Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/1.2.1.3/95-1.2.1.3-c

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**182**]. This is test number [95].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	96.70 (176)	3.30 (6)
Mathematica	93.41 (170)	6.59 (12)
Maple	85.71 (156)	14.29 (26)
Fricas	70.33 (128)	29.67 (54)
Giac	52.20 (95)	47.80 (87)
Reduce	47.80 (87)	52.20 (95)
Mupad	46.70 (85)	53.30 (97)
Maxima	31.87 (58)	68.13 (124)
Sympy	27.47 (50)	72.53 (132)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

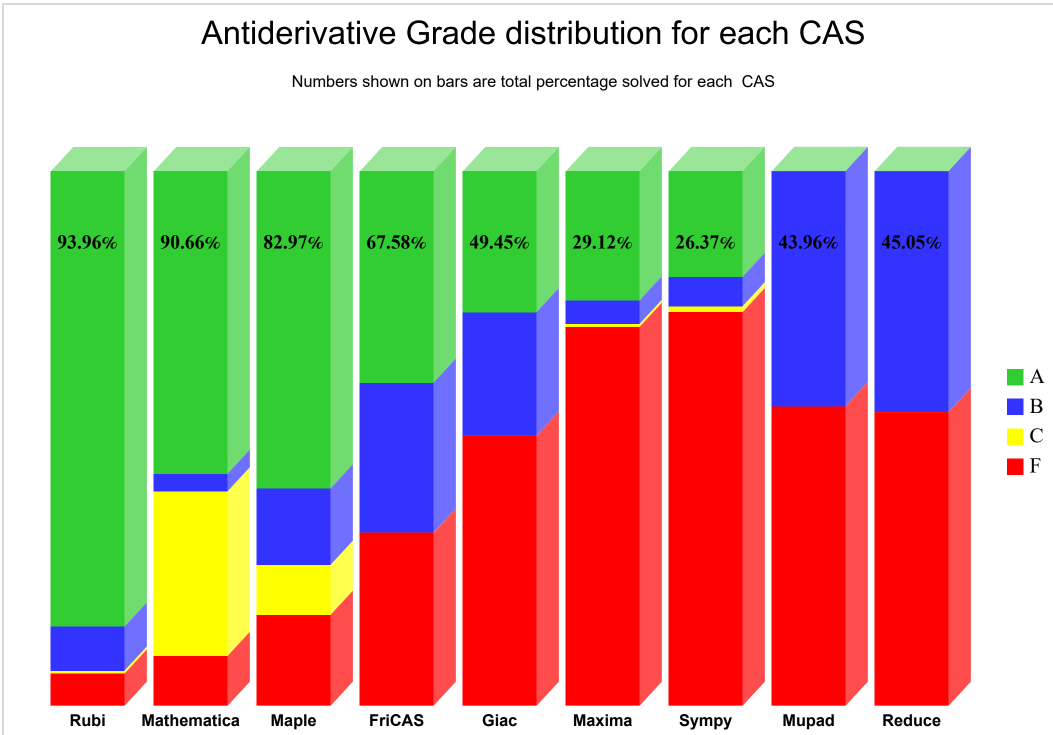
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

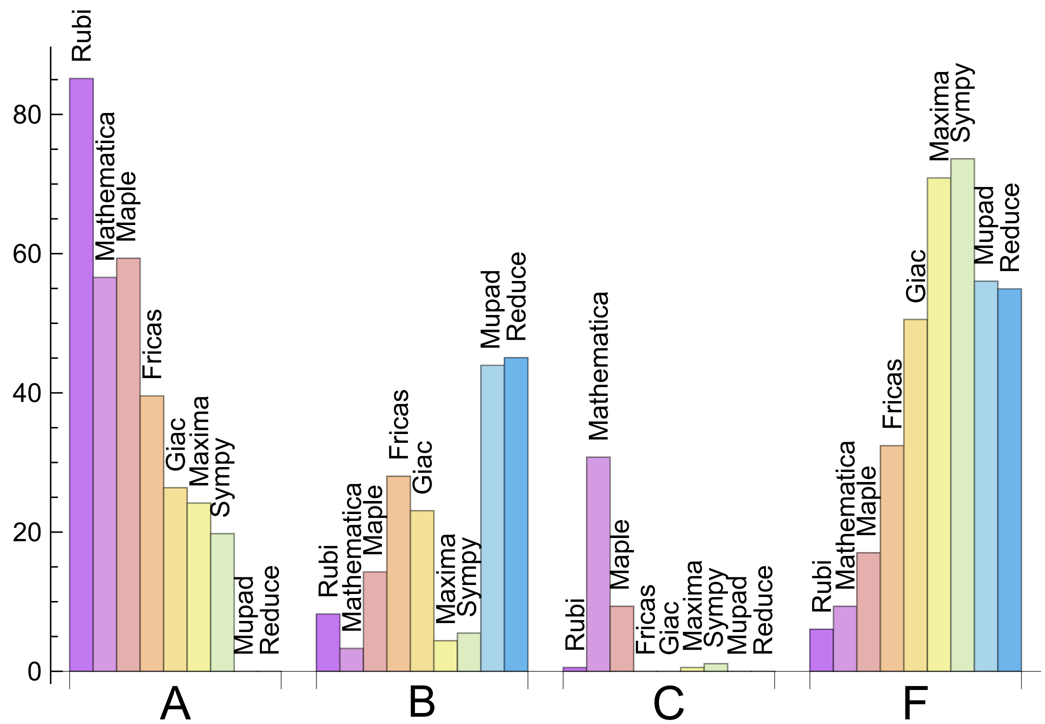
System	% A grade	% B grade	% C grade	% F grade
Rubi	85.165	8.242	0.549	6.044
Maple	59.341	14.286	9.341	17.033
Mathematica	56.593	3.297	30.769	9.341
Fricas	39.560	28.022	0.000	32.418
Giac	26.374	23.077	0.000	50.549
Maxima	24.176	4.396	0.549	70.879
Sympy	19.780	5.495	1.099	73.626
Mupad	0.000	43.956	0.000	56.044
Reduce	0.000	45.055	0.000	54.945

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	6	100.00	0.00	0.00
Mathematica	12	100.00	0.00	0.00
Maple	26	100.00	0.00	0.00
Fricas	54	59.26	40.74	0.00
Giac	87	79.31	18.39	2.30
Reduce	95	100.00	0.00	0.00
Mupad	97	0.00	100.00	0.00
Maxima	124	79.84	0.00	20.16
Sympy	132	68.94	28.03	3.03

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.06
Rubi	1.08
Giac	1.09
Maple	2.08
Fricas	2.86
Reduce	4.85
Mupad	5.54
Mathematica	7.97
Sympy	8.97

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	229.48	1.30	164.00	1.20
Mathematica	379.78	1.22	225.00	1.02
Rubi	409.60	1.23	235.00	1.00
Reduce	605.16	5.42	363.00	1.74
Giac	622.02	3.82	280.00	1.69
Mupad	799.59	4.87	282.00	1.23
Fricas	932.01	3.65	340.00	1.67
Maple	1055.23	3.31	326.50	1.18
Sympy	1673.66	6.85	313.00	1.71

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

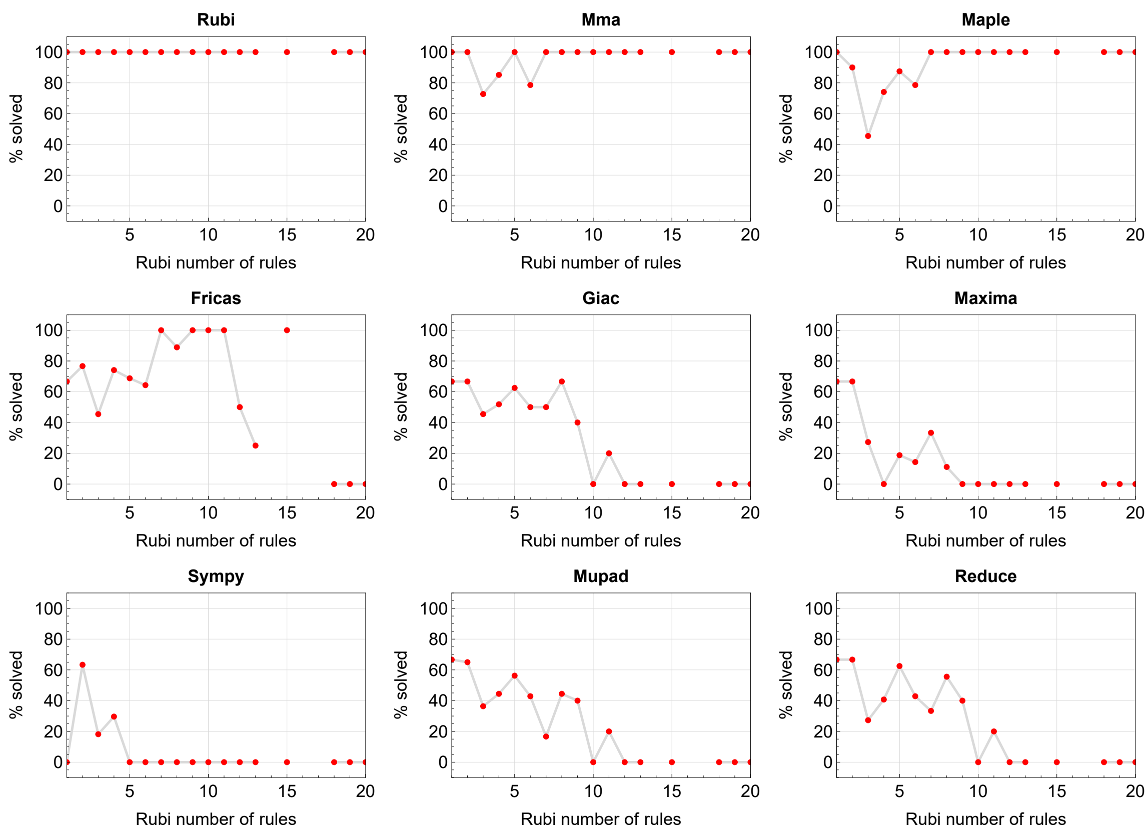


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

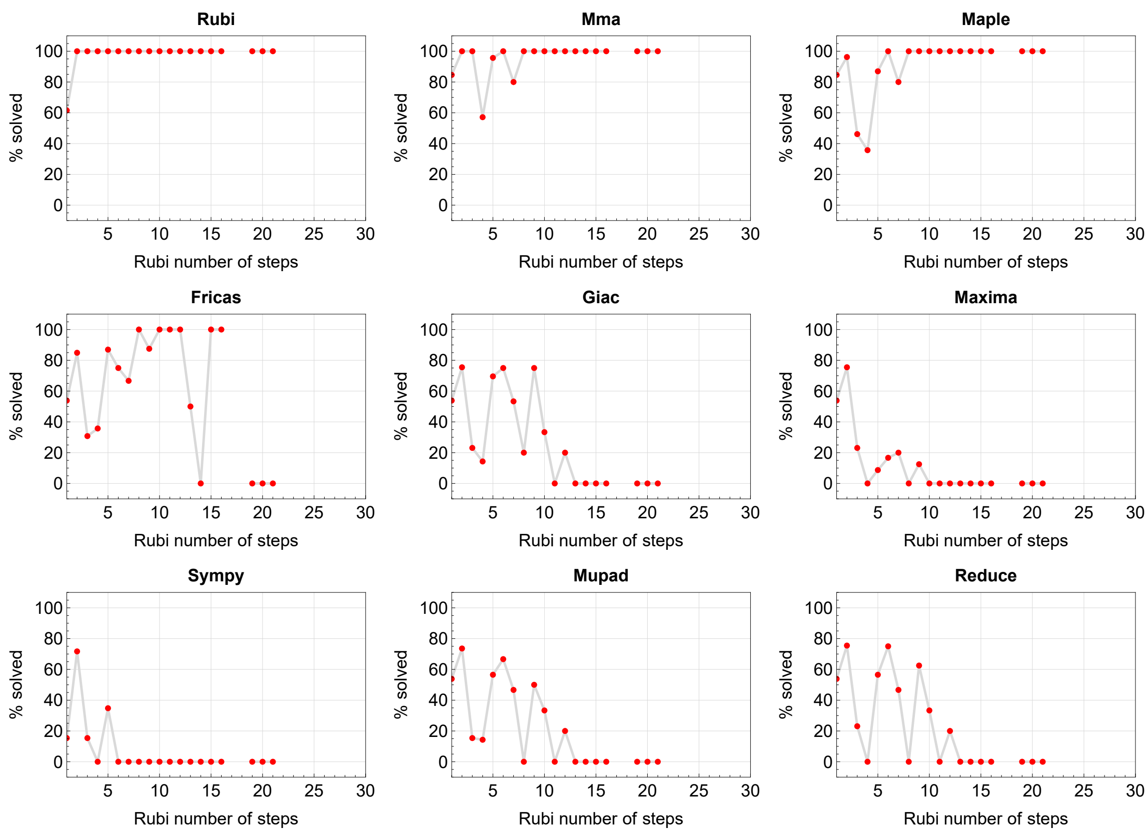


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

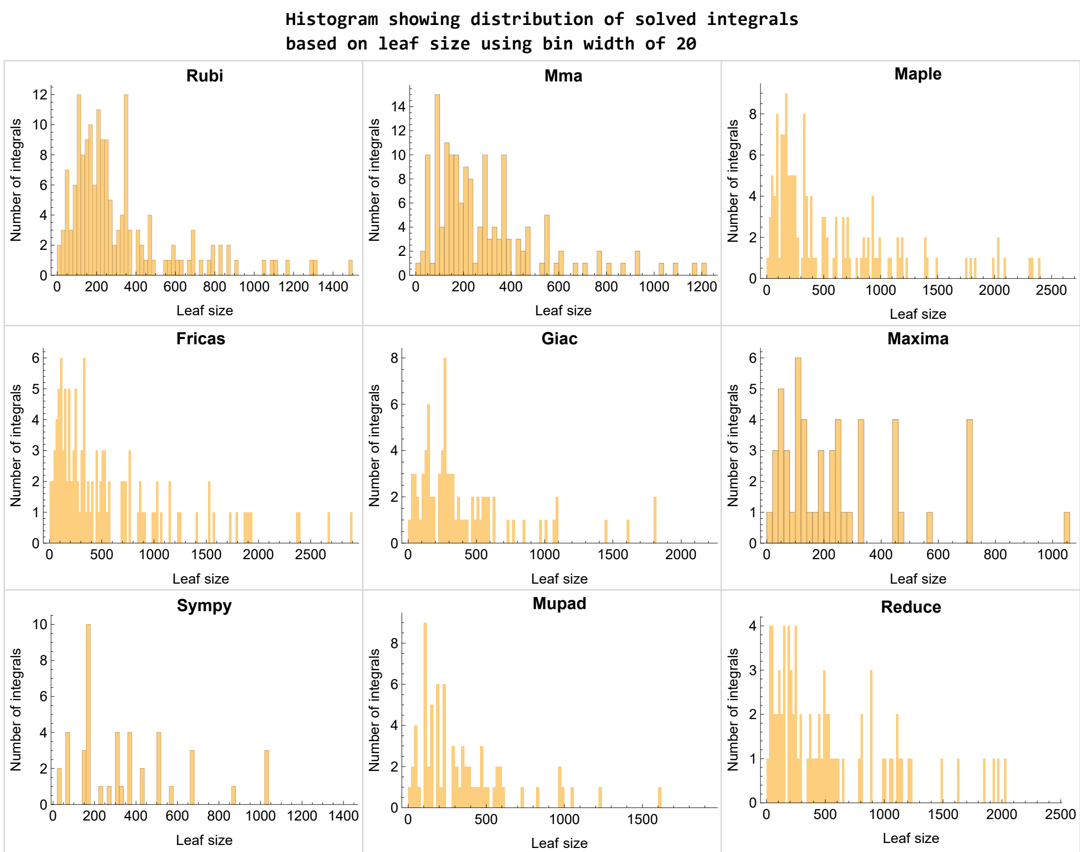


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

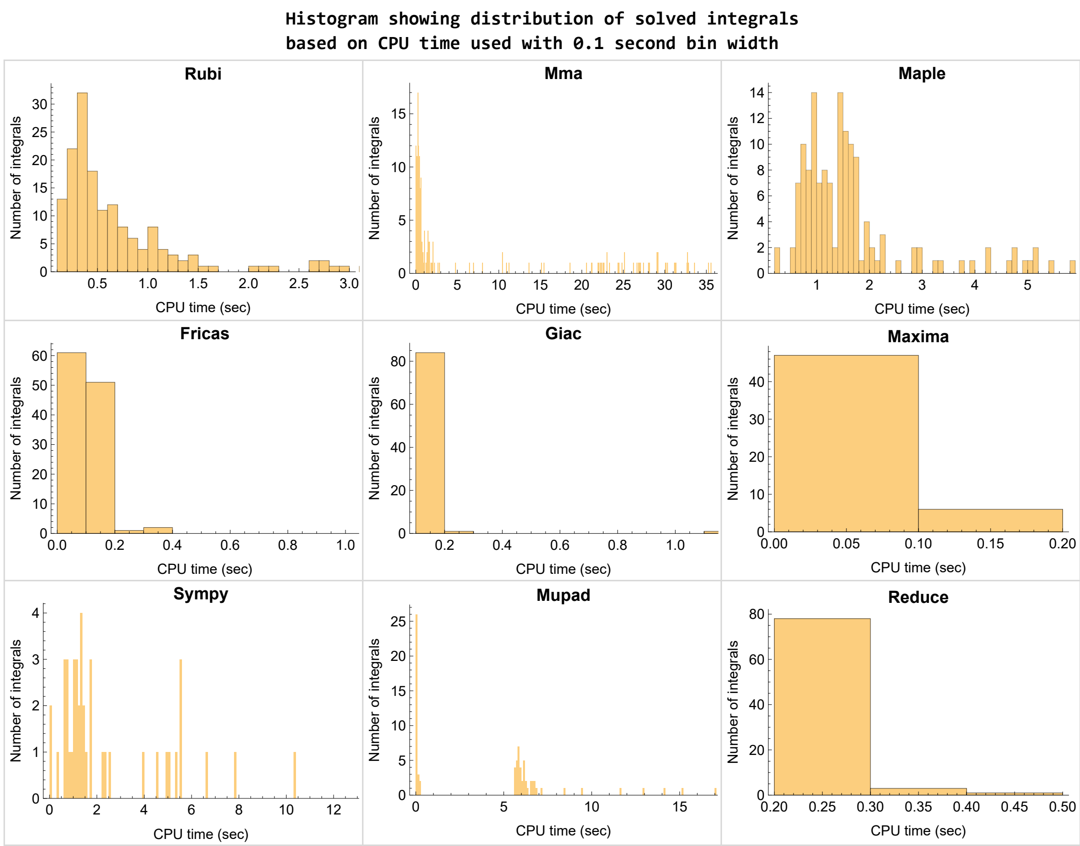


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

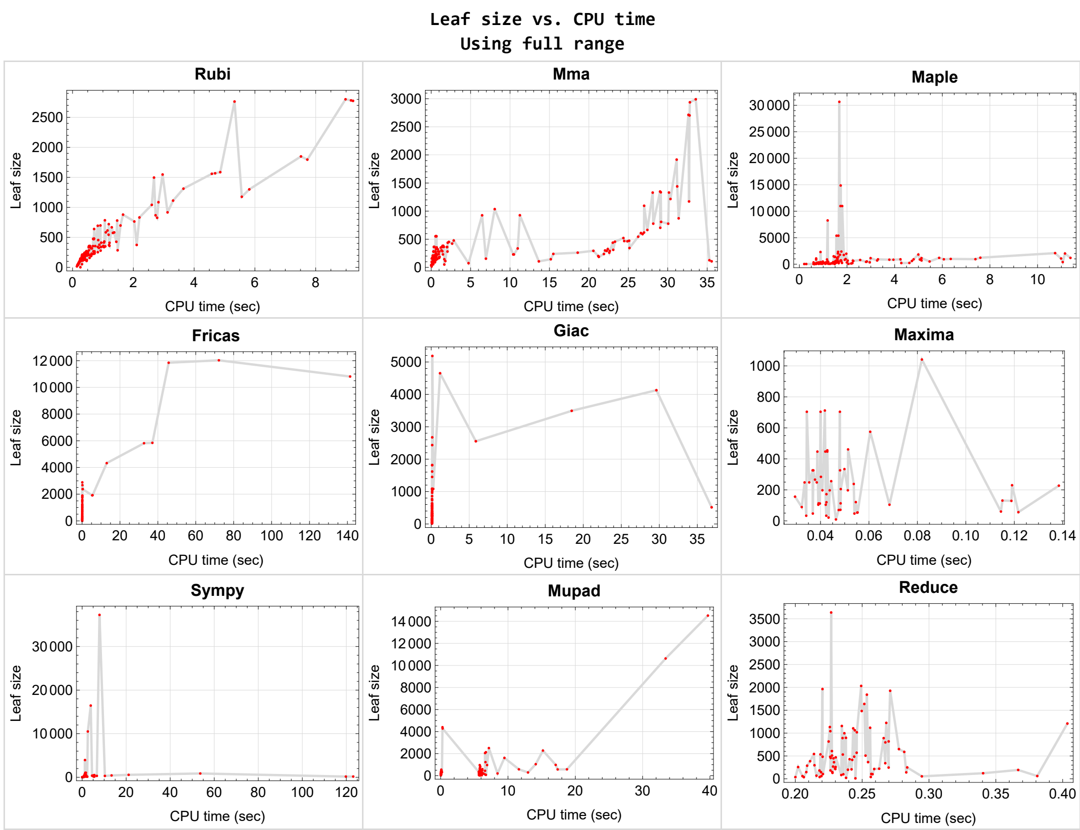


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{165, 169, 176, 181, 182\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{110, 111, 112, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 172\}$

Mathematica $\{14, 15, 16, 20, 21, 31, 131, 145, 147, 164\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```python
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

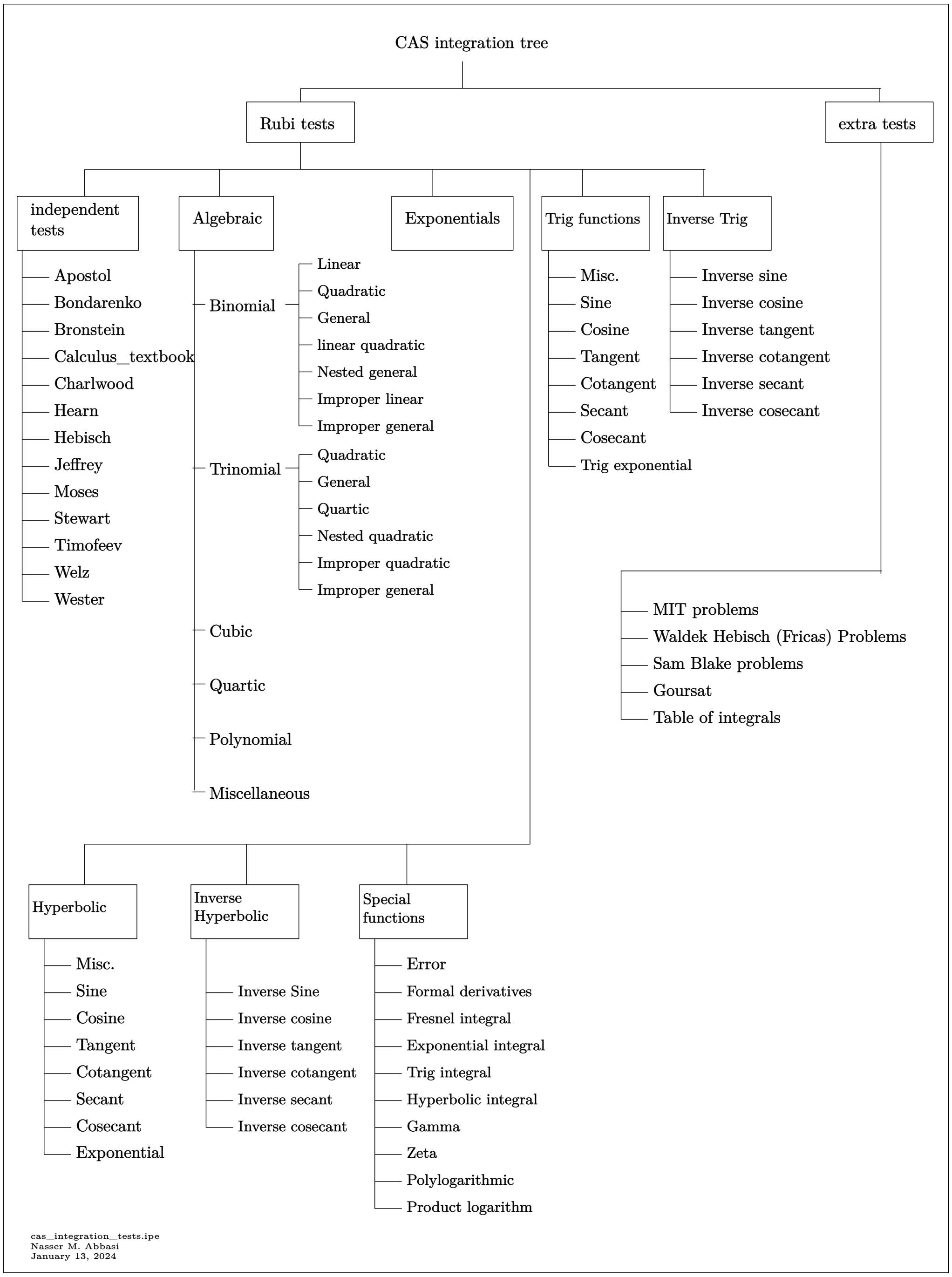
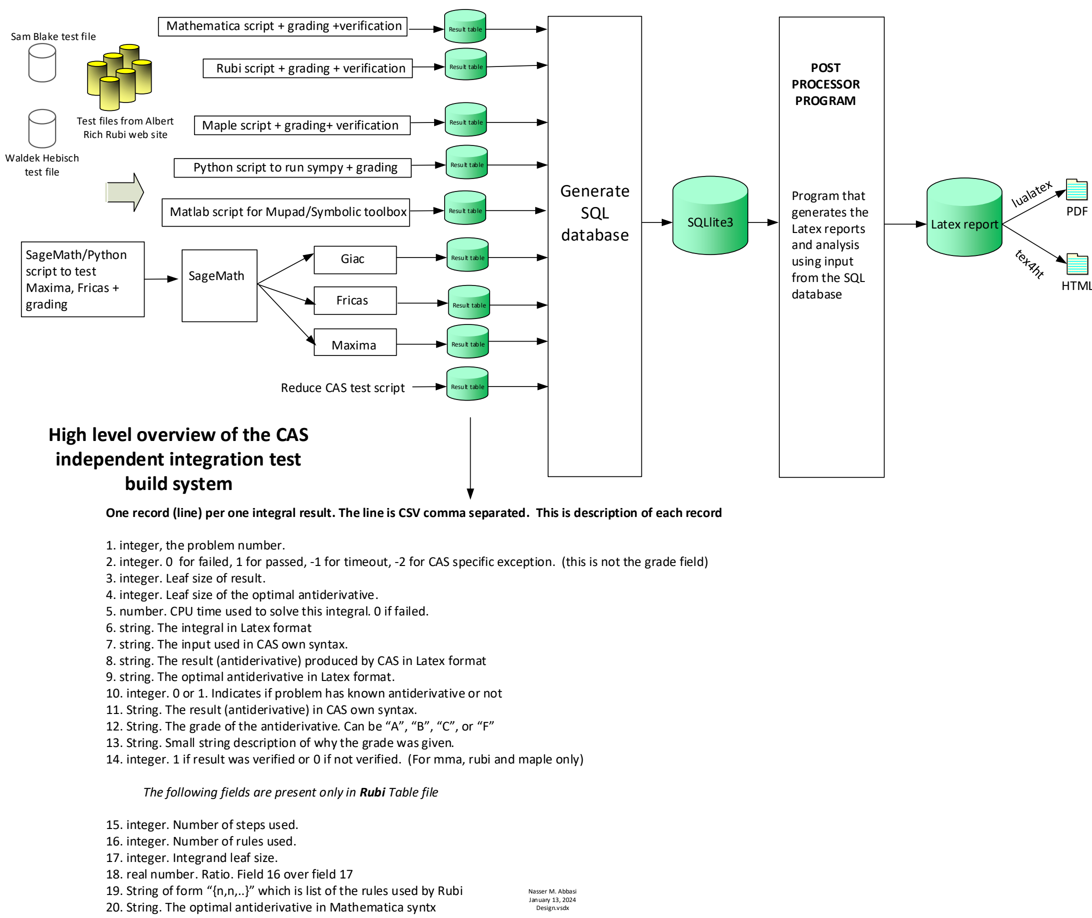


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 117, 118, 119, 120, 124, 125, 126, 127, 131, 133, 134, 135, 136, 141, 143, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 170, 171, 172, 173, 174, 175, 179, 180 }

B grade { 28, 115, 116, 121, 122, 123, 128, 129, 130, 132, 137, 138, 139, 140, 142 }

C grade { 145 }

F normal fail { 114, 144, 148, 149, 177, 178 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 99, 103, 106, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 164, 168, 170, 171, 172, 175, 180 }

B grade { 6, 7, 12, 13, 31, 147 }

C grade { 25, 26, 27, 28, 29, 30, 93, 94, 97, 98, 100, 101, 102, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 149 }

F normal fail { 32, 33, 34, 162, 163, 166, 167, 173, 174, 177, 178, 179 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 14, 15, 25, 26, 27, 29, 30, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 170, 171 }

B grade { 9, 16, 20, 21, 28, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 147, 148, 149, 150, 151, 156, 157 }

C grade { 5, 6, 7, 8, 10, 11, 12, 13, 17, 18, 19, 22, 23, 24, 143, 144, 145 }

F normal fail { 31, 32, 33, 34, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 172, 173, 174, 175, 177, 178, 179, 180 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 14, 15, 16, 17, 25, 26, 27, 28, 29, 30, 35, 36, 37, 38, 39, 44, 45, 46, 47, 49, 50, 51, 52, 53, 56, 57, 58, 59, 63, 64, 65, 66, 67, 73, 77, 78, 79, 80, 81, 84, 85, 86, 87, 95, 96, 110, 111, 112, 113, 117, 118, 119, 120, 124, 125, 126, 127, 133, 134, 135, 136, 147, 170, 171 }

B grade { 7, 8, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 40, 41, 42, 43, 48, 54, 55, 60, 61, 62, 68, 69, 70, 71, 72, 74, 75, 76, 82, 83, 88, 89, 90, 91, 92, 93, 94, 99, 100, 101, 103, 104, 105, 107, 108, 150, 151, 156, 157 }

C grade { }

F normal fail { 31, 32, 33, 34, 143, 144, 145, 146, 148, 149, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 172, 173, 174, 175, 177, 178, 179, 180 }

F(-1) timedout fail { 97, 98, 102, 106, 109, 114, 115, 116, 121, 122, 123, 128, 129, 130, 131, 132, 137, 138, 139, 140, 141, 142 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 10, 11, 15, 22, 35, 36, 37, 38, 42, 43, 44, 45, 49, 50, 51, 52, 56, 57, 58, 59, 63, 64, 65, 66, 70, 71, 72, 73, 77, 78, 79, 80, 84, 85, 86, 87, 151, 170, 171 }

B grade { 9, 14, 16, 20, 21, 150, 156, 157 }

C grade { 96 }

F normal fail { 6, 7, 8, 12, 13, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 91, 92, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 172, 173, 174, 175, 177, 178, 179, 180 }

F(-1) timedout fail { }

F(-2) exception fail { 39, 40, 41, 46, 47, 48, 53, 54, 55, 60, 61, 62, 67, 68, 69, 74, 75, 76, 81, 82, 83, 88, 89, 90, 95 }

Giac

A grade { 1, 2, 3, 4, 5, 14, 15, 16, 17, 39, 40, 41, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 93, 94, 95, 96, 170, 171 }

B grade { 6, 7, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 35, 36, 37, 38, 42, 43, 44, 45, 56, 63, 64, 65, 66, 70, 71, 72, 73, 74, 75, 84, 85, 89, 90, 91, 92, 150, 151, 156, 157 }

C grade { }

F normal fail { 23, 24, 27, 31, 32, 33, 34, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 172, 173, 174, 175, 177, 178, 179, 180 }

F(-1) timedout fail { 25, 26, 28, 29, 30, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109 }

F(-2) exception fail { 97, 98 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 14, 15, 16, 17, 18, 19, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 150, 151, 156, 157, 170, 171 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 8, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 172, 173, 174, 175, 177, 178, 179, 180 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 36, 37, 38, 39, 43, 44, 45, 46, 50, 51, 52, 53, 56, 57, 58, 59, 60, 64, 65, 66, 67, 71, 72, 73, 74, 78, 79, 80, 81, 84, 85, 86, 87, 88 }

B grade { 35, 42, 49, 63, 70, 77, 150, 151, 156, 157 }

C grade { 11, 22 }

F normal fail { 6, 7, 8, 9, 10, 12, 13, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 40, 54, 91, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 152, 154, 155, 158, 160, 162, 163, 164, 166, 167, 168, 171, 180 }

F(-1) timedout fail { 3, 4, 5, 14, 15, 16, 31, 32, 33, 34, 41, 47, 48, 55, 61, 62, 68, 69, 75, 76, 82, 83, 89, 90, 96, 123, 139, 170, 173, 174, 175, 176, 177, 178, 179, 181, 182 }

F(-2) exception fail { 153, 159, 161, 172 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 9, 10, 11, 14, 15, 16, 20, 21, 22, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 150, 151, 156, 157, 170, 171 }

C grade { }

F normal fail { 6, 7, 8, 12, 13, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 172, 173, 174, 175, 177, 178, 179, 180 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	33	33	32	34	37	34
N.S.	1	1.00	1.00	0.92	0.89	0.89	0.86	0.92	1.00	0.92
time (sec)	N/A	0.171	0.009	0.289	0.034	0.070	0.019	0.109	0.206	0.027

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	33	33	32	34	37	34
N.S.	1	1.00	1.00	0.92	0.89	0.89	0.86	0.92	1.00	0.92
time (sec)	N/A	0.174	0.004	0.205	0.042	0.065	0.024	0.107	0.200	0.020

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	171	131	201	230	145	0	190	228	1044
N.S.	1	1.15	0.88	1.35	1.54	0.97	0.00	1.28	1.53	7.01
time (sec)	N/A	0.328	0.419	1.161	0.119	0.101	0.000	0.125	0.230	14.124

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	102	93	156	131	99	0	113	130	580
N.S.	1	1.10	1.00	1.68	1.41	1.06	0.00	1.22	1.40	6.24
time (sec)	N/A	0.234	0.258	0.924	0.115	0.087	0.000	0.125	0.228	11.620

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	58	99	60	61	0	56	59	203
N.S.	1	1.00	1.35	2.30	1.40	1.42	0.00	1.30	1.37	4.72
time (sec)	N/A	0.171	0.138	0.883	0.114	0.100	0.000	0.126	0.205	8.448

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	235	322	0	152	0	1081	38	1615
N.S.	1	1.00	4.70	6.44	0.00	3.04	0.00	21.62	0.76	32.30
time (sec)	N/A	0.193	0.960	1.993	0.000	0.083	0.000	0.272	0.223	9.471

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	926	973	0	376	0	2553	62	0
N.S.	1	1.00	8.99	9.45	0.00	3.65	0.00	24.79	0.60	0.00
time (sec)	N/A	0.237	6.461	1.944	0.000	0.116	0.000	5.875	0.616	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	182	144	1982	0	731	0	4129	27	0
N.S.	1	1.10	0.87	12.01	0.00	4.43	0.00	25.02	0.16	0.00
time (sec)	N/A	0.308	15.170	1.971	0.000	0.131	0.000	29.624	200.058	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	161	129	350	227	188	0	375	194	0
N.S.	1	1.30	1.04	2.82	1.83	1.52	0.00	3.02	1.56	0.00
time (sec)	N/A	0.317	0.510	1.434	0.138	0.114	0.000	0.148	0.367	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	91	90	232	129	132	0	249	122	0
N.S.	1	1.20	1.18	3.05	1.70	1.74	0.00	3.28	1.61	0.00
time (sec)	N/A	0.227	0.321	1.009	0.119	0.095	0.000	0.134	0.340	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	54	124	56	84	168	152	61	0
N.S.	1	1.00	1.46	3.35	1.51	2.27	4.54	4.11	1.65	0.00
time (sec)	N/A	0.169	0.200	0.849	0.122	0.111	123.150	0.132	0.381	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	281	672	0	337	0	4653	84	0
N.S.	1	1.00	3.51	8.40	0.00	4.21	0.00	58.16	1.05	0.00
time (sec)	N/A	0.216	2.072	2.030	0.000	0.101	0.000	1.170	1.589	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	155	1036	1742	0	695	0	3493	27	0
N.S.	1	1.06	7.10	11.93	0.00	4.76	0.00	23.92	0.18	0.00
time (sec)	N/A	0.290	8.061	1.865	0.000	0.131	0.000	18.484	200.049	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	213	130	168	284	139	0	236	246	980
N.S.	1	1.32	0.81	1.04	1.76	0.86	0.00	1.47	1.53	6.09
time (sec)	N/A	0.340	0.436	0.768	0.040	0.098	0.000	0.129	0.270	17.088

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	145	90	123	172	93	0	142	145	577
N.S.	1	1.44	0.89	1.22	1.70	0.92	0.00	1.41	1.44	5.71
time (sec)	N/A	0.255	0.295	0.815	0.042	0.100	0.000	0.122	0.219	18.760

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	58	91	89	55	0	69	69	293
N.S.	1	1.00	1.23	1.94	1.89	1.17	0.00	1.47	1.47	6.23
time (sec)	N/A	0.167	0.164	0.714	0.032	0.087	0.000	0.125	0.216	12.973

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	54	329	0	257	0	69	36	2280
N.S.	1	1.00	0.68	4.11	0.00	3.21	0.00	0.86	0.45	28.50
time (sec)	N/A	0.204	0.260	1.560	0.000	0.127	0.000	0.118	0.223	15.192

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	133	111	996	0	547	0	243	59	14522
N.S.	1	0.97	0.81	7.27	0.00	3.99	0.00	1.77	0.43	106.00
time (sec)	N/A	0.254	1.800	1.606	0.000	0.108	0.000	0.148	0.307	39.698

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	211	154	2029	0	1024	0	514	82	10632
N.S.	1	1.04	0.76	10.00	0.00	5.04	0.00	2.53	0.40	52.37
time (sec)	N/A	0.316	6.944	1.617	0.000	0.155	0.000	36.889	0.707	33.414

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	203	129	321	266	251	0	251	448	0
N.S.	1	1.48	0.94	2.34	1.94	1.83	0.00	1.83	3.27	0.00
time (sec)	N/A	0.335	0.501	0.785	0.038	0.104	0.000	0.154	0.229	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	125	89	238	156	172	0	157	287	0
N.S.	1	1.47	1.05	2.80	1.84	2.02	0.00	1.85	3.38	0.00
time (sec)	N/A	0.384	0.313	0.768	0.029	0.116	0.000	0.142	0.209	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	130	70	100	165	92	147	0
N.S.	1	1.00	1.34	3.17	1.71	2.44	4.02	2.24	3.59	0.00
time (sec)	N/A	0.165	0.239	0.658	0.047	0.082	119.814	0.127	0.208	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	112	90	687	0	506	0	0	78	0
N.S.	1	0.97	0.78	5.92	0.00	4.36	0.00	0.00	0.67	0.00
time (sec)	N/A	0.236	0.483	1.600	0.000	0.119	0.000	0.000	0.269	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	144	1781	0	999	0	0	127	0
N.S.	1	1.00	0.78	9.68	0.00	5.43	0.00	0.00	0.69	0.00
time (sec)	N/A	0.303	1.671	1.706	0.000	0.127	0.000	0.000	0.506	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	354	291	511	0	237	0	0	434	0
N.S.	1	1.02	0.84	1.47	0.00	0.68	0.00	0.00	1.25	0.00
time (sec)	N/A	0.578	22.260	5.467	0.000	0.106	0.000	0.000	4.983	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	237	351	0	157	0	0	223	0
N.S.	1	1.00	0.88	1.30	0.00	0.58	0.00	0.00	0.83	0.00
time (sec)	N/A	0.456	21.905	2.955	0.000	0.087	0.000	0.000	1.909	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	219	106	198	0	111	0	0	33	0
N.S.	1	1.16	0.56	1.05	0.00	0.59	0.00	0.00	0.17	0.00
time (sec)	N/A	0.375	13.656	1.442	0.000	0.109	0.000	0.000	0.388	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	251	201	234	0	134	0	0	44	0
N.S.	1	2.56	2.05	2.39	0.00	1.37	0.00	0.00	0.45	0.00
time (sec)	N/A	0.429	21.189	1.790	0.000	0.108	0.000	0.000	6.588	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	355	290	532	0	378	0	0	55	0
N.S.	1	1.04	0.85	1.56	0.00	1.11	0.00	0.00	0.16	0.00
time (sec)	N/A	0.541	22.650	1.758	0.000	0.103	0.000	0.000	25.018	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	480	325	711	0	770	0	0	66	0
N.S.	1	1.04	0.70	1.54	0.00	1.67	0.00	0.00	0.14	0.00
time (sec)	N/A	0.708	22.399	1.753	0.000	0.118	0.000	0.000	61.251	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	167	0	0	0	0	0	1407	0
N.S.	1	1.00	3.09	0.00	0.00	0.00	0.00	0.00	26.06	0.00
time (sec)	N/A	0.190	0.385	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	1395	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	17.01	0.00
time (sec)	N/A	0.222	0.000	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0	1629	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	18.30	0.00
time (sec)	N/A	0.223	0.000	0.000	0.000	0.000	0.000	0.000	0.255	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	1665	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	18.10	0.00
time (sec)	N/A	0.234	0.000	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	283	244	326	419	503	859	483	222
N.S.	1	1.00	1.17	1.01	1.35	1.73	2.08	3.55	2.00	0.92
time (sec)	N/A	0.420	0.308	1.404	0.037	0.081	1.370	0.121	0.220	0.075

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	178	154	197	265	313	561	297	159
N.S.	1	1.00	1.01	0.87	1.11	1.50	1.77	3.17	1.68	0.90
time (sec)	N/A	0.343	0.182	1.344	0.041	0.073	1.241	0.120	0.214	5.805

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	94	78	104	143	167	310	147	100
N.S.	1	1.00	0.82	0.68	0.90	1.24	1.45	2.70	1.28	0.87
time (sec)	N/A	0.261	0.105	1.297	0.042	0.089	1.277	0.114	0.228	0.042

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	40	47	62	76	127	59	44
N.S.	1	1.00	0.70	0.63	0.75	0.98	1.21	2.02	0.94	0.70
time (sec)	N/A	0.195	0.048	0.592	0.037	0.095	0.601	0.106	0.220	5.988

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	135	131	132	0	306	178	165	207	192
N.S.	1	1.03	1.00	1.01	0.00	2.34	1.36	1.26	1.58	1.47
time (sec)	N/A	0.328	0.449	2.177	0.000	0.096	5.583	0.115	0.242	5.750

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	158	130	133	0	537	0	153	469	156
N.S.	1	1.16	0.96	0.98	0.00	3.95	0.00	1.12	3.45	1.15
time (sec)	N/A	0.364	0.546	0.964	0.000	0.100	0.000	0.118	0.230	5.694

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	237	184	184	0	876	0	271	896	230
N.S.	1	1.27	0.98	0.98	0.00	4.68	0.00	1.45	4.79	1.23
time (sec)	N/A	0.453	1.075	1.045	0.000	0.112	0.000	0.120	0.238	5.710

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	282	244	326	517	503	1452	607	222
N.S.	1	1.00	1.17	1.01	1.35	2.14	2.08	6.00	2.51	0.92
time (sec)	N/A	0.399	0.317	1.431	0.037	0.099	1.791	0.127	0.227	0.047

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	179	155	197	337	313	964	385	159
N.S.	1	1.00	1.01	0.88	1.11	1.90	1.77	5.45	2.18	0.90
time (sec)	N/A	0.330	0.196	1.418	0.051	0.083	1.358	0.130	0.211	0.035

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	95	79	104	190	167	548	199	100
N.S.	1	1.00	0.83	0.69	0.90	1.65	1.45	4.77	1.73	0.87
time (sec)	N/A	0.251	0.114	1.264	0.068	0.080	1.046	0.118	0.239	0.041

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	41	47	85	76	229	83	44
N.S.	1	1.00	0.70	0.65	0.75	1.35	1.21	3.63	1.32	0.70
time (sec)	N/A	0.193	0.055	0.698	0.054	0.093	0.640	0.119	0.234	5.691

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	173	162	170	0	492	233	266	414	363
N.S.	1	1.04	0.98	1.02	0.00	2.96	1.40	1.60	2.49	2.19
time (sec)	N/A	0.377	0.440	0.941	0.000	0.093	5.011	0.129	0.243	5.698

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	204	180	194	0	539	0	269	485	338
N.S.	1	1.18	1.04	1.12	0.00	3.12	0.00	1.55	2.80	1.95
time (sec)	N/A	0.546	0.654	0.944	0.000	0.099	0.000	0.121	0.244	5.663

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	267	182	176	0	858	0	282	891	282
N.S.	1	1.32	0.90	0.87	0.00	4.23	0.00	1.39	4.39	1.39
time (sec)	N/A	0.687	1.004	0.967	0.000	0.130	0.000	0.122	0.235	5.916

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	282	243	326	324	502	380	363	222
N.S.	1	1.00	1.18	1.01	1.36	1.35	2.09	1.58	1.51	0.92
time (sec)	N/A	0.401	0.289	1.536	0.048	0.093	1.105	0.114	0.229	6.261

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	177	155	197	197	311	243	213	159
N.S.	1	1.00	1.01	0.89	1.13	1.13	1.78	1.39	1.22	0.91
time (sec)	N/A	0.331	0.179	1.344	0.044	0.075	1.024	0.113	0.230	0.040

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	94	79	104	100	165	132	99	100
N.S.	1	1.00	0.83	0.70	0.92	0.88	1.46	1.17	0.88	0.88
time (sec)	N/A	0.258	0.125	1.214	0.039	0.081	0.896	0.113	0.236	0.041

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	44	40	53	40	70	53	39	44
N.S.	1	1.00	0.72	0.66	0.87	0.66	1.15	0.87	0.64	0.72
time (sec)	N/A	0.196	0.049	0.618	0.055	0.089	0.396	0.112	0.218	6.114

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	106	92	82	0	297	143	109	183	107
N.S.	1	1.02	0.88	0.79	0.00	2.86	1.38	1.05	1.76	1.03
time (sec)	N/A	0.276	0.308	0.947	0.000	0.112	2.327	0.104	0.226	6.126

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	164	133	138	0	539	0	151	456	128
N.S.	1	1.30	1.06	1.10	0.00	4.28	0.00	1.20	3.62	1.02
time (sec)	N/A	0.327	0.616	0.929	0.000	0.102	0.000	0.117	0.226	6.135

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	221	166	166	0	896	0	285	890	224
N.S.	1	1.21	0.91	0.91	0.00	4.92	0.00	1.57	4.89	1.23
time (sec)	N/A	0.380	1.005	0.979	0.000	0.109	0.000	0.122	0.266	6.220

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	278	247	334	333	420	465	365	292
N.S.	1	1.00	1.17	1.04	1.40	1.40	1.76	1.95	1.53	1.23
time (sec)	N/A	0.391	0.315	0.727	0.050	0.095	13.378	0.123	0.254	0.063

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	177	154	205	206	264	277	215	199
N.S.	1	1.00	1.02	0.89	1.18	1.19	1.53	1.60	1.24	1.15
time (sec)	N/A	0.321	0.236	0.775	0.048	0.092	5.505	0.125	0.259	0.039

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	92	86	112	110	150	137	101	111
N.S.	1	1.00	0.83	0.77	1.01	0.99	1.35	1.23	0.91	1.00
time (sec)	N/A	0.257	0.108	0.708	0.040	0.084	2.267	0.111	0.258	0.044

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	43	39	54	49	75	56	40	44
N.S.	1	1.00	0.73	0.66	0.92	0.83	1.27	0.95	0.68	0.75
time (sec)	N/A	0.196	0.054	0.647	0.055	0.082	0.731	0.109	0.256	0.031

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	114	112	0	492	151	114	221	141
N.S.	1	1.00	1.02	1.00	0.00	4.39	1.35	1.02	1.97	1.26
time (sec)	N/A	0.336	0.490	0.968	0.000	0.094	5.342	0.115	0.263	0.083

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	172	148	152	0	906	0	228	535	187
N.S.	1	1.19	1.03	1.06	0.00	6.29	0.00	1.58	3.72	1.30
time (sec)	N/A	0.390	0.783	0.969	0.000	0.117	0.000	0.116	0.219	6.384

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	260	230	230	0	1539	0	368	1062	310
N.S.	1	1.21	1.07	1.07	0.00	7.19	0.00	1.72	4.96	1.45
time (sec)	N/A	0.504	1.493	0.957	0.000	0.155	0.000	0.126	0.244	0.192

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	554	500	704	865	1032	1619	1018	479
N.S.	1	1.00	1.17	1.05	1.49	1.82	2.18	3.42	2.15	1.01
time (sec)	N/A	0.678	0.602	1.598	0.034	0.085	1.715	0.129	0.246	0.112

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	363	325	447	567	666	1090	650	343
N.S.	1	1.00	1.07	0.96	1.31	1.67	1.96	3.21	1.91	1.01
time (sec)	N/A	0.545	0.373	1.550	0.043	0.098	1.176	0.122	0.278	6.062

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	207	172	248	320	372	633	341	197
N.S.	1	1.00	0.97	0.80	1.16	1.50	1.74	2.96	1.59	0.92
time (sec)	N/A	0.393	0.209	1.497	0.033	0.085	1.008	0.118	0.267	0.054

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	96	88	113	143	177	274	141	114
N.S.	1	1.00	0.76	0.69	0.89	1.13	1.39	2.16	1.11	0.90
time (sec)	N/A	0.257	0.083	0.810	0.039	0.072	0.751	0.112	0.283	0.026

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	245	282	260	0	766	423	426	589	596
N.S.	1	0.99	1.14	1.05	0.00	3.10	1.71	1.72	2.38	2.41
time (sec)	N/A	0.489	0.548	1.079	0.000	0.126	4.987	0.119	0.282	6.195

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	265	281	274	0	1229	0	345	1117	552
N.S.	1	1.08	1.15	1.12	0.00	5.02	0.00	1.41	4.56	2.25
time (sec)	N/A	0.724	0.877	1.144	0.000	0.112	0.000	0.123	0.256	0.094

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	370	381	361	0	1865	0	524	1926	483
N.S.	1	1.23	1.27	1.20	0.00	6.20	0.00	1.74	6.40	1.60
time (sec)	N/A	1.199	1.511	1.135	0.000	0.193	0.000	0.122	0.271	6.187

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	554	499	704	1028	1032	2674	1224	479
N.S.	1	1.00	1.17	1.05	1.49	2.17	2.18	5.64	2.58	1.01
time (sec)	N/A	0.694	0.674	1.497	0.040	0.102	1.582	0.139	0.268	6.077

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	365	325	447	686	666	1819	797	343
N.S.	1	1.00	1.07	0.96	1.31	2.02	1.96	5.35	2.34	1.01
time (sec)	N/A	0.545	0.393	1.446	0.042	0.090	1.442	0.138	0.267	5.969

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	207	172	248	396	372	1072	429	197
N.S.	1	1.00	0.97	0.80	1.16	1.85	1.74	5.01	2.00	0.92
time (sec)	N/A	0.387	0.225	1.412	0.038	0.084	1.330	0.122	0.241	0.048

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	97	88	113	181	177	471	182	114
N.S.	1	1.00	0.76	0.69	0.89	1.43	1.39	3.71	1.43	0.90
time (sec)	N/A	0.262	0.090	0.692	0.048	0.091	0.779	0.123	0.218	5.858

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	285	322	323	0	1150	515	596	997	838
N.S.	1	1.00	1.13	1.14	0.00	4.05	1.81	2.10	3.51	2.95
time (sec)	N/A	0.541	0.848	1.500	0.000	0.115	5.551	0.129	0.237	5.877

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	324	351	396	0	1255	0	552	1156	965
N.S.	1	1.11	1.20	1.35	0.00	4.28	0.00	1.88	3.95	3.29
time (sec)	N/A	0.869	1.215	1.211	0.000	0.141	0.000	0.129	0.235	5.803

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	426	376	385	0	1897	0	580	1964	723
N.S.	1	1.26	1.11	1.14	0.00	5.61	0.00	1.72	5.81	2.14
time (sec)	N/A	1.440	1.535	1.150	0.000	0.152	0.000	0.130	0.220	5.846

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	472	472	553	499	704	706	1030	734	816	479
N.S.	1	1.00	1.17	1.06	1.49	1.50	2.18	1.56	1.73	1.01
time (sec)	N/A	0.685	0.550	1.648	0.048	0.097	1.422	0.127	0.225	0.087

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	363	328	447	452	665	489	507	343
N.S.	1	1.00	1.07	0.97	1.32	1.34	1.97	1.45	1.50	1.01
time (sec)	N/A	0.542	0.343	1.651	0.039	0.090	1.162	0.126	0.253	0.075

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	207	176	248	247	371	280	257	197
N.S.	1	1.00	0.98	0.83	1.17	1.17	1.75	1.32	1.21	0.93
time (sec)	N/A	0.399	0.201	1.414	0.035	0.091	0.988	0.112	0.202	5.714

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	96	88	120	107	175	120	104	114
N.S.	1	1.00	0.77	0.70	0.96	0.86	1.40	0.96	0.83	0.91
time (sec)	N/A	0.265	0.074	0.691	0.042	0.078	0.641	0.117	0.221	0.025

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	214	193	190	0	735	371	333	543	381
N.S.	1	0.98	0.89	0.87	0.00	3.37	1.70	1.53	2.49	1.75
time (sec)	N/A	0.443	0.482	1.651	0.000	0.109	4.509	0.118	0.214	0.051

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	278	279	216	0	1157	0	329	1042	376
N.S.	1	1.24	1.25	0.96	0.00	5.17	0.00	1.47	4.65	1.68
time (sec)	N/A	0.923	1.024	1.060	0.000	0.147	0.000	0.118	0.226	0.078

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	359	362	351	0	1791	0	512	1841	395
N.S.	1	1.27	1.28	1.24	0.00	6.35	0.00	1.82	6.53	1.40
time (sec)	N/A	1.215	1.346	1.122	0.000	0.154	0.000	0.125	0.254	0.153

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	550	500	712	715	872	1014	818	617
N.S.	1	1.00	1.17	1.06	1.51	1.52	1.86	2.16	1.74	1.31
time (sec)	N/A	0.700	0.630	0.987	0.042	0.096	53.669	0.138	0.270	5.802

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	363	328	455	461	563	637	509	431
N.S.	1	1.00	1.08	0.98	1.35	1.37	1.68	1.90	1.51	1.28
time (sec)	N/A	0.530	0.418	0.833	0.043	0.092	21.141	0.124	0.227	5.764

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	206	177	256	257	323	335	259	237
N.S.	1	1.00	0.98	0.84	1.22	1.22	1.54	1.60	1.23	1.13
time (sec)	N/A	0.397	0.223	0.758	0.044	0.088	6.685	0.111	0.230	0.052

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	97	88	121	117	160	140	106	128
N.S.	1	1.00	0.79	0.72	0.98	0.95	1.30	1.14	0.86	1.04
time (sec)	N/A	0.263	0.085	0.803	0.055	0.095	1.756	0.112	0.257	0.027

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	196	249	211	0	1062	314	272	571	319
N.S.	1	0.97	1.23	1.04	0.00	5.26	1.55	1.35	2.83	1.58
time (sec)	N/A	0.480	0.605	1.149	0.000	0.136	10.341	0.129	0.246	0.076

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	252	302	245	0	1736	0	407	1133	405
N.S.	1	1.21	1.45	1.18	0.00	8.35	0.00	1.96	5.45	1.95
time (sec)	N/A	0.831	1.309	1.214	0.000	0.139	0.000	0.120	0.226	5.779

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	349	453	393	0	2674	0	573	2032	519
N.S.	1	1.20	1.56	1.36	0.00	9.22	0.00	1.98	7.01	1.79
time (sec)	N/A	1.078	2.245	1.417	0.000	0.210	0.000	0.129	0.249	5.949

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	191	211	190	0	1530	0	316	268	4278
N.S.	1	1.06	1.17	1.06	0.00	8.50	0.00	1.76	1.49	23.77
time (sec)	N/A	0.425	0.544	1.096	0.000	0.124	0.000	0.153	0.228	0.293

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	161	189	132	0	2373	0	267	531	2058
N.S.	1	1.05	1.23	0.86	0.00	15.41	0.00	1.73	3.45	13.36
time (sec)	N/A	0.300	0.599	0.961	0.000	0.321	0.000	0.141	0.236	6.596

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	780	230	720	0	1574	0	318	1104	4400
N.S.	1	1.68	0.50	1.56	0.00	3.40	0.00	0.69	2.38	9.50
time (sec)	N/A	1.468	0.719	1.735	0.000	0.125	0.000	0.154	0.243	0.270

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	720	212	673	0	2385	0	275	1483	2133
N.S.	1	1.61	0.47	1.51	0.00	5.34	0.00	0.62	3.32	4.77
time (sec)	N/A	1.192	0.689	1.543	0.000	0.312	0.000	0.145	0.250	6.719

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	150	123	306	0	336	0	165	249	569
N.S.	1	1.02	0.84	2.08	0.00	2.29	0.00	1.12	1.69	3.87
time (sec)	N/A	0.311	0.519	1.013	0.000	0.145	0.000	0.134	0.284	17.379

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	9	12	0	12	10	16
N.S.	1	1.00	1.00	0.81	0.56	0.75	0.00	0.75	0.62	1.00
time (sec)	N/A	0.144	0.059	0.629	0.046	0.084	0.000	0.112	0.245	5.826

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	409	438	2385	0	0	0	0	54	0
N.S.	1	0.97	1.04	5.67	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.217	2.099	1.763	0.000	0.000	0.000	0.000	1.543	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	341	363	1495	0	0	0	0	24	0
N.S.	1	1.00	1.06	4.37	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.013	1.428	1.440	0.000	0.000	0.000	0.000	0.439	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	229	1383	0	1921	0	0	31	0
N.S.	1	1.00	0.95	5.76	0.00	8.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.482	10.495	1.476	0.000	5.556	0.000	0.000	0.374	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	340	361	5383	0	5816	0	0	55	0
N.S.	1	0.97	1.03	15.34	0.00	16.57	0.00	0.00	0.16	0.00
time (sec)	N/A	0.937	1.996	1.559	0.000	32.689	0.000	0.000	1.015	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	582	422	14861	0	10812	0	0	26	0
N.S.	1	1.33	0.97	34.08	0.00	24.80	0.00	0.00	0.06	0.00
time (sec)	N/A	1.370	2.663	1.734	0.000	141.415	0.000	0.000	200.033	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	363	2336	0	0	0	0	68	0
N.S.	1	1.00	1.08	6.93	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.039	1.607	1.625	0.000	0.000	0.000	0.000	0.707	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	229	1383	0	1913	0	0	31	0
N.S.	1	1.00	0.95	5.76	0.00	7.97	0.00	0.00	0.13	0.00
time (sec)	N/A	0.465	10.404	1.500	0.000	5.353	0.000	0.000	0.345	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	286	1415	0	4325	0	0	36	0
N.S.	1	1.00	1.24	6.15	0.00	18.80	0.00	0.00	0.16	0.00
time (sec)	N/A	0.432	1.420	1.637	0.000	13.013	0.000	0.000	0.330	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	354	383	10977	0	11846	0	0	72	0
N.S.	1	1.02	1.10	31.63	0.00	34.14	0.00	0.00	0.21	0.00
time (sec)	N/A	0.673	1.535	1.798	0.000	45.693	0.000	0.000	0.924	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	574	336	8264	0	0	0	0	116	0
N.S.	1	1.39	0.81	19.96	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	1.328	10.974	1.187	0.000	0.000	0.000	0.000	3.338	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	340	361	5383	0	5844	0	0	55	0
N.S.	1	0.97	1.03	15.34	0.00	16.65	0.00	0.00	0.16	0.00
time (sec)	N/A	0.831	2.031	1.638	0.000	37.100	0.000	0.000	0.895	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	354	383	10977	0	12028	0	0	72	0
N.S.	1	1.02	1.10	31.63	0.00	34.66	0.00	0.00	0.21	0.00
time (sec)	N/A	0.746	1.449	1.725	0.000	72.182	0.000	0.000	0.843	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	543	477	30656	0	0	0	0	150	0
N.S.	1	1.31	1.15	73.69	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	1.186	2.862	1.678	0.000	0.000	0.000	0.000	29.600	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1063	1311	1172	1824	0	765	0	0	0	0
N.S.	1	1.23	1.10	1.72	0.00	0.72	0.00	0.00	0.00	0.00
time (sec)	N/A	3.649	32.700	4.994	0.000	0.094	0.000	0.000	15.665	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	809	1041	809	1142	0	510	0	0	1044	0
N.S.	1	1.29	1.00	1.41	0.00	0.63	0.00	0.00	1.29	0.00
time (sec)	N/A	2.607	29.158	2.990	0.000	0.099	0.000	0.000	7.679	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	784	610	794	0	343	0	0	515	0
N.S.	1	1.31	1.02	1.32	0.00	0.57	0.00	0.00	0.86	0.00
time (sec)	N/A	1.065	26.601	2.553	0.000	0.104	0.000	0.000	3.421	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	693	521	602	0	229	0	0	237	0
N.S.	1	1.36	1.02	1.18	0.00	0.45	0.00	0.00	0.46	0.00
time (sec)	N/A	0.830	24.376	1.520	0.000	0.086	0.000	0.000	1.342	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	725	0	1216	922	0	0	0	0	26	0
N.S.	1	0.00	1.68	1.27	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	30.346	4.244	0.000	0.000	0.000	0.000	200.017	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	720	1547	1331	895	0	0	0	0	0	0
N.S.	1	2.15	1.85	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.963	30.154	1.161	0.000	0.000	0.000	0.000	22.245	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	963	2763	2703	1161	0	0	0	0	0	0
N.S.	1	2.87	2.81	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.330	32.769	1.579	0.000	0.000	0.000	0.000	84.182	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	871	1085	872	1156	0	578	0	0	1170	0
N.S.	1	1.25	1.00	1.33	0.00	0.66	0.00	0.00	1.34	0.00
time (sec)	N/A	2.827	31.386	5.128	0.000	0.096	0.000	0.000	11.308	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	672	878	665	848	0	409	0	0	638	0
N.S.	1	1.31	0.99	1.26	0.00	0.61	0.00	0.00	0.95	0.00
time (sec)	N/A	1.661	27.419	3.949	0.000	0.090	0.000	0.000	5.523	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	698	545	678	0	268	0	0	343	0
N.S.	1	1.34	1.05	1.30	0.00	0.52	0.00	0.00	0.66	0.00
time (sec)	N/A	0.918	26.253	2.247	0.000	0.100	0.000	0.000	2.586	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	639	456	584	0	208	0	0	24	0
N.S.	1	1.38	0.99	1.26	0.00	0.45	0.00	0.00	0.05	0.00
time (sec)	N/A	0.710	23.305	1.550	0.000	0.085	0.000	0.000	0.654	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	670	1496	1096	833	0	0	0	0	26	0
N.S.	1	2.23	1.64	1.24	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.680	27.007	1.905	0.000	0.000	0.000	0.000	200.027	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	743	1559	1336	921	0	0	0	0	61	0
N.S.	1	2.10	1.80	1.24	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	4.583	29.119	3.300	0.000	0.000	0.000	0.000	11.407	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	965	2774	2714	1196	0	0	0	0	85	0
N.S.	1	2.87	2.81	1.24	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	9.228	32.629	5.868	0.000	0.000	0.000	0.000	48.322	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	742	916	777	882	0	445	0	0	852	0
N.S.	1	1.23	1.05	1.19	0.00	0.60	0.00	0.00	1.15	0.00
time (sec)	N/A	3.126	28.163	5.059	0.000	0.108	0.000	0.000	8.510	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	764	591	700	0	300	0	0	547	0
N.S.	1	1.28	0.99	1.17	0.00	0.50	0.00	0.00	0.92	0.00
time (sec)	N/A	2.033	26.805	3.266	0.000	0.092	0.000	0.000	4.145	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	478	667	464	602	0	227	0	0	279	0
N.S.	1	1.40	0.97	1.26	0.00	0.47	0.00	0.00	0.58	0.00
time (sec)	N/A	1.291	24.885	2.263	0.000	0.093	0.000	0.000	2.097	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	583	294	396	0	173	0	0	26	0
N.S.	1	1.38	0.70	0.94	0.00	0.41	0.00	0.00	0.06	0.00
time (sec)	N/A	1.093	20.571	0.897	0.000	0.085	0.000	0.000	0.770	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	1112	300	439	0	0	0	0	39	0
N.S.	1	2.68	0.72	1.06	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	3.308	22.052	2.800	0.000	0.000	0.000	0.000	77.285	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	754	1567	1330	922	0	0	0	0	63	0
N.S.	1	2.08	1.76	1.22	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	4.691	28.092	4.797	0.000	0.000	0.000	0.000	19.665	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	983	2782	2937	1224	0	0	0	0	87	0
N.S.	1	2.83	2.99	1.25	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	9.162	32.809	7.598	0.000	0.000	0.000	0.000	77.517	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	726	1299	927	852	0	0	0	0	84	0
N.S.	1	1.79	1.28	1.17	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	5.815	11.249	3.777	0.000	0.000	0.000	0.000	103.856	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	740	1848	1440	948	0	0	0	0	26	0
N.S.	1	2.50	1.95	1.28	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	7.515	31.200	7.392	0.000	0.000	0.000	0.000	200.037	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	606	831	619	711	0	319	0	0	487	0
N.S.	1	1.37	1.02	1.17	0.00	0.53	0.00	0.00	0.80	0.00
time (sec)	N/A	2.201	26.992	5.128	0.000	0.121	0.000	0.000	4.616	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	489	697	473	614	0	246	0	0	212	0
N.S.	1	1.43	0.97	1.26	0.00	0.50	0.00	0.00	0.43	0.00
time (sec)	N/A	1.578	25.104	4.726	0.000	0.124	0.000	0.000	2.122	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	614	439	520	0	180	0	0	84	0
N.S.	1	1.45	1.04	1.23	0.00	0.42	0.00	0.00	0.20	0.00
time (sec)	N/A	1.151	23.034	2.832	0.000	0.139	0.000	0.000	1.317	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	233	186	200	0	66	0	0	39	0
N.S.	1	1.21	0.96	1.04	0.00	0.34	0.00	0.00	0.20	0.00
time (sec)	N/A	0.514	21.262	2.226	0.000	0.115	0.000	0.000	0.624	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	867	311	235	0	0	0	0	26	0
N.S.	1	3.80	1.36	1.03	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.732	23.088	4.618	0.000	0.000	0.000	0.000	200.033	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	778	1586	1349	995	0	0	0	0	121	0
N.S.	1	2.04	1.73	1.28	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	4.859	29.028	6.355	0.000	0.000	0.000	0.000	14.995	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	995	2800	2990	1192	0	0	0	0	173	0
N.S.	1	2.81	3.01	1.20	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	8.981	33.552	11.381	0.000	0.000	0.000	0.000	48.358	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	826	261	215	0	0	0	0	26	0
N.S.	1	4.21	1.33	1.10	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	2.784	22.589	4.280	0.000	0.000	0.000	0.000	200.038	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	740	1176	468	929	0	0	0	0	121	0
N.S.	1	1.59	0.63	1.26	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	5.568	24.483	6.065	0.000	0.000	0.000	0.000	10.783	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	892	1795	1917	1079	0	0	0	0	173	0
N.S.	1	2.01	2.15	1.21	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	7.725	31.132	10.987	0.000	0.000	0.000	0.000	30.105	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	126	140	0	0	0	0	43	0
N.S.	1	1.00	0.64	0.71	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.436	35.281	1.209	0.000	0.000	0.000	0.000	1.186	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	0	261	497	0	0	0	0	53	0
N.S.	1	0.00	0.61	1.16	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	18.581	1.202	0.000	0.000	0.000	0.000	1.231	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	119	239	160	0	0	0	0	38	0
N.S.	1	0.22	0.43	0.29	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.342	15.493	0.926	0.000	0.000	0.000	0.000	1.203	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	454	344	401	0	0	0	0	26	0
N.S.	1	0.98	0.75	0.87	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.845	25.148	11.057	0.000	0.000	0.000	0.000	200.042	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	52	107	58	0	4	0	0	33	0
N.S.	1	1.68	3.45	1.87	0.00	0.13	0.00	0.00	1.06	0.00
time (sec)	N/A	0.310	35.566	2.062	0.000	0.084	0.000	0.000	0.262	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	625	0	704	2020	0	0	0	0	27	0
N.S.	1	0.00	1.13	3.23	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	29.053	11.152	0.000	0.000	0.000	0.000	200.043	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	985	0	777	2091	0	0	0	0	26	0
N.S.	1	0.00	0.79	2.12	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	30.058	10.737	0.000	0.000	0.000	0.000	200.066	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	167	871	461	1019	10508	1814	1209	965
N.S.	1	1.00	0.89	4.63	2.45	5.42	55.89	9.65	6.43	5.13
time (sec)	N/A	0.613	0.297	0.704	0.051	0.093	2.593	0.130	0.404	6.893

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	122	342	238	443	3958	768	481	446
N.S.	1	1.00	0.99	2.78	1.93	3.60	32.18	6.24	3.91	3.63
time (sec)	N/A	0.470	0.318	0.595	0.054	0.087	1.304	0.118	0.226	6.592

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	95	0	0	0	0	0	729	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	6.18	0.00
time (sec)	N/A	0.449	0.256	0.000	0.000	0.000	0.000	0.000	0.262	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	157	122	0	0	0	0	0	0	0
N.S.	1	1.18	0.92	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.560	0.217	0.000	0.000	0.000	0.000	0.000	0.262	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	183	219	145	0	0	0	0	0	0	0
N.S.	1	1.20	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.618	0.238	0.000	0.000	0.000	0.000	0.000	0.421	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	221	140	0	0	0	0	0	0	0
N.S.	1	1.14	0.72	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.665	0.329	0.000	0.000	0.000	0.000	0.000	0.364	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	325	2306	1041	2887	37262	5187	3638	2508
N.S.	1	1.00	0.91	6.46	2.92	8.09	104.38	14.53	10.19	7.03
time (sec)	N/A	1.012	0.609	0.887	0.082	0.125	7.898	0.158	0.227	7.163

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	354	1085	575	1419	16458	2435	1638	1229
N.S.	1	1.00	1.55	4.76	2.52	6.22	72.18	10.68	7.18	5.39
time (sec)	N/A	0.720	0.704	0.705	0.061	0.103	3.913	0.135	0.252	6.656

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	215	0	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.725	0.428	0.000	0.000	0.000	0.000	0.000	0.279	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	284	216	0	0	0	0	0	0	0
N.S.	1	1.16	0.88	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.483	0.471	0.000	0.000	0.000	0.000	0.000	0.295	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	373	221	0	0	0	0	0	0	0
N.S.	1	1.24	0.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.111	0.681	0.000	0.000	0.000	0.000	0.000	0.498	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	228	159	0	0	0	0	0	0	0
N.S.	1	1.13	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.629	0.457	0.000	0.000	0.000	0.000	0.000	0.690	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	411	413	0	0	0	0	0	0	26	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.031	0.000	0.000	0.000	0.000	0.000	0.000	200.037	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	0	0	0	0	0	0	42	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.610	0.000	0.000	0.000	0.000	0.000	0.000	0.721	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	159	0	0	0	0	0	18	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.399	0.269	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	26	22	26	26	26
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.85	1.00	1.00	1.00
time (sec)	N/A	0.290	1.302	0.972	0.081	0.079	7.601	0.118	200.030	6.481

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	404	411	0	0	0	0	0	0	77	0
N.S.	1	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.943	0.000	0.000	0.000	0.000	0.000	0.000	0.507	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	0	0	0	0	0	0	46	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.617	0.000	0.000	0.000	0.000	0.000	0.000	0.346	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	159	0	0	0	0	0	20	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.396	0.286	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	41	22	26	26	26
N.S.	1	1.00	1.08	0.92	1.00	1.58	0.85	1.00	1.00	1.00
time (sec)	N/A	0.269	2.749	0.990	0.106	0.086	6.890	0.121	200.035	10.166

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	106	74	75	71	93	0	181	54	114
N.S.	1	0.99	0.69	0.70	0.66	0.87	0.00	1.69	0.50	1.07
time (sec)	N/A	0.463	4.741	1.093	0.048	0.083	0.000	0.128	0.295	6.604

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	1	50	43	21	72	0	27	21	74
N.S.	1	0.01	0.60	0.51	0.25	0.86	0.00	0.32	0.25	0.88
time (sec)	N/A	0.259	1.668	1.483	0.043	0.085	0.000	0.116	0.238	6.725

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	233	248	175	0	0	0	0	0	0	0
N.S.	1	1.06	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.719	0.246	0.000	0.000	0.000	0.000	0.000	0.850	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.046	0.000	0.000	0.000	0.000	0.000	0.000	1.567	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.637	0.000	0.000	0.000	0.000	0.000	0.000	0.480	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	157	0	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.401	0.082	0.000	0.000	0.000	0.000	0.000	0.266	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	6068	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	252.83	1.08
time (sec)	N/A	0.270	1.249	1.128	0.085	0.089	0.000	0.118	0.985	6.031

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	726	0	0	0	0	0	0	0	420	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	467	0	0	0	0	0	0	0	251	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	0	0	0	0	0	0	122	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.882	0.000	0.000	0.000	0.000	0.000	0.000	0.312	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	160	0	0	0	0	0	38	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.401	0.155	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	28	30	30	0	30	38	30
N.S.	1	1.00	1.08	1.08	1.15	1.15	0.00	1.15	1.46	1.15
time (sec)	N/A	0.262	1.212	1.220	0.087	0.090	0.000	0.116	0.237	7.041

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	41	0	30	122	30
N.S.	1	1.00	1.07	1.00	1.07	1.46	0.00	1.07	4.36	1.07
time (sec)	N/A	0.267	1.239	1.178	0.099	0.237	0.000	0.226	0.246	8.751

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [130] had the largest ratio of [.7142859999999999976]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	20	0.100
2	A	3	3	1.00	19	0.158
3	A	7	7	1.15	29	0.241
4	A	5	5	1.10	29	0.172
5	A	3	3	1.00	27	0.111
6	A	4	3	1.00	29	0.103
7	A	5	4	1.00	29	0.138
8	A	9	8	1.10	29	0.276
9	A	7	7	1.30	29	0.241
10	A	5	5	1.20	29	0.172
11	A	3	3	1.00	27	0.111
12	A	5	4	1.00	29	0.138
13	A	8	7	1.06	29	0.241
14	A	7	6	1.32	28	0.214
15	A	6	5	1.44	28	0.179
16	A	2	2	1.00	26	0.077
17	A	4	3	1.00	28	0.107
18	A	5	4	0.97	28	0.143
19	A	7	6	1.04	28	0.214
20	A	9	8	1.48	28	0.286
21	A	6	6	1.47	28	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	26	0.077
23	A	5	4	0.97	28	0.143
24	A	8	7	1.00	28	0.250
25	A	12	12	1.02	31	0.387
26	A	10	10	1.00	31	0.323
27	A	8	8	1.16	31	0.258
28	B	11	11	2.56	31	0.355
29	A	13	13	1.04	31	0.419
30	A	15	15	1.04	31	0.484
31	A	3	3	1.00	25	0.120
32	A	4	4	1.00	24	0.167
33	A	4	4	1.00	25	0.160
34	A	4	4	1.00	28	0.143
35	A	2	2	1.00	24	0.083
36	A	2	2	1.00	24	0.083
37	A	2	2	1.00	22	0.091
38	A	2	2	1.00	17	0.118
39	A	5	4	1.03	24	0.167
40	A	6	5	1.16	24	0.208
41	A	9	8	1.27	24	0.333
42	A	2	2	1.00	24	0.083
43	A	2	2	1.00	24	0.083
44	A	2	2	1.00	22	0.091
45	A	2	2	1.00	17	0.118
46	A	5	4	1.04	24	0.167
47	A	5	4	1.18	24	0.167
48	A	7	6	1.32	24	0.250
49	A	2	2	1.00	24	0.083
50	A	2	2	1.00	24	0.083
51	A	2	2	1.00	22	0.091
52	A	2	2	1.00	17	0.118
53	A	5	4	1.02	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	6	5	1.30	24	0.208
55	A	7	6	1.21	24	0.250
56	A	2	2	1.00	24	0.083
57	A	2	2	1.00	24	0.083
58	A	2	2	1.00	22	0.091
59	A	2	2	1.00	17	0.118
60	A	5	4	1.00	24	0.167
61	A	7	6	1.19	24	0.250
62	A	10	9	1.21	24	0.375
63	A	2	2	1.00	26	0.077
64	A	2	2	1.00	26	0.077
65	A	2	2	1.00	24	0.083
66	A	2	2	1.00	19	0.105
67	A	5	4	0.99	26	0.154
68	A	6	5	1.08	26	0.192
69	A	9	8	1.23	26	0.308
70	A	2	2	1.00	26	0.077
71	A	2	2	1.00	26	0.077
72	A	2	2	1.00	24	0.083
73	A	2	2	1.00	19	0.105
74	A	5	4	1.00	26	0.154
75	A	5	4	1.11	26	0.154
76	A	7	6	1.26	26	0.231
77	A	2	2	1.00	26	0.077
78	A	2	2	1.00	26	0.077
79	A	2	2	1.00	24	0.083
80	A	2	2	1.00	19	0.105
81	A	5	4	0.98	26	0.154
82	A	6	5	1.24	26	0.192
83	A	9	8	1.27	26	0.308
84	A	2	2	1.00	26	0.077
85	A	2	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.00	24	0.083
87	A	2	2	1.00	19	0.105
88	A	5	4	0.97	26	0.154
89	A	6	5	1.21	26	0.192
90	A	9	8	1.20	26	0.308
91	A	6	5	1.06	25	0.200
92	A	5	4	1.05	25	0.160
93	A	12	11	1.68	24	0.458
94	A	10	9	1.61	24	0.375
95	A	6	5	1.02	26	0.192
96	A	1	1	1.00	22	0.045
97	A	7	6	0.97	28	0.214
98	A	6	5	1.00	28	0.179
99	A	2	2	1.00	28	0.071
100	A	4	4	0.97	28	0.143
101	A	5	5	1.33	28	0.179
102	A	2	2	1.00	28	0.071
103	A	2	2	1.00	28	0.071
104	A	2	2	1.00	28	0.071
105	A	2	2	1.02	28	0.071
106	A	7	6	1.39	28	0.214
107	A	4	4	0.97	28	0.143
108	A	2	2	1.02	28	0.071
109	A	2	2	1.31	28	0.071
110	A	16	15	1.23	28	0.536
111	A	12	11	1.29	28	0.393
112	A	10	9	1.31	26	0.346
113	A	9	8	1.36	21	0.381
114	F	0	0	N/A	0.000	N/A
115	B	13	12	2.15	28	0.429
116	B	19	18	2.87	28	0.643
117	A	13	12	1.25	28	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	12	11	1.31	28	0.393
119	A	8	7	1.34	26	0.269
120	A	7	6	1.38	21	0.286
121	B	13	12	2.23	28	0.429
122	B	14	13	2.10	28	0.464
123	B	20	19	2.87	28	0.679
124	A	12	11	1.23	28	0.393
125	A	10	9	1.28	28	0.321
126	A	7	6	1.40	26	0.231
127	A	5	4	1.38	21	0.190
128	B	9	8	2.68	28	0.286
129	B	14	13	2.08	28	0.464
130	B	21	20	2.83	28	0.714
131	A	2	2	1.79	28	0.071
132	B	2	2	2.50	28	0.071
133	A	10	9	1.37	28	0.321
134	A	8	7	1.43	28	0.250
135	A	5	4	1.45	26	0.154
136	A	3	2	1.21	21	0.095
137	B	6	5	3.80	28	0.179
138	B	14	13	2.04	28	0.464
139	B	19	18	2.81	28	0.643
140	B	6	5	4.21	28	0.179
141	A	2	2	1.59	28	0.071
142	B	2	2	2.01	28	0.071
143	A	3	2	1.00	28	0.071
144	F	0	0	N/A	0.000	N/A
145	C	1	1	0.22	28	0.036
146	A	3	2	0.98	30	0.067
147	A	4	4	1.68	26	0.154
148	F	0	0	N/A	0.000	N/A
149	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	2	2	1.00	22	0.091
151	A	2	2	1.00	20	0.100
152	A	2	2	1.00	22	0.091
153	A	4	4	1.18	22	0.182
154	A	3	3	1.20	22	0.136
155	A	3	3	1.14	24	0.125
156	A	2	2	1.00	24	0.083
157	A	2	2	1.00	22	0.091
158	A	2	2	1.00	24	0.083
159	A	4	4	1.16	24	0.167
160	A	5	5	1.24	24	0.208
161	A	5	5	1.13	24	0.208
162	A	7	6	1.00	26	0.231
163	A	4	3	1.00	24	0.125
164	A	3	2	1.00	19	0.105
165	N/A	1	0	1.00	26	0.000
166	A	7	6	1.02	26	0.231
167	A	4	3	1.00	24	0.125
168	A	3	2	1.00	19	0.105
169	N/A	1	0	1.00	26	0.000
170	A	2	2	0.99	36	0.056
171	A	1	1	0.01	36	0.028
172	A	4	4	1.06	22	0.182
173	A	7	6	1.00	24	0.250
174	A	4	3	1.00	22	0.136
175	A	3	2	1.00	17	0.118
176	N/A	1	0	1.00	24	0.000
177	F	0	0	N/A	0.000	N/A
178	F	0	0	N/A	0.000	N/A
179	A	5	4	1.00	26	0.154
180	A	3	2	1.00	21	0.095
181	N/A	1	0	1.00	26	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	N/A	1	0	1.00	28	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+bx^2)(c^2-d^2x^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 37

$$\int (a+bx^2)(c^2-d^2x^2)\,dx = ac^2x+\frac{1}{3}(bc^2-ad^2)x^3-\frac{1}{5}bd^2x^5$$

output
```
a*c^2*x+1/3*(-a*d^2+b*c^2)*x^3-1/5*b*d^2*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c^2-d^2x^2)\,dx = ac^2x+\frac{1}{3}(bc^2-ad^2)x^3-\frac{1}{5}bd^2x^5$$

input
```
Integrate[(a + b*x^2)*(c^2 - d^2*x^2),x]
```

output
```
a*c^2*x + ((b*c^2 - a*d^2)*x^3)/3 - (b*d^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(c^2-d^2x^2)\,dx$$

$$\Big\downarrow \text{290}$$

$$\int (x^2(bc^2-ad^2)+ac^2-bd^2x^4)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(bc^2-ad^2)+ac^2x-\frac{1}{5}bd^2x^5$$

input
```
Int[(a + b*x^2)*(c^2 - d^2*x^2),x]
```

output
```
a*c^2*x + ((b*c^2 - a*d^2)*x^3)/3 - (b*d^2*x^5)/5
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
default	$a\,c^{2} x +\frac{\left(-a\,d^{2}+b\,c^{2}\right) x^{3}}{3}-\frac{b\,d^{2} x^{5}}{5}$	34
norman	$-\frac{b\,d^{2} x^{5}}{5}+\left(-\frac{a\,d^{2}}{3}+\frac{b\,c^{2}}{3}\right) x^{3}+a\,c^{2} x$	34
risch	$a\,c^{2} x -\frac{1}{3} a\,d^{2} x^{3}+\frac{1}{3} b\,c^{2} x^{3}-\frac{1}{5} b\,d^{2} x^{5}$	35
parallelrisch	$a\,c^{2} x -\frac{1}{3} a\,d^{2} x^{3}+\frac{1}{3} b\,c^{2} x^{3}-\frac{1}{5} b\,d^{2} x^{5}$	35
gosper	$\frac{x \left(-3 b\,d^{2} x^{4}-5 a\,d^{2} x^{2}+5 b\,c^{2} x^{2}+15 a\,c^{2}\right)}{15}$	38
orering	$\frac{x \left(-3 b\,d^{2} x^{4}-5 a\,d^{2} x^{2}+5 b\,c^{2} x^{2}+15 a\,c^{2}\right) \left(-d^{2} x^{2}+c^{2}\right)}{15 \left(-d x +c \right) \left(d x +c \right)}$	65

input
```
int((b*x^2+a)*(-d^2*x^2+c^2),x,method=_RETURNVERBOSE)
```

output
```
a*c^2*x+1/3*(-a*d^2+b*c^2)*x^3-1/5*b*d^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2\right) \left(c^2-d^2x^2\right) \, dx=-\frac{1}{5} \, bd^{2} x^{5} + ac^{2} x + \frac{1}{3} \, {\left(bc^{2} - ad^{2}\right)} x^{3}$$

input
```
integrate((b*x^2+a)*(-d^2*x^2+c^2),x, algorithm="fricas")
```

output
```
-1/5*b*d^2*x^5 + a*c^2*x + 1/3*(b*c^2 - a*d^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int (a+bx^2)(c^2-d^2x^2)\,dx = ac^2x - \frac{bd^2x^5}{5} + x^3\left(-\frac{ad^2}{3}+\frac{bc^2}{3}\right)$$

input
```
integrate((b*x**2+a)*(-d**2*x**2+c**2),x)
```

output
```
a*c**2*x - b*d**2*x**5/5 + x**3*(-a*d**2/3 + b*c**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int (a+bx^2)(c^2-d^2x^2)\,dx = -\frac{1}{5}bd^2x^5 + ac^2x + \frac{1}{3}(bc^2-ad^2)x^3$$

input
```
integrate((b*x^2+a)*(-d^2*x^2+c^2),x, algorithm="maxima")
```

output
```
-1/5*b*d^2*x^5 + a*c^2*x + 1/3*(b*c^2 - a*d^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int (a+bx^2)(c^2-d^2x^2)\,dx = -\frac{1}{5}bd^2x^5 + \frac{1}{3}bc^2x^3 - \frac{1}{3}ad^2x^3 + ac^2x$$

input
```
integrate((b*x^2+a)*(-d^2*x^2+c^2),x, algorithm="giac")
```

output
```
-1/5*b*d^2*x^5 + 1/3*b*c^2*x^3 - 1/3*a*d^2*x^3 + a*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int (a+bx^2)(c^2-d^2x^2)\,dx=a\,c^2\,x-\frac{b\,d^2\,x^5}{5}-x^3\left(\frac{a\,d^2}{3}-\frac{b\,c^2}{3}\right)$$

input
```
int((c^2 - d^2*x^2)*(a + b*x^2),x)
```

output
```
a*c^2*x - (b*d^2*x^5)/5 - x^3*((a*d^2)/3 - (b*c^2)/3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c^2-d^2x^2)\,dx=\frac{x(-3b\,d^2x^4-5a\,d^2x^2+5b\,c^2x^2+15a\,c^2)}{15}$$

input
```
int((b*x^2+a)*(-d^2*x^2+c^2),x)
```

output
```
(x*(15*a*c**2 - 5*a*d**2*x**2 + 5*b*c**2*x**2 - 3*b*d**2*x**4))/15
```

3.2 $\int (c-dx)(c+dx)\left(a+bx^2\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int (c-dx)(c+dx)\left(a+bx^2\right)\,dx = ac^2x+\frac{1}{3}\left(bc^2-ad^2\right)x^3-\frac{1}{5}bd^2x^5$$

output
```
a*c^2*x+1/3*(-a*d^2+b*c^2)*x^3-1/5*b*d^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (c-dx)(c+dx)\left(a+bx^2\right)\,dx = ac^2x+\frac{1}{3}\left(bc^2-ad^2\right)x^3-\frac{1}{5}bd^2x^5$$

input
```
Integrate[(c - d*x)*(c + d*x)*(a + b*x^2),x]
```

output
```
a*c^2*x + ((b*c^2 - a*d^2)*x^3)/3 - (b*d^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {643, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)(c-dx)(c+dx)\,dx$$

$$\Big\downarrow \text{643}$$

$$\int \left(a+bx^2\right)\left(c^2-d^2x^2\right)dx$$

$$\Big\downarrow \text{290}$$

$$\int \left(x^2\left(bc^2-ad^2\right)+ac^2-bd^2x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\left(bc^2-ad^2\right)+ac^2x-\frac{1}{5}bd^2x^5$$

input
```
Int[(c - d*x)*(c + d*x)*(a + b*x^2),x]
```

output
```
a*c^2*x + ((b*c^2 - a*d^2)*x^3)/3 - (b*d^2*x^5)/5
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
default	$a\,c^{2} x +\frac{\left(-a\,d^{2}+b\,c^{2}\right) x^{3}}{3}-\frac{b\,d^{2} x^{5}}{5}$	34
norman	$-\frac{b\,d^{2} x^{5}}{5}+\left(-\frac{a\,d^{2}}{3}+\frac{b\,c^{2}}{3}\right) x^{3}+a\,c^{2} x$	34
gosper	$a\,c^{2} x -\frac{1}{3} a\,d^{2} x^{3}+\frac{1}{3} b\,c^{2} x^{3}-\frac{1}{5} b\,d^{2} x^{5}$	35
risch	$a\,c^{2} x -\frac{1}{3} a\,d^{2} x^{3}+\frac{1}{3} b\,c^{2} x^{3}-\frac{1}{5} b\,d^{2} x^{5}$	35
parallelrisch	$a\,c^{2} x -\frac{1}{3} a\,d^{2} x^{3}+\frac{1}{3} b\,c^{2} x^{3}-\frac{1}{5} b\,d^{2} x^{5}$	35
orering	$\frac{x \left(-3 b\,d^{2} x^{4}-5 a\,d^{2} x^{2}+5 b\,c^{2} x^{2}+15 a\,c^{2}\right)}{15}$	38

input
```
int((-d*x+c)*(d*x+c)*(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
a*c^2*x+1/3*(-a*d^2+b*c^2)*x^3-1/5*b*d^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int (c-dx)(c+dx)\left(a+bx^2\right)\,dx=-\frac{1}{5}\,bd^2x^5+ac^2x+\frac{1}{3}\,(bc^2-ad^2)x^3$$

input
```
integrate((-d*x+c)*(d*x+c)*(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/5*b*d^2*x^5 + a*c^2*x + 1/3*(b*c^2 - a*d^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int (c-dx)(c+dx)\left(a+bx^2\right)\,dx=ac^2x-\frac{bd^2x^5}{5}+x^3\left(-\frac{ad^2}{3}+\frac{bc^2}{3}\right)$$

input
```
integrate((-d*x+c)*(d*x+c)*(b*x**2+a),x)
```

output
```
a*c**2*x - b*d**2*x**5/5 + x**3*(-a*d**2/3 + b*c**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int (c-dx)(c+dx)\left(a+bx^2\right)\,dx=-\frac{1}{5}\,bd^2x^5+ac^2x+\frac{1}{3}\left(bc^2-ad^2\right)x^3$$

input
```
integrate((-d*x+c)*(d*x+c)*(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/5*b*d^2*x^5 + a*c^2*x + 1/3*(b*c^2 - a*d^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int (c-dx)(c+dx)\left(a+bx^2\right)\,dx=-\frac{1}{5}\,bd^2x^5+\frac{1}{3}\,bc^2x^3-\frac{1}{3}\,ad^2x^3+ac^2x$$

input
```
integrate((-d*x+c)*(d*x+c)*(b*x^2+a),x, algorithm="giac")
```

output
```
-1/5*b*d^2*x^5 + 1/3*b*c^2*x^3 - 1/3*a*d^2*x^3 + a*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int (c-dx)(c+dx)\left(a+bx^2\right)\,dx = a\,c^2\,x - \frac{b\,d^2\,x^5}{5} - x^3\left(\frac{a\,d^2}{3} - \frac{b\,c^2}{3}\right)$$

input
```
int((a + b*x^2)*(c + d*x)*(c - d*x),x)
```

output
```
a*c^2*x - (b*d^2*x^5)/5 - x^3*((a*d^2)/3 - (b*c^2)/3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (c-dx)(c+dx)\left(a+bx^2\right)\,dx = \frac{x(-3b\,d^2x^4 - 5a\,d^2x^2 + 5b\,c^2x^2 + 15a\,c^2)}{15}$$

input
```
int((-d*x+c)*(d*x+c)*(b*x^2+a),x)
```

output
```
(x*(15*a*c**2 - 5*a*d**2*x**2 + 5*b*c**2*x**2 - 3*b*d**2*x**4))/15
```

3.3 $\int \frac{(a+cx^2)^3}{\sqrt{1-ex}\sqrt{1+ex}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 149

$$\int \frac{(a+cx^2)^3}{\sqrt{1-ex}\sqrt{1+ex}} \, dx = -\frac{c(5c^2+18ace^2+24a^2e^4)\,x\sqrt{1-e^2x^2}}{16e^6} - \frac{c^2(5c+18ae^2)\,x^3\sqrt{1-e^2x^2}}{24e^4} - \frac{c^3x^5\sqrt{1-e^2x^2}}{6e^2} + \frac{(c+2ae^2)\,(5c^2+8ace^2+8a^2e^4)\arcsin(ex)}{16e^7}$$

output
```
-1/16*c*(24*a^2*e^4+18*a*c*e^2+5*c^2)*x*(-e^2*x^2+1)^(1/2)/e^6-1/24*c^2*(1
8*a*e^2+5*c)*x^3*(-e^2*x^2+1)^(1/2)/e^4-1/6*c^3*x^5*(-e^2*x^2+1)^(1/2)/e^2
+1/16*(2*a*e^2+c)*(8*a^2*e^4+8*a*c*e^2+5*c^2)*arcsin(e*x)/e^7
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.88

$$\int \frac{(a+cx^2)^3}{\sqrt{1-ex}\sqrt{1+ex}} \, dx = \frac{-cex\sqrt{1-e^2x^2}(72a^2e^4+18ace^2(3+2e^2x^2)+c^2(15+10e^2x^2+8e^4x^4))+6(5c^3+18ac^2e^2+24a^2ce^4+}{48e^7}$$

input
```
Integrate[(a + c*x^2)^3/(Sqrt[1 - e*x]*Sqrt[1 + e*x]),x]
```

output
```
(-(c*e*x*Sqrt[1 - e^2*x^2]*(72*a^2*e^4 + 18*a*c*e^2*(3 + 2*e^2*x^2) + c^2*
(15 + 10*e^2*x^2 + 8*e^4*x^4))) + 6*(5*c^3 + 18*a*c^2*e^2 + 24*a^2*c*e^4 +
 16*a^3*e^6)*ArcTan[(e*x)/(-1 + Sqrt[1 - e^2*x^2])])/(48*e^7)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {643, 318, 25, 403, 25, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{\sqrt{1-ex}\sqrt{ex+1}}dx$$

$$\Big\downarrow \; 643$$

$$\int \frac{(a+cx^2)^3}{\sqrt{1-e^2x^2}}dx$$

$$\Big\downarrow \; 318$$

$$-\frac{\int -\frac{(cx^2+a)(5c(2ae^2+c)x^2+a(6ae^2+c))}{\sqrt{1-e^2x^2}}dx}{6e^2}-\frac{cx\sqrt{1-e^2x^2}(a+cx^2)^2}{6e^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(cx^2+a)(5c(2ae^2+c)x^2+a(6ae^2+c))}{\sqrt{1-e^2x^2}}dx}{6e^2}-\frac{cx\sqrt{1-e^2x^2}(a+cx^2)^2}{6e^2}$$

$$\Big\downarrow \; 403$$

$$\frac{-\frac{\int -\frac{c(44a^2e^4+44ace^2+15c^2)x^2+a(24a^2e^4+14ace^2+5c^2)}{\sqrt{1-e^2x^2}}dx}{4e^2}-\frac{5cx\sqrt{1-e^2x^2}(2ae^2+c)(a+cx^2)}{4e^2}}{6e^2}-\frac{cx\sqrt{1-e^2x^2}(a+cx^2)^2}{6e^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{c(44a^2e^4+44ace^2+15c^2)x^2+a(24a^2e^4+14ace^2+5c^2)}{\sqrt{1-e^2x^2}}dx}{4e^2}-\frac{5cx\sqrt{1-e^2x^2}(2ae^2+c)(a+cx^2)}{4e^2}}{6e^2}-\frac{cx\sqrt{1-e^2x^2}(a+cx^2)^2}{6e^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\frac{3(2ae^2+c)(8a^2e^4+8ace^2+5c^2)\int \frac{1}{\sqrt{1-e^2x^2}}dx}{2e^2}-\frac{cx\sqrt{1-e^2x^2}(44a^2e^4+44ace^2+15c^2)}{2e^2}}{4e^2}-\frac{5cx\sqrt{1-e^2x^2}(2ae^2+c)(a+cx^2)}{4e^2}}{6e^2}-\frac{cx\sqrt{1-e^2x^2}(a+cx^2)^2}{6e^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{\frac{3(2ae^2+c)(8a^2e^4+8ace^2+5c^2)\arcsin(ex)}{2e^3}-\frac{cx\sqrt{1-e^2x^2}(44a^2e^4+44ace^2+15c^2)}{2e^2}}{4e^2}-\frac{5cx\sqrt{1-e^2x^2}(2ae^2+c)(a+cx^2)}{4e^2}}{6e^2}-\frac{cx\sqrt{1-e^2x^2}(a+cx^2)^2}{6e^2}$$

input
```
Int[(a + c*x^2)^3/(Sqrt[1 - e*x]*Sqrt[1 + e*x]),x]
```

output
```
-1/6*(c*x*(a + c*x^2)^2*Sqrt[1 - e^2*x^2])/e^2 + ((-5*c*(c + 2*a*e^2)*x*(a
 + c*x^2)*Sqrt[1 - e^2*x^2])/(4*e^2) + (-1/2*(c*(15*c^2 + 44*a*c*e^2 + 44*
a^2*e^4)*x*Sqrt[1 - e^2*x^2])/e^2 + (3*(c + 2*a*e^2)*(5*c^2 + 8*a*c*e^2 +
8*a^2*e^4)*ArcSin[e*x])/(2*e^3))/(4*e^2))/(6*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.35

method	result
risch	$\frac{cx\left(8c^2x^4e^4+36x^2ac\,e^4+72a^2e^4+10c^2e^2x^2+54ac\,e^2+15c^2\right)(ex-1)\sqrt{ex+1}\,\sqrt{(-ex+1)(ex+1)}}{48e^6\sqrt{-(ex-1)(ex+1)}\,\sqrt{-ex+1}}+\frac{\left(16e^6a^3+24e^4a^2c+18c^2a\,e^2+5c\right.}{16e^6\sqrt{e}}$
default	$-\frac{\sqrt{-ex+1}\,\sqrt{ex+1}\left(8\,\mathrm{csgn}(e)c^3e^5x^5\sqrt{-e^2x^2+1}+36\,\mathrm{csgn}(e)a\,c^2e^5x^3\sqrt{-e^2x^2+1}+72\sqrt{-e^2x^2+1}\,\mathrm{csgn}(e)e^5a^2cx+10\sqrt{-e^2x^2+1}\right.}{}$

input
```
int((c*x^2+a)^3/(-e*x+1)^(1/2)/(e*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*c*x*(8*c^2*e^4*x^4+36*a*c*e^4*x^2+72*a^2*e^4+10*c^2*e^2*x^2+54*a*c*e^
2+15*c^2)*(e*x-1)*(e*x+1)^(1/2)/e^6/(-(e*x-1)*(e*x+1))^(1/2)*((-e*x+1)*(e*
x+1))^(1/2)/(-e*x+1)^(1/2)+1/16*(16*a^3*e^6+24*a^2*c*e^4+18*a*c^2*e^2+5*c^
3)/e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+1)^(1/2))*((-e*x+1)*(e*x
+1))^(1/2)/(-e*x+1)^(1/2)/(e*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{(a+cx^2)^3}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = -\frac{(8\,c^3e^5x^5+2\,(18\,ac^2e^5+5\,c^3e^3)x^3+3\,(24\,a^2ce^5+18\,ac^2e^3+5\,c^3e)x)\sqrt{ex+1}\sqrt{-ex+1}+6\,(16\,a^3e^6}{48\,e^7}$$

input
```
integrate((c*x^2+a)^3/(-e*x+1)^(1/2)/(e*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/48*((8*c^3*e^5*x^5 + 2*(18*a*c^2*e^5 + 5*c^3*e^3)*x^3 + 3*(24*a^2*c*e^5
 + 18*a*c^2*e^3 + 5*c^3*e)*x)*sqrt(e*x + 1)*sqrt(-e*x + 1) + 6*(16*a^3*e^6
 + 24*a^2*c*e^4 + 18*a*c^2*e^2 + 5*c^3)*arctan((sqrt(e*x + 1)*sqrt(-e*x +
1) - 1)/(e*x)))/e^7
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^3}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**3/(-e*x+1)**(1/2)/(e*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.54

$$\int \frac{(a+cx^2)^3}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = -\frac{\sqrt{-e^2x^2+1}c^3x^5}{6\,e^2} - \frac{3\sqrt{-e^2x^2+1}ac^2x^3}{4\,e^2} + \frac{a^3\arcsin\left(\frac{e^2x}{\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{3\sqrt{-e^2x^2+1}a^2cx}{2\,e^2} - \frac{5\sqrt{-e^2x^2+1}c^3x^3}{24\,e^4} + \frac{3\,a^2c\arcsin\left(\frac{e^2x}{\sqrt{e^2}}\right)}{2\sqrt{e^2}e^2} - \frac{9\sqrt{-e^2x^2+1}ac^2x}{8\,e^4} + \frac{9\,ac^2\arcsin\left(\frac{e^2x}{\sqrt{e^2}}\right)}{8\sqrt{e^2}e^4} - \frac{5\sqrt{-e^2x^2+1}c^3x}{16\,e^6} + \frac{5\,c^3\arcsin\left(\frac{e^2x}{\sqrt{e^2}}\right)}{16\sqrt{e^2}e^6}$$

input
```
integrate((c*x^2+a)^3/(-e*x+1)^(1/2)/(e*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(-e^2*x^2 + 1)*c^3*x^5/e^2 - 3/4*sqrt(-e^2*x^2 + 1)*a*c^2*x^3/e^2
 + a^3*arcsin(e^2*x/sqrt(e^2))/sqrt(e^2) - 3/2*sqrt(-e^2*x^2 + 1)*a^2*c*x/
e^2 - 5/24*sqrt(-e^2*x^2 + 1)*c^3*x^3/e^4 + 3/2*a^2*c*arcsin(e^2*x/sqrt(e^
2))/(sqrt(e^2)*e^2) - 9/8*sqrt(-e^2*x^2 + 1)*a*c^2*x/e^4 + 9/8*a*c^2*arcsi
n(e^2*x/sqrt(e^2))/(sqrt(e^2)*e^4) - 5/16*sqrt(-e^2*x^2 + 1)*c^3*x/e^6 + 5
/16*c^3*arcsin(e^2*x/sqrt(e^2))/(sqrt(e^2)*e^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.28

$$\int \frac{(a+cx^2)^3}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = \frac{(72\,a^2ce^4 + 90\,ac^2e^2 + 33\,c^3 - (72\,a^2ce^4 + 162\,ac^2e^2 + 85\,c^3 - 2\,(54\,ac^2e^2 + 55\,c^3 - (18\,ac^2e^2 + 45\,c^3 + 4}{}$$

input
```
integrate((c*x^2+a)^3/(-e*x+1)^(1/2)/(e*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/48*((72*a^2*c*e^4 + 90*a*c^2*e^2 + 33*c^3 - (72*a^2*c*e^4 + 162*a*c^2*e^
2 + 85*c^3 - 2*(54*a*c^2*e^2 + 55*c^3 - (18*a*c^2*e^2 + 45*c^3 + 4*((e*x +
 1)*c^3 - 5*c^3)*(e*x + 1))*(e*x + 1))*(e*x + 1))*(e*x + 1))*sqrt(e*x + 1)
*sqrt(-e*x + 1) + 6*(16*a^3*e^6 + 24*a^2*c*e^4 + 18*a*c^2*e^2 + 5*c^3)*arc
sin(1/2*sqrt(2)*sqrt(e*x + 1)))/e^7
```

Mupad [B] (verification not implemented)

Time = 14.12 (sec) , antiderivative size = 1044, normalized size of antiderivative = 7.01

$$\int \frac{(a+cx^2)^3}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = \text{Too large to display}$$

input
```
int((a + c*x^2)^3/((1 - e*x)^(1/2)*(e*x + 1)^(1/2)),x)
```

output
```
((((1 - e*x)^(1/2) - 1)*((5*c^3)/4 + (9*a*c^2*e^2)/2 + 6*a^2*c*e^4))/((e*x
 + 1)^(1/2) - 1) - (((1 - e*x)^(1/2) - 1)^23*((5*c^3)/4 + (9*a*c^2*e^2)/2
+ 6*a^2*c*e^4))/((e*x + 1)^(1/2) - 1)^23 + (((1 - e*x)^(1/2) - 1)^3*((175*
c^3)/12 + (105*a*c^2*e^2)/2 + 6*a^2*c*e^4))/((e*x + 1)^(1/2) - 1)^3 - (((1
 - e*x)^(1/2) - 1)^21*((175*c^3)/12 + (105*a*c^2*e^2)/2 + 6*a^2*c*e^4))/((
e*x + 1)^(1/2) - 1)^21 - (((1 - e*x)^(1/2) - 1)^5*((669*a*c^2*e^2)/2 - (31
1*c^3)/4 + 126*a^2*c*e^4))/((e*x + 1)^(1/2) - 1)^5 + (((1 - e*x)^(1/2) - 1
)^19*((669*a*c^2*e^2)/2 - (311*c^3)/4 + 126*a^2*c*e^4))/((e*x + 1)^(1/2) -
 1)^19 - (((1 - e*x)^(1/2) - 1)^7*((8361*c^3)/4 + (1533*a*c^2*e^2)/2 + 510
*a^2*c*e^4))/((e*x + 1)^(1/2) - 1)^7 + (((1 - e*x)^(1/2) - 1)^17*((8361*c^
3)/4 + (1533*a*c^2*e^2)/2 + 510*a^2*c*e^4))/((e*x + 1)^(1/2) - 1)^17 - (((
1 - e*x)^(1/2) - 1)^11*((25295*c^3)/2 - 549*a*c^2*e^2 + 420*a^2*c*e^4))/((
e*x + 1)^(1/2) - 1)^11 + (((1 - e*x)^(1/2) - 1)^13*((25295*c^3)/2 - 549*a*
c^2*e^2 + 420*a^2*c*e^4))/((e*x + 1)^(1/2) - 1)^13 + (((1 - e*x)^(1/2) - 1
)^9*((42259*c^3)/6 + 165*a*c^2*e^2 - 804*a^2*c*e^4))/((e*x + 1)^(1/2) - 1)
^9 - (((1 - e*x)^(1/2) - 1)^15*((42259*c^3)/6 + 165*a*c^2*e^2 - 804*a^2*c*
e^4))/((e*x + 1)^(1/2) - 1)^15)/(e^7 + (12*e^7*((1 - e*x)^(1/2) - 1)^2)/((
e*x + 1)^(1/2) - 1)^2 + (66*e^7*((1 - e*x)^(1/2) - 1)^4)/((e*x + 1)^(1/2)
- 1)^4 + (220*e^7*((1 - e*x)^(1/2) - 1)^6)/((e*x + 1)^(1/2) - 1)^6 + (495*
e^7*((1 - e*x)^(1/2) - 1)^8)/((e*x + 1)^(1/2) - 1)^8 + (792*e^7*((1 - e...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.53

$$\int \frac{(a+cx^2)^3}{\sqrt{1-ex}\sqrt{1+ex}}\,dx$$
$$= \frac{-96asin\left(\frac{\sqrt{-ex+1}}{\sqrt{2}}\right)a^3e^6 - 144asin\left(\frac{\sqrt{-ex+1}}{\sqrt{2}}\right)a^2c\,e^4 - 108asin\left(\frac{\sqrt{-ex+1}}{\sqrt{2}}\right)a\,c^2e^2 - 30asin\left(\frac{\sqrt{-ex+1}}{\sqrt{2}}\right)c^3 - 7}{}$$

input
```
int((c*x^2+a)^3/(-e*x+1)^(1/2)/(e*x+1)^(1/2),x)
```

output
```
( - 96*asin(sqrt( - e*x + 1)/sqrt(2))*a**3*e**6 - 144*asin(sqrt( - e*x + 1
)/sqrt(2))*a**2*c*e**4 - 108*asin(sqrt( - e*x + 1)/sqrt(2))*a*c**2*e**2 -
30*asin(sqrt( - e*x + 1)/sqrt(2))*c**3 - 72*sqrt(e*x + 1)*sqrt( - e*x + 1)
*a**2*c*e**5*x - 36*sqrt(e*x + 1)*sqrt( - e*x + 1)*a*c**2*e**5*x**3 - 54*s
qrt(e*x + 1)*sqrt( - e*x + 1)*a*c**2*e**3*x - 8*sqrt(e*x + 1)*sqrt( - e*x
+ 1)*c**3*e**5*x**5 - 10*sqrt(e*x + 1)*sqrt( - e*x + 1)*c**3*e**3*x**3 - 1
5*sqrt(e*x + 1)*sqrt( - e*x + 1)*c**3*e*x)/(48*e**7)
```

3.4 $\int \frac{(a+cx^2)^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 93

$$\int \frac{(a+cx^2)^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = -\frac{c(3c+8ae^2)\,x\sqrt{1-e^2x^2}}{8e^4} - \frac{c^2x^3\sqrt{1-e^2x^2}}{4e^2} + \frac{(3c^2+8ace^2+8a^2e^4)\arcsin(ex)}{8e^5}$$

output
```
-1/8*c*(8*a*e^2+3*c)*x*(-e^2*x^2+1)^(1/2)/e^4-1/4*c^2*x^3*(-e^2*x^2+1)^(1/
2)/e^2+1/8*(8*a^2*e^4+8*a*c*e^2+3*c^2)*arcsin(e*x)/e^5
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{(a+cx^2)^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = -\frac{cx\sqrt{1-e^2x^2}(3c+8ae^2+2ce^2x^2)}{8e^4} + \frac{(3c^2+8ace^2+8a^2e^4)\arctan\left(\frac{ex}{-1+\sqrt{1-e^2x^2}}\right)}{4e^5}$$

input
```
Integrate[(a + c*x^2)^2/(Sqrt[1 - e*x]*Sqrt[1 + e*x]),x]
```

output
```
-1/8*(c*x*Sqrt[1 - e^2*x^2]*(3*c + 8*a*e^2 + 2*c*e^2*x^2))/e^4 + ((3*c^2 +
 8*a*c*e^2 + 8*a^2*e^4)*ArcTan[(e*x)/(-1 + Sqrt[1 - e^2*x^2])])/(4*e^5)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {643, 318, 25, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{\sqrt{1-ex}\sqrt{ex+1}}dx$$

$$\Big\downarrow \text{643}$$

$$\int \frac{(a+cx^2)^2}{\sqrt{1-e^2x^2}}dx$$

$$\Big\downarrow \text{318}$$

$$-\frac{\int -\frac{3c(2ae^2+c)x^2+a(4ae^2+c)}{\sqrt{1-e^2x^2}}dx}{4e^2}-\frac{cx\sqrt{1-e^2x^2}(a+cx^2)}{4e^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3c(2ae^2+c)x^2+a(4ae^2+c)}{\sqrt{1-e^2x^2}}dx}{4e^2}-\frac{cx\sqrt{1-e^2x^2}(a+cx^2)}{4e^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{(8a^2e^4+8ace^2+3c^2)\int \frac{1}{\sqrt{1-e^2x^2}}dx}{2e^2}-\frac{3cx\sqrt{1-e^2x^2}(2ae^2+c)}{2e^2}}{4e^2}-\frac{cx\sqrt{1-e^2x^2}(a+cx^2)}{4e^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{(8a^2e^4+8ace^2+3c^2)\arcsin(ex)}{2e^3}-\frac{3cx\sqrt{1-e^2x^2}(2ae^2+c)}{2e^2}}{4e^2}-\frac{cx\sqrt{1-e^2x^2}(a+cx^2)}{4e^2}$$

input
```
Int[(a + c*x^2)^2/(Sqrt[1 - e*x]*Sqrt[1 + e*x]),x]
```

output
```
-1/4*(c*x*(a + c*x^2)*Sqrt[1 - e^2*x^2])/e^2 + ((-3*c*(c + 2*a*e^2)*x*Sqrt
[1 - e^2*x^2])/(2*e^2) + ((3*c^2 + 8*a*c*e^2 + 8*a^2*e^4)*ArcSin[e*x])/(2*
e^3))/(4*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

method	result
risch	$\frac{cx\left(2x^{2}c\,e^{2}+8a\,e^{2}+3c\right)\left(ex-1\right)\sqrt{ex+1}\,\sqrt{\left(-ex+1\right)\left(ex+1\right)}}{8e^{4}\sqrt{-\left(ex-1\right)\left(ex+1\right)}\,\sqrt{-ex+1}}+\frac{\left(8a^{2}e^{4}+8ac\,e^{2}+3c^{2}\right)\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+1}}\right)\sqrt{\left(-ex+1\right)\left(ex+1\right)}}{8e^{4}\sqrt{e^{2}}\,\sqrt{-ex+1}\,\sqrt{ex+1}}$
default	$-\frac{\sqrt{-ex+1}\,\sqrt{ex+1}\,\left(2\,\mathrm{csgn}\left(e\right)c^{2}e^{3}x^{3}\sqrt{-e^{2}x^{2}+1}+8\sqrt{-e^{2}x^{2}+1}\,\mathrm{csgn}\left(e\right)e^{3}acx-8\arctan\left(\frac{\mathrm{csgn}\left(e\right)ex}{\sqrt{-e^{2}x^{2}+1}}\right)a^{2}e^{4}+3\sqrt{-e^{2}x^{2}+1}\,\mathrm{csgn}\left(e\right)}{8e^{5}\sqrt{-e^{2}x^{2}+1}}$

input
```
int((c*x^2+a)^2/(-e*x+1)^(1/2)/(e*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*c*x*(2*c*e^2*x^2+8*a*e^2+3*c)*(e*x-1)*(e*x+1)^(1/2)/e^4/(-(e*x-1)*(e*x
+1))^(1/2)*((-e*x+1)*(e*x+1))^(1/2)/(-e*x+1)^(1/2)+1/8*(8*a^2*e^4+8*a*c*e^
2+3*c^2)/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+1)^(1/2))*((-e*x+1
)*(e*x+1))^(1/2)/(-e*x+1)^(1/2)/(e*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+cx^2\right)^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = -\frac{\left(2\,c^2e^3x^3+\left(8\,ace^3+3\,c^2e\right)x\right)\sqrt{ex+1}\sqrt{-ex+1}+2\left(8\,a^2e^4+8\,ace^2+3\,c^2\right)\arctan\left(\frac{\sqrt{ex+1}\sqrt{-ex+1}-1}{ex}\right)}{8\,e^5}$$

input
```
integrate((c*x^2+a)^2/(-e*x+1)^(1/2)/(e*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/8*((2*c^2*e^3*x^3 + (8*a*c*e^3 + 3*c^2*e)*x)*sqrt(e*x + 1)*sqrt(-e*x +
1) + 2*(8*a^2*e^4 + 8*a*c*e^2 + 3*c^2)*arctan((sqrt(e*x + 1)*sqrt(-e*x + 1
) - 1)/(e*x)))/e^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**2/(-e*x+1)**(1/2)/(e*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.41

$$\int \frac{(a+cx^2)^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = -\frac{\sqrt{-e^2x^2+1}c^2x^3}{4\,e^2} + \frac{a^2\arcsin\left(\frac{e^2x}{\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{\sqrt{-e^2x^2+1}acx}{e^2} + \frac{ac\arcsin\left(\frac{e^2x}{\sqrt{e^2}}\right)}{\sqrt{e^2}e^2} - \frac{3\sqrt{-e^2x^2+1}c^2x}{8\,e^4} + \frac{3\,c^2\arcsin\left(\frac{e^2x}{\sqrt{e^2}}\right)}{8\,\sqrt{e^2}e^4}$$

input
```
integrate((c*x^2+a)^2/(-e*x+1)^(1/2)/(e*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(-e^2*x^2 + 1)*c^2*x^3/e^2 + a^2*arcsin(e^2*x/sqrt(e^2))/sqrt(e^2
) - sqrt(-e^2*x^2 + 1)*a*c*x/e^2 + a*c*arcsin(e^2*x/sqrt(e^2))/(sqrt(e^2)*
e^2) - 3/8*sqrt(-e^2*x^2 + 1)*c^2*x/e^4 + 3/8*c^2*arcsin(e^2*x/sqrt(e^2))/
(sqrt(e^2)*e^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.22

$$\int \frac{(a+cx^2)^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = \frac{(8\,ace^2-(8\,ace^2+2\,((ex+1)c^2-3\,c^2)(ex+1)+9\,c^2)(ex+1)+5\,c^2)\sqrt{ex+1}\sqrt{-ex+1}+2\,(8\,a^2e^4+}{8\,e^5}$$

input
```
integrate((c*x^2+a)^2/(-e*x+1)^(1/2)/(e*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/8*((8*a*c*e^2 - (8*a*c*e^2 + 2*((e*x + 1)*c^2 - 3*c^2)*(e*x + 1) + 9*c^2
)*(e*x + 1) + 5*c^2)*sqrt(e*x + 1)*sqrt(-e*x + 1) + 2*(8*a^2*e^4 + 8*a*c*e
^2 + 3*c^2)*arcsin(1/2*sqrt(2)*sqrt(e*x + 1)))/e^5
```

Mupad [B] (verification not implemented)

Time = 11.62 (sec) , antiderivative size = 580, normalized size of antiderivative = 6.24

$$\int \frac{(a+cx^2)^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = -\frac{\frac{\left(\sqrt{1-e\,x}-1\right)^{15}\left(\frac{3\,c^2}{2}+4\,a\,c\,e^2\right)}{\left(\sqrt{e\,x+1}-1\right)^{15}}-\frac{\left(\sqrt{1-e\,x}-1\right)^{3}\left(\frac{23\,c^2}{2}-12\,a\,c\,e^2\right)}{\left(\sqrt{e\,x+1}-1\right)^{3}}+\frac{\left(\sqrt{1-e\,x}-1\right)^{13}\left(\frac{23\,c^2}{2}-12\,a\,c\,e^2\right)}{\left(\sqrt{e\,x+1}-1\right)^{13}}+\frac{\left(\sqrt{1-e\,x}-1\right)^{5}\left(\frac{333\,c^2}{2}+60\,a\right.}{\left(\sqrt{e\,x+1}-1\right)^{5}}}{e^5+\frac{8\,e^5\left(\sqrt{1-e\,x}-1\right)^2}{\left(\sqrt{e\,x+1}-1\right)^2}+\frac{28\,e^5\left(\sqrt{1-e\,x}-1\right)^4}{\left(\sqrt{e\,x+1}-1\right)^4}+\frac{56\,e^5\left(\sqrt{1-e\,x}-1\right)^6}{\left(\sqrt{e\,x+1}-1\right)^6}+\frac{70\,e^5\left(\sqrt{1-\right.}{\left(\sqrt{e\,x+1}\right.}} - \frac{\mathrm{atan}\left(\frac{\sqrt{1-e\,x}-1}{\sqrt{e\,x+1}-1}\right)\left(8\,a^2\,e^4+8\,a\,c\,e^2+3\,c^2\right)}{2\,e^5}$$

input
```
int((a + c*x^2)^2/((1 - e*x)^(1/2)*(e*x + 1)^(1/2)),x)
```

output
```
- ((((1 - e*x)^(1/2) - 1)^15*((3*c^2)/2 + 4*a*c*e^2))/((e*x + 1)^(1/2) - 1
)^15 - (((1 - e*x)^(1/2) - 1)^3*((23*c^2)/2 - 12*a*c*e^2))/((e*x + 1)^(1/2
) - 1)^3 + (((1 - e*x)^(1/2) - 1)^13*((23*c^2)/2 - 12*a*c*e^2))/((e*x + 1)
^(1/2) - 1)^13 + (((1 - e*x)^(1/2) - 1)^5*((333*c^2)/2 + 60*a*c*e^2))/((e*
x + 1)^(1/2) - 1)^5 - (((1 - e*x)^(1/2) - 1)^11*((333*c^2)/2 + 60*a*c*e^2)
)/((e*x + 1)^(1/2) - 1)^11 - (((1 - e*x)^(1/2) - 1)^7*((671*c^2)/2 - 44*a*
c*e^2))/((e*x + 1)^(1/2) - 1)^7 + (((1 - e*x)^(1/2) - 1)^9*((671*c^2)/2 -
44*a*c*e^2))/((e*x + 1)^(1/2) - 1)^9 - (((1 - e*x)^(1/2) - 1)*((3*c^2)/2 +
 4*a*c*e^2))/((e*x + 1)^(1/2) - 1))/(e^5 + (8*e^5*((1 - e*x)^(1/2) - 1)^2)
/((e*x + 1)^(1/2) - 1)^2 + (28*e^5*((1 - e*x)^(1/2) - 1)^4)/((e*x + 1)^(1/
2) - 1)^4 + (56*e^5*((1 - e*x)^(1/2) - 1)^6)/((e*x + 1)^(1/2) - 1)^6 + (70
*e^5*((1 - e*x)^(1/2) - 1)^8)/((e*x + 1)^(1/2) - 1)^8 + (56*e^5*((1 - e*x)
^(1/2) - 1)^10)/((e*x + 1)^(1/2) - 1)^10 + (28*e^5*((1 - e*x)^(1/2) - 1)^1
2)/((e*x + 1)^(1/2) - 1)^12 + (8*e^5*((1 - e*x)^(1/2) - 1)^14)/((e*x + 1)^
(1/2) - 1)^14 + (e^5*((1 - e*x)^(1/2) - 1)^16)/((e*x + 1)^(1/2) - 1)^16) -
 (atan(((1 - e*x)^(1/2) - 1)/((e*x + 1)^(1/2) - 1))*(3*c^2 + 8*a^2*e^4 + 8
*a*c*e^2))/(2*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.40

$$\int \frac{(a+cx^2)^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx$$
$$= \frac{-16asin\left(\frac{\sqrt{-ex+1}}{\sqrt{2}}\right)a^2e^4 - 16asin\left(\frac{\sqrt{-ex+1}}{\sqrt{2}}\right)ac\,e^2 - 6asin\left(\frac{\sqrt{-ex+1}}{\sqrt{2}}\right)c^2 - 8\sqrt{ex+1}\,\sqrt{-ex+1}\,ac\,e^3x - 2}{8e^5}$$

input
```
int((c*x^2+a)^2/(-e*x+1)^(1/2)/(e*x+1)^(1/2),x)
```

output
```
( - 16*asin(sqrt( - e*x + 1)/sqrt(2))*a**2*e**4 - 16*asin(sqrt( - e*x + 1)
/sqrt(2))*a*c*e**2 - 6*asin(sqrt( - e*x + 1)/sqrt(2))*c**2 - 8*sqrt(e*x +
1)*sqrt( - e*x + 1)*a*c*e**3*x - 2*sqrt(e*x + 1)*sqrt( - e*x + 1)*c**2*e**
3*x**3 - 3*sqrt(e*x + 1)*sqrt( - e*x + 1)*c**2*e*x)/(8*e**5)
```

3.5 $\int \frac{a+cx^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 43

$$\int \frac{a+cx^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = -\frac{cx\sqrt{1-e^2x^2}}{2e^2}+\frac{(c+2ae^2)\arcsin(ex)}{2e^3}$$

output
```
-1/2*c*x*(-e^2*x^2+1)^(1/2)/e^2+1/2*(2*a*e^2+c)*arcsin(e*x)/e^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

$$\int \frac{a+cx^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = -\frac{cx\sqrt{1-e^2x^2}}{2e^2}+\frac{(c+2ae^2)\arctan\left(\frac{ex}{-1+\sqrt{1-e^2x^2}}\right)}{e^3}$$

input
```
Integrate[(a + c*x^2)/(Sqrt[1 - e*x]*Sqrt[1 + e*x]),x]
```

output
```
-1/2*(c*x*Sqrt[1 - e^2*x^2])/e^2 + ((c + 2*a*e^2)*ArcTan[(e*x)/(-1 + Sqrt[
1 - e^2*x^2])])/e^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {643, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{\sqrt{1-ex}\sqrt{ex+1}}dx$$

$$\Big\downarrow \text{643}$$

$$\int \frac{a+cx^2}{\sqrt{1-e^2x^2}}dx$$

$$\Big\downarrow \text{299}$$

$$\frac{(2ae^2+c)\int \frac{1}{\sqrt{1-e^2x^2}}dx}{2e^2}-\frac{cx\sqrt{1-e^2x^2}}{2e^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{(2ae^2+c)\arcsin(ex)}{2e^3}-\frac{cx\sqrt{1-e^2x^2}}{2e^2}$$

input
```
Int[(a + c*x^2)/(Sqrt[1 - e*x]*Sqrt[1 + e*x]),x]
```

output
```
-1/2*(c*x*Sqrt[1 - e^2*x^2])/e^2 + ((c + 2*a*e^2)*ArcSin[e*x])/(2*e^3)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.88 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

method	result	size
default	$-\frac{\sqrt{-ex+1}\,\sqrt{ex+1}\,\left(\sqrt{-e^{2}x^{2}+1}\,\operatorname{csgn}(e)ecx-2\arctan\left(\frac{\operatorname{csgn}(e)ex}{\sqrt{-e^{2}x^{2}+1}}\right)a\,e^{2}-\arctan\left(\frac{\operatorname{csgn}(e)ex}{\sqrt{-e^{2}x^{2}+1}}\right)c\right)\operatorname{csgn}(e)}{2e^{3}\sqrt{-e^{2}x^{2}+1}}$	99
risch	$\frac{cx(ex-1)\sqrt{ex+1}\,\sqrt{(-ex+1)(ex+1)}}{2e^{2}\sqrt{-(ex-1)(ex+1)}\,\sqrt{-ex+1}}+\frac{(2a\,e^{2}+c)\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+1}}\right)\sqrt{(-ex+1)(ex+1)}}{2e^{2}\sqrt{e^{2}}\,\sqrt{-ex+1}\,\sqrt{ex+1}}$	124

input
```
int((c*x^2+a)/(-e*x+1)^(1/2)/(e*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-e*x+1)^(1/2)*(e*x+1)^(1/2)/e^3*((-e^2*x^2+1)^(1/2)*csgn(e)*e*c*x-2*
arctan(csgn(e)*e*x/(-e^2*x^2+1)^(1/2))*a*e^2-arctan(csgn(e)*e*x/(-e^2*x^2+
1)^(1/2))*c)/(-e^2*x^2+1)^(1/2)*csgn(e)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{a+cx^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = -\frac{\sqrt{ex+1}\sqrt{-ex+1}cex + 2\left(2\,ae^2+c\right)\arctan\left(\frac{\sqrt{ex+1}\sqrt{-ex+1}-1}{ex}\right)}{2\,e^3}$$

input
```
integrate((c*x^2+a)/(-e*x+1)^(1/2)/(e*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(e*x + 1)*sqrt(-e*x + 1)*c*e*x + 2*(2*a*e^2 + c)*arctan((sqrt(e*
x + 1)*sqrt(-e*x + 1) - 1)/(e*x)))/e^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+cx^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)/(-e*x+1)**(1/2)/(e*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \frac{a+cx^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = \frac{a\arcsin\left(\frac{e^2x}{\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{\sqrt{-e^2x^2+1}cx}{2\,e^2} + \frac{c\arcsin\left(\frac{e^2x}{\sqrt{e^2}}\right)}{2\,\sqrt{e^2}e^2}$$

input
```
integrate((c*x^2+a)/(-e*x+1)^(1/2)/(e*x+1)^(1/2),x, algorithm="maxima")
```

output
```
a*arcsin(e^2*x/sqrt(e^2))/sqrt(e^2) - 1/2*sqrt(-e^2*x^2 + 1)*c*x/e^2 + 1/2
*c*arcsin(e^2*x/sqrt(e^2))/(sqrt(e^2)*e^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \frac{a+cx^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = -\frac{((ex+1)c-c)\sqrt{ex+1}\sqrt{-ex+1}-2\left(2\,ae^2+c\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{ex+1}\right)}{2\,e^3}$$

input
```
integrate((c*x^2+a)/(-e*x+1)^(1/2)/(e*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(((e*x + 1)*c - c)*sqrt(e*x + 1)*sqrt(-e*x + 1) - 2*(2*a*e^2 + c)*arc
sin(1/2*sqrt(2)*sqrt(e*x + 1)))/e^3
```

Mupad [B] (verification not implemented)

Time = 8.45 (sec) , antiderivative size = 203, normalized size of antiderivative = 4.72

$$\int \frac{a+cx^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = -\frac{4\,a\,\mathrm{atan}\left(\frac{e\,\left(\sqrt{1-e\,x}-1\right)}{\left(\sqrt{e\,x+1}-1\right)\,\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{2\,c\,\mathrm{atan}\left(\frac{\sqrt{1-e\,x}-1}{\sqrt{e\,x+1}-1}\right)}{e^3} - \frac{\frac{14\,c\,{\left(\sqrt{1-e\,x}-1\right)}^3}{{\left(\sqrt{e\,x+1}-1\right)}^3}-\frac{14\,c\,{\left(\sqrt{1-e\,x}-1\right)}^5}{{\left(\sqrt{e\,x+1}-1\right)}^5}+\frac{2\,c\,{\left(\sqrt{1-e\,x}-1\right)}^7}{{\left(\sqrt{e\,x+1}-1\right)}^7}-\frac{2\,c\,\left(\sqrt{1-e\,x}-1\right)}{\sqrt{e\,x+1}-1}}{e^3\,{\left(\frac{{\left(\sqrt{1-e\,x}-1\right)}^2}{{\left(\sqrt{e\,x+1}-1\right)}^2}+1\right)}^4}$$

input
```
int((a + c*x^2)/((1 - e*x)^(1/2)*(e*x + 1)^(1/2)),x)
```

output
```
- (4*a*atan((e*((1 - e*x)^(1/2) - 1))/(((e*x + 1)^(1/2) - 1)*(e^2)^(1/2)))
)/(e^2)^(1/2) - (2*c*atan(((1 - e*x)^(1/2) - 1)/((e*x + 1)^(1/2) - 1)))/e^
3 - ((14*c*((1 - e*x)^(1/2) - 1)^3)/((e*x + 1)^(1/2) - 1)^3 - (14*c*((1 -
e*x)^(1/2) - 1)^5)/((e*x + 1)^(1/2) - 1)^5 + (2*c*((1 - e*x)^(1/2) - 1)^7)
/((e*x + 1)^(1/2) - 1)^7 - (2*c*((1 - e*x)^(1/2) - 1))/((e*x + 1)^(1/2) -
1))/(e^3*(((1 - e*x)^(1/2) - 1)^2/((e*x + 1)^(1/2) - 1)^2 + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

$$\int \frac{a+cx^2}{\sqrt{1-ex}\sqrt{1+ex}}\,dx = \frac{-4asin\left(\frac{\sqrt{-ex+1}}{\sqrt{2}}\right)a\,e^2-2asin\left(\frac{\sqrt{-ex+1}}{\sqrt{2}}\right)c-\sqrt{ex+1}\,\sqrt{-ex+1}\,cex}{2e^3}$$

input
```
int((c*x^2+a)/(-e*x+1)^(1/2)/(e*x+1)^(1/2),x)
```

output
```
( - 4*asin(sqrt( - e*x + 1)/sqrt(2))*a*e**2 - 2*asin(sqrt( - e*x + 1)/sqrt
(2))*c - sqrt(e*x + 1)*sqrt( - e*x + 1)*c*e*x)/(2*e**3)
```

3.6 $\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 50

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{c+ae^2}\,x}{\sqrt{a}\sqrt{1-e^2x^2}}\right)}{\sqrt{a}\sqrt{c+ae^2}}$$

output
```
arctan((a*e^2+c)^(1/2)*x/a^(1/2)/(-e^2*x^2+1)^(1/2))/a^(1/2)/(a*e^2+c)^(1/
2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 235 vs. 2(50) = 100.

Time = 0.96 (sec) , antiderivative size = 235, normalized size of antiderivative = 4.70

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)}\,dx$$
$$= \frac{\left(\sqrt{c}+\sqrt{c+ae^2}\right)\sqrt{2c+ae^2-2\sqrt{c}\sqrt{c+ae^2}}\arctan\left(\frac{\sqrt{2c+ae^2-2\sqrt{c}\sqrt{c+ae^2}}\,x}{\sqrt{a}\left(-1+\sqrt{1-e^2x^2}\right)}\right)+\left(-\sqrt{c}+\sqrt{c+ae^2}\right)\sqrt{2c+}}{a^{3/2}e^2\sqrt{c+ae^2}}$$

input
```
Integrate[1/(Sqrt[1 - e*x]*Sqrt[1 + e*x]*(a + c*x^2)),x]
```

output
```
((Sqrt[c] + Sqrt[c + a*e^2])*Sqrt[2*c + a*e^2 - 2*Sqrt[c]*Sqrt[c + a*e^2]]
*ArcTan[(Sqrt[2*c + a*e^2 - 2*Sqrt[c]*Sqrt[c + a*e^2]]*x)/(Sqrt[a]*(-1 + S
qrt[1 - e^2*x^2]))] + (-Sqrt[c] + Sqrt[c + a*e^2])*Sqrt[2*c + a*e^2 + 2*Sq
rt[c]*Sqrt[c + a*e^2]]*ArcTan[(Sqrt[2*c + a*e^2 + 2*Sqrt[c]*Sqrt[c + a*e^2
]]*x)/(Sqrt[a]*(-1 + Sqrt[1 - e^2*x^2]))])/(a^(3/2)*e^2*Sqrt[c + a*e^2])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {643, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-ex}\sqrt{ex+1}\,(a+cx^2)}dx$$

$$\Big\downarrow \text{643}$$

$$\int \frac{1}{\sqrt{1-e^2x^2}\,(a+cx^2)}dx$$

$$\Big\downarrow \text{291}$$

$$\int \frac{1}{a-\frac{x^2(-ae^2-c)}{1-e^2x^2}}d\frac{x}{\sqrt{1-e^2x^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{x\sqrt{ae^2+c}}{\sqrt{a}\sqrt{1-e^2x^2}}\right)}{\sqrt{a}\sqrt{ae^2+c}}$$

input
```
Int[1/(Sqrt[1 - e*x]*Sqrt[1 + e*x]*(a + c*x^2)),x]
```

output
```
ArcTan[(Sqrt[c + a*e^2]*x)/(Sqrt[a]*Sqrt[1 - e^2*x^2])]/(Sqrt[a]*Sqrt[c +
a*e^2])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.99 (sec) , antiderivative size = 322, normalized size of antiderivative = 6.44

method	result
default	$\frac{\sqrt{-ex+1}\,\sqrt{ex+1}\,c\,\mathrm{csgn}(e)^{2}\left(-\ln\left(\frac{-2\sqrt{-ac}\,e^{2}x+2\sqrt{\frac{a\,e^{2}+c}{c}}\,\sqrt{-e^{2}x^{2}+1}\,c+2c}{cx-\sqrt{-ac}}\right)a\,e^{2}+\ln\left(\frac{2\sqrt{-ac}\,e^{2}x+2\sqrt{\frac{a\,e^{2}+c}{c}}\,\sqrt{-e^{2}x^{2}+1}\,c+2c}{cx+\sqrt{-ac}}\right)a\,e^{2}\right.}{2\sqrt{-e^{2}x^{2}+1}\,\left(-e\sqrt{-ac}+c\right)\left(e\sqrt{-ac}+c\right)\sqrt{-a}}$

input
```
int(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-e*x+1)^(1/2)*(e*x+1)^(1/2)*c*csgn(e)^2*(-ln(2*(-(-a*c)^(1/2)*e^2*x+(
(a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x-(-a*c)^(1/2)))*a*e^2+ln(2*
((-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c
)^(1/2)))*a*e^2-ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)
^(1/2)*c+c)/(c*x-(-a*c)^(1/2)))*c+ln(2*((-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(
1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c)^(1/2)))*c)/(-e^2*x^2+1)^(1/2)/(-e
*(-a*c)^(1/2)+c)/(e*(-a*c)^(1/2)+c)/(-a*c)^(1/2)/((a*e^2+c)/c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 3.04

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)}\,dx = \left[-\frac{\sqrt{-a^2e^2-ac}\log\left(-\frac{(2\,ae^2+c)x^2-2\,\sqrt{-a^2e^2-ac}\sqrt{ex+1}\sqrt{-ex+1}x-a}{cx^2+a}\right)}{2\,(a^2e^2+ac)},\; -\frac{\arctan\left(\frac{\sqrt{a^2e^2+ac}\sqrt{ex+1}\sqrt{-ex+1}x}{ae^2x^2-a}\right)}{\sqrt{a^2e^2+ac}}\right]$$

input
```
integrate(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2*e^2 - a*c)*log(-((2*a*e^2 + c)*x^2 - 2*sqrt(-a^2*e^2 - a*c
)*sqrt(e*x + 1)*sqrt(-e*x + 1)*x - a)/(c*x^2 + a))/(a^2*e^2 + a*c), -arcta
n(sqrt(a^2*e^2 + a*c)*sqrt(e*x + 1)*sqrt(-e*x + 1)*x/(a*e^2*x^2 - a))/sqrt
(a^2*e^2 + a*c)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)}\,dx = \int \frac{1}{(a+cx^2)\,\sqrt{-ex+1}\sqrt{ex+1}}\,dx$$

input
```
integrate(1/(-e*x+1)**(1/2)/(e*x+1)**(1/2)/(c*x**2+a),x)
```

output
```
Integral(1/((a + c*x**2)*sqrt(-e*x + 1)*sqrt(e*x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)}\,dx = \int \frac{1}{(cx^2+a)\sqrt{ex+1}\sqrt{-ex+1}}\,dx$$

input
```
integrate(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)*sqrt(e*x + 1)*sqrt(-e*x + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1081 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 1081, normalized size of antiderivative = 21.62

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
-e*((2*(a^3*c*e^2 + a^2*c^2*e^2 + a^2*c^2 + a*c^3)*sqrt(a^2*e^2 + a*c)*e^2
*sgn(a*e^2 + c) - 2*(a^4*e^3 + a^3*c*e^3 + a^3*c*e + a^2*c^2*e)*sqrt(-a*c*
e^2 - c^2)*e^2 + (a^4*e^4 + a^3*c*e^4 + 2*a^3*c*e^2 + 2*a^2*c^2*e^2 + a^2*
c^2 + a*c^3)*sqrt(-a*c*e^2 - c^2)*abs(e)*sgn(a*e^2 + c) + (a^4*e^5 + a^3*c
*e^5 + 2*a^3*c*e^3 + 2*a^2*c^2*e^3 + a^2*c^2*e + a*c^3*e)*sqrt(a^2*e^2 + a
*c)*abs(e) + (a^3*c*e^4 + a^2*c^2*e^4 - a*c^3 - c^4)*sqrt(a^2*e^2 + a*c)*s
gn(a*e^2 + c) - (a^4*e^5 + a^3*c*e^5 - a^2*c^2*e - a*c^3*e)*sqrt(-a*c*e^2
- c^2))*arctan(-1/2*((sqrt(2) - sqrt(-e*x + 1))/sqrt(e*x + 1) - sqrt(e*x +
 1)/(sqrt(2) - sqrt(-e*x + 1)))/sqrt((a*e^2 - c + sqrt(-(a*e^2 + c)^2 + (a
*e^2 - c)^2))/(a*e^2 + c)))/(((a^6 + a^5*c)*e^8 + 4*(a^5*c + a^4*c^2)*e^6
+ a^2*c^4 + a*c^5 + 6*(a^4*c^2 + a^3*c^3)*e^4 + 4*(a^3*c^3 + a^2*c^4)*e^2)
*abs(e)) - (2*(a^3*c*e^2 + a^2*c^2*e^2 + a^2*c^2 + a*c^3)*sqrt(a^2*e^2 + a
*c)*e^2*sgn(a*e^2 + c) - 2*(a^4*e^3 + a^3*c*e^3 + a^3*c*e + a^2*c^2*e)*sqr
t(-a*c*e^2 - c^2)*e^2 + (a^4*e^4 + a^3*c*e^4 + 2*a^3*c*e^2 + 2*a^2*c^2*e^2
 + a^2*c^2 + a*c^3)*sqrt(-a*c*e^2 - c^2)*abs(e)*sgn(a*e^2 + c) + (a^4*e^5
+ a^3*c*e^5 + 2*a^3*c*e^3 + 2*a^2*c^2*e^3 + a^2*c^2*e + a*c^3*e)*sqrt(a^2*
e^2 + a*c)*abs(e) + (a^3*c*e^4 + a^2*c^2*e^4 - a*c^3 - c^4)*sqrt(a^2*e^2 +
 a*c)*sgn(a*e^2 + c) - (a^4*e^5 + a^3*c*e^5 - a^2*c^2*e - a*c^3*e)*sqrt(-a
*c*e^2 - c^2))*arctan(-1/2*((sqrt(2) - sqrt(-e*x + 1))/sqrt(e*x + 1) - sqr
t(e*x + 1)/(sqrt(2) - sqrt(-e*x + 1)))/sqrt((a*e^2 - c - sqrt(-(a*e^2 +...
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 1615, normalized size of antiderivative = 32.30

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^2)*(1 - e*x)^(1/2)*(e*x + 1)^(1/2)),x)
```

output
```
-(atan((a*e*((1 - e*x)^(1/2) - 1))/(4*(a*(c + a*e^2))^(1/2)*((e*x + 1)^(1/
2) - 1))) - atan(80*a*e*(a*(c + a*e^2))^(1/2)*(((1/(320*a*(c + a*e^2)) - 1
/(100*a^2*e^2))*((1 - e*x)^(1/2) - 1))/((e*x + 1)^(1/2) - 1) - ((1 - e*x)^
(1/2) - 1)^3/(1600*a*(c + a*e^2)*((e*x + 1)^(1/2) - 1)^3))) + atan(16*a^3*
c*e^4*(a*(c + a*e^2))^(1/2)*(((1/(16*a^4*c*e^5) - 27/(640*a^3*c*e^3*(c + a
*e^2)))*((1 - e*x)^(1/2) - 1)^3)/((e*x + 1)^(1/2) - 1)^3 - ((1/(20*a^4*c*e
^5) - 29/(256*a^3*c*e^3*(c + a*e^2)))*((1 - e*x)^(1/2) - 1))/((e*x + 1)^(1
/2) - 1) + ((1 - e*x)^(1/2) - 1)^5/(256*a^3*c*e^3*(c + a*e^2)*((e*x + 1)^(
1/2) - 1)^5))) - atan((((1 - e*x)^(1/2) - 1)^3*(a*c + a^2*e^2)^2*(((40*a^2
*c^4*e^5 + 45*a^3*c^3*e^7)/(a*c + a^2*e^2)^(3/2) - (40*a*c^3*e^5)/(a*(c +
a*e^2))^(1/2))/(a^3*c*e^4) - ((416*a*c^3*e^5 + 40*a^2*c^2*e^7)/(a*(c + a*e
^2))^(1/2) - (416*a^2*c^4*e^5 + 460*a^3*c^3*e^7 + 40*a^4*c^2*e^9)/(a*c + a
^2*e^2)^(3/2))/(a^3*c*e^4) + (((45*a^3*c^4*e^6)/4 + (75*a^4*c^3*e^8)/4 + (
125*a^5*c^2*e^10)/16)/(a*c + a^2*e^2)^2 - ((35*a^2*c^3*e^6)/2 + (225*a^3*c
^2*e^8)/16)/(a*c + a^2*e^2) + (25*a*c^2*e^6)/4)/(a^2*c*e^3*(a*(c + a*e^2))
^(1/2)) - ((696*a^2*c^5*e^4 + 1363*a^3*c^4*e^6 + (3219*a^4*c^3*e^8)/4 + (1
015*a^5*c^2*e^10)/8)/(a*c + a^2*e^2)^2 - (696*a*c^4*e^4 + (1479*a^2*c^3*e^
6)/2 + (1595*a^3*c^2*e^8)/8)/(a*c + a^2*e^2) + (145*a*c^2*e^6)/2)/(a^2*c*e
^3*(a*(c + a*e^2))^(1/2))))/(a*c^2*e^4*((e*x + 1)^(1/2) - 1)^3) - (((1 - e
*x)^(1/2) - 1)*(a*c + a^2*e^2)^2*(((524288*a^2*c^4*e^5 + 589824*a^3*c^3...
```

Reduce [F]

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)}\,dx = \int \frac{1}{\sqrt{ex+1}\,\sqrt{-ex+1}\,a+\sqrt{ex+1}\,\sqrt{-ex+1}\,c\,x^2}dx$$

input
```
int(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a),x)
```

output
```
int(1/(sqrt(e*x + 1)*sqrt( - e*x + 1)*a + sqrt(e*x + 1)*sqrt( - e*x + 1)*c
*x**2),x)
```

3.7 $\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\left(a+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 103

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)^2}\,dx = \frac{cx\sqrt{1-e^2x^2}}{2a\,(c+ae^2)\,(a+cx^2)} + \frac{(c+2ae^2)\arctan\left(\frac{\sqrt{c+ae^2}x}{\sqrt{a}\sqrt{1-e^2x^2}}\right)}{2a^{3/2}\,(c+ae^2)^{3/2}}$$

output
```
1/2*c*x*(-e^2*x^2+1)^(1/2)/a/(a*e^2+c)/(c*x^2+a)+1/2*(2*a*e^2+c)*arctan((a
*e^2+c)^(1/2)*x/a^(1/2)/(-e^2*x^2+1)^(1/2))/a^(3/2)/(a*e^2+c)^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 926 vs. 2(103) = 206.

Time = 6.46 (sec) , antiderivative size = 926, normalized size of antiderivative = 8.99

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)^2}\,dx$$

$$= \frac{\frac{2\sqrt{a}cx}{(c+ae^2)(a+cx^2)\left(-2+e^2x^2+2\sqrt{1-e^2x^2}\right)} - \frac{2\sqrt{a}ce^2x^3}{(c+ae^2)(a+cx^2)\left(-2+e^2x^2+2\sqrt{1-e^2x^2}\right)} - \frac{2\sqrt{a}cx\sqrt{1-e^2x^2}}{(c+ae^2)(a+cx^2)\left(-2+e^2x^2+2\sqrt{1-e^2x^2}\right)} + \frac{\cdots}{(c+\cdots}}{\cdots}$$

input
```
Integrate[1/(Sqrt[1 - e*x]*Sqrt[1 + e*x]*(a + c*x^2)^2),x]
```

output
```
((2*Sqrt[a]*c*x)/((c + a*e^2)*(a + c*x^2)*(-2 + e^2*x^2 + 2*Sqrt[1 - e^2*x
^2])) - (2*Sqrt[a]*c*e^2*x^3)/((c + a*e^2)*(a + c*x^2)*(-2 + e^2*x^2 + 2*S
qrt[1 - e^2*x^2])) - (2*Sqrt[a]*c*x*Sqrt[1 - e^2*x^2])/((c + a*e^2)*(a + c
*x^2)*(-2 + e^2*x^2 + 2*Sqrt[1 - e^2*x^2])) + (Sqrt[a]*c*e^2*x^3*Sqrt[1 -
e^2*x^2])/((c + a*e^2)*(a + c*x^2)*(-2 + e^2*x^2 + 2*Sqrt[1 - e^2*x^2])) +
 (c^(3/2)*ArcTan[(Sqrt[2*c + a*e^2 - 2*Sqrt[c]*Sqrt[c + a*e^2]]*x)/(Sqrt[a
]*(1 - Sqrt[1 - e^2*x^2]))])/((c + a*e^2)^(3/2)*Sqrt[2*c + a*e^2 - 2*Sqrt[
c]*Sqrt[c + a*e^2]]) + ((c*Sqrt[c + a*e^2] + 2*a*e^2*(-Sqrt[c] + Sqrt[c +
a*e^2]))*ArcTan[(Sqrt[2*c + a*e^2 - 2*Sqrt[c]*Sqrt[c + a*e^2]]*x)/(Sqrt[a]
*(-1 + Sqrt[1 - e^2*x^2]))])/((c + a*e^2)^(3/2)*Sqrt[2*c + a*e^2 - 2*Sqrt[
c]*Sqrt[c + a*e^2]]) + (c^(3/2)*ArcTan[(Sqrt[2*c + a*e^2 + 2*Sqrt[c]*Sqrt[
c + a*e^2]]*x)/(Sqrt[a]*(-1 + Sqrt[1 - e^2*x^2]))])/((c + a*e^2)^(3/2)*Sqr
t[2*c + a*e^2 + 2*Sqrt[c]*Sqrt[c + a*e^2]]) + (2*a*Sqrt[c]*e^2*ArcTan[(Sqr
t[2*c + a*e^2 + 2*Sqrt[c]*Sqrt[c + a*e^2]]*x)/(Sqrt[a]*(-1 + Sqrt[1 - e^2*
x^2]))])/((c + a*e^2)^(3/2)*Sqrt[2*c + a*e^2 + 2*Sqrt[c]*Sqrt[c + a*e^2]])
 + (c*ArcTan[(Sqrt[2*c + a*e^2 + 2*Sqrt[c]*Sqrt[c + a*e^2]]*x)/(Sqrt[a]*(-
1 + Sqrt[1 - e^2*x^2]))])/((c + a*e^2)*Sqrt[2*c + a*e^2 + 2*Sqrt[c]*Sqrt[c
 + a*e^2]]) + (2*a*e^2*ArcTan[(Sqrt[2*c + a*e^2 + 2*Sqrt[c]*Sqrt[c + a*e^2
]]*x)/(Sqrt[a]*(-1 + Sqrt[1 - e^2*x^2]))])/((c + a*e^2)*Sqrt[2*c + a*e^2 +
 2*Sqrt[c]*Sqrt[c + a*e^2]]))/(2*a^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {643, 296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-ex}\sqrt{ex+1}\,(a+cx^2)^2}\,dx$$

$$\Big\downarrow \; 643$$

$$\int \frac{1}{\sqrt{1-e^2x^2}\,(a+cx^2)^2}dx$$

$$\Big\downarrow \ 296$$

$$\frac{(2ae^2+c)\int \frac{1}{(cx^2+a)\sqrt{1-e^2x^2}}dx}{2a(ae^2+c)}+\frac{cx\sqrt{1-e^2x^2}}{2a(ae^2+c)(a+cx^2)}$$

$$\Big\downarrow \ 291$$

$$\frac{(2ae^2+c)\int \frac{1}{a-\frac{(-ae^2-c)x^2}{1-e^2x^2}}d\frac{x}{\sqrt{1-e^2x^2}}}{2a(ae^2+c)}+\frac{cx\sqrt{1-e^2x^2}}{2a(ae^2+c)(a+cx^2)}$$

$$\Big\downarrow \ 218$$

$$\frac{(2ae^2+c)\arctan\left(\frac{x\sqrt{ae^2+c}}{\sqrt{a}\sqrt{1-e^2x^2}}\right)}{2a^{3/2}(ae^2+c)^{3/2}}+\frac{cx\sqrt{1-e^2x^2}}{2a(ae^2+c)(a+cx^2)}$$

input
```
Int[1/(Sqrt[1 - e*x]*Sqrt[1 + e*x]*(a + c*x^2)^2),x]
```

output
```
(c*x*Sqrt[1 - e^2*x^2])/(2*a*(c + a*e^2)*(a + c*x^2)) + ((c + 2*a*e^2)*Arc
Tan[(Sqrt[c + a*e^2]*x)/(Sqrt[a]*Sqrt[1 - e^2*x^2])])/(2*a^(3/2)*(c + a*e^
2)^(3/2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.94 (sec) , antiderivative size = 973, normalized size of antiderivative = 9.45

method	result
default	$-\frac{\left(2\ln\left(\frac{-2\sqrt{-ac}\,e^{2}x+2\sqrt{\frac{a\,e^{2}+c}{c}}\,\sqrt{-e^{2}x^{2}+1}\,c+2c}{cx-\sqrt{-ac}}\right)a^{2}c\,e^{4}x^{2}-2\ln\left(\frac{2\sqrt{-ac}\,e^{2}x+2\sqrt{\frac{a\,e^{2}+c}{c}}\,\sqrt{-e^{2}x^{2}+1}\,c+2c}{cx+\sqrt{-ac}}\right)a^{2}c\,e^{4}x^{2}+2\ln\left(\frac{-2\sqrt{-a}\cdots}{\cdots}\right.\cdots}{\cdots}$

input
```
int(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c
+c)/(c*x-(-a*c)^(1/2)))*a^2*c*e^4*x^2-2*ln(2*((-a*c)^(1/2)*e^2*x+((a*e^2+c
)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c)^(1/2)))*a^2*c*e^4*x^2+2*ln(
2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x-(-
a*c)^(1/2)))*a^3*e^4+3*ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2
*x^2+1)^(1/2)*c+c)/(c*x-(-a*c)^(1/2)))*a*c^2*e^2*x^2-2*ln(2*((-a*c)^(1/2)*
e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c)^(1/2)))*a^3*
e^4-3*ln(2*((-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)
/(c*x+(-a*c)^(1/2)))*a*c^2*e^2*x^2-2*a*c*e^2*x*(-a*c)^(1/2)*(-e^2*x^2+1)^(
1/2)*((a*e^2+c)/c)^(1/2)+3*ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(
-e^2*x^2+1)^(1/2)*c+c)/(c*x-(-a*c)^(1/2)))*a^2*c*e^2+ln(2*(-(-a*c)^(1/2)*e
^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x-(-a*c)^(1/2)))*c^3*x
^2-3*ln(2*((-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/
(c*x+(-a*c)^(1/2)))*a^2*c*e^2-ln(2*((-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)
*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c)^(1/2)))*c^3*x^2-2*c^2*x*(-a*c)^(1/2)*
(-e^2*x^2+1)^(1/2)*((a*e^2+c)/c)^(1/2)+ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c
)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x-(-a*c)^(1/2)))*a*c^2-ln(2*((-a*c)^
(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c)^(1/2))
)*a*c^2)*csgn(e)^2*c^3*(e*x+1)^(1/2)*(-e*x+1)^(1/2)/(-a*c)^(1/2)/(c*x+(-a*
c)^(1/2))/((a*e^2+c)/c)^(1/2)/(c*x-(-a*c)^(1/2))/a/(-e*(-a*c)^(1/2)+c)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(87) = 174$.

Time = 0.12 (sec) , antiderivative size = 376, normalized size of antiderivative = 3.65

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)^2}\,dx$$
$$= \left[\frac{2\,(a^2ce^2+ac^2)\sqrt{ex+1}\sqrt{-ex+1}x-(2\,a^2e^2+(2\,ace^2+c^2)x^2+ac)\sqrt{-a^2e^2-ac}\log\left(-\frac{(2\,ae^2+c)x^2-2\,\sqrt{\ldots}}{\ldots}\right.}{4\,(a^5e^4+2\,a^4ce^2+a^3c^2+(a^4ce^4+2\,a^3c^2e^2+a^2c^3)x^2)}\right.$$

input
```
integrate(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^2,x, algorithm="fricas"
)
```

output
```
[1/4*(2*(a^2*c*e^2 + a*c^2)*sqrt(e*x + 1)*sqrt(-e*x + 1)*x - (2*a^2*e^2 +
(2*a*c*e^2 + c^2)*x^2 + a*c)*sqrt(-a^2*e^2 - a*c)*log(-((2*a*e^2 + c)*x^2
- 2*sqrt(-a^2*e^2 - a*c)*sqrt(e*x + 1)*sqrt(-e*x + 1)*x - a)/(c*x^2 + a)))
/(a^5*e^4 + 2*a^4*c*e^2 + a^3*c^2 + (a^4*c*e^4 + 2*a^3*c^2*e^2 + a^2*c^3)*
x^2), 1/2*((a^2*c*e^2 + a*c^2)*sqrt(e*x + 1)*sqrt(-e*x + 1)*x - (2*a^2*e^2
 + (2*a*c*e^2 + c^2)*x^2 + a*c)*sqrt(a^2*e^2 + a*c)*arctan(sqrt(a^2*e^2 +
a*c)*sqrt(e*x + 1)*sqrt(-e*x + 1)*x/(a*e^2*x^2 - a)))/(a^5*e^4 + 2*a^4*c*e
^2 + a^3*c^2 + (a^4*c*e^4 + 2*a^3*c^2*e^2 + a^2*c^3)*x^2)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\left(a+cx^2\right)^2}\,dx=\int \frac{1}{\left(a+cx^2\right)^2\sqrt{-ex+1}\sqrt{ex+1}}\,dx$$

input
```
integrate(1/(-e*x+1)**(1/2)/(e*x+1)**(1/2)/(c*x**2+a)**2,x)
```

output
```
Integral(1/((a + c*x**2)**2*sqrt(-e*x + 1)*sqrt(e*x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\left(a+cx^2\right)^2}\,dx=\int \frac{1}{\left(cx^2+a\right)^2\sqrt{ex+1}\sqrt{-ex+1}}\,dx$$

input
```
integrate(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^2,x, algorithm="maxima"
)
```

output
```
integrate(1/((c*x^2 + a)^2*sqrt(e*x + 1)*sqrt(-e*x + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2553 vs. $2(87) = 174$.

Time = 5.87 (sec) , antiderivative size = 2553, normalized size of antiderivative = 24.79

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*e^3*((2*(2*a^3*c*e^4 + 2*a^2*c^2*e^4 + 3*a^2*c^2*e^2 + 3*a*c^3*e^2 +
a*c^3 + c^4)*(a^2*e^5 + a*c*e^3)^2*sqrt(a^2*e^2 + a*c)*sgn(a^3*e^6 + 2*a^2
*c*e^4 + a*c^2*e^2) - 2*(2*a^4*e^5 + 2*a^3*c*e^5 + 3*a^3*c*e^3 + 3*a^2*c^2
*e^3 + a^2*c^2*e + a*c^3*e)*(a^2*e^5 + a*c*e^3)^2*sqrt(-a*c*e^2 - c^2) - (
2*a^6*e^10 + 2*a^5*c*e^10 + 7*a^5*c*e^8 + 7*a^4*c^2*e^8 + 9*a^4*c^2*e^6 +
9*a^3*c^3*e^6 + 5*a^3*c^3*e^4 + 5*a^2*c^4*e^4 + a^2*c^4*e^2 + a*c^5*e^2)*s
qrt(-a*c*e^2 - c^2)*abs(-a^2*e^5 - a*c*e^3)*sgn(a^3*e^6 + 2*a^2*c*e^4 + a*
c^2*e^2) + (2*a^6*e^11 + 2*a^5*c*e^11 + 7*a^5*c*e^9 + 7*a^4*c^2*e^9 + 9*a^
4*c^2*e^7 + 9*a^3*c^3*e^7 + 5*a^3*c^3*e^5 + 5*a^2*c^4*e^5 + a^2*c^4*e^3 +
a*c^5*e^3)*sqrt(a^2*e^2 + a*c)*abs(-a^2*e^5 - a*c*e^3) + (2*a^7*c*e^14 + 2
*a^6*c^2*e^14 + 5*a^6*c^2*e^12 + 5*a^5*c^3*e^12 + 2*a^5*c^3*e^10 + 2*a^4*c
^4*e^10 - 4*a^4*c^4*e^8 - 4*a^3*c^5*e^8 - 4*a^3*c^5*e^6 - 4*a^2*c^6*e^6 -
a^2*c^6*e^4 - a*c^7*e^4)*sqrt(a^2*e^2 + a*c)*sgn(a^3*e^6 + 2*a^2*c*e^4 + a
*c^2*e^2) - (2*a^8*e^15 + 2*a^7*c*e^15 + 5*a^7*c*e^13 + 5*a^6*c^2*e^13 + 2
*a^6*c^2*e^11 + 2*a^5*c^3*e^11 - 4*a^5*c^3*e^9 - 4*a^4*c^4*e^9 - 4*a^4*c^4
*e^7 - 4*a^3*c^5*e^7 - a^3*c^5*e^5 - a^2*c^6*e^5)*sqrt(-a*c*e^2 - c^2))*ar
ctan(-1/2*((sqrt(2) - sqrt(-e*x + 1))/sqrt(e*x + 1) - sqrt(e*x + 1)/(sqrt(
2) - sqrt(-e*x + 1)))/sqrt((a^3*e^6 - a*c^2*e^2 + sqrt(-(a^3*e^6 + 2*a^2*c
*e^4 + a*c^2*e^2)^2 + (a^3*e^6 - a*c^2*e^2)^2))/(a^3*e^6 + 2*a^2*c*e^4 + a
*c^2*e^2)))/(((a^9 + a^8*c)*e^16 + 6*(a^8*c + a^7*c^2)*e^14 + 15*(a^7*c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\left(a+cx^2\right)^2}\,dx = \text{Hanged}$$

input
```
int(1/((a + c*x^2)^2*(1 - e*x)^(1/2)*(e*x + 1)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\left(a+cx^2\right)^2}\,dx$$
$$= \int \frac{1}{\sqrt{ex+1}\,\sqrt{-ex+1}\,a^2+2\sqrt{ex+1}\,\sqrt{-ex+1}\,ac\,x^2+\sqrt{ex+1}\,\sqrt{-ex+1}\,c^2x^4}dx$$

input
```
int(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^2,x)
```

output
```
int(1/(sqrt(e*x + 1)*sqrt( - e*x + 1)*a**2 + 2*sqrt(e*x + 1)*sqrt( - e*x +
 1)*a*c*x**2 + sqrt(e*x + 1)*sqrt( - e*x + 1)*c**2*x**4),x)
```

3.8 $\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}(a+cx^2)^3} dx$

Optimal result

Integrand size = 29, antiderivative size = 165

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}(a+cx^2)^3} dx = \frac{cx\sqrt{1-e^2x^2}}{4a(c+ae^2)(a+cx^2)^2} + \frac{3c(c+2ae^2)x\sqrt{1-e^2x^2}}{8a^2(c+ae^2)^2(a+cx^2)} + \frac{(3c^2+8ace^2+8a^2e^4)\arctan\left(\frac{\sqrt{c+ae^2}x}{\sqrt{a}\sqrt{1-e^2x^2}}\right)}{8a^{5/2}(c+ae^2)^{5/2}}$$

output
```
1/4*c*x*(-e^2*x^2+1)^(1/2)/a/(a*e^2+c)/(c*x^2+a)^2+3/8*c*(2*a*e^2+c)*x*(-e
^2*x^2+1)^(1/2)/a^2/(a*e^2+c)^2/(c*x^2+a)+1/8*(8*a^2*e^4+8*a*c*e^2+3*c^2)*
arctan((a*e^2+c)^(1/2)*x/a^(1/2)/(-e^2*x^2+1)^(1/2))/a^(5/2)/(a*e^2+c)^(5/
2)
```

Mathematica [A] (verified)

Time = 15.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}(a+cx^2)^3} dx = \frac{\frac{\sqrt{a}cx\sqrt{1-e^2x^2}(2a(c+ae^2)+3(c+2ae^2)(a+cx^2))}{(c+ae^2)^2(a+cx^2)^2} + \frac{(3c^2+8ace^2+8a^2e^4)\arctan\left(\frac{\sqrt{c+ae^2}x}{\sqrt{a}\sqrt{1-e^2x^2}}\right)}{(c+ae^2)^{5/2}}}{8a^{5/2}}$$

input
```
Integrate[1/(Sqrt[1 - e*x]*Sqrt[1 + e*x]*(a + c*x^2)^3),x]
```

output
```
((Sqrt[a]*c*x*Sqrt[1 - e^2*x^2]*(2*a*(c + a*e^2) + 3*(c + 2*a*e^2)*(a + c*
x^2)))/((c + a*e^2)^2*(a + c*x^2)^2) + ((3*c^2 + 8*a*c*e^2 + 8*a^2*e^4)*Ar
cTan[(Sqrt[c + a*e^2]*x)/(Sqrt[a]*Sqrt[1 - e^2*x^2])])/(c + a*e^2)^(5/2))/
(8*a^(5/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {643, 316, 25, 402, 25, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-ex}\sqrt{ex+1}\,(a+cx^2)^3}\,dx$$

$$\Big\downarrow \text{643}$$

$$\int \frac{1}{\sqrt{1-e^2x^2}\,(a+cx^2)^3}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{cx\sqrt{1-e^2x^2}}{4a\,(ae^2+c)\,(a+cx^2)^2}-\frac{\int -\frac{-2cx^2e^2+4ae^2+3c}{(cx^2+a)^2\sqrt{1-e^2x^2}}dx}{4a\,(ae^2+c)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-2cx^2e^2+4ae^2+3c}{(cx^2+a)^2\sqrt{1-e^2x^2}}dx}{4a\,(ae^2+c)}+\frac{cx\sqrt{1-e^2x^2}}{4a\,(ae^2+c)\,(a+cx^2)^2}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{3cx\sqrt{1-e^2x^2}(2ae^2+c)}{2a(ae^2+c)(a+cx^2)}-\frac{\int -\frac{8a^2e^4+8ace^2+3c^2}{\left(cx^2+a\right)\sqrt{1-e^2x^2}}dx}{2a(ae^2+c)}}{4a\,(ae^2+c)}+\frac{cx\sqrt{1-e^2x^2}}{4a\,(ae^2+c)\,(a+cx^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{8a^2e^4+8ace^2+3c^2}{(cx^2+a)\sqrt{1-e^2x^2}}dx}{2a(ae^2+c)}+\frac{3cx\sqrt{1-e^2x^2}(2ae^2+c)}{2a(ae^2+c)(a+cx^2)}}{4a(ae^2+c)}+\frac{cx\sqrt{1-e^2x^2}}{4a(ae^2+c)(a+cx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(8a^2e^4+8ace^2+3c^2)\int \frac{1}{(cx^2+a)\sqrt{1-e^2x^2}}dx}{2a(ae^2+c)}+\frac{3cx\sqrt{1-e^2x^2}(2ae^2+c)}{2a(ae^2+c)(a+cx^2)}}{4a(ae^2+c)}+\frac{cx\sqrt{1-e^2x^2}}{4a(ae^2+c)(a+cx^2)^2}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{(8a^2e^4+8ace^2+3c^2)\int \frac{1}{a-\frac{(-ae^2-c)x^2}{1-e^2x^2}}d\frac{x}{\sqrt{1-e^2x^2}}}{2a(ae^2+c)}+\frac{3cx\sqrt{1-e^2x^2}(2ae^2+c)}{2a(ae^2+c)(a+cx^2)}}{4a(ae^2+c)}+\frac{cx\sqrt{1-e^2x^2}}{4a(ae^2+c)(a+cx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(8a^2e^4+8ace^2+3c^2)\arctan\left(\frac{x\sqrt{ae^2+c}}{\sqrt{a}\sqrt{1-e^2x^2}}\right)}{2a^{3/2}(ae^2+c)^{3/2}}+\frac{3cx\sqrt{1-e^2x^2}(2ae^2+c)}{2a(ae^2+c)(a+cx^2)}}{4a(ae^2+c)}+\frac{cx\sqrt{1-e^2x^2}}{4a(ae^2+c)(a+cx^2)^2}$$

input
```
Int[1/(Sqrt[1 - e*x]*Sqrt[1 + e*x]*(a + c*x^2)^3),x]
```

output
```
(c*x*Sqrt[1 - e^2*x^2])/(4*a*(c + a*e^2)*(a + c*x^2)^2) + ((3*c*(c + 2*a*e
^2)*x*Sqrt[1 - e^2*x^2])/(2*a*(c + a*e^2)*(a + c*x^2)) + ((3*c^2 + 8*a*c*e
^2 + 8*a^2*e^4)*ArcTan[(Sqrt[c + a*e^2]*x)/(Sqrt[a]*Sqrt[1 - e^2*x^2])])/(
2*a^(3/2)*(c + a*e^2)^(3/2)))/(4*a*(c + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.97 (sec) , antiderivative size = 1982, normalized size of antiderivative = 12.01

method	result	size
default	Expression too large to display	1982

input
```
int(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-12*a^2*c^2*e^4*x^3*(-a*c)^(1/2)*((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(
1/2)-16*a^3*c*e^4*x*(-a*c)^(1/2)*((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)-18
*a*c^3*e^2*x^3*(-a*c)^(1/2)*((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)-26*a^2*
c^2*e^2*x*(-a*c)^(1/2)*((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)+32*ln(2*(-(-
a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x-(-a*c)^(
1/2)))*a^3*c^2*e^4*x^2+11*ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-
e^2*x^2+1)^(1/2)*c+c)/(c*x-(-a*c)^(1/2)))*a*c^4*e^2*x^4-22*ln(2*((-a*c)^(1
/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c)^(1/2)))*
a^2*c^3*e^2*x^2+22*ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2
+1)^(1/2)*c+c)/(c*x-(-a*c)^(1/2)))*a^2*c^3*e^2*x^2-6*c^4*x^3*(-a*c)^(1/2)*
((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)-8*ln(2*((-a*c)^(1/2)*e^2*x+((a*e^2+
c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c)^(1/2)))*a^3*c^2*e^6*x^4+8*
ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x
-(-a*c)^(1/2)))*a^3*c^2*e^6*x^4-16*ln(2*((-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^
(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c)^(1/2)))*a^4*c*e^6*x^2-16*ln(2*((
-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c)^
(1/2)))*a^2*c^3*e^4*x^4+16*ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(
-e^2*x^2+1)^(1/2)*c+c)/(c*x-(-a*c)^(1/2)))*a^4*c*e^6*x^2+16*ln(2*(-(-a*c)^
(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x-(-a*c)^(1/2))
)*a^2*c^3*e^4*x^4-32*ln(2*((-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(145) = 290$.

Time = 0.13 (sec) , antiderivative size = 731, normalized size of antiderivative = 4.43

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)^3}\,dx$$
$$= \left[-\frac{(8a^4e^4+8a^3ce^2+(8a^2c^2e^4+8ac^3e^2+3c^4)x^4+3a^2c^2+2(8a^3ce^4+8a^2c^2e^2+3ac^3)x^2)\sqrt{-a^2e^2}}{16(a^8e^6+3a^7ce^4+3a^6c^2e^2+a^5c^3+(a^6}\right.$$
$$-\frac{(8a^4e^4+8a^3ce^2+(8a^2c^2e^4+8ac^3e^2+3c^4)x^4+3a^2c^2+2(8a^3ce^4+8a^2c^2e^2+3ac^3)x^2)\sqrt{a^2e^2+a}}{8(a^8e^6+3a^7ce^4+3a^6c^2e^2+a^5c^3+(a^6c^2e^6+3a}$$

input
```
integrate(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^3,x, algorithm="fricas"
)
```

output
```
[-1/16*((8*a^4*e^4 + 8*a^3*c*e^2 + (8*a^2*c^2*e^4 + 8*a*c^3*e^2 + 3*c^4)*x
^4 + 3*a^2*c^2 + 2*(8*a^3*c*e^4 + 8*a^2*c^2*e^2 + 3*a*c^3)*x^2)*sqrt(-a^2*
e^2 - a*c)*log(-((2*a*e^2 + c)*x^2 - 2*sqrt(-a^2*e^2 - a*c)*sqrt(e*x + 1)*
sqrt(-e*x + 1)*x - a)/(c*x^2 + a)) - 2*(3*(2*a^3*c^2*e^4 + 3*a^2*c^3*e^2 +
 a*c^4)*x^3 + (8*a^4*c*e^4 + 13*a^3*c^2*e^2 + 5*a^2*c^3)*x)*sqrt(e*x + 1)*
sqrt(-e*x + 1))/(a^8*e^6 + 3*a^7*c*e^4 + 3*a^6*c^2*e^2 + a^5*c^3 + (a^6*c^
2*e^6 + 3*a^5*c^3*e^4 + 3*a^4*c^4*e^2 + a^3*c^5)*x^4 + 2*(a^7*c*e^6 + 3*a^
6*c^2*e^4 + 3*a^5*c^3*e^2 + a^4*c^4)*x^2), -1/8*((8*a^4*e^4 + 8*a^3*c*e^2
+ (8*a^2*c^2*e^4 + 8*a*c^3*e^2 + 3*c^4)*x^4 + 3*a^2*c^2 + 2*(8*a^3*c*e^4 +
 8*a^2*c^2*e^2 + 3*a*c^3)*x^2)*sqrt(a^2*e^2 + a*c)*arctan(sqrt(a^2*e^2 + a
*c)*sqrt(e*x + 1)*sqrt(-e*x + 1)*x/(a*e^2*x^2 - a)) - (3*(2*a^3*c^2*e^4 +
3*a^2*c^3*e^2 + a*c^4)*x^3 + (8*a^4*c*e^4 + 13*a^3*c^2*e^2 + 5*a^2*c^3)*x)
*sqrt(e*x + 1)*sqrt(-e*x + 1))/(a^8*e^6 + 3*a^7*c*e^4 + 3*a^6*c^2*e^2 + a^
5*c^3 + (a^6*c^2*e^6 + 3*a^5*c^3*e^4 + 3*a^4*c^4*e^2 + a^3*c^5)*x^4 + 2*(a
^7*c*e^6 + 3*a^6*c^2*e^4 + 3*a^5*c^3*e^2 + a^4*c^4)*x^2)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)^3}\,dx = \int \frac{1}{(a+cx^2)^3\sqrt{-ex+1}\sqrt{ex+1}}\,dx$$

input
```
integrate(1/(-e*x+1)**(1/2)/(e*x+1)**(1/2)/(c*x**2+a)**3,x)
```

output
```
Integral(1/((a + c*x**2)**3*sqrt(-e*x + 1)*sqrt(e*x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)^3}\,dx = \int \frac{1}{(cx^2+a)^3\sqrt{ex+1}\sqrt{-ex+1}}\,dx$$

input
```
integrate(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^3,x, algorithm="maxima"
)
```

output
```
integrate(1/((c*x^2 + a)^3*sqrt(e*x + 1)*sqrt(-e*x + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4129 vs. $2(145) = 290$.

Time = 29.62 (sec) , antiderivative size = 4129, normalized size of antiderivative = 25.02

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/8*e^5*((2*(a^4*e^9 + 2*a^3*c*e^7 + a^2*c^2*e^5)^2*(8*a^4*c*e^6 + 8*a^3*
c^2*e^6 + 16*a^3*c^2*e^4 + 16*a^2*c^3*e^4 + 11*a^2*c^3*e^2 + 11*a*c^4*e^2
+ 3*a*c^4 + 3*c^5)*sqrt(a^2*e^2 + a*c)*sgn(4*a^5*e^10 + 12*a^4*c*e^8 + 12*
a^3*c^2*e^6 + 4*a^2*c^3*e^4) - 2*(a^4*e^9 + 2*a^3*c*e^7 + a^2*c^2*e^5)^2*(
8*a^5*e^7 + 8*a^4*c*e^7 + 16*a^4*c*e^5 + 16*a^3*c^2*e^5 + 11*a^3*c^2*e^3 +
 11*a^2*c^3*e^3 + 3*a^2*c^3*e + 3*a*c^4*e)*sqrt(-a*c*e^2 - c^2) + (8*a^9*e
^16 + 8*a^8*c*e^16 + 40*a^8*c*e^14 + 40*a^7*c^2*e^14 + 83*a^7*c^2*e^12 + 8
3*a^6*c^3*e^12 + 92*a^6*c^3*e^10 + 92*a^5*c^4*e^10 + 58*a^5*c^4*e^8 + 58*a
^4*c^5*e^8 + 20*a^4*c^5*e^6 + 20*a^3*c^6*e^6 + 3*a^3*c^6*e^4 + 3*a^2*c^7*e
^4)*sqrt(-a*c*e^2 - c^2)*abs(a^4*e^9 + 2*a^3*c*e^7 + a^2*c^2*e^5)*sgn(4*a^
5*e^10 + 12*a^4*c*e^8 + 12*a^3*c^2*e^6 + 4*a^2*c^3*e^4) + (8*a^9*e^17 + 8*
a^8*c*e^17 + 40*a^8*c*e^15 + 40*a^7*c^2*e^15 + 83*a^7*c^2*e^13 + 83*a^6*c^
3*e^13 + 92*a^6*c^3*e^11 + 92*a^5*c^4*e^11 + 58*a^5*c^4*e^9 + 58*a^4*c^5*e
^9 + 20*a^4*c^5*e^7 + 20*a^3*c^6*e^7 + 3*a^3*c^6*e^5 + 3*a^2*c^7*e^5)*sqrt
(a^2*e^2 + a*c)*abs(a^4*e^9 + 2*a^3*c*e^7 + a^2*c^2*e^5) + (8*a^12*c*e^24
+ 8*a^11*c^2*e^24 + 40*a^11*c^2*e^22 + 40*a^10*c^3*e^22 + 75*a^10*c^3*e^20
 + 75*a^9*c^4*e^20 + 52*a^9*c^4*e^18 + 52*a^8*c^5*e^18 - 25*a^8*c^5*e^16 -
 25*a^7*c^6*e^16 - 72*a^7*c^6*e^14 - 72*a^6*c^7*e^14 - 55*a^6*c^7*e^12 - 5
5*a^5*c^8*e^12 - 20*a^5*c^8*e^10 - 20*a^4*c^9*e^10 - 3*a^4*c^9*e^8 - 3*a^3
*c^10*e^8)*sqrt(a^2*e^2 + a*c)*sgn(4*a^5*e^10 + 12*a^4*c*e^8 + 12*a^3*c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)^3}\,dx = \text{Hanged}$$

input
```
int(1/((a + c*x^2)^3*(1 - e*x)^(1/2)*(e*x + 1)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{\sqrt{1-ex}\sqrt{1+ex}\,(a+cx^2)^3}\,dx = \int \frac{1}{\sqrt{-ex+1}\,\sqrt{ex+1}\,(c\,x^2+a)^3}dx$$

input
```
int(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^3,x)
```

output
```
int(1/(-e*x+1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^3,x)
```

3.9 $\int \frac{(a+cx^2)^3}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 124

$$\int \frac{(a+cx^2)^3}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{(c+ae^2)^3 x}{e^6\sqrt{1-e^2x^2}} + \frac{c^2(7c+12ae^2)\,x\sqrt{1-e^2x^2}}{8e^6} + \frac{c^3x^3\sqrt{1-e^2x^2}}{4e^4} - \frac{3c(5c^2+12ace^2+8a^2e^4)\arcsin(ex)}{8e^7}$$

output
```
(a*e^2+c)^3*x/e^6/(-e^2*x^2+1)^(1/2)+1/8*c^2*(12*a*e^2+7*c)*x*(-e^2*x^2+1)
^(1/2)/e^6+1/4*c^3*x^3*(-e^2*x^2+1)^(1/2)/e^4-3/8*c*(8*a^2*e^4+12*a*c*e^2+
5*c^2)*arcsin(e*x)/e^7
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04

$$\int \frac{(a+cx^2)^3}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{\frac{ex(24a^2ce^4+8a^3e^6-12ac^2e^2(-3+e^2x^2)+c^3(15-5e^2x^2-2e^4x^4))}{\sqrt{1-e^2x^2}} - 6c(5c^2+12ace^2+8a^2e^4}{8e^7}$$

input
```
Integrate[(a + c*x^2)^3/((1 - e*x)^(3/2)*(1 + e*x)^(3/2)),x]
```

output
```
((e*x*(24*a^2*c*e^4 + 8*a^3*e^6 - 12*a*c^2*e^2*(-3 + e^2*x^2) + c^3*(15 -
5*e^2*x^2 - 2*e^4*x^4)))/Sqrt[1 - e^2*x^2] - 6*c*(5*c^2 + 12*a*c*e^2 + 8*a
^2*e^4)*ArcTan[(e*x)/(-1 + Sqrt[1 - e^2*x^2])])/(8*e^7)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {643, 315, 27, 403, 25, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{(1-ex)^{3/2}(ex+1)^{3/2}}dx$$

$$\Big\downarrow \text{643}$$

$$\int \frac{(a+cx^2)^3}{(1-e^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{315}$$

$$\frac{x(ae^2+c)(a+cx^2)^2}{e^2\sqrt{1-e^2x^2}}-\frac{\int \frac{c(cx^2+a)((4ae^2+5c)x^2+a)}{\sqrt{1-e^2x^2}}dx}{e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(ae^2+c)(a+cx^2)^2}{e^2\sqrt{1-e^2x^2}}-\frac{c\int \frac{(cx^2+a)((4ae^2+5c)x^2+a)}{\sqrt{1-e^2x^2}}dx}{e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{x(ae^2+c)(a+cx^2)^2}{e^2\sqrt{1-e^2x^2}}-\frac{c\left(-\frac{\int -\frac{(2ae^2+5c)(4ae^2+3c)x^2+a(8ae^2+5c)}{\sqrt{1-e^2x^2}}dx}{4e^2}-\frac{1}{4}x\sqrt{1-e^2x^2}\left(4a+\frac{5c}{e^2}\right)(a+cx^2)\right)}{e^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{x(ae^2+c)(a+cx^2)^2}{e^2\sqrt{1-e^2x^2}}-\frac{c\left(\frac{\int\frac{(2ae^2+5c)(4ae^2+3c)x^2+a(8ae^2+5c)}{\sqrt{1-e^2x^2}}dx}{4e^2}-\frac{1}{4}x\sqrt{1-e^2x^2}\left(4a+\frac{5c}{e^2}\right)(a+cx^2)\right)}{e^2}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{x(ae^2+c)(a+cx^2)^2}{e^2\sqrt{1-e^2x^2}}-\frac{c\left(\frac{\frac{3(8a^2e^4+12ace^2+5c^2)\int\frac{1}{\sqrt{1-e^2x^2}}dx}{2e^2}-\frac{x\sqrt{1-e^2x^2}(2ae^2+5c)(4ae^2+3c)}{2e^2}}{4e^2}-\frac{1}{4}x\sqrt{1-e^2x^2}\left(4a+\frac{5c}{e^2}\right)(a+cx^2)\right)}{e^2}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{x(ae^2+c)(a+cx^2)^2}{e^2\sqrt{1-e^2x^2}}-\frac{c\left(\frac{\frac{3(8a^2e^4+12ace^2+5c^2)\arcsin(ex)}{2e^3}-\frac{x\sqrt{1-e^2x^2}(2ae^2+5c)(4ae^2+3c)}{2e^2}}{4e^2}-\frac{1}{4}x\sqrt{1-e^2x^2}\left(4a+\frac{5c}{e^2}\right)(a+cx^2)\right)}{e^2}$$

input
```
Int[(a + c*x^2)^3/((1 - e*x)^(3/2)*(1 + e*x)^(3/2)),x]
```

output
```
((c + a*e^2)*x*(a + c*x^2)^2)/(e^2*Sqrt[1 - e^2*x^2]) - (c*(-1/4*((4*a + (
5*c)/e^2)*x*(a + c*x^2)*Sqrt[1 - e^2*x^2]) + (-1/2*((5*c + 2*a*e^2)*(3*c +
 4*a*e^2)*x*Sqrt[1 - e^2*x^2])/e^2 + (3*(5*c^2 + 12*a*c*e^2 + 8*a^2*e^4)*A
rcSin[e*x])/(2*e^3))/(4*e^2)))/e^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(112) = 224$.

Time = 1.43 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.82

method	result
risch	$-\frac{c^2x(2x^2ce^2+12ae^2+7c)(ex-1)\sqrt{ex+1}\,\sqrt{(-ex+1)(ex+1)}}{8e^6\sqrt{-(ex-1)(ex+1)}\,\sqrt{-ex+1}}-\frac{\left(-\frac{(-4e^6a^3-12e^4a^2c-12c^2ae^2-4c^3)\sqrt{-e^2\left(x+\frac{1}{e}\right)^2+2e\left(x+\frac{1}{e}\right)}}{e^2\left(x+\frac{1}{e}\right)}+\cdots\right.}{\cdots}$
default	$\frac{\left(-2\,\mathrm{csgn}(e)c^3e^5x^5\sqrt{-e^2x^2+1}+8\,\mathrm{csgn}(e)e^7\sqrt{-e^2x^2+1}\,a^3x-12\,\mathrm{csgn}(e)ac^2e^5x^3\sqrt{-e^2x^2+1}+24\arctan\left(\frac{\mathrm{csgn}(e)ex}{\sqrt{-e^2x^2+1}}\right)a^2ce^6x^2+2\cdots\right.}{\cdots}$

input
```
int((c*x^2+a)^3/(-e*x+1)^(3/2)/(e*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*c^2*x*(2*c*e^2*x^2+12*a*e^2+7*c)*(e*x-1)*(e*x+1)^(1/2)/e^6/(-(e*x-1)*
(e*x+1))^(1/2)*((-e*x+1)*(e*x+1))^(1/2)/(-e*x+1)^(1/2)-1/8/e^6*(-(-4*a^3*e
^6-12*a^2*c*e^4-12*a*c^2*e^2-4*c^3)/e^2/(x+1/e)*(-e^2*(x+1/e)^2+2*e*(x+1/e
))^(1/2)+(4*a^3*e^6+12*a^2*c*e^4+12*a*c^2*e^2+4*c^3)/e^2/(x-1/e)*(-e^2*(x-
1/e)^2-2*e*(x-1/e))^(1/2)+15*c^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^
2+1)^(1/2))+36*c^2*a*e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+1)^(1/
2))+24*e^4*a^2*c/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+1)^(1/2)))*((-
e*x+1)*(e*x+1))^(1/2)/(-e*x+1)^(1/2)/(e*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.52

$$\int \frac{(a+cx^2)^3}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{(2\,c^3e^5x^5+(12\,ac^2e^5+5\,c^3e^3)x^3-(8\,a^3e^7+24\,a^2ce^5+36\,ac^2e^3+15\,c^3e)x\cdots}{\cdots}$$

input
```
integrate((c*x^2+a)^3/(-e*x+1)^(3/2)/(e*x+1)^(3/2),x, algorithm="fricas")
```

output
```
1/8*((2*c^3*e^5*x^5 + (12*a*c^2*e^5 + 5*c^3*e^3)*x^3 - (8*a^3*e^7 + 24*a^2
*c*e^5 + 36*a*c^2*e^3 + 15*c^3*e)*x)*sqrt(e*x + 1)*sqrt(-e*x + 1) - 6*(8*a
^2*c*e^4 + 12*a*c^2*e^2 + 5*c^3 - (8*a^2*c*e^6 + 12*a*c^2*e^4 + 5*c^3*e^2)
*x^2)*arctan((sqrt(e*x + 1)*sqrt(-e*x + 1) - 1)/(e*x)))/(e^9*x^2 - e^7)
```

Sympy [F]

$$\int \frac{(a+cx^2)^3}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = \int \frac{(a+cx^2)^3}{(-ex+1)^{\frac{3}{2}}\,(ex+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**2+a)**3/(-e*x+1)**(3/2)/(e*x+1)**(3/2),x)
```

output
```
Integral((a + c*x**2)**3/((-e*x + 1)**(3/2)*(e*x + 1)**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(112) = 224.

Time = 0.14 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.83

$$\int \frac{(a+cx^2)^3}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = -\frac{c^3x^5}{4\sqrt{-e^2x^2+1}e^2} + \frac{a^3x}{\sqrt{-e^2x^2+1}} - \frac{3\,ac^2x^3}{2\sqrt{-e^2x^2+1}e^2} + \frac{3\,a^2cx}{\sqrt{-e^2x^2+1}e^2} - \frac{5\,c^3x^3}{8\sqrt{-e^2x^2+1}e^4} - \frac{3\,a^2c\arcsin\left(\frac{e^2x}{\sqrt{e^2}}\right)}{\sqrt{e^2}e^2} + \frac{9\,ac^2x}{2\sqrt{-e^2x^2+1}e^4} - \frac{9\,ac^2\arcsin\left(\frac{e^2x}{\sqrt{e^2}}\right)}{2\sqrt{e^2}e^4} + \frac{15\,c^3x}{8\sqrt{-e^2x^2+1}e^6} - \frac{15\,c^3\arcsin\left(\frac{e^2x}{\sqrt{e^2}}\right)}{8\sqrt{e^2}e^6}$$

input
```
integrate((c*x^2+a)^3/(-e*x+1)^(3/2)/(e*x+1)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*c^3*x^5/(sqrt(-e^2*x^2 + 1)*e^2) + a^3*x/sqrt(-e^2*x^2 + 1) - 3/2*a*c
^2*x^3/(sqrt(-e^2*x^2 + 1)*e^2) + 3*a^2*c*x/(sqrt(-e^2*x^2 + 1)*e^2) - 5/8
*c^3*x^3/(sqrt(-e^2*x^2 + 1)*e^4) - 3*a^2*c*arcsin(e^2*x/sqrt(e^2))/(sqrt(
e^2)*e^2) + 9/2*a*c^2*x/(sqrt(-e^2*x^2 + 1)*e^4) - 9/2*a*c^2*arcsin(e^2*x/
sqrt(e^2))/(sqrt(e^2)*e^4) + 15/8*c^3*x/(sqrt(-e^2*x^2 + 1)*e^6) - 15/8*c^
3*arcsin(e^2*x/sqrt(e^2))/(sqrt(e^2)*e^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(112) = 224$.

Time = 0.15 (sec) , antiderivative size = 375, normalized size of antiderivative = 3.02

$$\int \frac{(a+cx^2)^3}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{\frac{\left(\left((ex+1)\left(2\,(ex+1)\left(\frac{(ex+1)c^3}{e^6}-\frac{5\,c^3}{e^6}\right)+\frac{12\,ac^2e^{32}+25\,c^3e^{30}}{e^{36}}\right)-\frac{36\,ac^2e^{32}+35\,c^3e^{30}}{e^{36}}\right)(ex+1)-\frac{2\left(2\,a^3e^{36}\right.}{}}{ex-1}}{}$$

input
```
integrate((c*x^2+a)^3/(-e*x+1)^(3/2)/(e*x+1)^(3/2),x, algorithm="giac")
```

output
```
1/8*((((e*x + 1)*(2*(e*x + 1)*((e*x + 1)*c^3/e^6 - 5*c^3/e^6) + (12*a*c^2*
e^32 + 25*c^3*e^30)/e^36) - (36*a*c^2*e^32 + 35*c^3*e^30)/e^36)*(e*x + 1)
- 2*(2*a^3*e^36 + 6*a^2*c*e^34 - 6*a*c^2*e^32 - 7*c^3*e^30)/e^36)*sqrt(e*x
 + 1)*sqrt(-e*x + 1)/(e*x - 1) - 6*(8*a^2*c*e^4 + 12*a*c^2*e^2 + 5*c^3)*ar
csin(1/2*sqrt(2)*sqrt(e*x + 1))/e^6 + 2*(a^3*e^6*(sqrt(2) - sqrt(-e*x + 1)
)/sqrt(e*x + 1) + 3*a^2*c*e^4*(sqrt(2) - sqrt(-e*x + 1))/sqrt(e*x + 1) + 3
*a*c^2*e^2*(sqrt(2) - sqrt(-e*x + 1))/sqrt(e*x + 1) + c^3*(sqrt(2) - sqrt(
-e*x + 1))/sqrt(e*x + 1))/e^6 - 2*(a^3*e^6 + 3*a^2*c*e^4 + 3*a*c^2*e^2 + c
^3)*sqrt(e*x + 1)/(e^6*(sqrt(2) - sqrt(-e*x + 1))))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^3}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = \int \frac{{\left(c\,x^2+a\right)}^3}{{\left(1-e\,x\right)}^{3/2}\,{\left(e\,x+1\right)}^{3/2}}\,dx$$

input
```
int((a + c*x^2)^3/((1 - e*x)^(3/2)*(e*x + 1)^(3/2)),x)
```

output
```
int((a + c*x^2)^3/((1 - e*x)^(3/2)*(e*x + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.56

$$\int \frac{(a+cx^2)^3}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{48\sqrt{ex+1}\,\sqrt{-ex+1}\,asin\left(\frac{\sqrt{-ex+1}}{\sqrt{2}}\right)a^2c\,e^4+72\sqrt{ex+1}\,\sqrt{-ex+1}\,asin\left(\sqrt{}\right.}{}$$

input
```
int((c*x^2+a)^3/(-e*x+1)^(3/2)/(e*x+1)^(3/2),x)
```

output
```
(48*sqrt(e*x + 1)*sqrt( - e*x + 1)*asin(sqrt( - e*x + 1)/sqrt(2))*a**2*c*e
**4 + 72*sqrt(e*x + 1)*sqrt( - e*x + 1)*asin(sqrt( - e*x + 1)/sqrt(2))*a*c
**2*e**2 + 30*sqrt(e*x + 1)*sqrt( - e*x + 1)*asin(sqrt( - e*x + 1)/sqrt(2)
)*c**3 + 8*a**3*e**7*x + 24*a**2*c*e**5*x - 12*a*c**2*e**5*x**3 + 36*a*c**
2*e**3*x - 2*c**3*e**5*x**5 - 5*c**3*e**3*x**3 + 15*c**3*e*x)/(8*sqrt(e*x
+ 1)*sqrt( - e*x + 1)*e**7)
```

3.10 $\int \frac{(a+cx^2)^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 76

$$\int \frac{(a+cx^2)^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx=\frac{(c+ae^2)^2x}{e^4\sqrt{1-e^2x^2}}+\frac{c^2x\sqrt{1-e^2x^2}}{2e^4}-\frac{c(3c+4ae^2)\arcsin(ex)}{2e^5}$$

output
```
(a*e^2+c)^2*x/e^4/(-e^2*x^2+1)^(1/2)+1/2*c^2*x*(-e^2*x^2+1)^(1/2)/e^4-1/2*
c*(4*a*e^2+3*c)*arcsin(e*x)/e^5
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.18

$$\int \frac{(a+cx^2)^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx=\frac{\frac{ex(4ace^2+2a^2e^4+c^2(3-e^2x^2))}{\sqrt{1-e^2x^2}}-2c(3c+4ae^2)\arctan\left(\frac{ex}{-1+\sqrt{1-e^2x^2}}\right)}{2e^5}$$

input
```
Integrate[(a + c*x^2)^2/((1 - e*x)^(3/2)*(1 + e*x)^(3/2)),x]
```

output
```
((e*x*(4*a*c*e^2 + 2*a^2*e^4 + c^2*(3 - e^2*x^2)))/Sqrt[1 - e^2*x^2] - 2*c
*(3*c + 4*a*e^2)*ArcTan[(e*x)/(-1 + Sqrt[1 - e^2*x^2])])/(2*e^5)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {643, 315, 27, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(1-ex)^{3/2}(ex+1)^{3/2}}dx$$

$$\Big\downarrow \text{643}$$

$$\int \frac{(a+cx^2)^2}{(1-e^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{315}$$

$$\frac{x(ae^2+c)(a+cx^2)}{e^2\sqrt{1-e^2x^2}}-\frac{\int \frac{c((2ae^2+3c)x^2+a)}{\sqrt{1-e^2x^2}}dx}{e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(ae^2+c)(a+cx^2)}{e^2\sqrt{1-e^2x^2}}-\frac{c\int \frac{(2ae^2+3c)x^2+a}{\sqrt{1-e^2x^2}}dx}{e^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{x(ae^2+c)(a+cx^2)}{e^2\sqrt{1-e^2x^2}}-\frac{c\left(\frac{1}{2}\left(4a+\frac{3c}{e^2}\right)\int \frac{1}{\sqrt{1-e^2x^2}}dx-\frac{1}{2}x\sqrt{1-e^2x^2}\left(2a+\frac{3c}{e^2}\right)\right)}{e^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{x(ae^2+c)(a+cx^2)}{e^2\sqrt{1-e^2x^2}}-\frac{c\left(\frac{\left(4a+\frac{3c}{e^2}\right)\arcsin(ex)}{2e}-\frac{1}{2}x\sqrt{1-e^2x^2}\left(2a+\frac{3c}{e^2}\right)\right)}{e^2}$$

input
```
Int[(a + c*x^2)^2/((1 - e*x)^(3/2)*(1 + e*x)^(3/2)),x]
```

output
```
((c + a*e^2)*x*(a + c*x^2))/(e^2*Sqrt[1 - e^2*x^2]) - (c*(-1/2*((2*a + (3*
c)/e^2)*x*Sqrt[1 - e^2*x^2]) + ((4*a + (3*c)/e^2)*ArcSin[e*x])/(2*e)))/e^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.01 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.05

method	result
default	$\frac{\left(2\,\mathrm{csgn}(e)e^5\sqrt{-e^2x^2+1}\,a^2x-\mathrm{csgn}(e)c^2e^3x^3\sqrt{-e^2x^2+1}+4\arctan\left(\frac{\mathrm{csgn}(e)ex}{\sqrt{-e^2x^2+1}}\right)ac\,e^4x^2+4\sqrt{-e^2x^2+1}\,\mathrm{csgn}(e)e^3acx+3\arctan\left(\right.}{2\sqrt{-ex+1}\,\sqrt{-e^2x^2+1}\,\sqrt{e}}$
risch	$-\frac{c^2x(ex-1)\sqrt{ex+1}\,\sqrt{(-ex+1)(ex+1)}}{2e^4\sqrt{-(ex-1)(ex+1)}\,\sqrt{-ex+1}}-\frac{\left(-\frac{\left(-a^2e^4-2ac\,e^2-c^2\right)\sqrt{-e^2\left(x+\frac{1}{e}\right)^2+2e\left(x+\frac{1}{e}\right)}}{e^2\left(x+\frac{1}{e}\right)}+\frac{\left(a^2e^4+2ac\,e^2+c^2\right)\sqrt{-e^2\left(x-\frac{1}{e}\right)^2-2e\left(\right.}}{e^2\left(x-\frac{1}{e}\right)}\right.}{2e^4\sqrt{-ex+1}}$

input
```
int((c*x^2+a)^2/(-e*x+1)^(3/2)/(e*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*csgn(e)*e^5*(-e^2*x^2+1)^(1/2)*a^2*x-csgn(e)*c^2*e^3*x^3*(-e^2*x^2+
1)^(1/2)+4*arctan(csgn(e)*e*x/(-e^2*x^2+1)^(1/2))*a*c*e^4*x^2+4*(-e^2*x^2+
1)^(1/2)*csgn(e)*e^3*a*c*x+3*arctan(csgn(e)*e*x/(-e^2*x^2+1)^(1/2))*c^2*e^
2*x^2+3*(-e^2*x^2+1)^(1/2)*csgn(e)*e*c^2*x-4*arctan(csgn(e)*e*x/(-e^2*x^2+
1)^(1/2))*a*c*e^2-3*arctan(csgn(e)*e*x/(-e^2*x^2+1)^(1/2))*c^2)/(-e*x+1)^(
1/2)/(-e^2*x^2+1)^(1/2)/(e*x+1)^(1/2)*csgn(e)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.74

$$\int \frac{(a+cx^2)^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx=\frac{(c^2e^3x^3-(2\,a^2e^5+4\,ace^3+3\,c^2e)x)\sqrt{ex+1}\sqrt{-ex+1}-2\,(4\,ace^2-(4\,ac}{2\,(e^7x^2-e^5)}$$

input
```
integrate((c*x^2+a)^2/(-e*x+1)^(3/2)/(e*x+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((c^2*e^3*x^3 - (2*a^2*e^5 + 4*a*c*e^3 + 3*c^2*e)*x)*sqrt(e*x + 1)*sqr
t(-e*x + 1) - 2*(4*a*c*e^2 - (4*a*c*e^4 + 3*c^2*e^2)*x^2 + 3*c^2)*arctan((
sqrt(e*x + 1)*sqrt(-e*x + 1) - 1)/(e*x)))/(e^7*x^2 - e^5)
```

Sympy [F]

$$\int \frac{(a+cx^2)^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = \int \frac{(a+cx^2)^2}{(-ex+1)^{\frac{3}{2}}\,(ex+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**2+a)**2/(-e*x+1)**(3/2)/(e*x+1)**(3/2),x)
```

output
```
Integral((a + c*x**2)**2/((-e*x + 1)**(3/2)*(e*x + 1)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.70

$$\int \frac{(a+cx^2)^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{a^2x}{\sqrt{-e^2x^2+1}} - \frac{c^2x^3}{2\sqrt{-e^2x^2+1}e^2} + \frac{2\,acx}{\sqrt{-e^2x^2+1}e^2} - \frac{2\,ac\arcsin\left(\frac{e^2x}{\sqrt{e^2}}\right)}{\sqrt{e^2}e^2} + \frac{3\,c^2x}{2\sqrt{-e^2x^2+1}e^4} - \frac{3\,c^2\arcsin\left(\frac{e^2x}{\sqrt{e^2}}\right)}{2\sqrt{e^2}e^4}$$

input
```
integrate((c*x^2+a)^2/(-e*x+1)^(3/2)/(e*x+1)^(3/2),x, algorithm="maxima")
```

output
```
a^2*x/sqrt(-e^2*x^2 + 1) - 1/2*c^2*x^3/(sqrt(-e^2*x^2 + 1)*e^2) + 2*a*c*x/
(sqrt(-e^2*x^2 + 1)*e^2) - 2*a*c*arcsin(e^2*x/sqrt(e^2))/(sqrt(e^2)*e^2) +
 3/2*c^2*x/(sqrt(-e^2*x^2 + 1)*e^4) - 3/2*c^2*arcsin(e^2*x/sqrt(e^2))/(sqr
t(e^2)*e^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(68) = 136.

Time = 0.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.28

$$\int \frac{(a+cx^2)^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{\frac{2\sqrt{ex+1}\sqrt{-ex+1}\left((ex+1)\left(\frac{(ex+1)c^2}{e^4}-\frac{3\,c^2}{e^4}\right)-\frac{a^2e^{16}+2\,ace^{14}-c^2e^{12}}{e^{16}}\right)}{ex-1} - \frac{4\left(4\,ace^2+3\,c^2\right)\arcsin\left(\frac{1}{2}\sqrt{2}\right.}{e^4}}{}$$

input
```
integrate((c*x^2+a)^2/(-e*x+1)^(3/2)/(e*x+1)^(3/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(e*x + 1)*sqrt(-e*x + 1)*((e*x + 1)*((e*x + 1)*c^2/e^4 - 3*c^2/
e^4) - (a^2*e^16 + 2*a*c*e^14 - c^2*e^12)/e^16)/(e*x - 1) - 4*(4*a*c*e^2 +
 3*c^2)*arcsin(1/2*sqrt(2)*sqrt(e*x + 1))/e^4 + (a^2*e^4*(sqrt(2) - sqrt(-
e*x + 1))/sqrt(e*x + 1) + 2*a*c*e^2*(sqrt(2) - sqrt(-e*x + 1))/sqrt(e*x +
1) + c^2*(sqrt(2) - sqrt(-e*x + 1))/sqrt(e*x + 1))/e^4 - (a^2*e^4 + 2*a*c*
e^2 + c^2)*sqrt(e*x + 1)/(e^4*(sqrt(2) - sqrt(-e*x + 1))))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx=\int \frac{\left(c\,x^2+a\right)^2}{\left(1-e\,x\right)^{3/2}\,\left(e\,x+1\right)^{3/2}}\,dx$$

input
```
int((a + c*x^2)^2/((1 - e*x)^(3/2)*(e*x + 1)^(3/2)),x)
```

output
```
int((a + c*x^2)^2/((1 - e*x)^(3/2)*(e*x + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.61

$$\int \frac{(a+cx^2)^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx=\frac{8\sqrt{ex+1}\,\sqrt{-ex+1}\,asin\left(\frac{\sqrt{-ex+1}}{\sqrt{2}}\right)ac\,e^2+6\sqrt{ex+1}\,\sqrt{-ex+1}\,asin\left(\frac{\sqrt{-ex}}{\sqrt{2}}\right.}{2\sqrt{ex+1}\,\sqrt{-ex+1}\,e^5}$$

input
```
int((c*x^2+a)^2/(-e*x+1)^(3/2)/(e*x+1)^(3/2),x)
```

output
```
(8*sqrt(e*x + 1)*sqrt( - e*x + 1)*asin(sqrt( - e*x + 1)/sqrt(2))*a*c*e**2
+ 6*sqrt(e*x + 1)*sqrt( - e*x + 1)*asin(sqrt( - e*x + 1)/sqrt(2))*c**2 + 2
*a**2*e**5*x + 4*a*c*e**3*x - c**2*e**3*x**3 + 3*c**2*e*x)/(2*sqrt(e*x + 1
)*sqrt( - e*x + 1)*e**5)
```

3.11 $\int \frac{a+cx^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 37

$$\int \frac{a+cx^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx=\frac{(c+ae^2)\,x}{e^2\sqrt{1-e^2x^2}}-\frac{c\arcsin(ex)}{e^3}$$

output
```
(a*e^2+c)*x/e^2/(-e^2*x^2+1)^(1/2)-c*arcsin(e*x)/e^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \frac{a+cx^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx=\frac{\frac{e(c+ae^2)x}{\sqrt{1-e^2x^2}}-2c\arctan\left(\frac{ex}{-1+\sqrt{1-e^2x^2}}\right)}{e^3}$$

input
```
Integrate[(a + c*x^2)/((1 - e*x)^(3/2)*(1 + e*x)^(3/2)),x]
```

output
```
((e*(c + a*e^2)*x)/Sqrt[1 - e^2*x^2] - 2*c*ArcTan[(e*x)/(-1 + Sqrt[1 - e^2
*x^2])])/e^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {643, 298, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(1-ex)^{3/2}(ex+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 643}$$

$$\int \frac{a+cx^2}{(1-e^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 298}$$

$$\frac{x(ae^2+c)}{e^2\sqrt{1-e^2x^2}}-\frac{c\int \frac{1}{\sqrt{1-e^2x^2}}dx}{e^2}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{x(ae^2+c)}{e^2\sqrt{1-e^2x^2}}-\frac{c\arcsin(ex)}{e^3}$$

input
```
Int[(a + c*x^2)/((1 - e*x)^(3/2)*(1 + e*x)^(3/2)),x]
```

output
```
((c + a*e^2)*x)/(e^2*Sqrt[1 - e^2*x^2]) - (c*ArcSin[e*x])/e^3
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.85 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.35

method	result	s
default	$\frac{\left(\sqrt{-e^{2} x^{2}+1}\, \operatorname{csgn}\left(e \right) e^{3} a x +\arctan\left(\frac{\operatorname{csgn}\left(e \right) e x}{\sqrt{-\left(e x -1\right) \left(e x +1\right)}}\right) c\, e^{2} x^{2}+\sqrt{-e^{2} x^{2}+1}\, \operatorname{csgn}\left(e \right) e c x -\arctan\left(\frac{\operatorname{csgn}\left(e \right) e x}{\sqrt{-\left(e x -1\right) \left(e x +1\right)}}\right) c \right) \operatorname{csgn}\left(e \right)}{\sqrt{-e x +1}\, \sqrt{-e^{2} x^{2}+1}\, \sqrt{e x +1}\, e^{3}}$	1

input
```
int((c*x^2+a)/(-e*x+1)^(3/2)/(e*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-e^2*x^2+1)^(1/2)*csgn(e)*e^3*a*x+arctan(csgn(e)*e*x/(-(e*x-1)*(e*x+1))^
(1/2))*c*e^2*x^2+(-e^2*x^2+1)^(1/2)*csgn(e)*e*c*x-arctan(csgn(e)*e*x/(-(e*
x-1)*(e*x+1))^(1/2))*c)*csgn(e)/(-e*x+1)^(1/2)/(-e^2*x^2+1)^(1/2)/(e*x+1)^
(1/2)/e^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(35) = 70$.

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.27

$$\int \frac{a+cx^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = -\frac{(ae^3+ce)\sqrt{ex+1}\sqrt{-ex+1}x - 2\left(ce^2x^2-c\right)\arctan\left(\frac{\sqrt{ex+1}\sqrt{-ex+1}-1}{ex}\right)}{e^5x^2-e^3}$$

input
```
integrate((c*x^2+a)/(-e*x+1)^(3/2)/(e*x+1)^(3/2),x, algorithm="fricas")
```

output
```
-((a*e^3 + c*e)*sqrt(e*x + 1)*sqrt(-e*x + 1)*x - 2*(c*e^2*x^2 - c)*arctan(
(sqrt(e*x + 1)*sqrt(-e*x + 1) - 1)/(e*x)))/(e^5*x^2 - e^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 123.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 4.54

$$\int \frac{a+cx^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = a\left(-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{1}{e^2x^2}\right)}{2\pi^{\frac{3}{2}}e}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{e^2x^2}\right)}{2\pi^{\frac{3}{2}}e}\right) + c\left(\frac{iG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & -\frac{1}{2},\frac{1}{2},1,1\\ -\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1,0 & \end{matrix}\middle|\frac{1}{e^2x^2}\right)}{2\pi^{\frac{3}{2}}e^3}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},1 & \\ -\frac{3}{4},-\frac{1}{4} & -\frac{3}{2},-1,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{e^2x^2}\right)}{2\pi^{\frac{3}{2}}e^3}\right)$$

input
```
integrate((c*x**2+a)/(-e*x+1)**(3/2)/(e*x+1)**(3/2),x)
```

output
```
a*(-I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)
), 1/(e**2*x**2))/(2*pi**(3/2)*e) + meijerg(((-1/2, 0, 1/4, 1/2, 3/4, 1),
()), ((1/4, 3/4), (-1/2, 0, 1, 0)), exp_polar(-2*I*pi)/(e**2*x**2))/(2*pi*
*(3/2)*e)) + c*(I*meijerg(((-1/4, 1/4), (-1/2, 1/2, 1, 1)), ((-1/4, 0, 1/4
, 1/2, 1, 0), ()), 1/(e**2*x**2))/(2*pi**(3/2)*e**3) + meijerg(((-3/2, -1,
 -3/4, -1/2, -1/4, 1), ()), ((-3/4, -1/4), (-3/2, -1, 0, 0)), exp_polar(-2
*I*pi)/(e**2*x**2))/(2*pi**(3/2)*e**3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{a+cx^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{ax}{\sqrt{-e^2x^2+1}} + \frac{cx}{\sqrt{-e^2x^2+1}e^2} - \frac{c\arcsin\left(\frac{e^2x}{\sqrt{e^2}}\right)}{\sqrt{e^2}e^2}$$

input
```
integrate((c*x^2+a)/(-e*x+1)^(3/2)/(e*x+1)^(3/2),x, algorithm="maxima")
```

output
```
a*x/sqrt(-e^2*x^2 + 1) + c*x/(sqrt(-e^2*x^2 + 1)*e^2) - c*arcsin(e^2*x/sqr
t(e^2))/(sqrt(e^2)*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(35) = 70.

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 4.11

$$\int \frac{a+cx^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = -\frac{\frac{8\,c\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{ex+1}\right)}{e^2} - \frac{\frac{ae^2\left(\sqrt{2}-\sqrt{-ex+1}\right)}{\sqrt{ex+1}} + \frac{c\left(\sqrt{2}-\sqrt{-ex+1}\right)}{\sqrt{ex+1}}}{e^2} + \frac{(ae^2+c)\sqrt{ex+1}}{e^2\left(\sqrt{2}-\sqrt{-ex+1}\right)} + \frac{2\,(ae^4+ce^2)\sqrt{ex+1}\sqrt{-ex+1}}{(ex-1)e^4}}{4\,e}$$

input
```
integrate((c*x^2+a)/(-e*x+1)^(3/2)/(e*x+1)^(3/2),x, algorithm="giac")
```

output
```
-1/4*(8*c*arcsin(1/2*sqrt(2)*sqrt(e*x + 1))/e^2 - (a*e^2*(sqrt(2) - sqrt(-
e*x + 1))/sqrt(e*x + 1) + c*(sqrt(2) - sqrt(-e*x + 1))/sqrt(e*x + 1))/e^2
+ (a*e^2 + c)*sqrt(e*x + 1)/(e^2*(sqrt(2) - sqrt(-e*x + 1))) + 2*(a*e^4 +
c*e^2)*sqrt(e*x + 1)*sqrt(-e*x + 1)/((e*x - 1)*e^4))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+cx^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = \int \frac{c\,x^2+a}{(1-e\,x)^{3/2}\,(e\,x+1)^{3/2}}\,dx$$

input
```
int((a + c*x^2)/((1 - e*x)^(3/2)*(e*x + 1)^(3/2)),x)
```

output
```
int((a + c*x^2)/((1 - e*x)^(3/2)*(e*x + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int \frac{a+cx^2}{(1-ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{2\sqrt{ex+1}\,\sqrt{-ex+1}\,asin\left(\frac{\sqrt{-ex+1}}{\sqrt{2}}\right)c+a\,e^3x+cex}{\sqrt{ex+1}\,\sqrt{-ex+1}\,e^3}$$

input
```
int((c*x^2+a)/(-e*x+1)^(3/2)/(e*x+1)^(3/2),x)
```

output
```
(2*sqrt(e*x + 1)*sqrt( - e*x + 1)*asin(sqrt( - e*x + 1)/sqrt(2))*c + a*e**
3*x + c*e*x)/(sqrt(e*x + 1)*sqrt( - e*x + 1)*e**3)
```

3.12 $\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}(a+cx^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 80

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}(a+cx^2)}\,dx=\frac{e^2x}{(c+ae^2)\sqrt{1-e^2x^2}}+\frac{c\arctan\left(\frac{\sqrt{c+ae^2}x}{\sqrt{a}\sqrt{1-e^2x^2}}\right)}{\sqrt{a}\,(c+ae^2)^{3/2}}$$

output

```
e^2*x/(a*e^2+c)/(-e^2*x^2+1)^(1/2)+c*arctan((a*e^2+c)^(1/2)*x/a^(1/2)/(-e^
2*x^2+1)^(1/2))/a^(1/2)/(a*e^2+c)^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(80) = 160.

Time = 2.07 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.51

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}(a+cx^2)}\,dx=\frac{\frac{e^4x}{(c+ae^2)\sqrt{1-e^2x^2}}+\frac{c\left(\sqrt{c}+\sqrt{c+ae^2}\right)\sqrt{2c+ae^2-2\sqrt{c}\sqrt{c+ae^2}}\arctan\left(\frac{\sqrt{2c+ae^2-2\sqrt{c}\sqrt{c}}}{\sqrt{a}\left(-1+\sqrt{1-e^2}\right.}\right.}{a^{3/2}(c+ae^2)^{3/2}}}{e}$$

input

```
Integrate[1/((1 - e*x)^(3/2)*(1 + e*x)^(3/2)*(a + c*x^2)),x]
```

output
```
((e^4*x)/((c + a*e^2)*Sqrt[1 - e^2*x^2]) + (c*(Sqrt[c] + Sqrt[c + a*e^2])*
Sqrt[2*c + a*e^2 - 2*Sqrt[c]*Sqrt[c + a*e^2]]*ArcTan[(Sqrt[2*c + a*e^2 - 2
*Sqrt[c]*Sqrt[c + a*e^2]]*x)/(Sqrt[a]*(-1 + Sqrt[1 - e^2*x^2]))]])/(a^(3/2)
*(c + a*e^2)^(3/2)) + (c*(-Sqrt[c] + Sqrt[c + a*e^2])*Sqrt[2*c + a*e^2 + 2
*Sqrt[c]*Sqrt[c + a*e^2]]*ArcTan[(Sqrt[2*c + a*e^2 + 2*Sqrt[c]*Sqrt[c + a*
e^2]]*x)/(Sqrt[a]*(-1 + Sqrt[1 - e^2*x^2]))]])/(a^(3/2)*(c + a*e^2)^(3/2)))
/e^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {643, 296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-ex)^{3/2}(ex+1)^{3/2}(a+cx^2)}dx$$

$$\Big\downarrow \text{643}$$

$$\int \frac{1}{(1-e^2x^2)^{3/2}(a+cx^2)}dx$$

$$\Big\downarrow \text{296}$$

$$\frac{c\int \frac{1}{(cx^2+a)\sqrt{1-e^2x^2}}dx}{ae^2+c}+\frac{e^2x}{\sqrt{1-e^2x^2}(ae^2+c)}$$

$$\Big\downarrow \text{291}$$

$$\frac{c\int \frac{1}{a-\frac{(-ae^2-c)x^2}{1-e^2x^2}}d\frac{x}{\sqrt{1-e^2x^2}}}{ae^2+c}+\frac{e^2x}{\sqrt{1-e^2x^2}(ae^2+c)}$$

$$\Big\downarrow \text{218}$$

$$\frac{c\arctan\left(\frac{x\sqrt{ae^2+c}}{\sqrt{a}\sqrt{1-e^2x^2}}\right)}{\sqrt{a}(ae^2+c)^{3/2}}+\frac{e^2x}{\sqrt{1-e^2x^2}(ae^2+c)}$$

input
```
Int[1/((1 - e*x)^(3/2)*(1 + e*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(e^2*x)/((c + a*e^2)*Sqrt[1 - e^2*x^2]) + (c*ArcTan[(Sqrt[c + a*e^2]*x)/(S
qrt[a]*Sqrt[1 - e^2*x^2])])/(Sqrt[a]*(c + a*e^2)^(3/2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.03 (sec) , antiderivative size = 672, normalized size of antiderivative = 8.40

method	result
default	$-\frac{\left(-\ln\left(\frac{-2\sqrt{-ac}\,e^2x+2\sqrt{\frac{a\,e^2+c}{c}}\,\sqrt{-e^2x^2+1}\,c+2c}{cx-\sqrt{-ac}}\right)ac\,e^4x^2+\ln\left(\frac{2\sqrt{-ac}\,e^2x+2\sqrt{\frac{a\,e^2+c}{c}}\,\sqrt{-e^2x^2+1}\,c+2c}{cx+\sqrt{-ac}}\right)ac\,e^4x^2-2a\sqrt{-ac}\,\sqrt{-e^2x}$

input
```
int(1/(-e*x+1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+
c)/(c*x-(-a*c)^(1/2)))*a*c*e^4*x^2+ln(2*((-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^
(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c)^(1/2)))*a*c*e^4*x^2-2*a*(-a*c)^(
1/2)*(-e^2*x^2+1)^(1/2)*((a*e^2+c)/c)^(1/2)*e^4*x-ln(2*(-(-a*c)^(1/2)*e^2*
x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x-(-a*c)^(1/2)))*c^2*e^2*
x^2+ln(2*((-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(
c*x+(-a*c)^(1/2)))*c^2*e^2*x^2-2*(-a*c)^(1/2)*(-e^2*x^2+1)^(1/2)*((a*e^2+c
)/c)^(1/2)*c*e^2*x+ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2
+1)^(1/2)*c+c)/(c*x-(-a*c)^(1/2)))*a*c*e^2-ln(2*((-a*c)^(1/2)*e^2*x+((a*e^
2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c)^(1/2)))*a*c*e^2+ln(2*(-(
-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x-(-a*c)^
(1/2)))*c^2-ln(2*((-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2
)*c+c)/(c*x+(-a*c)^(1/2)))*c^2)*c^2*csgn(e)^2/(-e*x+1)^(1/2)/(-e*(-a*c)^(1
/2)+c)^2/(e*(-a*c)^(1/2)+c)^2/(-e^2*x^2+1)^(1/2)/(e*x+1)^(1/2)/((a*e^2+c)/
c)^(1/2)/(-a*c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(68) = 136.

Time = 0.10 (sec) , antiderivative size = 337, normalized size of antiderivative = 4.21

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}\,(a+cx^2)}\,dx = \left[\frac{2\left(a^2e^4+ace^2\right)\sqrt{ex+1}\sqrt{-ex+1}x+\left(ce^2x^2-c\right)\sqrt{-a^2e^2-ac}\,\mathrm{l}}{2\left(a^3e^4+2\,a^2ce^2+ac^2-\left(a^3e^6+2\,a\right.\right.}\right.$$

input
```
integrate(1/(-e*x+1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*(a^2*e^4 + a*c*e^2)*sqrt(e*x + 1)*sqrt(-e*x + 1)*x + (c*e^2*x^2 -
c)*sqrt(-a^2*e^2 - a*c)*log(-((2*a*e^2 + c)*x^2 - 2*sqrt(-a^2*e^2 - a*c)*s
qrt(e*x + 1)*sqrt(-e*x + 1)*x - a)/(c*x^2 + a)))/(a^3*e^4 + 2*a^2*c*e^2 +
a*c^2 - (a^3*e^6 + 2*a^2*c*e^4 + a*c^2*e^2)*x^2), ((a^2*e^4 + a*c*e^2)*sqr
t(e*x + 1)*sqrt(-e*x + 1)*x + (c*e^2*x^2 - c)*sqrt(a^2*e^2 + a*c)*arctan(s
qrt(a^2*e^2 + a*c)*sqrt(e*x + 1)*sqrt(-e*x + 1)*x/(a*e^2*x^2 - a)))/(a^3*e
^4 + 2*a^2*c*e^2 + a*c^2 - (a^3*e^6 + 2*a^2*c*e^4 + a*c^2*e^2)*x^2)]
```

Sympy [F]

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)}\,dx=\int \frac{1}{\left(a+cx^2\right)\left(-ex+1\right)^{\frac{3}{2}}\left(ex+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-e*x+1)**(3/2)/(e*x+1)**(3/2)/(c*x**2+a),x)
```

output
```
Integral(1/((a + c*x**2)*(-e*x + 1)**(3/2)*(e*x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)}\,dx=\int \frac{1}{(cx^2+a)(ex+1)^{\frac{3}{2}}(-ex+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-e*x+1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)*(e*x + 1)^(3/2)*(-e*x + 1)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4653 vs. $2(68) = 136$.

Time = 1.17 (sec) , antiderivative size = 4653, normalized size of antiderivative = 58.16

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-e*x+1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
1/4*e*((a*e^2*((sqrt(2) - sqrt(-e*x + 1))/sqrt(e*x + 1) - sqrt(e*x + 1)/(s
qrt(2) - sqrt(-e*x + 1))) + c*((sqrt(2) - sqrt(-e*x + 1))/sqrt(e*x + 1) -
sqrt(e*x + 1)/(sqrt(2) - sqrt(-e*x + 1))))/(a^2*e^4 + 2*a*c*e^2 + c^2) - 4
*(sqrt(a^2*e^2 + a*c)*sqrt(-a*c)*a^9*c*e^17 + sqrt(a^2*e^2 + a*c)*sqrt(-a*
c)*a^8*c^2*e^17 + sqrt(-a*c*e^2 - c^2)*sqrt(-a*c)*a^9*c*e^16 + sqrt(-a*c*e
^2 - c^2)*sqrt(-a*c)*a^8*c^2*e^16 + 6*sqrt(a^2*e^2 + a*c)*sqrt(-a*c)*a^8*c
^2*e^15 + 6*sqrt(a^2*e^2 + a*c)*sqrt(-a*c)*a^7*c^3*e^15 + 6*sqrt(-a*c*e^2
- c^2)*sqrt(-a*c)*a^8*c^2*e^14 + 6*sqrt(-a*c*e^2 - c^2)*sqrt(-a*c)*a^7*c^3
*e^14 + 14*sqrt(a^2*e^2 + a*c)*sqrt(-a*c)*a^7*c^3*e^13 + 14*sqrt(a^2*e^2 +
 a*c)*sqrt(-a*c)*a^6*c^4*e^13 + 14*sqrt(-a*c*e^2 - c^2)*sqrt(-a*c)*a^7*c^3
*e^12 + 14*sqrt(-a*c*e^2 - c^2)*sqrt(-a*c)*a^6*c^4*e^12 + 14*sqrt(a^2*e^2
+ a*c)*sqrt(-a*c)*a^6*c^4*e^11 + 14*sqrt(a^2*e^2 + a*c)*sqrt(-a*c)*a^5*c^5
*e^11 + 14*sqrt(-a*c*e^2 - c^2)*sqrt(-a*c)*a^6*c^4*e^10 + 14*sqrt(-a*c*e^2
 - c^2)*sqrt(-a*c)*a^5*c^5*e^10 - 14*sqrt(a^2*e^2 + a*c)*sqrt(-a*c)*a^4*c^
6*e^7 - 14*sqrt(a^2*e^2 + a*c)*sqrt(-a*c)*a^3*c^7*e^7 - 14*sqrt(-a*c*e^2 -
 c^2)*sqrt(-a*c)*a^4*c^6*e^6 - 14*sqrt(-a*c*e^2 - c^2)*sqrt(-a*c)*a^3*c^7*
e^6 - 14*sqrt(a^2*e^2 + a*c)*sqrt(-a*c)*a^3*c^7*e^5 - 14*sqrt(a^2*e^2 + a*
c)*sqrt(-a*c)*a^2*c^8*e^5 - 14*sqrt(-a*c*e^2 - c^2)*sqrt(-a*c)*a^3*c^7*e^4
 - 14*sqrt(-a*c*e^2 - c^2)*sqrt(-a*c)*a^2*c^8*e^4 - 6*sqrt(a^2*e^2 + a*c)*
sqrt(-a*c)*a^2*c^8*e^3 - 6*sqrt(a^2*e^2 + a*c)*sqrt(-a*c)*a*c^9*e^3 - 6...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)}\,dx = \int \frac{1}{\left(c\,x^2+a\right)\,\left(1-e\,x\right)^{3/2}\,\left(e\,x+1\right)^{3/2}}\,dx$$

input
```
int(1/((a + c*x^2)*(1 - e*x)^(3/2)*(e*x + 1)^(3/2)),x)
```

output
```
int(1/((a + c*x^2)*(1 - e*x)^(3/2)*(e*x + 1)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)}\,dx =$$
$$-\left(\int \frac{1}{\sqrt{ex+1}\,\sqrt{-ex+1}\,a\,e^2x^2-\sqrt{ex+1}\,\sqrt{-ex+1}\,a+\sqrt{ex+1}\,\sqrt{-ex+1}\,c\,e^2x^4-\sqrt{ex+1}\,\sqrt{-ex+}\right.$$

input
```
int(1/(-e*x+1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a),x)
```

output
```
 - int(1/(sqrt(e*x + 1)*sqrt( - e*x + 1)*a*e**2*x**2 - sqrt(e*x + 1)*sqrt(
 - e*x + 1)*a + sqrt(e*x + 1)*sqrt( - e*x + 1)*c*e**2*x**4 - sqrt(e*x + 1)
*sqrt( - e*x + 1)*c*x**2),x)
```

3.13 $\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 146

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = -\frac{e^2(c-2ae^2)\,x}{2a\left(c+ae^2\right)^2\sqrt{1-e^2x^2}} + \frac{cx}{2a\left(c+ae^2\right)\left(a+cx^2\right)\sqrt{1-e^2x^2}} + \frac{c(c+4ae^2)\arctan\left(\frac{\sqrt{c+ae^2}x}{\sqrt{a}\sqrt{1-e^2x^2}}\right)}{2a^{3/2}\left(c+ae^2\right)^{5/2}}$$

output

```
-1/2*e^2*(-2*a*e^2+c)*x/a/(a*e^2+c)^2/(-e^2*x^2+1)^(1/2)+1/2*c*x/a/(a*e^2+
c)/(c*x^2+a)/(-e^2*x^2+1)^(1/2)+1/2*c*(4*a*e^2+c)*arctan((a*e^2+c)^(1/2)*x
/a^(1/2)/(-e^2*x^2+1)^(1/2))/a^(3/2)/(a*e^2+c)^(5/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1036 vs. 2(146) = 292.

Time = 8.06 (sec) , antiderivative size = 1036, normalized size of antiderivative = 7.10

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \frac{\frac{\sqrt{a}x\left(2a^2e^4+2ace^4x^2+c^2\left(1-e^2x^2\right)\right)\left(-16+28e^2x^2-13e^4x^4+e^6x^6+\sqrt{1-e^2x^2}\left(16-20e^2\right.\right.}{\left(c+ae^2\right)^2\left(a+cx^2\right)\left(16-36e^2x^2+25e^4x^4-5e^6x^6+\sqrt{1-e^2x^2}\left(-16+28e^2x^2-13e^4x^4+\right.\right.}}{}$$

input
```
Integrate[1/((1 - e*x)^(3/2)*(1 + e*x)^(3/2)*(a + c*x^2)^2),x]
```

output
```
((Sqrt[a]*x*(2*a^2*e^4 + 2*a*c*e^4*x^2 + c^2*(1 - e^2*x^2))*(-16 + 28*e^2*
x^2 - 13*e^4*x^4 + e^6*x^6 + Sqrt[1 - e^2*x^2]*(16 - 20*e^2*x^2 + 5*e^4*x^
4)))/((c + a*e^2)^2*(a + c*x^2)*(16 - 36*e^2*x^2 + 25*e^4*x^4 - 5*e^6*x^6
+ Sqrt[1 - e^2*x^2]*(-16 + 28*e^2*x^2 - 13*e^4*x^4 + e^6*x^6))) + (c^(5/2)
*ArcTan[(Sqrt[2*c + a*e^2 - 2*Sqrt[c]*Sqrt[c + a*e^2]]*x)/(Sqrt[a]*(1 - Sq
rt[1 - e^2*x^2]))])/((c + a*e^2)^(5/2)*Sqrt[2*c + a*e^2 - 2*Sqrt[c]*Sqrt[c
 + a*e^2]]) - (4*a*c^(3/2)*e^2*ArcTan[(Sqrt[2*c + a*e^2 - 2*Sqrt[c]*Sqrt[c
 + a*e^2]]*x)/(Sqrt[a]*(-1 + Sqrt[1 - e^2*x^2]))])/((c + a*e^2)^(5/2)*Sqrt
[2*c + a*e^2 - 2*Sqrt[c]*Sqrt[c + a*e^2]]) + (c^2*ArcTan[(Sqrt[2*c + a*e^2
 - 2*Sqrt[c]*Sqrt[c + a*e^2]]*x)/(Sqrt[a]*(-1 + Sqrt[1 - e^2*x^2]))])/((c
+ a*e^2)^2*Sqrt[2*c + a*e^2 - 2*Sqrt[c]*Sqrt[c + a*e^2]]) + (4*a*c*e^2*Arc
Tan[(Sqrt[2*c + a*e^2 - 2*Sqrt[c]*Sqrt[c + a*e^2]]*x)/(Sqrt[a]*(-1 + Sqrt[
1 - e^2*x^2]))])/((c + a*e^2)^2*Sqrt[2*c + a*e^2 - 2*Sqrt[c]*Sqrt[c + a*e^
2]]) + (c^(5/2)*ArcTan[(Sqrt[2*c + a*e^2 + 2*Sqrt[c]*Sqrt[c + a*e^2]]*x)/(
Sqrt[a]*(-1 + Sqrt[1 - e^2*x^2]))])/((c + a*e^2)^(5/2)*Sqrt[2*c + a*e^2 +
2*Sqrt[c]*Sqrt[c + a*e^2]]) + (4*a*c^(3/2)*e^2*ArcTan[(Sqrt[2*c + a*e^2 +
2*Sqrt[c]*Sqrt[c + a*e^2]]*x)/(Sqrt[a]*(-1 + Sqrt[1 - e^2*x^2]))])/((c + a
*e^2)^(5/2)*Sqrt[2*c + a*e^2 + 2*Sqrt[c]*Sqrt[c + a*e^2]]) + (c^2*ArcTan[(
Sqrt[2*c + a*e^2 + 2*Sqrt[c]*Sqrt[c + a*e^2]]*x)/(Sqrt[a]*(-1 + Sqrt[1 - e
^2*x^2]))])/((c + a*e^2)^2*Sqrt[2*c + a*e^2 + 2*Sqrt[c]*Sqrt[c + a*e^2]...
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {643, 316, 25, 402, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-ex)^{3/2}(ex+1)^{3/2}\left(a+cx^2\right)^2}dx$$

$$\Big\downarrow \text{643}$$

$$\int \frac{1}{\left(1-e^2x^2\right)^{3/2}\left(a+cx^2\right)^2}dx$$

$$\Big\downarrow \; 316$$

$$\frac{cx}{2a\sqrt{1-e^2x^2}\,(ae^2+c)\,(a+cx^2)}-\frac{\int -\frac{-2cx^2e^2+2ae^2+c}{(cx^2+a)(1-e^2x^2)^{3/2}}dx}{2a\,(ae^2+c)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-2cx^2e^2+2ae^2+c}{(cx^2+a)(1-e^2x^2)^{3/2}}dx}{2a\,(ae^2+c)}+\frac{cx}{2a\sqrt{1-e^2x^2}\,(ae^2+c)\,(a+cx^2)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\int \frac{c\left(4ae^2+c\right)}{\left(cx^2+a\right)\sqrt{1-e^2x^2}}dx}{ae^2+c}-\frac{e^2x(c-2ae^2)}{\sqrt{1-e^2x^2}(ae^2+c)}}{2a\,(ae^2+c)}+\frac{cx}{2a\sqrt{1-e^2x^2}\,(ae^2+c)\,(a+cx^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{c(4ae^2+c)\int \frac{1}{\left(cx^2+a\right)\sqrt{1-e^2x^2}}dx}{ae^2+c}-\frac{e^2x(c-2ae^2)}{\sqrt{1-e^2x^2}(ae^2+c)}}{2a\,(ae^2+c)}+\frac{cx}{2a\sqrt{1-e^2x^2}\,(ae^2+c)\,(a+cx^2)}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{c(4ae^2+c)\int \frac{1}{a-\frac{\left(-ae^2-c\right)x^2}{1-e^2x^2}}d\frac{x}{\sqrt{1-e^2x^2}}}{ae^2+c}-\frac{e^2x(c-2ae^2)}{\sqrt{1-e^2x^2}(ae^2+c)}}{2a\,(ae^2+c)}+\frac{cx}{2a\sqrt{1-e^2x^2}\,(ae^2+c)\,(a+cx^2)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{c(4ae^2+c)\arctan\left(\frac{x\sqrt{ae^2+c}}{\sqrt{a}\sqrt{1-e^2x^2}}\right)}{\sqrt{a}(ae^2+c)^{3/2}}-\frac{e^2x(c-2ae^2)}{\sqrt{1-e^2x^2}(ae^2+c)}}{2a\,(ae^2+c)}+\frac{cx}{2a\sqrt{1-e^2x^2}\,(ae^2+c)\,(a+cx^2)}$$

input
```
Int[1/((1 - e*x)^(3/2)*(1 + e*x)^(3/2)*(a + c*x^2)^2),x]
```

output
```
(c*x)/(2*a*(c + a*e^2)*(a + c*x^2)*Sqrt[1 - e^2*x^2]) + (-((e^2*(c - 2*a*e
^2)*x)/((c + a*e^2)*Sqrt[1 - e^2*x^2])) + (c*(c + 4*a*e^2)*ArcTan[(Sqrt[c
+ a*e^2]*x)/(Sqrt[a]*Sqrt[1 - e^2*x^2])])/(Sqrt[a]*(c + a*e^2)^(3/2)))/(2*
a*(c + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.86 (sec) , antiderivative size = 1742, normalized size of antiderivative = 11.93

method	result	size
default	Expression too large to display	1742

input
```
int(1/(-e*x+1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-4*ln(2*((-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+
c)/(c*x+(-a*c)^(1/2)))*a^2*c^2*e^6*x^4+4*ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2
+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x-(-a*c)^(1/2)))*a^2*c^2*e^6*x^4+4
*ln(2*((-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x
+(-a*c)^(1/2)))*a^3*c*e^4-4*ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*
(-e^2*x^2+1)^(1/2)*c+c)/(c*x-(-a*c)^(1/2)))*a^3*c*e^4+5*ln(2*((-a*c)^(1/2)
*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c)^(1/2)))*a^2
*c^2*e^2-5*ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2
)*c+c)/(c*x-(-a*c)^(1/2)))*a^2*c^2*e^2-5*ln(2*((-a*c)^(1/2)*e^2*x+((a*e^2+
c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c)^(1/2)))*a*c^3*e^4*x^4+4*ln
(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x-(
-a*c)^(1/2)))*a^3*c*e^6*x^2+5*ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2
)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x-(-a*c)^(1/2)))*a*c^3*e^4*x^4-ln(2*((-a*c)^(
1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c)^(1/2)))
*a^2*c^2*e^4*x^2+ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1
)^(1/2)*c+c)/(c*x-(-a*c)^(1/2)))*a^2*c^2*e^4*x^2+4*ln(2*((-a*c)^(1/2)*e^2*
x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c+c)/(c*x+(-a*c)^(1/2)))*a*c^3*e^
2*x^2-4*ln(2*(-(-a*c)^(1/2)*e^2*x+((a*e^2+c)/c)^(1/2)*(-e^2*x^2+1)^(1/2)*c
+c)/(c*x-(-a*c)^(1/2)))*a*c^3*e^2*x^2+4*a^2*c*e^6*x^3*(-a*c)^(1/2)*(-e^2*x
^2+1)^(1/2)*((a*e^2+c)/c)^(1/2)+2*a*c^2*e^4*x^3*(-a*c)^(1/2)*(-e^2*x^2+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(128) = 256$.

Time = 0.13 (sec) , antiderivative size = 695, normalized size of antiderivative = 4.76

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \left[-\frac{(4\,a^2ce^2-(4\,ac^2e^4+c^3e^2)x^4+ac^2-(4\,a^2ce^4-3\,ac^2e^2-c^3)}{4\,(a^6e^6+3\,a}\right.$$

input
```
integrate(1/(-e*x+1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a)^2,x, algorithm="fricas"
)
```

output
```
[-1/4*((4*a^2*c*e^2 - (4*a*c^2*e^4 + c^3*e^2)*x^4 + a*c^2 - (4*a^2*c*e^4 -
 3*a*c^2*e^2 - c^3)*x^2)*sqrt(-a^2*e^2 - a*c)*log(-((2*a*e^2 + c)*x^2 - 2*
sqrt(-a^2*e^2 - a*c)*sqrt(e*x + 1)*sqrt(-e*x + 1)*x - a)/(c*x^2 + a)) - 2*
((2*a^3*c*e^6 + a^2*c^2*e^4 - a*c^3*e^2)*x^3 + (2*a^4*e^6 + 2*a^3*c*e^4 +
a^2*c^2*e^2 + a*c^3)*x)*sqrt(e*x + 1)*sqrt(-e*x + 1))/(a^6*e^6 + 3*a^5*c*e
^4 + 3*a^4*c^2*e^2 + a^3*c^3 - (a^5*c*e^8 + 3*a^4*c^2*e^6 + 3*a^3*c^3*e^4
+ a^2*c^4*e^2)*x^4 - (a^6*e^8 + 2*a^5*c*e^6 - 2*a^3*c^3*e^2 - a^2*c^4)*x^2
), -1/2*((4*a^2*c*e^2 - (4*a*c^2*e^4 + c^3*e^2)*x^4 + a*c^2 - (4*a^2*c*e^4
 - 3*a*c^2*e^2 - c^3)*x^2)*sqrt(a^2*e^2 + a*c)*arctan(sqrt(a^2*e^2 + a*c)*
sqrt(e*x + 1)*sqrt(-e*x + 1)*x/(a*e^2*x^2 - a)) - ((2*a^3*c*e^6 + a^2*c^2*
e^4 - a*c^3*e^2)*x^3 + (2*a^4*e^6 + 2*a^3*c*e^4 + a^2*c^2*e^2 + a*c^3)*x)*
sqrt(e*x + 1)*sqrt(-e*x + 1))/(a^6*e^6 + 3*a^5*c*e^4 + 3*a^4*c^2*e^2 + a^3
*c^3 - (a^5*c*e^8 + 3*a^4*c^2*e^6 + 3*a^3*c^3*e^4 + a^2*c^4*e^2)*x^4 - (a^
6*e^8 + 2*a^5*c*e^6 - 2*a^3*c^3*e^2 - a^2*c^4)*x^2)]
```

Sympy [F]

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \int \frac{1}{\left(a+cx^2\right)^2\left(-ex+1\right)^{\frac{3}{2}}\left(ex+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-e*x+1)**(3/2)/(e*x+1)**(3/2)/(c*x**2+a)**2,x)
```

output
```
Integral(1/((a + c*x**2)**2*(-e*x + 1)**(3/2)*(e*x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx=\int \frac{1}{\left(cx^2+a\right)^2(ex+1)^{\frac{3}{2}}(-ex+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-e*x+1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a)^2,x, algorithm="maxima"
)
```

output
```
integrate(1/((c*x^2 + a)^2*(e*x + 1)^(3/2)*(-e*x + 1)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3493 vs. $2(128) = 256$.

Time = 18.48 (sec) , antiderivative size = 3493, normalized size of antiderivative = 23.92

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate(1/(-e*x+1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/4*e^3*((a^2*e^4*((sqrt(2) - sqrt(-e*x + 1))/sqrt(e*x + 1) - sqrt(e*x + 1
)/(sqrt(2) - sqrt(-e*x + 1))) + 2*a*c*e^2*((sqrt(2) - sqrt(-e*x + 1))/sqrt
(e*x + 1) - sqrt(e*x + 1)/(sqrt(2) - sqrt(-e*x + 1))) + c^2*((sqrt(2) - sq
rt(-e*x + 1))/sqrt(e*x + 1) - sqrt(e*x + 1)/(sqrt(2) - sqrt(-e*x + 1))))/(
a^4*e^8 + 4*a^3*c*e^6 + 6*a^2*c^2*e^4 + 4*a*c^3*e^2 + c^4) + 2*(2*(a^3*e^7
 + 2*a^2*c*e^5 + a*c^2*e^3)^2*(4*a^3*c^2*e^4 + 4*a^2*c^3*e^4 + 5*a^2*c^3*e
^2 + 5*a*c^4*e^2 + a*c^4 + c^5)*sqrt(a^2*e^2 + a*c)*sgn(a^4*e^8 + 3*a^3*c*
e^6 + 3*a^2*c^2*e^4 + a*c^3*e^2) - 2*(4*a^4*c*e^5 + 4*a^3*c^2*e^5 + 5*a^3*
c^2*e^3 + 5*a^2*c^3*e^3 + a^2*c^3*e + a*c^4*e)*(a^3*e^7 + 2*a^2*c*e^5 + a*
c^2*e^3)^2*sqrt(-a*c*e^2 - c^2) + (4*a^7*c*e^12 + 4*a^6*c^2*e^12 + 17*a^6*
c^2*e^10 + 17*a^5*c^3*e^10 + 28*a^5*c^3*e^8 + 28*a^4*c^4*e^8 + 22*a^4*c^4*
e^6 + 22*a^3*c^5*e^6 + 8*a^3*c^5*e^4 + 8*a^2*c^6*e^4 + a^2*c^6*e^2 + a*c^7
*e^2)*sqrt(-a*c*e^2 - c^2)*abs(a^3*e^7 + 2*a^2*c*e^5 + a*c^2*e^3)*sgn(a^4*
e^8 + 3*a^3*c*e^6 + 3*a^2*c^2*e^4 + a*c^3*e^2) + (4*a^7*c*e^13 + 4*a^6*c^2
*e^13 + 17*a^6*c^2*e^11 + 17*a^5*c^3*e^11 + 28*a^5*c^3*e^9 + 28*a^4*c^4*e^
9 + 22*a^4*c^4*e^7 + 22*a^3*c^5*e^7 + 8*a^3*c^5*e^5 + 8*a^2*c^6*e^5 + a^2*
c^6*e^3 + a*c^7*e^3)*sqrt(a^2*e^2 + a*c)*abs(a^3*e^7 + 2*a^2*c*e^5 + a*c^2
*e^3) + (4*a^9*c^2*e^18 + 4*a^8*c^3*e^18 + 17*a^8*c^3*e^16 + 17*a^7*c^4*e^
16 + 24*a^7*c^4*e^14 + 24*a^6*c^5*e^14 + 5*a^6*c^5*e^12 + 5*a^5*c^6*e^12 -
 20*a^5*c^6*e^10 - 20*a^4*c^7*e^10 - 21*a^4*c^7*e^8 - 21*a^3*c^8*e^8 - ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \int \frac{1}{\left(c\,x^2+a\right)^2\,\left(1-e\,x\right)^{3/2}\,\left(e\,x+1\right)^{3/2}}\,dx$$

input
```
int(1/((a + c*x^2)^2*(1 - e*x)^(3/2)*(e*x + 1)^(3/2)),x)
```

output
```
int(1/((a + c*x^2)^2*(1 - e*x)^(3/2)*(e*x + 1)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1-ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \int \frac{1}{\left(-ex+1\right)^{\frac{3}{2}}\left(ex+1\right)^{\frac{3}{2}}\left(c\,x^2+a\right)^2}dx$$

input
```
int(1/(-e*x+1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a)^2,x)
```

output
```
int(1/(-e*x+1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a)^2,x)
```

3.14 $\int \frac{(a+cx^2)^3}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 161

$$\int \frac{(a+cx^2)^3}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx = \frac{c(5c^2+18ace^2+24a^2e^4)\,x\sqrt{-1+ex}\sqrt{1+ex}}{16e^6} + \frac{c^2(5c+18ae^2)\,x^3\sqrt{-1+ex}\sqrt{1+ex}}{24e^4} + \frac{c^3x^5\sqrt{-1+ex}\sqrt{1+ex}}{6e^2} + \frac{(c+2ae^2)\,(5c^2+8ace^2+8a^2e^4)\,\mathrm{arccosh}(ex)}{16e^7}$$

output
```
1/16*c*(24*a^2*e^4+18*a*c*e^2+5*c^2)*x*(e*x-1)^(1/2)*(e*x+1)^(1/2)/e^6+1/2
4*c^2*(18*a*e^2+5*c)*x^3*(e*x-1)^(1/2)*(e*x+1)^(1/2)/e^4+1/6*c^3*x^5*(e*x-
1)^(1/2)*(e*x+1)^(1/2)/e^2+1/16*(2*a*e^2+c)*(8*a^2*e^4+8*a*c*e^2+5*c^2)*ar
ccosh(e*x)/e^7
```

Mathematica [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

$$\int \frac{(a+cx^2)^3}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx = \frac{cex\sqrt{-1+ex}\sqrt{1+ex}(72a^2e^4+18ace^2(3+2e^2x^2)+c^2(15+10e^2x^2+8e^4x^4))+6(5c^3+18ac^2e^2+24a}{48e^7}$$

input
```
Integrate[(a + c*x^2)^3/(Sqrt[-1 + e*x]*Sqrt[1 + e*x]),x]
```

output
```
(c*e*x*Sqrt[-1 + e*x]*Sqrt[1 + e*x]*(72*a^2*e^4 + 18*a*c*e^2*(3 + 2*e^2*x^
2) + c^2*(15 + 10*e^2*x^2 + 8*e^4*x^4)) + 6*(5*c^3 + 18*a*c^2*e^2 + 24*a^2
*c*e^4 + 16*a^3*e^6)*ArcTanh[Sqrt[(-1 + e*x)/(1 + e*x)]])/(48*e^7)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {648, 318, 403, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{\sqrt{ex-1}\sqrt{ex+1}}\,dx$$

$$\Big\downarrow \ 648$$

$$\frac{\sqrt{e^2x^2-1}\int \frac{(cx^2+a)^3}{\sqrt{e^2x^2-1}}dx}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \ 318$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\int \frac{(cx^2+a)(5c(2ae^2+c)x^2+a(6ae^2+c))}{\sqrt{e^2x^2-1}}dx}{6e^2}+\frac{cx\sqrt{e^2x^2-1}(a+cx^2)^2}{6e^2}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{\int \frac{c\left(44a^2e^4+44ace^2+15c^2\right)x^2+a\left(24a^2e^4+14ace^2+5c^2\right)}{\sqrt{e^2x^2-1}}dx}{4e^2}+\frac{5cx\sqrt{e^2x^2-1}\left(2ae^2+c\right)\left(a+cx^2\right)}{4e^2}}{6e^2}+\frac{cx\sqrt{e^2x^2-1}\left(a+cx^2\right)^2}{6e^2}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{\frac{3\left(2ae^2+c\right)\left(8a^2e^4+8ace^2+5c^2\right)\int \frac{1}{\sqrt{e^2x^2-1}}dx}{2e^2}+\frac{cx\sqrt{e^2x^2-1}\left(44a^2e^4+44ace^2+15c^2\right)}{2e^2}}{4e^2}+\frac{5cx\sqrt{e^2x^2-1}\left(2ae^2+c\right)\left(a+cx^2\right)}{4e^2}}{6e^2}+\frac{cx\sqrt{e^2x^2-1}}{6e}\right.}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{\frac{3\left(2ae^2+c\right)\left(8a^2e^4+8ace^2+5c^2\right)\int \frac{1}{1-\frac{e^2x^2}{e^2x^2-1}}d\frac{x}{\sqrt{e^2x^2-1}}}{2e^2}+\frac{cx\sqrt{e^2x^2-1}\left(44a^2e^4+44ace^2+15c^2\right)}{2e^2}}{4e^2}+\frac{5cx\sqrt{e^2x^2-1}\left(2ae^2+c\right)\left(a+cx^2\right)}{4e^2}}{6e^2}+\right.}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{\frac{3\left(2ae^2+c\right)\left(8a^2e^4+8ace^2+5c^2\right)\text{arctanh}\left(\frac{ex}{\sqrt{e^2x^2-1}}\right)}{2e^3}+\frac{cx\sqrt{e^2x^2-1}\left(44a^2e^4+44ace^2+15c^2\right)}{2e^2}}{4e^2}+\frac{5cx\sqrt{e^2x^2-1}\left(2ae^2+c\right)\left(a+cx^2\right)}{4e^2}}{6e^2}+\right.}{\sqrt{ex-1}\sqrt{ex+1}}$$

input
```
Int[(a + c*x^2)^3/(Sqrt[-1 + e*x]*Sqrt[1 + e*x]),x]
```

output
```
(Sqrt[-1 + e^2*x^2]*((c*x*(a + c*x^2)^2*Sqrt[-1 + e^2*x^2])/(6*e^2) + ((5*
c*(c + 2*a*e^2)*x*(a + c*x^2)*Sqrt[-1 + e^2*x^2])/(4*e^2) + ((c*(15*c^2 +
44*a*c*e^2 + 44*a^2*e^4)*x*Sqrt[-1 + e^2*x^2])/(2*e^2) + (3*(c + 2*a*e^2)*
(5*c^2 + 8*a*c*e^2 + 8*a^2*e^4)*ArcTanh[(e*x)/Sqrt[-1 + e^2*x^2]])/(2*e^3)
)/(4*e^2))/(6*e^2)))/(Sqrt[-1 + e*x]*Sqrt[1 + e*x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.04

method	result
risch	$\frac{cx(8c^2x^4e^4+36x^2ac\,e^4+72a^2e^4+10c^2e^2x^2+54ac\,e^2+15c^2)\sqrt{ex-1}\,\sqrt{ex+1}}{48e^6}+\frac{(16e^6a^3+24e^4a^2c+18c^2a\,e^2+5c^3)\ln\left(\frac{e^2x}{\sqrt{e^2}}+\sqrt{e^2x}\right.}{16e^6\sqrt{e^2}\,\sqrt{ex-1}\,\sqrt{ex+1}}$
default	$\frac{\sqrt{ex-1}\,\sqrt{ex+1}\,\left(8\,\mathrm{csgn}(e)c^3e^5x^5\sqrt{e^2x^2-1}+36\,\mathrm{csgn}(e)a\,c^2e^5x^3\sqrt{e^2x^2-1}+72\sqrt{e^2x^2-1}\,\mathrm{csgn}(e)e^5a^2cx+10\sqrt{e^2x^2-1}\,\mathrm{csgn}(e)e^3c^3x\right.}{}$

input
```
int((c*x^2+a)^3/(e*x-1)^(1/2)/(e*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*c*x*(8*c^2*e^4*x^4+36*a*c*e^4*x^2+72*a^2*e^4+10*c^2*e^2*x^2+54*a*c*e^
2+15*c^2)*(e*x-1)^(1/2)*(e*x+1)^(1/2)/e^6+1/16*(16*a^3*e^6+24*a^2*c*e^4+18
*a*c^2*e^2+5*c^3)/e^6*ln(e^2*x/(e^2)^(1/2)+(e^2*x^2-1)^(1/2))/(e^2)^(1/2)*
((e*x-1)*(e*x+1))^(1/2)/(e*x-1)^(1/2)/(e*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

$$\int \frac{(a+cx^2)^3}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx = \frac{(8\,c^3e^5x^5+2\,(18\,ac^2e^5+5\,c^3e^3)x^3+3\,(24\,a^2ce^5+18\,ac^2e^3+5\,c^3e)x)\sqrt{ex+1}\sqrt{ex-1}-3\,(16\,a^3e^6+24}{48\,e^7}$$

input
```
integrate((c*x^2+a)^3/(e*x-1)^(1/2)/(e*x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/48*((8*c^3*e^5*x^5 + 2*(18*a*c^2*e^5 + 5*c^3*e^3)*x^3 + 3*(24*a^2*c*e^5
+ 18*a*c^2*e^3 + 5*c^3*e)*x)*sqrt(e*x + 1)*sqrt(e*x - 1) - 3*(16*a^3*e^6 +
 24*a^2*c*e^4 + 18*a*c^2*e^2 + 5*c^3)*log(-e*x + sqrt(e*x + 1)*sqrt(e*x -
1)))/e^7
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^3}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**3/(e*x-1)**(1/2)/(e*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(141) = 282$.

Time = 0.04 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.76

$$\begin{aligned}\int \frac{(a+cx^2)^3}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx &= \frac{\sqrt{e^2x^2-1}c^3x^5}{6\,e^2} + \frac{3\sqrt{e^2x^2-1}ac^2x^3}{4\,e^2} \\ &+ \frac{a^3\log\left(2\,e^2x+2\sqrt{e^2x^2-1}\sqrt{e^2}\right)}{\sqrt{e^2}} + \frac{3\sqrt{e^2x^2-1}a^2cx}{2\,e^2} \\ &+ \frac{5\sqrt{e^2x^2-1}c^3x^3}{24\,e^4} + \frac{3\,a^2c\log\left(2\,e^2x+2\sqrt{e^2x^2-1}\sqrt{e^2}\right)}{2\sqrt{e^2}e^2} \\ &+ \frac{9\sqrt{e^2x^2-1}ac^2x}{8\,e^4} + \frac{9\,ac^2\log\left(2\,e^2x+2\sqrt{e^2x^2-1}\sqrt{e^2}\right)}{8\sqrt{e^2}e^4} \\ &+ \frac{5\sqrt{e^2x^2-1}c^3x}{16\,e^6} + \frac{5\,c^3\log\left(2\,e^2x+2\sqrt{e^2x^2-1}\sqrt{e^2}\right)}{16\sqrt{e^2}e^6}\end{aligned}$$

input
```
integrate((c*x^2+a)^3/(e*x-1)^(1/2)/(e*x+1)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(e^2*x^2 - 1)*c^3*x^5/e^2 + 3/4*sqrt(e^2*x^2 - 1)*a*c^2*x^3/e^2 +
a^3*log(2*e^2*x + 2*sqrt(e^2*x^2 - 1)*sqrt(e^2))/sqrt(e^2) + 3/2*sqrt(e^2*
x^2 - 1)*a^2*c*x/e^2 + 5/24*sqrt(e^2*x^2 - 1)*c^3*x^3/e^4 + 3/2*a^2*c*log(
2*e^2*x + 2*sqrt(e^2*x^2 - 1)*sqrt(e^2))/(sqrt(e^2)*e^2) + 9/8*sqrt(e^2*x^
2 - 1)*a*c^2*x/e^4 + 9/8*a*c^2*log(2*e^2*x + 2*sqrt(e^2*x^2 - 1)*sqrt(e^2)
)/(sqrt(e^2)*e^4) + 5/16*sqrt(e^2*x^2 - 1)*c^3*x/e^6 + 5/16*c^3*log(2*e^2*
x + 2*sqrt(e^2*x^2 - 1)*sqrt(e^2))/(sqrt(e^2)*e^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.47

$$\int \frac{(a+cx^2)^3}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx$$
$$= \frac{\left(\left(2\left((ex+1)\left(4\,(ex+1)\left(\frac{(ex+1)c^3}{e^6}-\frac{5\,c^3}{e^6}\right)+\frac{9\,(2\,ac^2e^{38}+5\,c^3e^{36})}{e^{42}}\right)-\frac{54\,ac^2e^{38}+55\,c^3e^{36}}{e^{42}}\right)(ex+1)+\frac{72\,a^2ce^{40}+162\,}{e}\right.}{}$$

input
```
integrate((c*x^2+a)^3/(e*x-1)^(1/2)/(e*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/48*(((2*((e*x + 1)*(4*(e*x + 1)*((e*x + 1)*c^3/e^6 - 5*c^3/e^6) + 9*(2*a
*c^2*e^38 + 5*c^3*e^36)/e^42) - (54*a*c^2*e^38 + 55*c^3*e^36)/e^42)*(e*x +
 1) + (72*a^2*c*e^40 + 162*a*c^2*e^38 + 85*c^3*e^36)/e^42)*(e*x + 1) - 3*(
24*a^2*c*e^40 + 30*a*c^2*e^38 + 11*c^3*e^36)/e^42)*sqrt(e*x + 1)*sqrt(e*x
- 1) - 6*(16*a^3*e^6 + 24*a^2*c*e^4 + 18*a*c^2*e^2 + 5*c^3)*log(sqrt(e*x +
 1) - sqrt(e*x - 1))/e^6)/e
```

Mupad [B] (verification not implemented)

Time = 17.09 (sec) , antiderivative size = 980, normalized size of antiderivative = 6.09

$$\int \frac{(a+cx^2)^3}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx = \text{Too large to display}$$

input
```
int((a + c*x^2)^3/((e*x - 1)^(1/2)*(e*x + 1)^(1/2)),x)
```

output
```
(atanh(((e*x - 1)^(1/2) - 1i)/((e*x + 1)^(1/2) - 1))*(c + 2*a*e^2)*(5*c^2
+ 8*a^2*e^4 + 8*a*c*e^2))/(4*e^7) - ((((e*x - 1)^(1/2) - 1i)*((5*c^3)/4 +
(9*a*c^2*e^2)/2 + 6*a^2*c*e^4))/((e*x + 1)^(1/2) - 1) + (((e*x - 1)^(1/2)
- 1i)^23*((5*c^3)/4 + (9*a*c^2*e^2)/2 + 6*a^2*c*e^4))/((e*x + 1)^(1/2) - 1
)^23 - (((e*x - 1)^(1/2) - 1i)^3*((175*c^3)/12 + (105*a*c^2*e^2)/2 + 6*a^2
*c*e^4))/((e*x + 1)^(1/2) - 1)^3 - (((e*x - 1)^(1/2) - 1i)^21*((175*c^3)/1
2 + (105*a*c^2*e^2)/2 + 6*a^2*c*e^4))/((e*x + 1)^(1/2) - 1)^21 - (((e*x -
1)^(1/2) - 1i)^5*((669*a*c^2*e^2)/2 - (311*c^3)/4 + 126*a^2*c*e^4))/((e*x
+ 1)^(1/2) - 1)^5 - (((e*x - 1)^(1/2) - 1i)^19*((669*a*c^2*e^2)/2 - (311*c
^3)/4 + 126*a^2*c*e^4))/((e*x + 1)^(1/2) - 1)^19 + (((e*x - 1)^(1/2) - 1i)
^7*((8361*c^3)/4 + (1533*a*c^2*e^2)/2 + 510*a^2*c*e^4))/((e*x + 1)^(1/2) -
 1)^7 + (((e*x - 1)^(1/2) - 1i)^17*((8361*c^3)/4 + (1533*a*c^2*e^2)/2 + 51
0*a^2*c*e^4))/((e*x + 1)^(1/2) - 1)^17 + (((e*x - 1)^(1/2) - 1i)^11*((2529
5*c^3)/2 - 549*a*c^2*e^2 + 420*a^2*c*e^4))/((e*x + 1)^(1/2) - 1)^11 + (((e
*x - 1)^(1/2) - 1i)^13*((25295*c^3)/2 - 549*a*c^2*e^2 + 420*a^2*c*e^4))/((
e*x + 1)^(1/2) - 1)^13 + (((e*x - 1)^(1/2) - 1i)^9*((42259*c^3)/6 + 165*a*
c^2*e^2 - 804*a^2*c*e^4))/((e*x + 1)^(1/2) - 1)^9 + (((e*x - 1)^(1/2) - 1i
)^15*((42259*c^3)/6 + 165*a*c^2*e^2 - 804*a^2*c*e^4))/((e*x + 1)^(1/2) - 1
)^15)/(e^7 - (12*e^7*((e*x - 1)^(1/2) - 1i)^2)/((e*x + 1)^(1/2) - 1)^2 + (
66*e^7*((e*x - 1)^(1/2) - 1i)^4)/((e*x + 1)^(1/2) - 1)^4 - (220*e^7*((e...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.53

$$\int \frac{(a+cx^2)^3}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx$$
$$= \frac{72\sqrt{ex+1}\,\sqrt{ex-1}\,a^2c\,e^5x+36\sqrt{ex+1}\,\sqrt{ex-1}\,a\,c^2e^5x^3+54\sqrt{ex+1}\,\sqrt{ex-1}\,a\,c^2e^3x+8\sqrt{ex+1}\,\sqrt{}}{}$$

input
```
int((c*x^2+a)^3/(e*x-1)^(1/2)/(e*x+1)^(1/2),x)
```

output
```
(72*sqrt(e*x + 1)*sqrt(e*x - 1)*a**2*c*e**5*x + 36*sqrt(e*x + 1)*sqrt(e*x
- 1)*a*c**2*e**5*x**3 + 54*sqrt(e*x + 1)*sqrt(e*x - 1)*a*c**2*e**3*x + 8*s
qrt(e*x + 1)*sqrt(e*x - 1)*c**3*e**5*x**5 + 10*sqrt(e*x + 1)*sqrt(e*x - 1)
*c**3*e**3*x**3 + 15*sqrt(e*x + 1)*sqrt(e*x - 1)*c**3*e*x + 96*log((sqrt(e
*x - 1) + sqrt(e*x + 1))/sqrt(2))*a**3*e**6 + 144*log((sqrt(e*x - 1) + sqr
t(e*x + 1))/sqrt(2))*a**2*c*e**4 + 108*log((sqrt(e*x - 1) + sqrt(e*x + 1))
/sqrt(2))*a*c**2*e**2 + 30*log((sqrt(e*x - 1) + sqrt(e*x + 1))/sqrt(2))*c*
*3)/(48*e**7)
```

3.15 $\int \frac{(a+cx^2)^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 101

$$\int \frac{(a+cx^2)^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx = \frac{c(3c+8ae^2)\,x\sqrt{-1+ex}\sqrt{1+ex}}{8e^4}+\frac{c^2x^3\sqrt{-1+ex}\sqrt{1+ex}}{4e^2} + \frac{(3c^2+8ace^2+8a^2e^4)\operatorname{arccosh}(ex)}{8e^5}$$

output
```
1/8*c*(8*a*e^2+3*c)*x*(e*x-1)^(1/2)*(e*x+1)^(1/2)/e^4+1/4*c^2*x^3*(e*x-1)^
(1/2)*(e*x+1)^(1/2)/e^2+1/8*(8*a^2*e^4+8*a*c*e^2+3*c^2)*arccosh(e*x)/e^5
```

Mathematica [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

$$\int \frac{(a+cx^2)^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx = \frac{cex\sqrt{-1+ex}\sqrt{1+ex}(8ae^2+c(3+2e^2x^2))+2(3c^2+8ace^2+8a^2e^4)\operatorname{arctanh}\left(\sqrt{\frac{-1+ex}{1+ex}}\right)}{8e^5}$$

input
```
Integrate[(a + c*x^2)^2/(Sqrt[-1 + e*x]*Sqrt[1 + e*x]),x]
```

output
```
(c*e*x*Sqrt[-1 + e*x]*Sqrt[1 + e*x]*(8*a*e^2 + c*(3 + 2*e^2*x^2)) + 2*(3*c
^2 + 8*a*c*e^2 + 8*a^2*e^4)*ArcTanh[Sqrt[(-1 + e*x)/(1 + e*x)]])/(8*e^5)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {648, 318, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{\sqrt{ex-1}\sqrt{ex+1}}dx$$

$$\Big\downarrow \text{ 648}$$

$$\frac{\sqrt{e^2x^2-1}\int \frac{(cx^2+a)^2}{\sqrt{e^2x^2-1}}dx}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{ 318}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\int \frac{3c(2ae^2+c)x^2+a(4ae^2+c)}{\sqrt{e^2x^2-1}}dx}{4e^2}+\frac{cx\sqrt{e^2x^2-1}(a+cx^2)}{4e^2}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{(8a^2e^4+8ace^2+3c^2)\int \frac{1}{\sqrt{e^2x^2-1}}dx}{2e^2}+\frac{3cx\sqrt{e^2x^2-1}(2ae^2+c)}{2e^2}}{4e^2}+\frac{cx\sqrt{e^2x^2-1}(a+cx^2)}{4e^2}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{(8a^2e^4+8ace^2+3c^2)\int \frac{1}{1-\frac{e^2x^2}{e^2x^2-1}}d\frac{x}{\sqrt{e^2x^2-1}}}{2e^2}+\frac{3cx\sqrt{e^2x^2-1}(2ae^2+c)}{2e^2}}{4e^2}+\frac{cx\sqrt{e^2x^2-1}(a+cx^2)}{4e^2}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{(8a^2e^4+8ace^2+3c^2)\operatorname{arctanh}\left(\frac{ex}{\sqrt{e^2x^2-1}}\right)}{2e^3}+\frac{3cx\sqrt{e^2x^2-1}(2ae^2+c)}{2e^2}}{4e^2}+\frac{cx\sqrt{e^2x^2-1}(a+cx^2)}{4e^2}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

input
```
Int[(a + c*x^2)^2/(Sqrt[-1 + e*x]*Sqrt[1 + e*x]),x]
```

output
```
(Sqrt[-1 + e^2*x^2]*((c*x*(a + c*x^2)*Sqrt[-1 + e^2*x^2])/(4*e^2) + ((3*c*
(c + 2*a*e^2)*x*Sqrt[-1 + e^2*x^2])/(2*e^2) + ((3*c^2 + 8*a*c*e^2 + 8*a^2*
e^4)*ArcTanh[(e*x)/Sqrt[-1 + e^2*x^2]])/(2*e^3))/(4*e^2)))/(Sqrt[-1 + e*x]
*Sqrt[1 + e*x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.22

method	result
risch	$\frac{cx(2x^2ce^2+8ae^2+3c)\sqrt{ex-1}\,\sqrt{ex+1}}{8e^4}+\frac{(8a^2e^4+8ace^2+3c^2)\ln\left(\frac{e^2x}{\sqrt{e^2}}+\sqrt{e^2x^2-1}\right)\sqrt{(ex-1)(ex+1)}}{8e^4\sqrt{e^2}\,\sqrt{ex-1}\,\sqrt{ex+1}}$
default	$\frac{\sqrt{ex-1}\,\sqrt{ex+1}\left(2\,\mathrm{csgn}(e)c^2e^3x^3\sqrt{e^2x^2-1}+8\,\mathrm{csgn}(e)e^3\sqrt{e^2x^2-1}\,acx+8\ln\left(\left(\sqrt{e^2x^2-1}\,\mathrm{csgn}(e)+ex\right)\mathrm{csgn}(e)\right)a^2e^4+3\,\mathrm{csgn}(e)e\sqrt{}\right.}{8e^5\sqrt{e^2x^2-1}}$

input
```
int((c*x^2+a)^2/(e*x-1)^(1/2)/(e*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*c*x*(2*c*e^2*x^2+8*a*e^2+3*c)*(e*x-1)^(1/2)*(e*x+1)^(1/2)/e^4+1/8*(8*a
^2*e^4+8*a*c*e^2+3*c^2)/e^4*ln(e^2*x/(e^2)^(1/2)+(e^2*x^2-1)^(1/2))/(e^2)^
(1/2)*((e*x-1)*(e*x+1))^(1/2)/(e*x-1)^(1/2)/(e*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{(a+cx^2)^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx$$
$$=\frac{(2\,c^2e^3x^3+(8\,ace^3+3\,c^2e)x)\sqrt{ex+1}\sqrt{ex-1}-(8\,a^2e^4+8\,ace^2+3\,c^2)\log\left(-ex+\sqrt{ex+1}\sqrt{ex-1}\right)}{8\,e^5}$$

input
```
integrate((c*x^2+a)^2/(e*x-1)^(1/2)/(e*x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*((2*c^2*e^3*x^3 + (8*a*c*e^3 + 3*c^2*e)*x)*sqrt(e*x + 1)*sqrt(e*x - 1)
 - (8*a^2*e^4 + 8*a*c*e^2 + 3*c^2)*log(-e*x + sqrt(e*x + 1)*sqrt(e*x - 1))
)/e^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**2/(e*x-1)**(1/2)/(e*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.70

$$\begin{aligned}\int \frac{(a+cx^2)^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx &= \frac{\sqrt{e^2x^2-1}c^2x^3}{4\,e^2} + \frac{a^2\log\left(2\,e^2x+2\,\sqrt{e^2x^2-1}\sqrt{e^2}\right)}{\sqrt{e^2}} \\ &\quad + \frac{\sqrt{e^2x^2-1}acx}{e^2} + \frac{ac\log\left(2\,e^2x+2\,\sqrt{e^2x^2-1}\sqrt{e^2}\right)}{\sqrt{e^2}e^2} \\ &\quad + \frac{3\,\sqrt{e^2x^2-1}c^2x}{8\,e^4} + \frac{3\,c^2\log\left(2\,e^2x+2\,\sqrt{e^2x^2-1}\sqrt{e^2}\right)}{8\,\sqrt{e^2}e^4}\end{aligned}$$

input
```
integrate((c*x^2+a)^2/(e*x-1)^(1/2)/(e*x+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(e^2*x^2 - 1)*c^2*x^3/e^2 + a^2*log(2*e^2*x + 2*sqrt(e^2*x^2 - 1)*
sqrt(e^2))/sqrt(e^2) + sqrt(e^2*x^2 - 1)*a*c*x/e^2 + a*c*log(2*e^2*x + 2*s
qrt(e^2*x^2 - 1)*sqrt(e^2))/(sqrt(e^2)*e^2) + 3/8*sqrt(e^2*x^2 - 1)*c^2*x/
e^4 + 3/8*c^2*log(2*e^2*x + 2*sqrt(e^2*x^2 - 1)*sqrt(e^2))/(sqrt(e^2)*e^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.41

$$\int \frac{(a+cx^2)^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx$$
$$= \frac{\left((ex+1)\left(2\,(ex+1)\left(\frac{(ex+1)c^2}{e^4}-\frac{3\,c^2}{e^4}\right)+\frac{8\,ace^{18}+9\,c^2e^{16}}{e^{20}}\right)-\frac{8\,ace^{18}+5\,c^2e^{16}}{e^{20}}\right)\sqrt{ex+1}\sqrt{ex-1}-\frac{2\,(8\,a^2e^4+8\,ace^2+}{}}{8\,e}$$

input
```
integrate((c*x^2+a)^2/(e*x-1)^(1/2)/(e*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/8*(((e*x + 1)*(2*(e*x + 1)*((e*x + 1)*c^2/e^4 - 3*c^2/e^4) + (8*a*c*e^18
 + 9*c^2*e^16)/e^20) - (8*a*c*e^18 + 5*c^2*e^16)/e^20)*sqrt(e*x + 1)*sqrt(
e*x - 1) - 2*(8*a^2*e^4 + 8*a*c*e^2 + 3*c^2)*log(sqrt(e*x + 1) - sqrt(e*x
- 1))/e^4)/e
```

Mupad [B] (verification not implemented)

Time = 18.76 (sec) , antiderivative size = 577, normalized size of antiderivative = 5.71

$$\int \frac{(a+cx^2)^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx$$
$$= \frac{-\frac{(\sqrt{e\,x-1}-\mathrm{i})^{15}\left(\frac{3\,c^2}{2}+4\,a\,c\,e^2\right)}{(\sqrt{e\,x+1}-1)^{15}}+\frac{(\sqrt{e\,x-1}-\mathrm{i})^{3}\left(\frac{23\,c^2}{2}-12\,a\,c\,e^2\right)}{(\sqrt{e\,x+1}-1)^{3}}+\frac{(\sqrt{e\,x-1}-\mathrm{i})^{13}\left(\frac{23\,c^2}{2}-12\,a\,c\,e^2\right)}{(\sqrt{e\,x+1}-1)^{13}}+\frac{(\sqrt{e\,x-1}-\mathrm{i})^{5}\left(\frac{333\,c^2}{2}+60\,a\,c\,e^{}\right)}{(\sqrt{e\,x+1}-1)^{5}}}{e^5-\frac{8\,e^5\,(\sqrt{e\,x-1}-\mathrm{i})^2}{(\sqrt{e\,x+1}-1)^2}+\frac{28\,e^5\,(\sqrt{e\,x-1}-\mathrm{i})^4}{(\sqrt{e\,x+1}-1)^4}-\frac{56\,e^5\,(\sqrt{e\,x-1}-\mathrm{i})^6}{(\sqrt{e\,x+1}-1)^6}+\frac{70\,e^5\,(\sqrt{e\,x-1}-}{(\sqrt{e\,x+1}-1)}}$$
$$+\frac{\mathrm{atanh}\left(\frac{\sqrt{e\,x-1}-\mathrm{i}}{\sqrt{e\,x+1}-1}\right)\,(8\,a^2\,e^4+8\,a\,c\,e^2+3\,c^2)}{2\,e^5}$$

input
```
int((a + c*x^2)^2/((e*x - 1)^(1/2)*(e*x + 1)^(1/2)),x)
```

output
```
((((e*x - 1)^(1/2) - 1i)^3*((23*c^2)/2 - 12*a*c*e^2))/((e*x + 1)^(1/2) - 1
)^3 - (((e*x - 1)^(1/2) - 1i)^15*((3*c^2)/2 + 4*a*c*e^2))/((e*x + 1)^(1/2)
 - 1)^15 + (((e*x - 1)^(1/2) - 1i)^13*((23*c^2)/2 - 12*a*c*e^2))/((e*x + 1
)^(1/2) - 1)^13 + (((e*x - 1)^(1/2) - 1i)^5*((333*c^2)/2 + 60*a*c*e^2))/((
e*x + 1)^(1/2) - 1)^5 + (((e*x - 1)^(1/2) - 1i)^11*((333*c^2)/2 + 60*a*c*e
^2))/((e*x + 1)^(1/2) - 1)^11 + (((e*x - 1)^(1/2) - 1i)^7*((671*c^2)/2 - 4
4*a*c*e^2))/((e*x + 1)^(1/2) - 1)^7 + (((e*x - 1)^(1/2) - 1i)^9*((671*c^2)
/2 - 44*a*c*e^2))/((e*x + 1)^(1/2) - 1)^9 - (((e*x - 1)^(1/2) - 1i)*((3*c^
2)/2 + 4*a*c*e^2))/((e*x + 1)^(1/2) - 1))/(e^5 - (8*e^5*((e*x - 1)^(1/2) -
 1i)^2)/((e*x + 1)^(1/2) - 1)^2 + (28*e^5*((e*x - 1)^(1/2) - 1i)^4)/((e*x
+ 1)^(1/2) - 1)^4 - (56*e^5*((e*x - 1)^(1/2) - 1i)^6)/((e*x + 1)^(1/2) - 1
)^6 + (70*e^5*((e*x - 1)^(1/2) - 1i)^8)/((e*x + 1)^(1/2) - 1)^8 - (56*e^5*
((e*x - 1)^(1/2) - 1i)^10)/((e*x + 1)^(1/2) - 1)^10 + (28*e^5*((e*x - 1)^(
1/2) - 1i)^12)/((e*x + 1)^(1/2) - 1)^12 - (8*e^5*((e*x - 1)^(1/2) - 1i)^14
)/((e*x + 1)^(1/2) - 1)^14 + (e^5*((e*x - 1)^(1/2) - 1i)^16)/((e*x + 1)^(1
/2) - 1)^16) + (atanh(((e*x - 1)^(1/2) - 1i)/((e*x + 1)^(1/2) - 1))*(3*c^2
 + 8*a^2*e^4 + 8*a*c*e^2))/(2*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.44

$$\int \frac{(a+cx^2)^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx = \frac{8\sqrt{ex+1}\,\sqrt{ex-1}\,ace^3x+2\sqrt{ex+1}\,\sqrt{ex-1}\,c^2e^3x^3+3\sqrt{ex+1}\,\sqrt{ex-1}\,c^2ex+16\log\left(\frac{\sqrt{ex-1}+\sqrt{ex+1}}{\sqrt{2}}\right)}{8e^5}$$

input
```
int((c*x^2+a)^2/(e*x-1)^(1/2)/(e*x+1)^(1/2),x)
```

output
```
(8*sqrt(e*x + 1)*sqrt(e*x - 1)*a*c*e**3*x + 2*sqrt(e*x + 1)*sqrt(e*x - 1)*
c**2*e**3*x**3 + 3*sqrt(e*x + 1)*sqrt(e*x - 1)*c**2*e*x + 16*log((sqrt(e*x
 - 1) + sqrt(e*x + 1))/sqrt(2))*a**2*e**4 + 16*log((sqrt(e*x - 1) + sqrt(e
*x + 1))/sqrt(2))*a*c*e**2 + 6*log((sqrt(e*x - 1) + sqrt(e*x + 1))/sqrt(2)
)*c**2)/(8*e**5)
```

3.16 $\int \frac{a+cx^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 47

$$\int \frac{a+cx^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx=\frac{cx\sqrt{-1+ex}\sqrt{1+ex}}{2e^2}+\frac{(c+2ae^2)\,\text{arccosh}(ex)}{2e^3}$$

output
```
1/2*c*x*(e*x-1)^(1/2)*(e*x+1)^(1/2)/e^2+1/2*(2*a*e^2+c)*arccosh(e*x)/e^3
```

Mathematica [A] (warning: unable to verify)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{a+cx^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx=\frac{cex\sqrt{-1+ex}\sqrt{1+ex}+2(c+2ae^2)\,\text{arctanh}\left(\sqrt{\frac{-1+ex}{1+ex}}\right)}{2e^3}$$

input
```
Integrate[(a + c*x^2)/(Sqrt[-1 + e*x]*Sqrt[1 + e*x]),x]
```

output
```
(c*e*x*Sqrt[-1 + e*x]*Sqrt[1 + e*x] + 2*(c + 2*a*e^2)*ArcTanh[Sqrt[(-1 + e
*x)/(1 + e*x)]])/(2*e^3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {646, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{\sqrt{ex-1}\sqrt{ex+1}}dx$$

$$\Big\downarrow \text{ 646}$$

$$\frac{(2ae^2+c)\int \frac{1}{\sqrt{ex-1}\sqrt{ex+1}}dx}{2e^2}+\frac{cx\sqrt{ex-1}\sqrt{ex+1}}{2e^2}$$

$$\Big\downarrow \text{ 43}$$

$$\frac{(2ae^2+c)\,\text{arccosh}(ex)}{2e^3}+\frac{cx\sqrt{ex-1}\sqrt{ex+1}}{2e^2}$$

input
```
Int[(a + c*x^2)/(Sqrt[-1 + e*x]*Sqrt[1 + e*x]),x]
```

output
```
(c*x*Sqrt[-1 + e*x]*Sqrt[1 + e*x])/(2*e^2) + ((c + 2*a*e^2)*ArcCosh[e*x])/
(2*e^3)
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 646
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[b*x*(c + d*x)^(m + 1)*((e + f*x)^(n + 1)/(d*f*(2*m +
 3))), x] - Simp[(b*c*e - a*d*f*(2*m + 3))/(d*f*(2*m + 3))   Int[(c + d*x)^
m*(e + f*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m, n] &&
 EqQ[d*e + c*f, 0] &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(39) = 78.

Time = 0.71 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.94

method	result
risch	$\frac{cx\sqrt{ex-1}\,\sqrt{ex+1}}{2e^2}+\frac{(2a\,e^2+c)\ln\left(\frac{e^2x}{\sqrt{e^2}}+\sqrt{e^2x^2-1}\right)\sqrt{(ex-1)(ex+1)}}{2e^2\sqrt{e^2}\,\sqrt{ex-1}\,\sqrt{ex+1}}$
default	$\frac{\sqrt{ex-1}\,\sqrt{ex+1}\,\left(\operatorname{csgn}(e)e\sqrt{e^2x^2-1}\,cx+2\ln\left(\left(\sqrt{e^2x^2-1}\,\operatorname{csgn}(e)+ex\right)\operatorname{csgn}(e)\right)a\,e^2+\ln\left(\left(\sqrt{e^2x^2-1}\,\operatorname{csgn}(e)+ex\right)\operatorname{csgn}(e)\right)c\right)\operatorname{csgn}(}{2e^3\sqrt{e^2x^2-1}}$

input
```
int((c*x^2+a)/(e*x-1)^(1/2)/(e*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*c*x*(e*x-1)^(1/2)*(e*x+1)^(1/2)/e^2+1/2*(2*a*e^2+c)/e^2*ln(e^2*x/(e^2)
^(1/2)+(e^2*x^2-1)^(1/2))/(e^2)^(1/2)*((e*x-1)*(e*x+1))^(1/2)/(e*x-1)^(1/2
)/(e*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.17

$$\int \frac{a+cx^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx = \frac{\sqrt{ex+1}\sqrt{ex-1}cex-(2\,ae^2+c)\log\left(-ex+\sqrt{ex+1}\sqrt{ex-1}\right)}{2\,e^3}$$

input
```
integrate((c*x^2+a)/(e*x-1)^(1/2)/(e*x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(e*x + 1)*sqrt(e*x - 1)*c*e*x - (2*a*e^2 + c)*log(-e*x + sqrt(e*x
 + 1)*sqrt(e*x - 1)))/e^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+cx^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)/(e*x-1)**(1/2)/(e*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(39) = 78$.

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.89

$$\int \frac{a+cx^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx = \frac{a\log\left(2\,e^2x+2\,\sqrt{e^2x^2-1}\sqrt{e^2}\right)}{\sqrt{e^2}} + \frac{\sqrt{e^2x^2-1}cx}{2\,e^2} + \frac{c\log\left(2\,e^2x+2\,\sqrt{e^2x^2-1}\sqrt{e^2}\right)}{2\,\sqrt{e^2}e^2}$$

input
```
integrate((c*x^2+a)/(e*x-1)^(1/2)/(e*x+1)^(1/2),x, algorithm="maxima")
```

output
```
a*log(2*e^2*x + 2*sqrt(e^2*x^2 - 1)*sqrt(e^2))/sqrt(e^2) + 1/2*sqrt(e^2*x^
2 - 1)*c*x/e^2 + 1/2*c*log(2*e^2*x + 2*sqrt(e^2*x^2 - 1)*sqrt(e^2))/(sqrt(
e^2)*e^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.47

$$\int \frac{a+cx^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx = \frac{\sqrt{ex+1}\sqrt{ex-1}\left(\frac{(ex+1)c}{e^2}-\frac{c}{e^2}\right)-\frac{2\,(2\,ae^2+c)\log\left(\sqrt{ex+1}-\sqrt{ex-1}\right)}{e^2}}{2\,e}$$

input
```
integrate((c*x^2+a)/(e*x-1)^(1/2)/(e*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(e*x + 1)*sqrt(e*x - 1)*((e*x + 1)*c/e^2 - c/e^2) - 2*(2*a*e^2 +
c)*log(sqrt(e*x + 1) - sqrt(e*x - 1))/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 12.97 (sec) , antiderivative size = 293, normalized size of antiderivative = 6.23

$$\int \frac{a+cx^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx$$
$$= -\frac{\frac{14\,c\,\left(\sqrt{e\,x-1}-\mathrm{i}\right)^3}{\left(\sqrt{e\,x+1}-1\right)^3}+\frac{14\,c\,\left(\sqrt{e\,x-1}-\mathrm{i}\right)^5}{\left(\sqrt{e\,x+1}-1\right)^5}+\frac{2\,c\,\left(\sqrt{e\,x-1}-\mathrm{i}\right)^7}{\left(\sqrt{e\,x+1}-1\right)^7}+\frac{2\,c\,\left(\sqrt{e\,x-1}-\mathrm{i}\right)}{\sqrt{e\,x+1}-1}}{e^3-\frac{4\,e^3\,\left(\sqrt{e\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{e\,x+1}-1\right)^2}+\frac{6\,e^3\,\left(\sqrt{e\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{e\,x+1}-1\right)^4}-\frac{4\,e^3\,\left(\sqrt{e\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{e\,x+1}-1\right)^6}+\frac{e^3\,\left(\sqrt{e\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{e\,x+1}-1\right)^8}}$$
$$-\frac{4\,a\,\mathrm{atan}\left(\frac{e\,\left(\sqrt{e\,x-1}-\mathrm{i}\right)}{\left(\sqrt{e\,x+1}-1\right)\,\sqrt{-e^2}}\right)}{\sqrt{-e^2}}+\frac{2\,c\,\mathrm{atanh}\left(\frac{\sqrt{e\,x-1}-\mathrm{i}}{\sqrt{e\,x+1}-1}\right)}{e^3}$$

input
```
int((a + c*x^2)/((e*x - 1)^(1/2)*(e*x + 1)^(1/2)),x)
```

output
```
(2*c*atanh(((e*x - 1)^(1/2) - 1i)/((e*x + 1)^(1/2) - 1)))/e^3 - (4*a*atan(
(e*((e*x - 1)^(1/2) - 1i))/(((e*x + 1)^(1/2) - 1)*(-e^2)^(1/2))))/(-e^2)^(
1/2) - ((14*c*((e*x - 1)^(1/2) - 1i)^3)/((e*x + 1)^(1/2) - 1)^3 + (14*c*((
e*x - 1)^(1/2) - 1i)^5)/((e*x + 1)^(1/2) - 1)^5 + (2*c*((e*x - 1)^(1/2) -
1i)^7)/((e*x + 1)^(1/2) - 1)^7 + (2*c*((e*x - 1)^(1/2) - 1i))/((e*x + 1)^(
1/2) - 1))/(e^3 - (4*e^3*((e*x - 1)^(1/2) - 1i)^2)/((e*x + 1)^(1/2) - 1)^2
 + (6*e^3*((e*x - 1)^(1/2) - 1i)^4)/((e*x + 1)^(1/2) - 1)^4 - (4*e^3*((e*x
 - 1)^(1/2) - 1i)^6)/((e*x + 1)^(1/2) - 1)^6 + (e^3*((e*x - 1)^(1/2) - 1i)
^8)/((e*x + 1)^(1/2) - 1)^8)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.47

$$\int \frac{a+cx^2}{\sqrt{-1+ex}\sqrt{1+ex}}\,dx$$
$$= \frac{\sqrt{ex+1}\,\sqrt{ex-1}\,cex+4\log\left(\frac{\sqrt{ex-1}+\sqrt{ex+1}}{\sqrt{2}}\right)a\,e^2+2\log\left(\frac{\sqrt{ex-1}+\sqrt{ex+1}}{\sqrt{2}}\right)c}{2e^3}$$

input
```
int((c*x^2+a)/(e*x-1)^(1/2)/(e*x+1)^(1/2),x)
```

output
```
(sqrt(e*x + 1)*sqrt(e*x - 1)*c*e*x + 4*log((sqrt(e*x - 1) + sqrt(e*x + 1))
/sqrt(2))*a*e**2 + 2*log((sqrt(e*x - 1) + sqrt(e*x + 1))/sqrt(2))*c)/(2*e*
*3)
```

3.17 $\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 80

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)}\,dx=\frac{\sqrt{-1+e^2x^2}\,\text{arctanh}\left(\frac{\sqrt{c+ae^2}x}{\sqrt{a}\sqrt{-1+e^2x^2}}\right)}{\sqrt{a}\sqrt{c+ae^2}\sqrt{-1+ex}\sqrt{1+ex}}$$

output
```
(e^2*x^2-1)^(1/2)*arctanh((a*e^2+c)^(1/2)*x/a^(1/2)/(e^2*x^2-1)^(1/2))/a^(
1/2)/(a*e^2+c)^(1/2)/(e*x-1)^(1/2)/(e*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)}\,dx=\frac{\text{arctanh}\left(\frac{\sqrt{c+ae^2}x}{\sqrt{a}\sqrt{-1+ex}\sqrt{1+ex}}\right)}{\sqrt{a}\sqrt{c+ae^2}}$$

input
```
Integrate[1/(Sqrt[-1 + e*x]*Sqrt[1 + e*x]*(a + c*x^2)),x]
```

output
```
ArcTanh[(Sqrt[c + a*e^2]*x)/(Sqrt[a]*Sqrt[-1 + e*x]*Sqrt[1 + e*x])]/(Sqrt[
a]*Sqrt[c + a*e^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {648, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ex-1}\sqrt{ex+1}\,(a+cx^2)}\,dx$$
$$\Big\downarrow \text{648}$$
$$\frac{\sqrt{e^2x^2-1}\int \frac{1}{(cx^2+a)\sqrt{e^2x^2-1}}dx}{\sqrt{ex-1}\sqrt{ex+1}}$$
$$\Big\downarrow \text{291}$$
$$\frac{\sqrt{e^2x^2-1}\int \frac{1}{a-\frac{(ae^2+c)x^2}{e^2x^2-1}}d\frac{x}{\sqrt{e^2x^2-1}}}{\sqrt{ex-1}\sqrt{ex+1}}$$
$$\Big\downarrow \text{221}$$
$$\frac{\sqrt{e^2x^2-1}\text{arctanh}\left(\frac{x\sqrt{ae^2+c}}{\sqrt{a}\sqrt{e^2x^2-1}}\right)}{\sqrt{a}\sqrt{ex-1}\sqrt{ex+1}\sqrt{ae^2+c}}$$

input
```
Int[1/(Sqrt[-1 + e*x]*Sqrt[1 + e*x]*(a + c*x^2)),x]
```

output
```
(Sqrt[-1 + e^2*x^2]*ArcTanh[(Sqrt[c + a*e^2]*x)/(Sqrt[a]*Sqrt[-1 + e^2*x^2
])])/(Sqrt[a]*Sqrt[c + a*e^2]*Sqrt[-1 + e*x]*Sqrt[1 + e*x])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 329, normalized size of antiderivative = 4.11

method	result
default	$-\frac{\sqrt{ex-1}\,\sqrt{ex+1}\,c\,\mathrm{csgn}(e)^2\left(\ln\left(\frac{2\sqrt{-ac}\,e^2x+2\sqrt{e^2x^2-1}\,\sqrt{-\frac{a\,e^2+c}{c}}\,c-2c}{cx-\sqrt{-ac}}\right)a\,e^2-\ln\left(\frac{-2\sqrt{-ac}\,e^2x+2\sqrt{e^2x^2-1}\,\sqrt{-\frac{a\,e^2+c}{c}}\,c-2c}{cx+\sqrt{-ac}}\right)a\,e^2+\right.}{2\sqrt{e^2x^2-1}\,(-e\sqrt{-ac}+c)\,(e\sqrt{-ac}+c)\,\sqrt{-ac}}$

input
```
int(1/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(e*x-1)^(1/2)*(e*x+1)^(1/2)*c*csgn(e)^2*(ln(2*((-a*c)^(1/2)*e^2*x+(e^
2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-a*c)^(1/2)))*a*e^2-ln(2*(-
(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x+(-a*c)
^(1/2)))*a*e^2+ln(2*((-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(
1/2)*c-c)/(c*x-(-a*c)^(1/2)))*c-ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2
)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*c)/(e^2*x^2-1)^(1/2)/(-e*(
-a*c)^(1/2)+c)/(e*(-a*c)^(1/2)+c)/(-a*c)^(1/2)/(-(a*e^2+c)/c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.21

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)}\,dx$$
$$= \left[\frac{\log\left(-\frac{2\,a^2e^2-(4\,a^2e^4+4\,ace^2+c^2)x^2-2\left(\sqrt{a^2e^2+ac}(2\,ae^2+c)x+2\,(a^2e^3+ace)x\right)\sqrt{ex+1}\sqrt{ex-1}+ac-2\,\sqrt{a^2e^2+ac}((2\,ae^3+ce)x^2-ae)}{cx^2+a}\right)}{2\,\sqrt{a^2e^2+ac}} \right.$$

input
```
integrate(1/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*log(-(2*a^2*e^2 - (4*a^2*e^4 + 4*a*c*e^2 + c^2)*x^2 - 2*(sqrt(a^2*e^2
 + a*c)*(2*a*e^2 + c)*x + 2*(a^2*e^3 + a*c*e)*x)*sqrt(e*x + 1)*sqrt(e*x -
1) + a*c - 2*sqrt(a^2*e^2 + a*c)*((2*a*e^3 + c*e)*x^2 - a*e))/(c*x^2 + a))
/sqrt(a^2*e^2 + a*c), sqrt(-a^2*e^2 - a*c)*arctan((sqrt(-a^2*e^2 - a*c)*sq
rt(e*x + 1)*sqrt(e*x - 1)*c*x - sqrt(-a^2*e^2 - a*c)*(c*e*x^2 + a*e))/(a^2
*e^2 + a*c))/(a^2*e^2 + a*c)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)}\,dx = \int \frac{1}{(a+cx^2)\,\sqrt{ex-1}\sqrt{ex+1}}\,dx$$

input
```
integrate(1/(e*x-1)**(1/2)/(e*x+1)**(1/2)/(c*x**2+a),x)
```

output
```
Integral(1/((a + c*x**2)*sqrt(e*x - 1)*sqrt(e*x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)}\,dx = \int \frac{1}{(cx^2+a)\sqrt{ex+1}\sqrt{ex-1}}\,dx$$

input
```
integrate(1/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)*sqrt(e*x + 1)*sqrt(e*x - 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)}\,dx = -\frac{\arctan\left(\frac{c\left(\sqrt{ex+1}-\sqrt{ex-1}\right)^4+8\,ae^2+4\,c}{8\,\sqrt{-a^2e^2-ac}e}\right)}{\sqrt{-a^2e^2-ac}}$$

input
```
integrate(1/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
-arctan(1/8*(c*(sqrt(e*x + 1) - sqrt(e*x - 1))^4 + 8*a*e^2 + 4*c)/(sqrt(-a
^2*e^2 - a*c)*e))/sqrt(-a^2*e^2 - a*c)
```

Mupad [B] (verification not implemented)

Time = 15.19 (sec) , antiderivative size = 2280, normalized size of antiderivative = 28.50

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^2)*(e*x - 1)^(1/2)*(e*x + 1)^(1/2)),x)
```

output
```
-(atan((((((13631488*a*c^3*e^5 + 1310720*a^2*c^2*e^7)*((e*x - 1)^(1/2) - 1
i)^3)/((e*x + 1)^(1/2) - 1)^3 - (((((e*x - 1)^(1/2) - 1i)*(4194304*a^2*c^4
*e^5 + 4718592*a^3*c^3*e^7))/((e*x + 1)^(1/2) - 1) + (((e*x - 1)^(1/2) - 1
i)^3*(54525952*a^2*c^4*e^5 + 60293120*a^3*c^3*e^7 + 5242880*a^4*c^2*e^9))/
((e*x + 1)^(1/2) - 1)^3 + (9437184*a^3*c^4*e^6 + 15728640*a^4*c^3*e^8 + 65
53600*a^5*c^2*e^10 - (((e*x - 1)^(1/2) - 1i)^4*(100663296*a^2*c^5*e^4 + 15
6237824*a^3*c^4*e^6 + 33554432*a^4*c^3*e^8 - 22282240*a^5*c^2*e^10))/((e*x
 + 1)^(1/2) - 1)^4 - (((e*x - 1)^(1/2) - 1i)^2*(100663296*a^2*c^5*e^4 + 19
7132288*a^3*c^4*e^6 + 116391936*a^4*c^3*e^8 + 18350080*a^5*c^2*e^10))/((e*
x + 1)^(1/2) - 1)^2)/(2*a^(1/2)*(c + a*e^2)^(1/2)))/(2*a^(1/2)*(c + a*e^2)
^(1/2)) - 3670016*a^2*c^3*e^6 - 2949120*a^3*c^2*e^8 + (((e*x - 1)^(1/2) -
1i)^4*(25165824*a*c^4*e^4 + 7340032*a^2*c^3*e^6 - 12124160*a^3*c^2*e^8))/(
(e*x + 1)^(1/2) - 1)^4 + (((e*x - 1)^(1/2) - 1i)^2*(25165824*a*c^4*e^4 + 2
6738688*a^2*c^3*e^6 + 7208960*a^3*c^2*e^8))/((e*x + 1)^(1/2) - 1)^2)/(2*a^
(1/2)*(c + a*e^2)^(1/2)) + (1048576*a*c^3*e^5*((e*x - 1)^(1/2) - 1i))/((e*
x + 1)^(1/2) - 1))*1i)/(2*a^(1/2)*(c + a*e^2)^(1/2)) + ((((13631488*a*c^3*
e^5 + 1310720*a^2*c^2*e^7)*((e*x - 1)^(1/2) - 1i)^3)/((e*x + 1)^(1/2) - 1)
^3 - (((((e*x - 1)^(1/2) - 1i)*(4194304*a^2*c^4*e^5 + 4718592*a^3*c^3*e^7)
)/((e*x + 1)^(1/2) - 1) + (((e*x - 1)^(1/2) - 1i)^3*(54525952*a^2*c^4*e^5
+ 60293120*a^3*c^3*e^7 + 5242880*a^4*c^2*e^9))/((e*x + 1)^(1/2) - 1)^3 ...
```

Reduce [F]

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)}\,dx = \int \frac{1}{\sqrt{ex+1}\,\sqrt{ex-1}\,a+\sqrt{ex+1}\,\sqrt{ex-1}\,c\,x^2}dx$$

input
```
int(1/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a),x)
```

output
```
int(1/(sqrt(e*x + 1)*sqrt(e*x - 1)*a + sqrt(e*x + 1)*sqrt(e*x - 1)*c*x**2)
,x)
```

3.18 $\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 137

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)^2}\,dx = -\frac{cx\sqrt{-1+ex}\sqrt{1+ex}}{2a\,(c+ae^2)\,(a+cx^2)} + \frac{(c+2ae^2)\sqrt{-1+e^2x^2}\,\text{arctanh}\left(\frac{\sqrt{c+ae^2}\,x}{\sqrt{a}\sqrt{-1+e^2x^2}}\right)}{2a^{3/2}\,(c+ae^2)^{3/2}\sqrt{-1+ex}\sqrt{1+ex}}$$

output
```
-1/2*c*x*(e*x-1)^(1/2)*(e*x+1)^(1/2)/a/(a*e^2+c)/(c*x^2+a)+1/2*(2*a*e^2+c)
*(e^2*x^2-1)^(1/2)*arctanh((a*e^2+c)^(1/2)*x/a^(1/2)/(e^2*x^2-1)^(1/2))/a^
(3/2)/(a*e^2+c)^(3/2)/(e*x-1)^(1/2)/(e*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)^2}\,dx = -\frac{cx\sqrt{-1+ex}\sqrt{1+ex}}{2a\,(c+ae^2)\,(a+cx^2)} + \frac{(c+2ae^2)\,\text{arctanh}\left(\frac{\sqrt{c+ae^2}\,x}{\sqrt{a}\sqrt{-1+ex}\sqrt{1+ex}}\right)}{2a^{3/2}\,(c+ae^2)^{3/2}}$$

input
```
Integrate[1/(Sqrt[-1 + e*x]*Sqrt[1 + e*x]*(a + c*x^2)^2),x]
```

output
```
-1/2*(c*x*Sqrt[-1 + e*x]*Sqrt[1 + e*x])/(a*(c + a*e^2)*(a + c*x^2)) + ((c
+ 2*a*e^2)*ArcTanh[(Sqrt[c + a*e^2]*x)/(Sqrt[a]*Sqrt[-1 + e*x]*Sqrt[1 + e*
x])])/(2*a^(3/2)*(c + a*e^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {648, 296, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ex-1}\sqrt{ex+1}\left(a+cx^2\right)^2}dx$$

$$\Big\downarrow \text{648}$$

$$\frac{\sqrt{e^2x^2-1}\int \frac{1}{\left(cx^2+a\right)^2\sqrt{e^2x^2-1}}dx}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{296}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\left(2ae^2+c\right)\int \frac{1}{\left(cx^2+a\right)\sqrt{e^2x^2-1}}dx}{2a\left(ae^2+c\right)}-\frac{cx\sqrt{e^2x^2-1}}{2a\left(ae^2+c\right)\left(a+cx^2\right)}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\left(2ae^2+c\right)\int \frac{1}{a-\frac{\left(ae^2+c\right)x^2}{e^2x^2-1}}d\frac{x}{\sqrt{e^2x^2-1}}}{2a\left(ae^2+c\right)}-\frac{cx\sqrt{e^2x^2-1}}{2a\left(ae^2+c\right)\left(a+cx^2\right)}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\left(2ae^2+c\right)\text{arctanh}\left(\frac{x\sqrt{ae^2+c}}{\sqrt{a}\sqrt{e^2x^2-1}}\right)}{2a^{3/2}\left(ae^2+c\right)^{3/2}}-\frac{cx\sqrt{e^2x^2-1}}{2a\left(ae^2+c\right)\left(a+cx^2\right)}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

input
```
Int[1/(Sqrt[-1 + e*x]*Sqrt[1 + e*x]*(a + c*x^2)^2),x]
```

output
```
(Sqrt[-1 + e^2*x^2]*(-1/2*(c*x*Sqrt[-1 + e^2*x^2])/(a*(c + a*e^2)*(a + c*x
^2)) + ((c + 2*a*e^2)*ArcTanh[(Sqrt[c + a*e^2]*x)/(Sqrt[a]*Sqrt[-1 + e^2*x
^2])])/(2*a^(3/2)*(c + a*e^2)^(3/2))))/(Sqrt[-1 + e*x]*Sqrt[1 + e*x])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.61 (sec) , antiderivative size = 996, normalized size of antiderivative = 7.27

method	result
default	$\frac{\left(2\ln\left(\frac{-2\sqrt{-ac}\,e^2x+2\sqrt{e^2x^2-1}\,\sqrt{-\frac{a e^2+c}{c}}\,c-2c}{cx+\sqrt{-ac}}\right)a^2 c e^4 x^2-2\ln\left(\frac{2\sqrt{-ac}\,e^2x+2\sqrt{e^2x^2-1}\,\sqrt{-\frac{a e^2+c}{c}}\,c-2c}{cx-\sqrt{-ac}}\right)a^2 c e^4 x^2+2\ln\left(\frac{-2\sqrt{-ac}\,e \cdots\right.\right.}{}$

input
```
int(1/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-
c)/(c*x+(-a*c)^(1/2)))*a^2*c*e^4*x^2-2*ln(2*((-a*c)^(1/2)*e^2*x+(e^2*x^2-1
)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-a*c)^(1/2)))*a^2*c*e^4*x^2+2*ln(2
*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x+(-a
*c)^(1/2)))*a^3*e^4+3*ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2
+c)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*a*c^2*e^2*x^2-2*ln(2*((-a*c)^(1/2)*e
^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-a*c)^(1/2)))*a^3*e
^4-3*ln(2*((-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/
(c*x-(-a*c)^(1/2)))*a*c^2*e^2*x^2-2*a*c*e^2*x*(-a*c)^(1/2)*(e^2*x^2-1)^(1/
2)*(-(a*e^2+c)/c)^(1/2)+3*ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a
*e^2+c)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*a^2*c*e^2+ln(2*(-(-a*c)^(1/2)*e^
2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*c^3*x^
2-3*ln(2*((-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(
c*x-(-a*c)^(1/2)))*a^2*c*e^2-ln(2*((-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-
(a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-a*c)^(1/2)))*c^3*x^2-2*c^2*x*(-a*c)^(1/2)*(
e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)+ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1
)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*a*c^2-ln(2*((-a*c)^(
1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-a*c)^(1/2)))
*a*c^2)*c^3*csgn(e)^2*(e*x+1)^(1/2)*(e*x-1)^(1/2)/(-a*c)^(1/2)/(c*x+(-a*c)
^(1/2))/(-(a*e^2+c)/c)^(1/2)/(c*x-(-a*c)^(1/2))/a/(-e*(-a*c)^(1/2)+c)^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(113) = 226$.

