}{10(2x+3)}-\frac{1}{24}(310-153x)\left(3x^2+2\right)^{3/2}$$

$\Big\downarrow$ 27

$$-\frac{7}{4}\int \frac{(23-243x)\sqrt{3x^2+2}}{2x+3}dx-\frac{(x+34)\left(3x^2+2\right)^{5/2}}{10(2x+3)}-\frac{1}{24}(310-153x)\left(3x^2+2\right)^{3/2}$$

$\Big\downarrow$ 682

$$-\frac{7}{4}\left(\frac{1}{24}\int \frac{6(1642-7947x)}{(2x+3)\sqrt{3x^2+2}}dx+\frac{1}{4}\sqrt{3x^2+2}(775-243x)\right)-\frac{(x+34)\left(3x^2+2\right)^{5/2}}{10(2x+3)}-\frac{1}{24}(310-153x)\left(3x^2+2\right)^{3/2}$$

$\Big\downarrow$ 27

$$-\frac{7}{4}\left(\frac{1}{4}\int \frac{1642-7947x}{(2x+3)\sqrt{3x^2+2}}dx+\frac{1}{4}\sqrt{3x^2+2}(775-243x)\right)-\frac{(x+34)\left(3x^2+2\right)^{5/2}}{10(2x+3)}-\frac{1}{24}(310-153x)\left(3x^2+2\right)^{3/2}$$

$\Big\downarrow$ 719

$$-\frac{7}{4}\left(\frac{1}{4}\left(\frac{27125}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{7947}{2}\int \frac{1}{\sqrt{3x^2+2}}dx\right)+\frac{1}{4}\sqrt{3x^2+2}(775-243x)\right)-\frac{(x+34)\left(3x^2+2\right)^{5/2}}{10(2x+3)}-\frac{1}{24}(310-153x)\left(3x^2+2\right)^{3/2}$$

$\Big\downarrow$ 222

$$-\frac{7}{4}\left(\frac{1}{4}\left(\frac{27125}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{2649}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\right)+\frac{1}{4}\sqrt{3x^2+2}(775-243x)\right)-\frac{(x+34)\left(3x^2+2\right)^{5/2}}{10(2x+3)}-\frac{1}{24}(310-153x)\left(3x^2+2\right)^{3/2}$$

$\Big\downarrow$ 488

$$-\frac{7}{4}\left(\frac{1}{4}\left(-\frac{27125}{2}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{2649}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\right)+\frac{1}{4}\sqrt{3x^2+2}(775-243x)\right)-\frac{(x+34)\left(3x^2+2\right)^{5/2}}{10(2x+3)}-\frac{1}{24}(310-153x)\left(3x^2+2\right)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{7}{4}\left(\frac{1}{4}\left(-\frac{2649}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-\frac{775}{2}\sqrt{35}\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)\right)+\frac{1}{4}\sqrt{3x^2+2}(775-243x)\right)-\frac{(x+34)\left(3x^2+2\right)^{5/2}}{10(2x+3)}-\frac{1}{24}(310-153x)\left(3x^2+2\right)^{3/2}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^2,x]
```

output
```
-1/24*((310 - 153*x)*(2 + 3*x^2)^(3/2)) - ((34 + x)*(2 + 3*x^2)^(5/2))/(10
*(3 + 2*x)) - (7*(((775 - 243*x)*Sqrt[2 + 3*x^2])/4 + ((-2649*Sqrt[3]*ArcS
inh[Sqrt[3/2]*x])/2 - (775*Sqrt[35]*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3
*x^2])])/2)/4))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{648x^7-5508x^6+15786x^5-62046x^4+278799x^3+759051x^2+179042x+531978}{240(2x+3)\sqrt{3x^2+2}}+\frac{18543 \operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{32}+\frac{5425\sqrt{35} \operatorname{arctan}}{}$
trager	$-\frac{(216x^5-1836x^4+5118x^3-19458x^2+89521x+265989)\sqrt{3x^2+2}}{240(2x+3)}-\frac{21 \operatorname{RootOf}\left(\_Z^2-2339067\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2-2339067\right.\right.}{32}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{7}{2}}}{70\left(x+\frac{3}{2}\right)}-\frac{31\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{35}+\frac{51x\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{8}+\frac{1701x\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}{16}+\frac{1854}{}$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(2*x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/240*(648*x^7-5508*x^6+15786*x^5-62046*x^4+278799*x^3+759051*x^2+179042*
x+531978)/(2*x+3)/(3*x^2+2)^(1/2)+18543/32*arcsinh(1/2*6^(1/2)*x)*3^(1/2)+
5425/32*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2
))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.12

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^2}\,dx = \frac{278145\sqrt{3}(2x+3)\log\left(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)+81375\sqrt{35}(2x+3)\,\mathrm{lo}}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^2,x, algorithm="fricas")
```

output
```
1/960*(278145*sqrt(3)*(2*x + 3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1
) + 81375*sqrt(35)*(2*x + 3)*log((sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) - 93*
x^2 + 36*x - 43)/(4*x^2 + 12*x + 9)) - 4*(216*x^5 - 1836*x^4 + 5118*x^3 -
19458*x^2 + 89521*x + 265989)*sqrt(3*x^2 + 2))/(2*x + 3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^2}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(5/2)/(3+2*x)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^2}\,dx &= -\frac{1}{20}\left(3x^2+2\right)^{\frac{5}{2}} + \frac{51}{8}\left(3x^2+2\right)^{\frac{3}{2}}x - \frac{155}{12}\left(3x^2+2\right)^{\frac{3}{2}} \\ &\quad - \frac{13\left(3x^2+2\right)^{\frac{5}{2}}}{4(2x+3)} + \frac{1701}{16}\sqrt{3x^2+2}x + \frac{18543}{32}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) \\ &\quad - \frac{5425}{32}\sqrt{35}\operatorname{arsinh}\left(\frac{3\sqrt{6}x}{2|2x+3|} - \frac{2\sqrt{6}}{3|2x+3|}\right) - \frac{5425}{16}\sqrt{3x^2+2}\end{aligned}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^2,x, algorithm="maxima")
```

output
```
-1/20*(3*x^2 + 2)^(5/2) + 51/8*(3*x^2 + 2)^(3/2)*x - 155/12*(3*x^2 + 2)^(3
/2) - 13/4*(3*x^2 + 2)^(5/2)/(2*x + 3) + 1701/16*sqrt(3*x^2 + 2)*x + 18543
/32*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 5425/32*sqrt(35)*arcsinh(3/2*sqrt(6)*
x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3)) - 5425/16*sqrt(3*x^2 + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(91) = 182$.

Time = 0.40 (sec) , antiderivative size = 665, normalized size of antiderivative = 5.68

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^2}\,dx = \text{Too large to display}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^2,x, algorithm="giac")
```

output
```
5425/32*sqrt(35)*log(sqrt(35)*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) +
sqrt(35)/(2*x + 3)) - 9)*sgn(1/(2*x + 3)) - 18543/32*sqrt(3)*log(1/2*abs(-
2*sqrt(3) + 2*sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + 2*sqrt(35)/(2*x +
 3))/(sqrt(3) + sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x +
 3)))*sgn(1/(2*x + 3)) - 15925/128*sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3
)*sgn(1/(2*x + 3)) + 9/320*(238455*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 +
3) + sqrt(35)/(2*x + 3))^9*sgn(1/(2*x + 3)) - 149045*sqrt(35)*(sqrt(-18/(2
*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3))^8*sgn(1/(2*x + 3)) - 6
97600*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3))^7*sg
n(1/(2*x + 3)) + 719040*sqrt(35)*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3)
 + sqrt(35)/(2*x + 3))^6*sgn(1/(2*x + 3)) + 4150566*(sqrt(-18/(2*x + 3) +
35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3))^5*sgn(1/(2*x + 3)) - 2707250*sqr
t(35)*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3))^4*sg
n(1/(2*x + 3)) - 6756120*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(
35)/(2*x + 3))^3*sgn(1/(2*x + 3)) + 4557000*sqrt(35)*(sqrt(-18/(2*x + 3) +
 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3))^2*sgn(1/(2*x + 3)) + 3563595*(s
qrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3))*sgn(1/(2*x +
 3)) - 2833425*sqrt(35)*sgn(1/(2*x + 3)))/((sqrt(-18/(2*x + 3) + 35/(2*x +
 3)^2 + 3) + sqrt(35)/(2*x + 3))^2 - 3)^5
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^2}\,dx = \frac{18543\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{32} - \frac{275027\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{960} - \frac{5425\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{32} + \frac{5425\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{32} - \frac{1393\sqrt{3}\,x^2\sqrt{x^2+\frac{2}{3}}}{80} + \frac{9\sqrt{3}\,x^3\sqrt{x^2+\frac{2}{3}}}{2} - \frac{9\sqrt{3}\,x^4\sqrt{x^2+\frac{2}{3}}}{20} - \frac{15925\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{128\left(x+\frac{3}{2}\right)} + \frac{2133\sqrt{3}\,x\sqrt{x^2+\frac{2}{3}}}{32}$$

input
```
int(-((3*x^2 + 2)^(5/2)*(x - 5))/(2*x + 3)^2,x)
```

output
```
(18543*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/32 - (275027*3^(1/2)*(x^2 + 2
/3)^(1/2))/960 - (5425*35^(1/2)*log(x + 3/2))/32 + (5425*35^(1/2)*log(x -
(3^(1/2)*35^(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/32 - (1393*3^(1/2)*x^2*(x^2
 + 2/3)^(1/2))/80 + (9*3^(1/2)*x^3*(x^2 + 2/3)^(1/2))/2 - (9*3^(1/2)*x^4*(
x^2 + 2/3)^(1/2))/20 - (15925*3^(1/2)*(x^2 + 2/3)^(1/2))/(128*(x + 3/2)) +
 (2133*3^(1/2)*x*(x^2 + 2/3)^(1/2))/32
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.91

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^2}\,dx = \frac{-864\sqrt{3x^2+2}\,x^5+7344\sqrt{3x^2+2}\,x^4-20472\sqrt{3x^2+2}\,x^3+77832\sqrt{3x^2+2}}{}$$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^2,x)
```

output
```
( - 864*sqrt(3*x**2 + 2)*x**5 + 7344*sqrt(3*x**2 + 2)*x**4 - 20472*sqrt(3*
x**2 + 2)*x**3 + 77832*sqrt(3*x**2 + 2)*x**2 - 358084*sqrt(3*x**2 + 2)*x -
 1063956*sqrt(3*x**2 + 2) + 325500*sqrt(35)*log( - sqrt(3*x**2 + 2)*sqrt(3
5) + 9*x - 4)*x + 488250*sqrt(35)*log( - sqrt(3*x**2 + 2)*sqrt(35) + 9*x -
 4) - 325500*sqrt(35)*log(2*x + 3)*x - 488250*sqrt(35)*log(2*x + 3) - 5562
90*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x - 834435*sqrt(3)*log(sqrt(3
*x**2 + 2) - sqrt(3)*x) + 556290*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)
*x + 834435*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x))/(960*(2*x + 3))
```

3.227 $\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 119

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^3}\,dx=\frac{15}{64}(859-267x)\sqrt{2+3x^2}+\frac{1}{224}(1718-807x)\left(2+3x^2\right)^{3/2}+\frac{(151+131x)\left(2+3x^2\right)^{5/2}}{28(3+2x)^2}-\frac{43995}{128}\sqrt{3}\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-\frac{12885}{128}\sqrt{35}\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)$$

output
```
15/64*(859-267*x)*(3*x^2+2)^(1/2)+1/224*(1718-807*x)*(3*x^2+2)^(3/2)+1/28*
(151+131*x)*(3*x^2+2)^(5/2)/(3+2*x)^2-43995/128*arcsinh(1/2*x*6^(1/2))*3^(
1/2)-12885/128*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^3}\,dx=\frac{1}{128}\left(-\frac{2\sqrt{2+3x^2}(-126181-127403x-19268x^2+2826x^3-696x^4+72x^5)}{(3+2x)^2}\right.$$
$$\left.+25770\sqrt{35}\text{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)+43995\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)\right)$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^3,x]
```

output
```
((-2*Sqrt[2 + 3*x^2]*(-126181 - 127403*x - 19268*x^2 + 2826*x^3 - 696*x^4
+ 72*x^5))/(3 + 2*x)^2 + 25770*Sqrt[35]*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x -
 2*Sqrt[2 + 3*x^2])/Sqrt[35]] + 43995*Sqrt[3]*Log[-(Sqrt[3]*x) + Sqrt[2 +
3*x^2]])/128
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {681, 27, 681, 27, 682, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{5/2}}{(2x+3)^3}\,dx$$

$$\Big\downarrow \ 681$$

$$-\frac{5}{64}\int \frac{4(4-87x)\left(3x^2+2\right)^{3/2}}{(2x+3)^2}dx-\frac{(2x+29)\left(3x^2+2\right)^{5/2}}{16(2x+3)^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{5}{16}\int \frac{(4-87x)\left(3x^2+2\right)^{3/2}}{(2x+3)^2}dx-\frac{(2x+29)\left(3x^2+2\right)^{5/2}}{16(2x+3)^2}$$

$$\Big\downarrow \text{681}$$

$$-\frac{5}{16}\left(-\frac{1}{8}\int \frac{24(29-267x)\sqrt{3x^2+2}}{2x+3}dx-\frac{(29x+178)\left(3x^2+2\right)^{3/2}}{2(2x+3)}\right)-\frac{(2x+29)\left(3x^2+2\right)^{5/2}}{16(2x+3)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5}{16}\left(-3\int \frac{(29-267x)\sqrt{3x^2+2}}{2x+3}dx-\frac{(29x+178)\left(3x^2+2\right)^{3/2}}{2(2x+3)}\right)-\frac{(2x+29)\left(3x^2+2\right)^{5/2}}{16(2x+3)^2}$$

$$\Big\downarrow \text{682}$$

$$-\frac{5}{16}\left(-3\left(\frac{1}{24}\int \frac{42(262-1257x)}{(2x+3)\sqrt{3x^2+2}}dx+\frac{1}{4}\sqrt{3x^2+2}(859-267x)\right)-\frac{(29x+178)\left(3x^2+2\right)^{3/2}}{2(2x+3)}\right)-\frac{(2x+29)\left(3x^2+2\right)^{5/2}}{16(2x+3)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5}{16}\left(-3\left(\frac{7}{4}\int \frac{262-1257x}{(2x+3)\sqrt{3x^2+2}}dx+\frac{1}{4}\sqrt{3x^2+2}(859-267x)\right)-\frac{(29x+178)\left(3x^2+2\right)^{3/2}}{2(2x+3)}\right)-\frac{(2x+29)\left(3x^2+2\right)^{5/2}}{16(2x+3)^2}$$

$$\Big\downarrow \text{719}$$

$$-\frac{5}{16}\left(-3\left(\frac{7}{4}\left(\frac{4295}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{1257}{2}\int \frac{1}{\sqrt{3x^2+2}}dx\right)+\frac{1}{4}\sqrt{3x^2+2}(859-267x)\right)-\frac{(29x+17}{2(}\right.$$
$$\frac{(2x+29)\left(3x^2+2\right)^{5/2}}{16(2x+3)^2}$$

$$\Big\downarrow \text{222}$$

$$-\frac{5}{16}\left(-3\left(\frac{7}{4}\left(\frac{4295}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{419}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\right)+\frac{1}{4}\sqrt{3x^2+2}(859-267x)\right)-\frac{(29x+}{}\right.$$
$$\frac{(2x+29)\left(3x^2+2\right)^{5/2}}{16(2x+3)^2}$$

$$\Big\downarrow \text{488}$$

$$-\frac{5}{16}\left(-3\left(\frac{7}{4}\left(-\frac{4295}{2}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{419}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\right)+\frac{1}{4}\sqrt{3x^2+2}(859-267x)\right)-\frac{(2\right.$$
$$\frac{(2x+29)\left(3x^2+2\right)^{5/2}}{16(2x+3)^2}$$

$$\Big\downarrow \ 219$$

$$-\frac{5}{16}\left(-3\left(\frac{7}{4}\left(-\frac{419}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-\frac{859}{2}\sqrt{\frac{5}{7}}\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)\right)+\frac{1}{4}\sqrt{3x^2+2}(859-267x)\right)-\frac{(\right.$$
$$\frac{(2x+29)\left(3x^2+2\right)^{5/2}}{16(2x+3)^2}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^3,x]
```

output
```
-1/16*((29 + 2*x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^2 - (5*(-1/2*((178 + 29*x)*
(2 + 3*x^2)^(3/2))/(3 + 2*x) - 3*(((859 - 267*x)*Sqrt[2 + 3*x^2])/4 + (7*(
(-419*Sqrt[3]*ArcSinh[Sqrt[3/2]*x])/2 - (859*Sqrt[5/7]*ArcTanh[(4 - 9*x)/(
Sqrt[35]*Sqrt[2 + 3*x^2])])/2))/4)))/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{216x^7-2088x^6+8622x^5-59196x^4-376557x^3-417079x^2-254806x-252362}{64(2x+3)^2\sqrt{3x^2+2}}-\frac{43995\,\text{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{128}-\frac{12885\sqrt{35}\,\text{arctan}}{}$
trager	$-\frac{(72x^5-696x^4+2826x^3-19268x^2-127403x-126181)\sqrt{3x^2+2}}{64(2x+3)^2}+\frac{43995\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\text{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+\right.}{128}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{7}{2}}}{280\left(x+\frac{3}{2}\right)^2}+\frac{421\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{7}{2}}}{4900\left(x+\frac{3}{2}\right)}+\frac{2577\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{4900}-\frac{807x\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{224}-\frac{40}{}$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(2*x+3)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/64*(216*x^7-2088*x^6+8622*x^5-59196*x^4-376557*x^3-417079*x^2-254806*x-
252362)/(2*x+3)^2/(3*x^2+2)^(1/2)-43995/128*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
-12885/128*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.23

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^3}\,dx = \frac{43995\sqrt{3}(4x^2+12x+9)\log\left(\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)+12885\sqrt{35}(4x^2}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^3,x, algorithm="fricas")
```

output
```
1/256*(43995*sqrt(3)*(4*x^2 + 12*x + 9)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*
x^2 - 1) + 12885*sqrt(35)*(4*x^2 + 12*x + 9)*log(-(sqrt(35)*sqrt(3*x^2 + 2
)*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9)) - 4*(72*x^5 - 696*x^
4 + 2826*x^3 - 19268*x^2 - 127403*x - 126181)*sqrt(3*x^2 + 2))/(4*x^2 + 12
*x + 9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^3}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(5/2)/(3+2*x)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^3}\,dx &= \frac{39}{280}\left(3x^2+2\right)^{\frac{5}{2}} - \frac{13\left(3x^2+2\right)^{\frac{7}{2}}}{70\left(4x^2+12x+9\right)} \\ &\quad - \frac{807}{224}\left(3x^2+2\right)^{\frac{3}{2}}x + \frac{859}{112}\left(3x^2+2\right)^{\frac{3}{2}} + \frac{421\left(3x^2+2\right)^{\frac{5}{2}}}{280\left(2x+3\right)} \\ &\quad - \frac{4005}{64}\sqrt{3x^2+2}x - \frac{43995}{128}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) \\ &\quad + \frac{12885}{128}\sqrt{35}\operatorname{arsinh}\left(\frac{3\sqrt{6}x}{2\left|2x+3\right|} - \frac{2\sqrt{6}}{3\left|2x+3\right|}\right) + \frac{12885}{64}\sqrt{3x^2+2}\end{aligned}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^3,x, algorithm="maxima")
```

output
```
39/280*(3*x^2 + 2)^(5/2) - 13/70*(3*x^2 + 2)^(7/2)/(4*x^2 + 12*x + 9) - 80
7/224*(3*x^2 + 2)^(3/2)*x + 859/112*(3*x^2 + 2)^(3/2) + 421/280*(3*x^2 + 2
)^(5/2)/(2*x + 3) - 4005/64*sqrt(3*x^2 + 2)*x - 43995/128*sqrt(3)*arcsinh(
1/2*sqrt(6)*x) + 12885/128*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2
/3*sqrt(6)/abs(2*x + 3)) + 12885/64*sqrt(3*x^2 + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(93) = 186$.

Time = 0.15 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.93

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^3}\,dx = -\frac{1}{32}\left(3\left(\left(3\,x-38\right)x+225\right)x-4177\right)\sqrt{3\,x^2+2}$$
$$+\frac{43995}{128}\,\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$
$$+\frac{12885}{128}\,\sqrt{35}\log\left(-\frac{\left|-2\,\sqrt{3}x-\sqrt{35}-3\,\sqrt{3}+2\,\sqrt{3\,x^2+2}\right|}{2\,\sqrt{3}x-\sqrt{35}+3\,\sqrt{3}-2\,\sqrt{3\,x^2+2}}\right)$$
$$+\frac{35\left(11472\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^3+25829\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2-57912\,\sqrt{3}x+8984\,\sqrt{3}+57912\,\sqrt{3\,x^2}\right.}{256\left(\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2+3\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)-2\right)^2}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^3,x, algorithm="giac")
```

output
```
-1/32*(3*((3*x - 38)*x + 225)*x - 4177)*sqrt(3*x^2 + 2) + 43995/128*sqrt(3
)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 12885/128*sqrt(35)*log(-abs(-2*sqrt(
3)*x - sqrt(35) - 3*sqrt(3) + 2*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) +
 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) + 35/256*(11472*(sqrt(3)*x - sqrt(3*x^2 +
 2))^3 + 25829*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 - 57912*sqrt(3)*x +
 8984*sqrt(3) + 57912*sqrt(3*x^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 +
3*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^2
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.24

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^3}\,dx = \frac{12885\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{128}+\frac{4177\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{32} - \frac{43995\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{128}-\frac{12885\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{128} + \frac{57\sqrt{3}\,x^2\sqrt{x^2+\frac{2}{3}}}{16}-\frac{9\sqrt{3}\,x^3\sqrt{x^2+\frac{2}{3}}}{32}+\frac{39305\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{256\left(x+\frac{3}{2}\right)} - \frac{15925\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{512\left(x^2+3\,x+\frac{9}{4}\right)}-\frac{675\sqrt{3}\,x\sqrt{x^2+\frac{2}{3}}}{32}$$

input
```
int(-((3*x^2 + 2)^(5/2)*(x - 5))/(2*x + 3)^3,x)
```

output
```
(12885*35^(1/2)*log(x + 3/2))/128 + (4177*3^(1/2)*(x^2 + 2/3)^(1/2))/32 -
(43995*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/128 - (12885*35^(1/2)*log(x -
 (3^(1/2)*35^(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/128 + (57*3^(1/2)*x^2*(x^2
 + 2/3)^(1/2))/16 - (9*3^(1/2)*x^3*(x^2 + 2/3)^(1/2))/32 + (39305*3^(1/2)*
(x^2 + 2/3)^(1/2))/(256*(x + 3/2)) - (15925*3^(1/2)*(x^2 + 2/3)^(1/2))/(51
2*(3*x + x^2 + 9/4)) - (675*3^(1/2)*x*(x^2 + 2/3)^(1/2))/32
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.58

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^3}\,dx = \frac{-288\sqrt{3x^2+2}\,x^5+2784\sqrt{3x^2+2}\,x^4-11304\sqrt{3x^2+2}\,x^3+77072\sqrt{3x^2+2}}{}$$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^3,x)
```

output
```
( - 288*sqrt(3*x**2 + 2)*x**5 + 2784*sqrt(3*x**2 + 2)*x**4 - 11304*sqrt(3*
x**2 + 2)*x**3 + 77072*sqrt(3*x**2 + 2)*x**2 + 509612*sqrt(3*x**2 + 2)*x +
 504724*sqrt(3*x**2 + 2) + 103080*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) +
 9*x - 4)*x**2 + 309240*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*
x + 231930*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4) - 103080*sqrt
(35)*log(2*x + 3)*x**2 - 309240*sqrt(35)*log(2*x + 3)*x - 231930*sqrt(35)*
log(2*x + 3) + 175980*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x**2 + 527
940*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x + 395955*sqrt(3)*log(sqrt(
3*x**2 + 2) - sqrt(3)*x) - 175980*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x
)*x**2 - 527940*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x - 395955*sqrt(
3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x))/(256*(4*x**2 + 12*x + 9))
```

3.228 $\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 128

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^4}\,dx = -\frac{3}{56}(1219-381x)\sqrt{2+3x^2}-\frac{(127+19x)\left(2+3x^2\right)^{3/2}}{14(3+2x)}+\frac{(58+69x)\left(2+3x^2\right)^{5/2}}{42(3+2x)^3} + \frac{1785}{16}\sqrt{3}\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)+\frac{3657}{16}\sqrt{\frac{5}{7}}\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)$$

output
```
-3/56*(1219-381*x)*(3*x^2+2)^(1/2)-(127+19*x)*(3*x^2+2)^(3/2)/(42+28*x)+1/
42*(58+69*x)*(3*x^2+2)^(5/2)/(3+2*x)^3+1785/16*arcsinh(1/2*x*6^(1/2))*3^(1
/2)+3657/112*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^4}\,dx = -\frac{\sqrt{2+3x^2}(46103+77061x+37974x^2+3408x^3-432x^4+36x^5)}{24(3+2x)^3} - \frac{3657}{8}\sqrt{\frac{5}{7}}\text{arctanh}\left(3\sqrt{\frac{3}{35}}+2\sqrt{\frac{3}{35}}x-\frac{2\sqrt{2+3x^2}}{\sqrt{35}}\right) - \frac{1785}{16}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^4,x]
```

output
```
-1/24*(Sqrt[2 + 3*x^2]*(46103 + 77061*x + 37974*x^2 + 3408*x^3 - 432*x^4 +
 36*x^5))/(3 + 2*x)^3 - (3657*Sqrt[5/7]*ArcTanh[3*Sqrt[3/35] + 2*Sqrt[3/35
]*x - (2*Sqrt[2 + 3*x^2])/Sqrt[35]])/8 - (1785*Sqrt[3]*Log[-(Sqrt[3]*x) +
Sqrt[2 + 3*x^2]])/16
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {681, 27, 681, 27, 681, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{5/2}}{(2x+3)^4}dx$$

$$\Big\downarrow \text{ 681}$$

$$-\frac{5}{72}\int \frac{24(1-12x)\left(3x^2+2\right)^{3/2}}{(2x+3)^3}dx-\frac{(x+8)\left(3x^2+2\right)^{5/2}}{6(2x+3)^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{5}{3}\int \frac{(1-12x)\left(3x^2+2\right)^{3/2}}{(2x+3)^3}dx-\frac{(x+8)\left(3x^2+2\right)^{5/2}}{6(2x+3)^3}$$

$$\Big\downarrow \ 681$$

$$-\frac{5}{3}\left(-\frac{3}{32}\int \frac{24(8-37x)\sqrt{3x^2+2}}{(2x+3)^2}dx-\frac{(12x+37)\left(3x^2+2\right)^{3/2}}{4(2x+3)^2}\right)-\frac{(x+8)\left(3x^2+2\right)^{5/2}}{6(2x+3)^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{5}{3}\left(-\frac{9}{4}\int \frac{(8-37x)\sqrt{3x^2+2}}{(2x+3)^2}dx-\frac{(12x+37)\left(3x^2+2\right)^{3/2}}{4(2x+3)^2}\right)-\frac{(x+8)\left(3x^2+2\right)^{5/2}}{6(2x+3)^3}$$

$$\Big\downarrow \ 681$$

$$-\frac{5}{3}\left(-\frac{9}{4}\left(-\frac{1}{8}\int \frac{4(74-357x)}{(2x+3)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}(37x+119)}{2(2x+3)}\right)-\frac{(12x+37)\left(3x^2+2\right)^{3/2}}{4(2x+3)^2}\right)-\frac{(x+8)\left(3x^2+2\right)^{5/2}}{6(2x+3)^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{5}{3}\left(-\frac{9}{4}\left(-\frac{1}{2}\int \frac{74-357x}{(2x+3)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}(37x+119)}{2(2x+3)}\right)-\frac{(12x+37)\left(3x^2+2\right)^{3/2}}{4(2x+3)^2}\right)-\frac{(x+8)\left(3x^2+2\right)^{5/2}}{6(2x+3)^3}$$

$$\Big\downarrow \ 719$$

$$-\frac{5}{3}\left(-\frac{9}{4}\left(\frac{1}{2}\left(\frac{357}{2}\int \frac{1}{\sqrt{3x^2+2}}dx-\frac{1219}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx\right)-\frac{(37x+119)\sqrt{3x^2+2}}{2(2x+3)}\right)-\frac{(12x+37)\left(3x\right.}{4(2x+\ldots}\right.$$
$$\frac{(x+8)\left(3x^2+2\right)^{5/2}}{6(2x+3)^3}$$

$$\Big\downarrow \ 222$$

$$-\frac{5}{3}\left(-\frac{9}{4}\left(\frac{1}{2}\left(\frac{119}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-\frac{1219}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx\right)-\frac{(37x+119)\sqrt{3x^2+2}}{2(2x+3)}\right)-\frac{(12x+37\ldots}{4(2\ldots}\right.$$
$$\frac{(x+8)\left(3x^2+2\right)^{5/2}}{6(2x+3)^3}$$

$$\Big\downarrow \text{488}$$

$$-\frac{5}{3}\left(-\frac{9}{4}\left(\frac{1}{2}\left(\frac{1219}{2}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}+\frac{119}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\right)-\frac{(37x+119)\sqrt{3x^2+2}}{2(2x+3)}\right)-\frac{(12x+\cdots}{4\cdots}\right.$$

$$\frac{(x+8)\left(3x^2+2\right)^{5/2}}{6(2x+3)^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{5}{3}\left(-\frac{9}{4}\left(\frac{1}{2}\left(\frac{119}{2}\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)+\frac{1219\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{2\sqrt{35}}\right)-\frac{(37x+119)\sqrt{3x^2+2}}{2(2x+3)}\right)-\frac{(12x+37)\left(\cdots\right.}{4(2x\cdots}\right.$$

$$\frac{(x+8)\left(3x^2+2\right)^{5/2}}{6(2x+3)^3}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^4,x]
```

output
```
-1/6*((8 + x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^3 - (5*(-1/4*((37 + 12*x)*(2 +
3*x^2)^(3/2))/(3 + 2*x)^2 - (9*(-1/2*((119 + 37*x)*Sqrt[2 + 3*x^2])/(3 + 2
*x) + ((119*Sqrt[3]*ArcSinh[Sqrt[3/2]*x])/2 + (1219*ArcTanh[(4 - 9*x)/(Sqr
t[35]*Sqrt[2 + 3*x^2])])/(2*Sqrt[35]))/2))/4))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{108x^7-1296x^6+10296x^5+113058x^4+237999x^3+214257x^2+154122x+92206}{24(2x+3)^3\sqrt{3x^2+2}}+\frac{1785\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{16}+\frac{3657\sqrt{35}\,\mathrm{arctanh}}{}$
trager	$-\frac{(36x^5-432x^4+3408x^3+37974x^2+77061x+46103)\sqrt{3x^2+2}}{24(2x+3)^3}-\frac{1785\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{16}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{7}{2}}}{840\left(x+\frac{3}{2}\right)^3}+\frac{37\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{7}{2}}}{4900\left(x+\frac{3}{2}\right)^2}-\frac{2819\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{7}{2}}}{85750\left(x+\frac{3}{2}\right)}-\frac{7314\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{42875}+\frac{591}{}$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(2*x+3)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(108*x^7-1296*x^6+10296*x^5+113058*x^4+237999*x^3+214257*x^2+154122*
x+92206)/(2*x+3)^3/(3*x^2+2)^(1/2)+1785/16*arcsinh(1/2*6^(1/2)*x)*3^(1/2)+
3657/112*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.27

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^4}\,dx = \frac{5355\,\sqrt{3}(8\,x^3+36\,x^2+54\,x+27)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+10971}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^4,x, algorithm="fricas")
```

output
```
1/96*(5355*sqrt(3)*(8*x^3 + 36*x^2 + 54*x + 27)*log(-sqrt(3)*sqrt(3*x^2 +
2)*x - 3*x^2 - 1) + 10971*sqrt(5/7)*(8*x^3 + 36*x^2 + 54*x + 27)*log((7*sq
rt(5/7)*sqrt(3*x^2 + 2)*(9*x - 4) - 93*x^2 + 36*x - 43)/(4*x^2 + 12*x + 9)
) - 4*(36*x^5 - 432*x^4 + 3408*x^3 + 37974*x^2 + 77061*x + 46103)*sqrt(3*x
^2 + 2))/(8*x^3 + 36*x^2 + 54*x + 27)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^4}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(5/2)/(3+2*x)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.35

$$\int \frac{(5-x)(2+3x^2)^{5/2}}{(3+2x)^4}\,dx = -\frac{111}{4900}\left(3\,x^2+2\right)^{\frac{5}{2}} - \frac{13\left(3\,x^2+2\right)^{\frac{7}{2}}}{105\left(8\,x^3+36\,x^2+54\,x+27\right)} + \frac{37\left(3\,x^2+2\right)^{\frac{7}{2}}}{1225\left(4\,x^2+12\,x+9\right)} + \frac{591}{490}\left(3\,x^2+2\right)^{\frac{3}{2}}x - \frac{1219}{490}\left(3\,x^2+2\right)^{\frac{3}{2}} - \frac{2819\left(3\,x^2+2\right)^{\frac{5}{2}}}{4900\left(2\,x+3\right)} + \frac{1143}{56}\sqrt{3\,x^2+2}x + \frac{1785}{16}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) - \frac{3657}{112}\sqrt{35}\,\mathrm{arsinh}\left(\frac{3\sqrt{6}x}{2\left|2\,x+3\right|} - \frac{2\sqrt{6}}{3\left|2\,x+3\right|}\right) - \frac{3657}{56}\sqrt{3\,x^2+2}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^4,x, algorithm="maxima")
```

output
```
-111/4900*(3*x^2 + 2)^(5/2) - 13/105*(3*x^2 + 2)^(7/2)/(8*x^3 + 36*x^2 + 5
4*x + 27) + 37/1225*(3*x^2 + 2)^(7/2)/(4*x^2 + 12*x + 9) + 591/490*(3*x^2
+ 2)^(3/2)*x - 1219/490*(3*x^2 + 2)^(3/2) - 2819/4900*(3*x^2 + 2)^(5/2)/(2
*x + 3) + 1143/56*sqrt(3*x^2 + 2)*x + 1785/16*sqrt(3)*arcsinh(1/2*sqrt(6)*
x) - 3657/112*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/ab
s(2*x + 3)) - 3657/56*sqrt(3*x^2 + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(100) = 200$.

Time = 0.15 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.15

$$\int \frac{(5-x)(2+3x^2)^{5/2}}{(3+2x)^4}\,dx = -\frac{1}{32}\left(3\left(2\,x-33\right)x+973\right)\sqrt{3\,x^2+2} - \frac{1785}{16}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) - \frac{3657}{112}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3\,x^2+2}}\right) - \frac{\sqrt{3}\left(40667\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^5+589140\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^4+467730\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^3-1\right.}{128\left(\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2+3\sqrt{3}\left(\sqrt{3}x-\sqrt{3}\right.\right.}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^4,x, algorithm="giac")
```

output
```
-1/32*(3*(2*x - 33)*x + 973)*sqrt(3*x^2 + 2) - 1785/16*sqrt(3)*log(-sqrt(3
)*x + sqrt(3*x^2 + 2)) - 3657/112*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35
) - 3*sqrt(3) + 2*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2
*sqrt(3*x^2 + 2))) - 1/128*sqrt(3)*(40667*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2
+ 2))^5 + 589140*(sqrt(3)*x - sqrt(3*x^2 + 2))^4 + 467730*sqrt(3)*(sqrt(3)
*x - sqrt(3*x^2 + 2))^3 - 1939920*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 585700
*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 166304)/((sqrt(3)*x - sqrt(3*x^2
+ 2))^2 + 3*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^3
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.26

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^4}\,dx = \frac{1785\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{16} - \frac{973\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{32} - \frac{3657\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{112} + \frac{3657\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{112} - \frac{3\sqrt{3}\,x^2\sqrt{x^2+\frac{2}{3}}}{16} - \frac{5197\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{64\left(x+\frac{3}{2}\right)} + \frac{9485\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{256\left(x^2+3x+\frac{9}{4}\right)} + \frac{99\sqrt{3}\,x\sqrt{x^2+\frac{2}{3}}}{32} - \frac{15925\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1536\left(x^3+\frac{9x^2}{2}+\frac{27x}{4}+\frac{27}{8}\right)}$$

input
```
int(-((3*x^2 + 2)^(5/2)*(x - 5))/(2*x + 3)^4,x)
```

output
```
(1785*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/16 - (973*3^(1/2)*(x^2 + 2/3)^
(1/2))/32 - (3657*35^(1/2)*log(x + 3/2))/112 + (3657*35^(1/2)*log(x - (3^(
1/2)*35^(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/112 - (3*3^(1/2)*x^2*(x^2 + 2/3
)^(1/2))/16 - (5197*3^(1/2)*(x^2 + 2/3)^(1/2))/(64*(x + 3/2)) + (9485*3^(1
/2)*(x^2 + 2/3)^(1/2))/(256*(3*x + x^2 + 9/4)) + (99*3^(1/2)*x*(x^2 + 2/3)
^(1/2))/32 - (15925*3^(1/2)*(x^2 + 2/3)^(1/2))/(1536*((27*x)/4 + (9*x^2)/2
 + x^3 + 27/8))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.09

$$\int \frac{(5-x)\,(2+3x^2)^{5/2}}{(3+2x)^4}\,dx = \frac{-1008\sqrt{3x^2+2}\,x^5+12096\sqrt{3x^2+2}\,x^4-95424\sqrt{3x^2+2}\,x^3-1063272\sqrt{3x^2}}{}$$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^4,x)
```

output
```
( - 1008*sqrt(3*x**2 + 2)*x**5 + 12096*sqrt(3*x**2 + 2)*x**4 - 95424*sqrt(
3*x**2 + 2)*x**3 - 1063272*sqrt(3*x**2 + 2)*x**2 - 2157708*sqrt(3*x**2 + 2
)*x - 1290884*sqrt(3*x**2 + 2) + 175536*sqrt(35)*log( - sqrt(3*x**2 + 2)*s
qrt(35) + 9*x - 4)*x**3 + 789912*sqrt(35)*log( - sqrt(3*x**2 + 2)*sqrt(35)
 + 9*x - 4)*x**2 + 1184868*sqrt(35)*log( - sqrt(3*x**2 + 2)*sqrt(35) + 9*x
 - 4)*x + 592434*sqrt(35)*log( - sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4) - 17
5536*sqrt(35)*log(2*x + 3)*x**3 - 789912*sqrt(35)*log(2*x + 3)*x**2 - 1184
868*sqrt(35)*log(2*x + 3)*x - 592434*sqrt(35)*log(2*x + 3) - 299880*sqrt(3
)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x**3 - 1349460*sqrt(3)*log(sqrt(3*x**2
 + 2) - sqrt(3)*x)*x**2 - 2024190*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x
)*x - 1012095*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x) + 299880*sqrt(3)*l
og(sqrt(3*x**2 + 2) + sqrt(3)*x)*x**3 + 1349460*sqrt(3)*log(sqrt(3*x**2 +
2) + sqrt(3)*x)*x**2 + 2024190*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x
 + 1012095*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x))/(672*(8*x**3 + 36*x*
*2 + 54*x + 27))
```

3.229 $\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^5}\,dx = \frac{9(20931-6499x)\sqrt{2+3x^2}}{15680} + \frac{(36126+32449x)\left(2+3x^2\right)^{3/2}}{7840(3+2x)^2} + \frac{(81+145x)\left(2+3x^2\right)^{5/2}}{168(3+2x)^4} - \frac{2625}{128}\sqrt{3}\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right) - \frac{188379\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{896\sqrt{35}}$$

output
```
9/15680*(20931-6499*x)*(3*x^2+2)^(1/2)+1/7840*(36126+32449*x)*(3*x^2+2)^(3
/2)/(3+2*x)^2+1/168*(81+145*x)*(3*x^2+2)^(5/2)/(3+2*x)^4-2625/128*arcsinh(
1/2*x*6^(1/2))*3^(1/2)-188379/31360*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)
/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.96

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^5}\,dx = \frac{-\frac{70\sqrt{2+3x^2}\left(-1421955-3335009x-2762820x^2-898734x^3-57456x^4+3024x^5\right)}{(3+2x)^4}+1130274\sqrt{35}\text{arc}}{94080}$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^5,x]
```

output
```
((-70*Sqrt[2 + 3*x^2]*(-1421955 - 3335009*x - 2762820*x^2 - 898734*x^3 - 5
7456*x^4 + 3024*x^5))/(3 + 2*x)^4 + 1130274*Sqrt[35]*ArcTanh[(3*Sqrt[3] +
2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]] + 1929375*Sqrt[3]*Log[-(Sqrt[3]
*x) + Sqrt[2 + 3*x^2]])/94080
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {681, 27, 680, 27, 681, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{5/2}}{(2x+3)^5}dx$$

$$\Big\downarrow \text{ 681}$$

$$-\frac{5}{64}\int \frac{4(8-57x)\left(3x^2+2\right)^{3/2}}{(2x+3)^4}dx-\frac{(4x+19)\left(3x^2+2\right)^{5/2}}{16(2x+3)^4}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{5}{16}\int \frac{(8-57x)\left(3x^2+2\right)^{3/2}}{(2x+3)^4}dx-\frac{(4x+19)\left(3x^2+2\right)^{5/2}}{16(2x+3)^4}$$

$$\Big\downarrow \text{ 680}$$

$$-\frac{5}{16}\left(\frac{(5517x+5003)\left(3x^2+2\right)^{3/2}}{210(2x+3)^3}-\frac{1}{560}\int -\frac{24(374-1917x)\sqrt{3x^2+2}}{(2x+3)^2}dx\right)-\frac{(4x+19)\left(3x^2+2\right)^{5/2}}{16(2x+3)^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{5}{16}\left(\frac{3}{70}\int \frac{(374-1917x)\sqrt{3x^2+2}}{(2x+3)^2}dx+\frac{(5517x+5003)\left(3x^2+2\right)^{3/2}}{210(2x+3)^3}\right)-\frac{(4x+19)\left(3x^2+2\right)^{5/2}}{16(2x+3)^4}$$

$$\Big\downarrow \; 681$$

$$-\frac{5}{16}\left(\frac{3}{70}\left(-\frac{1}{8}\int \frac{12(1278-6125x)}{(2x+3)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}(1917x+6125)}{2(2x+3)}\right)+\frac{(5517x+5003)\left(3x^2+2\right)^{3/2}}{210(2x+3)^3}\right)-\frac{(4x+19)\left(3x^2+2\right)^{5/2}}{16(2x+3)^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{5}{16}\left(\frac{3}{70}\left(-\frac{3}{2}\int \frac{1278-6125x}{(2x+3)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}(1917x+6125)}{2(2x+3)}\right)+\frac{(5517x+5003)\left(3x^2+2\right)^{3/2}}{210(2x+3)^3}\right)-\frac{(4x+19)\left(3x^2+2\right)^{5/2}}{16(2x+3)^4}$$

$$\Big\downarrow \; 719$$

$$-\frac{5}{16}\left(\frac{3}{70}\left(-\frac{3}{2}\left(\frac{20931}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{6125}{2}\int \frac{1}{\sqrt{3x^2+2}}dx\right)-\frac{\sqrt{3x^2+2}(1917x+6125)}{2(2x+3)}\right)+\frac{(5517\cdots\right.$$
$$-\frac{(4x+19)\left(3x^2+2\right)^{5/2}}{16(2x+3)^4}$$

$$\Big\downarrow \; 222$$

$$-\frac{5}{16}\left(\frac{3}{70}\left(-\frac{3}{2}\left(\frac{20931}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{6125\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)-\frac{\sqrt{3x^2+2}(1917x+6125)}{2(2x+3)}\right)+\frac{(5517\cdots\right.$$
$$-\frac{(4x+19)\left(3x^2+2\right)^{5/2}}{16(2x+3)^4}$$

$$\Big\downarrow \; 488$$

$$-\frac{5}{16}\left(\frac{3}{70}\left(-\frac{3}{2}\left(-\frac{20931}{2}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{6125\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)-\frac{\sqrt{3x^2+2}(1917x+6125)}{2(2x+3)}\right)+\right.$$

$$\frac{(4x+19)\left(3x^2+2\right)^{5/2}}{16(2x+3)^4}$$

$$\Big\downarrow \; 219$$

$$-\frac{5}{16}\left(\frac{3}{70}\left(-\frac{3}{2}\left(-\frac{6125\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{20931\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{2\sqrt{35}}\right)-\frac{\sqrt{3x^2+2}(1917x+6125)}{2(2x+3)}\right)+\frac{(5517x}{}\right.$$

$$\frac{(4x+19)\left(3x^2+2\right)^{5/2}}{16(2x+3)^4}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^5,x]
```

output
```
-1/16*((19 + 4*x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^4 - (5*(((5003 + 5517*x)*(2
 + 3*x^2)^(3/2))/(210*(3 + 2*x)^3) + (3*(-1/2*((6125 + 1917*x)*Sqrt[2 + 3*
x^2])/(3 + 2*x) - (3*((-6125*ArcSinh[Sqrt[3/2]*x])/(2*Sqrt[3]) - (20931*Ar
cTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(2*Sqrt[35])))/2))/70))/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{9072x^7-172368x^6-2690154x^5-8403372x^4-11802495x^3-9791505x^2-6670018x-2843910}{1344(2x+3)^4\sqrt{3x^2+2}}-\frac{2625\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{128}-$
trager	$-\frac{(3024x^5-57456x^4-898734x^3-2762820x^2-3335009x-1421955)\sqrt{3x^2+2}}{1344(2x+3)^4}+\frac{188379\,\operatorname{RootOf}\left(\_Z^2-35\right)\ln\left(\frac{9\,\operatorname{RootOf}\left(\_Z^2-\right.}{}\right.}{31360}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{7}{2}}}{2240\left(x+\frac{3}{2}\right)^4}+\frac{23\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{7}{2}}}{117600\left(x+\frac{3}{2}\right)^3}-\frac{1041\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{7}{2}}}{343000\left(x+\frac{3}{2}\right)^2}+\frac{29717\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{7}{2}}}{6002500\left(x+\frac{3}{2}\right)}+$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(2*x+3)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/1344*(9072*x^7-172368*x^6-2690154*x^5-8403372*x^4-11802495*x^3-9791505*
x^2-6670018*x-2843910)/(2*x+3)^4/(3*x^2+2)^(1/2)-2625/128*arcsinh(1/2*6^(1
/2)*x)*3^(1/2)-188379/31360*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+
3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.40

$$\int \frac{(5-x)\,(2+3x^2)^{5/2}}{(3+2x)^5}\,dx = \frac{1929375\,\sqrt{3}(16\,x^4+96\,x^3+216\,x^2+216\,x+81)\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-\right.}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^5,x, algorithm="fricas")
```

output
```
1/188160*(1929375*sqrt(3)*(16*x^4 + 96*x^3 + 216*x^2 + 216*x + 81)*log(sqr
t(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + 565137*sqrt(35)*(16*x^4 + 96*x^3 + 2
16*x^2 + 216*x + 81)*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) + 93*x^2 - 3
6*x + 43)/(4*x^2 + 12*x + 9)) - 140*(3024*x^5 - 57456*x^4 - 898734*x^3 - 2
762820*x^2 - 3335009*x - 1421955)*sqrt(3*x^2 + 2))/(16*x^4 + 96*x^3 + 216*
x^2 + 216*x + 81)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^5}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(5/2)/(3+2*x)**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(100) = 200$.

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.63

$$\begin{aligned}\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^5}\,dx &= \frac{3123}{343000}\left(3x^2+2\right)^{\frac{5}{2}} \\ &- \frac{13\left(3x^2+2\right)^{\frac{7}{2}}}{140\left(16x^4+96x^3+216x^2+216x+81\right)} + \frac{23\left(3x^2+2\right)^{\frac{7}{2}}}{14700\left(8x^3+36x^2+54x+27\right)} \\ &- \frac{1041\left(3x^2+2\right)^{\frac{7}{2}}}{85750\left(4x^2+12x+9\right)} - \frac{58629}{274400}\left(3x^2+2\right)^{\frac{3}{2}}x + \frac{62793}{137200}\left(3x^2+2\right)^{\frac{3}{2}} \\ &+ \frac{29717\left(3x^2+2\right)^{\frac{5}{2}}}{343000\left(2x+3\right)} - \frac{58491}{15680}\sqrt{3x^2+2}x - \frac{2625}{128}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) \\ &+ \frac{188379}{31360}\sqrt{35}\,\operatorname{arsinh}\left(\frac{3\sqrt{6}x}{2\left|2x+3\right|} - \frac{2\sqrt{6}}{3\left|2x+3\right|}\right) + \frac{188379}{15680}\sqrt{3x^2+2}\end{aligned}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^5,x, algorithm="maxima")
```

output
```
3123/343000*(3*x^2 + 2)^(5/2) - 13/140*(3*x^2 + 2)^(7/2)/(16*x^4 + 96*x^3
+ 216*x^2 + 216*x + 81) + 23/14700*(3*x^2 + 2)^(7/2)/(8*x^3 + 36*x^2 + 54*
x + 27) - 1041/85750*(3*x^2 + 2)^(7/2)/(4*x^2 + 12*x + 9) - 58629/274400*(
3*x^2 + 2)^(3/2)*x + 62793/137200*(3*x^2 + 2)^(3/2) + 29717/343000*(3*x^2
+ 2)^(5/2)/(2*x + 3) - 58491/15680*sqrt(3*x^2 + 2)*x - 2625/128*sqrt(3)*ar
csinh(1/2*sqrt(6)*x) + 188379/31360*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x
 + 3) - 2/3*sqrt(6)/abs(2*x + 3)) + 188379/15680*sqrt(3*x^2 + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(100) = 200$.

Time = 0.31 (sec) , antiderivative size = 440, normalized size of antiderivative = 3.49

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^5}\,dx =$$

$$-\frac{188379}{31360}\sqrt{35}\log\left(\sqrt{35}\left(\sqrt{-\frac{18}{2\,x+3}+\frac{35}{(2\,x+3)^2}+3}+\frac{\sqrt{35}}{2\,x+3}\right)-9\right)\operatorname{sgn}\left(\frac{1}{2\,x+3}\right)$$

$$+\frac{2625}{128}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{2\sqrt{35}}{2x+3}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)}\right)\operatorname{sgn}\left(\frac{1}{2\,x+3}\right)$$

$$-\frac{1}{10752}\left(\frac{7\left(\frac{35\left(\frac{1365\operatorname{sgn}\left(\frac{1}{2x+3}\right)}{2x+3}-2129\operatorname{sgn}\left(\frac{1}{2x+3}\right)\right)}{2x+3}+57681\operatorname{sgn}\left(\frac{1}{2x+3}\right)\right)}{2\,x+3}-242979\operatorname{sgn}\left(\frac{1}{2\,x+3}\right)\right)\sqrt{-\frac{18}{2\,x+3}}$$

$$-\frac{9\left(256\left(\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)^3\operatorname{sgn}\left(\frac{1}{2x+3}\right)-93\sqrt{35}\left(\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)^2\operatorname{sgn}\left(\frac{1}{2x}\right.}{64\left(\left(\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right.\right.}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^5,x, algorithm="giac")
```

output
```
-188379/31360*sqrt(35)*log(sqrt(35)*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 +
 3) + sqrt(35)/(2*x + 3)) - 9)*sgn(1/(2*x + 3)) + 2625/128*sqrt(3)*log(1/2
*abs(-2*sqrt(3) + 2*sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + 2*sqrt(35)/
(2*x + 3))/(sqrt(3) + sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(35)/
(2*x + 3)))*sgn(1/(2*x + 3)) - 1/10752*(7*(35*(1365*sgn(1/(2*x + 3))/(2*x
+ 3) - 2129*sgn(1/(2*x + 3)))/(2*x + 3) + 57681*sgn(1/(2*x + 3)))/(2*x + 3
) - 242979*sgn(1/(2*x + 3)))*sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) - 9/
64*(256*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3))^3*
sgn(1/(2*x + 3)) - 93*sqrt(35)*(sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) +
 sqrt(35)/(2*x + 3))^2*sgn(1/(2*x + 3)) - 582*(sqrt(-18/(2*x + 3) + 35/(2*
x + 3)^2 + 3) + sqrt(35)/(2*x + 3))*sgn(1/(2*x + 3)) + 225*sqrt(35)*sgn(1/
(2*x + 3)))/((sqrt(-18/(2*x + 3) + 35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3
))^2 - 3)^2
```

Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.43

$$
\begin{aligned}
\int \frac{(5-x)\,(2+3x^2)^{5/2}}{(3+2x)^5}\,dx &= \frac{188379\,\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{31360}+\frac{225\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{64}\\
&-\frac{2625\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{128}-\frac{188379\,\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{35}\,\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{31360}\\
&-\frac{15925\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{4096\,\left(x^4+6\,x^3+\frac{27\,x^2}{2}+\frac{27\,x}{2}+\frac{81}{16}\right)}+\frac{80993\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{3584\,\left(x+\frac{3}{2}\right)}\\
&-\frac{19227\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{1024\,\left(x^2+3\,x+\frac{9}{4}\right)}-\frac{9\,\sqrt{3}\,x\,\sqrt{x^2+\frac{2}{3}}}{64}+\frac{74515\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{6144\,\left(x^3+\frac{9\,x^2}{2}+\frac{27\,x}{4}+\frac{27}{8}\right)}
\end{aligned}
$$

input
```
int(-((3*x^2 + 2)^(5/2)*(x - 5))/(2*x + 3)^5,x)
```

output
```
(188379*35^(1/2)*log(x + 3/2))/31360 + (225*3^(1/2)*(x^2 + 2/3)^(1/2))/64
- (2625*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/128 - (188379*35^(1/2)*log(x
 - (3^(1/2)*35^(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/31360 - (15925*3^(1/2)*(
x^2 + 2/3)^(1/2))/(4096*((27*x)/2 + (27*x^2)/2 + 6*x^3 + x^4 + 81/16)) + (
80993*3^(1/2)*(x^2 + 2/3)^(1/2))/(3584*(x + 3/2)) - (19227*3^(1/2)*(x^2 +
2/3)^(1/2))/(1024*(3*x + x^2 + 9/4)) - (9*3^(1/2)*x*(x^2 + 2/3)^(1/2))/64
+ (74515*3^(1/2)*(x^2 + 2/3)^(1/2))/(6144*((27*x)/4 + (9*x^2)/2 + x^3 + 27
/8))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.79

$$\int \frac{(5-x)\,(2+3x^2)^{5/2}}{(3+2x)^5}\,dx = \frac{199073700\sqrt{3x^2+2}+30870000\sqrt{3}\log\left(\sqrt{3x^2+2}-\sqrt{3}\,x\right)x^4-30870000\sqrt{3}$$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^5,x)
```

output
```
( - 423360*sqrt(3*x**2 + 2)*x**5 + 8043840*sqrt(3*x**2 + 2)*x**4 + 1258227
60*sqrt(3*x**2 + 2)*x**3 + 386794800*sqrt(3*x**2 + 2)*x**2 + 466901260*sqr
t(3*x**2 + 2)*x + 199073700*sqrt(3*x**2 + 2) + 18084384*sqrt(35)*log(sqrt(
3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**4 + 108506304*sqrt(35)*log(sqrt(3*x**2
+ 2)*sqrt(35) + 9*x - 4)*x**3 + 244139184*sqrt(35)*log(sqrt(3*x**2 + 2)*sq
rt(35) + 9*x - 4)*x**2 + 244139184*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35)
+ 9*x - 4)*x + 91552194*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)
- 18084384*sqrt(35)*log(2*x + 3)*x**4 - 108506304*sqrt(35)*log(2*x + 3)*x*
*3 - 244139184*sqrt(35)*log(2*x + 3)*x**2 - 244139184*sqrt(35)*log(2*x + 3
)*x - 91552194*sqrt(35)*log(2*x + 3) + 30870000*sqrt(3)*log(sqrt(3*x**2 +
2) - sqrt(3)*x)*x**4 + 185220000*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)
*x**3 + 416745000*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x**2 + 4167450
00*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x + 156279375*sqrt(3)*log(sqr
t(3*x**2 + 2) - sqrt(3)*x) - 30870000*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(
3)*x)*x**4 - 185220000*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x**3 - 41
6745000*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x**2 - 416745000*sqrt(3)
*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x - 156279375*sqrt(3)*log(sqrt(3*x**2 +
 2) + sqrt(3)*x))/(188160*(16*x**4 + 96*x**3 + 216*x**2 + 216*x + 81))
```

3.230 $\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^6}\,dx = -\frac{9(8575+2643x)\sqrt{2+3x^2}}{19600(3+2x)} + \frac{(6637+8193x)\left(2+3x^2\right)^{3/2}}{9800(3+2x)^3} + \frac{(23+76x)\left(2+3x^2\right)^{5/2}}{140(3+2x)^5} + \frac{63}{32}\sqrt{3}\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right) + \frac{789723\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{39200\sqrt{35}}$$

output
```
-9*(8575+2643*x)*(3*x^2+2)^(1/2)/(58800+39200*x)+1/9800*(6637+8193*x)*(3*x
^2+2)^(3/2)/(3+2*x)^3+1/140*(23+76*x)*(3*x^2+2)^(5/2)/(3+2*x)^5+63/32*arcs
inh(1/2*x*6^(1/2))*3^(1/2)+789723/1372000*35^(1/2)*arctanh(1/35*(4-9*x)*35
^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^6}\,dx = -\frac{\sqrt{2+3x^2}(5999363+17940463x+20911298x^2+11367738x^3+2740188x^4+88200x^5)}{19600(3+2x)^5} - \frac{789723\text{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{19600\sqrt{35}} - \frac{63}{32}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^6,x]
```

output
```
-1/19600*(Sqrt[2 + 3*x^2]*(5999363 + 17940463*x + 20911298*x^2 + 11367738*
x^3 + 2740188*x^4 + 88200*x^5))/(3 + 2*x)^5 - (789723*ArcTanh[(3*Sqrt[3] +
 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]])/(19600*Sqrt[35]) - (63*Sqrt[3
]*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/32
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {680, 27, 680, 27, 681, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{5/2}}{(2x+3)^6}\,dx$$

$$\Big\downarrow \; 680$$

$$\frac{(76x+23)\left(3x^2+2\right)^{5/2}}{140(2x+3)^5} - \frac{\int -\frac{24(52-73x)\left(3x^2+2\right)^{3/2}}{(2x+3)^4}dx}{1120}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{140}\int \frac{(52-73x)\left(3x^2+2\right)^{3/2}}{(2x+3)^4}dx+\frac{(76x+23)\left(3x^2+2\right)^{5/2}}{140(2x+3)^5}$$

$$\Big\downarrow \; 680$$

$$\frac{3}{140}\left(\frac{(8193x+6637)\left(3x^2+2\right)^{3/2}}{210(2x+3)^3}-\frac{1}{560}\int -\frac{24(646-2643x)\sqrt{3x^2+2}}{(2x+3)^2}dx\right)+\frac{(76x+23)\left(3x^2+2\right)^{5/2}}{140(2x+3)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{140}\left(\frac{3}{70}\int \frac{(646-2643x)\sqrt{3x^2+2}}{(2x+3)^2}dx+\frac{(8193x+6637)\left(3x^2+2\right)^{3/2}}{210(2x+3)^3}\right)+\frac{(76x+23)\left(3x^2+2\right)^{5/2}}{140(2x+3)^5}$$

$$\Big\downarrow \; 681$$

$$\frac{3}{140}\left(\frac{3}{70}\left(-\frac{1}{8}\int \frac{12(1762-8575x)}{(2x+3)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}(2643x+8575)}{2(2x+3)}\right)+\frac{(8193x+6637)\left(3x^2+2\right)^{3/2}}{210(2x+3)^3}\right)+\frac{(76x+23)\left(3x^2+2\right)^{5/2}}{140(2x+3)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{140}\left(\frac{3}{70}\left(-\frac{3}{2}\int \frac{1762-8575x}{(2x+3)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}(2643x+8575)}{2(2x+3)}\right)+\frac{(8193x+6637)\left(3x^2+2\right)^{3/2}}{210(2x+3)^3}\right)+\frac{(76x+23)\left(3x^2+2\right)^{5/2}}{140(2x+3)^5}$$

$$\Big\downarrow \; 719$$

$$\frac{3}{140}\left(\frac{3}{70}\left(-\frac{3}{2}\left(\frac{29249}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{8575}{2}\int \frac{1}{\sqrt{3x^2+2}}dx\right)-\frac{\sqrt{3x^2+2}(2643x+8575)}{2(2x+3)}\right)+\frac{(8193x\ldots\right)+\frac{(76x+23)\left(3x^2+2\right)^{5/2}}{140(2x+3)^5}$$

$$\Big\downarrow \; 222$$

$$\frac{3}{140}\left(\frac{3}{70}\left(-\frac{3}{2}\left(\frac{29249}{2}\int\frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{8575\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)-\frac{\sqrt{3x^2+2}(2643x+8575)}{2(2x+3)}\right)+\frac{(8193}{}\right.\right.$$
$$\frac{(76x+23)\left(3x^2+2\right)^{5/2}}{140(2x+3)^5}$$

$\Big\downarrow$ 488

$$\frac{3}{140}\left(\frac{3}{70}\left(-\frac{3}{2}\left(-\frac{29249}{2}\int\frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{8575\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)-\frac{\sqrt{3x^2+2}(2643x+8575)}{2(2x+3)}\right)+\frac{(}{}\right.\right.$$
$$\frac{(76x+23)\left(3x^2+2\right)^{5/2}}{140(2x+3)^5}$$

$\Big\downarrow$ 219

$$\frac{3}{140}\left(\frac{3}{70}\left(-\frac{3}{2}\left(-\frac{8575\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{29249\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{2\sqrt{35}}\right)-\frac{\sqrt{3x^2+2}(2643x+8575)}{2(2x+3)}\right)+\frac{(8193x}{}\right.\right.$$
$$\frac{(76x+23)\left(3x^2+2\right)^{5/2}}{140(2x+3)^5}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^6,x]
```

output
```
((23 + 76*x)*(2 + 3*x^2)^(5/2))/(140*(3 + 2*x)^5) + (3*(((6637 + 8193*x)*(
2 + 3*x^2)^(3/2))/(210*(3 + 2*x)^3) + (3*(-1/2*((8575 + 2643*x)*Sqrt[2 + 3
*x^2])/(3 + 2*x) - (3*((-8575*ArcSinh[Sqrt[3/2]*x])/(2*Sqrt[3]) - (29249*A
rcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(2*Sqrt[35])))/2))/70))/140
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{264600x^7+8220564x^6+34279614x^5+68214270x^4+76556865x^3+59820685x^2+35880926x+11998726}{19600(2x+3)^5\sqrt{3x^2+2}}+\frac{63\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{32}$
trager	$-\frac{(88200x^5+2740188x^4+11367738x^3+20911298x^2+17940463x+5999363)\sqrt{3x^2+2}}{19600(2x+3)^5}-\frac{789723\,\mathrm{RootOf}\left(\_Z^2-35\right)\ln\left(\frac{9\,\mathrm{RootOf}}{}\right.}{}$
default	$-\frac{13\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{7}{2}}}{5600\left(x+\frac{3}{2}\right)^5}-\frac{789723\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}{1372000}-\frac{789723\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{262609375}-\frac{263241\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)}{6002500}$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(2*x+3)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/19600*(264600*x^7+8220564*x^6+34279614*x^5+68214270*x^4+76556865*x^3+59
820685*x^2+35880926*x+11998726)/(2*x+3)^5/(3*x^2+2)^(1/2)+63/32*arcsinh(1/
2*6^(1/2)*x)*3^(1/2)+789723/1372000*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)
/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.44

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^6}\,dx = \frac{2701125\sqrt{3}(32x^5+240x^4+720x^3+1080x^2+810x+243)\log\left(-\sqrt{3}\sqrt{3x\right.}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^6,x, algorithm="fricas")
```

output
```
1/2744000*(2701125*sqrt(3)*(32*x^5 + 240*x^4 + 720*x^3 + 1080*x^2 + 810*x
+ 243)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + 789723*sqrt(35)*(32*x
^5 + 240*x^4 + 720*x^3 + 1080*x^2 + 810*x + 243)*log((sqrt(35)*sqrt(3*x^2
+ 2)*(9*x - 4) - 93*x^2 + 36*x - 43)/(4*x^2 + 12*x + 9)) - 140*(88200*x^5
+ 2740188*x^4 + 11367738*x^3 + 20911298*x^2 + 17940463*x + 5999363)*sqrt(3
*x^2 + 2))/(32*x^5 + 240*x^4 + 720*x^3 + 1080*x^2 + 810*x + 243)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^6}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(5/2)/(3+2*x)**6,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(107) = 214$.

Time = 0.14 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.83

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^6}\,dx = \frac{6723}{30012500}\left(3\,x^2+2\right)^{\frac{5}{2}} - \frac{13\left(3\,x^2+2\right)^{\frac{7}{2}}}{175\left(32\,x^5+240\,x^4+720\,x^3+1080\,x^2+810\,x+243\right)} - \frac{44\left(3\,x^2+2\right)^{\frac{7}{2}}}{6125\left(16\,x^4+96\,x^3+216\,x^2+216\,x+81\right)} - \frac{1042\left(3\,x^2+2\right)^{\frac{7}{2}}}{214375\left(8\,x^3+36\,x^2+54\,x+27\right)} - \frac{2241\left(3\,x^2+2\right)^{\frac{7}{2}}}{7503125\left(4\,x^2+12\,x+9\right)} + \frac{267723}{12005000}\left(3\,x^2+2\right)^{\frac{3}{2}}x - \frac{263241}{6002500}\left(3\,x^2+2\right)^{\frac{3}{2}} - \frac{377133\left(3\,x^2+2\right)^{\frac{5}{2}}}{30012500\,(2\,x+3)} + \frac{248967}{686000}\sqrt{3\,x^2+2}x + \frac{63}{32}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) - \frac{789723}{1372000}\sqrt{35}\,\mathrm{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|} - \frac{2\sqrt{6}}{3\,|2\,x+3|}\right) - \frac{789723}{686000}\sqrt{3\,x^2+2}$$

input

```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^6,x, algorithm="maxima")
```

output

```
6723/30012500*(3*x^2 + 2)^(5/2) - 13/175*(3*x^2 + 2)^(7/2)/(32*x^5 + 240*x
^4 + 720*x^3 + 1080*x^2 + 810*x + 243) - 44/6125*(3*x^2 + 2)^(7/2)/(16*x^4
 + 96*x^3 + 216*x^2 + 216*x + 81) - 1042/214375*(3*x^2 + 2)^(7/2)/(8*x^3 +
 36*x^2 + 54*x + 27) - 2241/7503125*(3*x^2 + 2)^(7/2)/(4*x^2 + 12*x + 9) +
 267723/12005000*(3*x^2 + 2)^(3/2)*x - 263241/6002500*(3*x^2 + 2)^(3/2) -
377133/30012500*(3*x^2 + 2)^(5/2)/(2*x + 3) + 248967/686000*sqrt(3*x^2 + 2
)*x + 63/32*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 789723/1372000*sqrt(35)*arcsi
nh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3)) - 789723/686000*
sqrt(3*x^2 + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(107) = 214$.

Time = 0.15 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.67

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^6}\,dx = -\frac{63}{32}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) - \frac{789723}{1372000}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3\,x^2+2}}\right) - \frac{9}{64}\sqrt{3\,x^2+2} - \frac{3\sqrt{3}\left(1034487\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^9+28143036\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^8+94364251\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2\cdots\right.}{\cdots}$$

input

```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^6,x, algorithm="giac")
```

output

```
-63/32*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 789723/1372000*sqrt(35)
*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqrt(3*x^2 + 2))/(2*sqrt
(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) - 9/64*sqrt(3*x^2 + 2)
- 3/156800*sqrt(3)*(1034487*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^9 + 2814
3036*(sqrt(3)*x - sqrt(3*x^2 + 2))^8 + 94364251*sqrt(3)*(sqrt(3)*x - sqrt(
3*x^2 + 2))^7 + 328235733*(sqrt(3)*x - sqrt(3*x^2 + 2))^6 - 120044232*sqrt
(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^5 - 774358774*(sqrt(3)*x - sqrt(3*x^2 +
2))^4 + 578739476*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 - 495467552*(sqr
t(3)*x - sqrt(3*x^2 + 2))^2 + 66595728*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2
)) - 11086336)/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 3*sqrt(3)*(sqrt(3)*x - s
qrt(3*x^2 + 2)) - 2)^5
```

Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.55

$$\int \frac{(5-x)(2+3x^2)^{5/2}}{(3+2x)^6}\,dx = \frac{63\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{32} - \frac{9\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{64} - \frac{789723\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{1372000} + \frac{789723\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{1372000} + \frac{2303\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{512\left(x^4+6x^3+\frac{27x^2}{2}+\frac{27x}{2}+\frac{81}{16}\right)} - \frac{3185\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{2048\left(x^5+\frac{15x^4}{2}+\frac{45x^3}{2}+\frac{135x^2}{4}+\frac{405x}{16}+\frac{243}{32}\right)} - \frac{64959\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{19600\left(x+\frac{3}{2}\right)} + \frac{44127\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{8960\left(x^2+3x+\frac{9}{4}\right)} - \frac{15397\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{2560\left(x^3+\frac{9x^2}{2}+\frac{27x}{4}+\frac{27}{8}\right)}$$

input

```
int(-((3*x^2 + 2)^(5/2)*(x - 5))/(2*x + 3)^6,x)
```

output

```
(63*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/32 - (9*3^(1/2)*(x^2 + 2/3)^(1/2
))/64 - (789723*35^(1/2)*log(x + 3/2))/1372000 + (789723*35^(1/2)*log(x -
(3^(1/2)*35^(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/1372000 + (2303*3^(1/2)*(x^
2 + 2/3)^(1/2))/(512*((27*x)/2 + (27*x^2)/2 + 6*x^3 + x^4 + 81/16)) - (318
5*3^(1/2)*(x^2 + 2/3)^(1/2))/(2048*((405*x)/16 + (135*x^2)/4 + (45*x^3)/2
+ (15*x^4)/2 + x^5 + 243/32)) - (64959*3^(1/2)*(x^2 + 2/3)^(1/2))/(19600*(
x + 3/2)) + (44127*3^(1/2)*(x^2 + 2/3)^(1/2))/(8960*(3*x + x^2 + 9/4)) - (
15397*3^(1/2)*(x^2 + 2/3)^(1/2))/(2560*((27*x)/4 + (9*x^2)/2 + x^3 + 27/8)
)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 568, normalized size of antiderivative = 4.27

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^6}\,dx = \text{Too large to display}$$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^6,x)
```

output
```
( - 12348000*sqrt(3*x**2 + 2)*x**5 - 383626320*sqrt(3*x**2 + 2)*x**4 - 159
1483320*sqrt(3*x**2 + 2)*x**3 - 2927581720*sqrt(3*x**2 + 2)*x**2 - 2511664
820*sqrt(3*x**2 + 2)*x - 839910820*sqrt(3*x**2 + 2) + 50542272*sqrt(35)*lo
g( - sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**5 + 379067040*sqrt(35)*log( -
 sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**4 + 1137201120*sqrt(35)*log( - sq
rt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**3 + 1705801680*sqrt(35)*log( - sqrt(
3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**2 + 1279351260*sqrt(35)*log( - sqrt(3*x
**2 + 2)*sqrt(35) + 9*x - 4)*x + 383805378*sqrt(35)*log( - sqrt(3*x**2 + 2
)*sqrt(35) + 9*x - 4) - 50542272*sqrt(35)*log(2*x + 3)*x**5 - 379067040*sq
rt(35)*log(2*x + 3)*x**4 - 1137201120*sqrt(35)*log(2*x + 3)*x**3 - 1705801
680*sqrt(35)*log(2*x + 3)*x**2 - 1279351260*sqrt(35)*log(2*x + 3)*x - 3838
05378*sqrt(35)*log(2*x + 3) - 86436000*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt
(3)*x)*x**5 - 648270000*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x**4 - 1
944810000*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x**3 - 2917215000*sqrt
(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x**2 - 2187911250*sqrt(3)*log(sqrt(3
*x**2 + 2) - sqrt(3)*x)*x - 656373375*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(
3)*x) + 86436000*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x**5 + 64827000
0*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x**4 + 1944810000*sqrt(3)*log(
sqrt(3*x**2 + 2) + sqrt(3)*x)*x**3 + 2917215000*sqrt(3)*log(sqrt(3*x**2 +
2) + sqrt(3)*x)*x**2 + 2187911250*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3...
```

3.231 $\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^7}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^7}\,dx = \frac{9(4373+5167x)\sqrt{2+3x^2}}{109760(3+2x)^2} + \frac{(202+403x)\left(2+3x^2\right)^{3/2}}{1568(3+2x)^4} + \frac{(11+159x)\left(2+3x^2\right)^{5/2}}{420(3+2x)^6} - \frac{9}{128}\sqrt{3}\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right) - \frac{159759\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{219520\sqrt{35}}$$

output

```
9/109760*(4373+5167*x)*(3*x^2+2)^(1/2)/(3+2*x)^2+1/1568*(202+403*x)*(3*x^2
+2)^(3/2)/(3+2*x)^4+1/420*(11+159*x)*(3*x^2+2)^(5/2)/(3+2*x)^6-9/128*arcsi
nh(1/2*x*6^(1/2))*3^(1/2)-159759/7683200*35^(1/2)*arctanh(1/35*(4-9*x)*35^
(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 3.95 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^7}\,dx = \frac{\frac{70\sqrt{2+3x^2}(10361807+39843609x+59256588x^2+47453802x^3+18915336x^4+4369608x^5)}{(3+2x)^6}+958554\sqrt{\ldots}}{2304960\ldots}$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^7,x]
```

output
```
((70*Sqrt[2 + 3*x^2]*(10361807 + 39843609*x + 59256588*x^2 + 47453802*x^3
+ 18915336*x^4 + 4369608*x^5))/(3 + 2*x)^6 + 958554*Sqrt[35]*ArcTanh[(3*Sq
rt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]] + 1620675*Sqrt[3]*Log[-
(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/23049600
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {680, 27, 680, 27, 680, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{5/2}}{(2x+3)^7}dx$$

$$\Big\downarrow \text{680}$$

$$\frac{(159x+11)\left(3x^2+2\right)^{5/2}}{420(2x+3)^6}-\frac{\int -\frac{60(26-21x)\left(3x^2+2\right)^{3/2}}{(2x+3)^5}dx}{1680}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{28}\int \frac{(26-21x)\left(3x^2+2\right)^{3/2}}{(2x+3)^5}dx+\frac{(159x+11)\left(3x^2+2\right)^{5/2}}{420(2x+3)^6}$$

$$\Big\downarrow \text{680}$$

$$\frac{1}{28}\left(\frac{(403x+202)\left(3x^2+2\right)^{3/2}}{56(2x+3)^4}-\frac{\int -\frac{360(23-49x)\sqrt{3x^2+2}}{(2x+3)^3}dx}{1120}\right)+\frac{(159x+11)\left(3x^2+2\right)^{5/2}}{420(2x+3)^6}$$

$\big\downarrow$ 27

$$\frac{1}{28}\left(\frac{9}{28}\int \frac{(23-49x)\sqrt{3x^2+2}}{(2x+3)^3}dx+\frac{(403x+202)\left(3x^2+2\right)^{3/2}}{56(2x+3)^4}\right)+\frac{(159x+11)\left(3x^2+2\right)^{5/2}}{420(2x+3)^6}$$

$\big\downarrow$ 680

$$\frac{1}{28}\left(\frac{9}{28}\left(\frac{(5167x+4373)\sqrt{3x^2+2}}{140(2x+3)^2}-\frac{1}{560}\int -\frac{12(386-1715x)}{(2x+3)\sqrt{3x^2+2}}dx\right)+\frac{(403x+202)\left(3x^2+2\right)^{3/2}}{56(2x+3)^4}\right)+\frac{(159x+11)\left(3x^2+2\right)^{5/2}}{420(2x+3)^6}$$

$\big\downarrow$ 27

$$\frac{1}{28}\left(\frac{9}{28}\left(\frac{3}{140}\int \frac{386-1715x}{(2x+3)\sqrt{3x^2+2}}dx+\frac{\sqrt{3x^2+2}(5167x+4373)}{140(2x+3)^2}\right)+\frac{(403x+202)\left(3x^2+2\right)^{3/2}}{56(2x+3)^4}\right)+\frac{(159x+11)\left(3x^2+2\right)^{5/2}}{420(2x+3)^6}$$

$\big\downarrow$ 719

$$\frac{1}{28}\left(\frac{9}{28}\left(\frac{3}{140}\left(\frac{5917}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{1715}{2}\int \frac{1}{\sqrt{3x^2+2}}dx\right)+\frac{\sqrt{3x^2+2}(5167x+4373)}{140(2x+3)^2}\right)+\frac{(403x+}{5}\right.$$
$$\frac{(159x+11)\left(3x^2+2\right)^{5/2}}{420(2x+3)^6}$$

$\big\downarrow$ 222

$$\frac{1}{28}\left(\frac{9}{28}\left(\frac{3}{140}\left(\frac{5917}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{1715\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)+\frac{\sqrt{3x^2+2}(5167x+4373)}{140(2x+3)^2}\right)+\frac{(403x+}{5}\right.$$
$$\frac{(159x+11)\left(3x^2+2\right)^{5/2}}{420(2x+3)^6}$$

$\big\downarrow$ 488

$$\frac{1}{28}\left(\frac{9}{28}\left(\frac{3}{140}\left(-\frac{5917}{2}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{1715\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}\right)+\frac{\sqrt{3x^2+2}(5167x+4373)}{140(2x+3)^2}\right)+\frac{(40}{}\right.$$
$$\frac{(159x+11)\left(3x^2+2\right)^{5/2}}{420(2x+3)^6}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{28}\left(\frac{9}{28}\left(\frac{3}{140}\left(-\frac{1715\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{5917\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{2\sqrt{35}}\right)+\frac{\sqrt{3x^2+2}(5167x+4373)}{140(2x+3)^2}\right)+\frac{(403x+}{56}\right.$$
$$\frac{(159x+11)\left(3x^2+2\right)^{5/2}}{420(2x+3)^6}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^7,x]
```

output
```
((11 + 159*x)*(2 + 3*x^2)^(5/2))/(420*(3 + 2*x)^6) + (((202 + 403*x)*(2 +
3*x^2)^(3/2))/(56*(3 + 2*x)^4) + (9*(((4373 + 5167*x)*Sqrt[2 + 3*x^2])/(14
0*(3 + 2*x)^2) + (3*((-1715*ArcSinh[Sqrt[3/2]*x])/(2*Sqrt[3]) - (5917*ArcT
anh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(2*Sqrt[35])))/140))/28)/28
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.73

method	result
risch	$\frac{13108824x^7+56746008x^6+151100622x^5+215600436x^4+214438431x^3+149598597x^2+79687218x+20723614}{329280(2x+3)^6\sqrt{3x^2+2}}-\frac{9\,\text{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{128}$
trager	$\frac{(4369608x^5+18915336x^4+47453802x^3+59256588x^2+39843609x+10361807)\sqrt{3x^2+2}}{329280(2x+3)^6}+\frac{9\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\text{RootOf}\left(\_\right.\right.}{128}$
default	$-\frac{\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{7}{2}}}{3136\left(x+\frac{3}{2}\right)^5}+\frac{159759\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}{7683200}+\frac{159759\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{1470612500}+\frac{53253\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{33614000}$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(2*x+3)^7,x,method=_RETURNVERBOSE)
```

output
```
1/329280*(13108824*x^7+56746008*x^6+151100622*x^5+215600436*x^4+214438431*
x^3+149598597*x^2+79687218*x+20723614)/(2*x+3)^6/(3*x^2+2)^(1/2)-9/128*arc
sinh(1/2*6^(1/2)*x)*3^(1/2)-159759/7683200*35^(1/2)*arctanh(2/35*(4-9*x)*3
5^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.55

$$\int \frac{(5-x)(2+3x^2)^{5/2}}{(3+2x)^7}\,dx = \frac{1620675\sqrt{3}(64x^6+576x^5+2160x^4+4320x^3+4860x^2+2916x+729)\log}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^7,x, algorithm="fricas")
```

output
```
1/46099200*(1620675*sqrt(3)*(64*x^6 + 576*x^5 + 2160*x^4 + 4320*x^3 + 4860
*x^2 + 2916*x + 729)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + 479277*s
qrt(35)*(64*x^6 + 576*x^5 + 2160*x^4 + 4320*x^3 + 4860*x^2 + 2916*x + 729)
*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 1
2*x + 9)) + 140*(4369608*x^5 + 18915336*x^4 + 47453802*x^3 + 59256588*x^2
+ 39843609*x + 10361807)*sqrt(3*x^2 + 2))/(64*x^6 + 576*x^5 + 2160*x^4 + 4
320*x^3 + 4860*x^2 + 2916*x + 729)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)(2+3x^2)^{5/2}}{(3+2x)^7}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(5/2)/(3+2*x)**7,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(107) = 214$.

Time = 0.15 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.16

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^7}\,dx = \frac{19683}{84035000}\left(3x^2+2\right)^{\frac{5}{2}} - \frac{13\left(3x^2+2\right)^{\frac{7}{2}}}{210\left(64x^6+576x^5+2160x^4+4320x^3+4860x^2+2916x+729\right)} - \frac{\left(3x^2+2\right)^{\frac{7}{2}}}{98\left(32x^5+240x^4+720x^3+1080x^2+810x+243\right)} - \frac{113\left(3x^2+2\right)^{\frac{7}{2}}}{34300\left(16x^4+96x^3+216x^2+216x+81\right)} - \frac{1039\left(3x^2+2\right)^{\frac{7}{2}}}{1200500\left(8x^3+36x^2+54x+27\right)} - \frac{6561\left(3x^2+2\right)^{\frac{7}{2}}}{21008750\left(4x^2+12x+9\right)} - \frac{27009}{67228000}\left(3x^2+2\right)^{\frac{3}{2}}x + \frac{53253}{33614000}\left(3x^2+2\right)^{\frac{3}{2}} - \frac{41043\left(3x^2+2\right)^{\frac{5}{2}}}{84035000\left(2x+3\right)} - \frac{45711}{3841600}\sqrt{3x^2+2}\,x - \frac{9}{128}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{159759}{7683200}\sqrt{35}\operatorname{arsinh}\left(\frac{3\sqrt{6}x}{2\left|2x+3\right|} - \frac{2\sqrt{6}}{3\left|2x+3\right|}\right) + \frac{159759}{3841600}\sqrt{3x^2+2}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^7,x, algorithm="maxima")
```

output
```
19683/84035000*(3*x^2 + 2)^(5/2) - 13/210*(3*x^2 + 2)^(7/2)/(64*x^6 + 576*
x^5 + 2160*x^4 + 4320*x^3 + 4860*x^2 + 2916*x + 729) - 1/98*(3*x^2 + 2)^(7
/2)/(32*x^5 + 240*x^4 + 720*x^3 + 1080*x^2 + 810*x + 243) - 113/34300*(3*x
^2 + 2)^(7/2)/(16*x^4 + 96*x^3 + 216*x^2 + 216*x + 81) - 1039/1200500*(3*x
^2 + 2)^(7/2)/(8*x^3 + 36*x^2 + 54*x + 27) - 6561/21008750*(3*x^2 + 2)^(7/
2)/(4*x^2 + 12*x + 9) - 27009/67228000*(3*x^2 + 2)^(3/2)*x + 53253/3361400
0*(3*x^2 + 2)^(3/2) - 41043/84035000*(3*x^2 + 2)^(5/2)/(2*x + 3) - 45711/3
841600*sqrt(3*x^2 + 2)*x - 9/128*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 159759/7
683200*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x +
 3)) + 159759/3841600*sqrt(3*x^2 + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(107) = 214$.

Time = 0.16 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.92

$$\int \frac{(5-x)(2+3x^2)^{5/2}}{(3+2x)^7}\,dx = \frac{9}{128}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$
$$+\frac{159759}{7683200}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3\,x^2+2}}\right)$$
$$+\frac{3\sqrt{3}\Big(566976\sqrt{3}\big(\sqrt{3}x-\sqrt{3\,x^2+2}\big)^{11}+16427322\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^{10}+70792520\sqrt{3}\big(\sqrt{3}x-\sqrt{3\,x^2-}}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^7,x, algorithm="giac")
```

output
```
9/128*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 159759/7683200*sqrt(35)*
log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqrt(3*x^2 + 2))/(2*sqrt(
3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) + 3/878080*sqrt(3)*(5669
76*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^11 + 16427322*(sqrt(3)*x - sqrt(3
*x^2 + 2))^10 + 70792520*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^9 + 4213780
65*(sqrt(3)*x - sqrt(3*x^2 + 2))^8 + 244013814*sqrt(3)*(sqrt(3)*x - sqrt(3
*x^2 + 2))^7 - 879808433*(sqrt(3)*x - sqrt(3*x^2 + 2))^6 - 512612604*sqrt(
3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^5 + 2079633300*(sqrt(3)*x - sqrt(3*x^2 +
2))^4 - 831934400*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 + 500387712*(sqr
t(3)*x - sqrt(3*x^2 + 2))^2 - 51770496*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2
)) + 7768192)/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 3*sqrt(3)*(sqrt(3)*x - sq
rt(3*x^2 + 2)) - 2)^6
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.79

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^7}\,dx = \frac{159759\sqrt{35}\ \ln\left(x+\frac{3}{2}\right)}{7683200} - \frac{9\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{128} - \frac{159759\sqrt{35}\ \ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{7683200} - \frac{9019\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{4096\left(x^4+6x^3+\frac{27x^2}{2}+\frac{27x}{2}+\frac{81}{16}\right)} + \frac{7315\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{4096\left(x^5+\frac{15x^4}{2}+\frac{45x^3}{2}+\frac{135x^2}{4}+\frac{405x}{16}+\frac{243}{32}\right)} + \frac{182067\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{878080\left(x+\frac{3}{2}\right)} - \frac{15925\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{24576\left(x^6+9x^5+\frac{135x^4}{4}+\frac{135x^3}{2}+\frac{1215x^2}{16}+\frac{729x}{16}+\frac{729}{64}\right)} - \frac{164961\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{250880\left(x^2+3x+\frac{9}{4}\right)} + \frac{109789\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{71680\left(x^3+\frac{9x^2}{2}+\frac{27x}{4}+\frac{27}{8}\right)}$$

input
```
int(-((3*x^2 + 2)^(5/2)*(x - 5))/(2*x + 3)^7,x)
```

output
```
(159759*35^(1/2)*log(x + 3/2))/7683200 - (9*3^(1/2)*asinh((2^(1/2)*3^(1/2)
*x)/2))/128 - (159759*35^(1/2)*log(x - (3^(1/2)*35^(1/2)*(x^2 + 2/3)^(1/2)
)/9 - 4/9))/7683200 - (9019*3^(1/2)*(x^2 + 2/3)^(1/2))/(4096*((27*x)/2 + (
27*x^2)/2 + 6*x^3 + x^4 + 81/16)) + (7315*3^(1/2)*(x^2 + 2/3)^(1/2))/(4096
*((405*x)/16 + (135*x^2)/4 + (45*x^3)/2 + (15*x^4)/2 + x^5 + 243/32)) + (1
82067*3^(1/2)*(x^2 + 2/3)^(1/2))/(878080*(x + 3/2)) - (15925*3^(1/2)*(x^2
+ 2/3)^(1/2))/(24576*((729*x)/16 + (1215*x^2)/16 + (135*x^3)/2 + (135*x^4)
/4 + 9*x^5 + x^6 + 729/64)) - (164961*3^(1/2)*(x^2 + 2/3)^(1/2))/(250880*(
3*x + x^2 + 9/4)) + (109789*3^(1/2)*(x^2 + 2/3)^(1/2))/(71680*((27*x)/4 +
(9*x^2)/2 + x^3 + 27/8))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 647, normalized size of antiderivative = 4.86

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^7}\,dx = \text{Too large to display}$$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^7,x)
```

output
```
(611745120*sqrt(3*x**2 + 2)*x**5 + 2648147040*sqrt(3*x**2 + 2)*x**4 + 6643
532280*sqrt(3*x**2 + 2)*x**3 + 8295922320*sqrt(3*x**2 + 2)*x**2 + 55781052
60*sqrt(3*x**2 + 2)*x + 1450652980*sqrt(3*x**2 + 2) + 61347456*sqrt(35)*lo
g(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**6 + 552127104*sqrt(35)*log(sqrt(
3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**5 + 2070476640*sqrt(35)*log(sqrt(3*x**2
 + 2)*sqrt(35) + 9*x - 4)*x**4 + 4140953280*sqrt(35)*log(sqrt(3*x**2 + 2)*
sqrt(35) + 9*x - 4)*x**3 + 4658572440*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(3
5) + 9*x - 4)*x**2 + 2795143464*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9
*x - 4)*x + 698785866*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4) -
61347456*sqrt(35)*log(2*x + 3)*x**6 - 552127104*sqrt(35)*log(2*x + 3)*x**5
 - 2070476640*sqrt(35)*log(2*x + 3)*x**4 - 4140953280*sqrt(35)*log(2*x + 3
)*x**3 - 4658572440*sqrt(35)*log(2*x + 3)*x**2 - 2795143464*sqrt(35)*log(2
*x + 3)*x - 698785866*sqrt(35)*log(2*x + 3) + 103723200*sqrt(3)*log(sqrt(3
*x**2 + 2) - sqrt(3)*x)*x**6 + 933508800*sqrt(3)*log(sqrt(3*x**2 + 2) - sq
rt(3)*x)*x**5 + 3500658000*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x**4
+ 7001316000*sqrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x**3 + 7876480500*s
qrt(3)*log(sqrt(3*x**2 + 2) - sqrt(3)*x)*x**2 + 4725888300*sqrt(3)*log(sqr
t(3*x**2 + 2) - sqrt(3)*x)*x + 1181472075*sqrt(3)*log(sqrt(3*x**2 + 2) - s
qrt(3)*x) - 103723200*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x**6 - 933
508800*sqrt(3)*log(sqrt(3*x**2 + 2) + sqrt(3)*x)*x**5 - 3500658000*sqrt...
```

3.232 $\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 136

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^8}\,dx = -\frac{369(4-9x)\sqrt{2+3x^2}}{1200500(3+2x)^2} - \frac{41(4-9x)\left(2+3x^2\right)^{3/2}}{34300(3+2x)^4} - \frac{41(4-9x)\left(2+3x^2\right)^{5/2}}{7350(3+2x)^6} - \frac{13\left(2+3x^2\right)^{7/2}}{245(3+2x)^7} - \frac{1107\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{600250\sqrt{35}}$$

output
```
-369/1200500*(4-9*x)*(3*x^2+2)^(1/2)/(3+2*x)^2-41/34300*(4-9*x)*(3*x^2+2)^
(3/2)/(3+2*x)^4-41/7350*(4-9*x)*(3*x^2+2)^(5/2)/(3+2*x)^6-13/245*(3*x^2+2)
^(7/2)/(3+2*x)^7-1107/21008750*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x
^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 5.56 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^8}\,dx = \frac{-\frac{35\sqrt{2+3x^2}\left(4499004-593639x+3488490x^2-15015225x^3-2997810x^4-9455994x^5+656424x^6\right)}{(3+2x)^7}+132}{126052500}$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^8,x]
```

output
```
((-35*Sqrt[2 + 3*x^2]*(4499004 - 593639*x + 3488490*x^2 - 15015225*x^3 - 2
997810*x^4 - 9455994*x^5 + 656424*x^6))/(3 + 2*x)^7 + 13284*Sqrt[35]*ArcTa
nh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]])/126052500
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {679, 486, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{5/2}}{(2x+3)^8}dx$$

$$\Big\downarrow \text{679}$$

$$\frac{41}{35}\int \frac{\left(3x^2+2\right)^{5/2}}{(2x+3)^7}dx-\frac{13\left(3x^2+2\right)^{7/2}}{245(2x+3)^7}$$

$$\Big\downarrow \text{486}$$

$$\frac{41}{35}\left(\frac{1}{7}\int \frac{\left(3x^2+2\right)^{3/2}}{(2x+3)^5}dx-\frac{(4-9x)\left(3x^2+2\right)^{5/2}}{210(2x+3)^6}\right)-\frac{13\left(3x^2+2\right)^{7/2}}{245(2x+3)^7}$$

$$\Big\downarrow \text{486}$$

$$\frac{41}{35}\left(\frac{1}{7}\left(\frac{9}{70}\int \frac{\sqrt{3x^2+2}}{(2x+3)^3}dx-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{(4-9x)\left(3x^2+2\right)^{5/2}}{210(2x+3)^6}\right)-\frac{13\left(3x^2+2\right)^{7/2}}{245(2x+3)^7}$$

$$\Big\downarrow \text{486}$$

$$\frac{41}{35}\left(\frac{1}{7}\left(\frac{9}{70}\left(\frac{3}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{(4-9x)\sqrt{3x^2+2}}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{(4-9x)\left(3x^2+2\right)^{5}}{210(2x+3)^6}\right.$$
$$\frac{13\left(3x^2+2\right)^{7/2}}{245(2x+3)^7}$$

$$\Big\downarrow \text{488}$$

$$\frac{41}{35}\left(\frac{1}{7}\left(\frac{9}{70}\left(-\frac{3}{35}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{(4-9x)\left(3x^2\right.}{210(2x+}\right.$$

$$\frac{13\left(3x^2+2\right)^{7/2}}{245(2x+3)^7}$$

$$\Big\downarrow \text{219}$$

$$\frac{41}{35}\left(\frac{1}{7}\left(\frac{9}{70}\left(-\frac{3\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{(4-9x)\left(3x^2+2\right)^{5/}}{210(2x+3)^6}\right.$$

$$\frac{13\left(3x^2+2\right)^{7/2}}{245(2x+3)^7}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^8,x]
```

output
```
(-13*(2 + 3*x^2)^(7/2))/(245*(3 + 2*x)^7) + (41*(-1/210*((4 - 9*x)*(2 + 3*
x^2)^(5/2))/(3 + 2*x)^6 + (-1/140*((4 - 9*x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^
4 + (9*(-1/70*((4 - 9*x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^2 - (3*ArcTanh[(4 - 9*
x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(35*Sqrt[35])))/70)/7))/35
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{1969272x^8-28367982x^7-7680582x^6-63957663x^5+4469850x^4-31811367x^3+20473992x^2-1187278x+8998008}{3601500(2x+3)^7\sqrt{3x^2+2}}-\frac{1107\sqrt{35}\ \text{a}}{}$
trager	$-\frac{(656424x^6-9455994x^5-2997810x^4-15015225x^3+3488490x^2-593639x+4499004)\sqrt{3x^2+2}}{3601500(2x+3)^7}+\frac{1107\,\text{RootOf}\left(\_Z^2-35\right)\ln\left(\right.}{}$
default	$-\frac{123\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{7}{2}}}{1372000\left(x+\frac{3}{2}\right)^5}+\frac{1107\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}{21008750}+\frac{17712\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{64339296875}+\frac{1476\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}{367653125}+$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(2*x+3)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/3601500*(1969272*x^8-28367982*x^7-7680582*x^6-63957663*x^5+4469850*x^4-
31811367*x^3+20473992*x^2-1187278*x+8998008)/(2*x+3)^7/(3*x^2+2)^(1/2)-110
7/21008750*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^8}\,dx = \frac{3321\sqrt{35}(128\,x^7+1344\,x^6+6048\,x^5+15120\,x^4+22680\,x^3+20412\,x^2+10}{12605}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^8,x, algorithm="fricas")
```

output
```
1/126052500*(3321*sqrt(35)*(128*x^7 + 1344*x^6 + 6048*x^5 + 15120*x^4 + 22
680*x^3 + 20412*x^2 + 10206*x + 2187)*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x
- 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9)) - 35*(656424*x^6 - 9455994*
x^5 - 2997810*x^4 - 15015225*x^3 + 3488490*x^2 - 593639*x + 4499004)*sqrt(
3*x^2 + 2))/(128*x^7 + 1344*x^6 + 6048*x^5 + 15120*x^4 + 22680*x^3 + 20412
*x^2 + 10206*x + 2187)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^8}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(5/2)/(3+2*x)**8,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(113) = 226.

Time = 0.16 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.38

$$\int \frac{(5-x)(2+3x^2)^{5/2}}{(3+2x)^8}\,dx = \frac{397413}{7353062500}\left(3x^2+2\right)^{\frac{5}{2}} - \frac{13\left(3x^2+2\right)^{\frac{7}{2}}}{245\left(128x^7+1344x^6+6048x^5+15120x^4+22680x^3+20412x^2+10206x+2187\right)} - \frac{41\left(3x^2+2\right)^{\frac{7}{2}}}{3675\left(64x^6+576x^5+2160x^4+4320x^3+4860x^2+2916x+729\right)} - \frac{123\left(3x^2+2\right)^{\frac{7}{2}}}{42875\left(32x^5+240x^4+720x^3+1080x^2+810x+243\right)} - \frac{1189\left(3x^2+2\right)^{\frac{7}{2}}}{1500625\left(16x^4+96x^3+216x^2+216x+81\right)} - \frac{12177\left(3x^2+2\right)^{\frac{7}{2}}}{52521875\left(8x^3+36x^2+54x+27\right)} - \frac{132471\left(3x^2+2\right)^{\frac{7}{2}}}{1838265625\left(4x^2+12x+9\right)} + \frac{129519}{1470612500}\left(3x^2+2\right)^{\frac{3}{2}}x + \frac{1476}{367653125}\left(3x^2+2\right)^{\frac{3}{2}} - \frac{1537623\left(3x^2+2\right)^{\frac{5}{2}}}{7353062500\left(2x+3\right)} + \frac{9963}{42017500}\sqrt{3x^2+2}\,x + \frac{1107}{21008750}\sqrt{35}\,\mathrm{arsinh}\left(\frac{3\sqrt{6}x}{2\left|2x+3\right|} - \frac{2\sqrt{6}}{3\left|2x+3\right|}\right) + \frac{1107}{10504375}\sqrt{3x^2+2}$$

input

```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^8,x, algorithm="maxima")
```

output

```
397413/7353062500*(3*x^2 + 2)^(5/2) - 13/245*(3*x^2 + 2)^(7/2)/(128*x^7 +
1344*x^6 + 6048*x^5 + 15120*x^4 + 22680*x^3 + 20412*x^2 + 10206*x + 2187)
- 41/3675*(3*x^2 + 2)^(7/2)/(64*x^6 + 576*x^5 + 2160*x^4 + 4320*x^3 + 4860
*x^2 + 2916*x + 729) - 123/42875*(3*x^2 + 2)^(7/2)/(32*x^5 + 240*x^4 + 720
*x^3 + 1080*x^2 + 810*x + 243) - 1189/1500625*(3*x^2 + 2)^(7/2)/(16*x^4 +
96*x^3 + 216*x^2 + 216*x + 81) - 12177/52521875*(3*x^2 + 2)^(7/2)/(8*x^3 +
 36*x^2 + 54*x + 27) - 132471/1838265625*(3*x^2 + 2)^(7/2)/(4*x^2 + 12*x +
 9) + 129519/1470612500*(3*x^2 + 2)^(3/2)*x + 1476/367653125*(3*x^2 + 2)^(
3/2) - 1537623/7353062500*(3*x^2 + 2)^(5/2)/(2*x + 3) + 9963/42017500*sqrt
(3*x^2 + 2)*x + 1107/21008750*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3)
- 2/3*sqrt(6)/abs(2*x + 3)) + 1107/10504375*sqrt(3*x^2 + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(113) = 226.

Time = 0.15 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.00

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^8}\,dx = \frac{1107}{21008750}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3x^2+2}}\right)$$
$$-\frac{9\left(908247\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^{13}+3755004\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^{12}+52905908\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^{11}\right.}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^8,x, algorithm="giac")
```

output
```
1107/21008750*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sq
rt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) -
 9/38416000*(908247*(sqrt(3)*x - sqrt(3*x^2 + 2))^13 + 3755004*sqrt(3)*(sq
rt(3)*x - sqrt(3*x^2 + 2))^12 + 52905908*(sqrt(3)*x - sqrt(3*x^2 + 2))^11
+ 114259794*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^10 + 422075810*(sqrt(3)*
x - sqrt(3*x^2 + 2))^9 - 16674486*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^8
- 1093657086*(sqrt(3)*x - sqrt(3*x^2 + 2))^7 - 205745364*sqrt(3)*(sqrt(3)*
x - sqrt(3*x^2 + 2))^6 + 1886581864*(sqrt(3)*x - sqrt(3*x^2 + 2))^5 - 1023
977040*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^4 + 660654976*(sqrt(3)*x - sq
rt(3*x^2 + 2))^3 - 94952448*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 - 9114
816*sqrt(3)*x - 1555968*sqrt(3) + 9114816*sqrt(3*x^2 + 2))/((sqrt(3)*x - s
qrt(3*x^2 + 2))^2 + 3*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^7
```

Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.00

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^8}\,dx = \frac{1107\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{21008750} - \frac{1107\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{21008750} + \frac{34571\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{62720\left(x^4+6x^3+\frac{27x^2}{2}+\frac{27x}{2}+\frac{81}{16}\right)} - \frac{6213\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{7168\left(x^5+\frac{15x^4}{2}+\frac{45x^3}{2}+\frac{135x^2}{4}+\frac{405x}{16}+\frac{243}{32}\right)} - \frac{27351\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{19208000\left(x+\frac{3}{2}\right)} + \frac{9095\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{12288\left(x^6+9x^5+\frac{135x^4}{4}+\frac{135x^3}{2}+\frac{1215x^2}{16}+\frac{729x}{16}+\frac{729}{64}\right)} + \frac{73161\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{2195200\left(x^2+3x+\frac{9}{4}\right)} - \frac{2275\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{8192\left(x^7+\frac{21x^6}{2}+\frac{189x^5}{4}+\frac{945x^4}{8}+\frac{2835x^3}{16}+\frac{5103x^2}{32}+\frac{5103x}{64}+\frac{2187}{128}\right)} - \frac{122553\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{627200\left(x^3+\frac{9x^2}{2}+\frac{27x}{4}+\frac{27}{8}\right)}$$

input
```
int(-((3*x^2 + 2)^(5/2)*(x - 5))/(2*x + 3)^8,x)
```

output
```
(1107*35^(1/2)*log(x + 3/2))/21008750 - (1107*35^(1/2)*log(x - (3^(1/2)*35
^(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/21008750 + (34571*3^(1/2)*(x^2 + 2/3)^
(1/2))/(62720*((27*x)/2 + (27*x^2)/2 + 6*x^3 + x^4 + 81/16)) - (6213*3^(1/
2)*(x^2 + 2/3)^(1/2))/(7168*((405*x)/16 + (135*x^2)/4 + (45*x^3)/2 + (15*x
^4)/2 + x^5 + 243/32)) - (27351*3^(1/2)*(x^2 + 2/3)^(1/2))/(19208000*(x +
3/2)) + (9095*3^(1/2)*(x^2 + 2/3)^(1/2))/(12288*((729*x)/16 + (1215*x^2)/1
6 + (135*x^3)/2 + (135*x^4)/4 + 9*x^5 + x^6 + 729/64)) + (73161*3^(1/2)*(x
^2 + 2/3)^(1/2))/(2195200*(3*x + x^2 + 9/4)) - (2275*3^(1/2)*(x^2 + 2/3)^(
1/2))/(8192*((5103*x)/64 + (5103*x^2)/32 + (2835*x^3)/16 + (945*x^4)/8 + (
189*x^5)/4 + (21*x^6)/2 + x^7 + 2187/128)) - (122553*3^(1/2)*(x^2 + 2/3)^(
1/2))/(627200*((27*x)/4 + (9*x^2)/2 + x^3 + 27/8))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 411, normalized size of antiderivative = 3.02

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^8}\,dx = \frac{-22974840\sqrt{3x^2+2}\,x^6+330959790\sqrt{3x^2+2}\,x^5+104923350\sqrt{3x^2+2}\,x^4+}{}$$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^8,x)
```

output
```
( - 22974840*sqrt(3*x**2 + 2)*x**6 + 330959790*sqrt(3*x**2 + 2)*x**5 + 104
923350*sqrt(3*x**2 + 2)*x**4 + 525532875*sqrt(3*x**2 + 2)*x**3 - 122097150
*sqrt(3*x**2 + 2)*x**2 + 20777365*sqrt(3*x**2 + 2)*x - 157465140*sqrt(3*x*
*2 + 2) + 850176*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**7 +
8926848*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**6 + 40170816*
sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**5 + 100427040*sqrt(35
)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**4 + 150640560*sqrt(35)*log(s
qrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**3 + 135576504*sqrt(35)*log(sqrt(3*x
**2 + 2)*sqrt(35) + 9*x - 4)*x**2 + 67788252*sqrt(35)*log(sqrt(3*x**2 + 2)
*sqrt(35) + 9*x - 4)*x + 14526054*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) +
 9*x - 4) - 850176*sqrt(35)*log(2*x + 3)*x**7 - 8926848*sqrt(35)*log(2*x +
 3)*x**6 - 40170816*sqrt(35)*log(2*x + 3)*x**5 - 100427040*sqrt(35)*log(2*
x + 3)*x**4 - 150640560*sqrt(35)*log(2*x + 3)*x**3 - 135576504*sqrt(35)*lo
g(2*x + 3)*x**2 - 67788252*sqrt(35)*log(2*x + 3)*x - 14526054*sqrt(35)*log
(2*x + 3))/(126052500*(128*x**7 + 1344*x**6 + 6048*x**5 + 15120*x**4 + 226
80*x**3 + 20412*x**2 + 10206*x + 2187))
```

3.233 $\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^9}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^9}\,dx=\frac{48141\sqrt{2+3x^2}}{153664000(3+2x)^2}+\frac{24813\sqrt{2+3x^2}}{5378240000(3+2x)}-\frac{9(11323+4037x)\sqrt{2+3x^2}}{10976000(3+2x)^4}-\frac{(13106-16521x)\left(2+3x^2\right)^{3/2}}{548800(3+2x)^6}-\frac{(59-173x)\left(2+3x^2\right)^{5/2}}{784(3+2x)^8}-\frac{18873\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{42017500\sqrt{35}}$$

output

```
48141/153664000*(3*x^2+2)^(1/2)/(3+2*x)^2+24813*(3*x^2+2)^(1/2)/(161347200
00+10756480000*x)-9/10976000*(11323+4037*x)*(3*x^2+2)^(1/2)/(3+2*x)^4-1/54
8800*(13106-16521*x)*(3*x^2+2)^(3/2)/(3+2*x)^6-1/784*(59-173*x)*(3*x^2+2)^
(5/2)/(3+2*x)^8-18873/1470612500*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3
*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 5.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.65

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^9}\,dx = \frac{\frac{35\sqrt{2+3x^2}\left(-104577556-38788883x-178164896x^2+226355535x^3+33613440x^4+210306726x^5+2206008\right.}{(3+2x)^8}}{2941225000}$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^9,x]
```

output
```
((35*Sqrt[2 + 3*x^2]*(-104577556 - 38788883*x - 178164896*x^2 + 226355535*
x^3 + 33613440*x^4 + 210306726*x^5 + 2206008*x^6 + 49626*x^7))/(3 + 2*x)^8
 + 75492*Sqrt[35]*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sq
rt[35]])/2941225000
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {688, 25, 679, 486, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{5/2}}{(2x+3)^9}dx$$

$$\Big\downarrow \; 688$$

$$-\frac{1}{280}\int -\frac{(328-39x)\left(3x^2+2\right)^{5/2}}{(2x+3)^8}dx-\frac{13\left(3x^2+2\right)^{7/2}}{280(2x+3)^8}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{280}\int \frac{(328-39x)\left(3x^2+2\right)^{5/2}}{(2x+3)^8}dx-\frac{13\left(3x^2+2\right)^{7/2}}{280(2x+3)^8}$$

$$\Big\downarrow \; 679$$

$$\frac{1}{280}\left(\frac{2796}{35}\int \frac{\left(3x^2+2\right)^{5/2}}{(2x+3)^7}dx-\frac{773\left(3x^2+2\right)^{7/2}}{245(2x+3)^7}\right)-\frac{13\left(3x^2+2\right)^{7/2}}{280(2x+3)^8}$$

$$\Big\downarrow \; 486$$

$$\frac{1}{280}\left(\frac{2796}{35}\left(\frac{1}{7}\int \frac{\left(3x^2+2\right)^{3/2}}{(2x+3)^5}dx-\frac{(4-9x)\left(3x^2+2\right)^{5/2}}{210(2x+3)^6}\right)-\frac{773\left(3x^2+2\right)^{7/2}}{245(2x+3)^7}\right)-\frac{13\left(3x^2+2\right)^{7/2}}{280(2x+3)^8}$$

$$\Big\downarrow \; 486$$

$$\frac{1}{280}\left(\frac{2796}{35}\left(\frac{1}{7}\left(\frac{9}{70}\int \frac{\sqrt{3x^2+2}}{(2x+3)^3}dx-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{(4-9x)\left(3x^2+2\right)^{5/2}}{210(2x+3)^6}\right)-\frac{773\left(3x^2+2\right)^{7/2}}{245(2x+3)^7}\right)-\frac{13\left(3x^2+2\right)^{7/2}}{280(2x+3)^8}$$

$$\Big\downarrow \; 486$$

$$\frac{1}{280}\left(\frac{2796}{35}\left(\frac{1}{7}\left(\frac{9}{70}\left(\frac{3}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{(4-9x)\sqrt{3x^2+2}}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{(4-9x)\left(3\right.}{210(2x}\right.\right.\right.$$
$$\frac{13\left(3x^2+2\right)^{7/2}}{280(2x+3)^8}$$

$$\Big\downarrow \; 488$$

$$\frac{1}{280}\left(\frac{2796}{35}\left(\frac{1}{7}\left(\frac{9}{70}\left(-\frac{3}{35}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{(4-9}{21}\right.\right.\right.$$
$$\frac{13\left(3x^2+2\right)^{7/2}}{280(2x+3)^8}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{280}\left(\frac{2796}{35}\left(\frac{1}{7}\left(\frac{9}{70}\left(-\frac{3\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{(4-9x)\left(3x\right.}{210(2x}\right.\right.\right.$$
$$\frac{13\left(3x^2+2\right)^{7/2}}{280(2x+3)^8}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^9,x]
```

output
```
(-13*(2 + 3*x^2)^(7/2))/(280*(3 + 2*x)^8) + ((-773*(2 + 3*x^2)^(7/2))/(245
*(3 + 2*x)^7) + (2796*(-1/210*((4 - 9*x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^6 +
(-1/140*((4 - 9*x)*(2 + 3*x^2)^(3/2))/(3 + 2*x)^4 + (9*(-1/70*((4 - 9*x)*S
qrt[2 + 3*x^2])/(3 + 2*x)^2 - (3*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^
2])])/(35*Sqrt[35])))/70)/7))/35)/280
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.60

method	result
risch	$\frac{148878x^9+6618024x^8+631019430x^7+105252336x^6+1099680057x^5-467267808x^4+336344421x^3-670062460x^2-77577766x-209}{84035000(2x+3)^8\sqrt{3x^2+2}}$
trager	$\frac{(49626x^7+2206008x^6+210306726x^5+33613440x^4+226355535x^3-178164896x^2-38788883x-104577556)\sqrt{3x^2+2}}{84035000(2x+3)^8}+\frac{18873\,\mathrm{Roo}}{}$
default	$-\frac{2097\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{7}{2}}}{96040000\left(x+\frac{3}{2}\right)^5}+\frac{18873\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}{1470612500}+\frac{150984\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{2251875390625}+\frac{12582\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)}{12867859375}$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(2*x+3)^9,x,method=_RETURNVERBOSE)
```

output
```
1/84035000*(148878*x^9+6618024*x^8+631019430*x^7+105252336*x^6+1099680057*
x^5-467267808*x^4+336344421*x^3-670062460*x^2-77577766*x-209155112)/(2*x+3
)^8/(3*x^2+2)^(1/2)-18873/1470612500*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2
)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.13

$$\int \frac{(5-x)(2+3x^2)^{5/2}}{(3+2x)^9}\,dx = \frac{18873\sqrt{35}(256\,x^8+3072\,x^7+16128\,x^6+48384\,x^5+90720\,x^4+108864\,x^3+\cdots}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^9,x, algorithm="fricas")
```

output
```
1/2941225000*(18873*sqrt(35)*(256*x^8 + 3072*x^7 + 16128*x^6 + 48384*x^5 +
 90720*x^4 + 108864*x^3 + 81648*x^2 + 34992*x + 6561)*log(-(sqrt(35)*sqrt(
3*x^2 + 2)*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9)) + 35*(49626
*x^7 + 2206008*x^6 + 210306726*x^5 + 33613440*x^4 + 226355535*x^3 - 178164
896*x^2 - 38788883*x - 104577556)*sqrt(3*x^2 + 2))/(256*x^8 + 3072*x^7 + 1
6128*x^6 + 48384*x^5 + 90720*x^4 + 108864*x^3 + 81648*x^2 + 34992*x + 6561
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)(2+3x^2)^{5/2}}{(3+2x)^9}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(5/2)/(3+2*x)**9,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(131) = 262.

Time = 0.17 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.38

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^9}\,dx = \frac{6775407}{514714375000}\left(3\,x^2+2\right)^{\frac{5}{2}} - \frac{13\left(3\,x^2+2\right)^{\frac{7}{2}}}{280\left(256\,x^8+3072\,x^7+16128\,x^6+48384\,x^5+90720\,x^4+108864\,x^3+81648\,x^2+34992\,x+6561\right)} - \frac{773\left(3\,x^2+2\right)^{\frac{7}{2}}}{68600\left(128\,x^7+1344\,x^6+6048\,x^5+15120\,x^4+22680\,x^3+20412\,x^2+10206\,x+2187\right)} - \frac{233\left(3\,x^2+2\right)^{\frac{7}{2}}}{85750\left(64\,x^6+576\,x^5+2160\,x^4+4320\,x^3+4860\,x^2+2916\,x+729\right)} - \frac{2097\left(3\,x^2+2\right)^{\frac{7}{2}}}{3001250\left(32\,x^5+240\,x^4+720\,x^3+1080\,x^2+810\,x+243\right)} - \frac{20271\left(3\,x^2+2\right)^{\frac{7}{2}}}{105043750\left(16\,x^4+96\,x^3+216\,x^2+216\,x+81\right)} - \frac{207603\left(3\,x^2+2\right)^{\frac{7}{2}}}{3676531250\left(8\,x^3+36\,x^2+54\,x+27\right)} - \frac{2258469\left(3\,x^2+2\right)^{\frac{7}{2}}}{128678593750\left(4\,x^2+12\,x+9\right)} + \frac{2208141}{102942875000}\left(3\,x^2+2\right)^{\frac{3}{2}}x + \frac{12582}{12867859375}\left(3\,x^2+2\right)^{\frac{3}{2}} - \frac{26214597\left(3\,x^2+2\right)^{\frac{5}{2}}}{514714375000\left(2\,x+3\right)} + \frac{169857}{2941225000}\sqrt{3\,x^2+2}x + \frac{18873}{1470612500}\sqrt{35}\,\operatorname{arsinh}\left(\frac{3\sqrt{6}x}{2\left|2\,x+3\right|} - \frac{2\sqrt{6}}{3\left|2\,x+3\right|}\right) + \frac{18873}{735306250}\sqrt{3\,x^2+2}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^9,x, algorithm="maxima")
```

output
```
6775407/514714375000*(3*x^2 + 2)^(5/2) - 13/280*(3*x^2 + 2)^(7/2)/(256*x^8
 + 3072*x^7 + 16128*x^6 + 48384*x^5 + 90720*x^4 + 108864*x^3 + 81648*x^2 +
 34992*x + 6561) - 773/68600*(3*x^2 + 2)^(7/2)/(128*x^7 + 1344*x^6 + 6048*
x^5 + 15120*x^4 + 22680*x^3 + 20412*x^2 + 10206*x + 2187) - 233/85750*(3*x
^2 + 2)^(7/2)/(64*x^6 + 576*x^5 + 2160*x^4 + 4320*x^3 + 4860*x^2 + 2916*x
+ 729) - 2097/3001250*(3*x^2 + 2)^(7/2)/(32*x^5 + 240*x^4 + 720*x^3 + 1080
*x^2 + 810*x + 243) - 20271/105043750*(3*x^2 + 2)^(7/2)/(16*x^4 + 96*x^3 +
 216*x^2 + 216*x + 81) - 207603/3676531250*(3*x^2 + 2)^(7/2)/(8*x^3 + 36*x
^2 + 54*x + 27) - 2258469/128678593750*(3*x^2 + 2)^(7/2)/(4*x^2 + 12*x + 9
) + 2208141/102942875000*(3*x^2 + 2)^(3/2)*x + 12582/12867859375*(3*x^2 +
2)^(3/2) - 26214597/514714375000*(3*x^2 + 2)^(5/2)/(2*x + 3) + 169857/2941
225000*sqrt(3*x^2 + 2)*x + 18873/1470612500*sqrt(35)*arcsinh(3/2*sqrt(6)*x
/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3)) + 18873/735306250*sqrt(3*x^2 + 2
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(131) = 262$.

Time = 0.16 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.89

$$\int \frac{(5-x)(2+3x^2)^{5/2}}{(3+2x)^9}\,dx = \frac{18873}{1470612500}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3x^2+2}}\right)$$
$$-\frac{9\sqrt{3}\Big(178944\sqrt{3}\big(\sqrt{3}x-\sqrt{3x^2+2}\big)^{15}+138131220\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^{14}+30787400\sqrt{3}\big(\sqrt{3}x-\sqrt{3x^2}}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^9,x, algorithm="giac")
```

output
```
18873/1470612500*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2
*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))
) - 9/10756480000*sqrt(3)*(178944*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^15
 + 138131220*(sqrt(3)*x - sqrt(3*x^2 + 2))^14 + 30787400*sqrt(3)*(sqrt(3)*
x - sqrt(3*x^2 + 2))^13 + 573375810*(sqrt(3)*x - sqrt(3*x^2 + 2))^12 - 332
8877720*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^11 - 8681082564*(sqrt(3)*x -
 sqrt(3*x^2 + 2))^10 - 13787031160*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^9
 + 1566458475*(sqrt(3)*x - sqrt(3*x^2 + 2))^8 - 28541438480*sqrt(3)*(sqrt(
3)*x - sqrt(3*x^2 + 2))^7 + 30582301680*(sqrt(3)*x - sqrt(3*x^2 + 2))^6 -
23140527424*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^5 - 12885596640*(sqrt(3)
*x - sqrt(3*x^2 + 2))^4 + 1726278400*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))
^3 - 9101541120*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 39843840*sqrt(3)*(sqrt(3
)*x - sqrt(3*x^2 + 2)) - 1411584)/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 3*sqr
t(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^8
```

Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.06

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^9}\,dx = \frac{18873\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{1470612500} - \frac{18873\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{1470612500} - \frac{15925\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{131072\left(x^8+12x^7+63x^6+189x^5+\frac{2835x^4}{8}+\frac{1701x^3}{4}+\frac{5103x^2}{16}+\frac{2187x}{16}+\frac{6561}{256}\right)} - \frac{4816641\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{70246400\left(x^4+6x^3+\frac{27x^2}{2}+\frac{27x}{2}+\frac{81}{16}\right)} + \frac{861381\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{4014080\left(x^5+\frac{15x^4}{2}+\frac{45x^3}{2}+\frac{135x^2}{4}+\frac{405x}{16}+\frac{243}{32}\right)} + \frac{24813\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{10756480000\left(x+\frac{3}{2}\right)} - \frac{81899\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{229376\left(x^6+9x^5+\frac{135x^4}{4}+\frac{135x^3}{2}+\frac{1215x^2}{16}+\frac{729x}{16}+\frac{729}{64}\right)} + \frac{48141\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{614656000\left(x^2+3x+\frac{9}{4}\right)} + \frac{20705\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{65536\left(x^7+\frac{21x^6}{2}+\frac{189x^5}{4}+\frac{945x^4}{8}+\frac{2835x^3}{16}+\frac{5103x^2}{32}+\frac{5103x}{64}+\frac{2187}{128}\right)} + \frac{1573857\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{175616000\left(x^3+\frac{9x^2}{2}+\frac{27x}{4}+\frac{27}{8}\right)}$$

input
```
int(-((3*x^2 + 2)^(5/2)*(x - 5))/(2*x + 3)^9,x)
```

output
```
(18873*35^(1/2)*log(x + 3/2))/1470612500 - (18873*35^(1/2)*log(x - (3^(1/2
)*35^(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/1470612500 - (15925*3^(1/2)*(x^2 +
 2/3)^(1/2))/(131072*((2187*x)/16 + (5103*x^2)/16 + (1701*x^3)/4 + (2835*x
^4)/8 + 189*x^5 + 63*x^6 + 12*x^7 + x^8 + 6561/256)) - (4816641*3^(1/2)*(x
^2 + 2/3)^(1/2))/(70246400*((27*x)/2 + (27*x^2)/2 + 6*x^3 + x^4 + 81/16))
+ (861381*3^(1/2)*(x^2 + 2/3)^(1/2))/(4014080*((405*x)/16 + (135*x^2)/4 +
(45*x^3)/2 + (15*x^4)/2 + x^5 + 243/32)) + (24813*3^(1/2)*(x^2 + 2/3)^(1/2
))/(10756480000*(x + 3/2)) - (81899*3^(1/2)*(x^2 + 2/3)^(1/2))/(229376*((7
29*x)/16 + (1215*x^2)/16 + (135*x^3)/2 + (135*x^4)/4 + 9*x^5 + x^6 + 729/6
4)) + (48141*3^(1/2)*(x^2 + 2/3)^(1/2))/(614656000*(3*x + x^2 + 9/4)) + (2
0705*3^(1/2)*(x^2 + 2/3)^(1/2))/(65536*((5103*x)/64 + (5103*x^2)/32 + (283
5*x^3)/16 + (945*x^4)/8 + (189*x^5)/4 + (21*x^6)/2 + x^7 + 2187/128)) + (1
573857*3^(1/2)*(x^2 + 2/3)^(1/2))/(175616000*((27*x)/4 + (9*x^2)/2 + x^3 +
 27/8))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.95

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^9}\,dx = \frac{-9662976\sqrt{35}\log(2x+3)\,x^8 + 9662976\sqrt{35}\log\left(\sqrt{3x^2+2}\,\sqrt{35}+9x-4\right)x^8}{}$$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^9,x)
```

output

```
(1736910*sqrt(3*x**2 + 2)*x**7 + 77210280*sqrt(3*x**2 + 2)*x**6 + 73607354
10*sqrt(3*x**2 + 2)*x**5 + 1176470400*sqrt(3*x**2 + 2)*x**4 + 7922443725*s
qrt(3*x**2 + 2)*x**3 - 6235771360*sqrt(3*x**2 + 2)*x**2 - 1357610905*sqrt(
3*x**2 + 2)*x - 3660214460*sqrt(3*x**2 + 2) + 9662976*sqrt(35)*log(sqrt(3*
x**2 + 2)*sqrt(35) + 9*x - 4)*x**8 + 115955712*sqrt(35)*log(sqrt(3*x**2 +
2)*sqrt(35) + 9*x - 4)*x**7 + 608767488*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt
(35) + 9*x - 4)*x**6 + 1826302464*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) +
 9*x - 4)*x**5 + 3424317120*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x -
 4)*x**4 + 4109180544*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x*
*3 + 3081885408*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**2 + 1
320808032*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x + 247651506*
sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4) - 9662976*sqrt(35)*log(2
*x + 3)*x**8 - 115955712*sqrt(35)*log(2*x + 3)*x**7 - 608767488*sqrt(35)*l
og(2*x + 3)*x**6 - 1826302464*sqrt(35)*log(2*x + 3)*x**5 - 3424317120*sqrt
(35)*log(2*x + 3)*x**4 - 4109180544*sqrt(35)*log(2*x + 3)*x**3 - 308188540
8*sqrt(35)*log(2*x + 3)*x**2 - 1320808032*sqrt(35)*log(2*x + 3)*x - 247651
506*sqrt(35)*log(2*x + 3))/(2941225000*(256*x**8 + 3072*x**7 + 16128*x**6
+ 48384*x**5 + 90720*x**4 + 108864*x**3 + 81648*x**2 + 34992*x + 6561))
```

3.234 $\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^{10}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 180

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^{10}}\,dx = \frac{55473\sqrt{2+3x^2}}{192080000(3+2x)^3} + \frac{80649\sqrt{2+3x^2}}{1344560000(3+2x)^2} - \frac{152343\sqrt{2+3x^2}}{47059600000(3+2x)} - \frac{3(95869+112866x)\sqrt{2+3x^2}}{38416000(3+2x)^5} - \frac{(2497-447x)\left(2+3x^2\right)^{3/2}}{82320(3+2x)^7} - \frac{(47-90x)\left(2+3x^2\right)^{5/2}}{504(3+2x)^9} - \frac{76869\,\mathrm{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{735306250\sqrt{35}}$$

output
```
55473/192080000*(3*x^2+2)^(1/2)/(3+2*x)^3+80649/1344560000*(3*x^2+2)^(1/2)
/(3+2*x)^2-152343*(3*x^2+2)^(1/2)/(141178800000+94119200000*x)-3/38416000*
(95869+112866*x)*(3*x^2+2)^(1/2)/(3+2*x)^5-1/82320*(2497-447*x)*(3*x^2+2)^
(3/2)/(3+2*x)^7-1/504*(47-90*x)*(3*x^2+2)^(5/2)/(3+2*x)^9-76869/2573571875
0*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 3.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.60

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^{10}}\,dx = \frac{-\frac{35\sqrt{2+3x^2}\,(15948113036+11990965797x+42455611758x^2-11567526201x^3+9750269970x^4-25197346\ldots}{(3+2x)^9}}{46324\ldots}$$

input
```
Integrate[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^10,x]
```

output
```
((-35*Sqrt[2 + 3*x^2]*(15948113036 + 11990965797*x + 42455611758*x^2 - 115
67526201*x^3 + 9750269970*x^4 - 25197346566*x^5 - 620594352*x^6 + 30006612
*x^7 + 10968696*x^8))/(3 + 2*x)^9 + 2767284*Sqrt[35]*ArcTanh[(3*Sqrt[3] +
2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]])/463242937500
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {688, 27, 688, 27, 679, 486, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)\left(3x^2+2\right)^{5/2}}{(2x+3)^{10}}dx$$

$$\Big\downarrow \text{688}$$

$$-\frac{1}{315}\int -\frac{3(123-26x)\left(3x^2+2\right)^{5/2}}{(2x+3)^9}dx-\frac{13\left(3x^2+2\right)^{7/2}}{315(2x+3)^9}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{105}\int \frac{(123-26x)\left(3x^2+2\right)^{5/2}}{(2x+3)^9}dx-\frac{13\left(3x^2+2\right)^{7/2}}{315(2x+3)^9}$$

$$\Big\downarrow \text{688}$$

$$\frac{1}{105}\left(-\frac{1}{280}\int -\frac{4(2006-243x)\left(3x^2+2\right)^{5/2}}{(2x+3)^8}dx-\frac{81\left(3x^2+2\right)^{7/2}}{70(2x+3)^8}\right)-\frac{13\left(3x^2+2\right)^{7/2}}{315(2x+3)^9}$$

$\Big\downarrow$ 27

$$\frac{1}{105}\left(\frac{1}{70}\int \frac{(2006-243x)\left(3x^2+2\right)^{5/2}}{(2x+3)^8}dx-\frac{81\left(3x^2+2\right)^{7/2}}{70(2x+3)^8}\right)-\frac{13\left(3x^2+2\right)^{7/2}}{315(2x+3)^9}$$

$\Big\downarrow$ 679

$$\frac{1}{105}\left(\frac{1}{70}\left(\frac{17082}{35}\int \frac{\left(3x^2+2\right)^{5/2}}{(2x+3)^7}dx-\frac{4741\left(3x^2+2\right)^{7/2}}{245(2x+3)^7}\right)-\frac{81\left(3x^2+2\right)^{7/2}}{70(2x+3)^8}\right)-\frac{13\left(3x^2+2\right)^{7/2}}{315(2x+3)^9}$$

$\Big\downarrow$ 486

$$\frac{1}{105}\left(\frac{1}{70}\left(\frac{17082}{35}\left(\frac{1}{7}\int \frac{\left(3x^2+2\right)^{3/2}}{(2x+3)^5}dx-\frac{(4-9x)\left(3x^2+2\right)^{5/2}}{210(2x+3)^6}\right)-\frac{4741\left(3x^2+2\right)^{7/2}}{245(2x+3)^7}\right)-\frac{81\left(3x^2+2\right)^{7/2}}{70(2x+3)^8}\right)-\frac{13\left(3x^2+2\right)^{7/2}}{315(2x+3)^9}$$

$\Big\downarrow$ 486

$$\frac{1}{105}\left(\frac{1}{70}\left(\frac{17082}{35}\left(\frac{1}{7}\left(\frac{9}{70}\int \frac{\sqrt{3x^2+2}}{(2x+3)^3}dx-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{(4-9x)\left(3x^2+2\right)^{5/2}}{210(2x+3)^6}\right)-\frac{4741\left(3x^2+2\right)}{245(2x+3)^7}\right.\right.$$
$$\frac{13\left(3x^2+2\right)^{7/2}}{315(2x+3)^9}$$

$\Big\downarrow$ 486

$$\frac{1}{105}\left(\frac{1}{70}\left(\frac{17082}{35}\left(\frac{1}{7}\left(\frac{9}{70}\left(\frac{3}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{(4-9x)\sqrt{3x^2+2}}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-(4-\right.\right.\right.$$
$$\frac{13\left(3x^2+2\right)^{7/2}}{315(2x+3)^9}$$

$\Big\downarrow$ 488

$$\frac{1}{105}\left(\frac{1}{70}\left(\frac{17082}{35}\left(\frac{1}{7}\left(\frac{9}{70}\left(-\frac{3}{35}\int\frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\right.\right.\right.\right.$$
$$\frac{13\left(3x^2+2\right)^{7/2}}{315(2x+3)^9}$$

$\Big\downarrow$ 219

$$\frac{1}{105}\left(\frac{1}{70}\left(\frac{17082}{35}\left(\frac{1}{7}\left(\frac{9}{70}\left(-\frac{3\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}-\frac{\sqrt{3x^2+2}(4-9x)}{70(2x+3)^2}\right)-\frac{(4-9x)\left(3x^2+2\right)^{3/2}}{140(2x+3)^4}\right)-\frac{(4-9}{21}\right.\right.\right.\right.$$
$$\frac{13\left(3x^2+2\right)^{7/2}}{315(2x+3)^9}$$

input
```
Int[((5 - x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^10,x]
```

output
```
(-13*(2 + 3*x^2)^(7/2))/(315*(3 + 2*x)^9) + ((-81*(2 + 3*x^2)^(7/2))/(70*(
3 + 2*x)^8) + ((-4741*(2 + 3*x^2)^(7/2))/(245*(3 + 2*x)^7) + (17082*(-1/21
0*((4 - 9*x)*(2 + 3*x^2)^(5/2))/(3 + 2*x)^6 + (-1/140*((4 - 9*x)*(2 + 3*x^
2)^(3/2))/(3 + 2*x)^4 + (9*(-1/70*((4 - 9*x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^2
- (3*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(35*Sqrt[35])))/70)/7)
)/35)/70)/105
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.56

method	result
risch	$-\frac{32906088x^{10}+90019836x^9-1839845664x^8-75532026474x^7+28009621206x^6-85097271735x^5+146867375214x^4+12837844989}{13235512500(2x+3)^9\sqrt{3x^2+2}}$
trager	$-\frac{(10968696x^8+30006612x^7-620594352x^6-25197346566x^5+9750269970x^4-11567526201x^3+42455611758x^2+11990965797x+}{13235512500(2x+3)^9}$
default	$-\frac{8541\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{7}{2}}}{1680700000\left(x+\frac{3}{2}\right)^5}+\frac{76869\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}{25735718750}+\frac{1229904\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{5}{2}}}{78815638671875}+\frac{102492\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right.}{450375078125}$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(2*x+3)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/13235512500*(32906088*x^10+90019836*x^9-1839845664*x^8-75532026474*x^7+
28009621206*x^6-85097271735*x^5+146867375214*x^4+12837844989*x^3+132755562
624*x^2+23981931594*x+31896226072)/(2*x+3)^9/(3*x^2+2)^(1/2)-76869/2573571
8750*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.08

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^{10}}\,dx = \frac{691821\sqrt{35}(512\,x^9+6912\,x^8+41472\,x^7+145152\,x^6+326592\,x^5+489888\,x}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^10,x, algorithm="fricas")
```

output
```
1/463242937500*(691821*sqrt(35)*(512*x^9 + 6912*x^8 + 41472*x^7 + 145152*x
^6 + 326592*x^5 + 489888*x^4 + 489888*x^3 + 314928*x^2 + 118098*x + 19683)
*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 1
2*x + 9)) - 35*(10968696*x^8 + 30006612*x^7 - 620594352*x^6 - 25197346566*
x^5 + 9750269970*x^4 - 11567526201*x^3 + 42455611758*x^2 + 11990965797*x +
 15948113036)*sqrt(3*x^2 + 2))/(512*x^9 + 6912*x^8 + 41472*x^7 + 145152*x^
6 + 326592*x^5 + 489888*x^4 + 489888*x^3 + 314928*x^2 + 118098*x + 19683)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(5-x)\,(2+3x^2)^{5/2}}{(3+2x)^{10}}\,dx = \text{Timed out}$$

input
```
integrate((5-x)*(3*x**2+2)**(5/2)/(3+2*x)**10,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(149) = 298$.