Time = 0.11 (sec) , antiderivative size = 547, normalized size of antiderivative = 3.99

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)^2}\,dx$$

$$= \left[-\frac{2\,a^3e^3 + 2\,a^2ce + 2\,(a^2ce^2 + ac^2)\sqrt{ex+1}\sqrt{ex-1}x + 2\,(a^2ce^3 + ac^2e)x^2 - (2\,a^2e^2 + (2\,ace^2 + c^2)x^2}{4\,(a^5e^4 + 2\,a^4ce^2}\right.$$

$$-\frac{a^3e^3 + a^2ce + (a^2ce^2 + ac^2)\sqrt{ex+1}\sqrt{ex-1}x + (a^2ce^3 + ac^2e)x^2 - (2\,a^2e^2 + (2\,ace^2 + c^2)x^2 + ac)\sqrt{}}{2\,(a^5e^4 + 2\,a^4ce^2 + a^3c^2 + (a^4ce^4 + 2\,a^3c^2e^2 + }$$

input
```
integrate(1/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a^3*e^3 + 2*a^2*c*e + 2*(a^2*c*e^2 + a*c^2)*sqrt(e*x + 1)*sqrt(e*
x - 1)*x + 2*(a^2*c*e^3 + a*c^2*e)*x^2 - (2*a^2*e^2 + (2*a*c*e^2 + c^2)*x^
2 + a*c)*sqrt(a^2*e^2 + a*c)*log(-(2*a^2*e^2 - (4*a^2*e^4 + 4*a*c*e^2 + c^
2)*x^2 - 2*(sqrt(a^2*e^2 + a*c)*(2*a*e^2 + c)*x + 2*(a^2*e^3 + a*c*e)*x)*s
qrt(e*x + 1)*sqrt(e*x - 1) + a*c - 2*sqrt(a^2*e^2 + a*c)*((2*a*e^3 + c*e)*
x^2 - a*e))/(c*x^2 + a)))/(a^5*e^4 + 2*a^4*c*e^2 + a^3*c^2 + (a^4*c*e^4 +
2*a^3*c^2*e^2 + a^2*c^3)*x^2), -1/2*(a^3*e^3 + a^2*c*e + (a^2*c*e^2 + a*c^
2)*sqrt(e*x + 1)*sqrt(e*x - 1)*x + (a^2*c*e^3 + a*c^2*e)*x^2 - (2*a^2*e^2
+ (2*a*c*e^2 + c^2)*x^2 + a*c)*sqrt(-a^2*e^2 - a*c)*arctan((sqrt(-a^2*e^2
- a*c)*sqrt(e*x + 1)*sqrt(e*x - 1)*c*x - sqrt(-a^2*e^2 - a*c)*(c*e*x^2 + a
*e))/(a^2*e^2 + a*c)))/(a^5*e^4 + 2*a^4*c*e^2 + a^3*c^2 + (a^4*c*e^4 + 2*a
^3*c^2*e^2 + a^2*c^3)*x^2)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)^2}\,dx = \int \frac{1}{(a+cx^2)^2\sqrt{ex-1}\sqrt{ex+1}}\,dx$$

input
```
integrate(1/(e*x-1)**(1/2)/(e*x+1)**(1/2)/(c*x**2+a)**2,x)
```

output
```
Integral(1/((a + c*x**2)**2*sqrt(e*x - 1)*sqrt(e*x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)^2}\,dx = \int \frac{1}{(cx^2+a)^2\sqrt{ex+1}\sqrt{ex-1}}\,dx$$

input
```
integrate(1/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)^2*sqrt(e*x + 1)*sqrt(e*x - 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(113) = 226.

Time = 0.15 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)^2}\,dx = -\frac{1}{2}e^3\left(\frac{(2\,ae^2+c)\arctan\left(\frac{c(\sqrt{ex+1}-\sqrt{ex-1})^4+8\,ae^2+4\,c}{8\sqrt{-a^2e^2-ace}}\right)}{(a^2e^4+ace^2)\sqrt{-a^2e^2-ace}} + \frac{8\left(2\,ae^2(\sqrt{ex+1}-\sqrt{ex}\right.}{\left(c(\sqrt{ex+1}-\sqrt{ex-1})^8+16\,ae^2(\sqrt{ex+1}-\right.}\right.$$

input
```
integrate(1/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*e^3*((2*a*e^2 + c)*arctan(1/8*(c*(sqrt(e*x + 1) - sqrt(e*x - 1))^4 +
8*a*e^2 + 4*c)/(sqrt(-a^2*e^2 - a*c)*e))/((a^2*e^4 + a*c*e^2)*sqrt(-a^2*e^
2 - a*c)*e) + 8*(2*a*e^2*(sqrt(e*x + 1) - sqrt(e*x - 1))^4 + c*(sqrt(e*x +
 1) - sqrt(e*x - 1))^4 + 4*c)/((c*(sqrt(e*x + 1) - sqrt(e*x - 1))^8 + 16*a
*e^2*(sqrt(e*x + 1) - sqrt(e*x - 1))^4 + 8*c*(sqrt(e*x + 1) - sqrt(e*x - 1
))^4 + 16*c)*(a^2*e^4 + a*c*e^2)))
```

Mupad [B] (verification not implemented)

Time = 39.70 (sec) , antiderivative size = 14522, normalized size of antiderivative = 106.00

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^2)^2*(e*x - 1)^(1/2)*(e*x + 1)^(1/2)),x)
```

output
```
-((c^2*atan(((a^(1/2)*c^5*(c + a*e^2)^(3/2)*((e*x - 1)^(1/2) - 1i)*32i)/((
e*x + 1)^(1/2) - 1) - (a^(1/2)*c^2*(c + a*e^2)^(9/2)*((e*x - 1)^(1/2) - 1i
)*32i)/((e*x + 1)^(1/2) - 1) + (a^(1/2)*c^5*(c + a*e^2)^(3/2)*((e*x - 1)^(
1/2) - 1i)^3*416i)/((e*x + 1)^(1/2) - 1)^3 - (a^(1/2)*c^2*(c + a*e^2)^(9/2
)*((e*x - 1)^(1/2) - 1i)^3*416i)/((e*x + 1)^(1/2) - 1)^3 - (a^(5/2)*e^4*(c
 + a*e^2)^(9/2)*((e*x - 1)^(1/2) - 1i)^3*40i)/((e*x + 1)^(1/2) - 1)^3 + (a
^(11/2)*e^10*(c + a*e^2)^(3/2)*((e*x - 1)^(1/2) - 1i)^3*40i)/((e*x + 1)^(1
/2) - 1)^3 - (a^(3/2)*c*e^2*(c + a*e^2)^(9/2)*((e*x - 1)^(1/2) - 1i)*32i)/
((e*x + 1)^(1/2) - 1) + (a^(9/2)*c*e^8*(c + a*e^2)^(3/2)*((e*x - 1)^(1/2)
- 1i)*36i)/((e*x + 1)^(1/2) - 1) + (a^(3/2)*c^4*e^2*(c + a*e^2)^(3/2)*((e*
x - 1)^(1/2) - 1i)*132i)/((e*x + 1)^(1/2) - 1) + (a^(5/2)*c^3*e^4*(c + a*e
^2)^(3/2)*((e*x - 1)^(1/2) - 1i)*204i)/((e*x + 1)^(1/2) - 1) + (a^(7/2)*c^
2*e^6*(c + a*e^2)^(3/2)*((e*x - 1)^(1/2) - 1i)*140i)/((e*x + 1)^(1/2) - 1)
 - (a^(3/2)*c*e^2*(c + a*e^2)^(9/2)*((e*x - 1)^(1/2) - 1i)^3*456i)/((e*x +
 1)^(1/2) - 1)^3 + (a^(9/2)*c*e^8*(c + a*e^2)^(3/2)*((e*x - 1)^(1/2) - 1i)
^3*580i)/((e*x + 1)^(1/2) - 1)^3 + (a^(3/2)*c^4*e^2*(c + a*e^2)^(3/2)*((e*
x - 1)^(1/2) - 1i)^3*1708i)/((e*x + 1)^(1/2) - 1)^3 + (a^(5/2)*c^3*e^4*(c
+ a*e^2)^(3/2)*((e*x - 1)^(1/2) - 1i)^3*2668i)/((e*x + 1)^(1/2) - 1)^3 + (
a^(7/2)*c^2*e^6*(c + a*e^2)^(3/2)*((e*x - 1)^(1/2) - 1i)^3*1916i)/((e*x +
1)^(1/2) - 1)^3)/(a^6*c*e^11 + a^2*c^5*e^3 + 4*a^3*c^4*e^5 + 6*a^4*c^3*...
```

Reduce [F]

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)^2}\,dx$$
$$= \int \frac{1}{\sqrt{ex+1}\,\sqrt{ex-1}\,a^2+2\sqrt{ex+1}\,\sqrt{ex-1}\,ac\,x^2+\sqrt{ex+1}\,\sqrt{ex-1}\,c^2x^4}dx$$

input
```
int(1/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^2,x)
```

output
```
int(1/(sqrt(e*x + 1)*sqrt(e*x - 1)*a**2 + 2*sqrt(e*x + 1)*sqrt(e*x - 1)*a*
c*x**2 + sqrt(e*x + 1)*sqrt(e*x - 1)*c**2*x**4),x)
```

3.19 $\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 203

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)^3}\,dx = -\frac{cx\sqrt{-1+ex}\sqrt{1+ex}}{4a(c+ae^2)(a+cx^2)^2} - \frac{3c(c+2ae^2)\,x\sqrt{-1+ex}\sqrt{1+ex}}{8a^2(c+ae^2)^2(a+cx^2)} + \frac{(3c^2+8ace^2+8a^2e^4)\sqrt{-1+e^2x^2}\,\text{arctanh}\left(\frac{\sqrt{c+ae^2}\,x}{\sqrt{a}\sqrt{-1+e^2x^2}}\right)}{8a^{5/2}(c+ae^2)^{5/2}\sqrt{-1+ex}\sqrt{1+ex}}$$

output
```
-1/4*c*x*(e*x-1)^(1/2)*(e*x+1)^(1/2)/a/(a*e^2+c)/(c*x^2+a)^2-3/8*c*(2*a*e^
2+c)*x*(e*x-1)^(1/2)*(e*x+1)^(1/2)/a^2/(a*e^2+c)^2/(c*x^2+a)+1/8*(8*a^2*e^
4+8*a*c*e^2+3*c^2)*(e^2*x^2-1)^(1/2)*arctanh((a*e^2+c)^(1/2)*x/a^(1/2)/(e^
2*x^2-1)^(1/2))/a^(5/2)/(a*e^2+c)^(5/2)/(e*x-1)^(1/2)/(e*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 6.94 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)^3}\,dx$$
$$= -\frac{cx\sqrt{-1+ex}\sqrt{1+ex}(8a^2e^2+3c^2x^2+ac(5+6e^2x^2))}{8a^2\,(c+ae^2)^2\,(a+cx^2)^2} + \frac{(3c^2+8ace^2+8a^2e^4)\,\text{arctanh}\left(\frac{\sqrt{c+ae^2}x}{\sqrt{a}\sqrt{-1+ex}\sqrt{1+ex}}\right)}{8a^{5/2}\,(c+ae^2)^{5/2}}$$

input
```
Integrate[1/(Sqrt[-1 + e*x]*Sqrt[1 + e*x]*(a + c*x^2)^3),x]
```

output
```
-1/8*(c*x*Sqrt[-1 + e*x]*Sqrt[1 + e*x]*(8*a^2*e^2 + 3*c^2*x^2 + a*c*(5 + 6
*e^2*x^2)))/(a^2*(c + a*e^2)^2*(a + c*x^2)^2) + ((3*c^2 + 8*a*c*e^2 + 8*a^
2*e^4)*ArcTanh[(Sqrt[c + a*e^2]*x)/(Sqrt[a]*Sqrt[-1 + e*x]*Sqrt[1 + e*x])]
)/(8*a^(5/2)*(c + a*e^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {648, 316, 402, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ex-1}\sqrt{ex+1}\,(a+cx^2)^3}\,dx$$
$$\Big\downarrow \text{ 648}$$
$$\frac{\sqrt{e^2x^2-1}\int \frac{1}{(cx^2+a)^3\sqrt{e^2x^2-1}}dx}{\sqrt{ex-1}\sqrt{ex+1}}$$
$$\Big\downarrow \text{ 316}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\int \frac{-2cx^2e^2+4ae^2+3c}{\left(cx^2+a\right)^2\sqrt{e^2x^2-1}}dx}{4a(ae^2+c)}-\frac{cx\sqrt{e^2x^2-1}}{4a(ae^2+c)(a+cx^2)^2}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{\int \frac{8a^2e^4+8ace^2+3c^2}{\left(cx^2+a\right)\sqrt{e^2x^2-1}}dx}{2a\left(ae^2+c\right)}-\frac{3cx\sqrt{e^2x^2-1}\left(2ae^2+c\right)}{2a\left(ae^2+c\right)\left(a+cx^2\right)}}{4a(ae^2+c)}-\frac{cx\sqrt{e^2x^2-1}}{4a(ae^2+c)(a+cx^2)^2}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{\left(8a^2e^4+8ace^2+3c^2\right)\int \frac{1}{\left(cx^2+a\right)\sqrt{e^2x^2-1}}dx}{2a\left(ae^2+c\right)}-\frac{3cx\sqrt{e^2x^2-1}\left(2ae^2+c\right)}{2a\left(ae^2+c\right)\left(a+cx^2\right)}}{4a(ae^2+c)}-\frac{cx\sqrt{e^2x^2-1}}{4a(ae^2+c)(a+cx^2)^2}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \; 291$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{\left(8a^2e^4+8ace^2+3c^2\right)\int \frac{1}{a-\frac{\left(ae^2+c\right)x^2}{e^2x^2-1}}d\frac{x}{\sqrt{e^2x^2-1}}}{2a\left(ae^2+c\right)}-\frac{3cx\sqrt{e^2x^2-1}\left(2ae^2+c\right)}{2a\left(ae^2+c\right)\left(a+cx^2\right)}}{4a(ae^2+c)}-\frac{cx\sqrt{e^2x^2-1}}{4a(ae^2+c)(a+cx^2)^2}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{\left(8a^2e^4+8ace^2+3c^2\right)\text{arctanh}\left(\frac{x\sqrt{ae^2+c}}{\sqrt{a}\sqrt{e^2x^2-1}}\right)}{2a^{3/2}\left(ae^2+c\right)^{3/2}}-\frac{3cx\sqrt{e^2x^2-1}\left(2ae^2+c\right)}{2a\left(ae^2+c\right)\left(a+cx^2\right)}}{4a(ae^2+c)}-\frac{cx\sqrt{e^2x^2-1}}{4a(ae^2+c)(a+cx^2)^2}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

input
```
Int[1/(Sqrt[-1 + e*x]*Sqrt[1 + e*x]*(a + c*x^2)^3),x]
```

output
```
(Sqrt[-1 + e^2*x^2]*(-1/4*(c*x*Sqrt[-1 + e^2*x^2])/(a*(c + a*e^2)*(a + c*x
^2)^2) + ((-3*c*(c + 2*a*e^2)*x*Sqrt[-1 + e^2*x^2])/(2*a*(c + a*e^2)*(a +
c*x^2)) + ((3*c^2 + 8*a*c*e^2 + 8*a^2*e^4)*ArcTanh[(Sqrt[c + a*e^2]*x)/(Sq
rt[a]*Sqrt[-1 + e^2*x^2])])/(2*a^(3/2)*(c + a*e^2)^(3/2)))/(4*a*(c + a*e^2
))))/(Sqrt[-1 + e*x]*Sqrt[1 + e*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 2029, normalized size of antiderivative = 10.00

method	result	size
default	Expression too large to display	2029

input
```
int(1/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(8*ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c
-c)/(c*x+(-a*c)^(1/2)))*a^3*c^2*e^6*x^4-8*ln(2*((-a*c)^(1/2)*e^2*x+(e^2*x^
2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-a*c)^(1/2)))*a^3*c^2*e^6*x^4+1
6*ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c
*x+(-a*c)^(1/2)))*a^4*c*e^6*x^2+16*ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(
1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*a^2*c^3*e^4*x^4-16*ln(2
*((-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-a*
c)^(1/2)))*a^4*c*e^6*x^2-16*ln(2*((-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(
a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-a*c)^(1/2)))*a^2*c^3*e^4*x^4+32*ln(2*(-(-a*c
)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2
)))*a^3*c^2*e^4*x^2+11*ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^
2+c)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*a*c^4*e^2*x^4-32*ln(2*((-a*c)^(1/2)
*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-a*c)^(1/2)))*a^3
*c^2*e^4*x^2-11*ln(2*((-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^
(1/2)*c-c)/(c*x-(-a*c)^(1/2)))*a*c^4*e^2*x^4+22*ln(2*(-(-a*c)^(1/2)*e^2*x+
(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*a^2*c^3*e^
2*x^2-10*a*c^3*x*(-a*c)^(1/2)*(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)-22*ln
(2*((-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-
a*c)^(1/2)))*a^2*c^3*e^2*x^2-12*a^2*c^2*e^4*x^3*(-a*c)^(1/2)*(e^2*x^2-1)^(
1/2)*(-(a*e^2+c)/c)^(1/2)-16*a^3*c*e^4*x*(-a*c)^(1/2)*(e^2*x^2-1)^(1/2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(173) = 346$.

Time = 0.15 (sec) , antiderivative size = 1024, normalized size of antiderivative = 5.04

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(12*a^5*e^5 + 18*a^4*c*e^3 + 6*a^3*c^2*e + 6*(2*a^3*c^2*e^5 + 3*a^2
*c^3*e^3 + a*c^4*e)*x^4 + 12*(2*a^4*c*e^5 + 3*a^3*c^2*e^3 + a^2*c^3*e)*x^2
 - (8*a^4*e^4 + 8*a^3*c*e^2 + (8*a^2*c^2*e^4 + 8*a*c^3*e^2 + 3*c^4)*x^4 +
3*a^2*c^2 + 2*(8*a^3*c*e^4 + 8*a^2*c^2*e^2 + 3*a*c^3)*x^2)*sqrt(a^2*e^2 +
a*c)*log(-(2*a^2*e^2 - (4*a^2*e^4 + 4*a*c*e^2 + c^2)*x^2 - 2*(sqrt(a^2*e^2
 + a*c)*(2*a*e^2 + c)*x + 2*(a^2*e^3 + a*c*e)*x)*sqrt(e*x + 1)*sqrt(e*x -
1) + a*c - 2*sqrt(a^2*e^2 + a*c)*((2*a*e^3 + c*e)*x^2 - a*e))/(c*x^2 + a))
 + 2*(3*(2*a^3*c^2*e^4 + 3*a^2*c^3*e^2 + a*c^4)*x^3 + (8*a^4*c*e^4 + 13*a^
3*c^2*e^2 + 5*a^2*c^3)*x)*sqrt(e*x + 1)*sqrt(e*x - 1))/(a^8*e^6 + 3*a^7*c*
e^4 + 3*a^6*c^2*e^2 + a^5*c^3 + (a^6*c^2*e^6 + 3*a^5*c^3*e^4 + 3*a^4*c^4*e
^2 + a^3*c^5)*x^4 + 2*(a^7*c*e^6 + 3*a^6*c^2*e^4 + 3*a^5*c^3*e^2 + a^4*c^4
)*x^2), -1/8*(6*a^5*e^5 + 9*a^4*c*e^3 + 3*a^3*c^2*e + 3*(2*a^3*c^2*e^5 + 3
*a^2*c^3*e^3 + a*c^4*e)*x^4 + 6*(2*a^4*c*e^5 + 3*a^3*c^2*e^3 + a^2*c^3*e)*
x^2 - (8*a^4*e^4 + 8*a^3*c*e^2 + (8*a^2*c^2*e^4 + 8*a*c^3*e^2 + 3*c^4)*x^4
 + 3*a^2*c^2 + 2*(8*a^3*c*e^4 + 8*a^2*c^2*e^2 + 3*a*c^3)*x^2)*sqrt(-a^2*e^
2 - a*c)*arctan((sqrt(-a^2*e^2 - a*c)*sqrt(e*x + 1)*sqrt(e*x - 1)*c*x - sq
rt(-a^2*e^2 - a*c)*(c*e*x^2 + a*e))/(a^2*e^2 + a*c)) + (3*(2*a^3*c^2*e^4 +
 3*a^2*c^3*e^2 + a*c^4)*x^3 + (8*a^4*c*e^4 + 13*a^3*c^2*e^2 + 5*a^2*c^3)*x
)*sqrt(e*x + 1)*sqrt(e*x - 1))/(a^8*e^6 + 3*a^7*c*e^4 + 3*a^6*c^2*e^2 + a^
5*c^3 + (a^6*c^2*e^6 + 3*a^5*c^3*e^4 + 3*a^4*c^4*e^2 + a^3*c^5)*x^4 + 2...
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\left(a+cx^2\right)^3}\,dx=\int \frac{1}{\left(a+cx^2\right)^3\sqrt{ex-1}\sqrt{ex+1}}\,dx$$

input
```
integrate(1/(e*x-1)**(1/2)/(e*x+1)**(1/2)/(c*x**2+a)**3,x)
```

output
```
Integral(1/((a + c*x**2)**3*sqrt(e*x - 1)*sqrt(e*x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\left(a+cx^2\right)^3}\,dx=\int \frac{1}{\left(cx^2+a\right)^3\sqrt{ex+1}\sqrt{ex-1}}\,dx$$

input
```
integrate(1/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)^3*sqrt(e*x + 1)*sqrt(e*x - 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(173) = 346$.

Time = 36.89 (sec) , antiderivative size = 514, normalized size of antiderivative = 2.53

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\left(a+cx^2\right)^3}\,dx=-\frac{1}{8}e^5\left(\frac{\left(8a^2e^4+8ace^2+3c^2\right)\arctan\left(\frac{c\left(\sqrt{ex+1}-\sqrt{ex-1}\right)^4+8ae^2+4c}{8\sqrt{-a^2e^2-ace}}\right)}{\left(a^4e^8+2a^3ce^6+a^2c^2e^4\right)\sqrt{-a^2e^2-ace}}+\frac{8\left(8a^2ce^4\left(\sqrt{ex+1}-\sqrt{ex-1}\right)^{12}\right.}{}\right.$$

input
```
integrate(1/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/8*e^5*((8*a^2*e^4 + 8*a*c*e^2 + 3*c^2)*arctan(1/8*(c*(sqrt(e*x + 1) - s
qrt(e*x - 1))^4 + 8*a*e^2 + 4*c)/(sqrt(-a^2*e^2 - a*c)*e))/((a^4*e^8 + 2*a
^3*c*e^6 + a^2*c^2*e^4)*sqrt(-a^2*e^2 - a*c)*e) + 8*(8*a^2*c*e^4*(sqrt(e*x
 + 1) - sqrt(e*x - 1))^12 + 192*a^3*e^6*(sqrt(e*x + 1) - sqrt(e*x - 1))^8
+ 8*a*c^2*e^2*(sqrt(e*x + 1) - sqrt(e*x - 1))^12 + 288*a^2*c*e^4*(sqrt(e*x
 + 1) - sqrt(e*x - 1))^8 + 3*c^3*(sqrt(e*x + 1) - sqrt(e*x - 1))^12 + 168*
a*c^2*e^2*(sqrt(e*x + 1) - sqrt(e*x - 1))^8 + 640*a^2*c*e^4*(sqrt(e*x + 1)
 - sqrt(e*x - 1))^4 + 36*c^3*(sqrt(e*x + 1) - sqrt(e*x - 1))^8 + 640*a*c^2
*e^2*(sqrt(e*x + 1) - sqrt(e*x - 1))^4 + 144*c^3*(sqrt(e*x + 1) - sqrt(e*x
 - 1))^4 + 384*a*c^2*e^2 + 192*c^3)/((a^4*e^8 + 2*a^3*c*e^6 + a^2*c^2*e^4)
*(c*(sqrt(e*x + 1) - sqrt(e*x - 1))^8 + 16*a*e^2*(sqrt(e*x + 1) - sqrt(e*x
 - 1))^4 + 8*c*(sqrt(e*x + 1) - sqrt(e*x - 1))^4 + 16*c)^2))
```

Mupad [B] (verification not implemented)

Time = 33.41 (sec) , antiderivative size = 10632, normalized size of antiderivative = 52.37

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^2)^3*(e*x - 1)^(1/2)*(e*x + 1)^(1/2)),x)
```

output
```
- ((3*((e*x - 1)^(1/2) - 1i)^3*(16*c^3*e + 27*a*c^2*e^3 - 8*a^2*c*e^5))/(2
*a^2*((e*x + 1)^(1/2) - 1)^3*(c^2 + a^2*e^4 + 2*a*c*e^2)) + (3*((e*x - 1)^
(1/2) - 1i)^13*(16*c^3*e + 27*a*c^2*e^3 - 8*a^2*c*e^5))/(2*a^2*((e*x + 1)^
(1/2) - 1)^13*(c^2 + a^2*e^4 + 2*a*c*e^2)) + (11*((e*x - 1)^(1/2) - 1i)^7*
(96*c^3*e + 187*a*c^2*e^3 - 8*a^2*c*e^5))/(2*a^2*((e*x + 1)^(1/2) - 1)^7*(
c^2 + a^2*e^4 + 2*a*c*e^2)) + (11*((e*x - 1)^(1/2) - 1i)^9*(96*c^3*e + 187
*a*c^2*e^3 - 8*a^2*c*e^5))/(2*a^2*((e*x + 1)^(1/2) - 1)^9*(c^2 + a^2*e^4 +
 2*a*c*e^2)) + (3*((e*x - 1)^(1/2) - 1i)^5*(144*c^3*e + 313*a*c^2*e^3 + 40
*a^2*c*e^5))/(2*a^2*((e*x + 1)^(1/2) - 1)^5*(c^2 + a^2*e^4 + 2*a*c*e^2)) +
 (3*((e*x - 1)^(1/2) - 1i)^11*(144*c^3*e + 313*a*c^2*e^3 + 40*a^2*c*e^5))/
(2*a^2*((e*x + 1)^(1/2) - 1)^11*(c^2 + a^2*e^4 + 2*a*c*e^2)) - (e^3*((e*x
- 1)^(1/2) - 1i)^15*(5*c^2 + 8*a*c*e^2))/(2*a*((e*x + 1)^(1/2) - 1)^15*(c^
2 + a^2*e^4 + 2*a*c*e^2)) - (e^3*((e*x - 1)^(1/2) - 1i)*(5*c^2 + 8*a*c*e^2
))/(2*a*((e*x + 1)^(1/2) - 1)*(c^2 + a^2*e^4 + 2*a*c*e^2)))/(a^2*e^4 + (((
e*x - 1)^(1/2) - 1i)^4*(256*c^2 + 28*a^2*e^4 + 64*a*c*e^2))/((e*x + 1)^(1/
2) - 1)^4 + (((e*x - 1)^(1/2) - 1i)^12*(256*c^2 + 28*a^2*e^4 + 64*a*c*e^2)
)/((e*x + 1)^(1/2) - 1)^12 + (((e*x - 1)^(1/2) - 1i)^6*(1024*c^2 - 56*a^2*
e^4 + 32*a*c*e^2))/((e*x + 1)^(1/2) - 1)^6 + (((e*x - 1)^(1/2) - 1i)^10*(1
024*c^2 - 56*a^2*e^4 + 32*a*c*e^2))/((e*x + 1)^(1/2) - 1)^10 + (((e*x - 1)
^(1/2) - 1i)^8*(1536*c^2 + 70*a^2*e^4 - 128*a*c*e^2))/((e*x + 1)^(1/2) ...
```

Reduce [F]

$$\int \frac{1}{\sqrt{-1+ex}\sqrt{1+ex}\,(a+cx^2)^3}\,dx$$
$$= \int \frac{1}{\sqrt{ex+1}\,\sqrt{ex-1}\,a^3+3\sqrt{ex+1}\,\sqrt{ex-1}\,a^2c\,x^2+3\sqrt{ex+1}\,\sqrt{ex-1}\,a\,c^2x^4+\sqrt{ex+1}\,\sqrt{ex-1}\,c^3x^6}$$

input
```
int(1/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)^3,x)
```

output
```
int(1/(sqrt(e*x + 1)*sqrt(e*x - 1)*a**3 + 3*sqrt(e*x + 1)*sqrt(e*x - 1)*a*
*2*c*x**2 + 3*sqrt(e*x + 1)*sqrt(e*x - 1)*a*c**2*x**4 + sqrt(e*x + 1)*sqrt
(e*x - 1)*c**3*x**6),x)
```

3.20 $\int \frac{(a+cx^2)^3}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 137

$$\begin{aligned}\int \frac{(a+cx^2)^3}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx &= -\frac{(c+ae^2)^3\,x}{e^6\sqrt{-1+ex}\sqrt{1+ex}}\\ &\quad+\frac{c^2(7c+12ae^2)\,x\sqrt{-1+ex}\sqrt{1+ex}}{8e^6}+\frac{c^3x^3\sqrt{-1+ex}\sqrt{1+ex}}{4e^4}\\ &\quad+\frac{3c(5c^2+12ace^2+8a^2e^4)\operatorname{arccosh}(ex)}{8e^7}\end{aligned}$$

output
```
-(a*e^2+c)^3*x/e^6/(e*x-1)^(1/2)/(e*x+1)^(1/2)+1/8*c^2*(12*a*e^2+7*c)*x*(e
*x-1)^(1/2)*(e*x+1)^(1/2)/e^6+1/4*c^3*x^3*(e*x-1)^(1/2)*(e*x+1)^(1/2)/e^4+
3/8*c*(8*a^2*e^4+12*a*c*e^2+5*c^2)*arccosh(e*x)/e^7
```

Mathematica [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.94

$$\int \frac{(a+cx^2)^3}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{\frac{ex(-24a^2ce^4-8a^3e^6+12ac^2e^2(-3+e^2x^2)+c^3(-15+5e^2x^2+2e^4x^4))}{\sqrt{-1+ex}\sqrt{1+ex}}+6c(5c^2+12ace^2+8}{8e^7}$$

input
```
Integrate[(a + c*x^2)^3/((-1 + e*x)^(3/2)*(1 + e*x)^(3/2)),x]
```

output
```
((e*x*(-24*a^2*c*e^4 - 8*a^3*e^6 + 12*a*c^2*e^2*(-3 + e^2*x^2) + c^3*(-15
+ 5*e^2*x^2 + 2*e^4*x^4)))/(Sqrt[-1 + e*x]*Sqrt[1 + e*x]) + 6*c*(5*c^2 + 1
2*a*c*e^2 + 8*a^2*e^4)*ArcTanh[Sqrt[(-1 + e*x)/(1 + e*x)]])/(8*e^7)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.48, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {648, 315, 25, 27, 403, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3}{(ex-1)^{3/2}(ex+1)^{3/2}}\,dx$$

$$\Big\downarrow \text{648}$$

$$\frac{\sqrt{e^2x^2-1}\int \frac{(cx^2+a)^3}{(e^2x^2-1)^{3/2}}dx}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{e^2x^2-1}\left(-\frac{\int -\frac{c(cx^2+a)((4ae^2+5c)x^2+a)}{\sqrt{e^2x^2-1}}dx}{e^2}-\frac{x(ae^2+c)(a+cx^2)^2}{e^2\sqrt{e^2x^2-1}}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\int \frac{c(cx^2+a)((4ae^2+5c)x^2+a)}{\sqrt{e^2x^2-1}}dx}{e^2}-\frac{x(ae^2+c)(a+cx^2)^2}{e^2\sqrt{e^2x^2-1}}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{c\int \frac{(cx^2+a)((4ae^2+5c)x^2+a)}{\sqrt{e^2x^2-1}}dx}{e^2}-\frac{x(ae^2+c)(a+cx^2)^2}{e^2\sqrt{e^2x^2-1}}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{c\left(\frac{\int \frac{(2ae^2+5c)(4ae^2+3c)x^2+a(8ae^2+5c)}{\sqrt{e^2x^2-1}}dx}{4e^2}+\frac{1}{4}x\sqrt{e^2x^2-1}\left(4a+\frac{5c}{e^2}\right)(a+cx^2)\right)}{e^2}-\frac{x(ae^2+c)(a+cx^2)^2}{e^2\sqrt{e^2x^2-1}}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$\Big\downarrow$ 299

$$\frac{\sqrt{e^2x^2-1}\left(\frac{c\left(\frac{\frac{3(8a^2e^4+12ace^2+5c^2)\int \frac{1}{\sqrt{e^2x^2-1}}dx}{2e^2}+\frac{x\sqrt{e^2x^2-1}(2ae^2+5c)(4ae^2+3c)}{2e^2}}{4e^2}+\frac{1}{4}x\sqrt{e^2x^2-1}\left(4a+\frac{5c}{e^2}\right)(a+cx^2)\right)}{e^2}-\frac{x(ae^2+c)(a+cx^2)^2}{e^2\sqrt{e^2x^2-1}}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$\Big\downarrow$ 224

$$\frac{\sqrt{e^2x^2-1}\left(\frac{c\left(\frac{\frac{3(8a^2e^4+12ace^2+5c^2)\int \frac{1}{1-\frac{e^2x^2}{e^2x^2-1}}d\frac{x}{\sqrt{e^2x^2-1}}}{2e^2}+\frac{x\sqrt{e^2x^2-1}(2ae^2+5c)(4ae^2+3c)}{2e^2}}{4e^2}+\frac{1}{4}x\sqrt{e^2x^2-1}\left(4a+\frac{5c}{e^2}\right)(a+cx^2)\right)}{e^2}-\frac{x(ae^2+c)(a+cx^2)^2}{e^2\sqrt{e^2x^2-1}}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{e^2x^2-1}\left(\frac{c\left(\frac{\frac{3(8a^2e^4+12ace^2+5c^2)\operatorname{arctanh}\left(\frac{ex}{\sqrt{e^2x^2-1}}\right)}{2e^3}+\frac{x\sqrt{e^2x^2-1}(2ae^2+5c)(4ae^2+3c)}{2e^2}}{4e^2}+\frac{1}{4}x\sqrt{e^2x^2-1}\left(4a+\frac{5c}{e^2}\right)(a+cx^2)\right)}{e^2}-\frac{x(ae^2+c)(a+cx^2)^2}{e^2\sqrt{e^2x^2-1}}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

input
```
Int[(a + c*x^2)^3/((-1 + e*x)^(3/2)*(1 + e*x)^(3/2)),x]
```

output
```
(Sqrt[-1 + e^2*x^2]*(-(((c + a*e^2)*x*(a + c*x^2)^2)/(e^2*Sqrt[-1 + e^2*x^
2])) + (c*(((4*a + (5*c)/e^2)*x*(a + c*x^2)*Sqrt[-1 + e^2*x^2])/4 + (((5*c
 + 2*a*e^2)*(3*c + 4*a*e^2)*x*Sqrt[-1 + e^2*x^2])/(2*e^2) + (3*(5*c^2 + 12
*a*c*e^2 + 8*a^2*e^4)*ArcTanh[(e*x)/Sqrt[-1 + e^2*x^2]])/(2*e^3))/(4*e^2))
)/e^2))/(Sqrt[-1 + e*x]*Sqrt[1 + e*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(119) = 238$.

Time = 0.78 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.34

method	result
risch	$\frac{c^2 x\left(2x^2 c\,e^2+12a\,e^2+7c\right)\sqrt{ex-1}\,\sqrt{ex+1}}{8e^6}+\frac{\left(\frac{\left(-4e^6a^3-12e^4a^2c-12c^2a\,e^2-4c^3\right)\sqrt{e^2\left(x+\frac{1}{e}\right)^2-2e\left(x+\frac{1}{e}\right)}}{e^2\left(x+\frac{1}{e}\right)}-\frac{\left(4e^6a^3+12e^4a^2c+12c^2a\,e^2+\right.}{e^2\left(\right.}\right.}{}$
default	$-\frac{\left(-2\,\mathrm{csgn}(e)c^3e^5x^5\sqrt{e^2x^2-1}+8\,\mathrm{csgn}(e)e^7\sqrt{e^2x^2-1}\,a^3x-12\,\mathrm{csgn}(e)a\,c^2e^5x^3\sqrt{e^2x^2-1}-24\ln\left(\left(\sqrt{e^2x^2-1}\,\mathrm{csgn}(e)+ex\right)\mathrm{csgn}(e)\right.\right.}{}$

input
```
int((c*x^2+a)^3/(e*x-1)^(3/2)/(e*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*c^2*x*(2*c*e^2*x^2+12*a*e^2+7*c)*(e*x-1)^(1/2)*(e*x+1)^(1/2)/e^6+1/8/e
^6*((-4*a^3*e^6-12*a^2*c*e^4-12*a*c^2*e^2-4*c^3)/e^2/(x+1/e)*(e^2*(x+1/e)^
2-2*e*(x+1/e))^(1/2)-(4*a^3*e^6+12*a^2*c*e^4+12*a*c^2*e^2+4*c^3)/e^2/(x-1/
e)*(e^2*(x-1/e)^2+2*e*(x-1/e))^(1/2)+15*c^3*ln(e^2*x/(e^2)^(1/2)+(e^2*x^2-
1)^(1/2))/(e^2)^(1/2)+36*c^2*a*e^2*ln(e^2*x/(e^2)^(1/2)+(e^2*x^2-1)^(1/2))
/(e^2)^(1/2)+24*e^4*a^2*c*ln(e^2*x/(e^2)^(1/2)+(e^2*x^2-1)^(1/2))/(e^2)^(1
/2))*((e*x-1)*(e*x+1))^(1/2)/(e*x-1)^(1/2)/(e*x+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(119) = 238$.

Time = 0.10 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.83

$$\int \frac{(a+cx^2)^3}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{8\,a^3e^6 + 24\,a^2ce^4 + 24\,ac^2e^2 + 8\,c^3 - 8\,(a^3e^8 + 3\,a^2ce^6 + 3\,ac^2e^4 + c^3e^2)x^2}{}$$

input
```
integrate((c*x^2+a)^3/(e*x-1)^(3/2)/(e*x+1)^(3/2),x, algorithm="fricas")
```

output
```
1/8*(8*a^3*e^6 + 24*a^2*c*e^4 + 24*a*c^2*e^2 + 8*c^3 - 8*(a^3*e^8 + 3*a^2*
c*e^6 + 3*a*c^2*e^4 + c^3*e^2)*x^2 + (2*c^3*e^5*x^5 + (12*a*c^2*e^5 + 5*c^
3*e^3)*x^3 - (8*a^3*e^7 + 24*a^2*c*e^5 + 36*a*c^2*e^3 + 15*c^3*e)*x)*sqrt(
e*x + 1)*sqrt(e*x - 1) + 3*(8*a^2*c*e^4 + 12*a*c^2*e^2 + 5*c^3 - (8*a^2*c*
e^6 + 12*a*c^2*e^4 + 5*c^3*e^2)*x^2)*log(-e*x + sqrt(e*x + 1)*sqrt(e*x - 1
)))/(e^9*x^2 - e^7)
```

Sympy [F]

$$\int \frac{(a+cx^2)^3}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = \int \frac{(a+cx^2)^3}{(ex-1)^{\frac{3}{2}}\,(ex+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**2+a)**3/(e*x-1)**(3/2)/(e*x+1)**(3/2),x)
```

output
```
Integral((a + c*x**2)**3/((e*x - 1)**(3/2)*(e*x + 1)**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(119) = 238$.

Time = 0.04 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.94

$$\int \frac{(a+cx^2)^3}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{c^3x^5}{4\sqrt{e^2x^2-1}e^2} - \frac{a^3x}{\sqrt{e^2x^2-1}} + \frac{3\,ac^2x^3}{2\sqrt{e^2x^2-1}e^2} - \frac{3\,a^2cx}{\sqrt{e^2x^2-1}e^2} + \frac{5\,c^3x^3}{8\sqrt{e^2x^2-1}e^4} + \frac{3\,a^2c\log\left(2\,e^2x+2\sqrt{e^2x^2-1}\sqrt{e^2}\right)}{\sqrt{e^2}e^2} - \frac{9\,ac^2x}{2\sqrt{e^2x^2-1}e^4} + \frac{9\,ac^2\log\left(2\,e^2x+2\sqrt{e^2x^2-1}\sqrt{e^2}\right)}{2\sqrt{e^2}e^4} - \frac{15\,c^3x}{8\sqrt{e^2x^2-1}e^6} + \frac{15\,c^3\log\left(2\,e^2x+2\sqrt{e^2x^2-1}\sqrt{e^2}\right)}{8\sqrt{e^2}e^6}$$

input
```
integrate((c*x^2+a)^3/(e*x-1)^(3/2)/(e*x+1)^(3/2),x, algorithm="maxima")
```

output
```
1/4*c^3*x^5/(sqrt(e^2*x^2 - 1)*e^2) - a^3*x/sqrt(e^2*x^2 - 1) + 3/2*a*c^2*
x^3/(sqrt(e^2*x^2 - 1)*e^2) - 3*a^2*c*x/(sqrt(e^2*x^2 - 1)*e^2) + 5/8*c^3*
x^3/(sqrt(e^2*x^2 - 1)*e^4) + 3*a^2*c*log(2*e^2*x + 2*sqrt(e^2*x^2 - 1)*sq
rt(e^2))/(sqrt(e^2)*e^2) - 9/2*a*c^2*x/(sqrt(e^2*x^2 - 1)*e^4) + 9/2*a*c^2
*log(2*e^2*x + 2*sqrt(e^2*x^2 - 1)*sqrt(e^2))/(sqrt(e^2)*e^4) - 15/8*c^3*x
/(sqrt(e^2*x^2 - 1)*e^6) + 15/8*c^3*log(2*e^2*x + 2*sqrt(e^2*x^2 - 1)*sqrt
(e^2))/(sqrt(e^2)*e^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(119) = 238$.

Time = 0.15 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.83

$$\int \frac{(a+cx^2)^3}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{\left(\left((ex+1)\left(2\,(ex+1)\left(\frac{(ex+1)c^3}{e^7} - \frac{5\,c^3}{e^7}\right) + \frac{12\,ac^2e^{37}+25\,c^3e^{35}}{e^{42}}\right) - \frac{36\,ac^2e^{37}+35\,c^3 \cdots}{e^{42}} \cdots}{8\sqrt{ex-1} \cdots} - \frac{3\,(8\,a^2ce^4+12\,ac^2e^2+5\,c^3)\log\left(\left(\sqrt{ex+1}-\sqrt{ex-1}\right)^2\right)}{8\,e^7} - \frac{2\,(a^3e^6+3\,a^2ce^4+3\,ac^2e^2+c^3)}{\left(\left(\sqrt{ex+1}-\sqrt{ex-1}\right)^2+2\right)e^7}$$

input
```
integrate((c*x^2+a)^3/(e*x-1)^(3/2)/(e*x+1)^(3/2),x, algorithm="giac")
```

output
```
1/8*(((e*x + 1)*(2*(e*x + 1)*((e*x + 1)*c^3/e^7 - 5*c^3/e^7) + (12*a*c^2*e
^37 + 25*c^3*e^35)/e^42) - (36*a*c^2*e^37 + 35*c^3*e^35)/e^42)*(e*x + 1) -
 2*(2*a^3*e^41 + 6*a^2*c*e^39 - 6*a*c^2*e^37 - 7*c^3*e^35)/e^42)*sqrt(e*x
+ 1)/sqrt(e*x - 1) - 3/8*(8*a^2*c*e^4 + 12*a*c^2*e^2 + 5*c^3)*log((sqrt(e*
x + 1) - sqrt(e*x - 1))^2)/e^7 - 2*(a^3*e^6 + 3*a^2*c*e^4 + 3*a*c^2*e^2 +
c^3)/(((sqrt(e*x + 1) - sqrt(e*x - 1))^2 + 2)*e^7)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^3}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx=\int \frac{(c\,x^2+a)^3}{(e\,x-1)^{3/2}\,(e\,x+1)^{3/2}}\,dx$$

input
```
int((a + c*x^2)^3/((e*x - 1)^(3/2)*(e*x + 1)^(3/2)),x)
```

output
```
int((a + c*x^2)^3/((e*x - 1)^(3/2)*(e*x + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 448, normalized size of antiderivative = 3.27

$$\int \frac{(a+cx^2)^3}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx=\frac{48\sqrt{ex-1}\log\left(\frac{\sqrt{ex-1}+\sqrt{ex+1}}{\sqrt{2}}\right)a^2c\,e^5x+48\sqrt{ex-1}\log\left(\frac{\sqrt{ex-1}+\sqrt{ex+1}}{\sqrt{2}}\right)a^2c}{}$$

input
```
int((c*x^2+a)^3/(e*x-1)^(3/2)/(e*x+1)^(3/2),x)
```

output
```
(48*sqrt(e*x - 1)*log((sqrt(e*x - 1) + sqrt(e*x + 1))/sqrt(2))*a**2*c*e**5
*x + 48*sqrt(e*x - 1)*log((sqrt(e*x - 1) + sqrt(e*x + 1))/sqrt(2))*a**2*c*
e**4 + 72*sqrt(e*x - 1)*log((sqrt(e*x - 1) + sqrt(e*x + 1))/sqrt(2))*a*c**
2*e**3*x + 72*sqrt(e*x - 1)*log((sqrt(e*x - 1) + sqrt(e*x + 1))/sqrt(2))*a
*c**2*e**2 + 30*sqrt(e*x - 1)*log((sqrt(e*x - 1) + sqrt(e*x + 1))/sqrt(2))
*c**3*e*x + 30*sqrt(e*x - 1)*log((sqrt(e*x - 1) + sqrt(e*x + 1))/sqrt(2))*
c**3 - 8*sqrt(e*x - 1)*a**3*e**7*x - 8*sqrt(e*x - 1)*a**3*e**6 - 24*sqrt(e
*x - 1)*a**2*c*e**5*x - 24*sqrt(e*x - 1)*a**2*c*e**4 - 27*sqrt(e*x - 1)*a*
c**2*e**3*x - 27*sqrt(e*x - 1)*a*c**2*e**2 - 10*sqrt(e*x - 1)*c**3*e*x - 1
0*sqrt(e*x - 1)*c**3 - 8*sqrt(e*x + 1)*a**3*e**7*x - 24*sqrt(e*x + 1)*a**2
*c*e**5*x + 12*sqrt(e*x + 1)*a*c**2*e**5*x**3 - 36*sqrt(e*x + 1)*a*c**2*e*
*3*x + 2*sqrt(e*x + 1)*c**3*e**5*x**5 + 5*sqrt(e*x + 1)*c**3*e**3*x**3 - 1
5*sqrt(e*x + 1)*c**3*e*x)/(8*sqrt(e*x - 1)*e**7*(e*x + 1))
```

3.21 $\int \frac{(a+cx^2)^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{(a+cx^2)^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = -\frac{(c+ae^2)^2 x}{e^4\sqrt{-1+ex}\sqrt{1+ex}} + \frac{c^2x\sqrt{-1+ex}\sqrt{1+ex}}{2e^4} + \frac{c(3c+4ae^2)\operatorname{arccosh}(ex)}{2e^5}$$

output
```
-(a*e^2+c)^2*x/e^4/(e*x-1)^(1/2)/(e*x+1)^(1/2)+1/2*c^2*x*(e*x-1)^(1/2)*(e*
x+1)^(1/2)/e^4+1/2*c*(4*a*e^2+3*c)*arccosh(e*x)/e^5
```

Mathematica [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \frac{(a+cx^2)^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{\frac{ex(-4ace^2-2a^2e^4+c^2(-3+e^2x^2))}{\sqrt{-1+ex}\sqrt{1+ex}} + 2c(3c+4ae^2)\operatorname{arctanh}\left(\sqrt{\frac{-1+ex}{1+ex}}\right)}{2e^5}$$

input
```
Integrate[(a + c*x^2)^2/((-1 + e*x)^(3/2)*(1 + e*x)^(3/2)),x]
```

output
```
((e*x*(-4*a*c*e^2 - 2*a^2*e^4 + c^2*(-3 + e^2*x^2)))/(Sqrt[-1 + e*x]*Sqrt[
1 + e*x]) + 2*c*(3*c + 4*a*e^2)*ArcTanh[Sqrt[(-1 + e*x)/(1 + e*x)]])/(2*e^
5)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.47, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {650, 2124, 25, 27, 646, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(ex-1)^{3/2}(ex+1)^{3/2}}\,dx$$

$$\Big\downarrow \text{650}$$

$$-\frac{\int \frac{-c^2x^3-\frac{c^2x^2}{e}-c\left(2a+\frac{c}{e^2}\right)x+a^2e}{\sqrt{ex-1}(ex+1)^{3/2}}dx}{e}-\frac{(ae^2+c)^2}{e^5\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{2124}$$

$$-\frac{\frac{\int -\frac{c\left(cx^2+2a+\frac{c}{e^2}\right)}{\sqrt{ex-1}\sqrt{ex+1}}dx}{e}+\frac{\sqrt{ex-1}(ae^2+c)^2}{e^4\sqrt{ex+1}}}{e}-\frac{(ae^2+c)^2}{e^5\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\sqrt{ex-1}(ae^2+c)^2}{e^4\sqrt{ex+1}}-\frac{\int \frac{c\left(cx^2+2a+\frac{c}{e^2}\right)}{\sqrt{ex-1}\sqrt{ex+1}}dx}{e}}{e}-\frac{(ae^2+c)^2}{e^5\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\sqrt{ex-1}(ae^2+c)^2}{e^4\sqrt{ex+1}}-\frac{c\int \frac{cx^2+2a+\frac{c}{e^2}}{\sqrt{ex-1}\sqrt{ex+1}}dx}{e}}{e}-\frac{(ae^2+c)^2}{e^5\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{646}$$

$$-\frac{\frac{\sqrt{ex-1}(ae^2+c)^2}{e^4\sqrt{ex+1}}-\frac{c\left(\frac{1}{2}\left(4a+\frac{3c}{e^2}\right)\int\frac{1}{\sqrt{ex-1}\sqrt{ex+1}}dx+\frac{cx\sqrt{ex-1}\sqrt{ex+1}}{2e^2}\right)}{e}}{e}-\frac{(ae^2+c)^2}{e^5\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{43}$$

$$-\frac{\frac{\sqrt{ex-1}(ae^2+c)^2}{e^4\sqrt{ex+1}}-\frac{c\left(\frac{\left(4a+\frac{3c}{e^2}\right)\operatorname{arccosh}(ex)}{2e}+\frac{cx\sqrt{ex-1}\sqrt{ex+1}}{2e^2}\right)}{e}}{e}-\frac{(ae^2+c)^2}{e^5\sqrt{ex-1}\sqrt{ex+1}}$$

input
```
Int[(a + c*x^2)^2/((-1 + e*x)^(3/2)*(1 + e*x)^(3/2)),x]
```

output
```
-((c + a*e^2)^2/(e^5*Sqrt[-1 + e*x]*Sqrt[1 + e*x])) - (((c + a*e^2)^2*Sqrt
[-1 + e*x])/(e^4*Sqrt[1 + e*x]) - (c*((c*x*Sqrt[-1 + e*x]*Sqrt[1 + e*x])/(
2*e^2) + ((4*a + (3*c)/e^2)*ArcCosh[e*x])/(2*e)))/e)/e
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 646
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[b*x*(c + d*x)^(m + 1)*((e + f*x)^(n + 1)/(d*f*(2*m +
 3))), x] - Simp[(b*c*e - a*d*f*(2*m + 3))/(d*f*(2*m + 3))   Int[(c + d*x)^
m*(e + f*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m, n] &&
 EqQ[d*e + c*f, 0] &&  !LtQ[m, -1]
```

rule 650
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^2)^p, d + e*x
, x], R = PolynomialRemainder[(a + c*x^2)^p, d + e*x, x]}, Simp[R*(d + e*x)
^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))), x] + Simp[1/((m + 1)*(e
*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*ExpandToSum[(m + 1)*(e*f - d
*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g, n}, x] &&
 IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(73) = 146$.

Time = 0.77 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.80

method	result
risch	$\frac{c^2 x\sqrt{ex-1}\,\sqrt{ex+1}}{2e^4}+\frac{\left(\frac{\left(-a^2e^4-2ac\,e^2-c^2\right)\sqrt{e^2\left(x+\frac{1}{e}\right)^2-2e\left(x+\frac{1}{e}\right)}}{e^2\left(x+\frac{1}{e}\right)}-\frac{\left(a^2e^4+2ac\,e^2+c^2\right)\sqrt{e^2\left(x-\frac{1}{e}\right)^2+2e\left(x-\frac{1}{e}\right)}}{e^2\left(x-\frac{1}{e}\right)}+\frac{3c^2\ln\left(\frac{e^2x}{\sqrt{e^2}}+\sqrt{e^2x^2}\right.}{\sqrt{e^2}}\right.}{2e^4\sqrt{ex-1}\,\sqrt{ex+1}}$
default	$-\frac{\left(2\operatorname{csgn}(e)e^5\sqrt{e^2x^2-1}\,a^2x-\operatorname{csgn}(e)c^2e^3x^3\sqrt{e^2x^2-1}-4\ln\left(\left(\sqrt{e^2x^2-1}\,\operatorname{csgn}(e)+ex\right)\operatorname{csgn}(e)\right)ac\,e^4x^2+4\operatorname{csgn}(e)e^3\sqrt{e^2x^2-1}\,acx\right.}{}$

input
```
int((c*x^2+a)^2/(e*x-1)^(3/2)/(e*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*c^2*x*(e*x-1)^(1/2)*(e*x+1)^(1/2)/e^4+1/2/e^4*((-a^2*e^4-2*a*c*e^2-c^2
)/e^2/(x+1/e)*(e^2*(x+1/e)^2-2*e*(x+1/e))^(1/2)-(a^2*e^4+2*a*c*e^2+c^2)/e^
2/(x-1/e)*(e^2*(x-1/e)^2+2*e*(x-1/e))^(1/2)+3*c^2*ln(e^2*x/(e^2)^(1/2)+(e^
2*x^2-1)^(1/2))/(e^2)^(1/2)+4*a*c*e^2*ln(e^2*x/(e^2)^(1/2)+(e^2*x^2-1)^(1/
2))/(e^2)^(1/2))*((e*x-1)*(e*x+1))^(1/2)/(e*x-1)^(1/2)/(e*x+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(73) = 146$.

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.02

$$\int \frac{(a+cx^2)^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{2\,a^2e^4 + 4\,ace^2 - 2\,(a^2e^6 + 2\,ace^4 + c^2e^2)x^2 + (c^2e^3x^3 - (2\,a^2e^5 + 4\,ace^3}{}$$

input
```
integrate((c*x^2+a)^2/(e*x-1)^(3/2)/(e*x+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*a^2*e^4 + 4*a*c*e^2 - 2*(a^2*e^6 + 2*a*c*e^4 + c^2*e^2)*x^2 + (c^2*
e^3*x^3 - (2*a^2*e^5 + 4*a*c*e^3 + 3*c^2*e)*x)*sqrt(e*x + 1)*sqrt(e*x - 1)
 + 2*c^2 + (4*a*c*e^2 - (4*a*c*e^4 + 3*c^2*e^2)*x^2 + 3*c^2)*log(-e*x + sq
rt(e*x + 1)*sqrt(e*x - 1)))/(e^7*x^2 - e^5)
```

Sympy [F]

$$\int \frac{(a+cx^2)^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = \int \frac{(a+cx^2)^2}{(ex-1)^{\frac{3}{2}}\,(ex+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**2+a)**2/(e*x-1)**(3/2)/(e*x+1)**(3/2),x)
```

output
```
Integral((a + c*x**2)**2/((e*x - 1)**(3/2)*(e*x + 1)**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(73) = 146$.

Time = 0.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.84

$$\int \frac{(a+cx^2)^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = -\frac{a^2x}{\sqrt{e^2x^2-1}} + \frac{c^2x^3}{2\sqrt{e^2x^2-1}e^2} - \frac{2\,acx}{\sqrt{e^2x^2-1}e^2} + \frac{2\,ac\log\left(2\,e^2x+2\,\sqrt{e^2x^2-1}\sqrt{e^2}\right)}{\sqrt{e^2}e^2} - \frac{3\,c^2x}{2\,\sqrt{e^2x^2-1}e^4} + \frac{3\,c^2\log\left(2\,e^2x+2\,\sqrt{e^2x^2-1}\sqrt{e^2}\right)}{2\,\sqrt{e^2}e^4}$$

input
```
integrate((c*x^2+a)^2/(e*x-1)^(3/2)/(e*x+1)^(3/2),x, algorithm="maxima")
```

output
```
-a^2*x/sqrt(e^2*x^2 - 1) + 1/2*c^2*x^3/(sqrt(e^2*x^2 - 1)*e^2) - 2*a*c*x/(
sqrt(e^2*x^2 - 1)*e^2) + 2*a*c*log(2*e^2*x + 2*sqrt(e^2*x^2 - 1)*sqrt(e^2)
)/(sqrt(e^2)*e^2) - 3/2*c^2*x/(sqrt(e^2*x^2 - 1)*e^4) + 3/2*c^2*log(2*e^2*
x + 2*sqrt(e^2*x^2 - 1)*sqrt(e^2))/(sqrt(e^2)*e^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(73) = 146.

Time = 0.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.85

$$\int \frac{(a+cx^2)^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{\sqrt{ex+1}\left((ex+1)\left(\frac{(ex+1)c^2}{e^5}-\frac{3\,c^2}{e^5}\right)-\frac{a^2e^{19}+2\,ace^{17}-c^2e^{15}}{e^{20}}\right)}{2\,\sqrt{ex-1}} - \frac{(4\,ace^2+3\,c^2)\log\left(\left(\sqrt{ex+1}-\sqrt{ex-1}\right)^2\right)}{2\,e^5} - \frac{2\,(a^2e^4+2\,ace^2+c^2)}{\left(\left(\sqrt{ex+1}-\sqrt{ex-1}\right)^2+2\right)e^5}$$

input
```
integrate((c*x^2+a)^2/(e*x-1)^(3/2)/(e*x+1)^(3/2),x, algorithm="giac")
```

output
```
1/2*sqrt(e*x + 1)*((e*x + 1)*((e*x + 1)*c^2/e^5 - 3*c^2/e^5) - (a^2*e^19 +
 2*a*c*e^17 - c^2*e^15)/e^20)/sqrt(e*x - 1) - 1/2*(4*a*c*e^2 + 3*c^2)*log(
(sqrt(e*x + 1) - sqrt(e*x - 1))^2)/e^5 - 2*(a^2*e^4 + 2*a*c*e^2 + c^2)/(((
sqrt(e*x + 1) - sqrt(e*x - 1))^2 + 2)*e^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = \int \frac{{\left(c\,x^2+a\right)}^2}{{\left(e\,x-1\right)}^{3/2}\,{\left(e\,x+1\right)}^{3/2}}\,dx$$

input
```
int((a + c*x^2)^2/((e*x - 1)^(3/2)*(e*x + 1)^(3/2)),x)
```

output
```
int((a + c*x^2)^2/((e*x - 1)^(3/2)*(e*x + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.38

$$\int \frac{(a+cx^2)^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{32\sqrt{ex-1}\log\left(\frac{\sqrt{ex-1}+\sqrt{ex+1}}{\sqrt{2}}\right)ac\,e^3x + 32\sqrt{ex-1}\log\left(\frac{\sqrt{ex-1}+\sqrt{ex+1}}{\sqrt{2}}\right)ac\,e}{}$$

input
```
int((c*x^2+a)^2/(e*x-1)^(3/2)/(e*x+1)^(3/2),x)
```

output
```
(32*sqrt(e*x - 1)*log((sqrt(e*x - 1) + sqrt(e*x + 1))/sqrt(2))*a*c*e**3*x
+ 32*sqrt(e*x - 1)*log((sqrt(e*x - 1) + sqrt(e*x + 1))/sqrt(2))*a*c*e**2 +
 24*sqrt(e*x - 1)*log((sqrt(e*x - 1) + sqrt(e*x + 1))/sqrt(2))*c**2*e*x +
24*sqrt(e*x - 1)*log((sqrt(e*x - 1) + sqrt(e*x + 1))/sqrt(2))*c**2 - 8*sqr
t(e*x - 1)*a**2*e**5*x - 8*sqrt(e*x - 1)*a**2*e**4 - 16*sqrt(e*x - 1)*a*c*
e**3*x - 16*sqrt(e*x - 1)*a*c*e**2 - 9*sqrt(e*x - 1)*c**2*e*x - 9*sqrt(e*x
 - 1)*c**2 - 8*sqrt(e*x + 1)*a**2*e**5*x - 16*sqrt(e*x + 1)*a*c*e**3*x + 4
*sqrt(e*x + 1)*c**2*e**3*x**3 - 12*sqrt(e*x + 1)*c**2*e*x)/(8*sqrt(e*x - 1
)*e**5*(e*x + 1))
```

3.22 $\int \frac{a+cx^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{a+cx^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx=-\frac{(c+ae^2)\,x}{e^2\sqrt{-1+ex}\sqrt{1+ex}}+\frac{c\,\text{arccosh}(ex)}{e^3}$$

output
```
-(a*e^2+c)*x/e^2/(e*x-1)^(1/2)/(e*x+1)^(1/2)+c*arccosh(e*x)/e^3
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \frac{a+cx^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx=\frac{-\frac{e(c+ae^2)x}{\sqrt{-1+ex}\sqrt{1+ex}}+2c\,\text{arctanh}\left(\sqrt{\frac{-1+ex}{1+ex}}\right)}{e^3}$$

input
```
Integrate[(a + c*x^2)/((-1 + e*x)^(3/2)*(1 + e*x)^(3/2)),x]
```

output
```
(-((e*(c + a*e^2)*x)/(Sqrt[-1 + e*x]*Sqrt[1 + e*x])) + 2*c*ArcTanh[Sqrt[(-
1 + e*x)/(1 + e*x)]])/e^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {645, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(ex-1)^{3/2}(ex+1)^{3/2}}dx$$

$$\Big\downarrow \text{645}$$

$$\frac{c\int \frac{1}{\sqrt{ex-1}\sqrt{ex+1}}dx}{e^2}-\frac{x(ae^2+c)}{e^2\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{43}$$

$$\frac{c\,\text{arccosh}(ex)}{e^3}-\frac{x(ae^2+c)}{e^2\sqrt{ex-1}\sqrt{ex+1}}$$

input
```
Int[(a + c*x^2)/((-1 + e*x)^(3/2)*(1 + e*x)^(3/2)),x]
```

output
```
-(((c + a*e^2)*x)/(e^2*Sqrt[-1 + e*x]*Sqrt[1 + e*x])) + (c*ArcCosh[e*x])/e
^3
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 645
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[(b*c*e - a*d*f)*x*(c + d*x)^(m + 1)*((e + f*x)^(n +
1)/(2*c*d*e*f*(m + 1))), x] - Simp[(b*c*e - a*d*f*(2*m + 3))/(2*c*d*e*f*(m
+ 1))   Int[(c + d*x)^(m + 1)*(e + f*x)^(n + 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.17

method	result
default	$\frac{\left(-\sqrt{e^2x^2-1}\,\mathrm{csgn}(e)e^3ax+\ln\left(\left(\mathrm{csgn}(e)\sqrt{(ex-1)(ex+1)}+ex\right)\mathrm{csgn}(e)\right)c\,e^2x^2-\mathrm{csgn}(e)e\sqrt{e^2x^2-1}\,cx-\ln\left(\left(\mathrm{csgn}(e)\sqrt{(ex-1)(ex+1}\right.\right.\right.}{\sqrt{e^2x^2-1}\,e^3\sqrt{ex+1}\,\sqrt{ex-1}}$

input
```
int((c*x^2+a)/(e*x-1)^(3/2)/(e*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(e^2*x^2-1)^(1/2)*csgn(e)*e^3*a*x+ln((csgn(e)*((e*x-1)*(e*x+1))^(1/2)+e*
x)*csgn(e))*c*e^2*x^2-csgn(e)*e*(e^2*x^2-1)^(1/2)*c*x-ln((csgn(e)*((e*x-1)
*(e*x+1))^(1/2)+e*x)*csgn(e))*c)*csgn(e)/(e^2*x^2-1)^(1/2)/e^3/(e*x+1)^(1/
2)/(e*x-1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(37) = 74.

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.44

$$\int \frac{a+cx^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{ae^2-(ae^3+ce)\sqrt{ex+1}\sqrt{ex-1}x-(ae^4+ce^2)x^2-(ce^2x^2-c)\log\left(-e\right.}{e^5x^2-e^3}$$

input
```
integrate((c*x^2+a)/(e*x-1)^(3/2)/(e*x+1)^(3/2),x, algorithm="fricas")
```

output
```
(a*e^2 - (a*e^3 + c*e)*sqrt(e*x + 1)*sqrt(e*x - 1)*x - (a*e^4 + c*e^2)*x^2
 - (c*e^2*x^2 - c)*log(-e*x + sqrt(e*x + 1)*sqrt(e*x - 1)) + c)/(e^5*x^2 -
 e^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 119.81 (sec) , antiderivative size = 165, normalized size of antiderivative = 4.02

$$\int \frac{a+cx^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = a\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2 \\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0 \end{matrix}\middle| \frac{1}{e^2x^2}\right)}{2\pi^{\frac{3}{2}}e} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0 \end{matrix}\right.}{2\pi^{\frac{3}{2}}e}\right.$$

$$+c\left(\frac{G_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},\frac{1}{2},1,1 \\ -\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1,0 & \end{matrix}\middle| \frac{1}{e^2x^2}\right)}{2\pi^{\frac{3}{2}}e^3} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{3}{2},-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},1 & \\ -\frac{3}{4},-\frac{1}{4} & -\frac{3}{2},-1,0,0 \end{matrix}\middle| \frac{e^{2i\pi}}{e^2x^2}\right)}{2\pi^{\frac{3}{2}}e^3}\right)$$

input
```
integrate((c*x**2+a)/(e*x-1)**(3/2)/(e*x+1)**(3/2),x)
```

output
```
a*(-meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)),
 1/(e**2*x**2))/(2*pi**(3/2)*e) + I*meijerg(((-1/2, 0, 1/4, 1/2, 3/4, 1),
()), ((1/4, 3/4), (-1/2, 0, 1, 0)), exp_polar(2*I*pi)/(e**2*x**2))/(2*pi**
(3/2)*e)) + c*(meijerg(((-1/4, 1/4), (-1/2, 1/2, 1, 1)), ((-1/4, 0, 1/4, 1
/2, 1, 0), ()), 1/(e**2*x**2))/(2*pi**(3/2)*e**3) + I*meijerg(((-3/2, -1,
-3/4, -1/2, -1/4, 1), ()), ((-3/4, -1/4), (-3/2, -1, 0, 0)), exp_polar(2*I
*pi)/(e**2*x**2))/(2*pi**(3/2)*e**3))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.71

$$\int \frac{a+cx^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = -\frac{ax}{\sqrt{e^2x^2-1}} - \frac{cx}{\sqrt{e^2x^2-1}e^2} + \frac{c\log\left(2\,e^2x+2\,\sqrt{e^2x^2-1}\sqrt{e^2}\right)}{\sqrt{e^2}e^2}$$

input
```
integrate((c*x^2+a)/(e*x-1)^(3/2)/(e*x+1)^(3/2),x, algorithm="maxima")
```

output
```
-a*x/sqrt(e^2*x^2 - 1) - c*x/(sqrt(e^2*x^2 - 1)*e^2) + c*log(2*e^2*x + 2*s
qrt(e^2*x^2 - 1)*sqrt(e^2))/(sqrt(e^2)*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(37) = 74$.

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.24

$$\int \frac{a+cx^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = -\frac{c\log\left(\left(\sqrt{ex+1}-\sqrt{ex-1}\right)^2\right)}{e^3} - \frac{2\,(ae^2+c)}{\left(\left(\sqrt{ex+1}-\sqrt{ex-1}\right)^2+2\right)e^3} - \frac{(ae^5+ce^3)\sqrt{ex+1}}{2\,\sqrt{ex-1}e^6}$$

input
```
integrate((c*x^2+a)/(e*x-1)^(3/2)/(e*x+1)^(3/2),x, algorithm="giac")
```

output
```
-c*log((sqrt(e*x + 1) - sqrt(e*x - 1))^2)/e^3 - 2*(a*e^2 + c)/(((sqrt(e*x
+ 1) - sqrt(e*x - 1))^2 + 2)*e^3) - 1/2*(a*e^5 + c*e^3)*sqrt(e*x + 1)/(sqr
t(e*x - 1)*e^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+cx^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = \int \frac{c\,x^2+a}{{\left(e\,x-1\right)}^{3/2}\,{\left(e\,x+1\right)}^{3/2}}\,dx$$

input
```
int((a + c*x^2)/((e*x - 1)^(3/2)*(e*x + 1)^(3/2)),x)
```

output
```
int((a + c*x^2)/((e*x - 1)^(3/2)*(e*x + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.59

$$\int \frac{a+cx^2}{(-1+ex)^{3/2}(1+ex)^{3/2}}\,dx = \frac{2\sqrt{ex-1}\log\left(\frac{\sqrt{ex-1}+\sqrt{ex+1}}{\sqrt{2}}\right)cex+2\sqrt{ex-1}\log\left(\frac{\sqrt{ex-1}+\sqrt{ex+1}}{\sqrt{2}}\right)c-\sqrt{ex}}{\sqrt{ex}}$$

input
```
int((c*x^2+a)/(e*x-1)^(3/2)/(e*x+1)^(3/2),x)
```

output
```
(2*sqrt(e*x - 1)*log((sqrt(e*x - 1) + sqrt(e*x + 1))/sqrt(2))*c*e*x + 2*sq
rt(e*x - 1)*log((sqrt(e*x - 1) + sqrt(e*x + 1))/sqrt(2))*c - sqrt(e*x - 1)
*a*e**3*x - sqrt(e*x - 1)*a*e**2 - sqrt(e*x - 1)*c*e*x - sqrt(e*x - 1)*c -
 sqrt(e*x + 1)*a*e**3*x - sqrt(e*x + 1)*c*e*x)/(sqrt(e*x - 1)*e**3*(e*x +
1))
```

3.23 $\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)}\,dx = -\frac{e^2x}{(c+ae^2)\sqrt{-1+ex}\sqrt{1+ex}} - \frac{c\sqrt{-1+e^2x^2}\,\text{arctanh}\left(\frac{\sqrt{c+ae^2}\,x}{\sqrt{a}\sqrt{-1+e^2x^2}}\right)}{\sqrt{a}\left(c+ae^2\right)^{3/2}\sqrt{-1+ex}\sqrt{1+ex}}$$

output
```
-e^2*x/(a*e^2+c)/(e*x-1)^(1/2)/(e*x+1)^(1/2)-c*(e^2*x^2-1)^(1/2)*arctanh((
a*e^2+c)^(1/2)*x/a^(1/2)/(e^2*x^2-1)^(1/2))/a^(1/2)/(a*e^2+c)^(3/2)/(e*x-1
)^(1/2)/(e*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)}\,dx = -\frac{e^2x}{(c+ae^2)\sqrt{-1+ex}\sqrt{1+ex}} - \frac{c\,\text{arctanh}\left(\frac{\sqrt{c+ae^2}\,x}{\sqrt{a}\sqrt{-1+ex}\sqrt{1+ex}}\right)}{\sqrt{a}\left(c+ae^2\right)^{3/2}}$$

input
```
Integrate[1/((-1 + e*x)^(3/2)*(1 + e*x)^(3/2)*(a + c*x^2)),x]
```

output
```
-((e^2*x)/((c + a*e^2)*Sqrt[-1 + e*x]*Sqrt[1 + e*x])) - (c*ArcTanh[(Sqrt[c
 + a*e^2]*x)/(Sqrt[a]*Sqrt[-1 + e*x]*Sqrt[1 + e*x])])/(Sqrt[a]*(c + a*e^2)
^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {648, 296, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex-1)^{3/2}(ex+1)^{3/2}(a+cx^2)}dx$$

$$\Big\downarrow \text{648}$$

$$\frac{\sqrt{e^2x^2-1}\int \frac{1}{(cx^2+a)(e^2x^2-1)^{3/2}}dx}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{296}$$

$$\frac{\sqrt{e^2x^2-1}\left(-\frac{c\int \frac{1}{\left(cx^2+a\right)\sqrt{e^2x^2-1}}dx}{ae^2+c}-\frac{e^2x}{\sqrt{e^2x^2-1}(ae^2+c)}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{e^2x^2-1}\left(-\frac{c\int \frac{1}{a-\frac{\left(ae^2+c\right)x^2}{e^2x^2-1}}d\frac{x}{\sqrt{e^2x^2-1}}}{ae^2+c}-\frac{e^2x}{\sqrt{e^2x^2-1}(ae^2+c)}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{e^2x^2-1}\left(-\frac{c\,\text{arctanh}\left(\frac{x\sqrt{ae^2+c}}{\sqrt{a}\sqrt{e^2x^2-1}}\right)}{\sqrt{a}(ae^2+c)^{3/2}}-\frac{e^2x}{\sqrt{e^2x^2-1}(ae^2+c)}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

input
```
Int[1/((-1 + e*x)^(3/2)*(1 + e*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(Sqrt[-1 + e^2*x^2]*(-((e^2*x)/((c + a*e^2)*Sqrt[-1 + e^2*x^2])) - (c*ArcT
anh[(Sqrt[c + a*e^2]*x)/(Sqrt[a]*Sqrt[-1 + e^2*x^2])])/(Sqrt[a]*(c + a*e^2
)^(3/2))))/(Sqrt[-1 + e*x]*Sqrt[1 + e*x])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.60 (sec) , antiderivative size = 687, normalized size of antiderivative = 5.92

method	result
default	$\frac{c^2 \operatorname{csgn}(e)^2 \left(\ln\left(\frac{2\sqrt{-ac}\, e^2 x+2\sqrt{e^2x^2-1}\,\sqrt{-\frac{a\,e^2+c}{c}}\, c-2c}{cx-\sqrt{-ac}}\right) ac\,e^4 x^2-\ln\left(\frac{-2\sqrt{-ac}\, e^2 x+2\sqrt{e^2x^2-1}\,\sqrt{-\frac{a\,e^2+c}{c}}\, c-2c}{cx+\sqrt{-ac}}\right) ac\,e^4x^2-2a\sqrt{-ac}\,+}{}$

input
```
int(1/(e*x-1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*c^2*csgn(e)^2*(ln(2*((-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/
c)^(1/2)*c-c)/(c*x-(-a*c)^(1/2)))*a*c*e^4*x^2-ln(2*(-(-a*c)^(1/2)*e^2*x+(e
^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*a*c*e^4*x^2-
2*a*(-a*c)^(1/2)*(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*e^4*x+ln(2*((-a*c)
^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-a*c)^(1/2)
))*c^2*e^2*x^2-ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^
(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*c^2*e^2*x^2-2*(-a*c)^(1/2)*(e^2*x^2-1)^(1/2
)*(-(a*e^2+c)/c)^(1/2)*c*e^2*x-ln(2*((-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*
(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-a*c)^(1/2)))*a*c*e^2+ln(2*(-(-a*c)^(1/2)*
e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*a*c*
e^2-ln(2*((-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(
c*x-(-a*c)^(1/2)))*c^2+ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^
2+c)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*c^2)/(-(a*e^2+c)/c)^(1/2)/(-a*c)^(1
/2)/(-e*(-a*c)^(1/2)+c)^2/(e*(-a*c)^(1/2)+c)^2/(e^2*x^2-1)^(1/2)/(e*x+1)^(
1/2)/(e*x-1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(96) = 192.

Time = 0.12 (sec) , antiderivative size = 506, normalized size of antiderivative = 4.36

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}\,(a+cx^2)}\,dx = \left[-\frac{2\,a^2e^3+2\,ace-2\,(a^2e^4+ace^2)\sqrt{ex+1}\sqrt{ex-1}x-2\,(a^2e^5}{}\right.$$

input
```
integrate(1/(e*x-1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*(2*a^2*e^3 + 2*a*c*e - 2*(a^2*e^4 + a*c*e^2)*sqrt(e*x + 1)*sqrt(e*x
- 1)*x - 2*(a^2*e^5 + a*c*e^3)*x^2 + (c*e^2*x^2 - c)*sqrt(a^2*e^2 + a*c)*l
og(-(2*a^2*e^2 - (4*a^2*e^4 + 4*a*c*e^2 + c^2)*x^2 + 2*(sqrt(a^2*e^2 + a*c
)*(2*a*e^2 + c)*x - 2*(a^2*e^3 + a*c*e)*x)*sqrt(e*x + 1)*sqrt(e*x - 1) + a
*c + 2*sqrt(a^2*e^2 + a*c)*((2*a*e^3 + c*e)*x^2 - a*e))/(c*x^2 + a)))/(a^3
*e^4 + 2*a^2*c*e^2 + a*c^2 - (a^3*e^6 + 2*a^2*c*e^4 + a*c^2*e^2)*x^2), -(a
^2*e^3 + a*c*e - (a^2*e^4 + a*c*e^2)*sqrt(e*x + 1)*sqrt(e*x - 1)*x - (a^2*
e^5 + a*c*e^3)*x^2 - (c*e^2*x^2 - c)*sqrt(-a^2*e^2 - a*c)*arctan((sqrt(-a^
2*e^2 - a*c)*sqrt(e*x + 1)*sqrt(e*x - 1)*c*x - sqrt(-a^2*e^2 - a*c)*(c*e*x
^2 + a*e))/(a^2*e^2 + a*c)))/(a^3*e^4 + 2*a^2*c*e^2 + a*c^2 - (a^3*e^6 + 2
*a^2*c*e^4 + a*c^2*e^2)*x^2)]
```

Sympy [F]

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}\,(a+cx^2)}\,dx = \int \frac{1}{(a+cx^2)\,(ex-1)^{\frac{3}{2}}\,(ex+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x-1)**(3/2)/(e*x+1)**(3/2)/(c*x**2+a),x)
```

output
```
Integral(1/((a + c*x**2)*(e*x - 1)**(3/2)*(e*x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}\,(a+cx^2)}\,dx = \int \frac{1}{(cx^2+a)(ex+1)^{\frac{3}{2}}(ex-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x-1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)*(e*x + 1)^(3/2)*(e*x - 1)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}(a+cx^2)}\,dx = \int \frac{1}{(cx^2+a)(ex+1)^{\frac{3}{2}}(ex-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x-1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}(a+cx^2)}\,dx = \int \frac{1}{(c\,x^2+a)\,(e\,x-1)^{3/2}\,(e\,x+1)^{3/2}}\,dx$$

input
```
int(1/((a + c*x^2)*(e*x - 1)^(3/2)*(e*x + 1)^(3/2)),x)
```

output
```
int(1/((a + c*x^2)*(e*x - 1)^(3/2)*(e*x + 1)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}(a+cx^2)}\,dx = \int \frac{1}{\sqrt{ex+1}\,\sqrt{ex-1}\,a\,e^2x^2 - \sqrt{ex+1}\,\sqrt{ex-1}\,a + \sqrt{ex+1}\,\sqrt{e}}$$

input
```
int(1/(e*x-1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a),x)
```

output
```
int(1/(sqrt(e*x + 1)*sqrt(e*x - 1)*a*e**2*x**2 - sqrt(e*x + 1)*sqrt(e*x -
1)*a + sqrt(e*x + 1)*sqrt(e*x - 1)*c*e**2*x**4 - sqrt(e*x + 1)*sqrt(e*x -
1)*c*x**2),x)
```

3.24 $\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}(a+cx^2)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 184

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \frac{e^2(c-2ae^2)\,x}{2a\left(c+ae^2\right)^2\sqrt{-1+ex}\sqrt{1+ex}} - \frac{cx}{2a\left(c+ae^2\right)\sqrt{-1+ex}\sqrt{1+ex}\left(a+cx^2\right)} - \frac{c(c+4ae^2)\sqrt{-1+e^2x^2}\,\text{arctanh}\left(\frac{\sqrt{c+ae^2}\,x}{\sqrt{a}\sqrt{-1+e^2x^2}}\right)}{2a^{3/2}\left(c+ae^2\right)^{5/2}\sqrt{-1+ex}\sqrt{1+ex}}$$

output

```
1/2*e^2*(-2*a*e^2+c)*x/a/(a*e^2+c)^2/(e*x-1)^(1/2)/(e*x+1)^(1/2)-1/2*c*x/a
/(a*e^2+c)/(e*x-1)^(1/2)/(e*x+1)^(1/2)/(c*x^2+a)-1/2*c*(4*a*e^2+c)*(e^2*x^
2-1)^(1/2)*arctanh((a*e^2+c)^(1/2)*x/a^(1/2)/(e^2*x^2-1)^(1/2))/a^(3/2)/(a
*e^2+c)^(5/2)/(e*x-1)^(1/2)/(e*x+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.78

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = -\frac{x(2a^2e^4+2ace^4x^2+c^2(1-e^2x^2))}{2a\left(c+ae^2\right)^2\sqrt{-1+ex}\sqrt{1+ex}\left(a+cx^2\right)} - \frac{c(c+4ae^2)\operatorname{arctanh}\left(\frac{\sqrt{c+ae^2}x}{\sqrt{a}\sqrt{-1+ex}\sqrt{1+ex}}\right)}{2a^{3/2}\left(c+ae^2\right)^{5/2}}$$

input
```
Integrate[1/((-1 + e*x)^(3/2)*(1 + e*x)^(3/2)*(a + c*x^2)^2),x]
```

output
```
-1/2*(x*(2*a^2*e^4 + 2*a*c*e^4*x^2 + c^2*(1 - e^2*x^2)))/(a*(c + a*e^2)^2*
Sqrt[-1 + e*x]*Sqrt[1 + e*x]*(a + c*x^2)) - (c*(c + 4*a*e^2)*ArcTanh[(Sqrt
[c + a*e^2]*x)/(Sqrt[a]*Sqrt[-1 + e*x]*Sqrt[1 + e*x])])/(2*a^(3/2)*(c + a*
e^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {648, 316, 402, 25, 27, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex-1)^{3/2}(ex+1)^{3/2}\left(a+cx^2\right)^2}dx$$

$$\Big\downarrow \text{648}$$

$$\frac{\sqrt{e^2x^2-1}\int \frac{1}{\left(cx^2+a\right)^2\left(e^2x^2-1\right)^{3/2}}dx}{\sqrt{ex-1}\sqrt{ex+1}}$$

$$\Big\downarrow \text{316}$$

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\int \frac{-2cx^2e^2+2ae^2+c}{\left(cx^2+a\right)\left(e^2x^2-1\right)^{3/2}}dx}{2a(ae^2+c)}-\frac{cx}{2a\sqrt{e^2x^2-1}(ae^2+c)(a+cx^2)}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$\Big\downarrow$ 402

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{\int -\frac{c\left(4ae^2+c\right)}{\left(cx^2+a\right)\sqrt{e^2x^2-1}}dx}{ae^2+c}+\frac{e^2x\left(c-2ae^2\right)}{\sqrt{e^2x^2-1}\left(ae^2+c\right)}}{2a(ae^2+c)}-\frac{cx}{2a\sqrt{e^2x^2-1}(ae^2+c)(a+cx^2)}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{e^2x\left(c-2ae^2\right)}{\sqrt{e^2x^2-1}\left(ae^2+c\right)}-\frac{\int \frac{c\left(4ae^2+c\right)}{\left(cx^2+a\right)\sqrt{e^2x^2-1}}dx}{ae^2+c}}{2a(ae^2+c)}-\frac{cx}{2a\sqrt{e^2x^2-1}(ae^2+c)(a+cx^2)}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{e^2x\left(c-2ae^2\right)}{\sqrt{e^2x^2-1}\left(ae^2+c\right)}-\frac{c\left(4ae^2+c\right)\int \frac{1}{\left(cx^2+a\right)\sqrt{e^2x^2-1}}dx}{ae^2+c}}{2a(ae^2+c)}-\frac{cx}{2a\sqrt{e^2x^2-1}(ae^2+c)(a+cx^2)}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$\Big\downarrow$ 291

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{e^2x\left(c-2ae^2\right)}{\sqrt{e^2x^2-1}\left(ae^2+c\right)}-\frac{c\left(4ae^2+c\right)\int \frac{1}{a-\frac{\left(ae^2+c\right)x^2}{e^2x^2-1}}d\frac{x}{\sqrt{e^2x^2-1}}}{ae^2+c}}{2a(ae^2+c)}-\frac{cx}{2a\sqrt{e^2x^2-1}(ae^2+c)(a+cx^2)}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

$\Big\downarrow$ 221

$$\frac{\sqrt{e^2x^2-1}\left(\frac{\frac{e^2x\left(c-2ae^2\right)}{\sqrt{e^2x^2-1}\left(ae^2+c\right)}-\frac{c\left(4ae^2+c\right)\operatorname{arctanh}\left(\frac{x\sqrt{ae^2+c}}{\sqrt{a}\sqrt{e^2x^2-1}}\right)}{\sqrt{a}\left(ae^2+c\right)^{3/2}}}{2a(ae^2+c)}-\frac{cx}{2a\sqrt{e^2x^2-1}(ae^2+c)(a+cx^2)}\right)}{\sqrt{ex-1}\sqrt{ex+1}}$$

input
```
Int[1/((-1 + e*x)^(3/2)*(1 + e*x)^(3/2)*(a + c*x^2)^2),x]
```

output
```
(Sqrt[-1 + e^2*x^2]*(-1/2*(c*x)/(a*(c + a*e^2)*(a + c*x^2)*Sqrt[-1 + e^2*x
^2]) + ((e^2*(c - 2*a*e^2)*x)/((c + a*e^2)*Sqrt[-1 + e^2*x^2]) - (c*(c + 4
*a*e^2)*ArcTanh[(Sqrt[c + a*e^2]*x)/(Sqrt[a]*Sqrt[-1 + e^2*x^2])])/(Sqrt[a
]*(c + a*e^2)^(3/2)))/(2*a*(c + a*e^2))))/(Sqrt[-1 + e*x]*Sqrt[1 + e*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.71 (sec) , antiderivative size = 1781, normalized size of antiderivative = 9.68

method	result	size
default	Expression too large to display	1781

input
```
int(1/(e*x-1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-4*a^2*c*e^4*x*(-a*c)^(1/2)*(-(a*e^2+c)/c)^(1/2)*(e^2*x^2-1)^(1/2)-2*
a*c^2*e^2*x*(-a*c)^(1/2)*(-(a*e^2+c)/c)^(1/2)*(e^2*x^2-1)^(1/2)+ln(2*((-a*
c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-a*c)^(1/
2)))*c^4*e^2*x^4-4*a^2*c*e^6*x^3*(-a*c)^(1/2)*(-(a*e^2+c)/c)^(1/2)*(e^2*x^
2-1)^(1/2)-2*a*c^2*e^4*x^3*(-a*c)^(1/2)*(-(a*e^2+c)/c)^(1/2)*(e^2*x^2-1)^(
1/2)+4*ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-
c)/(c*x+(-a*c)^(1/2)))*a*c^3*e^2*x^2-ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)
^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*c^4*e^2*x^4-4*ln(2*((
-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-a*c)^
(1/2)))*a^3*c*e^4+4*ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c
)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*a^3*c*e^4-5*ln(2*((-a*c)^(1/2)*e^2*x+(
e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-a*c)^(1/2)))*a^2*c^2*e^2
+5*ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(
c*x+(-a*c)^(1/2)))*a^2*c^2*e^2-4*ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/
2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*a^2*c^2*e^6*x^4+4*ln(2*((
-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x-(-a*c)^
(1/2)))*a^3*c*e^6*x^2+5*ln(2*((-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^
2+c)/c)^(1/2)*c-c)/(c*x-(-a*c)^(1/2)))*a*c^3*e^4*x^4-4*ln(2*(-(-a*c)^(1/2)
*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)^(1/2)*c-c)/(c*x+(-a*c)^(1/2)))*a^3
*c*e^6*x^2-5*ln(2*(-(-a*c)^(1/2)*e^2*x+(e^2*x^2-1)^(1/2)*(-(a*e^2+c)/c)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(156) = 312$.

Time = 0.13 (sec) , antiderivative size = 999, normalized size of antiderivative = 5.43

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x-1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a^4*e^5 + 2*a^3*c*e^3 - 2*a^2*c^2*e - 2*(2*a^3*c*e^7 + a^2*c^2*e^
5 - a*c^3*e^3)*x^4 - 2*(2*a^4*e^7 - a^3*c*e^5 - 2*a^2*c^2*e^3 + a*c^3*e)*x
^2 - (4*a^2*c*e^2 - (4*a*c^2*e^4 + c^3*e^2)*x^4 + a*c^2 - (4*a^2*c*e^4 - 3
*a*c^2*e^2 - c^3)*x^2)*sqrt(a^2*e^2 + a*c)*log(-(2*a^2*e^2 - (4*a^2*e^4 +
4*a*c*e^2 + c^2)*x^2 + 2*(sqrt(a^2*e^2 + a*c)*(2*a*e^2 + c)*x - 2*(a^2*e^3
 + a*c*e)*x)*sqrt(e*x + 1)*sqrt(e*x - 1) + a*c + 2*sqrt(a^2*e^2 + a*c)*((2
*a*e^3 + c*e)*x^2 - a*e))/(c*x^2 + a)) - 2*((2*a^3*c*e^6 + a^2*c^2*e^4 - a
*c^3*e^2)*x^3 + (2*a^4*e^6 + 2*a^3*c*e^4 + a^2*c^2*e^2 + a*c^3)*x)*sqrt(e*
x + 1)*sqrt(e*x - 1))/(a^6*e^6 + 3*a^5*c*e^4 + 3*a^4*c^2*e^2 + a^3*c^3 - (
a^5*c*e^8 + 3*a^4*c^2*e^6 + 3*a^3*c^3*e^4 + a^2*c^4*e^2)*x^4 - (a^6*e^8 +
2*a^5*c*e^6 - 2*a^3*c^3*e^2 - a^2*c^4)*x^2), -1/2*(2*a^4*e^5 + a^3*c*e^3 -
 a^2*c^2*e - (2*a^3*c*e^7 + a^2*c^2*e^5 - a*c^3*e^3)*x^4 - (2*a^4*e^7 - a^
3*c*e^5 - 2*a^2*c^2*e^3 + a*c^3*e)*x^2 + (4*a^2*c*e^2 - (4*a*c^2*e^4 + c^3
*e^2)*x^4 + a*c^2 - (4*a^2*c*e^4 - 3*a*c^2*e^2 - c^3)*x^2)*sqrt(-a^2*e^2 -
 a*c)*arctan((sqrt(-a^2*e^2 - a*c)*sqrt(e*x + 1)*sqrt(e*x - 1)*c*x - sqrt(
-a^2*e^2 - a*c)*(c*e*x^2 + a*e))/(a^2*e^2 + a*c)) - ((2*a^3*c*e^6 + a^2*c^
2*e^4 - a*c^3*e^2)*x^3 + (2*a^4*e^6 + 2*a^3*c*e^4 + a^2*c^2*e^2 + a*c^3)*x
)*sqrt(e*x + 1)*sqrt(e*x - 1))/(a^6*e^6 + 3*a^5*c*e^4 + 3*a^4*c^2*e^2 + a^
3*c^3 - (a^5*c*e^8 + 3*a^4*c^2*e^6 + 3*a^3*c^3*e^4 + a^2*c^4*e^2)*x^4 - (a
^6*e^8 + 2*a^5*c*e^6 - 2*a^3*c^3*e^2 - a^2*c^4)*x^2)]
```

Sympy [F]

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx=\int \frac{1}{\left(a+cx^2\right)^2\left(ex-1\right)^{\frac{3}{2}}\left(ex+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x-1)**(3/2)/(e*x+1)**(3/2)/(c*x**2+a)**2,x)
```

output
```
Integral(1/((a + c*x**2)**2*(e*x - 1)**(3/2)*(e*x + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \int \frac{1}{(cx^2+a)^2(ex+1)^{\frac{3}{2}}(ex-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x-1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)^2*(e*x + 1)^(3/2)*(e*x - 1)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \int \frac{1}{(cx^2+a)^2(ex+1)^{\frac{3}{2}}(ex-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x-1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \int \frac{1}{\left(c\,x^2+a\right)^2\,\left(e\,x-1\right)^{3/2}\,\left(e\,x+1\right)^{3/2}}\,dx$$

input
```
int(1/((a + c*x^2)^2*(e*x - 1)^(3/2)*(e*x + 1)^(3/2)),x)
```

output
```
int(1/((a + c*x^2)^2*(e*x - 1)^(3/2)*(e*x + 1)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(-1+ex)^{3/2}(1+ex)^{3/2}\left(a+cx^2\right)^2}\,dx = \int \frac{}{\sqrt{ex+1}\,\sqrt{ex-1}\,a^2e^2x^2-\sqrt{ex+1}\,\sqrt{ex-1}\,a^2+2\sqrt{ex+1}}$$

input
```
int(1/(e*x-1)^(3/2)/(e*x+1)^(3/2)/(c*x^2+a)^2,x)
```

output
```
int(1/(sqrt(e*x + 1)*sqrt(e*x - 1)*a**2*e**2*x**2 - sqrt(e*x + 1)*sqrt(e*x
 - 1)*a**2 + 2*sqrt(e*x + 1)*sqrt(e*x - 1)*a*c*e**2*x**4 - 2*sqrt(e*x + 1)
*sqrt(e*x - 1)*a*c*x**2 + sqrt(e*x + 1)*sqrt(e*x - 1)*c**2*e**2*x**6 - sqr
t(e*x + 1)*sqrt(e*x - 1)*c**2*x**4),x)
```

3.25 $\int \sqrt{d-ex}\sqrt{d+ex}\left(a+cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 347

$$\int \sqrt{d-ex}\sqrt{d+ex}\left(a+cx^2\right)^{3/2}\,dx = \frac{1}{15}\left(3a-\frac{cd^2}{e^2}\right)x\sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2} + \frac{1}{5}x\sqrt{d-ex}\sqrt{d+ex}\left(a+cx^2\right)^{3/2} + \frac{d(2c^2d^4+7acd^2e^2-3a^2e^4)\sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}E\left(\arcsin\left(\frac{ex}{d}\right)|-\frac{cd^2}{ae^2}\right)}{15ce^3\sqrt{1+\frac{cx^2}{a}}\,(d^2-e^2x^2)} - \frac{ad(cd^2-3ae^2)(cd^2+ae^2)\sqrt{d-ex}\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{15ce^3\sqrt{a+cx^2}\,(d^2-e^2x^2)}$$

output

```
1/15*(3*a-c*d^2/e^2)*x*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)+1/5*x*
(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(3/2)+1/15*d*(-3*a^2*e^4+7*a*c*d^2*
e^2+2*c^2*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)*(1-e^2*x^2/d^2
)^(1/2)*EllipticE(e*x/d,(-c*d^2/a/e^2)^(1/2))/c/e^3/(1+c*x^2/a)^(1/2)/(-e^
2*x^2+d^2)-1/15*a*d*(-3*a*e^2+c*d^2)*(a*e^2+c*d^2)*(-e*x+d)^(1/2)*(e*x+d)^
(1/2)*(1+c*x^2/a)^(1/2)*(1-e^2*x^2/d^2)^(1/2)*EllipticF(e*x/d,(-c*d^2/a/e^
2)^(1/2))/c/e^3/(c*x^2+a)^(1/2)/(-e^2*x^2+d^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.26 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.84

$$\int \sqrt{d-ex}\sqrt{d+ex}\left(a+cx^2\right)^{3/2}\,dx = \frac{-\sqrt{\frac{c}{a}}e^2x(a+cx^2)\left(-d^2+e^2x^2\right)\left(-cd^2+6ae^2+3ce^2x^2\right)-id^2\left(-2c^2d^4-7acd^2e^2+3a^2e^4\right)\sqrt{\cdots}}{\cdots}$$

input
```
Integrate[Sqrt[d - e*x]*Sqrt[d + e*x]*(a + c*x^2)^(3/2),x]
```

output
```
(-(Sqrt[c/a]*e^2*x*(a + c*x^2)*(-d^2 + e^2*x^2)*(-(c*d^2) + 6*a*e^2 + 3*c*
e^2*x^2)) - I*d^2*(-2*c^2*d^4 - 7*a*c*d^2*e^2 + 3*a^2*e^4)*Sqrt[1 + (c*x^2
)/a]*Sqrt[1 - (e^2*x^2)/d^2]*EllipticE[I*ArcSinh[Sqrt[c/a]*x], -((a*e^2)/(
c*d^2))] - (2*I)*d^2*(c^2*d^4 + 4*a*c*d^2*e^2 + 3*a^2*e^4)*Sqrt[1 + (c*x^2
)/a]*Sqrt[1 - (e^2*x^2)/d^2]*EllipticF[I*ArcSinh[Sqrt[c/a]*x], -((a*e^2)/(
c*d^2))])/(15*Sqrt[c/a]*e^4*Sqrt[d - e*x]*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {648, 318, 25, 403, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^{3/2}\sqrt{d-ex}\sqrt{d+ex}\,dx$$

$$\Big\downarrow \text{648}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\int \left(cx^2+a\right)^{3/2}\sqrt{d^2-e^2x^2}dx}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{318}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(-\frac{\int -\frac{\sqrt{d^2-e^2x^2}\left(2c\left(cd^2+3ae^2\right)x^2+a\left(cd^2+5ae^2\right)\right)}{\sqrt{cx^2+a}}dx}{5e^2}-\frac{cx\sqrt{a+cx^2}\left(d^2-e^2x^2\right)^{3/2}}{5e^2}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\int \frac{\sqrt{d^2-e^2x^2}\left(2c\left(cd^2+3ae^2\right)x^2+a\left(cd^2+5ae^2\right)\right)}{\sqrt{cx^2+a}}dx}{5e^2}-\frac{cx\sqrt{a+cx^2}\left(d^2-e^2x^2\right)^{3/2}}{5e^2}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 403$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{\int \frac{c\left(a\left(cd^2+9ae^2\right)d^2+\left(2c^2d^4+7ace^2d^2-3a^2e^4\right)x^2\right)}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{3c}+\frac{2}{3}x\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}\left(3ae^2+cd^2\right)}{5e^2}-\frac{cx\sqrt{a+cx^2}\left(d^2-e^2x^2\right)^{3/2}}{5e^2}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{1}{3}\int \frac{a\left(cd^2+9ae^2\right)d^2+\left(2c^2d^4+7ace^2d^2-3a^2e^4\right)x^2}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx+\frac{2}{3}x\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}\left(3ae^2+cd^2\right)}{5e^2}-\frac{cx\sqrt{a+cx^2}\left(d^2-e^2x^2\right)^{3/2}}{5e^2}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{1}{3}\left(\frac{\left(-3a^2e^4+7acd^2e^2+2c^2d^4\right)\int \frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{a\left(cd^2-3ae^2\right)\left(ae^2+cd^2\right)\int \frac{1}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{c}\right)+\frac{2}{3}x\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}{5e^2}\right.}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 323$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{1}{3}\left(\frac{\left(-3a^2e^4+7acd^2e^2+2c^2d^4\right)\int\frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{a\sqrt{1-\frac{e^2x^2}{d^2}}\left(cd^2-3ae^2\right)\left(ae^2+cd^2\right)\int\frac{1}{\sqrt{cx^2+a}\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{d^2-e^2x^2}}\right)+\frac{2}{3}x\sqrt{a+cx^2}\sqrt{\cdots}}{5e^2}\cdots\right)}{\sqrt{d^2-e^2x^2}}$$

$\downarrow$ 323

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{1}{3}\left(\frac{\left(-3a^2e^4+7acd^2e^2+2c^2d^4\right)\int\frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{a\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(cd^2-3ae^2\right)\left(ae^2+cd^2\right)\int\frac{1}{\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)+\frac{2}{3}x\sqrt{\cdots}}{5e^2}\cdots\right)}{\sqrt{d^2-e^2x^2}}$$

$\downarrow$ 321

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{1}{3}\left(\frac{\left(-3a^2e^4+7acd^2e^2+2c^2d^4\right)\int\frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(cd^2-3ae^2\right)\left(ae^2+cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)+\cdots}{5e^2}\cdots\right)}{\sqrt{d^2-e^2x^2}}$$

$\downarrow$ 331

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{1}{3}\left(\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\left(-3a^2e^4+7acd^2e^2+2c^2d^4\right)\int\frac{\sqrt{cx^2+a}}{\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{d^2-e^2x^2}}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(cd^2-3ae^2\right)\left(ae^2+cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),\cdots\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\cdots\right)}{5e^2}\cdots\right)}{\sqrt{d^2-e^2x^2}}$$

$\downarrow$ 330

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{1}{3}\left(\frac{\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}\left(-3a^2e^4+7acd^2e^2+2c^2d^4\right)\int\frac{\sqrt{\frac{cx^2}{a}+1}}{\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{\frac{cx^2}{a}+1}\sqrt{d^2-e^2x^2}}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(cd^2-3ae^2\right)\left(ae^2+cd^2\right)\text{EllipticF}\left(\text{arcsi}\right.}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)}{5e^2}\right)}{\sqrt{d^2-e^2x^2}}$$

$\downarrow$ 327

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{1}{3}\left(\frac{d\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}\left(-3a^2e^4+7acd^2e^2+2c^2d^4\right)E\left(\arcsin\left(\frac{ex}{d}\right)|-\frac{cd^2}{ae^2}\right)}{ce\sqrt{\frac{cx^2}{a}+1}\sqrt{d^2-e^2x^2}}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(cd^2-3ae^2\right)\left(ae^2+cd^2\right)\text{Elliptic}}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)}{5e^2}\right)}{\sqrt{d^2-e^2x^2}}$$

input
```
Int[Sqrt[d - e*x]*Sqrt[d + e*x]*(a + c*x^2)^(3/2),x]
```

output
```
(Sqrt[d - e*x]*Sqrt[d + e*x]*(-1/5*(c*x*Sqrt[a + c*x^2]*(d^2 - e^2*x^2)^(3
/2))/e^2 + ((2*(c*d^2 + 3*a*e^2)*x*Sqrt[a + c*x^2]*Sqrt[d^2 - e^2*x^2])/3
+ ((d*(2*c^2*d^4 + 7*a*c*d^2*e^2 - 3*a^2*e^4)*Sqrt[a + c*x^2]*Sqrt[1 - (e^
2*x^2)/d^2]*EllipticE[ArcSin[(e*x)/d], -((c*d^2)/(a*e^2))])/(c*e*Sqrt[1 +
(c*x^2)/a]*Sqrt[d^2 - e^2*x^2]) - (a*d*(c*d^2 - 3*a*e^2)*(c*d^2 + a*e^2)*S
qrt[1 + (c*x^2)/a]*Sqrt[1 - (e^2*x^2)/d^2]*EllipticF[ArcSin[(e*x)/d], -((c
*d^2)/(a*e^2))])/(c*e*Sqrt[a + c*x^2]*Sqrt[d^2 - e^2*x^2]))/3)/(5*e^2)))/S
qrt[d^2 - e^2*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [A] (verified)

Time = 5.47 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.47

method	result
risch	$\frac{x(3x^2ce^2+6ae^2-cd^2)\sqrt{cx^2+a}\,\sqrt{-ex+d}\,\sqrt{ex+d}}{15e^2}+\frac{\left(\frac{(3a^2e^4-7acd^2e^2-2c^2d^4)a\sqrt{1-\frac{e^2x^2}{d^2}}\,\sqrt{1+\frac{cx^2}{a}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{e^2}{d^2}},\sqrt{-1-\frac{-ae^2}{a}}\right.\right.}{\sqrt{\frac{e^2}{d^2}}\,\sqrt{-ce^2x^4-ae^2x^2+cd^2x^2+a}}\right.}{}$
elliptic	$\frac{\sqrt{(-e^2x^2+d^2)(cx^2+a)}\left(\frac{cx^3\sqrt{-ce^2x^4-ae^2x^2+cd^2x^2+ad^2}}{5}-\frac{\left(-2ace^2+c^2d^2-\frac{c(-4ae^2+4cd^2)}{5}\right)x\sqrt{-ce^2x^4-ae^2x^2+cd^2x^2+ad^2}}{3ce^2}+\right.}{}$
default	$\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\sqrt{cx^2+a}\,\left(-3\sqrt{\frac{e^2}{d^2}}\,c^3e^4x^7-9\sqrt{\frac{e^2}{d^2}}\,ac^2e^4x^5+4\sqrt{\frac{e^2}{d^2}}\,c^3d^2e^2x^5-6\sqrt{\frac{e^2}{d^2}}\,a^2ce^4x^3+10\sqrt{\frac{e^2}{d^2}}\,ac^2d^2e^2x^3-\sqrt{\frac{e^2}{d^2}}\,c^3d^4x\right.}{}$

input
```
int((-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/e^2*x*(3*c*e^2*x^2+6*a*e^2-c*d^2)*(c*x^2+a)^(1/2)*(-e*x+d)^(1/2)*(e*x
+d)^(1/2)+1/15/e^2*((3*a^2*e^4-7*a*c*d^2*e^2-2*c^2*d^4)*a/(e^2/d^2)^(1/2)*
(1-e^2*x^2/d^2)^(1/2)*(1+c/a*x^2)^(1/2)/(-c*e^2*x^4-a*e^2*x^2+c*d^2*x^2+a*
d^2)^(1/2)/c*(EllipticF(x*(e^2/d^2)^(1/2),(-1-(-a*e^2+c*d^2)/a/e^2)^(1/2))
-EllipticE(x*(e^2/d^2)^(1/2),(-1-(-a*e^2+c*d^2)/a/e^2)^(1/2)))+a*c*d^4/(e^
2/d^2)^(1/2)*(1-e^2*x^2/d^2)^(1/2)*(1+c/a*x^2)^(1/2)/(-c*e^2*x^4-a*e^2*x^2
+c*d^2*x^2+a*d^2)^(1/2)*EllipticF(x*(e^2/d^2)^(1/2),(-1-(-a*e^2+c*d^2)/a/e
^2)^(1/2))+9*d^2*e^2*a^2/(e^2/d^2)^(1/2)*(1-e^2*x^2/d^2)^(1/2)*(1+c/a*x^2)
^(1/2)/(-c*e^2*x^4-a*e^2*x^2+c*d^2*x^2+a*d^2)^(1/2)*EllipticF(x*(e^2/d^2)^
(1/2),(-1-(-a*e^2+c*d^2)/a/e^2)^(1/2)))*((c*x^2+a)*(-e*x+d)*(e*x+d))^(1/2)
/(-e*x+d)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.68

$$\int \sqrt{d-ex}\sqrt{d+ex}\left(a+cx^2\right)^{3/2}\,dx=\frac{\left(3\,c^2e^7x^4-2\,c^2d^4e^3-7\,acd^2e^5+3\,a^2e^7-(c^2d^2e^5-6\,ace^7)x^2\right)\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{-ex+d}}{}$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(3/2),x, algorithm="frica
s")
```

output
```
1/15*((3*c^2*e^7*x^4 - 2*c^2*d^4*e^3 - 7*a*c*d^2*e^5 + 3*a^2*e^7 - (c^2*d^
2*e^5 - 6*a*c*e^7)*x^2)*sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(-e*x + d) - sqr
t(-c*e^2)*((2*c^2*d^7 + 7*a*c*d^5*e^2 - 3*a^2*d^3*e^4)*x*elliptic_e(arcsin
(d/(e*x)), -a*e^2/(c*d^2)) - (2*c^2*d^7 + 7*a*c*d^5*e^2 + 9*a^2*d*e^6 - (3
*a^2 - a*c)*d^3*e^4)*x*elliptic_f(arcsin(d/(e*x)), -a*e^2/(c*d^2))))/(c*e^
7*x)
```

Sympy [F]

$$\int \sqrt{d-ex}\sqrt{d+ex}\left(a+cx^2\right)^{3/2}\,dx=\int \left(a+cx^2\right)^{\frac{3}{2}}\sqrt{d-ex}\sqrt{d+ex}\,dx$$

input
```
integrate((-e*x+d)**(1/2)*(e*x+d)**(1/2)*(c*x**2+a)**(3/2),x)
```

output
```
Integral((a + c*x**2)**(3/2)*sqrt(d - e*x)*sqrt(d + e*x), x)
```

Maxima [F]

$$\int \sqrt{d-ex}\sqrt{d+ex}\left(a+cx^2\right)^{3/2}\,dx=\int \left(cx^2+a\right)^{\frac{3}{2}}\sqrt{ex+d}\sqrt{-ex+d}\,dx$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((c*x^2 + a)^(3/2)*sqrt(e*x + d)*sqrt(-e*x + d), x)
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{d-ex}\sqrt{d+ex}\left(a+cx^2\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(3/2),x, algorithm="giac"
)
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d-ex}\sqrt{d+ex}\left(a+cx^2\right)^{3/2}\,dx=\int \left(c\,x^2+a\right)^{3/2}\,\sqrt{d+e\,x}\,\sqrt{d-e\,x}\,dx$$

input
```
int((a + c*x^2)^(3/2)*(d + e*x)^(1/2)*(d - e*x)^(1/2),x)
```

output
```
int((a + c*x^2)^(3/2)*(d + e*x)^(1/2)*(d - e*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d-ex}\sqrt{d+ex}(a+cx^2)^{3/2}\,dx=\frac{6\sqrt{ex+d}\,\sqrt{-ex+d}\,\sqrt{c\,x^2+a}\,a\,e^2x-\sqrt{ex+d}\,\sqrt{-ex+d}\,\sqrt{c\,x^2+a}\,c\,d^2x+3\sqrt{ex+d}\,\sqrt{-}}{}$$

input
```
int((-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(3/2),x)
```

output
```
(6*sqrt(d + e*x)*sqrt(d - e*x)*sqrt(a + c*x**2)*a*e**2*x - sqrt(d + e*x)*s
qrt(d - e*x)*sqrt(a + c*x**2)*c*d**2*x + 3*sqrt(d + e*x)*sqrt(d - e*x)*sqr
t(a + c*x**2)*c*e**2*x**3 - 3*int((sqrt(d + e*x)*sqrt(d - e*x)*sqrt(a + c*
x**2)*x**2)/(a*d**2 - a*e**2*x**2 + c*d**2*x**2 - c*e**2*x**4),x)*a**2*e**
4 + 7*int((sqrt(d + e*x)*sqrt(d - e*x)*sqrt(a + c*x**2)*x**2)/(a*d**2 - a*
e**2*x**2 + c*d**2*x**2 - c*e**2*x**4),x)*a*c*d**2*e**2 + 2*int((sqrt(d +
e*x)*sqrt(d - e*x)*sqrt(a + c*x**2)*x**2)/(a*d**2 - a*e**2*x**2 + c*d**2*x
**2 - c*e**2*x**4),x)*c**2*d**4 + 9*int((sqrt(d + e*x)*sqrt(d - e*x)*sqrt(
a + c*x**2))/(a*d**2 - a*e**2*x**2 + c*d**2*x**2 - c*e**2*x**4),x)*a**2*d*
*2*e**2 + int((sqrt(d + e*x)*sqrt(d - e*x)*sqrt(a + c*x**2))/(a*d**2 - a*e
**2*x**2 + c*d**2*x**2 - c*e**2*x**4),x)*a*c*d**4)/(15*e**2)
```

3.26 $\int \sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 270

$$\int \sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2}\,dx = \frac{1}{3}x\sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2} + \frac{d\left(d^2-\frac{ae^2}{c}\right)\sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}E\left(\arcsin\left(\frac{ex}{d}\right)|-\frac{cd^2}{ae^2}\right)}{3e\sqrt{1+\frac{cx^2}{a}}(d^2-e^2x^2)} + \frac{ad(cd^2+ae^2)\sqrt{d-ex}\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\sqrt{1-\frac{e^2x^2}{d^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{3ce\sqrt{a+cx^2}(d^2-e^2x^2)}$$

output
```
1/3*x*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)+1/3*d*(d^2-a*e^2/c)*(-e
*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)*(1-e^2*x^2/d^2)^(1/2)*EllipticE(
e*x/d,(-c*d^2/a/e^2)^(1/2))/e/(1+c*x^2/a)^(1/2)/(-e^2*x^2+d^2)+1/3*a*d*(a*
e^2+c*d^2)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(1+c*x^2/a)^(1/2)*(1-e^2*x^2/d^2)^
(1/2)*EllipticF(e*x/d,(-c*d^2/a/e^2)^(1/2))/c/e/(c*x^2+a)^(1/2)/(-e^2*x^2+
d^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.90 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.88

$$\int \sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2}\,dx$$
$$= \frac{\sqrt{\frac{c}{a}}e^2x(a+cx^2)\left(d^2-e^2x^2\right)-id^2(-cd^2+ae^2)\sqrt{1+\frac{cx^2}{a}}\sqrt{1-\frac{e^2x^2}{d^2}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{c}{a}}x\right)|-\frac{ae^2}{cd^2}\right)-id^2(cd^2}{3\sqrt{\frac{c}{a}}e^2\sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2}}$$

input
```
Integrate[Sqrt[d - e*x]*Sqrt[d + e*x]*Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[c/a]*e^2*x*(a + c*x^2)*(d^2 - e^2*x^2) - I*d^2*(-(c*d^2) + a*e^2)*Sq
rt[1 + (c*x^2)/a]*Sqrt[1 - (e^2*x^2)/d^2]*EllipticE[I*ArcSinh[Sqrt[c/a]*x]
, -((a*e^2)/(c*d^2))] - I*d^2*(c*d^2 + a*e^2)*Sqrt[1 + (c*x^2)/a]*Sqrt[1 -
 (e^2*x^2)/d^2]*EllipticF[I*ArcSinh[Sqrt[c/a]*x], -((a*e^2)/(c*d^2))])/(3*
Sqrt[c/a]*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]*Sqrt[a + c*x^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {648, 319, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}\sqrt{d-ex}\sqrt{d+ex}\,dx$$
$$\Big\downarrow \text{648}$$
$$\frac{\sqrt{d-ex}\sqrt{d+ex}\int \sqrt{cx^2+a}\sqrt{d^2-e^2x^2}dx}{\sqrt{d^2-e^2x^2}}$$
$$\Big\downarrow \text{319}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{2}{3}\int \frac{2ad^2+\left(cd^2-ae^2\right)x^2}{2\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx+\frac{1}{3}x\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{1}{3}\int \frac{2ad^2+\left(cd^2-ae^2\right)x^2}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx+\frac{1}{3}x\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{1}{3}\left(\frac{a\left(ae^2+cd^2\right)\int \frac{1}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{c}+\frac{\left(cd^2-ae^2\right)\int \frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}\right)+\frac{1}{3}x\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 323$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{1}{3}\left(\frac{\left(cd^2-ae^2\right)\int \frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}+\frac{a\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\int \frac{1}{\sqrt{cx^2+a}\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{d^2-e^2x^2}}\right)+\frac{1}{3}x\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}\right.}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 323$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{1}{3}\left(\frac{\left(cd^2-ae^2\right)\int \frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}+\frac{a\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\int \frac{1}{\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)+\frac{1}{3}x\sqrt{a+cx^2}\sqrt{d^2}\right.}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 321$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{1}{3}\left(\frac{\left(cd^2-ae^2\right)\int \frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}+\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)+\frac{1}{3}x\sqrt{a+cx^2}\right.}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 331$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{1}{3}\left(\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\left(cd^2-ae^2\right)\int \frac{\sqrt{cx^2+a}}{\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{d^2-e^2x^2}}+\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)+\frac{1}{3}x\right.}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 330$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{1}{3}\left(\frac{\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}\left(cd^2-ae^2\right)\int \frac{\sqrt{\frac{cx^2}{a}+1}}{\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{\frac{cx^2}{a}+1}\sqrt{d^2-e^2x^2}}+\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)\right.}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{1}{3}\left(\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}+\frac{d\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}\left(cd^2-ae^2\right)E\left(\arcsin\left(\frac{ex}{d}\right)\right|-}{ce\sqrt{\frac{cx^2}{a}+1}\sqrt{d^2-e^2x^2}}\right.\right.}{\sqrt{d^2-e^2x^2}}$$

input
```
Int[Sqrt[d - e*x]*Sqrt[d + e*x]*Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[d - e*x]*Sqrt[d + e*x]*((x*Sqrt[a + c*x^2]*Sqrt[d^2 - e^2*x^2])/3 +
((d*(c*d^2 - a*e^2)*Sqrt[a + c*x^2]*Sqrt[1 - (e^2*x^2)/d^2]*EllipticE[ArcS
in[(e*x)/d], -((c*d^2)/(a*e^2))])/(c*e*Sqrt[1 + (c*x^2)/a]*Sqrt[d^2 - e^2*
x^2]) + (a*d*(c*d^2 + a*e^2)*Sqrt[1 + (c*x^2)/a]*Sqrt[1 - (e^2*x^2)/d^2]*E
llipticF[ArcSin[(e*x)/d], -((c*d^2)/(a*e^2))])/(c*e*Sqrt[a + c*x^2]*Sqrt[d
^2 - e^2*x^2]))/3))/Sqrt[d^2 - e^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 319
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[x*(a + b*x^2)^p*((c + d*x^2)^q/(2*(p + q) + 1)), x] + Simp[2/(2*(p + q) +
 1)   Int[(a + b*x^2)^(p - 1)*(c + d*x^2)^(q - 1)*Simp[a*c*(p + q) + (q*(b*
c - a*d) + a*d*(p + q))*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 2, p, q,
x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.30

method	result
risch	$\frac{x\sqrt{-ex+d}\,\sqrt{ex+d}\,\sqrt{c\,x^2+a}}{3}+\frac{\left(\frac{\left(a\,e^2-c\,d^2\right)a\sqrt{1-\frac{e^2x^2}{d^2}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{e^2}{d^2}},\sqrt{-1-\frac{-a\,e^2+c\,d^2}{a\,e^2}}\right)-\text{EllipticE}\left(x\sqrt{\frac{e^2}{d^2}},\sqrt{-1-\frac{-a\,e}{c}}\right.\right.}{3\sqrt{\frac{e^2}{d^2}}\,\sqrt{-c\,e^2x^4-a\,e^2x^2+c\,d^2x^2+a\,d^2}\,c}}{\sqrt{-ex+d}\,\sqrt{}}$
elliptic	$\frac{\sqrt{(-e^2x^2+d^2)(c\,x^2+a)}\left(\frac{x\sqrt{-c\,e^2x^4-a\,e^2x^2+c\,d^2x^2+a\,d^2}}{3}+\frac{2a\,d^2\sqrt{1-\frac{e^2x^2}{d^2}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\text{EllipticF}\left(x\sqrt{\frac{e^2}{d^2}},\sqrt{-1-\frac{-a\,e^2+c\,d^2}{a\,e^2}}\right)}{3\sqrt{\frac{e^2}{d^2}}\,\sqrt{-c\,e^2x^4-a\,e^2x^2+c\,d^2x^2+a\,d^2}}-\frac{\left(-\frac{a\,e^2}{3}+\right.}{}\right.}{\sqrt{-ex+d}\,\sqrt{ex+d}\,\sqrt{c\,x^2+a}}$
default	$\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\sqrt{c\,x^2+a}\left(-\sqrt{\frac{e^2}{d^2}}\,c^2e^2x^5-\sqrt{\frac{e^2}{d^2}}\,ac\,e^2x^3+\sqrt{\frac{e^2}{d^2}}\,c^2d^2x^3+\sqrt{\frac{-e^2x^2+d^2}{d^2}}\,\sqrt{\frac{c\,x^2+a}{a}}\,\text{EllipticF}\left(x\sqrt{\frac{e^2}{d^2}},\sqrt{-\frac{c\,d^2}{a\,e^2}}\right)a^2e^2\right.}{}$

input
```
int((-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)+(1/3*(a*e^2-c*d^2)*a/(e
^2/d^2)^(1/2)*(1-e^2*x^2/d^2)^(1/2)*(1+c/a*x^2)^(1/2)/(-c*e^2*x^4-a*e^2*x^
2+c*d^2*x^2+a*d^2)^(1/2)/c*(EllipticF(x*(e^2/d^2)^(1/2),(-1-(-a*e^2+c*d^2)
/a/e^2)^(1/2))-EllipticE(x*(e^2/d^2)^(1/2),(-1-(-a*e^2+c*d^2)/a/e^2)^(1/2)
))+2/3*a*d^2/(e^2/d^2)^(1/2)*(1-e^2*x^2/d^2)^(1/2)*(1+c/a*x^2)^(1/2)/(-c*e
^2*x^4-a*e^2*x^2+c*d^2*x^2+a*d^2)^(1/2)*EllipticF(x*(e^2/d^2)^(1/2),(-1-(-
a*e^2+c*d^2)/a/e^2)^(1/2)))*((c*x^2+a)*(-e*x+d)*(e*x+d))^(1/2)/(-e*x+d)^(1
/2)/(e*x+d)^(1/2)/(c*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.58

$$\int \sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2}\,dx$$
$$=\frac{(ce^5x^2-cd^2e^3+ae^5)\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{-ex+d}-\sqrt{-ce^2}\Big((cd^5-ad^3e^2)xE(\arcsin\left(\frac{d}{ex}\right)\mid-\frac{ae^2}{cd^2})-(cd}{3\,ce^5x}$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="frica
s")
```

output
```
1/3*((c*e^5*x^2 - c*d^2*e^3 + a*e^5)*sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(-e
*x + d) - sqrt(-c*e^2)*((c*d^5 - a*d^3*e^2)*x*elliptic_e(arcsin(d/(e*x)),
-a*e^2/(c*d^2)) - (c*d^5 - a*d^3*e^2 + 2*a*d*e^4)*x*elliptic_f(arcsin(d/(e
*x)), -a*e^2/(c*d^2))))/(c*e^5*x)
```

Sympy [F]

$$\int \sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2}\,dx=\int \sqrt{a+cx^2}\sqrt{d-ex}\sqrt{d+ex}\,dx$$

input
```
integrate((-e*x+d)**(1/2)*(e*x+d)**(1/2)*(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*sqrt(d - e*x)*sqrt(d + e*x), x)
```

Maxima [F]

$$\int \sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2}\,dx=\int \sqrt{cx^2+a}\sqrt{ex+d}\sqrt{-ex+d}\,dx$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(-e*x + d), x)
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2}\,dx=\text{Timed out}$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac"
)
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2}\,dx=\int \sqrt{c\,x^2+a}\,\sqrt{d+e\,x}\,\sqrt{d-e\,x}\,dx$$

input
```
int((a + c*x^2)^(1/2)*(d + e*x)^(1/2)*(d - e*x)^(1/2),x)
```

output
```
int((a + c*x^2)^(1/2)*(d + e*x)^(1/2)*(d - e*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2}\,dx=\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\sqrt{c\,x^2+a}\,x}{3}-\frac{\left(\int \frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\sqrt{c\,x^2+a}\,x^2}{-c\,e^2x^4-a\,e^2x^2+c\,d^2x^2+a\,d^2}dx\right)a\,e^2}{3}+\frac{\left(\int \frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\sqrt{c\,x^2+a}\,x^2}{-c\,e^2x^4-a\,e^2x^2+c\,d^2x^2+a\,d^2}dx\right)c\,d^2}{3}+\frac{2\left(\int \frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\sqrt{c\,x^2+a}}{-c\,e^2x^4-a\,e^2x^2+c\,d^2x^2+a\,d^2}dx\right)a\,d^2}{3}$$

input
```
int((-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2),x)
```

output
```
(sqrt(d + e*x)*sqrt(d - e*x)*sqrt(a + c*x**2)*x - int((sqrt(d + e*x)*sqrt(
d - e*x)*sqrt(a + c*x**2)*x**2)/(a*d**2 - a*e**2*x**2 + c*d**2*x**2 - c*e*
*2*x**4),x)*a*e**2 + int((sqrt(d + e*x)*sqrt(d - e*x)*sqrt(a + c*x**2)*x**
2)/(a*d**2 - a*e**2*x**2 + c*d**2*x**2 - c*e**2*x**4),x)*c*d**2 + 2*int((s
qrt(d + e*x)*sqrt(d - e*x)*sqrt(a + c*x**2))/(a*d**2 - a*e**2*x**2 + c*d**
2*x**2 - c*e**2*x**4),x)*a*d**2)/3
```

3.27 $\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 189

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx = -\frac{ae\sqrt{d-ex}\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}E\left(\arcsin\left(\frac{ex}{d}\right)|-\frac{cd^2}{ae^2}\right)}{cd\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{\left(\frac{d}{e}+\frac{ae}{cd}\right)\sqrt{d-ex}\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}}$$

output
```
-a*e*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(1+c*x^2/a)^(1/2)*EllipticE(e*x/d,(-c*d^
2/a/e^2)^(1/2))/c/d/(c*x^2+a)^(1/2)/(1-e^2*x^2/d^2)^(1/2)+(d/e+a*e/c/d)*(-
e*x+d)^(1/2)*(e*x+d)^(1/2)*(1+c*x^2/a)^(1/2)*EllipticF(e*x/d,(-c*d^2/a/e^2
)^(1/2))/(c*x^2+a)^(1/2)/(1-e^2*x^2/d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.66 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx = -\frac{id^2\sqrt{1+\frac{cx^2}{a}}\sqrt{1-\frac{e^2x^2}{d^2}}E\left(i\text{arcsinh}\left(\sqrt{\frac{c}{a}}x\right)|-\frac{ae^2}{cd^2}\right)}{\sqrt{\frac{c}{a}}\sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2}}$$

input
```
Integrate[(Sqrt[d - e*x]*Sqrt[d + e*x])/Sqrt[a + c*x^2],x]
```

output
```
((-I)*d^2*Sqrt[1 + (c*x^2)/a]*Sqrt[1 - (e^2*x^2)/d^2]*EllipticE[I*ArcSinh[
Sqrt[c/a]*x], -((a*e^2)/(c*d^2))])/(Sqrt[c/a]*Sqrt[d - e*x]*Sqrt[d + e*x]*
Sqrt[a + c*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {648, 326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx$$

$$\Big\downarrow \text{648}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\int \frac{\sqrt{d^2-e^2x^2}}{\sqrt{cx^2+a}}dx}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{326}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{(ae^2+cd^2)\int \frac{1}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{c}-\frac{e^2\int \frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\sqrt{1-\frac{e^2x^2}{d^2}}(ae^2+cd^2)\int\frac{1}{\sqrt{cx^2+a}\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{d^2-e^2x^2}}-\frac{e^2\int\frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \textbf{323}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}(ae^2+cd^2)\int\frac{1}{\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}-\frac{e^2\int\frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \textbf{321}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{d\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}(ae^2+cd^2)\,\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}-\frac{e^2\int\frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \textbf{331}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{d\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}(ae^2+cd^2)\,\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}-\frac{e^2\sqrt{1-\frac{e^2x^2}{d^2}}\int\frac{\sqrt{cx^2+a}}{\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{d^2-e^2x^2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \textbf{330}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{d\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}(ae^2+cd^2)\,\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}-\frac{e^2\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}\int\frac{\sqrt{\frac{cx^2}{a}+1}}{\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{\frac{cx^2}{a}+1}\sqrt{d^2-e^2x^2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \textbf{327}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{d\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}(ae^2+cd^2)\,\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}-\frac{de\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}E\left(\arcsin\left(\frac{ex}{d}\right)|-\frac{cd^2}{ae^2}\right)}{c\sqrt{\frac{cx^2}{a}+1}\sqrt{d^2-e^2x^2}}\right)}{\sqrt{d^2-e^2x^2}}$$

input
```
Int[(Sqrt[d - e*x]*Sqrt[d + e*x])/Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[d - e*x]*Sqrt[d + e*x]*(-((d*e*Sqrt[a + c*x^2]*Sqrt[1 - (e^2*x^2)/d^
2]*EllipticE[ArcSin[(e*x)/d], -((c*d^2)/(a*e^2))])/(c*Sqrt[1 + (c*x^2)/a]*
Sqrt[d^2 - e^2*x^2])) + (d*(c*d^2 + a*e^2)*Sqrt[1 + (c*x^2)/a]*Sqrt[1 - (e
^2*x^2)/d^2]*EllipticF[ArcSin[(e*x)/d], -((c*d^2)/(a*e^2))])/(c*e*Sqrt[a +
 c*x^2]*Sqrt[d^2 - e^2*x^2])))/Sqrt[d^2 - e^2*x^2]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.05

method	result
default	$\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\sqrt{c\,x^2+a}\,\sqrt{\frac{-e^2x^2+d^2}{d^2}}\,\sqrt{\frac{c\,x^2+a}{a}}\left(a\,e^2\,\mathrm{EllipticF}\left(x\sqrt{\frac{e^2}{d^2}},\sqrt{-\frac{c\,d^2}{a\,e^2}}\right)+d^2\,\mathrm{EllipticF}\left(x\sqrt{\frac{e^2}{d^2}},\sqrt{-\frac{c\,d^2}{a\,e^2}}\right)c-a\,e^2\,\mathrm{Ellipti}\right.}{\left(-c\,e^2x^4-a\,e^2x^2+c\,d^2x^2+a\,d^2\right)\sqrt{\frac{e^2}{d^2}}\,c}$
elliptic	$\frac{\sqrt{\left(-e^2x^2+d^2\right)\left(c\,x^2+a\right)}\left(\frac{d^2\sqrt{1-\frac{e^2x^2}{d^2}}\,\sqrt{1+\frac{c\,x^2}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e^2}{d^2}},\sqrt{-1-\frac{-a\,e^2+c\,d^2}{a\,e^2}}\right)}{\sqrt{\frac{e^2}{d^2}}\,\sqrt{-c\,e^2x^4-a\,e^2x^2+c\,d^2x^2+a\,d^2}}+\frac{e^2a\sqrt{1-\frac{e^2x^2}{d^2}}\,\sqrt{1+\frac{c\,x^2}{a}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e^2}{d^2}},\sqrt{-1-}\right.\right.}{\sqrt{\frac{e^2}{d^2}}\,\sqrt{-c\,e^2x^4-}}\right.}{\sqrt{-ex+d}\,\sqrt{ex+d}\,\sqrt{c\,x^2+a}}$

input
```
int((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)*((-e^2*x^2+d^2)/d^2)^(1/2)*((
c*x^2+a)/a)^(1/2)*(a*e^2*EllipticF(x*(e^2/d^2)^(1/2),(-c*d^2/a/e^2)^(1/2))
+d^2*EllipticF(x*(e^2/d^2)^(1/2),(-c*d^2/a/e^2)^(1/2))*c-a*e^2*EllipticE(x
*(e^2/d^2)^(1/2),(-c*d^2/a/e^2)^(1/2)))/(-c*e^2*x^4-a*e^2*x^2+c*d^2*x^2+a*
d^2)/(e^2/d^2)^(1/2)/c
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx = \frac{\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{-ex+d}e^3 + \left(d^3xE\left(\arcsin\left(\frac{d}{ex}\right) \mid -\frac{ae^2}{cd^2}\right) - (d^3-de^2)xF\left(\arcsin\left(\frac{d}{ex}\right) \mid -\frac{ae^2}{cd^2}\right)\right)\sqrt{-ce^2}}{ce^3x}$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="frica
s")
```

output
```
(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(-e*x + d)*e^3 + (d^3*x*elliptic_e(arcs
in(d/(e*x)), -a*e^2/(c*d^2)) - (d^3 - d*e^2)*x*elliptic_f(arcsin(d/(e*x)),
 -a*e^2/(c*d^2)))*sqrt(-c*e^2))/(c*e^3*x)
```

Sympy [F]

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{d-ex}\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx$$

input
```
integrate((-e*x+d)**(1/2)*(e*x+d)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(d - e*x)*sqrt(d + e*x)/sqrt(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{ex+d}\sqrt{-ex+d}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(e*x + d)*sqrt(-e*x + d)/sqrt(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{ex+d}\sqrt{-ex+d}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(e*x + d)*sqrt(-e*x + d)/sqrt(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{d+e\,x}\,\sqrt{d-e\,x}}{\sqrt{c\,x^2+a}}\,dx$$

input
```
int(((d + e*x)^(1/2)*(d - e*x)^(1/2))/(a + c*x^2)^(1/2),x)
```

output
```
int(((d + e*x)^(1/2)*(d - e*x)^(1/2))/(a + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\sqrt{c\,x^2+a}}{c\,x^2+a}dx$$

input
```
int((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x)*sqrt(d - e*x)*sqrt(a + c*x**2))/(a + c*x**2),x)
```

3.28 $\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{3/2}} dx$

Optimal result

Integrand size = 31, antiderivative size = 98

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{3/2}} dx = \frac{\sqrt{d-ex}\sqrt{d+ex}E\left(\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)|1+\frac{ae^2}{cd^2}\right)}{\sqrt{a}\sqrt{c}\sqrt{a+cx^2}\sqrt{\frac{a(d^2-e^2x^2)}{d^2(a+cx^2)}}}$$

output
```
(-e*x+d)^(1/2)*(e*x+d)^(1/2)*EllipticE(c^(1/2)*x/a^(1/2)/(1+c*x^2/a)^(1/2)
,(1+a*e^2/c/d^2)^(1/2))/a^(1/2)/c^(1/2)/(c*x^2+a)^(1/2)/(a*(-e^2*x^2+d^2)/
d^2/(c*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.19 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{3/2}} dx = \frac{\sqrt{\frac{c}{a}}\left(\sqrt{\frac{c}{a}}x(d^2-e^2x^2)+id^2\sqrt{1+\frac{cx^2}{a}}\sqrt{1-\frac{e^2x^2}{d^2}}E\left(i\text{arcsinh}\left(\sqrt{\frac{c}{a}}x\right)|-\frac{ae^2}{cd^2}\right)-id^2\cdots\right.}{c\sqrt{d-ex}\sqrt{d+ex}\sqrt{a+c\cdots}}$$

input
```
Integrate[(Sqrt[d - e*x]*Sqrt[d + e*x])/(a + c*x^2)^(3/2),x]
```

output
```
(Sqrt[c/a]*(Sqrt[c/a]*x*(d^2 - e^2*x^2) + I*d^2*Sqrt[1 + (c*x^2)/a]*Sqrt[1
 - (e^2*x^2)/d^2]*EllipticE[I*ArcSinh[Sqrt[c/a]*x], -((a*e^2)/(c*d^2))] -
I*d^2*Sqrt[1 + (c*x^2)/a]*Sqrt[1 - (e^2*x^2)/d^2]*EllipticF[I*ArcSinh[Sqrt
[c/a]*x], -((a*e^2)/(c*d^2))]))/(c*Sqrt[d - e*x]*Sqrt[d + e*x]*Sqrt[a + c*
x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 251 vs. $2(98) = 196$.

Time = 0.43 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.56, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {648, 314, 25, 27, 389, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{648}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\int \frac{\sqrt{d^2-e^2x^2}}{(cx^2+a)^{3/2}}dx}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{x\sqrt{d^2-e^2x^2}}{a\sqrt{a+cx^2}}-\frac{\int -\frac{e^2x^2}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{a}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\int \frac{e^2x^2}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{a}+\frac{x\sqrt{d^2-e^2x^2}}{a\sqrt{a+cx^2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{e^2\int \frac{x^2}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{a}+\frac{x\sqrt{d^2-e^2x^2}}{a\sqrt{a+cx^2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{389}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{e^2\left(\frac{\int \frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{a\int \frac{1}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{c}\right)}{a}+\frac{x\sqrt{d^2-e^2x^2}}{a\sqrt{a+cx^2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{e^2\left(\frac{\int \frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{a\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{1}{\sqrt{cx^2+a}\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{d^2-e^2x^2}}\right)}{a}+\frac{x\sqrt{d^2-e^2x^2}}{a\sqrt{a+cx^2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{e^2\left(\frac{\int \frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{a\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\int \frac{1}{\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)}{a}+\frac{x\sqrt{d^2-e^2x^2}}{a\sqrt{a+cx^2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{e^2\left(\frac{\int \frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)}{a}+\frac{x\sqrt{d^2-e^2x^2}}{a\sqrt{a+cx^2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{331}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{e^2\left(\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\int\frac{\sqrt{cx^2+a}}{\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{d^2-e^2x^2}}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)}{a}+\frac{x\sqrt{d^2-e^2x^2}}{a\sqrt{a+cx^2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{330}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{e^2\left(\frac{\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}\int\frac{\sqrt{\frac{cx^2}{a}+1}}{\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{\frac{cx^2}{a}+1}\sqrt{d^2-e^2x^2}}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)}{a}+\frac{x\sqrt{d^2-e^2x^2}}{a\sqrt{a+cx^2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{e^2\left(\frac{d\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}E\left(\arcsin\left(\frac{ex}{d}\right)|-\frac{cd^2}{ae^2}\right)}{ce\sqrt{\frac{cx^2}{a}+1}\sqrt{d^2-e^2x^2}}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)}{a}+\frac{x\sqrt{d^2-e^2x^2}}{a\sqrt{a+cx^2}}\right)}{\sqrt{d^2-e^2x^2}}$$

input

```
Int[(Sqrt[d - e*x]*Sqrt[d + e*x])/(a + c*x^2)^(3/2),x]
```

output

```
(Sqrt[d - e*x]*Sqrt[d + e*x]*((x*Sqrt[d^2 - e^2*x^2])/(a*Sqrt[a + c*x^2])
+ (e^2*((d*Sqrt[a + c*x^2]*Sqrt[1 - (e^2*x^2)/d^2]*EllipticE[ArcSin[(e*x)/
d], -((c*d^2)/(a*e^2))])/(c*e*Sqrt[1 + (c*x^2)/a]*Sqrt[d^2 - e^2*x^2]) - (
a*d*Sqrt[1 + (c*x^2)/a]*Sqrt[1 - (e^2*x^2)/d^2]*EllipticF[ArcSin[(e*x)/d],
 -((c*d^2)/(a*e^2))])/(c*e*Sqrt[a + c*x^2]*Sqrt[d^2 - e^2*x^2])))/a))/Sqrt
[d^2 - e^2*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(95) = 190$.

Time = 1.79 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.39

method	result
default	$\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\sqrt{c\,x^{2}+a}\,\left(-\sqrt{\frac{e^{2}}{d^{2}}}\,c\,e^{2}x^{3}-\sqrt{\frac{-e^{2}x^{2}+d^{2}}{d^{2}}}\,\sqrt{\frac{c\,x^{2}+a}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e^{2}}{d^{2}}},\sqrt{-\frac{c\,d^{2}}{a\,e^{2}}}\right)a\,e^{2}+\sqrt{\frac{-e^{2}x^{2}+d^{2}}{d^{2}}}\,\sqrt{\frac{c\,x^{2}+a}{a}}\,\text{Ellipt}\right.}{\left(-c\,e^{2}x^{4}-a\,e^{2}x^{2}+c\,d^{2}x^{2}+a\,d^{2}\right)ca\sqrt{\frac{e^{2}}{d^{2}}}}$
elliptic	$\frac{\sqrt{\left(-e^{2}x^{2}+d^{2}\right)\left(c\,x^{2}+a\right)}\,\left(\frac{\left(-x^{2}c\,e^{2}+c\,d^{2}\right)x}{ca\sqrt{\left(x^{2}+\frac{a}{c}\right)\left(-x^{2}c\,e^{2}+c\,d^{2}\right)}}+\frac{\left(-\frac{e^{2}}{c}+\frac{a\,e^{2}+c\,d^{2}}{ac}-\frac{d^{2}}{a}\right)\sqrt{1-\frac{e^{2}x^{2}}{d^{2}}}\,\sqrt{1+\frac{c\,x^{2}}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{e^{2}}{d^{2}}},\sqrt{-1-\frac{-a\,e^{2}+c\,d^{2}}{a\,e^{2}}}\right.}{\sqrt{\frac{e^{2}}{d^{2}}}\,\sqrt{-c\,e^{2}x^{4}-a\,e^{2}x^{2}+c\,d^{2}x^{2}+a\,d^{2}}}\right.}{\sqrt{-ex+d}\,\sqrt{ex+d}\,\sqrt{c\,x^{2}+a}}$

input
```
int((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)*(-(e^2/d^2)^(1/2)*c*e^2*x^3-(
(-e^2*x^2+d^2)/d^2)^(1/2)*((c*x^2+a)/a)^(1/2)*EllipticF(x*(e^2/d^2)^(1/2),
(-c*d^2/a/e^2)^(1/2))*a*e^2+((-e^2*x^2+d^2)/d^2)^(1/2)*((c*x^2+a)/a)^(1/2)
*EllipticE(x*(e^2/d^2)^(1/2),(-c*d^2/a/e^2)^(1/2))*a*e^2+(e^2/d^2)^(1/2)*c
*d^2*x)/(-c*e^2*x^4-a*e^2*x^2+c*d^2*x^2+a*d^2)/c/a/(e^2/d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{3/2}}\,dx = \frac{\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{-ex+d}cd^3x + \sqrt{ad^2}\Big((ce^3x^2+ae^3)E(\arcsin\left(\frac{ex}{d}\right)\,|-\frac{cd^2}{ae^2}) -}{ac^2d^3x^2+a^2cd^3}$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(3/2),x, algorithm="frica
s")
```

output
```
(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(-e*x + d)*c*d^3*x + sqrt(a*d^2)*((c*e^
3*x^2 + a*e^3)*elliptic_e(arcsin(e*x/d), -c*d^2/(a*e^2)) - (c*e^3*x^2 + a*
e^3)*elliptic_f(arcsin(e*x/d), -c*d^2/(a*e^2))))/(a*c^2*d^3*x^2 + a^2*c*d^
3)
```

Sympy [F]

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{3/2}}\,dx = \int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x+d)**(1/2)*(e*x+d)**(1/2)/(c*x**2+a)**(3/2),x)
```

output
```
Integral(sqrt(d - e*x)*sqrt(d + e*x)/(a + c*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{3/2}}\,dx = \int \frac{\sqrt{ex+d}\sqrt{-ex+d}}{(cx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(e*x + d)*sqrt(-e*x + d)/(c*x^2 + a)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(3/2),x, algorithm="giac"
)
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{3/2}}\,dx = \int \frac{\sqrt{d+e\,x}\,\sqrt{d-e\,x}}{(c\,x^2+a)^{3/2}}\,dx$$

input
```
int(((d + e*x)^(1/2)*(d - e*x)^(1/2))/(a + c*x^2)^(3/2),x)
```

output
```
int(((d + e*x)^(1/2)*(d - e*x)^(1/2))/(a + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{3/2}}\,dx = \int \frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\sqrt{c\,x^2+a}}{c^2x^4+2ac\,x^2+a^2}dx$$

input
```
int((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(3/2),x)
```

output
```
int((sqrt(d + e*x)*sqrt(d - e*x)*sqrt(a + c*x**2))/(a**2 + 2*a*c*x**2 + c*
*2*x**4),x)
```

3.29 $\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 341

$$\begin{aligned}\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{5/2}}\,dx &= \frac{x\sqrt{d-ex}\sqrt{d+ex}}{3a\,(a+cx^2)^{3/2}}+\frac{(2cd^2+ae^2)\,x\sqrt{d-ex}\sqrt{d+ex}}{3a^2\,(cd^2+ae^2)\,\sqrt{a+cx^2}}\\ &+\frac{de(2cd^2+ae^2)\,\sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}\,E\left(\arcsin\left(\frac{ex}{d}\right)|-\frac{cd^2}{ae^2}\right)}{3a^2c\,(cd^2+ae^2)\,\sqrt{1+\frac{cx^2}{a}}\,(d^2-e^2x^2)}\\ &-\frac{de\sqrt{d-ex}\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{3ac\sqrt{a+cx^2}\,(d^2-e^2x^2)}\end{aligned}$$

output
```
1/3*x*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/a/(c*x^2+a)^(3/2)+1/3*(a*e^2+2*c*d^2)*x
*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/a^2/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)+1/3*d*e*(a
*e^2+2*c*d^2)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)*(1-e^2*x^2/d^2)
^(1/2)*EllipticE(e*x/d,(-c*d^2/a/e^2)^(1/2))/a^2/c/(a*e^2+c*d^2)/(1+c*x^2/
a)^(1/2)/(-e^2*x^2+d^2)-1/3*d*e*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(1+c*x^2/a)^(
1/2)*(1-e^2*x^2/d^2)^(1/2)*EllipticF(e*x/d,(-c*d^2/a/e^2)^(1/2))/a/c/(c*x^
2+a)^(1/2)/(-e^2*x^2+d^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.65 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{c}{a}}x(d^2-e^2x^2)\left(2a^2e^2+2c^2d^2x^2+ac(3d^2+e^2x^2)\right)+id^2(2cd^2+ae^2)\left(a+cx^2\right.}{}$$

input
```
Integrate[(Sqrt[d - e*x]*Sqrt[d + e*x])/(a + c*x^2)^(5/2),x]
```

output
```
(Sqrt[c/a]*x*(d^2 - e^2*x^2)*(2*a^2*e^2 + 2*c^2*d^2*x^2 + a*c*(3*d^2 + e^2
*x^2)) + I*d^2*(2*c*d^2 + a*e^2)*(a + c*x^2)*Sqrt[1 + (c*x^2)/a]*Sqrt[1 -
(e^2*x^2)/d^2]*EllipticE[I*ArcSinh[Sqrt[c/a]*x], -((a*e^2)/(c*d^2))] - (2*
I)*d^2*(c*d^2 + a*e^2)*(a + c*x^2)*Sqrt[1 + (c*x^2)/a]*Sqrt[1 - (e^2*x^2)/
d^2]*EllipticF[I*ArcSinh[Sqrt[c/a]*x], -((a*e^2)/(c*d^2))])/(3*a^2*Sqrt[c/
a]*(c*d^2 + a*e^2)*Sqrt[d - e*x]*Sqrt[d + e*x]*(a + c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {648, 314, 25, 402, 25, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{648}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\int \frac{\sqrt{d^2-e^2x^2}}{(cx^2+a)^{5/2}}dx}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{x\sqrt{d^2-e^2x^2}}{3a(a+cx^2)^{3/2}}-\frac{\int -\frac{2d^2-e^2x^2}{\left(cx^2+a\right)^{3/2}\sqrt{d^2-e^2x^2}}dx}{3a}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\int \frac{2d^2-e^2x^2}{\left(cx^2+a\right)^{3/2}\sqrt{d^2-e^2x^2}}dx}{3a}+\frac{x\sqrt{d^2-e^2x^2}}{3a(a+cx^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{x\sqrt{d^2-e^2x^2}\left(ae^2+2cd^2\right)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{\int -\frac{e^2\left(ad^2+\left(2cd^2+ae^2\right)x^2\right)}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{a\left(ae^2+cd^2\right)}}{3a}+\frac{x\sqrt{d^2-e^2x^2}}{3a(a+cx^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{\int \frac{e^2\left(ad^2+\left(2cd^2+ae^2\right)x^2\right)}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-e^2x^2}\left(ae^2+2cd^2\right)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}}{3a}+\frac{x\sqrt{d^2-e^2x^2}}{3a(a+cx^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{e^2\int \frac{ad^2+\left(2cd^2+ae^2\right)x^2}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-e^2x^2}\left(ae^2+2cd^2\right)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}}{3a}+\frac{x\sqrt{d^2-e^2x^2}}{3a(a+cx^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{ 399}$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{e^2\left(\frac{\left(ae^2+2cd^2\right)\int \frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{a\left(ae^2+cd^2\right)\int \frac{1}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{c}\right)}{a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-e^2x^2}\left(ae^2+2cd^2\right)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}}{3a}+\frac{x\sqrt{d^2-e^2x^2}}{3a(a+cx^2)^{3/2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\downarrow 323$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{e^2\left(\frac{\left(ae^2+2cd^2\right)\int\frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{a\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\int\frac{1}{\sqrt{cx^2+a}\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{d^2-e^2x^2}}\right)}{a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-e^2x^2}\left(ae^2+2cd^2\right)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}}{3a}+\frac{x\sqrt{d^2-e^2\cdots}}{3a(a+cx^2)^{\cdots}}\cdots\right.}{\sqrt{d^2-e^2x^2}}$$

$$\downarrow 323$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{e^2\left(\frac{\left(ae^2+2cd^2\right)\int\frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{a\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\int\frac{1}{\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)}{a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-e^2x^2}\left(ae^2+2cd^2\right)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}}{3a}+\frac{x\cdots}{3a\cdots}\cdots\right.}{\sqrt{d^2-e^2x^2}}$$

$$\downarrow 321$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{e^2\left(\frac{\left(ae^2+2cd^2\right)\int\frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)}{a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-e^2x^2}\left(ae^2+2cd^2\right)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}}{3a}\cdots\right.}{\sqrt{d^2-e^2x^2}}$$

$$\downarrow 331$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{e^2\left(\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+2cd^2\right)\int \frac{\sqrt{cx^2+a}}{\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{d^2-e^2x^2}}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)}{a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-e^2x^2}\left(ae^{\cdots}\right.}{a\sqrt{a+cx^2}\left(ae^{2\cdots}\right.}}{3a}\cdots\right.}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 330$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{e^2\left(\frac{\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+2cd^2\right)\int \frac{\sqrt{\frac{cx^2}{a}+1}}{\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{\frac{cx^2}{a}+1}\sqrt{d^2-e^2x^2}}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)}{a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-\cdots}}{a\sqrt{a+\cdots}}}{3a}\cdots\right.}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{e^2\left(\frac{d\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+2cd^2\right)E\left(\arcsin\left(\frac{ex}{d}\right)|-\frac{cd^2}{ae^2}\right)}{ce\sqrt{\frac{cx^2}{a}+1}\sqrt{d^2-e^2x^2}}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)}{a\left(ae^2+cd^2\right)}+\cdots}{3a}\cdots\right.}{\sqrt{d^2-e^2x^2}}$$

input
```
Int[(Sqrt[d - e*x]*Sqrt[d + e*x])/(a + c*x^2)^(5/2),x]
```

output
```
(Sqrt[d - e*x]*Sqrt[d + e*x]*((x*Sqrt[d^2 - e^2*x^2])/(3*a*(a + c*x^2)^(3/
2)) + (((2*c*d^2 + a*e^2)*x*Sqrt[d^2 - e^2*x^2])/(a*(c*d^2 + a*e^2)*Sqrt[a
 + c*x^2]) + (e^2*((d*(2*c*d^2 + a*e^2)*Sqrt[a + c*x^2]*Sqrt[1 - (e^2*x^2)
/d^2]*EllipticE[ArcSin[(e*x)/d], -((c*d^2)/(a*e^2))])/(c*e*Sqrt[1 + (c*x^2
)/a]*Sqrt[d^2 - e^2*x^2]) - (a*d*(c*d^2 + a*e^2)*Sqrt[1 + (c*x^2)/a]*Sqrt[
1 - (e^2*x^2)/d^2]*EllipticF[ArcSin[(e*x)/d], -((c*d^2)/(a*e^2))])/(c*e*Sq
rt[a + c*x^2]*Sqrt[d^2 - e^2*x^2])))/(a*(c*d^2 + a*e^2)))/(3*a)))/Sqrt[d^2
 - e^2*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.56

method	result
elliptic	$\sqrt{(-e^2x^2+d^2)(c\,x^2+a)}\left(\frac{x\sqrt{-c\,e^2x^4-a\,e^2x^2+c\,d^2x^2+a\,d^2}}{3c^2a\left(x^2+\frac{a}{c}\right)^2}+\frac{\left(-x^2c\,e^2+c\,d^2\right)x\left(a\,e^2+2c\,d^2\right)}{3c\,a^2\left(a\,e^2+c\,d^2\right)\sqrt{\left(x^2+\frac{a}{c}\right)\left(-x^2c\,e^2+c\,d^2\right)}}+\frac{\left(-\frac{e^2}{3ac}+\frac{a\,e^2+2c\,d^2}{3c\,a^2}-\frac{d^2\left(a\,e^2\cdots\right.}{3a^2\left(a\,e\cdots\right.}\cdots\right.}{\cdots}\right.\cdots$
default	$-\frac{\left(\sqrt{\frac{e^2}{d^2}}\,a\,c^2e^4x^5+2\sqrt{\frac{e^2}{d^2}}\,c^3d^2e^2x^5+\sqrt{\frac{-e^2x^2+d^2}{d^2}}\,\sqrt{\frac{c\,x^2+a}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e^2}{d^2}},\sqrt{-\frac{c\,d^2}{a\,e^2}}\right)a^2c\,e^4x^2+\sqrt{\frac{-e^2x^2+d^2}{d^2}}\,\sqrt{\frac{c\,x^2+a}{a}}\,\mathrm{Ellipti}\cdots\right.}{\cdots}$

input
```
int((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((-e^2*x^2+d^2)*(c*x^2+a))^(1/2)/(-e*x+d)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a)^(1
/2)*(1/3/c^2/a*x*(-c*e^2*x^4-a*e^2*x^2+c*d^2*x^2+a*d^2)^(1/2)/(x^2+a/c)^2+
1/3*(-c*e^2*x^2+c*d^2)/c/a^2/(a*e^2+c*d^2)*x*(a*e^2+2*c*d^2)/((x^2+a/c)*(-
c*e^2*x^2+c*d^2))^(1/2)+(-1/3/a/c*e^2+1/3/c*(a*e^2+2*c*d^2)/a^2-1/3*d^2/a^
2/(a*e^2+c*d^2)*(a*e^2+2*c*d^2))/(e^2/d^2)^(1/2)*(1-e^2*x^2/d^2)^(1/2)*(1+
c/a*x^2)^(1/2)/(-c*e^2*x^4-a*e^2*x^2+c*d^2*x^2+a*d^2)^(1/2)*EllipticF(x*(e
^2/d^2)^(1/2),(-1-(-a*e^2+c*d^2)/a/e^2)^(1/2))-1/3*e^2*(a*e^2+2*c*d^2)/a/(
a*e^2+c*d^2)/(e^2/d^2)^(1/2)*(1-e^2*x^2/d^2)^(1/2)*(1+c/a*x^2)^(1/2)/(-c*e
^2*x^4-a*e^2*x^2+c*d^2*x^2+a*d^2)^(1/2)/c*(EllipticF(x*(e^2/d^2)^(1/2),(-1
-(-a*e^2+c*d^2)/a/e^2)^(1/2))-EllipticE(x*(e^2/d^2)^(1/2),(-1-(-a*e^2+c*d^
2)/a/e^2)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{\left(a+cx^2\right)^{5/2}}\,dx=\frac{\left(\left(2\,c^3d^5+ac^2d^3e^2\right)x^3+\left(3\,ac^2d^5+2\,a^2cd^3e^2\right)x\right)\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{-ex+d}+\cdots}{\cdots}$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(5/2),x, algorithm="frica
s")
```

output
```
1/3*(((2*c^3*d^5 + a*c^2*d^3*e^2)*x^3 + (3*a*c^2*d^5 + 2*a^2*c*d^3*e^2)*x)
*sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(-e*x + d) + sqrt(a*d^2)*((2*a^2*c*d^2*
e^3 + a^3*e^5 + (2*c^3*d^2*e^3 + a*c^2*e^5)*x^4 + 2*(2*a*c^2*d^2*e^3 + a^2
*c*e^5)*x^2)*elliptic_e(arcsin(e*x/d), -c*d^2/(a*e^2)) + (a^2*c*d^4*e - 2*
a^2*c*d^2*e^3 - a^3*e^5 + (c^3*d^4*e - 2*c^3*d^2*e^3 - a*c^2*e^5)*x^4 + 2*
(a*c^2*d^4*e - 2*a*c^2*d^2*e^3 - a^2*c*e^5)*x^2)*elliptic_f(arcsin(e*x/d),
 -c*d^2/(a*e^2))))/(a^4*c^2*d^5 + a^5*c*d^3*e^2 + (a^2*c^4*d^5 + a^3*c^3*d
^3*e^2)*x^4 + 2*(a^3*c^3*d^5 + a^4*c^2*d^3*e^2)*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{5/2}}\,dx = \int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x+d)**(1/2)*(e*x+d)**(1/2)/(c*x**2+a)**(5/2),x)
```

output
```
Integral(sqrt(d - e*x)*sqrt(d + e*x)/(a + c*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{5/2}}\,dx = \int \frac{\sqrt{ex+d}\sqrt{-ex+d}}{(cx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(e*x + d)*sqrt(-e*x + d)/(c*x^2 + a)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(5/2),x, algorithm="giac"
)
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{5/2}}\,dx = \int \frac{\sqrt{d+e\,x}\,\sqrt{d-e\,x}}{(c\,x^2+a)^{5/2}}\,dx$$

input
```
int(((d + e*x)^(1/2)*(d - e*x)^(1/2))/(a + c*x^2)^(5/2),x)
```

output
```
int(((d + e*x)^(1/2)*(d - e*x)^(1/2))/(a + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{5/2}}\,dx = \int \frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\sqrt{c\,x^2+a}}{c^3x^6+3a\,c^2x^4+3a^2c\,x^2+a^3}dx$$

input
```
int((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(5/2),x)
```

output
```
int((sqrt(d + e*x)*sqrt(d - e*x)*sqrt(a + c*x**2))/(a**3 + 3*a**2*c*x**2 +
 3*a*c**2*x**4 + c**3*x**6),x)
```

3.30 $\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 461

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{7/2}}\,dx = \frac{x\sqrt{d-ex}\sqrt{d+ex}}{5a(a+cx^2)^{5/2}} + \frac{(4cd^2+3ae^2)\,x\sqrt{d-ex}\sqrt{d+ex}}{15a^2(cd^2+ae^2)(a+cx^2)^{3/2}}$$
$$+ \frac{(8c^2d^4+13acd^2e^2+3a^2e^4)\,x\sqrt{d-ex}\sqrt{d+ex}}{15a^3(cd^2+ae^2)^2\sqrt{a+cx^2}}$$
$$+ \frac{de(8c^2d^4+13acd^2e^2+3a^2e^4)\sqrt{d-ex}\sqrt{d+ex}\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}\,E\left(\arcsin\left(\frac{ex}{d}\right)|-\frac{cd^2}{ae^2}\right)}{15a^3c(cd^2+ae^2)^2\sqrt{1+\frac{cx^2}{a}}\,(d^2-e^2x^2)}$$
$$- \frac{de(4cd^2+3ae^2)\sqrt{d-ex}\sqrt{d+ex}\sqrt{1+\frac{cx^2}{a}}\sqrt{1-\frac{e^2x^2}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{15a^2c(cd^2+ae^2)\sqrt{a+cx^2}\,(d^2-e^2x^2)}$$

output
```
1/5*x*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/a/(c*x^2+a)^(5/2)+1/15*(3*a*e^2+4*c*d^2
)*x*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/a^2/(a*e^2+c*d^2)/(c*x^2+a)^(3/2)+1/15*(3
*a^2*e^4+13*a*c*d^2*e^2+8*c^2*d^4)*x*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/a^3/(a*e
^2+c*d^2)^2/(c*x^2+a)^(1/2)+1/15*d*e*(3*a^2*e^4+13*a*c*d^2*e^2+8*c^2*d^4)*
(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(c*x^2+a)^(1/2)*(1-e^2*x^2/d^2)^(1/2)*Ellipti
cE(e*x/d,(-c*d^2/a/e^2)^(1/2))/a^3/c/(a*e^2+c*d^2)^2/(1+c*x^2/a)^(1/2)/(-e
^2*x^2+d^2)-1/15*d*e*(3*a*e^2+4*c*d^2)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(1+c*x
^2/a)^(1/2)*(1-e^2*x^2/d^2)^(1/2)*EllipticF(e*x/d,(-c*d^2/a/e^2)^(1/2))/a^
2/c/(a*e^2+c*d^2)/(c*x^2+a)^(1/2)/(-e^2*x^2+d^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.40 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{7/2}}\,dx = \frac{x(d-ex)(d+ex)\left(3a^2(cd^2+ae^2)^2+a(cd^2+ae^2)(4cd^2+3ae^2)(a+cx^2)+(8\right.}{}$$

input
```
Integrate[(Sqrt[d - e*x]*Sqrt[d + e*x])/(a + c*x^2)^(7/2),x]
```

output
```
(x*(d - e*x)*(d + e*x)*(3*a^2*(c*d^2 + a*e^2)^2 + a*(c*d^2 + a*e^2)*(4*c*d
^2 + 3*a*e^2)*(a + c*x^2) + (8*c^2*d^4 + 13*a*c*d^2*e^2 + 3*a^2*e^4)*(a +
c*x^2)^2) + (I*d^2*(a + c*x^2)^2*Sqrt[1 + (c*x^2)/a]*Sqrt[1 - (e^2*x^2)/d^
2]*((8*c^2*d^4 + 13*a*c*d^2*e^2 + 3*a^2*e^4)*EllipticE[I*ArcSinh[Sqrt[c/a]
*x], -((a*e^2)/(c*d^2))] - (8*c^2*d^4 + 17*a*c*d^2*e^2 + 9*a^2*e^4)*Ellipt
icF[I*ArcSinh[Sqrt[c/a]*x], -((a*e^2)/(c*d^2))]))/Sqrt[c/a])/(15*a^3*(c*d^
2 + a*e^2)^2*Sqrt[d - e*x]*Sqrt[d + e*x]*(a + c*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {648, 314, 25, 402, 25, 402, 25, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{7/2}}\,dx$$

$$\Big\downarrow \; 648$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\int \frac{\sqrt{d^2-e^2x^2}}{(cx^2+a)^{7/2}}dx}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 314$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{x\sqrt{d^2-e^2x^2}}{5a(a+cx^2)^{5/2}}-\frac{\int -\frac{4d^2-3e^2x^2}{\left(cx^2+a\right)^{5/2}\sqrt{d^2-e^2x^2}}dx}{5a}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\int \frac{4d^2-3e^2x^2}{\left(cx^2+a\right)^{5/2}\sqrt{d^2-e^2x^2}}dx}{5a}+\frac{x\sqrt{d^2-e^2x^2}}{5a(a+cx^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{x\sqrt{d^2-e^2x^2}\left(3ae^2+4cd^2\right)}{3a\left(a+cx^2\right)^{3/2}\left(ae^2+cd^2\right)}-\frac{\int -\frac{d^2\left(8cd^2+9ae^2\right)-e^2\left(4cd^2+3ae^2\right)x^2}{\left(cx^2+a\right)^{3/2}\sqrt{d^2-e^2x^2}}dx}{3a\left(ae^2+cd^2\right)}}{5a}+\frac{x\sqrt{d^2-e^2x^2}}{5a(a+cx^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{\int \frac{d^2\left(8cd^2+9ae^2\right)-e^2\left(4cd^2+3ae^2\right)x^2}{\left(cx^2+a\right)^{3/2}\sqrt{d^2-e^2x^2}}dx}{3a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-e^2x^2}\left(3ae^2+4cd^2\right)}{3a\left(a+cx^2\right)^{3/2}\left(ae^2+cd^2\right)}}{5a}+\frac{x\sqrt{d^2-e^2x^2}}{5a(a+cx^2)^{5/2}}\right)}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 402$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{\frac{x\sqrt{d^2-e^2x^2}\left(3a^2e^4+13acd^2e^2+8c^2d^4\right)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}-\frac{\int -\frac{e^2\left(2a\left(2cd^2+3ae^2\right)d^2+\left(8c^2d^4+13ace^2d^2+3a^2e^4\right)x^2\right)}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{a\left(ae^2+cd^2\right)}}{3a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-e^2x^2}\left(3ae^2+4cd^2\right)}{3a\left(a+cx^2\right)^{3/2}\left(ae^2+cd^2\right)}}{5a}\right.}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{\frac{\int \frac{e^2\left(2a\left(2cd^2+3ae^2\right)d^2+\left(8c^2d^4+13ace^2d^2+3a^2e^4\right)x^2\right)}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-e^2x^2}\left(3a^2e^4+13acd^2e^2+8c^2d^4\right)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}}{3a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-e^2x^2}\left(3ae^2+4cd^2\right)}{3a\left(a+cx^2\right)^{3/2}\left(ae^2+cd^2\right)}}{5a}\cdots\right.}{\sqrt{d^2-e^2x^2}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{\frac{e^2\int \frac{2a\left(2cd^2+3ae^2\right)d^2+\left(8c^2d^4+13ace^2d^2+3a^2e^4\right)x^2}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-e^2x^2}\left(3a^2e^4+13acd^2e^2+8c^2d^4\right)}{a\sqrt{a+cx^2}\left(ae^2+cd^2\right)}}{3a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-e^2x^2}\left(3ae^2+4cd^2\right)}{3a\left(a+cx^2\right)^{3/2}\left(ae^2+cd^2\right)}}{5a}\cdots\right.}{\sqrt{d^2-e^2x^2}}$$

$\Big\downarrow$ 399

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{\frac{e^2\left(\frac{\left(3a^2e^4+13acd^2e^2+8c^2d^4\right)\int \frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{a\left(ae^2+cd^2\right)\left(3ae^2+4cd^2\right)\int \frac{1}{\sqrt{cx^2+a}\sqrt{d^2-e^2x^2}}dx}{c}\right)}{a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-e^2x^2}\left(3a^2e^4+13\cdots\right.}{a\sqrt{a+cx^2}\left(ae^2\cdots\right.}}{3a\left(ae^2+cd^2\right)}\cdots}{5a}\cdots\right.}{\sqrt{d^2-e^2x^2}}$$

$\Big\downarrow$ 323

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{\frac{e^2\left(\frac{\left(3a^2e^4+13acd^2e^2+8c^2d^4\right)\int \frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{a\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\left(3ae^2+4cd^2\right)\int \frac{1}{\sqrt{cx^2+a}\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{d^2-e^2x^2}}\right)}{a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d^2-e^2x^2}\left(\cdots\right)}{a\sqrt{a\cdots}}}{3a\left(ae^2+cd^2\right)}}{5a}\cdots\right)}{\sqrt{d^2-e^2x^2}}$$

$\Big\downarrow$ 323

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{\frac{e^2\left(\frac{\left(3a^2e^4+13acd^2e^2+8c^2d^4\right)\int \frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{a\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\left(3ae^2+4cd^2\right)\int \frac{1}{\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)}{a\left(ae^2+cd^2\right)}+\frac{x\sqrt{d\cdots}}{\cdots}}{3a\left(ae^2+cd^2\right)}}{5a}\cdots\right)}{\sqrt{d^2-e^2x^2}}$$

$\Big\downarrow$ 321

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{\frac{e^2\left(\frac{\left(3a^2e^4+13acd^2e^2+8c^2d^4\right)\int \frac{\sqrt{cx^2+a}}{\sqrt{d^2-e^2x^2}}dx}{c}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\left(3ae^2+4cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),-\frac{cd^2}{ae^2}\right)}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\right)}{a\left(ae^2+cd^2\right)}\cdots}{3a\left(ae^2+cd^2\right)}}{5a}\cdots\right)}{\sqrt{d^2-e^2x^2}}$$

$\Big\downarrow$ 331

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{e^2\left(\frac{\sqrt{1-\frac{e^2x^2}{d^2}}\left(3a^2e^4+13acd^2e^2+8c^2d^4\right)\int \frac{\sqrt{cx^2+a}}{\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{d^2-e^2x^2}}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\left(3ae^2+4cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{ex}{d}\right),\cdots\right.}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\cdots\right)}{a\left(ae^2+cd^2\right)}\cdots}{3a\left(ae^2+cd^2\right)}\cdots}{5a}\cdots\right)}{\sqrt{d^2-e^2x^2}}$$

$\downarrow$ 330

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{e^2\left(\frac{\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}\left(3a^2e^4+13acd^2e^2+8c^2d^4\right)\int \frac{\sqrt{\frac{cx^2}{a}+1}}{\sqrt{1-\frac{e^2x^2}{d^2}}}dx}{c\sqrt{\frac{cx^2}{a}+1}\sqrt{d^2-e^2x^2}}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\left(3ae^2+4cd^2\right)\text{EllipticF}\left(\text{arcs}\cdots\right.}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\cdots\right)}{a\left(ae^2+cd^2\right)}\cdots}{3a\left(ae^2+cd^2\right)}\cdots}{5a}\cdots\right)}{\sqrt{d^2-e^2x^2}}$$

$\downarrow$ 327

$$\frac{\sqrt{d-ex}\sqrt{d+ex}\left(\frac{\frac{e^2\left(\frac{d\sqrt{a+cx^2}\sqrt{1-\frac{e^2x^2}{d^2}}\left(3a^2e^4+13acd^2e^2+8c^2d^4\right)E\left(\arcsin\left(\frac{ex}{d}\right)|-\frac{cd^2}{ae^2}\right)}{ce\sqrt{\frac{cx^2}{a}+1}\sqrt{d^2-e^2x^2}}-\frac{ad\sqrt{\frac{cx^2}{a}+1}\sqrt{1-\frac{e^2x^2}{d^2}}\left(ae^2+cd^2\right)\left(3ae^2+4cd^2\right)\text{Ellipt}\cdots}{ce\sqrt{a+cx^2}\sqrt{d^2-e^2x^2}}\cdots\right)}{a\left(ae^2+cd^2\right)}\cdots}{3a\left(ae^2+cd^2\right)}\cdots}{5a}\cdots\right)}{\sqrt{d^2-e^2x^2}}$$

input
```
Int[(Sqrt[d - e*x]*Sqrt[d + e*x])/(a + c*x^2)^(7/2),x]
```

output
```
(Sqrt[d - e*x]*Sqrt[d + e*x]*((x*Sqrt[d^2 - e^2*x^2])/(5*a*(a + c*x^2)^(5/
2)) + (((4*c*d^2 + 3*a*e^2)*x*Sqrt[d^2 - e^2*x^2])/(3*a*(c*d^2 + a*e^2)*(a
 + c*x^2)^(3/2)) + (((8*c^2*d^4 + 13*a*c*d^2*e^2 + 3*a^2*e^4)*x*Sqrt[d^2 -
 e^2*x^2])/(a*(c*d^2 + a*e^2)*Sqrt[a + c*x^2]) + (e^2*((d*(8*c^2*d^4 + 13*
a*c*d^2*e^2 + 3*a^2*e^4)*Sqrt[a + c*x^2]*Sqrt[1 - (e^2*x^2)/d^2]*EllipticE
[ArcSin[(e*x)/d], -((c*d^2)/(a*e^2))])/(c*e*Sqrt[1 + (c*x^2)/a]*Sqrt[d^2 -
 e^2*x^2]) - (a*d*(c*d^2 + a*e^2)*(4*c*d^2 + 3*a*e^2)*Sqrt[1 + (c*x^2)/a]*
Sqrt[1 - (e^2*x^2)/d^2]*EllipticF[ArcSin[(e*x)/d], -((c*d^2)/(a*e^2))])/(c
*e*Sqrt[a + c*x^2]*Sqrt[d^2 - e^2*x^2])))/(a*(c*d^2 + a*e^2)))/(3*a*(c*d^2
 + a*e^2)))/(5*a)))/Sqrt[d^2 - e^2*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.54

method	result
elliptic	$\frac{\sqrt{(-e^2x^2+d^2)(c\,x^2+a)}\left(\frac{x\sqrt{-c\,e^2x^4-a\,e^2x^2+c\,d^2x^2+a\,d^2}}{5c^3a\left(x^2+\frac{a}{c}\right)^3}+\frac{\left(3a\,e^2+4c\,d^2\right)x\sqrt{-c\,e^2x^4-a\,e^2x^2+c\,d^2x^2+a\,d^2}}{15a^2\left(a\,e^2+c\,d^2\right)c^2\left(x^2+\frac{a}{c}\right)^2}+\frac{\left(-x^2c\,e^2+c\,d^2\right)x\left(3a^2e^4+\right.}{15c\,a^3\left(a\,e^2+c\,d^2\right)^2\sqrt{\left(x^2+\right.}}\right.}{}$
default	Expression too large to display

input
```
int((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((-e^2*x^2+d^2)*(c*x^2+a))^(1/2)/(-e*x+d)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a)^(1
/2)*(1/5/c^3/a*x*(-c*e^2*x^4-a*e^2*x^2+c*d^2*x^2+a*d^2)^(1/2)/(x^2+a/c)^3+
1/15*(3*a*e^2+4*c*d^2)/a^2/(a*e^2+c*d^2)/c^2*x*(-c*e^2*x^4-a*e^2*x^2+c*d^2
*x^2+a*d^2)^(1/2)/(x^2+a/c)^2+1/15*(-c*e^2*x^2+c*d^2)/c/a^3/(a*e^2+c*d^2)^
2*x*(3*a^2*e^4+13*a*c*d^2*e^2+8*c^2*d^4)/((x^2+a/c)*(-c*e^2*x^2+c*d^2))^(1
/2)+(-1/15*e^2*(3*a*e^2+4*c*d^2)/c/a^2/(a*e^2+c*d^2)+1/15/(a*e^2+c*d^2)/c*
(3*a^2*e^4+13*a*c*d^2*e^2+8*c^2*d^4)/a^3-1/15*d^2/a^3/(a*e^2+c*d^2)^2*(3*a
^2*e^4+13*a*c*d^2*e^2+8*c^2*d^4))/(e^2/d^2)^(1/2)*(1-e^2*x^2/d^2)^(1/2)*(1
+c/a*x^2)^(1/2)/(-c*e^2*x^4-a*e^2*x^2+c*d^2*x^2+a*d^2)^(1/2)*EllipticF(x*(
e^2/d^2)^(1/2),(-1-(-a*e^2+c*d^2)/a/e^2)^(1/2))-1/15*e^2*(3*a^2*e^4+13*a*c
*d^2*e^2+8*c^2*d^4)/a^2/(a*e^2+c*d^2)^2/(e^2/d^2)^(1/2)*(1-e^2*x^2/d^2)^(1
/2)*(1+c/a*x^2)^(1/2)/(-c*e^2*x^4-a*e^2*x^2+c*d^2*x^2+a*d^2)^(1/2)/c*(Elli
pticF(x*(e^2/d^2)^(1/2),(-1-(-a*e^2+c*d^2)/a/e^2)^(1/2))-EllipticE(x*(e^2/
d^2)^(1/2),(-1-(-a*e^2+c*d^2)/a/e^2)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 770, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(7/2),x, algorithm="frica
s")
```

output
```
1/15*(((8*c^5*d^7 + 13*a*c^4*d^5*e^2 + 3*a^2*c^3*d^3*e^4)*x^5 + (20*a*c^4*
d^7 + 33*a^2*c^3*d^5*e^2 + 9*a^3*c^2*d^3*e^4)*x^3 + (15*a^2*c^3*d^7 + 26*a
^3*c^2*d^5*e^2 + 9*a^4*c*d^3*e^4)*x)*sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(-e
*x + d) + sqrt(a*d^2)*((8*a^3*c^2*d^4*e^3 + 13*a^4*c*d^2*e^5 + 3*a^5*e^7 +
 (8*c^5*d^4*e^3 + 13*a*c^4*d^2*e^5 + 3*a^2*c^3*e^7)*x^6 + 3*(8*a*c^4*d^4*e
^3 + 13*a^2*c^3*d^2*e^5 + 3*a^3*c^2*e^7)*x^4 + 3*(8*a^2*c^3*d^4*e^3 + 13*a
^3*c^2*d^2*e^5 + 3*a^4*c*e^7)*x^2)*elliptic_e(arcsin(e*x/d), -c*d^2/(a*e^2
)) + (4*a^3*c^2*d^6*e - 13*a^4*c*d^2*e^5 - 3*a^5*e^7 + 2*(3*a^4*c - 4*a^3*
c^2)*d^4*e^3 + (4*c^5*d^6*e - 13*a*c^4*d^2*e^5 - 3*a^2*c^3*e^7 + 2*(3*a*c^
4 - 4*c^5)*d^4*e^3)*x^6 + 3*(4*a*c^4*d^6*e - 13*a^2*c^3*d^2*e^5 - 3*a^3*c^
2*e^7 + 2*(3*a^2*c^3 - 4*a*c^4)*d^4*e^3)*x^4 + 3*(4*a^2*c^3*d^6*e - 13*a^3
*c^2*d^2*e^5 - 3*a^4*c*e^7 + 2*(3*a^3*c^2 - 4*a^2*c^3)*d^4*e^3)*x^2)*ellip
tic_f(arcsin(e*x/d), -c*d^2/(a*e^2))))/(a^6*c^3*d^7 + 2*a^7*c^2*d^5*e^2 +
a^8*c*d^3*e^4 + (a^3*c^6*d^7 + 2*a^4*c^5*d^5*e^2 + a^5*c^4*d^3*e^4)*x^6 +
3*(a^4*c^5*d^7 + 2*a^5*c^4*d^5*e^2 + a^6*c^3*d^3*e^4)*x^4 + 3*(a^5*c^4*d^7
 + 2*a^6*c^3*d^5*e^2 + a^7*c^2*d^3*e^4)*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{7/2}}\,dx = \int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-e*x+d)**(1/2)*(e*x+d)**(1/2)/(c*x**2+a)**(7/2),x)
```

output
```
Integral(sqrt(d - e*x)*sqrt(d + e*x)/(a + c*x**2)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{7/2}}\,dx = \int \frac{\sqrt{ex+d}\sqrt{-ex+d}}{(cx^2+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(7/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(e*x + d)*sqrt(-e*x + d)/(c*x^2 + a)^(7/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(7/2),x, algorithm="giac"
)
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{7/2}}\,dx = \int \frac{\sqrt{d+e\,x}\,\sqrt{d-e\,x}}{(c\,x^2+a)^{7/2}}\,dx$$

input
```
int(((d + e*x)^(1/2)*(d - e*x)^(1/2))/(a + c*x^2)^(7/2),x)
```

output
```
int(((d + e*x)^(1/2)*(d - e*x)^(1/2))/(a + c*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d-ex}\sqrt{d+ex}}{(a+cx^2)^{7/2}}\,dx = \int \frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\sqrt{c\,x^2+a}}{c^4x^8+4a\,c^3x^6+6a^2c^2x^4+4a^3c\,x^2+a^4}dx$$

input
```
int((-e*x+d)^(1/2)*(e*x+d)^(1/2)/(c*x^2+a)^(7/2),x)
```

output
```
int((sqrt(d + e*x)*sqrt(d - e*x)*sqrt(a + c*x**2))/(a**4 + 4*a**3*c*x**2 +
 6*a**2*c**2*x**4 + 4*a*c**3*x**6 + c**4*x**8),x)
```

3.31 $\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 54

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p \, dx = x\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},e^2x^2\right)$$

output
```
x*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2,-c*x^2/a)/((1+c*x^2/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 167 vs. 2(54) = 108.

Time = 0.39 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.09

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p \, dx$$
$$= \frac{3ax\left(a+cx^2\right)^p \left(1-e^2x^2\right)^m \text{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},e^2x^2\right)}{3a \,\text{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},e^2x^2\right)+2x^2\left(cp \,\text{AppellF1}\left(\frac{3}{2},1-p,-m,\frac{5}{2},-\frac{cx^2}{a},e^2x^2\right)-ae^2m \,\text{Appell}\right.}$$

input
```
Integrate[(1 - e*x)^m*(1 + e*x)^m*(a + c*x^2)^p,x]
```

output
```
(3*a*x*(a + c*x^2)^p*(1 - e^2*x^2)^m*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/
a), e^2*x^2])/(3*a*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2] + 2*x
^2*(c*p*AppellF1[3/2, 1 - p, -m, 5/2, -((c*x^2)/a), e^2*x^2] - a*e^2*m*App
ellF1[3/2, -p, 1 - m, 5/2, -((c*x^2)/a), e^2*x^2]))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {643, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-ex)^m(ex+1)^m \left(a+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{643}$$

$$\int \left(1-e^2x^2\right)^m \left(a+cx^2\right)^p dx$$

$$\Big\downarrow \text{334}$$

$$\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \int \left(\frac{cx^2}{a}+1\right)^p \left(1-e^2x^2\right)^m dx$$

$$\Big\downarrow \text{333}$$

$$x\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},e^2x^2\right)$$

input
```
Int[(1 - e*x)^m*(1 + e*x)^m*(a + c*x^2)^p,x]
```

output
```
(x*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2])/(1 + (
c*x^2)/a)^p
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

Maple [F]

$$\int (-ex+1)^m (ex+1)^m \left(c\,x^2+a\right)^p dx$$

input
```
int((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
```

output
```
int((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p \,dx = \int \left(cx^2+a\right)^p(ex+1)^m(-ex+1)^m \,dx$$

input
```
integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)^p*(e*x + 1)^m*(-e*x + 1)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((-e*x+1)**m*(e*x+1)**m*(c*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p \, dx = \int (cx^2+a)^p(ex+1)^m(-ex+1)^m \, dx$$

input
```
integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^p*(e*x + 1)^m*(-e*x + 1)^m, x)
```

Giac [F]

$$\int (1-ex)^m(1+ex)^m \left(a+cx^2\right)^p \, dx = \int (cx^2+a)^p(ex+1)^m(-ex+1)^m \, dx$$

input
```
integrate((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^p*(e*x + 1)^m*(-e*x + 1)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1-ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx=\int \left(c\,x^2+a\right)^p\,(1-e\,x)^m\,(e\,x+1)^m\,dx$$

input
```
int((a + c*x^2)^p*(1 - e*x)^m*(e*x + 1)^m,x)
```

output
```
int((a + c*x^2)^p*(1 - e*x)^m*(e*x + 1)^m, x)
```

Reduce [F]

$$\int (1-ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx=\text{Too large to display}$$

input
```
int((-e*x+1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
```

output
```
((a + c*x**2)**p*(e*x + 1)**m*( - e*x + 1)**m*x + 4*int(((a + c*x**2)**p*(
e*x + 1)**m*( - e*x + 1)**m*x**2)/(2*a*e**2*m*x**2 + 2*a*e**2*p*x**2 + a*e
**2*x**2 - 2*a*m - 2*a*p - a + 2*c*e**2*m*x**4 + 2*c*e**2*p*x**4 + c*e**2*
x**4 - 2*c*m*x**2 - 2*c*p*x**2 - c*x**2),x)*a*e**2*m*p + 4*int(((a + c*x**
2)**p*(e*x + 1)**m*( - e*x + 1)**m*x**2)/(2*a*e**2*m*x**2 + 2*a*e**2*p*x**
2 + a*e**2*x**2 - 2*a*m - 2*a*p - a + 2*c*e**2*m*x**4 + 2*c*e**2*p*x**4 +
c*e**2*x**4 - 2*c*m*x**2 - 2*c*p*x**2 - c*x**2),x)*a*e**2*p**2 + 2*int(((a
 + c*x**2)**p*(e*x + 1)**m*( - e*x + 1)**m*x**2)/(2*a*e**2*m*x**2 + 2*a*e*
*2*p*x**2 + a*e**2*x**2 - 2*a*m - 2*a*p - a + 2*c*e**2*m*x**4 + 2*c*e**2*p
*x**4 + c*e**2*x**4 - 2*c*m*x**2 - 2*c*p*x**2 - c*x**2),x)*a*e**2*p - 4*in
t(((a + c*x**2)**p*(e*x + 1)**m*( - e*x + 1)**m*x**2)/(2*a*e**2*m*x**2 + 2
*a*e**2*p*x**2 + a*e**2*x**2 - 2*a*m - 2*a*p - a + 2*c*e**2*m*x**4 + 2*c*e
**2*p*x**4 + c*e**2*x**4 - 2*c*m*x**2 - 2*c*p*x**2 - c*x**2),x)*c*m**2 - 4
*int(((a + c*x**2)**p*(e*x + 1)**m*( - e*x + 1)**m*x**2)/(2*a*e**2*m*x**2
+ 2*a*e**2*p*x**2 + a*e**2*x**2 - 2*a*m - 2*a*p - a + 2*c*e**2*m*x**4 + 2*
c*e**2*p*x**4 + c*e**2*x**4 - 2*c*m*x**2 - 2*c*p*x**2 - c*x**2),x)*c*m*p -
 2*int(((a + c*x**2)**p*(e*x + 1)**m*( - e*x + 1)**m*x**2)/(2*a*e**2*m*x**
2 + 2*a*e**2*p*x**2 + a*e**2*x**2 - 2*a*m - 2*a*p - a + 2*c*e**2*m*x**4 +
2*c*e**2*p*x**4 + c*e**2*x**4 - 2*c*m*x**2 - 2*c*p*x**2 - c*x**2),x)*c*m -
 4*int(((a + c*x**2)**p*(e*x + 1)**m*( - e*x + 1)**m)/(2*a*e**2*m*x**2 ...
```

3.32 $\int (-1+ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int (-1+ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx=x(-1+ex)^m(1+ex)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-e^2x^2\right)^{-m}\operatorname{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},e^2x^2\right)$$

output
```
x*(e*x-1)^m*(e*x+1)^m*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2,-c*x^2/a)
/((1+c*x^2/a)^p)/((-e^2*x^2+1)^m)
```

Mathematica [F]

$$\int (-1+ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx=\int (-1+ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx$$

input
```
Integrate[(-1 + e*x)^m*(1 + e*x)^m*(a + c*x^2)^p,x]
```

output
```
Integrate[(-1 + e*x)^m*(1 + e*x)^m*(a + c*x^2)^p, x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {648, 334, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex-1)^m(ex+1)^m \left(a+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{ 648}$$

$$(ex-1)^m(ex+1)^m \left(e^2x^2-1\right)^{-m} \int \left(cx^2+a\right)^p \left(e^2x^2-1\right)^m dx$$

$$\Big\downarrow \text{ 334}$$

$$(ex-1)^m(ex+1)^m \left(e^2x^2-1\right)^{-m} \left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \int \left(\frac{cx^2}{a}+1\right)^p \left(e^2x^2-1\right)^m dx$$

$$\Big\downarrow \text{ 334}$$

$$(ex-1)^m(ex+1)^m \left(1-e^2x^2\right)^{-m} \left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \int \left(\frac{cx^2}{a}+1\right)^p \left(1-e^2x^2\right)^m dx$$

$$\Big\downarrow \text{ 333}$$

$$x(ex-1)^m(ex+1)^m \left(1-e^2x^2\right)^{-m} \left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},e^2x^2\right)$$

input
```
Int[(-1 + e*x)^m*(1 + e*x)^m*(a + c*x^2)^p,x]
```

output
```
(x*(-1 + e*x)^m*(1 + e*x)^m*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*
x^2)/a), e^2*x^2])/((1 + (c*x^2)/a)^p*(1 - e^2*x^2)^m)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [F]

$$\int (ex-1)^m (ex+1)^m (c\,x^2+a)^p \,dx$$

input
```
int((e*x-1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
```

output
```
int((e*x-1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (-1+ex)^m(1+ex)^m \left(a+cx^2\right)^p \,dx = \int \left(cx^2+a\right)^p(ex+1)^m(ex-1)^m \,dx$$

input
```
integrate((e*x-1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)^p*(e*x + 1)^m*(e*x - 1)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (-1+ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x-1)**m*(e*x+1)**m*(c*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (-1+ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx=\int (cx^2+a)^p(ex+1)^m(ex-1)^m\,dx$$

input
```
integrate((e*x-1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^p*(e*x + 1)^m*(e*x - 1)^m, x)
```

Giac [F]

$$\int (-1+ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx=\int (cx^2+a)^p(ex+1)^m(ex-1)^m\,dx$$

input
```
integrate((e*x-1)^m*(e*x+1)^m*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^p*(e*x + 1)^m*(e*x - 1)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (-1+ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx=\int \left(c\,x^2+a\right)^p\,\left(e\,x-1\right)^m\,\left(e\,x+1\right)^m\,dx$$

input
```
int((a + c*x^2)^p*(e*x - 1)^m*(e*x + 1)^m,x)
```

output
```
int((a + c*x^2)^p*(e*x - 1)^m*(e*x + 1)^m, x)
```

Reduce [F]

$$\int (-1+ex)^m(1+ex)^m\left(a+cx^2\right)^p\,dx=\text{Too large to display}$$

input
```
int((e*x-1)^m*(e*x+1)^m*(c*x^2+a)^p,x)
```

output
```
((a + c*x**2)**p*(e*x + 1)**m*(e*x - 1)**m*x + 4*int(((a + c*x**2)**p*(e*x
 + 1)**m*(e*x - 1)**m*x**2)/(2*a*e**2*m*x**2 + 2*a*e**2*p*x**2 + a*e**2*x*
*2 - 2*a*m - 2*a*p - a + 2*c*e**2*m*x**4 + 2*c*e**2*p*x**4 + c*e**2*x**4 -
 2*c*m*x**2 - 2*c*p*x**2 - c*x**2),x)*a*e**2*m*p + 4*int(((a + c*x**2)**p*
(e*x + 1)**m*(e*x - 1)**m*x**2)/(2*a*e**2*m*x**2 + 2*a*e**2*p*x**2 + a*e**
2*x**2 - 2*a*m - 2*a*p - a + 2*c*e**2*m*x**4 + 2*c*e**2*p*x**4 + c*e**2*x*
*4 - 2*c*m*x**2 - 2*c*p*x**2 - c*x**2),x)*a*e**2*p**2 + 2*int(((a + c*x**2
)**p*(e*x + 1)**m*(e*x - 1)**m*x**2)/(2*a*e**2*m*x**2 + 2*a*e**2*p*x**2 +
a*e**2*x**2 - 2*a*m - 2*a*p - a + 2*c*e**2*m*x**4 + 2*c*e**2*p*x**4 + c*e*
*2*x**4 - 2*c*m*x**2 - 2*c*p*x**2 - c*x**2),x)*a*e**2*p - 4*int(((a + c*x*
*2)**p*(e*x + 1)**m*(e*x - 1)**m*x**2)/(2*a*e**2*m*x**2 + 2*a*e**2*p*x**2
+ a*e**2*x**2 - 2*a*m - 2*a*p - a + 2*c*e**2*m*x**4 + 2*c*e**2*p*x**4 + c*
e**2*x**4 - 2*c*m*x**2 - 2*c*p*x**2 - c*x**2),x)*c*m**2 - 4*int(((a + c*x*
*2)**p*(e*x + 1)**m*(e*x - 1)**m*x**2)/(2*a*e**2*m*x**2 + 2*a*e**2*p*x**2
+ a*e**2*x**2 - 2*a*m - 2*a*p - a + 2*c*e**2*m*x**4 + 2*c*e**2*p*x**4 + c*
e**2*x**4 - 2*c*m*x**2 - 2*c*p*x**2 - c*x**2),x)*c*m*p - 2*int(((a + c*x**
2)**p*(e*x + 1)**m*(e*x - 1)**m*x**2)/(2*a*e**2*m*x**2 + 2*a*e**2*p*x**2 +
 a*e**2*x**2 - 2*a*m - 2*a*p - a + 2*c*e**2*m*x**4 + 2*c*e**2*p*x**4 + c*e
**2*x**4 - 2*c*m*x**2 - 2*c*p*x**2 - c*x**2),x)*c*m - 4*int(((a + c*x**2)*
*p*(e*x + 1)**m*(e*x - 1)**m)/(2*a*e**2*m*x**2 + 2*a*e**2*p*x**2 + a*e*...
```

3.33 $\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx=x(d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

output
```
x*(-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2/d^2,-c*x
^2/a)/((1+c*x^2/a)^p)/((1-e^2*x^2/d^2)^m)
```

Mathematica [F]

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx=\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx$$

input
```
Integrate[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p,x]
```

output
```
Integrate[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p, x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {648, 334, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^p (d-ex)^m (d+ex)^m \, dx$$

$$\Big\downarrow \text{648}$$

$$(d-ex)^m (d+ex)^m \left(d^2-e^2x^2\right)^{-m} \int (cx^2+a)^p \left(d^2-e^2x^2\right)^m dx$$

$$\Big\downarrow \text{334}$$

$$(a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d-ex)^m (d+ex)^m \left(d^2-e^2x^2\right)^{-m} \int \left(\frac{cx^2}{a}+1\right)^p \left(d^2-e^2x^2\right)^m dx$$

$$\Big\downarrow \text{334}$$

$$(a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d-ex)^m (d+ex)^m \left(1-\frac{e^2x^2}{d^2}\right)^{-m} \int \left(\frac{cx^2}{a}+1\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^m dx$$

$$\Big\downarrow \text{333}$$

$$x (a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d-ex)^m (d+ex)^m \left(1-\frac{e^2x^2}{d^2}\right)^{-m} \text{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

input
```
Int[(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p,x]
```

output
```
(x*(d - e*x)^m*(d + e*x)^m*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x
^2)/a), (e^2*x^2)/d^2])/((1 + (c*x^2)/a)^p*(1 - (e^2*x^2)/d^2)^m)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [F]

$$\int (-ex+d)^m (ex+d)^m (c x^2+a)^p \, dx$$

input
```
int((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x)
```

output
```
int((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (d-ex)^m(d+ex)^m \left(a+cx^2\right)^p \, dx = \int \left(cx^2+a\right)^p (ex+d)^m (-ex+d)^m \, dx$$

input
```
integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((-e*x+d)**m*(e*x+d)**m*(c*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx=\int \left(cx^2+a\right)^p(ex+d)^m(-ex+d)^m\,dx$$

input
```
integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)
```

Giac [F]

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx=\int \left(cx^2+a\right)^p(ex+d)^m(-ex+d)^m\,dx$$

input
```
integrate((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx=\int \left(c\,x^2+a\right)^p\,\left(d+e\,x\right)^m\,\left(d-e\,x\right)^m\,dx$$

input
```
int((a + c*x^2)^p*(d + e*x)^m*(d - e*x)^m,x)
```

output
```
int((a + c*x^2)^p*(d + e*x)^m*(d - e*x)^m, x)
```

Reduce [F]

$$\int (d-ex)^m(d+ex)^m\left(a+cx^2\right)^p\,dx=\text{Too large to display}$$

input
```
int((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x)
```

output
```
((d + e*x)**m*(d - e*x)**m*(a + c*x**2)**p*x - 4*int(((d + e*x)**m*(d - e*
x)**m*(a + c*x**2)**p*x**2)/(2*a*d**2*m + 2*a*d**2*p + a*d**2 - 2*a*e**2*m
*x**2 - 2*a*e**2*p*x**2 - a*e**2*x**2 + 2*c*d**2*m*x**2 + 2*c*d**2*p*x**2
+ c*d**2*x**2 - 2*c*e**2*m*x**4 - 2*c*e**2*p*x**4 - c*e**2*x**4),x)*a*e**2
*m*p - 4*int(((d + e*x)**m*(d - e*x)**m*(a + c*x**2)**p*x**2)/(2*a*d**2*m
+ 2*a*d**2*p + a*d**2 - 2*a*e**2*m*x**2 - 2*a*e**2*p*x**2 - a*e**2*x**2 +
2*c*d**2*m*x**2 + 2*c*d**2*p*x**2 + c*d**2*x**2 - 2*c*e**2*m*x**4 - 2*c*e*
*2*p*x**4 - c*e**2*x**4),x)*a*e**2*p**2 - 2*int(((d + e*x)**m*(d - e*x)**m
*(a + c*x**2)**p*x**2)/(2*a*d**2*m + 2*a*d**2*p + a*d**2 - 2*a*e**2*m*x**2
 - 2*a*e**2*p*x**2 - a*e**2*x**2 + 2*c*d**2*m*x**2 + 2*c*d**2*p*x**2 + c*d
**2*x**2 - 2*c*e**2*m*x**4 - 2*c*e**2*p*x**4 - c*e**2*x**4),x)*a*e**2*p +
4*int(((d + e*x)**m*(d - e*x)**m*(a + c*x**2)**p*x**2)/(2*a*d**2*m + 2*a*d
**2*p + a*d**2 - 2*a*e**2*m*x**2 - 2*a*e**2*p*x**2 - a*e**2*x**2 + 2*c*d**
2*m*x**2 + 2*c*d**2*p*x**2 + c*d**2*x**2 - 2*c*e**2*m*x**4 - 2*c*e**2*p*x*
*4 - c*e**2*x**4),x)*c*d**2*m**2 + 4*int(((d + e*x)**m*(d - e*x)**m*(a + c
*x**2)**p*x**2)/(2*a*d**2*m + 2*a*d**2*p + a*d**2 - 2*a*e**2*m*x**2 - 2*a*
e**2*p*x**2 - a*e**2*x**2 + 2*c*d**2*m*x**2 + 2*c*d**2*p*x**2 + c*d**2*x**
2 - 2*c*e**2*m*x**4 - 2*c*e**2*p*x**4 - c*e**2*x**4),x)*c*d**2*m*p + 2*int
(((d + e*x)**m*(d - e*x)**m*(a + c*x**2)**p*x**2)/(2*a*d**2*m + 2*a*d**2*p
 + a*d**2 - 2*a*e**2*m*x**2 - 2*a*e**2*p*x**2 - a*e**2*x**2 + 2*c*d**2*...
```

3.34 $\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx=x(d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(1-\frac{e^2x^2}{d^2}\right)^{-m}\operatorname{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

output
```
x*(e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p*AppellF1(1/2,-m,-p,3/2,e^2*x^2/d^2,
-c*x^2/a)/((1+c*x^2/a)^p)/((1-e^2*x^2/d^2)^m)
```

Mathematica [F]

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx=\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx$$

input
```
Integrate[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p,x]
```

output
```
Integrate[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p, x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {648, 334, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^p (d+ex)^m (df-efx)^m \, dx$$

$$\Big\downarrow \text{648}$$

$$(d+ex)^m (df-efx)^m \left(d^2f-e^2fx^2\right)^{-m} \int \left(cx^2+a\right)^p \left(d^2f-e^2fx^2\right)^m dx$$

$$\Big\downarrow \text{334}$$

$$\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d+ex)^m (df-efx)^m \left(d^2f-e^2fx^2\right)^{-m} \int \left(\frac{cx^2}{a}+1\right)^p \left(d^2f-e^2fx^2\right)^m dx$$

$$\Big\downarrow \text{334}$$

$$\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d+ex)^m \left(1-\frac{e^2x^2}{d^2}\right)^{-m} (df-efx)^m \int \left(\frac{cx^2}{a}+1\right)^p \left(1-\frac{e^2x^2}{d^2}\right)^m dx$$

$$\Big\downarrow \text{333}$$

$$x\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d+ex)^m \left(1-\frac{e^2x^2}{d^2}\right)^{-m} (df-efx)^m \text{AppellF1}\left(\frac{1}{2},-p,-m,\frac{3}{2},-\frac{cx^2}{a},\frac{e^2x^2}{d^2}\right)$$

input
```
Int[(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p,x]
```

output
```
(x*(d + e*x)^m*(d*f - e*f*x)^m*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -(
(c*x^2)/a), (e^2*x^2)/d^2])/((1 + (c*x^2)/a)^p*(1 - (e^2*x^2)/d^2)^m)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [F]

$$\int (ex+d)^m (-efx+df)^m (c\,x^2+a)^p\,dx$$

input
```
int((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x)
```

output
```
int((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx = \int (-efx+df)^m(cx^2+a)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(df-efx)^m \left(a+cx^2\right)^p \,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(-e*f*x+d*f)**m*(c*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^m(df-efx)^m \left(a+cx^2\right)^p \,dx = \int (-efx+df)^m \left(cx^2+a\right)^p (ex+d)^m \,dx$$

input
```
integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m(df-efx)^m \left(a+cx^2\right)^p \,dx = \int (-efx+df)^m \left(cx^2+a\right)^p (ex+d)^m \,dx$$

input
```
integrate((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx=\int (d\,f-e\,f\,x)^m\,\left(c\,x^2+a\right)^p\,(d+e\,x)^m\,dx$$

input
```
int((d*f - e*f*x)^m*(a + c*x^2)^p*(d + e*x)^m,x)
```

output
```
int((d*f - e*f*x)^m*(a + c*x^2)^p*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m(df-efx)^m\left(a+cx^2\right)^p\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x)
```

output

```
((d + e*x)**m*(a + c*x**2)**p*(d*f - e*f*x)**m*x - 4*int(((d + e*x)**m*(a
+ c*x**2)**p*(d*f - e*f*x)**m*x**2)/(2*a*d**2*m + 2*a*d**2*p + a*d**2 - 2*
a*e**2*m*x**2 - 2*a*e**2*p*x**2 - a*e**2*x**2 + 2*c*d**2*m*x**2 + 2*c*d**2
*p*x**2 + c*d**2*x**2 - 2*c*e**2*m*x**4 - 2*c*e**2*p*x**4 - c*e**2*x**4),x
)*a*e**2*m*p - 4*int(((d + e*x)**m*(a + c*x**2)**p*(d*f - e*f*x)**m*x**2)/
(2*a*d**2*m + 2*a*d**2*p + a*d**2 - 2*a*e**2*m*x**2 - 2*a*e**2*p*x**2 - a*
e**2*x**2 + 2*c*d**2*m*x**2 + 2*c*d**2*p*x**2 + c*d**2*x**2 - 2*c*e**2*m*x
**4 - 2*c*e**2*p*x**4 - c*e**2*x**4),x)*a*e**2*p**2 - 2*int(((d + e*x)**m*
(a + c*x**2)**p*(d*f - e*f*x)**m*x**2)/(2*a*d**2*m + 2*a*d**2*p + a*d**2 -
 2*a*e**2*m*x**2 - 2*a*e**2*p*x**2 - a*e**2*x**2 + 2*c*d**2*m*x**2 + 2*c*d
**2*p*x**2 + c*d**2*x**2 - 2*c*e**2*m*x**4 - 2*c*e**2*p*x**4 - c*e**2*x**4
),x)*a*e**2*p + 4*int(((d + e*x)**m*(a + c*x**2)**p*(d*f - e*f*x)**m*x**2)
/(2*a*d**2*m + 2*a*d**2*p + a*d**2 - 2*a*e**2*m*x**2 - 2*a*e**2*p*x**2 - a
*e**2*x**2 + 2*c*d**2*m*x**2 + 2*c*d**2*p*x**2 + c*d**2*x**2 - 2*c*e**2*m*
x**4 - 2*c*e**2*p*x**4 - c*e**2*x**4),x)*c*d**2*m**2 + 4*int(((d + e*x)**m
*(a + c*x**2)**p*(d*f - e*f*x)**m*x**2)/(2*a*d**2*m + 2*a*d**2*p + a*d**2
- 2*a*e**2*m*x**2 - 2*a*e**2*p*x**2 - a*e**2*x**2 + 2*c*d**2*m*x**2 + 2*c*
d**2*p*x**2 + c*d**2*x**2 - 2*c*e**2*m*x**4 - 2*c*e**2*p*x**4 - c*e**2*x**
4),x)*c*d**2*m*p + 2*int(((d + e*x)**m*(a + c*x**2)**p*(d*f - e*f*x)**m*x*
*2)/(2*a*d**2*m + 2*a*d**2*p + a*d**2 - 2*a*e**2*m*x**2 - 2*a*e**2*p*x*...
```

3.35 $\int (d+ex)^3\sqrt{f+gx}(a+cx^2)\ dx$

Optimal result

Integrand size = 24, antiderivative size = 242

$$\begin{aligned}\int (d+ex)^3\sqrt{f+gx}(a+cx^2)\ dx &= -\frac{2(ef-dg)^3(cf^2+ag^2)(f+gx)^{3/2}}{3g^6} \\ &\quad+\frac{2(ef-dg)^2(3aeg^2+cf(5ef-2dg))(f+gx)^{5/2}}{5g^6} \\ &\quad-\frac{2(ef-dg)(3ae^2g^2+c(10e^2f^2-8defg+d^2g^2))(f+gx)^{7/2}}{7g^6} \\ &\quad+\frac{2e(ae^2g^2+c(10e^2f^2-12defg+3d^2g^2))(f+gx)^{9/2}}{9g^6} \\ &\quad-\frac{2ce^2(5ef-3dg)(f+gx)^{11/2}}{11g^6}+\frac{2ce^3(f+gx)^{13/2}}{13g^6}\end{aligned}$$

output
```
-2/3*(-d*g+e*f)^3*(a*g^2+c*f^2)*(g*x+f)^(3/2)/g^6+2/5*(-d*g+e*f)^2*(3*a*e*
g^2+c*f*(-2*d*g+5*e*f))*(g*x+f)^(5/2)/g^6-2/7*(-d*g+e*f)*(3*a*e^2*g^2+c*(d
^2*g^2-8*d*e*f*g+10*e^2*f^2))*(g*x+f)^(7/2)/g^6+2/9*e*(a*e^2*g^2+c*(3*d^2*
g^2-12*d*e*f*g+10*e^2*f^2))*(g*x+f)^(9/2)/g^6-2/11*c*e^2*(-3*d*g+5*e*f)*(g
*x+f)^(11/2)/g^6+2/13*c*e^3*(g*x+f)^(13/2)/g^6
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.17

$$\int (d+ex)^3\sqrt{f+gx}\left(a+cx^2\right)\,dx$$
$$=\frac{2(f+gx)^{3/2}\left(143ag^2(105d^3g^3+63d^2eg^2(-2f+3gx)+9de^2g(8f^2-12fgx+15g^2x^2)+e^3(-16f^3+24\right.}{}$$

input
```
Integrate[(d + e*x)^3*Sqrt[f + g*x]*(a + c*x^2),x]
```

output
```
(2*(f + g*x)^(3/2)*(143*a*g^2*(105*d^3*g^3 + 63*d^2*e*g^2*(-2*f + 3*g*x) +
 9*d*e^2*g*(8*f^2 - 12*f*g*x + 15*g^2*x^2) + e^3*(-16*f^3 + 24*f^2*g*x - 3
0*f*g^2*x^2 + 35*g^3*x^3)) + c*(429*d^3*g^3*(8*f^2 - 12*f*g*x + 15*g^2*x^2
) + 429*d^2*e*g^2*(-16*f^3 + 24*f^2*g*x - 30*f*g^2*x^2 + 35*g^3*x^3) + 39*
d*e^2*g*(128*f^4 - 192*f^3*g*x + 240*f^2*g^2*x^2 - 280*f*g^3*x^3 + 315*g^4
*x^4) - 5*e^3*(256*f^5 - 384*f^4*g*x + 480*f^3*g^2*x^2 - 560*f^2*g^3*x^3 +
 630*f*g^4*x^4 - 693*g^5*x^5))))/(45045*g^6)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(d+ex)^3\sqrt{f+gx}\,dx$$

$$\Big\downarrow \; 652$$

$$\int \left(\frac{e(f+gx)^{7/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{g^5}+\frac{(f+gx)^{5/2}(ef-dg)\left(-3ae^2g^2-c\left(d^2g^2-8defg+\right.\right.}{g^5}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{2e(f+gx)^{9/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{9g^6}-\frac{2(f+gx)^{7/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{7g^6}-\frac{2(f+gx)^{3/2}\left(ag^2+cf^2\right)(ef-dg)^3}{3g^6}+\frac{2(f+gx)^{5/2}(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)}{5g^6}-\frac{2ce^2(f+gx)^{11/2}(5ef-3dg)}{11g^6}+\frac{2ce^3(f+gx)^{13/2}}{13g^6}$$

input
```
Int[(d + e*x)^3*Sqrt[f + g*x]*(a + c*x^2),x]
```

output
```
(-2*(e*f - d*g)^3*(c*f^2 + a*g^2)*(f + g*x)^(3/2))/(3*g^6) + (2*(e*f - d*g
)^2*(3*a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x)^(5/2))/(5*g^6) - (2*(e*f -
 d*g)*(3*a*e^2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(7/2)
)/(7*g^6) + (2*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f
+ g*x)^(9/2))/(9*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(11/2))/(11*g^6
) + (2*c*e^3*(f + g*x)^(13/2))/(13*g^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{2e^3c(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left(3(dg-ef)e^2c-2fe^3c\right)(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(3(dg-ef)^2ec-6(dg-ef)e^2cf+e^3\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left((dg-ef)^3c\right.}{}$
default	$\frac{2e^3c(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left(3(dg-ef)e^2c-2fe^3c\right)(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(3(dg-ef)^2ec-6(dg-ef)e^2cf+e^3\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left((dg-ef)^3c\right.}{}$
pseudoelliptic	$\frac{2\left(\left(\frac{\left(\frac{9cx^2}{13}+a\right)x^3e^3}{3}+\frac{9\left(\frac{7cx^2}{11}+a\right)x^2de^2}{7}+\frac{9\left(\frac{5cx^2}{9}+a\right)xd^2e}{5}+d^3\left(a+\frac{3cx^2}{7}\right)\right)g^5-\frac{6f\left(\left(\frac{25}{143}cx^4+\frac{5}{21}ax^2\right)e^3+\frac{6xd\left(\frac{70cx}{99}\right.}{7}\right.}{}\right.}{}$
gosper	$\frac{2(gx+f)^{\frac{3}{2}}\left(3465e^3cx^5g^5+12285cde^2g^5x^4-3150ce^3fg^4x^4+5005ae^3g^5x^3+15015cd^2eg^5x^3-10920cde^2fg^4x^3+280\right.}{}$
orering	$\frac{2(gx+f)^{\frac{3}{2}}\left(3465e^3cx^5g^5+12285cde^2g^5x^4-3150ce^3fg^4x^4+5005ae^3g^5x^3+15015cd^2eg^5x^3-10920cde^2fg^4x^3+280\right.}{}$
trager	$\frac{2\left(3465e^3cg^6x^6+12285cde^2g^6x^5+315ce^3fg^5x^5+5005ae^3g^6x^4+15015cd^2eg^6x^4+1365cde^2fg^5x^4-350ce^3f^2g^4x^4\right.}{}$
risch	$\frac{2\left(3465e^3cg^6x^6+12285cde^2g^6x^5+315ce^3fg^5x^5+5005ae^3g^6x^4+15015cd^2eg^6x^4+1365cde^2fg^5x^4-350ce^3f^2g^4x^4\right.}{}$

input
```
int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/g^6*(1/13*e^3*c*(g*x+f)^(13/2)+1/11*(3*(d*g-e*f)*e^2*c-2*f*e^3*c)*(g*x+f
)^(11/2)+1/9*(3*(d*g-e*f)^2*e*c-6*(d*g-e*f)*e^2*c*f+e^3*(a*g^2+c*f^2))*(g*
x+f)^(9/2)+1/7*((d*g-e*f)^3*c-6*(d*g-e*f)^2*e*c*f+3*(d*g-e*f)*e^2*(a*g^2+c
*f^2))*(g*x+f)^(7/2)+1/5*(-2*(d*g-e*f)^3*c*f+3*(d*g-e*f)^2*e*(a*g^2+c*f^2)
)*(g*x+f)^(5/2)+1/3*(d*g-e*f)^3*(a*g^2+c*f^2)*(g*x+f)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.73

$$\int (d+ex)^3\sqrt{f+gx}\left(a+cx^2\right)\,dx$$
$$=\frac{2\left(3465\,ce^3g^6x^6-1280\,ce^3f^6+4992\,cde^2f^5g-18018\,ad^2ef^2g^4+15015\,ad^3fg^5-2288\left(3\,cd^2e+ae^3\right)f^4\right.}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a),x, algorithm="fricas")
```

output
```
2/45045*(3465*c*e^3*g^6*x^6 - 1280*c*e^3*f^6 + 4992*c*d*e^2*f^5*g - 18018*
a*d^2*e*f^2*g^4 + 15015*a*d^3*f*g^5 - 2288*(3*c*d^2*e + a*e^3)*f^4*g^2 + 3
432*(c*d^3 + 3*a*d*e^2)*f^3*g^3 + 315*(c*e^3*f*g^5 + 39*c*d*e^2*g^6)*x^5 -
 35*(10*c*e^3*f^2*g^4 - 39*c*d*e^2*f*g^5 - 143*(3*c*d^2*e + a*e^3)*g^6)*x^
4 + 5*(80*c*e^3*f^3*g^3 - 312*c*d*e^2*f^2*g^4 + 143*(3*c*d^2*e + a*e^3)*f*
g^5 + 1287*(c*d^3 + 3*a*d*e^2)*g^6)*x^3 - 3*(160*c*e^3*f^4*g^2 - 624*c*d*e
^2*f^3*g^3 - 9009*a*d^2*e*g^6 + 286*(3*c*d^2*e + a*e^3)*f^2*g^4 - 429*(c*d
^3 + 3*a*d*e^2)*f*g^5)*x^2 + (640*c*e^3*f^5*g - 2496*c*d*e^2*f^4*g^2 + 900
9*a*d^2*e*f*g^5 + 15015*a*d^3*g^6 + 1144*(3*c*d^2*e + a*e^3)*f^3*g^3 - 171
6*(c*d^3 + 3*a*d*e^2)*f^2*g^4)*x)*sqrt(g*x + f)/g^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(243) = 486$.

Time = 1.37 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.08

$$\int (d+ex)^3\sqrt{f+gx}\left(a+cx^2\right)\,dx$$
$$= \begin{cases} \frac{2\left(\frac{ce^3(f+gx)^{\frac{13}{2}}}{13g^5}+\frac{(f+gx)^{\frac{11}{2}}\cdot\left(3cde^2g-5ce^3f\right)}{11g^5}+\frac{(f+gx)^{\frac{9}{2}}\left(ae^3g^2+3cd^2eg^2-12cde^2fg+10ce^3f^2\right)}{9g^5}+\frac{(f+gx)^{\frac{7}{2}}\cdot\left(3ade^2g^3-3ae^3fg^2+cd^3g^3-9cd^2efg^2+1\right.}{7g^5}}{} \\ \sqrt{f}\left(ad^3x+\frac{3ad^2ex^2}{2}+\frac{3cde^2x^5}{5}+\frac{ce^3x^6}{6}+\frac{x^4\left(ae^3+3cd^2e\right)}{4}+\frac{x^3\cdot\left(3ade^2+cd^3\right)}{3}\right) \end{cases}$$

input
```
integrate((e*x+d)**3*(g*x+f)**(1/2)*(c*x**2+a),x)
```

output
```
Piecewise((2*(c*e**3*(f + g*x)**(13/2)/(13*g**5) + (f + g*x)**(11/2)*(3*c*
d*e**2*g - 5*c*e**3*f)/(11*g**5) + (f + g*x)**(9/2)*(a*e**3*g**2 + 3*c*d**
2*e*g**2 - 12*c*d*e**2*f*g + 10*c*e**3*f**2)/(9*g**5) + (f + g*x)**(7/2)*(
3*a*d*e**2*g**3 - 3*a*e**3*f*g**2 + c*d**3*g**3 - 9*c*d**2*e*f*g**2 + 18*c
*d*e**2*f**2*g - 10*c*e**3*f**3)/(7*g**5) + (f + g*x)**(5/2)*(3*a*d**2*e*g
**4 - 6*a*d*e**2*f*g**3 + 3*a*e**3*f**2*g**2 - 2*c*d**3*f*g**3 + 9*c*d**2*
e*f**2*g**2 - 12*c*d*e**2*f**3*g + 5*c*e**3*f**4)/(5*g**5) + (f + g*x)**(3
/2)*(a*d**3*g**5 - 3*a*d**2*e*f*g**4 + 3*a*d*e**2*f**2*g**3 - a*e**3*f**3*
g**2 + c*d**3*f**2*g**3 - 3*c*d**2*e*f**3*g**2 + 3*c*d*e**2*f**4*g - c*e**
3*f**5)/(3*g**5))/g, Ne(g, 0)), (sqrt(f)*(a*d**3*x + 3*a*d**2*e*x**2/2 + 3
*c*d*e**2*x**5/5 + c*e**3*x**6/6 + x**4*(a*e**3 + 3*c*d**2*e)/4 + x**3*(3*
a*d*e**2 + c*d**3)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.35

$$\int (d+ex)^3\sqrt{f+gx}\,(a+cx^2)\,dx$$

$$= \frac{2\left(3465\,(gx+f)^{\frac{13}{2}}ce^3 - 4095\,(5\,ce^3f-3\,cde^2g)(gx+f)^{\frac{11}{2}} + 5005\,(10\,ce^3f^2-12\,cde^2fg+(3\,cd^2e+ae\right.}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a),x, algorithm="maxima")
```

output
```
2/45045*(3465*(g*x + f)^(13/2)*c*e^3 - 4095*(5*c*e^3*f - 3*c*d*e^2*g)*(g*x
 + f)^(11/2) + 5005*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2*e + a*e^3)*g
^2)*(g*x + f)^(9/2) - 6435*(10*c*e^3*f^3 - 18*c*d*e^2*f^2*g + 3*(3*c*d^2*e
 + a*e^3)*f*g^2 - (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(7/2) + 9009*(5*c*e^3
*f^4 - 12*c*d*e^2*f^3*g + 3*a*d^2*e*g^4 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 -
2*(c*d^3 + 3*a*d*e^2)*f*g^3)*(g*x + f)^(5/2) - 15015*(c*e^3*f^5 - 3*c*d*e^
2*f^4*g + 3*a*d^2*e*f*g^4 - a*d^3*g^5 + (3*c*d^2*e + a*e^3)*f^3*g^2 - (c*d
^3 + 3*a*d*e^2)*f^2*g^3)*(g*x + f)^(3/2))/g^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(218) = 436$.

Time = 0.12 (sec) , antiderivative size = 859, normalized size of antiderivative = 3.55

$$\int (d+ex)^3\sqrt{f+gx}(a+cx^2)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(g*x + f)*a*d^3*f + 15015*((g*x + f)^(3/2) - 3*sqrt(g*x
 + f)*f)*a*d^3 + 45045*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d^2*e*f/g +
 3003*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*
d^3*f/g^2 + 9009*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x +
 f)*f^2)*a*d*e^2*f/g^2 + 9009*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f +
15*sqrt(g*x + f)*f^2)*a*d^2*e/g + 3861*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(
5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^2*e*f/g^3 + 12
87*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35
*sqrt(g*x + f)*f^3)*a*e^3*f/g^3 + 1287*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(
5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^3/g^2 + 3861*(
5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqr
t(g*x + f)*f^3)*a*d*e^2/g^2 + 429*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2
)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f
)*f^4)*c*d*e^2*f/g^4 + 429*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 3
78*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*
c*d^2*e/g^3 + 143*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x +
 f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*a*e^3/g^3
 + 65*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x + f)^(7/2)*f
^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*sqrt(g*x +
f)*f^5)*c*e^3*f/g^5 + 195*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f ...
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.92

$$\int (d+ex)^3\sqrt{f+gx}(a+cx^2)\,dx = \frac{(f+g\,x)^{9/2}\,(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+20\,c\,e^3\,f^2+2\,a\,e^3\,g^2)}{9\,g^6} + \frac{2\,(f+g\,x)^{3/2}\,(c\,f^2+a\,g^2)\,(d\,g-e\,f)^3}{3\,g^6} + \frac{2\,c\,e^3\,(f+g\,x)^{13/2}}{13\,g^6} + \frac{2\,(f+g\,x)^{5/2}\,(d\,g-e\,f)^2\,(5\,c\,e\,f^2-2\,c\,d\,f\,g+3\,a\,e\,g^2)}{5\,g^6} + \frac{2\,(f+g\,x)^{7/2}\,(d\,g-e\,f)\,(c\,d^2\,g^2-8\,c\,d\,e\,f\,g+10\,c\,e^2\,f^2+3\,a\,e^2\,g^2)}{7\,g^6} + \frac{2\,c\,e^2\,(f+g\,x)^{11/2}\,(3\,d\,g-5\,e\,f)}{11\,g^6}$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^3,x)
```

output
```
((f + g*x)^(9/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*
f*g))/(9*g^6) + (2*(f + g*x)^(3/2)*(a*g^2 + c*f^2)*(d*g - e*f)^3)/(3*g^6)
+ (2*c*e^3*(f + g*x)^(13/2))/(13*g^6) + (2*(f + g*x)^(5/2)*(d*g - e*f)^2*(
3*a*e*g^2 + 5*c*e*f^2 - 2*c*d*f*g))/(5*g^6) + (2*(f + g*x)^(7/2)*(d*g - e*
f)*(3*a*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 - 8*c*d*e*f*g))/(7*g^6) + (2*c*
e^2*(f + g*x)^(11/2)*(3*d*g - 5*e*f))/(11*g^6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.00

$$\int (d+ex)^3\sqrt{f+gx}(a+cx^2)\,dx = \frac{2\sqrt{gx+f}\,(3465c\,e^3g^6x^6+12285cd\,e^2g^6x^5+315c\,e^3f\,g^5x^5+5005a\,e^3g^6x^4+15015c\,d^2e\,g^6x^4+1365cd\,e}{}$$

input
```
int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a),x)
```

output
```
(2*sqrt(f + g*x)*(15015*a*d**3*f*g**5 + 15015*a*d**3*g**6*x - 18018*a*d**2
*e*f**2*g**4 + 9009*a*d**2*e*f*g**5*x + 27027*a*d**2*e*g**6*x**2 + 10296*a
*d*e**2*f**3*g**3 - 5148*a*d*e**2*f**2*g**4*x + 3861*a*d*e**2*f*g**5*x**2
+ 19305*a*d*e**2*g**6*x**3 - 2288*a*e**3*f**4*g**2 + 1144*a*e**3*f**3*g**3
*x - 858*a*e**3*f**2*g**4*x**2 + 715*a*e**3*f*g**5*x**3 + 5005*a*e**3*g**6
*x**4 + 3432*c*d**3*f**3*g**3 - 1716*c*d**3*f**2*g**4*x + 1287*c*d**3*f*g*
*5*x**2 + 6435*c*d**3*g**6*x**3 - 6864*c*d**2*e*f**4*g**2 + 3432*c*d**2*e*
f**3*g**3*x - 2574*c*d**2*e*f**2*g**4*x**2 + 2145*c*d**2*e*f*g**5*x**3 + 1
5015*c*d**2*e*g**6*x**4 + 4992*c*d*e**2*f**5*g - 2496*c*d*e**2*f**4*g**2*x
 + 1872*c*d*e**2*f**3*g**3*x**2 - 1560*c*d*e**2*f**2*g**4*x**3 + 1365*c*d*
e**2*f*g**5*x**4 + 12285*c*d*e**2*g**6*x**5 - 1280*c*e**3*f**6 + 640*c*e**
3*f**5*g*x - 480*c*e**3*f**4*g**2*x**2 + 400*c*e**3*f**3*g**3*x**3 - 350*c
*e**3*f**2*g**4*x**4 + 315*c*e**3*f*g**5*x**5 + 3465*c*e**3*g**6*x**6))/(4
5045*g**6)
```

3.36 $\int (d+ex)^2\sqrt{f+gx}(a+cx^2)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 177

$$\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)\,dx=\frac{2(ef-dg)^2\left(cf^2+ag^2\right)(f+gx)^{3/2}}{3g^5}-\frac{4(ef-dg)\left(aeg^2+cf(2ef-dg)\right)(f+gx)^{5/2}}{5g^5}+\frac{2\left(ae^2g^2+c(6e^2f^2-6defg+d^2g^2)\right)(f+gx)^{7/2}}{7g^5}-\frac{4ce(2ef-dg)(f+gx)^{9/2}}{9g^5}+\frac{2ce^2(f+gx)^{11/2}}{11g^5}$$

output
```
2/3*(-d*g+e*f)^2*(a*g^2+c*f^2)*(g*x+f)^(3/2)/g^5-4/5*(-d*g+e*f)*(a*e*g^2+c
*f*(-d*g+2*e*f))*(g*x+f)^(5/2)/g^5+2/7*(a*e^2*g^2+c*(d^2*g^2-6*d*e*f*g+6*e
^2*f^2))*(g*x+f)^(7/2)/g^5-4/9*c*e*(-d*g+2*e*f)*(g*x+f)^(9/2)/g^5+2/11*c*e
^2*(g*x+f)^(11/2)/g^5
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.01

$$\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)\,dx$$
$$= \frac{2(f+gx)^{3/2}\left(33ag^2(35d^2g^2+14deg(-2f+3gx)+e^2(8f^2-12fgx+15g^2x^2))+c(33d^2g^2(8f^2-12fg\right.}{}$$

input
```
Integrate[(d + e*x)^2*Sqrt[f + g*x]*(a + c*x^2),x]
```

output
```
(2*(f + g*x)^(3/2)*(33*a*g^2*(35*d^2*g^2 + 14*d*e*g*(-2*f + 3*g*x) + e^2*(
8*f^2 - 12*f*g*x + 15*g^2*x^2)) + c*(33*d^2*g^2*(8*f^2 - 12*f*g*x + 15*g^2
*x^2) + 22*d*e*g*(-16*f^3 + 24*f^2*g*x - 30*f*g^2*x^2 + 35*g^3*x^3) + e^2*
(128*f^4 - 192*f^3*g*x + 240*f^2*g^2*x^2 - 280*f*g^3*x^3 + 315*g^4*x^4)))
/(3465*g^5)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(d+ex)^2\sqrt{f+gx}\,dx$$
$$\Big\downarrow \text{652}$$
$$\int \left(\frac{(f+gx)^{5/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{g^4}+\frac{\sqrt{f+gx}\left(ag^2+cf^2\right)(dg-ef)^2}{g^4}+\frac{2(f+gx)^{3/2}(ef-dg\right.}{}$$
$$\Big\downarrow \text{2009}$$

$$\frac{2(f+gx)^{7/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{7g^5}+\frac{2(f+gx)^{3/2}\left(ag^2+cf^2\right)(ef-dg)^2}{3g^5}-\frac{4(f+gx)^{5/2}(ef-dg)\left(aeg^2+cf(2ef-dg)\right)}{5g^5}-\frac{4ce(f+gx)^{9/2}(2ef-dg)}{9g^5}+\frac{2ce^2(f+gx)^{11/2}}{11g^5}$$

input
```
Int[(d + e*x)^2*Sqrt[f + g*x]*(a + c*x^2),x]
```

output
```
(2*(e*f - d*g)^2*(c*f^2 + a*g^2)*(f + g*x)^(3/2))/(3*g^5) - (4*(e*f - d*g)
*(a*e*g^2 + c*f*(2*e*f - d*g))*(f + g*x)^(5/2))/(5*g^5) + (2*(a*e^2*g^2 +
c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(7/2))/(7*g^5) - (4*c*e*(2*
e*f - d*g)*(f + g*x)^(9/2))/(9*g^5) + (2*c*e^2*(f + g*x)^(11/2))/(11*g^5)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{2(gx+f)^{\frac{3}{2}}\left(\left(\frac{3\left(\frac{7c\,x^2}{11}+a\right)x^2e^2}{7}+\frac{6\left(\frac{5c\,x^2}{9}+a\right)xde}{5}+d^2\left(a+\frac{3c\,x^2}{7}\right)\right)g^4-\frac{4f\left(\frac{3x\left(\frac{70c\,x^2}{99}+a\right)e^2}{7}+d\left(\frac{5c\,x^2}{7}+a\right)e+\frac{3c\,d^2x}{7}\right)g^3}{5}}{3g^5}$
derivativedivides	$\frac{\frac{2c\,e^2(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(2e(dg-ef)c-2fc\,e^2\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left((dg-ef)^2c-4e(dg-ef)cf+e^2\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left(-2(dg-ef)^2cf+\right.}{}}{g^5}$
default	$\frac{\frac{2c\,e^2(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(2e(dg-ef)c-2fc\,e^2\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left((dg-ef)^2c-4e(dg-ef)cf+e^2\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left(-2(dg-ef)^2cf+\right.}{}}{g^5}$
gosper	$2(gx+f)^{\frac{3}{2}}\left(315c\,e^2x^4g^4+770cde\,g^4x^3-280c\,e^2f\,g^3x^3+495a\,e^2g^4x^2+495c\,d^2g^4x^2-660cdef\,g^3x^2+240c\,e^2f^2g^2x^2+13\right.$
orering	$2(gx+f)^{\frac{3}{2}}\left(315c\,e^2x^4g^4+770cde\,g^4x^3-280c\,e^2f\,g^3x^3+495a\,e^2g^4x^2+495c\,d^2g^4x^2-660cdef\,g^3x^2+240c\,e^2f^2g^2x^2+13\right.$
trager	$2\left(315c\,e^2g^5x^5+770cde\,g^5x^4+35c\,e^2f\,g^4x^4+495a\,e^2g^5x^3+495c\,d^2g^5x^3+110cdef\,g^4x^3-40c\,e^2f^2g^3x^3+1386ade\,g^5x^2\right.$
risch	$2\left(315c\,e^2g^5x^5+770cde\,g^5x^4+35c\,e^2f\,g^4x^4+495a\,e^2g^5x^3+495c\,d^2g^5x^3+110cdef\,g^4x^3-40c\,e^2f^2g^3x^3+1386ade\,g^5x^2\right.$

input
```
int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*(g*x+f)^(3/2)*((3/7*(7/11*c*x^2+a)*x^2*e^2+6/5*(5/9*c*x^2+a)*x*d*e+d^2
*(a+3/7*c*x^2))*g^4-4/5*f*(3/7*x*(70/99*c*x^2+a)*e^2+d*(5/7*c*x^2+a)*e+3/7
*c*d^2*x)*g^3+8/35*f^2*((10/11*c*x^2+a)*e^2+2*c*d*x*e+c*d^2)*g^2-32/105*e*
f^3*(6/11*e*x+d)*c*g+128/1155*c*e^2*f^4)/g^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.50

$$\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)\,dx$$
$$=\frac{2\left(315\,ce^2g^5x^5+128\,ce^2f^5-352\,cdef^4g-924\,adef^2g^3+1155\,ad^2fg^4+264\left(cd^2+ae^2\right)f^3g^2+35\left(ce^2f\right.\right.}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a),x, algorithm="fricas")
```

output
```
2/3465*(315*c*e^2*g^5*x^5 + 128*c*e^2*f^5 - 352*c*d*e*f^4*g - 924*a*d*e*f^
2*g^3 + 1155*a*d^2*f*g^4 + 264*(c*d^2 + a*e^2)*f^3*g^2 + 35*(c*e^2*f*g^4 +
 22*c*d*e*g^5)*x^4 - 5*(8*c*e^2*f^2*g^3 - 22*c*d*e*f*g^4 - 99*(c*d^2 + a*e
^2)*g^5)*x^3 + 3*(16*c*e^2*f^3*g^2 - 44*c*d*e*f^2*g^3 + 462*a*d*e*g^5 + 33
*(c*d^2 + a*e^2)*f*g^4)*x^2 - (64*c*e^2*f^4*g - 176*c*d*e*f^3*g^2 - 462*a*
d*e*f*g^4 - 1155*a*d^2*g^5 + 132*(c*d^2 + a*e^2)*f^2*g^3)*x)*sqrt(g*x + f)
/g^5
```

Sympy [A] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.77

$$\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)\,dx$$

$$= \begin{cases} \frac{2\left(\frac{ce^2(f+gx)^{\frac{11}{2}}}{11g^4}+\frac{(f+gx)^{\frac{9}{2}}\cdot\left(2cdeg-4ce^2f\right)}{9g^4}+\frac{(f+gx)^{\frac{7}{2}}\left(ae^2g^2+cd^2g^2-6cdefg+6ce^2f^2\right)}{7g^4}+\frac{(f+gx)^{\frac{5}{2}}\cdot\left(2adeg^3-2ae^2fg^2-2cd^2fg^2+6cdef^2g-4ce^2f^3\right)}{5g^4} - \right.}{g} \\ \sqrt{f}\left(ad^2x+adex^2+\frac{cdex^4}{2}+\frac{ce^2x^5}{5}+\frac{x^3\left(ae^2+cd^2\right)}{3}\right) \end{cases}$$

input
```
integrate((e*x+d)**2*(g*x+f)**(1/2)*(c*x**2+a),x)
```

output
```
Piecewise((2*(c*e**2*(f + g*x)**(11/2)/(11*g**4) + (f + g*x)**(9/2)*(2*c*d
*e*g - 4*c*e**2*f)/(9*g**4) + (f + g*x)**(7/2)*(a*e**2*g**2 + c*d**2*g**2
- 6*c*d*e*f*g + 6*c*e**2*f**2)/(7*g**4) + (f + g*x)**(5/2)*(2*a*d*e*g**3 -
 2*a*e**2*f*g**2 - 2*c*d**2*f*g**2 + 6*c*d*e*f**2*g - 4*c*e**2*f**3)/(5*g*
*4) + (f + g*x)**(3/2)*(a*d**2*g**4 - 2*a*d*e*f*g**3 + a*e**2*f**2*g**2 +
c*d**2*f**2*g**2 - 2*c*d*e*f**3*g + c*e**2*f**4)/(3*g**4))/g, Ne(g, 0)), (
sqrt(f)*(a*d**2*x + a*d*e*x**2 + c*d*e*x**4/2 + c*e**2*x**5/5 + x**3*(a*e*
*2 + c*d**2)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.11

$$\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)\,dx$$
$$= \frac{2\left(315\,(gx+f)^{\frac{11}{2}}ce^2 - 770\,(2\,ce^2f - cdeg)(gx+f)^{\frac{9}{2}} + 495\,(6\,ce^2f^2 - 6\,cdefg + (cd^2+ae^2)g^2)(gx+f$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a),x, algorithm="maxima")
```

output
```
2/3465*(315*(g*x + f)^(11/2)*c*e^2 - 770*(2*c*e^2*f - c*d*e*g)*(g*x + f)^(
9/2) + 495*(6*c*e^2*f^2 - 6*c*d*e*f*g + (c*d^2 + a*e^2)*g^2)*(g*x + f)^(7/
2) - 1386*(2*c*e^2*f^3 - 3*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 + a*e^2)*f*g^2
)*(g*x + f)^(5/2) + 1155*(c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^
2*g^4 + (c*d^2 + a*e^2)*f^2*g^2)*(g*x + f)^(3/2))/g^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(157) = 314.

Time = 0.12 (sec) , antiderivative size = 561, normalized size of antiderivative = 3.17

$$\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(g*x + f)*a*d^2*f + 1155*((g*x + f)^(3/2) - 3*sqrt(g*x +
f)*f)*a*d^2 + 2310*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d*e*f/g + 231*(
3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d^2*f/g
^2 + 231*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)
*a*e^2*f/g^2 + 462*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x
 + f)*f^2)*a*d*e/g + 198*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g
*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d*e*f/g^3 + 99*(5*(g*x + f)^(7
/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3
)*c*d^2/g^2 + 99*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^
(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*e^2/g^2 + 11*(35*(g*x + f)^(9/2) - 180
*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 3
15*sqrt(g*x + f)*f^4)*c*e^2*f/g^4 + 22*(35*(g*x + f)^(9/2) - 180*(g*x + f)
^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*
x + f)*f^4)*c*d*e/g^3 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 9
90*(g*x + f)^(7/2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f
^4 - 693*sqrt(g*x + f)*f^5)*c*e^2/g^4)/g
```

Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

$$
\begin{aligned}
\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)\,dx
&= \frac{(f+g\,x)^{7/2}\,\left(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+12\,c\,e^2\,f^2+2\,a\,e^2\,g^2\right)}{7\,g^5} \\
&\quad+\frac{2\,(f+g\,x)^{3/2}\,\left(c\,f^2+a\,g^2\right)\,{\left(d\,g-e\,f\right)}^2}{3\,g^5} \\
&\quad+\frac{4\,(f+g\,x)^{5/2}\,\left(d\,g-e\,f\right)\,\left(2\,c\,e\,f^2-c\,d\,f\,g+a\,e\,g^2\right)}{5\,g^5} \\
&\quad+\frac{2\,c\,e^2\,(f+g\,x)^{11/2}}{11\,g^5}+\frac{4\,c\,e\,(f+g\,x)^{9/2}\,\left(d\,g-2\,e\,f\right)}{9\,g^5}
\end{aligned}
$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^2,x)
```

output
```
((f + g*x)^(7/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 - 12*c*d*e*f*g)
)/(7*g^5) + (2*(f + g*x)^(3/2)*(a*g^2 + c*f^2)*(d*g - e*f)^2)/(3*g^5) + (4
*(f + g*x)^(5/2)*(d*g - e*f)*(a*e*g^2 + 2*c*e*f^2 - c*d*f*g))/(5*g^5) + (2
*c*e^2*(f + g*x)^(11/2))/(11*g^5) + (4*c*e*(f + g*x)^(9/2)*(d*g - 2*e*f))/
(9*g^5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.68

$$\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)\,dx$$
$$=\frac{2\sqrt{gx+f}\,(315c\,e^2g^5x^5+770cde\,g^5x^4+35c\,e^2f\,g^4x^4+495a\,e^2g^5x^3+495c\,d^2g^5x^3+110cdef\,g^4x^3-40c}{}$$

input
```
int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a),x)
```

output
```
(2*sqrt(f + g*x)*(1155*a*d**2*f*g**4 + 1155*a*d**2*g**5*x - 924*a*d*e*f**2
*g**3 + 462*a*d*e*f*g**4*x + 1386*a*d*e*g**5*x**2 + 264*a*e**2*f**3*g**2 -
 132*a*e**2*f**2*g**3*x + 99*a*e**2*f*g**4*x**2 + 495*a*e**2*g**5*x**3 + 2
64*c*d**2*f**3*g**2 - 132*c*d**2*f**2*g**3*x + 99*c*d**2*f*g**4*x**2 + 495
*c*d**2*g**5*x**3 - 352*c*d*e*f**4*g + 176*c*d*e*f**3*g**2*x - 132*c*d*e*f
**2*g**3*x**2 + 110*c*d*e*f*g**4*x**3 + 770*c*d*e*g**5*x**4 + 128*c*e**2*f
**5 - 64*c*e**2*f**4*g*x + 48*c*e**2*f**3*g**2*x**2 - 40*c*e**2*f**2*g**3*
x**3 + 35*c*e**2*f*g**4*x**4 + 315*c*e**2*g**5*x**5))/(3465*g**5)
```

3.37 $\int (d+ex)\sqrt{f+gx}(a+cx^2)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int (d+ex)\sqrt{f+gx}\left(a+cx^2\right)\,dx = -\frac{2(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{3/2}}{3g^4} + \frac{2\left(aeg^2+cf(3ef-2dg)\right)(f+gx)^{5/2}}{5g^4} - \frac{2c(3ef-dg)(f+gx)^{7/2}}{7g^4} + \frac{2ce(f+gx)^{9/2}}{9g^4}$$

output
```
-2/3*(-d*g+e*f)*(a*g^2+c*f^2)*(g*x+f)^(3/2)/g^4+2/5*(a*e*g^2+c*f*(-2*d*g+3
*e*f))*(g*x+f)^(5/2)/g^4-2/7*c*(-d*g+3*e*f)*(g*x+f)^(7/2)/g^4+2/9*c*e*(g*x
+f)^(9/2)/g^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int (d+ex)\sqrt{f+gx}\left(a+cx^2\right)\,dx$$

$$= \frac{2(f+gx)^{3/2}\left(21ag^2(-2ef+5dg+3egx)+3cdg(8f^2-12fgx+15g^2x^2)+ce(-16f^3+24f^2gx-30fg\right.}{315g^4}$$

input
```
Integrate[(d + e*x)*Sqrt[f + g*x]*(a + c*x^2),x]
```

output
```
(2*(f + g*x)^(3/2)*(21*a*g^2*(-2*e*f + 5*d*g + 3*e*g*x) + 3*c*d*g*(8*f^2 -
 12*f*g*x + 15*g^2*x^2) + c*e*(-16*f^3 + 24*f^2*g*x - 30*f*g^2*x^2 + 35*g^
3*x^3)))/(315*g^4)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(d+ex)\sqrt{f+gx}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{\sqrt{f+gx}\left(ag^2+cf^2\right)(dg-ef)}{g^3}+\frac{(f+gx)^{3/2}\left(aeg^2+cf(3ef-2dg)\right)}{g^3}+\frac{c(f+gx)^{5/2}(dg-3ef)}{g^3}+\frac{ce(f+}{g}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(f+gx)^{3/2}\left(ag^2+cf^2\right)(ef-dg)}{3g^4}+\frac{2(f+gx)^{5/2}\left(aeg^2+cf(3ef-2dg)\right)}{5g^4}-\frac{2c(f+gx)^{7/2}(3ef-dg)}{7g^4}+\frac{2ce(f+gx)^{9/2}}{9g^4}$$

input
```
Int[(d + e*x)*Sqrt[f + g*x]*(a + c*x^2),x]
```

output
```
(-2*(e*f - d*g)*(c*f^2 + a*g^2)*(f + g*x)^(3/2))/(3*g^4) + (2*(a*e*g^2 + c
*f*(3*e*f - 2*d*g))*(f + g*x)^(5/2))/(5*g^4) - (2*c*(3*e*f - d*g)*(f + g*x
)^(7/2))/(7*g^4) + (2*c*e*(f + g*x)^(9/2))/(9*g^4)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{2\left(\left(\frac{3\left(\frac{7ex}{9}+d\right)x^2c}{7}+a\left(\frac{3ex}{5}+d\right)\right)g^3-\frac{2f\left(\frac{6\left(\frac{5ex}{6}+d\right)xc}{7}+ae\right)g^2}{5}+\frac{8cf^2(ex+d)g}{35}-\frac{16cef^3}{105}\right)(gx+f)^{\frac{3}{2}}}{3g^4}$
gosper	$\frac{2(gx+f)^{\frac{3}{2}}\left(35ce\,x^3g^3+45cd\,g^3x^2-30cef\,g^2x^2+63ae\,g^3x-36cdf\,g^2x+24ce\,f^2gx+105ad\,g^3-42aef\,g^2+24cd\,f^2g-16c\right.}{315g^4}$
orering	$\frac{2(gx+f)^{\frac{3}{2}}\left(35ce\,x^3g^3+45cd\,g^3x^2-30cef\,g^2x^2+63ae\,g^3x-36cdf\,g^2x+24ce\,f^2gx+105ad\,g^3-42aef\,g^2+24cd\,f^2g-16c\right.}{315g^4}$
derivativedivides	$\frac{\frac{2ce(gx+f)^{\frac{9}{2}}}{9}+\frac{2((dg-ef)c-2fce)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left(-2(dg-ef)cf+e\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2(dg-ef)\left(a\,g^2+c\,f^2\right)(gx+f)^{\frac{3}{2}}}{3}}{g^4}$
default	$\frac{\frac{2ce(gx+f)^{\frac{9}{2}}}{9}+\frac{2((dg-ef)c-2fce)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left(-2(dg-ef)cf+e\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2(dg-ef)\left(a\,g^2+c\,f^2\right)(gx+f)^{\frac{3}{2}}}{3}}{g^4}$
trager	$\frac{2\left(35ce\,g^4x^4+45cd\,g^4x^3+5cef\,g^3x^3+63ae\,g^4x^2+9cdf\,g^3x^2-6ce\,f^2g^2x^2+105ad\,g^4x+21aef\,g^3x-12cd\,f^2g^2x+8ce\,f^3g\right.}{315g^4}$
risch	$\frac{2\left(35ce\,g^4x^4+45cd\,g^4x^3+5cef\,g^3x^3+63ae\,g^4x^2+9cdf\,g^3x^2-6ce\,f^2g^2x^2+105ad\,g^4x+21aef\,g^3x-12cd\,f^2g^2x+8ce\,f^3g\right.}{315g^4}$

input
```
int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*((3/7*(7/9*e*x+d)*x^2*c+a*(3/5*e*x+d))*g^3-2/5*f*(6/7*(5/6*e*x+d)*x*c+
a*e)*g^2+8/35*c*f^2*(e*x+d)*g-16/105*c*e*f^3)*(g*x+f)^(3/2)/g^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.24

$$\int (d+ex)\sqrt{f+gx}\left(a+cx^2\right)\,dx$$
$$=\frac{2\,(35\,ceg^4x^4-16\,cef^4+24\,cdf^3g-42\,aef^2g^2+105\,adfg^3+5\,(cefg^3+9\,cdg^4)x^3-3\,(2\,cef^2g^2-3\,cdfg}{315\,g^4}$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a),x, algorithm="fricas")
```

output
```
2/315*(35*c*e*g^4*x^4 - 16*c*e*f^4 + 24*c*d*f^3*g - 42*a*e*f^2*g^2 + 105*a
*d*f*g^3 + 5*(c*e*f*g^3 + 9*c*d*g^4)*x^3 - 3*(2*c*e*f^2*g^2 - 3*c*d*f*g^3
- 21*a*e*g^4)*x^2 + (8*c*e*f^3*g - 12*c*d*f^2*g^2 + 21*a*e*f*g^3 + 105*a*d
*g^4)*x)*sqrt(g*x + f)/g^4
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.45

$$\int (d+ex)\sqrt{f+gx}\left(a+cx^2\right)\,dx$$
$$=\begin{cases}\frac{2\left(\frac{ce(f+gx)^{\frac{9}{2}}}{9g^3}+\frac{(f+gx)^{\frac{7}{2}}(cdg-3cef)}{7g^3}+\frac{(f+gx)^{\frac{5}{2}}\left(aeg^2-2cdfg+3cef^2\right)}{5g^3}+\frac{(f+gx)^{\frac{3}{2}}\left(adg^3-aefg^2+cdf^2g-cef^3\right)}{3g^3}\right)}{g} & \text{for } g\neq 0\\ \sqrt{f}\left(adx+\frac{aex^2}{2}+\frac{cdx^3}{3}+\frac{cex^4}{4}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+a),x)
```

output
```
Piecewise((2*(c*e*(f + g*x)**(9/2)/(9*g**3) + (f + g*x)**(7/2)*(c*d*g - 3*
c*e*f)/(7*g**3) + (f + g*x)**(5/2)*(a*e*g**2 - 2*c*d*f*g + 3*c*e*f**2)/(5*
g**3) + (f + g*x)**(3/2)*(a*d*g**3 - a*e*f*g**2 + c*d*f**2*g - c*e*f**3)/(
3*g**3))/g, Ne(g, 0)), (sqrt(f)*(a*d*x + a*e*x**2/2 + c*d*x**3/3 + c*e*x**
4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int (d+ex)\sqrt{f+gx}(a+cx^2)\,dx = \frac{2\left(35\,(gx+f)^{\frac{9}{2}}ce-45\,(3\,cef-cdg)(gx+f)^{\frac{7}{2}}+63\,(3\,cef^2-2\,cdfg+aeg^2)(gx+f)^{\frac{5}{2}}-105\,(cef^3-c\right.}{315\,g^4}$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a),x, algorithm="maxima")
```

output
```
2/315*(35*(g*x + f)^(9/2)*c*e - 45*(3*c*e*f - c*d*g)*(g*x + f)^(7/2) + 63*
(3*c*e*f^2 - 2*c*d*f*g + a*e*g^2)*(g*x + f)^(5/2) - 105*(c*e*f^3 - c*d*f^2
*g + a*e*f*g^2 - a*d*g^3)*(g*x + f)^(3/2))/g^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(99) = 198.

Time = 0.11 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.70

$$\int (d+ex)\sqrt{f+gx}(a+cx^2)\,dx = \frac{2\left(315\,\sqrt{gx+f}adf+105\left((gx+f)^{\frac{3}{2}}-3\,\sqrt{gx+f}f\right)ad+\frac{105\left((gx+f)^{\frac{3}{2}}-3\,\sqrt{gx+f}f\right)aef}{g}+\frac{21\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx\right.}{}\right.}{}$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(g*x + f)*a*d*f + 105*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)
*a*d + 105*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*e*f/g + 21*(3*(g*x + f)
^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d*f/g^2 + 21*(3*(g
*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a*e/g + 9*(5*
(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(
g*x + f)*f^3)*c*e*f/g^3 + 9*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35
*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d/g^2 + (35*(g*x + f)^(9/2)
 - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f
^3 + 315*sqrt(g*x + f)*f^4)*c*e/g^3)/g
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int (d+ex)\sqrt{f+gx}\left(a+cx^2\right)\,dx = \frac{(f+g\,x)^{5/2}\,(6\,c\,e\,f^2-4\,c\,d\,f\,g+2\,a\,e\,g^2)}{5\,g^4} + \frac{2\,c\,e\,(f+g\,x)^{9/2}}{9\,g^4} + \frac{2\,c\,(f+g\,x)^{7/2}\,(d\,g-3\,e\,f)}{7\,g^4} + \frac{2\,(f+g\,x)^{3/2}\,(c\,f^2+a\,g^2)\,(d\,g-e\,f)}{3\,g^4}$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x),x)
```

output
```
((f + g*x)^(5/2)*(2*a*e*g^2 + 6*c*e*f^2 - 4*c*d*f*g))/(5*g^4) + (2*c*e*(f
+ g*x)^(9/2))/(9*g^4) + (2*c*(f + g*x)^(7/2)*(d*g - 3*e*f))/(7*g^4) + (2*(
f + g*x)^(3/2)*(a*g^2 + c*f^2)*(d*g - e*f))/(3*g^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.28

$$\int (d+ex)\sqrt{f+gx}\left(a+cx^2\right)\,dx = \frac{2\sqrt{gx+f}\,(35ce\,g^4x^4+45cd\,g^4x^3+5cef\,g^3x^3+63ae\,g^4x^2+9cdf\,g^3x^2-6ce\,f^2g^2x^2+105ad\,g^4x+21ae}{315g^4}$$

input
```
int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a),x)
```

output
```
(2*sqrt(f + g*x)*(105*a*d*f*g**3 + 105*a*d*g**4*x - 42*a*e*f**2*g**2 + 21*
a*e*f*g**3*x + 63*a*e*g**4*x**2 + 24*c*d*f**3*g - 12*c*d*f**2*g**2*x + 9*c
*d*f*g**3*x**2 + 45*c*d*g**4*x**3 - 16*c*e*f**4 + 8*c*e*f**3*g*x - 6*c*e*f
**2*g**2*x**2 + 5*c*e*f*g**3*x**3 + 35*c*e*g**4*x**4))/(315*g**4)
```

3.38 $\int \sqrt{f+gx}(a+cx^2)\ dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \sqrt{f+gx}(a+cx^2)\ dx = \frac{2(cf^2+ag^2)\,(f+gx)^{3/2}}{3g^3} - \frac{4cf(f+gx)^{5/2}}{5g^3} + \frac{2c(f+gx)^{7/2}}{7g^3}$$

output
```
2/3*(a*g^2+c*f^2)*(g*x+f)^(3/2)/g^3-4/5*c*f*(g*x+f)^(5/2)/g^3+2/7*c*(g*x+f
)^(7/2)/g^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \sqrt{f+gx}(a+cx^2)\ dx = \frac{2(f+gx)^{3/2}\,(35ag^2+c(8f^2-12fgx+15g^2x^2))}{105g^3}$$

input
```
Integrate[Sqrt[f + g*x]*(a + c*x^2),x]
```

output
```
(2*(f + g*x)^(3/2)*(35*a*g^2 + c*(8*f^2 - 12*f*g*x + 15*g^2*x^2)))/(105*g^
3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)\sqrt{f+gx}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{\sqrt{f+gx}(ag^2+cf^2)}{g^2}+\frac{c(f+gx)^{5/2}}{g^2}-\frac{2cf(f+gx)^{3/2}}{g^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(f+gx)^{3/2}(ag^2+cf^2)}{3g^3}+\frac{2c(f+gx)^{7/2}}{7g^3}-\frac{4cf(f+gx)^{5/2}}{5g^3}$$

input
```
Int[Sqrt[f + g*x]*(a + c*x^2),x]
```

output
```
(2*(c*f^2 + a*g^2)*(f + g*x)^(3/2))/(3*g^3) - (4*c*f*(f + g*x)^(5/2))/(5*g
^3) + (2*c*(f + g*x)^(7/2))/(7*g^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$\frac{\left(\left(30 c \,x^{2}+70 a \right) g^{2}-24 c f x g +16 c \,f^{2}\right) \left(g x +f \right)^{\frac{3}{2}}}{105 g^{3}}$	40
gosper	$\frac{2 \left(g x +f \right)^{\frac{3}{2}} \left(15 c \,x^{2} g^{2}-12 c f x g +35 a \,g^{2}+8 c \,f^{2}\right)}{105 g^{3}}$	41
orering	$\frac{2 \left(g x +f \right)^{\frac{3}{2}} \left(15 c \,x^{2} g^{2}-12 c f x g +35 a \,g^{2}+8 c \,f^{2}\right)}{105 g^{3}}$	41
derivativedivides	$\frac{\frac{2 c \left(g x +f \right)^{\frac{7}{2}}}{7}-\frac{4 c f \left(g x +f \right)^{\frac{5}{2}}}{5}+\frac{2 \left(a \,g^{2}+c \,f^{2}\right) \left(g x +f \right)^{\frac{3}{2}}}{3}}{g^{3}}$	48
default	$\frac{\frac{2 c \left(g x +f \right)^{\frac{7}{2}}}{7}-\frac{4 c f \left(g x +f \right)^{\frac{5}{2}}}{5}+\frac{2 \left(a \,g^{2}+c \,f^{2}\right) \left(g x +f \right)^{\frac{3}{2}}}{3}}{g^{3}}$	48
trager	$\frac{2 \left(15 c \,g^{3} x^{3}+3 c f \,g^{2} x^{2}+35 a \,g^{3} x -4 c \,f^{2} g x +35 a f \,g^{2}+8 c \,f^{3}\right) \sqrt{g x +f}}{105 g^{3}}$	61
risch	$\frac{2 \left(15 c \,g^{3} x^{3}+3 c f \,g^{2} x^{2}+35 a \,g^{3} x -4 c \,f^{2} g x +35 a f \,g^{2}+8 c \,f^{3}\right) \sqrt{g x +f}}{105 g^{3}}$	61

input
```
int((g*x+f)^(1/2)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/105*((30*c*x^2+70*a)*g^2-24*c*f*x*g+16*c*f^2)*(g*x+f)^(3/2)/g^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \sqrt{f+gx}\left(a+cx^2\right)\,dx = \frac{2\left(15\,cg^3x^3+3\,cfg^2x^2+8\,cf^3+35\,afg^2-(4\,cf^2g-35\,ag^3)x\right)\sqrt{gx+f}}{105\,g^3}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a),x, algorithm="fricas")
```

output
```
2/105*(15*c*g^3*x^3 + 3*c*f*g^2*x^2 + 8*c*f^3 + 35*a*f*g^2 - (4*c*f^2*g -
35*a*g^3)*x)*sqrt(g*x + f)/g^3
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \sqrt{f+gx}\left(a+cx^2\right)\,dx=\begin{cases} \frac{2\left(-\frac{2cf(f+gx)^{\frac{5}{2}}}{5g^2}+\frac{c(f+gx)^{\frac{7}{2}}}{7g^2}+\frac{(f+gx)^{\frac{3}{2}}\left(ag^2+cf^2\right)}{3g^2}\right)}{g} & \text{for } g\neq 0\\ \sqrt{f}\left(ax+\frac{cx^3}{3}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a),x)
```

output
```
Piecewise((2*(-2*c*f*(f + g*x)**(5/2)/(5*g**2) + c*(f + g*x)**(7/2)/(7*g**
2) + (f + g*x)**(3/2)*(a*g**2 + c*f**2)/(3*g**2))/g, Ne(g, 0)), (sqrt(f)*(
a*x + c*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

$$\int \sqrt{f+gx}\left(a+cx^2\right)\,dx=\frac{2\left(15\,(gx+f)^{\frac{7}{2}}c-42\,(gx+f)^{\frac{5}{2}}cf+35\,(cf^2+ag^2)(gx+f)^{\frac{3}{2}}\right)}{105\,g^3}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a),x, algorithm="maxima")
```

output
```
2/105*(15*(g*x + f)^(7/2)*c - 42*(g*x + f)^(5/2)*c*f + 35*(c*f^2 + a*g^2)*
(g*x + f)^(3/2))/g^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(51) = 102$.

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.02

$$\int \sqrt{f+gx}\left(a+cx^2\right)\,dx = \frac{2\left(105\sqrt{gx+f}af + 35\left((gx+f)^{\frac{3}{2}} - 3\sqrt{gx+f}f\right)a + \frac{7\left(3(gx+f)^{\frac{5}{2}} - 10(gx+f)^{\frac{3}{2}}f + 15\sqrt{gx+f}f^2\right)cf}{g^2} + \frac{3\left(5(gx+f)\right.}{}\right.}{105\,g}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a),x, algorithm="giac")
```

output
```
2/105*(105*sqrt(g*x + f)*a*f + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a
+ 7*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*f/
g^2 + 3*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2
 - 35*sqrt(g*x + f)*f^3)*c/g^2)/g
```

Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \sqrt{f+gx}\left(a+cx^2\right)\,dx = \frac{2\,(f+g\,x)^{3/2}\,\left(15\,c\,(f+g\,x)^2+35\,a\,g^2+35\,c\,f^2-42\,c\,f\,(f+g\,x)\right)}{105\,g^3}$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2),x)
```

output
```
(2*(f + g*x)^(3/2)*(15*c*(f + g*x)^2 + 35*a*g^2 + 35*c*f^2 - 42*c*f*(f + g
*x)))/(105*g^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \sqrt{f+gx}\left(a+cx^2\right)\,dx = \frac{2\sqrt{gx+f}\,(15c\,g^3x^3+3cf\,g^2x^2+35a\,g^3x-4c\,f^2gx+35af\,g^2+8c\,f^3)}{105g^3}$$

input
```
int((g*x+f)^(1/2)*(c*x^2+a),x)
```

output
```
(2*sqrt(f + g*x)*(35*a*f*g**2 + 35*a*g**3*x + 8*c*f**3 - 4*c*f**2*g*x + 3*
c*f*g**2*x**2 + 15*c*g**3*x**3))/(105*g**3)
```

3.39 $\int \frac{\sqrt{f+gx}\left(a+cx^2\right)}{d+ex} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{d+ex} \, dx = \frac{2(cd^2+ae^2)\sqrt{f+gx}}{e^3} - \frac{2c(ef+dg)(f+gx)^{3/2}}{3e^2g^2} + \frac{2c(f+gx)^{5/2}}{5eg^2} - \frac{2(cd^2+ae^2)\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{7/2}}$$

output
```
2*(a*e^2+c*d^2)*(g*x+f)^(1/2)/e^3-2/3*c*(d*g+e*f)*(g*x+f)^(3/2)/e^2/g^2+2/
5*c*(g*x+f)^(5/2)/e/g^2-2*(a*e^2+c*d^2)*(-d*g+e*f)^(1/2)*arctanh(e^(1/2)*(
g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{d+ex} \, dx = \frac{2\sqrt{f+gx}(15ae^2g^2+c(15d^2g^2-5deg(f+gx)+e^2(-2f^2+fgx+3g^2x^2)))}{15e^3g^2} - \frac{2(cd^2+ae^2)\sqrt{-ef+dg}\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{7/2}}$$

input
```
Integrate[(Sqrt[f + g*x]*(a + c*x^2))/(d + e*x),x]
```

output
```
(2*Sqrt[f + g*x]*(15*a*e^2*g^2 + c*(15*d^2*g^2 - 5*d*e*g*(f + g*x) + e^2*(
-2*f^2 + f*g*x + 3*g^2*x^2))))/(15*e^3*g^2) - (2*(c*d^2 + a*e^2)*Sqrt[-(e*
f) + d*g]*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/e^(7/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {649, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)\sqrt{f+gx}}{d+ex}\,dx$$

$$\Big\downarrow \text{649}$$

$$\frac{2\int -\frac{(f+gx)(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{(f+gx)(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{1584}$$

$$-\frac{2\int \left(-\frac{(cd^2+ae^2)g^2}{e^3}-\frac{c(f+gx)^2}{e}+\frac{c(ef+dg)(f+gx)}{e^2}+\frac{afg^2e^3-adg^3e^2+cd^2fg^2e-cd^3g^3}{e^3(ef-dg-e(f+gx))}\right)d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{g^2(ae^2+cd^2)\sqrt{ef-dg}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{7/2}}+\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{e^3}-\frac{c(f+gx)^{3/2}(dg+ef)}{3e^2}+\frac{c(f+gx)^{5/2}}{5e}\right)}{g^2}$$

input
```
Int[(Sqrt[f + g*x]*(a + c*x^2))/(d + e*x),x]
```

output
```
(2*(((c*d^2 + a*e^2)*g^2*Sqrt[f + g*x])/e^3 - (c*(e*f + d*g)*(f + g*x)^(3/
2))/(3*e^2) + (c*(f + g*x)^(5/2))/(5*e) - ((c*d^2 + a*e^2)*g^2*Sqrt[e*f -
d*g]*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/e^(7/2)))/g^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{2\left(-\left(\left(\left(-\frac{2\left(-\frac{3gx}{2}+f\right)(gx+f)c}{15}+a\,g^2\right)e^2-\frac{cdeg(gx+f)}{3}+c\,d^2g^2\right)\sqrt{(dg-ef)e}\,\sqrt{gx+f}+g^2(dg-ef)\left(a\,e^2+c\,d^2\right)\arctan\right.\right.}{\sqrt{(dg-ef)e}\,g^2e^3}$
risch	$\frac{2\left(3c\,e^2g^2x^2-5cde\,g^2x+c\,e^2fgx+15a\,e^2g^2+15c\,d^2g^2-5cdefg-2c\,e^2f^2\right)\sqrt{gx+f}}{15g^2e^3}-\frac{2\left(ad\,e^2g-a\,e^3f+c\,d^3g-c\,d^2ef\right)a}{e^3\sqrt{(dg-ef)}}$
derivativedivides	$\frac{\frac{2\left(\frac{c(gx+f)^{\frac{5}{2}}e^2}{5}-\frac{cdeg(gx+f)^{\frac{3}{2}}}{3}-\frac{c\,e^2f(gx+f)^{\frac{3}{2}}}{3}+a\,e^2g^2\sqrt{gx+f}+c\,d^2g^2\sqrt{gx+f}\right)}{e^3}-\frac{2g^2\left(ad\,e^2g-a\,e^3f+c\,d^3g-c\,d^2ef\right)\arctan\left(-\right.}{e^3\sqrt{(dg-ef)e}}}{g^2}$
default	$\frac{\frac{2\left(\frac{c(gx+f)^{\frac{5}{2}}e^2}{5}-\frac{cdeg(gx+f)^{\frac{3}{2}}}{3}-\frac{c\,e^2f(gx+f)^{\frac{3}{2}}}{3}+a\,e^2g^2\sqrt{gx+f}+c\,d^2g^2\sqrt{gx+f}\right)}{e^3}-\frac{2g^2\left(ad\,e^2g-a\,e^3f+c\,d^3g-c\,d^2ef\right)\arctan\left(-\right.}{e^3\sqrt{(dg-ef)e}}}{g^2}$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-2*(-((-2/15*(-3/2*g*x+f)*(g*x+f)*c+a*g^2)*e^2-1/3*c*d*e*g*(g*x+f)+c*d^2*g
^2)*((d*g-e*f)*e)^(1/2)*(g*x+f)^(1/2)+g^2*(d*g-e*f)*(a*e^2+c*d^2)*arctan(e
*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))/((d*g-e*f)*e)^(1/2)/g^2/e^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{d+ex}\,dx$$

$$= \left[\frac{15\,(cd^2+ae^2)g^2\sqrt{\frac{ef-dg}{e}}\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{gx+f}e\sqrt{\frac{ef-dg}{e}}}{ex+d}\right)+2\,(3\,ce^2g^2x^2-2\,ce^2f^2-5\,cdefg+15\,(cd^2}{15\,e^3g^2}\right.$$

$$-\frac{2\left(15\,(cd^2+ae^2)g^2\sqrt{-\frac{ef-dg}{e}}\arctan\left(-\frac{\sqrt{gx+f}e\sqrt{-\frac{ef-dg}{e}}}{ef-dg}\right)-(3\,ce^2g^2x^2-2\,ce^2f^2-5\,cdefg+15\,(cd^2\right.}{15\,e^3g^2}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/15*(15*(c*d^2 + a*e^2)*g^2*sqrt((e*f - d*g)/e)*log((e*g*x + 2*e*f - d*g
 - 2*sqrt(g*x + f)*e*sqrt((e*f - d*g)/e))/(e*x + d)) + 2*(3*c*e^2*g^2*x^2
- 2*c*e^2*f^2 - 5*c*d*e*f*g + 15*(c*d^2 + a*e^2)*g^2 + (c*e^2*f*g - 5*c*d*
e*g^2)*x)*sqrt(g*x + f))/(e^3*g^2), -2/15*(15*(c*d^2 + a*e^2)*g^2*sqrt(-(e
*f - d*g)/e)*arctan(-sqrt(g*x + f)*e*sqrt(-(e*f - d*g)/e)/(e*f - d*g)) - (
3*c*e^2*g^2*x^2 - 2*c*e^2*f^2 - 5*c*d*e*f*g + 15*(c*d^2 + a*e^2)*g^2 + (c*
e^2*f*g - 5*c*d*e*g^2)*x)*sqrt(g*x + f))/(e^3*g^2)]
```

Sympy [A] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{d+ex}\,dx = \begin{cases} \frac{2\left(\frac{c(f+gx)^{\frac{5}{2}}}{5eg} + \frac{(f+gx)^{\frac{3}{2}}(-cdg-cef)}{3e^2g} + \frac{\sqrt{f+gx}\left(ae^2g+cd^2g\right)}{e^3} - \frac{g\left(ae^2+cd^2\right)(dg-ef)\,\mathrm{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^4\sqrt{\frac{dg-ef}{e}}}\right)}{g} & \text{for } g \neq 0 \\ \sqrt{f}\left(-\frac{cdx}{e^2} + \frac{cx^2}{2e} + \frac{\left(ae^2+cd^2\right)\left(\begin{cases} \frac{x}{d} & \text{for } e = 0 \\ \frac{\log(d+ex)}{e} & \text{otherwise} \end{cases}\right)}{e^2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a)/(e*x+d),x)
```

output
```
Piecewise((2*(c*(f + g*x)**(5/2)/(5*e*g) + (f + g*x)**(3/2)*(-c*d*g - c*e*
f)/(3*e**2*g) + sqrt(f + g*x)*(a*e**2*g + c*d**2*g)/e**3 - g*(a*e**2 + c*d
**2)*(d*g - e*f)*atan(sqrt(f + g*x)/sqrt((d*g - e*f)/e))/(e**4*sqrt((d*g -
 e*f)/e)))/g, Ne(g, 0)), (sqrt(f)*(-c*d*x/e**2 + c*x**2/(2*e) + (a*e**2 +
c*d**2)*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{d+ex}\,dx = \frac{2\left(cd^2ef+ae^3f-cd^3g-ade^2g\right)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{\sqrt{-e^2f+deg}e^3} + \frac{2\left(3\,(gx+f)^{\frac{5}{2}}ce^4g^8-5\,(gx+f)^{\frac{3}{2}}ce^4fg^8-5\,(gx+f)^{\frac{3}{2}}cde^3g^9+15\,\sqrt{gx+f}cd^2e^2g^{10}+15\,\sqrt{gx+f}ae\right.}{15\,e^5g^{10}}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)/(e*x+d),x, algorithm="giac")
```

output
```
2*(c*d^2*e*f + a*e^3*f - c*d^3*g - a*d*e^2*g)*arctan(sqrt(g*x + f)*e/sqrt(
-e^2*f + d*e*g))/(sqrt(-e^2*f + d*e*g)*e^3) + 2/15*(3*(g*x + f)^(5/2)*c*e^
4*g^8 - 5*(g*x + f)^(3/2)*c*e^4*f*g^8 - 5*(g*x + f)^(3/2)*c*d*e^3*g^9 + 15
*sqrt(g*x + f)*c*d^2*e^2*g^10 + 15*sqrt(g*x + f)*a*e^4*g^10)/(e^5*g^10)
```

Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{d+ex}\,dx = \sqrt{f+g\,x}\left(\frac{2\,c\,f^2+2\,a\,g^2}{e\,g^2} + \frac{\left(\frac{2\,c\,(d\,g^3-e\,f\,g^2)}{e^2\,g^4}+\frac{4\,c\,f}{e\,g^2}\right)\,(d\,g^3-e\,f\,g^2)}{e\,g^2}\right) - (f+g\,x)^{3/2}\left(\frac{2\,c\,(d\,g^3-e\,f\,g^2)}{3\,e^2\,g^4}+\frac{4\,c\,f}{3\,e\,g^2}\right)+\frac{2\,c\,(f+g\,x)^{5/2}}{5\,e\,g^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}\,1\mathrm{i}}{\sqrt{e\,f-d\,g}}\right)\,(c\,d^2+a\,e^2)\,\sqrt{e\,f-d\,g}\,2\mathrm{i}}{e^{7/2}}$$

input
```
int(((f + g*x)^(1/2)*(a + c*x^2))/(d + e*x),x)
```

output
```
(f + g*x)^(1/2)*((2*a*g^2 + 2*c*f^2)/(e*g^2) + (((2*c*(d*g^3 - e*f*g^2))/(
e^2*g^4) + (4*c*f)/(e*g^2))*(d*g^3 - e*f*g^2))/(e*g^2)) - (f + g*x)^(3/2)*
((2*c*(d*g^3 - e*f*g^2))/(3*e^2*g^4) + (4*c*f)/(3*e*g^2)) + (atan((e^(1/2)
*(f + g*x)^(1/2)*1i)/(e*f - d*g)^(1/2))*(a*e^2 + c*d^2)*(e*f - d*g)^(1/2)*
2i)/e^(7/2) + (2*c*(f + g*x)^(5/2))/(5*e*g^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{d+ex}\,dx = \frac{-2\sqrt{e}\,\sqrt{dg-ef}\,atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-ef}}\right)a\,e^2g^2 - 2\sqrt{e}\,\sqrt{dg-ef}\,atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-ef}}\right)c\,d^2g^2 + 2\sqrt{gx+f}\,a\,e^3g^2 + 2}{e^4g^2}$$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)/(e*x+d),x)
```

output
```
(2*( - 15*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g
 - e*f)))*a*e**2*g**2 - 15*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/
(sqrt(e)*sqrt(d*g - e*f)))*c*d**2*g**2 + 15*sqrt(f + g*x)*a*e**3*g**2 + 15
*sqrt(f + g*x)*c*d**2*e*g**2 - 5*sqrt(f + g*x)*c*d*e**2*f*g - 5*sqrt(f + g
*x)*c*d*e**2*g**2*x - 2*sqrt(f + g*x)*c*e**3*f**2 + sqrt(f + g*x)*c*e**3*f
*g*x + 3*sqrt(f + g*x)*c*e**3*g**2*x**2))/(15*e**4*g**2)
```

3.40 $\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 136

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^2}\,dx = -\frac{4cd\sqrt{f+gx}}{e^3} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{e^3(d+ex)} + \frac{2c(f+gx)^{3/2}}{3e^2g} - \frac{(ae^2g-cd(4ef-5dg))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{7/2}\sqrt{ef-dg}}$$

output
```
-4*c*d*(g*x+f)^(1/2)/e^3-(a*e^2+c*d^2)*(g*x+f)^(1/2)/e^3/(e*x+d)+2/3*c*(g*
x+f)^(3/2)/e^2/g-(a*e^2*g-c*d*(-5*d*g+4*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2
)/(-d*g+e*f)^(1/2))/e^(7/2)/(-d*g+e*f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^2}\,dx = \frac{\sqrt{f+gx}(-3ae^2g+c(-15d^2g+2de(f-5gx)+2e^2x(f+gx)))}{3e^3g(d+ex)} + \frac{(ae^2g+cd(-4ef+5dg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{7/2}\sqrt{-ef+dg}}$$

input
```
Integrate[(Sqrt[f + g*x]*(a + c*x^2))/(d + e*x)^2,x]
```

output
```
(Sqrt[f + g*x]*(-3*a*e^2*g + c*(-15*d^2*g + 2*d*e*(f - 5*g*x) + 2*e^2*x*(f
 + g*x))))/(3*e^3*g*(d + e*x)) + ((a*e^2*g + c*d*(-4*e*f + 5*d*g))*ArcTan[
(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(7/2)*Sqrt[-(e*f) + d*g])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {649, 1580, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)\sqrt{f+gx}}{(d+ex)^2}dx$$

$$\Big\downarrow \ 649$$

$$\frac{2\int \frac{(f+gx)(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{g}$$

$$\Big\downarrow \ 1580$$

$$\frac{2\left(\frac{\int -\frac{(cd^2+ae^2)g^2+2ce^2(f+gx)^2-2ce(ef+dg)(f+gx)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{2e^3}+\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{2e^3(-dg-e(f+gx)+ef)}\right)}{g}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{2e^3(-dg-e(f+gx)+ef)}-\frac{\int \frac{(cd^2+ae^2)g^2+2ce^2(f+gx)^2-2ce(ef+dg)(f+gx)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{2e^3}\right)}{g}$$

$$\Big\downarrow \ 1467$$

$$\frac{2\left(\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{2e^3(-dg-e(f+gx)+ef)}-\frac{\int \left(4cdg-2ce(f+gx)+\frac{5cd^2g^2+ae^2g^2-4cdefg}{ef-dg-e(f+gx)}\right)d\sqrt{f+gx}}{2e^3}\right)}{g}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{2e^3(-dg-e(f+gx)+ef)}-\frac{\frac{g\left(ae^2g-cd(4ef-5dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}\sqrt{ef-dg}}+4cdg\sqrt{f+gx}-\frac{2}{3}ce(f+gx)^{3/2}}{2e^3}\right)}{g}$$

input
```
Int[(Sqrt[f + g*x]*(a + c*x^2))/(d + e*x)^2,x]
```

output
```
(2*(((c*d^2 + a*e^2)*g^2*Sqrt[f + g*x])/(2*e^3*(e*f - d*g - e*(f + g*x)))
- (4*c*d*g*Sqrt[f + g*x] - (2*c*e*(f + g*x)^(3/2))/3 + (g*(a*e^2*g - c*d*(
4*e*f - 5*d*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(Sqrt[e]
*Sqrt[e*f - d*g]))/(2*e^3)))/g
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
 - d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{2c(-egx+6dg-ef)\sqrt{gx+f}}{3g\,e^{3}}+\frac{\frac{2\left(-\frac{1}{2}a\,e^{2}g-\frac{1}{2}c\,d^{2}g\right)\sqrt{gx+f}}{e(gx+f)+dg-ef}+\frac{\left(a\,e^{2}g+5c\,d^{2}g-4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{\sqrt{(dg-ef)e}}}{e^{3}}$
derivativedivides	$\frac{-\frac{2c\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+2dg\sqrt{gx+f}\right)}{e^{3}}+\frac{2g\left(\frac{\left(-\frac{1}{2}a\,e^{2}g-\frac{1}{2}c\,d^{2}g\right)\sqrt{gx+f}}{e(gx+f)+dg-ef}+\frac{\left(a\,e^{2}g+5c\,d^{2}g-4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2\sqrt{(dg-ef)e}}\right)}{e^{3}}}{g}$
default	$\frac{-\frac{2c\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+2dg\sqrt{gx+f}\right)}{e^{3}}+\frac{2g\left(\frac{\left(-\frac{1}{2}a\,e^{2}g-\frac{1}{2}c\,d^{2}g\right)\sqrt{gx+f}}{e(gx+f)+dg-ef}+\frac{\left(a\,e^{2}g+5c\,d^{2}g-4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2\sqrt{(dg-ef)e}}\right)}{e^{3}}}{g}$
pseudoelliptic	$-\frac{\left(\left(\left(-\frac{2c\,x^{2}}{3}+a\right)g-\frac{2cfx}{3}\right)e^{2}-\frac{2cd(-5gx+f)e}{3}+5c\,d^{2}g\right)\sqrt{gx+f}\,\sqrt{(dg-ef)e}+g(ex+d)\left(a\,e^{2}g+5c\,d^{2}g-4cdef\right)\arctan\left(\right.}{\sqrt{(dg-ef)e}\,g\,e^{3}(ex+d)}$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*c*(-e*g*x+6*d*g-e*f)*(g*x+f)^(1/2)/g/e^3+1/e^3*(2*(-1/2*a*e^2*g-1/2*c
*d^2*g)*(g*x+f)^(1/2)/(e*(g*x+f)+d*g-e*f)+(a*e^2*g+5*c*d^2*g-4*c*d*e*f)/((
d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(118) = 236$.

Time = 0.10 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.95

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^2}\,dx$$
$$= \left[-\frac{3\,(4\,cd^2efg-(5\,cd^3+ade^2)g^2+(4\,cde^2fg-(5\,cd^2e+ae^3)g^2)x)\sqrt{e^2f-deg}\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{e^2f-}}{ex+d}\right.}{}\right.$$
$$-\frac{3\,(4\,cd^2efg-(5\,cd^3+ade^2)g^2+(4\,cde^2fg-(5\,cd^2e+ae^3)g^2)x)\sqrt{-e^2f+deg}\arctan\left(\frac{\sqrt{-e^2f+deg}\sqrt{gx-}}{egx+ef}\right.}{3\,(d}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[-1/6*(3*(4*c*d^2*e*f*g - (5*c*d^3 + a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (5*c*
d^2*e + a*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g - 2*sq
rt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^2 - (17*c*d^2
*e^2 + 3*a*e^4)*f*g + 3*(5*c*d^3*e + a*d*e^3)*g^2 + 2*(c*e^4*f*g - c*d*e^3
*g^2)*x^2 + 2*(c*e^4*f^2 - 6*c*d*e^3*f*g + 5*c*d^2*e^2*g^2)*x)*sqrt(g*x +
f))/(d*e^5*f*g - d^2*e^4*g^2 + (e^6*f*g - d*e^5*g^2)*x), -1/3*(3*(4*c*d^2*
e*f*g - (5*c*d^3 + a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (5*c*d^2*e + a*e^3)*g^2
)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x
 + e*f)) - (2*c*d*e^3*f^2 - (17*c*d^2*e^2 + 3*a*e^4)*f*g + 3*(5*c*d^3*e +
a*d*e^3)*g^2 + 2*(c*e^4*f*g - c*d*e^3*g^2)*x^2 + 2*(c*e^4*f^2 - 6*c*d*e^3*
f*g + 5*c*d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f*g - d^2*e^4*g^2 + (e^6*f
*g - d*e^5*g^2)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^2}\,dx = \int \frac{(a+cx^2)\,\sqrt{f+gx}}{(d+ex)^2}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a)/(e*x+d)**2,x)
```

output
```
Integral((a + c*x**2)*sqrt(f + g*x)/(d + e*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^2}\,dx = -\frac{(4\,cdef-5\,cd^2g-ae^2g)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{\sqrt{-e^2f+deg}e^3} - \frac{\sqrt{gx+f}cd^2g+\sqrt{gx+f}ae^2g}{((gx+f)e-ef+dg)e^3} + \frac{2\left((gx+f)^{\frac{3}{2}}ce^4g^2-6\sqrt{gx+f}cde^3g^3\right)}{3\,e^6g^3}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)/(e*x+d)^2,x, algorithm="giac")
```

output
```
-(4*c*d*e*f - 5*c*d^2*g - a*e^2*g)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*
e*g))/(sqrt(-e^2*f + d*e*g)*e^3) - (sqrt(g*x + f)*c*d^2*g + sqrt(g*x + f)*
a*e^2*g)/(((g*x + f)*e - e*f + d*g)*e^3) + 2/3*((g*x + f)^(3/2)*c*e^4*g^2
- 6*sqrt(g*x + f)*c*d*e^3*g^3)/(e^6*g^3)
```

Mupad [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{e}\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(5\,c\,g\,d^2-4\,c\,f\,d\,e+a\,g\,e^2\right)}{e^{7/2}\,\sqrt{d\,g-e\,f}} - \frac{\sqrt{f+g\,x}\,\left(c\,g\,d^2+a\,g\,e^2\right)}{e^4\,\left(f+g\,x\right)-e^4\,f+d\,e^3\,g} - \sqrt{f+g\,x}\,\left(\frac{4\,c\,\left(d\,g-e\,f\right)}{e^3\,g}+\frac{4\,c\,f}{e^2\,g}\right)+\frac{2\,c\,{\left(f+g\,x\right)}^{3/2}}{3\,e^2\,g}$$

input
```
int(((f + g*x)^(1/2)*(a + c*x^2))/(d + e*x)^2,x)
```

output
```
(atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2*g + 5*c*d^2*g -
4*c*d*e*f))/(e^(7/2)*(d*g - e*f)^(1/2)) - ((f + g*x)^(1/2)*(a*e^2*g + c*d^
2*g))/(e^4*(f + g*x) - e^4*f + d*e^3*g) - (f + g*x)^(1/2)*((4*c*(d*g - e*f
))/(e^3*g) + (4*c*f)/(e^2*g)) + (2*c*(f + g*x)^(3/2))/(3*e^2*g)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 469, normalized size of antiderivative = 3.45

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^2}\,dx = \frac{3\sqrt{e}\,\sqrt{dg-ef}\,atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-ef}}\right)ad\,e^2g^2+3\sqrt{e}\,\sqrt{dg-ef}\,atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-ef}}\right)a\,e^3g^2x+15\sqrt{e}\,\sqrt{dg-ef}\,ata}{}$$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)/(e*x+d)^2,x)
```

output
```
(3*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)
))*a*d*e**2*g**2 + 3*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(
e)*sqrt(d*g - e*f)))*a*e**3*g**2*x + 15*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt
(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**3*g**2 - 12*sqrt(e)*sqrt(d*g
- e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**2*e*f*g + 15
*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))
*c*d**2*e*g**2*x - 12*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt
(e)*sqrt(d*g - e*f)))*c*d*e**2*f*g*x - 3*sqrt(f + g*x)*a*d*e**3*g**2 + 3*s
qrt(f + g*x)*a*e**4*f*g - 15*sqrt(f + g*x)*c*d**3*e*g**2 + 17*sqrt(f + g*x
)*c*d**2*e**2*f*g - 10*sqrt(f + g*x)*c*d**2*e**2*g**2*x - 2*sqrt(f + g*x)*
c*d*e**3*f**2 + 12*sqrt(f + g*x)*c*d*e**3*f*g*x + 2*sqrt(f + g*x)*c*d*e**3
*g**2*x**2 - 2*sqrt(f + g*x)*c*e**4*f**2*x - 2*sqrt(f + g*x)*c*e**4*f*g*x*
*2)/(3*e**4*g*(d**2*g - d*e*f + d*e*g*x - e**2*f*x))
```

3.41 $\int \frac{\sqrt{f+gx}\,(a+cx^2)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 187

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^3}\,dx = \frac{2c\sqrt{f+gx}}{e^3} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{2e^3(d+ex)^2} - \frac{(ae^2g-cd(8ef-9dg))\sqrt{f+gx}}{4e^3(ef-dg)(d+ex)} + \frac{(ae^2g^2-c(8e^2f^2-24defg+15d^2g^2))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{7/2}(ef-dg)^{3/2}}$$

output
```
2*c*(g*x+f)^(1/2)/e^3-1/2*(a*e^2+c*d^2)*(g*x+f)^(1/2)/e^3/(e*x+d)^2-1/4*(a
*e^2*g-c*d*(-9*d*g+8*e*f))*(g*x+f)^(1/2)/e^3/(-d*g+e*f)/(e*x+d)+1/4*(a*e^2
*g^2-c*(15*d^2*g^2-24*d*e*f*g+8*e^2*f^2))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-
d*g+e*f)^(1/2))/e^(7/2)/(-d*g+e*f)^(3/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{\frac{\sqrt{e}\sqrt{f+gx}(c(-15d^3g+8e^3fx^2+d^2e(14f-25gx)-8de^2x(-3f+gx))+ae^2(dg-e(2f+gx)))}{(ef-dg)(d+ex)^2} - \frac{(-ae^2g^2+c(8e^2f^2-24defg+15d^2g^2))\arctan\left(\cdots\right.}{(-ef+dg)^{3/2}}}{4e^{7/2}}$$

input
```
Integrate[(Sqrt[f + g*x]*(a + c*x^2))/(d + e*x)^3,x]
```

output
```
((Sqrt[e]*Sqrt[f + g*x]*(c*(-15*d^3*g + 8*e^3*f*x^2 + d^2*e*(14*f - 25*g*x
) - 8*d*e^2*x*(-3*f + g*x)) + a*e^2*(d*g - e*(2*f + g*x))))/((e*f - d*g)*(
d + e*x)^2) - ((-(a*e^2*g^2) + c*(8*e^2*f^2 - 24*d*e*f*g + 15*d^2*g^2))*Ar
cTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(-(e*f) + d*g)^(3/2))/(4
*e^(7/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {649, 25, 1580, 25, 1471, 25, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)\sqrt{f+gx}}{(d+ex)^3}\,dx$$

$$\Big\downarrow \text{649}$$

$$2\int -\frac{(f+gx)\left(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2\right)}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{(f+gx)\left(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2\right)}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$

$\downarrow$ 1580

$$2\left(-\frac{\int -\frac{(cd^2+ae^2)g^2+4ce^2(f+gx)^2-4ce(ef+dg)(f+gx)}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4e^3}-\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{4e^3(-dg-e(f+gx)+ef)^2}\right)$$

$\downarrow$ 25

$$2\left(\frac{\int \frac{(cd^2+ae^2)g^2+4ce^2(f+gx)^2-4ce(ef+dg)(f+gx)}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4e^3}-\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{4e^3(-dg-e(f+gx)+ef)^2}\right)$$

$\downarrow$ 1471

$$2\left(\frac{\frac{g\sqrt{f+gx}(ae^2g-cd(8ef-9dg))}{2(ef-dg)(-dg-e(f+gx)+ef)}-\frac{\int -\frac{g\left(age^2+cd(8ef-7dg)\right)-8ce(ef-dg)(f+gx)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{2(ef-dg)}}{4e^3}-\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{4e^3(-dg-e(f+gx)+ef)^2}\right)$$

$\downarrow$ 25

$$2\left(\frac{\frac{\int \frac{g\left(age^2+cd(8ef-7dg)\right)-8ce(ef-dg)(f+gx)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{2(ef-dg)}+\frac{g\sqrt{f+gx}(ae^2g-cd(8ef-9dg))}{2(ef-dg)(-dg-e(f+gx)+ef)}}{4e^3}-\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{4e^3(-dg-e(f+gx)+ef)^2}\right)$$

$\downarrow$ 299

$$2\left(\frac{\frac{(ae^2g^2-c(15d^2g^2-24defg+8e^2f^2))\int \frac{1}{ef-dg-e(f+gx)}d\sqrt{f+gx}+8c\sqrt{f+gx}(ef-dg)}{2(ef-dg)}+\frac{g\sqrt{f+gx}(ae^2g-cd(8ef-9dg))}{2(ef-dg)(-dg-e(f+gx)+ef)}}{4e^3}-\frac{g^2\sqrt{f+g}}{4e^3(-dg-e}\right.$$

$\downarrow$ 221

$$2\left(\frac{\frac{\frac{\left(ae^2g^2-c\left(15d^2g^2-24defg+8e^2f^2\right)\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}\sqrt{ef-dg}}+8c\sqrt{f+gx}(ef-dg)}{2(ef-dg)}+\frac{g\sqrt{f+gx}(ae^2g-cd(8ef-9dg))}{2(ef-dg)(-dg-e(f+gx)+ef)}}{4e^3}-\frac{g^2\sqrt{f+gx}(ae}{4e^3(-dg-e(f+}\right.$$

input
```
Int[(Sqrt[f + g*x]*(a + c*x^2))/(d + e*x)^3,x]
```

output
```
2*(-1/4*((c*d^2 + a*e^2)*g^2*Sqrt[f + g*x])/(e^3*(e*f - d*g - e*(f + g*x))
^2) + ((g*(a*e^2*g - c*d*(8*e*f - 9*d*g))*Sqrt[f + g*x])/(2*(e*f - d*g)*(e
*f - d*g - e*(f + g*x))) + (8*c*(e*f - d*g)*Sqrt[f + g*x] + ((a*e^2*g^2 -
c*(8*e^2*f^2 - 24*d*e*f*g + 15*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/S
qrt[e*f - d*g]])/(Sqrt[e]*Sqrt[e*f - d*g]))/(2*(e*f - d*g)))/(4*e^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$-\frac{-\left(\left(a\,g^2-8c\,f^2\right)e^2+24cdefg-15c\,d^2g^2\right)(ex+d)^2\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)+\sqrt{gx+f}\,\left(\left(8cf\,x^2-2\left(\frac{gx}{2}+f\right)a\right)e^3+d\left(\left(-8g\right.\right.\right.}{4\sqrt{(dg-ef)e}\,e^3(ex+d)^2(dg-ef)}$
derivativedivides	$\frac{2c\sqrt{gx+f}}{e^3}+\frac{\frac{2\left(\frac{eg\left(a\,e^2g+9c\,d^2g-8cdef\right)(gx+f)^{\frac{3}{2}}}{8dg-8ef}-\frac{g\left(a\,e^2g-7c\,d^2g+8cdef\right)\sqrt{gx+f}}{8}\right)}{(e(gx+f)+dg-ef)^2}+\frac{\left(a\,e^2g^2-15c\,d^2g^2+24cdefg-8c\,e^2f\right.}{4(dg-ef)\sqrt{(dg-\right.}}}{e^3}$
default	$\frac{2c\sqrt{gx+f}}{e^3}+\frac{\frac{2\left(\frac{eg\left(a\,e^2g+9c\,d^2g-8cdef\right)(gx+f)^{\frac{3}{2}}}{8dg-8ef}-\frac{g\left(a\,e^2g-7c\,d^2g+8cdef\right)\sqrt{gx+f}}{8}\right)}{(e(gx+f)+dg-ef)^2}+\frac{\left(a\,e^2g^2-15c\,d^2g^2+24cdefg-8c\,e^2f\right.}{4(dg-ef)\sqrt{(dg-\right.}}}{e^3}$
risch	$\frac{2c\sqrt{gx+f}}{e^3}-\frac{\frac{-\frac{eg\left(a\,e^2g+9c\,d^2g-8cdef\right)(gx+f)^{\frac{3}{2}}}{4(dg-ef)}+\frac{g\left(a\,e^2g-7c\,d^2g+8cdef\right)\sqrt{gx+f}}{4}}{(e(gx+f)+dg-ef)^2}-\frac{\left(a\,e^2g^2-15c\,d^2g^2+24cdefg-8c\,e^2f^2\right)}{4(dg-ef)\sqrt{(dg-ef\right.}}}{e^3}$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-((a*g^2-8*c*f^2)*e^2+24*c*d*e*f*g-15*c*d^2*g^2)*(e*x+d)^2*arctan(e*
(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))+(g*x+f)^(1/2)*((8*c*f*x^2-2*(1/2*g*x+f)
*a)*e^3+d*((-8*g*x^2+24*f*x)*c+a*g)*e^2+14*(-25/14*g*x+f)*c*d^2*e-15*c*d^3
*g)*((d*g-e*f)*e)^(1/2))/((d*g-e*f)*e)^(1/2)/e^3/(e*x+d)^2/(d*g-e*f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(165) = 330$.

Time = 0.11 (sec) , antiderivative size = 876, normalized size of antiderivative = 4.68

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
[1/8*((8*c*d^2*e^2*f^2 - 24*c*d^3*e*f*g + (15*c*d^4 - a*d^2*e^2)*g^2 + (8*
c*e^4*f^2 - 24*c*d*e^3*f*g + (15*c*d^2*e^2 - a*e^4)*g^2)*x^2 + 2*(8*c*d*e^
3*f^2 - 24*c*d^2*e^2*f*g + (15*c*d^3*e - a*d*e^3)*g^2)*x)*sqrt(e^2*f - d*e
*g)*log((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x +
 d)) + 2*(2*(7*c*d^2*e^3 - a*e^5)*f^2 - (29*c*d^3*e^2 - 3*a*d*e^4)*f*g + (
15*c*d^4*e - a*d^2*e^3)*g^2 + 8*(c*e^5*f^2 - 2*c*d*e^4*f*g + c*d^2*e^3*g^2
)*x^2 + (24*c*d*e^4*f^2 - (49*c*d^2*e^3 + a*e^5)*f*g + (25*c*d^3*e^2 + a*d
*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^2 - 2*d^3*e^5*f*g + d^4*e^4*g^2 +
(e^8*f^2 - 2*d*e^7*f*g + d^2*e^6*g^2)*x^2 + 2*(d*e^7*f^2 - 2*d^2*e^6*f*g +
 d^3*e^5*g^2)*x), 1/4*((8*c*d^2*e^2*f^2 - 24*c*d^3*e*f*g + (15*c*d^4 - a*d
^2*e^2)*g^2 + (8*c*e^4*f^2 - 24*c*d*e^3*f*g + (15*c*d^2*e^2 - a*e^4)*g^2)*
x^2 + 2*(8*c*d*e^3*f^2 - 24*c*d^2*e^2*f*g + (15*c*d^3*e - a*d*e^3)*g^2)*x)
*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e
*f)) + (2*(7*c*d^2*e^3 - a*e^5)*f^2 - (29*c*d^3*e^2 - 3*a*d*e^4)*f*g + (15
*c*d^4*e - a*d^2*e^3)*g^2 + 8*(c*e^5*f^2 - 2*c*d*e^4*f*g + c*d^2*e^3*g^2)*
x^2 + (24*c*d*e^4*f^2 - (49*c*d^2*e^3 + a*e^5)*f*g + (25*c*d^3*e^2 + a*d*e
^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^2 - 2*d^3*e^5*f*g + d^4*e^4*g^2 + (e
^8*f^2 - 2*d*e^7*f*g + d^2*e^6*g^2)*x^2 + 2*(d*e^7*f^2 - 2*d^2*e^6*f*g + d
^3*e^5*g^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a)/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{(8\,ce^2f^2 - 24\,cdefg + 15\,cd^2g^2 - ae^2g^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{4\,(e^4f - de^3g)\sqrt{-e^2f+deg}} + \frac{2\,\sqrt{gx+f}c}{e^3}$$
$$+ \frac{8\,(gx+f)^{\frac{3}{2}}cde^2fg - 8\,\sqrt{gx+f}cde^2f^2g - 9\,(gx+f)^{\frac{3}{2}}cd^2eg^2 - (gx+f)^{\frac{3}{2}}ae^3g^2 + 15\,\sqrt{gx+f}cd^2efg^2}{4\,(e^4f - de^3g)((gx+f)e - ef + dg)^2}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)/(e*x+d)^3,x, algorithm="giac")
```

output
```
1/4*(8*c*e^2*f^2 - 24*c*d*e*f*g + 15*c*d^2*g^2 - a*e^2*g^2)*arctan(sqrt(g*
x + f)*e/sqrt(-e^2*f + d*e*g))/((e^4*f - d*e^3*g)*sqrt(-e^2*f + d*e*g)) +
2*sqrt(g*x + f)*c/e^3 + 1/4*(8*(g*x + f)^(3/2)*c*d*e^2*f*g - 8*sqrt(g*x +
f)*c*d*e^2*f^2*g - 9*(g*x + f)^(3/2)*c*d^2*e*g^2 - (g*x + f)^(3/2)*a*e^3*g
^2 + 15*sqrt(g*x + f)*c*d^2*e*f*g^2 - sqrt(g*x + f)*a*e^3*f*g^2 - 7*sqrt(g
*x + f)*c*d^3*g^3 + sqrt(g*x + f)*a*d*e^2*g^3)/((e^4*f - d*e^3*g)*((g*x +
f)*e - e*f + d*g)^2)
```

Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{2\,c\,\sqrt{f+g\,x}}{e^3} - \frac{\sqrt{f+g\,x}\,\left(-\frac{7\,c\,d^2\,g^2}{4}+2\,c\,f\,d\,e\,g+\frac{a\,e^2\,g^2}{4}\right)-\frac{{\left(f+g\,x\right)}^{3/2}\,\left(9\,c\,d^2\,e\,g^2-8\,c\,f\,d\,e^2\,g+a\,e^3\,g^2\right)}{4\,\left(d\,g-e\,f\right)}}{e^5\,{\left(f+g\,x\right)}^2-\left(f+g\,x\right)\,\left(2\,e^5\,f-2\,d\,e^4\,g\right)+e^5\,f^2+d^2\,e^3\,g^2-2\,d\,e^4\,f\,g} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(-15\,c\,d^2\,g^2+24\,c\,d\,e\,f\,g-8\,c\,e^2\,f^2+a\,e^2\,g^2\right)}{4\,e^{7/2}\,{\left(d\,g-e\,f\right)}^{3/2}}$$

input
```
int(((f + g*x)^(1/2)*(a + c*x^2))/(d + e*x)^3,x)
```

output
```
(2*c*(f + g*x)^(1/2))/e^3 - ((f + g*x)^(1/2)*((a*e^2*g^2)/4 - (7*c*d^2*g^2
)/4 + 2*c*d*e*f*g) - ((f + g*x)^(3/2)*(a*e^3*g^2 + 9*c*d^2*e*g^2 - 8*c*d*e
^2*f*g))/(4*(d*g - e*f)))/(e^5*(f + g*x)^2 - (f + g*x)*(2*e^5*f - 2*d*e^4*
g) + e^5*f^2 + d^2*e^3*g^2 - 2*d*e^4*f*g) + (atan((e^(1/2)*(f + g*x)^(1/2)
)/(d*g - e*f)^(1/2))*(a*e^2*g^2 - 15*c*d^2*g^2 - 8*c*e^2*f^2 + 24*c*d*e*f*
g))/(4*e^(7/2)*(d*g - e*f)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 896, normalized size of antiderivative = 4.79

$$\int \frac{\sqrt{f+gx}(a+cx^2)}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)/(e*x+d)^3,x)
```

output
```
(sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))
*a*d**2*e**2*g**2 + 2*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt
(e)*sqrt(d*g - e*f)))*a*d*e**3*g**2*x + sqrt(e)*sqrt(d*g - e*f)*atan((sqrt
(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*e**4*g**2*x**2 - 15*sqrt(e)*sqrt
(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**4*g**2
+ 24*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*
f)))*c*d**3*e*f*g - 30*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqr
t(e)*sqrt(d*g - e*f)))*c*d**3*e*g**2*x - 8*sqrt(e)*sqrt(d*g - e*f)*atan((s
qrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**2*e**2*f**2 + 48*sqrt(e)*s
qrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**2*e*
*2*f*g*x - 15*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt
(d*g - e*f)))*c*d**2*e**2*g**2*x**2 - 16*sqrt(e)*sqrt(d*g - e*f)*atan((sqr
t(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d*e**3*f**2*x + 24*sqrt(e)*sqrt
(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d*e**3*f*g
*x**2 - 8*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g
 - e*f)))*c*e**4*f**2*x**2 - sqrt(f + g*x)*a*d**2*e**3*g**2 + 3*sqrt(f + g
*x)*a*d*e**4*f*g + sqrt(f + g*x)*a*d*e**4*g**2*x - 2*sqrt(f + g*x)*a*e**5*
f**2 - sqrt(f + g*x)*a*e**5*f*g*x + 15*sqrt(f + g*x)*c*d**4*e*g**2 - 29*sq
rt(f + g*x)*c*d**3*e**2*f*g + 25*sqrt(f + g*x)*c*d**3*e**2*g**2*x + 14*sqr
t(f + g*x)*c*d**2*e**3*f**2 - 49*sqrt(f + g*x)*c*d**2*e**3*f*g*x + 8*sq...
```

3.42 $\int (d+ex)^3(f+gx)^{3/2} \left(a+cx^2\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 242

$$\begin{aligned}\int (d+ex)^3(f+gx)^{3/2} \left(a+cx^2\right) \, dx = &-\frac{2(ef-dg)^3 \left(cf^2+ag^2\right) (f+gx)^{5/2}}{5g^6}\\ &+\frac{2(ef-dg)^2 \left(3aeg^2+cf(5ef-2dg)\right) (f+gx)^{7/2}}{7g^6}\\ &-\frac{2(ef-dg) \left(3ae^2g^2+c(10e^2f^2-8defg+d^2g^2)\right) (f+gx)^{9/2}}{9g^6}\\ &+\frac{2e(ae^2g^2+c(10e^2f^2-12defg+3d^2g^2)) (f+gx)^{11/2}}{11g^6}\\ &-\frac{2ce^2(5ef-3dg)(f+gx)^{13/2}}{13g^6}+\frac{2ce^3(f+gx)^{15/2}}{15g^6}\end{aligned}$$

output
```
-2/5*(-d*g+e*f)^3*(a*g^2+c*f^2)*(g*x+f)^(5/2)/g^6+2/7*(-d*g+e*f)^2*(3*a*e*
g^2+c*f*(-2*d*g+5*e*f))*(g*x+f)^(7/2)/g^6-2/9*(-d*g+e*f)*(3*a*e^2*g^2+c*(d
^2*g^2-8*d*e*f*g+10*e^2*f^2))*(g*x+f)^(9/2)/g^6+2/11*e*(a*e^2*g^2+c*(3*d^2
*g^2-12*d*e*f*g+10*e^2*f^2))*(g*x+f)^(11/2)/g^6-2/13*c*e^2*(-3*d*g+5*e*f)*
(g*x+f)^(13/2)/g^6+2/15*c*e^3*(g*x+f)^(15/2)/g^6
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.17

$$\int (d+ex)^3(f+gx)^{3/2}\left(a+cx^2\right)\,dx = \frac{2(f+gx)^{5/2}\left(39ag^2(231d^3g^3+99d^2eg^2(-2f+5gx)+11de^2g(8f^2-20fgx+35g^2x^2)+e^3(-\right.}{}$$

input
```
Integrate[(d + e*x)^3*(f + g*x)^(3/2)*(a + c*x^2),x]
```

output
```
(2*(f + g*x)^(5/2)*(39*a*g^2*(231*d^3*g^3 + 99*d^2*e*g^2*(-2*f + 5*g*x) +
11*d*e^2*g*(8*f^2 - 20*f*g*x + 35*g^2*x^2) + e^3*(-16*f^3 + 40*f^2*g*x - 7
0*f*g^2*x^2 + 105*g^3*x^3)) + c*(143*d^3*g^3*(8*f^2 - 20*f*g*x + 35*g^2*x^
2) + 117*d^2*e*g^2*(-16*f^3 + 40*f^2*g*x - 70*f*g^2*x^2 + 105*g^3*x^3) + 9
*d*e^2*g*(128*f^4 - 320*f^3*g*x + 560*f^2*g^2*x^2 - 840*f*g^3*x^3 + 1155*g
^4*x^4) + e^3*(-256*f^5 + 640*f^4*g*x - 1120*f^3*g^2*x^2 + 1680*f^2*g^3*x^
3 - 2310*f*g^4*x^4 + 3003*g^5*x^5))))/(45045*g^6)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(d+ex)^3(f+gx)^{3/2}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{e(f+gx)^{9/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{g^5}+\frac{(f+gx)^{7/2}(ef-dg)\left(-3ae^2g^2-c\left(d^2g^2-8defg+\right.\right.}{g^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2e(f+gx)^{11/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{11g^6}-\frac{2(f+gx)^{9/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{9g^6}-\frac{2(f+gx)^{5/2}\left(ag^2+cf^2\right)(ef-dg)^3}{5g^6}+\frac{2(f+gx)^{7/2}(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)}{7g^6}-\frac{2ce^2(f+gx)^{13/2}(5ef-3dg)}{13g^6}+\frac{2ce^3(f+gx)^{15/2}}{15g^6}$$

input
```
Int[(d + e*x)^3*(f + g*x)^(3/2)*(a + c*x^2),x]
```

output
```
(-2*(e*f - d*g)^3*(c*f^2 + a*g^2)*(f + g*x)^(5/2))/(5*g^6) + (2*(e*f - d*g
)^2*(3*a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x)^(7/2))/(7*g^6) - (2*(e*f -
 d*g)*(3*a*e^2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(9/2)
)/(9*g^6) + (2*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f
+ g*x)^(11/2))/(11*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(13/2))/(13*g
^6) + (2*c*e^3*(f + g*x)^(15/2))/(15*g^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{2e^3c(gx+f)^{\frac{15}{2}}}{15}+\frac{2\left(3(dg-ef)e^2c-2f\,e^3c\right)(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left(3(dg-ef)^2ec-6(dg-ef)e^2cf+e^3\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left((dg-ef)^3\right.}{}$
default	$\frac{2e^3c(gx+f)^{\frac{15}{2}}}{15}+\frac{2\left(3(dg-ef)e^2c-2f\,e^3c\right)(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left(3(dg-ef)^2ec-6(dg-ef)e^2cf+e^3\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left((dg-ef)^3\right.}{}$
pseudoelliptic	$\frac{2\left(\left(\left(\frac{1}{3}c\,x^5+\frac{5}{11}a\,x^3\right)e^3+\frac{5\left(\frac{9c\,x^2}{13}+a\right)x^2d\,e^2}{3}+\frac{15\left(\frac{7c\,x^2}{11}+a\right)x\,d^2e}{7}+d^3\left(\frac{5c\,x^2}{9}+a\right)\right)g^5-\frac{6\left(\frac{35x^2\left(\frac{11c\,x^2}{13}+a\right)e^3}{99}+\frac{10\left(\frac{126c}{14}\right.}{}\right.}{}\right.}{}$
gosper	$\frac{2(gx+f)^{\frac{5}{2}}\left(3003e^3c\,x^5g^5+10395cd\,e^2g^5x^4-2310c\,e^3f\,g^4x^4+4095a\,e^3g^5x^3+12285c\,d^2e\,g^5x^3-7560cd\,e^2f\,g^4x^3+1680\right.}{}$
orering	$\frac{2(gx+f)^{\frac{5}{2}}\left(3003e^3c\,x^5g^5+10395cd\,e^2g^5x^4-2310c\,e^3f\,g^4x^4+4095a\,e^3g^5x^3+12285c\,d^2e\,g^5x^3-7560cd\,e^2f\,g^4x^3+1680\right.}{}$
trager	$\frac{2\left(3003c\,e^3g^7x^7+10395cd\,e^2g^7x^6+3696c\,e^3f\,g^6x^6+4095a\,e^3g^7x^5+12285c\,d^2e\,g^7x^5+13230cd\,e^2f\,g^6x^5+63c\,e^3f^2g^5x^5\right.}{}$
risch	$\frac{2\left(3003c\,e^3g^7x^7+10395cd\,e^2g^7x^6+3696c\,e^3f\,g^6x^6+4095a\,e^3g^7x^5+12285c\,d^2e\,g^7x^5+13230cd\,e^2f\,g^6x^5+63c\,e^3f^2g^5x^5\right.}{}$

input

```
int((e*x+d)^3*(g*x+f)^(3/2)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output

```
2/g^6*(1/15*e^3*c*(g*x+f)^(15/2)+1/13*(3*(d*g-e*f)*e^2*c-2*f*e^3*c)*(g*x+f
)^(13/2)+1/11*(3*(d*g-e*f)^2*e*c-6*(d*g-e*f)*e^2*c*f+e^3*(a*g^2+c*f^2))*(g
*x+f)^(11/2)+1/9*((d*g-e*f)^3*c-6*(d*g-e*f)^2*e*c*f+3*(d*g-e*f)*e^2*(a*g^2
+c*f^2))*(g*x+f)^(9/2)+1/7*(-2*(d*g-e*f)^3*c*f+3*(d*g-e*f)^2*e*(a*g^2+c*f^
2))*(g*x+f)^(7/2)+1/5*(d*g-e*f)^3*(a*g^2+c*f^2)*(g*x+f)^(5/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(218) = 436$.

Time = 0.10 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.14

$$\int (d+ex)^3(f+gx)^{3/2}\left(a+cx^2\right)\,dx = \frac{2\,(3003\,ce^3g^7x^7 - 256\,ce^3f^7 + 1152\,cde^2f^6g - 7722\,ad^2ef^3g^4 + 9009\,ad^3f^2g^5 - 624\,(3\,cd^2e + a\cdots}{\cdots}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(3/2)*(c*x^2+a),x, algorithm="fricas")
```

output
```
2/45045*(3003*c*e^3*g^7*x^7 - 256*c*e^3*f^7 + 1152*c*d*e^2*f^6*g - 7722*a*
d^2*e*f^3*g^4 + 9009*a*d^3*f^2*g^5 - 624*(3*c*d^2*e + a*e^3)*f^5*g^2 + 114
4*(c*d^3 + 3*a*d*e^2)*f^4*g^3 + 231*(16*c*e^3*f*g^6 + 45*c*d*e^2*g^7)*x^6
+ 63*(c*e^3*f^2*g^5 + 210*c*d*e^2*f*g^6 + 65*(3*c*d^2*e + a*e^3)*g^7)*x^5
- 35*(2*c*e^3*f^3*g^4 - 9*c*d*e^2*f^2*g^5 - 156*(3*c*d^2*e + a*e^3)*f*g^6
- 143*(c*d^3 + 3*a*d*e^2)*g^7)*x^4 + 5*(16*c*e^3*f^4*g^3 - 72*c*d*e^2*f^3*
g^4 + 3861*a*d^2*e*g^7 + 39*(3*c*d^2*e + a*e^3)*f^2*g^5 + 1430*(c*d^3 + 3*
a*d*e^2)*f*g^6)*x^3 - 3*(32*c*e^3*f^5*g^2 - 144*c*d*e^2*f^4*g^3 - 10296*a*
d^2*e*f*g^6 - 3003*a*d^3*g^7 + 78*(3*c*d^2*e + a*e^3)*f^3*g^4 - 143*(c*d^3
 + 3*a*d*e^2)*f^2*g^5)*x^2 + (128*c*e^3*f^6*g - 576*c*d*e^2*f^5*g^2 + 3861
*a*d^2*e*f^2*g^5 + 18018*a*d^3*f*g^6 + 312*(3*c*d^2*e + a*e^3)*f^4*g^3 - 5
72*(c*d^3 + 3*a*d*e^2)*f^3*g^4)*x)*sqrt(g*x + f)/g^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(243) = 486$.

Time = 1.79 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.08

$$\int (d+ex)^3(f+gx)^{3/2}\left(a+cx^2\right)\,dx = \begin{cases} \frac{2\left(\frac{ce^3(f+gx)^{\frac{15}{2}}}{15g^5} + \frac{(f+gx)^{\frac{13}{2}}\cdot\left(3cde^2g-5ce^3f\right)}{13g^5} + \frac{(f+gx)^{\frac{11}{2}}\left(ae^3g^2+3cd^2eg^2-12cde^2fg+10ce^3f^2\right)}{11g^5} + \frac{(f+gx)^{\frac{9}{2}}\cdot\left(3ade^2g^3-3ae^3fg^2+cd^3g\cdots\right.}{9g\cdots}\cdots\right.}{\cdots} \\ f^{\frac{3}{2}}\left(ad^3x + \frac{3ad^2ex^2}{2} + \frac{3cde^2x^5}{5} + \frac{ce^3x^6}{6} + \frac{x^4\left(ae^3+3cd^2e\right)}{4} + \frac{x^3\cdot\left(3ade^2+cd^3\right)}{3}\right) \end{cases}$$

input
```
integrate((e*x+d)**3*(g*x+f)**(3/2)*(c*x**2+a),x)
```

output
```
Piecewise((2*(c*e**3*(f + g*x)**(15/2)/(15*g**5) + (f + g*x)**(13/2)*(3*c*
d*e**2*g - 5*c*e**3*f)/(13*g**5) + (f + g*x)**(11/2)*(a*e**3*g**2 + 3*c*d*
*2*e*g**2 - 12*c*d*e**2*f*g + 10*c*e**3*f**2)/(11*g**5) + (f + g*x)**(9/2)
*(3*a*d*e**2*g**3 - 3*a*e**3*f*g**2 + c*d**3*g**3 - 9*c*d**2*e*f*g**2 + 18
*c*d*e**2*f**2*g - 10*c*e**3*f**3)/(9*g**5) + (f + g*x)**(7/2)*(3*a*d**2*e
*g**4 - 6*a*d*e**2*f*g**3 + 3*a*e**3*f**2*g**2 - 2*c*d**3*f*g**3 + 9*c*d**
2*e*f**2*g**2 - 12*c*d*e**2*f**3*g + 5*c*e**3*f**4)/(7*g**5) + (f + g*x)**
(5/2)*(a*d**3*g**5 - 3*a*d**2*e*f*g**4 + 3*a*d*e**2*f**2*g**3 - a*e**3*f**
3*g**2 + c*d**3*f**2*g**3 - 3*c*d**2*e*f**3*g**2 + 3*c*d*e**2*f**4*g - c*e
**3*f**5)/(5*g**5))/g, Ne(g, 0)), (f**(3/2)*(a*d**3*x + 3*a*d**2*e*x**2/2
+ 3*c*d*e**2*x**5/5 + c*e**3*x**6/6 + x**4*(a*e**3 + 3*c*d**2*e)/4 + x**3*
(3*a*d*e**2 + c*d**3)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.35

$$\int (d+ex)^3(f+gx)^{3/2} \left(a+cx^2\right) dx = \frac{2 \left(3003 \, (gx+f)^{\frac{15}{2}} ce^3 - 3465 \, (5 \, ce^3 f - 3 \, cde^2 g)(gx+f)^{\frac{13}{2}} + 4095 \, (10 \, ce^3 f^2 - 12 \, cde^2 fg + (3\right.}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(3/2)*(c*x^2+a),x, algorithm="maxima")
```

output
```
2/45045*(3003*(g*x + f)^(15/2)*c*e^3 - 3465*(5*c*e^3*f - 3*c*d*e^2*g)*(g*x
 + f)^(13/2) + 4095*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2*e + a*e^3)*g
^2)*(g*x + f)^(11/2) - 5005*(10*c*e^3*f^3 - 18*c*d*e^2*f^2*g + 3*(3*c*d^2*
e + a*e^3)*f*g^2 - (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(9/2) + 6435*(5*c*e^
3*f^4 - 12*c*d*e^2*f^3*g + 3*a*d^2*e*g^4 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 -
 2*(c*d^3 + 3*a*d*e^2)*f*g^3)*(g*x + f)^(7/2) - 9009*(c*e^3*f^5 - 3*c*d*e^
2*f^4*g + 3*a*d^2*e*f*g^4 - a*d^3*g^5 + (3*c*d^2*e + a*e^3)*f^3*g^2 - (c*d
^3 + 3*a*d*e^2)*f^2*g^3)*(g*x + f)^(5/2))/g^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1452 vs. $2(218) = 436$.

Time = 0.13 (sec) , antiderivative size = 1452, normalized size of antiderivative = 6.00

$$\int (d+ex)^3(f+gx)^{3/2}\left(a+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(3/2)*(c*x^2+a),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(g*x + f)*a*d^3*f^2 + 30030*((g*x + f)^(3/2) - 3*sqrt(g
*x + f)*f)*a*d^3*f + 45045*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d^2*e*f
^2/g + 3003*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f
^2)*a*d^3 + 3003*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x +
 f)*f^2)*c*d^3*f^2/g^2 + 9009*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f +
15*sqrt(g*x + f)*f^2)*a*d*e^2*f^2/g^2 + 18018*(3*(g*x + f)^(5/2) - 10*(g*x
 + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a*d^2*e*f/g + 3861*(5*(g*x + f)^(7/2
) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*
c*d^2*e*f^2/g^3 + 1287*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x
 + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*e^3*f^2/g^3 + 2574*(5*(g*x + f)^
(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f
^3)*c*d^3*f/g^2 + 7722*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x
 + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*d*e^2*f/g^2 + 3861*(5*(g*x + f)^
(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f
^3)*a*d^2*e/g + 429*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x
 + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c*d*e^2
*f^2/g^4 + 858*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)
^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c*d^2*e*f/g^
3 + 286*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*
f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*a*e^3*f/g^3 + 14...
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.92

$$\int (d+ex)^3(f+gx)^{3/2}\left(a+cx^2\right)\,dx=\frac{(f+g\,x)^{11/2}\,(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+20\,c\,e^3\,f^2+2\,a\,e^3\,g^2)}{11\,g^6}+\frac{2\,(f+g\,x)^{5/2}\,(c\,f^2+a\,g^2)\,(d\,g-e\,f)^3}{5\,g^6}+\frac{2\,c\,e^3\,(f+g\,x)^{15/2}}{15\,g^6}+\frac{2\,(f+g\,x)^{7/2}\,(d\,g-e\,f)^2\,(5\,c\,e\,f^2-2\,c\,d\,f\,g+3\,a\,e\,g^2)}{7\,g^6}+\frac{2\,(f+g\,x)^{9/2}\,(d\,g-e\,f)\,(c\,d^2\,g^2-8\,c\,d\,e\,f\,g+10\,c\,e^2\,f^2+3\,a\,e^2\,g^2)}{9\,g^6}+\frac{2\,c\,e^2\,(f+g\,x)^{13/2}\,(3\,d\,g-5\,e\,f)}{13\,g^6}$$

input
```
int((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^3,x)
```

output
```
((f + g*x)^(11/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2
*f*g))/(11*g^6) + (2*(f + g*x)^(5/2)*(a*g^2 + c*f^2)*(d*g - e*f)^3)/(5*g^6
) + (2*c*e^3*(f + g*x)^(15/2))/(15*g^6) + (2*(f + g*x)^(7/2)*(d*g - e*f)^2
*(3*a*e*g^2 + 5*c*e*f^2 - 2*c*d*f*g))/(7*g^6) + (2*(f + g*x)^(9/2)*(d*g -
e*f)*(3*a*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 - 8*c*d*e*f*g))/(9*g^6) + (2*
c*e^2*(f + g*x)^(13/2)*(3*d*g - 5*e*f))/(13*g^6)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.51

$$\int (d+ex)^3(f+gx)^{3/2}\left(a+cx^2\right)\,dx=\frac{2\sqrt{gx+f}\,(3003c\,e^3g^7x^7+10395cd\,e^2g^7x^6+3696c\,e^3f\,g^6x^6+4095a\,e^3g^7x^5+12285c\,d^2e\,g^7x^5}{}$$

input
```
int((e*x+d)^3*(g*x+f)^(3/2)*(c*x^2+a),x)
```

output

```
(2*sqrt(f + g*x)*(9009*a*d**3*f**2*g**5 + 18018*a*d**3*f*g**6*x + 9009*a*d
**3*g**7*x**2 - 7722*a*d**2*e*f**3*g**4 + 3861*a*d**2*e*f**2*g**5*x + 3088
8*a*d**2*e*f*g**6*x**2 + 19305*a*d**2*e*g**7*x**3 + 3432*a*d*e**2*f**4*g**
3 - 1716*a*d*e**2*f**3*g**4*x + 1287*a*d*e**2*f**2*g**5*x**2 + 21450*a*d*e
**2*f*g**6*x**3 + 15015*a*d*e**2*g**7*x**4 - 624*a*e**3*f**5*g**2 + 312*a*
e**3*f**4*g**3*x - 234*a*e**3*f**3*g**4*x**2 + 195*a*e**3*f**2*g**5*x**3 +
 5460*a*e**3*f*g**6*x**4 + 4095*a*e**3*g**7*x**5 + 1144*c*d**3*f**4*g**3 -
 572*c*d**3*f**3*g**4*x + 429*c*d**3*f**2*g**5*x**2 + 7150*c*d**3*f*g**6*x
**3 + 5005*c*d**3*g**7*x**4 - 1872*c*d**2*e*f**5*g**2 + 936*c*d**2*e*f**4*
g**3*x - 702*c*d**2*e*f**3*g**4*x**2 + 585*c*d**2*e*f**2*g**5*x**3 + 16380
*c*d**2*e*f*g**6*x**4 + 12285*c*d**2*e*g**7*x**5 + 1152*c*d*e**2*f**6*g -
576*c*d*e**2*f**5*g**2*x + 432*c*d*e**2*f**4*g**3*x**2 - 360*c*d*e**2*f**3
*g**4*x**3 + 315*c*d*e**2*f**2*g**5*x**4 + 13230*c*d*e**2*f*g**6*x**5 + 10
395*c*d*e**2*g**7*x**6 - 256*c*e**3*f**7 + 128*c*e**3*f**6*g*x - 96*c*e**3
*f**5*g**2*x**2 + 80*c*e**3*f**4*g**3*x**3 - 70*c*e**3*f**3*g**4*x**4 + 63
*c*e**3*f**2*g**5*x**5 + 3696*c*e**3*f*g**6*x**6 + 3003*c*e**3*g**7*x**7))
/(45045*g**6)
```

3.43 $\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 177

$$\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)\,dx = \frac{2(ef-dg)^2\left(cf^2+ag^2\right)(f+gx)^{5/2}}{5g^5} - \frac{4(ef-dg)\left(aeg^2+cf(2ef-dg)\right)(f+gx)^{7/2}}{7g^5} + \frac{2\left(ae^2g^2+c\left(6e^2f^2-6defg+d^2g^2\right)\right)(f+gx)^{9/2}}{9g^5} - \frac{4ce(2ef-dg)(f+gx)^{11/2}}{11g^5} + \frac{2ce^2(f+gx)^{13/2}}{13g^5}$$

output
```
2/5*(-d*g+e*f)^2*(a*g^2+c*f^2)*(g*x+f)^(5/2)/g^5-4/7*(-d*g+e*f)*(a*e*g^2+c
*f*(-d*g+2*e*f))*(g*x+f)^(7/2)/g^5+2/9*(a*e^2*g^2+c*(d^2*g^2-6*d*e*f*g+6*e
^2*f^2))*(g*x+f)^(9/2)/g^5-4/11*c*e*(-d*g+2*e*f)*(g*x+f)^(11/2)/g^5+2/13*c
*e^2*(g*x+f)^(13/2)/g^5
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.01

$$\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)\,dx=\frac{2(f+gx)^{5/2}\left(143ag^2(63d^2g^2+18deg(-2f+5gx)+e^2(8f^2-20fgx+35g^2x^2))+c(143d^2g^2(\right.}{}$$

input
```
Integrate[(d + e*x)^2*(f + g*x)^(3/2)*(a + c*x^2),x]
```

output
```
(2*(f + g*x)^(5/2)*(143*a*g^2*(63*d^2*g^2 + 18*d*e*g*(-2*f + 5*g*x) + e^2*
(8*f^2 - 20*f*g*x + 35*g^2*x^2)) + c*(143*d^2*g^2*(8*f^2 - 20*f*g*x + 35*g
^2*x^2) + 78*d*e*g*(-16*f^3 + 40*f^2*g*x - 70*f*g^2*x^2 + 105*g^3*x^3) + 3
*e^2*(128*f^4 - 320*f^3*g*x + 560*f^2*g^2*x^2 - 840*f*g^3*x^3 + 1155*g^4*x
^4))))/(45045*g^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(d+ex)^2(f+gx)^{3/2}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{(f+gx)^{7/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{g^4}+\frac{(f+gx)^{3/2}\left(ag^2+cf^2\right)(dg-ef)^2}{g^4}+\frac{2(f+gx)^{5/2}(ef-\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(f+gx)^{9/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{9g^5}+\frac{2(f+gx)^{5/2}\left(ag^2+cf^2\right)(ef-dg)^2}{5g^5}-\frac{4(f+gx)^{7/2}(ef-dg)\left(aeg^2+cf(2ef-dg)\right)}{7g^5}-\frac{4ce(f+gx)^{11/2}(2ef-dg)}{11g^5}+\frac{2ce^2(f+gx)^{13/2}}{13g^5}$$

input
```
Int[(d + e*x)^2*(f + g*x)^(3/2)*(a + c*x^2),x]
```

output
```
(2*(e*f - d*g)^2*(c*f^2 + a*g^2)*(f + g*x)^(5/2))/(5*g^5) - (4*(e*f - d*g)
*(a*e*g^2 + c*f*(2*e*f - d*g))*(f + g*x)^(7/2))/(7*g^5) + (2*(a*e^2*g^2 +
c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(9/2))/(9*g^5) - (4*c*e*(2*
e*f - d*g)*(f + g*x)^(11/2))/(11*g^5) + (2*c*e^2*(f + g*x)^(13/2))/(13*g^5
)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{2(gx+f)^{\frac{5}{2}}\left(\left(\left(\frac{5\left(\frac{9c\,x^{2}}{13}+a\right)x^{2}e^{2}}{9}+\frac{10\left(\frac{7c\,x^{2}}{11}+a\right)xde}{7}+d^{2}\left(\frac{5c\,x^{2}}{9}+a\right)\right)g^{4}-\frac{4f\left(\left(\frac{70}{143}c\,x^{3}+\frac{5}{9}ax\right)e^{2}+d\left(\frac{35c\,x^{2}}{33}+a\right)e+\frac{5c\,d^{2}x}{9}\right.}{7}\right.}{5g^{5}}$
derivativedivides	$\frac{\frac{2c\,e^{2}(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left(2e(dg-ef)c-2fc\,e^{2}\right)(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left((dg-ef)^{2}c-4e(dg-ef)cf+e^{2}\left(a\,g^{2}+c\,f^{2}\right)\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(-2(dg-ef)^{2}cf\right.}{}}{g^{5}}$
default	$\frac{\frac{2c\,e^{2}(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left(2e(dg-ef)c-2fc\,e^{2}\right)(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left((dg-ef)^{2}c-4e(dg-ef)cf+e^{2}\left(a\,g^{2}+c\,f^{2}\right)\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(-2(dg-ef)^{2}cf\right.}{}}{g^{5}}$
gosper	$\frac{2(gx+f)^{\frac{5}{2}}\left(3465c\,e^{2}x^{4}g^{4}+8190cde\,g^{4}x^{3}-2520c\,e^{2}f\,g^{3}x^{3}+5005a\,e^{2}g^{4}x^{2}+5005c\,d^{2}g^{4}x^{2}-5460cdef\,g^{3}x^{2}+1680c\,e^{2}f^{2}g\right.}{}$
orering	$\frac{2(gx+f)^{\frac{5}{2}}\left(3465c\,e^{2}x^{4}g^{4}+8190cde\,g^{4}x^{3}-2520c\,e^{2}f\,g^{3}x^{3}+5005a\,e^{2}g^{4}x^{2}+5005c\,d^{2}g^{4}x^{2}-5460cdef\,g^{3}x^{2}+1680c\,e^{2}f^{2}g\right.}{}$
trager	$\frac{2\left(3465c\,e^{2}g^{6}x^{6}+8190cde\,g^{6}x^{5}+4410c\,e^{2}f\,g^{5}x^{5}+5005a\,e^{2}g^{6}x^{4}+5005c\,d^{2}g^{6}x^{4}+10920cdef\,g^{5}x^{4}+105c\,e^{2}f^{2}g^{4}x^{4}+128\right.}{}$
risch	$\frac{2\left(3465c\,e^{2}g^{6}x^{6}+8190cde\,g^{6}x^{5}+4410c\,e^{2}f\,g^{5}x^{5}+5005a\,e^{2}g^{6}x^{4}+5005c\,d^{2}g^{6}x^{4}+10920cdef\,g^{5}x^{4}+105c\,e^{2}f^{2}g^{4}x^{4}+128\right.}{}$

input
```
int((e*x+d)^2*(g*x+f)^(3/2)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/5*(g*x+f)^(5/2)*((5/9*(9/13*c*x^2+a)*x^2*e^2+10/7*(7/11*c*x^2+a)*x*d*e+d
^2*(5/9*c*x^2+a))*g^4-4/7*f*((70/143*c*x^3+5/9*a*x)*e^2+d*(35/33*c*x^2+a)*
e+5/9*c*d^2*x)*g^3+8/63*f^2*((210/143*c*x^2+a)*e^2+30/11*c*d*x*e+c*d^2)*g^
2-32/231*(10/13*e*x+d)*e*f^3*c*g+128/3003*c*e^2*f^4)/g^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(157) = 314$.

Time = 0.08 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.90

$$\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)\,dx=\frac{2\left(3465\,ce^2g^6x^6+384\,ce^2f^6-1248\,cdef^5g-5148\,adef^3g^3+9009\,ad^2f^2g^4+1144\,(cd^2+ae^2)\right.}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(3/2)*(c*x^2+a),x, algorithm="fricas")
```

output
```
2/45045*(3465*c*e^2*g^6*x^6 + 384*c*e^2*f^6 - 1248*c*d*e*f^5*g - 5148*a*d*
e*f^3*g^3 + 9009*a*d^2*f^2*g^4 + 1144*(c*d^2 + a*e^2)*f^4*g^2 + 630*(7*c*e
^2*f*g^5 + 13*c*d*e*g^6)*x^5 + 35*(3*c*e^2*f^2*g^4 + 312*c*d*e*f*g^5 + 143
*(c*d^2 + a*e^2)*g^6)*x^4 - 10*(12*c*e^2*f^3*g^3 - 39*c*d*e*f^2*g^4 - 1287
*a*d*e*g^6 - 715*(c*d^2 + a*e^2)*f*g^5)*x^3 + 3*(48*c*e^2*f^4*g^2 - 156*c*
d*e*f^3*g^3 + 6864*a*d*e*f*g^5 + 3003*a*d^2*g^6 + 143*(c*d^2 + a*e^2)*f^2*
g^4)*x^2 - 2*(96*c*e^2*f^5*g - 312*c*d*e*f^4*g^2 - 1287*a*d*e*f^2*g^4 - 90
09*a*d^2*f*g^5 + 286*(c*d^2 + a*e^2)*f^3*g^3)*x)*sqrt(g*x + f)/g^5
```

Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.77

$$\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)\,dx=\begin{cases}\frac{2\left(\frac{ce^2(f+gx)^{\frac{13}{2}}}{13g^4}+\frac{(f+gx)^{\frac{11}{2}}\cdot\left(2cdeg-4ce^2f\right)}{11g^4}+\frac{(f+gx)^{\frac{9}{2}}\left(ae^2g^2+cd^2g^2-6cdefg+6ce^2f^2\right)}{9g^4}+\frac{(f+gx)^{\frac{7}{2}}\cdot\left(2adeg^3-2ae^2fg^2-2cd^2fg^2+6cd\cdots\right.}{7g^4}\cdots\right.}{g} \\ f^{\frac{3}{2}}\left(ad^2x+adex^2+\frac{cdex^4}{2}+\frac{ce^2x^5}{5}+\frac{x^3\left(ae^2+cd^2\right)}{3}\right)\end{cases}$$

input
```
integrate((e*x+d)**2*(g*x+f)**(3/2)*(c*x**2+a),x)
```

output
```
Piecewise((2*(c*e**2*(f + g*x)**(13/2)/(13*g**4) + (f + g*x)**(11/2)*(2*c*
d*e*g - 4*c*e**2*f)/(11*g**4) + (f + g*x)**(9/2)*(a*e**2*g**2 + c*d**2*g**
2 - 6*c*d*e*f*g + 6*c*e**2*f**2)/(9*g**4) + (f + g*x)**(7/2)*(2*a*d*e*g**3
 - 2*a*e**2*f*g**2 - 2*c*d**2*f*g**2 + 6*c*d*e*f**2*g - 4*c*e**2*f**3)/(7*
g**4) + (f + g*x)**(5/2)*(a*d**2*g**4 - 2*a*d*e*f*g**3 + a*e**2*f**2*g**2
+ c*d**2*f**2*g**2 - 2*c*d*e*f**3*g + c*e**2*f**4)/(5*g**4))/g, Ne(g, 0)),
 (f**(3/2)*(a*d**2*x + a*d*e*x**2 + c*d*e*x**4/2 + c*e**2*x**5/5 + x**3*(a
*e**2 + c*d**2)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.11

$$\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)\,dx = \frac{2\left(3465\,(gx+f)^{\frac{13}{2}}ce^2 - 8190\,(2\,ce^2f - cdeg)(gx+f)^{\frac{11}{2}} + 5005\,(6\,ce^2f^2 - 6\,cdefg + (cd^2+a\right.}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(3/2)*(c*x^2+a),x, algorithm="maxima")
```

output
```
2/45045*(3465*(g*x + f)^(13/2)*c*e^2 - 8190*(2*c*e^2*f - c*d*e*g)*(g*x + f
)^(11/2) + 5005*(6*c*e^2*f^2 - 6*c*d*e*f*g + (c*d^2 + a*e^2)*g^2)*(g*x + f
)^(9/2) - 12870*(2*c*e^2*f^3 - 3*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 + a*e^2)
*f*g^2)*(g*x + f)^(7/2) + 9009*(c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3
+ a*d^2*g^4 + (c*d^2 + a*e^2)*f^2*g^2)*(g*x + f)^(5/2))/g^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 964 vs. 2(157) = 314.

Time = 0.13 (sec) , antiderivative size = 964, normalized size of antiderivative = 5.45

$$\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(3/2)*(c*x^2+a),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(g*x + f)*a*d^2*f^2 + 30030*((g*x + f)^(3/2) - 3*sqrt(g
*x + f)*f)*a*d^2*f + 30030*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*d*e*f^2
/g + 3003*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2
)*a*d^2 + 3003*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f
)*f^2)*c*d^2*f^2/g^2 + 3003*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15
*sqrt(g*x + f)*f^2)*a*e^2*f^2/g^2 + 12012*(3*(g*x + f)^(5/2) - 10*(g*x + f
)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a*d*e*f/g + 2574*(5*(g*x + f)^(7/2) - 21
*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d*e*
f^2/g^3 + 2574*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3
/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^2*f/g^2 + 2574*(5*(g*x + f)^(7/2) - 21
*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*e^2*
f/g^2 + 2574*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2
)*f^2 - 35*sqrt(g*x + f)*f^3)*a*d*e/g + 143*(35*(g*x + f)^(9/2) - 180*(g*x
 + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sq
rt(g*x + f)*f^4)*c*e^2*f^2/g^4 + 572*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(
7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x
+ f)*f^4)*c*d*e*f/g^3 + 143*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f +
378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)
*c*d^2/g^2 + 143*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x +
f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*a*e^2/g...
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

$$\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)\,dx = \frac{(f+g\,x)^{9/2}\,\left(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+12\,c\,e^2\,f^2+2\,a\,e^2\,g^2\right)}{9\,g^5} + \frac{2\,(f+g\,x)^{5/2}\,\left(c\,f^2+a\,g^2\right)\,{\left(d\,g-e\,f\right)}^2}{5\,g^5} + \frac{4\,(f+g\,x)^{7/2}\,\left(d\,g-e\,f\right)\,\left(2\,c\,e\,f^2-c\,d\,f\,g+a\,e\,g^2\right)}{7\,g^5} + \frac{2\,c\,e^2\,(f+g\,x)^{13/2}}{13\,g^5} + \frac{4\,c\,e\,(f+g\,x)^{11/2}\,\left(d\,g-2\,e\,f\right)}{11\,g^5}$$

input
```
int((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^2,x)
```

output
```
((f + g*x)^(9/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 - 12*c*d*e*f*g)
)/(9*g^5) + (2*(f + g*x)^(5/2)*(a*g^2 + c*f^2)*(d*g - e*f)^2)/(5*g^5) + (4
*(f + g*x)^(7/2)*(d*g - e*f)*(a*e*g^2 + 2*c*e*f^2 - c*d*f*g))/(7*g^5) + (2
*c*e^2*(f + g*x)^(13/2))/(13*g^5) + (4*c*e*(f + g*x)^(11/2)*(d*g - 2*e*f))
/(11*g^5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.18

$$\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)\,dx = \frac{2\sqrt{gx+f}\,(3465c\,e^2g^6x^6+8190cde\,g^6x^5+4410c\,e^2f\,g^5x^5+5005a\,e^2g^6x^4+5005c\,d^2g^6x^4+10}{}$$

input
```
int((e*x+d)^2*(g*x+f)^(3/2)*(c*x^2+a),x)
```

output
```
(2*sqrt(f + g*x)*(9009*a*d**2*f**2*g**4 + 18018*a*d**2*f*g**5*x + 9009*a*d
**2*g**6*x**2 - 5148*a*d*e*f**3*g**3 + 2574*a*d*e*f**2*g**4*x + 20592*a*d*
e*f*g**5*x**2 + 12870*a*d*e*g**6*x**3 + 1144*a*e**2*f**4*g**2 - 572*a*e**2
*f**3*g**3*x + 429*a*e**2*f**2*g**4*x**2 + 7150*a*e**2*f*g**5*x**3 + 5005*
a*e**2*g**6*x**4 + 1144*c*d**2*f**4*g**2 - 572*c*d**2*f**3*g**3*x + 429*c*
d**2*f**2*g**4*x**2 + 7150*c*d**2*f*g**5*x**3 + 5005*c*d**2*g**6*x**4 - 12
48*c*d*e*f**5*g + 624*c*d*e*f**4*g**2*x - 468*c*d*e*f**3*g**3*x**2 + 390*c
*d*e*f**2*g**4*x**3 + 10920*c*d*e*f*g**5*x**4 + 8190*c*d*e*g**6*x**5 + 384
*c*e**2*f**6 - 192*c*e**2*f**5*g*x + 144*c*e**2*f**4*g**2*x**2 - 120*c*e**
2*f**3*g**3*x**3 + 105*c*e**2*f**2*g**4*x**4 + 4410*c*e**2*f*g**5*x**5 + 3
465*c*e**2*g**6*x**6))/(45045*g**5)
```

3.44 $\int (d+ex)(f+gx)^{3/2}\left(a+cx^2\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int (d+ex)(f+gx)^{3/2}\left(a+cx^2\right)\,dx = -\frac{2(ef-dg)\left(cf^2+ag^2\right)(f+gx)^{5/2}}{5g^4} + \frac{2\left(aeg^2+cf(3ef-2dg)\right)(f+gx)^{7/2}}{7g^4} - \frac{2c(3ef-dg)(f+gx)^{9/2}}{9g^4} + \frac{2ce(f+gx)^{11/2}}{11g^4}$$

output
```
-2/5*(-d*g+e*f)*(a*g^2+c*f^2)*(g*x+f)^(5/2)/g^4+2/7*(a*e*g^2+c*f*(-2*d*g+3
*e*f))*(g*x+f)^(7/2)/g^4-2/9*c*(-d*g+3*e*f)*(g*x+f)^(9/2)/g^4+2/11*c*e*(g*
x+f)^(11/2)/g^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int (d+ex)(f+gx)^{3/2}\left(a+cx^2\right)\,dx = \frac{2(f+gx)^{5/2}\left(99ag^2(-2ef+7dg+5egx)+11cdg(8f^2-20fgx+35g^2x^2)-3ce(16f^3-40f^2\right.}{3465g^4}$$

input
```
Integrate[(d + e*x)*(f + g*x)^(3/2)*(a + c*x^2),x]
```

output
```
(2*(f + g*x)^(5/2)*(99*a*g^2*(-2*e*f + 7*d*g + 5*e*g*x) + 11*c*d*g*(8*f^2
- 20*f*g*x + 35*g^2*x^2) - 3*c*e*(16*f^3 - 40*f^2*g*x + 70*f*g^2*x^2 - 105
*g^3*x^3)))/(3465*g^4)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)\,(d+ex)(f+gx)^{3/2}\,dx$$

$$\Big\downarrow \; 652$$

$$\int \left(\frac{(f+gx)^{3/2}\,(ag^2+cf^2)\,(dg-ef)}{g^3}+\frac{(f+gx)^{5/2}\,(aeg^2+cf(3ef-2dg))}{g^3}+\frac{c(f+gx)^{7/2}(dg-3ef)}{g^3}+\frac{ce(f$$

$$\Big\downarrow \; 2009$$

$$-\frac{2(f+gx)^{5/2}\,(ag^2+cf^2)\,(ef-dg)}{5g^4}+\frac{2(f+gx)^{7/2}\,(aeg^2+cf(3ef-2dg))}{7g^4}-\frac{2c(f+gx)^{9/2}(3ef-dg)}{9g^4}+\frac{2ce(f+gx)^{11/2}}{11g^4}$$

input
```
Int[(d + e*x)*(f + g*x)^(3/2)*(a + c*x^2),x]
```

output
```
(-2*(e*f - d*g)*(c*f^2 + a*g^2)*(f + g*x)^(5/2))/(5*g^4) + (2*(a*e*g^2 + c
*f*(3*e*f - 2*d*g))*(f + g*x)^(7/2))/(7*g^4) - (2*c*(3*e*f - d*g)*(f + g*x
)^(9/2))/(9*g^4) + (2*c*e*(f + g*x)^(11/2))/(11*g^4)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{2(gx+f)^{\frac{5}{2}}\left(\left(\frac{5x^2\left(\frac{9ex}{11}+d\right)c}{9}+a\left(\frac{5ex}{7}+d\right)\right)g^3-\frac{2f\left(\frac{10\left(\frac{21ex}{22}+d\right)xc}{9}+ae\right)g^2}{7}+\frac{8f^2c\left(\frac{15ex}{11}+d\right)g}{63}-\frac{16cef^3}{231}\right)}{5g^4}$
gosper	$\frac{2(gx+f)^{\frac{5}{2}}(315cex^3g^3+385cdg^3x^2-210cefg^2x^2+495aeg^3x-220cdfg^2x+120cef^2gx+693adg^3-198aefg^2+88cdf}{3465g^4}$
orering	$\frac{2(gx+f)^{\frac{5}{2}}(315cex^3g^3+385cdg^3x^2-210cefg^2x^2+495aeg^3x-220cdfg^2x+120cef^2gx+693adg^3-198aefg^2+88cdf}{3465g^4}$
derivativedivides	$\frac{\frac{2ce(gx+f)^{\frac{11}{2}}}{11}+\frac{2((dg-ef)c-2fce)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(-2(dg-ef)cf+e\left(ag^2+cf^2\right)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2(dg-ef)\left(ag^2+cf^2\right)(gx+f)^{\frac{5}{2}}}{5}}{g^4}$
default	$\frac{\frac{2ce(gx+f)^{\frac{11}{2}}}{11}+\frac{2((dg-ef)c-2fce)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(-2(dg-ef)cf+e\left(ag^2+cf^2\right)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2(dg-ef)\left(ag^2+cf^2\right)(gx+f)^{\frac{5}{2}}}{5}}{g^4}$
trager	$2(315ceg^5x^5+385cdg^5x^4+420cefg^4x^4+495aeg^5x^3+550cdfg^4x^3+15cef^2g^3x^3+693adg^5x^2+792aefg^4x^2+33cdf^2$
risch	$2(315ceg^5x^5+385cdg^5x^4+420cefg^4x^4+495aeg^5x^3+550cdfg^4x^3+15cef^2g^3x^3+693adg^5x^2+792aefg^4x^2+33cdf^2$

input
```
int((e*x+d)*(g*x+f)^(3/2)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/5*(g*x+f)^(5/2)*((5/9*x^2*(9/11*e*x+d)*c+a*(5/7*e*x+d))*g^3-2/7*f*(10/9*
(21/22*e*x+d)*x*c+a*e)*g^2+8/63*f^2*c*(15/11*e*x+d)*g-16/231*c*e*f^3)/g^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.65

$$\int (d+ex)(f+gx)^{3/2}\left(a+cx^2\right)\,dx = \frac{2\,(315\,ceg^5x^5 - 48\,cef^5 + 88\,cdf^4g - 198\,aef^3g^2 + 693\,adf^2g^3 + 35\,(12\,cefg^4 + 11\,cdg^5)x^4 + 5$$

input
```
integrate((e*x+d)*(g*x+f)^(3/2)*(c*x^2+a),x, algorithm="fricas")
```

output
```
2/3465*(315*c*e*g^5*x^5 - 48*c*e*f^5 + 88*c*d*f^4*g - 198*a*e*f^3*g^2 + 69
3*a*d*f^2*g^3 + 35*(12*c*e*f*g^4 + 11*c*d*g^5)*x^4 + 5*(3*c*e*f^2*g^3 + 11
0*c*d*f*g^4 + 99*a*e*g^5)*x^3 - 3*(6*c*e*f^3*g^2 - 11*c*d*f^2*g^3 - 264*a*
e*f*g^4 - 231*a*d*g^5)*x^2 + (24*c*e*f^4*g - 44*c*d*f^3*g^2 + 99*a*e*f^2*g
^3 + 1386*a*d*f*g^4)*x)*sqrt(g*x + f)/g^4
```

Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.45

$$\int (d+ex)(f+gx)^{3/2}\left(a+cx^2\right)\,dx = \begin{cases} \frac{2\left(\frac{ce(f+gx)^{\frac{11}{2}}}{11g^3} + \frac{(f+gx)^{\frac{9}{2}}(cdg-3cef)}{9g^3} + \frac{(f+gx)^{\frac{7}{2}}\left(aeg^2-2cdfg+3cef^2\right)}{7g^3} + \frac{(f+gx)^{\frac{5}{2}}\left(adg^3-aefg^2+cdf^2g-cef^3\right)}{5g^3}\right)}{g} & \text{for } g \neq 0 \\ f^{\frac{3}{2}}\left(adx + \frac{aex^2}{2} + \frac{cdx^3}{3} + \frac{cex^4}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((e*x+d)*(g*x+f)**(3/2)*(c*x**2+a),x)
```

output
```
Piecewise((2*(c*e*(f + g*x)**(11/2)/(11*g**3) + (f + g*x)**(9/2)*(c*d*g -
3*c*e*f)/(9*g**3) + (f + g*x)**(7/2)*(a*e*g**2 - 2*c*d*f*g + 3*c*e*f**2)/(
7*g**3) + (f + g*x)**(5/2)*(a*d*g**3 - a*e*f*g**2 + c*d*f**2*g - c*e*f**3)
/(5*g**3))/g, Ne(g, 0)), (f**(3/2)*(a*d*x + a*e*x**2/2 + c*d*x**3/3 + c*e*
x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int (d+ex)(f+gx)^{3/2}\left(a+cx^2\right)\,dx = \frac{2\left(315\,(gx+f)^{\frac{11}{2}}ce - 385\,(3\,cef - cdg)(gx+f)^{\frac{9}{2}} + 495\,(3\,cef^2 - 2\,cdfg + aeg^2)(gx+f)^{\frac{7}{2}} - \right.}{3465\,g^4}$$

input
```
integrate((e*x+d)*(g*x+f)^(3/2)*(c*x^2+a),x, algorithm="maxima")
```

output
```
2/3465*(315*(g*x + f)^(11/2)*c*e - 385*(3*c*e*f - c*d*g)*(g*x + f)^(9/2) +
 495*(3*c*e*f^2 - 2*c*d*f*g + a*e*g^2)*(g*x + f)^(7/2) - 693*(c*e*f^3 - c*
d*f^2*g + a*e*f*g^2 - a*d*g^3)*(g*x + f)^(5/2))/g^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. 2(99) = 198.

Time = 0.12 (sec) , antiderivative size = 548, normalized size of antiderivative = 4.77

$$\int (d+ex)(f+gx)^{3/2}\left(a+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(g*x+f)^(3/2)*(c*x^2+a),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(g*x + f)*a*d*f^2 + 2310*((g*x + f)^(3/2) - 3*sqrt(g*x +
f)*f)*a*d*f + 1155*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*e*f^2/g + 231*(
3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a*d + 231
*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*c*d*f^2
/g^2 + 462*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^
2)*a*e*f/g + 99*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(
3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*e*f^2/g^3 + 198*(5*(g*x + f)^(7/2) - 21
*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d*f/
g^2 + 99*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^
2 - 35*sqrt(g*x + f)*f^3)*a*e/g + 22*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(
7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x
+ f)*f^4)*c*e*f/g^3 + 11*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378
*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c*
d/g^2 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x + f)^(7/
2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*sqrt(g*
x + f)*f^5)*c*e/g^3)/g
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int (d+ex)(f+gx)^{3/2}\left(a+cx^2\right)\,dx=\frac{\left(f+g\,x\right)^{7/2}\left(6\,c\,e\,f^2-4\,c\,d\,f\,g+2\,a\,e\,g^2\right)}{7\,g^4}+\frac{2\,c\,e\,\left(f+g\,x\right)^{11/2}}{11\,g^4}+\frac{2\,c\,\left(f+g\,x\right)^{9/2}\left(d\,g-3\,e\,f\right)}{9\,g^4}+\frac{2\,\left(f+g\,x\right)^{5/2}\left(c\,f^2+a\,g^2\right)\,\left(d\,g-e\,f\right)}{5\,g^4}$$

input
```
int((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x),x)
```

output
```
((f + g*x)^(7/2)*(2*a*e*g^2 + 6*c*e*f^2 - 4*c*d*f*g))/(7*g^4) + (2*c*e*(f
+ g*x)^(11/2))/(11*g^4) + (2*c*(f + g*x)^(9/2)*(d*g - 3*e*f))/(9*g^4) + (2
*(f + g*x)^(5/2)*(a*g^2 + c*f^2)*(d*g - e*f))/(5*g^4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.73

$$\int (d+ex)(f+gx)^{3/2} \left(a +cx^2\right) \, dx = \frac{2\sqrt{gx+f}\,(315ce\,g^5x^5+385cd\,g^5x^4+420cef\,g^4x^4+495ae\,g^5x^3+550cdf\,g^4x^3+15ce\,f^2g^3x^3}{}$$

input
```
int((e*x+d)*(g*x+f)^(3/2)*(c*x^2+a),x)
```

output
```
(2*sqrt(f + g*x)*(693*a*d*f**2*g**3 + 1386*a*d*f*g**4*x + 693*a*d*g**5*x**
2 - 198*a*e*f**3*g**2 + 99*a*e*f**2*g**3*x + 792*a*e*f*g**4*x**2 + 495*a*e
*g**5*x**3 + 88*c*d*f**4*g - 44*c*d*f**3*g**2*x + 33*c*d*f**2*g**3*x**2 +
550*c*d*f*g**4*x**3 + 385*c*d*g**5*x**4 - 48*c*e*f**5 + 24*c*e*f**4*g*x -
18*c*e*f**3*g**2*x**2 + 15*c*e*f**2*g**3*x**3 + 420*c*e*f*g**4*x**4 + 315*
c*e*g**5*x**5))/(3465*g**4)
```

3.45 $\int (f+gx)^{3/2}(a+cx^2)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int (f+gx)^{3/2}\left(a+cx^2\right)\,dx=\frac{2(cf^2+ag^2)\,(f+gx)^{5/2}}{5g^3}-\frac{4cf(f+gx)^{7/2}}{7g^3}+\frac{2c(f+gx)^{9/2}}{9g^3}$$

output
```
2/5*(a*g^2+c*f^2)*(g*x+f)^(5/2)/g^3-4/7*c*f*(g*x+f)^(7/2)/g^3+2/9*c*(g*x+f
)^(9/2)/g^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int (f+gx)^{3/2}\left(a+cx^2\right)\,dx=\frac{2(f+gx)^{5/2}\,(63ag^2+c(8f^2-20fgx+35g^2x^2))}{315g^3}$$

input
```
Integrate[(f + g*x)^(3/2)*(a + c*x^2),x]
```

output
```
(2*(f + g*x)^(5/2)*(63*a*g^2 + c*(8*f^2 - 20*f*g*x + 35*g^2*x^2)))/(315*g^
3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)(f+gx)^{3/2}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{(f+gx)^{3/2}(ag^2+cf^2)}{g^2}+\frac{c(f+gx)^{7/2}}{g^2}-\frac{2cf(f+gx)^{5/2}}{g^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(f+gx)^{5/2}(ag^2+cf^2)}{5g^3}+\frac{2c(f+gx)^{9/2}}{9g^3}-\frac{4cf(f+gx)^{7/2}}{7g^3}$$

input
```
Int[(f + g*x)^(3/2)*(a + c*x^2),x]
```

output
```
(2*(c*f^2 + a*g^2)*(f + g*x)^(5/2))/(5*g^3) - (4*c*f*(f + g*x)^(7/2))/(7*g
^3) + (2*c*(f + g*x)^(9/2))/(9*g^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{2(gx+f)^{\frac{5}{2}}(35c\,x^{2}g^{2}-20cfxg+63a\,g^{2}+8c\,f^{2})}{315g^{3}}$	41
pseudoelliptic	$\frac{2(gx+f)^{\frac{5}{2}}(35c\,x^{2}g^{2}-20cfxg+63a\,g^{2}+8c\,f^{2})}{315g^{3}}$	41
orering	$\frac{2(gx+f)^{\frac{5}{2}}(35c\,x^{2}g^{2}-20cfxg+63a\,g^{2}+8c\,f^{2})}{315g^{3}}$	41
derivativedivides	$\frac{\frac{2c(gx+f)^{\frac{9}{2}}}{9}-\frac{4cf(gx+f)^{\frac{7}{2}}}{7}+\frac{2(a\,g^{2}+c\,f^{2})(gx+f)^{\frac{5}{2}}}{5}}{g^{3}}$	48
default	$\frac{\frac{2c(gx+f)^{\frac{9}{2}}}{9}-\frac{4cf(gx+f)^{\frac{7}{2}}}{7}+\frac{2(a\,g^{2}+c\,f^{2})(gx+f)^{\frac{5}{2}}}{5}}{g^{3}}$	48
trager	$\frac{2(35c\,g^{4}x^{4}+50cf\,g^{3}x^{3}+63a\,g^{4}x^{2}+3c\,f^{2}g^{2}x^{2}+126af\,g^{3}x-4c\,f^{3}gx+63a\,f^{2}g^{2}+8c\,f^{4})\sqrt{gx+f}}{315g^{3}}$	85
risch	$\frac{2(35c\,g^{4}x^{4}+50cf\,g^{3}x^{3}+63a\,g^{4}x^{2}+3c\,f^{2}g^{2}x^{2}+126af\,g^{3}x-4c\,f^{3}gx+63a\,f^{2}g^{2}+8c\,f^{4})\sqrt{gx+f}}{315g^{3}}$	85

input
```
int((g*x+f)^(3/2)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/315*(g*x+f)^(5/2)*(35*c*g^2*x^2-20*c*f*g*x+63*a*g^2+8*c*f^2)/g^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.35

$$\int (f+gx)^{3/2}\left(a+cx^2\right)\,dx=\frac{2\left(35\,cg^4x^4+50\,cfg^3x^3+8\,cf^4+63\,af^2g^2+3\,(cf^2g^2+21\,ag^4)x^2-2\,(2\,cf^3g-63\,afg^3)x\right)\sqrt{g}}{315\,g^3}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a),x, algorithm="fricas")
```

output
```
2/315*(35*c*g^4*x^4 + 50*c*f*g^3*x^3 + 8*c*f^4 + 63*a*f^2*g^2 + 3*(c*f^2*g
^2 + 21*a*g^4)*x^2 - 2*(2*c*f^3*g - 63*a*f*g^3)*x)*sqrt(g*x + f)/g^3
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int (f+gx)^{3/2}\left(a+cx^2\right)\,dx=\begin{cases}\frac{2\left(-\frac{2cf(f+gx)^{\frac{7}{2}}}{7g^2}+\frac{c(f+gx)^{\frac{9}{2}}}{9g^2}+\frac{(f+gx)^{\frac{5}{2}}\left(ag^2+cf^2\right)}{5g^2}\right)}{g} & \text{for } g\neq 0\\ f^{\frac{3}{2}}\left(ax+\frac{cx^3}{3}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((g*x+f)**(3/2)*(c*x**2+a),x)
```

output
```
Piecewise((2*(-2*c*f*(f + g*x)**(7/2)/(7*g**2) + c*(f + g*x)**(9/2)/(9*g**
2) + (f + g*x)**(5/2)*(a*g**2 + c*f**2)/(5*g**2))/g, Ne(g, 0)), (f**(3/2)*
(a*x + c*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

$$\int (f+gx)^{3/2}\left(a+cx^2\right)\,dx=\frac{2\left(35\,(gx+f)^{\frac{9}{2}}c-90\,(gx+f)^{\frac{7}{2}}cf+63\,(cf^2+ag^2)(gx+f)^{\frac{5}{2}}\right)}{315\,g^3}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a),x, algorithm="maxima")
```

output
```
2/315*(35*(g*x + f)^(9/2)*c - 90*(g*x + f)^(7/2)*c*f + 63*(c*f^2 + a*g^2)*
(g*x + f)^(5/2))/g^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(51) = 102$.

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.63

$$\int (f+gx)^{3/2} \left(a+cx^2\right) \, dx = \frac{2 \left(315 \sqrt{gx+f} a f^2 + 210 \left((gx+f)^{\frac{3}{2}} - 3 \sqrt{gx+f} f\right) a f + 21 \left(3 (gx+f)^{\frac{5}{2}} - 10 (gx+f)^{\frac{3}{2}} f \right.\right.}{}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(g*x + f)*a*f^2 + 210*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)
*a*f + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2
)*a + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)
*c*f^2/g^2 + 18*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(
3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*f/g^2 + (35*(g*x + f)^(9/2) - 180*(g*x
+ f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqr
t(g*x + f)*f^4)*c/g^2)/g
```

Mupad [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int (f+gx)^{3/2} \left(a+cx^2\right) \, dx = \frac{2 \left(f+g\,x\right)^{5/2} \left(35\,c\,{\left(f+g\,x\right)}^2+63\,a\,g^2+63\,c\,f^2-90\,c\,f\,\left(f+g\,x\right)\right)}{315\,g^3}$$

input
```
int((f + g*x)^(3/2)*(a + c*x^2),x)
```

output
```
(2*(f + g*x)^(5/2)*(35*c*(f + g*x)^2 + 63*a*g^2 + 63*c*f^2 - 90*c*f*(f + g
*x)))/(315*g^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int (f+gx)^{3/2}\left(a+cx^2\right)\,dx=\frac{2\sqrt{gx+f}\,(35c\,g^4x^4+50cf\,g^3x^3+63a\,g^4x^2+3c\,f^2g^2x^2+126af\,g^3x-4c\,f^3gx+63a\,f^2g^2+}{315g^3}$$

input
```
int((g*x+f)^(3/2)*(c*x^2+a),x)
```

output
```
(2*sqrt(f + g*x)*(63*a*f**2*g**2 + 126*a*f*g**3*x + 63*a*g**4*x**2 + 8*c*f
**4 - 4*c*f**3*g*x + 3*c*f**2*g**2*x**2 + 50*c*f*g**3*x**3 + 35*c*g**4*x**
4))/(315*g**3)
```

3.46 $\int \frac{(f+gx)^{3/2}(a+cx^2)}{d+ex} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 166

$$\int \frac{(f+gx)^{3/2}(a+cx^2)}{d+ex} \, dx = \frac{2(cd^2+ae^2)(ef-dg)\sqrt{f+gx}}{e^4} + \frac{2(cd^2+ae^2)(f+gx)^{3/2}}{3e^3} - \frac{2c(ef+dg)(f+gx)^{5/2}}{5e^2g^2} + \frac{2c(f+gx)^{7/2}}{7eg^2} - \frac{2(cd^2+ae^2)(ef-dg)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{9/2}}$$

output
```
2*(a*e^2+c*d^2)*(-d*g+e*f)*(g*x+f)^(1/2)/e^4+2/3*(a*e^2+c*d^2)*(g*x+f)^(3/
2)/e^3-2/5*c*(d*g+e*f)*(g*x+f)^(5/2)/e^2/g^2+2/7*c*(g*x+f)^(7/2)/e/g^2-2*(
a*e^2+c*d^2)*(-d*g+e*f)^(3/2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/
2))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.98

$$\int \frac{(f+gx)^{3/2}\left(a+cx^2\right)}{d+ex}\,dx = \frac{2\sqrt{f+gx}(35ae^2g^2(4ef-3dg+egx)+c(-105d^3g^3-21de^2g(f+gx)^2-3}{105e^4g^2}$$
$$+\frac{2\left(cd^2+ae^2\right)(-ef+dg)^{3/2}\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{9/2}}$$

input
```
Integrate[((f + g*x)^(3/2)*(a + c*x^2))/(d + e*x),x]
```

output
```
(2*Sqrt[f + g*x]*(35*a*e^2*g^2*(4*e*f - 3*d*g + e*g*x) + c*(-105*d^3*g^3 -
 21*d*e^2*g*(f + g*x)^2 - 3*e^3*(2*f - 5*g*x)*(f + g*x)^2 + 35*d^2*e*g^2*(
4*f + g*x))))/(105*e^4*g^2) + (2*(c*d^2 + a*e^2)*(-(e*f) + d*g)^(3/2)*ArcT
an[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/e^(9/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {649, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)(f+gx)^{3/2}}{d+ex}\,dx$$

$$\Big\downarrow \text{ 649}$$

$$\frac{2\int -\frac{(f+gx)^2\left(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2\right)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2\int \frac{(f+gx)^2\left(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2\right)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{ 1584}$$

$$-\frac{2\int\left(-\frac{c(f+gx)^3}{e}+\frac{c(ef+dg)(f+gx)^2}{e^2}-\frac{(cd^2+ae^2)g^2(f+gx)}{e^3}-\frac{(cd^2+ae^2)g^2(ef-dg)}{e^4}+\frac{af^2g^2e^4-2adfg^3e^3+ad^2g^4e^2+cd^2f^2g^2e^2-2}{e^4(ef-dg-e(f+gx))}\right.}{g^2}$$

$\downarrow$ 2009

$$\frac{2\left(-\frac{g^2(ae^2+cd^2)(ef-dg)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{9/2}}+\frac{g^2\sqrt{f+gx}(ae^2+cd^2)(ef-dg)}{e^4}+\frac{g^2(f+gx)^{3/2}(ae^2+cd^2)}{3e^3}-\frac{c(f+gx)^{5/2}(dg+ef)}{5e^2}+\right.}{g^2}$$

input
```
Int[((f + g*x)^(3/2)*(a + c*x^2))/(d + e*x),x]
```

output
```
(2*(((c*d^2 + a*e^2)*g^2*(e*f - d*g)*Sqrt[f + g*x])/e^4 + ((c*d^2 + a*e^2)
*g^2*(f + g*x)^(3/2))/(3*e^3) - (c*(e*f + d*g)*(f + g*x)^(5/2))/(5*e^2) +
(c*(f + g*x)^(7/2))/(7*e) - ((c*d^2 + a*e^2)*g^2*(e*f - d*g)^(3/2)*ArcTanh
[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/e^(9/2)))/g^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$-\frac{2\left(\sqrt{(dg-ef)e}\left(\left(\left(\frac{2(gx+f)^2\left(-\frac{5gx}{2}+f\right)c}{35}-\frac{4g^2a\left(\frac{gx}{4}+f\right)}{3}\right)e^3+g\left(\frac{(gx+f)^2c}{5}+a\,g^2\right)d\,e^2-\frac{4g^2c\left(\frac{gx}{4}+f\right)d^2e}{3}+c\,d^3g^3\right)\sqrt{}\right.\right.}{\sqrt{(dg-ef)e}\,g^2e^4}$
derivativedivides	$-\frac{\frac{2\left(-\frac{c(gx+f)^{\frac{7}{2}}e^3}{7}+\frac{cd\,e^2g(gx+f)^{\frac{5}{2}}}{5}+\frac{c\,e^3f(gx+f)^{\frac{5}{2}}}{5}-\frac{a\,e^3g^2(gx+f)^{\frac{3}{2}}}{3}-\frac{c\,d^2e\,g^2(gx+f)^{\frac{3}{2}}}{3}+ad\,e^2g^3\sqrt{gx+f}-a\,e^3f\,g^2\sqrt{gx+f}+\right.}{e^4}}{g^2}$
default	$-\frac{\frac{2\left(-\frac{c(gx+f)^{\frac{7}{2}}e^3}{7}+\frac{cd\,e^2g(gx+f)^{\frac{5}{2}}}{5}+\frac{c\,e^3f(gx+f)^{\frac{5}{2}}}{5}-\frac{a\,e^3g^2(gx+f)^{\frac{3}{2}}}{3}-\frac{c\,d^2e\,g^2(gx+f)^{\frac{3}{2}}}{3}+ad\,e^2g^3\sqrt{gx+f}-a\,e^3f\,g^2\sqrt{gx+f}+\right.}{e^4}}{g^2}$
risch	$-\frac{2(-15c\,g^3e^3x^3+21cd\,e^2g^3x^2-24c\,e^3f\,g^2x^2-35a\,e^3g^3x-35c\,d^2e\,g^3x+42cd\,e^2f\,g^2x-3c\,e^3f^2gx+105ad\,e^2g^3-140a}{105g^2e^4}$

input
```
int((g*x+f)^(3/2)*(c*x^2+a)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-2*(((d*g-e*f)*e)^(1/2)*((2/35*(g*x+f)^2*(-5/2*g*x+f)*c-4/3*g^2*a*(1/4*g*x
+f))*e^3+g*(1/5*(g*x+f)^2*c+a*g^2)*d*e^2-4/3*g^2*c*(1/4*g*x+f)*d^2*e+c*d^3
*g^3)*(g*x+f)^(1/2)-g^2*(d*g-e*f)^2*(a*e^2+c*d^2)*arctan(e*(g*x+f)^(1/2)/(
(d*g-e*f)*e)^(1/2)))/((d*g-e*f)*e)^(1/2)/g^2/e^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.96

$$\int \frac{(f+gx)^{3/2}\,(a+cx^2)}{d+ex}\,dx = \left[-\frac{105\left((cd^2e+ae^3)fg^2-(cd^3+ade^2)g^3\right)\sqrt{\frac{ef-dg}{e}}\log\left(\frac{egx+2\,ef-dg+2\,\sqrt{gx+f}e\sqrt{}}{ex+d}\right.}{}\right.$$

$$-\frac{2\left(105\left((cd^2e+ae^3)fg^2-(cd^3+ade^2)g^3\right)\sqrt{-\frac{ef-dg}{e}}\arctan\left(-\frac{\sqrt{gx+f}e\sqrt{-\frac{ef-dg}{e}}}{ef-dg}\right)-(15\,ce^3g^3x^3-6\,ce^3f^3\right.}{}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)/(e*x+d),x, algorithm="fricas")
```

output
```
[-1/105*(105*((c*d^2*e + a*e^3)*f*g^2 - (c*d^3 + a*d*e^2)*g^3)*sqrt((e*f -
 d*g)/e)*log((e*g*x + 2*e*f - d*g + 2*sqrt(g*x + f)*e*sqrt((e*f - d*g)/e))
/(e*x + d)) - 2*(15*c*e^3*g^3*x^3 - 6*c*e^3*f^3 - 21*c*d*e^2*f^2*g + 140*(
c*d^2*e + a*e^3)*f*g^2 - 105*(c*d^3 + a*d*e^2)*g^3 + 3*(8*c*e^3*f*g^2 - 7*
c*d*e^2*g^3)*x^2 + (3*c*e^3*f^2*g - 42*c*d*e^2*f*g^2 + 35*(c*d^2*e + a*e^3
)*g^3)*x)*sqrt(g*x + f))/(e^4*g^2), -2/105*(105*((c*d^2*e + a*e^3)*f*g^2 -
 (c*d^3 + a*d*e^2)*g^3)*sqrt(-(e*f - d*g)/e)*arctan(-sqrt(g*x + f)*e*sqrt(
-(e*f - d*g)/e)/(e*f - d*g)) - (15*c*e^3*g^3*x^3 - 6*c*e^3*f^3 - 21*c*d*e^
2*f^2*g + 140*(c*d^2*e + a*e^3)*f*g^2 - 105*(c*d^3 + a*d*e^2)*g^3 + 3*(8*c
*e^3*f*g^2 - 7*c*d*e^2*g^3)*x^2 + (3*c*e^3*f^2*g - 42*c*d*e^2*f*g^2 + 35*(
c*d^2*e + a*e^3)*g^3)*x)*sqrt(g*x + f))/(e^4*g^2)]
```

Sympy [A] (verification not implemented)

Time = 5.01 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.40

$$\int \frac{(f+gx)^{3/2}\,(a+cx^2)}{d+ex}\,dx = \begin{cases} \frac{2\left(\frac{c(f+gx)^{\frac{7}{2}}}{7eg} + \frac{(f+gx)^{\frac{5}{2}}(-cdg-cef)}{5e^2g} + \frac{(f+gx)^{\frac{3}{2}}\left(ae^2g+cd^2g\right)}{3e^3} + \frac{\sqrt{f+gx}\left(-ade^2g^2+ae^3fg-cd^3g^2+cd^2efg\right)}{e^4} + \right.}{g} \\ f^{\frac{3}{2}}\left(-\frac{cdx}{e^2} + \frac{cx^2}{2e} + \frac{(ae^2+cd^2)\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^2}\right) \end{cases}$$

input
```
integrate((g*x+f)**(3/2)*(c*x**2+a)/(e*x+d),x)
```

output
```
Piecewise((2*(c*(f + g*x)**(7/2)/(7*e*g) + (f + g*x)**(5/2)*(-c*d*g - c*e*
f)/(5*e**2*g) + (f + g*x)**(3/2)*(a*e**2*g + c*d**2*g)/(3*e**3) + sqrt(f +
 g*x)*(-a*d*e**2*g**2 + a*e**3*f*g - c*d**3*g**2 + c*d**2*e*f*g)/e**4 + g*
(a*e**2 + c*d**2)*(d*g - e*f)**2*atan(sqrt(f + g*x)/sqrt((d*g - e*f)/e))/(
e**5*sqrt((d*g - e*f)/e)))/g, Ne(g, 0)), (f**(3/2)*(-c*d*x/e**2 + c*x**2/(
2*e) + (a*e**2 + c*d**2)*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True)
)/e**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^{3/2}(a+cx^2)}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.60

$$\int \frac{(f+gx)^{3/2}(a+cx^2)}{d+ex}\,dx = \frac{2\,(cd^2e^2f^2 + ae^4f^2 - 2\,cd^3efg - 2\,ade^3fg + cd^4g^2 + ad^2e^2g^2)\arctan\left(\frac{\sqrt{gx}}{\sqrt{-e^2f}}\right.}{\sqrt{-e^2f+dege}e^4} + \frac{2\left(15\,(gx+f)^{\frac{7}{2}}ce^6g^{12} - 21\,(gx+f)^{\frac{5}{2}}ce^6fg^{12} - 21\,(gx+f)^{\frac{5}{2}}cde^5g^{13} + 35\,(gx+f)^{\frac{3}{2}}cd^2e^4g^{14} + 35\,(gx+\right.}{105}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)/(e*x+d),x, algorithm="giac")
```

output
```
2*(c*d^2*e^2*f^2 + a*e^4*f^2 - 2*c*d^3*e*f*g - 2*a*d*e^3*f*g + c*d^4*g^2 +
 a*d^2*e^2*g^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e^2*f
+ d*e*g)*e^4) + 2/105*(15*(g*x + f)^(7/2)*c*e^6*g^12 - 21*(g*x + f)^(5/2)*
c*e^6*f*g^12 - 21*(g*x + f)^(5/2)*c*d*e^5*g^13 + 35*(g*x + f)^(3/2)*c*d^2*
e^4*g^14 + 35*(g*x + f)^(3/2)*a*e^6*g^14 + 105*sqrt(g*x + f)*c*d^2*e^4*f*g
^14 + 105*sqrt(g*x + f)*a*e^6*f*g^14 - 105*sqrt(g*x + f)*c*d^3*e^3*g^15 -
105*sqrt(g*x + f)*a*d*e^5*g^15)/(e^7*g^14)
```

Mupad [B] (verification not implemented)

Time = 5.70 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.19

$$\int \frac{(f+gx)^{3/2}(a+cx^2)}{d+ex}\,dx = (f+g\,x)^{3/2}\left(\frac{2\,c\,f^2+2\,a\,g^2}{3\,e\,g^2}+\frac{\left(\frac{2\,c\,(d\,g^3-e\,f\,g^2)}{e^2\,g^4}+\frac{4\,c\,f}{e\,g^2}\right)\,(d\,g^3-e\,f\,g^2)}{3\,e\,g^2}\right)$$

$$-(f+g\,x)^{5/2}\left(\frac{2\,c\,(d\,g^3-e\,f\,g^2)}{5\,e^2\,g^4}+\frac{4\,c\,f}{5\,e\,g^2}\right)+\frac{2\,c\,(f+g\,x)^{7/2}}{7\,e\,g^2}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}\,(c\,d^2+a\,e^2)\,(d\,g-e\,f)^{3/2}}{c\,d^4\,g^2-2\,c\,d^3\,e\,f\,g+c\,d^2\,e^2\,f^2+a\,d^2\,e^2\,g^2-2\,a\,d\,e}\right.}{e^{9/2}}$$

input
```
int(((f + g*x)^(3/2)*(a + c*x^2))/(d + e*x),x)
```

output
```
(f + g*x)^(3/2)*((2*a*g^2 + 2*c*f^2)/(3*e*g^2) + (((2*c*(d*g^3 - e*f*g^2))
/(e^2*g^4) + (4*c*f)/(e*g^2))*(d*g^3 - e*f*g^2))/(3*e*g^2)) - (f + g*x)^(5
/2)*((2*c*(d*g^3 - e*f*g^2))/(5*e^2*g^4) + (4*c*f)/(5*e*g^2)) + (2*c*(f +
g*x)^(7/2))/(7*e*g^2) + (2*atan((e^(1/2)*(f + g*x)^(1/2)*(a*e^2 + c*d^2)*(
d*g - e*f)^(3/2))/(a*e^4*f^2 + c*d^4*g^2 + a*d^2*e^2*g^2 + c*d^2*e^2*f^2 -
 2*a*d*e^3*f*g - 2*c*d^3*e*f*g))*(a*e^2 + c*d^2)*(d*g - e*f)^(3/2))/e^(9/2
) - ((f + g*x)^(1/2)*((2*a*g^2 + 2*c*f^2)/(e*g^2) + (((2*c*(d*g^3 - e*f*g^
2))/(e^2*g^4) + (4*c*f)/(e*g^2))*(d*g^3 - e*f*g^2))/(e*g^2))*(d*g^3 - e*f*
g^2))/(e*g^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.49

$$\int \frac{(f+gx)^{3/2}\,(a+cx^2)}{d+ex}\,dx = \frac{2\sqrt{e}\,\sqrt{dg-ef}\,atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-ef}}\right) a d\,e^2 g^3 - 2\sqrt{e}\,\sqrt{dg-ef}\,atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-ef}}\right) a}{}$$

input
```
int((g*x+f)^(3/2)*(c*x^2+a)/(e*x+d),x)
```

output
```
(2*(105*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g -
 e*f)))*a*d*e**2*g**3 - 105*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)
/(sqrt(e)*sqrt(d*g - e*f)))*a*e**3*f*g**2 + 105*sqrt(e)*sqrt(d*g - e*f)*at
an((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**3*g**3 - 105*sqrt(e)*
sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**2*e
*f*g**2 - 105*sqrt(f + g*x)*a*d*e**3*g**3 + 140*sqrt(f + g*x)*a*e**4*f*g**
2 + 35*sqrt(f + g*x)*a*e**4*g**3*x - 105*sqrt(f + g*x)*c*d**3*e*g**3 + 140
*sqrt(f + g*x)*c*d**2*e**2*f*g**2 + 35*sqrt(f + g*x)*c*d**2*e**2*g**3*x -
21*sqrt(f + g*x)*c*d*e**3*f**2*g - 42*sqrt(f + g*x)*c*d*e**3*f*g**2*x - 21
*sqrt(f + g*x)*c*d*e**3*g**3*x**2 - 6*sqrt(f + g*x)*c*e**4*f**3 + 3*sqrt(f
 + g*x)*c*e**4*f**2*g*x + 24*sqrt(f + g*x)*c*e**4*f*g**2*x**2 + 15*sqrt(f
+ g*x)*c*e**4*g**3*x**3))/(105*e**5*g**2)
```

3.47 $\int \frac{(f+gx)^{3/2}(a+cx^2)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{(f+gx)^{3/2}\,(a+cx^2)}{(d+ex)^2}\,dx = \frac{(3ae^2g-cd(4ef-7dg))\,\sqrt{f+gx}}{e^4} - \frac{4cd(f+gx)^{3/2}}{3e^3} - \frac{(cd^2+ae^2)\,(f+gx)^{3/2}}{e^3(d+ex)} + \frac{2c(f+gx)^{5/2}}{5e^2g} - \frac{\sqrt{ef-dg}(3ae^2g-cd(4ef-7dg))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{9/2}}$$

output

```
(3*a*e^2*g-c*d*(-7*d*g+4*e*f))*(g*x+f)^(1/2)/e^4-4/3*c*d*(g*x+f)^(3/2)/e^3
-(a*e^2+c*d^2)*(g*x+f)^(3/2)/e^3/(e*x+d)+2/5*c*(g*x+f)^(5/2)/e^2/g-(-d*g+e
*f)^(1/2)*(3*a*e^2*g-c*d*(-7*d*g+4*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d
*g+e*f)^(1/2))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.04

$$\int \frac{(f+gx)^{3/2}(a+cx^2)}{(d+ex)^2}\,dx = \frac{\sqrt{f+gx}(15ae^2g(-ef+3dg+2egx)+c(105d^3g^2+6e^3x(f+gx)^2+5d^2eg}{15e^4g(d+ex)}$$
$$-\frac{\sqrt{-ef+dg}(3ae^2g+cd(-4ef+7dg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{9/2}}$$

input
```
Integrate[((f + g*x)^(3/2)*(a + c*x^2))/(d + e*x)^2,x]
```

output
```
(Sqrt[f + g*x]*(15*a*e^2*g*(-(e*f) + 3*d*g + 2*e*g*x) + c*(105*d^3*g^2 + 6
*e^3*x*(f + g*x)^2 + 5*d^2*e*g*(-19*f + 14*g*x) + 2*d*e^2*(3*f^2 - 34*f*g*
x - 7*g^2*x^2))))/(15*e^4*g*(d + e*x)) - (Sqrt[-(e*f) + d*g]*(3*a*e^2*g +
c*d*(-4*e*f + 7*d*g))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/
e^(9/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {649, 1580, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(f+gx)^{3/2}}{(d+ex)^2}\,dx$$
$$\Big\downarrow \text{649}$$
$$\frac{2\int \frac{(f+gx)^2(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{g}$$
$$\Big\downarrow \text{1580}$$

$$\frac{2\left(\frac{g^2\sqrt{f+gx}(ae^2+cd^2)(ef-dg)}{2e^4(-dg-e(f+gx)+ef)}-\frac{\int \frac{2ce^3(f+gx)^3-2ce^2(ef+dg)(f+gx)^2+2e\left(cd^2+ae^2\right)g^2(f+gx)+\left(cd^2+ae^2\right)g^2(ef-dg)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{2e^4}\right)}{g}$$

$\Big\downarrow$ 2341

$$\frac{2\left(\frac{g^2\sqrt{f+gx}(ae^2+cd^2)(ef-dg)}{2e^4(-dg-e(f+gx)+ef)}-\frac{\int \left(-2ce^2(f+gx)^2+4cdeg(f+gx)-2g\left(ae^2g-cd(2ef-3dg)\right)+\frac{3afg^2e^3-3adg^3e^2-4cdf^2ge^2+11cd^2fg^2e-7cd^3g}{ef-dg-e(f+gx)}\right.}{2e^4}\right.}{g}$$

$\Big\downarrow$ 2009

$$\frac{2\left(\frac{g^2\sqrt{f+gx}(ae^2+cd^2)(ef-dg)}{2e^4(-dg-e(f+gx)+ef)}-\frac{\frac{g\sqrt{ef-dg}\left(3ae^2g-cd(4ef-7dg)\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}}-2g\sqrt{f+gx}\left(ae^2g-cd(2ef-3dg)\right)+\frac{4}{3}cdeg(f+gx)^{3/2}}{2e^4}\right.}{g}$$

input
```
Int[((f + g*x)^(3/2)*(a + c*x^2))/(d + e*x)^2,x]
```

output
```
(2*(((c*d^2 + a*e^2)*g^2*(e*f - d*g)*Sqrt[f + g*x])/(2*e^4*(e*f - d*g - e*
(f + g*x))) - (-2*g*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f + g*x] + (4*c*d
*e*g*(f + g*x)^(3/2))/3 - (2*c*e^2*(f + g*x)^(5/2))/5 + (g*Sqrt[e*f - d*g]
*(3*a*e^2*g - c*d*(4*e*f - 7*d*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*
f - d*g]])/Sqrt[e])/(2*e^4)))/g
```

Defintions of rubi rules used

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.12

method	result
risch	$\frac{2\left(3c\,x^{2}e^{2}g^{2}-10cde\,g^{2}x+6c\,e^{2}fgx+15a\,e^{2}g^{2}+45c\,d^{2}g^{2}-40cdefg+3c\,e^{2}f^{2}\right)\sqrt{gx+f}}{15g\,e^{4}}-\frac{(2dg-2ef)\left(\frac{\left(-\frac{1}{2}a\,e^{2}g-\frac{1}{2}c\,d^{2}\cdots\right.}{e(gx+f)+dg\cdots}\right.}{\cdots}$
pseudoelliptic	$\frac{-3g\left(a\,e^{2}g+\frac{7}{3}c\,d^{2}g-\frac{4}{3}cdef\right)(ex+d)(dg-ef)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)+3\sqrt{gx+f}\left(\frac{\left(2\left(\frac{c\,x^{2}}{5}+a\right)x\,g^{2}-f\left(-\frac{4c\,x^{2}}{5}+a\right)g+\frac{2c\,f^{2}}{5}\cdots\right.}{3}\cdots\right.}{e^{4}(ex+d)g\sqrt{(dg-ef)e}\cdots}$
derivativedivides	$\frac{\frac{2\left(\frac{c(gx+f)^{\frac{5}{2}}e^{2}}{5}-\frac{2cdeg(gx+f)^{\frac{3}{2}}}{3}+a\,e^{2}g^{2}\sqrt{gx+f}+3c\,d^{2}g^{2}\sqrt{gx+f}-2\sqrt{gx+f}\,cdefg\right)}{e^{4}}-\frac{2g\left(\frac{\left(-\frac{1}{2}ad\,e^{2}g^{2}+\frac{1}{2}a\,e^{3}fg-\frac{1}{2}c\,d^{3}g^{2}+\frac{1}{2}c\cdots\right.}{e(gx+f)+dg-ef}\cdots\right.}{\cdots}}{g}$
default	$\frac{\frac{2\left(\frac{c(gx+f)^{\frac{5}{2}}e^{2}}{5}-\frac{2cdeg(gx+f)^{\frac{3}{2}}}{3}+a\,e^{2}g^{2}\sqrt{gx+f}+3c\,d^{2}g^{2}\sqrt{gx+f}-2\sqrt{gx+f}\,cdefg\right)}{e^{4}}-\frac{2g\left(\frac{\left(-\frac{1}{2}ad\,e^{2}g^{2}+\frac{1}{2}a\,e^{3}fg-\frac{1}{2}c\,d^{3}g^{2}+\frac{1}{2}c\cdots\right.}{e(gx+f)+dg-ef}\cdots\right.}{\cdots}}{g}$

input
```
int((g*x+f)^(3/2)*(c*x^2+a)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
2/15/g*(3*c*e^2*g^2*x^2-10*c*d*e*g^2*x+6*c*e^2*f*g*x+15*a*e^2*g^2+45*c*d^2
*g^2-40*c*d*e*f*g+3*c*e^2*f^2)*(g*x+f)^(1/2)/e^4-1/e^4*(2*d*g-2*e*f)*((-1/
2*a*e^2*g-1/2*c*d^2*g)*(g*x+f)^(1/2)/(e*(g*x+f)+d*g-e*f)+1/2*(3*a*e^2*g+7*
c*d^2*g-4*c*d*e*f)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e
)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 539, normalized size of antiderivative = 3.12

$$\int \frac{(f+gx)^{3/2}(a+cx^2)}{(d+ex)^2}\,dx = \left[-\frac{15\,(4\,cd^2efg-(7\,cd^3+3\,ade^2)g^2+(4\,cde^2fg-(7\,cd^2e+3\,ae^3)g^2)x)\sqrt{e\cdots}}{\cdots}\right.$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[-1/30*(15*(4*c*d^2*e*f*g - (7*c*d^3 + 3*a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (
7*c*d^2*e + 3*a*e^3)*g^2)*x)*sqrt((e*f - d*g)/e)*log((e*g*x + 2*e*f - d*g
- 2*sqrt(g*x + f)*e*sqrt((e*f - d*g)/e))/(e*x + d)) - 2*(6*c*e^3*g^2*x^3 +
 6*c*d*e^2*f^2 - 5*(19*c*d^2*e + 3*a*e^3)*f*g + 15*(7*c*d^3 + 3*a*d*e^2)*g
^2 + 2*(6*c*e^3*f*g - 7*c*d*e^2*g^2)*x^2 + 2*(3*c*e^3*f^2 - 34*c*d*e^2*f*g
 + 5*(7*c*d^2*e + 3*a*e^3)*g^2)*x)*sqrt(g*x + f))/(e^5*g*x + d*e^4*g), 1/1
5*(15*(4*c*d^2*e*f*g - (7*c*d^3 + 3*a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (7*c*d
^2*e + 3*a*e^3)*g^2)*x)*sqrt(-(e*f - d*g)/e)*arctan(-sqrt(g*x + f)*e*sqrt(
-(e*f - d*g)/e)/(e*f - d*g)) + (6*c*e^3*g^2*x^3 + 6*c*d*e^2*f^2 - 5*(19*c*
d^2*e + 3*a*e^3)*f*g + 15*(7*c*d^3 + 3*a*d*e^2)*g^2 + 2*(6*c*e^3*f*g - 7*c
*d*e^2*g^2)*x^2 + 2*(3*c*e^3*f^2 - 34*c*d*e^2*f*g + 5*(7*c*d^2*e + 3*a*e^3
)*g^2)*x)*sqrt(g*x + f))/(e^5*g*x + d*e^4*g)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}(a+cx^2)}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**(3/2)*(c*x**2+a)/(e*x+d)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^{3/2}(a+cx^2)}{(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.55

$$\int \frac{(f+gx)^{3/2}(a+cx^2)}{(d+ex)^2}\,dx = -\frac{(4\,cde^2f^2-11\,cd^2efg-3\,ae^3fg+7\,cd^3g^2+3\,ade^2g^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{\sqrt{-e^2f+deg}e^4} - \frac{\sqrt{gx+f}cd^2efg+\sqrt{gx+f}ae^3fg-\sqrt{gx+f}cd^3g^2-\sqrt{gx+f}ade^2g^2}{((gx+f)e-ef+dg)e^4} + \frac{2\left(3\,(gx+f)^{\frac{5}{2}}ce^8g^4-10\,(gx+f)^{\frac{3}{2}}cde^7g^5-30\,\sqrt{gx+f}cde^7fg^5+45\,\sqrt{gx+f}cd^2e^6g^6+15\,\sqrt{gx+f}ae^8\right.}{15\,e^{10}g^5}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)/(e*x+d)^2,x, algorithm="giac")
```

output
```
-(4*c*d*e^2*f^2 - 11*c*d^2*e*f*g - 3*a*e^3*f*g + 7*c*d^3*g^2 + 3*a*d*e^2*g
^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e^2*f + d*e*g)*e^4
) - (sqrt(g*x + f)*c*d^2*e*f*g + sqrt(g*x + f)*a*e^3*f*g - sqrt(g*x + f)*c
*d^3*g^2 - sqrt(g*x + f)*a*d*e^2*g^2)/(((g*x + f)*e - e*f + d*g)*e^4) + 2/
15*(3*(g*x + f)^(5/2)*c*e^8*g^4 - 10*(g*x + f)^(3/2)*c*d*e^7*g^5 - 30*sqrt
(g*x + f)*c*d*e^7*f*g^5 + 45*sqrt(g*x + f)*c*d^2*e^6*g^6 + 15*sqrt(g*x + f
)*a*e^8*g^6)/(e^10*g^5)
```

Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.95

$$\int \frac{(f+gx)^{3/2}(a+cx^2)}{(d+ex)^2}\,dx = \sqrt{f+g\,x}\left(\frac{2\,c\,f^2+2\,a\,g^2}{e^2\,g} + \frac{2\left(\frac{4\,c\,(d\,g-e\,f)}{e^3\,g}+\frac{4\,c\,f}{e^2\,g}\right)(d\,g-e\,f)}{e} - \frac{2\,c\,(d\,g-e\,f)^2}{e^4\,g}\right) - (f+g\,x)^{3/2}\left(\frac{4\,c\,(d\,g-e\,f)}{3\,e^3\,g}+\frac{4\,c\,f}{3\,e^2\,g}\right) + \frac{\sqrt{f+g\,x}\,(c\,d^3\,g^2-c\,f\,d^2\,e\,g+a\,d\,e^2\,g^2-a\,f\,e^3\,g)}{e^5\,(f+g\,x)-e^5\,f+d\,e^4\,g} + \frac{2\,c\,(f+g\,x)^{5/2}}{5\,e^2\,g} - \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}\,\sqrt{d\,g-e\,f}\,(7\,c\,g\,d^2-4\,c\,f\,d\,e+3\,a\,g\,e^2)}{7\,c\,d^3\,g^2-11\,c\,d^2\,e\,f\,g+4\,c\,d\,e^2\,f^2+3\,a\,d\,e^2\,g^2-3\,a\,e^3\,f\,g}\right)\sqrt{d\,g-e\,f}\,(7\,c\,g\,d^2-4\,c\,f\,d\,e+3\,a\,g\,e^2)}{e^{9/2}}$$

input
```
int(((f + g*x)^(3/2)*(a + c*x^2))/(d + e*x)^2,x)
```

output
```
(f + g*x)^(1/2)*((2*a*g^2 + 2*c*f^2)/(e^2*g) + (2*((4*c*(d*g - e*f))/(e^3*
g) + (4*c*f)/(e^2*g))*(d*g - e*f))/e - (2*c*(d*g - e*f)^2)/(e^4*g)) - (f +
 g*x)^(3/2)*((4*c*(d*g - e*f))/(3*e^3*g) + (4*c*f)/(3*e^2*g)) + ((f + g*x)
^(1/2)*(c*d^3*g^2 - a*e^3*f*g + a*d*e^2*g^2 - c*d^2*e*f*g))/(e^5*(f + g*x)
 - e^5*f + d*e^4*g) + (2*c*(f + g*x)^(5/2))/(5*e^2*g) - (atan((e^(1/2)*(f
+ g*x)^(1/2)*(d*g - e*f)^(1/2)*(3*a*e^2*g + 7*c*d^2*g - 4*c*d*e*f))/(7*c*d
^3*g^2 - 3*a*e^3*f*g + 3*a*d*e^2*g^2 + 4*c*d*e^2*f^2 - 11*c*d^2*e*f*g))*(d
*g - e*f)^(1/2)*(3*a*e^2*g + 7*c*d^2*g - 4*c*d*e*f))/e^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.80

$$\int \frac{(f+gx)^{3/2}\,(a+cx^2)}{(d+ex)^2}\,dx = \frac{-45\sqrt{e}\,\sqrt{dg-ef}\,atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-ef}}\right)ad\,e^2g^2-45\sqrt{e}\,\sqrt{dg-ef}\,atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-e}}\right.}{}$$

input
```
int((g*x+f)^(3/2)*(c*x^2+a)/(e*x+d)^2,x)
```

output
```
( - 45*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g -
e*f)))*a*d*e**2*g**2 - 45*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(
sqrt(e)*sqrt(d*g - e*f)))*a*e**3*g**2*x - 105*sqrt(e)*sqrt(d*g - e*f)*atan
((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**3*g**2 + 60*sqrt(e)*sqr
t(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**2*e*f*
g - 105*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g -
 e*f)))*c*d**2*e*g**2*x + 60*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e
)/(sqrt(e)*sqrt(d*g - e*f)))*c*d*e**2*f*g*x + 45*sqrt(f + g*x)*a*d*e**3*g*
*2 - 15*sqrt(f + g*x)*a*e**4*f*g + 30*sqrt(f + g*x)*a*e**4*g**2*x + 105*sq
rt(f + g*x)*c*d**3*e*g**2 - 95*sqrt(f + g*x)*c*d**2*e**2*f*g + 70*sqrt(f +
 g*x)*c*d**2*e**2*g**2*x + 6*sqrt(f + g*x)*c*d*e**3*f**2 - 68*sqrt(f + g*x
)*c*d*e**3*f*g*x - 14*sqrt(f + g*x)*c*d*e**3*g**2*x**2 + 6*sqrt(f + g*x)*c
*e**4*f**2*x + 12*sqrt(f + g*x)*c*e**4*f*g*x**2 + 6*sqrt(f + g*x)*c*e**4*g
**2*x**3)/(15*e**5*g*(d + e*x))
```

3.48 $\int \frac{(f+gx)^{3/2}(a+cx^2)}{(d+ex)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 203

$$\int \frac{(f+gx)^{3/2}(a+cx^2)}{(d+ex)^3} dx = \frac{2c(ef-3dg)\sqrt{f+gx}}{e^4} - \frac{(3ae^2g-cd(8ef-11dg))\sqrt{f+gx}}{4e^4(d+ex)} + \frac{2c(f+gx)^{3/2}}{3e^3} - \frac{(cd^2+ae^2)(f+gx)^{3/2}}{2e^3(d+ex)^2} - \frac{(3ae^2g^2+c(8e^2f^2-40defg+35d^2g^2))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{9/2}\sqrt{ef-dg}}$$

output

```
2*c*(-3*d*g+e*f)*(g*x+f)^(1/2)/e^4-1/4*(3*a*e^2*g-c*d*(-11*d*g+8*e*f))*(g*
x+f)^(1/2)/e^4/(e*x+d)+2/3*c*(g*x+f)^(3/2)/e^3-1/2*(a*e^2+c*d^2)*(g*x+f)^(
3/2)/e^3/(e*x+d)^2-1/4*(3*a*e^2*g^2+c*(35*d^2*g^2-40*d*e*f*g+8*e^2*f^2))*a
rctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(9/2)/(-d*g+e*f)^(1/2)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.90

$$\int \frac{(f+gx)^{3/2}(a+cx^2)}{(d+ex)^3}\,dx = \frac{\sqrt{f+gx}(-3ae^2(2ef+3dg+5egx)+c(-105d^3g+25d^2e(2f-7gx)+8de}{12e^4(d+ex)^2}$$
$$+\frac{(3ae^2g^2+c(8e^2f^2-40defg+35d^2g^2))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{4e^{9/2}\sqrt{-ef+dg}}$$

input
```
Integrate[((f + g*x)^(3/2)*(a + c*x^2))/(d + e*x)^3,x]
```

output
```
(Sqrt[f + g*x]*(-3*a*e^2*(2*e*f + 3*d*g + 5*e*g*x) + c*(-105*d^3*g + 25*d^
2*e*(2*f - 7*g*x) + 8*d*e^2*x*(11*f - 7*g*x) + 8*e^3*x^2*(4*f + g*x))))/(1
2*e^4*(d + e*x)^2) + ((3*a*e^2*g^2 + c*(8*e^2*f^2 - 40*d*e*f*g + 35*d^2*g^
2))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(4*e^(9/2)*Sqrt[-(
e*f) + d*g])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {649, 25, 1580, 2345, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(f+gx)^{3/2}}{(d+ex)^3}\,dx$$
$$\Big\downarrow \; 649$$
$$2\int -\frac{(f+gx)^2(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$
$$\Big\downarrow \; 25$$
$$-2\int \frac{(f+gx)^2(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$

$$\Big\downarrow \text{1580}$$

$$2\left(\frac{\int \frac{4ce^3(f+gx)^3-4ce^2(ef+dg)(f+gx)^2+4e(cd^2+ae^2)g^2(f+gx)+(cd^2+ae^2)g^2(ef-dg)}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4e^4}-\frac{g^2\sqrt{f+gx}(ae^2+cd^2)\,(ef}{4e^4(-dg-e(f+gx)+e}\right.$$

$$\Big\downarrow \text{2345}$$

$$2\left(\frac{\frac{g\sqrt{f+gx}(5ae^2g-cd(8ef-13dg))}{2(-dg-e(f+gx)+ef)}-\frac{\int \frac{8ce^2(ef-dg)(f+gx)^2-16cdeg(ef-dg)(f+gx)+g(ef-dg)\left(3ae^2g-cd(8ef-11dg)\right)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{2(ef-dg)}}{4e^4}-\frac{g^2\sqrt{f+g}}{4e^4(-dg}\right.$$

$$\Big\downarrow \text{1467}$$

$$2\left(\frac{\frac{g\sqrt{f+gx}(5ae^2g-cd(8ef-13dg))}{2(-dg-e(f+gx)+ef)}-\frac{\int\left(-8c(ef-3dg)(ef-dg)-8ce(f+gx)(ef-dg)+\frac{8cf^3e^3+3afg^2e^3-3adg^3e^2-48cdf^2ge^2+75cd^2fg^2e-35cd^3}{ef-dg-e(f+gx)}\right.}{2(ef-dg)}}{4e^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\frac{g\sqrt{f+gx}(5ae^2g-cd(8ef-13dg))}{2(-dg-e(f+gx)+ef)}-\frac{\frac{\sqrt{ef-dg}\left(3ae^2g^2+c\left(35d^2g^2-40defg+8e^2f^2\right)\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}}-\frac{8}{3}ce(f+gx)^{3/2}(ef-dg)-8c\sqrt{f+gx}}{2(ef-dg)}}{4e^4}\right.$$

input

```
Int[((f + g*x)^(3/2)*(a + c*x^2))/(d + e*x)^3,x]
```

output

```
2*(-1/4*((c*d^2 + a*e^2)*g^2*(e*f - d*g)*Sqrt[f + g*x])/(e^4*(e*f - d*g -
e*(f + g*x))^2) + ((g*(5*a*e^2*g - c*d*(8*e*f - 13*d*g))*Sqrt[f + g*x])/(2
*(e*f - d*g - e*(f + g*x))) - (-8*c*(e*f - 3*d*g)*(e*f - d*g)*Sqrt[f + g*x
] - (8*c*e*(e*f - d*g)*(f + g*x)^(3/2))/3 + (Sqrt[e*f - d*g]*(3*a*e^2*g^2
+ c*(8*e^2*f^2 - 40*d*e*f*g + 35*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])
/Sqrt[e*f - d*g]])/Sqrt[e])/(2*(e*f - d*g)))/(4*e^4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{3\left(-\left(\left(\left(a\,g^{2}+\frac{8c\,f^{2}}{3}\right)e^{2}-\frac{40cdefg}{3}+\frac{35c\,d^{2}g^{2}}{3}\right)(ex+d)^{2}\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)+\sqrt{gx+f}\left(\left(\left(-\frac{32x^{2}\left(\frac{gx}{4}+f\right)c}{9}+\frac{2\left(\frac{5gx}{2}+\cdots\right.}{3}\cdots\right.\right.\right.\right.}{4\sqrt{(dg-ef)e}\,e^{4}(ex+d)^{2}}$
risch	$-\frac{2c(-egx+9dg-4ef)\sqrt{gx+f}}{3e^{4}}+\frac{\frac{2\left(-\frac{5}{8}a\,e^{3}g^{2}-\frac{13}{8}c\,d^{2}e\,g^{2}+cd\,e^{2}fg\right)(gx+f)^{\frac{3}{2}}-\frac{g\left(3ad\,e^{2}g^{2}-3a\,e^{3}fg+11c\,d^{3}g^{2}-19c\,d^{2}efg\cdots\right.}{4}\cdots}{(e(gx+f)+dg-ef)^{2}}\cdots}{e^{4}}$
derivativedivides	$-\frac{2c\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+3dg\sqrt{gx+f}-ef\sqrt{gx+f}\right)}{e^{4}}+\frac{2\left(\left(-\frac{5}{8}a\,e^{3}g^{2}-\frac{13}{8}c\,d^{2}e\,g^{2}+cd\,e^{2}fg\right)(gx+f)^{\frac{3}{2}}-\frac{g\left(3ad\,e^{2}g^{2}-3a\,e^{3}fg+11\cdots\right.}{\cdots}\right.}{(e(gx+f)+dg-ef)^{2}}\cdots$
default	$-\frac{2c\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+3dg\sqrt{gx+f}-ef\sqrt{gx+f}\right)}{e^{4}}+\frac{2\left(\left(-\frac{5}{8}a\,e^{3}g^{2}-\frac{13}{8}c\,d^{2}e\,g^{2}+cd\,e^{2}fg\right)(gx+f)^{\frac{3}{2}}-\frac{g\left(3ad\,e^{2}g^{2}-3a\,e^{3}fg+11\cdots\right.}{\cdots}\right.}{(e(gx+f)+dg-ef)^{2}}\cdots$

input
```
int((g*x+f)^(3/2)*(c*x^2+a)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/4/((d*g-e*f)*e)^(1/2)*(-((a*g^2+8/3*c*f^2)*e^2-40/3*c*d*e*f*g+35/3*c*d^
2*g^2)*(e*x+d)^2*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))+(g*x+f)^(1/2)
*((-32/9*x^2*(1/4*g*x+f)*c+2/3*(5/2*g*x+f)*a)*e^3+(-88/9*x*(-7/11*g*x+f)*c
+a*g)*d*e^2-50/9*(-7/2*g*x+f)*c*d^2*e+35/3*c*d^3*g)*((d*g-e*f)*e)^(1/2))/e
^4/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(177) = 354.

Time = 0.13 (sec) , antiderivative size = 858, normalized size of antiderivative = 4.23

$$\int \frac{(f+gx)^{3/2}\left(a+cx^{2}\right)}{(d+ex)^{3}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
[1/24*(3*(8*c*d^2*e^2*f^2 - 40*c*d^3*e*f*g + (35*c*d^4 + 3*a*d^2*e^2)*g^2
+ (8*c*e^4*f^2 - 40*c*d*e^3*f*g + (35*c*d^2*e^2 + 3*a*e^4)*g^2)*x^2 + 2*(8
*c*d*e^3*f^2 - 40*c*d^2*e^2*f*g + (35*c*d^3*e + 3*a*d*e^3)*g^2)*x)*sqrt(e^
2*f - d*e*g)*log((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f
))/(e*x + d)) + 2*(8*(c*e^5*f*g - c*d*e^4*g^2)*x^3 + 2*(25*c*d^2*e^3 - 3*a
*e^5)*f^2 - (155*c*d^3*e^2 + 3*a*d*e^4)*f*g + 3*(35*c*d^4*e + 3*a*d^2*e^3)
*g^2 + 8*(4*c*e^5*f^2 - 11*c*d*e^4*f*g + 7*c*d^2*e^3*g^2)*x^2 + (88*c*d*e^
4*f^2 - (263*c*d^2*e^3 + 15*a*e^5)*f*g + 5*(35*c*d^3*e^2 + 3*a*d*e^4)*g^2)
*x)*sqrt(g*x + f))/(d^2*e^6*f - d^3*e^5*g + (e^8*f - d*e^7*g)*x^2 + 2*(d*e
^7*f - d^2*e^6*g)*x), 1/12*(3*(8*c*d^2*e^2*f^2 - 40*c*d^3*e*f*g + (35*c*d^
4 + 3*a*d^2*e^2)*g^2 + (8*c*e^4*f^2 - 40*c*d*e^3*f*g + (35*c*d^2*e^2 + 3*a
*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2 - 40*c*d^2*e^2*f*g + (35*c*d^3*e + 3*a*d
*e^3)*g^2)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x +
f)/(e*g*x + e*f)) + (8*(c*e^5*f*g - c*d*e^4*g^2)*x^3 + 2*(25*c*d^2*e^3 - 3
*a*e^5)*f^2 - (155*c*d^3*e^2 + 3*a*d*e^4)*f*g + 3*(35*c*d^4*e + 3*a*d^2*e^
3)*g^2 + 8*(4*c*e^5*f^2 - 11*c*d*e^4*f*g + 7*c*d^2*e^3*g^2)*x^2 + (88*c*d*
e^4*f^2 - (263*c*d^2*e^3 + 15*a*e^5)*f*g + 5*(35*c*d^3*e^2 + 3*a*d*e^4)*g^
2)*x)*sqrt(g*x + f))/(d^2*e^6*f - d^3*e^5*g + (e^8*f - d*e^7*g)*x^2 + 2*(d
*e^7*f - d^2*e^6*g)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}\,(a+cx^2)}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**(3/2)*(c*x**2+a)/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^{3/2}\,(a+cx^2)}{(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.39

$$\int \frac{(f+gx)^{3/2}\,(a+cx^2)}{(d+ex)^3}\,dx = \frac{(8\,ce^2f^2-40\,cdefg+35\,cd^2g^2+3\,ae^2g^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{4\sqrt{-e^2f+deg}e^4}$$
$$+\frac{8\,(gx+f)^{\frac{3}{2}}cde^2fg-8\sqrt{gx+f}cde^2f^2g-13\,(gx+f)^{\frac{3}{2}}cd^2eg^2-5\,(gx+f)^{\frac{3}{2}}ae^3g^2+19\sqrt{gx+f}cd^2efg^2}{4\,((gx+f)e-ef+dg)^2e^4}$$
$$+\frac{2\left((gx+f)^{\frac{3}{2}}ce^6+3\sqrt{gx+f}ce^6f-9\sqrt{gx+f}cde^5g\right)}{3\,e^9}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)/(e*x+d)^3,x, algorithm="giac")
```

output
```
1/4*(8*c*e^2*f^2 - 40*c*d*e*f*g + 35*c*d^2*g^2 + 3*a*e^2*g^2)*arctan(sqrt(
g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e^2*f + d*e*g)*e^4) + 1/4*(8*(g*x
+ f)^(3/2)*c*d*e^2*f*g - 8*sqrt(g*x + f)*c*d*e^2*f^2*g - 13*(g*x + f)^(3/2
)*c*d^2*e*g^2 - 5*(g*x + f)^(3/2)*a*e^3*g^2 + 19*sqrt(g*x + f)*c*d^2*e*f*g
^2 + 3*sqrt(g*x + f)*a*e^3*f*g^2 - 11*sqrt(g*x + f)*c*d^3*g^3 - 3*sqrt(g*x
 + f)*a*d*e^2*g^3)/(((g*x + f)*e - e*f + d*g)^2*e^4) + 2/3*((g*x + f)^(3/2
)*c*e^6 + 3*sqrt(g*x + f)*c*e^6*f - 9*sqrt(g*x + f)*c*d*e^5*g)/e^9
```

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.39

$$\int \frac{(f+gx)^{3/2}\,(a+cx^2)}{(d+ex)^3}\,dx = \sqrt{f+g\,x}\left(\frac{2\,c\,(3\,e^3\,f-3\,d\,e^2\,g)}{e^6}-\frac{4\,c\,f}{e^3}\right)$$
$$-\frac{(f+g\,x)^{3/2}\left(\frac{13\,c\,d^2\,e\,g^2}{4}-2\,c\,f\,d\,e^2\,g+\frac{5\,a\,e^3\,g^2}{4}\right)+\sqrt{f+g\,x}\left(\frac{11\,c\,d^3\,g^3}{4}-\frac{19\,c\,d^2\,e\,f\,g^2}{4}+2\,c\,d\,e^2\,f^2\,g+\frac{3\,a\,d\,e^2\,g^3}{4}\right.}{e^6\,(f+g\,x)^2-(f+g\,x)\,(2\,e^6\,f-2\,d\,e^5\,g)+e^6\,f^2+d^2\,e^4\,g^2-2\,d\,e^5\,f\,g}$$
$$+\frac{2\,c\,(f+g\,x)^{3/2}}{3\,e^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,(35\,c\,d^2\,g^2-40\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2+3\,a\,e^2\,g^2)}{4\,e^{9/2}\,\sqrt{d\,g-e\,f}}$$

input
```
int(((f + g*x)^(3/2)*(a + c*x^2))/(d + e*x)^3,x)
```

output
```
(f + g*x)^(1/2)*((2*c*(3*e^3*f - 3*d*e^2*g))/e^6 - (4*c*f)/e^3) - ((f + g*
x)^(3/2)*((5*a*e^3*g^2)/4 + (13*c*d^2*e*g^2)/4 - 2*c*d*e^2*f*g) + (f + g*x
)^(1/2)*((11*c*d^3*g^3)/4 + (3*a*d*e^2*g^3)/4 - (3*a*e^3*f*g^2)/4 + 2*c*d*
e^2*f^2*g - (19*c*d^2*e*f*g^2)/4))/(e^6*(f + g*x)^2 - (f + g*x)*(2*e^6*f -
 2*d*e^5*g) + e^6*f^2 + d^2*e^4*g^2 - 2*d*e^5*f*g) + (2*c*(f + g*x)^(3/2))
/(3*e^3) + (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(3*a*e^2*g^2
 + 35*c*d^2*g^2 + 8*c*e^2*f^2 - 40*c*d*e*f*g))/(4*e^(9/2)*(d*g - e*f)^(1/2
))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 891, normalized size of antiderivative = 4.39

$$\int \frac{(f+gx)^{3/2}\,(a+cx^2)}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^(3/2)*(c*x^2+a)/(e*x+d)^3,x)
```

output
```
(9*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)
))*a*d**2*e**2*g**2 + 18*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(s
qrt(e)*sqrt(d*g - e*f)))*a*d*e**3*g**2*x + 9*sqrt(e)*sqrt(d*g - e*f)*atan(
(sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*e**4*g**2*x**2 + 105*sqrt(e
)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**4
*g**2 - 120*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d
*g - e*f)))*c*d**3*e*f*g + 210*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)
*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**3*e*g**2*x + 24*sqrt(e)*sqrt(d*g - e*f
)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**2*e**2*f**2 - 240
*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))
*c*d**2*e**2*f*g*x + 105*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(s
qrt(e)*sqrt(d*g - e*f)))*c*d**2*e**2*g**2*x**2 + 48*sqrt(e)*sqrt(d*g - e*f
)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d*e**3*f**2*x - 120*
sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*
c*d*e**3*f*g*x**2 + 24*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqr
t(e)*sqrt(d*g - e*f)))*c*e**4*f**2*x**2 - 9*sqrt(f + g*x)*a*d**2*e**3*g**2
 + 3*sqrt(f + g*x)*a*d*e**4*f*g - 15*sqrt(f + g*x)*a*d*e**4*g**2*x + 6*sqr
t(f + g*x)*a*e**5*f**2 + 15*sqrt(f + g*x)*a*e**5*f*g*x - 105*sqrt(f + g*x)
*c*d**4*e*g**2 + 155*sqrt(f + g*x)*c*d**3*e**2*f*g - 175*sqrt(f + g*x)*c*d
**3*e**2*g**2*x - 50*sqrt(f + g*x)*c*d**2*e**3*f**2 + 263*sqrt(f + g*x)...
```

3.49 $\int \frac{(d+ex)^3(a+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 240

$$\begin{aligned}\int \frac{(d+ex)^3(a+cx^2)}{\sqrt{f+gx}}\,dx &= -\frac{2(ef-dg)^3(cf^2+ag^2)\sqrt{f+gx}}{g^6} \\ &\quad+\frac{2(ef-dg)^2(3aeg^2+cf(5ef-2dg))(f+gx)^{3/2}}{3g^6} \\ &\quad-\frac{2(ef-dg)(3ae^2g^2+c(10e^2f^2-8defg+d^2g^2))(f+gx)^{5/2}}{5g^6} \\ &\quad+\frac{2e(ae^2g^2+c(10e^2f^2-12defg+3d^2g^2))(f+gx)^{7/2}}{7g^6} \\ &\quad-\frac{2ce^2(5ef-3dg)(f+gx)^{9/2}}{9g^6}+\frac{2ce^3(f+gx)^{11/2}}{11g^6}\end{aligned}$$

output
```
-2*(-d*g+e*f)^3*(a*g^2+c*f^2)*(g*x+f)^(1/2)/g^6+2/3*(-d*g+e*f)^2*(3*a*e*g^
2+c*f*(-2*d*g+5*e*f))*(g*x+f)^(3/2)/g^6-2/5*(-d*g+e*f)*(3*a*e^2*g^2+c*(d^2
*g^2-8*d*e*f*g+10*e^2*f^2))*(g*x+f)^(5/2)/g^6+2/7*e*(a*e^2*g^2+c*(3*d^2*g^
2-12*d*e*f*g+10*e^2*f^2))*(g*x+f)^(7/2)/g^6-2/9*c*e^2*(-3*d*g+5*e*f)*(g*x+
f)^(9/2)/g^6+2/11*c*e^3*(g*x+f)^(11/2)/g^6
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^3 (a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\sqrt{f+gx}(99ag^2(35d^3g^3+35d^2eg^2(-2f+gx)+7de^2g(8f^2-4fgx+3g^2x^2)+e^3(-16f^3+8f^2gx-6}{}$$

input
```
Integrate[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(99*a*g^2*(35*d^3*g^3 + 35*d^2*e*g^2*(-2*f + g*x) + 7*d*e
^2*g*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + e^3*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^
2 + 5*g^3*x^3)) + c*(231*d^3*g^3*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 297*d^2*e
*g^2*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + 33*d*e^2*g*(128*f^4
 - 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4) - 5*e^3*(256*f
^5 - 128*f^4*g*x + 96*f^3*g^2*x^2 - 80*f^2*g^3*x^3 + 70*f*g^4*x^4 - 63*g^5
*x^5))))/(3465*g^6)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(d+ex)^3}{\sqrt{f+gx}}\,dx$$
$$\Big\downarrow \text{652}$$
$$\int \left(\frac{e(f+gx)^{5/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{g^5}+\frac{(f+gx)^{3/2}(ef-dg)\left(-3ae^2g^2-c\left(d^2g^2-8defg+\right.\right.}{g^5}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{2e(f+gx)^{7/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{7g^6}-\frac{2(f+gx)^{5/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{5g^6}-\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)^3}{g^6}+\frac{2(f+gx)^{3/2}(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)}{3g^6}-\frac{2ce^2(f+gx)^{9/2}(5ef-3dg)}{9g^6}+\frac{2ce^3(f+gx)^{11/2}}{11g^6}$$

input
```
Int[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(-2*(e*f - d*g)^3*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^6 + (2*(e*f - d*g)^2*(3
*a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^6) - (2*(e*f - d*g)*
(3*a*e^2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g
^6) + (2*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)
^(7/2))/(7*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(9/2))/(9*g^6) + (2*c
*e^3*(f + g*x)^(11/2))/(11*g^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{2e^3c(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(3(dg-ef)e^2c-2fe^3c\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(3(dg-ef)^2ec-6(dg-ef)e^2cf+e^3\left(a g^2+c f^2\right)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^3c-\right.}{}$
default	$\frac{2e^3c(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(3(dg-ef)e^2c-2fe^3c\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(3(dg-ef)^2ec-6(dg-ef)e^2cf+e^3\left(a g^2+c f^2\right)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^3c-\right.}{}$
pseudoelliptic	$\frac{2\sqrt{gx+f}\left(\left(\left(\frac{\left(\frac{7c\,x^2}{11}+a\right)x^3e^3}{7}+\frac{3\left(\frac{5c\,x^2}{9}+a\right)x^2d\,e^2}{5}+d^2x\left(a+\frac{3c\,x^2}{7}\right)e+d^3\left(\frac{c\,x^2}{5}+a\right)\right)g^5-2f\left(\left(\frac{5}{99}c\,x^4+\frac{3}{35}a\,x^2\right)e^3+\frac{2xc}{}\right.\right.\right.}{}$
gosper	$\frac{2\sqrt{gx+f}\,(315e^3cx^5g^5+1155cd\,e^2g^5x^4-350c\,e^3f\,g^4x^4+495a\,e^3g^5x^3+1485c\,d^2e\,g^5x^3-1320cd\,e^2f\,g^4x^3+400c\,e^3f^2g^3}{}$
trager	$\frac{2\sqrt{gx+f}\,(315e^3cx^5g^5+1155cd\,e^2g^5x^4-350c\,e^3f\,g^4x^4+495a\,e^3g^5x^3+1485c\,d^2e\,g^5x^3-1320cd\,e^2f\,g^4x^3+400c\,e^3f^2g^3}{}$
risch	$\frac{2\sqrt{gx+f}\,(315e^3cx^5g^5+1155cd\,e^2g^5x^4-350c\,e^3f\,g^4x^4+495a\,e^3g^5x^3+1485c\,d^2e\,g^5x^3-1320cd\,e^2f\,g^4x^3+400c\,e^3f^2g^3}{}$
orering	$\frac{2\sqrt{gx+f}\,(315e^3cx^5g^5+1155cd\,e^2g^5x^4-350c\,e^3f\,g^4x^4+495a\,e^3g^5x^3+1485c\,d^2e\,g^5x^3-1320cd\,e^2f\,g^4x^3+400c\,e^3f^2g^3}{}$

input
```
int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/g^6*(1/11*e^3*c*(g*x+f)^(11/2)+1/9*(3*(d*g-e*f)*e^2*c-2*f*e^3*c)*(g*x+f)
^(9/2)+1/7*(3*(d*g-e*f)^2*e*c-6*(d*g-e*f)*e^2*c*f+e^3*(a*g^2+c*f^2))*(g*x+
f)^(7/2)+1/5*((d*g-e*f)^3*c-6*(d*g-e*f)^2*e*c*f+3*(d*g-e*f)*e^2*(a*g^2+c*f
^2))*(g*x+f)^(5/2)+1/3*(-2*(d*g-e*f)^3*c*f+3*(d*g-e*f)^2*e*(a*g^2+c*f^2))*
(g*x+f)^(3/2)+(d*g-e*f)^3*(a*g^2+c*f^2)*(g*x+f)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^3\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$=\frac{2\,(315\,ce^3g^5x^5-1280\,ce^3f^5+4224\,cde^2f^4g-6930\,ad^2efg^4+3465\,ad^3g^5-1584\,(3\,cd^2e+ae^3)f^3g^2+1}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*c*e^3*g^5*x^5 - 1280*c*e^3*f^5 + 4224*c*d*e^2*f^4*g - 6930*a*d
^2*e*f*g^4 + 3465*a*d^3*g^5 - 1584*(3*c*d^2*e + a*e^3)*f^3*g^2 + 1848*(c*d
^3 + 3*a*d*e^2)*f^2*g^3 - 35*(10*c*e^3*f*g^4 - 33*c*d*e^2*g^5)*x^4 + 5*(80
*c*e^3*f^2*g^3 - 264*c*d*e^2*f*g^4 + 99*(3*c*d^2*e + a*e^3)*g^5)*x^3 - 3*(
160*c*e^3*f^3*g^2 - 528*c*d*e^2*f^2*g^3 + 198*(3*c*d^2*e + a*e^3)*f*g^4 -
231*(c*d^3 + 3*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f^4*g - 2112*c*d*e^2*f^3*g^2
 + 3465*a*d^2*e*g^5 + 792*(3*c*d^2*e + a*e^3)*f^2*g^3 - 924*(c*d^3 + 3*a*d
*e^2)*f*g^4)*x)*sqrt(g*x + f)/g^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(241) = 482$.

Time = 1.11 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.09

$$\int \frac{(d+ex)^3\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{ce^3(f+gx)^{\frac{11}{2}}}{11g^5} + \frac{(f+gx)^{\frac{9}{2}}\cdot\left(3cde^2g-5ce^3f\right)}{9g^5} + \frac{(f+gx)^{\frac{7}{2}}\left(ae^3g^2+3cd^2eg^2-12cde^2fg+10ce^3f^2\right)}{7g^5} + \frac{(f+gx)^{\frac{5}{2}}\cdot\left(3ade^2g^3-3ae^3fg^2+cd^3g^3-9cd^2efg^2+18\right.}{5g^5}}{} \\ \frac{ad^3x + \frac{3ad^2ex^2}{2} + \frac{3cde^2x^5}{5} + \frac{ce^3x^6}{6} + \frac{x^4\left(ae^3+3cd^2e\right)}{4} + \frac{x^3\cdot\left(3ade^2+cd^3\right)}{3}}{\sqrt{f}} \end{cases}$$

input
```
integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c*e**3*(f + g*x)**(11/2)/(11*g**5) + (f + g*x)**(9/2)*(3*c*d
*e**2*g - 5*c*e**3*f)/(9*g**5) + (f + g*x)**(7/2)*(a*e**3*g**2 + 3*c*d**2*
e*g**2 - 12*c*d*e**2*f*g + 10*c*e**3*f**2)/(7*g**5) + (f + g*x)**(5/2)*(3*
a*d*e**2*g**3 - 3*a*e**3*f*g**2 + c*d**3*g**3 - 9*c*d**2*e*f*g**2 + 18*c*d
*e**2*f**2*g - 10*c*e**3*f**3)/(5*g**5) + (f + g*x)**(3/2)*(3*a*d**2*e*g**
4 - 6*a*d*e**2*f*g**3 + 3*a*e**3*f**2*g**2 - 2*c*d**3*f*g**3 + 9*c*d**2*e*
f**2*g**2 - 12*c*d*e**2*f**3*g + 5*c*e**3*f**4)/(3*g**5) + sqrt(f + g*x)*(
a*d**3*g**5 - 3*a*d**2*e*f*g**4 + 3*a*d*e**2*f**2*g**3 - a*e**3*f**3*g**2
+ c*d**3*f**2*g**3 - 3*c*d**2*e*f**3*g**2 + 3*c*d*e**2*f**4*g - c*e**3*f**
5)/g**5)/g, Ne(g, 0)), ((a*d**3*x + 3*a*d**2*e*x**2/2 + 3*c*d*e**2*x**5/5
+ c*e**3*x**6/6 + x**4*(a*e**3 + 3*c*d**2*e)/4 + x**3*(3*a*d*e**2 + c*d**3
)/3)/sqrt(f), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^3(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(315\,(gx+f)^{\frac{11}{2}}ce^3 - 385\,(5\,ce^3f - 3\,cde^2g)(gx+f)^{\frac{9}{2}} + 495\,(10\,ce^3f^2 - 12\,cde^2fg + (3\,cd^2e + ae^3)g^2)\right.}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/3465*(315*(g*x + f)^(11/2)*c*e^3 - 385*(5*c*e^3*f - 3*c*d*e^2*g)*(g*x +
f)^(9/2) + 495*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2*e + a*e^3)*g^2)*(
g*x + f)^(7/2) - 693*(10*c*e^3*f^3 - 18*c*d*e^2*f^2*g + 3*(3*c*d^2*e + a*e
^3)*f*g^2 - (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(5/2) + 1155*(5*c*e^3*f^4 -
 12*c*d*e^2*f^3*g + 3*a*d^2*e*g^4 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 - 2*(c*d
^3 + 3*a*d*e^2)*f*g^3)*(g*x + f)^(3/2) - 3465*(c*e^3*f^5 - 3*c*d*e^2*f^4*g
 + 3*a*d^2*e*f*g^4 - a*d^3*g^5 + (3*c*d^2*e + a*e^3)*f^3*g^2 - (c*d^3 + 3*
a*d*e^2)*f^2*g^3)*sqrt(g*x + f))/g^6
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^3 (a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(3465\sqrt{gx+f}ad^3 + \frac{3465\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)ad^2e}{g} + \frac{231\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)cd^3}{g^2} + \frac{693\left(3(gx+f)\right.}{}\right.}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(g*x + f)*a*d^3 + 3465*((g*x + f)^(3/2) - 3*sqrt(g*x + f)
*f)*a*d^2*e/g + 231*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*
x + f)*f^2)*c*d^3/g^2 + 693*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15
*sqrt(g*x + f)*f^2)*a*d*e^2/g^2 + 297*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5
/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d^2*e/g^3 + 99*(5
*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt
(g*x + f)*f^3)*a*e^3/g^3 + 33*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f
+ 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^
4)*c*d*e^2/g^4 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x
 + f)^(7/2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 69
3*sqrt(g*x + f)*f^5)*c*e^3/g^5)/g
```

Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^3(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{(f+gx)^{7/2}(6cd^2eg^2-24cde^2fg+20ce^3f^2+2ae^3g^2)}{7g^6} + \frac{2\sqrt{f+gx}(cf^2+ag^2)(dg-ef)^3}{g^6} + \frac{2ce^3(f+gx)^{11/2}}{11g^6} + \frac{2(f+gx)^{3/2}(dg-ef)^2(5cef^2-2cdfg+3aeg^2)}{3g^6} + \frac{2(f+gx)^{5/2}(dg-ef)(cd^2g^2-8cdefg+10ce^2f^2+3ae^2g^2)}{5g^6} + \frac{2ce^2(f+gx)^{9/2}(3dg-5ef)}{9g^6}$$

input
```
int(((a + c*x^2)*(d + e*x)^3)/(f + g*x)^(1/2),x)
```

output
```
((f + g*x)^(7/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*
f*g))/(7*g^6) + (2*(f + g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f)^3)/g^6 + (2
*c*e^3*(f + g*x)^(11/2))/(11*g^6) + (2*(f + g*x)^(3/2)*(d*g - e*f)^2*(3*a*
e*g^2 + 5*c*e*f^2 - 2*c*d*f*g))/(3*g^6) + (2*(f + g*x)^(5/2)*(d*g - e*f)*(
3*a*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 - 8*c*d*e*f*g))/(5*g^6) + (2*c*e^2*
(f + g*x)^(9/2)*(3*d*g - 5*e*f))/(9*g^6)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^3(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\sqrt{gx+f}\,(315ce^3g^5x^5+1155cde^2g^5x^4-350ce^3fg^4x^4+495ae^3g^5x^3+1485cd^2eg^5x^3-1320cde^2fg^4}{}$$

input
```
int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x)
```

output
```
(2*sqrt(f + g*x)*(3465*a*d**3*g**5 - 6930*a*d**2*e*f*g**4 + 3465*a*d**2*e*
g**5*x + 5544*a*d*e**2*f**2*g**3 - 2772*a*d*e**2*f*g**4*x + 2079*a*d*e**2*
g**5*x**2 - 1584*a*e**3*f**3*g**2 + 792*a*e**3*f**2*g**3*x - 594*a*e**3*f*
g**4*x**2 + 495*a*e**3*g**5*x**3 + 1848*c*d**3*f**2*g**3 - 924*c*d**3*f*g*
*4*x + 693*c*d**3*g**5*x**2 - 4752*c*d**2*e*f**3*g**2 + 2376*c*d**2*e*f**2
*g**3*x - 1782*c*d**2*e*f*g**4*x**2 + 1485*c*d**2*e*g**5*x**3 + 4224*c*d*e
**2*f**4*g - 2112*c*d*e**2*f**3*g**2*x + 1584*c*d*e**2*f**2*g**3*x**2 - 13
20*c*d*e**2*f*g**4*x**3 + 1155*c*d*e**2*g**5*x**4 - 1280*c*e**3*f**5 + 640
*c*e**3*f**4*g*x - 480*c*e**3*f**3*g**2*x**2 + 400*c*e**3*f**2*g**3*x**3 -
 350*c*e**3*f*g**4*x**4 + 315*c*e**3*g**5*x**5))/(3465*g**6)
```

3.50 $\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 175

$$\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2(ef-dg)^2(cf^2+ag^2)\sqrt{f+gx}}{g^5} - \frac{4(ef-dg)(aeg^2+cf(2ef-dg))(f+gx)^{3/2}}{3g^5} + \frac{2(ae^2g^2+c(6e^2f^2-6defg+d^2g^2))(f+gx)^{5/2}}{5g^5} - \frac{4ce(2ef-dg)(f+gx)^{7/2}}{7g^5} + \frac{2ce^2(f+gx)^{9/2}}{9g^5}$$

output
```
2*(-d*g+e*f)^2*(a*g^2+c*f^2)*(g*x+f)^(1/2)/g^5-4/3*(-d*g+e*f)*(a*e*g^2+c*f
*(-d*g+2*e*f))*(g*x+f)^(3/2)/g^5+2/5*(a*e^2*g^2+c*(d^2*g^2-6*d*e*f*g+6*e^2
*f^2))*(g*x+f)^(5/2)/g^5-4/7*c*e*(-d*g+2*e*f)*(g*x+f)^(7/2)/g^5+2/9*c*e^2*
(g*x+f)^(9/2)/g^5
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}(21ag^2(15d^2g^2+10deg(-2f+gx)+e^2(8f^2-4fgx+3g^2x^2))+c(21d^2g^2(8f^2-4fgx+3g^2}{31}$$

input
```
Integrate[((d + e*x)^2*(a + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(21*a*g^2*(15*d^2*g^2 + 10*d*e*g*(-2*f + g*x) + e^2*(8*f^
2 - 4*f*g*x + 3*g^2*x^2)) + c*(21*d^2*g^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2) +
18*d*e*g*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + e^2*(128*f^4 -
64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4))))/(315*g^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(d+ex)^2}{\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{(f+gx)^{3/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{g^4}+\frac{(ag^2+cf^2)(dg-ef)^2}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}(ef-dg)\left(-aeg^2-\right.}{g^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(f+gx)^{5/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{5g^5}+\frac{2\sqrt{f+gx}(ag^2+cf^2)(ef-dg)^2}{g^5}-\frac{4(f+gx)^{3/2}(ef-dg)\left(aeg^2+cf(2ef-dg)\right)}{3g^5}-\frac{4ce(f+gx)^{7/2}(2ef-dg)}{7g^5}+\frac{2ce^2(f+gx)^{9/2}}{9g^5}$$

input
```
Int[((d + e*x)^2*(a + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(2*(e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^5 - (4*(e*f - d*g)*(a*e*
g^2 + c*f*(2*e*f - d*g))*(f + g*x)^(3/2))/(3*g^5) + (2*(a*e^2*g^2 + c*(6*e
^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*g^5) - (4*c*e*(2*e*f -
d*g)*(f + g*x)^(7/2))/(7*g^5) + (2*c*e^2*(f + g*x)^(9/2))/(9*g^5)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$\frac{2\sqrt{gx+f}\left(\left(\left(\frac{\left(\frac{5c\,x^2}{9}+a\right)x^2e^2}{5}+\frac{2\left(a+\frac{3c\,x^2}{7}\right)xde}{3}+d^2\left(\frac{c\,x^2}{5}+a\right)\right)g^4-\frac{4\left(\left(\frac{2}{21}c\,x^3+\frac{1}{5}a x\right)e^2+d\left(\frac{9c\,x^2}{35}+a\right)e+\frac{c\,d^2x}{5}\right)f\,g^3}{3}+\cdots\right.\right.}{g^5}$
derivativedivides	$\frac{\frac{2c\,e^2(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2e(dg-ef)c-2fc\,e^2\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^2c-4e(dg-ef)cf+e^2\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)^2cf+\cdots\right.}{}}{g^5}$
default	$\frac{\frac{2c\,e^2(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2e(dg-ef)c-2fc\,e^2\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)^2c-4e(dg-ef)cf+e^2\left(a\,g^2+c\,f^2\right)\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)^2cf+\cdots\right.}{}}{g^5}$
gosper	$2\sqrt{gx+f}\,(35c\,e^2x^4g^4+90cde\,g^4x^3-40c\,e^2f\,g^3x^3+63a\,e^2g^4x^2+63c\,d^2g^4x^2-108cdef\,g^3x^2+48c\,e^2f^2g^2x^2+210ade\,g^4x\cdots$
trager	$2\sqrt{gx+f}\,(35c\,e^2x^4g^4+90cde\,g^4x^3-40c\,e^2f\,g^3x^3+63a\,e^2g^4x^2+63c\,d^2g^4x^2-108cdef\,g^3x^2+48c\,e^2f^2g^2x^2+210ade\,g^4x\cdots$
risch	$2\sqrt{gx+f}\,(35c\,e^2x^4g^4+90cde\,g^4x^3-40c\,e^2f\,g^3x^3+63a\,e^2g^4x^2+63c\,d^2g^4x^2-108cdef\,g^3x^2+48c\,e^2f^2g^2x^2+210ade\,g^4x\cdots$
orering	$2\sqrt{gx+f}\,(35c\,e^2x^4g^4+90cde\,g^4x^3-40c\,e^2f\,g^3x^3+63a\,e^2g^4x^2+63c\,d^2g^4x^2-108cdef\,g^3x^2+48c\,e^2f^2g^2x^2+210ade\,g^4x\cdots$

input
```
int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(g*x+f)^(1/2)*((1/5*(5/9*c*x^2+a)*x^2*e^2+2/3*(a+3/7*c*x^2)*x*d*e+d^2*(1
/5*c*x^2+a))*g^4-4/3*((2/21*c*x^3+1/5*a*x)*e^2+d*(9/35*c*x^2+a)*e+1/5*c*d^
2*x)*f*g^3+8/15*f^2*((2/7*c*x^2+a)*e^2+6/7*c*d*x*e+c*d^2)*g^2-32/35*(2/9*e
*x+d)*e*f^3*c*g+128/315*c*e^2*f^4)/g^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(35\,ce^2g^4x^4+128\,ce^2f^4-288\,cdef^3g-420\,adefg^3+315\,ad^2g^4+168\,(cd^2+ae^2)f^2g^2-10\,(4\,ce^2fg^3\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*c*e^2*g^4*x^4 + 128*c*e^2*f^4 - 288*c*d*e*f^3*g - 420*a*d*e*f*g^
3 + 315*a*d^2*g^4 + 168*(c*d^2 + a*e^2)*f^2*g^2 - 10*(4*c*e^2*f*g^3 - 9*c*
d*e*g^4)*x^3 + 3*(16*c*e^2*f^2*g^2 - 36*c*d*e*f*g^3 + 21*(c*d^2 + a*e^2)*g
^4)*x^2 - 2*(32*c*e^2*f^3*g - 72*c*d*e*f^2*g^2 - 105*a*d*e*g^4 + 42*(c*d^2
 + a*e^2)*f*g^3)*x)*sqrt(g*x + f)/g^5
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.78

$$\int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{ce^2(f+gx)^{\frac{9}{2}}}{9g^4}+\frac{(f+gx)^{\frac{7}{2}}\cdot\left(2cdeg-4ce^2f\right)}{7g^4}+\frac{(f+gx)^{\frac{5}{2}}\left(ae^2g^2+cd^2g^2-6cdefg+6ce^2f^2\right)}{5g^4}+\frac{(f+gx)^{\frac{3}{2}}\cdot\left(2adeg^3-2ae^2fg^2-2cd^2fg^2+6cdef^2g-4ce^2f^3\right)}{3g^4}+\right.}{g} \\ \frac{ad^2x+adex^2+\frac{cdex^4}{2}+\frac{ce^2x^5}{5}+\frac{x^3\left(ae^2+cd^2\right)}{3}}{\sqrt{f}} \end{cases}$$

input
```
integrate((e*x+d)**2*(c*x**2+a)/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c*e**2*(f + g*x)**(9/2)/(9*g**4) + (f + g*x)**(7/2)*(2*c*d*e
*g - 4*c*e**2*f)/(7*g**4) + (f + g*x)**(5/2)*(a*e**2*g**2 + c*d**2*g**2 -
6*c*d*e*f*g + 6*c*e**2*f**2)/(5*g**4) + (f + g*x)**(3/2)*(2*a*d*e*g**3 - 2
*a*e**2*f*g**2 - 2*c*d**2*f*g**2 + 6*c*d*e*f**2*g - 4*c*e**2*f**3)/(3*g**4
) + sqrt(f + g*x)*(a*d**2*g**4 - 2*a*d*e*f*g**3 + a*e**2*f**2*g**2 + c*d**
2*f**2*g**2 - 2*c*d*e*f**3*g + c*e**2*f**4)/g**4)/g, Ne(g, 0)), ((a*d**2*x
 + a*d*e*x**2 + c*d*e*x**4/2 + c*e**2*x**5/5 + x**3*(a*e**2 + c*d**2)/3)/s
qrt(f), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^2 (a+cx^2)}{\sqrt{f+gx}} \, dx$$
$$= \frac{2 \left(35 (gx+f)^{\frac{9}{2}} ce^2 - 90 (2 ce^2 f - cdeg)(gx+f)^{\frac{7}{2}} + 63 (6 ce^2 f^2 - 6 cdefg + (cd^2+ae^2)g^2)(gx+f)^{\frac{5}{2}} - \right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*(g*x + f)^(9/2)*c*e^2 - 90*(2*c*e^2*f - c*d*e*g)*(g*x + f)^(7/2)
 + 63*(6*c*e^2*f^2 - 6*c*d*e*f*g + (c*d^2 + a*e^2)*g^2)*(g*x + f)^(5/2) -
210*(2*c*e^2*f^3 - 3*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 + a*e^2)*f*g^2)*(g*x
 + f)^(3/2) + 315*(c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g^4 +
 (c*d^2 + a*e^2)*f^2*g^2)*sqrt(g*x + f))/g^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^2 (a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\left(315\sqrt{gx+f}ad^2 + \frac{210\left((gx+f)^{\frac{3}{2}} - 3\sqrt{gx+f}f\right)ade}{g} + \frac{21\left(3(gx+f)^{\frac{5}{2}} - 10(gx+f)^{\frac{3}{2}}f + 15\sqrt{gx+f}f^2\right)cd^2}{g^2} + \frac{21\left(3(gx+f)^{\frac{5}{2}} - 10\right.}{}\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(g*x + f)*a*d^2 + 210*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)
*a*d*e/g + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)
*f^2)*c*d^2/g^2 + 21*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g
*x + f)*f^2)*a*e^2/g^2 + 18*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35
*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*c*d*e/g^3 + (35*(g*x + f)^(9/
2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)
*f^3 + 315*sqrt(g*x + f)*f^4)*c*e^2/g^4)/g
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^2 (a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{(f+g\,x)^{5/2}\,(2\,c\,d^2\,g^2 - 12\,c\,d\,e\,f\,g + 12\,c\,e^2\,f^2 + 2\,a\,e^2\,g^2)}{5\,g^5} + \frac{2\sqrt{f+g\,x}\,(c\,f^2 + a\,g^2)\,(d\,g - e\,f)^2}{g^5} + \frac{4\,(f+g\,x)^{3/2}\,(d\,g - e\,f)\,(2\,c\,e\,f^2 - c\,d\,f\,g + a\,e\,g^2)}{3\,g^5} + \frac{2\,c\,e^2\,(f+g\,x)^{9/2}}{9\,g^5} + \frac{4\,c\,e\,(f+g\,x)^{7/2}\,(d\,g - 2\,e\,f)}{7\,g^5}$$

input
```
int(((a + c*x^2)*(d + e*x)^2)/(f + g*x)^(1/2),x)
```

output
```
((f + g*x)^(5/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 - 12*c*d*e*f*g)
)/(5*g^5) + (2*(f + g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f)^2)/g^5 + (4*(f
+ g*x)^(3/2)*(d*g - e*f)*(a*e*g^2 + 2*c*e*f^2 - c*d*f*g))/(3*g^5) + (2*c*e
^2*(f + g*x)^(9/2))/(9*g^5) + (4*c*e*(f + g*x)^(7/2)*(d*g - 2*e*f))/(7*g^5
)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)^2\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\sqrt{gx+f}\,(35c\,e^2g^4x^4+90cde\,g^4x^3-40c\,e^2f\,g^3x^3+63a\,e^2g^4x^2+63c\,d^2g^4x^2-108cdef\,g^3x^2+48c\,e^2f^2}{}$$

input
```
int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(1/2),x)
```

output
```
(2*sqrt(f + g*x)*(315*a*d**2*g**4 - 420*a*d*e*f*g**3 + 210*a*d*e*g**4*x +
168*a*e**2*f**2*g**2 - 84*a*e**2*f*g**3*x + 63*a*e**2*g**4*x**2 + 168*c*d*
*2*f**2*g**2 - 84*c*d**2*f*g**3*x + 63*c*d**2*g**4*x**2 - 288*c*d*e*f**3*g
 + 144*c*d*e*f**2*g**2*x - 108*c*d*e*f*g**3*x**2 + 90*c*d*e*g**4*x**3 + 12
8*c*e**2*f**4 - 64*c*e**2*f**3*g*x + 48*c*e**2*f**2*g**2*x**2 - 40*c*e**2*
f*g**3*x**3 + 35*c*e**2*g**4*x**4))/(315*g**5)
```

3.51 $\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{(d+ex)\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx = -\frac{2(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}}{g^4} + \frac{2\left(aeg^2+cf(3ef-2dg)\right)(f+gx)^{3/2}}{3g^4} - \frac{2c(3ef-dg)(f+gx)^{5/2}}{5g^4} + \frac{2ce(f+gx)^{7/2}}{7g^4}$$

output
```
-2*(-d*g+e*f)*(a*g^2+c*f^2)*(g*x+f)^(1/2)/g^4+2/3*(a*e*g^2+c*f*(-2*d*g+3*e
*f))*(g*x+f)^(3/2)/g^4-2/5*c*(-d*g+3*e*f)*(g*x+f)^(5/2)/g^4+2/7*c*e*(g*x+f
)^(7/2)/g^4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)\left(a+cx^2\right)}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}\left(35ag^2(-2ef+3dg+egx)+7cdg\left(8f^2-4fgx+3g^2x^2\right)-3ce\left(16f^3-8f^2gx+6fg^2x^2-5g^3\right.\right.}{105g^4}$$

input
```
Integrate[((d + e*x)*(a + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(35*a*g^2*(-2*e*f + 3*d*g + e*g*x) + 7*c*d*g*(8*f^2 - 4*f
*g*x + 3*g^2*x^2) - 3*c*e*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3)))
/(105*g^4)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(d+ex)}{\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{(ag^2+cf^2)(dg-ef)}{g^3\sqrt{f+gx}}+\frac{\sqrt{f+gx}(aeg^2+cf(3ef-2dg))}{g^3}+\frac{c(f+gx)^{3/2}(dg-3ef)}{g^3}+\frac{ce(f+gx)^{5/2}}{g^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{f+gx}(ag^2+cf^2)(ef-dg)}{g^4}+\frac{2(f+gx)^{3/2}(aeg^2+cf(3ef-2dg))}{3g^4}-\frac{2c(f+gx)^{5/2}(3ef-dg)}{5g^4}+\frac{2ce(f+gx)^{7/2}}{7g^4}$$

input
```
Int[((d + e*x)*(a + c*x^2))/Sqrt[f + g*x],x]
```

output
```
(-2*(e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^4 + (2*(a*e*g^2 + c*f*(3*
e*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^4) - (2*c*(3*e*f - d*g)*(f + g*x)^(5/2
))/(5*g^4) + (2*c*e*(f + g*x)^(7/2))/(7*g^4)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{2\sqrt{gx+f}\left(\left(\frac{x^2\left(\frac{5ex}{7}+d\right)c}{5}+a\left(\frac{ex}{3}+d\right)\right)g^3-\frac{2\left(\frac{2\left(\frac{9ex}{14}+d\right)xc}{5}+ae\right)fg^2}{3}+\frac{8\left(\frac{3ex}{7}+d\right)f^2cg}{15}-\frac{16cef^3}{35}\right)}{g^4}$
gosper	$\frac{2\sqrt{gx+f}\,(15ce\,x^3g^3+21cd\,g^3x^2-18cef\,g^2x^2+35ae\,g^3x-28cdf\,g^2x+24ce\,f^2gx+105ad\,g^3-70aef\,g^2+56cd\,f^2g-48ce}{105g^4}$
trager	$\frac{2\sqrt{gx+f}\,(15ce\,x^3g^3+21cd\,g^3x^2-18cef\,g^2x^2+35ae\,g^3x-28cdf\,g^2x+24ce\,f^2gx+105ad\,g^3-70aef\,g^2+56cd\,f^2g-48ce}{105g^4}$
risch	$\frac{2\sqrt{gx+f}\,(15ce\,x^3g^3+21cd\,g^3x^2-18cef\,g^2x^2+35ae\,g^3x-28cdf\,g^2x+24ce\,f^2gx+105ad\,g^3-70aef\,g^2+56cd\,f^2g-48ce}{105g^4}$
orering	$\frac{2\sqrt{gx+f}\,(15ce\,x^3g^3+21cd\,g^3x^2-18cef\,g^2x^2+35ae\,g^3x-28cdf\,g^2x+24ce\,f^2gx+105ad\,g^3-70aef\,g^2+56cd\,f^2g-48ce}{105g^4}$
derivativedivides	$\frac{\frac{2ce(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)c-2fce)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)cf+e\left(ag^2+cf^2\right)\right)(gx+f)^{\frac{3}{2}}}{3}+2(dg-ef)\left(ag^2+cf^2\right)\sqrt{gx+f}}{g^4}$
default	$\frac{\frac{2ce(gx+f)^{\frac{7}{2}}}{7}+\frac{2((dg-ef)c-2fce)(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(-2(dg-ef)cf+e\left(ag^2+cf^2\right)\right)(gx+f)^{\frac{3}{2}}}{3}+2(dg-ef)\left(ag^2+cf^2\right)\sqrt{gx+f}}{g^4}$

input
```
int((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(g*x+f)^(1/2)*((1/5*x^2*(5/7*e*x+d)*c+a*(1/3*e*x+d))*g^3-2/3*(2/5*(9/14*
e*x+d)*x*c+a*e)*f*g^2+8/15*(3/7*e*x+d)*f^2*c*g-16/35*c*e*f^3)/g^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\,(15\,ceg^3x^3 - 48\,cef^3 + 56\,cdf^2g - 70\,aefg^2 + 105\,adg^3 - 3\,(6\,cefg^2 - 7\,cdg^3)x^2 + (24\,cef^2g - 28\,cdfg}{105\,g^4}$$

input
```
integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*c*e*g^3*x^3 - 48*c*e*f^3 + 56*c*d*f^2*g - 70*a*e*f*g^2 + 105*a*d
*g^3 - 3*(6*c*e*f*g^2 - 7*c*d*g^3)*x^2 + (24*c*e*f^2*g - 28*c*d*f*g^2 + 35
*a*e*g^3)*x)*sqrt(g*x + f)/g^4
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{ce(f+gx)^{\frac{7}{2}}}{7g^3} + \frac{(f+gx)^{\frac{5}{2}}(cdg-3cef)}{5g^3} + \frac{(f+gx)^{\frac{3}{2}}\left(aeg^2-2cdfg+3cef^2\right)}{3g^3} + \frac{\sqrt{f+gx}\left(adg^3-aefg^2+cdf^2g-cef^3\right)}{g^3}\right)}{g} & \text{for } g \neq 0 \\ \frac{adx+\frac{aex^2}{2}+\frac{cdx^3}{3}+\frac{cex^4}{4}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

input
```
integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c*e*(f + g*x)**(7/2)/(7*g**3) + (f + g*x)**(5/2)*(c*d*g - 3*
c*e*f)/(5*g**3) + (f + g*x)**(3/2)*(a*e*g**2 - 2*c*d*f*g + 3*c*e*f**2)/(3*
g**3) + sqrt(f + g*x)*(a*d*g**3 - a*e*f*g**2 + c*d*f**2*g - c*e*f**3)/g**3
)/g, Ne(g, 0)), ((a*d*x + a*e*x**2/2 + c*d*x**3/3 + c*e*x**4/4)/sqrt(f), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(15\,(gx+f)^{\frac{7}{2}}ce - 21\,(3\,cef - cdg)(gx+f)^{\frac{5}{2}} + 35\,(3\,cef^2 - 2\,cdfg + aeg^2)(gx+f)^{\frac{3}{2}} - 105\,(cef^3 - c\right.}{105\,g^4}$$

input
```
integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/105*(15*(g*x + f)^(7/2)*c*e - 21*(3*c*e*f - c*d*g)*(g*x + f)^(5/2) + 35*
(3*c*e*f^2 - 2*c*d*f*g + a*e*g^2)*(g*x + f)^(3/2) - 105*(c*e*f^3 - c*d*f^2
*g + a*e*f*g^2 - a*d*g^3)*sqrt(g*x + f))/g^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)\,(a+cx^2)}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(105\,\sqrt{gx+f}ad + \frac{35\left((gx+f)^{\frac{3}{2}} - 3\,\sqrt{gx+f}f\right)ae}{g} + \frac{7\left(3\,(gx+f)^{\frac{5}{2}} - 10\,(gx+f)^{\frac{3}{2}}f + 15\,\sqrt{gx+f}f^2\right)cd}{g^2} + \frac{3\left(5\,(gx+f)^{\frac{7}{2}} - 21\,(gx+f)\right.}{}\right.}{105\,g}$$

input
```
integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2/105*(105*sqrt(g*x + f)*a*d + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*
e/g + 7*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*
c*d/g^2 + 3*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)
*f^2 - 35*sqrt(g*x + f)*f^3)*c*e/g^3)/g
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{(f+g\,x)^{3/2}\,(6\,c\,e\,f^2-4\,c\,d\,f\,g+2\,a\,e\,g^2)}{3\,g^4} + \frac{2\,c\,e\,(f+g\,x)^{7/2}}{7\,g^4} + \frac{2\,c\,(f+g\,x)^{5/2}\,(d\,g-3\,e\,f)}{5\,g^4} + \frac{2\,\sqrt{f+g\,x}\,(c\,f^2+a\,g^2)\,(d\,g-e\,f)}{g^4}$$

input
```
int(((a + c*x^2)*(d + e*x))/(f + g*x)^(1/2),x)
```

output
```
((f + g*x)^(3/2)*(2*a*e*g^2 + 6*c*e*f^2 - 4*c*d*f*g))/(3*g^4) + (2*c*e*(f
+ g*x)^(7/2))/(7*g^4) + (2*c*(f + g*x)^(5/2)*(d*g - 3*e*f))/(5*g^4) + (2*(
f + g*x)^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f))/g^4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{gx+f}\,(15ce\,g^3x^3+21cd\,g^3x^2-18cef\,g^2x^2+35ae\,g^3x-28cdf\,g^2x+24ce\,f^2gx+105ad\,g^3-70aef}{105g^4}$$

input
```
int((e*x+d)*(c*x^2+a)/(g*x+f)^(1/2),x)
```

output
```
(2*sqrt(f + g*x)*(105*a*d*g**3 - 70*a*e*f*g**2 + 35*a*e*g**3*x + 56*c*d*f*
*2*g - 28*c*d*f*g**2*x + 21*c*d*g**3*x**2 - 48*c*e*f**3 + 24*c*e*f**2*g*x
- 18*c*e*f*g**2*x**2 + 15*c*e*g**3*x**3))/(105*g**4)
```

3.52 $\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \frac{2(cf^2+ag^2)\sqrt{f+gx}}{g^3} - \frac{4cf(f+gx)^{3/2}}{3g^3} + \frac{2c(f+gx)^{5/2}}{5g^3}$$

output
```
2*(a*g^2+c*f^2)*(g*x+f)^(1/2)/g^3-4/3*c*f*(g*x+f)^(3/2)/g^3+2/5*c*(g*x+f)^
(5/2)/g^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}(15ag^2+c(8f^2-4fgx+3g^2x^2))}{15g^3}$$

input
```
Integrate[(a + c*x^2)/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(15*a*g^2 + c*(8*f^2 - 4*f*g*x + 3*g^2*x^2)))/(15*g^3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{ag^2+cf^2}{g^2\sqrt{f+gx}}+\frac{c(f+gx)^{3/2}}{g^2}-\frac{2cf\sqrt{f+gx}}{g^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{f+gx}(ag^2+cf^2)}{g^3}+\frac{2c(f+gx)^{5/2}}{5g^3}-\frac{4cf(f+gx)^{3/2}}{3g^3}$$

input
```
Int[(a + c*x^2)/Sqrt[f + g*x],x]
```

output
```
(2*(c*f^2 + a*g^2)*Sqrt[f + g*x])/g^3 - (4*c*f*(f + g*x)^(3/2))/(3*g^3) +
(2*c*(f + g*x)^(5/2))/(5*g^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$\frac{2\left(3\left(c\,x^{2}+5a\right)g^{2}-4cfxg+8c\,f^{2}\right)\sqrt{gx+f}}{15g^{3}}$	40
gosper	$\frac{2\sqrt{gx+f}\,\left(3c\,x^{2}g^{2}-4cfxg+15a\,g^{2}+8c\,f^{2}\right)}{15g^{3}}$	41
trager	$\frac{2\sqrt{gx+f}\,\left(3c\,x^{2}g^{2}-4cfxg+15a\,g^{2}+8c\,f^{2}\right)}{15g^{3}}$	41
risch	$\frac{2\sqrt{gx+f}\,\left(3c\,x^{2}g^{2}-4cfxg+15a\,g^{2}+8c\,f^{2}\right)}{15g^{3}}$	41
orering	$\frac{2\sqrt{gx+f}\,\left(3c\,x^{2}g^{2}-4cfxg+15a\,g^{2}+8c\,f^{2}\right)}{15g^{3}}$	41
derivativedivides	$\frac{\frac{2c(gx+f)^{\frac{5}{2}}}{5}-\frac{4cf(gx+f)^{\frac{3}{2}}}{3}+2a\,g^{2}\sqrt{gx+f}+2c\,f^{2}\sqrt{gx+f}}{g^{3}}$	52
default	$\frac{\frac{2c(gx+f)^{\frac{5}{2}}}{5}-\frac{4cf(gx+f)^{\frac{3}{2}}}{3}+2a\,g^{2}\sqrt{gx+f}+2c\,f^{2}\sqrt{gx+f}}{g^{3}}$	52

input
```
int((c*x^2+a)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*(c*x^2+5*a)*g^2-4*c*f*x*g+8*c*f^2)*(g*x+f)^(1/2)/g^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\,(3\,cg^2x^2-4\,cfgx+8\,cf^2+15\,ag^2)\sqrt{gx+f}}{15\,g^3}$$

input
```
integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*c*g^2*x^2 - 4*c*f*g*x + 8*c*f^2 + 15*a*g^2)*sqrt(g*x + f)/g^3
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \begin{cases} \frac{2a\sqrt{f+gx}+\frac{2c\left(f^2\sqrt{f+gx}-\frac{2f(f+gx)^{\frac{3}{2}}}{3}+\frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}}{g} & \text{for } g\neq 0 \\ \frac{ax+\frac{cx^3}{3}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)/(g*x+f)**(1/2),x)
```

output
```
Piecewise(((2*a*sqrt(f + g*x) + 2*c*(f**2*sqrt(f + g*x) - 2*f*(f + g*x)**(
3/2)/3 + (f + g*x)**(5/2)/5)/g**2)/g, Ne(g, 0)), ((a*x + c*x**3/3)/sqrt(f)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\left(15\sqrt{gx+f}a+\frac{\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)c}{g^2}\right)}{15g}$$

input
```
integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(15*sqrt(g*x + f)*a + (3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*
sqrt(g*x + f)*f^2)*c/g^2)/g
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\left(15\sqrt{gx+f}a+\frac{\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\,\sqrt{gx+f}f^2\right)c}{g^2}\right)}{15\,g}$$

input
```
integrate((c*x^2+a)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2/15*(15*sqrt(g*x + f)*a + (3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*
sqrt(g*x + f)*f^2)*c/g^2)/g
```

Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+g\,x}\,\left(3\,c\,{(f+g\,x)}^2+15\,a\,g^2+15\,c\,f^2-10\,c\,f\,(f+g\,x)\right)}{15\,g^3}$$

input
```
int((a + c*x^2)/(f + g*x)^(1/2),x)
```

output
```
(2*(f + g*x)^(1/2)*(3*c*(f + g*x)^2 + 15*a*g^2 + 15*c*f^2 - 10*c*f*(f + g*
x)))/(15*g^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{a+cx^2}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{gx+f}\,(3c\,g^2x^2-4cfgx+15a\,g^2+8c\,f^2)}{15g^3}$$

input
```
int((c*x^2+a)/(g*x+f)^(1/2),x)
```

output
```
(2*sqrt(f + g*x)*(15*a*g**2 + 8*c*f**2 - 4*c*f*g*x + 3*c*g**2*x**2))/(15*g
**3)
```

3.53 $\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = -\frac{2c(ef+dg)\sqrt{f+gx}}{e^2g^2}+\frac{2c(f+gx)^{3/2}}{3eg^2} - \frac{2(cd^2+ae^2)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

output
```
-2*c*(d*g+e*f)*(g*x+f)^(1/2)/e^2/g^2+2/3*c*(g*x+f)^(3/2)/e/g^2-2*(a*e^2+c*
d^2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(5/2)/(-d*g+e*f)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2c\sqrt{f+gx}(-2ef-3dg+egx)}{3e^2g^2}+\frac{2(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{5/2}\sqrt{-ef+dg}}$$

input
```
Integrate[(a + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

output
```
(2*c*Sqrt[f + g*x]*(-2*e*f - 3*d*g + e*g*x))/(3*e^2*g^2) + (2*(c*d^2 + a*e
^2)*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(5/2)*Sqrt[-(e*
f) + d*g])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {649, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \; 649$$

$$\frac{2\int -\frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\int \frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \; 1467$$

$$-\frac{2\int \left(\frac{c(ef+dg)}{e^2}-\frac{c(f+gx)}{e}+\frac{cd^2g^2+ae^2g^2}{e^2(ef-dg-e(f+gx))}\right)d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(-\frac{g^2(ae^2+cd^2)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}-\frac{c\sqrt{f+gx}(dg+ef)}{e^2}+\frac{c(f+gx)^{3/2}}{3e}\right)}{g^2}$$

input
```
Int[(a + c*x^2)/((d + e*x)*Sqrt[f + g*x]),x]
```

output
```
(2*(-((c*(e*f + d*g)*Sqrt[f + g*x])/e^2) + (c*(f + g*x)^(3/2))/(3*e) - ((c
*d^2 + a*e^2)*g^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/
2)*Sqrt[e*f - d*g])))/g^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{2c(-egx+3dg+2ef)\sqrt{gx+f}}{3g^2e^2}+\frac{2(a\,e^2+c\,d^2)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e^2\sqrt{(dg-ef)e}}$	82
pseudoelliptic	$\frac{-\frac{2c\sqrt{gx+f}\,(-egx+3dg+2ef)}{3}+\frac{2g^2\left(a\,e^2+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{\sqrt{(dg-ef)e}}}{e^2g^2}$	83
derivativedivides	$\frac{-\frac{2c\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+dg\sqrt{gx+f}+ef\sqrt{gx+f}\right)}{e^2}+\frac{2g^2\left(a\,e^2+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e^2\sqrt{(dg-ef)e}}}{g^2}$	96
default	$\frac{-\frac{2c\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+dg\sqrt{gx+f}+ef\sqrt{gx+f}\right)}{e^2}+\frac{2g^2\left(a\,e^2+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e^2\sqrt{(dg-ef)e}}}{g^2}$	96

input
```
int((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*c*(-e*g*x+3*d*g+2*e*f)*(g*x+f)^(1/2)/g^2/e^2+2*(a*e^2+c*d^2)/e^2/((d*
g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.86

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx$$
$$= \left[\frac{3\left(cd^2+ae^2\right)\sqrt{e^2f-deg}g^2\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\left(2\,ce^3f^2+cde^2fg-3\,cd^2eg^2-(ce^3f\right.}{3\left(e^4fg^2-de^3g^3\right)}\right.$$

input
```
integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(c*d^2 + a*e^2)*sqrt(e^2*f - d*e*g)*g^2*log((e*g*x + 2*e*f - d*g -
 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*e^3*f^2 + c*d*e^
2*f*g - 3*c*d^2*e*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f
*g^2 - d*e^3*g^3), 2/3*(3*(c*d^2 + a*e^2)*sqrt(-e^2*f + d*e*g)*g^2*arctan(
sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (2*c*e^3*f^2 + c*d*e^2
*f*g - 3*c*d^2*e*g^2 - (c*e^3*f*g - c*d*e^2*g^2)*x)*sqrt(g*x + f))/(e^4*f*
g^2 - d*e^3*g^3)]
```

Sympy [A] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.38

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \begin{cases} \frac{2\left(\frac{c(f+gx)^{\frac{3}{2}}}{3eg}+\frac{\sqrt{f+gx}(-cdg-cef)}{e^2g}+\frac{g(ae^2+cd^2)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^3\sqrt{\frac{dg-ef}{e}}}\right)}{g} & \text{for } g\neq 0 \\ \frac{-\frac{cdx}{e^2}+\frac{cx^2}{2e}+\frac{(ae^2+cd^2)\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^2}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)/(e*x+d)/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c*(f + g*x)**(3/2)/(3*e*g) + sqrt(f + g*x)*(-c*d*g - c*e*f)/
(e**2*g) + g*(a*e**2 + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g - e*f)/e))/(e*
*3*sqrt((d*g - e*f)/e)))/g, Ne(g, 0)), ((-c*d*x/e**2 + c*x**2/(2*e) + (a*e
**2 + c*d**2)*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**2)/sqr
t(f), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.05

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\,(cd^2+ae^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{\sqrt{-e^2f+deg}e^2} + \frac{2\left((gx+f)^{\frac{3}{2}}ce^2g^4 - 3\sqrt{gx+f}ce^2fg^4 - 3\sqrt{gx+f}cdeg^5\right)}{3\,e^3g^6}$$

input
```
integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2*(c*d^2 + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e^2*
f + d*e*g)*e^2) + 2/3*((g*x + f)^(3/2)*c*e^2*g^4 - 3*sqrt(g*x + f)*c*e^2*f
*g^4 - 3*sqrt(g*x + f)*c*d*e*g^5)/(e^3*g^6)
```

Mupad [B] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(c\,d^2+a\,e^2\right)}{e^{5/2}\,\sqrt{d\,g-e\,f}} - \sqrt{f+g\,x}\,\left(\frac{2\,c\,\left(d\,g^3-e\,f\,g^2\right)}{e^2\,g^4}+\frac{4\,c\,f}{e\,g^2}\right)+\frac{2\,c\,{\left(f+g\,x\right)}^{3/2}}{3\,e\,g^2}$$

input
```
int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)),x)
```

output
```
(2*atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2 + c*d^2))/(e^(
5/2)*(d*g - e*f)^(1/2)) - (f + g*x)^(1/2)*((2*c*(d*g^3 - e*f*g^2))/(e^2*g^
4) + (4*c*f)/(e*g^2)) + (2*c*(f + g*x)^(3/2))/(3*e*g^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.76

$$\int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\sqrt{e}\,\sqrt{dg-ef}\;atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-ef}}\right)a\,e^2g^2+2\sqrt{e}\,\sqrt{dg-ef}\;atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-ef}}\right)c\,d^2g^2-2\sqrt{gx+f}\,c\,d^2e\,g^2+2}{e^3g^2\,(dg-ef)}$$

input
```
int((c*x^2+a)/(e*x+d)/(g*x+f)^(1/2),x)
```

output
```
(2*(3*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e
*f)))*a*e**2*g**2 + 3*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt
(e)*sqrt(d*g - e*f)))*c*d**2*g**2 - 3*sqrt(f + g*x)*c*d**2*e*g**2 + sqrt(f
 + g*x)*c*d*e**2*f*g + sqrt(f + g*x)*c*d*e**2*g**2*x + 2*sqrt(f + g*x)*c*e
**3*f**2 - sqrt(f + g*x)*c*e**3*f*g*x))/(3*e**3*g**2*(d*g - e*f))
```

3.54 $\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx=\frac{2c\sqrt{f+gx}}{e^2g}-\frac{(cd^2+ae^2)\sqrt{f+gx}}{e^2(ef-dg)(d+ex)}+\frac{(ae^2g+cd(4ef-3dg))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}$$

output
```
2*c*(g*x+f)^(1/2)/e^2/g-(a*e^2+c*d^2)*(g*x+f)^(1/2)/e^2/(-d*g+e*f)/(e*x+d)
+(a*e^2*g+c*d*(-3*d*g+4*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/
2))/e^(5/2)/(-d*g+e*f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx=\frac{\sqrt{f+gx}(-ae^2g+c(-3d^2g+2e^2fx+2de(f-gx)))}{e^2g(ef-dg)(d+ex)}+\frac{(ae^2g+cd(4ef-3dg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{5/2}(-ef+dg)^{3/2}}$$

input
```
Integrate[(a + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

output
```
(Sqrt[f + g*x]*(-(a*e^2*g) + c*(-3*d^2*g + 2*e^2*f*x + 2*d*e*(f - g*x))))/
(e^2*g*(e*f - d*g)*(d + e*x)) + ((a*e^2*g + c*d*(4*e*f - 3*d*g))*ArcTan[(S
qrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(5/2)*(-(e*f) + d*g)^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {649, 1471, 25, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{649}$$

$$\frac{2\int \frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{g}$$

$$\Big\downarrow \text{1471}$$

$$\frac{2\left(\frac{g^2\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{2(ef-dg)(-dg-e(f+gx)+ef)}-\frac{\int -\frac{2cf^2+ag^2-\frac{cd^2g^2}{e^2}-2c\left(f-\frac{dg}{e}\right)(f+gx)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{2(ef-dg)}\right)}{g}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(\frac{\int \frac{2cf^2+ag^2-\frac{cd^2g^2}{e^2}-2c\left(f-\frac{dg}{e}\right)(f+gx)}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{2(ef-dg)}+\frac{g^2\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{2(ef-dg)(-dg-e(f+gx)+ef)}\right)}{g}$$

$$\Big\downarrow \text{299}$$

$$\frac{2\left(\frac{\frac{g\left(ae^2g+cd(4ef-3dg)\right)\int \frac{1}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{e^2}+\frac{2c\sqrt{f+gx}(ef-dg)}{e^2}}{2(ef-dg)}+\frac{g^2\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{2(ef-dg)(-dg-e(f+gx)+ef)}\right)}{g}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\left(\frac{\frac{g\left(ae^2g+cd(4ef-3dg)\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}+\frac{2c\sqrt{f+gx}(ef-dg)}{e^2}}{2(ef-dg)}+\frac{g^2\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{2(ef-dg)(-dg-e(f+gx)+ef)}\right)}{g}$$

input
```
Int[(a + c*x^2)/((d + e*x)^2*Sqrt[f + g*x]),x]
```

output
```
(2*(((a + (c*d^2)/e^2)*g^2*Sqrt[f + g*x])/(2*(e*f - d*g)*(e*f - d*g - e*(f
 + g*x))) + ((2*c*(e*f - d*g)*Sqrt[f + g*x])/e^2 + (g*(a*e^2*g + c*d*(4*e*
f - 3*d*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*Sqr
t[e*f - d*g]))/(2*(e*f - d*g))))/g
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10

method	result
risch	$\frac{2c\sqrt{gx+f}}{e^{2}g}-\frac{-\frac{g\left(a\,e^{2}+c\,d^{2}\right)\sqrt{gx+f}}{(dg-ef)(e(gx+f)+dg-ef)}-\frac{\left(a\,e^{2}g-3c\,d^{2}g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}}}{e^{2}}$
derivativedivides	$\frac{\frac{2c\sqrt{gx+f}}{e^{2}}+\frac{2g\left(\frac{g\left(a\,e^{2}+c\,d^{2}\right)\sqrt{gx+f}}{2(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a\,e^{2}g-3c\,d^{2}g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2(dg-ef)\sqrt{(dg-ef)e}}\right)}{e^{2}}}{g}$
default	$\frac{\frac{2c\sqrt{gx+f}}{e^{2}}+\frac{2g\left(\frac{g\left(a\,e^{2}+c\,d^{2}\right)\sqrt{gx+f}}{2(dg-ef)(e(gx+f)+dg-ef)}+\frac{\left(a\,e^{2}g-3c\,d^{2}g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2(dg-ef)\sqrt{(dg-ef)e}}\right)}{e^{2}}}{g}$
pseudoelliptic	$\frac{g(ex+d)\left(a\,e^{2}g-3c\,d^{2}g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)+\sqrt{gx+f}\,\sqrt{(dg-ef)e}\,\left((-2cfx+ag)e^{2}-2cd(-gx+f)e+3c\,d^{2}g\right)}{\sqrt{(dg-ef)e}\,g\,e^{2}(dg-ef)(ex+d)}$

input
```
int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*c*(g*x+f)^(1/2)/e^2/g-1/e^2*(-g*(a*e^2+c*d^2)/(d*g-e*f)*(g*x+f)^(1/2)/(e
*(g*x+f)+d*g-e*f)-(a*e^2*g-3*c*d^2*g+4*c*d*e*f)/(d*g-e*f)/((d*g-e*f)*e)^(1
/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(112) = 224$.

Time = 0.10 (sec) , antiderivative size = 539, normalized size of antiderivative = 4.28

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$$
$$= \left[-\frac{(4\,cd^2efg-(3\,cd^3-ade^2)g^2+(4\,cde^2fg-(3\,cd^2e-ae^3)g^2)x)\sqrt{e^2f-deg}\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{e^2f-de}}{ex+d}\right.}{2\,(de^5f^2g-2\,d^2e^4fg^2+d^3e^3}\right.$$
$$-\frac{(4\,cd^2efg-(3\,cd^3-ade^2)g^2+(4\,cde^2fg-(3\,cd^2e-ae^3)g^2)x)\sqrt{-e^2f+deg}\arctan\left(\frac{\sqrt{-e^2f+deg}\sqrt{gx+f}}{egx+ef}\right.}{de^5f^2g-2\,d^2e^4fg^2+d^3e^3g^3+}$$

input
```
integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((4*c*d^2*e*f*g - (3*c*d^3 - a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (3*c*d^
2*e - a*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g - 2*sqrt
(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^2 - (5*c*d^2*e^
2 + a*e^4)*f*g + (3*c*d^3*e + a*d*e^3)*g^2 + 2*(c*e^4*f^2 - 2*c*d*e^3*f*g
+ c*d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^2*g - 2*d^2*e^4*f*g^2 + d^3*e^
3*g^3 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4*g^3)*x), -((4*c*d^2*e*f*g - (
3*c*d^3 - a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (3*c*d^2*e - a*e^3)*g^2)*x)*sqrt
(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f))
- (2*c*d*e^3*f^2 - (5*c*d^2*e^2 + a*e^4)*f*g + (3*c*d^3*e + a*d*e^3)*g^2 +
 2*(c*e^4*f^2 - 2*c*d*e^3*f*g + c*d^2*e^2*g^2)*x)*sqrt(g*x + f))/(d*e^5*f^
2*g - 2*d^2*e^4*f*g^2 + d^3*e^3*g^3 + (e^6*f^2*g - 2*d*e^5*f*g^2 + d^2*e^4
*g^3)*x)]
```

Sympy [F]

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+a)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

output
```
Integral((a + c*x**2)/((d + e*x)**2*sqrt(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.20

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = -\frac{(4\,cdef-3\,cd^2g+ae^2g)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^3f-de^2g)\sqrt{-e^2f+deg}} - \frac{\sqrt{gx+f}cd^2g+\sqrt{gx+f}ae^2g}{(e^3f-de^2g)((gx+f)e-ef+dg)} + \frac{2\sqrt{gx+f}c}{e^2g}$$

input
```
integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
-(4*c*d*e*f - 3*c*d^2*g + a*e^2*g)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*
e*g))/((e^3*f - d*e^2*g)*sqrt(-e^2*f + d*e*g)) - (sqrt(g*x + f)*c*d^2*g +
sqrt(g*x + f)*a*e^2*g)/((e^3*f - d*e^2*g)*((g*x + f)*e - e*f + d*g)) + 2*s
qrt(g*x + f)*c/(e^2*g)
```

Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(-3\,c\,g\,d^2+4\,c\,f\,d\,e+a\,g\,e^2\right)}{e^{5/2}\,{\left(d\,g-e\,f\right)}^{3/2}} + \frac{\sqrt{f+g\,x}\,\left(c\,g\,d^2+a\,g\,e^2\right)}{\left(d\,g-e\,f\right)\,\left(e^3\,\left(f+g\,x\right)-e^3\,f+d\,e^2\,g\right)} + \frac{2\,c\,\sqrt{f+g\,x}}{e^2\,g}$$

input
```
int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

output
```
(atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(a*e^2*g - 3*c*d^2*g +
4*c*d*e*f))/(e^(5/2)*(d*g - e*f)^(3/2)) + ((f + g*x)^(1/2)*(a*e^2*g + c*d^
2*g))/((d*g - e*f)*(e^3*(f + g*x) - e^3*f + d*e^2*g)) + (2*c*(f + g*x)^(1/
2))/(e^2*g)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.62

$$\int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{\sqrt{e}\,\sqrt{dg-ef}\,atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-ef}}\right)ad\,e^2g^2+\sqrt{e}\,\sqrt{dg-ef}\,atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-ef}}\right)a\,e^3g^2x-3\sqrt{e}\,\sqrt{dg-ef}\,atan\left(\right.}{}$$

input
```
int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x)
```

output
```
(sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))
*a*d*e**2*g**2 + sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*s
qrt(d*g - e*f)))*a*e**3*g**2*x - 3*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f +
g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**3*g**2 + 4*sqrt(e)*sqrt(d*g - e*f)
*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**2*e*f*g - 3*sqrt(e
)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**2
*e*g**2*x + 4*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt
(d*g - e*f)))*c*d*e**2*f*g*x + sqrt(f + g*x)*a*d*e**3*g**2 - sqrt(f + g*x)
*a*e**4*f*g + 3*sqrt(f + g*x)*c*d**3*e*g**2 - 5*sqrt(f + g*x)*c*d**2*e**2*
f*g + 2*sqrt(f + g*x)*c*d**2*e**2*g**2*x + 2*sqrt(f + g*x)*c*d*e**3*f**2 -
 4*sqrt(f + g*x)*c*d*e**3*f*g*x + 2*sqrt(f + g*x)*c*e**4*f**2*x)/(e**3*g*(
d**3*g**2 - 2*d**2*e*f*g + d**2*e*g**2*x + d*e**2*f**2 - 2*d*e**2*f*g*x +
e**3*f**2*x))
```

3.55 $\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 182

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = -\frac{(cd^2+ae^2)\sqrt{f+gx}}{2e^2(ef-dg)(d+ex)^2}+\frac{(3ae^2g+cd(8ef-5dg))\sqrt{f+gx}}{4e^2(ef-dg)^2(d+ex)} - \frac{(3ae^2g^2+c(8e^2f^2-8defg+3d^2g^2))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}}$$

output
```
-1/2*(a*e^2+c*d^2)*(g*x+f)^(1/2)/e^2/(-d*g+e*f)/(e*x+d)^2+1/4*(3*a*e^2*g+c
*d*(-5*d*g+8*e*f))*(g*x+f)^(1/2)/e^2/(-d*g+e*f)^2/(e*x+d)-1/4*(3*a*e^2*g^2
+c*(3*d^2*g^2-8*d*e*f*g+8*e^2*f^2))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*
f)^(1/2))/e^(5/2)/(-d*g+e*f)^(5/2)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.91

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{\frac{\sqrt{e}\sqrt{f+gx}(ae^2(-2ef+5dg+3egx)+cd(-3d^2g+8e^2fx+de(6f-5gx)))}{(ef-dg)^2(d+ex)^2}+\frac{(3ae^2g^2+c(8e^2f^2-8defg+3d^2g^2))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{(-ef+dg)^{5/2}}}{4e^{5/2}}$$

input
```
Integrate[(a + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

output
```
((Sqrt[e]*Sqrt[f + g*x]*(a*e^2*(-2*e*f + 5*d*g + 3*e*g*x) + c*d*(-3*d^2*g
+ 8*e^2*f*x + d*e*(6*f - 5*g*x))))/((e*f - d*g)^2*(d + e*x)^2) + ((3*a*e^2
*g^2 + c*(8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2))*ArcTan[(Sqrt[e]*Sqrt[f + g*x
])/Sqrt[-(e*f) + d*g]])/(-(e*f) + d*g)^(5/2))/(4*e^(5/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {649, 25, 1471, 25, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{649}$$

$$2\int -\frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$

$$\Big\downarrow \text{1471}$$

$$2\left(\frac{\int -\frac{4cf^2+3ag^2-\frac{cd^2g^2}{e^2}-4c\left(f-\frac{dg}{e}\right)(f+gx)}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4(ef-dg)}-\frac{g^2\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{4(ef-dg)(-dg-e(f+gx)+ef)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(-\frac{\int \frac{4cf^2+3ag^2-\frac{cd^2g^2}{e^2}-4c\left(f-\frac{dg}{e}\right)(f+gx)}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4(ef-dg)}-\frac{g^2\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{4(ef-dg)(-dg-e(f+gx)+ef)^2}\right)$$

$$\Big\downarrow \text{298}$$

$$2\left(-\frac{\frac{\left(3ae^2g^2+c\left(3d^2g^2-8defg+8e^2f^2\right)\right)\int \frac{1}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{2e^2(ef-dg)}+\frac{g\sqrt{f+gx}\left(3ae^2g+cd(8ef-5dg)\right)}{2e^2(ef-dg)(-dg-e(f+gx)+ef)}}{4(ef-dg)}-\frac{g^2\sqrt{f+gx}\left(a+\frac{c}{e}\right.}{4(ef-dg)(-dg-e(f+}\right.$$

$$\Big\downarrow \text{221}$$

$$2\left(-\frac{\frac{\left(3ae^2g^2+c\left(3d^2g^2-8defg+8e^2f^2\right)\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{2e^{5/2}(ef-dg)^{3/2}}+\frac{g\sqrt{f+gx}\left(3ae^2g+cd(8ef-5dg)\right)}{2e^2(ef-dg)(-dg-e(f+gx)+ef)}}{4(ef-dg)}-\frac{g^2\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{4(ef-dg)(-dg-e(f+gx)}\right.$$

input
```
Int[(a + c*x^2)/((d + e*x)^3*Sqrt[f + g*x]),x]
```

output
```
2*(-1/4*((a + (c*d^2)/e^2)*g^2*Sqrt[f + g*x])/((e*f - d*g)*(e*f - d*g - e*
(f + g*x))^2) - ((g*(3*a*e^2*g + c*d*(8*e*f - 5*d*g))*Sqrt[f + g*x])/(2*e^
2*(e*f - d*g)*(e*f - d*g - e*(f + g*x))) + ((3*a*e^2*g^2 + c*(8*e^2*f^2 -
8*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/
(2*e^(5/2)*(e*f - d*g)^(3/2)))/(4*(e*f - d*g)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeqQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$\frac{\frac{3\left(\left(a\,g^2+\frac{8c\,f^2}{3}\right)e^2-\frac{8cdefg}{3}+c\,d^2g^2\right)(ex+d)^2\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{4}+\frac{5\left(-\frac{2\left(-\frac{3gx}{2}+f\right)a\,e^3}{5}+d\left(\frac{8cfx}{5}+ag\right)e^2+\frac{6\left(-\frac{5gx}{6}+f\right)c\,d \cdots}{5}\cdots\right)}{4}}{(dg-ef)^2\sqrt{(dg-ef)e}\,(ex+d)^2e^2}$
derivativedivides	$\frac{\frac{g\left(3a\,e^2g-5c\,d^2g+8cdef\right)(gx+f)^{\frac{3}{2}}}{4e\left(d^2g^2-2defg+e^2f^2\right)}+\frac{\left(5a\,e^2g-3c\,d^2g+8cdef\right)g\sqrt{gx+f}}{4e^2(dg-ef)}}{(e(gx+f)+dg-ef)^2}+\frac{(3a\,e^2g^2+3c\,d^2g^2-8cdefg+8c\,e^2f^2)\arctan\left(\frac{e \cdots}{\sqrt{\cdots}}\right)}{4(d^2g^2-2defg+e^2f^2)e^2\sqrt{(dg-ef)e}}$
default	$\frac{\frac{g\left(3a\,e^2g-5c\,d^2g+8cdef\right)(gx+f)^{\frac{3}{2}}}{4e\left(d^2g^2-2defg+e^2f^2\right)}+\frac{\left(5a\,e^2g-3c\,d^2g+8cdef\right)g\sqrt{gx+f}}{4e^2(dg-ef)}}{(e(gx+f)+dg-ef)^2}+\frac{(3a\,e^2g^2+3c\,d^2g^2-8cdefg+8c\,e^2f^2)\arctan\left(\frac{e \cdots}{\sqrt{\cdots}}\right)}{4(d^2g^2-2defg+e^2f^2)e^2\sqrt{(dg-ef)e}}$

input
```
int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
5/4*(3/5*((a*g^2+8/3*c*f^2)*e^2-8/3*c*d*e*f*g+c*d^2*g^2)*(e*x+d)^2*arctan(
e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))+(-2/5*(-3/2*g*x+f)*a*e^3+d*(8/5*c*f*x
+a*g)*e^2+6/5*(-5/6*g*x+f)*c*d^2*e-3/5*c*d^3*g)*(g*x+f)^(1/2)*((d*g-e*f)*e
)^(1/2))/(d*g-e*f)^2/((d*g-e*f)*e)^(1/2)/(e*x+d)^2/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(162) = 324$.

Time = 0.11 (sec) , antiderivative size = 896, normalized size of antiderivative = 4.92

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((8*c*d^2*e^2*f^2 - 8*c*d^3*e*f*g + 3*(c*d^4 + a*d^2*e^2)*g^2 + (8*c*
e^4*f^2 - 8*c*d*e^3*f*g + 3*(c*d^2*e^2 + a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^
2 - 8*c*d^2*e^2*f*g + 3*(c*d^3*e + a*d*e^3)*g^2)*x)*sqrt(e^2*f - d*e*g)*lo
g((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) +
 2*(2*(3*c*d^2*e^3 - a*e^5)*f^2 - (9*c*d^3*e^2 - 7*a*d*e^4)*f*g + (3*c*d^4
*e - 5*a*d^2*e^3)*g^2 + (8*c*d*e^4*f^2 - (13*c*d^2*e^3 - 3*a*e^5)*f*g + (5
*c*d^3*e^2 - 3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*f^
2*g + 3*d^4*e^4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e^6
*f*g^2 - d^3*e^5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f*g
^2 - d^4*e^4*g^3)*x), 1/4*((8*c*d^2*e^2*f^2 - 8*c*d^3*e*f*g + 3*(c*d^4 + a
*d^2*e^2)*g^2 + (8*c*e^4*f^2 - 8*c*d*e^3*f*g + 3*(c*d^2*e^2 + a*e^4)*g^2)*
x^2 + 2*(8*c*d*e^3*f^2 - 8*c*d^2*e^2*f*g + 3*(c*d^3*e + a*d*e^3)*g^2)*x)*s
qrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f
)) + (2*(3*c*d^2*e^3 - a*e^5)*f^2 - (9*c*d^3*e^2 - 7*a*d*e^4)*f*g + (3*c*d
^4*e - 5*a*d^2*e^3)*g^2 + (8*c*d*e^4*f^2 - (13*c*d^2*e^3 - 3*a*e^5)*f*g +
(5*c*d^3*e^2 - 3*a*d*e^4)*g^2)*x)*sqrt(g*x + f))/(d^2*e^6*f^3 - 3*d^3*e^5*
f^2*g + 3*d^4*e^4*f*g^2 - d^5*e^3*g^3 + (e^8*f^3 - 3*d*e^7*f^2*g + 3*d^2*e
^6*f*g^2 - d^3*e^5*g^3)*x^2 + 2*(d*e^7*f^3 - 3*d^2*e^6*f^2*g + 3*d^3*e^5*f
*g^2 - d^4*e^4*g^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + cx^2}{(d + ex)^3\sqrt{f + gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.57

$$\int \frac{a + cx^2}{(d + ex)^3\sqrt{f + gx}}\,dx = \frac{(8\,ce^2f^2 - 8\,cdefg + 3\,cd^2g^2 + 3\,ae^2g^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{4\,(e^4f^2 - 2\,de^3fg + d^2e^2g^2)\sqrt{-e^2f + deg}} + \frac{8\,(gx + f)^{\frac{3}{2}}cde^2fg - 8\,\sqrt{gx + f}cde^2f^2g - 5\,(gx + f)^{\frac{3}{2}}cd^2eg^2 + 3\,(gx + f)^{\frac{3}{2}}ae^3g^2 + 11\,\sqrt{gx + f}cd^2ef}{4\,(e^4f^2 - 2\,de^3fg + d^2e^2g^2)((gx + f)e - ef +}$$

input
```
integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
1/4*(8*c*e^2*f^2 - 8*c*d*e*f*g + 3*c*d^2*g^2 + 3*a*e^2*g^2)*arctan(sqrt(g*
x + f)*e/sqrt(-e^2*f + d*e*g))/((e^4*f^2 - 2*d*e^3*f*g + d^2*e^2*g^2)*sqrt
(-e^2*f + d*e*g)) + 1/4*(8*(g*x + f)^(3/2)*c*d*e^2*f*g - 8*sqrt(g*x + f)*c
*d*e^2*f^2*g - 5*(g*x + f)^(3/2)*c*d^2*e*g^2 + 3*(g*x + f)^(3/2)*a*e^3*g^2
 + 11*sqrt(g*x + f)*c*d^2*e*f*g^2 - 5*sqrt(g*x + f)*a*e^3*f*g^2 - 3*sqrt(g
*x + f)*c*d^3*g^3 + 5*sqrt(g*x + f)*a*d*e^2*g^3)/((e^4*f^2 - 2*d*e^3*f*g +
 d^2*e^2*g^2)*((g*x + f)*e - e*f + d*g)^2)
```

Mupad [B] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.23

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{\frac{\sqrt{f+g\,x}\,\left(-3\,c\,d^2\,g^2+8\,c\,f\,d\,e\,g+5\,a\,e^2\,g^2\right)}{4\,e^2\,\left(d\,g-e\,f\right)}+\frac{{\left(f+g\,x\right)}^{3/2}\,\left(-5\,c\,d^2\,g^2+8\,c\,f\,d\,e\,g+3\,a\,e^2\,g^2\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^2-\left(f+g\,x\right)\,\left(2\,e^2\,f-2\,d\,e\,g\right)+d^2\,g^2+e^2\,f^2-2\,d\,e\,f\,g} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(3\,c\,d^2\,g^2-8\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2+3\,a\,e^2\,g^2\right)}{4\,e^{5/2}\,{\left(d\,g-e\,f\right)}^{5/2}}$$

input
```
int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

output
```
(((f + g*x)^(1/2)*(5*a*e^2*g^2 - 3*c*d^2*g^2 + 8*c*d*e*f*g))/(4*e^2*(d*g -
 e*f)) + ((f + g*x)^(3/2)*(3*a*e^2*g^2 - 5*c*d^2*g^2 + 8*c*d*e*f*g))/(4*e*
(d*g - e*f)^2))/(e^2*(f + g*x)^2 - (f + g*x)*(2*e^2*f - 2*d*e*g) + d^2*g^2
 + e^2*f^2 - 2*d*e*f*g) + (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2
))*(3*a*e^2*g^2 + 3*c*d^2*g^2 + 8*c*e^2*f^2 - 8*c*d*e*f*g))/(4*e^(5/2)*(d*
g - e*f)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 890, normalized size of antiderivative = 4.89

$$\int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(1/2),x)
```

output

```
(3*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)
))*a*d**2*e**2*g**2 + 6*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sq
rt(e)*sqrt(d*g - e*f)))*a*d*e**3*g**2*x + 3*sqrt(e)*sqrt(d*g - e*f)*atan((
sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*e**4*g**2*x**2 + 3*sqrt(e)*s
qrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**4*g*
*2 - 8*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g -
e*f)))*c*d**3*e*f*g + 6*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sq
rt(e)*sqrt(d*g - e*f)))*c*d**3*e*g**2*x + 8*sqrt(e)*sqrt(d*g - e*f)*atan((
sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**2*e**2*f**2 - 16*sqrt(e)*
sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**2*e
**2*f*g*x + 3*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt
(d*g - e*f)))*c*d**2*e**2*g**2*x**2 + 16*sqrt(e)*sqrt(d*g - e*f)*atan((sqr
t(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d*e**3*f**2*x - 8*sqrt(e)*sqrt(
d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d*e**3*f*g*
x**2 + 8*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g
- e*f)))*c*e**4*f**2*x**2 + 5*sqrt(f + g*x)*a*d**2*e**3*g**2 - 7*sqrt(f +
g*x)*a*d*e**4*f*g + 3*sqrt(f + g*x)*a*d*e**4*g**2*x + 2*sqrt(f + g*x)*a*e*
*5*f**2 - 3*sqrt(f + g*x)*a*e**5*f*g*x - 3*sqrt(f + g*x)*c*d**4*e*g**2 + 9
*sqrt(f + g*x)*c*d**3*e**2*f*g - 5*sqrt(f + g*x)*c*d**3*e**2*g**2*x - 6*sq
rt(f + g*x)*c*d**2*e**3*f**2 + 13*sqrt(f + g*x)*c*d**2*e**3*f*g*x - 8*s...
```

3.56 $\int \frac{(d+ex)^3(a+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 238

$$\begin{aligned}\int \frac{(d+ex)^3\,(a+cx^2)}{(f+gx)^{3/2}}\,dx &= \frac{2(ef-dg)^3\,(cf^2+ag^2)}{g^6\sqrt{f+gx}} \\ &+ \frac{2(ef-dg)^2\,(3aeg^2+cf(5ef-2dg))\,\sqrt{f+gx}}{g^6} \\ &- \frac{2(ef-dg)\,(3ae^2g^2+c(10e^2f^2-8defg+d^2g^2))\,(f+gx)^{3/2}}{3g^6} \\ &+ \frac{2e(ae^2g^2+c(10e^2f^2-12defg+3d^2g^2))\,(f+gx)^{5/2}}{5g^6} \\ &- \frac{2ce^2(5ef-3dg)(f+gx)^{7/2}}{7g^6}+\frac{2ce^3(f+gx)^{9/2}}{9g^6}\end{aligned}$$

output
```
2*(-d*g+e*f)^3*(a*g^2+c*f^2)/g^6/(g*x+f)^(1/2)+2*(-d*g+e*f)^2*(3*a*e*g^2+c
*f*(-2*d*g+5*e*f))*(g*x+f)^(1/2)/g^6-2/3*(-d*g+e*f)*(3*a*e^2*g^2+c*(d^2*g^
2-8*d*e*f*g+10*e^2*f^2))*(g*x+f)^(3/2)/g^6+2/5*e*(a*e^2*g^2+c*(3*d^2*g^2-1
2*d*e*f*g+10*e^2*f^2))*(g*x+f)^(5/2)/g^6-2/7*c*e^2*(-3*d*g+5*e*f)*(g*x+f)^
(7/2)/g^6+2/9*c*e^3*(g*x+f)^(9/2)/g^6
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^3(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2(63ag^2(-5d^3g^3+15d^2eg^2(2f+gx)+5de^2g(-8f^2-4fgx+g^2x^2)+e^3(16f}{}$$

input
```
Integrate[((d + e*x)^3*(a + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(2*(63*a*g^2*(-5*d^3*g^3 + 15*d^2*e*g^2*(2*f + g*x) + 5*d*e^2*g*(-8*f^2 -
4*f*g*x + g^2*x^2) + e^3*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3)) + c
*(105*d^3*g^3*(-8*f^2 - 4*f*g*x + g^2*x^2) + 189*d^2*e*g^2*(16*f^3 + 8*f^2
*g*x - 2*f*g^2*x^2 + g^3*x^3) + 27*d*e^2*g*(-128*f^4 - 64*f^3*g*x + 16*f^2
*g^2*x^2 - 8*f*g^3*x^3 + 5*g^4*x^4) + 5*e^3*(256*f^5 + 128*f^4*g*x - 32*f^
3*g^2*x^2 + 16*f^2*g^3*x^3 - 10*f*g^4*x^4 + 7*g^5*x^5))))/(315*g^6*Sqrt[f
+ g*x])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(d+ex)^3}{(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \; 652$$

$$\int \left(\frac{e(f+gx)^{3/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{g^5}+\frac{\sqrt{f+gx}(ef-dg)\left(-3ae^2g^2-c\left(d^2g^2-8defg+1\right.\right.}{g^5}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{2e(f+gx)^{5/2}\left(ae^2g^2+c\left(3d^2g^2-12defg+10e^2f^2\right)\right)}{5g^6}-\frac{2(f+gx)^{3/2}(ef-dg)\left(3ae^2g^2+c\left(d^2g^2-8defg+10e^2f^2\right)\right)}{3g^6}+\frac{2\left(ag^2+cf^2\right)(ef-dg)^3}{g^6\sqrt{f+gx}}+\frac{2\sqrt{f+gx}(ef-dg)^2\left(3aeg^2+cf(5ef-2dg)\right)}{g^6}-\frac{2ce^2(f+gx)^{7/2}(5ef-3dg)}{7g^6}+\frac{2ce^3(f+gx)^{9/2}}{9g^6}$$

input
```
Int[((d + e*x)^3*(a + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(2*(e*f - d*g)^3*(c*f^2 + a*g^2))/(g^6*Sqrt[f + g*x]) + (2*(e*f - d*g)^2*(
3*a*e*g^2 + c*f*(5*e*f - 2*d*g))*Sqrt[f + g*x])/g^6 - (2*(e*f - d*g)*(3*a*
e^2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^6) +
 (2*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2
))/(5*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^(7/2))/(7*g^6) + (2*c*e^3*
(f + g*x)^(9/2))/(9*g^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(-\frac{\left(\frac{5cx^2}{9}+a\right)x^3e^3}{5}-\left(a+\frac{3cx^2}{7}\right)x^2de^2-3\left(\frac{cx^2}{5}+a\right)xd^2e+d^3\left(a-\frac{cx^2}{3}\right)\right)g^5-6\left(-\frac{x^2\left(\frac{25cx^2}{63}+a\right)e^3}{15}-\frac{2\left(\frac{6cx^2}{35}+\cdots\right.}{3}\right.\right.\right.}{\cdots}$
risch	$\frac{2(35cx^4e^3g^4+135cd e^2g^4x^3-85ce^3fg^3x^3+63ae^3g^4x^2+189cd^2eg^4x^2-351cd e^2fg^3x^2+165ce^3f^2g^2x^2+315ade^2g^4\cdots}{\cdots}$
gosper	$-\frac{2(-35e^3cx^5g^5-135cde^2g^5x^4+50ce^3fg^4x^4-63ae^3g^5x^3-189cd^2eg^5x^3+216cde^2fg^4x^3-80ce^3f^2g^3x^3-315ade\cdots}{\cdots}$
trager	$-\frac{2(-35e^3cx^5g^5-135cde^2g^5x^4+50ce^3fg^4x^4-63ae^3g^5x^3-189cd^2eg^5x^3+216cde^2fg^4x^3-80ce^3f^2g^3x^3-315ade\cdots}{\cdots}$
orering	$-\frac{2(-35e^3cx^5g^5-135cde^2g^5x^4+50ce^3fg^4x^4-63ae^3g^5x^3-189cd^2eg^5x^3+216cde^2fg^4x^3-80ce^3f^2g^3x^3-315ade\cdots}{\cdots}$
derivativedivides	$\frac{\frac{2e^3c(gx+f)^{\frac{9}{2}}}{9}+\frac{6cde^2g(gx+f)^{\frac{7}{2}}}{7}-\frac{10ce^3f(gx+f)^{\frac{7}{2}}}{7}+\frac{2ae^3g^2(gx+f)^{\frac{5}{2}}}{5}+\frac{6cd^2eg^2(gx+f)^{\frac{5}{2}}}{5}-\frac{24cde^2fg(gx+f)^{\frac{5}{2}}}{5}+4ce^3f^2(gx\cdots}{\cdots}$
default	$\frac{\frac{2e^3c(gx+f)^{\frac{9}{2}}}{9}+\frac{6cde^2g(gx+f)^{\frac{7}{2}}}{7}-\frac{10ce^3f(gx+f)^{\frac{7}{2}}}{7}+\frac{2ae^3g^2(gx+f)^{\frac{5}{2}}}{5}+\frac{6cd^2eg^2(gx+f)^{\frac{5}{2}}}{5}-\frac{24cde^2fg(gx+f)^{\frac{5}{2}}}{5}+4ce^3f^2(gx\cdots}{\cdots}$

input
```
int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(g*x+f)^(1/2)*((-1/5*(5/9*c*x^2+a)*x^3*e^3-(a+3/7*c*x^2)*x^2*d*e^2-3*(1
/5*c*x^2+a)*x*d^2*e+d^3*(a-1/3*c*x^2))*g^5-6*(-1/15*x^2*(25/63*c*x^2+a)*e^
3-2/3*(6/35*c*x^2+a)*x*d*e^2+d^2*(-1/5*c*x^2+a)*e-2/9*d^3*c*x)*f*g^4+8*((-
2/63*c*x^3-1/5*a*x)*e^3+d*(-6/35*c*x^2+a)*e^2-3/5*c*d^2*e*x+1/3*c*d^3)*f^2
*g^3-16/5*e*f^3*((-10/63*c*x^2+a)*e^2-12/7*c*d*x*e+3*c*d^2)*g^2+384/35*e^2
*f^4*c*(-5/27*e*x+d)*g-256/63*c*e^3*f^5)/g^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)^3\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(35\,ce^3g^5x^5+1280\,ce^3f^5-3456\,cde^2f^4g+1890\,ad^2efg^4-315\,ad^3g^5+1008\cdots}{\cdots}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*c*e^3*g^5*x^5 + 1280*c*e^3*f^5 - 3456*c*d*e^2*f^4*g + 1890*a*d^2
*e*f*g^4 - 315*a*d^3*g^5 + 1008*(3*c*d^2*e + a*e^3)*f^3*g^2 - 840*(c*d^3 +
 3*a*d*e^2)*f^2*g^3 - 5*(10*c*e^3*f*g^4 - 27*c*d*e^2*g^5)*x^4 + (80*c*e^3*
f^2*g^3 - 216*c*d*e^2*f*g^4 + 63*(3*c*d^2*e + a*e^3)*g^5)*x^3 - (160*c*e^3
*f^3*g^2 - 432*c*d*e^2*f^2*g^3 + 126*(3*c*d^2*e + a*e^3)*f*g^4 - 105*(c*d^
3 + 3*a*d*e^2)*g^5)*x^2 + (640*c*e^3*f^4*g - 1728*c*d*e^2*f^3*g^2 + 945*a*
d^2*e*g^5 + 504*(3*c*d^2*e + a*e^3)*f^2*g^3 - 420*(c*d^3 + 3*a*d*e^2)*f*g^
4)*x)*sqrt(g*x + f)/(g^7*x + f*g^6)
```

Sympy [A] (verification not implemented)

Time = 13.38 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex)^3(a+cx^2)}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{ce^3(f+gx)^{\frac{9}{2}}}{9g^5} + \frac{(f+gx)^{\frac{7}{2}}\cdot\left(3cde^2g-5ce^3f\right)}{7g^5} + \frac{(f+gx)^{\frac{5}{2}}\left(ae^3g^2+3cd^2eg^2-12cde^2fg+10ce^3f^2\right)}{5g^5} + \frac{(f+gx)^{\frac{3}{2}}\cdot\left(3ad\right.}{}\right.}{} \\ \frac{ad^3x + \frac{3ad^2ex^2}{2} + \frac{3cde^2x^5}{5} + \frac{ce^3x^6}{6} + \frac{x^4\left(ae^3+3cd^2e\right)}{4} + \frac{x^3\cdot\left(3ade^2+cd^3\right)}{3}}{f^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c*e**3*(f + g*x)**(9/2)/(9*g**5) + (f + g*x)**(7/2)*(3*c*d*e
**2*g - 5*c*e**3*f)/(7*g**5) + (f + g*x)**(5/2)*(a*e**3*g**2 + 3*c*d**2*e*
g**2 - 12*c*d*e**2*f*g + 10*c*e**3*f**2)/(5*g**5) + (f + g*x)**(3/2)*(3*a*
d*e**2*g**3 - 3*a*e**3*f*g**2 + c*d**3*g**3 - 9*c*d**2*e*f*g**2 + 18*c*d*e
**2*f**2*g - 10*c*e**3*f**3)/(3*g**5) + sqrt(f + g*x)*(3*a*d**2*e*g**4 - 6
*a*d*e**2*f*g**3 + 3*a*e**3*f**2*g**2 - 2*c*d**3*f*g**3 + 9*c*d**2*e*f**2*
g**2 - 12*c*d*e**2*f**3*g + 5*c*e**3*f**4)/g**5 - (a*g**2 + c*f**2)*(d*g -
 e*f)**3/(g**5*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*d**3*x + 3*a*d**2*e*x**2/
2 + 3*c*d*e**2*x**5/5 + c*e**3*x**6/6 + x**4*(a*e**3 + 3*c*d**2*e)/4 + x**
3*(3*a*d*e**2 + c*d**3)/3)/f**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)^3\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{35\,(gx+f)^{\frac{9}{2}}ce^3-45\,(5\,ce^3f-3\,cde^2g)(gx+f)^{\frac{7}{2}}+63\,(10\,ce^3f^2-12\,cde^2fg+(3\,cd^2e+ae^3)g^2)(gx+f)^{\frac{5}{2}}-}{}\right.}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/315*((35*(g*x + f)^(9/2)*c*e^3 - 45*(5*c*e^3*f - 3*c*d*e^2*g)*(g*x + f)^
(7/2) + 63*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2*e + a*e^3)*g^2)*(g*x
+ f)^(5/2) - 105*(10*c*e^3*f^3 - 18*c*d*e^2*f^2*g + 3*(3*c*d^2*e + a*e^3)*
f*g^2 - (c*d^3 + 3*a*d*e^2)*g^3)*(g*x + f)^(3/2) + 315*(5*c*e^3*f^4 - 12*c
*d*e^2*f^3*g + 3*a*d^2*e*g^4 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 - 2*(c*d^3 +
3*a*d*e^2)*f*g^3)*sqrt(g*x + f))/g^5 + 315*(c*e^3*f^5 - 3*c*d*e^2*f^4*g +
3*a*d^2*e*f*g^4 - a*d^3*g^5 + (3*c*d^2*e + a*e^3)*f^3*g^2 - (c*d^3 + 3*a*d
*e^2)*f^2*g^3)/(sqrt(g*x + f)*g^5))/g
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(218) = 436$.