Time = 0.16 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.41

$$\int \frac{(5-x)\,(2+3x^2)^{5/2}}{(3+2x)^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^10,x, algorithm="maxima")
```

output
```
27595971/9007501562500*(3*x^2 + 2)^(5/2) - 13/315*(3*x^2 + 2)^(7/2)/(512*x
^9 + 6912*x^8 + 41472*x^7 + 145152*x^6 + 326592*x^5 + 489888*x^4 + 489888*
x^3 + 314928*x^2 + 118098*x + 19683) - 27/2450*(3*x^2 + 2)^(7/2)/(256*x^8
+ 3072*x^7 + 16128*x^6 + 48384*x^5 + 90720*x^4 + 108864*x^3 + 81648*x^2 +
34992*x + 6561) - 4741/1800750*(3*x^2 + 2)^(7/2)/(128*x^7 + 1344*x^6 + 604
8*x^5 + 15120*x^4 + 22680*x^3 + 20412*x^2 + 10206*x + 2187) - 949/1500625*
(3*x^2 + 2)^(7/2)/(64*x^6 + 576*x^5 + 2160*x^4 + 4320*x^3 + 4860*x^2 + 291
6*x + 729) - 8541/52521875*(3*x^2 + 2)^(7/2)/(32*x^5 + 240*x^4 + 720*x^3 +
 1080*x^2 + 810*x + 243) - 82563/1838265625*(3*x^2 + 2)^(7/2)/(16*x^4 + 96
*x^3 + 216*x^2 + 216*x + 81) - 845559/64339296875*(3*x^2 + 2)^(7/2)/(8*x^3
 + 36*x^2 + 54*x + 27) - 9198657/2251875390625*(3*x^2 + 2)^(7/2)/(4*x^2 +
12*x + 9) + 8993673/1801500312500*(3*x^2 + 2)^(3/2)*x + 102492/45037507812
5*(3*x^2 + 2)^(3/2) - 106771041/9007501562500*(3*x^2 + 2)^(5/2)/(2*x + 3)
+ 691821/51471437500*sqrt(3*x^2 + 2)*x + 76869/25735718750*sqrt(35)*arcsin
h(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3)) + 76869/128678593
75*sqrt(3*x^2 + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(149) = 298.

Time = 0.15 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.79

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^{10}}\,dx = \frac{76869}{25735718750}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3x^2+2}}\right)$$
$$-\frac{9\sqrt{3}\left(364416\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^{17}+27877824\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^{16}+1042205258\sqrt{3}\left(\sqrt{3}x-\sqrt{3x\right.\right.}{}$$

input
```
integrate((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^10,x, algorithm="giac")
```

output
```
76869/25735718750*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) +
2*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2)
)) - 9/94119200000*sqrt(3)*(364416*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^1
7 + 27877824*(sqrt(3)*x - sqrt(3*x^2 + 2))^16 + 1042205258*sqrt(3)*(sqrt(3
)*x - sqrt(3*x^2 + 2))^15 - 956098170*(sqrt(3)*x - sqrt(3*x^2 + 2))^14 + 1
003625490*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^13 - 85987901496*(sqrt(3)*
x - sqrt(3*x^2 + 2))^12 - 60468401868*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)
)^11 - 331045664193*(sqrt(3)*x - sqrt(3*x^2 + 2))^10 - 22913148915*sqrt(3)
*(sqrt(3)*x - sqrt(3*x^2 + 2))^9 - 544736640510*(sqrt(3)*x - sqrt(3*x^2 +
2))^8 + 284856270864*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^7 - 90885012422
4*(sqrt(3)*x - sqrt(3*x^2 + 2))^6 + 90616216992*sqrt(3)*(sqrt(3)*x - sqrt(
3*x^2 + 2))^5 - 115517223360*(sqrt(3)*x - sqrt(3*x^2 + 2))^4 - 52895204480
*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 - 565618176*(sqrt(3)*x - sqrt(3*x
^2 + 2))^2 + 140708352*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 17333248)/(
(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 3*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))
- 2)^9
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.14

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^{10}}\,dx = \frac{76869\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{25735718750}$$
$$-\frac{76869\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{35}\,\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{25735718750}$$
$$+\frac{4515\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{32768\,\left(x^8+12\,x^7+63\,x^6+189\,x^5+\frac{2835\,x^4}{8}+\frac{1701\,x^3}{4}+\frac{5103\,x^2}{16}+\frac{2187\,x}{16}+\frac{6561}{256}\right)}$$
$$+\frac{1838301\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{614656000\,\left(x^4+6\,x^3+\frac{27\,x^2}{2}+\frac{27\,x}{2}+\frac{81}{16}\right)}$$
$$-\frac{15925\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{294912\,\left(x^9+\frac{27\,x^8}{2}+81\,x^7+\frac{567\,x^6}{2}+\frac{5103\,x^5}{8}+\frac{15309\,x^4}{16}+\frac{15309\,x^3}{16}+\frac{19683\,x^2}{32}+\frac{59049\,x}{256}+\frac{19683}{512}\right)}$$
$$-\frac{923241\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{35123200\,\left(x^5+\frac{15\,x^4}{2}+\frac{45\,x^3}{2}+\frac{135\,x^2}{4}+\frac{405\,x}{16}+\frac{243}{32}\right)}-\frac{152343\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{94119200000\,\left(x+\frac{3}{2}\right)}$$
$$+\frac{35213\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{401408\,\left(x^6+9\,x^5+\frac{135\,x^4}{4}+\frac{135\,x^3}{2}+\frac{1215\,x^2}{16}+\frac{729\,x}{16}+\frac{729}{64}\right)}$$
$$+\frac{80649\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{5378240000\,\left(x^2+3\,x+\frac{9}{4}\right)}$$
$$-\frac{52201\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{344064\,\left(x^7+\frac{21\,x^6}{2}+\frac{189\,x^5}{4}+\frac{945\,x^4}{8}+\frac{2835\,x^3}{16}+\frac{5103\,x^2}{32}+\frac{5103\,x}{64}+\frac{2187}{128}\right)}$$
$$+\frac{55473\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{1536640000\,\left(x^3+\frac{9\,x^2}{2}+\frac{27\,x}{4}+\frac{27}{8}\right)}$$

input
```
int(-((3*x^2 + 2)^(5/2)*(x - 5))/(2*x + 3)^10,x)
```

output
```
(76869*35^(1/2)*log(x + 3/2))/25735718750 - (76869*35^(1/2)*log(x - (3^(1/
2)*35^(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/25735718750 + (4515*3^(1/2)*(x^2
+ 2/3)^(1/2))/(32768*((2187*x)/16 + (5103*x^2)/16 + (1701*x^3)/4 + (2835*x
^4)/8 + 189*x^5 + 63*x^6 + 12*x^7 + x^8 + 6561/256)) + (1838301*3^(1/2)*(x
^2 + 2/3)^(1/2))/(614656000*((27*x)/2 + (27*x^2)/2 + 6*x^3 + x^4 + 81/16))
 - (15925*3^(1/2)*(x^2 + 2/3)^(1/2))/(294912*((59049*x)/256 + (19683*x^2)/
32 + (15309*x^3)/16 + (15309*x^4)/16 + (5103*x^5)/8 + (567*x^6)/2 + 81*x^7
 + (27*x^8)/2 + x^9 + 19683/512)) - (923241*3^(1/2)*(x^2 + 2/3)^(1/2))/(35
123200*((405*x)/16 + (135*x^2)/4 + (45*x^3)/2 + (15*x^4)/2 + x^5 + 243/32)
) - (152343*3^(1/2)*(x^2 + 2/3)^(1/2))/(94119200000*(x + 3/2)) + (35213*3^
(1/2)*(x^2 + 2/3)^(1/2))/(401408*((729*x)/16 + (1215*x^2)/16 + (135*x^3)/2
 + (135*x^4)/4 + 9*x^5 + x^6 + 729/64)) + (80649*3^(1/2)*(x^2 + 2/3)^(1/2)
)/(5378240000*(3*x + x^2 + 9/4)) - (52201*3^(1/2)*(x^2 + 2/3)^(1/2))/(3440
64*((5103*x)/64 + (5103*x^2)/32 + (2835*x^3)/16 + (945*x^4)/8 + (189*x^5)/
4 + (21*x^6)/2 + x^7 + 2187/128)) + (55473*3^(1/2)*(x^2 + 2/3)^(1/2))/(153
6640000*((27*x)/4 + (9*x^2)/2 + x^3 + 27/8))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.89

$$\int \frac{(5-x)\left(2+3x^2\right)^{5/2}}{(3+2x)^{10}}\,dx = \text{Too large to display}$$

input
```
int((5-x)*(3*x^2+2)^(5/2)/(3+2*x)^10,x)
```

output
```
( - 383904360*sqrt(3*x**2 + 2)*x**8 - 1050231420*sqrt(3*x**2 + 2)*x**7 + 2
1720802320*sqrt(3*x**2 + 2)*x**6 + 881907129810*sqrt(3*x**2 + 2)*x**5 - 34
1259448950*sqrt(3*x**2 + 2)*x**4 + 404863417035*sqrt(3*x**2 + 2)*x**3 - 14
85946411530*sqrt(3*x**2 + 2)*x**2 - 419683802895*sqrt(3*x**2 + 2)*x - 5581
83956260*sqrt(3*x**2 + 2) + 708424704*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(3
5) + 9*x - 4)*x**9 + 9563733504*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9
*x - 4)*x**8 + 57382401024*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x -
4)*x**7 + 200838403584*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x
**6 + 451886408064*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**5
+ 677829612096*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**4 + 67
7829612096*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**3 + 435747
607776*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**2 + 1634053529
16*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x + 27234225486*sqrt(
35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4) - 708424704*sqrt(35)*log(2*x
+ 3)*x**9 - 9563733504*sqrt(35)*log(2*x + 3)*x**8 - 57382401024*sqrt(35)*l
og(2*x + 3)*x**7 - 200838403584*sqrt(35)*log(2*x + 3)*x**6 - 451886408064*
sqrt(35)*log(2*x + 3)*x**5 - 677829612096*sqrt(35)*log(2*x + 3)*x**4 - 677
829612096*sqrt(35)*log(2*x + 3)*x**3 - 435747607776*sqrt(35)*log(2*x + 3)*
x**2 - 163405352916*sqrt(35)*log(2*x + 3)*x - 27234225486*sqrt(35)*log(2*x
 + 3))/(463242937500*(512*x**9 + 6912*x**8 + 41472*x**7 + 145152*x**6 +...
```

3.235 $\int \frac{(5-x)(3+2x)^4}{\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 106

$$\int \frac{(5-x)(3+2x)^4}{\sqrt{2+3x^2}}\,dx = \frac{1477}{270}(3+2x)^2\sqrt{2+3x^2} + \frac{19}{30}(3+2x)^3\sqrt{2+3x^2} - \frac{1}{15}(3+2x)^4\sqrt{2+3x^2} + \frac{49}{81}(383+99x)\sqrt{2+3x^2} + \frac{343\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
1477/270*(3+2*x)^2*(3*x^2+2)^(1/2)+19/30*(3+2*x)^3*(3*x^2+2)^(1/2)-1/15*(3
+2*x)^4*(3*x^2+2)^(1/2)+49/81*(383+99*x)*(3*x^2+2)^(1/2)+343/9*arcsinh(1/2
*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int \frac{(5-x)(3+2x)^4}{\sqrt{2+3x^2}}\,dx = -\frac{1}{405}\sqrt{2+3x^2}\left(-118513-58860x-12264x^2+540x^3+432x^4\right) - \frac{343\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((5 - x)*(3 + 2*x)^4)/Sqrt[2 + 3*x^2],x]
```

output
```
-1/405*(Sqrt[2 + 3*x^2]*(-118513 - 58860*x - 12264*x^2 + 540*x^3 + 432*x^4
)) - (343*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {687, 687, 27, 687, 27, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)(2x+3)^4}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{15}\int \frac{(2x+3)^3(114x+241)}{\sqrt{3x^2+2}}dx-\frac{1}{15}(2x+3)^4\sqrt{3x^2+2}$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{15}\left(\frac{1}{12}\int \frac{42(2x+3)^2(211x+174)}{\sqrt{3x^2+2}}dx+\frac{19}{2}\sqrt{3x^2+2}(2x+3)^3\right)-\frac{1}{15}(2x+3)^4\sqrt{3x^2+2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}\left(\frac{7}{2}\int \frac{(2x+3)^2(211x+174)}{\sqrt{3x^2+2}}dx+\frac{19}{2}\sqrt{3x^2+2}(2x+3)^3\right)-\frac{1}{15}(2x+3)^4\sqrt{3x^2+2}$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{15}\left(\frac{7}{2}\left(\frac{1}{9}\int \frac{70(2x+3)(99x+43)}{\sqrt{3x^2+2}}dx+\frac{211}{9}\sqrt{3x^2+2}(2x+3)^2\right)+\frac{19}{2}\sqrt{3x^2+2}(2x+3)^3\right)-\frac{1}{15}(2x+3)^4\sqrt{3x^2+2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}\left(\frac{7}{2}\left(\frac{70}{9}\int \frac{(2x+3)(99x+43)}{\sqrt{3x^2+2}}dx+\frac{211}{9}\sqrt{3x^2+2}(2x+3)^2\right)+\frac{19}{2}\sqrt{3x^2+2}(2x+3)^3\right)-\frac{1}{15}(2x+3)^4\sqrt{3x^2+2}$$

$$\Big\downarrow \text{676}$$

$$\frac{1}{15}\left(\frac{7}{2}\left(\frac{70}{9}\left(63\int \frac{1}{\sqrt{3x^2+2}}dx+33\sqrt{3x^2+2}x+\frac{383}{3}\sqrt{3x^2+2}\right)+\frac{211}{9}\sqrt{3x^2+2}(2x+3)^2\right)+\frac{19}{2}\sqrt{3x^2+2}(2x\right.$$
$$\frac{1}{15}(2x+3)^4\sqrt{3x^2+2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{15}\left(\frac{7}{2}\left(\frac{70}{9}\left(21\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)+33\sqrt{3x^2+2}x+\frac{383}{3}\sqrt{3x^2+2}\right)+\frac{211}{9}\sqrt{3x^2+2}(2x+3)^2\right)+\frac{19}{2}\sqrt{3x^2+\right.$$
$$\frac{1}{15}(2x+3)^4\sqrt{3x^2+2}$$

input
```
Int[((5 - x)*(3 + 2*x)^4)/Sqrt[2 + 3*x^2],x]
```

output
```
-1/15*((3 + 2*x)^4*Sqrt[2 + 3*x^2]) + ((19*(3 + 2*x)^3*Sqrt[2 + 3*x^2])/2
+ (7*((211*(3 + 2*x)^2*Sqrt[2 + 3*x^2])/9 + (70*((383*Sqrt[2 + 3*x^2])/3 +
 33*x*Sqrt[2 + 3*x^2] + 21*Sqrt[3]*ArcSinh[Sqrt[3/2]*x]))/9))/2)/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

method	result
risch	$-\frac{\left(432 x^{4}+540 x^{3}-12264 x^{2}-58860 x -118513\right) \sqrt{3 x^{2}+2}}{405}+\frac{343 \,\operatorname{arcsinh}\left(\frac{\sqrt{6}\, x}{2}\right) \sqrt{3}}{9}$
trager	$\left(-\frac{16}{15} x^{4}-\frac{4}{3} x^{3}+\frac{4088}{135} x^{2}+\frac{436}{3} x +\frac{118513}{405}\right) \sqrt{3 x^{2}+2}-\frac{343 \operatorname{RootOf}\left(\_Z^{2}-3\right) \ln\left(-\operatorname{RootOf}\left(\_Z^{2}-3\right) \sqrt{3 x^{2}+2}\right.}{9}$
default	$\frac{343 \,\operatorname{arcsinh}\left(\frac{\sqrt{6}\, x}{2}\right) \sqrt{3}}{9}+\frac{118513 \sqrt{3 x^{2}+2}}{405}+\frac{436 x \sqrt{3 x^{2}+2}}{3}+\frac{4088 x^{2} \sqrt{3 x^{2}+2}}{135}-\frac{4 x^{3} \sqrt{3 x^{2}+2}}{3}-\frac{16 x^{4} \sqrt{3 x^{2}+2}}{15}$
meijerg	$135 \sqrt{3}\, \operatorname{arcsinh}\left(\frac{x \sqrt{2}\, \sqrt{3}}{2}\right)-\frac{32 \sqrt{3}\, \left(-\frac{\sqrt{\pi}\, x \sqrt{2}\, \sqrt{3}\, \left(-15 x^{2}+15\right) \sqrt{\frac{3 x^{2}}{2}+1}}{40}+\frac{3 \sqrt{\pi}\, \operatorname{arcsinh}\left(\frac{x \sqrt{2}\, \sqrt{3}}{2}\right)}{4}\right)}{27 \sqrt{\pi}}+\frac{88 \sqrt{2}\, \left(\frac{4 \sqrt{\pi}}{3}-\frac{\sqrt{\pi}\, \left(-\right.}{}\right.}{3 \sqrt{}}$

input
```
int((5-x)*(2*x+3)^4/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/405*(432*x^4+540*x^3-12264*x^2-58860*x-118513)*(3*x^2+2)^(1/2)+343/9*ar
csinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.57

$$
\begin{aligned}
\int \frac{(5-x)(3+2x)^4}{\sqrt{2+3x^2}}\,dx
&= -\frac{1}{405}\left(432\,x^4+540\,x^3-12264\,x^2-58860\,x-118513\right)\sqrt{3\,x^2+2}\\
&\quad+\frac{343}{18}\,\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)
\end{aligned}
$$

input
```
integrate((5-x)*(3+2*x)^4/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/405*(432*x^4 + 540*x^3 - 12264*x^2 - 58860*x - 118513)*sqrt(3*x^2 + 2)
+ 343/18*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{(5-x)(3+2x)^4}{\sqrt{2+3x^2}}\,dx = -\frac{16x^4\sqrt{3x^2+2}}{15} - \frac{4x^3\sqrt{3x^2+2}}{3} + \frac{4088x^2\sqrt{3x^2+2}}{135} + \frac{436x\sqrt{3x^2+2}}{3} + \frac{118513\sqrt{3x^2+2}}{405} + \frac{343\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

input
```
integrate((5-x)*(3+2*x)**4/(3*x**2+2)**(1/2),x)
```

output
```
-16*x**4*sqrt(3*x**2 + 2)/15 - 4*x**3*sqrt(3*x**2 + 2)/3 + 4088*x**2*sqrt(
3*x**2 + 2)/135 + 436*x*sqrt(3*x**2 + 2)/3 + 118513*sqrt(3*x**2 + 2)/405 +
 343*sqrt(3)*asinh(sqrt(6)*x/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

$$\int \frac{(5-x)(3+2x)^4}{\sqrt{2+3x^2}}\,dx = -\frac{16}{15}\sqrt{3x^2+2}x^4 - \frac{4}{3}\sqrt{3x^2+2}x^3 + \frac{4088}{135}\sqrt{3x^2+2}x^2 + \frac{436}{3}\sqrt{3x^2+2}x + \frac{343}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{118513}{405}\sqrt{3x^2+2}$$

input
```
integrate((5-x)*(3+2*x)^4/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
-16/15*sqrt(3*x^2 + 2)*x^4 - 4/3*sqrt(3*x^2 + 2)*x^3 + 4088/135*sqrt(3*x^2
 + 2)*x^2 + 436/3*sqrt(3*x^2 + 2)*x + 343/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x)
 + 118513/405*sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50

$$\int \frac{(5-x)(3+2x)^4}{\sqrt{2+3x^2}}\,dx = -\frac{1}{405}\left(12\left(\left(9\left(4x+5\right)x-1022\right)x-4905\right)x-118513\right)\sqrt{3x^2+2} - \frac{343}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)$$

input
```
integrate((5-x)*(3+2*x)^4/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-1/405*(12*((9*(4*x + 5)*x - 1022)*x - 4905)*x - 118513)*sqrt(3*x^2 + 2) -
 343/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

$$\int \frac{(5-x)(3+2x)^4}{\sqrt{2+3x^2}}\,dx = \frac{343\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(-\frac{16x^4}{5}-4x^3+\frac{4088x^2}{45}+436x+\frac{118513}{135}\right)}{3}$$

input
```
int(-((2*x + 3)^4*(x - 5))/(3*x^2 + 2)^(1/2),x)
```

output
```
(343*3^(1/2)*asinh((6^(1/2)*x)/2))/9 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(436*x +
 (4088*x^2)/45 - 4*x^3 - (16*x^4)/5 + 118513/135))/3
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

$$\int \frac{(5-x)(3+2x)^4}{\sqrt{2+3x^2}}\,dx = -\frac{16\sqrt{3x^2+2}\,x^4}{15} - \frac{4\sqrt{3x^2+2}\,x^3}{3} + \frac{4088\sqrt{3x^2+2}\,x^2}{135} + \frac{436\sqrt{3x^2+2}\,x}{3} + \frac{118513\sqrt{3x^2+2}}{405} + \frac{343\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{9}$$

input

```
int((5-x)*(3+2*x)^4/(3*x^2+2)^(1/2),x)
```

output

```
( - 432*sqrt(3*x**2 + 2)*x**4 - 540*sqrt(3*x**2 + 2)*x**3 + 12264*sqrt(3*x
**2 + 2)*x**2 + 58860*sqrt(3*x**2 + 2)*x + 118513*sqrt(3*x**2 + 2) + 15435
*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/405
```

3.236 $\int \frac{(5-x)(3+2x)^3}{\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 84

$$\int \frac{(5-x)(3+2x)^3}{\sqrt{2+3x^2}}\,dx = \frac{31}{36}(3+2x)^2\sqrt{2+3x^2}-\frac{1}{12}(3+2x)^3\sqrt{2+3x^2}+\frac{5}{54}(809+171x)\sqrt{2+3x^2}+\frac{275\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
31/36*(3+2*x)^2*(3*x^2+2)^(1/2)-1/12*(3+2*x)^3*(3*x^2+2)^(1/2)+5/54*(809+1
71*x)*(3*x^2+2)^(1/2)+275/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{(5-x)(3+2x)^3}{\sqrt{2+3x^2}}\,dx = -\frac{1}{27}\sqrt{2+3x^2}\left(-2171-585x-12x^2+18x^3\right)-\frac{275\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((5 - x)*(3 + 2*x)^3)/Sqrt[2 + 3*x^2],x]
```

output
```
-1/27*(Sqrt[2 + 3*x^2]*(-2171 - 585*x - 12*x^2 + 18*x^3)) - (275*Log[-(Sqr
t[3]*x) + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {687, 27, 687, 27, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)(2x+3)^3}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{12}\int \frac{3(2x+3)^2(31x+64)}{\sqrt{3x^2+2}}dx-\frac{1}{12}(2x+3)^3\sqrt{3x^2+2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{(2x+3)^2(31x+64)}{\sqrt{3x^2+2}}dx-\frac{1}{12}(2x+3)^3\sqrt{3x^2+2}$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{4}\left(\frac{1}{9}\int \frac{10(2x+3)(171x+148)}{\sqrt{3x^2+2}}dx+\frac{31}{9}\sqrt{3x^2+2}(2x+3)^2\right)-\frac{1}{12}(2x+3)^3\sqrt{3x^2+2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{10}{9}\int \frac{(2x+3)(171x+148)}{\sqrt{3x^2+2}}dx+\frac{31}{9}\sqrt{3x^2+2}(2x+3)^2\right)-\frac{1}{12}(2x+3)^3\sqrt{3x^2+2}$$

$$\Big\downarrow \text{676}$$

$$\frac{1}{4}\left(\frac{10}{9}\left(330\int \frac{1}{\sqrt{3x^2+2}}dx+57\sqrt{3x^2+2}x+\frac{809}{3}\sqrt{3x^2+2}\right)+\frac{31}{9}\sqrt{3x^2+2}(2x+3)^2\right)-\frac{1}{12}(2x+3)^3\sqrt{3x^2+2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{4}\left(\frac{10}{9}\left(110\sqrt{3}\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)+57\sqrt{3x^2+2}x+\frac{809}{3}\sqrt{3x^2+2}\right)+\frac{31}{9}\sqrt{3x^2+2}(2x+3)^2\right)-\frac{1}{12}(2x+3)^3\sqrt{3x^2+2}$$

input
```
Int[((5 - x)*(3 + 2*x)^3)/Sqrt[2 + 3*x^2],x]
```

output
```
-1/12*((3 + 2*x)^3*Sqrt[2 + 3*x^2]) + ((31*(3 + 2*x)^2*Sqrt[2 + 3*x^2])/9
+ (10*((809*Sqrt[2 + 3*x^2])/3 + 57*x*Sqrt[2 + 3*x^2] + 110*Sqrt[3]*ArcSin
h[Sqrt[3/2]*x]))/9)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.48

method	result
risch	$-\frac{(18x^3-12x^2-585x-2171)\sqrt{3x^2+2}}{27}+\frac{275\,\text{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}$
trager	$\left(-\frac{2}{3}x^3+\frac{4}{9}x^2+\frac{65}{3}x+\frac{2171}{27}\right)\sqrt{3x^2+2}+\frac{275\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(\text{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
default	$\frac{275\,\text{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}+\frac{2171\sqrt{3x^2+2}}{27}+\frac{65x\sqrt{3x^2+2}}{3}+\frac{4x^2\sqrt{3x^2+2}}{9}-\frac{2x^3\sqrt{3x^2+2}}{3}$
meijerg	$45\sqrt{3}\,\text{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)+\frac{4\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-6x^2+8\right)\sqrt{\frac{3x^2}{2}+1}}{6}\right)}{9\sqrt{\pi}}+\frac{14\sqrt{3}\left(\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{3x^2}{2}+1}}{2}-\sqrt{\pi}\,\text{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right.}{\sqrt{\pi}}$

input
```
int((5-x)*(2*x+3)^3/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27*(18*x^3-12*x^2-585*x-2171)*(3*x^2+2)^(1/2)+275/9*arcsinh(1/2*6^(1/2)
*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

$$\int \frac{(5-x)(3+2x)^3}{\sqrt{2+3x^2}}\,dx = -\frac{1}{27}\left(18\,x^3-12\,x^2-585\,x-2171\right)\sqrt{3\,x^2+2} + \frac{275}{18}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/27*(18*x^3 - 12*x^2 - 585*x - 2171)*sqrt(3*x^2 + 2) + 275/18*sqrt(3)*lo
g(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{(5-x)(3+2x)^3}{\sqrt{2+3x^2}}\,dx = -\frac{2x^3\sqrt{3x^2+2}}{3}+\frac{4x^2\sqrt{3x^2+2}}{9}+\frac{65x\sqrt{3x^2+2}}{3}+\frac{2171\sqrt{3x^2+2}}{27}+\frac{275\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

input
```
integrate((5-x)*(3+2*x)**3/(3*x**2+2)**(1/2),x)
```

output
```
-2*x**3*sqrt(3*x**2 + 2)/3 + 4*x**2*sqrt(3*x**2 + 2)/9 + 65*x*sqrt(3*x**2
+ 2)/3 + 2171*sqrt(3*x**2 + 2)/27 + 275*sqrt(3)*asinh(sqrt(6)*x/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \frac{(5-x)(3+2x)^3}{\sqrt{2+3x^2}}\,dx = -\frac{2}{3}\sqrt{3\,x^2+2}x^3+\frac{4}{9}\sqrt{3\,x^2+2}x^2+\frac{65}{3}\sqrt{3\,x^2+2}x+\frac{275}{9}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)+\frac{2171}{27}\sqrt{3\,x^2+2}$$

input
```
integrate((5-x)*(3+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*sqrt(3*x^2 + 2)*x^3 + 4/9*sqrt(3*x^2 + 2)*x^2 + 65/3*sqrt(3*x^2 + 2)*
x + 275/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 2171/27*sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.58

$$\int \frac{(5-x)(3+2x)^3}{\sqrt{2+3x^2}}\,dx = -\frac{1}{27}\left(3\left(2\left(3x-2\right)x-195\right)x-2171\right)\sqrt{3x^2+2} - \frac{275}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)$$

input
```
integrate((5-x)*(3+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-1/27*(3*(2*(3*x - 2)*x - 195)*x - 2171)*sqrt(3*x^2 + 2) - 275/9*sqrt(3)*l
og(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.48

$$\int \frac{(5-x)(3+2x)^3}{\sqrt{2+3x^2}}\,dx = \frac{275\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(-2x^3+\frac{4x^2}{3}+65x+\frac{2171}{9}\right)}{3}$$

input
```
int(-((2*x + 3)^3*(x - 5))/(3*x^2 + 2)^(1/2),x)
```

output
```
(275*3^(1/2)*asinh((6^(1/2)*x)/2))/9 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(65*x +
(4*x^2)/3 - 2*x^3 + 2171/9))/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int \frac{(5-x)(3+2x)^3}{\sqrt{2+3x^2}}\,dx = -\frac{2\sqrt{3x^2+2}\,x^3}{3} + \frac{4\sqrt{3x^2+2}\,x^2}{9} + \frac{65\sqrt{3x^2+2}\,x}{3} + \frac{2171\sqrt{3x^2+2}}{27} + \frac{275\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{9}$$

input
```
int((5-x)*(3+2*x)^3/(3*x^2+2)^(1/2),x)
```

output
```
( - 18*sqrt(3*x**2 + 2)*x**3 + 12*sqrt(3*x**2 + 2)*x**2 + 585*sqrt(3*x**2
+ 2)*x + 2171*sqrt(3*x**2 + 2) + 825*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(
3)*x)/sqrt(2)))/27
```

3.237 $\int \frac{(5-x)(3+2x)^2}{\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{(5-x)(3+2x)^2}{\sqrt{2+3x^2}}\,dx = -\frac{1}{9}(3+2x)^2\sqrt{2+3x^2} + \frac{2}{27}(251+36x)\sqrt{2+3x^2} + \frac{127\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
-1/9*(3+2*x)^2*(3*x^2+2)^(1/2)+2/27*(251+36*x)*(3*x^2+2)^(1/2)+127/9*arcsi
nh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{(5-x)(3+2x)^2}{\sqrt{2+3x^2}}\,dx = -\frac{1}{27}\sqrt{2+3x^2}\left(-475-36x+12x^2\right) - \frac{127\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((5 - x)*(3 + 2*x)^2)/Sqrt[2 + 3*x^2],x]
```

output
```
-1/27*(Sqrt[2 + 3*x^2]*(-475 - 36*x + 12*x^2)) - (127*Log[-(Sqrt[3]*x) + S
qrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {687, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)(2x+3)^2}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{9}\int \frac{(2x+3)(72x+143)}{\sqrt{3x^2+2}}dx-\frac{1}{9}(2x+3)^2\sqrt{3x^2+2}$$

$$\Big\downarrow \text{676}$$

$$\frac{1}{9}\left(381\int \frac{1}{\sqrt{3x^2+2}}dx+24\sqrt{3x^2+2}x+\frac{502}{3}\sqrt{3x^2+2}\right)-\frac{1}{9}(2x+3)^2\sqrt{3x^2+2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{9}\left(127\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)+24\sqrt{3x^2+2}x+\frac{502}{3}\sqrt{3x^2+2}\right)-\frac{1}{9}(2x+3)^2\sqrt{3x^2+2}$$

input
```
Int[((5 - x)*(3 + 2*x)^2)/Sqrt[2 + 3*x^2],x]
```

output
```
-1/9*((3 + 2*x)^2*Sqrt[2 + 3*x^2]) + ((502*Sqrt[2 + 3*x^2])/3 + 24*x*Sqrt[
2 + 3*x^2] + 127*Sqrt[3]*ArcSinh[Sqrt[3/2]*x])/9
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

method	result
risch	$-\frac{\left(12 x^{2}-36 x -475\right) \sqrt{3 x^{2}+2}}{27}+\frac{127 \,\operatorname{arcsinh}\left(\frac{\sqrt{6}\, x}{2}\right) \sqrt{3}}{9}$
default	$\frac{127 \,\operatorname{arcsinh}\left(\frac{\sqrt{6}\, x}{2}\right) \sqrt{3}}{9}+\frac{475 \sqrt{3 x^{2}+2}}{27}+\frac{4 x \sqrt{3 x^{2}+2}}{3}-\frac{4 x^{2} \sqrt{3 x^{2}+2}}{9}$
trager	$\left(-\frac{4}{9} x^{2}+\frac{4}{3} x +\frac{475}{27}\right) \sqrt{3 x^{2}+2}-\frac{127 \operatorname{RootOf}\left(\_Z^{2}-3\right) \ln\left(-\operatorname{RootOf}\left(\_Z^{2}-3\right) \sqrt{3 x^{2}+2}+3 x \right)}{9}$
meijerg	$15 \sqrt{3}\, \operatorname{arcsinh}\left(\frac{x \sqrt{2}\, \sqrt{3}}{2}\right)+\frac{8 \sqrt{3}\, \left(\frac{\sqrt{\pi}\, x \sqrt{2}\, \sqrt{3}\, \sqrt{\frac{3 x^{2}}{2}+1}}{2}-\sqrt{\pi}\, \operatorname{arcsinh}\left(\frac{x \sqrt{2}\, \sqrt{3}}{2}\right)\right)}{9 \sqrt{\pi}}+\frac{17 \sqrt{2}\, \left(-2 \sqrt{\pi}+2 \sqrt{\pi}\, \sqrt{\frac{3 x^{2}}{2}+1}\right)}{2 \sqrt{\pi}}-$

input
```
int((5-x)*(2*x+3)^2/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27*(12*x^2-36*x-475)*(3*x^2+2)^(1/2)+127/9*arcsinh(1/2*6^(1/2)*x)*3^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{(5-x)(3+2x)^2}{\sqrt{2+3x^2}}\,dx = -\frac{1}{27}\left(12\,x^2-36\,x-475\right)\sqrt{3\,x^2+2} + \frac{127}{18}\,\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/27*(12*x^2 - 36*x - 475)*sqrt(3*x^2 + 2) + 127/18*sqrt(3)*log(-sqrt(3)*
sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{(5-x)(3+2x)^2}{\sqrt{2+3x^2}}\,dx = -\frac{4x^2\sqrt{3x^2+2}}{9} + \frac{4x\sqrt{3x^2+2}}{3} + \frac{475\sqrt{3x^2+2}}{27} + \frac{127\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

input
```
integrate((5-x)*(3+2*x)**2/(3*x**2+2)**(1/2),x)
```

output
```
-4*x**2*sqrt(3*x**2 + 2)/9 + 4*x*sqrt(3*x**2 + 2)/3 + 475*sqrt(3*x**2 + 2)
/27 + 127*sqrt(3)*asinh(sqrt(6)*x/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{(5-x)(3+2x)^2}{\sqrt{2+3x^2}}\,dx = -\frac{4}{9}\sqrt{3\,x^2+2}x^2+\frac{4}{3}\sqrt{3\,x^2+2}x + \frac{127}{9}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)+\frac{475}{27}\sqrt{3\,x^2+2}$$

input
```
integrate((5-x)*(3+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
-4/9*sqrt(3*x^2 + 2)*x^2 + 4/3*sqrt(3*x^2 + 2)*x + 127/9*sqrt(3)*arcsinh(1
/2*sqrt(6)*x) + 475/27*sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

$$\int \frac{(5-x)(3+2x)^2}{\sqrt{2+3x^2}}\,dx = -\frac{1}{27}(12\,(x-3)x-475)\sqrt{3\,x^2+2} - \frac{127}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

input
```
integrate((5-x)*(3+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-1/27*(12*(x - 3)*x - 475)*sqrt(3*x^2 + 2) - 127/9*sqrt(3)*log(-sqrt(3)*x
+ sqrt(3*x^2 + 2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int \frac{(5-x)(3+2x)^2}{\sqrt{2+3x^2}}\,dx = \frac{127\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9} + \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(-\frac{4\,x^2}{3}+4\,x+\frac{475}{9}\right)}{3}$$

input
```
int(-((2*x + 3)^2*(x - 5))/(3*x^2 + 2)^(1/2),x)
```

output
```
(127*3^(1/2)*asinh((6^(1/2)*x)/2))/9 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(4*x - (
4*x^2)/3 + 475/9))/3
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{(5-x)(3+2x)^2}{\sqrt{2+3x^2}}\,dx = -\frac{4\sqrt{3x^2+2}\,x^2}{9} + \frac{4\sqrt{3x^2+2}\,x}{3} + \frac{475\sqrt{3x^2+2}}{27} + \frac{127\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{9}$$

input
```
int((5-x)*(3+2*x)^2/(3*x^2+2)^(1/2),x)
```

output
```
( - 12*sqrt(3*x**2 + 2)*x**2 + 36*sqrt(3*x**2 + 2)*x + 475*sqrt(3*x**2 + 2
) + 381*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)))/27
```

3.238 $\int \frac{(5-x)(3+2x)}{\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \frac{(5-x)(3+2x)}{\sqrt{2+3x^2}}\,dx = \frac{1}{3}(7-x)\sqrt{2+3x^2} + \frac{47\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
1/3*(7-x)*(3*x^2+2)^(1/2)+47/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \frac{(5-x)(3+2x)}{\sqrt{2+3x^2}}\,dx = -\frac{1}{3}(-7+x)\sqrt{2+3x^2} - \frac{47\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((5 - x)*(3 + 2*x))/Sqrt[2 + 3*x^2],x]
```

output
```
-1/3*((-7 + x)*Sqrt[2 + 3*x^2]) - (47*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])
/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)(2x+3)}{\sqrt{3x^2+2}}\,dx$$

$$\Big\downarrow \text{ 676}$$

$$\frac{47}{3}\int \frac{1}{\sqrt{3x^2+2}}dx-\frac{1}{3}\sqrt{3x^2+2}x+\frac{7}{3}\sqrt{3x^2+2}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{47\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}-\frac{1}{3}\sqrt{3x^2+2}x+\frac{7}{3}\sqrt{3x^2+2}$$

input
```
Int[((5 - x)*(3 + 2*x))/Sqrt[2 + 3*x^2],x]
```

output
```
(7*Sqrt[2 + 3*x^2])/3 - (x*Sqrt[2 + 3*x^2])/3 + (47*ArcSinh[Sqrt[3/2]*x])/
(3*Sqrt[3])
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] && !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{(-7+x)\sqrt{3x^2+2}}{3}+\frac{47\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}$	28
default	$\frac{47\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}+\frac{7\sqrt{3x^2+2}}{3}-\frac{x\sqrt{3x^2+2}}{3}$	37
trager	$\left(\frac{7}{3}-\frac{x}{3}\right)\sqrt{3x^2+2}-\frac{47\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$	47
meijerg	$5\sqrt{3}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)+\frac{7\sqrt{2}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{\frac{3x^2}{2}+1}\right)}{6\sqrt{\pi}}-\frac{2\sqrt{3}\left(\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{3x^2}{2}+1}}{2}-\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{9\sqrt{\pi}}$	90

input
```
int((5-x)*(2*x+3)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-7+x)*(3*x^2+2)^(1/2)+47/9*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{(5-x)(3+2x)}{\sqrt{2+3x^2}}\,dx=-\frac{1}{3}\sqrt{3\,x^2+2}(x-7)+\frac{47}{18}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((5-x)*(3+2*x)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3*x^2 + 2)*(x - 7) + 47/18*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*
x - 3*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{(5-x)(3+2x)}{\sqrt{2+3x^2}}\,dx = -\frac{x\sqrt{3x^2+2}}{3} + \frac{7\sqrt{3x^2+2}}{3} + \frac{47\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

input
```
integrate((5-x)*(3+2*x)/(3*x**2+2)**(1/2),x)
```

output
```
-x*sqrt(3*x**2 + 2)/3 + 7*sqrt(3*x**2 + 2)/3 + 47*sqrt(3)*asinh(sqrt(6)*x/
2)/9
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{(5-x)(3+2x)}{\sqrt{2+3x^2}}\,dx = -\frac{1}{3}\sqrt{3x^2+2}x + \frac{47}{9}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{7}{3}\sqrt{3x^2+2}$$

input
```
integrate((5-x)*(3+2*x)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3*x^2 + 2)*x + 47/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 7/3*sqrt(3*
x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(5-x)(3+2x)}{\sqrt{2+3x^2}}\,dx = -\frac{1}{3}\sqrt{3x^2+2}(x-7) - \frac{47}{9}\sqrt{3}\log\left(-\sqrt{3}x + \sqrt{3x^2+2}\right)$$

input
```
integrate((5-x)*(3+2*x)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(3*x^2 + 2)*(x - 7) - 47/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 +
2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{(5-x)(3+2x)}{\sqrt{2+3x^2}}\,dx = \frac{47\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9} - \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,(x-7)}{3}$$

input
```
int(-((2*x + 3)*(x - 5))/(3*x^2 + 2)^(1/2),x)
```

output
```
(47*3^(1/2)*asinh((6^(1/2)*x)/2))/9 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(x - 7))/
3
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{(5-x)(3+2x)}{\sqrt{2+3x^2}}\,dx = -\frac{\sqrt{3x^2+2}\,x}{3} + \frac{7\sqrt{3x^2+2}}{3} + \frac{47\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{9}$$

input
```
int((5-x)*(3+2*x)/(3*x^2+2)^(1/2),x)
```

output
```
( - 3*sqrt(3*x**2 + 2)*x + 21*sqrt(3*x**2 + 2) + 47*sqrt(3)*log((sqrt(3*x*
*2 + 2) + sqrt(3)*x)/sqrt(2)))/9
```

3.239 $\int \frac{5-x}{\sqrt{2+3x^2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{5-x}{\sqrt{2+3x^2}} \, dx = -\frac{1}{3}\sqrt{2+3x^2} + \frac{5\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

output
```
-1/3*(3*x^2+2)^(1/2)+5/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{5-x}{\sqrt{2+3x^2}} \, dx = -\frac{1}{3}\sqrt{2+3x^2} - \frac{5\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[(5 - x)/Sqrt[2 + 3*x^2],x]
```

output
```
-1/3*Sqrt[2 + 3*x^2] - (5*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5-x}{\sqrt{3x^2+2}}dx$$
$$\Big\downarrow \text{455}$$
$$5\int \frac{1}{\sqrt{3x^2+2}}dx-\frac{1}{3}\sqrt{3x^2+2}$$
$$\Big\downarrow \text{222}$$
$$\frac{5\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}-\frac{1}{3}\sqrt{3x^2+2}$$

input
```
Int[(5 - x)/Sqrt[2 + 3*x^2],x]
```

output
```
-1/3*Sqrt[2 + 3*x^2] + (5*ArcSinh[Sqrt[3/2]*x])/Sqrt[3]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{\sqrt{3x^2+2}}{3}+\frac{5\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{3}$	25
risch	$-\frac{\sqrt{3x^2+2}}{3}+\frac{5\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{3}$	25
trager	$-\frac{\sqrt{3x^2+2}}{3}+\frac{5\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{3}$	42
meijerg	$\frac{5\sqrt{3}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{3}-\frac{\sqrt{2}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{\frac{3x^2}{2}+1}\right)}{6\sqrt{\pi}}$	45

input
```
int((5-x)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*x^2+2)^(1/2)+5/3*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \frac{5-x}{\sqrt{2+3x^2}}\,dx=\frac{5}{6}\,\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)-\frac{1}{3}\,\sqrt{3\,x^2+2}$$

input
```
integrate((5-x)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
5/6*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) - 1/3*sqrt(3*x^2 +
 2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{5-x}{\sqrt{2+3x^2}}\,dx = -\frac{\sqrt{3x^2+2}}{3} + \frac{5\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{3}$$

input
```
integrate((5-x)/(3*x**2+2)**(1/2),x)
```

output
```
-sqrt(3*x**2 + 2)/3 + 5*sqrt(3)*asinh(sqrt(6)*x/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{5-x}{\sqrt{2+3x^2}}\,dx = \frac{5}{3}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) - \frac{1}{3}\sqrt{3\,x^2+2}$$

input
```
integrate((5-x)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
5/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 1/3*sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{5-x}{\sqrt{2+3x^2}}\,dx = -\frac{5}{3}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) - \frac{1}{3}\sqrt{3\,x^2+2}$$

input
```
integrate((5-x)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-5/3*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 1/3*sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{5-x}{\sqrt{2+3x^2}}\,dx = \frac{5\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{3} - \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{3}$$

input
```
int(-(x - 5)/(3*x^2 + 2)^(1/2),x)
```

output
```
(5*3^(1/2)*asinh((6^(1/2)*x)/2))/3 - (3^(1/2)*(x^2 + 2/3)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{5-x}{\sqrt{2+3x^2}}\,dx = -\frac{\sqrt{3x^2+2}}{3} + \frac{5\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{3}$$

input
```
int((5-x)/(3*x^2+2)^(1/2),x)
```

output
```
( - sqrt(3*x**2 + 2) + 5*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2
)))/3
```

3.240 $\int \frac{5-x}{(3+2x)\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 52

$$\int \frac{5-x}{(3+2x)\sqrt{2+3x^2}}\,dx = -\frac{\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}} - \frac{13\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{2\sqrt{35}}$$

output
```
-1/6*arcsinh(1/2*x*6^(1/2))*3^(1/2)-13/70*35^(1/2)*arctanh(1/35*(4-9*x)*35
^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

$$\int \frac{5-x}{(3+2x)\sqrt{2+3x^2}}\,dx = \frac{13\operatorname{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{\sqrt{35}} + \frac{\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{2\sqrt{3}}$$

input
```
Integrate[(5 - x)/((3 + 2*x)*Sqrt[2 + 3*x^2]),x]
```

output
```
(13*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]])/Sqrt[
35] + Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]]/(2*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5-x}{(2x+3)\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{719}$$

$$\frac{13}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{1}{2}\int \frac{1}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{222}$$

$$\frac{13}{2}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}$$

$$\Big\downarrow \text{488}$$

$$-\frac{13}{2}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{13\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{2\sqrt{35}}$$

input
```
Int[(5 - x)/((3 + 2*x)*Sqrt[2 + 3*x^2]),x]
```

output
```
-1/2*ArcSinh[Sqrt[3/2]*x]/Sqrt[3] - (13*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2
 + 3*x^2])])/(2*Sqrt[35])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

method	result
default	$-\frac{\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{6}-\frac{13\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^{2}-36x-19}}\right)}{70}$
trager	$-\frac{13\,\operatorname{RootOf}\left(\_Z^{2}-35\right)\ln\left(-\frac{9\,\operatorname{RootOf}\left(\_Z^{2}-35\right)x-4\,\operatorname{RootOf}\left(\_Z^{2}-35\right)-35\sqrt{3x^{2}+2}}{2x+3}\right)}{70}+\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(-\operatorname{RootOf}\left(\_\right.\right.}{6}$

input
```
int((5-x)/(2*x+3)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*arcsinh(1/2*6^(1/2)*x)*3^(1/2)-13/70*35^(1/2)*arctanh(2/35*(4-9*x)*35
^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.46

$$\int \frac{5-x}{(3+2x)\sqrt{2+3x^2}}\,dx = \frac{1}{12}\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right) + \frac{13}{140}\sqrt{35}\log\left(-\frac{\sqrt{35}\sqrt{3\,x^2+2}(9\,x-4)+93\,x^2-36\,x+43}{4\,x^2+12\,x+9}\right)$$

input
```
integrate((5-x)/(3+2*x)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + 13/140*sqrt(35)*
log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12
*x + 9))
```

Sympy [F]

$$\int \frac{5-x}{(3+2x)\sqrt{2+3x^2}}\,dx = -\int \frac{x}{2x\sqrt{3x^2+2}+3\sqrt{3x^2+2}}\,dx - \int \left(-\frac{5}{2x\sqrt{3x^2+2}+3\sqrt{3x^2+2}}\right)dx$$

input
```
integrate((5-x)/(3+2*x)/(3*x**2+2)**(1/2),x)
```

output
```
-Integral(x/(2*x*sqrt(3*x**2 + 2) + 3*sqrt(3*x**2 + 2)), x) - Integral(-5/
(2*x*sqrt(3*x**2 + 2) + 3*sqrt(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \frac{5-x}{(3+2x)\sqrt{2+3x^2}}\,dx = -\frac{1}{6}\sqrt{3}\,\text{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{13}{70}\sqrt{35}\,\text{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|} - \frac{2\sqrt{6}}{3\,|2\,x+3|}\right)$$

input
```
integrate((5-x)/(3+2*x)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 13/70*sqrt(35)*arcsinh(3/2*sqrt(6)*x
/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x + 3))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(38) = 76.

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.73

$$\int \frac{5-x}{(3+2x)\sqrt{2+3x^2}}\,dx = \frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{13}{70}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3\,x^2+2}}\right)$$

input
```
integrate((5-x)/(3+2*x)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 13/70*sqrt(35)*log(-abs(-2
*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt
(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2)))
```

Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{5-x}{(3+2x)\sqrt{2+3x^2}}\,dx = \frac{\sqrt{35}\left(26\,\ln\left(x+\frac{3}{2}\right)-26\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)\right)}{140} - \frac{\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{6}$$

input
```
int(-(x - 5)/((2*x + 3)*(3*x^2 + 2)^(1/2)),x)
```

output
```
(35^(1/2)*(26*log(x + 3/2) - 26*log(x - (3^(1/2)*35^(1/2)*(x^2 + 2/3)^(1/2
))/9 - 4/9)))/140 - (3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/6
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.29

$$\int \frac{5-x}{(3+2x)\sqrt{2+3x^2}}\,dx = \frac{13\sqrt{35}\,atan\left(\frac{2\sqrt{3x^2+2}\,i+2\sqrt{3}\,ix}{\sqrt{35}-3\sqrt{3}}\right)i}{70} + \frac{13\sqrt{35}\log\left(4\sqrt{3x^2+2}\,\sqrt{3}\,x+3\sqrt{105}+12x^2-27\right)}{140} - \frac{13\sqrt{35}\log\left(\frac{2\sqrt{3x^2+2}+\sqrt{35}+2\sqrt{3}\,x+3\sqrt{3}}{\sqrt{2}}\right)}{70} - \frac{\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)}{6}$$

input
```
int((5-x)/(3+2*x)/(3*x^2+2)^(1/2),x)
```

output
```
(78*sqrt(35)*atan((2*sqrt(3*x**2 + 2)*i + 2*sqrt(3)*i*x)/(sqrt(35) - 3*sqr
t(3)))*i + 39*sqrt(35)*log(4*sqrt(3*x**2 + 2)*sqrt(3)*x + 3*sqrt(105) + 12
*x**2 - 27) - 78*sqrt(35)*log((2*sqrt(3*x**2 + 2) + sqrt(35) + 2*sqrt(3)*x
 + 3*sqrt(3))/sqrt(2)) - 70*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqr
t(2)))/420
```

3.241 $\int \frac{5-x}{(3+2x)^2\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{5-x}{(3+2x)^2\sqrt{2+3x^2}}\,dx = -\frac{13\sqrt{2+3x^2}}{35(3+2x)} - \frac{41\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{35\sqrt{35}}$$

output
```
-13*(3*x^2+2)^(1/2)/(105+70*x)-41/1225*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1
/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int \frac{5-x}{(3+2x)^2\sqrt{2+3x^2}}\,dx = -\frac{13\sqrt{2+3x^2}}{35(3+2x)} + \frac{82\operatorname{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{35\sqrt{35}}$$

input
```
Integrate[(5 - x)/((3 + 2*x)^2*Sqrt[2 + 3*x^2]),x]
```

output
```
(-13*Sqrt[2 + 3*x^2])/(35*(3 + 2*x)) + (82*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*
x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]])/(35*Sqrt[35])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5-x}{(2x+3)^2\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{ 679}$$

$$\frac{41}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{13\sqrt{3x^2+2}}{35(2x+3)}$$

$$\Big\downarrow \text{ 488}$$

$$-\frac{41}{35}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{13\sqrt{3x^2+2}}{35(2x+3)}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{41\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}-\frac{13\sqrt{3x^2+2}}{35(2x+3)}$$

input
```
Int[(5 - x)/((3 + 2*x)^2*Sqrt[2 + 3*x^2]),x]
```

output
```
(-13*Sqrt[2 + 3*x^2])/(35*(3 + 2*x)) - (41*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqr
t[2 + 3*x^2])])/(35*Sqrt[35])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{13\sqrt{3x^2+2}}{35(2x+3)}-\frac{41\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{1225}$	50
default	$-\frac{13\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}{70\left(x+\frac{3}{2}\right)}-\frac{41\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{1225}$	53
trager	$-\frac{13\sqrt{3x^2+2}}{35(2x+3)}+\frac{41\,\operatorname{RootOf}\left(\_Z^2-35\right)\ln\left(\frac{9\,\operatorname{RootOf}\left(\_Z^2-35\right)x-4\,\operatorname{RootOf}\left(\_Z^2-35\right)+35\sqrt{3x^2+2}}{2x+3}\right)}{1225}$	66

input
```
int((5-x)/(2*x+3)^2/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-13/35*(3*x^2+2)^(1/2)/(2*x+3)-41/1225*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1
/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \frac{5-x}{(3+2x)^2\sqrt{2+3x^2}}\,dx$$
$$= \frac{41\sqrt{35}(2x+3)\log\left(-\frac{\sqrt{35}\sqrt{3x^2+2}(9x-4)+93x^2-36x+43}{4x^2+12x+9}\right)-910\sqrt{3x^2+2}}{2450\,(2x+3)}$$

input
```
integrate((5-x)/(3+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2450*(41*sqrt(35)*(2*x + 3)*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) + 9
3*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9)) - 910*sqrt(3*x^2 + 2))/(2*x + 3)
```

Sympy [F]

$$\int \frac{5-x}{(3+2x)^2\sqrt{2+3x^2}}\,dx = -\int \frac{x}{4x^2\sqrt{3x^2+2}+12x\sqrt{3x^2+2}+9\sqrt{3x^2+2}}\,dx$$
$$-\int\left(-\frac{5}{4x^2\sqrt{3x^2+2}+12x\sqrt{3x^2+2}+9\sqrt{3x^2+2}}\right)dx$$

input
```
integrate((5-x)/(3+2*x)**2/(3*x**2+2)**(1/2),x)
```

output
```
-Integral(x/(4*x**2*sqrt(3*x**2 + 2) + 12*x*sqrt(3*x**2 + 2) + 9*sqrt(3*x*
*2 + 2)), x) - Integral(-5/(4*x**2*sqrt(3*x**2 + 2) + 12*x*sqrt(3*x**2 + 2
) + 9*sqrt(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{5-x}{(3+2x)^2\sqrt{2+3x^2}}\,dx = \frac{41}{1225}\sqrt{35}\,\mathrm{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|}-\frac{2\sqrt{6}}{3\,|2\,x+3|}\right)-\frac{13\sqrt{3\,x^2+2}}{35\,(2\,x+3)}$$

input
```
integrate((5-x)/(3+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
41/1225*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x
+ 3)) - 13/35*sqrt(3*x^2 + 2)/(2*x + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(44) = 88$.

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.27

$$\begin{aligned}&\int \frac{5-x}{(3+2x)^2\sqrt{2+3x^2}}\,dx\\&=\frac{1}{2450}\sqrt{35}\left(13\sqrt{35}\sqrt{3}+82\,\log\left(\sqrt{35}\sqrt{3}-9\right)\right)\mathrm{sgn}\left(\frac{1}{2\,x+3}\right)\\&\quad-\frac{41\sqrt{35}\log\left(\sqrt{35}\left(\sqrt{-\frac{18}{2\,x+3}+\frac{35}{(2\,x+3)^2}+3}+\frac{\sqrt{35}}{2\,x+3}\right)-9\right)}{1225\,\mathrm{sgn}\left(\frac{1}{2\,x+3}\right)}\\&\quad-\frac{13\sqrt{-\frac{18}{2\,x+3}+\frac{35}{(2\,x+3)^2}+3}}{70\,\mathrm{sgn}\left(\frac{1}{2\,x+3}\right)}\end{aligned}$$

input
```
integrate((5-x)/(3+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
1/2450*sqrt(35)*(13*sqrt(35)*sqrt(3) + 82*log(sqrt(35)*sqrt(3) - 9))*sgn(1
/(2*x + 3)) - 41/1225*sqrt(35)*log(sqrt(35)*(sqrt(-18/(2*x + 3) + 35/(2*x
+ 3)^2 + 3) + sqrt(35)/(2*x + 3)) - 9)/sgn(1/(2*x + 3)) - 13/70*sqrt(-18/(
2*x + 3) + 35/(2*x + 3)^2 + 3)/sgn(1/(2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{5-x}{(3+2x)^2\sqrt{2+3x^2}}\,dx = \frac{41\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{1225} - \frac{41\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{35}\,\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{1225} - \frac{13\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{70\,\left(x+\frac{3}{2}\right)}$$

input
```
int(-(x - 5)/((2*x + 3)^2*(3*x^2 + 2)^(1/2)),x)
```

output
```
(41*35^(1/2)*log(x + 3/2))/1225 - (41*35^(1/2)*log(x - (3^(1/2)*35^(1/2)*(
x^2 + 2/3)^(1/2))/9 - 4/9))/1225 - (13*3^(1/2)*(x^2 + 2/3)^(1/2))/(70*(x +
 3/2))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{5-x}{(3+2x)^2\sqrt{2+3x^2}}\,dx = \frac{-455\sqrt{3x^2+2}+82\sqrt{35}\log\left(\sqrt{3x^2+2}\,\sqrt{35}+9x-4\right)x+123\sqrt{35}\log\left(\sqrt{3x^2+2}\,\sqrt{35}+9x-4\right)-82\sqrt{}}{2450x+3675}$$

input
```
int((5-x)/(3+2*x)^2/(3*x^2+2)^(1/2),x)
```

output
```
( - 455*sqrt(3*x**2 + 2) + 82*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x
 - 4)*x + 123*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4) - 82*sqrt(
35)*log(2*x + 3)*x - 123*sqrt(35)*log(2*x + 3))/(1225*(2*x + 3))
```

3.242 $\int \frac{5-x}{(3+2x)^3\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{5-x}{(3+2x)^3\sqrt{2+3x^2}}\,dx = -\frac{13\sqrt{2+3x^2}}{70(3+2x)^2}-\frac{281\sqrt{2+3x^2}}{2450(3+2x)}-\frac{291\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{1225\sqrt{35}}$$

output
```
-13/70*(3*x^2+2)^(1/2)/(3+2*x)^2-281*(3*x^2+2)^(1/2)/(7350+4900*x)-291/428
75*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{5-x}{(3+2x)^3\sqrt{2+3x^2}}\,dx = \frac{-\frac{35(649+281x)\sqrt{2+3x^2}}{(3+2x)^2}+582\sqrt{35}\text{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{42875}$$

input
```
Integrate[(5 - x)/((3 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

output
```
((-35*(649 + 281*x)*Sqrt[2 + 3*x^2])/(3 + 2*x)^2 + 582*Sqrt[35]*ArcTanh[(3
*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]])/42875
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {688, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5-x}{(2x+3)^3\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{688}$$

$$-\frac{1}{70}\int -\frac{82-39x}{(2x+3)^2\sqrt{3x^2+2}}dx-\frac{13\sqrt{3x^2+2}}{70(2x+3)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{70}\int \frac{82-39x}{(2x+3)^2\sqrt{3x^2+2}}dx-\frac{13\sqrt{3x^2+2}}{70(2x+3)^2}$$

$$\Big\downarrow \text{679}$$

$$\frac{1}{70}\left(\frac{582}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{281\sqrt{3x^2+2}}{35(2x+3)}\right)-\frac{13\sqrt{3x^2+2}}{70(2x+3)^2}$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{70}\left(-\frac{582}{35}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{281\sqrt{3x^2+2}}{35(2x+3)}\right)-\frac{13\sqrt{3x^2+2}}{70(2x+3)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{70}\left(-\frac{582\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}-\frac{281\sqrt{3x^2+2}}{35(2x+3)}\right)-\frac{13\sqrt{3x^2+2}}{70(2x+3)^2}$$

input
```
Int[(5 - x)/((3 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

output
```
(-13*Sqrt[2 + 3*x^2])/(70*(3 + 2*x)^2) + ((-281*Sqrt[2 + 3*x^2])/(35*(3 +
2*x)) - (582*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(35*Sqrt[35]))
/70
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

method	result	size
risch	$-\frac{843x^3+1947x^2+562x+1298}{1225(2x+3)^2\sqrt{3x^2+2}}-\frac{291\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{42875}$	65
trager	$-\frac{(281x+649)\sqrt{3x^2+2}}{1225(2x+3)^2}+\frac{291\,\mathrm{RootOf}\left(\_Z^2-35\right)\ln\left(\frac{9\,\mathrm{RootOf}\left(\_Z^2-35\right)x-4\,\mathrm{RootOf}\left(\_Z^2-35\right)+35\sqrt{3x^2+2}}{2x+3}\right)}{42875}$	71
default	$-\frac{13\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}{280\left(x+\frac{3}{2}\right)^2}-\frac{281\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}{4900\left(x+\frac{3}{2}\right)}-\frac{291\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{42875}$	74

input
```
int((5-x)/(2*x+3)^3/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1225*(843*x^3+1947*x^2+562*x+1298)/(2*x+3)^2/(3*x^2+2)^(1/2)-291/42875*
35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \frac{5-x}{(3+2x)^3\sqrt{2+3x^2}}\,dx = \frac{291\,\sqrt{35}(4\,x^2+12\,x+9)\log\left(-\frac{\sqrt{35}\sqrt{3\,x^2+2}(9\,x-4)+93\,x^2-36\,x+43}{4\,x^2+12\,x+9}\right)-70\,\sqrt{3\,x^2+2}(281\,x+649)}{85750\,(4\,x^2+12\,x+9)}$$

input
```
integrate((5-x)/(3+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/85750*(291*sqrt(35)*(4*x^2 + 12*x + 9)*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9
*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9)) - 70*sqrt(3*x^2 + 2)*(28
1*x + 649))/(4*x^2 + 12*x + 9)
```

Sympy [F]

$$\int \frac{5-x}{(3+2x)^3\sqrt{2+3x^2}}\,dx$$
$$= -\int \frac{x}{8x^3\sqrt{3x^2+2}+36x^2\sqrt{3x^2+2}+54x\sqrt{3x^2+2}+27\sqrt{3x^2+2}}\,dx$$
$$-\int \left(-\frac{5}{8x^3\sqrt{3x^2+2}+36x^2\sqrt{3x^2+2}+54x\sqrt{3x^2+2}+27\sqrt{3x^2+2}}\right)dx$$

input
```
integrate((5-x)/(3+2*x)**3/(3*x**2+2)**(1/2),x)
```

output
```
-Integral(x/(8*x**3*sqrt(3*x**2 + 2) + 36*x**2*sqrt(3*x**2 + 2) + 54*x*sqr
t(3*x**2 + 2) + 27*sqrt(3*x**2 + 2)), x) - Integral(-5/(8*x**3*sqrt(3*x**2
 + 2) + 36*x**2*sqrt(3*x**2 + 2) + 54*x*sqrt(3*x**2 + 2) + 27*sqrt(3*x**2
+ 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{5-x}{(3+2x)^3\sqrt{2+3x^2}}\,dx = \frac{291}{42875}\sqrt{35}\,\mathrm{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|}-\frac{2\sqrt{6}}{3\,|2\,x+3|}\right) - \frac{13\sqrt{3\,x^2+2}}{70\,(4\,x^2+12\,x+9)} - \frac{281\sqrt{3\,x^2+2}}{2450\,(2\,x+3)}$$

input
```
integrate((5-x)/(3+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
291/42875*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*
x + 3)) - 13/70*sqrt(3*x^2 + 2)/(4*x^2 + 12*x + 9) - 281/2450*sqrt(3*x^2 +
 2)/(2*x + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(62) = 124$.

Time = 0.14 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.38

$$\int \frac{5-x}{(3+2x)^3\sqrt{2+3x^2}}\,dx = \frac{291}{42875}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3x^2+2}}\right) - \frac{1164\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^3+6463\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2-17904\sqrt{3}x+2248\sqrt{3}+17904\sqrt{3x^2+2}}{4900\left(\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2+3\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)-2\right)^2}$$

input
```
integrate((5-x)/(3+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
291/42875*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqrt(3
*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) - 1/4
900*(1164*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 + 6463*sqrt(3)*(sqrt(3)*x - sqrt
(3*x^2 + 2))^2 - 17904*sqrt(3)*x + 2248*sqrt(3) + 17904*sqrt(3*x^2 + 2))/(
(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 3*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))
- 2)^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{5-x}{(3+2x)^3\sqrt{2+3x^2}}\,dx = \frac{291\sqrt{35}\ln\left(x+\frac{3}{2}\right)}{42875} - \frac{291\sqrt{35}\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{42875} - \frac{281\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{4900\left(x+\frac{3}{2}\right)} - \frac{13\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{280\left(x^2+3x+\frac{9}{4}\right)}$$

input
```
int(-(x - 5)/((2*x + 3)^3*(3*x^2 + 2)^(1/2)),x)
```

output
```
(291*35^(1/2)*log(x + 3/2))/42875 - (291*35^(1/2)*log(x - (3^(1/2)*35^(1/2
)*(x^2 + 2/3)^(1/2))/9 - 4/9))/42875 - (281*3^(1/2)*(x^2 + 2/3)^(1/2))/(49
00*(x + 3/2)) - (13*3^(1/2)*(x^2 + 2/3)^(1/2))/(280*(3*x + x^2 + 9/4))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.77

$$\int \frac{5-x}{(3+2x)^3\sqrt{2+3x^2}}\,dx$$
$$= \frac{-9835\sqrt{3x^2+2}\,x-22715\sqrt{3x^2+2}+1164\sqrt{35}\log\left(\sqrt{3x^2+2}\,\sqrt{35}+9x-4\right)x^2+3492\sqrt{35}\log\left(\sqrt{3x^2+\right.}}{}$$

input
```
int((5-x)/(3+2*x)^3/(3*x^2+2)^(1/2),x)
```

output
```
( - 9835*sqrt(3*x**2 + 2)*x - 22715*sqrt(3*x**2 + 2) + 1164*sqrt(35)*log(s
qrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**2 + 3492*sqrt(35)*log(sqrt(3*x**2 +
 2)*sqrt(35) + 9*x - 4)*x + 2619*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) +
9*x - 4) - 1164*sqrt(35)*log(2*x + 3)*x**2 - 3492*sqrt(35)*log(2*x + 3)*x
- 2619*sqrt(35)*log(2*x + 3))/(42875*(4*x**2 + 12*x + 9))
```

3.243 $\int \frac{5-x}{(3+2x)^4\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{5-x}{(3+2x)^4\sqrt{2+3x^2}}\,dx=-\frac{13\sqrt{2+3x^2}}{105(3+2x)^3}-\frac{16\sqrt{2+3x^2}}{245(3+2x)^2}-\frac{10\sqrt{2+3x^2}}{343(3+2x)}-\frac{57\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{1715\sqrt{35}}$$

output
```
-13/105*(3*x^2+2)^(1/2)/(3+2*x)^3-16/245*(3*x^2+2)^(1/2)/(3+2*x)^2-10*(3*x
^2+2)^(1/2)/(1029+686*x)-57/60025*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(
3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.79

$$\int \frac{5-x}{(3+2x)^4\sqrt{2+3x^2}}\,dx=\frac{-\frac{35\sqrt{2+3x^2}(2995+2472x+600x^2)}{(3+2x)^3}+342\sqrt{35}\operatorname{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{180075}$$

input
```
Integrate[(5 - x)/((3 + 2*x)^4*Sqrt[2 + 3*x^2]),x]
```

output
```
((-35*Sqrt[2 + 3*x^2]*(2995 + 2472*x + 600*x^2))/(3 + 2*x)^3 + 342*Sqrt[35
]*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]])/180075
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {688, 27, 688, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5-x}{(2x+3)^4\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{688}$$

$$-\frac{1}{105}\int -\frac{3(41-26x)}{(2x+3)^3\sqrt{3x^2+2}}dx-\frac{13\sqrt{3x^2+2}}{105(2x+3)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{35}\int \frac{41-26x}{(2x+3)^3\sqrt{3x^2+2}}dx-\frac{13\sqrt{3x^2+2}}{105(2x+3)^3}$$

$$\Big\downarrow \text{688}$$

$$\frac{1}{35}\left(-\frac{1}{70}\int -\frac{10(53-48x)}{(2x+3)^2\sqrt{3x^2+2}}dx-\frac{16\sqrt{3x^2+2}}{7(2x+3)^2}\right)-\frac{13\sqrt{3x^2+2}}{105(2x+3)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{35}\left(\frac{1}{7}\int \frac{53-48x}{(2x+3)^2\sqrt{3x^2+2}}dx-\frac{16\sqrt{3x^2+2}}{7(2x+3)^2}\right)-\frac{13\sqrt{3x^2+2}}{105(2x+3)^3}$$

$$\Big\downarrow \text{679}$$

$$\frac{1}{35}\left(\frac{1}{7}\left(\frac{57}{7}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{50\sqrt{3x^2+2}}{7(2x+3)}\right)-\frac{16\sqrt{3x^2+2}}{7(2x+3)^2}\right)-\frac{13\sqrt{3x^2+2}}{105(2x+3)^3}$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{35}\left(\frac{1}{7}\left(-\frac{57}{7}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{50\sqrt{3x^2+2}}{7(2x+3)}\right)-\frac{16\sqrt{3x^2+2}}{7(2x+3)^2}\right)-\frac{13\sqrt{3x^2+2}}{105(2x+3)^3}$$

$$\Big\downarrow 219$$

$$\frac{1}{35}\left(\frac{1}{7}\left(-\frac{57\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{7\sqrt{35}}-\frac{50\sqrt{3x^2+2}}{7(2x+3)}\right)-\frac{16\sqrt{3x^2+2}}{7(2x+3)^2}\right)-\frac{13\sqrt{3x^2+2}}{105(2x+3)^3}$$

input
```
Int[(5 - x)/((3 + 2*x)^4*Sqrt[2 + 3*x^2]),x]
```

output
```
(-13*Sqrt[2 + 3*x^2])/(105*(3 + 2*x)^3) + ((-16*Sqrt[2 + 3*x^2])/(7*(3 + 2
*x)^2) + ((-50*Sqrt[2 + 3*x^2])/(7*(3 + 2*x)) - (57*ArcTanh[(4 - 9*x)/(Sqr
t[35]*Sqrt[2 + 3*x^2])])/(7*Sqrt[35]))/7)/35
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

method	result	s
risch	$-\frac{1800x^4+7416x^3+10185x^2+4944x+5990}{5145(2x+3)^3\sqrt{3x^2+2}}-\frac{57\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{60025}$	7
trager	$-\frac{(600x^2+2472x+2995)\sqrt{3x^2+2}}{5145(2x+3)^3}-\frac{57\,\operatorname{RootOf}\left(\_Z^2-35\right)\ln\left(-\frac{9\,\operatorname{RootOf}\left(\_Z^2-35\right)x-4\,\operatorname{RootOf}\left(\_Z^2-35\right)-35\sqrt{3x^2+2}}{2x+3}\right)}{60025}$	7
default	$-\frac{13\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}{840\left(x+\frac{3}{2}\right)^3}-\frac{4\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}{245\left(x+\frac{3}{2}\right)^2}-\frac{5\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}{343\left(x+\frac{3}{2}\right)}-\frac{57\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{60025}$	9

input
```
int((5-x)/(2*x+3)^4/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5145*(1800*x^4+7416*x^3+10185*x^2+4944*x+5990)/(2*x+3)^3/(3*x^2+2)^(1/2
)-57/60025*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05

$$\int \frac{5-x}{(3+2x)^4\sqrt{2+3x^2}}\,dx$$
$$= \frac{171\sqrt{35}(8x^3+36x^2+54x+27)\log\left(-\frac{\sqrt{35}\sqrt{3x^2+2}(9x-4)+93x^2-36x+43}{4x^2+12x+9}\right)-70(600x^2+2472x+2995)\sqrt{}}{360150(8x^3+36x^2+54x+27)}$$

input
```
integrate((5-x)/(3+2*x)^4/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/360150*(171*sqrt(35)*(8*x^3 + 36*x^2 + 54*x + 27)*log(-(sqrt(35)*sqrt(3*
x^2 + 2)*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9)) - 70*(600*x^2
 + 2472*x + 2995)*sqrt(3*x^2 + 2))/(8*x^3 + 36*x^2 + 54*x + 27)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{5-x}{(3+2x)^4\sqrt{2+3x^2}}\,dx = \text{Timed out}$$

input
```
integrate((5-x)/(3+2*x)**4/(3*x**2+2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05

$$\int \frac{5-x}{(3+2x)^4\sqrt{2+3x^2}}\,dx = \frac{57}{60025}\sqrt{35}\,\text{arsinh}\left(\frac{3\sqrt{6}x}{2|2x+3|}-\frac{2\sqrt{6}}{3|2x+3|}\right) - \frac{13\sqrt{3x^2+2}}{105(8x^3+36x^2+54x+27)} - \frac{16\sqrt{3x^2+2}}{245(4x^2+12x+9)} - \frac{10\sqrt{3x^2+2}}{343(2x+3)}$$

input
```
integrate((5-x)/(3+2*x)^4/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
57/60025*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x
 + 3)) - 13/105*sqrt(3*x^2 + 2)/(8*x^3 + 36*x^2 + 54*x + 27) - 16/245*sqrt
(3*x^2 + 2)/(4*x^2 + 12*x + 9) - 10/343*sqrt(3*x^2 + 2)/(2*x + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(80) = 160$.

Time = 0.14 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.34

$$\int \frac{5-x}{(3+2x)^4\sqrt{2+3x^2}}\,dx = \frac{57}{60025}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3x^2+2}}\right) - \frac{\sqrt{3}\left(38\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^5+855\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^4+2250\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^3-13290\left(\right.}{3430\left(\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2+3\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+\right.\right.}$$

input
```
integrate((5-x)/(3+2*x)^4/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
57/60025*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqrt(3*
x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) - 1/34
30*sqrt(3)*(38*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^5 + 855*(sqrt(3)*x -
sqrt(3*x^2 + 2))^4 + 2250*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 - 13290*
(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 3448*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2
)) - 800)/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 3*sqrt(3)*(sqrt(3)*x - sqrt(3
*x^2 + 2)) - 2)^3
```

Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int \frac{5-x}{(3+2x)^4\sqrt{2+3x^2}}\,dx = \frac{57\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{60025} - \frac{57\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{60025} - \frac{5\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{343\left(x+\frac{3}{2}\right)} - \frac{4\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{245\left(x^2+3x+\frac{9}{4}\right)} - \frac{13\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{840\left(x^3+\frac{9x^2}{2}+\frac{27x}{4}+\frac{27}{8}\right)}$$

input
```
int(-(x - 5)/((2*x + 3)^4*(3*x^2 + 2)^(1/2)),x)
```

output
```
(57*35^(1/2)*log(x + 3/2))/60025 - (57*35^(1/2)*log(x - (3^(1/2)*35^(1/2)*
(x^2 + 2/3)^(1/2))/9 - 4/9))/60025 - (5*3^(1/2)*(x^2 + 2/3)^(1/2))/(343*(x
 + 3/2)) - (4*3^(1/2)*(x^2 + 2/3)^(1/2))/(245*(3*x + x^2 + 9/4)) - (13*3^(
1/2)*(x^2 + 2/3)^(1/2))/(840*((27*x)/4 + (9*x^2)/2 + x^3 + 27/8))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.93

$$\int \frac{5-x}{(3+2x)^4\sqrt{2+3x^2}}\,dx = \frac{-21000\sqrt{3x^2+2}\,x^2-86520\sqrt{3x^2+2}\,x-104825\sqrt{3x^2+2}+1368\sqrt{35}\log\left(\sqrt{3x^2+2}\,\sqrt{35}+9x-4\right)x^3}{}$$

input
```
int((5-x)/(3+2*x)^4/(3*x^2+2)^(1/2),x)
```

output
```
( - 21000*sqrt(3*x**2 + 2)*x**2 - 86520*sqrt(3*x**2 + 2)*x - 104825*sqrt(3
*x**2 + 2) + 1368*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**3 +
 6156*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**2 + 9234*sqrt(3
5)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x + 4617*sqrt(35)*log(sqrt(3*x
**2 + 2)*sqrt(35) + 9*x - 4) - 1368*sqrt(35)*log(2*x + 3)*x**3 - 6156*sqrt
(35)*log(2*x + 3)*x**2 - 9234*sqrt(35)*log(2*x + 3)*x - 4617*sqrt(35)*log(
2*x + 3))/(180075*(8*x**3 + 36*x**2 + 54*x + 27))
```

3.244 $\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{3/2}}\,dx = -\frac{7(2-7x)(3+2x)^3}{6\sqrt{2+3x^2}} - \frac{151}{27}(3+2x)^2\sqrt{2+3x^2} - \frac{10}{81}(185+207x)\sqrt{2+3x^2} + \frac{880\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
-7/6*(2-7*x)*(3+2*x)^3/(3*x^2+2)^(1/2)-151/27*(3+2*x)^2*(3*x^2+2)^(1/2)-10
/81*(185+207*x)*(3*x^2+2)^(1/2)+880/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.74

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{3/2}}\,dx = -\frac{33914+14715x-15024x^2+432x^3+288x^4}{162\sqrt{2+3x^2}} - \frac{880\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((5 - x)*(3 + 2*x)^4)/(2 + 3*x^2)^(3/2),x]
```

output
```
-1/162*(33914 + 14715*x - 15024*x^2 + 432*x^3 + 288*x^4)/Sqrt[2 + 3*x^2] -
 (880*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {684, 27, 687, 27, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)(2x+3)^4}{(3x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{684}$$

$$\frac{1}{6}\int \frac{2(36-151x)(2x+3)^2}{\sqrt{3x^2+2}}dx-\frac{7(2-7x)(2x+3)^3}{6\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(36-151x)(2x+3)^2}{\sqrt{3x^2+2}}dx-\frac{7(2-7x)(2x+3)^3}{6\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{3}\left(\frac{1}{9}\int \frac{10(218-207x)(2x+3)}{\sqrt{3x^2+2}}dx-\frac{151}{9}(2x+3)^2\sqrt{3x^2+2}\right)-\frac{7(2-7x)(2x+3)^3}{6\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{10}{9}\int \frac{(218-207x)(2x+3)}{\sqrt{3x^2+2}}dx-\frac{151}{9}(2x+3)^2\sqrt{3x^2+2}\right)-\frac{7(2-7x)(2x+3)^3}{6\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{676}$$

$$\frac{1}{3}\left(\frac{10}{9}\left(792\int \frac{1}{\sqrt{3x^2+2}}dx-69\sqrt{3x^2+2}x-\frac{185}{3}\sqrt{3x^2+2}\right)-\frac{151}{9}(2x+3)^2\sqrt{3x^2+2}\right)-\frac{7(2-7x)(2x+3)^3}{6\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{3}\left(\frac{10}{9}\left(264\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-69\sqrt{3x^2+2}x-\frac{185}{3}\sqrt{3x^2+2}\right)-\frac{151}{9}(2x+3)^2\sqrt{3x^2+2}\right)-\frac{7(2-7x)(2x+3)^3}{6\sqrt{3x^2+2}}$$

input
```
Int[((5 - x)*(3 + 2*x)^4)/(2 + 3*x^2)^(3/2),x]
```

output
```
(-7*(2 - 7*x)*(3 + 2*x)^3)/(6*Sqrt[2 + 3*x^2]) + ((-151*(3 + 2*x)^2*Sqrt[2
 + 3*x^2])/9 + (10*((-185*Sqrt[2 + 3*x^2])/3 - 69*x*Sqrt[2 + 3*x^2] + 264*
Sqrt[3]*ArcSinh[Sqrt[3/2]*x]))/9)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.51

method	result
risch	$-\frac{288x^4+432x^3-15024x^2+14715x+33914}{162\sqrt{3x^2+2}}+\frac{880\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}$
trager	$-\frac{288x^4+432x^3-15024x^2+14715x+33914}{162\sqrt{3x^2+2}}-\frac{880\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
default	$-\frac{545x}{6\sqrt{3x^2+2}}+\frac{880\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}+\frac{2504x^2}{27\sqrt{3x^2+2}}-\frac{16957}{81\sqrt{3x^2+2}}-\frac{8x^3}{3\sqrt{3x^2+2}}-\frac{16x^4}{9\sqrt{3x^2+2}}$
meijerg	$\frac{405\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}}-\frac{32\sqrt{3}\left(\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\left(\frac{15x^2}{2}+15\right)}{20\sqrt{\frac{3x^2}{2}+1}}-\frac{3\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{2}\right)}{27\sqrt{\pi}}+\frac{88\sqrt{2}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(6x^2+8\right)}{4\sqrt{\frac{3x^2}{2}+1}}\right)}{3\sqrt{\pi}}+\frac{96\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{}}{2\sqrt{\frac{3x^2}{2}+}}\right.}{}$

input
```
int((5-x)*(2*x+3)^4/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/162*(288*x^4+432*x^3-15024*x^2+14715*x+33914)/(3*x^2+2)^(1/2)+880/9*arc
sinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{3/2}}\,dx = \frac{7920\,\sqrt{3}(3\,x^2+2)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)-(288\,x^4+432\,x^3-15024}{162\,(3\,x^2+2)}$$

input
```
integrate((5-x)*(3+2*x)^4/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/162*(7920*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1
) - (288*x^4 + 432*x^3 - 15024*x^2 + 14715*x + 33914)*sqrt(3*x^2 + 2))/(3*
x^2 + 2)
```

Sympy [F]

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{3/2}}\,dx = -\int \left(-\frac{999x}{3x^2\sqrt{3x^2+2}+2\sqrt{3x^2+2}}\right)dx - \int \left(-\frac{864x^2}{3x^2\sqrt{3x^2+2}+2\sqrt{3x^2+2}}\right)dx - \int \left(-\frac{264x^3}{3x^2\sqrt{3x^2+2}+2\sqrt{3x^2+2}}\right)dx - \int \frac{16x^4}{3x^2\sqrt{3x^2+2}+2\sqrt{3x^2+2}}\,dx - \int \frac{16x^5}{3x^2\sqrt{3x^2+2}+2\sqrt{3x^2+2}}\,dx - \int \left(-\frac{405}{3x^2\sqrt{3x^2+2}+2\sqrt{3x^2+2}}\right)dx$$

input
```
integrate((5-x)*(3+2*x)**4/(3*x**2+2)**(3/2),x)
```

output
```
-Integral(-999*x/(3*x**2*sqrt(3*x**2 + 2) + 2*sqrt(3*x**2 + 2)), x) - Inte
gral(-864*x**2/(3*x**2*sqrt(3*x**2 + 2) + 2*sqrt(3*x**2 + 2)), x) - Integr
al(-264*x**3/(3*x**2*sqrt(3*x**2 + 2) + 2*sqrt(3*x**2 + 2)), x) - Integral
(16*x**4/(3*x**2*sqrt(3*x**2 + 2) + 2*sqrt(3*x**2 + 2)), x) - Integral(16*
x**5/(3*x**2*sqrt(3*x**2 + 2) + 2*sqrt(3*x**2 + 2)), x) - Integral(-405/(3
*x**2*sqrt(3*x**2 + 2) + 2*sqrt(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{3/2}}\,dx = -\frac{16\,x^4}{9\,\sqrt{3\,x^2+2}} - \frac{8\,x^3}{3\,\sqrt{3\,x^2+2}} + \frac{2504\,x^2}{27\,\sqrt{3\,x^2+2}} + \frac{880}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) - \frac{545\,x}{6\,\sqrt{3\,x^2+2}} - \frac{16957}{81\,\sqrt{3\,x^2+2}}$$

input
```
integrate((5-x)*(3+2*x)^4/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
-16/9*x^4/sqrt(3*x^2 + 2) - 8/3*x^3/sqrt(3*x^2 + 2) + 2504/27*x^2/sqrt(3*x
^2 + 2) + 880/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 545/6*x/sqrt(3*x^2 + 2) -
 16957/81/sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{3/2}}\,dx = -\frac{880}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)-\frac{3\,(16\,(3\,(2\,x+3)x-313)x+4905)x+33914}{162\,\sqrt{3\,x^2+2}}$$

input
```
integrate((5-x)*(3+2*x)^4/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
-880/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 1/162*(3*(16*(3*(2*x +
3)*x - 313)*x + 4905)*x + 33914)/sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{3/2}}\,dx = \frac{880\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9} - \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{16\,x^2}{9}+\frac{8\,x}{3}-\frac{2536}{27}\right)}{3} + \frac{\sqrt{3}\,\sqrt{6}\,\left(-44058+\sqrt{6}\,4809\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\,\sqrt{6}\,\left(44058+\sqrt{6}\,4809\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(-((2*x + 3)^4*(x - 5))/(3*x^2 + 2)^(3/2),x)
```

output
```
(880*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/9 - (3^(1/2)*(x^2 + 2/3)^(1/2)*
((8*x)/3 + (16*x^2)/9 - 2536/27))/3 + (3^(1/2)*6^(1/2)*(6^(1/2)*4809i - 44
058)*(x^2 + 2/3)^(1/2)*1i)/(1944*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*
(6^(1/2)*4809i + 44058)*(x^2 + 2/3)^(1/2)*1i)/(1944*(x - (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.47

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{3/2}}\,dx = \frac{-288\sqrt{3x^2+2}\,x^4-432\sqrt{3x^2+2}\,x^3+15024\sqrt{3x^2+2}\,x^2-14715\sqrt{3x^2+2}\,x-}{}$$

input
```
int((5-x)*(3+2*x)^4/(3*x^2+2)^(3/2),x)
```

output
```
( - 288*sqrt(3*x**2 + 2)*x**4 - 432*sqrt(3*x**2 + 2)*x**3 + 15024*sqrt(3*x
**2 + 2)*x**2 - 14715*sqrt(3*x**2 + 2)*x - 33914*sqrt(3*x**2 + 2) + 47520*
sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2))*x**2 + 31680*sqrt(3)*l
og((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)) - 14499*sqrt(3)*x**2 - 9666*sqr
t(3))/(162*(3*x**2 + 2))
```

3.245 $\int \frac{(5-x)(3+2x)^3}{(2+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{(5-x)(3+2x)^3}{(2+3x^2)^{3/2}}\,dx = -\frac{7(2-7x)(3+2x)^2}{6\sqrt{2+3x^2}} - \frac{2}{9}(131+51x)\sqrt{2+3x^2} + \frac{134\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
-7/6*(2-7*x)*(3+2*x)^2/(3*x^2+2)^(1/2)-2/9*(131+51*x)*(3*x^2+2)^(1/2)+134/
9*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{(5-x)(3+2x)^3}{(2+3x^2)^{3/2}}\,dx = -\frac{1426-411x-24x^2+24x^3}{18\sqrt{2+3x^2}} - \frac{134\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((5 - x)*(3 + 2*x)^3)/(2 + 3*x^2)^(3/2),x]
```

output
```
-1/18*(1426 - 411*x - 24*x^2 + 24*x^3)/Sqrt[2 + 3*x^2] - (134*Log[-(Sqrt[3
]*x) + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {684, 27, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)(2x+3)^3}{(3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{684}$$

$$\frac{1}{6}\int \frac{4(11-51x)(2x+3)}{\sqrt{3x^2+2}}dx-\frac{7(2-7x)(2x+3)^2}{6\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{(11-51x)(2x+3)}{\sqrt{3x^2+2}}dx-\frac{7(2-7x)(2x+3)^2}{6\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{676}$$

$$\frac{2}{3}\left(67\int \frac{1}{\sqrt{3x^2+2}}dx-17\sqrt{3x^2+2}x-\frac{131}{3}\sqrt{3x^2+2}\right)-\frac{7(2-7x)(2x+3)^2}{6\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{2}{3}\left(\frac{67\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}-17\sqrt{3x^2+2}x-\frac{131}{3}\sqrt{3x^2+2}\right)-\frac{7(2-7x)(2x+3)^2}{6\sqrt{3x^2+2}}$$

input
```
Int[((5 - x)*(3 + 2*x)^3)/(2 + 3*x^2)^(3/2),x]
```

output
```
(-7*(2 - 7*x)*(3 + 2*x)^2)/(6*Sqrt[2 + 3*x^2]) + (2*((-131*Sqrt[2 + 3*x^2]
)/3 - 17*x*Sqrt[2 + 3*x^2] + (67*ArcSinh[Sqrt[3/2]*x])/Sqrt[3]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

method	result
risch	$-\frac{24x^3-24x^2-411x+1426}{18\sqrt{3x^2+2}}+\frac{134\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}$
trager	$-\frac{24x^3-24x^2-411x+1426}{18\sqrt{3x^2+2}}+\frac{134\,\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
default	$\frac{137x}{6\sqrt{3x^2+2}}+\frac{134\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}+\frac{4x^2}{3\sqrt{3x^2+2}}-\frac{713}{9\sqrt{3x^2+2}}-\frac{4x^3}{3\sqrt{3x^2+2}}$
meijerg	$\frac{135\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}}+\frac{4\sqrt{2}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(6x^2+8\right)}{4\sqrt{\frac{3x^2}{2}+1}}\right)}{9\sqrt{\pi}}+\frac{14\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}}{2\sqrt{\frac{3x^2}{2}+1}}+\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{\sqrt{\pi}}+\frac{81\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^2}{2}+1}}\right)}{2\sqrt{\pi}}-$

input
```
int((5-x)*(2*x+3)^3/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/18*(24*x^3-24*x^2-411*x+1426)/(3*x^2+2)^(1/2)+134/9*arcsinh(1/2*6^(1/2)
*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{(5-x)(3+2x)^3}{(2+3x^2)^{3/2}}\,dx = \frac{134\sqrt{3}(3x^2+2)\log\left(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)-(24x^3-24x^2-411x+14}{18(3x^2+2)}$$

input
```
integrate((5-x)*(3+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/18*(134*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
- (24*x^3 - 24*x^2 - 411*x + 1426)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

Sympy [F]

$$\int \frac{(5-x)(3+2x)^3}{(2+3x^2)^{3/2}}\,dx = -\int \left(-\frac{243x}{3x^2\sqrt{3x^2+2}+2\sqrt{3x^2+2}}\right)dx - \int \left(-\frac{126x^2}{3x^2\sqrt{3x^2+2}+2\sqrt{3x^2+2}}\right)dx - \int \left(-\frac{4x^3}{3x^2\sqrt{3x^2+2}+2\sqrt{3x^2+2}}\right)dx - \int \frac{8x^4}{3x^2\sqrt{3x^2+2}+2\sqrt{3x^2+2}}\,dx - \int \left(-\frac{135}{3x^2\sqrt{3x^2+2}+2\sqrt{3x^2+2}}\right)dx$$

input
```
integrate((5-x)*(3+2*x)**3/(3*x**2+2)**(3/2),x)
```

output
```
-Integral(-243*x/(3*x**2*sqrt(3*x**2 + 2) + 2*sqrt(3*x**2 + 2)), x) - Inte
gral(-126*x**2/(3*x**2*sqrt(3*x**2 + 2) + 2*sqrt(3*x**2 + 2)), x) - Integr
al(-4*x**3/(3*x**2*sqrt(3*x**2 + 2) + 2*sqrt(3*x**2 + 2)), x) - Integral(8
*x**4/(3*x**2*sqrt(3*x**2 + 2) + 2*sqrt(3*x**2 + 2)), x) - Integral(-135/(
3*x**2*sqrt(3*x**2 + 2) + 2*sqrt(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{(5-x)(3+2x)^3}{(2+3x^2)^{3/2}}\,dx = -\frac{4\,x^3}{3\,\sqrt{3\,x^2+2}} + \frac{4\,x^2}{3\,\sqrt{3\,x^2+2}} + \frac{134}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{137\,x}{6\,\sqrt{3\,x^2+2}} - \frac{713}{9\,\sqrt{3\,x^2+2}}$$

input
```
integrate((5-x)*(3+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
-4/3*x^3/sqrt(3*x^2 + 2) + 4/3*x^2/sqrt(3*x^2 + 2) + 134/9*sqrt(3)*arcsinh
(1/2*sqrt(6)*x) + 137/6*x/sqrt(3*x^2 + 2) - 713/9/sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{(5-x)(3+2x)^3}{(2+3x^2)^{3/2}}\,dx = -\frac{134}{9}\,\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) - \frac{3\,(8\,(x-1)x-137)x+1426}{18\,\sqrt{3\,x^2+2}}$$

input
```
integrate((5-x)*(3+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
-134/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 1/18*(3*(8*(x - 1)*x -
137)*x + 1426)/sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.57

$$\int \frac{(5-x)(3+2x)^3}{(2+3x^2)^{3/2}}\,dx = \frac{134\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{9} - \frac{\sqrt{3}\left(\frac{4x}{3}-\frac{4}{3}\right)\sqrt{x^2+\frac{2}{3}}}{3} - \frac{\sqrt{3}\sqrt{6}\left(-12978+\sqrt{6}\,1281\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\left(12978+\sqrt{6}\,1281\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(-((2*x + 3)^3*(x - 5))/(3*x^2 + 2)^(3/2),x)
```

output
```
(134*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/9 - (3^(1/2)*((4*x)/3 - 4/3)*(x
^2 + 2/3)^(1/2))/3 - (3^(1/2)*6^(1/2)*(6^(1/2)*1281i - 12978)*(x^2 + 2/3)^
(1/2)*1i)/(1944*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*1281i +
12978)*(x^2 + 2/3)^(1/2)*1i)/(1944*(x + (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int \frac{(5-x)(3+2x)^3}{(2+3x^2)^{3/2}}\,dx = \frac{-24\sqrt{3x^2+2}\,x^3+24\sqrt{3x^2+2}\,x^2+411\sqrt{3x^2+2}\,x-1426\sqrt{3x^2+2}+804\sqrt{3}\,\mathrm{lo}}{54x^2+36}$$

input
```
int((5-x)*(3+2*x)^3/(3*x^2+2)^(3/2),x)
```

output
```
( - 24*sqrt(3*x**2 + 2)*x**3 + 24*sqrt(3*x**2 + 2)*x**2 + 411*sqrt(3*x**2
+ 2)*x - 1426*sqrt(3*x**2 + 2) + 804*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(
3)*x)/sqrt(2))*x**2 + 536*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(
2)) + 423*sqrt(3)*x**2 + 282*sqrt(3))/(18*(3*x**2 + 2))
```

3.246 $\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{3/2}}\,dx=-\frac{7(2-7x)(3+2x)}{6\sqrt{2+3x^2}}-\frac{53}{9}\sqrt{2+3x^2}+\frac{8\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
-7/6*(2-7*x)*(3+2*x)/(3*x^2+2)^(1/2)-53/9*(3*x^2+2)^(1/2)+8/9*arcsinh(1/2*
x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{3/2}}\,dx=-\frac{338-357x+24x^2}{18\sqrt{2+3x^2}}-\frac{8\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((5 - x)*(3 + 2*x)^2)/(2 + 3*x^2)^(3/2),x]
```

output
```
-1/18*(338 - 357*x + 24*x^2)/Sqrt[2 + 3*x^2] - (8*Log[-(Sqrt[3]*x) + Sqrt[
2 + 3*x^2]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {684, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)(2x+3)^2}{(3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{684}$$

$$\frac{1}{6}\int \frac{2(8-53x)}{\sqrt{3x^2+2}}dx-\frac{7(2-7x)(2x+3)}{6\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{8-53x}{\sqrt{3x^2+2}}dx-\frac{7(2-7x)(2x+3)}{6\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{3}\left(8\int \frac{1}{\sqrt{3x^2+2}}dx-\frac{53}{3}\sqrt{3x^2+2}\right)-\frac{7(2-7x)(2x+3)}{6\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{3}\left(\frac{8\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}-\frac{53}{3}\sqrt{3x^2+2}\right)-\frac{7(2-7x)(2x+3)}{6\sqrt{3x^2+2}}$$

input
```
Int[((5 - x)*(3 + 2*x)^2)/(2 + 3*x^2)^(3/2),x]
```

output
```
(-7*(2 - 7*x)*(3 + 2*x))/(6*Sqrt[2 + 3*x^2]) + ((-53*Sqrt[2 + 3*x^2])/3 +
(8*ArcSinh[Sqrt[3/2]*x])/Sqrt[3])/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

method	result	siz
risch	$-\frac{24x^{2}-357x+338}{18\sqrt{3x^{2}+2}}+\frac{8\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}$	35
default	$\frac{119x}{6\sqrt{3x^{2}+2}}+\frac{8\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}-\frac{4x^{2}}{3\sqrt{3x^{2}+2}}-\frac{169}{9\sqrt{3x^{2}+2}}$	51
trager	$-\frac{24x^{2}-357x+338}{18\sqrt{3x^{2}+2}}+\frac{8\,\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}-3\right)\sqrt{3x^{2}+2}+3x\right)}{9}$	52
meijerg	$\frac{45\sqrt{2}\,x}{4\sqrt{\frac{3x^{2}}{2}+1}}+\frac{8\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}}{2\sqrt{\frac{3x^{2}}{2}+1}}+\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{17\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^{2}}{2}+1}}\right)}{2\sqrt{\pi}}-\frac{4\sqrt{2}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\,\left(6x^{2}+8\right)}{4\sqrt{\frac{3x^{2}}{2}+1}}\right)}{9\sqrt{\pi}}$	12

input
```
int((5-x)*(2*x+3)^2/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/18*(24*x^2-357*x+338)/(3*x^2+2)^(1/2)+8/9*arcsinh(1/2*6^(1/2)*x)*3^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{3/2}}\,dx = \frac{8\sqrt{3}(3x^2+2)\log\left(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)-(24x^2-357x+338)\sqrt{3x^2+}}{18(3x^2+2)}$$

input
```
integrate((5-x)*(3+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/18*(8*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) -
(24*x^2 - 357*x + 338)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

Sympy [F]

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{3/2}}\,dx = -\int\left(-\frac{51x}{3x^2\sqrt{3x^2+2}+2\sqrt{3x^2+2}}\right)dx-\int\left(-\frac{8x^2}{3x^2\sqrt{3x^2+2}+2\sqrt{3x^2+2}}\right)dx - \int\frac{4x^3}{3x^2\sqrt{3x^2+2}+2\sqrt{3x^2+2}}\,dx-\int\left(-\frac{45}{3x^2\sqrt{3x^2+2}+2\sqrt{3x^2+2}}\right)dx$$

input
```
integrate((5-x)*(3+2*x)**2/(3*x**2+2)**(3/2),x)
```

output
```
-Integral(-51*x/(3*x**2*sqrt(3*x**2 + 2) + 2*sqrt(3*x**2 + 2)), x) - Integ
ral(-8*x**2/(3*x**2*sqrt(3*x**2 + 2) + 2*sqrt(3*x**2 + 2)), x) - Integral(
4*x**3/(3*x**2*sqrt(3*x**2 + 2) + 2*sqrt(3*x**2 + 2)), x) - Integral(-45/(
3*x**2*sqrt(3*x**2 + 2) + 2*sqrt(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{3/2}}\,dx = -\frac{4\,x^2}{3\,\sqrt{3\,x^2+2}} + \frac{8}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{119\,x}{6\,\sqrt{3\,x^2+2}} - \frac{169}{9\,\sqrt{3\,x^2+2}}$$

input
```
integrate((5-x)*(3+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
-4/3*x^2/sqrt(3*x^2 + 2) + 8/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 119/6*x/sq
rt(3*x^2 + 2) - 169/9/sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{3/2}}\,dx = -\frac{8}{9}\,\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) - \frac{3\,(8\,x-119)x+338}{18\,\sqrt{3\,x^2+2}}$$

input
```
integrate((5-x)*(3+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
-8/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 1/18*(3*(8*x - 119)*x + 3
38)/sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{3/2}}\,dx = \frac{8\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9} - \frac{4\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{9} - \frac{\sqrt{3}\,\sqrt{6}\,\left(-966+\sqrt{6}\,357\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{648\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\,\sqrt{6}\,\left(966+\sqrt{6}\,357\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{648\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(-((2*x + 3)^2*(x - 5))/(3*x^2 + 2)^(3/2),x)
```

output
```
(8*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/9 - (4*3^(1/2)*(x^2 + 2/3)^(1/2))
/9 - (3^(1/2)*6^(1/2)*(6^(1/2)*357i - 966)*(x^2 + 2/3)^(1/2)*1i)/(648*(x -
 (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*357i + 966)*(x^2 + 2/3)^(1/2
)*1i)/(648*(x + (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.75

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{3/2}}\,dx = \frac{-24\sqrt{3x^2+2}\,x^2+357\sqrt{3x^2+2}\,x-338\sqrt{3x^2+2}+48\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)x^2}{54x^2+36}$$

input
```
int((5-x)*(3+2*x)^2/(3*x^2+2)^(3/2),x)
```

output
```
( - 24*sqrt(3*x**2 + 2)*x**2 + 357*sqrt(3*x**2 + 2)*x - 338*sqrt(3*x**2 +
2) + 48*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2))*x**2 + 32*sqrt
(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)) + 357*sqrt(3)*x**2 + 238*s
qrt(3))/(18*(3*x**2 + 2))
```

3.247 $\int \frac{(5-x)(3+2x)}{\left(2+3x^2\right)^{3/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \frac{(5-x)(3+2x)}{\left(2+3x^2\right)^{3/2}} \, dx = -\frac{7(2-7x)}{6\sqrt{2+3x^2}}-\frac{2\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
1/6*(-14+49*x)/(3*x^2+2)^(1/2)-2/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \frac{(5-x)(3+2x)}{\left(2+3x^2\right)^{3/2}} \, dx = \frac{7(-2+7x)}{6\sqrt{2+3x^2}}+\frac{2 \log \left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((5 - x)*(3 + 2*x))/(2 + 3*x^2)^(3/2),x]
```

output
```
(7*(-2 + 7*x))/(6*Sqrt[2 + 3*x^2]) + (2*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]
])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{675, 222\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)(2x+3)}{(3x^2+2)^{3/2}} \, dx$$

$$\Big\downarrow \text{675}$$

$$-\frac{2}{3}\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{49x}{6\sqrt{3x^2+2}}-\frac{7}{3\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{2\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}+\frac{49x}{6\sqrt{3x^2+2}}-\frac{7}{3\sqrt{3x^2+2}}$$

input
```
Int[((5 - x)*(3 + 2*x))/(2 + 3*x^2)^(3/2),x]
```

output
```
-7/(3*Sqrt[2 + 3*x^2]) + (49*x)/(6*Sqrt[2 + 3*x^2]) - (2*ArcSinh[Sqrt[3/2]
*x])/(3*Sqrt[3])
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{-\frac{7}{3}+\frac{49x}{6}}{\sqrt{3x^2+2}}-\frac{2\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}$	30
default	$\frac{49x}{6\sqrt{3x^2+2}}-\frac{2\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{9}-\frac{7}{3\sqrt{3x^2+2}}$	37
trager	$\frac{-\frac{7}{3}+\frac{49x}{6}}{\sqrt{3x^2+2}}-\frac{2\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$	47
meijerg	$\frac{15\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}}+\frac{7\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^2}{2}+1}}\right)}{6\sqrt{\pi}}-\frac{2\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}}{2\sqrt{\frac{3x^2}{2}+1}}+\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)\right)}{9\sqrt{\pi}}$	87

input
```
int((5-x)*(2*x+3)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
7/6*(-2+7*x)/(3*x^2+2)^(1/2)-2/9*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.55

$$\int \frac{(5-x)(3+2x)}{(2+3x^2)^{3/2}}\,dx = \frac{2\sqrt{3}(3x^2+2)\log\left(\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)+21\sqrt{3x^2+2}(7x-2)}{18\,(3x^2+2)}$$

input
```
integrate((5-x)*(3+2*x)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/18*(2*sqrt(3)*(3*x^2 + 2)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + 2
1*sqrt(3*x^2 + 2)*(7*x - 2))/(3*x^2 + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(36) = 72$.

Time = 6.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.48

$$\int \frac{(5-x)(3+2x)}{(2+3x^2)^{3/2}}\,dx = -\frac{6\sqrt{3}x^2\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27x^2+18} + \frac{6x\sqrt{3x^2+2}}{27x^2+18} + \frac{15x}{2\sqrt{3x^2+2}} - \frac{4\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27x^2+18} - \frac{7}{3\sqrt{3x^2+2}}$$

input
```
integrate((5-x)*(3+2*x)/(3*x**2+2)**(3/2),x)
```

output
```
-6*sqrt(3)*x**2*asinh(sqrt(6)*x/2)/(27*x**2 + 18) + 6*x*sqrt(3*x**2 + 2)/(
27*x**2 + 18) + 15*x/(2*sqrt(3*x**2 + 2)) - 4*sqrt(3)*asinh(sqrt(6)*x/2)/(
27*x**2 + 18) - 7/(3*sqrt(3*x**2 + 2))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{(5-x)(3+2x)}{(2+3x^2)^{3/2}}\,dx = -\frac{2}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{49\,x}{6\sqrt{3\,x^2+2}} - \frac{7}{3\sqrt{3\,x^2+2}}$$

input
```
integrate((5-x)*(3+2*x)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 49/6*x/sqrt(3*x^2 + 2) - 7/3/sqrt(3*
x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{(5-x)(3+2x)}{(2+3x^2)^{3/2}}\,dx = \frac{2}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)+\frac{7\,(7\,x-2)}{6\,\sqrt{3\,x^2+2}}$$

input
```
integrate((5-x)*(3+2*x)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
2/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 7/6*(7*x - 2)/sqrt(3*x^2 +
 2)
```

Mupad [B] (verification not implemented)

Time = 5.97 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.20

$$\int \frac{(5-x)(3+2x)}{(2+3x^2)^{3/2}}\,dx = -\frac{2\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9} - \frac{\sqrt{3}\,\sqrt{6}\,\left(-126+\sqrt{6}\,147\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{648\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\,\sqrt{6}\,\left(126+\sqrt{6}\,147\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{648\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(-((2*x + 3)*(x - 5))/(3*x^2 + 2)^(3/2),x)
```

output
```
- (2*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/9 - (3^(1/2)*6^(1/2)*(6^(1/2)*1
47i - 126)*(x^2 + 2/3)^(1/2)*1i)/(648*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(
1/2)*(6^(1/2)*147i + 126)*(x^2 + 2/3)^(1/2)*1i)/(648*(x + (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int \frac{(5-x)(3+2x)}{(2+3x^2)^{3/2}}\,dx = \frac{147\sqrt{3x^2+2}\,x - 42\sqrt{3x^2+2} - 12\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}\,x}{\sqrt{2}}\right)x^2 - 8\sqrt{3}\log\left(\frac{\sqrt{3x^2+2}+\sqrt{3}}{\sqrt{2}}\right.}{54x^2+36}$$

input

```
int((5-x)*(3+2*x)/(3*x^2+2)^(3/2),x)
```

output

```
(147*sqrt(3*x**2 + 2)*x - 42*sqrt(3*x**2 + 2) - 12*sqrt(3)*log((sqrt(3*x**
2 + 2) + sqrt(3)*x)/sqrt(2))*x**2 - 8*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt
(3)*x)/sqrt(2)) + 147*sqrt(3)*x**2 + 98*sqrt(3))/(18*(3*x**2 + 2))
```

3.248 $\int \frac{5-x}{(2+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{5-x}{(2+3x^2)^{3/2}}\,dx = \frac{2+15x}{6\sqrt{2+3x^2}}$$

output
```
1/6*(2+15*x)/(3*x^2+2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{5-x}{(2+3x^2)^{3/2}}\,dx = \frac{2+15x}{6\sqrt{2+3x^2}}$$

input
```
Integrate[(5 - x)/(2 + 3*x^2)^(3/2),x]
```

output
```
(2 + 15*x)/(6*Sqrt[2 + 3*x^2])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5-x}{(3x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{453}$$

$$\frac{15x+2}{6\sqrt{3x^2+2}}$$

input
```
Int[(5 - x)/(2 + 3*x^2)^(3/2),x]
```

output
```
(2 + 15*x)/(6*Sqrt[2 + 3*x^2])
```

Defintions of rubi rules used

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{2+15x}{6\sqrt{3x^2+2}}$	17
trager	$\frac{2+15x}{6\sqrt{3x^2+2}}$	17
risch	$\frac{2+15x}{6\sqrt{3x^2+2}}$	17
default	$\frac{1}{3\sqrt{3x^2+2}}+\frac{5x}{2\sqrt{3x^2+2}}$	25
orering	$-\frac{(2+15x)(5-x)}{6\sqrt{3x^2+2}\,(-5+x)}$	27
meijerg	$\frac{5\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}}-\frac{\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^2}{2}+1}}\right)}{6\sqrt{\pi}}$	43

input
```
int((5-x)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2+15*x)/(3*x^2+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{5-x}{(2+3x^2)^{3/2}}\,dx=\frac{15\,x+2}{6\,\sqrt{3\,x^2+2}}$$

input
```
integrate((5-x)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(15*x + 2)/sqrt(3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 6.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{5-x}{(2+3x^2)^{3/2}}\,dx = \frac{5x}{2\sqrt{3x^2+2}} + \frac{1}{3\sqrt{3x^2+2}}$$

input
```
integrate((5-x)/(3*x**2+2)**(3/2),x)
```

output
```
5*x/(2*sqrt(3*x**2 + 2)) + 1/(3*sqrt(3*x**2 + 2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{5-x}{(2+3x^2)^{3/2}}\,dx = \frac{5\,x}{2\,\sqrt{3\,x^2+2}} + \frac{1}{3\,\sqrt{3\,x^2+2}}$$

input
```
integrate((5-x)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
5/2*x/sqrt(3*x^2 + 2) + 1/3/sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{5-x}{(2+3x^2)^{3/2}}\,dx = \frac{15\,x+2}{6\,\sqrt{3\,x^2+2}}$$

input
```
integrate((5-x)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
1/6*(15*x + 2)/sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{5-x}{(2+3x^2)^{3/2}}\,dx = \frac{\frac{5\,x}{2}+\frac{1}{3}}{\sqrt{3\,x^2+2}}$$

input
```
int(-(x - 5)/(3*x^2 + 2)^(3/2),x)
```

output
```
((5*x)/2 + 1/3)/(3*x^2 + 2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \frac{5-x}{(2+3x^2)^{3/2}}\,dx = \frac{15\sqrt{3x^2+2}\,x+2\sqrt{3x^2+2}+15\sqrt{3}\,x^2+10\sqrt{3}}{18x^2+12}$$

input
```
int((5-x)/(3*x^2+2)^(3/2),x)
```

output
```
(15*sqrt(3*x**2 + 2)*x + 2*sqrt(3*x**2 + 2) + 15*sqrt(3)*x**2 + 10*sqrt(3)
)/(6*(3*x**2 + 2))
```

3.249 $\int \frac{5-x}{(3+2x)(2+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 53

$$\int \frac{5-x}{(3+2x)(2+3x^2)^{3/2}}\,dx = \frac{26+41x}{70\sqrt{2+3x^2}} - \frac{26\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{35\sqrt{35}}$$

output
```
1/70*(26+41*x)/(3*x^2+2)^(1/2)-26/1225*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1
/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \frac{5-x}{(3+2x)(2+3x^2)^{3/2}}\,dx = \frac{26+41x}{70\sqrt{2+3x^2}} + \frac{52\operatorname{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{35\sqrt{35}}$$

input
```
Integrate[(5 - x)/((3 + 2*x)*(2 + 3*x^2)^(3/2)),x]
```

output
```
(26 + 41*x)/(70*Sqrt[2 + 3*x^2]) + (52*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x -
2*Sqrt[2 + 3*x^2])/Sqrt[35]])/(35*Sqrt[35])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {686, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5-x}{(2x+3)\left(3x^2+2\right)^{3/2}}\,dx$$

$$\Big\downarrow \ 686$$

$$\frac{41x+26}{70\sqrt{3x^2+2}}-\frac{1}{210}\int -\frac{156}{(2x+3)\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{26}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx+\frac{41x+26}{70\sqrt{3x^2+2}}$$

$$\Big\downarrow \ 488$$

$$\frac{41x+26}{70\sqrt{3x^2+2}}-\frac{26}{35}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}$$

$$\Big\downarrow \ 219$$

$$\frac{41x+26}{70\sqrt{3x^2+2}}-\frac{26\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}$$

input
```
Int[(5 - x)/((3 + 2*x)*(2 + 3*x^2)^(3/2)),x]
```

output
```
(26 + 41*x)/(70*Sqrt[2 + 3*x^2]) - (26*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2
+ 3*x^2])])/(35*Sqrt[35])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{26+41x}{70\sqrt{3x^2+2}}-\frac{26\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{1225}$	48
trager	$\frac{26+41x}{70\sqrt{3x^2+2}}+\frac{26\operatorname{RootOf}\left(\_Z^2-35\right)\ln\left(\frac{9\operatorname{RootOf}\left(\_Z^2-35\right)x-4\operatorname{RootOf}\left(\_Z^2-35\right)+35\sqrt{3x^2+2}}{2x+3}\right)}{1225}$	64
default	$-\frac{x}{4\sqrt{3x^2+2}}+\frac{13}{35\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}+\frac{117x}{140\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}-\frac{26\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{1225}$	77

input
```
int((5-x)/(2*x+3)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/70*(26+41*x)/(3*x^2+2)^(1/2)-26/1225*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1
/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.57

$$\int \frac{5-x}{(3+2x)\left(2+3x^2\right)^{3/2}}\,dx = \frac{26\sqrt{35}(3\,x^2+2)\log\left(-\frac{\sqrt{35}\sqrt{3\,x^2+2}(9\,x-4)+93\,x^2-36\,x+43}{4\,x^2+12\,x+9}\right)+35\sqrt{3\,x^2+2}(41\,x+}{2450\,(3\,x^2+2)}$$

input
```
integrate((5-x)/(3+2*x)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/2450*(26*sqrt(35)*(3*x^2 + 2)*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) +
 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9)) + 35*sqrt(3*x^2 + 2)*(41*x + 26))
/(3*x^2 + 2)
```

Sympy [F]

$$\int \frac{5-x}{(3+2x)\left(2+3x^2\right)^{3/2}}\,dx = -\int \frac{x}{6x^3\sqrt{3x^2+2}+9x^2\sqrt{3x^2+2}+4x\sqrt{3x^2+2}+6\sqrt{3x^2+2}}\,dx - \int \left(-\frac{5}{6x^3\sqrt{3x^2+2}+9x^2\sqrt{3x^2+2}+4x\sqrt{3x^2+2}+6\sqrt{3x^2+2}}\right)dx$$

input
```
integrate((5-x)/(3+2*x)/(3*x**2+2)**(3/2),x)
```

output
```
-Integral(x/(6*x**3*sqrt(3*x**2 + 2) + 9*x**2*sqrt(3*x**2 + 2) + 4*x*sqrt(
3*x**2 + 2) + 6*sqrt(3*x**2 + 2)), x) - Integral(-5/(6*x**3*sqrt(3*x**2 +
2) + 9*x**2*sqrt(3*x**2 + 2) + 4*x*sqrt(3*x**2 + 2) + 6*sqrt(3*x**2 + 2)),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{5-x}{(3+2x)\left(2+3x^2\right)^{3/2}}\,dx = \frac{26}{1225}\,\sqrt{35}\,\operatorname{arsinh}\left(\frac{3\,\sqrt{6}x}{2\,|2\,x+3|}-\frac{2\,\sqrt{6}}{3\,|2\,x+3|}\right) + \frac{41\,x}{70\,\sqrt{3\,x^2+2}}+\frac{13}{35\,\sqrt{3\,x^2+2}}$$

input
```
integrate((5-x)/(3+2*x)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
26/1225*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*x
+ 3)) + 41/70*x/sqrt(3*x^2 + 2) + 13/35/sqrt(3*x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.58

$$\int \frac{5-x}{(3+2x)\left(2+3x^2\right)^{3/2}}\,dx = \frac{26}{1225}\,\sqrt{35}\log\left(-\frac{\left|-2\,\sqrt{3}x-\sqrt{35}-3\,\sqrt{3}+2\,\sqrt{3\,x^2+2}\right|}{2\,\sqrt{3}x-\sqrt{35}+3\,\sqrt{3}-2\,\sqrt{3\,x^2+2}}\right) + \frac{41\,x+26}{70\,\sqrt{3\,x^2+2}}$$

input
```
integrate((5-x)/(3+2*x)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
26/1225*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqrt(3*x
^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/70*
(41*x + 26)/sqrt(3*x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.00

$$\int \frac{5-x}{(3+2x)\left(2+3x^2\right)^{3/2}}\,dx = \frac{\sqrt{35}\left(26\ln\left(x+\frac{3}{2}\right)-26\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)\right)}{1225} - \frac{\sqrt{3}\sqrt{6}\left(-234+\sqrt{6}\,123\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{7560\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\left(234+\sqrt{6}\,123\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{7560\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(-(x - 5)/((2*x + 3)*(3*x^2 + 2)^(3/2)),x)
```

output
```
(35^(1/2)*(26*log(x + 3/2) - 26*log(x - (3^(1/2)*35^(1/2)*(x^2 + 2/3)^(1/2
))/9 - 4/9)))/1225 - (3^(1/2)*6^(1/2)*(6^(1/2)*123i - 234)*(x^2 + 2/3)^(1/
2)*1i)/(7560*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*123i + 234)
*(x^2 + 2/3)^(1/2)*1i)/(7560*(x - (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.57

$$\int \frac{5-x}{(3+2x)\left(2+3x^2\right)^{3/2}}\,dx = \frac{468\sqrt{35}\,atan\left(\frac{2\sqrt{3x^2+2}\,i+2\sqrt{3}\,ix}{\sqrt{35}-3\sqrt{3}}\right)i\,x^2+312\sqrt{35}\,atan\left(\frac{2\sqrt{3x^2+2}\,i+2\sqrt{3}\,ix}{\sqrt{35}-3\sqrt{3}}\right)i+4305}{}$$

input
```
int((5-x)/(3+2*x)/(3*x^2+2)^(3/2),x)
```

output
```
(468*sqrt(35)*atan((2*sqrt(3*x**2 + 2)*i + 2*sqrt(3)*i*x)/(sqrt(35) - 3*sq
rt(3)))*i*x**2 + 312*sqrt(35)*atan((2*sqrt(3*x**2 + 2)*i + 2*sqrt(3)*i*x)/
(sqrt(35) - 3*sqrt(3)))*i + 4305*sqrt(3*x**2 + 2)*x + 2730*sqrt(3*x**2 + 2
) + 234*sqrt(35)*log(4*sqrt(3*x**2 + 2)*sqrt(3)*x + 3*sqrt(105) + 12*x**2
- 27)*x**2 + 156*sqrt(35)*log(4*sqrt(3*x**2 + 2)*sqrt(3)*x + 3*sqrt(105) +
 12*x**2 - 27) - 468*sqrt(35)*log((2*sqrt(3*x**2 + 2) + sqrt(35) + 2*sqrt(
3)*x + 3*sqrt(3))/sqrt(2))*x**2 - 312*sqrt(35)*log((2*sqrt(3*x**2 + 2) + s
qrt(35) + 2*sqrt(3)*x + 3*sqrt(3))/sqrt(2)) + 4305*sqrt(3)*x**2 + 2870*sqr
t(3))/(7350*(3*x**2 + 2))
```

3.250 $\int \frac{5-x}{(3+2x)^2(2+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{5-x}{(3+2x)^2(2+3x^2)^{3/2}}\,dx = -\frac{13}{35(3+2x)\sqrt{2+3x^2}} + \frac{632+57x}{2450\sqrt{2+3x^2}} - \frac{632\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{1225\sqrt{35}}$$

output
```
-13/35/(3+2*x)/(3*x^2+2)^(1/2)+1/2450*(632+57*x)/(3*x^2+2)^(1/2)-632/42875
*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int \frac{5-x}{(3+2x)^2(2+3x^2)^{3/2}}\,dx = \frac{\frac{35(986+1435x+114x^2)}{(3+2x)\sqrt{2+3x^2}} - 1264\sqrt{35}\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{85750}$$

input
```
Integrate[(5 - x)/((3 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```

output
```
((35*(986 + 1435*x + 114*x^2))/((3 + 2*x)*Sqrt[2 + 3*x^2]) - 1264*Sqrt[35]
*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/85750
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {686, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5-x}{(2x+3)^2 (3x^2+2)^{3/2}} dx$$

$$\Big\downarrow \text{686}$$

$$\frac{41x+26}{70(2x+3)\sqrt{3x^2+2}} - \frac{1}{210}\int -\frac{6(41x+52)}{(2x+3)^2\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{35}\int \frac{41x+52}{(2x+3)^2\sqrt{3x^2+2}}dx + \frac{41x+26}{70(2x+3)\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{679}$$

$$\frac{1}{35}\left(\frac{632}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx + \frac{19\sqrt{3x^2+2}}{35(2x+3)}\right) + \frac{41x+26}{70(2x+3)\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{35}\left(\frac{19\sqrt{3x^2+2}}{35(2x+3)} - \frac{632}{35}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}\right) + \frac{41x+26}{70(2x+3)\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{35}\left(\frac{19\sqrt{3x^2+2}}{35(2x+3)} - \frac{632\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}\right) + \frac{41x+26}{70(2x+3)\sqrt{3x^2+2}}$$

input
```
Int[(5 - x)/((3 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```

output
```
(26 + 41*x)/(70*(3 + 2*x)*Sqrt[2 + 3*x^2]) + ((19*Sqrt[2 + 3*x^2])/(35*(3
+ 2*x)) - (632*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(35*Sqrt[35]
))/35
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result
risch	$\frac{114x^2+1435x+986}{2450(2x+3)\sqrt{3x^2+2}}-\frac{632\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{42875}$
default	$-\frac{13}{70\left(x+\frac{3}{2}\right)\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}+\frac{316}{1225\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}+\frac{57x}{2450\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}-\frac{632\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2}}\right)}{42875}$
trager	$\frac{(114x^2+1435x+986)\sqrt{3x^2+2}}{14700x^3+22050x^2+9800x+14700}+\frac{632\,\mathrm{RootOf}\left(\_Z^2-35\right)\ln\left(\frac{9\,\mathrm{RootOf}\left(\_Z^2-35\right)x-4\,\mathrm{RootOf}\left(\_Z^2-35\right)+35\sqrt{3x^2+2}}{2x+3}\right)}{42875}$

input
```
int((5-x)/(2*x+3)^2/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2450*(114*x^2+1435*x+986)/(2*x+3)/(3*x^2+2)^(1/2)-632/42875*35^(1/2)*arc
tanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.39

$$\int \frac{5-x}{(3+2x)^2\left(2+3x^2\right)^{3/2}}\,dx=\frac{632\sqrt{35}(6x^3+9x^2+4x+6)\log\left(-\frac{\sqrt{35}\sqrt{3x^2+2}(9x-4)+93x^2-36x+43}{4x^2+12x+9}\right)+35\,(1}{85750\,(6x^3+9x^2+4x+6)}$$

input
```
integrate((5-x)/(3+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/85750*(632*sqrt(35)*(6*x^3 + 9*x^2 + 4*x + 6)*log(-(sqrt(35)*sqrt(3*x^2
+ 2)*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9)) + 35*(114*x^2 + 1
435*x + 986)*sqrt(3*x^2 + 2))/(6*x^3 + 9*x^2 + 4*x + 6)
```

Sympy [F]

$$\int \frac{5-x}{(3+2x)^2(2+3x^2)^{3/2}}\,dx = -\int \frac{x}{12x^4\sqrt{3x^2+2}+36x^3\sqrt{3x^2+2}+35x^2\sqrt{3x^2+2}+24x\sqrt{3x^2+2}+18\sqrt{3x^2+2}}\,dx - \int \left(-\frac{5}{12x^4\sqrt{3x^2+2}+36x^3\sqrt{3x^2+2}+35x^2\sqrt{3x^2+2}+24x\sqrt{3x^2+2}+18\sqrt{3x^2+2}}\right)dx$$

input
```
integrate((5-x)/(3+2*x)**2/(3*x**2+2)**(3/2),x)
```

output
```
-Integral(x/(12*x**4*sqrt(3*x**2 + 2) + 36*x**3*sqrt(3*x**2 + 2) + 35*x**2
*sqrt(3*x**2 + 2) + 24*x*sqrt(3*x**2 + 2) + 18*sqrt(3*x**2 + 2)), x) - Int
egral(-5/(12*x**4*sqrt(3*x**2 + 2) + 36*x**3*sqrt(3*x**2 + 2) + 35*x**2*sq
rt(3*x**2 + 2) + 24*x*sqrt(3*x**2 + 2) + 18*sqrt(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.15

$$\int \frac{5-x}{(3+2x)^2(2+3x^2)^{3/2}}\,dx = \frac{632}{42875}\sqrt{35}\,\mathrm{arsinh}\left(\frac{3\sqrt{6}x}{2|2x+3|}-\frac{2\sqrt{6}}{3|2x+3|}\right) + \frac{57x}{2450\sqrt{3x^2+2}} + \frac{316}{1225\sqrt{3x^2+2}} - \frac{13}{35\left(2\sqrt{3x^2+2}x+3\sqrt{3x^2+2}\right)}$$

input
```
integrate((5-x)/(3+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
632/42875*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*
x + 3)) + 57/2450*x/sqrt(3*x^2 + 2) + 316/1225/sqrt(3*x^2 + 2) - 13/35/(2*
sqrt(3*x^2 + 2)*x + 3*sqrt(3*x^2 + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(60) = 120$.

Time = 0.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.24

$$\int \frac{5-x}{(3+2x)^2 (2+3x^2)^{3/2}}\,dx = -\frac{1}{85750}\sqrt{35}\left(19\sqrt{35}\sqrt{3}-1264\log\left(\sqrt{35}\sqrt{3}-9\right)\right)\mathrm{sgn}\left(\frac{1}{2x+3}\right) + \frac{\frac{\frac{1093}{\mathrm{sgn}\left(\frac{1}{2x+3}\right)}-\frac{1820}{(2x+3)\mathrm{sgn}\left(\frac{1}{2x+3}\right)}}{2x+3}+\frac{57}{\mathrm{sgn}\left(\frac{1}{2x+3}\right)}}{2450\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}} - \frac{632\sqrt{35}\log\left(\sqrt{35}\left(\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)-9\right)}{42875\,\mathrm{sgn}\left(\frac{1}{2x+3}\right)}$$

input
```
integrate((5-x)/(3+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
-1/85750*sqrt(35)*(19*sqrt(35)*sqrt(3) - 1264*log(sqrt(35)*sqrt(3) - 9))*s
gn(1/(2*x + 3)) + 1/2450*((1093/sgn(1/(2*x + 3)) - 1820/((2*x + 3)*sgn(1/(
2*x + 3))))/(2*x + 3) + 57/sgn(1/(2*x + 3)))/sqrt(-18/(2*x + 3) + 35/(2*x
+ 3)^2 + 3) - 632/42875*sqrt(35)*log(sqrt(35)*(sqrt(-18/(2*x + 3) + 35/(2*
x + 3)^2 + 3) + sqrt(35)/(2*x + 3)) - 9)/sgn(1/(2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.09

$$\int \frac{5-x}{(3+2x)^2 (2+3x^2)^{3/2}}\,dx = \frac{632\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{42875} - \frac{632\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{42875} + \frac{71\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{4900\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{71\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{4900\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{26\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1225\left(x+\frac{3}{2}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,199\mathrm{i}}{14700\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,199\mathrm{i}}{14700\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(-(x - 5)/((2*x + 3)^2*(3*x^2 + 2)^(3/2)),x)
```

output
```
(632*35^(1/2)*log(x + 3/2))/42875 - (632*35^(1/2)*log(x - (3^(1/2)*35^(1/2
)*(x^2 + 2/3)^(1/2))/9 - 4/9))/42875 + (71*3^(1/2)*(x^2 + 2/3)^(1/2))/(490
0*(x - (6^(1/2)*1i)/3)) + (71*3^(1/2)*(x^2 + 2/3)^(1/2))/(4900*(x + (6^(1/
2)*1i)/3)) - (26*3^(1/2)*(x^2 + 2/3)^(1/2))/(1225*(x + 3/2)) - (3^(1/2)*6^
(1/2)*(x^2 + 2/3)^(1/2)*199i)/(14700*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1
/2)*(x^2 + 2/3)^(1/2)*199i)/(14700*(x + (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.55

$$\int \frac{5-x}{(3+2x)^2 (2+3x^2)^{3/2}}\,dx = \frac{3990\sqrt{3x^2+2}\,x^2 + 50225\sqrt{3x^2+2}\,x + 34510\sqrt{3x^2+2} + 7584\sqrt{35}\log\left(\sqrt{3\right.}$$

input
```
int((5-x)/(3+2*x)^2/(3*x^2+2)^(3/2),x)
```

output
```
(3990*sqrt(3*x**2 + 2)*x**2 + 50225*sqrt(3*x**2 + 2)*x + 34510*sqrt(3*x**2
 + 2) + 7584*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**3 + 1137
6*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**2 + 5056*sqrt(35)*l
og(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x + 7584*sqrt(35)*log(sqrt(3*x**2
+ 2)*sqrt(35) + 9*x - 4) - 7584*sqrt(35)*log(2*x + 3)*x**3 - 11376*sqrt(35
)*log(2*x + 3)*x**2 - 5056*sqrt(35)*log(2*x + 3)*x - 7584*sqrt(35)*log(2*x
 + 3))/(85750*(6*x**3 + 9*x**2 + 4*x + 6))
```

3.251 $\int \frac{5-x}{(3+2x)^3\left(2+3x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{5-x}{(3+2x)^3\left(2+3x^2\right)^{3/2}}\,dx=\frac{3(654-331x)}{17150\sqrt{2+3x^2}}-\frac{13}{70(3+2x)^2\sqrt{2+3x^2}}-\frac{103}{490(3+2x)\sqrt{2+3x^2}}-\frac{1962\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{8575\sqrt{35}}$$

output
```
3/17150*(654-331*x)/(3*x^2+2)^(1/2)-13/70/(3+2*x)^2/(3*x^2+2)^(1/2)-103/49
0/(3+2*x)/(3*x^2+2)^(1/2)-1962/300125*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/
2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{5-x}{(3+2x)^3\left(2+3x^2\right)^{3/2}}\,dx=\frac{\frac{35(3658+7397x-4068x^2-3972x^3)}{(3+2x)^2\sqrt{2+3x^2}}+7848\sqrt{35}\operatorname{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{35}}\right)}{600250}$$

input
```
Integrate[(5 - x)/((3 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
```

output
```
((35*(3658 + 7397*x - 4068*x^2 - 3972*x^3))/((3 + 2*x)^2*Sqrt[2 + 3*x^2])
+ 7848*Sqrt[35]*ArcTanh[(3*Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt
[35]])/600250
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {686, 27, 688, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5-x}{(2x+3)^3 \left(3x^2+2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{686}$$

$$\frac{41x+26}{70(2x+3)^2\sqrt{3x^2+2}}-\frac{1}{210}\int -\frac{12(41x+39)}{(2x+3)^3\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{35}\int \frac{41x+39}{(2x+3)^3\sqrt{3x^2+2}}dx+\frac{41x+26}{70(2x+3)^2\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{688}$$

$$\frac{2}{35}\left(\frac{9\sqrt{3x^2+2}}{14(2x+3)^2}-\frac{1}{70}\int -\frac{5(27x+206)}{(2x+3)^2\sqrt{3x^2+2}}dx\right)+\frac{41x+26}{70(2x+3)^2\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{35}\left(\frac{1}{14}\int \frac{27x+206}{(2x+3)^2\sqrt{3x^2+2}}dx+\frac{9\sqrt{3x^2+2}}{14(2x+3)^2}\right)+\frac{41x+26}{70(2x+3)^2\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{679}$$

$$\frac{2}{35}\left(\frac{1}{14}\left(\frac{1962}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx-\frac{331\sqrt{3x^2+2}}{35(2x+3)}\right)+\frac{9\sqrt{3x^2+2}}{14(2x+3)^2}\right)+\frac{41x+26}{70(2x+3)^2\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{488}$$

$$\frac{2}{35}\left(\frac{1}{14}\left(-\frac{1962}{35}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}-\frac{331\sqrt{3x^2+2}}{35(2x+3)}\right)+\frac{9\sqrt{3x^2+2}}{14(2x+3)^2}\right)+\frac{41x+26}{70(2x+3)^2\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{35}\left(\frac{1}{14}\left(-\frac{1962\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}-\frac{331\sqrt{3x^2+2}}{35(2x+3)}\right)+\frac{9\sqrt{3x^2+2}}{14(2x+3)^2}\right)+\frac{41x+26}{70(2x+3)^2\sqrt{3x^2+2}}$$

input
```
Int[(5 - x)/((3 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
```

output
```
(26 + 41*x)/(70*(3 + 2*x)^2*Sqrt[2 + 3*x^2]) + (2*((9*Sqrt[2 + 3*x^2])/(14
*(3 + 2*x)^2) + ((-331*Sqrt[2 + 3*x^2])/(35*(3 + 2*x)) - (1962*ArcTanh[(4
- 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(35*Sqrt[35]))/14))/35
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{3972x^3+4068x^2-7397x-3658}{17150(2x+3)^2\sqrt{3x^2+2}}-\frac{1962\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{300125}$
trager	$-\frac{3972x^3+4068x^2-7397x-3658}{17150(2x+3)^2\sqrt{3x^2+2}}-\frac{1962\,\operatorname{RootOf}\left(\_Z^2-35\right)\ln\left(-\frac{9\,\operatorname{RootOf}\left(\_Z^2-35\right)x-4\,\operatorname{RootOf}\left(\_Z^2-35\right)-35\sqrt{3x^2+2}}{2x+3}\right)}{300125}$
default	$-\frac{13}{280\left(x+\frac{3}{2}\right)^2\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}-\frac{103}{980\left(x+\frac{3}{2}\right)\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}+\frac{981}{8575\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}-\frac{993x}{17150\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-}}$

input
```
int((5-x)/(2*x+3)^3/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/17150*(3972*x^3+4068*x^2-7397*x-3658)/(2*x+3)^2/(3*x^2+2)^(1/2)-1962/30
0125*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.23

$$\int \frac{5-x}{(3+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \frac{1962\sqrt{35}(12\,x^4+36\,x^3+35\,x^2+24\,x+18)\log\left(-\frac{\sqrt{35}\sqrt{3\,x^2+2}(9\,x-4)+93\,x^2-36}{4\,x^2+12\,x+9}\right.}{600250\,(12\,x^4+36\,x^3+35\,x^2}$$

input
```
integrate((5-x)/(3+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/600250*(1962*sqrt(35)*(12*x^4 + 36*x^3 + 35*x^2 + 24*x + 18)*log(-(sqrt(
35)*sqrt(3*x^2 + 2)*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9)) -
35*(3972*x^3 + 4068*x^2 - 7397*x - 3658)*sqrt(3*x^2 + 2))/(12*x^4 + 36*x^3
 + 35*x^2 + 24*x + 18)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{5-x}{(3+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((5-x)/(3+2*x)**3/(3*x**2+2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.32

$$\int \frac{5-x}{(3+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \frac{1962}{300125}\sqrt{35}\,\operatorname{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|}-\frac{2\sqrt{6}}{3\,|2\,x+3|}\right) - \frac{993\,x}{17150\sqrt{3\,x^2+2}} + \frac{981}{8575\sqrt{3\,x^2+2}} - \frac{13}{70\left(4\sqrt{3\,x^2+2}x^2+12\sqrt{3\,x^2+2}x+9\sqrt{3\,x^2+2}\right)} - \frac{103}{490\left(2\sqrt{3\,x^2+2}x+3\sqrt{3\,x^2+2}\right)}$$

input
```
integrate((5-x)/(3+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
1962/300125*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(
2*x + 3)) - 993/17150*x/sqrt(3*x^2 + 2) + 981/8575/sqrt(3*x^2 + 2) - 13/70
/(4*sqrt(3*x^2 + 2)*x^2 + 12*sqrt(3*x^2 + 2)*x + 9*sqrt(3*x^2 + 2)) - 103/
490/(2*sqrt(3*x^2 + 2)*x + 3*sqrt(3*x^2 + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(78) = 156$.

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.05

$$\int \frac{5-x}{(3+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \frac{1962}{300125}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3\,x^2+2}}\right) - \frac{3\,(157\,x-1478)}{85750\sqrt{3\,x^2+2}} - \frac{768\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^3+2461\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2-6168\sqrt{3}x+856\sqrt{3}+6168\sqrt{3\,x^2+2}}{6125\left(\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2+3\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)-2\right)^2}$$

input
```
integrate((5-x)/(3+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
1962/300125*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqrt
(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) - 3
/85750*(157*x - 1478)/sqrt(3*x^2 + 2) - 1/6125*(768*(sqrt(3)*x - sqrt(3*x^
2 + 2))^3 + 2461*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 - 6168*sqrt(3)*x
+ 856*sqrt(3) + 6168*sqrt(3*x^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 3
*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.87

$$\int \frac{5-x}{(3+2x)^3\,(2+3x^2)^{3/2}}\,dx = \frac{1962\sqrt{35}\,\ln\left(x+\frac{3}{2}\right)}{300125} - \frac{1962\sqrt{35}\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{300125} - \frac{157\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{171500\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{157\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{171500\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{107\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{6125\left(x+\frac{3}{2}\right)} - \frac{13\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{2450\left(x^2+3x+\frac{9}{4}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,739\mathrm{i}}{171500\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,739\mathrm{i}}{171500\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(-(x - 5)/((2*x + 3)^3*(3*x^2 + 2)^(3/2)),x)
```

output
```
(1962*35^(1/2)*log(x + 3/2))/300125 - (1962*35^(1/2)*log(x - (3^(1/2)*35^(
1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/300125 - (157*3^(1/2)*(x^2 + 2/3)^(1/2))
/(171500*(x - (6^(1/2)*1i)/3)) - (157*3^(1/2)*(x^2 + 2/3)^(1/2))/(171500*(
x + (6^(1/2)*1i)/3)) - (107*3^(1/2)*(x^2 + 2/3)^(1/2))/(6125*(x + 3/2)) -
(13*3^(1/2)*(x^2 + 2/3)^(1/2))/(2450*(3*x + x^2 + 9/4)) - (3^(1/2)*6^(1/2)
*(x^2 + 2/3)^(1/2)*739i)/(171500*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*
(x^2 + 2/3)^(1/2)*739i)/(171500*(x + (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.54

$$\int \frac{5-x}{(3+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \frac{-139020\sqrt{3x^2+2}\,x^3-142380\sqrt{3x^2+2}\,x^2+258895\sqrt{3x^2+2}\,x+128030}{}$$

input
```
int((5-x)/(3+2*x)^3/(3*x^2+2)^(3/2),x)
```

output
```
( - 139020*sqrt(3*x**2 + 2)*x**3 - 142380*sqrt(3*x**2 + 2)*x**2 + 258895*s
qrt(3*x**2 + 2)*x + 128030*sqrt(3*x**2 + 2) + 47088*sqrt(35)*log(sqrt(3*x*
*2 + 2)*sqrt(35) + 9*x - 4)*x**4 + 141264*sqrt(35)*log(sqrt(3*x**2 + 2)*sq
rt(35) + 9*x - 4)*x**3 + 137340*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9
*x - 4)*x**2 + 94176*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x +
 70632*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4) - 47088*sqrt(35)*
log(2*x + 3)*x**4 - 141264*sqrt(35)*log(2*x + 3)*x**3 - 137340*sqrt(35)*lo
g(2*x + 3)*x**2 - 94176*sqrt(35)*log(2*x + 3)*x - 70632*sqrt(35)*log(2*x +
 3))/(600250*(12*x**4 + 36*x**3 + 35*x**2 + 24*x + 18))
```

3.252 $\int \frac{(5-x)(3+2x)^6}{(2+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{(5-x)(3+2x)^6}{(2+3x^2)^{5/2}}\,dx = -\frac{7(2-7x)(3+2x)^5}{18\,(2+3x^2)^{3/2}} + \frac{(3+2x)^3(158+2427x)}{54\sqrt{2+3x^2}} - \frac{2639}{81}(3+2x)^2\sqrt{2+3x^2} - \frac{70}{243}(2167+801x)\sqrt{2+3x^2} + \frac{20720\,\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{27\sqrt{3}}$$

output
```
-7/18*(2-7*x)*(3+2*x)^5/(3*x^2+2)^(3/2)+1/54*(3+2*x)^3*(158+2427*x)/(3*x^2
+2)^(1/2)-2639/81*(3+2*x)^2*(3*x^2+2)^(1/2)-70/243*(2167+801*x)*(3*x^2+2)^
(1/2)+20720/81*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\int \frac{(5-x)(3+2x)^6}{(2+3x^2)^{5/2}}\,dx = -\frac{1798610-139815x+2363976x^2-1125999x^3-130464x^4+20736x^5+3456x^6}{486\,(2+3x^2)^{3/2}} - \frac{20720\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{27\sqrt{3}}$$

input
```
Integrate[((5 - x)*(3 + 2*x)^6)/(2 + 3*x^2)^(5/2),x]
```

output
```
-1/486*(1798610 - 139815*x + 2363976*x^2 - 1125999*x^3 - 130464*x^4 + 2073
6*x^5 + 3456*x^6)/(2 + 3*x^2)^(3/2) - (20720*Log[-(Sqrt[3]*x) + Sqrt[2 + 3
*x^2]])/(27*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {684, 27, 684, 27, 687, 27, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)(2x+3)^6}{(3x^2+2)^{5/2}}dx$$

$$\Big\downarrow \text{684}$$

$$\frac{1}{18}\int \frac{2(199-159x)(2x+3)^4}{(3x^2+2)^{3/2}}dx-\frac{7(2-7x)(2x+3)^5}{18(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \frac{(199-159x)(2x+3)^4}{(3x^2+2)^{3/2}}dx-\frac{7(2-7x)(2x+3)^5}{18(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{684}$$

$$\frac{1}{9}\left(\frac{1}{6}\int -\frac{42(2x+3)^2(377x+68)}{\sqrt{3x^2+2}}dx+\frac{(2427x+158)(2x+3)^3}{6\sqrt{3x^2+2}}\right)-\frac{7(2-7x)(2x+3)^5}{18(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\frac{(2x+3)^3(2427x+158)}{6\sqrt{3x^2+2}}-7\int \frac{(2x+3)^2(377x+68)}{\sqrt{3x^2+2}}dx\right)-\frac{7(2-7x)(2x+3)^5}{18(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{9}\left(\frac{(2x+3)^3(2427x+158)}{6\sqrt{3x^2+2}}-7\left(\frac{1}{9}\int -\frac{10(118-801x)(2x+3)}{\sqrt{3x^2+2}}dx+\frac{377}{9}\sqrt{3x^2+2}(2x+3)^2\right)\right)-\frac{7(2-7x)(2x+3)^5}{18\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{9}\left(\frac{(2x+3)^3(2427x+158)}{6\sqrt{3x^2+2}}-7\left(\frac{377}{9}(2x+3)^2\sqrt{3x^2+2}-\frac{10}{9}\int \frac{(118-801x)(2x+3)}{\sqrt{3x^2+2}}dx\right)\right)-\frac{7(2-7x)(2x+3)^5}{18\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \ 676$$

$$\frac{1}{9}\left(\frac{(2x+3)^3(2427x+158)}{6\sqrt{3x^2+2}}-7\left(\frac{377}{9}(2x+3)^2\sqrt{3x^2+2}-\frac{10}{9}\left(888\int \frac{1}{\sqrt{3x^2+2}}dx-267\sqrt{3x^2+2}x-\frac{2167}{3}\sqrt{3\right.\right.\right.$$
$$-\frac{7(2-7x)(2x+3)^5}{18\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \ 222$$

$$\frac{1}{9}\left(\frac{(2x+3)^3(2427x+158)}{6\sqrt{3x^2+2}}-7\left(\frac{377}{9}(2x+3)^2\sqrt{3x^2+2}-\frac{10}{9}\left(296\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-267\sqrt{3x^2+2}x-\frac{216}{3}\right.\right.\right.$$
$$-\frac{7(2-7x)(2x+3)^5}{18\left(3x^2+2\right)^{3/2}}$$

input
```
Int[((5 - x)*(3 + 2*x)^6)/(2 + 3*x^2)^(5/2),x]
```

output
```
(-7*(2 - 7*x)*(3 + 2*x)^5)/(18*(2 + 3*x^2)^(3/2)) + (((3 + 2*x)^3*(158 + 2
427*x))/(6*Sqrt[2 + 3*x^2]) - 7*((377*(3 + 2*x)^2*Sqrt[2 + 3*x^2])/9 - (10
*((-2167*Sqrt[2 + 3*x^2])/3 - 267*x*Sqrt[2 + 3*x^2] + 296*Sqrt[3]*ArcSinh[
Sqrt[3/2]*x]))/9))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

method	result
risch	$-\frac{3456x^6+20736x^5-130464x^4-1125999x^3+2363976x^2-139815x+1798610}{486(3x^2+2)^{\frac{3}{2}}}+\frac{20720\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{81}$
trager	$-\frac{3456x^6+20736x^5-130464x^4-1125999x^3+2363976x^2-139815x+1798610}{486(3x^2+2)^{\frac{3}{2}}}-\frac{20720\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right.\right.}{81}$
default	$-\frac{3537x}{2(3x^2+2)^{\frac{3}{2}}}+\frac{55517x}{54\sqrt{3x^2+2}}-\frac{899305}{243(3x^2+2)^{\frac{3}{2}}}-\frac{131332x^2}{27(3x^2+2)^{\frac{3}{2}}}-\frac{20720x^3}{27(3x^2+2)^{\frac{3}{2}}}+\frac{20720\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{81}+\frac{2416x^4}{9(3x^2+2)^{\frac{3}{2}}}$
meijerg	$\frac{1215\sqrt{2}\,x(3x^2+3)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}-\frac{1024\sqrt{3}\left(\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\,\left(\frac{189}{4}x^4+210x^2+105\right)}{56\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}-\frac{15\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{4}\right)}{243\sqrt{\pi}}+\frac{160\sqrt{2}\left(-4\sqrt{\pi}+\frac{\sqrt{\pi}\,\left(\frac{27}{2}x^4+36x^2+\right.}{4\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right.}{9\sqrt{\pi}}$

input
```
int((5-x)*(2*x+3)^6/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/486*(3456*x^6+20736*x^5-130464*x^4-1125999*x^3+2363976*x^2-139815*x+179
8610)/(3*x^2+2)^(3/2)+20720/81*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \frac{(5-x)(3+2x)^6}{(2+3x^2)^{5/2}}\,dx=\frac{62160\,\sqrt{3}(9\,x^4+12\,x^2+4)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)-(3456\,x^6+20736}{486\,(9\,x^4+1}$$

input
```
integrate((5-x)*(3+2*x)^6/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/486*(62160*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x -
 3*x^2 - 1) - (3456*x^6 + 20736*x^5 - 130464*x^4 - 1125999*x^3 + 2363976*x
^2 - 139815*x + 1798610)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x^2 + 4)
```

Sympy [F]

$$\int \frac{(5-x)(3+2x)^6}{(2+3x^2)^{5/2}}\,dx = -\int \left(-\frac{13851x}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int \left(-\frac{21384x^2}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int \left(-\frac{16740x^3}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int \left(-\frac{6480x^4}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int \left(-\frac{720x^5}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int \frac{256x^6}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\,dx - \int \frac{64x^7}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\,dx - \int \left(-\frac{3645}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx$$

input
```
integrate((5-x)*(3+2*x)**6/(3*x**2+2)**(5/2),x)
```

output
```
-Integral(-13851*x/(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4
*sqrt(3*x**2 + 2)), x) - Integral(-21384*x**2/(9*x**4*sqrt(3*x**2 + 2) + 1
2*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x) - Integral(-16740*x**3/(
9*x**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)),
x) - Integral(-6480*x**4/(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 +
2) + 4*sqrt(3*x**2 + 2)), x) - Integral(-720*x**5/(9*x**4*sqrt(3*x**2 + 2)
 + 12*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x) - Integral(256*x**6/
(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)),
 x) - Integral(64*x**7/(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2)
 + 4*sqrt(3*x**2 + 2)), x) - Integral(-3645/(9*x**4*sqrt(3*x**2 + 2) + 12*
x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \frac{(5-x)(3+2x)^6}{(2+3x^2)^{5/2}}\,dx = -\frac{64\,x^6}{9\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{128\,x^5}{3\,(3\,x^2+2)^{\frac{3}{2}}} + \frac{2416\,x^4}{9\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{20720}{81}\,x\left(\frac{9\,x^2}{(3\,x^2+2)^{\frac{3}{2}}} + \frac{4}{(3\,x^2+2)^{\frac{3}{2}}}\right) + \frac{20720}{81}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{249431\,x}{162\,\sqrt{3\,x^2+2}} - \frac{131332\,x^2}{27\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{3537\,x}{2\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{899305}{243\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)*(3+2*x)^6/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
-64/9*x^6/(3*x^2 + 2)^(3/2) - 128/3*x^5/(3*x^2 + 2)^(3/2) + 2416/9*x^4/(3*
x^2 + 2)^(3/2) - 20720/81*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2)^(3/2)
) + 20720/81*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 249431/162*x/sqrt(3*x^2 + 2)
 - 131332/27*x^2/(3*x^2 + 2)^(3/2) - 3537/2*x/(3*x^2 + 2)^(3/2) - 899305/2
43/(3*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \frac{(5-x)(3+2x)^6}{(2+3x^2)^{5/2}}\,dx = -\frac{20720}{81}\,\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) - \frac{9\,(((96\,(4\,(x+6)x-151)x-125111)x+262664)x-15535)x+1798610}{486\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)*(3+2*x)^6/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
-20720/81*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 1/486*(9*(((96*(4*(x
 + 6)*x - 151)*x - 125111)*x + 262664)*x - 15535)*x + 1798610)/(3*x^2 + 2)
^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.91

$$\int \frac{(5-x)(3+2x)^6}{(2+3x^2)^{5/2}}\,dx = \frac{20720\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{81} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{64\,x^2}{27}+\frac{128\,x}{9}-\frac{7504}{81}\right)}{3} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{206689}{144}+\frac{\sqrt{6}\,81809\mathrm{i}}{432}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(-\frac{206689}{216}+\frac{\sqrt{6}\,81809\mathrm{i}}{648}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{206689}{144}+\frac{\sqrt{6}\,81809\mathrm{i}}{432}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(\frac{206689}{216}+\frac{\sqrt{6}\,81809\mathrm{i}}{648}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\sqrt{6}\left(-3390048+\sqrt{6}\,719421\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{23328\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\left(3390048+\sqrt{6}\,719421\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{23328\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(-((2*x + 3)^6*(x - 5))/(3*x^2 + 2)^(5/2),x)
```

output
```
(20720*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/81 - (3^(1/2)*(x^2 + 2/3)^(1/
2)*((128*x)/9 + (64*x^2)/27 - 7504/81))/3 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((
6^(1/2)*81809i)/432 - 206689/144)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2
)*81809i)/648 - 206689/216)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)
*(x^2 + 2/3)^(1/2)*(((6^(1/2)*81809i)/432 + 206689/144)/(x + (6^(1/2)*1i)/
3) + (6^(1/2)*((6^(1/2)*81809i)/648 + 206689/216)*1i)/(2*(x + (6^(1/2)*1i)
/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*719421i - 3390048)*(x^2 + 2/3)^(1/
2)*1i)/(23328*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*719421i +
3390048)*(x^2 + 2/3)^(1/2)*1i)/(23328*(x + (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.68

$$\int \frac{(5-x)(3+2x)^6}{(2+3x^2)^{5/2}}\,dx = \frac{-3456\sqrt{3x^2+2}\,x^6 - 20736\sqrt{3x^2+2}\,x^5 + 130464\sqrt{3x^2+2}\,x^4 + 1125999\sqrt{3x^2}}{}$$

input
```
int((5-x)*(3+2*x)^6/(3*x^2+2)^(5/2),x)
```

output
```
( - 3456*sqrt(3*x**2 + 2)*x**6 - 20736*sqrt(3*x**2 + 2)*x**5 + 130464*sqrt
(3*x**2 + 2)*x**4 + 1125999*sqrt(3*x**2 + 2)*x**3 - 2363976*sqrt(3*x**2 +
2)*x**2 + 139815*sqrt(3*x**2 + 2)*x - 1798610*sqrt(3*x**2 + 2) + 1118880*s
qrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2))*x**4 + 1491840*sqrt(3)*
log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2))*x**2 + 497280*sqrt(3)*log((sqr
t(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)) + 857889*sqrt(3)*x**4 + 1143852*sqrt(3
)*x**2 + 381284*sqrt(3))/(486*(9*x**4 + 12*x**2 + 4))
```

3.253 $\int \frac{(5-x)(3+2x)^5}{(2+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{(5-x)(3+2x)^5}{(2+3x^2)^{5/2}}\,dx = -\frac{7(2-7x)(3+2x)^4}{18(2+3x^2)^{3/2}} - \frac{5(16-421x)(3+2x)^2}{54\sqrt{2+3x^2}} - \frac{50}{81}(299+93x)\sqrt{2+3x^2} + \frac{1600\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{27\sqrt{3}}$$

output
```
-7/18*(2-7*x)*(3+2*x)^4/(3*x^2+2)^(3/2)-5/54*(16-421*x)*(3+2*x)^2/(3*x^2+2
)^(1/2)-50/81*(299+93*x)*(3*x^2+2)^(1/2)+1600/81*arcsinh(1/2*x*6^(1/2))*3^
(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.76

$$\int \frac{(5-x)(3+2x)^5}{(2+3x^2)^{5/2}}\,dx = -\frac{134126-79215x+147600x^2-183945x^3+4320x^4+864x^5}{162(2+3x^2)^{3/2}} - \frac{1600\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{27\sqrt{3}}$$

input
```
Integrate[((5 - x)*(3 + 2*x)^5)/(2 + 3*x^2)^(5/2),x]
```

output
```
-1/162*(134126 - 79215*x + 147600*x^2 - 183945*x^3 + 4320*x^4 + 864*x^5)/(
2 + 3*x^2)^(3/2) - (1600*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(27*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {684, 27, 684, 27, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)(2x+3)^5}{(3x^2+2)^{5/2}}dx$$

$$\Big\downarrow \text{684}$$

$$\frac{1}{18}\int \frac{10(37-22x)(2x+3)^3}{(3x^2+2)^{3/2}}dx-\frac{7(2-7x)(2x+3)^4}{18(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{9}\int \frac{(37-22x)(2x+3)^3}{(3x^2+2)^{3/2}}dx-\frac{7(2-7x)(2x+3)^4}{18(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{684}$$

$$\frac{5}{9}\left(\frac{1}{6}\int -\frac{20(2x+3)(93x+10)}{\sqrt{3x^2+2}}dx-\frac{(16-421x)(2x+3)^2}{6\sqrt{3x^2+2}}\right)-\frac{7(2-7x)(2x+3)^4}{18(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{9}\left(-\frac{10}{3}\int \frac{(2x+3)(93x+10)}{\sqrt{3x^2+2}}dx-\frac{(16-421x)(2x+3)^2}{6\sqrt{3x^2+2}}\right)-\frac{7(2-7x)(2x+3)^4}{18(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{676}$$

$$\frac{5}{9}\left(-\frac{10}{3}\left(-32\int \frac{1}{\sqrt{3x^2+2}}dx+31\sqrt{3x^2+2}x+\frac{299}{3}\sqrt{3x^2+2}\right)-\frac{(16-421x)(2x+3)^2}{6\sqrt{3x^2+2}}\right)-\frac{7(2-7x)(2x+3)^4}{18(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{5}{9}\left(-\frac{10}{3}\left(-\frac{32\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+31\sqrt{3x^2+2}x+\frac{299}{3}\sqrt{3x^2+2}\right)-\frac{(16-421x)(2x+3)^2}{6\sqrt{3x^2+2}}\right)-\frac{7(2-7x)(2x+3)^4}{18\left(3x^2+2\right)^{3/2}}$$

input
```
Int[((5 - x)*(3 + 2*x)^5)/(2 + 3*x^2)^(5/2),x]
```

output
```
(-7*(2 - 7*x)*(3 + 2*x)^4)/(18*(2 + 3*x^2)^(3/2)) + (5*(-1/6*((16 - 421*x)
*(3 + 2*x)^2)/Sqrt[2 + 3*x^2] - (10*((299*Sqrt[2 + 3*x^2])/3 + 31*x*Sqrt[2
 + 3*x^2] - (32*ArcSinh[Sqrt[3/2]*x])/Sqrt[3]))/3))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.53

method	result
risch	$-\frac{864x^5+4320x^4-183945x^3+147600x^2-79215x+134126}{162(3x^2+2)^{\frac{3}{2}}}+\frac{1600\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{81}$
trager	$-\frac{864x^5+4320x^4-183945x^3+147600x^2-79215x+134126}{162(3x^2+2)^{\frac{3}{2}}}-\frac{1600\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{81}$
default	$-\frac{615x}{2(3x^2+2)^{\frac{3}{2}}}+\frac{21505x}{54\sqrt{3x^2+2}}-\frac{67063}{81(3x^2+2)^{\frac{3}{2}}}-\frac{8200x^2}{9(3x^2+2)^{\frac{3}{2}}}-\frac{1600x^3}{27(3x^2+2)^{\frac{3}{2}}}+\frac{1600\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{81}-\frac{80x^4}{3(3x^2+2)^{\frac{3}{2}}}-$
meijerg	$\frac{405\sqrt{2}\,x(3x^2+3)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}-\frac{160\sqrt{2}\left(-4\sqrt{\pi}+\frac{\sqrt{\pi}\left(\frac{27}{2}x^4+36x^2+16\right)}{4\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{81\sqrt{\pi}}+\frac{320\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\left(30x^2+15\right)}{20\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{2}\right)}{27\sqrt{\pi}}+$

input
```
int((5-x)*(2*x+3)^5/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/162*(864*x^5+4320*x^4-183945*x^3+147600*x^2-79215*x+134126)/(3*x^2+2)^(
3/2)+1600/81*arcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \frac{(5-x)(3+2x)^5}{(2+3x^2)^{5/2}}\,dx = \frac{1600\sqrt{3}(9x^4+12x^2+4)\log\left(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)-(864x^5+4320x^4}{162(9x^4+12x^2+4)}$$

input
```
integrate((5-x)*(3+2*x)^5/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/162*(1600*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x -
3*x^2 - 1) - (864*x^5 + 4320*x^4 - 183945*x^3 + 147600*x^2 - 79215*x + 134
126)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x^2 + 4)
```

Sympy [F]

$$\int \frac{(5-x)(3+2x)^5}{(2+3x^2)^{5/2}}\,dx = -\int\left(-\frac{3807x}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int\left(-\frac{4590x^2}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int\left(-\frac{2520x^3}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int\left(-\frac{480x^4}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int\frac{80x^5}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\,dx - \int\frac{32x^6}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\,dx - \int\left(-\frac{1215}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx$$

input
```
integrate((5-x)*(3+2*x)**5/(3*x**2+2)**(5/2),x)
```

output
```
-Integral(-3807*x/(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*
sqrt(3*x**2 + 2)), x) - Integral(-4590*x**2/(9*x**4*sqrt(3*x**2 + 2) + 12*
x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x) - Integral(-2520*x**3/(9*x
**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x)
- Integral(-480*x**4/(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) +
 4*sqrt(3*x**2 + 2)), x) - Integral(80*x**5/(9*x**4*sqrt(3*x**2 + 2) + 12*
x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x) - Integral(32*x**6/(9*x**4
*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x) - I
ntegral(-1215/(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*sqrt
(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.27

$$\int \frac{(5-x)(3+2x)^5}{(2+3x^2)^{5/2}}\,dx = -\frac{16\,x^5}{3\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{80\,x^4}{3\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{1600}{81}\,x\left(\frac{9\,x^2}{(3\,x^2+2)^{\frac{3}{2}}} + \frac{4}{(3\,x^2+2)^{\frac{3}{2}}}\right) + \frac{1600}{81}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{70915\,x}{162\,\sqrt{3\,x^2+2}} - \frac{8200\,x^2}{9\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{615\,x}{2\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{67063}{81\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)*(3+2*x)^5/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
-16/3*x^5/(3*x^2 + 2)^(3/2) - 80/3*x^4/(3*x^2 + 2)^(3/2) - 1600/81*x*(9*x^
2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2)^(3/2)) + 1600/81*sqrt(3)*arcsinh(1/2*s
qrt(6)*x) + 70915/162*x/sqrt(3*x^2 + 2) - 8200/9*x^2/(3*x^2 + 2)^(3/2) - 6
15/2*x/(3*x^2 + 2)^(3/2) - 67063/81/(3*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

$$\int \frac{(5-x)(3+2x)^5}{(2+3x^2)^{5/2}}\,dx = -\frac{1600}{81}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) - \frac{3\left(\left(\left(288\,(x+5)x-61315\right)x+49200\right)x-26405\right)x+134126}{162\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)*(3+2*x)^5/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
-1600/81*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 1/162*(3*(((288*(x +
5)*x - 61315)*x + 49200)*x - 26405)*x + 134126)/(3*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.31

$$\int \frac{(5-x)(3+2x)^5}{(2+3x^2)^{5/2}}\,dx = \frac{1600\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{81} - \frac{\sqrt{3}\left(\frac{16\,x}{9}+\frac{80}{9}\right)\sqrt{x^2+\frac{2}{3}}}{3} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{43799}{144}+\frac{\sqrt{6}\,18823\mathrm{i}}{144}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}} - \frac{\sqrt{6}\left(-\frac{43799}{216}+\frac{\sqrt{6}\,18823\mathrm{i}}{216}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{43799}{144}+\frac{\sqrt{6}\,18823\mathrm{i}}{144}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}} + \frac{\sqrt{6}\left(\frac{43799}{216}+\frac{\sqrt{6}\,18823\mathrm{i}}{216}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\sqrt{6}\left(-567360+\sqrt{6}\,290595\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{23328\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\left(567360+\sqrt{6}\,290595\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{23328\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(-((2*x + 3)^5*(x - 5))/(3*x^2 + 2)^(5/2),x)
```

output
```
(1600*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/81 - (3^(1/2)*((16*x)/9 + 80/9
)*(x^2 + 2/3)^(1/2))/3 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*18823i)/144
 - 43799/144)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*18823i)/216 - 4379
9/216)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*(x^2 + 2/3)^(1/2)*((
(6^(1/2)*18823i)/144 + 43799/144)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2
)*18823i)/216 + 43799/216)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*
6^(1/2)*(6^(1/2)*290595i - 567360)*(x^2 + 2/3)^(1/2)*1i)/(23328*(x - (6^(1
/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*290595i + 567360)*(x^2 + 2/3)^(1/2
)*1i)/(23328*(x + (6^(1/2)*1i)/3))
```

Reduce [F]

$$\int \frac{(5-x)(3+2x)^5}{(2+3x^2)^{5/2}}\,dx = \int \frac{(5-x)\,(2x+3)^5}{(3x^2+2)^{\frac{5}{2}}}dx$$

input
```
int((5-x)*(3+2*x)^5/(3*x^2+2)^(5/2),x)
```

output
```
int((5-x)*(3+2*x)^5/(3*x^2+2)^(5/2),x)
```

3.254 $\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{5/2}}\,dx = -\frac{7(2-7x)(3+2x)^3}{18\,(2+3x^2)^{3/2}} - \frac{(318-1783x)(3+2x)}{54\sqrt{2+3x^2}} - \frac{2027}{81}\sqrt{2+3x^2} - \frac{16\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}$$

output
```
-7/18*(2-7*x)*(3+2*x)^3/(3*x^2+2)^(3/2)-1/54*(318-1783*x)*(3+2*x)/(3*x^2+2
)^(1/2)-2027/81*(3*x^2+2)^(1/2)-16/27*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.76

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{5/2}}\,dx = -\frac{25342-33381x+16560x^2-57285x^3+864x^4}{162\,(2+3x^2)^{3/2}} + \frac{16\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{9\sqrt{3}}$$

input
```
Integrate[((5 - x)*(3 + 2*x)^4)/(2 + 3*x^2)^(5/2),x]
```

output
```
-1/162*(25342 - 33381*x + 16560*x^2 - 57285*x^3 + 864*x^4)/(2 + 3*x^2)^(3/
2) + (16*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(9*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {684, 27, 684, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)(2x+3)^4}{(3x^2+2)^{5/2}}dx$$

$$\Big\downarrow \text{684}$$

$$\frac{1}{18}\int \frac{2(171-61x)(2x+3)^2}{(3x^2+2)^{3/2}}dx-\frac{7(2-7x)(2x+3)^3}{18(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \frac{(171-61x)(2x+3)^2}{(3x^2+2)^{3/2}}dx-\frac{7(2-7x)(2x+3)^3}{18(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{684}$$

$$\frac{1}{9}\left(\frac{1}{6}\int -\frac{2(2027x+48)}{\sqrt{3x^2+2}}dx-\frac{(318-1783x)(2x+3)}{6\sqrt{3x^2+2}}\right)-\frac{7(2-7x)(2x+3)^3}{18(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(-\frac{1}{3}\int \frac{2027x+48}{\sqrt{3x^2+2}}dx-\frac{(318-1783x)(2x+3)}{6\sqrt{3x^2+2}}\right)-\frac{7(2-7x)(2x+3)^3}{18(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{9}\left(\frac{1}{3}\left(-48\int \frac{1}{\sqrt{3x^2+2}}dx-\frac{2027}{3}\sqrt{3x^2+2}\right)-\frac{(318-1783x)(2x+3)}{6\sqrt{3x^2+2}}\right)-\frac{7(2-7x)(2x+3)^3}{18(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{9}\left(\frac{1}{3}\left(-16\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-\frac{2027}{3}\sqrt{3x^2+2}\right)-\frac{(318-1783x)(2x+3)}{6\sqrt{3x^2+2}}\right)-\frac{7(2-7x)(2x+3)^3}{18\left(3x^2+2\right)^{3/2}}$$

input
```
Int[((5 - x)*(3 + 2*x)^4)/(2 + 3*x^2)^(5/2),x]
```

output
```
(-7*(2 - 7*x)*(3 + 2*x)^3)/(18*(2 + 3*x^2)^(3/2)) + (-1/6*((318 - 1783*x)*
(3 + 2*x))/Sqrt[2 + 3*x^2] + ((-2027*Sqrt[2 + 3*x^2])/3 - 16*Sqrt[3]*ArcSi
nh[Sqrt[3/2]*x])/3)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.52

method	result
risch	$-\frac{864x^4-57285x^3+16560x^2-33381x+25342}{162(3x^2+2)^{\frac{3}{2}}}-\frac{16\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{27}$
trager	$-\frac{864x^4-57285x^3+16560x^2-33381x+25342}{162(3x^2+2)^{\frac{3}{2}}}-\frac{16\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{27}$
default	$-\frac{57x}{2(3x^2+2)^{\frac{3}{2}}}+\frac{2111x}{18\sqrt{3x^2+2}}-\frac{12671}{81(3x^2+2)^{\frac{3}{2}}}-\frac{920x^2}{9(3x^2+2)^{\frac{3}{2}}}+\frac{16x^3}{9(3x^2+2)^{\frac{3}{2}}}-\frac{16\,\mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{27}-\frac{16x^4}{3(3x^2+2)^{\frac{3}{2}}}$
meijerg	$\frac{135\sqrt{2}\,x(3x^2+3)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}-\frac{32\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\,(30x^2+15)}{20\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{3\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{2}\,\sqrt{3}}{2}\right)}{2}\right)}{81\sqrt{\pi}}+\frac{88\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}\,(18x^2+8)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{9\sqrt{\pi}}+\frac{36\sqrt{2}\,x^3}{\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}$

input
```
int((5-x)*(2*x+3)^4/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/162*(864*x^4-57285*x^3+16560*x^2-33381*x+25342)/(3*x^2+2)^(3/2)-16/27*a
rcsinh(1/2*6^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{5/2}}\,dx = \frac{48\sqrt{3}(9x^4+12x^2+4)\log\left(\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)-(864x^4-57285x^3+}{162\,(9x^4+12x^2+4)}$$

input
```
integrate((5-x)*(3+2*x)^4/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/162*(48*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x
^2 - 1) - (864*x^4 - 57285*x^3 + 16560*x^2 - 33381*x + 25342)*sqrt(3*x^2 +
 2))/(9*x^4 + 12*x^2 + 4)
```

Sympy [F]

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{5/2}}\,dx = -\int \left(-\frac{999x}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int \left(-\frac{864x^2}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int \left(-\frac{264x^3}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int \frac{16x^4}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\,dx - \int \frac{16x^5}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\,dx - \int \left(-\frac{405}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx$$

input
```
integrate((5-x)*(3+2*x)**4/(3*x**2+2)**(5/2),x)
```

output
```
-Integral(-999*x/(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*s
qrt(3*x**2 + 2)), x) - Integral(-864*x**2/(9*x**4*sqrt(3*x**2 + 2) + 12*x*
*2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x) - Integral(-264*x**3/(9*x**4
*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x) - I
ntegral(16*x**4/(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*sq
rt(3*x**2 + 2)), x) - Integral(16*x**5/(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*
sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x) - Integral(-405/(9*x**4*sqrt(3*
x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.21

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{5/2}}\,dx = -\frac{16\,x^4}{3\,(3\,x^2+2)^{\frac{3}{2}}} + \frac{16}{27}\,x\left(\frac{9\,x^2}{(3\,x^2+2)^{\frac{3}{2}}} + \frac{4}{(3\,x^2+2)^{\frac{3}{2}}}\right) - \frac{16}{27}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{6269\,x}{54\sqrt{3\,x^2+2}} - \frac{920\,x^2}{9\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{57\,x}{2\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{12671}{81\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)*(3+2*x)^4/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
-16/3*x^4/(3*x^2 + 2)^(3/2) + 16/27*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2
+ 2)^(3/2)) - 16/27*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 6269/54*x/sqrt(3*x^2
+ 2) - 920/9*x^2/(3*x^2 + 2)^(3/2) - 57/2*x/(3*x^2 + 2)^(3/2) - 12671/81/(
3*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{5/2}}\,dx = \frac{16}{27}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) - \frac{9\,(((96\,x-6365)x+1840)x-3709)x+25342}{162\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)*(3+2*x)^4/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
16/27*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 1/162*(9*(((96*x - 6365)
*x + 1840)*x - 3709)*x + 25342)/(3*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.44

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{5/2}}\,dx = -\frac{16\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{27} - \frac{16\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{27} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{1603}{48}+\frac{\sqrt{6}\,7343\mathrm{i}}{144}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}} - \frac{\sqrt{6}\left(-\frac{1603}{72}+\frac{\sqrt{6}\,7343\mathrm{i}}{216}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{1603}{48}+\frac{\sqrt{6}\,7343\mathrm{i}}{144}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}} + \frac{\sqrt{6}\left(\frac{1603}{72}+\frac{\sqrt{6}\,7343\mathrm{i}}{216}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\sqrt{6}\left(-20544+\sqrt{6}\,27063\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{7776\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\left(20544+\sqrt{6}\,27063\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{7776\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input

```
int(-((2*x + 3)^4*(x - 5))/(3*x^2 + 2)^(5/2),x)
```

output

```
(3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*7343i)/144 - 1603/48)/(x - (6^(1/2)*
1i)/3) - (6^(1/2)*((6^(1/2)*7343i)/216 - 1603/72)*1i)/(2*(x - (6^(1/2)*1i)
/3)^2)))/27 - (16*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/27 - (16*3^(1/2)*(
x^2 + 2/3)^(1/2))/27 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*7343i)/144 +
1603/48)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*7343i)/216 + 1603/72)*1
i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*27063i - 20
544)*(x^2 + 2/3)^(1/2)*1i)/(7776*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*
(6^(1/2)*27063i + 20544)*(x^2 + 2/3)^(1/2)*1i)/(7776*(x + (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.94

$$\int \frac{(5-x)(3+2x)^4}{(2+3x^2)^{5/2}}\,dx = \frac{-864\sqrt{3x^2+2}\,x^4+57285\sqrt{3x^2+2}\,x^3-16560\sqrt{3x^2+2}\,x^2+33381\sqrt{3x^2+2}\,x}{}$$

input
```
int((5-x)*(3+2*x)^4/(3*x^2+2)^(5/2),x)
```

output
```
( - 864*sqrt(3*x**2 + 2)*x**4 + 57285*sqrt(3*x**2 + 2)*x**3 - 16560*sqrt(3
*x**2 + 2)*x**2 + 33381*sqrt(3*x**2 + 2)*x - 25342*sqrt(3*x**2 + 2) - 864*
sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2))*x**4 - 1152*sqrt(3)*lo
g((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2))*x**2 - 384*sqrt(3)*log((sqrt(3*x
**2 + 2) + sqrt(3)*x)/sqrt(2)) - 9477*sqrt(3)*x**4 - 12636*sqrt(3)*x**2 -
4212*sqrt(3))/(162*(9*x**4 + 12*x**2 + 4))
```

3.255 $\int \frac{(5-x)(3+2x)^3}{\left(2+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{(5-x)(3+2x)^3}{\left(2+3x^2\right)^{5/2}}\,dx=-\frac{7(2-7x)(3+2x)^2}{18\left(2+3x^2\right)^{3/2}}-\frac{556-1461x}{54\sqrt{2+3x^2}}-\frac{8\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}$$

output
```
-7/18*(2-7*x)*(3+2*x)^2/(3*x^2+2)^(3/2)-1/54*(556-1461*x)/(3*x^2+2)^(1/2)-
8/27*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{(5-x)(3+2x)^3}{\left(2+3x^2\right)^{5/2}}\,dx=-\frac{1490-3741x+72x^2-4971x^3}{54\left(2+3x^2\right)^{3/2}}+\frac{8\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{9\sqrt{3}}$$

input
```
Integrate[((5 - x)*(3 + 2*x)^3)/(2 + 3*x^2)^(5/2),x]
```

output
```
-1/54*(1490 - 3741*x + 72*x^2 - 4971*x^3)/(2 + 3*x^2)^(3/2) + (8*Log[-(Sqr
t[3]*x) + Sqrt[2 + 3*x^2]])/(9*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {684, 27, 675, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)(2x+3)^3}{(3x^2+2)^{5/2}} \, dx$$

$$\Big\downarrow \text{684}$$

$$\frac{1}{18} \int \frac{2(157-12x)(2x+3)}{(3x^2+2)^{3/2}}dx - \frac{7(2-7x)(2x+3)^2}{18\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9} \int \frac{(157-12x)(2x+3)}{(3x^2+2)^{3/2}}dx - \frac{7(2-7x)(2x+3)^2}{18\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{675}$$

$$\frac{1}{9} \left(-8 \int \frac{1}{\sqrt{3x^2+2}}dx + \frac{487x}{2\sqrt{3x^2+2}} - \frac{278}{3\sqrt{3x^2+2}}\right) - \frac{7(2-7x)(2x+3)^2}{18\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{9} \left(-\frac{8\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}} + \frac{487x}{2\sqrt{3x^2+2}} - \frac{278}{3\sqrt{3x^2+2}}\right) - \frac{7(2-7x)(2x+3)^2}{18\left(3x^2+2\right)^{3/2}}$$

input
```
Int[((5 - x)*(3 + 2*x)^3)/(2 + 3*x^2)^(5/2),x]
```

output
```
(-7*(2 - 7*x)*(3 + 2*x)^2)/(18*(2 + 3*x^2)^(3/2)) + (-278/(3*Sqrt[2 + 3*x^
2]) + (487*x)/(2*Sqrt[2 + 3*x^2]) - (8*ArcSinh[Sqrt[3/2]*x])/Sqrt[3])/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

method	result
risch	$\frac{4971x^3-72x^2+3741x-1490}{54(3x^2+2)^{\frac{3}{2}}}-\frac{8\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{27}$
trager	$\frac{4971x^3-72x^2+3741x-1490}{54(3x^2+2)^{\frac{3}{2}}}+\frac{8\,\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{27}$
default	$\frac{17x}{2(3x^2+2)^{\frac{3}{2}}}+\frac{547x}{18\sqrt{3x^2+2}}-\frac{745}{27(3x^2+2)^{\frac{3}{2}}}-\frac{4x^2}{3(3x^2+2)^{\frac{3}{2}}}+\frac{8x^3}{9(3x^2+2)^{\frac{3}{2}}}-\frac{8\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)\sqrt{3}}{27}$
meijerg	$\frac{45\sqrt{2}\,x(3x^2+3)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{4\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}\left(18x^2+8\right)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{27\sqrt{\pi}}+\frac{21\sqrt{2}\,x^3}{4\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{27\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{2\sqrt{\pi}}-\frac{16\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\,(30x^2-}{20\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right.}{8}$

input
```
int((5-x)*(2*x+3)^3/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/54*(4971*x^3-72*x^2+3741*x-1490)/(3*x^2+2)^(3/2)-8/27*arcsinh(1/2*6^(1/2
)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\int \frac{(5-x)(3+2x)^3}{\left(2+3x^2\right)^{5/2}}\,dx=\frac{8\sqrt{3}(9\,x^4+12\,x^2+4)\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+(4971\,x^3-72\,x^2+374}{54\,(9\,x^4+12\,x^2+4)}$$

input
```
integrate((5-x)*(3+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/54*(8*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2
 - 1) + (4971*x^3 - 72*x^2 + 3741*x - 1490)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x
^2 + 4)
```

Sympy [F]

$$\int \frac{(5-x)(3+2x)^3}{(2+3x^2)^{5/2}}\,dx = -\int \left(-\frac{243x}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int \left(-\frac{126x^2}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int \left(-\frac{4x^3}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int \frac{8x^4}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\,dx - \int \left(-\frac{135}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx$$

input
```
integrate((5-x)*(3+2*x)**3/(3*x**2+2)**(5/2),x)
```

output
```
-Integral(-243*x/(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*s
qrt(3*x**2 + 2)), x) - Integral(-126*x**2/(9*x**4*sqrt(3*x**2 + 2) + 12*x*
*2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x) - Integral(-4*x**3/(9*x**4*s
qrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x) - Int
egral(8*x**4/(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*sqrt(
3*x**2 + 2)), x) - Integral(-135/(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3
*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.36

$$\int \frac{(5-x)(3+2x)^3}{(2+3x^2)^{5/2}}\,dx = \frac{8}{27}\,x\left(\frac{9\,x^2}{(3\,x^2+2)^{\frac{3}{2}}} + \frac{4}{(3\,x^2+2)^{\frac{3}{2}}}\right) - \frac{8}{27}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) + \frac{1609\,x}{54\,\sqrt{3\,x^2+2}} - \frac{4\,x^2}{3\,(3\,x^2+2)^{\frac{3}{2}}} + \frac{17\,x}{2\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{745}{27\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)*(3+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
8/27*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2)^(3/2)) - 8/27*sqrt(3)*arcs
inh(1/2*sqrt(6)*x) + 1609/54*x/sqrt(3*x^2 + 2) - 4/3*x^2/(3*x^2 + 2)^(3/2)
 + 17/2*x/(3*x^2 + 2)^(3/2) - 745/27/(3*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{(5-x)(3+2x)^3}{(2+3x^2)^{5/2}}\,dx = \frac{8}{27}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{3\,((1657\,x-24)x+1247)x-1490}{54\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)*(3+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
8/27*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/54*(3*((1657*x - 24)*x
+ 1247)*x - 1490)/(3*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 6.20 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.99

$$\int \frac{(5-x)(3+2x)^3}{(2+3x^2)^{5/2}}\,dx = -\frac{8\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{27} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{427}{48}+\frac{\sqrt{6}\,721\mathrm{i}}{48}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{427}{72}+\frac{\sqrt{6}\,721\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{427}{48}+\frac{\sqrt{6}\,721\mathrm{i}}{48}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(\frac{427}{72}+\frac{\sqrt{6}\,721\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\sqrt{6}\left(-96+\sqrt{6}\,2067\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\left(96+\sqrt{6}\,2067\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(-((2*x + 3)^3*(x - 5))/(3*x^2 + 2)^(5/2),x)
```

output
```
(3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*721i)/48 + 427/48)/(x - (6^(1/2)*1i)
/3) - (6^(1/2)*((6^(1/2)*721i)/72 + 427/72)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)
))/27 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*721i)/48 - 427/48)/(x + (6^(
1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*721i)/72 - 427/72)*1i)/(2*(x + (6^(1/2)*1
i)/3)^2)))/27 - (8*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/27 - (3^(1/2)*6^(
1/2)*(6^(1/2)*2067i - 96)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x - (6^(1/2)*1i)/3)
) - (3^(1/2)*6^(1/2)*(6^(1/2)*2067i + 96)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x +
 (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.33

$$\int \frac{(5-x)(3+2x)^3}{(2+3x^2)^{5/2}}\,dx = \frac{4971\sqrt{3x^2+2}\,x^3 - 72\sqrt{3x^2+2}\,x^2 + 3741\sqrt{3x^2+2}\,x - 1490\sqrt{3x^2+2} - 144\sqrt{3}\cdots}{\cdots}$$

input
```
int((5-x)*(3+2*x)^3/(3*x^2+2)^(5/2),x)
```

output
```
(4971*sqrt(3*x**2 + 2)*x**3 - 72*sqrt(3*x**2 + 2)*x**2 + 3741*sqrt(3*x**2
+ 2)*x - 1490*sqrt(3*x**2 + 2) - 144*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(
3)*x)/sqrt(2))*x**4 - 192*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(
2))*x**2 - 64*sqrt(3)*log((sqrt(3*x**2 + 2) + sqrt(3)*x)/sqrt(2)) - 2511*s
qrt(3)*x**4 - 3348*sqrt(3)*x**2 - 1116*sqrt(3))/(54*(9*x**4 + 12*x**2 + 4)
)
```

3.256 $\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{5/2}} \, dx = -\frac{7(2-7x)(3+2x)}{18(2+3x^2)^{3/2}} - \frac{74-429x}{54\sqrt{2+3x^2}}$$

output
```
-7/18*(2-7*x)*(3+2*x)/(3*x^2+2)^(3/2)-1/54*(74-429*x)/(3*x^2+2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{5/2}} \, dx = -\frac{274-1215x-72x^2-1287x^3}{54(2+3x^2)^{3/2}}$$

input
```
Integrate[((5 - x)*(3 + 2*x)^2)/(2 + 3*x^2)^(5/2),x]
```

output
```
-1/54*(274 - 1215*x - 72*x^2 - 1287*x^3)/(2 + 3*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {678, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)(2x+3)^2}{(3x^2+2)^{5/2}}dx$$

$$\Big\downarrow \text{678}$$

$$\frac{41}{9}\int \frac{2x+3}{(3x^2+2)^{3/2}}dx+\frac{(15x+2)(2x+3)^2}{18\,(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{453}$$

$$\frac{(2x+3)^2(15x+2)}{18\,(3x^2+2)^{3/2}}-\frac{41(4-9x)}{54\sqrt{3x^2+2}}$$

input
```
Int[((5 - x)*(3 + 2*x)^2)/(2 + 3*x^2)^(5/2),x]
```

output
```
((3 + 2*x)^2*(2 + 15*x))/(18*(2 + 3*x^2)^(3/2)) - (41*(4 - 9*x))/(54*Sqrt[
2 + 3*x^2])
```

Defintions of rubi rules used

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 678
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[m*((c*d*f + a*e*g)/(2*a*c*(p + 1)))   Int[(d + e*x)^(
m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && EqQ[S
implify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

method	result	size
gosper	$\frac{1287x^3+72x^2+1215x-274}{54(3x^2+2)^{\frac{3}{2}}}$	27
trager	$\frac{1287x^3+72x^2+1215x-274}{54(3x^2+2)^{\frac{3}{2}}}$	27
risch	$\frac{1287x^3+72x^2+1215x-274}{54(3x^2+2)^{\frac{3}{2}}}$	27
orering	$-\frac{(1287x^3+72x^2+1215x-274)(5-x)}{54(3x^2+2)^{\frac{3}{2}}(-5+x)}$	37
default	$\frac{119x}{18(3x^2+2)^{\frac{3}{2}}}+\frac{143x}{18\sqrt{3x^2+2}}-\frac{137}{27(3x^2+2)^{\frac{3}{2}}}+\frac{4x^2}{3(3x^2+2)^{\frac{3}{2}}}$	51
meijerg	$\frac{15\sqrt{2}\,x(3x^2+3)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{\sqrt{2}\,x^3}{3\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{17\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{6\sqrt{\pi}}-\frac{4\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}\left(18x^2+8\right)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{27\sqrt{\pi}}$	102

input
```
int((5-x)*(2*x+3)^2/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/54*(1287*x^3+72*x^2+1215*x-274)/(3*x^2+2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{5/2}}\,dx = \frac{(1287\,x^3+72\,x^2+1215\,x-274)\sqrt{3\,x^2+2}}{54\,(9\,x^4+12\,x^2+4)}$$

input
```
integrate((5-x)*(3+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/54*(1287*x^3 + 72*x^2 + 1215*x - 274)*sqrt(3*x^2 + 2)/(9*x^4 + 12*x^2 +
4)
```

Sympy [F]

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{5/2}}\,dx = -\int \left(-\frac{51x}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int \left(-\frac{8x^2}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx - \int \frac{4x^3}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\,dx - \int \left(-\frac{45}{9x^4\sqrt{3x^2+2}+12x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}\right)dx$$

input
```
integrate((5-x)*(3+2*x)**2/(3*x**2+2)**(5/2),x)
```

output
```
-Integral(-51*x/(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*sq
rt(3*x**2 + 2)), x) - Integral(-8*x**2/(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*
sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x) - Integral(4*x**3/(9*x**4*sqrt(
3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)), x) - Integra
l(-45/(9*x**4*sqrt(3*x**2 + 2) + 12*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2
+ 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{5/2}}\,dx = \frac{143\,x}{18\,\sqrt{3\,x^2+2}} + \frac{4\,x^2}{3\,(3\,x^2+2)^{\frac{3}{2}}} + \frac{119\,x}{18\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{137}{27\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)*(3+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
143/18*x/sqrt(3*x^2 + 2) + 4/3*x^2/(3*x^2 + 2)^(3/2) + 119/18*x/(3*x^2 + 2
)^(3/2) - 137/27/(3*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.54

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{5/2}}\,dx = \frac{9\,((143\,x+8)x+135)x-274}{54\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)*(3+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
1/54*(9*((143*x + 8)*x + 135)*x - 274)/(3*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 185, normalized size of antiderivative = 4.02

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{5/2}}\,dx = -\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{119}{16}+\frac{\sqrt{6}\,161\mathrm{i}}{48}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{119}{24}+\frac{\sqrt{6}\,161\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{119}{16}+\frac{\sqrt{6}\,161\mathrm{i}}{48}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(\frac{119}{24}+\frac{\sqrt{6}\,161\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\,\sqrt{6}\left(-96+\sqrt{6}\,453\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\,\sqrt{6}\left(96+\sqrt{6}\,453\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(-((2*x + 3)^2*(x - 5))/(3*x^2 + 2)^(5/2),x)
```

output
```
(3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*161i)/48 + 119/16)/(x - (6^(1/2)*1i)
/3) - (6^(1/2)*((6^(1/2)*161i)/72 + 119/24)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)
))/27 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*161i)/48 - 119/16)/(x + (6^(
1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*161i)/72 - 119/24)*1i)/(2*(x + (6^(1/2)*1
i)/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*453i - 96)*(x^2 + 2/3)^(1/2)*1i)
/(2592*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*453i + 96)*(x^2 +
 2/3)^(1/2)*1i)/(2592*(x - (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int \frac{(5-x)(3+2x)^2}{(2+3x^2)^{5/2}}\,dx = \frac{1287\sqrt{3x^2+2}\,x^3+72\sqrt{3x^2+2}\,x^2+1215\sqrt{3x^2+2}\,x-274\sqrt{3x^2+2}-1143\sqrt{3}}{486x^4+648x^2+216}$$

input
```
int((5-x)*(3+2*x)^2/(3*x^2+2)^(5/2),x)
```

output
```
(1287*sqrt(3*x**2 + 2)*x**3 + 72*sqrt(3*x**2 + 2)*x**2 + 1215*sqrt(3*x**2
+ 2)*x - 274*sqrt(3*x**2 + 2) - 1143*sqrt(3)*x**4 - 1524*sqrt(3)*x**2 - 50
8*sqrt(3))/(54*(9*x**4 + 12*x**2 + 4))
```

3.257 $\int \frac{(5-x)(3+2x)}{(2+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 37

$$\int \frac{(5-x)(3+2x)}{(2+3x^2)^{5/2}}\,dx = -\frac{7(2-7x)}{18(2+3x^2)^{3/2}} + \frac{43x}{18\sqrt{2+3x^2}}$$

output
```
1/18*(-14+49*x)/(3*x^2+2)^(3/2)+43/18*x/(3*x^2+2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{(5-x)(3+2x)}{(2+3x^2)^{5/2}}\,dx = -\frac{14-135x-129x^3}{18(2+3x^2)^{3/2}}$$

input
```
Integrate[((5 - x)*(3 + 2*x))/(2 + 3*x^2)^(5/2),x]
```

output
```
-1/18*(14 - 135*x - 129*x^3)/(2 + 3*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {675, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5-x)(2x+3)}{(3x^2+2)^{5/2}}dx$$

$$\Big\downarrow \text{675}$$

$$\frac{43}{9}\int \frac{1}{(3x^2+2)^{3/2}}dx+\frac{49x}{18\,(3x^2+2)^{3/2}}-\frac{7}{9\,(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{43x}{18\sqrt{3x^2+2}}+\frac{49x}{18\,(3x^2+2)^{3/2}}-\frac{7}{9\,(3x^2+2)^{3/2}}$$

input
```
Int[((5 - x)*(3 + 2*x))/(2 + 3*x^2)^(5/2),x]
```

output
```
-7/(9*(2 + 3*x^2)^(3/2)) + (49*x)/(18*(2 + 3*x^2)^(3/2)) + (43*x)/(18*Sqrt
[2 + 3*x^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))    Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

method	result	size
gosper	$\frac{129x^3+135x-14}{18(3x^2+2)^{\frac{3}{2}}}$	22
trager	$\frac{129x^3+135x-14}{18(3x^2+2)^{\frac{3}{2}}}$	22
risch	$\frac{129x^3+135x-14}{18(3x^2+2)^{\frac{3}{2}}}$	22
orering	$-\frac{(129x^3+135x-14)(5-x)}{18(3x^2+2)^{\frac{3}{2}}(-5+x)}$	32
default	$\frac{49x}{18(3x^2+2)^{\frac{3}{2}}}+\frac{43x}{18\sqrt{3x^2+2}}-\frac{7}{9(3x^2+2)^{\frac{3}{2}}}$	37
meijerg	$\frac{5\sqrt{2}\,x(3x^2+3)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{7\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{18\sqrt{\pi}}-\frac{\sqrt{2}\,x^3}{12\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}$	69

input
```
int((5-x)*(2*x+3)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*(129*x^3+135*x-14)/(3*x^2+2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(5-x)(3+2x)}{(2+3x^2)^{5/2}}\,dx=\frac{(129\,x^3+135\,x-14)\sqrt{3\,x^2+2}}{18\,(9\,x^4+12\,x^2+4)}$$

input
```
integrate((5-x)*(3+2*x)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/18*(129*x^3 + 135*x - 14)*sqrt(3*x^2 + 2)/(9*x^4 + 12*x^2 + 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(32) = 64$.

Time = 22.51 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.30

$$\int \frac{(5-x)(3+2x)}{(2+3x^2)^{5/2}}\,dx = -\frac{2x^3}{18x^2\sqrt{3x^2+2}+12\sqrt{3x^2+2}} + \frac{15x^3}{6x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}} + \frac{15x}{6x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}} - \frac{7}{27x^2\sqrt{3x^2+2}+18\sqrt{3x^2+2}}$$

input
```
integrate((5-x)*(3+2*x)/(3*x**2+2)**(5/2),x)
```

output
```
-2*x**3/(18*x**2*sqrt(3*x**2 + 2) + 12*sqrt(3*x**2 + 2)) + 15*x**3/(6*x**2
*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) + 15*x/(6*x**2*sqrt(3*x**2 + 2) +
4*sqrt(3*x**2 + 2)) - 7/(27*x**2*sqrt(3*x**2 + 2) + 18*sqrt(3*x**2 + 2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{(5-x)(3+2x)}{(2+3x^2)^{5/2}}\,dx = \frac{43\,x}{18\,\sqrt{3\,x^2+2}} + \frac{49\,x}{18\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{7}{9\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)*(3+2*x)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
43/18*x/sqrt(3*x^2 + 2) + 49/18*x/(3*x^2 + 2)^(3/2) - 7/9/(3*x^2 + 2)^(3/2
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{(5-x)(3+2x)}{(2+3x^2)^{5/2}}\,dx = \frac{3\,(43\,x^2+45)x-14}{18\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)*(3+2*x)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
1/18*(3*(43*x^2 + 45)*x - 14)/(3*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 6.52 (sec) , antiderivative size = 161, normalized size of antiderivative = 4.35

$$\int \frac{(5-x)(3+2x)}{(2+3x^2)^{5/2}}\,dx = \frac{41\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{144\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{41\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{144\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{49}{16}+\frac{\sqrt{6}\,7\mathrm{i}}{16}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{49}{24}+\frac{\sqrt{6}\,7\mathrm{i}}{24}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{49}{16}+\frac{\sqrt{6}\,7\mathrm{i}}{16}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(\frac{49}{24}+\frac{\sqrt{6}\,7\mathrm{i}}{24}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}$$

input
```
int(-((2*x + 3)*(x - 5))/(3*x^2 + 2)^(5/2),x)
```

output
```
(41*3^(1/2)*(x^2 + 2/3)^(1/2))/(144*(x - (6^(1/2)*1i)/3)) + (41*3^(1/2)*(x
^2 + 2/3)^(1/2))/(144*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*(x^2 + 2/3)^(1/2)*(
((6^(1/2)*7i)/16 - 49/16)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*7i)/24
 - 49/24)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)
*(((6^(1/2)*7i)/16 + 49/16)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*7i)/
24 + 49/24)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.84

$$\int \frac{(5-x)(3+2x)}{(2+3x^2)^{5/2}}\,dx = \frac{387\sqrt{3x^2+2}\,x^3+405\sqrt{3x^2+2}\,x-42\sqrt{3x^2+2}-423\sqrt{3}\,x^4-564\sqrt{3}\,x^2-188\surd}{486x^4+648x^2+216}$$

input
```
int((5-x)*(3+2*x)/(3*x^2+2)^(5/2),x)
```

output
```
(387*sqrt(3*x**2 + 2)*x**3 + 405*sqrt(3*x**2 + 2)*x - 42*sqrt(3*x**2 + 2)
- 423*sqrt(3)*x**4 - 564*sqrt(3)*x**2 - 188*sqrt(3))/(54*(9*x**4 + 12*x**2
 + 4))
```

3.258 $\int \frac{5-x}{(2+3x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{5-x}{(2+3x^2)^{5/2}} \, dx = \frac{2+15x}{18(2+3x^2)^{3/2}} + \frac{5x}{6\sqrt{2+3x^2}}$$

output
```
1/18*(2+15*x)/(3*x^2+2)^(3/2)+5/6*x/(3*x^2+2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{5-x}{(2+3x^2)^{5/2}} \, dx = \frac{2+45x+45x^3}{18(2+3x^2)^{3/2}}$$

input
```
Integrate[(5 - x)/(2 + 3*x^2)^(5/2),x]
```

output
```
(2 + 45*x + 45*x^3)/(18*(2 + 3*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {454, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5-x}{(3x^2+2)^{5/2}}\,dx$$

$$\Big\downarrow \text{454}$$

$$\frac{5}{3}\int \frac{1}{(3x^2+2)^{3/2}}dx+\frac{15x+2}{18\,(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{5x}{6\sqrt{3x^2+2}}+\frac{15x+2}{18\,(3x^2+2)^{3/2}}$$

input
```
Int[(5 - x)/(2 + 3*x^2)^(5/2),x]
```

output
```
(2 + 15*x)/(18*(2 + 3*x^2)^(3/2)) + (5*x)/(6*Sqrt[2 + 3*x^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

method	result	size
gosper	$\frac{45x^3+45x+2}{18(3x^2+2)^{\frac{3}{2}}}$	22
trager	$\frac{45x^3+45x+2}{18(3x^2+2)^{\frac{3}{2}}}$	22
risch	$\frac{45x^3+45x+2}{18(3x^2+2)^{\frac{3}{2}}}$	22
orering	$-\frac{(45x^3+45x+2)(5-x)}{18(3x^2+2)^{\frac{3}{2}}(-5+x)}$	32
default	$\frac{1}{9(3x^2+2)^{\frac{3}{2}}}+\frac{5x}{6(3x^2+2)^{\frac{3}{2}}}+\frac{5x}{6\sqrt{3x^2+2}}$	37
meijerg	$\frac{5\sqrt{2}\,x(3x^2+3)}{24\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{18\sqrt{\pi}}$	52

input
```
int((5-x)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*(45*x^3+45*x+2)/(3*x^2+2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{5-x}{(2+3x^2)^{5/2}}\,dx=\frac{(45\,x^3+45\,x+2)\sqrt{3\,x^2+2}}{18\,(9\,x^4+12\,x^2+4)}$$

input
```
integrate((5-x)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/18*(45*x^3 + 45*x + 2)*sqrt(3*x^2 + 2)/(9*x^4 + 12*x^2 + 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(32) = 64$.

Time = 15.97 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.43

$$\int \frac{5-x}{(2+3x^2)^{5/2}}\,dx = \frac{5x^3}{6x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}} + \frac{5x}{6x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}} + \frac{1}{27x^2\sqrt{3x^2+2}+18\sqrt{3x^2+2}}$$

input
```
integrate((5-x)/(3*x**2+2)**(5/2),x)
```

output
```
5*x**3/(6*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) + 5*x/(6*x**2*sqrt(3
*x**2 + 2) + 4*sqrt(3*x**2 + 2)) + 1/(27*x**2*sqrt(3*x**2 + 2) + 18*sqrt(3
*x**2 + 2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{5-x}{(2+3x^2)^{5/2}}\,dx = \frac{5\,x}{6\,\sqrt{3\,x^2+2}} + \frac{5\,x}{6\,(3\,x^2+2)^{\frac{3}{2}}} + \frac{1}{9\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
5/6*x/sqrt(3*x^2 + 2) + 5/6*x/(3*x^2 + 2)^(3/2) + 1/9/(3*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

$$\int \frac{5-x}{(2+3x^2)^{5/2}}\,dx = \frac{45\,(x^2+1)x+2}{18\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
1/18*(45*(x^2 + 1)*x + 2)/(3*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 161, normalized size of antiderivative = 4.35

$$\int \frac{5-x}{(2+3x^2)^{5/2}}\,dx = \frac{5\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{48\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{5\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{48\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{15}{16}+\frac{\sqrt{6}\,1\mathrm{i}}{16}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}} - \frac{\sqrt{6}\left(-\frac{5}{8}+\frac{\sqrt{6}\,1\mathrm{i}}{24}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{15}{16}+\frac{\sqrt{6}\,1\mathrm{i}}{16}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}} + \frac{\sqrt{6}\left(\frac{5}{8}+\frac{\sqrt{6}\,1\mathrm{i}}{24}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}$$

input
```
int(-(x - 5)/(3*x^2 + 2)^(5/2),x)
```

output
```
(5*3^(1/2)*(x^2 + 2/3)^(1/2))/(48*(x - (6^(1/2)*1i)/3)) + (5*3^(1/2)*(x^2
+ 2/3)^(1/2))/(48*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^
(1/2)*1i)/16 - 15/16)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*1i)/24 - 5
/8)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^
(1/2)*1i)/16 + 15/16)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*1i)/24 + 5
/8)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27
```

Reduce [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.84

$$\int \frac{5-x}{(2+3x^2)^{5/2}}\,dx = \frac{45\sqrt{3x^2+2}\,x^3+45\sqrt{3x^2+2}\,x+2\sqrt{3x^2+2}-45\sqrt{3}\,x^4-60\sqrt{3}\,x^2-20\sqrt{3}}{162x^4+216x^2+72}$$

input
```
int((5-x)/(3*x^2+2)^(5/2),x)
```

output
```
(45*sqrt(3*x**2 + 2)*x**3 + 45*sqrt(3*x**2 + 2)*x + 2*sqrt(3*x**2 + 2) - 4
5*sqrt(3)*x**4 - 60*sqrt(3)*x**2 - 20*sqrt(3))/(18*(9*x**4 + 12*x**2 + 4))
```

3.259 $\int \frac{5-x}{(3+2x)(2+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 73

$$\int \frac{5-x}{(3+2x)(2+3x^2)^{5/2}}\,dx=\frac{26+41x}{210(2+3x^2)^{3/2}}+\frac{312+2137x}{7350\sqrt{2+3x^2}}-\frac{104\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{1225\sqrt{35}}$$

output
```
1/210*(26+41*x)/(3*x^2+2)^(3/2)+1/7350*(312+2137*x)/(3*x^2+2)^(1/2)-104/42
875*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{5-x}{(3+2x)(2+3x^2)^{5/2}}\,dx=\frac{\frac{35(1534+5709x+936x^2+6411x^3)}{(2+3x^2)^{3/2}}-624\sqrt{35}\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{257250}$$

input
```
Integrate[(5 - x)/((3 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

output
```
((35*(1534 + 5709*x + 936*x^2 + 6411*x^3))/(2 + 3*x^2)^(3/2) - 624*Sqrt[35
]*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/257250
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {686, 27, 686, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5-x}{(2x+3)\left(3x^2+2\right)^{5/2}}dx$$

$$\Big\downarrow \text{686}$$

$$\frac{41x+26}{210\left(3x^2+2\right)^{3/2}}-\frac{1}{630}\int -\frac{6(82x+201)}{(2x+3)\left(3x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{105}\int \frac{82x+201}{(2x+3)\left(3x^2+2\right)^{3/2}}dx+\frac{41x+26}{210\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{686}$$

$$\frac{1}{105}\left(\frac{2137x+312}{70\sqrt{3x^2+2}}-\frac{1}{210}\int -\frac{1872}{(2x+3)\sqrt{3x^2+2}}dx\right)+\frac{41x+26}{210\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{105}\left(\frac{312}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx+\frac{2137x+312}{70\sqrt{3x^2+2}}\right)+\frac{41x+26}{210\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{105}\left(\frac{2137x+312}{70\sqrt{3x^2+2}}-\frac{312}{35}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}\right)+\frac{41x+26}{210\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{105}\left(\frac{2137x+312}{70\sqrt{3x^2+2}}-\frac{312\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}\right)+\frac{41x+26}{210\left(3x^2+2\right)^{3/2}}$$

input
```
Int[(5 - x)/((3 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

output
```
(26 + 41*x)/(210*(2 + 3*x^2)^(3/2)) + ((312 + 2137*x)/(70*Sqrt[2 + 3*x^2])
 - (312*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(35*Sqrt[35]))/105
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.88 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

method	result
trager	$\frac{6411x^3+936x^2+5709x+1534}{7350(3x^2+2)^{\frac{3}{2}}} - \frac{104\,\mathrm{RootOf}\left(\_Z^2-35\right)\ln\left(-\frac{9\,\mathrm{RootOf}\left(\_Z^2-35\right)x-4\,\mathrm{RootOf}\left(\_Z^2-35\right)-35\sqrt{3x^2+2}}{2x+3}\right)}{42875}$
default	$-\frac{x}{12(3x^2+2)^{\frac{3}{2}}} - \frac{x}{12\sqrt{3x^2+2}} + \frac{13}{105\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}} + \frac{39x}{140\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}} + \frac{1833x}{4900\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}} + \frac{}{12}$

input
```
int((5-x)/(2*x+3)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/7350*(6411*x^3+936*x^2+5709*x+1534)/(3*x^2+2)^(3/2)-104/42875*RootOf(_Z^
2-35)*ln(-(9*RootOf(_Z^2-35)*x-4*RootOf(_Z^2-35)-35*(3*x^2+2)^(1/2))/(2*x+
3))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.41

$$\int \frac{5-x}{(3+2x)\left(2+3x^2\right)^{5/2}}\,dx = \frac{312\sqrt{35}(9\,x^4+12\,x^2+4)\log\left(-\frac{\sqrt{35}\sqrt{3\,x^2+2}(9\,x-4)+93\,x^2-36\,x+43}{4\,x^2+12\,x+9}\right)+35\,(6411\,x}{257250\,(9\,x^4+12\,x^2+4)}$$

input
```
integrate((5-x)/(3+2*x)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/257250*(312*sqrt(35)*(9*x^4 + 12*x^2 + 4)*log(-(sqrt(35)*sqrt(3*x^2 + 2)
*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12*x + 9)) + 35*(6411*x^3 + 936*
x^2 + 5709*x + 1534)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x^2 + 4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{5-x}{(3+2x)\left(2+3x^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((5-x)/(3+2*x)/(3*x**2+2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int \frac{5-x}{(3+2x)\left(2+3x^2\right)^{5/2}}\,dx = \frac{104}{42875}\,\sqrt{35}\,\operatorname{arsinh}\left(\frac{3\,\sqrt{6}x}{2\,|2\,x+3|}-\frac{2\,\sqrt{6}}{3\,|2\,x+3|}\right) + \frac{2137\,x}{7350\,\sqrt{3\,x^2+2}}+\frac{52}{1225\,\sqrt{3\,x^2+2}}+\frac{41\,x}{210\,(3\,x^2+2)^{\frac{3}{2}}}+\frac{13}{105\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)/(3+2*x)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
104/42875*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*
x + 3)) + 2137/7350*x/sqrt(3*x^2 + 2) + 52/1225/sqrt(3*x^2 + 2) + 41/210*x
/(3*x^2 + 2)^(3/2) + 13/105/(3*x^2 + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \frac{5-x}{(3+2x)\left(2+3x^2\right)^{5/2}}\,dx = \frac{104}{42875}\,\sqrt{35}\log\left(-\frac{\left|-2\,\sqrt{3}x-\sqrt{35}-3\,\sqrt{3}+2\,\sqrt{3\,x^2+2}\right|}{2\,\sqrt{3}x-\sqrt{35}+3\,\sqrt{3}-2\,\sqrt{3\,x^2+2}}\right) + \frac{3\,((2137\,x+312)x+1903)x+1534}{7350\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)/(3+2*x)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
104/42875*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqrt(3
*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/7
350*(3*((2137*x + 312)*x + 1903)*x + 1534)/(3*x^2 + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.99

$$\int \frac{5-x}{(3+2x)\left(2+3x^2\right)^{5/2}}\,dx = \frac{\sqrt{35}\left(104\,\ln\left(x+\frac{3}{2}\right)-104\,\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)\right)}{42875}$$
$$-\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{123}{560}+\frac{\sqrt{6}\,39\mathrm{i}}{560}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(-\frac{41}{280}+\frac{\sqrt{6}\,13\mathrm{i}}{280}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}$$
$$+\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{123}{560}+\frac{\sqrt{6}\,39\mathrm{i}}{560}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(\frac{41}{280}+\frac{\sqrt{6}\,13\mathrm{i}}{280}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}$$
$$-\frac{\sqrt{3}\sqrt{6}\left(-3744+\sqrt{6}\,7113\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1058400\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\sqrt{6}\left(3744+\sqrt{6}\,7113\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1058400\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(-(x - 5)/((2*x + 3)*(3*x^2 + 2)^(5/2)),x)
```

output
```
(35^(1/2)*(104*log(x + 3/2) - 104*log(x - (3^(1/2)*35^(1/2)*(x^2 + 2/3)^(1
/2))/9 - 4/9)))/42875 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*39i)/560 - 1
23/560)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*13i)/280 - 41/280)*1i)/(
2*(x - (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*39i
)/560 + 123/560)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*13i)/280 + 41/2
80)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*7113i
- 3744)*(x^2 + 2/3)^(1/2)*1i)/(1058400*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^
(1/2)*(6^(1/2)*7113i + 3744)*(x^2 + 2/3)^(1/2)*1i)/(1058400*(x - (6^(1/2)*
1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 384, normalized size of antiderivative = 5.26

$$\int \frac{5-x}{(3+2x)\left(2+3x^2\right)^{5/2}}\,dx = \frac{16848\sqrt{35}\, atan\left(\frac{2\sqrt{3x^2+2}\,i+2\sqrt{3}\,ix}{\sqrt{35}-3\sqrt{3}}\right) i\,x^4+22464\sqrt{35}\, atan\left(\frac{2\sqrt{3x^2+2}\,i+2\sqrt{3}\,ix}{\sqrt{35}-3\sqrt{3}}\right) i\,x^2}{}$$

input

```
int((5-x)/(3+2*x)/(3*x^2+2)^(5/2),x)
```

output

```
(16848*sqrt(35)*atan((2*sqrt(3*x**2 + 2)*i + 2*sqrt(3)*i*x)/(sqrt(35) - 3*
sqrt(3)))*i*x**4 + 22464*sqrt(35)*atan((2*sqrt(3*x**2 + 2)*i + 2*sqrt(3)*i
*x)/(sqrt(35) - 3*sqrt(3)))*i*x**2 + 7488*sqrt(35)*atan((2*sqrt(3*x**2 + 2
)*i + 2*sqrt(3)*i*x)/(sqrt(35) - 3*sqrt(3)))*i + 673155*sqrt(3*x**2 + 2)*x
**3 + 98280*sqrt(3*x**2 + 2)*x**2 + 599445*sqrt(3*x**2 + 2)*x + 161070*sqr
t(3*x**2 + 2) + 8424*sqrt(35)*log(4*sqrt(3*x**2 + 2)*sqrt(3)*x + 3*sqrt(10
5) + 12*x**2 - 27)*x**4 + 11232*sqrt(35)*log(4*sqrt(3*x**2 + 2)*sqrt(3)*x
 + 3*sqrt(105) + 12*x**2 - 27)*x**2 + 3744*sqrt(35)*log(4*sqrt(3*x**2 + 2)*
sqrt(3)*x + 3*sqrt(105) + 12*x**2 - 27) - 16848*sqrt(35)*log((2*sqrt(3*x**
2 + 2) + sqrt(35) + 2*sqrt(3)*x + 3*sqrt(3))/sqrt(2))*x**4 - 22464*sqrt(35
)*log((2*sqrt(3*x**2 + 2) + sqrt(35) + 2*sqrt(3)*x + 3*sqrt(3))/sqrt(2))*x
**2 - 7488*sqrt(35)*log((2*sqrt(3*x**2 + 2) + sqrt(35) + 2*sqrt(3)*x + 3*s
qrt(3))/sqrt(2)) - 525735*sqrt(3)*x**4 - 700980*sqrt(3)*x**2 - 233660*sqrt
(3))/(771750*(9*x**4 + 12*x**2 + 4))
```

3.260 $\int \frac{5-x}{(3+2x)^2(2+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{5-x}{(3+2x)^2(2+3x^2)^{5/2}}\,dx = \frac{220-51x}{1470(2+3x^2)^{3/2}} - \frac{13}{35(3+2x)(2+3x^2)^{3/2}} + \frac{176+277x}{3430\sqrt{2+3x^2}} - \frac{176\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{1715\sqrt{35}}$$

output
```
1/1470*(220-51*x)/(3*x^2+2)^(3/2)-13/35/(3+2*x)/(3*x^2+2)^(3/2)+1/3430*(17
6+277*x)/(3*x^2+2)^(1/2)-176/60025*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/
(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \frac{5-x}{(3+2x)^2(2+3x^2)^{5/2}}\,dx = \frac{35(3966+9107x+7362x^2+10647x^3+4986x^4)-1056\sqrt{35}\sqrt{2+3x^2}(6+}{360150(3+2x)(2+3x^2)^{3/2}}$$

input
```
Integrate[(5 - x)/((3 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```

output
```
(35*(3966 + 9107*x + 7362*x^2 + 10647*x^3 + 4986*x^4) - 1056*Sqrt[35]*Sqrt
[2 + 3*x^2]*(6 + 4*x + 9*x^2 + 6*x^3)*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[2 +
 3*x^2])])/(360150*(3 + 2*x)*(2 + 3*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {686, 27, 686, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5-x}{(2x+3)^2 (3x^2+2)^{5/2}} dx$$

$$\Big\downarrow \text{686}$$

$$\frac{41x+26}{210(2x+3)(3x^2+2)^{3/2}}-\frac{1}{630}\int -\frac{6(123x+227)}{(2x+3)^2 (3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{105}\int \frac{123x+227}{(2x+3)^2 (3x^2+2)^{3/2}}dx+\frac{41x+26}{210(2x+3)(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{686}$$

$$\frac{1}{105}\left(\frac{507x+34}{14(2x+3)\sqrt{3x^2+2}}-\frac{1}{210}\int -\frac{30(507x+68)}{(2x+3)^2\sqrt{3x^2+2}}dx\right)+\frac{41x+26}{210(2x+3)(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{105}\left(\frac{1}{7}\int \frac{507x+68}{(2x+3)^2\sqrt{3x^2+2}}dx+\frac{507x+34}{14(2x+3)\sqrt{3x^2+2}}\right)+\frac{41x+26}{210(2x+3)(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{679}$$

$$\frac{1}{105}\left(\frac{1}{7}\left(\frac{528}{7}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx+\frac{277\sqrt{3x^2+2}}{7(2x+3)}\right)+\frac{507x+34}{14(2x+3)\sqrt{3x^2+2}}\right)+\frac{41x+26}{210(2x+3)(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{105}\left(\frac{1}{7}\left(\frac{277\sqrt{3x^2+2}}{7(2x+3)}-\frac{528}{7}\int\frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}\right)+\frac{507x+34}{14(2x+3)\sqrt{3x^2+2}}\right)+\frac{41x+26}{210(2x+3)\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{105}\left(\frac{1}{7}\left(\frac{277\sqrt{3x^2+2}}{7(2x+3)}-\frac{528\operatorname{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{7\sqrt{35}}\right)+\frac{507x+34}{14(2x+3)\sqrt{3x^2+2}}\right)+\frac{41x+26}{210(2x+3)\left(3x^2+2\right)^{3/2}}$$

input
```
Int[(5 - x)/((3 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```

output
```
(26 + 41*x)/(210*(3 + 2*x)*(2 + 3*x^2)^(3/2)) + ((34 + 507*x)/(14*(3 + 2*x
)*Sqrt[2 + 3*x^2]) + ((277*Sqrt[2 + 3*x^2])/(7*(3 + 2*x)) - (528*ArcTanh[(
4 - 9*x)/(Sqrt[35]*Sqrt[2 + 3*x^2])])/(7*Sqrt[35]))/7)/105
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

method	result
risch	$\frac{4986x^4+10647x^3+7362x^2+9107x+3966}{10290(3x^2+2)^{\frac{3}{2}}(2x+3)}-\frac{176\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{60025}$
trager	$\frac{4986x^4+10647x^3+7362x^2+9107x+3966}{10290(3x^2+2)^{\frac{3}{2}}(2x+3)}+\frac{176\operatorname{RootOf}\left(\_Z^2-35\right)\ln\left(\frac{9\operatorname{RootOf}\left(\_Z^2-35\right)x-4\operatorname{RootOf}\left(\_Z^2-35\right)+35\sqrt{3x^2+2}}{2x+3}\right)}{60025}$
default	$-\frac{13}{70\left(x+\frac{3}{2}\right)\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}+\frac{22}{147\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}-\frac{17x}{490\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}+\frac{277x}{3430\sqrt{3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}}}+$

input
```
int((5-x)/(2*x+3)^2/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/10290*(4986*x^4+10647*x^3+7362*x^2+9107*x+3966)/(3*x^2+2)^(3/2)/(2*x+3)-
176/60025*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(x+3/2)^2-36*x-19)^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.41

$$\int \frac{5-x}{(3+2x)^2 (2+3x^2)^{5/2}}\, dx = \frac{528 \sqrt{35}(18\, x^5 + 27\, x^4 + 24\, x^3 + 36\, x^2 + 8\, x + 12) \log\left(-\frac{\sqrt{35}\sqrt{3\, x^2+2}(9\, x-4)+9}{4\, x^2+12\, x+}\right.}{360150\,(18\, x^5 + 27\, x^4 +}$$

input
```
integrate((5-x)/(3+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/360150*(528*sqrt(35)*(18*x^5 + 27*x^4 + 24*x^3 + 36*x^2 + 8*x + 12)*log(
-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) + 93*x^2 - 36*x + 43)/(4*x^2 + 12*x +
 9)) + 35*(4986*x^4 + 10647*x^3 + 7362*x^2 + 9107*x + 3966)*sqrt(3*x^2 + 2
))/(18*x^5 + 27*x^4 + 24*x^3 + 36*x^2 + 8*x + 12)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{5-x}{(3+2x)^2 (2+3x^2)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((5-x)/(3+2*x)**2/(3*x**2+2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.15

$$\begin{aligned}\int \frac{5-x}{(3+2x)^2 (2+3x^2)^{5/2}}\, dx = {} & \frac{176}{60025}\sqrt{35}\,\operatorname{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|} - \frac{2\sqrt{6}}{3\,|2\,x+3|}\right) \\ & + \frac{277\,x}{3430\sqrt{3\,x^2+2}} + \frac{88}{1715\sqrt{3\,x^2+2}} - \frac{17\,x}{490\,(3\,x^2+2)^{\frac{3}{2}}} \\ & - \frac{13}{35\left(2\,(3\,x^2+2)^{\frac{3}{2}}x + 3\,(3\,x^2+2)^{\frac{3}{2}}\right)} + \frac{22}{147\,(3\,x^2+2)^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate((5-x)/(3+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
176/60025*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs(2*
x + 3)) + 277/3430*x/sqrt(3*x^2 + 2) + 88/1715/sqrt(3*x^2 + 2) - 17/490*x/
(3*x^2 + 2)^(3/2) - 13/35/(2*(3*x^2 + 2)^(3/2)*x + 3*(3*x^2 + 2)^(3/2)) +
22/147/(3*x^2 + 2)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(76) = 152$.

Time = 0.14 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.45

$$\int \frac{5-x}{(3+2x)^2 (2+3x^2)^{5/2}}\,dx = -\frac{1}{360150}\sqrt{35}\left(277\sqrt{35}\sqrt{3} - 1056\log\left(\sqrt{35}\sqrt{3}-9\right)\right)\operatorname{sgn}\left(\frac{1}{2x+3}\right) - \frac{176\sqrt{35}\log\left(\sqrt{35}\left(\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}+\frac{\sqrt{35}}{2x+3}\right)-9\right)}{60025\operatorname{sgn}\left(\frac{1}{2x+3}\right)} + \frac{\frac{\frac{\frac{7\left(\frac{4813}{\operatorname{sgn}\left(\frac{1}{2x+3}\right)}+\frac{4368}{(2x+3)\operatorname{sgn}\left(\frac{1}{2x+3}\right)}\right)}{2x+3}-\frac{53523}{\operatorname{sgn}\left(\frac{1}{2x+3}\right)}}{2x+3}+\frac{19269}{\operatorname{sgn}\left(\frac{1}{2x+3}\right)}}{2x+3}-\frac{2493}{\operatorname{sgn}\left(\frac{1}{2x+3}\right)}}{10290\left(\frac{18}{2x+3}-\frac{35}{(2x+3)^2}-3\right)\sqrt{-\frac{18}{2x+3}+\frac{35}{(2x+3)^2}+3}}$$

input
```
integrate((5-x)/(3+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
-1/360150*sqrt(35)*(277*sqrt(35)*sqrt(3) - 1056*log(sqrt(35)*sqrt(3) - 9))
*sgn(1/(2*x + 3)) - 176/60025*sqrt(35)*log(sqrt(35)*(sqrt(-18/(2*x + 3) +
35/(2*x + 3)^2 + 3) + sqrt(35)/(2*x + 3)) - 9)/sgn(1/(2*x + 3)) + 1/10290*
(((7*(4813/sgn(1/(2*x + 3)) + 4368/((2*x + 3)*sgn(1/(2*x + 3))))/(2*x + 3)
 - 53523/sgn(1/(2*x + 3)))/(2*x + 3) + 19269/sgn(1/(2*x + 3)))/(2*x + 3) -
 2493/sgn(1/(2*x + 3)))/((18/(2*x + 3) - 35/(2*x + 3)^2 - 3)*sqrt(-18/(2*x
 + 3) + 35/(2*x + 3)^2 + 3))
```

Mupad [B] (verification not implemented)

Time = 6.62 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.84

$$\int \frac{5-x}{(3+2x)^2\,(2+3x^2)^{5/2}}\,dx = \frac{\sqrt{35}\left(3464\,\ln\left(x+\frac{3}{2}\right)-3464\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{35}\,\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)\right)}{1500625} + \frac{\sqrt{35}\left(\frac{1872\,\ln\left(x+\frac{3}{2}\right)}{42875}-\frac{1872\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{35}\,\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{42875}\right)}{70} - \frac{104\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{42875\,\left(x+\frac{3}{2}\right)} - \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{639}{19600}+\frac{\sqrt{6}\,597\mathrm{i}}{19600}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(-\frac{213}{9800}+\frac{\sqrt{6}\,199\mathrm{i}}{9800}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{639}{19600}+\frac{\sqrt{6}\,597\mathrm{i}}{19600}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(\frac{213}{9800}+\frac{\sqrt{6}\,199\mathrm{i}}{9800}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\,\sqrt{6}\left(-41568+\sqrt{6}\,27711\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{12348000\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\,\sqrt{6}\left(41568+\sqrt{6}\,27711\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{12348000\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(-(x - 5)/((2*x + 3)^2*(3*x^2 + 2)^(5/2)),x)
```

output
```
(35^(1/2)*(3464*log(x + 3/2) - 3464*log(x - (3^(1/2)*35^(1/2)*(x^2 + 2/3)^
(1/2))/9 - 4/9)))/1500625 + (35^(1/2)*((1872*log(x + 3/2))/42875 - (1872*l
og(x - (3^(1/2)*35^(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/42875))/70 - (104*3^
(1/2)*(x^2 + 2/3)^(1/2))/(42875*(x + 3/2)) - (3^(1/2)*(x^2 + 2/3)^(1/2)*((
(6^(1/2)*597i)/19600 - 639/19600)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2
)*199i)/9800 - 213/9800)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x
^2 + 2/3)^(1/2)*(((6^(1/2)*597i)/19600 + 639/19600)/(x + (6^(1/2)*1i)/3) +
 (6^(1/2)*((6^(1/2)*199i)/9800 + 213/9800)*1i)/(2*(x + (6^(1/2)*1i)/3)^2))
)/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*27711i - 41568)*(x^2 + 2/3)^(1/2)*1i)/(12
348000*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*27711i + 41568)*(
x^2 + 2/3)^(1/2)*1i)/(12348000*(x - (6^(1/2)*1i)/3))
```

Reduce [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.17

$$\int \frac{5-x}{(3+2x)^2\left(2+3x^2\right)^{5/2}}\,dx = \frac{174510\sqrt{3x^2+2}\,x^4+372645\sqrt{3x^2+2}\,x^3+257670\sqrt{3x^2+2}\,x^2+318745}{}$$

input
```
int((5-x)/(3+2*x)^2/(3*x^2+2)^(5/2),x)
```

output
```
(174510*sqrt(3*x**2 + 2)*x**4 + 372645*sqrt(3*x**2 + 2)*x**3 + 257670*sqrt
(3*x**2 + 2)*x**2 + 318745*sqrt(3*x**2 + 2)*x + 138810*sqrt(3*x**2 + 2) +
19008*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**5 + 28512*sqrt(
35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**4 + 25344*sqrt(35)*log(sqr
t(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**3 + 38016*sqrt(35)*log(sqrt(3*x**2 +
2)*sqrt(35) + 9*x - 4)*x**2 + 8448*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35)
+ 9*x - 4)*x + 12672*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4) - 1
9008*sqrt(35)*log(2*x + 3)*x**5 - 28512*sqrt(35)*log(2*x + 3)*x**4 - 25344
*sqrt(35)*log(2*x + 3)*x**3 - 38016*sqrt(35)*log(2*x + 3)*x**2 - 8448*sqrt
(35)*log(2*x + 3)*x - 12672*sqrt(35)*log(2*x + 3))/(360150*(18*x**5 + 27*x
**4 + 24*x**3 + 36*x**2 + 8*x + 12))
```

3.261 $\int \frac{5-x}{(3+2x)^3\left(2+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{5-x}{(3+2x)^3\left(2+3x^2\right)^{5/2}}\,dx=\frac{256-173x}{2450\left(2+3x^2\right)^{3/2}}-\frac{13}{70(3+2x)^2\left(2+3x^2\right)^{3/2}}-\frac{107}{350(3+2x)\left(2+3x^2\right)^{3/2}}+\frac{3072+857x}{85750\sqrt{2+3x^2}}-\frac{3072\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{2+3x^2}}\right)}{42875\sqrt{35}}$$

output
```
1/2450*(256-173*x)/(3*x^2+2)^(3/2)-13/70/(3+2*x)^2/(3*x^2+2)^(3/2)-107/350
/(3+2*x)/(3*x^2+2)^(3/2)+1/85750*(3072+857*x)/(3*x^2+2)^(1/2)-3072/1500625
*35^(1/2)*arctanh(1/35*(4-9*x)*35^(1/2)/(3*x^2+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$\int \frac{5-x}{(3+2x)^3\left(2+3x^2\right)^{5/2}}\,dx=\frac{\frac{35\left(41366+89749x+91268x^2+116367x^3+67716x^4+10284x^5\right)}{(3+2x)^2\left(2+3x^2\right)^{3/2}}+12288\sqrt{35}\text{arctanh}\left(\frac{3\sqrt{3}+2\sqrt{3}\ldots}{\ldots}\right.}{3001250}$$

input
```
Integrate[(5 - x)/((3 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
```

output
```
((35*(41366 + 89749*x + 91268*x^2 + 116367*x^3 + 67716*x^4 + 10284*x^5))/(
(3 + 2*x)^2*(2 + 3*x^2)^(3/2)) + 12288*Sqrt[35]*ArcTanh[(3*Sqrt[3] + 2*Sqr
t[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[35]])/3001250
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {686, 27, 686, 27, 688, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5-x}{(2x+3)^3 (3x^2+2)^{5/2}} \, dx$$

$$\Big\downarrow \text{686}$$

$$\frac{41x+26}{210(2x+3)^2 (3x^2+2)^{3/2}}-\frac{1}{630}\int -\frac{6(164x+253)}{(2x+3)^3 (3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{105}\int \frac{164x+253}{(2x+3)^3 (3x^2+2)^{3/2}}dx+\frac{41x+26}{210(2x+3)^2 (3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{686}$$

$$\frac{1}{105}\left(\frac{419x+4}{10(2x+3)^2\sqrt{3x^2+2}}-\frac{1}{210}\int -\frac{84(419x+6)}{(2x+3)^3\sqrt{3x^2+2}}dx\right)+\frac{41x+26}{210(2x+3)^2 (3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{105}\left(\frac{2}{5}\int \frac{419x+6}{(2x+3)^3\sqrt{3x^2+2}}dx+\frac{419x+4}{10(2x+3)^2\sqrt{3x^2+2}}\right)+\frac{41x+26}{210(2x+3)^2 (3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{688}$$

$$\frac{1}{105}\left(\frac{2}{5}\left(\frac{249\sqrt{3x^2+2}}{14(2x+3)^2}-\frac{1}{70}\int -\frac{5(747x+692)}{(2x+3)^2\sqrt{3x^2+2}}dx\right)+\frac{419x+4}{10(2x+3)^2\sqrt{3x^2+2}}\right)+\frac{41x+26}{210(2x+3)^2 (3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{105}\left(\frac{2}{5}\left(\frac{1}{14}\int \frac{747x+692}{(2x+3)^2\sqrt{3x^2+2}}dx+\frac{249\sqrt{3x^2+2}}{14(2x+3)^2}\right)+\frac{419x+4}{10(2x+3)^2\sqrt{3x^2+2}}\right)+\frac{41x+26}{210(2x+3)^2\left(3x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 679

$$\frac{1}{105}\left(\frac{2}{5}\left(\frac{1}{14}\left(\frac{9216}{35}\int \frac{1}{(2x+3)\sqrt{3x^2+2}}dx+\frac{857\sqrt{3x^2+2}}{35(2x+3)}\right)+\frac{249\sqrt{3x^2+2}}{14(2x+3)^2}\right)+\frac{419x+4}{10(2x+3)^2\sqrt{3x^2+2}}\right)+\frac{41x+26}{210(2x+3)^2\left(3x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 488

$$\frac{1}{105}\left(\frac{2}{5}\left(\frac{1}{14}\left(\frac{857\sqrt{3x^2+2}}{35(2x+3)}-\frac{9216}{35}\int \frac{1}{35-\frac{(4-9x)^2}{3x^2+2}}d\frac{4-9x}{\sqrt{3x^2+2}}\right)+\frac{249\sqrt{3x^2+2}}{14(2x+3)^2}\right)+\frac{419x+4}{10(2x+3)^2\sqrt{3x^2+2}}\right)+\frac{41x+26}{210(2x+3)^2\left(3x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{1}{105}\left(\frac{2}{5}\left(\frac{1}{14}\left(\frac{857\sqrt{3x^2+2}}{35(2x+3)}-\frac{9216\text{arctanh}\left(\frac{4-9x}{\sqrt{35}\sqrt{3x^2+2}}\right)}{35\sqrt{35}}\right)+\frac{249\sqrt{3x^2+2}}{14(2x+3)^2}\right)+\frac{419x+4}{10(2x+3)^2\sqrt{3x^2+2}}\right)+\frac{41x+26}{210(2x+3)^2\left(3x^2+2\right)^{3/2}}$$

input
```
Int[(5 - x)/((3 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
```

output
```
(26 + 41*x)/(210*(3 + 2*x)^2*(2 + 3*x^2)^(3/2)) + ((4 + 419*x)/(10*(3 + 2*
x)^2*Sqrt[2 + 3*x^2]) + (2*((249*Sqrt[2 + 3*x^2])/(14*(3 + 2*x)^2) + ((857
*Sqrt[2 + 3*x^2])/(35*(3 + 2*x)) - (9216*ArcTanh[(4 - 9*x)/(Sqrt[35]*Sqrt[
2 + 3*x^2])])/(35*Sqrt[35]))/14))/5)/105
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

method	result
risch	$\frac{10284x^5+67716x^4+116367x^3+91268x^2+89749x+41366}{85750(2x+3)^2(3x^2+2)^{\frac{3}{2}}}-\frac{3072\sqrt{35}\,\operatorname{arctanh}\left(\frac{2(4-9x)\sqrt{35}}{35\sqrt{12\left(x+\frac{3}{2}\right)^2-36x-19}}\right)}{1500625}$
trager	$\frac{(10284x^5+67716x^4+116367x^3+91268x^2+89749x+41366)\sqrt{3x^2+2}}{85750(6x^3+9x^2+4x+6)^2}+\frac{3072\,\mathrm{RootOf}\left(\_Z^2-35\right)\ln\left(\frac{9\,\mathrm{RootOf}\left(\_Z^2-35\right)x-4\,\mathrm{Roo}}{2x}\right.}{1500625}$
default	$-\frac{13}{280\left(x+\frac{3}{2}\right)^2\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}-\frac{107}{700\left(x+\frac{3}{2}\right)\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}+\frac{128}{1225\left(3\left(x+\frac{3}{2}\right)^2-9x-\frac{19}{4}\right)^{\frac{3}{2}}}-\frac{173x}{2450\left(3\left(x+\frac{3}{2}\right)^2-9\right.}$

input
```
int((5-x)/(2*x+3)^3/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/85750*(10284*x^5+67716*x^4+116367*x^3+91268*x^2+89749*x+41366)/(2*x+3)^2
/(3*x^2+2)^(3/2)-3072/1500625*35^(1/2)*arctanh(2/35*(4-9*x)*35^(1/2)/(12*(
x+3/2)^2-36*x-19)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.27

$$\int \frac{5-x}{(3+2x)^3\,(2+3x^2)^{5/2}}\,dx = \frac{3072\,\sqrt{35}(36\,x^6+108\,x^5+129\,x^4+144\,x^3+124\,x^2+48\,x+36)\log\left(-\frac{\sqrt{3}}{}\right.}{3001250\,(36\,x^6}$$

input
```
integrate((5-x)/(3+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/3001250*(3072*sqrt(35)*(36*x^6 + 108*x^5 + 129*x^4 + 144*x^3 + 124*x^2 +
 48*x + 36)*log(-(sqrt(35)*sqrt(3*x^2 + 2)*(9*x - 4) + 93*x^2 - 36*x + 43)
/(4*x^2 + 12*x + 9)) + 35*(10284*x^5 + 67716*x^4 + 116367*x^3 + 91268*x^2
+ 89749*x + 41366)*sqrt(3*x^2 + 2))/(36*x^6 + 108*x^5 + 129*x^4 + 144*x^3
+ 124*x^2 + 48*x + 36)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{5-x}{(3+2x)^3 (2+3x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((5-x)/(3+2*x)**3/(3*x**2+2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.29

$$\int \frac{5-x}{(3+2x)^3 (2+3x^2)^{5/2}}\,dx = \frac{3072}{1500625}\sqrt{35}\,\operatorname{arsinh}\left(\frac{3\sqrt{6}x}{2\,|2\,x+3|}-\frac{2\sqrt{6}}{3\,|2\,x+3|}\right) + \frac{857\,x}{85750\sqrt{3\,x^2+2}} + \frac{1536}{42875\sqrt{3\,x^2+2}} - \frac{173\,x}{2450\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{13}{70\left(4\,(3\,x^2+2)^{\frac{3}{2}}x^2+12\,(3\,x^2+2)^{\frac{3}{2}}x+9\,(3\,x^2+2)^{\frac{3}{2}}\right)} - \frac{107}{350\left(2\,(3\,x^2+2)^{\frac{3}{2}}x+3\,(3\,x^2+2)^{\frac{3}{2}}\right)} + \frac{128}{1225\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((5-x)/(3+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
3072/1500625*sqrt(35)*arcsinh(3/2*sqrt(6)*x/abs(2*x + 3) - 2/3*sqrt(6)/abs
(2*x + 3)) + 857/85750*x/sqrt(3*x^2 + 2) + 1536/42875/sqrt(3*x^2 + 2) - 17
3/2450*x/(3*x^2 + 2)^(3/2) - 13/70/(4*(3*x^2 + 2)^(3/2)*x^2 + 12*(3*x^2 +
2)^(3/2)*x + 9*(3*x^2 + 2)^(3/2)) - 107/350/(2*(3*x^2 + 2)^(3/2)*x + 3*(3*
x^2 + 2)^(3/2)) + 128/1225/(3*x^2 + 2)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(94) = 188$.

Time = 0.14 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.78

$$\int \frac{5-x}{(3+2x)^3 (2+3x^2)^{5/2}}\,dx = \frac{3072}{1500625}\sqrt{35}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{35}-3\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{35}+3\sqrt{3}-2\sqrt{3x^2+2}}\right) + \frac{3((59203x+69168)x+37637)x+190066}{3001250(3x^2+2)^{\frac{3}{2}}} - \frac{4\left(9588\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^3+27991\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2-68448\sqrt{3}x+9736\sqrt{3}+68448\sqrt{3x^2+2}\right)}{1500625\left(\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2+3\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)-2\right)^2}$$

input
```
integrate((5-x)/(3+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
3072/1500625*sqrt(35)*log(-abs(-2*sqrt(3)*x - sqrt(35) - 3*sqrt(3) + 2*sqr
t(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(35) + 3*sqrt(3) - 2*sqrt(3*x^2 + 2))) +
1/3001250*(3*((59203*x + 69168)*x + 37637)*x + 190066)/(3*x^2 + 2)^(3/2) -
 4/1500625*(9588*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 + 27991*sqrt(3)*(sqrt(3)*
x - sqrt(3*x^2 + 2))^2 - 68448*sqrt(3)*x + 9736*sqrt(3) + 68448*sqrt(3*x^2
 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 3*sqrt(3)*(sqrt(3)*x - sqrt(3*x^
2 + 2)) - 2)^2
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.57

$$\int \frac{5-x}{(3+2x)^3 (2+3x^2)^{5/2}}\,dx = \frac{3072\sqrt{35}\ln\left(x+\frac{3}{2}\right)}{1500625} - \frac{3072\sqrt{35}\ln\left(x-\frac{\sqrt{3}\sqrt{35}\sqrt{x^2+\frac{2}{3}}}{9}-\frac{4}{9}\right)}{1500625} - \frac{739\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1029000\left(x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}-\frac{2}{3}\right)} + \frac{59203\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{18007500\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{59203\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{18007500\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{739\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1029000\left(-x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}+\frac{2}{3}\right)} - \frac{4868\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1500625\left(x+\frac{3}{2}\right)} - \frac{26\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{42875\left(x^2+3x+\frac{9}{4}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,157\mathrm{i}}{6174000\left(x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}-\frac{2}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,164201\mathrm{i}}{72030000\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,164201\mathrm{i}}{72030000\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,157\mathrm{i}}{6174000\left(-x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}+\frac{2}{3}\right)}$$

input
```
int(-(x - 5)/((2*x + 3)^3*(3*x^2 + 2)^(5/2)),x)
```

output
```
(3072*35^(1/2)*log(x + 3/2))/1500625 - (3072*35^(1/2)*log(x - (3^(1/2)*35^
(1/2)*(x^2 + 2/3)^(1/2))/9 - 4/9))/1500625 - (739*3^(1/2)*(x^2 + 2/3)^(1/2
))/(1029000*((6^(1/2)*x*2i)/3 + x^2 - 2/3)) + (59203*3^(1/2)*(x^2 + 2/3)^(
1/2))/(18007500*(x - (6^(1/2)*1i)/3)) + (59203*3^(1/2)*(x^2 + 2/3)^(1/2))/
(18007500*(x + (6^(1/2)*1i)/3)) + (739*3^(1/2)*(x^2 + 2/3)^(1/2))/(1029000
*((6^(1/2)*x*2i)/3 - x^2 + 2/3)) - (4868*3^(1/2)*(x^2 + 2/3)^(1/2))/(15006
25*(x + 3/2)) - (26*3^(1/2)*(x^2 + 2/3)^(1/2))/(42875*(3*x + x^2 + 9/4)) -
 (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*157i)/(6174000*((6^(1/2)*x*2i)/3 + x^2
 - 2/3)) - (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*164201i)/(72030000*(x - (6^(
1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*164201i)/(72030000*(x +
(6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*157i)/(6174000*((6^(
1/2)*x*2i)/3 - x^2 + 2/3))
```

Reduce [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.04

$$\int \frac{5-x}{(3+2x)^3 (2+3x^2)^{5/2}}\,dx = \frac{359940\sqrt{3x^2+2}\,x^5+2370060\sqrt{3x^2+2}\,x^4+4072845\sqrt{3x^2+2}\,x^3+31943}{}$$

input
```
int((5-x)/(3+2*x)^3/(3*x^2+2)^(5/2),x)
```

output
```
(359940*sqrt(3*x**2 + 2)*x**5 + 2370060*sqrt(3*x**2 + 2)*x**4 + 4072845*sq
rt(3*x**2 + 2)*x**3 + 3194380*sqrt(3*x**2 + 2)*x**2 + 3141215*sqrt(3*x**2
+ 2)*x + 1447810*sqrt(3*x**2 + 2) + 221184*sqrt(35)*log(sqrt(3*x**2 + 2)*s
qrt(35) + 9*x - 4)*x**6 + 663552*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) +
9*x - 4)*x**5 + 792576*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x
**4 + 884736*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**3 + 7618
56*sqrt(35)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x**2 + 294912*sqrt(35
)*log(sqrt(3*x**2 + 2)*sqrt(35) + 9*x - 4)*x + 221184*sqrt(35)*log(sqrt(3*
x**2 + 2)*sqrt(35) + 9*x - 4) - 221184*sqrt(35)*log(2*x + 3)*x**6 - 663552
*sqrt(35)*log(2*x + 3)*x**5 - 792576*sqrt(35)*log(2*x + 3)*x**4 - 884736*s
qrt(35)*log(2*x + 3)*x**3 - 761856*sqrt(35)*log(2*x + 3)*x**2 - 294912*sqr
t(35)*log(2*x + 3)*x - 221184*sqrt(35)*log(2*x + 3))/(3001250*(36*x**6 + 1
08*x**5 + 129*x**4 + 144*x**3 + 124*x**2 + 48*x + 36))
```

3.262 $\int (A+Bx)\sqrt{d+ex}\sqrt{a-cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 433

$$\int (A+Bx)\sqrt{d+ex}\sqrt{a-cx^2}\,dx = \frac{4(4Bcd^2-7Acde-5aBe^2)\sqrt{d+ex}\sqrt{a-cx^2}}{105ce^2} - \frac{2(d+ex)^{3/2}(4Bd-7Ae-5Bex)\sqrt{a-cx^2}}{35e^2} + \frac{4\sqrt{a}(4Bcd^3-7Acd^2e-8aBde^2-21aAe^3)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{105\sqrt{c}e^3\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} - \frac{4\sqrt{a}(cd^2-ae^2)(4Bcd^2-7Acde-5aBe^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{105c^{3/2}e^3\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
4/105*(-7*A*c*d*e-5*B*a*e^2+4*B*c*d^2)*(e*x+d)^(1/2)*(-c*x^2+a)^(1/2)/c/e^
2-2/35*(e*x+d)^(3/2)*(-5*B*e*x-7*A*e+4*B*d)*(-c*x^2+a)^(1/2)/e^2+4/105*a^(
1/2)*(-21*A*a*e^3-7*A*c*d^2*e-8*B*a*d*e^2+4*B*c*d^3)*(e*x+d)^(1/2)*(1-c*x^
2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1
/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/c^(1/2)/e^3/(c^(1/2)*(e*x+d)/(c^(1/2)*
d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)-4/105*a^(1/2)*(-a*e^2+c*d^2)*(-7*A*c*
d*e-5*B*a*e^2+4*B*c*d^2)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-
c*x^2/a)^(1/2)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(
a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/c^(3/2)/e^3/(e*x+d)^(1/2)/(-c*x^2+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.74 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.35

$$\int (A+Bx)\sqrt{d+ex}\sqrt{a-cx^2}\,dx$$

$$= \frac{2\sqrt{a-cx^2}\left(-((d+ex)\left(10aBe^2-7Ace(d+3ex)+Bc(4d^2-3dex-15e^2x^2)\right)) - \frac{2\left(e^2\sqrt{-d+\frac{\sqrt{a}e}{\sqrt{c}}}\left(-7Ae(c\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(A + B*x)*Sqrt[d + e*x]*Sqrt[a - c*x^2],x]
```

output
```
(2*Sqrt[a - c*x^2]*(-((d + e*x)*(10*a*B*e^2 - 7*A*c*e*(d + 3*e*x) + B*c*(4
*d^2 - 3*d*e*x - 15*e^2*x^2))) - (2*(e^2*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-
7*A*e*(c*d^2 + 3*a*e^2) + 4*B*(c*d^3 - 2*a*d*e^2))*(-a + c*x^2) + I*Sqrt[c
]*(Sqrt[c]*d - Sqrt[a]*e)*(-4*B*c*d^3 + 7*A*c*d^2*e + 8*a*B*d*e^2 + 21*a*A
*e^3)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c
] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d + (Sqrt[a
]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]
*e)] + I*Sqrt[a]*e*(-(Sqrt[c]*d) + Sqrt[a]*e)*(4*B*c*d^2 + 3*Sqrt[a]*B*Sqr
t[c]*d*e - 7*A*c*d*e - 5*a*B*e^2 + 21*Sqrt[a]*A*Sqrt[c]*e^2)*Sqrt[(e*(Sqrt
[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))
]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[
d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)]))/(e^2*Sqrt[-d
 + (Sqrt[a]*e)/Sqrt[c]]*(a - c*x^2))))/(105*c*e^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {687, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-cx^2}(A+Bx)\sqrt{d+ex}\,dx$$

$$\Big\downarrow \text{687}$$

$$-\frac{2\int -\frac{(7Acd+aBe+c(Bd+7Ae)x)\sqrt{a-cx^2}}{2\sqrt{d+ex}}dx}{7c}-\frac{2B\left(a-cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(7Acd+aBe+c(Bd+7Ae)x)\sqrt{a-cx^2}}{\sqrt{d+ex}}dx}{7c}-\frac{2B\left(a-cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$$\Big\downarrow \text{682}$$

$$\frac{-\frac{4\int \frac{c\left(ae\left(Bcd^2-28Aced-5aBe^2\right)+c\left(4Bcd^3-7Aced^2-8aBe^2d-21aAe^3\right)x\right)}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{15ce^2}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(-5aBe^2-3cex(7Ae+Bd)-7Acde+4Bcd^2)}{15e^2}}{7c}-\frac{2B\left(a-cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{2\int \frac{ae\left(Bcd^2-28Aced-5aBe^2\right)+c\left(4Bcd^3-7Aced^2-8aBe^2d-21aAe^3\right)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{15e^2}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(-5aBe^2-3cex(7Ae+Bd)-7Acde+4Bcd^2)}{15e^2}}{7c}-\frac{2B\left(a-cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$$\Big\downarrow \; 600$$

$$\frac{-\frac{2\left(\frac{c\left(-21aAe^3-8aBde^2-7Acd^2e+4Bcd^3\right)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{\left(cd^2-ae^2\right)\left(-5aBe^2-7Acde+4Bcd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{15e^2}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(-5aBe^2}{}}{7c}$$

$$\frac{2B\left(a-cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$$\Big\downarrow \; 509$$

$$\frac{-\frac{2\left(\frac{c\sqrt{1-\frac{cx^2}{a}}\left(-21aAe^3-8aBde^2-7Acd^2e+4Bcd^3\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{\left(cd^2-ae^2\right)\left(-5aBe^2-7Acde+4Bcd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{15e^2}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}}{}}{7c}$$

$$\frac{2B\left(a-cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$$\Big\downarrow \; 508$$

$$-\frac{\frac{2\left(-\frac{\left(cd^2-ae^2\right)\left(-5aBe^2-7Acde+4Bcd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(-21aAe^3-8aBde^2-7Acd^2e+4Bcd^3\right)\int \frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\ldots}}{\ldots}}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\ldots\right)}{15e^2}\ldots}{7c}$$

$$\frac{2B\left(a-cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$$\Big\downarrow \; 327$$

$$-\frac{\frac{2\left(-\frac{\left(cd^2-ae^2\right)\left(-5aBe^2-7Acde+4Bcd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(-21aAe^3-8aBde^2-7Acd^2e+4Bcd^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\ldots\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\ldots\right)}{15e^2}\ldots}{7c}$$

$$\frac{2B\left(a-cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$$\Big\downarrow \; 512$$

$$-\frac{\frac{2\left(-\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\left(-5aBe^2-7Acde+4Bcd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(-21aAe^3-8aBde^2-7Acd^2e+4Bcd^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\ldots}}{\ldots}\right)\ldots\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\ldots\right)}{15e^2}\ldots}{7c}$$

$$\frac{2B\left(a-cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$$\Big\downarrow \; 511$$

$$-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{cd}}}\left(-5aBe^2-7Acde+4Bcd^2\right)\int\frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{cd}}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(-21aAe^3-\right.}{}\right)}{15e^2}$$

$$\overline{7c}$$

$$\frac{2B\left(a-cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$\downarrow$ 321

$$-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{cd}}}\left(-5aBe^2-7Acde+4Bcd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{cd}}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(-21aAe^3-8aBde^2\right.}{e\sqrt{a-}}\right)}{15e^2}$$

$$\overline{7c}$$

$$\frac{2B\left(a-cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

input
```
Int[(A + B*x)*Sqrt[d + e*x]*Sqrt[a - c*x^2],x]
```

output
```
(-2*B*Sqrt[d + e*x]*(a - c*x^2)^(3/2))/(7*c) + ((-2*Sqrt[d + e*x]*(4*B*c*d
^2 - 7*A*c*d*e - 5*a*B*e^2 - 3*c*e*(B*d + 7*A*e)*x)*Sqrt[a - c*x^2])/(15*e
^2) - (2*((-2*Sqrt[a]*Sqrt[c]*(4*B*c*d^3 - 7*A*c*d^2*e - 8*a*B*d*e^2 - 21*
a*A*e^3)*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt
[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(e*Sqrt[(Sqrt[
c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) + (2*Sqrt[a]*(c*d^
2 - a*e^2)*(4*B*c*d^2 - 7*A*c*d*e - 5*a*B*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(S
qrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt
[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqr
t[d + e*x]*Sqrt[a - c*x^2])))/(15*e^2))/(7*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(363) = 726$.