Time = 0.12 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.95

$$\int \frac{(d+ex)^3\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(ce^3f^5-3\,cde^2f^4g+3\,cd^2ef^3g^2+ae^3f^3g^2-cd^3f^2g^3-3\,ade^2f^2g^3+3\,ad^2e}{\sqrt{gx+f}\,g^6}$$
$$+\frac{2\left(35\,(gx+f)^{\frac{9}{2}}ce^3g^{48}-225\,(gx+f)^{\frac{7}{2}}ce^3fg^{48}+630\,(gx+f)^{\frac{5}{2}}ce^3f^2g^{48}-1050\,(gx+f)^{\frac{3}{2}}ce^3f^3g^{48}+157\right.}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
2*(c*e^3*f^5 - 3*c*d*e^2*f^4*g + 3*c*d^2*e*f^3*g^2 + a*e^3*f^3*g^2 - c*d^3
*f^2*g^3 - 3*a*d*e^2*f^2*g^3 + 3*a*d^2*e*f*g^4 - a*d^3*g^5)/(sqrt(g*x + f)
*g^6) + 2/315*(35*(g*x + f)^(9/2)*c*e^3*g^48 - 225*(g*x + f)^(7/2)*c*e^3*f
*g^48 + 630*(g*x + f)^(5/2)*c*e^3*f^2*g^48 - 1050*(g*x + f)^(3/2)*c*e^3*f^
3*g^48 + 1575*sqrt(g*x + f)*c*e^3*f^4*g^48 + 135*(g*x + f)^(7/2)*c*d*e^2*g
^49 - 756*(g*x + f)^(5/2)*c*d*e^2*f*g^49 + 1890*(g*x + f)^(3/2)*c*d*e^2*f^
2*g^49 - 3780*sqrt(g*x + f)*c*d*e^2*f^3*g^49 + 189*(g*x + f)^(5/2)*c*d^2*e
*g^50 + 63*(g*x + f)^(5/2)*a*e^3*g^50 - 945*(g*x + f)^(3/2)*c*d^2*e*f*g^50
 - 315*(g*x + f)^(3/2)*a*e^3*f*g^50 + 2835*sqrt(g*x + f)*c*d^2*e*f^2*g^50
+ 945*sqrt(g*x + f)*a*e^3*f^2*g^50 + 105*(g*x + f)^(3/2)*c*d^3*g^51 + 315*
(g*x + f)^(3/2)*a*d*e^2*g^51 - 630*sqrt(g*x + f)*c*d^3*f*g^51 - 1890*sqrt(
g*x + f)*a*d*e^2*f*g^51 + 945*sqrt(g*x + f)*a*d^2*e*g^52)/g^54
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^3\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{(f+g\,x)^{5/2}\,(6\,c\,d^2\,e\,g^2-24\,c\,d\,e^2\,f\,g+20\,c\,e^3\,f^2+2\,a\,e^3\,g^2)}{5\,g^6}$$
$$-\frac{2\,c\,d^3\,f^2\,g^3+2\,a\,d^3\,g^5-6\,c\,d^2\,e\,f^3\,g^2-6\,a\,d^2\,e\,f\,g^4+6\,c\,d\,e^2\,f^4\,g+6\,a\,d\,e^2\,f^2\,g^3-2\,c\,e^3\,f^5-2\,a\,e^3\,f^3\,g^2}{g^6\,\sqrt{f+g\,x}}$$
$$+\frac{2\,c\,e^3\,(f+g\,x)^{9/2}}{9\,g^6}+\frac{2\,\sqrt{f+g\,x}\,(d\,g-e\,f)^2\,(5\,c\,e\,f^2-2\,c\,d\,f\,g+3\,a\,e\,g^2)}{g^6}$$
$$+\frac{2\,(f+g\,x)^{3/2}\,(d\,g-e\,f)\,(c\,d^2\,g^2-8\,c\,d\,e\,f\,g+10\,c\,e^2\,f^2+3\,a\,e^2\,g^2)}{3\,g^6}$$
$$+\frac{2\,c\,e^2\,(f+g\,x)^{7/2}\,(3\,d\,g-5\,e\,f)}{7\,g^6}$$

input
```
int(((a + c*x^2)*(d + e*x)^3)/(f + g*x)^(3/2),x)
```

output
```
((f + g*x)^(5/2)*(2*a*e^3*g^2 + 20*c*e^3*f^2 + 6*c*d^2*e*g^2 - 24*c*d*e^2*
f*g))/(5*g^6) - (2*a*d^3*g^5 - 2*c*e^3*f^5 - 2*a*e^3*f^3*g^2 + 2*c*d^3*f^2
*g^3 - 6*a*d^2*e*f*g^4 + 6*c*d*e^2*f^4*g + 6*a*d*e^2*f^2*g^3 - 6*c*d^2*e*f
^3*g^2)/(g^6*(f + g*x)^(1/2)) + (2*c*e^3*(f + g*x)^(9/2))/(9*g^6) + (2*(f
+ g*x)^(1/2)*(d*g - e*f)^2*(3*a*e*g^2 + 5*c*e*f^2 - 2*c*d*f*g))/g^6 + (2*(
f + g*x)^(3/2)*(d*g - e*f)*(3*a*e^2*g^2 + c*d^2*g^2 + 10*c*e^2*f^2 - 8*c*d
*e*f*g))/(3*g^6) + (2*c*e^2*(f + g*x)^(7/2)*(3*d*g - 5*e*f))/(7*g^6)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^3\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{-2a\,d^3g^5+\frac{512}{63}c\,e^3f^5+12a\,d^2ef\,g^4+6a\,d^2e\,g^5x-16ad\,e^2f^2g^3+2ad\,e^2g^5x^2+}{}$$

input
```
int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(3/2),x)
```

output
```
(2*( - 315*a*d**3*g**5 + 1890*a*d**2*e*f*g**4 + 945*a*d**2*e*g**5*x - 2520
*a*d*e**2*f**2*g**3 - 1260*a*d*e**2*f*g**4*x + 315*a*d*e**2*g**5*x**2 + 10
08*a*e**3*f**3*g**2 + 504*a*e**3*f**2*g**3*x - 126*a*e**3*f*g**4*x**2 + 63
*a*e**3*g**5*x**3 - 840*c*d**3*f**2*g**3 - 420*c*d**3*f*g**4*x + 105*c*d**
3*g**5*x**2 + 3024*c*d**2*e*f**3*g**2 + 1512*c*d**2*e*f**2*g**3*x - 378*c*
d**2*e*f*g**4*x**2 + 189*c*d**2*e*g**5*x**3 - 3456*c*d*e**2*f**4*g - 1728*
c*d*e**2*f**3*g**2*x + 432*c*d*e**2*f**2*g**3*x**2 - 216*c*d*e**2*f*g**4*x
**3 + 135*c*d*e**2*g**5*x**4 + 1280*c*e**3*f**5 + 640*c*e**3*f**4*g*x - 16
0*c*e**3*f**3*g**2*x**2 + 80*c*e**3*f**2*g**3*x**3 - 50*c*e**3*f*g**4*x**4
 + 35*c*e**3*g**5*x**5))/(315*sqrt(f + g*x)*g**6)
```

3.57 $\int \frac{(d+ex)^2(a+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{(d+ex)^2\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = -\frac{2(ef-dg)^2\,(cf^2+ag^2)}{g^5\sqrt{f+gx}} - \frac{4(ef-dg)\,(aeg^2+cf(2ef-dg))\,\sqrt{f+gx}}{g^5} + \frac{2(ae^2g^2+c(6e^2f^2-6defg+d^2g^2))\,(f+gx)^{3/2}}{3g^5} - \frac{4ce(2ef-dg)(f+gx)^{5/2}}{5g^5} + \frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

output
```
-2*(-d*g+e*f)^2*(a*g^2+c*f^2)/g^5/(g*x+f)^(1/2)-4*(-d*g+e*f)*(a*e*g^2+c*f*
(-d*g+2*e*f))*(g*x+f)^(1/2)/g^5+2/3*(a*e^2*g^2+c*(d^2*g^2-6*d*e*f*g+6*e^2*
f^2))*(g*x+f)^(3/2)/g^5-4/5*c*e*(-d*g+2*e*f)*(g*x+f)^(5/2)/g^5+2/7*c*e^2*(
g*x+f)^(7/2)/g^5
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^2\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{-70ag^2(3d^2g^2-6deg(2f+gx)+e^2(8f^2+4fgx-g^2x^2))+2c(35d^2g^2(-8f^2$$

input
```
Integrate[((d + e*x)^2*(a + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(-70*a*g^2*(3*d^2*g^2 - 6*d*e*g*(2*f + g*x) + e^2*(8*f^2 + 4*f*g*x - g^2*x
^2)) + 2*c*(35*d^2*g^2*(-8*f^2 - 4*f*g*x + g^2*x^2) + 42*d*e*g*(16*f^3 + 8
*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) - 3*e^2*(128*f^4 + 64*f^3*g*x - 16*f^2*g
^2*x^2 + 8*f*g^3*x^3 - 5*g^4*x^4)))/(105*g^5*Sqrt[f + g*x])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)\,(d+ex)^2}{(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \; 652$$

$$\int \left(\frac{\sqrt{f+gx}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{g^4}+\frac{(ag^2+cf^2)\,(dg-ef)^2}{g^4(f+gx)^{3/2}}+\frac{2(ef-dg)\,(-aeg^2-cf(2ef-d}{g^4\sqrt{f+gx}}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{2(f+gx)^{3/2}\left(ae^2g^2+c\left(d^2g^2-6defg+6e^2f^2\right)\right)}{3g^5}-\frac{2\left(ag^2+cf^2\right)(ef-dg)^2}{g^5\sqrt{f+gx}}-\frac{4\sqrt{f+gx}(ef-dg)\left(aeg^2+cf(2ef-dg)\right)}{g^5}-\frac{4ce(f+gx)^{5/2}(2ef-dg)}{5g^5}+\frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

input
```
Int[((d + e*x)^2*(a + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(-2*(e*f - d*g)^2*(c*f^2 + a*g^2))/(g^5*Sqrt[f + g*x]) - (4*(e*f - d*g)*(a
*e*g^2 + c*f*(2*e*f - d*g))*Sqrt[f + g*x])/g^5 + (2*(a*e^2*g^2 + c*(6*e^2*
f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(3/2))/(3*g^5) - (4*c*e*(2*e*f - d*g
)*(f + g*x)^(5/2))/(5*g^5) + (2*c*e^2*(f + g*x)^(7/2))/(7*g^5)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(-\frac{\left(a+\frac{3c x^2}{7}\right)x^2e^2}{3}-2\left(\frac{c x^2}{5}+a\right)xde+d^2\left(a-\frac{c x^2}{3}\right)\right)g^4-4f\left(-\frac{\left(\frac{6c x^2}{35}+a\right)x e^2}{3}+d\left(-\frac{c x^2}{5}+a\right)e-\frac{c d^2 x}{3}\right)g^3+\frac{8f^2}{}\right.\right.}{\sqrt{gx+f}\,g^5}$
risch	$\frac{2\left(15g^3c e^2x^3+42g^3cd x^2e-39f g^2x^2c e^2+35g^3a e^2x+35g^3c d^2x-126f g^2cdxe+87f^2gc e^2x+210ade g^3-175a e^2f g^2-\right.}{105g^5}$
gosper	$-\frac{2\left(-15c e^2x^4g^4-42cde g^4x^3+24c e^2f g^3x^3-35a e^2g^4x^2-35c d^2g^4x^2+84cdef g^3x^2-48c e^2f^2g^2x^2-210ade g^4x+14\right.}{}$
trager	$-\frac{2\left(-15c e^2x^4g^4-42cde g^4x^3+24c e^2f g^3x^3-35a e^2g^4x^2-35c d^2g^4x^2+84cdef g^3x^2-48c e^2f^2g^2x^2-210ade g^4x+14\right.}{}$
orering	$-\frac{2\left(-15c e^2x^4g^4-42cde g^4x^3+24c e^2f g^3x^3-35a e^2g^4x^2-35c d^2g^4x^2+84cdef g^3x^2-48c e^2f^2g^2x^2-210ade g^4x+14\right.}{}$
derivativedivides	$\frac{\frac{2c e^2(gx+f)^{\frac{7}{2}}}{7}+\frac{4cdeg(gx+f)^{\frac{5}{2}}}{5}-\frac{8c e^2f(gx+f)^{\frac{5}{2}}}{5}+\frac{2a e^2g^2(gx+f)^{\frac{3}{2}}}{3}+\frac{2c d^2g^2(gx+f)^{\frac{3}{2}}}{3}-4cdefg(gx+f)^{\frac{3}{2}}+4c e^2f^2(gx+f)}{}$
default	$\frac{\frac{2c e^2(gx+f)^{\frac{7}{2}}}{7}+\frac{4cdeg(gx+f)^{\frac{5}{2}}}{5}-\frac{8c e^2f(gx+f)^{\frac{5}{2}}}{5}+\frac{2a e^2g^2(gx+f)^{\frac{3}{2}}}{3}+\frac{2c d^2g^2(gx+f)^{\frac{3}{2}}}{3}-4cdefg(gx+f)^{\frac{3}{2}}+4c e^2f^2(gx+f)}{}$

input
```
int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(g*x+f)^(1/2)*((-1/3*(a+3/7*c*x^2)*x^2*e^2-2*(1/5*c*x^2+a)*x*d*e+d^2*(a
-1/3*c*x^2))*g^4-4*f*(-1/3*(6/35*c*x^2+a)*x*e^2+d*(-1/5*c*x^2+a)*e-1/3*c*d
^2*x)*g^3+8/3*f^2*((-6/35*c*x^2+a)*e^2-6/5*c*d*x*e+c*d^2)*g^2-32/5*e*f^3*c
*(-2/7*e*x+d)*g+128/35*c*e^2*f^4)/g^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)^2 (a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(15\,ce^2g^4x^4 - 384\,ce^2f^4 + 672\,cdef^3g + 420\,adefg^3 - 105\,ad^2g^4 - 280\,(cd^2 +$$

input
```
integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/105*(15*c*e^2*g^4*x^4 - 384*c*e^2*f^4 + 672*c*d*e*f^3*g + 420*a*d*e*f*g^
3 - 105*a*d^2*g^4 - 280*(c*d^2 + a*e^2)*f^2*g^2 - 6*(4*c*e^2*f*g^3 - 7*c*d
*e*g^4)*x^3 + (48*c*e^2*f^2*g^2 - 84*c*d*e*f*g^3 + 35*(c*d^2 + a*e^2)*g^4)
*x^2 - 2*(96*c*e^2*f^3*g - 168*c*d*e*f^2*g^2 - 105*a*d*e*g^4 + 70*(c*d^2 +
 a*e^2)*f*g^3)*x)*sqrt(g*x + f)/(g^6*x + f*g^5)
```

Sympy [A] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^2 (a+cx^2)}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{ce^2(f+gx)^{\frac{7}{2}}}{7g^4} + \frac{(f+gx)^{\frac{5}{2}}\cdot\left(2cdeg-4ce^2f\right)}{5g^4} + \frac{(f+gx)^{\frac{3}{2}}\left(ae^2g^2+cd^2g^2-6cdefg+6ce^2f^2\right)}{3g^4} + \frac{\sqrt{f+gx}\left(2adeg^3-2ae^2\right.}{}\right.}{g} \\ \frac{ad^2x+adex^2+\frac{cdex^4}{2}+\frac{ce^2x^5}{5}+\frac{x^3\left(ae^2+cd^2\right)}{3}}{f^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((e*x+d)**2*(c*x**2+a)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c*e**2*(f + g*x)**(7/2)/(7*g**4) + (f + g*x)**(5/2)*(2*c*d*e
*g - 4*c*e**2*f)/(5*g**4) + (f + g*x)**(3/2)*(a*e**2*g**2 + c*d**2*g**2 -
6*c*d*e*f*g + 6*c*e**2*f**2)/(3*g**4) + sqrt(f + g*x)*(2*a*d*e*g**3 - 2*a*
e**2*f*g**2 - 2*c*d**2*f*g**2 + 6*c*d*e*f**2*g - 4*c*e**2*f**3)/g**4 - (a*
g**2 + c*f**2)*(d*g - e*f)**2/(g**4*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*d**2
*x + a*d*e*x**2 + c*d*e*x**4/2 + c*e**2*x**5/5 + x**3*(a*e**2 + c*d**2)/3)
/f**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^2\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{15\,(gx+f)^{\frac{7}{2}}ce^2-42\,(2\,ce^2f-cdeg)(gx+f)^{\frac{5}{2}}+35\,(6\,ce^2f^2-6\,cdefg+(cd^2+ae^2)g^2)(gx+f)^{\frac{3}{2}}-210\,(2\,ce\cdots}{g^4}\cdots\right.}{\cdots}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/105*((15*(g*x + f)^(7/2)*c*e^2 - 42*(2*c*e^2*f - c*d*e*g)*(g*x + f)^(5/2
) + 35*(6*c*e^2*f^2 - 6*c*d*e*f*g + (c*d^2 + a*e^2)*g^2)*(g*x + f)^(3/2) -
 210*(2*c*e^2*f^3 - 3*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 + a*e^2)*f*g^2)*sqr
t(g*x + f))/g^4 - 105*(c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g
^4 + (c*d^2 + a*e^2)*f^2*g^2)/(sqrt(g*x + f)*g^4))/g
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^2\,(a+cx^2)}{(f+gx)^{3/2}}\,dx =$$

$$-\frac{2\,(ce^2f^4-2\,cdef^3g+cd^2f^2g^2+ae^2f^2g^2-2\,adefg^3+ad^2g^4)}{\sqrt{gx+f}\,g^5} + \frac{2\left(15\,(gx+f)^{\frac{7}{2}}ce^2g^{30}-84\,(gx+f)^{\frac{5}{2}}ce^2fg^{30}+210\,(gx+f)^{\frac{3}{2}}ce^2f^2g^{30}-420\,\sqrt{gx+f}\,ce^2f^3g^{30}+42\,(gx\cdots\right.}{\cdots}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
-2*(c*e^2*f^4 - 2*c*d*e*f^3*g + c*d^2*f^2*g^2 + a*e^2*f^2*g^2 - 2*a*d*e*f*
g^3 + a*d^2*g^4)/(sqrt(g*x + f)*g^5) + 2/105*(15*(g*x + f)^(7/2)*c*e^2*g^3
0 - 84*(g*x + f)^(5/2)*c*e^2*f*g^30 + 210*(g*x + f)^(3/2)*c*e^2*f^2*g^30 -
 420*sqrt(g*x + f)*c*e^2*f^3*g^30 + 42*(g*x + f)^(5/2)*c*d*e*g^31 - 210*(g
*x + f)^(3/2)*c*d*e*f*g^31 + 630*sqrt(g*x + f)*c*d*e*f^2*g^31 + 35*(g*x +
f)^(3/2)*c*d^2*g^32 + 35*(g*x + f)^(3/2)*a*e^2*g^32 - 210*sqrt(g*x + f)*c*
d^2*f*g^32 - 210*sqrt(g*x + f)*a*e^2*f*g^32 + 210*sqrt(g*x + f)*a*d*e*g^33
)/g^35
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^2 (a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{(f+g\,x)^{3/2}\,(2\,c\,d^2\,g^2-12\,c\,d\,e\,f\,g+12\,c\,e^2\,f^2+2\,a\,e^2\,g^2)}{3\,g^5} - \frac{2\,c\,d^2\,f^2\,g^2+2\,a\,d^2\,g^4-4\,c\,d\,e\,f^3\,g-4\,a\,d\,e\,f\,g^3+2\,c\,e^2\,f^4+2\,a\,e^2\,f^2\,g^2}{g^5\,\sqrt{f+g\,x}} + \frac{4\,\sqrt{f+g\,x}\,(d\,g-e\,f)\,(2\,c\,e\,f^2-c\,d\,f\,g+a\,e\,g^2)}{g^5} + \frac{2\,c\,e^2\,(f+g\,x)^{7/2}}{7\,g^5} + \frac{4\,c\,e\,(f+g\,x)^{5/2}\,(d\,g-2\,e\,f)}{5\,g^5}$$

input
```
int(((a + c*x^2)*(d + e*x)^2)/(f + g*x)^(3/2),x)
```

output
```
((f + g*x)^(3/2)*(2*a*e^2*g^2 + 2*c*d^2*g^2 + 12*c*e^2*f^2 - 12*c*d*e*f*g)
)/(3*g^5) - (2*a*d^2*g^4 + 2*c*e^2*f^4 + 2*a*e^2*f^2*g^2 + 2*c*d^2*f^2*g^2
 - 4*a*d*e*f*g^3 - 4*c*d*e*f^3*g)/(g^5*(f + g*x)^(1/2)) + (4*(f + g*x)^(1/
2)*(d*g - e*f)*(a*e*g^2 + 2*c*e*f^2 - c*d*f*g))/g^5 + (2*c*e^2*(f + g*x)^(
7/2))/(7*g^5) + (4*c*e*(f + g*x)^(5/2)*(d*g - 2*e*f))/(5*g^5)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^2 (a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{\frac{2}{7}c\,e^2g^4x^4+\frac{4}{5}cde\,g^4x^3-\frac{16}{35}c\,e^2f\,g^3x^3+\frac{2}{3}a\,e^2g^4x^2+\frac{2}{3}c\,d^2g^4x^2-\frac{8}{5}cdef\,g^3x^2+}{}$$

input
```
int((e*x+d)^2*(c*x^2+a)/(g*x+f)^(3/2),x)
```

output
```
(2*( - 105*a*d**2*g**4 + 420*a*d*e*f*g**3 + 210*a*d*e*g**4*x - 280*a*e**2*
f**2*g**2 - 140*a*e**2*f*g**3*x + 35*a*e**2*g**4*x**2 - 280*c*d**2*f**2*g*
*2 - 140*c*d**2*f*g**3*x + 35*c*d**2*g**4*x**2 + 672*c*d*e*f**3*g + 336*c*
d*e*f**2*g**2*x - 84*c*d*e*f*g**3*x**2 + 42*c*d*e*g**4*x**3 - 384*c*e**2*f
**4 - 192*c*e**2*f**3*g*x + 48*c*e**2*f**2*g**2*x**2 - 24*c*e**2*f*g**3*x*
*3 + 15*c*e**2*g**4*x**4))/(105*sqrt(f + g*x)*g**5)
```

3.58 $\int \frac{(d+ex)(a+cx^2)}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{(d+ex)(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2(ef-dg)(cf^2+ag^2)}{g^4\sqrt{f+gx}} + \frac{2(aeg^2+cf(3ef-2dg))\sqrt{f+gx}}{g^4} - \frac{2c(3ef-dg)(f+gx)^{3/2}}{3g^4} + \frac{2ce(f+gx)^{5/2}}{5g^4}$$

output
```
2*(-d*g+e*f)*(a*g^2+c*f^2)/g^4/(g*x+f)^(1/2)+2*(a*e*g^2+c*f*(-2*d*g+3*e*f)
)*(g*x+f)^(1/2)/g^4-2/3*c*(-d*g+3*e*f)*(g*x+f)^(3/2)/g^4+2/5*c*e*(g*x+f)^(
5/2)/g^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{30ag^2(2ef-dg+egx)+10cdg(-8f^2-4fgx+g^2x^2)+6ce(16f^3+8f^2gx-}{15g^4\sqrt{f+gx}}$$

input
```
Integrate[((d + e*x)*(a + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(30*a*g^2*(2*e*f - d*g + e*g*x) + 10*c*d*g*(-8*f^2 - 4*f*g*x + g^2*x^2) +
6*c*e*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3))/(15*g^4*Sqrt[f + g*x])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(d+ex)}{(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{(ag^2+cf^2)(dg-ef)}{g^3(f+gx)^{3/2}}+\frac{aeg^2+cf(3ef-2dg)}{g^3\sqrt{f+gx}}+\frac{c\sqrt{f+gx}(dg-3ef)}{g^3}+\frac{ce(f+gx)^{3/2}}{g^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(ag^2+cf^2)(ef-dg)}{g^4\sqrt{f+gx}}+\frac{2\sqrt{f+gx}(aeg^2+cf(3ef-2dg))}{g^4}-\frac{2c(f+gx)^{3/2}(3ef-dg)}{3g^4}+\frac{2ce(f+gx)^{5/2}}{5g^4}$$

input
```
Int[((d + e*x)*(a + c*x^2))/(f + g*x)^(3/2),x]
```

output
```
(2*(e*f - d*g)*(c*f^2 + a*g^2))/(g^4*Sqrt[f + g*x]) + (2*(a*e*g^2 + c*f*(3
*e*f - 2*d*g))*Sqrt[f + g*x])/g^4 - (2*c*(3*e*f - d*g)*(f + g*x)^(3/2))/(3
*g^4) + (2*c*e*(f + g*x)^(5/2))/(5*g^4)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{\left(\left(6x^{3}e+10d\,x^{2}\right)c-30a\left(-ex+d\right)\right)g^{3}+60f\left(\left(\left(-\frac{1}{5}e\,x^{2}-\frac{2}{3}dx\right)c+ae\right)g^{2}-80f^{2}\left(-\frac{3ex}{5}+d\right)cg+96ce\,f^{3}}{15\sqrt{gx+f}\,g^{4}}$
gosper	$-\frac{2\left(-3ce\,x^{3}g^{3}-5cd\,g^{3}x^{2}+6cef\,g^{2}x^{2}-15ae\,g^{3}x+20cdf\,g^{2}x-24ce\,f^{2}gx+15ad\,g^{3}-30aef\,g^{2}+40cd\,f^{2}g-48ce\,f^{3}\right)}{15\sqrt{gx+f}\,g^{4}}$
trager	$-\frac{2\left(-3ce\,x^{3}g^{3}-5cd\,g^{3}x^{2}+6cef\,g^{2}x^{2}-15ae\,g^{3}x+20cdf\,g^{2}x-24ce\,f^{2}gx+15ad\,g^{3}-30aef\,g^{2}+40cd\,f^{2}g-48ce\,f^{3}\right)}{15\sqrt{gx+f}\,g^{4}}$
risch	$\frac{2\left(3g^{2}ce\,x^{2}+5g^{2}cdx-9fgcex+15ae\,g^{2}-25cdfg+33ce\,f^{2}\right)\sqrt{gx+f}}{15g^{4}}-\frac{2\left(ad\,g^{3}-aef\,g^{2}+cd\,f^{2}g-ce\,f^{3}\right)}{g^{4}\sqrt{gx+f}}$
orering	$-\frac{2\left(-3ce\,x^{3}g^{3}-5cd\,g^{3}x^{2}+6cef\,g^{2}x^{2}-15ae\,g^{3}x+20cdf\,g^{2}x-24ce\,f^{2}gx+15ad\,g^{3}-30aef\,g^{2}+40cd\,f^{2}g-48ce\,f^{3}\right)}{15\sqrt{gx+f}\,g^{4}}$
derivativedivides	$\frac{\frac{2ce(gx+f)^{\frac{5}{2}}}{5}+\frac{2cdg(gx+f)^{\frac{3}{2}}}{3}-2cef(gx+f)^{\frac{3}{2}}+2ae\,g^{2}\sqrt{gx+f}-4cdfg\sqrt{gx+f}+6ce\,f^{2}\sqrt{gx+f}-\frac{2\left(ad\,g^{3}-aef\,g^{2}+cd\,f^{2}g-ce}{\sqrt{gx+f}}}{g^{4}}$
default	$\frac{\frac{2ce(gx+f)^{\frac{5}{2}}}{5}+\frac{2cdg(gx+f)^{\frac{3}{2}}}{3}-2cef(gx+f)^{\frac{3}{2}}+2ae\,g^{2}\sqrt{gx+f}-4cdfg\sqrt{gx+f}+6ce\,f^{2}\sqrt{gx+f}-\frac{2\left(ad\,g^{3}-aef\,g^{2}+cd\,f^{2}g-ce}{\sqrt{gx+f}}}{g^{4}}$

input
```
int((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(((6*e*x^3+10*d*x^2)*c-30*a*(-e*x+d))*g^3+60*f*((-1/5*e*x^2-2/3*d*x)*
c+a*e)*g^2-80*f^2*(-3/5*e*x+d)*c*g+96*c*e*f^3)/(g*x+f)^(1/2)/g^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(3\,ceg^3x^3+48\,cef^3-40\,cdf^2g+30\,aefg^2-15\,adg^3-(6\,cefg^2-5\,cdg^3)x^2+}{15\,(g^5x+fg^4)}$$

input
```
integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*c*e*g^3*x^3 + 48*c*e*f^3 - 40*c*d*f^2*g + 30*a*e*f*g^2 - 15*a*d*g^
3 - (6*c*e*f*g^2 - 5*c*d*g^3)*x^2 + (24*c*e*f^2*g - 20*c*d*f*g^2 + 15*a*e*
g^3)*x)*sqrt(g*x + f)/(g^5*x + f*g^4)
```

Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{ce(f+gx)^{\frac{5}{2}}}{5g^3}+\frac{(f+gx)^{\frac{3}{2}}(cdg-3cef)}{3g^3}+\frac{\sqrt{f+gx}\left(aeg^2-2cdfg+3cef^2\right)}{g^3}-\frac{\left(ag^2+cf^2\right)(dg-ef)}{g^3\sqrt{f+gx}}\right)}{g} & \text{for } g\neq 0 \\ \frac{adx+\frac{aex^2}{2}+\frac{cdx^3}{3}+\frac{cex^4}{4}}{f^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c*e*(f + g*x)**(5/2)/(5*g**3) + (f + g*x)**(3/2)*(c*d*g - 3*
c*e*f)/(3*g**3) + sqrt(f + g*x)*(a*e*g**2 - 2*c*d*f*g + 3*c*e*f**2)/g**3 -
 (a*g**2 + c*f**2)*(d*g - e*f)/(g**3*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*d*x
 + a*e*x**2/2 + c*d*x**3/3 + c*e*x**4/4)/f**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{3\,(gx+f)^{\frac{5}{2}}ce-5\,(3\,cef-cdg)(gx+f)^{\frac{3}{2}}+15\,(3\,cef^2-2\,cdfg+aeg^2)\sqrt{gx+f}}{g^3}+\frac{15\,(cef^3-cdf^2g+aefg^2-adg^3)}{\sqrt{gx+f}\,g^3}\right)}{15\,g}$$

input
```
integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/15*((3*(g*x + f)^(5/2)*c*e - 5*(3*c*e*f - c*d*g)*(g*x + f)^(3/2) + 15*(3
*c*e*f^2 - 2*c*d*f*g + a*e*g^2)*sqrt(g*x + f))/g^3 + 15*(c*e*f^3 - c*d*f^2
*g + a*e*f*g^2 - a*d*g^3)/(sqrt(g*x + f)*g^3))/g
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{2\,(cef^3-cdf^2g+aefg^2-adg^3)}{\sqrt{gx+f}\,g^4} + \frac{2\left(3\,(gx+f)^{\frac{5}{2}}ceg^{16}-15\,(gx+f)^{\frac{3}{2}}cefg^{16}+45\,\sqrt{gx+f}cef^2g^{16}+5\,(gx+f)^{\frac{3}{2}}cdg^{17}-30\,\sqrt{gx+f}cdfg^{17}+15\,\sqrt{gx+f}aeg^{18}\right)}{15\,g^{20}}$$

input
```
integrate((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
2*(c*e*f^3 - c*d*f^2*g + a*e*f*g^2 - a*d*g^3)/(sqrt(g*x + f)*g^4) + 2/15*(
3*(g*x + f)^(5/2)*c*e*g^16 - 15*(g*x + f)^(3/2)*c*e*f*g^16 + 45*sqrt(g*x +
 f)*c*e*f^2*g^16 + 5*(g*x + f)^(3/2)*c*d*g^17 - 30*sqrt(g*x + f)*c*d*f*g^1
7 + 15*sqrt(g*x + f)*a*e*g^18)/g^20
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{\sqrt{f+g\,x}\,\left(6\,c\,e\,f^2-4\,c\,d\,f\,g+2\,a\,e\,g^2\right)}{g^4} - \frac{-2\,c\,e\,f^3+2\,c\,d\,f^2\,g-2\,a\,e\,f\,g^2+2\,a\,d\,g^3}{g^4\,\sqrt{f+g\,x}} + \frac{2\,c\,e\,{\left(f+g\,x\right)}^{5/2}}{5\,g^4} + \frac{2\,c\,{\left(f+g\,x\right)}^{3/2}\,\left(d\,g-3\,e\,f\right)}{3\,g^4}$$

input
```
int(((a + c*x^2)*(d + e*x))/(f + g*x)^(3/2),x)
```

output
```
((f + g*x)^(1/2)*(2*a*e*g^2 + 6*c*e*f^2 - 4*c*d*f*g))/g^4 - (2*a*d*g^3 - 2
*c*e*f^3 - 2*a*e*f*g^2 + 2*c*d*f^2*g)/(g^4*(f + g*x)^(1/2)) + (2*c*e*(f +
g*x)^(5/2))/(5*g^4) + (2*c*(f + g*x)^(3/2)*(d*g - 3*e*f))/(3*g^4)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)\,(a+cx^2)}{(f+gx)^{3/2}}\,dx = \frac{\frac{2}{5}ce\,g^3x^3+\frac{2}{3}cd\,g^3x^2-\frac{4}{5}cef\,g^2x^2+2ae\,g^3x-\frac{8}{3}cdf\,g^2x+\frac{16}{5}ce\,f^2gx-2ad\,g^3+}{\sqrt{gx+f}\,g^4}$$

input
```
int((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x)
```

output
```
(2*( - 15*a*d*g**3 + 30*a*e*f*g**2 + 15*a*e*g**3*x - 40*c*d*f**2*g - 20*c*
d*f*g**2*x + 5*c*d*g**3*x**2 + 48*c*e*f**3 + 24*c*e*f**2*g*x - 6*c*e*f*g**
2*x**2 + 3*c*e*g**3*x**3))/(15*sqrt(f + g*x)*g**4)
```

3.59 $\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx = -\frac{2(cf^2+ag^2)}{g^3\sqrt{f+gx}} - \frac{4cf\sqrt{f+gx}}{g^3} + \frac{2c(f+gx)^{3/2}}{3g^3}$$

output
```
(-2*a*g^2-2*c*f^2)/g^3/(g*x+f)^(1/2)-4*c*f*(g*x+f)^(1/2)/g^3+2/3*c*(g*x+f)
^(3/2)/g^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx = \frac{2(-3ag^2+c(-8f^2-4fgx+g^2x^2))}{3g^3\sqrt{f+gx}}$$

input
```
Integrate[(a + c*x^2)/(f + g*x)^(3/2),x]
```

output
```
(2*(-3*a*g^2 + c*(-8*f^2 - 4*f*g*x + g^2*x^2)))/(3*g^3*Sqrt[f + g*x])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(f+gx)^{3/2}} \, dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{ag^2+cf^2}{g^2(f+gx)^{3/2}}+\frac{c\sqrt{f+gx}}{g^2}-\frac{2cf}{g^2\sqrt{f+gx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(ag^2+cf^2)}{g^3\sqrt{f+gx}}+\frac{2c(f+gx)^{3/2}}{3g^3}-\frac{4cf\sqrt{f+gx}}{g^3}$$

input
```
Int[(a + c*x^2)/(f + g*x)^(3/2),x]
```

output
```
(-2*(c*f^2 + a*g^2))/(g^3*Sqrt[f + g*x]) - (4*c*f*Sqrt[f + g*x])/g^3 + (2*
c*(f + g*x)^(3/2))/(3*g^3)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$\frac{\frac{2\left(c\,x^{2}-3a\right)g^{2}}{3}-\frac{8cfxg}{3}-\frac{16c\,f^{2}}{3}}{\sqrt{gx+f}\,g^{3}}$	39
gosper	$-\frac{2\left(-c\,x^{2} g^{2}+4cfxg+3a\,g^{2}+8c\,f^{2}\right)}{3\sqrt{gx+f}\,g^{3}}$	41
trager	$-\frac{2\left(-c\,x^{2} g^{2}+4cfxg+3a\,g^{2}+8c\,f^{2}\right)}{3\sqrt{gx+f}\,g^{3}}$	41
orering	$-\frac{2\left(-c\,x^{2} g^{2}+4cfxg+3a\,g^{2}+8c\,f^{2}\right)}{3\sqrt{gx+f}\,g^{3}}$	41
risch	$-\frac{2c(-gx+5f)\sqrt{gx+f}}{3g^{3}}-\frac{2\left(a\,g^{2}+c\,f^{2}\right)}{g^{3}\sqrt{gx+f}}$	46
derivativedivides	$\frac{\frac{2(gx+f)^{\frac{3}{2}}c}{3}-4cf\sqrt{gx+f}-\frac{2\left(a\,g^{2}+c\,f^{2}\right)}{\sqrt{gx+f}}}{g^{3}}$	48
default	$\frac{\frac{2(gx+f)^{\frac{3}{2}}c}{3}-4cf\sqrt{gx+f}-\frac{2\left(a\,g^{2}+c\,f^{2}\right)}{\sqrt{gx+f}}}{g^{3}}$	48

input
```
int((c*x^2+a)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((c*x^2-3*a)*g^2-4*c*f*x*g-8*c*f^2)/(g*x+f)^(1/2)/g^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx=\frac{2\,(cg^2x^2-4\,cfgx-8\,cf^2-3\,ag^2)\sqrt{gx+f}}{3\,(g^4x+fg^3)}$$

input
```
integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(c*g^2*x^2 - 4*c*f*g*x - 8*c*f^2 - 3*a*g^2)*sqrt(g*x + f)/(g^4*x + f*g
^3)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{a + cx^2}{(f + gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{2cf\sqrt{f+gx}}{g^2} + \frac{c(f+gx)^{\frac{3}{2}}}{3g^2} - \frac{ag^2+cf^2}{g^2\sqrt{f+gx}}\right)}{g} & \text{for } g \neq 0 \\ \frac{ax+\frac{cx^3}{3}}{f^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(-2*c*f*sqrt(f + g*x)/g**2 + c*(f + g*x)**(3/2)/(3*g**2) - (a
*g**2 + c*f**2)/(g**2*sqrt(f + g*x)))/g, Ne(g, 0)), ((a*x + c*x**3/3)/f**(
3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{a + cx^2}{(f + gx)^{3/2}}\,dx = \frac{2\left(\frac{(gx+f)^{\frac{3}{2}}c - 6\sqrt{gx+f}cf}{g^2} - \frac{3(cf^2+ag^2)}{\sqrt{gx+f}g^2}\right)}{3g}$$

input
```
integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/3*(((g*x + f)^(3/2)*c - 6*sqrt(g*x + f)*c*f)/g^2 - 3*(c*f^2 + a*g^2)/(sq
rt(g*x + f)*g^2))/g
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx = -\frac{2\,(cf^2+ag^2)}{\sqrt{gx+f}\,g^3} + \frac{2\left((gx+f)^{\frac{3}{2}}cg^6 - 6\sqrt{gx+f}\,cfg^6\right)}{3\,g^9}$$

input
```
integrate((c*x^2+a)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
-2*(c*f^2 + a*g^2)/(sqrt(g*x + f)*g^3) + 2/3*((g*x + f)^(3/2)*c*g^6 - 6*sq
rt(g*x + f)*c*f*g^6)/g^9
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx = -\frac{6\,a\,g^2 - 2\,c\,(f+g\,x)^2 + 6\,c\,f^2 + 12\,c\,f\,(f+g\,x)}{3\,g^3\,\sqrt{f+g\,x}}$$

input
```
int((a + c*x^2)/(f + g*x)^(3/2),x)
```

output
```
-(6*a*g^2 - 2*c*(f + g*x)^2 + 6*c*f^2 + 12*c*f*(f + g*x))/(3*g^3*(f + g*x)
^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{a+cx^2}{(f+gx)^{3/2}}\,dx = \frac{\frac{2}{3}c\,g^2x^2 - \frac{8}{3}cf\,gx - 2a\,g^2 - \frac{16}{3}c\,f^2}{\sqrt{gx+f}\,g^3}$$

input
```
int((c*x^2+a)/(g*x+f)^(3/2),x)
```

output
```
(2*( - 3*a*g**2 - 8*c*f**2 - 4*c*f*g*x + c*g**2*x**2))/(3*sqrt(f + g*x)*g*
*3)
```

3.60 $\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2(cf^2+ag^2)}{g^2(ef-dg)\sqrt{f+gx}} + \frac{2c\sqrt{f+gx}}{eg^2} - \frac{2(cd^2+ae^2)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}$$

output
```
2*(a*g^2+c*f^2)/g^2/(-d*g+e*f)/(g*x+f)^(1/2)+2*c*(g*x+f)^(1/2)/e/g^2-2*(a*
e^2+c*d^2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/(-d*g+e
*f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = -\frac{2(aeg^2-cdg(f+gx)+cef(2f+gx))}{eg^2(-ef+dg)\sqrt{f+gx}} - \frac{2(cd^2+ae^2)\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{3/2}(-ef+dg)^{3/2}}$$

input
```
Integrate[(a + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

output
```
(-2*(a*e*g^2 - c*d*g*(f + g*x) + c*e*f*(2*f + g*x)))/(e*g^2*(-(e*f) + d*g)
*Sqrt[f + g*x]) - (2*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-
(e*f) + d*g]])/(e^(3/2)*(-(e*f) + d*g)^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {649, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{ 649}$$

$$\frac{2\int -\frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2\int \frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{ 1584}$$

$$-\frac{2\int \left(\frac{(cd^2+ae^2)g^2}{e(ef-dg)(ef-dg-e(f+gx))}-\frac{c}{e}+\frac{cf^2+ag^2}{(ef-dg)(f+gx)}\right)d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\left(-\frac{g^2(ae^2+cd^2)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}+\frac{ag^2+cf^2}{\sqrt{f+gx}(ef-dg)}+\frac{c\sqrt{f+gx}}{e}\right)}{g^2}$$

input
```
Int[(a + c*x^2)/((d + e*x)*(f + g*x)^(3/2)),x]
```

output
```
(2*((c*f^2 + a*g^2)/((e*f - d*g)*Sqrt[f + g*x]) + (c*Sqrt[f + g*x])/e - ((
c*d^2 + a*e^2)*g^2*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(3
/2)*(e*f - d*g)^(3/2))))/g^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2g^2\left(a\,e^2+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e(dg-ef)\sqrt{(dg-ef)e}}-\frac{2\left(a\,g^2+c\,f^2\right)}{(dg-ef)\sqrt{gx+f}}}{g^2}$	112
default	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2g^2\left(a\,e^2+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e(dg-ef)\sqrt{(dg-ef)e}}-\frac{2\left(a\,g^2+c\,f^2\right)}{(dg-ef)\sqrt{gx+f}}}{g^2}$	112
pseudoelliptic	$\frac{\frac{2c\sqrt{gx+f}}{e}-\frac{2g^2\left(a\,e^2+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{e(dg-ef)\sqrt{(dg-ef)e}}-\frac{2\left(a\,g^2+c\,f^2\right)}{(dg-ef)\sqrt{gx+f}}}{g^2}$	112
risch	$\frac{2c\sqrt{gx+f}}{e\,g^2}-\frac{2\left(\frac{g^2\left(a\,e^2+c\,d^2\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{(dg-ef)\sqrt{(dg-ef)e}}+\frac{e\left(a\,g^2+c\,f^2\right)}{(dg-ef)\sqrt{gx+f}}\right)}{e\,g^2}$	116

input
```
int((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/g^2*(c/e*(g*x+f)^(1/2)-1/e/(d*g-e*f)*g^2*(a*e^2+c*d^2)/((d*g-e*f)*e)^(1/
2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))-(a*g^2+c*f^2)/(d*g-e*f)/(g*
x+f)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(98) = 196$.

Time = 0.09 (sec) , antiderivative size = 492, normalized size of antiderivative = 4.39

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \left[-\frac{((cd^2+ae^2)g^3x+(cd^2+ae^2)fg^2)\sqrt{e^2f-deg}\log\left(\frac{egx+2\,ef-dg+2\,\sqrt{e^2f-deg}\sqrt{}}{ex+d}\right.}{e^4f^3g^2-2\,de^3f^2}\right.$$

input
```
integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
[-((((c*d^2 + a*e^2)*g^3*x + (c*d^2 + a*e^2)*f*g^2)*sqrt(e^2*f - d*e*g)*log
((e*g*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) -
2*(2*c*e^3*f^3 - 3*c*d*e^2*f^2*g - a*d*e^2*g^3 + (c*d^2*e + a*e^3)*f*g^2 +
 (c*e^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*
g^2 - 2*d*e^3*f^2*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2
*e^2*g^5)*x), 2*(((c*d^2 + a*e^2)*g^3*x + (c*d^2 + a*e^2)*f*g^2)*sqrt(-e^2
*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*
c*e^3*f^3 - 3*c*d*e^2*f^2*g - a*d*e^2*g^3 + (c*d^2*e + a*e^3)*f*g^2 + (c*e
^3*f^2*g - 2*c*d*e^2*f*g^2 + c*d^2*e*g^3)*x)*sqrt(g*x + f))/(e^4*f^3*g^2 -
 2*d*e^3*f^2*g^3 + d^2*e^2*f*g^4 + (e^4*f^2*g^3 - 2*d*e^3*f*g^4 + d^2*e^2*
g^5)*x)]
```

Sympy [A] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.35

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c\sqrt{f+gx}}{eg} - \frac{ag^2+cf^2}{g\sqrt{f+gx}(dg-ef)} - \frac{g\left(ae^2+cd^2\right)\operatorname{atan}\left(\frac{\sqrt{f+gx}}{\sqrt{\frac{dg-ef}{e}}}\right)}{e^2\sqrt{\frac{dg-ef}{e}}(dg-ef)}\right)}{g} & \text{for } g \neq 0 \\ \frac{-\frac{cdx}{e^2} + \frac{cx^2}{2e} + \frac{\left(ae^2+cd^2\right)\left(\begin{cases} \frac{x}{d} & \text{for } e = 0 \\ \frac{\log(d+ex)}{e} & \text{otherwise} \end{cases}\right)}{e^2}}{f^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)/(e*x+d)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c*sqrt(f + g*x)/(e*g) - (a*g**2 + c*f**2)/(g*sqrt(f + g*x)*(
d*g - e*f)) - g*(a*e**2 + c*d**2)*atan(sqrt(f + g*x)/sqrt((d*g - e*f)/e))/
(e**2*sqrt((d*g - e*f)/e)*(d*g - e*f)))/g, Ne(g, 0)), ((-c*d*x/e**2 + c*x*
*2/(2*e) + (a*e**2 + c*d**2)*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, T
rue))/e**2)/f**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,(cd^2+ae^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^2f-deg)\sqrt{-e^2f+deg}} + \frac{2\,(cf^2+ag^2)}{(efg^2-dg^3)\sqrt{gx+f}} + \frac{2\,\sqrt{gx+f}c}{eg^2}$$

input
```
integrate((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
2*(c*d^2 + a*e^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/((e^2*f - d
*e*g)*sqrt(-e^2*f + d*e*g)) + 2*(c*f^2 + a*g^2)/((e*f*g^2 - d*g^3)*sqrt(g*
x + f)) + 2*sqrt(g*x + f)*c/(e*g^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.26

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{2\sqrt{f+g\,x}\,\left(c\,d^2+a\,e^2\right)\,\left(e^2\,f-d\,e\,g\right)}{\sqrt{e}\,\left(2\,c\,d^2+2\,a\,e^2\right)\,{\left(d\,g-e\,f\right)}^{3/2}}\right)\,\left(c\,d^2+a\,e^2\right)}{e^{3/2}\,{\left(d\,g-e\,f\right)}^{3/2}} + \frac{2\,c\,\sqrt{f+g\,x}}{e\,g^2} - \frac{2\,\left(c\,e\,f^2+a\,e\,g^2\right)}{e\,g^2\,\sqrt{f+g\,x}\,\left(d\,g-e\,f\right)}$$

input
```
int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)),x)
```

output
```
(2*atan((2*(f + g*x)^(1/2)*(a*e^2 + c*d^2)*(e^2*f - d*e*g))/(e^(1/2)*(2*a*
e^2 + 2*c*d^2)*(d*g - e*f)^(3/2)))*(a*e^2 + c*d^2))/(e^(3/2)*(d*g - e*f)^(
3/2)) + (2*c*(f + g*x)^(1/2))/(e*g^2) - (2*(a*e*g^2 + c*e*f^2))/(e*g^2*(f
+ g*x)^(1/2)*(d*g - e*f))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.97

$$\int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{-2\sqrt{e}\,\sqrt{gx+f}\,\sqrt{dg-ef}\,atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-ef}}\right)a\,e^2g^2-2\sqrt{e}\,\sqrt{gx+f}\,\sqrt{dg-ef}\,c}{}$$

input
```
int((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x)
```

output
```
(2*( - sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(
e)*sqrt(d*g - e*f)))*a*e**2*g**2 - sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*a
tan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**2*g**2 - a*d*e**2*g*
*3 + a*e**3*f*g**2 + c*d**2*e*f*g**2 + c*d**2*e*g**3*x - 3*c*d*e**2*f**2*g
 - 2*c*d*e**2*f*g**2*x + 2*c*e**3*f**3 + c*e**3*f**2*g*x))/(sqrt(f + g*x)*
e**2*g**2*(d*g - e*f)*(d**2*g**2 - 2*d*e*f*g + e**2*f**2))
```

3.61 $\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 144

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = -\frac{2(cf^2+ag^2)}{g(ef-dg)^2\sqrt{f+gx}} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{e(ef-dg)^2(d+ex)} + \frac{(3ae^2g+cd(4ef-dg))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}}$$

output
```
(-2*a*g^2-2*c*f^2)/g/(-d*g+e*f)^2/(g*x+f)^(1/2)-(a*e^2+c*d^2)*(g*x+f)^(1/2
)/e/(-d*g+e*f)^2/(e*x+d)+(3*a*e^2*g+c*d*(-d*g+4*e*f))*arctanh(e^(1/2)*(g*x
+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/(-d*g+e*f)^(5/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{-c(2def^2+2e^2f^2x+d^2g(f+gx))-aeg(2dg+e(f+3gx))}{eg(ef-dg)^2(d+ex)\sqrt{f+gx}} + \frac{(-3ae^2g+cd(-4ef+dg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{3/2}(-ef+dg)^{5/2}}$$

input
```
Integrate[(a + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

output
```
(-(c*(2*d*e*f^2 + 2*e^2*f^2*x + d^2*g*(f + g*x))) - a*e*g*(2*d*g + e*(f +
3*g*x)))/(e*g*(e*f - d*g)^2*(d + e*x)*Sqrt[f + g*x]) + ((-3*a*e^2*g + c*d*
(-4*e*f + d*g))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(3/
2)*(-(e*f) + d*g)^(5/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {649, 1582, 25, 27, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{649}$$

$$\frac{2\int \frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(f+gx)(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{g}$$

$$\Big\downarrow \text{1582}$$

$$\frac{2\left(\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{2e(ef-dg)^2(-dg-e(f+gx)+ef)}-\frac{\int -\frac{e\left(2e(ef-dg)\left(cf^2+ag^2\right)+\left(ae^2g^2-c\left(2e^2f^2-4degf+d^2g^2\right)\right)(f+gx)\right)}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{2e^2(ef-dg)^2}\right)}{g}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(\frac{\int \frac{e\left(2e(ef-dg)\left(cf^2+ag^2\right)+\left(ae^2g^2-c\left(2e^2f^2-4degf+d^2g^2\right)\right)(f+gx)\right)}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{2e^2(ef-dg)^2}+\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{2e(ef-dg)^2(-dg-e(f+gx)+ef)}\right)}{g}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{\int \frac{2e(ef-dg)\left(cf^2+ag^2\right)+\left(ae^2g^2-c\left(2e^2f^2-4degf+d^2g^2\right)\right)(f+gx)}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{2e(ef-dg)^2}+\frac{g^2\sqrt{f+gx}\left(ae^2+cd^2\right)}{2e(ef-dg)^2(-dg-e(f+gx)+ef)}\right)}{g}$$

$$\Big\downarrow \textbf{359}$$

$$\frac{2\left(\frac{g\left(3ae^2g+cd(4ef-dg)\right)\int \frac{1}{ef-dg-e(f+gx)}d\sqrt{f+gx}-\frac{2e\left(ag^2+cf^2\right)}{\sqrt{f+gx}}}{2e(ef-dg)^2}+\frac{g^2\sqrt{f+gx}\left(ae^2+cd^2\right)}{2e(ef-dg)^2(-dg-e(f+gx)+ef)}\right)}{g}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{2\left(\frac{\frac{g\left(3ae^2g+cd(4ef-dg)\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}\sqrt{ef-dg}}-\frac{2e\left(ag^2+cf^2\right)}{\sqrt{f+gx}}}{2e(ef-dg)^2}+\frac{g^2\sqrt{f+gx}\left(ae^2+cd^2\right)}{2e(ef-dg)^2(-dg-e(f+gx)+ef)}\right)}{g}$$

input

```
Int[(a + c*x^2)/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

output

```
(2*(((c*d^2 + a*e^2)*g^2*Sqrt[f + g*x])/(2*e*(e*f - d*g)^2*(e*f - d*g - e*
(f + g*x))) + ((-2*e*(c*f^2 + a*g^2))/Sqrt[f + g*x] + (g*(3*a*e^2*g + c*d*
(4*e*f - d*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(Sqrt[e]*
Sqrt[e*f - d*g]))/(2*e*(e*f - d*g)^2)))/g
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{-\frac{2g\left(\frac{g\left(a\,e^{2}+c\,d^{2}\right)\sqrt{gx+f}}{2e\left(e\left(gx+f\right)+dg-ef\right)}+\frac{\left(3a\,e^{2}g-c\,d^{2}g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{\left(dg-ef\right)e}}\right)}{2e\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^{2}}-\frac{2\left(a\,g^{2}+c\,f^{2}\right)}{\left(dg-ef\right)^{2}\sqrt{gx+f}}}{g}$
default	$\frac{-\frac{2g\left(\frac{g\left(a\,e^{2}+c\,d^{2}\right)\sqrt{gx+f}}{2e\left(e\left(gx+f\right)+dg-ef\right)}+\frac{\left(3a\,e^{2}g-c\,d^{2}g+4cdef\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{\left(dg-ef\right)e}}\right)}{2e\sqrt{\left(dg-ef\right)e}}\right)}{\left(dg-ef\right)^{2}}-\frac{2\left(a\,g^{2}+c\,f^{2}\right)}{\left(dg-ef\right)^{2}\sqrt{gx+f}}}{g}$
pseudoelliptic	$-\frac{2\left(\frac{3\left(a\,e^{2}g-\frac{1}{3}c\,d^{2}g+\frac{4}{3}cdef\right)g\left(ex+d\right)\sqrt{gx+f}\,\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{\left(dg-ef\right)e}}\right)}{2}+\sqrt{\left(dg-ef\right)e}\left(\left(\frac{3}{2}a\,g^{2}x+c\,f^{2}x+\frac{1}{2}afg\right)e^{2}+d\left(a\,g^{2}+c\right.\right.\right.}{\sqrt{gx+f}\,\sqrt{\left(dg-ef\right)e}\,g\left(ex+d\right)\left(dg-ef\right)^{2}e}$

input
```
int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/g*(-(a*g^2+c*f^2)/(d*g-e*f)^2/(g*x+f)^(1/2)-g/(d*g-e*f)^2*(1/2*g*(a*e^2+
c*d^2)/e*(g*x+f)^(1/2)/(e*(g*x+f)+d*g-e*f)+1/2*(3*a*e^2*g-c*d^2*g+4*c*d*e*
f)/e/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(130) = 260$.

Time = 0.12 (sec) , antiderivative size = 906, normalized size of antiderivative = 6.29

$$\int \frac{a + cx^2}{(d + ex)^2 (f + gx)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((4*c*d^2*e*f^2*g - (c*d^3 - 3*a*d*e^2)*f*g^2 + (4*c*d*e^2*f*g^2 - (c
*d^2*e - 3*a*e^3)*g^3)*x^2 + (4*c*d*e^2*f^2*g + 3*(c*d^2*e + a*e^3)*f*g^2
- (c*d^3 - 3*a*d*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g
 + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(2*c*d*e^3*f^3 - 2*
a*d^2*e^2*g^3 - (c*d^2*e^2 - a*e^4)*f^2*g - (c*d^3*e - a*d*e^3)*f*g^2 + (2
*c*e^4*f^3 - 2*c*d*e^3*f^2*g + (c*d^2*e^2 + 3*a*e^4)*f*g^2 - (c*d^3*e + 3*
a*d*e^3)*g^3)*x)*sqrt(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e
^3*f^2*g^3 - d^4*e^2*f*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*
g^4 - d^3*e^3*g^5)*x^2 + (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 -
d^4*e^2*g^5)*x), -((4*c*d^2*e*f^2*g - (c*d^3 - 3*a*d*e^2)*f*g^2 + (4*c*d*e
^2*f*g^2 - (c*d^2*e - 3*a*e^3)*g^3)*x^2 + (4*c*d*e^2*f^2*g + 3*(c*d^2*e +
a*e^3)*f*g^2 - (c*d^3 - 3*a*d*e^2)*g^3)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqr
t(-e^2*f + d*e*g)*sqrt(g*x + f)/(e*g*x + e*f)) + (2*c*d*e^3*f^3 - 2*a*d^2*
e^2*g^3 - (c*d^2*e^2 - a*e^4)*f^2*g - (c*d^3*e - a*d*e^3)*f*g^2 + (2*c*e^4
*f^3 - 2*c*d*e^3*f^2*g + (c*d^2*e^2 + 3*a*e^4)*f*g^2 - (c*d^3*e + 3*a*d*e^
3)*g^3)*x)*sqrt(g*x + f))/(d*e^5*f^4*g - 3*d^2*e^4*f^3*g^2 + 3*d^3*e^3*f^2
*g^3 - d^4*e^2*f*g^4 + (e^6*f^3*g^2 - 3*d*e^5*f^2*g^3 + 3*d^2*e^4*f*g^4 -
d^3*e^3*g^5)*x^2 + (e^6*f^4*g - 2*d*e^5*f^3*g^2 + 2*d^3*e^3*f*g^4 - d^4*e^
2*g^5)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)/(e*x+d)**2/(g*x+f)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.58

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = -\frac{(4\,cdef-cd^2g+3\,ae^2g)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^3f^2-2\,de^2fg+d^2eg^2)\sqrt{-e^2f+deg}} - \frac{2\,(gx+f)ce^2f^2-2\,ce^2f^3+2\,cdef^2g+(gx+f)cd^2g^2+3\,(gx+f)ae^2g^2-2\,ae^2fg^2+2\,adeg^3}{(e^3f^2g-2\,de^2fg^2+d^2eg^3)\left((gx+f)^{\frac{3}{2}}e-\sqrt{gx+f}ef+\sqrt{gx+f}dg\right)}$$

input
```
integrate((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
-(4*c*d*e*f - c*d^2*g + 3*a*e^2*g)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*
e*g))/((e^3*f^2 - 2*d*e^2*f*g + d^2*e*g^2)*sqrt(-e^2*f + d*e*g)) - (2*(g*x
 + f)*c*e^2*f^2 - 2*c*e^2*f^3 + 2*c*d*e*f^2*g + (g*x + f)*c*d^2*g^2 + 3*(g
*x + f)*a*e^2*g^2 - 2*a*e^2*f*g^2 + 2*a*d*e*g^3)/((e^3*f^2*g - 2*d*e^2*f*g
^2 + d^2*e*g^3)*((g*x + f)^(3/2)*e - sqrt(g*x + f)*e*f + sqrt(g*x + f)*d*g
))
```

Mupad [B] (verification not implemented)

Time = 6.38 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.30

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = -\frac{\frac{2\,(c\,f^2+a\,g^2)}{d\,g-e\,f}+\frac{(f+g\,x)\,(c\,d^2\,g^2+2\,c\,e^2\,f^2+3\,a\,e^2\,g^2)}{e\,{(d\,g-e\,f)}^2}}{\sqrt{f+g\,x}\,(d\,g^2-e\,f\,g)+e\,g\,{(f+g\,x)}^{3/2}} - \frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,(d^2\,e\,g^2-2\,d\,e^2\,f\,g+e^3\,f^2)}{\sqrt{e}\,{(d\,g-e\,f)}^{5/2}}\right)\,(-c\,g\,d^2+4\,c\,f\,d\,e+3\,a\,g\,e^2)}{e^{3/2}\,{(d\,g-e\,f)}^{5/2}}$$

input
```
int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^2),x)
```

output
```
- ((2*(a*g^2 + c*f^2))/(d*g - e*f) + ((f + g*x)*(3*a*e^2*g^2 + c*d^2*g^2 +
 2*c*e^2*f^2))/(e*(d*g - e*f)^2))/((f + g*x)^(1/2)*(d*g^2 - e*f*g) + e*g*(
f + g*x)^(3/2)) - (atan(((f + g*x)^(1/2)*(e^3*f^2 + d^2*e*g^2 - 2*d*e^2*f*
g))/(e^(1/2)*(d*g - e*f)^(5/2)))*(3*a*e^2*g - c*d^2*g + 4*c*d*e*f))/(e^(3/
2)*(d*g - e*f)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 535, normalized size of antiderivative = 3.72

$$\int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{-3\sqrt{e}\,\sqrt{gx+f}\,\sqrt{dg-ef}\,atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-ef}}\right)ad\,e^2g^2-3\sqrt{e}\,\sqrt{gx+f}\,\sqrt{dg-e}}{}$$

input
```
int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2),x)
```

output
```
( - 3*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e
)*sqrt(d*g - e*f)))*a*d*e**2*g**2 - 3*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f
)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*e**3*g**2*x + sqrt(e
)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g -
 e*f)))*c*d**3*g**2 - 4*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f
 + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**2*e*f*g + sqrt(e)*sqrt(f + g*x)
*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**2*
e*g**2*x - 4*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/
(sqrt(e)*sqrt(d*g - e*f)))*c*d*e**2*f*g*x - 2*a*d**2*e**2*g**3 + a*d*e**3*
f*g**2 - 3*a*d*e**3*g**3*x + a*e**4*f**2*g + 3*a*e**4*f*g**2*x - c*d**3*e*
f*g**2 - c*d**3*e*g**3*x - c*d**2*e**2*f**2*g + c*d**2*e**2*f*g**2*x + 2*c
*d*e**3*f**3 - 2*c*d*e**3*f**2*g*x + 2*c*e**4*f**3*x)/(sqrt(f + g*x)*e**2*
g*(d**4*g**3 - 3*d**3*e*f*g**2 + d**3*e*g**3*x + 3*d**2*e**2*f**2*g - 3*d*
*2*e**2*f*g**2*x - d*e**3*f**3 + 3*d*e**3*f**2*g*x - e**4*f**3*x))
```

3.62 $\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 214

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{2(cf^2+ag^2)}{(ef-dg)^3\sqrt{f+gx}} - \frac{(cd^2+ae^2)\sqrt{f+gx}}{2e(ef-dg)^2(d+ex)^2} + \frac{(7ae^2g+cd(8ef-dg))\sqrt{f+gx}}{4e(ef-dg)^3(d+ex)} - \frac{(15ae^2g^2+c(8e^2f^2+8defg-d^2g^2))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}}$$

output
```
2*(a*g^2+c*f^2)/(-d*g+e*f)^3/(g*x+f)^(1/2)-1/2*(a*e^2+c*d^2)*(g*x+f)^(1/2)
/e/(-d*g+e*f)^2/(e*x+d)^2+1/4*(7*a*e^2*g+c*d*(-d*g+8*e*f))*(g*x+f)^(1/2)/e
/(-d*g+e*f)^3/(e*x+d)-1/4*(15*a*e^2*g^2+c*(-d^2*g^2+8*d*e*f*g+8*e^2*f^2))*
arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(3/2)/(-d*g+e*f)^(7/2)
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.07

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{\frac{\sqrt{e}\left(c\left(8e^3f^2x^2+d^3g(f+gx)+8de^2fx(3f+gx)+d^2e\left(14f^2+5fgx-g^2x^2\right)\right)+ae\left(8d^2g^2+deg(9f+25gx)+e^2 \cdots\right.\right.}{(ef-dg)^3(d+ex)^2\sqrt{f+gx}}}{4e^{3/2}}$$

input
```
Integrate[(a + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

output
```
((Sqrt[e]*(c*(8*e^3*f^2*x^2 + d^3*g*(f + g*x) + 8*d*e^2*f*x*(3*f + g*x) +
d^2*e*(14*f^2 + 5*f*g*x - g^2*x^2)) + a*e*(8*d^2*g^2 + d*e*g*(9*f + 25*g*x
) + e^2*(-2*f^2 + 5*f*g*x + 15*g^2*x^2))))/((e*f - d*g)^3*(d + e*x)^2*Sqrt
[f + g*x]) - ((15*a*e^2*g^2 + c*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2))*ArcTan[
(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(-(e*f) + d*g)^(7/2))/(4*e^(3
/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {649, 25, 1582, 25, 27, 361, 25, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}dx$$

$$\Big\downarrow \text{649}$$

$$2\int -\frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(f+gx)(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{cf^2-2c(f+gx)f+ag^2+c(f+gx)^2}{(f+gx)(ef-dg-e(f+gx))^3}d\sqrt{f+gx}$$

$$\Big\downarrow \text{1582}$$

$$2\left(\frac{\int -\frac{e\left(4e(ef-dg)\left(cf^2+ag^2\right)+\left(3ae^2g^2-c\left(4e^2f^2-8degf+d^2g^2\right)\right)(f+gx)\right)}{(f+gx)(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4e^2(ef-dg)^2}-\frac{g^2\sqrt{f+gx}\left(ae^2+cd^2\right)}{4e(ef-dg)^2(-dg-e(f+gx)+ef)^2}\right.$$

$$\Big\downarrow \; 25$$

$$2\left(-\frac{\int \frac{e\left(4e(ef-dg)\left(cf^2+ag^2\right)+\left(3ae^2g^2-c\left(4e^2f^2-8degf+d^2g^2\right)\right)(f+gx)\right)}{(f+gx)(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4e^2(ef-dg)^2}-\frac{g^2\sqrt{f+gx}\left(ae^2+cd^2\right)}{4e(ef-dg)^2(-dg-e(f+gx)+ef)^2}\right.$$

$$\Big\downarrow \; 27$$

$$2\left(-\frac{\int \frac{4e(ef-dg)\left(cf^2+ag^2\right)+\left(3ae^2g^2-c\left(4e^2f^2-8degf+d^2g^2\right)\right)(f+gx)}{(f+gx)(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{4e(ef-dg)^2}-\frac{g^2\sqrt{f+gx}\left(ae^2+cd^2\right)}{4e(ef-dg)^2(-dg-e(f+gx)+ef)^2}\right)$$

$$\Big\downarrow \; 361$$

$$2\left(-\frac{\frac{g\sqrt{f+gx}\left(7ae^2g+cd(8ef-dg)\right)}{2(ef-dg)(-dg-e(f+gx)+ef)}-\frac{1}{2}\int -\frac{8e\left(cf^2+ag^2\right)+\frac{g\left(7age^2+cd(8ef-dg)\right)(f+gx)}{ef-dg}}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{4e(ef-dg)^2}-\frac{g^2\sqrt{f+gx}\left(ae^2+c\right.}{4e(ef-dg)^2(-dg-e(f+}\right.$$

$$\Big\downarrow \; 25$$

$$2\left(-\frac{\frac{1}{2}\int \frac{8e\left(cf^2+ag^2\right)+\frac{g\left(7age^2+cd(8ef-dg)\right)(f+gx)}{ef-dg}}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}+\frac{g\sqrt{f+gx}\left(7ae^2g+cd(8ef-dg)\right)}{2(ef-dg)(-dg-e(f+gx)+ef)}}{4e(ef-dg)^2}-\frac{g^2\sqrt{f+gx}\left(ae^2+cd\right.}{4e(ef-dg)^2(-dg-e(f+g}\right.$$

$$\Big\downarrow \; 359$$

$$2\left(-\frac{\frac{1}{2}\left(\frac{\left(15ae^2g^2+c\left(-d^2g^2+8defg+8e^2f^2\right)\right)\int \frac{1}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{ef-dg}-\frac{8e\left(ag^2+cf^2\right)}{\sqrt{f+gx}(ef-dg)}\right)+\frac{g\sqrt{f+gx}\left(7ae^2g+cd(8ef-dg)\right)}{2(ef-dg)(-dg-e(f+gx)+ef)}}{4e(ef-dg)^2}-\frac{}{4e(e}\right.$$

$$\Big\downarrow \; 221$$

$$2\left(-\frac{\frac{1}{2}\left(\frac{(15ae^2g^2+c(-d^2g^2+8defg+8e^2f^2))\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}(ef-dg)^{3/2}}-\frac{8e(ag^2+cf^2)}{\sqrt{f+gx}(ef-dg)}\right)+\frac{g\sqrt{f+gx}(7ae^2g+cd(8ef-dg))}{2(ef-dg)(-dg-e(f+gx)+ef)}}{4e(ef-dg)^2}-\frac{}{4e(ef-}\right.$$

input
```
Int[(a + c*x^2)/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

output
```
2*(-1/4*((c*d^2 + a*e^2)*g^2*Sqrt[f + g*x])/(e*(e*f - d*g)^2*(e*f - d*g -
e*(f + g*x))^2) - ((g*(7*a*e^2*g + c*d*(8*e*f - d*g))*Sqrt[f + g*x])/(2*(e
*f - d*g)*(e*f - d*g - e*(f + g*x))) + ((-8*e*(c*f^2 + a*g^2))/((e*f - d*g
)*Sqrt[f + g*x]) + ((15*a*e^2*g^2 + c*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2))*A
rcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(Sqrt[e]*(e*f - d*g)^(3/2
)))/2)/(4*e*(e*f - d*g)^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 649

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1582

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.07

method	result
derivativedivides	$-\frac{2(a\,g^2+c\,f^2)}{(dg-ef)^3\sqrt{gx+f}}-\frac{2\left(\frac{\left(\frac{7}{8}a\,e^2g^2-\frac{1}{8}c\,d^2g^2+cdefg\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9ad\,e^2g^2-9a\,e^3fg+c\,d^3g^2+7c\,d^2efg-8cd\,e^2f^2\right)\sqrt{gx+f}}{8e}}{(e(gx+f)+dg-ef)^2}\right.}{(dg-ef)^3}$
default	$-\frac{2(a\,g^2+c\,f^2)}{(dg-ef)^3\sqrt{gx+f}}-\frac{2\left(\frac{\left(\frac{7}{8}a\,e^2g^2-\frac{1}{8}c\,d^2g^2+cdefg\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9ad\,e^2g^2-9a\,e^3fg+c\,d^3g^2+7c\,d^2efg-8cd\,e^2f^2\right)\sqrt{gx+f}}{8e}}{(e(gx+f)+dg-ef)^2}\right.}{(dg-ef)^3}$
pseudoelliptic	$-\frac{2\left(\frac{15(ex+d)^2\sqrt{gx+f}\left(\left(a\,g^2+\frac{8c\,f^2}{15}\right)e^2+\frac{8cdefg}{15}-\frac{c\,d^2g^2}{15}\right)\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{8}+\left(\left(\frac{15a\,g^2x^2}{8}+\frac{5afgx}{8}-\frac{f^2\left(-4c\,x^2+a\right)}{4}\right.\right.\right.}{\sqrt{gx+f}\,\sqrt{(dg-ef)e}\,(d}$

input
```
int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a*g^2+c*f^2)/(d*g-e*f)^3/(g*x+f)^(1/2)-2/(d*g-e*f)^3*(((7/8*a*e^2*g^2-
1/8*c*d^2*g^2+c*d*e*f*g)*(g*x+f)^(3/2)+1/8*g*(9*a*d*e^2*g^2-9*a*e^3*f*g+c*
d^3*g^2+7*c*d^2*e*f*g-8*c*d*e^2*f^2)/e*(g*x+f)^(1/2))/(e*(g*x+f)+d*g-e*f)^
2+1/8*(15*a*e^2*g^2-c*d^2*g^2+8*c*d*e*f*g+8*c*e^2*f^2)/e/((d*g-e*f)*e)^(1/
2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(192) = 384$.

Time = 0.16 (sec) , antiderivative size = 1539, normalized size of antiderivative = 7.19

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*((8*c*d^2*e^2*f^3 + 8*c*d^3*e*f^2*g - (c*d^4 - 15*a*d^2*e^2)*f*g^2 +
 (8*c*e^4*f^2*g + 8*c*d*e^3*f*g^2 - (c*d^2*e^2 - 15*a*e^4)*g^3)*x^3 + (8*c
*e^4*f^3 + 24*c*d*e^3*f^2*g + 15*(c*d^2*e^2 + a*e^4)*f*g^2 - 2*(c*d^3*e -
15*a*d*e^3)*g^3)*x^2 + (16*c*d*e^3*f^3 + 24*c*d^2*e^2*f^2*g + 6*(c*d^3*e +
 5*a*d*e^3)*f*g^2 - (c*d^4 - 15*a*d^2*e^2)*g^3)*x)*sqrt(e^2*f - d*e*g)*log
((e*g*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) +
2*(8*a*d^3*e^2*g^3 - 2*(7*c*d^2*e^3 - a*e^5)*f^3 + (13*c*d^3*e^2 - 11*a*d*
e^4)*f^2*g + (c*d^4*e + a*d^2*e^3)*f*g^2 - (8*c*e^5*f^3 - 3*(3*c*d^2*e^3 -
 5*a*e^5)*f*g^2 + (c*d^3*e^2 - 15*a*d*e^4)*g^3)*x^2 - (24*c*d*e^4*f^3 - (1
9*c*d^2*e^3 - 5*a*e^5)*f^2*g - 4*(c*d^3*e^2 - 5*a*d*e^4)*f*g^2 - (c*d^4*e
+ 25*a*d^2*e^3)*g^3)*x)*sqrt(g*x + f))/(d^2*e^6*f^5 - 4*d^3*e^5*f^4*g + 6*
d^4*e^4*f^3*g^2 - 4*d^5*e^3*f^2*g^3 + d^6*e^2*f*g^4 + (e^8*f^4*g - 4*d*e^7
*f^3*g^2 + 6*d^2*e^6*f^2*g^3 - 4*d^3*e^5*f*g^4 + d^4*e^4*g^5)*x^3 + (e^8*f
^5 - 2*d*e^7*f^4*g - 2*d^2*e^6*f^3*g^2 + 8*d^3*e^5*f^2*g^3 - 7*d^4*e^4*f*g
^4 + 2*d^5*e^3*g^5)*x^2 + (2*d*e^7*f^5 - 7*d^2*e^6*f^4*g + 8*d^3*e^5*f^3*g
^2 - 2*d^4*e^4*f^2*g^3 - 2*d^5*e^3*f*g^4 + d^6*e^2*g^5)*x), 1/4*((8*c*d^2*
e^2*f^3 + 8*c*d^3*e*f^2*g - (c*d^4 - 15*a*d^2*e^2)*f*g^2 + (8*c*e^4*f^2*g
+ 8*c*d*e^3*f*g^2 - (c*d^2*e^2 - 15*a*e^4)*g^3)*x^3 + (8*c*e^4*f^3 + 24*c*
d*e^3*f^2*g + 15*(c*d^2*e^2 + a*e^4)*f*g^2 - 2*(c*d^3*e - 15*a*d*e^3)*g^3)
*x^2 + (16*c*d*e^3*f^3 + 24*c*d^2*e^2*f^2*g + 6*(c*d^3*e + 5*a*d*e^3)*f...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + cx^2}{(d + ex)^3 (f + gx)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)/(e*x+d)**3/(g*x+f)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.72

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{(8\,ce^2f^2+8\,cdefg-cd^2g^2+15\,ae^2g^2)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{4\,(e^4f^3-3\,de^3f^2g+3\,d^2e^2fg^2-d^3eg^3)\sqrt{-e^2f+deg}} + \frac{2\,(cf^2+ag^2)}{(e^3f^3-3\,de^2f^2g+3\,d^2efg^2-d^3g^3)\sqrt{gx+f}} + \frac{8\,(gx+f)^{\frac{3}{2}}cde^2fg-8\,\sqrt{gx+f}cde^2f^2g-(gx+f)^{\frac{3}{2}}cd^2eg^2+7\,(gx+f)^{\frac{3}{2}}ae^3g^2+7\,\sqrt{gx+f}cd^2efg^2-9}{4\,(e^4f^3-3\,de^3f^2g+3\,d^2e^2fg^2-d^3eg^3)((gx+f)e-ef+}$$

input
```
integrate((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
1/4*(8*c*e^2*f^2 + 8*c*d*e*f*g - c*d^2*g^2 + 15*a*e^2*g^2)*arctan(sqrt(g*x
 + f)*e/sqrt(-e^2*f + d*e*g))/((e^4*f^3 - 3*d*e^3*f^2*g + 3*d^2*e^2*f*g^2
- d^3*e*g^3)*sqrt(-e^2*f + d*e*g)) + 2*(c*f^2 + a*g^2)/((e^3*f^3 - 3*d*e^2
*f^2*g + 3*d^2*e*f*g^2 - d^3*g^3)*sqrt(g*x + f)) + 1/4*(8*(g*x + f)^(3/2)*
c*d*e^2*f*g - 8*sqrt(g*x + f)*c*d*e^2*f^2*g - (g*x + f)^(3/2)*c*d^2*e*g^2
+ 7*(g*x + f)^(3/2)*a*e^3*g^2 + 7*sqrt(g*x + f)*c*d^2*e*f*g^2 - 9*sqrt(g*x
 + f)*a*e^3*f*g^2 + sqrt(g*x + f)*c*d^3*g^3 + 9*sqrt(g*x + f)*a*d*e^2*g^3)
/((e^4*f^3 - 3*d*e^3*f^2*g + 3*d^2*e^2*f*g^2 - d^3*e*g^3)*((g*x + f)*e - e
*f + d*g)^2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.45

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{f+g\,x}\,\left(-d^3\,e\,g^3+3\,d^2\,e^2\,f\,g^2-3\,d\,e^3\,f^2\,g+e^4\,f^3\right)}{\sqrt{e}\,{\left(d\,g-e\,f\right)}^{7/2}}\right)\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g+8\,c\,e^2\,f\right.}{4\,e^{3/2}\,{\left(d\,g-e\,f\right)}^{7/2}} - \frac{\frac{2\,\left(c\,f^2+a\,g^2\right)}{d\,g-e\,f}+\frac{{\left(f+g\,x\right)}^2\,\left(-c\,d^2\,g^2+8\,c\,d\,e\,f\,g+8\,c\,e^2\,f^2+15\,a\,e^2\,g^2\right)}{4\,{\left(d\,g-e\,f\right)}^3}+\frac{\left(f+g\,x\right)\,\left(c\,d^2\,g^2+8\,c\,d\,e\,f\,g+16\,c\,e^2\,f^2+25\,a\,e^2\,g^2\right)}{4\,e\,{\left(d\,g-e\,f\right)}^2}}{e^2\,{\left(f+g\,x\right)}^{5/2}-{\left(f+g\,x\right)}^{3/2}\,\left(2\,e^2\,f-2\,d\,e\,g\right)+\sqrt{f+g\,x}\,\left(d^2\,g^2-2\,d\,e\,f\,g+e^2\,f^2\right)}$$

input
```
int((a + c*x^2)/((f + g*x)^(3/2)*(d + e*x)^3),x)
```

output
```
(atan(((f + g*x)^(1/2)*(e^4*f^3 - d^3*e*g^3 + 3*d^2*e^2*f*g^2 - 3*d*e^3*f^
2*g))/(e^(1/2)*(d*g - e*f)^(7/2)))*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2
 + 8*c*d*e*f*g))/(4*e^(3/2)*(d*g - e*f)^(7/2)) - ((2*(a*g^2 + c*f^2))/(d*g
 - e*f) + ((f + g*x)^2*(15*a*e^2*g^2 - c*d^2*g^2 + 8*c*e^2*f^2 + 8*c*d*e*f
*g))/(4*(d*g - e*f)^3) + ((f + g*x)*(25*a*e^2*g^2 + c*d^2*g^2 + 16*c*e^2*f
^2 + 8*c*d*e*f*g))/(4*e*(d*g - e*f)^2))/(e^2*(f + g*x)^(5/2) - (f + g*x)^(
3/2)*(2*e^2*f - 2*d*e*g) + (f + g*x)^(1/2)*(d^2*g^2 + e^2*f^2 - 2*d*e*f*g)
)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1062, normalized size of antiderivative = 4.96

$$\int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+a)/(e*x+d)^3/(g*x+f)^(3/2),x)
```

output
```
( - 15*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(
e)*sqrt(d*g - e*f)))*a*d**2*e**2*g**2 - 30*sqrt(e)*sqrt(f + g*x)*sqrt(d*g
- e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*d*e**3*g**2*x -
 15*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*
sqrt(d*g - e*f)))*a*e**4*g**2*x**2 + sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)
*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**4*g**2 - 8*sqrt(e)
*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g -
e*f)))*c*d**3*e*f*g + 2*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f
 + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**3*e*g**2*x - 8*sqrt(e)*sqrt(f +
 g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*
d**2*e**2*f**2 - 16*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g
*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**2*e**2*f*g*x + sqrt(e)*sqrt(f + g*x
)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d**2
*e**2*g**2*x**2 - 16*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f +
g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d*e**3*f**2*x - 8*sqrt(e)*sqrt(f + g*
x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*d*e
**3*f*g*x**2 - 8*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)
*e)/(sqrt(e)*sqrt(d*g - e*f)))*c*e**4*f**2*x**2 - 8*a*d**3*e**2*g**3 - a*d
**2*e**3*f*g**2 - 25*a*d**2*e**3*g**3*x + 11*a*d*e**4*f**2*g + 20*a*d*e**4
*f*g**2*x - 15*a*d*e**4*g**3*x**2 - 2*a*e**5*f**3 + 5*a*e**5*f**2*g*x +...
```

3.63 $\int (d+ex)^3\sqrt{f+gx}\left(a+cx^2\right)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 474

$$
\begin{aligned}
\int (d+ex)^3\sqrt{f+gx}\left(a+cx^2\right)^2\,dx = {} & -\frac{2(ef-dg)^3\left(cf^2+ag^2\right)^2(f+gx)^{3/2}}{3g^8} \\
& + \frac{2(ef-dg)^2\left(cf^2+ag^2\right)\left(3aeg^2+cf(7ef-4dg)\right)(f+gx)^{5/2}}{5g^8} \\
& - \frac{2(ef-dg)\left(3a^2e^2g^4+2acg^2(10e^2f^2-8defg+d^2g^2)+3c^2f^2(7e^2f^2-8defg+2d^2g^2)\right)(f+gx)^{7/2}}{7g^8} \\
& + \frac{2\left(a^2e^3g^4+2aceg^2(10e^2f^2-12defg+3d^2g^2)+c^2f(35e^3f^3-60de^2f^2g+30d^2efg^2-4d^3g^3)\right)(f+g}{9g^8} \\
& - \frac{2c\left(2ae^2g^2(5ef-3dg)+c(35e^3f^3-45de^2f^2g+15d^2efg^2-d^3g^3)\right)(f+gx)^{11/2}}{11g^8} \\
& + \frac{2ce\left(2ae^2g^2+3c(7e^2f^2-6defg+d^2g^2)\right)(f+gx)^{13/2}}{13g^8} \\
& - \frac{2c^2e^2(7ef-3dg)(f+gx)^{15/2}}{15g^8} + \frac{2c^2e^3(f+gx)^{17/2}}{17g^8}
\end{aligned}
$$

output
```
-2/3*(-d*g+e*f)^3*(a*g^2+c*f^2)^2*(g*x+f)^(3/2)/g^8+2/5*(-d*g+e*f)^2*(a*g^
2+c*f^2)*(3*a*e*g^2+c*f*(-4*d*g+7*e*f))*(g*x+f)^(5/2)/g^8-2/7*(-d*g+e*f)*(
3*a^2*e^2*g^4+2*a*c*g^2*(d^2*g^2-8*d*e*f*g+10*e^2*f^2)+3*c^2*f^2*(2*d^2*g^
2-8*d*e*f*g+7*e^2*f^2))*(g*x+f)^(7/2)/g^8+2/9*(a^2*e^3*g^4+2*a*c*e*g^2*(3*
d^2*g^2-12*d*e*f*g+10*e^2*f^2)+c^2*f*(-4*d^3*g^3+30*d^2*e*f*g^2-60*d*e^2*f
^2*g+35*e^3*f^3))*(g*x+f)^(9/2)/g^8-2/11*c*(2*a*e^2*g^2*(-3*d*g+5*e*f)+c*(
-d^3*g^3+15*d^2*e*f*g^2-45*d*e^2*f^2*g+35*e^3*f^3))*(g*x+f)^(11/2)/g^8+2/1
3*c*e*(2*a*e^2*g^2+3*c*(d^2*g^2-6*d*e*f*g+7*e^2*f^2))*(g*x+f)^(13/2)/g^8-2
/15*c^2*e^2*(-3*d*g+7*e*f)*(g*x+f)^(15/2)/g^8+2/17*c^2*e^3*(g*x+f)^(17/2)/
g^8
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.17

$$\int (d+ex)^3\sqrt{f+gx}\left(a+cx^2\right)^2\,dx$$
$$= \frac{2(f+gx)^{3/2}\left(2431a^2g^4(105d^3g^3+63d^2eg^2(-2f+3gx)+9de^2g(8f^2-12fgx+15g^2x^2)+e^3(-16f^3+\right.}{}$$

input
```
Integrate[(d + e*x)^3*Sqrt[f + g*x]*(a + c*x^2)^2,x]
```

output
```
(2*(f + g*x)^(3/2)*(2431*a^2*g^4*(105*d^3*g^3 + 63*d^2*e*g^2*(-2*f + 3*g*x
) + 9*d*e^2*g*(8*f^2 - 12*f*g*x + 15*g^2*x^2) + e^3*(-16*f^3 + 24*f^2*g*x
- 30*f*g^2*x^2 + 35*g^3*x^3)) + 34*a*c*g^2*(429*d^3*g^3*(8*f^2 - 12*f*g*x
+ 15*g^2*x^2) + 429*d^2*e*g^2*(-16*f^3 + 24*f^2*g*x - 30*f*g^2*x^2 + 35*g^
3*x^3) + 39*d*e^2*g*(128*f^4 - 192*f^3*g*x + 240*f^2*g^2*x^2 - 280*f*g^3*x
^3 + 315*g^4*x^4) - 5*e^3*(256*f^5 - 384*f^4*g*x + 480*f^3*g^2*x^2 - 560*f
^2*g^3*x^3 + 630*f*g^4*x^4 - 693*g^5*x^5)) + c^2*(221*d^3*g^3*(128*f^4 - 1
92*f^3*g*x + 240*f^2*g^2*x^2 - 280*f*g^3*x^3 + 315*g^4*x^4) + 255*d^2*e*g^
2*(-256*f^5 + 384*f^4*g*x - 480*f^3*g^2*x^2 + 560*f^2*g^3*x^3 - 630*f*g^4*
x^4 + 693*g^5*x^5) + 51*d*e^2*g*(1024*f^6 - 1536*f^5*g*x + 1920*f^4*g^2*x^
2 - 2240*f^3*g^3*x^3 + 2520*f^2*g^4*x^4 - 2772*f*g^5*x^5 + 3003*g^6*x^6) -
 7*e^3*(2048*f^7 - 3072*f^6*g*x + 3840*f^5*g^2*x^2 - 4480*f^4*g^3*x^3 + 50
40*f^3*g^4*x^4 - 5544*f^2*g^5*x^5 + 6006*f*g^6*x^6 - 6435*g^7*x^7))))/(765
765*g^8)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^2 (d+ex)^3 \sqrt{f+gx}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{(f+gx)^{5/2}(ef-dg)\left(-3a^2e^2g^4-2acg^2\left(d^2g^2-8defg+10e^2f^2\right)-3c^2f^2\left(2d^2g^2-8defg+7e^2f^2\right)\right)}{g^7}+\frac{(f$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(f+gx)^{7/2}(ef-dg)\left(3a^2e^2g^4+2acg^2\left(d^2g^2-8defg+10e^2f^2\right)+3c^2f^2\left(2d^2g^2-8defg+7e^2f^2\right)\right)}{7g^8}+\frac{2(f+gx)^{9/2}\left(a^2e^3g^4+2aceg^2\left(3d^2g^2-12defg+10e^2f^2\right)+c^2f\left(-4d^3g^3+30d^2efg^2-60de^2f^2g+35e^3f^3\right)\right)}{9g^8}+\frac{2ce(f+gx)^{13/2}\left(2ae^2g^2+3c\left(d^2g^2-6defg+7e^2f^2\right)\right)}{13g^8}-\frac{2c(f+gx)^{11/2}\left(2ae^2g^2(5ef-3dg)+c\left(-d^3g^3+15d^2efg^2-45de^2f^2g+35e^3f^3\right)\right)}{11g^8}+\frac{2(f+gx)^{5/2}\left(ag^2+cf^2\right)(ef-dg)^2\left(3aeg^2+cf(7ef-4dg)\right)}{5g^8}-\frac{2(f+gx)^{3/2}\left(ag^2+cf^2\right)^2(ef-dg)^3}{3g^8}-\frac{2c^2e^2(f+gx)^{15/2}(7ef-3dg)}{15g^8}+\frac{2c^2e^3(f+gx)^{17/2}}{17g^8}$$

input
```
Int[(d + e*x)^3*Sqrt[f + g*x]*(a + c*x^2)^2,x]
```

output
```
(-2*(e*f - d*g)^3*(c*f^2 + a*g^2)^2*(f + g*x)^(3/2))/(3*g^8) + (2*(e*f - d
*g)^2*(c*f^2 + a*g^2)*(3*a*e*g^2 + c*f*(7*e*f - 4*d*g))*(f + g*x)^(5/2))/(
5*g^8) - (2*(e*f - d*g)*(3*a^2*e^2*g^4 + 2*a*c*g^2*(10*e^2*f^2 - 8*d*e*f*g
 + d^2*g^2) + 3*c^2*f^2*(7*e^2*f^2 - 8*d*e*f*g + 2*d^2*g^2))*(f + g*x)^(7/
2))/(7*g^8) + (2*(a^2*e^3*g^4 + 2*a*c*e*g^2*(10*e^2*f^2 - 12*d*e*f*g + 3*d
^2*g^2) + c^2*f*(35*e^3*f^3 - 60*d*e^2*f^2*g + 30*d^2*e*f*g^2 - 4*d^3*g^3)
)*(f + g*x)^(9/2))/(9*g^8) - (2*c*(2*a*e^2*g^2*(5*e*f - 3*d*g) + c*(35*e^3
*f^3 - 45*d*e^2*f^2*g + 15*d^2*e*f*g^2 - d^3*g^3))*(f + g*x)^(11/2))/(11*g
^8) + (2*c*e*(2*a*e^2*g^2 + 3*c*(7*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*
x)^(13/2))/(13*g^8) - (2*c^2*e^2*(7*e*f - 3*d*g)*(f + g*x)^(15/2))/(15*g^8
) + (2*c^2*e^3*(f + g*x)^(17/2))/(17*g^8)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$2\left(\left(\left(\frac{6}{13}ac\,x^5+\frac{3}{17}c^2x^7+\frac{1}{3}a^2x^3\right)e^3+\frac{9\left(\frac{7}{15}c^2x^4+\frac{14}{11}ac\,x^2+a^2\right)x^2d\,e^2}{7}+\frac{9x\left(\frac{5}{13}c^2x^4+\frac{10}{9}ac\,x^2+a^2\right)d^2e}{5}+d^3\left(\frac{6}{7}ac\,x^2+\frac{3}{11}c^2x\right.\right.\right.$
derivativedivides	$\frac{2c^2e^3(gx+f)^{\frac{17}{2}}}{17}+\frac{2\left(3(dg-ef)e^2c^2-4f\,c^2e^3\right)(gx+f)^{\frac{15}{2}}}{15}+\frac{2\left(3(dg-ef)^2e\,c^2-12(dg-ef)e^2c^2f+e^3\left(2\left(a\,g^2+c\,f^2\right)c+4c^2f^2\right)\right)(gx+\right.}{13}$
default	$\frac{2c^2e^3(gx+f)^{\frac{17}{2}}}{17}+\frac{2\left(3(dg-ef)e^2c^2-4f\,c^2e^3\right)(gx+f)^{\frac{15}{2}}}{15}+\frac{2\left(3(dg-ef)^2e\,c^2-12(dg-ef)e^2c^2f+e^3\left(2\left(a\,g^2+c\,f^2\right)c+4c^2f^2\right)\right)(gx+\right.}{13}$
gosper	$2(gx+f)^{\frac{3}{2}}\left(45045c^2e^3x^7g^7+153153c^2d\,e^2g^7x^6-42042c^2e^3f\,g^6x^6+117810ac\,e^3g^7x^5+176715c^2d^2e\,g^7x^5-141372c^2d\,e\right.$
orering	$2(gx+f)^{\frac{3}{2}}\left(45045c^2e^3x^7g^7+153153c^2d\,e^2g^7x^6-42042c^2e^3f\,g^6x^6+117810ac\,e^3g^7x^5+176715c^2d^2e\,g^7x^5-141372c^2d\,e\right.$
trager	Expression too large to display
risch	Expression too large to display

input

```
int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output

```
2/3*(((6/13*a*c*x^5+3/17*c^2*x^7+1/3*a^2*x^3)*e^3+9/7*(7/15*c^2*x^4+14/11*
a*c*x^2+a^2)*x^2*d*e^2+9/5*x*(5/13*c^2*x^4+10/9*a*c*x^2+a^2)*d^2*e+d^3*(6/
7*a*c*x^2+3/11*c^2*x^4+a^2))*g^7-6/5*((7/51*c^2*x^6+50/143*a*c*x^4+5/21*a^
2*x^2)*e^3+6/7*x*d*(7/13*c^2*x^4+140/99*a*c*x^2+a^2)*e^2+d^2*(75/143*c^2*x
^4+10/7*a*c*x^2+a^2)*e+4/7*c*x*d^3*(35/99*c*x^2+a))*f*g^6+24/35*f^2*((49/2
21*c^2*x^5+700/1287*a*c*x^3+1/3*a^2*x)*e^3+d*(105/143*c^2*x^4+20/11*a*c*x^
2+a^2)*e^2+2*(175/429*c*x^2+a)*c*x*d^2*e+2/3*c*(5/11*c*x^2+a)*d^3)*g^5-16/
105*f^3*((2205/2431*c^2*x^4+300/143*a*c*x^2+a^2)*e^3+72/11*c*x*d*(35/78*c*
x^2+a)*e^2+6*(75/143*c*x^2+a)*c*d^2*e+12/11*c^2*d^3*x)*g^4+256/385*f^4*c*(
5/13*(49/102*c*x^2+a)*x*e^3+d*(15/26*c*x^2+a)*e^2+15/26*c*d^2*e*x+1/6*c*d^
3)*g^3-512/3003*e*((21/34*c*x^2+a)*e^2+9/5*c*d*x*e+3/2*c*d^2)*f^5*c*g^2+10
24/5005*e^2*(7/17*e*x+d)*f^6*c^2*g-2048/36465*c^2*e^3*f^7)*(g*x+f)^(3/2)/g
^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 865, normalized size of antiderivative = 1.82

$$\int (d+ex)^3\sqrt{f+gx}\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/765765*(45045*c^2*e^3*g^8*x^8 - 14336*c^2*e^3*f^8 + 52224*c^2*d*e^2*f^7*
g - 306306*a^2*d^2*e*f^2*g^6 + 255255*a^2*d^3*f*g^7 - 21760*(3*c^2*d^2*e +
 2*a*c*e^3)*f^6*g^2 + 28288*(c^2*d^3 + 6*a*c*d*e^2)*f^5*g^3 - 38896*(6*a*c
*d^2*e + a^2*e^3)*f^4*g^4 + 58344*(2*a*c*d^3 + 3*a^2*d*e^2)*f^3*g^5 + 3003
*(c^2*e^3*f*g^7 + 51*c^2*d*e^2*g^8)*x^7 - 231*(14*c^2*e^3*f^2*g^6 - 51*c^2
*d*e^2*f*g^7 - 255*(3*c^2*d^2*e + 2*a*c*e^3)*g^8)*x^6 + 63*(56*c^2*e^3*f^3
*g^5 - 204*c^2*d*e^2*f^2*g^6 + 85*(3*c^2*d^2*e + 2*a*c*e^3)*f*g^7 + 1105*(
c^2*d^3 + 6*a*c*d*e^2)*g^8)*x^5 - 35*(112*c^2*e^3*f^4*g^4 - 408*c^2*d*e^2*
f^3*g^5 + 170*(3*c^2*d^2*e + 2*a*c*e^3)*f^2*g^6 - 221*(c^2*d^3 + 6*a*c*d*e
^2)*f*g^7 - 2431*(6*a*c*d^2*e + a^2*e^3)*g^8)*x^4 + 5*(896*c^2*e^3*f^5*g^3
 - 3264*c^2*d*e^2*f^4*g^4 + 1360*(3*c^2*d^2*e + 2*a*c*e^3)*f^3*g^5 - 1768*
(c^2*d^3 + 6*a*c*d*e^2)*f^2*g^6 + 2431*(6*a*c*d^2*e + a^2*e^3)*f*g^7 + 218
79*(2*a*c*d^3 + 3*a^2*d*e^2)*g^8)*x^3 - 3*(1792*c^2*e^3*f^6*g^2 - 6528*c^2
*d*e^2*f^5*g^3 - 153153*a^2*d^2*e*g^8 + 2720*(3*c^2*d^2*e + 2*a*c*e^3)*f^4
*g^4 - 3536*(c^2*d^3 + 6*a*c*d*e^2)*f^3*g^5 + 4862*(6*a*c*d^2*e + a^2*e^3)
*f^2*g^6 - 7293*(2*a*c*d^3 + 3*a^2*d*e^2)*f*g^7)*x^2 + (7168*c^2*e^3*f^7*g
 - 26112*c^2*d*e^2*f^6*g^2 + 153153*a^2*d^2*e*f*g^7 + 255255*a^2*d^3*g^8 +
 10880*(3*c^2*d^2*e + 2*a*c*e^3)*f^5*g^3 - 14144*(c^2*d^3 + 6*a*c*d*e^2)*f
^4*g^4 + 19448*(6*a*c*d^2*e + a^2*e^3)*f^3*g^5 - 29172*(2*a*c*d^3 + 3*a^2*
d*e^2)*f^2*g^6)*x)*sqrt(g*x + f)/g^8
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1032 vs. 2(495) = 990.

Time = 1.72 (sec) , antiderivative size = 1032, normalized size of antiderivative = 2.18

$$\int (d+ex)^3\sqrt{f+gx}\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(g*x+f)**(1/2)*(c*x**2+a)**2,x)
```

output
```
Piecewise((2*(c**2*e**3*(f + g*x)**(17/2)/(17*g**7) + (f + g*x)**(15/2)*(3
*c**2*d*e**2*g - 7*c**2*e**3*f)/(15*g**7) + (f + g*x)**(13/2)*(2*a*c*e**3*
g**2 + 3*c**2*d**2*e*g**2 - 18*c**2*d*e**2*f*g + 21*c**2*e**3*f**2)/(13*g*
*7) + (f + g*x)**(11/2)*(6*a*c*d*e**2*g**3 - 10*a*c*e**3*f*g**2 + c**2*d**
3*g**3 - 15*c**2*d**2*e*f*g**2 + 45*c**2*d*e**2*f**2*g - 35*c**2*e**3*f**3
)/(11*g**7) + (f + g*x)**(9/2)*(a**2*e**3*g**4 + 6*a*c*d**2*e*g**4 - 24*a*
c*d*e**2*f*g**3 + 20*a*c*e**3*f**2*g**2 - 4*c**2*d**3*f*g**3 + 30*c**2*d**
2*e*f**2*g**2 - 60*c**2*d*e**2*f**3*g + 35*c**2*e**3*f**4)/(9*g**7) + (f +
 g*x)**(7/2)*(3*a**2*d*e**2*g**5 - 3*a**2*e**3*f*g**4 + 2*a*c*d**3*g**5 -
18*a*c*d**2*e*f*g**4 + 36*a*c*d*e**2*f**2*g**3 - 20*a*c*e**3*f**3*g**2 + 6
*c**2*d**3*f**2*g**3 - 30*c**2*d**2*e*f**3*g**2 + 45*c**2*d*e**2*f**4*g -
21*c**2*e**3*f**5)/(7*g**7) + (f + g*x)**(5/2)*(3*a**2*d**2*e*g**6 - 6*a**
2*d*e**2*f*g**5 + 3*a**2*e**3*f**2*g**4 - 4*a*c*d**3*f*g**5 + 18*a*c*d**2*
e*f**2*g**4 - 24*a*c*d*e**2*f**3*g**3 + 10*a*c*e**3*f**4*g**2 - 4*c**2*d**
3*f**3*g**3 + 15*c**2*d**2*e*f**4*g**2 - 18*c**2*d*e**2*f**5*g + 7*c**2*e*
*3*f**6)/(5*g**7) + (f + g*x)**(3/2)*(a**2*d**3*g**7 - 3*a**2*d**2*e*f*g**
6 + 3*a**2*d*e**2*f**2*g**5 - a**2*e**3*f**3*g**4 + 2*a*c*d**3*f**2*g**5 -
 6*a*c*d**2*e*f**3*g**4 + 6*a*c*d*e**2*f**4*g**3 - 2*a*c*e**3*f**5*g**2 +
c**2*d**3*f**4*g**3 - 3*c**2*d**2*e*f**5*g**2 + 3*c**2*d*e**2*f**6*g - c**
2*e**3*f**7)/(3*g**7))/g, Ne(g, 0)), (sqrt(f)*(a**2*d**3*x + 3*a**2*d**...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.49

$$\int (d+ex)^3\sqrt{f+gx}\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/765765*(45045*(g*x + f)^(17/2)*c^2*e^3 - 51051*(7*c^2*e^3*f - 3*c^2*d*e^
2*g)*(g*x + f)^(15/2) + 58905*(21*c^2*e^3*f^2 - 18*c^2*d*e^2*f*g + (3*c^2*
d^2*e + 2*a*c*e^3)*g^2)*(g*x + f)^(13/2) - 69615*(35*c^2*e^3*f^3 - 45*c^2*
d*e^2*f^2*g + 5*(3*c^2*d^2*e + 2*a*c*e^3)*f*g^2 - (c^2*d^3 + 6*a*c*d*e^2)*
g^3)*(g*x + f)^(11/2) + 85085*(35*c^2*e^3*f^4 - 60*c^2*d*e^2*f^3*g + 10*(3
*c^2*d^2*e + 2*a*c*e^3)*f^2*g^2 - 4*(c^2*d^3 + 6*a*c*d*e^2)*f*g^3 + (6*a*c
*d^2*e + a^2*e^3)*g^4)*(g*x + f)^(9/2) - 109395*(21*c^2*e^3*f^5 - 45*c^2*d
*e^2*f^4*g + 10*(3*c^2*d^2*e + 2*a*c*e^3)*f^3*g^2 - 6*(c^2*d^3 + 6*a*c*d*e
^2)*f^2*g^3 + 3*(6*a*c*d^2*e + a^2*e^3)*f*g^4 - (2*a*c*d^3 + 3*a^2*d*e^2)*
g^5)*(g*x + f)^(7/2) + 153153*(7*c^2*e^3*f^6 - 18*c^2*d*e^2*f^5*g + 3*a^2*
d^2*e*g^6 + 5*(3*c^2*d^2*e + 2*a*c*e^3)*f^4*g^2 - 4*(c^2*d^3 + 6*a*c*d*e^2
)*f^3*g^3 + 3*(6*a*c*d^2*e + a^2*e^3)*f^2*g^4 - 2*(2*a*c*d^3 + 3*a^2*d*e^2
)*f*g^5)*(g*x + f)^(5/2) - 255255*(c^2*e^3*f^7 - 3*c^2*d*e^2*f^6*g + 3*a^2
*d^2*e*f*g^6 - a^2*d^3*g^7 + (3*c^2*d^2*e + 2*a*c*e^3)*f^5*g^2 - (c^2*d^3
+ 6*a*c*d*e^2)*f^4*g^3 + (6*a*c*d^2*e + a^2*e^3)*f^3*g^4 - (2*a*c*d^3 + 3*
a^2*d*e^2)*f^2*g^5)*(g*x + f)^(3/2))/g^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1619 vs. $2(442) = 884$.

Time = 0.13 (sec) , antiderivative size = 1619, normalized size of antiderivative = 3.42

$$\int (d+ex)^3\sqrt{f+gx}\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^2,x, algorithm="giac")
```

output

```
2/765765*(765765*sqrt(g*x + f)*a^2*d^3*f + 255255*((g*x + f)^(3/2) - 3*sqr
t(g*x + f)*f)*a^2*d^3 + 765765*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a^2*d
^2*e*f/g + 102102*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x
+ f)*f^2)*a*c*d^3*f/g^2 + 153153*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f
 + 15*sqrt(g*x + f)*f^2)*a^2*d*e^2*f/g^2 + 153153*(3*(g*x + f)^(5/2) - 10*
(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a^2*d^2*e/g + 131274*(5*(g*x + f
)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)
*f^3)*a*c*d^2*e*f/g^3 + 21879*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f +
35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a^2*e^3*f/g^3 + 43758*(5*(g
*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*
x + f)*f^3)*a*c*d^3/g^2 + 65637*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f
+ 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a^2*d*e^2/g^2 + 2431*(35*
(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g
*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c^2*d^3*f/g^4 + 14586*(35*(g*x
+ f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x +
f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*a*c*d*e^2*f/g^4 + 14586*(35*(g*x + f
)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^
(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*a*c*d^2*e/g^3 + 2431*(35*(g*x + f)^(9/2
) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*
f^3 + 315*sqrt(g*x + f)*f^4)*a^2*e^3/g^3 + 3315*(63*(g*x + f)^(11/2) - ...
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.01

$$\int (d+ex)^3\sqrt{f+gx}\left(a+cx^2\right)^2\,dx$$
$$= \frac{(f+g\,x)^{11/2}\,(2\,c^2\,d^3\,g^3-30\,c^2\,d^2\,e\,f\,g^2+90\,c^2\,d\,e^2\,f^2\,g-70\,c^2\,e^3\,f^3+12\,a\,c\,d\,e^2\,g^3-20\,a\,c\,e^3\,f\,g^2)}{11\,g^8}$$
$$+\frac{(f+g\,x)^{9/2}\,(2\,a^2\,e^3\,g^4+12\,a\,c\,d^2\,e\,g^4-48\,a\,c\,d\,e^2\,f\,g^3+40\,a\,c\,e^3\,f^2\,g^2-8\,c^2\,d^3\,f\,g^3+60\,c^2\,d^2\,e\,f^2\,g^2}{9\,g^8}$$
$$+\frac{2\,c^2\,e^3\,(f+g\,x)^{17/2}}{17\,g^8}+\frac{2\,(f+g\,x)^{3/2}\,(c\,f^2+a\,g^2)^2\,(d\,g-e\,f)^3}{3\,g^8}$$
$$+\frac{2\,(f+g\,x)^{7/2}\,(d\,g-e\,f)\,(3\,a^2\,e^2\,g^4+2\,a\,c\,d^2\,g^4-16\,a\,c\,d\,e\,f\,g^3+20\,a\,c\,e^2\,f^2\,g^2+6\,c^2\,d^2\,f^2\,g^2-24\,c^2}{7\,g^8}$$
$$+\frac{2\,(f+g\,x)^{5/2}\,(c\,f^2+a\,g^2)\,(d\,g-e\,f)^2\,(7\,c\,e\,f^2-4\,c\,d\,f\,g+3\,a\,e\,g^2)}{5\,g^8}$$
$$+\frac{2\,c^2\,e^2\,(f+g\,x)^{15/2}\,(3\,d\,g-7\,e\,f)}{15\,g^8}$$
$$+\frac{2\,c\,e\,(f+g\,x)^{13/2}\,(3\,c\,d^2\,g^2-18\,c\,d\,e\,f\,g+21\,c\,e^2\,f^2+2\,a\,e^2\,g^2)}{13\,g^8}$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2)^2*(d + e*x)^3,x)
```

output
```
((f + g*x)^(11/2)*(2*c^2*d^3*g^3 - 70*c^2*e^3*f^3 + 12*a*c*d*e^2*g^3 - 20*
a*c*e^3*f*g^2 + 90*c^2*d*e^2*f^2*g - 30*c^2*d^2*e*f*g^2))/(11*g^8) + ((f +
 g*x)^(9/2)*(2*a^2*e^3*g^4 + 70*c^2*e^3*f^4 - 8*c^2*d^3*f*g^3 + 12*a*c*d^2
*e*g^4 + 40*a*c*e^3*f^2*g^2 - 120*c^2*d*e^2*f^3*g + 60*c^2*d^2*e*f^2*g^2 -
 48*a*c*d*e^2*f*g^3))/(9*g^8) + (2*c^2*e^3*(f + g*x)^(17/2))/(17*g^8) + (2
*(f + g*x)^(3/2)*(a*g^2 + c*f^2)^2*(d*g - e*f)^3)/(3*g^8) + (2*(f + g*x)^(
7/2)*(d*g - e*f)*(3*a^2*e^2*g^4 + 21*c^2*e^2*f^4 + 6*c^2*d^2*f^2*g^2 + 2*a
*c*d^2*g^4 - 24*c^2*d*e*f^3*g + 20*a*c*e^2*f^2*g^2 - 16*a*c*d*e*f*g^3))/(7
*g^8) + (2*(f + g*x)^(5/2)*(a*g^2 + c*f^2)*(d*g - e*f)^2*(3*a*e*g^2 + 7*c*
e*f^2 - 4*c*d*f*g))/(5*g^8) + (2*c^2*e^2*(f + g*x)^(15/2)*(3*d*g - 7*e*f))
/(15*g^8) + (2*c*e*(f + g*x)^(13/2)*(2*a*e^2*g^2 + 3*c*d^2*g^2 + 21*c*e^2*
f^2 - 18*c*d*e*f*g))/(13*g^8)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1018, normalized size of antiderivative = 2.15

$$\int (d+ex)^3\sqrt{f+gx}\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^2,x)
```

output
```
(2*sqrt(f + g*x)*(255255*a**2*d**3*f*g**7 + 255255*a**2*d**3*g**8*x - 3063
06*a**2*d**2*e*f**2*g**6 + 153153*a**2*d**2*e*f*g**7*x + 459459*a**2*d**2*
e*g**8*x**2 + 175032*a**2*d*e**2*f**3*g**5 - 87516*a**2*d*e**2*f**2*g**6*x
 + 65637*a**2*d*e**2*f*g**7*x**2 + 328185*a**2*d*e**2*g**8*x**3 - 38896*a*
*2*e**3*f**4*g**4 + 19448*a**2*e**3*f**3*g**5*x - 14586*a**2*e**3*f**2*g**
6*x**2 + 12155*a**2*e**3*f*g**7*x**3 + 85085*a**2*e**3*g**8*x**4 + 116688*
a*c*d**3*f**3*g**5 - 58344*a*c*d**3*f**2*g**6*x + 43758*a*c*d**3*f*g**7*x*
*2 + 218790*a*c*d**3*g**8*x**3 - 233376*a*c*d**2*e*f**4*g**4 + 116688*a*c*
d**2*e*f**3*g**5*x - 87516*a*c*d**2*e*f**2*g**6*x**2 + 72930*a*c*d**2*e*f*
g**7*x**3 + 510510*a*c*d**2*e*g**8*x**4 + 169728*a*c*d*e**2*f**5*g**3 - 84
864*a*c*d*e**2*f**4*g**4*x + 63648*a*c*d*e**2*f**3*g**5*x**2 - 53040*a*c*d
*e**2*f**2*g**6*x**3 + 46410*a*c*d*e**2*f*g**7*x**4 + 417690*a*c*d*e**2*g*
*8*x**5 - 43520*a*c*e**3*f**6*g**2 + 21760*a*c*e**3*f**5*g**3*x - 16320*a*
c*e**3*f**4*g**4*x**2 + 13600*a*c*e**3*f**3*g**5*x**3 - 11900*a*c*e**3*f**
2*g**6*x**4 + 10710*a*c*e**3*f*g**7*x**5 + 117810*a*c*e**3*g**8*x**6 + 282
88*c**2*d**3*f**5*g**3 - 14144*c**2*d**3*f**4*g**4*x + 10608*c**2*d**3*f**
3*g**5*x**2 - 8840*c**2*d**3*f**2*g**6*x**3 + 7735*c**2*d**3*f*g**7*x**4 +
 69615*c**2*d**3*g**8*x**5 - 65280*c**2*d**2*e*f**6*g**2 + 32640*c**2*d**2
*e*f**5*g**3*x - 24480*c**2*d**2*e*f**4*g**4*x**2 + 20400*c**2*d**2*e*f**3
*g**5*x**3 - 17850*c**2*d**2*e*f**2*g**6*x**4 + 16065*c**2*d**2*e*f*g**...
```

3.64 $\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 340

$$
\begin{aligned}
\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)^2\,dx &= \frac{2(ef-dg)^2\left(cf^2+ag^2\right)^2(f+gx)^{3/2}}{3g^7} \\
&- \frac{4(ef-dg)\left(cf^2+ag^2\right)\left(aeg^2+cf(3ef-2dg)\right)(f+gx)^{5/2}}{5g^7} \\
&+ \frac{2\left(a^2e^2g^4+2acg^2\left(6e^2f^2-6defg+d^2g^2\right)+c^2f^2\left(15e^2f^2-20defg+6d^2g^2\right)\right)(f+gx)^{7/2}}{7g^7} \\
&- \frac{8c\left(aeg^2(2ef-dg)+cf\left(5e^2f^2-5defg+d^2g^2\right)\right)(f+gx)^{9/2}}{9g^7} \\
&+ \frac{2c\left(2ae^2g^2+c\left(15e^2f^2-10defg+d^2g^2\right)\right)(f+gx)^{11/2}}{11g^7} \\
&- \frac{4c^2e(3ef-dg)(f+gx)^{13/2}}{13g^7} + \frac{2c^2e^2(f+gx)^{15/2}}{15g^7}
\end{aligned}
$$

output

```
2/3*(-d*g+e*f)^2*(a*g^2+c*f^2)^2*(g*x+f)^(3/2)/g^7-4/5*(-d*g+e*f)*(a*g^2+c
*f^2)*(a*e*g^2+c*f*(-2*d*g+3*e*f))*(g*x+f)^(5/2)/g^7+2/7*(a^2*e^2*g^4+2*a*
c*g^2*(d^2*g^2-6*d*e*f*g+6*e^2*f^2)+c^2*f^2*(6*d^2*g^2-20*d*e*f*g+15*e^2*f
^2))*(g*x+f)^(7/2)/g^7-8/9*c*(a*e*g^2*(-d*g+2*e*f)+c*f*(d^2*g^2-5*d*e*f*g+
5*e^2*f^2))*(g*x+f)^(9/2)/g^7+2/11*c*(2*a*e^2*g^2+c*(d^2*g^2-10*d*e*f*g+15
*e^2*f^2))*(g*x+f)^(11/2)/g^7-4/13*c^2*e*(-d*g+3*e*f)*(g*x+f)^(13/2)/g^7+2
/15*c^2*e^2*(g*x+f)^(15/2)/g^7
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.07

$$\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)^2\,dx$$
$$= \frac{2(f+gx)^{3/2}\left(429a^2g^4(35d^2g^2+14deg(-2f+3gx)+e^2(8f^2-12fgx+15g^2x^2))+26acg^2(33d^2g^2(8f^2\right.}{}$$

input
```
Integrate[(d + e*x)^2*Sqrt[f + g*x]*(a + c*x^2)^2,x]
```

output
```
(2*(f + g*x)^(3/2)*(429*a^2*g^4*(35*d^2*g^2 + 14*d*e*g*(-2*f + 3*g*x) + e^
2*(8*f^2 - 12*f*g*x + 15*g^2*x^2)) + 26*a*c*g^2*(33*d^2*g^2*(8*f^2 - 12*f*
g*x + 15*g^2*x^2) + 22*d*e*g*(-16*f^3 + 24*f^2*g*x - 30*f*g^2*x^2 + 35*g^3
*x^3) + e^2*(128*f^4 - 192*f^3*g*x + 240*f^2*g^2*x^2 - 280*f*g^3*x^3 + 315
*g^4*x^4)) + c^2*(13*d^2*g^2*(128*f^4 - 192*f^3*g*x + 240*f^2*g^2*x^2 - 28
0*f*g^3*x^3 + 315*g^4*x^4) + 10*d*e*g*(-256*f^5 + 384*f^4*g*x - 480*f^3*g^
2*x^2 + 560*f^2*g^3*x^3 - 630*f*g^4*x^4 + 693*g^5*x^5) + e^2*(1024*f^6 - 1
536*f^5*g*x + 1920*f^4*g^2*x^2 - 2240*f^3*g^3*x^3 + 2520*f^2*g^4*x^4 - 277
2*f*g^5*x^5 + 3003*g^6*x^6))))/(45045*g^7)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2(d+ex)^2\sqrt{f+gx}\,dx$$
$$\Big\downarrow \ 652$$
$$\int \left(\frac{(f+gx)^{5/2}\left(a^2e^2g^4+2acg^2\left(d^2g^2-6defg+6e^2f^2\right)+c^2f^2\left(6d^2g^2-20defg+15e^2f^2\right)\right)}{g^6}+\frac{c(f+gx)^{9/2}\left(2\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(f+gx)^{7/2}\left(a^2e^2g^4+2acg^2\left(d^2g^2-6defg+6e^2f^2\right)+c^2f^2\left(6d^2g^2-20defg+15e^2f^2\right)\right)}{7g^7}+\frac{2c(f+gx)^{11/2}\left(2ae^2g^2+c\left(d^2g^2-10defg+15e^2f^2\right)\right)}{11g^7}-\frac{8c(f+gx)^{9/2}\left(aeg^2(2ef-dg)+cf\left(d^2g^2-5defg+5e^2f^2\right)\right)}{9g^7}-\frac{4(f+gx)^{5/2}\left(ag^2+cf^2\right)(ef-dg)\left(aeg^2+cf(3ef-2dg)\right)}{5g^7}+\frac{2(f+gx)^{3/2}\left(ag^2+cf^2\right)^2(ef-dg)^2}{3g^7}-\frac{4c^2e(f+gx)^{13/2}(3ef-dg)}{13g^7}+\frac{2c^2e^2(f+gx)^{15/2}}{15g^7}$$

input
```
Int[(d + e*x)^2*Sqrt[f + g*x]*(a + c*x^2)^2,x]
```

output
```
(2*(e*f - d*g)^2*(c*f^2 + a*g^2)^2*(f + g*x)^(3/2))/(3*g^7) - (4*(e*f - d*
g)*(c*f^2 + a*g^2)*(a*e*g^2 + c*f*(3*e*f - 2*d*g))*(f + g*x)^(5/2))/(5*g^7
) + (2*(a^2*e^2*g^4 + 2*a*c*g^2*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2) + c^2*f^
2*(15*e^2*f^2 - 20*d*e*f*g + 6*d^2*g^2))*(f + g*x)^(7/2))/(7*g^7) - (8*c*(
a*e*g^2*(2*e*f - d*g) + c*f*(5*e^2*f^2 - 5*d*e*f*g + d^2*g^2))*(f + g*x)^(
9/2))/(9*g^7) + (2*c*(2*a*e^2*g^2 + c*(15*e^2*f^2 - 10*d*e*f*g + d^2*g^2))
*(f + g*x)^(11/2))/(11*g^7) - (4*c^2*e*(3*e*f - d*g)*(f + g*x)^(13/2))/(13
*g^7) + (2*c^2*e^2*(f + g*x)^(15/2))/(15*g^7)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{2(gx+f)^{\frac{3}{2}}\left(\left(\left(\left(\frac{1}{5}e^2x^6+\frac{3}{11}d^2x^4+\frac{6}{13}dex^5\right)c^2+\frac{6\left(\frac{7}{11}e^2x^2+\frac{14}{9}dex+d^2\right)x^2ac}{7}+a^2\left(d^2+\frac{3}{7}e^2x^2+\frac{6}{5}dex\right)\right)g^6-\frac{4f\left(\left(\frac{3}{13}e^2x^5+\right.\right.}{}\right.\right.}{}$
derivativedivides	$\frac{2c^2e^2(gx+f)^{\frac{15}{2}}}{15}+\frac{2\left(2e(dg-ef)c^2-4fc^2e^2\right)(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left((dg-ef)^2c^2-8e(dg-ef)c^2f+e^2\left(2\left(ag^2+cf^2\right)c+4c^2f^2\right)\right)(gx+f)^{\frac{11}{2}}}{11}$
default	$\frac{2c^2e^2(gx+f)^{\frac{15}{2}}}{15}+\frac{2\left(2e(dg-ef)c^2-4fc^2e^2\right)(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left((dg-ef)^2c^2-8e(dg-ef)c^2f+e^2\left(2\left(ag^2+cf^2\right)c+4c^2f^2\right)\right)(gx+f)^{\frac{11}{2}}}{11}$
gosper	$\frac{2(gx+f)^{\frac{3}{2}}\left(3003c^2e^2x^6g^6+6930c^2deg^6x^5-2772c^2e^2fg^5x^5+8190ace^2g^6x^4+4095c^2d^2g^6x^4-6300c^2defg^5x^4+2520c\right.}{}$
orering	$\frac{2(gx+f)^{\frac{3}{2}}\left(3003c^2e^2x^6g^6+6930c^2deg^6x^5-2772c^2e^2fg^5x^5+8190ace^2g^6x^4+4095c^2d^2g^6x^4-6300c^2defg^5x^4+2520c\right.}{}$
trager	$\frac{2\left(3003c^2e^2g^7x^7+6930c^2deg^7x^6+231c^2e^2fg^6x^6+8190ace^2g^7x^5+4095c^2d^2g^7x^5+630c^2defg^6x^5-252c^2e^2f^2g^5x^5+2\right.}{}$
risch	$\frac{2\left(3003c^2e^2g^7x^7+6930c^2deg^7x^6+231c^2e^2fg^6x^6+8190ace^2g^7x^5+4095c^2d^2g^7x^5+630c^2defg^6x^5-252c^2e^2f^2g^5x^5+2\right.}{}$

input
```
int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(g*x+f)^(3/2)*(((1/5*e^2*x^6+3/11*d^2*x^4+6/13*d*e*x^5)*c^2+6/7*(7/11*
e^2*x^2+14/9*d*e*x+d^2)*x^2*a*c+a^2*(d^2+3/7*e^2*x^2+6/5*d*e*x))*g^6-4/5*f
*((3/13*e^2*x^5+75/143*d*e*x^4+10/33*d^2*x^3)*c^2+6/7*x*a*(70/99*e^2*x^2+5
/3*d*e*x+d^2)*c+a^2*e*(3/7*e*x+d))*g^5+8/35*f^2*(10/11*x^2*(21/26*e^2*x^2+
70/39*d*e*x+d^2)*c^2+2*(10/11*e^2*x^2+2*d*e*x+d^2)*a*c+e^2*a^2)*g^4-64/105
*(3/11*x*(35/39*e^2*x^2+25/13*d*e*x+d^2)*c+e*a*(6/11*e*x+d))*f^3*c*g^3+256
/1155*((15/26*e^2*x^2+15/13*d*e*x+1/2*d^2)*c+a*e^2)*f^4*c*g^2-512/3003*e*f
^5*(3/5*e*x+d)*c^2*g+1024/15015*c^2*e^2*f^6)/g^7
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.67

$$\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)^2\,dx$$
$$= \frac{2\,(3003\,c^2e^2g^7x^7+1024\,c^2e^2f^7-2560\,c^2def^6g-9152\,acdef^4g^3-12012\,a^2def^2g^5+15015\,a^2d^2fg^6+1\cdots}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/45045*(3003*c^2*e^2*g^7*x^7 + 1024*c^2*e^2*f^7 - 2560*c^2*d*e*f^6*g - 91
52*a*c*d*e*f^4*g^3 - 12012*a^2*d*e*f^2*g^5 + 15015*a^2*d^2*f*g^6 + 1664*(c
^2*d^2 + 2*a*c*e^2)*f^5*g^2 + 3432*(2*a*c*d^2 + a^2*e^2)*f^3*g^4 + 231*(c^
2*e^2*f*g^6 + 30*c^2*d*e*g^7)*x^6 - 63*(4*c^2*e^2*f^2*g^5 - 10*c^2*d*e*f*g
^6 - 65*(c^2*d^2 + 2*a*c*e^2)*g^7)*x^5 + 35*(8*c^2*e^2*f^3*g^4 - 20*c^2*d*
e*f^2*g^5 + 572*a*c*d*e*g^7 + 13*(c^2*d^2 + 2*a*c*e^2)*f*g^6)*x^4 - 5*(64*
c^2*e^2*f^4*g^3 - 160*c^2*d*e*f^3*g^4 - 572*a*c*d*e*f*g^6 + 104*(c^2*d^2 +
 2*a*c*e^2)*f^2*g^5 - 1287*(2*a*c*d^2 + a^2*e^2)*g^7)*x^3 + 3*(128*c^2*e^2
*f^5*g^2 - 320*c^2*d*e*f^4*g^3 - 1144*a*c*d*e*f^2*g^5 + 6006*a^2*d*e*g^7 +
 208*(c^2*d^2 + 2*a*c*e^2)*f^3*g^4 + 429*(2*a*c*d^2 + a^2*e^2)*f*g^6)*x^2
- (512*c^2*e^2*f^6*g - 1280*c^2*d*e*f^5*g^2 - 4576*a*c*d*e*f^3*g^4 - 6006*
a^2*d*e*f*g^6 - 15015*a^2*d^2*g^7 + 832*(c^2*d^2 + 2*a*c*e^2)*f^4*g^3 + 17
16*(2*a*c*d^2 + a^2*e^2)*f^2*g^5)*x)*sqrt(g*x + f)/g^7
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.96

$$\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)^2\,dx$$
$$= \begin{cases} \frac{2\left(\frac{c^2e^2(f+gx)^{\frac{15}{2}}}{15g^6}+\frac{(f+gx)^{\frac{13}{2}}\cdot\left(2c^2deg-6c^2e^2f\right)}{13g^6}+\frac{(f+gx)^{\frac{11}{2}}\cdot\left(2ace^2g^2+c^2d^2g^2-10c^2defg+15c^2e^2f^2\right)}{11g^6}+\frac{(f+gx)^{\frac{9}{2}}\cdot\left(4acdeg^3-8ace^2fg^2-4c^2d^2fg^2+2\cdots\right.}{9g^6}\cdots\right.}{\cdots} \\ \sqrt{f}\left(a^2d^2x+a^2dex^2+acdex^4+\frac{c^2dex^6}{3}+\frac{c^2e^2x^7}{7}+\frac{x^5\cdot\left(2ace^2+c^2d^2\right)}{5}+\frac{x^3\left(a^2e^2+2acd^2\right)}{3}\right) \end{cases}$$

input
```
integrate((e*x+d)**2*(g*x+f)**(1/2)*(c*x**2+a)**2,x)
```

output
```
Piecewise((2*(c**2*e**2*(f + g*x)**(15/2)/(15*g**6) + (f + g*x)**(13/2)*(2
*c**2*d*e*g - 6*c**2*e**2*f)/(13*g**6) + (f + g*x)**(11/2)*(2*a*c*e**2*g**
2 + c**2*d**2*g**2 - 10*c**2*d*e*f*g + 15*c**2*e**2*f**2)/(11*g**6) + (f +
 g*x)**(9/2)*(4*a*c*d*e*g**3 - 8*a*c*e**2*f*g**2 - 4*c**2*d**2*f*g**2 + 20
*c**2*d*e*f**2*g - 20*c**2*e**2*f**3)/(9*g**6) + (f + g*x)**(7/2)*(a**2*e*
*2*g**4 + 2*a*c*d**2*g**4 - 12*a*c*d*e*f*g**3 + 12*a*c*e**2*f**2*g**2 + 6*
c**2*d**2*f**2*g**2 - 20*c**2*d*e*f**3*g + 15*c**2*e**2*f**4)/(7*g**6) + (
f + g*x)**(5/2)*(2*a**2*d*e*g**5 - 2*a**2*e**2*f*g**4 - 4*a*c*d**2*f*g**4
+ 12*a*c*d*e*f**2*g**3 - 8*a*c*e**2*f**3*g**2 - 4*c**2*d**2*f**3*g**2 + 10
*c**2*d*e*f**4*g - 6*c**2*e**2*f**5)/(5*g**6) + (f + g*x)**(3/2)*(a**2*d**
2*g**6 - 2*a**2*d*e*f*g**5 + a**2*e**2*f**2*g**4 + 2*a*c*d**2*f**2*g**4 -
4*a*c*d*e*f**3*g**3 + 2*a*c*e**2*f**4*g**2 + c**2*d**2*f**4*g**2 - 2*c**2*
d*e*f**5*g + c**2*e**2*f**6)/(3*g**6))/g, Ne(g, 0)), (sqrt(f)*(a**2*d**2*x
 + a**2*d*e*x**2 + a*c*d*e*x**4 + c**2*d*e*x**6/3 + c**2*e**2*x**7/7 + x**
5*(2*a*c*e**2 + c**2*d**2)/5 + x**3*(a**2*e**2 + 2*a*c*d**2)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.31

$$\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)^2\,dx$$
$$= \frac{2\left(3003\,(gx+f)^{\frac{15}{2}}c^2e^2 - 6930\,(3\,c^2e^2f - c^2deg)(gx+f)^{\frac{13}{2}} + 4095\,(15\,c^2e^2f^2 - 10\,c^2defg + (c^2d^2 + 2\,a\right.}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/45045*(3003*(g*x + f)^(15/2)*c^2*e^2 - 6930*(3*c^2*e^2*f - c^2*d*e*g)*(g
*x + f)^(13/2) + 4095*(15*c^2*e^2*f^2 - 10*c^2*d*e*f*g + (c^2*d^2 + 2*a*c*
e^2)*g^2)*(g*x + f)^(11/2) - 20020*(5*c^2*e^2*f^3 - 5*c^2*d*e*f^2*g - a*c*
d*e*g^3 + (c^2*d^2 + 2*a*c*e^2)*f*g^2)*(g*x + f)^(9/2) + 6435*(15*c^2*e^2*
f^4 - 20*c^2*d*e*f^3*g - 12*a*c*d*e*f*g^3 + 6*(c^2*d^2 + 2*a*c*e^2)*f^2*g^
2 + (2*a*c*d^2 + a^2*e^2)*g^4)*(g*x + f)^(7/2) - 18018*(3*c^2*e^2*f^5 - 5*
c^2*d*e*f^4*g - 6*a*c*d*e*f^2*g^3 - a^2*d*e*g^5 + 2*(c^2*d^2 + 2*a*c*e^2)*
f^3*g^2 + (2*a*c*d^2 + a^2*e^2)*f*g^4)*(g*x + f)^(5/2) + 15015*(c^2*e^2*f^
6 - 2*c^2*d*e*f^5*g - 4*a*c*d*e*f^3*g^3 - 2*a^2*d*e*f*g^5 + a^2*d^2*g^6 +
(c^2*d^2 + 2*a*c*e^2)*f^4*g^2 + (2*a*c*d^2 + a^2*e^2)*f^2*g^4)*(g*x + f)^(
3/2))/g^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1090 vs. $2(312) = 624$.

Time = 0.12 (sec) , antiderivative size = 1090, normalized size of antiderivative = 3.21

$$\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(g*x + f)*a^2*d^2*f + 15015*((g*x + f)^(3/2) - 3*sqrt(g
*x + f)*f)*a^2*d^2 + 30030*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a^2*d*e*f
/g + 6006*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2
)*a*c*d^2*f/g^2 + 3003*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt
(g*x + f)*f^2)*a^2*e^2*f/g^2 + 6006*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2
)*f + 15*sqrt(g*x + f)*f^2)*a^2*d*e/g + 5148*(5*(g*x + f)^(7/2) - 21*(g*x
+ f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*c*d*e*f/g^
3 + 2574*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^
2 - 35*sqrt(g*x + f)*f^3)*a*c*d^2/g^2 + 1287*(5*(g*x + f)^(7/2) - 21*(g*x
+ f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a^2*e^2/g^2
+ 143*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^
2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c^2*d^2*f/g^4 + 286*(
35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420
*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*a*c*e^2*f/g^4 + 572*(35*(g*x
 + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x +
 f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*a*c*d*e/g^3 + 130*(63*(g*x + f)^(11
/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x + f)^(7/2)*f^2 - 1386*(g*x + f)^(5/
2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*sqrt(g*x + f)*f^5)*c^2*d*e*f/g^5 +
 65*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x + f)^(7/2)*f^2
 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*sqrt(g*x +...
```

Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.01

$$\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)^2\,dx$$

$$= \frac{(f+g\,x)^{7/2}\,(2\,a^2\,e^2\,g^4+4\,a\,c\,d^2\,g^4-24\,a\,c\,d\,e\,f\,g^3+24\,a\,c\,e^2\,f^2\,g^2+12\,c^2\,d^2\,f^2\,g^2-40\,c^2\,d\,e\,f^3\,g+30\,c}{7\,g^7}$$

$$-\frac{(f+g\,x)^{9/2}\,(8\,c^2\,d^2\,f\,g^2-40\,c^2\,d\,e\,f^2\,g+40\,c^2\,e^2\,f^3-8\,a\,c\,d\,e\,g^3+16\,a\,c\,e^2\,f\,g^2)}{9\,g^7}$$

$$+\frac{(f+g\,x)^{11/2}\,(2\,c^2\,d^2\,g^2-20\,c^2\,d\,e\,f\,g+30\,c^2\,e^2\,f^2+4\,a\,c\,e^2\,g^2)}{11\,g^7}$$

$$+\frac{2\,c^2\,e^2\,(f+g\,x)^{15/2}}{15\,g^7}+\frac{2\,(f+g\,x)^{3/2}\,(c\,f^2+a\,g^2)^2\,(d\,g-e\,f)^2}{3\,g^7}$$

$$+\frac{4\,c^2\,e\,(f+g\,x)^{13/2}\,(d\,g-3\,e\,f)}{13\,g^7}$$

$$+\frac{4\,(f+g\,x)^{5/2}\,(c\,f^2+a\,g^2)\,(d\,g-e\,f)\,(3\,c\,e\,f^2-2\,c\,d\,f\,g+a\,e\,g^2)}{5\,g^7}$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2)^2*(d + e*x)^2,x)
```

output
```
((f + g*x)^(7/2)*(2*a^2*e^2*g^4 + 30*c^2*e^2*f^4 + 12*c^2*d^2*f^2*g^2 + 4*
a*c*d^2*g^4 - 40*c^2*d*e*f^3*g + 24*a*c*e^2*f^2*g^2 - 24*a*c*d*e*f*g^3))/(
7*g^7) - ((f + g*x)^(9/2)*(40*c^2*e^2*f^3 + 8*c^2*d^2*f*g^2 + 16*a*c*e^2*f
*g^2 - 40*c^2*d*e*f^2*g - 8*a*c*d*e*g^3))/(9*g^7) + ((f + g*x)^(11/2)*(2*c
^2*d^2*g^2 + 30*c^2*e^2*f^2 + 4*a*c*e^2*g^2 - 20*c^2*d*e*f*g))/(11*g^7) +
(2*c^2*e^2*(f + g*x)^(15/2))/(15*g^7) + (2*(f + g*x)^(3/2)*(a*g^2 + c*f^2)
^2*(d*g - e*f)^2)/(3*g^7) + (4*c^2*e*(f + g*x)^(13/2)*(d*g - 3*e*f))/(13*g
^7) + (4*(f + g*x)^(5/2)*(a*g^2 + c*f^2)*(d*g - e*f)*(a*e*g^2 + 3*c*e*f^2
- 2*c*d*f*g))/(5*g^7)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.91

$$\int (d+ex)^2\sqrt{f+gx}\left(a+cx^2\right)^2\,dx$$
$$= \frac{2\sqrt{gx+f}\,(3003c^2e^2g^7x^7 + 6930c^2de\,g^7x^6 + 231c^2e^2f\,g^6x^6 + 8190ac\,e^2g^7x^5 + 4095c^2d^2g^7x^5 + 630c^2def}{}$$

input
```
int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^2,x)
```

output
```
(2*sqrt(f + g*x)*(15015*a**2*d**2*f*g**6 + 15015*a**2*d**2*g**7*x - 12012*
a**2*d*e*f**2*g**5 + 6006*a**2*d*e*f*g**6*x + 18018*a**2*d*e*g**7*x**2 + 3
432*a**2*e**2*f**3*g**4 - 1716*a**2*e**2*f**2*g**5*x + 1287*a**2*e**2*f*g*
*6*x**2 + 6435*a**2*e**2*g**7*x**3 + 6864*a*c*d**2*f**3*g**4 - 3432*a*c*d*
*2*f**2*g**5*x + 2574*a*c*d**2*f*g**6*x**2 + 12870*a*c*d**2*g**7*x**3 - 91
52*a*c*d*e*f**4*g**3 + 4576*a*c*d*e*f**3*g**4*x - 3432*a*c*d*e*f**2*g**5*x
**2 + 2860*a*c*d*e*f*g**6*x**3 + 20020*a*c*d*e*g**7*x**4 + 3328*a*c*e**2*f
**5*g**2 - 1664*a*c*e**2*f**4*g**3*x + 1248*a*c*e**2*f**3*g**4*x**2 - 1040
*a*c*e**2*f**2*g**5*x**3 + 910*a*c*e**2*f*g**6*x**4 + 8190*a*c*e**2*g**7*x
**5 + 1664*c**2*d**2*f**5*g**2 - 832*c**2*d**2*f**4*g**3*x + 624*c**2*d**2
*f**3*g**4*x**2 - 520*c**2*d**2*f**2*g**5*x**3 + 455*c**2*d**2*f*g**6*x**4
 + 4095*c**2*d**2*g**7*x**5 - 2560*c**2*d*e*f**6*g + 1280*c**2*d*e*f**5*g*
*2*x - 960*c**2*d*e*f**4*g**3*x**2 + 800*c**2*d*e*f**3*g**4*x**3 - 700*c**
2*d*e*f**2*g**5*x**4 + 630*c**2*d*e*f*g**6*x**5 + 6930*c**2*d*e*g**7*x**6
+ 1024*c**2*e**2*f**7 - 512*c**2*e**2*f**6*g*x + 384*c**2*e**2*f**5*g**2*x
**2 - 320*c**2*e**2*f**4*g**3*x**3 + 280*c**2*e**2*f**3*g**4*x**4 - 252*c*
*2*e**2*f**2*g**5*x**5 + 231*c**2*e**2*f*g**6*x**6 + 3003*c**2*e**2*g**7*x
**7))/(45045*g**7)
```

3.65 $\int (d+ex)\sqrt{f+gx}(a+cx^2)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 214

$$\begin{aligned}\int (d+ex)\sqrt{f+gx}(a+cx^2)^2\,dx = &-\frac{2(ef-dg)\,(cf^2+ag^2)^2\,(f+gx)^{3/2}}{3g^6}\\ &+\frac{2(cf^2+ag^2)\,(aeg^2+cf(5ef-4dg))\,(f+gx)^{5/2}}{5g^6}\\ &-\frac{4c(cf^2(5ef-3dg)+ag^2(3ef-dg))\,(f+gx)^{7/2}}{7g^6}\\ &+\frac{4c(aeg^2+cf(5ef-2dg))\,(f+gx)^{9/2}}{9g^6}\\ &-\frac{2c^2(5ef-dg)(f+gx)^{11/2}}{11g^6}+\frac{2c^2e(f+gx)^{13/2}}{13g^6}\end{aligned}$$

output

```
-2/3*(-d*g+e*f)*(a*g^2+c*f^2)^2*(g*x+f)^(3/2)/g^6+2/5*(a*g^2+c*f^2)*(a*e*g
^2+c*f*(-4*d*g+5*e*f))*(g*x+f)^(5/2)/g^6-4/7*c*(c*f^2*(-3*d*g+5*e*f)+a*g^2
*(-d*g+3*e*f))*(g*x+f)^(7/2)/g^6+4/9*c*(a*e*g^2+c*f*(-2*d*g+5*e*f))*(g*x+f
)^(9/2)/g^6-2/11*c^2*(-d*g+5*e*f)*(g*x+f)^(11/2)/g^6+2/13*c^2*e*(g*x+f)^(1
3/2)/g^6
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.97

$$\int (d+ex)\sqrt{f+gx}\left(a+cx^2\right)^2\,dx$$
$$= \frac{2(f+gx)^{3/2}\left(3003a^2g^4(-2ef+5dg+3egx)-286acg^2\left(-3dg\left(8f^2-12fgx+15g^2x^2\right)+e\left(16f^3-24f^2\right.\right.\right.}{}$$

input
```
Integrate[(d + e*x)*Sqrt[f + g*x]*(a + c*x^2)^2,x]
```

output
```
(2*(f + g*x)^(3/2)*(3003*a^2*g^4*(-2*e*f + 5*d*g + 3*e*g*x) - 286*a*c*g^2*
(-3*d*g*(8*f^2 - 12*f*g*x + 15*g^2*x^2) + e*(16*f^3 - 24*f^2*g*x + 30*f*g^
2*x^2 - 35*g^3*x^3)) + c^2*(13*d*g*(128*f^4 - 192*f^3*g*x + 240*f^2*g^2*x^
2 - 280*f*g^3*x^3 + 315*g^4*x^4) - 5*e*(256*f^5 - 384*f^4*g*x + 480*f^3*g^
2*x^2 - 560*f^2*g^3*x^3 + 630*f*g^4*x^4 - 693*g^5*x^5))))/(45045*g^6)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2(d+ex)\sqrt{f+gx}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{2c(f+gx)^{5/2}\left(-ag^2(3ef-dg)-cf^2(5ef-3dg)\right)}{g^5}+\frac{(f+gx)^{3/2}\left(ag^2+cf^2\right)\left(aeg^2+cf(5ef-4dg)\right)}{g^5}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4c(f+gx)^{7/2}\left(ag^2(3ef-dg)+cf^2(5ef-3dg)\right)}{7g^6}+\frac{2(f+gx)^{5/2}\left(ag^2+cf^2\right)\left(aeg^2+cf(5ef-4dg)\right)}{5g^6}-\frac{2(f+gx)^{3/2}\left(ag^2+cf^2\right)^2(ef-dg)}{3g^6}+\frac{4c(f+gx)^{9/2}\left(aeg^2+cf(5ef-2dg)\right)}{9g^6}-\frac{2c^2(f+gx)^{11/2}(5ef-dg)}{11g^6}+\frac{2c^2e(f+gx)^{13/2}}{13g^6}$$

input
```
Int[(d + e*x)*Sqrt[f + g*x]*(a + c*x^2)^2,x]
```

output
```
(-2*(e*f - d*g)*(c*f^2 + a*g^2)^2*(f + g*x)^(3/2))/(3*g^6) + (2*(c*f^2 + a
*g^2)*(a*e*g^2 + c*f*(5*e*f - 4*d*g))*(f + g*x)^(5/2))/(5*g^6) - (4*c*(c*f
^2*(5*e*f - 3*d*g) + a*g^2*(3*e*f - d*g))*(f + g*x)^(7/2))/(7*g^6) + (4*c*
(a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x)^(9/2))/(9*g^6) - (2*c^2*(5*e*f -
 d*g)*(f + g*x)^(11/2))/(11*g^6) + (2*c^2*e*(f + g*x)^(13/2))/(13*g^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{2(gx+f)^{\frac{3}{2}}\left(\left(\left(\frac{3}{13}x^5e+\frac{3}{11}dx^4\right)c^2+\frac{6\left(\frac{7ex}{9}+d\right)x^2ac}{7}+a^2\left(\frac{3ex}{5}+d\right)\right)g^5-\frac{2\left(\frac{20\left(\frac{45ex}{52}+d\right)x^3c^2}{33}+\frac{12\left(\frac{5ex}{6}+d\right)xac}{7}+a^2e\right)fg^4}{5}\right.}{3g^6}$
derivativedivides	$\frac{2ec^2(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left((dg-ef)c^2-4fec^2\right)(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(-4(dg-ef)c^2f+e\left(2\left(ag^2+cf^2\right)c+4c^2f^2\right)\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left((dg-ef)\left(2\left(ag\right.\right.\right.}{}$
default	$\frac{2ec^2(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left((dg-ef)c^2-4fec^2\right)(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(-4(dg-ef)c^2f+e\left(2\left(ag^2+cf^2\right)c+4c^2f^2\right)\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left((dg-ef)\left(2\left(ag\right.\right.\right.}{}$
gosper	$\frac{2(gx+f)^{\frac{3}{2}}\left(3465c^2ex^5g^5+4095c^2dg^5x^4-3150c^2efg^4x^4+10010aceg^5x^3-3640c^2dfg^4x^3+2800c^2ef^2g^3x^3+12870ac\right.}{}$
orering	$\frac{2(gx+f)^{\frac{3}{2}}\left(3465c^2ex^5g^5+4095c^2dg^5x^4-3150c^2efg^4x^4+10010aceg^5x^3-3640c^2dfg^4x^3+2800c^2ef^2g^3x^3+12870ac\right.}{}$
trager	$\frac{2\left(3465ec^2g^6x^6+4095c^2dg^6x^5+315c^2efg^5x^5+10010aceg^6x^4+455c^2dfg^5x^4-350c^2ef^2g^4x^4+12870acdg^6x^3+1430\right.}{}$
risch	$\frac{2\left(3465ec^2g^6x^6+4095c^2dg^6x^5+315c^2efg^5x^5+10010aceg^6x^4+455c^2dfg^5x^4-350c^2ef^2g^4x^4+12870acdg^6x^3+1430\right.}{}$

input
```
int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(g*x+f)^(3/2)*(((3/13*x^5*e+3/11*d*x^4)*c^2+6/7*(7/9*e*x+d)*x^2*a*c+a^
2*(3/5*e*x+d))*g^5-2/5*(20/33*(45/52*e*x+d)*x^3*c^2+12/7*(5/6*e*x+d)*x*a*c
+a^2*e)*f*g^4+16/35*f^2*(5/11*(35/39*e*x+d)*x^2*c+a*(e*x+d))*c*g^3-32/105*
f^3*c*(6/11*x*(25/26*e*x+d)*c+a*e)*g^2+128/1155*f^4*c^2*(15/13*e*x+d)*g-25
6/3003*c^2*e*f^5)/g^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.50

$$\int(d+ex)\sqrt{f+gx}\left(a+cx^2\right)^2\,dx$$
$$=\frac{2\left(3465\,c^2eg^6x^6-1280\,c^2ef^6+1664\,c^2df^5g-4576\,acef^4g^2+6864\,acdf^3g^3-6006\,a^2ef^2g^4+15015\,a^2d\right.}{}$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/45045*(3465*c^2*e*g^6*x^6 - 1280*c^2*e*f^6 + 1664*c^2*d*f^5*g - 4576*a*c
*e*f^4*g^2 + 6864*a*c*d*f^3*g^3 - 6006*a^2*e*f^2*g^4 + 15015*a^2*d*f*g^5 +
 315*(c^2*e*f*g^5 + 13*c^2*d*g^6)*x^5 - 35*(10*c^2*e*f^2*g^4 - 13*c^2*d*f*
g^5 - 286*a*c*e*g^6)*x^4 + 10*(40*c^2*e*f^3*g^3 - 52*c^2*d*f^2*g^4 + 143*a
*c*e*f*g^5 + 1287*a*c*d*g^6)*x^3 - 3*(160*c^2*e*f^4*g^2 - 208*c^2*d*f^3*g^
3 + 572*a*c*e*f^2*g^4 - 858*a*c*d*f*g^5 - 3003*a^2*e*g^6)*x^2 + (640*c^2*e
*f^5*g - 832*c^2*d*f^4*g^2 + 2288*a*c*e*f^3*g^3 - 3432*a*c*d*f^2*g^4 + 300
3*a^2*e*f*g^5 + 15015*a^2*d*g^6)*x)*sqrt(g*x + f)/g^6
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.74

$$\int (d+ex)\sqrt{f+gx}\left(a+cx^2\right)^2\,dx$$
$$=\begin{cases} \frac{2\left(\frac{c^2e(f+gx)^{\frac{13}{2}}}{13g^5}+\frac{(f+gx)^{\frac{11}{2}}\left(c^2dg-5c^2ef\right)}{11g^5}+\frac{(f+gx)^{\frac{9}{2}}\cdot\left(2aceg^2-4c^2dfg+10c^2ef^2\right)}{9g^5}+\frac{(f+gx)^{\frac{7}{2}}\cdot\left(2acdg^3-6acefg^2+6c^2df^2g-10c^2ef^3\right)}{7g^5}+\frac{(f+gx)^{\frac{5}{2}}\left(a^2 \cdots\right.}{\cdots}\right.}{g} \\ \sqrt{f}\left(a^2dx+\frac{a^2ex^2}{2}+\frac{2acdx^3}{3}+\frac{acex^4}{2}+\frac{c^2dx^5}{5}+\frac{c^2ex^6}{6}\right) \end{cases}$$

input
```
integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+a)**2,x)
```

output
```
Piecewise((2*(c**2*e*(f + g*x)**(13/2)/(13*g**5) + (f + g*x)**(11/2)*(c**2
*d*g - 5*c**2*e*f)/(11*g**5) + (f + g*x)**(9/2)*(2*a*c*e*g**2 - 4*c**2*d*f
*g + 10*c**2*e*f**2)/(9*g**5) + (f + g*x)**(7/2)*(2*a*c*d*g**3 - 6*a*c*e*f
*g**2 + 6*c**2*d*f**2*g - 10*c**2*e*f**3)/(7*g**5) + (f + g*x)**(5/2)*(a**
2*e*g**4 - 4*a*c*d*f*g**3 + 6*a*c*e*f**2*g**2 - 4*c**2*d*f**3*g + 5*c**2*e
*f**4)/(5*g**5) + (f + g*x)**(3/2)*(a**2*d*g**5 - a**2*e*f*g**4 + 2*a*c*d*
f**2*g**3 - 2*a*c*e*f**3*g**2 + c**2*d*f**4*g - c**2*e*f**5)/(3*g**5))/g,
Ne(g, 0)), (sqrt(f)*(a**2*d*x + a**2*e*x**2/2 + 2*a*c*d*x**3/3 + a*c*e*x**
4/2 + c**2*d*x**5/5 + c**2*e*x**6/6), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.16

$$\int (d+ex)\sqrt{f+gx}\left(a+cx^2\right)^2\,dx$$
$$= \frac{2\left(3465\,(gx+f)^{\frac{13}{2}}c^2e - 4095\,(5\,c^2ef - c^2dg)(gx+f)^{\frac{11}{2}} + 10010\,(5\,c^2ef^2 - 2\,c^2dfg + aceg^2)(gx+f)^{\frac{9}{2}} \cdot\right.}{}$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/45045*(3465*(g*x + f)^(13/2)*c^2*e - 4095*(5*c^2*e*f - c^2*d*g)*(g*x + f
)^(11/2) + 10010*(5*c^2*e*f^2 - 2*c^2*d*f*g + a*c*e*g^2)*(g*x + f)^(9/2) -
 12870*(5*c^2*e*f^3 - 3*c^2*d*f^2*g + 3*a*c*e*f*g^2 - a*c*d*g^3)*(g*x + f)
^(7/2) + 9009*(5*c^2*e*f^4 - 4*c^2*d*f^3*g + 6*a*c*e*f^2*g^2 - 4*a*c*d*f*g
^3 + a^2*e*g^4)*(g*x + f)^(5/2) - 15015*(c^2*e*f^5 - c^2*d*f^4*g + 2*a*c*e
*f^3*g^2 - 2*a*c*d*f^2*g^3 + a^2*e*f*g^4 - a^2*d*g^5)*(g*x + f)^(3/2))/g^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. 2(190) = 380.

Time = 0.12 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.96

$$\int (d+ex)\sqrt{f+gx}\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(g*x + f)*a^2*d*f + 15015*((g*x + f)^(3/2) - 3*sqrt(g*x
 + f)*f)*a^2*d + 15015*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a^2*e*f/g + 6
006*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a*c*
d*f/g^2 + 3003*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f
)*f^2)*a^2*e/g + 2574*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x
+ f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*c*e*f/g^3 + 2574*(5*(g*x + f)^(7/
2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)
*a*c*d/g^2 + 143*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x +
f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c^2*d*f/g^
4 + 286*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*
f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*a*c*e/g^3 + 65*(63*
(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x + f)^(7/2)*f^2 - 1386*
(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*sqrt(g*x + f)*f^5)*c^
2*e*f/g^5 + 65*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x + f
)^(7/2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*sq
rt(g*x + f)*f^5)*c^2*d/g^4 + 15*(231*(g*x + f)^(13/2) - 1638*(g*x + f)^(11
/2)*f + 5005*(g*x + f)^(9/2)*f^2 - 8580*(g*x + f)^(7/2)*f^3 + 9009*(g*x +
f)^(5/2)*f^4 - 6006*(g*x + f)^(3/2)*f^5 + 3003*sqrt(g*x + f)*f^6)*c^2*e/g^
5)/g
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.92

$$\int (d+ex)\sqrt{f+gx}\left(a+cx^2\right)^2\,dx = \frac{(f+g\,x)^{9/2}\,\left(20\,e\,c^2\,f^2-8\,d\,c^2\,f\,g+4\,a\,e\,c\,g^2\right)}{9\,g^6} + \frac{2\,(f+g\,x)^{3/2}\,\left(c\,f^2+a\,g^2\right)^2\,(d\,g-e\,f)}{3\,g^6} + \frac{2\,(f+g\,x)^{5/2}\,\left(c\,f^2+a\,g^2\right)\,\left(5\,c\,e\,f^2-4\,c\,d\,f\,g+a\,e\,g^2\right)}{5\,g^6} + \frac{2\,c^2\,e\,(f+g\,x)^{13/2}}{13\,g^6} + \frac{4\,c\,(f+g\,x)^{7/2}\,\left(-5\,c\,e\,f^3+3\,c\,d\,f^2\,g-3\,a\,e\,f\,g^2+a\,d\,g^3\right)}{7\,g^6} + \frac{2\,c^2\,(f+g\,x)^{11/2}\,(d\,g-5\,e\,f)}{11\,g^6}$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2)^2*(d + e*x),x)
```

output
```
((f + g*x)^(9/2)*(20*c^2*e*f^2 + 4*a*c*e*g^2 - 8*c^2*d*f*g))/(9*g^6) + (2*
(f + g*x)^(3/2)*(a*g^2 + c*f^2)^2*(d*g - e*f))/(3*g^6) + (2*(f + g*x)^(5/2
)*(a*g^2 + c*f^2)*(a*e*g^2 + 5*c*e*f^2 - 4*c*d*f*g))/(5*g^6) + (2*c^2*e*(f
 + g*x)^(13/2))/(13*g^6) + (4*c*(f + g*x)^(7/2)*(a*d*g^3 - 5*c*e*f^3 - 3*a
*e*f*g^2 + 3*c*d*f^2*g))/(7*g^6) + (2*c^2*(f + g*x)^(11/2)*(d*g - 5*e*f))/
(11*g^6)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.59

$$\int (d+ex)\sqrt{f+gx}\left(a+cx^2\right)^2\,dx$$

$$= \frac{2\sqrt{gx+f}\,(3465c^2e\,g^6x^6+4095c^2d\,g^6x^5+315c^2ef\,g^5x^5+10010ace\,g^6x^4+455c^2df\,g^5x^4-350c^2e\,f^2g^4x}{}$$

input
```
int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^2,x)
```

output
```
(2*sqrt(f + g*x)*(15015*a**2*d*f*g**5 + 15015*a**2*d*g**6*x - 6006*a**2*e*
f**2*g**4 + 3003*a**2*e*f*g**5*x + 9009*a**2*e*g**6*x**2 + 6864*a*c*d*f**3
*g**3 - 3432*a*c*d*f**2*g**4*x + 2574*a*c*d*f*g**5*x**2 + 12870*a*c*d*g**6
*x**3 - 4576*a*c*e*f**4*g**2 + 2288*a*c*e*f**3*g**3*x - 1716*a*c*e*f**2*g*
*4*x**2 + 1430*a*c*e*f*g**5*x**3 + 10010*a*c*e*g**6*x**4 + 1664*c**2*d*f**
5*g - 832*c**2*d*f**4*g**2*x + 624*c**2*d*f**3*g**3*x**2 - 520*c**2*d*f**2
*g**4*x**3 + 455*c**2*d*f*g**5*x**4 + 4095*c**2*d*g**6*x**5 - 1280*c**2*e*
f**6 + 640*c**2*e*f**5*g*x - 480*c**2*e*f**4*g**2*x**2 + 400*c**2*e*f**3*g
**3*x**3 - 350*c**2*e*f**2*g**4*x**4 + 315*c**2*e*f*g**5*x**5 + 3465*c**2*
e*g**6*x**6))/(45045*g**6)
```

3.66 $\int \sqrt{f+gx}(a+cx^2)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 127

$$\int \sqrt{f+gx}(a+cx^2)^2\,dx = \frac{2(cf^2+ag^2)^2(f+gx)^{3/2}}{3g^5} - \frac{8cf(cf^2+ag^2)(f+gx)^{5/2}}{5g^5} + \frac{4c(3cf^2+ag^2)(f+gx)^{7/2}}{7g^5} - \frac{8c^2f(f+gx)^{9/2}}{9g^5} + \frac{2c^2(f+gx)^{11/2}}{11g^5}$$

output
```
2/3*(a*g^2+c*f^2)^2*(g*x+f)^(3/2)/g^5-8/5*c*f*(a*g^2+c*f^2)*(g*x+f)^(5/2)/
g^5+4/7*c*(a*g^2+3*c*f^2)*(g*x+f)^(7/2)/g^5-8/9*c^2*f*(g*x+f)^(9/2)/g^5+2/
11*c^2*(g*x+f)^(11/2)/g^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \sqrt{f+gx}(a+cx^2)^2\,dx$$
$$= \frac{2(f+gx)^{3/2}(1155a^2g^4+66acg^2(8f^2-12fgx+15g^2x^2)+c^2(128f^4-192f^3gx+240f^2g^2x^2-280fg^3}{3465g^5}$$

input
```
Integrate[Sqrt[f + g*x]*(a + c*x^2)^2,x]
```

output
```
(2*(f + g*x)^(3/2)*(1155*a^2*g^4 + 66*a*c*g^2*(8*f^2 - 12*f*g*x + 15*g^2*x
^2) + c^2*(128*f^4 - 192*f^3*g*x + 240*f^2*g^2*x^2 - 280*f*g^3*x^3 + 315*g
^4*x^4)))/(3465*g^5)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2 \sqrt{f+gx}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{2c(f+gx)^{5/2}\left(ag^2+3cf^2\right)}{g^4}-\frac{4cf(f+gx)^{3/2}\left(ag^2+cf^2\right)}{g^4}+\frac{\sqrt{f+gx}\left(ag^2+cf^2\right)^2}{g^4}+\frac{c^2(f+gx)^{9/2}}{g^4}-\frac{4c^2 \ldots$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c(f+gx)^{7/2}\left(ag^2+3cf^2\right)}{7g^5}-\frac{8cf(f+gx)^{5/2}\left(ag^2+cf^2\right)}{5g^5}+\frac{2(f+gx)^{3/2}\left(ag^2+cf^2\right)^2}{3g^5}+\frac{2c^2(f+gx)^{11/2}}{11g^5}-\frac{8c^2f(f+gx)^{9/2}}{9g^5}$$

input
```
Int[Sqrt[f + g*x]*(a + c*x^2)^2,x]
```

output
```
(2*(c*f^2 + a*g^2)^2*(f + g*x)^(3/2))/(3*g^5) - (8*c*f*(c*f^2 + a*g^2)*(f
+ g*x)^(5/2))/(5*g^5) + (4*c*(3*c*f^2 + a*g^2)*(f + g*x)^(7/2))/(7*g^5) -
(8*c^2*f*(f + g*x)^(9/2))/(9*g^5) + (2*c^2*(f + g*x)^(11/2))/(11*g^5)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{2(gx+f)^{\frac{3}{2}}\left(\left(\frac{6}{7}ac\,x^{2}+\frac{3}{11}c^{2}x^{4}+a^{2}\right)g^{4}-\frac{24xc\left(\frac{35c\,x^{2}}{99}+a\right)f\,g^{3}}{35}+\frac{16c\left(\frac{5c\,x^{2}}{11}+a\right)f^{2}g^{2}}{35}-\frac{64c^{2}f^{3}gx}{385}+\frac{128c^{2}f^{4}}{1155}\right)}{3g^{5}}$
gosper	$\frac{2(gx+f)^{\frac{3}{2}}\left(315c^{2}x^{4}g^{4}-280c^{2}f\,x^{3}g^{3}+990ac\,g^{4}x^{2}+240c^{2}f^{2}g^{2}x^{2}-792acf\,g^{3}x-192c^{2}f^{3}gx+1155a^{2}g^{4}+528ac\,f^{2}g^{2}+1\right.}{3465g^{5}}$
orering	$\frac{2(gx+f)^{\frac{3}{2}}\left(315c^{2}x^{4}g^{4}-280c^{2}f\,x^{3}g^{3}+990ac\,g^{4}x^{2}+240c^{2}f^{2}g^{2}x^{2}-792acf\,g^{3}x-192c^{2}f^{3}gx+1155a^{2}g^{4}+528ac\,f^{2}g^{2}+1\right.}{3465g^{5}}$
derivativedivides	$\frac{\frac{2c^{2}(gx+f)^{\frac{11}{2}}}{11}-\frac{8c^{2}f(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2\left(a\,g^{2}+c\,f^{2}\right)c+4c^{2}f^{2}\right)(gx+f)^{\frac{7}{2}}}{7}-\frac{8\left(a\,g^{2}+c\,f^{2}\right)cf(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(a\,g^{2}+c\,f^{2}\right)^{2}(gx+f)^{\frac{3}{2}}}{3}}{g^{5}}$
default	$\frac{\frac{2c^{2}(gx+f)^{\frac{11}{2}}}{11}-\frac{8c^{2}f(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(2\left(a\,g^{2}+c\,f^{2}\right)c+4c^{2}f^{2}\right)(gx+f)^{\frac{7}{2}}}{7}-\frac{8\left(a\,g^{2}+c\,f^{2}\right)cf(gx+f)^{\frac{5}{2}}}{5}+\frac{2\left(a\,g^{2}+c\,f^{2}\right)^{2}(gx+f)^{\frac{3}{2}}}{3}}{g^{5}}$
trager	$\frac{2\left(315c^{2}g^{5}x^{5}+35c^{2}f\,g^{4}x^{4}+990ac\,g^{5}x^{3}-40c^{2}f^{2}g^{3}x^{3}+198acf\,g^{4}x^{2}+48c^{2}f^{3}g^{2}x^{2}+1155a^{2}g^{5}x-264ac\,f^{2}g^{3}x-64c^{2}f^{4}g\right.}{3465g^{5}}$
risch	$\frac{2\left(315c^{2}g^{5}x^{5}+35c^{2}f\,g^{4}x^{4}+990ac\,g^{5}x^{3}-40c^{2}f^{2}g^{3}x^{3}+198acf\,g^{4}x^{2}+48c^{2}f^{3}g^{2}x^{2}+1155a^{2}g^{5}x-264ac\,f^{2}g^{3}x-64c^{2}f^{4}g\right.}{3465g^{5}}$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(g*x+f)^(3/2)*((6/7*a*c*x^2+3/11*c^2*x^4+a^2)*g^4-24/35*x*c*(35/99*c*x
^2+a)*f*g^3+16/35*c*(5/11*c*x^2+a)*f^2*g^2-64/385*c^2*f^3*g*x+128/1155*c^2
*f^4)/g^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13

$$\int \sqrt{f+gx}\left(a+cx^2\right)^2 dx$$
$$= \frac{2\,(315\,c^2g^5x^5+35\,c^2fg^4x^4+128\,c^2f^5+528\,acf^3g^2+1155\,a^2fg^4-10\,(4\,c^2f^2g^3-99\,acg^5)x^3+6\,(8\,c^2f}{3465\,g^5}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/3465*(315*c^2*g^5*x^5 + 35*c^2*f*g^4*x^4 + 128*c^2*f^5 + 528*a*c*f^3*g^2
 + 1155*a^2*f*g^4 - 10*(4*c^2*f^2*g^3 - 99*a*c*g^5)*x^3 + 6*(8*c^2*f^3*g^2
 + 33*a*c*f*g^4)*x^2 - (64*c^2*f^4*g + 264*a*c*f^2*g^3 - 1155*a^2*g^5)*x)*
sqrt(g*x + f)/g^5
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.39

$$\int \sqrt{f+gx}\left(a+cx^2\right)^2 dx$$
$$= \begin{cases} \frac{2\left(-\frac{4c^2f(f+gx)^{\frac{9}{2}}}{9g^4}+\frac{c^2(f+gx)^{\frac{11}{2}}}{11g^4}+\frac{(f+gx)^{\frac{7}{2}}\cdot(2acg^2+6c^2f^2)}{7g^4}+\frac{(f+gx)^{\frac{5}{2}}(-4acfg^2-4c^2f^3)}{5g^4}+\frac{(f+gx)^{\frac{3}{2}}(a^2g^4+2acf^2g^2+c^2f^4)}{3g^4}\right)}{g} & \text{for } g\neq 0 \\ \sqrt{f}\left(a^2x+\frac{2acx^3}{3}+\frac{c^2x^5}{5}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a)**2,x)
```

output
```
Piecewise((2*(-4*c**2*f*(f + g*x)**(9/2)/(9*g**4) + c**2*(f + g*x)**(11/2)
/(11*g**4) + (f + g*x)**(7/2)*(2*a*c*g**2 + 6*c**2*f**2)/(7*g**4) + (f + g
*x)**(5/2)*(-4*a*c*f*g**2 - 4*c**2*f**3)/(5*g**4) + (f + g*x)**(3/2)*(a**2
*g**4 + 2*a*c*f**2*g**2 + c**2*f**4)/(3*g**4))/g, Ne(g, 0)), (sqrt(f)*(a**
2*x + 2*a*c*x**3/3 + c**2*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.89

$$\int \sqrt{f+gx}\left(a+cx^2\right)^2\,dx$$
$$=\frac{2\left(315\,(gx+f)^{\frac{11}{2}}c^2-1540\,(gx+f)^{\frac{9}{2}}c^2f+990\,(3\,c^2f^2+acg^2)(gx+f)^{\frac{7}{2}}-2772\,(c^2f^3+acfg^2)(gx+f\right.}{3465\,g^5}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/3465*(315*(g*x + f)^(11/2)*c^2 - 1540*(g*x + f)^(9/2)*c^2*f + 990*(3*c^2
*f^2 + a*c*g^2)*(g*x + f)^(7/2) - 2772*(c^2*f^3 + a*c*f*g^2)*(g*x + f)^(5/
2) + 1155*(c^2*f^4 + 2*a*c*f^2*g^2 + a^2*g^4)*(g*x + f)^(3/2))/g^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(107) = 214$.

Time = 0.11 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.16

$$\int \sqrt{f+gx}\left(a+cx^2\right)^2\,dx$$
$$=\frac{2\left(3465\,\sqrt{gx+f}a^2f+1155\left((gx+f)^{\frac{3}{2}}-3\,\sqrt{gx+f}f\right)a^2+\frac{462\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\,\sqrt{gx+f}ff^2\right)acf}{g^2}+\right.}{}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(g*x + f)*a^2*f + 1155*((g*x + f)^(3/2) - 3*sqrt(g*x + f)
*f)*a^2 + 462*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)
*f^2)*a*c*f/g^2 + 198*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x
+ f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*c/g^2 + 11*(35*(g*x + f)^(9/2) -
180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3
+ 315*sqrt(g*x + f)*f^4)*c^2*f/g^4 + 5*(63*(g*x + f)^(11/2) - 385*(g*x + f
)^(9/2)*f + 990*(g*x + f)^(7/2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x
 + f)^(3/2)*f^4 - 693*sqrt(g*x + f)*f^5)*c^2/g^4)/g
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int \sqrt{f+gx}\left(a+cx^2\right)^2\,dx=\frac{2\,c^2\,{\left(f+g\,x\right)}^{11/2}}{11\,g^5}-\frac{{\left(f+g\,x\right)}^{5/2}\,\left(8\,c^2\,f^3+8\,a\,c\,f\,g^2\right)}{5\,g^5}+\frac{2\,{\left(f+g\,x\right)}^{3/2}\,{\left(c\,f^2+a\,g^2\right)}^2}{3\,g^5}+\frac{{\left(f+g\,x\right)}^{7/2}\,\left(12\,c^2\,f^2+4\,a\,c\,g^2\right)}{7\,g^5}-\frac{8\,c^2\,f\,{\left(f+g\,x\right)}^{9/2}}{9\,g^5}$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2)^2,x)
```

output
```
(2*c^2*(f + g*x)^(11/2))/(11*g^5) - ((f + g*x)^(5/2)*(8*c^2*f^3 + 8*a*c*f*
g^2))/(5*g^5) + (2*(f + g*x)^(3/2)*(a*g^2 + c*f^2)^2)/(3*g^5) + ((f + g*x)
^(7/2)*(12*c^2*f^2 + 4*a*c*g^2))/(7*g^5) - (8*c^2*f*(f + g*x)^(9/2))/(9*g^
5)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.11

$$\int \sqrt{f+gx}\left(a+cx^2\right)^2\,dx=\frac{2\sqrt{gx+f}\,(315c^2g^5x^5+35c^2f\,g^4x^4+990ac\,g^5x^3-40c^2f^2g^3x^3+198acf\,g^4x^2+48c^2f^3g^2x^2+1155a^2g^5}{3465g^5}$$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^2,x)
```

output
```
(2*sqrt(f + g*x)*(1155*a**2*f*g**4 + 1155*a**2*g**5*x + 528*a*c*f**3*g**2
- 264*a*c*f**2*g**3*x + 198*a*c*f*g**4*x**2 + 990*a*c*g**5*x**3 + 128*c**2
*f**5 - 64*c**2*f**4*g*x + 48*c**2*f**3*g**2*x**2 - 40*c**2*f**2*g**3*x**3
 + 35*c**2*f*g**4*x**4 + 315*c**2*g**5*x**5))/(3465*g**5)
```

3.67 $\int \frac{\sqrt{f+gx}\,(a+cx^2)^2}{d+ex}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 247

$$\begin{aligned}\int \frac{\sqrt{f+gx}(a+cx^2)^2}{d+ex}\,dx =& \frac{2(cd^2+ae^2)^2\sqrt{f+gx}}{e^5} \\ &- \frac{2c(ef+dg)\left(ce^2f^2+cd^2g^2+2ae^2g^2\right)(f+gx)^{3/2}}{3e^4g^4} \\ &+ \frac{2c\left(2ae^2g^2+c(3e^2f^2+2defg+d^2g^2)\right)(f+gx)^{5/2}}{5e^3g^4} \\ &- \frac{2c^2(3ef+dg)(f+gx)^{7/2}}{7e^2g^4} + \frac{2c^2(f+gx)^{9/2}}{9eg^4} \\ &- \frac{2(cd^2+ae^2)^2\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{11/2}}\end{aligned}$$

output
```
2*(a*e^2+c*d^2)^2*(g*x+f)^(1/2)/e^5-2/3*c*(d*g+e*f)*(2*a*e^2*g^2+c*d^2*g^2
+c*e^2*f^2)*(g*x+f)^(3/2)/e^4/g^4+2/5*c*(2*a*e^2*g^2+c*(d^2*g^2+2*d*e*f*g+
3*e^2*f^2))*(g*x+f)^(5/2)/e^3/g^4-2/7*c^2*(d*g+3*e*f)*(g*x+f)^(7/2)/e^2/g^
4+2/9*c^2*(g*x+f)^(9/2)/e/g^4-2*(a*e^2+c*d^2)^2*(-d*g+e*f)^(1/2)*arctanh(e
^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(11/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{f+gx}(a+cx^2)^2}{d+ex}\,dx$$
$$= \frac{2\sqrt{f+gx}(315a^2e^4g^4-42ace^2g^2(-15d^2g^2+5deg(f+gx)+e^2(2f^2-fgx-3g^2x^2))+c^2(315d^4g^4-10}{}$$
$$-\frac{2(cd^2+ae^2)^2\sqrt{-ef+dg}\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{11/2}}$$

input
```
Integrate[(Sqrt[f + g*x]*(a + c*x^2)^2)/(d + e*x),x]
```

output
```
(2*Sqrt[f + g*x]*(315*a^2*e^4*g^4 - 42*a*c*e^2*g^2*(-15*d^2*g^2 + 5*d*e*g*
(f + g*x) + e^2*(2*f^2 - f*g*x - 3*g^2*x^2)) + c^2*(315*d^4*g^4 - 105*d^3*
e*g^3*(f + g*x) + 21*d^2*e^2*g^2*(-2*f^2 + f*g*x + 3*g^2*x^2) - 3*d*e^3*g*
(8*f^3 - 4*f^2*g*x + 3*f*g^2*x^2 + 15*g^3*x^3) + e^4*(-16*f^4 + 8*f^3*g*x
- 6*f^2*g^2*x^2 + 5*f*g^3*x^3 + 35*g^4*x^4))))/(315*e^5*g^4) - (2*(c*d^2 +
 a*e^2)^2*Sqrt[-(e*f) + d*g]*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) +
d*g]])/e^(11/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {649, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2\sqrt{f+gx}}{d+ex}\,dx$$
$$\Big\downarrow \; 649$$
$$\frac{2\int -\frac{(f+gx)(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)^2}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\int \frac{(f+gx)\left(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2\right)^2}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^4}$$

$$\Big\downarrow \; 1584$$

$$-\frac{2\int \left(-\frac{(cd^2+ae^2)^2g^4}{e^5}-\frac{c^2(f+gx)^4}{e}+\frac{c^2(3ef+dg)(f+gx)^3}{e^2}-\frac{c(2ae^2g^2+c(3e^2f^2+2degf+d^2g^2))(f+gx)^2}{e^3}+\frac{c(ef+dg)(ce^2f^2+cd^2g^2-}{e^4}\right.}{g^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(-\frac{g^4(ae^2+cd^2)^2\sqrt{ef-dg}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{11/2}}+\frac{g^4\sqrt{f+gx}(ae^2+cd^2)^2}{e^5}-\frac{c(f+gx)^{3/2}(dg+ef)(2ae^2g^2+cd^2g^2+ce^2f^2)}{3e^4}+\frac{c(f+gx)^{5/2}}{}\right.}{g^4}$$

input
```
Int[(Sqrt[f + g*x]*(a + c*x^2)^2)/(d + e*x),x]
```

output
```
(2*(((c*d^2 + a*e^2)^2*g^4*Sqrt[f + g*x])/e^5 - (c*(e*f + d*g)*(c*e^2*f^2
+ c*d^2*g^2 + 2*a*e^2*g^2)*(f + g*x)^(3/2))/(3*e^4) + (c*(2*a*e^2*g^2 + c*
(3*e^2*f^2 + 2*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*e^3) - (c^2*(3*e*f
+ d*g)*(f + g*x)^(7/2))/(7*e^2) + (c^2*(f + g*x)^(9/2))/(9*e) - ((c*d^2 +
a*e^2)^2*g^4*Sqrt[e*f - d*g]*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*
g]])/e^(11/2)))/g^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{2\left(\left(\left(-\frac{16(gx+f)\left(-\frac{35}{16}x^3g^3+\frac{15}{8}fg^2x^2-\frac{3}{2}xf^2g+f^3\right)e^4}{315}-\frac{8g(gx+f)d\left(\frac{15}{8}g^2x^2-\frac{3}{2}fgx+f^2\right)e^3}{105}-\frac{2g^2\left(-\frac{3gx}{2}+f\right)(gx+f)d^2e^2}{15}-\cdots\right.\right.\right.}{\cdots}$
derivativedivides	$\frac{2\left(\frac{c^2(gx+f)^{\frac{9}{2}}e^4}{9}-\frac{c^2de^3g(gx+f)^{\frac{7}{2}}}{7}-\frac{3c^2e^4f(gx+f)^{\frac{7}{2}}}{7}+\frac{2ace^4g^2(gx+f)^{\frac{5}{2}}}{5}+\frac{c^2d^2e^2g^2(gx+f)^{\frac{5}{2}}}{5}+\frac{2c^2de^3fg(gx+f)^{\frac{5}{2}}}{5}+\frac{3c^2e^4f^2\cdots}{\cdots}\right.}{\cdots}$
default	$\frac{2\left(\frac{c^2(gx+f)^{\frac{9}{2}}e^4}{9}-\frac{c^2de^3g(gx+f)^{\frac{7}{2}}}{7}-\frac{3c^2e^4f(gx+f)^{\frac{7}{2}}}{7}+\frac{2ace^4g^2(gx+f)^{\frac{5}{2}}}{5}+\frac{c^2d^2e^2g^2(gx+f)^{\frac{5}{2}}}{5}+\frac{2c^2de^3fg(gx+f)^{\frac{5}{2}}}{5}+\frac{3c^2e^4f^2\cdots}{\cdots}\right.}{\cdots}$
risch	$\frac{2\left(35c^2e^4g^4x^4-45c^2de^3g^4x^3+5c^2e^4fg^3x^3+126ace^4g^4x^2+63c^2d^2e^2g^4x^2-9c^2de^3fg^3x^2-6c^2e^4f^2g^2x^2-210acde^3g\cdots\right.}{\cdots}$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
2*(((-16/315*(g*x+f)*(-35/16*x^3*g^3+15/8*f*g^2*x^2-3/2*x*f^2*g+f^3)*e^4-8
/105*g*(g*x+f)*d*(15/8*g^2*x^2-3/2*f*g*x+f^2)*e^3-2/15*g^2*(-3/2*g*x+f)*(g
*x+f)*d^2*e^2-1/3*d^3*g^3*(g*x+f)*e+g^4*d^4)*c^2+2*(-2/15*(-3/2*g*x+f)*(g*
x+f)*e^2-1/3*d*g*(g*x+f)*e+d^2*g^2)*e^2*g^2*a*c+a^2*e^4*g^4)*((d*g-e*f)*e)
^(1/2)*(g*x+f)^(1/2)-g^4*(a*e^2+c*d^2)^2*(d*g-e*f)*arctan(e*(g*x+f)^(1/2)/
((d*g-e*f)*e)^(1/2)))/((d*g-e*f)*e)^(1/2)/g^4/e^5
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 766, normalized size of antiderivative = 3.10

$$\int \frac{\sqrt{f+gx}(a+cx^2)^2}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^2/(e*x+d),x, algorithm="fricas")
```

output
```
[1/315*(315*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*g^4*sqrt((e*f - d*g)/e)*lo
g((e*g*x + 2*e*f - d*g - 2*sqrt(g*x + f)*e*sqrt((e*f - d*g)/e))/(e*x + d))
 + 2*(35*c^2*e^4*g^4*x^4 - 16*c^2*e^4*f^4 - 24*c^2*d*e^3*f^3*g - 42*(c^2*d
^2*e^2 + 2*a*c*e^4)*f^2*g^2 - 105*(c^2*d^3*e + 2*a*c*d*e^3)*f*g^3 + 315*(c
^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*g^4 + 5*(c^2*e^4*f*g^3 - 9*c^2*d*e^3*g^4
)*x^3 - 3*(2*c^2*e^4*f^2*g^2 + 3*c^2*d*e^3*f*g^3 - 21*(c^2*d^2*e^2 + 2*a*c
*e^4)*g^4)*x^2 + (8*c^2*e^4*f^3*g + 12*c^2*d*e^3*f^2*g^2 + 21*(c^2*d^2*e^2
 + 2*a*c*e^4)*f*g^3 - 105*(c^2*d^3*e + 2*a*c*d*e^3)*g^4)*x)*sqrt(g*x + f))
/(e^5*g^4), -2/315*(315*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*g^4*sqrt(-(e*f
 - d*g)/e)*arctan(-sqrt(g*x + f)*e*sqrt(-(e*f - d*g)/e)/(e*f - d*g)) - (35
*c^2*e^4*g^4*x^4 - 16*c^2*e^4*f^4 - 24*c^2*d*e^3*f^3*g - 42*(c^2*d^2*e^2 +
 2*a*c*e^4)*f^2*g^2 - 105*(c^2*d^3*e + 2*a*c*d*e^3)*f*g^3 + 315*(c^2*d^4 +
 2*a*c*d^2*e^2 + a^2*e^4)*g^4 + 5*(c^2*e^4*f*g^3 - 9*c^2*d*e^3*g^4)*x^3 -
3*(2*c^2*e^4*f^2*g^2 + 3*c^2*d*e^3*f*g^3 - 21*(c^2*d^2*e^2 + 2*a*c*e^4)*g^
4)*x^2 + (8*c^2*e^4*f^3*g + 12*c^2*d*e^3*f^2*g^2 + 21*(c^2*d^2*e^2 + 2*a*c
*e^4)*f*g^3 - 105*(c^2*d^3*e + 2*a*c*d*e^3)*g^4)*x)*sqrt(g*x + f))/(e^5*g^
4)]
```

Sympy [A] (verification not implemented)

Time = 4.99 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{f+gx}\,(a+cx^2)^2}{d+ex}\,dx$$

$$= \begin{cases} \frac{2\left(\frac{c^2(f+gx)^{\frac{9}{2}}}{9eg^3}+\frac{(f+gx)^{\frac{7}{2}}\left(-c^2dg-3c^2ef\right)}{7e^2g^3}+\frac{(f+gx)^{\frac{5}{2}}\cdot\left(2ace^2g^2+c^2d^2g^2+2c^2defg+3c^2e^2f^2\right)}{5e^3g^3}+\frac{(f+gx)^{\frac{3}{2}}\left(-2acde^2g^3-2ace^3fg^2-c^2d^3g^3-c^2d^2efg^2-\right.}{3e^4g^3}\right.}{g} \\ \sqrt{f}\left(-\frac{c^2dx^3}{3e^2}+\frac{c^2x^4}{4e}+\frac{x^2\cdot\left(2ace^2+c^2d^2\right)}{2e^3}+\frac{x\left(-2acde^2-c^2d^3\right)}{e^4}+\frac{\left(ae^2+cd^2\right)^2\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^4}\right) \end{cases}$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a)**2/(e*x+d),x)
```

output
```
Piecewise((2*(c**2*(f + g*x)**(9/2)/(9*e*g**3) + (f + g*x)**(7/2)*(-c**2*d
*g - 3*c**2*e*f)/(7*e**2*g**3) + (f + g*x)**(5/2)*(2*a*c*e**2*g**2 + c**2*
d**2*g**2 + 2*c**2*d*e*f*g + 3*c**2*e**2*f**2)/(5*e**3*g**3) + (f + g*x)**
(3/2)*(-2*a*c*d*e**2*g**3 - 2*a*c*e**3*f*g**2 - c**2*d**3*g**3 - c**2*d**2
*e*f*g**2 - c**2*d*e**2*f**2*g - c**2*e**3*f**3)/(3*e**4*g**3) + sqrt(f +
g*x)*(a**2*e**4*g + 2*a*c*d**2*e**2*g + c**2*d**4*g)/e**5 - g*(a*e**2 + c*
d**2)**2*(d*g - e*f)*atan(sqrt(f + g*x)/sqrt((d*g - e*f)/e))/(e**6*sqrt((d
*g - e*f)/e)))/g, Ne(g, 0)), (sqrt(f)*(-c**2*d*x**3/(3*e**2) + c**2*x**4/(
4*e) + x**2*(2*a*c*e**2 + c**2*d**2)/(2*e**3) + x*(-2*a*c*d*e**2 - c**2*d*
*3)/e**4 + (a*e**2 + c*d**2)**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e
, True))/e**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{f+gx}(a+cx^2)^2}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^2/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.72

$$\int \frac{\sqrt{f+gx}(a+cx^2)^2}{d+ex}\,dx = \frac{2\,(c^2d^4ef+2\,acd^2e^3f+a^2e^5f-c^2d^5g-2\,acd^3e^2g-a^2de^4g)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{\sqrt{-e^2f+deg}\,e^5} + \frac{2\left(35\,(gx+f)^{\frac{9}{2}}c^2e^8g^{32}-135\,(gx+f)^{\frac{7}{2}}c^2e^8fg^{32}+189\,(gx+f)^{\frac{5}{2}}c^2e^8f^2g^{32}-105\,(gx+f)^{\frac{3}{2}}c^2e^8f^3g^{32}\right.}{}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^2/(e*x+d),x, algorithm="giac")
```

output
```
2*(c^2*d^4*e*f + 2*a*c*d^2*e^3*f + a^2*e^5*f - c^2*d^5*g - 2*a*c*d^3*e^2*g
 - a^2*d*e^4*g)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e^2*f
+ d*e*g)*e^5) + 2/315*(35*(g*x + f)^(9/2)*c^2*e^8*g^32 - 135*(g*x + f)^(7/
2)*c^2*e^8*f*g^32 + 189*(g*x + f)^(5/2)*c^2*e^8*f^2*g^32 - 105*(g*x + f)^(
3/2)*c^2*e^8*f^3*g^32 - 45*(g*x + f)^(7/2)*c^2*d*e^7*g^33 + 126*(g*x + f)^
(5/2)*c^2*d*e^7*f*g^33 - 105*(g*x + f)^(3/2)*c^2*d*e^7*f^2*g^33 + 63*(g*x
+ f)^(5/2)*c^2*d^2*e^6*g^34 + 126*(g*x + f)^(5/2)*a*c*e^8*g^34 - 105*(g*x
+ f)^(3/2)*c^2*d^2*e^6*f*g^34 - 210*(g*x + f)^(3/2)*a*c*e^8*f*g^34 - 105*(
g*x + f)^(3/2)*c^2*d^3*e^5*g^35 - 210*(g*x + f)^(3/2)*a*c*d*e^7*g^35 + 315
*sqrt(g*x + f)*c^2*d^4*e^4*g^36 + 630*sqrt(g*x + f)*a*c*d^2*e^6*g^36 + 315
*sqrt(g*x + f)*a^2*e^8*g^36)/(e^9*g^36)
```

Mupad [B] (verification not implemented)

Time = 6.20 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.41

$$\int \frac{\sqrt{f+gx}(a+cx^2)^2}{d+ex}\,dx = \sqrt{f+g\,x}\left(\frac{2\,{\left(c\,f^2+a\,g^2\right)}^2}{e\,g^4} + \frac{\left(d\,g^5-e\,f\,g^4\right)\left(\frac{8\,c^2\,f^3+8\,a\,c\,f\,g^2}{e\,g^4}+\frac{\left(\frac{12\,c^2\,f^2+4\,a\,c\,g^2}{e\,g^4}+\frac{\left(d\,g^5-e\,f\,g^4\right)\left(\frac{8\,c^2\,f}{e\,g^4}+\frac{2\,c^2\,\left(d\,g^5-e\,f\,g^4\right)}{e^2\,g^8}\right)}{e\,g^4}\right)\left(d\,g^5-e\,f\,g^4\right)}{e\,g^4}\right)}{e\,g^4}\right)$$

$$- {\left(f+g\,x\right)}^{3/2}\left(\frac{8\,c^2\,f^3+8\,a\,c\,f\,g^2}{3\,e\,g^4} + \frac{\left(\frac{12\,c^2\,f^2+4\,a\,c\,g^2}{e\,g^4}+\frac{\left(d\,g^5-e\,f\,g^4\right)\left(\frac{8\,c^2\,f}{e\,g^4}+\frac{2\,c^2\,\left(d\,g^5-e\,f\,g^4\right)}{e^2\,g^8}\right)}{e\,g^4}\right)\left(d\,g^5-e\,f\,g^4\right)}{3\,e\,g^4}\right)$$

$$- {\left(f+g\,x\right)}^{7/2}\left(\frac{8\,c^2\,f}{7\,e\,g^4}+\frac{2\,c^2\,\left(d\,g^5-e\,f\,g^4\right)}{7\,e^2\,g^8}\right)$$

$$+ {\left(f+g\,x\right)}^{5/2}\left(\frac{12\,c^2\,f^2+4\,a\,c\,g^2}{5\,e\,g^4}+\frac{\left(d\,g^5-e\,f\,g^4\right)\left(\frac{8\,c^2\,f}{e\,g^4}+\frac{2\,c^2\,\left(d\,g^5-e\,f\,g^4\right)}{e^2\,g^8}\right)}{5\,e\,g^4}\right) + \frac{2\,c^2\,{\left(f+g\,x\right)}^{9/2}}{9\,e\,g^4} + \frac{\mathrm{atan}\left(\right.}{}$$

input
```
int(((f + g*x)^(1/2)*(a + c*x^2)^2)/(d + e*x),x)
```

output
```
(f + g*x)^(1/2)*((2*(a*g^2 + c*f^2)^2)/(e*g^4) + ((d*g^5 - e*f*g^4)*((8*c^
2*f^3 + 8*a*c*f*g^2)/(e*g^4) + (((12*c^2*f^2 + 4*a*c*g^2)/(e*g^4) + ((d*g^
5 - e*f*g^4)*((8*c^2*f)/(e*g^4) + (2*c^2*(d*g^5 - e*f*g^4))/(e^2*g^8)))/(e
*g^4))*(d*g^5 - e*f*g^4))/(e*g^4)))/(e*g^4)) - (f + g*x)^(3/2)*((8*c^2*f^3
 + 8*a*c*f*g^2)/(3*e*g^4) + (((12*c^2*f^2 + 4*a*c*g^2)/(e*g^4) + ((d*g^5 -
 e*f*g^4)*((8*c^2*f)/(e*g^4) + (2*c^2*(d*g^5 - e*f*g^4))/(e^2*g^8)))/(e*g^
4))*(d*g^5 - e*f*g^4))/(3*e*g^4)) - (f + g*x)^(7/2)*((8*c^2*f)/(7*e*g^4) +
 (2*c^2*(d*g^5 - e*f*g^4))/(7*e^2*g^8)) + (f + g*x)^(5/2)*((12*c^2*f^2 + 4
*a*c*g^2)/(5*e*g^4) + ((d*g^5 - e*f*g^4)*((8*c^2*f)/(e*g^4) + (2*c^2*(d*g^
5 - e*f*g^4))/(e^2*g^8)))/(5*e*g^4)) + (2*c^2*(f + g*x)^(9/2))/(9*e*g^4) +
 (atan((e^(1/2)*(f + g*x)^(1/2)*(a*e^2 + c*d^2)^2*(e*f - d*g)^(1/2)*1i)/(a
^2*e^5*f - c^2*d^5*g - a^2*d*e^4*g + c^2*d^4*e*f + 2*a*c*d^2*e^3*f - 2*a*c
*d^3*e^2*g))*(a*e^2 + c*d^2)^2*(e*f - d*g)^(1/2)*2i)/e^(11/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{f+gx}\,(a+cx^2)^2}{d+ex}\,dx = \frac{2\sqrt{gx+f}\,a^2e^5g^4 - \frac{32\sqrt{gx+f}\,c^2e^5f^4}{315} - \frac{4\sqrt{gx+f}\,acd\,e^4fg^3}{3} - \frac{4\sqrt{gx+f}\,acd\,e^4g^4x}{3} + \frac{4\sqrt{gx+f}\,ac\,e^5fg^3x}{15} + \frac{2\sqrt{gx+f}\,c^2d^2e^3fg^3x}{15}}{}$$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^2/(e*x+d),x)
```

output
```
(2*( - 315*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*
g - e*f)))*a**2*e**4*g**4 - 630*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x
)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**2*e**2*g**4 - 315*sqrt(e)*sqrt(d*g
- e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**4*g**4 +
315*sqrt(f + g*x)*a**2*e**5*g**4 + 630*sqrt(f + g*x)*a*c*d**2*e**3*g**4 -
210*sqrt(f + g*x)*a*c*d*e**4*f*g**3 - 210*sqrt(f + g*x)*a*c*d*e**4*g**4*x
- 84*sqrt(f + g*x)*a*c*e**5*f**2*g**2 + 42*sqrt(f + g*x)*a*c*e**5*f*g**3*x
 + 126*sqrt(f + g*x)*a*c*e**5*g**4*x**2 + 315*sqrt(f + g*x)*c**2*d**4*e*g*
*4 - 105*sqrt(f + g*x)*c**2*d**3*e**2*f*g**3 - 105*sqrt(f + g*x)*c**2*d**3
*e**2*g**4*x - 42*sqrt(f + g*x)*c**2*d**2*e**3*f**2*g**2 + 21*sqrt(f + g*x
)*c**2*d**2*e**3*f*g**3*x + 63*sqrt(f + g*x)*c**2*d**2*e**3*g**4*x**2 - 24
*sqrt(f + g*x)*c**2*d*e**4*f**3*g + 12*sqrt(f + g*x)*c**2*d*e**4*f**2*g**2
*x - 9*sqrt(f + g*x)*c**2*d*e**4*f*g**3*x**2 - 45*sqrt(f + g*x)*c**2*d*e**
4*g**4*x**3 - 16*sqrt(f + g*x)*c**2*e**5*f**4 + 8*sqrt(f + g*x)*c**2*e**5*
f**3*g*x - 6*sqrt(f + g*x)*c**2*e**5*f**2*g**2*x**2 + 5*sqrt(f + g*x)*c**2
*e**5*f*g**3*x**3 + 35*sqrt(f + g*x)*c**2*e**5*g**4*x**4))/(315*e**6*g**4)
```

3.68 $\int \frac{\sqrt{f+gx}\left(a+cx^2\right)^2}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 245

$$\begin{aligned}\int \frac{\sqrt{f+gx}\left(a+cx^2\right)^2}{(d+ex)^2}\,dx = &-\frac{8cd(cd^2+ae^2)\sqrt{f+gx}}{e^5}-\frac{(cd^2+ae^2)^2\sqrt{f+gx}}{e^5(d+ex)} \\ &+\frac{2c(2ae^2g^2+c(e^2f^2+2defg+3d^2g^2))\,(f+gx)^{3/2}}{3e^4g^3} \\ &-\frac{4c^2(ef+dg)(f+gx)^{5/2}}{5e^3g^3}+\frac{2c^2(f+gx)^{7/2}}{7e^2g^3} \\ &-\frac{(cd^2+ae^2)\,(ae^2g-cd(8ef-9dg))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{11/2}\sqrt{ef-dg}}\end{aligned}$$

output
```
-8*c*d*(a*e^2+c*d^2)*(g*x+f)^(1/2)/e^5-(a*e^2+c*d^2)^2*(g*x+f)^(1/2)/e^5/(
e*x+d)+2/3*c*(2*a*e^2*g^2+c*(3*d^2*g^2+2*d*e*f*g+e^2*f^2))*(g*x+f)^(3/2)/e
^4/g^3-4/5*c^2*(d*g+e*f)*(g*x+f)^(5/2)/e^3/g^3+2/7*c^2*(g*x+f)^(7/2)/e^2/g
^3-(a*e^2+c*d^2)*(a*e^2*g-c*d*(-9*d*g+8*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2
)/(-d*g+e*f)^(1/2))/e^(11/2)/(-d*g+e*f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{f+gx}(a+cx^2)^2}{(d+ex)^2}\,dx$$
$$= \frac{\sqrt{f+gx}(-105a^2e^4g^3-70ace^2g^2(15d^2g-2de(f-5gx)-2e^2x(f+gx))+c^2(-945d^4g^3+210d^3eg^2(f$$
$$+\frac{(cd^2+ae^2)(ae^2g+cd(-8ef+9dg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{11/2}\sqrt{-ef+dg}}$$

input
```
Integrate[(Sqrt[f + g*x]*(a + c*x^2)^2)/(d + e*x)^2,x]
```

output
```
(Sqrt[f + g*x]*(-105*a^2*e^4*g^3 - 70*a*c*e^2*g^2*(15*d^2*g - 2*d*e*(f - 5
*g*x) - 2*e^2*x*(f + g*x)) + c^2*(-945*d^4*g^3 + 210*d^3*e*g^2*(f - 3*g*x)
 + 14*d^2*e^2*g*(4*f^2 + 13*f*g*x + 9*g^2*x^2) + 2*d*e^3*(8*f^3 + 24*f^2*g
*x - 11*f*g^2*x^2 - 27*g^3*x^3) + 2*e^4*x*(8*f^3 - 4*f^2*g*x + 3*f*g^2*x^2
 + 15*g^3*x^3))))/(105*e^5*g^3*(d + e*x)) + ((c*d^2 + a*e^2)*(a*e^2*g + c*
d*(-8*e*f + 9*d*g))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e
^(11/2)*Sqrt[-(e*f) + d*g])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {649, 1580, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2\sqrt{f+gx}}{(d+ex)^2}\,dx$$
$$\Big\downarrow \; 649$$
$$\frac{2\int \frac{(f+gx)(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)^2}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{g^3}$$

$$\big\downarrow \text{ 1580}$$

$$2\left(\frac{\int -\frac{\frac{\left(cd^2+ae^2\right)^2g^4+2c^2e^4(f+gx)^4-2c^2e^3(3ef+dg)(f+gx)^3+2ce^2\left(2ae^2g^2+c\left(3e^2f^2+2degf+d^2g^2\right)\right)(f+gx)^2-2ce(ef+dg)\left(ce^2f^2+cd^2g^2+2ae^2g^2\right)(f+g}{ef-dg-e(f+gx)}}{2e^5}}{g^3}\right.$$

$$\big\downarrow \text{ 25}$$

$$2\left(\frac{\frac{g^4\sqrt{f+gx}\left(ae^2+cd^2\right)^2}{2e^5(-dg-e(f+gx)+ef)}-\frac{\int\frac{\left(cd^2+ae^2\right)^2g^4+2c^2e^4(f+gx)^4-2c^2e^3(3ef+dg)(f+gx)^3+2ce^2\left(2ae^2g^2+c\left(3e^2f^2+2degf+d^2g^2\right)\right)(f+gx)^2-2ce(ef+dg)}{ef-dg-e(f+gx)}}{2e^5}}{g^3}\right.$$

$$\big\downarrow \text{ 2341}$$

$$2\left(\frac{\frac{g^4\sqrt{f+gx}\left(ae^2+cd^2\right)^2}{2e^5(-dg-e(f+gx)+ef)}-\frac{\int\left(8cd\left(cd^2+ae^2\right)g^3-2c^2e^3(f+gx)^3+4c^2e^2(ef+dg)(f+gx)^2-2ce\left(2ae^2g^2+c\left(e^2f^2+2degf+3d^2g^2\right)\right)(f+gx)+\frac{9}{}\right.}{2e^5}}{g^3}\right.$$

$$\big\downarrow \text{ 2009}$$

$$2\left(\frac{\frac{g^4\sqrt{f+gx}\left(ae^2+cd^2\right)^2}{2e^5(-dg-e(f+gx)+ef)}-\frac{\frac{g^3\left(ae^2+cd^2\right)\left(ae^2g-cd(8ef-9dg)\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}\sqrt{ef-dg}}-\frac{2}{3}ce(f+gx)^{3/2}\left(2ae^2g^2+c\left(3d^2g^2+2defg+e^2f^2\right)\right)+8c}{2e^5}}{g^3}\right.$$

input
```
Int[(Sqrt[f + g*x]*(a + c*x^2)^2)/(d + e*x)^2,x]
```

output
```
(2*(((c*d^2 + a*e^2)^2*g^4*Sqrt[f + g*x])/(2*e^5*(e*f - d*g - e*(f + g*x))
) - (8*c*d*(c*d^2 + a*e^2)*g^3*Sqrt[f + g*x] - (2*c*e*(2*a*e^2*g^2 + c*(e^
2*f^2 + 2*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(3/2))/3 + (4*c^2*e^2*(e*f + d*g
)*(f + g*x)^(5/2))/5 - (2*c^2*e^3*(f + g*x)^(7/2))/7 + ((c*d^2 + a*e^2)*g^
3*(a*e^2*g - c*d*(8*e*f - 9*d*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f
 - d*g]])/(Sqrt[e]*Sqrt[e*f - d*g]))/(2*e^5)))/g^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{\left(\left(\left(\left(\frac{18}{35} d e^3 x^3-\frac{6}{5} d^2 e^2 x^2+6 d^3 e x+9 d^4-\frac{2}{7} e^4 x^4\right) g^3-2 e f\left(\frac{1}{35} e^2 x^2-\frac{2}{15} d e x+d^2\right)(e x+d) g^2-\frac{8 e^2\left(-\frac{e x}{7}+d\right) f^2(e x+d) g}{15}-16\right.\right.\right.}{}$
risch	$-\frac{2c\left(-15x^3 c e^3 g^3+42cd e^2 g^3 x^2-3c e^3 f g^2 x^2-70a e^3 g^3 x-105c d^2 e g^3 x+14cd e^2 f g^2 x+4c e^3 f^2 g x+420ad e^2 g^3-70a\right.}{105 g^3 e^5}$
derivativedivides	$-\frac{2c\left(-\frac{c(gx+f)^{\frac{7}{2}} e^3}{7}+\frac{2cd e^2 g(gx+f)^{\frac{5}{2}}}{5}+\frac{2c e^3 f(gx+f)^{\frac{5}{2}}}{5}-\frac{2a e^3 g^2(gx+f)^{\frac{3}{2}}}{3}-c d^2 e g^2(gx+f)^{\frac{3}{2}}-\frac{2cd e^2 f g(gx+f)^{\frac{3}{2}}}{3}-\frac{c e^3 f^2(gx}{3}\right.}{e^5}$
default	$-\frac{2c\left(-\frac{c(gx+f)^{\frac{7}{2}} e^3}{7}+\frac{2cd e^2 g(gx+f)^{\frac{5}{2}}}{5}+\frac{2c e^3 f(gx+f)^{\frac{5}{2}}}{5}-\frac{2a e^3 g^2(gx+f)^{\frac{3}{2}}}{3}-c d^2 e g^2(gx+f)^{\frac{3}{2}}-\frac{2cd e^2 f g(gx+f)^{\frac{3}{2}}}{3}-\frac{c e^3 f^2(gx}{3}\right.}{e^5}$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
(-((((18/35*d*e^3*x^3-6/5*d^2*e^2*x^2+6*d^3*e*x+9*d^4-2/7*e^4*x^4)*g^3-2*e*
f*(1/35*e^2*x^2-2/15*d*e*x+d^2)*(e*x+d)*g^2-8/15*e^2*(-1/7*e*x+d)*f^2*(e*x
+d)*g-16/105*e^3*f^3*(e*x+d))*c^2+10*e^2*g^2*((-2/15*e^2*x^2+2/3*d*e*x+d^2
)*g-2/15*e*f*(e*x+d))*a*c+a^2*e^4*g^3)*((d*g-e*f)*e)^(1/2)*(g*x+f)^(1/2)+(
a*e^2+c*d^2)*g^3*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*(e*x+d)*((9*d
^2*g-8*d*e*f)*c+a*e^2*g))/((d*g-e*f)*e)^(1/2)/g^3/e^5/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(219) = 438$.

Time = 0.11 (sec) , antiderivative size = 1229, normalized size of antiderivative = 5.02

$$\int \frac{\sqrt{f+gx}(a+cx^2)^2}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^2/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[-1/210*(105*(8*(c^2*d^4*e + a*c*d^2*e^3)*f*g^3 - (9*c^2*d^5 + 10*a*c*d^3*
e^2 + a^2*d*e^4)*g^4 + (8*(c^2*d^3*e^2 + a*c*d*e^4)*f*g^3 - (9*c^2*d^4*e +
 10*a*c*d^2*e^3 + a^2*e^5)*g^4)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f
- d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(16*c^2*d*e^5*
f^4 + 40*c^2*d^2*e^4*f^3*g + 14*(11*c^2*d^3*e^3 + 10*a*c*d*e^5)*f^2*g^2 -
35*(33*c^2*d^4*e^2 + 34*a*c*d^2*e^4 + 3*a^2*e^6)*f*g^3 + 105*(9*c^2*d^5*e
+ 10*a*c*d^3*e^3 + a^2*d*e^5)*g^4 + 30*(c^2*e^6*f*g^3 - c^2*d*e^5*g^4)*x^4
 + 6*(c^2*e^6*f^2*g^2 - 10*c^2*d*e^5*f*g^3 + 9*c^2*d^2*e^4*g^4)*x^3 - 2*(4
*c^2*e^6*f^3*g + 7*c^2*d*e^5*f^2*g^2 - 2*(37*c^2*d^2*e^4 + 35*a*c*e^6)*f*g
^3 + 7*(9*c^2*d^3*e^3 + 10*a*c*d*e^5)*g^4)*x^2 + 2*(8*c^2*e^6*f^4 + 16*c^2
*d*e^5*f^3*g + (67*c^2*d^2*e^4 + 70*a*c*e^6)*f^2*g^2 - 14*(29*c^2*d^3*e^3
+ 30*a*c*d*e^5)*f*g^3 + 35*(9*c^2*d^4*e^2 + 10*a*c*d^2*e^4)*g^4)*x)*sqrt(g
*x + f))/(d*e^7*f*g^3 - d^2*e^6*g^4 + (e^8*f*g^3 - d*e^7*g^4)*x), -1/105*(
105*(8*(c^2*d^4*e + a*c*d^2*e^3)*f*g^3 - (9*c^2*d^5 + 10*a*c*d^3*e^2 + a^2
*d*e^4)*g^4 + (8*(c^2*d^3*e^2 + a*c*d*e^4)*f*g^3 - (9*c^2*d^4*e + 10*a*c*d
^2*e^3 + a^2*e^5)*g^4)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)
*sqrt(g*x + f)/(e*g*x + e*f)) - (16*c^2*d*e^5*f^4 + 40*c^2*d^2*e^4*f^3*g +
 14*(11*c^2*d^3*e^3 + 10*a*c*d*e^5)*f^2*g^2 - 35*(33*c^2*d^4*e^2 + 34*a*c*
d^2*e^4 + 3*a^2*e^6)*f*g^3 + 105*(9*c^2*d^5*e + 10*a*c*d^3*e^3 + a^2*d*e^5
)*g^4 + 30*(c^2*e^6*f*g^3 - c^2*d*e^5*g^4)*x^4 + 6*(c^2*e^6*f^2*g^2 - 1...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\left(a+cx^2\right)^2}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a)**2/(e*x+d)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{f+gx}\left(a+cx^2\right)^2}{(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^2/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{f+gx}\left(a+cx^2\right)^2}{(d+ex)^2}\,dx = -\frac{\left(8\,c^2d^3ef+8\,acde^3f-9\,c^2d^4g-10\,acd^2e^2g-a^2e^4g\right)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{\sqrt{-e^2f+deg}e^5} - \frac{\sqrt{gx+f}c^2d^4g+2\sqrt{gx+f}acd^2e^2g+\sqrt{gx+f}a^2e^4g}{((gx+f)e-ef+dg)e^5} + \frac{2\left(15\,(gx+f)^{\frac{7}{2}}c^2e^{12}g^{18}-42\,(gx+f)^{\frac{5}{2}}c^2e^{12}fg^{18}+35\,(gx+f)^{\frac{3}{2}}c^2e^{12}f^2g^{18}-42\,(gx+f)^{\frac{5}{2}}c^2de^{11}g^{19}+\right.}{}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^2/(e*x+d)^2,x, algorithm="giac")
```

output
```
-(8*c^2*d^3*e*f + 8*a*c*d*e^3*f - 9*c^2*d^4*g - 10*a*c*d^2*e^2*g - a^2*e^4
*g)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e^2*f + d*e*g)*e^5
) - (sqrt(g*x + f)*c^2*d^4*g + 2*sqrt(g*x + f)*a*c*d^2*e^2*g + sqrt(g*x +
f)*a^2*e^4*g)/((((g*x + f)*e - e*f + d*g)*e^5) + 2/105*(15*(g*x + f)^(7/2)*
c^2*e^12*g^18 - 42*(g*x + f)^(5/2)*c^2*e^12*f*g^18 + 35*(g*x + f)^(3/2)*c^
2*e^12*f^2*g^18 - 42*(g*x + f)^(5/2)*c^2*d*e^11*g^19 + 70*(g*x + f)^(3/2)*
c^2*d*e^11*f*g^19 + 105*(g*x + f)^(3/2)*c^2*d^2*e^10*g^20 + 70*(g*x + f)^(
3/2)*a*c*e^12*g^20 - 420*sqrt(g*x + f)*c^2*d^3*e^9*g^21 - 420*sqrt(g*x + f
)*a*c*d*e^11*g^21)/(e^14*g^21)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{f+gx}(a+cx^2)^2}{(d+ex)^2}\,dx$$

$$= (f+g\,x)^{3/2}\left(\frac{12\,c^2\,f^2+4\,a\,c\,g^2}{3\,e^2\,g^3}+\frac{2\,(d\,g-e\,f)\,\left(\frac{8\,c^2\,f}{e^2\,g^3}+\frac{4\,c^2\,(d\,g-e\,f)}{e^3\,g^3}\right)}{3\,e}-\frac{2\,c^2\,(d\,g-e\,f)^2}{3\,e^4\,g^3}\right)-(f+g\,x)^{5/2}\left(\frac{8\,c^2\,f}{5\,e^2\,g^3}+\frac{4\,c^2\,(d\,g-e\,f)}{5\,e^3\,g^3}\right)$$

$$-\sqrt{f+g\,x}\left(\frac{8\,c^2\,f^3+8\,a\,c\,f\,g^2}{e^2\,g^3}-\frac{(d\,g-e\,f)^2\,\left(\frac{8\,c^2\,f}{e^2\,g^3}+\frac{4\,c^2\,(d\,g-e\,f)}{e^3\,g^3}\right)}{e^2}+\frac{2\,(d\,g-e\,f)\,\left(\frac{12\,c^2\,f^2+4\,a\,c\,g^2}{e^2\,g^3}+\frac{2\,(d\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{\sqrt{f+g\,x}\,(g\,a^2\,e^4+2\,g\,a\,c\,d^2\,e^2+g\,c^2\,d^4)}{e^6\,(f+g\,x)-e^6\,f+d\,e^5\,g}+\frac{2\,c^2\,(f+g\,x)^{7/2}}{7\,e^2\,g^3}$$

$$+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}\,(c\,d^2+a\,e^2)\,(9\,c\,g\,d^2-8\,c\,f\,d\,e+a\,g\,e^2)}{\sqrt{d\,g-e\,f}\,(g\,a^2\,e^4+10\,g\,a\,c\,d^2\,e^2-8\,f\,a\,c\,d\,e^3+9\,g\,c^2\,d^4-8\,f\,c^2\,d^3\,e)}\right)\,(c\,d^2+a\,e^2)\,(9\,c\,g\,d^2-8\,c\,f\,d\,e+a\,g\,e^2)}{e^{11/2}\,\sqrt{d\,g-e\,f}}$$

input
```
int(((f + g*x)^(1/2)*(a + c*x^2)^2)/(d + e*x)^2,x)
```

output
```
(f + g*x)^(3/2)*((12*c^2*f^2 + 4*a*c*g^2)/(3*e^2*g^3) + (2*(d*g - e*f)*((8
*c^2*f)/(e^2*g^3) + (4*c^2*(d*g - e*f))/(e^3*g^3)))/(3*e) - (2*c^2*(d*g -
e*f)^2)/(3*e^4*g^3)) - (f + g*x)^(5/2)*((8*c^2*f)/(5*e^2*g^3) + (4*c^2*(d*
g - e*f))/(5*e^3*g^3)) - (f + g*x)^(1/2)*((8*c^2*f^3 + 8*a*c*f*g^2)/(e^2*g
^3) - ((d*g - e*f)^2*((8*c^2*f)/(e^2*g^3) + (4*c^2*(d*g - e*f))/(e^3*g^3))
)/e^2 + (2*(d*g - e*f)*((12*c^2*f^2 + 4*a*c*g^2)/(e^2*g^3) + (2*(d*g - e*f
)*((8*c^2*f)/(e^2*g^3) + (4*c^2*(d*g - e*f))/(e^3*g^3)))/e - (2*c^2*(d*g -
 e*f)^2)/(e^4*g^3)))/e) - ((f + g*x)^(1/2)*(a^2*e^4*g + c^2*d^4*g + 2*a*c*
d^2*e^2*g))/(e^6*(f + g*x) - e^6*f + d*e^5*g) + (2*c^2*(f + g*x)^(7/2))/(7
*e^2*g^3) + (atan((e^(1/2)*(f + g*x)^(1/2)*(a*e^2 + c*d^2)*(a*e^2*g + 9*c*
d^2*g - 8*c*d*e*f))/((d*g - e*f)^(1/2)*(a^2*e^4*g + 9*c^2*d^4*g - 8*c^2*d^
3*e*f + 10*a*c*d^2*e^2*g - 8*a*c*d*e^3*f)))*(a*e^2 + c*d^2)*(a*e^2*g + 9*c
*d^2*g - 8*c*d*e*f))/(e^(11/2)*(d*g - e*f)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1117, normalized size of antiderivative = 4.56

$$\int \frac{\sqrt{f+gx}(a+cx^2)^2}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^2/(e*x+d)^2,x)
```

output

```
(105*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*
f)))*a**2*d*e**4*g**4 + 105*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)
/(sqrt(e)*sqrt(d*g - e*f)))*a**2*e**5*g**4*x + 1050*sqrt(e)*sqrt(d*g - e*f
)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**3*e**2*g**4 - 8
40*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)
))*a*c*d**2*e**3*f*g**3 + 1050*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)
*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**2*e**3*g**4*x - 840*sqrt(e)*sqrt(d*g
 - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d*e**4*f*g**
3*x + 945*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g
 - e*f)))*c**2*d**5*g**4 - 840*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)
*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**4*e*f*g**3 + 945*sqrt(e)*sqrt(d*g -
 e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**4*e*g**4*x
 - 840*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g -
e*f)))*c**2*d**3*e**2*f*g**3*x - 105*sqrt(f + g*x)*a**2*d*e**5*g**4 + 105*
sqrt(f + g*x)*a**2*e**6*f*g**3 - 1050*sqrt(f + g*x)*a*c*d**3*e**3*g**4 + 1
190*sqrt(f + g*x)*a*c*d**2*e**4*f*g**3 - 700*sqrt(f + g*x)*a*c*d**2*e**4*g
**4*x - 140*sqrt(f + g*x)*a*c*d*e**5*f**2*g**2 + 840*sqrt(f + g*x)*a*c*d*e
**5*f*g**3*x + 140*sqrt(f + g*x)*a*c*d*e**5*g**4*x**2 - 140*sqrt(f + g*x)*
a*c*e**6*f**2*g**2*x - 140*sqrt(f + g*x)*a*c*e**6*f*g**3*x**2 - 945*sqrt(f
 + g*x)*c**2*d**5*e*g**4 + 1155*sqrt(f + g*x)*c**2*d**4*e**2*f*g**3 - 6...
```

3.69 $\int \frac{\sqrt{f+gx}\left(a+cx^2\right)^2}{(d+ex)^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 301

$$\int \frac{\sqrt{f+gx}\left(a+cx^2\right)^2}{(d+ex)^3} \, dx = \frac{4c(3cd^2+ae^2)\sqrt{f+gx}}{e^5} - \frac{(cd^2+ae^2)^2\sqrt{f+gx}}{2e^5(d+ex)^2} - \frac{(cd^2+ae^2)(ae^2g-cd(16ef-17dg))\sqrt{f+gx}}{4e^5(ef-dg)(d+ex)} - \frac{2c^2(ef+3dg)(f+gx)^{3/2}}{3e^4g^2} + \frac{2c^2(f+gx)^{5/2}}{5e^3g^2} + \frac{(a^2e^4g^2-2ace^2(8e^2f^2-24defg+15d^2g^2)-c^2d^2(48e^2f^2-112defg+63d^2g^2))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{11/2}(ef-dg)^{3/2}}$$

output
```
4*c*(a*e^2+3*c*d^2)*(g*x+f)^(1/2)/e^5-1/2*(a*e^2+c*d^2)^2*(g*x+f)^(1/2)/e^
5/(e*x+d)^2-1/4*(a*e^2+c*d^2)*(a*e^2*g-c*d*(-17*d*g+16*e*f))*(g*x+f)^(1/2)
/e^5/(-d*g+e*f)/(e*x+d)-2/3*c^2*(3*d*g+e*f)*(g*x+f)^(3/2)/e^4/g^2+2/5*c^2*
(g*x+f)^(5/2)/e^3/g^2+1/4*(a^2*e^4*g^2-2*a*c*e^2*(15*d^2*g^2-24*d*e*f*g+8*
e^2*f^2)-c^2*d^2*(63*d^2*g^2-112*d*e*f*g+48*e^2*f^2))*arctanh(e^(1/2)*(g*x
+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(11/2)/(-d*g+e*f)^(3/2)
```

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{f+gx}(a+cx^2)^2}{(d+ex)^3}\,dx = -\frac{\sqrt{f+gx}(15a^2e^4g^2(2ef-dg+egx)+30ace^2g^2(15d^3g-8e^3fx^2+8de^2x(-3f+gx)+d^2e(-14f+2}{}$$

$$-\frac{(-a^2e^4g^2+2ace^2(8e^2f^2-24defg+15d^2g^2)+c^2d^2(48e^2f^2-112defg+63d^2g^2))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{4e^{11/2}(-ef+dg)^{3/2}}$$

input
```
Integrate[(Sqrt[f + g*x]*(a + c*x^2)^2)/(d + e*x)^3,x]
```

output
```
-1/60*(Sqrt[f + g*x]*(15*a^2*e^4*g^2*(2*e*f - d*g + e*g*x) + 30*a*c*e^2*g^
2*(15*d^3*g - 8*e^3*f*x^2 + 8*d*e^2*x*(-3*f + g*x) + d^2*e*(-14*f + 25*g*x
)) + c^2*(945*d^5*g^3 + 525*d^4*e*g^2*(-2*f + 3*g*x) + 8*d*e^4*x*(f + g*x)
^2*(4*f + 3*g*x) - 8*e^5*f*x^2*(-2*f^2 + f*g*x + 3*g^2*x^2) + 8*d^3*e^2*g*
(13*f^2 - 224*f*g*x + 63*g^2*x^2) + 8*d^2*e^3*(2*f^3 + 25*f^2*g*x - 76*f*g
^2*x^2 - 9*g^3*x^3))))/(e^5*g^2*(e*f - d*g)*(d + e*x)^2) - ((-(a^2*e^4*g^2
) + 2*a*c*e^2*(8*e^2*f^2 - 24*d*e*f*g + 15*d^2*g^2) + c^2*d^2*(48*e^2*f^2
- 112*d*e*f*g + 63*d^2*g^2))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) +
d*g]])/(4*e^(11/2)*(-(e*f) + d*g)^(3/2))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {649, 25, 1580, 25, 2345, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2\sqrt{f+gx}}{(d+ex)^3}\,dx$$

$$\Big\downarrow \text{649}$$

$$\frac{2\int -\frac{(f+gx)\left(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2\right)^2}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}}{g^2}$$

$\downarrow$ 25

$$-\frac{2\int \frac{(f+gx)\left(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2\right)^2}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}}{g^2}$$

$\downarrow$ 1580

$$\frac{2\left(-\frac{\int -\frac{\left(cd^2+ae^2\right)^2g^4+4c^2e^4(f+gx)^4-4c^2e^3(3ef+dg)(f+gx)^3+4ce^2\left(2ae^2g^2+c\left(3e^2f^2+2degf+d^2g^2\right)\right)(f+gx)^2-4ce(ef+dg)\left(ce^2f^2+cd^2g^2+2ae^2g^2\right)(f}{(ef-dg-e(f+gx))^2}}{4e^5}\right.}{g^2}$$

$\downarrow$ 25

$$\frac{2\left(\frac{\int \frac{\left(cd^2+ae^2\right)^2g^4+4c^2e^4(f+gx)^4-4c^2e^3(3ef+dg)(f+gx)^3+4ce^2\left(2ae^2g^2+c\left(3e^2f^2+2degf+d^2g^2\right)\right)(f+gx)^2-4ce(ef+dg)\left(ce^2f^2+cd^2g^2+2ae^2g^2\right)(f+gx)}{(ef-dg-e(f+gx))^2}}{4e^5}\right.}{g^2}$$

$\downarrow$ 2345

$$\frac{2\left(\frac{\frac{g^3\sqrt{f+gx}\left(ae^2+cd^2\right)\left(ae^2g-cd(16ef-17dg)\right)}{2(ef-dg)(-dg-e(f+gx)+ef)}-\frac{\int -\frac{\left(cd^2+ae^2\right)\left(age^2+cd(16ef-15dg)\right)g^3-8c^2e^3(ef-dg)(f+gx)^3+16c^2e^2(ef-dg)(ef+dg)(f+gx)^2-8ce(}{ef-dg-e(f+gx)}}{2(ef-dg)}}{4e^5}\right.}{g^2}$$

$\downarrow$ 25

$$\frac{2\left(\frac{\frac{\int \frac{\left(cd^2+ae^2\right)\left(age^2+cd(16ef-15dg)\right)g^3-8c^2e^3(ef-dg)(f+gx)^3+16c^2e^2(ef-dg)(ef+dg)(f+gx)^2-8ce(ef-dg)\left(2ae^2g^2+c\left(e^2f^2+2degf+3d^2g^2\right)\right)(f+gx)}{ef-dg-e(f+gx)}}{2(ef-dg)}}{4e^5}\right.}{g^2}$$

$\downarrow$ 2341

$$\frac{2\left(\frac{\frac{\int \left(8e^2(ef-dg)(f+gx)^2c^2-8e(ef-dg)(ef+3dg)(f+gx)c^2+16\left(3cd^2+ae^2\right)g^2(ef-dg)c+\frac{a^2g^4e^4-16acf^2g^2e^4+48acdfg^3e^3-30acd^2g^4e^2-48c^2d^2f^2g^2e^2+1}{ef-dg-e(f+gx)}\right.}{2(ef-dg)}}{4e^5}\right.}{g^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{\frac{\frac{g^2\left(a^2e^4g^2-2ace^2\left(15d^2g^2-24defg+8e^2f^2\right)-c^2d^2\left(63d^2g^2-112defg+48e^2f^2\right)\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}\sqrt{ef-dg}}+16cg^2\sqrt{f+gx}\left(ae^2+3cd^2\right)(ef-dg)+\frac{8}{5}c^2e^2(f+\cdots}{2(ef-dg)}}{4e^5}\cdots\right)}{g^2}$$

input
```
Int[(Sqrt[f + g*x]*(a + c*x^2)^2)/(d + e*x)^3,x]
```

output
```
(2*(-1/4*((c*d^2 + a*e^2)^2*g^4*Sqrt[f + g*x])/(e^5*(e*f - d*g - e*(f + g*
x))^2) + (((c*d^2 + a*e^2)*g^3*(a*e^2*g - c*d*(16*e*f - 17*d*g))*Sqrt[f +
g*x])/(2*(e*f - d*g)*(e*f - d*g - e*(f + g*x))) + (16*c*(3*c*d^2 + a*e^2)*
g^2*(e*f - d*g)*Sqrt[f + g*x] - (8*c^2*e*(e*f - d*g)*(e*f + 3*d*g)*(f + g*
x)^(3/2))/3 + (8*c^2*e^2*(e*f - d*g)*(f + g*x)^(5/2))/5 + (g^2*(a^2*e^4*g^
2 - 2*a*c*e^2*(8*e^2*f^2 - 24*d*e*f*g + 15*d^2*g^2) - c^2*d^2*(48*e^2*f^2
- 112*d*e*f*g + 63*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*
g]])/(Sqrt[e]*Sqrt[e*f - d*g]))/(2*(e*f - d*g)))/(4*e^5)))/g^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
 - d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.20

method	result
risch	$\frac{2c(3c\,x^{2}e^{2}g^{2}-15cde\,g^{2}x+c\,e^{2}fgx+30a\,e^{2}g^{2}+90c\,d^{2}g^{2}-15cdefg-2c\,e^{2}f^{2})\sqrt{gx+f}}{15g^{2}e^{5}}-\frac{-\frac{eg\left(a^{2}e^{4}g+18ac\,d^{2}e^{2}g-16acd\right.}{4(}}{}$
derivativedivides	$\frac{\frac{2c\left(\frac{c(gx+f)^{\frac{5}{2}}e^{2}}{5}-cdeg(gx+f)^{\frac{3}{2}}-\frac{c\,e^{2}f(gx+f)^{\frac{3}{2}}}{3}+2a\,e^{2}g^{2}\sqrt{gx+f}+6c\,d^{2}g^{2}\sqrt{gx+f}\right)}{e^{5}}+\frac{2g^{2}\left(\frac{eg\left(a^{2}e^{4}g+18ac\,d^{2}e^{2}g-16acd\,e^{3}f+\right.}{8dg-8e}\right.}{}}{}$
default	$\frac{\frac{2c\left(\frac{c(gx+f)^{\frac{5}{2}}e^{2}}{5}-cdeg(gx+f)^{\frac{3}{2}}-\frac{c\,e^{2}f(gx+f)^{\frac{3}{2}}}{3}+2a\,e^{2}g^{2}\sqrt{gx+f}+6c\,d^{2}g^{2}\sqrt{gx+f}\right)}{e^{5}}+\frac{2g^{2}\left(\frac{eg\left(a^{2}e^{4}g+18ac\,d^{2}e^{2}g-16acd\,e^{3}f+\right.}{8dg-8e}\right.}{}}{}$
pseudoelliptic	$-\frac{\sqrt{(dg-ef)e}\left(\left(\left(\left(-63d^{5}-\frac{8}{5}d\,e^{4}x^{4}+\frac{24}{5}d^{2}e^{3}x^{3}-\frac{168}{5}d^{3}e^{2}x^{2}-105d^{4}ex\right)g^{3}+70ef\left(\frac{4}{175}e^{4}x^{4}-\frac{8}{105}d\,e^{3}x^{3}+\frac{304}{525}d^{2}e^{2}x^{2}+\right.\right.\right.\right.}{}$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
2/15*c*(3*c*e^2*g^2*x^2-15*c*d*e*g^2*x+c*e^2*f*g*x+30*a*e^2*g^2+90*c*d^2*g
^2-15*c*d*e*f*g-2*c*e^2*f^2)*(g*x+f)^(1/2)/g^2/e^5-1/e^5*(2*(-1/8*e*g*(a^2
*e^4*g+18*a*c*d^2*e^2*g-16*a*c*d*e^3*f+17*c^2*d^4*g-16*c^2*d^3*e*f)/(d*g-e
*f)*(g*x+f)^(3/2)+1/8*g*(a^2*e^4*g-14*a*c*d^2*e^2*g+16*a*c*d*e^3*f-15*c^2*
d^4*g+16*c^2*d^3*e*f)*(g*x+f)^(1/2))/(e*(g*x+f)+d*g-e*f)^2-1/4*(a^2*e^4*g^
2-30*a*c*d^2*e^2*g^2+48*a*c*d*e^3*f*g-16*a*c*e^4*f^2-63*c^2*d^4*g^2+112*c^
2*d^3*e*f*g-48*c^2*d^2*e^2*f^2)/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*
x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 926 vs. $2(271) = 542$.

Time = 0.19 (sec) , antiderivative size = 1865, normalized size of antiderivative = 6.20

$$\int \frac{\sqrt{f+gx}(a+cx^2)^2}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^2/(e*x+d)^3,x, algorithm="fricas")
```

output
```
[1/120*(15*(16*(3*c^2*d^4*e^2 + a*c*d^2*e^4)*f^2*g^2 - 16*(7*c^2*d^5*e + 3
*a*c*d^3*e^3)*f*g^3 + (63*c^2*d^6 + 30*a*c*d^4*e^2 - a^2*d^2*e^4)*g^4 + (1
6*(3*c^2*d^2*e^4 + a*c*e^6)*f^2*g^2 - 16*(7*c^2*d^3*e^3 + 3*a*c*d*e^5)*f*g
^3 + (63*c^2*d^4*e^2 + 30*a*c*d^2*e^4 - a^2*e^6)*g^4)*x^2 + 2*(16*(3*c^2*d
^3*e^3 + a*c*d*e^5)*f^2*g^2 - 16*(7*c^2*d^4*e^2 + 3*a*c*d^2*e^4)*f*g^3 + (
63*c^2*d^5*e + 30*a*c*d^3*e^3 - a^2*d*e^5)*g^4)*x)*sqrt(e^2*f - d*e*g)*log
((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) -
2*(16*c^2*d^2*e^5*f^4 + 88*c^2*d^3*e^4*f^3*g - 2*(577*c^2*d^4*e^3 + 210*a*
c*d^2*e^5 - 15*a^2*e^7)*f^2*g^2 + 15*(133*c^2*d^5*e^2 + 58*a*c*d^3*e^4 - 3
*a^2*d*e^6)*f*g^3 - 15*(63*c^2*d^6*e + 30*a*c*d^4*e^3 - a^2*d^2*e^5)*g^4 -
 24*(c^2*e^7*f^2*g^2 - 2*c^2*d*e^6*f*g^3 + c^2*d^2*e^5*g^4)*x^4 - 8*(c^2*e
^7*f^3*g - 11*c^2*d*e^6*f^2*g^2 + 19*c^2*d^2*e^5*f*g^3 - 9*c^2*d^3*e^4*g^4
)*x^3 + 8*(2*c^2*e^7*f^4 + 9*c^2*d*e^6*f^3*g - 3*(29*c^2*d^2*e^5 + 10*a*c*
e^7)*f^2*g^2 + (139*c^2*d^3*e^4 + 60*a*c*d*e^6)*f*g^3 - 3*(21*c^2*d^4*e^3
+ 10*a*c*d^2*e^5)*g^4)*x^2 + (32*c^2*d*e^6*f^4 + 168*c^2*d^2*e^5*f^3*g - 2
4*(83*c^2*d^3*e^4 + 30*a*c*d*e^6)*f^2*g^2 + (3367*c^2*d^4*e^3 + 1470*a*c*d
^2*e^5 + 15*a^2*e^7)*f*g^3 - 15*(105*c^2*d^5*e^2 + 50*a*c*d^3*e^4 + a^2*d*
e^6)*g^4)*x)*sqrt(g*x + f))/(d^2*e^8*f^2*g^2 - 2*d^3*e^7*f*g^3 + d^4*e^6*g
^4 + (e^10*f^2*g^2 - 2*d*e^9*f*g^3 + d^2*e^8*g^4)*x^2 + 2*(d*e^9*f^2*g^2 -
 2*d^2*e^8*f*g^3 + d^3*e^7*g^4)*x), 1/60*(15*(16*(3*c^2*d^4*e^2 + a*c*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}(a+cx^2)^2}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a)**2/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{f+gx}(a+cx^2)^2}{(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^2/(e*x+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt{f+gx}(a+cx^2)^2}{(d+ex)^3}\,dx$$

$$= \frac{(48\,c^2d^2e^2f^2 + 16\,ace^4f^2 - 112\,c^2d^3efg - 48\,acde^3fg + 63\,c^2d^4g^2 + 30\,acd^2e^2g^2 - a^2e^4g^2)\arctan\left(\frac{\sqrt{gx}}{\sqrt{-e^2}}\right.}{4\,(e^6f - de^5g)\sqrt{-e^2f+deg}}$$

$$+ \frac{16\,(gx+f)^{\frac{3}{2}}c^2d^3e^2fg + 16\,(gx+f)^{\frac{3}{2}}acde^4fg - 16\,\sqrt{gx+f}c^2d^3e^2f^2g - 16\,\sqrt{gx+f}acde^4f^2g - 17\,(g}{}$$

$$+ \frac{2\left(3\,(gx+f)^{\frac{5}{2}}c^2e^{12}g^8 - 5\,(gx+f)^{\frac{3}{2}}c^2e^{12}fg^8 - 15\,(gx+f)^{\frac{3}{2}}c^2de^{11}g^9 + 90\,\sqrt{gx+f}c^2d^2e^{10}g^{10} + 30\,\sqrt{}\right.}{15\,e^{15}g^{10}}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^2/(e*x+d)^3,x, algorithm="giac")
```

output
```
1/4*(48*c^2*d^2*e^2*f^2 + 16*a*c*e^4*f^2 - 112*c^2*d^3*e*f*g - 48*a*c*d*e^
3*f*g + 63*c^2*d^4*g^2 + 30*a*c*d^2*e^2*g^2 - a^2*e^4*g^2)*arctan(sqrt(g*x
 + f)*e/sqrt(-e^2*f + d*e*g))/((e^6*f - d*e^5*g)*sqrt(-e^2*f + d*e*g)) + 1
/4*(16*(g*x + f)^(3/2)*c^2*d^3*e^2*f*g + 16*(g*x + f)^(3/2)*a*c*d*e^4*f*g
- 16*sqrt(g*x + f)*c^2*d^3*e^2*f^2*g - 16*sqrt(g*x + f)*a*c*d*e^4*f^2*g -
17*(g*x + f)^(3/2)*c^2*d^4*e*g^2 - 18*(g*x + f)^(3/2)*a*c*d^2*e^3*g^2 - (g
*x + f)^(3/2)*a^2*e^5*g^2 + 31*sqrt(g*x + f)*c^2*d^4*e*f*g^2 + 30*sqrt(g*x
 + f)*a*c*d^2*e^3*f*g^2 - sqrt(g*x + f)*a^2*e^5*f*g^2 - 15*sqrt(g*x + f)*c
^2*d^5*g^3 - 14*sqrt(g*x + f)*a*c*d^3*e^2*g^3 + sqrt(g*x + f)*a^2*d*e^4*g^
3)/((e^6*f - d*e^5*g)*((g*x + f)*e - e*f + d*g)^2) + 2/15*(3*(g*x + f)^(5/
2)*c^2*e^12*g^8 - 5*(g*x + f)^(3/2)*c^2*e^12*f*g^8 - 15*(g*x + f)^(3/2)*c^
2*d*e^11*g^9 + 90*sqrt(g*x + f)*c^2*d^2*e^10*g^10 + 30*sqrt(g*x + f)*a*c*e
^12*g^10)/(e^15*g^10)
```

Mupad [B] (verification not implemented)

Time = 6.19 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{f+gx}\,(a+cx^2)^2}{(d+ex)^3}\,dx$$

$$= \sqrt{f+g\,x}\left(\frac{12\,c^2\,f^2+4\,a\,c\,g^2}{e^3\,g^2}+\frac{3\,(d\,g-e\,f)\,\left(\frac{8\,c^2\,f}{e^3\,g^2}+\frac{6\,c^2\,(d\,g-e\,f)}{e^4\,g^2}\right)}{e}-\frac{6\,c^2\,(d\,g-e\,f)^2}{e^5\,g^2}\right)-(f+g\,x)^{3/2}\left(\frac{8\,c^2\,f}{3\,e^3\,g^2}+\frac{2\,c^2\,(d\,g-e\,f)}{e^4\,g^2}\right)$$

$$-\frac{\sqrt{f+g\,x}\,\left(\frac{a^2\,e^4\,g^2}{4}-\frac{7\,a\,c\,d^2\,e^2\,g^2}{2}+4\,f\,a\,c\,d\,e^3\,g-\frac{15\,c^2\,d^4\,g^2}{4}+4\,f\,c^2\,d^3\,e\,g\right)-\frac{(f+g\,x)^{3/2}\,(a^2\,e^5\,g^2+18\,a\,c\,d^2\,e^3\,g^2-\cdots}{4\,(\cdots}}{e^7\,(f+g\,x)^2-(f+g\,x)\,(2\,e^7\,f-2\,d\,e^6\,g)+e^7\,f^2+d^2\,e^5\,g^2-2\,d\,e^6\,f\cdots}$$

$$+\frac{2\,c^2\,(f+g\,x)^{5/2}}{5\,e^3\,g^2}$$

$$-\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,(-a^2\,e^4\,g^2+30\,a\,c\,d^2\,e^2\,g^2-48\,a\,c\,d\,e^3\,f\,g+16\,a\,c\,e^4\,f^2+63\,c^2\,d^4\,g^2-112\,c^2\,d^3\,e\,f\,g\cdots}{4\,e^{11/2}\,(d\,g-e\,f)^{3/2}}$$

input
```
int(((f + g*x)^(1/2)*(a + c*x^2)^2)/(d + e*x)^3,x)
```

output
```
(f + g*x)^(1/2)*((12*c^2*f^2 + 4*a*c*g^2)/(e^3*g^2) + (3*(d*g - e*f)*((8*c
^2*f)/(e^3*g^2) + (6*c^2*(d*g - e*f))/(e^4*g^2)))/e - (6*c^2*(d*g - e*f)^2
)/(e^5*g^2)) - (f + g*x)^(3/2)*((8*c^2*f)/(3*e^3*g^2) + (2*c^2*(d*g - e*f)
)/(e^4*g^2)) - ((f + g*x)^(1/2)*((a^2*e^4*g^2)/4 - (15*c^2*d^4*g^2)/4 + 4*
c^2*d^3*e*f*g - (7*a*c*d^2*e^2*g^2)/2 + 4*a*c*d*e^3*f*g) - ((f + g*x)^(3/2
)*(a^2*e^5*g^2 + 17*c^2*d^4*e*g^2 + 18*a*c*d^2*e^3*g^2 - 16*c^2*d^3*e^2*f*
g - 16*a*c*d*e^4*f*g))/(4*(d*g - e*f)))/(e^7*(f + g*x)^2 - (f + g*x)*(2*e^
7*f - 2*d*e^6*g) + e^7*f^2 + d^2*e^5*g^2 - 2*d*e^6*f*g) + (2*c^2*(f + g*x)
^(5/2))/(5*e^3*g^2) - (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(
63*c^2*d^4*g^2 - a^2*e^4*g^2 + 48*c^2*d^2*e^2*f^2 + 16*a*c*e^4*f^2 - 112*c
^2*d^3*e*f*g + 30*a*c*d^2*e^2*g^2 - 48*a*c*d*e^3*f*g))/(4*e^(11/2)*(d*g -
e*f)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1926, normalized size of antiderivative = 6.40

$$\int \frac{\sqrt{f+gx}\,(a+cx^2)^2}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^2/(e*x+d)^3,x)
```

output

```
(15*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f
)))*a**2*d**2*e**4*g**4 + 30*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e
)/(sqrt(e)*sqrt(d*g - e*f)))*a**2*d*e**5*g**4*x + 15*sqrt(e)*sqrt(d*g - e*
f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a**2*e**6*g**4*x**2 -
 450*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*
f)))*a*c*d**4*e**2*g**4 + 720*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*
e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**3*e**3*f*g**3 - 900*sqrt(e)*sqrt(d*g
- e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**3*e**3*g**
4*x - 240*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g
 - e*f)))*a*c*d**2*e**4*f**2*g**2 + 1440*sqrt(e)*sqrt(d*g - e*f)*atan((sqr
t(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**2*e**4*f*g**3*x - 450*sqrt
(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*
d**2*e**4*g**4*x**2 - 480*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(
sqrt(e)*sqrt(d*g - e*f)))*a*c*d*e**5*f**2*g**2*x + 720*sqrt(e)*sqrt(d*g -
e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d*e**5*f*g**3*x
**2 - 240*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g
 - e*f)))*a*c*e**6*f**2*g**2*x**2 - 945*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt
(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**6*g**4 + 1680*sqrt(e)*sqrt
(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**5*e*
f*g**3 - 1890*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*s...
```

3.70 $\int (d+ex)^3(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 474

$$\int (d+ex)^3(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx = -\frac{2(ef-dg)^3\left(cf^2+ag^2\right)^2(f+gx)^{5/2}}{5g^8}$$
$$+\frac{2(ef-dg)^2\left(cf^2+ag^2\right)\left(3aeg^2+cf(7ef-4dg)\right)(f+gx)^{7/2}}{7g^8}$$
$$-\frac{2(ef-dg)\left(3a^2e^2g^4+2acg^2(10e^2f^2-8defg+d^2g^2)+3c^2f^2(7e^2f^2-8defg+2d^2g^2)\right)(f+gx)^{9/2}}{9g^8}$$
$$+\frac{2\left(a^2e^3g^4+2aceg^2(10e^2f^2-12defg+3d^2g^2)+c^2f(35e^3f^3-60de^2f^2g+30d^2efg^2-4d^3g^3)\right)(f+gx)^{11/2}}{11g^8}$$
$$-\frac{2c\left(2ae^2g^2(5ef-3dg)+c(35e^3f^3-45de^2f^2g+15d^2efg^2-d^3g^3)\right)(f+gx)^{13/2}}{13g^8}$$
$$+\frac{2ce\left(2ae^2g^2+3c(7e^2f^2-6defg+d^2g^2)\right)(f+gx)^{15/2}}{15g^8}$$
$$-\frac{2c^2e^2(7ef-3dg)(f+gx)^{17/2}}{17g^8}+\frac{2c^2e^3(f+gx)^{19/2}}{19g^8}$$

output
```
-2/5*(-d*g+e*f)^3*(a*g^2+c*f^2)^2*(g*x+f)^(5/2)/g^8+2/7*(-d*g+e*f)^2*(a*g^
2+c*f^2)*(3*a*e*g^2+c*f*(-4*d*g+7*e*f))*(g*x+f)^(7/2)/g^8-2/9*(-d*g+e*f)*(
3*a^2*e^2*g^4+2*a*c*g^2*(d^2*g^2-8*d*e*f*g+10*e^2*f^2)+3*c^2*f^2*(2*d^2*g^
2-8*d*e*f*g+7*e^2*f^2))*(g*x+f)^(9/2)/g^8+2/11*(a^2*e^3*g^4+2*a*c*e*g^2*(3
*d^2*g^2-12*d*e*f*g+10*e^2*f^2)+c^2*f*(-4*d^3*g^3+30*d^2*e*f*g^2-60*d*e^2*
f^2*g+35*e^3*f^3))*(g*x+f)^(11/2)/g^8-2/13*c*(2*a*e^2*g^2*(-3*d*g+5*e*f)+c
*(-d^3*g^3+15*d^2*e*f*g^2-45*d*e^2*f^2*g+35*e^3*f^3))*(g*x+f)^(13/2)/g^8+2
/15*c*e*(2*a*e^2*g^2+3*c*(d^2*g^2-6*d*e*f*g+7*e^2*f^2))*(g*x+f)^(15/2)/g^8
-2/17*c^2*e^2*(-3*d*g+7*e*f)*(g*x+f)^(17/2)/g^8+2/19*c^2*e^3*(g*x+f)^(19/2
)/g^8
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.17

$$\int (d+ex)^3(f+gx)^{3/2} \left(a+cx^2\right)^2 \, dx = \frac{2(f+gx)^{5/2} \left(12597a^2g^4(231d^3g^3+99d^2eg^2(-2f+5gx)+11de^2g(8f^2-20fgx+35g^2x^2)+\right.}{}$$

input
```
Integrate[(d + e*x)^3*(f + g*x)^(3/2)*(a + c*x^2)^2,x]
```

output
```
(2*(f + g*x)^(5/2)*(12597*a^2*g^4*(231*d^3*g^3 + 99*d^2*e*g^2*(-2*f + 5*g*
x) + 11*d*e^2*g*(8*f^2 - 20*f*g*x + 35*g^2*x^2) + e^3*(-16*f^3 + 40*f^2*g*
x - 70*f*g^2*x^2 + 105*g^3*x^3)) + 646*a*c*g^2*(143*d^3*g^3*(8*f^2 - 20*f*
g*x + 35*g^2*x^2) + 117*d^2*e*g^2*(-16*f^3 + 40*f^2*g*x - 70*f*g^2*x^2 + 1
05*g^3*x^3) + 9*d*e^2*g*(128*f^4 - 320*f^3*g*x + 560*f^2*g^2*x^2 - 840*f*g
^3*x^3 + 1155*g^4*x^4) + e^3*(-256*f^5 + 640*f^4*g*x - 1120*f^3*g^2*x^2 +
1680*f^2*g^3*x^3 - 2310*f*g^4*x^4 + 3003*g^5*x^5)) - 3*c^2*(-323*d^3*g^3*(
128*f^4 - 320*f^3*g*x + 560*f^2*g^2*x^2 - 840*f*g^3*x^3 + 1155*g^4*x^4) +
323*d^2*e*g^2*(256*f^5 - 640*f^4*g*x + 1120*f^3*g^2*x^2 - 1680*f^2*g^3*x^3
 + 2310*f*g^4*x^4 - 3003*g^5*x^5) - 57*d*e^2*g*(1024*f^6 - 2560*f^5*g*x +
4480*f^4*g^2*x^2 - 6720*f^3*g^3*x^3 + 9240*f^2*g^4*x^4 - 12012*f*g^5*x^5 +
 15015*g^6*x^6) + 7*e^3*(2048*f^7 - 5120*f^6*g*x + 8960*f^5*g^2*x^2 - 1344
0*f^4*g^3*x^3 + 18480*f^3*g^4*x^4 - 24024*f^2*g^5*x^5 + 30030*f*g^6*x^6 -
36465*g^7*x^7))))/(14549535*g^8)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2 (d+ex)^3(f+gx)^{3/2} \, dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{(f+gx)^{7/2}(ef-dg)\left(-3a^2e^2g^4-2acg^2\left(d^2g^2-8defg+10e^2f^2\right)-3c^2f^2\left(2d^2g^2-8defg+7e^2f^2\right)\right)}{g^7}+\frac{(f$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(f+gx)^{9/2}(ef-dg)\left(3a^2e^2g^4+2acg^2\left(d^2g^2-8defg+10e^2f^2\right)+3c^2f^2\left(2d^2g^2-8defg+7e^2f^2\right)\right)}{9g^8}+\frac{2(f+gx)^{11/2}\left(a^2e^3g^4+2aceg^2\left(3d^2g^2-12defg+10e^2f^2\right)+c^2f\left(-4d^3g^3+30d^2efg^2-60de^2f^2g+35e^3f^3\right)\right)}{11g^8}+\frac{2ce(f+gx)^{15/2}\left(2ae^2g^2+3c\left(d^2g^2-6defg+7e^2f^2\right)\right)}{15g^8}-\frac{2c(f+gx)^{13/2}\left(2ae^2g^2(5ef-3dg)+c\left(-d^3g^3+15d^2efg^2-45de^2f^2g+35e^3f^3\right)\right)}{13g^8}+\frac{2(f+gx)^{7/2}\left(ag^2+cf^2\right)(ef-dg)^2\left(3aeg^2+cf(7ef-4dg)\right)}{7g^8}-\frac{2(f+gx)^{5/2}\left(ag^2+cf^2\right)^2(ef-dg)^3}{5g^8}-\frac{2c^2e^2(f+gx)^{17/2}(7ef-3dg)}{17g^8}+\frac{2c^2e^3(f+gx)^{19/2}}{19g^8}$$

input
```
Int[(d + e*x)^3*(f + g*x)^(3/2)*(a + c*x^2)^2,x]
```

output
```
(-2*(e*f - d*g)^3*(c*f^2 + a*g^2)^2*(f + g*x)^(5/2))/(5*g^8) + (2*(e*f - d
*g)^2*(c*f^2 + a*g^2)*(3*a*e*g^2 + c*f*(7*e*f - 4*d*g))*(f + g*x)^(7/2))/(
7*g^8) - (2*(e*f - d*g)*(3*a^2*e^2*g^4 + 2*a*c*g^2*(10*e^2*f^2 - 8*d*e*f*g
 + d^2*g^2) + 3*c^2*f^2*(7*e^2*f^2 - 8*d*e*f*g + 2*d^2*g^2))*(f + g*x)^(9/
2))/(9*g^8) + (2*(a^2*e^3*g^4 + 2*a*c*e*g^2*(10*e^2*f^2 - 12*d*e*f*g + 3*d
^2*g^2) + c^2*f*(35*e^3*f^3 - 60*d*e^2*f^2*g + 30*d^2*e*f*g^2 - 4*d^3*g^3)
)*(f + g*x)^(11/2))/(11*g^8) - (2*c*(2*a*e^2*g^2*(5*e*f - 3*d*g) + c*(35*e
^3*f^3 - 45*d*e^2*f^2*g + 15*d^2*e*f*g^2 - d^3*g^3))*(f + g*x)^(13/2))/(13
*g^8) + (2*c*e*(2*a*e^2*g^2 + 3*c*(7*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f +
g*x)^(15/2))/(15*g^8) - (2*c^2*e^2*(7*e*f - 3*d*g)*(f + g*x)^(17/2))/(17*g
^8) + (2*c^2*e^3*(f + g*x)^(19/2))/(19*g^8)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{2(gx+f)^{\frac{5}{2}}\left(\left(\left(\frac{5}{19}c^2x^7+\frac{5}{11}a^2x^3+\frac{2}{3}ac\,x^5\right)e^3+\frac{5x^2\left(\frac{9}{17}c^2x^4+\frac{18}{13}ac\,x^2+a^2\right)d\,e^2}{3}+\frac{15\left(\frac{7}{15}c^2x^4+\frac{14}{11}ac\,x^2+a^2\right)x\,d^2e}{7}+d^3\left(\frac{5}{13}c^2\right.\right.\right.}{}$
derivativedivides	$\frac{2c^2e^3(gx+f)^{\frac{19}{2}}}{19}+\frac{2\left(3(dg-ef)e^2c^2-4f\,c^2e^3\right)(gx+f)^{\frac{17}{2}}}{17}+\frac{2\left(3(dg-ef)^2e\,c^2-12(dg-ef)e^2c^2f+e^3\left(2\left(a\,g^2+c\,f^2\right)c+4c^2f^2\right)\right)(gx+}{15}$
default	$\frac{2c^2e^3(gx+f)^{\frac{19}{2}}}{19}+\frac{2\left(3(dg-ef)e^2c^2-4f\,c^2e^3\right)(gx+f)^{\frac{17}{2}}}{17}+\frac{2\left(3(dg-ef)^2e\,c^2-12(dg-ef)e^2c^2f+e^3\left(2\left(a\,g^2+c\,f^2\right)c+4c^2f^2\right)\right)(gx+}{15}$
gosper	$\frac{2(gx+f)^{\frac{5}{2}}\left(765765c^2e^3x^7g^7+2567565c^2d\,e^2g^7x^6-630630c^2e^3f\,g^6x^6+1939938ac\,e^3g^7x^5+2909907c^2d^2e\,g^7x^5-20540\right.}{}$
orering	$\frac{2(gx+f)^{\frac{5}{2}}\left(765765c^2e^3x^7g^7+2567565c^2d\,e^2g^7x^6-630630c^2e^3f\,g^6x^6+1939938ac\,e^3g^7x^5+2909907c^2d^2e\,g^7x^5-20540\right.}{}$
trager	Expression too large to display
risch	Expression too large to display

input

```
int((e*x+d)^3*(g*x+f)^(3/2)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output

```
2/5*(g*x+f)^(5/2)*(((5/19*c^2*x^7+5/11*a^2*x^3+2/3*a*c*x^5)*e^3+5/3*x^2*(9
/17*c^2*x^4+18/13*a*c*x^2+a^2)*d*e^2+15/7*(7/15*c^2*x^4+14/11*a*c*x^2+a^2)
*x*d^2*e+d^3*(5/13*c^2*x^4+10/9*a*c*x^2+a^2))*g^7-6/7*(35/99*x^2*(231/323*
c^2*x^4+22/13*a*c*x^2+a^2)*e^3+10/9*x*(63/85*c^2*x^4+252/143*a*c*x^2+a^2)*
d*e^2+d^2*(35/39*c^2*x^4+70/33*a*c*x^2+a^2)*e+20/27*c*x*(63/143*c*x^2+a)*d
^3)*f*g^6+8/21*f^2*((147/323*c^2*x^5+140/143*a*c*x^3+5/11*a^2*x)*e^3+d*(31
5/221*c^2*x^4+420/143*a*c*x^2+a^2)*e^2+30/11*(7/13*c*x^2+a)*c*x*d^2*e+2/3*
(105/143*c*x^2+a)*c*d^3)*g^5-16/231*((8085/4199*c^2*x^4+140/39*a*c*x^2+a^2
)*e^3+120/13*c*x*(21/34*c*x^2+a)*d*e^2+6*(35/39*c*x^2+a)*c*d^2*e+20/13*c^2
*d^3*x)*f^3*g^4+256/1001*f^4*c*(5/9*x*(441/646*c*x^2+a)*e^3+d*(35/34*c*x^2
+a)*e^2+5/6*c*d^2*e*x+1/6*c*d^3)*g^3-512/9009*((735/646*c*x^2+a)*e^2+45/17
*c*d*x*e+3/2*c*d^2)*e*f^5*c*g^2+1024/17017*(35/57*e*x+d)*e^2*f^6*c^2*g-204
8/138567*c^2*e^3*f^7)/g^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(442) = 884$.

Time = 0.10 (sec) , antiderivative size = 1028, normalized size of antiderivative = 2.17

$$\int (d+ex)^3(f+gx)^{3/2} \left(a+cx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(3/2)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/14549535*(765765*c^2*e^3*g^9*x^9 - 43008*c^2*e^3*f^9 + 175104*c^2*d*e^2*
f^8*g - 2494206*a^2*d^2*e*f^3*g^6 + 2909907*a^2*d^3*f^2*g^7 - 82688*(3*c^2
*d^2*e + 2*a*c*e^3)*f^7*g^2 + 124032*(c^2*d^3 + 6*a*c*d*e^2)*f^6*g^3 - 201
552*(6*a*c*d^2*e + a^2*e^3)*f^5*g^4 + 369512*(2*a*c*d^3 + 3*a^2*d*e^2)*f^4
*g^5 + 45045*(20*c^2*e^3*f*g^8 + 57*c^2*d*e^2*g^9)*x^8 + 3003*(3*c^2*e^3*f
^2*g^7 + 1026*c^2*d*e^2*f*g^8 + 323*(3*c^2*d^2*e + 2*a*c*e^3)*g^9)*x^7 - 2
31*(42*c^2*e^3*f^3*g^6 - 171*c^2*d*e^2*f^2*g^7 - 5168*(3*c^2*d^2*e + 2*a*c
*e^3)*f*g^8 - 4845*(c^2*d^3 + 6*a*c*d*e^2)*g^9)*x^6 + 63*(168*c^2*e^3*f^4*
g^5 - 684*c^2*d*e^2*f^3*g^6 + 323*(3*c^2*d^2*e + 2*a*c*e^3)*f^2*g^7 + 2261
0*(c^2*d^3 + 6*a*c*d*e^2)*f*g^8 + 20995*(6*a*c*d^2*e + a^2*e^3)*g^9)*x^5 -
 35*(336*c^2*e^3*f^5*g^4 - 1368*c^2*d*e^2*f^4*g^5 + 646*(3*c^2*d^2*e + 2*a
*c*e^3)*f^3*g^6 - 969*(c^2*d^3 + 6*a*c*d*e^2)*f^2*g^7 - 50388*(6*a*c*d^2*e
 + a^2*e^3)*f*g^8 - 46189*(2*a*c*d^3 + 3*a^2*d*e^2)*g^9)*x^4 + 5*(2688*c^2
*e^3*f^6*g^3 - 10944*c^2*d*e^2*f^5*g^4 + 1247103*a^2*d^2*e*g^9 + 5168*(3*c
^2*d^2*e + 2*a*c*e^3)*f^4*g^5 - 7752*(c^2*d^3 + 6*a*c*d*e^2)*f^3*g^6 + 125
97*(6*a*c*d^2*e + a^2*e^3)*f^2*g^7 + 461890*(2*a*c*d^3 + 3*a^2*d*e^2)*f*g^
8)*x^3 - 3*(5376*c^2*e^3*f^7*g^2 - 21888*c^2*d*e^2*f^6*g^3 - 3325608*a^2*d
^2*e*f*g^8 - 969969*a^2*d^3*g^9 + 10336*(3*c^2*d^2*e + 2*a*c*e^3)*f^5*g^4
- 15504*(c^2*d^3 + 6*a*c*d*e^2)*f^4*g^5 + 25194*(6*a*c*d^2*e + a^2*e^3)*f^
3*g^6 - 46189*(2*a*c*d^3 + 3*a^2*d*e^2)*f^2*g^7)*x^2 + (21504*c^2*e^3*f...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1032 vs. $2(495) = 990$.

Time = 1.58 (sec) , antiderivative size = 1032, normalized size of antiderivative = 2.18

$$\int (d+ex)^3(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(g*x+f)**(3/2)*(c*x**2+a)**2,x)
```

output
```
Piecewise((2*(c**2*e**3*(f + g*x)**(19/2)/(19*g**7) + (f + g*x)**(17/2)*(3
*c**2*d*e**2*g - 7*c**2*e**3*f)/(17*g**7) + (f + g*x)**(15/2)*(2*a*c*e**3*
g**2 + 3*c**2*d**2*e*g**2 - 18*c**2*d*e**2*f*g + 21*c**2*e**3*f**2)/(15*g*
*7) + (f + g*x)**(13/2)*(6*a*c*d*e**2*g**3 - 10*a*c*e**3*f*g**2 + c**2*d**
3*g**3 - 15*c**2*d**2*e*f*g**2 + 45*c**2*d*e**2*f**2*g - 35*c**2*e**3*f**3
)/(13*g**7) + (f + g*x)**(11/2)*(a**2*e**3*g**4 + 6*a*c*d**2*e*g**4 - 24*a
*c*d*e**2*f*g**3 + 20*a*c*e**3*f**2*g**2 - 4*c**2*d**3*f*g**3 + 30*c**2*d*
*2*e*f**2*g**2 - 60*c**2*d*e**2*f**3*g + 35*c**2*e**3*f**4)/(11*g**7) + (f
 + g*x)**(9/2)*(3*a**2*d*e**2*g**5 - 3*a**2*e**3*f*g**4 + 2*a*c*d**3*g**5
- 18*a*c*d**2*e*f*g**4 + 36*a*c*d*e**2*f**2*g**3 - 20*a*c*e**3*f**3*g**2 +
 6*c**2*d**3*f**2*g**3 - 30*c**2*d**2*e*f**3*g**2 + 45*c**2*d*e**2*f**4*g
- 21*c**2*e**3*f**5)/(9*g**7) + (f + g*x)**(7/2)*(3*a**2*d**2*e*g**6 - 6*a
**2*d*e**2*f*g**5 + 3*a**2*e**3*f**2*g**4 - 4*a*c*d**3*f*g**5 + 18*a*c*d**
2*e*f**2*g**4 - 24*a*c*d*e**2*f**3*g**3 + 10*a*c*e**3*f**4*g**2 - 4*c**2*d
**3*f**3*g**3 + 15*c**2*d**2*e*f**4*g**2 - 18*c**2*d*e**2*f**5*g + 7*c**2*
e**3*f**6)/(7*g**7) + (f + g*x)**(5/2)*(a**2*d**3*g**7 - 3*a**2*d**2*e*f*g
**6 + 3*a**2*d*e**2*f**2*g**5 - a**2*e**3*f**3*g**4 + 2*a*c*d**3*f**2*g**5
 - 6*a*c*d**2*e*f**3*g**4 + 6*a*c*d*e**2*f**4*g**3 - 2*a*c*e**3*f**5*g**2
+ c**2*d**3*f**4*g**3 - 3*c**2*d**2*e*f**5*g**2 + 3*c**2*d*e**2*f**6*g - c
**2*e**3*f**7)/(5*g**7))/g, Ne(g, 0)), (f**(3/2)*(a**2*d**3*x + 3*a**2*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.49

$$\int (d+ex)^3(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(3/2)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/14549535*(765765*(g*x + f)^(19/2)*c^2*e^3 - 855855*(7*c^2*e^3*f - 3*c^2*
d*e^2*g)*(g*x + f)^(17/2) + 969969*(21*c^2*e^3*f^2 - 18*c^2*d*e^2*f*g + (3
*c^2*d^2*e + 2*a*c*e^3)*g^2)*(g*x + f)^(15/2) - 1119195*(35*c^2*e^3*f^3 -
45*c^2*d*e^2*f^2*g + 5*(3*c^2*d^2*e + 2*a*c*e^3)*f*g^2 - (c^2*d^3 + 6*a*c*
d*e^2)*g^3)*(g*x + f)^(13/2) + 1322685*(35*c^2*e^3*f^4 - 60*c^2*d*e^2*f^3*
g + 10*(3*c^2*d^2*e + 2*a*c*e^3)*f^2*g^2 - 4*(c^2*d^3 + 6*a*c*d*e^2)*f*g^3
 + (6*a*c*d^2*e + a^2*e^3)*g^4)*(g*x + f)^(11/2) - 1616615*(21*c^2*e^3*f^5
 - 45*c^2*d*e^2*f^4*g + 10*(3*c^2*d^2*e + 2*a*c*e^3)*f^3*g^2 - 6*(c^2*d^3
+ 6*a*c*d*e^2)*f^2*g^3 + 3*(6*a*c*d^2*e + a^2*e^3)*f*g^4 - (2*a*c*d^3 + 3*
a^2*d*e^2)*g^5)*(g*x + f)^(9/2) + 2078505*(7*c^2*e^3*f^6 - 18*c^2*d*e^2*f^
5*g + 3*a^2*d^2*e*g^6 + 5*(3*c^2*d^2*e + 2*a*c*e^3)*f^4*g^2 - 4*(c^2*d^3 +
 6*a*c*d*e^2)*f^3*g^3 + 3*(6*a*c*d^2*e + a^2*e^3)*f^2*g^4 - 2*(2*a*c*d^3 +
 3*a^2*d*e^2)*f*g^5)*(g*x + f)^(7/2) - 2909907*(c^2*e^3*f^7 - 3*c^2*d*e^2*
f^6*g + 3*a^2*d^2*e*f*g^6 - a^2*d^3*g^7 + (3*c^2*d^2*e + 2*a*c*e^3)*f^5*g^
2 - (c^2*d^3 + 6*a*c*d*e^2)*f^4*g^3 + (6*a*c*d^2*e + a^2*e^3)*f^3*g^4 - (2
*a*c*d^3 + 3*a^2*d*e^2)*f^2*g^5)*(g*x + f)^(5/2))/g^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2674 vs. 2(442) = 884.

Time = 0.14 (sec) , antiderivative size = 2674, normalized size of antiderivative = 5.64

$$\int (d+ex)^3(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(3/2)*(c*x^2+a)^2,x, algorithm="giac")
```

output

```
2/14549535*(14549535*sqrt(g*x + f)*a^2*d^3*f^2 + 9699690*((g*x + f)^(3/2)
- 3*sqrt(g*x + f)*f)*a^2*d^3*f + 14549535*((g*x + f)^(3/2) - 3*sqrt(g*x +
f)*f)*a^2*d^2*e*f^2/g + 969969*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f +
 15*sqrt(g*x + f)*f^2)*a^2*d^3 + 1939938*(3*(g*x + f)^(5/2) - 10*(g*x + f)
^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a*c*d^3*f^2/g^2 + 2909907*(3*(g*x + f)^(5
/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a^2*d*e^2*f^2/g^2 + 581
9814*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a^2
*d^2*e*f/g + 2494206*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x +
 f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*c*d^2*e*f^2/g^3 + 415701*(5*(g*x +
 f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x +
f)*f^3)*a^2*e^3*f^2/g^3 + 1662804*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*
f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*c*d^3*f/g^2 + 2494206
*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*s
qrt(g*x + f)*f^3)*a^2*d*e^2*f/g^2 + 1247103*(5*(g*x + f)^(7/2) - 21*(g*x +
 f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a^2*d^2*e/g +
 46189*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f
^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c^2*d^3*f^2/g^4 + 27
7134*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2
 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*a*c*d*e^2*f^2/g^4 + 55
4268*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*...
```

Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.01

$$\int (d+ex)^3(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx=\frac{(f+g\,x)^{13/2}\,\left(2\,c^2\,d^3\,g^3-30\,c^2\,d^2\,e\,f\,g^2+90\,c^2\,d\,e^2\,f^2\,g-70\,c^2\,e^3\,f^3+12\,a\,c\,d\,e^2\,g^3-20\,a\,c\,e^3\right.}{13\,g^8}$$
$$+\frac{(f+g\,x)^{11/2}\,\left(2\,a^2\,e^3\,g^4+12\,a\,c\,d^2\,e\,g^4-48\,a\,c\,d\,e^2\,f\,g^3+40\,a\,c\,e^3\,f^2\,g^2-8\,c^2\,d^3\,f\,g^3+60\,c^2\,d^2\,e\,f^2\,g^2-\right.}{11\,g^8}$$
$$+\frac{2\,c^2\,e^3\,(f+g\,x)^{19/2}}{19\,g^8}+\frac{2\,(f+g\,x)^{5/2}\,\left(c\,f^2+a\,g^2\right)^2\,(d\,g-e\,f)^3}{5\,g^8}$$
$$+\frac{2\,(f+g\,x)^{9/2}\,(d\,g-e\,f)\,\left(3\,a^2\,e^2\,g^4+2\,a\,c\,d^2\,g^4-16\,a\,c\,d\,e\,f\,g^3+20\,a\,c\,e^2\,f^2\,g^2+6\,c^2\,d^2\,f^2\,g^2-24\,c^2\,d\,e\right.}{9\,g^8}$$
$$+\frac{2\,(f+g\,x)^{7/2}\,\left(c\,f^2+a\,g^2\right)\,(d\,g-e\,f)^2\,\left(7\,c\,e\,f^2-4\,c\,d\,f\,g+3\,a\,e\,g^2\right)}{7\,g^8}$$
$$+\frac{2\,c^2\,e^2\,(f+g\,x)^{17/2}\,(3\,d\,g-7\,e\,f)}{17\,g^8}$$
$$+\frac{2\,c\,e\,(f+g\,x)^{15/2}\,\left(3\,c\,d^2\,g^2-18\,c\,d\,e\,f\,g+21\,c\,e^2\,f^2+2\,a\,e^2\,g^2\right)}{15\,g^8}$$

input
```
int((f + g*x)^(3/2)*(a + c*x^2)^2*(d + e*x)^3,x)
```

output
```
((f + g*x)^(13/2)*(2*c^2*d^3*g^3 - 70*c^2*e^3*f^3 + 12*a*c*d*e^2*g^3 - 20*
a*c*e^3*f*g^2 + 90*c^2*d*e^2*f^2*g - 30*c^2*d^2*e*f*g^2))/(13*g^8) + ((f +
 g*x)^(11/2)*(2*a^2*e^3*g^4 + 70*c^2*e^3*f^4 - 8*c^2*d^3*f*g^3 + 12*a*c*d^
2*e*g^4 + 40*a*c*e^3*f^2*g^2 - 120*c^2*d*e^2*f^3*g + 60*c^2*d^2*e*f^2*g^2
- 48*a*c*d*e^2*f*g^3))/(11*g^8) + (2*c^2*e^3*(f + g*x)^(19/2))/(19*g^8) +
(2*(f + g*x)^(5/2)*(a*g^2 + c*f^2)^2*(d*g - e*f)^3)/(5*g^8) + (2*(f + g*x)
^(9/2)*(d*g - e*f)*(3*a^2*e^2*g^4 + 21*c^2*e^2*f^4 + 6*c^2*d^2*f^2*g^2 + 2
*a*c*d^2*g^4 - 24*c^2*d*e*f^3*g + 20*a*c*e^2*f^2*g^2 - 16*a*c*d*e*f*g^3))/
(9*g^8) + (2*(f + g*x)^(7/2)*(a*g^2 + c*f^2)*(d*g - e*f)^2*(3*a*e*g^2 + 7*
c*e*f^2 - 4*c*d*f*g))/(7*g^8) + (2*c^2*e^2*(f + g*x)^(17/2)*(3*d*g - 7*e*f
))/(17*g^8) + (2*c*e*(f + g*x)^(15/2)*(2*a*e^2*g^2 + 3*c*d^2*g^2 + 21*c*e^
2*f^2 - 18*c*d*e*f*g))/(15*g^8)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1224, normalized size of antiderivative = 2.58

$$\int (d+ex)^3(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^3*(g*x+f)^(3/2)*(c*x^2+a)^2,x)
```

output
```
(2*sqrt(f + g*x)*(2909907*a**2*d**3*f**2*g**7 + 5819814*a**2*d**3*f*g**8*x
 + 2909907*a**2*d**3*g**9*x**2 - 2494206*a**2*d**2*e*f**3*g**6 + 1247103*a
**2*d**2*e*f**2*g**7*x + 9976824*a**2*d**2*e*f*g**8*x**2 + 6235515*a**2*d*
*2*e*g**9*x**3 + 1108536*a**2*d*e**2*f**4*g**5 - 554268*a**2*d*e**2*f**3*g
**6*x + 415701*a**2*d*e**2*f**2*g**7*x**2 + 6928350*a**2*d*e**2*f*g**8*x**
3 + 4849845*a**2*d*e**2*g**9*x**4 - 201552*a**2*e**3*f**5*g**4 + 100776*a*
*2*e**3*f**4*g**5*x - 75582*a**2*e**3*f**3*g**6*x**2 + 62985*a**2*e**3*f**
2*g**7*x**3 + 1763580*a**2*e**3*f*g**8*x**4 + 1322685*a**2*e**3*g**9*x**5
+ 739024*a*c*d**3*f**4*g**5 - 369512*a*c*d**3*f**3*g**6*x + 277134*a*c*d**
3*f**2*g**7*x**2 + 4618900*a*c*d**3*f*g**8*x**3 + 3233230*a*c*d**3*g**9*x*
*4 - 1209312*a*c*d**2*e*f**5*g**4 + 604656*a*c*d**2*e*f**4*g**5*x - 453492
*a*c*d**2*e*f**3*g**6*x**2 + 377910*a*c*d**2*e*f**2*g**7*x**3 + 10581480*a
*c*d**2*e*f*g**8*x**4 + 7936110*a*c*d**2*e*g**9*x**5 + 744192*a*c*d*e**2*f
**6*g**3 - 372096*a*c*d*e**2*f**5*g**4*x + 279072*a*c*d*e**2*f**4*g**5*x**
2 - 232560*a*c*d*e**2*f**3*g**6*x**3 + 203490*a*c*d*e**2*f**2*g**7*x**4 +
8546580*a*c*d*e**2*f*g**8*x**5 + 6715170*a*c*d*e**2*g**9*x**6 - 165376*a*c
*e**3*f**7*g**2 + 82688*a*c*e**3*f**6*g**3*x - 62016*a*c*e**3*f**5*g**4*x*
*2 + 51680*a*c*e**3*f**4*g**5*x**3 - 45220*a*c*e**3*f**3*g**6*x**4 + 40698
*a*c*e**3*f**2*g**7*x**5 + 2387616*a*c*e**3*f*g**8*x**6 + 1939938*a*c*e**3
*g**9*x**7 + 124032*c**2*d**3*f**6*g**3 - 62016*c**2*d**3*f**5*g**4*x +...
```

3.71 $\int (d+ex)^2(f+gx)^{3/2} \left(a+cx^2\right)^2 \, dx$

Optimal result

Integrand size = 26, antiderivative size = 340

$$\int (d+ex)^2(f+gx)^{3/2} \left(a+cx^2\right)^2 \, dx = \frac{2(ef-dg)^2 \left(cf^2+ag^2\right)^2 (f+gx)^{5/2}}{5g^7} - \frac{4(ef-dg) \left(cf^2+ag^2\right) \left(aeg^2+cf(3ef-2dg)\right) (f+gx)^{7/2}}{7g^7} + \frac{2\left(a^2e^2g^4+2acg^2\left(6e^2f^2-6defg+d^2g^2\right)+c^2f^2\left(15e^2f^2-20defg+6d^2g^2\right)\right) (f+gx)^{9/2}}{9g^7} - \frac{8c\left(aeg^2(2ef-dg)+cf\left(5e^2f^2-5defg+d^2g^2\right)\right) (f+gx)^{11/2}}{11g^7} + \frac{2c\left(2ae^2g^2+c\left(15e^2f^2-10defg+d^2g^2\right)\right) (f+gx)^{13/2}}{13g^7} - \frac{4c^2e(3ef-dg)(f+gx)^{15/2}}{15g^7} + \frac{2c^2e^2(f+gx)^{17/2}}{17g^7}$$

output
```
2/5*(-d*g+e*f)^2*(a*g^2+c*f^2)^2*(g*x+f)^(5/2)/g^7-4/7*(-d*g+e*f)*(a*g^2+c
*f^2)*(a*e*g^2+c*f*(-2*d*g+3*e*f))*(g*x+f)^(7/2)/g^7+2/9*(a^2*e^2*g^4+2*a*
c*g^2*(d^2*g^2-6*d*e*f*g+6*e^2*f^2)+c^2*f^2*(6*d^2*g^2-20*d*e*f*g+15*e^2*f
^2))*(g*x+f)^(9/2)/g^7-8/11*c*(a*e*g^2*(-d*g+2*e*f)+c*f*(d^2*g^2-5*d*e*f*g
+5*e^2*f^2))*(g*x+f)^(11/2)/g^7+2/13*c*(2*a*e^2*g^2+c*(d^2*g^2-10*d*e*f*g+
15*e^2*f^2))*(g*x+f)^(13/2)/g^7-4/15*c^2*e*(-d*g+3*e*f)*(g*x+f)^(15/2)/g^7
+2/17*c^2*e^2*(g*x+f)^(17/2)/g^7
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.07

$$\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx=\frac{2(f+gx)^{5/2}\left(2431a^2g^4(63d^2g^2+18deg(-2f+5gx)+e^2(8f^2-20fgx+35g^2x^2))+34acg^2(\cdots\right.}{\cdots}$$

input
```
Integrate[(d + e*x)^2*(f + g*x)^(3/2)*(a + c*x^2)^2,x]
```

output
```
(2*(f + g*x)^(5/2)*(2431*a^2*g^4*(63*d^2*g^2 + 18*d*e*g*(-2*f + 5*g*x) + e
^2*(8*f^2 - 20*f*g*x + 35*g^2*x^2)) + 34*a*c*g^2*(143*d^2*g^2*(8*f^2 - 20*
f*g*x + 35*g^2*x^2) + 78*d*e*g*(-16*f^3 + 40*f^2*g*x - 70*f*g^2*x^2 + 105*
g^3*x^3) + 3*e^2*(128*f^4 - 320*f^3*g*x + 560*f^2*g^2*x^2 - 840*f*g^3*x^3
+ 1155*g^4*x^4)) + c^2*(51*d^2*g^2*(128*f^4 - 320*f^3*g*x + 560*f^2*g^2*x^
2 - 840*f*g^3*x^3 + 1155*g^4*x^4) + 34*d*e*g*(-256*f^5 + 640*f^4*g*x - 112
0*f^3*g^2*x^2 + 1680*f^2*g^3*x^3 - 2310*f*g^4*x^4 + 3003*g^5*x^5) + 3*e^2*
(1024*f^6 - 2560*f^5*g*x + 4480*f^4*g^2*x^2 - 6720*f^3*g^3*x^3 + 9240*f^2*
g^4*x^4 - 12012*f*g^5*x^5 + 15015*g^6*x^6))))/(765765*g^7)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2(d+ex)^2(f+gx)^{3/2}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{(f+gx)^{7/2}\left(a^2e^2g^4+2acg^2\left(d^2g^2-6defg+6e^2f^2\right)+c^2f^2\left(6d^2g^2-20defg+15e^2f^2\right)\right)}{g^6}+\frac{c(f+gx)^{11/2}(\cdots}{\cdots}\right.$$

$$\downarrow \text{2009}$$

$$\frac{2(f+gx)^{9/2}\left(a^2e^2g^4+2acg^2\left(d^2g^2-6defg+6e^2f^2\right)+c^2f^2\left(6d^2g^2-20defg+15e^2f^2\right)\right)}{9g^7}+\frac{2c(f+gx)^{13/2}\left(2ae^2g^2+c\left(d^2g^2-10defg+15e^2f^2\right)\right)}{13g^7}-\frac{8c(f+gx)^{11/2}\left(aeg^2(2ef-dg)+cf\left(d^2g^2-5defg+5e^2f^2\right)\right)}{11g^7}-\frac{4(f+gx)^{7/2}\left(ag^2+cf^2\right)(ef-dg)\left(aeg^2+cf(3ef-2dg)\right)}{7g^7}+\frac{2(f+gx)^{5/2}\left(ag^2+cf^2\right)^2(ef-dg)^2}{5g^7}-\frac{4c^2e(f+gx)^{15/2}(3ef-dg)}{15g^7}+\frac{2c^2e^2(f+gx)^{17/2}}{17g^7}$$

input
```
Int[(d + e*x)^2*(f + g*x)^(3/2)*(a + c*x^2)^2,x]
```

output
```
(2*(e*f - d*g)^2*(c*f^2 + a*g^2)^2*(f + g*x)^(5/2))/(5*g^7) - (4*(e*f - d*
g)*(c*f^2 + a*g^2)*(a*e*g^2 + c*f*(3*e*f - 2*d*g))*(f + g*x)^(7/2))/(7*g^7
) + (2*(a^2*e^2*g^4 + 2*a*c*g^2*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2) + c^2*f^
2*(15*e^2*f^2 - 20*d*e*f*g + 6*d^2*g^2))*(f + g*x)^(9/2))/(9*g^7) - (8*c*(
a*e*g^2*(2*e*f - d*g) + c*f*(5*e^2*f^2 - 5*d*e*f*g + d^2*g^2))*(f + g*x)^(
11/2))/(11*g^7) + (2*c*(2*a*e^2*g^2 + c*(15*e^2*f^2 - 10*d*e*f*g + d^2*g^2
))*(f + g*x)^(13/2))/(13*g^7) - (4*c^2*e*(3*e*f - d*g)*(f + g*x)^(15/2))/(
15*g^7) + (2*c^2*e^2*(f + g*x)^(17/2))/(17*g^7)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{2(gx+f)^{\frac{5}{2}}\left(\left(\left(\frac{5}{17}e^2x^6+\frac{5}{13}d^2x^4+\frac{2}{3}de\,x^5\right)c^2+\frac{10\left(\frac{9}{13}e^2x^2+\frac{18}{11}dex+d^2\right)x^2ac}{9}+a^2\left(d^2+\frac{5}{9}e^2x^2+\frac{10}{7}dex\right)\right)g^6-\frac{4f\left(\left(\frac{7}{17}e^2x^5\right.\right.}{}\right.}{}$
derivativedivides	$\frac{2c^2e^2(gx+f)^{\frac{17}{2}}}{17}+\frac{2\left(2e(dg-ef)c^2-4f\,c^2e^2\right)(gx+f)^{\frac{15}{2}}}{15}+\frac{2\left((dg-ef)^2c^2-8e(dg-ef)c^2f+e^2\left(2\left(a\,g^2+c\,f^2\right)c+4c^2f^2\right)\right)(gx+f)^{\frac{13}{2}}}{13}$
default	$\frac{2c^2e^2(gx+f)^{\frac{17}{2}}}{17}+\frac{2\left(2e(dg-ef)c^2-4f\,c^2e^2\right)(gx+f)^{\frac{15}{2}}}{15}+\frac{2\left((dg-ef)^2c^2-8e(dg-ef)c^2f+e^2\left(2\left(a\,g^2+c\,f^2\right)c+4c^2f^2\right)\right)(gx+f)^{\frac{13}{2}}}{13}$
gosper	$2(gx+f)^{\frac{5}{2}}\left(45045c^2e^2x^6g^6+102102c^2de\,g^6x^5-36036c^2e^2f\,g^5x^5+117810ac\,e^2g^6x^4+58905c^2d^2g^6x^4-78540c^2def\,g^5x\right.$
orering	$2(gx+f)^{\frac{5}{2}}\left(45045c^2e^2x^6g^6+102102c^2de\,g^6x^5-36036c^2e^2f\,g^5x^5+117810ac\,e^2g^6x^4+58905c^2d^2g^6x^4-78540c^2def\,g^5x\right.$
trager	$2\left(45045c^2e^2g^8x^8+102102c^2de\,g^8x^7+54054c^2e^2f\,g^7x^7+117810ac\,e^2g^8x^6+58905c^2d^2g^8x^6+125664c^2def\,g^7x^6+693c\right.$
risch	$2\left(45045c^2e^2g^8x^8+102102c^2de\,g^8x^7+54054c^2e^2f\,g^7x^7+117810ac\,e^2g^8x^6+58905c^2d^2g^8x^6+125664c^2def\,g^7x^6+693c\right.$

input
```
int((e*x+d)^2*(g*x+f)^(3/2)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/5*(g*x+f)^(5/2)*(((5/17*e^2*x^6+5/13*d^2*x^4+2/3*d*e*x^5)*c^2+10/9*(9/13
*e^2*x^2+18/11*d*e*x+d^2)*x^2*a*c+a^2*(d^2+5/9*e^2*x^2+10/7*d*e*x))*g^6-4/
7*f*((7/17*e^2*x^5+35/39*d*e*x^4+70/143*d^2*x^3)*c^2+10/9*(126/143*e^2*x^2
+21/11*d*e*x+d^2)*x*a*c+a^2*e*(5/9*e*x+d))*g^5+8/63*f^2*(210/143*(33/34*e^
2*x^2+2*d*e*x+d^2)*x^2*c^2+2*(210/143*e^2*x^2+30/11*d*e*x+d^2)*a*c+e^2*a^2
)*g^4-64/231*(5/13*x*(21/17*e^2*x^2+7/3*d*e*x+d^2)*c+e*a*(10/13*e*x+d))*f^
3*c*g^3+256/3003*f^4*c*((35/34*e^2*x^2+5/3*d*e*x+1/2*d^2)*c+a*e^2)*g^2-512
/9009*e*f^5*(15/17*e*x+d)*c^2*g+1024/51051*c^2*e^2*f^6)/g^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(312) = 624$.

Time = 0.09 (sec) , antiderivative size = 686, normalized size of antiderivative = 2.02

$$\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx = \frac{2\left(45045\,c^2e^2g^8x^8+3072\,c^2e^2f^8-8704\,c^2def^7g-42432\,acdef^5g^3-87516\,a^2def^3g^5+15315\right.}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(3/2)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/765765*(45045*c^2*e^2*g^8*x^8 + 3072*c^2*e^2*f^8 - 8704*c^2*d*e*f^7*g -
42432*a*c*d*e*f^5*g^3 - 87516*a^2*d*e*f^3*g^5 + 153153*a^2*d^2*f^2*g^6 + 6
528*(c^2*d^2 + 2*a*c*e^2)*f^6*g^2 + 19448*(2*a*c*d^2 + a^2*e^2)*f^4*g^4 +
6006*(9*c^2*e^2*f*g^7 + 17*c^2*d*e*g^8)*x^7 + 231*(3*c^2*e^2*f^2*g^6 + 544
*c^2*d*e*f*g^7 + 255*(c^2*d^2 + 2*a*c*e^2)*g^8)*x^6 - 126*(6*c^2*e^2*f^3*g
^5 - 17*c^2*d*e*f^2*g^6 - 2210*a*c*d*e*g^8 - 595*(c^2*d^2 + 2*a*c*e^2)*f*g
^7)*x^5 + 35*(24*c^2*e^2*f^4*g^4 - 68*c^2*d*e*f^3*g^5 + 10608*a*c*d*e*f*g^
7 + 51*(c^2*d^2 + 2*a*c*e^2)*f^2*g^6 + 2431*(2*a*c*d^2 + a^2*e^2)*g^8)*x^4
 - 10*(96*c^2*e^2*f^5*g^3 - 272*c^2*d*e*f^4*g^4 - 1326*a*c*d*e*f^2*g^6 - 2
1879*a^2*d*e*g^8 + 204*(c^2*d^2 + 2*a*c*e^2)*f^3*g^5 - 12155*(2*a*c*d^2 +
a^2*e^2)*f*g^7)*x^3 + 3*(384*c^2*e^2*f^6*g^2 - 1088*c^2*d*e*f^5*g^3 - 5304
*a*c*d*e*f^3*g^5 + 116688*a^2*d*e*f*g^7 + 51051*a^2*d^2*g^8 + 816*(c^2*d^2
 + 2*a*c*e^2)*f^4*g^4 + 2431*(2*a*c*d^2 + a^2*e^2)*f^2*g^6)*x^2 - 2*(768*c
^2*e^2*f^7*g - 2176*c^2*d*e*f^6*g^2 - 10608*a*c*d*e*f^4*g^4 - 21879*a^2*d*
e*f^2*g^6 - 153153*a^2*d^2*f*g^7 + 1632*(c^2*d^2 + 2*a*c*e^2)*f^5*g^3 + 48
62*(2*a*c*d^2 + a^2*e^2)*f^3*g^5)*x)*sqrt(g*x + f)/g^7
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.96

$$\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx = \begin{cases} \frac{2\left(\frac{c^2e^2(f+gx)^{\frac{17}{2}}}{17g^6}+\frac{(f+gx)^{\frac{15}{2}}\cdot\left(2c^2deg-6c^2e^2f\right)}{15g^6}+\frac{(f+gx)^{\frac{13}{2}}\cdot\left(2ace^2g^2+c^2d^2g^2-10c^2defg+15c^2e^2f^2\right)}{13g^6}+\frac{(f+gx)^{\frac{11}{2}}\cdot\left(4acdeg^3-8ace^2\right.}{}\right.}{} \\ f^{\frac{3}{2}}\left(a^2d^2x+a^2dex^2+acdex^4+\frac{c^2dex^6}{3}+\frac{c^2e^2x^7}{7}+\frac{x^5\cdot\left(2ace^2+c^2d^2\right)}{5}+\frac{x^3\left(a^2e^2+2acd^2\right)}{3}\right) \end{cases}$$

input
```
integrate((e*x+d)**2*(g*x+f)**(3/2)*(c*x**2+a)**2,x)
```

output
```
Piecewise((2*(c**2*e**2*(f + g*x)**(17/2)/(17*g**6) + (f + g*x)**(15/2)*(2
*c**2*d*e*g - 6*c**2*e**2*f)/(15*g**6) + (f + g*x)**(13/2)*(2*a*c*e**2*g**
2 + c**2*d**2*g**2 - 10*c**2*d*e*f*g + 15*c**2*e**2*f**2)/(13*g**6) + (f +
 g*x)**(11/2)*(4*a*c*d*e*g**3 - 8*a*c*e**2*f*g**2 - 4*c**2*d**2*f*g**2 + 2
0*c**2*d*e*f**2*g - 20*c**2*e**2*f**3)/(11*g**6) + (f + g*x)**(9/2)*(a**2*
e**2*g**4 + 2*a*c*d**2*g**4 - 12*a*c*d*e*f*g**3 + 12*a*c*e**2*f**2*g**2 +
6*c**2*d**2*f**2*g**2 - 20*c**2*d*e*f**3*g + 15*c**2*e**2*f**4)/(9*g**6) +
 (f + g*x)**(7/2)*(2*a**2*d*e*g**5 - 2*a**2*e**2*f*g**4 - 4*a*c*d**2*f*g**
4 + 12*a*c*d*e*f**2*g**3 - 8*a*c*e**2*f**3*g**2 - 4*c**2*d**2*f**3*g**2 +
10*c**2*d*e*f**4*g - 6*c**2*e**2*f**5)/(7*g**6) + (f + g*x)**(5/2)*(a**2*d
**2*g**6 - 2*a**2*d*e*f*g**5 + a**2*e**2*f**2*g**4 + 2*a*c*d**2*f**2*g**4
- 4*a*c*d*e*f**3*g**3 + 2*a*c*e**2*f**4*g**2 + c**2*d**2*f**4*g**2 - 2*c**
2*d*e*f**5*g + c**2*e**2*f**6)/(5*g**6))/g, Ne(g, 0)), (f**(3/2)*(a**2*d**
2*x + a**2*d*e*x**2 + a*c*d*e*x**4 + c**2*d*e*x**6/3 + c**2*e**2*x**7/7 +
x**5*(2*a*c*e**2 + c**2*d**2)/5 + x**3*(a**2*e**2 + 2*a*c*d**2)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.31

$$\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx = \frac{2\left(45045\,(gx+f)^{\frac{17}{2}}c^2e^2 - 102102\,(3\,c^2e^2f - c^2deg)(gx+f)^{\frac{15}{2}} + 58905\,(15\,c^2e^2f^2 - 10\,c^2def\right.}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(3/2)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/765765*(45045*(g*x + f)^(17/2)*c^2*e^2 - 102102*(3*c^2*e^2*f - c^2*d*e*g
)*(g*x + f)^(15/2) + 58905*(15*c^2*e^2*f^2 - 10*c^2*d*e*f*g + (c^2*d^2 + 2
*a*c*e^2)*g^2)*(g*x + f)^(13/2) - 278460*(5*c^2*e^2*f^3 - 5*c^2*d*e*f^2*g
- a*c*d*e*g^3 + (c^2*d^2 + 2*a*c*e^2)*f*g^2)*(g*x + f)^(11/2) + 85085*(15*
c^2*e^2*f^4 - 20*c^2*d*e*f^3*g - 12*a*c*d*e*f*g^3 + 6*(c^2*d^2 + 2*a*c*e^2
)*f^2*g^2 + (2*a*c*d^2 + a^2*e^2)*g^4)*(g*x + f)^(9/2) - 218790*(3*c^2*e^2
*f^5 - 5*c^2*d*e*f^4*g - 6*a*c*d*e*f^2*g^3 - a^2*d*e*g^5 + 2*(c^2*d^2 + 2*
a*c*e^2)*f^3*g^2 + (2*a*c*d^2 + a^2*e^2)*f*g^4)*(g*x + f)^(7/2) + 153153*(
c^2*e^2*f^6 - 2*c^2*d*e*f^5*g - 4*a*c*d*e*f^3*g^3 - 2*a^2*d*e*f*g^5 + a^2*
d^2*g^6 + (c^2*d^2 + 2*a*c*e^2)*f^4*g^2 + (2*a*c*d^2 + a^2*e^2)*f^2*g^4)*(
g*x + f)^(5/2))/g^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1819 vs. $2(312) = 624$.

Time = 0.14 (sec) , antiderivative size = 1819, normalized size of antiderivative = 5.35

$$\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(3/2)*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
2/765765*(765765*sqrt(g*x + f)*a^2*d^2*f^2 + 510510*((g*x + f)^(3/2) - 3*s
qrt(g*x + f)*f)*a^2*d^2*f + 510510*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a
^2*d*e*f^2/g + 51051*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g
*x + f)*f^2)*a^2*d^2 + 102102*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f +
15*sqrt(g*x + f)*f^2)*a*c*d^2*f^2/g^2 + 51051*(3*(g*x + f)^(5/2) - 10*(g*x
 + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a^2*e^2*f^2/g^2 + 204204*(3*(g*x + f
)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2)*a^2*d*e*f/g + 87516
*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*s
qrt(g*x + f)*f^3)*a*c*d*e*f^2/g^3 + 87516*(5*(g*x + f)^(7/2) - 21*(g*x + f
)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*c*d^2*f/g^2 +
 43758*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2
- 35*sqrt(g*x + f)*f^3)*a^2*e^2*f/g^2 + 43758*(5*(g*x + f)^(7/2) - 21*(g*x
 + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a^2*d*e/g +
 2431*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^
2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c^2*d^2*f^2/g^4 + 486
2*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 -
420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*a*c*e^2*f^2/g^4 + 19448*(
35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420
*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*a*c*d*e*f/g^3 + 4862*(35*(g*
x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g...
```

Mupad [B] (verification not implemented)

Time = 5.97 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.01

$$\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx=\frac{(f+g\,x)^{9/2}\,\left(2\,a^2\,e^2\,g^4+4\,a\,c\,d^2\,g^4-24\,a\,c\,d\,e\,f\,g^3+24\,a\,c\,e^2\,f^2\,g^2+12\,c^2\,d^2\,f^2\,g^2-40\,c^2\,d\,e\right.}{9\,g^7}-\frac{(f+g\,x)^{11/2}\,\left(8\,c^2\,d^2\,f\,g^2-40\,c^2\,d\,e\,f^2\,g+40\,c^2\,e^2\,f^3-8\,a\,c\,d\,e\,g^3+16\,a\,c\,e^2\,f\,g^2\right)}{11\,g^7}+\frac{(f+g\,x)^{13/2}\,\left(2\,c^2\,d^2\,g^2-20\,c^2\,d\,e\,f\,g+30\,c^2\,e^2\,f^2+4\,a\,c\,e^2\,g^2\right)}{13\,g^7}+\frac{2\,c^2\,e^2\,(f+g\,x)^{17/2}}{17\,g^7}+\frac{2\,(f+g\,x)^{5/2}\,\left(c\,f^2+a\,g^2\right)^2\,(d\,g-e\,f)^2}{5\,g^7}+\frac{4\,c^2\,e\,(f+g\,x)^{15/2}\,(d\,g-3\,e\,f)}{15\,g^7}+\frac{4\,(f+g\,x)^{7/2}\,\left(c\,f^2+a\,g^2\right)\,(d\,g-e\,f)\,\left(3\,c\,e\,f^2-2\,c\,d\,f\,g+a\,e\,g^2\right)}{7\,g^7}$$

input
```
int((f + g*x)^(3/2)*(a + c*x^2)^2*(d + e*x)^2,x)
```

output
```
((f + g*x)^(9/2)*(2*a^2*e^2*g^4 + 30*c^2*e^2*f^4 + 12*c^2*d^2*f^2*g^2 + 4*
a*c*d^2*g^4 - 40*c^2*d*e*f^3*g + 24*a*c*e^2*f^2*g^2 - 24*a*c*d*e*f*g^3))/(
9*g^7) - ((f + g*x)^(11/2)*(40*c^2*e^2*f^3 + 8*c^2*d^2*f*g^2 + 16*a*c*e^2*
f*g^2 - 40*c^2*d*e*f^2*g - 8*a*c*d*e*g^3))/(11*g^7) + ((f + g*x)^(13/2)*(2
*c^2*d^2*g^2 + 30*c^2*e^2*f^2 + 4*a*c*e^2*g^2 - 20*c^2*d*e*f*g))/(13*g^7)
+ (2*c^2*e^2*(f + g*x)^(17/2))/(17*g^7) + (2*(f + g*x)^(5/2)*(a*g^2 + c*f^
2)^2*(d*g - e*f)^2)/(5*g^7) + (4*c^2*e*(f + g*x)^(15/2)*(d*g - 3*e*f))/(15
*g^7) + (4*(f + g*x)^(7/2)*(a*g^2 + c*f^2)*(d*g - e*f)*(a*e*g^2 + 3*c*e*f^
2 - 2*c*d*f*g))/(7*g^7)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 797, normalized size of antiderivative = 2.34

$$\int (d+ex)^2(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx = \frac{2\sqrt{gx+f}\,(45045c^2e^2g^8x^8+102102c^2de\,g^8x^7+54054c^2e^2f\,g^7x^7+117810ac\,e^2g^8x^6+58905c$$

input
```
int((e*x+d)^2*(g*x+f)^(3/2)*(c*x^2+a)^2,x)
```

output
```
(2*sqrt(f + g*x)*(153153*a**2*d**2*f**2*g**6 + 306306*a**2*d**2*f*g**7*x +
 153153*a**2*d**2*g**8*x**2 - 87516*a**2*d*e*f**3*g**5 + 43758*a**2*d*e*f*
*2*g**6*x + 350064*a**2*d*e*f*g**7*x**2 + 218790*a**2*d*e*g**8*x**3 + 1944
8*a**2*e**2*f**4*g**4 - 9724*a**2*e**2*f**3*g**5*x + 7293*a**2*e**2*f**2*g
**6*x**2 + 121550*a**2*e**2*f*g**7*x**3 + 85085*a**2*e**2*g**8*x**4 + 3889
6*a*c*d**2*f**4*g**4 - 19448*a*c*d**2*f**3*g**5*x + 14586*a*c*d**2*f**2*g*
*6*x**2 + 243100*a*c*d**2*f*g**7*x**3 + 170170*a*c*d**2*g**8*x**4 - 42432*
a*c*d*e*f**5*g**3 + 21216*a*c*d*e*f**4*g**4*x - 15912*a*c*d*e*f**3*g**5*x*
*2 + 13260*a*c*d*e*f**2*g**6*x**3 + 371280*a*c*d*e*f*g**7*x**4 + 278460*a*
c*d*e*g**8*x**5 + 13056*a*c*e**2*f**6*g**2 - 6528*a*c*e**2*f**5*g**3*x + 4
896*a*c*e**2*f**4*g**4*x**2 - 4080*a*c*e**2*f**3*g**5*x**3 + 3570*a*c*e**2
*f**2*g**6*x**4 + 149940*a*c*e**2*f*g**7*x**5 + 117810*a*c*e**2*g**8*x**6
+ 6528*c**2*d**2*f**6*g**2 - 3264*c**2*d**2*f**5*g**3*x + 2448*c**2*d**2*f
**4*g**4*x**2 - 2040*c**2*d**2*f**3*g**5*x**3 + 1785*c**2*d**2*f**2*g**6*x
**4 + 74970*c**2*d**2*f*g**7*x**5 + 58905*c**2*d**2*g**8*x**6 - 8704*c**2*
d*e*f**7*g + 4352*c**2*d*e*f**6*g**2*x - 3264*c**2*d*e*f**5*g**3*x**2 + 27
20*c**2*d*e*f**4*g**4*x**3 - 2380*c**2*d*e*f**3*g**5*x**4 + 2142*c**2*d*e*
f**2*g**6*x**5 + 125664*c**2*d*e*f*g**7*x**6 + 102102*c**2*d*e*g**8*x**7 +
 3072*c**2*e**2*f**8 - 1536*c**2*e**2*f**7*g*x + 1152*c**2*e**2*f**6*g**2*
x**2 - 960*c**2*e**2*f**5*g**3*x**3 + 840*c**2*e**2*f**4*g**4*x**4 - 75...
```

3.72 $\int (d+ex)(f+gx)^{3/2} \left(a+cx^2\right)^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 214

$$\begin{aligned}\int (d+ex)(f+gx)^{3/2} \left(a+cx^2\right)^2 \, dx = &-\frac{2(ef-dg) \left(cf^2+ag^2\right)^2 (f+gx)^{5/2}}{5g^6} \\ &+ \frac{2(cf^2+ag^2) \left(aeg^2+cf(5ef-4dg)\right) (f+gx)^{7/2}}{7g^6} \\ &- \frac{4c(cf^2(5ef-3dg)+ag^2(3ef-dg)) (f+gx)^{9/2}}{9g^6} \\ &+ \frac{4c(aeg^2+cf(5ef-2dg)) (f+gx)^{11/2}}{11g^6} \\ &- \frac{2c^2(5ef-dg)(f+gx)^{13/2}}{13g^6} + \frac{2c^2e(f+gx)^{15/2}}{15g^6}\end{aligned}$$

output

```
-2/5*(-d*g+e*f)*(a*g^2+c*f^2)^2*(g*x+f)^(5/2)/g^6+2/7*(a*g^2+c*f^2)*(a*e*g
^2+c*f*(-4*d*g+5*e*f))*(g*x+f)^(7/2)/g^6-4/9*c*(c*f^2*(-3*d*g+5*e*f)+a*g^2
*(-d*g+3*e*f))*(g*x+f)^(9/2)/g^6+4/11*c*(a*e*g^2+c*f*(-2*d*g+5*e*f))*(g*x+
f)^(11/2)/g^6-2/13*c^2*(-d*g+5*e*f)*(g*x+f)^(13/2)/g^6+2/15*c^2*e*(g*x+f)^
(15/2)/g^6
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.97

$$\int (d+ex)(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx=\frac{2(f+gx)^{5/2}\left(1287a^2g^4(-2ef+7dg+5egx)-26acg^2(-11dg(8f^2-20fgx+35g^2x^2)+3e(\ldots$$

input
```
Integrate[(d + e*x)*(f + g*x)^(3/2)*(a + c*x^2)^2,x]
```

output
```
(2*(f + g*x)^(5/2)*(1287*a^2*g^4*(-2*e*f + 7*d*g + 5*e*g*x) - 26*a*c*g^2*(
-11*d*g*(8*f^2 - 20*f*g*x + 35*g^2*x^2) + 3*e*(16*f^3 - 40*f^2*g*x + 70*f*
g^2*x^2 - 105*g^3*x^3)) + c^2*(3*d*g*(128*f^4 - 320*f^3*g*x + 560*f^2*g^2*
x^2 - 840*f*g^3*x^3 + 1155*g^4*x^4) + e*(-256*f^5 + 640*f^4*g*x - 1120*f^3
*g^2*x^2 + 1680*f^2*g^3*x^3 - 2310*f*g^4*x^4 + 3003*g^5*x^5))))/(45045*g^6
)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2(d+ex)(f+gx)^{3/2}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{2c(f+gx)^{7/2}\left(-ag^2(3ef-dg)-cf^2(5ef-3dg)\right)}{g^5}+\frac{(f+gx)^{5/2}\left(ag^2+cf^2\right)\left(aeg^2+cf(5ef-4dg)\right)}{g^5}+\ldots$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4c(f+gx)^{9/2}\left(ag^2(3ef-dg)+cf^2(5ef-3dg)\right)}{9g^6}+\frac{2(f+gx)^{7/2}\left(ag^2+cf^2\right)\left(aeg^2+cf(5ef-4dg)\right)}{7g^6}-\frac{2(f+gx)^{5/2}\left(ag^2+cf^2\right)^2(ef-dg)}{5g^6}+\frac{4c(f+gx)^{11/2}\left(aeg^2+cf(5ef-2dg)\right)}{11g^6}-\frac{2c^2(f+gx)^{13/2}(5ef-dg)}{13g^6}+\frac{2c^2e(f+gx)^{15/2}}{15g^6}$$

input
```
Int[(d + e*x)*(f + g*x)^(3/2)*(a + c*x^2)^2,x]
```

output
```
(-2*(e*f - d*g)*(c*f^2 + a*g^2)^2*(f + g*x)^(5/2))/(5*g^6) + (2*(c*f^2 + a
*g^2)*(a*e*g^2 + c*f*(5*e*f - 4*d*g))*(f + g*x)^(7/2))/(7*g^6) - (4*c*(c*f
^2*(5*e*f - 3*d*g) + a*g^2*(3*e*f - d*g))*(f + g*x)^(9/2))/(9*g^6) + (4*c*
(a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x)^(11/2))/(11*g^6) - (2*c^2*(5*e*f
 - d*g)*(f + g*x)^(13/2))/(13*g^6) + (2*c^2*e*(f + g*x)^(15/2))/(15*g^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{2(gx+f)^{\frac{5}{2}}\left(\left(\left(\left(\frac{1}{3}x^5e+\frac{5}{13}dx^4\right)c^2+\frac{10x^2a\left(\frac{9ex}{11}+d\right)c}{9}+a^2\left(\frac{5ex}{7}+d\right)\right)g^5-\frac{2f\left(\frac{140x^3\left(\frac{11ex}{12}+d\right)c^2}{143}+\frac{20\left(\frac{21ex}{22}+d\right)xac}{9}+a^2e\right)g^4}{7}\cdots}{5g^6}$
derivativedivides	$\frac{2ec^2(gx+f)^{\frac{15}{2}}}{15}+\frac{2\left((dg-ef)c^2-4fec^2\right)(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left(-4(dg-ef)c^2f+e\left(2\left(ag^2+cf^2\right)c+4c^2f^2\right)\right)(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left((dg-ef)\left(2\left(a\cdots\right.\right.\right.}{\cdots}$
default	$\frac{2ec^2(gx+f)^{\frac{15}{2}}}{15}+\frac{2\left((dg-ef)c^2-4fec^2\right)(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left(-4(dg-ef)c^2f+e\left(2\left(ag^2+cf^2\right)c+4c^2f^2\right)\right)(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left((dg-ef)\left(2\left(a\cdots\right.\right.\right.}{\cdots}$
gosper	$\frac{2(gx+f)^{\frac{5}{2}}\left(3003c^2ex^5g^5+3465c^2dg^5x^4-2310c^2efg^4x^4+8190aceg^5x^3-2520c^2dfg^4x^3+1680c^2ef^2g^3x^3+10010acd\cdots\right.}{\cdots}$
orering	$\frac{2(gx+f)^{\frac{5}{2}}\left(3003c^2ex^5g^5+3465c^2dg^5x^4-2310c^2efg^4x^4+8190aceg^5x^3-2520c^2dfg^4x^3+1680c^2ef^2g^3x^3+10010acd\cdots\right.}{\cdots}$
trager	$\frac{2\left(3003c^2eg^7x^7+3465c^2dg^7x^6+3696c^2efg^6x^6+8190aceg^7x^5+4410c^2dfg^6x^5+63c^2ef^2g^5x^5+10010acdg^7x^4+10920\cdots\right.}{\cdots}$
risch	$\frac{2\left(3003c^2eg^7x^7+3465c^2dg^7x^6+3696c^2efg^6x^6+8190aceg^7x^5+4410c^2dfg^6x^5+63c^2ef^2g^5x^5+10010acdg^7x^4+10920\cdots\right.}{\cdots}$

input
```
int((e*x+d)*(g*x+f)^(3/2)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/5*(g*x+f)^(5/2)*(((1/3*x^5*e+5/13*d*x^4)*c^2+10/9*x^2*a*(9/11*e*x+d)*c+a
^2*(5/7*e*x+d))*g^5-2/7*f*(140/143*x^3*(11/12*e*x+d)*c^2+20/9*(21/22*e*x+d
)*x*a*c+a^2*e)*g^4+16/63*(105/143*x^2*(e*x+d)*c+a*(15/11*e*x+d))*f^2*c*g^3
-32/231*f^3*(10/13*(7/6*e*x+d)*x*c+a*e)*c*g^2+128/3003*(5/3*e*x+d)*f^4*c^2
*g-256/9009*c^2*e*f^5)/g^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(190) = 380$.

Time = 0.08 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.85

$$\int (d+ex)(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx=\frac{2\left(3003\,c^2eg^7x^7-256\,c^2ef^7+384\,c^2df^6g-1248\,acef^5g^2+2288\,acdf^4g^3-2574\,a^2ef^3g^4+90\cdots\right.}{\cdots}$$

input
```
integrate((e*x+d)*(g*x+f)^(3/2)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/45045*(3003*c^2*e*g^7*x^7 - 256*c^2*e*f^7 + 384*c^2*d*f^6*g - 1248*a*c*e
*f^5*g^2 + 2288*a*c*d*f^4*g^3 - 2574*a^2*e*f^3*g^4 + 9009*a^2*d*f^2*g^5 +
231*(16*c^2*e*f*g^6 + 15*c^2*d*g^7)*x^6 + 63*(c^2*e*f^2*g^5 + 70*c^2*d*f*g
^6 + 130*a*c*e*g^7)*x^5 - 35*(2*c^2*e*f^3*g^4 - 3*c^2*d*f^2*g^5 - 312*a*c*
e*f*g^6 - 286*a*c*d*g^7)*x^4 + 5*(16*c^2*e*f^4*g^3 - 24*c^2*d*f^3*g^4 + 78
*a*c*e*f^2*g^5 + 2860*a*c*d*f*g^6 + 1287*a^2*e*g^7)*x^3 - 3*(32*c^2*e*f^5*
g^2 - 48*c^2*d*f^4*g^3 + 156*a*c*e*f^3*g^4 - 286*a*c*d*f^2*g^5 - 3432*a^2*
e*f*g^6 - 3003*a^2*d*g^7)*x^2 + (128*c^2*e*f^6*g - 192*c^2*d*f^5*g^2 + 624
*a*c*e*f^4*g^3 - 1144*a*c*d*f^3*g^4 + 1287*a^2*e*f^2*g^5 + 18018*a^2*d*f*g
^6)*x)*sqrt(g*x + f)/g^6
```

Sympy [A] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.74

$$\int (d+ex)(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx=\begin{cases}\frac{2\left(\frac{c^2e(f+gx)^{\frac{15}{2}}}{15g^5}+\frac{(f+gx)^{\frac{13}{2}}\left(c^2dg-5c^2ef\right)}{13g^5}+\frac{(f+gx)^{\frac{11}{2}}\cdot\left(2aceg^2-4c^2dfg+10c^2ef^2\right)}{11g^5}+\frac{(f+gx)^{\frac{9}{2}}\cdot\left(2acdg^3-6acefg^2+6c^2df^2g-10c^2ef^3\right.}{9g^5}\right.}{g} \\ f^{\frac{3}{2}}\left(a^2dx+\frac{a^2ex^2}{2}+\frac{2acdx^3}{3}+\frac{acex^4}{2}+\frac{c^2dx^5}{5}+\frac{c^2ex^6}{6}\right)\end{cases}$$

input
```
integrate((e*x+d)*(g*x+f)**(3/2)*(c*x**2+a)**2,x)
```

output
```
Piecewise((2*(c**2*e*(f + g*x)**(15/2)/(15*g**5) + (f + g*x)**(13/2)*(c**2
*d*g - 5*c**2*e*f)/(13*g**5) + (f + g*x)**(11/2)*(2*a*c*e*g**2 - 4*c**2*d*
f*g + 10*c**2*e*f**2)/(11*g**5) + (f + g*x)**(9/2)*(2*a*c*d*g**3 - 6*a*c*e
*f*g**2 + 6*c**2*d*f**2*g - 10*c**2*e*f**3)/(9*g**5) + (f + g*x)**(7/2)*(a
**2*e*g**4 - 4*a*c*d*f*g**3 + 6*a*c*e*f**2*g**2 - 4*c**2*d*f**3*g + 5*c**2
*e*f**4)/(7*g**5) + (f + g*x)**(5/2)*(a**2*d*g**5 - a**2*e*f*g**4 + 2*a*c*
d*f**2*g**3 - 2*a*c*e*f**3*g**2 + c**2*d*f**4*g - c**2*e*f**5)/(5*g**5))/g
, Ne(g, 0)), (f**(3/2)*(a**2*d*x + a**2*e*x**2/2 + 2*a*c*d*x**3/3 + a*c*e*
x**4/2 + c**2*d*x**5/5 + c**2*e*x**6/6), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.16

$$\int (d+ex)(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx = \frac{2\left(3003\,(gx+f)^{\frac{15}{2}}c^2e-3465\,(5\,c^2ef-c^2dg)(gx+f)^{\frac{13}{2}}+8190\,(5\,c^2ef^2-2\,c^2dfg+aceg^2)(g$$

input
```
integrate((e*x+d)*(g*x+f)^(3/2)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/45045*(3003*(g*x + f)^(15/2)*c^2*e - 3465*(5*c^2*e*f - c^2*d*g)*(g*x + f
)^(13/2) + 8190*(5*c^2*e*f^2 - 2*c^2*d*f*g + a*c*e*g^2)*(g*x + f)^(11/2) -
 10010*(5*c^2*e*f^3 - 3*c^2*d*f^2*g + 3*a*c*e*f*g^2 - a*c*d*g^3)*(g*x + f)
^(9/2) + 6435*(5*c^2*e*f^4 - 4*c^2*d*f^3*g + 6*a*c*e*f^2*g^2 - 4*a*c*d*f*g
^3 + a^2*e*g^4)*(g*x + f)^(7/2) - 9009*(c^2*e*f^5 - c^2*d*f^4*g + 2*a*c*e*
f^3*g^2 - 2*a*c*d*f^2*g^3 + a^2*e*f*g^4 - a^2*d*g^5)*(g*x + f)^(5/2))/g^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1072 vs. 2(190) = 380.

Time = 0.12 (sec) , antiderivative size = 1072, normalized size of antiderivative = 5.01

$$\int (d+ex)(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(g*x+f)^(3/2)*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(g*x + f)*a^2*d*f^2 + 30030*((g*x + f)^(3/2) - 3*sqrt(g
*x + f)*f)*a^2*d*f + 15015*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a^2*e*f^2
/g + 3003*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f)*f^2
)*a^2*d + 6006*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x + f
)*f^2)*a*c*d*f^2/g^2 + 6006*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15
*sqrt(g*x + f)*f^2)*a^2*e*f/g + 2574*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/
2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*c*e*f^2/g^3 + 5148
*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*s
qrt(g*x + f)*f^3)*a*c*d*f/g^2 + 1287*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/
2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a^2*e/g + 143*(35*(g
*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x
 + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c^2*d*f^2/g^4 + 572*(35*(g*x + f)
^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(
3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*a*c*e*f/g^3 + 286*(35*(g*x + f)^(9/2) -
180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3
+ 315*sqrt(g*x + f)*f^4)*a*c*d/g^2 + 65*(63*(g*x + f)^(11/2) - 385*(g*x +
f)^(9/2)*f + 990*(g*x + f)^(7/2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*
x + f)^(3/2)*f^4 - 693*sqrt(g*x + f)*f^5)*c^2*e*f^2/g^5 + 130*(63*(g*x + f
)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x + f)^(7/2)*f^2 - 1386*(g*x + f
)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*sqrt(g*x + f)*f^5)*c^2*d*f...
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.92

$$\int (d+ex)(f+gx)^{3/2}\left(a+cx^2\right)^2\,dx=\frac{(f+g\,x)^{11/2}\,(20\,e\,c^2\,f^2-8\,d\,c^2\,f\,g+4\,a\,e\,c\,g^2)}{11\,g^6}+\frac{2\,(f+g\,x)^{5/2}\,(c\,f^2+a\,g^2)^2\,(d\,g-e\,f)}{5\,g^6}+\frac{2\,(f+g\,x)^{7/2}\,(c\,f^2+a\,g^2)\,(5\,c\,e\,f^2-4\,c\,d\,f\,g+a\,e\,g^2)}{7\,g^6}+\frac{2\,c^2\,e\,(f+g\,x)^{15/2}}{15\,g^6}+\frac{4\,c\,(f+g\,x)^{9/2}\,(-5\,c\,e\,f^3+3\,c\,d\,f^2\,g-3\,a\,e\,f\,g^2+a\,d\,g^3)}{9\,g^6}+\frac{2\,c^2\,(f+g\,x)^{13/2}\,(d\,g-5\,e\,f)}{13\,g^6}$$

input
```
int((f + g*x)^(3/2)*(a + c*x^2)^2*(d + e*x),x)
```

output
```
((f + g*x)^(11/2)*(20*c^2*e*f^2 + 4*a*c*e*g^2 - 8*c^2*d*f*g))/(11*g^6) + (
2*(f + g*x)^(5/2)*(a*g^2 + c*f^2)^2*(d*g - e*f))/(5*g^6) + (2*(f + g*x)^(7
/2)*(a*g^2 + c*f^2)*(a*e*g^2 + 5*c*e*f^2 - 4*c*d*f*g))/(7*g^6) + (2*c^2*e*
(f + g*x)^(15/2))/(15*g^6) + (4*c*(f + g*x)^(9/2)*(a*d*g^3 - 5*c*e*f^3 - 3
*a*e*f*g^2 + 3*c*d*f^2*g))/(9*g^6) + (2*c^2*(f + g*x)^(13/2)*(d*g - 5*e*f)
)/(13*g^6)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.00

$$\int (d+ex)(f+gx)^{3/2} (a+cx^2)^2 \, dx = \frac{2\sqrt{gx+f}\,(3003c^2e\,g^7x^7+3465c^2d\,g^7x^6+3696c^2ef\,g^6x^6+8190ace\,g^7x^5+4410c^2df\,g^6x^5+6}{}$$

input
```
int((e*x+d)*(g*x+f)^(3/2)*(c*x^2+a)^2,x)
```

output
```
(2*sqrt(f + g*x)*(9009*a**2*d*f**2*g**5 + 18018*a**2*d*f*g**6*x + 9009*a**
2*d*g**7*x**2 - 2574*a**2*e*f**3*g**4 + 1287*a**2*e*f**2*g**5*x + 10296*a*
*2*e*f*g**6*x**2 + 6435*a**2*e*g**7*x**3 + 2288*a*c*d*f**4*g**3 - 1144*a*c
*d*f**3*g**4*x + 858*a*c*d*f**2*g**5*x**2 + 14300*a*c*d*f*g**6*x**3 + 1001
0*a*c*d*g**7*x**4 - 1248*a*c*e*f**5*g**2 + 624*a*c*e*f**4*g**3*x - 468*a*c
*e*f**3*g**4*x**2 + 390*a*c*e*f**2*g**5*x**3 + 10920*a*c*e*f*g**6*x**4 + 8
190*a*c*e*g**7*x**5 + 384*c**2*d*f**6*g - 192*c**2*d*f**5*g**2*x + 144*c**
2*d*f**4*g**3*x**2 - 120*c**2*d*f**3*g**4*x**3 + 105*c**2*d*f**2*g**5*x**4
 + 4410*c**2*d*f*g**6*x**5 + 3465*c**2*d*g**7*x**6 - 256*c**2*e*f**7 + 128
*c**2*e*f**6*g*x - 96*c**2*e*f**5*g**2*x**2 + 80*c**2*e*f**4*g**3*x**3 - 7
0*c**2*e*f**3*g**4*x**4 + 63*c**2*e*f**2*g**5*x**5 + 3696*c**2*e*f*g**6*x*
*6 + 3003*c**2*e*g**7*x**7))/(45045*g**6)
```

3.73 $\int (f+gx)^{3/2} \left(a+cx^2\right)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 127

$$\int (f+gx)^{3/2} \left(a+cx^2\right)^2 \, dx = \frac{2\left(cf^2+ag^2\right)^2 (f+gx)^{5/2}}{5g^5} - \frac{8cf\left(cf^2+ag^2\right)(f+gx)^{7/2}}{7g^5} + \frac{4c\left(3cf^2+ag^2\right)(f+gx)^{9/2}}{9g^5} - \frac{8c^2f(f+gx)^{11/2}}{11g^5} + \frac{2c^2(f+gx)^{13/2}}{13g^5}$$

output
```
2/5*(a*g^2+c*f^2)^2*(g*x+f)^(5/2)/g^5-8/7*c*f*(a*g^2+c*f^2)*(g*x+f)^(7/2)/
g^5+4/9*c*(a*g^2+3*c*f^2)*(g*x+f)^(9/2)/g^5-8/11*c^2*f*(g*x+f)^(11/2)/g^5+
2/13*c^2*(g*x+f)^(13/2)/g^5
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.76

$$\int (f+gx)^{3/2} (a +cx^2)^2 \, dx = \frac{2(f+gx)^{5/2} \left(9009a^2g^4+286acg^2\left(8f^2-20fgx+35g^2x^2\right)+3c^2\left(128f^4-320f^3gx+560f^2g^2\right.\right.}{45045g^5}$$

input
```
Integrate[(f + g*x)^(3/2)*(a + c*x^2)^2,x]
```

output
```
(2*(f + g*x)^(5/2)*(9009*a^2*g^4 + 286*a*c*g^2*(8*f^2 - 20*f*g*x + 35*g^2*
x^2) + 3*c^2*(128*f^4 - 320*f^3*g*x + 560*f^2*g^2*x^2 - 840*f*g^3*x^3 + 11
55*g^4*x^4)))/(45045*g^5)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^2 (f+gx)^{3/2}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{2c(f+gx)^{7/2}(ag^2+3cf^2)}{g^4} - \frac{4cf(f+gx)^{5/2}(ag^2+cf^2)}{g^4} + \frac{(f+gx)^{3/2}(ag^2+cf^2)^2}{g^4} + \frac{c^2(f+gx)^{11/2}}{g^4} - \right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c(f+gx)^{9/2}(ag^2+3cf^2)}{9g^5} - \frac{8cf(f+gx)^{7/2}(ag^2+cf^2)}{7g^5} + \frac{2(f+gx)^{5/2}(ag^2+cf^2)^2}{5g^5} + \frac{2c^2(f+gx)^{13/2}}{13g^5} - \frac{8c^2f(f+gx)^{11/2}}{11g^5}$$

input
```
Int[(f + g*x)^(3/2)*(a + c*x^2)^2,x]
```

output
```
(2*(c*f^2 + a*g^2)^2*(f + g*x)^(5/2))/(5*g^5) - (8*c*f*(c*f^2 + a*g^2)*(f
+ g*x)^(7/2))/(7*g^5) + (4*c*(3*c*f^2 + a*g^2)*(f + g*x)^(9/2))/(9*g^5) -
(8*c^2*f*(f + g*x)^(11/2))/(11*g^5) + (2*c^2*(f + g*x)^(13/2))/(13*g^5)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{2(gx+f)^{\frac{5}{2}}\left(\left(\frac{5}{13}c^2x^4+\frac{10}{9}acx^2+a^2\right)g^4-\frac{40x\left(\frac{63cx^2}{143}+a\right)cfg^3}{63}+\frac{16\left(\frac{105cx^2}{143}+a\right)cf^2g^2}{63}-\frac{320c^2f^3gx}{3003}+\frac{128c^2f^4}{3003}\right)}{5g^5}$
gosper	$\frac{2(gx+f)^{\frac{5}{2}}(3465c^2x^4g^4-2520c^2fx^3g^3+10010acg^4x^2+1680c^2f^2g^2x^2-5720acfg^3x-960c^2f^3gx+9009a^2g^4+2288ac}{45045g^5}$
orering	$\frac{2(gx+f)^{\frac{5}{2}}(3465c^2x^4g^4-2520c^2fx^3g^3+10010acg^4x^2+1680c^2f^2g^2x^2-5720acfg^3x-960c^2f^3gx+9009a^2g^4+2288ac}{45045g^5}$
derivativedivides	$\frac{\frac{2c^2(gx+f)^{\frac{13}{2}}}{13}-\frac{8c^2f(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(2\left(ag^2+cf^2\right)c+4c^2f^2\right)(gx+f)^{\frac{9}{2}}}{9}-\frac{8\left(ag^2+cf^2\right)cf(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left(ag^2+cf^2\right)^2(gx+f)^{\frac{5}{2}}}{5}}{g^5}$
default	$\frac{\frac{2c^2(gx+f)^{\frac{13}{2}}}{13}-\frac{8c^2f(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left(2\left(ag^2+cf^2\right)c+4c^2f^2\right)(gx+f)^{\frac{9}{2}}}{9}-\frac{8\left(ag^2+cf^2\right)cf(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left(ag^2+cf^2\right)^2(gx+f)^{\frac{5}{2}}}{5}}{g^5}$
trager	$\frac{2(3465c^2g^6x^6+4410c^2fg^5x^5+10010acg^6x^4+105c^2f^2g^4x^4+14300acfg^5x^3-120c^2f^3g^3x^3+9009a^2g^6x^2+858acf^2g^4}{45045g^5}$
risch	$\frac{2(3465c^2g^6x^6+4410c^2fg^5x^5+10010acg^6x^4+105c^2f^2g^4x^4+14300acfg^5x^3-120c^2f^3g^3x^3+9009a^2g^6x^2+858acf^2g^4}{45045g^5}$

input
```
int((g*x+f)^(3/2)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/5*(g*x+f)^(5/2)*((5/13*c^2*x^4+10/9*a*c*x^2+a^2)*g^4-40/63*x*(63/143*c*x
^2+a)*c*f*g^3+16/63*(105/143*c*x^2+a)*c*f^2*g^2-320/3003*c^2*f^3*g*x+128/3
003*c^2*f^4)/g^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.43

$$\int (f + gx)^{3/2} (a + cx^2)^2 \, dx = \frac{2 (3465 c^2 g^6 x^6 + 4410 c^2 f g^5 x^5 + 384 c^2 f^6 + 2288 acf^4 g^2 + 9009 a^2 f^2 g^4 + 35 (3 c^2 f^2 g^4 + 286 a}{}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/45045*(3465*c^2*g^6*x^6 + 4410*c^2*f*g^5*x^5 + 384*c^2*f^6 + 2288*a*c*f^
4*g^2 + 9009*a^2*f^2*g^4 + 35*(3*c^2*f^2*g^4 + 286*a*c*g^6)*x^4 - 20*(6*c^
2*f^3*g^3 - 715*a*c*f*g^5)*x^3 + 3*(48*c^2*f^4*g^2 + 286*a*c*f^2*g^4 + 300
3*a^2*g^6)*x^2 - 2*(96*c^2*f^5*g + 572*a*c*f^3*g^3 - 9009*a^2*f*g^5)*x)*sq
rt(g*x + f)/g^5
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.39

$$\int (f + gx)^{3/2} (a + cx^2)^2 \, dx = \begin{cases} \frac{2 \left(- \frac{4c^2 f (f+gx)^{\frac{11}{2}}}{11g^4} + \frac{c^2 (f+gx)^{\frac{13}{2}}}{13g^4} + \frac{(f+gx)^{\frac{9}{2}} \cdot (2acg^2 + 6c^2 f^2)}{9g^4} + \frac{(f+gx)^{\frac{7}{2}} (-4acfg^2 - 4c^2 f^3)}{7g^4} + \frac{(f+gx)^{\frac{5}{2}} (a^2 g^4 + 2acf^2 g^2 + c^2 f^4)}{5g^4} \right)}{g} \\ f^{\frac{3}{2}} \left(a^2 x + \frac{2acx^3}{3} + \frac{c^2 x^5}{5} \right) \end{cases}$$

input
```
integrate((g*x+f)**(3/2)*(c*x**2+a)**2,x)
```

output
```
Piecewise((2*(-4*c**2*f*(f + g*x)**(11/2)/(11*g**4) + c**2*(f + g*x)**(13/
2)/(13*g**4) + (f + g*x)**(9/2)*(2*a*c*g**2 + 6*c**2*f**2)/(9*g**4) + (f +
 g*x)**(7/2)*(-4*a*c*f*g**2 - 4*c**2*f**3)/(7*g**4) + (f + g*x)**(5/2)*(a*
*2*g**4 + 2*a*c*f**2*g**2 + c**2*f**4)/(5*g**4))/g, Ne(g, 0)), (f**(3/2)*(
a**2*x + 2*a*c*x**3/3 + c**2*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.89

$$\int (f+gx)^{3/2}\left(a+cx^2\right)^2\,dx = \frac{2\left(3465\,(gx+f)^{\frac{13}{2}}c^2 - 16380\,(gx+f)^{\frac{11}{2}}c^2f + 10010\,(3\,c^2f^2+acg^2)(gx+f)^{\frac{9}{2}} - 25740\,(c^2f^3\right.}{45045\,g^5}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/45045*(3465*(g*x + f)^(13/2)*c^2 - 16380*(g*x + f)^(11/2)*c^2*f + 10010*
(3*c^2*f^2 + a*c*g^2)*(g*x + f)^(9/2) - 25740*(c^2*f^3 + a*c*f*g^2)*(g*x +
 f)^(7/2) + 9009*(c^2*f^4 + 2*a*c*f^2*g^2 + a^2*g^4)*(g*x + f)^(5/2))/g^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(107) = 214.

Time = 0.12 (sec) , antiderivative size = 471, normalized size of antiderivative = 3.71

$$\int (f+gx)^{3/2}\left(a+cx^2\right)^2\,dx = \frac{2\left(45045\,\sqrt{gx+f}a^2f^2 + 30030\left((gx+f)^{\frac{3}{2}} - 3\,\sqrt{gx+f}f\right)a^2f + 3003\left(3\,(gx+f)^{\frac{5}{2}} - 10\,(g\right.\right.}{}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(g*x + f)*a^2*f^2 + 30030*((g*x + f)^(3/2) - 3*sqrt(g*x
 + f)*f)*a^2*f + 3003*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(
g*x + f)*f^2)*a^2 + 6006*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sq
rt(g*x + f)*f^2)*a*c*f^2/g^2 + 5148*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2
)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*c*f/g^2 + 143*(35*(
g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*
x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c^2*f^2/g^4 + 286*(35*(g*x + f)^
(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3
/2)*f^3 + 315*sqrt(g*x + f)*f^4)*a*c/g^2 + 130*(63*(g*x + f)^(11/2) - 385*
(g*x + f)^(9/2)*f + 990*(g*x + f)^(7/2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1
155*(g*x + f)^(3/2)*f^4 - 693*sqrt(g*x + f)*f^5)*c^2*f/g^4 + 15*(231*(g*x
+ f)^(13/2) - 1638*(g*x + f)^(11/2)*f + 5005*(g*x + f)^(9/2)*f^2 - 8580*(g
*x + f)^(7/2)*f^3 + 9009*(g*x + f)^(5/2)*f^4 - 6006*(g*x + f)^(3/2)*f^5 +
3003*sqrt(g*x + f)*f^6)*c^2/g^4)/g
```

Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int (f+gx)^{3/2}\left(a+cx^2\right)^2\,dx = \frac{2\,c^2\,{\left(f+g\,x\right)}^{13/2}}{13\,g^5} - \frac{{\left(f+g\,x\right)}^{7/2}\,\left(8\,c^2\,f^3+8\,a\,c\,f\,g^2\right)}{7\,g^5} + \frac{2\,{\left(f+g\,x\right)}^{5/2}\,{\left(c\,f^2+a\,g^2\right)}^2}{5\,g^5} + \frac{{\left(f+g\,x\right)}^{9/2}\,\left(12\,c^2\,f^2+4\,a\,c\,g^2\right)}{9\,g^5} - \frac{8\,c^2\,f\,{\left(f+g\,x\right)}^{11/2}}{11\,g^5}$$

input
```
int((f + g*x)^(3/2)*(a + c*x^2)^2,x)
```

output
```
(2*c^2*(f + g*x)^(13/2))/(13*g^5) - ((f + g*x)^(7/2)*(8*c^2*f^3 + 8*a*c*f*
g^2))/(7*g^5) + (2*(f + g*x)^(5/2)*(a*g^2 + c*f^2)^2)/(5*g^5) + ((f + g*x)
^(9/2)*(12*c^2*f^2 + 4*a*c*g^2))/(9*g^5) - (8*c^2*f*(f + g*x)^(11/2))/(11*
g^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.43

$$\int (f+gx)^{3/2}\left(a+cx^2\right)^2\,dx=\frac{2\sqrt{gx+f}\,(3465c^2g^6x^6+4410c^2f\,g^5x^5+10010ac\,g^6x^4+105c^2f^2g^4x^4+14300acf\,g^5x^3-120}{}$$

input
```
int((g*x+f)^(3/2)*(c*x^2+a)^2,x)
```

output
```
(2*sqrt(f + g*x)*(9009*a**2*f**2*g**4 + 18018*a**2*f*g**5*x + 9009*a**2*g*
*6*x**2 + 2288*a*c*f**4*g**2 - 1144*a*c*f**3*g**3*x + 858*a*c*f**2*g**4*x*
*2 + 14300*a*c*f*g**5*x**3 + 10010*a*c*g**6*x**4 + 384*c**2*f**6 - 192*c**
2*f**5*g*x + 144*c**2*f**4*g**2*x**2 - 120*c**2*f**3*g**3*x**3 + 105*c**2*
f**2*g**4*x**4 + 4410*c**2*f*g**5*x**5 + 3465*c**2*g**6*x**6))/(45045*g**5
)
```

3.74 $\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{d+ex}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 284

$$\begin{aligned}\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{d+ex}\,dx &= \frac{2(cd^2+ae^2)^2(ef-dg)\sqrt{f+gx}}{e^6} \\ &+ \frac{2(cd^2+ae^2)^2(f+gx)^{3/2}}{3e^5} - \frac{2c(ef+dg)(ce^2f^2+cd^2g^2+2ae^2g^2)(f+gx)^{5/2}}{5e^4g^4} \\ &+ \frac{2c(2ae^2g^2+c(3e^2f^2+2defg+d^2g^2))(f+gx)^{7/2}}{7e^3g^4} - \frac{2c^2(3ef+dg)(f+gx)^{9/2}}{9e^2g^4} \\ &+ \frac{2c^2(f+gx)^{11/2}}{11eg^4} - \frac{2(cd^2+ae^2)^2(ef-dg)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{13/2}}\end{aligned}$$

output
```
2*(a*e^2+c*d^2)^2*(-d*g+e*f)*(g*x+f)^(1/2)/e^6+2/3*(a*e^2+c*d^2)^2*(g*x+f)
^(3/2)/e^5-2/5*c*(d*g+e*f)*(2*a*e^2*g^2+c*d^2*g^2+c*e^2*f^2)*(g*x+f)^(5/2)
/e^4/g^4+2/7*c*(2*a*e^2*g^2+c*(d^2*g^2+2*d*e*f*g+3*e^2*f^2))*(g*x+f)^(7/2)
/e^3/g^4-2/9*c^2*(d*g+3*e*f)*(g*x+f)^(9/2)/e^2/g^4+2/11*c^2*(g*x+f)^(11/2)
/e/g^4-2*(a*e^2+c*d^2)^2*(-d*g+e*f)^(3/2)*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-
d*g+e*f)^(1/2))/e^(13/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.13

$$\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{d+ex}\,dx = \frac{2\sqrt{f+gx}(1155a^2e^4g^4(4ef-3dg+egx)-66ace^2g^2(105d^3g^3+21de^2g(f-$$

$$+\frac{2(cd^2+ae^2)^2(-ef+dg)^{3/2}\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{13/2}}$$

input
```
Integrate[((f + g*x)^(3/2)*(a + c*x^2)^2)/(d + e*x),x]
```

output
```
(2*Sqrt[f + g*x]*(1155*a^2*e^4*g^4*(4*e*f - 3*d*g + e*g*x) - 66*a*c*e^2*g^
2*(105*d^3*g^3 + 21*d*e^2*g*(f + g*x)^2 + 3*e^3*(2*f - 5*g*x)*(f + g*x)^2
- 35*d^2*e*g^2*(4*f + g*x)) + c^2*(-3465*d^5*g^5 - 693*d^3*e^2*g^3*(f + g*
x)^2 + 1155*d^4*e*g^4*(4*f + g*x) + 99*d^2*e^3*g^2*(f + g*x)^2*(-2*f + 5*g
*x) - 11*d*e^4*g*(f + g*x)^2*(8*f^2 - 20*f*g*x + 35*g^2*x^2) - 3*e^5*(f +
g*x)^2*(16*f^3 - 40*f^2*g*x + 70*f*g^2*x^2 - 105*g^3*x^3))))/(3465*e^6*g^4
) + (2*(c*d^2 + a*e^2)^2*(-(e*f) + d*g)^(3/2)*ArcTan[(Sqrt[e]*Sqrt[f + g*x
])/Sqrt[-(e*f) + d*g]])/e^(13/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {649, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2(f+gx)^{3/2}}{d+ex}\,dx$$

$$\Big\downarrow \ 649$$

$$\frac{2\int -\frac{(f+gx)^2(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)^2}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{2\int \frac{(f+gx)^2\left(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2\right)^2}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^4}$$

$\Big\downarrow$ 1584

$$-\frac{2\int \left(-\frac{c^2(f+gx)^5}{e}+\frac{c^2(3ef+dg)(f+gx)^4}{e^2}-\frac{c\left(2ae^2g^2+c\left(3e^2f^2+2degf+d^2g^2\right)\right)(f+gx)^3}{e^3}+\frac{c(ef+dg)\left(ce^2f^2+cd^2g^2+2ae^2g^2\right)(f+gx)^2}{e^4}\right.}{}$$

$\Big\downarrow$ 2009

$$\frac{2\left(-\frac{g^4\left(ae^2+cd^2\right)^2(ef-dg)^{3/2}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{13/2}}+\frac{g^4\sqrt{f+gx}\left(ae^2+cd^2\right)^2(ef-dg)}{e^6}+\frac{g^4(f+gx)^{3/2}\left(ae^2+cd^2\right)^2}{3e^5}-\frac{c(f+gx)^{5/2}(dg+ef}{}\right.}{g^4}$$

input
```
Int[((f + g*x)^(3/2)*(a + c*x^2)^2)/(d + e*x),x]
```

output
```
(2*((((c*d^2 + a*e^2)^2*g^4*(e*f - d*g)*Sqrt[f + g*x])/e^6 + ((c*d^2 + a*e^
2)^2*g^4*(f + g*x)^(3/2))/(3*e^5) - (c*(e*f + d*g)*(c*e^2*f^2 + c*d^2*g^2
+ 2*a*e^2*g^2)*(f + g*x)^(5/2))/(5*e^4) + (c*(2*a*e^2*g^2 + c*(3*e^2*f^2 +
 2*d*e*f*g + d^2*g^2))*(f + g*x)^(7/2))/(7*e^3) - (c^2*(3*e*f + d*g)*(f +
g*x)^(9/2))/(9*e^2) + (c^2*(f + g*x)^(11/2))/(11*e) - ((c*d^2 + a*e^2)^2*g
^4*(e*f - d*g)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/e^(
13/2)))/g^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$-2\Big(\Big(\Big(\frac{16(gx+f)^2\left(-\frac{105}{16}x^3g^3+\frac{35}{8}fg^2x^2-\frac{5}{2}xf^2g+f^3\right)e^5}{1155}+\frac{8g(gx+f)^2\left(\frac{35}{8}g^2x^2-\frac{5}{2}fgx+f^2\right)de^4}{315}+\frac{2g^2\left(-\frac{5gx}{2}+f\right)(gx+f)^2d^2e\ldots}{35}$
derivativedivides	$-\frac{2\Big(-\frac{c^2(gx+f)^{\frac{11}{2}}e^5}{11}+\frac{\left(-\left(-cdeg-fce^2\right)e^3c+2c^2e^5f\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{\left(-\left(ae^2g^2+cd^2g^2\right)e^3c+2\left(-cdeg-fce^2\right)e^3fc+ce^2\left(-ae^3g\ldots\right.\right.}{7}}{}$
default	$-\frac{2\Big(-\frac{c^2(gx+f)^{\frac{11}{2}}e^5}{11}+\frac{\left(-\left(-cdeg-fce^2\right)e^3c+2c^2e^5f\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{\left(-\left(ae^2g^2+cd^2g^2\right)e^3c+2\left(-cdeg-fce^2\right)e^3fc+ce^2\left(-ae^3g\ldots\right.\right.}{7}}{}$
risch	$-\frac{2\left(-315c^2g^5e^5x^5+385c^2de^4g^5x^4-420c^2e^5fg^4x^4-990ace^5g^5x^3-495c^2d^2e^3g^5x^3+550c^2de^4fg^4x^3-15c^2e^5f^2g^3x\ldots\right.}{}$

input
```
int((g*x+f)^(3/2)*(c*x^2+a)^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
2*(-((16/1155*(g*x+f)^2*(-105/16*x^3*g^3+35/8*f*g^2*x^2-5/2*x*f^2*g+f^3)*e
^5+8/315*g*(g*x+f)^2*(35/8*g^2*x^2-5/2*f*g*x+f^2)*d*e^4+2/35*g^2*(-5/2*g*x
+f)*(g*x+f)^2*d^2*e^3+1/5*d^3*g^3*(g*x+f)^2*e^2-4/3*g^4*(1/4*g*x+f)*d^4*e+
g^5*d^5)*c^2+2*e^2*g^2*(2/35*(-5/2*g*x+f)*(g*x+f)^2*e^3+1/5*d*g*(g*x+f)^2*
e^2-4/3*g^2*(1/4*g*x+f)*d^2*e+d^3*g^3)*a*c+e^4*g^4*(1/3*(-g*x-4*f)*e+d*g)*
a^2)*((d*g-e*f)*e)^(1/2)*(g*x+f)^(1/2)+g^4*(d*g-e*f)^2*(a*e^2+c*d^2)^2*arc
tan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)))/((d*g-e*f)*e)^(1/2)/g^4/e^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(252) = 504$.

Time = 0.12 (sec) , antiderivative size = 1150, normalized size of antiderivative = 4.05

$$\int \frac{(f+gx)^{3/2}\left(a+cx^2\right)^2}{d+ex}\,dx = \text{Too large to display}$$

input

```
integrate((g*x+f)^(3/2)*(c*x^2+a)^2/(e*x+d),x, algorithm="fricas")
```

output

```
[-1/3465*(3465*((c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*f*g^4 - (c^2*d^5 + 2
*a*c*d^3*e^2 + a^2*d*e^4)*g^5)*sqrt((e*f - d*g)/e)*log((e*g*x + 2*e*f - d*
g + 2*sqrt(g*x + f)*e*sqrt((e*f - d*g)/e))/(e*x + d)) - 2*(315*c^2*e^5*g^5
*x^5 - 48*c^2*e^5*f^5 - 88*c^2*d*e^4*f^4*g - 198*(c^2*d^2*e^3 + 2*a*c*e^5)
*f^3*g^2 - 693*(c^2*d^3*e^2 + 2*a*c*d*e^4)*f^2*g^3 + 4620*(c^2*d^4*e + 2*a
*c*d^2*e^3 + a^2*e^5)*f*g^4 - 3465*(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*g
^5 + 35*(12*c^2*e^5*f*g^4 - 11*c^2*d*e^4*g^5)*x^4 + 5*(3*c^2*e^5*f^2*g^3 -
 110*c^2*d*e^4*f*g^4 + 99*(c^2*d^2*e^3 + 2*a*c*e^5)*g^5)*x^3 - 3*(6*c^2*e^
5*f^3*g^2 + 11*c^2*d*e^4*f^2*g^3 - 264*(c^2*d^2*e^3 + 2*a*c*e^5)*f*g^4 + 2
31*(c^2*d^3*e^2 + 2*a*c*d*e^4)*g^5)*x^2 + (24*c^2*e^5*f^4*g + 44*c^2*d*e^4
*f^3*g^2 + 99*(c^2*d^2*e^3 + 2*a*c*e^5)*f^2*g^3 - 1386*(c^2*d^3*e^2 + 2*a*
c*d*e^4)*f*g^4 + 1155*(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*g^5)*x)*sqrt(g
*x + f))/(e^6*g^4), -2/3465*(3465*((c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*f
*g^4 - (c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*g^5)*sqrt(-(e*f - d*g)/e)*arc
tan(-sqrt(g*x + f)*e*sqrt(-(e*f - d*g)/e)/(e*f - d*g)) - (315*c^2*e^5*g^5*
x^5 - 48*c^2*e^5*f^5 - 88*c^2*d*e^4*f^4*g - 198*(c^2*d^2*e^3 + 2*a*c*e^5)*
f^3*g^2 - 693*(c^2*d^3*e^2 + 2*a*c*d*e^4)*f^2*g^3 + 4620*(c^2*d^4*e + 2*a*
c*d^2*e^3 + a^2*e^5)*f*g^4 - 3465*(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*g^
5 + 35*(12*c^2*e^5*f*g^4 - 11*c^2*d*e^4*g^5)*x^4 + 5*(3*c^2*e^5*f^2*g^3 -
110*c^2*d*e^4*f*g^4 + 99*(c^2*d^2*e^3 + 2*a*c*e^5)*g^5)*x^3 - 3*(6*c^2*...
```

Sympy [A] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.81

$$\int \frac{(f+gx)^{3/2}\left(a+cx^2\right)^2}{d+ex}\,dx = \begin{cases} \frac{2\left(\frac{c^2(f+gx)^{\frac{11}{2}}}{11eg^3}+\frac{(f+gx)^{\frac{9}{2}}\left(-c^2dg-3c^2ef\right)}{9e^2g^3}+\frac{(f+gx)^{\frac{7}{2}}\cdot\left(2ace^2g^2+c^2d^2g^2+2c^2defg+3c^2e^2f^2\right)}{7e^3g^3}+\frac{(f+gx)^{\frac{5}{2}}(\cdots}{\cdots}\right.}{\cdots} \\ f^{\frac{3}{2}}\left(-\frac{c^2dx^3}{3e^2}+\frac{c^2x^4}{4e}+\frac{x^2\cdot(2ace^2+c^2d^2)}{2e^3}+\frac{x(-2acde^2-c^2d^3)}{e^4}+\frac{(ae^2+cd^2)^2\left(\begin{cases}\frac{x}{d} \\ \frac{\log(d+ex\cdots}{e}\end{cases}\right.}{e^4}\right. \end{cases}$$

input
```
integrate((g*x+f)**(3/2)*(c*x**2+a)**2/(e*x+d),x)
```

output
```
Piecewise((2*(c**2*(f + g*x)**(11/2)/(11*e*g**3) + (f + g*x)**(9/2)*(-c**2
*d*g - 3*c**2*e*f)/(9*e**2*g**3) + (f + g*x)**(7/2)*(2*a*c*e**2*g**2 + c**
2*d**2*g**2 + 2*c**2*d*e*f*g + 3*c**2*e**2*f**2)/(7*e**3*g**3) + (f + g*x)
**(5/2)*(-2*a*c*d*e**2*g**3 - 2*a*c*e**3*f*g**2 - c**2*d**3*g**3 - c**2*d*
*2*e*f*g**2 - c**2*d*e**2*f**2*g - c**2*e**3*f**3)/(5*e**4*g**3) + (f + g*
x)**(3/2)*(a**2*e**4*g + 2*a*c*d**2*e**2*g + c**2*d**4*g)/(3*e**5) + sqrt(
f + g*x)*(-a**2*d*e**4*g**2 + a**2*e**5*f*g - 2*a*c*d**3*e**2*g**2 + 2*a*c
*d**2*e**3*f*g - c**2*d**5*g**2 + c**2*d**4*e*f*g)/e**6 + g*(a*e**2 + c*d*
*2)**2*(d*g - e*f)**2*atan(sqrt(f + g*x)/sqrt((d*g - e*f)/e))/(e**7*sqrt((
d*g - e*f)/e)))/g, Ne(g, 0)), (f**(3/2)*(-c**2*d*x**3/(3*e**2) + c**2*x**4
/(4*e) + x**2*(2*a*c*e**2 + c**2*d**2)/(2*e**3) + x*(-2*a*c*d*e**2 - c**2*
d**3)/e**4 + (a*e**2 + c*d**2)**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)
/e, True))/e**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^{3/2}\left(a+cx^2\right)^2}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)^2/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(252) = 504$.

Time = 0.13 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.10

$$\int \frac{(f+gx)^{3/2}\left(a+cx^2\right)^2}{d+ex}\,dx = \frac{2\left(c^2d^4e^2f^2+2\,acd^2e^4f^2+a^2e^6f^2-2\,c^2d^5efg-4\,acd^3e^3fg-2\,a^2de^5fg+\right.}{\sqrt{-e^2f+dege^6}} + \frac{2\left(315\,(gx+f)^{\frac{11}{2}}c^2e^{10}g^{40}-1155\,(gx+f)^{\frac{9}{2}}c^2e^{10}fg^{40}+1485\,(gx+f)^{\frac{7}{2}}c^2e^{10}f^2g^{40}-693\,(gx+f)^{\frac{5}{2}}c^2e^{10}f\right.}{}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)^2/(e*x+d),x, algorithm="giac")
```

output
```
2*(c^2*d^4*e^2*f^2 + 2*a*c*d^2*e^4*f^2 + a^2*e^6*f^2 - 2*c^2*d^5*e*f*g - 4
*a*c*d^3*e^3*f*g - 2*a^2*d*e^5*f*g + c^2*d^6*g^2 + 2*a*c*d^4*e^2*g^2 + a^2
*d^2*e^4*g^2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e^2*f +
d*e*g)*e^6) + 2/3465*(315*(g*x + f)^(11/2)*c^2*e^10*g^40 - 1155*(g*x + f)^
(9/2)*c^2*e^10*f*g^40 + 1485*(g*x + f)^(7/2)*c^2*e^10*f^2*g^40 - 693*(g*x
+ f)^(5/2)*c^2*e^10*f^3*g^40 - 385*(g*x + f)^(9/2)*c^2*d*e^9*g^41 + 990*(g
*x + f)^(7/2)*c^2*d*e^9*f*g^41 - 693*(g*x + f)^(5/2)*c^2*d*e^9*f^2*g^41 +
495*(g*x + f)^(7/2)*c^2*d^2*e^8*g^42 + 990*(g*x + f)^(7/2)*a*c*e^10*g^42 -
 693*(g*x + f)^(5/2)*c^2*d^2*e^8*f*g^42 - 1386*(g*x + f)^(5/2)*a*c*e^10*f*
g^42 - 693*(g*x + f)^(5/2)*c^2*d^3*e^7*g^43 - 1386*(g*x + f)^(5/2)*a*c*d*e
^9*g^43 + 1155*(g*x + f)^(3/2)*c^2*d^4*e^6*g^44 + 2310*(g*x + f)^(3/2)*a*c
*d^2*e^8*g^44 + 1155*(g*x + f)^(3/2)*a^2*e^10*g^44 + 3465*sqrt(g*x + f)*c^
2*d^4*e^6*f*g^44 + 6930*sqrt(g*x + f)*a*c*d^2*e^8*f*g^44 + 3465*sqrt(g*x +
 f)*a^2*e^10*f*g^44 - 3465*sqrt(g*x + f)*c^2*d^5*e^5*g^45 - 6930*sqrt(g*x
+ f)*a*c*d^3*e^7*g^45 - 3465*sqrt(g*x + f)*a^2*d*e^9*g^45)/(e^11*g^44)
```

Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 838, normalized size of antiderivative = 2.95

$$\int \frac{(f+gx)^{3/2}\left(a+cx^2\right)^2}{d+ex}\,dx = \text{Too large to display}$$

input
```
int(((f + g*x)^(3/2)*(a + c*x^2)^2)/(d + e*x),x)
```

output
```
(f + g*x)^(3/2)*((2*(a*g^2 + c*f^2)^2)/(3*e*g^4) + ((d*g^5 - e*f*g^4)*((8*
c^2*f^3 + 8*a*c*f*g^2)/(e*g^4) + (((12*c^2*f^2 + 4*a*c*g^2)/(e*g^4) + ((d*
g^5 - e*f*g^4)*((8*c^2*f)/(e*g^4) + (2*c^2*(d*g^5 - e*f*g^4))/(e^2*g^8)))/
(e*g^4))*(d*g^5 - e*f*g^4))/(e*g^4)))/(3*e*g^4)) - (f + g*x)^(5/2)*((8*c^2
*f^3 + 8*a*c*f*g^2)/(5*e*g^4) + (((12*c^2*f^2 + 4*a*c*g^2)/(e*g^4) + ((d*g
^5 - e*f*g^4)*((8*c^2*f)/(e*g^4) + (2*c^2*(d*g^5 - e*f*g^4))/(e^2*g^8)))/(
e*g^4))*(d*g^5 - e*f*g^4))/(5*e*g^4)) - (f + g*x)^(9/2)*((8*c^2*f)/(9*e*g^
4) + (2*c^2*(d*g^5 - e*f*g^4))/(9*e^2*g^8)) + (f + g*x)^(7/2)*((12*c^2*f^2
 + 4*a*c*g^2)/(7*e*g^4) + ((d*g^5 - e*f*g^4)*((8*c^2*f)/(e*g^4) + (2*c^2*(
d*g^5 - e*f*g^4))/(e^2*g^8)))/(7*e*g^4)) + (2*c^2*(f + g*x)^(11/2))/(11*e*
g^4) + (2*atan((e^(1/2)*(f + g*x)^(1/2)*(a*e^2 + c*d^2)^2*(d*g - e*f)^(3/2
))/(a^2*e^6*f^2 + c^2*d^6*g^2 + a^2*d^2*e^4*g^2 + c^2*d^4*e^2*f^2 - 2*a^2*
d*e^5*f*g - 2*c^2*d^5*e*f*g + 2*a*c*d^2*e^4*f^2 + 2*a*c*d^4*e^2*g^2 - 4*a*
c*d^3*e^3*f*g))*(a*e^2 + c*d^2)^2*(d*g - e*f)^(3/2))/e^(13/2) - ((f + g*x)
^(1/2)*((2*(a*g^2 + c*f^2)^2)/(e*g^4) + ((d*g^5 - e*f*g^4)*((8*c^2*f^3 + 8
*a*c*f*g^2)/(e*g^4) + (((12*c^2*f^2 + 4*a*c*g^2)/(e*g^4) + ((d*g^5 - e*f*g
^4)*((8*c^2*f)/(e*g^4) + (2*c^2*(d*g^5 - e*f*g^4))/(e^2*g^8)))/(e*g^4))*(d
*g^5 - e*f*g^4))/(e*g^4)))/(e*g^4))*(d*g^5 - e*f*g^4))/(e*g^4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 997, normalized size of antiderivative = 3.51

$$\int \frac{(f+gx)^{3/2}\left(a+cx^2\right)^2}{d+ex}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^(3/2)*(c*x^2+a)^2/(e*x+d),x)
```

output

```
(2*(3465*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g
- e*f)))*a**2*d*e**4*g**5 - 3465*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*
x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a**2*e**5*f*g**4 + 6930*sqrt(e)*sqrt(d*g
- e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**3*e**2*g**
5 - 6930*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g
- e*f)))*a*c*d**2*e**3*f*g**4 + 3465*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f
+ g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**5*g**5 - 3465*sqrt(e)*sqrt(d*
g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**4*e*f*g
**4 - 3465*sqrt(f + g*x)*a**2*d*e**5*g**5 + 4620*sqrt(f + g*x)*a**2*e**6*f
*g**4 + 1155*sqrt(f + g*x)*a**2*e**6*g**5*x - 6930*sqrt(f + g*x)*a*c*d**3*
e**3*g**5 + 9240*sqrt(f + g*x)*a*c*d**2*e**4*f*g**4 + 2310*sqrt(f + g*x)*a
*c*d**2*e**4*g**5*x - 1386*sqrt(f + g*x)*a*c*d*e**5*f**2*g**3 - 2772*sqrt(
f + g*x)*a*c*d*e**5*f*g**4*x - 1386*sqrt(f + g*x)*a*c*d*e**5*g**5*x**2 - 3
96*sqrt(f + g*x)*a*c*e**6*f**3*g**2 + 198*sqrt(f + g*x)*a*c*e**6*f**2*g**3
*x + 1584*sqrt(f + g*x)*a*c*e**6*f*g**4*x**2 + 990*sqrt(f + g*x)*a*c*e**6*
g**5*x**3 - 3465*sqrt(f + g*x)*c**2*d**5*e*g**5 + 4620*sqrt(f + g*x)*c**2*
d**4*e**2*f*g**4 + 1155*sqrt(f + g*x)*c**2*d**4*e**2*g**5*x - 693*sqrt(f +
 g*x)*c**2*d**3*e**3*f**2*g**3 - 1386*sqrt(f + g*x)*c**2*d**3*e**3*f*g**4*
x - 693*sqrt(f + g*x)*c**2*d**3*e**3*g**5*x**2 - 198*sqrt(f + g*x)*c**2*d*
*2*e**4*f**3*g**2 + 99*sqrt(f + g*x)*c**2*d**2*e**4*f**2*g**3*x + 792*s...
```

3.75 $\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{(d+ex)^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 293

$$\begin{aligned}\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{(d+ex)^2}\,dx &= \frac{(cd^2+ae^2)\left(3ae^2g-cd(8ef-11dg)\right)\sqrt{f+gx}}{e^6} \\ &- \frac{8cd(cd^2+ae^2)(f+gx)^{3/2}}{3e^5} - \frac{(cd^2+ae^2)^2(f+gx)^{3/2}}{e^5(d+ex)} \\ &+ \frac{2c\left(2ae^2g^2+c(e^2f^2+2defg+3d^2g^2)\right)(f+gx)^{5/2}}{5e^4g^3} \\ &- \frac{4c^2(ef+dg)(f+gx)^{7/2}}{7e^3g^3} + \frac{2c^2(f+gx)^{9/2}}{9e^2g^3} \\ &- \frac{(cd^2+ae^2)\sqrt{ef-dg}\left(3ae^2g-cd(8ef-11dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{13/2}}\end{aligned}$$

output
```
(a*e^2+c*d^2)*(3*a*e^2*g-c*d*(-11*d*g+8*e*f))*(g*x+f)^(1/2)/e^6-8/3*c*d*(a
*e^2+c*d^2)*(g*x+f)^(3/2)/e^5-(a*e^2+c*d^2)^2*(g*x+f)^(3/2)/e^5/(e*x+d)+2/
5*c*(2*a*e^2*g^2+c*(3*d^2*g^2+2*d*e*f*g+e^2*f^2))*(g*x+f)^(5/2)/e^4/g^3-4/
7*c^2*(d*g+e*f)*(g*x+f)^(7/2)/e^3/g^3+2/9*c^2*(g*x+f)^(9/2)/e^2/g^3-(a*e^2
+c*d^2)*(-d*g+e*f)^(1/2)*(3*a*e^2*g-c*d*(-11*d*g+8*e*f))*arctanh(e^(1/2)*(
g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(13/2)
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.20

$$\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{(d+ex)^2}\,dx = \frac{\sqrt{f+gx}(315a^2e^4g^3(-ef+3dg+2egx)+42ace^2g^2(105d^3g^2+6e^3x(f+g$$
$$-\frac{(cd^2+ae^2)\sqrt{-ef+dg}(3ae^2g+cd(-8ef+11dg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{13/2}}$$

input
```
Integrate[((f + g*x)^(3/2)*(a + c*x^2)^2)/(d + e*x)^2,x]
```

output
```
(Sqrt[f + g*x]*(315*a^2*e^4*g^3*(-(e*f) + 3*d*g + 2*e*g*x) + 42*a*c*e^2*g^
2*(105*d^3*g^2 + 6*e^3*x*(f + g*x)^2 + 5*d^2*e*g*(-19*f + 14*g*x) + 2*d*e^
2*(3*f^2 - 34*f*g*x - 7*g^2*x^2)) + c^2*(3465*d^5*g^4 + 18*d^2*e^3*g*(f +
g*x)^2*(4*f + 11*g*x) + 105*d^4*e*g^3*(-35*f + 22*g*x) + 2*d*e^4*(f + g*x)
^2*(8*f^2 + 16*f*g*x - 55*g^2*x^2) - 42*d^3*e^2*g^2*(-9*f^2 + 62*f*g*x + 1
1*g^2*x^2) + 2*e^5*x*(f + g*x)^2*(8*f^2 - 20*f*g*x + 35*g^2*x^2))))/(315*e
^6*g^3*(d + e*x)) - ((c*d^2 + a*e^2)*Sqrt[-(e*f) + d*g]*(3*a*e^2*g + c*d*(
-8*e*f + 11*d*g))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/e^(1
3/2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {649, 1580, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2(f+gx)^{3/2}}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{649}$$

$$\frac{2\int \frac{(f+gx)^2(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)^2}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{g^3}$$

$$\downarrow \text{1580}$$

$$\frac{2\left(\frac{g^4\sqrt{f+gx}(ae^2+cd^2)^2(ef-dg)}{2e^6(-dg-e(f+gx)+ef)}-\frac{\int \frac{2c^2e^5(f+gx)^5-2c^2e^4(3ef+dg)(f+gx)^4+2ce^3\left(2ae^2g^2+c\left(3e^2f^2+2degf+d^2g^2\right)\right)(f+gx)^3-2ce^2(ef+dg)\left(ce^2f^2+\right.}{ef-dg-e(f+gx)}}{2e^6}\right.}{g^3}$$

$$\downarrow \text{2341}$$

$$\frac{2\left(\frac{g^4\sqrt{f+gx}(ae^2+cd^2)^2(ef-dg)}{2e^6(-dg-e(f+gx)+ef)}-\frac{\int\left(-2c^2e^4(f+gx)^4+4c^2e^3(ef+dg)(f+gx)^3-2ce^2(2ae^2g^2+c(e^2f^2+2degf+3d^2g^2))(f+gx)^2+8cde(cd^2\right.}{}\right.}{}$$

$$\downarrow \text{2009}$$

$$\frac{2\left(\frac{g^4\sqrt{f+gx}(ae^2+cd^2)^2(ef-dg)}{2e^6(-dg-e(f+gx)+ef)}-\frac{\frac{g^3(ae^2+cd^2)\sqrt{ef-dg}\left(3ae^2g-cd(8ef-11dg)\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{e}}-\frac{2}{5}ce^2(f+gx)^{5/2}(2ae^2g^2+c(3d^2g^2+2d\right.}{}\right.}{g^3}$$

input
```
Int[((f + g*x)^(3/2)*(a + c*x^2)^2)/(d + e*x)^2,x]
```

output
```
(2*(((c*d^2 + a*e^2)^2*g^4*(e*f - d*g)*Sqrt[f + g*x])/(2*e^6*(e*f - d*g -
e*(f + g*x))) - (-2*(c*d^2 + a*e^2)*g^3*(a*e^2*g - c*d*(4*e*f - 5*d*g))*Sq
rt[f + g*x] + (8*c*d*e*(c*d^2 + a*e^2)*g^3*(f + g*x)^(3/2))/3 - (2*c*e^2*(
2*a*e^2*g^2 + c*(e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2))*(f + g*x)^(5/2))/5 + (4
*c^2*e^3*(e*f + d*g)*(f + g*x)^(7/2))/7 - (2*c^2*e^4*(f + g*x)^(9/2))/9 +
((c*d^2 + a*e^2)*g^3*Sqrt[e*f - d*g]*(3*a*e^2*g - c*d*(8*e*f - 11*d*g))*Ar
cTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/Sqrt[e])/(2*e^6)))/g^3
```

Defintions of rubi rules used

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.35

method	result
pseudoelliptic	$3\sqrt{(dg-ef)e}\left(\left(\left(\frac{2}{27}e^5x^5-\frac{22}{189}d\,e^4x^4+\frac{22}{105}d^2e^3x^3-\frac{22}{45}d^3e^2x^2+\frac{22}{9}d^4ex+\frac{11}{3}d^5\right)g^4-\frac{35ef\left(-\frac{4}{147}e^4x^4+\frac{188}{3675}d\,e^3x^3-\frac{156}{1225}\right.}{9}\right.\right.$
risch	$2(35c^2e^4g^4x^4-90c^2d\,e^3g^4x^3+50c^2e^4f\,g^3x^3+126ac\,e^4g^4x^2+189c^2d^2e^2g^4x^2-144c^2d\,e^3f\,g^3x^2+3c^2e^4f^2g^2x^2-420ac$
derivativedivides	$2\left(\frac{c^2(gx+f)^{\frac{9}{2}}e^4}{9}-\frac{2c^2d\,e^3g(gx+f)^{\frac{7}{2}}}{7}-\frac{2c^2e^4f(gx+f)^{\frac{7}{2}}}{7}+\frac{2ac\,e^4g^2(gx+f)^{\frac{5}{2}}}{5}+\frac{3c^2d^2e^2g^2(gx+f)^{\frac{5}{2}}}{5}+\frac{2c^2d\,e^3fg(gx+f)^{\frac{5}{2}}}{5}+\frac{c^2e^4f}{}\right.$
default	$2\left(\frac{c^2(gx+f)^{\frac{9}{2}}e^4}{9}-\frac{2c^2d\,e^3g(gx+f)^{\frac{7}{2}}}{7}-\frac{2c^2e^4f(gx+f)^{\frac{7}{2}}}{7}+\frac{2ac\,e^4g^2(gx+f)^{\frac{5}{2}}}{5}+\frac{3c^2d^2e^2g^2(gx+f)^{\frac{5}{2}}}{5}+\frac{2c^2d\,e^3fg(gx+f)^{\frac{5}{2}}}{5}+\frac{c^2e^4f}{}\right.$

input
```
int((g*x+f)^(3/2)*(c*x^2+a)^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
3*(((d*g-e*f)*e)^(1/2)*(((2/27*e^5*x^5-22/189*d*e^4*x^4+22/105*d^2*e^3*x^3
-22/45*d^3*e^2*x^2+22/9*d^4*e*x+11/3*d^5)*g^4-35/9*e*f*(-4/147*e^4*x^4+188
/3675*d*e^3*x^3-156/1225*d^2*e^2*x^2+124/175*d^3*e*x+d^4)*g^3+2/5*e^2*f^2*
(1/63*e^2*x^2-2/21*d*e*x+d^2)*(e*x+d)*g^2+8/105*(-1/9*e*x+d)*e^3*f^3*(e*x+
d)*g+16/945*e^4*f^4*(e*x+d))*c^2+14/3*e^2*g^2*((2/35*e^3*x^3-2/15*d*e^2*x^
2+2/3*d^2*e*x+d^3)*g^2-19/21*e*f*(-12/95*e^2*x^2+68/95*d*e*x+d^2)*g+2/35*e
^2*f^2*(e*x+d))*a*c+e^4*g^3*((2/3*e*x+d)*g-1/3*e*f)*a^2)*(g*x+f)^(1/2)-(a*
e^2+c*d^2)*g^3*((11/3*d^2*g-8/3*d*e*f)*c+a*e^2*g)*arctan(e*(g*x+f)^(1/2)/(
(d*g-e*f)*e)^(1/2))*(e*x+d)*(d*g-e*f))/((d*g-e*f)*e)^(1/2)/g^3/e^6/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(263) = 526$.

Time = 0.14 (sec) , antiderivative size = 1255, normalized size of antiderivative = 4.28

$$\int \frac{(f+gx)^{3/2}\left(a+cx^2\right)^2}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)^2/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[-1/630*(315*(8*(c^2*d^4*e + a*c*d^2*e^3)*f*g^3 - (11*c^2*d^5 + 14*a*c*d^3
*e^2 + 3*a^2*d*e^4)*g^4 + (8*(c^2*d^3*e^2 + a*c*d*e^4)*f*g^3 - (11*c^2*d^4
*e + 14*a*c*d^2*e^3 + 3*a^2*e^5)*g^4)*x)*sqrt((e*f - d*g)/e)*log((e*g*x +
2*e*f - d*g - 2*sqrt(g*x + f)*e*sqrt((e*f - d*g)/e))/(e*x + d)) - 2*(70*c^
2*e^5*g^4*x^5 + 16*c^2*d*e^4*f^4 + 72*c^2*d^2*e^3*f^3*g + 126*(3*c^2*d^3*e
^2 + 2*a*c*d*e^4)*f^2*g^2 - 105*(35*c^2*d^4*e + 38*a*c*d^2*e^3 + 3*a^2*e^5
)*f*g^3 + 315*(11*c^2*d^5 + 14*a*c*d^3*e^2 + 3*a^2*d*e^4)*g^4 + 10*(10*c^2
*e^5*f*g^3 - 11*c^2*d*e^4*g^4)*x^4 + 2*(3*c^2*e^5*f^2*g^2 - 94*c^2*d*e^4*f
*g^3 + 9*(11*c^2*d^2*e^3 + 14*a*c*e^5)*g^4)*x^3 - 2*(4*c^2*e^5*f^3*g + 15*
c^2*d*e^4*f^2*g^2 - 18*(13*c^2*d^2*e^3 + 14*a*c*e^5)*f*g^3 + 21*(11*c^2*d^
3*e^2 + 14*a*c*d*e^4)*g^4)*x^2 + 2*(8*c^2*e^5*f^4 + 32*c^2*d*e^4*f^3*g + 9
*(19*c^2*d^2*e^3 + 14*a*c*e^5)*f^2*g^2 - 42*(31*c^2*d^3*e^2 + 34*a*c*d*e^4
)*f*g^3 + 105*(11*c^2*d^4*e + 14*a*c*d^2*e^3 + 3*a^2*e^5)*g^4)*x)*sqrt(g*x
 + f))/(e^7*g^3*x + d*e^6*g^3), 1/315*(315*(8*(c^2*d^4*e + a*c*d^2*e^3)*f*
g^3 - (11*c^2*d^5 + 14*a*c*d^3*e^2 + 3*a^2*d*e^4)*g^4 + (8*(c^2*d^3*e^2 +
a*c*d*e^4)*f*g^3 - (11*c^2*d^4*e + 14*a*c*d^2*e^3 + 3*a^2*e^5)*g^4)*x)*sqr
t(-(e*f - d*g)/e)*arctan(-sqrt(g*x + f)*e*sqrt(-(e*f - d*g)/e)/(e*f - d*g)
) + (70*c^2*e^5*g^4*x^5 + 16*c^2*d*e^4*f^4 + 72*c^2*d^2*e^3*f^3*g + 126*(3
*c^2*d^3*e^2 + 2*a*c*d*e^4)*f^2*g^2 - 105*(35*c^2*d^4*e + 38*a*c*d^2*e^3 +
 3*a^2*e^5)*f*g^3 + 315*(11*c^2*d^5 + 14*a*c*d^3*e^2 + 3*a^2*d*e^4)*g^4...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**(3/2)*(c*x**2+a)**2/(e*x+d)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)^2/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(263) = 526.

Time = 0.13 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.88

$$\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{(d+ex)^2}\,dx =$$

$$-\frac{(8\,c^2d^3e^2f^2+8\,acde^4f^2-19\,c^2d^4efg-22\,acd^2e^3fg-3\,a^2e^5fg+11\,c^2d^5g^2+14\,acd^3e^2g^2+3\,a^2de^4g^2)\,a}{\sqrt{-e^2f+dege^6}}$$

$$-\frac{\sqrt{gx+f}c^2d^4efg+2\sqrt{gx+f}acd^2e^3fg+\sqrt{gx+f}a^2e^5fg-\sqrt{gx+f}c^2d^5g^2-2\sqrt{gx+f}acd^3e^2g^2-\sqrt{g}}{((gx+f)e-ef+dg)e^6}$$

$$+\frac{2\left(35\,(gx+f)^{\frac{9}{2}}c^2e^{16}g^{24}-90\,(gx+f)^{\frac{7}{2}}c^2e^{16}fg^{24}+63\,(gx+f)^{\frac{5}{2}}c^2e^{16}f^2g^{24}-90\,(gx+f)^{\frac{7}{2}}c^2de^{15}g^{25}+12\right.}{}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)^2/(e*x+d)^2,x, algorithm="giac")
```

output
```
-(8*c^2*d^3*e^2*f^2 + 8*a*c*d*e^4*f^2 - 19*c^2*d^4*e*f*g - 22*a*c*d^2*e^3*
f*g - 3*a^2*e^5*f*g + 11*c^2*d^5*g^2 + 14*a*c*d^3*e^2*g^2 + 3*a^2*d*e^4*g^
2)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e^2*f + d*e*g)*e^6)
 - (sqrt(g*x + f)*c^2*d^4*e*f*g + 2*sqrt(g*x + f)*a*c*d^2*e^3*f*g + sqrt(g
*x + f)*a^2*e^5*f*g - sqrt(g*x + f)*c^2*d^5*g^2 - 2*sqrt(g*x + f)*a*c*d^3*
e^2*g^2 - sqrt(g*x + f)*a^2*d*e^4*g^2)/(((g*x + f)*e - e*f + d*g)*e^6) + 2
/315*(35*(g*x + f)^(9/2)*c^2*e^16*g^24 - 90*(g*x + f)^(7/2)*c^2*e^16*f*g^2
4 + 63*(g*x + f)^(5/2)*c^2*e^16*f^2*g^24 - 90*(g*x + f)^(7/2)*c^2*d*e^15*g
^25 + 126*(g*x + f)^(5/2)*c^2*d*e^15*f*g^25 + 189*(g*x + f)^(5/2)*c^2*d^2*
e^14*g^26 + 126*(g*x + f)^(5/2)*a*c*e^16*g^26 - 420*(g*x + f)^(3/2)*c^2*d^
3*e^13*g^27 - 420*(g*x + f)^(3/2)*a*c*d*e^15*g^27 - 1260*sqrt(g*x + f)*c^2
*d^3*e^13*f*g^27 - 1260*sqrt(g*x + f)*a*c*d*e^15*f*g^27 + 1575*sqrt(g*x +
f)*c^2*d^4*e^12*g^28 + 1890*sqrt(g*x + f)*a*c*d^2*e^14*g^28 + 315*sqrt(g*x
 + f)*a^2*e^16*g^28)/(e^18*g^27)
```

Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 965, normalized size of antiderivative = 3.29

$$\int \frac{(f+gx)^{3/2}\left(a+cx^2\right)^2}{(d+ex)^2}\,dx=\text{Too large to display}$$

input
```
int(((f + g*x)^(3/2)*(a + c*x^2)^2)/(d + e*x)^2,x)
```

output
```
(f + g*x)^(1/2)*((2*(d*g - e*f)*((8*c^2*f^3 + 8*a*c*f*g^2)/(e^2*g^3) - ((d
*g - e*f)^2*((8*c^2*f)/(e^2*g^3) + (4*c^2*(d*g - e*f))/(e^3*g^3)))/e^2 + (
2*(d*g - e*f)*((12*c^2*f^2 + 4*a*c*g^2)/(e^2*g^3) + (2*(d*g - e*f)*((8*c^2
*f)/(e^2*g^3) + (4*c^2*(d*g - e*f))/(e^3*g^3)))/e - (2*c^2*(d*g - e*f)^2)/
(e^4*g^3)))/e))/e - ((d*g - e*f)^2*((12*c^2*f^2 + 4*a*c*g^2)/(e^2*g^3) + (
2*(d*g - e*f)*((8*c^2*f)/(e^2*g^3) + (4*c^2*(d*g - e*f))/(e^3*g^3)))/e - (
2*c^2*(d*g - e*f)^2)/(e^4*g^3)))/e^2 + (2*(a*g^2 + c*f^2)^2)/(e^2*g^3)) -
(f + g*x)^(3/2)*((8*c^2*f^3 + 8*a*c*f*g^2)/(3*e^2*g^3) - ((d*g - e*f)^2*((
8*c^2*f)/(e^2*g^3) + (4*c^2*(d*g - e*f))/(e^3*g^3)))/(3*e^2) + (2*(d*g - e
*f)*((12*c^2*f^2 + 4*a*c*g^2)/(e^2*g^3) + (2*(d*g - e*f)*((8*c^2*f)/(e^2*g
^3) + (4*c^2*(d*g - e*f))/(e^3*g^3)))/e - (2*c^2*(d*g - e*f)^2)/(e^4*g^3))
)/(3*e)) - (f + g*x)^(7/2)*((8*c^2*f)/(7*e^2*g^3) + (4*c^2*(d*g - e*f))/(7
*e^3*g^3)) + (f + g*x)^(5/2)*((12*c^2*f^2 + 4*a*c*g^2)/(5*e^2*g^3) + (2*(d
*g - e*f)*((8*c^2*f)/(e^2*g^3) + (4*c^2*(d*g - e*f))/(e^3*g^3)))/(5*e) - (
2*c^2*(d*g - e*f)^2)/(5*e^4*g^3)) + ((f + g*x)^(1/2)*(c^2*d^5*g^2 + a^2*d*
e^4*g^2 - a^2*e^5*f*g - c^2*d^4*e*f*g + 2*a*c*d^3*e^2*g^2 - 2*a*c*d^2*e^3*
f*g))/(e^7*(f + g*x) - e^7*f + d*e^6*g) + (2*c^2*(f + g*x)^(9/2))/(9*e^2*g
^3) - (atan((e^(1/2)*(f + g*x)^(1/2)*(a*e^2 + c*d^2)*(d*g - e*f)^(1/2)*(3*
a*e^2*g + 11*c*d^2*g - 8*c*d*e*f))/(11*c^2*d^5*g^2 + 3*a^2*d*e^4*g^2 + 8*c
^2*d^3*e^2*f^2 - 3*a^2*e^5*f*g + 8*a*c*d*e^4*f^2 - 19*c^2*d^4*e*f*g + 1...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1156, normalized size of antiderivative = 3.95

$$\int \frac{(f+gx)^{3/2}\,(a+cx^2)^2}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^(3/2)*(c*x^2+a)^2/(e*x+d)^2,x)
```

output

```
( - 945*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g -
 e*f)))*a**2*d*e**4*g**4 - 945*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)
*e)/(sqrt(e)*sqrt(d*g - e*f)))*a**2*e**5*g**4*x - 4410*sqrt(e)*sqrt(d*g -
e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**3*e**2*g**4
+ 2520*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g -
e*f)))*a*c*d**2*e**3*f*g**3 - 4410*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f +
g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**2*e**3*g**4*x + 2520*sqrt(e)*sqr
t(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d*e**4*
f*g**3*x - 3465*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sq
rt(d*g - e*f)))*c**2*d**5*g**4 + 2520*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f
 + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**4*e*f*g**3 - 3465*sqrt(e)*sq
rt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**4*
e*g**4*x + 2520*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sq
rt(d*g - e*f)))*c**2*d**3*e**2*f*g**3*x + 945*sqrt(f + g*x)*a**2*d*e**5*g*
*4 - 315*sqrt(f + g*x)*a**2*e**6*f*g**3 + 630*sqrt(f + g*x)*a**2*e**6*g**4
*x + 4410*sqrt(f + g*x)*a*c*d**3*e**3*g**4 - 3990*sqrt(f + g*x)*a*c*d**2*e
**4*f*g**3 + 2940*sqrt(f + g*x)*a*c*d**2*e**4*g**4*x + 252*sqrt(f + g*x)*a
*c*d*e**5*f**2*g**2 - 2856*sqrt(f + g*x)*a*c*d*e**5*f*g**3*x - 588*sqrt(f
+ g*x)*a*c*d*e**5*g**4*x**2 + 252*sqrt(f + g*x)*a*c*e**6*f**2*g**2*x + 504
*sqrt(f + g*x)*a*c*e**6*f*g**3*x**2 + 252*sqrt(f + g*x)*a*c*e**6*g**4*x...
```

3.76 $\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 338

$$\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{(d+ex)^3}\,dx = \frac{4c(cd^2(3ef-5dg)+ae^2(ef-3dg))\sqrt{f+gx}}{e^6} - \frac{(cd^2+ae^2)(3ae^2g-cd(16ef-19dg))\sqrt{f+gx}}{4e^6(d+ex)} + \frac{4c(3cd^2+ae^2)(f+gx)^{3/2}}{3e^5} - \frac{(cd^2+ae^2)^2(f+gx)^{3/2}}{2e^5(d+ex)^2} - \frac{2c^2(ef+3dg)(f+gx)^{5/2}}{5e^4g^2} + \frac{2c^2(f+gx)^{7/2}}{7e^3g^2} - \frac{(3a^2e^4g^2+3c^2d^2(16e^2f^2-48defg+33d^2g^2)+2ace^2(8e^2f^2-40defg+35d^2g^2))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{13/2}\sqrt{ef-dg}}$$

output

```
4*c*(c*d^2*(-5*d*g+3*e*f)+a*e^2*(-3*d*g+e*f))*(g*x+f)^(1/2)/e^6-1/4*(a*e^2
+c*d^2)*(3*a*e^2*g-c*d*(-19*d*g+16*e*f))*(g*x+f)^(1/2)/e^6/(e*x+d)+4/3*c*(
a*e^2+3*c*d^2)*(g*x+f)^(3/2)/e^5-1/2*(a*e^2+c*d^2)^2*(g*x+f)^(3/2)/e^5/(e*
x+d)^2-2/5*c^2*(3*d*g+e*f)*(g*x+f)^(5/2)/e^4/g^2+2/7*c^2*(g*x+f)^(7/2)/e^3
/g^2-1/4*(3*a^2*e^4*g^2+3*c^2*d^2*(33*d^2*g^2-48*d*e*f*g+16*e^2*f^2)+2*a*c
*e^2*(35*d^2*g^2-40*d*e*f*g+8*e^2*f^2))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*
g+e*f)^(1/2))/e^(13/2)/(-d*g+e*f)^(1/2)
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.11

$$\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{(d+ex)^3}\,dx =$$
$$-\frac{\sqrt{f+gx}(105a^2e^4g^2(2ef+3dg+5egx)+70ace^2g^2(105d^3g-25d^2e(2f-7gx)-8e^3x^2(4f+gx)+8de^2\ldots}{}$$
$$+\frac{(3a^2e^4g^2+3c^2d^2(16e^2f^2-48defg+33d^2g^2)+2ace^2(8e^2f^2-40defg+35d^2g^2))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{4e^{13/2}\sqrt{-ef+dg}}$$

input
```
Integrate[((f + g*x)^(3/2)*(a + c*x^2)^2)/(d + e*x)^3,x]
```

output
```
-1/420*(Sqrt[f + g*x]*(105*a^2*e^4*g^2*(2*e*f + 3*d*g + 5*e*g*x) + 70*a*c*
e^2*g^2*(105*d^3*g - 25*d^2*e*(2*f - 7*g*x) - 8*e^3*x^2*(4*f + g*x) + 8*d*
e^2*x*(-11*f + 7*g*x)) + 3*c^2*(3465*d^5*g^3 + 8*e^5*x^2*(2*f - 5*g*x)*(f
+ g*x)^2 + 8*d*e^4*x*(f + g*x)^2*(4*f + 11*g*x) + 105*d^4*e*g^2*(-26*f + 5
5*g*x) + 168*d^3*e^2*g*(f^2 - 28*f*g*x + 11*g^2*x^2) + 8*d^2*e^3*(2*f^3 +
41*f^2*g*x - 204*f*g^2*x^2 - 33*g^3*x^3))))/(e^6*g^2*(d + e*x)^2) + ((3*a^
2*e^4*g^2 + 3*c^2*d^2*(16*e^2*f^2 - 48*d*e*f*g + 33*d^2*g^2) + 2*a*c*e^2*(
8*e^2*f^2 - 40*d*e*f*g + 35*d^2*g^2))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[
-(e*f) + d*g]])/(4*e^(13/2)*Sqrt[-(e*f) + d*g])
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {649, 25, 1580, 2345, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2(f+gx)^{3/2}}{(d+ex)^3}\,dx$$
$$\Big\downarrow \; 649$$

$$\frac{2\int -\frac{(f+gx)^2(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)^2}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}}{g^2}$$

$\downarrow$ 25

$$-\frac{2\int \frac{(f+gx)^2(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)^2}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}}{g^2}$$

$\downarrow$ 1580

$$\frac{2\left(\frac{\int \frac{4c^2e^5(f+gx)^5-4c^2e^4(3ef+dg)(f+gx)^4+4ce^3\left(2ae^2g^2+c\left(3e^2f^2+2degf+d^2g^2\right)\right)(f+gx)^3-4ce^2(ef+dg)\left(ce^2f^2+cd^2g^2+2ae^2g^2\right)(f+gx)^2+4e\left(cd^2+ae^2\right.}{(ef-dg-e(f+gx))^2}}{4e^6}\right.}{g^2}$$

$\downarrow$ 2345

$$\frac{2\left(\frac{\frac{g^3\sqrt{f+gx}\left(ae^2+cd^2\right)\left(5ae^2g-cd(16ef-21dg)\right)}{2(-dg-e(f+gx)+ef)}-\frac{\int \frac{8c^2e^4(ef-dg)(f+gx)^4-16c^2e^3(ef-dg)(ef+dg)(f+gx)^3+8ce^2(ef-dg)\left(2ae^2g^2+c\left(e^2f^2+2degf+3d^2g\right.\right.}{ef-dg-e(}}{2(ef}}{4e^6}\right.}{g^2}$$

$\downarrow$ 2341

$$2\left(\frac{\frac{g^3\sqrt{f+gx}\left(ae^2+cd^2\right)\left(5ae^2g-cd(16ef-21dg)\right)}{2(-dg-e(f+gx)+ef)}-\frac{\int\left(-8c^2e^3(ef-dg)(f+gx)^3+8c^2e^2(ef-dg)(ef+3dg)(f+gx)^2-16ce\left(3cd^2+ae^2\right)g^2(ef-dg)(f+gx)-16cg^2\right.}{}}{}\right.$$

$\downarrow$ 2009

$$2\left(\frac{\frac{g^3\sqrt{f+gx}\left(ae^2+cd^2\right)\left(5ae^2g-cd(16ef-21dg)\right)}{2(-dg-e(f+gx)+ef)}-\frac{\frac{g^2\sqrt{ef-dg}\left(3a^2e^4g^2+2ace^2\left(35d^2g^2-40defg+8e^2f^2\right)+3c^2d^2\left(33d^2g^2-48defg+16e^2f^2\right)\right)\text{arctanh}\left(\right.}{\sqrt{e}}}{}}{}\right.$$

input
```
Int[((f + g*x)^(3/2)*(a + c*x^2)^2)/(d + e*x)^3,x]
```

output
```
(2*(-1/4*((c*d^2 + a*e^2)^2*g^4*(e*f - d*g)*Sqrt[f + g*x])/(e^6*(e*f - d*g
 - e*(f + g*x))^2) + (((c*d^2 + a*e^2)*g^3*(5*a*e^2*g - c*d*(16*e*f - 21*d
*g))*Sqrt[f + g*x])/(2*(e*f - d*g - e*(f + g*x))) - (-16*c*g^2*(e*f - d*g)
*(c*d^2*(3*e*f - 5*d*g) + a*e^2*(e*f - 3*d*g))*Sqrt[f + g*x] - (16*c*e*(3*
c*d^2 + a*e^2)*g^2*(e*f - d*g)*(f + g*x)^(3/2))/3 + (8*c^2*e^2*(e*f - d*g)
*(e*f + 3*d*g)*(f + g*x)^(5/2))/5 - (8*c^2*e^3*(e*f - d*g)*(f + g*x)^(7/2)
)/7 + (g^2*Sqrt[e*f - d*g]*(3*a^2*e^4*g^2 + 3*c^2*d^2*(16*e^2*f^2 - 48*d*e
*f*g + 33*d^2*g^2) + 2*a*c*e^2*(8*e^2*f^2 - 40*d*e*f*g + 35*d^2*g^2))*ArcT
anh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/Sqrt[e])/(2*(e*f - d*g)))/(4
*e^6)))/g^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$-\frac{3\left(\left(\left(33d^5-\frac{8}{21}e^5x^5+55d^4ex+\frac{88}{105}de^4x^4-\frac{88}{35}d^2e^3x^3+\frac{88}{5}d^3e^2x^2\right)g^3-26ef\left(\frac{32}{1365}e^4x^4-\frac{8}{105}de^3x^3+\frac{272}{455}d^2e^2x^2+\frac{112}{65}d^3ex+d^4\right.\right.\right.}{}$
derivativedivides	$-\frac{2c\left(-\frac{c(gx+f)^{\frac{7}{2}}e^3}{7}+\frac{3cde^2g(gx+f)^{\frac{5}{2}}}{5}+\frac{ce^3f(gx+f)^{\frac{5}{2}}}{5}-\frac{2ae^3g^2(gx+f)^{\frac{3}{2}}}{3}-2cd^2eg^2(gx+f)^{\frac{3}{2}}+6ade^2g^3\sqrt{gx+f}-2ae^3fg^2\sqrt{gx\right.}{e^6}$
default	$-\frac{2c\left(-\frac{c(gx+f)^{\frac{7}{2}}e^3}{7}+\frac{3cde^2g(gx+f)^{\frac{5}{2}}}{5}+\frac{ce^3f(gx+f)^{\frac{5}{2}}}{5}-\frac{2ae^3g^2(gx+f)^{\frac{3}{2}}}{3}-2cd^2eg^2(gx+f)^{\frac{3}{2}}+6ade^2g^3\sqrt{gx+f}-2ae^3fg^2\sqrt{gx\right.}{e^6}$
risch	$-\frac{2c(-15x^3ce^3g^3+63cde^2g^3x^2-24ce^3fg^2x^2-70ae^3g^3x-210cd^2eg^3x+126cde^2fg^2x-3ce^3f^2gx+630ade^2g^3-28}{105g^2e^6}$

input
```
int((g*x+f)^(3/2)*(c*x^2+a)^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
3/4*(-(((33*d^5-8/21*e^5*x^5+55*d^4*e*x+88/105*d*e^4*x^4-88/35*d^2*e^3*x^3
+88/5*d^3*e^2*x^2)*g^3-26*e*f*(32/1365*e^4*x^4-8/105*d*e^3*x^3+272/455*d^2
*e^2*x^2+112/65*d^3*e*x+d^4)*g^2+8/5*e^2*(-1/21*e*x+d)*f^2*(e*x+d)^2*g+16/
105*e^3*f^3*(e*x+d)^2)*c^2+70/3*e^2*g^2*a*((-8/105*e^3*x^3+8/15*d*e^2*x^2+
5/3*d^2*e*x+d^3)*g-10/21*e*(16/25*e^2*x^2+44/25*d*e*x+d^2)*f)*c+e^4*g^2*((
5/3*e*x+d)*g+2/3*e*f)*a^2)*((d*g-e*f)*e)^(1/2)*(g*x+f)^(1/2)+g^2*((33*d^4*
g^2-48*d^3*e*f*g+16*d^2*e^2*f^2)*c^2+70/3*e^2*a*(d^2*g^2-8/7*d*e*f*g+8/35*
e^2*f^2)*c+a^2*e^4*g^2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))*(e*x+d
)^2)/((d*g-e*f)*e)^(1/2)/e^6/(e*x+d)^2/g^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 942 vs. $2(304) = 608$.

Time = 0.15 (sec) , antiderivative size = 1897, normalized size of antiderivative = 5.61

$$\int \frac{(f+gx)^{3/2}\left(a+cx^2\right)^2}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)^2/(e*x+d)^3,x, algorithm="fricas")
```

output
```
[1/840*(105*(16*(3*c^2*d^4*e^2 + a*c*d^2*e^4)*f^2*g^2 - 16*(9*c^2*d^5*e +
5*a*c*d^3*e^3)*f*g^3 + (99*c^2*d^6 + 70*a*c*d^4*e^2 + 3*a^2*d^2*e^4)*g^4 +
 (16*(3*c^2*d^2*e^4 + a*c*e^6)*f^2*g^2 - 16*(9*c^2*d^3*e^3 + 5*a*c*d*e^5)*
f*g^3 + (99*c^2*d^4*e^2 + 70*a*c*d^2*e^4 + 3*a^2*e^6)*g^4)*x^2 + 2*(16*(3*
c^2*d^3*e^3 + a*c*d*e^5)*f^2*g^2 - 16*(9*c^2*d^4*e^2 + 5*a*c*d^2*e^4)*f*g^
3 + (99*c^2*d^5*e + 70*a*c*d^3*e^3 + 3*a^2*d*e^5)*g^4)*x)*sqrt(e^2*f - d*e
*g)*log((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x +
 d)) - 2*(48*c^2*d^2*e^5*f^4 + 456*c^2*d^3*e^4*f^3*g - 120*(c^2*e^7*f*g^3
- c^2*d*e^6*g^4)*x^5 - 14*(621*c^2*d^4*e^3 + 250*a*c*d^2*e^5 - 15*a^2*e^7)
*f^2*g^2 + 35*(531*c^2*d^5*e^2 + 310*a*c*d^3*e^4 + 3*a^2*d*e^6)*f*g^3 - 10
5*(99*c^2*d^6*e + 70*a*c*d^4*e^3 + 3*a^2*d^2*e^5)*g^4 - 24*(8*c^2*e^7*f^2*
g^2 - 19*c^2*d*e^6*f*g^3 + 11*c^2*d^2*e^5*g^4)*x^4 - 8*(3*c^2*e^7*f^3*g -
81*c^2*d*e^6*f^2*g^2 + (177*c^2*d^2*e^5 + 70*a*c*e^7)*f*g^3 - (99*c^2*d^3*
e^4 + 70*a*c*d*e^6)*g^4)*x^3 + 8*(6*c^2*e^7*f^4 + 51*c^2*d*e^6*f^3*g - (66
9*c^2*d^2*e^5 + 280*a*c*e^7)*f^2*g^2 + 5*(261*c^2*d^3*e^4 + 154*a*c*d*e^6)
*f*g^3 - 7*(99*c^2*d^4*e^3 + 70*a*c*d^2*e^5)*g^4)*x^2 + (96*c^2*d*e^6*f^4
+ 888*c^2*d^2*e^5*f^3*g - 8*(1887*c^2*d^3*e^4 + 770*a*c*d*e^6)*f^2*g^2 + 7
*(4491*c^2*d^4*e^3 + 2630*a*c*d^2*e^5 + 75*a^2*e^7)*f*g^3 - 175*(99*c^2*d^
5*e^2 + 70*a*c*d^3*e^4 + 3*a^2*d*e^6)*g^4)*x)*sqrt(g*x + f))/(d^2*e^8*f*g^
2 - d^3*e^7*g^3 + (e^10*f*g^2 - d*e^9*g^3)*x^2 + 2*(d*e^9*f*g^2 - d^2*e...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**(3/2)*(c*x**2+a)**2/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)^2/(e*x+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.72

$$\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{(d+ex)^3}\,dx = \frac{(48\,c^2d^2e^2f^2 + 16\,ace^4f^2 - 144\,c^2d^3efg - 80\,acde^3fg + 99\,c^2d^4g^2 + 70\,acd^2}{4\sqrt{-e^2f+dege^6}}$$
$$+ \frac{16\,(gx+f)^{\frac{3}{2}}c^2d^3e^2fg + 16\,(gx+f)^{\frac{3}{2}}acde^4fg - 16\,\sqrt{gx+f}c^2d^3e^2f^2g - 16\,\sqrt{gx+f}acde^4f^2g - 21\,(gx+}{}$$
$$+ \frac{2\left(15\,(gx+f)^{\frac{7}{2}}c^2e^{18}g^{12} - 21\,(gx+f)^{\frac{5}{2}}c^2e^{18}fg^{12} - 63\,(gx+f)^{\frac{5}{2}}c^2de^{17}g^{13} + 210\,(gx+f)^{\frac{3}{2}}c^2d^2e^{16}g^{14} + 7\right.}{}$$

input
```
integrate((g*x+f)^(3/2)*(c*x^2+a)^2/(e*x+d)^3,x, algorithm="giac")
```

output
```
1/4*(48*c^2*d^2*e^2*f^2 + 16*a*c*e^4*f^2 - 144*c^2*d^3*e*f*g - 80*a*c*d*e^
3*f*g + 99*c^2*d^4*g^2 + 70*a*c*d^2*e^2*g^2 + 3*a^2*e^4*g^2)*arctan(sqrt(g
*x + f)*e/sqrt(-e^2*f + d*e*g))/(sqrt(-e^2*f + d*e*g)*e^6) + 1/4*(16*(g*x
+ f)^(3/2)*c^2*d^3*e^2*f*g + 16*(g*x + f)^(3/2)*a*c*d*e^4*f*g - 16*sqrt(g*
x + f)*c^2*d^3*e^2*f^2*g - 16*sqrt(g*x + f)*a*c*d*e^4*f^2*g - 21*(g*x + f)
^(3/2)*c^2*d^4*e*g^2 - 26*(g*x + f)^(3/2)*a*c*d^2*e^3*g^2 - 5*(g*x + f)^(3
/2)*a^2*e^5*g^2 + 35*sqrt(g*x + f)*c^2*d^4*e*f*g^2 + 38*sqrt(g*x + f)*a*c*
d^2*e^3*f*g^2 + 3*sqrt(g*x + f)*a^2*e^5*f*g^2 - 19*sqrt(g*x + f)*c^2*d^5*g
^3 - 22*sqrt(g*x + f)*a*c*d^3*e^2*g^3 - 3*sqrt(g*x + f)*a^2*d*e^4*g^3)/(((
g*x + f)*e - e*f + d*g)^2*e^6) + 2/105*(15*(g*x + f)^(7/2)*c^2*e^18*g^12 -
 21*(g*x + f)^(5/2)*c^2*e^18*f*g^12 - 63*(g*x + f)^(5/2)*c^2*d*e^17*g^13 +
 210*(g*x + f)^(3/2)*c^2*d^2*e^16*g^14 + 70*(g*x + f)^(3/2)*a*c*e^18*g^14
+ 630*sqrt(g*x + f)*c^2*d^2*e^16*f*g^14 + 210*sqrt(g*x + f)*a*c*e^18*f*g^1
4 - 1050*sqrt(g*x + f)*c^2*d^3*e^15*g^15 - 630*sqrt(g*x + f)*a*c*d*e^17*g^
15)/(e^21*g^14)
```

Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 723, normalized size of antiderivative = 2.14

$$\int \frac{(f+gx)^{3/2}(a+cx^2)^2}{(d+ex)^3}\,dx = (f+g\,x)^{3/2}\left(\frac{12\,c^2\,f^2+4\,a\,c\,g^2}{3\,e^3\,g^2}+\frac{(d\,g-e\,f)\,\left(\frac{8\,c^2\,f}{e^3\,g^2}+\frac{6\,c^2\,(d\,g-e\,f)}{e^4\,g^2}\right)}{e}-\frac{2\,c^2\,(d\ \cdots}{e\ \cdots}\right.$$

$$-\sqrt{f+g\,x}\left(\frac{8\,c^2\,f^3+8\,a\,c\,f\,g^2}{e^3\,g^2}-\frac{3\,(d\,g-e\,f)^2\,\left(\frac{8\,c^2\,f}{e^3\,g^2}+\frac{6\,c^2\,(d\,g-e\,f)}{e^4\,g^2}\right)}{e^2}+\frac{3\,(d\,g-e\,f)\,\left(\frac{12\,c^2\,f^2+4\,a\,c\,g^2}{e^3\,g^2}+\frac{3\,(d\,g\ \cdots}{\cdots}\right.}{\cdots}\right.$$

$$-(f+g\,x)^{5/2}\left(\frac{8\,c^2\,f}{5\,e^3\,g^2}+\frac{6\,c^2\,(d\,g-e\,f)}{5\,e^4\,g^2}\right)-\frac{(f+g\,x)^{3/2}\left(\frac{5\,a^2\,e^5\,g^2}{4}+\frac{13\,a\,c\,d^2\,e^3\,g^2}{2}-4\,f\,a\,c\,d\,e^4\,g+\frac{21\,c^2\,d^4\,e\,g^2}{4}\ \cdots\right.}{\cdots}$$

input
```
int(((f + g*x)^(3/2)*(a + c*x^2)^2)/(d + e*x)^3,x)
```

output
```
(f + g*x)^(3/2)*((12*c^2*f^2 + 4*a*c*g^2)/(3*e^3*g^2) + ((d*g - e*f)*((8*c
^2*f)/(e^3*g^2) + (6*c^2*(d*g - e*f))/(e^4*g^2)))/e - (2*c^2*(d*g - e*f)^2
)/(e^5*g^2)) - (f + g*x)^(1/2)*((8*c^2*f^3 + 8*a*c*f*g^2)/(e^3*g^2) - (3*(
d*g - e*f)^2*((8*c^2*f)/(e^3*g^2) + (6*c^2*(d*g - e*f))/(e^4*g^2)))/e^2 +
(3*(d*g - e*f)*((12*c^2*f^2 + 4*a*c*g^2)/(e^3*g^2) + (3*(d*g - e*f)*((8*c^
2*f)/(e^3*g^2) + (6*c^2*(d*g - e*f))/(e^4*g^2)))/e - (6*c^2*(d*g - e*f)^2)
/(e^5*g^2)))/e + (2*c^2*(d*g - e*f)^3)/(e^6*g^2)) - (f + g*x)^(5/2)*((8*c^
2*f)/(5*e^3*g^2) + (6*c^2*(d*g - e*f))/(5*e^4*g^2)) - ((f + g*x)^(3/2)*((5
*a^2*e^5*g^2)/4 + (21*c^2*d^4*e*g^2)/4 + (13*a*c*d^2*e^3*g^2)/2 - 4*c^2*d^
3*e^2*f*g - 4*a*c*d*e^4*f*g) + (f + g*x)^(1/2)*((19*c^2*d^5*g^3)/4 + (3*a^
2*d*e^4*g^3)/4 - (3*a^2*e^5*f*g^2)/4 + (11*a*c*d^3*e^2*g^3)/2 - (35*c^2*d^
4*e*f*g^2)/4 + 4*c^2*d^3*e^2*f^2*g + 4*a*c*d*e^4*f^2*g - (19*a*c*d^2*e^3*f
*g^2)/2))/(e^8*(f + g*x)^2 - (f + g*x)*(2*e^8*f - 2*d*e^7*g) + e^8*f^2 + d
^2*e^6*g^2 - 2*d*e^7*f*g) + (2*c^2*(f + g*x)^(7/2))/(7*e^3*g^2) + (atan((e
^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(3*a^2*e^4*g^2 + 99*c^2*d^4*g^2
 + 48*c^2*d^2*e^2*f^2 + 16*a*c*e^4*f^2 - 144*c^2*d^3*e*f*g + 70*a*c*d^2*e^
2*g^2 - 80*a*c*d*e^3*f*g))/(4*e^(13/2)*(d*g - e*f)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1964, normalized size of antiderivative = 5.81

$$\int \frac{(f+gx)^{3/2}\,(a+cx^2)^2}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^(3/2)*(c*x^2+a)^2/(e*x+d)^3,x)
```

output
```
(315*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*
f)))*a**2*d**2*e**4*g**4 + 630*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)
*e)/(sqrt(e)*sqrt(d*g - e*f)))*a**2*d*e**5*g**4*x + 315*sqrt(e)*sqrt(d*g -
 e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a**2*e**6*g**4*x**
2 + 7350*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g
- e*f)))*a*c*d**4*e**2*g**4 - 8400*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f +
g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**3*e**3*f*g**3 + 14700*sqrt(e)*sq
rt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**3*e
**3*g**4*x + 1680*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*
sqrt(d*g - e*f)))*a*c*d**2*e**4*f**2*g**2 - 16800*sqrt(e)*sqrt(d*g - e*f)*
atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**2*e**4*f*g**3*x +
 7350*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e
*f)))*a*c*d**2*e**4*g**4*x**2 + 3360*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f
+ g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d*e**5*f**2*g**2*x - 8400*sqrt(e)
*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d*e
**5*f*g**3*x**2 + 1680*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqr
t(e)*sqrt(d*g - e*f)))*a*c*e**6*f**2*g**2*x**2 + 10395*sqrt(e)*sqrt(d*g -
e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**6*g**4 - 15
120*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f
)))*c**2*d**5*e*f*g**3 + 20790*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g...
```

3.77 $\int \frac{(d+ex)^3 (a+cx^2)^2}{\sqrt{f+gx}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 472

$$
\begin{aligned}
\int \frac{(d+ex)^3 (a+cx^2)^2}{\sqrt{f+gx}} \, dx = &-\frac{2(ef-dg)^3 (cf^2+ag^2)^2 \sqrt{f+gx}}{g^8} \\
&+ \frac{2(ef-dg)^2 (cf^2+ag^2) (3aeg^2+cf(7ef-4dg)) (f+gx)^{3/2}}{3g^8} \\
&- \frac{2(ef-dg) (3a^2e^2g^4+2acg^2(10e^2f^2-8defg+d^2g^2)+3c^2f^2(7e^2f^2-8defg+2d^2g^2)) (f+gx)^{5/2}}{5g^8} \\
&+ \frac{2(a^2e^3g^4+2aceg^2(10e^2f^2-12defg+3d^2g^2)+c^2f(35e^3f^3-60de^2f^2g+30d^2efg^2-4d^3g^3)) (f+g}{7g^8} \\
&- \frac{2c(2ae^2g^2(5ef-3dg)+c(35e^3f^3-45de^2f^2g+15d^2efg^2-d^3g^3)) (f+gx)^{9/2}}{9g^8} \\
&+ \frac{2ce(2ae^2g^2+3c(7e^2f^2-6defg+d^2g^2)) (f+gx)^{11/2}}{11g^8} \\
&- \frac{2c^2e^2(7ef-3dg)(f+gx)^{13/2}}{13g^8} + \frac{2c^2e^3(f+gx)^{15/2}}{15g^8}
\end{aligned}
$$

output
```
-2*(-d*g+e*f)^3*(a*g^2+c*f^2)^2*(g*x+f)^(1/2)/g^8+2/3*(-d*g+e*f)^2*(a*g^2+
c*f^2)*(3*a*e*g^2+c*f*(-4*d*g+7*e*f))*(g*x+f)^(3/2)/g^8-2/5*(-d*g+e*f)*(3*
a^2*e^2*g^4+2*a*c*g^2*(d^2*g^2-8*d*e*f*g+10*e^2*f^2)+3*c^2*f^2*(2*d^2*g^2-
8*d*e*f*g+7*e^2*f^2))*(g*x+f)^(5/2)/g^8+2/7*(a^2*e^3*g^4+2*a*c*e*g^2*(3*d^
2*g^2-12*d*e*f*g+10*e^2*f^2)+c^2*f*(-4*d^3*g^3+30*d^2*e*f*g^2-60*d*e^2*f^2
*g+35*e^3*f^3))*(g*x+f)^(7/2)/g^8-2/9*c*(2*a*e^2*g^2*(-3*d*g+5*e*f)+c*(-d^
3*g^3+15*d^2*e*f*g^2-45*d*e^2*f^2*g+35*e^3*f^3))*(g*x+f)^(9/2)/g^8+2/11*c*
e*(2*a*e^2*g^2+3*c*(d^2*g^2-6*d*e*f*g+7*e^2*f^2))*(g*x+f)^(11/2)/g^8-2/13*
c^2*e^2*(-3*d*g+7*e*f)*(g*x+f)^(13/2)/g^8+2/15*c^2*e^3*(g*x+f)^(15/2)/g^8
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^3 (a+cx^2)^2}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\sqrt{f+gx}(1287a^2g^4(35d^3g^3+35d^2eg^2(-2f+gx)+7de^2g(8f^2-4fgx+3g^2x^2)+e^3(-16f^3+8f^2gx-$$

input
```
Integrate[((d + e*x)^3*(a + c*x^2)^2)/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(1287*a^2*g^4*(35*d^3*g^3 + 35*d^2*e*g^2*(-2*f + g*x) + 7
*d*e^2*g*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + e^3*(-16*f^3 + 8*f^2*g*x - 6*f*g^
2*x^2 + 5*g^3*x^3)) + 26*a*c*g^2*(231*d^3*g^3*(8*f^2 - 4*f*g*x + 3*g^2*x^2
) + 297*d^2*e*g^2*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + 33*d*e
^2*g*(128*f^4 - 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4) -
 5*e^3*(256*f^5 - 128*f^4*g*x + 96*f^3*g^2*x^2 - 80*f^2*g^3*x^3 + 70*f*g^4
*x^4 - 63*g^5*x^5)) + c^2*(143*d^3*g^3*(128*f^4 - 64*f^3*g*x + 48*f^2*g^2*
x^2 - 40*f*g^3*x^3 + 35*g^4*x^4) + 195*d^2*e*g^2*(-256*f^5 + 128*f^4*g*x -
 96*f^3*g^2*x^2 + 80*f^2*g^3*x^3 - 70*f*g^4*x^4 + 63*g^5*x^5) + 45*d*e^2*g
*(1024*f^6 - 512*f^5*g*x + 384*f^4*g^2*x^2 - 320*f^3*g^3*x^3 + 280*f^2*g^4
*x^4 - 252*f*g^5*x^5 + 231*g^6*x^6) - 7*e^3*(2048*f^7 - 1024*f^6*g*x + 768
*f^5*g^2*x^2 - 640*f^4*g^3*x^3 + 560*f^3*g^4*x^4 - 504*f^2*g^5*x^5 + 462*f
*g^6*x^6 - 429*g^7*x^7))))/(45045*g^8)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2 (d+ex)^3}{\sqrt{f+gx}} \, dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{(f+gx)^{3/2}(ef-dg)\left(-3a^2e^2g^4-2acg^2\left(d^2g^2-8defg+10e^2f^2\right)-3c^2f^2\left(2d^2g^2-8defg+7e^2f^2\right)\right)}{g^7}+\frac{(f$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(f+gx)^{5/2}(ef-dg)\left(3a^2e^2g^4+2acg^2\left(d^2g^2-8defg+10e^2f^2\right)+3c^2f^2\left(2d^2g^2-8defg+7e^2f^2\right)\right)}{5g^8}+\frac{2(f+gx)^{7/2}\left(a^2e^3g^4+2aceg^2\left(3d^2g^2-12defg+10e^2f^2\right)+c^2f\left(-4d^3g^3+30d^2efg^2-60de^2f^2g+35e^3f^3\right)\right)}{7g^8}+\frac{2ce(f+gx)^{11/2}\left(2ae^2g^2+3c\left(d^2g^2-6defg+7e^2f^2\right)\right)}{11g^8}-\frac{2c(f+gx)^{9/2}\left(2ae^2g^2(5ef-3dg)+c\left(-d^3g^3+15d^2efg^2-45de^2f^2g+35e^3f^3\right)\right)}{9g^8}+\frac{2(f+gx)^{3/2}\left(ag^2+cf^2\right)(ef-dg)^2\left(3aeg^2+cf(7ef-4dg)\right)}{3g^8}-\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)^2(ef-dg)^3}{g^8}-\frac{2c^2e^2(f+gx)^{13/2}(7ef-3dg)}{13g^8}+\frac{2c^2e^3(f+gx)^{15/2}}{15g^8}$$

input
```
Int[((d + e*x)^3*(a + c*x^2)^2)/Sqrt[f + g*x],x]
```

output
```
(-2*(e*f - d*g)^3*(c*f^2 + a*g^2)^2*Sqrt[f + g*x])/g^8 + (2*(e*f - d*g)^2*
(c*f^2 + a*g^2)*(3*a*e*g^2 + c*f*(7*e*f - 4*d*g))*(f + g*x)^(3/2))/(3*g^8)
 - (2*(e*f - d*g)*(3*a^2*e^2*g^4 + 2*a*c*g^2*(10*e^2*f^2 - 8*d*e*f*g + d^2
*g^2) + 3*c^2*f^2*(7*e^2*f^2 - 8*d*e*f*g + 2*d^2*g^2))*(f + g*x)^(5/2))/(5
*g^8) + (2*(a^2*e^3*g^4 + 2*a*c*e*g^2*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2
) + c^2*f*(35*e^3*f^3 - 60*d*e^2*f^2*g + 30*d^2*e*f*g^2 - 4*d^3*g^3))*(f +
 g*x)^(7/2))/(7*g^8) - (2*c*(2*a*e^2*g^2*(5*e*f - 3*d*g) + c*(35*e^3*f^3 -
 45*d*e^2*f^2*g + 15*d^2*e*f*g^2 - d^3*g^3))*(f + g*x)^(9/2))/(9*g^8) + (2
*c*e*(2*a*e^2*g^2 + 3*c*(7*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(11/2
))/(11*g^8) - (2*c^2*e^2*(7*e*f - 3*d*g)*(f + g*x)^(13/2))/(13*g^8) + (2*c
^2*e^3*(f + g*x)^(15/2))/(15*g^8)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$\frac{2\sqrt{gx+f}\left(\left(\left(\frac{1}{15}c^2x^7+\frac{1}{7}a^2x^3+\frac{2}{11}acx^5\right)e^3+\frac{3x^2\left(\frac{5}{13}c^2x^4+\frac{10}{9}acx^2+a^2\right)de^2}{5}+d^2x\left(\frac{6}{7}acx^2+\frac{3}{11}c^2x^4+a^2\right)e+d^3\left(\frac{2}{5}acx^2+\right.\right.\right.}{}$
derivativedivides	$\frac{2c^2e^3(gx+f)^{\frac{15}{2}}}{15}+\frac{2\left(3(dg-ef)e^2c^2-4fc^2e^3\right)(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left(3(dg-ef)^2ec^2-12(dg-ef)e^2c^2f+e^3\left(2\left(ag^2+cf^2\right)c+4c^2f^2\right)\right)(gx+}{11}$
default	$\frac{2c^2e^3(gx+f)^{\frac{15}{2}}}{15}+\frac{2\left(3(dg-ef)e^2c^2-4fc^2e^3\right)(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left(3(dg-ef)^2ec^2-12(dg-ef)e^2c^2f+e^3\left(2\left(ag^2+cf^2\right)c+4c^2f^2\right)\right)(gx+}{11}$
gosper	$\frac{2\sqrt{gx+f}\,(3003c^2e^3x^7g^7+10395c^2de^2g^7x^6-3234c^2e^3fg^6x^6+8190ace^3g^7x^5+12285c^2d^2eg^7x^5-11340c^2de^2fg^6x^5+}{}$
trager	$\frac{2\sqrt{gx+f}\,(3003c^2e^3x^7g^7+10395c^2de^2g^7x^6-3234c^2e^3fg^6x^6+8190ace^3g^7x^5+12285c^2d^2eg^7x^5-11340c^2de^2fg^6x^5+}{}$
risch	$\frac{2\sqrt{gx+f}\,(3003c^2e^3x^7g^7+10395c^2de^2g^7x^6-3234c^2e^3fg^6x^6+8190ace^3g^7x^5+12285c^2d^2eg^7x^5-11340c^2de^2fg^6x^5+}{}$
orering	$\frac{2\sqrt{gx+f}\,(3003c^2e^3x^7g^7+10395c^2de^2g^7x^6-3234c^2e^3fg^6x^6+8190ace^3g^7x^5+12285c^2d^2eg^7x^5-11340c^2de^2fg^6x^5+}{}$

input

```
int((e*x+d)^3*(c*x^2+a)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2*(g*x+f)^(1/2)*(((1/15*c^2*x^7+1/7*a^2*x^3+2/11*a*c*x^5)*e^3+3/5*x^2*(5/1
3*c^2*x^4+10/9*a*c*x^2+a^2)*d*e^2+d^2*x*(6/7*a*c*x^2+3/11*c^2*x^4+a^2)*e+d
^3*(2/5*a*c*x^2+a^2+1/9*c^2*x^4))*g^7-2*f*((7/195*c^2*x^6+10/99*a*c*x^4+3/
35*a^2*x^2)*e^3+2/5*(45/143*c^2*x^4+20/21*a*c*x^2+a^2)*x*d*e^2+d^2*(5/33*c
^2*x^4+18/35*a*c*x^2+a^2)*e+4/15*c*x*(5/21*c*x^2+a)*d^3)*g^6+8/5*f^2*((7/1
43*c^2*x^5+100/693*a*c*x^3+1/7*a^2*x)*e^3+d*(25/143*c^2*x^4+4/7*a*c*x^2+a^
2)*e^2+6/7*c*x*d^2*(25/99*c*x^2+a)*e+2/3*c*d^3*(1/7*c*x^2+a))*g^5-16/35*f^
3*((245/1287*c^2*x^4+20/33*a*c*x^2+a^2)*e^3+8/3*(75/286*c*x^2+a)*c*x*d*e^2
+6*(5/33*c*x^2+a)*c*d^2*e+4/9*c^2*d^3*x)*g^4+256/105*(5/33*x*(7/26*c*x^2+a
)*e^3+d*(45/286*c*x^2+a)*e^2+5/22*c*d^2*e*x+1/6*c*d^3)*f^4*c*g^3-512/693*e
*f^5*c*((21/130*c*x^2+a)*e^2+9/13*c*d*x*e+3/2*c*d^2)*g^2+1024/1001*e^2*f^6
*c^2*(7/45*e*x+d)*g-2048/6435*c^2*e^3*f^7)/g^8
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex)^3 (a+cx^2)^2}{\sqrt{f+gx}} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/45045*(3003*c^2*e^3*g^7*x^7 - 14336*c^2*e^3*f^7 + 46080*c^2*d*e^2*f^6*g
- 90090*a^2*d^2*e*f*g^6 + 45045*a^2*d^3*g^7 - 16640*(3*c^2*d^2*e + 2*a*c*e
^3)*f^5*g^2 + 18304*(c^2*d^3 + 6*a*c*d*e^2)*f^4*g^3 - 20592*(6*a*c*d^2*e +
 a^2*e^3)*f^3*g^4 + 24024*(2*a*c*d^3 + 3*a^2*d*e^2)*f^2*g^5 - 231*(14*c^2*
e^3*f*g^6 - 45*c^2*d*e^2*g^7)*x^6 + 63*(56*c^2*e^3*f^2*g^5 - 180*c^2*d*e^2
*f*g^6 + 65*(3*c^2*d^2*e + 2*a*c*e^3)*g^7)*x^5 - 35*(112*c^2*e^3*f^3*g^4 -
 360*c^2*d*e^2*f^2*g^5 + 130*(3*c^2*d^2*e + 2*a*c*e^3)*f*g^6 - 143*(c^2*d^
3 + 6*a*c*d*e^2)*g^7)*x^4 + 5*(896*c^2*e^3*f^4*g^3 - 2880*c^2*d*e^2*f^3*g^
4 + 1040*(3*c^2*d^2*e + 2*a*c*e^3)*f^2*g^5 - 1144*(c^2*d^3 + 6*a*c*d*e^2)*
f*g^6 + 1287*(6*a*c*d^2*e + a^2*e^3)*g^7)*x^3 - 3*(1792*c^2*e^3*f^5*g^2 -
5760*c^2*d*e^2*f^4*g^3 + 2080*(3*c^2*d^2*e + 2*a*c*e^3)*f^3*g^4 - 2288*(c^
2*d^3 + 6*a*c*d*e^2)*f^2*g^5 + 2574*(6*a*c*d^2*e + a^2*e^3)*f*g^6 - 3003*(
2*a*c*d^3 + 3*a^2*d*e^2)*g^7)*x^2 + (7168*c^2*e^3*f^6*g - 23040*c^2*d*e^2*
f^5*g^2 + 45045*a^2*d^2*e*g^7 + 8320*(3*c^2*d^2*e + 2*a*c*e^3)*f^4*g^3 - 9
152*(c^2*d^3 + 6*a*c*d*e^2)*f^3*g^4 + 10296*(6*a*c*d^2*e + a^2*e^3)*f^2*g^
5 - 12012*(2*a*c*d^3 + 3*a^2*d*e^2)*f*g^6)*x)*sqrt(g*x + f)/g^8
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. 2(493) = 986.

Time = 1.42 (sec) , antiderivative size = 1030, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex)^3 (a+cx^2)^2}{\sqrt{f+gx}} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(c*x**2+a)**2/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c**2*e**3*(f + g*x)**(15/2)/(15*g**7) + (f + g*x)**(13/2)*(3
*c**2*d*e**2*g - 7*c**2*e**3*f)/(13*g**7) + (f + g*x)**(11/2)*(2*a*c*e**3*
g**2 + 3*c**2*d**2*e*g**2 - 18*c**2*d*e**2*f*g + 21*c**2*e**3*f**2)/(11*g*
*7) + (f + g*x)**(9/2)*(6*a*c*d*e**2*g**3 - 10*a*c*e**3*f*g**2 + c**2*d**3
*g**3 - 15*c**2*d**2*e*f*g**2 + 45*c**2*d*e**2*f**2*g - 35*c**2*e**3*f**3)
/(9*g**7) + (f + g*x)**(7/2)*(a**2*e**3*g**4 + 6*a*c*d**2*e*g**4 - 24*a*c*
d*e**2*f*g**3 + 20*a*c*e**3*f**2*g**2 - 4*c**2*d**3*f*g**3 + 30*c**2*d**2*
e*f**2*g**2 - 60*c**2*d*e**2*f**3*g + 35*c**2*e**3*f**4)/(7*g**7) + (f + g
*x)**(5/2)*(3*a**2*d*e**2*g**5 - 3*a**2*e**3*f*g**4 + 2*a*c*d**3*g**5 - 18
*a*c*d**2*e*f*g**4 + 36*a*c*d*e**2*f**2*g**3 - 20*a*c*e**3*f**3*g**2 + 6*c
**2*d**3*f**2*g**3 - 30*c**2*d**2*e*f**3*g**2 + 45*c**2*d*e**2*f**4*g - 21
*c**2*e**3*f**5)/(5*g**7) + (f + g*x)**(3/2)*(3*a**2*d**2*e*g**6 - 6*a**2*
d*e**2*f*g**5 + 3*a**2*e**3*f**2*g**4 - 4*a*c*d**3*f*g**5 + 18*a*c*d**2*e*
f**2*g**4 - 24*a*c*d*e**2*f**3*g**3 + 10*a*c*e**3*f**4*g**2 - 4*c**2*d**3*
f**3*g**3 + 15*c**2*d**2*e*f**4*g**2 - 18*c**2*d*e**2*f**5*g + 7*c**2*e**3
*f**6)/(3*g**7) + sqrt(f + g*x)*(a**2*d**3*g**7 - 3*a**2*d**2*e*f*g**6 + 3
*a**2*d*e**2*f**2*g**5 - a**2*e**3*f**3*g**4 + 2*a*c*d**3*f**2*g**5 - 6*a*
c*d**2*e*f**3*g**4 + 6*a*c*d*e**2*f**4*g**3 - 2*a*c*e**3*f**5*g**2 + c**2*
d**3*f**4*g**3 - 3*c**2*d**2*e*f**5*g**2 + 3*c**2*d*e**2*f**6*g - c**2*e**
3*f**7)/g**7)/g, Ne(g, 0)), ((a**2*d**3*x + 3*a**2*d**2*e*x**2/2 + 3*c*...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^3\left(a+cx^2\right)^2}{\sqrt{f+gx}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/45045*(3003*(g*x + f)^(15/2)*c^2*e^3 - 3465*(7*c^2*e^3*f - 3*c^2*d*e^2*g
)*(g*x + f)^(13/2) + 4095*(21*c^2*e^3*f^2 - 18*c^2*d*e^2*f*g + (3*c^2*d^2*
e + 2*a*c*e^3)*g^2)*(g*x + f)^(11/2) - 5005*(35*c^2*e^3*f^3 - 45*c^2*d*e^2
*f^2*g + 5*(3*c^2*d^2*e + 2*a*c*e^3)*f*g^2 - (c^2*d^3 + 6*a*c*d*e^2)*g^3)*
(g*x + f)^(9/2) + 6435*(35*c^2*e^3*f^4 - 60*c^2*d*e^2*f^3*g + 10*(3*c^2*d^
2*e + 2*a*c*e^3)*f^2*g^2 - 4*(c^2*d^3 + 6*a*c*d*e^2)*f*g^3 + (6*a*c*d^2*e
+ a^2*e^3)*g^4)*(g*x + f)^(7/2) - 9009*(21*c^2*e^3*f^5 - 45*c^2*d*e^2*f^4*
g + 10*(3*c^2*d^2*e + 2*a*c*e^3)*f^3*g^2 - 6*(c^2*d^3 + 6*a*c*d*e^2)*f^2*g
^3 + 3*(6*a*c*d^2*e + a^2*e^3)*f*g^4 - (2*a*c*d^3 + 3*a^2*d*e^2)*g^5)*(g*x
 + f)^(5/2) + 15015*(7*c^2*e^3*f^6 - 18*c^2*d*e^2*f^5*g + 3*a^2*d^2*e*g^6
+ 5*(3*c^2*d^2*e + 2*a*c*e^3)*f^4*g^2 - 4*(c^2*d^3 + 6*a*c*d*e^2)*f^3*g^3
+ 3*(6*a*c*d^2*e + a^2*e^3)*f^2*g^4 - 2*(2*a*c*d^3 + 3*a^2*d*e^2)*f*g^5)*(
g*x + f)^(3/2) - 45045*(c^2*e^3*f^7 - 3*c^2*d*e^2*f^6*g + 3*a^2*d^2*e*f*g^
6 - a^2*d^3*g^7 + (3*c^2*d^2*e + 2*a*c*e^3)*f^5*g^2 - (c^2*d^3 + 6*a*c*d*e
^2)*f^4*g^3 + (6*a*c*d^2*e + a^2*e^3)*f^3*g^4 - (2*a*c*d^3 + 3*a^2*d*e^2)*
f^2*g^5)*sqrt(g*x + f))/g^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.56

$$\int \frac{(d+ex)^3 (a+cx^2)^2}{\sqrt{f+gx}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

output

```
2/45045*(45045*sqrt(g*x + f)*a^2*d^3 + 45045*((g*x + f)^(3/2) - 3*sqrt(g*x
 + f)*f)*a^2*d^2*e/g + 6006*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15
*sqrt(g*x + f)*f^2)*a*c*d^3/g^2 + 9009*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(
3/2)*f + 15*sqrt(g*x + f)*f^2)*a^2*d*e^2/g^2 + 7722*(5*(g*x + f)^(7/2) - 2
1*(g*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*c*d
^2*e/g^3 + 1287*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(g*x + f)^(
3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a^2*e^3/g^3 + 143*(35*(g*x + f)^(9/2) - 1
80*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 +
 315*sqrt(g*x + f)*f^4)*c^2*d^3/g^4 + 858*(35*(g*x + f)^(9/2) - 180*(g*x +
 f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt
(g*x + f)*f^4)*a*c*d*e^2/g^4 + 195*(63*(g*x + f)^(11/2) - 385*(g*x + f)^(9
/2)*f + 990*(g*x + f)^(7/2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 1155*(g*x + f
)^(3/2)*f^4 - 693*sqrt(g*x + f)*f^5)*c^2*d^2*e/g^5 + 130*(63*(g*x + f)^(11
/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x + f)^(7/2)*f^2 - 1386*(g*x + f)^(5/
2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*sqrt(g*x + f)*f^5)*a*c*e^3/g^5 + 4
5*(231*(g*x + f)^(13/2) - 1638*(g*x + f)^(11/2)*f + 5005*(g*x + f)^(9/2)*f
^2 - 8580*(g*x + f)^(7/2)*f^3 + 9009*(g*x + f)^(5/2)*f^4 - 6006*(g*x + f)^
(3/2)*f^5 + 3003*sqrt(g*x + f)*f^6)*c^2*d*e^2/g^6 + 7*(429*(g*x + f)^(15/2
) - 3465*(g*x + f)^(13/2)*f + 12285*(g*x + f)^(11/2)*f^2 - 25025*(g*x + f)
^(9/2)*f^3 + 32175*(g*x + f)^(7/2)*f^4 - 27027*(g*x + f)^(5/2)*f^5 + 15...
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^3(a+cx^2)^2}{\sqrt{f+gx}}\,dx$$
$$= \frac{(f+g\,x)^{9/2}\,(2\,c^2\,d^3\,g^3-30\,c^2\,d^2\,e\,f\,g^2+90\,c^2\,d\,e^2\,f^2\,g-70\,c^2\,e^3\,f^3+12\,a\,c\,d\,e^2\,g^3-20\,a\,c\,e^3\,f\,g^2)}{9\,g^8}$$
$$+\frac{(f+g\,x)^{7/2}\,(2\,a^2\,e^3\,g^4+12\,a\,c\,d^2\,e\,g^4-48\,a\,c\,d\,e^2\,f\,g^3+40\,a\,c\,e^3\,f^2\,g^2-8\,c^2\,d^3\,f\,g^3+60\,c^2\,d^2\,e\,f^2\,g^2+}{7\,g^8}$$
$$+\frac{2\,c^2\,e^3\,(f+g\,x)^{15/2}}{15\,g^8}+\frac{2\,\sqrt{f+g\,x}\,(c\,f^2+a\,g^2)^2\,(d\,g-e\,f)^3}{g^8}$$
$$+\frac{2\,(f+g\,x)^{5/2}\,(d\,g-e\,f)\,\left(3\,a^2\,e^2\,g^4+2\,a\,c\,d^2\,g^4-16\,a\,c\,d\,e\,f\,g^3+20\,a\,c\,e^2\,f^2\,g^2+6\,c^2\,d^2\,f^2\,g^2-24\,c^2\right.}{5\,g^8}$$
$$+\frac{2\,(f+g\,x)^{3/2}\,(c\,f^2+a\,g^2)\,(d\,g-e\,f)^2\,(7\,c\,e\,f^2-4\,c\,d\,f\,g+3\,a\,e\,g^2)}{3\,g^8}$$
$$+\frac{2\,c^2\,e^2\,(f+g\,x)^{13/2}\,(3\,d\,g-7\,e\,f)}{13\,g^8}$$
$$+\frac{2\,c\,e\,(f+g\,x)^{11/2}\,(3\,c\,d^2\,g^2-18\,c\,d\,e\,f\,g+21\,c\,e^2\,f^2+2\,a\,e^2\,g^2)}{11\,g^8}$$

input
```
int(((a + c*x^2)^2*(d + e*x)^3)/(f + g*x)^(1/2),x)
```

output
```
((f + g*x)^(9/2)*(2*c^2*d^3*g^3 - 70*c^2*e^3*f^3 + 12*a*c*d*e^2*g^3 - 20*a
*c*e^3*f*g^2 + 90*c^2*d*e^2*f^2*g - 30*c^2*d^2*e*f*g^2))/(9*g^8) + ((f + g
*x)^(7/2)*(2*a^2*e^3*g^4 + 70*c^2*e^3*f^4 - 8*c^2*d^3*f*g^3 + 12*a*c*d^2*e
*g^4 + 40*a*c*e^3*f^2*g^2 - 120*c^2*d*e^2*f^3*g + 60*c^2*d^2*e*f^2*g^2 - 4
8*a*c*d*e^2*f*g^3))/(7*g^8) + (2*c^2*e^3*(f + g*x)^(15/2))/(15*g^8) + (2*(
f + g*x)^(1/2)*(a*g^2 + c*f^2)^2*(d*g - e*f)^3)/g^8 + (2*(f + g*x)^(5/2)*(
d*g - e*f)*(3*a^2*e^2*g^4 + 21*c^2*e^2*f^4 + 6*c^2*d^2*f^2*g^2 + 2*a*c*d^2
*g^4 - 24*c^2*d*e*f^3*g + 20*a*c*e^2*f^2*g^2 - 16*a*c*d*e*f*g^3))/(5*g^8)
+ (2*(f + g*x)^(3/2)*(a*g^2 + c*f^2)*(d*g - e*f)^2*(3*a*e*g^2 + 7*c*e*f^2
- 4*c*d*f*g))/(3*g^8) + (2*c^2*e^2*(f + g*x)^(13/2)*(3*d*g - 7*e*f))/(13*g
^8) + (2*c*e*(f + g*x)^(11/2)*(2*a*e^2*g^2 + 3*c*d^2*g^2 + 21*c*e^2*f^2 -
18*c*d*e*f*g))/(11*g^8)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 816, normalized size of antiderivative = 1.73

$$\int \frac{(d+ex)^3 (a+cx^2)^2}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\sqrt{gx+f}\,(3003c^2e^3g^7x^7 + 10395c^2d\,e^2g^7x^6 - 3234c^2e^3f\,g^6x^6 + 8190ac\,e^3g^7x^5 + 12285c^2d^2e\,g^7x^5 - 113}{}$$

input
```
int((e*x+d)^3*(c*x^2+a)^2/(g*x+f)^(1/2),x)
```

output
```
(2*sqrt(f + g*x)*(45045*a**2*d**3*g**7 - 90090*a**2*d**2*e*f*g**6 + 45045*
a**2*d**2*e*g**7*x + 72072*a**2*d*e**2*f**2*g**5 - 36036*a**2*d*e**2*f*g**
6*x + 27027*a**2*d*e**2*g**7*x**2 - 20592*a**2*e**3*f**3*g**4 + 10296*a**2
*e**3*f**2*g**5*x - 7722*a**2*e**3*f*g**6*x**2 + 6435*a**2*e**3*g**7*x**3
+ 48048*a*c*d**3*f**2*g**5 - 24024*a*c*d**3*f*g**6*x + 18018*a*c*d**3*g**7
*x**2 - 123552*a*c*d**2*e*f**3*g**4 + 61776*a*c*d**2*e*f**2*g**5*x - 46332
*a*c*d**2*e*f*g**6*x**2 + 38610*a*c*d**2*e*g**7*x**3 + 109824*a*c*d*e**2*f
**4*g**3 - 54912*a*c*d*e**2*f**3*g**4*x + 41184*a*c*d*e**2*f**2*g**5*x**2
- 34320*a*c*d*e**2*f*g**6*x**3 + 30030*a*c*d*e**2*g**7*x**4 - 33280*a*c*e*
*3*f**5*g**2 + 16640*a*c*e**3*f**4*g**3*x - 12480*a*c*e**3*f**3*g**4*x**2
+ 10400*a*c*e**3*f**2*g**5*x**3 - 9100*a*c*e**3*f*g**6*x**4 + 8190*a*c*e**
3*g**7*x**5 + 18304*c**2*d**3*f**4*g**3 - 9152*c**2*d**3*f**3*g**4*x + 686
4*c**2*d**3*f**2*g**5*x**2 - 5720*c**2*d**3*f*g**6*x**3 + 5005*c**2*d**3*g
**7*x**4 - 49920*c**2*d**2*e*f**5*g**2 + 24960*c**2*d**2*e*f**4*g**3*x - 1
8720*c**2*d**2*e*f**3*g**4*x**2 + 15600*c**2*d**2*e*f**2*g**5*x**3 - 13650
*c**2*d**2*e*f*g**6*x**4 + 12285*c**2*d**2*e*g**7*x**5 + 46080*c**2*d*e**2
*f**6*g - 23040*c**2*d*e**2*f**5*g**2*x + 17280*c**2*d*e**2*f**4*g**3*x**2
 - 14400*c**2*d*e**2*f**3*g**4*x**3 + 12600*c**2*d*e**2*f**2*g**5*x**4 - 1
1340*c**2*d*e**2*f*g**6*x**5 + 10395*c**2*d*e**2*g**7*x**6 - 14336*c**2*e*
*3*f**7 + 7168*c**2*e**3*f**6*g*x - 5376*c**2*e**3*f**5*g**2*x**2 + 448...
```

3.78 $\int \frac{(d+ex)^2(a+cx^2)^2}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 338

$$\begin{aligned}\int \frac{(d+ex)^2\,(a+cx^2)^2}{\sqrt{f+gx}}\,dx &= \frac{2(ef-dg)^2\,(cf^2+ag^2)^2\sqrt{f+gx}}{g^7}\\ &- \frac{4(ef-dg)\,(cf^2+ag^2)\,(aeg^2+cf(3ef-2dg))\,(f+gx)^{3/2}}{3g^7}\\ &+ \frac{2(a^2e^2g^4+2acg^2(6e^2f^2-6defg+d^2g^2)+c^2f^2(15e^2f^2-20defg+6d^2g^2))\,(f+gx)^{5/2}}{5g^7}\\ &- \frac{8c(aeg^2(2ef-dg)+cf(5e^2f^2-5defg+d^2g^2))\,(f+gx)^{7/2}}{7g^7}\\ &+ \frac{2c(2ae^2g^2+c(15e^2f^2-10defg+d^2g^2))\,(f+gx)^{9/2}}{9g^7}\\ &- \frac{4c^2e(3ef-dg)(f+gx)^{11/2}}{11g^7}+\frac{2c^2e^2(f+gx)^{13/2}}{13g^7}\end{aligned}$$

output
```
2*(-d*g+e*f)^2*(a*g^2+c*f^2)^2*(g*x+f)^(1/2)/g^7-4/3*(-d*g+e*f)*(a*g^2+c*f
^2)*(a*e*g^2+c*f*(-2*d*g+3*e*f))*(g*x+f)^(3/2)/g^7+2/5*(a^2*e^2*g^4+2*a*c*
g^2*(d^2*g^2-6*d*e*f*g+6*e^2*f^2)+c^2*f^2*(6*d^2*g^2-20*d*e*f*g+15*e^2*f^2
))*(g*x+f)^(5/2)/g^7-8/7*c*(a*e*g^2*(-d*g+2*e*f)+c*f*(d^2*g^2-5*d*e*f*g+5*
e^2*f^2))*(g*x+f)^(7/2)/g^7+2/9*c*(2*a*e^2*g^2+c*(d^2*g^2-10*d*e*f*g+15*e^
2*f^2))*(g*x+f)^(9/2)/g^7-4/11*c^2*e*(-d*g+3*e*f)*(g*x+f)^(11/2)/g^7+2/13*
c^2*e^2*(g*x+f)^(13/2)/g^7
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^2 (a+cx^2)^2}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\sqrt{f+gx}(3003a^2g^4(15d^2g^2+10deg(-2f+gx)+e^2(8f^2-4fgx+3g^2x^2))+286acg^2(21d^2g^2(8f^2-4\ldots}{}$$

input
```
Integrate[((d + e*x)^2*(a + c*x^2)^2)/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(3003*a^2*g^4*(15*d^2*g^2 + 10*d*e*g*(-2*f + g*x) + e^2*(
8*f^2 - 4*f*g*x + 3*g^2*x^2)) + 286*a*c*g^2*(21*d^2*g^2*(8*f^2 - 4*f*g*x +
 3*g^2*x^2) + 18*d*e*g*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + e
^2*(128*f^4 - 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4)) +
c^2*(143*d^2*g^2*(128*f^4 - 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 3
5*g^4*x^4) + 130*d*e*g*(-256*f^5 + 128*f^4*g*x - 96*f^3*g^2*x^2 + 80*f^2*g
^3*x^3 - 70*f*g^4*x^4 + 63*g^5*x^5) + 15*e^2*(1024*f^6 - 512*f^5*g*x + 384
*f^4*g^2*x^2 - 320*f^3*g^3*x^3 + 280*f^2*g^4*x^4 - 252*f*g^5*x^5 + 231*g^6
*x^6))))/(45045*g^7)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2 (d+ex)^2}{\sqrt{f+gx}}\,dx$$
$$\Big\downarrow \; 652$$
$$\int \left(\frac{(f+gx)^{3/2}\left(a^2e^2g^4+2acg^2\left(d^2g^2-6defg+6e^2f^2\right)+c^2f^2\left(6d^2g^2-20defg+15e^2f^2\right)\right)}{g^6}+\frac{c(f+gx)^{7/2}(2\ldots}{}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{2(f+gx)^{5/2}\left(a^2e^2g^4+2acg^2\left(d^2g^2-6defg+6e^2f^2\right)+c^2f^2\left(6d^2g^2-20defg+15e^2f^2\right)\right)}{5g^7}+\frac{2c(f+gx)^{9/2}\left(2ae^2g^2+c\left(d^2g^2-10defg+15e^2f^2\right)\right)}{9g^7}-\frac{8c(f+gx)^{7/2}\left(aeg^2(2ef-dg)+cf\left(d^2g^2-5defg+5e^2f^2\right)\right)}{7g^7}-\frac{4(f+gx)^{3/2}\left(ag^2+cf^2\right)(ef-dg)\left(aeg^2+cf(3ef-2dg)\right)}{3g^7}+\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)^2(ef-dg)^2}{g^7}-\frac{4c^2e(f+gx)^{11/2}(3ef-dg)}{11g^7}+\frac{2c^2e^2(f+gx)^{13/2}}{13g^7}$$

input
```
Int[((d + e*x)^2*(a + c*x^2)^2)/Sqrt[f + g*x],x]
```

output
```
(2*(e*f - d*g)^2*(c*f^2 + a*g^2)^2*Sqrt[f + g*x])/g^7 - (4*(e*f - d*g)*(c*
f^2 + a*g^2)*(a*e*g^2 + c*f*(3*e*f - 2*d*g))*(f + g*x)^(3/2))/(3*g^7) + (2
*(a^2*e^2*g^4 + 2*a*c*g^2*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2) + c^2*f^2*(15*
e^2*f^2 - 20*d*e*f*g + 6*d^2*g^2))*(f + g*x)^(5/2))/(5*g^7) - (8*c*(a*e*g^
2*(2*e*f - d*g) + c*f*(5*e^2*f^2 - 5*d*e*f*g + d^2*g^2))*(f + g*x)^(7/2))/
(7*g^7) + (2*c*(2*a*e^2*g^2 + c*(15*e^2*f^2 - 10*d*e*f*g + d^2*g^2))*(f +
g*x)^(9/2))/(9*g^7) - (4*c^2*e*(3*e*f - d*g)*(f + g*x)^(11/2))/(11*g^7) +
(2*c^2*e^2*(f + g*x)^(13/2))/(13*g^7)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{2\sqrt{gx+f}\left(\left(\left(\frac{1}{13}e^{2}x^{6}+\frac{1}{9}d^{2}x^{4}+\frac{2}{11}de\,x^{5}\right)c^{2}+\frac{2x^{2}a\left(d^{2}+\frac{5}{9}e^{2}x^{2}+\frac{10}{7}dex\right)c}{5}+a^{2}\left(\frac{1}{5}e^{2}x^{2}+d^{2}+\frac{2}{3}dex\right)\right)g^{6}-\frac{4\left(\left(\frac{9}{143}e^{2}x^{5}+\frac{5}{33}\right.\right.}{}\right.}{}$
derivativedivides	$\frac{2c^{2}e^{2}(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left(2e(dg-ef)c^{2}-4f\,c^{2}e^{2}\right)(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left((dg-ef)^{2}c^{2}-8e(dg-ef)c^{2}f+e^{2}\left(2\left(a\,g^{2}+c\,f^{2}\right)c+4c^{2}f^{2}\right)\right)(gx+f)^{\frac{9}{2}}}{9}+$
default	$\frac{2c^{2}e^{2}(gx+f)^{\frac{13}{2}}}{13}+\frac{2\left(2e(dg-ef)c^{2}-4f\,c^{2}e^{2}\right)(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left((dg-ef)^{2}c^{2}-8e(dg-ef)c^{2}f+e^{2}\left(2\left(a\,g^{2}+c\,f^{2}\right)c+4c^{2}f^{2}\right)\right)(gx+f)^{\frac{9}{2}}}{9}+$
gosper	$\frac{2\sqrt{gx+f}\,(3465c^{2}e^{2}x^{6}g^{6}+8190c^{2}de\,g^{6}x^{5}-3780c^{2}e^{2}f\,g^{5}x^{5}+10010ac\,e^{2}g^{6}x^{4}+5005c^{2}d^{2}g^{6}x^{4}-9100c^{2}def\,g^{5}x^{4}+4200c^{2}}{}$
trager	$\frac{2\sqrt{gx+f}\,(3465c^{2}e^{2}x^{6}g^{6}+8190c^{2}de\,g^{6}x^{5}-3780c^{2}e^{2}f\,g^{5}x^{5}+10010ac\,e^{2}g^{6}x^{4}+5005c^{2}d^{2}g^{6}x^{4}-9100c^{2}def\,g^{5}x^{4}+4200c^{2}}{}$
risch	$\frac{2\sqrt{gx+f}\,(3465c^{2}e^{2}x^{6}g^{6}+8190c^{2}de\,g^{6}x^{5}-3780c^{2}e^{2}f\,g^{5}x^{5}+10010ac\,e^{2}g^{6}x^{4}+5005c^{2}d^{2}g^{6}x^{4}-9100c^{2}def\,g^{5}x^{4}+4200c^{2}}{}$
orering	$\frac{2\sqrt{gx+f}\,(3465c^{2}e^{2}x^{6}g^{6}+8190c^{2}de\,g^{6}x^{5}-3780c^{2}e^{2}f\,g^{5}x^{5}+10010ac\,e^{2}g^{6}x^{4}+5005c^{2}d^{2}g^{6}x^{4}-9100c^{2}def\,g^{5}x^{4}+4200c^{2}}{}$

input
```
int((e*x+d)^2*(c*x^2+a)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(g*x+f)^(1/2)*(((1/13*e^2*x^6+1/9*d^2*x^4+2/11*d*e*x^5)*c^2+2/5*x^2*a*(d
^2+5/9*e^2*x^2+10/7*d*e*x)*c+a^2*(1/5*e^2*x^2+d^2+2/3*d*e*x))*g^6-4/3*((9/
143*e^2*x^5+5/33*d*e*x^4+2/21*d^2*x^3)*c^2+2/5*(10/21*e^2*x^2+9/7*d*e*x+d^
2)*x*a*c+a^2*e*(1/5*e*x+d))*f*g^5+8/15*f^2*(2/7*(175/286*e^2*x^2+50/33*d*e
*x+d^2)*x^2*c^2+2*(2/7*e^2*x^2+6/7*d*e*x+d^2)*a*c+e^2*a^2)*g^4-64/35*f^3*(
(25/429*e^2*x^3+5/33*d*e*x^2+1/9*d^2*x)*c+e*a*(2/9*e*x+d))*c*g^3+256/315*f
^4*c*((45/286*e^2*x^2+5/11*d*e*x+1/2*d^2)*c+a*e^2)*g^2-512/693*e*f^5*c^2*(
3/13*e*x+d)*g+1024/3003*c^2*e^2*f^6)/g^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex)^2 \left(a+cx^2\right)^2}{\sqrt{f+gx}}\,dx = \frac{2\left(3465\,c^2e^2g^6x^6+15360\,c^2e^2f^6-33280\,c^2def^5g-82368\,acdef^3g^3-60060\,a^2defg^5+45045\,a^2d^2g^6+\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/45045*(3465*c^2*e^2*g^6*x^6 + 15360*c^2*e^2*f^6 - 33280*c^2*d*e*f^5*g -
82368*a*c*d*e*f^3*g^3 - 60060*a^2*d*e*f*g^5 + 45045*a^2*d^2*g^6 + 18304*(c
^2*d^2 + 2*a*c*e^2)*f^4*g^2 + 24024*(2*a*c*d^2 + a^2*e^2)*f^2*g^4 - 630*(6
*c^2*e^2*f*g^5 - 13*c^2*d*e*g^6)*x^5 + 35*(120*c^2*e^2*f^2*g^4 - 260*c^2*d
*e*f*g^5 + 143*(c^2*d^2 + 2*a*c*e^2)*g^6)*x^4 - 20*(240*c^2*e^2*f^3*g^3 -
520*c^2*d*e*f^2*g^4 - 1287*a*c*d*e*g^6 + 286*(c^2*d^2 + 2*a*c*e^2)*f*g^5)*
x^3 + 3*(1920*c^2*e^2*f^4*g^2 - 4160*c^2*d*e*f^3*g^3 - 10296*a*c*d*e*f*g^5
 + 2288*(c^2*d^2 + 2*a*c*e^2)*f^2*g^4 + 3003*(2*a*c*d^2 + a^2*e^2)*g^6)*x^
2 - 2*(3840*c^2*e^2*f^5*g - 8320*c^2*d*e*f^4*g^2 - 20592*a*c*d*e*f^2*g^4 -
 15015*a^2*d*e*g^6 + 4576*(c^2*d^2 + 2*a*c*e^2)*f^3*g^3 + 6006*(2*a*c*d^2
+ a^2*e^2)*f*g^5)*x)*sqrt(g*x + f)/g^7
```

Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex)^2 \left(a+cx^2\right)^2}{\sqrt{f+gx}}\,dx = \begin{cases} \frac{2\left(\frac{c^2e^2(f+gx)^{\frac{13}{2}}}{13g^6}+\frac{(f+gx)^{\frac{11}{2}}\cdot\left(2c^2deg-6c^2e^2f\right)}{11g^6}+\frac{(f+gx)^{\frac{9}{2}}\cdot\left(2ace^2g^2+c^2d^2g^2-10c^2defg+15c^2e^2f^2\right)}{9g^6}+\frac{(f+gx)^{\frac{7}{2}}\cdot\left(4acdeg^3-8ace^2fg^2-4c^2d^2fg^2+20\right.}{7g^6}\right.}{} \\ \frac{a^2d^2x+a^2dex^2+acdex^4+\frac{c^2dex^6}{3}+\frac{c^2e^2x^7}{7}+\frac{x^5\cdot\left(2ace^2+c^2d^2\right)}{5}+\frac{x^3\left(a^2e^2+2acd^2\right)}{3}}{\sqrt{f}} \end{cases}$$

input
```
integrate((e*x+d)**2*(c*x**2+a)**2/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c**2*e**2*(f + g*x)**(13/2)/(13*g**6) + (f + g*x)**(11/2)*(2
*c**2*d*e*g - 6*c**2*e**2*f)/(11*g**6) + (f + g*x)**(9/2)*(2*a*c*e**2*g**2
 + c**2*d**2*g**2 - 10*c**2*d*e*f*g + 15*c**2*e**2*f**2)/(9*g**6) + (f + g
*x)**(7/2)*(4*a*c*d*e*g**3 - 8*a*c*e**2*f*g**2 - 4*c**2*d**2*f*g**2 + 20*c
**2*d*e*f**2*g - 20*c**2*e**2*f**3)/(7*g**6) + (f + g*x)**(5/2)*(a**2*e**2
*g**4 + 2*a*c*d**2*g**4 - 12*a*c*d*e*f*g**3 + 12*a*c*e**2*f**2*g**2 + 6*c*
*2*d**2*f**2*g**2 - 20*c**2*d*e*f**3*g + 15*c**2*e**2*f**4)/(5*g**6) + (f
+ g*x)**(3/2)*(2*a**2*d*e*g**5 - 2*a**2*e**2*f*g**4 - 4*a*c*d**2*f*g**4 +
12*a*c*d*e*f**2*g**3 - 8*a*c*e**2*f**3*g**2 - 4*c**2*d**2*f**3*g**2 + 10*c
**2*d*e*f**4*g - 6*c**2*e**2*f**5)/(3*g**6) + sqrt(f + g*x)*(a**2*d**2*g**
6 - 2*a**2*d*e*f*g**5 + a**2*e**2*f**2*g**4 + 2*a*c*d**2*f**2*g**4 - 4*a*c
*d*e*f**3*g**3 + 2*a*c*e**2*f**4*g**2 + c**2*d**2*f**4*g**2 - 2*c**2*d*e*f
**5*g + c**2*e**2*f**6)/g**6)/g, Ne(g, 0)), ((a**2*d**2*x + a**2*d*e*x**2
+ a*c*d*e*x**4 + c**2*d*e*x**6/3 + c**2*e**2*x**7/7 + x**5*(2*a*c*e**2 + c
**2*d**2)/5 + x**3*(a**2*e**2 + 2*a*c*d**2)/3)/sqrt(f), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)^2 (a+cx^2)^2}{\sqrt{f+gx}}\,dx$$

$$= \frac{2\left(3465\,(gx+f)^{\frac{13}{2}}c^2e^2 - 8190\,(3\,c^2e^2f - c^2deg)(gx+f)^{\frac{11}{2}} + 5005\,(15\,c^2e^2f^2 - 10\,c^2defg + (c^2d^2 + 2\,a\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/45045*(3465*(g*x + f)^(13/2)*c^2*e^2 - 8190*(3*c^2*e^2*f - c^2*d*e*g)*(g
*x + f)^(11/2) + 5005*(15*c^2*e^2*f^2 - 10*c^2*d*e*f*g + (c^2*d^2 + 2*a*c*
e^2)*g^2)*(g*x + f)^(9/2) - 25740*(5*c^2*e^2*f^3 - 5*c^2*d*e*f^2*g - a*c*d
*e*g^3 + (c^2*d^2 + 2*a*c*e^2)*f*g^2)*(g*x + f)^(7/2) + 9009*(15*c^2*e^2*f
^4 - 20*c^2*d*e*f^3*g - 12*a*c*d*e*f*g^3 + 6*(c^2*d^2 + 2*a*c*e^2)*f^2*g^2
 + (2*a*c*d^2 + a^2*e^2)*g^4)*(g*x + f)^(5/2) - 30030*(3*c^2*e^2*f^5 - 5*c
^2*d*e*f^4*g - 6*a*c*d*e*f^2*g^3 - a^2*d*e*g^5 + 2*(c^2*d^2 + 2*a*c*e^2)*f
^3*g^2 + (2*a*c*d^2 + a^2*e^2)*f*g^4)*(g*x + f)^(3/2) + 45045*(c^2*e^2*f^6
 - 2*c^2*d*e*f^5*g - 4*a*c*d*e*f^3*g^3 - 2*a^2*d*e*f*g^5 + a^2*d^2*g^6 + (
c^2*d^2 + 2*a*c*e^2)*f^4*g^2 + (2*a*c*d^2 + a^2*e^2)*f^2*g^4)*sqrt(g*x + f
))/g^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.45

$$\int \frac{(d+ex)^2\left(a+cx^2\right)^2}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(45045\sqrt{gx+f}a^2d^2 + \frac{30030\left((gx+f)^{\frac{3}{2}}-3\sqrt{gx+f}f\right)a^2de}{g} + \frac{6006\left(3(gx+f)^{\frac{5}{2}}-10(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)acd^2}{g^2} + \frac{3003\left(3\right.}{}\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(g*x + f)*a^2*d^2 + 30030*((g*x + f)^(3/2) - 3*sqrt(g*x
 + f)*f)*a^2*d*e/g + 6006*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*s
qrt(g*x + f)*f^2)*a*c*d^2/g^2 + 3003*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/
2)*f + 15*sqrt(g*x + f)*f^2)*a^2*e^2/g^2 + 5148*(5*(g*x + f)^(7/2) - 21*(g
*x + f)^(5/2)*f + 35*(g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*c*d*e/g
^3 + 143*(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)
*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*c^2*d^2/g^4 + 286*
(35*(g*x + f)^(9/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 42
0*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x + f)*f^4)*a*c*e^2/g^4 + 130*(63*(g*x
+ f)^(11/2) - 385*(g*x + f)^(9/2)*f + 990*(g*x + f)^(7/2)*f^2 - 1386*(g*x
+ f)^(5/2)*f^3 + 1155*(g*x + f)^(3/2)*f^4 - 693*sqrt(g*x + f)*f^5)*c^2*d*e
/g^5 + 15*(231*(g*x + f)^(13/2) - 1638*(g*x + f)^(11/2)*f + 5005*(g*x + f)
^(9/2)*f^2 - 8580*(g*x + f)^(7/2)*f^3 + 9009*(g*x + f)^(5/2)*f^4 - 6006*(g
*x + f)^(3/2)*f^5 + 3003*sqrt(g*x + f)*f^6)*c^2*e^2/g^6)/g
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2 (a+cx^2)^2}{\sqrt{f+gx}}\,dx$$
$$= \frac{(f+g\,x)^{5/2}\,(2\,a^2\,e^2\,g^4+4\,a\,c\,d^2\,g^4-24\,a\,c\,d\,e\,f\,g^3+24\,a\,c\,e^2\,f^2\,g^2+12\,c^2\,d^2\,f^2\,g^2-40\,c^2\,d\,e\,f^3\,g+30\,c}{5\,g^7}$$
$$-\frac{(f+g\,x)^{7/2}\,(8\,c^2\,d^2\,f\,g^2-40\,c^2\,d\,e\,f^2\,g+40\,c^2\,e^2\,f^3-8\,a\,c\,d\,e\,g^3+16\,a\,c\,e^2\,f\,g^2)}{7\,g^7}$$
$$+\frac{(f+g\,x)^{9/2}\,(2\,c^2\,d^2\,g^2-20\,c^2\,d\,e\,f\,g+30\,c^2\,e^2\,f^2+4\,a\,c\,e^2\,g^2)}{9\,g^7}$$
$$+\frac{2\,c^2\,e^2\,(f+g\,x)^{13/2}}{13\,g^7}+\frac{2\,\sqrt{f+g\,x}\,(c\,f^2+a\,g^2)^2\,(d\,g-e\,f)^2}{g^7}$$
$$+\frac{4\,c^2\,e\,(f+g\,x)^{11/2}\,(d\,g-3\,e\,f)}{11\,g^7}$$
$$+\frac{4\,(f+g\,x)^{3/2}\,(c\,f^2+a\,g^2)\,(d\,g-e\,f)\,(3\,c\,e\,f^2-2\,c\,d\,f\,g+a\,e\,g^2)}{3\,g^7}$$

input
```
int(((a + c*x^2)^2*(d + e*x)^2)/(f + g*x)^(1/2),x)
```

output
```
((f + g*x)^(5/2)*(2*a^2*e^2*g^4 + 30*c^2*e^2*f^4 + 12*c^2*d^2*f^2*g^2 + 4*
a*c*d^2*g^4 - 40*c^2*d*e*f^3*g + 24*a*c*e^2*f^2*g^2 - 24*a*c*d*e*f*g^3))/(
5*g^7) - ((f + g*x)^(7/2)*(40*c^2*e^2*f^3 + 8*c^2*d^2*f*g^2 + 16*a*c*e^2*f
*g^2 - 40*c^2*d*e*f^2*g - 8*a*c*d*e*g^3))/(7*g^7) + ((f + g*x)^(9/2)*(2*c^
2*d^2*g^2 + 30*c^2*e^2*f^2 + 4*a*c*e^2*g^2 - 20*c^2*d*e*f*g))/(9*g^7) + (2
*c^2*e^2*(f + g*x)^(13/2))/(13*g^7) + (2*(f + g*x)^(1/2)*(a*g^2 + c*f^2)^2
*(d*g - e*f)^2)/g^7 + (4*c^2*e*(f + g*x)^(11/2)*(d*g - 3*e*f))/(11*g^7) +
(4*(f + g*x)^(3/2)*(a*g^2 + c*f^2)*(d*g - e*f)*(a*e*g^2 + 3*c*e*f^2 - 2*c*
d*f*g))/(3*g^7)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex)^2 (a+cx^2)^2}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\sqrt{gx+f}\,(3465c^2e^2g^6x^6 + 8190c^2de\,g^6x^5 - 3780c^2e^2f\,g^5x^5 + 10010ac\,e^2g^6x^4 + 5005c^2d^2g^6x^4 - 9100c^2d}{}$$

input
```
int((e*x+d)^2*(c*x^2+a)^2/(g*x+f)^(1/2),x)
```

output
```
(2*sqrt(f + g*x)*(45045*a**2*d**2*g**6 - 60060*a**2*d*e*f*g**5 + 30030*a**
2*d*e*g**6*x + 24024*a**2*e**2*f**2*g**4 - 12012*a**2*e**2*f*g**5*x + 9009
*a**2*e**2*g**6*x**2 + 48048*a*c*d**2*f**2*g**4 - 24024*a*c*d**2*f*g**5*x
+ 18018*a*c*d**2*g**6*x**2 - 82368*a*c*d*e*f**3*g**3 + 41184*a*c*d*e*f**2*
g**4*x - 30888*a*c*d*e*f*g**5*x**2 + 25740*a*c*d*e*g**6*x**3 + 36608*a*c*e
**2*f**4*g**2 - 18304*a*c*e**2*f**3*g**3*x + 13728*a*c*e**2*f**2*g**4*x**2
 - 11440*a*c*e**2*f*g**5*x**3 + 10010*a*c*e**2*g**6*x**4 + 18304*c**2*d**2
*f**4*g**2 - 9152*c**2*d**2*f**3*g**3*x + 6864*c**2*d**2*f**2*g**4*x**2 -
5720*c**2*d**2*f*g**5*x**3 + 5005*c**2*d**2*g**6*x**4 - 33280*c**2*d*e*f**
5*g + 16640*c**2*d*e*f**4*g**2*x - 12480*c**2*d*e*f**3*g**3*x**2 + 10400*c
**2*d*e*f**2*g**4*x**3 - 9100*c**2*d*e*f*g**5*x**4 + 8190*c**2*d*e*g**6*x*
*5 + 15360*c**2*e**2*f**6 - 7680*c**2*e**2*f**5*g*x + 5760*c**2*e**2*f**4*
g**2*x**2 - 4800*c**2*e**2*f**3*g**3*x**3 + 4200*c**2*e**2*f**2*g**4*x**4
- 3780*c**2*e**2*f*g**5*x**5 + 3465*c**2*e**2*g**6*x**6))/(45045*g**7)
```

3.79 $\int \frac{(d+ex)(a+cx^2)^2}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 212

$$\begin{aligned}\int \frac{(d+ex)(a+cx^2)^2}{\sqrt{f+gx}}\,dx = & -\frac{2(ef-dg)(cf^2+ag^2)^2\sqrt{f+gx}}{g^6} \\ & +\frac{2(cf^2+ag^2)(aeg^2+cf(5ef-4dg))(f+gx)^{3/2}}{3g^6} \\ & -\frac{4c(cf^2(5ef-3dg)+ag^2(3ef-dg))(f+gx)^{5/2}}{5g^6} \\ & +\frac{4c(aeg^2+cf(5ef-2dg))(f+gx)^{7/2}}{7g^6} \\ & -\frac{2c^2(5ef-dg)(f+gx)^{9/2}}{9g^6}+\frac{2c^2e(f+gx)^{11/2}}{11g^6}\end{aligned}$$

output
```
-2*(-d*g+e*f)*(a*g^2+c*f^2)^2*(g*x+f)^(1/2)/g^6+2/3*(a*g^2+c*f^2)*(a*e*g^2
+c*f*(-4*d*g+5*e*f))*(g*x+f)^(3/2)/g^6-4/5*c*(c*f^2*(-3*d*g+5*e*f)+a*g^2*(
-d*g+3*e*f))*(g*x+f)^(5/2)/g^6+4/7*c*(a*e*g^2+c*f*(-2*d*g+5*e*f))*(g*x+f)^
(7/2)/g^6-2/9*c^2*(-d*g+5*e*f)*(g*x+f)^(9/2)/g^6+2/11*c^2*e*(g*x+f)^(11/2)
/g^6
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)\left(a+cx^2\right)^2}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}(1155a^2g^4(-2ef+3dg+egx)-66acg^2(-7dg(8f^2-4fgx+3g^2x^2)+3e(16f^3-8f^2gx+6f}{}$$

input

```
Integrate[((d + e*x)*(a + c*x^2)^2)/Sqrt[f + g*x],x]
```

output

```
(2*Sqrt[f + g*x]*(1155*a^2*g^4*(-2*e*f + 3*d*g + e*g*x) - 66*a*c*g^2*(-7*d
*g*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 3*e*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 -
 5*g^3*x^3)) + c^2*(11*d*g*(128*f^4 - 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g
^3*x^3 + 35*g^4*x^4) - 5*e*(256*f^5 - 128*f^4*g*x + 96*f^3*g^2*x^2 - 80*f^
2*g^3*x^3 + 70*f*g^4*x^4 - 63*g^5*x^5))))/(3465*g^6)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2(d+ex)}{\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{2c(f+gx)^{3/2}\left(-ag^2(3ef-dg)-cf^2(5ef-3dg)\right)}{g^5}+\frac{\sqrt{f+gx}\left(ag^2+cf^2\right)\left(aeg^2+cf(5ef-4dg)\right)}{g^5}+\frac{(ag^2}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4c(f+gx)^{5/2}\left(ag^2(3ef-dg)+cf^2(5ef-3dg)\right)}{5g^6}+\frac{2(f+gx)^{3/2}\left(ag^2+cf^2\right)\left(aeg^2+cf(5ef-4dg)\right)}{3g^6}-\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)^2(ef-dg)}{g^6}+\frac{4c(f+gx)^{7/2}\left(aeg^2+cf(5ef-2dg)\right)}{7g^6}-\frac{2c^2(f+gx)^{9/2}(5ef-dg)}{9g^6}+\frac{2c^2e(f+gx)^{11/2}}{11g^6}$$

input
```
Int[((d + e*x)*(a + c*x^2)^2)/Sqrt[f + g*x],x]
```

output
```
(-2*(e*f - d*g)*(c*f^2 + a*g^2)^2*Sqrt[f + g*x])/g^6 + (2*(c*f^2 + a*g^2)*
(a*e*g^2 + c*f*(5*e*f - 4*d*g))*(f + g*x)^(3/2))/(3*g^6) - (4*c*(c*f^2*(5*
e*f - 3*d*g) + a*g^2*(3*e*f - d*g))*(f + g*x)^(5/2))/(5*g^6) + (4*c*(a*e*g
^2 + c*f*(5*e*f - 2*d*g))*(f + g*x)^(7/2))/(7*g^6) - (2*c^2*(5*e*f - d*g)*
(f + g*x)^(9/2))/(9*g^6) + (2*c^2*e*(f + g*x)^(11/2))/(11*g^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{2\left(\left(\frac{x^4\left(\frac{9ex}{11}+d\right)c^2}{9}+\frac{2x^2\left(\frac{5ex}{7}+d\right)ac}{5}+a^2\left(\frac{ex}{3}+d\right)\right)g^5-\frac{2f\left(\left(\frac{5}{33}x^4e+\frac{4}{21}d\,x^3\right)c^2+\frac{4\left(\frac{9ex}{14}+d\right)xac}{5}+a^2e\right)g^4}{3}+\frac{16f^2\left(\frac{x^2\left(\frac{25ex}{33}\right.}{7}\right.}{}\right.}{g^6}$
derivativedivides	$\frac{2ec^2(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left((dg-ef)c^2-4fec^2\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(-4(dg-ef)c^2f+e\left(2\left(ag^2+cf^2\right)c+4c^2f^2\right)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)\left(2\left(ag^2\right.\right.\right.}{}$
default	$\frac{2ec^2(gx+f)^{\frac{11}{2}}}{11}+\frac{2\left((dg-ef)c^2-4fec^2\right)(gx+f)^{\frac{9}{2}}}{9}+\frac{2\left(-4(dg-ef)c^2f+e\left(2\left(ag^2+cf^2\right)c+4c^2f^2\right)\right)(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left((dg-ef)\left(2\left(ag^2\right.\right.\right.}{}$
gosper	$\frac{2\sqrt{gx+f}\,(315c^2e\,x^5g^5+385c^2d\,g^5x^4-350c^2ef\,g^4x^4+990ace\,g^5x^3-440c^2df\,g^4x^3+400c^2e\,f^2g^3x^3+1386acd\,g^5x^2-118}{}$
trager	$\frac{2\sqrt{gx+f}\,(315c^2e\,x^5g^5+385c^2d\,g^5x^4-350c^2ef\,g^4x^4+990ace\,g^5x^3-440c^2df\,g^4x^3+400c^2e\,f^2g^3x^3+1386acd\,g^5x^2-118}{}$
risch	$\frac{2\sqrt{gx+f}\,(315c^2e\,x^5g^5+385c^2d\,g^5x^4-350c^2ef\,g^4x^4+990ace\,g^5x^3-440c^2df\,g^4x^3+400c^2e\,f^2g^3x^3+1386acd\,g^5x^2-118}{}$
orering	$\frac{2\sqrt{gx+f}\,(315c^2e\,x^5g^5+385c^2d\,g^5x^4-350c^2ef\,g^4x^4+990ace\,g^5x^3-440c^2df\,g^4x^3+400c^2e\,f^2g^3x^3+1386acd\,g^5x^2-118}{}$

input
```
int((e*x+d)*(c*x^2+a)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((1/9*x^4*(9/11*e*x+d)*c^2+2/5*x^2*(5/7*e*x+d)*a*c+a^2*(1/3*e*x+d))*g^5-
2/3*f*((5/33*x^4*e+4/21*d*x^3)*c^2+4/5*(9/14*e*x+d)*x*a*c+a^2*e)*g^4+16/15
*f^2*(1/7*x^2*(25/33*e*x+d)*c+a*(3/7*e*x+d))*c*g^3-32/35*f^3*c*((5/33*e*x^
2+2/9*d*x)*c+a*e)*g^2+128/315*(5/11*e*x+d)*f^4*c^2*g-256/693*c^2*e*f^5)*(g
*x+f)^(1/2)/g^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)\left(a+cx^2\right)^2}{\sqrt{f+gx}}\,dx$$
$$=\frac{2\left(315\,c^2eg^5x^5-1280\,c^2ef^5+1408\,c^2df^4g-3168\,acef^3g^2+3696\,acdf^2g^3-2310\,a^2efg^4+3465\,a^2dg^5\right.}{}$$

input
```
integrate((e*x+d)*(c*x^2+a)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*c^2*e*g^5*x^5 - 1280*c^2*e*f^5 + 1408*c^2*d*f^4*g - 3168*a*c*e
*f^3*g^2 + 3696*a*c*d*f^2*g^3 - 2310*a^2*e*f*g^4 + 3465*a^2*d*g^5 - 35*(10
*c^2*e*f*g^4 - 11*c^2*d*g^5)*x^4 + 10*(40*c^2*e*f^2*g^3 - 44*c^2*d*f*g^4 +
 99*a*c*e*g^5)*x^3 - 6*(80*c^2*e*f^3*g^2 - 88*c^2*d*f^2*g^3 + 198*a*c*e*f*
g^4 - 231*a*c*d*g^5)*x^2 + (640*c^2*e*f^4*g - 704*c^2*d*f^3*g^2 + 1584*a*c
*e*f^2*g^3 - 1848*a*c*d*f*g^4 + 1155*a^2*e*g^5)*x)*sqrt(g*x + f)/g^6
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.75

$$\int \frac{(d+ex)\left(a+cx^2\right)^2}{\sqrt{f+gx}}\,dx$$

$$= \begin{cases} \frac{2\left(\frac{c^2e(f+gx)^{\frac{11}{2}}}{11g^5}+\frac{(f+gx)^{\frac{9}{2}}\left(c^2dg-5c^2ef\right)}{9g^5}+\frac{(f+gx)^{\frac{7}{2}}\cdot\left(2aceg^2-4c^2dfg+10c^2ef^2\right)}{7g^5}+\frac{(f+gx)^{\frac{5}{2}}\cdot\left(2acdg^3-6acefg^2+6c^2df^2g-10c^2ef^3\right)}{5g^5}+\frac{(f+gx)^{\frac{3}{2}}\left(a^2e g\right.}{}\right.}{g} \\ \frac{a^2dx+\frac{a^2ex^2}{2}+\frac{2acdx^3}{3}+\frac{acex^4}{2}+\frac{c^2dx^5}{5}+\frac{c^2ex^6}{6}}{\sqrt{f}} \end{cases}$$

input
```
integrate((e*x+d)*(c*x**2+a)**2/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c**2*e*(f + g*x)**(11/2)/(11*g**5) + (f + g*x)**(9/2)*(c**2*
d*g - 5*c**2*e*f)/(9*g**5) + (f + g*x)**(7/2)*(2*a*c*e*g**2 - 4*c**2*d*f*g
 + 10*c**2*e*f**2)/(7*g**5) + (f + g*x)**(5/2)*(2*a*c*d*g**3 - 6*a*c*e*f*g
**2 + 6*c**2*d*f**2*g - 10*c**2*e*f**3)/(5*g**5) + (f + g*x)**(3/2)*(a**2*
e*g**4 - 4*a*c*d*f*g**3 + 6*a*c*e*f**2*g**2 - 4*c**2*d*f**3*g + 5*c**2*e*f
**4)/(3*g**5) + sqrt(f + g*x)*(a**2*d*g**5 - a**2*e*f*g**4 + 2*a*c*d*f**2*
g**3 - 2*a*c*e*f**3*g**2 + c**2*d*f**4*g - c**2*e*f**5)/g**5)/g, Ne(g, 0))
, ((a**2*d*x + a**2*e*x**2/2 + 2*a*c*d*x**3/3 + a*c*e*x**4/2 + c**2*d*x**5
/5 + c**2*e*x**6/6)/sqrt(f), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)\left(a+cx^2\right)^2}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(315\,(gx+f)^{\frac{11}{2}}c^2e-385\,(5\,c^2ef-c^2dg)(gx+f)^{\frac{9}{2}}+990\,(5\,c^2ef^2-2\,c^2dfg+aceg^2)(gx+f)^{\frac{7}{2}}-138\right.}{}$$

input
```
integrate((e*x+d)*(c*x^2+a)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/3465*(315*(g*x + f)^(11/2)*c^2*e - 385*(5*c^2*e*f - c^2*d*g)*(g*x + f)^(
9/2) + 990*(5*c^2*e*f^2 - 2*c^2*d*f*g + a*c*e*g^2)*(g*x + f)^(7/2) - 1386*
(5*c^2*e*f^3 - 3*c^2*d*f^2*g + 3*a*c*e*f*g^2 - a*c*d*g^3)*(g*x + f)^(5/2)
+ 1155*(5*c^2*e*f^4 - 4*c^2*d*f^3*g + 6*a*c*e*f^2*g^2 - 4*a*c*d*f*g^3 + a^
2*e*g^4)*(g*x + f)^(3/2) - 3465*(c^2*e*f^5 - c^2*d*f^4*g + 2*a*c*e*f^3*g^2
 - 2*a*c*d*f^2*g^3 + a^2*e*f*g^4 - a^2*d*g^5)*sqrt(g*x + f))/g^6
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)\left(a+cx^2\right)^2}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(3465\,\sqrt{gx+f}a^2d+\frac{1155\left((gx+f)^{\frac{3}{2}}-3\,\sqrt{gx+f}f\right)a^2e}{g}+\frac{462\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\,\sqrt{gx+f}f^2\right)acd}{g^2}+\frac{198\left(5\,(gx+f)^{\frac{7}{2}}\right.}{}\right.}{}$$

input
```
integrate((e*x+d)*(c*x^2+a)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(g*x + f)*a^2*d + 1155*((g*x + f)^(3/2) - 3*sqrt(g*x + f)
*f)*a^2*e/g + 462*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*f + 15*sqrt(g*x
+ f)*f^2)*a*c*d/g^2 + 198*(5*(g*x + f)^(7/2) - 21*(g*x + f)^(5/2)*f + 35*(
g*x + f)^(3/2)*f^2 - 35*sqrt(g*x + f)*f^3)*a*c*e/g^3 + 11*(35*(g*x + f)^(9
/2) - 180*(g*x + f)^(7/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2
)*f^3 + 315*sqrt(g*x + f)*f^4)*c^2*d/g^4 + 5*(63*(g*x + f)^(11/2) - 385*(g
*x + f)^(9/2)*f + 990*(g*x + f)^(7/2)*f^2 - 1386*(g*x + f)^(5/2)*f^3 + 115
5*(g*x + f)^(3/2)*f^4 - 693*sqrt(g*x + f)*f^5)*c^2*e/g^5)/g
```

Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)\,(a+cx^2)^2}{\sqrt{f+gx}}\,dx = \frac{(f+g\,x)^{7/2}\,(20\,e\,c^2\,f^2-8\,d\,c^2\,f\,g+4\,a\,e\,c\,g^2)}{7\,g^6} + \frac{2\,\sqrt{f+g\,x}\,(c\,f^2+a\,g^2)^2\,(d\,g-e\,f)}{g^6} + \frac{2\,(f+g\,x)^{3/2}\,(c\,f^2+a\,g^2)\,(5\,c\,e\,f^2-4\,c\,d\,f\,g+a\,e\,g^2)}{3\,g^6} + \frac{2\,c^2\,e\,(f+g\,x)^{11/2}}{11\,g^6} + \frac{4\,c\,(f+g\,x)^{5/2}\,(-5\,c\,e\,f^3+3\,c\,d\,f^2\,g-3\,a\,e\,f\,g^2+a\,d\,g^3)}{5\,g^6} + \frac{2\,c^2\,(f+g\,x)^{9/2}\,(d\,g-5\,e\,f)}{9\,g^6}$$

input
```
int(((a + c*x^2)^2*(d + e*x))/(f + g*x)^(1/2),x)
```

output
```
((f + g*x)^(7/2)*(20*c^2*e*f^2 + 4*a*c*e*g^2 - 8*c^2*d*f*g))/(7*g^6) + (2*
(f + g*x)^(1/2)*(a*g^2 + c*f^2)^2*(d*g - e*f))/g^6 + (2*(f + g*x)^(3/2)*(a
*g^2 + c*f^2)*(a*e*g^2 + 5*c*e*f^2 - 4*c*d*f*g))/(3*g^6) + (2*c^2*e*(f + g
*x)^(11/2))/(11*g^6) + (4*c*(f + g*x)^(5/2)*(a*d*g^3 - 5*c*e*f^3 - 3*a*e*f
*g^2 + 3*c*d*f^2*g))/(5*g^6) + (2*c^2*(f + g*x)^(9/2)*(d*g - 5*e*f))/(9*g^
6)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)\left(a+cx^2\right)^2}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\sqrt{gx+f}\,(315c^2e\,g^5x^5+385c^2d\,g^5x^4-350c^2ef\,g^4x^4+990ace\,g^5x^3-440c^2df\,g^4x^3+400c^2e\,f^2g^3x^3+\cdots}{}$$

input
```
int((e*x+d)*(c*x^2+a)^2/(g*x+f)^(1/2),x)
```

output
```
(2*sqrt(f + g*x)*(3465*a**2*d*g**5 - 2310*a**2*e*f*g**4 + 1155*a**2*e*g**5
*x + 3696*a*c*d*f**2*g**3 - 1848*a*c*d*f*g**4*x + 1386*a*c*d*g**5*x**2 - 3
168*a*c*e*f**3*g**2 + 1584*a*c*e*f**2*g**3*x - 1188*a*c*e*f*g**4*x**2 + 99
0*a*c*e*g**5*x**3 + 1408*c**2*d*f**4*g - 704*c**2*d*f**3*g**2*x + 528*c**2
*d*f**2*g**3*x**2 - 440*c**2*d*f*g**4*x**3 + 385*c**2*d*g**5*x**4 - 1280*c
**2*e*f**5 + 640*c**2*e*f**4*g*x - 480*c**2*e*f**3*g**2*x**2 + 400*c**2*e*
f**2*g**3*x**3 - 350*c**2*e*f*g**4*x**4 + 315*c**2*e*g**5*x**5))/(3465*g**
6)
```

3.80 $\int \frac{(a+cx^2)^2}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 125

$$\int \frac{(a+cx^2)^2}{\sqrt{f+gx}}\,dx = \frac{2(cf^2+ag^2)^2\sqrt{f+gx}}{g^5} - \frac{8cf(cf^2+ag^2)(f+gx)^{3/2}}{3g^5} + \frac{4c(3cf^2+ag^2)(f+gx)^{5/2}}{5g^5} - \frac{8c^2f(f+gx)^{7/2}}{7g^5} + \frac{2c^2(f+gx)^{9/2}}{9g^5}$$

output
```
2*(a*g^2+c*f^2)^2*(g*x+f)^(1/2)/g^5-8/3*c*f*(a*g^2+c*f^2)*(g*x+f)^(3/2)/g^
5+4/5*c*(a*g^2+3*c*f^2)*(g*x+f)^(5/2)/g^5-8/7*c^2*f*(g*x+f)^(7/2)/g^5+2/9*
c^2*(g*x+f)^(9/2)/g^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{(a+cx^2)^2}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{f+gx}(315a^2g^4+42acg^2(8f^2-4fgx+3g^2x^2)+c^2(128f^4-64f^3gx+48f^2g^2x^2-40fg^3x^3+35g^4}{315g^5}$$

input
```
Integrate[(a + c*x^2)^2/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(315*a^2*g^4 + 42*a*c*g^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2) +
 c^2*(128*f^4 - 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4)))
/(315*g^5)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{2c(f+gx)^{3/2}(ag^2+3cf^2)}{g^4}-\frac{4cf\sqrt{f+gx}(ag^2+cf^2)}{g^4}+\frac{(ag^2+cf^2)^2}{g^4\sqrt{f+gx}}+\frac{c^2(f+gx)^{7/2}}{g^4}-\frac{4c^2f(f+gx)^{5/2}}{g^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c(f+gx)^{5/2}(ag^2+3cf^2)}{5g^5}-\frac{8cf(f+gx)^{3/2}(ag^2+cf^2)}{3g^5}+\frac{2\sqrt{f+gx}(ag^2+cf^2)^2}{g^5}+\frac{2c^2(f+gx)^{9/2}}{9g^5}-\frac{8c^2f(f+gx)^{7/2}}{7g^5}$$

input
```
Int[(a + c*x^2)^2/Sqrt[f + g*x],x]
```

output
```
(2*(c*f^2 + a*g^2)^2*Sqrt[f + g*x])/g^5 - (8*c*f*(c*f^2 + a*g^2)*(f + g*x)
^(3/2))/(3*g^5) + (4*c*(3*c*f^2 + a*g^2)*(f + g*x)^(5/2))/(5*g^5) - (8*c^2
*f*(f + g*x)^(7/2))/(7*g^5) + (2*c^2*(f + g*x)^(9/2))/(9*g^5)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{2\sqrt{gx+f}\left(\left(\frac{2}{5}ac\,x^2+a^2+\frac{1}{9}c^2x^4\right)g^4-\frac{8x\left(\frac{5c\,x^2}{21}+a\right)cf\,g^3}{15}+\frac{16c\,f^2\left(\frac{c\,x^2}{7}+a\right)g^2}{15}-\frac{64c^2f^3gx}{315}+\frac{128c^2f^4}{315}\right)}{g^5}$
gosper	$\frac{2\sqrt{gx+f}\,(35c^2x^4g^4-40c^2f\,x^3g^3+126ac\,g^4x^2+48c^2f^2g^2x^2-168acf\,g^3x-64c^2f^3gx+315a^2g^4+336ac\,f^2g^2+128c^2f^4}{315g^5}$
trager	$\frac{2\sqrt{gx+f}\,(35c^2x^4g^4-40c^2f\,x^3g^3+126ac\,g^4x^2+48c^2f^2g^2x^2-168acf\,g^3x-64c^2f^3gx+315a^2g^4+336ac\,f^2g^2+128c^2f^4}{315g^5}$
risch	$\frac{2\sqrt{gx+f}\,(35c^2x^4g^4-40c^2f\,x^3g^3+126ac\,g^4x^2+48c^2f^2g^2x^2-168acf\,g^3x-64c^2f^3gx+315a^2g^4+336ac\,f^2g^2+128c^2f^4}{315g^5}$
orering	$\frac{2\sqrt{gx+f}\,(35c^2x^4g^4-40c^2f\,x^3g^3+126ac\,g^4x^2+48c^2f^2g^2x^2-168acf\,g^3x-64c^2f^3gx+315a^2g^4+336ac\,f^2g^2+128c^2f^4}{315g^5}$
derivativedivides	$\frac{\frac{2c^2(gx+f)^{\frac{9}{2}}}{9}-\frac{8c^2f(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left(2\left(a\,g^2+c\,f^2\right)c+4c^2f^2\right)(gx+f)^{\frac{5}{2}}}{5}-\frac{8\left(a\,g^2+c\,f^2\right)cf(gx+f)^{\frac{3}{2}}}{3}+2\left(a\,g^2+c\,f^2\right)^2\sqrt{gx+f}}{g^5}$
default	$\frac{\frac{2c^2(gx+f)^{\frac{9}{2}}}{9}-\frac{8c^2f(gx+f)^{\frac{7}{2}}}{7}+\frac{2\left(2\left(a\,g^2+c\,f^2\right)c+4c^2f^2\right)(gx+f)^{\frac{5}{2}}}{5}-\frac{8\left(a\,g^2+c\,f^2\right)cf(gx+f)^{\frac{3}{2}}}{3}+2\left(a\,g^2+c\,f^2\right)^2\sqrt{gx+f}}{g^5}$

input
```
int((c*x^2+a)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(g*x+f)^(1/2)*((2/5*a*c*x^2+a^2+1/9*c^2*x^4)*g^4-8/15*x*(5/21*c*x^2+a)*c
*f*g^3+16/15*c*f^2*(1/7*c*x^2+a)*g^2-64/315*c^2*f^3*g*x+128/315*c^2*f^4)/g
^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int \frac{(a+cx^2)^2}{\sqrt{f+gx}}\,dx = \frac{2\,(35\,c^2g^4x^4 - 40\,c^2fg^3x^3 + 128\,c^2f^4 + 336\,acf^2g^2 + 315\,a^2g^4 + 6\,(8\,c^2f^2g^2 + 21\,acg^4)x^2 - 8\,(8\,c^2f^3g +}{315\,g^5}$$

input
```
integrate((c*x^2+a)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*c^2*g^4*x^4 - 40*c^2*f*g^3*x^3 + 128*c^2*f^4 + 336*a*c*f^2*g^2 +
 315*a^2*g^4 + 6*(8*c^2*f^2*g^2 + 21*a*c*g^4)*x^2 - 8*(8*c^2*f^3*g + 21*a*
c*f*g^3)*x)*sqrt(g*x + f)/g^5
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.40

$$\int \frac{(a+cx^2)^2}{\sqrt{f+gx}}\,dx = \begin{cases} \frac{2\left(-\frac{4c^2f(f+gx)^{\frac{7}{2}}}{7g^4} + \frac{c^2(f+gx)^{\frac{9}{2}}}{9g^4} + \frac{(f+gx)^{\frac{5}{2}}\cdot\left(2acg^2+6c^2f^2\right)}{5g^4} + \frac{(f+gx)^{\frac{3}{2}}\left(-4acfg^2-4c^2f^3\right)}{3g^4} + \frac{\sqrt{f+gx}\left(a^2g^4+2acf^2g^2+c^2f^4\right)}{g^4}\right)}{g} & \text{for } g \neq 0 \\ \frac{a^2x + \frac{2acx^3}{3} + \frac{c^2x^5}{5}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)**2/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(-4*c**2*f*(f + g*x)**(7/2)/(7*g**4) + c**2*(f + g*x)**(9/2)/
(9*g**4) + (f + g*x)**(5/2)*(2*a*c*g**2 + 6*c**2*f**2)/(5*g**4) + (f + g*x
)**(3/2)*(-4*a*c*f*g**2 - 4*c**2*f**3)/(3*g**4) + sqrt(f + g*x)*(a**2*g**4
 + 2*a*c*f**2*g**2 + c**2*f**4)/g**4)/g, Ne(g, 0)), ((a**2*x + 2*a*c*x**3/
3 + c**2*x**5/5)/sqrt(f), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96

$$\int \frac{(a+cx^2)^2}{\sqrt{f+gx}}\,dx = \frac{2\left(315\sqrt{gx+f}a^2 + \frac{42\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)ac}{g^2} + \frac{\left(35\,(gx+f)^{\frac{9}{2}}-180\,(gx+f)^{\frac{7}{2}}f+378\,(gx+f)^{\frac{5}{2}}f^2-420\,(gx+f\cdots\right.}{g^4}\right.}{315\,g}$$

input
```
integrate((c*x^2+a)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(315*sqrt(g*x + f)*a^2 + 42*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*
f + 15*sqrt(g*x + f)*f^2)*a*c/g^2 + (35*(g*x + f)^(9/2) - 180*(g*x + f)^(7
/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x +
 f)*f^4)*c^2/g^4)/g
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96

$$\int \frac{(a+cx^2)^2}{\sqrt{f+gx}}\,dx = \frac{2\left(315\sqrt{gx+f}a^2 + \frac{42\left(3\,(gx+f)^{\frac{5}{2}}-10\,(gx+f)^{\frac{3}{2}}f+15\sqrt{gx+f}f^2\right)ac}{g^2} + \frac{\left(35\,(gx+f)^{\frac{9}{2}}-180\,(gx+f)^{\frac{7}{2}}f+378\,(gx+f)^{\frac{5}{2}}f^2-420\,(gx+f\cdots\right.}{g^4}\right.}{315\,g}$$

input
```
integrate((c*x^2+a)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(g*x + f)*a^2 + 42*(3*(g*x + f)^(5/2) - 10*(g*x + f)^(3/2)*
f + 15*sqrt(g*x + f)*f^2)*a*c/g^2 + (35*(g*x + f)^(9/2) - 180*(g*x + f)^(7
/2)*f + 378*(g*x + f)^(5/2)*f^2 - 420*(g*x + f)^(3/2)*f^3 + 315*sqrt(g*x +
 f)*f^4)*c^2/g^4)/g
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.91

$$\int \frac{(a+cx^2)^2}{\sqrt{f+gx}}\,dx = \frac{2\,c^2\,(f+g\,x)^{9/2}}{9\,g^5} - \frac{(f+g\,x)^{3/2}\,(8\,c^2\,f^3+8\,a\,c\,f\,g^2)}{3\,g^5} + \frac{2\,\sqrt{f+g\,x}\,(c\,f^2+a\,g^2)^2}{g^5} + \frac{(f+g\,x)^{5/2}\,(12\,c^2\,f^2+4\,a\,c\,g^2)}{5\,g^5} - \frac{8\,c^2\,f\,(f+g\,x)^{7/2}}{7\,g^5}$$

input
```
int((a + c*x^2)^2/(f + g*x)^(1/2),x)
```

output
```
(2*c^2*(f + g*x)^(9/2))/(9*g^5) - ((f + g*x)^(3/2)*(8*c^2*f^3 + 8*a*c*f*g^
2))/(3*g^5) + (2*(f + g*x)^(1/2)*(a*g^2 + c*f^2)^2)/g^5 + ((f + g*x)^(5/2)
*(12*c^2*f^2 + 4*a*c*g^2))/(5*g^5) - (8*c^2*f*(f + g*x)^(7/2))/(7*g^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{(a+cx^2)^2}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{gx+f}\,(35c^2g^4x^4-40c^2f\,g^3x^3+126ac\,g^4x^2+48c^2f^2g^2x^2-168acf\,g^3x-64c^2f^3gx+315a^2g^4+33}{315g^5}$$

input
```
int((c*x^2+a)^2/(g*x+f)^(1/2),x)
```

output
```
(2*sqrt(f + g*x)*(315*a**2*g**4 + 336*a*c*f**2*g**2 - 168*a*c*f*g**3*x + 1
26*a*c*g**4*x**2 + 128*c**2*f**4 - 64*c**2*f**3*g*x + 48*c**2*f**2*g**2*x*
*2 - 40*c**2*f*g**3*x**3 + 35*c**2*g**4*x**4))/(315*g**5)
```

3.81 $\int \frac{(a+cx^2)^2}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 218

$$\begin{aligned}\int \frac{(a+cx^2)^2}{(d+ex)\sqrt{f+gx}}\,dx = &-\frac{2c(ef+dg)\left(ce^2f^2+cd^2g^2+2ae^2g^2\right)\sqrt{f+gx}}{e^4g^4}\\ &+\frac{2c(2ae^2g^2+c(3e^2f^2+2defg+d^2g^2))\,(f+gx)^{3/2}}{3e^3g^4}\\ &-\frac{2c^2(3ef+dg)(f+gx)^{5/2}}{5e^2g^4}+\frac{2c^2(f+gx)^{7/2}}{7eg^4}\\ &-\frac{2(cd^2+ae^2)^2\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{9/2}\sqrt{ef-dg}}\end{aligned}$$

output
```
-2*c*(d*g+e*f)*(2*a*e^2*g^2+c*d^2*g^2+c*e^2*f^2)*(g*x+f)^(1/2)/e^4/g^4+2/3
*c*(2*a*e^2*g^2+c*(d^2*g^2+2*d*e*f*g+3*e^2*f^2))*(g*x+f)^(3/2)/e^3/g^4-2/5
*c^2*(d*g+3*e*f)*(g*x+f)^(5/2)/e^2/g^4+2/7*c^2*(g*x+f)^(7/2)/e/g^4-2*(a*e^
2+c*d^2)^2*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(9/2)/(-d*g+e
*f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.89

$$\int \frac{(a+cx^2)^2}{(d+ex)\sqrt{f+gx}}\,dx = -\frac{2c\sqrt{f+gx}(70ae^2g^2(2ef+3dg-egx)+c(105d^3g^3-35d^2eg^2(-2f+gx)+7de^2g(8f^2-4fgx+3g^2}{105e^4g^4} + \frac{2(cd^2+ae^2)^2\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{9/2}\sqrt{-ef+dg}}$$

input
```
Integrate[(a + c*x^2)^2/((d + e*x)*Sqrt[f + g*x]),x]
```

output
```
(-2*c*Sqrt[f + g*x]*(70*a*e^2*g^2*(2*e*f + 3*d*g - e*g*x) + c*(105*d^3*g^3
 - 35*d^2*e*g^2*(-2*f + g*x) + 7*d*e^2*g*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 3
*e^3*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3))))/(105*e^4*g^4) + (2*
(c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(
9/2)*Sqrt[-(e*f) + d*g])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {649, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \; 649$$

$$\frac{2\int -\frac{(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)^2}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\int \frac{\left(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2\right)^2}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{g^4}$$

$\downarrow$ 1467

$$-\frac{2\int \left(-\frac{c^2(f+gx)^3}{e}+\frac{c^2(3ef+dg)(f+gx)^2}{e^2}-\frac{c\left(2ae^2g^2+c\left(3e^2f^2+2degf+d^2g^2\right)\right)(f+gx)}{e^3}+\frac{c(ef+dg)\left(ce^2f^2+cd^2g^2+2ae^2g^2\right)}{e^4}+\frac{c^2d^4g^4}{e^4(}\right.}{g^4}$$

$\downarrow$ 2009

$$\frac{2\left(-\frac{g^4\left(ae^2+cd^2\right)^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{9/2}\sqrt{ef-dg}}-\frac{c\sqrt{f+gx}(dg+ef)\left(2ae^2g^2+cd^2g^2+ce^2f^2\right)}{e^4}+\frac{c(f+gx)^{3/2}\left(2ae^2g^2+c\left(d^2g^2+2defg+3e^2f^2\right)\right)}{3e^3}-\right.}{g^4}$$

input
```
Int[(a + c*x^2)^2/((d + e*x)*Sqrt[f + g*x]),x]
```

output
```
(2*(-((c*(e*f + d*g)*(c*e^2*f^2 + c*d^2*g^2 + 2*a*e^2*g^2)*Sqrt[f + g*x])/
e^4) + (c*(2*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g + d^2*g^2))*(f + g*x)^(3
/2))/(3*e^3) - (c^2*(3*e*f + d*g)*(f + g*x)^(5/2))/(5*e^2) + (c^2*(f + g*x
)^(7/2))/(7*e) - ((c*d^2 + a*e^2)^2*g^4*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sq
rt[e*f - d*g]])/(e^(9/2)*Sqrt[e*f - d*g])))/g^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{2g^4(a e^2+c d^2)^2 \arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)-4\sqrt{gx+f}\, c\left(\left(\frac{\left(-\frac{1}{7}e^3x^3+\frac{1}{5}d e^2x^2-\frac{1}{3}d^2ex+d^3\right)c}{2}+a e^2\left(-\frac{ex}{3}+d\right)\right)g^3+\frac{2ef\left(\left(\frac{9}{70}e \cdots\right.\right.}{\cdots}\cdots\right)}{g^4e^4\sqrt{(dg-ef)e}}$
derivativedivides	$-\frac{\frac{2c\left(-\frac{c(gx+f)^{\frac{7}{2}}e^3}{7}+\frac{\left((dg+ef)c e^2+2f e^3c\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{\left(-2(dg+ef)c e^2f-e\left(2a e^2g^2+c d^2g^2+c e^2f^2\right)\right)(gx+f)^{\frac{3}{2}}}{3}+(dg+ef)\left(2a \cdots\right.\right)}{e^4}\cdots}{g^4}$
default	$-\frac{\frac{2c\left(-\frac{c(gx+f)^{\frac{7}{2}}e^3}{7}+\frac{\left((dg+ef)c e^2+2f e^3c\right)(gx+f)^{\frac{5}{2}}}{5}+\frac{\left(-2(dg+ef)c e^2f-e\left(2a e^2g^2+c d^2g^2+c e^2f^2\right)\right)(gx+f)^{\frac{3}{2}}}{3}+(dg+ef)\left(2a \cdots\right.\right)}{e^4}\cdots}{g^4}$
risch	$-\frac{2c\left(-15x^3c e^3g^3+21cd e^2g^3x^2+18c e^3f g^2x^2-70a e^3g^3x-35c d^2e g^3x-28cd e^2f g^2x-24c e^3f^2gx+210ad e^2g^3+140\cdots\right.}{105g^4e^4}$

input
```
int((c*x^2+a)^2/(e*x+d)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(g^4*(a*e^2+c*d^2)^2*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))-2*(g*x+
f)^(1/2)*c*((1/2*(-1/7*e^3*x^3+1/5*d*e^2*x^2-1/3*d^2*e*x+d^3)*c+a*e^2*(-1/
3*e*x+d))*g^3+2/3*e*f*((9/70*e^2*x^2-1/5*d*e*x+1/2*d^2)*c+a*e^2)*g^2+4/15*
e^2*(-3/7*e*x+d)*f^2*c*g+8/35*c*e^3*f^3)*((d*g-e*f)*e)^(1/2))/((d*g-e*f)*e
)^(1/2)/g^4/e^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 735, normalized size of antiderivative = 3.37

$$\int \frac{(a+cx^2)^2}{(d+ex)\sqrt{f+gx}}\,dx$$
$$= \left[\frac{105\,(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{e^2f-deg}g^4\log\left(\frac{egx+2\,ef-dg-2\,\sqrt{e^2f-deg}\sqrt{gx+f}}{ex+d}\right)-2\,(48\,c^2e^5f^4+8\,c^2de^4f^3\right.$$

input
```
integrate((c*x^2+a)^2/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[1/105*(105*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(e^2*f - d*e*g)*g^4*lo
g((e*g*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) -
 2*(48*c^2*e^5*f^4 + 8*c^2*d*e^4*f^3*g + 14*(c^2*d^2*e^3 + 10*a*c*e^5)*f^2
*g^2 + 35*(c^2*d^3*e^2 + 2*a*c*d*e^4)*f*g^3 - 105*(c^2*d^4*e + 2*a*c*d^2*e
^3)*g^4 - 15*(c^2*e^5*f*g^3 - c^2*d*e^4*g^4)*x^3 + 3*(6*c^2*e^5*f^2*g^2 +
c^2*d*e^4*f*g^3 - 7*c^2*d^2*e^3*g^4)*x^2 - (24*c^2*e^5*f^3*g + 4*c^2*d*e^4
*f^2*g^2 + 7*(c^2*d^2*e^3 + 10*a*c*e^5)*f*g^3 - 35*(c^2*d^3*e^2 + 2*a*c*d*
e^4)*g^4)*x)*sqrt(g*x + f))/(e^6*f*g^4 - d*e^5*g^5), 2/105*(105*(c^2*d^4 +
 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-e^2*f + d*e*g)*g^4*arctan(sqrt(-e^2*f + d*
e*g)*sqrt(g*x + f)/(e*g*x + e*f)) - (48*c^2*e^5*f^4 + 8*c^2*d*e^4*f^3*g +
14*(c^2*d^2*e^3 + 10*a*c*e^5)*f^2*g^2 + 35*(c^2*d^3*e^2 + 2*a*c*d*e^4)*f*g
^3 - 105*(c^2*d^4*e + 2*a*c*d^2*e^3)*g^4 - 15*(c^2*e^5*f*g^3 - c^2*d*e^4*g
^4)*x^3 + 3*(6*c^2*e^5*f^2*g^2 + c^2*d*e^4*f*g^3 - 7*c^2*d^2*e^3*g^4)*x^2
- (24*c^2*e^5*f^3*g + 4*c^2*d*e^4*f^2*g^2 + 7*(c^2*d^2*e^3 + 10*a*c*e^5)*f
*g^3 - 35*(c^2*d^3*e^2 + 2*a*c*d*e^4)*g^4)*x)*sqrt(g*x + f))/(e^6*f*g^4 -
d*e^5*g^5)]
```

Sympy [A] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.70

$$\int \frac{(a+cx^2)^2}{(d+ex)\sqrt{f+gx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{c^2(f+gx)^{\frac{7}{2}}}{7eg^3}+\frac{(f+gx)^{\frac{5}{2}}(-c^2dg-3c^2ef)}{5e^2g^3}+\frac{(f+gx)^{\frac{3}{2}}\cdot(2ace^2g^2+c^2d^2g^2+2c^2defg+3c^2e^2f^2)}{3e^3g^3}+\frac{\sqrt{f+gx}(-2acde^2g^3-2ace^3fg^2-c^2d^3g^3-c^2d^2efg^2-c^2}{e^4g^3}\right.}{g} \\ \frac{-\frac{c^2dx^3}{3e^2}+\frac{c^2x^4}{4e}+\frac{x^2\cdot(2ace^2+c^2d^2)}{2e^3}+\frac{x(-2acde^2-c^2d^3)}{e^4}+\frac{(ae^2+cd^2)^2\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^4}}{\sqrt{f}} \end{cases}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)/(g*x+f)**(1/2),x)
```

output
```
Piecewise((2*(c**2*(f + g*x)**(7/2)/(7*e*g**3) + (f + g*x)**(5/2)*(-c**2*d
*g - 3*c**2*e*f)/(5*e**2*g**3) + (f + g*x)**(3/2)*(2*a*c*e**2*g**2 + c**2*
d**2*g**2 + 2*c**2*d*e*f*g + 3*c**2*e**2*f**2)/(3*e**3*g**3) + sqrt(f + g*
x)*(-2*a*c*d*e**2*g**3 - 2*a*c*e**3*f*g**2 - c**2*d**3*g**3 - c**2*d**2*e*
f*g**2 - c**2*d*e**2*f**2*g - c**2*e**3*f**3)/(e**4*g**3) + g*(a*e**2 + c*
d**2)**2*atan(sqrt(f + g*x)/sqrt((d*g - e*f)/e))/(e**5*sqrt((d*g - e*f)/e)
))/g, Ne(g, 0)), ((-c**2*d*x**3/(3*e**2) + c**2*x**4/(4*e) + x**2*(2*a*c*e
**2 + c**2*d**2)/(2*e**3) + x*(-2*a*c*d*e**2 - c**2*d**3)/e**4 + (a*e**2 +
 c*d**2)**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**4)/sqrt(
f), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^2}{(d+ex)\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.53

$$\int \frac{(a+cx^2)^2}{(d+ex)\sqrt{f+gx}}\,dx = \frac{2\,(c^2d^4+2\,acd^2e^2+a^2e^4)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{\sqrt{-e^2f+deg}e^4} + \frac{2\left(15\,(gx+f)^{\frac{7}{2}}c^2e^6g^{24}-63\,(gx+f)^{\frac{5}{2}}c^2e^6fg^{24}+105\,(gx+f)^{\frac{3}{2}}c^2e^6f^2g^{24}-105\,\sqrt{gx+f}c^2e^6f^3g^{24}-\right.}{}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
2*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f +
 d*e*g))/(sqrt(-e^2*f + d*e*g)*e^4) + 2/105*(15*(g*x + f)^(7/2)*c^2*e^6*g^
24 - 63*(g*x + f)^(5/2)*c^2*e^6*f*g^24 + 105*(g*x + f)^(3/2)*c^2*e^6*f^2*g
^24 - 105*sqrt(g*x + f)*c^2*e^6*f^3*g^24 - 21*(g*x + f)^(5/2)*c^2*d*e^5*g^
25 + 70*(g*x + f)^(3/2)*c^2*d*e^5*f*g^25 - 105*sqrt(g*x + f)*c^2*d*e^5*f^2
*g^25 + 35*(g*x + f)^(3/2)*c^2*d^2*e^4*g^26 + 70*(g*x + f)^(3/2)*a*c*e^6*g
^26 - 105*sqrt(g*x + f)*c^2*d^2*e^4*f*g^26 - 210*sqrt(g*x + f)*a*c*e^6*f*g
^26 - 105*sqrt(g*x + f)*c^2*d^3*e^3*g^27 - 210*sqrt(g*x + f)*a*c*d*e^5*g^2
7)/(e^7*g^28)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.75

$$\int \frac{(a+cx^2)^2}{(d+ex)\sqrt{f+gx}}\,dx$$
$$= (f+g\,x)^{3/2}\left(\frac{12\,c^2\,f^2+4\,a\,c\,g^2}{3\,e\,g^4}+\frac{(d\,g^5-e\,f\,g^4)\,\left(\frac{8\,c^2\,f}{e\,g^4}+\frac{2\,c^2\,(d\,g^5-e\,f\,g^4)}{e^2\,g^8}\right)}{3\,e\,g^4}\right)$$
$$-(f+g\,x)^{5/2}\left(\frac{8\,c^2\,f}{5\,e\,g^4}+\frac{2\,c^2\,(d\,g^5-e\,f\,g^4)}{5\,e^2\,g^8}\right)$$
$$-\sqrt{f+g\,x}\left(\frac{8\,c^2\,f^3+8\,a\,c\,f\,g^2}{e\,g^4}+\frac{\left(\frac{12\,c^2\,f^2+4\,a\,c\,g^2}{e\,g^4}+\frac{(d\,g^5-e\,f\,g^4)\,\left(\frac{8\,c^2\,f}{e\,g^4}+\frac{2\,c^2\,(d\,g^5-e\,f\,g^4)}{e^2\,g^8}\right)}{e\,g^4}\right)(d\,g^5-e\,f\,g^4)}{e\,g^4}\right)$$
$$+\frac{2\,c^2\,(f+g\,x)^{7/2}}{7\,e\,g^4}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}\,(c\,d^2+a\,e^2)^2}{\sqrt{d\,g-e\,f}\,(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4)}\right)\,(c\,d^2+a\,e^2)^2}{e^{9/2}\,\sqrt{d\,g-e\,f}}$$

input
```
int((a + c*x^2)^2/((f + g*x)^(1/2)*(d + e*x)),x)
```

output
```
(f + g*x)^(3/2)*((12*c^2*f^2 + 4*a*c*g^2)/(3*e*g^4) + ((d*g^5 - e*f*g^4)*(
(8*c^2*f)/(e*g^4) + (2*c^2*(d*g^5 - e*f*g^4))/(e^2*g^8)))/(3*e*g^4)) - (f
+ g*x)^(5/2)*((8*c^2*f)/(5*e*g^4) + (2*c^2*(d*g^5 - e*f*g^4))/(5*e^2*g^8))
 - (f + g*x)^(1/2)*((8*c^2*f^3 + 8*a*c*f*g^2)/(e*g^4) + (((12*c^2*f^2 + 4*
a*c*g^2)/(e*g^4) + ((d*g^5 - e*f*g^4)*((8*c^2*f)/(e*g^4) + (2*c^2*(d*g^5 -
 e*f*g^4))/(e^2*g^8)))/(e*g^4))*(d*g^5 - e*f*g^4))/(e*g^4)) + (2*c^2*(f +
g*x)^(7/2))/(7*e*g^4) + (2*atan((e^(1/2)*(f + g*x)^(1/2)*(a*e^2 + c*d^2)^2
)/((d*g - e*f)^(1/2)*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))*(a*e^2 + c*d^2)
^2)/(e^(9/2)*(d*g - e*f)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.49

$$\int \frac{(a+cx^2)^2}{(d+ex)\sqrt{f+gx}}\,dx$$
$$= \frac{2\sqrt{e}\,\sqrt{dg-ef}\,atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-ef}}\right)a^2e^4g^4+4\sqrt{e}\,\sqrt{dg-ef}\,atan\left(\frac{\sqrt{gx+f}\,e}{\sqrt{e}\,\sqrt{dg-ef}}\right)ac\,d^2e^2g^4+2\sqrt{e}\,\sqrt{dg-ef}\,ata}{}$$

input
```
int((c*x^2+a)^2/(e*x+d)/(g*x+f)^(1/2),x)
```

output
```
(2*(105*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g -
 e*f)))*a**2*e**4*g**4 + 210*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e
)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**2*e**2*g**4 + 105*sqrt(e)*sqrt(d*g - e
*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**4*g**4 - 210
*sqrt(f + g*x)*a*c*d**2*e**3*g**4 + 70*sqrt(f + g*x)*a*c*d*e**4*f*g**3 + 7
0*sqrt(f + g*x)*a*c*d*e**4*g**4*x + 140*sqrt(f + g*x)*a*c*e**5*f**2*g**2 -
 70*sqrt(f + g*x)*a*c*e**5*f*g**3*x - 105*sqrt(f + g*x)*c**2*d**4*e*g**4 +
 35*sqrt(f + g*x)*c**2*d**3*e**2*f*g**3 + 35*sqrt(f + g*x)*c**2*d**3*e**2*
g**4*x + 14*sqrt(f + g*x)*c**2*d**2*e**3*f**2*g**2 - 7*sqrt(f + g*x)*c**2*
d**2*e**3*f*g**3*x - 21*sqrt(f + g*x)*c**2*d**2*e**3*g**4*x**2 + 8*sqrt(f
+ g*x)*c**2*d*e**4*f**3*g - 4*sqrt(f + g*x)*c**2*d*e**4*f**2*g**2*x + 3*sq
rt(f + g*x)*c**2*d*e**4*f*g**3*x**2 + 15*sqrt(f + g*x)*c**2*d*e**4*g**4*x*
*3 + 48*sqrt(f + g*x)*c**2*e**5*f**4 - 24*sqrt(f + g*x)*c**2*e**5*f**3*g*x
 + 18*sqrt(f + g*x)*c**2*e**5*f**2*g**2*x**2 - 15*sqrt(f + g*x)*c**2*e**5*
f*g**3*x**3))/(105*e**5*g**4*(d*g - e*f))
```

3.82 $\int \frac{(a+cx^2)^2}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 224

$$\int \frac{(a+cx^2)^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \frac{2c(2ae^2g^2+c(e^2f^2+2defg+3d^2g^2))\sqrt{f+gx}}{e^4g^3} - \frac{(cd^2+ae^2)^2\sqrt{f+gx}}{e^4(ef-dg)(d+ex)} - \frac{4c^2(ef+dg)(f+gx)^{3/2}}{3e^3g^3} + \frac{2c^2(f+gx)^{5/2}}{5e^2g^3} + \frac{(cd^2+ae^2)\,(ae^2g+cd(8ef-7dg))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{9/2}(ef-dg)^{3/2}}$$

output

```
2*c*(2*a*e^2*g^2+c*(3*d^2*g^2+2*d*e*f*g+e^2*f^2))*(g*x+f)^(1/2)/e^4/g^3-(a
*e^2+c*d^2)^2*(g*x+f)^(1/2)/e^4/(-d*g+e*f)/(e*x+d)-4/3*c^2*(d*g+e*f)*(g*x+
f)^(3/2)/e^3/g^3+2/5*c^2*(g*x+f)^(5/2)/e^2/g^3+(a*e^2+c*d^2)*(a*e^2*g+c*d*
(-7*d*g+8*e*f))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(9/2)/(-
d*g+e*f)^(3/2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.25

$$\int \frac{(a+cx^2)^2}{(d+ex)^2\sqrt{f+gx}}\,dx$$
$$= \frac{\sqrt{f+gx}(-15a^2e^4g^3+30ace^2g^2(-3d^2g+2e^2fx+2de(f-gx))+c^2(-105d^4g^3+10d^3eg^2(5f-7gx)+}{15e^4g^3(ef-}$$
$$+\frac{(cd^2+ae^2)(ae^2g+cd(8ef-7dg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{9/2}(-ef+dg)^{3/2}}$$

input
```
Integrate[(a + c*x^2)^2/((d + e*x)^2*Sqrt[f + g*x]),x]
```

output
```
(Sqrt[f + g*x]*(-15*a^2*e^4*g^3 + 30*a*c*e^2*g^2*(-3*d^2*g + 2*e^2*f*x + 2
*d*e*(f - g*x)) + c^2*(-105*d^4*g^3 + 10*d^3*e*g^2*(5*f - 7*g*x) + 2*e^4*f
*x*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 2*d^2*e^2*g*(12*f^2 + 19*f*g*x + 7*g^2*
x^2) + 2*d*e^3*(8*f^3 + 8*f^2*g*x - 3*f*g^2*x^2 - 3*g^3*x^3))))/(15*e^4*g^
3*(e*f - d*g)*(d + e*x)) + ((c*d^2 + a*e^2)*(a*e^2*g + c*d*(8*e*f - 7*d*g)
)*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(e^(9/2)*(-(e*f) + d
*g)^(3/2))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {649, 1471, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)^2\sqrt{f+gx}}\,dx$$
$$\Big\downarrow \; 649$$
$$\frac{2\int \frac{(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)^2}{(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{g^3}$$

$\downarrow$ 1471

$$2\left(\frac{g^4\sqrt{f+gx}(ae^2+cd^2)^2}{2e^4(ef-dg)(-dg-e(f+gx)+ef)}-\frac{\int -\frac{a^2g^4-2c^2\left(f-\frac{dg}{e}\right)(f+gx)^3+\frac{2c^2(ef-dg)(3ef+dg)(f+gx)^2}{e^2}+c^2\left(2f^4-\frac{d^4g^4}{e^4}\right)+ac\left(4f^2g^2-\frac{2d^2g^4}{e^2}\right)-\frac{2c(ef-}{}}{ef-dg-e(f+gx)}}{2(ef-dg)}\right.$$
$$\overline{\qquad g^3 \qquad}$$

$\downarrow$ 25

$$2\left(\frac{\int \frac{a^2g^4-2c^2\left(f-\frac{dg}{e}\right)(f+gx)^3+\frac{2c^2(ef-dg)(3ef+dg)(f+gx)^2}{e^2}+c^2\left(2f^4-\frac{d^4g^4}{e^4}\right)+ac\left(4f^2g^2-\frac{2d^2g^4}{e^2}\right)-\frac{2c(ef-dg)\left(2ae^2g^2+c\left(3e^2f^2+2degf+d^2g^2\right)\right)(f+}{e^3}}{ef-dg-e(f+gx)}}{2(ef-dg)}\right.$$
$$\overline{\qquad g^3 \qquad}$$

$\downarrow$ 2341

$$2\left(\frac{\int\left(\frac{2(ef-dg)(f+gx)^2c^2}{e^2}-\frac{4(ef-dg)(ef+dg)(f+gx)c^2}{e^3}+\frac{2(ef-dg)\left(2ae^2g^2+c\left(e^2f^2+2degf+3d^2g^2\right)\right)c}{e^4}+\frac{-7c^2d^4g^4+a^2e^4g^4-6acd^2e^2g^4+8acde^3fg^3+8c^2}{e^4(ef-dg-e(f+gx))}\right.}{2(ef-dg)}\right.$$
$$\overline{\qquad g^3 \qquad}$$

$\downarrow$ 2009

$$2\left(\frac{\frac{g^3\left(ae^2+cd^2\right)\left(ae^2g+cd(8ef-7dg)\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{9/2}\sqrt{ef-dg}}+\frac{2c\sqrt{f+gx}(ef-dg)\left(2ae^2g^2+c\left(3d^2g^2+2defg+e^2f^2\right)\right)}{e^4}-\frac{4c^2(f+gx)^{3/2}(ef-dg)(dg+ef)}{3e^3}+\frac{2}{}}{2(ef-dg)}\right.$$
$$\overline{\qquad g^3 \qquad}$$

input
```
Int[(a + c*x^2)^2/((d + e*x)^2*Sqrt[f + g*x]),x]
```

output
```
(2*(((c*d^2 + a*e^2)^2*g^4*Sqrt[f + g*x])/(2*e^4*(e*f - d*g)*(e*f - d*g -
e*(f + g*x))) + ((2*c*(e*f - d*g)*(2*a*e^2*g^2 + c*(e^2*f^2 + 2*d*e*f*g +
3*d^2*g^2))*Sqrt[f + g*x])/e^4 - (4*c^2*(e*f - d*g)*(e*f + d*g)*(f + g*x)^
(3/2))/(3*e^3) + (2*c^2*(e*f - d*g)*(f + g*x)^(5/2))/(5*e^2) + ((c*d^2 + a
*e^2)*g^3*(a*e^2*g + c*d*(8*e*f - 7*d*g))*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/
Sqrt[e*f - d*g]])/(e^(9/2)*Sqrt[e*f - d*g]))/(2*(e*f - d*g))))/g^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96

method	result
risch	$\frac{2c\left(3c\,x^2e^2g^2-10cde\,g^2x-4c\,e^2fgx+30a\,e^2g^2+45c\,d^2g^2+20cdefg+8c\,e^2f^2\right)\sqrt{gx+f}}{15g^3e^4} - \frac{\left(2a\,e^2+2c\,d^2\right)\left(-\frac{g\left(a\,e^2+ \cdots\right.}{2(dg-ef)(e \cdots}\right.}{\cdots}$
pseudoelliptic	$\frac{\sqrt{(dg-ef)e}\left(\left(\left(7d\left(\frac{2}{35}e^3x^3-\frac{2}{15}d\,e^2x^2+\frac{2}{3}d^2ex+d^3\right)g^3-\frac{10ef(ex+d)\left(\frac{3}{25}e^2x^2-\frac{6}{25}dex+d^2\right)g^2}{3}-\frac{8e^2f^2\left(-\frac{ex}{3}+d\right)(ex+d)g}{5}-\cdots\right.\right.\right.}{\sqrt{(d \cdots}}$
derivativedivides	$\frac{\frac{2c\left(\frac{c(gx+f)^{\frac{5}{2}}e^2}{5}-\frac{2cdeg(gx+f)^{\frac{3}{2}}}{3}-\frac{2c\,e^2f(gx+f)^{\frac{3}{2}}}{3}+2a\,e^2g^2\sqrt{gx+f}+3c\,d^2g^2\sqrt{gx+f}+2\sqrt{gx+f}\,cdefg+c\,e^2f^2\sqrt{gx+f}\right)}{e^4}+\frac{2g^3\left(\cdots\right.}{\cdots}}{g^3}$
default	$\frac{\frac{2c\left(\frac{c(gx+f)^{\frac{5}{2}}e^2}{5}-\frac{2cdeg(gx+f)^{\frac{3}{2}}}{3}-\frac{2c\,e^2f(gx+f)^{\frac{3}{2}}}{3}+2a\,e^2g^2\sqrt{gx+f}+3c\,d^2g^2\sqrt{gx+f}+2\sqrt{gx+f}\,cdefg+c\,e^2f^2\sqrt{gx+f}\right)}{e^4}+\frac{2g^3\left(\cdots\right.}{\cdots}}{g^3}$

input
```
int((c*x^2+a)^2/(e*x+d)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*c*(3*c*e^2*g^2*x^2-10*c*d*e*g^2*x-4*c*e^2*f*g*x+30*a*e^2*g^2+45*c*d^2
*g^2+20*c*d*e*f*g+8*c*e^2*f^2)*(g*x+f)^(1/2)/g^3/e^4-1/e^4*(2*a*e^2+2*c*d^
2)*(-1/2*g*(a*e^2+c*d^2)/(d*g-e*f)*(g*x+f)^(1/2)/(e*(g*x+f)+d*g-e*f)-1/2*(
a*e^2*g-7*c*d^2*g+8*c*d*e*f)/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f
)^(1/2)/((d*g-e*f)*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(202) = 404$.

Time = 0.15 (sec) , antiderivative size = 1157, normalized size of antiderivative = 5.17

$$\int \frac{(a+cx^2)^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[-1/30*(15*(8*(c^2*d^4*e + a*c*d^2*e^3)*f*g^3 - (7*c^2*d^5 + 6*a*c*d^3*e^2
 - a^2*d*e^4)*g^4 + (8*(c^2*d^3*e^2 + a*c*d*e^4)*f*g^3 - (7*c^2*d^4*e + 6*
a*c*d^2*e^3 - a^2*e^5)*g^4)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*
g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(16*c^2*d*e^5*f^4
+ 8*c^2*d^2*e^4*f^3*g + 2*(13*c^2*d^3*e^3 + 30*a*c*d*e^5)*f^2*g^2 - 5*(31*
c^2*d^4*e^2 + 30*a*c*d^2*e^4 + 3*a^2*e^6)*f*g^3 + 15*(7*c^2*d^5*e + 6*a*c*
d^3*e^3 + a^2*d*e^5)*g^4 + 6*(c^2*e^6*f^2*g^2 - 2*c^2*d*e^5*f*g^3 + c^2*d^
2*e^4*g^4)*x^3 - 2*(4*c^2*e^6*f^3*g - c^2*d*e^5*f^2*g^2 - 10*c^2*d^2*e^4*f
*g^3 + 7*c^2*d^3*e^3*g^4)*x^2 + 2*(8*c^2*e^6*f^4 + (11*c^2*d^2*e^4 + 30*a*
c*e^6)*f^2*g^2 - 6*(9*c^2*d^3*e^3 + 10*a*c*d*e^5)*f*g^3 + 5*(7*c^2*d^4*e^2
 + 6*a*c*d^2*e^4)*g^4)*x)*sqrt(g*x + f))/(d*e^7*f^2*g^3 - 2*d^2*e^6*f*g^4
+ d^3*e^5*g^5 + (e^8*f^2*g^3 - 2*d*e^7*f*g^4 + d^2*e^6*g^5)*x), -1/15*(15*
(8*(c^2*d^4*e + a*c*d^2*e^3)*f*g^3 - (7*c^2*d^5 + 6*a*c*d^3*e^2 - a^2*d*e^
4)*g^4 + (8*(c^2*d^3*e^2 + a*c*d*e^4)*f*g^3 - (7*c^2*d^4*e + 6*a*c*d^2*e^3
 - a^2*e^5)*g^4)*x)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(
g*x + f)/(e*g*x + e*f)) - (16*c^2*d*e^5*f^4 + 8*c^2*d^2*e^4*f^3*g + 2*(13*
c^2*d^3*e^3 + 30*a*c*d*e^5)*f^2*g^2 - 5*(31*c^2*d^4*e^2 + 30*a*c*d^2*e^4 +
 3*a^2*e^6)*f*g^3 + 15*(7*c^2*d^5*e + 6*a*c*d^3*e^3 + a^2*d*e^5)*g^4 + 6*(
c^2*e^6*f^2*g^2 - 2*c^2*d*e^5*f*g^3 + c^2*d^2*e^4*g^4)*x^3 - 2*(4*c^2*e^6*
f^3*g - c^2*d*e^5*f^2*g^2 - 10*c^2*d^2*e^4*f*g^3 + 7*c^2*d^3*e^3*g^4)*x...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + cx^2)^2}{(d + ex)^2 \sqrt{f + gx}} \, dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)**2/(g*x+f)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.47

$$\int \frac{(a+cx^2)^2}{(d+ex)^2\sqrt{f+gx}}\,dx$$
$$= -\frac{(8\,c^2d^3ef + 8\,acde^3f - 7\,c^2d^4g - 6\,acd^2e^2g + a^2e^4g)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^5f - de^4g)\sqrt{-e^2f+deg}}$$
$$- \frac{\sqrt{gx+f}c^2d^4g + 2\sqrt{gx+f}acd^2e^2g + \sqrt{gx+f}a^2e^4g}{(e^5f - de^4g)((gx+f)e - ef + dg)}$$
$$+ \frac{2\left(3\,(gx+f)^{\frac{5}{2}}c^2e^8g^{12} - 10\,(gx+f)^{\frac{3}{2}}c^2e^8fg^{12} + 15\sqrt{gx+f}c^2e^8f^2g^{12} - 10\,(gx+f)^{\frac{3}{2}}c^2de^7g^{13} + 30\,\sqrt{}\right.}{15\,e^{10}g^{15}}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
-(8*c^2*d^3*e*f + 8*a*c*d*e^3*f - 7*c^2*d^4*g - 6*a*c*d^2*e^2*g + a^2*e^4*
g)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/((e^5*f - d*e^4*g)*sqrt(-e
^2*f + d*e*g)) - (sqrt(g*x + f)*c^2*d^4*g + 2*sqrt(g*x + f)*a*c*d^2*e^2*g
+ sqrt(g*x + f)*a^2*e^4*g)/((e^5*f - d*e^4*g)*((g*x + f)*e - e*f + d*g)) +
 2/15*(3*(g*x + f)^(5/2)*c^2*e^8*g^12 - 10*(g*x + f)^(3/2)*c^2*e^8*f*g^12
+ 15*sqrt(g*x + f)*c^2*e^8*f^2*g^12 - 10*(g*x + f)^(3/2)*c^2*d*e^7*g^13 +
30*sqrt(g*x + f)*c^2*d*e^7*f*g^13 + 45*sqrt(g*x + f)*c^2*d^2*e^6*g^14 + 30
*sqrt(g*x + f)*a*c*e^8*g^14)/(e^10*g^15)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.68

$$\int \frac{(a+cx^2)^2}{(d+ex)^2\sqrt{f+gx}}\,dx$$
$$= \sqrt{f+g\,x}\left(\frac{12\,c^2\,f^2+4\,a\,c\,g^2}{e^2\,g^3}+\frac{2\,(d\,g-e\,f)\,\left(\frac{8\,c^2\,f}{e^2\,g^3}+\frac{4\,c^2\,(d\,g-e\,f)}{e^3\,g^3}\right)}{e}-\frac{2\,c^2\,(d\,g-e\,f)^2}{e^4\,g^3}\right)-(f+g\,x)^{3/2}\left(\frac{8\,c^2\,f}{3\,e^2\,g^3}+\frac{4\,c^2\,(d\,g-e\,f)}{3\,e^3\,g^3}\right)$$
$$+\frac{2\,c^2\,(f+g\,x)^{5/2}}{5\,e^2\,g^3}+\frac{\sqrt{f+g\,x}\,(g\,a^2\,e^4+2\,g\,a\,c\,d^2\,e^2+g\,c^2\,d^4)}{(d\,g-e\,f)\,(e^5\,(f+g\,x)-e^5\,f+d\,e^4\,g)}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}\,(c\,d^2+a\,e^2)\,(-7\,c\,g\,d^2+8\,c\,f\,d\,e+a\,g\,e^2)}{\sqrt{d\,g-e\,f}\,(g\,a^2\,e^4-6\,g\,a\,c\,d^2\,e^2+8\,f\,a\,c\,d\,e^3-7\,g\,c^2\,d^4+8\,f\,c^2\,d^3\,e)}\right)\,(c\,d^2+a\,e^2)\,(-7\,c\,g\,d^2+8\,c\,f\,d\,e+a\,g\,e^2)}{e^{9/2}\,(d\,g-e\,f)^{3/2}}$$

input
```
int((a + c*x^2)^2/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

output
```
(f + g*x)^(1/2)*((12*c^2*f^2 + 4*a*c*g^2)/(e^2*g^3) + (2*(d*g - e*f)*((8*c
^2*f)/(e^2*g^3) + (4*c^2*(d*g - e*f))/(e^3*g^3)))/e - (2*c^2*(d*g - e*f)^2
)/(e^4*g^3)) - (f + g*x)^(3/2)*((8*c^2*f)/(3*e^2*g^3) + (4*c^2*(d*g - e*f)
)/(3*e^3*g^3)) + (2*c^2*(f + g*x)^(5/2))/(5*e^2*g^3) + ((f + g*x)^(1/2)*(a
^2*e^4*g + c^2*d^4*g + 2*a*c*d^2*e^2*g))/((d*g - e*f)*(e^5*(f + g*x) - e^5
*f + d*e^4*g)) + (atan((e^(1/2)*(f + g*x)^(1/2)*(a*e^2 + c*d^2)*(a*e^2*g -
 7*c*d^2*g + 8*c*d*e*f))/((d*g - e*f)^(1/2)*(a^2*e^4*g - 7*c^2*d^4*g + 8*c
^2*d^3*e*f - 6*a*c*d^2*e^2*g + 8*a*c*d*e^3*f)))*(a*e^2 + c*d^2)*(a*e^2*g -
 7*c*d^2*g + 8*c*d*e*f))/(e^(9/2)*(d*g - e*f)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1042, normalized size of antiderivative = 4.65

$$\int \frac{(a+cx^2)^2}{(d+ex)^2\sqrt{f+gx}}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+a)^2/(e*x+d)^2/(g*x+f)^(1/2),x)
```

output

```
(15*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f
)))*a**2*d*e**4*g**4 + 15*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(
sqrt(e)*sqrt(d*g - e*f)))*a**2*e**5*g**4*x - 90*sqrt(e)*sqrt(d*g - e*f)*at
an((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**3*e**2*g**4 + 120*s
qrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a
*c*d**2*e**3*f*g**3 - 90*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(s
qrt(e)*sqrt(d*g - e*f)))*a*c*d**2*e**3*g**4*x + 120*sqrt(e)*sqrt(d*g - e*f
)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d*e**4*f*g**3*x -
105*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f
)))*c**2*d**5*g**4 + 120*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(s
qrt(e)*sqrt(d*g - e*f)))*c**2*d**4*e*f*g**3 - 105*sqrt(e)*sqrt(d*g - e*f)*
atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**4*e*g**4*x + 120
*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))
*c**2*d**3*e**2*f*g**3*x + 15*sqrt(f + g*x)*a**2*d*e**5*g**4 - 15*sqrt(f +
 g*x)*a**2*e**6*f*g**3 + 90*sqrt(f + g*x)*a*c*d**3*e**3*g**4 - 150*sqrt(f
+ g*x)*a*c*d**2*e**4*f*g**3 + 60*sqrt(f + g*x)*a*c*d**2*e**4*g**4*x + 60*s
qrt(f + g*x)*a*c*d*e**5*f**2*g**2 - 120*sqrt(f + g*x)*a*c*d*e**5*f*g**3*x
+ 60*sqrt(f + g*x)*a*c*e**6*f**2*g**2*x + 105*sqrt(f + g*x)*c**2*d**5*e*g*
*4 - 155*sqrt(f + g*x)*c**2*d**4*e**2*f*g**3 + 70*sqrt(f + g*x)*c**2*d**4*
e**2*g**4*x + 26*sqrt(f + g*x)*c**2*d**3*e**3*f**2*g**2 - 108*sqrt(f + ...
```

3.83 $\int \frac{\left(a+cx^2\right)^2}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 282

$$\begin{aligned}\int \frac{\left(a+cx^2\right)^2}{(d+ex)^3\sqrt{f+gx}}\,dx &= -\frac{2c^2(ef+3dg)\sqrt{f+gx}}{e^4g^2}-\frac{\left(cd^2+ae^2\right)^2\sqrt{f+gx}}{2e^4(ef-dg)(d+ex)^2}\\ &+\frac{\left(cd^2+ae^2\right)\left(3ae^2g+cd(16ef-13dg)\right)\sqrt{f+gx}}{4e^4(ef-dg)^2(d+ex)}+\frac{2c^2(f+gx)^{3/2}}{3e^3g^2}\\ &-\frac{\left(3a^2e^4g^2+2ace^2(8e^2f^2-8defg+3d^2g^2)+c^2d^2(48e^2f^2-80defg+35d^2g^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{9/2}(ef-dg)^{5/2}}\end{aligned}$$

output
```
-2*c^2*(3*d*g+e*f)*(g*x+f)^(1/2)/e^4/g^2-1/2*(a*e^2+c*d^2)^2*(g*x+f)^(1/2)
/e^4/(-d*g+e*f)/(e*x+d)^2+1/4*(a*e^2+c*d^2)*(3*a*e^2*g+c*d*(-13*d*g+16*e*f
))*(g*x+f)^(1/2)/e^4/(-d*g+e*f)^2/(e*x+d)+2/3*c^2*(g*x+f)^(3/2)/e^3/g^2-1/
4*(3*a^2*e^4*g^2+2*a*c*e^2*(3*d^2*g^2-8*d*e*f*g+8*e^2*f^2)+c^2*d^2*(35*d^2
*g^2-80*d*e*f*g+48*e^2*f^2))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2
))/e^(9/2)/(-d*g+e*f)^(5/2)
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.28

$$\int \frac{(a+cx^2)^2}{(d+ex)^3\sqrt{f+gx}}\,dx =$$
$$-\frac{\sqrt{f+gx}(-3a^2e^4g^2(-2ef+5dg+3egx)+6acde^2g^2(-6def+3d^2g-8e^2fx+5degx)+c^2(105d^5g^3}{}$$
$$+\frac{(3a^2e^4g^2+2ace^2(8e^2f^2-8defg+3d^2g^2)+c^2d^2(48e^2f^2-80defg+35d^2g^2))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{4e^{9/2}(-ef+dg)^{5/2}}$$

input
```
Integrate[(a + c*x^2)^2/((d + e*x)^3*Sqrt[f + g*x]),x]
```

output
```
-1/12*(Sqrt[f + g*x]*(-3*a^2*e^4*g^2*(-2*e*f + 5*d*g + 3*e*g*x) + 6*a*c*d*
e^2*g^2*(-6*d*e*f + 3*d^2*g - 8*e^2*f*x + 5*d*e*g*x) + c^2*(105*d^5*g^3 +
8*e^5*f^2*x^2*(2*f - g*x) + 5*d^4*e*g^2*(-34*f + 35*g*x) + 8*d*e^4*f*x*(4*
f^2 + 3*f*g*x + 2*g^2*x^2) + 8*d^3*e^2*g*(5*f^2 - 36*f*g*x + 7*g^2*x^2) +
8*d^2*e^3*(2*f^3 + 9*f^2*g*x - 12*f*g^2*x^2 - g^3*x^3))))/(e^4*g^2*(e*f -
d*g)^2*(d + e*x)^2) + ((3*a^2*e^4*g^2 + 2*a*c*e^2*(8*e^2*f^2 - 8*d*e*f*g +
 3*d^2*g^2) + c^2*d^2*(48*e^2*f^2 - 80*d*e*f*g + 35*d^2*g^2))*ArcTan[(Sqrt
[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(4*e^(9/2)*(-(e*f) + d*g)^(5/2))
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {649, 25, 1471, 25, 2345, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)^3\sqrt{f+gx}}\,dx$$
$$\Big\downarrow \text{ 649}$$

$$\frac{2\int -\frac{\left(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2\right)^2}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2\int \frac{\left(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2\right)^2}{(ef-dg-e(f+gx))^3}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \text{ 1471}$$

$$\frac{2\left(\frac{\int -\frac{3a^2g^4-4c^2\left(f-\frac{dg}{e}\right)(f+gx)^3+\frac{4c^2(ef-dg)(3ef+dg)(f+gx)^2}{e^2}+c^2\left(4f^4-\frac{d^4g^4}{e^4}\right)+ac\left(8f^2g^2-\frac{2d^2g^4}{e^2}\right)-\frac{4c(ef-dg)\left(2ae^2g^2+c\left(3e^2f^2+2degf+d^2g^2\right)\right)(\cdots}{e^3}\cdots}{(ef-dg-e(f+gx))^2}\cdots}{4(ef-dg)}\cdots\right.}{g^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\left(-\frac{\int \frac{3a^2g^4-4c^2\left(f-\frac{dg}{e}\right)(f+gx)^3+\frac{4c^2(ef-dg)(3ef+dg)(f+gx)^2}{e^2}+c^2\left(4f^4-\frac{d^4g^4}{e^4}\right)+ac\left(8f^2g^2-\frac{2d^2g^4}{e^2}\right)-\frac{4c(ef-dg)\left(2ae^2g^2+c\left(3e^2f^2+2degf+d^2g^2\right)\right)(\cdots}{e^3}\cdots}{(ef-dg-e(f+gx))^2}\cdots}{4(ef-dg)}\cdots\right.}{g^2}$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{2\left(-\frac{\frac{g^3\sqrt{f+gx}\left(ae^2+cd^2\right)\left(3ae^2g+cd(16ef-13dg)\right)}{2e^4(ef-dg)(-dg-e(f+gx)+ef)}-\frac{\int -\frac{3a^2g^4+\frac{2ac\left(8e^2f^2-8degf+3d^2g^2\right)g^2}{e^2}+\frac{8c^2(ef-dg)^2(f+gx)^2}{e^2}+c^2\left(8f^4-\frac{16d^3g^3f}{e^3}+\frac{11d^4g^4}{e^4}\right)-\frac{16\cdots}{\cdots}}{ef-dg-e(f+gx)}\cdots}{2(ef-dg)}}{4(ef-dg)}\cdots\right.}{g^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\left(-\frac{\frac{\int \frac{3a^2g^4+\frac{2ac\left(8e^2f^2-8degf+3d^2g^2\right)g^2}{e^2}+\frac{8c^2(ef-dg)^2(f+gx)^2}{e^2}+\frac{c^2\left(8e^4f^4-16d^3eg^3f+11d^4g^4\right)}{e^4}-\frac{16c^2(ef-dg)^2(ef+dg)(f+gx)}{e^3}}{ef-dg-e(f+gx)}d\sqrt{f+gx}}{2(ef-dg)}+\frac{g^3\sqrt{f+gx}\cdots}{2e\cdots}}{4(ef-dg)}\cdots\right.}{g^2}$$

$$\Big\downarrow \text{ 1467}$$

$$\frac{2\left(-\frac{\frac{\int \left(\frac{8c^2(ef+3dg)(ef-dg)^2}{e^4}-\frac{8c^2(f+gx)(ef-dg)^2}{e^3}+\frac{3a^2g^4e^4+16acf^2g^2e^4-16acdfg^3e^3+6acd^2g^4e^2+48c^2d^2f^2g^2e^2-80c^2d^3fg^3e+35c^2d^4g^4}{e^4(ef-dg-e(f+gx))}\right)d\sqrt{f+gx}}{2(ef-dg)} -}{4(ef-dg)}\right.}{g^2}$$

$\downarrow$ 2009

$$\frac{2\left(-\frac{\frac{\frac{g^2\left(3a^2e^4g^2+2ace^2\left(3d^2g^2-8defg+8e^2f^2\right)+c^2d^2\left(35d^2g^2-80defg+48e^2f^2\right)\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{9/2}\sqrt{ef-dg}}+\frac{8c^2\sqrt{f+gx}(ef-dg)^2(3dg+ef)}{e^4}-\frac{8c^2(f+gx)^{3/}}{3e}}{2(ef-dg)}}{4(ef-dg)}\right.}{g^2}$$

input
```
Int[(a + c*x^2)^2/((d + e*x)^3*Sqrt[f + g*x]),x]
```

output
```
(2*(-1/4*((c*d^2 + a*e^2)^2*g^4*Sqrt[f + g*x])/(e^4*(e*f - d*g)*(e*f - d*g
 - e*(f + g*x))^2) - (((c*d^2 + a*e^2)*g^3*(3*a*e^2*g + c*d*(16*e*f - 13*d
*g))*Sqrt[f + g*x])/(2*e^4*(e*f - d*g)*(e*f - d*g - e*(f + g*x))) + ((8*c^
2*(e*f - d*g)^2*(e*f + 3*d*g)*Sqrt[f + g*x])/e^4 - (8*c^2*(e*f - d*g)^2*(f
 + g*x)^(3/2))/(3*e^3) + (g^2*(3*a^2*e^4*g^2 + 2*a*c*e^2*(8*e^2*f^2 - 8*d*
e*f*g + 3*d^2*g^2) + c^2*d^2*(48*e^2*f^2 - 80*d*e*f*g + 35*d^2*g^2))*ArcTa
nh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(9/2)*Sqrt[e*f - d*g]))/(2
*(e*f - d*g)))/(4*(e*f - d*g))))/g^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
 - d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{2c^2(-egx+9dg+2ef)\sqrt{gx+f}}{3g^2e^4}+\frac{\frac{2eg\left(3a^2e^4g-10ac\,d^2e^2g+16acd\,e^3f-13c^2d^4g+16c^2d^3ef\right)(gx+f)^{\frac{3}{2}}}{8d^2g^2-16defg+8e^2f^2}+\frac{2\left(5a^2e^4g-6ac\,d^2e\right.}{}}{(e(gx+f)+dg-ef)^2}$
pseudoelliptic	$\frac{5\left(\left(\left(-7\left(-\frac{8}{105}e^3x^3+\frac{8}{15}d\,e^2x^2+\frac{5}{3}d^2ex+d^3\right)d^2c^2-\frac{6\left(\frac{5ex}{3}+d\right)e^2a\,d^2c}{5}+a^2e^4\left(\frac{3ex}{5}+d\right)\right)g^3-\frac{2ef\left(\left(\frac{8}{3}d\,e^3x^3-16d^2e^2x^2-48d\right.\right.}{}\right.\right.}{}$
derivativedivides	$\frac{-\frac{2c^2\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+3dg\sqrt{gx+f}+ef\sqrt{gx+f}\right)}{e^4}+\frac{2g^2\left(\frac{\frac{eg\left(3a^2e^4g-10ac\,d^2e^2g+16acd\,e^3f-13c^2d^4g+16c^2d^3ef\right)(gx+f)^{\frac{3}{2}}}{8d^2g^2-16defg+8e^2f^2}+\frac{\left(5a^2\right.}{}}{(e(gx+f)+dg-ef)}\right.}{}}{}$
default	$\frac{-\frac{2c^2\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+3dg\sqrt{gx+f}+ef\sqrt{gx+f}\right)}{e^4}+\frac{2g^2\left(\frac{\frac{eg\left(3a^2e^4g-10ac\,d^2e^2g+16acd\,e^3f-13c^2d^4g+16c^2d^3ef\right)(gx+f)^{\frac{3}{2}}}{8d^2g^2-16defg+8e^2f^2}+\frac{\left(5a^2\right.}{}}{(e(gx+f)+dg-ef)}\right.}{}}{}$

input
```
int((c*x^2+a)^2/(e*x+d)^3/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*c^2*(-e*g*x+9*d*g+2*e*f)*(g*x+f)^(1/2)/g^2/e^4+1/e^4*(2*(1/8*e*g*(3*a
^2*e^4*g-10*a*c*d^2*e^2*g+16*a*c*d*e^3*f-13*c^2*d^4*g+16*c^2*d^3*e*f)/(d^2
*g^2-2*d*e*f*g+e^2*f^2)*(g*x+f)^(3/2)+1/8*(5*a^2*e^4*g-6*a*c*d^2*e^2*g+16*
a*c*d*e^3*f-11*c^2*d^4*g+16*c^2*d^3*e*f)*g/(d*g-e*f)*(g*x+f)^(1/2))/(e*(g*
x+f)+d*g-e*f)^2+1/4*(3*a^2*e^4*g^2+6*a*c*d^2*e^2*g^2-16*a*c*d*e^3*f*g+16*a
*c*e^4*f^2+35*c^2*d^4*g^2-80*c^2*d^3*e*f*g+48*c^2*d^2*e^2*f^2)/(d^2*g^2-2*
d*e*f*g+e^2*f^2)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^
(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(256) = 512$.

Time = 0.15 (sec) , antiderivative size = 1791, normalized size of antiderivative = 6.35

$$\int \frac{(a+cx^2)^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(16*(3*c^2*d^4*e^2 + a*c*d^2*e^4)*f^2*g^2 - 16*(5*c^2*d^5*e + a*c
*d^3*e^3)*f*g^3 + (35*c^2*d^6 + 6*a*c*d^4*e^2 + 3*a^2*d^2*e^4)*g^4 + (16*(
3*c^2*d^2*e^4 + a*c*e^6)*f^2*g^2 - 16*(5*c^2*d^3*e^3 + a*c*d*e^5)*f*g^3 +
(35*c^2*d^4*e^2 + 6*a*c*d^2*e^4 + 3*a^2*e^6)*g^4)*x^2 + 2*(16*(3*c^2*d^3*e
^3 + a*c*d*e^5)*f^2*g^2 - 16*(5*c^2*d^4*e^2 + a*c*d^2*e^4)*f*g^3 + (35*c^2
*d^5*e + 6*a*c*d^3*e^3 + 3*a^2*d*e^5)*g^4)*x)*sqrt(e^2*f - d*e*g)*log((e*g
*x + 2*e*f - d*g - 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(16
*c^2*d^2*e^5*f^4 + 24*c^2*d^3*e^4*f^3*g - 6*(35*c^2*d^4*e^3 + 6*a*c*d^2*e^
5 - a^2*e^7)*f^2*g^2 + (275*c^2*d^5*e^2 + 54*a*c*d^3*e^4 - 21*a^2*d*e^6)*f
*g^3 - 3*(35*c^2*d^6*e + 6*a*c*d^4*e^3 - 5*a^2*d^2*e^5)*g^4 - 8*(c^2*e^7*f
^3*g - 3*c^2*d*e^6*f^2*g^2 + 3*c^2*d^2*e^5*f*g^3 - c^2*d^3*e^4*g^4)*x^3 +
8*(2*c^2*e^7*f^4 + c^2*d*e^6*f^3*g - 15*c^2*d^2*e^5*f^2*g^2 + 19*c^2*d^3*e
^4*f*g^3 - 7*c^2*d^4*e^3*g^4)*x^2 + (32*c^2*d*e^6*f^4 + 40*c^2*d^2*e^5*f^3
*g - 24*(15*c^2*d^3*e^4 + 2*a*c*d*e^6)*f^2*g^2 + (463*c^2*d^4*e^3 + 78*a*c
*d^2*e^5 - 9*a^2*e^7)*f*g^3 - (175*c^2*d^5*e^2 + 30*a*c*d^3*e^4 - 9*a^2*d*
e^6)*g^4)*x)*sqrt(g*x + f))/(d^2*e^8*f^3*g^2 - 3*d^3*e^7*f^2*g^3 + 3*d^4*e
^6*f*g^4 - d^5*e^5*g^5 + (e^10*f^3*g^2 - 3*d*e^9*f^2*g^3 + 3*d^2*e^8*f*g^4
 - d^3*e^7*g^5)*x^2 + 2*(d*e^9*f^3*g^2 - 3*d^2*e^8*f^2*g^3 + 3*d^3*e^7*f*g
^4 - d^4*e^6*g^5)*x), 1/12*(3*(16*(3*c^2*d^4*e^2 + a*c*d^2*e^4)*f^2*g^2 -
16*(5*c^2*d^5*e + a*c*d^3*e^3)*f*g^3 + (35*c^2*d^6 + 6*a*c*d^4*e^2 + 3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)**3/(g*x+f)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.82

$$\int \frac{(a+cx^2)^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \frac{(48\,c^2d^2e^2f^2 + 16\,ace^4f^2 - 80\,c^2d^3efg - 16\,acde^3fg + 35\,c^2d^4g^2 + 6\,acd^2e^2g^2 + 3\,a^2e^4g^2)\arctan\left(\frac{\sqrt{gx-}}{\sqrt{-e^2f}}\right.}{4\,(e^6f^2 - 2\,de^5fg + d^2e^4g^2)\sqrt{-e^2f+deg}} + \frac{16\,(gx+f)^{\frac{3}{2}}c^2d^3e^2fg + 16\,(gx+f)^{\frac{3}{2}}acde^4fg - 16\,\sqrt{gx+f}c^2d^3e^2f^2g - 16\,\sqrt{gx+f}acde^4f^2g - 13\,(g}{} + \frac{2\left((gx+f)^{\frac{3}{2}}c^2e^6g^4 - 3\,\sqrt{gx+f}c^2e^6fg^4 - 9\,\sqrt{gx+f}c^2de^5g^5\right)}{3\,e^9g^6}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
1/4*(48*c^2*d^2*e^2*f^2 + 16*a*c*e^4*f^2 - 80*c^2*d^3*e*f*g - 16*a*c*d*e^3
*f*g + 35*c^2*d^4*g^2 + 6*a*c*d^2*e^2*g^2 + 3*a^2*e^4*g^2)*arctan(sqrt(g*x
 + f)*e/sqrt(-e^2*f + d*e*g))/((e^6*f^2 - 2*d*e^5*f*g + d^2*e^4*g^2)*sqrt(
-e^2*f + d*e*g)) + 1/4*(16*(g*x + f)^(3/2)*c^2*d^3*e^2*f*g + 16*(g*x + f)^
(3/2)*a*c*d*e^4*f*g - 16*sqrt(g*x + f)*c^2*d^3*e^2*f^2*g - 16*sqrt(g*x + f
)*a*c*d*e^4*f^2*g - 13*(g*x + f)^(3/2)*c^2*d^4*e*g^2 - 10*(g*x + f)^(3/2)*
a*c*d^2*e^3*g^2 + 3*(g*x + f)^(3/2)*a^2*e^5*g^2 + 27*sqrt(g*x + f)*c^2*d^4
*e*f*g^2 + 22*sqrt(g*x + f)*a*c*d^2*e^3*f*g^2 - 5*sqrt(g*x + f)*a^2*e^5*f*
g^2 - 11*sqrt(g*x + f)*c^2*d^5*g^3 - 6*sqrt(g*x + f)*a*c*d^3*e^2*g^3 + 5*s
qrt(g*x + f)*a^2*d*e^4*g^3)/((e^6*f^2 - 2*d*e^5*f*g + d^2*e^4*g^2)*((g*x +
 f)*e - e*f + d*g)^2) + 2/3*((g*x + f)^(3/2)*c^2*e^6*g^4 - 3*sqrt(g*x + f)
*c^2*e^6*f*g^4 - 9*sqrt(g*x + f)*c^2*d*e^5*g^5)/(e^9*g^6)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.40

$$\int \frac{(a+cx^2)^2}{(d+ex)^3\sqrt{f+gx}}\,dx$$
$$= \frac{\frac{\sqrt{f+g\,x}\,\left(5\,a^2\,e^4\,g^2-6\,a\,c\,d^2\,e^2\,g^2+16\,f\,a\,c\,d\,e^3\,g-11\,c^2\,d^4\,g^2+16\,f\,c^2\,d^3\,e\,g\right)}{4\,(d\,g-e\,f)}+\frac{(f+g\,x)^{3/2}\,\left(3\,a^2\,e^5\,g^2-10\,a\,c\,d^2\,e^3\,g^2+16\,f\,a\,c\,d\,e^4\,g-13\,c^2\,d^4\right.}{4\,(d\,g-e\,f)^2}}{e^6\,(f+g\,x)^2-(f+g\,x)\,\left(2\,e^6\,f-2\,d\,e^5\,g\right)+e^6\,f^2+d^2\,e^4\,g^2-2\,d\,e^5\,f\,g}$$
$$-\sqrt{f+g\,x}\,\left(\frac{8\,c^2\,f}{e^3\,g^2}+\frac{6\,c^2\,(d\,g-e\,f)}{e^4\,g^2}\right)+\frac{2\,c^2\,(f+g\,x)^{3/2}}{3\,e^3\,g^2}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,\sqrt{f+g\,x}}{\sqrt{d\,g-e\,f}}\right)\,\left(3\,a^2\,e^4\,g^2+6\,a\,c\,d^2\,e^2\,g^2-16\,a\,c\,d\,e^3\,f\,g+16\,a\,c\,e^4\,f^2+35\,c^2\,d^4\,g^2-80\,c^2\,d^3\,e\,f\,g+\right.}{4\,e^{9/2}\,(d\,g-e\,f)^{5/2}}$$

input
```
int((a + c*x^2)^2/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

output
```
(((f + g*x)^(1/2)*(5*a^2*e^4*g^2 - 11*c^2*d^4*g^2 + 16*c^2*d^3*e*f*g - 6*a
*c*d^2*e^2*g^2 + 16*a*c*d*e^3*f*g))/(4*(d*g - e*f)) + ((f + g*x)^(3/2)*(3*
a^2*e^5*g^2 - 13*c^2*d^4*e*g^2 - 10*a*c*d^2*e^3*g^2 + 16*c^2*d^3*e^2*f*g +
 16*a*c*d*e^4*f*g))/(4*(d*g - e*f)^2))/(e^6*(f + g*x)^2 - (f + g*x)*(2*e^6
*f - 2*d*e^5*g) + e^6*f^2 + d^2*e^4*g^2 - 2*d*e^5*f*g) - (f + g*x)^(1/2)*(
(8*c^2*f)/(e^3*g^2) + (6*c^2*(d*g - e*f))/(e^4*g^2)) + (2*c^2*(f + g*x)^(3
/2))/(3*e^3*g^2) + (atan((e^(1/2)*(f + g*x)^(1/2))/(d*g - e*f)^(1/2))*(3*a
^2*e^4*g^2 + 35*c^2*d^4*g^2 + 48*c^2*d^2*e^2*f^2 + 16*a*c*e^4*f^2 - 80*c^2
*d^3*e*f*g + 6*a*c*d^2*e^2*g^2 - 16*a*c*d*e^3*f*g))/(4*e^(9/2)*(d*g - e*f)
^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1841, normalized size of antiderivative = 6.53

$$\int \frac{(a+cx^2)^2}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+a)^2/(e*x+d)^3/(g*x+f)^(1/2),x)
```

output

```
(9*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)
))*a**2*d**2*e**4*g**4 + 18*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)
/(sqrt(e)*sqrt(d*g - e*f)))*a**2*d*e**5*g**4*x + 9*sqrt(e)*sqrt(d*g - e*f)
*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a**2*e**6*g**4*x**2 + 1
8*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f))
)*a*c*d**4*e**2*g**4 - 48*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(
sqrt(e)*sqrt(d*g - e*f)))*a*c*d**3*e**3*f*g**3 + 36*sqrt(e)*sqrt(d*g - e*f
)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**3*e**3*g**4*x +
 48*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f
)))*a*c*d**2*e**4*f**2*g**2 - 96*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*
x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**2*e**4*f*g**3*x + 18*sqrt(e)*sqrt(
d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**2*e**4
*g**4*x**2 + 96*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sq
rt(d*g - e*f)))*a*c*d*e**5*f**2*g**2*x - 48*sqrt(e)*sqrt(d*g - e*f)*atan((
sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d*e**5*f*g**3*x**2 + 48*sq
rt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*
c*e**6*f**2*g**2*x**2 + 105*sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)
/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**6*g**4 - 240*sqrt(e)*sqrt(d*g - e*f)*a
tan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**5*e*f*g**3 + 210*
sqrt(e)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)...
```

3.84 $\int \frac{(d+ex)^3(a+cx^2)^2}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 470

$$\begin{aligned}
\int \frac{(d+ex)^3\,(a+cx^2)^2}{(f+gx)^{3/2}}\,dx &= \frac{2(ef-dg)^3\,(cf^2+ag^2)^2}{g^8\sqrt{f+gx}} \\
&+ \frac{2(ef-dg)^2\,(cf^2+ag^2)\,(3aeg^2+cf(7ef-4dg))\,\sqrt{f+gx}}{g^8} \\
&- \frac{2(ef-dg)\,(3a^2e^2g^4+2acg^2(10e^2f^2-8defg+d^2g^2)+3c^2f^2(7e^2f^2-8defg+2d^2g^2))\,(f+gx)^{3/2}}{3g^8} \\
&+ \frac{2(a^2e^3g^4+2aceg^2(10e^2f^2-12defg+3d^2g^2)+c^2f(35e^3f^3-60de^2f^2g+30d^2efg^2-4d^3g^3))\,(f+gx)^{5/2}}{5g^8} \\
&- \frac{2c(2ae^2g^2(5ef-3dg)+c(35e^3f^3-45de^2f^2g+15d^2efg^2-d^3g^3))\,(f+gx)^{7/2}}{7g^8} \\
&+ \frac{2ce(2ae^2g^2+3c(7e^2f^2-6defg+d^2g^2))\,(f+gx)^{9/2}}{9g^8} \\
&- \frac{2c^2e^2(7ef-3dg)(f+gx)^{11/2}}{11g^8} + \frac{2c^2e^3(f+gx)^{13/2}}{13g^8}
\end{aligned}$$

output
```
2*(-d*g+e*f)^3*(a*g^2+c*f^2)^2/g^8/(g*x+f)^(1/2)+2*(-d*g+e*f)^2*(a*g^2+c*f
^2)*(3*a*e*g^2+c*f*(-4*d*g+7*e*f))*(g*x+f)^(1/2)/g^8-2/3*(-d*g+e*f)*(3*a^2
*e^2*g^4+2*a*c*g^2*(d^2*g^2-8*d*e*f*g+10*e^2*f^2)+3*c^2*f^2*(2*d^2*g^2-8*d
*e*f*g+7*e^2*f^2))*(g*x+f)^(3/2)/g^8+2/5*(a^2*e^3*g^4+2*a*c*e*g^2*(3*d^2*g
^2-12*d*e*f*g+10*e^2*f^2)+c^2*f*(-4*d^3*g^3+30*d^2*e*f*g^2-60*d*e^2*f^2*g+
35*e^3*f^3))*(g*x+f)^(5/2)/g^8-2/7*c*(2*a*e^2*g^2*(-3*d*g+5*e*f)+c*(-d^3*g
^3+15*d^2*e*f*g^2-45*d*e^2*f^2*g+35*e^3*f^3))*(g*x+f)^(7/2)/g^8+2/9*c*e*(2
*a*e^2*g^2+3*c*(d^2*g^2-6*d*e*f*g+7*e^2*f^2))*(g*x+f)^(9/2)/g^8-2/11*c^2*e
^2*(-3*d*g+7*e*f)*(g*x+f)^(11/2)/g^8+2/13*c^2*e^3*(g*x+f)^(13/2)/g^8
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^3 (a+cx^2)^2}{(f+gx)^{3/2}} dx = \frac{18018a^2g^4(-5d^3g^3+15d^2eg^2(2f+gx)+5de^2g(-8f^2-4fgx+g^2x^2)+e^3(1$$

input
```
Integrate[((d + e*x)^3*(a + c*x^2)^2)/(f + g*x)^(3/2),x]
```

output
```
(18018*a^2*g^4*(-5*d^3*g^3 + 15*d^2*e*g^2*(2*f + g*x) + 5*d*e^2*g*(-8*f^2
- 4*f*g*x + g^2*x^2) + e^3*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3)) +
 572*a*c*g^2*(105*d^3*g^3*(-8*f^2 - 4*f*g*x + g^2*x^2) + 189*d^2*e*g^2*(16
*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) + 27*d*e^2*g*(-128*f^4 - 64*f^3*
g*x + 16*f^2*g^2*x^2 - 8*f*g^3*x^3 + 5*g^4*x^4) + 5*e^3*(256*f^5 + 128*f^4
*g*x - 32*f^3*g^2*x^2 + 16*f^2*g^3*x^3 - 10*f*g^4*x^4 + 7*g^5*x^5)) + 6*c^
2*(429*d^3*g^3*(-128*f^4 - 64*f^3*g*x + 16*f^2*g^2*x^2 - 8*f*g^3*x^3 + 5*g
^4*x^4) + 715*d^2*e*g^2*(256*f^5 + 128*f^4*g*x - 32*f^3*g^2*x^2 + 16*f^2*g
^3*x^3 - 10*f*g^4*x^4 + 7*g^5*x^5) + 195*d*e^2*g*(-1024*f^6 - 512*f^5*g*x
+ 128*f^4*g^2*x^2 - 64*f^3*g^3*x^3 + 40*f^2*g^4*x^4 - 28*f*g^5*x^5 + 21*g^
6*x^6) + 35*e^3*(2048*f^7 + 1024*f^6*g*x - 256*f^5*g^2*x^2 + 128*f^4*g^3*x
^3 - 80*f^3*g^4*x^4 + 56*f^2*g^5*x^5 - 42*f*g^6*x^6 + 33*g^7*x^7)))/(45045
*g^8*Sqrt[f + g*x])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2 (d+ex)^3}{(f+gx)^{3/2}} \, dx$$

$$\Big\downarrow \; 652$$

$$\int \left(\frac{\sqrt{f+gx}(ef-dg)\left(-3a^2e^2g^4-2acg^2\left(d^2g^2-8defg+10e^2f^2\right)-3c^2f^2\left(2d^2g^2-8defg+7e^2f^2\right)\right)}{g^7} + \frac{(f+$$

$$\Big\downarrow \; 2009$$

$$-\frac{2(f+gx)^{3/2}(ef-dg)\left(3a^2e^2g^4+2acg^2\left(d^2g^2-8defg+10e^2f^2\right)+3c^2f^2\left(2d^2g^2-8defg+7e^2f^2\right)\right)}{3g^8}+\frac{2(f+gx)^{5/2}\left(a^2e^3g^4+2aceg^2\left(3d^2g^2-12defg+10e^2f^2\right)+c^2f\left(-4d^3g^3+30d^2efg^2-60de^2f^2g+35e^3f^3\right)\right)}{5g^8}+\frac{2ce(f+gx)^{9/2}\left(2ae^2g^2+3c\left(d^2g^2-6defg+7e^2f^2\right)\right)}{9g^8}-\frac{2c(f+gx)^{7/2}\left(2ae^2g^2(5ef-3dg)+c\left(-d^3g^3+15d^2efg^2-45de^2f^2g+35e^3f^3\right)\right)}{7g^8}+\frac{2\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)^2\left(3aeg^2+cf(7ef-4dg)\right)}{g^8}+\frac{2\left(ag^2+cf^2\right)^2(ef-dg)^3}{g^8\sqrt{f+gx}}-\frac{2c^2e^2(f+gx)^{11/2}(7ef-3dg)}{11g^8}+\frac{2c^2e^3(f+gx)^{13/2}}{13g^8}$$

input
```
Int[((d + e*x)^3*(a + c*x^2)^2)/(f + g*x)^(3/2),x]
```

output
```
(2*(e*f - d*g)^3*(c*f^2 + a*g^2)^2)/(g^8*Sqrt[f + g*x]) + (2*(e*f - d*g)^2
*(c*f^2 + a*g^2)*(3*a*e*g^2 + c*f*(7*e*f - 4*d*g))*Sqrt[f + g*x])/g^8 - (2
*(e*f - d*g)*(3*a^2*e^2*g^4 + 2*a*c*g^2*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2)
 + 3*c^2*f^2*(7*e^2*f^2 - 8*d*e*f*g + 2*d^2*g^2))*(f + g*x)^(3/2))/(3*g^8)
 + (2*(a^2*e^3*g^4 + 2*a*c*e*g^2*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2) + c
^2*f*(35*e^3*f^3 - 60*d*e^2*f^2*g + 30*d^2*e*f*g^2 - 4*d^3*g^3))*(f + g*x)
^(5/2))/(5*g^8) - (2*c*(2*a*e^2*g^2*(5*e*f - 3*d*g) + c*(35*e^3*f^3 - 45*d
*e^2*f^2*g + 15*d^2*e*f*g^2 - d^3*g^3))*(f + g*x)^(7/2))/(7*g^8) + (2*c*e*
(2*a*e^2*g^2 + 3*c*(7*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^(9/2))/(9*
g^8) - (2*c^2*e^2*(7*e*f - 3*d*g)*(f + g*x)^(11/2))/(11*g^8) + (2*c^2*e^3*
(f + g*x)^(13/2))/(13*g^8)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(-\frac{1}{13}c^2x^7-\frac{1}{5}a^2x^3-\frac{2}{9}acx^5\right)e^3-x^2d\left(\frac{6}{7}acx^2+\frac{3}{11}c^2x^4+a^2\right)e^2-3\left(\frac{2}{5}acx^2+a^2+\frac{1}{9}c^2x^4\right)xd^2e+d^3\left(-\frac{1}{7}c^2x^4-\frac{2}{3}ac\right.\right.\right.}{}$
risch	$\frac{2(3465g^6c^2e^3x^6+12285g^6c^2de^2x^5-7875fg^5c^2x^5e^3+10010g^6ace^3x^4+15015g^6d^2ec^2x^4-28665fg^5c^2dx^4e^2+13755}{}$
gosper	$-\frac{2(-3465c^2e^3x^7g^7-12285c^2de^2g^7x^6+4410c^2e^3fg^6x^6-10010ace^3g^7x^5-15015c^2d^2eg^7x^5+16380c^2de^2fg^6x^5-58}{}$
trager	$-\frac{2(-3465c^2e^3x^7g^7-12285c^2de^2g^7x^6+4410c^2e^3fg^6x^6-10010ace^3g^7x^5-15015c^2d^2eg^7x^5+16380c^2de^2fg^6x^5-58}{}$
orering	$-\frac{2(-3465c^2e^3x^7g^7-12285c^2de^2g^7x^6+4410c^2e^3fg^6x^6-10010ace^3g^7x^5-15015c^2d^2eg^7x^5+16380c^2de^2fg^6x^5-58}{}$
derivativedivides	$\frac{-\frac{40ace^3f^3g^2(gx+f)^{\frac{3}{2}}}{3}+30c^2d^2ef^4g^2\sqrt{gx+f}-24c^2de^2f^3g(gx+f)^{\frac{5}{2}}-12acd^2efg^4(gx+f)^{\frac{3}{2}}-2a^2e^3fg^4(gx+f)^{\frac{3}{2}}-12}{}$
default	$\frac{-\frac{40ace^3f^3g^2(gx+f)^{\frac{3}{2}}}{3}+30c^2d^2ef^4g^2\sqrt{gx+f}-24c^2de^2f^3g(gx+f)^{\frac{5}{2}}-12acd^2efg^4(gx+f)^{\frac{3}{2}}-2a^2e^3fg^4(gx+f)^{\frac{3}{2}}-12}{}$

input
```
int((e*x+d)^3*(c*x^2+a)^2/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(g*x+f)^(1/2)*(((-1/13*c^2*x^7-1/5*a^2*x^3-2/9*a*c*x^5)*e^3-x^2*d*(6/7*
a*c*x^2+3/11*c^2*x^4+a^2)*e^2-3*(2/5*a*c*x^2+a^2+1/9*c^2*x^4)*x*d^2*e+d^3*
(-1/7*c^2*x^4-2/3*a*c*x^2+a^2))*g^7-6*f*((-7/429*c^2*x^6-10/189*a*c*x^4-1/
15*a^2*x^2)*e^3-2/3*x*(1/11*c^2*x^4+12/35*a*c*x^2+a^2)*d*e^2+d^2*(-5/63*c^
2*x^4-2/5*a*c*x^2+a^2)*e-4/9*c*x*d^3*(3/35*c*x^2+a))*g^6+8*f^2*((-7/429*c^
2*x^5-4/63*a*c*x^3-1/5*a^2*x)*e^3+d*(-5/77*c^2*x^4-12/35*a*c*x^2+a^2)*e^2-
6/5*c*x*d^2*(5/63*c*x^2+a)*e+2/3*(-3/35*c*x^2+a)*c*d^3)*g^5-16/5*f^3*((-25
/429*c^2*x^4-20/63*a*c*x^2+a^2)*e^3-24/7*c*(5/66*c*x^2+a)*x*d*e^2+6*c*(-5/
63*c*x^2+a)*d^2*e-4/7*c^2*d^3*x)*g^4+768/35*(-5/27*(21/286*c*x^2+a)*x*e^3+
d*(-5/66*c*x^2+a)*e^2-5/18*c*d^2*e*x+1/6*c*d^3)*f^4*c*g^3-512/63*e*((-21/2
86*c*x^2+a)*e^2-9/11*c*d*x*e+3/2*c*d^2)*f^5*c*g^2+1024/77*e^2*f^6*c^2*(-7/
39*e*x+d)*g-2048/429*c^2*e^3*f^7)/g^8
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 715, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex)^3 (a+cx^2)^2}{(f+gx)^{3/2}} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/45045*(3465*c^2*e^3*g^7*x^7 + 215040*c^2*e^3*f^7 - 599040*c^2*d*e^2*f^6*
g + 270270*a^2*d^2*e*f*g^6 - 45045*a^2*d^3*g^7 + 183040*(3*c^2*d^2*e + 2*a
*c*e^3)*f^5*g^2 - 164736*(c^2*d^3 + 6*a*c*d*e^2)*f^4*g^3 + 144144*(6*a*c*d
^2*e + a^2*e^3)*f^3*g^4 - 120120*(2*a*c*d^3 + 3*a^2*d*e^2)*f^2*g^5 - 315*(
14*c^2*e^3*f*g^6 - 39*c^2*d*e^2*g^7)*x^6 + 35*(168*c^2*e^3*f^2*g^5 - 468*c
^2*d*e^2*f*g^6 + 143*(3*c^2*d^2*e + 2*a*c*e^3)*g^7)*x^5 - 5*(1680*c^2*e^3*
f^3*g^4 - 4680*c^2*d*e^2*f^2*g^5 + 1430*(3*c^2*d^2*e + 2*a*c*e^3)*f*g^6 -
1287*(c^2*d^3 + 6*a*c*d*e^2)*g^7)*x^4 + (13440*c^2*e^3*f^4*g^3 - 37440*c^2
*d*e^2*f^3*g^4 + 11440*(3*c^2*d^2*e + 2*a*c*e^3)*f^2*g^5 - 10296*(c^2*d^3
+ 6*a*c*d*e^2)*f*g^6 + 9009*(6*a*c*d^2*e + a^2*e^3)*g^7)*x^3 - (26880*c^2*
e^3*f^5*g^2 - 74880*c^2*d*e^2*f^4*g^3 + 22880*(3*c^2*d^2*e + 2*a*c*e^3)*f^
3*g^4 - 20592*(c^2*d^3 + 6*a*c*d*e^2)*f^2*g^5 + 18018*(6*a*c*d^2*e + a^2*e
^3)*f*g^6 - 15015*(2*a*c*d^3 + 3*a^2*d*e^2)*g^7)*x^2 + (107520*c^2*e^3*f^6
*g - 299520*c^2*d*e^2*f^5*g^2 + 135135*a^2*d^2*e*g^7 + 91520*(3*c^2*d^2*e
+ 2*a*c*e^3)*f^4*g^3 - 82368*(c^2*d^3 + 6*a*c*d*e^2)*f^3*g^4 + 72072*(6*a*
c*d^2*e + a^2*e^3)*f^2*g^5 - 60060*(2*a*c*d^3 + 3*a^2*d*e^2)*f*g^6)*x)*sqr
t(g*x + f)/(g^9*x + f*g^8)
```

Sympy [A] (verification not implemented)

Time = 53.67 (sec) , antiderivative size = 872, normalized size of antiderivative = 1.86

$$\int \frac{(d+ex)^3 (a+cx^2)^2}{(f+gx)^{3/2}} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(c*x**2+a)**2/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c**2*e**3*(f + g*x)**(13/2)/(13*g**7) + (f + g*x)**(11/2)*(3
*c**2*d*e**2*g - 7*c**2*e**3*f)/(11*g**7) + (f + g*x)**(9/2)*(2*a*c*e**3*g
**2 + 3*c**2*d**2*e*g**2 - 18*c**2*d*e**2*f*g + 21*c**2*e**3*f**2)/(9*g**7
) + (f + g*x)**(7/2)*(6*a*c*d*e**2*g**3 - 10*a*c*e**3*f*g**2 + c**2*d**3*g
**3 - 15*c**2*d**2*e*f*g**2 + 45*c**2*d*e**2*f**2*g - 35*c**2*e**3*f**3)/(
7*g**7) + (f + g*x)**(5/2)*(a**2*e**3*g**4 + 6*a*c*d**2*e*g**4 - 24*a*c*d*
e**2*f*g**3 + 20*a*c*e**3*f**2*g**2 - 4*c**2*d**3*f*g**3 + 30*c**2*d**2*e*
f**2*g**2 - 60*c**2*d*e**2*f**3*g + 35*c**2*e**3*f**4)/(5*g**7) + (f + g*x
)**(3/2)*(3*a**2*d*e**2*g**5 - 3*a**2*e**3*f*g**4 + 2*a*c*d**3*g**5 - 18*a
*c*d**2*e*f*g**4 + 36*a*c*d*e**2*f**2*g**3 - 20*a*c*e**3*f**3*g**2 + 6*c**
2*d**3*f**2*g**3 - 30*c**2*d**2*e*f**3*g**2 + 45*c**2*d*e**2*f**4*g - 21*c
**2*e**3*f**5)/(3*g**7) + sqrt(f + g*x)*(3*a**2*d**2*e*g**6 - 6*a**2*d*e**
2*f*g**5 + 3*a**2*e**3*f**2*g**4 - 4*a*c*d**3*f*g**5 + 18*a*c*d**2*e*f**2*
g**4 - 24*a*c*d*e**2*f**3*g**3 + 10*a*c*e**3*f**4*g**2 - 4*c**2*d**3*f**3*
g**3 + 15*c**2*d**2*e*f**4*g**2 - 18*c**2*d*e**2*f**5*g + 7*c**2*e**3*f**6
)/g**7 - (a*g**2 + c*f**2)**2*(d*g - e*f)**3/(g**7*sqrt(f + g*x)))/g, Ne(g
, 0)), ((a**2*d**3*x + 3*a**2*d**2*e*x**2/2 + 3*c**2*d*e**2*x**7/7 + c**2*
e**3*x**8/8 + x**6*(2*a*c*e**3 + 3*c**2*d**2*e)/6 + x**5*(6*a*c*d*e**2 + c
**2*d**3)/5 + x**4*(a**2*e**3 + 6*a*c*d**2*e)/4 + x**3*(3*a**2*d*e**2 + 2*
a*c*d**3)/3)/f**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 712, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^3 (a+cx^2)^2}{(f+gx)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/45045*((3465*(g*x + f)^(13/2)*c^2*e^3 - 4095*(7*c^2*e^3*f - 3*c^2*d*e^2*
g)*(g*x + f)^(11/2) + 5005*(21*c^2*e^3*f^2 - 18*c^2*d*e^2*f*g + (3*c^2*d^2
*e + 2*a*c*e^3)*g^2)*(g*x + f)^(9/2) - 6435*(35*c^2*e^3*f^3 - 45*c^2*d*e^2
*f^2*g + 5*(3*c^2*d^2*e + 2*a*c*e^3)*f*g^2 - (c^2*d^3 + 6*a*c*d*e^2)*g^3)*
(g*x + f)^(7/2) + 9009*(35*c^2*e^3*f^4 - 60*c^2*d*e^2*f^3*g + 10*(3*c^2*d^
2*e + 2*a*c*e^3)*f^2*g^2 - 4*(c^2*d^3 + 6*a*c*d*e^2)*f*g^3 + (6*a*c*d^2*e
+ a^2*e^3)*g^4)*(g*x + f)^(5/2) - 15015*(21*c^2*e^3*f^5 - 45*c^2*d*e^2*f^4
*g + 10*(3*c^2*d^2*e + 2*a*c*e^3)*f^3*g^2 - 6*(c^2*d^3 + 6*a*c*d*e^2)*f^2*
g^3 + 3*(6*a*c*d^2*e + a^2*e^3)*f*g^4 - (2*a*c*d^3 + 3*a^2*d*e^2)*g^5)*(g*
x + f)^(3/2) + 45045*(7*c^2*e^3*f^6 - 18*c^2*d*e^2*f^5*g + 3*a^2*d^2*e*g^6
 + 5*(3*c^2*d^2*e + 2*a*c*e^3)*f^4*g^2 - 4*(c^2*d^3 + 6*a*c*d*e^2)*f^3*g^3
 + 3*(6*a*c*d^2*e + a^2*e^3)*f^2*g^4 - 2*(2*a*c*d^3 + 3*a^2*d*e^2)*f*g^5)*
sqrt(g*x + f))/g^7 + 45045*(c^2*e^3*f^7 - 3*c^2*d*e^2*f^6*g + 3*a^2*d^2*e*
f*g^6 - a^2*d^3*g^7 + (3*c^2*d^2*e + 2*a*c*e^3)*f^5*g^2 - (c^2*d^3 + 6*a*c
*d*e^2)*f^4*g^3 + (6*a*c*d^2*e + a^2*e^3)*f^3*g^4 - (2*a*c*d^3 + 3*a^2*d*e
^2)*f^2*g^5)/(sqrt(g*x + f)*g^7))/g
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. $2(442) = 884$.