Time = 4.13 (sec) , antiderivative size = 792, normalized size of antiderivative = 1.83

method	result
elliptic	$\dfrac{\sqrt{(ex+d)(-c\,x^2+a)}\left(\frac{2B\,x^2\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}{7}-\frac{2\left(-Ace-\frac{1}{7}Bcd\right)x\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}{5ce}-\frac{2\left(-Acd+\frac{2Bae}{7}-\frac{4d\left(-Ace-\frac{1}{7}Bcd\right)}{5e}\right)}{3ce}\right.}{}$
risch	$\frac{2(15e^2Bc\,x^2+21Ac\,e^2x+3Bcdex+7Acde-10Ba\,e^2-4Bc\,d^2)\sqrt{ex+d}\,\sqrt{-c\,x^2+a}}{105c\,e^2}+\frac{2\left(\frac{\left(21Aa\,e^3+7Ac\,d^2e+8Bad\,e^2-4Bc\,d^3\right)\sqrt{ac}\,\sqrt{2}}{}\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)*(e*x+d)^(1/2)*(-c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(2/7*B*x^2*(-c*e
*x^3-c*d*x^2+a*e*x+a*d)^(1/2)-2/5*(-A*c*e-1/7*B*c*d)/c/e*x*(-c*e*x^3-c*d*x
^2+a*e*x+a*d)^(1/2)-2/3*(-A*c*d+2/7*B*a*e-4/5*d/e*(-A*c*e-1/7*B*c*d))/c/e*
(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)+2*(A*a*d+2/5*a/c*d/e*(-A*c*e-1/7*B*c*d)
+1/3*a/c*(-A*c*d+2/7*B*a*e-4/5*d/e*(-A*c*e-1/7*B*c*d)))*(d/e-1/c*(a*c)^(1/
2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(
a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*
e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(
1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+2*(a*A*e+3/7*B
*a*d+3/5*a/c*(-A*c*e-1/7*B*c*d)-2/3*d/e*(-A*c*d+2/7*B*a*e-4/5*d/e*(-A*c*e-
1/7*B*c*d)))*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*(
(x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d
/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(
a*c)^(1/2))*EllipticE(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*
c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+1/c*(a*c)^(1/2)*EllipticF(((x+d/e
)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/
2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.80

$$\int (A+Bx)\sqrt{d+ex}\sqrt{a-cx^2}\,dx =$$
$$-\frac{2\left(2\left(4Bc^2d^4-7Ac^2d^3e-11Bacd^2e^2+63Aacde^3+15Ba^2e^4\right)\sqrt{-ce}\,\text{weierstrassPInverse}\left(\frac{4\left(cd^2+3ae^2\right)}{3ce^2}\right.\right.}{}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)*(-c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*(2*(4*B*c^2*d^4 - 7*A*c^2*d^3*e - 11*B*a*c*d^2*e^2 + 63*A*a*c*d*e^3
 + 15*B*a^2*e^4)*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e
^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 6*(4*B*c^2*d^
3*e - 7*A*c^2*d^2*e^2 - 8*B*a*c*d*e^3 - 21*A*a*c*e^4)*sqrt(-c*e)*weierstra
ssZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), w
eierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2
)/(c*e^3), 1/3*(3*e*x + d)/e)) - 3*(15*B*c^2*e^4*x^2 - 4*B*c^2*d^2*e^2 + 7
*A*c^2*d*e^3 - 10*B*a*c*e^4 + 3*(B*c^2*d*e^3 + 7*A*c^2*e^4)*x)*sqrt(-c*x^2
 + a)*sqrt(e*x + d))/(c^2*e^4)
```

Sympy [F]

$$\int (A+Bx)\sqrt{d+ex}\sqrt{a-cx^2}\,dx = \int (A+Bx)\sqrt{a-cx^2}\sqrt{d+ex}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)**(1/2)*(-c*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)*sqrt(a - c*x**2)*sqrt(d + e*x), x)
```

Maxima [F]

$$\int (A+Bx)\sqrt{d+ex}\sqrt{a-cx^2}\,dx = \int \sqrt{-cx^2+a}(Bx+A)\sqrt{ex+d}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)*(-c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^2 + a)*(B*x + A)*sqrt(e*x + d), x)
```

Giac [F]

$$\int (A+Bx)\sqrt{d+ex}\sqrt{a-cx^2}\,dx = \int \sqrt{-cx^2+a}(Bx+A)\sqrt{ex+d}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)*(-c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^2 + a)*(B*x + A)*sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)\sqrt{d+ex}\sqrt{a-cx^2}\,dx=\int \sqrt{a-c\,x^2}\,(A+B\,x)\,\sqrt{d+e\,x}\,dx$$

input
```
int((a - c*x^2)^(1/2)*(A + B*x)*(d + e*x)^(1/2),x)
```

output
```
int((a - c*x^2)^(1/2)*(A + B*x)*(d + e*x)^(1/2), x)
```

Reduce [F]

$$\int (A+Bx)\sqrt{d+ex}\sqrt{a-cx^2}\,dx$$
$$=\frac{-14\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\,a^2e^2-12\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\,abde+14\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\,acdex+2\sqrt{ex+}}{}$$

input
```
int((B*x+A)*(e*x+d)^(1/2)*(-c*x^2+a)^(1/2),x)
```

output
```
( - 14*sqrt(d + e*x)*sqrt(a - c*x**2)*a**2*e**2 - 12*sqrt(d + e*x)*sqrt(a
- c*x**2)*a*b*d*e + 14*sqrt(d + e*x)*sqrt(a - c*x**2)*a*c*d*e*x + 2*sqrt(d
 + e*x)*sqrt(a - c*x**2)*b*c*d**2*x + 10*sqrt(d + e*x)*sqrt(a - c*x**2)*b*
c*d*e*x**2 - 21*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d + a*e*x - c
*d*x**2 - c*e*x**3),x)*a**2*c*e**3 - 8*int((sqrt(d + e*x)*sqrt(a - c*x**2)
*x**2)/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a*b*c*d*e**2 - 7*int((sqrt(d
 + e*x)*sqrt(a - c*x**2)*x**2)/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a*c*
*2*d**2*e + 4*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d + a*e*x - c*d
*x**2 - c*e*x**3),x)*b*c**2*d**3 + 7*int((sqrt(d + e*x)*sqrt(a - c*x**2))/
(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a**3*e**3 + 6*int((sqrt(d + e*x)*sq
rt(a - c*x**2))/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a**2*b*d*e**2 + 21*
int((sqrt(d + e*x)*sqrt(a - c*x**2))/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x
)*a**2*c*d**2*e - 2*int((sqrt(d + e*x)*sqrt(a - c*x**2))/(a*d + a*e*x - c*
d*x**2 - c*e*x**3),x)*a*b*c*d**3)/(35*c*d*e)
```

3.263 $\int \frac{(A+Bx)\sqrt{a-cx^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 359

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{\sqrt{d+ex}}\,dx = -\frac{2\sqrt{d+ex}(4Bd-5Ae-3Bex)\sqrt{a-cx^2}}{15e^2} + \frac{4\sqrt{a}(4Bcd^2-5Acde-3aBe^2)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{15\sqrt{c}e^3\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} - \frac{4\sqrt{a}(4Bd-5Ae)(cd^2-ae^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{15\sqrt{c}e^3\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
-2/15*(e*x+d)^(1/2)*(-3*B*e*x-5*A*e+4*B*d)*(-c*x^2+a)^(1/2)/e^2+4/15*a^(1/
2)*(-5*A*c*d*e-3*B*a*e^2+4*B*c*d^2)*(e*x+d)^(1/2)*(1-c*x^2/a)^(1/2)*Ellipt
icE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+
a^(1/2)*e))^(1/2))/c^(1/2)/e^3/(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/
2)/(-c*x^2+a)^(1/2)-4/15*a^(1/2)*(-5*A*e+4*B*d)*(-a*e^2+c*d^2)*(c^(1/2)*(e
*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2)*EllipticF(1/2*(1-c^(1
/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/
2))/c^(1/2)/e^3/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.05 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.45

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{\sqrt{d+ex}}\,dx$$

$$= \frac{\sqrt{a-cx^2}\left(\frac{2(d+ex)(-4Bd+5Ae+3Bex)}{e^2}+\frac{4\left(e^2\sqrt{-d+\frac{\sqrt{a}e}{\sqrt{c}}}\left(4Bcd^2-5Acde-3aBe^2\right)\left(-a+cx^2\right)+i\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)\left(-4Bcd^2+5Acde+3a\right.\right.}{}\right.}{}$$

input
```
Integrate[((A + B*x)*Sqrt[a - c*x^2])/Sqrt[d + e*x],x]
```

output
```
(Sqrt[a - c*x^2]*((2*(d + e*x)*(-4*B*d + 5*A*e + 3*B*e*x))/e^2 + (4*(e^2*S
qrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(4*B*c*d^2 - 5*A*c*d*e - 3*a*B*e^2)*(-a + c*
x^2) + I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(-4*B*c*d^2 + 5*A*c*d*e + 3*a*B*e
^2)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c]
- e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d + (Sqrt[a]*
e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e
)] + I*Sqrt[a]*Sqrt[c]*e*(Sqrt[c]*d - Sqrt[a]*e)*(-4*B*Sqrt[c]*d - 3*Sqrt[
a]*B*e + 5*A*Sqrt[c]*e)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-((
(Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh
[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(S
qrt[c]*d - Sqrt[a]*e)]))/(c*e^4*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2
))))/(15*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^2}(A+Bx)}{\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{ 682}$$

$$-\frac{4\int \frac{c(ae(Bd-5Ae)+(4Bcd^2-5Aced-3aBe^2)x)}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{15ce^2}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(-5Ae+4Bd-3Bex)}{15e^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2\int \frac{ae(Bd-5Ae)+(4Bcd^2-5Aced-3aBe^2)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{15e^2}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(-5Ae+4Bd-3Bex)}{15e^2}$$

$$\Big\downarrow \text{ 600}$$

$$-\frac{2\left(\frac{(-3aBe^2-5Acde+4Bcd^2)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{(cd^2-ae^2)(4Bd-5Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{15e^2}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(-5Ae+4Bd-3Bex)}{15e^2}$$

$$\Big\downarrow \text{ 509}$$

$$-\frac{2\left(\frac{\sqrt{1-\frac{cx^2}{a}}(-3aBe^2-5Acde+4Bcd^2)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{(cd^2-ae^2)(4Bd-5Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{15e^2}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(-5Ae+4Bd-3Bex)}{15e^2}$$

$$\Big\downarrow \text{ 508}$$

$$-\frac{2\left(-\frac{(cd^2-ae^2)(4Bd-5Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(-3aBe^2-5Acde+4Bcd^2)\int \frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{15e^2}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(-5Ae+4Bd-3Bex)}{15e^2}$$

$$\Big\downarrow \text{ 327}$$

$$-\frac{2\left(-\frac{(cd^2-ae^2)(4Bd-5Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(-3aBe^2-5Acde+4Bcd^2)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{15e^2}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(-5Ae+4Bd-3Bex)}{15e^2}$$

$$\Big\downarrow \text{512}$$

$$-\frac{2\left(-\frac{\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)(4Bd-5Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(-3aBe^2-5Acde+4Bcd^2)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+\ldots}\right.}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\ldots\right.}{15e^2}\ldots$$
$$\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(-5Ae+4Bd-3Bex)}{15e^2}$$

$$\Big\downarrow \text{511}$$

$$-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)(4Bd-5Ae)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\int \frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(-3aBe^2-5Acde\ldots}{\sqrt{c}e\sqrt{a-c\ldots}}\ldots\right.}{15e^2}\ldots$$
$$\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(-5Ae+4Bd-3Bex)}{15e^2}$$

$$\Big\downarrow \text{321}$$

$$-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)(4Bd-5Ae)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(-3aBe^2-5Acde+4B\ldots}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\ldots}}\ldots\right.}{15e^2}\ldots$$
$$\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(-5Ae+4Bd-3Bex)}{15e^2}$$

input
```
Int[((A + B*x)*Sqrt[a - c*x^2])/Sqrt[d + e*x],x]
```

output
```
(-2*Sqrt[d + e*x]*(4*B*d - 5*A*e - 3*B*e*x)*Sqrt[a - c*x^2])/(15*e^2) - (2
*((-2*Sqrt[a]*(4*B*c*d^2 - 5*A*c*d*e - 3*a*B*e^2)*Sqrt[d + e*x]*Sqrt[1 - (
c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/(
(Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d
+ Sqrt[a]*e)]*Sqrt[a - c*x^2]) + (2*Sqrt[a]*(4*B*d - 5*A*e)*(c*d^2 - a*e^2
)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]*El
lipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/
Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[a - c*x^2])))/(15*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(295) = 590$.

Time = 3.58 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.86

method	result
elliptic	$\frac{\sqrt{(ex+d)(-cx^2+a)}\left(\frac{2Bx\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}{5e}-\frac{2\left(-Ac+\frac{4cdB}{5e}\right)\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}{3ce}+\frac{2\left(Aa-\frac{2adB}{5e}+\frac{a\left(-Ac+\frac{4cdB}{5e}\right)}{3c}\right)\left(\frac{d}{e}-\frac{\sqrt{ac}}{c}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
risch	$\frac{2(3Bex+5Ae-4Bd)\sqrt{ex+d}\,\sqrt{-c\,x^2+a}}{15e^2}+\frac{2\left(\frac{\left(5Acde+3Ba\,e^2-4Bc\,d^2\right)\sqrt{ac}\,\sqrt{2}\sqrt{\frac{\left(x+\frac{\sqrt{ac}}{c}\right)c}{\sqrt{ac}}}\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{\sqrt{ac}}{c}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{ac}}{c}\right)c}{\sqrt{ac}}}\left(\left(\frac{d}{e}-\frac{\sqrt{ac}}{c}\right)\cdots\right)}{c\sqrt{-c\cdots}}\cdots\right)}{\cdots}$
default	Expression too large to display

input
```
int((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(2/5*B/e*x*(-c*e
*x^3-c*d*x^2+a*e*x+a*d)^(1/2)-2/3*(-A*c+4/5*c*d/e*B)/c/e*(-c*e*x^3-c*d*x^2
+a*e*x+a*d)^(1/2)+2*(A*a-2/5*a*d/e*B+1/3*a/c*(-A*c+4/5*c*d/e*B))*(d/e-1/c*
(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-
d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(
1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^
(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+2*(2/
5*B*a-2/3*d/e*(-A*c+4/5*c*d/e*B))*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*
(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x
+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*
d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))*EllipticE(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))
^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+1/c*(a*c)^(1
/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2)
)/(-d/e-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.75

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{\sqrt{d+ex}}\,dx = -\frac{2\left(2\left(4\,Bcd^3-5\,Acd^2e-6\,Bade^2+15\,Aae^3\right)\sqrt{-ce}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(cd^2+3\,ae^2)}{3\,ce^2},-\frac{8\,(cd^3-9\,ade^2)}{27\,ce^3},\frac{3\,e}{3}\right.\right.}{}$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(2*(4*B*c*d^3 - 5*A*c*d^2*e - 6*B*a*d*e^2 + 15*A*a*e^3)*sqrt(-c*e)*w
eierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2
)/(c*e^3), 1/3*(3*e*x + d)/e) + 6*(4*B*c*d^2*e - 5*A*c*d*e^2 - 3*B*a*e^3)*
sqrt(-c*e)*weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9
*a*d*e^2)/(c*e^3), weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/2
7*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e)) - 3*(3*B*c*e^3*x - 4*B*
c*d*e^2 + 5*A*c*e^3)*sqrt(-c*x^2 + a)*sqrt(e*x + d))/(c*e^4)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{\sqrt{d+ex}}\,dx = \int \frac{(A+Bx)\sqrt{a-cx^2}}{\sqrt{d+ex}}\,dx$$

input
```
integrate((B*x+A)*(-c*x**2+a)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Integral((A + B*x)*sqrt(a - c*x**2)/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{-cx^2+a}(Bx+A)}{\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^2 + a)*(B*x + A)/sqrt(e*x + d), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{-cx^2+a}(Bx+A)}{\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^2 + a)*(B*x + A)/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{a-c\,x^2}\,(A+B\,x)}{\sqrt{d+e\,x}}\,dx$$

input
```
int(((a - c*x^2)^(1/2)*(A + B*x))/(d + e*x)^(1/2),x)
```

output
```
int(((a - c*x^2)^(1/2)*(A + B*x))/(d + e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{\sqrt{d+ex}}\,dx$$
$$=\frac{-2\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\,abe+2\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\,bcdx-3\left(\int \frac{\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\,x^2}{-ce\,x^3-cd\,x^2+aex+ad}dx\right)abc\,e^2-5\left(\int \frac{\sqrt{e}}{-ce}\right.}{}$$

input
```
int((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
( - 2*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*e + 2*sqrt(d + e*x)*sqrt(a - c*x*
*2)*b*c*d*x - 3*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d + a*e*x - c
*d*x**2 - c*e*x**3),x)*a*b*c*e**2 - 5*int((sqrt(d + e*x)*sqrt(a - c*x**2)*
x**2)/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a*c**2*d*e + 4*int((sqrt(d +
e*x)*sqrt(a - c*x**2)*x**2)/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*b*c**2*
d**2 + int((sqrt(d + e*x)*sqrt(a - c*x**2))/(a*d + a*e*x - c*d*x**2 - c*e*
x**3),x)*a**2*b*e**2 + 5*int((sqrt(d + e*x)*sqrt(a - c*x**2))/(a*d + a*e*x
 - c*d*x**2 - c*e*x**3),x)*a**2*c*d*e - 2*int((sqrt(d + e*x)*sqrt(a - c*x*
*2))/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a*b*c*d**2)/(5*c*d*e)
```

3.264 $\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 346

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{3/2}}\,dx=\frac{2(4Bd-3Ae+Bex)\sqrt{a-cx^2}}{3e^2\sqrt{d+ex}}-\frac{4\sqrt{a}\sqrt{c}(4Bd-3Ae)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{3e^3\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}}+\frac{4\sqrt{a}(4Bcd^2-3Acde-aBe^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{3\sqrt{c}e^3\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
2/3*(B*e*x-3*A*e+4*B*d)*(-c*x^2+a)^(1/2)/e^2/(e*x+d)^(1/2)-4/3*a^(1/2)*c^(
1/2)*(-3*A*e+4*B*d)*(e*x+d)^(1/2)*(1-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/
2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2
))/e^3/(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)+4/3*
a^(1/2)*(-3*A*c*d*e-B*a*e^2+4*B*c*d^2)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)
*e))^(1/2)*(1-c*x^2/a)^(1/2)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(
1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/c^(1/2)/e^3/(e*x+d)^
(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.64 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{a-cx^2}\left(3Bd-3Ae+B(d+ex)-\frac{2\left(e^2(4Bd-3Ae)\sqrt{-d+\frac{\sqrt{a}e}{\sqrt{c}}}(a-cx^2)+i\sqrt{c}(\sqrt{c}d-\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((A + B*x)*Sqrt[a - c*x^2])/(d + e*x)^(3/2),x]
```

output
```
(2*Sqrt[a - c*x^2]*(3*B*d - 3*A*e + B*(d + e*x) - (2*(e^2*(4*B*d - 3*A*e)*
Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(a - c*x^2) + I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a
]*e)*(4*B*d - 3*A*e)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sq
rt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sq
rt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt
[c]*d - Sqrt[a]*e)] - I*Sqrt[a]*e*(-4*B*Sqrt[c]*d + Sqrt[a]*B*e + 3*A*Sqrt
[c]*e)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[
c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d + (Sqrt[
a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a
]*e)]))/(e^2*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(a - c*x^2))))/(3*e^2*Sqrt[d +
 e*x])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {681, 25, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^2}(A+Bx)}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{681}$$

$$\frac{2\sqrt{a-cx^2}(-3Ae+4Bd+Bex)}{3e^2\sqrt{d+ex}}-\frac{2\int -\frac{aBe+c(4Bd-3Ae)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{3e^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{aBe+c(4Bd-3Ae)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{3e^2}+\frac{2\sqrt{a-cx^2}(-3Ae+4Bd+Bex)}{3e^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{600}$$

$$\frac{2\left(\frac{(aBe^2-cd(4Bd-3Ae))\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}+\frac{c(4Bd-3Ae)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}\right)}{3e^2}+\frac{2\sqrt{a-cx^2}(-3Ae+4Bd+Bex)}{3e^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{509}$$

$$\frac{2\left(\frac{(aBe^2-cd(4Bd-3Ae))\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}+\frac{c\sqrt{1-\frac{cx^2}{a}}(4Bd-3Ae)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}\right)}{3e^2}+\frac{2\sqrt{a-cx^2}(-3Ae+4Bd+Bex)}{3e^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{508}$$

$$\frac{2\left(\frac{(aBe^2-cd(4Bd-3Ae))\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(4Bd-3Ae)\int \frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{3e^2}+\frac{2\sqrt{a-cx^2}(-3Ae+4Bd+Bex)}{3e^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{327}$$

$$\frac{2\left(\frac{\left(aBe^2-cd(4Bd-3Ae)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(4Bd-3Ae)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{3e^2}+\frac{2\sqrt{a-cx^2}(-3Ae+4Bd+Bex)}{3e^2\sqrt{d+ex}}$$

$$\Big\downarrow \; 512$$

$$\frac{2\left(\frac{\sqrt{1-\frac{cx^2}{a}}\left(aBe^2-cd(4Bd-3Ae)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(4Bd-3Ae)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{3e^2}+\frac{2\sqrt{a-cx^2}(-3Ae+4Bd+Bex)}{3e^2\sqrt{d+ex}}$$

$$\Big\downarrow \; 511$$

$$\frac{2\left(-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(aBe^2-cd(4Bd-3Ae)\right)\int \frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(4Bd-3Ae)E}{e\sqrt{a-cx^2}\sqrt{\cdots}}\cdots}{3e^2}$$

$$\frac{2\sqrt{a-cx^2}(-3Ae+4Bd+Bex)}{3e^2\sqrt{d+ex}}$$

$$\Big\downarrow \; 321$$

$$\frac{2\left(-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(aBe^2-cd(4Bd-3Ae)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(4Bd-3Ae)E\left(\text{ar}\cdots\right.}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+\cdots}{\sqrt{a}e+\cdots}}}\cdots}{3e^2}$$

$$\frac{2\sqrt{a-cx^2}(-3Ae+4Bd+Bex)}{3e^2\sqrt{d+ex}}$$

input
```
Int[((A + B*x)*Sqrt[a - c*x^2])/(d + e*x)^(3/2),x]
```

output
```
(2*(4*B*d - 3*A*e + B*e*x)*Sqrt[a - c*x^2])/(3*e^2*Sqrt[d + e*x]) + (2*((-
2*Sqrt[a]*Sqrt[c]*(4*B*d - 3*A*e)*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*Ellipt
icE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt
[a] + e)])/(e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c
*x^2]) - (2*Sqrt[a]*(a*B*e^2 - c*d*(4*B*d - 3*A*e))*Sqrt[(Sqrt[c]*(d + e*x
))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 -
(Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*
e*Sqrt[d + e*x]*Sqrt[a - c*x^2])))/(3*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(282) = 564$.

Time = 6.83 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.97

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c\,x^2+a)}\left(-\frac{2\left(-ce\,x^2+ae\right)(Ae-Bd)}{e^3\sqrt{\left(x+\frac{d}{e}\right)\left(-ce\,x^2+ae\right)}}+\frac{2B\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}{3e^2}+\frac{2\left(\frac{Acde+Ba\,e^2-Bc\,d^2}{e^3}-\frac{(Ae-Bd)cd}{e^3}-\frac{Ba}{3e}\right)\left(\frac{d}{e}-\frac{\sqrt{ac}}{c}\right)\sqrt{}}{\sqrt{-}}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(-2*(-c*e*x^2+a*
e)*(A*e-B*d)/e^3/((x+d/e)*(-c*e*x^2+a*e))^(1/2)+2/3*B/e^2*(-c*e*x^3-c*d*x^
2+a*e*x+a*d)^(1/2)+2*((A*c*d*e+B*a*e^2-B*c*d^2)/e^3-(A*e-B*d)*c/e^3*d-1/3*
B*a/e)*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c
*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c
*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*EllipticF(((x+d/e)
/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2
)))^(1/2))+2*(-2*(A*e-B*d)*c/e^2+2/3*d/e^2*B*c)*(d/e-1/c*(a*c)^(1/2))*((x+
d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/
2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*
d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))*EllipticE(((x+d/e)/(d/e-1/c
*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)
)+1/c*(a*c)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1
/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{3/2}}\,dx = \frac{2\left(2\,(4\,Bcd^3-3\,Acd^2e-3\,Bade^2+(4\,Bcd^2e-3\,Acde^2-3\,Bae^3)x)\sqrt{-ce}\mathrm{we}\right.}{}$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
2/9*(2*(4*B*c*d^3 - 3*A*c*d^2*e - 3*B*a*d*e^2 + (4*B*c*d^2*e - 3*A*c*d*e^2
 - 3*B*a*e^3)*x)*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e
^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 6*(4*B*c*d^2*
e - 3*A*c*d*e^2 + (4*B*c*d*e^2 - 3*A*c*e^3)*x)*sqrt(-c*e)*weierstrassZeta(
4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), weierstr
assPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^
3), 1/3*(3*e*x + d)/e)) + 3*(B*c*e^3*x + 4*B*c*d*e^2 - 3*A*c*e^3)*sqrt(-c*
x^2 + a)*sqrt(e*x + d))/(c*e^5*x + c*d*e^4)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{3/2}}\,dx = \int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x**2+a)**(1/2)/(e*x+d)**(3/2),x)
```

output
```
Integral((A + B*x)*sqrt(a - c*x**2)/(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{-cx^2+a}(Bx+A)}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^2 + a)*(B*x + A)/(e*x + d)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{-cx^2+a}(Bx+A)}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^2 + a)*(B*x + A)/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{a-c\,x^2}\,(A+B\,x)}{(d+e\,x)^{3/2}}\,dx$$

input
```
int(((a - c*x^2)^(1/2)*(A + B*x))/(d + e*x)^(3/2),x)
```

output
```
int(((a - c*x^2)^(1/2)*(A + B*x))/(d + e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(3/2),x)
```

output

```
( - 2*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*e + 2*sqrt(d + e*x)*sqrt(a - c*x*
*2)*b*c*d*x - int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**2 + 2*a*d*e*
x + a*e**2*x**2 - c*d**2*x**2 - 2*c*d*e*x**3 - c*e**2*x**4),x)*a*b*c*d*e**
2 - int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**2 + 2*a*d*e*x + a*e**2
*x**2 - c*d**2*x**2 - 2*c*d*e*x**3 - c*e**2*x**4),x)*a*b*c*e**3*x - 3*int(
(sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**2 + 2*a*d*e*x + a*e**2*x**2 -
c*d**2*x**2 - 2*c*d*e*x**3 - c*e**2*x**4),x)*a*c**2*d**2*e - 3*int((sqrt(d
 + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**2 + 2*a*d*e*x + a*e**2*x**2 - c*d**2*
x**2 - 2*c*d*e*x**3 - c*e**2*x**4),x)*a*c**2*d*e**2*x + 4*int((sqrt(d + e*
x)*sqrt(a - c*x**2)*x**2)/(a*d**2 + 2*a*d*e*x + a*e**2*x**2 - c*d**2*x**2
- 2*c*d*e*x**3 - c*e**2*x**4),x)*b*c**2*d**3 + 4*int((sqrt(d + e*x)*sqrt(a
 - c*x**2)*x**2)/(a*d**2 + 2*a*d*e*x + a*e**2*x**2 - c*d**2*x**2 - 2*c*d*e
*x**3 - c*e**2*x**4),x)*b*c**2*d**2*e*x - int((sqrt(d + e*x)*sqrt(a - c*x*
*2))/(a*d**2 + 2*a*d*e*x + a*e**2*x**2 - c*d**2*x**2 - 2*c*d*e*x**3 - c*e*
*2*x**4),x)*a**2*b*d*e**2 - int((sqrt(d + e*x)*sqrt(a - c*x**2))/(a*d**2 +
 2*a*d*e*x + a*e**2*x**2 - c*d**2*x**2 - 2*c*d*e*x**3 - c*e**2*x**4),x)*a*
*2*b*e**3*x + 3*int((sqrt(d + e*x)*sqrt(a - c*x**2))/(a*d**2 + 2*a*d*e*x +
 a*e**2*x**2 - c*d**2*x**2 - 2*c*d*e*x**3 - c*e**2*x**4),x)*a**2*c*d**2*e
+ 3*int((sqrt(d + e*x)*sqrt(a - c*x**2))/(a*d**2 + 2*a*d*e*x + a*e**2*x**2
 - c*d**2*x**2 - 2*c*d*e*x**3 - c*e**2*x**4),x)*a**2*c*d*e**2*x - 2*int...
```

3.265 $\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 424

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{5/2}}\,dx = -\frac{4(4Bcd^2-Acde-3aBe^2)\sqrt{a-cx^2}}{3e^2(cd^2-ae^2)\sqrt{d+ex}} + \frac{2(4Bd-Ae+3Bex)\sqrt{a-cx^2}}{3e^2(d+ex)^{3/2}} + \frac{4\sqrt{a}\sqrt{c}(4Bcd^2-Acde-3aBe^2)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{3e^3(cd^2-ae^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} - \frac{4\sqrt{a}\sqrt{c}(4Bd-Ae)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{3e^3\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
-4/3*(-A*c*d*e-3*B*a*e^2+4*B*c*d^2)*(-c*x^2+a)^(1/2)/e^2/(-a*e^2+c*d^2)/(e
*x+d)^(1/2)+2/3*(3*B*e*x-A*e+4*B*d)*(-c*x^2+a)^(1/2)/e^2/(e*x+d)^(3/2)+4/3
*a^(1/2)*c^(1/2)*(-A*c*d*e-3*B*a*e^2+4*B*c*d^2)*(e*x+d)^(1/2)*(1-c*x^2/a)^
(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e
/(c^(1/2)*d+a^(1/2)*e))^(1/2))/e^3/(-a*e^2+c*d^2)/(c^(1/2)*(e*x+d)/(c^(1/2
)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)-4/3*a^(1/2)*c^(1/2)*(-A*e+4*B*d)*(c
^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2)*EllipticF(1/
2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2
)*e))^(1/2))/e^3/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.86 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{5/2}}\,dx = \frac{2\sqrt{a-cx^2}\left(3Bcd^2-3aBe^2+\frac{(Bd-Ae)(cd^2-ae^2)}{d+ex}+\frac{2i\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)\left(-4Bcd^2+Acde+3a\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[((A + B*x)*Sqrt[a - c*x^2])/(d + e*x)^(5/2),x]
```

output
```
(2*Sqrt[a - c*x^2]*(3*B*c*d^2 - 3*a*B*e^2 + ((B*d - A*e)*(c*d^2 - a*e^2))/
(d + e*x) + ((2*I)*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(-4*B*c*d^2 + A*c*d*e +
 3*a*B*e^2)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/
Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d + (
Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - S
qrt[a]*e)])/(e^2*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2)) + ((2*I)*Sqr
t[a]*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(-4*B*Sqrt[c]*d - 3*Sqrt[a]*B*e + A*S
qrt[c]*e)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sq
rt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d + (Sq
rt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqr
t[a]*e)])/(e*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2))))/(3*e^2*(c*d^2
- a*e^2)*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {680, 25, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^2}(A+Bx)}{(d+ex)^{5/2}} \, dx$$

$$\Big\downarrow \text{680}$$

$$\frac{2\int -\frac{c(ae(Bd-Ae)+(4Bcd^2-Aced-3aBe^2)x)}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{3e^2(cd^2-ae^2)}-\frac{2\sqrt{a-cx^2}\left(ex\left(-3aBe^2-2Acde+5Bcd^2\right)-aAe^3-2aBde^2-Acd^2e+4Bcd^3\right)}{3e^2(d+ex)^{3/2}(cd^2-ae^2)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{c(ae(Bd-Ae)+(4Bcd^2-Aced-3aBe^2)x)}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{3e^2(cd^2-ae^2)}-\frac{2\sqrt{a-cx^2}\left(ex\left(-3aBe^2-2Acde+5Bcd^2\right)-aAe^3-2aBde^2-Acd^2e+4Bcd^3\right)}{3e^2(d+ex)^{3/2}(cd^2-ae^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2c\int \frac{ae(Bd-Ae)+(4Bcd^2-Aced-3aBe^2)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{3e^2(cd^2-ae^2)}-\frac{2\sqrt{a-cx^2}\left(ex\left(-3aBe^2-2Acde+5Bcd^2\right)-aAe^3-2aBde^2-Acd^2e+4Bcd^3\right)}{3e^2(d+ex)^{3/2}(cd^2-ae^2)}$$

$$\Big\downarrow \text{600}$$

$$-\frac{2c\left(\frac{(-3aBe^2-Acde+4Bcd^2)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{(cd^2-ae^2)(4Bd-Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{3e^2(cd^2-ae^2)}-\frac{2\sqrt{a-cx^2}\left(ex\left(-3aBe^2-2Acde+5Bcd^2\right)-aAe^3-2aBde^2-Acd^2e+4Bcd^3\right)}{3e^2(d+ex)^{3/2}(cd^2-ae^2)}$$

$$\Big\downarrow \text{509}$$

$$-\frac{2c\left(\frac{\sqrt{1-\frac{cx^2}{a}}\left(-3aBe^2-Acde+4Bcd^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{\left(cd^2-ae^2\right)(4Bd-Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{3e^2\left(cd^2-ae^2\right)}-\frac{2\sqrt{a-cx^2}\left(ex\left(-3aBe^2-2Acde+5Bcd^2\right)-aAe^3-2aBde^2-Acd^2e+4Bcd^3\right)}{3e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$\big\downarrow$ 508

$$-\frac{2c\left(-\frac{\left(cd^2-ae^2\right)(4Bd-Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(-3aBe^2-Acde+4Bcd^2\right)\int \frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{3e^2\left(cd^2-ae^2\right)}-\frac{2\sqrt{a-cx^2}\left(ex\left(-3aBe^2-2Acde+5Bcd^2\right)-aAe^3-2aBde^2-Acd^2e+4Bcd^3\right)}{3e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$\big\downarrow$ 327

$$-\frac{2c\left(-\frac{\left(cd^2-ae^2\right)(4Bd-Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(-3aBe^2-Acde+4Bcd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{3e^2\left(cd^2-ae^2\right)}-\frac{2\sqrt{a-cx^2}\left(ex\left(-3aBe^2-2Acde+5Bcd^2\right)-aAe^3-2aBde^2-Acd^2e+4Bcd^3\right)}{3e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$\big\downarrow$ 512

$$-\frac{2c\left(-\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)(4Bd-Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(-3aBe^2-Acde+4Bcd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{3e^2\left(cd^2-ae^2\right)}-\frac{2\sqrt{a-cx^2}\left(ex\left(-3aBe^2-2Acde+5Bcd^2\right)-aAe^3-2aBde^2-Acd^2e+4Bcd^3\right)}{3e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$\big\downarrow$ 511

$$-\frac{2c\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)(4Bd-Ae)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\int\frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(-3aBe^2-Acde+}{\sqrt{c}e\sqrt{a-c}}\right.}{3e^2(cd^2-ae^2)}$$

$$\frac{2\sqrt{a-cx^2}\left(ex(-3aBe^2-2Acde+5Bcd^2)-aAe^3-2aBde^2-Acd^2e+4Bcd^3\right)}{3e^2(d+ex)^{3/2}(cd^2-ae^2)}$$

$\downarrow$ 321

$$-\frac{2c\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)(4Bd-Ae)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(-3aBe^2-Acde+4Bc}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{}}\right.}{3e^2(cd^2-ae^2)}$$

$$\frac{2\sqrt{a-cx^2}\left(ex(-3aBe^2-2Acde+5Bcd^2)-aAe^3-2aBde^2-Acd^2e+4Bcd^3\right)}{3e^2(d+ex)^{3/2}(cd^2-ae^2)}$$

input

```
Int[((A + B*x)*Sqrt[a - c*x^2])/(d + e*x)^(5/2),x]
```

output

```
(-2*(4*B*c*d^3 - A*c*d^2*e - 2*a*B*d*e^2 - a*A*e^3 + e*(5*B*c*d^2 - 2*A*c*
d*e - 3*a*B*e^2)*x)*Sqrt[a - c*x^2])/(3*e^2*(c*d^2 - a*e^2)*(d + e*x)^(3/2
)) - (2*c*((-2*Sqrt[a]*(4*B*c*d^2 - A*c*d*e - 3*a*B*e^2)*Sqrt[d + e*x]*Sqr
t[1 - (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]],
(2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqr
t[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) + (2*Sqrt[a]*(4*B*d - A*e)*(c*d^2 -
a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/
a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c
]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[a - c*x^2])))/(3*e^2*(c*
d^2 - a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(354) = 708$.

Time = 4.40 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.83

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c x^2+a)}\left(-\frac{2(Ae-Bd)\sqrt{-ce x^3-cd x^2+aex+ad}}{3e^4\left(x+\frac{d}{e}\right)^2}-\frac{2\left(-ce x^2+ae\right)\left(2Acde+3Ba e^2-5Bc d^2\right)}{3e^3\left(a e^2-c d^2\right)\sqrt{\left(x+\frac{d}{e}\right)\left(-ce x^2+ae\right)}}+\frac{2\left(-\frac{c(Ae-2Bd)}{e^3}+\frac{(Ae-Bd)c}{3e^3}-\frac{cd}{}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(-2/3*(A*e-B*d)/
e^4*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)/(x+d/e)^2-2/3*(-c*e*x^2+a*e)/e^3/(a
*e^2-c*d^2)*(2*A*c*d*e+3*B*a*e^2-5*B*c*d^2)/((x+d/e)*(-c*e*x^2+a*e))^(1/2)
+2*(-c*(A*e-2*B*d)/e^3+1/3*(A*e-B*d)*c/e^3-1/3*c/e^3*d*(2*A*c*d*e+3*B*a*e^
2-5*B*c*d^2)/(a*e^2-c*d^2))*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^
(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(
a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/
2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))
/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+2*(-B*c/e^2-1/3*c/e^2*(2*A*c*d*e+3*B*a*e^2
-5*B*c*d^2)/(a*e^2-c*d^2))*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(
1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a
*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2
)*((-d/e-1/c*(a*c)^(1/2))*EllipticE(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),
((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+1/c*(a*c)^(1/2)*Ell
ipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e
-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{5/2}}\,dx =$$

$$-\frac{2\left(2\left(4\,Bcd^5-Acd^4e-6\,Bad^3e^2+3\,Aad^2e^3+(4\,Bcd^3e^2-Acd^2e^3-6\,Bade^4+3\,Aae^5)x^2+2\left(4\,Bcd^4e\right.\right.\right.}{}$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
-2/9*(2*(4*B*c*d^5 - A*c*d^4*e - 6*B*a*d^3*e^2 + 3*A*a*d^2*e^3 + (4*B*c*d^
3*e^2 - A*c*d^2*e^3 - 6*B*a*d*e^4 + 3*A*a*e^5)*x^2 + 2*(4*B*c*d^4*e - A*c*
d^3*e^2 - 6*B*a*d^2*e^3 + 3*A*a*d*e^4)*x)*sqrt(-c*e)*weierstrassPInverse(4
/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*
x + d)/e) + 6*(4*B*c*d^4*e - A*c*d^3*e^2 - 3*B*a*d^2*e^3 + (4*B*c*d^2*e^3
- A*c*d*e^4 - 3*B*a*e^5)*x^2 + 2*(4*B*c*d^3*e^2 - A*c*d^2*e^3 - 3*B*a*d*e^
4)*x)*sqrt(-c*e)*weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d
^3 - 9*a*d*e^2)/(c*e^3), weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2)
, -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e)) + 3*(4*B*c*d^3*e^
2 - A*c*d^2*e^3 - 2*B*a*d*e^4 - A*a*e^5 + (5*B*c*d^2*e^3 - 2*A*c*d*e^4 - 3
*B*a*e^5)*x)*sqrt(-c*x^2 + a)*sqrt(e*x + d))/(c*d^4*e^4 - a*d^2*e^6 + (c*d
^2*e^6 - a*e^8)*x^2 + 2*(c*d^3*e^5 - a*d*e^7)*x)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{5/2}}\,dx = \int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x**2+a)**(1/2)/(e*x+d)**(5/2),x)
```

output
```
Integral((A + B*x)*sqrt(a - c*x**2)/(d + e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{5/2}}\,dx = \int \frac{\sqrt{-cx^2+a}(Bx+A)}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^2 + a)*(B*x + A)/(e*x + d)^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{5/2}}\,dx = \int \frac{\sqrt{-cx^2+a}(Bx+A)}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^2 + a)*(B*x + A)/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{5/2}}\,dx = \int \frac{\sqrt{a-c\,x^2}\,(A+B\,x)}{(d+e\,x)^{5/2}}\,dx$$

input
```
int(((a - c*x^2)^(1/2)*(A + B*x))/(d + e*x)^(5/2),x)
```

output
```
int(((a - c*x^2)^(1/2)*(A + B*x))/(d + e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(5/2),x)
```

output
```
( - 2*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*e + 2*sqrt(d + e*x)*sqrt(a - c*x*
*2)*b*c*d*x + int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**3 + 3*a*d**2
*e*x + 3*a*d*e**2*x**2 + a*e**3*x**3 - c*d**3*x**2 - 3*c*d**2*e*x**3 - 3*c
*d*e**2*x**4 - c*e**3*x**5),x)*a*b*c*d**2*e**2 + 2*int((sqrt(d + e*x)*sqrt
(a - c*x**2)*x**2)/(a*d**3 + 3*a*d**2*e*x + 3*a*d*e**2*x**2 + a*e**3*x**3
- c*d**3*x**2 - 3*c*d**2*e*x**3 - 3*c*d*e**2*x**4 - c*e**3*x**5),x)*a*b*c*
d*e**3*x + int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**3 + 3*a*d**2*e*
x + 3*a*d*e**2*x**2 + a*e**3*x**3 - c*d**3*x**2 - 3*c*d**2*e*x**3 - 3*c*d*
e**2*x**4 - c*e**3*x**5),x)*a*b*c*e**4*x**2 - int((sqrt(d + e*x)*sqrt(a -
c*x**2)*x**2)/(a*d**3 + 3*a*d**2*e*x + 3*a*d*e**2*x**2 + a*e**3*x**3 - c*d
**3*x**2 - 3*c*d**2*e*x**3 - 3*c*d*e**2*x**4 - c*e**3*x**5),x)*a*c**2*d**3
*e - 2*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**3 + 3*a*d**2*e*x +
3*a*d*e**2*x**2 + a*e**3*x**3 - c*d**3*x**2 - 3*c*d**2*e*x**3 - 3*c*d*e**2
*x**4 - c*e**3*x**5),x)*a*c**2*d**2*e**2*x - int((sqrt(d + e*x)*sqrt(a - c
*x**2)*x**2)/(a*d**3 + 3*a*d**2*e*x + 3*a*d*e**2*x**2 + a*e**3*x**3 - c*d*
*3*x**2 - 3*c*d**2*e*x**3 - 3*c*d*e**2*x**4 - c*e**3*x**5),x)*a*c**2*d*e**
3*x**2 + 4*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**3 + 3*a*d**2*e*
x + 3*a*d*e**2*x**2 + a*e**3*x**3 - c*d**3*x**2 - 3*c*d**2*e*x**3 - 3*c*d*
e**2*x**4 - c*e**3*x**5),x)*b*c**2*d**4 + 8*int((sqrt(d + e*x)*sqrt(a - c*
x**2)*x**2)/(a*d**3 + 3*a*d**2*e*x + 3*a*d*e**2*x**2 + a*e**3*x**3 - c*...
```

3.266 $\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 529

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{7/2}}\,dx = \frac{4(4Bcd^2+Acde-5aBe^2)\sqrt{a-cx^2}}{15e^2(cd^2-ae^2)(d+ex)^{3/2}} + \frac{4c(4Bcd^3+Acd^2e-8aBde^2+3aAe^3)\sqrt{a-cx^2}}{15e^2(cd^2-ae^2)^2\sqrt{d+ex}} - \frac{2(4Bd+Ae+5Bex)\sqrt{a-cx^2}}{5e^2(d+ex)^{5/2}} - \frac{4\sqrt{a}c^{3/2}(4Bcd^3+Acd^2e-8aBde^2+3aAe^3)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{15e^3(cd^2-ae^2)^2\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} + \frac{4\sqrt{a}\sqrt{c}(4Bcd^2+Acde-5aBe^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{15e^3(cd^2-ae^2)\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
4/15*(A*c*d*e-5*B*a*e^2+4*B*c*d^2)*(-c*x^2+a)^(1/2)/e^2/(-a*e^2+c*d^2)/(e*
x+d)^(3/2)+4/15*c*(3*A*a*e^3+A*c*d^2*e-8*B*a*d*e^2+4*B*c*d^3)*(-c*x^2+a)^(
1/2)/e^2/(-a*e^2+c*d^2)^2/(e*x+d)^(1/2)-2/5*(5*B*e*x+A*e+4*B*d)*(-c*x^2+a)
^(1/2)/e^2/(e*x+d)^(5/2)-4/15*a^(1/2)*c^(3/2)*(3*A*a*e^3+A*c*d^2*e-8*B*a*d
*e^2+4*B*c*d^3)*(e*x+d)^(1/2)*(1-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x
/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/e
^3/(-a*e^2+c*d^2)^2/(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+
a)^(1/2)+4/15*a^(1/2)*c^(1/2)*(A*c*d*e-5*B*a*e^2+4*B*c*d^2)*(c^(1/2)*(e*x+
d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2)*EllipticF(1/2*(1-c^(1/2)
*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))
/e^3/(-a*e^2+c*d^2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.13 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{7/2}}\,dx = \frac{2\sqrt{a-cx^2}\left(\frac{3(Bd-Ae)\left(cd^2-ae^2\right)^2}{(d+ex)^2}-\frac{\left(cd^2-ae^2\right)\left(7Bcd^2-2Acde-5aBe^2\right)}{d+ex}+\frac{2ic^{3/2}\left(\sqrt{c}d-\sqrt{a}e\right)(4}{}\right.}{}$$

input
```
Integrate[((A + B*x)*Sqrt[a - c*x^2])/(d + e*x)^(7/2),x]
```

output
```
(2*Sqrt[a - c*x^2]*((3*(B*d - A*e)*(c*d^2 - a*e^2)^2)/(d + e*x)^2 - ((c*d^
2 - a*e^2)*(7*B*c*d^2 - 2*A*c*d*e - 5*a*B*e^2))/(d + e*x) + ((2*I)*c^(3/2)
*(Sqrt[c]*d - Sqrt[a]*e)*(4*B*c*d^3 + A*c*d^2*e - 8*a*B*d*e^2 + 3*a*A*e^3)
*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e
*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/
Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)])
/(e^2*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2)) + ((2*I)*Sqrt[a]*c*(Sqr
t[c]*d - Sqrt[a]*e)*(4*B*c*d^2 + 3*Sqrt[a]*B*Sqrt[c]*d*e + A*c*d*e - 5*a*B
*e^2 - 3*Sqrt[a]*A*Sqrt[c]*e^2)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*
Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I
*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[
a]*e)/(Sqrt[c]*d - Sqrt[a]*e)])/(e*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*
x^2))))/(15*(c*d^2*e - a*e^3)^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 524, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {680, 25, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^2}(A+Bx)}{(d+ex)^{7/2}}dx$$

$$\Big\downarrow \text{680}$$

$$\frac{2\int -\frac{c(3ae(Bd-Ae)+(4Bcd^2+Aced-5aBe^2)x)}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{15e^2(cd^2-ae^2)}-\frac{2\sqrt{a-cx^2}\left(ex\left(-5aBe^2-2Acde+7Bcd^2\right)-3aAe^3-2aBde^2+Acd^2e+4Bcd^3\right)}{15e^2(d+ex)^{5/2}(cd^2-ae^2)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{c(3ae(Bd-Ae)+(4Bcd^2+Aced-5aBe^2)x)}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{15e^2(cd^2-ae^2)}-\frac{2\sqrt{a-cx^2}\left(ex\left(-5aBe^2-2Acde+7Bcd^2\right)-3aAe^3-2aBde^2+Acd^2e+4Bcd^3\right)}{15e^2(d+ex)^{5/2}(cd^2-ae^2)}$$

$$\Big\downarrow \ 27$$

$$-\frac{2c\int \frac{3ae(Bd-Ae)+\left(4Bcd^2+Aced-5aBe^2\right)x}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{15e^2\left(cd^2-ae^2\right)}-\frac{2\sqrt{a-cx^2}\left(ex\left(-5aBe^2-2Acde+7Bcd^2\right)-3aAe^3-2aBde^2+Acd^2e+4Bcd^3\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 688$$

$$-\frac{2c\left(\frac{2\int -\frac{ae\left(Bcd^2+4Aced-5aBe^2\right)+c\left(4Bcd^3+Aced^2-8aBe^2d+3aAe^3\right)x}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\left(3aAe^3-8aBde^2+Acd^2e+4Bcd^3\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{15e^2\left(cd^2-ae^2\right)}-\frac{2\sqrt{a-cx^2}\left(ex\left(-5aBe^2-2Acde+7Bcd^2\right)-3aAe^3-2aBde^2+Acd^2e+4Bcd^3\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 27$$

$$-\frac{2c\left(-\frac{\int \frac{ae\left(Bcd^2+4Aced-5aBe^2\right)+c\left(4Bcd^3+Aced^2-8aBe^2d+3aAe^3\right)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\left(3aAe^3-8aBde^2+Acd^2e+4Bcd^3\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{15e^2\left(cd^2-ae^2\right)}-\frac{2\sqrt{a-cx^2}\left(ex\left(-5aBe^2-2Acde+7Bcd^2\right)-3aAe^3-2aBde^2+Acd^2e+4Bcd^3\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 600$$

$$-\frac{2c\left(-\frac{\frac{c\left(3aAe^3-8aBde^2+Acd^2e+4Bcd^3\right)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{\left(cd^2-ae^2\right)\left(-5aBe^2+Acde+4Bcd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\left(3aAe^3-8aBde^2+\cdots\right.}{\sqrt{d+ex}\left(cd^2-ae\cdots\right.}\right.}{15e^2\left(cd^2-ae^2\right)}-\frac{2\sqrt{a-cx^2}\left(ex\left(-5aBe^2-2Acde+7Bcd^2\right)-3aAe^3-2aBde^2+Acd^2e+4Bcd^3\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 509$$

$$-\frac{2c\left(-\frac{\frac{c\sqrt{1-\frac{cx^2}{a}}\left(3aAe^3-8aBde^2+Acd^2e+4Bcd^3\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{\left(cd^2-ae^2\right)\left(-5aBe^2+Acde+4Bcd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}(3aAe^3-8}{\sqrt{d+ex}}\right.}{15e^2\left(cd^2-ae^2\right)}$$

$$\frac{2\sqrt{a-cx^2}\left(ex\left(-5aBe^2-2Acde+7Bcd^2\right)-3aAe^3-2aBde^2+Acd^2e+4Bcd^3\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{508}$$

$$-\frac{2c\left(-\frac{-\frac{\left(cd^2-ae^2\right)\left(-5aBe^2+Acde+4Bcd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(3aAe^3-8aBde^2+Acd^2e+4Bcd^3\right)\int\frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\sqrt{1-}}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{cd^2-ae^2}\right.}{15e^2\left(cd^2-ae^2\right)}$$

$$\frac{2\sqrt{a-cx^2}\left(ex\left(-5aBe^2-2Acde+7Bcd^2\right)-3aAe^3-2aBde^2+Acd^2e+4Bcd^3\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{327}$$

$$-\frac{2c\left(-\frac{-\frac{\left(cd^2-ae^2\right)\left(-5aBe^2+Acde+4Bcd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(3aAe^3-8aBde^2+Acd^2e+4Bcd^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\right.}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{cd^2-ae^2}\right.}{15e^2\left(cd^2-ae^2\right)}$$

$$\frac{2\sqrt{a-cx^2}\left(ex\left(-5aBe^2-2Acde+7Bcd^2\right)-3aAe^3-2aBde^2+Acd^2e+4Bcd^3\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{512}$$

$$-\frac{2c\left(-\frac{-\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\left(-5aBe^2+Acde+4Bcd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(3aAe^3-8aBde^2+Acd^2e+4Bcd^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\cdots}}{\sqrt{\cdots}}\cdots\right)\cdots\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\cdots}{cd^2-ae^2}\cdots\right)}{15e^2\left(cd^2-ae^2\right)\cdots}$$

$$\frac{2\sqrt{a-cx^2}\left(ex\left(-5aBe^2-2Acde+7Bcd^2\right)-3aAe^3-2aBde^2+Acd^2e+4Bcd^3\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 511$$

$$-\frac{2c\left(-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(-5aBe^2+Acde+4Bcd^2\right)\int \frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(3aAe^3\cdots\right.}{\cdots}}{cd^2-ae^2}\cdots\right)}{15e^2\left(cd^2-ae^2\right)\cdots}$$

$$\frac{2\sqrt{a-cx^2}\left(ex\left(-5aBe^2-2Acde+7Bcd^2\right)-3aAe^3-2aBde^2+Acd^2e+4Bcd^3\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 321$$

$$-\frac{2c\left(-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(-5aBe^2+Acde+4Bcd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(3aAe^3-8aBde\cdots\right.}{e\sqrt{a}\cdots}}{cd^2-ae^2}\cdots\right)}{15e^2\left(cd^2-ae^2\right)\cdots}$$

$$\frac{2\sqrt{a-cx^2}\left(ex\left(-5aBe^2-2Acde+7Bcd^2\right)-3aAe^3-2aBde^2+Acd^2e+4Bcd^3\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}$$

input

```
Int[((A + B*x)*Sqrt[a - c*x^2])/(d + e*x)^(7/2),x]
```

output
```
(-2*(4*B*c*d^3 + A*c*d^2*e - 2*a*B*d*e^2 - 3*a*A*e^3 + e*(7*B*c*d^2 - 2*A*
c*d*e - 5*a*B*e^2)*x)*Sqrt[a - c*x^2])/(15*e^2*(c*d^2 - a*e^2)*(d + e*x)^(
5/2)) - (2*c*((-2*(4*B*c*d^3 + A*c*d^2*e - 8*a*B*d*e^2 + 3*a*A*e^3)*Sqrt[a
 - c*x^2])/((c*d^2 - a*e^2)*Sqrt[d + e*x]) - ((-2*Sqrt[a]*Sqrt[c]*(4*B*c*d
^3 + A*c*d^2*e - 8*a*B*d*e^2 + 3*a*A*e^3)*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a
]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]
*d)/Sqrt[a] + e)])/(e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sq
rt[a - c*x^2]) + (2*Sqrt[a]*(c*d^2 - a*e^2)*(4*B*c*d^2 + A*c*d*e - 5*a*B*e
^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]*
EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d
)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[a - c*x^2]))/(c*d^2 - a*e^2
)))/(15*e^2*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 5.62 (sec) , antiderivative size = 890, normalized size of antiderivative = 1.68

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c\,x^2+a)}\left(-\frac{2(Ae-Bd)\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}{5e^5\left(x+\frac{d}{e}\right)^3}-\frac{2\left(2Acde+5Ba\,e^2-7Bc\,d^2\right)\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}{15\left(a\,e^2-c\,d^2\right)e^4\left(x+\frac{d}{e}\right)^2}+\frac{4\left(-ce\,x^2+ae\right)c\left(3Aa\,e\right.}{15e^3\left(a\,e^2-c\,d^2\right)^2}\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(-2/5*(A*e-B*d)/
e^5*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)/(x+d/e)^3-2/15*(2*A*c*d*e+5*B*a*e^2
-7*B*c*d^2)/(a*e^2-c*d^2)/e^4*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)/(x+d/e)^2
+4/15*(-c*e*x^2+a*e)/e^3/(a*e^2-c*d^2)^2*c*(3*A*a*e^3+A*c*d^2*e-8*B*a*d*e^
2+4*B*c*d^3)/((x+d/e)*(-c*e*x^2+a*e))^(1/2)+2*(-B*c/e^3+1/15*c*(2*A*c*d*e+
5*B*a*e^2-7*B*c*d^2)/e^3/(a*e^2-c*d^2)+2/15*c^2/e^3*d*(3*A*a*e^3+A*c*d^2*e
-8*B*a*d*e^2+4*B*c*d^3)/(a*e^2-c*d^2)^2)*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d
/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1
/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a
*e*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c
*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+4/15*c^2/e^2*(3*A*a*e^3+A*c*d
^2*e-8*B*a*d*e^2+4*B*c*d^3)/(a*e^2-c*d^2)^2*(d/e-1/c*(a*c)^(1/2))*((x+d/e)
/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))
^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^
2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))*EllipticE(((x+d/e)/(d/e-1/c*(a*
c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+1/
c*(a*c)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(
a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 903, normalized size of antiderivative = 1.71

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
2/45*(2*(4*B*c^2*d^7 + A*c^2*d^6*e - 11*B*a*c*d^5*e^2 - 9*A*a*c*d^4*e^3 +
15*B*a^2*d^3*e^4 + (4*B*c^2*d^4*e^3 + A*c^2*d^3*e^4 - 11*B*a*c*d^2*e^5 - 9
*A*a*c*d*e^6 + 15*B*a^2*e^7)*x^3 + 3*(4*B*c^2*d^5*e^2 + A*c^2*d^4*e^3 - 11
*B*a*c*d^3*e^4 - 9*A*a*c*d^2*e^5 + 15*B*a^2*d*e^6)*x^2 + 3*(4*B*c^2*d^6*e
+ A*c^2*d^5*e^2 - 11*B*a*c*d^4*e^3 - 9*A*a*c*d^3*e^4 + 15*B*a^2*d^2*e^5)*x
)*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d
^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 6*(4*B*c^2*d^6*e + A*c^2*d^5
*e^2 - 8*B*a*c*d^4*e^3 + 3*A*a*c*d^3*e^4 + (4*B*c^2*d^3*e^4 + A*c^2*d^2*e^
5 - 8*B*a*c*d*e^6 + 3*A*a*c*e^7)*x^3 + 3*(4*B*c^2*d^4*e^3 + A*c^2*d^3*e^4
- 8*B*a*c*d^2*e^5 + 3*A*a*c*d*e^6)*x^2 + 3*(4*B*c^2*d^5*e^2 + A*c^2*d^4*e^
3 - 8*B*a*c*d^3*e^4 + 3*A*a*c*d^2*e^5)*x)*sqrt(-c*e)*weierstrassZeta(4/3*(
c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), weierstrassPI
nverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1
/3*(3*e*x + d)/e)) + 3*(4*B*c^2*d^5*e^2 + A*c^2*d^4*e^3 - 10*B*a*c*d^3*e^4
 + 10*A*a*c*d^2*e^5 - 2*B*a^2*d*e^6 - 3*A*a^2*e^7 + 2*(4*B*c^2*d^3*e^4 + A
*c^2*d^2*e^5 - 8*B*a*c*d*e^6 + 3*A*a*c*e^7)*x^2 + (9*B*c^2*d^4*e^3 + 6*A*c
^2*d^3*e^4 - 20*B*a*c*d^2*e^5 + 10*A*a*c*d*e^6 - 5*B*a^2*e^7)*x)*sqrt(-c*x
^2 + a)*sqrt(e*x + d))/(c^2*d^7*e^4 - 2*a*c*d^5*e^6 + a^2*d^3*e^8 + (c^2*d
^4*e^7 - 2*a*c*d^2*e^9 + a^2*e^11)*x^3 + 3*(c^2*d^5*e^6 - 2*a*c*d^3*e^8 +
a^2*d*e^10)*x^2 + 3*(c^2*d^6*e^5 - 2*a*c*d^4*e^7 + a^2*d^2*e^9)*x)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{7/2}}\,dx = \int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x**2+a)**(1/2)/(e*x+d)**(7/2),x)
```

output
```
Integral((A + B*x)*sqrt(a - c*x**2)/(d + e*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{7/2}}\,dx = \int \frac{\sqrt{-cx^2+a}(Bx+A)}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^2 + a)*(B*x + A)/(e*x + d)^(7/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{7/2}}\,dx = \int \frac{\sqrt{-cx^2+a}(Bx+A)}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^2 + a)*(B*x + A)/(e*x + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{7/2}}\,dx = \int \frac{\sqrt{a-c\,x^2}\,(A+B\,x)}{(d+e\,x)^{7/2}}\,dx$$

input
```
int(((a - c*x^2)^(1/2)*(A + B*x))/(d + e*x)^(7/2),x)
```

output
```
int(((a - c*x^2)^(1/2)*(A + B*x))/(d + e*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt{a-cx^2}}{(d+ex)^{7/2}}\,dx = \text{too large to display}$$

input

```
int((B*x+A)*(-c*x^2+a)^(1/2)/(e*x+d)^(7/2),x)
```

output

```
(2*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*e - 2*sqrt(d + e*x)*sqrt(a - c*x**2)
*b*c*d*x - 3*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**4 + 4*a*d**3*
e*x + 6*a*d**2*e**2*x**2 + 4*a*d*e**3*x**3 + a*e**4*x**4 - c*d**4*x**2 - 4
*c*d**3*e*x**3 - 6*c*d**2*e**2*x**4 - 4*c*d*e**3*x**5 - c*e**4*x**6),x)*a*
b*c*d**3*e**2 - 9*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**4 + 4*a*
d**3*e*x + 6*a*d**2*e**2*x**2 + 4*a*d*e**3*x**3 + a*e**4*x**4 - c*d**4*x**
2 - 4*c*d**3*e*x**3 - 6*c*d**2*e**2*x**4 - 4*c*d*e**3*x**5 - c*e**4*x**6),
x)*a*b*c*d**2*e**3*x - 9*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**4
 + 4*a*d**3*e*x + 6*a*d**2*e**2*x**2 + 4*a*d*e**3*x**3 + a*e**4*x**4 - c*d
**4*x**2 - 4*c*d**3*e*x**3 - 6*c*d**2*e**2*x**4 - 4*c*d*e**3*x**5 - c*e**4
*x**6),x)*a*b*c*d*e**4*x**2 - 3*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/
(a*d**4 + 4*a*d**3*e*x + 6*a*d**2*e**2*x**2 + 4*a*d*e**3*x**3 + a*e**4*x**
4 - c*d**4*x**2 - 4*c*d**3*e*x**3 - 6*c*d**2*e**2*x**4 - 4*c*d*e**3*x**5 -
 c*e**4*x**6),x)*a*b*c*e**5*x**3 - int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**
2)/(a*d**4 + 4*a*d**3*e*x + 6*a*d**2*e**2*x**2 + 4*a*d*e**3*x**3 + a*e**4*
x**4 - c*d**4*x**2 - 4*c*d**3*e*x**3 - 6*c*d**2*e**2*x**4 - 4*c*d*e**3*x**
5 - c*e**4*x**6),x)*a*c**2*d**4*e - 3*int((sqrt(d + e*x)*sqrt(a - c*x**2)*
x**2)/(a*d**4 + 4*a*d**3*e*x + 6*a*d**2*e**2*x**2 + 4*a*d*e**3*x**3 + a*e*
*4*x**4 - c*d**4*x**2 - 4*c*d**3*e*x**3 - 6*c*d**2*e**2*x**4 - 4*c*d*e**3*
x**5 - c*e**4*x**6),x)*a*c**2*d**3*e**2*x - 3*int((sqrt(d + e*x)*sqrt(a...
```

3.267 $\int (A+Bx)\sqrt{d+ex}\left(a-cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 603

$$\int (A+Bx)\sqrt{d+ex}\left(a-cx^2\right)^{3/2}\,dx=\frac{8(44Acde(cd^2-3ae^2)-B(32c^2d^4-69acd^2e^2+45a^2e^4))\sqrt{d+ex}\sqrt{a-cx^2}}{3465ce^4}+\frac{4(d+ex)^{3/2}\left(32Bcd^3-44Acd^2e-29aBde^2+77aAe^3-5e(8Bcd^2-11Acde-9aBe^2)x\right)\sqrt{a-cx^2}}{1155e^4}-\frac{2(d+ex)^{3/2}(8Bd-11Ae-9Bex)\left(a-cx^2\right)^{3/2}}{99e^2}-\frac{8\sqrt{a}(32Bc^2d^5-44Ac^2d^4e-93aBcd^3e^2+165aAcd^2e^3+93a^2Bde^4+231a^2Ae^5)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}E\left(a\ldots\right.}{3465\sqrt{c}e^5\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}}-\frac{8\sqrt{a}(cd^2-ae^2)\left(44Acde(cd^2-3ae^2)-B(32c^2d^4-69acd^2e^2+45a^2e^4)\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\ldots\right.}{3465c^{3/2}e^5\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
8/3465*(44*A*c*d*e*(-3*a*e^2+c*d^2)-B*(45*a^2*e^4-69*a*c*d^2*e^2+32*c^2*d^
4))*(e*x+d)^(1/2)*(-c*x^2+a)^(1/2)/c/e^4+4/1155*(e*x+d)^(3/2)*(32*B*c*d^3-
44*A*c*d^2*e-29*B*a*d*e^2+77*A*a*e^3-5*e*(-11*A*c*d*e-9*B*a*e^2+8*B*c*d^2)
*x)*(-c*x^2+a)^(1/2)/e^4-2/99*(e*x+d)^(3/2)*(-9*B*e*x-11*A*e+8*B*d)*(-c*x^
2+a)^(3/2)/e^2-8/3465*a^(1/2)*(231*A*a^2*e^5+165*A*a*c*d^2*e^3-44*A*c^2*d^
4*e+93*B*a^2*d*e^4-93*B*a*c*d^3*e^2+32*B*c^2*d^5)*(e*x+d)^(1/2)*(1-c*x^2/a
)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)
*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/c^(1/2)/e^5/(c^(1/2)*(e*x+d)/(c^(1/2)*d+a
^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)-8/3465*a^(1/2)*(-a*e^2+c*d^2)*(44*A*c*d*
e*(-3*a*e^2+c*d^2)-B*(45*a^2*e^4-69*a*c*d^2*e^2+32*c^2*d^4))*(c^(1/2)*(e*x
+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2)*EllipticF(1/2*(1-c^(1/2
)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2)
)/c^(3/2)/e^5/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.85 (sec) , antiderivative size = 801, normalized size of antiderivative = 1.33

$$\int (A+Bx)\sqrt{d+ex}(a-cx^2)^{3/2}\,dx = \frac{2\sqrt{a-cx^2}\left((d+ex)\left(11Ace(ae^2(29d+77ex)-c(8d^3-6d^2ex+5de^2x^2+35e^3x^3))+B(-\right.\right.}{}$$

input
```
Integrate[(A + B*x)*Sqrt[d + e*x]*(a - c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[a - c*x^2]*((d + e*x)*(11*A*c*e*(a*e^2*(29*d + 77*e*x) - c*(8*d^3
- 6*d^2*e*x + 5*d*e^2*x^2 + 35*e^3*x^3)) + B*(-180*a^2*e^4 + a*c*e^2*(-178
*d^2 + 131*d*e*x + 585*e^2*x^2) + c^2*(64*d^4 - 48*d^3*e*x + 40*d^2*e^2*x^
2 - 35*d*e^3*x^3 - 315*e^4*x^4))) - (4*(e^2*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]
*(11*A*e*(-4*c^2*d^4 + 15*a*c*d^2*e^2 + 21*a^2*e^4) + B*(32*c^2*d^5 - 93*a
*c*d^3*e^2 + 93*a^2*d*e^4))*(a - c*x^2) + I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e
)*(11*A*e*(-4*c^2*d^4 + 15*a*c*d^2*e^2 + 21*a^2*e^4) + B*(32*c^2*d^5 - 93*
a*c*d^3*e^2 + 93*a^2*d*e^4))*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqr
t[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*Ar
cSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*
e)/(Sqrt[c]*d - Sqrt[a]*e)] - I*Sqrt[a]*e*(Sqrt[c]*d - Sqrt[a]*e)*(11*A*Sq
rt[c]*e*(4*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e - 12*a*Sqrt[c]*d*e^2 + 21*a^(3/
2)*e^3) + B*(-32*c^2*d^4 - 24*Sqrt[a]*c^(3/2)*d^3*e + 69*a*c*d^2*e^2 + 48*
a^(3/2)*Sqrt[c]*d*e^3 - 45*a^2*e^4))*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e
*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*Ellipt
icF[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d +
Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)]))/(e^2*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*
(a - c*x^2))))/(3465*c*e^4*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {687, 27, 682, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^2\right)^{3/2} (A+Bx)\sqrt{d+ex}\,dx$$

$$\Big\downarrow \text{ 687}$$

$$-\frac{2\int -\frac{(11Acd+aBe+c(Bd+11Ae)x)\left(a-cx^2\right)^{3/2}}{2\sqrt{d+ex}}dx}{11c}-\frac{2B\left(a-cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(11Acd+aBe+c(Bd+11Ae)x)\left(a-cx^2\right)^{3/2}}{\sqrt{d+ex}}dx}{11c}-\frac{2B\left(a-cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\Big\downarrow$ 682

$$-\frac{\frac{4\int \frac{c\left(ae\left(Bcd^2-88Aced-9aBe^2\right)+c\left(8Bcd^3-11Aced^2-16aBe^2d-77aAe^3\right)x\right)\sqrt{a-cx^2}}{2\sqrt{d+ex}}dx}{21ce^2}-\frac{2(a-cx^2)^{3/2}\sqrt{d+ex}(-9aBe^2-7cex(11Ae+Bd)-11Ac}{63e^2}}{11c}$$

$$\frac{2B\left(a-cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\Big\downarrow$ 27

$$-\frac{\frac{2\int \frac{\left(ae\left(Bcd^2-88Aced-9aBe^2\right)+c\left(8Bcd^3-11Aced^2-16aBe^2d-77aAe^3\right)x\right)\sqrt{a-cx^2}}{\sqrt{d+ex}}dx}{21e^2}-\frac{2(a-cx^2)^{3/2}\sqrt{d+ex}(-9aBe^2-7cex(11Ae+Bd)-11Acd}{63e^2}}{11c}$$

$$\frac{2B\left(a-cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\Big\downarrow$ 682

$$-\frac{2\left(\frac{2\sqrt{a-cx^2}\sqrt{d+ex}\left(-B\left(45a^2e^4-69acd^2e^2+32c^2d^4\right)+3cex\left(-77aAe^3-16aBde^2-11Acd^2e+8Bcd^3\right)+44Acde\left(cd^2-3ae^2\right)\right)}{15e^2}-\frac{4\int -\frac{c\left(ae\left(11Acde\left(cd^2-33a\right.\right.\right.}{}}{}\right.}{21e^2}$$

$$\frac{2B\left(a-cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\Big\downarrow$ 27

$$-\frac{2\left(\frac{2\int \frac{ae\left(11Acde\left(cd^2-33ae^2\right)-B\left(8c^2d^4-21ace^2d^2+45a^2e^4\right)\right)+c\left(11Ae\left(4c^2d^4-15ace^2d^2-21a^2e^4\right)-B\left(32c^2d^5-93ace^2d^3+93a^2e^4d\right)\right)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{15e^2}+\frac{2\sqrt{a-cx^2}\sqrt{}}{}\right.}{21e^2}$$

$$\frac{2B\left(a-cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\Big\downarrow$ 600

$$-\frac{2\left(\frac{2\left(-\frac{\left(cd^2-ae^2\right)\left(44Acde\left(cd^2-3ae^2\right)-B\left(45a^2e^4-69acd^2e^2+32c^2d^4\right)\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{c\left(231a^2Ae^5+93a^2Bde^4+165aAcd^2e^3-93aBcd^3e^2-44A\cdots\right.}{e}\right.}{15e^2}\right.}{2\cdots}$$

$$\frac{2B\left(a-cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\downarrow$ 509

$$-\frac{2\left(\frac{2\left(-\frac{\left(cd^2-ae^2\right)\left(44Acde\left(cd^2-3ae^2\right)-B\left(45a^2e^4-69acd^2e^2+32c^2d^4\right)\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{c\sqrt{1-\frac{cx^2}{a}}\left(231a^2Ae^5+93a^2Bde^4+165aAcd^2e^3-93aBcd^3\cdots\right.}{e\sqrt{a-cx^2}}\right.}{15e^2}\right.}{\cdots}$$

$$\frac{2B\left(a-cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\downarrow$ 508

$$-\frac{2\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(231a^2Ae^5+93a^2Bde^4+165aAcd^2e^3-93aBcd^3e^2-44Ac^2d^4e+32Bc^2d^5\right)\int\frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{\left(cd^2-ae^2\right)\left(44Ac\cdots\right.}{\cdots}\right)}{15e^2}\right.}{\cdots}$$

$$\frac{2B\left(a-cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$$\downarrow \text{327}$$

$$-\frac{2\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(231a^2Ae^5+93a^2Bde^4+165aAcd^2e^3-93aBcd^3e^2-44Ac^2d^4e+32Bc^2d^5\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{\left(cd^2-ae^2\right)\left(44A\cdots\right.}{\cdots}\cdots\right)}{15e^2}\cdots\right)}{\cdots}$$

$$\frac{2B\left(a-cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$$\downarrow \text{512}$$

$$-\frac{2\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(231a^2Ae^5+93a^2Bde^4+165aAcd^2e^3-93aBcd^3e^2-44Ac^2d^4e+32Bc^2d^5\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-\cdots\right.}{\cdots}\cdots\right)}{15e^2}\cdots\right)}{\cdots}$$

$$\frac{2B\left(a-cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$$\downarrow \text{511}$$

$$-\frac{2\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(44Acde\left(cd^2-3ae^2\right)-B\left(45a^2e^4-69acd^2e^2+32c^2d^4\right)\right)\int \frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}+\frac{2\sqrt{a}\cdots}{\cdots}\right)}{15e^2}\cdots\right)}{\cdots} - \cdots$$

$$\frac{2B\left(a-cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\Big\downarrow$ 321

$$-\frac{2\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(44Acde\left(cd^2-3ae^2\right)-B\left(45a^2e^4-69acd^2e^2+32c^2d^4\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}+\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\cdots}}{\cdots}\right)}{15e^2}\cdots\right)}{\cdots} - \cdots$$

$$\frac{2B\left(a-cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

input
```
Int[(A + B*x)*Sqrt[d + e*x]*(a - c*x^2)^(3/2),x]
```

output
```
(-2*B*Sqrt[d + e*x]*(a - c*x^2)^(5/2))/(11*c) + ((-2*Sqrt[d + e*x]*(8*B*c*
d^2 - 11*A*c*d*e - 9*a*B*e^2 - 7*c*e*(B*d + 11*A*e)*x)*(a - c*x^2)^(3/2))/
(63*e^2) - (2*((2*Sqrt[d + e*x]*(44*A*c*d*e*(c*d^2 - 3*a*e^2) - B*(32*c^2*
d^4 - 69*a*c*d^2*e^2 + 45*a^2*e^4) + 3*c*e*(8*B*c*d^3 - 11*A*c*d^2*e - 16*
a*B*d*e^2 - 77*a*A*e^3)*x)*Sqrt[a - c*x^2])/(15*e^2) + (2*((2*Sqrt[a]*Sqrt
[c]*(32*B*c^2*d^5 - 44*A*c^2*d^4*e - 93*a*B*c*d^3*e^2 + 165*a*A*c*d^2*e^3
+ 93*a^2*B*d*e^4 + 231*a^2*A*e^5)*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*Ellipt
icE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt
[a] + e)])/(e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c
*x^2]) + (2*Sqrt[a]*(c*d^2 - a*e^2)*(44*A*c*d*e*(c*d^2 - 3*a*e^2) - B*(32*
c^2*d^4 - 69*a*c*d^2*e^2 + 45*a^2*e^4))*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*
d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/
Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e
*x]*Sqrt[a - c*x^2])))/(15*e^2)))/(21*e^2))/(11*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1380 vs. 2(527) = 1054.

Time = 5.95 (sec) , antiderivative size = 1381, normalized size of antiderivative = 2.29

method	result	size
risch	Expression too large to display	1381
elliptic	Expression too large to display	1519
default	Expression too large to display	3551

input
```
int((B*x+A)*(e*x+d)^(1/2)*(-c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3465/c*(-315*B*c^2*e^4*x^4-385*A*c^2*e^4*x^3-35*B*c^2*d*e^3*x^3-55*A*c^2
*d*e^3*x^2+585*B*a*c*e^4*x^2+40*B*c^2*d^2*e^2*x^2+847*A*a*c*e^4*x+66*A*c^2
*d^2*e^2*x+131*B*a*c*d*e^3*x-48*B*c^2*d^3*e*x+319*A*a*c*d*e^3-88*A*c^2*d^3
*e-180*B*a^2*e^4-178*B*a*c*d^2*e^2+64*B*c^2*d^4)*(e*x+d)^(1/2)/e^4*(-c*x^2
+a)^(1/2)+4/3465/e^4/c*((231*A*a^2*e^5+165*A*a*c*d^2*e^3-44*A*c^2*d^4*e+93
*B*a^2*d*e^4-93*B*a*c*d^3*e^2+32*B*c^2*d^5)*(a*c)^(1/2)*2^(1/2)*((x+1/c*(a
*c)^(1/2))*c/(a*c)^(1/2))^(1/2)*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*(-2*
(x-1/c*(a*c)^(1/2))*c/(a*c)^(1/2))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2
)*((d/e-1/c*(a*c)^(1/2))*EllipticE(1/2*2^(1/2)*((x+1/c*(a*c)^(1/2))*c/(a*c
)^(1/2))^(1/2),(-2/c*(a*c)^(1/2)/(d/e-1/c*(a*c)^(1/2)))^(1/2))-d/e*Ellipti
cF(1/2*2^(1/2)*((x+1/c*(a*c)^(1/2))*c/(a*c)^(1/2))^(1/2),(-2/c*(a*c)^(1/2)
/(d/e-1/c*(a*c)^(1/2)))^(1/2)))+45*B*e^5*a^3/c*(a*c)^(1/2)*2^(1/2)*((x+1/c
*(a*c)^(1/2))*c/(a*c)^(1/2))^(1/2)*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*(
-2*(x-1/c*(a*c)^(1/2))*c/(a*c)^(1/2))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(
1/2)*EllipticF(1/2*2^(1/2)*((x+1/c*(a*c)^(1/2))*c/(a*c)^(1/2))^(1/2),(-2/c
*(a*c)^(1/2)/(d/e-1/c*(a*c)^(1/2)))^(1/2))-11*A*a*c*d^3*e^2*(a*c)^(1/2)*2^
(1/2)*((x+1/c*(a*c)^(1/2))*c/(a*c)^(1/2))^(1/2)*((x+d/e)/(d/e-1/c*(a*c)^(1
/2)))^(1/2)*(-2*(x-1/c*(a*c)^(1/2))*c/(a*c)^(1/2))^(1/2)/(-c*e*x^3-c*d*x^2
+a*e*x+a*d)^(1/2)*EllipticF(1/2*2^(1/2)*((x+1/c*(a*c)^(1/2))*c/(a*c)^(1/2)
)^(1/2),(-2/c*(a*c)^(1/2)/(d/e-1/c*(a*c)^(1/2)))^(1/2))+363*A*a^2*d*e^4...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 522, normalized size of antiderivative = 0.87

$$\int (A+Bx)\sqrt{d+ex}(a-cx^2)^{3/2}\,dx = \frac{2\left(4\left(32\,Bc^3d^6-44\,Ac^3d^5e-117\,Bac^2d^4e^2+198\,Aac^2d^3e^3+156\,Ba^2cd^2e^4-858\,Aa^2cde^5\right.\right.}{}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)*(-c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
2/10395*(4*(32*B*c^3*d^6 - 44*A*c^3*d^5*e - 117*B*a*c^2*d^4*e^2 + 198*A*a*
c^2*d^3*e^3 + 156*B*a^2*c*d^2*e^4 - 858*A*a^2*c*d*e^5 - 135*B*a^3*e^6)*sqr
t(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 -
9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 12*(32*B*c^3*d^5*e - 44*A*c^3*d^4
*e^2 - 93*B*a*c^2*d^3*e^3 + 165*A*a*c^2*d^2*e^4 + 93*B*a^2*c*d*e^5 + 231*A
*a^2*c*e^6)*sqrt(-c*e)*weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/2
7*(c*d^3 - 9*a*d*e^2)/(c*e^3), weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(
c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e)) - 3*(315*B*
c^3*e^6*x^4 - 64*B*c^3*d^4*e^2 + 88*A*c^3*d^3*e^3 + 178*B*a*c^2*d^2*e^4 -
319*A*a*c^2*d*e^5 + 180*B*a^2*c*e^6 + 35*(B*c^3*d*e^5 + 11*A*c^3*e^6)*x^3
- 5*(8*B*c^3*d^2*e^4 - 11*A*c^3*d*e^5 + 117*B*a*c^2*e^6)*x^2 + (48*B*c^3*d
^3*e^3 - 66*A*c^3*d^2*e^4 - 131*B*a*c^2*d*e^5 - 847*A*a*c^2*e^6)*x)*sqrt(-
c*x^2 + a)*sqrt(e*x + d))/(c^2*e^6)
```

Sympy [F]

$$\int (A+Bx)\sqrt{d+ex}\left(a-cx^2\right)^{3/2}\,dx=\int (A+Bx)\left(a-cx^2\right)^{\frac{3}{2}}\sqrt{d+ex}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)**(1/2)*(-c*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x)*(a - c*x**2)**(3/2)*sqrt(d + e*x), x)
```

Maxima [F]

$$\int (A+Bx)\sqrt{d+ex}\left(a-cx^2\right)^{3/2}\,dx=\int \left(-cx^2+a\right)^{\frac{3}{2}}(Bx+A)\sqrt{ex+d}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)*(-c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^2 + a)^(3/2)*(B*x + A)*sqrt(e*x + d), x)
```

Giac [F]

$$\int (A+Bx)\sqrt{d+ex}\left(a-cx^2\right)^{3/2}\,dx=\int \left(-cx^2+a\right)^{\frac{3}{2}}(Bx+A)\sqrt{ex+d}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)*(-c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*x^2 + a)^(3/2)*(B*x + A)*sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)\sqrt{d+ex}\left(a-cx^2\right)^{3/2}\,dx=\int \left(a-c\,x^2\right)^{3/2}\,(A+B\,x)\,\sqrt{d+e\,x}\,dx$$

input
```
int((a - c*x^2)^(3/2)*(A + B*x)*(d + e*x)^(1/2),x)
```

output
```
int((a - c*x^2)^(3/2)*(A + B*x)*(d + e*x)^(1/2), x)
```

Reduce [F]

$$\int (A+Bx)\sqrt{d+ex}\left(a-cx^2\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^(1/2)*(-c*x^2+a)^(3/2),x)
```

output
```
(2*( - 462*sqrt(d + e*x)*sqrt(a - c*x**2)*a**3*e**4 - 366*sqrt(d + e*x)*sq
rt(a - c*x**2)*a**2*b*d*e**3 - 11*sqrt(d + e*x)*sqrt(a - c*x**2)*a**2*c*d*
*2*e**2 + 847*sqrt(d + e*x)*sqrt(a - c*x**2)*a**2*c*d*e**3*x + 8*sqrt(d +
e*x)*sqrt(a - c*x**2)*a*b*c*d**3*e + 131*sqrt(d + e*x)*sqrt(a - c*x**2)*a*
b*c*d**2*e**2*x + 585*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*c*d*e**3*x**2 + 6
6*sqrt(d + e*x)*sqrt(a - c*x**2)*a*c**2*d**3*e*x - 55*sqrt(d + e*x)*sqrt(a
 - c*x**2)*a*c**2*d**2*e**2*x**2 - 385*sqrt(d + e*x)*sqrt(a - c*x**2)*a*c*
*2*d*e**3*x**3 - 48*sqrt(d + e*x)*sqrt(a - c*x**2)*b*c**2*d**4*x + 40*sqrt
(d + e*x)*sqrt(a - c*x**2)*b*c**2*d**3*e*x**2 - 35*sqrt(d + e*x)*sqrt(a -
c*x**2)*b*c**2*d**2*e**2*x**3 - 315*sqrt(d + e*x)*sqrt(a - c*x**2)*b*c**2*
d*e**3*x**4 - 693*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d + a*e*x -
 c*d*x**2 - c*e*x**3),x)*a**3*c*e**5 - 279*int((sqrt(d + e*x)*sqrt(a - c*x
**2)*x**2)/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a**2*b*c*d*e**4 - 495*in
t((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d + a*e*x - c*d*x**2 - c*e*x**3
),x)*a**2*c**2*d**2*e**3 + 279*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(
a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a*b*c**2*d**3*e**2 + 132*int((sqrt(d
 + e*x)*sqrt(a - c*x**2)*x**2)/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a*c*
*3*d**4*e - 96*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d + a*e*x - c*
d*x**2 - c*e*x**3),x)*b*c**3*d**5 + 231*int((sqrt(d + e*x)*sqrt(a - c*x**2
))/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a**4*e**5 + 183*int((sqrt(d +...
```

3.268 $\int \frac{(A+Bx)(a-cx^2)^{3/2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 493

$$\int \frac{(A+Bx)(a-cx^2)^{3/2}}{\sqrt{d+ex}}\,dx = \frac{4\sqrt{d+ex}(32Bcd^3-36Acd^2e-33aBde^2+45aAe^3-3e(8Bcd^2-9Acde-}{315e^4}$$

$$-\frac{2\sqrt{d+ex}(8Bd-9Ae-7Bex)(a-cx^2)^{3/2}}{63e^2}$$

$$+\frac{8\sqrt{a}(36Acde(cd^2-2ae^2)-B(32c^2d^4-57acd^2e^2+21a^2e^4))\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\right.}{315\sqrt{c}e^5\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}}$$

$$+\frac{8\sqrt{a}(cd^2-ae^2)(32Bcd^3-36Acd^2e-33aBde^2+45aAe^3)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-}}{\sqrt{2}}\right.\right.}{315\sqrt{c}e^5\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
4/315*(e*x+d)^(1/2)*(32*B*c*d^3-36*A*c*d^2*e-33*B*a*d*e^2+45*A*a*e^3-3*e*(
-9*A*c*d*e-7*B*a*e^2+8*B*c*d^2)*x)*(-c*x^2+a)^(1/2)/e^4-2/63*(e*x+d)^(1/2)
*(-7*B*e*x-9*A*e+8*B*d)*(-c*x^2+a)^(3/2)/e^2+8/315*a^(1/2)*(36*A*c*d*e*(-2
*a*e^2+c*d^2)-B*(21*a^2*e^4-57*a*c*d^2*e^2+32*c^2*d^4))*(e*x+d)^(1/2)*(1-c
*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a
^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/c^(1/2)/e^5/(c^(1/2)*(e*x+d)/(c^(1/
2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)+8/315*a^(1/2)*(-a*e^2+c*d^2)*(45*A
*a*e^3-36*A*c*d^2*e-33*B*a*d*e^2+32*B*c*d^3)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a
^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/
2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/c^(1/2)/e^5/(e
*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.33 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.41

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{\sqrt{d+ex}}\,dx = \frac{\sqrt{a-cx^2}\left(\frac{2(d+ex)\left(135aAe^3+aBe^2(-106d+77ex)-9Ace\left(8d^2-6dex+5e^2x^2\right)+Bc\left(64d^3-48d^2ex\right.\right.}{e^4}\right.}{}$$

input
```
Integrate[((A + B*x)*(a - c*x^2)^(3/2))/Sqrt[d + e*x],x]
```

output
```
(Sqrt[a - c*x^2]*((2*(d + e*x)*(135*a*A*e^3 + a*B*e^2*(-106*d + 77*e*x) -
9*A*c*e*(8*d^2 - 6*d*e*x + 5*e^2*x^2) + B*c*(64*d^3 - 48*d^2*e*x + 40*d*e^
2*x^2 - 35*e^3*x^3)))/e^4 + (8*(e^2*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-36*A*
c*d*e*(c*d^2 - 2*a*e^2) + B*(32*c^2*d^4 - 57*a*c*d^2*e^2 + 21*a^2*e^4))*(-
a + c*x^2) + I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(36*A*c*d*e*(c*d^2 - 2*a*e^
2) + B*(-32*c^2*d^4 + 57*a*c*d^2*e^2 - 21*a^2*e^4))*Sqrt[(e*(Sqrt[a]/Sqrt[
c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*
x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]]
, (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)] + I*Sqrt[a]*Sqrt[c]*e*(
Sqrt[c]*d - Sqrt[a]*e)*(9*A*Sqrt[c]*e*(4*c*d^2 + 3*Sqrt[a]*Sqrt[c]*d*e - 5
*a*e^2) + B*(-32*c^(3/2)*d^3 - 24*Sqrt[a]*c*d^2*e + 33*a*Sqrt[c]*d*e^2 + 2
1*a^(3/2)*e^3))*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]
*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d
 + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d
 - Sqrt[a]*e)]))/(c*e^6*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(a - c*x^2))))/(315
*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {682, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-cx^2\right)^{3/2}(A+Bx)}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{682}$$

$$-\frac{4\int \frac{c\left(ae(Bd-9Ae)+\left(8Bcd^2-9Aced-7aBe^2\right)x\right)\sqrt{a-cx^2}}{2\sqrt{d+ex}}dx}{21ce^2}-\frac{2\left(a-cx^2\right)^{3/2}\sqrt{d+ex}(-9Ae+8Bd-7Bex)}{63e^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int \frac{\left(ae(Bd-9Ae)+\left(8Bcd^2-9Aced-7aBe^2\right)x\right)\sqrt{a-cx^2}}{\sqrt{d+ex}}dx}{21e^2}-\frac{2\left(a-cx^2\right)^{3/2}\sqrt{d+ex}(-9Ae+8Bd-7Bex)}{63e^2}$$

$$\Big\downarrow \text{682}$$

$$-\frac{2\left(-\frac{4\int \frac{c\left(ae\left(8Bcd^3-9Aced^2-12aBe^2d+45aAe^3\right)-\left(36Acde\left(cd^2-2ae^2\right)-B\left(32c^2d^4-57ace^2d^2+21a^2e^4\right)\right)x\right)}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{15ce^2}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}\left(-3ex\left(-7aBe\cdots\right.\right.}{\cdots}\cdots\right)}{21e^2}-\frac{2\left(a-cx^2\right)^{3/2}\sqrt{d+ex}(-9Ae+8Bd-7Bex)}{63e^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\left(-\frac{2\int \frac{ae\left(8Bcd^3-9Aced^2-12aBe^2d+45aAe^3\right)-\left(36Acde\left(cd^2-2ae^2\right)-B\left(32c^2d^4-57ace^2d^2+21a^2e^4\right)\right)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{15e^2}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}\left(-3ex\left(-7aBe^2-\cdots\right.\right.}{\cdots}\cdots\right)}{21e^2}-\frac{2\left(a-cx^2\right)^{3/2}\sqrt{d+ex}(-9Ae+8Bd-7Bex)}{63e^2}$$

$$\Big\downarrow \text{600}$$

$$-\frac{2\left(-\frac{2\left(-\frac{\left(36Acde\left(cd^2-2ae^2\right)-B\left(21a^2e^4-57acd^2e^2+32c^2d^4\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{\left(cd^2-ae^2\right)\left(45aAe^3-33aBde^2-36Acd^2e+32Bcd^3\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}d\cdots}{e}\cdots\right)}{15e^2}\cdots\right)}{21e^2}-\frac{2\left(a-cx^2\right)^{3/2}\sqrt{d+ex}(-9Ae+8Bd-7Bex)}{63e^2}$$

$$\Big\downarrow \text{509}$$

$$-\frac{2\left(-\frac{2\left(-\frac{\sqrt{1-\frac{cx^2}{a}}\left(36Acde\left(cd^2-2ae^2\right)-B\left(21a^2e^4-57acd^2e^2+32c^2d^4\right)\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{\left(cd^2-ae^2\right)\left(45aAe^3-33aBde^2-36Acd^2e+32Bcd^3\right)\int\frac{}{\sqrt{d+ex}}}{e}\right.}{15e^2}\right.}{21e^2}$$

$$\frac{2\left(a-cx^2\right)^{3/2}\sqrt{d+ex}(-9Ae+8Bd-7Bex)}{63e^2}$$

$\Big\downarrow$ 508

$$-\frac{2\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(36Acde\left(cd^2-2ae^2\right)-B\left(21a^2e^4-57acd^2e^2+32c^2d^4\right)\right)\int\frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{\left(cd^2-ae^2\right)\left(45aAe^3-33aBde^2-3\right.}{}\right.}{15e^2}\right.}{21e^2}$$

$$\frac{2\left(a-cx^2\right)^{3/2}\sqrt{d+ex}(-9Ae+8Bd-7Bex)}{63e^2}$$

$\Big\downarrow$ 327

$$-\frac{2\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(36Acde\left(cd^2-2ae^2\right)-B\left(21a^2e^4-57acd^2e^2+32c^2d^4\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{cd}}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{cd}}}}-\frac{\left(cd^2-ae^2\right)\left(45aAe^3-33aBde^2-\cdots\right.}{\cdots}\right)}{15e^2}\cdots\right)}{21e\cdots}$$

$$\frac{2\left(a-cx^2\right)^{3/2}\sqrt{d+ex}(-9Ae+8Bd-7Bex)}{63e^2}$$

$\downarrow$ 512

$$-\frac{2\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(36Acde\left(cd^2-2ae^2\right)-B\left(21a^2e^4-57acd^2e^2+32c^2d^4\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{cd}}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{cd}}}}-\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\left(45aAe^3-3\cdots\right.}{\cdots}\right)}{15e^2}\cdots\right)}{\cdots}$$

$$\frac{2\left(a-cx^2\right)^{3/2}\sqrt{d+ex}(-9Ae+8Bd-7Bex)}{63e^2}$$

$\downarrow$ 511

$$-\frac{2\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(45aAe^3-33aBde^2-36Acd^2e+32Bcd^3\right)\int \frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}+\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{\cdots}}{\cdots}\right)}{15e^2}\cdots\right)}{\cdots}$$

$$\frac{2\left(a-cx^2\right)^{3/2}\sqrt{d+ex}(-9Ae+8Bd-7Bex)}{63e^2}$$

$$\Big\downarrow 321$$

$$-\frac{2\left(-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(36Acde\left(cd^2-2ae^2\right)-B\left(21a^2e^4-57acd^2e^2+32c^2d^4\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}+\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d\cdots}{\sqrt{a}e\cdots}}\cdots}{\cdots}\right)}{15e^2}\cdots\right)}{\cdots}$$

$$\frac{2\left(a-cx^2\right)^{3/2}\sqrt{d+ex}(-9Ae+8Bd-7Bex)}{63e^2}$$

input
```
Int[((A + B*x)*(a - c*x^2)^(3/2))/Sqrt[d + e*x],x]
```

output
```
(-2*Sqrt[d + e*x]*(8*B*d - 9*A*e - 7*B*e*x)*(a - c*x^2)^(3/2))/(63*e^2) -
(2*((-2*Sqrt[d + e*x]*(32*B*c*d^3 - 36*A*c*d^2*e - 33*a*B*d*e^2 + 45*a*A*e
^3 - 3*e*(8*B*c*d^2 - 9*A*c*d*e - 7*a*B*e^2)*x)*Sqrt[a - c*x^2])/(15*e^2)
- (2*((2*Sqrt[a]*(36*A*c*d*e*(c*d^2 - 2*a*e^2) - B*(32*c^2*d^4 - 57*a*c*d^
2*e^2 + 21*a^2*e^4))*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSin[Sq
rt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(S
qrt[c]*e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]
) + (2*Sqrt[a]*(c*d^2 - a*e^2)*(32*B*c*d^3 - 36*A*c*d^2*e - 33*a*B*d*e^2 +
 45*a*A*e^3)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((
Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[a - c*x^2])))/(15*
e^2)))/(21*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1049 vs. 2(423) = 846.

Time = 7.00 (sec) , antiderivative size = 1050, normalized size of antiderivative = 2.13

method	result	size
elliptic	Expression too large to display	1050
risch	Expression too large to display	1117
default	Expression too large to display	2966

input
```
int((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(-2/9*B/e*c*x^3*
(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)-2/7*(A*c^2-8/9*c^2*d/e*B)/c/e*x^2*(-c*e
*x^3-c*d*x^2+a*e*x+a*d)^(1/2)-2/5*(-11/9*a*B*c-6/7*d/e*(A*c^2-8/9*c^2*d/e*
B))/c/e*x*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)-2/3*(-2*A*a*c-4/5*d/e*(-11/9*
a*B*c-6/7*d/e*(A*c^2-8/9*c^2*d/e*B))+5/7*a/c*(A*c^2-8/9*c^2*d/e*B)+2/3*a*c
*d/e*B)/c/e*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)+2*(a^2*A+2/5*a/c*d/e*(-11/9
*a*B*c-6/7*d/e*(A*c^2-8/9*c^2*d/e*B))+1/3*a/c*(-2*A*a*c-4/5*d/e*(-11/9*a*B
*c-6/7*d/e*(A*c^2-8/9*c^2*d/e*B))+5/7*a/c*(A*c^2-8/9*c^2*d/e*B)+2/3*a*c*d/
e*B))*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*
(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*
(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*EllipticF(((x+d/e)/
(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)
))^(1/2))+2*(a^2*B+4/7*a/c*d/e*(A*c^2-8/9*c^2*d/e*B)+3/5*a/c*(-11/9*a*B*c-
6/7*d/e*(A*c^2-8/9*c^2*d/e*B))-2/3*d/e*(-2*A*a*c-4/5*d/e*(-11/9*a*B*c-6/7*
d/e*(A*c^2-8/9*c^2*d/e*B))+5/7*a/c*(A*c^2-8/9*c^2*d/e*B)+2/3*a*c*d/e*B))*(
d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(
1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(
1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))*El
lipticE(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/
e-1/c*(a*c)^(1/2)))^(1/2))+1/c*(a*c)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)(a-cx^2)^{3/2}}{\sqrt{d+ex}}\,dx = \frac{2\Big(4\,(32\,Bc^2d^5-36\,Ac^2d^4e-81\,Bacd^3e^2+99\,Aacd^2e^3+57\,Ba^2de^4-135$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/945*(4*(32*B*c^2*d^5 - 36*A*c^2*d^4*e - 81*B*a*c*d^3*e^2 + 99*A*a*c*d^2*
e^3 + 57*B*a^2*d*e^4 - 135*A*a^2*e^5)*sqrt(-c*e)*weierstrassPInverse(4/3*(
c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x +
d)/e) + 12*(32*B*c^2*d^4*e - 36*A*c^2*d^3*e^2 - 57*B*a*c*d^2*e^3 + 72*A*a*
c*d*e^4 + 21*B*a^2*e^5)*sqrt(-c*e)*weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(
c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), weierstrassPInverse(4/3*(c*d^2
+ 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e))
 - 3*(35*B*c^2*e^5*x^3 - 64*B*c^2*d^3*e^2 + 72*A*c^2*d^2*e^3 + 106*B*a*c*d
*e^4 - 135*A*a*c*e^5 - 5*(8*B*c^2*d*e^4 - 9*A*c^2*e^5)*x^2 + (48*B*c^2*d^2
*e^3 - 54*A*c^2*d*e^4 - 77*B*a*c*e^5)*x)*sqrt(-c*x^2 + a)*sqrt(e*x + d))/(
c*e^6)
```

Sympy [F]

$$\int \frac{(A+Bx)(a-cx^2)^{3/2}}{\sqrt{d+ex}}\,dx = \int \frac{(A+Bx)(a-cx^2)^{\frac{3}{2}}}{\sqrt{d+ex}}\,dx$$

input
```
integrate((B*x+A)*(-c*x**2+a)**(3/2)/(e*x+d)**(1/2),x)
```

output
```
Integral((A + B*x)*(a - c*x**2)**(3/2)/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{\sqrt{d+ex}}\,dx=\int \frac{(-cx^2+a)^{\frac{3}{2}}(Bx+A)}{\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^2 + a)^(3/2)*(B*x + A)/sqrt(e*x + d), x)
```

Giac [F]

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{\sqrt{d+ex}}\,dx=\int \frac{(-cx^2+a)^{\frac{3}{2}}(Bx+A)}{\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((-c*x^2 + a)^(3/2)*(B*x + A)/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{\sqrt{d+ex}}\,dx=\int \frac{\left(a-c\,x^2\right)^{3/2}(A+B\,x)}{\sqrt{d+e\,x}}\,dx$$

input
```
int(((a - c*x^2)^(3/2)*(A + B*x))/(d + e*x)^(1/2),x)
```

output
```
int(((a - c*x^2)^(3/2)*(A + B*x))/(d + e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{\sqrt{d+ex}}\,dx=\text{Too large to display}$$

input
```
int((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(1/2),x)
```

output
```
(2*( - 42*sqrt(d + e*x)*sqrt(a - c*x**2)*a**2*b*e**3 - 9*sqrt(d + e*x)*sqr
t(a - c*x**2)*a**2*c*d*e**2 + 8*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*c*d**2*
e + 77*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*c*d*e**2*x + 54*sqrt(d + e*x)*sq
rt(a - c*x**2)*a*c**2*d**2*e*x - 45*sqrt(d + e*x)*sqrt(a - c*x**2)*a*c**2*
d*e**2*x**2 - 48*sqrt(d + e*x)*sqrt(a - c*x**2)*b*c**2*d**3*x + 40*sqrt(d
+ e*x)*sqrt(a - c*x**2)*b*c**2*d**2*e*x**2 - 35*sqrt(d + e*x)*sqrt(a - c*x
**2)*b*c**2*d*e**2*x**3 - 63*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*
d + a*e*x - c*d*x**2 - c*e*x**3),x)*a**2*b*c*e**4 - 216*int((sqrt(d + e*x)
*sqrt(a - c*x**2)*x**2)/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a**2*c**2*d
*e**3 + 171*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d + a*e*x - c*d*x
**2 - c*e*x**3),x)*a*b*c**2*d**2*e**2 + 108*int((sqrt(d + e*x)*sqrt(a - c*
x**2)*x**2)/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a*c**3*d**3*e - 96*int(
(sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d + a*e*x - c*d*x**2 - c*e*x**3),
x)*b*c**3*d**4 + 21*int((sqrt(d + e*x)*sqrt(a - c*x**2))/(a*d + a*e*x - c*
d*x**2 - c*e*x**3),x)*a**3*b*e**4 + 162*int((sqrt(d + e*x)*sqrt(a - c*x**2
))/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a**3*c*d*e**3 - 81*int((sqrt(d +
 e*x)*sqrt(a - c*x**2))/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a**2*b*c*d*
*2*e**2 - 54*int((sqrt(d + e*x)*sqrt(a - c*x**2))/(a*d + a*e*x - c*d*x**2
- c*e*x**3),x)*a**2*c**2*d**3*e + 48*int((sqrt(d + e*x)*sqrt(a - c*x**2))/
(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a*b*c**2*d**4))/(315*c*d*e**3)
```

3.269 $\int \frac{(A+Bx)(a-cx^2)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 443

$$
\begin{aligned}
\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx &= \frac{4\sqrt{d+ex}\left(5aBe^2-4cd(8Bd-7Ae)+3ce(8Bd-7Ae)x\right)\sqrt{a-cx^2}}{35e^4} \\
&+ \frac{2(8Bd-7Ae+Bex)\left(a-cx^2\right)^{3/2}}{7e^2\sqrt{d+ex}} \\
&+ \frac{8\sqrt{a}\sqrt{c}\left(32Bcd^3-28Acd^2e-29aBde^2+21aAe^3\right)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{35e^5\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} \\
&- \frac{8\sqrt{a}\left(cd^2-ae^2\right)\left(32Bcd^2-28Acde-5aBe^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{35\sqrt{c}e^5\sqrt{d+ex}\sqrt{a-cx^2}}
\end{aligned}
$$

output
```
4/35*(e*x+d)^(1/2)*(5*B*a*e^2-4*c*d*(-7*A*e+8*B*d)+3*c*e*(-7*A*e+8*B*d)*x)
*(-c*x^2+a)^(1/2)/e^4+2/7*(B*e*x-7*A*e+8*B*d)*(-c*x^2+a)^(3/2)/e^2/(e*x+d)
^(1/2)+8/35*a^(1/2)*c^(1/2)*(21*A*a*e^3-28*A*c*d^2*e-29*B*a*d*e^2+32*B*c*d
^3)*(e*x+d)^(1/2)*(1-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1
/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/e^5/(c^(1/2)*
(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)-8/35*a^(1/2)*(-a*e^2
+c*d^2)*(-28*A*c*d*e-5*B*a*e^2+32*B*c*d^2)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(
1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)
*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/c^(1/2)/e^5/(e*x
+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.23 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.39

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{a-cx^2}\left(-35(Bd-Ae)\left(cd^2-ae^2\right)-\left(29Bcd^2-21Acde-15aBe^2\right)(c\right.}{}$$

input
```
Integrate[((A + B*x)*(a - c*x^2)^(3/2))/(d + e*x)^(3/2),x]
```

output
```
(2*Sqrt[a - c*x^2]*(-35*(B*d - A*e)*(c*d^2 - a*e^2) - (29*B*c*d^2 - 21*A*c
*d*e - 15*a*B*e^2)*(d + e*x) + c*e*(13*B*d - 7*A*e)*x*(d + e*x) - 5*B*c*e^
2*x^2*(d + e*x) + (4*(e^2*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(32*B*c*d^3 - 28*
A*c*d^2*e - 29*a*B*d*e^2 + 21*a*A*e^3)*(a - c*x^2) + I*Sqrt[c]*(Sqrt[c]*d
- Sqrt[a]*e)*(32*B*c*d^3 - 28*A*c*d^2*e - 29*a*B*d*e^2 + 21*a*A*e^3)*Sqrt[
(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d
 + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c
]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)] + I*Sq
rt[a]*e*(-(Sqrt[c]*d) + Sqrt[a]*e)*(B*(-32*c*d^2 - 24*Sqrt[a]*Sqrt[c]*d*e
+ 5*a*e^2) + 7*A*(4*c*d*e + 3*Sqrt[a]*Sqrt[c]*e^2))*Sqrt[(e*(Sqrt[a]/Sqrt[
c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*
x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]]
, (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)]))/(e^2*Sqrt[-d + (Sqrt[
a]*e)/Sqrt[c]]*(a - c*x^2))))/(35*e^4*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {681, 25, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^2)^{3/2}(A+Bx)}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{681}$$

$$\frac{2(a-cx^2)^{3/2}(-7Ae+8Bd+Bex)}{7e^2\sqrt{d+ex}}-\frac{6\int -\frac{(aBe+c(8Bd-7Ae)x)\sqrt{a-cx^2}}{\sqrt{d+ex}}dx}{7e^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{6\int \frac{(aBe+c(8Bd-7Ae)x)\sqrt{a-cx^2}}{\sqrt{d+ex}}dx}{7e^2}+\frac{2(a-cx^2)^{3/2}(-7Ae+8Bd+Bex)}{7e^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{682}$$

$$\frac{6\left(\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(5aBe^2+3cex(8Bd-7Ae)-4cd(8Bd-7Ae))}{15e^2}-\frac{4\int\frac{c\left(ae\left(8Bcd^2-7Aced-5aBe^2\right)+c\left(32Bcd^3-28Aced^2-29aBe^2d+21aAe^3\right)x\right)}{2\sqrt{d+ex}\sqrt{a-cx^2}}d}{15ce^2}\right.}{7e^2}$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-7Ae+8Bd+Bex)}{7e^2\sqrt{d+ex}}$$

$\Big\downarrow$ 27

$$\frac{6\left(\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(5aBe^2+3cex(8Bd-7Ae)-4cd(8Bd-7Ae))}{15e^2}-\frac{2\int\frac{ae\left(8Bcd^2-7Aced-5aBe^2\right)+c\left(32Bcd^3-28Aced^2-29aBe^2d+21aAe^3\right)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{15e^2}\right)}{7e^2}$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-7Ae+8Bd+Bex)}{7e^2\sqrt{d+ex}}$$

$\Big\downarrow$ 600

$$\frac{6\left(\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(5aBe^2+3cex(8Bd-7Ae)-4cd(8Bd-7Ae))}{15e^2}-\frac{2\left(\frac{c\left(21aAe^3-29aBde^2-28Acd^2e+32Bcd^3\right)\int\frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{\left(cd^2-ae^2\right)\left(-5aBe\right.}{}\right.}{15e^2}\right.}{7e^2}$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-7Ae+8Bd+Bex)}{7e^2\sqrt{d+ex}}$$

$\Big\downarrow$ 509

$$\frac{6\left(\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(5aBe^2+3cex(8Bd-7Ae)-4cd(8Bd-7Ae))}{15e^2}-\frac{2\left(\frac{c\sqrt{1-\frac{cx^2}{a}}\left(21aAe^3-29aBde^2-28Acd^2e+32Bcd^3\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{\left(cd^2-ae^2\right.}{}\right.}{15e^2}\right.}{7e^2}$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-7Ae+8Bd+Bex)}{7e^2\sqrt{d+ex}}$$

$\Big\downarrow$ 508

$$6\left(\frac{2\sqrt{a-cx^2}\sqrt{d+ex}\left(5aBe^2+3cex(8Bd-7Ae)-4cd(8Bd-7Ae)\right)}{15e^2}-\frac{2\left(-\frac{\left(cd^2-ae^2\right)\left(-5aBe^2-28Acde+32Bcd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-}}{}\right.}{}\right.$$

$$\overline{\qquad\qquad 7e^2 \qquad\qquad}$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-7Ae+8Bd+Bex)}{7e^2\sqrt{d+ex}}$$

$$\Big\downarrow \; 327$$

$$6\left(\frac{2\sqrt{a-cx^2}\sqrt{d+ex}\left(5aBe^2+3cex(8Bd-7Ae)-4cd(8Bd-7Ae)\right)}{15e^2}-\frac{2\left(-\frac{\left(cd^2-ae^2\right)\left(-5aBe^2-28Acde+32Bcd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-}}{}\right.}{}\right.$$

$$\overline{\qquad\qquad 7e^2 \qquad\qquad}$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-7Ae+8Bd+Bex)}{7e^2\sqrt{d+ex}}$$

$$\Big\downarrow \; 512$$

$$6\left(\frac{2\sqrt{a-cx^2}\sqrt{d+ex}\left(5aBe^2+3cex(8Bd-7Ae)-4cd(8Bd-7Ae)\right)}{15e^2}-\frac{2\left(-\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\left(-5aBe^2-28Acde+32Bcd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\ldots\right.}{\ldots}\right.$$

$$7e^2$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-7Ae+8Bd+Bex)}{7e^2\sqrt{d+ex}}$$

$\downarrow$ 511

$$6\left(\frac{2\sqrt{a-cx^2}\sqrt{d+ex}\left(5aBe^2+3cex(8Bd-7Ae)-4cd(8Bd-7Ae)\right)}{15e^2}-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(-5aBe^2-28Acde+32Bcd^2\right)\int\ldots}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}\ldots\right.}{\ldots}\right.$$

$$7e^2$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-7Ae+8Bd+Bex)}{7e^2\sqrt{d+ex}}$$

$\downarrow$ 321

$$6\left(\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(5aBe^2+3cex(8Bd-7Ae)-4cd(8Bd-7Ae))}{15e^2}-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(-5aBe^2-28Acde+32Bcd^2\right)\text{EllipticF}\left(\right.}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}\right.}{}\right)$$

$$7e^2$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-7Ae+8Bd+Bex)}{7e^2\sqrt{d+ex}}$$

input
```
Int[((A + B*x)*(a - c*x^2)^(3/2))/(d + e*x)^(3/2),x]
```

output
```
(2*(8*B*d - 7*A*e + B*e*x)*(a - c*x^2)^(3/2))/(7*e^2*Sqrt[d + e*x]) + (6*(
(2*Sqrt[d + e*x]*(5*a*B*e^2 - 4*c*d*(8*B*d - 7*A*e) + 3*c*e*(8*B*d - 7*A*e
)*x)*Sqrt[a - c*x^2])/(15*e^2) - (2*((-2*Sqrt[a]*Sqrt[c]*(32*B*c*d^3 - 28*
A*c*d^2*e - 29*a*B*d*e^2 + 21*a*A*e^3)*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*E
llipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)
/Sqrt[a] + e)])/(e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[
a - c*x^2]) + (2*Sqrt[a]*(c*d^2 - a*e^2)*(32*B*c*d^2 - 28*A*c*d*e - 5*a*B*
e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]
*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*
d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[a - c*x^2])))/(15*e^2)))/(
7*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1136 vs. $2(373) = 746$.

Time = 8.69 (sec) , antiderivative size = 1137, normalized size of antiderivative = 2.57

method	result	size
risch	Expression too large to display	1137
elliptic	Expression too large to display	1178
default	Expression too large to display	2439

input
```
int((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(-5*B*c*e^2*x^2-7*A*c*e^2*x+13*B*c*d*e*x+21*A*c*d*e+15*B*a*e^2-29*B*c
*d^2)*(e*x+d)^(1/2)/e^4*(-c*x^2+a)^(1/2)-1/35/e^4*((49*A*a*e^3-77*A*c*d^2*
e-81*B*a*d*e^2+93*B*c*d^3)*(a*c)^(1/2)*2^(1/2)*((x+1/c*(a*c)^(1/2))*c/(a*c
)^(1/2))^(1/2)*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*(-2*(x-1/c*(a*c)^(1/2
))*c/(a*c)^(1/2))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((d/e-1/c*(a*c)
^(1/2))*EllipticE(1/2*2^(1/2)*((x+1/c*(a*c)^(1/2))*c/(a*c)^(1/2))^(1/2),(-
2/c*(a*c)^(1/2)/(d/e-1/c*(a*c)^(1/2)))^(1/2))-d/e*EllipticF(1/2*2^(1/2)*((
x+1/c*(a*c)^(1/2))*c/(a*c)^(1/2))^(1/2),(-2/c*(a*c)^(1/2)/(d/e-1/c*(a*c)^(
1/2)))^(1/2)))-35*(A*a^2*e^5-2*A*a*c*d^2*e^3+A*c^2*d^4*e-B*a^2*d*e^4+2*B*a
*c*d^3*e^2-B*c^2*d^5)/e*(-2*(-c*e*x^2+a*e)/(a*e^2-c*d^2)/((x+d/e)*(-c*e*x^
2+a*e))^(1/2)-d/(a*e^2-c*d^2)*(a*c)^(1/2)*2^(1/2)*((x+1/c*(a*c)^(1/2))*c/(
a*c)^(1/2))^(1/2)*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*(-2*(x-1/c*(a*c)^(
1/2))*c/(a*c)^(1/2))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*EllipticF(1/
2*2^(1/2)*((x+1/c*(a*c)^(1/2))*c/(a*c)^(1/2))^(1/2),(-2/c*(a*c)^(1/2)/(d/e
-1/c*(a*c)^(1/2)))^(1/2))-e/(a*e^2-c*d^2)*(a*c)^(1/2)*2^(1/2)*((x+1/c*(a*c
)^(1/2))*c/(a*c)^(1/2))^(1/2)*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*(-2*(x
-1/c*(a*c)^(1/2))*c/(a*c)^(1/2))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*
((d/e-1/c*(a*c)^(1/2))*EllipticE(1/2*2^(1/2)*((x+1/c*(a*c)^(1/2))*c/(a*c)^
(1/2))^(1/2),(-2/c*(a*c)^(1/2)/(d/e-1/c*(a*c)^(1/2)))^(1/2))-d/e*EllipticF
(1/2*2^(1/2)*((x+1/c*(a*c)^(1/2))*c/(a*c)^(1/2))^(1/2),(-2/c*(a*c)^(1/2...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)(a-cx^2)^{3/2}}{(d+ex)^{3/2}}\,dx =$$

$$-\frac{2\left(4\left(32\,Bc^2d^5-28\,Ac^2d^4e-53\,Bacd^3e^2+42\,Aacd^2e^3+15\,Ba^2de^4+\left(32\,Bc^2d^4e-28\,Ac^2d^3e^2-53\,Bc\right.\right.\right.}{}$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
-2/105*(4*(32*B*c^2*d^5 - 28*A*c^2*d^4*e - 53*B*a*c*d^3*e^2 + 42*A*a*c*d^2
*e^3 + 15*B*a^2*d*e^4 + (32*B*c^2*d^4*e - 28*A*c^2*d^3*e^2 - 53*B*a*c*d^2*
e^3 + 42*A*a*c*d*e^4 + 15*B*a^2*e^5)*x)*sqrt(-c*e)*weierstrassPInverse(4/3
*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x
+ d)/e) + 12*(32*B*c^2*d^4*e - 28*A*c^2*d^3*e^2 - 29*B*a*c*d^2*e^3 + 21*A*
a*c*d*e^4 + (32*B*c^2*d^3*e^2 - 28*A*c^2*d^2*e^3 - 29*B*a*c*d*e^4 + 21*A*a
*c*e^5)*x)*sqrt(-c*e)*weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27
*(c*d^3 - 9*a*d*e^2)/(c*e^3), weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c
*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e)) + 3*(5*B*c^2
*e^5*x^3 + 64*B*c^2*d^3*e^2 - 56*A*c^2*d^2*e^3 - 50*B*a*c*d*e^4 + 35*A*a*c
*e^5 - (8*B*c^2*d*e^4 - 7*A*c^2*e^5)*x^2 + (16*B*c^2*d^2*e^3 - 14*A*c^2*d*
e^4 - 15*B*a*c*e^5)*x)*sqrt(-c*x^2 + a)*sqrt(e*x + d))/(c*e^7*x + c*d*e^6)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{(A+Bx)\left(a-cx^2\right)^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x**2+a)**(3/2)/(e*x+d)**(3/2),x)
```

output
```
Integral((A + B*x)*(a - c*x**2)**(3/2)/(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{\left(-cx^2+a\right)^{\frac{3}{2}}(Bx+A)}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^2 + a)^(3/2)*(B*x + A)/(e*x + d)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{\left(-cx^2+a\right)^{\frac{3}{2}}(Bx+A)}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*x^2 + a)^(3/2)*(B*x + A)/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{\left(a-c\,x^2\right)^{3/2}(A+B\,x)}{(d+e\,x)^{3/2}}\,dx$$

input
```
int(((a - c*x^2)^(3/2)*(A + B*x))/(d + e*x)^(3/2),x)
```

output
```
int(((a - c*x^2)^(3/2)*(A + B*x))/(d + e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(3/2),x)
```

output
```
(2*( - 10*sqrt(d + e*x)*sqrt(a - c*x**2)*a**2*b*e**3 - 7*sqrt(d + e*x)*sqr
t(a - c*x**2)*a**2*c*d*e**2 + 8*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*c*d**2*
e + 15*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*c*d*e**2*x + 14*sqrt(d + e*x)*sq
rt(a - c*x**2)*a*c**2*d**2*e*x - 7*sqrt(d + e*x)*sqrt(a - c*x**2)*a*c**2*d
*e**2*x**2 - 16*sqrt(d + e*x)*sqrt(a - c*x**2)*b*c**2*d**3*x + 8*sqrt(d +
e*x)*sqrt(a - c*x**2)*b*c**2*d**2*e*x**2 - 5*sqrt(d + e*x)*sqrt(a - c*x**2
)*b*c**2*d*e**2*x**3 - 5*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**2
 + 2*a*d*e*x + a*e**2*x**2 - c*d**2*x**2 - 2*c*d*e*x**3 - c*e**2*x**4),x)*
a**2*b*c*d*e**4 - 5*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**2 + 2*
a*d*e*x + a*e**2*x**2 - c*d**2*x**2 - 2*c*d*e*x**3 - c*e**2*x**4),x)*a**2*
b*c*e**5*x - 28*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**2 + 2*a*d*
e*x + a*e**2*x**2 - c*d**2*x**2 - 2*c*d*e*x**3 - c*e**2*x**4),x)*a**2*c**2
*d**2*e**3 - 28*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**2 + 2*a*d*
e*x + a*e**2*x**2 - c*d**2*x**2 - 2*c*d*e*x**3 - c*e**2*x**4),x)*a**2*c**2
*d*e**4*x + 37*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**2 + 2*a*d*e
*x + a*e**2*x**2 - c*d**2*x**2 - 2*c*d*e*x**3 - c*e**2*x**4),x)*a*b*c**2*d
**3*e**2 + 37*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**2 + 2*a*d*e*
x + a*e**2*x**2 - c*d**2*x**2 - 2*c*d*e*x**3 - c*e**2*x**4),x)*a*b*c**2*d*
*2*e**3*x + 28*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**2 + 2*a*d*e
*x + a*e**2*x**2 - c*d**2*x**2 - 2*c*d*e*x**3 - c*e**2*x**4),x)*a*c**3*...
```

3.270 $\int \frac{(A+Bx)(a-cx^2)^{3/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 432

$$
\begin{aligned}
\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx =
&-\frac{4(9aBe^2-4cd(8Bd-5Ae)-ce(8Bd-5Ae)x)\sqrt{a-cx^2}}{15e^4\sqrt{d+ex}} \\
&+\frac{2(8Bd-5Ae+3Bex)\left(a-cx^2\right)^{3/2}}{15e^2(d+ex)^{3/2}} \\
&+\frac{8\sqrt{a}\sqrt{c}(9aBe^2-4cd(8Bd-5Ae))\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{15e^5\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} \\
&+\frac{8\sqrt{a}\sqrt{c}(32Bcd^3-20Acd^2e-17aBde^2+5aAe^3)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\cdots}{\sqrt{cd\cdots}}\right.}{15e^5\sqrt{d+ex}\sqrt{a-cx^2}}
\end{aligned}
$$

output
```
-4/15*(9*B*a*e^2-4*c*d*(-5*A*e+8*B*d)-c*e*(-5*A*e+8*B*d)*x)*(-c*x^2+a)^(1/
2)/e^4/(e*x+d)^(1/2)+2/15*(3*B*e*x-5*A*e+8*B*d)*(-c*x^2+a)^(3/2)/e^2/(e*x+
d)^(3/2)+8/15*a^(1/2)*c^(1/2)*(9*B*a*e^2-4*c*d*(-5*A*e+8*B*d))*(e*x+d)^(1/
2)*(1-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(
1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/e^5/(c^(1/2)*(e*x+d)/(c^(1/2
)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)+8/15*a^(1/2)*c^(1/2)*(5*A*a*e^3-20*
A*c*d^2*e-17*B*a*d*e^2+32*B*c*d^3)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))
^(1/2)*(1-c*x^2/a)^(1/2)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2)
,2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/e^5/(e*x+d)^(1/2)/(-c*x^
2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.82 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{a-cx^2}\left(-\frac{2\left(5aAe^3+5aBe^2(2d+3ex)+5Ace\left(8d^2+10dex+e^2x^2\right)-Bc\left(64d^3+80d^2ex+8de^2x^2-\right.\right.}{e^4(d+ex)}\right.}{}$$

input
```
Integrate[((A + B*x)*(a - c*x^2)^(3/2))/(d + e*x)^(5/2),x]
```

output
```
(Sqrt[a - c*x^2]*((-2*(5*a*A*e^3 + 5*a*B*e^2*(2*d + 3*e*x) + 5*A*c*e*(8*d^
2 + 10*d*e*x + e^2*x^2) - B*c*(64*d^3 + 80*d^2*e*x + 8*d*e^2*x^2 - 3*e^3*x
^3)))/(e^4*(d + e*x)) - (8*(e^2*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(32*B*c*d^2
 - 20*A*c*d*e - 9*a*B*e^2)*(-a + c*x^2) + I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e
)*(-32*B*c*d^2 + 20*A*c*d*e + 9*a*B*e^2)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d
 + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*El
lipticE[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*
d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)] + I*Sqrt[a]*Sqrt[c]*e*(-32*B*c*d^2
 + 8*Sqrt[a]*B*Sqrt[c]*d*e + 20*A*c*d*e + 9*a*B*e^2 - 5*Sqrt[a]*A*Sqrt[c]*
e^2)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c]
 - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d + (Sqrt[a]
*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*
e)]))/(e^6*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2))))/(15*Sqrt[d + e*x
])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {681, 25, 681, 25, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-cx^2\right)^{3/2} (A+Bx)}{(d+ex)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 681}$$

$$\frac{2\left(a-cx^2\right)^{3/2} (-5Ae+8Bd+3Bex)}{15e^2(d+ex)^{3/2}}-\frac{2\int -\frac{(3aBe+c(8Bd-5Ae)x)\sqrt{a-cx^2}}{(d+ex)^{3/2}}dx}{5e^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\int \frac{(3aBe+c(8Bd-5Ae)x)\sqrt{a-cx^2}}{(d+ex)^{3/2}}dx}{5e^2}+\frac{2\left(a-cx^2\right)^{3/2} (-5Ae+8Bd+3Bex)}{15e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{ 681}$$

$$\frac{2\left(-\frac{2\int -\frac{c\left(ae(8Bd-5Ae)-\left(9aBe^2-4cd(8Bd-5Ae)\right)x\right)}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{3e^2}-\frac{2\sqrt{a-cx^2}(9aBe^2-cex(8Bd-5Ae)-4cd(8Bd-5Ae))}{3e^2\sqrt{d+ex}}\right)}{5e^2}+\frac{2\left(a-cx^2\right)^{3/2}(-5Ae+8Bd+3Bex)}{15e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(\frac{2\int \frac{c\left(ae(8Bd-5Ae)-\left(9aBe^2-4cd(8Bd-5Ae)\right)x\right)}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{3e^2}-\frac{2\sqrt{a-cx^2}(9aBe^2-cex(8Bd-5Ae)-4cd(8Bd-5Ae))}{3e^2\sqrt{d+ex}}\right)}{5e^2}+\frac{2\left(a-cx^2\right)^{3/2}(-5Ae+8Bd+3Bex)}{15e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\frac{2c\int \frac{ae(8Bd-5Ae)-\left(9aBe^2-4cd(8Bd-5Ae)\right)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{3e^2}-\frac{2\sqrt{a-cx^2}(9aBe^2-cex(8Bd-5Ae)-4cd(8Bd-5Ae))}{3e^2\sqrt{d+ex}}\right)}{5e^2}+\frac{2\left(a-cx^2\right)^{3/2}(-5Ae+8Bd+3Bex)}{15e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \; 600$$

$$\frac{2\left(\frac{2c\left(-\frac{\left(5aAe^3-17aBde^2-20Acd^2e+32Bcd^3\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{\left(9aBe^2-4cd(8Bd-5Ae)\right)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}\right)}{3e^2}-\frac{2\sqrt{a-cx^2}(9aBe^2-cex(8Bd-5Ae}{3e^2\sqrt{d+ex}}\right.}{5e^2}$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-5Ae+8Bd+3Bex)}{15e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \; 509$$

$$\frac{2\left(\frac{2c\left(-\frac{\left(5aAe^3-17aBde^2-20Acd^2e+32Bcd^3\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{\sqrt{1-\frac{cx^2}{a}}\left(9aBe^2-4cd(8Bd-5Ae)\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}\right)}{3e^2}-\frac{2\sqrt{a-cx^2}\left(9aBe^2-cex(8\right.}{3e^2}\right.}{5e^2}$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-5Ae+8Bd+3Bex)}{15e^2(d+ex)^{3/2}}$$

$\Big\downarrow$ 508

$$\frac{2\left(\frac{2c\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aBe^2-4cd(8Bd-5Ae)\right)\int \frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{\left(5aAe^3-17aBde^2-20Acd^2e+32Bcd^3\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{3e^2}-\right.}{5e^2}$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-5Ae+8Bd+3Bex)}{15e^2(d+ex)^{3/2}}$$

$\Big\downarrow$ 327

$$\frac{2\left(\frac{2c\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aBe^2-4cd(8Bd-5Ae)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{\left(5aAe^3-17aBde^2-20Acd^2e+32Bcd^3\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{3e^2}-\right.}{5e^2}$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-5Ae+8Bd+3Bex)}{15e^2(d+ex)^{3/2}}$$

$\downarrow$ 512

$$\frac{2\left(\frac{2c\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aBe^2-4cd(8Bd-5Ae)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{\sqrt{1-\frac{cx^2}{a}}\left(5aAe^3-17aBde^2-20Acd^2e+32Bcd^3\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}}{e\sqrt{a-cx^2}}\right.}{3e^2}\right.}{5e^2}$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-5Ae+8Bd+3Bex)}{15e^2(d+ex)^{3/2}}$$

$\downarrow$ 511

$$2\left(\frac{2c\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(5aAe^3-17aBde^2-20Acd^2e+32Bcd^3\right)\int \frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}+\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aBe^2-4\right.}{\sqrt{c}e}\right.}{3e^2}}{5e^2}$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-5Ae+8Bd+3Bex)}{15e^2(d+ex)^{3/2}}$$

$\downarrow$ 321

$$2\left(\frac{2c\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(5aAe^3-17aBde^2-20Acd^2e+32Bcd^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}+\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aBe^2-4cd(8Bd-\right.}{\sqrt{c}e\sqrt{a-cx^2}}\right.}{3e^2}}{5e^2}$$

$$\frac{2\left(a-cx^2\right)^{3/2}(-5Ae+8Bd+3Bex)}{15e^2(d+ex)^{3/2}}$$

input
```
Int[((A + B*x)*(a - c*x^2)^(3/2))/(d + e*x)^(5/2),x]
```

output
```
(2*(8*B*d - 5*A*e + 3*B*e*x)*(a - c*x^2)^(3/2))/(15*e^2*(d + e*x)^(3/2)) +
 (2*((-2*(9*a*B*e^2 - 4*c*d*(8*B*d - 5*A*e) - c*e*(8*B*d - 5*A*e)*x)*Sqrt[
a - c*x^2])/(3*e^2*Sqrt[d + e*x]) + (2*c*((2*Sqrt[a]*(9*a*B*e^2 - 4*c*d*(8
*B*d - 5*A*e))*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
 (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]
*e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) + (2
*Sqrt[a]*(32*B*c*d^3 - 20*A*c*d^2*e - 17*a*B*d*e^2 + 5*a*A*e^3)*Sqrt[(Sqrt
[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[ArcS
in[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)
])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[a - c*x^2])))/(3*e^2)))/(5*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 988 vs. $2(362) = 724$.