Time = 0.14 (sec) , antiderivative size = 1014, normalized size of antiderivative = 2.16

$$\int \frac{(d+ex)^3 (a+cx^2)^2}{(f+gx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

output

```
2*(c^2*e^3*f^7 - 3*c^2*d*e^2*f^6*g + 3*c^2*d^2*e*f^5*g^2 + 2*a*c*e^3*f^5*g
^2 - c^2*d^3*f^4*g^3 - 6*a*c*d*e^2*f^4*g^3 + 6*a*c*d^2*e*f^3*g^4 + a^2*e^3
*f^3*g^4 - 2*a*c*d^3*f^2*g^5 - 3*a^2*d*e^2*f^2*g^5 + 3*a^2*d^2*e*f*g^6 - a
^2*d^3*g^7)/(sqrt(g*x + f)*g^8) + 2/45045*(3465*(g*x + f)^(13/2)*c^2*e^3*g
^96 - 28665*(g*x + f)^(11/2)*c^2*e^3*f*g^96 + 105105*(g*x + f)^(9/2)*c^2*e
^3*f^2*g^96 - 225225*(g*x + f)^(7/2)*c^2*e^3*f^3*g^96 + 315315*(g*x + f)^(
5/2)*c^2*e^3*f^4*g^96 - 315315*(g*x + f)^(3/2)*c^2*e^3*f^5*g^96 + 315315*s
qrt(g*x + f)*c^2*e^3*f^6*g^96 + 12285*(g*x + f)^(11/2)*c^2*d*e^2*g^97 - 90
090*(g*x + f)^(9/2)*c^2*d*e^2*f*g^97 + 289575*(g*x + f)^(7/2)*c^2*d*e^2*f^
2*g^97 - 540540*(g*x + f)^(5/2)*c^2*d*e^2*f^3*g^97 + 675675*(g*x + f)^(3/2
)*c^2*d*e^2*f^4*g^97 - 810810*sqrt(g*x + f)*c^2*d*e^2*f^5*g^97 + 15015*(g*
x + f)^(9/2)*c^2*d^2*e*g^98 + 10010*(g*x + f)^(9/2)*a*c*e^3*g^98 - 96525*(
g*x + f)^(7/2)*c^2*d^2*e*f*g^98 - 64350*(g*x + f)^(7/2)*a*c*e^3*f*g^98 + 2
70270*(g*x + f)^(5/2)*c^2*d^2*e*f^2*g^98 + 180180*(g*x + f)^(5/2)*a*c*e^3*
f^2*g^98 - 450450*(g*x + f)^(3/2)*c^2*d^2*e*f^3*g^98 - 300300*(g*x + f)^(3
/2)*a*c*e^3*f^3*g^98 + 675675*sqrt(g*x + f)*c^2*d^2*e*f^4*g^98 + 450450*sq
rt(g*x + f)*a*c*e^3*f^4*g^98 + 6435*(g*x + f)^(7/2)*c^2*d^3*g^99 + 38610*(
g*x + f)^(7/2)*a*c*d*e^2*g^99 - 36036*(g*x + f)^(5/2)*c^2*d^3*f*g^99 - 216
216*(g*x + f)^(5/2)*a*c*d*e^2*f*g^99 + 90090*(g*x + f)^(3/2)*c^2*d^3*f^2*g
^99 + 540540*(g*x + f)^(3/2)*a*c*d*e^2*f^2*g^99 - 180180*sqrt(g*x + f)*...
```

Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^3 (a+cx^2)^2}{(f+gx)^{3/2}}\,dx = \frac{(f+g\,x)^{7/2}\,(2\,c^2\,d^3\,g^3-30\,c^2\,d^2\,e\,f\,g^2+90\,c^2\,d\,e^2\,f^2\,g-70\,c^2\,e^3\,f^3+12\,a\,c\,d}{7\,g^8}$$
$$-\frac{2\,a^2\,d^3\,g^7-6\,a^2\,d^2\,e\,f\,g^6+6\,a^2\,d\,e^2\,f^2\,g^5-2\,a^2\,e^3\,f^3\,g^4+4\,a\,c\,d^3\,f^2\,g^5-12\,a\,c\,d^2\,e\,f^3\,g^4+12\,a\,c\,d\,e^2\,f^4\,g}{g^8\,\sqrt{f+g\,x}}$$
$$+\frac{(f+g\,x)^{5/2}\,(2\,a^2\,e^3\,g^4+12\,a\,c\,d^2\,e\,g^4-48\,a\,c\,d\,e^2\,f\,g^3+40\,a\,c\,e^3\,f^2\,g^2-8\,c^2\,d^3\,f\,g^3+60\,c^2\,d^2\,e\,f^2\,g^2-1}{5\,g^8}$$
$$+\frac{2\,c^2\,e^3\,(f+g\,x)^{13/2}}{13\,g^8}$$
$$+\frac{2\,(f+g\,x)^{3/2}\,(d\,g-e\,f)\,\left(3\,a^2\,e^2\,g^4+2\,a\,c\,d^2\,g^4-16\,a\,c\,d\,e\,f\,g^3+20\,a\,c\,e^2\,f^2\,g^2+6\,c^2\,d^2\,f^2\,g^2-24\,c^2\,d\,e\right.}{3\,g^8}$$
$$+\frac{2\,\sqrt{f+g\,x}\,(c\,f^2+a\,g^2)\,(d\,g-e\,f)^2\,(7\,c\,e\,f^2-4\,c\,d\,f\,g+3\,a\,e\,g^2)}{g^8}$$
$$+\frac{2\,c^2\,e^2\,(f+g\,x)^{11/2}\,(3\,d\,g-7\,e\,f)}{11\,g^8}$$
$$+\frac{2\,c\,e\,(f+g\,x)^{9/2}\,(3\,c\,d^2\,g^2-18\,c\,d\,e\,f\,g+21\,c\,e^2\,f^2+2\,a\,e^2\,g^2)}{9\,g^8}$$

input
```
int(((a + c*x^2)^2*(d + e*x)^3)/(f + g*x)^(3/2),x)
```

output
```
((f + g*x)^(7/2)*(2*c^2*d^3*g^3 - 70*c^2*e^3*f^3 + 12*a*c*d*e^2*g^3 - 20*a
*c*e^3*f*g^2 + 90*c^2*d*e^2*f^2*g - 30*c^2*d^2*e*f*g^2))/(7*g^8) - (2*a^2*
d^3*g^7 - 2*c^2*e^3*f^7 - 2*a^2*e^3*f^3*g^4 + 2*c^2*d^3*f^4*g^3 + 4*a*c*d^
3*f^2*g^5 - 4*a*c*e^3*f^5*g^2 - 6*a^2*d^2*e*f*g^6 + 6*c^2*d*e^2*f^6*g + 6*
a^2*d*e^2*f^2*g^5 - 6*c^2*d^2*e*f^5*g^2 + 12*a*c*d*e^2*f^4*g^3 - 12*a*c*d^
2*e*f^3*g^4)/(g^8*(f + g*x)^(1/2)) + ((f + g*x)^(5/2)*(2*a^2*e^3*g^4 + 70*
c^2*e^3*f^4 - 8*c^2*d^3*f*g^3 + 12*a*c*d^2*e*g^4 + 40*a*c*e^3*f^2*g^2 - 12
0*c^2*d*e^2*f^3*g + 60*c^2*d^2*e*f^2*g^2 - 48*a*c*d*e^2*f*g^3))/(5*g^8) +
(2*c^2*e^3*(f + g*x)^(13/2))/(13*g^8) + (2*(f + g*x)^(3/2)*(d*g - e*f)*(3*
a^2*e^2*g^4 + 21*c^2*e^2*f^4 + 6*c^2*d^2*f^2*g^2 + 2*a*c*d^2*g^4 - 24*c^2*
d*e*f^3*g + 20*a*c*e^2*f^2*g^2 - 16*a*c*d*e*f*g^3))/(3*g^8) + (2*(f + g*x)
^(1/2)*(a*g^2 + c*f^2)*(d*g - e*f)^2*(3*a*e*g^2 + 7*c*e*f^2 - 4*c*d*f*g))/
g^8 + (2*c^2*e^2*(f + g*x)^(11/2)*(3*d*g - 7*e*f))/(11*g^8) + (2*c*e*(f +
g*x)^(9/2)*(2*a*e^2*g^2 + 3*c*d^2*g^2 + 21*c*e^2*f^2 - 18*c*d*e*f*g))/(9*g
^8)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex)^3 (a+cx^2)^2}{(f+gx)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int((e*x+d)^3*(c*x^2+a)^2/(g*x+f)^(3/2),x)
```

output
```
(2*( - 45045*a**2*d**3*g**7 + 270270*a**2*d**2*e*f*g**6 + 135135*a**2*d**2
*e*g**7*x - 360360*a**2*d*e**2*f**2*g**5 - 180180*a**2*d*e**2*f*g**6*x + 4
5045*a**2*d*e**2*g**7*x**2 + 144144*a**2*e**3*f**3*g**4 + 72072*a**2*e**3*
f**2*g**5*x - 18018*a**2*e**3*f*g**6*x**2 + 9009*a**2*e**3*g**7*x**3 - 240
240*a*c*d**3*f**2*g**5 - 120120*a*c*d**3*f*g**6*x + 30030*a*c*d**3*g**7*x*
*2 + 864864*a*c*d**2*e*f**3*g**4 + 432432*a*c*d**2*e*f**2*g**5*x - 108108*
a*c*d**2*e*f*g**6*x**2 + 54054*a*c*d**2*e*g**7*x**3 - 988416*a*c*d*e**2*f*
*4*g**3 - 494208*a*c*d*e**2*f**3*g**4*x + 123552*a*c*d*e**2*f**2*g**5*x**2
 - 61776*a*c*d*e**2*f*g**6*x**3 + 38610*a*c*d*e**2*g**7*x**4 + 366080*a*c*
e**3*f**5*g**2 + 183040*a*c*e**3*f**4*g**3*x - 45760*a*c*e**3*f**3*g**4*x*
*2 + 22880*a*c*e**3*f**2*g**5*x**3 - 14300*a*c*e**3*f*g**6*x**4 + 10010*a*
c*e**3*g**7*x**5 - 164736*c**2*d**3*f**4*g**3 - 82368*c**2*d**3*f**3*g**4*
x + 20592*c**2*d**3*f**2*g**5*x**2 - 10296*c**2*d**3*f*g**6*x**3 + 6435*c*
*2*d**3*g**7*x**4 + 549120*c**2*d**2*e*f**5*g**2 + 274560*c**2*d**2*e*f**4
*g**3*x - 68640*c**2*d**2*e*f**3*g**4*x**2 + 34320*c**2*d**2*e*f**2*g**5*x
**3 - 21450*c**2*d**2*e*f*g**6*x**4 + 15015*c**2*d**2*e*g**7*x**5 - 599040
*c**2*d*e**2*f**6*g - 299520*c**2*d*e**2*f**5*g**2*x + 74880*c**2*d*e**2*f
**4*g**3*x**2 - 37440*c**2*d*e**2*f**3*g**4*x**3 + 23400*c**2*d*e**2*f**2*
g**5*x**4 - 16380*c**2*d*e**2*f*g**6*x**5 + 12285*c**2*d*e**2*g**7*x**6 +
215040*c**2*e**3*f**7 + 107520*c**2*e**3*f**6*g*x - 26880*c**2*e**3*f**...
```

3.85 $\int \frac{(d+ex)^2\left(a+cx^2\right)^2}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 336

$$
\begin{aligned}
\int \frac{(d+ex)^2\left(a+cx^2\right)^2}{(f+gx)^{3/2}}\,dx = &-\frac{2(ef-dg)^2\left(cf^2+ag^2\right)^2}{g^7\sqrt{f+gx}} \\
&-\frac{4(ef-dg)\left(cf^2+ag^2\right)\left(aeg^2+cf(3ef-2dg)\right)\sqrt{f+gx}}{g^7} \\
&+\frac{2\left(a^2e^2g^4+2acg^2\left(6e^2f^2-6defg+d^2g^2\right)+c^2f^2\left(15e^2f^2-20defg+6d^2g^2\right)\right)(f+gx)^{3/2}}{3g^7} \\
&-\frac{8c\left(aeg^2(2ef-dg)+cf\left(5e^2f^2-5defg+d^2g^2\right)\right)(f+gx)^{5/2}}{5g^7} \\
&+\frac{2c\left(2ae^2g^2+c\left(15e^2f^2-10defg+d^2g^2\right)\right)(f+gx)^{7/2}}{7g^7} \\
&-\frac{4c^2e(3ef-dg)(f+gx)^{9/2}}{9g^7}+\frac{2c^2e^2(f+gx)^{11/2}}{11g^7}
\end{aligned}
$$

output

```
-2*(-d*g+e*f)^2*(a*g^2+c*f^2)^2/g^7/(g*x+f)^(1/2)-4*(-d*g+e*f)*(a*g^2+c*f^
2)*(a*e*g^2+c*f*(-2*d*g+3*e*f))*(g*x+f)^(1/2)/g^7+2/3*(a^2*e^2*g^4+2*a*c*g
^2*(d^2*g^2-6*d*e*f*g+6*e^2*f^2)+c^2*f^2*(6*d^2*g^2-20*d*e*f*g+15*e^2*f^2)
)*(g*x+f)^(3/2)/g^7-8/5*c*(a*e*g^2*(-d*g+2*e*f)+c*f*(d^2*g^2-5*d*e*f*g+5*e
^2*f^2))*(g*x+f)^(5/2)/g^7+2/7*c*(2*a*e^2*g^2+c*(d^2*g^2-10*d*e*f*g+15*e^2
*f^2))*(g*x+f)^(7/2)/g^7-4/9*c^2*e*(-d*g+3*e*f)*(g*x+f)^(9/2)/g^7+2/11*c^2
*e^2*(g*x+f)^(11/2)/g^7
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^2 (a+cx^2)^2}{(f+gx)^{3/2}} dx = \frac{2(-1155a^2g^4(3d^2g^2-6deg(2f+gx)+e^2(8f^2+4fgx-g^2x^2))-66acg^2(35$$

input
```
Integrate[((d + e*x)^2*(a + c*x^2)^2)/(f + g*x)^(3/2),x]
```

output
```
(2*(-1155*a^2*g^4*(3*d^2*g^2 - 6*d*e*g*(2*f + g*x) + e^2*(8*f^2 + 4*f*g*x
- g^2*x^2)) - 66*a*c*g^2*(35*d^2*g^2*(8*f^2 + 4*f*g*x - g^2*x^2) - 42*d*e*
g*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) + 3*e^2*(128*f^4 + 64*f^3*g
*x - 16*f^2*g^2*x^2 + 8*f*g^3*x^3 - 5*g^4*x^4)) + c^2*(99*d^2*g^2*(-128*f^
4 - 64*f^3*g*x + 16*f^2*g^2*x^2 - 8*f*g^3*x^3 + 5*g^4*x^4) + 110*d*e*g*(25
6*f^5 + 128*f^4*g*x - 32*f^3*g^2*x^2 + 16*f^2*g^3*x^3 - 10*f*g^4*x^4 + 7*g
^5*x^5) - 15*e^2*(1024*f^6 + 512*f^5*g*x - 128*f^4*g^2*x^2 + 64*f^3*g^3*x^
3 - 40*f^2*g^4*x^4 + 28*f*g^5*x^5 - 21*g^6*x^6))))/(3465*g^7*Sqrt[f + g*x]
)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2 (d+ex)^2}{(f+gx)^{3/2}} dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{\sqrt{f+gx}\left(a^2e^2g^4+2acg^2\left(d^2g^2-6defg+6e^2f^2\right)+c^2f^2\left(6d^2g^2-20defg+15e^2f^2\right)\right)}{g^6}+\frac{c(f+gx)^{5/2}\left(2ae^2\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(f+gx)^{3/2}\left(a^2e^2g^4+2acg^2\left(d^2g^2-6defg+6e^2f^2\right)+c^2f^2\left(6d^2g^2-20defg+15e^2f^2\right)\right)}{3g^7}+\frac{2c(f+gx)^{7/2}\left(2ae^2g^2+c\left(d^2g^2-10defg+15e^2f^2\right)\right)}{7g^7}-\frac{8c(f+gx)^{5/2}\left(aeg^2(2ef-dg)+cf\left(d^2g^2-5defg+5e^2f^2\right)\right)}{5g^7}-\frac{4\sqrt{f+gx}\left(ag^2+cf^2\right)(ef-dg)\left(aeg^2+cf(3ef-2dg)\right)}{g^7}-\frac{2\left(ag^2+cf^2\right)^2(ef-dg)^2}{g^7\sqrt{f+gx}}-\frac{4c^2e(f+gx)^{9/2}(3ef-dg)}{9g^7}+\frac{2c^2e^2(f+gx)^{11/2}}{11g^7}$$

input
```
Int[((d + e*x)^2*(a + c*x^2)^2)/(f + g*x)^(3/2),x]
```

output
```
(-2*(e*f - d*g)^2*(c*f^2 + a*g^2)^2)/(g^7*Sqrt[f + g*x]) - (4*(e*f - d*g)*
(c*f^2 + a*g^2)*(a*e*g^2 + c*f*(3*e*f - 2*d*g))*Sqrt[f + g*x])/g^7 + (2*(a
^2*e^2*g^4 + 2*a*c*g^2*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2) + c^2*f^2*(15*e^2
*f^2 - 20*d*e*f*g + 6*d^2*g^2))*(f + g*x)^(3/2))/(3*g^7) - (8*c*(a*e*g^2*(
2*e*f - d*g) + c*f*(5*e^2*f^2 - 5*d*e*f*g + d^2*g^2))*(f + g*x)^(5/2))/(5*
g^7) + (2*c*(2*a*e^2*g^2 + c*(15*e^2*f^2 - 10*d*e*f*g + d^2*g^2))*(f + g*x
)^(7/2))/(7*g^7) - (4*c^2*e*(3*e*f - d*g)*(f + g*x)^(9/2))/(9*g^7) + (2*c^
2*e^2*(f + g*x)^(11/2))/(11*g^7)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(-\frac{1}{11}e^{2}x^{6}-\frac{1}{7}d^{2}x^{4}-\frac{2}{9}de\,x^{5}\right)c^{2}-\frac{2\left(d^{2}+\frac{3}{7}e^{2}x^{2}+\frac{6}{5}dex\right)x^{2}ac}{3}+a^{2}\left(-\frac{1}{3}e^{2}x^{2}-2dex+d^{2}\right)\right)g^{6}-4f\left(\left(-\frac{1}{33}e^{2}x^{5}-\frac{5}{63}d\cdots\right.\right.\right.}{\cdots}$
risch	$\frac{2\left(315g^{5}c^{2}e^{2}x^{5}+770g^{5}c^{2}de\,x^{4}-735f\,g^{4}c^{2}x^{4}e^{2}+990g^{5}ac\,e^{2}x^{3}+495g^{5}d^{2}c^{2}x^{3}-1870f\,g^{4}c^{2}x^{3}de+1335f^{2}g^{3}c^{2}e^{2}x^{3}+27\cdots\right.}{\cdots}$
gosper	$-\frac{2\left(-315c^{2}e^{2}x^{6}g^{6}-770c^{2}de\,g^{6}x^{5}+420c^{2}e^{2}f\,g^{5}x^{5}-990ac\,e^{2}g^{6}x^{4}-495c^{2}d^{2}g^{6}x^{4}+1100c^{2}def\,g^{5}x^{4}-600c^{2}e^{2}f^{2}g^{4}x^{4}-\cdots\right.}{\cdots}$
trager	$-\frac{2\left(-315c^{2}e^{2}x^{6}g^{6}-770c^{2}de\,g^{6}x^{5}+420c^{2}e^{2}f\,g^{5}x^{5}-990ac\,e^{2}g^{6}x^{4}-495c^{2}d^{2}g^{6}x^{4}+1100c^{2}def\,g^{5}x^{4}-600c^{2}e^{2}f^{2}g^{4}x^{4}-\cdots\right.}{\cdots}$
orering	$-\frac{2\left(-315c^{2}e^{2}x^{6}g^{6}-770c^{2}de\,g^{6}x^{5}+420c^{2}e^{2}f\,g^{5}x^{5}-990ac\,e^{2}g^{6}x^{4}-495c^{2}d^{2}g^{6}x^{4}+1100c^{2}def\,g^{5}x^{4}-600c^{2}e^{2}f^{2}g^{4}x^{4}-\cdots\right.}{\cdots}$
derivativedivides	$\frac{-16ac\,e^{2}f^{3}g^{2}\sqrt{gx+f}+24acde\,f^{2}g^{3}\sqrt{gx+f}+4a^{2}de\,g^{5}\sqrt{gx+f}+8c^{2}de\,f^{2}g(gx+f)^{\frac{5}{2}}-\frac{16ac\,e^{2}f\,g^{2}(gx+f)^{\frac{5}{2}}}{5}-8ac\,d^{2}f\,g^{4}\sqrt{\cdots}}{\cdots}$
default	$\frac{-16ac\,e^{2}f^{3}g^{2}\sqrt{gx+f}+24acde\,f^{2}g^{3}\sqrt{gx+f}+4a^{2}de\,g^{5}\sqrt{gx+f}+8c^{2}de\,f^{2}g(gx+f)^{\frac{5}{2}}-\frac{16ac\,e^{2}f\,g^{2}(gx+f)^{\frac{5}{2}}}{5}-8ac\,d^{2}f\,g^{4}\sqrt{\cdots}}{\cdots}$

input
```
int((e*x+d)^2*(c*x^2+a)^2/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(g*x+f)^(1/2)*(((-1/11*e^2*x^6-1/7*d^2*x^4-2/9*d*e*x^5)*c^2-2/3*(d^2+3/
7*e^2*x^2+6/5*d*e*x)*x^2*a*c+a^2*(-1/3*e^2*x^2-2*d*e*x+d^2))*g^6-4*f*((-1/
33*e^2*x^5-5/63*d*e*x^4-2/35*d^2*x^3)*c^2-2/3*(6/35*e^2*x^2+3/5*d*e*x+d^2)
*x*a*c+a^2*e*(-1/3*e*x+d))*g^5+8/3*((-5/77*e^2*x^4-4/21*d*e*x^3-6/35*d^2*x
^2)*c^2+2*(-6/35*e^2*x^2-6/5*d*e*x+d^2)*a*c+e^2*a^2)*f^2*g^4-64/5*(-1/7*x*
(5/33*e^2*x^2+5/9*d*e*x+d^2)*c+e*a*(-2/7*e*x+d))*f^3*c*g^3+256/35*((-5/66*
e^2*x^2-5/9*d*e*x+1/2*d^2)*c+a*e^2)*f^4*c*g^2-512/63*e*f^5*c^2*(-3/11*e*x+
d)*g+1024/231*c^2*e^2*f^6)/g^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex)^2 (a+cx^2)^2}{(f+gx)^{3/2}}\,dx = \frac{2\,(315\,c^2e^2g^6x^6 - 15360\,c^2e^2f^6 + 28160\,c^2def^5g + 44352\,acdef^3g^3 + 13860\,a^2}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/3465*(315*c^2*e^2*g^6*x^6 - 15360*c^2*e^2*f^6 + 28160*c^2*d*e*f^5*g + 44
352*a*c*d*e*f^3*g^3 + 13860*a^2*d*e*f*g^5 - 3465*a^2*d^2*g^6 - 12672*(c^2*
d^2 + 2*a*c*e^2)*f^4*g^2 - 9240*(2*a*c*d^2 + a^2*e^2)*f^2*g^4 - 70*(6*c^2*
e^2*f*g^5 - 11*c^2*d*e*g^6)*x^5 + 5*(120*c^2*e^2*f^2*g^4 - 220*c^2*d*e*f*g
^5 + 99*(c^2*d^2 + 2*a*c*e^2)*g^6)*x^4 - 4*(240*c^2*e^2*f^3*g^3 - 440*c^2*
d*e*f^2*g^4 - 693*a*c*d*e*g^6 + 198*(c^2*d^2 + 2*a*c*e^2)*f*g^5)*x^3 + (19
20*c^2*e^2*f^4*g^2 - 3520*c^2*d*e*f^3*g^3 - 5544*a*c*d*e*f*g^5 + 1584*(c^2
*d^2 + 2*a*c*e^2)*f^2*g^4 + 1155*(2*a*c*d^2 + a^2*e^2)*g^6)*x^2 - 2*(3840*
c^2*e^2*f^5*g - 7040*c^2*d*e*f^4*g^2 - 11088*a*c*d*e*f^2*g^4 - 3465*a^2*d*
e*g^6 + 3168*(c^2*d^2 + 2*a*c*e^2)*f^3*g^3 + 2310*(2*a*c*d^2 + a^2*e^2)*f*
g^5)*x)*sqrt(g*x + f)/(g^8*x + f*g^7)
```

Sympy [A] (verification not implemented)

Time = 21.14 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.68

$$\int \frac{(d+ex)^2 (a+cx^2)^2}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^2e^2(f+gx)^{\frac{11}{2}}}{11g^6} + \frac{(f+gx)^{\frac{9}{2}}\cdot\left(2c^2deg-6c^2e^2f\right)}{9g^6} + \frac{(f+gx)^{\frac{7}{2}}\cdot\left(2ace^2g^2+c^2d^2g^2-10c^2defg+15c^2e^2f^2\right)}{7g^6} + \frac{(f+gx}{}\right.}{} \\ \frac{a^2d^2x+a^2dex^2+acdex^4+\frac{c^2dex^6}{3}+\frac{c^2e^2x^7}{7}+\frac{x^5\cdot\left(2ace^2+c^2d^2\right)}{5}+\frac{x^3\left(a^2e^2+2acd^2\right)}{3}}{f^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((e*x+d)**2*(c*x**2+a)**2/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c**2*e**2*(f + g*x)**(11/2)/(11*g**6) + (f + g*x)**(9/2)*(2*
c**2*d*e*g - 6*c**2*e**2*f)/(9*g**6) + (f + g*x)**(7/2)*(2*a*c*e**2*g**2 +
 c**2*d**2*g**2 - 10*c**2*d*e*f*g + 15*c**2*e**2*f**2)/(7*g**6) + (f + g*x
)**(5/2)*(4*a*c*d*e*g**3 - 8*a*c*e**2*f*g**2 - 4*c**2*d**2*f*g**2 + 20*c**
2*d*e*f**2*g - 20*c**2*e**2*f**3)/(5*g**6) + (f + g*x)**(3/2)*(a**2*e**2*g
**4 + 2*a*c*d**2*g**4 - 12*a*c*d*e*f*g**3 + 12*a*c*e**2*f**2*g**2 + 6*c**2
*d**2*f**2*g**2 - 20*c**2*d*e*f**3*g + 15*c**2*e**2*f**4)/(3*g**6) + sqrt(
f + g*x)*(2*a**2*d*e*g**5 - 2*a**2*e**2*f*g**4 - 4*a*c*d**2*f*g**4 + 12*a*
c*d*e*f**2*g**3 - 8*a*c*e**2*f**3*g**2 - 4*c**2*d**2*f**3*g**2 + 10*c**2*d
*e*f**4*g - 6*c**2*e**2*f**5)/g**6 - (a*g**2 + c*f**2)**2*(d*g - e*f)**2/(
g**6*sqrt(f + g*x)))/g, Ne(g, 0)), ((a**2*d**2*x + a**2*d*e*x**2 + a*c*d*e
*x**4 + c**2*d*e*x**6/3 + c**2*e**2*x**7/7 + x**5*(2*a*c*e**2 + c**2*d**2)
/5 + x**3*(a**2*e**2 + 2*a*c*d**2)/3)/f**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^2\left(a+cx^2\right)^2}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{315\,(gx+f)^{\frac{11}{2}}c^2e^2-770\,(3\,c^2e^2f-c^2deg)(gx+f)^{\frac{9}{2}}+495\,(15\,c^2e^2f^2-10\,c^2defg+(c^2d^2+2\,ace^2)g^2)(g}{}\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/3465*((315*(g*x + f)^(11/2)*c^2*e^2 - 770*(3*c^2*e^2*f - c^2*d*e*g)*(g*x
 + f)^(9/2) + 495*(15*c^2*e^2*f^2 - 10*c^2*d*e*f*g + (c^2*d^2 + 2*a*c*e^2)
*g^2)*(g*x + f)^(7/2) - 2772*(5*c^2*e^2*f^3 - 5*c^2*d*e*f^2*g - a*c*d*e*g^
3 + (c^2*d^2 + 2*a*c*e^2)*f*g^2)*(g*x + f)^(5/2) + 1155*(15*c^2*e^2*f^4 -
20*c^2*d*e*f^3*g - 12*a*c*d*e*f*g^3 + 6*(c^2*d^2 + 2*a*c*e^2)*f^2*g^2 + (2
*a*c*d^2 + a^2*e^2)*g^4)*(g*x + f)^(3/2) - 6930*(3*c^2*e^2*f^5 - 5*c^2*d*e
*f^4*g - 6*a*c*d*e*f^2*g^3 - a^2*d*e*g^5 + 2*(c^2*d^2 + 2*a*c*e^2)*f^3*g^2
 + (2*a*c*d^2 + a^2*e^2)*f*g^4)*sqrt(g*x + f))/g^6 - 3465*(c^2*e^2*f^6 - 2
*c^2*d*e*f^5*g - 4*a*c*d*e*f^3*g^3 - 2*a^2*d*e*f*g^5 + a^2*d^2*g^6 + (c^2*
d^2 + 2*a*c*e^2)*f^4*g^2 + (2*a*c*d^2 + a^2*e^2)*f^2*g^4)/(sqrt(g*x + f)*g
^6))/g
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(312) = 624$.

Time = 0.12 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.90

$$\int \frac{(d+ex)^2 (a+cx^2)^2}{(f+gx)^{3/2}}\,dx =$$
$$-\frac{2\,(c^2e^2f^6 - 2\,c^2def^5g + c^2d^2f^4g^2 + 2\,ace^2f^4g^2 - 4\,acdef^3g^3 + 2\,acd^2f^2g^4 + a^2e^2f^2g^4 - 2\,a^2defg^5 + a^2d}{\sqrt{gx+f}\,g^7}$$
$$+\frac{2\left(315\,(gx+f)^{\frac{11}{2}}c^2e^2g^{70} - 2310\,(gx+f)^{\frac{9}{2}}c^2e^2fg^{70} + 7425\,(gx+f)^{\frac{7}{2}}c^2e^2f^2g^{70} - 13860\,(gx+f)^{\frac{5}{2}}c^2e^2f^3\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
-2*(c^2*e^2*f^6 - 2*c^2*d*e*f^5*g + c^2*d^2*f^4*g^2 + 2*a*c*e^2*f^4*g^2 -
4*a*c*d*e*f^3*g^3 + 2*a*c*d^2*f^2*g^4 + a^2*e^2*f^2*g^4 - 2*a^2*d*e*f*g^5
+ a^2*d^2*g^6)/(sqrt(g*x + f)*g^7) + 2/3465*(315*(g*x + f)^(11/2)*c^2*e^2*
g^70 - 2310*(g*x + f)^(9/2)*c^2*e^2*f*g^70 + 7425*(g*x + f)^(7/2)*c^2*e^2*
f^2*g^70 - 13860*(g*x + f)^(5/2)*c^2*e^2*f^3*g^70 + 17325*(g*x + f)^(3/2)*
c^2*e^2*f^4*g^70 - 20790*sqrt(g*x + f)*c^2*e^2*f^5*g^70 + 770*(g*x + f)^(9
/2)*c^2*d*e*g^71 - 4950*(g*x + f)^(7/2)*c^2*d*e*f*g^71 + 13860*(g*x + f)^(
5/2)*c^2*d*e*f^2*g^71 - 23100*(g*x + f)^(3/2)*c^2*d*e*f^3*g^71 + 34650*sqr
t(g*x + f)*c^2*d*e*f^4*g^71 + 495*(g*x + f)^(7/2)*c^2*d^2*g^72 + 990*(g*x
+ f)^(7/2)*a*c*e^2*g^72 - 2772*(g*x + f)^(5/2)*c^2*d^2*f*g^72 - 5544*(g*x
+ f)^(5/2)*a*c*e^2*f*g^72 + 6930*(g*x + f)^(3/2)*c^2*d^2*f^2*g^72 + 13860*
(g*x + f)^(3/2)*a*c*e^2*f^2*g^72 - 13860*sqrt(g*x + f)*c^2*d^2*f^3*g^72 -
27720*sqrt(g*x + f)*a*c*e^2*f^3*g^72 + 2772*(g*x + f)^(5/2)*a*c*d*e*g^73 -
 13860*(g*x + f)^(3/2)*a*c*d*e*f*g^73 + 41580*sqrt(g*x + f)*a*c*d*e*f^2*g^
73 + 2310*(g*x + f)^(3/2)*a*c*d^2*g^74 + 1155*(g*x + f)^(3/2)*a^2*e^2*g^74
 - 13860*sqrt(g*x + f)*a*c*d^2*f*g^74 - 6930*sqrt(g*x + f)*a^2*e^2*f*g^74
+ 6930*sqrt(g*x + f)*a^2*d*e*g^75)/g^77
```

Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^2 (a+cx^2)^2}{(f+gx)^{3/2}} dx = \frac{(f+g\,x)^{3/2} \left(2\,a^2\,e^2\,g^4+4\,a\,c\,d^2\,g^4-24\,a\,c\,d\,e\,f\,g^3+24\,a\,c\,e^2\,f^2\,g^2+12\,c^2\,d^2 \right.}{3\,g^7}$$
$$-\frac{(f+g\,x)^{5/2} \left(8\,c^2\,d^2\,f\,g^2-40\,c^2\,d\,e\,f^2\,g+40\,c^2\,e^2\,f^3-8\,a\,c\,d\,e\,g^3+16\,a\,c\,e^2\,f\,g^2\right)}{5\,g^7}$$
$$+\frac{(f+g\,x)^{7/2} \left(2\,c^2\,d^2\,g^2-20\,c^2\,d\,e\,f\,g+30\,c^2\,e^2\,f^2+4\,a\,c\,e^2\,g^2\right)}{7\,g^7}$$
$$-\frac{2\,a^2\,d^2\,g^6-4\,a^2\,d\,e\,f\,g^5+2\,a^2\,e^2\,f^2\,g^4+4\,a\,c\,d^2\,f^2\,g^4-8\,a\,c\,d\,e\,f^3\,g^3+4\,a\,c\,e^2\,f^4\,g^2+2\,c^2\,d^2\,f^4\,g^2-4\,c^2}{g^7\,\sqrt{f+g\,x}}$$
$$+\frac{2\,c^2\,e^2\,(f+g\,x)^{11/2}}{11\,g^7}+\frac{4\,c^2\,e\,(f+g\,x)^{9/2}\,(d\,g-3\,e\,f)}{9\,g^7}$$
$$+\frac{4\,\sqrt{f+g\,x}\,(c\,f^2+a\,g^2)\,(d\,g-e\,f)\,(3\,c\,e\,f^2-2\,c\,d\,f\,g+a\,e\,g^2)}{g^7}$$

input
```
int(((a + c*x^2)^2*(d + e*x)^2)/(f + g*x)^(3/2),x)
```

output
```
((f + g*x)^(3/2)*(2*a^2*e^2*g^4 + 30*c^2*e^2*f^4 + 12*c^2*d^2*f^2*g^2 + 4*
a*c*d^2*g^4 - 40*c^2*d*e*f^3*g + 24*a*c*e^2*f^2*g^2 - 24*a*c*d*e*f*g^3))/(
3*g^7) - ((f + g*x)^(5/2)*(40*c^2*e^2*f^3 + 8*c^2*d^2*f*g^2 + 16*a*c*e^2*f
*g^2 - 40*c^2*d*e*f^2*g - 8*a*c*d*e*g^3))/(5*g^7) + ((f + g*x)^(7/2)*(2*c^
2*d^2*g^2 + 30*c^2*e^2*f^2 + 4*a*c*e^2*g^2 - 20*c^2*d*e*f*g))/(7*g^7) - (2
*a^2*d^2*g^6 + 2*c^2*e^2*f^6 + 2*a^2*e^2*f^2*g^4 + 2*c^2*d^2*f^4*g^2 - 4*a
^2*d*e*f*g^5 - 4*c^2*d*e*f^5*g + 4*a*c*d^2*f^2*g^4 + 4*a*c*e^2*f^4*g^2 - 8
*a*c*d*e*f^3*g^3)/(g^7*(f + g*x)^(1/2)) + (2*c^2*e^2*(f + g*x)^(11/2))/(11
*g^7) + (4*c^2*e*(f + g*x)^(9/2)*(d*g - 3*e*f))/(9*g^7) + (4*(f + g*x)^(1/
2)*(a*g^2 + c*f^2)*(d*g - e*f)*(a*e*g^2 + 3*c*e*f^2 - 2*c*d*f*g))/g^7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^2 (a+cx^2)^2}{(f+gx)^{3/2}}\,dx = \frac{-\frac{16}{3}ac\,d^2 f\,g^5 x + \frac{128}{5}acde\,f^3 g^3 + \frac{8}{5}acde\,g^6 x^3 - \frac{256}{35}ac\,e^2 f^3 g^3 x + \frac{64}{35}ac\,e^2 f^2 g^4 x^2}{}$$

input

```
int((e*x+d)^2*(c*x^2+a)^2/(g*x+f)^(3/2),x)
```

output

```
(2*( - 3465*a**2*d**2*g**6 + 13860*a**2*d*e*f*g**5 + 6930*a**2*d*e*g**6*x
- 9240*a**2*e**2*f**2*g**4 - 4620*a**2*e**2*f*g**5*x + 1155*a**2*e**2*g**6
*x**2 - 18480*a*c*d**2*f**2*g**4 - 9240*a*c*d**2*f*g**5*x + 2310*a*c*d**2*
g**6*x**2 + 44352*a*c*d*e*f**3*g**3 + 22176*a*c*d*e*f**2*g**4*x - 5544*a*c
*d*e*f*g**5*x**2 + 2772*a*c*d*e*g**6*x**3 - 25344*a*c*e**2*f**4*g**2 - 126
72*a*c*e**2*f**3*g**3*x + 3168*a*c*e**2*f**2*g**4*x**2 - 1584*a*c*e**2*f*g
**5*x**3 + 990*a*c*e**2*g**6*x**4 - 12672*c**2*d**2*f**4*g**2 - 6336*c**2*
d**2*f**3*g**3*x + 1584*c**2*d**2*f**2*g**4*x**2 - 792*c**2*d**2*f*g**5*x*
*3 + 495*c**2*d**2*g**6*x**4 + 28160*c**2*d*e*f**5*g + 14080*c**2*d*e*f**4
*g**2*x - 3520*c**2*d*e*f**3*g**3*x**2 + 1760*c**2*d*e*f**2*g**4*x**3 - 11
00*c**2*d*e*f*g**5*x**4 + 770*c**2*d*e*g**6*x**5 - 15360*c**2*e**2*f**6 -
7680*c**2*e**2*f**5*g*x + 1920*c**2*e**2*f**4*g**2*x**2 - 960*c**2*e**2*f*
*3*g**3*x**3 + 600*c**2*e**2*f**2*g**4*x**4 - 420*c**2*e**2*f*g**5*x**5 +
315*c**2*e**2*g**6*x**6))/(3465*sqrt(f + g*x)*g**7)
```

3.86 $\int \frac{(d+ex)(a+cx^2)^2}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 210

$$\begin{aligned}\int \frac{(d+ex)(a+cx^2)^2}{(f+gx)^{3/2}}\,dx &= \frac{2(ef-dg)(cf^2+ag^2)^2}{g^6\sqrt{f+gx}} \\ &+ \frac{2(cf^2+ag^2)(aeg^2+cf(5ef-4dg))\sqrt{f+gx}}{g^6} \\ &- \frac{4c(cf^2(5ef-3dg)+ag^2(3ef-dg))(f+gx)^{3/2}}{3g^6} \\ &+ \frac{4c(aeg^2+cf(5ef-2dg))(f+gx)^{5/2}}{5g^6} \\ &- \frac{2c^2(5ef-dg)(f+gx)^{7/2}}{7g^6} + \frac{2c^2e(f+gx)^{9/2}}{9g^6}\end{aligned}$$

output
```
2*(-d*g+e*f)*(a*g^2+c*f^2)^2/g^6/(g*x+f)^(1/2)+2*(a*g^2+c*f^2)*(a*e*g^2+c*
f*(-4*d*g+5*e*f))*(g*x+f)^(1/2)/g^6-4/3*c*(c*f^2*(-3*d*g+5*e*f)+a*g^2*(-d*
g+3*e*f))*(g*x+f)^(3/2)/g^6+4/5*c*(a*e*g^2+c*f*(-2*d*g+5*e*f))*(g*x+f)^(5/
2)/g^6-2/7*c^2*(-d*g+5*e*f)*(g*x+f)^(7/2)/g^6+2/9*c^2*e*(g*x+f)^(9/2)/g^6
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)\left(a+cx^2\right)^2}{(f+gx)^{3/2}}\,dx = \frac{630a^2g^4(2ef-dg+egx)+84acg^2(5dg(-8f^2-4fgx+g^2x^2)+3e(16f^3+8}{}$$

input
```
Integrate[((d + e*x)*(a + c*x^2)^2)/(f + g*x)^(3/2),x]
```

output
```
(630*a^2*g^4*(2*e*f - d*g + e*g*x) + 84*a*c*g^2*(5*d*g*(-8*f^2 - 4*f*g*x +
 g^2*x^2) + 3*e*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3)) + 2*c^2*(9*d
*g*(-128*f^4 - 64*f^3*g*x + 16*f^2*g^2*x^2 - 8*f*g^3*x^3 + 5*g^4*x^4) + 5*
e*(256*f^5 + 128*f^4*g*x - 32*f^3*g^2*x^2 + 16*f^2*g^3*x^3 - 10*f*g^4*x^4
+ 7*g^5*x^5)))/(315*g^6*Sqrt[f + g*x])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2(d+ex)}{(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \; 652$$

$$\int \left(\frac{2c\sqrt{f+gx}\left(-ag^2(3ef-dg)-cf^2(5ef-3dg)\right)}{g^5}+\frac{\left(ag^2+cf^2\right)\left(aeg^2+cf(5ef-4dg)\right)}{g^5\sqrt{f+gx}}+\frac{\left(ag^2+cf^2\right)^2(dg}{g^5(f+gx)^3}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{4c(f+gx)^{3/2}\left(ag^2(3ef-dg)+cf^2(5ef-3dg)\right)}{3g^6}+\frac{2\sqrt{f+gx}(ag^2+cf^2)\left(aeg^2+cf(5ef-4dg)\right)}{g^6}+\frac{2\left(ag^2+cf^2\right)^2(ef-dg)}{g^6\sqrt{f+gx}}+\frac{4c(f+gx)^{5/2}\left(aeg^2+cf(5ef-2dg)\right)}{5g^6}-\frac{2c^2(f+gx)^{7/2}(5ef-dg)}{7g^6}+\frac{2c^2e(f+gx)^{9/2}}{9g^6}$$

input
```
Int[((d + e*x)*(a + c*x^2)^2)/(f + g*x)^(3/2),x]
```

output
```
(2*(e*f - d*g)*(c*f^2 + a*g^2)^2)/(g^6*Sqrt[f + g*x]) + (2*(c*f^2 + a*g^2)
*(a*e*g^2 + c*f*(5*e*f - 4*d*g))*Sqrt[f + g*x])/g^6 - (4*c*(c*f^2*(5*e*f -
 3*d*g) + a*g^2*(3*e*f - d*g))*(f + g*x)^(3/2))/(3*g^6) + (4*c*(a*e*g^2 +
c*f*(5*e*f - 2*d*g))*(f + g*x)^(5/2))/(5*g^6) - (2*c^2*(5*e*f - d*g)*(f +
g*x)^(7/2))/(7*g^6) + (2*c^2*e*(f + g*x)^(9/2))/(9*g^6)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{\left(\left(70x^5e+90d\,x^4\right)c^2+420x^2\left(\frac{3ex}{5}+d\right)ac-630a^2(-ex+d)\right)g^5+1260f\left(\left(-\frac{5}{63}x^4e-\frac{4}{35}d\,x^3\right)c^2-\frac{4\left(\frac{3ex}{10}+d\right)xac}{3}+a^2e\right)g^4-}{315\sqrt{gx}}$
risch	$\frac{2\left(35g^4c^2e\,x^4+45g^4c^2d\,x^3-85f\,g^3e\,x^3c^2+126g^4ace\,x^2-117f\,g^3c^2d\,x^2+165f^2c^2g^2e\,x^2+210g^4adxc-378f\,g^3acex+26\right.}{315g^6}$
gosper	$-\frac{2\left(-35c^2e\,x^5g^5-45c^2d\,g^5x^4+50c^2ef\,g^4x^4-126ace\,g^5x^3+72c^2df\,g^4x^3-80c^2e\,f^2g^3x^3-210acd\,g^5x^2+252acef\,g^4x^2\right.}{}$
trager	$-\frac{2\left(-35c^2e\,x^5g^5-45c^2d\,g^5x^4+50c^2ef\,g^4x^4-126ace\,g^5x^3+72c^2df\,g^4x^3-80c^2e\,f^2g^3x^3-210acd\,g^5x^2+252acef\,g^4x^2\right.}{}$
orering	$-\frac{2\left(-35c^2e\,x^5g^5-45c^2d\,g^5x^4+50c^2ef\,g^4x^4-126ace\,g^5x^3+72c^2df\,g^4x^3-80c^2e\,f^2g^3x^3-210acd\,g^5x^2+252acef\,g^4x^2\right.}{}$
derivativedivides	$\frac{\frac{2e\,c^2(gx+f)^{\frac{9}{2}}}{9}+\frac{2c^2dg(gx+f)^{\frac{7}{2}}}{7}-\frac{10c^2ef(gx+f)^{\frac{7}{2}}}{7}+\frac{4ace\,g^2(gx+f)^{\frac{5}{2}}}{5}-\frac{8c^2dfg(gx+f)^{\frac{5}{2}}}{5}+4c^2e\,f^2(gx+f)^{\frac{5}{2}}+\frac{4acd\,g^3(gx+f)^{\frac{3}{2}}}{3}}{}$
default	$\frac{\frac{2e\,c^2(gx+f)^{\frac{9}{2}}}{9}+\frac{2c^2dg(gx+f)^{\frac{7}{2}}}{7}-\frac{10c^2ef(gx+f)^{\frac{7}{2}}}{7}+\frac{4ace\,g^2(gx+f)^{\frac{5}{2}}}{5}-\frac{8c^2dfg(gx+f)^{\frac{5}{2}}}{5}+4c^2e\,f^2(gx+f)^{\frac{5}{2}}+\frac{4acd\,g^3(gx+f)^{\frac{3}{2}}}{3}}{}$

input
```
int((e*x+d)*(c*x^2+a)^2/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(((70*e*x^5+90*d*x^4)*c^2+420*x^2*(3/5*e*x+d)*a*c-630*a^2*(-e*x+d))*
g^5+1260*f*((-5/63*x^4*e-4/35*d*x^3)*c^2-4/3*(3/10*e*x+d)*x*a*c+a^2*e)*g^4
-3360*f^2*c*(-3/35*x^2*(5/9*e*x+d)*c+a*(-3/5*e*x+d))*g^3+4032*f^3*(-2/7*x*
(5/18*e*x+d)*c+a*e)*c*g^2-2304*f^4*c^2*(-5/9*e*x+d)*g+2560*c^2*e*f^5)/(g*x
+f)^(1/2)/g^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)\left(a+cx^2\right)^2}{(f+gx)^{3/2}}\,dx = \frac{2\left(35\,c^2eg^5x^5+1280\,c^2ef^5-1152\,c^2df^4g+2016\,acef^3g^2-1680\,acdf^2g^3+63\right.}{}$$

input
```
integrate((e*x+d)*(c*x^2+a)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*c^2*e*g^5*x^5 + 1280*c^2*e*f^5 - 1152*c^2*d*f^4*g + 2016*a*c*e*f
^3*g^2 - 1680*a*c*d*f^2*g^3 + 630*a^2*e*f*g^4 - 315*a^2*d*g^5 - 5*(10*c^2*
e*f*g^4 - 9*c^2*d*g^5)*x^4 + 2*(40*c^2*e*f^2*g^3 - 36*c^2*d*f*g^4 + 63*a*c
*e*g^5)*x^3 - 2*(80*c^2*e*f^3*g^2 - 72*c^2*d*f^2*g^3 + 126*a*c*e*f*g^4 - 1
05*a*c*d*g^5)*x^2 + (640*c^2*e*f^4*g - 576*c^2*d*f^3*g^2 + 1008*a*c*e*f^2*
g^3 - 840*a*c*d*f*g^4 + 315*a^2*e*g^5)*x)*sqrt(g*x + f)/(g^7*x + f*g^6)
```

Sympy [A] (verification not implemented)

Time = 6.69 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)\left(a+cx^2\right)^2}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^2e(f+gx)^{\frac{9}{2}}}{9g^5}+\frac{(f+gx)^{\frac{7}{2}}\left(c^2dg-5c^2ef\right)}{7g^5}+\frac{(f+gx)^{\frac{5}{2}}\cdot\left(2aceg^2-4c^2dfg+10c^2ef^2\right)}{5g^5}+\frac{(f+gx)^{\frac{3}{2}}\cdot\left(2acdg^3-6acefg^2\right.}{3g^5}}{g} \\ \frac{a^2dx+\frac{a^2ex^2}{2}+\frac{2acdx^3}{3}+\frac{acex^4}{2}+\frac{c^2dx^5}{5}+\frac{c^2ex^6}{6}}{f^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((e*x+d)*(c*x**2+a)**2/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c**2*e*(f + g*x)**(9/2)/(9*g**5) + (f + g*x)**(7/2)*(c**2*d*
g - 5*c**2*e*f)/(7*g**5) + (f + g*x)**(5/2)*(2*a*c*e*g**2 - 4*c**2*d*f*g +
 10*c**2*e*f**2)/(5*g**5) + (f + g*x)**(3/2)*(2*a*c*d*g**3 - 6*a*c*e*f*g**
2 + 6*c**2*d*f**2*g - 10*c**2*e*f**3)/(3*g**5) + sqrt(f + g*x)*(a**2*e*g**
4 - 4*a*c*d*f*g**3 + 6*a*c*e*f**2*g**2 - 4*c**2*d*f**3*g + 5*c**2*e*f**4)/
g**5 - (a*g**2 + c*f**2)**2*(d*g - e*f)/(g**5*sqrt(f + g*x)))/g, Ne(g, 0))
, ((a**2*d*x + a**2*e*x**2/2 + 2*a*c*d*x**3/3 + a*c*e*x**4/2 + c**2*d*x**5
/5 + c**2*e*x**6/6)/f**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)\left(a+cx^2\right)^2}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{35(gx+f)^{\frac{9}{2}}c^2e-45\left(5c^2ef-c^2dg\right)(gx+f)^{\frac{7}{2}}+126\left(5c^2ef^2-2c^2dfg+aceg^2\right)(gx+f)^{\frac{5}{2}}-210\left(5c^2ef^3-3\right.}{g^5}}{}$$

input
```
integrate((e*x+d)*(c*x^2+a)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/315*((35*(g*x + f)^(9/2)*c^2*e - 45*(5*c^2*e*f - c^2*d*g)*(g*x + f)^(7/2
) + 126*(5*c^2*e*f^2 - 2*c^2*d*f*g + a*c*e*g^2)*(g*x + f)^(5/2) - 210*(5*c
^2*e*f^3 - 3*c^2*d*f^2*g + 3*a*c*e*f*g^2 - a*c*d*g^3)*(g*x + f)^(3/2) + 31
5*(5*c^2*e*f^4 - 4*c^2*d*f^3*g + 6*a*c*e*f^2*g^2 - 4*a*c*d*f*g^3 + a^2*e*g
^4)*sqrt(g*x + f))/g^5 + 315*(c^2*e*f^5 - c^2*d*f^4*g + 2*a*c*e*f^3*g^2 -
2*a*c*d*f^2*g^3 + a^2*e*f*g^4 - a^2*d*g^5)/(sqrt(g*x + f)*g^5))/g
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)\left(a+cx^2\right)^2}{(f+gx)^{3/2}}\,dx = \frac{2\left(c^2ef^5 - c^2df^4g + 2\,acef^3g^2 - 2\,acdf^2g^3 + a^2efg^4 - a^2dg^5\right)}{\sqrt{gx+f}\,g^6} + \frac{2\left(35\,(gx+f)^{\frac{9}{2}}c^2eg^{48} - 225\,(gx+f)^{\frac{7}{2}}c^2efg^{48} + 630\,(gx+f)^{\frac{5}{2}}c^2ef^2g^{48} - 1050\,(gx+f)^{\frac{3}{2}}c^2ef^3g^{48} + 157\right.}{}$$

input
```
integrate((e*x+d)*(c*x^2+a)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
2*(c^2*e*f^5 - c^2*d*f^4*g + 2*a*c*e*f^3*g^2 - 2*a*c*d*f^2*g^3 + a^2*e*f*g
^4 - a^2*d*g^5)/(sqrt(g*x + f)*g^6) + 2/315*(35*(g*x + f)^(9/2)*c^2*e*g^48
 - 225*(g*x + f)^(7/2)*c^2*e*f*g^48 + 630*(g*x + f)^(5/2)*c^2*e*f^2*g^48 -
 1050*(g*x + f)^(3/2)*c^2*e*f^3*g^48 + 1575*sqrt(g*x + f)*c^2*e*f^4*g^48 +
 45*(g*x + f)^(7/2)*c^2*d*g^49 - 252*(g*x + f)^(5/2)*c^2*d*f*g^49 + 630*(g
*x + f)^(3/2)*c^2*d*f^2*g^49 - 1260*sqrt(g*x + f)*c^2*d*f^3*g^49 + 126*(g*
x + f)^(5/2)*a*c*e*g^50 - 630*(g*x + f)^(3/2)*a*c*e*f*g^50 + 1890*sqrt(g*x
 + f)*a*c*e*f^2*g^50 + 210*(g*x + f)^(3/2)*a*c*d*g^51 - 1260*sqrt(g*x + f)
*a*c*d*f*g^51 + 315*sqrt(g*x + f)*a^2*e*g^52)/g^54
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)(a+cx^2)^2}{(f+gx)^{3/2}}\,dx = \frac{(f+g\,x)^{5/2}\,(20\,e\,c^2\,f^2-8\,d\,c^2\,f\,g+4\,a\,e\,c\,g^2)}{5\,g^6} - \frac{-2\,e\,a^2\,f\,g^4+2\,d\,a^2\,g^5-4\,e\,a\,c\,f^3\,g^2+4\,d\,a\,c\,f^2\,g^3-2\,e\,c^2\,f^5+2\,d\,c^2\,f^4\,g}{g^6\,\sqrt{f+g\,x}} + \frac{2\,\sqrt{f+g\,x}\,(c\,f^2+a\,g^2)\,(5\,c\,e\,f^2-4\,c\,d\,f\,g+a\,e\,g^2)}{g^6} + \frac{2\,c^2\,e\,(f+g\,x)^{9/2}}{9\,g^6} + \frac{4\,c\,(f+g\,x)^{3/2}\,(-5\,c\,e\,f^3+3\,c\,d\,f^2\,g-3\,a\,e\,f\,g^2+a\,d\,g^3)}{3\,g^6} + \frac{2\,c^2\,(f+g\,x)^{7/2}\,(d\,g-5\,e\,f)}{7\,g^6}$$

input
```
int(((a + c*x^2)^2*(d + e*x))/(f + g*x)^(3/2),x)
```

output
```
((f + g*x)^(5/2)*(20*c^2*e*f^2 + 4*a*c*e*g^2 - 8*c^2*d*f*g))/(5*g^6) - (2*
a^2*d*g^5 - 2*c^2*e*f^5 - 2*a^2*e*f*g^4 + 2*c^2*d*f^4*g + 4*a*c*d*f^2*g^3
- 4*a*c*e*f^3*g^2)/(g^6*(f + g*x)^(1/2)) + (2*(f + g*x)^(1/2)*(a*g^2 + c*f
^2)*(a*e*g^2 + 5*c*e*f^2 - 4*c*d*f*g))/g^6 + (2*c^2*e*(f + g*x)^(9/2))/(9*
g^6) + (4*c*(f + g*x)^(3/2)*(a*d*g^3 - 5*c*e*f^3 - 3*a*e*f*g^2 + 3*c*d*f^2
*g))/(3*g^6) + (2*c^2*(f + g*x)^(7/2)*(d*g - 5*e*f))/(7*g^6)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)(a+cx^2)^2}{(f+gx)^{3/2}}\,dx = \frac{\frac{2}{9}c^2e\,g^5x^5+\frac{2}{7}c^2d\,g^5x^4-\frac{20}{63}c^2ef\,g^4x^4+\frac{4}{5}ace\,g^5x^3-\frac{16}{35}c^2df\,g^4x^3+\frac{32}{63}c^2e\,f^2g^3x^3}{}$$

input
```
int((e*x+d)*(c*x^2+a)^2/(g*x+f)^(3/2),x)
```

output
```
(2*( - 315*a**2*d*g**5 + 630*a**2*e*f*g**4 + 315*a**2*e*g**5*x - 1680*a*c*
d*f**2*g**3 - 840*a*c*d*f*g**4*x + 210*a*c*d*g**5*x**2 + 2016*a*c*e*f**3*g
**2 + 1008*a*c*e*f**2*g**3*x - 252*a*c*e*f*g**4*x**2 + 126*a*c*e*g**5*x**3
 - 1152*c**2*d*f**4*g - 576*c**2*d*f**3*g**2*x + 144*c**2*d*f**2*g**3*x**2
 - 72*c**2*d*f*g**4*x**3 + 45*c**2*d*g**5*x**4 + 1280*c**2*e*f**5 + 640*c*
*2*e*f**4*g*x - 160*c**2*e*f**3*g**2*x**2 + 80*c**2*e*f**2*g**3*x**3 - 50*
c**2*e*f*g**4*x**4 + 35*c**2*e*g**5*x**5))/(315*sqrt(f + g*x)*g**6)
```

3.87 $\int \frac{(a+cx^2)^2}{(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{(a+cx^2)^2}{(f+gx)^{3/2}}\,dx = -\frac{2(cf^2+ag^2)^2}{g^5\sqrt{f+gx}} - \frac{8cf(cf^2+ag^2)\sqrt{f+gx}}{g^5} + \frac{4c(3cf^2+ag^2)(f+gx)^{3/2}}{3g^5} - \frac{8c^2f(f+gx)^{5/2}}{5g^5} + \frac{2c^2(f+gx)^{7/2}}{7g^5}$$

output
```
-2*(a*g^2+c*f^2)^2/g^5/(g*x+f)^(1/2)-8*c*f*(a*g^2+c*f^2)*(g*x+f)^(1/2)/g^5
+4/3*c*(a*g^2+3*c*f^2)*(g*x+f)^(3/2)/g^5-8/5*c^2*f*(g*x+f)^(5/2)/g^5+2/7*c
^2*(g*x+f)^(7/2)/g^5
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.79

$$\int \frac{(a+cx^2)^2}{(f+gx)^{3/2}}\,dx = -\frac{2(105a^2g^4+70acg^2(8f^2+4fgx-g^2x^2)+3c^2(128f^4+64f^3gx-16f^2g^2x^2+8fg^3x^3-5g^4x^4))}{105g^5\sqrt{f+gx}}$$

input
```
Integrate[(a + c*x^2)^2/(f + g*x)^(3/2),x]
```

output
```
(-2*(105*a^2*g^4 + 70*a*c*g^2*(8*f^2 + 4*f*g*x - g^2*x^2) + 3*c^2*(128*f^4
 + 64*f^3*g*x - 16*f^2*g^2*x^2 + 8*f*g^3*x^3 - 5*g^4*x^4)))/(105*g^5*Sqrt[
f + g*x])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{2c\sqrt{f+gx}(ag^2+3cf^2)}{g^4}-\frac{4cf(ag^2+cf^2)}{g^4\sqrt{f+gx}}+\frac{(ag^2+cf^2)^2}{g^4(f+gx)^{3/2}}+\frac{c^2(f+gx)^{5/2}}{g^4}-\frac{4c^2f(f+gx)^{3/2}}{g^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c(f+gx)^{3/2}(ag^2+3cf^2)}{3g^5}-\frac{8cf\sqrt{f+gx}(ag^2+cf^2)}{g^5}-\frac{2(ag^2+cf^2)^2}{g^5\sqrt{f+gx}}+\frac{2c^2(f+gx)^{7/2}}{7g^5}-\frac{8c^2f(f+gx)^{5/2}}{5g^5}$$

input
```
Int[(a + c*x^2)^2/(f + g*x)^(3/2),x]
```

output
```
(-2*(c*f^2 + a*g^2)^2)/(g^5*Sqrt[f + g*x]) - (8*c*f*(c*f^2 + a*g^2)*Sqrt[f
 + g*x])/g^5 + (4*c*(3*c*f^2 + a*g^2)*(f + g*x)^(3/2))/(3*g^5) - (8*c^2*f*
(f + g*x)^(5/2))/(5*g^5) + (2*c^2*(f + g*x)^(7/2))/(7*g^5)
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{2\left(\left(-\frac{1}{7}c^2x^4-\frac{2}{3}ac\,x^2+a^2\right)g^4+\frac{8x\left(\frac{3c\,x^2}{35}+a\right)cf\,g^3}{3}+\frac{16\left(-\frac{3c\,x^2}{35}+a\right)c\,f^2g^2}{3}+\frac{64c^2f^3gx}{35}+\frac{128c^2f^4}{35}\right)}{\sqrt{gx+f}\,g^5}$
risch	$-\frac{2c\left(-15c\,x^3g^3+39fc\,x^2g^2-70g^3ax-87cx\,f^2g+350af\,g^2+279c\,f^3\right)\sqrt{gx+f}}{105g^5}-\frac{2\left(a^2g^4+2ac\,f^2g^2+c^2f^4\right)}{g^5\sqrt{gx+f}}$
gosper	$-\frac{2\left(-15c^2x^4g^4+24c^2f\,x^3g^3-70ac\,g^4x^2-48c^2f^2g^2x^2+280acf\,g^3x+192c^2f^3gx+105a^2g^4+560ac\,f^2g^2+384c^2f^4\right)}{105\sqrt{gx+f}\,g^5}$
trager	$-\frac{2\left(-15c^2x^4g^4+24c^2f\,x^3g^3-70ac\,g^4x^2-48c^2f^2g^2x^2+280acf\,g^3x+192c^2f^3gx+105a^2g^4+560ac\,f^2g^2+384c^2f^4\right)}{105\sqrt{gx+f}\,g^5}$
orering	$-\frac{2\left(-15c^2x^4g^4+24c^2f\,x^3g^3-70ac\,g^4x^2-48c^2f^2g^2x^2+280acf\,g^3x+192c^2f^3gx+105a^2g^4+560ac\,f^2g^2+384c^2f^4\right)}{105\sqrt{gx+f}\,g^5}$
derivativedivides	$\frac{\frac{2c^2(gx+f)^{\frac{7}{2}}}{7}-\frac{8c^2f(gx+f)^{\frac{5}{2}}}{5}+\frac{4ac\,g^2(gx+f)^{\frac{3}{2}}}{3}+4c^2f^2(gx+f)^{\frac{3}{2}}-8acf\,g^2\sqrt{gx+f}-8c^2f^3\sqrt{gx+f}-\frac{2\left(a^2g^4+2ac\,f^2g^2+c^2f\right.}{\sqrt{gx+f}}}{g^5}$
default	$\frac{\frac{2c^2(gx+f)^{\frac{7}{2}}}{7}-\frac{8c^2f(gx+f)^{\frac{5}{2}}}{5}+\frac{4ac\,g^2(gx+f)^{\frac{3}{2}}}{3}+4c^2f^2(gx+f)^{\frac{3}{2}}-8acf\,g^2\sqrt{gx+f}-8c^2f^3\sqrt{gx+f}-\frac{2\left(a^2g^4+2ac\,f^2g^2+c^2f\right.}{\sqrt{gx+f}}}{g^5}$

input
```
int((c*x^2+a)^2/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(g*x+f)^(1/2)*((-1/7*c^2*x^4-2/3*a*c*x^2+a^2)*g^4+8/3*x*(3/35*c*x^2+a)*
c*f*g^3+16/3*(-3/35*c*x^2+a)*c*f^2*g^2+64/35*c^2*f^3*g*x+128/35*c^2*f^4)/g
^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

$$\int \frac{(a+cx^2)^2}{(f+gx)^{3/2}}\,dx = \frac{2\,(15\,c^2g^4x^4 - 24\,c^2fg^3x^3 - 384\,c^2f^4 - 560\,acf^2g^2 - 105\,a^2g^4 + 2\,(24\,c^2f^2g^2 + 35\,acg}{105\,(g^6x + fg^5)}$$

input
```
integrate((c*x^2+a)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
2/105*(15*c^2*g^4*x^4 - 24*c^2*f*g^3*x^3 - 384*c^2*f^4 - 560*a*c*f^2*g^2 -
 105*a^2*g^4 + 2*(24*c^2*f^2*g^2 + 35*a*c*g^4)*x^2 - 8*(24*c^2*f^3*g + 35*
a*c*f*g^3)*x)*sqrt(g*x + f)/(g^6*x + f*g^5)
```

Sympy [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.30

$$\int \frac{(a+cx^2)^2}{(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{4c^2f(f+gx)^{\frac{5}{2}}}{5g^4} + \frac{c^2(f+gx)^{\frac{7}{2}}}{7g^4} + \frac{(f+gx)^{\frac{3}{2}}\cdot\left(2acg^2+6c^2f^2\right)}{3g^4} + \frac{\sqrt{f+gx}\left(-4acfg^2-4c^2f^3\right)}{g^4} - \frac{\left(ag^2+cf^2\right)^2}{g^4\sqrt{f+gx}}\right)}{g} & \text{for } g \neq \\ \frac{a^2x+\frac{2acx^3}{3}+\frac{c^2x^5}{5}}{f^{\frac{3}{2}}} & \text{otherwis} \end{cases}$$

input
```
integrate((c*x**2+a)**2/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(-4*c**2*f*(f + g*x)**(5/2)/(5*g**4) + c**2*(f + g*x)**(7/2)/
(7*g**4) + (f + g*x)**(3/2)*(2*a*c*g**2 + 6*c**2*f**2)/(3*g**4) + sqrt(f +
 g*x)*(-4*a*c*f*g**2 - 4*c**2*f**3)/g**4 - (a*g**2 + c*f**2)**2/(g**4*sqrt
(f + g*x)))/g, Ne(g, 0)), ((a**2*x + 2*a*c*x**3/3 + c**2*x**5/5)/f**(3/2),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{(a+cx^2)^2}{(f+gx)^{3/2}}\,dx = \frac{2\left(\frac{15\,(gx+f)^{\frac{7}{2}}c^2-84\,(gx+f)^{\frac{5}{2}}c^2f+70\,(3\,c^2f^2+acg^2)(gx+f)^{\frac{3}{2}}-420\,(c^2f^3+acfg^2)\sqrt{gx+f}}{g^4} - \frac{105\,(c^2f^4+2\,acf^2g}{\sqrt{gx+f}g^4}\right.}{105\,g}$$

input
```
integrate((c*x^2+a)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
2/105*((15*(g*x + f)^(7/2)*c^2 - 84*(g*x + f)^(5/2)*c^2*f + 70*(3*c^2*f^2
+ a*c*g^2)*(g*x + f)^(3/2) - 420*(c^2*f^3 + a*c*f*g^2)*sqrt(g*x + f))/g^4
- 105*(c^2*f^4 + 2*a*c*f^2*g^2 + a^2*g^4)/(sqrt(g*x + f)*g^4))/g
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.14

$$\int \frac{(a+cx^2)^2}{(f+gx)^{3/2}}\,dx = -\frac{2\,(c^2f^4+2\,acf^2g^2+a^2g^4)}{\sqrt{gx+f}g^5} + \frac{2\left(15\,(gx+f)^{\frac{7}{2}}c^2g^{30}-84\,(gx+f)^{\frac{5}{2}}c^2fg^{30}+210\,(gx+f)^{\frac{3}{2}}c^2f^2g^{30}-420\,\sqrt{gx+f}c^2f^3g^{30}+70\,(gx+f\right.}{105\,g^{35}}$$

input
```
integrate((c*x^2+a)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
-2*(c^2*f^4 + 2*a*c*f^2*g^2 + a^2*g^4)/(sqrt(g*x + f)*g^5) + 2/105*(15*(g*
x + f)^(7/2)*c^2*g^30 - 84*(g*x + f)^(5/2)*c^2*f*g^30 + 210*(g*x + f)^(3/2
)*c^2*f^2*g^30 - 420*sqrt(g*x + f)*c^2*f^3*g^30 + 70*(g*x + f)^(3/2)*a*c*g
^32 - 420*sqrt(g*x + f)*a*c*f*g^32)/g^35
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+cx^2\right)^2}{(f+gx)^{3/2}}\,dx = \frac{2\,c^2\,\left(f+g\,x\right)^{7/2}}{7\,g^5} - \frac{\sqrt{f+g\,x}\,\left(8\,c^2\,f^3+8\,a\,c\,f\,g^2\right)}{g^5} - \frac{2\,a^2\,g^4+4\,a\,c\,f^2\,g^2+2\,c^2\,f^4}{g^5\,\sqrt{f+g\,x}} + \frac{\left(f+g\,x\right)^{3/2}\,\left(12\,c^2\,f^2+4\,a\,c\,g^2\right)}{3\,g^5} - \frac{8\,c^2\,f\,\left(f+g\,x\right)^{5/2}}{5\,g^5}$$

input
```
int((a + c*x^2)^2/(f + g*x)^(3/2),x)
```

output
```
(2*c^2*(f + g*x)^(7/2))/(7*g^5) - ((f + g*x)^(1/2)*(8*c^2*f^3 + 8*a*c*f*g^
2))/g^5 - (2*a^2*g^4 + 2*c^2*f^4 + 4*a*c*f^2*g^2)/(g^5*(f + g*x)^(1/2)) +
((f + g*x)^(3/2)*(12*c^2*f^2 + 4*a*c*g^2))/(3*g^5) - (8*c^2*f*(f + g*x)^(5
/2))/(5*g^5)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+cx^2\right)^2}{(f+gx)^{3/2}}\,dx = \frac{\frac{2}{7}c^2g^4x^4-\frac{16}{35}c^2f\,g^3x^3+\frac{4}{3}ac\,g^4x^2+\frac{32}{35}c^2f^2g^2x^2-\frac{16}{3}acf\,g^3x-\frac{128}{35}c^2f^3gx-2a^2g^4-\frac{32}{3}}{\sqrt{gx+f}\,g^5}$$

input
```
int((c*x^2+a)^2/(g*x+f)^(3/2),x)
```

output
```
(2*( - 105*a**2*g**4 - 560*a*c*f**2*g**2 - 280*a*c*f*g**3*x + 70*a*c*g**4*
x**2 - 384*c**2*f**4 - 192*c**2*f**3*g*x + 48*c**2*f**2*g**2*x**2 - 24*c**
2*f*g**3*x**3 + 15*c**2*g**4*x**4))/(105*sqrt(f + g*x)*g**5)
```

3.88 $\int \frac{(a+cx^2)^2}{(d+ex)(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 202

$$\int \frac{(a+cx^2)^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2(cf^2+ag^2)^2}{g^4(ef-dg)\sqrt{f+gx}} + \frac{2c(2ae^2g^2+c(3e^2f^2+2defg+d^2g^2))\sqrt{f+gx}}{e^3g^4} - \frac{2c^2(3ef+dg)(f+gx)^{3/2}}{3e^2g^4} + \frac{2c^2(f+gx)^{5/2}}{5eg^4} - \frac{2(cd^2+ae^2)^2\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{7/2}(ef-dg)^{3/2}}$$

output
```
2*(a*g^2+c*f^2)^2/g^4/(-d*g+e*f)/(g*x+f)^(1/2)+2*c*(2*a*e^2*g^2+c*(d^2*g^2
+2*d*e*f*g+3*e^2*f^2))*(g*x+f)^(1/2)/e^3/g^4-2/3*c^2*(d*g+3*e*f)*(g*x+f)^(
3/2)/e^2/g^4+2/5*c^2*(g*x+f)^(5/2)/e/g^4-2*(a*e^2+c*d^2)^2*arctanh(e^(1/2)
*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(7/2)/(-d*g+e*f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.23

$$\int \frac{(a+cx^2)^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{-30a^2e^3g^4 - 60ace^2g^2(-dg(f+gx)+ef(2f+gx)) + 2c^2(15d^3g^3(f+gx) + \cdots}{} - \frac{2(cd^2+ae^2)^2\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{7/2}(-ef+dg)^{3/2}}$$

input
```
Integrate[(a + c*x^2)^2/((d + e*x)*(f + g*x)^(3/2)),x]
```

output
```
(-30*a^2*e^3*g^4 - 60*a*c*e^2*g^2*(-(d*g*(f + g*x)) + e*f*(2*f + g*x)) + 2
*c^2*(15*d^3*g^3*(f + g*x) + 5*d^2*e*g^2*(2*f^2 + f*g*x - g^2*x^2) - 3*e^3
*f*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) + d*e^2*g*(8*f^3 + 4*f^2*g
*x - f*g^2*x^2 + 3*g^3*x^3)))/(15*e^3*g^4*(-(e*f) + d*g)*Sqrt[f + g*x]) -
(2*(c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-(e*f) + d*g]])/(
e^(7/2)*(-(e*f) + d*g)^(3/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {649, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{649}$$

$$\frac{2\int -\frac{(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)^2}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{g^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{\left(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2\right)^2}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{g^4}$$

$$\Big\downarrow \text{1584}$$

$$-\frac{2\int \left(\frac{\left(cd^2+ae^2\right)^2 g^4}{e^3(ef-dg)(ef-dg-e(f+gx))}-\frac{c^2(f+gx)^2}{e}+\frac{c\left(-2ae^2g^2-c\left(3e^2f^2+2degf+d^2g^2\right)\right)}{e^3}+\frac{c^2(3ef+dg)(f+gx)}{e^2}+\frac{\left(cf^2+ag^2\right)^2}{(ef-dg)(f+gx)}\right)\cdots}{g^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{g^4\left(ae^2+cd^2\right)^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{7/2}(ef-dg)^{3/2}}+\frac{c\sqrt{f+gx}\left(2ae^2g^2+c\left(d^2g^2+2defg+3e^2f^2\right)\right)}{e^3}+\frac{\left(ag^2+cf^2\right)^2}{\sqrt{f+gx}(ef-dg)}-\frac{c^2(f+gx)^{3/2}(dg+3ef)}{3e^2}+\cdots\right.}{g^4}$$

input
```
Int[(a + c*x^2)^2/((d + e*x)*(f + g*x)^(3/2)),x]
```

output
```
(2*((c*f^2 + a*g^2)^2/((e*f - d*g)*Sqrt[f + g*x]) + (c*(2*a*e^2*g^2 + c*(3
*e^2*f^2 + 2*d*e*f*g + d^2*g^2))*Sqrt[f + g*x])/e^3 - (c^2*(3*e*f + d*g)*(
f + g*x)^(3/2))/(3*e^2) + (c^2*(f + g*x)^(5/2))/(5*e) - ((c*d^2 + a*e^2)^2
*g^4*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(7/2)*(e*f - d*g
)^(3/2))))/g^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.04

method	result
risch	$\frac{2c\left(3c\,x^2e^2g^2-5cde\,g^2x-9c\,e^2fgx+30a\,e^2g^2+15c\,d^2g^2+25cdefg+33c\,e^2f^2\right)\sqrt{gx+f}}{15g^4e^3}-\frac{2\left(\frac{g^4\left(a^2e^4+2ac\,d^2e^2+c^2d^4\right)\ldots}{(dg-ef)\sqrt{(d\ldots}}\right.}{\ldots}$
derivativedivides	$\frac{\frac{2c\left(\frac{c(gx+f)^{\frac{5}{2}}e^2}{5}-\frac{cdeg(gx+f)^{\frac{3}{2}}}{3}-c\,e^2f(gx+f)^{\frac{3}{2}}+2a\,e^2g^2\sqrt{gx+f}+c\,d^2g^2\sqrt{gx+f}+2\sqrt{gx+f}\,cdefg+3c\,e^2f^2\sqrt{gx+f}\right)}{e^3}-\frac{2g^4\left(a^2e^4\ldots\right.}{\ldots}}{g^4}$
default	$\frac{\frac{2c\left(\frac{c(gx+f)^{\frac{5}{2}}e^2}{5}-\frac{cdeg(gx+f)^{\frac{3}{2}}}{3}-c\,e^2f(gx+f)^{\frac{3}{2}}+2a\,e^2g^2\sqrt{gx+f}+c\,d^2g^2\sqrt{gx+f}+2\sqrt{gx+f}\,cdefg+3c\,e^2f^2\sqrt{gx+f}\right)}{e^3}-\frac{2g^4\left(a^2e^4\ldots\right.}{\ldots}}{g^4}$
pseudoelliptic	$-\frac{2\left(g^4\sqrt{gx+f}\,\left(a\,e^2+c\,d^2\right)^2\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)+\sqrt{(dg-ef)e}\,\left(\left(\left(\frac{16f\left(\frac{1}{16}x^3g^3-\frac{1}{8}f\,g^2x^2+\frac{1}{2}x\,f^2g+f^3\right)c^2}{5}+4g^2f\left(\frac{gx}{2}+\ldots\right.\right.\right.\right.\right.}{\sqrt{gx+f}\,\sqrt{(dg-e\ldots}}$

input
```
int((c*x^2+a)^2/(e*x+d)/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*c*(3*c*e^2*g^2*x^2-5*c*d*e*g^2*x-9*c*e^2*f*g*x+30*a*e^2*g^2+15*c*d^2*
g^2+25*c*d*e*f*g+33*c*e^2*f^2)*(g*x+f)^(1/2)/g^4/e^3-2/e^3/g^4*(g^4*(a^2*e
^4+2*a*c*d^2*e^2+c^2*d^4)/(d*g-e*f)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(
1/2)/((d*g-e*f)*e)^(1/2))+(a^2*g^4+2*a*c*f^2*g^2+c^2*f^4)*e^3/(d*g-e*f)/(g
*x+f)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(180) = 360$.

Time = 0.14 (sec) , antiderivative size = 1062, normalized size of antiderivative = 5.26

$$\int \frac{(a + cx^2)^2}{(d + ex)(f + gx)^{3/2}} \, dx = \text{Too large to display}$$

input

```
integrate((c*x^2+a)^2/(e*x+d)/(g*x+f)^(3/2),x, algorithm="fricas")
```

output

```
[-1/15*(15*((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*g^5*x + (c^2*d^4 + 2*a*c*d
^2*e^2 + a^2*e^4)*f*g^4)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g + 2*
sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(48*c^2*e^5*f^5 - 56*c^2
*d*e^4*f^4*g - 15*a^2*d*e^4*g^5 - 2*(c^2*d^2*e^3 - 30*a*c*e^5)*f^3*g^2 - 5
*(c^2*d^3*e^2 + 18*a*c*d*e^4)*f^2*g^3 + 15*(c^2*d^4*e + 2*a*c*d^2*e^3 + a^
2*e^5)*f*g^4 + 3*(c^2*e^5*f^2*g^3 - 2*c^2*d*e^4*f*g^4 + c^2*d^2*e^3*g^5)*x
^3 - (6*c^2*e^5*f^3*g^2 - 7*c^2*d*e^4*f^2*g^3 - 4*c^2*d^2*e^3*f*g^4 + 5*c^
2*d^3*e^2*g^5)*x^2 + (24*c^2*e^5*f^4*g - 28*c^2*d*e^4*f^3*g^2 - (c^2*d^2*e
^3 - 30*a*c*e^5)*f^2*g^3 - 10*(c^2*d^3*e^2 + 6*a*c*d*e^4)*f*g^4 + 15*(c^2*
d^4*e + 2*a*c*d^2*e^3)*g^5)*x)*sqrt(g*x + f))/(e^6*f^3*g^4 - 2*d*e^5*f^2*g
^5 + d^2*e^4*f*g^6 + (e^6*f^2*g^5 - 2*d*e^5*f*g^6 + d^2*e^4*g^7)*x), 2/15*
(15*((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*g^5*x + (c^2*d^4 + 2*a*c*d^2*e^2
+ a^2*e^4)*f*g^4)*sqrt(-e^2*f + d*e*g)*arctan(sqrt(-e^2*f + d*e*g)*sqrt(g*
x + f)/(e*g*x + e*f)) + (48*c^2*e^5*f^5 - 56*c^2*d*e^4*f^4*g - 15*a^2*d*e^
4*g^5 - 2*(c^2*d^2*e^3 - 30*a*c*e^5)*f^3*g^2 - 5*(c^2*d^3*e^2 + 18*a*c*d*e
^4)*f^2*g^3 + 15*(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*f*g^4 + 3*(c^2*e^5*
f^2*g^3 - 2*c^2*d*e^4*f*g^4 + c^2*d^2*e^3*g^5)*x^3 - (6*c^2*e^5*f^3*g^2 -
7*c^2*d*e^4*f^2*g^3 - 4*c^2*d^2*e^3*f*g^4 + 5*c^2*d^3*e^2*g^5)*x^2 + (24*c
^2*e^5*f^4*g - 28*c^2*d*e^4*f^3*g^2 - (c^2*d^2*e^3 - 30*a*c*e^5)*f^2*g^3 -
 10*(c^2*d^3*e^2 + 6*a*c*d*e^4)*f*g^4 + 15*(c^2*d^4*e + 2*a*c*d^2*e^3)*...
```

Sympy [A] (verification not implemented)

Time = 10.34 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.55

$$\int \frac{(a+cx^2)^2}{(d+ex)(f+gx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^2(f+gx)^{\frac{5}{2}}}{5eg^3} - \frac{\left(ag^2+cf^2\right)^2}{g^3\sqrt{f+gx}(dg-ef)} + \frac{(f+gx)^{\frac{3}{2}}\left(-c^2dg-3c^2ef\right)}{3e^2g^3} + \frac{\sqrt{f+gx}\left(2ace^2g^2+c^2d^2g^2+2c^2defg+3c^2e^2f^2\right)}{e^3g^3}\right.}{g} \\ \frac{-\frac{c^2dx^3}{3e^2} + \frac{c^2x^4}{4e} + \frac{x^2\cdot\left(2ace^2+c^2d^2\right)}{2e^3} + \frac{x\left(-2acde^2-c^2d^3\right)}{e^4} + \frac{\left(ae^2+cd^2\right)^2\left(\begin{cases}\frac{x}{d} & \text{for } e=0\\ \frac{\log(d+ex)}{e} & \text{otherwise}\end{cases}\right)}{e^4}}{f^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)/(g*x+f)**(3/2),x)
```

output
```
Piecewise((2*(c**2*(f + g*x)**(5/2)/(5*e*g**3) - (a*g**2 + c*f**2)**2/(g**
3*sqrt(f + g*x)*(d*g - e*f)) + (f + g*x)**(3/2)*(-c**2*d*g - 3*c**2*e*f)/(
3*e**2*g**3) + sqrt(f + g*x)*(2*a*c*e**2*g**2 + c**2*d**2*g**2 + 2*c**2*d*
e*f*g + 3*c**2*e**2*f**2)/(e**3*g**3) - g*(a*e**2 + c*d**2)**2*atan(sqrt(f
 + g*x)/sqrt((d*g - e*f)/e))/(e**4*sqrt((d*g - e*f)/e)*(d*g - e*f)))/g, Ne
(g, 0)), ((-c**2*d*x**3/(3*e**2) + c**2*x**4/(4*e) + x**2*(2*a*c*e**2 + c*
*2*d**2)/(2*e**3) + x*(-2*a*c*d*e**2 - c**2*d**3)/e**4 + (a*e**2 + c*d**2)
**2*Piecewise((x/d, Eq(e, 0)), (log(d + e*x)/e, True))/e**4)/f**(3/2), Tru
e))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^2}{(d+ex)(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.35

$$\int \frac{(a+cx^2)^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{2\,(c^2d^4+2\,acd^2e^2+a^2e^4)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^4f-de^3g)\sqrt{-e^2f+deg}} + \frac{2\,(c^2f^4+2\,acf^2g^2+a^2g^4)}{(efg^4-dg^5)\sqrt{gx+f}} + \frac{2\left(3\,(gx+f)^{\frac{5}{2}}c^2e^4g^{16}-15\,(gx+f)^{\frac{3}{2}}c^2e^4fg^{16}+45\,\sqrt{gx+f}c^2e^4f^2g^{16}-5\,(gx+f)^{\frac{3}{2}}c^2de^3g^{17}+30\,\sqrt{gx+\cdots}\right.}{15\,e^5g^{20}}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
2*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f +
 d*e*g))/((e^4*f - d*e^3*g)*sqrt(-e^2*f + d*e*g)) + 2*(c^2*f^4 + 2*a*c*f^2
*g^2 + a^2*g^4)/((e*f*g^4 - d*g^5)*sqrt(g*x + f)) + 2/15*(3*(g*x + f)^(5/2
)*c^2*e^4*g^16 - 15*(g*x + f)^(3/2)*c^2*e^4*f*g^16 + 45*sqrt(g*x + f)*c^2*
e^4*f^2*g^16 - 5*(g*x + f)^(3/2)*c^2*d*e^3*g^17 + 30*sqrt(g*x + f)*c^2*d*e
^3*f*g^17 + 15*sqrt(g*x + f)*c^2*d^2*e^2*g^18 + 30*sqrt(g*x + f)*a*c*e^4*g
^18)/(e^5*g^20)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.58

$$\int \frac{(a+cx^2)^2}{(d+ex)(f+gx)^{3/2}}\,dx = \sqrt{f+g\,x}\,\left(\frac{12\,c^2\,f^2+4\,a\,c\,g^2}{e\,g^4} + \frac{\left(d\,g^5-e\,f\,g^4\right)\,\left(\frac{8\,c^2\,f}{e\,g^4}+\frac{2\,c^2\,\left(d\,g^5-e\,f\,g^4\right)}{e^2\,g^8}\right)}{e\,g^4}\right) - {\left(f+g\,x\right)}^{3/2}\,\left(\frac{8\,c^2\,f}{3\,e\,g^4}+\frac{2\,c^2\,\left(d\,g^5-e\,f\,g^4\right)}{3\,e^2\,g^8}\right)+\frac{2\,c^2\,{\left(f+g\,x\right)}^{5/2}}{5\,e\,g^4} + \frac{2\,\mathrm{atan}\left(\frac{2\,\sqrt{f+g\,x}\,{\left(c\,d^2+a\,e^2\right)}^2\,\left(e^4\,f-d\,e^3\,g\right)}{e^{5/2}\,{\left(d\,g-e\,f\right)}^{3/2}\,\left(2\,a^2\,e^4+4\,a\,c\,d^2\,e^2+2\,c^2\,d^4\right)}\right)\,{\left(c\,d^2+a\,e^2\right)}^2}{e^{7/2}\,{\left(d\,g-e\,f\right)}^{3/2}} - \frac{2\,\left(a^2\,e^3\,g^4+2\,a\,c\,e^3\,f^2\,g^2+c^2\,e^3\,f^4\right)}{e^3\,g^4\,\sqrt{f+g\,x}\,\left(d\,g-e\,f\right)}$$

input
```
int((a + c*x^2)^2/((f + g*x)^(3/2)*(d + e*x)),x)
```

output
```
(f + g*x)^(1/2)*((12*c^2*f^2 + 4*a*c*g^2)/(e*g^4) + ((d*g^5 - e*f*g^4)*((8
*c^2*f)/(e*g^4) + (2*c^2*(d*g^5 - e*f*g^4))/(e^2*g^8)))/(e*g^4)) - (f + g*
x)^(3/2)*((8*c^2*f)/(3*e*g^4) + (2*c^2*(d*g^5 - e*f*g^4))/(3*e^2*g^8)) + (
2*c^2*(f + g*x)^(5/2))/(5*e*g^4) + (2*atan((2*(f + g*x)^(1/2)*(a*e^2 + c*d
^2)^2*(e^4*f - d*e^3*g))/(e^(5/2)*(d*g - e*f)^(3/2)*(2*a^2*e^4 + 2*c^2*d^4
 + 4*a*c*d^2*e^2)))*(a*e^2 + c*d^2)^2)/(e^(7/2)*(d*g - e*f)^(3/2)) - (2*(a
^2*e^3*g^4 + c^2*e^3*f^4 + 2*a*c*e^3*f^2*g^2))/(e^3*g^4*(f + g*x)^(1/2)*(d
*g - e*f))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.83

$$\int \frac{(a+cx^2)^2}{(d+ex)(f+gx)^{3/2}}\,dx = \frac{-\frac{2c^2d^2e^3f^2g^3x}{15}+4ac\,d^2e^3f\,g^4+4ac\,d^2e^3g^5x-12acd\,e^4f^2g^3+4ac\,e^5f^2g^3x- \cdots}{\cdots}$$

input
```
int((c*x^2+a)^2/(e*x+d)/(g*x+f)^(3/2),x)
```

output

```
(2*( - 15*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sq
rt(e)*sqrt(d*g - e*f)))*a**2*e**4*g**4 - 30*sqrt(e)*sqrt(f + g*x)*sqrt(d*g
 - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**2*e**2*g*
*4 - 15*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt
(e)*sqrt(d*g - e*f)))*c**2*d**4*g**4 - 15*a**2*d*e**4*g**5 + 15*a**2*e**5*
f*g**4 + 30*a*c*d**2*e**3*f*g**4 + 30*a*c*d**2*e**3*g**5*x - 90*a*c*d*e**4
*f**2*g**3 - 60*a*c*d*e**4*f*g**4*x + 60*a*c*e**5*f**3*g**2 + 30*a*c*e**5*
f**2*g**3*x + 15*c**2*d**4*e*f*g**4 + 15*c**2*d**4*e*g**5*x - 5*c**2*d**3*
e**2*f**2*g**3 - 10*c**2*d**3*e**2*f*g**4*x - 5*c**2*d**3*e**2*g**5*x**2 -
 2*c**2*d**2*e**3*f**3*g**2 - c**2*d**2*e**3*f**2*g**3*x + 4*c**2*d**2*e**
3*f*g**4*x**2 + 3*c**2*d**2*e**3*g**5*x**3 - 56*c**2*d*e**4*f**4*g - 28*c*
*2*d*e**4*f**3*g**2*x + 7*c**2*d*e**4*f**2*g**3*x**2 - 6*c**2*d*e**4*f*g**
4*x**3 + 48*c**2*e**5*f**5 + 24*c**2*e**5*f**4*g*x - 6*c**2*e**5*f**3*g**2
*x**2 + 3*c**2*e**5*f**2*g**3*x**3))/(15*sqrt(f + g*x)*e**4*g**4*(d**2*g**
2 - 2*d*e*f*g + e**2*f**2))
```

3.89 $\int \frac{(a+cx^2)^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 208

$$\int \frac{(a+cx^2)^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = -\frac{2(cf^2+ag^2)^2}{g^3(ef-dg)^2\sqrt{f+gx}} - \frac{4c^2(ef+dg)\sqrt{f+gx}}{e^3g^3} - \frac{(cd^2+ae^2)^2\sqrt{f+gx}}{e^3(ef-dg)^2(d+ex)} + \frac{2c^2(f+gx)^{3/2}}{3e^2g^3} + \frac{(cd^2+ae^2)\,(3ae^2g+cd(8ef-5dg))\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{7/2}(ef-dg)^{5/2}}$$

output

```
-2*(a*g^2+c*f^2)^2/g^3/(-d*g+e*f)^2/(g*x+f)^(1/2)-4*c^2*(d*g+e*f)*(g*x+f)^
(1/2)/e^3/g^3-(a*e^2+c*d^2)^2*(g*x+f)^(1/2)/e^3/(-d*g+e*f)^2/(e*x+d)+2/3*c
^2*(g*x+f)^(3/2)/e^2/g^3+(a*e^2+c*d^2)*(3*a*e^2*g+c*d*(-5*d*g+8*e*f))*arct
anh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e*f)^(1/2))/e^(7/2)/(-d*g+e*f)^(5/2)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.45

$$\int \frac{(a+cx^2)^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{-6ace^2g^2(2def^2+2e^2f^2x+d^2g(f+gx))-3a^2e^3g^3(2dg+e(f+3gx))+c}{}$$
$$+\frac{(cd^2+ae^2)(-3ae^2g+cd(-8ef+5dg))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{e^{7/2}(-ef+dg)^{5/2}}$$

input
```
Integrate[(a + c*x^2)^2/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

output
```
(-6*a*c*e^2*g^2*(2*d*e*f^2 + 2*e^2*f^2*x + d^2*g*(f + g*x)) - 3*a^2*e^3*g^
3*(2*d*g + e*(f + 3*g*x)) + c^2*(-15*d^4*g^3*(f + g*x) + 2*d^2*e^2*g*(f +
g*x)^2*(4*f + g*x) + 2*d^3*e*g^2*(7*f^2 + 2*f*g*x - 5*g^2*x^2) + 2*e^4*f^2
*x*(-8*f^2 - 4*f*g*x + g^2*x^2) - 2*d*e^3*f*(8*f^3 - 3*f*g^2*x^2 + 2*g^3*x
^3)))/(3*e^3*g^3*(e*f - d*g)^2*(d + e*x)*Sqrt[f + g*x]) + ((c*d^2 + a*e^2)
*(-3*a*e^2*g + c*d*(-8*e*f + 5*d*g))*ArcTan[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[-
(e*f) + d*g]])/(e^(7/2)*(-(e*f) + d*g)^(5/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {649, 1582, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)^2(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \; 649$$

$$\frac{2\int \frac{(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2)^2}{(f+gx)(ef-dg-e(f+gx))^2}d\sqrt{f+gx}}{g^3}$$

$$\Big\downarrow \; 1582$$

$$2\left(\frac{\frac{g^4\sqrt{f+gx}\left(ae^2+cd^2\right)^2}{2e^3(ef-dg)^2(-dg-e(f+gx)+ef)}-\frac{\int -\frac{2(ef-dg)\left(cf^2+ag^2\right)^2e^4-2c^2(ef-dg)^2(f+gx)^3e^3+2c^2(ef-dg)^2(3ef+dg)(f+gx)^2e^2+\left(a^2g^4e^5-2acg^2\left(2e^2\right.\right.}{(f+gx)(ef-dg-e(f+gx))}}{2e^4(ef-dg)^2}}{g^3}\right.$$

$\Big\downarrow$ 25

$$2\left(\frac{\frac{\int \frac{2(ef-dg)\left(cf^2+ag^2\right)^2e^4-2c^2(ef-dg)^2(f+gx)^3e^3+2c^2(ef-dg)^2(3ef+dg)(f+gx)^2e^2+\left(a^2g^4e^5-2acg^2\left(2e^2f^2-4degf+d^2g^2\right)e^3-c^2\left(6f^4e^5-8df^3ge^4+\right.\right.}{(f+gx)(ef-dg-e(f+gx))}}{2e^4(ef-dg)^2}}{g^3}\right.$$

$\Big\downarrow$ 2333

$$2\left(\frac{\frac{\int \left(\frac{2\left(cf^2+ag^2\right)^2e^4}{f+gx}+2c^2(ef-dg)^2(f+gx)e^2-4c^2(ef-dg)^2(ef+dg)e+\frac{\left(cd^2+ae^2\right)g^3\left(3age^2+cd(8ef-5dg)\right)e}{ef-dg-e(f+gx)}\right)d\sqrt{f+gx}}{2e^4(ef-dg)^2}+\frac{g^4\sqrt{f+gx}\left(ae^2\right.}{2e^3(ef-dg)^2(-dg-}}{g^3}\right.$$

$\Big\downarrow$ 2009

$$2\left(\frac{\frac{\frac{\sqrt{e}g^3\left(ae^2+cd^2\right)\left(3ae^2g+cd(8ef-5dg)\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}-\frac{2e^4\left(ag^2+cf^2\right)^2}{\sqrt{f+gx}}+\frac{2}{3}c^2e^2(f+gx)^{3/2}(ef-dg)^2-4c^2e\sqrt{f+gx}(ef-dg)^2(dg+ef)}{2e^4(ef-dg)^2}+}{g^3}\right.$$

input
```
Int[(a + c*x^2)^2/((d + e*x)^2*(f + g*x)^(3/2)),x]
```

output
```
(2*(((c*d^2 + a*e^2)^2*g^4*Sqrt[f + g*x])/(2*e^3*(e*f - d*g)^2*(e*f - d*g
- e*(f + g*x))) + ((-2*e^4*(c*f^2 + a*g^2)^2)/Sqrt[f + g*x] - 4*c^2*e*(e*f
 - d*g)^2*(e*f + d*g)*Sqrt[f + g*x] + (2*c^2*e^2*(e*f - d*g)^2*(f + g*x)^(
3/2))/3 + (Sqrt[e]*(c*d^2 + a*e^2)*g^3*(3*a*e^2*g + c*d*(8*e*f - 5*d*g))*A
rcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/Sqrt[e*f - d*g])/(2*e^4*(
e*f - d*g)^2)))/g^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
- d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{2c^2(-egx+6dg+5ef)\sqrt{gx+f}}{3g^3e^3}+\frac{-\frac{2g^3\left(\frac{\left(\frac{1}{2}a^2e^4g+acd^2e^2g+\frac{1}{2}c^2d^4g\right)\sqrt{gx+f}}{e(gx+f)+dg-ef}+\frac{(3a^2e^4g-2acd^2e^2g+8acde^3f-5c^2d^4g+}{2\sqrt{(dg-ef}}\right.}{(dg-ef)^2}}{e^3g^3}$
derivativedivides	$\frac{-\frac{2c^2\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+2dg\sqrt{gx+f}+2ef\sqrt{gx+f}\right)}{e^3}-\frac{2g^3\left(\frac{\left(\frac{1}{2}a^2e^4g+acd^2e^2g+\frac{1}{2}c^2d^4g\right)\sqrt{gx+f}}{e(gx+f)+dg-ef}+\frac{(3a^2e^4g-2acd^2e^2g+8acde^3f-5}{2\surd}\right.}{(dg-ef)^2e^3}}{g^3}$
default	$\frac{-\frac{2c^2\left(-\frac{e(gx+f)^{\frac{3}{2}}}{3}+2dg\sqrt{gx+f}+2ef\sqrt{gx+f}\right)}{e^3}-\frac{2g^3\left(\frac{\left(\frac{1}{2}a^2e^4g+acd^2e^2g+\frac{1}{2}c^2d^4g\right)\sqrt{gx+f}}{e(gx+f)+dg-ef}+\frac{(3a^2e^4g-2acd^2e^2g+8acde^3f-5}{2\surd}\right.}{(dg-ef)^2e^3}}{g^3}$
pseudoelliptic	$-\frac{2\left(\frac{3(ae^2+cd^2)g^3(ex+d)\left(ae^2g-\frac{5}{3}cd^2g+\frac{8}{3}cdef\right)\sqrt{gx+f}\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)e}}\right)}{2}+\left(\left(\frac{3a^2g^4x}{2}+\frac{a^2fg^3}{2}+2f^2cx\left(-\frac{cx^2}{6}+a\right)\right.\right.\right.}{}$

input
```
int((c*x^2+a)^2/(e*x+d)^2/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*c^2*(-e*g*x+6*d*g+5*e*f)*(g*x+f)^(1/2)/g^3/e^3+2/e^3/g^3*(-g^3/(d*g-e
*f)^2*((1/2*a^2*e^4*g+a*c*d^2*e^2*g+1/2*c^2*d^4*g)*(g*x+f)^(1/2)/(e*(g*x+f
)+d*g-e*f)+1/2*(3*a^2*e^4*g-2*a*c*d^2*e^2*g+8*a*c*d*e^3*f-5*c^2*d^4*g+8*c^
2*d^3*e*f)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2))
)-e^3*(a^2*g^4+2*a*c*f^2*g^2+c^2*f^4)/(d*g-e*f)^2/(g*x+f)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(188) = 376$.

Time = 0.14 (sec) , antiderivative size = 1736, normalized size of antiderivative = 8.35

$$\int \frac{(a+cx^2)^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(8*(c^2*d^4*e + a*c*d^2*e^3)*f^2*g^3 - (5*c^2*d^5 + 2*a*c*d^3*e^2
- 3*a^2*d*e^4)*f*g^4 + (8*(c^2*d^3*e^2 + a*c*d*e^4)*f*g^4 - (5*c^2*d^4*e +
 2*a*c*d^2*e^3 - 3*a^2*e^5)*g^5)*x^2 + (8*(c^2*d^3*e^2 + a*c*d*e^4)*f^2*g^
3 + 3*(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5)*f*g^4 - (5*c^2*d^5 + 2*a*c*d^3
*e^2 - 3*a^2*d*e^4)*g^5)*x)*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g +
 2*sqrt(e^2*f - d*e*g)*sqrt(g*x + f))/(e*x + d)) - 2*(16*c^2*d*e^5*f^5 - 2
4*c^2*d^2*e^4*f^4*g - 6*a^2*d^2*e^4*g^5 - 6*(c^2*d^3*e^3 - 2*a*c*d*e^5)*f^
3*g^2 + (29*c^2*d^4*e^2 - 6*a*c*d^2*e^4 + 3*a^2*e^6)*f^2*g^3 - 3*(5*c^2*d^
5*e + 2*a*c*d^3*e^3 - a^2*d*e^5)*f*g^4 - 2*(c^2*e^6*f^3*g^2 - 3*c^2*d*e^5*
f^2*g^3 + 3*c^2*d^2*e^4*f*g^4 - c^2*d^3*e^3*g^5)*x^3 + 2*(4*c^2*e^6*f^4*g
- 7*c^2*d*e^5*f^3*g^2 - 3*c^2*d^2*e^4*f^2*g^3 + 11*c^2*d^3*e^3*f*g^4 - 5*c
^2*d^4*e^2*g^5)*x^2 + (16*c^2*e^6*f^5 - 16*c^2*d*e^5*f^4*g - 6*(3*c^2*d^2*
e^4 - 2*a*c*e^6)*f^3*g^2 + 2*(7*c^2*d^3*e^3 - 6*a*c*d*e^5)*f^2*g^3 + (19*c
^2*d^4*e^2 + 6*a*c*d^2*e^4 + 9*a^2*e^6)*f*g^4 - 3*(5*c^2*d^5*e + 2*a*c*d^3
*e^3 + 3*a^2*d*e^5)*g^5)*x)*sqrt(g*x + f))/(d*e^7*f^4*g^3 - 3*d^2*e^6*f^3*
g^4 + 3*d^3*e^5*f^2*g^5 - d^4*e^4*f*g^6 + (e^8*f^3*g^4 - 3*d*e^7*f^2*g^5 +
 3*d^2*e^6*f*g^6 - d^3*e^5*g^7)*x^2 + (e^8*f^4*g^3 - 2*d*e^7*f^3*g^4 + 2*d
^3*e^5*f*g^6 - d^4*e^4*g^7)*x), -1/3*(3*(8*(c^2*d^4*e + a*c*d^2*e^3)*f^2*g
^3 - (5*c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*f*g^4 + (8*(c^2*d^3*e^2 + a
*c*d*e^4)*f*g^4 - (5*c^2*d^4*e + 2*a*c*d^2*e^3 - 3*a^2*e^5)*g^5)*x^2 + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)**2/(g*x+f)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(188) = 376.

Time = 0.12 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.96

$$\int \frac{(a+cx^2)^2}{(d+ex)^2(f+gx)^{3/2}}\,dx =$$
$$-\frac{(8\,c^2d^3ef+8\,acde^3f-5\,c^2d^4g-2\,acd^2e^2g+3\,a^2e^4g)\arctan\left(\frac{\sqrt{gx+f}e}{\sqrt{-e^2f+deg}}\right)}{(e^5f^2-2\,de^4fg+d^2e^3g^2)\sqrt{-e^2f+deg}}$$
$$-\frac{2\,(gx+f)c^2e^4f^4-2\,c^2e^4f^5+2\,c^2de^3f^4g+4\,(gx+f)ace^4f^2g^2-4\,ace^4f^3g^2+4\,acde^3f^2g^3+(gx+f)c}{(e^5f^2g^3-2\,de^4fg^4+d^2e^3g^5)\left((gx+f)^{\frac{3}{2}}e-\sqrt{gx+}\right.}$$
$$+\frac{2\left((gx+f)^{\frac{3}{2}}c^2e^4g^6-6\sqrt{gx+f}c^2e^4fg^6-6\sqrt{gx+f}c^2de^3g^7\right)}{3\,e^6g^9}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^2/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
-(8*c^2*d^3*e*f + 8*a*c*d*e^3*f - 5*c^2*d^4*g - 2*a*c*d^2*e^2*g + 3*a^2*e^
4*g)*arctan(sqrt(g*x + f)*e/sqrt(-e^2*f + d*e*g))/((e^5*f^2 - 2*d*e^4*f*g
+ d^2*e^3*g^2)*sqrt(-e^2*f + d*e*g)) - (2*(g*x + f)*c^2*e^4*f^4 - 2*c^2*e^
4*f^5 + 2*c^2*d*e^3*f^4*g + 4*(g*x + f)*a*c*e^4*f^2*g^2 - 4*a*c*e^4*f^3*g^
2 + 4*a*c*d*e^3*f^2*g^3 + (g*x + f)*c^2*d^4*g^4 + 2*(g*x + f)*a*c*d^2*e^2*
g^4 + 3*(g*x + f)*a^2*e^4*g^4 - 2*a^2*e^4*f*g^4 + 2*a^2*d*e^3*g^5)/((e^5*f
^2*g^3 - 2*d*e^4*f*g^4 + d^2*e^3*g^5)*((g*x + f)^(3/2)*e - sqrt(g*x + f)*e
*f + sqrt(g*x + f)*d*g)) + 2/3*((g*x + f)^(3/2)*c^2*e^4*g^6 - 6*sqrt(g*x +
 f)*c^2*e^4*f*g^6 - 6*sqrt(g*x + f)*c^2*d*e^3*g^7)/(e^6*g^9)
```

Mupad [B] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.95

$$\int \frac{(a+cx^2)^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \frac{2\,c^2\,{(f+g\,x)}^{3/2}}{3\,e^2\,g^3} - \frac{\frac{2\,\left(a^2\,e^3\,g^4+2\,a\,c\,e^3\,f^2\,g^2+c^2\,e^3\,f^4\right)}{d\,g-e\,f}+\frac{(f+g\,x)\,\left(3\,a^2\,e^4\,g^4+2\,a\,c\,d^2\,e^2\,g^4+4\,a\,c\,e^4\,f^2\,g^2+c^2\,d^4\,g^4+2\,c^2\,e^4\,f^4\right)}{{(d\,g-e\,f)}^2}}{\sqrt{f+g\,x}\,\left(d\,e^3\,g^4-e^4\,f\,g^3\right)+e^4\,g^3\,{(f+g\,x)}^{3/2}} - \sqrt{f+g\,x}\,\left(\frac{8\,c^2\,f}{e^2\,g^3}+\frac{4\,c^2\,(d\,g-e\,f)}{e^3\,g^3}\right) - \frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,\left(c\,d^2+a\,e^2\right)\,\left(-5\,c\,g\,d^2+8\,c\,f\,d\,e+3\,a\,g\,e^2\right)\,\left(d^2\,e^3\,g^2-2\,d\,e^4\,f\,g+e^5\,f^2\right)}{e^{5/2}\,{(d\,g-e\,f)}^{5/2}\,\left(3\,g\,a^2\,e^4-2\,g\,a\,c\,d^2\,e^2+8\,f\,a\,c\,d\,e^3-5\,g\,c^2\,d^4+8\,f\,c^2\,d^3\,e\right)}\right)}{e^{7/2}\,{(d\,g-e\,f)}^{5/2}}$$

input
```
int((a + c*x^2)^2/((f + g*x)^(3/2)*(d + e*x)^2),x)
```

output
```
(2*c^2*(f + g*x)^(3/2))/(3*e^2*g^3) - ((2*(a^2*e^3*g^4 + c^2*e^3*f^4 + 2*a
*c*e^3*f^2*g^2))/(d*g - e*f) + ((f + g*x)*(3*a^2*e^4*g^4 + c^2*d^4*g^4 + 2
*c^2*e^4*f^4 + 2*a*c*d^2*e^2*g^4 + 4*a*c*e^4*f^2*g^2))/(d*g - e*f)^2)/((f
+ g*x)^(1/2)*(d*e^3*g^4 - e^4*f*g^3) + e^4*g^3*(f + g*x)^(3/2)) - (f + g*x
)^(1/2)*((8*c^2*f)/(e^2*g^3) + (4*c^2*(d*g - e*f))/(e^3*g^3)) - (atan(((f
+ g*x)^(1/2)*(a*e^2 + c*d^2)*(3*a*e^2*g - 5*c*d^2*g + 8*c*d*e*f)*(e^5*f^2
+ d^2*e^3*g^2 - 2*d*e^4*f*g))/(e^(5/2)*(d*g - e*f)^(5/2)*(3*a^2*e^4*g - 5*
c^2*d^4*g + 8*c^2*d^3*e*f - 2*a*c*d^2*e^2*g + 8*a*c*d*e^3*f)))*(a*e^2 + c*
d^2)*(3*a*e^2*g - 5*c*d^2*g + 8*c*d*e*f))/(e^(7/2)*(d*g - e*f)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1133, normalized size of antiderivative = 5.45

$$\int \frac{(a+cx^2)^2}{(d+ex)^2(f+gx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+a)^2/(e*x+d)^2/(g*x+f)^(3/2),x)
```

output
```
( - 9*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e
)*sqrt(d*g - e*f)))*a**2*d*e**4*g**4 - 9*sqrt(e)*sqrt(f + g*x)*sqrt(d*g -
e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a**2*e**5*g**4*x +
6*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sq
rt(d*g - e*f)))*a*c*d**3*e**2*g**4 - 24*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e
*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**2*e**3*f*g**3
 + 6*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)
*sqrt(d*g - e*f)))*a*c*d**2*e**3*g**4*x - 24*sqrt(e)*sqrt(f + g*x)*sqrt(d*
g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d*e**4*f*g*
*3*x + 15*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sq
rt(e)*sqrt(d*g - e*f)))*c**2*d**5*g**4 - 24*sqrt(e)*sqrt(f + g*x)*sqrt(d*g
 - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**4*e*f*g*
*3 + 15*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt
(e)*sqrt(d*g - e*f)))*c**2*d**4*e*g**4*x - 24*sqrt(e)*sqrt(f + g*x)*sqrt(d
*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**3*e**2
*f*g**3*x - 6*a**2*d**2*e**4*g**5 + 3*a**2*d*e**5*f*g**4 - 9*a**2*d*e**5*g
**5*x + 3*a**2*e**6*f**2*g**3 + 9*a**2*e**6*f*g**4*x - 6*a*c*d**3*e**3*f*g
**4 - 6*a*c*d**3*e**3*g**5*x - 6*a*c*d**2*e**4*f**2*g**3 + 6*a*c*d**2*e**4
*f*g**4*x + 12*a*c*d*e**5*f**3*g**2 - 12*a*c*d*e**5*f**2*g**3*x + 12*a*c*e
**6*f**3*g**2*x - 15*c**2*d**5*e*f*g**4 - 15*c**2*d**5*e*g**5*x + 29*c*...
```

3.90 $\int \frac{(a+cx^2)^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 290

$$\int \frac{(a+cx^2)^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{2(cf^2+ag^2)^2}{g^2(ef-dg)^3\sqrt{f+gx}} + \frac{2c^2\sqrt{f+gx}}{e^3g^2} - \frac{(cd^2+ae^2)^2\sqrt{f+gx}}{2e^3(ef-dg)^2(d+ex)^2} + \frac{(cd^2+ae^2)\left(7ae^2g+cd(16ef-9dg)\right)\sqrt{f+gx}}{4e^3(ef-dg)^3(d+ex)} - \frac{(15a^2e^4g^2+2ace^2(8e^2f^2+8defg-d^2g^2)+3c^2d^2(16e^2f^2-16defg+5d^2g^2))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{7/2}(ef-dg)^{7/2}}$$

output

```
2*(a*g^2+c*f^2)^2/g^2/(-d*g+e*f)^3/(g*x+f)^(1/2)+2*c^2*(g*x+f)^(1/2)/e^3/g
^2-1/2*(a*e^2+c*d^2)^2*(g*x+f)^(1/2)/e^3/(-d*g+e*f)^2/(e*x+d)^2+1/4*(a*e^2
+c*d^2)*(7*a*e^2*g+c*d*(-9*d*g+16*e*f))*(g*x+f)^(1/2)/e^3/(-d*g+e*f)^3/(e*
x+d)-1/4*(15*a^2*e^4*g^2+2*a*c*e^2*(-d^2*g^2+8*d*e*f*g+8*e^2*f^2)+3*c^2*d^
2*(5*d^2*g^2-16*d*e*f*g+16*e^2*f^2))*arctanh(e^(1/2)*(g*x+f)^(1/2)/(-d*g+e
*f)^(1/2))/e^(7/2)/(-d*g+e*f)^(7/2)
```

Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.56

$$\int \frac{(a+cx^2)^2}{(d+ex)^3(f+gx)^{3/2}}\,dx =$$

$$-\frac{2ace^2g^2(8e^3f^2x^2+d^3g(f+gx)+8de^2fx(3f+gx)+d^2e(14f^2+5fgx-g^2x^2))+a^2e^3g^2(8d^2g^2+deg(}{}$$

$$-\frac{(15a^2e^4g^2+2ace^2(8e^2f^2+8defg-d^2g^2)+3c^2d^2(16e^2f^2-16defg+5d^2g^2))\arctan\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{-ef+dg}}\right)}{4e^{7/2}(-ef+dg)^{7/2}}$$

input
```
Integrate[(a + c*x^2)^2/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

output
```
-1/4*(2*a*c*e^2*g^2*(8*e^3*f^2*x^2 + d^3*g*(f + g*x) + 8*d*e^2*f*x*(3*f +
g*x) + d^2*e*(14*f^2 + 5*f*g*x - g^2*x^2)) + a^2*e^3*g^2*(8*d^2*g^2 + d*e*
g*(9*f + 25*g*x) + e^2*(-2*f^2 + 5*f*g*x + 15*g^2*x^2)) + c^2*(-15*d^5*g^3
*(f + g*x) + 8*e^5*f^3*x^2*(2*f + g*x) + d^4*e*g^2*(38*f^2 + 13*f*g*x - 25
*g^2*x^2) - 8*d*e^4*f^2*x*(-4*f^2 + f*g*x + 3*g^2*x^2) - 8*d^3*e^2*g*(3*f^
3 - 5*f^2*g*x - 7*f*g^2*x^2 + g^3*x^3) + 8*d^2*e^3*f*(2*f^3 - 5*f^2*g*x -
3*f*g^2*x^2 + 3*g^3*x^3)))/(e^3*g^2*(-(e*f) + d*g)^3*(d + e*x)^2*Sqrt[f +
g*x]) - ((15*a^2*e^4*g^2 + 2*a*c*e^2*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2) + 3
*c^2*d^2*(16*e^2*f^2 - 16*d*e*f*g + 5*d^2*g^2))*ArcTan[(Sqrt[e]*Sqrt[f + g
*x])/Sqrt[-(e*f) + d*g]])/(4*e^(7/2)*(-(e*f) + d*g)^(7/2))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {649, 25, 1582, 25, 2336, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2}{(d+ex)^3(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \; 649$$

$$\frac{2\int -\frac{\left(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2\right)^2}{(f+gx)(ef-dg-e(f+gx))^3}d\sqrt{f+gx}}{g^2}$$

$\Big\downarrow$ 25

$$-\frac{2\int \frac{\left(cf^2-2c(f+gx)f+ag^2+c(f+gx)^2\right)^2}{(f+gx)(ef-dg-e(f+gx))^3}d\sqrt{f+gx}}{g^2}$$

$\Big\downarrow$ 1582

$$\frac{2\left(\frac{\int -\frac{4(ef-dg)\left(cf^2+ag^2\right)^2e^4-4c^2(ef-dg)^2(f+gx)^3e^3+4c^2(ef-dg)^2(3ef+dg)(f+gx)^2e^2+\left(3a^2g^4e^5-2acg^2\left(4e^2f^2-8degf+d^2g^2\right)e^3-c^2\left(12f^4e^5-16df^3\right.\right.}{(f+gx)(ef-dg-e(f+gx))^2}}{4e^4(ef-dg)^2}\right.}{g^2}$$

$\Big\downarrow$ 25

$$\frac{2\left(-\frac{\int \frac{4(ef-dg)\left(cf^2+ag^2\right)^2e^4-4c^2(ef-dg)^2(f+gx)^3e^3+4c^2(ef-dg)^2(3ef+dg)(f+gx)^2e^2+\left(3a^2g^4e^5-2acg^2\left(4e^2f^2-8degf+d^2g^2\right)e^3-c^2\left(12f^4e^5-16df^3\right.\right.}{(f+gx)(ef-dg-e(f+gx))^2}}{4e^4(ef-dg)^2}\right.}{g^2}$$

$\Big\downarrow$ 2336

$$\frac{2\left(-\frac{\frac{eg^3\sqrt{f+gx}\left(ae^2+cd^2\right)\left(7ae^2g+cd(16ef-9dg)\right)}{2(ef-dg)(-dg-e(f+gx)+ef)}-\frac{\int -\frac{8(ef-dg)\left(cf^2+ag^2\right)^2e^4+8c^2(ef-dg)^3(f+gx)^2e^2+\left(7a^2e^4g^4+2acde^2(8ef-dg)g^3-c^2\left(16e^4f^4-32\right.\right.}{(f+gx)(ef-dg-e(f+gx))}}{2(ef-dg)}}{4e^4(ef-dg)^2}\right.}{g^2}$$

$\Big\downarrow$ 25

$$\frac{2\left(-\frac{\frac{\int \frac{8(ef-dg)\left(cf^2+ag^2\right)^2e^4+8c^2(ef-dg)^3(f+gx)^2e^2+\left(7a^2e^4g^4+2acde^2(8ef-dg)g^3-c^2\left(16e^4f^4-32de^3gf^3+16d^3eg^3f-7d^4g^4\right)\right)(f+gx)e}{(f+gx)(ef-dg-e(f+gx))}d\sqrt{f+gx}}{2(ef-dg)}+\frac{eg}{\cdots}}{4e^4(ef-dg)^2}\right.}{g^2}$$

$\Big\downarrow$ 1584

$$2\left(-\frac{\frac{\int \left(\frac{8\left(cf^2+ag^2\right)^2e^4}{f+gx}-8c^2(ef-dg)^3e+\frac{g^2\left(15a^2g^2e^4+2ac\left(8e^2f^2+8degf-d^2g^2\right)e^2+3c^2d^2\left(16e^2f^2-16degf+5d^2g^2\right)\right)e}{ef-dg-e(f+gx)}\right)d\sqrt{f+gx}}{2(ef-dg)}+\frac{eg^3\sqrt{f+gx}\left(ae^2+cd\cdots\right.}{2(ef-dg)(-\cdots}}{4e^4(ef-dg)^2}\cdots\right.}{g^2}$$

$\big\downarrow$ 2009

$$2\left(-\frac{\frac{\frac{\sqrt{e}g^2\left(15a^2e^4g^2+2ace^2\left(-d^2g^2+8defg+8e^2f^2\right)+3c^2d^2\left(5d^2g^2-16defg+16e^2f^2\right)\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{\sqrt{ef-dg}}-\frac{8e^4\left(ag^2+cf^2\right)^2}{\sqrt{f+gx}}-8c^2e\sqrt{f+gx}(ef-dg)^3\cdots}{2(ef-dg)}}{4e^4(ef-dg)^2}\cdots\right.}{g^2}$$

input
```
Int[(a + c*x^2)^2/((d + e*x)^3*(f + g*x)^(3/2)),x]
```

output
```
(2*(-1/4*((c*d^2 + a*e^2)^2*g^4*Sqrt[f + g*x])/(e^3*(e*f - d*g)^2*(e*f - d
*g - e*(f + g*x))^2) - ((e*(c*d^2 + a*e^2)*g^3*(7*a*e^2*g + c*d*(16*e*f -
9*d*g))*Sqrt[f + g*x])/(2*(e*f - d*g)*(e*f - d*g - e*(f + g*x))) + ((-8*e^
4*(c*f^2 + a*g^2)^2)/Sqrt[f + g*x] - 8*c^2*e*(e*f - d*g)^3*Sqrt[f + g*x] +
 (Sqrt[e]*g^2*(15*a^2*e^4*g^2 + 2*a*c*e^2*(8*e^2*f^2 + 8*d*e*f*g - d^2*g^2
) + 3*c^2*d^2*(16*e^2*f^2 - 16*d*e*f*g + 5*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqrt
[f + g*x])/Sqrt[e*f - d*g]])/Sqrt[e*f - d*g])/(2*(e*f - d*g)))/(4*e^4*(e*f
 - d*g)^2)))/g^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 649
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*f
 - d*g + g*x^2)^n*(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x
]], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && Integ
erQ[m + 1/2]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{\frac{2c^2\sqrt{gx+f}}{e^3}-\frac{2g^2\left(\frac{\left(\frac{7}{8}a^2e^5g^2-\frac{1}{4}ac\,d^2e^3g^2+2acdfg\,e^4-\frac{9}{8}c^2d^4e\,g^2+2c^2d^3fg\,e^2\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9a^2d\,e^4g^2-9a^2e^5fg+2ac\,d^3e^2g^2+\right.}{}}{(e(gx+f)+dg-ef)^2}\right.}{}}{}$
default	$\frac{\frac{2c^2\sqrt{gx+f}}{e^3}-\frac{2g^2\left(\frac{\left(\frac{7}{8}a^2e^5g^2-\frac{1}{4}ac\,d^2e^3g^2+2acdfg\,e^4-\frac{9}{8}c^2d^4e\,g^2+2c^2d^3fg\,e^2\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9a^2d\,e^4g^2-9a^2e^5fg+2ac\,d^3e^2g^2+\right.}{}}{(e(gx+f)+dg-ef)^2}\right.}{}}{}$
risch	$\frac{2c^2\sqrt{gx+f}}{e^3g^2}-\frac{2\left(\frac{g^2\left(\frac{\left(\frac{7}{8}a^2e^5g^2-\frac{1}{4}ac\,d^2e^3g^2+2acdfg\,e^4-\frac{9}{8}c^2d^4e\,g^2+2c^2d^3fg\,e^2\right)(gx+f)^{\frac{3}{2}}+\frac{g\left(9a^2d\,e^4g^2-9a^2e^5fg+2ac\,d^3\right.}{}}{(e(gx+f)+dg-ef)^2}\right.}{}\right.}{}$
pseudoelliptic	$-\frac{2\left(\frac{15g^2\left(a\left(a\,g^2+\frac{16c\,f^2}{15}\right)e^4+\frac{16acd\,e^3fg}{15}-\frac{2c\,d^2\left(a\,g^2-24c\,f^2\right)e^2}{15}-\frac{16c^2d^3efg}{5}+c^2d^4g^2\right)(ex+d)^2\sqrt{gx+f}\,\arctan\left(\frac{e\sqrt{gx+f}}{\sqrt{(dg-ef)}}\right.}{8}\right.}{}$

input

```
int((c*x^2+a)^2/(e*x+d)^3/(g*x+f)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/g^2*(c^2/e^3*(g*x+f)^(1/2)-g^2/(d*g-e*f)^3/e^3*(((7/8*a^2*e^5*g^2-1/4*a*
c*d^2*e^3*g^2+2*a*c*d*f*g*e^4-9/8*c^2*d^4*e*g^2+2*c^2*d^3*f*g*e^2)*(g*x+f)
^(3/2)+1/8*g*(9*a^2*d*e^4*g^2-9*a^2*e^5*f*g+2*a*c*d^3*e^2*g^2+14*a*c*d^2*e
^3*f*g-16*a*c*d*e^4*f^2-7*c^2*d^5*g^2+23*c^2*d^4*e*f*g-16*c^2*d^3*e^2*f^2)
*(g*x+f)^(1/2))/(e*(g*x+f)+d*g-e*f)^2+1/8*(15*a^2*e^4*g^2-2*a*c*d^2*e^2*g^
2+16*a*c*d*e^3*f*g+16*a*c*e^4*f^2+15*c^2*d^4*g^2-48*c^2*d^3*e*f*g+48*c^2*d
^2*e^2*f^2)/((d*g-e*f)*e)^(1/2)*arctan(e*(g*x+f)^(1/2)/((d*g-e*f)*e)^(1/2)
))-(a^2*g^4+2*a*c*f^2*g^2+c^2*f^4)/(d*g-e*f)^3/(g*x+f)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1330 vs. $2(266) = 532$.

Time = 0.21 (sec) , antiderivative size = 2674, normalized size of antiderivative = 9.22

$$\int \frac{(a + cx^2)^2}{(d + ex)^3(f + gx)^{3/2}} \, dx = \text{Too large to display}$$

input

```
integrate((c*x^2+a)^2/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="fricas")
```

output

```
[-1/8*((16*(3*c^2*d^4*e^2 + a*c*d^2*e^4)*f^3*g^2 - 16*(3*c^2*d^5*e - a*c*d
^3*e^3)*f^2*g^3 + (15*c^2*d^6 - 2*a*c*d^4*e^2 + 15*a^2*d^2*e^4)*f*g^4 + (1
6*(3*c^2*d^2*e^4 + a*c*e^6)*f^2*g^3 - 16*(3*c^2*d^3*e^3 - a*c*d*e^5)*f*g^4
 + (15*c^2*d^4*e^2 - 2*a*c*d^2*e^4 + 15*a^2*e^6)*g^5)*x^3 + (16*(3*c^2*d^2
*e^4 + a*c*e^6)*f^3*g^2 + 48*(c^2*d^3*e^3 + a*c*d*e^5)*f^2*g^3 - 3*(27*c^2
*d^4*e^2 - 10*a*c*d^2*e^4 - 5*a^2*e^6)*f*g^4 + 2*(15*c^2*d^5*e - 2*a*c*d^3
*e^3 + 15*a^2*d*e^5)*g^5)*x^2 + (32*(3*c^2*d^3*e^3 + a*c*d*e^5)*f^3*g^2 -
48*(c^2*d^4*e^2 - a*c*d^2*e^4)*f^2*g^3 - 6*(3*c^2*d^5*e - 2*a*c*d^3*e^3 -
5*a^2*d*e^5)*f*g^4 + (15*c^2*d^6 - 2*a*c*d^4*e^2 + 15*a^2*d^2*e^4)*g^5)*x)
*sqrt(e^2*f - d*e*g)*log((e*g*x + 2*e*f - d*g + 2*sqrt(e^2*f - d*e*g)*sqrt
(g*x + f))/(e*x + d)) - 2*(16*c^2*d^2*e^5*f^5 - 40*c^2*d^3*e^4*f^4*g - 8*a
^2*d^3*e^4*g^5 + 2*(31*c^2*d^4*e^3 + 14*a*c*d^2*e^5 - a^2*e^7)*f^3*g^2 - (
53*c^2*d^5*e^2 + 26*a*c*d^3*e^4 - 11*a^2*d*e^6)*f^2*g^3 + (15*c^2*d^6*e -
2*a*c*d^4*e^3 - a^2*d^2*e^5)*f*g^4 + 8*(c^2*e^7*f^4*g - 4*c^2*d*e^6*f^3*g^
2 + 6*c^2*d^2*e^5*f^2*g^3 - 4*c^2*d^3*e^4*f*g^4 + c^2*d^4*e^3*g^5)*x^3 + (
16*c^2*e^7*f^5 - 24*c^2*d*e^6*f^4*g + 80*c^2*d^3*e^4*f^2*g^3 - 16*(c^2*d^2
*e^5 - a*c*e^7)*f^3*g^2 - 3*(27*c^2*d^4*e^3 + 6*a*c*d^2*e^5 - 5*a^2*e^7)*f
*g^4 + (25*c^2*d^5*e^2 + 2*a*c*d^3*e^4 - 15*a^2*d*e^6)*g^5)*x^2 + (32*c^2*
d*e^6*f^5 - 72*c^2*d^2*e^5*f^4*g + 16*(5*c^2*d^3*e^4 + 3*a*c*d*e^6)*f^3*g^
2 - (27*c^2*d^4*e^3 + 38*a*c*d^2*e^5 - 5*a^2*e^7)*f^2*g^3 - 4*(7*c^2*d^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**2/(e*x+d)**3/(g*x+f)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(d*g-e*f)>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(266) = 532$.

Time = 0.13 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.98

$$\int \frac{(a+cx^2)^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{(48\,c^2d^2e^2f^2+16\,ace^4f^2-48\,c^2d^3efg+16\,acde^3fg+15\,c^2d^4g^2-2\,acd^2e^2g}{4\,(e^6f^3-3\,de^5f^2g+3\,d^2e^4fg^2-d^3e^3g^3)\sqrt{-e^2f}}$$
$$+\frac{2\,(c^2f^4+2\,acf^2g^2+a^2g^4)}{(e^3f^3g^2-3\,de^2f^2g^3+3\,d^2efg^4-d^3g^5)\sqrt{gx+f}}$$
$$+\frac{16\,(gx+f)^{\frac{3}{2}}c^2d^3e^2fg+16\,(gx+f)^{\frac{3}{2}}acde^4fg-16\,\sqrt{gx+f}c^2d^3e^2f^2g-16\,\sqrt{gx+f}acde^4f^2g-9\,(gx+}{}$$
$$+\frac{2\,\sqrt{gx+f}c^2}{e^3g^2}$$

input
```
integrate((c*x^2+a)^2/(e*x+d)^3/(g*x+f)^(3/2),x, algorithm="giac")
```

output
```
1/4*(48*c^2*d^2*e^2*f^2 + 16*a*c*e^4*f^2 - 48*c^2*d^3*e*f*g + 16*a*c*d*e^3
*f*g + 15*c^2*d^4*g^2 - 2*a*c*d^2*e^2*g^2 + 15*a^2*e^4*g^2)*arctan(sqrt(g*
x + f)*e/sqrt(-e^2*f + d*e*g))/((e^6*f^3 - 3*d*e^5*f^2*g + 3*d^2*e^4*f*g^2
 - d^3*e^3*g^3)*sqrt(-e^2*f + d*e*g)) + 2*(c^2*f^4 + 2*a*c*f^2*g^2 + a^2*g
^4)/((e^3*f^3*g^2 - 3*d*e^2*f^2*g^3 + 3*d^2*e*f*g^4 - d^3*g^5)*sqrt(g*x +
f)) + 1/4*(16*(g*x + f)^(3/2)*c^2*d^3*e^2*f*g + 16*(g*x + f)^(3/2)*a*c*d*e
^4*f*g - 16*sqrt(g*x + f)*c^2*d^3*e^2*f^2*g - 16*sqrt(g*x + f)*a*c*d*e^4*f
^2*g - 9*(g*x + f)^(3/2)*c^2*d^4*e*g^2 - 2*(g*x + f)^(3/2)*a*c*d^2*e^3*g^2
 + 7*(g*x + f)^(3/2)*a^2*e^5*g^2 + 23*sqrt(g*x + f)*c^2*d^4*e*f*g^2 + 14*s
qrt(g*x + f)*a*c*d^2*e^3*f*g^2 - 9*sqrt(g*x + f)*a^2*e^5*f*g^2 - 7*sqrt(g*
x + f)*c^2*d^5*g^3 + 2*sqrt(g*x + f)*a*c*d^3*e^2*g^3 + 9*sqrt(g*x + f)*a^2
*d*e^4*g^3)/((e^6*f^3 - 3*d*e^5*f^2*g + 3*d^2*e^4*f*g^2 - d^3*e^3*g^3)*((g
*x + f)*e - e*f + d*g)^2) + 2*sqrt(g*x + f)*c^2/(e^3*g^2)
```

Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.79

$$\int \frac{(a+cx^2)^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \frac{2\,c^2\,\sqrt{f+g\,x}}{e^3\,g^2}$$
$$-\frac{\frac{2\,(a^2\,e^3\,g^4+2\,a\,c\,e^3\,f^2\,g^2+c^2\,e^3\,f^4)}{d\,g-e\,f}+\frac{(f+g\,x)\,(25\,a^2\,e^4\,g^4+2\,a\,c\,d^2\,e^2\,g^4+16\,a\,c\,d\,e^3\,f\,g^3+32\,a\,c\,e^4\,f^2\,g^2-7\,c^2\,d^4\,g^4+16\,c^2\,d^3\,e\,f\,g^3+16\,c^2\,e^4}{4\,(d\,g-e\,f)^2}}{\sqrt{f+g\,x}\,(d^2\,e^3\,g^4-2\,d\,e^4\,f\,g^3+e^5\,f^2\,g^2)+(f+g\,x)^{3/2}\,(}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{f+g\,x}\,(-d^3\,e^3\,g^3+3\,d^2\,e^4\,f\,g^2-3\,d\,e^5\,f^2\,g+e^6\,f^3)}{e^{5/2}\,(d\,g-e\,f)^{7/2}}\right)\,(15\,a^2\,e^4\,g^2-2\,a\,c\,d^2\,e^2\,g^2+16\,a\,c\,d\,e^3\,f\,g+16\,a\,c\,e^4\,f^2+}{4\,e^{7/2}\,(d\,g-e\,f)^{7/2}}$$

input
```
int((a + c*x^2)^2/((f + g*x)^(3/2)*(d + e*x)^3),x)
```

output
```
(2*c^2*(f + g*x)^(1/2))/(e^3*g^2) - ((2*(a^2*e^3*g^4 + c^2*e^3*f^4 + 2*a*c
*e^3*f^2*g^2))/(d*g - e*f) + ((f + g*x)*(25*a^2*e^4*g^4 - 7*c^2*d^4*g^4 +
16*c^2*e^4*f^4 + 2*a*c*d^2*e^2*g^4 + 32*a*c*e^4*f^2*g^2 + 16*c^2*d^3*e*f*g
^3 + 16*a*c*d*e^3*f*g^3))/(4*(d*g - e*f)^2) + ((f + g*x)^2*(15*a^2*e^5*g^4
 + 8*c^2*e^5*f^4 - 9*c^2*d^4*e*g^4 - 2*a*c*d^2*e^3*g^4 + 16*a*c*e^5*f^2*g^
2 + 16*c^2*d^3*e^2*f*g^3 + 16*a*c*d*e^4*f*g^3))/(4*(d*g - e*f)^3))/((f + g
*x)^(1/2)*(d^2*e^3*g^4 + e^5*f^2*g^2 - 2*d*e^4*f*g^3) + (f + g*x)^(3/2)*(2
*d*e^4*g^3 - 2*e^5*f*g^2) + e^5*g^2*(f + g*x)^(5/2)) + (atan(((f + g*x)^(1
/2)*(e^6*f^3 - d^3*e^3*g^3 + 3*d^2*e^4*f*g^2 - 3*d*e^5*f^2*g))/(e^(5/2)*(d
*g - e*f)^(7/2)))*(15*a^2*e^4*g^2 + 15*c^2*d^4*g^2 + 48*c^2*d^2*e^2*f^2 +
16*a*c*e^4*f^2 - 48*c^2*d^3*e*f*g - 2*a*c*d^2*e^2*g^2 + 16*a*c*d*e^3*f*g))
/(4*e^(7/2)*(d*g - e*f)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 2032, normalized size of antiderivative = 7.01

$$\int \frac{(a+cx^2)^2}{(d+ex)^3(f+gx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+a)^2/(e*x+d)^3/(g*x+f)^(3/2),x)
```

output
```
( - 15*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(
e)*sqrt(d*g - e*f)))*a**2*d**2*e**4*g**4 - 30*sqrt(e)*sqrt(f + g*x)*sqrt(d
*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a**2*d*e**5*g*
*4*x - 15*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sq
rt(e)*sqrt(d*g - e*f)))*a**2*e**6*g**4*x**2 + 2*sqrt(e)*sqrt(f + g*x)*sqrt
(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**4*e**
2*g**4 - 16*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(
sqrt(e)*sqrt(d*g - e*f)))*a*c*d**3*e**3*f*g**3 + 4*sqrt(e)*sqrt(f + g*x)*s
qrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**3*
e**3*g**4*x - 16*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)
*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**2*e**4*f**2*g**2 - 32*sqrt(e)*sqrt(f
 + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*
a*c*d**2*e**4*f*g**3*x + 2*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqr
t(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d**2*e**4*g**4*x**2 - 32*sqrt
(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g
 - e*f)))*a*c*d*e**5*f**2*g**2*x - 16*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f
)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*a*c*d*e**5*f*g**3*x**2
 - 16*sqrt(e)*sqrt(f + g*x)*sqrt(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e
)*sqrt(d*g - e*f)))*a*c*e**6*f**2*g**2*x**2 - 15*sqrt(e)*sqrt(f + g*x)*sqr
t(d*g - e*f)*atan((sqrt(f + g*x)*e)/(sqrt(e)*sqrt(d*g - e*f)))*c**2*d**...
```

3.91 $\int \frac{\sqrt{d+ex}(f+gx)}{a-cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 180

$$\int \frac{\sqrt{d+ex}(f+gx)}{a-cx^2}\,dx = -\frac{2g\sqrt{d+ex}}{c} - \frac{\sqrt{\sqrt{c}d-\sqrt{a}e}\left(\sqrt{c}f-\sqrt{a}g\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{\sqrt{a}c^{5/4}} + \frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\left(\sqrt{c}f+\sqrt{a}g\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{\sqrt{a}c^{5/4}}$$

output
```
-2*g*(e*x+d)^(1/2)/c-(c^(1/2)*d-a^(1/2)*e)^(1/2)*(c^(1/2)*f-a^(1/2)*g)*arc
tanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)^(1/2))/a^(1/2)/c^(5/4)+(c
^(1/2)*d+a^(1/2)*e)^(1/2)*(c^(1/2)*f+a^(1/2)*g)*arctanh(c^(1/4)*(e*x+d)^(1
/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/a^(1/2)/c^(5/4)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex}(f+gx)}{a-cx^2}\,dx$$
$$= \frac{-2\sqrt{a}\sqrt{c}g\sqrt{d+ex}-\sqrt{-cd-\sqrt{a}\sqrt{c}e}\left(\sqrt{c}f+\sqrt{a}g\right)\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)+\sqrt{-cd+\sqrt{a}\sqrt{c}e}\left(\sqrt{c}\right.}{\sqrt{a}c^{3/2}}$$

input
```
Integrate[(Sqrt[d + e*x]*(f + g*x))/(a - c*x^2),x]
```

output
```
(-2*Sqrt[a]*Sqrt[c]*g*Sqrt[d + e*x] - Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*(Sq
rt[c]*f + Sqrt[a]*g)*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x
])/(Sqrt[c]*d + Sqrt[a]*e)] + Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*(Sqrt[c]*f
- Sqrt[a]*g)*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x])/(Sqrt
[c]*d - Sqrt[a]*e)])/(Sqrt[a]*c^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {653, 25, 654, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)}{a-cx^2}\,dx$$
$$\Big\downarrow \; 653$$
$$-\frac{\int -\frac{cdf+aeg+c(ef+dg)x}{\sqrt{d+ex}(a-cx^2)}dx}{c}-\frac{2g\sqrt{d+ex}}{c}$$
$$\Big\downarrow \; 25$$
$$\frac{\int \frac{cdf+aeg+c(ef+dg)x}{\sqrt{d+ex}(a-cx^2)}dx}{c}-\frac{2g\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{654}$$

$$\frac{2\int \frac{(cd^2-ae^2)g-c(ef+dg)(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}-\frac{2g\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{1480}$$

$$\frac{2\left(\frac{\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\sqrt{c}f-\sqrt{a}g\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}}-\frac{\sqrt{c}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\sqrt{a}g+\sqrt{c}f\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{d+ex}}{2\sqrt{a}}\right)}{c}-\frac{2g\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\left(\frac{\sqrt{\sqrt{a}e+\sqrt{c}d}(\sqrt{a}g+\sqrt{c}f)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2\sqrt{a}\sqrt[4]{c}}-\frac{\sqrt{\sqrt{c}d-\sqrt{a}e}(\sqrt{c}f-\sqrt{a}g)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2\sqrt{a}\sqrt[4]{c}}\right)}{c}-\frac{2g\sqrt{d+ex}}{c}$$

input
```
Int[(Sqrt[d + e*x]*(f + g*x))/(a - c*x^2),x]
```

output
```
(-2*g*Sqrt[d + e*x])/c + (2*(-1/2*(Sqrt[Sqrt[c]*d - Sqrt[a]*e]*(Sqrt[c]*f
- Sqrt[a]*g)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d - Sqrt[a]*e]])
/(Sqrt[a]*c^(1/4)) + (Sqrt[Sqrt[c]*d + Sqrt[a]*e]*(Sqrt[c]*f + Sqrt[a]*g)*
ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e]])/(2*Sqrt[a]*c
^(1/4))))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{2g\sqrt{ex+d}}{c}+\frac{\left(a\,e^{2}g+cdef+\sqrt{ac\,e^{2}}\,dg+\sqrt{ac\,e^{2}}\,ef\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{ac\,e^{2}}\,\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}-\frac{\left(-a\,e^{2}g-cdef+\sqrt{ac\,e^{2}}\,dg+\cdots\right.}{\sqrt{ac\,e^{2}}\,\cdots}$
risch	$-\frac{2g\sqrt{ex+d}}{c}+\frac{\left(a\,e^{2}g+cdef+\sqrt{ac\,e^{2}}\,dg+\sqrt{ac\,e^{2}}\,ef\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{ac\,e^{2}}\,\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}-\frac{\left(-a\,e^{2}g-cdef+\sqrt{ac\,e^{2}}\,dg+\cdots\right.}{\sqrt{ac\,e^{2}}\,\cdots}$
pseudoelliptic	$-\frac{2g\sqrt{ex+d}}{c}+\frac{\left(a\,e^{2}g+cdef+\sqrt{ac\,e^{2}}\,dg+\sqrt{ac\,e^{2}}\,ef\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{ac\,e^{2}}\,\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}-\frac{\left(-a\,e^{2}g-cdef+\sqrt{ac\,e^{2}}\,dg+\cdots\right.}{\sqrt{ac\,e^{2}}\,\cdots}$
default	$-\frac{2g\sqrt{ex+d}}{c}-\frac{\left(-a\,e^{2}g-cdef-\sqrt{ac\,e^{2}}\,dg-\sqrt{ac\,e^{2}}\,ef\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{ac\,e^{2}}\,\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}+\frac{\left(a\,e^{2}g+cdef-\sqrt{ac\,e^{2}}\,dg-\cdots\right.}{\sqrt{ac\,e^{2}}\,\cdots}$

input
```
int((e*x+d)^(1/2)*(g*x+f)/(-c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2*g*(e*x+d)^(1/2)/c+(a*e^2*g+c*d*e*f+(a*c*e^2)^(1/2)*d*g+(a*c*e^2)^(1/2)*
e*f)/(a*c*e^2)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/
2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))-(-a*e^2*g-c*d*e*f+(a*c*e^2)^(1/2)*d*g+
(a*c*e^2)^(1/2)*e*f)/(a*c*e^2)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arct
an(c*(e*x+d)^(1/2)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1530 vs. $2(130) = 260$.

Time = 0.12 (sec) , antiderivative size = 1530, normalized size of antiderivative = 8.50

$$\int \frac{\sqrt{d+ex}(f+gx)}{a-cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)/(-c*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(c*sqrt((c*d*f^2 + 2*a*e*f*g + a*d*g^2 + a*c^2*sqrt((c^2*e^2*f^4 + 4*c
^2*d*e*f^3*g + 4*a*c*d*e*f*g^3 + a^2*e^2*g^4 + 2*(2*c^2*d^2 + a*c*e^2)*f^2
*g^2)/(a*c^5)))/(a*c^2))*log(-(c^2*e*f^4 + 2*c^2*d*f^3*g - 2*a*c*d*f*g^3 -
 a^2*e*g^4)*sqrt(e*x + d) + (a*c^2*e*f^2*g + 2*a*c^2*d*f*g^2 + a^2*c*e*g^3
 - a*c^4*f*sqrt((c^2*e^2*f^4 + 4*c^2*d*e*f^3*g + 4*a*c*d*e*f*g^3 + a^2*e^2
*g^4 + 2*(2*c^2*d^2 + a*c*e^2)*f^2*g^2)/(a*c^5)))*sqrt((c*d*f^2 + 2*a*e*f*
g + a*d*g^2 + a*c^2*sqrt((c^2*e^2*f^4 + 4*c^2*d*e*f^3*g + 4*a*c*d*e*f*g^3
+ a^2*e^2*g^4 + 2*(2*c^2*d^2 + a*c*e^2)*f^2*g^2)/(a*c^5)))/(a*c^2))) - c*s
qrt((c*d*f^2 + 2*a*e*f*g + a*d*g^2 + a*c^2*sqrt((c^2*e^2*f^4 + 4*c^2*d*e*f
^3*g + 4*a*c*d*e*f*g^3 + a^2*e^2*g^4 + 2*(2*c^2*d^2 + a*c*e^2)*f^2*g^2)/(a
*c^5)))/(a*c^2))*log(-(c^2*e*f^4 + 2*c^2*d*f^3*g - 2*a*c*d*f*g^3 - a^2*e*g
^4)*sqrt(e*x + d) - (a*c^2*e*f^2*g + 2*a*c^2*d*f*g^2 + a^2*c*e*g^3 - a*c^4
*f*sqrt((c^2*e^2*f^4 + 4*c^2*d*e*f^3*g + 4*a*c*d*e*f*g^3 + a^2*e^2*g^4 + 2
*(2*c^2*d^2 + a*c*e^2)*f^2*g^2)/(a*c^5)))*sqrt((c*d*f^2 + 2*a*e*f*g + a*d*
g^2 + a*c^2*sqrt((c^2*e^2*f^4 + 4*c^2*d*e*f^3*g + 4*a*c*d*e*f*g^3 + a^2*e^
2*g^4 + 2*(2*c^2*d^2 + a*c*e^2)*f^2*g^2)/(a*c^5)))/(a*c^2))) + c*sqrt((c*d
*f^2 + 2*a*e*f*g + a*d*g^2 - a*c^2*sqrt((c^2*e^2*f^4 + 4*c^2*d*e*f^3*g + 4
*a*c*d*e*f*g^3 + a^2*e^2*g^4 + 2*(2*c^2*d^2 + a*c*e^2)*f^2*g^2)/(a*c^5)))/
(a*c^2))*log(-(c^2*e*f^4 + 2*c^2*d*f^3*g - 2*a*c*d*f*g^3 - a^2*e*g^4)*sqrt
(e*x + d) + (a*c^2*e*f^2*g + 2*a*c^2*d*f*g^2 + a^2*c*e*g^3 + a*c^4*f*sq...
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)}{a-cx^2}\,dx = -\int \frac{f\sqrt{d+ex}}{-a+cx^2}\,dx - \int \frac{gx\sqrt{d+ex}}{-a+cx^2}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(g*x+f)/(-c*x**2+a),x)
```

output
```
-Integral(f*sqrt(d + e*x)/(-a + c*x**2), x) - Integral(g*x*sqrt(d + e*x)/(
-a + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}(f+gx)}{a-cx^2}\,dx=\int -\frac{\sqrt{ex+d}(gx+f)}{cx^2-a}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)/(-c*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(sqrt(e*x + d)*(g*x + f)/(c*x^2 - a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(130) = 260$.

Time = 0.15 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{d+ex}(f+gx)}{a-cx^2}\,dx=-\frac{2\sqrt{ex+d}g}{c}-\frac{\left(\sqrt{ac}c^3d^2ef-\sqrt{ac}ac^2e^3f+(ac^2d^2-a^2ce^2)g|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{c^2d+\sqrt{c^4d^2-(c^2d^2-ace^2)c^2}}{c^2}}}\right)}{\left(ac^3d-\sqrt{ac}ac^2e\right)\sqrt{-c^2d-\sqrt{ac}ce}|e|}+\frac{\left(\sqrt{ac}c^3d^2ef-\sqrt{ac}ac^2e^3f-(ac^2d^2-a^2ce^2)g|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{c^2d-\sqrt{c^4d^2-(c^2d^2-ace^2)c^2}}{c^2}}}\right)}{\left(ac^3d+\sqrt{ac}ac^2e\right)\sqrt{-c^2d+\sqrt{ac}ce}|e|}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)/(-c*x^2+a),x, algorithm="giac")
```

output
```
-2*sqrt(e*x + d)*g/c - (sqrt(a*c)*c^3*d^2*e*f - sqrt(a*c)*a*c^2*e^3*f + (a
*c^2*d^2 - a^2*c*e^2)*g*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(c^2*d +
 sqrt(c^4*d^2 - (c^2*d^2 - a*c*e^2)*c^2))/c^2))/((a*c^3*d - sqrt(a*c)*a*c^
2*e)*sqrt(-c^2*d - sqrt(a*c)*c*e)*abs(e)) + (sqrt(a*c)*c^3*d^2*e*f - sqrt(
a*c)*a*c^2*e^3*f - (a*c^2*d^2 - a^2*c*e^2)*g*abs(c)*abs(e))*arctan(sqrt(e*
x + d)/sqrt(-(c^2*d - sqrt(c^4*d^2 - (c^2*d^2 - a*c*e^2)*c^2))/c^2))/((a*c
^3*d + sqrt(a*c)*a*c^2*e)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 4278, normalized size of antiderivative = 23.77

$$\int \frac{\sqrt{d+ex}(f+gx)}{a-cx^2}\,dx = \text{Too large to display}$$

input
```
int(((f + g*x)*(d + e*x)^(1/2))/(a - c*x^2),x)
```

output
```
2*atanh((32*d*e^3*g^2*(a^3*c^5)^(1/2)*(d + e*x)^(1/2)*((d*g^2)/(4*c^2) + (
d*f^2)/(4*a*c) + (e*f*g)/(2*c^2) - (e*f^2*(a^3*c^5)^(1/2))/(4*a^2*c^4) - (
e*g^2*(a^3*c^5)^(1/2))/(4*a*c^5) - (d*f*g*(a^3*c^5)^(1/2))/(2*a^2*c^4))^(1
/2))/(16*a*c^2*e^5*f^3 - 16*c^3*d^2*e^3*f^3 - (16*a*e^5*g^3*(a^3*c^5)^(1/2
))/c^2 - (16*e^5*f^2*g*(a^3*c^5)^(1/2))/c + 16*a^2*c*e^5*f*g^2 + (16*d^2*e
^3*g^3*(a^3*c^5)^(1/2))/c - 32*c^3*d^3*e^2*f^2*g + (16*d^2*e^3*f^2*g*(a^3*
c^5)^(1/2))/a + (32*d^3*e^2*f*g^2*(a^3*c^5)^(1/2))/a - 16*a*c^2*d^2*e^3*f*
g^2 - (32*d*e^4*f*g^2*(a^3*c^5)^(1/2))/c + 32*a*c^2*d*e^4*f^2*g) + (32*a*c
^2*e^4*f^2*(d + e*x)^(1/2)*((d*g^2)/(4*c^2) + (d*f^2)/(4*a*c) + (e*f*g)/(2
*c^2) - (e*f^2*(a^3*c^5)^(1/2))/(4*a^2*c^4) - (e*g^2*(a^3*c^5)^(1/2))/(4*a
*c^5) - (d*f*g*(a^3*c^5)^(1/2))/(2*a^2*c^4))^(1/2))/(16*a^2*e^5*f*g^2 - 16
*c^2*d^2*e^3*f^3 + 16*a*c*e^5*f^3 - (16*a*e^5*g^3*(a^3*c^5)^(1/2))/c^3 - (
16*e^5*f^2*g*(a^3*c^5)^(1/2))/c^2 + (16*d^2*e^3*g^3*(a^3*c^5)^(1/2))/c^2 -
 32*c^2*d^3*e^2*f^2*g + 32*a*c*d*e^4*f^2*g - (32*d*e^4*f*g^2*(a^3*c^5)^(1/
2))/c^2 - 16*a*c*d^2*e^3*f*g^2 + (16*d^2*e^3*f^2*g*(a^3*c^5)^(1/2))/(a*c)
+ (32*d^3*e^2*f*g^2*(a^3*c^5)^(1/2))/(a*c)) + (32*a^2*c*e^4*g^2*(d + e*x)^
(1/2)*((d*g^2)/(4*c^2) + (d*f^2)/(4*a*c) + (e*f*g)/(2*c^2) - (e*f^2*(a^3*c
^5)^(1/2))/(4*a^2*c^4) - (e*g^2*(a^3*c^5)^(1/2))/(4*a*c^5) - (d*f*g*(a^3*c
^5)^(1/2))/(2*a^2*c^4))^(1/2))/(16*a^2*e^5*f*g^2 - 16*c^2*d^2*e^3*f^3 + 16
*a*c*e^5*f^3 - (16*a*e^5*g^3*(a^3*c^5)^(1/2))/c^3 - (16*e^5*f^2*g*(a^3*...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{d+ex}(f+gx)}{a-cx^2}\,dx = \frac{-2\sqrt{a}\sqrt{\sqrt{c}\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{a}\,e-cd}}\right)cf+2\sqrt{c}\sqrt{\sqrt{c}\sqrt{a}\,e-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{a}\,e-cd}}\right)ag-\sqrt{a}\sqrt{\cdots}}{\cdots}$$

input
```
int((e*x+d)^(1/2)*(g*x+f)/(-c*x^2+a),x)
```

output
```
( - 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c*f + 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e -
 c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*g
- sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*
d) + sqrt(c)*sqrt(d + e*x))*c*f + sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*lo
g(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c*f - sqrt(c)*sqr
t(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*
sqrt(d + e*x))*a*g + sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c
)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*g - 4*sqrt(d + e*x)*a*c*g)/(
2*a*c**2)
```

3.92 $\int \frac{f+gx}{\sqrt{d+ex}(a-cx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 154

$$\int \frac{f+gx}{\sqrt{d+ex}\,(a-cx^2)}\,dx = -\frac{\left(\frac{\sqrt{c}f}{\sqrt{a}}-g\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{c^{3/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}+\frac{\left(\frac{\sqrt{c}f}{\sqrt{a}}+g\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{a}e}}\right)}{c^{3/4}\sqrt{\sqrt{c}d+\sqrt{a}e}}$$

output
```
-(c^(1/2)*f/a^(1/2)-g)*arctanh(c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d-a^(1/2)*e)
^(1/2))/c^(3/4)/(c^(1/2)*d-a^(1/2)*e)^(1/2)+(c^(1/2)*f/a^(1/2)+g)*arctanh(
c^(1/4)*(e*x+d)^(1/2)/(c^(1/2)*d+a^(1/2)*e)^(1/2))/c^(3/4)/(c^(1/2)*d+a^(1
/2)*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.23

$$\int \frac{f+gx}{\sqrt{d+ex}\,(a-cx^2)}\,dx = \frac{\frac{(\sqrt{c}f+\sqrt{a}g)\arctan\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+\sqrt{a}e}\right)}{\sqrt{-cd-\sqrt{a}\sqrt{c}e}}-\frac{(\sqrt{c}f-\sqrt{a}g)\arctan\left(\frac{\sqrt{-cd+\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-\sqrt{a}e}\right)}{\sqrt{-cd+\sqrt{a}\sqrt{c}e}}}{\sqrt{a}\sqrt{c}}$$

input
```
Integrate[(f + g*x)/(Sqrt[d + e*x]*(a - c*x^2)),x]
```

output
```
(((Sqrt[c]*f + Sqrt[a]*g)*ArcTan[(Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e]*Sqrt[d
+ e*x])/(Sqrt[c]*d + Sqrt[a]*e)])/Sqrt[-(c*d) - Sqrt[a]*Sqrt[c]*e] - ((Sqr
t[c]*f - Sqrt[a]*g)*ArcTan[(Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e]*Sqrt[d + e*x]
)/(Sqrt[c]*d - Sqrt[a]*e)])/Sqrt[-(c*d) + Sqrt[a]*Sqrt[c]*e])/(Sqrt[a]*Sqr
t[c])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {654, 25, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(a-cx^2)\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{654}$$

$$2\int -\frac{ef-dg+g(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{ef-dg+g(d+ex)}{cd^2-2c(d+ex)d-ae^2+c(d+ex)^2}d\sqrt{d+ex}$$

$$\Big\downarrow \text{1480}$$

$$2\left(\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{a}}-g\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d-\sqrt{a}e\right)}d\sqrt{d+ex}-\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{a}}+g\right)\int \frac{1}{c(d+ex)-\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)}d\sqrt{}$$

$$\Big\downarrow \text{221}$$

$$2\left(\frac{\left(\frac{\sqrt{c}f}{\sqrt{a}}+g\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{a}e+\sqrt{c}d}}\right)}{2c^{3/4}\sqrt{\sqrt{a}e+\sqrt{c}d}}-\frac{\left(\frac{\sqrt{c}f}{\sqrt{a}}-g\right)\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)}{2c^{3/4}\sqrt{\sqrt{c}d-\sqrt{a}e}}\right)$$

input
```
Int[(f + g*x)/(Sqrt[d + e*x]*(a - c*x^2)),x]
```

output
```
2*(-1/2*(((Sqrt[c]*f)/Sqrt[a] - g)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sq
rt[c]*d - Sqrt[a]*e]])/(c^(3/4)*Sqrt[Sqrt[c]*d - Sqrt[a]*e]) + (((Sqrt[c]*
f)/Sqrt[a] + g)*ArcTanh[(c^(1/4)*Sqrt[d + e*x])/Sqrt[Sqrt[c]*d + Sqrt[a]*e
]])/(2*c^(3/4)*Sqrt[Sqrt[c]*d + Sqrt[a]*e]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.86

method	result	siz
pseudoelliptic	$-\frac{-\frac{\left(fce+\sqrt{ac\,e^{2}}\,g\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}+\frac{\left(-fce+\sqrt{ac\,e^{2}}\,g\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}}{\sqrt{ac\,e^{2}}}$	13
derivativedivides	$-2c\left(-\frac{\left(fce+\sqrt{ac\,e^{2}}\,g\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2c\sqrt{ac\,e^{2}}\,\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}+\frac{\left(-fce+\sqrt{ac\,e^{2}}\,g\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2c\sqrt{ac\,e^{2}}\,\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)$	14
default	$2c\left(-\frac{\left(-fce-\sqrt{ac\,e^{2}}\,g\right)\operatorname{arctanh}\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2c\sqrt{ac\,e^{2}}\,\sqrt{\left(cd+\sqrt{ac\,e^{2}}\right)c}}+\frac{\left(fce-\sqrt{ac\,e^{2}}\,g\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)}{2c\sqrt{ac\,e^{2}}\,\sqrt{\left(-cd+\sqrt{ac\,e^{2}}\right)c}}\right)$	15

input
```
int((g*x+f)/(e*x+d)^(1/2)/(-c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*c*e^2)^(1/2)*(-(f*c*e+(a*c*e^2)^(1/2)*g)/((c*d+(a*c*e^2)^(1/2))*c)^(
1/2)*arctanh(c*(e*x+d)^(1/2)/((c*d+(a*c*e^2)^(1/2))*c)^(1/2))+(-f*c*e+(a*c
*e^2)^(1/2)*g)/((-c*d+(a*c*e^2)^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)/((-
c*d+(a*c*e^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2373 vs. $2(110) = 220$.

Time = 0.32 (sec) , antiderivative size = 2373, normalized size of antiderivative = 15.41

$$\int \frac{f + gx}{\sqrt{d + ex}\,(a - cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)/(e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*sqrt((c*d*f^2 - 2*a*e*f*g + a*d*g^2 + (a*c^2*d^2 - a^2*c*e^2)*sqrt((c
^2*e^2*f^4 - 4*c^2*d*e*f^3*g - 4*a*c*d*e*f*g^3 + a^2*e^2*g^4 + 2*(2*c^2*d^
2 + a*c*e^2)*f^2*g^2)/(a*c^5*d^4 - 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4)))/(a*c
^2*d^2 - a^2*c*e^2))*log(-(c^2*e*f^4 - 2*c^2*d*f^3*g + 2*a*c*d*f*g^3 - a^2
*e*g^4)*sqrt(e*x + d) + (a*c^2*e^2*f^3 - 3*a*c^2*d*e*f^2*g - a^2*c*d*e*g^3
 + (2*a*c^2*d^2 + a^2*c*e^2)*f*g^2 - ((a*c^4*d^3 - a^2*c^3*d*e^2)*f - (a^2
*c^3*d^2*e - a^3*c^2*e^3)*g)*sqrt((c^2*e^2*f^4 - 4*c^2*d*e*f^3*g - 4*a*c*d
*e*f*g^3 + a^2*e^2*g^4 + 2*(2*c^2*d^2 + a*c*e^2)*f^2*g^2)/(a*c^5*d^4 - 2*a
^2*c^4*d^2*e^2 + a^3*c^3*e^4)))*sqrt((c*d*f^2 - 2*a*e*f*g + a*d*g^2 + (a*c
^2*d^2 - a^2*c*e^2)*sqrt((c^2*e^2*f^4 - 4*c^2*d*e*f^3*g - 4*a*c*d*e*f*g^3
+ a^2*e^2*g^4 + 2*(2*c^2*d^2 + a*c*e^2)*f^2*g^2)/(a*c^5*d^4 - 2*a^2*c^4*d^
2*e^2 + a^3*c^3*e^4)))/(a*c^2*d^2 - a^2*c*e^2))) + 1/2*sqrt((c*d*f^2 - 2*a
*e*f*g + a*d*g^2 + (a*c^2*d^2 - a^2*c*e^2)*sqrt((c^2*e^2*f^4 - 4*c^2*d*e*f
^3*g - 4*a*c*d*e*f*g^3 + a^2*e^2*g^4 + 2*(2*c^2*d^2 + a*c*e^2)*f^2*g^2)/(a
*c^5*d^4 - 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4)))/(a*c^2*d^2 - a^2*c*e^2))*log
(-(c^2*e*f^4 - 2*c^2*d*f^3*g + 2*a*c*d*f*g^3 - a^2*e*g^4)*sqrt(e*x + d) -
(a*c^2*e^2*f^3 - 3*a*c^2*d*e*f^2*g - a^2*c*d*e*g^3 + (2*a*c^2*d^2 + a^2*c*
e^2)*f*g^2 - ((a*c^4*d^3 - a^2*c^3*d*e^2)*f - (a^2*c^3*d^2*e - a^3*c^2*e^3
)*g)*sqrt((c^2*e^2*f^4 - 4*c^2*d*e*f^3*g - 4*a*c*d*e*f*g^3 + a^2*e^2*g^4 +
 2*(2*c^2*d^2 + a*c*e^2)*f^2*g^2)/(a*c^5*d^4 - 2*a^2*c^4*d^2*e^2 + a^3*...
```

Sympy [F]

$$\int \frac{f+gx}{\sqrt{d+ex}\,(a-cx^2)}\,dx = -\int \frac{f}{-a\sqrt{d+ex}+cx^2\sqrt{d+ex}}\,dx - \int \frac{gx}{-a\sqrt{d+ex}+cx^2\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)**(1/2)/(-c*x**2+a),x)
```

output
```
-Integral(f/(-a*sqrt(d + e*x) + c*x**2*sqrt(d + e*x)), x) - Integral(g*x/(
-a*sqrt(d + e*x) + c*x**2*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{f+gx}{\sqrt{d+ex}\,(a-cx^2)}\,dx = \int -\frac{gx+f}{(cx^2-a)\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((g*x + f)/((c*x^2 - a)*sqrt(e*x + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(110) = 220$.

Time = 0.14 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.73

$$\int \frac{f+gx}{\sqrt{d+ex}\,(a-cx^2)}\,dx$$
$$= \frac{\left(acef|c||e|-acdg|c||e|-\sqrt{ac}cdef|c|+\sqrt{ac}ae^2g|c|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{cd+\sqrt{c^2d^2-(cd^2-ae^2)c}}{c}}}\right)}{\left(ac^2d-\sqrt{ac}ace\right)\sqrt{-c^2d-\sqrt{ac}ce}|e|}$$
$$+\frac{\left(acef|c||e|-acdg|c||e|+\sqrt{ac}cdef|c|-\sqrt{ac}ae^2g|c|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{cd-\sqrt{c^2d^2-(cd^2-ae^2)c}}{c}}}\right)}{\left(ac^2d+\sqrt{ac}ace\right)\sqrt{-c^2d+\sqrt{ac}ce}|e|}$$

input
```
integrate((g*x+f)/(e*x+d)^(1/2)/(-c*x^2+a),x, algorithm="giac")
```

output
```
(a*c*e*f*abs(c)*abs(e) - a*c*d*g*abs(c)*abs(e) - sqrt(a*c)*c*d*e*f*abs(c)
+ sqrt(a*c)*a*e^2*g*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(c*d + sqrt(c^2*d^2
 - (c*d^2 - a*e^2)*c))/c))/((a*c^2*d - sqrt(a*c)*a*c*e)*sqrt(-c^2*d - sqrt
(a*c)*c*e)*abs(e)) + (a*c*e*f*abs(c)*abs(e) - a*c*d*g*abs(c)*abs(e) + sqrt
(a*c)*c*d*e*f*abs(c) - sqrt(a*c)*a*e^2*g*abs(c))*arctan(sqrt(e*x + d)/sqrt
(-(c*d - sqrt(c^2*d^2 - (c*d^2 - a*e^2)*c))/c))/((a*c^2*d + sqrt(a*c)*a*c*
e)*sqrt(-c^2*d + sqrt(a*c)*c*e)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 2058, normalized size of antiderivative = 13.36

$$\int \frac{f+gx}{\sqrt{d+ex}\,(a-cx^2)}\,dx = \text{Too large to display}$$

input
```
int((f + g*x)/((a - c*x^2)*(d + e*x)^(1/2)),x)
```

output
```
- atan((a^2*c^5*d^3*((a^2*c^2*d*g^2 + a*e*g^2*(a^3*c^3)^(1/2) + c*e*f^2*(a
^3*c^3)^(1/2) + a*c^3*d*f^2 - 2*a^2*c^2*e*f*g - 2*c*d*f*g*(a^3*c^3)^(1/2))
/(4*a^2*c^4*d^2 - 4*a^3*c^3*e^2))^(3/2)*(d + e*x)^(1/2)*8i - a^2*c^3*d^2*g
^2*((a^2*c^2*d*g^2 + a*e*g^2*(a^3*c^3)^(1/2) + c*e*f^2*(a^3*c^3)^(1/2) + a
*c^3*d*f^2 - 2*a^2*c^2*e*f*g - 2*c*d*f*g*(a^3*c^3)^(1/2))/(4*a^2*c^4*d^2 -
 4*a^3*c^3*e^2))^(1/2)*(d + e*x)^(1/2)*2i + a^2*c^3*e^2*f^2*((a^2*c^2*d*g^
2 + a*e*g^2*(a^3*c^3)^(1/2) + c*e*f^2*(a^3*c^3)^(1/2) + a*c^3*d*f^2 - 2*a^
2*c^2*e*f*g - 2*c*d*f*g*(a^3*c^3)^(1/2))/(4*a^2*c^4*d^2 - 4*a^3*c^3*e^2))^
(1/2)*(d + e*x)^(1/2)*2i + a^3*c^2*e^2*g^2*((a^2*c^2*d*g^2 + a*e*g^2*(a^3*
c^3)^(1/2) + c*e*f^2*(a^3*c^3)^(1/2) + a*c^3*d*f^2 - 2*a^2*c^2*e*f*g - 2*c
*d*f*g*(a^3*c^3)^(1/2))/(4*a^2*c^4*d^2 - 4*a^3*c^3*e^2))^(1/2)*(d + e*x)^(
1/2)*2i - a^3*c^4*d*e^2*((a^2*c^2*d*g^2 + a*e*g^2*(a^3*c^3)^(1/2) + c*e*f^
2*(a^3*c^3)^(1/2) + a*c^3*d*f^2 - 2*a^2*c^2*e*f*g - 2*c*d*f*g*(a^3*c^3)^(1
/2))/(4*a^2*c^4*d^2 - 4*a^3*c^3*e^2))^(3/2)*(d + e*x)^(1/2)*8i - a*c^4*d^2
*f^2*((a^2*c^2*d*g^2 + a*e*g^2*(a^3*c^3)^(1/2) + c*e*f^2*(a^3*c^3)^(1/2) +
 a*c^3*d*f^2 - 2*a^2*c^2*e*f*g - 2*c*d*f*g*(a^3*c^3)^(1/2))/(4*a^2*c^4*d^2
 - 4*a^3*c^3*e^2))^(1/2)*(d + e*x)^(1/2)*2i)/(a^3*c*e^2*g^3 - c*e^2*f^3*(a
^3*c^3)^(1/2) - a*c^3*d*e*f^3 + 2*a*c^3*d^2*f^2*g + a*d*e*g^3*(a^3*c^3)^(1
/2) + a^2*c^2*e^2*f^2*g - a*e^2*f*g^2*(a^3*c^3)^(1/2) - 2*c*d^2*f*g^2*(a^3
*c^3)^(1/2) - 3*a^2*c^2*d*e*f*g^2 + 3*c*d*e*f^2*g*(a^3*c^3)^(1/2)))*((a...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 531, normalized size of antiderivative = 3.45

$$\int \frac{f + gx}{\sqrt{d + ex}\,(a - cx^2)}\,dx$$
$$= \frac{-2\sqrt{a}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e - cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}}\right) aeg + 2\sqrt{a}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e - cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{a}\,e-cd}}\right) cdf - 2\sqrt{c}}{}$$

input
```
int((g*x+f)/(e*x+d)^(1/2)/(-c*x^2+a),x)
```

output
```
( - 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*e*g + 2*sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e
 - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*c*
d*f - 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt
(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*a*d*g + 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a)
*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e - c*d)))*
a*e*f - sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*
e + c*d) + sqrt(c)*sqrt(d + e*x))*a*e*g + sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e +
 c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c*d*f
+ sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d)
+ sqrt(c)*sqrt(d + e*x))*a*e*g - sqrt(a)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log
(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*c*d*f + sqrt(c)*sq
rt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)
*sqrt(d + e*x))*a*d*g - sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log( - sqrt(
sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*e*f - sqrt(c)*sqrt(sqr
t(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt(a)*e + c*d) + sqrt(c)*sqrt(d +
 e*x))*a*d*g + sqrt(c)*sqrt(sqrt(c)*sqrt(a)*e + c*d)*log(sqrt(sqrt(c)*sqrt
(a)*e + c*d) + sqrt(c)*sqrt(d + e*x))*a*e*f)/(2*a*c*(a*e**2 - c*d**2))
```

3.93 $\int \frac{\sqrt{d+ex}(f+gx)}{a+cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 463

$$\int \frac{\sqrt{d+ex}(f+gx)}{a+cx^2}\,dx = \frac{2g\sqrt{d+ex}}{c}+\frac{\left(\sqrt{cd^2+ae^2}\,g-\sqrt{c}(ef+dg)\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}c^{5/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}-\frac{\left(\sqrt{cd^2+ae^2}\,g-\sqrt{c}(ef+dg)\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}c^{5/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}-\frac{\left(\sqrt{cd^2+ae^2}\,g+\sqrt{c}(ef+dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{\sqrt{2}c^{5/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

output
```
2*g*(e*x+d)^(1/2)/c+1/2*((a*e^2+c*d^2)^(1/2)*g-c^(1/2)*(d*g+e*f))*arctan((
(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(
1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/c^(5/4)/(-c^(1/2)*d+(a*e^2+c*d^
2)^(1/2))^(1/2)-1/2*((a*e^2+c*d^2)^(1/2)*g-c^(1/2)*(d*g+e*f))*arctan(((c^(
1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)
*d+(a*e^2+c*d^2)^(1/2))^(1/2))*2^(1/2)/c^(5/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(
1/2))^(1/2)-1/2*((a*e^2+c*d^2)^(1/2)*g+c^(1/2)*(d*g+e*f))*arctanh(2^(1/2)*
c^(1/4)*(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+c*d^2)
^(1/2)+c^(1/2)*(e*x+d)))*2^(1/2)/c^(5/4)/(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{d+ex}(f+gx)}{a+cx^2}\,dx$$

$$= \frac{2\sqrt{a}\sqrt{c}g\sqrt{d+ex} + \sqrt{-cd-i\sqrt{a}\sqrt{c}e}\left(-i\sqrt{c}f+\sqrt{a}g\right)\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+i\sqrt{a}e}\right) + \sqrt{-cd+i\sqrt{a}\sqrt{c}e}(}{\sqrt{a}c^{3/2}}$$

input
```
Integrate[(Sqrt[d + e*x]*(f + g*x))/(a + c*x^2),x]
```

output
```
(2*Sqrt[a]*Sqrt[c]*g*Sqrt[d + e*x] + Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*((
-I)*Sqrt[c]*f + Sqrt[a]*g)*ArcTan[(Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*Sqrt
[d + e*x])/(Sqrt[c]*d + I*Sqrt[a]*e)] + Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]
*(I*Sqrt[c]*f + Sqrt[a]*g)*ArcTan[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e]*Sqrt
[d + e*x])/(Sqrt[c]*d - I*Sqrt[a]*e)])/(Sqrt[a]*c^(3/2))
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.68, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {653, 654, 25, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(f+gx)}{a+cx^2}\,dx$$

$$\Big\downarrow \text{653}$$

$$\frac{\int \frac{cdf-aeg+c(ef+dg)x}{\sqrt{d+ex}(cx^2+a)}dx}{c}+\frac{2g\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{654}$$

$$\frac{2\int -\frac{(cd^2+ae^2)g-c(ef+dg)(d+ex)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}+\frac{2g\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{2g\sqrt{d+ex}}{c}-\frac{2\int \frac{(cd^2+ae^2)g-c(ef+dg)(d+ex)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{1483}$$

$$\frac{2\left(-\frac{\int \frac{\sqrt{2}\left(cd^2+ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}g-\sqrt[4]{c}\left(cgd^2+ae^2g+\sqrt{c}\sqrt{cd^2+ae^2}(ef+dg)\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}-\frac{\int \frac{\sqrt{2}\left(cd^2+ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}g+\sqrt[4]{c}\left(cgd^2 \cdots\right.}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\cdots}}{\cdots}\right.}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\cdots}\cdots\right.}{c}$$

$$\frac{2g\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(-\frac{\int \frac{\sqrt{2}\left(cd^2+ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}g-\sqrt[4]{c}\left(cgd^2+ae^2g+\sqrt{c}\sqrt{cd^2+ae^2}(ef+dg)\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}-\frac{\int \frac{\sqrt{2}\left(cd^2+ae^2\right)\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}g+\sqrt[4]{c}\left(cgd^2\cdots\right.}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}\cdots}}{\cdots}}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\cdots}\cdots\right.}{c}$$

$$\frac{2g\sqrt{d+ex}}{c}$$

$\downarrow$ 1142

$$\frac{2\sqrt{d+ex}g}{c}+$$

$$\frac{2\left(-\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cgd^2+ae^2g-\sqrt{c}\sqrt{cd^2+ae^2}(ef+dg)\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{1}{2}\sqrt[4]{c}\left(cgd^2+ae^2g+\sqrt{c}\sqrt{cd^2\cdots}\right.}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right.}{\cdots}$$

$\downarrow$ 25

$$\frac{2\sqrt{d+ex}g}{c}+$$

$$\frac{2\left(-\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cgd^2+ae^2g-\sqrt{c}\sqrt{cd^2+ae^2}(ef+dg)\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}+\frac{1}{2}\sqrt[4]{c}\left(cgd^2+ae^2g+\sqrt{c}\sqrt{cd^2\cdots}\right.}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\cdots\right.}{\cdots}$$

$\downarrow$ 27

$$\frac{2\sqrt{d+ex}g}{c}+$$

$$2\left(-\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cgd^2+ae^2g-\sqrt{c}\sqrt{cd^2+ae^2}(ef+dg)\right)\int\frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}+\frac{\left(cgd^2+ae^2g+\sqrt{c}\sqrt{cd^2+ae^2}(ef\right.}{}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$\downarrow$ 1083

$$\frac{2\sqrt{d+ex}g}{c}+$$

$$2\left(-\frac{\frac{\left(cgd^2+ae^2g+\sqrt{c}\sqrt{cd^2+ae^2}(ef+dg)\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cgd^2+ae^2g-\sqrt{c}\sqrt{cd^2+\right.}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$\downarrow$ 219

$$\frac{2\sqrt{d+ex}g}{c}+$$

$$2\left(-\frac{\frac{\left(cgd^2+ae^2g+\sqrt{c}\sqrt{cd^2+ae^2}(ef+dg)\right)\int\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(cgd^2+ae^2g-\sqrt{c}\sqrt{cd^2+ae\right.}{\sqrt{}}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$\downarrow$ 1103

$$2\left(-\frac{-\frac{\sqrt[4]{c}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\,\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\right)\left(-\sqrt{c}\sqrt{ae^2+cd^2}(dg+ef)+ae^2g+cd^2g\right)}{\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}-\frac{1}{2}\sqrt[4]{c}\left(\sqrt{c}\sqrt{ae^2+cd^2}(dg+\right.}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right.$$

$$\frac{2g\sqrt{d+ex}}{c}$$

input
```
Int[(Sqrt[d + e*x]*(f + g*x))/(a + c*x^2),x]
```

output
```
(2*g*Sqrt[d + e*x])/c + (2*(-1/2*(-((c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 +
 a*e^2]]*(c*d^2*g + a*e^2*g - Sqrt[c]*Sqrt[c*d^2 + a*e^2]*(e*f + d*g))*Arc
Tanh[(c^(1/4)*(-((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4))
+ 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt
[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]]) - (c^(1/4)*(c*d^2*g + a*e^2*g + Sqrt[c]
*Sqrt[c*d^2 + a*e^2]*(e*f + d*g))*Log[Sqrt[c*d^2 + a*e^2] - Sqrt[2]*c^(1/4
)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)]
)/2)/(Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*
e^2]]) - (-((c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(c*d^2*g + a*e^
2*g - Sqrt[c]*Sqrt[c*d^2 + a*e^2]*(e*f + d*g))*ArcTanh[(c^(1/4)*((Sqrt[2]*
Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4) + 2*Sqrt[d + e*x]))/(Sqrt[2
]*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])])/Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a
*e^2]]) + (c^(1/4)*(c*d^2*g + a*e^2*g + Sqrt[c]*Sqrt[c*d^2 + a*e^2]*(e*f +
 d*g))*Log[Sqrt[c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d
^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqr
t[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]])))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 653
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int[(d + e*x)^(m
- 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fr
eeQ[{a, c, d, e, f, g}, x] && FractionQ[m] && GtQ[m, 0]
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.56

method	result
pseudoelliptic	$-\frac{\frac{\left(\left(-g\sqrt{a\,e^{2}+c\,d^{2}}\,\sqrt{c}-c(dg+ef)\right)\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}+\left(c^{\frac{3}{2}}\sqrt{a\,e^{2}+c\,d^{2}}\,g+c^{2}(dg+ef)\right)d\right)\sqrt{4\sqrt{a\,e^{2}+c\,d^{2}}\,\sqrt{c}-2\sqrt{\left(a\,e^{2}+c\,d^{2}\right)c}-2cd}}{4}}{}$
risch	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^(1/2)*(g*x+f)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/c^(5/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)
^(1/2)*(1/4*((-g*(a*e^2+c*d^2)^(1/2)*c^(1/2)-c*(d*g+e*f))*((a*e^2+c*d^2)*c
)^(1/2)+(c^(3/2)*(a*e^2+c*d^2)^(1/2)*g+c^2*(d*g+e*f))*d)*(4*(a*e^2+c*d^2)^
(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*(2*((a*e^2+c*d^2)*c)^
(1/2)+2*c*d)^(1/2)*ln(c^(1/2)*(e*x+d)-(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(
1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))-1/4*((-g*(a*e^2+c*d^2)^(1/2)*c^(1/2
)-c*(d*g+e*f))*((a*e^2+c*d^2)*c)^(1/2)+(c^(3/2)*(a*e^2+c*d^2)^(1/2)*g+c^2*
(d*g+e*f))*d)*(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c
*d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)*ln(c^(1/2)*(e*x+d)+(e*x+
d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))+e^2*
(-2*c^(3/2)*(e*x+d)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*
c)^(1/2)-2*c*d)^(1/2)*g+(arctan((2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)
*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)
^(1/2)-2*c*d)^(1/2))-arctan((-2*c^(1/2)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)
^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1
/2)-2*c*d)^(1/2)))*(c^(3/2)*(a*e^2+c*d^2)^(1/2)*g-c^2*(d*g+e*f)))*a)/a/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1574 vs. $2(370) = 740$.

Time = 0.13 (sec) , antiderivative size = 1574, normalized size of antiderivative = 3.40

$$\int \frac{\sqrt{d+ex}(f+gx)}{a+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)/(c*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(c*sqrt(-(c*d*f^2 - 2*a*e*f*g - a*d*g^2 + a*c^2*sqrt(-(c^2*e^2*f^4 + 4
*c^2*d*e*f^3*g - 4*a*c*d*e*f*g^3 + a^2*e^2*g^4 + 2*(2*c^2*d^2 - a*c*e^2)*f
^2*g^2)/(a*c^5)))/(a*c^2))*log(-(c^2*e*f^4 + 2*c^2*d*f^3*g + 2*a*c*d*f*g^3
 - a^2*e*g^4)*sqrt(e*x + d) + (a*c^2*e*f^2*g + 2*a*c^2*d*f*g^2 - a^2*c*e*g
^3 + a*c^4*f*sqrt(-(c^2*e^2*f^4 + 4*c^2*d*e*f^3*g - 4*a*c*d*e*f*g^3 + a^2*
e^2*g^4 + 2*(2*c^2*d^2 - a*c*e^2)*f^2*g^2)/(a*c^5)))*sqrt(-(c*d*f^2 - 2*a*
e*f*g - a*d*g^2 + a*c^2*sqrt(-(c^2*e^2*f^4 + 4*c^2*d*e*f^3*g - 4*a*c*d*e*f
*g^3 + a^2*e^2*g^4 + 2*(2*c^2*d^2 - a*c*e^2)*f^2*g^2)/(a*c^5)))/(a*c^2)))
- c*sqrt(-(c*d*f^2 - 2*a*e*f*g - a*d*g^2 + a*c^2*sqrt(-(c^2*e^2*f^4 + 4*c^
2*d*e*f^3*g - 4*a*c*d*e*f*g^3 + a^2*e^2*g^4 + 2*(2*c^2*d^2 - a*c*e^2)*f^2*
g^2)/(a*c^5)))/(a*c^2))*log(-(c^2*e*f^4 + 2*c^2*d*f^3*g + 2*a*c*d*f*g^3 -
a^2*e*g^4)*sqrt(e*x + d) - (a*c^2*e*f^2*g + 2*a*c^2*d*f*g^2 - a^2*c*e*g^3
+ a*c^4*f*sqrt(-(c^2*e^2*f^4 + 4*c^2*d*e*f^3*g - 4*a*c*d*e*f*g^3 + a^2*e^2
*g^4 + 2*(2*c^2*d^2 - a*c*e^2)*f^2*g^2)/(a*c^5)))*sqrt(-(c*d*f^2 - 2*a*e*f
*g - a*d*g^2 + a*c^2*sqrt(-(c^2*e^2*f^4 + 4*c^2*d*e*f^3*g - 4*a*c*d*e*f*g^
3 + a^2*e^2*g^4 + 2*(2*c^2*d^2 - a*c*e^2)*f^2*g^2)/(a*c^5)))/(a*c^2))) + c
*sqrt(-(c*d*f^2 - 2*a*e*f*g - a*d*g^2 - a*c^2*sqrt(-(c^2*e^2*f^4 + 4*c^2*d
*e*f^3*g - 4*a*c*d*e*f*g^3 + a^2*e^2*g^4 + 2*(2*c^2*d^2 - a*c*e^2)*f^2*g^2
)/(a*c^5)))/(a*c^2))*log(-(c^2*e*f^4 + 2*c^2*d*f^3*g + 2*a*c*d*f*g^3 - a^2
*e*g^4)*sqrt(e*x + d) + (a*c^2*e*f^2*g + 2*a*c^2*d*f*g^2 - a^2*c*e*g^3 ...
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(f+gx)}{a+cx^2}\,dx=\int \frac{\sqrt{d+ex}(f+gx)}{a+cx^2}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(g*x+f)/(c*x**2+a),x)
```

output
```
Integral(sqrt(d + e*x)*(f + g*x)/(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}(f+gx)}{a+cx^2}\,dx=\int \frac{\sqrt{ex+d}(gx+f)}{cx^2+a}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*(g*x + f)/(c*x^2 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{d+ex}(f+gx)}{a+cx^2}\,dx=\frac{2\sqrt{ex+d}g}{c}-\frac{\left(\sqrt{-ac}c^3d^2ef+\sqrt{-ac}ac^2e^3f-(ac^2d^2+a^2ce^2)g|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{c^2d+\sqrt{c^4d^2-(c^2d^2+ace^2)c^2}}{c^2}}}\right)}{(ac^3d-\sqrt{-ac}ac^2e)\sqrt{-c^2d-\sqrt{-ac}ce}|e|}+\frac{\left(\sqrt{-ac}c^3d^2ef+\sqrt{-ac}ac^2e^3f+(ac^2d^2+a^2ce^2)g|c||e|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{c^2d-\sqrt{c^4d^2-(c^2d^2+ace^2)c^2}}{c^2}}}\right)}{(ac^3d+\sqrt{-ac}ac^2e)\sqrt{-c^2d+\sqrt{-ac}ce}|e|}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)/(c*x^2+a),x, algorithm="giac")
```

output
```
2*sqrt(e*x + d)*g/c - (sqrt(-a*c)*c^3*d^2*e*f + sqrt(-a*c)*a*c^2*e^3*f - (
a*c^2*d^2 + a^2*c*e^2)*g*abs(c)*abs(e))*arctan(sqrt(e*x + d)/sqrt(-(c^2*d
+ sqrt(c^4*d^2 - (c^2*d^2 + a*c*e^2)*c^2))/c^2))/((a*c^3*d - sqrt(-a*c)*a*
c^2*e)*sqrt(-c^2*d - sqrt(-a*c)*c*e)*abs(e)) + (sqrt(-a*c)*c^3*d^2*e*f + s
qrt(-a*c)*a*c^2*e^3*f + (a*c^2*d^2 + a^2*c*e^2)*g*abs(c)*abs(e))*arctan(sq
rt(e*x + d)/sqrt(-(c^2*d - sqrt(c^4*d^2 - (c^2*d^2 + a*c*e^2)*c^2))/c^2))/
((a*c^3*d + sqrt(-a*c)*a*c^2*e)*sqrt(-c^2*d + sqrt(-a*c)*c*e)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 4400, normalized size of antiderivative = 9.50

$$\int \frac{\sqrt{d+ex}(f+gx)}{a+cx^2}\,dx = \text{Too large to display}$$

input
```
int(((f + g*x)*(d + e*x)^(1/2))/(a + c*x^2),x)
```

output
```
2*atanh((32*a*c^2*e^4*f^2*(d + e*x)^(1/2)*((d*g^2)/(4*c^2) - (d*f^2)/(4*a*
c) + (e*f*g)/(2*c^2) + (e*f^2*(-a^3*c^5)^(1/2))/(4*a^2*c^4) - (e*g^2*(-a^3
*c^5)^(1/2))/(4*a*c^5) + (d*f*g*(-a^3*c^5)^(1/2))/(2*a^2*c^4))^(1/2))/(16*
a^2*e^5*f*g^2 - 16*c^2*d^2*e^3*f^3 - 16*a*c*e^5*f^3 - (16*a*e^5*g^3*(-a^3*
c^5)^(1/2))/c^3 + (16*e^5*f^2*g*(-a^3*c^5)^(1/2))/c^2 - (16*d^2*e^3*g^3*(-
a^3*c^5)^(1/2))/c^2 - 32*c^2*d^3*e^2*f^2*g - 32*a*c*d*e^4*f^2*g + (32*d*e^
4*f*g^2*(-a^3*c^5)^(1/2))/c^2 + 16*a*c*d^2*e^3*f*g^2 + (16*d^2*e^3*f^2*g*(
-a^3*c^5)^(1/2))/(a*c) + (32*d^3*e^2*f*g^2*(-a^3*c^5)^(1/2))/(a*c)) - (32*
d*e^3*g^2*(-a^3*c^5)^(1/2)*(d + e*x)^(1/2)*((d*g^2)/(4*c^2) - (d*f^2)/(4*a
*c) + (e*f*g)/(2*c^2) + (e*f^2*(-a^3*c^5)^(1/2))/(4*a^2*c^4) - (e*g^2*(-a^
3*c^5)^(1/2))/(4*a*c^5) + (d*f*g*(-a^3*c^5)^(1/2))/(2*a^2*c^4))^(1/2))/(16
*a*c^2*e^5*f^3 + 16*c^3*d^2*e^3*f^3 + (16*a*e^5*g^3*(-a^3*c^5)^(1/2))/c^2
- (16*e^5*f^2*g*(-a^3*c^5)^(1/2))/c - 16*a^2*c*e^5*f*g^2 + (16*d^2*e^3*g^3
*(-a^3*c^5)^(1/2))/c + 32*c^3*d^3*e^2*f^2*g - (16*d^2*e^3*f^2*g*(-a^3*c^5)
^(1/2))/a - (32*d^3*e^2*f*g^2*(-a^3*c^5)^(1/2))/a - 16*a*c^2*d^2*e^3*f*g^2
 - (32*d*e^4*f*g^2*(-a^3*c^5)^(1/2))/c + 32*a*c^2*d*e^4*f^2*g) - (32*a^2*c
*e^4*g^2*(d + e*x)^(1/2)*((d*g^2)/(4*c^2) - (d*f^2)/(4*a*c) + (e*f*g)/(2*c
^2) + (e*f^2*(-a^3*c^5)^(1/2))/(4*a^2*c^4) - (e*g^2*(-a^3*c^5)^(1/2))/(4*a
*c^5) + (d*f*g*(-a^3*c^5)^(1/2))/(2*a^2*c^4))^(1/2))/(16*a^2*e^5*f*g^2 - 1
6*c^2*d^2*e^3*f^3 - 16*a*c*e^5*f^3 - (16*a*e^5*g^3*(-a^3*c^5)^(1/2))/c^...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1104, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{d+ex}(f+gx)}{a+cx^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(1/2)*(g*x+f)/(c*x^2+a),x)
```

output

```
( - 2*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt
(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sq
rt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*c*f + 2*
sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(
c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(s
qrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*e*g - 2*sqrt(c)*sqrt(sqrt(
c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c
*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2
 + c*d**2) - c*d)*sqrt(2)))*c*d*f + 2*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*s
qrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**
2) + c*d)*sqrt(2) + 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c
*d**2) - c*d)*sqrt(2)))*c*f - 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2)
 - c*d)*sqrt(2)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) +
2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2
)))*a*e*g + 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)*at
an((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) + 2*sqrt(c)*sqrt(d +
 e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*c*d*f + sqrt(a
*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2)*log( - s
qrt(d + e*x)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) + sqrt(a*e*
*2 + c*d**2) + sqrt(c)*d + sqrt(c)*e*x)*c*f - sqrt(a*e**2 + c*d**2)*sqr...
```

3.94 $\int \frac{f+gx}{\sqrt{d+ex}(a+cx^2)} dx$

Optimal result

Integrand size = 24, antiderivative size = 447

$$\int \frac{f+gx}{\sqrt{d+ex}\,(a+cx^2)}\,dx = -\frac{\left(g+\frac{\sqrt{c}(ef-dg)}{\sqrt{cd^2+ae^2}}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}c^{3/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} + \frac{\left(g+\frac{\sqrt{c}(ef-dg)}{\sqrt{cd^2+ae^2}}\right)\arctan\left(\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}+\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right)}{\sqrt{2}c^{3/4}\sqrt{-\sqrt{c}d+\sqrt{cd^2+ae^2}}} - \frac{\left(g-\frac{\sqrt{c}(ef-dg)}{\sqrt{cd^2+ae^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt{cd^2+ae^2}+\sqrt{c}(d+ex)}\right)}{\sqrt{2}c^{3/4}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}$$

output
```
-1/2*(g+c^(1/2)*(-d*g+e*f)/(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+
c*d^2)^(1/2))^(1/2)-2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^
2)^(1/2))^(1/2))*2^(1/2)/c^(3/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)+1/
2*(g+c^(1/2)*(-d*g+e*f)/(a*e^2+c*d^2)^(1/2))*arctan(((c^(1/2)*d+(a*e^2+c*d
^2)^(1/2))^(1/2)+2^(1/2)*c^(1/4)*(e*x+d)^(1/2))/(-c^(1/2)*d+(a*e^2+c*d^2)^
(1/2))^(1/2))*2^(1/2)/c^(3/4)/(-c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)-1/2*(
g-c^(1/2)*(-d*g+e*f)/(a*e^2+c*d^2)^(1/2))*arctanh(2^(1/2)*c^(1/4)*(c^(1/2)
*d+(a*e^2+c*d^2)^(1/2))^(1/2)*(e*x+d)^(1/2)/((a*e^2+c*d^2)^(1/2)+c^(1/2)*(
e*x+d)))*2^(1/2)/c^(3/4)/(c^(1/2)*d+(a*e^2+c*d^2)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.47

$$\int \frac{f+gx}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \frac{i\left(\frac{(\sqrt{c}f+i\sqrt{a}g)\arctan\left(\frac{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d+i\sqrt{a}e}\right)}{\sqrt{-cd-i\sqrt{a}\sqrt{c}e}}-\frac{(\sqrt{c}f-i\sqrt{a}g)\arctan\left(\frac{\sqrt{-cd+i\sqrt{a}\sqrt{c}e}\sqrt{d+ex}}{\sqrt{c}d-i\sqrt{a}e}\right)}{\sqrt{-cd+i\sqrt{a}\sqrt{c}e}}\right)}{\sqrt{a}\sqrt{c}}$$

input
```
Integrate[(f + g*x)/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

output
```
(I*(((Sqrt[c]*f + I*Sqrt[a]*g)*ArcTan[(Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]*e]*
Sqrt[d + e*x])/(Sqrt[c]*d + I*Sqrt[a]*e)])/Sqrt[-(c*d) - I*Sqrt[a]*Sqrt[c]
*e] - ((Sqrt[c]*f - I*Sqrt[a]*g)*ArcTan[(Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[c]*e
]*Sqrt[d + e*x])/(Sqrt[c]*d - I*Sqrt[a]*e)])/Sqrt[-(c*d) + I*Sqrt[a]*Sqrt[
c]*e]))/(Sqrt[a]*Sqrt[c])
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.61, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {654, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{(a+cx^2)\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 654}$$

$$2\int \frac{ef-dg+g(d+ex)}{cd^2-2c(d+ex)d+ae^2+c(d+ex)^2}d\sqrt{d+ex}$$

$$\Big\downarrow \text{ 1483}$$

$$2\left(\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}(ef-dg)+\sqrt[4]{c}\left(ef-dg-\frac{\sqrt{cd^2+ae^2}g}{\sqrt{c}}\right)\sqrt{d+ex}}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}\right)}d\sqrt{d+ex}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}(ef-dg)-\sqrt[4]{c}\left(ef-\left(d+\cdots\right.\right.}{\sqrt[4]{c}\left(d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2-\cdots}}}{\sqrt[4]{c}}\right.}}{2\sqrt{2}\sqrt[4]{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2\cdots}}}\right.$$

$$\Big\downarrow \text{ 27}$$

$$2\left(\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}(ef-dg)+\sqrt[4]{c}\left(ef-dg-\frac{\sqrt{cd^2+ae^2}g}{\sqrt{c}}\right)\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}(ef-dg)-\sqrt[4]{c}\left(ef-\left(d+\cdots\right.\right.}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+a\cdots}}}{\sqrt[4]{c}}}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2\cdots}}}\right.$$

$$\Big\downarrow \text{ 1142}$$

$$2\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(\sqrt{cd^2+ae^2}g+\sqrt{c}(ef-dg)\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt{c}}-\frac{1}{2}\sqrt[4]{c}\left(ef-\left(d+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right.\right.}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$$\Big\downarrow \ 25$$

$$2\left(\frac{\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(\sqrt{cd^2+ae^2}g+\sqrt{c}(ef-dg)\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt{c}}+\frac{1}{2}\sqrt[4]{c}\left(ef-\left(d+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right.\right.}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\left(g\sqrt{ae^2+cd^2}+\sqrt{c}(ef-dg)\right)\int \frac{1}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}\sqrt{c}}+\frac{\left(ef-g\left(\frac{\sqrt{ae^2+cd^2}}{\sqrt{c}}+d\right)\right)\int \frac{}{d+\cdots}}{}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right.$$

$$\Big\downarrow \ 1083$$

$$2\left(\frac{\frac{\left(ef-g\left(\frac{\sqrt{ae^2+cd^2}}{\sqrt{c}}+d\right)\right)\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\sqrt{2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\left(g\sqrt{ae^2+cd^2}+\sqrt{c}(ef-dg)\right)\int}{}}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}\right.$$

$$\Big\downarrow \ 219$$

$$2\left(\frac{\frac{\left(ef-\left(d+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}\right)g\right)\int \frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}-\sqrt{2}\sqrt[4]{c}\sqrt{d+ex}}{d+ex+\frac{\sqrt{cd^2+ae^2}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\sqrt{d+ex}}{\sqrt[4]{c}}}d\sqrt{d+ex}}{\sqrt{2}}-\frac{\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}\left(\sqrt{cd^2+ae^2}g+\sqrt{c}(ef-dg)\right)\text{arcta}}{\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{c}}}}{2\sqrt{2}\sqrt{c}\sqrt{cd^2+ae^2}\sqrt{\sqrt{c}d+\sqrt{cd^2+ae^2}}}\right.$$

$\downarrow$ 1103

$$2\left(\frac{-\frac{\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}\,\text{arctanh}\left(\frac{\sqrt[4]{c}\left(2\sqrt{d+ex}-\frac{\sqrt{2}\sqrt{\sqrt{ae^2+cd^2}+\sqrt{c}d}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}\right)\left(g\sqrt{ae^2+cd^2}+\sqrt{c}(ef-dg)\right)}{\sqrt[4]{c}\sqrt{\sqrt{c}d-\sqrt{ae^2+cd^2}}}-\frac{1}{2}\sqrt[4]{c}\left(ef-g\left(\frac{\sqrt{ae^2+cd^2}}{\sqrt{c}}+d\right.\right.}{2\sqrt{2}\sqrt{c}\sqrt{ae^2+cd^2}\sqrt{\sqrt{ae^2+cd^2}+}}\right.$$

input
```
Int[(f + g*x)/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

output
```
2*((-((Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(Sqrt[c*d^2 + a*e^2]*g + Sqrt
[c]*(e*f - d*g))*ArcTanh[(c^(1/4)*(-((Sqrt[2]*Sqrt[Sqrt[c]*d + Sqrt[c*d^2
+ a*e^2]])/c^(1/4)) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqrt[c]*d - Sqrt[c*d
^2 + a*e^2]])])/(c^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*e^2]])) - (c^(1/4
)*(e*f - (d + Sqrt[c*d^2 + a*e^2]/Sqrt[c])*g)*Log[Sqrt[c*d^2 + a*e^2] - Sq
rt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqrt[d + e*x] + Sqrt[c
]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]*Sqrt[Sqrt[c]*d + S
qrt[c*d^2 + a*e^2]]) + (-((Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*(Sqrt[c*d
^2 + a*e^2]*g + Sqrt[c]*(e*f - d*g))*ArcTanh[(c^(1/4)*((Sqrt[2]*Sqrt[Sqrt[
c]*d + Sqrt[c*d^2 + a*e^2]])/c^(1/4) + 2*Sqrt[d + e*x]))/(Sqrt[2]*Sqrt[Sqr
t[c]*d - Sqrt[c*d^2 + a*e^2]])])/(c^(1/4)*Sqrt[Sqrt[c]*d - Sqrt[c*d^2 + a*
e^2]])) + (c^(1/4)*(e*f - d*g - (Sqrt[c*d^2 + a*e^2]*g)/Sqrt[c])*Log[Sqrt[
c*d^2 + a*e^2] + Sqrt[2]*c^(1/4)*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]*Sqr
t[d + e*x] + Sqrt[c]*(d + e*x)])/2)/(2*Sqrt[2]*Sqrt[c]*Sqrt[c*d^2 + a*e^2]
*Sqrt[Sqrt[c]*d + Sqrt[c*d^2 + a*e^2]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 654
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*
x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.51

method	result
pseudoelliptic	$-\frac{\frac{\sqrt{4\sqrt{a e^2+c d^2}\,\sqrt{c}-2\sqrt{\left(a e^2+c d^2\right)c}-2cd}\left(\left(-\sqrt{a e^2+c d^2}\,g-\sqrt{c}\,(dg-ef)\right)\sqrt{\left(a e^2+c d^2\right)c}+\left(\sqrt{a e^2+c d^2}\,cg+c^{\frac{3}{2}}(dg-ef)\right)d\right)\sqrt{}}{4}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((g*x+f)/(e*x+d)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*e^2+c*d^2)^(1/2)/c^(3/2)/(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c*
d^2)*c)^(1/2)-2*c*d)^(1/2)*(1/4*(4*(a*e^2+c*d^2)^(1/2)*c^(1/2)-2*((a*e^2+c
*d^2)*c)^(1/2)-2*c*d)^(1/2)*((-(a*e^2+c*d^2)^(1/2)*g-c^(1/2)*(d*g-e*f))*((
a*e^2+c*d^2)*c)^(1/2)+((a*e^2+c*d^2)^(1/2)*c*g+c^(3/2)*(d*g-e*f))*d)*(2*((
a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)*ln(c^(1/2)*(e*x+d)-(e*x+d)^(1/2)*(2*((a
*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)^(1/2))-1/4*(4*(a*e^2+c*d^2
)^(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)*((-(a*e^2+c*d^2)^(1
/2)*g-c^(1/2)*(d*g-e*f))*((a*e^2+c*d^2)*c)^(1/2)+((a*e^2+c*d^2)^(1/2)*c*g+
c^(3/2)*(d*g-e*f))*d)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)*ln(c^(1/2)*(
e*x+d)+(e*x+d)^(1/2)*(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2)+(a*e^2+c*d^2)
^(1/2))+e^2*(-(a*e^2+c*d^2)^(1/2)*c*g+c^(3/2)*(d*g-e*f))*(arctan((2*c^(1/2
)*(e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^
(1/2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2))-arctan((-2*c^(1/2)*(
e*x+d)^(1/2)+(2*((a*e^2+c*d^2)*c)^(1/2)+2*c*d)^(1/2))/(4*(a*e^2+c*d^2)^(1/
2)*c^(1/2)-2*((a*e^2+c*d^2)*c)^(1/2)-2*c*d)^(1/2)))*a)/a/e^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2385 vs. $2(357) = 714$.

Time = 0.31 (sec) , antiderivative size = 2385, normalized size of antiderivative = 5.34

$$\int \frac{f + gx}{\sqrt{d + ex}\,(a + cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-(c*d*f^2 + 2*a*e*f*g - a*d*g^2 + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-
(c^2*e^2*f^4 - 4*c^2*d*e*f^3*g + 4*a*c*d*e*f*g^3 + a^2*e^2*g^4 + 2*(2*c^2*
d^2 - a*c*e^2)*f^2*g^2)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4)))/(a
*c^2*d^2 + a^2*c*e^2))*log(-(c^2*e*f^4 - 2*c^2*d*f^3*g - 2*a*c*d*f*g^3 - a
^2*e*g^4)*sqrt(e*x + d) + (a*c^2*e^2*f^3 - 3*a*c^2*d*e*f^2*g + a^2*c*d*e*g
^3 + (2*a*c^2*d^2 - a^2*c*e^2)*f*g^2 + ((a*c^4*d^3 + a^2*c^3*d*e^2)*f + (a
^2*c^3*d^2*e + a^3*c^2*e^3)*g)*sqrt(-(c^2*e^2*f^4 - 4*c^2*d*e*f^3*g + 4*a*
c*d*e*f*g^3 + a^2*e^2*g^4 + 2*(2*c^2*d^2 - a*c*e^2)*f^2*g^2)/(a*c^5*d^4 +
2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4)))*sqrt(-(c*d*f^2 + 2*a*e*f*g - a*d*g^2 +
(a*c^2*d^2 + a^2*c*e^2)*sqrt(-(c^2*e^2*f^4 - 4*c^2*d*e*f^3*g + 4*a*c*d*e*f
*g^3 + a^2*e^2*g^4 + 2*(2*c^2*d^2 - a*c*e^2)*f^2*g^2)/(a*c^5*d^4 + 2*a^2*c
^4*d^2*e^2 + a^3*c^3*e^4)))/(a*c^2*d^2 + a^2*c*e^2))) + 1/2*sqrt(-(c*d*f^2
 + 2*a*e*f*g - a*d*g^2 + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(c^2*e^2*f^4 - 4*c^
2*d*e*f^3*g + 4*a*c*d*e*f*g^3 + a^2*e^2*g^4 + 2*(2*c^2*d^2 - a*c*e^2)*f^2*
g^2)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4)))/(a*c^2*d^2 + a^2*c*e^
2))*log(-(c^2*e*f^4 - 2*c^2*d*f^3*g - 2*a*c*d*f*g^3 - a^2*e*g^4)*sqrt(e*x
+ d) - (a*c^2*e^2*f^3 - 3*a*c^2*d*e*f^2*g + a^2*c*d*e*g^3 + (2*a*c^2*d^2 -
 a^2*c*e^2)*f*g^2 + ((a*c^4*d^3 + a^2*c^3*d*e^2)*f + (a^2*c^3*d^2*e + a^3*
c^2*e^3)*g)*sqrt(-(c^2*e^2*f^4 - 4*c^2*d*e*f^3*g + 4*a*c*d*e*f*g^3 + a^2*e
^2*g^4 + 2*(2*c^2*d^2 - a*c*e^2)*f^2*g^2)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^...
```

Sympy [F]

$$\int \frac{f + gx}{\sqrt{d + ex}\,(a + cx^2)}\,dx = \int \frac{f + gx}{(a + cx^2)\,\sqrt{d + ex}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)**(1/2)/(c*x**2+a),x)
```

output
```
Integral((f + g*x)/((a + c*x**2)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{f+gx}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \int \frac{gx+f}{(cx^2+a)\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((g*x + f)/((c*x^2 + a)*sqrt(e*x + d)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.62

$$\int \frac{f+gx}{\sqrt{d+ex}\,(a+cx^2)}\,dx = -\frac{\left(acef|c||e| - acdg|c||e| + \sqrt{-ac}cdef|c| + \sqrt{-ac}ae^2g|c|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{cd+\sqrt{c^2d^2-(cd^2+ae^2)c}}{c}}}\right)}{\left(ac^2d - \sqrt{-ac}ace\right)\sqrt{-c^2d - \sqrt{-ac}ce}\,|e|} - \frac{\left(acef|c||e| - acdg|c||e| - \sqrt{-ac}cdef|c| - \sqrt{-ac}ae^2g|c|\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-\frac{cd-\sqrt{c^2d^2-(cd^2+ae^2)c}}{c}}}\right)}{\left(ac^2d + \sqrt{-ac}ace\right)\sqrt{-c^2d + \sqrt{-ac}ce}\,|e|}$$

input
```
integrate((g*x+f)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
-(a*c*e*f*abs(c)*abs(e) - a*c*d*g*abs(c)*abs(e) + sqrt(-a*c)*c*d*e*f*abs(c
) + sqrt(-a*c)*a*e^2*g*abs(c))*arctan(sqrt(e*x + d)/sqrt(-(c*d + sqrt(c^2*
d^2 - (c*d^2 + a*e^2)*c))/c))/((a*c^2*d - sqrt(-a*c)*a*c*e)*sqrt(-c^2*d -
sqrt(-a*c)*c*e)*abs(e)) - (a*c*e*f*abs(c)*abs(e) - a*c*d*g*abs(c)*abs(e) -
 sqrt(-a*c)*c*d*e*f*abs(c) - sqrt(-a*c)*a*e^2*g*abs(c))*arctan(sqrt(e*x +
d)/sqrt(-(c*d - sqrt(c^2*d^2 - (c*d^2 + a*e^2)*c))/c))/((a*c^2*d + sqrt(-a
*c)*a*c*e)*sqrt(-c^2*d + sqrt(-a*c)*c*e)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.72 (sec) , antiderivative size = 2133, normalized size of antiderivative = 4.77

$$\int \frac{f+gx}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((f + g*x)/((a + c*x^2)*(d + e*x)^(1/2)),x)
```

output
```
- atan((a^2*c^5*d^3*((a^2*c^2*d*g^2 + a*e*g^2*(-a^3*c^3)^(1/2) - c*e*f^2*(
-a^3*c^3)^(1/2) - a*c^3*d*f^2 - 2*a^2*c^2*e*f*g + 2*c*d*f*g*(-a^3*c^3)^(1/
2))/(4*a^2*c^4*d^2 + 4*a^3*c^3*e^2))^(3/2)*(d + e*x)^(1/2)*8i - a^2*c^3*d^
2*g^2*((a^2*c^2*d*g^2 + a*e*g^2*(-a^3*c^3)^(1/2) - c*e*f^2*(-a^3*c^3)^(1/2
) - a*c^3*d*f^2 - 2*a^2*c^2*e*f*g + 2*c*d*f*g*(-a^3*c^3)^(1/2))/(4*a^2*c^4
*d^2 + 4*a^3*c^3*e^2))^(1/2)*(d + e*x)^(1/2)*2i + a^2*c^3*e^2*f^2*((a^2*c^
2*d*g^2 + a*e*g^2*(-a^3*c^3)^(1/2) - c*e*f^2*(-a^3*c^3)^(1/2) - a*c^3*d*f^
2 - 2*a^2*c^2*e*f*g + 2*c*d*f*g*(-a^3*c^3)^(1/2))/(4*a^2*c^4*d^2 + 4*a^3*c
^3*e^2))^(1/2)*(d + e*x)^(1/2)*2i - a^3*c^2*e^2*g^2*((a^2*c^2*d*g^2 + a*e*
g^2*(-a^3*c^3)^(1/2) - c*e*f^2*(-a^3*c^3)^(1/2) - a*c^3*d*f^2 - 2*a^2*c^2*
e*f*g + 2*c*d*f*g*(-a^3*c^3)^(1/2))/(4*a^2*c^4*d^2 + 4*a^3*c^3*e^2))^(1/2)
*(d + e*x)^(1/2)*2i + a^3*c^4*d*e^2*((a^2*c^2*d*g^2 + a*e*g^2*(-a^3*c^3)^(
1/2) - c*e*f^2*(-a^3*c^3)^(1/2) - a*c^3*d*f^2 - 2*a^2*c^2*e*f*g + 2*c*d*f*
g*(-a^3*c^3)^(1/2))/(4*a^2*c^4*d^2 + 4*a^3*c^3*e^2))^(3/2)*(d + e*x)^(1/2)
*8i + a*c^4*d^2*f^2*((a^2*c^2*d*g^2 + a*e*g^2*(-a^3*c^3)^(1/2) - c*e*f^2*(
-a^3*c^3)^(1/2) - a*c^3*d*f^2 - 2*a^2*c^2*e*f*g + 2*c*d*f*g*(-a^3*c^3)^(1/
2))/(4*a^2*c^4*d^2 + 4*a^3*c^3*e^2))^(1/2)*(d + e*x)^(1/2)*2i)/(a^3*c*e^2*
g^3 - c*e^2*f^3*(-a^3*c^3)^(1/2) - a*c^3*d*e*f^3 + 2*a*c^3*d^2*f^2*g - a*d
*e*g^3*(-a^3*c^3)^(1/2) - a^2*c^2*e^2*f^2*g + a*e^2*f*g^2*(-a^3*c^3)^(1/2)
 - 2*c*d^2*f*g^2*(-a^3*c^3)^(1/2) + 3*a^2*c^2*d*e*f*g^2 + 3*c*d*e*f^2*g...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1483, normalized size of antiderivative = 3.32

$$\int \frac{f+gx}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)/(e*x+d)^(1/2)/(c*x^2+a),x)
```

output

```
(sqrt(2)*( - 2*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) -
c*d)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*s
qrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*e*g -
 2*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*atan((s
qrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x)
)/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*c*d*f - 2*sqrt(c)*s
qrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*atan((sqrt(sqrt(c)*sqrt(a*e**2 +
c*d**2) + c*d)*sqrt(2) - 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**
2 + c*d**2) - c*d)*sqrt(2)))*a*e**2*f - 2*sqrt(c)*sqrt(sqrt(c)*sqrt(a*e**2
 + c*d**2) - c*d)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2)
- 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt
(2)))*c*d**2*f + 2*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2
) - c*d)*atan((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) + 2*sqrt(
c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*a*e
*g + 2*sqrt(a*e**2 + c*d**2)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*ata
n((sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) + c*d)*sqrt(2) + 2*sqrt(c)*sqrt(d +
e*x))/(sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*sqrt(2)))*c*d*f + 2*sqrt(
c)*sqrt(sqrt(c)*sqrt(a*e**2 + c*d**2) - c*d)*atan((sqrt(sqrt(c)*sqrt(a*e**
2 + c*d**2) + c*d)*sqrt(2) + 2*sqrt(c)*sqrt(d + e*x))/(sqrt(sqrt(c)*sqrt(a
*e**2 + c*d**2) - c*d)*sqrt(2)))*a*e**2*f + 2*sqrt(c)*sqrt(sqrt(c)*sqrt...
```

3.95 $\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = -\frac{c(3ef+5dg)\sqrt{d+ex}\sqrt{f+gx}}{4e^2g^2}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}+\frac{(8ae^2g^2+c(3e^2f^2+2defg+3d^2g^2))\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}}$$

output
```
-1/4*c*(5*d*g+3*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2)/e^2/g^2+1/2*c*(e*x+d)^(3/
2)*(g*x+f)^(1/2)/e^2/g+1/4*(8*a*e^2*g^2+c*(3*d^2*g^2+2*d*e*f*g+3*e^2*f^2))
*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/e^(5/2)/g^(5/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.84

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{c\sqrt{d+ex}\sqrt{f+gx}(-3ef-3dg+2egx)}{4e^2g^2}+\frac{(8ae^2g^2+c(3e^2f^2+2defg+3d^2g^2))\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{4e^{5/2}g^{5/2}}$$

input
```
Integrate[(a + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

output
```
(c*Sqrt[d + e*x]*Sqrt[f + g*x]*(-3*e*f - 3*d*g + 2*e*g*x))/(4*e^2*g^2) + (
(8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2))*ArcTanh[(Sqrt[e]*Sqr
t[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/(4*e^(5/2)*g^(5/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {651, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{651}$$

$$\frac{\int \frac{4age^2-c(3ef+5dg)xe-cd(3ef+dg)}{2\sqrt{d+ex}\sqrt{f+gx}}dx}{2e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4age^2-c(3ef+5dg)xe-cd(3ef+dg)}{\sqrt{d+ex}\sqrt{f+gx}}dx}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{\left(8ae^2g^2+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}dx}{2g}-\frac{c\sqrt{d+ex}\sqrt{f+gx}(5dg+3ef)}{g}}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{66}$$

$$\frac{\frac{\left(8ae^2g^2+c\left(3d^2g^2+2defg+3e^2f^2\right)\right)\int \frac{1}{e-\frac{g(d+ex)}{f+gx}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{g}-\frac{c\sqrt{d+ex}\sqrt{f+gx}(5dg+3ef)}{g}}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{(8ae^2g^2+c(3d^2g^2+2defg+3e^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{e}g^{3/2}}-\frac{c\sqrt{d+ex}\sqrt{f+gx}(5dg+3ef)}{g}}{4e^2g}+\frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

input
```
Int[(a + c*x^2)/(Sqrt[d + e*x]*Sqrt[f + g*x]),x]
```

output
```
(c*(d + e*x)^(3/2)*Sqrt[f + g*x])/(2*e^2*g) + (-((c*(3*e*f + 5*d*g)*Sqrt[d
 + e*x]*Sqrt[f + g*x])/g) + ((8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g + 3*d
^2*g^2))*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[e
]*g^(3/2)))/(4*e^2*g)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 651
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2*p + 1))   Int
[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^
2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1)
, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] &&  !IntegerQ[m]
 &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(121) = 242$.

Time $= 1.01$ (sec) , antiderivative size $= 306$, normalized size of antiderivative $= 2.08$

method	result
default	$\frac{\left(8\ln\left(\frac{2egx+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)a\,e^{2}g^{2}+3\ln\left(\frac{2egx+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}+dg+ef}{2\sqrt{eg}}\right)c\,d^{2}g^{2}+2\ln\left(\frac{2egx+2\sqrt{(ex+d)(gx+f)}\,\sqrt{eg}\right.\right.}{2\sqrt{eg}}}{}$

input
```
int((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(8*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g
)^(1/2))*a*e^2*g^2+3*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)
+d*g+e*f)/(e*g)^(1/2))*c*d^2*g^2+2*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/
2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d*e*f*g+3*ln(1/2*(2*e*g*x+2*((e*x+d
)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*e^2*f^2+4*(e*g)^(1/2)
*((e*x+d)*(g*x+f))^(1/2)*c*e*g*x-6*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*d
*g-6*(e*g)^(1/2)*((e*x+d)*(g*x+f))^(1/2)*c*e*f)*(e*x+d)^(1/2)*(g*x+f)^(1/2
)/(e*g)^(1/2)/g^2/e^2/((e*x+d)*(g*x+f))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.29

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$= \left[\frac{(3\,ce^2f^2+2\,cdefg+(3\,cd^2+8\,ae^2)g^2)\sqrt{eg}\log\left(8\,e^2g^2x^2+e^2f^2+6\,defg+d^2g^2+4\,(2\,egx+ef+dg\right.}{16\,e^3g^3} \right.$$
$$-\frac{(3\,ce^2f^2+2\,cdefg+(3\,cd^2+8\,ae^2)g^2)\sqrt{-eg}\arctan\left(\frac{(2\,egx+ef+dg)\sqrt{-eg}\sqrt{ex+d}\sqrt{gx+f}}{2\,(e^2g^2x^2+defg+(e^2fg+deg^2)x)}\right)-2\,(2\,ce^2g^2x-3\,c}{8\,e^3g^3}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((3*c*e^2*f^2 + 2*c*d*e*f*g + (3*c*d^2 + 8*a*e^2)*g^2)*sqrt(e*g)*log
(8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 4*(2*e*g*x + e*f + d*g)*s
qrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) + 8*(e^2*f*g + d*e*g^2)*x) + 4*(2*c*e
^2*g^2*x - 3*c*e^2*f*g - 3*c*d*e*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^
3), -1/8*((3*c*e^2*f^2 + 2*c*d*e*f*g + (3*c*d^2 + 8*a*e^2)*g^2)*sqrt(-e*g)
*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(
e^2*g^2*x^2 + d*e*f*g + (e^2*f*g + d*e*g^2)*x)) - 2*(2*c*e^2*g^2*x - 3*c*e
^2*f*g - 3*c*d*e*g^2)*sqrt(e*x + d)*sqrt(g*x + f))/(e^3*g^3)]
```

Sympy [F]

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral((a + c*x**2)/(sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d*g-e*f>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.12

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx = \frac{\left(\sqrt{e^2f+(ex+d)eg-deg}\sqrt{ex+d}\left(\frac{2\,(ex+d)c}{e^3g}-\frac{3\,ce^6fg+5\,cde^5g^2}{e^8g^3}\right)-\frac{(3\,ce^2f^2+2\,cdefg+3\,cd^2g^2+8\,ae^2g^2)\log\left(\left|-\sqrt{eg}\sqrt{e\right.\right.}{\sqrt{eg}e^2g^2}}{4\,|e|}$$

input
```
integrate((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(e^2*f + (e*x + d)*e*g - d*e*g)*sqrt(e*x + d)*(2*(e*x + d)*c/(e^3
*g) - (3*c*e^6*f*g + 5*c*d*e^5*g^2)/(e^8*g^3)) - (3*c*e^2*f^2 + 2*c*d*e*f*
g + 3*c*d^2*g^2 + 8*a*e^2*g^2)*log(abs(-sqrt(e*g)*sqrt(e*x + d) + sqrt(e^2
*f + (e*x + d)*e*g - d*e*g)))/(sqrt(e*g)*e^2*g^2))*e/abs(e)
```

Mupad [B] (verification not implemented)

Time = 17.38 (sec) , antiderivative size = 569, normalized size of antiderivative = 3.87

$$\int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

$$= \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{g}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{e}\left(\sqrt{f+g\,x}-\sqrt{f}\right)}\right)\left(3\,d^2\,g^2+2\,d\,e\,f\,g+3\,e^2\,f^2\right)}{2\,e^{5/2}\,g^{5/2}} - \frac{4\,a\,\mathrm{atan}\left(\frac{e\left(\sqrt{f+g\,x}-\sqrt{f}\right)}{\sqrt{-e\,g}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{-e\,g}}$$

$$-\frac{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)\left(\frac{3\,c\,d^2\,e\,g^2}{2}+c\,d\,e^2\,f\,g+\frac{3\,c\,e^3\,f^2}{2}\right)}{g^6\left(\sqrt{f+g\,x}-\sqrt{f}\right)} - \frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3\left(\frac{11\,c\,d^2\,g^2}{2}+25\,c\,d\,e\,f\,g+\frac{11\,c\,e^2\,f^2}{2}\right)}{g^5\left(\sqrt{f+g\,x}-\sqrt{f}\right)^3} + \frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7\left(\frac{3\,c\,d^2\,g^2}{2}\cdots\right)}{e^2\,g^3\left(\sqrt{f+g\,x}\cdots\right.}}{\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^8}{\left(\sqrt{f+g\,x}-\sqrt{f}\right)^8}+\frac{e^4}{g^4}-\frac{4\,e\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{g\left(\sqrt{f+g\,x}-\sqrt{f}\right)^6}-\frac{4\,e^3}{g^3\,\cdots}}$$

input
```
int((a + c*x^2)/((f + g*x)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
(c*atanh((g^(1/2)*((d + e*x)^(1/2) - d^(1/2)))/(e^(1/2)*((f + g*x)^(1/2) -
 f^(1/2))))*(3*d^2*g^2 + 3*e^2*f^2 + 2*d*e*f*g))/(2*e^(5/2)*g^(5/2)) - (4*
a*atan((e*((f + g*x)^(1/2) - f^(1/2)))/((-e*g)^(1/2)*((d + e*x)^(1/2) - d^
(1/2)))))/(-e*g)^(1/2) - ((((d + e*x)^(1/2) - d^(1/2))*((3*c*e^3*f^2)/2 +
(3*c*d^2*e*g^2)/2 + c*d*e^2*f*g))/(g^6*((f + g*x)^(1/2) - f^(1/2))) - (((d
 + e*x)^(1/2) - d^(1/2))^3*((11*c*d^2*g^2)/2 + (11*c*e^2*f^2)/2 + 25*c*d*e
*f*g))/(g^5*((f + g*x)^(1/2) - f^(1/2))^3) + (((d + e*x)^(1/2) - d^(1/2))^
7*((3*c*d^2*g^2)/2 + (3*c*e^2*f^2)/2 + c*d*e*f*g))/(e^2*g^3*((f + g*x)^(1/
2) - f^(1/2))^7) - (((d + e*x)^(1/2) - d^(1/2))^5*((11*c*d^2*g^2)/2 + (11*
c*e^2*f^2)/2 + 25*c*d*e*f*g))/(e*g^4*((f + g*x)^(1/2) - f^(1/2))^5) + (d^(
1/2)*f^(1/2)*(32*c*d*g + 32*c*e*f)*((d + e*x)^(1/2) - d^(1/2))^4)/(g^4*((f
 + g*x)^(1/2) - f^(1/2))^4))/((((d + e*x)^(1/2) - d^(1/2))^8/((f + g*x)^(1/
2) - f^(1/2))^8 + e^4/g^4 - (4*e*((d + e*x)^(1/2) - d^(1/2))^6)/(g*((f + g
*x)^(1/2) - f^(1/2))^6) - (4*e^3*((d + e*x)^(1/2) - d^(1/2))^2)/(g^3*((f +
 g*x)^(1/2) - f^(1/2))^2) + (6*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(g^2*((f
 + g*x)^(1/2) - f^(1/2))^4))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.69

$$\int \frac{a + cx^2}{\sqrt{d + ex}\sqrt{f + gx}}\,dx$$
$$= \frac{-3\sqrt{gx+f}\,\sqrt{ex+d}\,cde\,g^2 - 3\sqrt{gx+f}\,\sqrt{ex+d}\,c\,e^2fg + 2\sqrt{gx+f}\,\sqrt{ex+d}\,c\,e^2g^2x + 8\sqrt{g}\,\sqrt{e}\log\left(\frac{\sqrt{g}}{}\right.}{}$$

input
```
int((c*x^2+a)/(e*x+d)^(1/2)/(g*x+f)^(1/2),x)
```

output
```
( - 3*sqrt(f + g*x)*sqrt(d + e*x)*c*d*e*g**2 - 3*sqrt(f + g*x)*sqrt(d + e*
x)*c*e**2*f*g + 2*sqrt(f + g*x)*sqrt(d + e*x)*c*e**2*g**2*x + 8*sqrt(g)*sq
rt(e)*log((sqrt(g)*sqrt(d + e*x) + sqrt(e)*sqrt(f + g*x))/sqrt(d*g - e*f))
*a*e**2*g**2 + 3*sqrt(g)*sqrt(e)*log((sqrt(g)*sqrt(d + e*x) + sqrt(e)*sqrt
(f + g*x))/sqrt(d*g - e*f))*c*d**2*g**2 + 2*sqrt(g)*sqrt(e)*log((sqrt(g)*s
qrt(d + e*x) + sqrt(e)*sqrt(f + g*x))/sqrt(d*g - e*f))*c*d*e*f*g + 3*sqrt(
g)*sqrt(e)*log((sqrt(g)*sqrt(d + e*x) + sqrt(e)*sqrt(f + g*x))/sqrt(d*g -
e*f))*c*e**2*f**2)/(4*e**3*g**3)
```

3.96 $\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx=\sqrt{-1+x}x\sqrt{1+x}$$

output
```
(-1+x)^(1/2)*x*(1+x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx=\sqrt{-1+x}x\sqrt{1+x}$$

input
```
Integrate[(-1 + 2*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]),x]
```

output
```
Sqrt[-1 + x]*x*Sqrt[1 + x]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {644}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2-1}{\sqrt{x-1}\sqrt{x+1}}\,dx$$

$$\Big\downarrow \; 644$$

$$\sqrt{x-1}x\sqrt{x+1}$$

input
```
Int[(-1 + 2*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]),x]
```

output
```
Sqrt[-1 + x]*x*Sqrt[1 + x]
```

Defintions of rubi rules used

rule 644
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[a*x*(c + d*x)^(m + 1)*((e + f*x)^(n + 1)/(c*e)), x]
/; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
EqQ[b*c*e - a*d*f*(2*m + 3), 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\sqrt{x-1}\,x\sqrt{x+1}$	13
default	$\sqrt{x-1}\,x\sqrt{x+1}$	13
risch	$\sqrt{x-1}\,x\sqrt{x+1}$	13
orering	$\sqrt{x-1}\,x\sqrt{x+1}$	13

input
```
int((2*x^2-1)/(x-1)^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x-1)^(1/2)*x*(x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{x-1}x$$

input
```
integrate((2*x^2-1)/(x-1)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x + 1)*sqrt(x - 1)*x
```

Sympy [F(-1)]

Timed out.

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \text{Timed out}$$

input
```
integrate((2*x**2-1)/(x-1)**(1/2)/(1+x)**(1/2),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \sqrt{x^2-1}x$$

input
```
integrate((2*x^2-1)/(x-1)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 - 1)*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{x-1}x$$

input
```
integrate((2*x^2-1)/(x-1)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x + 1)*sqrt(x - 1)*x
```

Mupad [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \frac{(x^2+x)\,\sqrt{x-1}}{\sqrt{x+1}}$$

input
```
int((2*x^2 - 1)/((x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

output
```
((x + x^2)*(x - 1)^(1/2))/(x + 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx = \sqrt{x+1}\,\sqrt{x-1}\,x$$

input
```
int((2*x^2-1)/(x-1)^(1/2)/(1+x)^(1/2),x)
```

output
```
sqrt(x + 1)*sqrt(x - 1)*x
```

3.97 $\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 421

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c}+\frac{\sqrt{e}(ef+3dg)\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}$$
$$+\frac{\left(\sqrt{-a}(ae^2g-cd(2ef+dg))+\sqrt{c}(cd^2f-ae(ef+2dg))\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c^{3/2}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}$$
$$+\frac{\left(\sqrt{-a}(ae^2g-cd(2ef+dg))-\sqrt{c}(cd^2f-ae(ef+2dg))\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c^{3/2}\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

output
```
e*(e*x+d)^(1/2)*(g*x+f)^(1/2)/c+e^(1/2)*(3*d*g+e*f)*arctanh(g^(1/2)*(e*x+d
)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/c/g^(1/2)+((-a)^(1/2)*(a*e^2*g-c*d*(d*g+2*e
*f))+c^(1/2)*(c*d^2*f-a*e*(2*d*g+e*f)))*arctanh((c^(1/2)*f-(-a)^(1/2)*g)^(
1/2)*(e*x+d)^(1/2)/(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2
)/c^(3/2)/(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)/(c^(1/2)*f-(-a)^(1/2)*g)^(1/2)+((
-a)^(1/2)*(a*e^2*g-c*d*(d*g+2*e*f))-c^(1/2)*(c*d^2*f-a*e*(2*d*g+e*f)))*arc
tanh((c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1/2)/(c^(1/2)*d+(-a)^(1/2)*e)
^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/c^(3/2)/(c^(1/2)*d+(-a)^(1/2)*e)^(1/2)/(c
^(1/2)*f+(-a)^(1/2)*g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.10 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \frac{\sqrt{c}e\sqrt{d+ex}\sqrt{f+gx} + \frac{(i\sqrt{c}d+\sqrt{a}e)\sqrt{cd^2+ae^2}(\sqrt{c}f-i\sqrt{a}g)\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}\cdots}}\right.}{\sqrt{a}\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g))}}\cdots}{\cdots}$$

input
```
Integrate[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]
```

output
```
(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[f + g*x] + ((I*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c
*d^2 + a*e^2]*(Sqrt[c]*f - I*Sqrt[a]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f
 + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqr
t[d + e*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqr
t[a]*g))]) + (((-I)*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*(Sqrt[c]*f
+ I*Sqrt[a]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]
*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*Sq
rt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + (Sqrt[c]*Sqr
t[e]*(e*f + 3*d*g)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])
])/Sqrt[g])/c^(3/2)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {658, 90, 66, 221, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx$$

$$\Big\downarrow \text{ 658}$$

$$\frac{\int \frac{cfd^2-ae(ef+2dg)-(ae^2g-cd(2ef+dg))x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{c}+\frac{e\int \frac{ef+2dg+egx}{\sqrt{d+ex}\sqrt{f+gx}}dx}{c}$$

$$\Big\downarrow \ 90$$

$$\frac{\int \frac{cfd^2-ae(ef+2dg)-(ae^2g-cd(2ef+dg))x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{c}+\frac{e\left(\frac{1}{2}(3dg+ef)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}dx+\sqrt{d+ex}\sqrt{f+gx}\right)}{c}$$

$$\Big\downarrow \ 66$$

$$\frac{\int \frac{cfd^2-ae(ef+2dg)-(ae^2g-cd(2ef+dg))x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{c}+\frac{e\left((3dg+ef)\int \frac{1}{e-\frac{g(d+ex)}{f+gx}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}+\sqrt{d+ex}\sqrt{f+gx}\right)}{c}$$

$$\Big\downarrow \ 221$$

$$\frac{\int \frac{cfd^2-ae(ef+2dg)-(ae^2g-cd(2ef+dg))x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{c}+\frac{e\left(\frac{(3dg+ef)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{e}\sqrt{g}}+\sqrt{d+ex}\sqrt{f+gx}\right)}{c}$$

$$\Big\downarrow \ 2348$$

$$\frac{\int \left(\frac{\sqrt{-a}(cd^2f-ae(ef+2dg))-\frac{a\left(cd(2ef+dg)-ae^2g\right)}{\sqrt{c}}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{\frac{a\left(cd(2ef+dg)-ae^2g\right)}{\sqrt{c}}+\sqrt{-a}(cd^2f-ae(ef+2dg))}{2a\left(\sqrt{c}x+\sqrt{-a}\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{c}+\frac{e\left(\frac{(3dg+ef)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{e}\sqrt{g}}+\sqrt{d+ex}\sqrt{f+gx}\right)}{c}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\left(\frac{a\left(ae^2g-cd(dg+2ef)\right)}{\sqrt{c}}-\sqrt{-a}(cd^2f-ae(2dg+ef))\right)\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{a\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}+\frac{\left(\sqrt{-a}(cd^2f-ae(2dg+ef))+\frac{a\left(ae^2g-cd(dg+2ef)\right)}{\sqrt{c}}\right)\text{ar}\cdots}{a\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\cdots}}}{c}+\frac{e\left(\frac{(3dg+ef)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{e}\sqrt{g}}+\sqrt{d+ex}\sqrt{f+gx}\right)}{c}$$

input

```
Int[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]
```

output
```
(e*(Sqrt[d + e*x]*Sqrt[f + g*x] + ((e*f + 3*d*g)*ArcTanh[(Sqrt[g]*Sqrt[d +
 e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[e]*Sqrt[g])))/c + ((((a*(a*e^2*g -
c*d*(2*e*f + d*g)))/Sqrt[c] - Sqrt[-a]*(c*d^2*f - a*e*(e*f + 2*d*g)))*ArcT
anh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a
]*e]*Sqrt[f + g*x])])/(a*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqr
t[-a]*g]) + (((a*(a*e^2*g - c*d*(2*e*f + d*g)))/Sqrt[c] + Sqrt[-a]*(c*d^2*
f - a*e*(e*f + 2*d*g)))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x
])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(a*Sqrt[Sqrt[c]*d + Sqrt
[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]))/c
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 658
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Simp[g/c   Int[Simp[2*e*f + d*g + e*g*x, x]*(d + e*x)^(m
 - 1)*(f + g*x)^(n - 2), x], x] + Simp[1/c   Int[Simp[c*d*f^2 - 2*a*e*f*g -
 a*d*g^2 + (c*e*f^2 + 2*c*d*f*g - a*e*g^2)*x, x]*(d + e*x)^(m - 1)*((f + g*
x)^(n - 2)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x] &&  !Intege
rQ[m] &&  !IntegerQ[n] && GtQ[m, 0] && GtQ[n, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2384 vs. $2(333) = 666$.

Time = 1.76 (sec) , antiderivative size = 2385, normalized size of antiderivative = 5.67

method	result	size
default	Expression too large to display	2385

input
```
int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(-2*(e*g)^(1/2)*a*c*(((-a*c)^(1/2)*d*g+(-a
*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*
f*x+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d
*f*c)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x+(-a*c)^(1
/2)))*d*e*g-(e*g)^(1/2)*a*c*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*
c)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^
(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)*c-(-a*c)^
(1/2)*d*g-(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x+(-a*c)^(1/2)))*e^2*f+(e*g)^(1/2)*
(-a*c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*ln(
(-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)
^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c
)^(1/2)*e*f+2*d*f*c)/(c*x+(-a*c)^(1/2)))*a*e^2*g+(e*g)^(1/2)*(((-a*c)^(1/2
)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d
*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*
f+a*e*g-d*f*c)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x+
(-a*c)^(1/2)))*c^2*d^2*f-(e*g)^(1/2)*(-a*c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c
)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*
x+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f
*c)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x+(-a*c)^(1/2
)))*c*d^2*g-2*(e*g)^(1/2)*(-a*c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}{a+cx^2}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(c*x**2+a),x)
```

output
```
Integral((d + e*x)**(3/2)*sqrt(f + g*x)/(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}{cx^2+a}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \int \frac{\sqrt{f+g\,x}\,(d+e\,x)^{3/2}}{c\,x^2+a}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(a + c*x^2),x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x)^(3/2))/(a + c*x^2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{3/2}\sqrt{f+gx}}{a+cx^2}\,dx = \left(\int \frac{\sqrt{gx+f}\,\sqrt{ex+d}\,x}{c\,x^2+a}dx\right)e + \left(\int \frac{\sqrt{gx+f}\,\sqrt{ex+d}}{c\,x^2+a}dx\right)d$$

input
```
int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x)
```

output
```
int((sqrt(f + g*x)*sqrt(d + e*x)*x)/(a + c*x**2),x)*e + int((sqrt(f + g*x)
*sqrt(d + e*x))/(a + c*x**2),x)*d
```

3.98 $\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 342

$$\begin{aligned}\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx &= \frac{2\sqrt{e}\sqrt{g}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c} \\ &\quad + \frac{\left(cdf-aeg-\sqrt{-a}\sqrt{c}(ef+dg)\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} \\ &\quad - \frac{\left(cdf-aeg+\sqrt{-a}\sqrt{c}(ef+dg)\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}\end{aligned}$$

output
```
2*e^(1/2)*g^(1/2)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/c+(
c*d*f-a*e*g-(-a)^(1/2)*c^(1/2)*(d*g+e*f))*arctanh((c^(1/2)*f-(-a)^(1/2)*g)
^(1/2)*(e*x+d)^(1/2)/(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1
/2)/c/(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)/(c^(1/2)*f-(-a)^(1/2)*g)^(1/2)-(c*d*f
-a*e*g+(-a)^(1/2)*c^(1/2)*(d*g+e*f))*arctanh((c^(1/2)*f+(-a)^(1/2)*g)^(1/2
)*(e*x+d)^(1/2)/(c^(1/2)*d+(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/c
/(c^(1/2)*d+(-a)^(1/2)*e)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.43 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx$$

$$= \frac{\frac{\sqrt{cd^2+ae^2}\left(i\sqrt{c}f+\sqrt{a}g\right)\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}} + \frac{\sqrt{cd^2+ae^2}\left(-i\sqrt{c}f+\sqrt{a}g\right)\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{d+\cdots}}\right.}{\sqrt{a}\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}}}{c}$$

input
```
Integrate[(Sqrt[d + e*x]*Sqrt[f + g*x])/(a + c*x^2),x]
```

output
```
((Sqrt[c*d^2 + a*e^2]*(I*Sqrt[c]*f + Sqrt[a]*g)*ArcTan[(Sqrt[c*d^2 + a*e^2
]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*
g))]*Sqrt[d + e*x])]])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f
 - I*Sqrt[a]*g))]) + (Sqrt[c*d^2 + a*e^2]*((-I)*Sqrt[c]*f + Sqrt[a]*g)*Arc
Tan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*
(Sqrt[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])]])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d -
I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + 2*Sqrt[e]*Sqrt[g]*ArcTanh[(Sqr
t[e]*Sqrt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/c
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {659, 66, 221, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx$$

$$\Big\downarrow \; 659$$

$$\frac{\int \frac{cdf-aeg+c(ef+dg)x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{c}+\frac{eg\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}dx}{c}$$

$$\Big\downarrow \; 66$$

$$\frac{\int \frac{cdf-aeg+c(ef+dg)x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{c}+\frac{2eg\int \frac{1}{e-\frac{g(d+ex)}{f+gx}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{c}$$

$$\Big\downarrow \; 221$$

$$\frac{\int \frac{cdf-aeg+c(ef+dg)x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{c}+\frac{2\sqrt{e}\sqrt{g}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c}$$

$$\Big\downarrow \; 2348$$

$$\frac{\int \left(\frac{\sqrt{-a}(cdf-aeg)-a\sqrt{c}(ef+dg)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{a\sqrt{c}(ef+dg)+\sqrt{-a}(cdf-aeg)}{2a\left(\sqrt{c}x+\sqrt{-a}\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{c}+\frac{2\sqrt{e}\sqrt{g}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\left(-\sqrt{-a}\sqrt{c}(dg+ef)-aeg+cdf\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\left(\sqrt{-a}\sqrt{c}(dg+ef)-aeg+cdf\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}}{c}+\frac{2\sqrt{e}\sqrt{g}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c}$$

input

```
Int[(Sqrt[d + e*x]*Sqrt[f + g*x])/(a + c*x^2),x]
```

output

```
(2*Sqrt[e]*Sqrt[g]*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])
])/c + (((c*d*f - a*e*g - Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt
[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f +
g*x])]])/(Sqrt[-a]*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g
]) - ((c*d*f - a*e*g + Sqrt[-a]*Sqrt[c]*(e*f + d*g))*ArcTanh[(Sqrt[Sqrt[c]
*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x
])])/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]))
/c
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 659
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Simp[e*(g/c)   Int[(d + e*x)^(m - 1)*(f + g*x)^(n - 1),
x], x] + Simp[1/c   Int[Simp[c*d*f - a*e*g + (c*e*f + c*d*g)*x, x]*(d + e*x
)^(m - 1)*((f + g*x)^(n - 1)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f,
g}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[m, 0] && GtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1494 vs. $2(262) = 524$.

Time = 1.44 (sec) , antiderivative size = 1495, normalized size of antiderivative = 4.37

method	result	size
default	Expression too large to display	1495

input
```
int((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(-2*(-a*c)^(1/2)*ln(1/2*(2*e*g*x+2*((e*x+
d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*(((-a*c)^(1/2)*d*g+(-a
*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a
*e*g-d*f*c)/c)^(1/2)*e*g+(-a*c)^(1/2)*(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c
)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*
x+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f
*c)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x+(-a*c)^(1/2
)))*d*g+(-a*c)^(1/2)*(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g
+d*f*c)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x
+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)*c-(-
a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x+(-a*c)^(1/2)))*e*f+(-a*c)^(1
/2)*(e*g)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f
))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*c+(-a*c
)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*
d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)*d*g+(-a*c)^(1/2)*(e*g)^(1/2)*ln
((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^
(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)
^(1/2)*e*f+2*d*f*c)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
*f+a*e*g-d*f*c)/c)^(1/2)*e*f+(e*g)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e
*f-a*e*g+d*f*c)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(g*x+f)**(1/2)/(c*x**2+a),x)
```

output
```
Integral(sqrt(d + e*x)*sqrt(f + g*x)/(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \int \frac{\sqrt{ex+d}\sqrt{gx+f}}{cx^2+a}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*sqrt(g*x + f)/(c*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \text{Hanged}$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^(1/2))/(a + c*x^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2}\,dx = \int \frac{\sqrt{gx+f}\,\sqrt{ex+d}}{c\,x^2+a}dx$$

input
```
int((e*x+d)^(1/2)*(g*x+f)^(1/2)/(c*x^2+a),x)
```

output
```
int((sqrt(f + g*x)*sqrt(d + e*x))/(a + c*x**2),x)
```

3.99 $\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 240

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}} - \frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{-a}e}}$$

output
```
(c^(1/2)*f-(-a)^(1/2)*g)^(1/2)*arctanh((c^(1/2)*f-(-a)^(1/2)*g)^(1/2)*(e*x
+d)^(1/2)/(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/c^(1/2)
/(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)-(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*arctanh((c^
(1/2)*f+(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1/2)/(c^(1/2)*d+(-a)^(1/2)*e)^(1/2)/(
g*x+f)^(1/2))/(-a)^(1/2)/c^(1/2)/(c^(1/2)*d+(-a)^(1/2)*e)^(1/2)
```

Mathematica [A] (verified)

Time = 10.49 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \frac{\frac{\sqrt{-\sqrt{c}f+\sqrt{-a}g}\,\text{arctanh}\left(\frac{\sqrt{-\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-\sqrt{c}d+\sqrt{-a}e}}-\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{\sqrt{c}d+\sqrt{-a}e}}}{\sqrt{-a}\sqrt{c}}$$

input
```
Integrate[Sqrt[f + g*x]/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

output
```
((Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/Sqrt[-(S
qrt[c]*d) + Sqrt[-a]*e] - (Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt
[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f +
g*x])]])/Sqrt[Sqrt[c]*d + Sqrt[-a]*e])/(Sqrt[-a]*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {661, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{(a+cx^2)\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{661}$$

$$\int \left(\frac{\sqrt{-a}f-\frac{ag}{\sqrt{c}}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{\frac{ag}{\sqrt{c}}+\sqrt{-a}f}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}}-\frac{\sqrt{\sqrt{-a}g+\sqrt{c}f}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-a}e+\sqrt{c}d}}$$

input
```
Int[Sqrt[f + g*x]/(Sqrt[d + e*x]*(a + c*x^2)),x]
```

output
```
(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
 + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[c]*
Sqrt[Sqrt[c]*d - Sqrt[-a]*e]) - (Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*ArcTanh[(Sqr
t[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqr
t[f + g*x])]])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e])
```

Defintions of rubi rules used

rule 661
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGt
Q[m + 1/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1382 vs. $2(176) = 352$.

Time = 1.48 (sec) , antiderivative size = 1383, normalized size of antiderivative = 5.76

method	result	size
default	Expression too large to display	1383

input
```
int((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(g*x+f)^(1/2)*(e*x+d)^(1/2)*((-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e
*g-d*f*c)/c)^(1/2)*(-a*c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2
*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/
c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x-(-a*c)^(1/2)))*
a*e^2*g+(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)*(-a*c)^
(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(
((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*c+(-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x-(-a*c)^(1/2)))*c*d^2*g+(-((-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+
c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*
g+d*f*c)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x-(-a*c)
^(1/2)))*a*c*e^2*f+(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1
/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(((
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+
(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x-(-a*c)^(1/2)))*c^2*d^2*f+(-a*c)^(1/2)*(((-a
*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e
*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^
(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*d*f*
c)/(c*x+(-a*c)^(1/2)))*a*e^2*g+(-a*c)^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2
)*e*f-a*e*g+d*f*c)/c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1921 vs. $2(176) = 352$.

Time = 5.56 (sec) , antiderivative size = 1921, normalized size of antiderivative = 8.00

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e
*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 +
 a^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2 + 2*(c*d*e*f - c*d^2*g - (a*c^2*d^2*e
 + a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2
*d^2*e^2 + a^3*c*e^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g +
 (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4
+ 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g -
 d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2
*d^3 + a*c*d*e^2)*g)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 +
 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*d
^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c
^2*d^2*e^2 + a^3*c*e^4)))/(a*c^2*d^2 + a^2*c*e^2))*log(-(e^2*f^2 - d^2*g^2
 - 2*(c*d*e*f - c*d^2*g - (a*c^2*d^2*e + a^2*c*e^3)*sqrt(-(e^2*f^2 - 2*d*e
*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*sqrt(e*x + d
)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*
f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/(
a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g - d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^
2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2*d^3 + a*c*d*e^2)*g)*x)*sqrt(-(e^2*f
^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))/x)
 - 1/4*sqrt(-(c*d*f + a*e*g - (a*c^2*d^2 + a^2*c*e^2)*sqrt(-(e^2*f^2 - ...
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx=\int \frac{\sqrt{f+gx}}{(a+cx^2)\,\sqrt{d+ex}}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)**(1/2)/(c*x**2+a),x)
```

output
```
Integral(sqrt(f + g*x)/((a + c*x**2)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \int \frac{\sqrt{gx+f}}{(cx^2+a)\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/((c*x^2 + a)*sqrt(e*x + d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \text{Hanged}$$

input
```
int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\,(a+cx^2)}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{ex+d}\,a+\sqrt{ex+d}\,c\,x^2}dx$$

input
```
int((g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a),x)
```

output
```
int(sqrt(f + g*x)/(sqrt(d + e*x)*a + sqrt(d + e*x)*c*x**2),x)
```

3.100 $\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 351

$$
\begin{aligned}
\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\,(a+cx^2)}\,dx &= -\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\sqrt{d+ex}} \\
&+ \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\,(cd^2+ae^2)\sqrt{\sqrt{c}f-\sqrt{-a}g}} \\
&- \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\,(cd^2+ae^2)\sqrt{\sqrt{c}f+\sqrt{-a}g}}
\end{aligned}
$$

output

```
-2*e*(g*x+f)^(1/2)/(a*e^2+c*d^2)/(e*x+d)^(1/2)+(c*d*f+a*e*g+(-a)^(1/2)*c^(
1/2)*(-d*g+e*f))*arctanh((c^(1/2)*f-(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1/2)/(c^(
1/2)*d-(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/(c^(1/2)*d-(-a)^(1/2)
*e)^(1/2)/(a*e^2+c*d^2)/(c^(1/2)*f-(-a)^(1/2)*g)^(1/2)-(c*d*f+a*e*g-(-a)^(
1/2)*c^(1/2)*(-d*g+e*f))*arctanh((c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1
/2)/(c^(1/2)*d+(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/(c^(1/2)*d+(-
a)^(1/2)*e)^(1/2)/(a*e^2+c*d^2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx = -\frac{2e\sqrt{f+gx}}{(cd^2+ae^2)\sqrt{d+ex}} - \frac{i\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d-i\sqrt{a}e\right)\sqrt{cd^2+ae^2}} + \frac{i\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d+i\sqrt{a}e\right)\sqrt{cd^2+ae^2}}$$

input
```
Integrate[Sqrt[f + g*x]/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) - (I*Sqrt[-((Sqrt[c]*
d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*ArcTan[(Sqrt[c*d^2 + a*e^2]*S
qrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))
]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d - I*Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2])
 + (I*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*ArcTan[
(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqr
t[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d + I*Sqrt[a]*e
)*Sqrt[c*d^2 + a*e^2])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {660, 48, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{(a+cx^2)(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{660}$$

$$\frac{\int \frac{cdf+aeg-c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{ae^2+cd^2}+\frac{e(ef-dg)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}}dx}{ae^2+cd^2}$$

$$\Big\downarrow \text{48}$$

$$\frac{\int \frac{cdf+aeg-c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{ae^2+cd^2}-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\,(ae^2+cd^2)}$$

$$\Big\downarrow \text{2348}$$

$$\frac{\int \left(\frac{\sqrt{-a}(cdf+aeg)-a\sqrt{c}(ef-dg)}{2a(\sqrt{c}x+\sqrt{-a})\sqrt{d+ex}\sqrt{f+gx}}+\frac{a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{ae^2+cd^2}-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\,(ae^2+cd^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}}{ae^2+cd^2}-\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}\,(ae^2+cd^2)}$$

input
```
Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(-2*e*Sqrt[f + g*x])/((c*d^2 + a*e^2)*Sqrt[d + e*x]) + (((c*d*f + a*e*g +
Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
 + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[Sqr
t[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((c*d*f + a*e*g - Sqr
t[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[Sqrt[c
]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]))/(c*d^2 + a*e^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 660
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Simp[(-g)*((e*f - d*g)/(c*f^2 + a*g^2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n, x], x] + Simp[1/(c*f^2 + a*g^2)   Int[Simp[c*d*f + a*e
*g + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)),
 x], x] /; FreeQ[{a, c, d, e, f, g}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &
& GtQ[m, 0] && LtQ[n, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5382 vs. $2(279) = 558$.

Time = 1.56 (sec) , antiderivative size = 5383, normalized size of antiderivative = 15.34

method	result	size
default	Expression too large to display	5383

input
```
int((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5816 vs. $2(279) = 558$.

Time = 32.69 (sec) , antiderivative size = 5816, normalized size of antiderivative = 16.57

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\,(a+cx^2)}\,dx = \int \frac{\sqrt{f+gx}}{(a+cx^2)\,(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)**(3/2)/(c*x**2+a),x)
```

output
```
Integral(sqrt(f + g*x)/((a + c*x**2)*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\,(a+cx^2)}\,dx = \int \frac{\sqrt{gx+f}}{(cx^2+a)(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/((c*x^2 + a)*(e*x + d)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx = \int \frac{\sqrt{f+g\,x}}{(c\,x^2+a)\,(d+e\,x)^{3/2}}\,dx$$

input
```
int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(3/2)),x)
```

output
```
int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{ex+d}\,ad+\sqrt{ex+d}\,aex+\sqrt{ex+d}\,cd\,x^2+\sqrt{ex+d}\,ce\,x^3}dx$$

input
```
int((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a),x)
```

output
```
int(sqrt(f + g*x)/(sqrt(d + e*x)*a*d + sqrt(d + e*x)*a*e*x + sqrt(d + e*x)
*c*d*x**2 + sqrt(d + e*x)*c*e*x**3),x)
```

3.101 $\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 436

$$\begin{aligned}\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}\,dx &= -\frac{2e\sqrt{f+gx}}{3(cd^2+ae^2)(d+ex)^{3/2}} \\ &- \frac{2e(ae^2g+cd(6ef-5dg))\sqrt{f+gx}}{3(cd^2+ae^2)^2(ef-dg)\sqrt{d+ex}} \\ &- \frac{\sqrt{c}\left(\sqrt{-a}cdf+\sqrt{-a}aeg-a\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{a\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}(cd^2+ae^2)\sqrt{\sqrt{c}f-\sqrt{-a}g}} \\ &+ \frac{\sqrt{c}\left(\sqrt{-a}cdf+\sqrt{-a}aeg+a\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{a\left(\sqrt{c}d+\sqrt{-a}e\right)^{3/2}(cd^2+ae^2)\sqrt{\sqrt{c}f+\sqrt{-a}g}}\end{aligned}$$

output
```
-2/3*e*(g*x+f)^(1/2)/(a*e^2+c*d^2)/(e*x+d)^(3/2)-2/3*e*(a*e^2*g+c*d*(-5*d*
g+6*e*f))*(g*x+f)^(1/2)/(a*e^2+c*d^2)^2/(-d*g+e*f)/(e*x+d)^(1/2)-c^(1/2)*(
(-a)^(1/2)*c*d*f+(-a)^(1/2)*a*e*g-a*c^(1/2)*(-d*g+e*f))*arctanh((c^(1/2)*f
-(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1/2)/(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)/(g*x+f)^
(1/2))/a/(c^(1/2)*d-(-a)^(1/2)*e)^(3/2)/(a*e^2+c*d^2)/(c^(1/2)*f-(-a)^(1/2
)*g)^(1/2)+c^(1/2)*((-a)^(1/2)*c*d*f+(-a)^(1/2)*a*e*g+a*c^(1/2)*(-d*g+e*f)
)*arctanh((c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1/2)/(c^(1/2)*d+(-a)^(1/
2)*e)^(1/2)/(g*x+f)^(1/2))/a/(c^(1/2)*d+(-a)^(1/2)*e)^(3/2)/(a*e^2+c*d^2)/
(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.66 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}\,dx = \frac{2\sqrt{f+gx}(ae^4(f+gx)+cde(-6d^2g+6e^2fx+de(7f-5gx)))}{3(cd^2+ae^2)^2(-ef+dg)(d+ex)^{3/2}} - \frac{i\sqrt{c}\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g))}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g))}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d-i\sqrt{a}e\right)^2\sqrt{cd^2+ae^2}} + \frac{i\sqrt{c}\sqrt{-((\sqrt{c}d-i\sqrt{a}e)(\sqrt{c}f+i\sqrt{a}g))}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-((\sqrt{c}d-i\sqrt{a}e)(\sqrt{c}f+i\sqrt{a}g))}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d+i\sqrt{a}e\right)^2\sqrt{cd^2+ae^2}}$$

input
```
Integrate[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

output
```
(2*Sqrt[f + g*x]*(a*e^4*(f + g*x) + c*d*e*(-6*d^2*g + 6*e^2*f*x + d*e*(7*f
 - 5*g*x))))/(3*(c*d^2 + a*e^2)^2*(-(e*f) + d*g)*(d + e*x)^(3/2)) - (I*Sqr
t[c]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*ArcTan[(
Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt
[c]*f - I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d - I*Sqrt[a]*e)
^2*Sqrt[c*d^2 + a*e^2]) + (I*Sqrt[c]*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqr
t[c]*f + I*Sqrt[a]*g))]*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-
((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(S
qrt[a]*(Sqrt[c]*d + I*Sqrt[a]*e)^2*Sqrt[c*d^2 + a*e^2])
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {660, 55, 48, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{(a+cx^2)(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \; 660$$

$$\frac{\int \frac{cdf+aeg-c(ef-dg)x}{(d+ex)^{3/2}\sqrt{f+gx}(cx^2+a)}dx}{ae^2+cd^2}+\frac{e(ef-dg)\int \frac{1}{(d+ex)^{5/2}\sqrt{f+gx}}dx}{ae^2+cd^2}$$

$$\Big\downarrow \; 55$$

$$\frac{\int \frac{cdf+aeg-c(ef-dg)x}{(d+ex)^{3/2}\sqrt{f+gx}(cx^2+a)}dx}{ae^2+cd^2}+\frac{e(ef-dg)\left(-\frac{2g\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}}dx}{3(ef-dg)}-\frac{2\sqrt{f+gx}}{3(d+ex)^{3/2}(ef-dg)}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \; 48$$

$$\frac{\int \frac{cdf+aeg-c(ef-dg)x}{(d+ex)^{3/2}\sqrt{f+gx}(cx^2+a)}dx}{ae^2+cd^2}+\frac{e(ef-dg)\left(\frac{4g\sqrt{f+gx}}{3\sqrt{d+ex}(ef-dg)^2}-\frac{2\sqrt{f+gx}}{3(d+ex)^{3/2}(ef-dg)}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \; 2348$$

$$\frac{\int \left(\frac{\sqrt{-a}(cdf+aeg)-a\sqrt{c}(ef-dg)}{2a\left(\sqrt{c}x+\sqrt{-a}\right)(d+ex)^{3/2}\sqrt{f+gx}}+\frac{a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a\left(\sqrt{-a}-\sqrt{c}x\right)(d+ex)^{3/2}\sqrt{f+gx}}\right)dx}{ae^2+cd^2}+\frac{e(ef-dg)\left(\frac{4g\sqrt{f+gx}}{3\sqrt{d+ex}(ef-dg)^2}-\frac{2\sqrt{f+gx}}{3(d+ex)^{3/2}(ef-dg)}\right)}{ae^2+cd^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\sqrt{c}\left(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf\right)\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}\sqrt{\sqrt{c}f-\sqrt{-a}g}}+\frac{\sqrt{c}\left(a\sqrt{c}(ef-dg)+\sqrt{-a}cdf+\sqrt{-a}aeg\right)\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{a\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}\sqrt{\sqrt{-a}g+\sqrt{c}f}}+}{ae^2+cd^2}$$

$$\frac{e(ef-dg)\left(\frac{4g\sqrt{f+gx}}{3\sqrt{d+ex}(ef-dg)^2}-\frac{2\sqrt{f+gx}}{3(d+ex)^{3/2}(ef-dg)}\right)}{ae^2+cd^2}$$

input
```
Int[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)),x]
```

output
```
(e*(e*f - d*g)*((-2*Sqrt[f + g*x])/(3*(e*f - d*g)*(d + e*x)^(3/2)) + (4*g*
Sqrt[f + g*x])/(3*(e*f - d*g)^2*Sqrt[d + e*x])))/(c*d^2 + a*e^2) + ((e*(c*
d*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt
[c]*d + Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]) - (e*(c*d*f + a*e*g + Sqrt[
-a]*Sqrt[c]*(e*f - d*g))*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)
*(e*f - d*g)*Sqrt[d + e*x]) + (Sqrt[c]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*(
e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqr
t[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)^(
3/2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) + (Sqrt[c]*(Sqrt[-a]*c*d*f + Sqrt[-a]*a
*e*g + a*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d
 + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(a*(Sqrt[c]*d + Sq
rt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]))/(c*d^2 + a*e^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 660
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Simp[(-g)*((e*f - d*g)/(c*f^2 + a*g^2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n, x], x] + Simp[1/(c*f^2 + a*g^2)   Int[Simp[c*d*f + a*e
*g + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)),
 x], x] /; FreeQ[{a, c, d, e, f, g}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &
& GtQ[m, 0] && LtQ[n, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 14860 vs. $2(352) = 704$.

Time = 1.73 (sec) , antiderivative size = 14861, normalized size of antiderivative = 34.08

method	result	size
default	Expression too large to display	14861

input
```
int((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10812 vs. $2(352) = 704$.

Time = 141.42 (sec) , antiderivative size = 10812, normalized size of antiderivative = 24.80

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\,(a+cx^2)}\,dx = \int \frac{\sqrt{f+gx}}{(a+cx^2)\,(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)**(5/2)/(c*x**2+a),x)
```

output
```
Integral(sqrt(f + g*x)/((a + c*x**2)*(d + e*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\,(a+cx^2)}\,dx = \int \frac{\sqrt{gx+f}}{(cx^2+a)(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/((c*x^2 + a)*(e*x + d)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\,(a+cx^2)}\,dx=\int \frac{\sqrt{f+g\,x}}{(c\,x^2+a)\,\left(d+e\,x\right)^{5/2}}\,dx$$

input
```
int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(5/2)),x)
```

output
```
int((f + g*x)^(1/2)/((a + c*x^2)*(d + e*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}\,(a+cx^2)}\,dx=\int \frac{\sqrt{gx+f}}{(ex+d)^{\frac{5}{2}}\,(c\,x^2+a)}dx$$

input
```
int((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x)
```

output
```
int((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x)
```

3.102 $\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 337

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{2e^{3/2}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} + \frac{\left(cd^2-2\sqrt{-a}\sqrt{c}de-ae^2\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\left(cd^2+2\sqrt{-a}\sqrt{c}de-ae^2\right)\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

output

```
2*e^(3/2)*arctanh(g^(1/2)*(e*x+d)^(1/2)/e^(1/2)/(g*x+f)^(1/2))/c/g^(1/2)+(
c*d^2-2*(-a)^(1/2)*c^(1/2)*d*e-a*e^2)*arctanh((c^(1/2)*f-(-a)^(1/2)*g)^(1/
2)*(e*x+d)^(1/2)/(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/
c/(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)/(c^(1/2)*f-(-a)^(1/2)*g)^(1/2)-(c*d^2+2*(
-a)^(1/2)*c^(1/2)*d*e-a*e^2)*arctanh((c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(e*x+d
)^(1/2)/(c^(1/2)*d+(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/c/(c^(1/2
)*d+(-a)^(1/2)*e)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.61 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{\frac{(i\sqrt{c}d+\sqrt{a}e)\sqrt{cd^2+ae^2}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g))}\sqrt{d+ex}}\right)}{\sqrt{a}\sqrt{-((\sqrt{c}d+i\sqrt{a}e)(\sqrt{c}f-i\sqrt{a}g))}} + \frac{(-i\sqrt{c}d+\sqrt{a}e)\sqrt{cd^2+ae^2}\arctan}{\sqrt{a}\sqrt{-((\sqrt{c}d-}}{c}$$

input
```
Integrate[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
(((I*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*ArcTan[(Sqrt[c*d^2 + a*e^2
]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*
g))]*Sqrt[d + e*x])]])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f
 - I*Sqrt[a]*g))]) + (((-I)*Sqrt[c]*d + Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*Arc
Tan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*
(Sqrt[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])]])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d -
I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]) + (2*e^(3/2)*ArcTanh[(Sqrt[e]*Sq
rt[f + g*x])/(Sqrt[g]*Sqrt[d + e*x])])/Sqrt[g])/c
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {661, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{661}$$

$$\int \left(\frac{-ae^2+cd^2+2cdex}{c(a+cx^2)\sqrt{d+ex}\sqrt{f+gx}}+\frac{e^2}{c\sqrt{d+ex}\sqrt{f+gx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}c\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\left(2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}c\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}+\frac{2e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}$$

input
```
Int[(d + e*x)^(3/2)/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
(2*e^(3/2)*ArcTanh[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(c*Sq
rt[g]) + ((c*d^2 - 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f
 - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])
])/(Sqrt[-a]*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g])
- ((c*d^2 + 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt
[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqr
t[-a]*c*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Defintions of rubi rules used

rule 661
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
+ e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGt
Q[m + 1/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2335 vs. $2(257) = 514$.

Time = 1.62 (sec) , antiderivative size = 2336, normalized size of antiderivative = 6.93

method	result	size
default	Expression too large to display	2336

input
```
int((e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*(ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+
2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)
/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x-(-a*c)^(1/2)))
*a^2*e^2*g^2*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)
/c)^(1/2)-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/
2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*c+(-a*c)^(1/2
)*d*g+(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x-(-a*c)^(1/2)))*a*c*d^2*g^2*(e*g)^(1/2
)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)+ln((2*(-a*c)^
(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+
2*d*f*c)/(c*x-(-a*c)^(1/2)))*a*c*e^2*f^2*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(
-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)-2*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c
*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g
+d*f*c)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x-(-a*c)^
(1/2)))*a*d*e*g^2*(-a*c)^(1/2)*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2
)*e*f+a*e*g-d*f*c)/c)^(1/2)-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e
*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(
1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x-(-a*c)^(1/2)))*c^2*
d^2*f^2*(e*g)^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(
1/2)-2*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{(a+cx^2)\,\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**(1/2)/(c*x**2+a),x)
```

output
```
Integral((d + e*x)**(3/2)/((a + c*x**2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*x^2 + a)*sqrt(g*x + f)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx=\int \frac{{\left(d+e\,x\right)}^{3/2}}{\sqrt{f+g\,x}\,\left(c\,x^2+a\right)}\,dx$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + c*x^2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)^(1/2)*(a + c*x^2)), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}\,(a+cx^2)}\,dx=\left(\int \frac{\sqrt{ex+d}}{\sqrt{gx+f}\,a+\sqrt{gx+f}\,c\,x^2}dx\right)d+\left(\int \frac{\sqrt{ex+d}\,x}{\sqrt{gx+f}\,a+\sqrt{gx+f}\,c\,x^2}dx\right)e$$

input
```
int((e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+a),x)
```

output
```
int(sqrt(d + e*x)/(sqrt(f + g*x)*a + sqrt(f + g*x)*c*x**2),x)*d + int((sqr
t(d + e*x)*x)/(sqrt(f + g*x)*a + sqrt(f + g*x)*c*x**2),x)*e
```

3.103 $\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 240

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\sqrt{\sqrt{c}d+\sqrt{-a}e}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

output
```
(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)*arctanh((c^(1/2)*f-(-a)^(1/2)*g)^(1/2)*(e*x
+d)^(1/2)/(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/c^(1/2)
/(c^(1/2)*f-(-a)^(1/2)*g)^(1/2)-(c^(1/2)*d+(-a)^(1/2)*e)^(1/2)*arctanh((c^
(1/2)*f+(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1/2)/(c^(1/2)*d+(-a)^(1/2)*e)^(1/2)/(
g*x+f)^(1/2))/(-a)^(1/2)/c^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)
```

Mathematica [A] (verified)

Time = 10.40 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{\frac{\sqrt{-\sqrt{c}d+\sqrt{-a}e}\,\text{arctanh}\left(\frac{\sqrt{-\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-\sqrt{c}f+\sqrt{-a}g}} - \frac{\sqrt{\sqrt{c}d+\sqrt{-a}e}\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}}{\sqrt{-a}\sqrt{c}}$$

input
```
Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
((Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + Sqrt[-a]*g]
*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]])/Sqrt[-(S
qrt[c]*f) + Sqrt[-a]*g] - (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTanh[(Sqrt[Sqrt
[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f +
g*x])]])/Sqrt[Sqrt[c]*f + Sqrt[-a]*g])/(Sqrt[-a]*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {661, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{661}$$

$$\int \left(\frac{\sqrt{-a}d-\frac{ae}{\sqrt{c}}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{\frac{ae}{\sqrt{c}}+\sqrt{-a}d}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\sqrt{\sqrt{-a}e+\sqrt{c}d}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

input
```
Int[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
 + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[c]*
Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*ArcTanh[(Sqr
t[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqr
t[f + g*x])]])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Defintions of rubi rules used

rule 661
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*((a_.) + (c_.)*(x_)
^2)), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[d + e*x]*Sqrt[f + g*x]), (d
 + e*x)^(m + 1/2)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGt
Q[m + 1/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1382 vs. $2(176) = 352$.

Time = 1.50 (sec) , antiderivative size = 1383, normalized size of antiderivative = 5.76

method	result	size
default	Expression too large to display	1383

input
```
int((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(e*x+d)^(1/2)*(g*x+f)^(1/2)*((-a*c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*
d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*
f-a*e*g+d*f*c)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x-
(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)*
a*e*g^2+(-a*c)^(1/2)*ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(
g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*c+
(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(
1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)*c*e*f^2+(-a*c)^(1/2)*ln((-
2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(
1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^
(1/2)*e*f+2*d*f*c)/(c*x+(-a*c)^(1/2)))*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f
-a*e*g+d*f*c)/c)^(1/2)*a*e*g^2+(-a*c)^(1/2)*ln((-2*(-a*c)^(1/2)*e*g*x+c*d*
g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f
+a*e*g-d*f*c)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x+(
-a*c)^(1/2)))*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*c*
e*f^2+ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(
((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*c+(-a*c)^(1/2)*d*
g+(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x-(-a*c)^(1/2)))*(-((-a*c)^(1/2)*d*g+(-a*c)
^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)*a*c*d*g^2+ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*
x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1913 vs. $2(176) = 352$.

Time = 5.35 (sec) , antiderivative size = 1913, normalized size of antiderivative = 7.97

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e
*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 +
 a^2*c*g^2))*log(-(e^2*f^2 - d^2*g^2 + 2*(c*e*f^2 - c*d*f*g + (a*c^2*f^2*g
 + a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2
*f^2*g^2 + a^3*c*g^4)))*sqrt(e*x + d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g +
 (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4
+ 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2)) + 2*(e^2*f*g -
 d*e*g^2)*x - (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*
c*e*f*g^2 + a*c*d*g^3)*x)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4
 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))/x) + 1/4*sqrt(-(c*d*f + a*e*g + (a*c^2
*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2
*c^2*f^2*g^2 + a^3*c*g^4)))/(a*c^2*f^2 + a^2*c*g^2))*log(-(e^2*f^2 - d^2*g
^2 - 2*(c*e*f^2 - c*d*f*g + (a*c^2*f^2*g + a^2*c*g^3)*sqrt(-(e^2*f^2 - 2*d
*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))*sqrt(e*x +
 d)*sqrt(g*x + f)*sqrt(-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^
2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)))
/(a*c^2*f^2 + a^2*c*g^2)) + 2*(e^2*f*g - d*e*g^2)*x - (2*c^2*d*f^3 + 2*a*c
*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*c*e*f*g^2 + a*c*d*g^3)*x)*sqrt(-(e
^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4))
)/x) - 1/4*sqrt(-(c*d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2)*sqrt(-(e^2*f^...
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{\sqrt{d+ex}}{(a+cx^2)\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+a),x)
```

output
```
Integral(sqrt(d + e*x)/((a + c*x**2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+a)\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/((c*x^2 + a)*sqrt(g*x + f)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Hanged}$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(1/2)*(a + c*x^2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{gx+f}\,a+\sqrt{gx+f}\,c\,x^2}dx$$

input
```
int((e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a),x)
```

output
```
int(sqrt(d + e*x)/(sqrt(f + g*x)*a + sqrt(f + g*x)*c*x**2),x)
```

3.104 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 230

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}}$$

output
```
arctanh((c^(1/2)*f-(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1/2)/(c^(1/2)*d-(-a)^(1/2)
*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)/(c^(1/2
)*f-(-a)^(1/2)*g)^(1/2)-arctanh((c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1/
2)/(c^(1/2)*d+(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/(c^(1/2)*d+(-a
)^(1/2)*e)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{\sqrt[4]{-1}\left(-\frac{\sqrt{-i\sqrt{c}d+\sqrt{a}e}\arctan\left(\frac{\sqrt[4]{-1}\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-i\sqrt{c}d+\sqrt{a}e}\sqrt{\sqrt{c}f-i\sqrt{a}g}\sqrt{d+ex}}\right)}{\sqrt{\sqrt{c}f-i\sqrt{a}g}}+\frac{\sqrt{i\sqrt{c}d+\sqrt{a}e}\,\text{arctanh}\left(\frac{\sqrt[4]{-1}\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{i\sqrt{c}d+\sqrt{a}e}\sqrt{\sqrt{c}f+i\sqrt{a}g}\sqrt{d+ex}}\right)}{\sqrt{\sqrt{c}f+i\sqrt{a}g}}\right)}{\sqrt{a}\sqrt{cd^2+ae^2}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
((-1)^(1/4)*(-((Sqrt[(-I)*Sqrt[c]*d + Sqrt[a]*e]*ArcTan[((-1)^(1/4)*Sqrt[c
*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[(-I)*Sqrt[c]*d + Sqrt[a]*e]*Sqrt[Sqrt[c
]*f - I*Sqrt[a]*g]*Sqrt[d + e*x])])/Sqrt[Sqrt[c]*f - I*Sqrt[a]*g]) + (Sqrt
[I*Sqrt[c]*d + Sqrt[a]*e]*ArcTanh[((-1)^(1/4)*Sqrt[c*d^2 + a*e^2]*Sqrt[f +
 g*x])/(Sqrt[I*Sqrt[c]*d + Sqrt[a]*e]*Sqrt[Sqrt[c]*f + I*Sqrt[a]*g]*Sqrt[d
 + e*x])])/Sqrt[Sqrt[c]*f + I*Sqrt[a]*g]))/(Sqrt[a]*Sqrt[c*d^2 + a*e^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {662, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \ 662$$

$$\int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}+\frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{f+gx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}$$

input
```
Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d - Sqr
t[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[
c]*f - Sqrt[-a]*g]) - ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])
/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]/(Sqrt[-a]*Sqrt[Sqrt[c]*d +
Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g])
```

Defintions of rubi rules used

rule 662
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && !IntegerQ[m] && !Inte
gerQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. $2(170) = 340$.

Time = 1.64 (sec) , antiderivative size = 1415, normalized size of antiderivative = 6.15

method	result	size
default	Expression too large to display	1415

input
```
int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*c^2*(ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/
2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)*c-(-a*c)^(1/
2)*d*g-(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x+(-a*c)^(1/2)))*a^2*e^2*g^2*(((-a*c)^
(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)+ln((-2*(-a*c)^(1/2)*e*g*x
+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2
)*e*f+a*e*g-d*f*c)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*d*f*c)/(
c*x+(-a*c)^(1/2)))*a*c*d^2*g^2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d
*f*c)/c)^(1/2)+ln((-2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f
))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)*c-(-a*
c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x+(-a*c)^(1/2)))*a*c*e^2*f^2*(((
-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)+ln((-2*(-a*c)^(1/2)
*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(-((-a*c)^(1/2)*d*g+(-a*c
)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)*c-(-a*c)^(1/2)*d*g-(-a*c)^(1/2)*e*f+2*d*
f*c)/(c*x+(-a*c)^(1/2)))*c^2*d^2*f^2*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a
*e*g+d*f*c)/c)^(1/2)-ln((2*(-a*c)^(1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(
g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*c+
(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+2*d*f*c)/(c*x-(-a*c)^(1/2)))*a^2*e^2*g^2
*(-((-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*f+a*e*g-d*f*c)/c)^(1/2)-ln((2*(-a*c)^(
1/2)*e*g*x+c*d*g*x+c*e*f*x+2*((e*x+d)*(g*x+f))^(1/2)*(((-a*c)^(1/2)*d*g+(-
a*c)^(1/2)*e*f-a*e*g+d*f*c)/c)^(1/2)*c+(-a*c)^(1/2)*d*g+(-a*c)^(1/2)*e*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4325 vs. $2(170) = 340$.

Time = 13.01 (sec) , antiderivative size = 4325, normalized size of antiderivative = 18.80

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{1}{(a+cx^2)\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+a),x)
```

output
```
Integral(1/((a + c*x**2)*sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{1}{(cx^2+a)\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Hanged}$$

input
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{1}{\sqrt{gx+f}\,\sqrt{ex+d}\,a+\sqrt{gx+f}\,\sqrt{ex+d}\,c\,x^2}dx$$

input
```
int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a),x)
```

output
```
int(1/(sqrt(f + g*x)*sqrt(d + e*x)*a + sqrt(f + g*x)*sqrt(d + e*x)*c*x**2)
,x)
```

3.105 $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 347

$$\begin{aligned}\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx &= -\frac{2e^2\sqrt{f+gx}}{(cd^2+ae^2)\,(ef-dg)\sqrt{d+ex}}\\
&+\frac{\sqrt{c}\big(\sqrt{c}d+\sqrt{-a}e\big)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\,(cd^2+ae^2)\sqrt{\sqrt{c}f-\sqrt{-a}g}}\\
&-\frac{\sqrt{c}\big(\sqrt{c}d-\sqrt{-a}e\big)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\,(cd^2+ae^2)\sqrt{\sqrt{c}f+\sqrt{-a}g}}\end{aligned}$$

output

```
-2*e^2*(g*x+f)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)^(1/2)+c^(1/2)*(c^(1/
2)*d+(-a)^(1/2)*e)*arctanh((c^(1/2)*f-(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1/2)/(c
^(1/2)*d-(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/(c^(1/2)*d-(-a)^(1/
2)*e)^(1/2)/(a*e^2+c*d^2)/(c^(1/2)*f-(-a)^(1/2)*g)^(1/2)-c^(1/2)*(c^(1/2)*
d-(-a)^(1/2)*e)*arctanh((c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1/2)/(c^(1
/2)*d+(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/(c^(1/2)*d+(-a)^(1/2)*
e)^(1/2)/(a*e^2+c*d^2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = \frac{2e^2\sqrt{f+gx}}{(cd^2+ae^2)\,(-ef+dg)\sqrt{d+ex}} + \frac{i\sqrt{c}\left(\sqrt{c}d+i\sqrt{a}e\right)^2\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\,(cd^2+ae^2)^{3/2}\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}} - \frac{i\sqrt{c}\left(\sqrt{c}d-i\sqrt{a}e\right)^2\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\,(cd^2+ae^2)^{3/2}\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}}$$

input
```
Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
(2*e^2*Sqrt[f + g*x])/((c*d^2 + a*e^2)*(-(e*f) + d*g)*Sqrt[d + e*x]) + (I*
Sqrt[c]*(Sqrt[c]*d + I*Sqrt[a]*e)^2*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g
*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqrt[d
+ e*x])])/(Sqrt[a]*(c*d^2 + a*e^2)^(3/2)*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*
(Sqrt[c]*f - I*Sqrt[a]*g))]) - (I*Sqrt[c]*(Sqrt[c]*d - I*Sqrt[a]*e)^2*ArcT
an[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(
Sqrt[c]*f + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(c*d^2 + a*e^2)^(3/2)
*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {662, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)(d+ex)^{3/2}\sqrt{f+gx}}\,dx$$
$$\Big\downarrow \text{662}$$
$$\int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)(d+ex)^{3/2}\sqrt{f+gx}}+\frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)(d+ex)^{3/2}\sqrt{f+gx}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}\sqrt{\sqrt{-a}g+\sqrt{c}f}}-\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)}+\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}\left(\sqrt{-a}e+\sqrt{c}d\right)(ef-dg)}$$

input
```
Int[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + c*x^2)),x]
```

output
```
-((e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d
+ e*x])) + (e*Sqrt[f + g*x])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g
)*Sqrt[d + e*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d +
 e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d
 - Sqrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - (Sqrt[c]*ArcTanh[(Sqr
t[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqr
t[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*Sqrt[Sqrt[c]*f + Sq
rt[-a]*g])
```

Defintions of rubi rules used

rule 662
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && !IntegerQ[m] && !Inte
gerQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10976 vs. 2(271) = 542.

Time = 1.80 (sec) , antiderivative size = 10977, normalized size of antiderivative = 31.63

method	result	size
default	Expression too large to display	10977

input
```
int(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11846 vs. 2(271) = 542.

Time = 45.69 (sec) , antiderivative size = 11846, normalized size of antiderivative = 34.14

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{1}{(a+cx^2)\,(d+ex)^{\frac{3}{2}}\,\sqrt{f+gx}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(c*x**2+a),x)
```

output
```
Integral(1/((a + c*x**2)*(d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx=\int \frac{1}{(cx^2+a)(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx=\int \frac{1}{\sqrt{f+g\,x}\,\left(c\,x^2+a\right)\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(3/2)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\,(a+cx^2)}\,dx = \int \frac{1}{\sqrt{gx+f}\,\sqrt{ex+d}\,ad+\sqrt{gx+f}\,\sqrt{ex+d}\,aex+\sqrt{gx+f}\,\sqrt{ex+}$$

input
```
int(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+a),x)
```

output
```
int(1/(sqrt(f + g*x)*sqrt(d + e*x)*a*d + sqrt(f + g*x)*sqrt(d + e*x)*a*e*x
 + sqrt(f + g*x)*sqrt(d + e*x)*c*d*x**2 + sqrt(f + g*x)*sqrt(d + e*x)*c*e*
x**3),x)
```

3.106 $\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 414

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \frac{2(ef-dg)\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} + \frac{\left(\sqrt{c}(cd^2f-ae(ef-2dg))-\sqrt{-a}(ae^2g+cd(2ef-dg))\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\,(cf^2+ag^2)} - \frac{\left(\sqrt{c}(cd^2f-ae(ef-2dg))+\sqrt{-a}(ae^2g+cd(2ef-dg))\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(cf^2+ag^2)}$$

output
```
2*(-d*g+e*f)*(e*x+d)^(1/2)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+(c^(1/2)*(c*d^2*f-a
*e*(-2*d*g+e*f))-(-a)^(1/2)*(a*e^2*g+c*d*(-d*g+2*e*f)))*arctanh((c^(1/2)*f
-(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1/2)/(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)/(g*x+f)^
(1/2))/(-a)^(1/2)/c^(1/2)/(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)/(c^(1/2)*f-(-a)^(
1/2)*g)^(1/2)/(a*g^2+c*f^2)-(c^(1/2)*(c*d^2*f-a*e*(-2*d*g+e*f))+(-a)^(1/2)
*(a*e^2*g+c*d*(-d*g+2*e*f)))*arctanh((c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(e*x+d
)^(1/2)/(c^(1/2)*d+(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/c^(1/2)/(
c^(1/2)*d+(-a)^(1/2)*e)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)/(a*g^2+c*f^2)
```

Mathematica [A] (verified)

Time = 10.97 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}\,(a+cx^2)}\,dx =$$
$$-\left(\left(\frac{d}{\sqrt{-a}}-\frac{e}{\sqrt{c}}\right)\left(\frac{\sqrt{d+ex}}{\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{f+gx}}+\frac{\sqrt{-\sqrt{c}d+\sqrt{-a}e}\,\text{arctanh}\left(\frac{\sqrt{-\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{-\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\left(-\sqrt{c}f+\sqrt{-a}g\right)^{3/2}}\right)\right)-\Bigg(\Big($$

input
```
Integrate[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

output
```
-((d/Sqrt[-a] - e/Sqrt[c])*(Sqrt[d + e*x]/((Sqrt[c]*f - Sqrt[-a]*g)*Sqrt[f
 + g*x]) + (Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*ArcTanh[(Sqrt[-(Sqrt[c]*f) + S
qrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[-(Sqrt[c]*d) + Sqrt[-a]*e]*Sqrt[f + g*x])]
)/(-(Sqrt[c]*f) + Sqrt[-a]*g)^(3/2))) - ((a*d)/(-a)^(3/2) - e/Sqrt[c])*(Sq
rt[d + e*x]/((Sqrt[c]*f + Sqrt[-a]*g)*Sqrt[f + g*x]) - (Sqrt[Sqrt[c]*d + S
qrt[-a]*e]*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt
[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {660, 57, 66, 221, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(a+cx^2)\,(f+gx)^{3/2}}\,dx$$
$$\Big\downarrow \; 660$$
$$\frac{\int \frac{\sqrt{d+ex}(cdf+aeg+c(ef-dg)x)}{\sqrt{f+gx}(cx^2+a)}dx}{ag^2+cf^2}-\frac{g(ef-dg)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}}dx}{ag^2+cf^2}$$
$$\Big\downarrow \; 57$$

$$\frac{\int \frac{\sqrt{d+ex}(cdf+aeg+c(ef-dg)x)}{\sqrt{f+gx}(cx^2+a)}dx}{ag^2+cf^2}-\frac{g(ef-dg)\left(\frac{e\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}}dx}{g}-\frac{2\sqrt{d+ex}}{g\sqrt{f+gx}}\right)}{ag^2+cf^2}$$

$$\Big\downarrow \textbf{66}$$

$$\frac{\int \frac{\sqrt{d+ex}(cdf+aeg+c(ef-dg)x)}{\sqrt{f+gx}(cx^2+a)}dx}{ag^2+cf^2}-\frac{g(ef-dg)\left(\frac{2e\int \frac{1}{e-\frac{g(d+ex)}{f+gx}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{g}-\frac{2\sqrt{d+ex}}{g\sqrt{f+gx}}\right)}{ag^2+cf^2}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{\int \frac{\sqrt{d+ex}(cdf+aeg+c(ef-dg)x)}{\sqrt{f+gx}(cx^2+a)}dx}{ag^2+cf^2}-\frac{g(ef-dg)\left(\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{g^{3/2}}-\frac{2\sqrt{d+ex}}{g\sqrt{f+gx}}\right)}{ag^2+cf^2}$$

$$\Big\downarrow \textbf{2348}$$

$$\frac{\int \left(\frac{\sqrt{d+ex}(\sqrt{-a}(cdf+aeg)-a\sqrt{c}(ef-dg))}{2a(\sqrt{-a}-\sqrt{c}x)\sqrt{f+gx}}+\frac{(a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg))\sqrt{d+ex}}{2a(\sqrt{c}x+\sqrt{-a})\sqrt{f+gx}}\right)dx}{ag^2+cf^2}-\frac{g(ef-dg)\left(\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{g^{3/2}}-\frac{2\sqrt{d+ex}}{g\sqrt{f+gx}}\right)}{ag^2+cf^2}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{-\frac{\sqrt{e}(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}}+\frac{\sqrt{e}(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}}+\frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}(-\sqrt{-}}{}}{ag^2+}-\frac{g(ef-dg)\left(\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{g^{3/2}}-\frac{2\sqrt{d+ex}}{g\sqrt{f+gx}}\right)}{ag^2+cf^2}$$

input
```
Int[(d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

output
```
-((g*(e*f - d*g)*((-2*Sqrt[d + e*x])/(g*Sqrt[f + g*x]) + (2*Sqrt[e]*ArcTan
h[(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/g^(3/2)))/(c*f^2 + a*g
^2)) + (-((Sqrt[e]*(c*d*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[
(Sqrt[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[g
])) + (Sqrt[e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqr
t[g]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[g]) +
 (Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*(c*d*f + a*e*g - Sqrt[-a]*Sqrt[c]*(e*f - d*
g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d -
 Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f - Sqrt[-a]*
g]) - (Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(c*d*f + a*e*g + Sqrt[-a]*Sqrt[c]*(e*f
 - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c
]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[c]*Sqrt[Sqrt[c]*f + Sqrt
[-a]*g]))/(c*f^2 + a*g^2)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2  Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 660
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Simp[(-g)*((e*f - d*g)/(c*f^2 + a*g^2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n, x], x] + Simp[1/(c*f^2 + a*g^2)   Int[Simp[c*d*f + a*e
*g + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)),
 x], x] /; FreeQ[{a, c, d, e, f, g}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &
& GtQ[m, 0] && LtQ[n, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8263 vs. $2(338) = 676$.

Time = 1.19 (sec) , antiderivative size = 8264, normalized size of antiderivative = 19.96

method	result	size
default	Expression too large to display	8264

input
```
int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{(a+cx^2)\,(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

output
```
Integral((d + e*x)**(3/2)/((a + c*x**2)*(f + g*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/((c*x^2 + a)*(g*x + f)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \int \frac{(d+e\,x)^{3/2}}{(f+g\,x)^{3/2}\,(c\,x^2+a)}\,dx$$

input
```
int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)),x)
```

output
```
int((d + e*x)^(3/2)/((f + g*x)^(3/2)*(a + c*x^2)), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \left(\int \frac{\sqrt{ex+d}}{\sqrt{gx+f}\,af+\sqrt{gx+f}\,agx+\sqrt{gx+f}\,cf\,x^2+\sqrt{gx+f}\,cg\,x^3}dx\right)d + \left(\int \frac{\sqrt{ex+d}\,x}{\sqrt{gx+f}\,af+\sqrt{gx+f}\,agx+\sqrt{gx+f}\,cf\,x^2+\sqrt{gx+f}\,cg\,x^3}dx\right)e$$

input
```
int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
```

output
```
int(sqrt(d + e*x)/(sqrt(f + g*x)*a*f + sqrt(f + g*x)*a*g*x + sqrt(f + g*x)
*c*f*x**2 + sqrt(f + g*x)*c*g*x**3),x)*d + int((sqrt(d + e*x)*x)/(sqrt(f +
 g*x)*a*f + sqrt(f + g*x)*a*g*x + sqrt(f + g*x)*c*f*x**2 + sqrt(f + g*x)*c
*g*x**3),x)*e
```

3.107 $\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 351

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx = -\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} + \frac{\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\,(cf^2+ag^2)} - \frac{\left(cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(cf^2+ag^2)}$$

output
```
-2*g*(e*x+d)^(1/2)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+(c*d*f+a*e*g-(-a)^(1/2)*c^(
1/2)*(-d*g+e*f))*arctanh((c^(1/2)*f-(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1/2)/(c^(
1/2)*d-(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/(c^(1/2)*d-(-a)^(1/2)
*e)^(1/2)/(c^(1/2)*f-(-a)^(1/2)*g)^(1/2)/(a*g^2+c*f^2)-(c*d*f+a*e*g+(-a)^(
1/2)*c^(1/2)*(-d*g+e*f))*arctanh((c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1
/2)/(c^(1/2)*d+(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/(c^(1/2)*d+(-
a)^(1/2)*e)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)/(a*g^2+c*f^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.03 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx = -\frac{2g\sqrt{d+ex}}{(cf^2+ag^2)\sqrt{f+gx}} + \frac{i\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cf^2+ag^2}\sqrt{d+ex}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{f+gx}}\right)}{\sqrt{a}\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{cf^2+ag^2}} - \frac{i\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cf^2+ag^2}\sqrt{d+ex}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{f+gx}}\right)}{\sqrt{a}\left(\sqrt{c}f-i\sqrt{a}g\right)\sqrt{cf^2+ag^2}}$$

input
```
Integrate[Sqrt[d + e*x]/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + (I*Sqrt[-((Sqrt[c]*
d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*ArcTan[(Sqrt[c*f^2 + a*g^2]*S
qrt[d + e*x])/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))
]*Sqrt[f + g*x])])/(Sqrt[a]*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[c*f^2 + a*g^2])
 - (I*Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*ArcTan[
(Sqrt[c*f^2 + a*g^2]*Sqrt[d + e*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqr
t[c]*f + I*Sqrt[a]*g))]*Sqrt[f + g*x])])/(Sqrt[a]*(Sqrt[c]*f - I*Sqrt[a]*g
)*Sqrt[c*f^2 + a*g^2])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {660, 48, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{660}$$

$$\frac{\int \frac{cdf+aeg+c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{ag^2+cf^2}-\frac{g(ef-dg)\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}}dx}{ag^2+cf^2}$$

$$\Big\downarrow \text{48}$$

$$\frac{\int \frac{cdf+aeg+c(ef-dg)x}{\sqrt{d+ex}\sqrt{f+gx}(cx^2+a)}dx}{ag^2+cf^2}-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\,(ag^2+cf^2)}$$

$$\Big\downarrow \text{2348}$$

$$\frac{\int \left(\frac{\sqrt{-a}(cdf+aeg)-a\sqrt{c}(ef-dg)}{2a(\sqrt{-a}-\sqrt{c}x)\sqrt{d+ex}\sqrt{f+gx}}+\frac{a\sqrt{c}(ef-dg)+\sqrt{-a}(cdf+aeg)}{2a(\sqrt{c}x+\sqrt{-a})\sqrt{d+ex}\sqrt{f+gx}}\right)dx}{ag^2+cf^2}-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\,(ag^2+cf^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}}-\frac{(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\sqrt{\sqrt{-a}g+\sqrt{c}f}}}{ag^2+cf^2}-\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}\,(ag^2+cf^2)}$$

input
```
Int[Sqrt[d + e*x]/((f + g*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(-2*g*Sqrt[d + e*x])/((c*f^2 + a*g^2)*Sqrt[f + g*x]) + (((c*d*f + a*e*g -
Sqrt[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d
 + e*x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[Sqr
t[c]*d - Sqrt[-a]*e]*Sqrt[Sqrt[c]*f - Sqrt[-a]*g]) - ((c*d*f + a*e*g + Sqr
t[-a]*Sqrt[c]*(e*f - d*g))*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g]*Sqrt[d +
e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])])/(Sqrt[-a]*Sqrt[Sqrt[c
]*d + Sqrt[-a]*e]*Sqrt[Sqrt[c]*f + Sqrt[-a]*g]))/(c*f^2 + a*g^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 660
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Simp[(-g)*((e*f - d*g)/(c*f^2 + a*g^2))   Int[(d + e*x)^
(m - 1)*(f + g*x)^n, x], x] + Simp[1/(c*f^2 + a*g^2)   Int[Simp[c*d*f + a*e
*g + c*(e*f - d*g)*x, x]*(d + e*x)^(m - 1)*((f + g*x)^(n + 1)/(a + c*x^2)),
 x], x] /; FreeQ[{a, c, d, e, f, g}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &
& GtQ[m, 0] && LtQ[n, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5382 vs. $2(279) = 558$.

Time = 1.64 (sec) , antiderivative size = 5383, normalized size of antiderivative = 15.34

method	result	size
default	Expression too large to display	5383

input
```
int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5844 vs. $2(279) = 558$.

Time = 37.10 (sec) , antiderivative size = 5844, normalized size of antiderivative = 16.65

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \int \frac{\sqrt{d+ex}}{(a+cx^2)(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

output
```
Integral(sqrt(d + e*x)/((a + c*x**2)*(f + g*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+a)(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/((c*x^2 + a)*(g*x + f)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \int \frac{\sqrt{d+e\,x}}{(f+g\,x)^{3/2}\,(c\,x^2+a)}\,dx$$

input
```
int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(a + c*x^2)),x)
```

output
```
int((d + e*x)^(1/2)/((f + g*x)^(3/2)*(a + c*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{gx+f}\,af+\sqrt{gx+f}\,agx+\sqrt{gx+f}\,cf\,x^2+\sqrt{gx+f}\,cg\,x^3}\,dx$$

input
```
int((e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
```

output
```
int(sqrt(d + e*x)/(sqrt(f + g*x)*a*f + sqrt(f + g*x)*a*g*x + sqrt(f + g*x)
*c*f*x**2 + sqrt(f + g*x)*c*g*x**3),x)
```

3.108 $\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 347

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}(a+cx^2)}\,dx = \frac{2g^2\sqrt{d+ex}}{(ef-dg)(cf^2+ag^2)\sqrt{f+gx}} + \frac{\sqrt{c}\left(\sqrt{c}f+\sqrt{-a}g\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{\sqrt{c}f-\sqrt{-a}g}\,(cf^2+ag^2)} - \frac{\sqrt{c}\left(\sqrt{c}f-\sqrt{-a}g\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(cf^2+ag^2)}$$

output

```
2*g^2*(e*x+d)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+c^(1/2)*(c^(1/2
)*f+(-a)^(1/2)*g)*arctanh((c^(1/2)*f-(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1/2)/(c^
(1/2)*d-(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/(c^(1/2)*d-(-a)^(1/2
)*e)^(1/2)/(c^(1/2)*f-(-a)^(1/2)*g)^(1/2)/(a*g^2+c*f^2)-c^(1/2)*(c^(1/2)*f
-(-a)^(1/2)*g)*arctanh((c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(e*x+d)^(1/2)/(c^(1/
2)*d+(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/(-a)^(1/2)/(c^(1/2)*d+(-a)^(1/2)*e
)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)/(a*g^2+c*f^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}(a+cx^2)}\,dx = \frac{2g^2\sqrt{d+ex}}{(ef-dg)(cf^2+ag^2)\sqrt{f+gx}} - \frac{i\sqrt{c}\left(\sqrt{c}f-i\sqrt{a}g\right)^2 \arctan\left(\frac{\sqrt{cf^2+ag^2}\sqrt{d+ex}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{f+gx}}\right)}{\sqrt{a}\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\left(cf^2+ag^2\right)^{3/2}} + \frac{i\sqrt{c}\left(\sqrt{c}f+i\sqrt{a}g\right)^2 \arctan\left(\frac{\sqrt{cf^2+ag^2}\sqrt{d+ex}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{f+gx}}\right)}{\sqrt{a}\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\left(cf^2+ag^2\right)^{3/2}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(2*g^2*Sqrt[d + e*x])/((e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]) - (I*Sqr
t[c]*(Sqrt[c]*f - I*Sqrt[a]*g)^2*ArcTan[(Sqrt[c*f^2 + a*g^2]*Sqrt[d + e*x]
)/(Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqrt[f + g
*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g)
)]*(c*f^2 + a*g^2)^(3/2)) + (I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)^2*ArcTan[
(Sqrt[c*f^2 + a*g^2]*Sqrt[d + e*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqr
t[c]*f + I*Sqrt[a]*g))]*Sqrt[f + g*x])])/(Sqrt[a]*Sqrt[-((Sqrt[c]*d - I*Sq
rt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*(c*f^2 + a*g^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {662, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)\sqrt{d+ex}(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 662}$$

$$\int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)\sqrt{d+ex}(f+gx)^{3/2}}+\frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)\sqrt{d+ex}(f+gx)^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(\sqrt{-a}g+\sqrt{c}f\right)^{3/2}}+\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)}-\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}\left(\sqrt{-a}g+\sqrt{c}f\right)(ef-dg)}$$

input
```
Int[1/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)*Sqrt[f +
g*x]) - (g*Sqrt[d + e*x])/(Sqrt[-a]*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)*S
qrt[f + g*x]) + (Sqrt[c]*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*
x])/(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*Sqrt[Sqrt[c]*
d - Sqrt[-a]*e]*(Sqrt[c]*f - Sqrt[-a]*g)^(3/2)) - (Sqrt[c]*ArcTanh[(Sqrt[S
qrt[c]*f + Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f
 + g*x])]])/(Sqrt[-a]*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*(Sqrt[c]*f + Sqrt[-a]*g)
^(3/2))
```

Defintions of rubi rules used

rule 662
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] &&  !IntegerQ[m] &&  !Inte
gerQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10976 vs. 2(271) = 542.

Time = 1.72 (sec) , antiderivative size = 10977, normalized size of antiderivative = 31.63

method	result	size
default	Expression too large to display	10977

input
```
int(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12028 vs. 2(271) = 542.

Time = 72.18 (sec) , antiderivative size = 12028, normalized size of antiderivative = 34.66

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = \int \frac{1}{\left(a+cx^2\right)\sqrt{d+ex}\,(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

output
```
Integral(1/((a + c*x**2)*sqrt(d + e*x)*(f + g*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{1}{(cx^2+a)\sqrt{ex+d}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)*sqrt(e*x + d)*(g*x + f)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{1}{{\left(f+g\,x\right)}^{3/2}\,\left(c\,x^2+a\right)\,\sqrt{d+e\,x}}\,dx$$

input
```
int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(1/2)),x)
```

output
```
int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{1}{\sqrt{gx+f}\,\sqrt{ex+d}\,af+\sqrt{gx+f}\,\sqrt{ex+d}\,agx+\sqrt{gx+f}\,\sqrt{ex+}$$

input
```
int(1/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
```

output
```
int(1/(sqrt(f + g*x)*sqrt(d + e*x)*a*f + sqrt(f + g*x)*sqrt(d + e*x)*a*g*x
 + sqrt(f + g*x)*sqrt(d + e*x)*c*f*x**2 + sqrt(f + g*x)*sqrt(d + e*x)*c*g*
x**3),x)
```

3.109 $\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}(a+cx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 416

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}(a+cx^2)}\,dx = -\frac{2e^2}{(cd^2+ae^2)(ef-dg)\sqrt{d+ex}\sqrt{f+gx}} - \frac{2g(ce^2f^2+cd^2g^2+2ae^2g^2)\sqrt{d+ex}}{(cd^2+ae^2)(ef-dg)^2(cf^2+ag^2)\sqrt{f+gx}} - \frac{c\left(\sqrt{-a}\sqrt{c}d-ae\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f-\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d-\sqrt{-a}e}\sqrt{f+gx}}\right)}{a\sqrt{\sqrt{c}d-\sqrt{-a}e}\,(cd^2+ae^2)\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}} + \frac{c\left(\sqrt{-a}\sqrt{c}d+ae\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{d+ex}}{\sqrt{\sqrt{c}d+\sqrt{-a}e}\sqrt{f+gx}}\right)}{a\sqrt{\sqrt{c}d+\sqrt{-a}e}\,(cd^2+ae^2)\left(\sqrt{c}f+\sqrt{-a}g\right)^{3/2}}$$

output
```
-2*e^2/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)^(1/2)/(g*x+f)^(1/2)-2*g*(2*a*e^2*g
^2+c*d^2*g^2+c*e^2*f^2)*(e*x+d)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)^2/(a*g^2+c*
f^2)/(g*x+f)^(1/2)-c*((-a)^(1/2)*c^(1/2)*d-a*e)*arctanh((c^(1/2)*f-(-a)^(1
/2)*g)^(1/2)*(e*x+d)^(1/2)/(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/a
/(c^(1/2)*d-(-a)^(1/2)*e)^(1/2)/(a*e^2+c*d^2)/(c^(1/2)*f-(-a)^(1/2)*g)^(3/
2)+c*((-a)^(1/2)*c^(1/2)*d+a*e)*arctanh((c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(e*
x+d)^(1/2)/(c^(1/2)*d+(-a)^(1/2)*e)^(1/2)/(g*x+f)^(1/2))/a/(c^(1/2)*d+(-a)
^(1/2)*e)^(1/2)/(a*e^2+c*d^2)/(c^(1/2)*f+(-a)^(1/2)*g)^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.86 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.15

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx = -\frac{2(c(d^3g^3+d^2eg^3x+e^3f^2(f+gx))+ae^2g^2(dg+e(f+2gx)))}{\left(cd^2+ae^2\right)(ef-dg)^2\left(cf^2+ag^2\right)\sqrt{d+ex}\sqrt{f+gx}} - \frac{ic\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d+i\sqrt{a}e\right)\left(\sqrt{c}f-i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d-i\sqrt{a}e\right)\sqrt{cd^2+ae^2}\left(\sqrt{c}f-i\sqrt{a}g\right)^2} + \frac{ic\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\arctan\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{-\left(\left(\sqrt{c}d-i\sqrt{a}e\right)\left(\sqrt{c}f+i\sqrt{a}g\right)\right)}\sqrt{d+ex}}\right)}{\sqrt{a}\left(\sqrt{c}d+i\sqrt{a}e\right)\sqrt{cd^2+ae^2}\left(\sqrt{c}f+i\sqrt{a}g\right)^2}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

output
```
(-2*(c*(d^3*g^3 + d^2*e*g^3*x + e^3*f^2*(f + g*x)) + a*e^2*g^2*(d*g + e*(f
 + 2*g*x))))/((c*d^2 + a*e^2)*(e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[d + e*x]*
Sqrt[f + g*x]) - (I*c*Sqrt[-((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt
[a]*g))]*ArcTan[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d + I
*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]
*d - I*Sqrt[a]*e)*Sqrt[c*d^2 + a*e^2]*(Sqrt[c]*f - I*Sqrt[a]*g)^2) + (I*c*
Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]*ArcTan[(Sqrt[
c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[-((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f
 + I*Sqrt[a]*g))]*Sqrt[d + e*x])])/(Sqrt[a]*(Sqrt[c]*d + I*Sqrt[a]*e)*Sqrt
[c*d^2 + a*e^2]*(Sqrt[c]*f + I*Sqrt[a]*g)^2)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {662, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)\,(d+ex)^{3/2}(f+gx)^{3/2}}\,dx$$

$$\Big\downarrow \text{662}$$

$$\int \left(\frac{\sqrt{-a}}{2a\left(\sqrt{-a}-\sqrt{c}x\right)(d+ex)^{3/2}(f+gx)^{3/2}}+\frac{\sqrt{-a}}{2a\left(\sqrt{-a}+\sqrt{c}x\right)(d+ex)^{3/2}(f+gx)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-\sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d-\sqrt{-a}e}}\right)}{\sqrt{-a}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}\left(\sqrt{c}f-\sqrt{-a}g\right)^{3/2}}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g+\sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e+\sqrt{c}d}}\right)}{\sqrt{-a}\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}\left(\sqrt{-a}g+\sqrt{c}f\right)^{3/2}}-\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{c}d-\sqrt{-a}e\right)(ef-dg)}+\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}\left(\sqrt{-a}e+\sqrt{c}d\right)(ef-dg)}+\frac{g\sqrt{d+ex}\left(2aeg-\sqrt{-a}\sqrt{c}(dg+ef)\right)}{a\sqrt{f+gx}\left(\sqrt{-a}e+\sqrt{c}d\right)\left(\sqrt{-a}g+\sqrt{c}f\right)(ef-dg)^2}+\frac{g\sqrt{d+ex}\left(\sqrt{-a}\sqrt{c}(dg+ef)+2aeg\right)}{a\sqrt{f+gx}\left(\sqrt{c}d-\sqrt{-a}e\right)\left(\sqrt{c}f-\sqrt{-a}g\right)(ef-dg)^2}$$

input
```
Int[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)),x]
```

output
```
-(e/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt[f +
g*x])) + e/(Sqrt[-a]*(Sqrt[c]*d + Sqrt[-a]*e)*(e*f - d*g)*Sqrt[d + e*x]*Sq
rt[f + g*x]) + (g*(2*a*e*g - Sqrt[-a]*Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/
(a*(Sqrt[c]*d + Sqrt[-a]*e)*(Sqrt[c]*f + Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f
+ g*x]) + (g*(2*a*e*g + Sqrt[-a]*Sqrt[c]*(e*f + d*g))*Sqrt[d + e*x])/(a*(S
qrt[c]*d - Sqrt[-a]*e)*(Sqrt[c]*f - Sqrt[-a]*g)*(e*f - d*g)^2*Sqrt[f + g*x
]) + (c*ArcTanh[(Sqrt[Sqrt[c]*f - Sqrt[-a]*g]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]
*d - Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*(Sqrt[c]*d - Sqrt[-a]*e)^(3/2)
*(Sqrt[c]*f - Sqrt[-a]*g)^(3/2)) - (c*ArcTanh[(Sqrt[Sqrt[c]*f + Sqrt[-a]*g
]*Sqrt[d + e*x])/(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*Sqrt[f + g*x])]])/(Sqrt[-a]*
(Sqrt[c]*d + Sqrt[-a]*e)^(3/2)*(Sqrt[c]*f + Sqrt[-a]*g)^(3/2))
```

Defintions of rubi rules used

rule 662
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && !IntegerQ[m] && !Inte
gerQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 30655 vs. $2(344) = 688$.

Time = 1.68 (sec) , antiderivative size = 30656, normalized size of antiderivative = 73.69

method	result	size
default	Expression too large to display	30656

input
```
int(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx=\int \frac{1}{\left(a+cx^2\right)\left(d+ex\right)^{\frac{3}{2}}\left(f+gx\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

output
```
Integral(1/((a + c*x**2)*(d + e*x)**(3/2)*(f + g*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\left(a+cx^2\right)}\,dx=\int \frac{1}{(cx^2+a)(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*(g*x + f)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{1}{(f+g\,x)^{3/2}\,\left(c\,x^2+a\right)\,(d+e\,x)^{3/2}}\,dx$$

input
```
int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(3/2)),x)
```

output
```
int(1/((f + g*x)^(3/2)*(a + c*x^2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\,(a+cx^2)}\,dx = \int \frac{}{\sqrt{gx+f}\,\sqrt{ex+d}\,adf+\sqrt{gx+f}\,\sqrt{ex+d}\,adgx+\sqrt{gx+f}\,\sqrt{}}$$

input
```
int(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x)
```

output
```
int(1/(sqrt(f + g*x)*sqrt(d + e*x)*a*d*f + sqrt(f + g*x)*sqrt(d + e*x)*a*d
*g*x + sqrt(f + g*x)*sqrt(d + e*x)*a*e*f*x + sqrt(f + g*x)*sqrt(d + e*x)*a
*e*g*x**2 + sqrt(f + g*x)*sqrt(d + e*x)*c*d*f*x**2 + sqrt(f + g*x)*sqrt(d
+ e*x)*c*d*g*x**3 + sqrt(f + g*x)*sqrt(d + e*x)*c*e*f*x**3 + sqrt(f + g*x)
*sqrt(d + e*x)*c*e*g*x**4),x)
```

3.110 $\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 1063

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx =$$

$$-\frac{4\left(7fg(33cd^3g+ae^2(2ef-33dg))+\frac{e(4cf^2+5ag^2)(15ae^2g^2-c(16e^2f^2-66defg+99d^2g^2))}{cg}\right)\sqrt{f+gx}\sqrt{a+cx^2}}{3465cg^3}$$

$$+\frac{2(f+gx)^{3/2}\left(ae^2g^2(74ef-231dg)-c(64e^3f^3-264de^2f^2g+396d^2efg^2-231d^3g^3)-5eg(15ae^2g^2\right.}{1155cg^4}$$

$$-\frac{2e^2(17ef-33dg)(f+gx)^{3/2}\left(a+cx^2\right)^{3/2}}{99cg^2}+\frac{2e^3(f+gx)^{5/2}\left(a+cx^2\right)^{3/2}}{11cg^2}$$

$$+\frac{4\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(3a^2e^2g^4(26ef+231dg)-c^2f^2(64e^3f^3-264de^2f^2g+396d^2efg^2-\right.}{}$$

$$+\frac{4\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(75a^3e^3g^5+3(-a)^{5/2}\sqrt{c}e^2g^4(26ef+231dg)-9a^2ceg^3(e^2f^2+66defg+55d^2g^2)-\right.}{}$$

output
```
-4/3465*(7*f*g*(33*c*d^3*g+a*e^2*(-33*d*g+2*e*f))+e*(5*a*g^2+4*c*f^2)*(15*
a*e^2*g^2-c*(99*d^2*g^2-66*d*e*f*g+16*e^2*f^2))/c/g)*(g*x+f)^(1/2)*(c*x^2+
a)^(1/2)/c/g^3+2/1155*(g*x+f)^(3/2)*(a*e^2*g^2*(-231*d*g+74*e*f)-c*(-231*d
^3*g^3+396*d^2*e*f*g^2-264*d*e^2*f^2*g+64*e^3*f^3)-5*e*g*(15*a*e^2*g^2-c*(
99*d^2*g^2-66*d*e*f*g+16*e^2*f^2))*x)*(c*x^2+a)^(1/2)/c/g^4-2/99*e^2*(-33*
d*g+17*e*f)*(g*x+f)^(3/2)*(c*x^2+a)^(3/2)/c/g^2+2/11*e^3*(g*x+f)^(5/2)*(c*
x^2+a)^(3/2)/c/g^2+4/3465*(c^(1/2)*f-(-a)^(1/2)*g)*(c^(1/2)*f+(-a)^(1/2)*g
)^(1/2)*(3*a^2*e^2*g^4*(231*d*g+26*e*f)-c^2*f^2*(-231*d^3*g^3+396*d^2*e*f*
g^2-264*d*e^2*f^2*g+64*e^3*f^3)-9*a*c*g^2*(77*d^3*g^3+88*d^2*e*f*g^2-33*d*
e^2*f^2*g+6*e^3*f^3))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(
1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticE(c^(1/4)*(g*x+f
)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*
f-(-a)^(1/2)*g))^(1/2))/c^(7/4)/g^6/(c*x^2+a)^(1/2)+4/3465*(c^(1/2)*f+(-a)
^(1/2)*g)^(1/2)*(75*a^3*e^3*g^5+3*(-a)^(5/2)*c^(1/2)*e^2*g^4*(231*d*g+26*e
*f)-9*a^2*c*e*g^3*(55*d^2*g^2+66*d*e*f*g+e^2*f^2)-a*c^2*f*g*(-924*d^3*g^3+
99*d^2*e*f*g^2-66*d*e^2*f^2*g+16*e^3*f^3)-(-a)^(1/2)*c^(5/2)*f^2*(-231*d^3
*g^3+396*d^2*e*f*g^2-264*d*e^2*f^2*g+64*e^3*f^3)+9*(-a)^(3/2)*c^(3/2)*g^2*
(77*d^3*g^3+88*d^2*e*f*g^2-33*d*e^2*f^2*g+6*e^3*f^3))*(1-c^(1/2)*(g*x+f)/(
c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g)
)^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.70 (sec) , antiderivative size = 1172, normalized size of antiderivative = 1.10

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

output
```
Sqrt[f + g*x]*Sqrt[a + c*x^2]*((2*(-64*c^2*e^3*f^4 + 264*c^2*d*e^2*f^3*g -
 396*c^2*d^2*e*f^2*g^2 - 46*a*c*e^3*f^2*g^2 + 231*c^2*d^3*f*g^3 + 264*a*c*
d*e^2*f*g^3 + 990*a*c*d^2*e*g^4 - 150*a^2*e^3*g^4))/(3465*c^2*g^4) + (2*(4
8*c*e^3*f^3 - 198*c*d*e^2*f^2*g + 297*c*d^2*e*f*g^2 + 32*a*e^3*f*g^2 + 693
*c*d^3*g^3 + 462*a*d*e^2*g^3)*x)/(3465*c*g^3) + (2*e*(-8*c*e^2*f^2 + 33*c*
d*e*f*g + 297*c*d^2*g^2 + 18*a*e^2*g^2)*x^2)/(693*c*g^2) + (2*e^2*(e*f + 3
3*d*g)*x^3)/(99*g) + (2*e^3*x^4)/11) - (4*(f + g*x)^(3/2)*(-(Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]]*(-3*a^2*e^2*g^4*(26*e*f + 231*d*g) + c^2*f^2*(64*e^3*f
^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) + 9*a*c*g^2*(6*e^3*f
^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*((a*g^2)/(f + g*x)^2 +
 c*(-1 + f/(f + g*x))^2)) + (I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(-3*a^2*e
^2*g^4*(26*e*f + 231*d*g) + c^2*f^2*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^
2*e*f*g^2 - 231*d^3*g^3) + 9*a*c*g^2*(6*e^3*f^3 - 33*d*e^2*f^2*g + 88*d^2*
e*f*g^2 + 77*d^3*g^3))*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f +
g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticE
[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I
*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + (Sqrt[a]*g*(Sqrt[c
]*f + I*Sqrt[a]*g)*(-75*a^2*e^3*g^4 + (3*I)*a^(3/2)*Sqrt[c]*e^2*g^3*(e*f +
 231*d*g) + 3*a*c*e*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2) + c^2*f*(64
*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) - (3*I)*Sqr...
```

Rubi [A] (warning: unable to verify)

Time = 3.65 (sec) , antiderivative size = 1311, normalized size of antiderivative = 1.23, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {722, 2185, 27, 2185, 27, 2185, 27, 2185, 25, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}\,dx$$

$$\Big\downarrow \text{722}$$

$$\frac{\int \frac{(d+ex)^3\left(c(ef-3dg)x^2-2(cdf-aeg)x+a(3ef-dg)\right)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{11e}+\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \text{2185}$$

$$\frac{2\int -\frac{-ce^2g^4\left(18ae^2g^2-c\left(29e^2f^2-96degf+81d^2g^2\right)\right)x^4-ceg^3\left(2ae^2g^2(10ef+33dg)-3c\left(11e^3f^3-33de^2gf^2+9d^2eg^2f+27d^3g^3\right)\right)x^3+3cg^2\left(ae^2\left(7e^2f^2-48deg\right.\right.}{}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$\big\downarrow$ 27

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4}-\frac{\int \frac{-ce^2g^4\left(18ae^2g^2-c\left(29e^2f^2-96degf+81d^2g^2\right)\right)x^4-ceg^3\left(2ae^2g^2(10ef+33dg)-3c\left(11e^3f^3-33de^2gf^2+9d^2eg^2f+2\right.\right.}{}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$\big\downarrow$ 2185

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4}-\frac{2\int \frac{c^2e\left(2ae^2g^2(74ef-231dg)-c\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)\right)x^3g^7+c\left(90a^2e^4g^4+2ace^2\left(100e^2f^2-\right.\right.}{}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$\big\downarrow$ 27

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4}-\frac{\int \frac{c^2e\left(2ae^2g^2(74ef-231dg)-c\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)\right)x^3g^7+c\left(90a^2e^4g^4+2ace^2\left(100e^2f^2-2\right.\right.}{}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$\big\downarrow$ 2185

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4}-\frac{2\int \frac{3\left(c^2\left(150a^2e^4g^4-6ace^2\left(2e^2f^2-33degf+165d^2g^2\right)g^2+c^2\left(187e^4f^4-732de^3gf^3+1098d^2e^2g^2f^2-798d^3eg^3f+315\right.\right.\right.}{}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$\big\downarrow$ 27

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4} - \frac{3\int \frac{c^2\left(150a^2e^4g^4-6ace^2\left(2e^2f^2-33degf+165d^2g^2\right)g^2+c^2\left(187e^4f^4-732de^3gf^3+1098d^2e^2g^2f^2-798d^3eg^3f+315d^4 \cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \; 2185$$

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4} - \frac{3\left(2\int -\frac{c^2eg^{10}\left(ag\left(75a^2e^3g^4-9ace\left(e^2f^2+66degf+55d^2g^2\right)g^2-c^2f\left(16e^3f^3-66de^2gf^2+99d^2eg^2f-924d^3g^3\right)\right)-c\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \; 25$$

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4} - \frac{3\left(\frac{2}{3}cg^8\sqrt{a+cx^2}\sqrt{f+gx}\left(150a^2e^4g^4-6ace^2g^2\left(165d^2g^2-33defg+2e^2f^2\right)+c^2\left(315d^4g^4-798d^3efg^3+1098d^2e^2f^2g^2-\cdots\right.\right.\right.}{\cdots}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \; 27$$

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4} - \frac{3\left(\frac{2}{3}cg^8\sqrt{a+cx^2}\sqrt{f+gx}\left(150a^2e^4g^4-6ace^2g^2\left(165d^2g^2-33defg+2e^2f^2\right)+c^2\left(315d^4g^4-798d^3efg^3+1098d^2e^2f^2g^2-\cdots\right.\right.\right.}{\cdots}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$$\Big\downarrow \; 599$$

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4} - \frac{3\left(\frac{4}{3}ceg^6\int -\frac{\left(cf^2+ag^2\right)\left(75a^2e^3g^4-3ace\left(2e^2f^2-33degf+165d^2g^2\right)g^2-c^2f\left(64e^3f^3-264de^2gf^2+396d^2eg^2f-231d\right.\right.}{}\right.}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$\downarrow$ 25

$$\frac{2e^3\sqrt{a+cx^2}(f+gx)^{7/2}(ef-3dg)}{9g^4} - \frac{3\left(\frac{2}{3}cg^8\sqrt{a+cx^2}\sqrt{f+gx}\left(150a^2e^4g^4-6ace^2g^2\left(165d^2g^2-33defg+2e^2f^2\right)+c^2\left(315d^4g^4-798d^3efg^3+1098d^2e^2f^2g^2-\right.\right.\right.}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^4\sqrt{f+gx}}{11e}$$

$\downarrow$ 1511

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^4}{11e} +$$

$$\frac{2e^3(ef-3dg)(f+gx)^{7/2}\sqrt{cx^2+a}}{9g^4} - \frac{\frac{2}{5}ce\left(2ae^2g^2(74ef-231dg)-c\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)\right)(f+gx)^{3/2}\sqrt{cx^2+a}g^5+\frac{3\left(\frac{2}{3}c\left(150a^2e^4g\right.\right.}{}}{}$$

$\downarrow$ 1416

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^4}{11e} +$$

$$\frac{2e^3(ef-3dg)(f+gx)^{7/2}\sqrt{cx^2+a}}{9g^4} - \frac{\frac{2}{5}ce\left(2ae^2g^2(74ef-231dg)-c\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)\right)(f+gx)^{3/2}\sqrt{cx^2+a}g^5+\frac{3\left(\frac{2}{3}c\left(150a^2e^4g\right.\right.}{}}{}$$

$\downarrow$ 1509

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^4}{11e}+$$

$$3\left(\frac{2}{3}c\left(150a^2e^4g\right.\right.$$

$$\frac{\frac{2e^3(ef-3dg)(f+gx)^{7/2}\sqrt{cx^2+a}}{9g^4}-\frac{\frac{2}{5}ce\left(2ae^2g^2(74ef-231dg)-c\left(233e^3f^3-843de^2gf^2+1107d^2eg^2f-567d^3g^3\right)\right)(f+gx)^{3/2}\sqrt{cx^2+a}g^5+\cdots}{\cdots}}{\cdots}$$

input

```
Int[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

output

```
(2*(d + e*x)^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(11*e) + ((2*e^3*(e*f - 3*d*
g)*(f + g*x)^(7/2)*Sqrt[a + c*x^2])/(9*g^4) - ((-2*e^2*g*(18*a*e^2*g^2 - c
*(29*e^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/
7 + ((2*c*e*g^5*(2*a*e^2*g^2*(74*e*f - 231*d*g) - c*(233*e^3*f^3 - 843*d*e
^2*f^2*g + 1107*d^2*e*f*g^2 - 567*d^3*g^3))*(f + g*x)^(3/2)*Sqrt[a + c*x^2
])/5 + (3*((2*c*g^8*(150*a^2*e^4*g^4 - 6*a*c*e^2*g^2*(2*e^2*f^2 - 33*d*e*f
*g + 165*d^2*g^2) + c^2*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*
g^2 - 798*d^3*e*f*g^3 + 315*d^4*g^4))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/3 + (
4*c*e*g^6*(-(Sqrt[c]*Sqrt[c*f^2 + a*g^2]*(3*a^2*e^2*g^4*(26*e*f + 231*d*g)
 - c^2*f^2*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3)
- 9*a*c*g^2*(6*e^3*f^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*(-
((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x
)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]
))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])
*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a
+ (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE
[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)
/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x)
)/g^2 + (c*(f + g*x)^2)/g^2]))) - ((c*f^2 + a*g^2)^(3/4)*(Sqrt[c*f^2 + a*g
^2]*(75*a^2*e^3*g^4 - 3*a*c*e*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 722
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/(e*(2*m + 5))), x] + Simp[1/(e*(2*m + 5))   Int[((d + e*x)^m/(Sqrt[f + g*
x]*Sqrt[a + c*x^2]))*Simp[3*a*e*f - a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f -
3*c*d*g)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegerQ[2*m
] &&  !LtQ[m, -1]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 4.99 (sec) , antiderivative size = 1824, normalized size of antiderivative = 1.72

method	result	size
elliptic	Expression too large to display	1824
risch	Expression too large to display	2562
default	Expression too large to display	6457

input
```
int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/11*e^3*x^4*(c*g
*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/9*(3*c*d*e^2*g+1/11*f*e^3*c)/c/g*x^3*(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/7*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9
/g*f*(3*c*d*e^2*g+1/11*f*e^3*c))/c/g*x^2*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)
+2/5*(3*a*d*e^2*g+3/11*a*e^3*f+c*d^3*g+3*c*d^2*e*f-6/7/g*f*(2/11*a*e^3*g+3
*c*d^2*e*g+3*c*d*e^2*f-8/9/g*f*(3*c*d*e^2*g+1/11*f*e^3*c))-7/9*a/c*(3*c*d*
e^2*g+1/11*f*e^3*c))/c/g*x*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(3*a*d^2*
e*g+3*a*d*e^2*f+c*d^3*f-4/5/g*f*(3*a*d*e^2*g+3/11*a*e^3*f+c*d^3*g+3*c*d^2*
e*f-6/7/g*f*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9/g*f*(3*c*d*e^2*g+1/1
1*f*e^3*c))-7/9*a/c*(3*c*d*e^2*g+1/11*f*e^3*c))-5/7*a/c*(2/11*a*e^3*g+3*c*
d^2*e*g+3*c*d*e^2*f-8/9/g*f*(3*c*d*e^2*g+1/11*f*e^3*c))-2/3*a*f/c/g*(3*c*d
*e^2*g+1/11*f*e^3*c))/c/g*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(a*d^3*f-2/5
*a*f/c/g*(3*a*d*e^2*g+3/11*a*e^3*f+c*d^3*g+3*c*d^2*e*f-6/7/g*f*(2/11*a*e^3
*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9/g*f*(3*c*d*e^2*g+1/11*f*e^3*c))-7/9*a/c*(3*
c*d*e^2*g+1/11*f*e^3*c))-1/3*a/c*(3*a*d^2*e*g+3*a*d*e^2*f+c*d^3*f-4/5/g*f*
(3*a*d*e^2*g+3/11*a*e^3*f+c*d^3*g+3*c*d^2*e*f-6/7/g*f*(2/11*a*e^3*g+3*c*d^
2*e*g+3*c*d*e^2*f-8/9/g*f*(3*c*d*e^2*g+1/11*f*e^3*c))-7/9*a/c*(3*c*d*e^2*g
+1/11*f*e^3*c))-5/7*a/c*(2/11*a*e^3*g+3*c*d^2*e*g+3*c*d*e^2*f-8/9/g*f*(3*c
*d*e^2*g+1/11*f*e^3*c))-2/3*a*f/c/g*(3*c*d*e^2*g+1/11*f*e^3*c)))*(f/g-(-a*
c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 765, normalized size of antiderivative = 0.72

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/10395*(2*(64*c^3*e^3*f^6 - 264*c^3*d*e^2*f^5*g + 6*(66*c^3*d^2*e + 17*a
*c^2*e^3)*f^4*g^2 - 33*(7*c^3*d^3 + 15*a*c^2*d*e^2)*f^3*g^3 + 3*(363*a*c^2
*d^2*e - 17*a^2*c*e^3)*f^2*g^4 - 99*(21*a*c^2*d^3 - 11*a^2*c*d*e^2)*f*g^5
+ 45*(33*a^2*c*d^2*e - 5*a^3*e^3)*g^6)*sqrt(c*g)*weierstrassPInverse(4/3*(
c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x +
f)/g) + 6*(64*c^3*e^3*f^5*g - 264*c^3*d*e^2*f^4*g^2 + 18*(22*c^3*d^2*e + 3
*a*c^2*e^3)*f^3*g^3 - 33*(7*c^3*d^3 + 9*a*c^2*d*e^2)*f^2*g^4 + 6*(132*a*c^
2*d^2*e - 13*a^2*c*e^3)*f*g^5 + 693*(a*c^2*d^3 - a^2*c*d*e^2)*g^6)*sqrt(c*
g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2
)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3
 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) - 3*(315*c^3*e^3*g^6*x^4 - 64*c
^3*e^3*f^4*g^2 + 264*c^3*d*e^2*f^3*g^3 - 2*(198*c^3*d^2*e + 23*a*c^2*e^3)*
f^2*g^4 + 33*(7*c^3*d^3 + 8*a*c^2*d*e^2)*f*g^5 + 30*(33*a*c^2*d^2*e - 5*a^
2*c*e^3)*g^6 + 35*(c^3*e^3*f*g^5 + 33*c^3*d*e^2*g^6)*x^3 - 5*(8*c^3*e^3*f^
2*g^4 - 33*c^3*d*e^2*f*g^5 - 9*(33*c^3*d^2*e + 2*a*c^2*e^3)*g^6)*x^2 + (48
*c^3*e^3*f^3*g^3 - 198*c^3*d*e^2*f^2*g^4 + (297*c^3*d^2*e + 32*a*c^2*e^3)*
f*g^5 + 231*(3*c^3*d^3 + 2*a*c^2*d*e^2)*g^6)*x)*sqrt(c*x^2 + a)*sqrt(g*x +
 f))/(c^3*g^6)
```

Sympy [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}\,dx$$

input
```
integrate((e*x+d)**3*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)**3*sqrt(f + g*x), x)
```

Maxima [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(ex+d)^3\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

Giac [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(ex+d)^3\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,(d+e\,x)^3\,dx$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3,x)
```

output
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3, x)
```

Reduce [F]

$$\int (d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \text{too large to display}$$

input
```
int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x)
```

output
```
( - 1386*sqrt(f + g*x)*sqrt(a + c*x**2)*a**2*d*e**2*g**4 - 456*sqrt(f + g*
x)*sqrt(a + c*x**2)*a**2*e**3*f*g**3 + 1386*sqrt(f + g*x)*sqrt(a + c*x**2)
*a*c*d**3*g**4 + 3564*sqrt(f + g*x)*sqrt(a + c*x**2)*a*c*d**2*e*f*g**3 - 6
6*sqrt(f + g*x)*sqrt(a + c*x**2)*a*c*d*e**2*f**2*g**2 + 924*sqrt(f + g*x)*
sqrt(a + c*x**2)*a*c*d*e**2*f*g**3*x + 16*sqrt(f + g*x)*sqrt(a + c*x**2)*a
*c*e**3*f**3*g + 64*sqrt(f + g*x)*sqrt(a + c*x**2)*a*c*e**3*f**2*g**2*x +
180*sqrt(f + g*x)*sqrt(a + c*x**2)*a*c*e**3*f*g**3*x**2 + 1386*sqrt(f + g*
x)*sqrt(a + c*x**2)*c**2*d**3*f*g**3*x + 594*sqrt(f + g*x)*sqrt(a + c*x**2
)*c**2*d**2*e*f**2*g**2*x + 2970*sqrt(f + g*x)*sqrt(a + c*x**2)*c**2*d**2*
e*f*g**3*x**2 - 396*sqrt(f + g*x)*sqrt(a + c*x**2)*c**2*d*e**2*f**3*g*x +
330*sqrt(f + g*x)*sqrt(a + c*x**2)*c**2*d*e**2*f**2*g**2*x**2 + 2310*sqrt(
f + g*x)*sqrt(a + c*x**2)*c**2*d*e**2*f*g**3*x**3 + 96*sqrt(f + g*x)*sqrt(
a + c*x**2)*c**2*e**3*f**4*x - 80*sqrt(f + g*x)*sqrt(a + c*x**2)*c**2*e**3
*f**3*g*x**2 + 70*sqrt(f + g*x)*sqrt(a + c*x**2)*c**2*e**3*f**2*g**2*x**3
+ 630*sqrt(f + g*x)*sqrt(a + c*x**2)*c**2*e**3*f*g**3*x**4 + 2079*int((sqr
t(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a
**2*c*d*e**2*g**5 + 234*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a
*g*x + c*f*x**2 + c*g*x**3),x)*a**2*c*e**3*f*g**4 - 2079*int((sqrt(f + g*x
)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*c**2*d**
3*g**5 - 2376*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + ...
```

3.111 $\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 809

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$

$$= -\frac{4(7(3cd^2-ae^2)fg^2+2e(ef-3dg)(4cf^2+5ag^2))\sqrt{f+gx}\sqrt{a+cx^2}}{315cg^3}$$

$$+\frac{2(f+gx)^{3/2}(7(3cd^2-ae^2)g^2+8cef(ef-3dg)-10ceg(ef-3dg)x)\sqrt{a+cx^2}}{105cg^3}$$

$$+\frac{2e^2(f+gx)^{3/2}(a+cx^2)^{3/2}}{9cg}$$

$$-\frac{4\left(\sqrt{-a}-\frac{\sqrt{c}f}{g}\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}(21a^2e^2g^4+3acg^2(3e^2f^2-16defg-21d^2g^2)+c^2f^2(8e^2f^2-24defg}{315c^{7/4}g^4\sqrt{a+cx^2}}$$

$$+\frac{4\sqrt{\sqrt{c}f+\sqrt{-a}g}(21(-a)^{5/2}e^2g^4-6a^2\sqrt{c}eg^3(3ef+5dg)-3(-a)^{3/2}cg^2(3e^2f^2-16defg-21d^2g^2)+}{}$$

output
```
-4/315*(7*(-a*e^2+3*c*d^2)*f*g^2+2*e*(-3*d*g+e*f)*(5*a*g^2+4*c*f^2))*(g*x+
f)^(1/2)*(c*x^2+a)^(1/2)/c/g^3+2/105*(g*x+f)^(3/2)*(7*(-a*e^2+3*c*d^2)*g^2
+8*c*e*f*(-3*d*g+e*f)-10*c*e*g*(-3*d*g+e*f)*x)*(c*x^2+a)^(1/2)/c/g^3+2/9*e
^2*(g*x+f)^(3/2)*(c*x^2+a)^(3/2)/c/g-4/315*((-a)^(1/2)-c^(1/2)*f/g)*(c^(1/
2)*f+(-a)^(1/2)*g)^(1/2)*(21*a^2*e^2*g^4+3*a*c*g^2*(-21*d^2*g^2-16*d*e*f*g
+3*e^2*f^2)+c^2*f^2*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2))*(1-c^(1/2)*(g*x+f)/
(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g
))^(1/2)*EllipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((
c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(7/4)/g^4/(c*x^
2+a)^(1/2)+4/315*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(21*(-a)^(5/2)*e^2*g^4-6*a
^2*c^(1/2)*e*g^3*(5*d*g+3*e*f)-3*(-a)^(3/2)*c*g^2*(-21*d^2*g^2-16*d*e*f*g+
3*e^2*f^2)+(-a)^(1/2)*c^2*f^2*(21*d^2*g^2-24*d*e*f*g+8*e^2*f^2)+2*a*c^(3/2
)*f*g*(42*d^2*g^2-3*d*e*f*g+e^2*f^2))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(
1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*Elliptic
F(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1
/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(7/4)/g^4/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.16 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.00

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)\left(2aeg^2(4ef+30dg+7egx)+c\left(21d^2g^2(f+3gx)+6deg\left(-4f^2+3fgx+15g^2x^2\right)+e^2\left(8f^3-6f^2gx+5fg^2x^2+35g^3x^3\right)\right)\right)}{cg^3} - \right.}{}$$

input
```
Integrate[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[f + g*x]*((2*(a + c*x^2)*(2*a*e*g^2*(4*e*f + 30*d*g + 7*e*g*x) + c*(
21*d^2*g^2*(f + 3*g*x) + 6*d*e*g*(-4*f^2 + 3*f*g*x + 15*g^2*x^2) + e^2*(8*
f^3 - 6*f^2*g*x + 5*f*g^2*x^2 + 35*g^3*x^3))))/(c*g^3) - (4*(g^2*Sqrt[-f -
 (I*Sqrt[a]*g)/Sqrt[c]]*(21*a^2*e^2*g^4 + c^2*f^2*(8*e^2*f^2 - 24*d*e*f*g
+ 21*d^2*g^2) - 3*a*c*g^2*(-3*e^2*f^2 + 16*d*e*f*g + 21*d^2*g^2))*(a + c*x
^2) - I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(21*a^2*e^2*g^4 + c^2*f^2*(8*e^2
*f^2 - 24*d*e*f*g + 21*d^2*g^2) - 3*a*c*g^2*(-3*e^2*f^2 + 16*d*e*f*g + 21*
d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]
*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[
c]*f + I*Sqrt[a]*g)] + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((21*I)
*a^(3/2)*e^2*g^3 - 3*a*Sqrt[c]*e*g^2*(e*f - 10*d*g) + c^(3/2)*f*(-8*e^2*f^
2 + 24*d*e*f*g - 21*d^2*g^2) - (3*I)*Sqrt[a]*c*g*(-2*e^2*f^2 + 6*d*e*f*g +
 21*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqr
t[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqr
t[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(S
qrt[c]*f + I*Sqrt[a]*g)]))/(c^2*g^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f +
g*x))))/(315*Sqrt[a + c*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 2.61 (sec) , antiderivative size = 1041, normalized size of antiderivative = 1.29, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {722, 2185, 27, 2185, 27, 2185, 27, 599, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}\,dx$$

$$\Big\downarrow \text{ 722}$$

$$\frac{\int \frac{(d+ex)^2\left(c(ef-3dg)x^2-2(cdf-aeg)x+a(3ef-dg)\right)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{9e}+\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\frac{2\int -\frac{-2ceg^3\left(7ae^2g^2-c\left(8e^2f^2-24degf+21d^2g^2\right)\right)x^3-cg^2\left(4ae^2g^2(4ef+9dg)-c\left(11e^3f^3-33de^2gf^2+21d^2eg^2f+21d^3g^3\right)\right)x^2+2cfg\left(ae^2(5ef-36dg)g^2+c\left(e^3\right.\right.}{2\sqrt{f+gx}\sqrt{cx^2+a}}}{7cg^4}}{9e}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(ef-3dg)}{7g^3}-\frac{\int \frac{-2ceg^3\left(7ae^2g^2-c\left(8e^2f^2-24degf+21d^2g^2\right)\right)x^3-cg^2\left(4ae^2g^2(4ef+9dg)-c\left(11e^3f^3-33de^2gf^2+21d^2eg^2f+21d^3g^3\right.\right.}{\sqrt{f+gx}\sqrt{cx^2}}}{7cg^4}}{9e}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(ef-3dg)}{7g^3}-\frac{2\int \frac{3c^2\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3\right)\right)x^2g^5+ac\left(42ae^3fg^2-c\left(23e^3f^3-69de^2gf^2+23\right.\right.}{2\sqrt{f}}}{}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(ef-3dg)}{7g^3}-\frac{\int \frac{3c^2\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3\right)\right)x^2g^5+ac\left(42ae^3fg^2-c\left(23e^3f^3-69de^2gf^2+231\right.\right.}{\sqrt{f+}}}{}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(ef-3dg)}{7g^3}-\frac{\frac{2\int \frac{3c^2eg^6\left(2ag\left(3aeg^2(3ef+5dg)-cf\left(e^2f^2-3degf+42d^2g^2\right)\right)+\left(21a^2e^2g^4+3ac\left(3e^2f^2-16degf-21d^2g^2\right)g^2+c^2f^2\left(8\right.\right.\right.}{\sqrt{f+gx}\sqrt{cx^2+a}}}{3cg^2}}{5}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$$\Big\downarrow \; 27$$

$$\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(ef-3dg)}{7g^3}-\frac{\frac{2ceg^4\int\frac{2ag\left(3aeg^2(3ef+5dg)-cf\left(e^2f^2-3degf+42d^2g^2\right)\right)+\left(21a^2e^2g^4+3ac\left(3e^2f^2-16degf-21d^2g^2\right)g^2+c^2f^2\left(8e^2f\right.\right.}{\sqrt{f+gx}\sqrt{cx^2+a}}}{5cg^3}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$\downarrow$ 599

$$\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(ef-3dg)}{7g^3}-\frac{\frac{2cg^4\sqrt{a+cx^2}\sqrt{f+gx}\left(6ae^2g^2(ef-10dg)-c\left(-35d^3g^3+63d^2efg^2-57de^2f^2g+19e^3f^3\right)\right)-4ceg^2\int\frac{\left(cf^2+ag^2\right)\left(3ae(ef-\right.}{}}{}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$\downarrow$ 1511

$$\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(ef-3dg)}{7g^3}-\frac{\frac{2cg^4\sqrt{a+cx^2}\sqrt{f+gx}\left(6ae^2g^2(ef-10dg)-c\left(-35d^3g^3+63d^2efg^2-57de^2f^2g+19e^3f^3\right)\right)-4ceg^2\left(\frac{\sqrt{ag^2+cf^2}\left(21a^2e^2g^4\right.}{}\right.}{}}{}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}$$

$\downarrow$ 1416

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^3}{9e}+$$

$$\frac{2e^2(ef-3dg)(f+gx)^{5/2}\sqrt{cx^2+a}}{7g^3}-\frac{\frac{2cg^4\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3\right)\right)\sqrt{f+gx}\sqrt{cx^2+a}-4ceg^2\left(\frac{\sqrt{cf^2+ag^2}\left(21a^2e^2g^4+\right.}{}\right.}{}}{}$$

$\downarrow$ 1509

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^3}{9e}+$$

$$\frac{\frac{2e^2(ef-3dg)(f+gx)^{5/2}\sqrt{cx^2+a}}{7g^3}-\frac{2cg^4\left(6ae^2g^2(ef-10dg)-c\left(19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3\right)\right)\sqrt{f+gx}\sqrt{cx^2+a}-4ceg^2\left(\frac{\sqrt{cf^2+ag^2}\left(21a^2e^2g^4+\right.}{}\right.}{}}{}$$

input
```
Int[(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

output
```
(2*(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(9*e) + ((2*e^2*(e*f - 3*d*g
)*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(7*g^3) - ((-4*e*g*(7*a*e^2*g^2 - c*(8*
e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/5 + (
2*c*g^4*(6*a*e^2*g^2*(e*f - 10*d*g) - c*(19*e^3*f^3 - 57*d*e^2*f^2*g + 63*
d^2*e*f*g^2 - 35*d^3*g^3))*Sqrt[f + g*x]*Sqrt[a + c*x^2] - 4*c*e*g^2*((Sqr
t[c*f^2 + a*g^2]*(21*a^2*e^2*g^4 + 3*a*c*g^2*(3*e^2*f^2 - 16*d*e*f*g - 21*
d^2*g^2) + c^2*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*(-((Sqrt[f + g*x
]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a
 + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2
+ a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*
f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)
*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(
1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 +
a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f
+ g*x)^2)/g^2]))))/Sqrt[c] - ((c*f^2 + a*g^2)^(3/4)*(21*a^2*e^2*g^4 + 3*a*c
*g^2*(3*e^2*f^2 - 16*d*e*f*g - 21*d^2*g^2) + c^2*f^2*(8*e^2*f^2 - 24*d*e*f
*g + 21*d^2*g^2) - Sqrt[c]*Sqrt[c*f^2 + a*g^2]*(3*a*e*g^2*(e*f - 10*d*g) +
 c*f*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)))*(1 + (Sqrt[c]*(f + g*x))/Sqrt
[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g
*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 722
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/(e*(2*m + 5))), x] + Simp[1/(e*(2*m + 5))   Int[((d + e*x)^m/(Sqrt[f + g*
x]*Sqrt[a + c*x^2]))*Simp[3*a*e*f - a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f -
3*c*d*g)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegerQ[2*m
] &&  !LtQ[m, -1]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 1142, normalized size of antiderivative = 1.41

method	result	size
elliptic	Expression too large to display	1142
risch	Expression too large to display	1677
default	Expression too large to display	4351

input
```
int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/9*e^2*x^3*(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/7*(2*c*d*e*g+1/9*f*c*e^2)/c/g*x^2*(c*g*x^3+
c*f*x^2+a*g*x+a*f)^(1/2)+2/5*(2/9*a*e^2*g+c*d^2*g+2*c*d*e*f-6/7*f/g*(2*c*d
*e*g+1/9*f*c*e^2))/c/g*x*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(2*a*d*e*g+
1/3*a*e^2*f+c*d^2*f-4/5*f/g*(2/9*a*e^2*g+c*d^2*g+2*c*d*e*f-6/7*f/g*(2*c*d*
e*g+1/9*f*c*e^2))-5/7*a/c*(2*c*d*e*g+1/9*f*c*e^2))/c/g*(c*g*x^3+c*f*x^2+a*
g*x+a*f)^(1/2)+2*(a*d^2*f-2/5*a*f/c/g*(2/9*a*e^2*g+c*d^2*g+2*c*d*e*f-6/7*f
/g*(2*c*d*e*g+1/9*f*c*e^2))-1/3*a/c*(2*a*d*e*g+1/3*a*e^2*f+c*d^2*f-4/5*f/g
*(2/9*a*e^2*g+c*d^2*g+2*c*d*e*f-6/7*f/g*(2*c*d*e*g+1/9*f*c*e^2))-5/7*a/c*(
2*c*d*e*g+1/9*f*c*e^2)))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c
))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)
/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*Ellipti
cF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c
)^(1/2)/c))^(1/2))+2*(a*d^2*g+2*a*d*e*f-4/7*a*f/c/g*(2*c*d*e*g+1/9*f*c*e^2
)-3/5*a/c*(2/9*a*e^2*g+c*d^2*g+2*c*d*e*f-6/7*f/g*(2*c*d*e*g+1/9*f*c*e^2))-
2/3*f/g*(2*a*d*e*g+1/3*a*e^2*f+c*d^2*f-4/5*f/g*(2/9*a*e^2*g+c*d^2*g+2*c*d*
e*f-6/7*f/g*(2*c*d*e*g+1/9*f*c*e^2))-5/7*a/c*(2*c*d*e*g+1/9*f*c*e^2)))*(f/
g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)
/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(
1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 510, normalized size of antiderivative = 0.63

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \frac{2\left(2\left(8c^2e^2f^5-24c^2def^4g-66acdef^2g^3-90a^2deg^5+3\left(7c^2d^2+5ace^2\right)f^3g^2+3\left(63acd^2-11a^2e^2\right)\ldots\right.\right.}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/945*(2*(8*c^2*e^2*f^5 - 24*c^2*d*e*f^4*g - 66*a*c*d*e*f^2*g^3 - 90*a^2*d
*e*g^5 + 3*(7*c^2*d^2 + 5*a*c*e^2)*f^3*g^2 + 3*(63*a*c*d^2 - 11*a^2*e^2)*f
*g^4)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(
c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 6*(8*c^2*e^2*f^4*g - 24*c
^2*d*e*f^3*g^2 - 48*a*c*d*e*f*g^4 + 3*(7*c^2*d^2 + 3*a*c*e^2)*f^2*g^3 - 21
*(3*a*c*d^2 - a^2*e^2)*g^5)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2
)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f
^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/
g)) + 3*(35*c^2*e^2*g^5*x^3 + 8*c^2*e^2*f^3*g^2 - 24*c^2*d*e*f^2*g^3 + 60*
a*c*d*e*g^5 + (21*c^2*d^2 + 8*a*c*e^2)*f*g^4 + 5*(c^2*e^2*f*g^4 + 18*c^2*d
*e*g^5)*x^2 - (6*c^2*e^2*f^2*g^3 - 18*c^2*d*e*f*g^4 - 7*(9*c^2*d^2 + 2*a*c
*e^2)*g^5)*x)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c^2*g^5)
```

Sympy [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}\,dx$$

input
```
integrate((e*x+d)**2*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)**2*sqrt(f + g*x), x)
```

Maxima [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{cx^2+a}(ex+d)^2\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

Giac [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{cx^2+a}(ex+d)^2\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx=\int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2\,dx$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2,x)
```

output
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2, x)
```

Reduce [F]

$$\int (d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x)
```

output

```
( - 42*sqrt(f + g*x)*sqrt(a + c*x**2)*a**2*e**2*g**3 + 126*sqrt(f + g*x)*s
qrt(a + c*x**2)*a*c*d**2*g**3 + 216*sqrt(f + g*x)*sqrt(a + c*x**2)*a*c*d*e
*f*g**2 - 2*sqrt(f + g*x)*sqrt(a + c*x**2)*a*c*e**2*f**2*g + 28*sqrt(f + g
*x)*sqrt(a + c*x**2)*a*c*e**2*f*g**2*x + 126*sqrt(f + g*x)*sqrt(a + c*x**2
)*c**2*d**2*f*g**2*x + 36*sqrt(f + g*x)*sqrt(a + c*x**2)*c**2*d*e*f**2*g*x
 + 180*sqrt(f + g*x)*sqrt(a + c*x**2)*c**2*d*e*f*g**2*x**2 - 12*sqrt(f + g
*x)*sqrt(a + c*x**2)*c**2*e**2*f**3*x + 10*sqrt(f + g*x)*sqrt(a + c*x**2)*
c**2*e**2*f**2*g*x**2 + 70*sqrt(f + g*x)*sqrt(a + c*x**2)*c**2*e**2*f*g**2
*x**3 + 63*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x*
*2 + c*g*x**3),x)*a**2*c*e**2*g**4 - 189*int((sqrt(f + g*x)*sqrt(a + c*x**
2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*c**2*d**2*g**4 - 144*int
((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3)
,x)*a*c**2*d*e*f*g**3 + 27*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f
+ a*g*x + c*f*x**2 + c*g*x**3),x)*a*c**2*e**2*f**2*g**2 + 63*int((sqrt(f +
 g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c**3*d
**2*f**2*g**2 - 72*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x
+ c*f*x**2 + c*g*x**3),x)*c**3*d*e*f**3*g + 24*int((sqrt(f + g*x)*sqrt(a +
 c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c**3*e**2*f**4 + 21*
int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x
)*a**3*e**2*g**4 - 63*int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x...
```

3.112 $\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 600

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \frac{4}{105}\left(\frac{5ae}{c}+\frac{f(4ef-7dg)}{g^2}\right)\sqrt{f+gx}\sqrt{a+cx^2} - \frac{2(f+gx)^{3/2}(4ef-7dg-5egx)\sqrt{a+cx^2}}{35g^2} + \frac{4\left(\sqrt{-a}-\frac{\sqrt{c}f}{g}\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(cf^2(4ef-7dg)+ag^2(8ef+21dg))\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}}{105c^{3/4}g^3\sqrt{a+cx^2}} - \frac{4\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(5a^2eg^3+acfg(ef-28dg)+\sqrt{-a}c^{3/2}f^2(4ef-7dg)+\sqrt{-a}a\sqrt{c}g^2(8ef+21dg)\right)}{105c^{5/4}g^3\sqrt{a+cx^2}}$$

output
```
4/105*(5*a*e/c+f*(-7*d*g+4*e*f)/g^2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)-2/35*(g
*x+f)^(3/2)*(-5*e*g*x-7*d*g+4*e*f)*(c*x^2+a)^(1/2)/g^2+4/105*((-a)^(1/2)-c
^(1/2)*f/g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(c*f^2*(-7*d*g+4*e*f)+a*g^2*(21
*d*g+8*e*f))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)
*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticE(c^(1/4)*(g*x+f)^(1/2)/(
c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1
/2)*g))^(1/2))/c^(3/4)/g^3/(c*x^2+a)^(1/2)-4/105*(c^(1/2)*f+(-a)^(1/2)*g)^
(1/2)*(5*a^2*e*g^3+a*c*f*g*(-28*d*g+e*f)+(-a)^(1/2)*c^(3/2)*f^2*(-7*d*g+4*
e*f)+(-a)^(1/2)*a*c^(1/2)*g^2*(21*d*g+8*e*f))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*
f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*
EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f
+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(5/4)/g^3/(c*x^2+a)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.60 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.02

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)(10aeg^2+7cdg(f+3gx)+ce(-4f^2+3fgx+15g^2x^2))}{cg^2}+\frac{4\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(cf^2(4ef-7dg)+ag^2(8ef+21dg))(a+cx^2)\right.}{}\right.}{}$$

input
```
Integrate[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[f + g*x]*((2*(a + c*x^2)*(10*a*e*g^2 + 7*c*d*g*(f + 3*g*x) + c*e*(-4
*f^2 + 3*f*g*x + 15*g^2*x^2)))/(c*g^2) + (4*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]]*(c*f^2*(4*e*f - 7*d*g) + a*g^2*(8*e*f + 21*d*g))*(a + c*x^2) + I*S
qrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(c*f^2*(-4*e*f + 7*d*g) - a*g^2*(8*e*f +
21*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*
g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)] + Sqrt[a]*g*(I*Sqrt[c]*f - Sqrt[a]*g)*((5*I)*a*e*g^2 +
 I*c*f*(4*e*f - 7*d*g) + 3*Sqrt[a]*Sqrt[c]*g*(e*f + 7*d*g))*Sqrt[(g*((I*Sq
rt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f +
g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]
]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(
c*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(105*Sqrt[a + c*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.06 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.31, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {687, 27, 682, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}(d+ex)\sqrt{f+gx}\,dx$$

$$\Big\downarrow \text{687}$$

$$\frac{2\int \frac{(7cdf-aeg+c(ef+7dg)x)\sqrt{cx^2+a}}{2\sqrt{f+gx}}dx}{7c}+\frac{2e\left(a+cx^2\right)^{3/2}\sqrt{f+gx}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(7cdf-aeg+c(ef+7dg)x)\sqrt{cx^2+a}}{\sqrt{f+gx}}dx}{7c}+\frac{2e\left(a+cx^2\right)^{3/2}\sqrt{f+gx}}{7c}$$

$$\Big\downarrow \text{682}$$

$$\frac{\frac{4\int -\frac{c\left(ag\left(5aeg^2+cf(ef-28dg)\right)-c\left(c(4ef-7dg)f^2+ag^2(8ef+21dg)\right)x\right)}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{15cg^2}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(5aeg^2-3cgx(7dg+ef)+cf(4ef-7dg))}{15g^2}}{7c}+\frac{2e\left(a+cx^2\right)^{3/2}\sqrt{f+gx}}{7c}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{2\int \frac{ag\left(5aeg^2+cf(ef-28dg)\right)-c\left(c(4ef-7dg)f^2+ag^2(8ef+21dg)\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{15g^2}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(5aeg^2-3cgx(7dg+ef)+cf(4ef-7dg))}{15g^2}}{7c}+\frac{2e\left(a+cx^2\right)^{3/2}\sqrt{f+gx}}{7c}$$

$$\Big\downarrow \; 599$$

$$\frac{\frac{4\int -\frac{\left(cf^2+ag^2\right)\left(5aeg^2+cf(4ef-7dg)\right)-c\left(c(4ef-7dg)f^2+ag^2(8ef+21dg)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{15g^4}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(5aeg^2-3cgx(7dg+ef)+cf(4ef-7d}{15g^2}}{7c}+\frac{2e\left(a+cx^2\right)^{3/2}\sqrt{f+gx}}{7c}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{4\int \frac{\left(cf^2+ag^2\right)\left(5aeg^2+cf(4ef-7dg)\right)-c\left(c(4ef-7dg)f^2+ag^2(8ef+21dg)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{15g^4}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(5aeg^2-3cgx(7dg+ef)+cf(4ef-7c}{15g^2}}{7c}+\frac{2e\left(a+cx^2\right)^{3/2}\sqrt{f+gx}}{7c}$$

$$\Big\downarrow \; 1511$$

$$\frac{\frac{4\left(-\sqrt{ag^2+cf^2}\left(\sqrt{ag^2+cf^2}(5aeg^2+cf(4ef-7dg))-\sqrt{c}\left(ag^2(21dg+8ef)+cf^2(4ef-7dg)\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\sqrt{c}\sqrt{a}\right.}{15g^4}}{}+\frac{2e\left(a+cx^2\right)^{3/2}\sqrt{f+gx}}{7c}$$

$$\Big\downarrow \; 1416$$

$$\frac{4\left(-\sqrt{c}\sqrt{ag^2+cf^2}\left(ag^2(21dg+8ef)+cf^2(4ef-7dg)\right)\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(\ldots}{\ldots}}{\left(a+\frac{cf^2}{g^2}\right)\left(\ldots\right.}}}{\ldots}\right.}{1\ldots}$$

$$\frac{2e\left(a+cx^2\right)^{3/2}\sqrt{f+gx}}{7c}$$

$\downarrow$ 1509

$$\frac{4\left(-\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(\sqrt{ag^2+cf^2}\left(5aeg^2+cf(4ef-7dg)\right)-\sqrt{c}\left(ag^2(21dg+8ef)+cf^2(4ef-7dg)\right)\right)\text{EllipticF}\ldots}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.}{\ldots}$$

$$\frac{2e\left(a+cx^2\right)^{3/2}\sqrt{f+gx}}{7c}$$

input
```
Int[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

output
```
(2*e*Sqrt[f + g*x]*(a + c*x^2)^(3/2))/(7*c) + ((-2*Sqrt[f + g*x]*(5*a*e*g^
2 + c*f*(4*e*f - 7*d*g) - 3*c*g*(e*f + 7*d*g)*x)*Sqrt[a + c*x^2])/(15*g^2)
 + (4*(-(Sqrt[c]*Sqrt[c*f^2 + a*g^2]*(c*f^2*(4*e*f - 7*d*g) + a*g^2*(8*e*f
 + 21*d*g))*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2
 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[
c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c
*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x
)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])
^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1
 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2
*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2]))) - ((c*f^2 + a*g^2)^(3/4)*(Sq
rt[c*f^2 + a*g^2]*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g)) - Sqrt[c]*(c*f^2*(4*e*
f - 7*d*g) + a*g^2*(8*e*f + 21*d*g)))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2
+ a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/
g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*
EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (S
qrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*
f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(15*g^4))/(7*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.32

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2e\,x^2\sqrt{cg\,x^3+cf\,x^2+agx+af}}{7}+\frac{2\left(cdg+\frac{1}{7}fce\right)x\sqrt{cg\,x^3+cf\,x^2+agx+af}}{5cg}+\frac{2\left(\frac{2aeg}{7}+dfc-\frac{4f\left(cdg+\frac{1}{7}fce\right)}{5g}\right)\sqrt{cg\,x^3+cf\,x}}{3cg}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/7*e*x^2*(c*g*x^
3+c*f*x^2+a*g*x+a*f)^(1/2)+2/5*(c*d*g+1/7*f*c*e)/c/g*x*(c*g*x^3+c*f*x^2+a*
g*x+a*f)^(1/2)+2/3*(2/7*a*e*g+d*f*c-4/5*f/g*(c*d*g+1/7*f*c*e))/c/g*(c*g*x^
3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(a*d*f-2/5*a/c*f/g*(c*d*g+1/7*f*c*e)-1/3*a/c*
(2/7*a*e*g+d*f*c-4/5*f/g*(c*d*g+1/7*f*c*e)))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)
/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1
/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*
x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^
(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(a*d*g+3/7*a*e*f-3/5*a/c*(c*d*g+1
/7*f*c*e)-2/3*f/g*(2/7*a*e*g+d*f*c-4/5*f/g*(c*d*g+1/7*f*c*e)))*(f/g-(-a*c)
^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(
-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(
f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(
1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g
+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.57

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = -\frac{2\left(2\left(4\,c^2ef^4-7\,c^2df^3g+11\,acef^2g^2-63\,acdfg^3+15\,a^2eg^4\right)\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\right.}{}$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*(2*(4*c^2*e*f^4 - 7*c^2*d*f^3*g + 11*a*c*e*f^2*g^2 - 63*a*c*d*f*g^3
 + 15*a^2*e*g^4)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^
2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 6*(4*c^2*e*f^3
*g - 7*c^2*d*f^2*g^2 + 8*a*c*e*f*g^3 + 21*a*c*d*g^4)*sqrt(c*g)*weierstrass
Zeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), wei
erstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/
(c*g^3), 1/3*(3*g*x + f)/g)) - 3*(15*c^2*e*g^4*x^2 - 4*c^2*e*f^2*g^2 + 7*c
^2*d*f*g^3 + 10*a*c*e*g^4 + 3*(c^2*e*f*g^3 + 7*c^2*d*g^4)*x)*sqrt(c*x^2 +
a)*sqrt(g*x + f))/(c^2*g^4)
```

Sympy [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{a+cx^2}(d+ex)\,\sqrt{f+gx}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)*sqrt(f + g*x), x)
```

Maxima [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(ex+d)\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f), x)
```

Giac [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(ex+d)\sqrt{gx+f}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,(d+e\,x)\,dx$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x),x)
```

output
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x), x)
```

Reduce [F]

$$\int (d+ex)\sqrt{f+gx}\sqrt{a+cx^2}\,dx$$
$$= \frac{14\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,ad\,g^2+12\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,aefg+14\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,cdfgx+2\sqrt{gx+f}\,\sqrt{c}}{}$$

input
```
int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2),x)
```

output
```
(14*sqrt(f + g*x)*sqrt(a + c*x**2)*a*d*g**2 + 12*sqrt(f + g*x)*sqrt(a + c*
x**2)*a*e*f*g + 14*sqrt(f + g*x)*sqrt(a + c*x**2)*c*d*f*g*x + 2*sqrt(f + g
*x)*sqrt(a + c*x**2)*c*e*f**2*x + 10*sqrt(f + g*x)*sqrt(a + c*x**2)*c*e*f*
g*x**2 - 21*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x
**2 + c*g*x**3),x)*a*c*d*g**3 - 8*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2
)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*c*e*f*g**2 + 7*int((sqrt(f + g*
x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c**2*d*f*
*2*g - 4*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2
 + c*g*x**3),x)*c**2*e*f**3 - 7*int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f
+ a*g*x + c*f*x**2 + c*g*x**3),x)*a**2*d*g**3 - 6*int((sqrt(f + g*x)*sqrt(
a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a**2*e*f*g**2 + 21*int
((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a
*c*d*f**2*g - 2*int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x*
*2 + c*g*x**3),x)*a*c*e*f**3)/(35*c*f*g)
```

3.113 $\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 511

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx = -\frac{4f\sqrt{f+gx}\sqrt{a+cx^2}}{15g}+\frac{2(f+gx)^{3/2}\sqrt{a+cx^2}}{5g}$$
$$+\frac{4\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(cf^2-3ag^2)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right.\right.}{15c^{3/4}g^3\sqrt{a+cx^2}}$$
$$+\frac{4\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(\sqrt{-a}cf^2+4a\sqrt{c}fg+3(-a)^{3/2}g^2\right)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticF}\left(\arcsin\right.}{15c^{3/4}g^2\sqrt{a+cx^2}}$$

output
```
-4/15*f*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+2/5*(g*x+f)^(3/2)*(c*x^2+a)^(1/2)/
g+4/15*(c^(1/2)*f-(-a)^(1/2)*g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(-3*a*g^2+c
*f^2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f
)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)
*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))
^(1/2))/c^(3/4)/g^3/(c*x^2+a)^(1/2)+4/15*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*((
-a)^(1/2)*c*f^2+4*a*c^(1/2)*f*g+3*(-a)^(3/2)*g^2)*(1-c^(1/2)*(g*x+f)/(c^(1
/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1
/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/
2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(3/4)/g^2/(c*x^2+a)^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.38 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.02

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(f+3gx)(a+cx^2)}{g} - \frac{4\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\,(cf^2-3ag^2)\,(a+cx^2)+\sqrt{c}(-ic^{3/2}f^3+\sqrt{a}cf^2g+3ia\sqrt{c}fg^2-3a^{3/2}g^3)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[f + g*x]*((2*(f + 3*g*x)*(a + c*x^2))/g - (4*(g^2*Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]*(c*f^2 - 3*a*g^2)*(a + c*x^2) + Sqrt[c]*((-I)*c^(3/2)*f^3 +
 Sqrt[a]*c*f^2*g + (3*I)*a*Sqrt[c]*f*g^2 - 3*a^(3/2)*g^3)*Sqrt[(g*((I*Sqrt
[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*
x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Sqr
t[a]*Sqrt[c]*g*(c*f^2 + (4*I)*Sqrt[a]*Sqrt[c]*f*g - 3*a*g^2)*Sqrt[(g*((I*S
qrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f +
 g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/
(c*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(15*Sqrt[a + c*x^2])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {493, 687, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}\sqrt{f+gx}\,dx$$

$$\Big\downarrow \text{493}$$

$$\frac{2\int \frac{(ag-cfx)\sqrt{f+gx}}{\sqrt{cx^2+a}}dx}{5g}+\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

$$\Big\downarrow \text{687}$$

$$\frac{2\left(\frac{2\int \frac{c\left(4afg-\left(cf^2-3ag^2\right)x\right)}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3c}-\frac{2}{3}f\sqrt{a+cx^2}\sqrt{f+gx}\right)}{5g}+\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{1}{3}\int \frac{4afg-(cf^2-3ag^2)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2}{3}f\sqrt{a+cx^2}\sqrt{f+gx}\right)}{5g}+\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

$$\Big\downarrow \text{599}$$

$$\frac{2\left(-\frac{2\int -\frac{f\left(cf^2+ag^2\right)-\left(cf^2-3ag^2\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^2}-\frac{2}{3}f\sqrt{a+cx^2}\sqrt{f+gx}\right)}{5g}+\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(\frac{2\int \frac{f\left(cf^2+ag^2\right)-\left(cf^2-3ag^2\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^2}-\frac{2}{3}f\sqrt{a+cx^2}\sqrt{f+gx}\right)}{5g}+\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

$$\Big\downarrow \text{1511}$$

$$\frac{2\left(-\frac{2\left(\frac{\sqrt{ag^2+cf^2}\left(-\sqrt{c}f\sqrt{ag^2+cf^2}-3ag^2+cf^2\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\left(cf^2-3ag^2\right)\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)}{g^2}}}}{\sqrt{c}}\right.}{3g^2}\right.}{5g}$$

$$\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

$$\Big\downarrow \; 1416$$

$$\frac{2\left(-\frac{2\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(-\sqrt{c}f\sqrt{ag^2+cf^2}-3ag^2+cf^2\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(-\right.\right.}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.}{3g^2}\right.}{5g}$$

$$\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

$$\Big\downarrow \; 1509$$

$$2\left(-\frac{2\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(-\sqrt{c}f\sqrt{ag^2+cf^2}-3ag^2+cf^2\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\cdots\right.\right.}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.}{\cdots}\right.$$

$$\frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g}$$

input
```
Int[Sqrt[f + g*x]*Sqrt[a + c*x^2],x]
```

output
```
(2*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(5*g) + (2*((-2*f*Sqrt[f + g*x]*Sqrt[a
 + c*x^2])/3 - (2*(-(((c*f^2 - 3*a*g^2)*Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g
*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/(
(a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^
2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (
c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^
2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c
^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2
+ a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(
f + g*x)^2)/g^2])))/Sqrt[c]) + ((c*f^2 + a*g^2)^(3/4)*(c*f^2 - 3*a*g^2 - S
qrt[c]*f*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]
)*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a
 + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f
)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(3/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g
*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(3*g^2)))/(5*g)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.18

method	result
risch	$\frac{2(3gx+f)\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{15g}+\frac{2\left(\frac{2\left(3a\,g^2-c\,f^2\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\left(\left(-\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\mathrm{EllipticE}\left(\sqrt{\cdots}\right.\right.}{\sqrt{c g\,x^3+c f\,x^2+a g x+a f}\cdots}\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2x\sqrt{c g\,x^3+c f\,x^2+a g x+a f}}{5}+\frac{2f\sqrt{c g\,x^3+c f\,x^2+a g x+a f}}{15g}+\frac{16af\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\cdots}{15\sqrt{c g\,x^3+c f\,x^2+a g x+\cdots}}\right.}{\cdots}$
default	Expression too large to display

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*g*x+f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+2/15/g*(2*(3*a*g^2-c*f^2)*(
f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/
c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))
^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(
((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(
1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1
/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+8*a*f*g*(f/g-(-a
*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/
g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/
(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))
^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))*((g*x+f)*(c*x
^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.45

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx = \frac{2\left(2\left(cf^3+9\,afg^2\right)\sqrt{cg}\,\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)+6\left(cf^2g-3\,ag^3\right)\sqrt{cg}\,\text{w}}{}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(2*(c*f^3 + 9*a*f*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a
*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 6*(
c*f^2*g - 3*a*g^3)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2)
, -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*
g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + 3*(
3*c*g^3*x + c*f*g^2)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c*g^3)
```

Sympy [F]

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{a+cx^2}\sqrt{f+gx}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*sqrt(f + g*x), x)
```

Maxima [F]

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}\sqrt{gx+f}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

Giac [F]

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}\sqrt{gx+f}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx = \int \sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,dx$$

input
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2),x)
```

output
```
int((f + g*x)^(1/2)*(a + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{f+gx}\sqrt{a+cx^2}\,dx$$
$$= \frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,ag+2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,cfx-3\left(\int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,x^2}{cg\,x^3+cf\,x^2+agx+af}dx\right)ac\,g^2+\left(\int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{cg\,x^3+cf\,x^2+agx}\right.}{5cf}$$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(f + g*x)*sqrt(a + c*x**2)*a*g + 2*sqrt(f + g*x)*sqrt(a + c*x**2)*c
*f*x - 3*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2
 + c*g*x**3),x)*a*c*g**2 + int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f
+ a*g*x + c*f*x**2 + c*g*x**3),x)*c**2*f**2 - int((sqrt(f + g*x)*sqrt(a +
c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a**2*g**2 + 3*int((sqrt(f
+ g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*c*f**2)/
(5*c*f)
```

3.114 $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 725

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3e}$$

$$-\frac{2\sqrt[4]{c}\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}(ef-3dg)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right.\right.}{3e^2g^2\sqrt{a+cx^2}}$$

$$+\frac{2\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(2ae^2g-\sqrt{-a}\sqrt{c}e(ef-3dg)-3cd(ef-dg)\right)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\mathrm{Elli}}{3\sqrt[4]{c}e^3g\sqrt{a+cx^2}}$$

$$-\frac{2(cd^2+ae^2)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\mathrm{EllipticPi}\left(\frac{e\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}{ef-dg},\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+g}}{\sqrt{\sqrt{c}f+\sqrt{-}}}\right.\right.}{\sqrt[4]{c}e^3\sqrt{a+cx^2}}$$

output
```
2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e-2/3*c^(1/4)*(c^(1/2)*f-(-a)^(1/2)*g)*(
c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(-3*d*g+e*f)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(
-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*Ell
ipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-
a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/e^2/g^2/(c*x^2+a)^(1/2)+2/3*(
c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(2*a*e^2*g-(-a)^(1/2)*c^(1/2)*e*(-3*d*g+e*f)
-3*c*d*(-d*g+e*f))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c
^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(
1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(
-a)^(1/2)*g))^(1/2))/c^(1/4)/e^3/g/(c*x^2+a)^(1/2)-2*(a*e^2+c*d^2)*(c^(1/2
)*f+(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)
*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticPi(c^(1/4)*(g*
x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),e*(f+(-a)^(1/2)*g/c^(1/2))/(-d*g
+e*f),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(1/4)/e
^3/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.35 (sec) , antiderivative size = 1216, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x),x]
```

output
```
(2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*e) + ((f + g*x)^(3/2)*(2*c*e^2*f*Sqrt
[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 6*c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
+ (2*c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (6*c*d*e*f^
2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (2*a*e^2*f*g^2*Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (6*a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]])/(f + g*x)^2 - (4*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])
/(f + g*x) + (12*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (
2*Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - 3*d*g)*Sqrt[1 - f/(f + g*x
) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g
)/(Sqrt[c]*(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]
]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sq
rt[f + g*x] + (2*e*(3*Sqrt[c]*d - I*Sqrt[a]*e)*g*((-I)*Sqrt[c]*f + Sqrt[a]
*g)*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(
f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((6*I)*c*d^2*g^2*Sqrt[1 - f/(f + g*x)
 - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)
/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*S
qrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sq
rt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((6*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{d+ex}\,dx$$

$$\Big\downarrow \text{722}$$

$$\frac{\int \frac{c(ef-3dg)x^2-2(cdf-aeg)x+a(3ef-dg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \text{2349}$$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}+\int \frac{\frac{3cgd^2}{e^2}-\frac{3cfd}{e}+2ag+\left(cf-\frac{3cdg}{e}\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \text{599}$$

$$\frac{\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3\left(ae^2+cd^2\right)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}}{}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}-\frac{2\int -\frac{2ag^2-c\left(f^2-\frac{3d^2g^2}{e^2}\right)+c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\int -\frac{cf^2-2ag^2-\frac{3cd^2g^2}{e^2}-c\left(f-\frac{3dg}{e}\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}+\frac{3(ae^2+cd^2)(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}}{3e}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e}$$

input
```
Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/(e*(2*m + 5))), x] + Simp[1/(e*(2*m + 5))   Int[((d + e*x)^m/(Sqrt[f + g*
x]*Sqrt[a + c*x^2]))*Simp[3*a*e*f - a*d*g - 2*(c*d*f - a*e*g)*x + (c*e*f -
3*c*d*g)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegerQ[2*m
] &&  !LtQ[m, -1]
```

rule 2349

```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.27

method	result
elliptic	$\frac{\sqrt{(gx+f)(c x^2+a)}\left(\frac{2\sqrt{cg\,x^3+cf\,x^2+agx+af}}{3e}+\frac{2\left(\frac{a\,e^2g+c\,d^2g-cdef}{e^3}-\frac{ag}{3e}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\ \mathrm{Ellipt}}{\sqrt{cg\,x^3+cf\,x^2+agx+af}}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input

```
int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/3/e*(c*g*x^3+c*
f*x^2+a*g*x+a*f)^(1/2)+2*((a*e^2*g+c*d^2*g-c*d*e*f)/e^3-1/3/e*a*g)*(f/g-(-
a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f
/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)
/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c)
)^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(-1/e^2*(d*
g-e*f)*c-2/3/e*c*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1
/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(
-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c
)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1
/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/
g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/
2)))-2*(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/e^4*(f/g-(-a*c)^(1/2)/c)*((x+
f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c)
)^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+
a*g*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2
),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/
2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{d+ex}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{ex+d}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{ex+d}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}{d+e\,x}\,dx$$

input
```
int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x),x)
```

output
```
int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{ex+d}dx$$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x)
```

output
```
int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d),x)
```

3.115 $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 720

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{e(d+ex)}$$

$$+\frac{3\sqrt[4]{c}\left(\sqrt{-a}-\frac{\sqrt{c}f}{g}\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right)|\frac{\sqrt{c}f+\sqrt{-a}g}{\sqrt{c}f-\sqrt{-a}g}\right)}{e^2\sqrt{a+cx^2}}$$

$$-\frac{\sqrt[4]{c}\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(3\sqrt{-a}eg-\sqrt{c}(2ef-3dg)\right)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticF}\left(\arcsin\left(\cdots\right)\right)}{e^3g\sqrt{a+cx^2}}$$

$$-\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(ae^2g-cd(2ef-3dg)\right)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticPi}\left(\frac{e\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}{ef-dg},\arcsin\cdots\right)}{\sqrt[4]{c}e^3(ef-dg)\sqrt{a+cx^2}}$$

output
```
-(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e/(e*x+d)+3*c^(1/4)*((-a)^(1/2)-c^(1/2)*f/g
)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*
g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticE(c^(
1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g
)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/e^2/(c*x^2+a)^(1/2)-c^(1/4)*(c^(1/2)*f+
(-a)^(1/2)*g)^(1/2)*(3*(-a)^(1/2)*e*g-c^(1/2)*(-3*d*g+2*e*f))*(1-c^(1/2)*(
g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^
(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(
1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/e^3/g/(c*x
^2+a)^(1/2)-(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(a*e^2*g-c*d*(-3*d*g+2*e*f))*(1
-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/
2)*f+(-a)^(1/2)*g))^(1/2)*EllipticPi(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)
^(1/2)*g)^(1/2),e*(f+(-a)^(1/2)*g/c^(1/2))/(-d*g+e*f),((c^(1/2)*f+(-a)^(1/
2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(1/4)/e^3/(-d*g+e*f)/(c*x^2+a)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.15 (sec) , antiderivative size = 1331, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^2,x]
```

output
```
(Sqrt[f + g*x]*(-((e^2*(a + c*x^2))/(d + e*x)) - (-3*c*e^2*f^3*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]] + 3*c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3
*a*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 3*a*d*e*g^3*Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]] + 6*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x
) - 6*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 3*c*e^2*f*Sqr
t[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + 3*c*d*e*g*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]]*(f + g*x)^2 + 3*Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f
) + d*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*
g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c
]*f + I*Sqrt[a]*g)] + e*(Sqrt[c]*f + I*Sqrt[a]*g)*(Sqrt[a]*e*g - I*Sqrt[c]
*(2*e*f - 3*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I
*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh
[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g
)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c
] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g
*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)),
I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*
Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - (3*I)*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a
])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g...
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1547 vs. $2(720) = 1440$.

Time = 2.96 (sec) , antiderivative size = 1547, normalized size of antiderivative = 2.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {721, 2349, 599, 27, 729, 25, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{ 721}$$

$$\frac{\int \frac{3cgx^2+2cfx+ag}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

$\downarrow$ 2349

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{\frac{2cf}{e}-\frac{3cdg}{e^2}+\frac{3cgx}{e}}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

$\downarrow$ 599

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int \frac{cg(ef+3dg-3e(f+gx))}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

$\downarrow$ 27

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2c\int \frac{ef+3dg-3e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g}}{2e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

$\downarrow$ 729

$$\frac{2\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\int \frac{ef+3dg-3e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g}}{2e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

$\downarrow$ 25

$$\frac{-\frac{2c\int \frac{ef+3dg-3e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g}-2\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{2e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

$\downarrow$ 1511

$$\frac{-2\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\left(\left(e\left(f-\frac{3\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)+3dg\right)\int\frac{\cdots}{\sqrt{\frac{cf^2}{g^2}-\cdots}}\cdots\right.}{2e}\cdots}{\cdots}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

$\downarrow$ 1416

$$\frac{-\frac{2c\left(\frac{3e\sqrt{ag^2+cf^2}\int\frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}+\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(e\left(f-\frac{3\sqrt{ag^2+c\cdots}}{\sqrt{c}}\cdots\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx\cdots}{g^2}\cdots}}\cdots\right)}{e^2g}\cdots}{\cdots}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

$\downarrow$ 1509

$$\frac{-2\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\left(\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{c\cdots}{\cdots}}{\left(a\cdots\right.}}}{\cdots}\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}$$

$$\downarrow 1540$$

$$2\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(age^2+cd(2ef-dg))}-\frac{\sqrt{c}\left(cef^2\cdots\right.}{\cdots}\right.$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{e(d+ex)}$$

$$\downarrow 1416$$

$$2\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(age^2+cd(2ef-dg))}-\frac{\sqrt[4]{c}\left(cef^2\cdots\right.}{\cdots}\right.$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{e(d+ex)}$$

$$\downarrow 2222$$

$$\frac{2\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\left(\frac{\left(e+\frac{\sqrt{c}(ef-dg)}{\sqrt{cf^2+ag^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}\right)}{2\sqrt{e}\sqrt{cd^2+ae^2}\sqrt{ef-dg}}\right.}{}\right.}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{e(d+ex)}$$

input
```
Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^2,x]
```

output
```
-((Sqrt[f + g*x]*Sqrt[a + c*x^2])/(e*(d + e*x))) + ((-2*c*((3*e*Sqrt[c*f^2
 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 +
 (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*
f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f
^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^
2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2
)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 +
 (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c
*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/Sqrt[c] + ((c*f^2 + a*g^2)^(1/
4)*(3*d*g + e*(f - (3*Sqrt[c*f^2 + a*g^2])/Sqrt[c]))*(1 + (Sqrt[c]*(f + g*
x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (
c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2
 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^
(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sqrt[a + (c*f
^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(e^2*g) + 2*(a*g
 - (c*d*(2*e*f - 3*d*g))/e^2)*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g^2 - Sqrt[c]*
(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g
^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/
((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*Ellip
ticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqr...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 721
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/
(e*(m + 1))), x] - Simp[1/(2*e*(m + 1))   Int[((d + e*x)^(m + 1)/(Sqrt[f +
g*x]*Sqrt[a + c*x^2]))*Simp[a*g + 2*c*f*x + 3*c*g*x^2, x], x], x] /; FreeQ[
{a, c, d, e, f, g}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.24

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(-\frac{\sqrt{cg\,x^3+cf\,x^2+agx+af}}{e(ex+d)}+\frac{2\left(-\frac{c(2dg-ef)}{e^3}+\frac{cdg}{2e^3}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\text{EllipticF}\left(\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+af}}\right.}{}$
default	Expression too large to display

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-1/e*(c*g*x^3+c*f
*x^2+a*g*x+a*f)^(1/2)/(e*x+d)+2*(-c*(2*d*g-e*f)/e^3+1/2*c*d/e^3*g)*(f/g-(-
a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f
/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)
/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c)
)^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+3*c/e^2*g*(f/
g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)
/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(
1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((
x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/
2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2
),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+1/e^4*(a*e^2*g+3*c
*d^2*g-2*c*d*e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2
)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f
/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*Ell
ipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d
/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)^2}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d)**2,x)
```

output
```
Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}{{\left(d+e\,x\right)}^2}\,dx$$

input
```
int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^2,x)
```

output
```
int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2}\,dx = \text{too large to display}$$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^2,x)
```

output
```
(2*sqrt(f + g*x)*sqrt(a + c*x**2)*f + 3*int((sqrt(f + g*x)*sqrt(a + c*x**2
)*x**3)/(a*d**2*f + a*d**2*g*x + 2*a*d*e*f*x + 2*a*d*e*g*x**2 + a*e**2*f*x
**2 + a*e**2*g*x**3 + c*d**2*f*x**2 + c*d**2*g*x**3 + 2*c*d*e*f*x**3 + 2*c
*d*e*g*x**4 + c*e**2*f*x**4 + c*e**2*g*x**5),x)*c*d**2*g**2 - int((sqrt(f
+ g*x)*sqrt(a + c*x**2)*x**3)/(a*d**2*f + a*d**2*g*x + 2*a*d*e*f*x + 2*a*d
*e*g*x**2 + a*e**2*f*x**2 + a*e**2*g*x**3 + c*d**2*f*x**2 + c*d**2*g*x**3
+ 2*c*d*e*f*x**3 + 2*c*d*e*g*x**4 + c*e**2*f*x**4 + c*e**2*g*x**5),x)*c*d*
e*f*g + 3*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**3)/(a*d**2*f + a*d**2*g*x
 + 2*a*d*e*f*x + 2*a*d*e*g*x**2 + a*e**2*f*x**2 + a*e**2*g*x**3 + c*d**2*f
*x**2 + c*d**2*g*x**3 + 2*c*d*e*f*x**3 + 2*c*d*e*g*x**4 + c*e**2*f*x**4 +
c*e**2*g*x**5),x)*c*d*e*g**2*x - int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**3)
/(a*d**2*f + a*d**2*g*x + 2*a*d*e*f*x + 2*a*d*e*g*x**2 + a*e**2*f*x**2 + a
*e**2*g*x**3 + c*d**2*f*x**2 + c*d**2*g*x**3 + 2*c*d*e*f*x**3 + 2*c*d*e*g*
x**4 + c*e**2*f*x**4 + c*e**2*g*x**5),x)*c*e**2*f*g*x + 3*int((sqrt(f + g*
x)*sqrt(a + c*x**2)*x)/(a*d**2*f + a*d**2*g*x + 2*a*d*e*f*x + 2*a*d*e*g*x*
*2 + a*e**2*f*x**2 + a*e**2*g*x**3 + c*d**2*f*x**2 + c*d**2*g*x**3 + 2*c*d
*e*f*x**3 + 2*c*d*e*g*x**4 + c*e**2*f*x**4 + c*e**2*g*x**5),x)*a*d**2*g**2
 + int((sqrt(f + g*x)*sqrt(a + c*x**2)*x)/(a*d**2*f + a*d**2*g*x + 2*a*d*e
*f*x + 2*a*d*e*g*x**2 + a*e**2*f*x**2 + a*e**2*g*x**3 + c*d**2*f*x**2 + c*
d**2*g*x**3 + 2*c*d*e*f*x**3 + 2*c*d*e*g*x**4 + c*e**2*f*x**4 + c*e**2*...
```

3.116 $\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 963

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3} \, dx$$

$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2e(d+ex)^2} - \frac{(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4e(cd^2+ae^2)(ef-dg)(d+ex)}$$

$$-\frac{\sqrt[4]{c}\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}(ae^2g-cd(2ef-3dg))\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\Big(\text{arcsin}}{4e^2(cd^2+ae^2)g(ef-dg)\sqrt{a+cx^2}}$$

$$+\frac{\sqrt[4]{c}\sqrt{\sqrt{c}f+\sqrt{-a}g}\left((-a)^{3/2}e^3g+a\sqrt{c}e^2(6ef-5dg)+\sqrt{-a}cde(2ef-3dg)+c^{3/2}d^2(4ef-3dg)\right)\sqrt{}}{4e^3(cd^2+ae^2)(ef-dg)\sqrt{a+}}$$

$$+\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}(a^2e^4g^2+c^2d^3g(4ef-3dg)-2ace^2(2e^2f^2-6defg+3d^2g^2))\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-}}{4\sqrt[4]{c}e^3(cd^2+ae^2)(ef-dg)^2\sqrt{a+c}}$$

output
```
-1/2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e/(e*x+d)^2-1/4*(a*e^2*g-c*d*(-3*d*g+2*
e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/e/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)-1/4
*c^(1/4)*(c^(1/2)*f-(-a)^(1/2)*g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(a*e^2*g-
c*d*(-3*d*g+2*e*f))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-
c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticE(c^(1/4)*(g*x+f)^
(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-
(-a)^(1/2)*g))^(1/2))/e^2/(a*e^2+c*d^2)/g/(-d*g+e*f)/(c*x^2+a)^(1/2)+1/4*c
^(1/4)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*((-a)^(3/2)*e^3*g+a*c^(1/2)*e^2*(-5*
d*g+6*e*f)+(-a)^(1/2)*c*d*e*(-3*d*g+2*e*f)+c^(3/2)*d^2*(-3*d*g+4*e*f))*(1-
c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2
)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(
1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/e
^3/(a*e^2+c*d^2)/(-d*g+e*f)/(c*x^2+a)^(1/2)+1/4*(c^(1/2)*f+(-a)^(1/2)*g)^(
1/2)*(a^2*e^4*g^2+c^2*d^3*g*(-3*d*g+4*e*f)-2*a*c*e^2*(3*d^2*g^2-6*d*e*f*g+
2*e^2*f^2))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*
(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticPi(c^(1/4)*(g*x+f)^(1/2)/(
c^(1/2)*f+(-a)^(1/2)*g)^(1/2),e*(f+(-a)^(1/2)*g/c^(1/2))/(-d*g+e*f),((c^(1
/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(1/4)/e^3/(a*e^2+c*
d^2)/(-d*g+e*f)^2/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.77 (sec) , antiderivative size = 2703, normalized size of antiderivative = 2.81

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^3,x]
```

output
```
Sqrt[f + g*x]*Sqrt[a + c*x^2]*(-1/2*1/(e*(d + e*x)^2) + (2*c*d*e*f - 3*c*d
^2*g - a*e^2*g)/(4*e*(c*d^2 + a*e^2)*(e*f - d*g)*(d + e*x))) + ((f + g*x)^
(3/2)*(-2*c^2*d*e^3*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 5*c^2*d^2*e^2*f
*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*c*e^4*f*g*Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]] - 3*c^2*d^3*e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*c*d*e^3*g^
2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - (2*c^2*d*e^3*f^4*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]])/(f + g*x)^2 + (5*c^2*d^2*e^2*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]])/(f + g*x)^2 + (a*c*e^4*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f
 + g*x)^2 - (3*c^2*d^3*e*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*
x)^2 - (3*a*c*d*e^3*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2
+ (5*a*c*d^2*e^2*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (a^
2*e^4*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3*a*c*d^3*e*g
^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (a^2*d*e^3*g^4*Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (4*c^2*d*e^3*f^3*Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]])/(f + g*x) - (10*c^2*d^2*e^2*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]])/(f + g*x) - (2*a*c*e^4*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/
(f + g*x) + (6*c^2*d^3*e*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)
 + (2*a*c*d*e^3*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (Sqrt[
c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - d*g)*(a*e^2*g + c*d*(-2*e*f + 3*d
*g))*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 -...
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2763 vs. $2(963) = 1926$.

Time = 5.33 (sec) , antiderivative size = 2763, normalized size of antiderivative = 2.87, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {721, 2349, 734, 2349, 27, 510, 599, 25, 27, 729, 25, 1416, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)^3} \, dx$$

$$\Big\downarrow \text{721}$$

$$\frac{\int \frac{3cgx^2+2cfx+ag}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \ 2349$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{\frac{2cf}{e}-\frac{3cdg}{e^2}+\frac{3cgx}{e}}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \ 734$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{\int \frac{-cgx^2e^2+age^2-2cdgxe-2cd(ef-dg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+\int \frac{\frac{2cf}{e}-\frac{3cdg}{e^2}+\frac{3cgx}{e}}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \ 2349$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+\frac{2c(ef-}{}}{4e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+\frac{2c(ef-}{}}{4e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \ 510$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+\frac{2c(ef-}{}}{4e}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \ 599$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int -\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\cdots}{(d+ex)(ae^2+c\cdots}\right.}{4e}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2\int \frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{\cdots}}{(d+ex)(ae^2+cd^2\cdots}\right.}{4e}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\cdots}{(d+ex)(ae^2+c\cdots}\right.}{4e}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \ 729$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{2\left(ae^2g-cd(2ef-3dg)\right)\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}+\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f\cdots}{}}}\cdots}{g}}{2(ae^2+cd^2)(ef-dg)}\right.}{\cdots}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+g}{g^2}}}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 1416$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+g}{g^2}}}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 1511$$

$$\frac{\left(ag-\frac{cd(2ef-3dg)}{e^2}\right)\left(-\frac{\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\left(dg-e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.}{g}}{2(ae^2+cd^2)(ef-d}\right.}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 1416$$

$$\frac{3c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}} - \frac{4c(ef}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2e(d+ex)^2}$$

$\big\downarrow$ 1509

$$\frac{3c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}} - \frac{4c(ef}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2e(d+ex)^2}$$

$$\downarrow 1540$$

$$\frac{3c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+\frac{4c(ef}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2e(d+ex)^2}$$

$\downarrow$ 1416

$$\frac{3c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+\frac{4c(ef}{}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2e(d+ex)^2}$$

$$\downarrow 2222$$

$4c(ef$

$$\frac{3c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2e(d+ex)^2}$$

input
```
Int[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^3,x]
```

output
```
-1/2*(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(e*(d + e*x)^2) + ((3*c^(3/4)*(c*f^2
+ a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*
f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)
*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(
1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 +
a*g^2])/2])/(e^2*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*
x)^2)/g^2]) + (4*c*(e*f - 3*d*g)*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g^2 - Sqrt[
c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 +
a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^
2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*El
lipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqr
t[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(g*(c*f^2 + a*g^2)^(1/4)*(a*e^2*g + c*d*(
2*e*f - d*g))*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^
2)/g^2]) + (e*Sqrt[c*f^2 + a*g^2]*(Sqrt[c]*(e*f - d*g) - e*Sqrt[c*f^2 + a*
g^2])*(((e + (Sqrt[c]*(e*f - d*g))/Sqrt[c*f^2 + a*g^2])*ArcTanh[(Sqrt[c*d^
2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt[a + (c*f^2)/g^2 -
(2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])])/(2*Sqrt[e]*Sqrt[c*d^2 + a*
e^2]*Sqrt[e*f - d*g]) - ((Sqrt[c]/e - Sqrt[c*f^2 + a*g^2]/(e*f - d*g))*(1
+ (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*
(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 510
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2/d   Subst[Int[1/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2
)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 721
```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/
(e*(m + 1))), x] - Simp[1/(2*e*(m + 1))   Int[((d + e*x)^(m + 1)/(Sqrt[f +
g*x]*Sqrt[a + c*x^2]))*Simp[a*g + 2*c*f*x + 3*c*g*x^2, x], x], x] /; FreeQ[
{a, c, d, e, f, g}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 734
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Simp[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^
2]))*Simp[2*c*d*(e*f - d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*c*e*(d*g*(m + 1
) - e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 1161, normalized size of antiderivative = 1.21

method	result	size
elliptic	Expression too large to display	1161
default	Expression too large to display	19181

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-1/2/e*(c*g*x^3+c
*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)^2+1/4*(a*e^2*g+3*c*d^2*g-2*c*d*e*f)/e/(a*d
*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d
)+2*(c*g/e^3-1/8*c*g*(3*a*d*e^2*g-2*a*e^3*f+5*c*d^3*g-4*c*d^2*e*f)/e^3/(a*
d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a
*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(
-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1
/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/
(-f/g-(-a*c)^(1/2)/c))^(1/2))-1/4*c/e^2*g*(a*e^2*g+3*c*d^2*g-2*c*d*e*f)/(a
*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a
*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(
-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1
/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),
((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*Ellipt
icF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*
c)^(1/2)/c))^(1/2)))+1/4*(a^2*e^4*g^2-6*a*c*d^2*e^2*g^2+12*a*c*d*e^3*f*g-4
*a*c*e^4*f^2-3*c^2*d^4*g^2+4*c^2*d^3*e*f*g)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d
^2*e*f)/e^4*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-
(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a
*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*Ellipt...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)^3}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d)**3,x)
```

output
```
Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{(ex+d)^3}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{cx^2+a}\sqrt{gx+f}}{(ex+d)^3}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}{(d+e\,x)^3}\,dx$$

input
```
int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^3,x)
```

output
```
int(((f + g*x)^(1/2)*(a + c*x^2)^(1/2))/(d + e*x)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3}\,dx = \text{too large to display}$$

input
```
int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x)
```

output

```
(2*sqrt(f + g*x)*sqrt(a + c*x**2)*f + 9*int((sqrt(f + g*x)*sqrt(a + c*x**2
)*x**3)/(3*a*d**4*f*g + 3*a*d**4*g**2*x - 2*a*d**3*e*f**2 + 7*a*d**3*e*f*g
*x + 9*a*d**3*e*g**2*x**2 - 6*a*d**2*e**2*f**2*x + 3*a*d**2*e**2*f*g*x**2
+ 9*a*d**2*e**2*g**2*x**3 - 6*a*d*e**3*f**2*x**2 - 3*a*d*e**3*f*g*x**3 + 3
*a*d*e**3*g**2*x**4 - 2*a*e**4*f**2*x**3 - 2*a*e**4*f*g*x**4 + 3*c*d**4*f*
g*x**2 + 3*c*d**4*g**2*x**3 - 2*c*d**3*e*f**2*x**2 + 7*c*d**3*e*f*g*x**3 +
 9*c*d**3*e*g**2*x**4 - 6*c*d**2*e**2*f**2*x**3 + 3*c*d**2*e**2*f*g*x**4 +
 9*c*d**2*e**2*g**2*x**5 - 6*c*d*e**3*f**2*x**4 - 3*c*d*e**3*f*g*x**5 + 3*
c*d*e**3*g**2*x**6 - 2*c*e**4*f**2*x**5 - 2*c*e**4*f*g*x**6),x)*c*d**4*g**
3 - 9*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**3)/(3*a*d**4*f*g + 3*a*d**4*g
**2*x - 2*a*d**3*e*f**2 + 7*a*d**3*e*f*g*x + 9*a*d**3*e*g**2*x**2 - 6*a*d*
*2*e**2*f**2*x + 3*a*d**2*e**2*f*g*x**2 + 9*a*d**2*e**2*g**2*x**3 - 6*a*d*
e**3*f**2*x**2 - 3*a*d*e**3*f*g*x**3 + 3*a*d*e**3*g**2*x**4 - 2*a*e**4*f**
2*x**3 - 2*a*e**4*f*g*x**4 + 3*c*d**4*f*g*x**2 + 3*c*d**4*g**2*x**3 - 2*c*
d**3*e*f**2*x**2 + 7*c*d**3*e*f*g*x**3 + 9*c*d**3*e*g**2*x**4 - 6*c*d**2*e
**2*f**2*x**3 + 3*c*d**2*e**2*f*g*x**4 + 9*c*d**2*e**2*g**2*x**5 - 6*c*d*e
**3*f**2*x**4 - 3*c*d*e**3*f*g*x**5 + 3*c*d*e**3*g**2*x**6 - 2*c*e**4*f**2
*x**5 - 2*c*e**4*f*g*x**6),x)*c*d**3*e*f*g**2 + 18*int((sqrt(f + g*x)*sqrt
(a + c*x**2)*x**3)/(3*a*d**4*f*g + 3*a*d**4*g**2*x - 2*a*d**3*e*f**2 + 7*a
*d**3*e*f*g*x + 9*a*d**3*e*g**2*x**2 - 6*a*d**2*e**2*f**2*x + 3*a*d**2*...
```

3.117 $\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 871

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{2\sqrt{f+gx}\,(9ae^2g^2(2ef-5dg) - c(64e^3f^3 - 216de^2f^2g + 252d^2efg^2 - 105d^3g^3) - 3eg(7ae^2g^2 - c(16e^2}{315cg^4}$$

$$- \frac{2e^2(5ef-9dg)\sqrt{f+gx}\,(a+cx^2)^{3/2}}{21cg^2} + \frac{2e^3(f+gx)^{3/2}\,(a+cx^2)^{3/2}}{9cg^2}$$

$$- \frac{4\left(\sqrt{-a}-\frac{\sqrt{c}f}{g}\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(5cfg^2(21cd^3g - ae^2(2ef+9dg)) + e(4cf^2+3ag^2)\,(7ae^2g^2 - c(16e^2}{315c^{7/4}g^5\sqrt{a}}$$

$$+ \frac{4\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(21(-a)^{5/2}e^3g^4 - 3a^2\sqrt{c}e^2g^3(ef+15dg) - 3\sqrt{-a}aceg^2(10e^2f^2 - 39defg + 63d^2g^2)}{}$$

output
```
2/315*(g*x+f)^(1/2)*(9*a*e^2*g^2*(-5*d*g+2*e*f)-c*(-105*d^3*g^3+252*d^2*e*
f*g^2-216*d*e^2*f^2*g+64*e^3*f^3)-3*e*g*(7*a*e^2*g^2-c*(63*d^2*g^2-54*d*e*
f*g+16*e^2*f^2))*x)*(c*x^2+a)^(1/2)/c/g^4-2/21*e^2*(-9*d*g+5*e*f)*(g*x+f)^
(1/2)*(c*x^2+a)^(3/2)/c/g^2+2/9*e^3*(g*x+f)^(3/2)*(c*x^2+a)^(3/2)/c/g^2-4/
315*((-a)^(1/2)-c^(1/2)*f/g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(5*c*f*g^2*(21
*c*d^3*g-a*e^2*(9*d*g+2*e*f))+e*(3*a*g^2+4*c*f^2)*(7*a*e^2*g^2-c*(63*d^2*g
^2-54*d*e*f*g+16*e^2*f^2)))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(
1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticE(c^(1/4)*
(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^
(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(7/4)/g^5/(c*x^2+a)^(1/2)+4/315*(c^(1/2)*f
+(-a)^(1/2)*g)^(1/2)*(21*(-a)^(5/2)*e^3*g^4-3*a^2*c^(1/2)*e^2*g^3*(15*d*g+
e*f)-3*(-a)^(1/2)*a*c*e*g^2*(63*d^2*g^2-39*d*e*f*g+10*e^2*f^2)-a*c^(3/2)*g
*(-105*d^3*g^3+63*d^2*e*f*g^2-54*d*e^2*f^2*g+16*e^3*f^3)-(-a)^(1/2)*c^2*f*
(-105*d^3*g^3+252*d^2*e*f*g^2-216*d*e^2*f^2*g+64*e^3*f^3))*(1-c^(1/2)*(g*x
+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/
2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2
),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(7/4)/g^5/(
c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.39 (sec) , antiderivative size = 872, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)\left(2ae^2g^2(-11ef+45dg+7egx)+c\left(105d^3g^3+63d^2eg^2(-4f+3gx)+27de^2g\left(8f^2-6fgx+5g^2x^2\right)+e^3\left(-64f^3+48f^2gx-4\right.\right.\right.}{cg^4}\right.}{}$$

input
```
Integrate[((d + e*x)^3*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

output
```
(Sqrt[f + g*x]*((2*(a + c*x^2)*(2*a*e^2*g^2*(-11*e*f + 45*d*g + 7*e*g*x) +
 c*(105*d^3*g^3 + 63*d^2*e*g^2*(-4*f + 3*g*x) + 27*d*e^2*g*(8*f^2 - 6*f*g*
x + 5*g^2*x^2) + e^3*(-64*f^3 + 48*f^2*g*x - 40*f*g^2*x^2 + 35*g^3*x^3))))
/(c*g^4) + (4*(f + g*x)*((g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-21*a^2*e^
3*g^4 + 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^2) + c^2*f*(64*e^3
*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3))*(a + c*x^2))/(f +
 g*x)^2 + (I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(21*a^2*e^3*g^4 - 3*a*c*e*g
^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^2) + c^2*f*(-64*e^3*f^3 + 216*d*e^2
*f^2*g - 252*d^2*e*f*g^2 + 105*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x)
)/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*EllipticE[I*
ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sq
rt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + (Sqrt[a]*Sqrt[c]*g*(S
qrt[c]*f + I*Sqrt[a]*g)*((-21*I)*a^(3/2)*e^3*g^3 + 9*a*Sqrt[c]*e^2*g^2*(2*
e*f - 5*d*g) + (3*I)*Sqrt[a]*c*e*g*(16*e^2*f^2 - 54*d*e*f*g + 63*d^2*g^2)
+ c^(3/2)*(-64*e^3*f^3 + 216*d*e^2*f^2*g - 252*d^2*e*f*g^2 + 105*d^3*g^3))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[
c] - g*x)/(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqr
t[f + g*x]))/(c^2*g^6*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])))/(315*Sqrt[a + c*
x^2])
```

Rubi [A] (warning: unable to verify)

Time = 2.83 (sec) , antiderivative size = 1085, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {723, 27, 2185, 27, 2185, 27, 2185, 27, 599, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}(d+ex)^3}{\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{723}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-\frac{\int \frac{2(d+ex)^2\left(c(4ef-3dg)x^2+(cdf-aeg)x+a(3ef-4dg)\right)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{9g}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-\frac{2\int\frac{(d+ex)^2\left(c(4ef-3dg)x^2+(cdf-aeg)x+a(3ef-4dg)\right)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{9g}$$

$\downarrow$ 2185

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-$$

$$\frac{2\left(\frac{2\int-\frac{ceg^3\left(7ae^2g^2+c\left(64e^2f^2-111degf+42d^2g^2\right)\right)x^3-cg^2\left(ae^2g^2(ef-27dg)-c\left(44e^3f^3-33de^2gf^2-42d^2eg^2f+21d^3g^3\right)\right)x^2+cg\left(ae\left(40e^2f^2-72degf+63\cdots\right.\right.}{2\sqrt{f+gx}\sqrt{cx^2+a}}}{7cg^4}\cdots\right.}{9g}$$

$\downarrow$ 27

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-$$

$$\frac{2\left(\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef-3dg)}{7g^3}-\frac{\int\frac{ceg^3\left(7ae^2g^2+c\left(64e^2f^2-111degf+42d^2g^2\right)\right)x^3-cg^2\left(ae^2g^2(ef-27dg)-c\left(44e^3f^3-33de^2gf^2-42d^2eg^2f+21\cdots\right.\right.}{\cdots}}{\cdots}\right.}{9g}$$

$\downarrow$ 2185

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-$$

$$\frac{2\left(\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef-3dg)}{7g^3}-\frac{2\int-\frac{3c^2\left(9ae^2(2ef-5dg)g^2+c\left(76e^3f^3-204de^2gf^2+168d^2eg^2f-35d^3g^3\right)\right)x^2g^5+ac\left(21afg^2e^3+c\left(92e^3f^3-258\cdots\right.\right.}{\cdots}}{\cdots}\right.}{\cdots}$$

$\downarrow$ 27

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-$$

$$\frac{2\left(\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef-3dg)}{7g^3}-\frac{\frac{2}{5}eg\sqrt{a+cx^2}(f+gx)^{3/2}\left(7ae^2g^2+c\left(42d^2g^2-111defg+64e^2f^2\right)\right)-\frac{\int\frac{3c^2\left(9ae^2(2ef-5dg)g^2+c\left(76e^3f^3-20\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}\right.}{\cdots}$$

$\downarrow$ 2185

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g} -$$

$$2\left(\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef-3dg)}{7g^3} - \frac{\frac{2}{5}eg\sqrt{a+cx^2}(f+gx)^{3/2}\left(7ae^2g^2+c\left(42d^2g^2-111defg+64e^2f^2\right)\right) - \frac{2\int \frac{3c^2g^6\left(ag\left(3ae^2(ef+15dg)g^2+c\left(16e\cdots\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 27

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g} -$$

$$2\left(\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef-3dg)}{7g^3} - \frac{\frac{2}{5}eg\sqrt{a+cx^2}(f+gx)^{3/2}\left(7ae^2g^2+c\left(42d^2g^2-111defg+64e^2f^2\right)\right) - \frac{cg^4\int \frac{ag\left(3ae^2(ef+15dg)g^2+c\left(16e^3f^3-\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 599

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g} -$$

$$2\left(\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef-3dg)}{7g^3} - \frac{\frac{2}{5}eg\sqrt{a+cx^2}(f+gx)^{3/2}\left(7ae^2g^2+c\left(42d^2g^2-111defg+64e^2f^2\right)\right) - \frac{2cg^4\sqrt{a+cx^2}\sqrt{f+gx}\left(9ae^2g^2(2ef-5dg)\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 1511

$$\frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}-$$

$$2\left(\frac{2e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef-3dg)}{7g^3}-\frac{\frac{2}{5}eg\sqrt{a+cx^2}(f+gx)^{3/2}\left(7ae^2g^2+c\left(42d^2g^2-111defg+64e^2f^2\right)\right)-\frac{2cg^4\sqrt{a+cx^2}\sqrt{f+gx}\left(9ae^2g^2(2ef-5dg)\right.}{}}{}\right.$$

$\downarrow$ 1416

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{cx^2+a}}{9g}-$$

$$2\left(\frac{2e^2(4ef-3dg)(f+gx)^{5/2}\sqrt{cx^2+a}}{7g^3}-\frac{\frac{2}{5}eg\left(7ae^2g^2+c\left(64e^2f^2-111degf+42d^2g^2\right)\right)(f+gx)^{3/2}\sqrt{cx^2+a}-\frac{2cg^4\left(9ae^2(2ef-5dg)g^2+c\left(76e^3f^3-20\right.\right.}{}}{}\right.$$

$\downarrow$ 1509

$$\frac{2(d+ex)^3\sqrt{f+gx}\sqrt{cx^2+a}}{9g}-$$

$$2\left(\frac{2e^2(4ef-3dg)(f+gx)^{5/2}\sqrt{cx^2+a}}{7g^3}-\frac{\frac{2}{5}eg(7ae^2g^2+c(64e^2f^2-111degf+42d^2g^2))(f+gx)^{3/2}\sqrt{cx^2+a}-\frac{2cg^4\left(9ae^2(2ef-5dg)g^2+c\left(76e^3f^3-204\right.\right.}{}}{}\right.$$

input
```
Int[((d + e*x)^3*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

output
```
(2*(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(9*g) - (2*((2*e^2*(4*e*f -
3*d*g)*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(7*g^3) - ((2*e*g*(7*a*e^2*g^2 + c
*(64*e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])
/5 - (2*c*g^4*(9*a*e^2*g^2*(2*e*f - 5*d*g) + c*(76*e^3*f^3 - 204*d*e^2*f^2
*g + 168*d^2*e*f*g^2 - 35*d^3*g^3))*Sqrt[f + g*x]*Sqrt[a + c*x^2] - 2*c*g^
2*((Sqrt[c*f^2 + a*g^2]*(21*a^2*e^3*g^4 - 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e
*f*g + 63*d^2*g^2) - c^2*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2
 - 105*d^3*g^3))*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x)
)/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/
Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/S
qrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f
+ g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*
g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)
], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2
 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/Sqrt[c] - ((c*f^2 + a*g
^2)^(3/4)*(21*a^2*e^3*g^4 - 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*
g^2) - c^2*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3
) - Sqrt[c]*Sqrt[c*f^2 + a*g^2]*(9*a*e^2*g^2*(2*e*f - 5*d*g) - c*(64*e^3*f
^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3)))*(1 + (Sqrt[c]*(f +
 g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 723
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[2*(d + e*x)^m*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/(g
*(2*m + 3))), x] - Simp[1/(g*(2*m + 3))   Int[((d + e*x)^(m - 1)/(Sqrt[f +
g*x]*Sqrt[a + c*x^2]))*Simp[2*a*(e*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g)
*x - (2*c*(d*g*m - e*f*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g
}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 1156, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	1156
risch	Expression too large to display	1937
default	Expression too large to display	5079

input
```
int((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/9/g*e^3*x^3*(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/7*(3*c*d*e^2-8/9*c*f/g*e^3)/c/g*x^2*(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/5*(2/9*a*e^3+3*c*d^2*e-6/7*f/g*(3*c*d*e^2-8
/9*c*f/g*e^3))/c/g*x*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(3*a*d*e^2+c*d^
3-4/5*f/g*(2/9*a*e^3+3*c*d^2*e-6/7*f/g*(3*c*d*e^2-8/9*c*f/g*e^3))-5/7*a/c*
(3*c*d*e^2-8/9*c*f/g*e^3)-2/3*a*f/g*e^3)/c/g*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(
1/2)+2*(a*d^3-2/5*a*f/c/g*(2/9*a*e^3+3*c*d^2*e-6/7*f/g*(3*c*d*e^2-8/9*c*f/
g*e^3))-1/3*a/c*(3*a*d*e^2+c*d^3-4/5*f/g*(2/9*a*e^3+3*c*d^2*e-6/7*f/g*(3*c
*d*e^2-8/9*c*f/g*e^3))-5/7*a/c*(3*c*d*e^2-8/9*c*f/g*e^3)-2/3*a*f/g*e^3))*(
f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/
c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))
^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1
/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(3*a*
d^2*e-4/7*a*f/c/g*(3*c*d*e^2-8/9*c*f/g*e^3)-3/5*a/c*(2/9*a*e^3+3*c*d^2*e-6
/7*f/g*(3*c*d*e^2-8/9*c*f/g*e^3))-2/3*f/g*(3*a*d*e^2+c*d^3-4/5*f/g*(2/9*a*
e^3+3*c*d^2*e-6/7*f/g*(3*c*d*e^2-8/9*c*f/g*e^3))-5/7*a/c*(3*c*d*e^2-8/9*c*
f/g*e^3)-2/3*a*f/g*e^3))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c
))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)
/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-
(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 578, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx =$$
$$-\frac{2\left(2\left(64\,c^2e^3f^5-216\,c^2de^2f^4g+6\left(42\,c^2d^2e+13\,ace^3\right)f^3g^2-3\left(35\,c^2d^3+93\,acde^2\right)f^2g^3+6\left(63\,acd^2\right.\right.\right.}{}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
-2/945*(2*(64*c^2*e^3*f^5 - 216*c^2*d*e^2*f^4*g + 6*(42*c^2*d^2*e + 13*a*c
*e^3)*f^3*g^2 - 3*(35*c^2*d^3 + 93*a*c*d*e^2)*f^2*g^3 + 6*(63*a*c*d^2*e -
2*a^2*e^3)*f*g^4 - 45*(7*a*c*d^3 - 3*a^2*d*e^2)*g^5)*sqrt(c*g)*weierstrass
PInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3),
 1/3*(3*g*x + f)/g) + 6*(64*c^2*e^3*f^4*g - 216*c^2*d*e^2*f^3*g^2 + 6*(42*
c^2*d^2*e + 5*a*c*e^3)*f^2*g^3 - 3*(35*c^2*d^3 + 39*a*c*d*e^2)*f*g^4 + 21*
(9*a*c*d^2*e - a^2*e^3)*g^5)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^
2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*
f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)
/g)) - 3*(35*c^2*e^3*g^5*x^3 - 64*c^2*e^3*f^3*g^2 + 216*c^2*d*e^2*f^2*g^3
- 2*(126*c^2*d^2*e + 11*a*c*e^3)*f*g^4 + 15*(7*c^2*d^3 + 6*a*c*d*e^2)*g^5
- 5*(8*c^2*e^3*f*g^4 - 27*c^2*d*e^2*g^5)*x^2 + (48*c^2*e^3*f^2*g^3 - 162*c
^2*d*e^2*f*g^4 + 7*(27*c^2*d^2*e + 2*a*c*e^3)*g^5)*x)*sqrt(c*x^2 + a)*sqrt
(g*x + f))/(c^2*g^6)
```

Sympy [F]

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}\,(d+ex)^3}{\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**3*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)**3/sqrt(f + g*x), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)^3}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

Giac [F]

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)^3}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+a}\,(d+e\,x)^3}{\sqrt{f+g\,x}}\,dx$$

input
```
int(((a + c*x^2)^(1/2)*(d + e*x)^3)/(f + g*x)^(1/2),x)
```

output
```
int(((a + c*x^2)^(1/2)*(d + e*x)^3)/(f + g*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^3\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^3*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x)
```

output
```
( - 42*sqrt(f + g*x)*sqrt(a + c*x**2)*a**2*e**3*g**3 + 378*sqrt(f + g*x)*s
qrt(a + c*x**2)*a*c*d**2*e*g**3 - 54*sqrt(f + g*x)*sqrt(a + c*x**2)*a*c*d*
e**2*f*g**2 + 16*sqrt(f + g*x)*sqrt(a + c*x**2)*a*c*e**3*f**2*g + 28*sqrt(
f + g*x)*sqrt(a + c*x**2)*a*c*e**3*f*g**2*x + 378*sqrt(f + g*x)*sqrt(a + c
*x**2)*c**2*d**2*e*f*g**2*x - 324*sqrt(f + g*x)*sqrt(a + c*x**2)*c**2*d*e*
*2*f**2*g*x + 270*sqrt(f + g*x)*sqrt(a + c*x**2)*c**2*d*e**2*f*g**2*x**2 +
 96*sqrt(f + g*x)*sqrt(a + c*x**2)*c**2*e**3*f**3*x - 80*sqrt(f + g*x)*sqr
t(a + c*x**2)*c**2*e**3*f**2*g*x**2 + 70*sqrt(f + g*x)*sqrt(a + c*x**2)*c*
*2*e**3*f*g**2*x**3 + 63*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f +
a*g*x + c*f*x**2 + c*g*x**3),x)*a**2*c*e**3*g**4 - 567*int((sqrt(f + g*x)*
sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*c**2*d**2*
e*g**4 + 351*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*
x**2 + c*g*x**3),x)*a*c**2*d*e**2*f*g**3 - 90*int((sqrt(f + g*x)*sqrt(a +
c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*c**2*e**3*f**2*g**2
 + 315*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 +
 c*g*x**3),x)*c**3*d**3*f*g**3 - 756*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x
**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c**3*d**2*e*f**2*g**2 + 648*in
t((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3
),x)*c**3*d*e**2*f**3*g - 192*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a
*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c**3*e**3*f**4 + 21*int((sqrt(f + ...
```

3.118 $\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 672

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{2\sqrt{f+gx}\left(5(7cd^2-ae^2)\,g^2+8cef(3ef-7dg)-6ceg(3ef-7dg)x\right)\sqrt{a+cx^2}}{105cg^3}$$

$$+ \frac{2e^2\sqrt{f+gx}(a+cx^2)^{3/2}}{7cg}$$

$$- \frac{4\left(\sqrt{-a}-\frac{\sqrt{c}f}{g}\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(5(7cd^2-ae^2)\,fg^2+2e(3ef-7dg)\,(4cf^2+3ag^2)\right)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{}}{105c^{3/4}g^4\sqrt{a+cx^2}}$$

$$- \frac{4\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(5a^2e^2g^3-\sqrt{-a}a\sqrt{c}eg^2(13ef-42dg)-\sqrt{-a}c^{3/2}f(24e^2f^2-56defg+35d^2g^2)-\right.}{105c^5}$$

output
```
2/105*(g*x+f)^(1/2)*(5*(-a*e^2+7*c*d^2)*g^2+8*c*e*f*(-7*d*g+3*e*f)-6*c*e*g
*(-7*d*g+3*e*f)*x)*(c*x^2+a)^(1/2)/c/g^3+2/7*e^2*(g*x+f)^(1/2)*(c*x^2+a)^(
3/2)/c/g-4/105*((-a)^(1/2)-c^(1/2)*f/g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(5*
(-a*e^2+7*c*d^2)*f*g^2+2*e*(-7*d*g+3*e*f)*(3*a*g^2+4*c*f^2))*(1-c^(1/2)*(g
*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(
1/2)*g))^(1/2)*EllipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1
/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(3/4)/g^4
/(c*x^2+a)^(1/2)-4/105*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(5*a^2*e^2*g^3-(-a)^
(1/2)*a*c^(1/2)*e*g^2*(-42*d*g+13*e*f)-(-a)^(1/2)*c^(3/2)*f*(35*d^2*g^2-56
*d*e*f*g+24*e^2*f^2)-a*c*g*(35*d^2*g^2-14*d*e*f*g+6*e^2*f^2))*(1-c^(1/2)*(
g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^
(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(
1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(5/4)/g^
4/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.42 (sec) , antiderivative size = 665, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)\left(10ae^2g^2+c\left(35d^2g^2+14deg(-4f+3gx)+3e^2\left(8f^2-6fgx+5g^2x^2\right)\right)\right)}{cg^3} - \frac{4\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\left(aeg^2(13ef-42dg)+cf(24\right.\right.}{}\right.}{}$$

input
```
Integrate[((d + e*x)^2*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

output
```
(Sqrt[f + g*x]*((2*(a + c*x^2)*(10*a*e^2*g^2 + c*(35*d^2*g^2 + 14*d*e*g*(-
4*f + 3*g*x) + 3*e^2*(8*f^2 - 6*f*g*x + 5*g^2*x^2))))/(c*g^3) - (4*(g^2*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^
2 - 56*d*e*f*g + 35*d^2*g^2))*(a + c*x^2) - I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[
a]*g)*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g
^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/S
qrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f
+ I*Sqrt[a]*g)] + Sqrt[a]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*(5*a*e^2*g^2 + (6*I)
*Sqrt[a]*Sqrt[c]*e*g*(3*e*f - 7*d*g) + c*(-24*e^2*f^2 + 56*d*e*f*g - 35*d^
2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g
)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f -
 (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]
*f + I*Sqrt[a]*g)]))/(c*g^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/
(105*Sqrt[a + c*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.66 (sec) , antiderivative size = 878, normalized size of antiderivative = 1.31, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {723, 27, 2185, 27, 2185, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}(d+ex)^2}{\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{ 723}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-\frac{\int \frac{2(d+ex)\left(c(3ef-2dg)x^2+(cdf-aeg)x+a(2ef-3dg)\right)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{7g}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-\frac{2\int \frac{(d+ex)\left(c(3ef-2dg)x^2+(cdf-aeg)x+a(2ef-3dg)\right)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{7g}$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-\frac{2\left(\frac{2\int -\frac{c\left(5ae^2g^2+c\left(21e^2f^2-34degf+10d^2g^2\right)\right)x^2g^2+ac\left(9e^2f^2-16degf+15d^2g^2\right)g^2-c\left(aeg^2(ef-14dg)-cf\left(6e^2f^2-4degf-5d^2g^2\right)\right)xg}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{5cg^3}+\frac{2e\sqrt{a+cx}\cdots}{\cdots}\cdots\right.}{7g}$$

$\downarrow$ 27

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-\frac{2\left(\frac{2e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{5g^2}-\frac{\int \frac{c\left(5ae^2g^2+c\left(21e^2f^2-34degf+10d^2g^2\right)\right)x^2g^2+ac\left(9e^2f^2-16degf+15d^2g^2\right)g^2-c\left(aeg^2(ef-14dg)-cf\left(6e^2f^2-\cdots\right.\right.}{\sqrt{f+gx}\sqrt{cx^2+a}}\cdots}{5cg^3}\cdots\right.}{7g}$$

$\downarrow$ 2185

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-\frac{2\left(\frac{2e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{5g^2}-\frac{\frac{2\int -\frac{cg^3\left(ag\left(5ae^2g^2-c\left(6e^2f^2-14degf+35d^2g^2\right)\right)+c\left(ae(13ef-42dg)g^2+cf\left(24e^2f^2-56degf+35d^2g^2\right)\right)x\right)}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3cg^2}+\cdots}{5cg^3}\cdots\right.}{7g}$$

$\downarrow$ 27

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-\frac{2\left(\frac{2e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{5g^2}-\frac{\frac{2}{3}g\sqrt{a+cx^2}\sqrt{f+gx}\left(5ae^2g^2+c\left(10d^2g^2-34defg+21e^2f^2\right)\right)-\frac{1}{3}g\int \frac{ag\left(5ae^2g^2-c\left(6e^2f^2-14degf+35d^2g^2\right)\right)\cdots}{\cdots}}{5cg^3}\cdots\right.}{7g}$$

$\downarrow$ 599

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-\frac{2\left(\frac{2e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{5g^2}-\frac{\frac{2\int -\frac{\left(cf^2+ag^2\right)\left(5ae^2g^2-c\left(24e^2f^2-56degf+35d^2g^2\right)\right)+c\left(ae(13ef-42dg)g^2+cf\left(24e^2f^2-56degf+35d^2g^2\right)\right)(f\cdots}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}\cdots}{3g}\cdots}{5cg^3}\cdots\right.}{7g}$$

$\downarrow$ 25

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-$$

$$\frac{2\left(\frac{2e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{5g^2}-\frac{\frac{2}{3}g\sqrt{a+cx^2}\sqrt{f+gx}\left(5ae^2g^2+c\left(10d^2g^2-34defg+21e^2f^2\right)\right)-\frac{2\int\frac{\left(cf^2+ag^2\right)\left(5ae^2g^2-c\left(24e^2f^2-56degf+3\cdots\right.\right.}{\sqrt{\frac{c\cdots}{g\cdots}}}}{\cdots}}{5cg^3}\cdots\right.}{7g}$$

$\downarrow$ 1511

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-$$

$$\frac{2\left(\frac{2e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{5g^2}-\frac{2\left(\sqrt{c}\sqrt{ag^2+cf^2}\left(aeg^2(13ef-42dg)+cf\left(35d^2g^2-56defg+24e^2f^2\right)\right)\int\frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+g\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$$

$\downarrow$ 1416

$$\frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}-$$

$$\frac{2\left(\frac{2e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{5g^2}-\frac{2\left(\sqrt{c}\sqrt{ag^2+cf^2}\left(aeg^2(13ef-42dg)+cf\left(35d^2g^2-56defg+24e^2f^2\right)\right)\int\frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+g\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$$

$\downarrow$ 1509

$$\frac{2(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}{7g} - \frac{2\left(\frac{2e(3ef-2dg)(f+gx)^{3/2}\sqrt{cx^2+a}}{5g^2} - \frac{\frac{2}{3}g\sqrt{f+gx}\sqrt{cx^2+a}\left(5ae^2g^2+c\left(21e^2f^2-34degf+10d^2g^2\right)\right) + \frac{2\left(\sqrt{c}\sqrt{cf^2+ag^2}\left(ae(13ef-42dg)g^2+cf\left(24e^2f\right.\right.\right.}{}}{}\right.}{}$$

input
```
Int[((d + e*x)^2*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

output
```
(2*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(7*g) - (2*((2*e*(3*e*f - 2*
d*g)*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(5*g^2) - ((2*g*(5*a*e^2*g^2 + c*(21
*e^2*f^2 - 34*d*e*f*g + 10*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/3 + (2
*(Sqrt[c]*Sqrt[c*f^2 + a*g^2]*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2
 - 56*d*e*f*g + 35*d^2*g^2))*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c
*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]
*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*
(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/
g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqr
t[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 +
a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a +
 (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])) - ((c*f^2 +
a*g^2)^(3/4)*(Sqrt[c*f^2 + a*g^2]*(5*a*e^2*g^2 - c*(24*e^2*f^2 - 56*d*e*f*
g + 35*d^2*g^2)) + Sqrt[c]*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2 -
56*d*e*f*g + 35*d^2*g^2)))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*S
qrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a +
(c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2
*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/S
qrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x)
)/g^2 + (c*(f + g*x)^2)/g^2])))/(3*g))/(5*c*g^3)))/(7*g)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 723
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[2*(d + e*x)^m*Sqrt[f + g*x]*(Sqrt[a + c*x^2]/(g
*(2*m + 3))), x] - Simp[1/(g*(2*m + 3))   Int[((d + e*x)^(m - 1)/(Sqrt[f +
g*x]*Sqrt[a + c*x^2]))*Simp[2*a*(e*f*m - d*g*(m + 1)) + (2*c*d*f - 2*a*e*g)
*x - (2*c*(d*g*m - e*f*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g
}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && !(EqQ[d, 0] &&
True) && !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 3.95 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.26

method	result
elliptic	$\sqrt{(gx+f)(cx^2+a)}\left(\frac{2e^2x^2\sqrt{cg\,x^3+cf\,x^2+agx+af}}{7g}+\frac{2\left(2dec-\frac{6cf\,e^2}{7g}\right)x\sqrt{cg\,x^3+cf\,x^2+agx+af}}{5cg}+\frac{2\left(\frac{2a\,e^2}{7}+c\,d^2-\frac{4f\left(2dec-\frac{6cf\,e^2}{7g}\right)}{5g}\right)\sqrt{cg}}{3cg}\right.$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/7*e^2/g*x^2*(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/5*(2*d*e*c-6/7*c*f/g*e^2)/c/g*x*(c*g*x^3+
c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(2/7*a*e^2+c*d^2-4/5*f/g*(2*d*e*c-6/7*c*f/g*e
^2))/c/g*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(a*d^2-2/5*a*f/c/g*(2*d*e*c-6
/7*c*f/g*e^2)-1/3*a/c*(2/7*a*e^2+c*d^2-4/5*f/g*(2*d*e*c-6/7*c*f/g*e^2)))*(
f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/
c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))
^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1
/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(2*a*
d*e-4/7*a*f/g*e^2-3/5*a/c*(2*d*e*c-6/7*c*f/g*e^2)-2/3*f/g*(2/7*a*e^2+c*d^2
-4/5*f/g*(2*d*e*c-6/7*c*f/g*e^2)))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*
c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-
a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/
2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(
(-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*Ellipti
cF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c
)^(1/2)/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.61

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\left(2\left(24\,c^2e^2f^4-56\,c^2def^3g-84\,acdefg^3+(35\,c^2d^2+31\,ace^2)f^2g^2+15\,(7\,acd^2-a^2e^2)g^4\right)\sqrt{cg}\text{weierst}\right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(2*(24*c^2*e^2*f^4 - 56*c^2*d*e*f^3*g - 84*a*c*d*e*f*g^3 + (35*c^2*d
^2 + 31*a*c*e^2)*f^2*g^2 + 15*(7*a*c*d^2 - a^2*e^2)*g^4)*sqrt(c*g)*weierst
rassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g
^3), 1/3*(3*g*x + f)/g) + 6*(24*c^2*e^2*f^3*g - 56*c^2*d*e*f^2*g^2 - 42*a*
c*d*e*g^4 + (35*c^2*d^2 + 13*a*c*e^2)*f*g^3)*sqrt(c*g)*weierstrassZeta(4/3
*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrass
PInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3),
 1/3*(3*g*x + f)/g)) + 3*(15*c^2*e^2*g^4*x^2 + 24*c^2*e^2*f^2*g^2 - 56*c^2
*d*e*f*g^3 + 5*(7*c^2*d^2 + 2*a*c*e^2)*g^4 - 6*(3*c^2*e^2*f*g^3 - 7*c^2*d*
e*g^4)*x)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c^2*g^5)
```

Sympy [F]

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}\,(d+ex)^2}{\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**2*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)**2/sqrt(f + g*x), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}\,(ex+d)^2}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

Giac [F]

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)^2}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^2/sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+a}\,(d+e\,x)^2}{\sqrt{f+g\,x}}\,dx$$

input
```
int(((a + c*x^2)^(1/2)*(d + e*x)^2)/(f + g*x)^(1/2),x)
```

output
```
int(((a + c*x^2)^(1/2)*(d + e*x)^2)/(f + g*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^2\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$
$$= \frac{28\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,ade\,g^2 - 2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,a\,e^2fg + 28\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,cdefgx - 12\sqrt{gx+}}{}$$

input
```
int((e*x+d)^2*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x)
```

output

```
(28*sqrt(f + g*x)*sqrt(a + c*x**2)*a*d*e*g**2 - 2*sqrt(f + g*x)*sqrt(a + c
*x**2)*a*e**2*f*g + 28*sqrt(f + g*x)*sqrt(a + c*x**2)*c*d*e*f*g*x - 12*sqr
t(f + g*x)*sqrt(a + c*x**2)*c*e**2*f**2*x + 10*sqrt(f + g*x)*sqrt(a + c*x*
*2)*c*e**2*f*g*x**2 - 42*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f +
a*g*x + c*f*x**2 + c*g*x**3),x)*a*c*d*e*g**3 + 13*int((sqrt(f + g*x)*sqrt(
a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*c*e**2*f*g**2 +
 35*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*
g*x**3),x)*c**2*d**2*f*g**2 - 56*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)
/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c**2*d*e*f**2*g + 24*int((sqrt(f +
 g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c**2*e
**2*f**3 - 14*int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2
 + c*g*x**3),x)*a**2*d*e*g**3 + int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f
+ a*g*x + c*f*x**2 + c*g*x**3),x)*a**2*e**2*f*g**2 + 35*int((sqrt(f + g*x)
*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*c*d**2*f*g**2
- 28*int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x*
*3),x)*a*c*d*e*f**2*g + 12*int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g
*x + c*f*x**2 + c*g*x**3),x)*a*c*e**2*f**3)/(35*c*f*g**2)
```

3.119 $\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 520

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = -\frac{2\sqrt{f+gx}(4ef-5dg-3egx)\sqrt{a+cx^2}}{15g^2}$$
$$-\frac{4\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(3aeg^2+cf(4ef-5dg))\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\Big(\arcsin}{15c^{3/4}g^4\sqrt{a+cx^2}}$$
$$-\frac{4\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(3\sqrt{-a}aeg^2+a\sqrt{c}g(ef-5dg)+\sqrt{-a}cf(4ef-5dg)\right)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+}{\sqrt{c}f+\sqrt{}}}}{15c^{3/4}g^3\sqrt{a+cx^2}}$$

output
```
-2/15*(g*x+f)^(1/2)*(-3*e*g*x-5*d*g+4*e*f)*(c*x^2+a)^(1/2)/g^2-4/15*(c^(1/
2)*f-(-a)^(1/2)*g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(3*a*e*g^2+c*f*(-5*d*g+4
*e*f))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+
f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2
)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g)
)^(1/2))/c^(3/4)/g^4/(c*x^2+a)^(1/2)-4/15*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(
3*(-a)^(1/2)*a*e*g^2+a*c^(1/2)*g*(-5*d*g+e*f)+(-a)^(1/2)*c*f*(-5*d*g+4*e*f
))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(
c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+
(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1
/2))/c^(3/4)/g^3/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.25 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(-4ef+5dg+3egx)(a+cx^2)}{g^2} + \frac{4\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\,(3aeg^2+cf(4ef-5dg))\,(a+cx^2)-\sqrt{c}(i\sqrt{c}f-\sqrt{a}g)\,(3aeg^2+cf(4ef-5dg))\,\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((d + e*x)*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

output
```
(Sqrt[f + g*x]*((2*(-4*e*f + 5*d*g + 3*e*g*x)*(a + c*x^2))/g^2 + (4*(g^2*S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*(a + c*x
^2) - Sqrt[c]*(I*Sqrt[c]*f - Sqrt[a]*g)*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*
Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
qrt[a]*g)] + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((3*I)*Sqrt[a]*e*
g + Sqrt[c]*(-4*e*f + 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)
]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Ellipti
cF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f -
 I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c*g^4*Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]]*(f + g*x))))/(15*Sqrt[a + c*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {682, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}(d+ex)}{\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{ 682}$$

$$\frac{4\int -\frac{c\left(ag(ef-5dg)-\left(3aeg^2+cf(4ef-5dg)\right)x\right)}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{15cg^2}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2\int \frac{ag(ef-5dg)-\left(3aeg^2+cf(4ef-5dg)\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{15g^2}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2}$$

$$\Big\downarrow \text{ 599}$$

$$\frac{4\int -\frac{(4ef-5dg)\left(cf^2+ag^2\right)-\left(3aeg^2+cf(4ef-5dg)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{15g^4}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{4\int \frac{(4ef-5dg)\left(cf^2+ag^2\right)-\left(3aeg^2+cf(4ef-5dg)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{15g^4}-\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2}$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{4\left(\frac{\sqrt{ag^2+cf^2}\left(-\sqrt{c}\sqrt{ag^2+cf^2}(4ef-5dg)+3aeg^2+cf(4ef-5dg)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt{ag^2+cf^2}\left(3aeg^2+cf(4ef-\cdots\right.}{\cdots}\right.}{15g^4}$$

$$\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2}$$

$$\Big\downarrow \text{ 1416}$$

$$4\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(-\sqrt{c}\sqrt{ag^2+cf^2}(4ef-5dg)+3aeg^2+cf(4ef-5dg)\right)\text{EllipticF}\left(2\arctan\left(-\right.\right.}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.$$

$$15g^4$$

$$\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2}$$

$\downarrow$ 1509

$$4\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(-\sqrt{c}\sqrt{ag^2+cf^2}(4ef-5dg)+3aeg^2+cf(4ef-5dg)\right)\text{EllipticF}\left(2\arctan\left(-\right.\right.}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.$$

$$\frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2}$$

input
```
Int[((d + e*x)*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

output
```
(-2*Sqrt[f + g*x]*(4*e*f - 5*d*g - 3*e*g*x)*Sqrt[a + c*x^2])/(15*g^2) + (4
*(-((Sqrt[c*f^2 + a*g^2]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*(-((Sqrt[f + g*
x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((
a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2
 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c
*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2
)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^
(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 +
 a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f
 + g*x)^2)/g^2])))/Sqrt[c]) + ((c*f^2 + a*g^2)^(3/4)*(3*a*e*g^2 + c*f*(4*e
*f - 5*d*g) - Sqrt[c]*(4*e*f - 5*d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(
f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g
^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt
[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a
*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(3/4)*Sqrt[a
+ (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(15*g^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.30

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^{2}+a)}\left(\frac{2ex\sqrt{cg\,x^{3}+cf\,x^{2}+agx+af}}{5g}+\frac{2\left(cd-\frac{4cfe}{5g}\right)\sqrt{cg\,x^{3}+cf\,x^{2}+agx+af}}{3cg}+\frac{2\left(ad-\frac{2afe}{5g}-\frac{a\left(cd-\frac{4cfe}{5g}\right)}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x}{\frac{f}{g}-}}}{}\right.}{}$
risch	$\frac{2(3egx+5dg-4ef)\sqrt{gx+f}\,\sqrt{c\,x^{2}+a}}{15g^{2}}+\frac{2\left(\frac{2\left(3ae\,g^{2}-5cdfg+4ce\,f^{2}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\left(-\frac{f}{g}-\sqrt{}\right.}{\sqrt{cg\,x^{3}+c}}\right.}{}$
default	Expression too large to display

input
```
int((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/5*e/g*x*(c*g*x^
3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(c*d-4/5*c*f/g*e)/c/g*(c*g*x^3+c*f*x^2+a*g*
x+a*f)^(1/2)+2*(a*d-2/5*a*f/g*e-1/3*a/c*(c*d-4/5*c*f/g*e))*(f/g-(-a*c)^(1/
2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c
)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^
3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),
((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(2/5*a*e-2/3*f/g*(c
*d-4/5*c*f/g*e))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)
*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/
g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(
1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)
/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(
-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))
))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = -\frac{2\left(2\left(4\,cef^3-5\,cdf^2g+6\,aefg^2-15\,adg^3\right)\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right.\right.}{}$$

input
```
integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(2*(4*c*e*f^3 - 5*c*d*f^2*g + 6*a*e*f*g^2 - 15*a*d*g^3)*sqrt(c*g)*we
ierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)
/(c*g^3), 1/3*(3*g*x + f)/g) + 6*(4*c*e*f^2*g - 5*c*d*f*g^2 + 3*a*e*g^3)*s
qrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a
*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*
(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) - 3*(3*c*e*g^3*x - 4*c*e*
f*g^2 + 5*c*d*g^3)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c*g^4)
```

Sympy [F]

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}(d+ex)}{\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)/sqrt(f + g*x), x)
```

Maxima [F]

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x)
```

Giac [F]

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)}{\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx=\int \frac{\sqrt{c\,x^2+a}\,(d+e\,x)}{\sqrt{f+g\,x}}\,dx$$

input
```
int(((a + c*x^2)^(1/2)*(d + e*x))/(f + g*x)^(1/2),x)
```

output
```
int(((a + c*x^2)^(1/2)*(d + e*x))/(f + g*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,aeg+2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,cefx-3\left(\int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,x^2}{cg\,x^3+cf\,x^2+agx+af}dx\right)ace\,g^2+5\left(\int \frac{\sqrt{gx+f}\,\sqrt{c}}{cg\,x^3+cf\,x^2}\right.}{}$$

input
```
int((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x)
```

output
```
(2*sqrt(f + g*x)*sqrt(a + c*x**2)*a*e*g + 2*sqrt(f + g*x)*sqrt(a + c*x**2)
*c*e*f*x - 3*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*
x**2 + c*g*x**3),x)*a*c*e*g**2 + 5*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**
2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c**2*d*f*g - 4*int((sqrt(f + g*x
)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c**2*e*f**
2 - int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**
3),x)*a**2*e*g**2 + 5*int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x +
c*f*x**2 + c*g*x**3),x)*a*c*d*f*g - 2*int((sqrt(f + g*x)*sqrt(a + c*x**2))
/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*c*e*f**2)/(5*c*f*g)
```

3.120 $\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 462

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}} \, dx = \frac{2\sqrt{f+gx}\sqrt{a+cx^2}}{3g} + \frac{4\sqrt[4]{c}f\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right)|\frac{\sqrt{c}f+\cdots}{\sqrt{c}f-\cdots}\right.}{3g^3\sqrt{a+cx^2}} + \frac{4\sqrt{-a}\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right.\right.}{3\sqrt[4]{c}g^2\sqrt{a+cx^2}}$$

output
```
2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+4/3*c^(1/4)*f*(c^(1/2)*f-(-a)^(1/2)*g)
*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g
))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticE(c^(1
/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)
/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/g^3/(c*x^2+a)^(1/2)+4/3*(-a)^(1/2)*(c^(1
/2)*f-(-a)^(1/2)*g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(
1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(
1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1
/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(1/4)/g^2/(c*x^2+a)
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.31 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$
$$= \frac{2\sqrt{f+gx}\left(g^2(a+cx^2)-\frac{2\left(fg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(a+cx^2)+\sqrt{c}f(-i\sqrt{c}f+\sqrt{a}g)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)^{3/2}E\left(i\operatorname{arcsinh}\left(\cdots\right.\right.\right.}{\cdots}\right.}{3g^3\sqrt{a\cdots}}$$

input
```
Integrate[Sqrt[a + c*x^2]/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*(g^2*(a + c*x^2) - (2*(f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]*(a + c*x^2) + Sqrt[c]*f*((-I)*Sqrt[c]*f + Sqrt[a]*g)*Sqrt[(g*((I*Sqrt
[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*
x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Sqr
t[a]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g
*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Elli
pticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*
f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]*(f + g*x))))/(3*g^3*Sqrt[a + c*x^2])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {493, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

$\downarrow$ 493

$$\frac{2\int \frac{ag-cfx}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3g}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g}$$

$\downarrow$ 599

$$\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g}-\frac{4\int -\frac{cf^2-c(f+gx)f+ag^2}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^3}$$

$\downarrow$ 25

$$\frac{4\int \frac{cf^2-c(f+gx)f+ag^2}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^3}+\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g}$$

$\downarrow$ 1511

$$\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g}-$$
$$\frac{4\left(\sqrt{ag^2+cf^2}\left(\sqrt{c}f-\sqrt{ag^2+cf^2}\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\sqrt{c}f\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f}{\sqrt{cf^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}}}\right.}{3g^3}$$

$\downarrow$ 1416

$$\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g}-$$
$$\frac{4\left(\frac{(ag^2+cf^2)^{3/4}\left(\sqrt{c}f-\sqrt{ag^2+cf^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.}{3g^3}$$

$\downarrow$ 1509

$$\frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g} - \frac{4\left(\frac{(ag^2+cf^2)^{3/4}\left(\sqrt{c}f-\sqrt{ag^2+cf^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}\cdots\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\cdots\right.}{\cdots}$$

input
```
Int[Sqrt[a + c*x^2]/Sqrt[f + g*x],x]
```

output
```
(2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*g) - (4*(-(Sqrt[c]*f*Sqrt[c*f^2 + a*g
^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f
 + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 +
a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a
*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2
)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*Ell
ipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt
[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f
+ g*x))/g^2 + (c*(f + g*x)^2)/g^2]))) + ((c*f^2 + a*g^2)^(3/4)*(Sqrt[c]*f
- Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[
(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f
^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[
c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^
2 + (c*(f + g*x)^2)/g^2])))/(3*g^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.26

method	result
risch	$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{3g}+\frac{2\left(-\frac{2cf\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\left(\left(-\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+}{-\frac{f}{g}-}}\right.\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+af}}\right.}{}$
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2\sqrt{cg\,x^3+cf\,x^2+agx+af}}{3g}+\frac{4a\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}}{-\frac{f}{g}}}\right.}{3\sqrt{cg\,x^3+cf\,x^2+agx+af}}\right.}{}$
default	$-\frac{2\sqrt{c\,x^2+a}\,\sqrt{gx+f}\left(2\sqrt{-ac}\,\sqrt{-\frac{(gx+f)c}{\sqrt{-ac}\,g-cf}}\,\sqrt{\frac{(-cx+\sqrt{-ac})g}{\sqrt{-ac}\,g+cf}}\,\sqrt{\frac{(cx+\sqrt{-ac})g}{\sqrt{-ac}\,g-cf}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{(gx+f)c}{\sqrt{-ac}\,g-cf}},\sqrt{-\frac{\sqrt{-ac}\,g-cf}{\sqrt{-ac}\,g+cf}}\right)a\,g^3+2\right.}{}$

input
```
int((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/g+2/3/g*(-2*c*f*(f/g-(-a*c)^(1/2)/c)*((x
+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c
))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2
+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1
/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^
(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)
/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+2*a*g*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/
g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*
((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*
f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2
)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))*((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2
)/(c*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \frac{2\left(6\sqrt{cg}cfg\text{weierstrassZeta}\left(\frac{4\left(cf^2-3\,ag^2\right)}{3\,cg^2},-\frac{8\left(cf^3+9\,afg^2\right)}{27\,cg^3},\text{weierstrassPInverse}\left(\frac{4\left(cf^2-3\,ag^2\right)}{3\,cg^2},-\frac{8\left(cf^3+9\,afg^2\right)}{27\,cg^3}\right.\right.\right.}{}$$

input
```
integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(6*sqrt(c*g)*c*f*g*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/2
7*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(
c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + 3*sqrt(c*
x^2 + a)*sqrt(g*x + f)*c*g^2 + 2*(c*f^2 + 3*a*g^2)*sqrt(c*g)*weierstrassPI
nverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1
/3*(3*g*x + f)/g))/(c*g^3)
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)/sqrt(f + g*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx=\int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}}\,dx$$

input
```
int((a + c*x^2)^(1/2)/(f + g*x)^(1/2),x)
```

output
```
int((a + c*x^2)^(1/2)/(f + g*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}}\,dx=\int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{gx+f}dx$$

input
```
int((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x)
```

output
```
int((sqrt(f + g*x)*sqrt(a + c*x**2))/(f + g*x),x)
```

3.121 $\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 670

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = -\frac{2\sqrt[4]{c}\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right)|\frac{\sqrt{c}f+\sqrt{}}{\sqrt{c}f-\sqrt{}}\right.}{eg^2\sqrt{a+cx^2}} - \frac{2\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{-a}e\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right)\right.}{e^2g\sqrt{a+cx^2}} - \frac{2(cd^2+ae^2)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\text{EllipticPi}\left(\frac{e\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}{ef-dg},\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+}}{\sqrt{\sqrt{c}f+\sqrt{-}}}\right.\right.}{\sqrt[4]{c}e^2(ef-dg)\sqrt{a+cx^2}}$$

output
```
-2*c^(1/4)*(c^(1/2)*f-(-a)^(1/2)*g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-c^(1
/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+
(-a)^(1/2)*g))^(1/2)*EllipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)
*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/e/g^2
/(c*x^2+a)^(1/2)-2*c^(1/4)*(c^(1/2)*d+(-a)^(1/2)*e)*(c^(1/2)*f+(-a)^(1/2)*
g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*
x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1
/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*
g))^(1/2))/e^2/g/(c*x^2+a)^(1/2)-2*(a*e^2+c*d^2)*(c^(1/2)*f+(-a)^(1/2)*g)^
(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f
)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticPi(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2
)*f+(-a)^(1/2)*g)^(1/2),e*(f+(-a)^(1/2)*g/c^(1/2))/(-d*g+e*f),((c^(1/2)*f+
(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(1/4)/e^2/(-d*g+e*f)/(c*x
^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.01 (sec) , antiderivative size = 1096, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

output
```
(-2*(-(c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f -
 (I*Sqrt[a]*g)/Sqrt[c]] - a*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a
*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]*(f + g*x) - 2*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f
 + g*x) - c*e^2*f*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f +
Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sq
rt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I
*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*S
qrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(I*Sqrt[c]*d + Sqrt[a]*e)*g*(Sqrt
[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(
((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcS
inh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a
]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - I*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]
 + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*
x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I
*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*S
qrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - I*a*e^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sq
rt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f
 + g*x)^(3/2)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a...
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1496 vs. $2(670) = 1340$.

Time = 2.68 (sec) , antiderivative size = 1496, normalized size of antiderivative = 2.23, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {724, 27, 599, 25, 729, 25, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{ 724}$$

$$\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{\int \frac{c(d-ex)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}$$

$$\Big\downarrow \text{27}$$

$$\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{c\int \frac{d-ex}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e^2}$$

$$\Big\downarrow \text{599}$$

$$\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2c\int -\frac{ef+dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g^2}$$

$$\Big\downarrow \text{25}$$

$$\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2c\int \frac{ef+dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g^2}$$

$$\Big\downarrow \text{729}$$

$$2\left(a+\frac{cd^2}{e^2}\right)\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\int \frac{ef+dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g^2}$$

$$\Big\downarrow \text{25}$$

$$-2\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\int \frac{ef+dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g^2}$$

$$\Big\downarrow \text{1511}$$

$$\frac{2c\left(-\left(\left(e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)+dg\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}\right)-\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+}}}{\sqrt{c}}\right)}{e^2g^2}$$

$$2\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}$$

$\downarrow$ 1416

$$\frac{2c\left(-\frac{e\sqrt{ag^2+cf^2}\int\frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(e\left(f-\sqrt{\cdots}\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\cdots}}\right.}{e^2g^2}$$

$$2\left(a+\frac{cd^2}{e^2}\right)\int\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}$$

$\downarrow$ 1509

$$\frac{2c\left(-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)+dg\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.}{\cdots}$$

$$2\left(a+\frac{cd^2}{e^2}\right)\int\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}$$

$\downarrow$ 1540

$$\frac{2c\left(-\frac{e\sqrt{cf^2+ag^2}\left(\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-\frac{\sqrt{f}\cdots}{\cdots}\right)}{\sqrt{c}}\cdots\right)}{2\left(\frac{cd^2}{e^2}+a\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int\frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+g\cdots}}{g\left(age^2+cd(2ef-dg)\right)}\cdots\right)}$$

$$\Big\downarrow \; 1416$$

$$\frac{2c\left(-\frac{e\sqrt{cf^2+ag^2}\left(\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-\frac{\sqrt{f}\cdots}{\cdots}\right)}{\sqrt{c}}\cdots\right)}{2\left(\frac{cd^2}{e^2}+a\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int\frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+g\cdots}}{g\left(age^2+cd(2ef-dg)\right)}\cdots\right)}$$

$$\downarrow \text{2222}$$

$$2c\left(-\frac{e\sqrt{cf^2+ag^2}\left(\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-\frac{\sqrt{f}\cdots}{\cdots}\right.}{\sqrt{c}}\right.$$

$$\overline{2\left(\frac{cd^2}{e^2}+a\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\left(\frac{\left(e+\frac{\sqrt{c}(ef-dg)}{\sqrt{cf^2+ag^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+\cdots}{g\cdots}}}\right.}{2\sqrt{e}\sqrt{cd^2+ae^2}\sqrt{ef-dg}}\right.}{\cdots}\right.}$$

input

```
Int[Sqrt[a + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

output
```
(2*c*(-((e*Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2
*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[
c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c
]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x)
)/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/S
qrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2
+ a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a
 + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2]))))/Sqrt[c])
- ((c*f^2 + a*g^2)^(1/4)*(d*g + e*(f - Sqrt[c*f^2 + a*g^2]/Sqrt[c]))*(1 +
(Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f
 + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f +
g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/
(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/
4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/
(e^2*g^2) + 2*(a + (c*d^2)/e^2)*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g^2 - Sqrt[c
]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a
*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2
)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*Ell
ipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt
[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(g*(c*f^2 + a*g^2)^(1/4)*(a*e^2*g + c*d...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 724
```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)
]), x_Symbol] :> Simp[(c*d^2 + a*e^2)/e^2   Int[1/((d + e*x)*Sqrt[f + g*x]*
Sqrt[a + c*x^2]), x], x] - Simp[1/e^2   Int[(c*d - c*e*x)/(Sqrt[f + g*x]*Sq
rt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 833, normalized size of antiderivative = 1.24

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(-\frac{2cd\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{e^2\sqrt{cg\,x^3+cf\,x^2+agx+af}}+\frac{2c\left(\frac{f}{g}-\frac{\sqrt{-}}{c}\cdots\right.}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-2*c*d/e^2*(f/g-(
-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-
f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2
)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c
))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*c/e*(f/g-(
-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-
f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2
)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f
/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/
c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(
(-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+2*(a*e^2+c*d^2)/e^3*(
f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/
c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))
^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f
/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(
1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}}{(d+ex)\,\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/(e*x+d)/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)/((d + e*x)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)),x)
```

output
```
int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}}\,dx = \int \frac{\sqrt{c\,x^2+a}}{(ex+d)\,\sqrt{gx+f}}dx$$

input
```
int((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x)
```

output
```
int((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x)
```

3.122 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 743

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(ef-dg)(d+ex)}$$

$$+\frac{\sqrt[4]{c}\left(\sqrt{-a}-\frac{\sqrt{c}f}{g}\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right)|\frac{\sqrt{c}f+\sqrt{-a}g}{\sqrt{c}f-\sqrt{-a}g}\cdots\right.}{e(ef-dg)\sqrt{a+cx^2}}$$

$$-\frac{\sqrt[4]{c}\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(\sqrt{-a}eg-\sqrt{c}(2ef-dg)\right)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{e^2g(ef-dg)\sqrt{a+cx^2}}$$

$$+\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(ae^2g+cd(2ef-dg)\right)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticPi}\left(\frac{e\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}{ef-dg},\arcsin\cdots\right.}{\sqrt[4]{c}e^2(ef-dg)^2\sqrt{a+cx^2}}$$

output
```
-(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(-d*g+e*f)/(e*x+d)+c^(1/4)*((-a)^(1/2)-c^(1
/2)*f/g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)
^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*Ellipt
icE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^
(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/e/(-d*g+e*f)/(c*x^2+a)^(1/2)-c^(
1/4)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*((-a)^(1/2)*e*g-c^(1/2)*(-d*g+2*e*f))*
(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(
1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a
)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)
)/e^2/g/(-d*g+e*f)/(c*x^2+a)^(1/2)+(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(a*e^2*g
+c*d*(-d*g+2*e*f))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c
^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticPi(c^(1/4)*(g*x+f)^
(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),e*(f+(-a)^(1/2)*g/c^(1/2))/(-d*g+e*f)
,((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(1/4)/e^2/(-
d*g+e*f)^2/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.12 (sec) , antiderivative size = 1336, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

output
```
(Sqrt[f + g*x]*((a + c*x^2)/(d + e*x) - (c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]] - c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*e^2*f*g^2*Sqr
t[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
 - 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 2*c*d*e*f*g*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + c*e^2*f*Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]]*(f + g*x)^2 - c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)
^2 + I*Sqrt[c]*e*(Sqrt[c]*f + I*Sqrt[a]*g)*(-(e*f) + d*g)*Sqrt[(g*((I*Sqrt
[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*
x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/
Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(
Sqrt[c]*f + I*Sqrt[a]*g)*(Sqrt[a]*e*g + I*Sqrt[c]*(2*e*f - d*g))*Sqrt[(g*(
(I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/
(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)
] - (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-((
(I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt[
c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
qrt[a]*g)] + I*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqr
t[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticP...
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1559 vs. $2(743) = 1486$.

Time = 4.58 (sec) , antiderivative size = 1559, normalized size of antiderivative = 2.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {725, 25, 2349, 599, 27, 729, 25, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{725}$$

$$\frac{\int -\frac{-cgx^2-2cfx+ag}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{-cgx^2-2cfx+ag}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 2349$$

$$-\frac{\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-\frac{2cf}{e}+\frac{cdg}{e^2}-\frac{cgx}{e}}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 599$$

$$-\frac{\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int \frac{cg(ef-dg+e(f+gx))}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2c\int \frac{ef-dg+e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e^2g}}{2(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 729$$

$$-\frac{2\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\int \frac{ef-dg+e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+}}{e^2g}}{2(ef-dg)}$$
$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$$\Big\downarrow \; 25$$

$$-\frac{-2\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\int \frac{ef-dg+e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+}}{e^2g}}{2(ef-dg)}$$
$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$\Big\downarrow$ 1511

$$-\frac{-2\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\left(-\left(\left(dg-e\left(\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}+f\right)\right)\right)\int \frac{\ldots}{\sqrt{\frac{cf^2}{g^2}\ldots}}\ldots\right.}{2(ef-dg)\ldots}}{\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}}$$

$\Big\downarrow$ 1416

$$-\frac{-\frac{2c\left(-\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(dg-e\left(\frac{\sqrt{ag^2\ldots}}{\ldots}\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+\ldots}{g^2}\ldots}}\ldots\right)}{e^2g}\ldots}{\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}}$$

$\Big\downarrow$ 1509

$$2c\left(-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\cdots}}{\cdots}\cdots\right.$$

$$-\frac{-2\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{\cdots}{\cdots}}{\cdots}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}$$

$\downarrow$ 1540

$$-\frac{2\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(age^2+cd(2ef-dg))}-\frac{\sqrt{c}\left(cef^2\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{(ef-dg)(d+ex)}$$

$\downarrow$ 1416

$$-\frac{2\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(age^2+cd(2ef-dg))}-\frac{\sqrt[4]{c}\left(ce\ldots\right.}{\ldots}\right.}{\ldots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{(ef-dg)(d+ex)}$$

$\downarrow$ 2222

$$-\frac{2\left(ag+\frac{cd(2ef-dg)}{e^2}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\left(\frac{\left(e+\frac{\sqrt{c}(ef-dg)}{\sqrt{cf^2+ag^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}\ldots\right.}{2\sqrt{e}\sqrt{cd^2+ae^2}\sqrt{ef-dg}}\ldots\right.}{\ldots}\right.}{\ldots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{(ef-dg)(d+ex)}$$

input
```
Int[Sqrt[a + c*x^2]/((d + e*x)^2*Sqrt[f + g*x]),x]
```

output
```
-((Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)*(d + e*x))) - ((-2*c*(-((e*
Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g
*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x
))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x)
)/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*
(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 +
 a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1
/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/
g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/Sqrt[c]) - ((c*f^2 +
 a*g^2)^(1/4)*(d*g - e*(f + Sqrt[c*f^2 + a*g^2]/Sqrt[c]))*(1 + (Sqrt[c]*(f
 + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^
2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[
c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*
g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sqrt[a +
 (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(e^2*g) + 2
*(a*g + (c*d*(2*e*f - d*g))/e^2)*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g^2 - Sqrt[
c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 +
a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^
2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*El
lipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 725
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/((m + 1)*(e*f - d*g))), x] - Simp[1/(2*(m + 1)*(e*f - d*g))   Int[((d + e
*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*g*(2*m + 3) + 2*(c*f)*x
 + c*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && Integer
Q[2*m] && LtQ[m, -1]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 921, normalized size of antiderivative = 1.24

method	result
elliptic	$\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{\sqrt{cg\,x^3+cf\,x^2+agx+af}}{(dg-ef)(ex+d)}+\frac{2\left(\frac{c}{e^2}-\frac{cdg}{2e^2(dg-ef)}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\ \mathrm{EllipticF}\left(\sqrt{\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+af}}\right.$
default	Expression too large to display

input
```
int((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(1/(d*g-e*f)*(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)+2*(c/e^2-1/2*c*d/e^2*g/(d*g-e*f))*(f/
g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)
/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(
1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2
)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-g*c/(d*g-
e*f)/e*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c
)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(
1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*El
lipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-
(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2
)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+(a*e^2*g
-c*d^2*g+2*c*d*e*f)/(d*g-e*f)/e^3*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c
)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a
*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2
)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^
(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)/((d + e*x)**2*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx = \int \frac{\sqrt{cx^2+a}}{(ex+d)^2\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx=\int \frac{\sqrt{cx^2+a}}{(ex+d)^2\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx=\int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}\,(d+e\,x)^2}\,dx$$

input
```
int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2),x)
```

output
```
int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}}\,dx=\int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{e^2g\,x^3+2deg\,x^2+e^2f\,x^2+d^2gx+2defx+d^2f}dx$$

input
```
int((c*x^2+a)^(1/2)/(e*x+d)^2/(g*x+f)^(1/2),x)
```

output
```
int((sqrt(f + g*x)*sqrt(a + c*x**2))/(d**2*f + d**2*g*x + 2*d*e*f*x + 2*d*
e*g*x**2 + e**2*f*x**2 + e**2*g*x**3),x)
```

3.123 $\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 965

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$$
$$= -\frac{\sqrt{f+gx}\sqrt{a+cx^2}}{2(ef-dg)(d+ex)^2} + \frac{(3ae^2g+cd(2ef+dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4(cd^2+ae^2)(ef-dg)^2(d+ex)}$$
$$- \frac{\sqrt[4]{c}\left(3ae^2+cd\left(d+\frac{2ef}{g}\right)\right)\left(\sqrt{-a}-\frac{\sqrt{c}f}{g}\right)g\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\left(\arcsin\left(\right.\right.}{4e(cd^2+ae^2)(ef-dg)^2\sqrt{a+cx^2}}$$
$$- \frac{\sqrt[4]{c}\sqrt{\sqrt{c}f+\sqrt{-a}g}(3(-a)^{3/2}e^3g+c^{3/2}d^2(4ef-dg)-\sqrt{-a}cde(2ef+dg)+a\sqrt{c}e^2(2ef+dg))\sqrt{1-}}{4e^2(cd^2+ae^2)(ef-dg)^2\sqrt{a+}}$$
$$- \frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}(3a^2e^4g^2+c^2d^3g(4ef-dg)+2ace^2(2e^2f^2-2defg+3d^2g^2))\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-}}{4\sqrt[4]{c}e^2(cd^2+ae^2)(ef-dg)^3\sqrt{a+c}}$$

output
```
-1/2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(-d*g+e*f)/(e*x+d)^2+1/4*(3*a*e^2*g+c*d
*(d*g+2*e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)^2/(e*
x+d)-1/4*c^(1/4)*(3*a*e^2+c*d*(d+2*e*f/g))*((-a)^(1/2)-c^(1/2)*f/g)*g*(c^(
1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1
/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticE(c^(1/4)*(
g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(
1/2)*f-(-a)^(1/2)*g))^(1/2))/e/(a*e^2+c*d^2)/(-d*g+e*f)^2/(c*x^2+a)^(1/2)-
1/4*c^(1/4)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(3*(-a)^(3/2)*e^3*g+c^(3/2)*d^2
*(-d*g+4*e*f)-(-a)^(1/2)*c*d*e*(d*g+2*e*f)+a*c^(1/2)*e^2*(d*g+2*e*f))*(1-c
^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)
*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1
/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/e^
2/(a*e^2+c*d^2)/(-d*g+e*f)^2/(c*x^2+a)^(1/2)-1/4*(c^(1/2)*f+(-a)^(1/2)*g)^
(1/2)*(3*a^2*e^4*g^2+c^2*d^3*g*(-d*g+4*e*f)+2*a*c*e^2*(3*d^2*g^2-2*d*e*f*g
+2*e^2*f^2))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)
*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticPi(c^(1/4)*(g*x+f)^(1/2)/
(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),e*(f+(-a)^(1/2)*g/c^(1/2))/(-d*g+e*f),((c^(
1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(1/4)/e^2/(a*e^2+c
*d^2)/(-d*g+e*f)^3/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.63 (sec) , antiderivative size = 2714, normalized size of antiderivative = 2.81

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

output
```
Sqrt[f + g*x]*Sqrt[a + c*x^2]*(1/(2*(-(e*f) + d*g)*(d + e*x)^2) + (2*c*d*e
*f + c*d^2*g + 3*a*e^2*g)/(4*(c*d^2 + a*e^2)*(e*f - d*g)^2*(d + e*x))) + (
(f + g*x)^(3/2)*(-2*c^2*d*e^3*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + c^2*d
^2*e^2*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*a*c*e^4*f*g*Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]] + c^2*d^3*e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 3*a*
c*d*e^3*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - (2*c^2*d*e^3*f^4*Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (c^2*d^2*e^2*f^3*g*Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3*a*c*e^4*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sq
rt[c]])/(f + g*x)^2 + (c^2*d^3*e*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])
/(f + g*x)^2 + (a*c*d*e^3*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g
*x)^2 + (a*c*d^2*e^2*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 -
 (3*a^2*e^4*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (a*c*d^3
*e*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (3*a^2*d*e^3*g^4*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (4*c^2*d*e^3*f^3*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) - (2*c^2*d^2*e^2*f^2*g*Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]])/(f + g*x) + (6*a*c*e^4*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[
c]])/(f + g*x) - (2*c^2*d^3*e*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f +
 g*x) - (6*a*c*d*e^3*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (
I*Sqrt[c]*e*(Sqrt[c]*f + I*Sqrt[a]*g)*(e*f - d*g)*(3*a*e^2*g + c*d*(2*e*f
+ d*g))*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[...
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2774 vs. $2(965) = 1930$.

Time = 9.23 (sec) , antiderivative size = 2774, normalized size of antiderivative = 2.87, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.679$, Rules used = {725, 25, 2349, 734, 2349, 27, 510, 599, 25, 27, 729, 25, 1416, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{725}$$

$$\frac{\int -\frac{cgx^2-2cfx+3ag}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{cgx^2-2cfx+3ag}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$$\Big\downarrow \; 2349$$

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-\frac{2cf}{e}-\frac{cdg}{e^2}+\frac{cgx}{e}}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$$\Big\downarrow \; 734$$

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{\int \frac{-cgx^2e^2+age^2-2cdgxe-2cd(ef-dg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+\int \frac{-\frac{2cf}{e}-\frac{cdg}{e^2}+\frac{cgx}{e}}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$$\Big\downarrow \; 2349$$

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)-2c(d}{4(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)-2c(d}{4(ef-dg)}-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$$\Big\downarrow \; 510$$

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int\frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)-\frac{2c(d\cdots}{\cdots}}{4(ef-dg)}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\Big\downarrow$ 599

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int-\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+c\cdots}}{(d+ex)(ae^2\cdots}\right.}{4(ef-dg)}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\Big\downarrow$ 25

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2\int\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx\cdots}}{(d+ex)(ae^2+\cdots}\right.}{4(ef-dg)}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\Big\downarrow$ 27

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2c\int\frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+c\cdots}}{(d+ex)(ae^2+\cdots}\right.}{4(ef-dg)}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\Big\downarrow$ 729

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{2\left(ae^2g-cd(2ef-3dg)\right)\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}+\frac{2c\int \frac{ef-dg-e(f+}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+}}}{g}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\big\downarrow$ 25

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f}{}}}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\big\downarrow$ 1416

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f}{}}}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\big\downarrow$ 1511

$$-\frac{\left(3ag+\frac{cd(dg+2ef)}{e^2}\right)\left(-\frac{\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int\frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\left(dg-e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)\right)\int\frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+g}{g^2}}}\right.}{g}}{2(ae^2+cd^2)(ef}\right.}{\,} - \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2(ef-dg)}$$

$\downarrow$ 1416

$$-\frac{\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+\frac{4c(e}{\,}}{\,} - \frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2(ef-dg)(d+ex)^2}$$

$\downarrow$ 1509

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+\frac{4c(e\ldots}{\ldots}$$

$$-\frac{\ldots}{\ldots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2(ef-dg)(d+ex)^2}$$

$\downarrow$ 1540

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}} - \frac{4c(e\ldots}{\ldots}$$

$$-\frac{\ldots}{\ldots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2(ef-dg)(d+ex)^2}$$

$\downarrow$ 1416

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}} - \frac{4c(e\ldots}{\ldots}$$

$$-\frac{\ldots}{\ldots}$$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2(ef-dg)(d+ex)^2}$$

$\downarrow$ 2222

$4c(e$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e^2\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}} - $$

$$- \frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2(ef-dg)(d+ex)^2}$$

input
```
Int[Sqrt[a + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

output
```
-1/2*(Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)*(d + e*x)^2) - ((c^(3/4)
*(c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[
(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f
^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[
c*f^2 + a*g^2])/2])/(e^2*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c
*(f + g*x)^2)/g^2]) - (4*c*(e*f + d*g)*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g^2 -
 Sqrt[c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*
f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)
^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^
2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1
+ (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(g*(c*f^2 + a*g^2)^(1/4)*(a*e^2*g +
 c*d*(2*e*f - d*g))*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f +
 g*x)^2)/g^2]) + (e*Sqrt[c*f^2 + a*g^2]*(Sqrt[c]*(e*f - d*g) - e*Sqrt[c*f^
2 + a*g^2])*(((e + (Sqrt[c]*(e*f - d*g))/Sqrt[c*f^2 + a*g^2])*ArcTanh[(Sqr
t[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt[a + (c*f^2)/
g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])])/(2*Sqrt[e]*Sqrt[c*d^
2 + a*e^2]*Sqrt[e*f - d*g]) - ((Sqrt[c]/e - Sqrt[c*f^2 + a*g^2]/(e*f - d*g
))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (
2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sq...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 510
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2/d   Subst[Int[1/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2
)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 725
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(a_) + (c_.)*(x_)^2])/Sqrt[(f_.) + (g_
.)*(x_)], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^2
]/((m + 1)*(e*f - d*g))), x] - Simp[1/(2*(m + 1)*(e*f - d*g))   Int[((d + e
*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*g*(2*m + 3) + 2*(c*f)*x
 + c*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && Integer
Q[2*m] && LtQ[m, -1]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 734
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Simp[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^
2]))*Simp[2*c*d*(e*f - d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*c*e*(d*g*(m + 1
) - e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 5.87 (sec) , antiderivative size = 1196, normalized size of antiderivative = 1.24

method	result	size
elliptic	Expression too large to display	1196
default	Expression too large to display	19187

input
```
int((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(1/2/(d*g-e*f)*(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)^2+1/4*(3*a*e^2*g+c*d^2*g+2*c*d*e*f)
/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(d*g-e*f)*(c*g*x^3+c*f*x^2+a*g*x+a*
f)^(1/2)/(e*x+d)-1/4*c*g*(a*d*e^2*g+2*a*e^3*f-c*d^3*g+4*c*d^2*e*f)/(a*d*e^
2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(d*g-e*f)/e^2*(f/g-(-a*c)^(1/2)/c)*((x+f/g)
/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1
/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*
x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^
(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-1/4*c*g*(3*a*e^2*g+c*d^2*g+2*c*d*e*
f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(d*g-e*f)/e*(f/g-(-a*c)^(1/2)/c)*
((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2
)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*
x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)
^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*
c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1
/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+1/4*(3*a^2*e^4*g^2+6*a*c*d^2*e^2*g^2
-4*a*c*d*e^3*f*g+4*a*c*e^4*f^2-c^2*d^4*g^2+4*c^2*d^3*e*f*g)/(a*d*e^2*g-a*e
^3*f+c*d^3*g-c*d^2*e*f)/e^3/(d*g-e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(
-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x
+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx=\text{Timed out}$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx=\text{Timed out}$$

input
```
integrate((c*x**2+a)**(1/2)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx=\int \frac{\sqrt{cx^2+a}}{(ex+d)^3\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx=\int \frac{\sqrt{cx^2+a}}{(ex+d)^3\sqrt{gx+f}}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)/((e*x + d)^3*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx=\int \frac{\sqrt{c\,x^2+a}}{\sqrt{f+g\,x}\,(d+e\,x)^3}\,dx$$

input
```
int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3),x)
```

output
```
int((a + c*x^2)^(1/2)/((f + g*x)^(1/2)*(d + e*x)^3), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}}\,dx$$
$$=\int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{e^3g\,x^4+3d\,e^2g\,x^3+e^3f\,x^3+3d^2eg\,x^2+3d\,e^2f\,x^2+d^3gx+3d^2efx+d^3f}dx$$

input
```
int((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x)
```

output
```
int((sqrt(f + g*x)*sqrt(a + c*x**2))/(d**3*f + d**3*g*x + 3*d**2*e*f*x + 3
*d**2*e*g*x**2 + 3*d*e**2*f*x**2 + 3*d*e**2*g*x**3 + e**3*f*x**3 + e**3*g*
x**4),x)
```

3.124 $\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 742

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

$$= -\frac{2e(25ae^2g^2-c(8e^2f^2-42defg+105d^2g^2))\sqrt{f+gx}\sqrt{a+cx^2}}{105c^2g^2}$$

$$-\frac{6e^2(3ef-7dg)(f+gx)^{3/2}\sqrt{a+cx^2}}{35cg^2}+\frac{2e^3(f+gx)^{5/2}\sqrt{a+cx^2}}{7cg^2}$$

$$+\frac{2\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}(ae^2g^2(19ef+189dg)-c(8e^3f^3-42de^2f^2g+105d^2efg^2+105d^3g}{105c^{7/4}g^4\sqrt{a+cx^2}}$$

$$+\frac{2\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(105c^2d^3fg^2+25a^2e^3g^3+\sqrt{-a}a\sqrt{c}e^2g^2(19ef+189dg)-aceg(2e^2f^2+147defg+\right.}{}$$

output
```
-2/105*e*(25*a*e^2*g^2-c*(105*d^2*g^2-42*d*e*f*g+8*e^2*f^2))*(g*x+f)^(1/2)
*(c*x^2+a)^(1/2)/c^2/g^2-6/35*e^2*(-7*d*g+3*e*f)*(g*x+f)^(3/2)*(c*x^2+a)^(
1/2)/c/g^2+2/7*e^3*(g*x+f)^(5/2)*(c*x^2+a)^(1/2)/c/g^2+2/105*(c^(1/2)*f-(-
a)^(1/2)*g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(a*e^2*g^2*(189*d*g+19*e*f)-c*(
105*d^3*g^3+105*d^2*e*f*g^2-42*d*e^2*f^2*g+8*e^3*f^3))*(1-c^(1/2)*(g*x+f)/
(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g
))^(1/2)*EllipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((
c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(7/4)/g^4/(c*x^
2+a)^(1/2)+2/105*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(105*c^2*d^3*f*g^2+25*a^2*
e^3*g^3+(-a)^(1/2)*a*c^(1/2)*e^2*g^2*(189*d*g+19*e*f)-a*c*e*g*(105*d^2*g^2
+147*d*e*f*g+2*e^2*f^2)-(-a)^(1/2)*c^(3/2)*(105*d^3*g^3+105*d^2*e*f*g^2-42
*d*e^2*f^2*g+8*e^3*f^3))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2
)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*
x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/
2)*f-(-a)^(1/2)*g))^(1/2))/c^(9/4)/g^3/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.16 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2(a+cx^2)\left(-25ae^3g^2+ce\left(105d^2g^2+21deg(f+3gx)+e^2\left(-4f^2+3fgx+15g^2x^2\right)\right)\right)}{c^2g^2} - \frac{2\left(g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\left(ae^2g^2(19ef+189dg)-c\right.\right.}{}\right.}{}$$

input
```
Integrate[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[f + g*x]*((2*(a + c*x^2)*(-25*a*e^3*g^2 + c*e*(105*d^2*g^2 + 21*d*e*
g*(f + 3*g*x) + e^2*(-4*f^2 + 3*f*g*x + 15*g^2*x^2))))/(c^2*g^2) - (2*(g^2
*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(a*e^2*g^2*(19*e*f + 189*d*g) - c*(8*e^3
*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g^3))*(a + c*x^2) - Sqrt
[c]*(I*a*Sqrt[c]*e^2*f*g^2*(19*e*f + 189*d*g) - a^(3/2)*e^2*g^3*(19*e*f +
189*d*g) - I*c^(3/2)*f*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105
*d^3*g^3) + Sqrt[a]*c*g*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 10
5*d^3*g^3))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[
a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqr
t[c]*f + I*Sqrt[a]*g)] - g*(Sqrt[c]*f + I*Sqrt[a]*g)*((105*I)*c^(3/2)*d^3*
g^2 + 25*a^(3/2)*e^3*g^2 + (3*I)*a*Sqrt[c]*e^2*g*(2*e*f - 63*d*g) + Sqrt[a
]*c*e*(-8*e^2*f^2 + 42*d*e*f*g - 105*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c
] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g
*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*
x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(c^2*g^4*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x))))/(105*Sqrt[a + c*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 3.13 (sec) , antiderivative size = 916, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {735, 25, 2185, 27, 2185, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}dx$$

$$\Big\downarrow \text{ 735}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}-\frac{\int -\frac{(d+ex)\left(7cfd^2+ce(ef+11dg)x^2-ae(4ef+dg)-\left(5ae^2g-cd(12ef+7dg)\right)x\right)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{7c}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{(d+ex)\left(7cfd^2+ce(ef+11dg)x^2-ae(4ef+dg)-\left(5ae^2g-cd(12ef+7dg)\right)x\right)}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{7c}+}{\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}}$$

$\downarrow$ 2185

$$2\int \frac{-ce\left(25ae^2g^2+c\left(7e^2f^2+12degf-90d^2g^2\right)\right)x^2g^2+c\left(35cd^3fg-ae\left(3e^2f^2+53degf+5d^2g^2\right)\right)g^2-c\left(ae^2(23ef+63dg)g^2+c\left(2e^3f^3+22de^2gf^2-95d^2eg^2f-35d\right.\right.}{2\sqrt{f+gx}\sqrt{cx^2+a}}$$

$$5cg^3$$

$$7c$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\downarrow$ 27

$$\int \frac{-ce\left(25ae^2g^2+c\left(7e^2f^2+12degf-90d^2g^2\right)\right)x^2g^2+c\left(35cd^3fg-ae\left(3e^2f^2+53degf+5d^2g^2\right)\right)g^2-c\left(ae^2(23ef+63dg)g^2+c\left(2e^3f^3+22de^2gf^2-95d^2eg^2f-35d^3g\right.\right.}{\sqrt{f+gx}\sqrt{cx^2+a}}$$

$$5cg^3$$

$$7c$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\downarrow$ 2185

$$\frac{2\int \frac{cg^3\left(g\left(105c^2fgd^3+25a^2e^3g^2-ace\left(2e^2f^2+147degf+105d^2g^2\right)\right)-c\left(ae^2g^2(19ef+189dg)-c\left(8e^3f^3-42de^2gf^2+105d^2eg^2f+105d^3g^3\right)\right)x\right)}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3cg^2}-\frac{2}{3}eg\sqrt{a+cx}$$

$$5cg^3$$

$$7c$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\downarrow$ 27

$$\frac{1}{3}g\int \frac{g\left(105c^2fgd^3+25a^2e^3g^2-ace\left(2e^2f^2+147degf+105d^2g^2\right)\right)-c\left(ae^2g^2(19ef+189dg)-c\left(8e^3f^3-42de^2gf^2+105d^2eg^2f+105d^3g^3\right)\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2}{3}eg\sqrt{a+cx^2}\sqrt{}$$

$$5cg^3$$

$$7c$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\downarrow$ 599

$$-\frac{\frac{-\frac{2\int -\frac{e\left(cf^2+ag^2\right)\left(25ae^2g^2-c\left(8e^2f^2-42degf+105d^2g^2\right)\right)-c\left(ae^2g^2(19ef+189dg)-c\left(8e^3f^3-42de^2gf^2+105d^2eg^2f+105d^3g^3\right)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g}-\frac{2}{3}eg\sqrt{a\cdots}}{5cg^3}}{7c}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{2\int \frac{e\left(cf^2+ag^2\right)\left(25ae^2g^2-c\left(8e^2f^2-42degf+105d^2g^2\right)\right)-c\left(ae^2g^2(19ef+189dg)-c\left(8e^3f^3-42de^2gf^2+105d^2eg^2f+105d^3g^3\right)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g}-\frac{2}{3}eg\sqrt{a+cx\cdots}}{5cg^3}}{7c}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\Big\downarrow$ 1511

$$-\frac{2\left(-\sqrt{ag^2+cf^2}\left(e\sqrt{ag^2+cf^2}\left(25ae^2g^2-c\left(105d^2g^2-42defg+8e^2f^2\right)\right)-\sqrt{c}\left(ae^2g^2(189dg+19ef)-c\left(105d^3g^3+105d^2efg^2-42de^2f^2g+8e^3f^3\right)\right)\right)\int \frac{\cdots}{\sqrt{\frac{cf^2}{g^2}-\frac{2\cdots}{}}}\cdots\right.}{\cdots}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\Big\downarrow$ 1416

$$-\frac{2\left(-\sqrt{c}\sqrt{ag^2+cf^2}\left(ae^2g^2(189dg+19ef)-c\left(105d^3g^3+105d^2efg^2-42de^2f^2g+8e^3f^3\right)\right)\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(\cdots}{\sqrt{ag\cdots}}\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

$\Big\downarrow$ 1509

$$\frac{2e\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^2}{7c}+$$

$$\frac{\frac{2(ef+11dg)(f+gx)^{3/2}\sqrt{cx^2+a}e^2}{5g^2}+\frac{-\frac{2}{3}eg\sqrt{f+gx}\sqrt{cx^2+a}\left(25ae^2g^2+c\left(7e^2f^2+12degf-90d^2g^2\right)\right)-\frac{2\left(-\sqrt{c}\sqrt{cf^2+ag^2}\left(ae^2g^2(19ef+189dg)-c\left(8e^3\right.\right.\right.}{}}{}$$

input
```
Int[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

output
```
(2*e*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(7*c) + ((2*e^2*(e*f + 11*
d*g)*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(5*g^2) + ((-2*e*g*(25*a*e^2*g^2 + c
*(7*e^2*f^2 + 12*d*e*f*g - 90*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/3 -
 (2*(-(Sqrt[c]*Sqrt[c*f^2 + a*g^2]*(a*e^2*g^2*(19*e*f + 189*d*g) - c*(8*e^
3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g^3))*(-((Sqrt[f + g*x]
*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a
+ (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 +
 a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f
^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*
(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1
/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a
*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f +
 g*x)^2)/g^2]))) - ((c*f^2 + a*g^2)^(3/4)*(e*Sqrt[c*f^2 + a*g^2]*(25*a*e^2
*g^2 - c*(8*e^2*f^2 - 42*d*e*f*g + 105*d^2*g^2)) - Sqrt[c]*(a*e^2*g^2*(19*
e*f + 189*d*g) - c*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3
*g^3)))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^
2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (
Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sq
rt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])
/2])/(2*c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + ...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 735
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/(c*(2*m + 1))), x] - Simp[1/(c*(2*m + 1))   Int[((d + e*x)^(m - 2)/(Sq
rt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m
 + 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f +
d*g*(4*m - 1))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && IntegerQ[
2*m] && GtQ[m, 1]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2e^3x^2\sqrt{cg\,x^3+cf\,x^2+agx+af}}{7c}+\frac{2\left(3d\,e^2g+\frac{1}{7}f\,e^3\right)x\sqrt{cg\,x^3+cf\,x^2+agx+af}}{5cg}+\frac{2\left(3g\,d^2e+3d\,e^2f-\frac{4f\left(3d\,e^2g+\frac{1}{7}f\,e^3\right)}{5g}-\right.}{3cg}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/7*e^3/c*x^2*(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/5*(3*d*e^2*g+1/7*f*e^3)/c/g*x*(c*g*x^3+c*
f*x^2+a*g*x+a*f)^(1/2)+2/3*(3*g*d^2*e+3*d*e^2*f-4/5*f/g*(3*d*e^2*g+1/7*f*e
^3)-5/7*a/c*g*e^3)/c/g*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(d^3*f-2/5*a*f/
c/g*(3*d*e^2*g+1/7*f*e^3)-1/3*a/c*(3*g*d^2*e+3*d*e^2*f-4/5*f/g*(3*d*e^2*g+
1/7*f*e^3)-5/7*a/c*g*e^3))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)
/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/
2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*Ellip
ticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a
*c)^(1/2)/c))^(1/2))+2*(d^3*g+3*d^2*e*f-4/7*a*f/c*e^3-3/5*a/c*(3*d*e^2*g+1
/7*f*e^3)-2/3*f/g*(3*g*d^2*e+3*d*e^2*f-4/5*f/g*(3*d*e^2*g+1/7*f*e^3)-5/7*a
/c*g*e^3))*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(
-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*
c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c
)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-
f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^
(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx =$$
$$-\frac{2\left((8\,c^2e^3f^4-42\,c^2de^2f^3g+(105\,c^2d^2e-13\,ace^3)f^2g^2-42\,(5\,c^2d^3-6\,acde^2)fg^3+15\,(21\,acd^2e-5\right.}{}$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*((8*c^2*e^3*f^4 - 42*c^2*d*e^2*f^3*g + (105*c^2*d^2*e - 13*a*c*e^3)
*f^2*g^2 - 42*(5*c^2*d^3 - 6*a*c*d*e^2)*f*g^3 + 15*(21*a*c*d^2*e - 5*a^2*e
^3)*g^4)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/2
7*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 3*(8*c^2*e^3*f^3*g - 4
2*c^2*d*e^2*f^2*g^2 + (105*c^2*d^2*e - 19*a*c*e^3)*f*g^3 + 21*(5*c^2*d^3 -
 9*a*c*d*e^2)*g^4)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2)
, -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*
g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) - 3*(
15*c^2*e^3*g^4*x^2 - 4*c^2*e^3*f^2*g^2 + 21*c^2*d*e^2*f*g^3 + 5*(21*c^2*d^
2*e - 5*a*c*e^3)*g^4 + 3*(c^2*e^3*f*g^3 + 21*c^2*d*e^2*g^4)*x)*sqrt(c*x^2
+ a)*sqrt(g*x + f))/(c^3*g^4)
```

Sympy [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

input
```
integrate((e*x+d)**3*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**3*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^3\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx=\int \frac{(ex+d)^3\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}\,(d+e\,x)^3}{\sqrt{c\,x^2+a}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + c*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x)^3)/(a + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^3\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^3*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

output
```
( - 126*sqrt(f + g*x)*sqrt(a + c*x**2)*a*d*e**2*g**2 - 46*sqrt(f + g*x)*sq
rt(a + c*x**2)*a*e**3*f*g + 70*sqrt(f + g*x)*sqrt(a + c*x**2)*c*d**3*g**2
+ 210*sqrt(f + g*x)*sqrt(a + c*x**2)*c*d**2*e*f*g + 84*sqrt(f + g*x)*sqrt(
a + c*x**2)*c*d*e**2*f*g*x + 4*sqrt(f + g*x)*sqrt(a + c*x**2)*c*e**3*f**2*
x + 20*sqrt(f + g*x)*sqrt(a + c*x**2)*c*e**3*f*g*x**2 + 189*int((sqrt(f +
g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*c*d*e
**2*g**3 + 19*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f
*x**2 + c*g*x**3),x)*a*c*e**3*f*g**2 - 105*int((sqrt(f + g*x)*sqrt(a + c*x
**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c**2*d**3*g**3 - 105*int
((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3)
,x)*c**2*d**2*e*f*g**2 + 42*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f
 + a*g*x + c*f*x**2 + c*g*x**3),x)*c**2*d*e**2*f**2*g - 8*int((sqrt(f + g*
x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c**2*e**3
*f**3 + 63*int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 +
c*g*x**3),x)*a**2*d*e**2*g**3 + 23*int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a
*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a**2*e**3*f*g**2 - 35*int((sqrt(f + g
*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*c*d**3*g**3
 - 105*int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*
x**3),x)*a*c*d**2*e*f*g**2 - 84*int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f
+ a*g*x + c*f*x**2 + c*g*x**3),x)*a*c*d*e**2*f**2*g - 4*int((sqrt(f + g...
```

3.125 $\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 596

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = -\frac{4e(ef-5dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg}+\frac{2e^2(f+gx)^{3/2}\sqrt{a+cx^2}}{5cg}$$

$$+\frac{2\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(9ae^2g^2+c(2e^2f^2-10defg-15d^2g^2)\right)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f}{\sqrt{c}f+}}}{15c^{7/4}g^3\sqrt{a+cx^2}}$$

$$+\frac{2\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(15c^{3/2}d^2fg+9\sqrt{-a}ae^2g^2-a\sqrt{c}eg(7ef+10dg)+\sqrt{-a}c(2e^2f^2-10defg-15d^2g\right.}{15c^{7/4}g^2\sqrt{a+cx^2}}$$

output
```
-4/15*e*(-5*d*g+e*f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g+2/5*e^2*(g*x+f)^(3/
2)*(c*x^2+a)^(1/2)/c/g+2/15*(c^(1/2)*f-(-a)^(1/2)*g)*(c^(1/2)*f+(-a)^(1/2)
*g)^(1/2)*(9*a*e^2*g^2+c*(-15*d^2*g^2-10*d*e*f*g+2*e^2*f^2))*(1-c^(1/2)*(g
*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(
1/2)*g))^(1/2)*EllipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1
/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(7/4)/g^3
/(c*x^2+a)^(1/2)+2/15*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(15*c^(3/2)*d^2*f*g+9
*(-a)^(1/2)*a*e^2*g^2-a*c^(1/2)*e*g*(10*d*g+7*e*f)+(-a)^(1/2)*c*(-15*d^2*g
^2-10*d*e*f*g+2*e^2*f^2))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/
2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g
*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1
/2)*f-(-a)^(1/2)*g))^(1/2))/c^(7/4)/g^2/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.80 (sec) , antiderivative size = 591, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2e(a+cx^2)(10dg+e(f+3gx))}{cg} + \frac{(f+gx)\left(\frac{2g^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\left(-9ae^2g^2+c\left(-2e^2f^2+10defg+15d^2g^2\right)\right)\left(a+cx^2\right)}{(f+gx)^2} + \frac{2\sqrt{c}(-i\sqrt{c}f+\sqrt{a}g)\left(- \right.}{}\right.}{}\right.}{}$$

input
```
Integrate[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[f + g*x]*((2*e*(a + c*x^2)*(10*d*g + e*(f + 3*g*x)))/(c*g) + ((f + g
*x)*((2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-9*a*e^2*g^2 + c*(-2*e^2*f^2
 + 10*d*e*f*g + 15*d^2*g^2))*(a + c*x^2))/(f + g*x)^2 + (2*Sqrt[c]*((-I)*S
qrt[c]*f + Sqrt[a]*g)*(-9*a*e^2*g^2 + c*(-2*e^2*f^2 + 10*d*e*f*g + 15*d^2*
g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/
Sqrt[c] - g*x)/(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]
)/Sqrt[f + g*x] + (2*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((15*I)*c*d^2*g -
 (9*I)*a*e^2*g + 2*Sqrt[a]*Sqrt[c]*e*(e*f - 5*d*g))*Sqrt[(g*((I*Sqrt[a])/S
qrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*E
llipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[
c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(c^2*g^3*S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])))/(15*Sqrt[a + c*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 2.03 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {735, 25, 2185, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}dx$$

$$\Big\downarrow \text{ 735}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}-\frac{\int -\frac{5cfd^2+ce(ef+7dg)x^2-ae(2ef+dg)-\left(3ae^2g-cd(8ef+5dg)\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{5c}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{5cfd^2+ce(ef+7dg)x^2-ae(2ef+dg)-\left(3ae^2g-cd(8ef+5dg)\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{5c}+\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\frac{2\int \frac{cg\left(g\left(15cd^2f-ae(7ef+10dg)\right)-\left(9ae^2g^2+c\left(2e^2f^2-10degf-15d^2g^2\right)\right)x\right)}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3cg^2}+\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg+ef)}{3g}}{5c}+\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{g\left(15cd^2f-ae(7ef+10dg)\right)-\left(9ae^2g^2+c\left(2e^2f^2-10degf-15d^2g^2\right)\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3g}+\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg+ef)}{3g}}{5c}+\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

$\big\downarrow$ 599

$$\frac{\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg+ef)}{3g}-\frac{2\int -\frac{2e(ef-5dg)\left(cf^2+ag^2\right)-\left(9ae^2g^2+c\left(2e^2f^2-10degf-15d^2g^2\right)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^3}}{5c}+\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

$\big\downarrow$ 25

$$\frac{\frac{2\int \frac{2e(ef-5dg)\left(cf^2+ag^2\right)-\left(9ae^2g^2+c\left(2e^2f^2-10degf-15d^2g^2\right)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^3}+\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg+ef)}{3g}}{5c}+\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

$\big\downarrow$ 1511

$$\frac{\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg+ef)}{3g}-\frac{2\left(\frac{\sqrt{ag^2+cf^2}\left(-2\sqrt{c}e\sqrt{ag^2+cf^2}(ef-5dg)+9ae^2g^2+c\left(-15d^2g^2-10defg+2e^2f^2\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}\cdots}}{\sqrt{c}}\cdots\right)}{3g^3}}{5c}+\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

$\big\downarrow$ 1416

$$\frac{\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg+ef)}{3g}-\frac{2\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(-2\sqrt{c}e\sqrt{ag^2+cf^2}(ef-5dg)+9ae^2g^2+c\left(-15d^2g^2\right.\right.}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx}{g^2}}}}{}}{}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

$\Big\downarrow$ 1509

$$\frac{\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg+ef)}{3g}-\frac{2\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(-2\sqrt{c}e\sqrt{ag^2+cf^2}(ef-5dg)+9ae^2g^2+c\left(-15d^2g^2\right.\right.}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx}{g^2}}}}{}}{}$$

$$\frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

input
```
Int[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

output
```
(2*e*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(5*c) + ((2*e*(e*f + 7*d*g)*
Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*g) - (2*(-((Sqrt[c*f^2 + a*g^2]*(9*a*e^2
*g^2 + c*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*(-((Sqrt[f + g*x]*Sqrt[a +
 (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)
/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(
1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 -
 (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqr
t[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[
f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2]
)/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/
g^2])))/Sqrt[c]) + ((c*f^2 + a*g^2)^(3/4)*(9*a*e^2*g^2 - 2*Sqrt[c]*e*(e*f
- 5*d*g)*Sqrt[c*f^2 + a*g^2] + c*(2*e^2*f^2 - 10*d*e*f*g - 15*d^2*g^2))*(1
 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f
*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f
 + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x
])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^
(3/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])
))/(3*g^3))/(5*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 735
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/(c*(2*m + 1))), x] - Simp[1/(c*(2*m + 1))   Int[((d + e*x)^(m - 2)/(Sq
rt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m
 + 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f +
d*g*(4*m - 1))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && IntegerQ[
2*m] && GtQ[m, 1]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 3.27 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.17

method	result
elliptic	$\frac{\sqrt{(gx+f)(c x^2+a)}\left(\frac{2e^2 x\sqrt{cg\,x^3+cf\,x^2+agx+af}}{5c}+\frac{2\left(2gde+\frac{1}{5}e^2 f\right)\sqrt{cg\,x^3+cf\,x^2+agx+af}}{3cg}+\frac{2\left(d^2 f-\frac{2af\,e^2}{5c}-\frac{a\left(2gde+\frac{1}{5}e^2 f\right)}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-a}}{c}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input

```
int((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/5*e^2/c*x*(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(2*g*d*e+1/5*e^2*f)/c/g*(c*g*x^3+c*f*x^2+
a*g*x+a*f)^(1/2)+2*(d^2*f-2/5*a*f/c*e^2-1/3*a/c*(2*g*d*e+1/5*e^2*f))*(f/g-
(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(
-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/
2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/
c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(d^2*g+2*
d*e*f-3/5*a/c*g*e^2-2/3*f/g*(2*g*d*e+1/5*e^2*f))*(f/g-(-a*c)^(1/2)/c)*((x+
f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c)
)^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+
a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/
2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(
1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/
c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.50

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{2\left(2\left(ce^2f^3-5\,cdef^2g-15\,adeg^3+3\left(5\,cd^2-2\,ae^2\right)fg^2\right)\sqrt{cg}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(cf^2-3\,ag^2\right)}{3\,cg^2},-\frac{8\left(cf^3+9\right.}{27\,cg}\right.}{}$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(2*(c*e^2*f^3 - 5*c*d*e*f^2*g - 15*a*d*e*g^3 + 3*(5*c*d^2 - 2*a*e^2)*
f*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*
(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 3*(2*c*e^2*f^2*g - 10*c*
d*e*f*g^2 - 3*(5*c*d^2 - 3*a*e^2)*g^3)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^
2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInver
se(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(
3*g*x + f)/g)) + 3*(3*c*e^2*g^3*x + c*e^2*f*g^2 + 10*c*d*e*g^3)*sqrt(c*x^2
 + a)*sqrt(g*x + f))/(c^2*g^3)
```

Sympy [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

input
```
integrate((e*x+d)**2*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**2*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx=\int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx=\int \frac{(ex+d)^2\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}\,(d+e\,x)^2}{\sqrt{c\,x^2+a}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + c*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x)^2)/(a + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^2\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{-6\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,a\,e^2g+10\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,c\,d^2g+20\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,cdef+4\sqrt{gx+f}\,\sqrt{c}$$

input

```
int((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

output

```
( - 6*sqrt(f + g*x)*sqrt(a + c*x**2)*a*e**2*g + 10*sqrt(f + g*x)*sqrt(a +
c*x**2)*c*d**2*g + 20*sqrt(f + g*x)*sqrt(a + c*x**2)*c*d*e*f + 4*sqrt(f +
g*x)*sqrt(a + c*x**2)*c*e**2*f*x + 9*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x
**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*c*e**2*g**2 - 15*int((sqrt(f
 + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c**2
*d**2*g**2 - 10*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c
*f*x**2 + c*g*x**3),x)*c**2*d*e*f*g + 2*int((sqrt(f + g*x)*sqrt(a + c*x**2
)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c**2*e**2*f**2 + 3*int((sqr
t(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a**2*e
**2*g**2 - 5*int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2
+ c*g*x**3),x)*a*c*d**2*g**2 - 10*int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*
f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*c*d*e*f*g - 4*int((sqrt(f + g*x)*sqr
t(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*c*e**2*f**2 + 10*i
nt((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)
*c**2*d**2*f**2)/(10*c**2*f)
```

3.126 $\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 478

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2e\sqrt{f+gx}\sqrt{a+cx^2}}{3c}$$
$$-\frac{2\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(ef+3dg)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right)\right.}{3c^{3/4}g^2\sqrt{a+cx^2}}$$
$$+\frac{2\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(3cdf-aeg-\sqrt{-a}\sqrt{c}(ef+3dg)\right)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticF}\left(\text{arcsi}\right.}{3c^{5/4}g\sqrt{a+cx^2}}$$

output
```
2/3*e*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c-2/3*(c^(1/2)*f-(-a)^(1/2)*g)*(c^(1/2
)*f+(-a)^(1/2)*g)^(1/2)*(3*d*g+e*f)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/
2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticE(
c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2
)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(3/4)/g^2/(c*x^2+a)^(1/2)+2/3*(c^(
1/2)*f+(-a)^(1/2)*g)^(1/2)*(3*c*d*f-a*e*g-(-a)^(1/2)*c^(1/2)*(3*d*g+e*f))*
(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(
1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a
)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)
)/c^(5/4)/g/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.89 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{2\sqrt{f+gx}\left(e(a+cx^2)+\frac{(ef+3dg)(a+cx^2)}{f+gx}+\frac{ic\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(ef+3dg)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f+gx}E\left(i\text{arcsinh}\left(\frac{\sqrt{-f-i}}{\sqrt{f+g}}\right.\right.}{g^2}\right.}{3c\sqrt{a}}$$

input
```
Integrate[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

output
```
(2*Sqrt[f + g*x]*(e*(a + c*x^2) + ((e*f + 3*d*g)*(a + c*x^2))/(f + g*x) +
(I*c*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f + 3*d*g)*Sqrt[(g*((I*Sqrt[a])/S
qrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*S
qrt[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
 g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/g^2 + (I*(3*
Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sq
rt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*Sq
rt[f + g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/(g*Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]])))/(3*c*Sqrt[a + c*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.29 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {687, 27, 599, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

$\Big\downarrow$ 687

$$\frac{2\int \frac{3cdf-aeg+c(ef+3dg)x}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3c}+\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{3cdf-aeg+c(ef+3dg)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3c}+\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c}$$

$\Big\downarrow$ 599

$$\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c}-\frac{2\int \frac{e(cf^2+ag^2)-c(ef+3dg)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3cg^2}$$

$\Big\downarrow$ 1511

$$\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c}-$$
$$\frac{2\left(\left(-\sqrt{c}\sqrt{ag^2+cf^2}(3dg+ef)+aeg^2+cef^2\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}+\sqrt{c}\sqrt{ag^2+cf^2}(3dg+\right.}{3cg^2}$$

$\Big\downarrow$ 1416

$$\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c}-$$
$$\frac{2\left(\sqrt{c}\sqrt{ag^2+cf^2}(3dg+ef)\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}+\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(}{\sqrt{ag}}\right.}}}{}\right.}{3cg^2}$$

$\Big\downarrow$ 1509

$$\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c} -$$

$$2\left(\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(-\sqrt{c}\sqrt{ag^2+cf^2}(3dg+ef)+aeg^2+cef^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f}}{\sqrt[4]{cf^2+}}\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.$$

input
```
Int[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]
```

output
```
(2*e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c) - (2*(Sqrt[c]*(e*f + 3*d*g)*Sqrt
[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))
/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/S
qrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sq
rt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f +
 g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g
^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)]
, (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2
- (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])) + ((c*f^2 + a*g^2)^(1/4)*
(c*e*f^2 + a*e*g^2 - Sqrt[c]*(e*f + 3*d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt
[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*
x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))
/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^
2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sq
rt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(3*c*
g^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.26

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2e\sqrt{cg\,x^3+cf\,x^2+agx+af}}{3c}+\frac{2\left(df-\frac{eag}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+af}}\right.}{}$
risch	$\frac{2e\sqrt{gx+f}\,\sqrt{cx^2+a}}{3c}-\frac{\left(-\frac{2c(3dg+ef)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\left(\left(-\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\right.\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+af}}\right.}{}$
default	Expression too large to display

input
```
int((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/3*e/c*(c*g*x^3+
c*f*x^2+a*g*x+a*f)^(1/2)+2*(d*f-1/3*e/c*a*g)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)
/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1
/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*
x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^
(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(d*g+1/3*e*f)*(f/g-(-a*c)^(1/2)/c
)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1
/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*
f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*
c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-
a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^
(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.47

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{2\left(3\sqrt{cx^2+a}\sqrt{gx+f}ceg^2-(cef^2-6\,cdfg+3\,aeg^2)\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg}{27\,cg^3}\right.\right.}{}$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(3*sqrt(c*x^2 + a)*sqrt(g*x + f)*c*e*g^2 - (c*e*f^2 - 6*c*d*f*g + 3*a*
e*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*
(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) - 3*(c*e*f*g + 3*c*d*g^2)*
sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*
a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27
*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)))/(c^2*g^2)
```

Sympy [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)*sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}\,(d+e\,x)}{\sqrt{c\,x^2+a}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(d + e*x))/(a + c*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^(1/2)*(d + e*x))/(a + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,dg+2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,ef-3\left(\int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,x^2}{cg\,x^3+cf\,x^2+agx+af}dx\right)cd\,g^2-\left(\int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,x}{cg\,x^3+cf\,x^2+agx+}\right.}{}$$

input
```
int((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(f + g*x)*sqrt(a + c*x**2)*d*g + 2*sqrt(f + g*x)*sqrt(a + c*x**2)*e
*f - 3*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 +
 c*g*x**3),x)*c*d*g**2 - int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f +
a*g*x + c*f*x**2 + c*g*x**3),x)*c*e*f*g - int((sqrt(f + g*x)*sqrt(a + c*x*
*2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*d*g**2 - int((sqrt(f + g*x)*
sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*e*f*g + 2*int((
sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c*d
*f**2)/(2*c*f)
```

3.127 $\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 423

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = -\frac{2\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right)|\frac{\sqrt{c}f+\sqrt{-a}g}{\sqrt{c}f-\sqrt{-a}g}\right)}{c^{3/4}g\sqrt{a+cx^2}} + \frac{2\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right),\frac{\sqrt{c}f+\sqrt{-a}g}{\sqrt{c}f-\sqrt{-a}g}\right)}{c^{3/4}g\sqrt{a+cx^2}}$$

output
```
-2*(c^(1/2)*f-(-a)^(1/2)*g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x
+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/
2)*g))^(1/2)*EllipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2
),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(3/4)/g/(c*
x^2+a)^(1/2)+2*(c^(1/2)*f-(-a)^(1/2)*g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-
c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2
)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(
1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c
^(3/4)/g/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.57 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \frac{2i\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{g(\sqrt{a}+i\sqrt{c}x)}{-i\sqrt{c}f+\sqrt{a}g}}\sqrt{f+gx}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right)|\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-\text{EllipticF}\left(i\operatorname{arcsinh}\left(\cdots\right.\right.\right.}{\sqrt{c}g\sqrt{\frac{\sqrt{c}(f+gx)}{g(i\sqrt{a}+\sqrt{c}x)}}\sqrt{a+cx^2}}$$

input
```
Integrate[Sqrt[f + g*x]/Sqrt[a + c*x^2],x]
```

output
```
((2*I)*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sq
rt[c]*f + Sqrt[a]*g)]*Sqrt[f + g*x]*(EllipticE[I*ArcSinh[Sqrt[-((Sqrt[c]*(
f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]
*f + I*Sqrt[a]*g)] - EllipticF[I*ArcSinh[Sqrt[-((Sqrt[c]*(f + g*x))/(Sqrt[
c]*f - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g
)]))/(Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(g*(I*Sqrt[a] + Sqrt[c]*x))]*Sqrt
[a + c*x^2])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {507, 1459, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

$$\Big\downarrow \ 507$$

$$\frac{2\int \frac{f+gx}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}$$

$$\Big\downarrow \; 1459$$

$$\frac{2\left(\frac{\sqrt{ag^2+cf^2}\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}\right)}{g}$$

$$\Big\downarrow \; 1416$$

$$\frac{2\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}-\frac{\sqrt{ag^2+c\cdots}}{\cdots}\cdots\right.}{g}$$

$$\Big\downarrow \; 1509$$

$$\frac{2\left(\frac{\left(ag^2+cf^2\right)^{3/4}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{2c^{3/4}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}-\frac{\sqrt{ag^2+c\cdots}}{\cdots}\cdots\right.}{\cdots}$$

input
```
Int[Sqrt[f + g*x]/Sqrt[a + c*x^2],x]
```

output
```
(2*(-((Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f
*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(
f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f
 + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^
2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[
c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*
g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (
c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2]))))/Sqrt[c]) + ((
c*f^2 + a*g^2)^(3/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a
 + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2
)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTa
n[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*
f^2 + a*g^2])/2])/(2*c^(3/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2
+ (c*(f + g*x)^2)/g^2])))/g
```

Defintions of rubi rules used

rule 507
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[2/
d   Subst[Int[x^2/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]
, x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.94

method	result
default	$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^{2}+a}\,(cf-\sqrt{-ac}\,g)\sqrt{-\frac{(gx+f)c}{\sqrt{-ac}\,g-cf}}\,\sqrt{\frac{(-cx+\sqrt{-ac})g}{\sqrt{-ac}\,g+cf}}\,\sqrt{\frac{(cx+\sqrt{-ac})g}{\sqrt{-ac}\,g-cf}}\left(\sqrt{-ac}\,\operatorname{EllipticF}\left(\sqrt{-\frac{(gx+f)c}{\sqrt{-ac}\,g-cf}},\sqrt{-\frac{\sqrt{-ac}\,g-c}{\sqrt{-ac}\,g+c}}\right.\right.}{g}$
elliptic	$\frac{\sqrt{(gx+f)(c\,x^{2}+a)}\left(\frac{2f\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{cg\,x^{3}+cf\,x^{2}+agx+af}}+\frac{2g\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right.}{}\right.}{\sqrt{}}$

input

```
int((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(c*f-(-a*c)^(1/2)*g)*(-(g*x+f)*c/((-a*c)^(
1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*
x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*((-a*c)^(1/2)*EllipticF((-(g
*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g
+c*f))^(1/2))*g-(-a*c)^(1/2)*EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(
1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*g+f*EllipticF((-(
g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*
g+c*f))^(1/2))*c-EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*
c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c*f)/g/(c*g*x^3+c*f*x^2+a*g*x
+a*f)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{2\Bigl(2\sqrt{cg}f\text{weierstrassPInverse}\Bigl(\frac{4(cf^2-3ag^2)}{3cg^2}, -\frac{8(cf^3+9afg^2)}{27cg^3}, \frac{3gx+f}{3g}\Bigr) - 3\sqrt{cg}g\text{weierstrassZeta}\Bigl(\frac{4(cf^2-3ag^2)}{3cg^2}}{3cg}$$

input
```
integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(2*sqrt(c*g)*f*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/2
7*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) - 3*sqrt(c*g)*g*weierstr
assZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3),
weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^
2)/(c*g^3), 1/3*(3*g*x + f)/g)))/(c*g)
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)/sqrt(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}}\,dx$$

input
```
int((f + g*x)^(1/2)/(a + c*x^2)^(1/2),x)
```

output
```
int((f + g*x)^(1/2)/(a + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{c\,x^2+a}dx$$

input
```
int((g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

output
```
int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a + c*x**2),x)
```

3.128 $\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 415

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{2\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right),\frac{\sqrt{c}f+\sqrt{-a}g}{\sqrt{c}f-\sqrt{-a}g}\right)}{\sqrt[4]{c}e\sqrt{a+cx^2}} - \frac{2\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticPi}\left(\frac{e\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}{ef-dg},\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right),\frac{\sqrt{c}f+\sqrt{}}{\sqrt{c}f-\sqrt{}}\right.}{\sqrt[4]{c}e\sqrt{a+cx^2}}$$

output
```
2*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*
g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(
1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g
)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(1/4)/e/(c*x^2+a)^(1/2)-2*(c^(1/2)*f+
(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-
c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticPi(c^(1/4)*(g*x+f)
^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),e*(f+(-a)^(1/2)*g/c^(1/2))/(-d*g+e*f
),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(1/4)/e/(c*
x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.05 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx =$$

$$-\frac{2i\sqrt{\frac{g(\sqrt{a}+i\sqrt{c}x)}{-i\sqrt{c}f+\sqrt{a}g}}\sqrt{f+gx}\left(\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\right),\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-\text{EllipticPi}\left(\frac{e\left(f-\frac{i\sqrt{a}g}{\sqrt{c}}\right)}{ef-dg},i\text{arc}\right.\right.}{e\sqrt{\frac{\sqrt{c}(f+gx)}{g(i\sqrt{a}+\sqrt{c}x)}}\sqrt{a+cx^2}}$$

input
```
Integrate[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
((-2*I)*Sqrt[(g*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqrt[c]*f + Sqrt[a]*g)]*Sqr
t[f + g*x]*(EllipticF[I*ArcSinh[Sqrt[-((Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I*
Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Elli
pticPi[(e*(f - (I*Sqrt[a]*g)/Sqrt[c]))/(e*f - d*g), I*ArcSinh[Sqrt[-((Sqrt
[c]*(f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g))]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sq
rt[c]*f + I*Sqrt[a]*g)]))/(e*Sqrt[(Sqrt[c]*(f + g*x))/(g*(I*Sqrt[a] + Sqrt
[c]*x))]*Sqrt[a + c*x^2])
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1112 vs. 2(415) = 830.

Time = 3.31 (sec) , antiderivative size = 1112, normalized size of antiderivative = 2.68, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {736, 510, 729, 25, 1416, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}(d+ex)}\,dx$$

$$\Big\downarrow \; 736$$

$$\frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e}+\frac{g\int \frac{1}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e}$$

$$\Big\downarrow \text{510}$$

$$\frac{(ef-dg)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{e}+\frac{2\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e}$$

$$\Big\downarrow \text{729}$$

$$\frac{2(ef-dg)\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e}+\frac{2\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e}-\frac{2(ef-dg)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}e\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}-\frac{2(ef-dg)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{e}$$

$$\Big\downarrow \text{1540}$$

$$2(ef-dg)\left(\frac{e\sqrt{ag^2+cf^2}\left(\sqrt{c}(ef-dg)-e\sqrt{ag^2+cf^2}\right)\int\frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(ae^2g+cd(2ef-dg))}-\frac{\sqrt{c}\left(-\sqrt{c}\sqrt{ag^2+cf^2}\right.}{\ldots}\right.$$

$$\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}e\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}$$

$\downarrow$ 1416

$$2(ef-dg)\left(\frac{e\sqrt{ag^2+cf^2}\left(\sqrt{c}(ef-dg)-e\sqrt{ag^2+cf^2}\right)\int\frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(ae^2g+cd(2ef-dg))}-\frac{\sqrt[4]{c}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\ldots}{\ldots}\right.$$

$$\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}e\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}$$

$\downarrow$ 2222

$$\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right.}{\sqrt[4]{c}e\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}$$

$$2(ef-dg)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\left(\frac{\left(e+\frac{\sqrt{c}(ef-dg)}{\sqrt{cf^2+ag^2}}\right)\text{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}\right)}{2\sqrt{e}\sqrt{cd^2+ae^2}\sqrt{ef-dg}}-\frac{\left(\frac{\sqrt{c}}{e}-\sqrt{\right.}{}\right.}{}\right.}{}\right.$$

input
```
Int[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[
(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f
^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[
c*f^2 + a*g^2])/2])/(c^(1/4)*e*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^
2 + (c*(f + g*x)^2)/g^2]) + (2*(e*f - d*g)*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g
^2 - Sqrt[c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqr
t[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f +
g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^
2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)],
 (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(g*(c*f^2 + a*g^2)^(1/4)*(a*e^2
*g + c*d*(2*e*f - d*g))*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*
(f + g*x)^2)/g^2]) + (e*Sqrt[c*f^2 + a*g^2]*(Sqrt[c]*(e*f - d*g) - e*Sqrt[
c*f^2 + a*g^2])*(((e + (Sqrt[c]*(e*f - d*g))/Sqrt[c*f^2 + a*g^2])*ArcTanh[
(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt[a + (c*f
^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])])/(2*Sqrt[e]*Sqrt[
c*d^2 + a*e^2]*Sqrt[e*f - d*g]) - ((Sqrt[c]/e - Sqrt[c*f^2 + a*g^2]/(e*f -
 d*g))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2
 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (S
qrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticPi[(Sqrt[c]*(e*f - d...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 510
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2/d   Subst[Int[1/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2
)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 736
```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Simp[g/e   Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
 Simp[(e*f - d*g)/e   Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x],
x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.06

method	result
default	$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^{2}+a}\,\sqrt{-\frac{(gx+f)c}{\sqrt{-ac}\,g-cf}}\,\sqrt{\frac{(-cx+\sqrt{-ac})g}{\sqrt{-ac}\,g+cf}}\,\sqrt{\frac{(cx+\sqrt{-ac})g}{\sqrt{-ac}\,g-cf}}\left(f\,\mathrm{EllipticF}\left(\sqrt{-\frac{(gx+f)c}{\sqrt{-ac}\,g-cf}},\sqrt{-\frac{\sqrt{-ac}\,g-cf}{\sqrt{-ac}\,g+cf}}\right)c-\sqrt{-ac}\,\mathrm{Ellipti}\right.}{}$
elliptic	$\frac{\sqrt{(gx+f)(c\,x^{2}+a)}\left(\frac{2g\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{e\sqrt{cg\,x^{3}+cf\,x^{2}+agx+af}}-\frac{2(dg-ef)\left(\frac{f}{g}-\right.}{}\right.}{\sqrt{gx+f}\,\sqrt{c\,x^{2}+a}}$

input
```
int((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*((
-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/((-
a*c)^(1/2)*g-c*f))^(1/2)*(f*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1
/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c-(-a*c)^(1/2)*Ell
ipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a
*c)^(1/2)*g+c*f))^(1/2))*g-EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1
/2),((-a*c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2
)*g+c*f))^(1/2))*c*f+(-a*c)^(1/2)*EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c
*f))^(1/2),((-a*c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*
c)^(1/2)*g+c*f))^(1/2))*g)/e/c/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

input
```
int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{ce\,x^3+cd\,x^2+aex+ad}dx$$

input
```
int((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x)
```

output
```
int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*d + a*e*x + c*d*x**2 + c*e*x**3),x
)
```

3.129 $\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 754

$$
\begin{aligned}
\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx &= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)\,(d+ex)} \\
&- \frac{\sqrt[4]{c}\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right)|\frac{\sqrt{c}f+\sqrt{-}}{\sqrt{c}f-\sqrt{-}}\right.}{(cd^2+ae^2)\,g\sqrt{a+cx^2}} \\
&+ \frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{-a}e\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right),\right.}{e\,(cd^2+ae^2)\sqrt{a+cx^2}} \\
&- \frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(ae^2g+cd(2ef-dg))\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\mathrm{EllipticPi}\left(\frac{e\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}{ef-dg},\arcsin\right.}{\sqrt[4]{c}e\,(cd^2+ae^2)\,(ef-dg)\sqrt{a+cx^2}}
\end{aligned}
$$

output
```
-e*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)-c^(1/4)*(c^(1/2)*f-
(-a)^(1/2)*g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f
-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*E
llipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+
(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/(a*e^2+c*d^2)/g/(c*x^2+a)^(
1/2)+c^(1/4)*(c^(1/2)*d-(-a)^(1/2)*e)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-c^
(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*
f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/
2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/e/(
a*e^2+c*d^2)/(c*x^2+a)^(1/2)-(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(a*e^2*g+c*d*(
-d*g+2*e*f))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)
*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticPi(c^(1/4)*(g*x+f)^(1/2)/
(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),e*(f+(-a)^(1/2)*g/c^(1/2))/(-d*g+e*f),((c^(
1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(1/4)/e/(a*e^2+c*d
^2)/(-d*g+e*f)/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.09 (sec) , antiderivative size = 1330, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
(Sqrt[f + g*x]*(-((e^2*(a + c*x^2))/(d + e*x)) - (-(c*e^2*f^3*Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - a*e
^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d*e*g^3*Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - 2*
c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - c*e^2*f*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]]*(f + g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*Sqr
t[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
 g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt
[a]*g)] + e*(I*Sqrt[c]*d + Sqrt[a]*e)*g*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*
((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)
/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g
)] - (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(
((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticPi[(Sqrt
[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*
Sqrt[a]*g)] + I*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sq
rt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*Elliptic...
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1567 vs. $2(754) = 1508$.

Time = 4.69 (sec) , antiderivative size = 1567, normalized size of antiderivative = 2.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {737, 25, 2349, 599, 27, 729, 25, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}(d+ex)^2}dx$$

$$\Big\downarrow \text{737}$$

$$-\frac{\int -\frac{cegx^2+2cdgx+2cdf+aeg}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{cegx^2+2cdgx+2cdf+aeg}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 2349$$

$$\frac{\left(aeg+\frac{cd(2ef-dg)}{e}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{cxg+\frac{cdg}{e}}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 599$$

$$\frac{\left(aeg+\frac{cd(2ef-dg)}{e}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int \frac{cg(ef-dg-e(f+gx))}{e\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(aeg+\frac{cd(2ef-dg)}{e}\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{eg}}{2\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 729$$

$$\frac{2\left(aeg+\frac{cd(2ef-dg)}{e}\right)\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+g}}{eg}}{2\left(ae^2+cd^2\right)}$$
$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{-2\left(aeg+\frac{cd(2ef-dg)}{e}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+g}}{eg}}{2\left(ae^2+cd^2\right)}$$
$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow 1511$$

$$\frac{-2\left(aeg+\frac{cd(2ef-dg)}{e}\right)\int\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int\frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+g}{g^2}}}}{\sqrt{c}}\right.}{2\left(ae^2+cd^2\right)}}{}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow 1416$$

$$-\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int\frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(dg-e\left(f-\frac{\sqrt{ag^2-}}{\sqrt{c}}\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}-}}\right.}{eg}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow 1509$$

$$\frac{-2\left(aeg+\frac{cd(2ef-dg)}{e}\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\left(\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+g}{\sqrt{ag^2+c}}\right.}{}\right.}{}\right.}{}}{}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)}$$

$\downarrow$ 1540

$$\frac{2\left(aeg+\frac{cd(2ef-dg)}{e}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(age^2+cd(2ef-dg))}-\frac{\sqrt{c}\left(cef^2\right.}{}\right.}{}$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{\left(cd^2+ae^2\right)(d+ex)}$$

$\downarrow$ 1416

$$\frac{2\left(aeg+\frac{cd(2ef-dg)}{e}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g(age^2+cd(2ef-dg))}-\frac{\sqrt[4]{c}\left(cef\ \cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{(cd^2+ae^2)(d+ex)}$$

$\Big\downarrow$ 2222

$$\frac{2\left(aeg+\frac{cd(2ef-dg)}{e}\right)\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\left(\frac{\left(e+\frac{\sqrt{c}(ef-dg)}{\sqrt{cf^2+ag^2}}\right)\text{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}\cdots\right)}{2\sqrt{e}\sqrt{cd^2+ae^2}\sqrt{ef-dg}}\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{(cd^2+ae^2)(d+ex)}$$

input
```
Int[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
-((e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x))) + ((-2*c*
((e*Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f
 + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f +
 g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f +
g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 +
 (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f
^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2
)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f
^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/Sqrt[c] - ((c*f^
2 + a*g^2)^(1/4)*(d*g - e*(f - Sqrt[c*f^2 + a*g^2]/Sqrt[c]))*(1 + (Sqrt[c]
*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))
/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sq
rt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 +
 a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sqrt[
a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(e*g) +
2*(a*e*g + (c*d*(2*e*f - d*g))/e)*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g^2 - Sqrt
[c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 +
 a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g
^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*E
llipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + ...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 737
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^
2]/((m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/(2*(m + 1)*(c*d^2 + a*e^2))   In
t[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[2*c*d*f*(m + 1)
- e*(a*g) + 2*c*(d*g*(m + 1) - e*f*(m + 2))*x - c*e*g*(2*m + 5)*x^2, x], x]
, x] /; FreeQ[{a, c, d, e, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.22

method	result
elliptic	$\sqrt{(gx+f)(c\,x^2+a)}\left(-\frac{e\sqrt{cg\,x^3+cf\,x^2+agx+af}}{\left(a\,e^2+c\,d^2\right)(ex+d)}+\frac{cdg\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\cdots}\right.}{\left(a\,e^2+c\,d^2\right)e\sqrt{cg\,x^3+cf\,x^2+agx+af}}\right.$
default	Expression too large to display

input

```
int((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-e/(a*e^2+c*d^2)*
(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)+c*d*g/(a*e^2+c*d^2)/e*(f/g-(-a*c
)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-
(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c
*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(
1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+c*g/(a*e^2+c*d^2
)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/
2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/
c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*Ellipti
cE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c
)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))
^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+(a*e^2*g-c*d^
2*g+2*c*d*e*f)/e^2/(a*e^2+c*d^2)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)
^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*
c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)
/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(
1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}\,(d+ex)^2}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)**2/(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^2), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^2}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\,(d+e\,x)^2}\,dx$$

input
```
int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{c\,e^2x^4+2cde\,x^3+a\,e^2x^2+c\,d^2x^2+2adex+a\,d^2}dx$$

input
```
int((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+a)^(1/2),x)
```

output
```
int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*d**2 + 2*a*d*e*x + a*e**2*x**2 + c
*d**2*x**2 + 2*c*d*e*x**3 + c*e**2*x**4),x)
```

3.130 $\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 983

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$$

$$= -\frac{e\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(d+ex)^2} - \frac{e(ae^2g+cd(6ef-5dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)(d+ex)}$$

$$- \frac{\sqrt[4]{c}\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}(ae^2g+cd(6ef-5dg))\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\Big(\arcsin}{4\left(cd^2+ae^2\right)^2g(ef-dg)\sqrt{a+cx^2}}$$

$$+ \frac{\sqrt[4]{c}\sqrt{\sqrt{c}f+\sqrt{-a}g}\left((-a)^{3/2}e^3g-\sqrt{-a}cde(6ef-5dg)-a\sqrt{c}e^2(2ef-3dg)+c^{3/2}d^2(4ef-3dg)\right)\sqrt{}}{4e\left(cd^2+ae^2\right)^2(ef-dg)\sqrt{a+}}$$

$$+ \frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}(a^2e^4g^2-c^2d^2(8e^2f^2-12defg+3d^2g^2)+2ace^2(2e^2f^2-6defg+5d^2g^2))\sqrt{1-\frac{\sqrt{c}}{\sqrt{c}f}}}{4\sqrt[4]{c}e\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{}}$$

output
```
-1/2*e*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(e*x+d)^2-1/4*e*(a*e^2*
g+c*d*(-5*d*g+6*e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)^2/(-d*g+
e*f)/(e*x+d)-1/4*c^(1/4)*(c^(1/2)*f-(-a)^(1/2)*g)*(c^(1/2)*f+(-a)^(1/2)*g)
^(1/2)*(a*e^2*g+c*d*(-5*d*g+6*e*f))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/
2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticE(
c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2
)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/(a*e^2+c*d^2)^2/g/(-d*g+e*f)/(c*x^2+
a)^(1/2)+1/4*c^(1/4)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*((-a)^(3/2)*e^3*g-(-a)
^(1/2)*c*d*e*(-5*d*g+6*e*f)-a*c^(1/2)*e^2*(-3*d*g+2*e*f)+c^(3/2)*d^2*(-3*d
*g+4*e*f))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(
g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^
(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2
)*g))^(1/2))/e/(a*e^2+c*d^2)^2/(-d*g+e*f)/(c*x^2+a)^(1/2)+1/4*(c^(1/2)*f+(
-a)^(1/2)*g)^(1/2)*(a^2*e^4*g^2-c^2*d^2*(3*d^2*g^2-12*d*e*f*g+8*e^2*f^2)+2
*a*c*e^2*(5*d^2*g^2-6*d*e*f*g+2*e^2*f^2))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-
a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*Elli
pticPi(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),e*(f+(-a)^(1/2
)*g/c^(1/2))/(-d*g+e*f),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g)
)^(1/2))/c^(1/4)/e/(a*e^2+c*d^2)^2/(-d*g+e*f)^2/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.81 (sec) , antiderivative size = 2937, normalized size of antiderivative = 2.99

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + c*x^2]),x]
```

output
```
Sqrt[f + g*x]*Sqrt[a + c*x^2]*(-1/2*e/((c*d^2 + a*e^2)*(d + e*x)^2) - (e*(
6*c*d*e*f - 5*c*d^2*g + a*e^2*g))/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)*(d + e*
x))) - ((f + g*x)^(3/2)*(6*c^2*d*e^3*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]
- 11*c^2*d^2*e^2*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*c*e^4*f*g*Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]] + 5*c^2*d^3*e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[
c]] - a*c*d*e^3*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + (6*c^2*d*e^3*f^4*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (11*c^2*d^2*e^2*f^3*g*Sqrt[-
f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (a*c*e^4*f^3*g*Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]])/(f + g*x)^2 + (5*c^2*d^3*e*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]])/(f + g*x)^2 + (5*a*c*d*e^3*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]])/(f + g*x)^2 - (11*a*c*d^2*e^2*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]
])/(f + g*x)^2 + (a^2*e^4*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x
)^2 + (5*a*c*d^3*e*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (a^
2*d*e^3*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (12*c^2*d*e^3*
f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (22*c^2*d^2*e^2*f^2*g*Sq
rt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) - (2*a*c*e^4*f^2*g*Sqrt[-f - (I*
Sqrt[a]*g)/Sqrt[c]])/(f + g*x) - (10*c^2*d^3*e*f*g^2*Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]])/(f + g*x) + (2*a*c*d*e^3*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]])/(f + g*x) + (Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - d*g)*(a*e^2
*g + c*d*(6*e*f - 5*d*g))*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]...
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2782 vs. $2(983) = 1966$.