Time = 15.36 (sec) , antiderivative size = 989, normalized size of antiderivative = 2.29

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c x^2+a)}\left(-\frac{2\left(A a e^3-A c d^2 e-B a d e^2+B c d^3\right)\sqrt{-c e x^3-c d x^2+a e x+a d}}{3 e^6\left(x+\frac{d}{e}\right)^2}-\frac{2\left(-c e x^2+a e\right)\left(8 A c d e+3 B a e^2-11 B c d^2\right)}{3 e^5\sqrt{\left(x+\frac{d}{e}\right)\left(-c e x^2+a e\right)}}-\frac{2 B c x\sqrt{-c}}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(-2/3*(A*a*e^3-A
*c*d^2*e-B*a*d*e^2+B*c*d^3)/e^6*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)/(x+d/e)
^2-2/3*(-c*e*x^2+a*e)*(8*A*c*d*e+3*B*a*e^2-11*B*c*d^2)/e^5/((x+d/e)*(-c*e*
x^2+a*e))^(1/2)-2/5*B*c/e^3*x*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)-2/3*(c^2/
e^3*(A*e-2*B*d)-4/5*B*c^2/e^3*d)/c/e*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)+2*
(-c*(2*A*a*e^3-3*A*c*d^2*e-4*B*a*d*e^2+4*B*c*d^3)/e^5+1/3*(A*a*e^3-A*c*d^2
*e-B*a*d*e^2+B*c*d^3)*c/e^5-1/3*(8*A*c*d*e+3*B*a*e^2-11*B*c*d^2)*c/e^5*d+2
/5*B*c/e^3*a*d+1/3*(c^2/e^3*(A*e-2*B*d)-4/5*B*c^2/e^3*d)/c*a)*(d/e-1/c*(a*
c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e
-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2
)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/
2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+2*(-c/e^
4*(2*A*c*d*e+2*B*a*e^2-3*B*c*d^2)-1/3*(8*A*c*d*e+3*B*a*e^2-11*B*c*d^2)*c/e
^4+3/5*B*c/e^2*a-2/3*(c^2/e^3*(A*e-2*B*d)-4/5*B*c^2/e^3*d)/e*d)*(d/e-1/c*(
a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d
/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1
/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))*EllipticE((
(x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*
c)^(1/2)))^(1/2))+1/c*(a*c)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2))
)^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(4\left(32\,Bcd^5-20\,Acd^4e-33\,Bad^3e^2+15\,Aad^2e^3+\left(32\,Bcd^3e^2-20\,Acd\right.\right.\right.}{}$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
2/45*(4*(32*B*c*d^5 - 20*A*c*d^4*e - 33*B*a*d^3*e^2 + 15*A*a*d^2*e^3 + (32
*B*c*d^3*e^2 - 20*A*c*d^2*e^3 - 33*B*a*d*e^4 + 15*A*a*e^5)*x^2 + 2*(32*B*c
*d^4*e - 20*A*c*d^3*e^2 - 33*B*a*d^2*e^3 + 15*A*a*d*e^4)*x)*sqrt(-c*e)*wei
erstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/
(c*e^3), 1/3*(3*e*x + d)/e) + 12*(32*B*c*d^4*e - 20*A*c*d^3*e^2 - 9*B*a*d^
2*e^3 + (32*B*c*d^2*e^3 - 20*A*c*d*e^4 - 9*B*a*e^5)*x^2 + 2*(32*B*c*d^3*e^
2 - 20*A*c*d^2*e^3 - 9*B*a*d*e^4)*x)*sqrt(-c*e)*weierstrassZeta(4/3*(c*d^2
 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), weierstrassPInvers
e(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3
*e*x + d)/e)) - 3*(3*B*c*e^5*x^3 - 64*B*c*d^3*e^2 + 40*A*c*d^2*e^3 + 10*B*
a*d*e^4 + 5*A*a*e^5 - (8*B*c*d*e^4 - 5*A*c*e^5)*x^2 - 5*(16*B*c*d^2*e^3 -
10*A*c*d*e^4 - 3*B*a*e^5)*x)*sqrt(-c*x^2 + a)*sqrt(e*x + d))/(e^8*x^2 + 2*
d*e^7*x + d^2*e^6)
```

Sympy [F]

$$\int \frac{(A+Bx)\,(a-cx^2)^{3/2}}{(d+ex)^{5/2}}\,dx=\int \frac{(A+Bx)\,(a-cx^2)^{\frac{3}{2}}}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x**2+a)**(3/2)/(e*x+d)**(5/2),x)
```

output
```
Integral((A + B*x)*(a - c*x**2)**(3/2)/(d + e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\,(a-cx^2)^{3/2}}{(d+ex)^{5/2}}\,dx=\int \frac{(-cx^2+a)^{\frac{3}{2}}(Bx+A)}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^2 + a)^(3/2)*(B*x + A)/(e*x + d)^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)(a-cx^2)^{3/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(-cx^2+a)^{\frac{3}{2}}(Bx+A)}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((-c*x^2 + a)^(3/2)*(B*x + A)/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(a-cx^2)^{3/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(a-c\,x^2)^{3/2}\,(A+B\,x)}{(d+e\,x)^{5/2}}\,dx$$

input
```
int(((a - c*x^2)^(3/2)*(A + B*x))/(d + e*x)^(5/2),x)
```

output
```
int(((a - c*x^2)^(3/2)*(A + B*x))/(d + e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)(a-cx^2)^{3/2}}{(d+ex)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(5/2),x)
```

output
```
(2*( - 18*sqrt(d + e*x)*sqrt(a - c*x**2)*a**2*b*e**3 - 25*sqrt(d + e*x)*sq
rt(a - c*x**2)*a**2*c*d*e**2 + 40*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*c*d**
2*e + 21*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*c*d*e**2*x + 30*sqrt(d + e*x)*
sqrt(a - c*x**2)*a*c**2*d**2*e*x - 5*sqrt(d + e*x)*sqrt(a - c*x**2)*a*c**2
*d*e**2*x**2 - 48*sqrt(d + e*x)*sqrt(a - c*x**2)*b*c**2*d**3*x + 8*sqrt(d
+ e*x)*sqrt(a - c*x**2)*b*c**2*d**2*e*x**2 - 3*sqrt(d + e*x)*sqrt(a - c*x*
*2)*b*c**2*d*e**2*x**3 + 9*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d*
*3 + 3*a*d**2*e*x + 3*a*d*e**2*x**2 + a*e**3*x**3 - c*d**3*x**2 - 3*c*d**2
*e*x**3 - 3*c*d*e**2*x**4 - c*e**3*x**5),x)*a**2*b*c*d**2*e**4 + 18*int((s
qrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**3 + 3*a*d**2*e*x + 3*a*d*e**2*x*
*2 + a*e**3*x**3 - c*d**3*x**2 - 3*c*d**2*e*x**3 - 3*c*d*e**2*x**4 - c*e**
3*x**5),x)*a**2*b*c*d*e**5*x + 9*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)
/(a*d**3 + 3*a*d**2*e*x + 3*a*d*e**2*x**2 + a*e**3*x**3 - c*d**3*x**2 - 3*
c*d**2*e*x**3 - 3*c*d*e**2*x**4 - c*e**3*x**5),x)*a**2*b*c*e**6*x**2 + 27*
int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**3 + 3*a*d**2*e*x + 3*a*d*e
**2*x**2 + a*e**3*x**3 - c*d**3*x**2 - 3*c*d**2*e*x**3 - 3*c*d*e**2*x**4 -
 c*e**3*x**5),x)*a*b*c**2*d**4*e**2 + 54*int((sqrt(d + e*x)*sqrt(a - c*x**
2)*x**2)/(a*d**3 + 3*a*d**2*e*x + 3*a*d*e**2*x**2 + a*e**3*x**3 - c*d**3*x
**2 - 3*c*d**2*e*x**3 - 3*c*d*e**2*x**4 - c*e**3*x**5),x)*a*b*c**2*d**3*e*
*3*x + 27*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**3 + 3*a*d**2*...
```

3.271 $\int \frac{(A+Bx)(a-cx^2)^{3/2}}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 520

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx = -\frac{8c(32Bcd^3-12Acd^2e-29aBde^2+9aAe^3)\sqrt{a-cx^2}}{15e^4\left(cd^2-ae^2\right)\sqrt{d+ex}}$$
$$-\frac{4(5aBe^2-4cd(8Bd-3Ae)-3ce(8Bd-3Ae)x)\sqrt{a-cx^2}}{15e^4(d+ex)^{3/2}}$$
$$+\frac{2(8Bd-3Ae+5Bex)\left(a-cx^2\right)^{3/2}}{15e^2(d+ex)^{5/2}}$$
$$+\frac{8\sqrt{a}c^{3/2}(32Bcd^3-12Acd^2e-29aBde^2+9aAe^3)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{15e^5\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}}$$
$$-\frac{8\sqrt{a}\sqrt{c}(32Bcd^2-12Acde-5aBe^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{15e^5\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
-8/15*c*(9*A*a*e^3-12*A*c*d^2*e-29*B*a*d*e^2+32*B*c*d^3)*(-c*x^2+a)^(1/2)/
e^4/(-a*e^2+c*d^2)/(e*x+d)^(1/2)-4/15*(5*B*a*e^2-4*c*d*(-3*A*e+8*B*d)-3*c*
e*(-3*A*e+8*B*d)*x)*(-c*x^2+a)^(1/2)/e^4/(e*x+d)^(3/2)+2/15*(5*B*e*x-3*A*e
+8*B*d)*(-c*x^2+a)^(3/2)/e^2/(e*x+d)^(5/2)+8/15*a^(1/2)*c^(3/2)*(9*A*a*e^3
-12*A*c*d^2*e-29*B*a*d*e^2+32*B*c*d^3)*(e*x+d)^(1/2)*(1-c*x^2/a)^(1/2)*Ell
ipticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)
*d+a^(1/2)*e))^(1/2))/e^5/(-a*e^2+c*d^2)/(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/
2)*e))^(1/2)/(-c*x^2+a)^(1/2)-8/15*a^(1/2)*c^(1/2)*(-12*A*c*d*e-5*B*a*e^2+
32*B*c*d^2)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2
)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^
(1/2)*d+a^(1/2)*e))^(1/2))/e^5/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.86 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.29

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx = \frac{\sqrt{a-cx^2}\left(\frac{2(d+ex)\left(-5Bc-\frac{3(Bd-Ae)\left(cd^2-ae^2\right)}{(d+ex)^3}+\frac{17Bcd^2-12Acde-5aBe^2}{(d+ex)^2}+\frac{c\left(73Bcd^3-33Acd^2e-6\cdots\right.}{\left(-cd^2+ae^2\right)\cdots}\right.}{e^4}\cdots\right.}{\cdots}$$

input
```
Integrate[((A + B*x)*(a - c*x^2)^(3/2))/(d + e*x)^(7/2),x]
```

output
```
(Sqrt[a - c*x^2]*((2*(d + e*x)*(-5*B*c - (3*(B*d - A*e)*(c*d^2 - a*e^2))/(
d + e*x)^3 + (17*B*c*d^2 - 12*A*c*d*e - 5*a*B*e^2)/(d + e*x)^2 + (c*(73*B*
c*d^3 - 33*A*c*d^2*e - 61*a*B*d*e^2 + 21*a*A*e^3))/((-(c*d^2) + a*e^2)*(d
+ e*x))))/e^4 + (8*c*(e^2*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(32*B*c*d^3 - 12*
A*c*d^2*e - 29*a*B*d*e^2 + 9*a*A*e^3)*(a - c*x^2) + I*Sqrt[c]*(Sqrt[c]*d -
 Sqrt[a]*e)*(32*B*c*d^3 - 12*A*c*d^2*e - 29*a*B*d*e^2 + 9*a*A*e^3)*Sqrt[(e
*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d +
 e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]
/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)] + I*Sqrt
[a]*e*(-(Sqrt[c]*d) + Sqrt[a]*e)*(-32*B*c*d^2 - 24*Sqrt[a]*B*Sqrt[c]*d*e +
 12*A*c*d*e + 5*a*B*e^2 + 9*Sqrt[a]*A*Sqrt[c]*e^2)*Sqrt[(e*(Sqrt[a]/Sqrt[c
] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x
)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]],
 (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)]))/(e^6*Sqrt[-d + (Sqrt[a
]*e)/Sqrt[c]]*(-(c*d^2) + a*e^2)*(-a + c*x^2))))/(15*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {680, 25, 27, 681, 25, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^2)^{3/2}(A+Bx)}{(d+ex)^{7/2}}dx$$

$$\Big\downarrow \; 680$$

$$\frac{2\int -\frac{c(3ae(Bd-Ae)+(8Bcd^2-3Aced-5aBe^2)x)\sqrt{a-cx^2}}{(d+ex)^{3/2}}dx}{5e^2(cd^2-ae^2)}+\frac{2(a-cx^2)^{3/2}(-ex(-5aBe^2-6Acde+11Bcd^2)+3Ae(ae^2+cd^2)-2B(4cd^3-ade^2))}{15e^2(d+ex)^{5/2}(cd^2-ae^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(a-cx^2\right)^{3/2}\left(-ex\left(-5aBe^2-6Acde+11Bcd^2\right)+3Ae\left(ae^2+cd^2\right)-2B\left(4cd^3-ade^2\right)\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}-\frac{2\int\frac{c\left(3ae(Bd-Ae)+\left(8Bcd^2-3Aced-5aBe^2\right)x\right)\sqrt{a-cx^2}}{(d+ex)^{3/2}}dx}{5e^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow 27$$

$$\frac{2\left(a-cx^2\right)^{3/2}\left(-ex\left(-5aBe^2-6Acde+11Bcd^2\right)+3Ae\left(ae^2+cd^2\right)-2B\left(4cd^3-ade^2\right)\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}-\frac{2c\int\frac{\left(3ae(Bd-Ae)+\left(8Bcd^2-3Aced-5aBe^2\right)x\right)\sqrt{a-cx^2}}{(d+ex)^{3/2}}dx}{5e^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow 681$$

$$\frac{2\left(a-cx^2\right)^{3/2}\left(-ex\left(-5aBe^2-6Acde+11Bcd^2\right)+3Ae\left(ae^2+cd^2\right)-2B\left(4cd^3-ade^2\right)\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}-\frac{2c\left(\frac{2\sqrt{a-cx^2}\left(ex\left(-5aBe^2-3Acde+8Bcd^2\right)+9aAe^3-29aBde^2-12Acd^2e+32Bcd^3\right)}{3e^2\sqrt{d+ex}}-\frac{2\int-\frac{ae\left(8Bcd^2-3Aced-5aBe^2\right)+c\left(32Bcd^3-12Aced^2-29\right.}{\sqrt{d+ex}\sqrt{a-cx^2}}}{3e^2}\right.}{5e^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow 25$$

$$\frac{2\left(a-cx^2\right)^{3/2}\left(-ex\left(-5aBe^2-6Acde+11Bcd^2\right)+3Ae\left(ae^2+cd^2\right)-2B\left(4cd^3-ade^2\right)\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}-\frac{2c\left(\frac{2\int\frac{ae\left(8Bcd^2-3Aced-5aBe^2\right)+c\left(32Bcd^3-12Aced^2-29aBe^2d+9aAe^3\right)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{3e^2}+\frac{2\sqrt{a-cx^2}\left(ex\left(-5aBe^2-3Acde+8Bcd^2\right)+9aAe^3-29aBde^2-12\right.}{3e^2\sqrt{d+ex}}\right.}{5e^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow 600$$

$$\frac{2\left(a-cx^2\right)^{3/2}\left(-ex\left(-5aBe^2-6Acde+11Bcd^2\right)+3Ae\left(ae^2+cd^2\right)-2B\left(4cd^3-ade^2\right)\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}-\frac{2c\left(\frac{2\left(\frac{c\left(9aAe^3-29aBde^2-12Acd^2e+32Bcd^3\right)\int\frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{\left(cd^2-ae^2\right)\left(-5aBe^2-12Acde+32Bcd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{3e^2}+\frac{2\sqrt{a-cx^2}\left(ex\left(-5aBe\right.\right.}{}\right.}{5e^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow 509$$

$$\frac{2\left(a-cx^2\right)^{3/2}\left(-ex\left(-5aBe^2-6Acde+11Bcd^2\right)+3Ae\left(ae^2+cd^2\right)-2B\left(4cd^3-ade^2\right)\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}-$$

$$\frac{2c\left(\frac{2\left(\frac{c\sqrt{1-\frac{cx^2}{a}}\left(9aAe^3-29aBde^2-12Acd^2e+32Bcd^3\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{\left(cd^2-ae^2\right)\left(-5aBe^2-12Acde+32Bcd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{3e^2}+\frac{2\sqrt{a-cx^2}(e\cdots}{\cdots}\right)}{5e^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{508}$$

$$\frac{2\left(a-cx^2\right)^{3/2}\left(-ex\left(-5aBe^2-6Acde+11Bcd^2\right)+3Ae\left(ae^2+cd^2\right)-2B\left(4cd^3-ade^2\right)\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}-$$

$$\frac{2c\left(\frac{2\left(-\frac{\left(cd^2-ae^2\right)\left(-5aBe^2-12Acde+32Bcd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aAe^3-29aBde^2-12Acd^2e+32Bcd^3\right)\int\frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\cdots}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1\cdots}}}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\cdots\right)}{3e^2}\cdots\right)}{5e^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{327}$$

$$\frac{2(a-cx^2)^{3/2}\left(-ex\left(-5aBe^2-6Acde+11Bcd^2\right)+3Ae\left(ae^2+cd^2\right)-2B\left(4cd^3-ade^2\right)\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}-$$

$$\frac{2c\left(\frac{2\left(-\frac{\left(cd^2-ae^2\right)\left(-5aBe^2-12Acde+32Bcd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aAe^3-29aBde^2-12Acd^2e+32Bcd^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{2}}\cdots\right.\right.}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{3e^2}\cdots\right)}{5e^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 512

$$\frac{2(a-cx^2)^{3/2}\left(-ex\left(-5aBe^2-6Acde+11Bcd^2\right)+3Ae\left(ae^2+cd^2\right)-2B\left(4cd^3-ade^2\right)\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}-$$

$$\frac{2c\left(\frac{2\left(-\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\left(-5aBe^2-12Acde+32Bcd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aAe^3-29aBde^2-12Acd^2e+32Bcd^3\right)E\left(\arcsin\cdots\right.}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{3e^2}\cdots\right)}{5e^2\left(cd^2-ae^2\right)}$$

$\downarrow$ 511

$$\frac{2\left(a-cx^2\right)^{3/2}\left(-ex\left(-5aBe^2-6Acde+11Bcd^2\right)+3Ae\left(ae^2+cd^2\right)-2B\left(4cd^3-ade^2\right)\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}-$$

$$\frac{2c\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(-5aBe^2-12Acde+32Bcd^2\right)\int\frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aAe\cdots\right.}{\cdots}\right)}{3e^2}\cdots\right)}{5e^2\left(\cdots\right.}$$

$\downarrow$ 321

$$\frac{2\left(a-cx^2\right)^{3/2}\left(-ex\left(-5aBe^2-6Acde+11Bcd^2\right)+3Ae\left(ae^2+cd^2\right)-2B\left(4cd^3-ade^2\right)\right)}{15e^2(d+ex)^{5/2}\left(cd^2-ae^2\right)}-$$

$$\frac{2c\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(-5aBe^2-12Acde+32Bcd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aAe^3-29aB\cdots\right.}{\cdots}\right)}{3e^2}\cdots\right)}{5e^2\left(cd^2\cdots\right.}$$

input
```
Int[((A + B*x)*(a - c*x^2)^(3/2))/(d + e*x)^(7/2),x]
```

output

```
(2*(3*A*e*(c*d^2 + a*e^2) - 2*B*(4*c*d^3 - a*d*e^2) - e*(11*B*c*d^2 - 6*A*
c*d*e - 5*a*B*e^2)*x)*(a - c*x^2)^(3/2))/(15*e^2*(c*d^2 - a*e^2)*(d + e*x)
^(5/2)) - (2*c*((2*(32*B*c*d^3 - 12*A*c*d^2*e - 29*a*B*d*e^2 + 9*a*A*e^3 +
 e*(8*B*c*d^2 - 3*A*c*d*e - 5*a*B*e^2)*x)*Sqrt[a - c*x^2])/(3*e^2*Sqrt[d +
 e*x]) + (2*((-2*Sqrt[a]*Sqrt[c]*(32*B*c*d^3 - 12*A*c*d^2*e - 29*a*B*d*e^2
 + 9*a*A*e^3)*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(e*Sqrt[(
Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) + (2*Sqrt[a]*
(c*d^2 - a*e^2)*(32*B*c*d^2 - 12*A*c*d*e - 5*a*B*e^2)*Sqrt[(Sqrt[c]*(d + e
*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1
- (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c
]*e*Sqrt[d + e*x]*Sqrt[a - c*x^2])))/(3*e^2)))/(5*e^2*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508

```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(444) = 888$.

Time = 17.20 (sec) , antiderivative size = 973, normalized size of antiderivative = 1.87

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c\,x^2+a)}\left(-\frac{2\left(Aa\,e^3-Ac\,d^2e-Bad\,e^2+Bc\,d^3\right)\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}{5e^7\left(x+\frac{d}{e}\right)^3}-\frac{2\left(12Acde+5Ba\,e^2-17Bc\,d^2\right)\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}{15e^6\left(x+\frac{d}{e}\right)^2}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(-2/5*(A*a*e^3-A
*c*d^2*e-B*a*d*e^2+B*c*d^3)/e^7*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)/(x+d/e)
^3-2/15*(12*A*c*d*e+5*B*a*e^2-17*B*c*d^2)/e^6*(-c*e*x^3-c*d*x^2+a*e*x+a*d)
^(1/2)/(x+d/e)^2+2/15*(-c*e*x^2+a*e)/e^5/(a*e^2-c*d^2)*c*(21*A*a*e^3-33*A*
c*d^2*e-61*B*a*d*e^2+73*B*c*d^3)/((x+d/e)*(-c*e*x^2+a*e))^(1/2)-2/3*B/e^4*
c*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)+2*(-c*(3*A*c*d*e+2*B*a*e^2-6*B*c*d^2)
/e^5+1/15*c*(12*A*c*d*e+5*B*a*e^2-17*B*c*d^2)/e^5+1/15*c^2/e^5*d*(21*A*a*e
^3-33*A*c*d^2*e-61*B*a*d*e^2+73*B*c*d^3)/(a*e^2-c*d^2)+1/3*B/e^3*c*a)*(d/e
-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2
))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2
)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(
a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+
2*(1/e^4*c^2*(A*e-3*B*d)+1/15*c^2/e^4*(21*A*a*e^3-33*A*c*d^2*e-61*B*a*d*e^
2+73*B*c*d^3)/(a*e^2-c*d^2)-2/3*B/e^4*c^2*d)*(d/e-1/c*(a*c)^(1/2))*((x+d/e
)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2))
)^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x
^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))*EllipticE(((x+d/e)/(d/e-1/c*(a
*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+1
/c*(a*c)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*
(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 885, normalized size of antiderivative = 1.70

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
-2/45*(4*(32*B*c^2*d^7 - 12*A*c^2*d^6*e - 53*B*a*c*d^5*e^2 + 18*A*a*c*d^4*
e^3 + 15*B*a^2*d^3*e^4 + (32*B*c^2*d^4*e^3 - 12*A*c^2*d^3*e^4 - 53*B*a*c*d
^2*e^5 + 18*A*a*c*d*e^6 + 15*B*a^2*e^7)*x^3 + 3*(32*B*c^2*d^5*e^2 - 12*A*c
^2*d^4*e^3 - 53*B*a*c*d^3*e^4 + 18*A*a*c*d^2*e^5 + 15*B*a^2*d*e^6)*x^2 + 3
*(32*B*c^2*d^6*e - 12*A*c^2*d^5*e^2 - 53*B*a*c*d^4*e^3 + 18*A*a*c*d^3*e^4
+ 15*B*a^2*d^2*e^5)*x)*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2
)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 12*(32*
B*c^2*d^6*e - 12*A*c^2*d^5*e^2 - 29*B*a*c*d^4*e^3 + 9*A*a*c*d^3*e^4 + (32*
B*c^2*d^3*e^4 - 12*A*c^2*d^2*e^5 - 29*B*a*c*d*e^6 + 9*A*a*c*e^7)*x^3 + 3*(
32*B*c^2*d^4*e^3 - 12*A*c^2*d^3*e^4 - 29*B*a*c*d^2*e^5 + 9*A*a*c*d*e^6)*x^
2 + 3*(32*B*c^2*d^5*e^2 - 12*A*c^2*d^4*e^3 - 29*B*a*c*d^3*e^4 + 9*A*a*c*d^
2*e^5)*x)*sqrt(-c*e)*weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*
(c*d^3 - 9*a*d*e^2)/(c*e^3), weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*
e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e)) + 3*(64*B*c^2
*d^5*e^2 - 24*A*c^2*d^4*e^3 - 50*B*a*c*d^3*e^4 + 15*A*a*c*d^2*e^5 - 2*B*a^
2*d*e^6 - 3*A*a^2*e^7 + 5*(B*c^2*d^2*e^5 - B*a*c*e^7)*x^3 + (88*B*c^2*d^3*
e^4 - 33*A*c^2*d^2*e^5 - 76*B*a*c*d*e^6 + 21*A*a*c*e^7)*x^2 + (144*B*c^2*d
^4*e^3 - 54*A*c^2*d^3*e^4 - 115*B*a*c*d^2*e^5 + 30*A*a*c*d*e^6 - 5*B*a^2*e
^7)*x)*sqrt(-c*x^2 + a)*sqrt(e*x + d))/(c*d^5*e^6 - a*d^3*e^8 + (c*d^2*e^9
 - a*e^11)*x^3 + 3*(c*d^3*e^8 - a*d*e^10)*x^2 + 3*(c*d^4*e^7 - a*d^2*e^...
```

Sympy [F]

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx = \int \frac{(A+Bx)\left(a-cx^2\right)^{\frac{3}{2}}}{(d+ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x**2+a)**(3/2)/(e*x+d)**(7/2),x)
```

output
```
Integral((A + B*x)*(a - c*x**2)**(3/2)/(d + e*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx=\int \frac{\left(-cx^2+a\right)^{\frac{3}{2}}(Bx+A)}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^2 + a)^(3/2)*(B*x + A)/(e*x + d)^(7/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx=\int \frac{\left(-cx^2+a\right)^{\frac{3}{2}}(Bx+A)}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
integrate((-c*x^2 + a)^(3/2)*(B*x + A)/(e*x + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx=\int \frac{\left(a-c\,x^2\right)^{3/2}\,\left(A+B\,x\right)}{{\left(d+e\,x\right)}^{7/2}}\,dx$$

input
```
int(((a - c*x^2)^(3/2)*(A + B*x))/(d + e*x)^(7/2),x)
```

output
```
int(((a - c*x^2)^(3/2)*(A + B*x))/(d + e*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(A + Bx)\left(a - cx^2\right)^{3/2}}{(d + ex)^{7/2}}\,dx = \text{too large to display}$$

input
```
int((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(7/2),x)
```

output
```
(2*(6*sqrt(d + e*x)*sqrt(a - c*x**2)*a**2*b*e**3 + 21*sqrt(d + e*x)*sqrt(a
 - c*x**2)*a**2*c*d*e**2 - 56*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*c*d**2*e
- 5*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*c*d*e**2*x - 18*sqrt(d + e*x)*sqrt(
a - c*x**2)*a*c**2*d**2*e*x - 3*sqrt(d + e*x)*sqrt(a - c*x**2)*a*c**2*d*e*
*2*x**2 + 48*sqrt(d + e*x)*sqrt(a - c*x**2)*b*c**2*d**3*x + 8*sqrt(d + e*x
)*sqrt(a - c*x**2)*b*c**2*d**2*e*x**2 - sqrt(d + e*x)*sqrt(a - c*x**2)*b*c
**2*d*e**2*x**3 - 9*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**4 + 4*
a*d**3*e*x + 6*a*d**2*e**2*x**2 + 4*a*d*e**3*x**3 + a*e**4*x**4 - c*d**4*x
**2 - 4*c*d**3*e*x**3 - 6*c*d**2*e**2*x**4 - 4*c*d*e**3*x**5 - c*e**4*x**6
),x)*a**2*b*c*d**3*e**4 - 27*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*
d**4 + 4*a*d**3*e*x + 6*a*d**2*e**2*x**2 + 4*a*d*e**3*x**3 + a*e**4*x**4 -
 c*d**4*x**2 - 4*c*d**3*e*x**3 - 6*c*d**2*e**2*x**4 - 4*c*d*e**3*x**5 - c*
e**4*x**6),x)*a**2*b*c*d**2*e**5*x - 27*int((sqrt(d + e*x)*sqrt(a - c*x**2
)*x**2)/(a*d**4 + 4*a*d**3*e*x + 6*a*d**2*e**2*x**2 + 4*a*d*e**3*x**3 + a*
e**4*x**4 - c*d**4*x**2 - 4*c*d**3*e*x**3 - 6*c*d**2*e**2*x**4 - 4*c*d*e**
3*x**5 - c*e**4*x**6),x)*a**2*b*c*d*e**6*x**2 - 9*int((sqrt(d + e*x)*sqrt(
a - c*x**2)*x**2)/(a*d**4 + 4*a*d**3*e*x + 6*a*d**2*e**2*x**2 + 4*a*d*e**3
*x**3 + a*e**4*x**4 - c*d**4*x**2 - 4*c*d**3*e*x**3 - 6*c*d**2*e**2*x**4 -
 4*c*d*e**3*x**5 - c*e**4*x**6),x)*a**2*b*c*e**7*x**3 - 36*int((sqrt(d + e
*x)*sqrt(a - c*x**2)*x**2)/(a*d**4 + 4*a*d**3*e*x + 6*a*d**2*e**2*x**2 ...
```

3.272 $\int \frac{(A+Bx)(a-cx^2)^{3/2}}{(d+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 654

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx = \frac{8c(32Bcd^3-4Acd^2e-33aBde^2+5aAe^3)\sqrt{a-cx^2}}{35e^4\left(cd^2-ae^2\right)(d+ex)^{3/2}}$$
$$-\frac{8c(4Acde(cd^2-2ae^2)-B(32c^2d^4-57acd^2e^2+21a^2e^4))\sqrt{a-cx^2}}{35e^4\left(cd^2-ae^2\right)^2\sqrt{d+ex}}$$
$$-\frac{12(7aBe^2+4cd(8Bd-Ae)+5ce(8Bd-Ae)x)\sqrt{a-cx^2}}{35e^4(d+ex)^{5/2}}$$
$$+\frac{2(8Bd-Ae+7Bex)\left(a-cx^2\right)^{3/2}}{7e^2(d+ex)^{7/2}}$$
$$+\frac{8\sqrt{a}c^{3/2}(4Acde(cd^2-2ae^2)-B(32c^2d^4-57acd^2e^2+21a^2e^4))\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)\right.}{35e^5\left(cd^2-ae^2\right)^2\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}}$$
$$+\frac{8\sqrt{a}c^{3/2}(32Bcd^3-4Acd^2e-33aBde^2+5aAe^3)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{}}{\sqrt{cd-}}\right.}{35e^5\left(cd^2-ae^2\right)\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
8/35*c*(5*A*a*e^3-4*A*c*d^2*e-33*B*a*d*e^2+32*B*c*d^3)*(-c*x^2+a)^(1/2)/e^
4/(-a*e^2+c*d^2)/(e*x+d)^(3/2)-8/35*c*(4*A*c*d*e*(-2*a*e^2+c*d^2)-B*(21*a^
2*e^4-57*a*c*d^2*e^2+32*c^2*d^4))*(-c*x^2+a)^(1/2)/e^4/(-a*e^2+c*d^2)^2/(e
*x+d)^(1/2)-12/35*(7*B*a*e^2+4*c*d*(-A*e+8*B*d)+5*c*e*(-A*e+8*B*d)*x)*(-c*
x^2+a)^(1/2)/e^4/(e*x+d)^(5/2)+2/7*(7*B*e*x-A*e+8*B*d)*(-c*x^2+a)^(3/2)/e^
2/(e*x+d)^(7/2)+8/35*a^(1/2)*c^(3/2)*(4*A*c*d*e*(-2*a*e^2+c*d^2)-B*(21*a^2
*e^4-57*a*c*d^2*e^2+32*c^2*d^4))*(e*x+d)^(1/2)*(1-c*x^2/a)^(1/2)*EllipticE
(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(
1/2)*e))^(1/2))/e^5/(-a*e^2+c*d^2)^2/(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e
))^(1/2)/(-c*x^2+a)^(1/2)+8/35*a^(1/2)*c^(3/2)*(5*A*a*e^3-4*A*c*d^2*e-33*B
*a*d*e^2+32*B*c*d^3)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^
2/a)^(1/2)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1
/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/e^5/(-a*e^2+c*d^2)/(e*x+d)^(1/2)/(-c*x
^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.34 (sec) , antiderivative size = 795, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx = \frac{2\sqrt{a-cx^2}\left(4c(4Acde(cd^2-2ae^2)+B(-32c^2d^4+57acd^2e^2-21a^2e^4))+\right.}{}$$

input
```
Integrate[((A + B*x)*(a - c*x^2)^(3/2))/(d + e*x)^(9/2),x]
```

output
```
(2*Sqrt[a - c*x^2]*(4*c*(4*A*c*d*e*(c*d^2 - 2*a*e^2) + B*(-32*c^2*d^4 + 57
*a*c*d^2*e^2 - 21*a^2*e^4)) + c*(-16*A*c*d*e*(c*d^2 - 2*a*e^2) + B*(93*c^2
*d^4 - 158*a*c*d^2*e^2 + 49*a^2*e^4)) - (5*(B*d - A*e)*(c*d^2 - a*e^2)^3)/
(d + e*x)^3 + ((c*d^2 - a*e^2)^2*(23*B*c*d^2 - 16*A*c*d*e - 7*a*B*e^2))/(d
 + e*x)^2 - (c*(c*d^2 - a*e^2)*(47*B*c*d^3 - 19*A*c*d^2*e - 43*a*B*d*e^2 +
 15*a*A*e^3))/(d + e*x) + ((4*I)*c^(3/2)*(Sqrt[c]*d - Sqrt[a]*e)*(-4*A*c*d
*e*(c*d^2 - 2*a*e^2) + B*(32*c^2*d^4 - 57*a*c*d^2*e^2 + 21*a^2*e^4))*Sqrt[
(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d
 + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c
]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)])/(e^2*
Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2)) - ((4*I)*Sqrt[a]*c^(3/2)*(Sqr
t[c]*d - Sqrt[a]*e)*(A*Sqrt[c]*e*(4*c*d^2 + 3*Sqrt[a]*Sqrt[c]*d*e - 5*a*e^
2) + B*(-32*c^(3/2)*d^3 - 24*Sqrt[a]*c*d^2*e + 33*a*Sqrt[c]*d*e^2 + 21*a^(
3/2)*e^3))*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/S
qrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d + (S
qrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sq
rt[a]*e)])/(e*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2))))/(35*e^4*(c*d^
2 - a*e^2)^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 650, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {680, 25, 27, 680, 25, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^2)^{3/2}(A+Bx)}{(d+ex)^{9/2}}dx$$

$$\Big\downarrow \text{680}$$

$$\frac{6\int -\frac{c(5ae(Bd-Ae)+(8Bcd^2-Aced-7aBe^2)x)\sqrt{a-cx^2}}{(d+ex)^{5/2}}dx}{35e^2(cd^2-ae^2)}-\frac{2(a-cx^2)^{3/2}(ex(-7aBe^2-6Acde+13Bcd^2)-5aAe^3-2aBde^2-Acd^2e+8Bcd^3)}{35e^2(d+ex)^{7/2}(cd^2-ae^2)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{6\int \frac{c(5ae(Bd-Ae)+(8Bcd^2-Aced-7aBe^2)x)\sqrt{a-cx^2}}{(d+ex)^{5/2}}dx}{35e^2(cd^2-ae^2)}-\frac{2(a-cx^2)^{3/2}\left(ex(-7aBe^2-6Acde+13Bcd^2)-5aAe^3-2aBde^2-Acd^2e+8Bcd^3\right)}{35e^2(d+ex)^{7/2}(cd^2-ae^2)}$$

$\Big\downarrow$ 27

$$-\frac{6c\int \frac{(5ae(Bd-Ae)+(8Bcd^2-Aced-7aBe^2)x)\sqrt{a-cx^2}}{(d+ex)^{5/2}}dx}{35e^2(cd^2-ae^2)}-\frac{2(a-cx^2)^{3/2}\left(ex(-7aBe^2-6Acde+13Bcd^2)-5aAe^3-2aBde^2-Acd^2e+8Bcd^3\right)}{35e^2(d+ex)^{7/2}(cd^2-ae^2)}$$

$\Big\downarrow$ 680

$$-\frac{6c\left(\frac{2\int -\frac{c\left(ae\left(8Bcd^3-Aced^2-12aBe^2d+5aAe^3\right)-\left(4Acde\left(cd^2-2ae^2\right)-B\left(32c^2d^4-57ace^2d^2+21a^2e^4\right)\right)x\right)}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{3e^2(cd^2-ae^2)}+\frac{2\sqrt{a-cx^2}\left(ex\left(Acde\left(5cd^2-13ae^2\right)\right.\right.}{}\right.}{35e^2(cd^2-ae^2)}-\frac{2(a-cx^2)^{3/2}\left(ex(-7aBe^2-6Acde+13Bcd^2)-5aAe^3-2aBde^2-Acd^2e+8Bcd^3\right)}{35e^2(d+ex)^{7/2}(cd^2-ae^2)}$$

$\Big\downarrow$ 25

$$-\frac{6c\left(\frac{2\sqrt{a-cx^2}\left(ex\left(Acde(5cd^2-13ae^2)-B(21a^2e^4-69acd^2e^2+40c^2d^4)\right)+Ae(-5a^2e^4-7acd^2e^2+4c^2d^4)-B(9a^2de^4-49acd^3e^2+32c^2d^5)\right)}{3e^2(d+ex)^{3/2}(cd^2-ae^2)}-\right.}{35e^2(cd^2-ae^2)}-\frac{2(a-cx^2)^{3/2}\left(ex(-7aBe^2-6Acde+13Bcd^2)-5aAe^3-2aBde^2-Acd^2e+8Bcd^3\right)}{35e^2(d+ex)^{7/2}(cd^2-ae^2)}$$

$\Big\downarrow$ 27

$$-\frac{6c\left(\frac{2\sqrt{a-cx^2}\left(ex\left(Acde(5cd^2-13ae^2)-B(21a^2e^4-69acd^2e^2+40c^2d^4)\right)+Ae(-5a^2e^4-7acd^2e^2+4c^2d^4)-B(9a^2de^4-49acd^3e^2+32c^2d^5)\right)}{3e^2(d+ex)^{3/2}(cd^2-ae^2)}-\right.}{35e^2(cd^2-ae^2)}-\frac{2(a-cx^2)^{3/2}\left(ex(-7aBe^2-6Acde+13Bcd^2)-5aAe^3-2aBde^2-Acd^2e+8Bcd^3\right)}{35e^2(d+ex)^{7/2}(cd^2-ae^2)}$$

$\Big\downarrow$ 600

$$-\frac{6c\left(\frac{2\sqrt{a-cx^2}\left(ex\left(Acde\left(5cd^2-13ae^2\right)-B\left(21a^2e^4-69acd^2e^2+40c^2d^4\right)\right)+Ae\left(-5a^2e^4-7acd^2e^2+4c^2d^4\right)-B\left(9a^2de^4-49acd^3e^2+32c^2d^5\right)\right)}{3e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}-\right.}{35e^2\left(cd\right.}$$

$$\frac{2\left(a-cx^2\right)^{3/2}\left(ex\left(-7aBe^2-6Acde+13Bcd^2\right)-5aAe^3-2aBde^2-Acd^2e+8Bcd^3\right)}{35e^2(d+ex)^{7/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{509}$$

$$-\frac{6c\left(\frac{2\sqrt{a-cx^2}\left(ex\left(Acde\left(5cd^2-13ae^2\right)-B\left(21a^2e^4-69acd^2e^2+40c^2d^4\right)\right)+Ae\left(-5a^2e^4-7acd^2e^2+4c^2d^4\right)-B\left(9a^2de^4-49acd^3e^2+32c^2d^5\right)\right)}{3e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}-\right.}{35e^2}$$

$$\frac{2\left(a-cx^2\right)^{3/2}\left(ex\left(-7aBe^2-6Acde+13Bcd^2\right)-5aAe^3-2aBde^2-Acd^2e+8Bcd^3\right)}{35e^2(d+ex)^{7/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{508}$$

$$-\frac{6c\left(\frac{2\sqrt{a-cx^2}\left(ex\left(Acde\left(5cd^2-13ae^2\right)-B\left(21a^2e^4-69acd^2e^2+40c^2d^4\right)\right)+Ae\left(-5a^2e^4-7acd^2e^2+4c^2d^4\right)-B\left(9a^2de^4-49acd^3e^2+32c^2d^5\right)\right)}{3e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}-\right.}{}$$

$$\frac{2\left(a-cx^2\right)^{3/2}\left(ex\left(-7aBe^2-6Acde+13Bcd^2\right)-5aAe^3-2aBde^2-Acd^2e+8Bcd^3\right)}{35e^2(d+ex)^{7/2}\left(cd^2-ae^2\right)}$$

$\downarrow$ 327

$$-\frac{6c\left(\frac{2\sqrt{a-cx^2}\left(ex\left(Acde\left(5cd^2-13ae^2\right)-B\left(21a^2e^4-69acd^2e^2+40c^2d^4\right)\right)+Ae\left(-5a^2e^4-7acd^2e^2+4c^2d^4\right)-B\left(9a^2de^4-49acd^3e^2+32c^2d^5\right)\right)}{3e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}-\right.}{}$$

$$\frac{2\left(a-cx^2\right)^{3/2}\left(ex\left(-7aBe^2-6Acde+13Bcd^2\right)-5aAe^3-2aBde^2-Acd^2e+8Bcd^3\right)}{35e^2(d+ex)^{7/2}\left(cd^2-ae^2\right)}$$

$\downarrow$ 512

$$-\frac{6c\left(\frac{2\sqrt{a-cx^2}\left(ex\left(Acde\left(5cd^2-13ae^2\right)-B\left(21a^2e^4-69acd^2e^2+40c^2d^4\right)\right)+Ae\left(-5a^2e^4-7acd^2e^2+4c^2d^4\right)-B\left(9a^2de^4-49acd^3e^2+32c^2d^5\right)\right)}{3e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}-\right.}{}$$

$$\frac{2\left(a-cx^2\right)^{3/2}\left(ex\left(-7aBe^2-6Acde+13Bcd^2\right)-5aAe^3-2aBde^2-Acd^2e+8Bcd^3\right)}{35e^2(d+ex)^{7/2}\left(cd^2-ae^2\right)}$$

$\downarrow$ 511

$$-\frac{6c\left(\frac{2\sqrt{a-cx^2}\left(ex\left(Acde\left(5cd^2-13ae^2\right)-B\left(21a^2e^4-69acd^2e^2+40c^2d^4\right)\right)+Ae\left(-5a^2e^4-7acd^2e^2+4c^2d^4\right)-B\left(9a^2de^4-49acd^3e^2+32c^2d^5\right)\right)}{3e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}-\right.}{}$$

$$\frac{2\left(a-cx^2\right)^{3/2}\left(ex\left(-7aBe^2-6Acde+13Bcd^2\right)-5aAe^3-2aBde^2-Acd^2e+8Bcd^3\right)}{35e^2(d+ex)^{7/2}\left(cd^2-ae^2\right)}$$

$\downarrow$ 321

$$-\frac{6c\left(\frac{2\sqrt{a-cx^2}\left(ex\left(Acde\left(5cd^2-13ae^2\right)-B\left(21a^2e^4-69acd^2e^2+40c^2d^4\right)\right)+Ae\left(-5a^2e^4-7acd^2e^2+4c^2d^4\right)-B\left(9a^2de^4-49acd^3e^2+32c^2d^5\right)\right)}{3e^2(d+ex)^{3/2}\left(cd^2-ae^2\right)}-\right.}{}$$

$$\frac{2\left(a-cx^2\right)^{3/2}\left(ex\left(-7aBe^2-6Acde+13Bcd^2\right)-5aAe^3-2aBde^2-Acd^2e+8Bcd^3\right)}{35e^2(d+ex)^{7/2}\left(cd^2-ae^2\right)}$$

input
```
Int[((A + B*x)*(a - c*x^2)^(3/2))/(d + e*x)^(9/2),x]
```

output
```
(-2*(8*B*c*d^3 - A*c*d^2*e - 2*a*B*d*e^2 - 5*a*A*e^3 + e*(13*B*c*d^2 - 6*A
*c*d*e - 7*a*B*e^2)*x)*(a - c*x^2)^(3/2))/(35*e^2*(c*d^2 - a*e^2)*(d + e*x
)^(7/2)) - (6*c*((2*(A*e*(4*c^2*d^4 - 7*a*c*d^2*e^2 - 5*a^2*e^4) - B*(32*c
^2*d^5 - 49*a*c*d^3*e^2 + 9*a^2*d*e^4) + e*(A*c*d*e*(5*c*d^2 - 13*a*e^2) -
 B*(40*c^2*d^4 - 69*a*c*d^2*e^2 + 21*a^2*e^4))*x)*Sqrt[a - c*x^2])/(3*e^2*
(c*d^2 - a*e^2)*(d + e*x)^(3/2)) - (2*c*((2*Sqrt[a]*(4*A*c*d*e*(c*d^2 - 2*
a*e^2) - B*(32*c^2*d^4 - 57*a*c*d^2*e^2 + 21*a^2*e^4))*Sqrt[d + e*x]*Sqrt[
1 - (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2
*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[
c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) + (2*Sqrt[a]*(c*d^2 - a*e^2)*(32*B*c*d
^3 - 4*A*c*d^2*e - 33*a*B*d*e^2 + 5*a*A*e^3)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqr
t[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c
]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[
d + e*x]*Sqrt[a - c*x^2])))/(3*e^2*(c*d^2 - a*e^2))))/(35*e^2*(c*d^2 - a*e
^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

Maple [A] (verified)

Time = 9.36 (sec) , antiderivative size = 1069, normalized size of antiderivative = 1.63

method	result	size
elliptic	Expression too large to display	1069
default	Expression too large to display	11309

input

```
int((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(9/2),x,method=_RETURNVERBOSE)
```

output

```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(-2/7*(A*a*e^3-A
*c*d^2*e-B*a*d*e^2+B*c*d^3)/e^8*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)/(x+d/e)
^4-2/35*(16*A*c*d*e+7*B*a*e^2-23*B*c*d^2)/e^7*(-c*e*x^3-c*d*x^2+a*e*x+a*d)
^(1/2)/(x+d/e)^3+2/35*c*(15*A*a*e^3-19*A*c*d^2*e-43*B*a*d*e^2+47*B*c*d^3)/
(a*e^2-c*d^2)/e^6*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)/(x+d/e)^2+2/35*(-c*e*
x^2+a*e)/e^5/(a*e^2-c*d^2)^2*c*(32*A*a*c*d*e^3-16*A*c^2*d^3*e+49*B*a^2*e^4
-158*B*a*c*d^2*e^2+93*B*c^2*d^4)/((x+d/e)*(-c*e*x^2+a*e))^(1/2)+2*(c^2*(A*
e-4*B*d)/e^5-1/35*c^2*(15*A*a*e^3-19*A*c*d^2*e-43*B*a*d*e^2+47*B*c*d^3)/(a
*e^2-c*d^2)/e^5+1/35*c^2/e^5*d*(32*A*a*c*d*e^3-16*A*c^2*d^3*e+49*B*a^2*e^4
-158*B*a*c*d^2*e^2+93*B*c^2*d^4)/(a*e^2-c*d^2)^2)*(d/e-1/c*(a*c)^(1/2))*((
x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(
1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-
c*d*x^2+a*e*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),(
(-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+2*(B*c^2/e^4+1/35*c^
2/e^4*(32*A*a*c*d*e^3-16*A*c^2*d^3*e+49*B*a^2*e^4-158*B*a*c*d^2*e^2+93*B*c
^2*d^4)/(a*e^2-c*d^2)^2)*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/
2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c
)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*
((-d/e-1/c*(a*c)^(1/2))*EllipticE(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((
-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+1/c*(a*c)^(1/2)*El...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1392 vs. $2(572) = 1144$.

Time = 0.21 (sec) , antiderivative size = 1392, normalized size of antiderivative = 2.13

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

output
```
2/105*(4*(32*B*c^3*d^9 - 4*A*c^3*d^8*e - 81*B*a*c^2*d^7*e^2 + 11*A*a*c^2*d
^6*e^3 + 57*B*a^2*c*d^5*e^4 - 15*A*a^2*c*d^4*e^5 + (32*B*c^3*d^5*e^4 - 4*A
*c^3*d^4*e^5 - 81*B*a*c^2*d^3*e^6 + 11*A*a*c^2*d^2*e^7 + 57*B*a^2*c*d*e^8
- 15*A*a^2*c*e^9)*x^4 + 4*(32*B*c^3*d^6*e^3 - 4*A*c^3*d^5*e^4 - 81*B*a*c^2
*d^4*e^5 + 11*A*a*c^2*d^3*e^6 + 57*B*a^2*c*d^2*e^7 - 15*A*a^2*c*d*e^8)*x^3
 + 6*(32*B*c^3*d^7*e^2 - 4*A*c^3*d^6*e^3 - 81*B*a*c^2*d^5*e^4 + 11*A*a*c^2
*d^4*e^5 + 57*B*a^2*c*d^3*e^6 - 15*A*a^2*c*d^2*e^7)*x^2 + 4*(32*B*c^3*d^8*
e - 4*A*c^3*d^7*e^2 - 81*B*a*c^2*d^6*e^3 + 11*A*a*c^2*d^5*e^4 + 57*B*a^2*c
*d^4*e^5 - 15*A*a^2*c*d^3*e^6)*x)*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^
2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e
) + 12*(32*B*c^3*d^8*e - 4*A*c^3*d^7*e^2 - 57*B*a*c^2*d^6*e^3 + 8*A*a*c^2*
d^5*e^4 + 21*B*a^2*c*d^4*e^5 + (32*B*c^3*d^4*e^5 - 4*A*c^3*d^3*e^6 - 57*B*
a*c^2*d^2*e^7 + 8*A*a*c^2*d*e^8 + 21*B*a^2*c*e^9)*x^4 + 4*(32*B*c^3*d^5*e^
4 - 4*A*c^3*d^4*e^5 - 57*B*a*c^2*d^3*e^6 + 8*A*a*c^2*d^2*e^7 + 21*B*a^2*c*
d*e^8)*x^3 + 6*(32*B*c^3*d^6*e^3 - 4*A*c^3*d^5*e^4 - 57*B*a*c^2*d^4*e^5 +
8*A*a*c^2*d^3*e^6 + 21*B*a^2*c*d^2*e^7)*x^2 + 4*(32*B*c^3*d^7*e^2 - 4*A*c^
3*d^6*e^3 - 57*B*a*c^2*d^5*e^4 + 8*A*a*c^2*d^4*e^5 + 21*B*a^2*c*d^3*e^6)*x
)*sqrt(-c*e)*weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 -
 9*a*d*e^2)/(c*e^3), weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8
/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e)) + 3*(64*B*c^3*d^7*...
```

Sympy [F]

$$\int \frac{(A+Bx)(a-cx^2)^{3/2}}{(d+ex)^{9/2}}\,dx = \int \frac{(A+Bx)(a-cx^2)^{\frac{3}{2}}}{(d+ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x**2+a)**(3/2)/(e*x+d)**(9/2),x)
```

output
```
Integral((A + B*x)*(a - c*x**2)**(3/2)/(d + e*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)(a-cx^2)^{3/2}}{(d+ex)^{9/2}}\,dx = \int \frac{(-cx^2+a)^{\frac{3}{2}}(Bx+A)}{(ex+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(9/2),x, algorithm="maxima")
```

output
```
integrate((-c*x^2 + a)^(3/2)*(B*x + A)/(e*x + d)^(9/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)(a-cx^2)^{3/2}}{(d+ex)^{9/2}}\,dx = \int \frac{(-cx^2+a)^{\frac{3}{2}}(Bx+A)}{(ex+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(9/2),x, algorithm="giac")
```

output
```
integrate((-c*x^2 + a)^(3/2)*(B*x + A)/(e*x + d)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx = \int \frac{\left(a-c\,x^2\right)^{3/2}\,(A+B\,x)}{\left(d+e\,x\right)^{9/2}}\,dx$$

input
```
int(((a - c*x^2)^(3/2)*(A + B*x))/(d + e*x)^(9/2),x)
```

output
```
int(((a - c*x^2)^(3/2)*(A + B*x))/(d + e*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\left(a-cx^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx = \text{too large to display}$$

input
```
int((B*x+A)*(-c*x^2+a)^(3/2)/(e*x+d)^(9/2),x)
```

output
```
(2*(2*sqrt(d + e*x)*sqrt(a - c*x**2)*a**2*b*e**3 - 3*sqrt(d + e*x)*sqrt(a
- c*x**2)*a**2*c*d*e**2 + 24*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*c*d**2*e -
 sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*c*d*e**2*x + 2*sqrt(d + e*x)*sqrt(a -
c*x**2)*a*c**2*d**2*e*x + sqrt(d + e*x)*sqrt(a - c*x**2)*a*c**2*d*e**2*x**
2 - 16*sqrt(d + e*x)*sqrt(a - c*x**2)*b*c**2*d**3*x - 8*sqrt(d + e*x)*sqrt
(a - c*x**2)*b*c**2*d**2*e*x**2 - sqrt(d + e*x)*sqrt(a - c*x**2)*b*c**2*d*
e**2*x**3 - 5*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**5 + 5*a*d**4
*e*x + 10*a*d**3*e**2*x**2 + 10*a*d**2*e**3*x**3 + 5*a*d*e**4*x**4 + a*e**
5*x**5 - c*d**5*x**2 - 5*c*d**4*e*x**3 - 10*c*d**3*e**2*x**4 - 10*c*d**2*e
**3*x**5 - 5*c*d*e**4*x**6 - c*e**5*x**7),x)*a**2*b*c*d**4*e**4 - 20*int((
sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d**5 + 5*a*d**4*e*x + 10*a*d**3*e*
*2*x**2 + 10*a*d**2*e**3*x**3 + 5*a*d*e**4*x**4 + a*e**5*x**5 - c*d**5*x**
2 - 5*c*d**4*e*x**3 - 10*c*d**3*e**2*x**4 - 10*c*d**2*e**3*x**5 - 5*c*d*e*
*4*x**6 - c*e**5*x**7),x)*a**2*b*c*d**3*e**5*x - 30*int((sqrt(d + e*x)*sqr
t(a - c*x**2)*x**2)/(a*d**5 + 5*a*d**4*e*x + 10*a*d**3*e**2*x**2 + 10*a*d*
*2*e**3*x**3 + 5*a*d*e**4*x**4 + a*e**5*x**5 - c*d**5*x**2 - 5*c*d**4*e*x*
*3 - 10*c*d**3*e**2*x**4 - 10*c*d**2*e**3*x**5 - 5*c*d*e**4*x**6 - c*e**5*
x**7),x)*a**2*b*c*d**2*e**6*x**2 - 20*int((sqrt(d + e*x)*sqrt(a - c*x**2)*
x**2)/(a*d**5 + 5*a*d**4*e*x + 10*a*d**3*e**2*x**2 + 10*a*d**2*e**3*x**3 +
 5*a*d*e**4*x**4 + a*e**5*x**5 - c*d**5*x**2 - 5*c*d**4*e*x**3 - 10*c*d...
```

3.273 $\int \frac{(A+Bx)(d+ex)^{3/2}}{\sqrt{a-cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 383

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{\sqrt{a-cx^2}}\,dx = -\frac{2(3Bd+5Ae)\sqrt{d+ex}\sqrt{a-cx^2}}{15c} - \frac{2B(d+ex)^{3/2}\sqrt{a-cx^2}}{5c} - \frac{2\sqrt{a}(3Bcd^2+20Acde+9aBe^2)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{15c^{3/2}e\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} + \frac{2\sqrt{a}(3Bd+5Ae)(cd^2-ae^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{15c^{3/2}e\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
-2/15*(5*A*e+3*B*d)*(e*x+d)^(1/2)*(-c*x^2+a)^(1/2)/c-2/5*B*(e*x+d)^(3/2)*(
-c*x^2+a)^(1/2)/c-2/15*a^(1/2)*(20*A*c*d*e+9*B*a*e^2+3*B*c*d^2)*(e*x+d)^(1
/2)*(1-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^
(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/c^(3/2)/e/(c^(1/2)*(e*x+d)/
(c^(1/2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)+2/15*a^(1/2)*(5*A*e+3*B*d)*(
-a*e^2+c*d^2)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1
/2)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(
c^(1/2)*d+a^(1/2)*e))^(1/2))/c^(3/2)/e/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.61 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{\sqrt{a-cx^2}}\,dx = \frac{2\sqrt{a-cx^2}\left(-3Bcd^2-20Acde-9aBe^2-c(d+ex)(6Bd+5Ae+3Bex)+\right.}{}$$

input
```
Integrate[((A + B*x)*(d + e*x)^(3/2))/Sqrt[a - c*x^2],x]
```

output
```
(2*Sqrt[a - c*x^2]*(-3*B*c*d^2 - 20*A*c*d*e - 9*a*B*e^2 - c*(d + e*x)*(6*B
*d + 5*A*e + 3*B*e*x) + (I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(3*B*c*d^2 + 20
*A*c*d*e + 9*a*B*e^2)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((S
qrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[S
qrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqr
t[c]*d - Sqrt[a]*e)])/(e^2*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2)) -
(I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(15*A*c*d + 9*a*B*e - Sqrt[a]*Sqrt[c]*(
3*B*d + 5*A*e))*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]
*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d
 + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d
 - Sqrt[a]*e)])/(e*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2))))/(15*c^2*
Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {687, 27, 687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \text{687}$$

$$-\frac{2\int -\frac{\sqrt{d+ex}(5Acd+3aBe+c(3Bd+5Ae)x)}{2\sqrt{a-cx^2}}dx}{5c}-\frac{2B\sqrt{a-cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{d+ex}(5Acd+3aBe+c(3Bd+5Ae)x)}{\sqrt{a-cx^2}}dx}{5c}-\frac{2B\sqrt{a-cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \text{687}$$

$$\frac{-\frac{2\int -\frac{c\left(12aBde+5A\left(3cd^2+ae^2\right)+\left(3Bcd^2+20Aced+9aBe^2\right)x\right)}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{3c}-\frac{2}{3}\sqrt{a-cx^2}\sqrt{d+ex}(5Ae+3Bd)}{5c}-\frac{2B\sqrt{a-cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{3}\int \frac{15Acd^2+12aBed+5aAe^2+\left(3Bcd^2+20Aced+9aBe^2\right)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx-\frac{2}{3}\sqrt{a-cx^2}\sqrt{d+ex}(5Ae+3Bd)}{5c}-\frac{2B\sqrt{a-cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \text{600}$$

$$\frac{\frac{1}{3}\left(\frac{\left(9aBe^2+20Acde+3Bcd^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{\left(cd^2-ae^2\right)(5Ae+3Bd)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)-\frac{2}{3}\sqrt{a-cx^2}\sqrt{d+ex}(5Ae+3Bd)}{5c}-\frac{2B\sqrt{a-cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \text{509}$$

$$\frac{\frac{1}{3}\left(\frac{\sqrt{1-\frac{cx^2}{a}}\left(9aBe^2+20Acde+3Bcd^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{\left(cd^2-ae^2\right)(5Ae+3Bd)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)-\frac{2}{3}\sqrt{a-cx^2}\sqrt{d+ex}(5Ae+3}{5c}$$

$$\frac{2B\sqrt{a-cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \; 508$$

$$\frac{\frac{1}{3}\left(-\frac{\left(cd^2-ae^2\right)(5Ae+3Bd)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aBe^2+20Acde+3Bcd^2\right)\int\frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)-\frac{2}{3}\sqrt{a}}{5c}$$

$$\frac{2B\sqrt{a-cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{1}{3}\left(-\frac{\left(cd^2-ae^2\right)(5Ae+3Bd)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aBe^2+20Acde+3Bcd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)-\frac{2}{3}}{5c}$$

$$\frac{2B\sqrt{a-cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \; 512$$

$$\frac{\frac{1}{3}\left(-\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)(5Ae+3Bd)\int\frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aBe^2+20Acde+3Bcd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right.}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right.}{5c}$$

$$\frac{2B\sqrt{a-cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \; 511$$

$$\frac{\frac{1}{3}\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)(5Ae+3Bd)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\int\frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(9aBe^2+20Acde+3}{\sqrt{c}e\sqrt{a-cx^2}}\right.}{5c}$$

$$\frac{2B\sqrt{a-cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \; 321$$

$$\frac{\frac{1}{3}\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)(5Ae+3Bd)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(9aBe^2+20Acde+3Bcd}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{}}{\sqrt{}}}}\right.}{5c}$$

$$\frac{2B\sqrt{a-cx^2}(d+ex)^{3/2}}{5c}$$

input

```
Int[((A + B*x)*(d + e*x)^(3/2))/Sqrt[a - c*x^2],x]
```

output

```
(-2*B*(d + e*x)^(3/2)*Sqrt[a - c*x^2])/(5*c) + ((-2*(3*B*d + 5*A*e)*Sqrt[d
 + e*x]*Sqrt[a - c*x^2])/3 + ((-2*Sqrt[a]*(3*B*c*d^2 + 20*A*c*d*e + 9*a*B*
e^2)*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*
x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[(S
qrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) + (2*Sqrt[a]*(
3*B*d + 5*A*e)*(c*d^2 - a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[
a]*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/
Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[
a - c*x^2]))/3)/(5*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(313) = 626$.

Time = 5.00 (sec) , antiderivative size = 682, normalized size of antiderivative = 1.78

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c\,x^2+a)}\left(-\frac{2Bex\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}{5c}-\frac{2\left(A\,e^2+\frac{6}{5}Bde\right)\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}{3ce}+\frac{2\left(A\,d^2+\frac{2adeB}{5c}+\frac{a\left(A\,e^2+\frac{6}{5}Bde\right)}{3c}\right)\left(\ldots\right.}{\ldots}\right.}{\ldots}$
risch	$-\frac{2(3Bex+5Ae+6Bd)\sqrt{ex+d}\,\sqrt{-c\,x^2+a}}{15c}+\frac{\left(\frac{\left(20Acde+9Ba\,e^2+3Bc\,d^2\right)\sqrt{ac}\,\sqrt{2}\,\sqrt{\frac{\left(x+\frac{\sqrt{ac}}{c}\right)c}{\sqrt{ac}}}\,\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{\sqrt{ac}}{c}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{ac}}{c}\right)c}{\sqrt{ac}}}\left(\left(\frac{d}{e}-\frac{\sqrt{\ldots}}{\ldots}\right.\right.\ldots}{c\sqrt{\ldots}}\right.\ldots}{\ldots}$
default	Expression too large to display

input
```
int((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(-2/5*B*e/c*x*(-
c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)-2/3*(A*e^2+6/5*B*d*e)/c/e*(-c*e*x^3-c*d*x
^2+a*e*x+a*d)^(1/2)+2*(A*d^2+2/5*a/c*d*e*B+1/3*a/c*(A*e^2+6/5*B*d*e))*(d/e
-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2
))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2
)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(
a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+
2*(2*A*d*e+B*d^2+3/5*a/c*e^2*B-2/3*d/e*(A*e^2+6/5*B*d*e))*(d/e-1/c*(a*c)^(
1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c
*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-
c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))*EllipticE(((x+d/e
)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/
2)))^(1/2))+1/c*(a*c)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2
),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.70

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{\sqrt{a-cx^2}}\,dx = \frac{2\left(\left(3\,Bcd^3-25\,Acd^2e-27\,Bade^2-15\,Aae^3\right)\sqrt{-ce}\,\text{weierstrassPInverse}\left(\frac{4\,(\cdots}{}\right.\right.}{}$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*((3*B*c*d^3 - 25*A*c*d^2*e - 27*B*a*d*e^2 - 15*A*a*e^3)*sqrt(-c*e)*we
ierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)
/(c*e^3), 1/3*(3*e*x + d)/e) + 3*(3*B*c*d^2*e + 20*A*c*d*e^2 + 9*B*a*e^3)*
sqrt(-c*e)*weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9
*a*d*e^2)/(c*e^3), weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/2
7*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e)) - 3*(3*B*c*e^3*x + 6*B*
c*d*e^2 + 5*A*c*e^3)*sqrt(-c*x^2 + a)*sqrt(e*x + d))/(c^2*e^2)
```

Sympy [F]

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{\sqrt{a-cx^2}}\,dx = \int \frac{(A+Bx)\,(d+ex)^{\frac{3}{2}}}{\sqrt{a-cx^2}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)**(3/2)/(-c*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)*(d + e*x)**(3/2)/sqrt(a - c*x**2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{\sqrt{a-cx^2}}\,dx = \int \frac{(Bx+A)(ex+d)^{\frac{3}{2}}}{\sqrt{-cx^2+a}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x + d)^(3/2)/sqrt(-c*x^2 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{\sqrt{a-cx^2}}\,dx = \int \frac{(Bx+A)(ex+d)^{\frac{3}{2}}}{\sqrt{-cx^2+a}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x + d)^(3/2)/sqrt(-c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{\sqrt{a-cx^2}}\,dx = \int \frac{(A+B\,x)\,\left(d+e\,x\right)^{3/2}}{\sqrt{a-c\,x^2}}\,dx$$

input
```
int(((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^(1/2),x)
```

output
```
int(((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{\sqrt{a-cx^2}}\,dx = \frac{-6\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\,ab\,e^2-20\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\,acde-10\sqrt{ex+d}\,\sqrt{}$$

input
```
int((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x)
```

output
```
( - 6*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*e**2 - 20*sqrt(d + e*x)*sqrt(a -
c*x**2)*a*c*d*e - 10*sqrt(d + e*x)*sqrt(a - c*x**2)*b*c*d**2 - 4*sqrt(d +
e*x)*sqrt(a - c*x**2)*b*c*d*e*x - 9*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x*
*2)/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a*b*c*e**3 - 20*int((sqrt(d + e
*x)*sqrt(a - c*x**2)*x**2)/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a*c**2*d
*e**2 - 3*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d + a*e*x - c*d*x**
2 - c*e*x**3),x)*b*c**2*d**2*e + 3*int((sqrt(d + e*x)*sqrt(a - c*x**2))/(a
*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a**2*b*e**3 + 10*int((sqrt(d + e*x)*s
qrt(a - c*x**2))/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a**2*c*d*e**2 + 9*
int((sqrt(d + e*x)*sqrt(a - c*x**2))/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x
)*a*b*c*d**2*e + 10*int((sqrt(d + e*x)*sqrt(a - c*x**2))/(a*d + a*e*x - c*
d*x**2 - c*e*x**3),x)*a*c**2*d**3)/(10*c**2*d)
```

3.274 $\int \frac{(A+Bx)\sqrt{d+ex}}{\sqrt{a-cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 325

$$\begin{aligned}\int \frac{(A+Bx)\sqrt{d+ex}}{\sqrt{a-cx^2}}\,dx &= -\frac{2B\sqrt{d+ex}\sqrt{a-cx^2}}{3c} \\ &\quad - \frac{2\sqrt{a}(Bd+3Ae)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{3\sqrt{c}e\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} \\ &\quad + \frac{2\sqrt{a}B(cd^2-ae^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{3c^{3/2}e\sqrt{d+ex}\sqrt{a-cx^2}}\end{aligned}$$

output
```
-2/3*B*(e*x+d)^(1/2)*(-c*x^2+a)^(1/2)/c-2/3*a^(1/2)*(3*A*e+B*d)*(e*x+d)^(1
/2)*(1-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^
(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/c^(1/2)/e/(c^(1/2)*(e*x+d)/
(c^(1/2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)+2/3*a^(1/2)*B*(-a*e^2+c*d^2)
*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2)*EllipticF
(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(
1/2)*e))^(1/2))/c^(3/2)/e/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.35 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.39

$$\int \frac{(A+Bx)\sqrt{d+ex}}{\sqrt{a-cx^2}}\,dx$$

$$= \frac{2\sqrt{a-cx^2}\left(-B(d+ex)+\frac{e^2(Bd+3Ae)\sqrt{-d+\frac{\sqrt{a}e}{\sqrt{c}}}(a-cx^2)+i\sqrt{c}(\sqrt{c}d-\sqrt{a}e)(Bd+3Ae)\sqrt{\frac{e\left(\frac{\sqrt{a}}{\sqrt{c}}+x\right)}{d+ex}}\sqrt{-\frac{\frac{\sqrt{a}e}{\sqrt{c}}-ex}{d+ex}}(d+ex)^{3/2}E\Big(i\cdots}{\cdots}\right)}{\cdots}$$

input
```
Integrate[((A + B*x)*Sqrt[d + e*x])/Sqrt[a - c*x^2],x]
```

output
```
(2*Sqrt[a - c*x^2]*(-(B*(d + e*x)) + (e^2*(B*d + 3*A*e)*Sqrt[-d + (Sqrt[a]
*e)/Sqrt[c]]*(a - c*x^2) + I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(B*d + 3*A*e)
*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e
*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/
Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)]
+ I*(Sqrt[a]*B - 3*A*Sqrt[c])*e*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[(e*(Sqrt[a]/S
qrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d
+ e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e
*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)])/(e^2*Sqrt[-d + (Sq
rt[a]*e)/Sqrt[c]]*(-a + c*x^2))))/(3*c*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{d+ex}}{\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \text{687}$$

$$-\frac{2\int -\frac{3Acd+aBe+c(Bd+3Ae)x}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{3c}-\frac{2B\sqrt{a-cx^2}\sqrt{d+ex}}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3Acd+aBe+c(Bd+3Ae)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{3c}-\frac{2B\sqrt{a-cx^2}\sqrt{d+ex}}{3c}$$

$$\Big\downarrow \text{600}$$

$$\frac{\frac{c(3Ae+Bd)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{B(cd^2-ae^2)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}}{3c}-\frac{2B\sqrt{a-cx^2}\sqrt{d+ex}}{3c}$$

$$\Big\downarrow \text{509}$$

$$\frac{\frac{c\sqrt{1-\frac{cx^2}{a}}(3Ae+Bd)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{B(cd^2-ae^2)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}}{3c}-\frac{2B\sqrt{a-cx^2}\sqrt{d+ex}}{3c}$$

$$\Big\downarrow \text{508}$$

$$\frac{-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(3Ae+Bd)\int \frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{B(cd^2-ae^2)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}}{3c}-\frac{2B\sqrt{a-cx^2}\sqrt{d+ex}}{3c}$$

$$\Big\downarrow \text{327}$$

$$\frac{-\frac{B(cd^2-ae^2)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(3Ae+Bd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{3c}-\frac{2B\sqrt{a-cx^2}\sqrt{d+ex}}{3c}$$

$$\Big\downarrow \text{512}$$

$$\frac{-\frac{B\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)\int\frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(3Ae+Bd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{3c}-\frac{2B\sqrt{a-cx^2}\sqrt{d+ex}}{3c}$$

$$\Big\downarrow \text{511}$$

$$\frac{\frac{2\sqrt{a}B\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\int\frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(3Ae+Bd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{3c}-\frac{2B\sqrt{a-cx^2}\sqrt{d+ex}}{3c}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{2\sqrt{a}B\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(3Ae+Bd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{3c}-\frac{2B\sqrt{a-cx^2}\sqrt{d+ex}}{3c}$$

input
```
Int[((A + B*x)*Sqrt[d + e*x])/Sqrt[a - c*x^2],x]
```

output
```
(-2*B*Sqrt[d + e*x]*Sqrt[a - c*x^2])/(3*c) + ((-2*Sqrt[a]*Sqrt[c]*(B*d + 3
*A*e)*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]
*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(e*Sqrt[(Sqrt[c]*
(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) + (2*Sqrt[a]*B*(c*d^2
 - a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^
2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqr
t[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[a - c*x^2]))/(3*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(261) = 522$.

Time = 2.81 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.81

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c\,x^2+a)}\left(-\frac{2B\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}{3c}+\frac{2\left(Ad+\frac{Bae}{3c}\right)\left(\frac{d}{e}-\frac{\sqrt{ac}}{c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{\sqrt{ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{ac}}{c}}{-\frac{d}{e}-\frac{\sqrt{ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{ac}}{c}}{-\frac{d}{e}+\frac{\sqrt{ac}}{c}}}\,\text{EllipticF}\left(\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{\sqrt{ac}}{c}}}\right.}{\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}\right.}{}$
risch	$-\frac{2B\sqrt{ex+d}\sqrt{-c\,x^2+a}}{3c}+\frac{\left(\frac{(3Ae+Bd)\sqrt{ac}\sqrt{2}\sqrt{\frac{\left(x+\frac{\sqrt{ac}}{c}\right)c}{\sqrt{ac}}}\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{\sqrt{ac}}{c}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{ac}}{c}\right)c}{\sqrt{ac}}}\left(\left(\frac{d}{e}-\frac{\sqrt{ac}}{c}\right)\text{EllipticE}\left(\frac{\sqrt{2}\sqrt{\frac{\left(x+\frac{\sqrt{ac}}{c}\right)c}{\sqrt{ac}}}}{2},\right.\right.}{\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(-2/3*B/c*(-c*e*
x^3-c*d*x^2+a*e*x+a*d)^(1/2)+2*(A*d+1/3*B/c*a*e)*(d/e-1/c*(a*c)^(1/2))*((x
+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1
/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c
*d*x^2+a*e*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((
-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+2*(A*e+1/3*B*d)*(d/e-
1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2)
)/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)
))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))*Ellipt
icE(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/
c*(a*c)^(1/2)))^(1/2))+1/c*(a*c)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(
1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.71

$$\int \frac{(A+Bx)\sqrt{d+ex}}{\sqrt{a-cx^2}}\,dx = -\frac{2\left(3\sqrt{-cx^2+a}\sqrt{ex+d}Bce^2-(Bcd^2-6\,Acde-3\,Bae^2)\sqrt{-ce}\text{weierstrassPInverse}\left(\frac{4\,(cd^2+3\,ae^2)}{3\,ce^2},-\frac{8}{}\right.\right.}{}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/9*(3*sqrt(-c*x^2 + a)*sqrt(e*x + d)*B*c*e^2 - (B*c*d^2 - 6*A*c*d*e - 3*
B*a*e^2)*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/
27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) - 3*(B*c*d*e + 3*A*c*e^
2)*sqrt(-c*e)*weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3
- 9*a*d*e^2)/(c*e^3), weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -
8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e)))/(c^2*e^2)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{\sqrt{a-cx^2}}\,dx = \int \frac{(A+Bx)\sqrt{d+ex}}{\sqrt{a-cx^2}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)**(1/2)/(-c*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)*sqrt(d + e*x)/sqrt(a - c*x**2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{\sqrt{a-cx^2}}\,dx = \int \frac{(Bx+A)\sqrt{ex+d}}{\sqrt{-cx^2+a}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*sqrt(e*x + d)/sqrt(-c*x^2 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{\sqrt{a-cx^2}}\,dx = \int \frac{(Bx+A)\sqrt{ex+d}}{\sqrt{-cx^2+a}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*sqrt(e*x + d)/sqrt(-c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{d+ex}}{\sqrt{a-cx^2}}\,dx = \int \frac{(A+B\,x)\,\sqrt{d+e\,x}}{\sqrt{a-c\,x^2}}\,dx$$

input
```
int(((A + B*x)*(d + e*x)^(1/2))/(a - c*x^2)^(1/2),x)
```

output
```
int(((A + B*x)*(d + e*x)^(1/2))/(a - c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{\sqrt{a-cx^2}}\,dx = \frac{-2\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\,ae-2\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\,bd-3\left(\int \frac{\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\,x^2}{-ce\,x^3-cd\,x^2+aex+ad}dx\right)ac\,e^2-\left(\int \frac{\sqrt{ex+d}\,\sqrt{-}}{-ce\,x^3-cd\,x}\right.}{}$$

input
```
int((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(d + e*x)*sqrt(a - c*x**2)*a*e - 2*sqrt(d + e*x)*sqrt(a - c*x**2
)*b*d - 3*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d + a*e*x - c*d*x**
2 - c*e*x**3),x)*a*c*e**2 - int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**2)/(a*d
 + a*e*x - c*d*x**2 - c*e*x**3),x)*b*c*d*e + int((sqrt(d + e*x)*sqrt(a - c
*x**2))/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a**2*e**2 + int((sqrt(d + e
*x)*sqrt(a - c*x**2))/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)*a*b*d*e + 2*i
nt((sqrt(d + e*x)*sqrt(a - c*x**2))/(a*d + a*e*x - c*d*x**2 - c*e*x**3),x)
*a*c*d**2)/(2*c*d)
```

3.275 $\int \frac{A+Bx}{\sqrt{d+ex}\sqrt{a-cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 280

$$\begin{aligned}\int \frac{A+Bx}{\sqrt{d+ex}\sqrt{a-cx^2}}\,dx &= -\frac{2\sqrt{a}B\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{c}e\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} \\ &\quad + \frac{2\sqrt{a}(Bd-Ae)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{c}e\sqrt{d+ex}\sqrt{a-cx^2}}\end{aligned}$$

output
```
-2*a^(1/2)*B*(e*x+d)^(1/2)*(1-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^
(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/c^(1
/2)/e/(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)+2*a^(
1/2)*(-A*e+B*d)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^
(1/2)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e
/(c^(1/2)*d+a^(1/2)*e))^(1/2))/c^(1/2)/e/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.69 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx}{\sqrt{d+ex}\sqrt{a-cx^2}}\,dx = -\frac{2\left(Be^2\sqrt{-d+\frac{\sqrt{a}e}{\sqrt{c}}}(a-cx^2)+iB\sqrt{c}(\sqrt{c}d-\sqrt{a}e)\sqrt{\frac{e\left(\frac{\sqrt{a}}{\sqrt{c}}+x\right)}{d+ex}}\sqrt{-\frac{\frac{\sqrt{a}e}{\sqrt{c}}-ex}{d+ex}}(d+ex)^{3/2}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\cdots}}{\sqrt{\cdots}}\right)\cdots\right)\cdots\right)}{ce^2\sqrt{\cdots}}$$

input
```
Integrate[(A + B*x)/(Sqrt[d + e*x]*Sqrt[a - c*x^2]),x]
```

output
```
(-2*(B*e^2*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(a - c*x^2) + I*B*Sqrt[c]*(Sqrt[
c]*d - Sqrt[a]*e)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[
a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[
-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]
*d - Sqrt[a]*e)] + I*(Sqrt[a]*B - A*Sqrt[c])*Sqrt[c]*e*Sqrt[(e*(Sqrt[a]/Sq
rt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d +
 e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*
x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)]))/(c*e^2*Sqrt[-d + (
Sqrt[a]*e)/Sqrt[c]]*Sqrt[d + e*x]*Sqrt[a - c*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a-cx^2}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \; 600$$

$$\frac{B\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}$$

$$\Big\downarrow \text{509}$$

$$\frac{B\sqrt{1-\frac{cx^2}{a}}\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}$$

$$\Big\downarrow \text{508}$$

$$-\frac{(Bd-Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}B\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}$$

$$\Big\downarrow \text{327}$$

$$-\frac{(Bd-Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}B\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}$$

$$\Big\downarrow \text{512}$$

$$-\frac{\sqrt{1-\frac{cx^2}{a}}(Bd-Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}B\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}$$

$$\Big\downarrow \text{511}$$

$$\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}(Bd-Ae)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\int \frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}B\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}(Bd-Ae)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}B\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}$$

input
```
Int[(A + B*x)/(Sqrt[d + e*x]*Sqrt[a - c*x^2]),x]
```

output
```
(-2*Sqrt[a]*B*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*
e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) + (2*
Sqrt[a]*(B*d - A*e)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt
[1 - (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (
2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[a - c*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(226) = 452$.

Time = 4.23 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.78

method	result
default	$\frac{2\left(A\,\mathrm{EllipticF}\left(\sqrt{-\frac{c(ex+d)}{\sqrt{ac}\,e-cd}},\sqrt{-\frac{\sqrt{ac}\,e-cd}{\sqrt{ac}\,e+cd}}\right)cde-A\sqrt{ac}\,\mathrm{EllipticF}\left(\sqrt{-\frac{c(ex+d)}{\sqrt{ac}\,e-cd}},\sqrt{-\frac{\sqrt{ac}\,e-cd}{\sqrt{ac}\,e+cd}}\right)e^2-B\,\mathrm{EllipticF}\left(\sqrt{-\frac{c(ex+d)}{\sqrt{ac}\,e-cd}},\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{(ex+d)(-cx^2+a)}\left(\frac{2A\left(\frac{d}{e}-\frac{\sqrt{ac}}{c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{\sqrt{ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{ac}}{c}}{-\frac{d}{e}-\frac{\sqrt{ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{ac}}{c}}{-\frac{d}{e}+\frac{\sqrt{ac}}{c}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{\sqrt{ac}}{c}}},\sqrt{\frac{-\frac{d}{e}+\frac{\sqrt{ac}}{c}}{-\frac{d}{e}-\frac{\sqrt{ac}}{c}}}\right)}{\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}+\frac{2B\left(\frac{d}{e}-\frac{\sqrt{ac}}{c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\cdots}}\cdots}{\cdots}\right.}{\sqrt{ex+d}\,\sqrt{-c\,x\cdots}}$

input
```
int((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(A*EllipticF((-c*(e*x+d)/((a*c)^(1/2)*e-c*d))^(1/2),(-((a*c)^(1/2)*e-c*d
)/((a*c)^(1/2)*e+c*d))^(1/2))*c*d*e-A*(a*c)^(1/2)*EllipticF((-c*(e*x+d)/((
a*c)^(1/2)*e-c*d))^(1/2),(-((a*c)^(1/2)*e-c*d)/((a*c)^(1/2)*e+c*d))^(1/2))
*e^2-B*EllipticF((-c*(e*x+d)/((a*c)^(1/2)*e-c*d))^(1/2),(-((a*c)^(1/2)*e-c
*d)/((a*c)^(1/2)*e+c*d))^(1/2))*a*e^2+B*(a*c)^(1/2)*EllipticF((-c*(e*x+d)/
((a*c)^(1/2)*e-c*d))^(1/2),(-((a*c)^(1/2)*e-c*d)/((a*c)^(1/2)*e+c*d))^(1/2
))*d*e+B*EllipticE((-c*(e*x+d)/((a*c)^(1/2)*e-c*d))^(1/2),(-((a*c)^(1/2)*e
-c*d)/((a*c)^(1/2)*e+c*d))^(1/2))*a*e^2-B*EllipticE((-c*(e*x+d)/((a*c)^(1/
2)*e-c*d))^(1/2),(-((a*c)^(1/2)*e-c*d)/((a*c)^(1/2)*e+c*d))^(1/2))*c*d^2)*
((c*x+(a*c)^(1/2))*e/((a*c)^(1/2)*e-c*d))^(1/2)*((-c*x+(a*c)^(1/2))*e/((a*
c)^(1/2)*e+c*d))^(1/2)*(-c*(e*x+d)/((a*c)^(1/2)*e-c*d))^(1/2)*(e*x+d)^(1/2
)*(-c*x^2+a)^(1/2)/c/e^2/(-c*e*x^3-c*d*x^2+a*e*x+a*d)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx}{\sqrt{d+ex}\sqrt{a-cx^2}}\,dx = \frac{2\left(3\sqrt{-ce}Be\,\text{weierstrassZeta}\left(\frac{4\,(cd^2+3\,ae^2)}{3\,ce^2},-\frac{8\,(cd^3-9\,ade^2)}{27\,ce^3},\text{weierstrassPInverse}\left(\frac{4\,(cd^2+3\,ae^2)}{3\,ce^2},-\frac{8\,(cd^3-9\,ade^2)}{27\,ce^3}\right.\right.\right.}{3\,ce^2}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(3*sqrt(-c*e)*B*e*weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27
*(c*d^3 - 9*a*d*e^2)/(c*e^3), weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c
*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e)) + (B*d - 3*A
*e)*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c
*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e))/(c*e^2)
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\sqrt{a-cx^2}}\,dx = \int \frac{A+Bx}{\sqrt{a-cx^2}\sqrt{d+ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)**(1/2)/(-c*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a - c*x**2)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\sqrt{a-cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{-cx^2+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(-c*x^2 + a)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\sqrt{a-cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{-cx^2+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(-c*x^2 + a)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{d+ex}\sqrt{a-cx^2}}\,dx=\int \frac{A+B\,x}{\sqrt{a-c\,x^2}\,\sqrt{d+e\,x}}\,dx$$

input
```
int((A + B*x)/((a - c*x^2)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
int((A + B*x)/((a - c*x^2)^(1/2)*(d + e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\sqrt{a-cx^2}}\,dx=\left(\int \frac{\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\,x}{-ce\,x^3-cd\,x^2+aex+ad}dx\right)b+\left(\int \frac{\sqrt{ex+d}\,\sqrt{-c\,x^2+a}}{-ce\,x^3-cd\,x^2+aex+ad}dx\right)a$$

input
```
int((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x)*sqrt(a - c*x**2)*x)/(a*d + a*e*x - c*d*x**2 - c*e*x**3)
,x)*b + int((sqrt(d + e*x)*sqrt(a - c*x**2))/(a*d + a*e*x - c*d*x**2 - c*e
*x**3),x)*a
```

3.276 $\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 339

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx = -\frac{2(Bd-Ae)\sqrt{a-cx^2}}{(cd^2-ae^2)\sqrt{d+ex}} + \frac{2\sqrt{a}\sqrt{c}(Bd-Ae)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{e(cd^2-ae^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} - \frac{2\sqrt{a}B\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{c}e\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
-2*(-A*e+B*d)*(-c*x^2+a)^(1/2)/(-a*e^2+c*d^2)/(e*x+d)^(1/2)+2*a^(1/2)*c^(1
/2)*(-A*e+B*d)*(e*x+d)^(1/2)*(1-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/
a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/e/
(-a*e^2+c*d^2)/(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1
/2)-2*a^(1/2)*B*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^
(1/2)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e
/(c^(1/2)*d+a^(1/2)*e))^(1/2))/c^(1/2)/e/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.01 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx = \frac{2i\sqrt{\frac{e\left(\frac{\sqrt{a}}{\sqrt{c}}+x\right)}{d+ex}}\sqrt{-\frac{\frac{\sqrt{a}e}{\sqrt{c}}-ex}{d+ex}}(d+ex)\left(\sqrt{c}(Bd-Ae)E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-d+\frac{\sqrt{a}e}{\sqrt{c}}}}{\sqrt{d+ex}}\right)|\right.\right.}{e^2\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{-}}$$

input
```
Integrate[(A + B*x)/((d + e*x)^(3/2)*Sqrt[a - c*x^2]),x]
```

output
```
((2*I)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[
c] - e*x)/(d + e*x))]*(d + e*x)*(Sqrt[c]*(B*d - A*e)*EllipticE[I*ArcSinh[S
qrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqr
t[c]*d - Sqrt[a]*e)] + (Sqrt[a]*B + A*Sqrt[c])*e*EllipticF[I*ArcSinh[Sqrt[
-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]
*d - Sqrt[a]*e)]))/(e^2*(Sqrt[c]*d + Sqrt[a]*e)*Sqrt[-d + (Sqrt[a]*e)/Sqrt
[c]]*Sqrt[a - c*x^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a-cx^2}(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{688}$$

$$\frac{2\int \frac{Acd-aBe-c(Bd-Ae)x}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}(Bd-Ae)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{Acd-aBe-c(Bd-Ae)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}(Bd-Ae)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 600$$

$$\frac{\frac{B\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{c(Bd-Ae)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}(Bd-Ae)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 509$$

$$\frac{\frac{B\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{c\sqrt{1-\frac{cx^2}{a}}(Bd-Ae)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}(Bd-Ae)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 508$$

$$\frac{\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(Bd-Ae)\int \frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}+\frac{B\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}(Bd-Ae)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 327$$

$$\frac{\frac{B\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}+\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(Bd-Ae)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}(Bd-Ae)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 512$$

$$\frac{\frac{B\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}+\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(Bd-Ae)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}(Bd-Ae)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 511$$

$$\frac{\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(Bd-Ae)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{2\sqrt{a}B\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\int\frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+\cdots}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}}{cd^2-ae^2}$$

$$\frac{2\sqrt{a-cx^2}(Bd-Ae)}{\sqrt{d+ex}\,(cd^2-ae^2)}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(Bd-Ae)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{2\sqrt{a}B\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\cdots\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}}{cd^2-ae^2}$$

$$\frac{2\sqrt{a-cx^2}(Bd-Ae)}{\sqrt{d+ex}\,(cd^2-ae^2)}$$

input
```
Int[(A + B*x)/((d + e*x)^(3/2)*Sqrt[a - c*x^2]),x]
```

output
```
(-2*(B*d - A*e)*Sqrt[a - c*x^2])/((c*d^2 - a*e^2)*Sqrt[d + e*x]) + ((2*Sqr
t[a]*Sqrt[c]*(B*d - A*e)*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSi
n[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)]
)/(e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) -
(2*Sqrt[a]*B*(c*d^2 - a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]
*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sq
rt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[a
- c*x^2]))/(c*d^2 - a*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(281) = 562$.

Time = 5.99 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.93

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c\,x^2+a)}\left(-\frac{2\left(-ce\,x^2+ae\right)(Ae-Bd)}{e\left(a\,e^2-c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)\left(-ce\,x^2+ae\right)}}+\frac{2\left(\frac{B}{e}-\frac{cd(Ae-Bd)}{e\left(a\,e^2-c\,d^2\right)}\right)\left(\frac{d}{e}-\frac{\sqrt{ac}}{c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{\sqrt{ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{ac}}{c}}{-\frac{d}{e}-\frac{\sqrt{ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{ac}}{c}}{-\frac{d}{e}+\frac{\sqrt{ac}}{c}}}\,\mathrm{Ell}}{\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(-2*(-c*e*x^2+a*
e)/e/(a*e^2-c*d^2)*(A*e-B*d)/((x+d/e)*(-c*e*x^2+a*e))^(1/2)+2*(B/e-c*d/e*(
A*e-B*d)/(a*e^2-c*d^2))*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2
)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)
^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*E
llipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d
/e-1/c*(a*c)^(1/2)))^(1/2))-2*(A*e-B*d)*c/(a*e^2-c*d^2)*(d/e-1/c*(a*c)^(1/
2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(
a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*
e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))*EllipticE(((x+d/e)/
(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)
))^(1/2))+1/c*(a*c)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),
((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx = -\frac{2\left(\left(Bcd^3+2\,Acd^2e-3\,Bade^2+\left(Bcd^2e+2\,Acde^2-3\,Bae^3\right)x\right)\sqrt{-ce}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(cd^2+3\,ae^2)}{3\,ce^2},\right.\right.}{}$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*((B*c*d^3 + 2*A*c*d^2*e - 3*B*a*d*e^2 + (B*c*d^2*e + 2*A*c*d*e^2 - 3*
B*a*e^3)*x)*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2),
-8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 3*(B*c*d^2*e - A*c
*d*e^2 + (B*c*d*e^2 - A*c*e^3)*x)*sqrt(-c*e)*weierstrassZeta(4/3*(c*d^2 +
3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), weierstrassPInverse(4
/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*
x + d)/e)) + 3*(B*c*d*e^2 - A*c*e^3)*sqrt(-c*x^2 + a)*sqrt(e*x + d))/(c^2*
d^3*e^2 - a*c*d*e^4 + (c^2*d^2*e^3 - a*c*e^5)*x)
```

Sympy [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx = \int \frac{A+Bx}{\sqrt{a-cx^2}\,(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)**(3/2)/(-c*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a - c*x**2)*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx=\int \frac{A+B\,x}{\sqrt{a-c\,x^2}\,(d+e\,x)^{3/2}}\,dx$$

input
```
int((A + B*x)/((a - c*x^2)^(1/2)*(d + e*x)^(3/2)),x)
```

output
```
int((A + B*x)/((a - c*x^2)^(1/2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx=\left(\int \frac{\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\,x}{-c\,e^2x^4-2cde\,x^3+a\,e^2x^2-c\,d^2x^2+2adex+a\,d^2}dx\right)b+\left(\int \frac{\sqrt{ex+d}\,\sqrt{-c\,x^2+a}}{-c\,e^2x^4-2cde\,x^3+a\,e^2x^2-c\,d^2x^2+2adex+a\,d^2}dx\right)a$$

input
```
int((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x)*sqrt(a - c*x**2)*x)/(a*d**2 + 2*a*d*e*x + a*e**2*x**2 -
 c*d**2*x**2 - 2*c*d*e*x**3 - c*e**2*x**4),x)*b + int((sqrt(d + e*x)*sqrt(
a - c*x**2))/(a*d**2 + 2*a*d*e*x + a*e**2*x**2 - c*d**2*x**2 - 2*c*d*e*x**
3 - c*e**2*x**4),x)*a
```

3.277 $\int \frac{A+Bx}{(d+ex)^{5/2}\sqrt{a-cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 437

$$\int \frac{A+Bx}{(d+ex)^{5/2}\sqrt{a-cx^2}}\,dx = -\frac{2(Bd-Ae)\sqrt{a-cx^2}}{3(cd^2-ae^2)(d+ex)^{3/2}} - \frac{2(Bcd^2-4Acde+3aBe^2)\sqrt{a-cx^2}}{3(cd^2-ae^2)^2\sqrt{d+ex}} + \frac{2\sqrt{a}\sqrt{c}(Bcd^2-4Acde+3aBe^2)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{3e(cd^2-ae^2)^2\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} - \frac{2\sqrt{a}\sqrt{c}(Bd-Ae)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{3e(cd^2-ae^2)\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
-2/3*(-A*e+B*d)*(-c*x^2+a)^(1/2)/(-a*e^2+c*d^2)/(e*x+d)^(3/2)-2/3*(-4*A*c*
d*e+3*B*a*e^2+B*c*d^2)*(-c*x^2+a)^(1/2)/(-a*e^2+c*d^2)^2/(e*x+d)^(1/2)+2/3
*a^(1/2)*c^(1/2)*(-4*A*c*d*e+3*B*a*e^2+B*c*d^2)*(e*x+d)^(1/2)*(1-c*x^2/a)^
(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e
/(c^(1/2)*d+a^(1/2)*e))^(1/2))/e/(-a*e^2+c*d^2)^2/(c^(1/2)*(e*x+d)/(c^(1/2
)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)-2/3*a^(1/2)*c^(1/2)*(-A*e+B*d)*(c^(
1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2)*EllipticF(1/2*
(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*
e))^(1/2))/e/(-a*e^2+c*d^2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.75 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx}{(d+ex)^{5/2}\sqrt{a-cx^2}}\,dx = \frac{2\left(-e^2(Bd-Ae)\sqrt{-d+\frac{\sqrt{a}e}{\sqrt{c}}}(cd^2-ae^2)\left(a-cx^2\right)+i\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)\left(Bc\right.\right.}{}$$

input
```
Integrate[(A + B*x)/((d + e*x)^(5/2)*Sqrt[a - c*x^2]),x]
```

output
```
(2*(-(e^2*(B*d - A*e)*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(c*d^2 - a*e^2)*(a -
c*x^2)) + I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(B*c*d^2 - 4*A*c*d*e + 3*a*B*e
^2)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c]
- e*x)/(d + e*x))]*(d + e*x)^(5/2)*EllipticE[I*ArcSinh[Sqrt[-d + (Sqrt[a]*
e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e
)] + I*Sqrt[c]*e*(Sqrt[c]*d - Sqrt[a]*e)*(3*A*c*d - 3*a*B*e + Sqrt[a]*Sqrt
[c]*(B*d - A*e))*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a
]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(5/2)*EllipticF[I*ArcSinh[Sqrt[-
d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*
d - Sqrt[a]*e)]))/(3*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(c*d^2*e - a*e^3)^2*(d
 + e*x)^(3/2)*Sqrt[a - c*x^2])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {688, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a-cx^2}(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{ 688}$$

$$\frac{2\int \frac{3(Acd-aBe)+c(Bd-Ae)x}{2(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{3(cd^2-ae^2)}-\frac{2\sqrt{a-cx^2}(Bd-Ae)}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3(Acd-aBe)+c(Bd-Ae)x}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{3(cd^2-ae^2)}-\frac{2\sqrt{a-cx^2}(Bd-Ae)}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

$$\Big\downarrow \text{ 688}$$

$$\frac{\frac{2\int \frac{c\left(3Acd^2-4aBed+aAe^2-\left(Bcd^2-4Aced+3aBe^2\right)x\right)}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}(3aBe^2-4Acde+Bcd^2)}{\sqrt{d+ex}(cd^2-ae^2)}}{3(cd^2-ae^2)}-\frac{2\sqrt{a-cx^2}(Bd-Ae)}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{c\int \frac{3Acd^2-4aBed+aAe^2-\left(Bcd^2-4Aced+3aBe^2\right)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}(3aBe^2-4Acde+Bcd^2)}{\sqrt{d+ex}(cd^2-ae^2)}}{3(cd^2-ae^2)}-\frac{2\sqrt{a-cx^2}(Bd-Ae)}{3(d+ex)^{3/2}(cd^2-ae^2)}$$

$$\Big\downarrow \text{ 600}$$

$$\frac{\frac{c\left(\frac{\left(cd^2-ae^2\right)(Bd-Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{\left(3aBe^2-4Acde+Bcd^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}\right)}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\left(3aBe^2-4Acde+Bcd^2\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}}{3\left(cd^2-ae^2\right)}-\frac{2\sqrt{a-cx^2}(Bd-Ae)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{509}$$

$$\frac{\frac{c\left(\frac{\left(cd^2-ae^2\right)(Bd-Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{\sqrt{1-\frac{cx^2}{a}}\left(3aBe^2-4Acde+Bcd^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}\right)}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\left(3aBe^2-4Acde+Bcd^2\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}}{3\left(cd^2-ae^2\right)}-\frac{2\sqrt{a-cx^2}(Bd-Ae)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{508}$$

$$\frac{\frac{c\left(\frac{\left(cd^2-ae^2\right)(Bd-Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}+\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(3aBe^2-4Acde+Bcd^2\right)\int \frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\left(3aBe^2-4Ac\right.}{\sqrt{d+ex}\left(cd^2-ae\right.}}{3\left(cd^2-ae^2\right)}$$

$$-\frac{2\sqrt{a-cx^2}(Bd-Ae)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{c\left(\frac{\left(cd^2-ae^2\right)(Bd-Ae)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}+\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(3aBe^2-4Acde+Bcd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\left(3aBe^2-4A\right.}{\sqrt{d+ex}\left(cd^2-\right.}}{3\left(cd^2-ae^2\right)}$$

$$-\frac{2\sqrt{a-cx^2}(Bd-Ae)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 512

$$\frac{\frac{c\left(\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)(Bd-Ae)\int\frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}+\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(3aBe^2-4Acde+Bcd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}(3a}{\sqrt{d+}}}{3\left(cd^2-ae^2\right)}$$

$$\frac{2\sqrt{a-cx^2}(Bd-Ae)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 511

$$\frac{\frac{c\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(3aBe^2-4Acde+Bcd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)(Bd-Ae)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\int\frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}(}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}}{cd^2-ae^2}}{3\left(cd^2-ae^2\right)}$$

$$\frac{2\sqrt{a-cx^2}(Bd-Ae)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 321

$$\frac{\frac{c\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(3aBe^2-4Acde+Bcd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)(Bd-Ae)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}}{\sqrt{}}}}{\sqrt{2}}\right.\right.}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}}{cd^2-ae^2}}{3\left(cd^2-ae^2\right)}$$

$$\frac{2\sqrt{a-cx^2}(Bd-Ae)}{3(d+ex)^{3/2}\left(cd^2-ae^2\right)}$$

input
```
Int[(A + B*x)/((d + e*x)^(5/2)*Sqrt[a - c*x^2]),x]
```

output
```
(-2*(B*d - A*e)*Sqrt[a - c*x^2])/(3*(c*d^2 - a*e^2)*(d + e*x)^(3/2)) + ((-
2*(B*c*d^2 - 4*A*c*d*e + 3*a*B*e^2)*Sqrt[a - c*x^2])/((c*d^2 - a*e^2)*Sqrt
[d + e*x]) + (c*((2*Sqrt[a]*(B*c*d^2 - 4*A*c*d*e + 3*a*B*e^2)*Sqrt[d + e*x
]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[
2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[(Sqrt[c]*(d + e*x))
/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) - (2*Sqrt[a]*(B*d - A*e)*(c*d^2
 - a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^
2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqr
t[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[a - c*x^2])))/(c*d^2
- a*e^2))/(3*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 775 vs. $2(367) = 734$.

Time = 8.34 (sec) , antiderivative size = 776, normalized size of antiderivative = 1.78

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c x^2+a)}\left(-\frac{2(Ae-Bd)\sqrt{-ce x^3-cd x^2+aex+ad}}{3e^2\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)^2}+\frac{2\left(-ce x^2+ae\right)\left(4Acde-3Ba e^2-Bc d^2\right)}{3e\left(a e^2-c d^2\right)^2\sqrt{\left(x+\frac{d}{e}\right)\left(-ce x^2+ae\right)}}+\frac{2\left(\frac{c(Ae-Bd)}{3e\left(a e^2-c d^2\right)}+\frac{cd\left(4Acde-3Ba\ldots\right.}{3e\left(a e^2-\ldots\right.}\ldots\right.}{\ldots}\right.}{\ldots}$
default	Expression too large to display

input
```
int((B*x+A)/(e*x+d)^(5/2)/(-c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(-2/3/e^2/(a*e^2
-c*d^2)*(A*e-B*d)*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)/(x+d/e)^2+2/3*(-c*e*x
^2+a*e)/e/(a*e^2-c*d^2)^2*(4*A*c*d*e-3*B*a*e^2-B*c*d^2)/((x+d/e)*(-c*e*x^2
+a*e))^(1/2)+2*(1/3*c/e*(A*e-B*d)/(a*e^2-c*d^2)+1/3*c*d/e*(4*A*c*d*e-3*B*a
*e^2-B*c*d^2)/(a*e^2-c*d^2)^2)*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*
c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/
c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^
(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/
2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+2/3*c*(4*A*c*d*e-3*B*a*e^2-B*c*d^2)/(a*
e^2-c*d^2)^2*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*(
(x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d
/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(
a*c)^(1/2))*EllipticE(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*
c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+1/c*(a*c)^(1/2)*EllipticF(((x+d/e
)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/
2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx}{(d+ex)^{5/2}\sqrt{a-cx^2}}\,dx =$$

$$-\frac{2\left(\left(Bcd^5+5\,Acd^4e-9\,Bad^3e^2+3\,Aad^2e^3+\left(Bcd^3e^2+5\,Acd^2e^3-9\,Bade^4+3\,Aae^5\right)x^2+2\,\left(Bcd^4e+\ldots\right.\right.\right.}{\ldots}$$

input
```
integrate((B*x+A)/(e*x+d)^(5/2)/(-c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/9*((B*c*d^5 + 5*A*c*d^4*e - 9*B*a*d^3*e^2 + 3*A*a*d^2*e^3 + (B*c*d^3*e^
2 + 5*A*c*d^2*e^3 - 9*B*a*d*e^4 + 3*A*a*e^5)*x^2 + 2*(B*c*d^4*e + 5*A*c*d^
3*e^2 - 9*B*a*d^2*e^3 + 3*A*a*d*e^4)*x)*sqrt(-c*e)*weierstrassPInverse(4/3
*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x
 + d)/e) + 3*(B*c*d^4*e - 4*A*c*d^3*e^2 + 3*B*a*d^2*e^3 + (B*c*d^2*e^3 - 4*
A*c*d*e^4 + 3*B*a*e^5)*x^2 + 2*(B*c*d^3*e^2 - 4*A*c*d^2*e^3 + 3*B*a*d*e^4)
*x)*sqrt(-c*e)*weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3
 - 9*a*d*e^2)/(c*e^3), weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2),
-8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e)) + 3*(2*B*c*d^3*e^2
 - 5*A*c*d^2*e^3 + 2*B*a*d*e^4 + A*a*e^5 + (B*c*d^2*e^3 - 4*A*c*d*e^4 + 3*B
*a*e^5)*x)*sqrt(-c*x^2 + a)*sqrt(e*x + d))/(c^2*d^6*e^2 - 2*a*c*d^4*e^4 +
a^2*d^2*e^6 + (c^2*d^4*e^4 - 2*a*c*d^2*e^6 + a^2*e^8)*x^2 + 2*(c^2*d^5*e^3
 - 2*a*c*d^3*e^5 + a^2*d*e^7)*x)
```

Sympy [F]

$$\int \frac{A+Bx}{(d+ex)^{5/2}\sqrt{a-cx^2}}\,dx = \int \frac{A+Bx}{\sqrt{a-cx^2}\,(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)**(5/2)/(-c*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a - c*x**2)*(d + e*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(d+ex)^{5/2}\sqrt{a-cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{-cx^2+a}(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(5/2)/(-c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(-c*x^2 + a)*(e*x + d)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(d+ex)^{5/2}\sqrt{a-cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{-cx^2+a}(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(5/2)/(-c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(-c*x^2 + a)*(e*x + d)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)^{5/2}\sqrt{a-cx^2}}\,dx = \int \frac{A+B\,x}{\sqrt{a-c\,x^2}\,{(d+e\,x)}^{5/2}}\,dx$$

input
```
int((A + B*x)/((a - c*x^2)^(1/2)*(d + e*x)^(5/2)),x)
```

output
```
int((A + B*x)/((a - c*x^2)^(1/2)*(d + e*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(d+ex)^{5/2}\sqrt{a-cx^2}}\,dx = \left(\int \frac{\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\,x}{-c\,e^3x^5-3cd\,e^2x^4+a\,e^3x^3-3c\,d^2e\,x^3+3ad\,e^2x^2-c\,d^3x^2+3a\,d^2ex+\cdots}\right.$$
$$+\left(\int \frac{\sqrt{ex+d}\,\sqrt{-c\,x^2+a}}{-c\,e^3x^5-3cd\,e^2x^4+a\,e^3x^3-3c\,d^2e\,x^3+3ad\,e^2x^2-c\,d^3x^2+3a\,d^2ex+a\,d^3}dx\right)a$$

input
```
int((B*x+A)/(e*x+d)^(5/2)/(-c*x^2+a)^(1/2),x)
```

output

```
int((sqrt(d + e*x)*sqrt(a - c*x**2)*x)/(a*d**3 + 3*a*d**2*e*x + 3*a*d*e**2
*x**2 + a*e**3*x**3 - c*d**3*x**2 - 3*c*d**2*e*x**3 - 3*c*d*e**2*x**4 - c*
e**3*x**5),x)*b + int((sqrt(d + e*x)*sqrt(a - c*x**2))/(a*d**3 + 3*a*d**2*
e*x + 3*a*d*e**2*x**2 + a*e**3*x**3 - c*d**3*x**2 - 3*c*d**2*e*x**3 - 3*c*
d*e**2*x**4 - c*e**3*x**5),x)*a
```

3.278 $\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 405

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{3/2}}\,dx = \frac{(d+ex)^{3/2}(a(Bd+Ae)+(Acd+aBe)x)}{ac\sqrt{a-cx^2}} + \frac{e(3Acd+5aBe)\sqrt{d+ex}\sqrt{a-cx^2}}{3ac^2} + \frac{(3Acd^2+20aBde+9aAe^2)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{3\sqrt{a}c^{3/2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} - \frac{(3Acd+5aBe)(cd^2-ae^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{3\sqrt{a}c^{5/2}\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
(e*x+d)^(3/2)*(a*(A*e+B*d)+(A*c*d+B*a*e)*x)/a/c/(-c*x^2+a)^(1/2)+1/3*e*(3*
A*c*d+5*B*a*e)*(e*x+d)^(1/2)*(-c*x^2+a)^(1/2)/a/c^2+1/3*(9*A*a*e^2+3*A*c*d
^2+20*B*a*d*e)*(e*x+d)^(1/2)*(1-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/
a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/a^
(1/2)/c^(3/2)/(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/
2)-1/3*(3*A*c*d+5*B*a*e)*(-a*e^2+c*d^2)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2
)*e))^(1/2)*(1-c*x^2/a)^(1/2)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^
(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/a^(1/2)/c^(5/2)/(e*
x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.54 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.40

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{3/2}}\,dx = \frac{e^2\sqrt{-d+\frac{\sqrt{a}e}{\sqrt{c}}}\left(3Acd^2+20aBde+9aAe^2\right)(a-cx^2)+e\sqrt{-d+\frac{\sqrt{a}e}{\sqrt{c}}}(d+ex)}{}$$

input
```
Integrate[((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2)^(3/2),x]
```

output
```
(e^2*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(3*A*c*d^2 + 20*a*B*d*e + 9*a*A*e^2)*(
a - c*x^2) + e*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(d + e*x)*(5*a^2*B*e^2 + 3*A
*c^2*d^2*x + a*c*(3*A*e*(2*d + e*x) + B*(3*d^2 + 6*d*e*x - 2*e^2*x^2))) +
I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(3*A*c*d^2 + 20*a*B*d*e + 9*a*A*e^2)*Sqr
t[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/
(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt
[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)] + I*
Sqrt[a]*e*(Sqrt[c]*d - Sqrt[a]*e)*(3*A*c*d + 5*a*B*e - 3*Sqrt[a]*Sqrt[c]*(
5*B*d + 3*A*e))*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]
*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d
 + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d
 - Sqrt[a]*e)])/(3*a*c^2*e*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*Sqrt[d + e*x]*Sq
rt[a - c*x^2])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {684, 27, 687, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{\left(a-cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{684}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{\int \frac{e\sqrt{d+ex}(a(5Bd+3Ae)+(3Acd+5aBe)x)}{2\sqrt{a-cx^2}}dx}{ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\int \frac{\sqrt{d+ex}(a(5Bd+3Ae)+(3Acd+5aBe)x)}{\sqrt{a-cx^2}}dx}{2ac}$$

$$\Big\downarrow \text{687}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(-\frac{2\int -\frac{a\left(12Acde+5B\left(3cd^2+ae^2\right)\right)+c\left(3Acd^2+20aBed+9aAe^2\right)x}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{3c}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(5aBe+3Acd)}{3c}\right)}{2ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{\int \frac{a\left(15Bcd^2+12Aced+5aBe^2\right)+c\left(3Acd^2+20aBed+9aAe^2\right)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{3c}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(5aBe+3Acd)}{3c}\right)}{2ac}$$

$$\Big\downarrow \text{600}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{\frac{c\left(9aAe^2+20aBde+3Acd^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{\left(cd^2-ae^2\right)(5aBe+3Acd)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}}{3c}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(5aBe+3Acd)}{3c}\right)}{2ac}$$

$\big\downarrow$ 509

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{\frac{c\sqrt{1-\frac{cx^2}{a}}\left(9aAe^2+20aBde+3Acd^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{\left(cd^2-ae^2\right)(5aBe+3Acd)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}}{3c}-\frac{2\sqrt{a-cx^2}\sqrt{d+ex}(5aBe+3Acd)}{3c}\right)}{2ac}$$

$\big\downarrow$ 508

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{-\frac{\left(cd^2-ae^2\right)(5aBe+3Acd)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aAe^2+20aBde+3Acd^2\right)\int\frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{3c}-2\sqrt{a-cx^2}\sqrt{d+\cdots}\right)}{2ac}$$

$\big\downarrow$ 327

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{-\frac{\left(cd^2-ae^2\right)(5aBe+3Acd)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aAe^2+20aBde+3Acd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{3c}-2\sqrt{a-cx^2}\sqrt{\cdots}\right)}{2ac}$$

$\big\downarrow$ 512

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{-\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)(5aBe+3Acd)\int\frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aAe^2+20aBde+3Acd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{3c}-2\sqrt{\cdots}\right)}{2ac}$$

$\downarrow$ 511

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}(5aBe+3Acd)\int\frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aAe^2+20aBde+3Acd^2\right)\cdots}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+\cdots}{\sqrt{a}e+\sqrt{\cdots}}}}}{3c}\cdots\right)}{2ac}$$

$\downarrow$ 321

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}(5aBe+3Acd)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(9aAe^2+20aBde+3Acd^2\right)E\left(\arcsin\cdots\right.}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{3c}\cdots\right)}{2ac}$$

input
```
Int[((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2)^(3/2),x]
```

output
```
((d + e*x)^(3/2)*(a*(B*d + A*e) + (A*c*d + a*B*e)*x))/(a*c*Sqrt[a - c*x^2]
) - (e*((-2*(3*A*c*d + 5*a*B*e)*Sqrt[d + e*x]*Sqrt[a - c*x^2])/(3*c) + ((-
2*Sqrt[a]*Sqrt[c]*(3*A*c*d^2 + 20*a*B*d*e + 9*a*A*e^2)*Sqrt[d + e*x]*Sqrt[
1 - (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2
*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + S
qrt[a]*e)]*Sqrt[a - c*x^2]) + (2*Sqrt[a]*(3*A*c*d + 5*a*B*e)*(c*d^2 - a*e^
2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]*E
llipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)
/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[a - c*x^2]))/(3*c)))/(2*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 844 vs. $2(337) = 674$.

Time = 10.78 (sec) , antiderivative size = 845, normalized size of antiderivative = 2.09

method	result
elliptic	$\sqrt{(ex+d)(-cx^2+a)}\left(-\frac{2(-cex-cd)\left(\frac{(Aae^2+Acd^2+2Bade)x}{2c^2a}+\frac{2Acde+Bae^2+Bcd^2}{2c^3}\right)}{\sqrt{\left(x^2-\frac{a}{c}\right)(-cex-cd)}}+\frac{2Be^2\sqrt{-cex^3-cdx^2+aex+ad}}{3c^2}+\frac{2\left(-\frac{e(3Acde+ \dots$
risch	Expression too large to display
default	Expression too large to display

input
```
int((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(-2*(-c*e*x-c*d)
*(1/2*(A*a*e^2+A*c*d^2+2*B*a*d*e)/c^2/a*x+1/2*(2*A*c*d*e+B*a*e^2+B*c*d^2)/
c^3)/((x^2-a/c)*(-c*e*x-c*d))^(1/2)+2/3*B*e^2/c^2*(-c*e*x^3-c*d*x^2+a*e*x+
a*d)^(1/2)+2*(-e*(3*A*c*d*e+B*a*e^2+3*B*c*d^2)/c^2+1/c^2*(3*A*a*c*d*e^2+A*
c^2*d^3+B*a^2*e^3+3*B*a*c*d^2*e)/a-1/2/c^2*e*(2*A*c*d*e+B*a*e^2+B*c*d^2)-1
/c*d*(A*a*e^2+A*c*d^2+2*B*a*d*e)/a-1/3*a/c^2*B*e^3)*(d/e-1/c*(a*c)^(1/2))*
((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)
^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^
3-c*d*x^2+a*e*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)
,((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+2*(-e^2*(A*e+3*B*d
)/c-1/2*(A*a*e^2+A*c*d^2+2*B*a*d*e)*e/a/c+2/3/c*d*e^2*B)*(d/e-1/c*(a*c)^(1
/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*
(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c
*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))*EllipticE(((x+d/e)
/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2
)))^(1/2))+1/c*(a*c)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)
,((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{3/2}}\,dx = \frac{(3\,Aac^2d^3 - 25\,Ba^2cd^2e - 27\,Aa^2cde^2 - 15\,Ba^3e^3 - (3\,Ac^3d^3 - 25\,Bac^2d^2e}{}$$

input
```
integrate((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/9*((3*A*a*c^2*d^3 - 25*B*a^2*c*d^2*e - 27*A*a^2*c*d*e^2 - 15*B*a^3*e^3 -
 (3*A*c^3*d^3 - 25*B*a*c^2*d^2*e - 27*A*a*c^2*d*e^2 - 15*B*a^2*c*e^3)*x^2)
*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^
3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 3*(3*A*a*c^2*d^2*e + 20*B*a^2
*c*d*e^2 + 9*A*a^2*c*e^3 - (3*A*c^3*d^2*e + 20*B*a*c^2*d*e^2 + 9*A*a*c^2*e
^3)*x^2)*sqrt(-c*e)*weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(
c*d^3 - 9*a*d*e^2)/(c*e^3), weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e
^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e)) + 3*(2*B*a*c^2
*e^3*x^2 - 3*B*a*c^2*d^2*e - 6*A*a*c^2*d*e^2 - 5*B*a^2*c*e^3 - 3*(A*c^3*d^
2*e + 2*B*a*c^2*d*e^2 + A*a*c^2*e^3)*x)*sqrt(-c*x^2 + a)*sqrt(e*x + d))/(a
*c^4*e*x^2 - a^2*c^3*e)
```

Sympy [F]

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{3/2}}\,dx = \int \frac{(A+Bx)\,(d+ex)^{\frac{5}{2}}}{(a-cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)**(5/2)/(-c*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x)*(d + e*x)**(5/2)/(a - c*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{3/2}}\,dx = \int \frac{(Bx+A)(ex+d)^{\frac{5}{2}}}{(-cx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x + d)^(5/2)/(-c*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{3/2}}\,dx = \int \frac{(Bx+A)(ex+d)^{\frac{5}{2}}}{(-cx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x + d)^(5/2)/(-c*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{3/2}}\,dx = \int \frac{(A+B\,x)\,(d+e\,x)^{5/2}}{(a-c\,x^2)^{3/2}}\,dx$$

input
```
int(((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2)^(3/2),x)
```

output
```
int(((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^(3/2),x)
```

output
```
(18*sqrt(d + e*x)*sqrt(a - c*x**2)*a**2*e**3 + 50*sqrt(d + e*x)*sqrt(a - c
*x**2)*a*b*d*e**2 + 18*sqrt(d + e*x)*sqrt(a - c*x**2)*a*c*d**2*e - 12*sqrt
(d + e*x)*sqrt(a - c*x**2)*a*c*d*e**2*x + 6*sqrt(d + e*x)*sqrt(a - c*x**2)
*b*c*d**3 - 28*sqrt(d + e*x)*sqrt(a - c*x**2)*b*c*d**2*e*x - 4*sqrt(d + e*
x)*sqrt(a - c*x**2)*b*c*d*e**2*x**2 - 9*int(sqrt(d + e*x)/(sqrt(a - c*x**2
)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c*d**2*x**2 + s
qrt(a - c*x**2)*c*e**2*x**4),x)*a**4*d*e**4 - 25*int(sqrt(d + e*x)/(sqrt(a
 - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c*d**2
*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)*a**3*b*d**2*e**3 + 3*int(sqrt(d +
 e*x)/(sqrt(a - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c
*x**2)*c*d**2*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)*a**3*c*d**3*e**2 + 9
*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2
 - sqrt(a - c*x**2)*c*d**2*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)*a**3*c*
d*e**4*x**2 + 25*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a*d**2 - sqrt(a - c*x
**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c*d**2*x**2 + sqrt(a - c*x**2)*c*e**2*
x**4),x)*a**2*b*c*d**4*e + 25*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a*d**2 -
 sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c*d**2*x**2 + sqrt(a - c*
x**2)*c*e**2*x**4),x)*a**2*b*c*d**2*e**3*x**2 + 6*int(sqrt(d + e*x)/(sqrt(
a - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c*d**
2*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)*a**2*c**2*d**5 - 3*int(sqrt(d...
```

3.279 $\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{3/2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 336

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{3/2}} dx = \frac{\sqrt{d+ex}(a(Bd+Ae)+(Acd+aBe)x)}{ac\sqrt{a-cx^2}} + \frac{(Acd+3aBe)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}c^{3/2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} - \frac{A(cd^2-ae^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}c^{3/2}\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
(e*x+d)^(1/2)*(a*(A*e+B*d)+(A*c*d+B*a*e)*x)/a/c/(-c*x^2+a)^(1/2)+(A*c*d+3*
B*a*e)*(e*x+d)^(1/2)*(1-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2))
^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/a^(1/2)/c^
(3/2)/(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)-A*(-a
*e^2+c*d^2)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2
)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^
(1/2)*d+a^(1/2)*e))^(1/2))/a^(1/2)/c^(3/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.56 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.48

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{3/2}}\,dx = \frac{\sqrt{a-cx^2}\left(-\frac{c(d+ex)(aAe+Acdx+aB(d+ex))}{-a+cx^2} - \frac{e^2(Acd+3aBe)\sqrt{-d+\frac{\sqrt{a}e}{\sqrt{c}}}(a-cx^2)+i\sqrt{c}(\sqrt{c}d-}{}\right.}{}$$

input
```
Integrate[((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^(3/2),x]
```

output
```
(Sqrt[a - c*x^2]*(-((c*(d + e*x)*(a*A*e + A*c*d*x + a*B*(d + e*x)))/(-a +
c*x^2)) - (e^2*(A*c*d + 3*a*B*e)*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(a - c*x^2
) + I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(A*c*d + 3*a*B*e)*Sqrt[(e*(Sqrt[a]/S
qrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d
+ e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e
*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)] + I*Sqrt[a]*(-3*Sqr
t[a]*B + A*Sqrt[c])*Sqrt[c]*e*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[(e*(Sqrt[a]/Sqr
t[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d +
e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x
]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)])/(e*Sqrt[-d + (Sqrt[a
]*e)/Sqrt[c]]*(-a + c*x^2))))/(a*c^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {684, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{684}$$

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{\int \frac{e(a(3Bd+Ae)+(Acd+3aBe)x)}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\int \frac{a(3Bd+Ae)+(Acd+3aBe)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{2ac}$$

$$\Big\downarrow \text{600}$$

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{(3aBe+Acd)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{A\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{2ac}$$

$$\Big\downarrow \text{509}$$

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{\sqrt{1-\frac{cx^2}{a}}(3aBe+Acd)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{A\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{2ac}$$

$$\Big\downarrow \text{508}$$

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(3aBe+Acd)\int \frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{A\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{2ac}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(-\frac{A(cd^2-ae^2)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(3aBe+Acd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2ac}$$

$\downarrow$ 512

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(-\frac{A\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)\int\frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(3aBe+Acd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2ac}$$

$\downarrow$ 511

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{2\sqrt{a}A\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\int\frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(3aBe+Acd)E\left(\arcsin\left(\frac{\sqrt{1-}}{\sqrt{}}\right.\right.}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right.}{2ac}$$

$\downarrow$ 321

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{2\sqrt{a}A\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(3aBe+Acd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right.}{2ac}$$

input
```
Int[((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^(3/2),x]
```

output
```
(Sqrt[d + e*x]*(a*(B*d + A*e) + (A*c*d + a*B*e)*x))/(a*c*Sqrt[a - c*x^2])
- (e*((-2*Sqrt[a]*(A*c*d + 3*a*B*e)*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*Elli
pticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sq
rt[a] + e)])/(Sqrt[c]*e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*
Sqrt[a - c*x^2]) + (2*Sqrt[a]*A*(c*d^2 - a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(
Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqr
t[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sq
rt[d + e*x]*Sqrt[a - c*x^2])))/(2*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 702 vs. $2(278) = 556$.

Time = 4.90 (sec) , antiderivative size = 703, normalized size of antiderivative = 2.09

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c\,x^2+a)}\left(-\frac{2(-cex-cd)\left(\frac{(Acd+Bae)x}{2a\,c^2}+\frac{Ae+Bd}{2c^2}\right)}{\sqrt{\left(x^2-\frac{a}{c}\right)(-cex-cd)}}+\frac{2\left(-\frac{e(Ae+2Bd)}{c}+\frac{Aa\,e^2+Ac\,d^2+2Bade}{ac}-\frac{e(Ae+Bd)}{2c}-\frac{d(Acd+Bae)}{ca}\right)\left(\frac{d}{e}-\right.}{\sqrt{-ce\,x^3}}\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(-2*(-c*e*x-c*d)
*(1/2*(A*c*d+B*a*e)/a/c^2*x+1/2*(A*e+B*d)/c^2)/((x^2-a/c)*(-c*e*x-c*d))^(1
/2)+2*(-e*(A*e+2*B*d)/c+(A*a*e^2+A*c*d^2+2*B*a*d*e)/a/c-1/2/c*e*(A*e+B*d)-
1/c*d*(A*c*d+B*a*e)/a)*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)
))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^
(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*El
lipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/
e-1/c*(a*c)^(1/2)))^(1/2))+2*(-B*e^2/c-1/2*(A*c*d+B*a*e)*e/a/c)*(d/e-1/c*(
a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d
/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1
/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))*EllipticE((
(x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*
c)^(1/2)))^(1/2))+1/c*(a*c)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2))
)^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{3/2}}\,dx = \frac{(Aacd^2-6\,Ba^2de-3\,Aa^2e^2-(Ac^2d^2-6\,Bacde-3\,Aace^2)x^2)\sqrt{-ce}\mathrm{weiers}}{}$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*((A*a*c*d^2 - 6*B*a^2*d*e - 3*A*a^2*e^2 - (A*c^2*d^2 - 6*B*a*c*d*e - 3
*A*a*c*e^2)*x^2)*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e
^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 3*(A*a*c*d*e
+ 3*B*a^2*e^2 - (A*c^2*d*e + 3*B*a*c*e^2)*x^2)*sqrt(-c*e)*weierstrassZeta(
4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), weierstr
assPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^
3), 1/3*(3*e*x + d)/e)) - 3*(B*a*c*d*e + A*a*c*e^2 + (A*c^2*d*e + B*a*c*e^
2)*x)*sqrt(-c*x^2 + a)*sqrt(e*x + d))/(a*c^3*e*x^2 - a^2*c^2*e)
```

Sympy [F]

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{3/2}}\,dx = \int \frac{(A+Bx)\,(d+ex)^{\frac{3}{2}}}{(a-cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)**(3/2)/(-c*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x)*(d + e*x)**(3/2)/(a - c*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{3/2}}\,dx = \int \frac{(Bx+A)(ex+d)^{\frac{3}{2}}}{(-cx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x + d)^(3/2)/(-c*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{3/2}}\,dx = \int \frac{(Bx+A)(ex+d)^{\frac{3}{2}}}{(-cx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x + d)^(3/2)/(-c*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{3/2}}\,dx = \int \frac{(A+B\,x)\,(d+e\,x)^{3/2}}{(a-c\,x^2)^{3/2}}\,dx$$

input
```
int(((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^(3/2),x)
```

output
```
int(((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^(3/2),x)
```

output
```
(6*sqrt(d + e*x)*sqrt(a - c*x**2)*a*b*e**2 + 4*sqrt(d + e*x)*sqrt(a - c*x*
*2)*a*c*d*e + 2*sqrt(d + e*x)*sqrt(a - c*x**2)*b*c*d**2 - 4*sqrt(d + e*x)*
sqrt(a - c*x**2)*b*c*d*e*x - 3*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a*d**2
- sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c*d**2*x**2 + sqrt(a - c
*x**2)*c*e**2*x**4),x)*a**3*b*d*e**3 - 2*int(sqrt(d + e*x)/(sqrt(a - c*x**
2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c*d**2*x**2 +
sqrt(a - c*x**2)*c*e**2*x**4),x)*a**3*c*d**2*e**2 + 3*int(sqrt(d + e*x)/(s
qrt(a - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c
*d**2*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)*a**2*b*c*d**3*e + 3*int(sqrt
(d + e*x)/(sqrt(a - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a
 - c*x**2)*c*d**2*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)*a**2*b*c*d*e**3*
x**2 + 2*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e
**2*x**2 - sqrt(a - c*x**2)*c*d**2*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)
*a**2*c**2*d**4 + 2*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a*d**2 - sqrt(a -
c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c*d**2*x**2 + sqrt(a - c*x**2)*c*e*
*2*x**4),x)*a**2*c**2*d**2*e**2*x**2 - 3*int(sqrt(d + e*x)/(sqrt(a - c*x**
2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c*d**2*x**2 +
sqrt(a - c*x**2)*c*e**2*x**4),x)*a*b*c**2*d**3*e*x**2 - 2*int(sqrt(d + e*x
)/(sqrt(a - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x**
2)*c*d**2*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)*a*c**3*d**4*x**2 - 3*...
```

3.280 $\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 311

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{3/2}}\,dx = \frac{(aB+Acx)\sqrt{d+ex}}{ac\sqrt{a-cx^2}} + \frac{A\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}\sqrt{c}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} - \frac{(Acd-aBe)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}c^{3/2}\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
(A*c*x+B*a)*(e*x+d)^(1/2)/a/c/(-c*x^2+a)^(1/2)+A*(e*x+d)^(1/2)*(1-c*x^2/a)
^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*
e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/a^(1/2)/c^(1/2)/(c^(1/2)*(e*x+d)/(c^(1/2)*
d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)-(A*c*d-B*a*e)*(c^(1/2)*(e*x+d)/(c^(1/
2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2
))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/a^(1/2)/
c^(3/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.55 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{3/2}}\,dx = \frac{\sqrt{a-cx^2}\left(Ae+\frac{(aB+Acx)(d+ex)}{a-cx^2}-\frac{iA\sqrt{c}(\sqrt{c}d-\sqrt{a}e)\sqrt{\frac{e\left(\frac{\sqrt{a}}{\sqrt{c}}+x\right)}{d+ex}}\sqrt{-\frac{\frac{\sqrt{a}e}{\sqrt{c}}-ex}{d+ex}}(d+ex)^{3/2}E\left(ia\right.}{e\sqrt{-d+\frac{\sqrt{a}e}{\sqrt{c}}}(-a+cx^2)}\right.}{}$$

input
```
Integrate[((A + B*x)*Sqrt[d + e*x])/(a - c*x^2)^(3/2),x]
```

output
```
(Sqrt[a - c*x^2]*(A*e + ((a*B + A*c*x)*(d + e*x))/(a - c*x^2) - (I*A*Sqrt[
c]*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[
-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcS
inh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)
/(Sqrt[c]*d - Sqrt[a]*e)])/(e*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2))
 + (I*Sqrt[a]*(Sqrt[a]*B - A*Sqrt[c])*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d +
e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*Ellip
ticF[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d +
 Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)])/(Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a
+ c*x^2))))/(a*c*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {685, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 685}$$

$$\frac{\sqrt{d+ex}(aB+Acx)}{ac\sqrt{a-cx^2}}-\frac{\int \frac{e(aB+Acx)}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{ac}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{d+ex}(aB+Acx)}{ac\sqrt{a-cx^2}}-\frac{e\int \frac{aB+Acx}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{2ac}$$

$\Big\downarrow$ 600

$$\frac{\sqrt{d+ex}(aB+Acx)}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{Ac\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{(Acd-aBe)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{2ac}$$

$\Big\downarrow$ 509

$$\frac{\sqrt{d+ex}(aB+Acx)}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{Ac\sqrt{1-\frac{cx^2}{a}}\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{(Acd-aBe)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{2ac}$$

$\Big\downarrow$ 508

$$\frac{\sqrt{d+ex}(aB+Acx)}{ac\sqrt{a-cx^2}}-\frac{e\left(-\frac{(Acd-aBe)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}A\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2ac}$$

$\Big\downarrow$ 327

$$\frac{\sqrt{d+ex}(aB+Acx)}{ac\sqrt{a-cx^2}}-\frac{e\left(-\frac{(Acd-aBe)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}A\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2ac}$$

$\Big\downarrow$ 512

$$\frac{\frac{\sqrt{d+ex}(aB+Acx)}{ac\sqrt{a-cx^2}}-\frac{e\left(-\frac{\sqrt{1-\frac{cx^2}{a}}(Acd-aBe)\int\frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}A\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2ac}}{}$$

$\downarrow$ 511

$$\frac{\sqrt{d+ex}(aB+Acx)}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}(Acd-aBe)\int\frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}A\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2ac}$$

$\downarrow$ 321

$$\frac{\sqrt{d+ex}(aB+Acx)}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}(Acd-aBe)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}A\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2ac}$$

input
```
Int[((A + B*x)*Sqrt[d + e*x])/(a - c*x^2)^(3/2),x]
```

output
```
((a*B + A*c*x)*Sqrt[d + e*x])/(a*c*Sqrt[a - c*x^2]) - (e*((-2*Sqrt[a]*A*Sq
rt[c]*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]
*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(e*Sqrt[(Sqrt[c]*
(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) + (2*Sqrt[a]*(A*c*d -
 a*B*e)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)
/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[
c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[a - c*x^2])))/(2*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 685
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^(m - 1)*(a + c*x^2)
^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m]
 || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(253) = 506$.

Time = 1.84 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.01

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c\,x^2+a)}\left(-\frac{2(-cex-cd)\left(\frac{Ax}{2ac}+\frac{B}{2c^2}\right)}{\sqrt{\left(x^2-\frac{a}{c}\right)(-cex-cd)}}+\frac{2\left(-\frac{3Be}{2c}+\frac{Acd+Bae}{ac}-\frac{Ad}{a}\right)\left(\frac{d}{e}-\frac{\sqrt{ac}}{c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{\sqrt{ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{ac}}{c}}{-\frac{d}{e}-\frac{\sqrt{ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{ac}}{c}}{-\frac{d}{e}+\frac{\sqrt{ac}}{c}}}\ \text{EllipticF}\left(\right.}{\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}\right.}{}$
default	$\frac{\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\left(A\sqrt{-\frac{c(ex+d)}{\sqrt{ac}\,e-cd}}\sqrt{\frac{(-cx+\sqrt{ac})e}{\sqrt{ac}\,e+cd}}\sqrt{\frac{(cx+\sqrt{ac})e}{\sqrt{ac}\,e-cd}}\ \text{EllipticF}\left(\sqrt{-\frac{c(ex+d)}{\sqrt{ac}\,e-cd}},\sqrt{-\frac{\sqrt{ac}\,e-cd}{\sqrt{ac}\,e+cd}}\right)ac\,e^2-A\sqrt{ac}\,\sqrt{-\frac{c(ex+}{\sqrt{ac}\,e-}}\right.}{}$

input
```
int((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(-2*(-c*e*x-c*d)
*(1/2*A/a/c*x+1/2*B/c^2)/((x^2-a/c)*(-c*e*x-c*d))^(1/2)+2*(-3/2*B*e/c+(A*c
*d+B*a*e)/a/c-A/a*d)*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))
^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1
/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*Elli
pticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-
1/c*(a*c)^(1/2)))^(1/2))-A*e/a*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*
c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/
c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^
(1/2)*((-d/e-1/c*(a*c)^(1/2))*EllipticE(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1
/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+1/c*(a*c)^(1/2)
*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(
-d/e-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{3/2}}\,dx = \frac{(Aacd-3\,Ba^2e-(Ac^2d-3\,Bace)x^2)\sqrt{-ce}\text{weierstrassPInverse}\left(\frac{4\,(cd^2+3\,ae^2)}{3\,ce^2},-\right.}{}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*((A*a*c*d - 3*B*a^2*e - (A*c^2*d - 3*B*a*c*e)*x^2)*sqrt(-c*e)*weierstr
assPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^
3), 1/3*(3*e*x + d)/e) - 3*(A*c^2*e*x^2 - A*a*c*e)*sqrt(-c*e)*weierstrassZ
eta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), weie
rstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(
c*e^3), 1/3*(3*e*x + d)/e)) - 3*(A*c^2*e*x + B*a*c*e)*sqrt(-c*x^2 + a)*sqr
t(e*x + d))/(a*c^3*e*x^2 - a^2*c^2*e)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{3/2}}\,dx = \int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)**(1/2)/(-c*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x)*sqrt(d + e*x)/(a - c*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{3/2}}\,dx = \int \frac{(Bx+A)\sqrt{ex+d}}{(-cx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*sqrt(e*x + d)/(-c*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{3/2}}\,dx = \int \frac{(Bx+A)\sqrt{ex+d}}{(-cx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*sqrt(e*x + d)/(-c*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{3/2}}\,dx = \int \frac{(A+B\,x)\,\sqrt{d+e\,x}}{(a-c\,x^2)^{3/2}}\,dx$$

input
```
int(((A + B*x)*(d + e*x)^(1/2))/(a - c*x^2)^(3/2),x)
```

output
```
int(((A + B*x)*(d + e*x)^(1/2))/(a - c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(d + e*x)*sqrt(a - c*x**2)*a*e + 2*sqrt(d + e*x)*sqrt(a - c*x**2)*b
*d - int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*
x**2 - sqrt(a - c*x**2)*c*d**2*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)*a**
3*d*e**2 - int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a
*e**2*x**2 - sqrt(a - c*x**2)*c*d**2*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),
x)*a**2*b*d**2*e + 2*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a*d**2 - sqrt(a -
 c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c*d**2*x**2 + sqrt(a - c*x**2)*c*e
**2*x**4),x)*a**2*c*d**3 + int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a*d**2 - sq
rt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c*d**2*x**2 + sqrt(a - c*x**
2)*c*e**2*x**4),x)*a**2*c*d*e**2*x**2 + int(sqrt(d + e*x)/(sqrt(a - c*x**2
)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c*d**2*x**2 + s
qrt(a - c*x**2)*c*e**2*x**4),x)*a*b*c*d**2*e*x**2 - 2*int(sqrt(d + e*x)/(s
qrt(a - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c
*d**2*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)*a*c**2*d**3*x**2 - int((sqrt
(d + e*x)*sqrt(a - c*x**2)*x**2)/(a**2*d + a**2*e*x - 2*a*c*d*x**2 - 2*a*c
*e*x**3 + c**2*d*x**4 + c**2*e*x**5),x)*a**2*c*e**2 + int((sqrt(d + e*x)*s
qrt(a - c*x**2)*x**2)/(a**2*d + a**2*e*x - 2*a*c*d*x**2 - 2*a*c*e*x**3 + c
**2*d*x**4 + c**2*e*x**5),x)*a*b*c*d*e + int((sqrt(d + e*x)*sqrt(a - c*x**
2)*x**2)/(a**2*d + a**2*e*x - 2*a*c*d*x**2 - 2*a*c*e*x**3 + c**2*d*x**4 +
c**2*e*x**5),x)*a*c**2*e**2*x**2 - int((sqrt(d + e*x)*sqrt(a - c*x**2)*...
```

3.281 $\int \frac{A+Bx}{\sqrt{d+ex}\left(a-cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 351

$$\int \frac{A+Bx}{\sqrt{d+ex}\left(a-cx^2\right)^{3/2}}\,dx=\frac{\sqrt{d+ex}(a(Bd-Ae)+(Acd-aBe)x)}{a\left(cd^2-ae^2\right)\sqrt{a-cx^2}}+\frac{(Acd-aBe)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}\sqrt{c}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}}-\frac{A\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}\sqrt{c}\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
(e*x+d)^(1/2)*(a*(-A*e+B*d)+(A*c*d-B*a*e)*x)/a/(-a*e^2+c*d^2)/(-c*x^2+a)^(
1/2)+(A*c*d-B*a*e)*(e*x+d)^(1/2)*(1-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2
)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2)
)/a^(1/2)/c^(1/2)/(-a*e^2+c*d^2)/(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(
1/2)/(-c*x^2+a)^(1/2)-A*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c
*x^2/a)^(1/2)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a
^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/a^(1/2)/c^(1/2)/(e*x+d)^(1/2)/(-c*x
^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.63 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{3/2}}\,dx = \frac{-e^2(Acd-aBe)\sqrt{-d+\frac{\sqrt{a}e}{\sqrt{c}}}(-a+cx^2)-ce\sqrt{-d+\frac{\sqrt{a}e}{\sqrt{c}}}(d+ex)(-Acdx+}{}$$

input
```
Integrate[(A + B*x)/(Sqrt[d + e*x]*(a - c*x^2)^(3/2)),x]
```

output
```
(-(e^2*(A*c*d - a*B*e)*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2)) - c*e*
Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(d + e*x)*(-(A*c*d*x) + a*(-(B*d) + A*e + B
*e*x)) + I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(A*c*d - a*B*e)*Sqrt[(e*(Sqrt[a
]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*
(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d
+ e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)] + I*Sqrt[a]*(Sqr
t[a]*B + A*Sqrt[c])*Sqrt[c]*e*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[(e*(Sqrt[a]/Sqr
t[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d +
e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x
]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)])/(a*c*e*Sqrt[-d + (Sq
rt[a]*e)/Sqrt[c]]*(c*d^2 - a*e^2)*Sqrt[d + e*x]*Sqrt[a - c*x^2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {686, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a-cx^2)^{3/2}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{686}$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{a\sqrt{a-cx^2}\left(cd^2-ae^2\right)}-\frac{\int -\frac{ce(a(Bd-Ae)-(Acd-aBe)x)}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{ac\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{a(Bd-Ae)-(Acd-aBe)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{2a\left(cd^2-ae^2\right)}+\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{a\sqrt{a-cx^2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{600}$$

$$\frac{e\left(\frac{A\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{(Acd-aBe)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}\right)}{2a\left(cd^2-ae^2\right)}+\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{a\sqrt{a-cx^2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{509}$$

$$\frac{e\left(\frac{A\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{\sqrt{1-\frac{cx^2}{a}}(Acd-aBe)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}\right)}{2a\left(cd^2-ae^2\right)}+\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{a\sqrt{a-cx^2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{508}$$

$$\frac{e\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(Acd-aBe)\int \frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}+\frac{A\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{2a\left(cd^2-ae^2\right)}+\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{a\sqrt{a-cx^2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{327}$$

$$\frac{e\left(\frac{A(cd^2-ae^2)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}+\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(Acd-aBe)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2a\left(cd^2-ae^2\right)}+\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{a\sqrt{a-cx^2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 512$$

$$\frac{e\left(\frac{A\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)\int\frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}+\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(Acd-aBe)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2a\left(cd^2-ae^2\right)}+\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{a\sqrt{a-cx^2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 511$$

$$\frac{e\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(Acd-aBe)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{2\sqrt{a}A\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\int\frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right.}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}}{2a\left(cd^2-ae^2\right)}$$

$$+\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{a\sqrt{a-cx^2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 321$$

$$\frac{e\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(Acd-aBe)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{2\sqrt{a}A\sqrt{1-\frac{cx^2}{a}}(cd^2-ae^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}}{2a\left(cd^2-ae^2\right)}$$

$$+\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{a\sqrt{a-cx^2}\left(cd^2-ae^2\right)}$$

input
```
Int[(A + B*x)/(Sqrt[d + e*x]*(a - c*x^2)^(3/2)),x]
```

output
```
(Sqrt[d + e*x]*(a*(B*d - A*e) + (A*c*d - a*B*e)*x))/(a*(c*d^2 - a*e^2)*Sqr
t[a - c*x^2]) + (e*((2*Sqrt[a]*(A*c*d - a*B*e)*Sqrt[d + e*x]*Sqrt[1 - (c*x
^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sq
rt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + S
qrt[a]*e)]*Sqrt[a - c*x^2]) - (2*Sqrt[a]*A*(c*d^2 - a*e^2)*Sqrt[(Sqrt[c]*(
d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[ArcSin[Sq
rt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(S
qrt[c]*e*Sqrt[d + e*x]*Sqrt[a - c*x^2])))/(2*a*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 719 vs. $2(293) = 586$.

Time = 6.30 (sec) , antiderivative size = 720, normalized size of antiderivative = 2.05

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c\,x^2+a)}\left(-\frac{2(-cex-cd)\left(-\frac{(Acd-Bae)x}{2\left(a\,e^2-c\,d^2\right)ac}+\frac{Ae-Bd}{2c\left(a\,e^2-c\,d^2\right)}\right)}{\sqrt{\left(x^2-\frac{a}{c}\right)(-cex-cd)}}+\frac{2\left(\frac{A}{a}-\frac{e(Ae-Bd)}{2\left(a\,e^2-c\,d^2\right)}+\frac{d(Acd-Bae)}{\left(a\,e^2-c\,d^2\right)a}\right)\left(\frac{d}{e}-\frac{\sqrt{ac}}{c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{\sqrt{ac}}{c}}}}{\sqrt{-ce\,x^3-c}}\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(-2*(-c*e*x-c*d)
*(-1/2*(A*c*d-B*a*e)/(a*e^2-c*d^2)/a/c*x+1/2*(A*e-B*d)/c/(a*e^2-c*d^2))/((
x^2-a/c)*(-c*e*x-c*d))^(1/2)+2*(A/a-1/2*e*(A*e-B*d)/(a*e^2-c*d^2)+d*(A*c*d
-B*a*e)/(a*e^2-c*d^2)/a)*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/
2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c
)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*
EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-
d/e-1/c*(a*c)^(1/2)))^(1/2))+e*(A*c*d-B*a*e)/(a*e^2-c*d^2)/a*(d/e-1/c*(a*c
)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-
1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)
/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))*EllipticE(((x+
d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^
(1/2)))^(1/2))+1/c*(a*c)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(
1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{3/2}}\,dx =$$

$$-\frac{(Aacd^2+2\,Ba^2de-3\,Aa^2e^2-(Ac^2d^2+2\,Bacde-3\,Aace^2)x^2)\sqrt{-ce}\text{weierstrassPInverse}\left(\frac{4\,(cd^2+3\,ae^2)}{3\,ce^2},\right.}{}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*((A*a*c*d^2 + 2*B*a^2*d*e - 3*A*a^2*e^2 - (A*c^2*d^2 + 2*B*a*c*d*e -
3*A*a*c*e^2)*x^2)*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*
e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 3*(A*a*c*d*e
 - B*a^2*e^2 - (A*c^2*d*e - B*a*c*e^2)*x^2)*sqrt(-c*e)*weierstrassZeta(4/3
*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), weierstrass
PInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3),
 1/3*(3*e*x + d)/e)) - 3*(B*a*c*d*e - A*a*c*e^2 + (A*c^2*d*e - B*a*c*e^2)*
x)*sqrt(-c*x^2 + a)*sqrt(e*x + d))/(a^2*c^2*d^2*e - a^3*c*e^3 - (a*c^3*d^2
*e - a^2*c^2*e^3)*x^2)
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{3/2}}\,dx = \int \frac{A+Bx}{(a-cx^2)^{\frac{3}{2}}\sqrt{d+ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)**(1/2)/(-c*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x)/((a - c*x**2)**(3/2)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{3/2}}\,dx = \int \frac{Bx+A}{(-cx^2+a)^{\frac{3}{2}}\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((-c*x^2 + a)^(3/2)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{3/2}}\,dx = \int \frac{Bx+A}{(-cx^2+a)^{\frac{3}{2}}\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/((-c*x^2 + a)^(3/2)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{3/2}}\,dx = \int \frac{A+B\,x}{(a-c\,x^2)^{3/2}\,\sqrt{d+e\,x}}\,dx$$

input
```
int((A + B*x)/((a - c*x^2)^(3/2)*(d + e*x)^(1/2)),x)
```

output
```
int((A + B*x)/((a - c*x^2)^(3/2)*(d + e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(d + e*x)*sqrt(a - c*x**2)*b*x + 3*int(sqrt(d + e*x)/(sqrt(a - c*x*
*2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c*d**2*x**2 +
 sqrt(a - c*x**2)*c*e**2*x**4),x)*a**3*d*e - 2*int(sqrt(d + e*x)/(sqrt(a -
 c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*c*d**2*x
**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)*a**2*b*d**2 - 3*int(sqrt(d + e*x)/(
sqrt(a - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x**2)*
c*d**2*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)*a**2*c*d*e*x**2 + 2*int(sqr
t(d + e*x)/(sqrt(a - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(
a - c*x**2)*c*d**2*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)*a*b*c*d**2*x**2
 + int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**3)/(a**2*d + a**2*e*x - 2*a*c*d*
x**2 - 2*a*c*e*x**3 + c**2*d*x**4 + c**2*e*x**5),x)*a*b*c*e - int((sqrt(d
+ e*x)*sqrt(a - c*x**2)*x**3)/(a**2*d + a**2*e*x - 2*a*c*d*x**2 - 2*a*c*e*
x**3 + c**2*d*x**4 + c**2*e*x**5),x)*b*c**2*e*x**2 - 3*int((sqrt(d + e*x)*
x)/(sqrt(a - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqrt(a - c*x*
*2)*c*d**2*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)*a**3*e**2 + 2*int((sqrt
(d + e*x)*x)/(sqrt(a - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e**2*x**2 - sqr
t(a - c*x**2)*c*d**2*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)*a**2*b*d*e +
3*int((sqrt(d + e*x)*x)/(sqrt(a - c*x**2)*a*d**2 - sqrt(a - c*x**2)*a*e**2
*x**2 - sqrt(a - c*x**2)*c*d**2*x**2 + sqrt(a - c*x**2)*c*e**2*x**4),x)*a*
*2*c*e**2*x**2 - 2*int((sqrt(d + e*x)*x)/(sqrt(a - c*x**2)*a*d**2 - sqr...
```

3.282 $\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 452

$$\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)^{3/2}}\,dx = -\frac{2(Bd-Ae)}{(cd^2-ae^2)\sqrt{d+ex}\sqrt{a-cx^2}} + \frac{\sqrt{d+ex}\left(a(3Bcd^2-4Acde+aBe^2)+c(Acd^2-4aBde+3aAe^2)x\right)}{a\left(cd^2-ae^2\right)^2\sqrt{a-cx^2}} + \frac{\sqrt{c}(Acd^2-4aBde+3aAe^2)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}\left(cd^2-ae^2\right)^2\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} - \frac{(Acd-aBe)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{a}\sqrt{c}\left(cd^2-ae^2\right)\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
(2*A*e-2*B*d)/(-a*e^2+c*d^2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)+(e*x+d)^(1/2)*
(a*(-4*A*c*d*e+B*a*e^2+3*B*c*d^2)+c*(3*A*a*e^2+A*c*d^2-4*B*a*d*e)*x)/a/(-a
*e^2+c*d^2)^2/(-c*x^2+a)^(1/2)+c^(1/2)*(3*A*a*e^2+A*c*d^2-4*B*a*d*e)*(e*x+
d)^(1/2)*(1-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/
2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/a^(1/2)/(-a*e^2+c*d^2)
^2/(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)-(A*c*d-B
*a*e)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2)*Elli
pticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*
d+a^(1/2)*e))^(1/2))/a^(1/2)/c^(1/2)/(-a*e^2+c*d^2)/(e*x+d)^(1/2)/(-c*x^2+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.92 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)^{3/2}}\,dx = \frac{\sqrt{a-cx^2}\left(2ae^2(Bd-Ae)+e(Acd^2-4aBde+3aAe^2)-\frac{(d+ex)(a^2Be^2+Ac}{}\right.}{}$$

input
```
Integrate[(A + B*x)/((d + e*x)^(3/2)*(a - c*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - c*x^2]*(2*a*e^2*(B*d - A*e) + e*(A*c*d^2 - 4*a*B*d*e + 3*a*A*e^2
) - ((d + e*x)*(a^2*B*e^2 + A*c^2*d^2*x + a*c*(B*d*(d - 2*e*x) + A*e*(-2*d
 + e*x))))/(-a + c*x^2) - (I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(A*c*d^2 - 4*
a*B*d*e + 3*a*A*e^2)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sq
rt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sq
rt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt
[c]*d - Sqrt[a]*e)])/(e*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2)) - (I*
Sqrt[a]*(Sqrt[c]*d - Sqrt[a]*e)*(A*c*d - a*B*e + 3*Sqrt[a]*Sqrt[c]*(B*d -
A*e))*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c
] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d + (Sqrt[a
]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]
*e)])/(Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2))))/(a*(c*d^2 - a*e^2)^2
*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {686, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a-cx^2)^{3/2}(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 686}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{a\sqrt{a-cx^2}\sqrt{d+ex}\,(cd^2-ae^2)}-\frac{\int -\frac{ce(3a(Bd-Ae)+(Acd-aBe)x)}{2(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ac\,(cd^2-ae^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\int \frac{3a(Bd-Ae)+(Acd-aBe)x}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{2a\,(cd^2-ae^2)}+\frac{x(Acd-aBe)+a(Bd-Ae)}{a\sqrt{a-cx^2}\sqrt{d+ex}\,(cd^2-ae^2)}$$

$$\Big\downarrow \text{ 688}$$

$$\frac{e\left(\frac{2\int \frac{a\left(3Bcd^2-4Aced+aBe^2\right)-c\left(Acd^2-4aBed+3aAe^2\right)x}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\,(3aAe^2-4aBde+Acd^2)}{\sqrt{d+ex}(cd^2-ae^2)}\right)}{2a\,(cd^2-ae^2)}+\frac{x(Acd-aBe)+a(Bd-Ae)}{a\sqrt{a-cx^2}\sqrt{d+ex}\,(cd^2-ae^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\left(\frac{\int \frac{a\left(3Bcd^2-4Aced+aBe^2\right)-c\left(Acd^2-4aBed+3aAe^2\right)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\,(3aAe^2-4aBde+Acd^2)}{\sqrt{d+ex}(cd^2-ae^2)}\right)}{2a\,(cd^2-ae^2)}+\frac{x(Acd-aBe)+a(Bd-Ae)}{a\sqrt{a-cx^2}\sqrt{d+ex}\,(cd^2-ae^2)}$$

$$\Big\downarrow \text{ 600}$$

$$\frac{e\left(\frac{\frac{\left(cd^2-ae^2\right)(Acd-aBe)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{c\left(3aAe^2-4aBde+Acd^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\left(3aAe^2-4aBde+Acd^2\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{2a\left(cd^2-ae^2\right)}+\frac{x(Acd-aBe)+a(Bd-Ae)}{a\sqrt{a-cx^2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 509

$$\frac{e\left(\frac{\frac{\left(cd^2-ae^2\right)(Acd-aBe)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{c\sqrt{1-\frac{cx^2}{a}}\left(3aAe^2-4aBde+Acd^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\left(3aAe^2-4aBde+Acd^2\right)}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right)}{2a\left(cd^2-ae^2\right)}+\frac{x(Acd-aBe)+a(Bd-Ae)}{a\sqrt{a-cx^2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 508

$$\frac{e\left(\frac{\frac{\left(cd^2-ae^2\right)(Acd-aBe)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}+\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(3aAe^2-4aBde+Acd^2\right)\int \frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\left(3aAe^2-4\cdots\right.}{\sqrt{d+ex}\left(cd^2\cdots\right.}\right.}{2a\left(cd^2-ae^2\right)}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{a\sqrt{a-cx^2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 327

$$\frac{e\left(\frac{\frac{\left(cd^2-ae^2\right)(Acd-aBe)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}+\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(3aAe^2-4aBde+Acd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\left(3aAe^2-\cdots\right.}{\sqrt{d+ex}\left(cd\cdots\right.}\cdots\right)}{2a\left(cd^2-ae^2\right)}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{a\sqrt{a-cx^2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$\downarrow$ 512

$$\frac{e\left(\frac{\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)(Acd-aBe)\int \frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}+\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(3aAe^2-4aBde+Acd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\cdots}{\sqrt{\cdots}}\cdots\right)}{2a\left(cd^2-ae^2\right)}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{a\sqrt{a-cx^2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$\downarrow$ 511

$$\frac{e\left(\frac{\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(3aAe^2-4aBde+Acd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}(Acd-aBe)\int \frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\cdots}\cdots}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}}{cd^2-ae^2}\cdots\right)}{2a\left(cd^2-ae^2\right)}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{a\sqrt{a-cx^2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$\downarrow$ 321

$$\frac{e\left(\frac{\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(3aAe^2-4aBde+Acd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}(Acd-aBe)\text{EllipticF}\left(\arcsin\left(\sqrt{\cdots}\right.\right.}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}}{cd^2-ae^2}\right.}{2a\left(cd^2-ae^2\right)}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{a\sqrt{a-cx^2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

input
```
Int[(A + B*x)/((d + e*x)^(3/2)*(a - c*x^2)^(3/2)),x]
```

output
```
(a*(B*d - A*e) + (A*c*d - a*B*e)*x)/(a*(c*d^2 - a*e^2)*Sqrt[d + e*x]*Sqrt[
a - c*x^2]) + (e*((-2*(A*c*d^2 - 4*a*B*d*e + 3*a*A*e^2)*Sqrt[a - c*x^2])/(
(c*d^2 - a*e^2)*Sqrt[d + e*x]) + ((2*Sqrt[a]*Sqrt[c]*(A*c*d^2 - 4*a*B*d*e
+ 3*a*A*e^2)*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (
Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(e*Sqrt[(S
qrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) - (2*Sqrt[a]*(
A*c*d - a*B*e)*(c*d^2 - a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[
a]*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/
Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[
a - c*x^2]))/(c*d^2 - a*e^2)))/(2*a*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 883 vs. $2(390) = 780$.

Time = 8.42 (sec) , antiderivative size = 884, normalized size of antiderivative = 1.96

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c\,x^2+a)}\left(\frac{2ce\left(\frac{\left(3Aa\,e^2+Ac\,d^2-4Bade\right)x^2}{2a\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)}-\frac{(Acd-Bae)x}{2ace\left(a\,e^2-c\,d^2\right)}-\frac{2Aa\,e^3+2Ac\,d^2e-3Bad\,e^2-Bc\,d^3}{2ce\left(a^2e^4-2ac\,d^2e^2+c^2d^4\right)}\right)}{\sqrt{-\left(x^3+\frac{d\,x^2}{e}-\frac{ax}{c}-\frac{ad}{ce}\right)ce}}+\frac{2\left(-\frac{6Aacd\,e^2-2A\,c^2d^3-}{2a\left(a^2e^4-2ac\right.}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*(2*c*e*(1/2*(3*A
*a*e^2+A*c*d^2-4*B*a*d*e)/a/(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)*x^2-1/2*(A*c*d
-B*a*e)/a/c/e/(a*e^2-c*d^2)*x-1/2*(2*A*a*e^3+2*A*c*d^2*e-3*B*a*d*e^2-B*c*d
^3)/c/e/(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4))/(-(x^3+d/e*x^2-a/c*x-a/c*d/e)*c*e
)^(1/2)+2*(-1/2*(6*A*a*c*d*e^2-2*A*c^2*d^3-3*B*a^2*e^3-B*a*c*d^2*e)/a/(a^2
*e^4-2*a*c*d^2*e^2+c^2*d^4)+(A*c*d-B*a*e)/a/(a*e^2-c*d^2))*(d/e-1/c*(a*c)^
(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/
c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(
-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2))
)^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))-c*e*(3*A*a*
e^2+A*c*d^2-4*B*a*d*e)/a/(a^2*e^4-2*a*c*d^2*e^2+c^2*d^4)*(d/e-1/c*(a*c)^(1
/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*
(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c
*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))*EllipticE(((x+d/e)
/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2
)))^(1/2))+1/c*(a*c)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)
,((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(390) = 780$.

Time = 0.12 (sec) , antiderivative size = 804, normalized size of antiderivative = 1.78

$$\int \frac{A+Bx}{(d+ex)^{3/2}\,(a-cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*((A*a*c^2*d^4 + 5*B*a^2*c*d^3*e - 9*A*a^2*c*d^2*e^2 + 3*B*a^3*d*e^3 -
 (A*c^3*d^3*e + 5*B*a*c^2*d^2*e^2 - 9*A*a*c^2*d*e^3 + 3*B*a^2*c*e^4)*x^3 -
 (A*c^3*d^4 + 5*B*a*c^2*d^3*e - 9*A*a*c^2*d^2*e^2 + 3*B*a^2*c*d*e^3)*x^2 +
 (A*a*c^2*d^3*e + 5*B*a^2*c*d^2*e^2 - 9*A*a^2*c*d*e^3 + 3*B*a^3*e^4)*x)*sq
rt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 -
 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 3*(A*a*c^2*d^3*e - 4*B*a^2*c*d^2
*e^2 + 3*A*a^2*c*d*e^3 - (A*c^3*d^2*e^2 - 4*B*a*c^2*d*e^3 + 3*A*a*c^2*e^4)
*x^3 - (A*c^3*d^3*e - 4*B*a*c^2*d^2*e^2 + 3*A*a*c^2*d*e^3)*x^2 + (A*a*c^2*
d^2*e^2 - 4*B*a^2*c*d*e^3 + 3*A*a^2*c*e^4)*x)*sqrt(-c*e)*weierstrassZeta(4
/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), weierstra
ssPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3
), 1/3*(3*e*x + d)/e)) - 3*(B*a*c^2*d^3*e - 2*A*a*c^2*d^2*e^2 + 3*B*a^2*c*
d*e^3 - 2*A*a^2*c*e^4 + (A*c^3*d^2*e^2 - 4*B*a*c^2*d*e^3 + 3*A*a*c^2*e^4)*
x^2 + (A*c^3*d^3*e - B*a*c^2*d^2*e^2 - A*a*c^2*d*e^3 + B*a^2*c*e^4)*x)*sqr
t(-c*x^2 + a)*sqrt(e*x + d))/(a^2*c^3*d^5*e - 2*a^3*c^2*d^3*e^3 + a^4*c*d*
e^5 - (a*c^4*d^4*e^2 - 2*a^2*c^3*d^2*e^4 + a^3*c^2*e^6)*x^3 - (a*c^4*d^5*e
 - 2*a^2*c^3*d^3*e^3 + a^3*c^2*d*e^5)*x^2 + (a^2*c^3*d^4*e^2 - 2*a^3*c^2*d
^2*e^4 + a^4*c*e^6)*x)
```

Sympy [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)^{3/2}}\,dx=\int \frac{A+Bx}{\left(a-cx^2\right)^{\frac{3}{2}}\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)**(3/2)/(-c*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x)/((a - c*x**2)**(3/2)*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\,(a-cx^2)^{3/2}}\,dx = \int \frac{Bx+A}{(-cx^2+a)^{\frac{3}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((-c*x^2 + a)^(3/2)*(e*x + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\,(a-cx^2)^{3/2}}\,dx = \int \frac{Bx+A}{(-cx^2+a)^{\frac{3}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/((-c*x^2 + a)^(3/2)*(e*x + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)^{3/2}\,(a-cx^2)^{3/2}}\,dx = \int \frac{A+B\,x}{(a-c\,x^2)^{3/2}\,(d+e\,x)^{3/2}}\,dx$$

input
```
int((A + B*x)/((a - c*x^2)^(3/2)*(d + e*x)^(3/2)),x)
```

output
```
int((A + B*x)/((a - c*x^2)^(3/2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a - c*x**2)*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**3)/(a**2*d**2
+ 2*a**2*d*e*x + a**2*e**2*x**2 - 2*a*c*d**2*x**2 - 4*a*c*d*e*x**3 - 2*a*c
*e**2*x**4 + c**2*d**2*x**4 + 2*c**2*d*e*x**5 + c**2*e**2*x**6),x)*a*c*d*e
**2 - sqrt(a - c*x**2)*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**3)/(a**2*d**
2 + 2*a**2*d*e*x + a**2*e**2*x**2 - 2*a*c*d**2*x**2 - 4*a*c*d*e*x**3 - 2*a
*c*e**2*x**4 + c**2*d**2*x**4 + 2*c**2*d*e*x**5 + c**2*e**2*x**6),x)*a*c*e
**3*x - 2*sqrt(a - c*x**2)*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**3)/(a**2
*d**2 + 2*a**2*d*e*x + a**2*e**2*x**2 - 2*a*c*d**2*x**2 - 4*a*c*d*e*x**3 -
 2*a*c*e**2*x**4 + c**2*d**2*x**4 + 2*c**2*d*e*x**5 + c**2*e**2*x**6),x)*b
*c*d**2*e - 2*sqrt(a - c*x**2)*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x**3)/(
a**2*d**2 + 2*a**2*d*e*x + a**2*e**2*x**2 - 2*a*c*d**2*x**2 - 4*a*c*d*e*x*
*3 - 2*a*c*e**2*x**4 + c**2*d**2*x**4 + 2*c**2*d*e*x**5 + c**2*e**2*x**6),
x)*b*c*d*e**2*x - sqrt(a - c*x**2)*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x)/
(a**2*d**2 + 2*a**2*d*e*x + a**2*e**2*x**2 - 2*a*c*d**2*x**2 - 4*a*c*d*e*x
**3 - 2*a*c*e**2*x**4 + c**2*d**2*x**4 + 2*c**2*d*e*x**5 + c**2*e**2*x**6)
,x)*a**2*d*e**2 - sqrt(a - c*x**2)*int((sqrt(d + e*x)*sqrt(a - c*x**2)*x)/
(a**2*d**2 + 2*a**2*d*e*x + a**2*e**2*x**2 - 2*a*c*d**2*x**2 - 4*a*c*d*e*x
**3 - 2*a*c*e**2*x**4 + c**2*d**2*x**4 + 2*c**2*d*e*x**5 + c**2*e**2*x**6)
,x)*a**2*e**3*x - 4*sqrt(a - c*x**2)*int((sqrt(d + e*x)*sqrt(a - c*x**2))/
(a**2*d**2 + 2*a**2*d*e*x + a**2*e**2*x**2 - 2*a*c*d**2*x**2 - 4*a*c*d*...
```

3.283 $\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 482

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^{5/2}}\,dx = \frac{(d+ex)^{5/2}(a(Bd+Ae)+(Acd+aBe)x)}{3ac\,(a-cx^2)^{3/2}}$$
$$+\frac{\sqrt{d+ex}\,(ae(3Acd^2-14aBde-5aAe^2)+(2Acd(2cd^2-3ae^2)-7aBe(cd^2+ae^2))\,x)}{6a^2c^2\sqrt{a-cx^2}}$$
$$+\frac{(4Acd(cd^2-2ae^2)-7aBe(cd^2+3ae^2))\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{6a^{3/2}c^{5/2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}}$$
$$-\frac{(cd^2-ae^2)(4Acd^2-7aBde-5aAe^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{6a^{3/2}c^{5/2}\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
1/3*(e*x+d)^(5/2)*(a*(A*e+B*d)+(A*c*d+B*a*e)*x)/a/c/(-c*x^2+a)^(3/2)+1/6*(
e*x+d)^(1/2)*(a*e*(-5*A*a*e^2+3*A*c*d^2-14*B*a*d*e)+(2*A*c*d*(-3*a*e^2+2*c
*d^2)-7*a*B*e*(a*e^2+c*d^2))*x)/a^2/c^2/(-c*x^2+a)^(1/2)+1/6*(4*A*c*d*(-2*
a*e^2+c*d^2)-7*a*B*e*(3*a*e^2+c*d^2))*(e*x+d)^(1/2)*(1-c*x^2/a)^(1/2)*Elli
pticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*
d+a^(1/2)*e))^(1/2))/a^(3/2)/c^(5/2)/(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e
))^(1/2)/(-c*x^2+a)^(1/2)-1/6*(-a*e^2+c*d^2)*(-5*A*a*e^2+4*A*c*d^2-7*B*a*d
*e)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2)*Ellipt
icF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+
a^(1/2)*e))^(1/2))/a^(3/2)/c^(5/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.36 (sec) , antiderivative size = 710, normalized size of antiderivative = 1.47

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^{5/2}}\,dx = \frac{\sqrt{a-cx^2}\left(-\frac{(d+ex)\left(4Ac^3d^3x^3+a^3e^2(5Ae+7B(2d+ex))-ac^2dx\left(7Bdex^2+A\left(6d^2+dex+8e^2x^2\right)\right)-\cdots}{a^2c^2(a-cx^2)^2}\cdots\right.}{\cdots}$$

input
```
Integrate[((A + B*x)*(d + e*x)^(7/2))/(a - c*x^2)^(5/2),x]
```

output
```
(Sqrt[a - c*x^2]*(-(((d + e*x)*(4*A*c^3*d^3*x^3 + a^3*e^2*(5*A*e + 7*B*(2*
d + e*x)) - a*c^2*d*x*(7*B*d*e*x^2 + A*(6*d^2 + d*e*x + 8*e^2*x^2)) - a^2*
c*(A*e*(5*d^2 - 2*d*e*x + 7*e^2*x^2) + B*(2*d^3 - d^2*e*x + 20*d*e^2*x^2 +
 9*e^3*x^3))))/(a^2*c^2*(a - c*x^2)^2)) - (e^2*Sqrt[-d + (Sqrt[a]*e)/Sqrt[
c]]*(4*A*c*d*(c*d^2 - 2*a*e^2) - 7*a*B*e*(c*d^2 + 3*a*e^2))*(a - c*x^2) +
I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(4*A*c*d*(c*d^2 - 2*a*e^2) - 7*a*B*e*(c*
d^2 + 3*a*e^2))*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]
*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d
 + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d
 - Sqrt[a]*e)] + I*Sqrt[a]*Sqrt[c]*e*(Sqrt[c]*d - Sqrt[a]*e)*(7*a*B*e*(-(S
qrt[c]*d) + 3*Sqrt[a]*e) + A*(4*c^(3/2)*d^2 + 3*Sqrt[a]*c*d*e - 5*a*Sqrt[c
]*e^2))*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt
[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d + (Sqrt
[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[
a]*e)])/(a^2*c^3*e*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2))))/(6*Sqrt[
d + e*x])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {684, 27, 684, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{684}$$

$$\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac(a-cx^2)^{3/2}}-\frac{\int -\frac{(d+ex)^{3/2}(4Acd^2-ae(7Bd+5Ae)-e(Acd+7aBe)x)}{2(a-cx^2)^{3/2}}dx}{3ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d+ex)^{3/2}(4Acd^2-ae(7Bd+5Ae)-e(Acd+7aBe)x)}{(a-cx^2)^{3/2}}dx}{6ac}+\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac(a-cx^2)^{3/2}}$$

$$\Big\downarrow \text{684}$$

$$\frac{\frac{\sqrt{d+ex}\left(x\left(2Acd\left(2cd^2-3ae^2\right)-7aBe\left(ae^2+cd^2\right)\right)+ae\left(-5aAe^2-14aBde+3Acd^2\right)\right)}{ac\sqrt{a-cx^2}}-\frac{\int \frac{e\left(ae\left(Acd^2-28aBed-5aAe^2\right)+\left(4Acd\left(cd^2-2ae^2\right)-7aBe\left(cd\cdots\right.\right.\right.}{2\sqrt{d+ex}\sqrt{a-cx^2}}\cdots}{ac}}{6ac}$$

$$\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{\sqrt{d+ex}\left(x\left(2Acd\left(2cd^2-3ae^2\right)-7aBe\left(ae^2+cd^2\right)\right)+ae\left(-5aAe^2-14aBde+3Acd^2\right)\right)}{ac\sqrt{a-cx^2}}-\frac{e\int \frac{ae\left(Acd^2-28aBed-5aAe^2\right)+\left(4Acd\left(cd^2-2ae^2\right)-7aBe\left(cd^2\cdots\right.\right.}{\sqrt{d+ex}\sqrt{a-cx^2}}\cdots}{2ac}}{6ac}$$

$$\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$\big\downarrow$ 600

$$\frac{\frac{\sqrt{d+ex}\left(x\left(2Acd\left(2cd^2-3ae^2\right)-7aBe\left(ae^2+cd^2\right)\right)+ae\left(-5aAe^2-14aBde+3Acd^2\right)\right)}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{\left(4Acd\left(cd^2-2ae^2\right)-7aBe\left(3ae^2+cd^2\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{\left(cd\cdots\right.}{\cdots}\cdots\right.}{2ac}}{6ac}$$

$$\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$\big\downarrow$ 509

$$\frac{\frac{\sqrt{d+ex}\left(x\left(2Acd\left(2cd^2-3ae^2\right)-7aBe\left(ae^2+cd^2\right)\right)+ae\left(-5aAe^2-14aBde+3Acd^2\right)\right)}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{\sqrt{1-\frac{cx^2}{a}}\left(4Acd\left(cd^2-2ae^2\right)-7aBe\left(3ae^2+cd^2\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}\cdots}{e\sqrt{a-cx^2}}\cdots\right.}{\cdots}}{6ac}$$

$$\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$\big\downarrow$ 508

$$\frac{\frac{\sqrt{d+ex}\left(x\left(2Acd\left(2cd^2-3ae^2\right)-7aBe\left(ae^2+cd^2\right)\right)+ae\left(-5aAe^2-14aBde+3Acd^2\right)\right)}{ac\sqrt{a-cx^2}}-\frac{e\left(-\frac{\left(cd^2-ae^2\right)\left(-5aAe^2-7aBde+4Acd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}d}{e}\right.}{}}{6ac}$$

$$\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$\Big\downarrow$ 327

$$\frac{\frac{\sqrt{d+ex}\left(x\left(2Acd\left(2cd^2-3ae^2\right)-7aBe\left(ae^2+cd^2\right)\right)+ae\left(-5aAe^2-14aBde+3Acd^2\right)\right)}{ac\sqrt{a-cx^2}}-\frac{e\left(-\frac{\left(cd^2-ae^2\right)\left(-5aAe^2-7aBde+4Acd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}d}{e}\right.}{}}{6ac}$$

$$\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$\Big\downarrow$ 512

$$\frac{\frac{\sqrt{d+ex}\left(x\left(2Acd\left(2cd^2-3ae^2\right)-7aBe\left(ae^2+cd^2\right)\right)+ae\left(-5aAe^2-14aBde+3Acd^2\right)\right)}{ac\sqrt{a-cx^2}}-\frac{e\left(-\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\left(-5aAe^2-7aBde+4Acd^2\right)\int\frac{}{\sqrt{d+ex}}}{e\sqrt{a-cx^2}}\right.}{}}{6ac}$$

$$\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$\Big\downarrow$ 511

$$\frac{\frac{\sqrt{d+ex}(x(2Acd(2cd^2-3ae^2)-7aBe(ae^2+cd^2))+ae(-5aAe^2-14aBde+3Acd^2))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(-5aAe^2-7aBde+4A\cdots\right.}{\sqrt{c}e\sqrt{a-cx^2}\cdots}\cdots\right)}{\cdots}}{6ac\cdots}+\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$\downarrow$ 321

$$\frac{\frac{\sqrt{d+ex}(x(2Acd(2cd^2-3ae^2)-7aBe(ae^2+cd^2))+ae(-5aAe^2-14aBde+3Acd^2))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(-5aAe^2-7aBde+4A\cdots\right.}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d\cdots}}\cdots\right)}{\cdots}}{6ac}+\frac{(d+ex)^{5/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

input

```
Int[((A + B*x)*(d + e*x)^(7/2))/(a - c*x^2)^(5/2),x]
```

output

```
((d + e*x)^(5/2)*(a*(B*d + A*e) + (A*c*d + a*B*e)*x))/(3*a*c*(a - c*x^2)^(
3/2)) + ((Sqrt[d + e*x]*(a*e*(3*A*c*d^2 - 14*a*B*d*e - 5*a*A*e^2) + (2*A*c
*d*(2*c*d^2 - 3*a*e^2) - 7*a*B*e*(c*d^2 + a*e^2))*x))/(a*c*Sqrt[a - c*x^2]
) - (e*((-2*Sqrt[a]*(4*A*c*d*(c*d^2 - 2*a*e^2) - 7*a*B*e*(c*d^2 + 3*a*e^2)
)*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/
Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[(Sqrt
[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) + (2*Sqrt[a]*(c*d
^2 - a*e^2)*(4*A*c*d^2 - 7*a*B*d*e - 5*a*A*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(
Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqr
t[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sq
rt[d + e*x]*Sqrt[a - c*x^2])))/(2*a*c))/(6*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 968 vs. $2(412) = 824$.

Time = 7.20 (sec), antiderivative size = 969, normalized size of antiderivative = 2.01

method	result
elliptic	$\dfrac{\sqrt{(ex+d)(-cx^2+a)}\left(\dfrac{\left(\frac{(3Aacde^2+Ac^2d^3+Be^3a^2+3Bacd^2e)x}{3ac^4}+\frac{Aae^3+3Acd^2e+3Bade^2+Bcd^3}{3c^4}\right)\sqrt{-cex^3-cdx^2+aex+ad}}{\left(x^2-\frac{a}{c}\right)^2}-\dfrac{2(-cex-c\cdots}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((B*x+A)*(e*x+d)^(7/2)/(-c*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*((1/3*(3*A*a*c*d
*e^2+A*c^2*d^3+B*a^2*e^3+3*B*a*c*d^2*e)/a/c^4*x+1/3*(A*a*e^3+3*A*c*d^2*e+3
*B*a*d*e^2+B*c*d^3)/c^4)*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)/(x^2-a/c)^2-2*
(-c*e*x-c*d)*(-1/12*(8*A*a*c*d*e^2-4*A*c^2*d^3+9*B*a^2*e^3+7*B*a*c*d^2*e)/
a^2/c^3*x-1/12*(7*A*a*e^2+A*c*d^2+20*B*a*d*e)*e/c^3/a)/((x^2-a/c)*(-c*e*x-
c*d))^(1/2)+2*(e^3*(A*e+4*B*d)/c^2-1/6/c^2*(7*A*a^2*e^4+9*A*a*c*d^2*e^2-4*
A*c^2*d^4+29*B*a^2*d*e^3+7*B*a*c*d^3*e)/a^2+1/12/c^2*e^2*(7*A*a*e^2+A*c*d^
2+20*B*a*d*e)/a+1/6/c^2*d*(8*A*a*c*d*e^2-4*A*c^2*d^3+9*B*a^2*e^3+7*B*a*c*d
^2*e)/a^2)*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x
-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e
+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*EllipticF(((x+
d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^
(1/2)))^(1/2))+2*(B*e^4/c^2+1/12*(8*A*a*c*d*e^2-4*A*c^2*d^3+9*B*a^2*e^3+7*
B*a*c*d^2*e)*e/a^2/c^2)*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2
)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)
^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*(
(-d/e-1/c*(a*c)^(1/2))*EllipticE(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-
d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+1/c*(a*c)^(1/2)*Ellipt
icF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/
c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 752, normalized size of antiderivative = 1.56

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^(7/2)/(-c*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/18*((4*A*a^2*c^2*d^4 - 7*B*a^3*c*d^3*e - 11*A*a^3*c*d^2*e^2 + 63*B*a^4*
d*e^3 + 15*A*a^4*e^4 + (4*A*c^4*d^4 - 7*B*a*c^3*d^3*e - 11*A*a*c^3*d^2*e^2
 + 63*B*a^2*c^2*d*e^3 + 15*A*a^2*c^2*e^4)*x^4 - 2*(4*A*a*c^3*d^4 - 7*B*a^2
*c^2*d^3*e - 11*A*a^2*c^2*d^2*e^2 + 63*B*a^3*c*d*e^3 + 15*A*a^3*c*e^4)*x^2
)*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d
^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 3*(4*A*a^2*c^2*d^3*e - 7*B*a
^3*c*d^2*e^2 - 8*A*a^3*c*d*e^3 - 21*B*a^4*e^4 + (4*A*c^4*d^3*e - 7*B*a*c^3
*d^2*e^2 - 8*A*a*c^3*d*e^3 - 21*B*a^2*c^2*e^4)*x^4 - 2*(4*A*a*c^3*d^3*e -
7*B*a^2*c^2*d^2*e^2 - 8*A*a^2*c^2*d*e^3 - 21*B*a^3*c*e^4)*x^2)*sqrt(-c*e)*
weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(
c*e^3), weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 -
9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e)) - 3*(2*B*a^2*c^2*d^3*e + 5*A*a^2*c
^2*d^2*e^2 - 14*B*a^3*c*d*e^3 - 5*A*a^3*c*e^4 - (4*A*c^4*d^3*e - 7*B*a*c^3
*d^2*e^2 - 8*A*a*c^3*d*e^3 - 9*B*a^2*c^2*e^4)*x^3 + (A*a*c^3*d^2*e^2 + 20*
B*a^2*c^2*d*e^3 + 7*A*a^2*c^2*e^4)*x^2 + (6*A*a*c^3*d^3*e - B*a^2*c^2*d^2*
e^2 - 2*A*a^2*c^2*d*e^3 - 7*B*a^3*c*e^4)*x)*sqrt(-c*x^2 + a)*sqrt(e*x + d)
)/(a^2*c^5*e*x^4 - 2*a^3*c^4*e*x^2 + a^4*c^3*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(e*x+d)**(7/2)/(-c*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)(ex+d)^{\frac{7}{2}}}{(-cx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(7/2)/(-c*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x + d)^(7/2)/(-c*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)(ex+d)^{\frac{7}{2}}}{(-cx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(7/2)/(-c*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x + d)^(7/2)/(-c*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^{5/2}}\,dx = \int \frac{(A+B\,x)\,(d+e\,x)^{7/2}}{(a-c\,x^2)^{5/2}}\,dx$$

input
```
int(((A + B*x)*(d + e*x)^(7/2))/(a - c*x^2)^(5/2),x)
```

output
```
int(((A + B*x)*(d + e*x)^(7/2))/(a - c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)(d+ex)^{7/2}}{(a-cx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((B*x+A)*(e*x+d)^(7/2)/(-c*x^2+a)^(5/2),x)
```

output
```
(21*sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**2 - sqrt(
a - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a - c*x**2)*a*c*d**2*x**2 + 2*sqrt(a -
 c*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**2)*c**2*d**2*x**4 - sqrt(a - c*x**2
)*c**2*e**2*x**6),x)*a**5*b*d*e**5 + 3*sqrt(a - c*x**2)*int(sqrt(d + e*x)/
(sqrt(a - c*x**2)*a**2*d**2 - sqrt(a - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a -
 c*x**2)*a*c*d**2*x**2 + 2*sqrt(a - c*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**
2)*c**2*d**2*x**4 - sqrt(a - c*x**2)*c**2*e**2*x**6),x)*a**5*c*d**2*e**4 -
 42*sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**2 - sqrt(
a - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a - c*x**2)*a*c*d**2*x**2 + 2*sqrt(a -
 c*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**2)*c**2*d**2*x**4 - sqrt(a - c*x**2
)*c**2*e**2*x**6),x)*a**4*b*c*d**3*e**3 - 42*sqrt(a - c*x**2)*int(sqrt(d +
 e*x)/(sqrt(a - c*x**2)*a**2*d**2 - sqrt(a - c*x**2)*a**2*e**2*x**2 - 2*sq
rt(a - c*x**2)*a*c*d**2*x**2 + 2*sqrt(a - c*x**2)*a*c*e**2*x**4 + sqrt(a -
 c*x**2)*c**2*d**2*x**4 - sqrt(a - c*x**2)*c**2*e**2*x**6),x)*a**4*b*c*d*e
**5*x**2 - 6*sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**
2 - sqrt(a - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a - c*x**2)*a*c*d**2*x**2 + 2
*sqrt(a - c*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**2)*c**2*d**2*x**4 - sqrt(a
 - c*x**2)*c**2*e**2*x**6),x)*a**4*c**2*d**4*e**2 - 6*sqrt(a - c*x**2)*int
(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**2 - sqrt(a - c*x**2)*a**2*e**2*x*
*2 - 2*sqrt(a - c*x**2)*a*c*d**2*x**2 + 2*sqrt(a - c*x**2)*a*c*e**2*x**...
```

3.284 $\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 434

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{5/2}}\,dx = \frac{(d+ex)^{3/2}(a(Bd+Ae)+(Acd+aBe)x)}{3ac\,(a-cx^2)^{3/2}} + \frac{\sqrt{d+ex}\,(ae(Acd-5aBe)+c(4Acd^2-5aBde-3aAe^2)\,x)}{6a^2c^2\sqrt{a-cx^2}} + \frac{(4Acd^2-5aBde-3aAe^2)\,\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{6a^{3/2}c^{3/2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} - \frac{(4Acd-5aBe)\,(cd^2-ae^2)\,\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{6a^{3/2}c^{5/2}\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
1/3*(e*x+d)^(3/2)*(a*(A*e+B*d)+(A*c*d+B*a*e)*x)/a/c/(-c*x^2+a)^(3/2)+1/6*(
e*x+d)^(1/2)*(a*e*(A*c*d-5*B*a*e)+c*(-3*A*a*e^2+4*A*c*d^2-5*B*a*d*e)*x)/a^
2/c^2/(-c*x^2+a)^(1/2)+1/6*(-3*A*a*e^2+4*A*c*d^2-5*B*a*d*e)*(e*x+d)^(1/2)*
(1-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2
)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/a^(3/2)/c^(3/2)/(c^(1/2)*(e*x+d
)/(c^(1/2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)-1/6*(4*A*c*d-5*B*a*e)*(-a*
e^2+c*d^2)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2)
*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(
1/2)*d+a^(1/2)*e))^(1/2))/a^(3/2)/c^(5/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.93 (sec) , antiderivative size = 606, normalized size of antiderivative = 1.40

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{5/2}}\,dx = \frac{\sqrt{a-cx^2}\left(-\frac{(d+ex)(5a^3Be^2+4Ac^3d^2x^3+a^2c(Ae(-3d+ex)+B(-2d^2+dex-7e^2x^2))-ac^2x(5Bde}{a^2c^2(a-cx^2)^2}\right.}{}$$

input
```
Integrate[((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2)^(5/2),x]
```

output
```
(Sqrt[a - c*x^2]*(-(((d + e*x)*(5*a^3*B*e^2 + 4*A*c^3*d^2*x^3 + a^2*c*(A*e
*(-3*d + e*x) + B*(-2*d^2 + d*e*x - 7*e^2*x^2)) - a*c^2*x*(5*B*d*e*x^2 + A
*(6*d^2 + d*e*x + 3*e^2*x^2))))/(a^2*c^2*(a - c*x^2)^2)) + (e^2*Sqrt[-d +
(Sqrt[a]*e)/Sqrt[c]]*(4*A*c*d^2 - 5*a*B*d*e - 3*a*A*e^2)*(-a + c*x^2) - I*
Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(4*A*c*d^2 - 5*a*B*d*e - 3*a*A*e^2)*Sqrt[(
e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d
+ e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]
]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)] + I*Sqr
t[a]*e*(-(Sqrt[c]*d) + Sqrt[a]*e)*(4*A*c*d - 5*a*B*e + 3*Sqrt[a]*A*Sqrt[c]
*e)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c]
- e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d + (Sqrt[a]*
e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e
)])/(a^2*c^2*e*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2))))/(6*Sqrt[d +
e*x])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {684, 27, 685, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{684}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac(a-cx^2)^{3/2}}-\frac{\int -\frac{\sqrt{d+ex}(4Acd^2-ae(5Bd+3Ae)+e(Acd-5aBe)x)}{2(a-cx^2)^{3/2}}dx}{3ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{d+ex}(4Acd^2-ae(5Bd+3Ae)+e(Acd-5aBe)x)}{(a-cx^2)^{3/2}}dx}{6ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac(a-cx^2)^{3/2}}$$

$$\Big\downarrow \text{685}$$

$$\frac{\frac{\sqrt{d+ex}(cx(4Acd^2-ae(3Ae+5Bd))+ae(Acd-5aBe))}{ac\sqrt{a-cx^2}}-\frac{\int \frac{e\left(ae(Acd-5aBe)+c\left(4Acd^2-ae(5Bd+3Ae)\right)x\right)}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{ac}}{6ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac(a-cx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sqrt{d+ex}(cx(4Acd^2-ae(3Ae+5Bd))+ae(Acd-5aBe))}{ac\sqrt{a-cx^2}}-\frac{e\int \frac{ae(Acd-5aBe)+c\left(4Acd^2-ae(5Bd+3Ae)\right)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{2ac}}{6ac}+\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac(a-cx^2)^{3/2}}$$

$$\Big\downarrow \text{600}$$

$$\frac{\frac{\sqrt{d+ex}\left(cx\left(4Acd^2-ae(3Ae+5Bd)\right)+ae(Acd-5aBe)\right)}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{c\left(4Acd^2-ae(3Ae+5Bd)\right)\int\frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{\left(cd^2-ae^2\right)(4Acd-5aBe)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{2ac}}{6ac}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$\Big\downarrow$ 509

$$\frac{\frac{\sqrt{d+ex}\left(cx\left(4Acd^2-ae(3Ae+5Bd)\right)+ae(Acd-5aBe)\right)}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{c\sqrt{1-\frac{cx^2}{a}}\left(4Acd^2-ae(3Ae+5Bd)\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{\left(cd^2-ae^2\right)(4Acd-5aBe)\int\frac{1}{\sqrt{d+ex}\sqrt{a-\cdots}}}{e}\cdots\right.}{2ac}}{6ac}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$\Big\downarrow$ 508

$$\frac{\frac{\sqrt{d+ex}\left(cx\left(4Acd^2-ae(3Ae+5Bd)\right)+ae(Acd-5aBe)\right)}{ac\sqrt{a-cx^2}}-\frac{e\left(-\frac{\left(cd^2-ae^2\right)(4Acd-5aBe)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(4Acd^2-ae(3Ae\cdots\right.}{e\sqrt{a-cx^2}\cdots}\right.}{2ac}}{6ac}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$\Big\downarrow$ 327

$$\frac{\frac{\sqrt{d+ex}\left(cx\left(4Acd^2-ae(3Ae+5Bd)\right)+ae(Acd-5aBe)\right)}{ac\sqrt{a-cx^2}}-\frac{e\left(-\frac{\left(cd^2-ae^2\right)(4Acd-5aBe)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(4Acd^2-ae(3Ae\cdots\right.}{e\sqrt{a-cx^2}\cdots}\right.}{2ac}}{6ac}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$\Big\downarrow$ 512

$$\frac{\frac{\sqrt{d+ex}(cx(4Acd^2-ae(3Ae+5Bd))+ae(Acd-5aBe))}{ac\sqrt{a-cx^2}}-\frac{e\left(-\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)(4Acd-5aBe)\int\frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(4Acd\right.}{}\right.}{2ac}}{6ac}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$\downarrow$ 511

$$\frac{\frac{\sqrt{d+ex}(cx(4Acd^2-ae(3Ae+5Bd))+ae(Acd-5aBe))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}(4Acd-5aBe)\int\frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}\right.}{}}{6ac}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$\downarrow$ 321

$$\frac{\frac{\sqrt{d+ex}(cx(4Acd^2-ae(3Ae+5Bd))+ae(Acd-5aBe))}{ac\sqrt{a-cx^2}}-\frac{e\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}(4Acd-5aBe)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right.}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}\right.}{2ac}}{6ac}$$

$$\frac{(d+ex)^{3/2}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

input
```
Int[((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2)^(5/2),x]
```

output
```
((d + e*x)^(3/2)*(a*(B*d + A*e) + (A*c*d + a*B*e)*x))/(3*a*c*(a - c*x^2)^(
3/2)) + ((Sqrt[d + e*x]*(a*e*(A*c*d - 5*a*B*e) + c*(4*A*c*d^2 - a*e*(5*B*d
 + 3*A*e))*x))/(a*c*Sqrt[a - c*x^2]) - (e*((-2*Sqrt[a]*Sqrt[c]*(4*A*c*d^2
- a*e*(5*B*d + 3*A*e))*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSin[
Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/
(e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) + (2
*Sqrt[a]*(4*A*c*d - 5*a*B*e)*(c*d^2 - a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqr
t[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c
]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[
d + e*x]*Sqrt[a - c*x^2])))/(2*a*c))/(6*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 685
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^(m - 1)*(a + c*x^2)
^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m]
 || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 844 vs. $2(364) = 728$.

Time = 6.30 (sec) , antiderivative size = 845, normalized size of antiderivative = 1.95

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c\,x^2+a)}\left(\frac{\left(\frac{\left(Aa\,e^2+Ac\,d^2+2Bade\right)x}{3c^3a}+\frac{2Acde+Ba\,e^2+Bc\,d^2}{3c^4}\right)\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}{\left(x^2-\frac{a}{c}\right)^2}-\frac{2(-cex-cd)\left(-\frac{\left(3Aa\,e^2-4Ac\,d^2+5B\right.}{12c^2a^2}\right.}{\sqrt{\left(x^2-\frac{a}{c}\right)(-cex}}\right.}{}$
default	Expression too large to display

input

```
int((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*((1/3*(A*a*e^2+A
*c*d^2+2*B*a*d*e)/c^3/a*x+1/3*(2*A*c*d*e+B*a*e^2+B*c*d^2)/c^4)*(-c*e*x^3-c
*d*x^2+a*e*x+a*d)^(1/2)/(x^2-a/c)^2-2*(-c*e*x-c*d)*(-1/12*(3*A*a*e^2-4*A*c
*d^2+5*B*a*d*e)/c^2/a^2*x-1/12*(A*c*d+7*B*a*e)*e/c^3/a)/((x^2-a/c)*(-c*e*x
-c*d))^(1/2)+2*(B*e^3/c^2-1/6/c^2*(4*A*a*c*d*e^2-4*A*c^2*d^3+7*B*a^2*e^3+5
*B*a*c*d^2*e)/a^2+1/12/c^2*e^2*(A*c*d+7*B*a*e)/a+1/6/c*d*(3*A*a*e^2-4*A*c*
d^2+5*B*a*d*e)/a^2)*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^
(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/
2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*Ellip
ticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1
/c*(a*c)^(1/2)))^(1/2))+1/6*(3*A*a*e^2-4*A*c*d^2+5*B*a*d*e)*e/a^2/c*(d/e-1
/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))
/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2))
)^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))*Ellipti
cE(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c
*(a*c)^(1/2)))^(1/2))+1/c*(a*c)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1
/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{5/2}}\,dx =$$

$$-\frac{(4\,Aa^2c^2d^3 - 5\,Ba^3cd^2e - 6\,Aa^3cde^2 + 15\,Ba^4e^3 + (4\,Ac^4d^3 - 5\,Bac^3d^2e - 6\,Aac^3de^2 + 15\,Ba^2c^2e^3)x^4 -}{}$$

input
```
integrate((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/18*((4*A*a^2*c^2*d^3 - 5*B*a^3*c*d^2*e - 6*A*a^3*c*d*e^2 + 15*B*a^4*e^3
 + (4*A*c^4*d^3 - 5*B*a*c^3*d^2*e - 6*A*a*c^3*d*e^2 + 15*B*a^2*c^2*e^3)*x^
4 - 2*(4*A*a*c^3*d^3 - 5*B*a^2*c^2*d^2*e - 6*A*a^2*c^2*d*e^2 + 15*B*a^3*c*
e^3)*x^2)*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8
/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 3*(4*A*a^2*c^2*d^2*e
 - 5*B*a^3*c*d*e^2 - 3*A*a^3*c*e^3 + (4*A*c^4*d^2*e - 5*B*a*c^3*d*e^2 - 3*
A*a*c^3*e^3)*x^4 - 2*(4*A*a*c^3*d^2*e - 5*B*a^2*c^2*d*e^2 - 3*A*a^2*c^2*e^
3)*x^2)*sqrt(-c*e)*weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c
*d^3 - 9*a*d*e^2)/(c*e^3), weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^
2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e)) - 3*(2*B*a^2*c^
2*d^2*e + 3*A*a^2*c^2*d*e^2 - 5*B*a^3*c*e^3 - (4*A*c^4*d^2*e - 5*B*a*c^3*d
*e^2 - 3*A*a*c^3*e^3)*x^3 + (A*a*c^3*d*e^2 + 7*B*a^2*c^2*e^3)*x^2 + (6*A*a
*c^3*d^2*e - B*a^2*c^2*d*e^2 - A*a^2*c^2*e^3)*x)*sqrt(-c*x^2 + a)*sqrt(e*x
 + d))/(a^2*c^5*e*x^4 - 2*a^3*c^4*e*x^2 + a^4*c^3*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(e*x+d)**(5/2)/(-c*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)(ex+d)^{\frac{5}{2}}}{(-cx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x + d)^(5/2)/(-c*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)(ex+d)^{\frac{5}{2}}}{(-cx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x + d)^(5/2)/(-c*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{5/2}}\,dx = \int \frac{(A+B\,x)\,\left(d+e\,x\right)^{5/2}}{\left(a-c\,x^2\right)^{5/2}}\,dx$$

input
```
int(((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2)^(5/2),x)
```

output
```
int(((A + B*x)*(d + e*x)^(5/2))/(a - c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)(d+ex)^{5/2}}{(a-cx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((B*x+A)*(e*x+d)^(5/2)/(-c*x^2+a)^(5/2),x)
```

output
```
(sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**2 - sqrt(a -
 c*x**2)*a**2*e**2*x**2 - 2*sqrt(a - c*x**2)*a*c*d**2*x**2 + 2*sqrt(a - c*
x**2)*a*c*e**2*x**4 + sqrt(a - c*x**2)*c**2*d**2*x**4 - sqrt(a - c*x**2)*c
**2*e**2*x**6),x)*a**5*d*e**4 + 5*sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sqrt
(a - c*x**2)*a**2*d**2 - sqrt(a - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a - c*x*
*2)*a*c*d**2*x**2 + 2*sqrt(a - c*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**2)*c*
*2*d**2*x**4 - sqrt(a - c*x**2)*c**2*e**2*x**6),x)*a**4*b*d**2*e**3 - 7*sq
rt(a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**2 - sqrt(a - c*
x**2)*a**2*e**2*x**2 - 2*sqrt(a - c*x**2)*a*c*d**2*x**2 + 2*sqrt(a - c*x**
2)*a*c*e**2*x**4 + sqrt(a - c*x**2)*c**2*d**2*x**4 - sqrt(a - c*x**2)*c**2
*e**2*x**6),x)*a**4*c*d**3*e**2 - 2*sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sq
rt(a - c*x**2)*a**2*d**2 - sqrt(a - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a - c*
x**2)*a*c*d**2*x**2 + 2*sqrt(a - c*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**2)*
c**2*d**2*x**4 - sqrt(a - c*x**2)*c**2*e**2*x**6),x)*a**4*c*d*e**4*x**2 -
5*sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**2 - sqrt(a
- c*x**2)*a**2*e**2*x**2 - 2*sqrt(a - c*x**2)*a*c*d**2*x**2 + 2*sqrt(a - c
*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**2)*c**2*d**2*x**4 - sqrt(a - c*x**2)*
c**2*e**2*x**6),x)*a**3*b*c*d**4*e - 10*sqrt(a - c*x**2)*int(sqrt(d + e*x)
/(sqrt(a - c*x**2)*a**2*d**2 - sqrt(a - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a
- c*x**2)*a*c*d**2*x**2 + 2*sqrt(a - c*x**2)*a*c*e**2*x**4 + sqrt(a - c...
```

3.285 $\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 403

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{5/2}}\,dx = \frac{\sqrt{d+ex}(a(Bd+Ae)+(Acd+aBe)x)}{3ac\,(a-cx^2)^{3/2}} - \frac{\sqrt{d+ex}(aAe-(4Acd-3aBe)x)}{6a^2c\sqrt{a-cx^2}} + \frac{(4Acd-3aBe)\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{6a^{3/2}c^{3/2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} - \frac{(4Acd^2-3aBde-aAe^2)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{6a^{3/2}c^{3/2}\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
1/3*(e*x+d)^(1/2)*(a*(A*e+B*d)+(A*c*d+B*a*e)*x)/a/c/(-c*x^2+a)^(3/2)-1/6*(
e*x+d)^(1/2)*(a*A*e-(4*A*c*d-3*B*a*e)*x)/a^2/c/(-c*x^2+a)^(1/2)+1/6*(4*A*c
*d-3*B*a*e)*(e*x+d)^(1/2)*(1-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(
1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/a^(3/
2)/c^(3/2)/(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)-
1/6*(-A*a*e^2+4*A*c*d^2-3*B*a*d*e)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))
^(1/2)*(1-c*x^2/a)^(1/2)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2)
,2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/a^(3/2)/c^(3/2)/(e*x+d)^
(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.77 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.31

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{5/2}}\,dx = \frac{\sqrt{a-cx^2}\left(\frac{(d+ex)\left(-4Ac^2dx^3+a^2(2Bd+Ae-Bex)+acx\left(6Ad+Aex+3Bex^2\right)\right)}{a^2c(a-cx^2)^2}+\frac{e^2(4Acd-3aBe)\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^(5/2),x]
```

output
```
(Sqrt[a - c*x^2]*(((d + e*x)*(-4*A*c^2*d*x^3 + a^2*(2*B*d + A*e - B*e*x) +
 a*c*x*(6*A*d + A*e*x + 3*B*e*x^2)))/(a^2*c*(a - c*x^2)^2) + (e^2*(4*A*c*d
 - 3*a*B*e)*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2) - I*Sqrt[c]*(Sqrt[
c]*d - Sqrt[a]*e)*(4*A*c*d - 3*a*B*e)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d +
e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*Ellip
ticE[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d +
 Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)] - I*Sqrt[a]*Sqrt[c]*e*(4*A*c*d - 3*a*
B*e - Sqrt[a]*A*Sqrt[c]*e)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[
-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcS
inh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)
/(Sqrt[c]*d - Sqrt[a]*e)])/(a^2*c^2*e*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a +
 c*x^2))))/(6*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {684, 27, 686, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{5/2}}dx$$

$$\Big\downarrow \ 684$$

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{3ac(a-cx^2)^{3/2}}-\frac{\int -\frac{4Acd^2-3aBed-aAe^2+3e(Acd-aBe)x}{2\sqrt{d+ex}(a-cx^2)^{3/2}}dx}{3ac}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{4Acd^2-3aBed-aAe^2+3e(Acd-aBe)x}{\sqrt{d+ex}(a-cx^2)^{3/2}}dx}{6ac}+\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{3ac(a-cx^2)^{3/2}}$$

$$\Big\downarrow \ 686$$

$$\frac{-\frac{\int \frac{ce\left(cd^2-ae^2\right)(aAe+(4Acd-3aBe)x)}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{ac(cd^2-ae^2)}-\frac{\sqrt{d+ex}\left(aAe\left(cd^2-ae^2\right)-x\left(cd^2-ae^2\right)(4Acd-3aBe)\right)}{a\sqrt{a-cx^2}(cd^2-ae^2)}}{6ac}+\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{3ac(a-cx^2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{e\int \frac{aAe+(4Acd-3aBe)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{2a}-\frac{\sqrt{d+ex}\left(aAe\left(cd^2-ae^2\right)-x\left(cd^2-ae^2\right)(4Acd-3aBe)\right)}{a\sqrt{a-cx^2}(cd^2-ae^2)}}{6ac}+\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{3ac(a-cx^2)^{3/2}}$$

$$\Big\downarrow \ 600$$

$$\frac{-\frac{e\left(\frac{(4Acd-3aBe)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{\left(-aAe^2-3aBde+4Acd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{2a}-\frac{\sqrt{d+ex}\left(aAe\left(cd^2-ae^2\right)-x\left(cd^2-ae^2\right)(4Acd-3aBe)\right)}{a\sqrt{a-cx^2}\left(cd^2-ae^2\right)}}{6ac}+\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 509$$

$$\frac{-\frac{e\left(\frac{\sqrt{1-\frac{cx^2}{a}}(4Acd-3aBe)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{\left(-aAe^2-3aBde+4Acd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{2a}-\frac{\sqrt{d+ex}\left(aAe\left(cd^2-ae^2\right)-x\left(cd^2-ae^2\right)(4Acd-3aBe)\right)}{a\sqrt{a-cx^2}\left(cd^2-ae^2\right)}}{6ac}+\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 508$$

$$\frac{-\frac{e\left(-\frac{\left(-aAe^2-3aBde+4Acd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(4Acd-3aBe)\int \frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2a}-\frac{\sqrt{d+ex}\left(aAe\left(cd^2-ae^2\right)-x\left(cd^2-ae^2\right)(4Acd-3aBe)\right)}{a\sqrt{a-cx^2}\left(cd^2-ae^2\right)}}{6ac}+\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 327$$

$$\frac{-\frac{e\left(-\frac{\left(-aAe^2-3aBde+4Acd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(4Acd-3aBe)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2a}-\frac{\sqrt{d+ex}\left(aAe\left(cd^2-ae^2\right)-x\left(cd^2-ae^2\right)(4Acd-3aBe)\right)}{a\sqrt{a-cx^2}\left(cd^2-ae^2\right)}}{6ac}+\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 512$$

$$-\frac{e\left(-\frac{\sqrt{1-\frac{cx^2}{a}}\left(-aAe^2-3aBde+4Acd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(4Acd-3aBe)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2a}-\frac{\sqrt{d+ex}(aAe(}{6ac}$$

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 511$$

$$-\frac{e\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(-aAe^2-3aBde+4Acd^2\right)\int\frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(4Acd-3aBe)E\left(\arcsin\left(\frac{\sqrt{}}{}\right.\right.}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2a}$$

$$6ac$$

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 321$$

$$-\frac{e\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(-aAe^2-3aBde+4Acd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}(4Acd-3aBe)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2a}$$

$$6ac$$

$$\frac{\sqrt{d+ex}(x(aBe+Acd)+a(Ae+Bd))}{3ac\left(a-cx^2\right)^{3/2}}$$

input
```
Int[((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^(5/2),x]
```

output
```
(Sqrt[d + e*x]*(a*(B*d + A*e) + (A*c*d + a*B*e)*x))/(3*a*c*(a - c*x^2)^(3/
2)) + (-((Sqrt[d + e*x]*(a*A*e*(c*d^2 - a*e^2) - (4*A*c*d - 3*a*B*e)*(c*d^
2 - a*e^2)*x))/(a*(c*d^2 - a*e^2)*Sqrt[a - c*x^2])) - (e*((-2*Sqrt[a]*(4*A
*c*d - 3*a*B*e)*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1
- (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c
]*e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) + (
2*Sqrt[a]*(4*A*c*d^2 - 3*a*B*d*e - a*A*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt
[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]
*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d
 + e*x]*Sqrt[a - c*x^2])))/(2*a))/(6*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(333) = 666$.

Time = 6.09 (sec) , antiderivative size = 750, normalized size of antiderivative = 1.86

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c\,x^2+a)}\left(\frac{\left(\frac{(Acd+Bae)x}{3c^3a}+\frac{Ae+Bd}{3c^3}\right)\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}{\left(x^2-\frac{a}{c}\right)^2}-\frac{2(-cex-cd)\left(\frac{(4Acd-3Bae)x}{12a^2c^2}-\frac{Ae}{12a\,c^2}\right)}{\sqrt{\left(x^2-\frac{a}{c}\right)(-cex-cd)}}+\frac{2\left(-\frac{Aa\,e^2-4Ac\,d^2+3}{6c\,a^2}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*((1/3*(A*c*d+B*a
*e)/c^3/a*x+1/3*(A*e+B*d)/c^3)*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)/(x^2-a/c
)^2-2*(-c*e*x-c*d)*(1/12*(4*A*c*d-3*B*a*e)/a^2/c^2*x-1/12*A*e/a/c^2)/((x^2
-a/c)*(-c*e*x-c*d))^(1/2)+2*(-1/6/c*(A*a*e^2-4*A*c*d^2+3*B*a*d*e)/a^2+1/12
/a/c*A*e^2-1/6/c*d*(4*A*c*d-3*B*a*e)/a^2)*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(
d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(
1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+
a*e*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/
c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))-1/6*(4*A*c*d-3*B*a*e)*e/a^2/
c*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c
)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c
)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))
*EllipticE(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(
-d/e-1/c*(a*c)^(1/2)))^(1/2))+1/c*(a*c)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*
(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))
))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{5/2}}\,dx =$$
$$-\frac{(4\,Aa^2cd^2-3\,Ba^3de-3\,Aa^3e^2+(4\,Ac^3d^2-3\,Bac^2de-3\,Aac^2e^2)x^4-2\,(4\,Aac^2d^2-3\,Ba^2cde-3\,Aa^2}{}$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/18*((4*A*a^2*c*d^2 - 3*B*a^3*d*e - 3*A*a^3*e^2 + (4*A*c^3*d^2 - 3*B*a*c
^2*d*e - 3*A*a*c^2*e^2)*x^4 - 2*(4*A*a*c^2*d^2 - 3*B*a^2*c*d*e - 3*A*a^2*c
*e^2)*x^2)*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -
8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 3*(4*A*a^2*c*d*e -
3*B*a^3*e^2 + (4*A*c^3*d*e - 3*B*a*c^2*e^2)*x^4 - 2*(4*A*a*c^2*d*e - 3*B*a
^2*c*e^2)*x^2)*sqrt(-c*e)*weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -
8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2
)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e)) - 3*(A*a
*c^2*e^2*x^2 + 2*B*a^2*c*d*e + A*a^2*c*e^2 - (4*A*c^3*d*e - 3*B*a*c^2*e^2)
*x^3 + (6*A*a*c^2*d*e - B*a^2*c*e^2)*x)*sqrt(-c*x^2 + a)*sqrt(e*x + d))/(a
^2*c^4*e*x^4 - 2*a^3*c^3*e*x^2 + a^4*c^2*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(e*x+d)**(3/2)/(-c*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)(ex+d)^{\frac{3}{2}}}{(-cx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x + d)^(3/2)/(-c*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)(ex+d)^{\frac{3}{2}}}{(-cx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x + d)^(3/2)/(-c*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{5/2}}\,dx = \int \frac{(A+B\,x)\,(d+e\,x)^{3/2}}{(a-c\,x^2)^{5/2}}\,dx$$

input
```
int(((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^(5/2),x)
```

output
```
int(((A + B*x)*(d + e*x)^(3/2))/(a - c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)(d+ex)^{3/2}}{(a-cx^2)^{5/2}}\,dx = \text{too large to display}$$

input

```
int((B*x+A)*(e*x+d)^(3/2)/(-c*x^2+a)^(5/2),x)
```

output

```
(3*sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**2 - sqrt(a
 - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a - c*x**2)*a*c*d**2*x**2 + 2*sqrt(a -
c*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**2)*c**2*d**2*x**4 - sqrt(a - c*x**2)
*c**2*e**2*x**6),x)*a**4*b*d*e**3 - 3*sqrt(a - c*x**2)*int(sqrt(d + e*x)/(
sqrt(a - c*x**2)*a**2*d**2 - sqrt(a - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a -
c*x**2)*a*c*d**2*x**2 + 2*sqrt(a - c*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**2
)*c**2*d**2*x**4 - sqrt(a - c*x**2)*c**2*e**2*x**6),x)*a**4*c*d**2*e**2 -
3*sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**2 - sqrt(a
- c*x**2)*a**2*e**2*x**2 - 2*sqrt(a - c*x**2)*a*c*d**2*x**2 + 2*sqrt(a - c
*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**2)*c**2*d**2*x**4 - sqrt(a - c*x**2)*
c**2*e**2*x**6),x)*a**3*b*c*d**3*e - 6*sqrt(a - c*x**2)*int(sqrt(d + e*x)/
(sqrt(a - c*x**2)*a**2*d**2 - sqrt(a - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a -
 c*x**2)*a*c*d**2*x**2 + 2*sqrt(a - c*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**
2)*c**2*d**2*x**4 - sqrt(a - c*x**2)*c**2*e**2*x**6),x)*a**3*b*c*d*e**3*x*
*2 + 3*sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**2 - sq
rt(a - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a - c*x**2)*a*c*d**2*x**2 + 2*sqrt(
a - c*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**2)*c**2*d**2*x**4 - sqrt(a - c*x
**2)*c**2*e**2*x**6),x)*a**3*c**2*d**4 + 6*sqrt(a - c*x**2)*int(sqrt(d + e
*x)/(sqrt(a - c*x**2)*a**2*d**2 - sqrt(a - c*x**2)*a**2*e**2*x**2 - 2*sqrt
(a - c*x**2)*a*c*d**2*x**2 + 2*sqrt(a - c*x**2)*a*c*e**2*x**4 + sqrt(a ...
```

3.286 $\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 438

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{5/2}}\,dx = \frac{(aB+Acx)\sqrt{d+ex}}{3ac\,(a-cx^2)^{3/2}} - \frac{\sqrt{d+ex}(ae(Acd-aBe)-c(4Acd^2-aBde-3aAe^2)\,x)}{6a^2c\,(cd^2-ae^2)\,\sqrt{a-cx^2}} + \frac{(4Acd^2-aBde-3aAe^2)\,\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{6a^{3/2}\sqrt{c}\,(cd^2-ae^2)\,\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} - \frac{(4Acd-aBe)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{6a^{3/2}c^{3/2}\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
1/3*(A*c*x+B*a)*(e*x+d)^(1/2)/a/c/(-c*x^2+a)^(3/2)-1/6*(e*x+d)^(1/2)*(a*e*
(A*c*d-B*a*e)-c*(-3*A*a*e^2+4*A*c*d^2-B*a*d*e)*x)/a^2/c/(-a*e^2+c*d^2)/(-c
*x^2+a)^(1/2)+1/6*(-3*A*a*e^2+4*A*c*d^2-B*a*d*e)*(e*x+d)^(1/2)*(1-c*x^2/a)
^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*
e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/a^(3/2)/c^(1/2)/(-a*e^2+c*d^2)/(c^(1/2)*(e
*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)-1/6*(4*A*c*d-B*a*e)*(c
^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2)*EllipticF(1/
2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2
)*e))^(1/2))/a^(3/2)/c^(3/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.79 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{5/2}}\,dx = \frac{\sqrt{a-cx^2}\left(\frac{(d+ex)\left(2a\left(cd^2-ae^2\right)(aB+Acx)+\left(a-cx^2\right)\left(a^2Be^2+4Ac^2d^2x-ace(Bdx+A(d+3ex))\right)\right)}{(a-cx^2)^2}+\cdots\right.}{\cdots}$$

input
```
Integrate[((A + B*x)*Sqrt[d + e*x])/(a - c*x^2)^(5/2),x]
```

output
```
(Sqrt[a - c*x^2]*(((d + e*x)*(2*a*(c*d^2 - a*e^2)*(a*B + A*c*x) + (a - c*x
^2)*(a^2*B*e^2 + 4*A*c^2*d^2*x - a*c*e*(B*d*x + A*(d + 3*e*x)))))/(a - c*x
^2)^2 + (e^2*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(4*A*c*d^2 - a*B*d*e - 3*a*A*e
^2)*(a - c*x^2) + I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(4*A*c*d^2 - a*B*d*e -
 3*a*A*e^2)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/
Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d + (
Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - S
qrt[a]*e)] + I*Sqrt[a]*e*(Sqrt[c]*d - Sqrt[a]*e)*(4*A*c*d - a*B*e + 3*Sqrt
[a]*A*Sqrt[c]*e)*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a
]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-
d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*
d - Sqrt[a]*e)])/(e*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(a - c*x^2))))/(6*a^2*c
*(c*d^2 - a*e^2)*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {685, 27, 686, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 685$$

$$\frac{\sqrt{d+ex}(aB+Acx)}{3ac\left(a-cx^2\right)^{3/2}}-\frac{\int -\frac{4Acd-aBe+3Acex}{2\sqrt{d+ex}(a-cx^2)^{3/2}}dx}{3ac}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{4Acd-aBe+3Acex}{\sqrt{d+ex}(a-cx^2)^{3/2}}dx}{6ac}+\frac{\sqrt{d+ex}(aB+Acx)}{3ac\left(a-cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 686$$

$$\frac{-\frac{\int \frac{ce\left(ae(Acd-aBe)+c\left(4Acd^2-aBed-3aAe^2\right)x\right)}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{ac(cd^2-ae^2)}-\frac{\sqrt{d+ex}\left(ae(Acd-aBe)-cx\left(-3aAe^2-aBde+4Acd^2\right)\right)}{a\sqrt{a-cx^2}(cd^2-ae^2)}}{6ac}+\frac{\sqrt{d+ex}(aB+Acx)}{3ac\left(a-cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{e\int \frac{ae(Acd-aBe)+c\left(4Acd^2-aBed-3aAe^2\right)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}\left(ae(Acd-aBe)-cx\left(-3aAe^2-aBde+4Acd^2\right)\right)}{a\sqrt{a-cx^2}(cd^2-ae^2)}}{6ac}+\frac{\sqrt{d+ex}(aB+Acx)}{3ac\left(a-cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 600$$

$$\frac{-\frac{e\left(\frac{c\left(-3aAe^2-aBde+4Acd^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{\left(cd^2-ae^2\right)(4Acd-aBe)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(Acd-aBe)-cx(-3aAe^2-aBde+4Acd^2\ldots}{a\sqrt{a-cx^2}(cd^2-ae^2)}}{6ac}$$

$$\frac{\sqrt{d+ex}(aB+Acx)}{3ac\left(a-cx^2\right)^{3/2}}$$

$\big\downarrow$ 509

$$\frac{-\frac{e\left(\frac{c\sqrt{1-\frac{cx^2}{a}}\left(-3aAe^2-aBde+4Acd^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{\left(cd^2-ae^2\right)(4Acd-aBe)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(Acd-aBe)-cx(-3aAe^2-aBd\ldots}{a\sqrt{a-cx^2}(cd^2-ae^2)}}{6ac}$$

$$\frac{\sqrt{d+ex}(aB+Acx)}{3ac\left(a-cx^2\right)^{3/2}}$$

$\big\downarrow$ 508

$$\frac{-\frac{e\left(-\frac{\left(cd^2-ae^2\right)(4Acd-aBe)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(-3aAe^2-aBde+4Acd^2\right)\int\frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(\ldots}{\ldots}}{6ac}$$

$$\frac{\sqrt{d+ex}(aB+Acx)}{3ac\left(a-cx^2\right)^{3/2}}$$

$\big\downarrow$ 327

$$\frac{-\frac{e\left(-\frac{\left(cd^2-ae^2\right)(4Acd-aBe)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(-3aAe^2-aBde+4Acd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae\ldots}{\ldots}}{6ac}$$

$$\frac{\sqrt{d+ex}(aB+Acx)}{3ac\left(a-cx^2\right)^{3/2}}$$

$\big\downarrow$ 512

$$-\frac{-\frac{e\left(-\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)(4Acd-aBe)\int \frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(-3aAe^2-aBde+4Acd^2\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2a(cd^2-ae^2)}-\frac{\sqrt{}}{}}{6ac}$$

$$\frac{\sqrt{d+ex}(aB+Acx)}{3ac\left(a-cx^2\right)^{3/2}}$$

$\downarrow$ 511

$$-\frac{-\frac{e\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}(4Acd-aBe)\int \frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(-3aAe^2-aBde+4Acd^2\right)}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+}{\sqrt{a}e+}}}\right.}{2a(cd^2-ae^2)}}{6ac}$$

$$\frac{\sqrt{d+ex}(aB+Acx)}{3ac\left(a-cx^2\right)^{3/2}}$$

$\downarrow$ 321

$$-\frac{-\frac{e\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}(4Acd-aBe)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(-3aAe^2-aBde+4Acd^2\right)E\left(\arcsi\right.}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right.}{2a(cd^2-ae^2)}}{6ac}$$

$$\frac{\sqrt{d+ex}(aB+Acx)}{3ac\left(a-cx^2\right)^{3/2}}$$

input
```
Int[((A + B*x)*Sqrt[d + e*x])/(a - c*x^2)^(5/2),x]
```

output
```
((a*B + A*c*x)*Sqrt[d + e*x])/(3*a*c*(a - c*x^2)^(3/2)) + (-((Sqrt[d + e*x
]*(a*e*(A*c*d - a*B*e) - c*(4*A*c*d^2 - a*B*d*e - 3*a*A*e^2)*x))/(a*(c*d^2
 - a*e^2)*Sqrt[a - c*x^2])) - (e*((-2*Sqrt[a]*Sqrt[c]*(4*A*c*d^2 - a*B*d*e
 - 3*a*A*e^2)*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(e*Sqrt[(
Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x^2]) + (2*Sqrt[a]*
(4*A*c*d - a*B*e)*(c*d^2 - a*e^2)*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sq
rt[a]*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a
]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sq
rt[a - c*x^2])))/(2*a*(c*d^2 - a*e^2)))/(6*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 685
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^(m - 1)*(a + c*x^2)
^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m]
 || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(368) = 736$.

Time = 1.87 (sec) , antiderivative size = 836, normalized size of antiderivative = 1.91

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c\,x^2+a)}\left(\frac{\left(\frac{Ax}{3c^2a}+\frac{B}{3c^3}\right)\sqrt{-ce\,x^3-cd\,x^2+aex+ad}}{\left(x^2-\frac{a}{c}\right)^2}-\frac{2(-cex-cd)\left(\frac{\left(3Aa\,e^2-4Ac\,d^2+Bade\right)x}{12c\,a^2\left(a\,e^2-c\,d^2\right)}+\frac{e(Acd-Bae)}{12c^2a\left(a\,e^2-c\,d^2\right)}\right)}{\sqrt{\left(x^2-\frac{a}{c}\right)(-cex-cd)}}+\frac{2\left(\frac{4Acd-}{6c\,a}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*((1/3*A/c^2/a*x+
1/3*B/c^3)*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)/(x^2-a/c)^2-2*(-c*e*x-c*d)*(
1/12/c*(3*A*a*e^2-4*A*c*d^2+B*a*d*e)/a^2/(a*e^2-c*d^2)*x+1/12*e*(A*c*d-B*a
*e)/c^2/a/(a*e^2-c*d^2))/((x^2-a/c)*(-c*e*x-c*d))^(1/2)+2*(1/6/c*(4*A*c*d-
B*a*e)/a^2-1/12/c*e^2*(A*c*d-B*a*e)/a/(a*e^2-c*d^2)-1/6*d*(3*A*a*e^2-4*A*c
*d^2+B*a*d*e)/a^2/(a*e^2-c*d^2))*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(
a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+
1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d
)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(
1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))-1/6*e*(3*A*a*e^2-4*A*c*d^2+B*a*d*e)/(
a*e^2-c*d^2)/a^2*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/
2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))
/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1
/c*(a*c)^(1/2))*EllipticE(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c
*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+1/c*(a*c)^(1/2)*EllipticF(((x
+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)
^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 662, normalized size of antiderivative = 1.51

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{5/2}}\,dx =$$
$$-\frac{(4\,Aa^2c^2d^3 - Ba^3cd^2e - 6\,Aa^3cde^2 + 3\,Ba^4e^3 + (4\,Ac^4d^3 - Bac^3d^2e - 6\,Aac^3de^2 + 3\,Ba^2c^2e^3)x^4 - 2\,(4\,}{}$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/18*((4*A*a^2*c^2*d^3 - B*a^3*c*d^2*e - 6*A*a^3*c*d*e^2 + 3*B*a^4*e^3 +
(4*A*c^4*d^3 - B*a*c^3*d^2*e - 6*A*a*c^3*d*e^2 + 3*B*a^2*c^2*e^3)*x^4 - 2*
(4*A*a*c^3*d^3 - B*a^2*c^2*d^2*e - 6*A*a^2*c^2*d*e^2 + 3*B*a^3*c*e^3)*x^2)
*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^
3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 3*(4*A*a^2*c^2*d^2*e - B*a^3*
c*d*e^2 - 3*A*a^3*c*e^3 + (4*A*c^4*d^2*e - B*a*c^3*d*e^2 - 3*A*a*c^3*e^3)*
x^4 - 2*(4*A*a*c^3*d^2*e - B*a^2*c^2*d*e^2 - 3*A*a^2*c^2*e^3)*x^2)*sqrt(-c
*e)*weierstrassZeta(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^
2)/(c*e^3), weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^
3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e)) - 3*(2*B*a^2*c^2*d^2*e - A*a^2
*c^2*d*e^2 - B*a^3*c*e^3 - (4*A*c^4*d^2*e - B*a*c^3*d*e^2 - 3*A*a*c^3*e^3)
*x^3 + (A*a*c^3*d*e^2 - B*a^2*c^2*e^3)*x^2 + (6*A*a*c^3*d^2*e - B*a^2*c^2*
d*e^2 - 5*A*a^2*c^2*e^3)*x)*sqrt(-c*x^2 + a)*sqrt(e*x + d))/(a^4*c^3*d^2*e
 - a^5*c^2*e^3 + (a^2*c^5*d^2*e - a^3*c^4*e^3)*x^4 - 2*(a^3*c^4*d^2*e - a^
4*c^3*e^3)*x^2)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{5/2}}\,dx = \int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)**(1/2)/(-c*x**2+a)**(5/2),x)
```

output
```
Integral((A + B*x)*sqrt(d + e*x)/(a - c*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)\sqrt{ex+d}}{(-cx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*sqrt(e*x + d)/(-c*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)\sqrt{ex+d}}{(-cx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*sqrt(e*x + d)/(-c*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{5/2}}\,dx = \int \frac{(A+B\,x)\,\sqrt{d+e\,x}}{(a-c\,x^2)^{5/2}}\,dx$$

input
```
int(((A + B*x)*(d + e*x)^(1/2))/(a - c*x^2)^(5/2),x)
```

output
```
int(((A + B*x)*(d + e*x)^(1/2))/(a - c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt{d+ex}}{(a-cx^2)^{5/2}}\,dx = \text{too large to display}$$

input

```
int((B*x+A)*(e*x+d)^(1/2)/(-c*x^2+a)^(5/2),x)
```

output

```
( - sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**2 - sqrt(
a - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a - c*x**2)*a*c*d**2*x**2 + 2*sqrt(a -
 c*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**2)*c**2*d**2*x**4 - sqrt(a - c*x**2
)*c**2*e**2*x**6),x)*a**4*d*e**2 - sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sqr
t(a - c*x**2)*a**2*d**2 - sqrt(a - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a - c*x
**2)*a*c*d**2*x**2 + 2*sqrt(a - c*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**2)*c
**2*d**2*x**4 - sqrt(a - c*x**2)*c**2*e**2*x**6),x)*a**3*b*d**2*e + 6*sqrt
(a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**2 - sqrt(a - c*x*
*2)*a**2*e**2*x**2 - 2*sqrt(a - c*x**2)*a*c*d**2*x**2 + 2*sqrt(a - c*x**2)
*a*c*e**2*x**4 + sqrt(a - c*x**2)*c**2*d**2*x**4 - sqrt(a - c*x**2)*c**2*e
**2*x**6),x)*a**3*c*d**3 + 2*sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a -
c*x**2)*a**2*d**2 - sqrt(a - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a - c*x**2)*a
*c*d**2*x**2 + 2*sqrt(a - c*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**2)*c**2*d*
*2*x**4 - sqrt(a - c*x**2)*c**2*e**2*x**6),x)*a**3*c*d*e**2*x**2 + 2*sqrt(
a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**2 - sqrt(a - c*x**
2)*a**2*e**2*x**2 - 2*sqrt(a - c*x**2)*a*c*d**2*x**2 + 2*sqrt(a - c*x**2)*
a*c*e**2*x**4 + sqrt(a - c*x**2)*c**2*d**2*x**4 - sqrt(a - c*x**2)*c**2*e*
*2*x**6),x)*a**2*b*c*d**2*e*x**2 - 12*sqrt(a - c*x**2)*int(sqrt(d + e*x)/(
sqrt(a - c*x**2)*a**2*d**2 - sqrt(a - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a -
c*x**2)*a*c*d**2*x**2 + 2*sqrt(a - c*x**2)*a*c*e**2*x**4 + sqrt(a - c*x...
```

3.287 $\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 519

$$
\begin{aligned}
\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{5/2}}\,dx &= \frac{\sqrt{d+ex}(a(Bd-Ae)+(Acd-aBe)x)}{3a\,(cd^2-ae^2)\,(a-cx^2)^{3/2}} \\
&- \frac{\sqrt{d+ex}(ae(Acd^2+4aBde-5aAe^2)-(4Acd(cd^2-2ae^2)+aBe(cd^2+3ae^2))\,x)}{6a^2\,(cd^2-ae^2)^2\,\sqrt{a-cx^2}} \\
&+ \frac{(4Acd(cd^2-2ae^2)+aBe(cd^2+3ae^2))\,\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{6a^{3/2}\sqrt{c}\,(cd^2-ae^2)^2\,\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}} \\
&- \frac{(4Acd^2+aBde-5aAe^2)\,\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{6a^{3/2}\sqrt{c}\,(cd^2-ae^2)\,\sqrt{d+ex}\sqrt{a-cx^2}}
\end{aligned}
$$

output
```
1/3*(e*x+d)^(1/2)*(a*(-A*e+B*d)+(A*c*d-B*a*e)*x)/a/(-a*e^2+c*d^2)/(-c*x^2+
a)^(3/2)-1/6*(e*x+d)^(1/2)*(a*e*(-5*A*a*e^2+A*c*d^2+4*B*a*d*e)-(4*A*c*d*(-
2*a*e^2+c*d^2)+a*B*e*(3*a*e^2+c*d^2))*x)/a^2/(-a*e^2+c*d^2)^2/(-c*x^2+a)^(
1/2)+1/6*(4*A*c*d*(-2*a*e^2+c*d^2)+a*B*e*(3*a*e^2+c*d^2))*(e*x+d)^(1/2)*(1
-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*
(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/a^(3/2)/c^(1/2)/(-a*e^2+c*d^2)^2/
(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(1/2)-1/6*(-5*A*a
*e^2+4*A*c*d^2+B*a*d*e)*(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c
*x^2/a)^(1/2)*EllipticF(1/2*(1-c^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a
^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/a^(3/2)/c^(1/2)/(-a*e^2+c*d^2)/(e*x
+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.77 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{5/2}}\,dx = \frac{\sqrt{a-cx^2}\left(\frac{(d+ex)(2a(cd^2-ae^2)(-aAe+Acdx+aB(d-ex))+(a-cx^2)(4Ac^2d^3x+a^2e^2(-4Bd+5Ae\cdots}{(a-cx^2)^2}\cdots\right.}{\cdots}$$

input
```
Integrate[(A + B*x)/(Sqrt[d + e*x]*(a - c*x^2)^(5/2)),x]
```

output
```
(Sqrt[a - c*x^2]*(((d + e*x)*(2*a*(c*d^2 - a*e^2)*(-(a*A*e) + A*c*d*x + a*
B*(d - e*x)) + (a - c*x^2)*(4*A*c^2*d^3*x + a^2*e^2*(-4*B*d + 5*A*e + 3*B*
e*x) + a*c*d*e*(B*d*x - A*(d + 8*e*x)))))/(a - c*x^2)^2 + (e^2*Sqrt[-d + (
Sqrt[a]*e)/Sqrt[c]]*(4*A*c*d*(c*d^2 - 2*a*e^2) + a*B*e*(c*d^2 + 3*a*e^2))*
(a - c*x^2) + I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(4*A*c*d*(c*d^2 - 2*a*e^2)
 + a*B*e*(c*d^2 + 3*a*e^2))*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt
[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*Arc
Sinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e
)/(Sqrt[c]*d - Sqrt[a]*e)] + I*Sqrt[a]*Sqrt[c]*e*(Sqrt[c]*d - Sqrt[a]*e)*(
a*B*e*(Sqrt[c]*d - 3*Sqrt[a]*e) + A*(4*c^(3/2)*d^2 + 3*Sqrt[a]*c*d*e - 5*a
*Sqrt[c]*e^2))*Sqrt[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*
e)/Sqrt[c] - e*x)/(d + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d
+ (Sqrt[a]*e)/Sqrt[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d
- Sqrt[a]*e)])/(c*e*Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(a - c*x^2))))/(6*a^2*(
c*d^2 - a*e^2)^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {686, 27, 686, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a-cx^2)^{5/2}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{686}$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{3a(a-cx^2)^{3/2}(cd^2-ae^2)}-\frac{\int -\frac{c(4Acd^2+aBed-5aAe^2+3e(Acd-aBe)x)}{2\sqrt{d+ex}(a-cx^2)^{3/2}}dx}{3ac(cd^2-ae^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4Acd^2+aBed-5aAe^2+3e(Acd-aBe)x}{\sqrt{d+ex}(a-cx^2)^{3/2}}dx}{6a(cd^2-ae^2)}+\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{3a(a-cx^2)^{3/2}(cd^2-ae^2)}$$

$$\Big\downarrow \text{686}$$

$$-\frac{-\frac{\int \frac{ce\left(ae\left(Acd^2+4aBed-5aAe^2\right)+\left(4Acd\left(cd^2-2ae^2\right)+aBe\left(cd^2+3ae^2\right)\right)x\right)}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{ac(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(-5aAe^2+4aBde+Acd^2)-x(4Acd(cd^2-2ae^2)+aBe(3}{a\sqrt{a-cx^2}(cd^2-ae^2)}}{6a\left(cd^2-ae^2\right)}$$
$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{3a\left(a-cx^2\right)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{e\int \frac{ae\left(Acd^2+4aBed-5aAe^2\right)+\left(4Acd\left(cd^2-2ae^2\right)+aBe\left(cd^2+3ae^2\right)\right)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(-5aAe^2+4aBde+Acd^2)-x(4Acd(cd^2-2ae^2)+aBe(3ae^2}{a\sqrt{a-cx^2}(cd^2-ae^2)}}{6a\left(cd^2-ae^2\right)}$$
$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{3a\left(a-cx^2\right)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 600$$

$$-\frac{-\frac{e\left(\frac{\left(4Acd\left(cd^2-2ae^2\right)+aBe\left(3ae^2+cd^2\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{\left(cd^2-ae^2\right)\left(-5aAe^2+aBde+4Acd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(-5aAe^2+4aBde}{}}{6a\left(cd^2-ae^2\right)}$$
$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{3a\left(a-cx^2\right)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 509$$

$$-\frac{-\frac{e\left(\frac{\sqrt{1-\frac{cx^2}{a}}\left(4Acd\left(cd^2-2ae^2\right)+aBe\left(3ae^2+cd^2\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{\left(cd^2-ae^2\right)\left(-5aAe^2+aBde+4Acd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}\right)}{2a(cd^2-ae^2)}-\frac{\sqrt{d+ex}(ae(-5aAe^2}{}}{6a\left(cd^2-ae^2\right)}$$
$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{3a\left(a-cx^2\right)^{3/2}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 508$$

$$-\frac{e\left(-\frac{\left(cd^2-ae^2\right)\left(-5aAe^2+aBde+4Acd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(4Acd\left(cd^2-2ae^2\right)+aBe\left(3ae^2+cd^2\right)\right)\int \frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2a(cd^2-ae^2)}$$

$$6a\left(cd^2-ae^2\right)$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{3a\left(a-cx^2\right)^{3/2}\left(cd^2-ae^2\right)}$$

$\downarrow$ 327

$$-\frac{e\left(-\frac{\left(cd^2-ae^2\right)\left(-5aAe^2+aBde+4Acd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(4Acd\left(cd^2-2ae^2\right)+aBe\left(3ae^2+cd^2\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+}\right.}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2a(cd^2-ae^2)}$$

$$6a\left(cd^2-ae^2\right)$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{3a\left(a-cx^2\right)^{3/2}\left(cd^2-ae^2\right)}$$

$\downarrow$ 512

$$-\frac{e\left(-\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\left(-5aAe^2+aBde+4Acd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(4Acd\left(cd^2-2ae^2\right)+aBe\left(3ae^2+cd^2\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}\right)}{2a(cd^2-ae^2)}$$

$$6a\left(cd^2-ae^2\right)$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{3a\left(a-cx^2\right)^{3/2}\left(cd^2-ae^2\right)}$$

$\downarrow$ 511

$$-\frac{e\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(-5aAe^2+aBde+4Acd^2\right)\int \frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(4Acd\left(cd^2-2ae\right.\right.}{\sqrt{}}}{2a(cd^2-ae^2)}}{6a\left(cd^2-\right.}$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{3a\left(a-cx^2\right)^{3/2}\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 321

$$-\frac{e\left(\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(-5aAe^2+aBde+4Acd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(4Acd\left(cd^2-2ae^2\right)+aBe\right.}{\sqrt{c}e\sqrt{a-cx}}}{2a(cd^2-ae^2)}}{6a\left(cd^2-ae^2\right.}$$

$$\frac{\sqrt{d+ex}(x(Acd-aBe)+a(Bd-Ae))}{3a\left(a-cx^2\right)^{3/2}\left(cd^2-ae^2\right)}$$

input
```
Int[(A + B*x)/(Sqrt[d + e*x]*(a - c*x^2)^(5/2)),x]
```

output
```
(Sqrt[d + e*x]*(a*(B*d - A*e) + (A*c*d - a*B*e)*x))/(3*a*(c*d^2 - a*e^2)*(
a - c*x^2)^(3/2)) + (-((Sqrt[d + e*x]*(a*e*(A*c*d^2 + 4*a*B*d*e - 5*a*A*e^
2) - (4*A*c*d*(c*d^2 - 2*a*e^2) + a*B*e*(c*d^2 + 3*a*e^2))*x))/(a*(c*d^2 -
 a*e^2)*Sqrt[a - c*x^2])) - (e*((-2*Sqrt[a]*(4*A*c*d*(c*d^2 - 2*a*e^2) + a
*B*e*(c*d^2 + 3*a*e^2))*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*EllipticE[ArcSin
[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a] + e)])
/(Sqrt[c]*e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a - c*x
^2]) + (2*Sqrt[a]*(c*d^2 - a*e^2)*(4*A*c*d^2 + a*B*d*e - 5*a*A*e^2)*Sqrt[(
Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[1 - (c*x^2)/a]*EllipticF[
ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqrt[a]
+ e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[a - c*x^2])))/(2*a*(c*d^2 - a*e^2)))/
(6*a*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 970 vs. $2(449) = 898$.

Time = 8.79 (sec) , antiderivative size = 971, normalized size of antiderivative = 1.87

method	result
elliptic	$\frac{\sqrt{(ex+d)(-c x^2+a)}\left(\frac{\left(-\frac{(Acd-Bae)x}{3a\left(a e^2-c d^2\right)c^2}+\frac{Ae-Bd}{3\left(a e^2-c d^2\right)c^2}\right)\sqrt{-ce x^3-cd x^2+aex+ad}}{\left(x^2-\frac{a}{c}\right)^2}-\frac{2(-cex-cd)\left(-\frac{\left(8Aacd e^2-4A c^2d^3-3B e^3a^2-B\right.}{12a^2\left(a e^2-c d^2\right)^2c}\right.}{\sqrt{\left(x^2-\frac{a}{c}\right)(-ce}}\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*((-1/3*(A*c*d-B*
a*e)/a/(a*e^2-c*d^2)/c^2*x+1/3*(A*e-B*d)/(a*e^2-c*d^2)/c^2)*(-c*e*x^3-c*d*
x^2+a*e*x+a*d)^(1/2)/(x^2-a/c)^2-2*(-c*e*x-c*d)*(-1/12*(8*A*a*c*d*e^2-4*A*
c^2*d^3-3*B*a^2*e^3-B*a*c*d^2*e)/a^2/(a*e^2-c*d^2)^2/c*x+1/12*e*(5*A*a*e^2
-A*c*d^2-4*B*a*d*e)/a/c/(a*e^2-c*d^2)^2)/((x^2-a/c)*(-c*e*x-c*d))^(1/2)+2*
(1/6/(a*e^2-c*d^2)*(5*A*a*e^2-4*A*c*d^2-B*a*d*e)/a^2-1/12*e^2*(5*A*a*e^2-A
*c*d^2-4*B*a*d*e)/a/(a*e^2-c*d^2)^2+1/6*d*(8*A*a*c*d*e^2-4*A*c^2*d^3-3*B*a
^2*e^3-B*a*c*d^2*e)/a^2/(a*e^2-c*d^2)^2)*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d
/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1
/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a
*e*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c
*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))+1/6*e*(8*A*a*c*d*e^2-4*A*c^2*
d^3-3*B*a^2*e^3-B*a*c*d^2*e)/a^2/(a*e^2-c*d^2)^2*(d/e-1/c*(a*c)^(1/2))*((x
+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1
/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(1/2)))^(1/2)/(-c*e*x^3-c
*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))*EllipticE(((x+d/e)/(d/e-1/
c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2
))+1/c*(a*c)^(1/2)*EllipticF(((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+
1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 828, normalized size of antiderivative = 1.60

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/18*((4*A*a^2*c^2*d^4 + B*a^3*c*d^3*e - 11*A*a^3*c*d^2*e^2 - 9*B*a^4*d*e
^3 + 15*A*a^4*e^4 + (4*A*c^4*d^4 + B*a*c^3*d^3*e - 11*A*a*c^3*d^2*e^2 - 9*
B*a^2*c^2*d*e^3 + 15*A*a^2*c^2*e^4)*x^4 - 2*(4*A*a*c^3*d^4 + B*a^2*c^2*d^3
*e - 11*A*a^2*c^2*d^2*e^2 - 9*B*a^3*c*d*e^3 + 15*A*a^3*c*e^4)*x^2)*sqrt(-c
*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*
d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 3*(4*A*a^2*c^2*d^3*e + B*a^3*c*d^2*e^
2 - 8*A*a^3*c*d*e^3 + 3*B*a^4*e^4 + (4*A*c^4*d^3*e + B*a*c^3*d^2*e^2 - 8*A
*a*c^3*d*e^3 + 3*B*a^2*c^2*e^4)*x^4 - 2*(4*A*a*c^3*d^3*e + B*a^2*c^2*d^2*e
^2 - 8*A*a^2*c^2*d*e^3 + 3*B*a^3*c*e^4)*x^2)*sqrt(-c*e)*weierstrassZeta(4/
3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), weierstras
sPInverse(4/3*(c*d^2 + 3*a*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3)
, 1/3*(3*e*x + d)/e)) - 3*(2*B*a^2*c^2*d^3*e - 3*A*a^2*c^2*d^2*e^2 - 6*B*a
^3*c*d*e^3 + 7*A*a^3*c*e^4 - (4*A*c^4*d^3*e + B*a*c^3*d^2*e^2 - 8*A*a*c^3*
d*e^3 + 3*B*a^2*c^2*e^4)*x^3 + (A*a*c^3*d^2*e^2 + 4*B*a^2*c^2*d*e^3 - 5*A*
a^2*c^2*e^4)*x^2 + (6*A*a*c^3*d^3*e - B*a^2*c^2*d^2*e^2 - 10*A*a^2*c^2*d*e
^3 + 5*B*a^3*c*e^4)*x)*sqrt(-c*x^2 + a)*sqrt(e*x + d))/(a^4*c^3*d^4*e - 2*
a^5*c^2*d^2*e^3 + a^6*c*e^5 + (a^2*c^5*d^4*e - 2*a^3*c^4*d^2*e^3 + a^4*c^3
*e^5)*x^4 - 2*(a^3*c^4*d^4*e - 2*a^4*c^3*d^2*e^3 + a^5*c^2*e^5)*x^2)
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{5/2}}\,dx = \int \frac{A+Bx}{(a-cx^2)^{\frac{5}{2}}\sqrt{d+ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)**(1/2)/(-c*x**2+a)**(5/2),x)
```

output
```
Integral((A + B*x)/((a - c*x**2)**(5/2)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{5/2}}\,dx = \int \frac{Bx+A}{(-cx^2+a)^{\frac{5}{2}}\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((-c*x^2 + a)^(5/2)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{5/2}}\,dx = \int \frac{Bx+A}{(-cx^2+a)^{\frac{5}{2}}\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/((-c*x^2 + a)^(5/2)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{5/2}}\,dx = \int \frac{A+B\,x}{(a-c\,x^2)^{5/2}\,\sqrt{d+e\,x}}\,dx$$

input
```
int((A + B*x)/((a - c*x^2)^(5/2)*(d + e*x)^(1/2)),x)
```

output
```
int((A + B*x)/((a - c*x^2)^(5/2)*(d + e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}\,(a-cx^2)^{5/2}}\,dx = \left(\int \frac{\sqrt{ex+d}}{\sqrt{-c\,x^2+a}\,a^2d^2-\sqrt{-c\,x^2+a}\,a^2e^2x^2-2\sqrt{-c\,x^2+a}\,ac\,d^2x^2+2\sqrt{-c\,x^2+a}\right.$$
$$+\left(\int \frac{\sqrt{ex+d}\,\sqrt{-c\,x^2+a}\,x}{-c^3e\,x^7-c^3d\,x^6+3a\,c^2e\,x^5+3a\,c^2d\,x^4-3a^2ce\,x^3-3a^2cd\,x^2+a^3ex+a^3d}dx\right)b$$
$$-\left(\int \frac{\sqrt{ex+d}\,x}{\sqrt{-c\,x^2+a}\,a^2d^2-\sqrt{-c\,x^2+a}\,a^2e^2x^2-2\sqrt{-c\,x^2+a}\,ac\,d^2x^2+2\sqrt{-c\,x^2+a}\,ac\,e^2x^4+\sqrt{-c\,x^2+a}\right.$$

input

```
int((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^(5/2),x)
```

output

```
int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**2 - sqrt(a - c*x**2)*a**2*e**2
*x**2 - 2*sqrt(a - c*x**2)*a*c*d**2*x**2 + 2*sqrt(a - c*x**2)*a*c*e**2*x**
4 + sqrt(a - c*x**2)*c**2*d**2*x**4 - sqrt(a - c*x**2)*c**2*e**2*x**6),x)*
a*d + int((sqrt(d + e*x)*sqrt(a - c*x**2)*x)/(a**3*d + a**3*e*x - 3*a**2*c
*d*x**2 - 3*a**2*c*e*x**3 + 3*a*c**2*d*x**4 + 3*a*c**2*e*x**5 - c**3*d*x**
6 - c**3*e*x**7),x)*b - int((sqrt(d + e*x)*x)/(sqrt(a - c*x**2)*a**2*d**2
- sqrt(a - c*x**2)*a**2*e**2*x**2 - 2*sqrt(a - c*x**2)*a*c*d**2*x**2 + 2*s
qrt(a - c*x**2)*a*c*e**2*x**4 + sqrt(a - c*x**2)*c**2*d**2*x**4 - sqrt(a -
 c*x**2)*c**2*e**2*x**6),x)*a*e
```

3.288 $\int \frac{A+Bx}{(d+ex)^{3/2}(a-cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 645

$$\int \frac{A+Bx}{(d+ex)^{3/2}(a-cx^2)^{5/2}}\,dx = -\frac{2(Bd-Ae)}{(cd^2-ae^2)\sqrt{d+ex}\,(a-cx^2)^{3/2}}$$
$$+\frac{\sqrt{d+ex}(a(7Bcd^2-8Acde+aBe^2)+c(Acd^2-8aBde+7aAe^2)\,x)}{3a\,(cd^2-ae^2)^2\,(a-cx^2)^{3/2}}$$
$$-\frac{\sqrt{d+ex}(ae(Acd(cd^2-33ae^2)+aBe(27cd^2+5ae^2))-c(aBde(3cd^2+29ae^2)+A(4c^2d^4-15acd^2e^2-}{6a^2\,(cd^2-ae^2)^3\sqrt{a-cx^2}}$$
$$+\frac{\sqrt{c}(aBde(3cd^2+29ae^2)+A(4c^2d^4-15acd^2e^2-21a^2e^4))\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{}}{\sqrt{cd+}}\right.}{6a^{3/2}\,(cd^2-ae^2)^3\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{a-cx^2}}$$
$$-\frac{(4Acd(cd^2-3ae^2)+aBe(3cd^2+5ae^2))\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{a}e}}\sqrt{1-\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}e}{\sqrt{c}d+\sqrt{a}e}\right)}{6a^{3/2}\sqrt{c}\,(cd^2-ae^2)^2\sqrt{d+ex}\sqrt{a-cx^2}}$$

output
```
(2*A*e-2*B*d)/(-a*e^2+c*d^2)/(e*x+d)^(1/2)/(-c*x^2+a)^(3/2)+1/3*(e*x+d)^(1
/2)*(a*(-8*A*c*d*e+B*a*e^2+7*B*c*d^2)+c*(7*A*a*e^2+A*c*d^2-8*B*a*d*e)*x)/a
/(-a*e^2+c*d^2)^2/(-c*x^2+a)^(3/2)-1/6*(e*x+d)^(1/2)*(a*e*(A*c*d*(-33*a*e^
2+c*d^2)+a*B*e*(5*a*e^2+27*c*d^2))-c*(a*B*d*e*(29*a*e^2+3*c*d^2)+A*(-21*a^
2*e^4-15*a*c*d^2*e^2+4*c^2*d^4))*x)/a^2/(-a*e^2+c*d^2)^3/(-c*x^2+a)^(1/2)+
1/6*c^(1/2)*(a*B*d*e*(29*a*e^2+3*c*d^2)+A*(-21*a^2*e^4-15*a*c*d^2*e^2+4*c^
2*d^4))*(e*x+d)^(1/2)*(1-c*x^2/a)^(1/2)*EllipticE(1/2*(1-c^(1/2)*x/a^(1/2)
)^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))/a^(3/2)/(
-a*e^2+c*d^2)^3/(c^(1/2)*(e*x+d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)/(-c*x^2+a)^(
1/2)-1/6*(4*A*c*d*(-3*a*e^2+c*d^2)+a*B*e*(5*a*e^2+3*c*d^2))*(c^(1/2)*(e*x+
d)/(c^(1/2)*d+a^(1/2)*e))^(1/2)*(1-c*x^2/a)^(1/2)*EllipticF(1/2*(1-c^(1/2)
*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*e/(c^(1/2)*d+a^(1/2)*e))^(1/2))
/a^(3/2)/c^(1/2)/(-a*e^2+c*d^2)^2/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.55 (sec) , antiderivative size = 789, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx}{(d+ex)^{3/2}\,(a-cx^2)^{5/2}}\,dx = \frac{\sqrt{a-cx^2}\left(12a^2e^4(-Bd+Ae)+e(aBde(3cd^2+29ae^2)+A(4c^2d^4-15a\right.}{}$$

input
```
Integrate[(A + B*x)/((d + e*x)^(3/2)*(a - c*x^2)^(5/2)),x]
```

output
```
(Sqrt[a - c*x^2]*(12*a^2*e^4*(-(B*d) + A*e) + e*(a*B*d*e*(3*c*d^2 + 29*a*e
^2) + A*(4*c^2*d^4 - 15*a*c*d^2*e^2 - 21*a^2*e^4)) - (2*a*(-(c*d^2) + a*e^
2)*(d + e*x)*(a^2*B*e^2 + A*c^2*d^2*x + a*c*(B*d*(d - 2*e*x) + A*e*(-2*d +
 e*x))))/(a - c*x^2)^2 - ((d + e*x)*(-5*a^3*B*e^4 + 4*A*c^3*d^4*x + a*c^2*
d^2*e*(3*B*d*x - A*(d + 15*e*x)) + a^2*c*e^2*(3*A*e*(7*d - 3*e*x) + B*d*(-
15*d + 17*e*x))))/(-a + c*x^2) - (I*Sqrt[c]*(Sqrt[c]*d - Sqrt[a]*e)*(a*B*d
*e*(3*c*d^2 + 29*a*e^2) + A*(4*c^2*d^4 - 15*a*c*d^2*e^2 - 21*a^2*e^4))*Sqr
t[(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/
(d + e*x))]*(d + e*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt
[c]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)])/(e*
Sqrt[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2)) - (I*Sqrt[a]*(Sqrt[c]*d - Sqr
t[a]*e)*(a*B*e*(3*c*d^2 - 24*Sqrt[a]*Sqrt[c]*d*e + 5*a*e^2) + A*(4*c^2*d^3
 + 3*Sqrt[a]*c^(3/2)*d^2*e - 12*a*c*d*e^2 + 21*a^(3/2)*Sqrt[c]*e^3))*Sqrt[
(e*(Sqrt[a]/Sqrt[c] + x))/(d + e*x)]*Sqrt[-(((Sqrt[a]*e)/Sqrt[c] - e*x)/(d
 + e*x))]*(d + e*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-d + (Sqrt[a]*e)/Sqrt[c
]]/Sqrt[d + e*x]], (Sqrt[c]*d + Sqrt[a]*e)/(Sqrt[c]*d - Sqrt[a]*e)])/(Sqrt
[-d + (Sqrt[a]*e)/Sqrt[c]]*(-a + c*x^2))))/(6*a^2*(c*d^2 - a*e^2)^3*Sqrt[d
 + e*x])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {686, 27, 686, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a-cx^2)^{5/2}(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{686}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{3a(a-cx^2)^{3/2}\sqrt{d+ex}(cd^2-ae^2)}-\frac{\int -\frac{c(4Acd^2+3aBed-7aAe^2+5e(Acd-aBe)x)}{2(d+ex)^{3/2}(a-cx^2)^{3/2}}dx}{3ac(cd^2-ae^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4Acd^2+3aBed-7aAe^2+5e(Acd-aBe)x}{(d+ex)^{3/2}(a-cx^2)^{3/2}}dx}{6a(cd^2-ae^2)}+\frac{x(Acd-aBe)+a(Bd-Ae)}{3a(a-cx^2)^{3/2}\sqrt{d+ex}(cd^2-ae^2)}$$

$$\Big\downarrow \; 686$$

$$\frac{\frac{x(4Acd(cd^2-3ae^2)+aBe(5ae^2+3cd^2))+ae(7aAe^2-8aBde+Acd^2)}{a\sqrt{a-cx^2}\sqrt{d+ex}(cd^2-ae^2)}-\frac{\int -\frac{ce\left(3ae\left(Acd^2-8aBed+7aAe^2\right)+\left(4Acd\left(cd^2-3ae^2\right)+aBe\left(3cd^2+5ae^2\right)\right)x\right)}{2(d+ex)^{3/2}\sqrt{a-cx^2}}}{ac(cd^2-ae^2)}}{6a\left(cd^2-ae^2\right)}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{3a\left(a-cx^2\right)^{3/2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{e\int \frac{3ae\left(Acd^2-8aBed+7aAe^2\right)+\left(4Acd\left(cd^2-3ae^2\right)+aBe\left(3cd^2+5ae^2\right)\right)x}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{2a(cd^2-ae^2)}+\frac{x(4Acd(cd^2-3ae^2)+aBe(5ae^2+3cd^2))+ae(7aAe^2-8aBde+Acd^2)}{a\sqrt{a-cx^2}\sqrt{d+ex}(cd^2-ae^2)}}{6a\left(cd^2-ae^2\right)}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{3a\left(a-cx^2\right)^{3/2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 688$$

$$\frac{\frac{e\left(\frac{2\int -\frac{ae\left(Acd\left(cd^2-33ae^2\right)+aBe\left(27cd^2+5ae^2\right)\right)+c\left(aBde\left(3cd^2+29ae^2\right)+A\left(4c^2d^4-15ace^2d^2-21a^2e^4\right)\right)x}{2\sqrt{d+ex}\sqrt{a-cx^2}}dx}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\left(A\left(-21a^2e^4-15acd^2e^2+4c^2d^4\right)\right.}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right.}{2a(cd^2-ae^2)}}{6a\left(cd^2-ae^2\right)}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{3a\left(a-cx^2\right)^{3/2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{e\left(-\frac{\int \frac{ae\left(Acd\left(cd^2-33ae^2\right)+aBe\left(27cd^2+5ae^2\right)\right)+c\left(aBde\left(3cd^2+29ae^2\right)+A\left(4c^2d^4-15ace^2d^2-21a^2e^4\right)\right)x}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{cd^2-ae^2}-\frac{2\sqrt{a-cx^2}\left(A\left(-21a^2e^4-15acd^2e^2+4c^2d^4\right)+\right.}{\sqrt{d+ex}\left(cd^2-ae^2\right)}\right.}{2a(cd^2-ae^2)}}{6a\left(cd^2-ae^2\right)}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{3a\left(a-cx^2\right)^{3/2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 600$$

$$e\left(-\frac{\frac{c\left(A\left(-21a^2e^4-15acd^2e^2+4c^2d^4\right)+aBde\left(29ae^2+3cd^2\right)\right)\int\frac{\sqrt{d+ex}}{\sqrt{a-cx^2}}dx}{e}-\frac{\left(cd^2-ae^2\right)\left(4Acd\left(cd^2-3ae^2\right)+aBe\left(5ae^2+3cd^2\right)\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}}{cd^2-ae^2}-\frac{2\sqrt{c}}{\cdots}\right.$$

$$\frac{\cdots}{2a(cd^2-ae^2)}$$

$$6a\left(cd^2-ae\cdots\right.$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{3a\left(a-cx^2\right)^{3/2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$\downarrow$ 509

$$e\left(-\frac{\frac{c\sqrt{1-\frac{cx^2}{a}}\left(A\left(-21a^2e^4-15acd^2e^2+4c^2d^4\right)+aBde\left(29ae^2+3cd^2\right)\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{\left(cd^2-ae^2\right)\left(4Acd\left(cd^2-3ae^2\right)+aBe\left(5ae^2+3cd^2\right)\right)\int\frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}\cdots}{e}}{cd^2-ae^2}\cdots\right.$$

$$\frac{\cdots}{2a(cd^2-ae^2)}$$

$$6a\left(cd^2-\cdots\right.$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{3a\left(a-cx^2\right)^{3/2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$\downarrow$ 508

$$e\left(-\frac{-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(A\left(-21a^2e^4-15acd^2e^2+4c^2d^4\right)+aBde\left(29ae^2+3cd^2\right)\right)\int\frac{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}-\frac{\left(cd^2-ae^2\right)\left(4Acd\left(cd^2-3ae^2\right)+aBe\cdots\right.}{e\cdots}}{cd^2-ae^2}\cdots\right.$$

$$\frac{\cdots}{2a(cd^2-ae^2)}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{3a\left(a-cx^2\right)^{3/2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$\downarrow$ 327

$$e\left(-\frac{-\frac{\left(cd^2-ae^2\right)\left(4Acd\left(cd^2-3ae^2\right)+aBe\left(5ae^2+3cd^2\right)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{a-cx^2}}dx}{e}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(A\left(-21a^2e^4-15acd^2e^2+4c^2d^4\right)+aBde\left(29ae^2+3cd^2\right.\right.}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}}}{cd^2-ae^2}\right.$$

$$2a(cd^2-ae^2)$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{3a\left(a-cx^2\right)^{3/2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 512$$

$$e\left(-\frac{-\frac{\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\left(4Acd\left(cd^2-3ae^2\right)+aBe\left(5ae^2+3cd^2\right)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-\frac{cx^2}{a}}}dx}{e\sqrt{a-cx^2}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(A\left(-21a^2e^4-15acd^2e^2+4c^2d^4\right)+aBde\left(29\right.\right.}{e\sqrt{a-cx^2}\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}}}}}{cd^2-ae^2}\right.$$

$$2a(cd^2-ae^2)$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{3a\left(a-cx^2\right)^{3/2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 511$$

$$e\left(-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(4Acd\left(cd^2-3ae^2\right)+aBe\left(5ae^2+3cd^2\right)\right)\int \frac{1}{\sqrt{1-\frac{e\left(1-\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\frac{\sqrt{c}d}{\sqrt{a}}+e}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{c}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}}{cd^2-ae^2}\right.$$

$$2a(c$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{3a\left(a-cx^2\right)^{3/2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 321$$

$$e\left(-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{cx^2}{a}}\left(cd^2-ae^2\right)\sqrt{\frac{\sqrt{c}(d+ex)}{\sqrt{a}e+\sqrt{c}d}}\left(4Acd\left(cd^2-3ae^2\right)+aBe\left(5ae^2+3cd^2\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2e}{\frac{\sqrt{c}d}{\sqrt{a}}+e}\right)}{\sqrt{c}e\sqrt{a-cx^2}\sqrt{d+ex}}-\frac{2\sqrt{a}\sqrt{c}\sqrt{1-\frac{cx^2}{a}}\sqrt{d+ex}\left(A\left(-21\right.\right.}{}}{cd^2-ae^2}\right.$$

$$\overline{\quad 2a(cd^2- \quad}$$

$$\frac{x(Acd-aBe)+a(Bd-Ae)}{3a\left(a-cx^2\right)^{3/2}\sqrt{d+ex}\left(cd^2-ae^2\right)}$$

input

```
Int[(A + B*x)/((d + e*x)^(3/2)*(a - c*x^2)^(5/2)),x]
```

output

```
(a*(B*d - A*e) + (A*c*d - a*B*e)*x)/(3*a*(c*d^2 - a*e^2)*Sqrt[d + e*x]*(a
- c*x^2)^(3/2)) + ((a*e*(A*c*d^2 - 8*a*B*d*e + 7*a*A*e^2) + (4*A*c*d*(c*d^
2 - 3*a*e^2) + a*B*e*(3*c*d^2 + 5*a*e^2))*x)/(a*(c*d^2 - a*e^2)*Sqrt[d + e
*x]*Sqrt[a - c*x^2]) + (e*((-2*(a*B*d*e*(3*c*d^2 + 29*a*e^2) + A*(4*c^2*d^
4 - 15*a*c*d^2*e^2 - 21*a^2*e^4))*Sqrt[a - c*x^2])/((c*d^2 - a*e^2)*Sqrt[d
 + e*x]) - ((-2*Sqrt[a]*Sqrt[c]*(a*B*d*e*(3*c*d^2 + 29*a*e^2) + A*(4*c^2*d
^4 - 15*a*c*d^2*e^2 - 21*a^2*e^4))*Sqrt[d + e*x]*Sqrt[1 - (c*x^2)/a]*Ellip
ticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*e)/((Sqrt[c]*d)/Sqr
t[a] + e)])/(e*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[a -
c*x^2]) + (2*Sqrt[a]*(c*d^2 - a*e^2)*(4*A*c*d*(c*d^2 - 3*a*e^2) + a*B*e*(3
*c*d^2 + 5*a*e^2))*Sqrt[(Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[a]*e)]*Sqrt[
1 - (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2
*e)/((Sqrt[c]*d)/Sqrt[a] + e)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[a - c*x^2]))
/(c*d^2 - a*e^2)))/(2*a*(c*d^2 - a*e^2)))/(6*a*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1194 vs. $2(571) = 1142$.

Time = 10.76 (sec) , antiderivative size = 1195, normalized size of antiderivative = 1.85

method	result	size
elliptic	Expression too large to display	1195
default	Expression too large to display	5787

input
```
int((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((e*x+d)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(-c*x^2+a)^(1/2)*((1/3*(A*a*e^2+A
*c*d^2-2*B*a*d*e)/c/a/(a*e^2-c*d^2)^2*x-1/3*(2*A*c*d*e-B*a*e^2-B*c*d^2)/(a
*e^2-c*d^2)^2/c^2)*(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)/(x^2-a/c)^2-2*(-c*e*
x-c*d)*(1/12*(9*A*a^2*e^4+15*A*a*c*d^2*e^2-4*A*c^2*d^4-17*B*a^2*d*e^3-3*B*
a*c*d^3*e)/(a*e^2-c*d^2)^3/a^2*x-1/12*e*(21*A*a*c*d*e^2-A*c^2*d^3-5*B*a^2*
e^3-15*B*a*c*d^2*e)/(a*e^2-c*d^2)^3/a/c)/((x^2-a/c)*(-c*e*x-c*d))^(1/2)-2*
(-c*e*x^2+a*e)*e^3/(a*e^2-c*d^2)^3*(A*e-B*d)/((x+d/e)*(-c*e*x^2+a*e))^(1/2
)+2*(-1/6/(a*e^2-c*d^2)^2*(12*A*a*c*d*e^2-4*A*c^2*d^3-5*B*a^2*e^3-3*B*a*c*
d^2*e)/a^2+1/12*e^2*(21*A*a*c*d*e^2-A*c^2*d^3-5*B*a^2*e^3-15*B*a*c*d^2*e)/
(a*e^2-c*d^2)^3/a-1/6*c*d*(9*A*a^2*e^4+15*A*a*c*d^2*e^2-4*A*c^2*d^4-17*B*a
^2*d*e^3-3*B*a*c*d^3*e)/(a*e^2-c*d^2)^3/a^2-e^3*c*d*(A*e-B*d)/(a*e^2-c*d^2
)^3)*(d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(
a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(
a*c)^(1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*EllipticF(((x+d/e)/(
d/e-1/c*(a*c)^(1/2)))^(1/2),((-d/e+1/c*(a*c)^(1/2))/(-d/e-1/c*(a*c)^(1/2))
)^(1/2))+2*(-1/12*c*e*(9*A*a^2*e^4+15*A*a*c*d^2*e^2-4*A*c^2*d^4-17*B*a^2*d
*e^3-3*B*a*c*d^3*e)/(a*e^2-c*d^2)^3/a^2-c*e^4*(A*e-B*d)/(a*e^2-c*d^2)^3)*(
d/e-1/c*(a*c)^(1/2))*((x+d/e)/(d/e-1/c*(a*c)^(1/2)))^(1/2)*((x-1/c*(a*c)^(
1/2))/(-d/e-1/c*(a*c)^(1/2)))^(1/2)*((x+1/c*(a*c)^(1/2))/(-d/e+1/c*(a*c)^(
1/2)))^(1/2)/(-c*e*x^3-c*d*x^2+a*e*x+a*d)^(1/2)*((-d/e-1/c*(a*c)^(1/2))...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1814 vs. $2(571) = 1142$.

Time = 0.24 (sec) , antiderivative size = 1814, normalized size of antiderivative = 2.81

$$\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/18*((4*A*a^2*c^3*d^6 + 3*B*a^3*c^2*d^5*e - 18*A*a^3*c^2*d^4*e^2 - 52*B*
a^4*c*d^3*e^3 + 78*A*a^4*c*d^2*e^4 - 15*B*a^5*d*e^5 + (4*A*c^5*d^5*e + 3*B
*a*c^4*d^4*e^2 - 18*A*a*c^4*d^3*e^3 - 52*B*a^2*c^3*d^2*e^4 + 78*A*a^2*c^3*
d*e^5 - 15*B*a^3*c^2*e^6)*x^5 + (4*A*c^5*d^6 + 3*B*a*c^4*d^5*e - 18*A*a*c^
4*d^4*e^2 - 52*B*a^2*c^3*d^3*e^3 + 78*A*a^2*c^3*d^2*e^4 - 15*B*a^3*c^2*d*e
^5)*x^4 - 2*(4*A*a*c^4*d^5*e + 3*B*a^2*c^3*d^4*e^2 - 18*A*a^2*c^3*d^3*e^3
- 52*B*a^3*c^2*d^2*e^4 + 78*A*a^3*c^2*d*e^5 - 15*B*a^4*c*e^6)*x^3 - 2*(4*A
*a*c^4*d^6 + 3*B*a^2*c^3*d^5*e - 18*A*a^2*c^3*d^4*e^2 - 52*B*a^3*c^2*d^3*e
^3 + 78*A*a^3*c^2*d^2*e^4 - 15*B*a^4*c*d*e^5)*x^2 + (4*A*a^2*c^3*d^5*e + 3
*B*a^3*c^2*d^4*e^2 - 18*A*a^3*c^2*d^3*e^3 - 52*B*a^4*c*d^2*e^4 + 78*A*a^4*
c*d*e^5 - 15*B*a^5*e^6)*x)*sqrt(-c*e)*weierstrassPInverse(4/3*(c*d^2 + 3*a
*e^2)/(c*e^2), -8/27*(c*d^3 - 9*a*d*e^2)/(c*e^3), 1/3*(3*e*x + d)/e) + 3*(
4*A*a^2*c^3*d^5*e + 3*B*a^3*c^2*d^4*e^2 - 15*A*a^3*c^2*d^3*e^3 + 29*B*a^4*
c*d^2*e^4 - 21*A*a^4*c*d*e^5 + (4*A*c^5*d^4*e^2 + 3*B*a*c^4*d^3*e^3 - 15*A
*a*c^4*d^2*e^4 + 29*B*a^2*c^3*d*e^5 - 21*A*a^2*c^3*e^6)*x^5 + (4*A*c^5*d^5
*e + 3*B*a*c^4*d^4*e^2 - 15*A*a*c^4*d^3*e^3 + 29*B*a^2*c^3*d^2*e^4 - 21*A*
a^2*c^3*d*e^5)*x^4 - 2*(4*A*a*c^4*d^4*e^2 + 3*B*a^2*c^3*d^3*e^3 - 15*A*a^2
*c^3*d^2*e^4 + 29*B*a^3*c^2*d*e^5 - 21*A*a^3*c^2*e^6)*x^3 - 2*(4*A*a*c^4*d
^5*e + 3*B*a^2*c^3*d^4*e^2 - 15*A*a^2*c^3*d^3*e^3 + 29*B*a^3*c^2*d^2*e^4 -
 21*A*a^3*c^2*d*e^5)*x^2 + (4*A*a^2*c^3*d^4*e^2 + 3*B*a^3*c^2*d^3*e^3 -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)^{3/2}\,(a-cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)**(3/2)/(-c*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)^{5/2}}\,dx = \int \frac{Bx+A}{\left(-cx^2+a\right)^{\frac{5}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((-c*x^2 + a)^(5/2)*(e*x + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)^{5/2}}\,dx = \int \frac{Bx+A}{\left(-cx^2+a\right)^{\frac{5}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/((-c*x^2 + a)^(5/2)*(e*x + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)^{5/2}}\,dx = \int \frac{A+B\,x}{\left(a-c\,x^2\right)^{5/2}\,\left(d+e\,x\right)^{3/2}}\,dx$$

input
```
int((A + B*x)/((a - c*x^2)^(5/2)*(d + e*x)^(3/2)),x)
```

output
```
int((A + B*x)/((a - c*x^2)^(5/2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\left(a-cx^2\right)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((B*x+A)/(e*x+d)^(3/2)/(-c*x^2+a)^(5/2),x)
```

output
```
(8*sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**4 - 2*sqrt
(a - c*x**2)*a**2*d**2*e**2*x**2 + sqrt(a - c*x**2)*a**2*e**4*x**4 - 2*sqr
t(a - c*x**2)*a*c*d**4*x**2 + 4*sqrt(a - c*x**2)*a*c*d**2*e**2*x**4 - 2*sq
rt(a - c*x**2)*a*c*e**4*x**6 + sqrt(a - c*x**2)*c**2*d**4*x**4 - 2*sqrt(a
- c*x**2)*c**2*d**2*e**2*x**6 + sqrt(a - c*x**2)*c**2*e**4*x**8),x)*a**2*b
*d**4*e + 8*sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a - c*x**2)*a**2*d**4
 - 2*sqrt(a - c*x**2)*a**2*d**2*e**2*x**2 + sqrt(a - c*x**2)*a**2*e**4*x**
4 - 2*sqrt(a - c*x**2)*a*c*d**4*x**2 + 4*sqrt(a - c*x**2)*a*c*d**2*e**2*x*
*4 - 2*sqrt(a - c*x**2)*a*c*e**4*x**6 + sqrt(a - c*x**2)*c**2*d**4*x**4 -
2*sqrt(a - c*x**2)*c**2*d**2*e**2*x**6 + sqrt(a - c*x**2)*c**2*e**4*x**8),
x)*a**2*b*d**3*e**2*x - 8*sqrt(a - c*x**2)*int(sqrt(d + e*x)/(sqrt(a - c*x
**2)*a**2*d**4 - 2*sqrt(a - c*x**2)*a**2*d**2*e**2*x**2 + sqrt(a - c*x**2)
*a**2*e**4*x**4 - 2*sqrt(a - c*x**2)*a*c*d**4*x**2 + 4*sqrt(a - c*x**2)*a*
c*d**2*e**2*x**4 - 2*sqrt(a - c*x**2)*a*c*e**4*x**6 + sqrt(a - c*x**2)*c**
2*d**4*x**4 - 2*sqrt(a - c*x**2)*c**2*d**2*e**2*x**6 + sqrt(a - c*x**2)*c*
*2*e**4*x**8),x)*a*b*c*d**4*e*x**2 - 8*sqrt(a - c*x**2)*int(sqrt(d + e*x)/
(sqrt(a - c*x**2)*a**2*d**4 - 2*sqrt(a - c*x**2)*a**2*d**2*e**2*x**2 + sqr
t(a - c*x**2)*a**2*e**4*x**4 - 2*sqrt(a - c*x**2)*a*c*d**4*x**2 + 4*sqrt(a
 - c*x**2)*a*c*d**2*e**2*x**4 - 2*sqrt(a - c*x**2)*a*c*e**4*x**6 + sqrt(a
- c*x**2)*c**2*d**4*x**4 - 2*sqrt(a - c*x**2)*c**2*d**2*e**2*x**6 + sqr...
```

3.289 $\int \frac{2+dx}{\sqrt{e+fx}\sqrt{4-d^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 91

$$\int \frac{2+dx}{\sqrt{e+fx}\sqrt{4-d^2x^2}}\,dx=-\frac{4\sqrt{de+2f}\sqrt{\frac{d(e+fx)}{de+2f}}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{2-dx}}{\sqrt{de+2f}}\right)|\frac{1}{4}\left(2+\frac{de}{f}\right)\right)}{d\sqrt{f}\sqrt{e+fx}}$$

output
```
-4*(d*e+2*f)^(1/2)*(d*(f*x+e)/(d*e+2*f))^(1/2)*EllipticE(f^(1/2)*(-d*x+2)^
(1/2)/(d*e+2*f)^(1/2),1/2*(2+d*e/f)^(1/2))/d/f^(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.16

$$\int \frac{2+dx}{\sqrt{e+fx}\sqrt{4-d^2x^2}}\,dx=$$
$$-\frac{2\left(f^2\sqrt{-e-\frac{2f}{d}}(4-d^2x^2)+id(de+2f)\sqrt{\frac{f(-2+dx)}{d(e+fx)}}\sqrt{\frac{f(2+dx)}{d(e+fx)}}(e+fx)^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{-e-\frac{2f}{d}}}{\sqrt{e+fx}}\right)|\frac{de-2f}{de+2f}\right.\right.}{df^2\sqrt{-e-\frac{2f}{d}}\sqrt{e+fx}\sqrt{4}}$$

input
```
Integrate[(2 + d*x)/(Sqrt[e + f*x]*Sqrt[4 - d^2*x^2]),x]
```

output
```
(-2*(f^2*Sqrt[-e - (2*f)/d]*(4 - d^2*x^2) + I*d*(d*e + 2*f)*Sqrt[(f*(-2 +
d*x))/(d*(e + f*x))]*Sqrt[(f*(2 + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Ell
ipticE[I*ArcSinh[Sqrt[-e - (2*f)/d]/Sqrt[e + f*x]], (d*e - 2*f)/(d*e + 2*f
)] - (4*I)*d*f*Sqrt[(f*(-2 + d*x))/(d*(e + f*x))]*Sqrt[(f*(2 + d*x))/(d*(e
 + f*x))]*(e + f*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-e - (2*f)/d]/Sqrt[e +
f*x]], (d*e - 2*f)/(d*e + 2*f)]))/(d*f^2*Sqrt[-e - (2*f)/d]*Sqrt[e + f*x]*
Sqrt[4 - d^2*x^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {600, 508, 327, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{dx+2}{\sqrt{4-d^2x^2}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{600}$$

$$\frac{d\int \frac{\sqrt{e+fx}}{\sqrt{4-d^2x^2}}dx}{f}-\frac{(de-2f)\int \frac{1}{\sqrt{e+fx}\sqrt{4-d^2x^2}}dx}{f}$$

$$\Big\downarrow \text{508}$$

$$-\frac{(de-2f)\int \frac{1}{\sqrt{e+fx}\sqrt{4-d^2x^2}}dx}{f}-\frac{2\sqrt{e+fx}\int \frac{\sqrt{1-\frac{f(2-dx)}{de+2f}}}{\sqrt{\frac{1}{4}(dx-2)+1}}d\left(\frac{1}{2}\sqrt{2-dx}\right)}{f\sqrt{\frac{d(e+fx)}{de+2f}}}$$

$$\Big\downarrow \text{327}$$

$$-\frac{(de-2f)\int \frac{1}{\sqrt{e+fx}\sqrt{4-d^2x^2}}dx}{f}-\frac{2\sqrt{e+fx}E\left(\arcsin\left(\frac{1}{2}\sqrt{2-dx}\right)|\frac{4f}{de+2f}\right)}{f\sqrt{\frac{d(e+fx)}{de+2f}}}$$

$$\Big\downarrow \text{511}$$

$$\frac{2(de-2f)\sqrt{\frac{d(e+fx)}{de+2f}}\int \frac{1}{\sqrt{1-\frac{f(2-dx)}{de+2f}}\sqrt{\frac{1}{4}(dx-2)+1}}d\left(\frac{1}{2}\sqrt{2-dx}\right)}{df\sqrt{e+fx}}-\frac{2\sqrt{e+fx}E\left(\arcsin\left(\frac{1}{2}\sqrt{2-dx}\right)|\frac{4f}{de+2f}\right)}{f\sqrt{\frac{d(e+fx)}{de+2f}}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2(de-2f)\sqrt{\frac{d(e+fx)}{de+2f}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{2-dx}\right),\frac{4f}{de+2f}\right)}{df\sqrt{e+fx}}-\frac{2\sqrt{e+fx}E\left(\arcsin\left(\frac{1}{2}\sqrt{2-dx}\right)|\frac{4f}{de+2f}\right)}{f\sqrt{\frac{d(e+fx)}{de+2f}}}$$

input

```
Int[(2 + d*x)/(Sqrt[e + f*x]*Sqrt[4 - d^2*x^2]),x]
```

output

```
(-2*Sqrt[e + f*x]*EllipticE[ArcSin[Sqrt[2 - d*x]/2], (4*f)/(d*e + 2*f)])/(
f*Sqrt[(d*(e + f*x))/(d*e + 2*f)]) + (2*(d*e - 2*f)*Sqrt[(d*(e + f*x))/(d*
e + 2*f)]*EllipticF[ArcSin[Sqrt[2 - d*x]/2], (4*f)/(d*e + 2*f)])/(d*f*Sqrt
[e + f*x])
```

Defintions of rubi rules used

rule 321

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(76) = 152$.

Time = 2.74 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.77

method	result
default	$\frac{2(d^2e^2-4f^2)\,\mathrm{EllipticE}\left(\sqrt{\frac{d(fx+e)}{de+2f}},\sqrt{\frac{de+2f}{de-2f}}\right)\sqrt{-\frac{f(dx-2)}{de+2f}}\,\sqrt{-\frac{(dx+2)f}{de-2f}}\,\sqrt{\frac{d(fx+e)}{de+2f}}\,\sqrt{fx+e}\,\sqrt{-d^2x^2+4}}{f^2d(d^2f\,x^3+d^2e\,x^2-4fx-4e)}$
elliptic	$\frac{\sqrt{-(fx+e)(d^2x^2-4)}\left(\frac{4\left(\frac{e}{f}+\frac{2}{d}\right)\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}+\frac{2}{d}}}\,\sqrt{\frac{x+\frac{2}{d}}{-\frac{e}{f}+\frac{2}{d}}}\,\sqrt{\frac{x-\frac{2}{d}}{-\frac{e}{f}-\frac{2}{d}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}+\frac{2}{d}}},\sqrt{\frac{-\frac{e}{f}-\frac{2}{d}}{-\frac{e}{f}+\frac{2}{d}}}\right)}{\sqrt{-d^2f\,x^3-d^2e\,x^2+4fx+4e}}+\frac{2d\left(\frac{e}{f}+\frac{2}{d}\right)\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}+\frac{2}{d}}}\,\sqrt{\frac{x+\frac{2}{d}}{-\frac{e}{f}+\frac{2}{d}}}\,\sqrt{\frac{x-\frac{2}{d}}{-\frac{e}{f}-\frac{2}{d}}}}{}\right.}{\sqrt{fx+e}\,\sqrt{-d^2x^2+4}}$

input
```
int((d*x+2)/(f*x+e)^(1/2)/(-d^2*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(d^2*e^2-4*f^2)*EllipticE((d*(f*x+e)/(d*e+2*f))^(1/2),((d*e+2*f)/(d*e-2*
f))^(1/2))*(-f*(d*x-2)/(d*e+2*f))^(1/2)*(-(d*x+2)*f/(d*e-2*f))^(1/2)*(d*(f
*x+e)/(d*e+2*f))^(1/2)*(f*x+e)^(1/2)*(-d^2*x^2+4)^(1/2)/f^2/d/(d^2*f*x^3+d
^2*e*x^2-4*f*x-4*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(76) = 152$.

Time = 0.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.10

$$\int \frac{2+dx}{\sqrt{e+fx}\sqrt{4-d^2x^2}}\,dx = \frac{2\left(3\sqrt{-d^2f}\,df\,\text{weierstrassZeta}\left(\frac{4\,(d^2e^2+12\,f^2)}{3\,d^2f^2},-\frac{8\,(d^2e^3-36\,ef^2)}{27\,d^2f^3},\text{weierstrassPInverse}\left(\frac{4\,(d^2e^2+12\,f^2)}{3\,d^2f^2},-\frac{8\,(d^2e^3-3\right.\right.\right.}{3\,d^2f^2}$$

input
```
integrate((d*x+2)/(f*x+e)^(1/2)/(-d^2*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(3*sqrt(-d^2*f)*d*f*weierstrassZeta(4/3*(d^2*e^2 + 12*f^2)/(d^2*f^2),
-8/27*(d^2*e^3 - 36*e*f^2)/(d^2*f^3), weierstrassPInverse(4/3*(d^2*e^2 + 1
2*f^2)/(d^2*f^2), -8/27*(d^2*e^3 - 36*e*f^2)/(d^2*f^3), 1/3*(3*f*x + e)/f)
) + sqrt(-d^2*f)*(d*e - 6*f)*weierstrassPInverse(4/3*(d^2*e^2 + 12*f^2)/(d
^2*f^2), -8/27*(d^2*e^3 - 36*e*f^2)/(d^2*f^3), 1/3*(3*f*x + e)/f))/(d^2*f^
2)
```

Sympy [F]

$$\int \frac{2+dx}{\sqrt{e+fx}\sqrt{4-d^2x^2}}\,dx = \int \frac{dx+2}{\sqrt{-\left(dx-2\right)\left(dx+2\right)}\sqrt{e+fx}}\,dx$$

input
```
integrate((d*x+2)/(f*x+e)**(1/2)/(-d**2*x**2+4)**(1/2),x)
```

output
```
Integral((d*x + 2)/(sqrt(-(d*x - 2)*(d*x + 2))*sqrt(e + f*x)), x)
```

Maxima [F]

$$\int \frac{2+dx}{\sqrt{e+fx}\sqrt{4-d^2x^2}}\,dx = \int \frac{dx+2}{\sqrt{-d^2x^2+4}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+2)/(f*x+e)^(1/2)/(-d^2*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + 2)/(sqrt(-d^2*x^2 + 4)*sqrt(f*x + e)), x)
```

Giac [F]

$$\int \frac{2+dx}{\sqrt{e+fx}\sqrt{4-d^2x^2}}\,dx = \int \frac{dx+2}{\sqrt{-d^2x^2+4}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+2)/(f*x+e)^(1/2)/(-d^2*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + 2)/(sqrt(-d^2*x^2 + 4)*sqrt(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+dx}{\sqrt{e+fx}\sqrt{4-d^2x^2}}\,dx = \int \frac{d\,x+2}{\sqrt{e+f\,x}\,\sqrt{4-d^2\,x^2}}\,dx$$

input
```
int((d*x + 2)/((e + f*x)^(1/2)*(4 - d^2*x^2)^(1/2)),x)
```

output
```
int((d*x + 2)/((e + f*x)^(1/2)*(4 - d^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{2+dx}{\sqrt{e+fx}\sqrt{4-d^2x^2}}\,dx = -\left(\int \frac{\sqrt{fx+e}\,\sqrt{-d^2x^2+4}}{df\,x^2+dex-2fx-2e}dx\right)$$

input
```
int((d*x+2)/(f*x+e)^(1/2)/(-d^2*x^2+4)^(1/2),x)
```

output
```
- int((sqrt(e + f*x)*sqrt( - d**2*x**2 + 4))/(d*e*x + d*f*x**2 - 2*e - 2*
f*x),x)
```

3.290 $\int \frac{\sqrt{4-d^2x^2}}{(2-dx)\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 91

$$\int \frac{\sqrt{4-d^2x^2}}{(2-dx)\sqrt{e+fx}}\,dx=-\frac{4\sqrt{de+2f}\sqrt{\frac{d(e+fx)}{de+2f}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{2-dx}}{\sqrt{de+2f}}\right)|\frac{1}{4}\left(2+\frac{de}{f}\right)\right)}{d\sqrt{f}\sqrt{e+fx}}$$

output
```
-4*(d*e+2*f)^(1/2)*(d*(f*x+e)/(d*e+2*f))^(1/2)*EllipticE(f^(1/2)*(-d*x+2)^
(1/2)/(d*e+2*f)^(1/2),1/2*(2+d*e/f)^(1/2))/d/f^(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.15 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.15

$$\int \frac{\sqrt{4-d^2x^2}}{(2-dx)\sqrt{e+fx}}\,dx$$
$$=\frac{2\left(f^2\sqrt{-e-\frac{2f}{d}}(-4+d^2x^2)-id(de+2f)\sqrt{\frac{f(-2+dx)}{d(e+fx)}}\sqrt{\frac{f(2+dx)}{d(e+fx)}}(e+fx)^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{-e-\frac{2f}{d}}}{\sqrt{e+fx}}\right)|\frac{de-2f}{de+2f}\right.\right.}{df^2\sqrt{-e-\frac{2f}{d}}\sqrt{e+fx}\sqrt{4-}}$$

input
```
Integrate[Sqrt[4 - d^2*x^2]/((2 - d*x)*Sqrt[e + f*x]),x]
```

output
```
(2*(f^2*Sqrt[-e - (2*f)/d]*(-4 + d^2*x^2) - I*d*(d*e + 2*f)*Sqrt[(f*(-2 +
d*x))/(d*(e + f*x))]*Sqrt[(f*(2 + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Ell
ipticE[I*ArcSinh[Sqrt[-e - (2*f)/d]/Sqrt[e + f*x]], (d*e - 2*f)/(d*e + 2*f
)] + (4*I)*d*f*Sqrt[(f*(-2 + d*x))/(d*(e + f*x))]*Sqrt[(f*(2 + d*x))/(d*(e
 + f*x))]*(e + f*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-e - (2*f)/d]/Sqrt[e +
f*x]], (d*e - 2*f)/(d*e + 2*f)]))/(d*f^2*Sqrt[-e - (2*f)/d]*Sqrt[e + f*x]*
Sqrt[4 - d^2*x^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.48, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {667, 600, 508, 327, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4-d^2x^2}}{(2-dx)\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{ 667}$$

$$\int \frac{dx+2}{\sqrt{4-d^2x^2}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{ 600}$$

$$\frac{d\int \frac{\sqrt{e+fx}}{\sqrt{4-d^2x^2}}dx}{f}-\frac{(de-2f)\int \frac{1}{\sqrt{e+fx}\sqrt{4-d^2x^2}}dx}{f}$$

$$\Big\downarrow \text{ 508}$$

$$-\frac{(de-2f)\int \frac{1}{\sqrt{e+fx}\sqrt{4-d^2x^2}}dx}{f}-\frac{2\sqrt{e+fx}\int \frac{\sqrt{1-\frac{f(2-dx)}{de+2f}}}{\sqrt{\frac{1}{4}(dx-2)+1}}d\left(\frac{1}{2}\sqrt{2-dx}\right)}{f\sqrt{\frac{d(e+fx)}{de+2f}}}$$

$$\Big\downarrow \text{ 327}$$

$$-\frac{(de-2f)\int \frac{1}{\sqrt{e+fx}\sqrt{4-d^2x^2}}dx}{f}-\frac{2\sqrt{e+fx}E\left(\arcsin\left(\frac{1}{2}\sqrt{2-dx}\right)|\frac{4f}{de+2f}\right)}{f\sqrt{\frac{d(e+fx)}{de+2f}}}$$

$$\Big\downarrow \text{ 511}$$

$$\frac{2(de-2f)\sqrt{\frac{d(e+fx)}{de+2f}}\int \frac{1}{\sqrt{1-\frac{f(2-dx)}{de+2f}}\sqrt{\frac{1}{4}(dx-2)+1}}d\left(\frac{1}{2}\sqrt{2-dx}\right)}{df\sqrt{e+fx}}-\frac{2\sqrt{e+fx}E\left(\arcsin\left(\frac{1}{2}\sqrt{2-dx}\right)|\frac{4f}{de+2f}\right)}{f\sqrt{\frac{d(e+fx)}{de+2f}}}$$

$$\Big\downarrow \textbf{321}$$

$$\frac{2(de-2f)\sqrt{\frac{d(e+fx)}{de+2f}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{2-dx}\right),\frac{4f}{de+2f}\right)}{df\sqrt{e+fx}}-\frac{2\sqrt{e+fx}E\left(\arcsin\left(\frac{1}{2}\sqrt{2-dx}\right)|\frac{4f}{de+2f}\right)}{f\sqrt{\frac{d(e+fx)}{de+2f}}}$$

input
```
Int[Sqrt[4 - d^2*x^2]/((2 - d*x)*Sqrt[e + f*x]),x]
```

output
```
(-2*Sqrt[e + f*x]*EllipticE[ArcSin[Sqrt[2 - d*x]/2], (4*f)/(d*e + 2*f)])/(
f*Sqrt[(d*(e + f*x))/(d*e + 2*f)]) + (2*(d*e - 2*f)*Sqrt[(d*(e + f*x))/(d*
e + 2*f)]*EllipticF[ArcSin[Sqrt[2 - d*x]/2], (4*f)/(d*e + 2*f)])/(d*f*Sqrt
[e + f*x])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 667
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(
x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + c*x^2)^(p - 1), x]
/; FreeQ[{a, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(76) = 152$.

Time = 2.23 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.77

method	result
default	$\frac{2(d^2e^2-4f^2)\,\mathrm{EllipticE}\left(\sqrt{\frac{d(fx+e)}{de+2f}},\sqrt{\frac{de+2f}{de-2f}}\right)\sqrt{-\frac{f(dx-2)}{de+2f}}\,\sqrt{-\frac{(dx+2)f}{de-2f}}\,\sqrt{\frac{d(fx+e)}{de+2f}}\,\sqrt{fx+e}\,\sqrt{-d^2x^2+4}}{f^2d(d^2f\,x^3+d^2e\,x^2-4fx-4e)}$
elliptic	$\frac{\sqrt{-(fx+e)(d^2x^2-4)}\left(\frac{4\left(\frac{e}{f}+\frac{2}{d}\right)\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}+\frac{2}{d}}}\,\sqrt{\frac{x+\frac{2}{d}}{-\frac{e}{f}+\frac{2}{d}}}\,\sqrt{\frac{x-\frac{2}{d}}{-\frac{e}{f}-\frac{2}{d}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}+\frac{2}{d}}},\sqrt{\frac{-\frac{e}{f}-\frac{2}{d}}{-\frac{e}{f}+\frac{2}{d}}}\right)}{\sqrt{-d^2f\,x^3-d^2e\,x^2+4fx+4e}}+\frac{2d\left(\frac{e}{f}+\frac{2}{d}\right)\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}+\frac{2}{d}}}\,\sqrt{\frac{x+\frac{2}{d}}{-\frac{e}{f}+\frac{2}{d}}}\,\sqrt{\frac{x-\frac{2}{d}}{-\frac{e}{f}-\frac{2}{d}}}}{}\right.}{\sqrt{fx+e}\,\sqrt{-d^2x^2+4}}$

input
```
int((-d^2*x^2+4)^(1/2)/(-d*x+2)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(d^2*e^2-4*f^2)*EllipticE((d*(f*x+e)/(d*e+2*f))^(1/2),((d*e+2*f)/(d*e-2*
f))^(1/2))*(-f*(d*x-2)/(d*e+2*f))^(1/2)*(-(d*x+2)*f/(d*e-2*f))^(1/2)*(d*(f
*x+e)/(d*e+2*f))^(1/2)*(f*x+e)^(1/2)*(-d^2*x^2+4)^(1/2)/f^2/d/(d^2*f*x^3+d
^2*e*x^2-4*f*x-4*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(76) = 152$.

Time = 0.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{4-d^2x^2}}{(2-dx)\sqrt{e+fx}}\,dx$$
$$= \frac{2\left(3\sqrt{-d^2f}\,df\,\mathrm{weierstrassZeta}\left(\frac{4\,(d^2e^2+12\,f^2)}{3\,d^2f^2},-\frac{8\,(d^2e^3-36\,ef^2)}{27\,d^2f^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(d^2e^2+12\,f^2)}{3\,d^2f^2},-\frac{8\,(d^2e^3-3}{27\,d^2f}\right.\right.\right.}{3\,d^2f^2}$$

input
```
integrate((-d^2*x^2+4)^(1/2)/(-d*x+2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(3*sqrt(-d^2*f)*d*f*weierstrassZeta(4/3*(d^2*e^2 + 12*f^2)/(d^2*f^2),
-8/27*(d^2*e^3 - 36*e*f^2)/(d^2*f^3), weierstrassPInverse(4/3*(d^2*e^2 + 1
2*f^2)/(d^2*f^2), -8/27*(d^2*e^3 - 36*e*f^2)/(d^2*f^3), 1/3*(3*f*x + e)/f)
) + sqrt(-d^2*f)*(d*e - 6*f)*weierstrassPInverse(4/3*(d^2*e^2 + 12*f^2)/(d
^2*f^2), -8/27*(d^2*e^3 - 36*e*f^2)/(d^2*f^3), 1/3*(3*f*x + e)/f))/(d^2*f^
2)
```

Sympy [F]

$$\int \frac{\sqrt{4-d^2x^2}}{(2-dx)\sqrt{e+fx}}\,dx=-\int \frac{\sqrt{-d^2x^2+4}}{dx\sqrt{e+fx}-2\sqrt{e+fx}}\,dx$$

input
```
integrate((-d**2*x**2+4)**(1/2)/(-d*x+2)/(f*x+e)**(1/2),x)
```

output
```
-Integral(sqrt(-d**2*x**2 + 4)/(d*x*sqrt(e + f*x) - 2*sqrt(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{4-d^2x^2}}{(2-dx)\sqrt{e+fx}}\,dx=\int -\frac{\sqrt{-d^2x^2+4}}{(dx-2)\sqrt{fx+e}}\,dx$$

input
```
integrate((-d^2*x^2+4)^(1/2)/(-d*x+2)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-d^2*x^2 + 4)/((d*x - 2)*sqrt(f*x + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{4-d^2x^2}}{(2-dx)\sqrt{e+fx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-d^2*x^2+4)^(1/2)/(-d*x+2)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4-d^2x^2}}{(2-dx)\sqrt{e+fx}}\,dx = -\int \frac{\sqrt{4-d^2\,x^2}}{\sqrt{e+f\,x}\,(d\,x-2)}\,dx$$

input
```
int(-(4 - d^2*x^2)^(1/2)/((e + f*x)^(1/2)*(d*x - 2)),x)
```

output
```
-int((4 - d^2*x^2)^(1/2)/((e + f*x)^(1/2)*(d*x - 2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{4-d^2x^2}}{(2-dx)\sqrt{e+fx}}\,dx = -\left(\int \frac{\sqrt{fx+e}\,\sqrt{-d^2x^2+4}}{df\,x^2+dex-2fx-2e}dx\right)$$

input
```
int((-d^2*x^2+4)^(1/2)/(-d*x+2)/(f*x+e)^(1/2),x)
```

output
```
 - int((sqrt(e + f*x)*sqrt( - d**2*x**2 + 4))/(d*e*x + d*f*x**2 - 2*e - 2*
f*x),x)
```

3.291 $\int \frac{\sqrt{2+dx}}{\sqrt{2-dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 91

$$\int \frac{\sqrt{2+dx}}{\sqrt{2-dx}\sqrt{e+fx}}\,dx = -\frac{4\sqrt{de+2f}\sqrt{\frac{d(e+fx)}{de+2f}}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{2-dx}}{\sqrt{de+2f}}\right)|\frac{1}{4}\left(2+\frac{de}{f}\right)\right)}{d\sqrt{f}\sqrt{e+fx}}$$

output
```
-4*(d*e+2*f)^(1/2)*(d*(f*x+e)/(d*e+2*f))^(1/2)*EllipticE(f^(1/2)*(-d*x+2)^
(1/2)/(d*e+2*f)^(1/2),1/2*(2+d*e/f)^(1/2))/d/f^(1/2)/(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 7.90 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{2+dx}}{\sqrt{2-dx}\sqrt{e+fx}}\,dx = \frac{2\sqrt{e+fx}\left(\frac{2+dx}{\sqrt{2-dx}}-\frac{2\sqrt{\frac{2+dx}{-2+dx}}\,E\left(\arcsin\left(\frac{2}{\sqrt{2-dx}}\right)|\frac{1}{4}\left(2+\frac{de}{f}\right)\right)}{\sqrt{\frac{d(e+fx)}{f(-2+dx)}}}\right)}{f\sqrt{2+dx}}$$

input
```
Integrate[Sqrt[2 + d*x]/(Sqrt[2 - d*x]*Sqrt[e + f*x]),x]
```

output
```
(2*Sqrt[e + f*x]*((2 + d*x)/Sqrt[2 - d*x] - (2*Sqrt[(2 + d*x)/(-2 + d*x)]*
EllipticE[ArcSin[2/Sqrt[2 - d*x]], (2 + (d*e)/f)/4])/Sqrt[(d*(e + f*x))/(f
*(-2 + d*x))]))/(f*Sqrt[2 + d*x])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {124, 27, 123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx+2}}{\sqrt{2-dx}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{124}$$

$$\frac{2\sqrt{\frac{d(e+fx)}{de+2f}}\int \frac{\sqrt{dx+2}}{2\sqrt{2-dx}\sqrt{\frac{de}{de+2f}+\frac{dfx}{de+2f}}}dx}{\sqrt{e+fx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\frac{d(e+fx)}{de+2f}}\int \frac{\sqrt{dx+2}}{\sqrt{2-dx}\sqrt{\frac{de}{de+2f}+\frac{dfx}{de+2f}}}dx}{\sqrt{e+fx}}$$

$$\Big\downarrow \text{123}$$

$$-\frac{4\sqrt{de+2f}\sqrt{\frac{d(e+fx)}{de+2f}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{2-dx}}{\sqrt{de+2f}}\right)|\frac{1}{4}\left(\frac{de}{f}+2\right)\right)}{d\sqrt{f}\sqrt{e+fx}}$$

input
```
Int[Sqrt[2 + d*x]/(Sqrt[2 - d*x]*Sqrt[e + f*x]),x]
```

output
```
(-4*Sqrt[d*e + 2*f]*Sqrt[(d*(e + f*x))/(d*e + 2*f)]*EllipticE[ArcSin[(Sqrt
[f]*Sqrt[2 - d*x])/Sqrt[d*e + 2*f]], (2 + (d*e)/f)/4])/(d*Sqrt[f]*Sqrt[e +
 f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(76) = 152$.

Time = 1.90 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.80

method	result
default	$\frac{2\left(d^{2} e^{2}-4 f^{2}\right) \operatorname{EllipticE}\left(\sqrt{\frac{d(f x+e)}{d e+2 f}}, \sqrt{\frac{d e+2 f}{d e-2 f}}\right) \sqrt{-\frac{f(d x-2)}{d e+2 f}}\, \sqrt{-\frac{(d x+2) f}{d e-2 f}}\, \sqrt{\frac{d(f x+e)}{d e+2 f}}\, \sqrt{d x+2}\, \sqrt{-d x+2}\, \sqrt{f x+e}}{f^{2} d\left(d^{2} f\, x^{3}+d^{2} e\, x^{2}-4 f x-4 e\right)}$
elliptic	$\frac{\sqrt{-(f x+e)\left(d^{2} x^{2}-4\right)}\left(\frac{4\left(\frac{e}{f}+\frac{2}{d}\right) \sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}+\frac{2}{d}}}\, \sqrt{\frac{x+\frac{2}{d}}{-\frac{e}{f}+\frac{2}{d}}}\, \sqrt{\frac{x-\frac{2}{d}}{-\frac{e}{f}-\frac{2}{d}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}+\frac{2}{d}}}, \sqrt{\frac{-\frac{e}{f}-\frac{2}{d}}{-\frac{e}{f}+\frac{2}{d}}}\right)}{\sqrt{-d^{2} f\, x^{3}-d^{2} e\, x^{2}+4 f x+4 e}}+\frac{2 d\left(\frac{e}{f}+\frac{2}{d}\right) \sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}+\frac{2}{d}}}\, \sqrt{\frac{x+\frac{2}{d}}{-\frac{e}{f}+\frac{2}{d}}}\, \sqrt{\frac{x-\frac{2}{d}}{-\frac{e}{f}-\frac{2}{d}}} \cdots}{\cdots}\right)}{\sqrt{d x+2}\, \sqrt{-d x+2}\, \sqrt{f x+e}}$

input
```
int((d*x+2)^(1/2)/(-d*x+2)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(d^2*e^2-4*f^2)*EllipticE((d*(f*x+e)/(d*e+2*f))^(1/2),((d*e+2*f)/(d*e-2*
f))^(1/2))*(-f*(d*x-2)/(d*e+2*f))^(1/2)*(-(d*x+2)*f/(d*e-2*f))^(1/2)*(d*(f
*x+e)/(d*e+2*f))^(1/2)*(d*x+2)^(1/2)*(-d*x+2)^(1/2)*(f*x+e)^(1/2)/f^2/d/(d
^2*f*x^3+d^2*e*x^2-4*f*x-4*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(76) = 152$.

Time = 0.07 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{2+dx}}{\sqrt{2-dx}\sqrt{e+fx}}\,dx = \frac{2\left(3\sqrt{-d^2f}\,df\,\text{weierstrassZeta}\left(\frac{4\,(d^2e^2+12\,f^2)}{3\,d^2f^2},-\frac{8\,(d^2e^3-36\,ef^2)}{27\,d^2f^3},\text{weierstrassPInverse}\left(\frac{4\,(d^2e^2+12\,f^2)}{3\,d^2f^2},-\frac{8\,(d^2e^3-3}{27\,d^2f}\right.\right.\right.}{3\,d^2f^2}$$

input
```
integrate((d*x+2)^(1/2)/(-d*x+2)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas"
)
```

output
```
2/3*(3*sqrt(-d^2*f)*d*f*weierstrassZeta(4/3*(d^2*e^2 + 12*f^2)/(d^2*f^2),
-8/27*(d^2*e^3 - 36*e*f^2)/(d^2*f^3), weierstrassPInverse(4/3*(d^2*e^2 + 1
2*f^2)/(d^2*f^2), -8/27*(d^2*e^3 - 36*e*f^2)/(d^2*f^3), 1/3*(3*f*x + e)/f)
) + sqrt(-d^2*f)*(d*e - 6*f)*weierstrassPInverse(4/3*(d^2*e^2 + 12*f^2)/(d
^2*f^2), -8/27*(d^2*e^3 - 36*e*f^2)/(d^2*f^3), 1/3*(3*f*x + e)/f))/(d^2*f^
2)
```

Sympy [F]

$$\int \frac{\sqrt{2+dx}}{\sqrt{2-dx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{dx+2}}{\sqrt{e+fx}\sqrt{-dx+2}}\,dx$$

input
```
integrate((d*x+2)**(1/2)/(-d*x+2)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(d*x + 2)/(sqrt(e + f*x)*sqrt(-d*x + 2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+dx}}{\sqrt{2-dx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{dx+2}}{\sqrt{-dx+2}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+2)^(1/2)/(-d*x+2)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(d*x + 2)/(sqrt(-d*x + 2)*sqrt(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{2+dx}}{\sqrt{2-dx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{dx+2}}{\sqrt{-dx+2}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+2)^(1/2)/(-d*x+2)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + 2)/(sqrt(-d*x + 2)*sqrt(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+dx}}{\sqrt{2-dx}\sqrt{e+fx}}\,dx=\int \frac{\sqrt{d\,x+2}}{\sqrt{e+f\,x}\,\sqrt{2-d\,x}}\,dx$$

input
```
int((d*x + 2)^(1/2)/((e + f*x)^(1/2)*(2 - d*x)^(1/2)),x)
```

output
```
int((d*x + 2)^(1/2)/((e + f*x)^(1/2)*(2 - d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{2+dx}}{\sqrt{2-dx}\sqrt{e+fx}}\,dx=-\left(\int \frac{\sqrt{fx+e}\,\sqrt{dx+2}\,\sqrt{-dx+2}}{df\,x^{2}+dex-2fx-2e}dx\right)$$

input
```
int((d*x+2)^(1/2)/(-d*x+2)^(1/2)/(f*x+e)^(1/2),x)
```

output
```
- int((sqrt(e + f*x)*sqrt(d*x + 2)*sqrt( - d*x + 2))/(d*e*x + d*f*x**2 -
2*e - 2*f*x),x)
```

3.292 $\int \frac{c+dx}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 137

$$\int \frac{c+dx}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}\,dx = -\frac{2\sqrt{2}\sqrt{de+cf}\sqrt{\frac{d(e+fx)}{de+cf}}\sqrt{c^2-d^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c-dx}}{\sqrt{de+cf}}\right)|\frac{1}{2}\left(1+\frac{de}{cf}\right)\right)}{d\sqrt{f}\sqrt{c-dx}\sqrt{1+\frac{dx}{c}}\sqrt{e+fx}}$$

output
```
-2*2^(1/2)*(c*f+d*e)^(1/2)*(d*(f*x+e)/(c*f+d*e))^(1/2)*(-d^2*x^2+c^2)^(1/2
)*EllipticE(f^(1/2)*(-d*x+c)^(1/2)/(c*f+d*e)^(1/2),1/2*(2+2*d*e/c/f)^(1/2)
)/d/f^(1/2)/(-d*x+c)^(1/2)/(1+d*x/c)^(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.24

$$\int \frac{c+dx}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}\,dx = -\frac{2\left(f^2\sqrt{-\frac{de+cf}{d}}(c^2-d^2x^2)+id(de+cf)\sqrt{\frac{f(-c+dx)}{d(e+fx)}}\sqrt{\frac{f(c+dx)}{d(e+fx)}}(e+fx)^{3/2}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{de+cf}{d}}}{\sqrt{e+fx}}\right)|\frac{de-cf}{de+cf}\right.\right.}{df^2\sqrt{-\frac{de+cf}{d}}\sqrt{e+fx}\sqrt{c^2}}$$

input
```
Integrate[(c + d*x)/(Sqrt[e + f*x]*Sqrt[c^2 - d^2*x^2]),x]
```

output
```
(-2*(f^2*Sqrt[-((d*e + c*f)/d)]*(c^2 - d^2*x^2) + I*d*(d*e + c*f)*Sqrt[(f*
(-c + d*x))/(d*(e + f*x))]*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/
2)*EllipticE[I*ArcSinh[Sqrt[-((d*e + c*f)/d)]/Sqrt[e + f*x]], (d*e - c*f)/
(d*e + c*f)] - (2*I)*c*d*f*Sqrt[(f*(-c + d*x))/(d*(e + f*x))]*Sqrt[(f*(c +
 d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-((d*e + c*
f)/d)]/Sqrt[e + f*x]], (d*e - c*f)/(d*e + c*f)]))/(d*f^2*Sqrt[-((d*e + c*f
)/d)]*Sqrt[e + f*x]*Sqrt[c^2 - d^2*x^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.58, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{c^2-d^2x^2}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{ 600}$$

$$\frac{d\int \frac{\sqrt{e+fx}}{\sqrt{c^2-d^2x^2}}dx}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}dx}{f}$$

$$\Big\downarrow \text{ 509}$$

$$\frac{d\sqrt{1-\frac{d^2x^2}{c^2}}\int \frac{\sqrt{e+fx}}{\sqrt{1-\frac{d^2x^2}{c^2}}}dx}{f\sqrt{c^2-d^2x^2}}-\frac{(de-cf)\int \frac{1}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}dx}{f}$$

$$\Big\downarrow \text{ 508}$$

$$-\frac{(de-cf)\int \frac{1}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}dx}{f}-\frac{2c\sqrt{1-\frac{d^2x^2}{c^2}}\sqrt{e+fx}\int \frac{\sqrt{1-\frac{cf\left(1-\frac{dx}{c}\right)}{de+cf}}}{\sqrt{\frac{1}{2}\left(\frac{dx}{c}-1\right)+1}}d\frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{2}}}{f\sqrt{c^2-d^2x^2}\sqrt{\frac{d(e+fx)}{cf+de}}}$$

$$\Big\downarrow \text{ 327}$$

$$-\frac{(de-cf)\int \frac{1}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}dx}{f}-\frac{2c\sqrt{1-\frac{d^2x^2}{c^2}}\sqrt{e+fx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{2}}\right)|\frac{2cf}{de+cf}\right)}{f\sqrt{c^2-d^2x^2}\sqrt{\frac{d(e+fx)}{cf+de}}}$$

$$\Big\downarrow \text{512}$$

$$-\frac{\sqrt{1-\frac{d^2x^2}{c^2}}(de-cf)\int \frac{1}{\sqrt{e+fx}\sqrt{1-\frac{d^2x^2}{c^2}}}dx}{f\sqrt{c^2-d^2x^2}}-\frac{2c\sqrt{1-\frac{d^2x^2}{c^2}}\sqrt{e+fx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{2}}\right)|\frac{2cf}{de+cf}\right)}{f\sqrt{c^2-d^2x^2}\sqrt{\frac{d(e+fx)}{cf+de}}}$$

$$\Big\downarrow \text{511}$$

$$\frac{2c\sqrt{1-\frac{d^2x^2}{c^2}}(de-cf)\sqrt{\frac{d(e+fx)}{cf+de}}\int \frac{1}{\sqrt{1-\frac{cf\left(1-\frac{dx}{c}\right)}{de+cf}}\sqrt{\frac{1}{2}\left(\frac{dx}{c}-1\right)+1}}d\frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{2}}}{df\sqrt{c^2-d^2x^2}\sqrt{e+fx}}-\frac{2c\sqrt{1-\frac{d^2x^2}{c^2}}\sqrt{e+fx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{2}}\right)|\frac{2cf}{de+cf}\right)}{f\sqrt{c^2-d^2x^2}\sqrt{\frac{d(e+fx)}{cf+de}}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2c\sqrt{1-\frac{d^2x^2}{c^2}}(de-cf)\sqrt{\frac{d(e+fx)}{cf+de}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{2}}\right),\frac{2cf}{de+cf}\right)}{df\sqrt{c^2-d^2x^2}\sqrt{e+fx}}-\frac{2c\sqrt{1-\frac{d^2x^2}{c^2}}\sqrt{e+fx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{2}}\right)|\frac{2cf}{de+cf}\right)}{f\sqrt{c^2-d^2x^2}\sqrt{\frac{d(e+fx)}{cf+de}}}$$

input
```
Int[(c + d*x)/(Sqrt[e + f*x]*Sqrt[c^2 - d^2*x^2]),x]
```

output
```
(-2*c*Sqrt[e + f*x]*Sqrt[1 - (d^2*x^2)/c^2]*EllipticE[ArcSin[Sqrt[1 - (d*x
)/c]/Sqrt[2]], (2*c*f)/(d*e + c*f)])/(f*Sqrt[(d*(e + f*x))/(d*e + c*f)]*Sq
rt[c^2 - d^2*x^2]) + (2*c*(d*e - c*f)*Sqrt[(d*(e + f*x))/(d*e + c*f)]*Sqrt
[1 - (d^2*x^2)/c^2]*EllipticF[ArcSin[Sqrt[1 - (d*x)/c]/Sqrt[2]], (2*c*f)/(
d*e + c*f)])/(d*f*Sqrt[e + f*x]*Sqrt[c^2 - d^2*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(115) = 230.

Time = 1.74 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.32

method	result
default	$-\frac{2\left(2\,\mathrm{EllipticF}\left(\sqrt{-\frac{d(fx+e)}{cf-de}},\sqrt{-\frac{cf-de}{cf+de}}\right)c^2f^2-2\,\mathrm{EllipticF}\left(\sqrt{-\frac{d(fx+e)}{cf-de}},\sqrt{-\frac{cf-de}{cf+de}}\right)cdef-\mathrm{EllipticE}\left(\sqrt{-\frac{d(fx+e)}{cf-de}},\sqrt{-\frac{cf-de}{cf+de}}\right)\cdots\right.}{f^2d(-d^2fx^3-d^2ex^2+c^2fx\cdots}$
elliptic	$\frac{\sqrt{(fx+e)(-d^2x^2+c^2)}\left(\frac{2c\left(\frac{e}{f}-\frac{c}{d}\right)\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}-\frac{c}{d}}}\sqrt{\frac{x-\frac{c}{d}}{-\frac{e}{f}-\frac{c}{d}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{e}{f}+\frac{c}{d}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}-\frac{c}{d}}},\sqrt{\frac{-\frac{e}{f}+\frac{c}{d}}{-\frac{e}{f}-\frac{c}{d}}}\right)}{\sqrt{-d^2fx^3-d^2ex^2+c^2fx+ec^2}}+\frac{2d\left(\frac{e}{f}-\frac{c}{d}\right)\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}-\frac{c}{d}}}\sqrt{\frac{x-\frac{c}{d}}{-\frac{e}{f}-\frac{c}{d}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{e}{f}+\frac{c}{d}}}\cdots}{\cdots}\right.}{\sqrt{fx+e}\sqrt{-d^2x^2+c^2}}$

input
```
int((d*x+c)/(f*x+e)^(1/2)/(-d^2*x^2+c^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(2*EllipticF((-d*(f*x+e)/(c*f-d*e))^(1/2),(-(c*f-d*e)/(c*f+d*e))^(1/2))
*c^2*f^2-2*EllipticF((-d*(f*x+e)/(c*f-d*e))^(1/2),(-(c*f-d*e)/(c*f+d*e))^(
1/2))*c*d*e*f-EllipticE((-d*(f*x+e)/(c*f-d*e))^(1/2),(-(c*f-d*e)/(c*f+d*e)
)^(1/2))*c^2*f^2+EllipticE((-d*(f*x+e)/(c*f-d*e))^(1/2),(-(c*f-d*e)/(c*f+d
*e))^(1/2))*d^2*e^2)*(f*(d*x+c)/(c*f-d*e))^(1/2)*(f*(-d*x+c)/(c*f+d*e))^(1
/2)*(-d*(f*x+e)/(c*f-d*e))^(1/2)/f^2/d*(f*x+e)^(1/2)*(-d^2*x^2+c^2)^(1/2)/
(-d^2*f*x^3-d^2*e*x^2+c^2*f*x+c^2*e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.53

$$\int \frac{c+dx}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}\,dx$$
$$= \frac{2\left(3\sqrt{-d^2f}df\text{weierstrassZeta}\left(\frac{4\,(d^2e^2+3\,c^2f^2)}{3\,d^2f^2},-\frac{8\,(d^2e^3-9\,c^2ef^2)}{27\,d^2f^3},\text{weierstrassPInverse}\left(\frac{4\,(d^2e^2+3\,c^2f^2)}{3\,d^2f^2},-\frac{8\,(d^2e^3}{27}\right.\right.\right.}{3\,d^2f^2}$$

input
```
integrate((d*x+c)/(f*x+e)^(1/2)/(-d^2*x^2+c^2)^(1/2),x, algorithm="fricas"
)
```

output
```
2/3*(3*sqrt(-d^2*f)*d*f*weierstrassZeta(4/3*(d^2*e^2 + 3*c^2*f^2)/(d^2*f^2
), -8/27*(d^2*e^3 - 9*c^2*e*f^2)/(d^2*f^3), weierstrassPInverse(4/3*(d^2*e
^2 + 3*c^2*f^2)/(d^2*f^2), -8/27*(d^2*e^3 - 9*c^2*e*f^2)/(d^2*f^3), 1/3*(3
*f*x + e)/f)) + sqrt(-d^2*f)*(d*e - 3*c*f)*weierstrassPInverse(4/3*(d^2*e^
2 + 3*c^2*f^2)/(d^2*f^2), -8/27*(d^2*e^3 - 9*c^2*e*f^2)/(d^2*f^3), 1/3*(3*
f*x + e)/f))/(d^2*f^2)
```

Sympy [F]

$$\int \frac{c+dx}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}\,dx = \int \frac{c+dx}{\sqrt{-\left(-c+dx\right)\left(c+dx\right)}\sqrt{e+fx}}\,dx$$

input
```
integrate((d*x+c)/(f*x+e)**(1/2)/(-d**2*x**2+c**2)**(1/2),x)
```

output
```
Integral((c + d*x)/(sqrt(-(-c + d*x)*(c + d*x))*sqrt(e + f*x)), x)
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}\,dx = \int \frac{dx+c}{\sqrt{-d^2x^2+c^2}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+c)/(f*x+e)^(1/2)/(-d^2*x^2+c^2)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((d*x + c)/(sqrt(-d^2*x^2 + c^2)*sqrt(f*x + e)), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}\,dx = \int \frac{dx+c}{\sqrt{-d^2x^2+c^2}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+c)/(f*x+e)^(1/2)/(-d^2*x^2+c^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(sqrt(-d^2*x^2 + c^2)*sqrt(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}\,dx = \int \frac{c+d\,x}{\sqrt{e+f\,x}\,\sqrt{c^2-d^2\,x^2}}\,dx$$

input
```
int((c + d*x)/((e + f*x)^(1/2)*(c^2 - d^2*x^2)^(1/2)),x)
```

output
```
int((c + d*x)/((e + f*x)^(1/2)*(c^2 - d^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}\,dx = \int \frac{\sqrt{fx+e}\,\sqrt{-d^2x^2+c^2}}{-df\,x^2+cfx-dex+ce}dx$$

input
```
int((d*x+c)/(f*x+e)^(1/2)/(-d^2*x^2+c^2)^(1/2),x)
```

output
```
int((sqrt(e + f*x)*sqrt(c**2 - d**2*x**2))/(c*e + c*f*x - d*e*x - d*f*x**2
),x)
```

3.293 $\int \frac{\sqrt{c^2-d^2x^2}}{(c-dx)\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 137

$$\int \frac{\sqrt{c^2-d^2x^2}}{(c-dx)\sqrt{e+fx}}\,dx = -\frac{2\sqrt{2}\sqrt{de+cf}\sqrt{\frac{d(e+fx)}{de+cf}}\sqrt{c^2-d^2x^2}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c-dx}}{\sqrt{de+cf}}\right)|\frac{1}{2}\left(1+\frac{de}{cf}\right)\right)}{d\sqrt{f}\sqrt{c-dx}\sqrt{1+\frac{dx}{c}}\sqrt{e+fx}}$$

output
```
-2*2^(1/2)*(c*f+d*e)^(1/2)*(d*(f*x+e)/(c*f+d*e))^(1/2)*(-d^2*x^2+c^2)^(1/2
)*EllipticE(f^(1/2)*(-d*x+c)^(1/2)/(c*f+d*e)^(1/2),1/2*(2+2*d*e/c/f)^(1/2)
)/d/f^(1/2)/(-d*x+c)^(1/2)/(1+d*x/c)^(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.55 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{c^2-d^2x^2}}{(c-dx)\sqrt{e+fx}}\,dx = -\frac{2\sqrt{c^2-d^2x^2}\left(f^2\sqrt{-\frac{de+cf}{d}}(-c^2+d^2x^2)-id(de+cf)\sqrt{\frac{f(-c+dx)}{d(e+fx)}}\sqrt{\frac{f(c+dx)}{d(e+fx)}}(e+fx)^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{-}}{\sqrt{}}\right.\right.\right.}{f^2\sqrt{-\frac{de+cf}{d}}\sqrt{e+fx}}$$

input
```
Integrate[Sqrt[c^2 - d^2*x^2]/((c - d*x)*Sqrt[e + f*x]),x]
```

output
```
(-2*Sqrt[c^2 - d^2*x^2]*(f^2*Sqrt[-((d*e + c*f)/d)]*(-c^2 + d^2*x^2) - I*d
*(d*e + c*f)*Sqrt[(f*(-c + d*x))/(d*(e + f*x))]*Sqrt[(f*(c + d*x))/(d*(e +
 f*x))]*(e + f*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-((d*e + c*f)/d)]/Sqrt[e
+ f*x]], (d*e - c*f)/(d*e + c*f)] + (2*I)*c*d*f*Sqrt[(f*(-c + d*x))/(d*(e
+ f*x))]*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*EllipticF[I*Arc
Sinh[Sqrt[-((d*e + c*f)/d)]/Sqrt[e + f*x]], (d*e - c*f)/(d*e + c*f)]))/(f^
2*Sqrt[-((d*e + c*f)/d)]*Sqrt[e + f*x]*(-(c^2*d) + d^3*x^2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.58, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {667, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2-d^2x^2}}{(c-dx)\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{ 667}$$

$$\int \frac{c+dx}{\sqrt{c^2-d^2x^2}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{ 600}$$

$$\frac{d\int \frac{\sqrt{e+fx}}{\sqrt{c^2-d^2x^2}}dx}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}dx}{f}$$

$$\Big\downarrow \ 509$$

$$\frac{d\sqrt{1-\frac{d^2x^2}{c^2}}\int \frac{\sqrt{e+fx}}{\sqrt{1-\frac{d^2x^2}{c^2}}}dx}{f\sqrt{c^2-d^2x^2}}-\frac{(de-cf)\int \frac{1}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}dx}{f}$$

$$\Big\downarrow \ 508$$

$$-\frac{(de-cf)\int \frac{1}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}dx}{f}-\frac{2c\sqrt{1-\frac{d^2x^2}{c^2}}\sqrt{e+fx}\int \frac{\sqrt{1-\frac{cf\left(1-\frac{dx}{c}\right)}{de+cf}}}{\sqrt{\frac{1}{2}\left(\frac{dx}{c}-1\right)+1}}d\frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{2}}}{f\sqrt{c^2-d^2x^2}\sqrt{\frac{d(e+fx)}{cf+de}}}$$

$$\Big\downarrow \ 327$$

$$-\frac{(de-cf)\int \frac{1}{\sqrt{e+fx}\sqrt{c^2-d^2x^2}}dx}{f}-\frac{2c\sqrt{1-\frac{d^2x^2}{c^2}}\sqrt{e+fx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{2}}\right)|\frac{2cf}{de+cf}\right)}{f\sqrt{c^2-d^2x^2}\sqrt{\frac{d(e+fx)}{cf+de}}}$$

$$\Big\downarrow \ 512$$

$$-\frac{\sqrt{1-\frac{d^2x^2}{c^2}}(de-cf)\int \frac{1}{\sqrt{e+fx}\sqrt{1-\frac{d^2x^2}{c^2}}}dx}{f\sqrt{c^2-d^2x^2}}-\frac{2c\sqrt{1-\frac{d^2x^2}{c^2}}\sqrt{e+fx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{2}}\right)|\frac{2cf}{de+cf}\right)}{f\sqrt{c^2-d^2x^2}\sqrt{\frac{d(e+fx)}{cf+de}}}$$

$$\Big\downarrow \ 511$$

$$\frac{2c\sqrt{1-\frac{d^2x^2}{c^2}}(de-cf)\sqrt{\frac{d(e+fx)}{cf+de}}\int \frac{1}{\sqrt{1-\frac{cf\left(1-\frac{dx}{c}\right)}{de+cf}}\sqrt{\frac{1}{2}\left(\frac{dx}{c}-1\right)+1}}d\frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{2}}}{df\sqrt{c^2-d^2x^2}\sqrt{e+fx}}-\frac{2c\sqrt{1-\frac{d^2x^2}{c^2}}\sqrt{e+fx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{2}}\right)|\frac{2cf}{de+cf}\right)}{f\sqrt{c^2-d^2x^2}\sqrt{\frac{d(e+fx)}{cf+de}}}$$

$$\Big\downarrow \ 321$$

$$\frac{2c\sqrt{1-\frac{d^2x^2}{c^2}}(de-cf)\sqrt{\frac{d(e+fx)}{cf+de}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{2}}\right),\frac{2cf}{de+cf}\right)}{df\sqrt{c^2-d^2x^2}\sqrt{e+fx}}-\frac{2c\sqrt{1-\frac{d^2x^2}{c^2}}\sqrt{e+fx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{dx}{c}}}{\sqrt{2}}\right)|\frac{2cf}{de+cf}\right)}{f\sqrt{c^2-d^2x^2}\sqrt{\frac{d(e+fx)}{cf+de}}}$$

input
```
Int[Sqrt[c^2 - d^2*x^2]/((c - d*x)*Sqrt[e + f*x]),x]
```

output
```
(-2*c*Sqrt[e + f*x]*Sqrt[1 - (d^2*x^2)/c^2]*EllipticE[ArcSin[Sqrt[1 - (d*x
)/c]/Sqrt[2]], (2*c*f)/(d*e + c*f)])/(f*Sqrt[(d*(e + f*x))/(d*e + c*f)]*Sq
rt[c^2 - d^2*x^2]) + (2*c*(d*e - c*f)*Sqrt[(d*(e + f*x))/(d*e + c*f)]*Sqrt
[1 - (d^2*x^2)/c^2]*EllipticF[ArcSin[Sqrt[1 - (d*x)/c]/Sqrt[2]], (2*c*f)/(
d*e + c*f)])/(d*f*Sqrt[e + f*x]*Sqrt[c^2 - d^2*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 667
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(
x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + c*x^2)^(p - 1), x]
/; FreeQ[{a, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(115) = 230$.

Time = 1.91 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.32

method	result
default	$-\frac{2\left(2\,\mathrm{EllipticF}\left(\sqrt{-\frac{d(fx+e)}{cf-de}},\sqrt{-\frac{cf-de}{cf+de}}\right)c^{2}f^{2}-2\,\mathrm{EllipticF}\left(\sqrt{-\frac{d(fx+e)}{cf-de}},\sqrt{-\frac{cf-de}{cf+de}}\right)cdef-\mathrm{EllipticE}\left(\sqrt{-\frac{d(fx+e)}{cf-de}},\sqrt{-\frac{cf-de}{cf+de}}\right)\right.}{f^{2}d\left(-d^{2}f\,x^{3}-d^{2}e\,x^{2}+c^{2}fx\right.}$
elliptic	$\frac{\sqrt{(fx+e)\left(-d^{2}x^{2}+c^{2}\right)}\left(\frac{2c\left(\frac{e}{f}-\frac{c}{d}\right)\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}-\frac{c}{d}}}\sqrt{\frac{x-\frac{c}{d}}{-\frac{e}{f}-\frac{c}{d}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{e}{f}+\frac{c}{d}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}-\frac{c}{d}}},\sqrt{\frac{-\frac{e}{f}+\frac{c}{d}}{-\frac{e}{f}-\frac{c}{d}}}\right)}{\sqrt{-d^{2}f\,x^{3}-d^{2}e\,x^{2}+c^{2}fx+e\,c^{2}}}+\frac{2d\left(\frac{e}{f}-\frac{c}{d}\right)\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}-\frac{c}{d}}}\sqrt{\frac{x-\frac{c}{d}}{-\frac{e}{f}-\frac{c}{d}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{e}{f}+\frac{c}{d}}}}{}\right.}{\sqrt{fx+e}\,\sqrt{-d^{2}x^{2}+c^{2}}}$

input

```
int((-d^2*x^2+c^2)^(1/2)/(-d*x+c)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2*(2*EllipticF((-d*(f*x+e)/(c*f-d*e))^(1/2),(-(c*f-d*e)/(c*f+d*e))^(1/2))
*c^2*f^2-2*EllipticF((-d*(f*x+e)/(c*f-d*e))^(1/2),(-(c*f-d*e)/(c*f+d*e))^(
1/2))*c*d*e*f-EllipticE((-d*(f*x+e)/(c*f-d*e))^(1/2),(-(c*f-d*e)/(c*f+d*e)
)^(1/2))*c^2*f^2+EllipticE((-d*(f*x+e)/(c*f-d*e))^(1/2),(-(c*f-d*e)/(c*f+d
*e))^(1/2))*d^2*e^2)*(f*(d*x+c)/(c*f-d*e))^(1/2)*(f*(-d*x+c)/(c*f+d*e))^(1
/2)*(-d*(f*x+e)/(c*f-d*e))^(1/2)/f^2/d*(f*x+e)^(1/2)*(-d^2*x^2+c^2)^(1/2)/
(-d^2*f*x^3-d^2*e*x^2+c^2*f*x+c^2*e)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{c^2-d^2x^2}}{(c-dx)\sqrt{e+fx}}\,dx = \frac{2\left(3\sqrt{-d^2f}\,df\,\mathrm{weierstrassZeta}\left(\frac{4\left(d^2e^2+3c^2f^2\right)}{3d^2f^2},-\frac{8\left(d^2e^3-9c^2ef^2\right)}{27d^2f^3},\mathrm{weierstrassPInverse}\left(\frac{4\left(d^2e^2+3c^2f^2\right)}{3d^2f^2},-\frac{8\left(d^2e^3\right.}{27}\right.\right.\right.}{3d^2f^2}$$

input

```
integrate((-d^2*x^2+c^2)^(1/2)/(-d*x+c)/(f*x+e)^(1/2),x, algorithm="fricas
")
```

output
```
2/3*(3*sqrt(-d^2*f)*d*f*weierstrassZeta(4/3*(d^2*e^2 + 3*c^2*f^2)/(d^2*f^2
), -8/27*(d^2*e^3 - 9*c^2*e*f^2)/(d^2*f^3), weierstrassPInverse(4/3*(d^2*e
^2 + 3*c^2*f^2)/(d^2*f^2), -8/27*(d^2*e^3 - 9*c^2*e*f^2)/(d^2*f^3), 1/3*(3
*f*x + e)/f)) + sqrt(-d^2*f)*(d*e - 3*c*f)*weierstrassPInverse(4/3*(d^2*e^
2 + 3*c^2*f^2)/(d^2*f^2), -8/27*(d^2*e^3 - 9*c^2*e*f^2)/(d^2*f^3), 1/3*(3*
f*x + e)/f))/(d^2*f^2)
```

Sympy [F]

$$\int \frac{\sqrt{c^2-d^2x^2}}{(c-dx)\sqrt{e+fx}}\,dx = -\int \frac{\sqrt{c^2-d^2x^2}}{-c\sqrt{e+fx}+dx\sqrt{e+fx}}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(1/2)/(-d*x+c)/(f*x+e)**(1/2),x)
```

output
```
-Integral(sqrt(c**2 - d**2*x**2)/(-c*sqrt(e + f*x) + d*x*sqrt(e + f*x)), x
)
```

Maxima [F]

$$\int \frac{\sqrt{c^2-d^2x^2}}{(c-dx)\sqrt{e+fx}}\,dx = \int -\frac{\sqrt{-d^2x^2+c^2}}{(dx-c)\sqrt{fx+e}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/2)/(-d*x+c)/(f*x+e)^(1/2),x, algorithm="maxima
")
```

output
```
-integrate(sqrt(-d^2*x^2 + c^2)/((d*x - c)*sqrt(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{c^2-d^2x^2}}{(c-dx)\sqrt{e+fx}}\,dx=\int -\frac{\sqrt{-d^2x^2+c^2}}{(dx-c)\sqrt{fx+e}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/2)/(-d*x+c)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(-d^2*x^2 + c^2)/((d*x - c)*sqrt(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c^2-d^2x^2}}{(c-dx)\sqrt{e+fx}}\,dx=\int \frac{\sqrt{c^2-d^2\,x^2}}{\sqrt{e+f\,x}\,(c-d\,x)}\,dx$$

input
```
int((c^2 - d^2*x^2)^(1/2)/((e + f*x)^(1/2)*(c - d*x)),x)
```

output
```
int((c^2 - d^2*x^2)^(1/2)/((e + f*x)^(1/2)*(c - d*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c^2-d^2x^2}}{(c-dx)\sqrt{e+fx}}\,dx=\int \frac{\sqrt{fx+e}\,\sqrt{-d^2x^2+c^2}}{-df\,x^2+cfx-dex+ce}dx$$

input
```
int((-d^2*x^2+c^2)^(1/2)/(-d*x+c)/(f*x+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x)*sqrt(c**2 - d**2*x**2))/(c*e + c*f*x - d*e*x - d*f*x**2
),x)
```

3.294 $\int \frac{\sqrt{c+dx}}{\sqrt{c-dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 120

$$\int \frac{\sqrt{c+dx}}{\sqrt{c-dx}\sqrt{e+fx}}\,dx = -\frac{2\sqrt{2}\sqrt{de+cf}\sqrt{c+dx}\sqrt{\frac{d(e+fx)}{de+cf}}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c-dx}}{\sqrt{de+cf}}\right)|\frac{1}{2}\left(1+\frac{de}{cf}\right)\right)}{d\sqrt{f}\sqrt{1+\frac{dx}{c}}\sqrt{e+fx}}$$

output
```
-2*2^(1/2)*(c*f+d*e)^(1/2)*(d*x+c)^(1/2)*(d*(f*x+e)/(c*f+d*e))^(1/2)*Ellip
ticE(f^(1/2)*(-d*x+c)^(1/2)/(c*f+d*e)^(1/2),1/2*(2+2*d*e/c/f)^(1/2))/d/f^(
1/2)/(1+d*x/c)^(1/2)/(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 8.68 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{c+dx}}{\sqrt{c-dx}\sqrt{e+fx}}\,dx = \frac{2\sqrt{e+fx}\left(\frac{c+dx}{\sqrt{c-dx}}-\frac{\sqrt{2}\sqrt{c}\sqrt{\frac{c+dx}{-c+dx}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{c-dx}}\right)|\frac{1}{2}\left(1+\frac{de}{cf}\right)\right)}{\sqrt{\frac{d(e+fx)}{f(-c+dx)}}}\right)}{f\sqrt{c+dx}}$$

input
```
Integrate[Sqrt[c + d*x]/(Sqrt[c - d*x]*Sqrt[e + f*x]),x]
```

output
```
(2*Sqrt[e + f*x]*((c + d*x)/Sqrt[c - d*x] - (Sqrt[2]*Sqrt[c]*Sqrt[(c + d*x
)/(-c + d*x)]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c])/Sqrt[c - d*x]], (1 + (d*e
)/(c*f))/2])/Sqrt[(d*(e + f*x))/(f*(-c + d*x))]))/(f*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {124, 27, 123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{\sqrt{c-dx}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{124}$$

$$\frac{\sqrt{2}\sqrt{c+dx}\sqrt{\frac{d(e+fx)}{cf+de}}\int \frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{2}\sqrt{c-dx}\sqrt{\frac{de}{de+cf}+\frac{dfx}{de+cf}}}dx}{\sqrt{\frac{c+dx}{c}}\sqrt{e+fx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx}\sqrt{\frac{d(e+fx)}{cf+de}}\int \frac{\sqrt{\frac{dx}{c}+1}}{\sqrt{c-dx}\sqrt{\frac{de}{de+cf}+\frac{dfx}{de+cf}}}dx}{\sqrt{\frac{c+dx}{c}}\sqrt{e+fx}}$$

$$\Big\downarrow \text{123}$$

$$-\frac{2\sqrt{2}\sqrt{c+dx}\sqrt{cf+de}\sqrt{\frac{d(e+fx)}{cf+de}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c-dx}}{\sqrt{de+cf}}\right)|\frac{1}{2}\left(\frac{de}{cf}+1\right)\right)}{d\sqrt{f}\sqrt{\frac{c+dx}{c}}\sqrt{e+fx}}$$

input
```
Int[Sqrt[c + d*x]/(Sqrt[c - d*x]*Sqrt[e + f*x]),x]
```

output
```
(-2*Sqrt[2]*Sqrt[d*e + c*f]*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(d*e + c*f)]*
EllipticE[ArcSin[(Sqrt[f]*Sqrt[c - d*x])/Sqrt[d*e + c*f]], (1 + (d*e)/(c*f
))/2])/(d*Sqrt[f]*Sqrt[(c + d*x)/c]*Sqrt[e + f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(100) = 200$.

Time = 1.99 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.66

method	result
default	$-\frac{2\left(2\,\mathrm{EllipticF}\left(\sqrt{-\frac{d(fx+e)}{cf-de}},\sqrt{-\frac{cf-de}{cf+de}}\right)c^{2}f^{2}-2\,\mathrm{EllipticF}\left(\sqrt{-\frac{d(fx+e)}{cf-de}},\sqrt{-\frac{cf-de}{cf+de}}\right)cdef-\mathrm{EllipticE}\left(\sqrt{-\frac{d(fx+e)}{cf-de}},\sqrt{-\frac{cf-de}{cf+de}}\right)\right.}{f^{2}d(-d^{2}fx^{3}-d^{2}ex^{2}+c^{2}}$
elliptic	$\frac{\sqrt{(fx+e)(-d^{2}x^{2}+c^{2})}\left(\frac{2c\left(\frac{e}{f}-\frac{c}{d}\right)\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}-\frac{c}{d}}}\sqrt{\frac{x-\frac{c}{d}}{-\frac{e}{f}-\frac{c}{d}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{e}{f}+\frac{c}{d}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}-\frac{c}{d}}},\sqrt{\frac{-\frac{e}{f}+\frac{c}{d}}{-\frac{e}{f}-\frac{c}{d}}}\right)}{\sqrt{-d^{2}fx^{3}-d^{2}ex^{2}+c^{2}fx+ec^{2}}}+\frac{2d\left(\frac{e}{f}-\frac{c}{d}\right)\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}-\frac{c}{d}}}\sqrt{\frac{x-\frac{c}{d}}{-\frac{e}{f}-\frac{c}{d}}}\sqrt{\frac{x+\frac{c}{d}}{-\frac{e}{f}+\frac{c}{d}}}}{}\right.}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{-dx+c}}$

input
```
int((d*x+c)^(1/2)/(-d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(2*EllipticF((-d*(f*x+e)/(c*f-d*e))^(1/2),(-(c*f-d*e)/(c*f+d*e))^(1/2))
*c^2*f^2-2*EllipticF((-d*(f*x+e)/(c*f-d*e))^(1/2),(-(c*f-d*e)/(c*f+d*e))^(
1/2))*c*d*e*f-EllipticE((-d*(f*x+e)/(c*f-d*e))^(1/2),(-(c*f-d*e)/(c*f+d*e)
)^(1/2))*c^2*f^2+EllipticE((-d*(f*x+e)/(c*f-d*e))^(1/2),(-(c*f-d*e)/(c*f+d
*e))^(1/2))*d^2*e^2)*(f*(d*x+c)/(c*f-d*e))^(1/2)*(f*(-d*x+c)/(c*f+d*e))^(1
/2)*(-d*(f*x+e)/(c*f-d*e))^(1/2)/f^2/d*(d*x+c)^(1/2)*(-d*x+c)^(1/2)*(f*x+e
)^(1/2)/(-d^2*f*x^3-d^2*e*x^2+c^2*f*x+c^2*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(100) = 200$.

Time = 0.08 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{c+dx}}{\sqrt{c-dx}\sqrt{e+fx}}\,dx = \frac{2\left(3\sqrt{-d^{2}f}\,df\,\mathrm{weierstrassZeta}\left(\frac{4\left(d^{2}e^{2}+3c^{2}f^{2}\right)}{3d^{2}f^{2}},-\frac{8\left(d^{2}e^{3}-9c^{2}ef^{2}\right)}{27d^{2}f^{3}},\mathrm{weierstrassPInverse}\left(\frac{4\left(d^{2}e^{2}+3c^{2}f^{2}\right)}{3d^{2}f^{2}},-\frac{8\left(d^{2}e^{3}\right.}{27}\right.\right.\right.}{3d^{2}f^{2}}$$

input
```
integrate((d*x+c)^(1/2)/(-d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas"
)
```

output
```
2/3*(3*sqrt(-d^2*f)*d*f*weierstrassZeta(4/3*(d^2*e^2 + 3*c^2*f^2)/(d^2*f^2
), -8/27*(d^2*e^3 - 9*c^2*e*f^2)/(d^2*f^3), weierstrassPInverse(4/3*(d^2*e
^2 + 3*c^2*f^2)/(d^2*f^2), -8/27*(d^2*e^3 - 9*c^2*e*f^2)/(d^2*f^3), 1/3*(3
*f*x + e)/f)) + sqrt(-d^2*f)*(d*e - 3*c*f)*weierstrassPInverse(4/3*(d^2*e^
2 + 3*c^2*f^2)/(d^2*f^2), -8/27*(d^2*e^3 - 9*c^2*e*f^2)/(d^2*f^3), 1/3*(3*
f*x + e)/f))/(d^2*f^2)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}}{\sqrt{c-dx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}}{\sqrt{c-dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((d*x+c)**(1/2)/(-d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)/(sqrt(c - d*x)*sqrt(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}}{\sqrt{c-dx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{dx+c}}{\sqrt{-dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(-d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(d*x + c)/(sqrt(-d*x + c)*sqrt(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}}{\sqrt{c-dx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{dx+c}}{\sqrt{-dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(-d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)/(sqrt(-d*x + c)*sqrt(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}}{\sqrt{c-dx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+d\,x}}{\sqrt{e+f\,x}\,\sqrt{c-d\,x}}\,dx$$

input
```
int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(c - d*x)^(1/2)),x)
```

output
```
int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(c - d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}}{\sqrt{c-dx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{fx+e}\,\sqrt{dx+c}\,\sqrt{-dx+c}}{-df\,x^2+cfx-dex+ce}\,dx$$

input
```
int((d*x+c)^(1/2)/(-d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x)*sqrt(c + d*x)*sqrt(c - d*x))/(c*e + c*f*x - d*e*x - d*f
*x**2),x)
```

3.295 $\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 372

$$\begin{aligned}\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^3 \, dx &= -\frac{(Bd-Ae)\left(cd^2+ae^2\right)^3(d+ex)^{1+m}}{e^8(1+m)}\\
&\quad+\frac{\left(cd^2+ae^2\right)^2\left(7Bcd^2-6Acde+aBe^2\right)(d+ex)^{2+m}}{e^8(2+m)}\\
&\quad-\frac{3c\left(cd^2+ae^2\right)\left(7Bcd^3-5Acd^2e+3aBde^2-aAe^3\right)(d+ex)^{3+m}}{e^8(3+m)}\\
&\quad-\frac{c\left(4Acde\left(5cd^2+3ae^2\right)-B\left(35c^2d^4+30acd^2e^2+3a^2e^4\right)\right)(d+ex)^{4+m}}{e^8(4+m)}\\
&\quad-\frac{c^2\left(35Bcd^3-15Acd^2e+15aBde^2-3aAe^3\right)(d+ex)^{5+m}}{e^8(5+m)}\\
&\quad+\frac{3c^2\left(7Bcd^2-2Acde+aBe^2\right)(d+ex)^{6+m}}{e^8(6+m)}\\
&\quad-\frac{c^3(7Bd-Ae)(d+ex)^{7+m}}{e^8(7+m)}+\frac{Bc^3(d+ex)^{8+m}}{e^8(8+m)}\end{aligned}$$

output
```
-(-A*e+B*d)*(a*e^2+c*d^2)^3*(e*x+d)^(1+m)/e^8/(1+m)+(a*e^2+c*d^2)^2*(-6*A*
c*d*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^(2+m)/e^8/(2+m)-3*c*(a*e^2+c*d^2)*(-A*a*e
^3-5*A*c*d^2*e+3*B*a*d*e^2+7*B*c*d^3)*(e*x+d)^(3+m)/e^8/(3+m)-c*(4*A*c*d*e
*(3*a*e^2+5*c*d^2)-B*(3*a^2*e^4+30*a*c*d^2*e^2+35*c^2*d^4))*(e*x+d)^(4+m)/
e^8/(4+m)-c^2*(-3*A*a*e^3-15*A*c*d^2*e+15*B*a*d*e^2+35*B*c*d^3)*(e*x+d)^(5
+m)/e^8/(5+m)+3*c^2*(-2*A*c*d*e+B*a*e^2+7*B*c*d^2)*(e*x+d)^(6+m)/e^8/(6+m)
-c^3*(-A*e+7*B*d)*(e*x+d)^(7+m)/e^8/(7+m)+B*c^3*(e*x+d)^(8+m)/e^8/(8+m)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 773 vs. $2(372) = 744$.

Time = 1.75 (sec) , antiderivative size = 773, normalized size of antiderivative = 2.08

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^3 \, dx$$

$$= \frac{(d+ex)^{1+m} \left(-\left((Bd-Ae)(8+m) \left(e^6(1+m)(2+m)(3+m)(4+m)(5+m)(6+m) \left(a+cx^2\right)^3 + 6(\right.\right.\right.}{}$$

input
```
Integrate[(A + B*x)*(d + e*x)^m*(a + c*x^2)^3,x]
```

output
```
((d + e*x)^(1 + m)*(-((B*d - A*e)*(8 + m)*(e^6*(1 + m)*(2 + m)*(3 + m)*(4
+ m)*(5 + m)*(6 + m)*(a + c*x^2)^3 + 6*(c*d^2 + a*e^2)*(6 + m)*(e^4*(1 + m
)*(2 + m)*(3 + m)*(4 + m)*(a + c*x^2)^2 + 4*(c*d^2 + a*e^2)*(4 + m)*(a*e^2
*(6 + 5*m + m^2) + c*(2*d^2 - 2*d*e*(1 + m)*x + e^2*(2 + 3*m + m^2)*x^2))
- 4*c*d*(1 + m)*(d + e*x)*(a*e^2*(12 + 7*m + m^2) + c*(2*d^2 - 2*d*e*(2 +
m)*x + e^2*(6 + 5*m + m^2)*x^2))) - 6*c*d*(1 + m)*(d + e*x)*(e^4*(2 + m)*(
3 + m)*(4 + m)*(5 + m)*(a + c*x^2)^2 + 4*(c*d^2 + a*e^2)*(5 + m)*(a*e^2*(1
2 + 7*m + m^2) + c*(2*d^2 - 2*d*e*(2 + m)*x + e^2*(6 + 5*m + m^2)*x^2)) -
4*c*d*(2 + m)*(d + e*x)*(a*e^2*(20 + 9*m + m^2) + c*(2*d^2 - 2*d*e*(3 + m)
*x + e^2*(12 + 7*m + m^2)*x^2))))) + B*(1 + m)*(d + e*x)*(e^6*(2 + m)*(3 +
 m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)*(a + c*x^2)^3 + 6*(c*d^2 + a*e^2)*(7 +
 m)*(e^4*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(a + c*x^2)^2 + 4*(c*d^2 + a*e^2)
*(5 + m)*(a*e^2*(12 + 7*m + m^2) + c*(2*d^2 - 2*d*e*(2 + m)*x + e^2*(6 + 5
*m + m^2)*x^2)) - 4*c*d*(2 + m)*(d + e*x)*(a*e^2*(20 + 9*m + m^2) + c*(2*d
^2 - 2*d*e*(3 + m)*x + e^2*(12 + 7*m + m^2)*x^2))) - 6*c*d*(2 + m)*(d + e*
x)*(e^4*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(a + c*x^2)^2 + 4*(c*d^2 + a*e^2)*
(6 + m)*(a*e^2*(20 + 9*m + m^2) + c*(2*d^2 - 2*d*e*(3 + m)*x + e^2*(12 + 7
*m + m^2)*x^2)) - 4*c*d*(3 + m)*(d + e*x)*(a*e^2*(30 + 11*m + m^2) + c*(2*
d^2 - 2*d*e*(4 + m)*x + e^2*(20 + 9*m + m^2)*x^2))))))/(e^8*(1 + m)*(2 + m
)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)*(8 + m))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3 (A+Bx)(d+ex)^m \, dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(-\frac{c(d+ex)^{m+3} \left(-3a^2Be^4+12aAcde^3-30aBcd^2e^2+20Ac^2d^3e-35Bc^2d^4\right)}{e^7}-\frac{3c^2(d+ex)^{m+5} \left(-aBe^2+\right.}{e^7}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c(d+ex)^{m+4}\left(4Acde(3ae^2+5cd^2)-B(3a^2e^4+30acd^2e^2+35c^2d^4)\right)}{e^8(m+4)}+\frac{3c^2(d+ex)^{m+6}\left(aBe^2-2Acde+7Bcd^2\right)}{e^8(m+6)}-\frac{c^2(d+ex)^{m+5}\left(-3aAe^3+15aBde^2-15Acd^2e+35Bcd^3\right)}{e^8(m+5)}-\frac{\left(ae^2+cd^2\right)^3(Bd-Ae)(d+ex)^{m+1}}{e^8(m+1)}+\frac{\left(ae^2+cd^2\right)^2(d+ex)^{m+2}\left(aBe^2-6Acde+7Bcd^2\right)}{e^8(m+2)}-\frac{3c\left(ae^2+cd^2\right)(d+ex)^{m+3}\left(-aAe^3+3aBde^2-5Acd^2e+7Bcd^3\right)}{e^8(m+3)}-\frac{c^3(7Bd-Ae)(d+ex)^{m+7}}{e^8(m+7)}+\frac{Bc^3(d+ex)^{m+8}}{e^8(m+8)}$$

input
```
Int[(A + B*x)*(d + e*x)^m*(a + c*x^2)^3,x]
```

output
```
-(((B*d - A*e)*(c*d^2 + a*e^2)^3*(d + e*x)^(1 + m))/(e^8*(1 + m))) + ((c*d
^2 + a*e^2)^2*(7*B*c*d^2 - 6*A*c*d*e + a*B*e^2)*(d + e*x)^(2 + m))/(e^8*(2
 + m)) - (3*c*(c*d^2 + a*e^2)*(7*B*c*d^3 - 5*A*c*d^2*e + 3*a*B*d*e^2 - a*A
*e^3)*(d + e*x)^(3 + m))/(e^8*(3 + m)) - (c*(4*A*c*d*e*(5*c*d^2 + 3*a*e^2)
 - B*(35*c^2*d^4 + 30*a*c*d^2*e^2 + 3*a^2*e^4))*(d + e*x)^(4 + m))/(e^8*(4
 + m)) - (c^2*(35*B*c*d^3 - 15*A*c*d^2*e + 15*a*B*d*e^2 - 3*a*A*e^3)*(d +
e*x)^(5 + m))/(e^8*(5 + m)) + (3*c^2*(7*B*c*d^2 - 2*A*c*d*e + a*B*e^2)*(d
+ e*x)^(6 + m))/(e^8*(6 + m)) - (c^3*(7*B*d - A*e)*(d + e*x)^(7 + m))/(e^8
*(7 + m)) + (B*c^3*(d + e*x)^(8 + m))/(e^8*(8 + m))
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2311 vs. $2(372) = 744$.

Time = 0.86 (sec) , antiderivative size = 2312, normalized size of antiderivative = 6.22

method	result	size
norman	Expression too large to display	2312
gosper	Expression too large to display	3176
orering	Expression too large to display	3179
risch	Expression too large to display	3864
parallelrisch	Expression too large to display	5537

input
```
int((B*x+A)*(e*x+d)^m*(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
B*c^3/(8+m)*x^8*exp(m*ln(e*x+d))+d*(A*a^3*e^7*m^7+35*A*a^3*e^7*m^6-B*a^3*d
*e^6*m^6+511*A*a^3*e^7*m^5+6*A*a^2*c*d^2*e^5*m^5-33*B*a^3*d*e^6*m^5+4025*A
*a^3*e^7*m^4+180*A*a^2*c*d^2*e^5*m^4-445*B*a^3*d*e^6*m^4-18*B*a^2*c*d^3*e^
4*m^4+18424*A*a^3*e^7*m^3+2130*A*a^2*c*d^2*e^5*m^3+72*A*a*c^2*d^4*e^3*m^3-
3135*B*a^3*d*e^6*m^3-468*B*a^2*c*d^3*e^4*m^3+48860*A*a^3*e^7*m^2+12420*A*a
^2*c*d^2*e^5*m^2+1512*A*a*c^2*d^4*e^3*m^2-12154*B*a^3*d*e^6*m^2-4518*B*a^2
*c*d^3*e^4*m^2-360*B*a*c^2*d^5*e^2*m^2+69264*A*a^3*e^7*m+35664*A*a^2*c*d^2
*e^5*m+10512*A*a*c^2*d^4*e^3*m+720*A*c^3*d^6*e*m-24552*B*a^3*d*e^6*m-19188
*B*a^2*c*d^3*e^4*m-5400*B*a*c^2*d^5*e^2*m+40320*A*a^3*e^7+40320*A*a^2*c*d^
2*e^5+24192*A*a*c^2*d^4*e^3+5760*A*c^3*d^6*e-20160*B*a^3*d*e^6-30240*B*a^2
*c*d^3*e^4-20160*B*a*c^2*d^5*e^2-5040*B*c^3*d^7)/e^8/(m^8+36*m^7+546*m^6+4
536*m^5+22449*m^4+67284*m^3+118124*m^2+109584*m+40320)*exp(m*ln(e*x+d))+(3
*A*a^2*c*d*e^5*m^6+B*a^3*e^6*m^6+90*A*a^2*c*d*e^5*m^5+33*B*a^3*e^6*m^5-9*B
*a^2*c*d^2*e^4*m^5+1065*A*a^2*c*d*e^5*m^4+36*A*a*c^2*d^3*e^3*m^4+445*B*a^3
*e^6*m^4-234*B*a^2*c*d^2*e^4*m^4+6210*A*a^2*c*d*e^5*m^3+756*A*a*c^2*d^3*e^
3*m^3+3135*B*a^3*e^6*m^3-2259*B*a^2*c*d^2*e^4*m^3-180*B*a*c^2*d^4*e^2*m^3+
17832*A*a^2*c*d*e^5*m^2+5256*A*a*c^2*d^3*e^3*m^2+360*A*c^3*d^5*e*m^2+12154
*B*a^3*e^6*m^2-9594*B*a^2*c*d^2*e^4*m^2-2700*B*a*c^2*d^4*e^2*m^2+20160*A*a
^2*c*d*e^5*m+12096*A*a*c^2*d^3*e^3*m+2880*A*c^3*d^5*e*m+24552*B*a^3*e^6*m-
15120*B*a^2*c*d^2*e^4*m-10080*B*a*c^2*d^4*e^2*m-2520*B*c^3*d^6*m+20160*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3116 vs. $2(372) = 744$.

Time = 0.18 (sec) , antiderivative size = 3116, normalized size of antiderivative = 8.38

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^m*(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46038 vs. $2(366) = 732$.

Time = 11.93 (sec) , antiderivative size = 46038, normalized size of antiderivative = 123.76

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)**m*(c*x**2+a)**3,x)
```

output
```
Piecewise((d**m*(A*a**3*x + A*a**2*c*x**3 + 3*A*a*c**2*x**5/5 + A*c**3*x**
7/7 + B*a**3*x**2/2 + 3*B*a**2*c*x**4/4 + B*a*c**2*x**6/2 + B*c**3*x**8/8)
, Eq(e, 0)), (-60*A*a**3*e**7/(420*d**7*e**8 + 2940*d**6*e**9*x + 8820*d**
5*e**10*x**2 + 14700*d**4*e**11*x**3 + 14700*d**3*e**12*x**4 + 8820*d**2*e
**13*x**5 + 2940*d*e**14*x**6 + 420*e**15*x**7) - 12*A*a**2*c*d**2*e**5/(4
20*d**7*e**8 + 2940*d**6*e**9*x + 8820*d**5*e**10*x**2 + 14700*d**4*e**11*
x**3 + 14700*d**3*e**12*x**4 + 8820*d**2*e**13*x**5 + 2940*d*e**14*x**6 +
420*e**15*x**7) - 84*A*a**2*c*d*e**6*x/(420*d**7*e**8 + 2940*d**6*e**9*x +
 8820*d**5*e**10*x**2 + 14700*d**4*e**11*x**3 + 14700*d**3*e**12*x**4 + 88
20*d**2*e**13*x**5 + 2940*d*e**14*x**6 + 420*e**15*x**7) - 252*A*a**2*c*e*
*7*x**2/(420*d**7*e**8 + 2940*d**6*e**9*x + 8820*d**5*e**10*x**2 + 14700*d
**4*e**11*x**3 + 14700*d**3*e**12*x**4 + 8820*d**2*e**13*x**5 + 2940*d*e**
14*x**6 + 420*e**15*x**7) - 12*A*a*c**2*d**4*e**3/(420*d**7*e**8 + 2940*d*
*6*e**9*x + 8820*d**5*e**10*x**2 + 14700*d**4*e**11*x**3 + 14700*d**3*e**1
2*x**4 + 8820*d**2*e**13*x**5 + 2940*d*e**14*x**6 + 420*e**15*x**7) - 84*A
*a*c**2*d**3*e**4*x/(420*d**7*e**8 + 2940*d**6*e**9*x + 8820*d**5*e**10*x*
*2 + 14700*d**4*e**11*x**3 + 14700*d**3*e**12*x**4 + 8820*d**2*e**13*x**5
+ 2940*d*e**14*x**6 + 420*e**15*x**7) - 252*A*a*c**2*d**2*e**5*x**2/(420*d
**7*e**8 + 2940*d**6*e**9*x + 8820*d**5*e**10*x**2 + 14700*d**4*e**11*x**3
 + 14700*d**3*e**12*x**4 + 8820*d**2*e**13*x**5 + 2940*d*e**14*x**6 + 4...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1104 vs. $2(372) = 744$.

Time = 0.09 (sec) , antiderivative size = 1104, normalized size of antiderivative = 2.97

$$\int (A + Bx)(d + ex)^m \left(a + cx^2\right)^3 dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^m*(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*B*a^3/((m^2 + 3*m + 2)*e^2)
+ (e*x + d)^(m + 1)*A*a^3/(e*(m + 1)) + 3*((m^2 + 3*m + 2)*e^3*x^3 + (m^2
+ m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*A*a^2*c/((m^3 + 6*m^2 +
11*m + 6)*e^3) + 3*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)
*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*B*
a^2*c/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + 3*((m^4 + 10*m^3 + 35*m^
2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 +
 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d
^5)*(e*x + d)^m*A*a*c^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e
^5) + 3*((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 +
10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m
)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*
x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x + d)^m*B*a*c^2/((m^6 + 21*m^5 + 175*m^
4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6) + ((m^6 + 21*m^5 + 175*m^4 + 7
35*m^3 + 1624*m^2 + 1764*m + 720)*e^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m
^3 + 274*m^2 + 120*m)*d*e^6*x^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m
)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3
*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2 + m)*d^5*e^2*x^2 - 720*d^6*e*m*x + 720*
d^7)*(e*x + d)^m*A*c^3/((m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13
132*m^2 + 13068*m + 5040)*e^7) + ((m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5562 vs. $2(372) = 744$.

Time = 0.15 (sec) , antiderivative size = 5562, normalized size of antiderivative = 14.95

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^m*(c*x^2+a)^3,x, algorithm="giac")
```

output
```
((e*x + d)^m*B*c^3*e^8*m^7*x^8 + (e*x + d)^m*B*c^3*d*e^7*m^7*x^7 + (e*x +
d)^m*A*c^3*e^8*m^7*x^7 + 28*(e*x + d)^m*B*c^3*e^8*m^6*x^8 + (e*x + d)^m*A*
c^3*d*e^7*m^7*x^6 + 3*(e*x + d)^m*B*a*c^2*e^8*m^7*x^6 + 21*(e*x + d)^m*B*c
^3*d*e^7*m^6*x^7 + 29*(e*x + d)^m*A*c^3*e^8*m^6*x^7 + 322*(e*x + d)^m*B*c^
3*e^8*m^5*x^8 + 3*(e*x + d)^m*B*a*c^2*d*e^7*m^7*x^5 + 3*(e*x + d)^m*A*a*c^
2*e^8*m^7*x^5 - 7*(e*x + d)^m*B*c^3*d^2*e^6*m^6*x^6 + 23*(e*x + d)^m*A*c^3
*d*e^7*m^6*x^6 + 90*(e*x + d)^m*B*a*c^2*e^8*m^6*x^6 + 175*(e*x + d)^m*B*c^
3*d*e^7*m^5*x^7 + 343*(e*x + d)^m*A*c^3*e^8*m^5*x^7 + 1960*(e*x + d)^m*B*c
^3*e^8*m^4*x^8 + 3*(e*x + d)^m*A*a*c^2*d*e^7*m^7*x^4 + 3*(e*x + d)^m*B*a^2
*c*e^8*m^7*x^4 - 6*(e*x + d)^m*A*c^3*d^2*e^6*m^6*x^5 + 75*(e*x + d)^m*B*a*
c^2*d*e^7*m^6*x^5 + 93*(e*x + d)^m*A*a*c^2*e^8*m^6*x^5 - 105*(e*x + d)^m*B
*c^3*d^2*e^6*m^5*x^6 + 205*(e*x + d)^m*A*c^3*d*e^7*m^5*x^6 + 1098*(e*x + d
)^m*B*a*c^2*e^8*m^5*x^6 + 735*(e*x + d)^m*B*c^3*d*e^7*m^4*x^7 + 2135*(e*x
+ d)^m*A*c^3*e^8*m^4*x^7 + 6769*(e*x + d)^m*B*c^3*e^8*m^3*x^8 + 3*(e*x + d
)^m*B*a^2*c*d*e^7*m^7*x^3 + 3*(e*x + d)^m*A*a^2*c*e^8*m^7*x^3 - 15*(e*x +
d)^m*B*a*c^2*d^2*e^6*m^6*x^4 + 81*(e*x + d)^m*A*a*c^2*d*e^7*m^6*x^4 + 96*(
e*x + d)^m*B*a^2*c*e^8*m^6*x^4 + 42*(e*x + d)^m*B*c^3*d^3*e^5*m^5*x^5 - 10
8*(e*x + d)^m*A*c^3*d^2*e^6*m^5*x^5 + 723*(e*x + d)^m*B*a*c^2*d*e^7*m^5*x^
5 + 1173*(e*x + d)^m*A*a*c^2*e^8*m^5*x^5 - 595*(e*x + d)^m*B*c^3*d^2*e^6*m
^4*x^6 + 905*(e*x + d)^m*A*c^3*d*e^7*m^4*x^6 + 7020*(e*x + d)^m*B*a*c^2...
```

Mupad [B] (verification not implemented)

Time = 6.93 (sec) , antiderivative size = 2585, normalized size of antiderivative = 6.95

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
int((a + c*x^2)^3*(A + B*x)*(d + e*x)^m,x)
```

output
```
((d + e*x)^m*(40320*A*a^3*d*e^7 - 5040*B*c^3*d^8 + 5760*A*c^3*d^7*e - 2016
0*B*a^3*d^2*e^6 + 24192*A*a*c^2*d^5*e^3 + 40320*A*a^2*c*d^3*e^5 - 20160*B*
a*c^2*d^6*e^2 - 30240*B*a^2*c*d^4*e^4 + 48860*A*a^3*d*e^7*m^2 + 18424*A*a^
3*d*e^7*m^3 + 4025*A*a^3*d*e^7*m^4 + 511*A*a^3*d*e^7*m^5 + 35*A*a^3*d*e^7*
m^6 + A*a^3*d*e^7*m^7 - 24552*B*a^3*d^2*e^6*m - 12154*B*a^3*d^2*e^6*m^2 -
3135*B*a^3*d^2*e^6*m^3 - 445*B*a^3*d^2*e^6*m^4 - 33*B*a^3*d^2*e^6*m^5 - B*
a^3*d^2*e^6*m^6 + 69264*A*a^3*d*e^7*m + 720*A*c^3*d^7*e*m + 1512*A*a*c^2*d
^5*e^3*m^2 + 12420*A*a^2*c*d^3*e^5*m^2 + 72*A*a*c^2*d^5*e^3*m^3 + 2130*A*a
^2*c*d^3*e^5*m^3 + 180*A*a^2*c*d^3*e^5*m^4 + 6*A*a^2*c*d^3*e^5*m^5 - 360*B
*a*c^2*d^6*e^2*m^2 - 4518*B*a^2*c*d^4*e^4*m^2 - 468*B*a^2*c*d^4*e^4*m^3 -
18*B*a^2*c*d^4*e^4*m^4 + 10512*A*a*c^2*d^5*e^3*m + 35664*A*a^2*c*d^3*e^5*m
 - 5400*B*a*c^2*d^6*e^2*m - 19188*B*a^2*c*d^4*e^4*m))/(e^8*(109584*m + 118
124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 4032
0)) + (x*(d + e*x)^m*(40320*A*a^3*e^8 + 69264*A*a^3*e^8*m + 48860*A*a^3*e^
8*m^2 + 18424*A*a^3*e^8*m^3 + 4025*A*a^3*e^8*m^4 + 511*A*a^3*e^8*m^5 + 35*
A*a^3*e^8*m^6 + A*a^3*e^8*m^7 + 24552*B*a^3*d*e^7*m^2 + 12154*B*a^3*d*e^7*
m^3 + 3135*B*a^3*d*e^7*m^4 + 445*B*a^3*d*e^7*m^5 + 33*B*a^3*d*e^7*m^6 + B*
a^3*d*e^7*m^7 - 5760*A*c^3*d^6*e^2*m - 720*A*c^3*d^6*e^2*m^2 + 20160*B*a^3
*d*e^7*m + 5040*B*c^3*d^7*e*m - 10512*A*a*c^2*d^4*e^4*m^2 - 35664*A*a^2*c*
d^2*e^6*m^2 - 1512*A*a*c^2*d^4*e^4*m^3 - 12420*A*a^2*c*d^2*e^6*m^3 - 72...
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 3854, normalized size of antiderivative = 10.36

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^m*(c*x^2+a)^3,x)
```

output

```
((d + e*x)**m*(a**4*d*e**7*m**7 + 35*a**4*d*e**7*m**6 + 511*a**4*d*e**7*m*
*5 + 4025*a**4*d*e**7*m**4 + 18424*a**4*d*e**7*m**3 + 48860*a**4*d*e**7*m*
*2 + 69264*a**4*d*e**7*m + 40320*a**4*d*e**7 + a**4*e**8*m**7*x + 35*a**4*
e**8*m**6*x + 511*a**4*e**8*m**5*x + 4025*a**4*e**8*m**4*x + 18424*a**4*e*
*8*m**3*x + 48860*a**4*e**8*m**2*x + 69264*a**4*e**8*m*x + 40320*a**4*e**8
*x - a**3*b*d**2*e**6*m**6 - 33*a**3*b*d**2*e**6*m**5 - 445*a**3*b*d**2*e*
*6*m**4 - 3135*a**3*b*d**2*e**6*m**3 - 12154*a**3*b*d**2*e**6*m**2 - 24552
*a**3*b*d**2*e**6*m - 20160*a**3*b*d**2*e**6 + a**3*b*d*e**7*m**7*x + 33*a
**3*b*d*e**7*m**6*x + 445*a**3*b*d*e**7*m**5*x + 3135*a**3*b*d*e**7*m**4*x
 + 12154*a**3*b*d*e**7*m**3*x + 24552*a**3*b*d*e**7*m**2*x + 20160*a**3*b*
d*e**7*m*x + a**3*b*e**8*m**7*x**2 + 34*a**3*b*e**8*m**6*x**2 + 478*a**3*b
*e**8*m**5*x**2 + 3580*a**3*b*e**8*m**4*x**2 + 15289*a**3*b*e**8*m**3*x**2
 + 36706*a**3*b*e**8*m**2*x**2 + 44712*a**3*b*e**8*m*x**2 + 20160*a**3*b*e
**8*x**2 + 6*a**3*c*d**3*e**5*m**5 + 180*a**3*c*d**3*e**5*m**4 + 2130*a**3
*c*d**3*e**5*m**3 + 12420*a**3*c*d**3*e**5*m**2 + 35664*a**3*c*d**3*e**5*m
 + 40320*a**3*c*d**3*e**5 - 6*a**3*c*d**2*e**6*m**6*x - 180*a**3*c*d**2*e*
*6*m**5*x - 2130*a**3*c*d**2*e**6*m**4*x - 12420*a**3*c*d**2*e**6*m**3*x -
 35664*a**3*c*d**2*e**6*m**2*x - 40320*a**3*c*d**2*e**6*m*x + 3*a**3*c*d*e
**7*m**7*x**2 + 93*a**3*c*d*e**7*m**6*x**2 + 1155*a**3*c*d*e**7*m**5*x**2
+ 7275*a**3*c*d*e**7*m**4*x**2 + 24042*a**3*c*d*e**7*m**3*x**2 + 37992*...
```

3.296 $\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 234

$$\begin{aligned}
\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^2 \, dx
&= -\frac{(Bd-Ae)\left(cd^2+ae^2\right)^2 (d+ex)^{1+m}}{e^6(1+m)} \\
&\quad + \frac{\left(cd^2+ae^2\right)\left(5Bcd^2-4Acde+aBe^2\right)(d+ex)^{2+m}}{e^6(2+m)} \\
&\quad - \frac{2c\left(5Bcd^3-3Acd^2e+3aBde^2-aAe^3\right)(d+ex)^{3+m}}{e^6(3+m)} \\
&\quad + \frac{2c\left(5Bcd^2-2Acde+aBe^2\right)(d+ex)^{4+m}}{e^6(4+m)} \\
&\quad - \frac{c^2(5Bd-Ae)(d+ex)^{5+m}}{e^6(5+m)} + \frac{Bc^2(d+ex)^{6+m}}{e^6(6+m)}
\end{aligned}$$

output
```
-(-A*e+B*d)*(a*e^2+c*d^2)^2*(e*x+d)^(1+m)/e^6/(1+m)+(a*e^2+c*d^2)*(-4*A*c*
d*e+B*a*e^2+5*B*c*d^2)*(e*x+d)^(2+m)/e^6/(2+m)-2*c*(-A*a*e^3-3*A*c*d^2*e+3
*B*a*d*e^2+5*B*c*d^3)*(e*x+d)^(3+m)/e^6/(3+m)+2*c*(-2*A*c*d*e+B*a*e^2+5*B*
c*d^2)*(e*x+d)^(4+m)/e^6/(4+m)-c^2*(-A*e+5*B*d)*(e*x+d)^(5+m)/e^6/(5+m)+B*
c^2*(e*x+d)^(6+m)/e^6/(6+m)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.52

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^2 dx$$
$$= \frac{(d+ex)^{1+m} \left(-\left((Bd-Ae)(6+m)\left(e^4(1+m)(2+m)(3+m)(4+m)\left(a+cx^2\right)^2+4(cd^2+ae^2)(4+\right.\right.\right.}{}$$

input
```
Integrate[(A + B*x)*(d + e*x)^m*(a + c*x^2)^2,x]
```

output
```
((d + e*x)^(1 + m)*(-((B*d - A*e)*(6 + m)*(e^4*(1 + m)*(2 + m)*(3 + m)*(4
+ m)*(a + c*x^2)^2 + 4*(c*d^2 + a*e^2)*(4 + m)*(a*e^2*(6 + 5*m + m^2) + c*
(2*d^2 - 2*d*e*(1 + m)*x + e^2*(2 + 3*m + m^2)*x^2)) - 4*c*d*(1 + m)*(d +
e*x)*(a*e^2*(12 + 7*m + m^2) + c*(2*d^2 - 2*d*e*(2 + m)*x + e^2*(6 + 5*m +
 m^2)*x^2)))) + B*(1 + m)*(d + e*x)*(e^4*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(
a + c*x^2)^2 + 4*(c*d^2 + a*e^2)*(5 + m)*(a*e^2*(12 + 7*m + m^2) + c*(2*d^
2 - 2*d*e*(2 + m)*x + e^2*(6 + 5*m + m^2)*x^2)) - 4*c*d*(2 + m)*(d + e*x)*
(a*e^2*(20 + 9*m + m^2) + c*(2*d^2 - 2*d*e*(3 + m)*x + e^2*(12 + 7*m + m^2
)*x^2)))))/(e^6*(1 + m)*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2 (A+Bx)(d+ex)^m dx$$
$$\Big\downarrow \text{652}$$
$$\int \left(\frac{\left(ae^2+cd^2\right)^2 (Ae-Bd)(d+ex)^m}{e^5}+\frac{\left(ae^2+cd^2\right)(d+ex)^{m+1}\left(aBe^2-4Acde+5Bcd^2\right)}{e^5}-\frac{2c(d+ex)^{m+3}\left(\right.}{}\right.$$

$$\downarrow \text{2009}$$

$$-\frac{(ae^2+cd^2)^2(Bd-Ae)(d+ex)^{m+1}}{e^6(m+1)}+\frac{(ae^2+cd^2)(d+ex)^{m+2}(aBe^2-4Acde+5Bcd^2)}{e^6(m+2)}+\frac{2c(d+ex)^{m+4}(aBe^2-2Acde+5Bcd^2)}{e^6(m+4)}-\frac{2c(d+ex)^{m+3}(-aAe^3+3aBde^2-3Acd^2e+5Bcd^3)}{e^6(m+3)}-\frac{c^2(5Bd-Ae)(d+ex)^{m+5}}{e^6(m+5)}+\frac{Bc^2(d+ex)^{m+6}}{e^6(m+6)}$$

input
```
Int[(A + B*x)*(d + e*x)^m*(a + c*x^2)^2,x]
```

output
```
-(((B*d - A*e)*(c*d^2 + a*e^2)^2*(d + e*x)^(1 + m))/(e^6*(1 + m))) + ((c*d
^2 + a*e^2)*(5*B*c*d^2 - 4*A*c*d*e + a*B*e^2)*(d + e*x)^(2 + m))/(e^6*(2 +
 m)) - (2*c*(5*B*c*d^3 - 3*A*c*d^2*e + 3*a*B*d*e^2 - a*A*e^3)*(d + e*x)^(3
 + m))/(e^6*(3 + m)) + (2*c*(5*B*c*d^2 - 2*A*c*d*e + a*B*e^2)*(d + e*x)^(4
 + m))/(e^6*(4 + m)) - (c^2*(5*B*d - A*e)*(d + e*x)^(5 + m))/(e^6*(5 + m))
 + (B*c^2*(d + e*x)^(6 + m))/(e^6*(6 + m))
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1083 vs. 2(234) = 468.

Time = 0.96 (sec) , antiderivative size = 1084, normalized size of antiderivative = 4.63

method	result	size
norman	Expression too large to display	1084
gosper	Expression too large to display	1271
orering	Expression too large to display	1274
risch	Expression too large to display	1639
parallelrisch	Expression too large to display	2517

input
```
int((B*x+A)*(e*x+d)^m*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
B*c^2/(6+m)*x^6*exp(m*ln(e*x+d))+d*(A*a^2*e^5*m^5+20*A*a^2*e^5*m^4-B*a^2*d
*e^4*m^4+155*A*a^2*e^5*m^3+4*A*a*c*d^2*e^3*m^3-18*B*a^2*d*e^4*m^3+580*A*a^
2*e^5*m^2+60*A*a*c*d^2*e^3*m^2-119*B*a^2*d*e^4*m^2-12*B*a*c*d^3*e^2*m^2+10
44*A*a^2*e^5*m+296*A*a*c*d^2*e^3*m+24*A*c^2*d^4*e*m-342*B*a^2*d*e^4*m-132*
B*a*c*d^3*e^2*m+720*A*a^2*e^5+480*A*a*c*d^2*e^3+144*A*c^2*d^4*e-360*B*a^2*
d*e^4-360*B*a*c*d^3*e^2-120*B*c^2*d^5)/e^6/(m^6+21*m^5+175*m^4+735*m^3+162
4*m^2+1764*m+720)*exp(m*ln(e*x+d))+(2*A*a*c*d*e^3*m^4+B*a^2*e^4*m^4+30*A*a
*c*d*e^3*m^3+18*B*a^2*e^4*m^3-6*B*a*c*d^2*e^2*m^3+148*A*a*c*d*e^3*m^2+12*A
*c^2*d^3*e*m^2+119*B*a^2*e^4*m^2-66*B*a*c*d^2*e^2*m^2+240*A*a*c*d*e^3*m+72
*A*c^2*d^3*e*m+342*B*a^2*e^4*m-180*B*a*c*d^2*e^2*m-60*B*c^2*d^4*m+360*B*a^
2*e^4)/e^4/(m^5+20*m^4+155*m^3+580*m^2+1044*m+720)*x^2*exp(m*ln(e*x+d))+(A
*a^2*e^5*m^5+B*a^2*d*e^4*m^5+20*A*a^2*e^5*m^4-4*A*a*c*d^2*e^3*m^4+18*B*a^2
*d*e^4*m^4+155*A*a^2*e^5*m^3-60*A*a*c*d^2*e^3*m^3+119*B*a^2*d*e^4*m^3+12*B
*a*c*d^3*e^2*m^3+580*A*a^2*e^5*m^2-296*A*a*c*d^2*e^3*m^2-24*A*c^2*d^4*e*m^
2+342*B*a^2*d*e^4*m^2+132*B*a*c*d^3*e^2*m^2+1044*A*a^2*e^5*m-480*A*a*c*d^2
*e^3*m-144*A*c^2*d^4*e*m+360*B*a^2*d*e^4*m+360*B*a*c*d^3*e^2*m+120*B*c^2*d
^5*m+720*A*a^2*e^5)/e^5/(m^6+21*m^5+175*m^4+735*m^3+1624*m^2+1764*m+720)*x
*exp(m*ln(e*x+d))+c*(A*c*d*e*m^2+2*B*a*e^2*m^2+6*A*c*d*e*m+22*B*a*e^2*m-5*
B*c*d^2*m+60*B*a*e^2)/e^2/(m^3+15*m^2+74*m+120)*x^4*exp(m*ln(e*x+d))+(A*e*
m+B*d*m+6*A*e)*c^2/e/(m^2+11*m+30)*x^5*exp(m*ln(e*x+d))+2*(A*a*e^3*m^3+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. $2(234) = 468$.

Time = 0.11 (sec) , antiderivative size = 1373, normalized size of antiderivative = 5.87

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^2 dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^m*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
(A*a^2*d*e^5*m^5 - 120*B*c^2*d^6 + 144*A*c^2*d^5*e - 360*B*a*c*d^4*e^2 + 4
80*A*a*c*d^3*e^3 - 360*B*a^2*d^2*e^4 + 720*A*a^2*d*e^5 + (B*c^2*e^6*m^5 +
15*B*c^2*e^6*m^4 + 85*B*c^2*e^6*m^3 + 225*B*c^2*e^6*m^2 + 274*B*c^2*e^6*m
+ 120*B*c^2*e^6)*x^6 + (144*A*c^2*e^6 + (B*c^2*d*e^5 + A*c^2*e^6)*m^5 + 2*
(5*B*c^2*d*e^5 + 8*A*c^2*e^6)*m^4 + 5*(7*B*c^2*d*e^5 + 19*A*c^2*e^6)*m^3 +
 10*(5*B*c^2*d*e^5 + 26*A*c^2*e^6)*m^2 + 12*(2*B*c^2*d*e^5 + 27*A*c^2*e^6)
*m)*x^5 - (B*a^2*d^2*e^4 - 20*A*a^2*d*e^5)*m^4 + (360*B*a*c*e^6 + (A*c^2*d
*e^5 + 2*B*a*c*e^6)*m^5 - (5*B*c^2*d^2*e^4 - 12*A*c^2*d*e^5 - 34*B*a*c*e^6
)*m^4 - (30*B*c^2*d^2*e^4 - 47*A*c^2*d*e^5 - 214*B*a*c*e^6)*m^3 - (55*B*c^
2*d^2*e^4 - 72*A*c^2*d*e^5 - 614*B*a*c*e^6)*m^2 - 6*(5*B*c^2*d^2*e^4 - 6*A
*c^2*d*e^5 - 132*B*a*c*e^6)*m)*x^4 + (4*A*a*c*d^3*e^3 - 18*B*a^2*d^2*e^4 +
 155*A*a^2*d*e^5)*m^3 + 2*(240*A*a*c*e^6 + (B*a*c*d*e^5 + A*a*c*e^6)*m^5 -
 2*(A*c^2*d^2*e^4 - 7*B*a*c*d*e^5 - 9*A*a*c*e^6)*m^4 + (10*B*c^2*d^3*e^3 -
 18*A*c^2*d^2*e^4 + 65*B*a*c*d*e^5 + 121*A*a*c*e^6)*m^3 + 2*(15*B*c^2*d^3*
e^3 - 20*A*c^2*d^2*e^4 + 56*B*a*c*d*e^5 + 186*A*a*c*e^6)*m^2 + 4*(5*B*c^2*
d^3*e^3 - 6*A*c^2*d^2*e^4 + 15*B*a*c*d*e^5 + 127*A*a*c*e^6)*m)*x^3 - (12*B
*a*c*d^4*e^2 - 60*A*a*c*d^3*e^3 + 119*B*a^2*d^2*e^4 - 580*A*a^2*d*e^5)*m^2
 + (360*B*a^2*e^6 + (2*A*a*c*d*e^5 + B*a^2*e^6)*m^5 - (6*B*a*c*d^2*e^4 - 3
2*A*a*c*d*e^5 - 19*B*a^2*e^6)*m^4 + (12*A*c^2*d^3*e^3 - 72*B*a*c*d^2*e^4 +
 178*A*a*c*d*e^5 + 137*B*a^2*e^6)*m^3 - (60*B*c^2*d^4*e^2 - 84*A*c^2*d^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16458 vs. $2(226) = 452$.

Time = 4.08 (sec) , antiderivative size = 16458, normalized size of antiderivative = 70.33

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)**m*(c*x**2+a)**2,x)
```

output
```
Piecewise((d**m*(A*a**2*x + 2*A*a*c*x**3/3 + A*c**2*x**5/5 + B*a**2*x**2/2
 + B*a*c*x**4/2 + B*c**2*x**6/6), Eq(e, 0)), (-12*A*a**2*e**5/(60*d**5*e**
6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**1
0*x**4 + 60*e**11*x**5) - 4*A*a*c*d**2*e**3/(60*d**5*e**6 + 300*d**4*e**7*
x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*
x**5) - 20*A*a*c*d*e**4*x/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*
x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 40*A*a*c*e
**5*x**2/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e
**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 12*A*c**2*d**4*e/(60*d**5*e
**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e*
*10*x**4 + 60*e**11*x**5) - 60*A*c**2*d**3*e**2*x/(60*d**5*e**6 + 300*d**4
*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*
e**11*x**5) - 120*A*c**2*d**2*e**3*x**2/(60*d**5*e**6 + 300*d**4*e**7*x +
600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5
) - 120*A*c**2*d*e**4*x**3/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8
*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 60*A*c**2
*e**5*x**4/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2
*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 3*B*a**2*d*e**4/(60*d**5*
e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e
**10*x**4 + 60*e**11*x**5) - 15*B*a**2*e**5*x/(60*d**5*e**6 + 300*d**4*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(234) = 468$.

Time = 0.08 (sec) , antiderivative size = 575, normalized size of antiderivative = 2.46

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^2 dx$$
$$= \frac{(e^2(m+1)x^2+demx-d^2)(ex+d)^m Ba^2}{(m^2+3m+2)e^2} + \frac{(ex+d)^{m+1}Aa^2}{e(m+1)}$$
$$+ \frac{2((m^2+3m+2)e^3x^3+(m^2+m)de^2x^2-2d^2emx+2d^3)(ex+d)^m Aac}{(m^3+6m^2+11m+6)e^3}$$
$$+ \frac{2((m^3+6m^2+11m+6)e^4x^4+(m^3+3m^2+2m)de^3x^3-3(m^2+m)d^2e^2x^2+6d^3emx-6d^4)(ex}{(m^4+10m^3+35m^2+50m+24)e^4}$$
$$+ \frac{((m^4+10m^3+35m^2+50m+24)e^5x^5+(m^4+6m^3+11m^2+6m)de^4x^4-4(m^3+3m^2+2m)d}{(m^5+15m^4+85m^3+225m^2+274m+1}$$
$$+ \frac{((m^5+15m^4+85m^3+225m^2+274m+120)e^6x^6+(m^5+10m^4+35m^3+50m^2+24m)de^5x^5}{(m^6+21m^5+17}$$

input
```
integrate((B*x+A)*(e*x+d)^m*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*B*a^2/((m^2 + 3*m + 2)*e^2)
+ (e*x + d)^(m + 1)*A*a^2/(e*(m + 1)) + 2*((m^2 + 3*m + 2)*e^3*x^3 + (m^2
+ m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*A*a*c/((m^3 + 6*m^2 + 11
*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d
*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*B*a*
c/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^4 + 10*m^3 + 35*m^2 + 50
*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2
 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e
*x + d)^m*A*c^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + ((
m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 3
5*m^3 + 50*m^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*
x^4 + 20*(m^3 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*
d^5*e*m*x - 120*d^6)*(e*x + d)^m*B*c^2/((m^6 + 21*m^5 + 175*m^4 + 735*m^3
+ 1624*m^2 + 1764*m + 720)*e^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2435 vs. $2(234) = 468$.

Time = 0.13 (sec) , antiderivative size = 2435, normalized size of antiderivative = 10.41

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^m*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
((e*x + d)^m*B*c^2*e^6*m^5*x^6 + (e*x + d)^m*B*c^2*d*e^5*m^5*x^5 + (e*x +
d)^m*A*c^2*e^6*m^5*x^5 + 15*(e*x + d)^m*B*c^2*e^6*m^4*x^6 + (e*x + d)^m*A*
c^2*d*e^5*m^5*x^4 + 2*(e*x + d)^m*B*a*c*e^6*m^5*x^4 + 10*(e*x + d)^m*B*c^2
*d*e^5*m^4*x^5 + 16*(e*x + d)^m*A*c^2*e^6*m^4*x^5 + 85*(e*x + d)^m*B*c^2*e
^6*m^3*x^6 + 2*(e*x + d)^m*B*a*c*d*e^5*m^5*x^3 + 2*(e*x + d)^m*A*a*c*e^6*m
^5*x^3 - 5*(e*x + d)^m*B*c^2*d^2*e^4*m^4*x^4 + 12*(e*x + d)^m*A*c^2*d*e^5*
m^4*x^4 + 34*(e*x + d)^m*B*a*c*e^6*m^4*x^4 + 35*(e*x + d)^m*B*c^2*d*e^5*m^
3*x^5 + 95*(e*x + d)^m*A*c^2*e^6*m^3*x^5 + 225*(e*x + d)^m*B*c^2*e^6*m^2*x
^6 + 2*(e*x + d)^m*A*a*c*d*e^5*m^5*x^2 + (e*x + d)^m*B*a^2*e^6*m^5*x^2 - 4
*(e*x + d)^m*A*c^2*d^2*e^4*m^4*x^3 + 28*(e*x + d)^m*B*a*c*d*e^5*m^4*x^3 +
36*(e*x + d)^m*A*a*c*e^6*m^4*x^3 - 30*(e*x + d)^m*B*c^2*d^2*e^4*m^3*x^4 +
47*(e*x + d)^m*A*c^2*d*e^5*m^3*x^4 + 214*(e*x + d)^m*B*a*c*e^6*m^3*x^4 + 5
0*(e*x + d)^m*B*c^2*d*e^5*m^2*x^5 + 260*(e*x + d)^m*A*c^2*e^6*m^2*x^5 + 27
4*(e*x + d)^m*B*c^2*e^6*m*x^6 + (e*x + d)^m*B*a^2*d*e^5*m^5*x + (e*x + d)^
m*A*a^2*e^6*m^5*x - 6*(e*x + d)^m*B*a*c*d^2*e^4*m^4*x^2 + 32*(e*x + d)^m*A
*a*c*d*e^5*m^4*x^2 + 19*(e*x + d)^m*B*a^2*e^6*m^4*x^2 + 20*(e*x + d)^m*B*c
^2*d^3*e^3*m^3*x^3 - 36*(e*x + d)^m*A*c^2*d^2*e^4*m^3*x^3 + 130*(e*x + d)^
m*B*a*c*d*e^5*m^3*x^3 + 242*(e*x + d)^m*A*a*c*e^6*m^3*x^3 - 55*(e*x + d)^m
*B*c^2*d^2*e^4*m^2*x^4 + 72*(e*x + d)^m*A*c^2*d*e^5*m^2*x^4 + 614*(e*x + d
)^m*B*a*c*e^6*m^2*x^4 + 24*(e*x + d)^m*B*c^2*d*e^5*m*x^5 + 324*(e*x + d...
```

Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 1229, normalized size of antiderivative = 5.25

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
int((a + c*x^2)^2*(A + B*x)*(d + e*x)^m,x)
```

output
```
((d + e*x)^m*(720*A*a^2*d*e^5 - 120*B*c^2*d^6 + 144*A*c^2*d^5*e - 360*B*a^
2*d^2*e^4 + 580*A*a^2*d*e^5*m^2 + 155*A*a^2*d*e^5*m^3 + 20*A*a^2*d*e^5*m^4
 + A*a^2*d*e^5*m^5 - 342*B*a^2*d^2*e^4*m - 119*B*a^2*d^2*e^4*m^2 - 18*B*a^
2*d^2*e^4*m^3 - B*a^2*d^2*e^4*m^4 + 480*A*a*c*d^3*e^3 - 360*B*a*c*d^4*e^2
+ 1044*A*a^2*d*e^5*m + 24*A*c^2*d^5*e*m + 296*A*a*c*d^3*e^3*m - 132*B*a*c*
d^4*e^2*m + 60*A*a*c*d^3*e^3*m^2 + 4*A*a*c*d^3*e^3*m^3 - 12*B*a*c*d^4*e^2*
m^2))/(e^6*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) +
 (x*(d + e*x)^m*(720*A*a^2*e^6 + 1044*A*a^2*e^6*m + 580*A*a^2*e^6*m^2 + 15
5*A*a^2*e^6*m^3 + 20*A*a^2*e^6*m^4 + A*a^2*e^6*m^5 + 342*B*a^2*d*e^5*m^2 +
 119*B*a^2*d*e^5*m^3 + 18*B*a^2*d*e^5*m^4 + B*a^2*d*e^5*m^5 - 144*A*c^2*d^
4*e^2*m - 24*A*c^2*d^4*e^2*m^2 + 360*B*a^2*d*e^5*m + 120*B*c^2*d^5*e*m - 4
80*A*a*c*d^2*e^4*m + 360*B*a*c*d^3*e^3*m - 296*A*a*c*d^2*e^4*m^2 - 60*A*a*
c*d^2*e^4*m^3 - 4*A*a*c*d^2*e^4*m^4 + 132*B*a*c*d^3*e^3*m^2 + 12*B*a*c*d^3
*e^3*m^3))/(e^6*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 72
0)) + (x^2*(m + 1)*(d + e*x)^m*(360*B*a^2*e^4 + 342*B*a^2*e^4*m - 60*B*c^2
*d^4*m + 119*B*a^2*e^4*m^2 + 18*B*a^2*e^4*m^3 + B*a^2*e^4*m^4 + 12*A*c^2*d
^3*e*m^2 + 72*A*c^2*d^3*e*m + 148*A*a*c*d*e^3*m^2 + 30*A*a*c*d*e^3*m^3 + 2
*A*a*c*d*e^3*m^4 - 180*B*a*c*d^2*e^2*m - 66*B*a*c*d^2*e^2*m^2 - 6*B*a*c*d^
2*e^2*m^3 + 240*A*a*c*d*e^3*m))/(e^4*(1764*m + 1624*m^2 + 735*m^3 + 175*m^
4 + 21*m^5 + m^6 + 720)) + (B*c^2*x^6*(d + e*x)^m*(274*m + 225*m^2 + 85...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1645, normalized size of antiderivative = 7.03

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^m*(c*x^2+a)^2,x)
```

output
```
((d + e*x)**m*(a**3*d*e**5*m**5 + 20*a**3*d*e**5*m**4 + 155*a**3*d*e**5*m*
*3 + 580*a**3*d*e**5*m**2 + 1044*a**3*d*e**5*m + 720*a**3*d*e**5 + a**3*e*
*6*m**5*x + 20*a**3*e**6*m**4*x + 155*a**3*e**6*m**3*x + 580*a**3*e**6*m**
2*x + 1044*a**3*e**6*m*x + 720*a**3*e**6*x - a**2*b*d**2*e**4*m**4 - 18*a*
*2*b*d**2*e**4*m**3 - 119*a**2*b*d**2*e**4*m**2 - 342*a**2*b*d**2*e**4*m -
 360*a**2*b*d**2*e**4 + a**2*b*d*e**5*m**5*x + 18*a**2*b*d*e**5*m**4*x + 1
19*a**2*b*d*e**5*m**3*x + 342*a**2*b*d*e**5*m**2*x + 360*a**2*b*d*e**5*m*x
 + a**2*b*e**6*m**5*x**2 + 19*a**2*b*e**6*m**4*x**2 + 137*a**2*b*e**6*m**3
*x**2 + 461*a**2*b*e**6*m**2*x**2 + 702*a**2*b*e**6*m*x**2 + 360*a**2*b*e*
*6*x**2 + 4*a**2*c*d**3*e**3*m**3 + 60*a**2*c*d**3*e**3*m**2 + 296*a**2*c*
d**3*e**3*m + 480*a**2*c*d**3*e**3 - 4*a**2*c*d**2*e**4*m**4*x - 60*a**2*c
*d**2*e**4*m**3*x - 296*a**2*c*d**2*e**4*m**2*x - 480*a**2*c*d**2*e**4*m*x
 + 2*a**2*c*d*e**5*m**5*x**2 + 32*a**2*c*d*e**5*m**4*x**2 + 178*a**2*c*d*e
**5*m**3*x**2 + 388*a**2*c*d*e**5*m**2*x**2 + 240*a**2*c*d*e**5*m*x**2 + 2
*a**2*c*e**6*m**5*x**3 + 36*a**2*c*e**6*m**4*x**3 + 242*a**2*c*e**6*m**3*x
**3 + 744*a**2*c*e**6*m**2*x**3 + 1016*a**2*c*e**6*m*x**3 + 480*a**2*c*e**
6*x**3 - 12*a*b*c*d**4*e**2*m**2 - 132*a*b*c*d**4*e**2*m - 360*a*b*c*d**4*
e**2 + 12*a*b*c*d**3*e**3*m**3*x + 132*a*b*c*d**3*e**3*m**2*x + 360*a*b*c*
d**3*e**3*m*x - 6*a*b*c*d**2*e**4*m**4*x**2 - 72*a*b*c*d**2*e**4*m**3*x**2
 - 246*a*b*c*d**2*e**4*m**2*x**2 - 180*a*b*c*d**2*e**4*m*x**2 + 2*a*b*c...
```

3.297 $\int (A+Bx)(d+ex)^m \left(a+cx^2\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 126

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right) \, dx = -\frac{(Bd-Ae)\left(cd^2+ae^2\right)(d+ex)^{1+m}}{e^4(1+m)} + \frac{\left(3Bcd^2-2Acde+aBe^2\right)(d+ex)^{2+m}}{e^4(2+m)} - \frac{c(3Bd-Ae)(d+ex)^{3+m}}{e^4(3+m)} + \frac{Bc(d+ex)^{4+m}}{e^4(4+m)}$$

output
```
-(-A*e+B*d)*(a*e^2+c*d^2)*(e*x+d)^(1+m)/e^4/(1+m)+(-2*A*c*d*e+B*a*e^2+3*B*
c*d^2)*(e*x+d)^(2+m)/e^4/(2+m)-c*(-A*e+3*B*d)*(e*x+d)^(3+m)/e^4/(3+m)+B*c*
(e*x+d)^(4+m)/e^4/(4+m)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.97

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right) \, dx = \frac{(d+ex)^{1+m}\left((-Bd+Ae)\left(\frac{cd^2+ae^2}{1+m}-\frac{2cd(d+ex)}{2+m}+\frac{c(d+ex)^2}{3+m}\right)+B(d+ex)\left(\frac{cd^2+ae^2}{2+m}-\frac{2cd(d+ex)}{3+m}+\frac{c(d+ex)^2}{4+m}\right)\right)}{e^4}$$

input
```
Integrate[(A + B*x)*(d + e*x)^m*(a + c*x^2),x]
```

output
```
((d + e*x)^(1 + m)*((-(B*d) + A*e)*((c*d^2 + a*e^2)/(1 + m) - (2*c*d*(d +
e*x))/(2 + m) + (c*(d + e*x)^2)/(3 + m)) + B*(d + e*x)*((c*d^2 + a*e^2)/(2
 + m) - (2*c*d*(d + e*x))/(3 + m) + (c*(d + e*x)^2)/(4 + m))))/e^4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(A+Bx)(d+ex)^m\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{\left(ae^2+cd^2\right)(Ae-Bd)(d+ex)^m}{e^3}+\frac{(d+ex)^{m+1}\left(aBe^2-2Acde+3Bcd^2\right)}{e^3}+\frac{c(Ae-3Bd)(d+ex)^{m+2}}{e^3}+$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(ae^2+cd^2\right)(Bd-Ae)(d+ex)^{m+1}}{e^4(m+1)}+\frac{(d+ex)^{m+2}\left(aBe^2-2Acde+3Bcd^2\right)}{e^4(m+2)}-\frac{c(3Bd-Ae)(d+ex)^{m+3}}{e^4(m+3)}+\frac{Bc(d+ex)^{m+4}}{e^4(m+4)}$$

input
```
Int[(A + B*x)*(d + e*x)^m*(a + c*x^2),x]
```

output
```
-(((B*d - A*e)*(c*d^2 + a*e^2)*(d + e*x)^(1 + m))/(e^4*(1 + m))) + ((3*B*c
*d^2 - 2*A*c*d*e + a*B*e^2)*(d + e*x)^(2 + m))/(e^4*(2 + m)) - (c*(3*B*d -
 A*e)*(d + e*x)^(3 + m))/(e^4*(3 + m)) + (B*c*(d + e*x)^(4 + m))/(e^4*(4 +
 m))
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(126) = 252$.

Time = 0.61 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.68

method	result
gosper	$\frac{(ex+d)^{1+m}\left(Bc\,e^3m^3x^3+Ac\,e^3m^3x^2+6Bc\,e^3m^2x^3+7Ac\,e^3m^2x^2+Ba\,e^3m^3x-3Bcd\,e^2m^2x^2+11Bc\,e^3m\,x^3+Aa\,e^3m^3-2A\right.}{}$
orering	$\frac{(ex+d)^m\left(Bc\,e^3m^3x^3+Ac\,e^3m^3x^2+6Bc\,e^3m^2x^3+7Ac\,e^3m^2x^2+Ba\,e^3m^3x-3Bcd\,e^2m^2x^2+11Bc\,e^3m\,x^3+Aa\,e^3m^3-2Acd\right.}{}$
norman	$\frac{Bc\,x^4\mathrm{e}^{m\ln(ex+d)}}{4+m}+\frac{d\left(Aa\,e^3m^3+9Aa\,e^3m^2-Bad\,e^2m^2+26Aa\,e^3m+2Ac\,d^2em-7Bad\,e^2m+24Aa\,e^3+8Ac\,d^2e-12Bad\,e^2\right.}{e^4(m^4+10m^3+35m^2+50m+24)}$
risch	$\frac{\left(Bc\,e^4m^3x^4+Ac\,e^4m^3x^3+Bcd\,e^3m^3x^3+6Bc\,e^4m^2x^4+Acd\,e^3m^3x^2+7Ac\,e^4m^2x^3+Ba\,e^4m^3x^2+3Bcd\,e^3m^2x^3+11Bc\,e^4m\right.}{}$
parallelrisch	$\frac{12B\,x^2(ex+d)^m a\,e^4+24Ax(ex+d)^m a\,e^4+24A(ex+d)^m ad\,e^3+8A(ex+d)^m c\,d^3e-12B(ex+d)^m a\,d^2e^2+5A\,x^2(ex+d)^m cd\,e^3}{}$

input
```
int((B*x+A)*(e*x+d)^m*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/e^4*(e*x+d)^(1+m)/(m^4+10*m^3+35*m^2+50*m+24)*(B*c*e^3*m^3*x^3+A*c*e^3*m
^3*x^2+6*B*c*e^3*m^2*x^3+7*A*c*e^3*m^2*x^2+B*a*e^3*m^3*x-3*B*c*d*e^2*m^2*x
^2+11*B*c*e^3*m*x^3+A*a*e^3*m^3-2*A*c*d*e^2*m^2*x+14*A*c*e^3*m*x^2+8*B*a*e
^3*m^2*x-9*B*c*d*e^2*m*x^2+6*B*c*e^3*x^3+9*A*a*e^3*m^2-10*A*c*d*e^2*m*x+8*
A*c*e^3*x^2-B*a*d*e^2*m^2+19*B*a*e^3*m*x+6*B*c*d^2*e*m*x-6*B*c*d*e^2*x^2+2
6*A*a*e^3*m+2*A*c*d^2*e*m-8*A*c*d*e^2*x-7*B*a*d*e^2*m+12*B*a*e^3*x+6*B*c*d
^2*e*x+24*A*a*e^3+8*A*c*d^2*e-12*B*a*d*e^2-6*B*c*d^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(126) = 252$.

Time = 0.13 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.44

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right)\,dx = \frac{(Aade^3m^3 - 6\,Bcd^4 + 8\,Acd^3e - 12\,Bad^2e^2 + 24\,Aade^3 + (Bce^4m^3 + 6\,Bce^4m^2 + 11\,Bce^4m + 6\,Bce^4)x}{}$$

input
```
integrate((B*x+A)*(e*x+d)^m*(c*x^2+a),x, algorithm="fricas")
```

output
```
(A*a*d*e^3*m^3 - 6*B*c*d^4 + 8*A*c*d^3*e - 12*B*a*d^2*e^2 + 24*A*a*d*e^3 +
 (B*c*e^4*m^3 + 6*B*c*e^4*m^2 + 11*B*c*e^4*m + 6*B*c*e^4)*x^4 + (8*A*c*e^4
 + (B*c*d*e^3 + A*c*e^4)*m^3 + (3*B*c*d*e^3 + 7*A*c*e^4)*m^2 + 2*(B*c*d*e^
3 + 7*A*c*e^4)*m)*x^3 - (B*a*d^2*e^2 - 9*A*a*d*e^3)*m^2 + (12*B*a*e^4 + (A
*c*d*e^3 + B*a*e^4)*m^3 - (3*B*c*d^2*e^2 - 5*A*c*d*e^3 - 8*B*a*e^4)*m^2 -
(3*B*c*d^2*e^2 - 4*A*c*d*e^3 - 19*B*a*e^4)*m)*x^2 + (2*A*c*d^3*e - 7*B*a*d
^2*e^2 + 26*A*a*d*e^3)*m + (24*A*a*e^4 + (B*a*d*e^3 + A*a*e^4)*m^3 - (2*A*
c*d^2*e^2 - 7*B*a*d*e^3 - 9*A*a*e^4)*m^2 + 2*(3*B*c*d^3*e - 4*A*c*d^2*e^2
+ 6*B*a*d*e^3 + 13*A*a*e^4)*m)*x)*(e*x + d)^m/(e^4*m^4 + 10*e^4*m^3 + 35*e
^4*m^2 + 50*e^4*m + 24*e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3958 vs. $2(114) = 228$.

Time = 1.22 (sec) , antiderivative size = 3958, normalized size of antiderivative = 31.41

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)**m*(c*x**2+a),x)
```

output
```
Piecewise((d**m*(A*a*x + A*c*x**3/3 + B*a*x**2/2 + B*c*x**4/4), Eq(e, 0)),
 (-2*A*a*e**3/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3
) - 2*A*c*d**2*e/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x
**3) - 6*A*c*d*e**2*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e
**7*x**3) - 6*A*c*e**3*x**2/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2
 + 6*e**7*x**3) - B*a*d*e**2/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**
2 + 6*e**7*x**3) - 3*B*a*e**3*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*
x**2 + 6*e**7*x**3) + 6*B*c*d**3*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*
x + 18*d*e**6*x**2 + 6*e**7*x**3) + 11*B*c*d**3/(6*d**3*e**4 + 18*d**2*e**
5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 18*B*c*d**2*e*x*log(d/e + x)/(6*d**3
*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 27*B*c*d**2*e*x/(
6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 18*B*c*d*e*
*2*x**2*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e*
*7*x**3) + 18*B*c*d*e**2*x**2/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x*
*2 + 6*e**7*x**3) + 6*B*c*e**3*x**3*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e*
*5*x + 18*d*e**6*x**2 + 6*e**7*x**3), Eq(m, -4)), (-A*a*e**3/(2*d**2*e**4
+ 4*d*e**5*x + 2*e**6*x**2) + 2*A*c*d**2*e*log(d/e + x)/(2*d**2*e**4 + 4*d
*e**5*x + 2*e**6*x**2) + 3*A*c*d**2*e/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x
**2) + 4*A*c*d*e**2*x*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2
) + 4*A*c*d*e**2*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*A*c*e**...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.89

$$\int (A+Bx)(d+ex)^m\left(a+cx^2\right)\,dx$$
$$= \frac{(e^2(m+1)x^2+demx-d^2)(ex+d)^mBa}{(m^2+3\,m+2)e^2} + \frac{(ex+d)^{m+1}Aa}{e(m+1)} + \frac{((m^2+3\,m+2)e^3x^3+(m^2+m)de^2x^2-2\,d^2emx+2\,d^3)(ex+d)^mAc}{(m^3+6\,m^2+11\,m+6)e^3} + \frac{((m^3+6\,m^2+11\,m+6)e^4x^4+(m^3+3\,m^2+2\,m)de^3x^3-3\,(m^2+m)d^2e^2x^2+6\,d^3emx-6\,d^4)(ex-}{(m^4+10\,m^3+35\,m^2+50\,m+24)e^4}$$

input
```
integrate((B*x+A)*(e*x+d)^m*(c*x^2+a),x, algorithm="maxima")
```

output
```
(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*B*a/((m^2 + 3*m + 2)*e^2) +
(e*x + d)^(m + 1)*A*a/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d
*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*A*c/((m^3 + 6*m^2 + 11*m + 6)*
e^3) + ((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 -
 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*B*c/((m^4 + 10
*m^3 + 35*m^2 + 50*m + 24)*e^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. $2(126) = 252$.

Time = 0.11 (sec) , antiderivative size = 768, normalized size of antiderivative = 6.10

$$\int (A + Bx)(d + ex)^m \left(a + cx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(e*x+d)^m*(c*x^2+a),x, algorithm="giac")
```

output
```
((e*x + d)^m*B*c*e^4*m^3*x^4 + (e*x + d)^m*B*c*d*e^3*m^3*x^3 + (e*x + d)^m
*A*c*e^4*m^3*x^3 + 6*(e*x + d)^m*B*c*e^4*m^2*x^4 + (e*x + d)^m*A*c*d*e^3*m
^3*x^2 + (e*x + d)^m*B*a*e^4*m^3*x^2 + 3*(e*x + d)^m*B*c*d*e^3*m^2*x^3 + 7
*(e*x + d)^m*A*c*e^4*m^2*x^3 + 11*(e*x + d)^m*B*c*e^4*m*x^4 + (e*x + d)^m*
B*a*d*e^3*m^3*x + (e*x + d)^m*A*a*e^4*m^3*x - 3*(e*x + d)^m*B*c*d^2*e^2*m^
2*x^2 + 5*(e*x + d)^m*A*c*d*e^3*m^2*x^2 + 8*(e*x + d)^m*B*a*e^4*m^2*x^2 +
2*(e*x + d)^m*B*c*d*e^3*m*x^3 + 14*(e*x + d)^m*A*c*e^4*m*x^3 + 6*(e*x + d)
^m*B*c*e^4*x^4 + (e*x + d)^m*A*a*d*e^3*m^3 - 2*(e*x + d)^m*A*c*d^2*e^2*m^2
*x + 7*(e*x + d)^m*B*a*d*e^3*m^2*x + 9*(e*x + d)^m*A*a*e^4*m^2*x - 3*(e*x
+ d)^m*B*c*d^2*e^2*m*x^2 + 4*(e*x + d)^m*A*c*d*e^3*m*x^2 + 19*(e*x + d)^m*
B*a*e^4*m*x^2 + 8*(e*x + d)^m*A*c*e^4*x^3 - (e*x + d)^m*B*a*d^2*e^2*m^2 +
9*(e*x + d)^m*A*a*d*e^3*m^2 + 6*(e*x + d)^m*B*c*d^3*e*m*x - 8*(e*x + d)^m*
A*c*d^2*e^2*m*x + 12*(e*x + d)^m*B*a*d*e^3*m*x + 26*(e*x + d)^m*A*a*e^4*m*
x + 12*(e*x + d)^m*B*a*e^4*x^2 + 2*(e*x + d)^m*A*c*d^3*e*m - 7*(e*x + d)^m
*B*a*d^2*e^2*m + 26*(e*x + d)^m*A*a*d*e^3*m + 24*(e*x + d)^m*A*a*e^4*x - 6
*(e*x + d)^m*B*c*d^4 + 8*(e*x + d)^m*A*c*d^3*e - 12*(e*x + d)^m*B*a*d^2*e^
2 + 24*(e*x + d)^m*A*a*d*e^3)/(e^4*m^4 + 10*e^4*m^3 + 35*e^4*m^2 + 50*e^4*
m + 24*e^4)
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 446, normalized size of antiderivative = 3.54

$$\int (A+Bx)(d+ex)^m \left(a+cx^2\right) \, dx$$
$$= \frac{(d+e\,x)^m\,(-6\,B\,c\,d^4+2\,A\,c\,d^3\,e\,m+8\,A\,c\,d^3\,e-B\,a\,d^2\,e^2\,m^2-7\,B\,a\,d^2\,e^2\,m-12\,B\,a\,d^2\,e^2+A\,a\,d\,e^3\,m}{e^4\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}$$
$$+\frac{x\,(d+e\,x)^m\,(6\,B\,c\,d^3\,e\,m-2\,A\,c\,d^2\,e^2\,m^2-8\,A\,c\,d^2\,e^2\,m+B\,a\,d\,e^3\,m^3+7\,B\,a\,d\,e^3\,m^2+12\,B\,a\,d\,e^3\,m}{e^4\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}$$
$$+\frac{x^2\,(m+1)\,\left(d+e\,x\right)^m\,(-3\,B\,c\,d^2\,m+A\,c\,d\,e\,m^2+4\,A\,c\,d\,e\,m+B\,a\,e^2\,m^2+7\,B\,a\,e^2\,m+12\,B\,a\,e^2)}{e^2\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}$$
$$+\frac{B\,c\,x^4\,(d+e\,x)^m\,(m^3+6\,m^2+11\,m+6)}{m^4+10\,m^3+35\,m^2+50\,m+24}$$
$$+\frac{c\,x^3\,(d+e\,x)^m\,(4\,A\,e+A\,e\,m+B\,d\,m)\,\left(m^2+3\,m+2\right)}{e\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}$$

input
```
int((a + c*x^2)*(A + B*x)*(d + e*x)^m,x)
```

output
```
((d + e*x)^m*(24*A*a*d*e^3 - 6*B*c*d^4 + 8*A*c*d^3*e - 12*B*a*d^2*e^2 - B*
a*d^2*e^2*m^2 + 26*A*a*d*e^3*m + 2*A*c*d^3*e*m + 9*A*a*d*e^3*m^2 + A*a*d*e
^3*m^3 - 7*B*a*d^2*e^2*m))/(e^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x*
(d + e*x)^m*(24*A*a*e^4 + 26*A*a*e^4*m + 9*A*a*e^4*m^2 + A*a*e^4*m^3 - 2*A
*c*d^2*e^2*m^2 + 12*B*a*d*e^3*m + 6*B*c*d^3*e*m + 7*B*a*d*e^3*m^2 + B*a*d*
e^3*m^3 - 8*A*c*d^2*e^2*m))/(e^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x
^2*(m + 1)*(d + e*x)^m*(12*B*a*e^2 + 7*B*a*e^2*m - 3*B*c*d^2*m + B*a*e^2*m
^2 + 4*A*c*d*e*m + A*c*d*e*m^2))/(e^2*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
 + (B*c*x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3
+ m^4 + 24) + (c*x^3*(d + e*x)^m*(4*A*e + A*e*m + B*d*m)*(3*m + m^2 + 2))/
(e*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 489, normalized size of antiderivative = 3.88

$$\int (A+Bx)(d+ex)^m\left(a+cx^2\right)\,dx$$
$$=\frac{(ex+d)^m\left(bc\,e^4m^3x^4+ac\,e^4m^3x^3+bcd\,e^3m^3x^3+6bc\,e^4m^2x^4+ab\,e^4m^3x^2+acd\,e^3m^3x^2+7ac\,e^4m^2x^3\right.}{}$$

input
```
int((B*x+A)*(e*x+d)^m*(c*x^2+a),x)
```

output
```
((d + e*x)**m*(a**2*d*e**3*m**3 + 9*a**2*d*e**3*m**2 + 26*a**2*d*e**3*m +
24*a**2*d*e**3 + a**2*e**4*m**3*x + 9*a**2*e**4*m**2*x + 26*a**2*e**4*m*x
+ 24*a**2*e**4*x - a*b*d**2*e**2*m**2 - 7*a*b*d**2*e**2*m - 12*a*b*d**2*e*
*2 + a*b*d*e**3*m**3*x + 7*a*b*d*e**3*m**2*x + 12*a*b*d*e**3*m*x + a*b*e**
4*m**3*x**2 + 8*a*b*e**4*m**2*x**2 + 19*a*b*e**4*m*x**2 + 12*a*b*e**4*x**2
 + 2*a*c*d**3*e*m + 8*a*c*d**3*e - 2*a*c*d**2*e**2*m**2*x - 8*a*c*d**2*e**
2*m*x + a*c*d*e**3*m**3*x**2 + 5*a*c*d*e**3*m**2*x**2 + 4*a*c*d*e**3*m*x**
2 + a*c*e**4*m**3*x**3 + 7*a*c*e**4*m**2*x**3 + 14*a*c*e**4*m*x**3 + 8*a*c
*e**4*x**3 - 6*b*c*d**4 + 6*b*c*d**3*e*m*x - 3*b*c*d**2*e**2*m**2*x**2 - 3
*b*c*d**2*e**2*m*x**2 + b*c*d*e**3*m**3*x**3 + 3*b*c*d*e**3*m**2*x**3 + 2*
b*c*d*e**3*m*x**3 + b*c*e**4*m**3*x**4 + 6*b*c*e**4*m**2*x**4 + 11*b*c*e**
4*m*x**4 + 6*b*c*e**4*x**4))/(e**4*(m**4 + 10*m**3 + 35*m**2 + 50*m + 24))
```

3.298 $\int \frac{(A+Bx)(d+ex)^m}{a+cx^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 212

$$\int \frac{(A+Bx)(d+ex)^m}{a+cx^2} dx = -\frac{\left(\sqrt{-a}B-A\sqrt{c}\right)(d+ex)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}\sqrt{c}\left(\sqrt{c}d-\sqrt{-a}e\right)(1+m)} - \frac{\left(\sqrt{-a}B+A\sqrt{c}\right)(d+ex)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}\sqrt{c}\left(\sqrt{c}d+\sqrt{-a}e\right)(1+m)}$$

output
```
-1/2*((-a)^(1/2)*B-A*c^(1/2))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],c^(1/
2)*(e*x+d)/(c^(1/2)*d-(-a)^(1/2)*e))/(-a)^(1/2)/c^(1/2)/(c^(1/2)*d-(-a)^(1
/2)*e)/(1+m)-1/2*((-a)^(1/2)*B+A*c^(1/2))*(e*x+d)^(1+m)*hypergeom([1, 1+m]
,[2+m],c^(1/2)*(e*x+d)/(c^(1/2)*d+(-a)^(1/2)*e))/(-a)^(1/2)/c^(1/2)/(c^(1/
2)*d+(-a)^(1/2)*e)/(1+m)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)(d+ex)^m}{a+cx^2}\,dx = \frac{(d+ex)^{1+m}\left(\frac{\left(aB+\sqrt{-a}A\sqrt{c}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{-\sqrt{c}d+\sqrt{-a}e}+\frac{\left(-aB+\sqrt{-a}A\sqrt{c}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m\right.}{\sqrt{c}d+\sqrt{-a}e}\right.}{2a\sqrt{c}(1+m)}$$

input
```
Integrate[((A + B*x)*(d + e*x)^m)/(a + c*x^2),x]
```

output
```
((d + e*x)^(1 + m)*(((a*B + Sqrt[-a]*A*Sqrt[c])*Hypergeometric2F1[1, 1 + m
, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(-(Sqrt[c]*d) + Sq
rt[-a]*e) + ((-(a*B) + Sqrt[-a]*A*Sqrt[c])*Hypergeometric2F1[1, 1 + m, 2 +
 m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e
)))/(2*a*Sqrt[c]*(1 + m))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^m}{a+cx^2}\,dx$$

$$\Big\downarrow \text{657}$$

$$\int \left(\frac{\left(\sqrt{-a}A-\frac{aB}{\sqrt{c}}\right)(d+ex)^m}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\left(\sqrt{-a}A+\frac{aB}{\sqrt{c}}\right)(d+ex)^m}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\sqrt{-a}A\sqrt{c}+aB\right)(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a\sqrt{c}(m+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{\left(\frac{\sqrt{-a}B}{\sqrt{c}}+A\right)(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}(m+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

input
```
Int[((A + B*x)*(d + e*x)^m)/(a + c*x^2),x]
```

output
```
-1/2*((a*B + Sqrt[-a]*A*Sqrt[c])*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1
+ m, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(a*Sqrt[c]*(Sqr
t[c]*d - Sqrt[-a]*e)*(1 + m)) - ((A + (Sqrt[-a]*B)/Sqrt[c])*(d + e*x)^(1 +
 m)*Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sq
rt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + m))
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(Bx+A)\,(ex+d)^m}{c\,x^2+a}dx$$

input
```
int((B*x+A)*(e*x+d)^m/(c*x^2+a),x)
```

output
```
int((B*x+A)*(e*x+d)^m/(c*x^2+a),x)
```

Fricas [F]

$$\int \frac{(A+Bx)(d+ex)^m}{a+cx^2}\,dx=\int \frac{(Bx+A)(ex+d)^m}{cx^2+a}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^m/(c*x^2+a),x, algorithm="fricas")
```

output
```
integral((B*x + A)*(e*x + d)^m/(c*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(A+Bx)(d+ex)^m}{a+cx^2}\,dx=\int \frac{(A+Bx)\,(d+ex)^m}{a+cx^2}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)**m/(c*x**2+a),x)
```

output
```
Integral((A + B*x)*(d + e*x)**m/(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^m}{a+cx^2}\,dx=\int \frac{(Bx+A)(ex+d)^m}{cx^2+a}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^m/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x + d)^m/(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx)(d+ex)^m}{a+cx^2}\,dx = \int \frac{(Bx+A)(ex+d)^m}{cx^2+a}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^m/(c*x^2+a),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x + d)^m/(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^m}{a+cx^2}\,dx = \int \frac{(A+B\,x)\,(d+e\,x)^m}{c\,x^2+a}\,dx$$

input
```
int(((A + B*x)*(d + e*x)^m)/(a + c*x^2),x)
```

output
```
int(((A + B*x)*(d + e*x)^m)/(a + c*x^2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)(d+ex)^m}{a+cx^2}\,dx = \frac{(ex+d)^m b - \left(\int \frac{(ex+d)^m}{ce\,x^3+cd\,x^2+aex+ad}dx\right) abem + \left(\int \frac{(ex+d)^m}{ce\,x^3+cd\,x^2+aex+ad}dx\right) acdm + \left(\int \frac{(ex+d)^m x}{ce\,x^3+cd\,x^2+aex+ad}dx\right)}{cm}$$

input
```
int((B*x+A)*(e*x+d)^m/(c*x^2+a),x)
```

output
```
((d + e*x)**m*b - int((d + e*x)**m/(a*d + a*e*x + c*d*x**2 + c*e*x**3),x)*
a*b*e*m + int((d + e*x)**m/(a*d + a*e*x + c*d*x**2 + c*e*x**3),x)*a*c*d*m
+ int(((d + e*x)**m*x)/(a*d + a*e*x + c*d*x**2 + c*e*x**3),x)*a*c*e*m + in
t(((d + e*x)**m*x)/(a*d + a*e*x + c*d*x**2 + c*e*x**3),x)*b*c*d*m)/(c*m)
```

3.299 $\int \frac{(A+Bx)(d+ex)^m}{(a+cx^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 365

$$\int \frac{(A+Bx)(d+ex)^m}{(a+cx^2)^2} dx = -\frac{(d+ex)^{1+m}(a(Bd-Ae)-(Acd+aBe)x)}{2a(cd^2+ae^2)(a+cx^2)}$$
$$-\frac{\left(\sqrt{-a}e(Acd+aBe)m+\sqrt{c}(Acd^2+aAe^2(1-m)+aBdem)\right)(d+ex)^{1+m}\text{Hypergeometric2F1}\left(1,1\right.}{4(-a)^{3/2}\sqrt{c}\left(\sqrt{c}d-\sqrt{-a}e\right)(cd^2+ae^2)(1+m)}$$
$$-\frac{\left(\sqrt{-a}e(Acd+aBe)m-\sqrt{c}(Acd^2+aAe^2(1-m)+aBdem)\right)(d+ex)^{1+m}\text{Hypergeometric2F1}\left(1,1\right.}{4(-a)^{3/2}\sqrt{c}\left(\sqrt{c}d+\sqrt{-a}e\right)(cd^2+ae^2)(1+m)}$$

output
```
-1/2*(e*x+d)^(1+m)*(a*(-A*e+B*d)-(A*c*d+B*a*e)*x)/a/(a*e^2+c*d^2)/(c*x^2+a
)-1/4*((-a)^(1/2)*e*(A*c*d+B*a*e)*m+c^(1/2)*(A*c*d^2+a*A*e^2*(1-m)+a*B*d*e
*m))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],c^(1/2)*(e*x+d)/(c^(1/2)*d-(-a
)^(1/2)*e))/(-a)^(3/2)/c^(1/2)/(c^(1/2)*d-(-a)^(1/2)*e)/(a*e^2+c*d^2)/(1+m
)-1/4*((-a)^(1/2)*e*(A*c*d+B*a*e)*m-c^(1/2)*(A*c*d^2+a*A*e^2*(1-m)+a*B*d*e
*m))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],c^(1/2)*(e*x+d)/(c^(1/2)*d+(-a
)^(1/2)*e))/(-a)^(3/2)/c^(1/2)/(c^(1/2)*d+(-a)^(1/2)*e)/(a*e^2+c*d^2)/(1+m
)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)(d+ex)^m}{(a+cx^2)^2}\,dx$$
$$= \frac{(d+ex)^{1+m}\left(\frac{2c(Acdx+a(-Bd+Ae+Bex))}{a+cx^2}+\frac{\sqrt{c}(ae(Acd+aBe)m-\sqrt{-a}\sqrt{c}(Acd^2-aAe^2(-1+m)+aBdem))\,\text{Hypergeometric2F1}\left(1,\right.}{a(\sqrt{c}d-\sqrt{-a}e)(1+m)}\right.}{4ac\,(cd^{}}$$

input
```
Integrate[((A + B*x)*(d + e*x)^m)/(a + c*x^2)^2,x]
```

output
```
((d + e*x)^(1 + m)*((2*c*(A*c*d*x + a*(-(B*d) + A*e + B*e*x)))/(a + c*x^2)
 + (Sqrt[c]*(a*e*(A*c*d + a*B*e)*m - Sqrt[-a]*Sqrt[c]*(A*c*d^2 - a*A*e^2*(
-1 + m) + a*B*d*e*m))*Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[c]*(d + e*x
))/(Sqrt[c]*d - Sqrt[-a]*e)])/(a*(Sqrt[c]*d - Sqrt[-a]*e)*(1 + m)) + (Sqrt
[c]*(a*e*(A*c*d + a*B*e)*m + Sqrt[-a]*Sqrt[c]*(A*c*d^2 - a*A*e^2*(-1 + m)
+ a*B*d*e*m))*Hypergeometric2F1[1, 1 + m, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt
[c]*d + Sqrt[-a]*e)])/(a*(Sqrt[c]*d + Sqrt[-a]*e)*(1 + m))))/(4*a*c*(c*d^2
 + a*e^2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {686, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^m}{(a+cx^2)^2}\,dx$$
$$\Big\downarrow \; 686$$

$$-\frac{\int -\frac{c(d+ex)^m\left(Acd^2+aBemd+aAe^2(1-m)-e(Acd+aBe)mx\right)}{cx^2+a}dx}{2ac\left(ae^2+cd^2\right)}-\frac{(d+ex)^{m+1}(a(Bd-Ae)-x(aBe+Acd))}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{c(d+ex)^m\left(Acd^2+aBemd+aAe^2(1-m)-e(Acd+aBe)mx\right)}{cx^2+a}dx}{2ac\left(ae^2+cd^2\right)}-\frac{(d+ex)^{m+1}(a(Bd-Ae)-x(aBe+Acd))}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(d+ex)^m\left(Acd^2+aBemd+aAe^2(1-m)-e(Acd+aBe)mx\right)}{cx^2+a}dx}{2a\left(ae^2+cd^2\right)}-\frac{(d+ex)^{m+1}(a(Bd-Ae)-x(aBe+Acd))}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 657$$

$$\frac{\int \left(\frac{\left(\frac{ae(Acd+aBe)m}{\sqrt{c}}+\sqrt{-a}\left(Acd^2+aBemd+aAe^2(1-m)\right)\right)(d+ex)^m}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\left(\sqrt{-a}\left(Acd^2+aBemd+aAe^2(1-m)\right)-\frac{ae(Acd+aBe)m}{\sqrt{c}}\right)(d+ex)^m}{2a\left(\sqrt{c}x+\sqrt{-a}\right)}\right)dx}{2a\left(ae^2+cd^2\right)}-\frac{(d+ex)^{m+1}(a(Bd-Ae)-x(aBe+Acd))}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{(d+ex)^{m+1}\left(\frac{aem(aBe+Acd)}{\sqrt{c}}-\sqrt{-a}\left(aAe^2(1-m)+aBdem+Acd^2\right)\right)\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a(m+1)\left(\sqrt{c}d-\sqrt{-a}e\right)}+\frac{(d+ex)^{m+1}\left(\sqrt{-a}\left(aAe^2\right.\right.}{}}{2a\left(ae^2+cd^2\right)}-\frac{(d+ex)^{m+1}(a(Bd-Ae)-x(aBe+Acd))}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

input
```
Int[((A + B*x)*(d + e*x)^m)/(a + c*x^2)^2,x]
```

output
```
-1/2*((d + e*x)^(1 + m)*(a*(B*d - A*e) - (A*c*d + a*B*e)*x))/(a*(c*d^2 + a
*e^2)*(a + c*x^2)) + ((((a*e*(A*c*d + a*B*e)*m)/Sqrt[c] - Sqrt[-a]*(A*c*d^
2 + a*A*e^2*(1 - m) + a*B*d*e*m))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1
 + m, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(2*a*(Sqrt[c]*
d - Sqrt[-a]*e)*(1 + m)) + (((a*e*(A*c*d + a*B*e)*m)/Sqrt[c] + Sqrt[-a]*(A
*c*d^2 + a*A*e^2*(1 - m) + a*B*d*e*m))*(d + e*x)^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(2*a*(Sqr
t[c]*d + Sqrt[-a]*e)*(1 + m)))/(2*a*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(Bx+A)\,(ex+d)^m}{(c\,x^2+a)^2}dx$$

input
```
int((B*x+A)*(e*x+d)^m/(c*x^2+a)^2,x)
```

output
```
int((B*x+A)*(e*x+d)^m/(c*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{(A+Bx)(d+ex)^m}{(a+cx^2)^2}\,dx = \int \frac{(Bx+A)(ex+d)^m}{(cx^2+a)^2}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^m/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(e*x + d)^m/(c^2*x^4 + 2*a*c*x^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^m}{(a+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(e*x+d)**m/(c*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^m}{(a+cx^2)^2}\,dx = \int \frac{(Bx+A)(ex+d)^m}{(cx^2+a)^2}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^m/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x + d)^m/(c*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{(A+Bx)(d+ex)^m}{(a+cx^2)^2}\,dx = \int \frac{(Bx+A)(ex+d)^m}{(cx^2+a)^2}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^m/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x + d)^m/(c*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^m}{(a+cx^2)^2}\,dx = \int \frac{(A+B\,x)\,(d+e\,x)^m}{(c\,x^2+a)^2}\,dx$$

input
```
int(((A + B*x)*(d + e*x)^m)/(a + c*x^2)^2,x)
```

output
```
int(((A + B*x)*(d + e*x)^m)/(a + c*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(A+Bx)(d+ex)^m}{(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((B*x+A)*(e*x+d)^m/(c*x^2+a)^2,x)
```

output
```
( - (d + e*x)**m*a*e - (d + e*x)**m*b*d + int((d + e*x)**m/(a**2*d + a**2*
e*x + 2*a*c*d*x**2 + 2*a*c*e*x**3 + c**2*d*x**4 + c**2*e*x**5),x)*a**3*e**
2*m + int((d + e*x)**m/(a**2*d + a**2*e*x + 2*a*c*d*x**2 + 2*a*c*e*x**3 +
c**2*d*x**4 + c**2*e*x**5),x)*a**2*b*d*e*m + 2*int((d + e*x)**m/(a**2*d +
a**2*e*x + 2*a*c*d*x**2 + 2*a*c*e*x**3 + c**2*d*x**4 + c**2*e*x**5),x)*a**
2*c*d**2 + int((d + e*x)**m/(a**2*d + a**2*e*x + 2*a*c*d*x**2 + 2*a*c*e*x*
*3 + c**2*d*x**4 + c**2*e*x**5),x)*a**2*c*e**2*m*x**2 + int((d + e*x)**m/(
a**2*d + a**2*e*x + 2*a*c*d*x**2 + 2*a*c*e*x**3 + c**2*d*x**4 + c**2*e*x**
5),x)*a*b*c*d*e*m*x**2 + 2*int((d + e*x)**m/(a**2*d + a**2*e*x + 2*a*c*d*x
**2 + 2*a*c*e*x**3 + c**2*d*x**4 + c**2*e*x**5),x)*a*c**2*d**2*x**2 + int(
((d + e*x)**m*x**2)/(a**2*d + a**2*e*x + 2*a*c*d*x**2 + 2*a*c*e*x**3 + c**
2*d*x**4 + c**2*e*x**5),x)*a**2*c*e**2*m - 2*int(((d + e*x)**m*x**2)/(a**2
*d + a**2*e*x + 2*a*c*d*x**2 + 2*a*c*e*x**3 + c**2*d*x**4 + c**2*e*x**5),x
)*a**2*c*e**2 + int(((d + e*x)**m*x**2)/(a**2*d + a**2*e*x + 2*a*c*d*x**2
+ 2*a*c*e*x**3 + c**2*d*x**4 + c**2*e*x**5),x)*a*b*c*d*e*m + int(((d + e*x
)**m*x**2)/(a**2*d + a**2*e*x + 2*a*c*d*x**2 + 2*a*c*e*x**3 + c**2*d*x**4
+ c**2*e*x**5),x)*a*c**2*e**2*m*x**2 - 2*int(((d + e*x)**m*x**2)/(a**2*d +
 a**2*e*x + 2*a*c*d*x**2 + 2*a*c*e*x**3 + c**2*d*x**4 + c**2*e*x**5),x)*a*
c**2*e**2*x**2 + int(((d + e*x)**m*x**2)/(a**2*d + a**2*e*x + 2*a*c*d*x**2
 + 2*a*c*e*x**3 + c**2*d*x**4 + c**2*e*x**5),x)*b*c**2*d*e*m*x**2)/(2*c...
```

3.300 $\int \frac{(A+Bx)(d+ex)^{1+m}}{a+cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 212

$$\int \frac{(A+Bx)(d+ex)^{1+m}}{a+cx^2}\,dx = -\frac{\left(\sqrt{-a}B-A\sqrt{c}\right)(d+ex)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2\sqrt{-a}\sqrt{c}\left(\sqrt{c}d-\sqrt{-a}e\right)(2+m)} - \frac{\left(\sqrt{-a}B+A\sqrt{c}\right)(d+ex)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}\sqrt{c}\left(\sqrt{c}d+\sqrt{-a}e\right)(2+m)}$$

output
```
-1/2*((-a)^(1/2)*B-A*c^(1/2))*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],c^(1/
2)*(e*x+d)/(c^(1/2)*d-(-a)^(1/2)*e))/(-a)^(1/2)/c^(1/2)/(c^(1/2)*d-(-a)^(1
/2)*e)/(2+m)-1/2*((-a)^(1/2)*B+A*c^(1/2))*(e*x+d)^(2+m)*hypergeom([1, 2+m]
,[3+m],c^(1/2)*(e*x+d)/(c^(1/2)*d+(-a)^(1/2)*e))/(-a)^(1/2)/c^(1/2)/(c^(1/
2)*d+(-a)^(1/2)*e)/(2+m)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)(d+ex)^{1+m}}{a+cx^2}\,dx = \frac{(d+ex)^{2+m}\left(\frac{(aB+\sqrt{-a}A\sqrt{c})\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{-\sqrt{c}d+\sqrt{-a}e}+\frac{(-aB+\sqrt{-a}A\sqrt{c})\,\text{Hypergeometric2F1}\left(1,2+m,3+m\right.}{\sqrt{c}d+\sqrt{-a}e}\right.}{2a\sqrt{c}(2+m)}$$

input
```
Integrate[((A + B*x)*(d + e*x)^(1 + m))/(a + c*x^2),x]
```

output
```
((d + e*x)^(2 + m)*(((a*B + Sqrt[-a]*A*Sqrt[c])*Hypergeometric2F1[1, 2 + m
, 3 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(-(Sqrt[c]*d) + Sq
rt[-a]*e) + ((-(a*B) + Sqrt[-a]*A*Sqrt[c])*Hypergeometric2F1[1, 2 + m, 3 +
 m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)])/(Sqrt[c]*d + Sqrt[-a]*e
)))/(2*a*Sqrt[c]*(2 + m))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(d+ex)^{m+1}}{a+cx^2}\,dx$$

$$\Big\downarrow \text{657}$$

$$\int \left(\frac{\left(\sqrt{-a}A-\frac{aB}{\sqrt{c}}\right)(d+ex)^{m+1}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)}+\frac{\left(\sqrt{-a}A+\frac{aB}{\sqrt{c}}\right)(d+ex)^{m+1}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\sqrt{-a}A\sqrt{c}+aB\right)(d+ex)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d-\sqrt{-a}e}\right)}{2a\sqrt{c}(m+2)\left(\sqrt{c}d-\sqrt{-a}e\right)}-\frac{\left(\frac{\sqrt{-a}B}{\sqrt{c}}+A\right)(d+ex)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{\sqrt{c}(d+ex)}{\sqrt{c}d+\sqrt{-a}e}\right)}{2\sqrt{-a}(m+2)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

input
```
Int[((A + B*x)*(d + e*x)^(1 + m))/(a + c*x^2),x]
```

output
```
-1/2*((a*B + Sqrt[-a]*A*Sqrt[c])*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2
+ m, 3 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)])/(a*Sqrt[c]*(Sqr
t[c]*d - Sqrt[-a]*e)*(2 + m)) - ((A + (Sqrt[-a]*B)/Sqrt[c])*(d + e*x)^(2 +
 m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sq
rt[-a]*e)])/(2*Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(2 + m))
```

Defintions of rubi rules used

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(Bx+A)\,(ex+d)^{1+m}}{c\,x^{2}+a}dx$$

input
```
int((B*x+A)*(e*x+d)^(1+m)/(c*x^2+a),x)
```

output
```
int((B*x+A)*(e*x+d)^(1+m)/(c*x^2+a),x)
```

Fricas [F]

$$\int \frac{(A+Bx)(d+ex)^{1+m}}{a+cx^2}\,dx = \int \frac{(Bx+A)(ex+d)^{m+1}}{cx^2+a}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1+m)/(c*x^2+a),x, algorithm="fricas")
```

output
```
integral((B*x + A)*(e*x + d)^(m + 1)/(c*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(A+Bx)(d+ex)^{1+m}}{a+cx^2}\,dx = \int \frac{(A+Bx)\,(d+ex)^{m+1}}{a+cx^2}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)**(1+m)/(c*x**2+a),x)
```

output
```
Integral((A + B*x)*(d + e*x)**(m + 1)/(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)(d+ex)^{1+m}}{a+cx^2}\,dx = \int \frac{(Bx+A)(ex+d)^{m+1}}{cx^2+a}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1+m)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(e*x + d)^(m + 1)/(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx)(d+ex)^{1+m}}{a+cx^2}\,dx=\int \frac{(Bx+A)(ex+d)^{m+1}}{cx^2+a}\,dx$$

input
```
integrate((B*x+A)*(e*x+d)^(1+m)/(c*x^2+a),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x + d)^(m + 1)/(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(d+ex)^{1+m}}{a+cx^2}\,dx=\int \frac{(A+B\,x)\,(d+e\,x)^{m+1}}{c\,x^2+a}\,dx$$

input
```
int(((A + B*x)*(d + e*x)^(m + 1))/(a + c*x^2),x)
```

output
```
int(((A + B*x)*(d + e*x)^(m + 1))/(a + c*x^2), x)
```

Reduce [F]

$$\int \frac{(A+Bx)(d+ex)^{1+m}}{a+cx^2}\,dx$$
$$=\frac{(ex+d)^m\,aem+(ex+d)^m\,ae+2(ex+d)^m\,bdm+(ex+d)^m\,bd+(ex+d)^m\,bemx-\left(\int \frac{(ex+d)^m}{ce\,x^3+cd\,x^2+aex-}\right.}{}$$

input
```
int((B*x+A)*(e*x+d)^(1+m)/(c*x^2+a),x)
```

output
```
((d + e*x)**m*a*e*m + (d + e*x)**m*a*e + 2*(d + e*x)**m*b*d*m + (d + e*x)*
*m*b*d + (d + e*x)**m*b*e*m*x - int((d + e*x)**m/(a*d + a*e*x + c*d*x**2 +
 c*e*x**3),x)*a**2*e**2*m**2 - int((d + e*x)**m/(a*d + a*e*x + c*d*x**2 +
c*e*x**3),x)*a**2*e**2*m - 2*int((d + e*x)**m/(a*d + a*e*x + c*d*x**2 + c*
e*x**3),x)*a*b*d*e*m**2 - 2*int((d + e*x)**m/(a*d + a*e*x + c*d*x**2 + c*e
*x**3),x)*a*b*d*e*m + int((d + e*x)**m/(a*d + a*e*x + c*d*x**2 + c*e*x**3)
,x)*a*c*d**2*m**2 + int((d + e*x)**m/(a*d + a*e*x + c*d*x**2 + c*e*x**3),x
)*a*c*d**2*m - int(((d + e*x)**m*x)/(a*d + a*e*x + c*d*x**2 + c*e*x**3),x)
*a*b*e**2*m**2 - int(((d + e*x)**m*x)/(a*d + a*e*x + c*d*x**2 + c*e*x**3),
x)*a*b*e**2*m + 2*int(((d + e*x)**m*x)/(a*d + a*e*x + c*d*x**2 + c*e*x**3)
,x)*a*c*d*e*m**2 + 2*int(((d + e*x)**m*x)/(a*d + a*e*x + c*d*x**2 + c*e*x*
*3),x)*a*c*d*e*m + int(((d + e*x)**m*x)/(a*d + a*e*x + c*d*x**2 + c*e*x**3
),x)*b*c*d**2*m**2 + int(((d + e*x)**m*x)/(a*d + a*e*x + c*d*x**2 + c*e*x*
*3),x)*b*c*d**2*m)/(c*m*(m + 1))
```

3.301 $\int (d+ex)(f+gx)\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 116

$$\int (d+ex)(f+gx)\left(a+cx^2\right)^p\,dx=\frac{((ef+dg)(3+2p)+2eg(1+p)x)\left(a+cx^2\right)^{1+p}}{2c\left(3+5p+2p^2\right)}+\left(df-\frac{aeg}{3c+2cp}\right)x\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{cx^2}{a}\right)$$

output
```
1/2*((d*g+e*f)*(3+2*p)+2*e*g*(p+1)*x)*(c*x^2+a)^(p+1)/c/(2*p^2+5*p+3)+(d*f
-a*e*g/(2*c*p+3*c))*x*(c*x^2+a)^p*hypergeom([1/2, -p],[3/2],-c*x^2/a)/((1+
c*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05

$$\int (d+ex)(f+gx)\left(a+cx^2\right)^p\,dx=\frac{\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(3(ef+dg)\left(a+cx^2\right)\left(1+\frac{cx^2}{a}\right)^p+6cdf(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\right.\right.}{6c(1+p)}$$

input
```
Integrate[(d + e*x)*(f + g*x)*(a + c*x^2)^p,x]
```

output
```
((a + c*x^2)^p*(3*(e*f + d*g)*(a + c*x^2)*(1 + (c*x^2)/a)^p + 6*c*d*f*(1 +
 p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((c*x^2)/a)] + 2*c*e*g*(1 + p)*x^3*
Hypergeometric2F1[3/2, -p, 5/2, -((c*x^2)/a)]))/(6*c*(1 + p)*(1 + (c*x^2)/
a)^p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {676, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)(f+gx)\left(a+cx^2\right)^p\,dx$$

$$\Big\downarrow \text{676}$$

$$\left(df-\frac{aeg}{2cp+3c}\right)\int \left(cx^2+a\right)^p dx+\frac{\left(a+cx^2\right)^{p+1}(dg+ef)}{2c(p+1)}+\frac{egx\left(a+cx^2\right)^{p+1}}{c(2p+3)}$$

$$\Big\downarrow \text{238}$$

$$\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(df-\frac{aeg}{2cp+3c}\right)\int \left(\frac{cx^2}{a}+1\right)^p dx+\frac{\left(a+cx^2\right)^{p+1}(dg+ef)}{2c(p+1)}+\frac{egx\left(a+cx^2\right)^{p+1}}{c(2p+3)}$$

$$\Big\downarrow \text{237}$$

$$x\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{cx^2}{a}\right)\left(df-\frac{aeg}{2cp+3c}\right)+\frac{\left(a+cx^2\right)^{p+1}(dg+ef)}{2c(p+1)}+\frac{egx\left(a+cx^2\right)^{p+1}}{c(2p+3)}$$

input
```
Int[(d + e*x)*(f + g*x)*(a + c*x^2)^p,x]
```

output
```
((e*f + d*g)*(a + c*x^2)^(1 + p))/(2*c*(1 + p)) + (e*g*x*(a + c*x^2)^(1 +
p))/(c*(3 + 2*p)) + ((d*f - (a*e*g)/(3*c + 2*c*p))*x*(a + c*x^2)^p*Hyperge
ometric2F1[1/2, -p, 3/2, -((c*x^2)/a)])/(1 + (c*x^2)/a)^p
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int (ex+d)\,(gx+f)\,\left(c\,x^2+a\right)^p dx$$

input
```
int((e*x+d)*(g*x+f)*(c*x^2+a)^p,x)
```

output
```
int((e*x+d)*(g*x+f)*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (d+ex)(f+gx)\left(a+cx^2\right)^p\,dx=\int (ex+d)(gx+f)\left(cx^2+a\right)^p\,dx$$

input
```
integrate((e*x+d)*(g*x+f)*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((e*g*x^2 + d*f + (e*f + d*g)*x)*(c*x^2 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.16

$$\int (d+ex)(f+gx)\left(a+cx^2\right)^p\,dx=a^pdfx{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{cx^2e^{i\pi}}{a}\right)+\frac{a^pegx^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{cx^2e^{i\pi}}{a}\right)}{3}+dg\left(\begin{cases}\frac{a^px^2}{2} & \text{for } c=0\\ \frac{\begin{cases}\frac{(a+cx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+cx^2\right) & \text{otherwise}\end{cases}}{2c} & \text{otherwise}\end{cases}\right)+ef\left(\begin{cases}\frac{a^px^2}{2} & \text{for } c=0\\ \frac{\begin{cases}\frac{(a+cx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+cx^2\right) & \text{otherwise}\end{cases}}{2c} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((e*x+d)*(g*x+f)*(c*x**2+a)**p,x)
```

output
```
a**p*d*f*x*hyper((1/2, -p), (3/2,), c*x**2*exp_polar(I*pi)/a) + a**p*e*g*x
**3*hyper((3/2, -p), (5/2,), c*x**2*exp_polar(I*pi)/a)/3 + d*g*Piecewise((
a**p*x**2/2, Eq(c, 0)), (Piecewise(((a + c*x**2)**(p + 1)/(p + 1), Ne(p, -
1)), (log(a + c*x**2), True))/(2*c), True)) + e*f*Piecewise((a**p*x**2/2,
Eq(c, 0)), (Piecewise(((a + c*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a +
 c*x**2), True))/(2*c), True))
```

Maxima [F]

$$\int (d+ex)(f+gx)\left(a+cx^2\right)^p\,dx = \int (ex+d)(gx+f)\left(cx^2+a\right)^p\,dx$$

input
```
integrate((e*x+d)*(g*x+f)*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(g*x + f)*(c*x^2 + a)^p, x)
```

Giac [F]

$$\int (d+ex)(f+gx)\left(a+cx^2\right)^p\,dx = \int (ex+d)(gx+f)\left(cx^2+a\right)^p\,dx$$

input
```
integrate((e*x+d)*(g*x+f)*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(g*x + f)*(c*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)(f+gx)\left(a+cx^2\right)^p\,dx=\int (f+g\,x)\,\left(c\,x^2+a\right)^p\,(d+e\,x)\,dx$$

input
```
int((f + g*x)*(a + c*x^2)^p*(d + e*x),x)
```

output
```
int((f + g*x)*(a + c*x^2)^p*(d + e*x), x)
```

Reduce [F]

$$\int (d+ex)(f+gx)\left(a+cx^2\right)^p\,dx=\text{Too large to display}$$

input
```
int((e*x+d)*(g*x+f)*(c*x^2+a)^p,x)
```

output

```
(4*(a + c*x**2)**p*a*d*g*p**2 + 8*(a + c*x**2)**p*a*d*g*p + 3*(a + c*x**2)
**p*a*d*g + 4*(a + c*x**2)**p*a*e*f*p**2 + 8*(a + c*x**2)**p*a*e*f*p + 3*(
a + c*x**2)**p*a*e*f + 4*(a + c*x**2)**p*a*e*g*p**2*x + 4*(a + c*x**2)**p*
a*e*g*p*x + 4*(a + c*x**2)**p*c*d*f*p**2*x + 10*(a + c*x**2)**p*c*d*f*p*x
+ 6*(a + c*x**2)**p*c*d*f*x + 4*(a + c*x**2)**p*c*d*g*p**2*x**2 + 8*(a + c
*x**2)**p*c*d*g*p*x**2 + 3*(a + c*x**2)**p*c*d*g*x**2 + 4*(a + c*x**2)**p*
c*e*f*p**2*x**2 + 8*(a + c*x**2)**p*c*e*f*p*x**2 + 3*(a + c*x**2)**p*c*e*f
*x**2 + 4*(a + c*x**2)**p*c*e*g*p**2*x**3 + 6*(a + c*x**2)**p*c*e*g*p*x**3
 + 2*(a + c*x**2)**p*c*e*g*x**3 - 16*int((a + c*x**2)**p/(4*a*p**2 + 8*a*p
 + 3*a + 4*c*p**2*x**2 + 8*c*p*x**2 + 3*c*x**2),x)*a**2*e*g*p**4 - 48*int(
(a + c*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*c*p**2*x**2 + 8*c*p*x**2 + 3*c
*x**2),x)*a**2*e*g*p**3 - 44*int((a + c*x**2)**p/(4*a*p**2 + 8*a*p + 3*a +
 4*c*p**2*x**2 + 8*c*p*x**2 + 3*c*x**2),x)*a**2*e*g*p**2 - 12*int((a + c*x
**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*c*p**2*x**2 + 8*c*p*x**2 + 3*c*x**2),x
)*a**2*e*g*p + 32*int((a + c*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*c*p**2*x
**2 + 8*c*p*x**2 + 3*c*x**2),x)*a*c*d*f*p**5 + 144*int((a + c*x**2)**p/(4*
a*p**2 + 8*a*p + 3*a + 4*c*p**2*x**2 + 8*c*p*x**2 + 3*c*x**2),x)*a*c*d*f*p
**4 + 232*int((a + c*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*c*p**2*x**2 + 8*
c*p*x**2 + 3*c*x**2),x)*a*c*d*f*p**3 + 156*int((a + c*x**2)**p/(4*a*p**2 +
 8*a*p + 3*a + 4*c*p**2*x**2 + 8*c*p*x**2 + 3*c*x**2),x)*a*c*d*f*p**2 +...
```

3.302 $\int (A+Bx)(c+dx)^m \left(a+bx^2\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 315

$$\int (A+Bx)(c+dx)^m \left(a+bx^2\right)^p \, dx =$$

$$-\frac{(Bc-Ad)(c+dx)^{1+m}\left(a+bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \text{AppellF1}\left(1+m,-p,-p,2+m\right.}{d^2(1+m)}$$

$$+\frac{B(c+dx)^{2+m}\left(a+bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \text{AppellF1}\left(2+m,-p,-p,3+m,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right.}{d^2(2+m)}$$

output
```
-(-A*d+B*c)*(d*x+c)^(1+m)*(b*x^2+a)^p*AppellF1(1+m,-p,-p,2+m,(d*x+c)/(c-(-
a)^(1/2)*d/b^(1/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d^2/(1+m)/((1-(d*x+c
)/(c-(-a)^(1/2)*d/b^(1/2)))^p)/((1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^p)+B*
(d*x+c)^(2+m)*(b*x^2+a)^p*AppellF1(2+m,-p,-p,3+m,(d*x+c)/(c-(-a)^(1/2)*d/b
^(1/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d^2/(2+m)/((1-(d*x+c)/(c-(-a)^(1
/2)*d/b^(1/2)))^p)/((1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^p)
```

Mathematica [F]

$$\int (A+Bx)(c+dx)^m \left(a+bx^2\right)^p \, dx = \int (A+Bx)(c+dx)^m \left(a+bx^2\right)^p \, dx$$

input
```
Integrate[(A + B*x)*(c + d*x)^m*(a + b*x^2)^p,x]
```

output
```
Integrate[(A + B*x)*(c + d*x)^m*(a + b*x^2)^p, x]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {719, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx) \left(a+bx^2\right)^p (c+dx)^m \, dx$$

$$\Big\downarrow \text{ 719}$$

$$\frac{B\int (c+dx)^{m+1}\left(bx^2+a\right)^p dx}{d}-\frac{(Bc-Ad)\int (c+dx)^m\left(bx^2+a\right)^p dx}{d}$$

$$\Big\downarrow \text{ 514}$$

$$\frac{B\left(a+bx^2\right)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-ad}}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{-ad}}{\sqrt{b}}+c}\right)^{-p}\int (c+dx)^{m+1}\left(1-\frac{c+dx}{c-\frac{\sqrt{-ad}}{\sqrt{b}}}\right)^p\left(1-\frac{c+dx}{c+\frac{\sqrt{-ad}}{\sqrt{b}}}\right)^p d(c+dx)}{d^2}-\frac{\left(a+bx^2\right)^p(Bc-Ad)\left(1-\frac{c+dx}{c-\frac{\sqrt{-ad}}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{-ad}}{\sqrt{b}}+c}\right)^{-p}\int (c+dx)^m\left(1-\frac{c+dx}{c-\frac{\sqrt{-ad}}{\sqrt{b}}}\right)^p\left(1-\frac{c+dx}{c+\frac{\sqrt{-ad}}{\sqrt{b}}}\right)^p d(c+dx)}{d^2}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{B(a+bx^2)^p(c+dx)^{m+2}\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p}\operatorname{AppellF1}\left(m+2,-p,-p,m+3,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{d^2(m+2)}$$
$$\frac{(a+bx^2)^p(Bc-Ad)(c+dx)^{m+1}\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p}\operatorname{AppellF1}\left(m+1,-p,-p,m+2,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right.}{d^2(m+1)}$$

input
```
Int[(A + B*x)*(c + d*x)^m*(a + b*x^2)^p,x]
```

output
```
-(((B*c - A*d)*(c + d*x)^(1 + m)*(a + b*x^2)^p*AppellF1[1 + m, -p, -p, 2 +
 m, (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt
[b])])/(d^2*(1 + m)*(1 - (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]))^p*(1 - (c +
 d*x)/(c + (Sqrt[-a]*d)/Sqrt[b]))^p)) + (B*(c + d*x)^(2 + m)*(a + b*x^2)^p
*AppellF1[2 + m, -p, -p, 3 + m, (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]), (c +
 d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])/(d^2*(2 + m)*(1 - (c + d*x)/(c - (Sqrt[
-a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int (Bx + A) (dx + c)^m (b x^2 + a)^p dx$$

input
```
int((B*x+A)*(d*x+c)^m*(b*x^2+a)^p,x)
```

output
```
int((B*x+A)*(d*x+c)^m*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (A + Bx)(c + dx)^m (a + bx^2)^p dx = \int (Bx + A)(bx^2 + a)^p (dx + c)^m dx$$

input
```
integrate((B*x+A)*(d*x+c)^m*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(b*x^2 + a)^p*(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (A + Bx)(c + dx)^m (a + bx^2)^p dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(d*x+c)**m*(b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (A+Bx)(c+dx)^m \left(a+bx^2\right)^p \, dx = \int (Bx+A)\left(bx^2+a\right)^p (dx+c)^m \, dx$$

input
```
integrate((B*x+A)*(d*x+c)^m*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(b*x^2 + a)^p*(d*x + c)^m, x)
```

Giac [F]

$$\int (A+Bx)(c+dx)^m \left(a+bx^2\right)^p \, dx = \int (Bx+A)\left(bx^2+a\right)^p (dx+c)^m \, dx$$

input
```
integrate((B*x+A)*(d*x+c)^m*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(b*x^2 + a)^p*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)(c+dx)^m \left(a+bx^2\right)^p \, dx = \int \left(b\,x^2+a\right)^p\,\left(A+B\,x\right)\,{\left(c+d\,x\right)}^m \, dx$$

input
```
int((a + b*x^2)^p*(A + B*x)*(c + d*x)^m,x)
```

output
```
int((a + b*x^2)^p*(A + B*x)*(c + d*x)^m, x)
```

Reduce [F]

$$\int (A + Bx)(c + dx)^m \left(a + bx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((B*x+A)*(d*x+c)^m*(b*x^2+a)^p,x)
```

output
```
((c + d*x)**m*(a + b*x**2)**p*a**2*d**2*m + 2*(c + d*x)**m*(a + b*x**2)**p
*a**2*d**2*p + 2*(c + d*x)**m*(a + b*x**2)**p*a**2*d**2 + (c + d*x)**m*(a
+ b*x**2)**p*a*b*c*d*m*x + 2*(c + d*x)**m*(a + b*x**2)**p*a*b*c*d*m + 2*(c
 + d*x)**m*(a + b*x**2)**p*a*b*c*d*p*x + 2*(c + d*x)**m*(a + b*x**2)**p*a*
b*c*d*p + 2*(c + d*x)**m*(a + b*x**2)**p*a*b*c*d*x + (c + d*x)**m*(a + b*x
**2)**p*a*b*c*d + (c + d*x)**m*(a + b*x**2)**p*b**2*c**2*m*x + (c + d*x)**
m*(a + b*x**2)**p*b**2*c*d*m*x**2 + 2*(c + d*x)**m*(a + b*x**2)**p*b**2*c*
d*p*x**2 + (c + d*x)**m*(a + b*x**2)**p*b**2*c*d*x**2 - int(((c + d*x)**m*
(a + b*x**2)**p*x**2)/(a*c*m**2 + 4*a*c*m*p + 3*a*c*m + 4*a*c*p**2 + 6*a*c
*p + 2*a*c + a*d*m**2*x + 4*a*d*m*p*x + 3*a*d*m*x + 4*a*d*p**2*x + 6*a*d*p
*x + 2*a*d*x + b*c*m**2*x**2 + 4*b*c*m*p*x**2 + 3*b*c*m*x**2 + 4*b*c*p**2*
x**2 + 6*b*c*p*x**2 + 2*b*c*x**2 + b*d*m**2*x**3 + 4*b*d*m*p*x**3 + 3*b*d*
m*x**3 + 4*b*d*p**2*x**3 + 6*b*d*p*x**3 + 2*b*d*x**3),x)*a**2*b*d**3*m**4
- 8*int(((c + d*x)**m*(a + b*x**2)**p*x**2)/(a*c*m**2 + 4*a*c*m*p + 3*a*c*
m + 4*a*c*p**2 + 6*a*c*p + 2*a*c + a*d*m**2*x + 4*a*d*m*p*x + 3*a*d*m*x +
4*a*d*p**2*x + 6*a*d*p*x + 2*a*d*x + b*c*m**2*x**2 + 4*b*c*m*p*x**2 + 3*b*
c*m*x**2 + 4*b*c*p**2*x**2 + 6*b*c*p*x**2 + 2*b*c*x**2 + b*d*m**2*x**3 + 4
*b*d*m*p*x**3 + 3*b*d*m*x**3 + 4*b*d*p**2*x**3 + 6*b*d*p*x**3 + 2*b*d*x**3
),x)*a**2*b*d**3*m**3*p - 5*int(((c + d*x)**m*(a + b*x**2)**p*x**2)/(a*c*m
**2 + 4*a*c*m*p + 3*a*c*m + 4*a*c*p**2 + 6*a*c*p + 2*a*c + a*d*m**2*x +...
```

3.303 $\int (d+ex)^{-5-2p}(e+fx)\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 26, antiderivative size = 510

$$\int (d+ex)^{-5-2p}(e+fx)\left(a+cx^2\right)^p\,dx$$
$$= -\frac{\left(ae^2f(2+p)-cd(df-e^2(3+p))\right)(d+ex)^{-3-2p}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)^2(2+p)(3+2p)}$$
$$+\frac{c\left(cd^2(df-e^2(9+8p+2p^2))+ae^2(e^2(3+2p)-df(11+10p+2p^2))\right)(d+ex)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2\left(cd^2+ae^2\right)^3(1+p)(2+p)(3+2p)}$$
$$-\frac{\left(e^2-df\right)(d+ex)^{-2(2+p)}\left(a+cx^2\right)^{1+p}}{2\left(cd^2+ae^2\right)(2+p)}$$
$$+\frac{ce(a^2e^2f-c^2d^3(3+2p)+acd(3e^2-df(5+2p)))\left(\sqrt{-a}-\sqrt{c}x\right)\left(-\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{-a}+\sqrt{c}x\right)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)^{-p}(d+ex)}{\left(\sqrt{c}d+\sqrt{-a}e\right)\left(cd^2+ae^2\right)^3(1+}$$

output
```
-(a*e^2*f*(2+p)-c*d*(d*f-e^2*(3+p)))*(e*x+d)^(-3-2*p)*(c*x^2+a)^(p+1)/(a*e
^2+c*d^2)^2/(2+p)/(3+2*p)+1/2*c*(c*d^2*(d*f-e^2*(2*p^2+8*p+9))+a*e^2*(e^2*
(3+2*p)-d*f*(2*p^2+10*p+11)))*(c*x^2+a)^(p+1)/(a*e^2+c*d^2)^3/(p+1)/(2+p)/
(3+2*p)/((e*x+d)^(2*p+2))-1/2*(-d*f+e^2)*(c*x^2+a)^(p+1)/(a*e^2+c*d^2)/(2+
p)/((e*x+d)^(4+2*p))+c*e*(a^2*e^2*f-c^2*d^3*(3+2*p)+a*c*d*(3*e^2-d*f*(5+2*
p)))*((-a)^(1/2)-c^(1/2)*x)*(e*x+d)^(-1-2*p)*(c*x^2+a)^p*hypergeom([-p, -1
-2*p],[-2*p],2*(-a)^(1/2)*c^(1/2)*(e*x+d)/(c^(1/2)*d-(-a)^(1/2)*e)/((-a)^(
1/2)-c^(1/2)*x))/(c^(1/2)*d+(-a)^(1/2)*e)/(a*e^2+c*d^2)^3/(1+2*p)/(3+2*p)/
((-(c^(1/2)*d+(-a)^(1/2)*e)*((-a)^(1/2)+c^(1/2)*x)/(c^(1/2)*d-(-a)^(1/2)*e
)/((-a)^(1/2)-c^(1/2)*x))^p)
```

Mathematica [F]

$$\int (d+ex)^{-5-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (d+ex)^{-5-2p}(e+fx)\left(a+cx^2\right)^p\,dx$$

input
```
Integrate[(d + e*x)^(-5 - 2*p)*(e + f*x)*(a + c*x^2)^p,x]
```

output
```
Integrate[(d + e*x)^(-5 - 2*p)*(e + f*x)*(a + c*x^2)^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+cx^2\right)^p(d+ex)^{-2p-5}\,dx$$

$$\Big\downarrow \text{689}$$

$$-\frac{\int -2(d+ex)^{-2(p+2)}\left(e(cd+af)(p+2)-c\left(e^2-df\right)x\right)\left(cx^2+a\right)^p dx}{2(p+2)\left(ae^2+cd^2\right)}-\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (d+ex)^{-2(p+2)}\left(e(cd+af)(p+2)-c\left(e^2-df\right)x\right)\left(cx^2+a\right)^p dx}{(p+2)\left(ae^2+cd^2\right)}-\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 689$$

$$\frac{-\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(cd^2(p+2)-a\left(e^2-df(p+3)\right)\right)-\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)\left(ae^2+cd^2\right)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f\ldots\right.}{(2p+3)\left(ae^2\ldots\right.}}{(p+2)\left(ae^2+cd^2\right)}-\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(-c(p+2)d^2-af(p+3)d+ae^2\right)+\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)\left(ae^2+cd^2\right)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f(\ldots\right.}{(2p+3)\left(ae^2+\ldots\right.}}{(p+2)\left(ae^2+cd^2\right)}-\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(cd^2(p+2)-a\left(e^2-df(p+3)\right)\right)-\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)\left(ae^2+cd^2\right)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f\ldots\right.}{(2p+3)\left(ae^2\ldots\right.}}{(p+2)\left(ae^2+cd^2\right)}-\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(-c(p+2)d^2-af(p+3)d+ae^2\right)+\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)\left(ae^2+cd^2\right)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f(\ldots\right.}{(2p+3)\left(ae^2+\ldots\right.}}{(p+2)\left(ae^2+cd^2\right)}-\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(cd^2(p+2)-a\left(e^2-df(p+3)\right)\right)-\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)\left(ae^2+cd^2\right)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f\ldots\right.}{(2p+3)\left(ae^2\ldots\right.}}{(p+2)\left(ae^2+cd^2\right)}-\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(-c(p+2)d^2-af(p+3)d+ae^2\right)+\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f(\ldots\right.}{(2p+3)(ae^2+\ldots}}{(p+2)\left(ae^2+cd^2\right)}$$
$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(cd^2(p+2)-a\left(e^2-df(p+3)\right)\right)-\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f\ldots\right.}{(2p+3)(ae^2\ldots}}{(p+2)\left(ae^2+cd^2\right)}$$
$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(-c(p+2)d^2-af(p+3)d+ae^2\right)+\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f(\ldots\right.}{(2p+3)(ae^2+\ldots}}{(p+2)\left(ae^2+cd^2\right)}$$
$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(cd^2(p+2)-a\left(e^2-df(p+3)\right)\right)-\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f\ldots\right.}{(2p+3)(ae^2\ldots}}{(p+2)\left(ae^2+cd^2\right)}$$
$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(-c(p+2)d^2-af(p+3)d+ae^2\right)+\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f(\ldots\right.}{(2p+3)(ae^2+\ldots}}{(p+2)\left(ae^2+cd^2\right)}$$
$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(cd^2(p+2)-a\left(e^2-df(p+3)\right)\right)-\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^pdx}{(2p+3)\left(ae^2+cd^2\right)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f\right.}{(2p+3)\left(ae^2\right.}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(-c(p+2)d^2-af(p+3)d+ae^2\right)+\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^pdx}{(2p+3)\left(ae^2+cd^2\right)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f(\right.}{(2p+3)\left(ae^2+\right.}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{-\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(cd^2(p+2)-a\left(e^2-df(p+3)\right)\right)-\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^pdx}{(2p+3)\left(ae^2+cd^2\right)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f\right.}{(2p+3)\left(ae^2\right.}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(-c(p+2)d^2-af(p+3)d+ae^2\right)+\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^pdx}{(2p+3)\left(ae^2+cd^2\right)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f(\right.}{(2p+3)\left(ae^2+\right.}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{-\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(cd^2(p+2)-a\left(e^2-df(p+3)\right)\right)-\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^pdx}{(2p+3)\left(ae^2+cd^2\right)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f\right.}{(2p+3)\left(ae^2\right.}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(-c(p+2)d^2-af(p+3)d+ae^2\right)+\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^pdx}{(2p+3)\left(ae^2+cd^2\right)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f(\right.}{(2p+3)\left(ae^2+\right.}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{-\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(cd^2(p+2)-a\left(e^2-df(p+3)\right)\right)-\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f\right.}{(2p+3)(ae^2}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(-c(p+2)d^2-af(p+3)d+ae^2\right)+\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f(\right.}{(2p+3)(ae^2+}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{-\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(cd^2(p+2)-a\left(e^2-df(p+3)\right)\right)-\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f\right.}{(2p+3)(ae^2}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(-c(p+2)d^2-af(p+3)d+ae^2\right)+\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f(\right.}{(2p+3)(ae^2+}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{-\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(cd^2(p+2)-a\left(e^2-df(p+3)\right)\right)-\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f\right.}{(2p+3)(ae^2}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(-c(p+2)d^2-af(p+3)d+ae^2\right)+\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f(\right.}{(2p+3)(ae^2+}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{-\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(cd^2(p+2)-a\left(e^2-df(p+3)\right)\right)-\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f\right.}{(2p+3)(ae^2}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(-c(p+2)d^2-af(p+3)d+ae^2\right)+\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f(\right.}{(2p+3)(ae^2+}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{-\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(cd^2(p+2)-a\left(e^2-df(p+3)\right)\right)-\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f\right.}{(2p+3)(ae^2}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(-c(p+2)d^2-af(p+3)d+ae^2\right)+\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f(\right.}{(2p+3)(ae^2+}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{-\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(cd^2(p+2)-a\left(e^2-df(p+3)\right)\right)-\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^p dx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f\right.}{(2p+3)(ae^2}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int -c(d+ex)^{-2p-3}\left(e(2p+3)\left(-c(p+2)d^2-af(p+3)d+ae^2\right)+\left(ae^2f(p+2)-cd\left(df-e^2(p+3)\right)\right)x\right)\left(cx^2+a\right)^pdx}{(2p+3)(ae^2+cd^2)}-\frac{\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}\left(ae^2f(\right.}{(2p+3)(ae^2+}}{(p+2)\left(ae^2+cd^2\right)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+2)}}{2(p+2)\left(ae^2+cd^2\right)}$$

input
```
Int[(d + e*x)^(-5 - 2*p)*(e + f*x)*(a + c*x^2)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 689
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && ILtQ[Sim
plify[m + 2*p + 3], 0] && NeQ[m, -1]
```

Maple [F]

$$\int \left(ex+d\right)^{-5-2p}\left(fx+e\right)\left(c\,x^{2}+a\right)^{p}dx$$

input
```
int((e*x+d)^(-5-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

output
```
int((e*x+d)^(-5-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^{-5-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (fx+e)\left(cx^2+a\right)^p(ex+d)^{-2\,p-5}\,dx$$

input
```
integrate((e*x+d)^(-5-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 5), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{-5-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(-5-2*p)*(f*x+e)*(c*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^{-5-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (fx+e)\left(cx^2+a\right)^p(ex+d)^{-2\,p-5}\,dx$$

input
```
integrate((e*x+d)^(-5-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 5), x)
```

Giac [F]

$$\int (d+ex)^{-5-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (fx+e)\left(cx^2+a\right)^p(ex+d)^{-2\,p-5}\,dx$$

input
```
integrate((e*x+d)^(-5-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{-5-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int \frac{(e+f\,x)\,\left(c\,x^2+a\right)^p}{(d+e\,x)^{2\,p+5}}\,dx$$

input
```
int(((e + f*x)*(a + c*x^2)^p)/(d + e*x)^(2*p + 5),x)
```

output
```
int(((e + f*x)*(a + c*x^2)^p)/(d + e*x)^(2*p + 5), x)
```

Reduce [F]

$$\int (d+ex)^{-5-2p}(e+fx)\left(a+cx^2\right)^p\,dx$$
$$=\left(\int \frac{(c\,x^2+a)^p}{(ex+d)^{2p}\,d^5+5\,(ex+d)^{2p}\,d^4ex+10\,(ex+d)^{2p}\,d^3e^2x^2+10\,(ex+d)^{2p}\,d^2e^3x^3+5\,(ex+d)^{2p}\,d\,e^4x^4\right.$$
$$+\left(\int \frac{(c\,x^2+a)^p\,x}{(ex+d)^{2p}\,d^5+5\,(ex+d)^{2p}\,d^4ex+10\,(ex+d)^{2p}\,d^3e^2x^2+10\,(ex+d)^{2p}\,d^2e^3x^3+5\,(ex+d)^{2p}\,d\,e^4\right.$$

input
```
int((e*x+d)^(-5-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

output
```
int((a + c*x**2)**p/((d + e*x)**(2*p)*d**5 + 5*(d + e*x)**(2*p)*d**4*e*x +
 10*(d + e*x)**(2*p)*d**3*e**2*x**2 + 10*(d + e*x)**(2*p)*d**2*e**3*x**3 +
 5*(d + e*x)**(2*p)*d*e**4*x**4 + (d + e*x)**(2*p)*e**5*x**5),x)*e + int((
(a + c*x**2)**p*x)/((d + e*x)**(2*p)*d**5 + 5*(d + e*x)**(2*p)*d**4*e*x +
10*(d + e*x)**(2*p)*d**3*e**2*x**2 + 10*(d + e*x)**(2*p)*d**2*e**3*x**3 +
5*(d + e*x)**(2*p)*d*e**4*x**4 + (d + e*x)**(2*p)*e**5*x**5),x)*f
```

3.304 $\int (d+ex)^{-4-2p}(e+fx)\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 26, antiderivative size = 387

$$\int (d+ex)^{-4-2p}(e+fx)\left(a+cx^2\right)^p\,dx = -\frac{\left(e^2-df\right)(d+ex)^{-3-2p}\left(a+cx^2\right)^{1+p}}{\left(cd^2+ae^2\right)(3+2p)} - \frac{\left(ae^2f(3+2p)-cd\left(df-2e^2(2+p)\right)\right)(d+ex)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2\left(cd^2+ae^2\right)^2(1+p)(3+2p)} + \frac{ce\left(ae^2-2adf(2+p)-cd^2(3+2p)\right)\left(\sqrt{-a}-\sqrt{c}x\right)\left(-\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{-a}+\sqrt{c}x\right)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)^{-p}(d+ex)^{-1-2p}\left(a+cx\right.}{\left(\sqrt{c}d+\sqrt{-a}e\right)\left(cd^2+ae^2\right)^2(1+2p)(3-}$$

output
```
-(-d*f+e^2)*(e*x+d)^(-3-2*p)*(c*x^2+a)^(p+1)/(a*e^2+c*d^2)/(3+2*p)-1/2*(a*
e^2*f*(3+2*p)-c*d*(d*f-2*e^2*(2+p)))*(c*x^2+a)^(p+1)/(a*e^2+c*d^2)^2/(p+1)
/(3+2*p)/((e*x+d)^(2*p+2))+c*e*(a*e^2-2*a*d*f*(2+p)-c*d^2*(3+2*p))*((-a)^(
1/2)-c^(1/2)*x)*(e*x+d)^(-1-2*p)*(c*x^2+a)^p*hypergeom([-p, -1-2*p],[-2*p]
,2*(-a)^(1/2)*c^(1/2)*(e*x+d)/(c^(1/2)*d-(-a)^(1/2)*e)/((-a)^(1/2)-c^(1/2)
*x))/(c^(1/2)*d+(-a)^(1/2)*e)/(a*e^2+c*d^2)^2/(1+2*p)/(3+2*p)/((-(c^(1/2)*
d+(-a)^(1/2)*e)*((-a)^(1/2)+c^(1/2)*x)/(c^(1/2)*d-(-a)^(1/2)*e)/((-a)^(1/2
)-c^(1/2)*x))^p)
```

Mathematica [F]

$$\int (d+ex)^{-4-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (d+ex)^{-4-2p}(e+fx)\left(a+cx^2\right)^p\,dx$$

input
```
Integrate[(d + e*x)^(-4 - 2*p)*(e + f*x)*(a + c*x^2)^p,x]
```

output
```
Integrate[(d + e*x)^(-4 - 2*p)*(e + f*x)*(a + c*x^2)^p, x]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {689, 25, 679, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+cx^2\right)^p(d+ex)^{-2p-4}dx$$

$$\Big\downarrow \text{689}$$

$$-\frac{\int -(d+ex)^{-2p-3}\left(e(cd+af)(2p+3)-c\left(e^2-df\right)x\right)\left(cx^2+a\right)^pdx}{(2p+3)\left(ae^2+cd^2\right)}-\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}}{(2p+3)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (d+ex)^{-2p-3}\left(e(cd+af)(2p+3)-c\left(e^2-df\right)x\right)\left(cx^2+a\right)^pdx}{(2p+3)\left(ae^2+cd^2\right)}-\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2p-3}}{(2p+3)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{679}$$

$$\frac{-\frac{ce(-2adf(p+2)+ae^2-cd^2(2p+3))\int (d+ex)^{-2(p+1)}(cx^2+a)^p dx}{ae^2+cd^2}-\frac{(a+cx^2)^{p+1}(d+ex)^{-2(p+1)}(ae^2f(2p+3)-cd(df-2e^2(p+2)))}{2(p+1)(ae^2+cd^2)}}{(2p+3)(ae^2+cd^2)}-\frac{(e^2-df)(a+cx^2)^{p+1}(d+ex)^{-2p-3}}{(2p+3)(ae^2+cd^2)}$$

$\Big\downarrow$ 489

$$\frac{\frac{ce(\sqrt{-a}-\sqrt{c}x)(a+cx^2)^p(d+ex)^{-2p-1}(-2adf(p+2)+ae^2-cd^2(2p+3))\left(-\frac{(\sqrt{-a}+\sqrt{c}x)(\sqrt{-a}e+\sqrt{c}d)}{(\sqrt{-a}-\sqrt{c}x)(\sqrt{c}d-\sqrt{-a}e)}\right)^{-p}\text{Hypergeometric2F1}\left(-2p-1,-p,-2p,\frac{(\sqrt{}}{}\right.}{(2p+1)(\sqrt{-a}e+\sqrt{c}d)(ae^2+cd^2)}}{(2p+3)(ae^2+cd^2)}$$

$$\frac{(e^2-df)(a+cx^2)^{p+1}(d+ex)^{-2p-3}}{(2p+3)(ae^2+cd^2)}$$

input
```
Int[(d + e*x)^(-4 - 2*p)*(e + f*x)*(a + c*x^2)^p,x]
```

output
```
-(((e^2 - d*f)*(d + e*x)^(-3 - 2*p)*(a + c*x^2)^(1 + p))/((c*d^2 + a*e^2)*
(3 + 2*p))) + (-1/2*((a*e^2*f*(3 + 2*p) - c*d*(d*f - 2*e^2*(2 + p)))*(a +
c*x^2)^(1 + p))/((c*d^2 + a*e^2)*(1 + p)*(d + e*x)^(2*(1 + p))) + (c*e*(a*
e^2 - 2*a*d*f*(2 + p) - c*d^2*(3 + 2*p))*(Sqrt[-a] - Sqrt[c]*x)*(d + e*x)^
(-1 - 2*p)*(a + c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]
*Sqrt[c]*(d + e*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]*x))])/((
Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + 2*p)*(-(((Sqrt[c]*d + Sqrt[-a
]*e)*(Sqrt[-a] + Sqrt[c]*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]
*x))))^p))/((c*d^2 + a*e^2)*(3 + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 689
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && ILtQ[Sim
plify[m + 2*p + 3], 0] && NeQ[m, -1]
```

Maple [F]

$$\int (ex+d)^{-4-2p} (fx+e) (c\,x^2+a)^p \,dx$$

input
```
int((e*x+d)^(-4-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

output
```
int((e*x+d)^(-4-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^{-4-2p}(e+fx)\left(a+cx^2\right)^p\,dx = \int (fx+e)\left(cx^2+a\right)^p(ex+d)^{-2\,p-4}\,dx$$

input
```
integrate((e*x+d)^(-4-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 4), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{-4-2p}(e+fx)\left(a+cx^2\right)^p\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(-4-2*p)*(f*x+e)*(c*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^{-4-2p}(e+fx)\left(a+cx^2\right)^p\,dx = \int (fx+e)(cx^2+a)^p(ex+d)^{-2p-4}\,dx$$

input
```
integrate((e*x+d)^(-4-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 4), x)
```

Giac [F]

$$\int (d+ex)^{-4-2p}(e+fx)\left(a+cx^2\right)^p\,dx = \int (fx+e)(cx^2+a)^p(ex+d)^{-2p-4}\,dx$$

input
```
integrate((e*x+d)^(-4-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{-4-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int \frac{(e+f\,x)\,\left(c\,x^2+a\right)^p}{(d+e\,x)^{2\,p+4}}\,dx$$

input
```
int(((e + f*x)*(a + c*x^2)^p)/(d + e*x)^(2*p + 4),x)
```

output
```
int(((e + f*x)*(a + c*x^2)^p)/(d + e*x)^(2*p + 4), x)
```

Reduce [F]

$$\int (d+ex)^{-4-2p}(e+fx)\left(a+cx^2\right)^p\,dx$$
$$=\left(\int \frac{\left(c\,x^2+a\right)^p}{(ex+d)^{2p}\,d^4+4\,(ex+d)^{2p}\,d^3ex+6\,(ex+d)^{2p}\,d^2e^2x^2+4\,(ex+d)^{2p}\,d\,e^3x^3+(ex+d)^{2p}\,e^4x^4}dx\right)e$$
$$+\left(\int \frac{\left(c\,x^2+a\right)^p x}{(ex+d)^{2p}\,d^4+4\,(ex+d)^{2p}\,d^3ex+6\,(ex+d)^{2p}\,d^2e^2x^2+4\,(ex+d)^{2p}\,d\,e^3x^3+(ex+d)^{2p}\,e^4x^4}dx\right)$$

input
```
int((e*x+d)^(-4-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

output
```
int((a + c*x**2)**p/((d + e*x)**(2*p)*d**4 + 4*(d + e*x)**(2*p)*d**3*e*x +
 6*(d + e*x)**(2*p)*d**2*e**2*x**2 + 4*(d + e*x)**(2*p)*d*e**3*x**3 + (d +
 e*x)**(2*p)*e**4*x**4),x)*e + int(((a + c*x**2)**p*x)/((d + e*x)**(2*p)*d
**4 + 4*(d + e*x)**(2*p)*d**3*e*x + 6*(d + e*x)**(2*p)*d**2*e**2*x**2 + 4*
(d + e*x)**(2*p)*d*e**3*x**3 + (d + e*x)**(2*p)*e**4*x**4),x)*f
```

3.305 $\int (d+ex)^{-3-2p}(e+fx)\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 26, antiderivative size = 283

$$\int (d+ex)^{-3-2p}(e+fx)\left(a+cx^2\right)^p\,dx=-\frac{\left(e^2-df\right)(d+ex)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2\left(cd^2+ae^2\right)(1+p)}-\frac{e(cd+af)\left(\sqrt{-a}-\sqrt{c}x\right)\left(-\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{-a}+\sqrt{c}x\right)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)^{-p}(d+ex)^{-1-2p}\left(a+cx^2\right)^p\,\text{Hypergeometric2F1}\left(\right.}{\left(\sqrt{c}d+\sqrt{-a}e\right)\left(cd^2+ae^2\right)(1+2p)}$$

output
```
-1/2*(-d*f+e^2)*(c*x^2+a)^(p+1)/(a*e^2+c*d^2)/(p+1)/((e*x+d)^(2*p+2))-e*(a
*f+c*d)*((-a)^(1/2)-c^(1/2)*x)*(e*x+d)^(-1-2*p)*(c*x^2+a)^p*hypergeom([-p,
 -1-2*p],[-2*p],2*(-a)^(1/2)*c^(1/2)*(e*x+d)/(c^(1/2)*d-(-a)^(1/2)*e)/((-a
)^(1/2)-c^(1/2)*x))/(c^(1/2)*d+(-a)^(1/2)*e)/(a*e^2+c*d^2)/(1+2*p)/((-(c^(
1/2)*d+(-a)^(1/2)*e)*((-a)^(1/2)+c^(1/2)*x)/(c^(1/2)*d-(-a)^(1/2)*e)/((-a)
^(1/2)-c^(1/2)*x))^p)
```

Mathematica [F]

$$\int (d+ex)^{-3-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (d+ex)^{-3-2p}(e+fx)\left(a+cx^2\right)^p\,dx$$

input
```
Integrate[(d + e*x)^(-3 - 2*p)*(e + f*x)*(a + c*x^2)^p,x]
```

output
```
Integrate[(d + e*x)^(-3 - 2*p)*(e + f*x)*(a + c*x^2)^p, x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {679, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+cx^2\right)^p(d+ex)^{-2p-3}\,dx$$

$$\Big\downarrow \text{679}$$

$$\frac{e(af+cd)\int (d+ex)^{-2(p+1)}\left(cx^2+a\right)^p\,dx}{ae^2+cd^2}-\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+1)}}{2(p+1)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{489}$$

$$-\frac{e\left(\sqrt{-a}-\sqrt{c}x\right)(af+cd)\left(a+cx^2\right)^p(d+ex)^{-2p-1}\left(-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}\right)^{-p}\text{Hypergeometric2F1}\left(-2p-\right.}{(2p+1)\left(\sqrt{-a}e+\sqrt{c}d\right)\left(ae^2+cd^2\right)}$$
$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^{p+1}(d+ex)^{-2(p+1)}}{2(p+1)\left(ae^2+cd^2\right)}$$

input
```
Int[(d + e*x)^(-3 - 2*p)*(e + f*x)*(a + c*x^2)^p,x]
```

output
```
-1/2*((e^2 - d*f)*(a + c*x^2)^(1 + p))/((c*d^2 + a*e^2)*(1 + p)*(d + e*x)^
(2*(1 + p))) - (e*(c*d + a*f)*(Sqrt[-a] - Sqrt[c]*x)*(d + e*x)^(-1 - 2*p)*
(a + c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[c]*(d
 + e*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]*x))])/((Sqrt[c]*d +
 Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + 2*p)*(-(((Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[
-a] + Sqrt[c]*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]*x))))^p)
```

Defintions of rubi rules used

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [F]

$$\int (ex+d)^{-3-2p} (fx+e) (c\,x^2+a)^p \,dx$$

input
```
int((e*x+d)^(-3-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

output
```
int((e*x+d)^(-3-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^{-3-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (fx+e)\left(cx^2+a\right)^p(ex+d)^{-2\,p-3}\,dx$$

input
```
integrate((e*x+d)^(-3-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{-3-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(-3-2*p)*(f*x+e)*(c*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^{-3-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (fx+e)\left(cx^2+a\right)^p(ex+d)^{-2\,p-3}\,dx$$

input
```
integrate((e*x+d)^(-3-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 3), x)
```

Giac [F]

$$\int (d+ex)^{-3-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (fx+e)\left(cx^2+a\right)^p(ex+d)^{-2\,p-3}\,dx$$

input
```
integrate((e*x+d)^(-3-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{-3-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int \frac{(e+f\,x)\,\left(c\,x^2+a\right)^p}{(d+e\,x)^{2\,p+3}}\,dx$$

input
```
int(((e + f*x)*(a + c*x^2)^p)/(d + e*x)^(2*p + 3),x)
```

output
```
int(((e + f*x)*(a + c*x^2)^p)/(d + e*x)^(2*p + 3), x)
```

Reduce [F]

$$\begin{aligned}&\int (d+ex)^{-3-2p}(e+fx)\left(a+cx^2\right)^p\,dx\\&=\left(\int \frac{\left(c\,x^2+a\right)^p}{(ex+d)^{2p}\,d^3+3\,(ex+d)^{2p}\,d^2ex+3\,(ex+d)^{2p}\,d\,e^2x^2+(ex+d)^{2p}\,e^3x^3}dx\right)e\\&\quad+\left(\int \frac{\left(c\,x^2+a\right)^p x}{(ex+d)^{2p}\,d^3+3\,(ex+d)^{2p}\,d^2ex+3\,(ex+d)^{2p}\,d\,e^2x^2+(ex+d)^{2p}\,e^3x^3}dx\right)f\end{aligned}$$

input
```
int((e*x+d)^(-3-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

output
```
int((a + c*x**2)**p/((d + e*x)**(2*p)*d**3 + 3*(d + e*x)**(2*p)*d**2*e*x +
 3*(d + e*x)**(2*p)*d*e**2*x**2 + (d + e*x)**(2*p)*e**3*x**3),x)*e + int((
(a + c*x**2)**p*x)/((d + e*x)**(2*p)*d**3 + 3*(d + e*x)**(2*p)*d**2*e*x +
3*(d + e*x)**(2*p)*d*e**2*x**2 + (d + e*x)**(2*p)*e**3*x**3),x)*f
```

3.306 $\int (d+ex)^{-2-2p}(e+fx)\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 26, antiderivative size = 377

$$\int (d+ex)^{-2-2p}(e+fx)\left(a+cx^2\right)^p\,dx =$$

$$-\frac{f(d+ex)^{-2p}\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{}{d}\right.}{2e^2p}$$

$$-\frac{\left(e^2-df\right)\left(\sqrt{-a}-\sqrt{c}x\right)\left(-\frac{\left(\sqrt{c}d+\sqrt{-a}e\right)\left(\sqrt{-a}+\sqrt{c}x\right)}{\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{-a}-\sqrt{c}x\right)}\right)^{-p}(d+ex)^{-1-2p}\left(a+cx^2\right)^p\text{Hypergeometric2F1}\left(-\right.}{e\left(\sqrt{c}d+\sqrt{-a}e\right)(1+2p)}$$

output

```
-1/2*f*(c*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(e*x+d)/(d-(-a)^(1/2)*e/c^(1/
2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e^2/p/((e*x+d)^(2*p))/((1-(e*x+d)/(d
-(-a)^(1/2)*e/c^(1/2)))^p)/((1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))^p)-(-d*f+
e^2)*((-a)^(1/2)-c^(1/2)*x)*(e*x+d)^(-1-2*p)*(c*x^2+a)^p*hypergeom([-p, -1
-2*p],[-2*p],2*(-a)^(1/2)*c^(1/2)*(e*x+d)/(c^(1/2)*d-(-a)^(1/2)*e)/((-a)^(
1/2)-c^(1/2)*x))/e/(c^(1/2)*d+(-a)^(1/2)*e)/(1+2*p)/((-(c^(1/2)*d+(-a)^(1/
2)*e)*((-a)^(1/2)+c^(1/2)*x)/(c^(1/2)*d-(-a)^(1/2)*e)/((-a)^(1/2)-c^(1/2)*
x))^p)
```

Mathematica [F]

$$\int (d+ex)^{-2-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (d+ex)^{-2-2p}(e+fx)\left(a+cx^2\right)^p\,dx$$

input
```
Integrate[(d + e*x)^(-2 - 2*p)*(e + f*x)*(a + c*x^2)^p,x]
```

output
```
Integrate[(d + e*x)^(-2 - 2*p)*(e + f*x)*(a + c*x^2)^p, x]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {719, 489, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+cx^2\right)^p (d+ex)^{-2p-2}\,dx$$

$$\Big\downarrow \text{719}$$

$$\frac{\left(e^2-df\right)\int (d+ex)^{-2(p+1)}\left(cx^2+a\right)^p\,dx}{e}+\frac{f\int (d+ex)^{-2p-1}\left(cx^2+a\right)^p\,dx}{e}$$

$$\Big\downarrow \text{489}$$

$$\frac{f\int (d+ex)^{-2p-1}\left(cx^2+a\right)^p\,dx}{e}-\frac{\left(\sqrt{-a}-\sqrt{c}x\right)\left(e^2-df\right)\left(a+cx^2\right)^p (d+ex)^{-2p-1}\left(-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}\right)^{-p}\text{Hypergeometric2F1}\left(-2p-1,-\right.}{e(2p+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

$$\Big\downarrow \text{514}$$

$$\frac{f\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p}\int(d+ex)^{-2p-1}\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^p\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^p d(d+ex)}{e^2}-\frac{\left(\sqrt{-a}-\sqrt{c}x\right)\left(e^2-df\right)\left(a+cx^2\right)^p(d+ex)^{-2p-1}\left(-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}\right)^{-p}\text{Hypergeometric2F1}\left(-2p-1,-\right.}{e(2p+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

$$\Big\downarrow 150$$

$$-\frac{f\left(a+cx^2\right)^p(d+ex)^{-2p}\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{2e^2p}$$
$$\frac{\left(\sqrt{-a}-\sqrt{c}x\right)\left(e^2-df\right)\left(a+cx^2\right)^p(d+ex)^{-2p-1}\left(-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}e+\sqrt{c}d\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}d-\sqrt{-a}e\right)}\right)^{-p}\text{Hypergeometric2F1}\left(-2p-1,-\right.}{e(2p+1)\left(\sqrt{-a}e+\sqrt{c}d\right)}$$

input
```
Int[(d + e*x)^(-2 - 2*p)*(e + f*x)*(a + c*x^2)^p,x]
```

output
```
-1/2*(f*(a + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (d + e*x)/(d - (Sqrt
[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e^2*p*(d + e*x)^
(2*p)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1 - (d + e*x)/(d + (Sq
rt[-a]*e)/Sqrt[c]))^p) - ((e^2 - d*f)*(Sqrt[-a] - Sqrt[c]*x)*(d + e*x)^(-1
 - 2*p)*(a + c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sq
rt[c]*(d + e*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]*x))])/(e*(S
qrt[c]*d + Sqrt[-a]*e)*(1 + 2*p)*(-(((Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[-a] +
Sqrt[c]*x))/((Sqrt[c]*d - Sqrt[-a]*e)*(Sqrt[-a] - Sqrt[c]*x))))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int (ex+d)^{-2p-2}(fx+e)\left(c\,x^2+a\right)^p dx$$

input
```
int((e*x+d)^(-2*p-2)*(f*x+e)*(c*x^2+a)^p,x)
```

output
```
int((e*x+d)^(-2*p-2)*(f*x+e)*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^{-2-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (fx+e)\left(cx^2+a\right)^p(ex+d)^{-2p-2}\,dx$$

input
```
integrate((e*x+d)^(-2-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{-2-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**(-2-2*p)*(f*x+e)*(c*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^{-2-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (fx+e)\left(cx^2+a\right)^p(ex+d)^{-2p-2}\,dx$$

input
```
integrate((e*x+d)^(-2-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 2), x)
```

Giac [F]

$$\int (d+ex)^{-2-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (fx+e)\left(cx^2+a\right)^p(ex+d)^{-2p-2}\,dx$$

input
```
integrate((e*x+d)^(-2-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{-2-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int \frac{(e+f\,x)\,\left(c\,x^2+a\right)^p}{(d+e\,x)^{2\,p+2}}\,dx$$

input
```
int(((e + f*x)*(a + c*x^2)^p)/(d + e*x)^(2*p + 2),x)
```

output
```
int(((e + f*x)*(a + c*x^2)^p)/(d + e*x)^(2*p + 2), x)
```

Reduce [F]

$$\begin{aligned}&\int (d+ex)^{-2-2p}(e+fx)\left(a+cx^2\right)^p\,dx\\&=\left(\int \frac{\left(c\,x^2+a\right)^p}{(ex+d)^{2p}\,d^2+2\,(ex+d)^{2p}\,dex+(ex+d)^{2p}\,e^2x^2}dx\right)e\\&\quad+\left(\int \frac{\left(c\,x^2+a\right)^p x}{(ex+d)^{2p}\,d^2+2\,(ex+d)^{2p}\,dex+(ex+d)^{2p}\,e^2x^2}dx\right)f\end{aligned}$$

input
```
int((e*x+d)^(-2-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

output
```
int((a + c*x**2)**p/((d + e*x)**(2*p)*d**2 + 2*(d + e*x)**(2*p)*d*e*x + (d
 + e*x)**(2*p)*e**2*x**2),x)*e + int(((a + c*x**2)**p*x)/((d + e*x)**(2*p)
*d**2 + 2*(d + e*x)**(2*p)*d*e*x + (d + e*x)**(2*p)*e**2*x**2),x)*f
```

3.307 $\int (d+ex)^{-1-2p}(e+fx)\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 26, antiderivative size = 325

$$\int (d+ex)^{-1-2p}(e+fx)\left(a+cx^2\right)^p\,dx$$

$$= \frac{f(d+ex)^{1-2p}\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\ldots\right.}{e^2(1-2p)}$$

$$-\frac{\left(e^2-df\right)(d+ex)^{-2p}\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\ldots\right.}{2e^2p}$$

output
```
f*(e*x+d)^(1-2*p)*(c*x^2+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,(e*x+d)/(d-(-a)^(
1/2)*e/c^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e^2/(1-2*p)/((1-(e*x+d)/
(d-(-a)^(1/2)*e/c^(1/2)))^p)/((1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))^p)-1/2*
(-d*f+e^2)*(c*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(e*x+d)/(d-(-a)^(1/2)*e/c
^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e^2/p/((e*x+d)^(2*p))/((1-(e*x+d
)/(d-(-a)^(1/2)*e/c^(1/2)))^p)/((1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.78

$$\int (d+ex)^{-1-2p}(e+fx)\left(a+cx^2\right)^p\,dx =$$

$$-\frac{\left(\frac{e\left(\sqrt{-\frac{a}{c}}-x\right)}{d+\sqrt{-\frac{a}{c}}e}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{c}}+x\right)}{-d+\sqrt{-\frac{a}{c}}e}\right)^{-p}(d+ex)^{-2p}\left(a+cx^2\right)^p\left(2fp(d+ex)\,\text{AppellF1}\left(1-2p,-p,-p,2-\right.\right.}{2e^2p(-1}$$

input
```
Integrate[(d + e*x)^(-1 - 2*p)*(e + f*x)*(a + c*x^2)^p,x]
```

output
```
-1/2*((a + c*x^2)^p*(2*f*p*(d + e*x)*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d
 + e*x)/(d - Sqrt[-(a/c)]*e), (d + e*x)/(d + Sqrt[-(a/c)]*e)] + (e^2 - d*f
)*(-1 + 2*p)*AppellF1[-2*p, -p, -p, 1 - 2*p, (d + e*x)/(d - Sqrt[-(a/c)]*e
), (d + e*x)/(d + Sqrt[-(a/c)]*e)]))/(e^2*p*(-1 + 2*p)*((e*(Sqrt[-(a/c)] -
 x))/(d + Sqrt[-(a/c)]*e))^p*((e*(Sqrt[-(a/c)] + x))/(-d + Sqrt[-(a/c)]*e)
)^p*(d + e*x)^(2*p))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {719, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+cx^2\right)^p(d+ex)^{-2p-1}dx$$

$$\Big\downarrow \ 719$$

$$\frac{\left(e^2-df\right)\int (d+ex)^{-2p-1}\left(cx^2+a\right)^pdx}{e}+\frac{f\int (d+ex)^{-2p}\left(cx^2+a\right)^pdx}{e}$$

$$\Big\downarrow \ 514$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p}\int(d+ex)^{-2p-1}\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^p\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^p d(d+}{e^2}$$

$$\frac{f\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p}\int(d+ex)^{-2p}\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^p\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^p d(d+ex)}{e^2}$$

$$\Big\downarrow \text{150}$$

$$\frac{f\left(a+cx^2\right)^p(d+ex)^{1-2p}\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right.}{e^2(1-2p)}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^p(d+ex)^{-2p}\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\right.}{2e^2p}$$

input
```
Int[(d + e*x)^(-1 - 2*p)*(e + f*x)*(a + c*x^2)^p,x]
```

output
```
(f*(d + e*x)^(1 - 2*p)*(a + c*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d
 + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])
/(e^2*(1 - 2*p)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1 - (d + e*x
)/(d + (Sqrt[-a]*e)/Sqrt[c]))^p) - ((e^2 - d*f)*(a + c*x^2)^p*AppellF1[-2*
p, -p, -p, 1 - 2*p, (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (
Sqrt[-a]*e)/Sqrt[c])])/(2*e^2*p*(d + e*x)^(2*p)*(1 - (d + e*x)/(d - (Sqrt[
-a]*e)/Sqrt[c]))^p*(1 - (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int (ex+d)^{-1-2p} (fx+e) (c\,x^2+a)^p\,dx$$

input
```
int((e*x+d)^(-1-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

output
```
int((e*x+d)^(-1-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^{-1-2p}(e+fx)\left(a+cx^2\right)^p\,dx = \int (fx+e)(cx^2+a)^p(ex+d)^{-2\,p-1}\,dx$$

input
```
integrate((e*x+d)^(-1-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{-1-2p}(e+fx)\left(a+cx^2\right)^p\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(-1-2*p)*(f*x+e)*(c*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^{-1-2p}(e+fx)\left(a+cx^2\right)^p\,dx = \int (fx+e)\left(cx^2+a\right)^p(ex+d)^{-2\,p-1}\,dx$$

input
```
integrate((e*x+d)^(-1-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 1), x)
```

Giac [F]

$$\int (d+ex)^{-1-2p}(e+fx)\left(a+cx^2\right)^p\,dx = \int (fx+e)\left(cx^2+a\right)^p(ex+d)^{-2\,p-1}\,dx$$

input
```
integrate((e*x+d)^(-1-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{-1-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int \frac{(e+f\,x)\,\left(c\,x^2+a\right)^p}{\left(d+e\,x\right)^{2\,p+1}}\,dx$$

input
```
int(((e + f*x)*(a + c*x^2)^p)/(d + e*x)^(2*p + 1),x)
```

output
```
int(((e + f*x)*(a + c*x^2)^p)/(d + e*x)^(2*p + 1), x)
```

Reduce [F]

$$\int (d+ex)^{-1-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\left(\int \frac{\left(c\,x^2+a\right)^p}{\left(ex+d\right)^{2p}d+\left(ex+d\right)^{2p}ex}dx\right)e+\left(\int \frac{\left(c\,x^2+a\right)^p x}{\left(ex+d\right)^{2p}d+\left(ex+d\right)^{2p}ex}dx\right)f$$

input
```
int((e*x+d)^(-1-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

output
```
int((a + c*x**2)**p/((d + e*x)**(2*p)*d + (d + e*x)**(2*p)*e*x),x)*e + int
(((a + c*x**2)**p*x)/((d + e*x)**(2*p)*d + (d + e*x)**(2*p)*e*x),x)*f
```

3.308 $\int (d+ex)^{-2p}(e+fx)\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 333

$$\int (d+ex)^{-2p}(e+fx)\left(a+cx^2\right)^p\,dx$$

$$= \frac{\left(e^2-df\right)(d+ex)^{1-2p}\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,\ldots\right)}{e^2(1-2p)}$$

$$+\frac{f(d+ex)^{2-2p}\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\operatorname{AppellF1}\left(2-2p,-p,-p,3-2p,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\ldots\right)}{2e^2(1-p)}$$

output
```
(-d*f+e^2)*(e*x+d)^(1-2*p)*(c*x^2+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,(e*x+d)/
(d-(-a)^(1/2)*e/c^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e^2/(1-2*p)/((1
-(e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)))^p)/((1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2))
)^p)+1/2*f*(e*x+d)^(2-2*p)*(c*x^2+a)^p*AppellF1(2-2*p,-p,-p,3-2*p,(e*x+d)/
(d-(-a)^(1/2)*e/c^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e^2/(1-p)/((1-(
e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)))^p)/((1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))^
p)
```

Mathematica [F]

$$\int (d+ex)^{-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (d+ex)^{-2p}(e+fx)\left(a+cx^2\right)^p\,dx$$

input
```
Integrate[((e + f*x)*(a + c*x^2)^p)/(d + e*x)^(2*p),x]
```

output
```
Integrate[((e + f*x)*(a + c*x^2)^p)/(d + e*x)^(2*p), x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {719, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+cx^2\right)^p(d+ex)^{-2p}dx$$

$$\Big\downarrow \text{719}$$

$$\frac{\left(e^2-df\right)\int (d+ex)^{-2p}\left(cx^2+a\right)^pdx}{e}+\frac{f\int (d+ex)^{1-2p}\left(cx^2+a\right)^pdx}{e}$$

$$\Big\downarrow \text{514}$$

$$\frac{f\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-ae}}{\sqrt{c}}+d}\right)^{-p}\int (d+ex)^{1-2p}\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^p\left(1-\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^pd(d+ex)}{e^2}+\frac{\left(e^2-df\right)\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-ae}}{\sqrt{c}}+d}\right)^{-p}\int (d+ex)^{-2p}\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^p\left(1-\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^pd(d+ex)}{e^2}$$

$$\Big\downarrow \text{150}$$

$$\frac{(e^2-df)\left(a+cx^2\right)^p(d+ex)^{1-2p}\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right.}{e^2(1-2p)}$$
$$\frac{f\left(a+cx^2\right)^p(d+ex)^{2-2p}\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p}\text{AppellF1}\left(2-2p,-p,-p,3-2p,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right.}{2e^2(1-p)}$$

input
```
Int[((e + f*x)*(a + c*x^2)^p)/(d + e*x)^(2*p),x]
```

output
```
((e^2 - d*f)*(d + e*x)^(1 - 2*p)*(a + c*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2
 - 2*p, (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/
Sqrt[c])])/(e^2*(1 - 2*p)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1
- (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c]))^p) + (f*(d + e*x)^(2 - 2*p)*(a + c
*x^2)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, (d + e*x)/(d - (Sqrt[-a]*e)/Sqr
t[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(2*e^2*(1 - p)*(1 - (d + e*x
)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1 - (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c]))
^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int (fx+e)\left(c\,x^{2}+a\right)^{p}(ex+d)^{-2p}\,dx$$

input
```
int((f*x+e)*(c*x^2+a)^p/((e*x+d)^(2*p)),x)
```

output
```
int((f*x+e)*(c*x^2+a)^p/((e*x+d)^(2*p)),x)
```

Fricas [F]

$$\int (d+ex)^{-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int \frac{(fx+e)(cx^2+a)^p}{(ex+d)^{2\,p}}\,dx$$

input
```
integrate((f*x+e)*(c*x^2+a)^p/((e*x+d)^(2*p)),x, algorithm="fricas")
```

output
```
integral((f*x + e)*(c*x^2 + a)^p/(e*x + d)^(2*p), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)*(c*x**2+a)**p/((e*x+d)**(2*p)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^{-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int \frac{(fx+e)(cx^2+a)^p}{(ex+d)^{2p}}\,dx$$

input
```
integrate((f*x+e)*(c*x^2+a)^p/((e*x+d)^(2*p)),x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(c*x^2 + a)^p/(e*x + d)^(2*p), x)
```

Giac [F]

$$\int (d+ex)^{-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int \frac{(fx+e)(cx^2+a)^p}{(ex+d)^{2p}}\,dx$$

input
```
integrate((f*x+e)*(c*x^2+a)^p/((e*x+d)^(2*p)),x, algorithm="giac")
```

output
```
integrate((f*x + e)*(c*x^2 + a)^p/(e*x + d)^(2*p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int \frac{(e+f\,x)\,\left(c\,x^2+a\right)^p}{{(d+e\,x)}^{2\,p}}\,dx$$

input
```
int(((e + f*x)*(a + c*x^2)^p)/(d + e*x)^(2*p),x)
```

output
```
int(((e + f*x)*(a + c*x^2)^p)/(d + e*x)^(2*p), x)
```

Reduce [F]

$$\int (d+ex)^{-2p}(e+fx)\left(a+cx^2\right)^p\,dx = \left(\int \frac{(c\,x^2+a)^p}{(ex+d)^{2p}}dx\right)e + \left(\int \frac{(c\,x^2+a)^p\,x}{(ex+d)^{2p}}dx\right)f$$

input
```
int((f*x+e)*(c*x^2+a)^p/((e*x+d)^(2*p)),x)
```

output
```
int((a + c*x**2)**p/(d + e*x)**(2*p),x)*e + int(((a + c*x**2)**p*x)/(d + e
*x)**(2*p),x)*f
```

3.309 $\int (d+ex)^{1-2p}(e+fx)\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 26, antiderivative size = 333

$$\int (d+ex)^{1-2p}(e+fx)\left(a+cx^2\right)^p\,dx$$
$$=\frac{\left(e^2-df\right)(d+ex)^{2-2p}\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\operatorname{AppellF1}\left(2-2p,-p,-p,3-2p,\ldots\right.}{2e^2(1-p)}$$
$$+\frac{f(d+ex)^{3-2p}\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\operatorname{AppellF1}\left(3-2p,-p,-p,4-2p,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\ldots\right.}{e^2(3-2p)}$$

output
```
1/2*(-d*f+e^2)*(e*x+d)^(2-2*p)*(c*x^2+a)^p*AppellF1(2-2*p,-p,-p,3-2*p,(e*x
+d)/(d-(-a)^(1/2)*e/c^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e^2/(1-p)/(
(1-(e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)))^p)/((1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2
)))^p)+f*(e*x+d)^(3-2*p)*(c*x^2+a)^p*AppellF1(3-2*p,-p,-p,4-2*p,(e*x+d)/(d
-(-a)^(1/2)*e/c^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e^2/(3-2*p)/((1-(
e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)))^p)/((1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))^
p)
```

Mathematica [F]

$$\int (d+ex)^{1-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (d+ex)^{1-2p}(e+fx)\left(a+cx^2\right)^p\,dx$$

input
```
Integrate[(d + e*x)^(1 - 2*p)*(e + f*x)*(a + c*x^2)^p,x]
```

output
```
Integrate[(d + e*x)^(1 - 2*p)*(e + f*x)*(a + c*x^2)^p, x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {719, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+cx^2\right)^p(d+ex)^{1-2p}\,dx$$

$$\Big\downarrow \text{719}$$

$$\frac{\left(e^2-df\right)\int (d+ex)^{1-2p}\left(cx^2+a\right)^p\,dx}{e}+\frac{f\int (d+ex)^{2-2p}\left(cx^2+a\right)^p\,dx}{e}$$

$$\Big\downarrow \text{514}$$

$$\frac{\left(e^2-df\right)\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-ae}}{\sqrt{c}}+d}\right)^{-p}\int (d+ex)^{1-2p}\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^p\left(1-\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^p d(d+ex)}{e^2}$$

$$\frac{f\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-ae}}{\sqrt{c}}+d}\right)^{-p}\int (d+ex)^{2-2p}\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^p\left(1-\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^p d(d+ex)}{e^2}$$

$$\Big\downarrow \text{150}$$

$$\frac{(e^2-df)\left(a+cx^2\right)^p(d+ex)^{2-2p}\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p}\text{AppellF1}\left(2-2p,-p,-p,3-2p,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right.}{2e^2(1-p)}$$

$$\frac{f\left(a+cx^2\right)^p(d+ex)^{3-2p}\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p}\text{AppellF1}\left(3-2p,-p,-p,4-2p,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right.}{e^2(3-2p)}$$

input
```
Int[(d + e*x)^(1 - 2*p)*(e + f*x)*(a + c*x^2)^p,x]
```

output
```
((e^2 - d*f)*(d + e*x)^(2 - 2*p)*(a + c*x^2)^p*AppellF1[2 - 2*p, -p, -p, 3
 - 2*p, (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/
Sqrt[c])])/(2*e^2*(1 - p)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1
- (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c]))^p) + (f*(d + e*x)^(3 - 2*p)*(a + c
*x^2)^p*AppellF1[3 - 2*p, -p, -p, 4 - 2*p, (d + e*x)/(d - (Sqrt[-a]*e)/Sqr
t[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e^2*(3 - 2*p)*(1 - (d + e*x
)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1 - (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c]))
^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

Maple [F]

$$\int (ex+d)^{1-2p} (fx+e) \left(c\,x^2+a\right)^p dx$$

input
```
int((e*x+d)^(1-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

output
```
int((e*x+d)^(1-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^{1-2p}(e+fx) \left(a+cx^2\right)^p \, dx = \int (fx+e)\left(cx^2+a\right)^p (ex+d)^{-2\,p+1} \, dx$$

input
```
integrate((e*x+d)^(1-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^{1-2p}(e+fx) \left(a+cx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(1-2*p)*(f*x+e)*(c*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^{1-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (fx+e)(cx^2+a)^p(ex+d)^{-2\,p+1}\,dx$$

input
```
integrate((e*x+d)^(1-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p + 1), x)
```

Giac [F]

$$\int (d+ex)^{1-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (fx+e)(cx^2+a)^p(ex+d)^{-2\,p+1}\,dx$$

input
```
integrate((e*x+d)^(1-2*p)*(f*x+e)*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(c*x^2 + a)^p*(e*x + d)^(-2*p + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{1-2p}(e+fx)\left(a+cx^2\right)^p\,dx=\int (e+f\,x)\,\left(c\,x^2+a\right)^p\,(d+e\,x)^{1-2\,p}\,dx$$

input
```
int((e + f*x)*(a + c*x^2)^p*(d + e*x)^(1 - 2*p),x)
```

output
```
int((e + f*x)*(a + c*x^2)^p*(d + e*x)^(1 - 2*p), x)
```

Reduce [F]

$$\int (d+ex)^{1-2p}(e+fx)\left(a+cx^2\right)^p\,dx = \left(\int \frac{(c\,x^2+a)^p}{(ex+d)^{2p}}dx\right)de + \left(\int \frac{(c\,x^2+a)^p\,x^2}{(ex+d)^{2p}}dx\right)ef + \left(\int \frac{(c\,x^2+a)^p\,x}{(ex+d)^{2p}}dx\right)df + \left(\int \frac{(c\,x^2+a)^p\,x}{(ex+d)^{2p}}dx\right)e^2$$

input
```
int((e*x+d)^(1-2*p)*(f*x+e)*(c*x^2+a)^p,x)
```

output
```
int((a + c*x**2)**p/(d + e*x)**(2*p),x)*d*e + int(((a + c*x**2)**p*x**2)/(
d + e*x)**(2*p),x)*e*f + int(((a + c*x**2)**p*x)/(d + e*x)**(2*p),x)*d*f +
 int(((a + c*x**2)**p*x)/(d + e*x)**(2*p),x)*e**2
```

3.310 $\int (-ae+cdx)(d+ex)^{-3-2p}\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 31

$$\int (-ae+cdx)(d+ex)^{-3-2p}\left(a+cx^2\right)^p\,dx=\frac{(d+ex)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2(1+p)}$$

output
```
1/2*(c*x^2+a)^(p+1)/(p+1)/((e*x+d)^(2*p+2))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (-ae+cdx)(d+ex)^{-3-2p}\left(a+cx^2\right)^p\,dx=\frac{(d+ex)^{-2-2p}\left(a+cx^2\right)^{1+p}}{2(1+p)}$$

input
```
Integrate[(-(a*e) + c*d*x)*(d + e*x)^(-3 - 2*p)*(a + c*x^2)^p,x]
```

output
```
((d + e*x)^(-2 - 2*p)*(a + c*x^2)^(1 + p))/(2*(1 + p))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {677}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^p (d+ex)^{-2p-3}(cdx-ae)\,dx$$

$$\Big\downarrow \text{677}$$

$$\frac{(a+cx^2)^{p+1} (d+ex)^{-2(p+1)}}{2(p+1)}$$

input
```
Int[(-(a*e) + c*d*x)*(d + e*x)^(-3 - 2*p)*(a + c*x^2)^p,x]
```

output
```
(a + c*x^2)^(1 + p)/(2*(1 + p)*(d + e*x)^(2*(1 + p)))
```

Defintions of rubi rules used

rule 677
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] &&
 EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*g, 0]
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result
gosper	$\frac{(c\,x^2+a)^{p+1}(ex+d)^{-2p-2}}{2+2p}$
orering	$-\frac{(ex+d)(c\,x^2+a)(cdx-ae)(ex+d)^{-3-2p}(c\,x^2+a)^p}{2(p+1)(-cdx+ae)}$
parallelrisch	$\frac{x^3(c\,x^2+a)^p(ex+d)^{-3-2p}c^2e^2+x^2(c\,x^2+a)^p(ex+d)^{-3-2p}c^2de+x(c\,x^2+a)^p(ex+d)^{-3-2p}ac\,e^2+(c\,x^2+a)^p(ex+d)^{-3-2p}a}{2(p+1)ce}$

input
```
int((c*d*x-a*e)*(e*x+d)^(-3-2*p)*(c*x^2+a)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2/(p+1)*(c*x^2+a)^(p+1)*(e*x+d)^(-2*p-2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int(-ae+cdx)(d+ex)^{-3-2p}\left(a+cx^2\right)^p\,dx = \frac{(cex^3+cdx^2+aex+ad)(cx^2+a)^p(ex+d)^{-2\,p-3}}{2\,(p+1)}$$

input
```
integrate((c*d*x-a*e)*(e*x+d)^(-3-2*p)*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
1/2*(c*e*x^3 + c*d*x^2 + a*e*x + a*d)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 3)/(
p + 1)
```

Sympy [F(-1)]

Timed out.

$$\int(-ae+cdx)(d+ex)^{-3-2p}\left(a+cx^2\right)^p\,dx = \text{Timed out}$$

input
```
integrate((c*d*x-a*e)*(e*x+d)**(-3-2*p)*(c*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int (-ae+cdx)(d+ex)^{-3-2p}\left(a+cx^2\right)^p\,dx=\frac{(cx^2+a)e^{\left(p\log\left(cx^2+a\right)-2\,p\log\left(ex+d\right)\right)}}{2\left(e^2(p+1)x^2+2\,de(p+1)x+d^2(p+1)\right)}$$

input
```
integrate((c*d*x-a*e)*(e*x+d)^(-3-2*p)*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
1/2*(c*x^2 + a)*e^(p*log(c*x^2 + a) - 2*p*log(e*x + d))/(e^2*(p + 1)*x^2 +
 2*d*e*(p + 1)*x + d^2*(p + 1))
```

Giac [F]

$$\int (-ae+cdx)(d+ex)^{-3-2p}\left(a+cx^2\right)^p\,dx=\int (cdx-ae)\left(cx^2+a\right)^p(ex+d)^{-2\,p-3}\,dx$$

input
```
integrate((c*d*x-a*e)*(e*x+d)^(-3-2*p)*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*d*x - a*e)*(c*x^2 + a)^p*(e*x + d)^(-2*p - 3), x)
```

Mupad [B] (verification not implemented)

Time = 5.68 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.16

$$\int (-ae+cdx)(d+ex)^{-3-2p}\left(a+cx^2\right)^p\,dx$$
$$=\frac{\frac{a\,d\,\left(c\,x^2+a\right)^p}{2\,p+2}+\frac{a\,e\,x\,\left(c\,x^2+a\right)^p}{2\,p+2}+\frac{c\,d\,x^2\,\left(c\,x^2+a\right)^p}{2\,p+2}+\frac{c\,e\,x^3\,\left(c\,x^2+a\right)^p}{2\,p+2}}{\left(d+e\,x\right)^{2\,p+3}}$$

input
```
int(-((a*e - c*d*x)*(a + c*x^2)^p)/(d + e*x)^(2*p + 3),x)
```

output
```
((a*d*(a + c*x^2)^p)/(2*p + 2) + (a*e*x*(a + c*x^2)^p)/(2*p + 2) + (c*d*x^
2*(a + c*x^2)^p)/(2*p + 2) + (c*e*x^3*(a + c*x^2)^p)/(2*p + 2))/(d + e*x)^
(2*p + 3)
```

Reduce [F]

$$\int (-ae + cdx)(d + ex)^{-3-2p} \left(a + cx^2\right)^p \, dx$$
$$= -\left(\int \frac{(c\,x^2 + a)^p}{(ex + d)^{2p}\, d^3 + 3\,(ex + d)^{2p}\, d^2ex + 3\,(ex + d)^{2p}\, d\,e^2x^2 + (ex + d)^{2p}\, e^3x^3}dx\right) ae$$
$$+ \left(\int \frac{(c\,x^2 + a)^p\, x}{(ex + d)^{2p}\, d^3 + 3\,(ex + d)^{2p}\, d^2ex + 3\,(ex + d)^{2p}\, d\,e^2x^2 + (ex + d)^{2p}\, e^3x^3}dx\right) cd$$

input
```
int((c*d*x-a*e)*(e*x+d)^(-3-2*p)*(c*x^2+a)^p,x)
```

output
```
 - int((a + c*x**2)**p/((d + e*x)**(2*p)*d**3 + 3*(d + e*x)**(2*p)*d**2*e*
x + 3*(d + e*x)**(2*p)*d*e**2*x**2 + (d + e*x)**(2*p)*e**3*x**3),x)*a*e +
int(((a + c*x**2)**p*x)/((d + e*x)**(2*p)*d**3 + 3*(d + e*x)**(2*p)*d**2*e
*x + 3*(d + e*x)**(2*p)*d*e**2*x**2 + (d + e*x)**(2*p)*e**3*x**3),x)*c*d
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral',
        'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
        'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file