Time = 9.16 (sec) , antiderivative size = 2782, normalized size of antiderivative = 2.83, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {737, 25, 2349, 734, 2349, 25, 27, 510, 599, 25, 27, 729, 25, 1416, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}(d+ex)^3}\,dx$$

$$\Big\downarrow \ 737$$

$$-\frac{\int -\frac{-cegx^2-2c(ef-2dg)x+4cdf+aeg}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int \frac{-cegx^2-2c(ef-2dg)x+4cdf+aeg}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \textbf{2349}$$

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-2cf+\frac{5cdg}{e}-cgx}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \textbf{734}$$

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\int \frac{-cgx^2e^2+age^2-2cdgxe-2cd(ef-dg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+\int \frac{-2cf+\frac{5cdg}{e}-cgx}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \textbf{2349}$$

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)-2c\Big(f \cdots}{4\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{(ae^2g-cd(2ef-3dg))\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)-2c\Big(f \cdots}{4\left(ae^2+cd^2\right)}-\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int\frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)-2c\Big(f}{4\left(ae^2+cd^2\right)}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 510

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int\frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)-2c\Big(f}{4\left(ae^2+cd^2\right)}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 599

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int-\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}}{(d+ex)(ae^2+}\right.}{4\left(ae^2+cd^2\right)}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2\int\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}}{(d+ex)(ae^2+c}\right.}{4\left(ae^2+cd^2\right)}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 27

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2c\int\frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}}{(d+ex)(ae^2+c}\right.}{4\left(ae^2+cd^2\right)}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 729

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{2\left(ae^2g-cd(2ef-3dg)\right)\int-\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}+\frac{2c\int\frac{ef-dg-e(f+g}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(}{}}}}{g}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 25

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\frac{2c\int\frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+}{g}}}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 1416

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+}{g}}}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 1511

$$\frac{\left(aeg+\frac{cd(6ef-5dg)}{e}\right)\left(-\frac{\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\left(dg-e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx}{g^2}}}\right.}{g}}{2(ae^2+cd^2)(ef-}\right.}{}$$

$$\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)}$$

$\downarrow$ 1416

$$-\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+4c\Big($$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{2\left(cd^2+ae^2\right)(d+ex)^2}$$

$\downarrow$ 1509

$$-\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+4c\Big($$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{2\left(cd^2+ae^2\right)(d+ex)^2}$$

$\downarrow$ 1540

$$-\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-4c\Big($$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{2\left(cd^2+ae^2\right)(d+ex)^2}$$

$$\Big\downarrow \text{ 1416}$$

$$-\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-4c\Big($$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{2\left(cd^2+ae^2\right)(d+ex)^2}$$

$$\Big\downarrow \; 2222$$

$$-\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{e\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-4c\Big($$

$$\frac{e\sqrt{f+gx}\sqrt{cx^2+a}}{2\left(cd^2+ae^2\right)(d+ex)^2}$$

input
```
Int[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + c*x^2]),x]
```

output
```
-1/2*(e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)^2) + (-(
(c^(3/4)*(c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2
])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((
a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*Ellipti
cF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*
f)/Sqrt[c*f^2 + a*g^2])/2])/(e*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^
2 + (c*(f + g*x)^2)/g^2])) - 4*c*(f - (3*d*g)/e)*(-1/2*(c^(1/4)*(c*e*f^2 +
 a*e*g^2 - Sqrt[c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x
))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c
*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2
+ a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(
1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(g*(c*f^2 + a*g^2)^(1/4)*
(a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2
 + (c*(f + g*x)^2)/g^2]) + (e*Sqrt[c*f^2 + a*g^2]*(Sqrt[c]*(e*f - d*g) - e
*Sqrt[c*f^2 + a*g^2])*(((e + (Sqrt[c]*(e*f - d*g))/Sqrt[c*f^2 + a*g^2])*Ar
cTanh[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt[a
+ (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])])/(2*Sqrt[e]
*Sqrt[c*d^2 + a*e^2]*Sqrt[e*f - d*g]) - ((Sqrt[c]/e - Sqrt[c*f^2 + a*g^2]/
(e*f - d*g))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^
2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 510
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2/d   Subst[Int[1/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2
)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 734
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Simp[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^
2]))*Simp[2*c*d*(e*f - d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*c*e*(d*g*(m + 1
) - e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 737
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*x^
2]/((m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/(2*(m + 1)*(c*d^2 + a*e^2))   In
t[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[2*c*d*f*(m + 1)
- e*(a*g) + 2*c*(d*g*(m + 1) - e*f*(m + 2))*x - c*e*g*(2*m + 5)*x^2, x], x]
, x] /; FreeQ[{a, c, d, e, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 7.60 (sec) , antiderivative size = 1224, normalized size of antiderivative = 1.25

method	result	size
elliptic	Expression too large to display	1224
default	Expression too large to display	20359

input
```
int((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-1/2*e/(a*e^2+c*d
^2)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)^2+1/4*e*(a*e^2*g-5*c*d^2*g+6
*c*d*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(a*e^2+c*d^2)*(c*g*x^3+c*f
*x^2+a*g*x+a*f)^(1/2)/(e*x+d)-1/4*c*g*(3*a*d*e^2*g-2*a*e^3*f-3*c*d^3*g+4*c
*d^2*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(a*e^2+c*d^2)/e*(f/g-(-a*c
)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-
(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c
*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(
1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-1/4*c*g*(a*e^2*g
-5*c*d^2*g+6*c*d*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(a*e^2+c*d^2)*
(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)
/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c)
)^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE
(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^
(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(
1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+1/4*(a^2*e^4*g^
2+10*a*c*d^2*e^2*g^2-12*a*c*d*e^3*f*g+4*a*c*e^4*f^2-3*c^2*d^4*g^2+12*c^2*d
^3*e*f*g-8*c^2*d^2*e^2*f^2)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/(a*e^2+c
*d^2)/e^2*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-
a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}\,(d+ex)^3}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)**3/(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^3}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^3), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^3}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\,(d+e\,x)^3}\,dx$$

input
```
int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^3), x)
```

Reduce [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3\sqrt{a+cx^2}}\,dx$$
$$=\int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{c\,e^3x^5+3cd\,e^2x^4+a\,e^3x^3+3c\,d^2e\,x^3+3ad\,e^2x^2+c\,d^3x^2+3a\,d^2ex+a\,d^3}dx$$

input
```
int((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x)
```

output
```
int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*d**3 + 3*a*d**2*e*x + 3*a*d*e**2*x
**2 + a*e**3*x**3 + c*d**3*x**2 + 3*c*d**2*e*x**3 + 3*c*d*e**2*x**4 + c*e*
*3*x**5),x)
```

3.131 $\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 726

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{2g^2\sqrt{f+gx}\sqrt{a+cx^2}}{\sqrt{c}e\left(\sqrt{cf^2+ag^2}+\sqrt{c}(f+gx)\right)}$$

$$-\frac{2(cf^2+ag^2)^{3/4}\sqrt{\frac{g^2(a+cx^2)}{(cf^2+ag^2)\left(1+\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}\right)^2}}\left(1+\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}\right)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(1+\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}\right)\right.}{c^{3/4}e\sqrt{a+cx^2}}$$

$$+\frac{2\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(cef^2+aeg^2+\sqrt{c}(ef-dg)\sqrt{cf^2+ag^2}\right)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticF}\left(\text{a}\right.}{c^{3/4}e^2\sqrt{cf^2+ag^2}\sqrt{a+cx^2}}$$

$$-\frac{2\sqrt{\sqrt{c}f+\sqrt{-a}g}(ef-dg)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticPi}\left(\frac{e\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}{ef-dg},\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right)\right.}{\sqrt[4]{c}e^2\sqrt{a+cx^2}}$$

output
```
2*g^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c^(1/2)/e/((a*g^2+c*f^2)^(1/2)+c^(1/2)
*(g*x+f))-2*(a*g^2+c*f^2)^(3/4)*(g^2*(c*x^2+a)/(a*g^2+c*f^2)/(1+c^(1/2)*(g
*x+f)/(a*g^2+c*f^2)^(1/2))^2)^(1/2)*(1+c^(1/2)*(g*x+f)/(a*g^2+c*f^2)^(1/2)
)*EllipticE(sin(2*arctan(c^(1/4)*(g*x+f)^(1/2)/(a*g^2+c*f^2)^(1/4))),1/2*(
2+2*c^(1/2)*f/(a*g^2+c*f^2)^(1/2))^(1/2))/c^(3/4)/e/(c*x^2+a)^(1/2)+2*(c^(
1/2)*f+(-a)^(1/2)*g)^(1/2)*(c*e*f^2+a*e*g^2+c^(1/2)*(-d*g+e*f)*(a*g^2+c*f^
2)^(1/2))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g
*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(
1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)
*g))^(1/2))/c^(3/4)/e^2/(a*g^2+c*f^2)^(1/2)/(c*x^2+a)^(1/2)-2*(c^(1/2)*f+(
-a)^(1/2)*g)^(1/2)*(-d*g+e*f)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))
^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticPi(c^(1/
4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),e*(f+(-a)^(1/2)*g/c^(1/2))
/(-d*g+e*f),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(
1/4)/e^2/(c*x^2+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.25 (sec) , antiderivative size = 927, normalized size of antiderivative = 1.28

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-i\sqrt{a}g}}\left(\frac{2i\sqrt{a}fg\sqrt{1+\frac{cx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{c}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}g}{i\sqrt{c}f+\sqrt{a}g}\right)}{\sqrt{c}e} - \frac{i\sqrt{a}dg^2\sqrt{1+\frac{cx^2}{a}}\,\text{Ellip}}{}\right.}{}$$

input
```
Integrate[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
(2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g)]*(((2*I)*Sqrt[a]*f*g
*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt
[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(Sqrt[c]*e) - (I*Sqrt[a]*d
*g^2*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/
Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)])/(Sqrt[c]*e^2) + (g*Sqr
t[(g*(Sqrt[a] + I*Sqrt[c]*x))/((-I)*Sqrt[c]*f + Sqrt[a]*g)]*(I*Sqrt[a] + S
qrt[c]*x)*((Sqrt[c]*f + I*Sqrt[a]*g)*EllipticE[ArcSin[Sqrt[(Sqrt[c]*(f + g
*x))/(Sqrt[c]*f - I*Sqrt[a]*g)]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I
*Sqrt[a]*g)] - I*Sqrt[a]*g*EllipticF[ArcSin[Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt
[c]*f - I*Sqrt[a]*g)]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g
)]))/(c*e*Sqrt[(g*(Sqrt[a] - I*Sqrt[c]*x))/(I*Sqrt[c]*f + Sqrt[a]*g)]) - (
Sqrt[a]*f^2*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sq
rt[a]*e), ArcSin[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(
I*Sqrt[c]*f + Sqrt[a]*g)])/(I*Sqrt[c]*d + Sqrt[a]*e) + (2*Sqrt[a]*d*f*g*Sq
rt[1 + (c*x^2)/a]*EllipticPi[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), ArcS
in[Sqrt[1 - (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f +
Sqrt[a]*g)])/(I*Sqrt[c]*d*e + Sqrt[a]*e^2) - (Sqrt[a]*d^2*g^2*Sqrt[1 + (c*
x^2)/a]*EllipticPi[(2*Sqrt[a]*e)/(I*Sqrt[c]*d + Sqrt[a]*e), ArcSin[Sqrt[1
- (I*Sqrt[c]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*g)/(I*Sqrt[c]*f + Sqrt[a]*g)
])/(e^2*(I*Sqrt[c]*d + Sqrt[a]*e))))/(Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 5.82 (sec) , antiderivative size = 1299, normalized size of antiderivative = 1.79, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {740, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{3/2}}{\sqrt{a+cx^2}(d+ex)}dx$$

$$\Big\downarrow \text{740}$$

$$\int \left(\frac{(ef-dg)^2}{e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}+\frac{g(ef-dg)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{g\sqrt{f+gx}}{e\sqrt{a+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}g^2}{\sqrt{c}e\sqrt{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)}-\frac{(ef-dg)^{3/2}\text{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{cx^2+a}}\right)}{e^{3/2}\sqrt{cd^2+ae^2}}-$$

$$\frac{2(cf^2+ag^2)^{3/4}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{c^{3/4}e\sqrt{cx^2+a}}+$$

$$\frac{(cf^2+ag^2)^{3/4}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right.\right.}{c^{3/4}e\sqrt{cx^2+a}}$$

$$\frac{(ef-dg)\sqrt[4]{cf^2+ag^2}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+a}}\right.\right.}{\sqrt[4]{c}e^2\sqrt{cx^2+a}}$$

$$\frac{\sqrt[4]{c}(ef-dg)^2\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(\right.}{e^2\left(age^2+cd(2ef-dg)\right)\sqrt{cx^2+ag}}$$

$$\frac{(ef-dg)\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)^2\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticPi}\left(\right.}{2\sqrt[4]{c}e^2\left(age^2+cd(2ef-dg)\right)\sqrt{cx^2+ag}}$$

input
```
Int[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
(2*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(Sqrt[c]*e*Sqrt[c*f^2 + a*g^2]*(1 +
(Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])) - ((e*f - d*g)^(3/2)*ArcTanh[(Sq
rt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt[a + c*x^2])
])/(e^(3/2)*Sqrt[c*d^2 + a*e^2]) - (2*(c*f^2 + a*g^2)^(3/4)*Sqrt[(g^2*(a +
 c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)
]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticE[2*ArcTan[(c^(1/4
)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g
^2])/2])/(c^(3/4)*e*Sqrt[a + c*x^2]) + ((e*f - d*g)*(c*f^2 + a*g^2)^(1/4)*
Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^
2 + a*g^2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticF[2*
ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sq
rt[c*f^2 + a*g^2])/2])/(c^(1/4)*e^2*Sqrt[a + c*x^2]) + ((c*f^2 + a*g^2)^(3
/4)*Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[
c*f^2 + a*g^2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f
)/Sqrt[c*f^2 + a*g^2])/2])/(c^(3/4)*e*Sqrt[a + c*x^2]) + (c^(1/4)*(e*f - d
*g)^2*(c*f^2 + a*g^2)^(1/4)*(Sqrt[c]*(e*f - d*g) - e*Sqrt[c*f^2 + a*g^2])*
Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^
2 + a*g^2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticF[2*
ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)...
```

Defintions of rubi rules used

rule 740
```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && Inte
gerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 852, normalized size of antiderivative = 1.17

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(-\frac{2g(dg-2ef)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{e^2\sqrt{cg\,x^3+cf\,x^2+agx+af}}+\frac{2g^2 \cdots}{\cdots}\right.}{\cdots}$
default	$\frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,\sqrt{-\frac{(gx+f)c}{\sqrt{-ac}\,g-cf}}\,\sqrt{\frac{(-cx+\sqrt{-ac})g}{\sqrt{-ac}\,g+cf}}\,\sqrt{\frac{(cx+\sqrt{-ac})g}{\sqrt{-ac}\,g-cf}}\left(\sqrt{-ac}\ \mathrm{EllipticF}\left(\sqrt{-\frac{(gx+f)c}{\sqrt{-ac}\,g-cf}},\sqrt{-\frac{\sqrt{-ac}\,g-cf}{\sqrt{-ac}\,g+cf}}\right)d\,g^2-\sqrt{-ac}\cdots\right.}{\cdots}$

input
```
int((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-2*g*(d*g-2*e*f)/
e^2*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(
1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2
)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*
c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*
g^2/e*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)
^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1
/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*Ell
ipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(
-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)
/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+2*(d^2*g^
2-2*d*e*f*g+e^2*f^2)/e^3*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c
))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)
/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d
/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/
(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(f+gx)^{\frac{3}{2}}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

input
```
integrate((g*x+f)**(3/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((f + g*x)**(3/2)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{3}{2}}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx=\int \frac{(gx+f)^{\frac{3}{2}}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx=\int \frac{(f+g\,x)^{3/2}}{\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

input
```
int((f + g*x)^(3/2)/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int((f + g*x)^(3/2)/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}}\,dx=\left(\int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,x}{ce\,x^3+cd\,x^2+aex+ad}dx\right)g+\left(\int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{ce\,x^3+cd\,x^2+aex+ad}dx\right)f$$

input
```
int((g*x+f)^(3/2)/(e*x+d)/(c*x^2+a)^(1/2),x)
```

output
```
int((sqrt(f + g*x)*sqrt(a + c*x**2)*x)/(a*d + a*e*x + c*d*x**2 + c*e*x**3)
,x)*g + int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*d + a*e*x + c*d*x**2 + c*e
*x**3),x)*f
```

3.132 $\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 740

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \frac{2g^2\sqrt{f+gx}\sqrt{a+cx^2}}{3ce}$$

$$-\frac{2\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}(7ef-3dg)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right)\right|}{3c^{3/4}e^2\sqrt{a+cx^2}}$$

$$-\frac{2\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(ae^2g^2+\sqrt{-a}\sqrt{c}eg(7ef-3dg)-3c(3e^2f^2-3defg+d^2g^2)\right)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-}}{3c^{5/4}e^3\sqrt{a+cx^2}}$$

$$-\frac{2\sqrt{\sqrt{c}f+\sqrt{-a}g}(ef-dg)^2\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticPi}\left(\frac{e\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}{ef-dg},\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right)\right.}{\sqrt[4]{c}e^3\sqrt{a+cx^2}}$$

output
```
2/3*g^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/e-2/3*(c^(1/2)*f-(-a)^(1/2)*g)*(c^
(1/2)*f+(-a)^(1/2)*g)^(1/2)*(-3*d*g+7*e*f)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(
-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*Ell
ipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-
a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(3/4)/e^2/(c*x^2+a)^(1/2)-2
/3*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(a*e^2*g^2+(-a)^(1/2)*c^(1/2)*e*g*(-3*d*
g+7*e*f)-3*c*(d^2*g^2-3*d*e*f*g+3*e^2*f^2))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-
(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*El
lipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(
-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(5/4)/e^3/(c*x^2+a)^(1/2)-
2*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(-d*g+e*f)^2*(1-c^(1/2)*(g*x+f)/(c^(1/2)*
f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*
EllipticPi(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),e*(f+(-a)^
(1/2)*g/c^(1/2))/(-d*g+e*f),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2
)*g))^(1/2))/c^(1/4)/e^3/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.20 (sec) , antiderivative size = 1440, normalized size of antiderivative = 1.95

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
(2*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*e) + (2*(f + g*x)^(3/2)*(7*c*e^
2*f*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]] + (7*c*e^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3*
c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (7*a*e^2*f*g^2
*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (3*a*d*e*g^3*Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (14*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]])/(f + g*x) + (6*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f +
g*x) + (Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(7*e*f - 3*d*g)*Sqrt[1 - f/
(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x) + (I*S
qrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)
/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]
*g)])/Sqrt[f + g*x] + (I*e*(Sqrt[c]*f + I*Sqrt[a]*g)*(I*Sqrt[a]*e*g + Sqrt
[c]*(6*e*f - 3*d*g))*Sqrt[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*
x))]*Sqrt[1 - f/(f + g*x) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticF[I
*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*S
qrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqrt[f + g*x] + ((3*I)*c*e^2*f^2*Sqr
t[1 - f/(f + g*x) - (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*Sqrt[1 - f/(f + g*x
) + (I*Sqrt[a]*g)/(Sqrt[c]*(f + g*x))]*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e
*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sq
rt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)])/Sqr...
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1848 vs. $2(740) = 1480$.

Time = 7.52 (sec) , antiderivative size = 1848, normalized size of antiderivative = 2.50, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {740, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^{5/2}}{\sqrt{a+cx^2}(d+ex)}dx$$

$$\Big\downarrow \text{740}$$

$$\int \left(\frac{(ef-dg)^3}{e^3\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}+\frac{g(ef-dg)^2}{e^3\sqrt{a+cx^2}\sqrt{f+gx}}+\frac{g\sqrt{f+gx}(ef-dg)}{e^2\sqrt{a+cx^2}}+\frac{g(f+gx)^{3/2}}{e\sqrt{a+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{c}\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\cdots\right.\right.}{e^3g\left(age^2+cd(2ef-dg)\right)\sqrt{cx^2+a}}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{cx^2+a}}\right)(ef-dg)^{5/2}}{e^{5/2}\sqrt{cd^2+ae^2}}+$$

$$\frac{\sqrt[4]{cf^2+ag^2}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}e^3\sqrt{cx^2+a}}$$

$$\frac{\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)^2\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticPi}\left(\frac{\left(\sqrt{cf^2+ag^2}e+\cdots\right.}{4\sqrt{c}e(ef-dg\cdots}\right.}{2\sqrt[4]{c}e^3g\left(age^2+cd(2ef-dg)\right)\sqrt{cx^2+a}}$$

$$\frac{2\left(cf^2+ag^2\right)^{3/4}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)(ef\cdots}{c^{3/4}e^2\sqrt{cx^2+a}}$$

$$\frac{\left(cf^2+ag^2\right)^{3/4}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\cdots\right.\right.}{c^{3/4}e^2\sqrt{cx^2+a}}$$

$$\frac{2g^2\sqrt{f+gx}\sqrt{cx^2+a}(ef-dg)}{\sqrt{c}e^2\sqrt{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)}-$$

$$\frac{8f\left(cf^2+ag^2\right)^{3/4}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{3c^{3/4}e\sqrt{cx^2+a}}-$$

$$\frac{\sqrt[4]{cf^2+ag^2}\left(cf^2-4\sqrt{c}\sqrt{cf^2+ag^2}f+ag^2\right)\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\cdots}{\sqrt[4]{\cdots}}\right.\right.}{3c^{5/4}e\sqrt{cx^2+a}}$$

$$\frac{2g^2\sqrt{f+gx}\sqrt{cx^2+a}}{3ce}+\frac{8fg^2\sqrt{f+gx}\sqrt{cx^2+a}}{3\sqrt{c}e\sqrt{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)}$$

input
```
Int[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + c*x^2]),x]
```

output
```
(2*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*e) + (8*f*g^2*Sqrt[f + g*x]*Sqr
t[a + c*x^2])/(3*Sqrt[c]*e*Sqrt[c*f^2 + a*g^2]*(1 + (Sqrt[c]*(f + g*x))/Sq
rt[c*f^2 + a*g^2])) + (2*g^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(S
qrt[c]*e^2*Sqrt[c*f^2 + a*g^2]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2
])) - ((e*f - d*g)^(5/2)*ArcTanh[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt
[e]*Sqrt[e*f - d*g]*Sqrt[a + c*x^2])])/(e^(5/2)*Sqrt[c*d^2 + a*e^2]) - (8*
f*(c*f^2 + a*g^2)^(3/4)*Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt
[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^
2 + a*g^2])*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/
4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(3*c^(3/4)*e*Sqrt[a + c*x^2
]) - (2*(e*f - d*g)*(c*f^2 + a*g^2)^(3/4)*Sqrt[(g^2*(a + c*x^2))/((c*f^2 +
 a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*(1 + (Sqrt[c]*(f
 + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(
c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(3/4)*
e^2*Sqrt[a + c*x^2]) + ((e*f - d*g)^2*(c*f^2 + a*g^2)^(1/4)*Sqrt[(g^2*(a +
 c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)
]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticF[2*ArcTan[(c^(1/4
)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g
^2])/2])/(c^(1/4)*e^3*Sqrt[a + c*x^2]) + ((e*f - d*g)*(c*f^2 + a*g^2)^(3/4
)*Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt...
```

Defintions of rubi rules used

rule 740
```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && Inte
gerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 7.39 (sec) , antiderivative size = 948, normalized size of antiderivative = 1.28

method	result
elliptic	$\dfrac{\sqrt{(gx+f)(c\,x^{2}+a)}\left(\dfrac{2g^{2}\sqrt{c g\,x^{3}+c f\,x^{2}+a g x+a f}}{3ec}+\dfrac{2\left(\frac{g\left(d^{2}g^{2}-3defg+3e^{2}f^{2}\right)}{e^{3}}-\frac{a\,g^{3}}{3ce}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-}}{c}}{-\frac{f}{g}+\sqrt{}}}}{\sqrt{c g\,x^{3}+c f\,x^{2}+a g x+a f}}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/3*g^2/e/c*(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(g*(d^2*g^2-3*d*e*f*g+3*e^2*f^2)/e^3-1/3*a/
c*g^3/e)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a
*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)
^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g
-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2
))+2*(-1/e^2*g^2*(d*g-3*e*f)-2/3*f*g^2/e)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f
/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)
*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a
*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^
(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*
EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/
g-(-a*c)^(1/2)/c))^(1/2)))-2*(d^3*g^3-3*d^2*e*f*g^2+3*d*e^2*f^2*g-e^3*f^3)
/e^4*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^
(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/
2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f
/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-
a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(f+gx)^{\frac{5}{2}}}{\sqrt{a+cx^2}\,(d+ex)}\,dx$$

input
```
integrate((g*x+f)**(5/2)/(e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((f + g*x)**(5/2)/(sqrt(a + c*x**2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{5}{2}}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{5}{2}}}{\sqrt{cx^2+a}(ex+d)}\,dx$$

input
```
integrate((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(f+g\,x)^{5/2}}{\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

input
```
int((f + g*x)^(5/2)/((a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int((f + g*x)^(5/2)/((a + c*x^2)^(1/2)*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)^{\frac{5}{2}}}{(ex+d)\sqrt{c\,x^2+a}}dx$$

input
```
int((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x)
```

output
```
int((g*x+f)^(5/2)/(e*x+d)/(c*x^2+a)^(1/2),x)
```

3.133 $\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 606

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$

$$= -\frac{2e^2(7ef-15dg)\sqrt{f+gx}\sqrt{a+cx^2}}{15cg^2}+\frac{2e^3(f+gx)^{3/2}\sqrt{a+cx^2}}{5cg^2}$$

$$+\frac{2e\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\,(9ae^2g^2-c(8e^2f^2-30defg+45d^2g^2))\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(}{\sqrt{c}f-}}}{15c^{7/4}g^4\sqrt{a+cx^2}}$$

$$+\frac{2\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(15c^{3/2}d^3g^2+9\sqrt{-a}ae^3g^2-a\sqrt{c}e^2g(2ef+15dg)-\sqrt{-a}ce(8e^2f^2-30defg+45d^2\right.}{15c^{7/4}g^3\sqrt{a+cx^2}}$$

output
```
-2/15*e^2*(-15*d*g+7*e*f)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2+2/5*e^3*(g*x
+f)^(3/2)*(c*x^2+a)^(1/2)/c/g^2+2/15*e*(c^(1/2)*f-(-a)^(1/2)*g)*(c^(1/2)*f
+(-a)^(1/2)*g)^(1/2)*(9*a*e^2*g^2-c*(45*d^2*g^2-30*d*e*f*g+8*e^2*f^2))*(1-
c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2
)*f+(-a)^(1/2)*g))^(1/2)*EllipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(
1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c
^(7/4)/g^4/(c*x^2+a)^(1/2)+2/15*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(15*c^(3/2)
*d^3*g^2+9*(-a)^(1/2)*a*e^3*g^2-a*c^(1/2)*e^2*g*(15*d*g+2*e*f)-(-a)^(1/2)*
c*e*(45*d^2*g^2-30*d*e*f*g+8*e^2*f^2))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^
(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*Ellipti
cF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(
1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(7/4)/g^3/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.99 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$

$$= \frac{\sqrt{f+gx}\left(\frac{2e^2(-4ef+15dg+3egx)(a+cx^2)}{cg^2} + \frac{2(f+gx)\left(\frac{eg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\left(-9ae^2g^2+c\left(8e^2f^2-30defg+45d^2g^2\right)\right)\left(a+cx^2\right)}{(f+gx)^2} + \frac{\sqrt{c}e(-i\sqrt{c}f+\sqrt{a}g)}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$$

input
```
Integrate[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
(Sqrt[f + g*x]*((2*e^2*(-4*e*f + 15*d*g + 3*e*g*x)*(a + c*x^2))/(c*g^2) +
(2*(f + g*x)*((e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-9*a*e^2*g^2 + c*(8
*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*(a + c*x^2))/(f + g*x)^2 + (Sqrt[c]*e
*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-9*a*e^2*g^2 + c*(8*e^2*f^2 - 30*d*e*f*g +
45*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt
[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]
*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt
[a]*g)])/Sqrt[f + g*x] + (Sqrt[c]*g*((15*I)*c^(3/2)*d^3*g^2 + 9*a^(3/2)*e^
3*g^2 - I*a*Sqrt[c]*e^2*g*(2*e*f + 15*d*g) + Sqrt[a]*c*e*(-8*e^2*f^2 + 30*
d*e*f*g - 45*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[
-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*EllipticF[I*ArcSinh[Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*
f + I*Sqrt[a]*g)])/Sqrt[f + g*x]))/(c^2*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]])))/(15*Sqrt[a + c*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 2.20 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.37, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {728, 25, 2185, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{728}$$

$$\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}-\frac{\int -\frac{5cgd^3-4ce^2(ef-3dg)x^2-ae^2(2ef+dg)-e(3age^2+cd(2ef-15dg))x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{5cg}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5cgd^3-4ce^2(ef-3dg)x^2-ae^2(2ef+dg)-e(3age^2+cd(2ef-15dg))x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{5cg}+\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\frac{2\int \frac{cg\left(g\left(15cd^3g-ae^2(2ef+15dg)\right)-e\left(9ae^2g^2-c\left(8e^2f^2-30degf+45d^2g^2\right)\right)x\right)}{2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3cg^2}-\frac{8e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef-3dg)}{3g}}{5cg}+\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{g\left(15cd^3g-ae^2(2ef+15dg)\right)-e\left(9ae^2g^2-c\left(8e^2f^2-30degf+45d^2g^2\right)\right)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3g}-\frac{8e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef-3dg)}{3g}}{5cg}+\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

$\Big\downarrow$ 599

$$\frac{-\frac{2\int -\frac{ae^2(7ef-15dg)g^2-c\left(8e^3f^3-30de^2gf^2+45d^2eg^2f-15d^3g^3\right)-e\left(9ae^2g^2-c\left(8e^2f^2-30degf+45d^2g^2\right)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^3}-\frac{8e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef}{3g}}{5cg}+\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

$\Big\downarrow$ 25

$$\frac{\frac{2\int \frac{ae^2(7ef-15dg)g^2-c\left(8e^3f^3-30de^2gf^2+45d^2eg^2f-15d^3g^3\right)-e\left(9ae^2g^2-c\left(8e^2f^2-30degf+45d^2g^2\right)\right)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3g^3}-\frac{8e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef-3d}{3g}}{5cg}+\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

$\Big\downarrow$ 1511

$$\frac{-\frac{2\left(-\frac{\left(-e\sqrt{ag^2+cf^2}\left(9ae^2g^2-c\left(45d^2g^2-30defg+8e^2f^2\right)\right)+a\sqrt{c}e^2g^2(7ef-15dg)-c^{3/2}\left(-15d^3g^3+45d^2efg^2-30de^2f^2g+8e^3f^3\right)\right)\int\frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\cdots}}}{\sqrt{c}}\cdots\right.}{3g^3}}{5\cdots}+\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

$\Big\downarrow$ 1416

$$-\frac{2\left(-\frac{e\sqrt{ag^2+cf^2}\left(9ae^2g^2-c\left(45d^2g^2-30defg+8e^2f^2\right)\right)\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2c\ldots}{}}{\left(a+\frac{cf^2}{g^2}\right)\ldots}}}{\ldots}\ldots\right)}{\ldots}$$

$$\frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

$\Big\downarrow$ 1509

$$\frac{2(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}e^2}{5cg}+$$

$$\frac{-\frac{8(ef-3dg)\sqrt{f+gx}\sqrt{cx^2+a}e^2}{3g}-\frac{2\left(-\frac{e\sqrt{cf^2+ag^2}\left(9ae^2g^2-c\left(8e^2f^2-30degf+45d^2g^2\right)\right)\left(\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(\ldots}{}}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+\ldots}{\sqrt{cf^2+\ldots}}\right)}}}{\sqrt[4]{c}\sqrt{\frac{cf^2}{g^2}-\frac{2c\ldots}{}}}\ldots\right)}{\ldots}\ldots\right)}{\ldots}}{\ldots}$$

input
```
Int[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
(2*e^2*(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(5*c*g) + ((-8*e^2*(e*f -
3*d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*g) - (2*(-((e*Sqrt[c*f^2 + a*g^2]
*(9*a*e^2*g^2 - c*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*(-((Sqrt[f + g*x]
*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a
+ (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 +
 a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f
^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*
(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1
/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a
*g^2])/2])/(c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f +
 g*x)^2)/g^2]))))/Sqrt[c]) - ((c*f^2 + a*g^2)^(1/4)*(a*Sqrt[c]*e^2*g^2*(7*e
*f - 15*d*g) - c^(3/2)*(8*e^3*f^3 - 30*d*e^2*f^2*g + 45*d^2*e*f*g^2 - 15*d
^3*g^3) - e*Sqrt[c*f^2 + a*g^2]*(9*a*e^2*g^2 - c*(8*e^2*f^2 - 30*d*e*f*g +
 45*d^2*g^2)))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*
f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)
*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(
1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 +
a*g^2])/2])/(2*c^(3/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(
f + g*x)^2)/g^2])))/(3*g^3))/(5*c*g)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 728
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Simp[1/(c*g*(2*m - 1))   Int[((d + e*x)^(m -
3)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^
3*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
 2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g
}, x] && IntegerQ[2*m] && GeQ[m, 2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.17

method	result
elliptic	$\dfrac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{2e^3x\sqrt{cg\,x^3+cf\,x^2+agx+af}}{5cg}+\frac{2\left(3d\,e^2-\frac{4f\,e^3}{5g}\right)\sqrt{cg\,x^3+cf\,x^2+agx+af}}{3cg}+\frac{2\left(d^3-\frac{2af\,e^3}{5cg}-\frac{a\left(3d\,e^2-\frac{4f\,e^3}{5g}\right)}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-}}{c}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/5*e^3/c/g*x*(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2/3*(3*d*e^2-4/5*f/g*e^3)/c/g*(c*g*x^3+c*f*
x^2+a*g*x+a*f)^(1/2)+2*(d^3-2/5*a*f/c/g*e^3-1/3*a/c*(3*d*e^2-4/5*f/g*e^3))
*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2
)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c
))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^
(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(3*
d^2*e-3/5*a/c*e^3-2/3*f/g*(3*d*e^2-4/5*f/g*e^3))*(f/g-(-a*c)^(1/2)/c)*((x+
f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c)
)^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+
a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/
2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(
1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/
c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx =$$
$$-\frac{2\left(\left(8\,ce^3f^3-30\,cde^2f^2g+3\,(15\,cd^2e-ae^3)fg^2-45\,(cd^3-ade^2)g^3\right)\sqrt{cg}\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3}{3\,cg}\right.\right.}{}$$

input
```
integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*((8*c*e^3*f^3 - 30*c*d*e^2*f^2*g + 3*(15*c*d^2*e - a*e^3)*f*g^2 - 45
*(c*d^3 - a*d*e^2)*g^3)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2
)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 3*(8*c*
e^3*f^2*g - 30*c*d*e^2*f*g^2 + 9*(5*c*d^2*e - a*e^3)*g^3)*sqrt(c*g)*weiers
trassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3)
, weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*
g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) - 3*(3*c*e^3*g^3*x - 4*c*e^3*f*g^2 + 15*
c*d*e^2*g^3)*sqrt(c*x^2 + a)*sqrt(g*x + f))/(c^2*g^4)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^3}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**3/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^3}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^3}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(d+e\,x)^3}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

input
```
int((d + e*x)^3/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^3/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$
$$= \frac{-6\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,a\,e^3g+30\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,c\,d^2eg+4\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,c\,e^3fx+9\Big(\int \frac{\sqrt{gx+f}\,\sqrt{}}{cg\,x^3+cf\,x}}{}$$

input
```
int((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

output
```
( - 6*sqrt(f + g*x)*sqrt(a + c*x**2)*a*e**3*g + 30*sqrt(f + g*x)*sqrt(a +
c*x**2)*c*d**2*e*g + 4*sqrt(f + g*x)*sqrt(a + c*x**2)*c*e**3*f*x + 9*int((
sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x
)*a*c*e**3*g**2 - 45*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*
x + c*f*x**2 + c*g*x**3),x)*c**2*d**2*e*g**2 + 30*int((sqrt(f + g*x)*sqrt(
a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c**2*d*e**2*f*g -
 8*int((sqrt(f + g*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g
*x**3),x)*c**2*e**3*f**2 + 3*int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a
*g*x + c*f*x**2 + c*g*x**3),x)*a**2*e**3*g**2 - 15*int((sqrt(f + g*x)*sqrt
(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*a*c*d**2*e*g**2 - 4*i
nt((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)
*a*c*e**3*f**2 + 10*int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*
f*x**2 + c*g*x**3),x)*c**2*d**3*f*g)/(10*c**2*f*g)
```

3.134 $\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 489

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{2e^2\sqrt{f+gx}\sqrt{a+cx^2}}{3cg}$$

$$+\frac{4e\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}(ef-3dg)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right.\right.}{3c^{3/4}g^3\sqrt{a+cx^2}}$$

$$+\frac{2\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(3cd^2g-ae^2g+2\sqrt{-a}\sqrt{c}e(ef-3dg)\right)\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticF}\left(\right.}{3c^{5/4}g^2\sqrt{a+cx^2}}$$

output
```
2/3*e^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g+4/3*e*(c^(1/2)*f-(-a)^(1/2)*g)*(
c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(-3*d*g+e*f)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(
-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*Ell
ipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-
a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(3/4)/g^3/(c*x^2+a)^(1/2)+2
/3*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(3*c*d^2*g-a*e^2*g+2*(-a)^(1/2)*c^(1/2)*
e*(-3*d*g+e*f))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1
/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2
)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)
^(1/2)*g))^(1/2))/c^(5/4)/g^2/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.10 (sec) , antiderivative size = 473, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$
$$= \frac{2\sqrt{f+gx}\left(e^2g^2(a+cx^2) - \frac{2eg^2(ef-3dg)(a+cx^2)}{f+gx} - 2ice\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(ef-3dg)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}\sqrt{f\right.}{}$$

input
```
Integrate[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
(2*Sqrt[f + g*x]*(e^2*g^2*(a + c*x^2) - (2*e*g^2*(e*f - 3*d*g)*(a + c*x^2)
)/(f + g*x) - (2*I)*c*e*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - 3*d*g)*Sqr
t[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] -
 g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g
)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a
]*g)] + (g*((3*I)*c*d^2*g - I*a*e^2*g + 2*Sqrt[a]*Sqrt[c]*e*(e*f - 3*d*g))
*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[
c] - g*x)/(f + g*x))]*Sqrt[f + g*x]*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sq
rt[a]*g)])/Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]))/(3*c*g^3*Sqrt[a + c*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.58 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.43, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {728, 25, 2004, 599, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{ 728}$$

$$\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}-\frac{\int -\frac{-2ce^2(ef-3dg)x^2-e(age^2+cd(2ef-9dg))x+d(3cd^2-ae^2)g}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3cg}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-2ce^2(ef-3dg)x^2-e(age^2+cd(2ef-9dg))x+d(3cd^2-ae^2)g}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3cg}+\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}$$

$$\Big\downarrow \text{ 2004}$$

$$\frac{\int \frac{(3cd^2-ae^2)g-2ce(ef-3dg)x}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{3cg}+\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}$$

$$\Big\downarrow \text{ 599}$$

$$\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}-\frac{2\int \frac{ae^2g^2-c(2e^2f^2-6degf+3d^2g^2)+2ce(ef-3dg)(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{3cg^3}$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}-$$

$$\frac{2\left(\left(2\sqrt{c}e\sqrt{ag^2+cf^2}(ef-3dg)+ae^2g^2-c\left(3d^2g^2-6defg+2e^2f^2\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}-\right.}{3cg^3}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}-$$

$$\frac{2\left(\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(2\sqrt{c}e\sqrt{ag^2+cf^2}(ef-3dg)+ae^2g^2-c\left(3d^2g^2-6defg+2e^2f^2\right)\right)\text{EllipticF}\left(\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.}{3c}$$

$$\Big\downarrow \text{ 1509}$$

$$\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg} - \frac{2\left(\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(2\sqrt{c}e\sqrt{ag^2+cf^2}(ef-3dg)+ae^2g^2-c\left(3d^2g^2-6defg+2e^2f^2\right)\right)\text{EllipticF}\left(\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}\right.}{}$$

input

```
Int[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output

```
(2*e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*c*g) - (2*(-2*Sqrt[c]*e*(e*f - 3*
d*g)*Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(
f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f
+ g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f +
 g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2
+ (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*
f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^
2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a + (c*
f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])) + ((c*f^2 + a*g^
2)^(1/4)*(a*e^2*g^2 + 2*Sqrt[c]*e*(e*f - 3*d*g)*Sqrt[c*f^2 + a*g^2] - c*(2
*e^2*f^2 - 6*d*e*f*g + 3*d^2*g^2))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a
*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2
)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*Ell
ipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt
[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(
f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(3*c*g^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 728
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Simp[1/(c*g*(2*m - 1))   Int[((d + e*x)^(m -
3)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^
3*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
 2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g
}, x] && IntegerQ[2*m] && GeQ[m, 2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.26

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(\frac{2e^2\sqrt{cg\,x^3+cf\,x^2+agx+af}}{3cg}+\frac{2\left(d^2-\frac{a\,e^2}{3c}\right)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x-}{\frac{f}{g}-}}\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+af}}\right.}{}$
risch	$\frac{2e^2\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{3cg}-\frac{\left(-\frac{4ce(3dg-ef)\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\left(\left(-\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right.\right.}{\sqrt{cg\,x^3+cf\,x^2+agx+af}}\right.}{}$
default	Expression too large to display

input
```
int((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(2/3*e^2/c/g*(c*g*
x^3+c*f*x^2+a*g*x+a*f)^(1/2)+2*(d^2-1/3*a*e^2/c)*(f/g-(-a*c)^(1/2)/c)*((x+
f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c)
)^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+
a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a
*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+2*(2*d*e-2/3*e^2*f/g)*(f/g-(-a*
c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g
-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(
c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)
/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))
^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f
/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.50

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$
$$= \frac{2\left(3\sqrt{cx^2+a}\sqrt{gx+f}ce^2g^2+(2ce^2f^2-6cdefg+3(3cd^2-ae^2)g^2)\sqrt{cg}\,\text{weierstrassPInverse}\left(\frac{4(cf^2-3a}{3cg^2}\right.\right.}{}$$

input
```
integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(3*sqrt(c*x^2 + a)*sqrt(g*x + f)*c*e^2*g^2 + (2*c*e^2*f^2 - 6*c*d*e*f*
g + 3*(3*c*d^2 - a*e^2)*g^2)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*
a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g) + 6*
(c*e^2*f*g - 3*c*d*e*g^2)*sqrt(c*g)*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/
(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2
 - 3*a*g^2)/(c*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)
))/(c^2*g^3)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^2}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**2/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{{\left(d+e\,x\right)}^2}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

input
```
int((d + e*x)^2/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^2/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{2\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,de-3\left(\int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,x^2}{cg\,x^3+cf\,x^2+agx+af}dx\right)cdeg+\left(\int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,x^2}{cg\,x^3+cf\,x^2+agx+af}dx\right)c\,e^2f-\left(\int \frac{\sqrt{gx+f}\ldots}{cg\,x^3+cf\,x^2\ldots}\right.}{cf}$$

input
```
int((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(f + g*x)*sqrt(a + c*x**2)*d*e - 3*int((sqrt(f + g*x)*sqrt(a + c*x*
*2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c*d*e*g + int((sqrt(f + g
*x)*sqrt(a + c*x**2)*x**2)/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x)*c*e**2*f
 - int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3
),x)*a*d*e*g + int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**
2 + c*g*x**3),x)*c*d**2*f)/(c*f)
```

3.135 $\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 424

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2e\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right)|\frac{\sqrt{c}f+\sqrt{-a}\ldots}{\sqrt{c}f-\sqrt{-a}\ldots}\right.}{c^{3/4}g^2\sqrt{a+cx^2}} + \frac{2\left(\sqrt{c}d-\sqrt{-a}e\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right),\ldots\right.}{c^{3/4}g\sqrt{a+cx^2}}$$

output
```
-2*e*(c^(1/2)*f-(-a)^(1/2)*g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g
*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(
1/2)*g))^(1/2)*EllipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1
/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(3/4)/g^2
/(c*x^2+a)^(1/2)+2*(c^(1/2)*d-(-a)^(1/2)*e)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)
*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^
(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-
a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2
))/c^(3/4)/g/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.03 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.04

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx =$$

$$-\frac{2\left(-eg^2\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(a+cx^2)+i\sqrt{c}e\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)^{3/2}E\left(i\text{arcsinh}\left(\right.\right.}{cg}$$

input
```
Integrate[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
(-2*(-(e*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(a + c*x^2)) + I*Sqrt[c]*e*(
Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqr
t[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*
ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sq
rt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + Sqrt[c]*((-I)*Sqrt[c]*d + Sqrt[a]*e)
*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqr
t[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
I*Sqrt[a]*g)]))/(c*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[f + g*x]*Sqrt
[a + c*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.15 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.45, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {599, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \; 599$$

$$-\frac{2\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}$$

$$\Big\downarrow \; 1511$$

$$-\frac{2\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\left(dg-e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+\cdots}\right.}{g^2}$$

$$\Big\downarrow \; 1416$$

$$-\frac{2\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(dg-e\left(f-\cdots\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2c\cdots}{}}}\right.}{g^2}$$

$$\Big\downarrow \; 1509$$

$$-\frac{2\left(\frac{e\sqrt{ag^2+cf^2}\left(\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}-\frac{\sqrt{f+\cdots}}{\cdots}\right.}{\sqrt{c}}\right.}{\cdots}$$

input
```
Int[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
(-2*((e*Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^2 - (2*c*
f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*
(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(
f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g
^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt
[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a
*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*Sqrt[a +
(c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/Sqrt[c] - ((
c*f^2 + a*g^2)^(1/4)*(d*g - e*(f - Sqrt[c*f^2 + a*g^2]/Sqrt[c]))*(1 + (Sqr
t[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g
*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x)
)/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f
^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*c^(1/4)*S
qrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2]))))/g^2
```

Defintions of rubi rules used

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.23

method	result
default	$\frac{2\left(\operatorname{EllipticF}\left(\sqrt{-\frac{(gx+f)c}{\sqrt{-ac}\,g-cf}},\sqrt{-\frac{\sqrt{-ac}\,g-cf}{\sqrt{-ac}\,g+cf}}\right)ae\,g^{2}+\operatorname{EllipticF}\left(\sqrt{-\frac{(gx+f)c}{\sqrt{-ac}\,g-cf}},\sqrt{-\frac{\sqrt{-ac}\,g-cf}{\sqrt{-ac}\,g+cf}}\right)cdfg-\sqrt{-ac}\,\operatorname{EllipticF}\left(\sqrt{-\frac{(gx}{\sqrt{-a}}}\right.\right.}{}$
elliptic	$\frac{\sqrt{(gx+f)(c\,x^{2}+a)}\left(\frac{2d\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{cg\,x^{3}+cf\,x^{2}+agx+af}}+\frac{2e\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right.}{}\right.}{\sqrt{}}$

input
```
int((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f
)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*e*g^2+EllipticF((-(g*x+f)*c/((-a*c)^(1/2)
*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*c*d*f*g
-(-a*c)^(1/2)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^
(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*d*g^2+(-a*c)^(1/2)*EllipticF((-(
g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*
g+c*f))^(1/2))*e*f*g-EllipticE((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-(
(-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*a*e*g^2-EllipticE((-(g*x+
f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*
f))^(1/2))*c*e*f^2)*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c
*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*(-(g*x+f)*c/((-a*c)^(1/2)*g
-c*f))^(1/2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2/(c*g*x^3+c*f*x^2+a*g*x+a*
f)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.42

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2\left(3\,\sqrt{cg}eg\text{weierstrassZeta}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\text{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3}\right.\right.\right.}{3\,cg^2}$$

input
```
integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(c*g)*e*g*weierstrassZeta(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27
*(c*f^3 + 9*a*f*g^2)/(c*g^3), weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c
*g^2), -8/27*(c*f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)) + (e*f - 3*d
*g)*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*
f^3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g))/(c*g^2)
```

Sympy [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{d+ex}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

input
```
integrate((e*x+d)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{ex+d}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{ex+d}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{d+e\,x}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

input
```
int((d + e*x)/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x)/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \left(\int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}\,x}{cg\,x^3+cf\,x^2+agx+af}dx\right)e + \left(\int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{cg\,x^3+cf\,x^2+agx+af}dx\right)d$$

input
```
int((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

output
```
int((sqrt(f + g*x)*sqrt(a + c*x**2)*x)/(a*f + a*g*x + c*f*x**2 + c*g*x**3)
,x)*e + int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g
*x**3),x)*d
```

3.136 $\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 193

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{2\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right),\frac{\sqrt{c}f+\sqrt{-a}g}{\sqrt{c}f-\sqrt{-a}g}\right)}{\sqrt[4]{c}g\sqrt{a+cx^2}}$$

output
```
2*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*
g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(
1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g
)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(1/4)/g/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{2i\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right),\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)}{g\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\sqrt{a+cx^2}}$$

input
```
Integrate[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
((2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqr
t[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*
Sqrt[a]*g)])/(g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[a + c*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {510, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \text{510}$$

$$\frac{2\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}g\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}$$

input
```
Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
((c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[
(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f
^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[
c*f^2 + a*g^2])/2])/(c^(1/4)*g*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^
2 + (c*(f + g*x)^2)/g^2])
```

Defintions of rubi rules used

rule 510
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2/d   Subst[Int[1/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2
)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.04

method	result	si
default	$\frac{2\left(cf-\sqrt{-ac}\,g\right)\operatorname{EllipticF}\left(\sqrt{-\frac{(gx+f)c}{\sqrt{-ac}\,g-cf}},\sqrt{-\frac{\sqrt{-ac}\,g-cf}{\sqrt{-ac}\,g+cf}}\right)\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{\sqrt{-ac}\,g-cf}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{\sqrt{-ac}\,g+cf}}\,\sqrt{-\frac{(gx+f)c}{\sqrt{-ac}\,g-cf}}\,\sqrt{c\,x^{2}+a}\,\sqrt{gx+f}}{cg\left(cg\,x^{3}+cf\,x^{2}+agx+af\right)}$	20
elliptic	$\frac{2\sqrt{(gx+f)\left(c\,x^{2}+a\right)}\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{gx+f}\,\sqrt{c\,x^{2}+a}\,\sqrt{cg\,x^{3}+cf\,x^{2}+agx+af}}$	24

input
```
int(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(c*f-(-a*c)^(1/2)*g)*EllipticF((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),(
-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+c*f))^(1/2))*((c*x+(-a*c)^(1/2))*g/(
(-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1
/2)*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2)*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)/
c/g/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \frac{2\,\sqrt{cg}\operatorname{weierstrassPInverse}\left(\frac{4\,(cf^2-3\,ag^2)}{3\,cg^2},-\frac{8\,(cf^3+9\,afg^2)}{27\,cg^3},\frac{3\,gx+f}{3\,g}\right)}{cg}$$

input
```
integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(c*g)*weierstrassPInverse(4/3*(c*f^2 - 3*a*g^2)/(c*g^2), -8/27*(c*f^
3 + 9*a*f*g^2)/(c*g^3), 1/3*(3*g*x + f)/g)/(c*g)
```

Sympy [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\sqrt{f+gx}}\,dx$$

input
```
integrate(1/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{cg\,x^3+cf\,x^2+agx+af}dx$$

input
```
int(1/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

output
```
int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*f + a*g*x + c*f*x**2 + c*g*x**3),x
)
```

3.137 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 228

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx =$$
$$-\frac{2\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\,\text{EllipticPi}\left(\frac{e\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}{ef-dg},\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right),\frac{\sqrt{c}f+\sqrt{}}{\sqrt{c}f-\sqrt{}}\right.}{\sqrt[4]{c}(ef-dg)\sqrt{a+cx^2}}$$

output
```
-2*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)
*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticPi(c
^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),e*(f+(-a)^(1/2)*g/c^(1
/2))/(-d*g+e*f),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))
/c^(1/4)/(-d*g+e*f)/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.09 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.36

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx =$$

$$-\frac{2i\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right),\frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g}\right)-\text{EllipticPi}\left(\frac{\sqrt{c}(}{e(\sqrt{c}}\right.\right.}{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}(ef-dg)\sqrt{a+cx^2}}$$

input
```
Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
((-2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g
)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*(EllipticF[I*ArcSinh[Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f +
I*Sqrt[a]*g)] - EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]
*g)), I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*
f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/(Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]*(e*f - d*g)*Sqrt[a + c*x^2])
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 867 vs. 2(228) = 456.

Time = 2.73 (sec) , antiderivative size = 867, normalized size of antiderivative = 3.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {729, 25, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \; 729$$

$$2\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}$$

$\downarrow$ 25

$$-2\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}$$

$\downarrow$ 1540

$$2\left(\frac{e\sqrt{ag^2+cf^2}\left(\sqrt{c}(ef-dg)-e\sqrt{ag^2+cf^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g\left(ae^2g+cd(2ef-dg)\right)}-\frac{\sqrt{c}\left(-\sqrt{\right.}}{}\right.$$

$\downarrow$ 1416

$$2\left(\frac{e\sqrt{ag^2+cf^2}\left(\sqrt{c}(ef-dg)-e\sqrt{ag^2+cf^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g\left(ae^2g+cd(2ef-dg)\right)}-\frac{\sqrt[4]{c}\left(\frac{\sqrt{c}}{\sqrt{a}}\right.}{}\right.$$

$\downarrow$ 2222

$$2\left(\frac{e\sqrt{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\left(\frac{\left(e+\frac{\sqrt{c}(ef-dg)}{\sqrt{cf^2+ag^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}\right)}{2\sqrt{e}\sqrt{cd^2+ae^2}\sqrt{ef-dg}}-\right.}{g\left(a\right.}\right.$$

input
```
Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
2*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g^2 - Sqrt[c]*(e*f - d*g)*Sqrt[c*f^2 + a*g
^2])*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 -
 (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqr
t[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[
f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2]
)/(g*(c*f^2 + a*g^2)^(1/4)*(a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[a + (c*f^2)/
g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2]) + (e*Sqrt[c*f^2 + a*g^
2]*(Sqrt[c]*(e*f - d*g) - e*Sqrt[c*f^2 + a*g^2])*(((e + (Sqrt[c]*(e*f - d*
g))/Sqrt[c*f^2 + a*g^2])*ArcTanh[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt
[e]*Sqrt[e*f - d*g]*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f +
 g*x)^2)/g^2])])/(2*Sqrt[e]*Sqrt[c*d^2 + a*e^2]*Sqrt[e*f - d*g]) - ((Sqrt[
c]/e - Sqrt[c*f^2 + a*g^2]/(e*f - d*g))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^
2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2
)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)
]*EllipticPi[(Sqrt[c]*(e*f - d*g) + e*Sqrt[c*f^2 + a*g^2])^2/(4*Sqrt[c]*e*
(e*f - d*g)*Sqrt[c*f^2 + a*g^2]), 2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2
+ a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(4*c^(1/4)*(c*f
^2 + a*g^2)^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g
*x)^2)/g^2])))/(g*(a*e^2*g + c*d*(2*e*f - d*g))))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.03

method	result
default	$\frac{2\left(cf-\sqrt{-ac}\,g\right)\operatorname{EllipticPi}\left(\sqrt{-\frac{(gx+f)c}{\sqrt{-ac}\,g-cf}},\frac{\left(\sqrt{-ac}\,g-cf\right)e}{c(dg-ef)},\sqrt{-\frac{\sqrt{-ac}\,g-cf}{\sqrt{-ac}\,g+cf}}\right)\sqrt{\frac{\left(cx+\sqrt{-ac}\right)g}{\sqrt{-ac}\,g-cf}}\,\sqrt{\frac{\left(-cx+\sqrt{-ac}\right)g}{\sqrt{-ac}\,g+cf}}\,\sqrt{-\frac{(gx+f)c}{\sqrt{-ac}\,g-cf}}\,\sqrt{c\,x^{2}+}}{c(dg-ef)\left(cg\,x^{3}+cf\,x^{2}+agx+af\right)}$
elliptic	$\frac{2\sqrt{(gx+f)(c\,x^{2}+a)}\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\,\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\operatorname{EllipticPi}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}},\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{d}{e}},\sqrt{\frac{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\right)}{\sqrt{gx+f}\,\sqrt{c\,x^{2}+a}\,e\sqrt{cg\,x^{3}+cf\,x^{2}+agx+af}\left(-\frac{f}{g}+\frac{d}{e}\right)}$

input
```
int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(c*f-(-a*c)^(1/2)*g)*EllipticPi((-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^(1/2),
((-a*c)^(1/2)*g-c*f)*e/c/(d*g-e*f),(-((-a*c)^(1/2)*g-c*f)/((-a*c)^(1/2)*g+
c*f))^(1/2))*((c*x+(-a*c)^(1/2))*g/((-a*c)^(1/2)*g-c*f))^(1/2)*((-c*x+(-a*
c)^(1/2))*g/((-a*c)^(1/2)*g+c*f))^(1/2)*(-(g*x+f)*c/((-a*c)^(1/2)*g-c*f))^
(1/2)*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)/c/(d*g-e*f)/(c*g*x^3+c*f*x^2+a*g*x+a*f
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex)\,\sqrt{f+gx}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**2)*(d + e*x)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{(ex+d)\,\sqrt{gx+f}\,\sqrt{c\,x^2+a}}dx$$

input
```
int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

output
```
int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

3.138 $\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 778

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{(cd^2+ae^2)(ef-dg)(d+ex)}$$

$$+\frac{\sqrt[4]{c}e\left(\sqrt{-a}-\frac{\sqrt{c}f}{g}\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right)|\frac{\sqrt{c}f+\sqrt{-}}{\sqrt{c}f-\sqrt{-}}\right.}{(cd^2+ae^2)(ef-dg)\sqrt{a+cx^2}}$$

$$+\frac{\sqrt[4]{c}(\sqrt{c}d-\sqrt{-a}e)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right),\right.}{(cd^2+ae^2)(ef-dg)\sqrt{a+cx^2}}$$

$$+\frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}(ae^2g-cd(2ef-3dg))\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\operatorname{EllipticPi}\left(\frac{e\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}{ef-dg},\arcsin\right.}{\sqrt[4]{c}(cd^2+ae^2)(ef-dg)^2\sqrt{a+cx^2}}$$

output
```
-e^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)+c^(1/4
)*e*((-a)^(1/2)-c^(1/2)*f/g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*
x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1
/2)*g))^(1/2)*EllipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/
2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/(a*e^2+c*d^2
)/(-d*g+e*f)/(c*x^2+a)^(1/2)+c^(1/4)*(c^(1/2)*d-(-a)^(1/2)*e)*(c^(1/2)*f+(
-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c
^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(
1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(
-a)^(1/2)*g))^(1/2))/(a*e^2+c*d^2)/(-d*g+e*f)/(c*x^2+a)^(1/2)+(c^(1/2)*f+(
-a)^(1/2)*g)^(1/2)*(a*e^2*g-c*d*(-3*d*g+2*e*f))*(1-c^(1/2)*(g*x+f)/(c^(1/2
)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2
)*EllipticPi(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),e*(f+(-a
)^(1/2)*g/c^(1/2))/(-d*g+e*f),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1
/2)*g))^(1/2))/c^(1/4)/(a*e^2+c*d^2)/(-d*g+e*f)^2/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.03 (sec) , antiderivative size = 1349, normalized size of antiderivative = 1.73

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
(Sqrt[f + g*x]*((-2*e^2*(a + c*x^2))/(d + e*x) + (2*(-(c*e^2*f^3*Sqrt[-f -
 (I*Sqrt[a]*g)/Sqrt[c]]) + c*d*e*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] -
a*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + a*d*e*g^3*Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]] + 2*c*e^2*f^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) -
 2*c*d*e*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) - c*e^2*f*Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + c*d*e*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqr
t[c]]*(f + g*x)^2 + Sqrt[c]*e*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(-(e*f) + d*g)*
Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c
] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*S
qrt[a]*g)] + (Sqrt[c]*d - I*Sqrt[a]*e)*g*(Sqrt[a]*e*g + I*Sqrt[c]*(e*f - 2
*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f -
(I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*
f + I*Sqrt[a]*g)] - (2*I)*c*d*e*f*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f
+ g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*E
llipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*ArcSinh[S
qrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/
(Sqrt[c]*f + I*Sqrt[a]*g)] + (3*I)*c*d^2*g^2*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]
+ x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + ...
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1586 vs. $2(778) = 1556$.

Time = 4.86 (sec) , antiderivative size = 1586, normalized size of antiderivative = 2.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {734, 2349, 599, 25, 27, 729, 25, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{734}$$

$$-\frac{\int \frac{-cgx^2e^2+age^2-2cdgxe-2cd(ef-dg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{2349}$$

$$-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \; 599$$

$$-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int -\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \; 25$$

$$-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2\int \frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}}{2\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \; 729$$

$$-\frac{2\left(ae^2g-cd(2ef-3dg)\right)\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}+\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+\cdots}}}{g}}{2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{}}{2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$\Big\downarrow$ 1511

$$-\frac{\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\left(dg-e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}\right)}{g}-2(ae}{2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$\Big\downarrow$ 1416

$$-\frac{\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}\left(dg-e\left(f-\frac{\sqrt{ag^2}}{\sqrt{}}\right.\right.}{2\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}}}\right.}{g}}{}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$\Big\downarrow$ 1509

$$2c\left(\frac{e\sqrt{ag^2+cf^2}\left(\frac{\sqrt[4]{ag^2+cf^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}{\left(a+\frac{cf^2}{g^2}\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{ag^2+cf^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{a+\frac{cf^2}{g^2}-\frac{2cf(f+gx)}{g^2}+\frac{c(f+gx)^2}{g^2}}}-\frac{\sqrt{f+g\cdots}}{\cdots}\cdots\right)}{\sqrt{c}}\cdots\right.$$

$$-\frac{\cdots}{\cdots}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)\left(ae^2+cd^2\right)(ef-dg)}$$

$\Big\downarrow$ 1540

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)}-$$

$$2c\left(\frac{e\sqrt{cf^2+ag^2}\left(\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-\frac{\sqrt{f+gx}\sqrt{\cdots}}{\left(\cdots\right)}\cdots\right)}{\sqrt{c}}\cdots\right.$$

$\Big\downarrow$ 1416

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)}-\frac{2c\left(\frac{e\sqrt{cf^2+ag^2}\left(\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-\frac{\sqrt{f+gx}\sqrt{\cdots}}{\left(\cdots\right)}\cdots\right)}{\sqrt{c}}\cdots\right)}{\cdots}$$

$\downarrow$ 2222

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{\left(cd^2+ae^2\right)(ef-dg)(d+ex)}-\frac{2c\left(\frac{e\sqrt{cf^2+ag^2}\left(\frac{\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt[4]{c}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-\frac{\sqrt{f+gx}\sqrt{\cdots}}{\left(\cdots\right)}\cdots\right)}{\sqrt{c}}\cdots\right)}{\cdots}$$

input
```
Int[1/((d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
-((e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(e*f - d*g)*(d + e*
x))) - ((2*c*((e*Sqrt[c*f^2 + a*g^2]*(-((Sqrt[f + g*x]*Sqrt[a + (c*f^2)/g^
2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])/((a + (c*f^2)/g^2)*(1 +
(Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]))) + ((c*f^2 + a*g^2)^(1/4)*(1 + (
Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f
+ g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g
*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(
c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(c^(1/4)*
Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/Sqr
t[c] - ((c*f^2 + a*g^2)^(1/4)*(d*g - e*(f - Sqrt[c*f^2 + a*g^2]/Sqrt[c]))*
(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c
*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*
(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g
*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(2*
c^(1/4)*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2
])))/g + 2*(a*e^2*g - c*d*(2*e*f - 3*d*g))*(-1/2*(c^(1/4)*(c*e*f^2 + a*e*g
^2 - Sqrt[c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f + g*x))/Sqr
t[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f +
g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^
2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 734
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Simp[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^
2]))*Simp[2*c*d*(e*f - d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*c*e*(d*g*(m + 1
) - e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 6.36 (sec) , antiderivative size = 995, normalized size of antiderivative = 1.28

method	result
elliptic	$\frac{\sqrt{(gx+f)(c\,x^2+a)}\left(\frac{e^2\sqrt{cg\,x^3+cf\,x^2+agx+af}}{\left(ad\,e^2g-a\,e^3f+c\,d^3g-c\,d^2ef\right)(ex+d)}-\frac{cdg\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x}{\frac{f}{g}-}}\right.}{\left(ad\,e^2g-a\,e^3f+c\,d^3g-c\,d^2ef\right)\sqrt{cg\,x^3+cf\,x^2+agx+af}}\right.}{}$
default	Expression too large to display

input
```
int(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(e^2/(a*d*e^2*g-a*
e^3*f+c*d^3*g-c*d^2*e*f)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)-c*d*g/(
a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-
a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+
(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(
1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)
/(-f/g-(-a*c)^(1/2)/c))^(1/2))-c*e*g/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)
*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2
)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c
))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*Elliptic
E(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)
^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^
(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+(a*e^2*g+3*c*d
^2*g-2*c*d*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)/e*(f/g-(-a*c)^(1/2)/
c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(
1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c
*f*x^2+a*g*x+a*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c
))^(1/2),(-f/g+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a
*c)^(1/2)/c))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex)^2\,\sqrt{f+gx}}\,dx$$

input
```
integrate(1/(e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**2)*(d + e*x)**2*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^2\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^2}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$
$$= \int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{c\,e^2g\,x^5+2cdeg\,x^4+c\,e^2f\,x^4+a\,e^2g\,x^3+c\,d^2g\,x^3+2cdef\,x^3+2adeg\,x^2+a\,e^2f\,x^2+c\,d^2f\,x^2+a\,d^2}$$

input
```
int(1/(e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

output
```
int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*d**2*f + a*d**2*g*x + 2*a*d*e*f*x
+ 2*a*d*e*g*x**2 + a*e**2*f*x**2 + a*e**2*g*x**3 + c*d**2*f*x**2 + c*d**2*
g*x**3 + 2*c*d*e*f*x**3 + 2*c*d*e*g*x**4 + c*e**2*f*x**4 + c*e**2*g*x**5),
x)
```

3.139 $\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 995

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$

$$= -\frac{e^2\sqrt{f+gx}\sqrt{a+cx^2}}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2} + \frac{3e^2(ae^2g-cd(2ef-3dg))\sqrt{f+gx}\sqrt{a+cx^2}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2(d+ex)}$$

$$+ \frac{3\sqrt[4]{c}e\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}(ae^2g-cd(2ef-3dg))\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\left(\text{arcs}\right.}{4\left(cd^2+ae^2\right)^2g(ef-dg)^2\sqrt{a+cx^2}}$$

$$+ \frac{\sqrt[4]{c}\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(3\sqrt{-a}ae^3g+c^{3/2}d^2(4ef-7dg)-3\sqrt{-a}cde(2ef-3dg)-a\sqrt{c}e^2(2ef+dg)\right)\sqrt{1}}{4\left(cd^2+ae^2\right)^2(ef-dg)^2\sqrt{a+}}$$

$$- \frac{\sqrt{\sqrt{c}f+\sqrt{-a}g}(3a^2e^4g^2-2ace^2(2e^2f^2-2defg-3d^2g^2)+c^2d^2(8e^2f^2-20defg+15d^2g^2))\sqrt{1-}}{4\sqrt[4]{c}\left(cd^2+ae^2\right)^2(ef-dg)^3}$$

output
```
-1/2*e^2*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+c*d^2)/(-d*g+e*f)/(e*x+d)^2+
3/4*e^2*(a*e^2*g-c*d*(-3*d*g+2*e*f))*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(a*e^2+
c*d^2)^2/(-d*g+e*f)^2/(e*x+d)+3/4*c^(1/4)*e*(c^(1/2)*f-(-a)^(1/2)*g)*(c^(1
/2)*f+(-a)^(1/2)*g)^(1/2)*(a*e^2*g-c*d*(-3*d*g+2*e*f))*(1-c^(1/2)*(g*x+f)/
(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g
))^(1/2)*EllipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((
c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/(a*e^2+c*d^2)^2/g
/(-d*g+e*f)^2/(c*x^2+a)^(1/2)+1/4*c^(1/4)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(
3*(-a)^(1/2)*a*e^3*g+c^(3/2)*d^2*(-7*d*g+4*e*f)-3*(-a)^(1/2)*c*d*e*(-3*d*g
+2*e*f)-a*c^(1/2)*e^2*(d*g+2*e*f))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2
)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c
^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)
*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/(a*e^2+c*d^2)^2/(-d*g+e*f)^2/(c*x^2+a
)^(1/2)-1/4*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(3*a^2*e^4*g^2-2*a*c*e^2*(-3*d^
2*g^2-2*d*e*f*g+2*e^2*f^2)+c^2*d^2*(15*d^2*g^2-20*d*e*f*g+8*e^2*f^2))*(1-c
^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)
*f+(-a)^(1/2)*g))^(1/2)*EllipticPi(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(
1/2)*g)^(1/2),e*(f+(-a)^(1/2)*g/c^(1/2))/(-d*g+e*f),((c^(1/2)*f+(-a)^(1/2)
*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(1/4)/(a*e^2+c*d^2)^2/(-d*g+e*f)^3/
(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.55 (sec) , antiderivative size = 2990, normalized size of antiderivative = 3.01

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
Sqrt[f + g*x]*Sqrt[a + c*x^2]*(-1/2*e^2/((c*d^2 + a*e^2)*(e*f - d*g)*(d +
e*x)^2) + (3*e^2*(-2*c*d*e*f + 3*c*d^2*g + a*e^2*g))/(4*(c*d^2 + a*e^2)^2*
(e*f - d*g)^2*(d + e*x))) - ((f + g*x)^(3/2)*(-6*c^2*d*e^3*f^2*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]] + 15*c^2*d^2*e^2*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]
] + 3*a*c*e^4*f*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 9*c^2*d^3*e*g^2*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]] - 3*a*c*d*e^3*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]] - (6*c^2*d*e^3*f^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (1
5*c^2*d^2*e^2*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (3*a*c
*e^4*f^3*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (9*c^2*d^3*e*f^
2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (9*a*c*d*e^3*f^2*g^2
*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (15*a*c*d^2*e^2*f*g^3*Sqr
t[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 + (3*a^2*e^4*f*g^3*Sqrt[-f - (I
*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)^2 - (9*a*c*d^3*e*g^4*Sqrt[-f - (I*Sqrt[a]*
g)/Sqrt[c]])/(f + g*x)^2 - (3*a^2*d*e^3*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]])/(f + g*x)^2 + (12*c^2*d*e^3*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f +
 g*x) - (30*c^2*d^2*e^2*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x)
- (6*a*c*e^4*f^2*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (18*c^2*d
^3*e*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (6*a*c*d*e^3*f*g^
2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]])/(f + g*x) + (3*Sqrt[c]*e*((-I)*Sqrt[c]
*f + Sqrt[a]*g)*(e*f - d*g)*(a*e^2*g + c*d*(-2*e*f + 3*d*g))*Sqrt[1 - f...
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2800 vs. $2(995) = 1990$.

Time = 8.98 (sec) , antiderivative size = 2800, normalized size of antiderivative = 2.81, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {734, 2349, 734, 2349, 27, 510, 599, 25, 27, 729, 25, 1416, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{734}$$

$$-\frac{\int \frac{cgx^2e^2+3age^2+2c(ef-2dg)xe-4cd(ef-dg)}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{2349}$$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(d+ex)^2\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{2cef-5cdg+cegx}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{4\left(ae^2+cd^2\right)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{734}$$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\int \frac{-cgx^2e^2+age^2-2cdgxe-2cd(ef-dg)}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)+\int \frac{2cef-5cdg+cegx}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}d}{4\left(ae^2+cd^2\right)(ef-dg)}$$
$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{2349}$$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)}{4\left(ae^2+cd^2\right)(ef-dg)}$$
$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)}{4\left(ae^2+cd^2\right)(ef-dg)}$$
$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{510}$$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\int \frac{-cdg-cexg}{\sqrt{f+gx}\sqrt{cx^2+a}}dx}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)}\right)}{4\left(ae^2+cd^2\right)(ef-dg)}$$
$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \text{599}$$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx-\frac{2\int-\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e\cdots}{(d+e\cdots}\right.}{4\left(ae^2+cd^2\right)(e\cdots}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow\ 25$$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2\int\frac{cg(ef-dg-e(f+gx))}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g^2}}{2(ae^2+cd^2)(ef-dg)}-\frac{e^2\cdots}{(d+ex\cdots}\right.}{4\left(ae^2+cd^2\right)(e\cdots}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow\ 27$$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\left(ae^2g-cd(2ef-3dg)\right)\int\frac{1}{(d+ex)\sqrt{f+gx}\sqrt{cx^2+a}}dx+\frac{2c\int\frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}}{2(ae^2+cd^2)(ef-dg)}-\frac{e\cdots}{(d+e\cdots}\right.}{4\left(ae^2+cd^2\right)(e\cdots}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow\ 729$$

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{2\left(ae^2g-cd(2ef-3dg)\right)\int-\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}+\frac{2c\int\frac{ef-d\cdots}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+\cdots}{g\cdots}}}\cdots}{\cdots}}{2(ae^2+cd^2)(ef-dg)}\right.}{\cdots}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$\downarrow$ 25

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx}{g^2}}}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$\downarrow$ 1416

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\frac{2c\int \frac{ef-dg-e(f+gx)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{g}-2\left(ae^2g-cd(2ef-3dg)\right)\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx}{g^2}}}}{2(ae^2+cd^2)(ef-dg)}\right.}{}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$\downarrow$ 1511

$$-\frac{3\left(ae^2g-cd(2ef-3dg)\right)\left(-\frac{\frac{2c\left(\frac{e\sqrt{ag^2+cf^2}\int \frac{1-\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}d\sqrt{f+gx}}{\sqrt{c}}-\left(dg-e\left(f-\frac{\sqrt{ag^2+cf^2}}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}}}\right.}{g}}{2(ae^2+}\right.}{}$$

$$\frac{e^2\sqrt{a+cx^2}\sqrt{f+gx}}{2(d+ex)^2\left(ae^2+cd^2\right)(ef-dg)}$$

$$\Big\downarrow \ 1416$$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-4c(ef$$

$$\Big\downarrow \ 1509$$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}-4c(ef$$

↓ 1540

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+4c(ef$$

↓ 1416

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+4c(ef$$

$\downarrow$ 2222

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+a}e^2}{2\left(cd^2+ae^2\right)(ef-dg)(d+ex)^2}-$$

$$\frac{c^{3/4}\sqrt[4]{cf^2+ag^2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\sqrt{\frac{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}{\left(\frac{cf^2}{g^2}+a\right)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+a}}+4c(ef$$

---

input
```
Int[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
-1/2*(e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(e*f - d*g)*(d +
 e*x)^2) - ((c^(3/4)*(c*f^2 + a*g^2)^(1/4)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c
*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x
)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])
^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1
 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f +
 g*x))/g^2 + (c*(f + g*x)^2)/g^2] + 4*c*(e*f - 3*d*g)*(-1/2*(c^(1/4)*(c*e*
f^2 + a*e*g^2 - Sqrt[c]*(e*f - d*g)*Sqrt[c*f^2 + a*g^2])*(1 + (Sqrt[c]*(f
+ g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2
 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2)/g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c
*f^2 + a*g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g
^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2])/(g*(c*f^2 + a*g^2)^(
1/4)*(a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x)
)/g^2 + (c*(f + g*x)^2)/g^2]) + (e*Sqrt[c*f^2 + a*g^2]*(Sqrt[c]*(e*f - d*g
) - e*Sqrt[c*f^2 + a*g^2])*(((e + (Sqrt[c]*(e*f - d*g))/Sqrt[c*f^2 + a*g^2
])*ArcTanh[(Sqrt[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sq
rt[a + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])])/(2*Sq
rt[e]*Sqrt[c*d^2 + a*e^2]*Sqrt[e*f - d*g]) - ((Sqrt[c]/e - Sqrt[c*f^2 + a*
g^2]/(e*f - d*g))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*Sqrt[(a +
(c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((a + (c*f^2...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 510
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2/d   Subst[Int[1/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2
)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 734
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/((m + 1)*(e*f - d*g)*(c*d^2 + a*e^2))), x] + Simp[1/(2*(m + 1)*(e*f -
d*g)*(c*d^2 + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + c*x^
2]))*Simp[2*c*d*(e*f - d*g)*(m + 1) - a*e^2*g*(2*m + 3) + 2*c*e*(d*g*(m + 1
) - e*f*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 11.38 (sec) , antiderivative size = 1192, normalized size of antiderivative = 1.20

method	result	size
elliptic	Expression too large to display	1192
default	Expression too large to display	20366

input
```
int(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(1/2*e^2/(a*d*e^2*
g-a*e^3*f+c*d^3*g-c*d^2*e*f)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)^2+3
/4*e^2*(a*e^2*g+3*c*d^2*g-2*c*d*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)
^2*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(e*x+d)-1/4*g*c*(a*d*e^2*g+2*a*e^3*f+
7*c*d^3*g-4*c*d^2*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)^2*(f/g-(-a*c)
^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(
-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1
/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-3/4*c*e*g*(a*e^2*
g+3*c*d^2*g-2*c*d*e*f)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)^2*(f/g-(-a*c)
^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(
-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*
g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(
f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(
1/2))+(-a*c)^(1/2)/c*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g
+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)))+1/4*(3*a^2*e^4*g^2+6*a*c*d
^2*e^2*g^2+4*a*c*d*e^3*f*g-4*a*c*e^4*f^2+15*c^2*d^4*g^2-20*c^2*d^3*e*f*g+8
*c^2*d^2*e^2*f^2)/(a*d*e^2*g-a*e^3*f+c*d^3*g-c*d^2*e*f)^2/e*(f/g-(-a*c)^(1
/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*
c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^3\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)^3\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^3}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^3), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$
$$= \int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2+a}}{c\,e^3g\,x^6+3cd\,e^2g\,x^5+c\,e^3f\,x^5+a\,e^3g\,x^4+3c\,d^2eg\,x^4+3cd\,e^2f\,x^4+3ad\,e^2g\,x^3+a\,e^3f\,x^3+c\,d^3g\,x^3}$$

input
```
int(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

output
```
int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*d**3*f + a*d**3*g*x + 3*a*d**2*e*f
*x + 3*a*d**2*e*g*x**2 + 3*a*d*e**2*f*x**2 + 3*a*d*e**2*g*x**3 + a*e**3*f*
x**3 + a*e**3*g*x**4 + c*d**3*f*x**2 + c*d**3*g*x**3 + 3*c*d**2*e*f*x**3 +
 3*c*d**2*e*g*x**4 + 3*c*d*e**2*f*x**4 + 3*c*d*e**2*g*x**5 + c*e**3*f*x**5
 + c*e**3*g*x**6),x)
```

3.140 $\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 196

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = -\frac{2\sqrt{cf+\sqrt{-c}g}\sqrt{1-\frac{c(f+gx)}{cf-\sqrt{-c}g}}\sqrt{1-\frac{c(f+gx)}{cf+\sqrt{-c}g}}\,\text{EllipticPi}\left(\frac{e(cf+\sqrt{-c}g)}{c(ef-dg)},\arcsin\left(\frac{\sqrt{c}\sqrt{f+gx}}{\sqrt{cf+\sqrt{-c}g}}\right),\frac{cf+\sqrt{-c}g}{cf-\sqrt{-c}g}\right)}{\sqrt{c}(ef-dg)\sqrt{1+cx^2}}$$

output
```
-2*(c*f+(-c)^(1/2)*g)^(1/2)*(1-c*(g*x+f)/(c*f-(-c)^(1/2)*g))^(1/2)*(1-c*(g
*x+f)/(c*f+(-c)^(1/2)*g))^(1/2)*EllipticPi(c^(1/2)*(g*x+f)^(1/2)/(c*f+(-c)
^(1/2)*g)^(1/2),e*(c*f+(-c)^(1/2)*g)/c/(-d*g+e*f),((c*f+(-c)^(1/2)*g)/(c*f
-(-c)^(1/2)*g))^(1/2))/c^(1/2)/(-d*g+e*f)/(c*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.59 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.33

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx =$$
$$-\frac{2i\sqrt{\frac{g\left(\frac{i}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{ig}{\sqrt{c}}-gx}{f+gx}}(f+gx)\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-f-\frac{ig}{\sqrt{c}}}}{\sqrt{f+gx}}\right),\frac{\sqrt{c}f-ig}{\sqrt{c}f+ig}\right)-\text{EllipticPi}\left(\frac{\sqrt{c}(ef-dg)}{e(\sqrt{c}f+ig)},ia\right.\right.}{\sqrt{-f-\frac{ig}{\sqrt{c}}}(ef-dg)\sqrt{1+cx^2}}$$

input
```
Integrate[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 + c*x^2]),x]
```

output
```
((-2*I)*Sqrt[(g*(I/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*g)/Sqrt[c] - g*x)/(
f + g*x))]*(f + g*x)*(EllipticF[I*ArcSinh[Sqrt[-f - (I*g)/Sqrt[c]]/Sqrt[f
+ g*x]], (Sqrt[c]*f - I*g)/(Sqrt[c]*f + I*g)] - EllipticPi[(Sqrt[c]*(e*f -
 d*g))/(e*(Sqrt[c]*f + I*g)), I*ArcSinh[Sqrt[-f - (I*g)/Sqrt[c]]/Sqrt[f +
g*x]], (Sqrt[c]*f - I*g)/(Sqrt[c]*f + I*g)]))/(Sqrt[-f - (I*g)/Sqrt[c]]*(e
*f - d*g)*Sqrt[1 + c*x^2])
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 826 vs. 2(196) = 392.

Time = 2.78 (sec) , antiderivative size = 826, normalized size of antiderivative = 4.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {729, 25, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx^2+1}(d+ex)\sqrt{f+gx}}dx$$
$$\Big\downarrow \ 729$$
$$2\int -\frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+1}}d\sqrt{f+gx}$$
$$\Big\downarrow \ 25$$

$$-2\int \frac{1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+1}}d\sqrt{f+gx}$$

$\downarrow$ 1540

$$2\left(\frac{e\sqrt{cf^2+g^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+g^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+g^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+1}}d\sqrt{f+gx}}{g\left(cd(2ef-dg)+e^2g\right)}-\frac{\sqrt{c}\left(-\sqrt{c}\right.}{}\right.$$

$\downarrow$ 1416

$$2\left(\frac{e\sqrt{cf^2+g^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+g^2}\right)\int \frac{\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+g^2}}+1}{(ef-dg-e(f+gx))\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+1}}d\sqrt{f+gx}}{g\left(cd(2ef-dg)+e^2g\right)}-\frac{\sqrt[4]{c}\left(\frac{\sqrt{c}(f+\right.}{\sqrt{cf^2+}}\right.$$

$\downarrow$ 2222

$$2\left(\frac{e\sqrt{cf^2+g^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+g^2}\right)\left(\frac{\left(e+\frac{\sqrt{c}(ef-dg)}{\sqrt{cf^2+g^2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{cd^2+e^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{\frac{cf^2}{g^2}-\frac{2c(f+gx)f}{g^2}+\frac{c(f+gx)^2}{g^2}+1}}\right)}{2\sqrt{e}\sqrt{cd^2+e^2}\sqrt{ef-dg}}-\right.}{g\left(ge^2\right.}\right.$$

input

```
Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 + c*x^2]),x]
```

output

```
2*(-1/2*(c^(1/4)*(c*e*f^2 + e*g^2 - Sqrt[c]*(e*f - d*g)*Sqrt[c*f^2 + g^2])
*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + g^2])*Sqrt[(1 + (c*f^2)/g^2 - (2*c*
f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((1 + (c*f^2)/g^2)*(1 + (Sqrt[c]*(
f + g*x))/Sqrt[c*f^2 + g^2])^2)]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x]
)/(c*f^2 + g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + g^2])/2]])/(g*(c*f^2
+ g^2)^(1/4)*(e^2*g + c*d*(2*e*f - d*g))*Sqrt[1 + (c*f^2)/g^2 - (2*c*f*(f
+ g*x))/g^2 + (c*(f + g*x)^2)/g^2]) + (e*Sqrt[c*f^2 + g^2]*(Sqrt[c]*(e*f -
 d*g) - e*Sqrt[c*f^2 + g^2])*(((e + (Sqrt[c]*(e*f - d*g))/Sqrt[c*f^2 + g^2
])*ArcTanh[(Sqrt[c*d^2 + e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt
[1 + (c*f^2)/g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])])/(2*Sqrt
[e]*Sqrt[c*d^2 + e^2]*Sqrt[e*f - d*g]) - ((Sqrt[c]/e - Sqrt[c*f^2 + g^2]/(
e*f - d*g))*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + g^2])*Sqrt[(1 + (c*f^2)/
g^2 - (2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2)/((1 + (c*f^2)/g^2)*(1 +
 (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + g^2])^2)]*EllipticPi[(Sqrt[c]*(e*f - d*g
) + e*Sqrt[c*f^2 + g^2])^2/(4*Sqrt[c]*e*(e*f - d*g)*Sqrt[c*f^2 + g^2]), 2*
ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt
[c*f^2 + g^2])/2])/(4*c^(1/4)*(c*f^2 + g^2)^(1/4)*Sqrt[1 + (c*f^2)/g^2 - (
2*c*f*(f + g*x))/g^2 + (c*(f + g*x)^2)/g^2])))/(g*(e^2*g + c*d*(2*e*f - d*
g))))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 729

```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.10

method	result
default	$\frac{2\left(f\sqrt{-c}+g\right)\operatorname{EllipticPi}\left(\sqrt{\frac{(gx+f)\sqrt{-c}}{f\sqrt{-c}+g}},-\frac{\left(f\sqrt{-c}+g\right)e}{\sqrt{-c}\,(dg-ef)},\sqrt{\frac{f\sqrt{-c}+g}{f\sqrt{-c}-g}}\right)\sqrt{-\frac{g\left(\sqrt{-c}\,x-1\right)}{f\sqrt{-c}+g}}\,\sqrt{-\frac{\left(\sqrt{-c}\,x+1\right)g}{f\sqrt{-c}-g}}\,\sqrt{\frac{(gx+f)\sqrt{-c}}{f\sqrt{-c}+g}}\,\sqrt{c\,x^{2}+1}\,\sqrt{gx+\ldots}}{\sqrt{-c}\,(dg-ef)\left(cg\,x^{3}+cf\,x^{2}+gx+f\right)}$
elliptic	$\frac{2\sqrt{(gx+f)\left(c\,x^{2}+1\right)}\left(\frac{f}{g}+\frac{1}{\sqrt{-c}}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}+\frac{1}{\sqrt{-c}}}}\,\sqrt{\frac{x+\frac{1}{\sqrt{-c}}}{-\frac{f}{g}+\frac{1}{\sqrt{-c}}}}\,\sqrt{\frac{x-\frac{1}{\sqrt{-c}}}{-\frac{f}{g}-\frac{1}{\sqrt{-c}}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}+\frac{1}{\sqrt{-c}}}},\frac{-\frac{f}{g}-\frac{1}{\sqrt{-c}}}{-\frac{f}{g}+\frac{d}{e}},\sqrt{\frac{-\frac{f}{g}-\frac{1}{\sqrt{-c}}}{-\frac{f}{g}+\frac{1}{\sqrt{-c}}}}\right)}{\sqrt{gx+f}\,\sqrt{c\,x^{2}+1}\,e\sqrt{cg\,x^{3}+cf\,x^{2}+gx+f}\left(-\frac{f}{g}+\frac{d}{e}\right)}$

input
```
int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(f*(-c)^(1/2)+g)/(-c)^(1/2)*EllipticPi(((g*x+f)*(-c)^(1/2)/(f*(-c)^(1/2)
+g))^(1/2),-(f*(-c)^(1/2)+g)*e/(-c)^(1/2)/(d*g-e*f),((f*(-c)^(1/2)+g)/(f*(
-c)^(1/2)-g))^(1/2))*(-g*((-c)^(1/2)*x-1)/(f*(-c)^(1/2)+g))^(1/2)*(-((-c)^
(1/2)*x+1)*g/(f*(-c)^(1/2)-g))^(1/2)*((g*x+f)*(-c)^(1/2)/(f*(-c)^(1/2)+g))
^(1/2)*(c*x^2+1)^(1/2)*(g*x+f)^(1/2)/(d*g-e*f)/(c*g*x^3+c*f*x^2+g*x+f)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{(d+ex)\,\sqrt{f+gx}\sqrt{cx^2+1}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+1)**(1/2),x)
```

output
```
Integral(1/((d + e*x)*sqrt(f + g*x)*sqrt(c*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+1}(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + 1)*(e*x + d)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+1}(ex+d)\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + 1)*(e*x + d)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+1}\,(d+e\,x)}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(c*x^2 + 1)^(1/2)*(d + e*x)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(c*x^2 + 1)^(1/2)*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}}\,dx = \int \frac{1}{(ex+d)\,\sqrt{gx+f}\,\sqrt{c\,x^2+1}}dx$$

input
```
int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x)
```

output
```
int(1/(e*x+d)/(g*x+f)^(1/2)/(c*x^2+1)^(1/2),x)
```

3.141 $\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 740

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \frac{2g^2\sqrt{a+cx^2}}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{f+gx}}$$
$$+\frac{2\sqrt[4]{c}\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right)|\frac{\sqrt{c}f+\sqrt{-a}g}{\sqrt{c}f-\sqrt{-a}g}\right)}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{a+cx^2}}$$
$$-\frac{2\sqrt[4]{c}\left(\sqrt{c}f-\sqrt{-a}g\right)\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right),\ldots\right)}{(ef-dg)\left(cf^2+ag^2\right)\sqrt{a+cx^2}}$$
$$-\frac{2e\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\operatorname{EllipticPi}\left(\frac{e\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}{ef-dg},\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right),\frac{\sqrt{c}f+\sqrt{-}\ldots}{\sqrt{c}f-\sqrt{-}\ldots}\right)}{\sqrt[4]{c}(ef-dg)^2\sqrt{a+cx^2}}$$

output
```
2*g^2*(c*x^2+a)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^2)/(g*x+f)^(1/2)+2*c^(1/4)*(c^
(1/2)*f-(-a)^(1/2)*g)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c
^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))
^(1/2)*EllipticE(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^
(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/(-d*g+e*f)/(a*g^2+c
*f^2)/(c*x^2+a)^(1/2)-2*c^(1/4)*(c^(1/2)*f-(-a)^(1/2)*g)*(c^(1/2)*f+(-a)^(
1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2
)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/
(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(
1/2)*g))^(1/2))/(-d*g+e*f)/(a*g^2+c*f^2)/(c*x^2+a)^(1/2)-2*e*(c^(1/2)*f+(-
a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^
(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticPi(c^(1/4)*(g*x+f)^(
1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),e*(f+(-a)^(1/2)*g/c^(1/2))/(-d*g+e*f),
((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(1/4)/(-d*g+e
*f)^2/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.48 (sec) , antiderivative size = 468, normalized size of antiderivative = 0.63

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \frac{2i\sqrt{\frac{g\left(\frac{i\sqrt{a}}{\sqrt{c}}+x\right)}{f+gx}}\sqrt{-\frac{\frac{i\sqrt{a}g}{\sqrt{c}}-gx}{f+gx}}(f+gx)\left(\sqrt{c}(ef-dg)E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-f\cdots}}{\sqrt{f\cdots}}\right.\right.\right.\cdots}{\cdots}$$

input
```
Integrate[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

output
```
((2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)
/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)*(Sqrt[c]*(e*f - d*g)*EllipticE[I*Arc
Sinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[
a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + (I*Sqrt[a]*e*g + Sqrt[c]*(-2*e*f + d*g)
)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sq
rt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + e*(Sqrt[c]*f - I*Sqrt[
a]*g)*EllipticPi[(Sqrt[c]*(e*f - d*g))/(e*(Sqrt[c]*f + I*Sqrt[a]*g)), I*Ar
cSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt
[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)]))/((Sqrt[c]*f - I*Sqrt[a]*g)*Sqrt[-f - (
I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)^2*Sqrt[a + c*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 5.57 (sec) , antiderivative size = 1176, normalized size of antiderivative = 1.59, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {740, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex)(f+gx)^{3/2}}dx$$
$$\Big\downarrow \text{740}$$
$$\int \left(\frac{e}{\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}(ef-dg)}-\frac{g}{\sqrt{a+cx^2}(f+gx)^{3/2}(ef-dg)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{cx^2+a}g^2}{(ef-dg)(cf^2+ag^2)\sqrt{f+gx}}-\frac{2\sqrt{c}\sqrt{f+gx}\sqrt{cx^2+a}g^2}{(ef-dg)(cf^2+ag^2)^{3/2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)}-$$
$$\frac{e^{3/2}\text{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{cx^2+a}}\right)}{\sqrt{cd^2+ae^2}(ef-dg)^{3/2}}+$$
$$\frac{2\sqrt[4]{c}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{(ef-dg)\sqrt[4]{cf^2+ag^2}\sqrt{cx^2+a}}-$$
$$\frac{\sqrt[4]{c}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{(ef-dg)\sqrt[4]{cf^2+ag^2}\sqrt{cx^2+a}}+$$
$$\frac{\sqrt[4]{c}e\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\right.\right.}{(ef-dg)(age^2+cd(2ef-dg))\sqrt{cx^2+ag}}$$
$$\frac{e\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)^2\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticPi}\left(\frac{\left(\sqrt{cf^2+ag^2}e\right.}{4\sqrt{c}e(ef-d}\right.}{2\sqrt[4]{c}(ef-dg)^2(age^2+cd(2ef-dg))\sqrt{cx^2+ag}}$$

input

```
Int[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

output

```
(2*g^2*Sqrt[a + c*x^2])/((e*f - d*g)*(c*f^2 + a*g^2)*Sqrt[f + g*x]) - (2*S
qrt[c]*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)*(c*f^2 + a*g^2)^(3/
2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])) - (e^(3/2)*ArcTanh[(Sqrt
[c*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt[a + c*x^2])])
/(Sqrt[c*d^2 + a*e^2]*(e*f - d*g)^(3/2)) + (2*c^(1/4)*Sqrt[(g^2*(a + c*x^2
))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*(1 +
 (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2
])/((e*f - d*g)*(c*f^2 + a*g^2)^(1/4)*Sqrt[a + c*x^2]) - (c^(1/4)*Sqrt[(g^
2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^
2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticF[2*ArcTan[(
c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2
 + a*g^2])/2])/((e*f - d*g)*(c*f^2 + a*g^2)^(1/4)*Sqrt[a + c*x^2]) + (c^(1
/4)*e*(c*f^2 + a*g^2)^(1/4)*(Sqrt[c]*(e*f - d*g) - e*Sqrt[c*f^2 + a*g^2])*
Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^
2 + a*g^2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticF[2*
ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sq
rt[c*f^2 + a*g^2])/2])/(g*(e*f - d*g)*(a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[a
 + c*x^2]) - (e*(c*f^2 + a*g^2)^(1/4)*(Sqrt[c]*(e*f - d*g) - e*Sqrt[c*f^2
+ a*g^2])^2*Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + ...
```

Defintions of rubi rules used

rule 740

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && Inte
gerQ[n + 1/2]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 6.06 (sec) , antiderivative size = 929, normalized size of antiderivative = 1.26

method	result
elliptic	$\frac{\sqrt{(gx+f)(cx^2+a)}\left(-\frac{2\left(cg\,x^2+ag\right)g}{\left(a\,g^2+c\,f^2\right)(dg-ef)\sqrt{\left(x+\frac{f}{g}\right)\left(cg\,x^2+ag\right)}}+\frac{2fgc\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{x+\frac{f}{g}}{\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x-\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}-\frac{\sqrt{-ac}}{c}}}\sqrt{\frac{x+\frac{\sqrt{-ac}}{c}}{-\frac{f}{g}+\frac{\sqrt{-ac}}{c}}}\,\mathrm{EllipticF}(}{(dg-ef)\left(a\,g^2+c\,f^2\right)\sqrt{cg\,x^3+cf\,x^2+agx+c}}$
default	Expression too large to display

input
```
int(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-2*(c*g*x^2+a*g)/
(a*g^2+c*f^2)*g/(d*g-e*f)/((x+f/g)*(c*g*x^2+a*g))^(1/2)+2*f*g*c/(d*g-e*f)/
(a*g^2+c*f^2)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((
x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(
-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)
/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))
^(1/2))+2*g^2*c/(d*g-e*f)/(a*g^2+c*f^2)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g
-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*(
(x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f
)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1
/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*El
lipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-
(-a*c)^(1/2)/c))^(1/2)))-2/(d*g-e*f)*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/g-(-
a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*((x+
(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)^(
1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g+(-a*
c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex)\,(f+gx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)**(3/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**2)*(d + e*x)*(f + g*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{(f+g\,x)^{3/2}\,\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

input
```
int(1/((f + g*x)^(3/2)*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(1/((f + g*x)^(3/2)*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}\,\sqrt{c\,x^2}}{ce\,g^2x^5+cd\,g^2x^4+2cefg\,x^4+ae\,g^2x^3+2cdfg\,x^3+ce\,f^2x^3+ad}$$

input
```
int(1/(e*x+d)/(g*x+f)^(3/2)/(c*x^2+a)^(1/2),x)
```

output
```
int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*d*f**2 + 2*a*d*f*g*x + a*d*g**2*x*
*2 + a*e*f**2*x + 2*a*e*f*g*x**2 + a*e*g**2*x**3 + c*d*f**2*x**2 + 2*c*d*f
*g*x**3 + c*d*g**2*x**4 + c*e*f**2*x**3 + 2*c*e*f*g*x**4 + c*e*g**2*x**5),
x)
```

3.142 $\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 892

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \frac{2g^2\sqrt{a+cx^2}}{3(ef-dg)(cf^2+ag^2)(f+gx)^{3/2}}$$
$$+\frac{2g^2(3aeg^2+cf(7ef-4dg))\sqrt{a+cx^2}}{3(ef-dg)^2(cf^2+ag^2)^2\sqrt{f+gx}}$$
$$+\frac{2\sqrt[4]{c}(\sqrt{c}f-\sqrt{-a}g)\sqrt{\sqrt{c}f+\sqrt{-a}g}(3aeg^2+cf(7ef-4dg))\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}E\left(\arcsin\right.}{3(ef-dg)^2(cf^2+ag^2)^2\sqrt{a+cx^2}}$$
$$-\frac{2\sqrt[4]{c}\sqrt{\sqrt{c}f+\sqrt{-a}g}\left(3(-a)^{3/2}eg^3-\sqrt{-a}cfg(7ef-4dg)+3c^{3/2}f^2(2ef-dg)+a\sqrt{c}g^2(2ef+dg)\right)\sqrt{1}}{3(ef-dg)^2(cf^2+ag^2)^2\sqrt{a+c}}$$
$$-\frac{2e^2\sqrt{\sqrt{c}f+\sqrt{-a}g}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f-\sqrt{-a}g}}\sqrt{1-\frac{\sqrt{c}(f+gx)}{\sqrt{c}f+\sqrt{-a}g}}\operatorname{EllipticPi}\left(\frac{e\left(f+\frac{\sqrt{-a}g}{\sqrt{c}}\right)}{ef-dg},\arcsin\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{-a}g}}\right),\frac{\sqrt{c}f+\sqrt{}}{\sqrt{c}f-\sqrt{}}\right.}{\sqrt[4]{c}(ef-dg)^3\sqrt{a+cx^2}}$$

output
```
2/3*g^2*(c*x^2+a)^(1/2)/(-d*g+e*f)/(a*g^2+c*f^2)/(g*x+f)^(3/2)+2/3*g^2*(3*
a*e*g^2+c*f*(-4*d*g+7*e*f))*(c*x^2+a)^(1/2)/(-d*g+e*f)^2/(a*g^2+c*f^2)^2/(
g*x+f)^(1/2)+2/3*c^(1/4)*(c^(1/2)*f-(-a)^(1/2)*g)*(c^(1/2)*f+(-a)^(1/2)*g)
^(1/2)*(3*a*e*g^2+c*f*(-4*d*g+7*e*f))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(
1/2)*g))^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*Elliptic
E(c^(1/4)*(g*x+f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1
/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/(-d*g+e*f)^2/(a*g^2+c*f^2)^2/(c*x^
2+a)^(1/2)-2/3*c^(1/4)*(c^(1/2)*f+(-a)^(1/2)*g)^(1/2)*(3*(-a)^(3/2)*e*g^3-
(-a)^(1/2)*c*f*g*(-4*d*g+7*e*f)+3*c^(3/2)*f^2*(-d*g+2*e*f)+a*c^(1/2)*g^2*(
d*g+2*e*f))*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(1-c^(1/2)*
(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticF(c^(1/4)*(g*x+f)^(1/2)/(c
^(1/2)*f+(-a)^(1/2)*g)^(1/2),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/
2)*g))^(1/2))/(-d*g+e*f)^2/(a*g^2+c*f^2)^2/(c*x^2+a)^(1/2)-2*e^2*(c^(1/2)*
f+(-a)^(1/2)*g)^(1/2)*(1-c^(1/2)*(g*x+f)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2)*(
1-c^(1/2)*(g*x+f)/(c^(1/2)*f+(-a)^(1/2)*g))^(1/2)*EllipticPi(c^(1/4)*(g*x+
f)^(1/2)/(c^(1/2)*f+(-a)^(1/2)*g)^(1/2),e*(f+(-a)^(1/2)*g/c^(1/2))/(-d*g+e
*f),((c^(1/2)*f+(-a)^(1/2)*g)/(c^(1/2)*f-(-a)^(1/2)*g))^(1/2))/c^(1/4)/(-d
*g+e*f)^3/(c*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.13 (sec) , antiderivative size = 1917, normalized size of antiderivative = 2.15

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + c*x^2]),x]
```

output
```
(2*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(e*f - d*g)*(a + c*x^2)*(a*g^2*(4
*e*f - d*g + 3*e*g*x) + c*f*(-(d*g*(5*f + 4*g*x)) + e*f*(8*f + 7*g*x))) -
(f + g*x)*(7*c^2*e^2*f^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 11*c^2*d*e*f^4
*g*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 4*c^2*d^2*f^3*g^2*Sqrt[-f - (I*Sqrt[
a]*g)/Sqrt[c]] + 10*a*c*e^2*f^3*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 14*
a*c*d*e*f^2*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] + 4*a*c*d^2*f*g^4*Sqrt[-f
 - (I*Sqrt[a]*g)/Sqrt[c]] + 3*a^2*e^2*f*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c
]] - 3*a^2*d*e*g^5*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]] - 14*c^2*e^2*f^4*Sqrt[
-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 22*c^2*d*e*f^3*g*Sqrt[-f - (I*Sqrt
[a]*g)/Sqrt[c]]*(f + g*x) - 8*c^2*d^2*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt
[c]]*(f + g*x) - 6*a*c*e^2*f^2*g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g
*x) + 6*a*c*d*e*f*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x) + 7*c^2*e
^2*f^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 - 11*c^2*d*e*f^2*g*Sqr
t[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + 4*c^2*d^2*f*g^2*Sqrt[-f - (I*S
qrt[a]*g)/Sqrt[c]]*(f + g*x)^2 + 3*a*c*e^2*f*g^2*Sqrt[-f - (I*Sqrt[a]*g)/S
qrt[c]]*(f + g*x)^2 - 3*a*c*d*e*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f +
g*x)^2 + Sqrt[c]*((-I)*Sqrt[c]*f + Sqrt[a]*g)*(e*f - d*g)*(3*a*e*g^2 + c*f
*(7*e*f - 4*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I
*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh
[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a...
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1795 vs. $2(892) = 1784$.

Time = 7.72 (sec) , antiderivative size = 1795, normalized size of antiderivative = 2.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {740, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex)(f+gx)^{5/2}}\,dx$$

$$\Big\downarrow \ 740$$

$$\int \left(\frac{e^2}{\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}(ef-dg)^2}-\frac{eg}{\sqrt{a+cx^2}(f+gx)^{3/2}(ef-dg)^2}-\frac{g}{\sqrt{a+cx^2}(f+gx)^{5/2}(ef-dg)}\right.$$

$\downarrow$ 2009

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cd^2+ae^2}\sqrt{f+gx}}{\sqrt{e}\sqrt{ef-dg}\sqrt{cx^2+a}}\right)e^{5/2}}{\sqrt{cd^2+ae^2}(ef-dg)^{5/2}}+$$

$$\frac{\sqrt[4]{c}\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\right.\right.}{g(ef-dg)^2\left(age^2+cd(2ef-dg)\right)\sqrt{cx^2+a}}$$

$$\frac{\sqrt[4]{cf^2+ag^2}\left(\sqrt{c}(ef-dg)-e\sqrt{cf^2+ag^2}\right)^2\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticPi}\left(\frac{\left(\sqrt{cf^2+ag^2}e+\right.}{4\sqrt{c}e(ef-dg)}\right.}{2\sqrt[4]{c}g(ef-dg)^3\left(age^2+cd(2ef-dg)\right)\sqrt{cx^2+a}}$$

$$\frac{2\sqrt[4]{c}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)e}{(ef-dg)^2\sqrt[4]{cf^2+ag^2}\sqrt{cx^2+a}}-$$

$$\frac{\sqrt[4]{c}\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right),\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)e}{(ef-dg)^2\sqrt[4]{cf^2+ag^2}\sqrt{cx^2+a}}+$$

$$\frac{2g^2\sqrt{cx^2+a}e}{(ef-dg)^2(cf^2+ag^2)\sqrt{f+gx}}-\frac{2\sqrt{c}g^2\sqrt{f+gx}\sqrt{cx^2+a}e}{(ef-dg)^2(cf^2+ag^2)^{3/2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)}+$$

$$\frac{8c^{5/4}f\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{c}f}{\sqrt{cf^2+ag^2}}+1\right)\right)}{3(ef-dg)(cf^2+ag^2)^{5/4}\sqrt{cx^2+a}}+$$

$$\frac{c^{3/4}\left(cf^2-4\sqrt{c}\sqrt{cf^2+ag^2}f+ag^2\right)\sqrt{\frac{g^2(cx^2+a)}{(cf^2+ag^2)\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)^2}}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{f+gx}}{\sqrt[4]{cf^2+a}}\right.\right.}{3(ef-dg)(cf^2+ag^2)^{7/4}\sqrt{cx^2+a}}$$

$$\frac{8cfg^2\sqrt{cx^2+a}}{3(ef-dg)(cf^2+ag^2)^2\sqrt{f+gx}}+\frac{2g^2\sqrt{cx^2+a}}{3(ef-dg)(cf^2+ag^2)(f+gx)^{3/2}}-\frac{8c^{3/2}fg^2\sqrt{f+gx}\sqrt{cx^2+a}}{3(ef-dg)(cf^2+ag^2)^{5/2}\left(\frac{\sqrt{c}(f+gx)}{\sqrt{cf^2+ag^2}}+1\right)}$$

input
```
Int[1/((d + e*x)*(f + g*x)^(5/2)*Sqrt[a + c*x^2]),x]
```

output
```
(2*g^2*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)*(f + g*x)^(3/2)) +
(8*c*f*g^2*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)^2*Sqrt[f + g*x]
) + (2*e*g^2*Sqrt[a + c*x^2])/((e*f - d*g)^2*(c*f^2 + a*g^2)*Sqrt[f + g*x]
) - (8*c^(3/2)*f*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*(e*f - d*g)*(c*f^2
+ a*g^2)^(5/2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])) - (2*Sqrt[c]
*e*g^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)^2*(c*f^2 + a*g^2)^(3/2)
*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])) - (e^(5/2)*ArcTanh[(Sqrt[c
*d^2 + a*e^2]*Sqrt[f + g*x])/(Sqrt[e]*Sqrt[e*f - d*g]*Sqrt[a + c*x^2])])/(
Sqrt[c*d^2 + a*e^2]*(e*f - d*g)^(5/2)) + (8*c^(5/4)*f*Sqrt[(g^2*(a + c*x^2
))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])^2)]*(1 +
 (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt[c*f^2 + a*g^2])/2
])/(3*(e*f - d*g)*(c*f^2 + a*g^2)^(5/4)*Sqrt[a + c*x^2]) + (2*c^(1/4)*e*Sq
rt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2
+ a*g^2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2])*EllipticE[2*Ar
cTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1 + (Sqrt[c]*f)/Sqrt
[c*f^2 + a*g^2])/2])/((e*f - d*g)^2*(c*f^2 + a*g^2)^(1/4)*Sqrt[a + c*x^2])
 - (c^(1/4)*e*Sqrt[(g^2*(a + c*x^2))/((c*f^2 + a*g^2)*(1 + (Sqrt[c]*(f + g
*x))/Sqrt[c*f^2 + a*g^2])^2)]*(1 + (Sqrt[c]*(f + g*x))/Sqrt[c*f^2 + a*g^2]
)*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[f + g*x])/(c*f^2 + a*g^2)^(1/4)], (1...
```

Defintions of rubi rules used

rule 740
```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && Inte
gerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 10.99 (sec) , antiderivative size = 1079, normalized size of antiderivative = 1.21

method	result	size
elliptic	Expression too large to display	1079
default	Expression too large to display	9409

input
```
int(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)*(-2/3/(a*g^2+c*f^2
)/(d*g-e*f)*(c*g*x^3+c*f*x^2+a*g*x+a*f)^(1/2)/(x+f/g)^2+2/3*(c*g*x^2+a*g)/
(a*g^2+c*f^2)^2*g*(3*a*e*g^2-4*c*d*f*g+7*c*e*f^2)/(d*g-e*f)^2/((x+f/g)*(c*
g*x^2+a*g))^(1/2)+2*(-1/3*c*g/(a*g^2+c*f^2)/(d*g-e*f)-1/3*c*f*g*(3*a*e*g^2
-4*c*d*f*g+7*c*e*f^2)/(a*g^2+c*f^2)^2/(d*g-e*f)^2)*(f/g-(-a*c)^(1/2)/c)*((
x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/
c))^(1/2)*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^
2+a*g*x+a*f)^(1/2)*EllipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(
-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))-2/3*c*g^2*(3*a*e*g^2-4*c*d*f*
g+7*c*e*f^2)/(a*g^2+c*f^2)^2/(d*g-e*f)^2*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f/
g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)*
((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a*
f)^(1/2)*((-f/g-(-a*c)^(1/2)/c)*EllipticE(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(
1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2))+(-a*c)^(1/2)/c*E
llipticF(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),((-f/g+(-a*c)^(1/2)/c)/(-f/g
-(-a*c)^(1/2)/c))^(1/2)))+2*e/(d*g-e*f)^2*(f/g-(-a*c)^(1/2)/c)*((x+f/g)/(f
/g-(-a*c)^(1/2)/c))^(1/2)*((x-(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c))^(1/2)
*((x+(-a*c)^(1/2)/c)/(-f/g+(-a*c)^(1/2)/c))^(1/2)/(c*g*x^3+c*f*x^2+a*g*x+a
*f)^(1/2)/(-f/g+d/e)*EllipticPi(((x+f/g)/(f/g-(-a*c)^(1/2)/c))^(1/2),(-f/g
+(-a*c)^(1/2)/c)/(-f/g+d/e),((-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex)\,(f+gx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)**(5/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**2)*(d + e*x)*(f + g*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}(ex+d)(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \int \frac{1}{(f+g\,x)^{5/2}\,\sqrt{c\,x^2+a}\,(d+e\,x)}\,dx$$

input
```
int(1/((f + g*x)^(5/2)*(a + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(1/((f + g*x)^(5/2)*(a + c*x^2)^(1/2)*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}}\,dx = \int \frac{}{ce\,g^3x^6+cd\,g^3x^5+3cef\,g^2x^5+ae\,g^3x^4+3cdf\,g^2x^4+3ce\,f^2g\,x^4}$$

input
```
int(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x)
```

output
```
int((sqrt(f + g*x)*sqrt(a + c*x**2))/(a*d*f**3 + 3*a*d*f**2*g*x + 3*a*d*f*
g**2*x**2 + a*d*g**3*x**3 + a*e*f**3*x + 3*a*e*f**2*g*x**2 + 3*a*e*f*g**2*
x**3 + a*e*g**3*x**4 + c*d*f**3*x**2 + 3*c*d*f**2*g*x**3 + 3*c*d*f*g**2*x*
*4 + c*d*g**3*x**5 + c*e*f**3*x**3 + 3*c*e*f**2*g*x**4 + 3*c*e*f*g**2*x**5
 + c*e*g**3*x**6),x)
```

3.143 $\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 196

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\sqrt{1+x^2}}\,dx = -\frac{(2+3x)\sqrt{\frac{1+x^2}{(2+3x)^2}}\left(41+\frac{\sqrt{533}(4+5x)}{2+3x}\right)\sqrt{\frac{41-\frac{46(4+5x)}{2+3x}+\frac{13(4+5x)^2}{(2+3x)^2}}{\left(41+\frac{\sqrt{533}(4+5x)}{2+3x}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{13}{41}}\sqrt{4+5x}}{\sqrt{2+3x}}\right),\frac{533+23\sqrt{}}{1066}\right.}{\sqrt[4]{533}\sqrt{1+x^2}\sqrt{41-\frac{46(4+5x)}{2+3x}+\frac{13(4+5x)^2}{(2+3x)^2}}}$$

output

```
-1/533*(2+3*x)*((x^2+1)/(2+3*x)^2)^(1/2)*(41+533^(1/2)*(4+5*x)/(2+3*x))*((
41-46*(4+5*x)/(2+3*x)+13*(4+5*x)^2/(2+3*x)^2)/(41+533^(1/2)*(4+5*x)/(2+3*x
))^2)^(1/2)*InverseJacobiAM(2*arctan(1/41*13^(1/4)*41^(3/4)*(4+5*x)^(1/2)/
(2+3*x)^(1/2)),1/1066*(568178+24518*533^(1/2))^(1/2))*533^(3/4)/(x^2+1)^(1
/2)/(41-46*(4+5*x)/(2+3*x)+13*(4+5*x)^2/(2+3*x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\sqrt{1+x^2}}\,dx = -\frac{2\sqrt{\frac{(4+5i)(i+x)}{2+3x}}\sqrt{2+3x}\sqrt{4+5x}\sqrt{-\frac{(5+4i)-(4-5i)x}{82+123x}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(23+2i)(4+5x)}{2+3x}}}{\sqrt{41}}\right),\frac{525}{533}-\frac{92i}{533}\right)}{\sqrt{\frac{(23+2i)(4+5x)}{2+3x}}\sqrt{1+x^2}}$$

input
```
Integrate[1/(Sqrt[2 + 3*x]*Sqrt[4 + 5*x]*Sqrt[1 + x^2]),x]
```

output
```
(-2*Sqrt[((4 + 5*I)*(I + x))/(2 + 3*x)]*Sqrt[2 + 3*x]*Sqrt[4 + 5*x]*Sqrt[-
(((5 + 4*I) - (4 - 5*I)*x)/(82 + 123*x))]*EllipticF[ArcSin[Sqrt[((23 + 2*I
)*(4 + 5*x))/(2 + 3*x)]/Sqrt[41]], 525/533 - (92*I)/533])/(Sqrt[((23 + 2*I
)*(4 + 5*x))/(2 + 3*x)]*Sqrt[1 + x^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {732, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x+2}\sqrt{5x+4}\sqrt{x^2+1}}\,dx$$

$$\Big\downarrow \text{ 732}$$

$$-\frac{2(3x+2)\sqrt{\frac{x^2+1}{(3x+2)^2}}\int \frac{1}{\sqrt{\frac{13(5x+4)^2}{41(3x+2)^2}-\frac{46(5x+4)}{41(3x+2)}+1}}d\frac{\sqrt{5x+4}}{\sqrt{3x+2}}}{\sqrt{41}\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 1416}$$

$$-\frac{(3x+2)\sqrt{\frac{x^2+1}{(3x+2)^2}}\left(\frac{\sqrt{533}(5x+4)}{3x+2}+41\right)\sqrt{\frac{\frac{13(5x+4)^2}{(3x+2)^2}-\frac{46(5x+4)}{3x+2}+41}{\left(\frac{\sqrt{533}(5x+4)}{3x+2}+41\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{13}{41}}\sqrt{5x+4}}{\sqrt{3x+2}}\right),\frac{533+23\sqrt{533}}{1066}\right)}{\sqrt[4]{533}\sqrt{x^2+1}\sqrt{\frac{13(5x+4)^2}{(3x+2)^2}-\frac{46(5x+4)}{3x+2}+41}}$$

input
```
Int[1/(Sqrt[2 + 3*x]*Sqrt[4 + 5*x]*Sqrt[1 + x^2]),x]
```

output
```
-(((2 + 3*x)*Sqrt[(1 + x^2)/(2 + 3*x)^2]*(41 + (Sqrt[533]*(4 + 5*x))/(2 +
3*x))*Sqrt[(41 - (46*(4 + 5*x))/(2 + 3*x) + (13*(4 + 5*x)^2)/(2 + 3*x)^2)/
(41 + (Sqrt[533]*(4 + 5*x))/(2 + 3*x))^2]*EllipticF[2*ArcTan[((13/41)^(1/4
)*Sqrt[4 + 5*x])/Sqrt[2 + 3*x]], (533 + 23*Sqrt[533])/1066])/(533^(1/4)*Sq
rt[1 + x^2]*Sqrt[41 - (46*(4 + 5*x))/(2 + 3*x) + (13*(4 + 5*x)^2)/(2 + 3*x
)^2]))
```

Defintions of rubi rules used

rule 732
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*
(x_)^2]), x_Symbol] :> Simp[-2*(c + d*x)*(Sqrt[(d*e - c*f)^2*((a + b*x^2)/(
(b*e^2 + a*f^2)*(c + d*x)^2))]/((d*e - c*f)*Sqrt[a + b*x^2]))   Subst[Int[1
/Sqrt[Simp[1 - (2*b*c*e + 2*a*d*f)*(x^2/(b*e^2 + a*f^2)) + (b*c^2 + a*d^2)*
(x^4/(b*e^2 + a*f^2)), x]], x], x, Sqrt[e + f*x]/Sqrt[c + d*x]], x] /; Free
Q[{a, b, c, d, e, f}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.71

method	result
elliptic	$\frac{\left(\frac{115}{123}-\frac{10i}{123}\right)\sqrt{(3x+2)(5x+4)(x^2+1)}\sqrt{\frac{\left(\frac{69}{65}+\frac{6i}{65}\right)\left(x+\frac{2}{3}\right)}{x+\frac{4}{5}}}\left(x+\frac{4}{5}\right)^2\sqrt{\frac{\left(-\frac{4}{65}+\frac{6i}{65}\right)(x-i)}{x+\frac{4}{5}}}\sqrt{\frac{\left(-\frac{4}{65}-\frac{6i}{65}\right)(x+i)}{x+\frac{4}{5}}}\sqrt{15}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{69}{65}+\frac{6i}{65}\right.}{x+}}\right.}{\sqrt{3x+2}\sqrt{5x+4}\sqrt{x^2+1}\sqrt{\left(x+\frac{2}{3}\right)\left(x+\frac{4}{5}\right)(x-i)(x+i)}}$
default	$\frac{\left(\frac{23}{41}-\frac{2i}{41}\right)\sqrt{3x+2}\,(5x+4)^{\frac{5}{2}}\sqrt{x^2+1}\sqrt{\frac{\left(\frac{23}{13}+\frac{2i}{13}\right)(3x+2)}{5x+4}}\sqrt{\frac{\left(\frac{4}{13}-\frac{6i}{13}\right)(-x+i)}{5x+4}}\sqrt{\frac{\left(-\frac{4}{13}-\frac{6i}{13}\right)(x+i)}{5x+4}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{23}{13}+\frac{2i}{13}\right)(3x+2)}{5x+4}},\frac{23\sqrt{533}}{533}\right.}{\sqrt{15x^4+22x^3+23x^2+22x+8}\sqrt{-(3x+2)(5x+4)(-x+i)(x+i)}}$

input
```
int(1/(3*x+2)^(1/2)/(5*x+4)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(115/123-10/123*I)*((3*x+2)*(5*x+4)*(x^2+1))^(1/2)/(3*x+2)^(1/2)/(5*x+4)^(
1/2)/(x^2+1)^(1/2)*((69/65+6/65*I)*(x+2/3)/(x+4/5))^(1/2)*(x+4/5)^2*((-4/6
5+6/65*I)*(x-I)/(x+4/5))^(1/2)*((-4/65-6/65*I)*(x+I)/(x+4/5))^(1/2)*15^(1/
2)/((x+2/3)*(x+4/5)*(x-I)*(x+I))^(1/2)*EllipticF(((69/65+6/65*I)*(x+2/3)/(
x+4/5))^(1/2),23/533*533^(1/2)-2/533*I*533^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{5\,x+4}\sqrt{3\,x+2}}\,dx$$

input
```
integrate(1/(2+3*x)^(1/2)/(4+5*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(x^2 + 1)*sqrt(5*x + 4)*sqrt(3*x + 2)/(15*x^4 + 22*x^3 + 23*x
^2 + 22*x + 8), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{3x+2}\sqrt{5x+4}\sqrt{x^2+1}}\,dx$$

input
```
integrate(1/(2+3*x)**(1/2)/(4+5*x)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(3*x + 2)*sqrt(5*x + 4)*sqrt(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{5\,x+4}\sqrt{3\,x+2}}\,dx$$

input
```
integrate(1/(2+3*x)^(1/2)/(4+5*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(x^2 + 1)*sqrt(5*x + 4)*sqrt(3*x + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{5\,x+4}\sqrt{3\,x+2}}\,dx$$

input
```
integrate(1/(2+3*x)^(1/2)/(4+5*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + 1)*sqrt(5*x + 4)*sqrt(3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{3\,x+2}\,\sqrt{5\,x+4}\,\sqrt{x^2+1}}\,dx$$

input
```
int(1/((3*x + 2)^(1/2)*(5*x + 4)^(1/2)*(x^2 + 1)^(1/2)),x)
```

output
```
int(1/((3*x + 2)^(1/2)*(5*x + 4)^(1/2)*(x^2 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{3x+2}\,\sqrt{5x+4}\,\sqrt{x^2+1}}{15x^4+22x^3+23x^2+22x+8}dx$$

input
```
int(1/(2+3*x)^(1/2)/(4+5*x)^(1/2)/(x^2+1)^(1/2),x)
```

output
```
int((sqrt(3*x + 2)*sqrt(5*x + 4)*sqrt(x**2 + 1))/(15*x**4 + 22*x**3 + 23*x
**2 + 22*x + 8),x)
```

3.144 $\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}(1+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 428

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\,(1+x^2)^{3/2}}\,dx=\frac{(22-7x)\sqrt{2+3x}\sqrt{4+5x}}{533\sqrt{1+x^2}}$$
$$+\frac{7\left(123+5\sqrt{533}\right)\sqrt{2+3x}\sqrt{4+5x}\sqrt{1+x^2}}{533\left(82+4\sqrt{533}+\left(123+5\sqrt{533}\right)x\right)}$$
$$+\frac{7(2+3x)\sqrt{\frac{1+x^2}{(2+3x)^2}}\left(41+\frac{\sqrt{533}(4+5x)}{2+3x}\right)\sqrt{\frac{41-\frac{46(4+5x)}{2+3x}+\frac{13(4+5x)^2}{(2+3x)^2}}{\left(41+\frac{\sqrt{533}(4+5x)}{2+3x}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{\frac{13}{41}}\sqrt{4+5x}}{\sqrt{2+3x}}\right)\middle|\frac{533+23\sqrt{533}}{1066}\right)}{533^{3/4}\sqrt{1+x^2}\sqrt{41-\frac{46(4+5x)}{2+3x}+\frac{13(4+5x)^2}{(2+3x)^2}}}$$
$$-\frac{\left(1763+79\sqrt{533}\right)\left(82+4\sqrt{533}+\left(123+5\sqrt{533}\right)x\right)\sqrt{\frac{1+x^2}{\left(82+4\sqrt{533}+\left(123+5\sqrt{533}\right)x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\right.\right.}{41\sqrt[4]{533}\left(65+3\sqrt{533}\right)\sqrt{1+x^2}}$$

output
```
1/533*(22-7*x)*(2+3*x)^(1/2)*(4+5*x)^(1/2)/(x^2+1)^(1/2)+7*(5*533^(1/2)+12
3)*(2+3*x)^(1/2)*(4+5*x)^(1/2)*(x^2+1)^(1/2)/(43706+2132*533^(1/2)+533*(5*
533^(1/2)+123)*x)+7/533*(2+3*x)*((x^2+1)/(2+3*x)^2)^(1/2)*(41+533^(1/2)*(4
+5*x)/(2+3*x))*((41-46*(4+5*x)/(2+3*x)+13*(4+5*x)^2/(2+3*x)^2)/(41+533^(1/
2)*(4+5*x)/(2+3*x))^2)^(1/2)*EllipticE(sin(2*arctan(1/41*13^(1/4)*41^(3/4)
*(4+5*x)^(1/2)/(2+3*x)^(1/2))),1/1066*(568178+24518*533^(1/2))^(1/2))*533^
(1/4)/(x^2+1)^(1/2)/(41-46*(4+5*x)/(2+3*x)+13*(4+5*x)^2/(2+3*x)^2)^(1/2)-1
/21853*(1763+79*533^(1/2))*((5*533^(1/2)+123)*x+4*533^(1/2)+82)*((x^2+1)/(
(5*533^(1/2)+123)*x+4*533^(1/2)+82)^2)^(1/2)*InverseJacobiAM(2*arctan(1/41
*13^(1/4)*41^(3/4)*(4+5*x)^(1/2)/(2+3*x)^(1/2)),1/1066*(568178+24518*533^(
1/2))^(1/2))*533^(3/4)/(65+3*533^(1/2))/(x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.58 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.61

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\,(1+x^2)^{3/2}}\,dx = \frac{\sqrt{2+3x}\sqrt{\frac{(23+2i)(4+5x)}{2+3x}}\left((23-2i)\sqrt{\frac{(4-5i)(-i+x)}{2+3x}}(5+4x)\sqrt{\frac{(23+2i)(4+5x}{2+3x}}\right.}{}$$

input
```
Integrate[1/(Sqrt[2 + 3*x]*Sqrt[4 + 5*x]*(1 + x^2)^(3/2)),x]
```

output
```
(Sqrt[2 + 3*x]*Sqrt[((23 + 2*I)*(4 + 5*x))/(2 + 3*x)]*((23 - 2*I)*Sqrt[((4
 - 5*I)*(-I + x))/(2 + 3*x)]*(5 + 4*x)*Sqrt[((23 + 2*I)*(4 + 5*x))/(2 + 3*
x)] + (28 - 35*I)*Sqrt[41]*(-I + x)*Sqrt[((4 + 5*I)*(I + x))/(2 + 3*x)]*El
lipticE[ArcSin[Sqrt[((23 + 2*I)*(4 + 5*x))/(2 + 3*x)]/Sqrt[41]], 525/533 -
 (92*I)/533] + (158 + 110*I)*Sqrt[41]*(1 + I*x)*Sqrt[((4 + 5*I)*(I + x))/(
2 + 3*x)]*EllipticF[ArcSin[Sqrt[((23 + 2*I)*(4 + 5*x))/(2 + 3*x)]/Sqrt[41]
], 525/533 - (92*I)/533]))/(21853*Sqrt[((4 - 5*I)*(-I + x))/(2 + 3*x)]*Sqr
t[4 + 5*x]*Sqrt[1 + x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x+2}\sqrt{5x+4}\,(x^2+1)^{3/2}}\,dx$$
$$\Big\downarrow \text{744}$$
$$\int \frac{1}{\sqrt{3x+2}\sqrt{5x+4}\,(x^2+1)^{3/2}}dx$$

input
```
Int[1/(Sqrt[2 + 3*x]*Sqrt[4 + 5*x]*(1 + x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.16

method	result	size
elliptic	Expression too large to display	497
default	Expression too large to display	1281

input
```
int(1/(3*x+2)^(1/2)/(5*x+4)^(1/2)/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((3*x+2)*(5*x+4)*(x^2+1))^(1/2)/(3*x+2)^(1/2)/(5*x+4)^(1/2)/(x^2+1)^(1/2)*
(-2*(15*x^2+22*x+8)*(-11/533+7/1066*x)/((x^2+1)*(15*x^2+22*x+8))^(1/2)+(39
905/65559-3470/65559*I)*((69/65+6/65*I)*(x+2/3)/(x+4/5))^(1/2)*(x+4/5)^2*(
(-4/65+6/65*I)*(x-I)/(x+4/5))^(1/2)*((-4/65-6/65*I)*(x+I)/(x+4/5))^(1/2)*1
5^(1/2)/((x+2/3)*(x+4/5)*(x-I)*(x+I))^(1/2)*EllipticF(((69/65+6/65*I)*(x+2
/3)/(x+4/5))^(1/2),23/533*533^(1/2)-2/533*I*533^(1/2))+(8855/65559-770/655
59*I)*((69/65+6/65*I)*(x+2/3)/(x+4/5))^(1/2)*(x+4/5)^2*((-4/65+6/65*I)*(x-
I)/(x+4/5))^(1/2)*((-4/65-6/65*I)*(x+I)/(x+4/5))^(1/2)*15^(1/2)/((x+2/3)*(
x+4/5)*(x-I)*(x+I))^(1/2)*(-4/5*EllipticF(((69/65+6/65*I)*(x+2/3)/(x+4/5))
^(1/2),23/533*533^(1/2)-2/533*I*533^(1/2))+2/15*EllipticPi(((69/65+6/65*I)
*(x+2/3)/(x+4/5))^(1/2),115/123-10/123*I,23/533*533^(1/2)-2/533*I*533^(1/2
)))+7/533*((x+2/3)*(x-I)*(x+I)+(-2/3+I)*((69/65+6/65*I)*(x+2/3)/(x+4/5))^(
1/2)*(x+4/5)^2*((-4/65+6/65*I)*(x-I)/(x+4/5))^(1/2)*((-4/65-6/65*I)*(x+I)/
(x+4/5))^(1/2)*((-151/41-240/41*I)*EllipticF(((69/65+6/65*I)*(x+2/3)/(x+4/
5))^(1/2),23/533*533^(1/2)-2/533*I*533^(1/2))+(5+15/2*I)*EllipticE(((69/65
+6/65*I)*(x+2/3)/(x+4/5))^(1/2),23/533*533^(1/2)-2/533*I*533^(1/2))+(88/12
3+110/123*I)*EllipticPi(((69/65+6/65*I)*(x+2/3)/(x+4/5))^(1/2),115/123-10/
123*I,23/533*533^(1/2)-2/533*I*533^(1/2))))*15^(1/2)/((x+2/3)*(x+4/5)*(x-I
)*(x+I))^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\,(1+x^2)^{3/2}}\,dx = \int \frac{1}{(x^2+1)^{\frac{3}{2}}\sqrt{5\,x+4}\sqrt{3\,x+2}}\,dx$$

input
```
integrate(1/(2+3*x)^(1/2)/(4+5*x)^(1/2)/(x^2+1)^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(x^2 + 1)*sqrt(5*x + 4)*sqrt(3*x + 2)/(15*x^6 + 22*x^5 + 38*x
^4 + 44*x^3 + 31*x^2 + 22*x + 8), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\,(1+x^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{3x+2}\sqrt{5x+4}\,(x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2+3*x)**(1/2)/(4+5*x)**(1/2)/(x**2+1)**(3/2),x)
```

output
```
Integral(1/(sqrt(3*x + 2)*sqrt(5*x + 4)*(x**2 + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\,(1+x^2)^{3/2}}\,dx = \int \frac{1}{(x^2+1)^{\frac{3}{2}}\sqrt{5\,x+4}\sqrt{3\,x+2}}\,dx$$

input
```
integrate(1/(2+3*x)^(1/2)/(4+5*x)^(1/2)/(x^2+1)^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((x^2 + 1)^(3/2)*sqrt(5*x + 4)*sqrt(3*x + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\,(1+x^2)^{3/2}}\,dx = \int \frac{1}{(x^2+1)^{\frac{3}{2}}\sqrt{5\,x+4}\sqrt{3\,x+2}}\,dx$$

input
```
integrate(1/(2+3*x)^(1/2)/(4+5*x)^(1/2)/(x^2+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^2 + 1)^(3/2)*sqrt(5*x + 4)*sqrt(3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\,(1+x^2)^{3/2}}\,dx=\int \frac{1}{\sqrt{3\,x+2}\,\sqrt{5\,x+4}\,(x^2+1)^{3/2}}\,dx$$

input
```
int(1/((3*x + 2)^(1/2)*(5*x + 4)^(1/2)*(x^2 + 1)^(3/2)),x)
```

output
```
int(1/((3*x + 2)^(1/2)*(5*x + 4)^(1/2)*(x^2 + 1)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{2+3x}\sqrt{4+5x}\,(1+x^2)^{3/2}}\,dx=\int \frac{\sqrt{3x+2}\,\sqrt{5x+4}\,\sqrt{x^2+1}}{15x^6+22x^5+38x^4+44x^3+31x^2+22x+8}dx$$

input
```
int(1/(2+3*x)^(1/2)/(4+5*x)^(1/2)/(x^2+1)^(3/2),x)
```

output
```
int((sqrt(3*x + 2)*sqrt(5*x + 4)*sqrt(x**2 + 1))/(15*x**6 + 22*x**5 + 38*x
**4 + 44*x**3 + 31*x**2 + 22*x + 8),x)
```

3.145 $\int \frac{\sqrt{4+5x}}{\sqrt{2+3x}\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 550

$$\int \frac{\sqrt{4+5x}}{\sqrt{2+3x}\sqrt{1+x^2}}\,dx = \frac{\sqrt{\frac{5}{3}}(4+5x)\sqrt{\frac{1+x^2}{(4+5x)^2}}\operatorname{arctanh}\left(\frac{2\sqrt{2+3x}}{\sqrt{15}\sqrt{4+5x}\sqrt{13+\frac{41(2+3x)^2}{(4+5x)^2}-\frac{46(2+3x)}{4+5x}}}\right)}{\sqrt{1+x^2}}$$
$$+\frac{2\ 41^{3/4}(4+5x)\sqrt{\frac{1+x^2}{(4+5x)^2}}\sqrt{\frac{13+\frac{41(2+3x)^2}{(4+5x)^2}-\frac{46(2+3x)}{4+5x}}{\left(13+\frac{\sqrt{533}(2+3x)}{4+5x}\right)^2}}\left(13+\frac{\sqrt{533}(2+3x)}{4+5x}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\frac{41}{13}}\sqrt{2+3x}}{\sqrt{4+5x}}\right),\right.}{\sqrt[4]{13}\left(123+5\sqrt{533}\right)\sqrt{1+x^2}\sqrt{13+\frac{41(2+3x)^2}{(4+5x)^2}-\frac{46(2+3x)}{4+5x}}}$$
$$+\frac{\sqrt[4]{41}\left(65-3\sqrt{533}\right)(4+5x)\sqrt{\frac{1+x^2}{(4+5x)^2}}\sqrt{\frac{13+\frac{41(2+3x)^2}{(4+5x)^2}-\frac{46(2+3x)}{4+5x}}{\left(13+\frac{\sqrt{533}(2+3x)}{4+5x}\right)^2}}\left(13+\frac{\sqrt{533}(2+3x)}{4+5x}\right)\operatorname{EllipticPi}\left(\frac{7995+347\sqrt{533}}{15990},\right.}{3\ 13^{3/4}\left(123+5\sqrt{533}\right)\sqrt{1+x^2}\sqrt{13+\frac{41(2+3x)^2}{(4+5x)^2}-\frac{46(2+3x)}{4+5x}}}$$

output
```
1/3*15^(1/2)*(4+5*x)*((x^2+1)/(4+5*x)^2)^(1/2)*arctanh(2/15*(2+3*x)^(1/2)*
15^(1/2)/(4+5*x)^(1/2)/(13+41*(2+3*x)^2/(4+5*x)^2-46*(2+3*x)/(4+5*x))^(1/2
))/(x^2+1)^(1/2)+2/13*41^(3/4)*(4+5*x)*((x^2+1)/(4+5*x)^2)^(1/2)*((13+41*(
2+3*x)^2/(4+5*x)^2-46*(2+3*x)/(4+5*x))/(13+533^(1/2)*(2+3*x)/(4+5*x))^2)^(
1/2)*(13+533^(1/2)*(2+3*x)/(4+5*x))*InverseJacobiAM(2*arctan(1/13*13^(3/4)
*41^(1/4)/(4+5*x)^(1/2)*(2+3*x)^(1/2)),1/1066*(568178+24518*533^(1/2))^(1/
2))*13^(3/4)/(5*533^(1/2)+123)/(x^2+1)^(1/2)/(13+41*(2+3*x)^2/(4+5*x)^2-46
*(2+3*x)/(4+5*x))^(1/2)+1/39*41^(1/4)*(65-3*533^(1/2))*(4+5*x)*((x^2+1)/(4
+5*x)^2)^(1/2)*((13+41*(2+3*x)^2/(4+5*x)^2-46*(2+3*x)/(4+5*x))/(13+533^(1/
2)*(2+3*x)/(4+5*x))^2)^(1/2)*(13+533^(1/2)*(2+3*x)/(4+5*x))*EllipticPi(sin
(2*arctan(1/13*13^(3/4)*41^(1/4)/(4+5*x)^(1/2)*(2+3*x)^(1/2))),1/2+347/159
90*533^(1/2),1/1066*(568178+24518*533^(1/2))^(1/2))*13^(1/4)/(5*533^(1/2)+
123)/(x^2+1)^(1/2)/(13+41*(2+3*x)^2/(4+5*x)^2-46*(2+3*x)/(4+5*x))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 15.49 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{4+5x}}{\sqrt{2+3x}\sqrt{1+x^2}}\,dx = -\frac{4\sqrt{\frac{(4+5i)(i+x)}{2+3x}}\sqrt{4+5x}\left((23+2i)\sqrt{\frac{(4-5i)(-i+x)}{2+3x}}(4+5x)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(23+2i)(4+5x)}{2+3x}}}{\sqrt{41}}\right),\frac{525}{533}-\frac{92i}{533}\right)\right.}{3\sqrt{41}\sqrt{2+3x}\left(\right.}$$

input
```
Integrate[Sqrt[4 + 5*x]/(Sqrt[2 + 3*x]*Sqrt[1 + x^2]),x]
```

output
```
(-4*Sqrt[((4 + 5*I)*(I + x))/(2 + 3*x)]*Sqrt[4 + 5*x]*((23 + 2*I)*Sqrt[((4
 - 5*I)*(-I + x))/(2 + 3*x)]*(4 + 5*x)*EllipticF[ArcSin[Sqrt[((23 + 2*I)*(
4 + 5*x))/(2 + 3*x)]/Sqrt[41]], 525/533 - (92*I)/533] - (2 + 3*x)*Sqrt[((2
3 + 2*I)*(4 + 5*x))/(2 + 3*x)]*Sqrt[((102 - 107*I)*(-4*I + (4 - 5*I)*x + 5
*x^2))/(2 + 3*x)^2]*EllipticPi[69/65 - (6*I)/65, ArcSin[Sqrt[((23 + 2*I)*(
4 + 5*x))/(2 + 3*x)]/Sqrt[41]], 525/533 - (92*I)/533]))/(3*Sqrt[41]*Sqrt[2
 + 3*x]*(((23 + 2*I)*(4 + 5*x))/(2 + 3*x))^(3/2)*Sqrt[1 + x^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.22, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{5x+4}}{\sqrt{3x+2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{726}$$

$$\frac{4\sqrt{\frac{(3+2i)(1+ix)}{5x+4}}\sqrt{-\frac{(2+3i)(x+i)}{5x+4}}(5x+4)\text{EllipticPi}\left(\frac{115}{123}-\frac{10i}{123},\arcsin\left(\frac{\sqrt{41}\sqrt{3x+2}}{\sqrt{23-2i}\sqrt{5x+4}}\right),\frac{525}{533}-\frac{92i}{533}\right)}{3\sqrt{13}\sqrt{23+2i}\sqrt{x^2+1}}$$

input
```
Int[Sqrt[4 + 5*x]/(Sqrt[2 + 3*x]*Sqrt[1 + x^2]),x]
```

output
```
(4*Sqrt[((3 + 2*I)*(1 + I*x))/(4 + 5*x)]*Sqrt[((-2 - 3*I)*(I + x))/(4 + 5*
x)]*(4 + 5*x)*EllipticPi[115/123 - (10*I)/123, ArcSin[(Sqrt[41]*Sqrt[2 + 3
*x])/(Sqrt[23 - 2*I]*Sqrt[4 + 5*x])], 525/533 - (92*I)/533])/(3*Sqrt[13]*S
qrt[23 + 2*I]*Sqrt[1 + x^2])
```

Defintions of rubi rules used

rule 726
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x
_)^2]), x_Symbol] :> With[{q = Rt[-4*a*c, 2]}, Simp[Sqrt[2]*Sqrt[2*c*f - g*
q]*Sqrt[-q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*((q + 2*c*x)/((2*c*f - g*q)*
(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a + q*x)/((q*f - 2*a*g)*(d + e*x)))]/(g*
Sqrt[2*c*d - e*q]*Sqrt[2*a*(c/q) + c*x]*Sqrt[a + c*x^2]))*EllipticPi[e*((2*
c*f - g*q)/(g*(2*c*d - e*q))), ArcSin[Sqrt[2*c*d - e*q]*(Sqrt[f + g*x]/(Sqr
t[2*c*f - g*q]*Sqrt[d + e*x]))], (q*d - 2*a*e)*((2*c*f - g*q)/((q*f - 2*a*g
)*(2*c*d - e*q)))], x]] /; FreeQ[{a, c, d, e, f, g}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.29

method	result
default	$\frac{\left(\frac{46}{123}-\frac{4i}{123}\right)(5x+4)^{\frac{5}{2}}\sqrt{3x+2}\,\sqrt{x^2+1}\,\sqrt{\frac{\left(\frac{23}{13}+\frac{2i}{13}\right)(3x+2)}{5x+4}}\,\sqrt{\frac{\left(\frac{4}{13}-\frac{6i}{13}\right)(-x+i)}{5x+4}}\,\sqrt{\frac{\left(-\frac{4}{13}-\frac{6i}{13}\right)(x+i)}{5x+4}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{\left(\frac{23}{13}+\frac{2i}{13}\right)(3x+2)}{5x+4}},\frac{115}{123}-\right.}{\sqrt{15x^4+22x^3+23x^2+22x+8}\,\sqrt{-(3x+2)(5x+4)(-x+i)(x+i)}}$
elliptic	$\frac{\sqrt{(3x+2)(5x+4)(x^2+1)}\left(\frac{\left(\frac{460}{123}-\frac{40i}{123}\right)\sqrt{\frac{\left(\frac{69}{65}+\frac{6i}{65}\right)\left(x+\frac{2}{3}\right)}{x+\frac{4}{5}}}\left(x+\frac{4}{5}\right)^2\sqrt{\frac{\left(-\frac{4}{65}+\frac{6i}{65}\right)(x-i)}{x+\frac{4}{5}}}\,\sqrt{\frac{\left(-\frac{4}{65}-\frac{6i}{65}\right)(x+i)}{x+\frac{4}{5}}}\,\sqrt{15}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(\frac{69}{65}+\frac{6i}{65}\right)(\ldots}{x+\frac{4}{5}}}\right.}{\sqrt{\left(x+\frac{2}{3}\right)\left(x+\frac{4}{5}\right)(x-i)(x+i)}}\right.}{}$

input
```
int((5*x+4)^(1/2)/(3*x+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(46/123-4/123*I)*(5*x+4)^(5/2)*(3*x+2)^(1/2)*(x^2+1)^(1/2)*((23/13+2/13*I)
*(3*x+2)/(5*x+4))^(1/2)*((4/13-6/13*I)*(-x+I)/(5*x+4))^(1/2)*((-4/13-6/13*
I)*(x+I)/(5*x+4))^(1/2)*EllipticPi(((23/13+2/13*I)*(3*x+2)/(5*x+4))^(1/2),
115/123-10/123*I,23/533*533^(1/2)-2/533*I*533^(1/2))/(15*x^4+22*x^3+23*x^2
+22*x+8)^(1/2)/(-(3*x+2)*(5*x+4)*(-x+I)*(x+I))^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{4+5x}}{\sqrt{2+3x}\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{5\,x+4}}{\sqrt{x^2+1}\sqrt{3\,x+2}}\,dx$$

input
```
integrate((4+5*x)^(1/2)/(2+3*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + 1)*sqrt(5*x + 4)*sqrt(3*x + 2)/(3*x^3 + 2*x^2 + 3*x +
2), x)
```

Sympy [F]

$$\int \frac{\sqrt{4+5x}}{\sqrt{2+3x}\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{5x+4}}{\sqrt{3x+2}\sqrt{x^2+1}}\,dx$$

input
```
integrate((4+5*x)**(1/2)/(2+3*x)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(5*x + 4)/(sqrt(3*x + 2)*sqrt(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{4+5x}}{\sqrt{2+3x}\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{5\,x+4}}{\sqrt{x^2+1}\sqrt{3\,x+2}}\,dx$$

input
```
integrate((4+5*x)^(1/2)/(2+3*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(5*x + 4)/(sqrt(x^2 + 1)*sqrt(3*x + 2)), x)
```

Giac [F]

$$\int \frac{\sqrt{4+5x}}{\sqrt{2+3x}\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{5\,x+4}}{\sqrt{x^2+1}\sqrt{3\,x+2}}\,dx$$

input
```
integrate((4+5*x)^(1/2)/(2+3*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(5*x + 4)/(sqrt(x^2 + 1)*sqrt(3*x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+5x}}{\sqrt{2+3x}\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{5\,x+4}}{\sqrt{3\,x+2}\,\sqrt{x^2+1}}\,dx$$

input
```
int((5*x + 4)^(1/2)/((3*x + 2)^(1/2)*(x^2 + 1)^(1/2)),x)
```

output
```
int((5*x + 4)^(1/2)/((3*x + 2)^(1/2)*(x^2 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{4+5x}}{\sqrt{2+3x}\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{3x+2}\,\sqrt{5x+4}\,\sqrt{x^2+1}}{3x^3+2x^2+3x+2}dx$$

input
```
int((4+5*x)^(1/2)/(2+3*x)^(1/2)/(x^2+1)^(1/2),x)
```

output
```
int((sqrt(3*x + 2)*sqrt(5*x + 4)*sqrt(x**2 + 1))/(3*x**3 + 2*x**2 + 3*x +
2),x)
```

3.146 $\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 461

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = -\frac{2\sqrt{cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)}(d+ex)\sqrt{\frac{(ef-dg)^2(a+cx^2)}{(cf^2+ag^2)(d+ex)^2}}\sqrt{1-\frac{(cd^2+ae^2)(f+gx)}{(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg))(d+ex)}}\sqrt{1-\frac{}{(c}}}{\sqrt{cd^2+ae^2}(ef-dg)\sqrt{a+cx^2}\sqrt{1-}}$$

output
```
-2*(c*d*f+a*e*g+(-a)^(1/2)*c^(1/2)*(-d*g+e*f))^(1/2)*(e*x+d)*((-d*g+e*f)^2
*(c*x^2+a)/(a*g^2+c*f^2)/(e*x+d)^2)^(1/2)*(1-(a*e^2+c*d^2)*(g*x+f)/(c*d*f+
a*e*g-(-a)^(1/2)*c^(1/2)*(-d*g+e*f))/(e*x+d))^(1/2)*(1-(a*e^2+c*d^2)*(g*x+
f)/(c*d*f+a*e*g+(-a)^(1/2)*c^(1/2)*(-d*g+e*f))/(e*x+d))^(1/2)*EllipticF((a
*e^2+c*d^2)^(1/2)*(g*x+f)^(1/2)/(c*d*f+a*e*g+(-a)^(1/2)*c^(1/2)*(-d*g+e*f)
)^(1/2)/(e*x+d)^(1/2),((c*d*f+a*e*g+(-a)^(1/2)*c^(1/2)*(-d*g+e*f))/(c*d*f+
a*e*g-(-a)^(1/2)*c^(1/2)*(-d*g+e*f)))^(1/2))/(a*e^2+c*d^2)^(1/2)/(-d*g+e*f
)/(c*x^2+a)^(1/2)/(1-2*(a*e*g+c*d*f)*(g*x+f)/(a*g^2+c*f^2)/(e*x+d)+(a*e^2+
c*d^2)*(g*x+f)^2/(a*g^2+c*f^2)/(e*x+d)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.15 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx$$
$$= \frac{\sqrt{2}\left(i\sqrt{a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{\frac{d-\frac{i\sqrt{a}e}{\sqrt{c}}+\frac{i\sqrt{c}dx}{\sqrt{a}}+ex}{d+ex}}\sqrt{\frac{(i\sqrt{c}d+\sqrt{a}e)(f+gx)}{(i\sqrt{c}f+\sqrt{a}g)(d+ex)}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(ef-dg)(i\sqrt{a}+\sqrt{c}x)}{(\sqrt{c}f-i\sqrt{a}g)(d+ex)}}\right),-\right.}{\left(\sqrt{c}d-i\sqrt{a}e\right)\sqrt{\frac{(ef-dg)(i\sqrt{a}+\sqrt{c}x)}{(\sqrt{c}f-i\sqrt{a}g)(d+ex)}}\sqrt{f+gx}\sqrt{a+cx^2}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
(Sqrt[2]*(I*Sqrt[a] + Sqrt[c]*x)*Sqrt[d + e*x]*Sqrt[(d - (I*Sqrt[a]*e)/Sqr
t[c] + (I*Sqrt[c]*d*x)/Sqrt[a] + e*x)/(d + e*x)]*Sqrt[((I*Sqrt[c]*d + Sqrt
[a]*e)*(f + g*x))/((I*Sqrt[c]*f + Sqrt[a]*g)*(d + e*x))]*EllipticF[ArcSin[
Sqrt[((e*f - d*g)*(I*Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f - I*Sqrt[a]*g)*(d +
 e*x))]], -(((I*Sqrt[c]*d*f)/Sqrt[a] - e*f + d*g + (I*Sqrt[a]*e*g)/Sqrt[c]
)/(2*e*f - 2*d*g))])/((Sqrt[c]*d - I*Sqrt[a]*e)*Sqrt[((e*f - d*g)*(I*Sqrt[
a] + Sqrt[c]*x))/((Sqrt[c]*f - I*Sqrt[a]*g)*(d + e*x))]*Sqrt[f + g*x]*Sqrt
[a + c*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {732, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}\sqrt{d+ex}\sqrt{f+gx}}\,dx$$
$$\downarrow \text{732}$$

$$-\frac{2(d+ex)\sqrt{\frac{(a+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2+cf^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2+ae^2\right)(f+gx)^2}{\left(cf^2+ag^2\right)(d+ex)^2}-\frac{2(cdf+aeg)(f+gx)}{\left(cf^2+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+cx^2}(ef-dg)}$$

$\Big\downarrow$ 1416

$$-\frac{(d+ex)\sqrt[4]{ag^2+cf^2}\sqrt{\frac{(a+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2\left(ae^2+cd^2\right)}{(d+ex)^2\left(ag^2+cf^2\right)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)\left(ag^2+cf^2\right)}+1}{\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)^2}}\text{EllipticF}\left(2\,\text{a}\right.}{\sqrt{a+cx^2}\sqrt[4]{ae^2+cd^2}(ef-dg)\sqrt{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg}{(d+ex)(ag^2}}}$$

input
```
Int[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]
```

output
```
-(((c*f^2 + a*g^2)^(1/4)*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + c*x^2))/((c*f^
2 + a*g^2)*(d + e*x)^2)]*(1 + (Sqrt[c*d^2 + a*e^2]*(f + g*x))/(Sqrt[c*f^2
+ a*g^2]*(d + e*x)))*Sqrt[(1 - (2*(c*d*f + a*e*g)*(f + g*x))/((c*f^2 + a*g
^2)*(d + e*x)) + ((c*d^2 + a*e^2)*(f + g*x)^2)/((c*f^2 + a*g^2)*(d + e*x)^
2))/(1 + (Sqrt[c*d^2 + a*e^2]*(f + g*x))/(Sqrt[c*f^2 + a*g^2]*(d + e*x)))^
2]*EllipticF[2*ArcTan[((c*d^2 + a*e^2)^(1/4)*Sqrt[f + g*x])/((c*f^2 + a*g^
2)^(1/4)*Sqrt[d + e*x])], (1 + (c*d*f + a*e*g)/(Sqrt[c*d^2 + a*e^2]*Sqrt[c
*f^2 + a*g^2]))/2])/((c*d^2 + a*e^2)^(1/4)*(e*f - d*g)*Sqrt[a + c*x^2]*Sqr
t[1 - (2*(c*d*f + a*e*g)*(f + g*x))/((c*f^2 + a*g^2)*(d + e*x)) + ((c*d^2
+ a*e^2)*(f + g*x)^2)/((c*f^2 + a*g^2)*(d + e*x)^2)]))
```

Defintions of rubi rules used

rule 732
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*
(x_)^2]), x_Symbol] :> Simp[-2*(c + d*x)*(Sqrt[(d*e - c*f)^2*((a + b*x^2)/(
(b*e^2 + a*f^2)*(c + d*x)^2))]/((d*e - c*f)*Sqrt[a + b*x^2]))   Subst[Int[1
/Sqrt[Simp[1 - (2*b*c*e + 2*a*d*f)*(x^2/(b*e^2 + a*f^2)) + (b*c^2 + a*d^2)*
(x^4/(b*e^2 + a*f^2)), x]], x], x, Sqrt[e + f*x]/Sqrt[c + d*x]], x] /; Free
Q[{a, b, c, d, e, f}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 11.06 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.87

method	result
default	$\frac{2\left(c e^2 f x^2-\sqrt{-ac}\, e^2 g x^2+2cdefx-2\sqrt{-ac}\, degx+c d^2 f-\sqrt{-ac}\, d^2 g\right) \operatorname{EllipticF}\left(\sqrt{\frac{(e\sqrt{-ac}-cd)(gx+f)}{(\sqrt{-ac}\, g-cf)(ex+d)}},\sqrt{\frac{(e\sqrt{-ac}+cd)(\sqrt{-ac}\, g-cf)}{(\sqrt{-ac}\, g+cf)(e\sqrt{-ac}-cd)}}\right)}{\sqrt{-\frac{(gx+f)(ex+d)(-cx+\sqrt{-ac})(cx+\sqrt{-ac})}{c}}\,(cd-e\sqrt{-ac})(dg-ef)}$
elliptic	$\frac{2\sqrt{(gx+f)(ex+d)(c x^2+a)}\left(-\frac{f}{g}+\frac{\sqrt{-ac}}{c}\right)\sqrt{\frac{\left(-\frac{\sqrt{-ac}}{c}+\frac{d}{e}\right)\left(x+\frac{f}{g}\right)}{\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\left(x+\frac{d}{e}\right)}}\left(x+\frac{d}{e}\right)^2\sqrt{\frac{\left(-\frac{d}{e}+\frac{f}{g}\right)\left(x-\frac{\sqrt{-ac}}{c}\right)}{\left(\frac{f}{g}+\frac{\sqrt{-ac}}{c}\right)\left(x+\frac{d}{e}\right)}}\sqrt{\frac{\left(-\frac{d}{e}+\frac{f}{g}\right)\left(x+\frac{\sqrt{-ac}}{c}\right)}{\left(\frac{f}{g}-\frac{\sqrt{-ac}}{c}\right)\left(x+\frac{d}{e}\right)}}\ \text{Ellipti}}{\sqrt{gx+f}\,\sqrt{ex+d}\,\sqrt{c x^2+a}\left(-\frac{\sqrt{-ac}}{c}+\frac{d}{e}\right)\left(-\frac{d}{e}+\frac{f}{g}\right)\sqrt{ceg\left(x+\frac{f}{g}\right)\left(x+\frac{d}{e}\right)\left(x-\frac{\sqrt{-ac}}{c}\right.}}$

input
```
int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(c*e^2*f*x^2-(-a*c)^(1/2)*e^2*g*x^2+2*c*d*e*f*x-2*(-a*c)^(1/2)*d*e*g*x+c
*d^2*f-(-a*c)^(1/2)*d^2*g)*EllipticF(((e*(-a*c)^(1/2)-c*d)*(g*x+f)/((-a*c)
^(1/2)*g-c*f)/(e*x+d))^(1/2),((e*(-a*c)^(1/2)+c*d)*((-a*c)^(1/2)*g-c*f)/((
-a*c)^(1/2)*g+c*f)/(e*(-a*c)^(1/2)-c*d))^(1/2))*((d*g-e*f)*(c*x+(-a*c)^(1/
2))/((-a*c)^(1/2)*g-c*f)/(e*x+d))^(1/2)*((d*g-e*f)*(-c*x+(-a*c)^(1/2))/((-
a*c)^(1/2)*g+c*f)/(e*x+d))^(1/2)*((e*(-a*c)^(1/2)-c*d)*(g*x+f)/((-a*c)^(1/
2)*g-c*f)/(e*x+d))^(1/2)*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)*(e*x+d)^(1/2)/(-1/c
*(g*x+f)*(e*x+d)*(-c*x+(-a*c)^(1/2))*(c*x+(-a*c)^(1/2)))^(1/2)/(c*d-e*(-a*
c)^(1/2))/(d*g-e*f)/((g*x+f)*(e*x+d)*(c*x^2+a))^(1/2)
```

Fricas [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)/(c*e*g*x^4 + (c*e*f +
 c*d*g)*x^3 + a*d*f + (c*d*f + a*e*g)*x^2 + (a*e*f + a*d*g)*x), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{a+cx^2}\sqrt{d+ex}\sqrt{f+gx}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**2)*sqrt(d + e*x)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+a}\sqrt{ex+d}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{c\,x^2+a}\,\sqrt{d+e\,x}}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a + c*x^2)^(1/2)*(d + e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}}\,dx = \int \frac{1}{\sqrt{ex+d}\,\sqrt{gx+f}\,\sqrt{c\,x^2+a}}dx$$

input
```
int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

output
```
int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)
```

3.147 $\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 31

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx=-\frac{\sqrt{1-x^2}\,\text{EllipticF}(\arccos(x),2)}{\sqrt{-1+x}\sqrt{1+x}}$$

output
```
-(-x^2+1)^(1/2)*InverseJacobiAM(arccos(x),2^(1/2))/(-1+x)^(1/2)/(1+x)^(1/2
)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 107 vs. 2(31) = 62.

Time = 35.57 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.45

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx =-\frac{2(-1+x)^{3/2}\sqrt{\frac{1+x}{1-x}}\sqrt{\frac{1-2x^2}{(-1+x)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2+\sqrt{2}+\frac{1}{-1+x}}}{2^{3/4}}\right),4(-4+3\sqrt{2})\right)}{\sqrt{3+2\sqrt{2}}\sqrt{1+x}\sqrt{-1+2x^2}}$$

input
```
Integrate[1/(Sqrt[-1 + x]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2]),x]
```

output
```
(-2*(-1 + x)^(3/2)*Sqrt[(1 + x)/(1 - x)]*Sqrt[(1 - 2*x^2)/(-1 + x)^2]*Elli
pticF[ArcSin[Sqrt[2 + Sqrt[2] + (-1 + x)^(-1)]/2^(3/4)], 4*(-4 + 3*Sqrt[2]
)])/(Sqrt[3 + 2*Sqrt[2]]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {648, 323, 323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}dx$$

$$\Big\downarrow \text{648}$$

$$\frac{\sqrt{x^2-1}\int \frac{1}{\sqrt{x^2-1}\sqrt{2x^2-1}}dx}{\sqrt{x-1}\sqrt{x+1}}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{1-2x^2}\sqrt{x^2-1}\int \frac{1}{\sqrt{1-2x^2}\sqrt{x^2-1}}dx}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{1-2x^2}\sqrt{1-x^2}\int \frac{1}{\sqrt{1-2x^2}\sqrt{1-x^2}}dx}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{1-2x^2}\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),2)}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}$$

input
```
Int[1/(Sqrt[-1 + x]*Sqrt[1 + x]*Sqrt[-1 + 2*x^2]),x]
```

output
```
(Sqrt[1 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[x], 2])/(Sqrt[-1 + x]*Sqrt
[1 + x]*Sqrt[-1 + 2*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 648
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(c + d*x)^FracPart[m]*((e + f*x)^FracPart[m]/(c
*e + d*f*x^2)^FracPart[m])   Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] &&
  !(EqQ[p, 2] && LtQ[m, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(27) = 54.

Time = 2.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

method	result	size
default	$\frac{\sqrt{x-1}\,\sqrt{x+1}\,\sqrt{2x^2-1}\,\sqrt{-x^2+1}\,\sqrt{-2x^2+1}\ \text{EllipticF}\left(x,\sqrt{2}\right)}{2x^4-3x^2+1}$	58
elliptic	$\frac{\sqrt{(2x^2-1)(x^2-1)}\,\sqrt{-x^2+1}\,\sqrt{-2x^2+1}\ \text{EllipticF}\left(x,\sqrt{2}\right)}{\sqrt{x-1}\,\sqrt{x+1}\,\sqrt{2x^2-1}\,\sqrt{2x^4-3x^2+1}}$	73

input
```
int(1/(x-1)^(1/2)/(x+1)^(1/2)/(2*x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x-1)^(1/2)*(x+1)^(1/2)*(2*x^2-1)^(1/2)/(2*x^4-3*x^2+1)*(-x^2+1)^(1/2)*(-2
*x^2+1)^(1/2)*EllipticF(x,2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.13

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = F(\arcsin{(x)}\,|2)$$

input
```
integrate(1/(x-1)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
elliptic_f(arcsin(x), 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}\,dx$$

input
```
integrate(1/(x-1)**(1/2)/(1+x)**(1/2)/(2*x**2-1)**(1/2),x)
```

output
```
Integral(1/(sqrt(x - 1)*sqrt(x + 1)*sqrt(2*x**2 - 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2-1}\sqrt{x+1}\sqrt{x-1}}\,dx$$

input
```
integrate(1/(x-1)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(2*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2-1}\sqrt{x+1}\sqrt{x-1}}\,dx$$

input
```
integrate(1/(x-1)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2-1}\,\sqrt{x-1}\,\sqrt{x+1}}\,dx$$

input
```
int(1/((2*x^2 - 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

output
```
int(1/((2*x^2 - 1)^(1/2)*(x - 1)^(1/2)*(x + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}}\,dx = \int \frac{\sqrt{x+1}\,\sqrt{x-1}\,\sqrt{2x^2-1}}{2x^4-3x^2+1}dx$$

input
```
int(1/(x-1)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x)
```

output
```
int((sqrt(x + 1)*sqrt(x - 1)*sqrt(2*x**2 - 1))/(2*x**4 - 3*x**2 + 1),x)
```

3.148 $\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 625

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx = \frac{2\sqrt{\sqrt{c}f+\sqrt{a}g}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{-\frac{(ef-dg)\left(\sqrt{a}-\sqrt{c}x\right)}{\left(\sqrt{c}f+\sqrt{a}g\right)(d+ex)}}E\left(\arcsin\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{a}g}\sqrt{d+ex}}\right)\right.}{\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{\sqrt{c}d+\sqrt{a}e}\sqrt{\frac{(ef-dg)\left(\sqrt{a}+\sqrt{c}x\right)}{\left(\sqrt{c}f-\sqrt{a}g\right)(d+ex)}}\sqrt{a-cx^2}}$$

$$-\frac{2\sqrt{\sqrt{c}f+\sqrt{a}g}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{-\frac{(ef-dg)\left(\sqrt{a}-\sqrt{c}x\right)}{\left(\sqrt{c}f+\sqrt{a}g\right)(d+ex)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{a}g}\sqrt{d+ex}}\right),\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\left(\sqrt{c}f+\sqrt{a}g\right)}{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{c}f-\sqrt{a}g\right)}\right.}{\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{\sqrt{c}d+\sqrt{a}e}\sqrt{\frac{(ef-dg)\left(\sqrt{a}+\sqrt{c}x\right)}{\left(\sqrt{c}f-\sqrt{a}g\right)(d+ex)}}\sqrt{a-cx^2}}$$

output
```
2*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(a^(1/2)+c^(1/2)*x)*(-(-d*g+e*f)*(a^(1/2)-c^
(1/2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d))^(1/2)*EllipticE((c^(1/2)*d+a^(1/2)
*e)^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^(1/2)*g)^(1/2)/(e*x+d)^(1/2),((c^(1/2
)*d-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(c^(1/2)*f-a^(1
/2)*g))^(1/2))/(c^(1/2)*d-a^(1/2)*e)/(c^(1/2)*d+a^(1/2)*e)^(1/2)/((-d*g+e*
f)*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)*g)/(e*x+d))^(1/2)/(-c*x^2+a)^(1/
2)-2*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(a^(1/2)+c^(1/2)*x)*(-(-d*g+e*f)*(a^(1/2)
-c^(1/2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d))^(1/2)*EllipticF((c^(1/2)*d+a^(1
/2)*e)^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^(1/2)*g)^(1/2)/(e*x+d)^(1/2),((c^(
1/2)*d-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(c^(1/2)*f-a
^(1/2)*g))^(1/2))/(c^(1/2)*d-a^(1/2)*e)/(c^(1/2)*d+a^(1/2)*e)^(1/2)/((-d*g
+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)*g)/(e*x+d))^(1/2)/(-c*x^2+a)^
(1/2)
```

Mathematica [A] (verified)

Time = 29.05 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx = \frac{\sqrt{d+ex}\sqrt{\frac{(\sqrt{c}d+\sqrt{a}e)(f+gx)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\sqrt{a-cx^2}\left(2e\left(\sqrt{c}f-\sqrt{a}g\right)\sqrt{\frac{(ef-dg)(\sqrt{a}+\sqrt{c}x)}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}E\left(\mathrm{a}\right.\right.}{}$$

input
```
Integrate[Sqrt[f + g*x]/((d + e*x)^(3/2)*Sqrt[a - c*x^2]),x]
```

output
```
(Sqrt[d + e*x]*Sqrt[((Sqrt[c]*d + Sqrt[a]*e)*(f + g*x))/((Sqrt[c]*f + Sqrt
[a]*g)*(d + e*x))]*Sqrt[a - c*x^2]*(2*e*(Sqrt[c]*f - Sqrt[a]*g)*Sqrt[((e*f
 - d*g)*(Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f - Sqrt[a]*g)*(d + e*x))]*Ellipt
icE[ArcSin[Sqrt[((Sqrt[c]*d + Sqrt[a]*e)*(f + g*x))/((Sqrt[c]*f + Sqrt[a]*
g)*(d + e*x))]], ((Sqrt[c]*d - Sqrt[a]*e)*(Sqrt[c]*f + Sqrt[a]*g))/((Sqrt[
c]*d + Sqrt[a]*e)*(Sqrt[c]*f - Sqrt[a]*g))] + Sqrt[2]*(Sqrt[c]*d - Sqrt[a]
*e)*g*Sqrt[(d + (Sqrt[a]*e)/Sqrt[c] + (Sqrt[c]*d*x)/Sqrt[a] + e*x)/(d + e*
x)]*EllipticF[ArcSin[Sqrt[((e*f - d*g)*(-Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f
 + Sqrt[a]*g)*(d + e*x))]], -(((Sqrt[c]*d*f)/Sqrt[a] - e*f + d*g - (Sqrt[a
]*e*g)/Sqrt[c])/(2*e*f - 2*d*g))] + 2*Sqrt[c]*(-(e*f) + d*g)*Sqrt[((e*f -
d*g)*(Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f - Sqrt[a]*g)*(d + e*x))]*EllipticF
[ArcSin[Sqrt[((Sqrt[c]*d + Sqrt[a]*e)*(f + g*x))/((Sqrt[c]*f + Sqrt[a]*g)*
(d + e*x))]], ((Sqrt[c]*d - Sqrt[a]*e)*(Sqrt[c]*f + Sqrt[a]*g))/((Sqrt[c]*
d + Sqrt[a]*e)*(Sqrt[c]*f - Sqrt[a]*g))]))/(e*(-(c*d^2) + a*e^2)*(Sqrt[a]
+ Sqrt[c]*x)*Sqrt[((e*f - d*g)*(-Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f + Sqrt[
a]*g)*(d + e*x))]*Sqrt[f + g*x])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a-cx^2}(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 744}$$

$$\int \frac{\sqrt{f+gx}}{\sqrt{a-cx^2}(d+ex)^{3/2}}dx$$

input
```
Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*Sqrt[a - c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2019 vs. $2(479) = 958$.

Time = 11.15 (sec) , antiderivative size = 2020, normalized size of antiderivative = 3.23

method	result	size
elliptic	Expression too large to display	2020
default	Expression too large to display	3968

input
```
int((g*x+f)^(1/2)/(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(-c*x^2+a)*(e*x+d))^(1/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2)/(e*x+d)^
(1/2)*(-2*(-c*e*g*x^3-c*e*f*x^2+a*e*g*x+a*e*f)/(a*e^2-c*d^2)/((x+d/e)*(-c*
e*g*x^3-c*e*f*x^2+a*e*g*x+a*e*f))^(1/2)+2*(g/e-(a*e^2*g-c*d^2*g+c*d*e*f)/e
/(a*e^2-c*d^2)+a*e*g/(a*e^2-c*d^2))*(-f/g+d/e)*((-d/e-1/c*(a*c)^(1/2))*(x+
f/g)/(-d/e+f/g)/(x-1/c*(a*c)^(1/2)))^(1/2)*(x-1/c*(a*c)^(1/2))^2*((1/c*(a*
c)^(1/2)+f/g)*(x+1/c*(a*c)^(1/2))/(-1/c*(a*c)^(1/2)+f/g)/(x-1/c*(a*c)^(1/2
)))^(1/2)*((1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-d/e+f/g)/(x-1/c*(a*c)^(1/2)))^(
1/2)/(-d/e-1/c*(a*c)^(1/2))/(1/c*(a*c)^(1/2)+f/g)/(-c*e*g*(x+f/g)*(x-1/c*(
a*c)^(1/2))*(x+1/c*(a*c)^(1/2))*(x+d/e))^(1/2)*EllipticF(((-d/e-1/c*(a*c)^
(1/2))*(x+f/g)/(-d/e+f/g)/(x-1/c*(a*c)^(1/2)))^(1/2),2^(1/2)*(1/c*(a*c)^(1
/2)*(-f/g+d/e)/(1/c*(a*c)^(1/2)-f/g)/(d/e+1/c*(a*c)^(1/2)))^(1/2))+2*(-(d*
g-e*f)*c/(a*e^2-c*d^2)-2*f*c*e/(a*e^2-c*d^2))*(-f/g+d/e)*((-d/e-1/c*(a*c)^
(1/2))*(x+f/g)/(-d/e+f/g)/(x-1/c*(a*c)^(1/2)))^(1/2)*(x-1/c*(a*c)^(1/2))^2
*((1/c*(a*c)^(1/2)+f/g)*(x+1/c*(a*c)^(1/2))/(-1/c*(a*c)^(1/2)+f/g)/(x-1/c*
(a*c)^(1/2)))^(1/2)*((1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-d/e+f/g)/(x-1/c*(a*c)
^(1/2)))^(1/2)/(-d/e-1/c*(a*c)^(1/2))/(1/c*(a*c)^(1/2)+f/g)/(-c*e*g*(x+f/g
)*(x-1/c*(a*c)^(1/2))*(x+1/c*(a*c)^(1/2))*(x+d/e))^(1/2)*(1/c*(a*c)^(1/2)*
EllipticF(((-d/e-1/c*(a*c)^(1/2))*(x+f/g)/(-d/e+f/g)/(x-1/c*(a*c)^(1/2)))^
(1/2),2^(1/2)*(1/c*(a*c)^(1/2)*(-f/g+d/e)/(1/c*(a*c)^(1/2)-f/g)/(d/e+1/c*(
a*c)^(1/2)))^(1/2))+(-1/c*(a*c)^(1/2)-f/g)*EllipticPi(((-d/e-1/c*(a*c)^...
```

Fricas [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x, algorithm="frica
s")
```

output
```
integral(-sqrt(-c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)/(c*e^2*x^4 + 2*c*d*
e*x^3 - 2*a*d*e*x - a*d^2 + (c*d^2 - a*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx = \int \frac{\sqrt{f+gx}}{\sqrt{a-cx^2}\,(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)**(3/2)/(-c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)/(sqrt(a - c*x**2)*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(sqrt(g*x + f)/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}}{\sqrt{a-c\,x^2}\,(d+e\,x)^{3/2}}\,dx$$

input
```
int((f + g*x)^(1/2)/((a - c*x^2)^(1/2)*(d + e*x)^(3/2)),x)
```

output
```
int((f + g*x)^(1/2)/((a - c*x^2)^(1/2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{(ex+d)^{\frac{3}{2}}\sqrt{-c\,x^2+a}}dx$$

input
```
int((g*x+f)^(1/2)/(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x)
```

output
```
int((g*x+f)^(1/2)/(e*x+d)^(3/2)/(-c*x^2+a)^(1/2),x)
```

3.149 $\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 985

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx = \frac{2\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\sqrt{cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)}(d+ex)\cdots}{(c\cdots}$$

$$-\frac{2\left(cdf+aeg-\sqrt{-a}\sqrt{c}(ef-dg)\right)\sqrt{cdf+aeg+\sqrt{-a}\sqrt{c}(ef-dg)}(d+ex)\sqrt{\frac{(ef-dg)^2(a+cx^2)}{(cf^2+ag^2)(d+ex)^2}}\sqrt{1-\frac{\cdots}{(cdf+\cdots}}}{\left(cd^2+ae^2\right)^{3/2}(ef-dg)\cdots}$$

output
```
2*(c*d*f+a*e*g-(-a)^(1/2)*c^(1/2)*(-d*g+e*f))*(c*d*f+a*e*g+(-a)^(1/2)*c^(1
/2)*(-d*g+e*f))^(1/2)*(e*x+d)*((-d*g+e*f)^2*(c*x^2+a)/(a*g^2+c*f^2)/(e*x+d
)^2)^(1/2)*(1-(a*e^2+c*d^2)*(g*x+f)/(c*d*f+a*e*g-(-a)^(1/2)*c^(1/2)*(-d*g+
e*f))/(e*x+d))^(1/2)*(1-(a*e^2+c*d^2)*(g*x+f)/(c*d*f+a*e*g+(-a)^(1/2)*c^(1
/2)*(-d*g+e*f))/(e*x+d))^(1/2)*EllipticE((a*e^2+c*d^2)^(1/2)*(g*x+f)^(1/2)
/(c*d*f+a*e*g+(-a)^(1/2)*c^(1/2)*(-d*g+e*f))^(1/2)/(e*x+d)^(1/2),((c*d*f+a
*e*g+(-a)^(1/2)*c^(1/2)*(-d*g+e*f))/(c*d*f+a*e*g-(-a)^(1/2)*c^(1/2)*(-d*g+
e*f)))^(1/2))/(a*e^2+c*d^2)^(3/2)/(-d*g+e*f)/(c*x^2+a)^(1/2)/(1-2*(a*e*g+c
*d*f)*(g*x+f)/(a*g^2+c*f^2)/(e*x+d)+(a*e^2+c*d^2)*(g*x+f)^2/(a*g^2+c*f^2)/
(e*x+d)^2)^(1/2)-2*(c*d*f+a*e*g-(-a)^(1/2)*c^(1/2)*(-d*g+e*f))*(c*d*f+a*e*
g+(-a)^(1/2)*c^(1/2)*(-d*g+e*f))^(1/2)*(e*x+d)*((-d*g+e*f)^2*(c*x^2+a)/(a*
g^2+c*f^2)/(e*x+d)^2)^(1/2)*(1-(a*e^2+c*d^2)*(g*x+f)/(c*d*f+a*e*g-(-a)^(1/
2)*c^(1/2)*(-d*g+e*f))/(e*x+d))^(1/2)*(1-(a*e^2+c*d^2)*(g*x+f)/(c*d*f+a*e*
g+(-a)^(1/2)*c^(1/2)*(-d*g+e*f))/(e*x+d))^(1/2)*EllipticF((a*e^2+c*d^2)^(1
/2)*(g*x+f)^(1/2)/(c*d*f+a*e*g+(-a)^(1/2)*c^(1/2)*(-d*g+e*f))^(1/2)/(e*x+d
)^(1/2),((c*d*f+a*e*g+(-a)^(1/2)*c^(1/2)*(-d*g+e*f))/(c*d*f+a*e*g-(-a)^(1/
2)*c^(1/2)*(-d*g+e*f)))^(1/2))/(a*e^2+c*d^2)^(3/2)/(-d*g+e*f)/(c*x^2+a)^(1
/2)/(1-2*(a*e*g+c*d*f)*(g*x+f)/(a*g^2+c*f^2)/(e*x+d)+(a*e^2+c*d^2)*(g*x+f)
^2/(a*g^2+c*f^2)/(e*x+d)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.06 (sec) , antiderivative size = 777, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx = \frac{\left(i\sqrt{a}+\sqrt{c}x\right)\sqrt{d+ex}\sqrt{\frac{\left(i\sqrt{c}d+\sqrt{a}e\right)(f+gx)}{\left(i\sqrt{c}f+\sqrt{a}g\right)(d+ex)}}\left(2e\left(\sqrt{c}f+i\sqrt{a}g\right)\sqrt{\frac{(ef-dg)\left(-i\sqrt{a}+\sqrt{\cdots}\right.}{\left(\sqrt{c}f+i\sqrt{a}g\right)(d+e\cdots}}\right.}{\cdots}$$

input
```
Integrate[Sqrt[f + g*x]/((d + e*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

output
```
((I*Sqrt[a] + Sqrt[c]*x)*Sqrt[d + e*x]*Sqrt[((I*Sqrt[c]*d + Sqrt[a]*e)*(f
+ g*x))/((I*Sqrt[c]*f + Sqrt[a]*g)*(d + e*x))]*(2*e*(Sqrt[c]*f + I*Sqrt[a]
*g)*Sqrt[((e*f - d*g)*((-I)*Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f + I*Sqrt[a]*
g)*(d + e*x))]*EllipticE[ArcSin[Sqrt[((Sqrt[c]*d - I*Sqrt[a]*e)*(f + g*x))
/((Sqrt[c]*f - I*Sqrt[a]*g)*(d + e*x))]], ((Sqrt[c]*d + I*Sqrt[a]*e)*(Sqrt
[c]*f - I*Sqrt[a]*g))/((Sqrt[c]*d - I*Sqrt[a]*e)*(Sqrt[c]*f + I*Sqrt[a]*g)
)] + Sqrt[2]*(Sqrt[c]*d + I*Sqrt[a]*e)*g*Sqrt[(d - (I*Sqrt[a]*e)/Sqrt[c] +
 (I*Sqrt[c]*d*x)/Sqrt[a] + e*x)/(d + e*x)]*EllipticF[ArcSin[Sqrt[((e*f - d
*g)*(I*Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f - I*Sqrt[a]*g)*(d + e*x))]], -(((
I*Sqrt[c]*d*f)/Sqrt[a] - e*f + d*g + (I*Sqrt[a]*e*g)/Sqrt[c])/(2*e*f - 2*d
*g))] + 2*Sqrt[c]*(-(e*f) + d*g)*Sqrt[((e*f - d*g)*((-I)*Sqrt[a] + Sqrt[c]
*x))/((Sqrt[c]*f + I*Sqrt[a]*g)*(d + e*x))]*EllipticF[ArcSin[Sqrt[((Sqrt[c
]*d - I*Sqrt[a]*e)*(f + g*x))/((Sqrt[c]*f - I*Sqrt[a]*g)*(d + e*x))]], ((S
qrt[c]*d + I*Sqrt[a]*e)*(Sqrt[c]*f - I*Sqrt[a]*g))/((Sqrt[c]*d - I*Sqrt[a]
*e)*(Sqrt[c]*f + I*Sqrt[a]*g))]))/((c*d^2*e + a*e^3)*Sqrt[((e*f - d*g)*(I*
Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f - I*Sqrt[a]*g)*(d + e*x))]*Sqrt[f + g*x]
*Sqrt[a + c*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \; 744$$

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}(d+ex)^{3/2}}dx$$

input
```
Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*Sqrt[a + c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2090 vs. $2(887) = 1774$.

Time = 10.74 (sec) , antiderivative size = 2091, normalized size of antiderivative = 2.12

method	result	size
elliptic	Expression too large to display	2091
default	Expression too large to display	3816

input
```
int((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((g*x+f)*(e*x+d)*(c*x^2+a))^(1/2)/(g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a)^(1
/2)*(-2*(c*e*g*x^3+c*e*f*x^2+a*e*g*x+a*e*f)/(a*e^2+c*d^2)/((x+d/e)*(c*e*g*
x^3+c*e*f*x^2+a*e*g*x+a*e*f))^(1/2)+2*(g/e-(a*e^2*g+c*d^2*g-c*d*e*f)/e/(a*
e^2+c*d^2)+a*e*g/(a*e^2+c*d^2))*(-f/g+(-a*c)^(1/2)/c)*((-(-a*c)^(1/2)/c+d/
e)*(x+f/g)/(f/g-(-a*c)^(1/2)/c)/(x+d/e))^(1/2)*(x+d/e)^2*((-d/e+f/g)*(x-(-
a*c)^(1/2)/c)/(f/g+(-a*c)^(1/2)/c)/(x+d/e))^(1/2)*((-d/e+f/g)*(x+(-a*c)^(1
/2)/c)/(f/g-(-a*c)^(1/2)/c)/(x+d/e))^(1/2)/(-(-a*c)^(1/2)/c+d/e)/(-d/e+f/g
)/(c*e*g*(x+f/g)*(x+d/e)*(x-(-a*c)^(1/2)/c)*(x+(-a*c)^(1/2)/c))^(1/2)*Elli
pticF(((-(-a*c)^(1/2)/c+d/e)*(x+f/g)/(f/g-(-a*c)^(1/2)/c)/(x+d/e))^(1/2),(
(-d/e-(-a*c)^(1/2)/c)*(-f/g+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c)/((-a*c)^
(1/2)/c-d/e))^(1/2))+2*((d*g-e*f)*c/(a*e^2+c*d^2)+2*f*c*e/(a*e^2+c*d^2))*(
-f/g+(-a*c)^(1/2)/c)*((-(-a*c)^(1/2)/c+d/e)*(x+f/g)/(f/g-(-a*c)^(1/2)/c)/(
x+d/e))^(1/2)*(x+d/e)^2*((-d/e+f/g)*(x-(-a*c)^(1/2)/c)/(f/g+(-a*c)^(1/2)/c
)/(x+d/e))^(1/2)*((-d/e+f/g)*(x+(-a*c)^(1/2)/c)/(f/g-(-a*c)^(1/2)/c)/(x+d/
e))^(1/2)/(-(-a*c)^(1/2)/c+d/e)/(-d/e+f/g)/(c*e*g*(x+f/g)*(x+d/e)*(x-(-a*c
)^(1/2)/c)*(x+(-a*c)^(1/2)/c))^(1/2)*(-d/e*EllipticF(((-(-a*c)^(1/2)/c+d/e
)*(x+f/g)/(f/g-(-a*c)^(1/2)/c)/(x+d/e))^(1/2),((-d/e-(-a*c)^(1/2)/c)*(-f/g
+(-a*c)^(1/2)/c)/(-f/g-(-a*c)^(1/2)/c)/((-a*c)^(1/2)/c-d/e))^(1/2))+(-f/g+
d/e)*EllipticPi(((-(-a*c)^(1/2)/c+d/e)*(x+f/g)/(f/g-(-a*c)^(1/2)/c)/(x+d/e
))^(1/2),(f/g-(-a*c)^(1/2)/c)/(-(-a*c)^(1/2)/c+d/e),((-d/e-(-a*c)^(1/2)...
```

Fricas [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)/(c*e^2*x^4 + 2*c*d*e*
x^3 + 2*a*d*e*x + a*d^2 + (c*d^2 + a*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}\,(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)**(1/2)/(e*x+d)**(3/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}}{\sqrt{c\,x^2+a}\,(d+e\,x)^{3/2}}\,dx$$

input
```
int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^(3/2)),x)
```

output
```
int((f + g*x)^(1/2)/((a + c*x^2)^(1/2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a+cx^2}}\,dx = \int \frac{\sqrt{gx+f}}{(ex+d)^{\frac{3}{2}}\sqrt{c\,x^2+a}}dx$$

input
```
int((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a)^(1/2),x)
```

output
```
int((g*x+f)^(1/2)/(e*x+d)^(3/2)/(c*x^2+a)^(1/2),x)
```

3.150 $\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 188

$$\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)\,dx = \frac{\left(cd^2+ae^2\right)(ef-dg)^2(d+ex)^{1+m}}{e^5(1+m)} + \frac{2(ef-dg)\left(ae^2g-cd(ef-2dg)\right)(d+ex)^{2+m}}{e^5(2+m)} + \frac{\left(ae^2g^2+c(e^2f^2-6defg+6d^2g^2)\right)(d+ex)^{3+m}}{e^5(3+m)} + \frac{2cg(ef-2dg)(d+ex)^{4+m}}{e^5(4+m)} + \frac{cg^2(d+ex)^{5+m}}{e^5(5+m)}$$

output
```
(a*e^2+c*d^2)*(-d*g+e*f)^2*(e*x+d)^(1+m)/e^5/(1+m)+2*(-d*g+e*f)*(a*e^2*g-c
*d*(-2*d*g+e*f))*(e*x+d)^(2+m)/e^5/(2+m)+(a*e^2*g^2+c*(6*d^2*g^2-6*d*e*f*g
+e^2*f^2))*(e*x+d)^(3+m)/e^5/(3+m)+2*c*g*(-2*d*g+e*f)*(e*x+d)^(4+m)/e^5/(4
+m)+c*g^2*(e*x+d)^(5+m)/e^5/(5+m)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.89

$$\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)\,dx$$
$$=\frac{(d+ex)^{1+m}\left(\frac{(cd^2+ae^2)(ef-dg)^2}{1+m}+\frac{2(ef-dg)(ae^2g+cd(-ef+2dg))(d+ex)}{2+m}+\frac{(ae^2g^2+c(e^2f^2-6defg+6d^2g^2))(d+ex)^2}{3+m}+\frac{2cg(ef-\dots}{\dots}\right.}{e^5}$$

input
```
Integrate[(d + e*x)^m*(f + g*x)^2*(a + c*x^2),x]
```

output
```
((d + e*x)^(1 + m)*(((c*d^2 + a*e^2)*(e*f - d*g)^2)/(1 + m) + (2*(e*f - d*
g)*(a*e^2*g + c*d*(-(e*f) + 2*d*g))*(d + e*x))/(2 + m) + ((a*e^2*g^2 + c*(
e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2))*(d + e*x)^2)/(3 + m) + (2*c*g*(e*f - 2*d
*g)*(d + e*x)^3)/(4 + m) + (c*g^2*(d + e*x)^4)/(5 + m)))/e^5
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(f+gx)^2(d+ex)^m\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{(d+ex)^{m+2}\left(ae^2g^2+c\left(6d^2g^2-6defg+e^2f^2\right)\right)}{e^4}+\frac{\left(ae^2+cd^2\right)(ef-dg)^2(d+ex)^m}{e^4}+\frac{2(ef-dg)(d+ex)\dots}{\dots}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^{m+3}\left(ae^2g^2+c\left(6d^2g^2-6defg+e^2f^2\right)\right)}{e^5(m+3)}+\frac{\left(ae^2+cd^2\right)(ef-dg)^2(d+ex)^{m+1}}{e^5(m+1)}+\frac{2(ef-dg)(d+ex)^{m+2}\left(ae^2g-cd(ef-2dg)\right)}{e^5(m+2)}+\frac{2cg(ef-2dg)(d+ex)^{m+4}}{e^5(m+4)}+\frac{cg^2(d+ex)^{m+5}}{e^5(m+5)}$$

input
```
Int[(d + e*x)^m*(f + g*x)^2*(a + c*x^2),x]
```

output
```
((c*d^2 + a*e^2)*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) + (2*(e*f
- d*g)*(a*e^2*g - c*d*(e*f - 2*d*g))*(d + e*x)^(2 + m))/(e^5*(2 + m)) + ((
a*e^2*g^2 + c*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2))*(d + e*x)^(3 + m))/(e^5*(
3 + m)) + (2*c*g*(e*f - 2*d*g)*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (c*g^2*(
d + e*x)^(5 + m))/(e^5*(5 + m))
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 870 vs. $2(188) = 376$.

Time = 0.70 (sec) , antiderivative size = 871, normalized size of antiderivative = 4.63

method	result
norman	$\frac{c\,g^2x^5\mathrm{e}^{m\ln(ex+d)}}{5+m}+\frac{d\left(a\,e^4f^2m^4-2ad\,e^3fg\,m^3+14a\,e^4f^2m^3+2a\,d^2e^2g^2m^2-24ad\,e^3fg\,m^2+71a\,e^4f^2m^2+2c\,d^2e^2f^2m^2+\cdots\right.}{}$
gosper	$\frac{(ex+d)^{1+m}\left(c\,e^4g^2m^4x^4+2c\,e^4fg\,m^4x^3+10c\,e^4g^2m^3x^4+a\,e^4g^2m^4x^2-4cd\,e^3g^2m^3x^3+c\,e^4f^2m^4x^2+22c\,e^4fg\,m^3x^3+35c\,e^4\cdots\right.}{}$
orering	$\frac{(ex+d)\left(c\,e^4g^2m^4x^4+2c\,e^4fg\,m^4x^3+10c\,e^4g^2m^3x^4+a\,e^4g^2m^4x^2-4cd\,e^3g^2m^3x^3+c\,e^4f^2m^4x^2+22c\,e^4fg\,m^3x^3+35c\,e^4g^2m\cdots\right.}{}$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
c*g^2/(5+m)*x^5*exp(m*ln(e*x+d))+d*(a*e^4*f^2*m^4-2*a*d*e^3*f*g*m^3+14*a*e
^4*f^2*m^3+2*a*d^2*e^2*g^2*m^2-24*a*d*e^3*f*g*m^2+71*a*e^4*f^2*m^2+2*c*d^2
*e^2*f^2*m^2+18*a*d^2*e^2*g^2*m-94*a*d*e^3*f*g*m+154*a*e^4*f^2*m-12*c*d^3*
e*f*g*m+18*c*d^2*e^2*f^2*m+40*a*d^2*e^2*g^2-120*a*d*e^3*f*g+120*a*e^4*f^2+
24*c*d^4*g^2-60*c*d^3*e*f*g+40*c*d^2*e^2*f^2)/e^5/(m^5+15*m^4+85*m^3+225*m
^2+274*m+120)*exp(m*ln(e*x+d))+(a*e^2*g^2*m^2+2*c*d*e*f*g*m^2+c*e^2*f^2*m^
2+9*a*e^2*g^2*m-4*c*d^2*g^2*m+10*c*d*e*f*g*m+9*c*e^2*f^2*m+20*a*e^2*g^2+20
*c*e^2*f^2)/e^2/(m^3+12*m^2+47*m+60)*x^3*exp(m*ln(e*x+d))+(a*d*e^2*g^2*m^3
+2*a*e^3*f*g*m^3+c*d*e^2*f^2*m^3+9*a*d*e^2*g^2*m^2+24*a*e^3*f*g*m^2-6*c*d^
2*e*f*g*m^2+9*c*d*e^2*f^2*m^2+20*a*d*e^2*g^2*m+94*a*e^3*f*g*m+12*c*d^3*g^2
*m-30*c*d^2*e*f*g*m+20*c*d*e^2*f^2*m+120*a*e^3*f*g)/e^3/(m^4+14*m^3+71*m^2
+154*m+120)*x^2*exp(m*ln(e*x+d))+(d*g*m+2*e*f*m+10*e*f)*c/e*g/(m^2+9*m+20)
*x^4*exp(m*ln(e*x+d))-(-2*a*d*e^3*f*g*m^4-a*e^4*f^2*m^4+2*a*d^2*e^2*g^2*m^
3-24*a*d*e^3*f*g*m^3-14*a*e^4*f^2*m^3+2*c*d^2*e^2*f^2*m^3+18*a*d^2*e^2*g^2
*m^2-94*a*d*e^3*f*g*m^2-71*a*e^4*f^2*m^2-12*c*d^3*e*f*g*m^2+18*c*d^2*e^2*f
^2*m^2+40*a*d^2*e^2*g^2*m-120*a*d*e^3*f*g*m-154*a*e^4*f^2*m+24*c*d^4*g^2*m
-60*c*d^3*e*f*g*m+40*c*d^2*e^2*f^2*m-120*a*e^4*f^2)/e^4/(m^5+15*m^4+85*m^3
+225*m^2+274*m+120)*x*exp(m*ln(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. $2(188) = 376$.

Time = 0.09 (sec) , antiderivative size = 1019, normalized size of antiderivative = 5.42

$$\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+a),x, algorithm="fricas")
```

output
```
(a*d*e^4*f^2*m^4 + (c*e^5*g^2*m^4 + 10*c*e^5*g^2*m^3 + 35*c*e^5*g^2*m^2 +
50*c*e^5*g^2*m + 24*c*e^5*g^2)*x^5 + (60*c*e^5*f*g + (2*c*e^5*f*g + c*d*e^
4*g^2)*m^4 + 2*(11*c*e^5*f*g + 3*c*d*e^4*g^2)*m^3 + (82*c*e^5*f*g + 11*c*d
*e^4*g^2)*m^2 + 2*(61*c*e^5*f*g + 3*c*d*e^4*g^2)*m)*x^4 + 2*(7*a*d*e^4*f^2
 - a*d^2*e^3*f*g)*m^3 + (40*c*e^5*f^2 + 40*a*e^5*g^2 + (c*e^5*f^2 + 2*c*d*
e^4*f*g + a*e^5*g^2)*m^4 + 4*(3*c*e^5*f^2 + 4*c*d*e^4*f*g - (c*d^2*e^3 - 3
*a*e^5)*g^2)*m^3 + (49*c*e^5*f^2 + 34*c*d*e^4*f*g - (12*c*d^2*e^3 - 49*a*e
^5)*g^2)*m^2 + 2*(39*c*e^5*f^2 + 10*c*d*e^4*f*g - (4*c*d^2*e^3 - 39*a*e^5)
*g^2)*m)*x^3 + 40*(c*d^3*e^2 + 3*a*d*e^4)*f^2 - 60*(c*d^4*e + 2*a*d^2*e^3)
*f*g + 8*(3*c*d^5 + 5*a*d^3*e^2)*g^2 - (24*a*d^2*e^3*f*g - 2*a*d^3*e^2*g^2
 - (2*c*d^3*e^2 + 71*a*d*e^4)*f^2)*m^2 + (120*a*e^5*f*g + (c*d*e^4*f^2 + 2
*a*e^5*f*g + a*d*e^4*g^2)*m^4 + 2*(5*c*d*e^4*f^2 + 5*a*d*e^4*g^2 - (3*c*d^
2*e^3 - 13*a*e^5)*f*g)*m^3 + (29*c*d*e^4*f^2 - 2*(18*c*d^2*e^3 - 59*a*e^5)
*f*g + (12*c*d^3*e^2 + 29*a*d*e^4)*g^2)*m^2 + 2*(10*c*d*e^4*f^2 - (15*c*d^
2*e^3 - 107*a*e^5)*f*g + 2*(3*c*d^3*e^2 + 5*a*d*e^4)*g^2)*m)*x^2 + 2*(9*a*
d^3*e^2*g^2 + (9*c*d^3*e^2 + 77*a*d*e^4)*f^2 - (6*c*d^4*e + 47*a*d^2*e^3)*
f*g)*m + (120*a*e^5*f^2 + (a*e^5*f^2 + 2*a*d*e^4*f*g)*m^4 + 2*(12*a*d*e^4*
f*g - a*d^2*e^3*g^2 - (c*d^2*e^3 - 7*a*e^5)*f^2)*m^3 - (18*a*d^2*e^3*g^2 +
 (18*c*d^2*e^3 - 71*a*e^5)*f^2 - 2*(6*c*d^3*e^2 + 47*a*d*e^4)*f*g)*m^2 - 2
*((20*c*d^2*e^3 - 77*a*e^5)*f^2 - 30*(c*d^3*e^2 + 2*a*d*e^4)*f*g + 4*(3...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10508 vs. $2(175) = 350$.

Time = 2.59 (sec) , antiderivative size = 10508, normalized size of antiderivative = 55.89

$$\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+a),x)
```

output

```
Piecewise((d**m*(a*f**2*x + a*f*g*x**2 + a*g**2*x**3/3 + c*f**2*x**3/3 + c
*f*g*x**4/2 + c*g**2*x**5/5), Eq(e, 0)), (-a*d**2*e**2*g**2/(12*d**4*e**5
+ 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 2*
a*d*e**3*f*g/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**
8*x**3 + 12*e**9*x**4) - 4*a*d*e**3*g**2*x/(12*d**4*e**5 + 48*d**3*e**6*x
+ 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 3*a*e**4*f**2/(12*d
**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x
**4) - 8*a*e**4*f*g*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 +
 48*d*e**8*x**3 + 12*e**9*x**4) - 6*a*e**4*g**2*x**2/(12*d**4*e**5 + 48*d*
*3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 12*c*d**4
*g**2*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48
*d*e**8*x**3 + 12*e**9*x**4) + 25*c*d**4*g**2/(12*d**4*e**5 + 48*d**3*e**6
*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 6*c*d**3*e*f*g/(
12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e*
*9*x**4) + 48*c*d**3*e*g**2*x*log(d/e + x)/(12*d**4*e**5 + 48*d**3*e**6*x
+ 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 88*c*d**3*e*g**2*x/
(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e
**9*x**4) - c*d**2*e**2*f**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7
*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 24*c*d**2*e**2*f*g*x/(12*d**4*e**
5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4)...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(188) = 376$.

Time = 0.05 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.45

$$\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)\,dx$$
$$= \frac{2\left(e^2(m+1)x^2+demx-d^2\right)(ex+d)^m afg}{(m^2+3m+2)e^2} + \frac{(ex+d)^{m+1}af^2}{e(m+1)}$$
$$+ \frac{((m^2+3m+2)e^3x^3+(m^2+m)de^2x^2-2d^2emx+2d^3)(ex+d)^m cf^2}{(m^3+6m^2+11m+6)e^3}$$
$$+ \frac{((m^2+3m+2)e^3x^3+(m^2+m)de^2x^2-2d^2emx+2d^3)(ex+d)^m ag^2}{(m^3+6m^2+11m+6)e^3}$$
$$+ \frac{2\left((m^3+6m^2+11m+6)e^4x^4+(m^3+3m^2+2m)de^3x^3-3(m^2+m)d^2e^2x^2+6d^3emx-6d^4\right)(ex}{(m^4+10m^3+35m^2+50m+24)e^4}$$
$$+ \frac{((m^4+10m^3+35m^2+50m+24)e^5x^5+(m^4+6m^3+11m^2+6m)de^4x^4-4(m^3+3m^2+2m)d}{(m^5+15m^4+85m^3+225m^2+274m+1}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+a),x, algorithm="maxima")
```

output
```
2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*f*g/((m^2 + 3*m + 2)*e^2
) + (e*x + d)^(m + 1)*a*f^2/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2
+ m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*c*f^2/((m^3 + 6*m^2 + 11
*m + 6)*e^3) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*
x + 2*d^3)*(e*x + d)^m*a*g^2/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^3 + 6*
m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*
e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*c*f*g/((m^4 + 10*m^3 + 35*m^2 +
 50*m + 24)*e^4) + ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6
*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m
^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*c*g^2/((m^5 + 15*
m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1814 vs. $2(188) = 376$.

Time = 0.13 (sec) , antiderivative size = 1814, normalized size of antiderivative = 9.65

$$\int (d+ex)^m (f+gx)^2 \left(a+cx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+a),x, algorithm="giac")
```

output
```
((e*x + d)^m*c*e^5*g^2*m^4*x^5 + 2*(e*x + d)^m*c*e^5*f*g*m^4*x^4 + (e*x +
d)^m*c*d*e^4*g^2*m^4*x^4 + 10*(e*x + d)^m*c*e^5*g^2*m^3*x^5 + (e*x + d)^m*
c*e^5*f^2*m^4*x^3 + 2*(e*x + d)^m*c*d*e^4*f*g*m^4*x^3 + (e*x + d)^m*a*e^5*
g^2*m^4*x^3 + 22*(e*x + d)^m*c*e^5*f*g*m^3*x^4 + 6*(e*x + d)^m*c*d*e^4*g^2
*m^3*x^4 + 35*(e*x + d)^m*c*e^5*g^2*m^2*x^5 + (e*x + d)^m*c*d*e^4*f^2*m^4*
x^2 + 2*(e*x + d)^m*a*e^5*f*g*m^4*x^2 + (e*x + d)^m*a*d*e^4*g^2*m^4*x^2 +
12*(e*x + d)^m*c*e^5*f^2*m^3*x^3 + 16*(e*x + d)^m*c*d*e^4*f*g*m^3*x^3 - 4*
(e*x + d)^m*c*d^2*e^3*g^2*m^3*x^3 + 12*(e*x + d)^m*a*e^5*g^2*m^3*x^3 + 82*
(e*x + d)^m*c*e^5*f*g*m^2*x^4 + 11*(e*x + d)^m*c*d*e^4*g^2*m^2*x^4 + 50*(e
*x + d)^m*c*e^5*g^2*m*x^5 + (e*x + d)^m*a*e^5*f^2*m^4*x + 2*(e*x + d)^m*a*
d*e^4*f*g*m^4*x + 10*(e*x + d)^m*c*d*e^4*f^2*m^3*x^2 - 6*(e*x + d)^m*c*d^2
*e^3*f*g*m^3*x^2 + 26*(e*x + d)^m*a*e^5*f*g*m^3*x^2 + 10*(e*x + d)^m*a*d*e
^4*g^2*m^3*x^2 + 49*(e*x + d)^m*c*e^5*f^2*m^2*x^3 + 34*(e*x + d)^m*c*d*e^4
*f*g*m^2*x^3 - 12*(e*x + d)^m*c*d^2*e^3*g^2*m^2*x^3 + 49*(e*x + d)^m*a*e^5
*g^2*m^2*x^3 + 122*(e*x + d)^m*c*e^5*f*g*m*x^4 + 6*(e*x + d)^m*c*d*e^4*g^2
*m*x^4 + 24*(e*x + d)^m*c*e^5*g^2*x^5 + (e*x + d)^m*a*d*e^4*f^2*m^4 - 2*(e
*x + d)^m*c*d^2*e^3*f^2*m^3*x + 14*(e*x + d)^m*a*e^5*f^2*m^3*x + 24*(e*x +
 d)^m*a*d*e^4*f*g*m^3*x - 2*(e*x + d)^m*a*d^2*e^3*g^2*m^3*x + 29*(e*x + d)
^m*c*d*e^4*f^2*m^2*x^2 - 36*(e*x + d)^m*c*d^2*e^3*f*g*m^2*x^2 + 118*(e*x +
 d)^m*a*e^5*f*g*m^2*x^2 + 12*(e*x + d)^m*c*d^3*e^2*g^2*m^2*x^2 + 29*(e*...
```

Mupad [B] (verification not implemented)

Time = 6.89 (sec) , antiderivative size = 965, normalized size of antiderivative = 5.13

$$\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)\,dx=\text{Too large to display}$$

input
```
int((f + g*x)^2*(a + c*x^2)*(d + e*x)^m,x)
```

output
```
((d + e*x)^m*(24*c*d^5*g^2 + 40*a*d^3*e^2*g^2 + 40*c*d^3*e^2*f^2 + 120*a*d
*e^4*f^2 - 120*a*d^2*e^3*f*g + 154*a*d*e^4*f^2*m + 71*a*d*e^4*f^2*m^2 + 14
*a*d*e^4*f^2*m^3 + a*d*e^4*f^2*m^4 + 18*a*d^3*e^2*g^2*m + 18*c*d^3*e^2*f^2
*m - 60*c*d^4*e*f*g + 2*a*d^3*e^2*g^2*m^2 + 2*c*d^3*e^2*f^2*m^2 - 12*c*d^4
*e*f*g*m - 94*a*d^2*e^3*f*g*m - 24*a*d^2*e^3*f*g*m^2 - 2*a*d^2*e^3*f*g*m^3
))/(e^5*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (x*(d + e*x)^m*
(120*a*e^5*f^2 + 71*a*e^5*f^2*m^2 + 14*a*e^5*f^2*m^3 + a*e^5*f^2*m^4 + 154
*a*e^5*f^2*m - 24*c*d^4*e*g^2*m - 40*a*d^2*e^3*g^2*m - 40*c*d^2*e^3*f^2*m
- 18*a*d^2*e^3*g^2*m^2 - 2*a*d^2*e^3*g^2*m^3 - 18*c*d^2*e^3*f^2*m^2 - 2*c*
d^2*e^3*f^2*m^3 + 120*a*d*e^4*f*g*m + 94*a*d*e^4*f*g*m^2 + 24*a*d*e^4*f*g*
m^3 + 2*a*d*e^4*f*g*m^4 + 60*c*d^3*e^2*f*g*m + 12*c*d^3*e^2*f*g*m^2))/(e^5
*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (c*g^2*x^5*(d + e*x)^m
*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))/(274*m + 225*m^2 + 85*m^3 + 15*m^4 +
 m^5 + 120) + (x^2*(m + 1)*(d + e*x)^m*(120*a*e^3*f*g + 12*c*d^3*g^2*m + 2
0*a*d*e^2*g^2*m + 20*c*d*e^2*f^2*m + 24*a*e^3*f*g*m^2 + 2*a*e^3*f*g*m^3 +
9*a*d*e^2*g^2*m^2 + a*d*e^2*g^2*m^3 + 9*c*d*e^2*f^2*m^2 + c*d*e^2*f^2*m^3
+ 94*a*e^3*f*g*m - 30*c*d^2*e*f*g*m - 6*c*d^2*e*f*g*m^2))/(e^3*(274*m + 22
5*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(
20*a*e^2*g^2 + 20*c*e^2*f^2 + a*e^2*g^2*m^2 + c*e^2*f^2*m^2 + 9*a*e^2*g^2*
m - 4*c*d^2*g^2*m + 9*c*e^2*f^2*m + 2*c*d*e*f*g*m^2 + 10*c*d*e*f*g*m))/...
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 1209, normalized size of antiderivative = 6.43

$$\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+a),x)
```

output

```
((d + e*x)**m*(2*a*d**3*e**2*g**2*m**2 + 18*a*d**3*e**2*g**2*m + 40*a*d**3
*e**2*g**2 - 2*a*d**2*e**3*f*g*m**3 - 24*a*d**2*e**3*f*g*m**2 - 94*a*d**2*
e**3*f*g*m - 120*a*d**2*e**3*f*g - 2*a*d**2*e**3*g**2*m**3*x - 18*a*d**2*e
**3*g**2*m**2*x - 40*a*d**2*e**3*g**2*m*x + a*d*e**4*f**2*m**4 + 14*a*d*e*
*4*f**2*m**3 + 71*a*d*e**4*f**2*m**2 + 154*a*d*e**4*f**2*m + 120*a*d*e**4*
f**2 + 2*a*d*e**4*f*g*m**4*x + 24*a*d*e**4*f*g*m**3*x + 94*a*d*e**4*f*g*m*
*2*x + 120*a*d*e**4*f*g*m*x + a*d*e**4*g**2*m**4*x**2 + 10*a*d*e**4*g**2*m
**3*x**2 + 29*a*d*e**4*g**2*m**2*x**2 + 20*a*d*e**4*g**2*m*x**2 + a*e**5*f
**2*m**4*x + 14*a*e**5*f**2*m**3*x + 71*a*e**5*f**2*m**2*x + 154*a*e**5*f*
*2*m*x + 120*a*e**5*f**2*x + 2*a*e**5*f*g*m**4*x**2 + 26*a*e**5*f*g*m**3*x
**2 + 118*a*e**5*f*g*m**2*x**2 + 214*a*e**5*f*g*m*x**2 + 120*a*e**5*f*g*x*
*2 + a*e**5*g**2*m**4*x**3 + 12*a*e**5*g**2*m**3*x**3 + 49*a*e**5*g**2*m**
2*x**3 + 78*a*e**5*g**2*m*x**3 + 40*a*e**5*g**2*x**3 + 24*c*d**5*g**2 - 12
*c*d**4*e*f*g*m - 60*c*d**4*e*f*g - 24*c*d**4*e*g**2*m*x + 2*c*d**3*e**2*f
**2*m**2 + 18*c*d**3*e**2*f**2*m + 40*c*d**3*e**2*f**2 + 12*c*d**3*e**2*f*
g*m**2*x + 60*c*d**3*e**2*f*g*m*x + 12*c*d**3*e**2*g**2*m**2*x**2 + 12*c*d
**3*e**2*g**2*m*x**2 - 2*c*d**2*e**3*f**2*m**3*x - 18*c*d**2*e**3*f**2*m**
2*x - 40*c*d**2*e**3*f**2*m*x - 6*c*d**2*e**3*f*g*m**3*x**2 - 36*c*d**2*e*
*3*f*g*m**2*x**2 - 30*c*d**2*e**3*f*g*m*x**2 - 4*c*d**2*e**3*g**2*m**3*x**
3 - 12*c*d**2*e**3*g**2*m**2*x**3 - 8*c*d**2*e**3*g**2*m*x**3 + c*d*e**...
```

3.151 $\int (d+ex)^m(f+gx)\left(a+cx^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 123

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)\,dx = \frac{\left(cd^2+ae^2\right)(ef-dg)(d+ex)^{1+m}}{e^4(1+m)} + \frac{\left(ae^2g-cd(2ef-3dg)\right)(d+ex)^{2+m}}{e^4(2+m)} + \frac{c(ef-3dg)(d+ex)^{3+m}}{e^4(3+m)} + \frac{cg(d+ex)^{4+m}}{e^4(4+m)}$$

output
```
(a*e^2+c*d^2)*(-d*g+e*f)*(e*x+d)^(1+m)/e^4/(1+m)+(a*e^2*g-c*d*(-3*d*g+2*e*
f))*(e*x+d)^(2+m)/e^4/(2+m)+c*(-3*d*g+e*f)*(e*x+d)^(3+m)/e^4/(3+m)+c*g*(e*
x+d)^(4+m)/e^4/(4+m)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)\,dx = \frac{(d+ex)^{1+m}\left((ef-dg)\left(\frac{cd^2+ae^2}{1+m}-\frac{2cd(d+ex)}{2+m}+\frac{c(d+ex)^2}{3+m}\right)+g(d+ex)\left(\frac{cd^2+ae^2}{2+m}-\frac{2cd(d+ex)}{3+m}+\frac{c(d+ex)^2}{4+m}\right)\right)}{e^4}$$

input
```
Integrate[(d + e*x)^m*(f + g*x)*(a + c*x^2),x]
```

output
```
((d + e*x)^(1 + m)*((e*f - d*g)*((c*d^2 + a*e^2)/(1 + m) - (2*c*d*(d + e*x
))/(2 + m) + (c*(d + e*x)^2)/(3 + m)) + g*(d + e*x)*((c*d^2 + a*e^2)/(2 +
m) - (2*c*d*(d + e*x))/(3 + m) + (c*(d + e*x)^2)/(4 + m))))/e^4
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(f+gx)(d+ex)^m\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{\left(ae^2+cd^2\right)(ef-dg)(d+ex)^m}{e^3}+\frac{(d+ex)^{m+1}\left(ae^2g-cd(2ef-3dg)\right)}{e^3}+\frac{c(ef-3dg)(d+ex)^{m+2}}{e^3}+\frac{cg(d$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(ae^2+cd^2\right)(ef-dg)(d+ex)^{m+1}}{e^4(m+1)}+\frac{(d+ex)^{m+2}\left(ae^2g-cd(2ef-3dg)\right)}{e^4(m+2)}+\frac{c(ef-3dg)(d+ex)^{m+3}}{e^4(m+3)}+\frac{cg(d+ex)^{m+4}}{e^4(m+4)}$$

input
```
Int[(d + e*x)^m*(f + g*x)*(a + c*x^2),x]
```

output
```
((c*d^2 + a*e^2)*(e*f - d*g)*(d + e*x)^(1 + m))/(e^4*(1 + m)) + ((a*e^2*g
- c*d*(2*e*f - 3*d*g))*(d + e*x)^(2 + m))/(e^4*(2 + m)) + (c*(e*f - 3*d*g)
*(d + e*x)^(3 + m))/(e^4*(3 + m)) + (c*g*(d + e*x)^(4 + m))/(e^4*(4 + m))
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(123) = 246.

Time = 0.60 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.78

method	result
gosper	$-\frac{(ex+d)^{1+m}(-c e^3 g m^3 x^3-c e^3 f m^3 x^2-6c e^3 g m^2 x^3-a e^3 g m^3 x+3cd e^2 g m^2 x^2-7c e^3 f m^2 x^2-11c e^3 gm x^3-a e^3 f m^3-8}{}$
orering	$-\frac{(ex+d)^m(-c e^3 g m^3 x^3-c e^3 f m^3 x^2-6c e^3 g m^2 x^3-a e^3 g m^3 x+3cd e^2 g m^2 x^2-7c e^3 f m^2 x^2-11c e^3 gm x^3-a e^3 f m^3-8a}{}$
norman	$\frac{cg x^4 e^{m\ln(ex+d)}}{4+m}+\frac{(a e^2 g m^2+cdef m^2+7a e^2 gm-3c d^2 gm+4cdefm+12a e^2 g) x^2 e^{m\ln(ex+d)}}{e^2(m^3+9m^2+26m+24)}+\frac{(ad e^2 g m^3+a e^3 f m^3+}{}$
risch	$-\frac{(-c e^4 g m^3 x^4-cd e^3 g m^3 x^3-c e^4 f m^3 x^3-6c e^4 g m^2 x^4-a e^4 g m^3 x^2-cd e^3 f m^3 x^2-3cd e^3 g m^2 x^3-7c e^4 f m^2 x^3-11c e^4 g}{}$
parallelrisch	$\frac{26x(ex+d)^m a e^4 fm-(ex+d)^m a d^2 e^2 g m^2+9(ex+d)^m ad e^3 f m^2-7(ex+d)^m a d^2 e^2 gm+26(ex+d)^m ad e^3 fm+4x^2(ex+d)^m}{}$

input
```
int((e*x+d)^m*(g*x+f)*(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/e^4*(e*x+d)^(1+m)/(m^4+10*m^3+35*m^2+50*m+24)*(-c*e^3*g*m^3*x^3-c*e^3*f
*m^3*x^2-6*c*e^3*g*m^2*x^3-a*e^3*g*m^3*x+3*c*d*e^2*g*m^2*x^2-7*c*e^3*f*m^2
*x^2-11*c*e^3*g*m*x^3-a*e^3*f*m^3-8*a*e^3*g*m^2*x+2*c*d*e^2*f*m^2*x+9*c*d*
e^2*g*m*x^2-14*c*e^3*f*m*x^2-6*c*e^3*g*x^3+a*d*e^2*g*m^2-9*a*e^3*f*m^2-19*
a*e^3*g*m*x-6*c*d^2*e*g*m*x+10*c*d*e^2*f*m*x+6*c*d*e^2*g*x^2-8*c*e^3*f*x^2
+7*a*d*e^2*g*m-26*a*e^3*f*m-12*a*e^3*g*x-2*c*d^2*e*f*m-6*c*d^2*e*g*x+8*c*d
*e^2*f*x+12*a*d*e^2*g-24*a*e^3*f+6*c*d^3*g-8*c*d^2*e*f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(123) = 246$.

Time = 0.09 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.60

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)\,dx$$
$$= \frac{(ade^3fm^3 + (ce^4gm^3 + 6\,ce^4gm^2 + 11\,ce^4gm + 6\,ce^4g)x^4 + (8\,ce^4f + (ce^4f + cde^3g)m^3 + (7\,ce^4f + 3\,c\cdots}{}$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+a),x, algorithm="fricas")
```

output
```
(a*d*e^3*f*m^3 + (c*e^4*g*m^3 + 6*c*e^4*g*m^2 + 11*c*e^4*g*m + 6*c*e^4*g)*
x^4 + (8*c*e^4*f + (c*e^4*f + c*d*e^3*g)*m^3 + (7*c*e^4*f + 3*c*d*e^3*g)*m
^2 + 2*(7*c*e^4*f + c*d*e^3*g)*m)*x^3 + (9*a*d*e^3*f - a*d^2*e^2*g)*m^2 +
(12*a*e^4*g + (c*d*e^3*f + a*e^4*g)*m^3 + (5*c*d*e^3*f - (3*c*d^2*e^2 - 8*
a*e^4)*g)*m^2 + (4*c*d*e^3*f - (3*c*d^2*e^2 - 19*a*e^4)*g)*m)*x^2 + 8*(c*d
^3*e + 3*a*d*e^3)*f - 6*(c*d^4 + 2*a*d^2*e^2)*g - (7*a*d^2*e^2*g - 2*(c*d^
3*e + 13*a*d*e^3)*f)*m + (24*a*e^4*f + (a*e^4*f + a*d*e^3*g)*m^3 + (7*a*d*
e^3*g - (2*c*d^2*e^2 - 9*a*e^4)*f)*m^2 - 2*((4*c*d^2*e^2 - 13*a*e^4)*f - 3
*(c*d^3*e + 2*a*d*e^3)*g)*m)*x)*(e*x + d)^m/(e^4*m^4 + 10*e^4*m^3 + 35*e^4
*m^2 + 50*e^4*m + 24*e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3958 vs. $2(110) = 220$.

Time = 1.30 (sec) , antiderivative size = 3958, normalized size of antiderivative = 32.18

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(g*x+f)*(c*x**2+a),x)
```

output
```
Piecewise((d**m*(a*f*x + a*g*x**2/2 + c*f*x**3/3 + c*g*x**4/4), Eq(e, 0)),
 (-a*d*e**2*g/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3
) - 2*a*e**3*f/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**
3) - 3*a*e**3*g*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*
x**3) + 6*c*d**3*g*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*
x**2 + 6*e**7*x**3) + 11*c*d**3*g/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**
6*x**2 + 6*e**7*x**3) - 2*c*d**2*e*f/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*
e**6*x**2 + 6*e**7*x**3) + 18*c*d**2*e*g*x*log(d/e + x)/(6*d**3*e**4 + 18*
d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 27*c*d**2*e*g*x/(6*d**3*e**4
 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) - 6*c*d*e**2*f*x/(6*d**3
*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 18*c*d*e**2*g*x**
2*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**
3) + 18*c*d*e**2*g*x**2/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6
*e**7*x**3) - 6*c*e**3*f*x**2/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x*
*2 + 6*e**7*x**3) + 6*c*e**3*g*x**3*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e*
*5*x + 18*d*e**6*x**2 + 6*e**7*x**3), Eq(m, -4)), (-a*d*e**2*g/(2*d**2*e**
4 + 4*d*e**5*x + 2*e**6*x**2) - a*e**3*f/(2*d**2*e**4 + 4*d*e**5*x + 2*e**
6*x**2) - 2*a*e**3*g*x/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 6*c*d**3
*g*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 9*c*d**3*g/(2*d
**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c*d**2*e*f*log(d/e + x)/(2*d**...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.93

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)\,dx$$
$$= \frac{(e^2(m+1)x^2+demx-d^2)(ex+d)^m ag}{(m^2+3m+2)e^2} + \frac{(ex+d)^{m+1}af}{e(m+1)} + \frac{((m^2+3m+2)e^3x^3+(m^2+m)de^2x^2-2d^2emx+2d^3)(ex+d)^m cf}{(m^3+6m^2+11m+6)e^3} + \frac{((m^3+6m^2+11m+6)e^4x^4+(m^3+3m^2+2m)de^3x^3-3(m^2+m)d^2e^2x^2+6d^3emx-6d^4)(ex-}{(m^4+10m^3+35m^2+50m+24)e^4}$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+a),x, algorithm="maxima")
```

output
```
(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a*g/((m^2 + 3*m + 2)*e^2) +
(e*x + d)^(m + 1)*a*f/(e*(m + 1)) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d
*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*c*f/((m^3 + 6*m^2 + 11*m + 6)*
e^3) + ((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 -
 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*c*g/((m^4 + 10
*m^3 + 35*m^2 + 50*m + 24)*e^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. $2(123) = 246$.

Time = 0.12 (sec) , antiderivative size = 768, normalized size of antiderivative = 6.24

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+a),x, algorithm="giac")
```

output
```
((e*x + d)^m*c*e^4*g*m^3*x^4 + (e*x + d)^m*c*e^4*f*m^3*x^3 + (e*x + d)^m*c
*d*e^3*g*m^3*x^3 + 6*(e*x + d)^m*c*e^4*g*m^2*x^4 + (e*x + d)^m*c*d*e^3*f*m
^3*x^2 + (e*x + d)^m*a*e^4*g*m^3*x^2 + 7*(e*x + d)^m*c*e^4*f*m^2*x^3 + 3*(
e*x + d)^m*c*d*e^3*g*m^2*x^3 + 11*(e*x + d)^m*c*e^4*g*m*x^4 + (e*x + d)^m*
a*e^4*f*m^3*x + (e*x + d)^m*a*d*e^3*g*m^3*x + 5*(e*x + d)^m*c*d*e^3*f*m^2*
x^2 - 3*(e*x + d)^m*c*d^2*e^2*g*m^2*x^2 + 8*(e*x + d)^m*a*e^4*g*m^2*x^2 +
14*(e*x + d)^m*c*e^4*f*m*x^3 + 2*(e*x + d)^m*c*d*e^3*g*m*x^3 + 6*(e*x + d)
^m*c*e^4*g*x^4 + (e*x + d)^m*a*d*e^3*f*m^3 - 2*(e*x + d)^m*c*d^2*e^2*f*m^2
*x + 9*(e*x + d)^m*a*e^4*f*m^2*x + 7*(e*x + d)^m*a*d*e^3*g*m^2*x + 4*(e*x
+ d)^m*c*d*e^3*f*m*x^2 - 3*(e*x + d)^m*c*d^2*e^2*g*m*x^2 + 19*(e*x + d)^m*
a*e^4*g*m*x^2 + 8*(e*x + d)^m*c*e^4*f*x^3 + 9*(e*x + d)^m*a*d*e^3*f*m^2 -
(e*x + d)^m*a*d^2*e^2*g*m^2 - 8*(e*x + d)^m*c*d^2*e^2*f*m*x + 26*(e*x + d)
^m*a*e^4*f*m*x + 6*(e*x + d)^m*c*d^3*e*g*m*x + 12*(e*x + d)^m*a*d*e^3*g*m*
x + 12*(e*x + d)^m*a*e^4*g*x^2 + 2*(e*x + d)^m*c*d^3*e*f*m + 26*(e*x + d)^
m*a*d*e^3*f*m - 7*(e*x + d)^m*a*d^2*e^2*g*m + 24*(e*x + d)^m*a*e^4*f*x + 8
*(e*x + d)^m*c*d^3*e*f + 24*(e*x + d)^m*a*d*e^3*f - 6*(e*x + d)^m*c*d^4*g
- 12*(e*x + d)^m*a*d^2*e^2*g)/(e^4*m^4 + 10*e^4*m^3 + 35*e^4*m^2 + 50*e^4*
m + 24*e^4)
```

Mupad [B] (verification not implemented)

Time = 6.59 (sec) , antiderivative size = 446, normalized size of antiderivative = 3.63

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)\,dx$$
$$= \frac{(d+e\,x)^m\,(-6\,c\,g\,d^4+2\,c\,f\,d^3\,e\,m+8\,c\,f\,d^3\,e-a\,g\,d^2\,e^2\,m^2-7\,a\,g\,d^2\,e^2\,m-12\,a\,g\,d^2\,e^2+a\,f\,d\,e^3\,m^3+}{e^4\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}$$
$$+\frac{x\,(d+e\,x)^m\,(6\,c\,g\,d^3\,e\,m-2\,c\,f\,d^2\,e^2\,m^2-8\,c\,f\,d^2\,e^2\,m+a\,g\,d\,e^3\,m^3+7\,a\,g\,d\,e^3\,m^2+12\,a\,g\,d\,e^3\,m+}{e^4\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}$$
$$+\frac{x^2\,(m+1)\,\,(d+e\,x)^m\,\left(-3\,c\,g\,d^2\,m+c\,f\,d\,e\,m^2+4\,c\,f\,d\,e\,m+a\,g\,e^2\,m^2+7\,a\,g\,e^2\,m+12\,a\,g\,e^2\right)}{e^2\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}$$
$$+\frac{c\,g\,x^4\,(d+e\,x)^m\,\left(m^3+6\,m^2+11\,m+6\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}$$
$$+\frac{c\,x^3\,(d+e\,x)^m\,(4\,e\,f+d\,g\,m+e\,f\,m)\,\left(m^2+3\,m+2\right)}{e\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}$$

input
```
int((f + g*x)*(a + c*x^2)*(d + e*x)^m,x)
```

output
```
((d + e*x)^m*(24*a*d*e^3*f - 6*c*d^4*g + 8*c*d^3*e*f - 12*a*d^2*e^2*g + 9*
a*d*e^3*f*m^2 + a*d*e^3*f*m^3 - 7*a*d^2*e^2*g*m - a*d^2*e^2*g*m^2 + 26*a*d
*e^3*f*m + 2*c*d^3*e*f*m))/(e^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x*
(d + e*x)^m*(24*a*e^4*f + 26*a*e^4*f*m + 9*a*e^4*f*m^2 + a*e^4*f*m^3 + 7*a
*d*e^3*g*m^2 + a*d*e^3*g*m^3 - 8*c*d^2*e^2*f*m - 2*c*d^2*e^2*f*m^2 + 12*a*
d*e^3*g*m + 6*c*d^3*e*g*m))/(e^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x
^2*(m + 1)*(d + e*x)^m*(12*a*e^2*g + 7*a*e^2*g*m - 3*c*d^2*g*m + a*e^2*g*m
^2 + 4*c*d*e*f*m + c*d*e*f*m^2))/(e^2*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
 + (c*g*x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3
+ m^4 + 24) + (c*x^3*(d + e*x)^m*(4*e*f + d*g*m + e*f*m)*(3*m + m^2 + 2))/
(e*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 481, normalized size of antiderivative = 3.91

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)\,dx$$
$$=\frac{(ex+d)^m\left(c\,e^4g\,m^3x^4+cd\,e^3g\,m^3x^3+c\,e^4f\,m^3x^3+6c\,e^4g\,m^2x^4+a\,e^4g\,m^3x^2+cd\,e^3f\,m^3x^2+3cd\,e^3g\,m\right.}{}$$

input
```
int((e*x+d)^m*(g*x+f)*(c*x^2+a),x)
```

output
```
((d + e*x)**m*( - a*d**2*e**2*g*m**2 - 7*a*d**2*e**2*g*m - 12*a*d**2*e**2*
g + a*d*e**3*f*m**3 + 9*a*d*e**3*f*m**2 + 26*a*d*e**3*f*m + 24*a*d*e**3*f
+ a*d*e**3*g*m**3*x + 7*a*d*e**3*g*m**2*x + 12*a*d*e**3*g*m*x + a*e**4*f*m
**3*x + 9*a*e**4*f*m**2*x + 26*a*e**4*f*m*x + 24*a*e**4*f*x + a*e**4*g*m**
3*x**2 + 8*a*e**4*g*m**2*x**2 + 19*a*e**4*g*m*x**2 + 12*a*e**4*g*x**2 - 6*
c*d**4*g + 2*c*d**3*e*f*m + 8*c*d**3*e*f + 6*c*d**3*e*g*m*x - 2*c*d**2*e**
2*f*m**2*x - 8*c*d**2*e**2*f*m*x - 3*c*d**2*e**2*g*m**2*x**2 - 3*c*d**2*e*
*2*g*m*x**2 + c*d*e**3*f*m**3*x**2 + 5*c*d*e**3*f*m**2*x**2 + 4*c*d*e**3*f
*m*x**2 + c*d*e**3*g*m**3*x**3 + 3*c*d*e**3*g*m**2*x**3 + 2*c*d*e**3*g*m*x
**3 + c*e**4*f*m**3*x**3 + 7*c*e**4*f*m**2*x**3 + 14*c*e**4*f*m*x**3 + 8*c
*e**4*f*x**3 + c*e**4*g*m**3*x**4 + 6*c*e**4*g*m**2*x**4 + 11*c*e**4*g*m*x
**4 + 6*c*e**4*g*x**4))/(e**4*(m**4 + 10*m**3 + 35*m**2 + 50*m + 24))
```

3.152 $\int \frac{(d+ex)^m (a+cx^2)}{f+gx} dx$

Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{(d+ex)^m (a+cx^2)}{f+gx} dx = -\frac{c(ef+dg)(d+ex)^{1+m}}{e^2g^2(1+m)} + \frac{c(d+ex)^{2+m}}{e^2g(2+m)} + \frac{(cf^2+ag^2)(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{g(d+ex)}{ef-dg}\right)}{g^2(ef-dg)(1+m)}$$

output

```
-c*(d*g+e*f)*(e*x+d)^(1+m)/e^2/g^2/(1+m)+c*(e*x+d)^(2+m)/e^2/g/(2+m)+(a*g^
2+c*f^2)*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f))/g^2
/(-d*g+e*f)/(1+m)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^m (a+cx^2)}{f+gx} dx = \frac{(d+ex)^{1+m}\left(\frac{c(-dg-ef(2+m)+eg(1+m)x)}{e^2(2+m)} + \frac{(cf^2+ag^2)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{g(d+ex)}{-ef+dg}\right)}{ef-dg}\right)}{g^2(1+m)}$$

input
```
Integrate[((d + e*x)^m*(a + c*x^2))/(f + g*x),x]
```

output
```
((d + e*x)^(1 + m)*((c*(-(d*g) - e*f*(2 + m) + e*g*(1 + m)*x))/(e^2*(2 + m
)) + ((c*f^2 + a*g^2)*Hypergeometric2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(
e*f) + d*g)])/(e*f - d*g)))/(g^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(d+ex)^m}{f+gx} \, dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{(ag^2+cf^2)(d+ex)^m}{g^2(f+gx)}-\frac{c(dg+ef)(d+ex)^m}{eg^2}+\frac{c(d+ex)^{m+1}}{eg}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ag^2+cf^2)(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{g(d+ex)}{ef-dg}\right)}{g^2(m+1)(ef-dg)}-\frac{c(dg+ef)(d+ex)^{m+1}}{e^2g^2(m+1)}+\frac{c(d+ex)^{m+2}}{e^2g(m+2)}$$

input
```
Int[((d + e*x)^m*(a + c*x^2))/(f + g*x),x]
```

output
```
-((c*(e*f + d*g)*(d + e*x)^(1 + m))/(e^2*g^2*(1 + m))) + (c*(d + e*x)^(2 +
 m))/(e^2*g*(2 + m)) + ((c*f^2 + a*g^2)*(d + e*x)^(1 + m)*Hypergeometric2F
1[1, 1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^2*(e*f - d*g)*(1 + m)
)
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex+d)^m (c\,x^2+a)}{gx+f}dx$$

input
```
int((e*x+d)^m*(c*x^2+a)/(g*x+f),x)
```

output
```
int((e*x+d)^m*(c*x^2+a)/(g*x+f),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m (a+cx^2)}{f+gx}\,dx = \int \frac{(cx^2+a)(ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)/(g*x+f),x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)*(e*x + d)^m/(g*x + f), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m (a+cx^2)}{f+gx} \, dx = \int \frac{(a+cx^2)(d+ex)^m}{f+gx} \, dx$$

input
```
integrate((e*x+d)**m*(c*x**2+a)/(g*x+f),x)
```

output
```
Integral((a + c*x**2)*(d + e*x)**m/(f + g*x), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m (a+cx^2)}{f+gx} \, dx = \int \frac{(cx^2+a)(ex+d)^m}{gx+f} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)/(g*x+f),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)*(e*x + d)^m/(g*x + f), x)
```

Giac [F]

$$\int \frac{(d+ex)^m (a+cx^2)}{f+gx} \, dx = \int \frac{(cx^2+a)(ex+d)^m}{gx+f} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)/(g*x+f),x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)*(e*x + d)^m/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+cx^2)}{f+gx}\,dx = \int \frac{(c\,x^2+a)\,(d+e\,x)^m}{f+g\,x}\,dx$$

input
```
int(((a + c*x^2)*(d + e*x)^m)/(f + g*x),x)
```

output
```
int(((a + c*x^2)*(d + e*x)^m)/(f + g*x), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m (a+cx^2)}{f+gx}\,dx$$
$$= \frac{(ex+d)^m\,ade\,g^2m^2+3(ex+d)^m\,ade\,g^2m+2(ex+d)^m\,ade\,g^2-(ex+d)^m\,c\,d^2fgm+(ex+d)^m\,cde\,f^2}{}$$

input
```
int((e*x+d)^m*(c*x^2+a)/(g*x+f),x)
```

output
```
((d + e*x)**m*a*d*e*g**2*m**2 + 3*(d + e*x)**m*a*d*e*g**2*m + 2*(d + e*x)*
*m*a*d*e*g**2 - (d + e*x)**m*c*d**2*f*g*m + (d + e*x)**m*c*d*e*f**2*m + 2*
(d + e*x)**m*c*d*e*f**2 + (d + e*x)**m*c*d*e*f*g*m**2*x - (d + e*x)**m*c*e
**2*f**2*m**2*x - 2*(d + e*x)**m*c*e**2*f**2*m*x + (d + e*x)**m*c*e**2*f*g
*m**2*x**2 + (d + e*x)**m*c*e**2*f*g*m*x**2 - int(((d + e*x)**m*x)/(d*f +
d*g*x + e*f*x + e*g*x**2),x)*a*d*e**2*g**3*m**3 - 3*int(((d + e*x)**m*x)/(
d*f + d*g*x + e*f*x + e*g*x**2),x)*a*d*e**2*g**3*m**2 - 2*int(((d + e*x)**
m*x)/(d*f + d*g*x + e*f*x + e*g*x**2),x)*a*d*e**2*g**3*m + int(((d + e*x)*
*m*x)/(d*f + d*g*x + e*f*x + e*g*x**2),x)*a*e**3*f*g**2*m**3 + 3*int(((d +
 e*x)**m*x)/(d*f + d*g*x + e*f*x + e*g*x**2),x)*a*e**3*f*g**2*m**2 + 2*int
(((d + e*x)**m*x)/(d*f + d*g*x + e*f*x + e*g*x**2),x)*a*e**3*f*g**2*m - in
t(((d + e*x)**m*x)/(d*f + d*g*x + e*f*x + e*g*x**2),x)*c*d*e**2*f**2*g*m**
3 - 3*int(((d + e*x)**m*x)/(d*f + d*g*x + e*f*x + e*g*x**2),x)*c*d*e**2*f*
*2*g*m**2 - 2*int(((d + e*x)**m*x)/(d*f + d*g*x + e*f*x + e*g*x**2),x)*c*d
*e**2*f**2*g*m + int(((d + e*x)**m*x)/(d*f + d*g*x + e*f*x + e*g*x**2),x)*
c*e**3*f**3*m**3 + 3*int(((d + e*x)**m*x)/(d*f + d*g*x + e*f*x + e*g*x**2)
,x)*c*e**3*f**3*m**2 + 2*int(((d + e*x)**m*x)/(d*f + d*g*x + e*f*x + e*g*x
**2),x)*c*e**3*f**3*m)/(e**2*f*g**2*m*(m**2 + 3*m + 2))
```

3.153 $\int \frac{(d+ex)^m(a+cx^2)}{(f+gx)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 133

$$\int \frac{(d+ex)^m(a+cx^2)}{(f+gx)^2} dx = \frac{c(d+ex)^{1+m}}{eg^2(1+m)} - \frac{2cf(d+ex)^{1+m}}{eg^2m(f+gx)} + \frac{(aeg^2m - cf(2dg - ef(2+m)))(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(2, 1+m, 2+m, -\frac{g(d+ex)}{ef-dg}\right)}{g^2(ef-dg)^2m(1+m)}$$

output
```
c*(e*x+d)^(1+m)/e/g^2/(1+m)-2*c*f*(e*x+d)^(1+m)/e/g^2/m/(g*x+f)+(a*e*g^2*m
-c*f*(2*d*g-e*f*(2+m)))*(e*x+d)^(1+m)*hypergeom([2, 1+m],[2+m],-g*(e*x+d)/
(-d*g+e*f))/g^2/(-d*g+e*f)^2/m/(1+m)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^m(a+cx^2)}{(f+gx)^2} dx = \frac{(d+ex)^{1+m}\left(c(ef-dg)^2 + 2cef(-ef+dg)\,\text{Hypergeometric2F1}\left(1, 1+m, 2+m, \frac{g(d+ex)}{-ef+dg}\right) + e^2(cf^2 + \cdots\right.}{eg^2(ef-dg)^2(1+m)}$$

input
```
Integrate[((d + e*x)^m*(a + c*x^2))/(f + g*x)^2,x]
```

output
```
((d + e*x)^(1 + m)*(c*(e*f - d*g)^2 + 2*c*e*f*(-(e*f) + d*g)*Hypergeometri
c2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e^2*(c*f^2 + a*g^2)*
Hypergeometric2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)]))/(e*g^2*
(e*f - d*g)^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {650, 25, 90, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(d+ex)^m}{(f+gx)^2}dx$$

$$\Big\downarrow \textbf{650}$$

$$\frac{\int -\frac{(d+ex)^m\left(aem-\frac{cf(dg-ef(m+1))}{g^2}+c\left(d-\frac{ef}{g}\right)x\right)}{f+gx}dx}{ef-dg}+\frac{\left(a+\frac{cf^2}{g^2}\right)(d+ex)^{m+1}}{(f+gx)(ef-dg)}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\left(a+\frac{cf^2}{g^2}\right)(d+ex)^{m+1}}{(f+gx)(ef-dg)}-\frac{\int \frac{(d+ex)^m\left(aem-\frac{cf(dg-ef(m+1))}{g^2}+c\left(d-\frac{ef}{g}\right)x\right)}{f+gx}dx}{ef-dg}$$

$$\Big\downarrow \textbf{90}$$

$$\frac{\left(a+\frac{cf^2}{g^2}\right)(d+ex)^{m+1}}{(f+gx)(ef-dg)}-\frac{\left(aem-\frac{cf(2dg-ef(m+2))}{g^2}\right)\int \frac{(d+ex)^m}{f+gx}dx-\frac{c(ef-dg)(d+ex)^{m+1}}{eg^2(m+1)}}{ef-dg}$$

$$\Big\downarrow \textbf{78}$$

$$\frac{\frac{\left(a+\frac{cf^2}{g^2}\right)(d+ex)^{m+1}}{(f+gx)(ef-dg)}-\frac{\frac{(d+ex)^{m+1}\left(aem-\frac{cf(2dg-ef(m+2))}{g^2}\right)\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{g(d+ex)}{ef-dg}\right)}{(m+1)(ef-dg)}-\frac{c(ef-dg)(d+ex)^{m+1}}{eg^2(m+1)}}{ef-dg}}{}$$

input
```
Int[((d + e*x)^m*(a + c*x^2))/(f + g*x)^2,x]
```

output
```
((a + (c*f^2)/g^2)*(d + e*x)^(1 + m))/((e*f - d*g)*(f + g*x)) - (-((c*(e*f
 - d*g)*(d + e*x)^(1 + m))/(e*g^2*(1 + m))) + ((a*e*m - (c*f*(2*d*g - e*f*
(2 + m)))/g^2)*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(
d + e*x))/(e*f - d*g))])/((e*f - d*g)*(1 + m)))/(e*f - d*g)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 650
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^2)^p, d + e*x
, x], R = PolynomialRemainder[(a + c*x^2)^p, d + e*x, x]}, Simp[R*(d + e*x)
^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))), x] + Simp[1/((m + 1)*(e
*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*ExpandToSum[(m + 1)*(e*f - d
*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g, n}, x] &&
 IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{(ex+d)^m (c\,x^2+a)}{(gx+f)^2}dx$$

input
```
int((e*x+d)^m*(c*x^2+a)/(g*x+f)^2,x)
```

output
```
int((e*x+d)^m*(c*x^2+a)/(g*x+f)^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m (a+cx^2)}{(f+gx)^2}\,dx = \int \frac{(cx^2+a)(ex+d)^m}{(gx+f)^2}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)*(e*x + d)^m/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m (a+cx^2)}{(f+gx)^2} dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*(c*x**2+a)/(g*x+f)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(d+ex)^m (a+cx^2)}{(f+gx)^2} dx = \int \frac{(cx^2+a)(ex+d)^m}{(gx+f)^2} dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)/(g*x+f)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)*(e*x + d)^m/(g*x + f)^2, x)
```

Giac [F]

$$\int \frac{(d+ex)^m (a+cx^2)}{(f+gx)^2} dx = \int \frac{(cx^2+a)(ex+d)^m}{(gx+f)^2} dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)*(e*x + d)^m/(g*x + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m(a+cx^2)}{(f+gx)^2}\,dx=\int \frac{\left(c\,x^2+a\right)\,\left(d+e\,x\right)^m}{\left(f+g\,x\right)^2}\,dx$$

input
```
int(((a + c*x^2)*(d + e*x)^m)/(f + g*x)^2,x)
```

output
```
int(((a + c*x^2)*(d + e*x)^m)/(f + g*x)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex)^m(a+cx^2)}{(f+gx)^2}\,dx=\text{too large to display}$$

input
```
int((e*x+d)^m*(c*x^2+a)/(g*x+f)^2,x)
```

output

```
( - (d + e*x)**m*a*d*e*g**2*m**2 - (d + e*x)**m*a*d*e*g**2*m + (d + e*x)**
m*c*d**2*f*g*m + (d + e*x)**m*c*d**2*g**2*m*x - (d + e*x)**m*c*d*e*f**2*m
- 2*(d + e*x)**m*c*d*e*f**2 - (d + e*x)**m*c*d*e*f*g*m**2*x - (d + e*x)**m
*c*d*e*f*g*m*x - 2*(d + e*x)**m*c*d*e*f*g*x + (d + e*x)**m*c*d*e*g**2*m*x*
*2 + (d + e*x)**m*c*e**2*f**2*m**2*x + 2*(d + e*x)**m*c*e**2*f**2*m*x - (d
 + e*x)**m*c*e**2*f*g*m**2*x**2 + int(((d + e*x)**m*x)/(d**2*f**2*g + 2*d*
*2*f*g**2*x + d**2*g**3*x**2 - d*e*f**3*m - 2*d*e*f**2*g*m*x + d*e*f**2*g*
x - d*e*f*g**2*m*x**2 + 2*d*e*f*g**2*x**2 + d*e*g**3*x**3 - e**2*f**3*m*x
- 2*e**2*f**2*g*m*x**2 - e**2*f*g**2*m*x**3),x)*a*d**2*e**2*f*g**4*m**3 +
int(((d + e*x)**m*x)/(d**2*f**2*g + 2*d**2*f*g**2*x + d**2*g**3*x**2 - d*e
*f**3*m - 2*d*e*f**2*g*m*x + d*e*f**2*g*x - d*e*f*g**2*m*x**2 + 2*d*e*f*g*
*2*x**2 + d*e*g**3*x**3 - e**2*f**3*m*x - 2*e**2*f**2*g*m*x**2 - e**2*f*g*
*2*m*x**3),x)*a*d**2*e**2*f*g**4*m**2 + int(((d + e*x)**m*x)/(d**2*f**2*g
+ 2*d**2*f*g**2*x + d**2*g**3*x**2 - d*e*f**3*m - 2*d*e*f**2*g*m*x + d*e*f
**2*g*x - d*e*f*g**2*m*x**2 + 2*d*e*f*g**2*x**2 + d*e*g**3*x**3 - e**2*f**
3*m*x - 2*e**2*f**2*g*m*x**2 - e**2*f*g**2*m*x**3),x)*a*d**2*e**2*g**5*m**
3*x + int(((d + e*x)**m*x)/(d**2*f**2*g + 2*d**2*f*g**2*x + d**2*g**3*x**2
 - d*e*f**3*m - 2*d*e*f**2*g*m*x + d*e*f**2*g*x - d*e*f*g**2*m*x**2 + 2*d*
e*f*g**2*x**2 + d*e*g**3*x**3 - e**2*f**3*m*x - 2*e**2*f**2*g*m*x**2 - e**
2*f*g**2*m*x**3),x)*a*d**2*e**2*g**5*m**2*x - int(((d + e*x)**m*x)/(d**...
```

3.154 $\int \frac{(d+ex)^m(a+cx^2)}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 183

$$\int \frac{(d+ex)^m(a+cx^2)}{(f+gx)^3}\,dx = \frac{(cf^2+ag^2)(d+ex)^{1+m}}{2g^2(ef-dg)(f+gx)^2} + \frac{c(d+ex)^{1+m}}{eg^2m(f+gx)} + \frac{(ae^2g^2(1-m)m - c(2d^2g^2-4defg(1+m)+e^2f^2(2+3m+m^2)))(d+ex)^{1+m}\,\text{Hypergeometric2F1}}{2g^2(ef-dg)^3m(1+m)}$$

output
```
1/2*(a*g^2+c*f^2)*(e*x+d)^(1+m)/g^2/(-d*g+e*f)/(g*x+f)^2+c*(e*x+d)^(1+m)/e
/g^2/m/(g*x+f)+1/2*(a*e^2*g^2*(1-m)*m-c*(2*d^2*g^2-4*d*e*f*g*(1+m)+e^2*f^2
*(m^2+3*m+2)))*(e*x+d)^(1+m)*hypergeom([2, 1+m],[2+m],-g*(e*x+d)/(-d*g+e*f
))/g^2/(-d*g+e*f)^3/m/(1+m)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^m(a+cx^2)}{(f+gx)^3}\,dx = -\frac{(d+ex)^{1+m}\left(c(ef-dg)^2\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{g(d+ex)}{-ef+dg}\right)+e\left(2cf(-ef+dg)\,\text{Hyperg}\right.\right.}{g^2(-ef+}$$

input
```
Integrate[((d + e*x)^m*(a + c*x^2))/(f + g*x)^3,x]
```

output
```
-((((d + e*x)^(1 + m)*(c*(e*f - d*g)^2*Hypergeometric2F1[1, 1 + m, 2 + m, (
g*(d + e*x))/(-(e*f) + d*g)] + e*(2*c*f*(-(e*f) + d*g)*Hypergeometric2F1[2
, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e*(c*f^2 + a*g^2)*Hypergeo
metric2F1[3, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])))/(g^2*(-(e*f) +
 d*g)^3*(1 + m)))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {650, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(d+ex)^m}{(f+gx)^3}dx$$

$$\Big\downarrow \text{650}$$

$$\frac{\int \frac{(d+ex)^m\left(a(e-em)+\frac{cf(2dg-ef(m+1))}{g^2}-2c\left(d-\frac{ef}{g}\right)x\right)}{(f+gx)^2}dx}{2(ef-dg)}+\frac{\left(a+\frac{cf^2}{g^2}\right)(d+ex)^{m+1}}{2(f+gx)^2(ef-dg)}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{(d+ex)^{m+1}(aeg^2(1-m)+cf(4dg-ef(m+3)))}{g^2(f+gx)(ef-dg)}-\frac{\left(ae^2g^2(1-m)m-c\left(2d^2g^2-4defg(m+1)+e^2f^2\left(m^2+3m+2\right)\right)\right)\int \frac{(d+ex)^m}{f+gx}dx}{g^2(ef-dg)}}{2(ef-dg)}+\frac{\left(a+\frac{cf^2}{g^2}\right)(d+ex)^{m+1}}{2(f+gx)^2(ef-dg)}$$

$$\Big\downarrow \text{78}$$

$$\frac{\frac{(d+ex)^{m+1}(aeg^2(1-m)+cf(4dg-ef(m+3)))}{g^2(f+gx)(ef-dg)}-\frac{(d+ex)^{m+1}(ae^2g^2(1-m)m-c(2d^2g^2-4defg(m+1)+e^2f^2(m^2+3m+2)))\,\text{Hypergeometric2}}{g^2(m+1)(ef-dg)^2}}{2(ef-dg)}$$

$$\frac{\left(a+\frac{cf^2}{g^2}\right)(d+ex)^{m+1}}{2(f+gx)^2(ef-dg)}$$

input
```
Int[((d + e*x)^m*(a + c*x^2))/(f + g*x)^3,x]
```

output
```
((a + (c*f^2)/g^2)*(d + e*x)^(1 + m))/(2*(e*f - d*g)*(f + g*x)^2) + (((a*e
*g^2*(1 - m) + c*f*(4*d*g - e*f*(3 + m)))*(d + e*x)^(1 + m))/(g^2*(e*f - d
*g)*(f + g*x)) - ((a*e^2*g^2*(1 - m)*m - c*(2*d^2*g^2 - 4*d*e*f*g*(1 + m)
+ e^2*f^2*(2 + 3*m + m^2)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m,
2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^2*(e*f - d*g)^2*(1 + m)))/(2*(e*f
 - d*g))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 650
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^2)^p, d + e*x
, x], R = PolynomialRemainder[(a + c*x^2)^p, d + e*x, x]}, Simp[R*(d + e*x)
^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))), x] + Simp[1/((m + 1)*(e
*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*ExpandToSum[(m + 1)*(e*f - d
*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g, n}, x] &&
 IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{(ex+d)^m (c\,x^2+a)}{(gx+f)^3}dx$$

input
```
int((e*x+d)^m*(c*x^2+a)/(g*x+f)^3,x)
```

output
```
int((e*x+d)^m*(c*x^2+a)/(g*x+f)^3,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m (a+cx^2)}{(f+gx)^3}\,dx = \int \frac{(cx^2+a)(ex+d)^m}{(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)/(g*x+f)^3,x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)*(e*x + d)^m/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)
, x)
```

Sympy [F]

$$\int \frac{(d+ex)^m(a+cx^2)}{(f+gx)^3}\,dx = \int \frac{(a+cx^2)(d+ex)^m}{(f+gx)^3}\,dx$$

input
```
integrate((e*x+d)**m*(c*x**2+a)/(g*x+f)**3,x)
```

output
```
Integral((a + c*x**2)*(d + e*x)**m/(f + g*x)**3, x)
```

Maxima [F]

$$\int \frac{(d+ex)^m(a+cx^2)}{(f+gx)^3}\,dx = \int \frac{(cx^2+a)(ex+d)^m}{(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)/(g*x+f)^3,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)*(e*x + d)^m/(g*x + f)^3, x)
```

Giac [F]

$$\int \frac{(d+ex)^m(a+cx^2)}{(f+gx)^3}\,dx = \int \frac{(cx^2+a)(ex+d)^m}{(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)/(g*x+f)^3,x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)*(e*x + d)^m/(g*x + f)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+cx^2)}{(f+gx)^3}\,dx = \int \frac{(c\,x^2+a)\,(d+e\,x)^m}{(f+g\,x)^3}\,dx$$

input
```
int(((a + c*x^2)*(d + e*x)^m)/(f + g*x)^3,x)
```

output
```
int(((a + c*x^2)*(d + e*x)^m)/(f + g*x)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex)^m (a+cx^2)}{(f+gx)^3}\,dx = \text{too large to display}$$

input
```
int((e*x+d)^m*(c*x^2+a)/(g*x+f)^3,x)
```

output

```
( - (d + e*x)**m*a*d*e*g**2*m**2 + (d + e*x)**m*a*d*e*g**2*m + (d + e*x)**
m*c*d**2*f*g*m + 2*(d + e*x)**m*c*d**2*g**2*m*x - (d + e*x)**m*c*d*e*f**2*
m - 2*(d + e*x)**m*c*d*e*f**2 - (d + e*x)**m*c*d*e*f*g*m**2*x - 2*(d + e*x
)**m*c*d*e*f*g*m*x - 4*(d + e*x)**m*c*d*e*f*g*x + 2*(d + e*x)**m*c*d*e*g**
2*m*x**2 - 2*(d + e*x)**m*c*d*e*g**2*x**2 + (d + e*x)**m*c*e**2*f**2*m**2*
x + 2*(d + e*x)**m*c*e**2*f**2*m*x - (d + e*x)**m*c*e**2*f*g*m**2*x**2 + (
d + e*x)**m*c*e**2*f*g*m*x**2 + 2*int(((d + e*x)**m*x)/(2*d**2*f**3*g*m -
2*d**2*f**3*g + 6*d**2*f**2*g**2*m*x - 6*d**2*f**2*g**2*x + 6*d**2*f*g**3*
m*x**2 - 6*d**2*f*g**3*x**2 + 2*d**2*g**4*m*x**3 - 2*d**2*g**4*x**3 - d*e*
f**4*m**2 + d*e*f**4*m - 3*d*e*f**3*g*m**2*x + 5*d*e*f**3*g*m*x - 2*d*e*f*
*3*g*x - 3*d*e*f**2*g**2*m**2*x**2 + 9*d*e*f**2*g**2*m*x**2 - 6*d*e*f**2*g
**2*x**2 - d*e*f*g**3*m**2*x**3 + 7*d*e*f*g**3*m*x**3 - 6*d*e*f*g**3*x**3
+ 2*d*e*g**4*m*x**4 - 2*d*e*g**4*x**4 - e**2*f**4*m**2*x + e**2*f**4*m*x -
 3*e**2*f**3*g*m**2*x**2 + 3*e**2*f**3*g*m*x**2 - 3*e**2*f**2*g**2*m**2*x*
*3 + 3*e**2*f**2*g**2*m*x**3 - e**2*f*g**3*m**2*x**4 + e**2*f*g**3*m*x**4)
,x)*a*d**2*e**2*f**2*g**4*m**4 - 4*int(((d + e*x)**m*x)/(2*d**2*f**3*g*m -
 2*d**2*f**3*g + 6*d**2*f**2*g**2*m*x - 6*d**2*f**2*g**2*x + 6*d**2*f*g**3
*m*x**2 - 6*d**2*f*g**3*x**2 + 2*d**2*g**4*m*x**3 - 2*d**2*g**4*x**3 - d*e
*f**4*m**2 + d*e*f**4*m - 3*d*e*f**3*g*m**2*x + 5*d*e*f**3*g*m*x - 2*d*e*f
**3*g*x - 3*d*e*f**2*g**2*m**2*x**2 + 9*d*e*f**2*g**2*m*x**2 - 6*d*e*f*...
```

3.155 $\int \frac{(d+ex)^{-2+m}(a+cx^2)}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 194

$$\int \frac{(d+ex)^{-2+m}(a+cx^2)}{(f+gx)^3}\,dx = \frac{(cf^2+ag^2)(d+ex)^{-1+m}}{2g^2(ef-dg)(f+gx)^2} - \frac{c(d+ex)^{-1+m}}{eg^2(2-m)(f+gx)} - \frac{(c(2d^2g^2+4defg(1-m)-e^2f^2(1-m)m)+ae^2g^2(6-5m+m^2))(d+ex)^{-1+m}\,\text{Hypergeometric2F}}{2g^2(ef-dg)^3(1-m)(2-m)}$$

output
```
1/2*(a*g^2+c*f^2)*(e*x+d)^(-1+m)/g^2/(-d*g+e*f)/(g*x+f)^2-c*(e*x+d)^(-1+m)
/e/g^2/(2-m)/(g*x+f)-1/2*(c*(2*d^2*g^2+4*d*e*f*g*(1-m)-e^2*f^2*(1-m)*m)+a*
e^2*g^2*(m^2-5*m+6))*(e*x+d)^(-1+m)*hypergeom([2, -1+m],[m],-g*(e*x+d)/(-d
*g+e*f))/g^2/(-d*g+e*f)^3/(1-m)/(2-m)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^{-2+m}(a+cx^2)}{(f+gx)^3}\,dx$$
$$= \frac{(d+ex)^{-1+m}\left(-c(ef-dg)^2\,\text{Hypergeometric2F1}\left(1,-1+m,m,\frac{g(d+ex)}{-ef+dg}\right)+e\Big(2cf(ef-dg)\,\text{Hypergeor}}{g^2(-ef+dg)^3(}$$

input
```
Integrate[((d + e*x)^(-2 + m)*(a + c*x^2))/(f + g*x)^3,x]
```

output
```
((d + e*x)^(-1 + m)*(-(c*(e*f - d*g)^2*Hypergeometric2F1[1, -1 + m, m, (g*
(d + e*x))/(-(e*f) + d*g)]) + e*(2*c*f*(e*f - d*g)*Hypergeometric2F1[2, -1
 + m, m, (g*(d + e*x))/(-(e*f) + d*g)] - e*(c*f^2 + a*g^2)*Hypergeometric2
F1[3, -1 + m, m, (g*(d + e*x))/(-(e*f) + d*g)])))/(g^2*(-(e*f) + d*g)^3*(-
1 + m))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {650, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)(d+ex)^{m-2}}{(f+gx)^3}dx$$

$$\Big\downarrow \text{650}$$

$$\frac{\int \frac{(d+ex)^{m-2}\left(ae(3-m)+\frac{cf(2dg+e(f-fm))}{g^2}-2c\left(d-\frac{ef}{g}\right)x\right)}{(f+gx)^2}dx}{2(ef-dg)}+\frac{\left(a+\frac{cf^2}{g^2}\right)(d+ex)^{m-1}}{2(f+gx)^2(ef-dg)}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{\left(ae^2g^2\left(m^2-5m+6\right)+c\left(2d^2g^2+4defg(1-m)-e^2f^2(1-m)m\right)\right)\int \frac{(d+ex)^{m-2}}{f+gx}dx}{g^2(ef-dg)}+\frac{(d+ex)^{m-1}\left(aeg^2(3-m)+cf(4dg-ef(m+1))\right)}{g^2(f+gx)(ef-dg)}}{2(ef-dg)}+\frac{\left(a+\frac{cf^2}{g^2}\right)(d+ex)^{m-1}}{2(f+gx)^2(ef-dg)}$$

$$\Big\downarrow \text{78}$$

$$\frac{\frac{(d+ex)^{m-1}(aeg^2(3-m)+cf(4dg-ef(m+1)))}{g^2(f+gx)(ef-dg)} - \frac{(d+ex)^{m-1}(ae^2g^2(m^2-5m+6)+c(2d^2g^2+4defg(1-m)-e^2f^2(1-m)m))\text{ Hypergeometric2F}}{g^2(1-m)(ef-dg)^2}}{2(ef-dg)}$$
$$\frac{\left(a+\frac{cf^2}{g^2}\right)(d+ex)^{m-1}}{2(f+gx)^2(ef-dg)}$$

input
```
Int[((d + e*x)^(-2 + m)*(a + c*x^2))/(f + g*x)^3,x]
```

output
```
((a + (c*f^2)/g^2)*(d + e*x)^(-1 + m))/(2*(e*f - d*g)*(f + g*x)^2) + (((a*
e*g^2*(3 - m) + c*f*(4*d*g - e*f*(1 + m)))*(d + e*x)^(-1 + m))/(g^2*(e*f -
 d*g)*(f + g*x)) - ((c*(2*d^2*g^2 + 4*d*e*f*g*(1 - m) - e^2*f^2*(1 - m)*m)
 + a*e^2*g^2*(6 - 5*m + m^2))*(d + e*x)^(-1 + m)*Hypergeometric2F1[1, -1 +
 m, m, -((g*(d + e*x))/(e*f - d*g))])/(g^2*(e*f - d*g)^2*(1 - m)))/(2*(e*f
 - d*g))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 650
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^2)^p, d + e*x
, x], R = PolynomialRemainder[(a + c*x^2)^p, d + e*x, x]}, Simp[R*(d + e*x)
^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))), x] + Simp[1/((m + 1)*(e
*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*ExpandToSum[(m + 1)*(e*f - d
*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g, n}, x] &&
 IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{(ex+d)^{-2+m} (c\,x^2+a)}{(gx+f)^3} dx$$

input
```
int((e*x+d)^(-2+m)*(c*x^2+a)/(g*x+f)^3,x)
```

output
```
int((e*x+d)^(-2+m)*(c*x^2+a)/(g*x+f)^3,x)
```

Fricas [F]

$$\int \frac{(d+ex)^{-2+m} (a+cx^2)}{(f+gx)^3}\, dx = \int \frac{(cx^2+a)(ex+d)^{m-2}}{(gx+f)^3}\, dx$$

input
```
integrate((e*x+d)^(-2+m)*(c*x^2+a)/(g*x+f)^3,x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)*(e*x + d)^(m - 2)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x
+ f^3), x)
```

Sympy [F]

$$\int \frac{(d+ex)^{-2+m}(a+cx^2)}{(f+gx)^3}\,dx = \int \frac{(a+cx^2)(d+ex)^{m-2}}{(f+gx)^3}\,dx$$

input
```
integrate((e*x+d)**(-2+m)*(c*x**2+a)/(g*x+f)**3,x)
```

output
```
Integral((a + c*x**2)*(d + e*x)**(m - 2)/(f + g*x)**3, x)
```

Maxima [F]

$$\int \frac{(d+ex)^{-2+m}(a+cx^2)}{(f+gx)^3}\,dx = \int \frac{(cx^2+a)(ex+d)^{m-2}}{(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^(-2+m)*(c*x^2+a)/(g*x+f)^3,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)*(e*x + d)^(m - 2)/(g*x + f)^3, x)
```

Giac [F]

$$\int \frac{(d+ex)^{-2+m}(a+cx^2)}{(f+gx)^3}\,dx = \int \frac{(cx^2+a)(ex+d)^{m-2}}{(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^(-2+m)*(c*x^2+a)/(g*x+f)^3,x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)*(e*x + d)^(m - 2)/(g*x + f)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{-2+m}(a+cx^2)}{(f+gx)^3}\,dx=\int \frac{(c\,x^2+a)\,(d+e\,x)^{m-2}}{(f+g\,x)^3}\,dx$$

input
```
int(((a + c*x^2)*(d + e*x)^(m - 2))/(f + g*x)^3,x)
```

output
```
int(((a + c*x^2)*(d + e*x)^(m - 2))/(f + g*x)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex)^{-2+m}(a+cx^2)}{(f+gx)^3}\,dx=\text{too large to display}$$

input
```
int((e*x+d)^(-2+m)*(c*x^2+a)/(g*x+f)^3,x)
```

output

```
( - (d + e*x)**m*a*e*g*m + 2*(d + e*x)**m*a*e*g + (d + e*x)**m*c*d*f + 2*(
d + e*x)**m*c*d*g*x - (d + e*x)**m*c*e*f*m*x + (d + e*x)**m*c*e*f*x + 2*in
t(((d + e*x)**m*x)/(2*d**3*f**3*g*m - 4*d**3*f**3*g + 6*d**3*f**2*g**2*m*x
 - 12*d**3*f**2*g**2*x + 6*d**3*f*g**3*m*x**2 - 12*d**3*f*g**3*x**2 + 2*d*
*3*g**4*m*x**3 - 4*d**3*g**4*x**3 - d**2*e*f**4*m**2 + 3*d**2*e*f**4*m - 2
*d**2*e*f**4 - 3*d**2*e*f**3*g*m**2*x + 13*d**2*e*f**3*g*m*x - 14*d**2*e*f
**3*g*x - 3*d**2*e*f**2*g**2*m**2*x**2 + 21*d**2*e*f**2*g**2*m*x**2 - 30*d
**2*e*f**2*g**2*x**2 - d**2*e*f*g**3*m**2*x**3 + 15*d**2*e*f*g**3*m*x**3 -
 26*d**2*e*f*g**3*x**3 + 4*d**2*e*g**4*m*x**4 - 8*d**2*e*g**4*x**4 - 2*d*e
**2*f**4*m**2*x + 6*d*e**2*f**4*m*x - 4*d*e**2*f**4*x - 6*d*e**2*f**3*g*m*
*2*x**2 + 20*d*e**2*f**3*g*m*x**2 - 16*d*e**2*f**3*g*x**2 - 6*d*e**2*f**2*
g**2*m**2*x**3 + 24*d*e**2*f**2*g**2*m*x**3 - 24*d*e**2*f**2*g**2*x**3 - 2
*d*e**2*f*g**3*m**2*x**4 + 12*d*e**2*f*g**3*m*x**4 - 16*d*e**2*f*g**3*x**4
 + 2*d*e**2*g**4*m*x**5 - 4*d*e**2*g**4*x**5 - e**3*f**4*m**2*x**2 + 3*e**
3*f**4*m*x**2 - 2*e**3*f**4*x**2 - 3*e**3*f**3*g*m**2*x**3 + 9*e**3*f**3*g
*m*x**3 - 6*e**3*f**3*g*x**3 - 3*e**3*f**2*g**2*m**2*x**4 + 9*e**3*f**2*g*
*2*m*x**4 - 6*e**3*f**2*g**2*x**4 - e**3*f*g**3*m**2*x**5 + 3*e**3*f*g**3*
m*x**5 - 2*e**3*f*g**3*x**5),x)*a*d**2*e**2*f**2*g**3*m**3 - 14*int(((d +
e*x)**m*x)/(2*d**3*f**3*g*m - 4*d**3*f**3*g + 6*d**3*f**2*g**2*m*x - 12*d*
*3*f**2*g**2*x + 6*d**3*f*g**3*m*x**2 - 12*d**3*f*g**3*x**2 + 2*d**3*g*...
```

3.156 $\int (d+ex)^m (f+gx)^2 \left(a+cx^2\right)^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 357

$$\int (d+ex)^m (f+gx)^2 \left(a+cx^2\right)^2 \, dx = \frac{\left(cd^2+ae^2\right)^2 (ef-dg)^2 (d+ex)^{1+m}}{e^7(1+m)} + \frac{2\left(cd^2+ae^2\right)(ef-dg)\left(ae^2g-cd(2ef-3dg)\right)(d+ex)^{2+m}}{e^7(2+m)} + \frac{\left(a^2e^4g^2+2ace^2(e^2f^2-6defg+6d^2g^2)+c^2d^2(6e^2f^2-20defg+15d^2g^2)\right)(d+ex)^{3+m}}{e^7(3+m)} + \frac{4c\left(ae^2g(ef-2dg)-cd(e^2f^2-5defg+5d^2g^2)\right)(d+ex)^{4+m}}{e^7(4+m)} + \frac{c\left(2ae^2g^2+c(e^2f^2-10defg+15d^2g^2)\right)(d+ex)^{5+m}}{e^7(5+m)} + \frac{2c^2g(ef-3dg)(d+ex)^{6+m}}{e^7(6+m)} + \frac{c^2g^2(d+ex)^{7+m}}{e^7(7+m)}$$

output
```
(a*e^2+c*d^2)^2*(-d*g+e*f)^2*(e*x+d)^(1+m)/e^7/(1+m)+2*(a*e^2+c*d^2)*(-d*g
+e*f)*(a*e^2*g-c*d*(-3*d*g+2*e*f))*(e*x+d)^(2+m)/e^7/(2+m)+(a^2*e^4*g^2+2*
a*c*e^2*(6*d^2*g^2-6*d*e*f*g+e^2*f^2)+c^2*d^2*(15*d^2*g^2-20*d*e*f*g+6*e^2
*f^2))*(e*x+d)^(3+m)/e^7/(3+m)+4*c*(a*e^2*g*(-2*d*g+e*f)-c*d*(5*d^2*g^2-5*
d*e*f*g+e^2*f^2))*(e*x+d)^(4+m)/e^7/(4+m)+c*(2*a*e^2*g^2+c*(15*d^2*g^2-10*
d*e*f*g+e^2*f^2))*(e*x+d)^(5+m)/e^7/(5+m)+2*c^2*g*(-3*d*g+e*f)*(e*x+d)^(6+
m)/e^7/(6+m)+c^2*g^2*(e*x+d)^(7+m)/e^7/(7+m)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.91

$$\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)^2\,dx$$
$$=\frac{(d+ex)^{1+m}\left(\frac{(cd^2+ae^2)^2(ef-dg)^2}{1+m}-\frac{2(cd^2+ae^2)(-ef+dg)(ae^2g+cd(-2ef+3dg))(d+ex)}{2+m}+\frac{(a^2e^4g^2+2ace^2(e^2f^2-6defg+6d^2g^2)\cdots}{3\cdots}\right.}{}$$

input
```
Integrate[(d + e*x)^m*(f + g*x)^2*(a + c*x^2)^2,x]
```

output
```
((d + e*x)^(1 + m)*(((c*d^2 + a*e^2)^2*(e*f - d*g)^2)/(1 + m) - (2*(c*d^2
+ a*e^2)*(-(e*f) + d*g)*(a*e^2*g + c*d*(-2*e*f + 3*d*g))*(d + e*x))/(2 + m
) + ((a^2*e^4*g^2 + 2*a*c*e^2*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2) + c^2*d^2*
(6*e^2*f^2 - 20*d*e*f*g + 15*d^2*g^2))*(d + e*x)^2)/(3 + m) + (4*c*(a*e^2*
g*(e*f - 2*d*g) - c*d*(e^2*f^2 - 5*d*e*f*g + 5*d^2*g^2))*(d + e*x)^3)/(4 +
 m) + (c*(2*a*e^2*g^2 + c*(e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2))*(d + e*x)^4
)/(5 + m) + (2*c^2*g*(e*f - 3*d*g)*(d + e*x)^5)/(6 + m) + (c^2*g^2*(d + e*
x)^6)/(7 + m)))/e^7
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2(f+gx)^2(d+ex)^m\,dx$$

$$\Big\downarrow \text{652}$$

$$\int \left(\frac{(d+ex)^{m+2}\left(a^2e^4g^2+2ace^2\left(6d^2g^2-6defg+e^2f^2\right)+c^2d^2\left(15d^2g^2-20defg+6e^2f^2\right)\right)}{e^6}+\frac{4c(d+ex)^{m+3}\cdots}{\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^{m+3}\left(a^2e^4g^2+2ace^2\left(6d^2g^2-6defg+e^2f^2\right)+c^2d^2\left(15d^2g^2-20defg+6e^2f^2\right)\right)}{e^7(m+3)}+\frac{4c(d+ex)^{m+4}\left(ae^2g(ef-2dg)-cd\left(5d^2g^2-5defg+e^2f^2\right)\right)}{e^7(m+4)}+\frac{c(d+ex)^{m+5}\left(2ae^2g^2+c\left(15d^2g^2-10defg+e^2f^2\right)\right)}{e^7(m+5)}+\frac{\left(ae^2+cd^2\right)^2(ef-dg)^2(d+ex)^{m+1}}{e^7(m+1)}+\frac{2\left(ae^2+cd^2\right)(ef-dg)(d+ex)^{m+2}\left(ae^2g-cd(2ef-3dg)\right)}{e^7(m+2)}+\frac{2c^2g(ef-3dg)(d+ex)^{m+6}}{e^7(m+6)}+\frac{c^2g^2(d+ex)^{m+7}}{e^7(m+7)}$$

input
```
Int[(d + e*x)^m*(f + g*x)^2*(a + c*x^2)^2,x]
```

output
```
((c*d^2 + a*e^2)^2*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^7*(1 + m)) + (2*(c*
d^2 + a*e^2)*(e*f - d*g)*(a*e^2*g - c*d*(2*e*f - 3*d*g))*(d + e*x)^(2 + m)
)/(e^7*(2 + m)) + ((a^2*e^4*g^2 + 2*a*c*e^2*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g
^2) + c^2*d^2*(6*e^2*f^2 - 20*d*e*f*g + 15*d^2*g^2))*(d + e*x)^(3 + m))/(e
^7*(3 + m)) + (4*c*(a*e^2*g*(e*f - 2*d*g) - c*d*(e^2*f^2 - 5*d*e*f*g + 5*d
^2*g^2))*(d + e*x)^(4 + m))/(e^7*(4 + m)) + (c*(2*a*e^2*g^2 + c*(e^2*f^2 -
 10*d*e*f*g + 15*d^2*g^2))*(d + e*x)^(5 + m))/(e^7*(5 + m)) + (2*c^2*g*(e*
f - 3*d*g)*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (c^2*g^2*(d + e*x)^(7 + m))/
(e^7*(7 + m))
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2305 vs. $2(357) = 714$.

Time = 0.89 (sec) , antiderivative size = 2306, normalized size of antiderivative = 6.46

method	result	size
norman	Expression too large to display	2306
gosper	Expression too large to display	2923
orering	Expression too large to display	2926
risch	Expression too large to display	3639
parallelrisch	Expression too large to display	5341

input
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d*(a^2*e^6*f^2*m^6-2*a^2*d*e^5*f*g*m^5+27*a^2*e^6*f^2*m^5+2*a^2*d^2*e^4*g^
2*m^4-50*a^2*d*e^5*f*g*m^4+295*a^2*e^6*f^2*m^4+4*a*c*d^2*e^4*f^2*m^4+44*a^
2*d^2*e^4*g^2*m^3-490*a^2*d*e^5*f*g*m^3+1665*a^2*e^6*f^2*m^3-24*a*c*d^3*e^
3*f*g*m^3+88*a*c*d^2*e^4*f^2*m^3+358*a^2*d^2*e^4*g^2*m^2-2350*a^2*d*e^5*f*
g*m^2+5104*a^2*e^6*f^2*m^2+48*a*c*d^4*e^2*g^2*m^2-432*a*c*d^3*e^3*f*g*m^2+
716*a*c*d^2*e^4*f^2*m^2+24*c^2*d^4*e^2*f^2*m^2+1276*a^2*d^2*e^4*g^2*m-5508
*a^2*d*e^5*f*g*m+8028*a^2*e^6*f^2*m+624*a*c*d^4*e^2*g^2*m-2568*a*c*d^3*e^3
*f*g*m+2552*a*c*d^2*e^4*f^2*m-240*c^2*d^5*e*f*g*m+312*c^2*d^4*e^2*f^2*m+16
80*a^2*d^2*e^4*g^2-5040*a^2*d*e^5*f*g+5040*a^2*e^6*f^2+2016*a*c*d^4*e^2*g^
2-5040*a*c*d^3*e^3*f*g+3360*a*c*d^2*e^4*f^2+720*c^2*d^6*g^2-1680*c^2*d^5*e
*f*g+1008*c^2*d^4*e^2*f^2)/e^7/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+13132
*m^2+13068*m+5040)*exp(m*ln(e*x+d))+g^2*c^2/(7+m)*x^7*exp(m*ln(e*x+d))+(a^
2*e^4*g^2*m^4+4*a*c*d*e^3*f*g*m^4+2*a*c*e^4*f^2*m^4+22*a^2*e^4*g^2*m^3-8*a
*c*d^2*e^2*g^2*m^3+72*a*c*d*e^3*f*g*m^3+44*a*c*e^4*f^2*m^3-4*c^2*d^2*e^2*f
^2*m^3+179*a^2*e^4*g^2*m^2-104*a*c*d^2*e^2*g^2*m^2+428*a*c*d*e^3*f*g*m^2+3
58*a*c*e^4*f^2*m^2+40*c^2*d^3*e*f*g*m^2-52*c^2*d^2*e^2*f^2*m^2+638*a^2*e^4
*g^2*m-336*a*c*d^2*e^2*g^2*m+840*a*c*d*e^3*f*g*m+1276*a*c*e^4*f^2*m-120*c^
2*d^4*g^2*m+280*c^2*d^3*e*f*g*m-168*c^2*d^2*e^2*f^2*m+840*a^2*e^4*g^2+1680
*a*c*e^4*f^2)/e^4/(m^5+25*m^4+245*m^3+1175*m^2+2754*m+2520)*x^3*exp(m*ln(e
*x+d))+(a^2*d*e^4*g^2*m^5+2*a^2*e^5*f*g*m^5+2*a*c*d*e^4*f^2*m^5+22*a^2*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2887 vs. $2(357) = 714$.

Time = 0.13 (sec) , antiderivative size = 2887, normalized size of antiderivative = 8.09

$$\int (d+ex)^m (f+gx)^2 \left(a+cx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
(a^2*d*e^6*f^2*m^6 + (c^2*e^7*g^2*m^6 + 21*c^2*e^7*g^2*m^5 + 175*c^2*e^7*g
^2*m^4 + 735*c^2*e^7*g^2*m^3 + 1624*c^2*e^7*g^2*m^2 + 1764*c^2*e^7*g^2*m +
 720*c^2*e^7*g^2)*x^7 + (1680*c^2*e^7*f*g + (2*c^2*e^7*f*g + c^2*d*e^6*g^2
)*m^6 + (44*c^2*e^7*f*g + 15*c^2*d*e^6*g^2)*m^5 + 5*(76*c^2*e^7*f*g + 17*c
^2*d*e^6*g^2)*m^4 + 5*(328*c^2*e^7*f*g + 45*c^2*d*e^6*g^2)*m^3 + 2*(1849*c
^2*e^7*f*g + 137*c^2*d*e^6*g^2)*m^2 + 4*(1019*c^2*e^7*f*g + 30*c^2*d*e^6*g
^2)*m)*x^6 + (27*a^2*d*e^6*f^2 - 2*a^2*d^2*e^5*f*g)*m^5 + (1008*c^2*e^7*f^
2 + 2016*a*c*e^7*g^2 + (c^2*e^7*f^2 + 2*c^2*d*e^6*f*g + 2*a*c*e^7*g^2)*m^6
 + (23*c^2*e^7*f^2 + 34*c^2*d*e^6*f*g - 2*(3*c^2*d^2*e^5 - 23*a*c*e^7)*g^2
)*m^5 + 3*(69*c^2*e^7*f^2 + 70*c^2*d*e^6*f*g - 2*(10*c^2*d^2*e^5 - 69*a*c*
e^7)*g^2)*m^4 + 5*(185*c^2*e^7*f^2 + 118*c^2*d*e^6*f*g - 2*(21*c^2*d^2*e^5
 - 185*a*c*e^7)*g^2)*m^3 + 4*(536*c^2*e^7*f^2 + 187*c^2*d*e^6*f*g - (75*c^
2*d^2*e^5 - 1072*a*c*e^7)*g^2)*m^2 + 12*(201*c^2*e^7*f^2 + 28*c^2*d*e^6*f*
g - 6*(2*c^2*d^2*e^5 - 67*a*c*e^7)*g^2)*m)*x^5 - (50*a^2*d^2*e^5*f*g - 2*a
^2*d^3*e^4*g^2 - (4*a*c*d^3*e^4 + 295*a^2*d*e^6)*f^2)*m^4 + (5040*a*c*e^7*
f*g + (c^2*d*e^6*f^2 + 4*a*c*e^7*f*g + 2*a*c*d*e^6*g^2)*m^6 + (19*c^2*d*e^
6*f^2 + 38*a*c*d*e^6*g^2 - 2*(5*c^2*d^2*e^5 - 48*a*c*e^7)*f*g)*m^5 + (131*
c^2*d*e^6*f^2 - 2*(65*c^2*d^2*e^5 - 452*a*c*e^7)*f*g + 2*(15*c^2*d^3*e^4 +
 131*a*c*d*e^6)*g^2)*m^4 + (401*c^2*d*e^6*f^2 - 2*(265*c^2*d^2*e^5 - 2112*
a*c*e^7)*f*g + 2*(90*c^2*d^3*e^4 + 401*a*c*d*e^6)*g^2)*m^3 + 10*(54*c^2...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37262 vs. $2(347) = 694$.

Time = 7.90 (sec) , antiderivative size = 37262, normalized size of antiderivative = 104.38

$$\int (d+ex)^m (f+gx)^2 \left(a+cx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+a)**2,x)
```

output
```
Piecewise((d**m*(a**2*f**2*x + a**2*f*g*x**2 + a**2*g**2*x**3/3 + 2*a*c*f*
*2*x**3/3 + a*c*f*g*x**4 + 2*a*c*g**2*x**5/5 + c**2*f**2*x**5/5 + c**2*f*g
*x**6/3 + c**2*g**2*x**7/7), Eq(e, 0)), (-a**2*d**2*e**4*g**2/(60*d**6*e**
7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2
*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 4*a**2*d*e**5*f*g/(60*d*
*6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 90
0*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 6*a**2*d*e**5*g**2
*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*
x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 10*a**2*e
**6*f**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*
e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 24*
a**2*e**6*f*g*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 120
0*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6
) - 15*a**2*e**6*g**2*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9
*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60
*e**13*x**6) - 4*a*c*d**4*e**2*g**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*
d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*
x**5 + 60*e**13*x**6) - 4*a*c*d**3*e**3*f*g/(60*d**6*e**7 + 360*d**5*e**8*
x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*
d*e**12*x**5 + 60*e**13*x**6) - 24*a*c*d**3*e**3*g**2*x/(60*d**6*e**7 +...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. 2(357) = 714.

Time = 0.08 (sec) , antiderivative size = 1041, normalized size of antiderivative = 2.92

$$\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
2*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a^2*f*g/((m^2 + 3*m + 2)*e
^2) + (e*x + d)^(m + 1)*a^2*f^2/(e*(m + 1)) + 2*((m^2 + 3*m + 2)*e^3*x^3 +
 (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a*c*f^2/((m^3 + 6*
m^2 + 11*m + 6)*e^3) + ((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*
d^2*e*m*x + 2*d^3)*(e*x + d)^m*a^2*g^2/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 4*
((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2
 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*a*c*f*g/((m^4 + 10*m^
3 + 35*m^2 + 50*m + 24)*e^4) + ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^
5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3
*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*c^2*f
^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + 2*((m^4 + 10*m^
3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 -
 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m
*x + 24*d^5)*(e*x + d)^m*a*c*g^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m
 + 120)*e^5) + 2*((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6
+ (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*
m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)
*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x + d)^m*c^2*f*g/((m^6 + 21*m^5
 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6) + ((m^6 + 21*m^5 + 17
5*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^7*x^7 + (m^6 + 15*m^5 + 85...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5187 vs. $2(357) = 714$.

Time = 0.16 (sec) , antiderivative size = 5187, normalized size of antiderivative = 14.53

$$\int (d+ex)^m (f+gx)^2 \left(a+cx^2\right)^2 \, dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^2,x, algorithm="giac")
```

output

```
((e*x + d)^m*c^2*e^7*g^2*m^6*x^7 + 2*(e*x + d)^m*c^2*e^7*f*g*m^6*x^6 + (e*
x + d)^m*c^2*d*e^6*g^2*m^6*x^6 + 21*(e*x + d)^m*c^2*e^7*g^2*m^5*x^7 + (e*x
 + d)^m*c^2*e^7*f^2*m^6*x^5 + 2*(e*x + d)^m*c^2*d*e^6*f*g*m^6*x^5 + 2*(e*x
 + d)^m*a*c*e^7*g^2*m^6*x^5 + 44*(e*x + d)^m*c^2*e^7*f*g*m^5*x^6 + 15*(e*x
 + d)^m*c^2*d*e^6*g^2*m^5*x^6 + 175*(e*x + d)^m*c^2*e^7*g^2*m^4*x^7 + (e*x
 + d)^m*c^2*d*e^6*f^2*m^6*x^4 + 4*(e*x + d)^m*a*c*e^7*f*g*m^6*x^4 + 2*(e*x
 + d)^m*a*c*d*e^6*g^2*m^6*x^4 + 23*(e*x + d)^m*c^2*e^7*f^2*m^5*x^5 + 34*(e
*x + d)^m*c^2*d*e^6*f*g*m^5*x^5 - 6*(e*x + d)^m*c^2*d^2*e^5*g^2*m^5*x^5 +
46*(e*x + d)^m*a*c*e^7*g^2*m^5*x^5 + 380*(e*x + d)^m*c^2*e^7*f*g*m^4*x^6 +
 85*(e*x + d)^m*c^2*d*e^6*g^2*m^4*x^6 + 735*(e*x + d)^m*c^2*e^7*g^2*m^3*x^
7 + 2*(e*x + d)^m*a*c*e^7*f^2*m^6*x^3 + 4*(e*x + d)^m*a*c*d*e^6*f*g*m^6*x^
3 + (e*x + d)^m*a^2*e^7*g^2*m^6*x^3 + 19*(e*x + d)^m*c^2*d*e^6*f^2*m^5*x^4
 - 10*(e*x + d)^m*c^2*d^2*e^5*f*g*m^5*x^4 + 96*(e*x + d)^m*a*c*e^7*f*g*m^5
*x^4 + 38*(e*x + d)^m*a*c*d*e^6*g^2*m^5*x^4 + 207*(e*x + d)^m*c^2*e^7*f^2*
m^4*x^5 + 210*(e*x + d)^m*c^2*d*e^6*f*g*m^4*x^5 - 60*(e*x + d)^m*c^2*d^2*e
^5*g^2*m^4*x^5 + 414*(e*x + d)^m*a*c*e^7*g^2*m^4*x^5 + 1640*(e*x + d)^m*c^
2*e^7*f*g*m^3*x^6 + 225*(e*x + d)^m*c^2*d*e^6*g^2*m^3*x^6 + 1624*(e*x + d)
^m*c^2*e^7*g^2*m^2*x^7 + 2*(e*x + d)^m*a*c*d*e^6*f^2*m^6*x^2 + 2*(e*x + d)
^m*a^2*e^7*f*g*m^6*x^2 + (e*x + d)^m*a^2*d*e^6*g^2*m^6*x^2 - 4*(e*x + d)^m
*c^2*d^2*e^5*f^2*m^5*x^3 + 50*(e*x + d)^m*a*c*e^7*f^2*m^5*x^3 + 84*(e*x...
```

Mupad [B] (verification not implemented)

Time = 7.16 (sec) , antiderivative size = 2508, normalized size of antiderivative = 7.03

$$\int (d+ex)^m (f+gx)^2 \left(a+cx^2\right)^2 dx = \text{Too large to display}$$

input
```
int((f + g*x)^2*(a + c*x^2)^2*(d + e*x)^m,x)
```

output
```
((d + e*x)^m*(720*c^2*d^7*g^2 + 5040*a^2*d*e^6*f^2 + 1680*a^2*d^3*e^4*g^2
+ 1008*c^2*d^5*e^2*f^2 - 1680*c^2*d^6*e*f*g + 358*a^2*d^3*e^4*g^2*m^2 + 44
*a^2*d^3*e^4*g^2*m^3 + 2*a^2*d^3*e^4*g^2*m^4 + 24*c^2*d^5*e^2*f^2*m^2 + 33
60*a*c*d^3*e^4*f^2 + 2016*a*c*d^5*e^2*g^2 - 5040*a^2*d^2*e^5*f*g + 8028*a^
2*d*e^6*f^2*m + 5104*a^2*d*e^6*f^2*m^2 + 1665*a^2*d*e^6*f^2*m^3 + 295*a^2*
d*e^6*f^2*m^4 + 27*a^2*d*e^6*f^2*m^5 + a^2*d*e^6*f^2*m^6 + 1276*a^2*d^3*e^
4*g^2*m + 312*c^2*d^5*e^2*f^2*m + 716*a*c*d^3*e^4*f^2*m^2 + 88*a*c*d^3*e^4
*f^2*m^3 + 4*a*c*d^3*e^4*f^2*m^4 + 48*a*c*d^5*e^2*g^2*m^2 - 2350*a^2*d^2*e
^5*f*g*m^2 - 490*a^2*d^2*e^5*f*g*m^3 - 50*a^2*d^2*e^5*f*g*m^4 - 2*a^2*d^2*
e^5*f*g*m^5 - 5040*a*c*d^4*e^3*f*g - 240*c^2*d^6*e*f*g*m + 2552*a*c*d^3*e^
4*f^2*m + 624*a*c*d^5*e^2*g^2*m - 5508*a^2*d^2*e^5*f*g*m - 432*a*c*d^4*e^3
*f*g*m^2 - 24*a*c*d^4*e^3*f*g*m^3 - 2568*a*c*d^4*e^3*f*g*m))/(e^7*(13068*m
 + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x*
(d + e*x)^m*(5040*a^2*e^7*f^2 + 8028*a^2*e^7*f^2*m + 5104*a^2*e^7*f^2*m^2
+ 1665*a^2*e^7*f^2*m^3 + 295*a^2*e^7*f^2*m^4 + 27*a^2*e^7*f^2*m^5 + a^2*e^
7*f^2*m^6 - 1276*a^2*d^2*e^5*g^2*m^2 - 358*a^2*d^2*e^5*g^2*m^3 - 44*a^2*d^
2*e^5*g^2*m^4 - 2*a^2*d^2*e^5*g^2*m^5 - 312*c^2*d^4*e^3*f^2*m^2 - 24*c^2*d
^4*e^3*f^2*m^3 - 720*c^2*d^6*e*g^2*m - 1680*a^2*d^2*e^5*g^2*m - 1008*c^2*d
^4*e^3*f^2*m - 2552*a*c*d^2*e^5*f^2*m^2 - 716*a*c*d^2*e^5*f^2*m^3 - 88*a*c
*d^2*e^5*f^2*m^4 - 4*a*c*d^2*e^5*f^2*m^5 - 624*a*c*d^4*e^3*g^2*m^2 - 48...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 3638, normalized size of antiderivative = 10.19

$$\int (d+ex)^m (f+gx)^2 \left(a+cx^2\right)^2 dx = \text{Too large to display}$$

input
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^2,x)
```

output
```
((d + e*x)**m*(2*a**2*d**3*e**4*g**2*m**4 + 44*a**2*d**3*e**4*g**2*m**3 +
358*a**2*d**3*e**4*g**2*m**2 + 1276*a**2*d**3*e**4*g**2*m + 1680*a**2*d**3
*e**4*g**2 - 2*a**2*d**2*e**5*f*g*m**5 - 50*a**2*d**2*e**5*f*g*m**4 - 490*
a**2*d**2*e**5*f*g*m**3 - 2350*a**2*d**2*e**5*f*g*m**2 - 5508*a**2*d**2*e*
*5*f*g*m - 5040*a**2*d**2*e**5*f*g - 2*a**2*d**2*e**5*g**2*m**5*x - 44*a**
2*d**2*e**5*g**2*m**4*x - 358*a**2*d**2*e**5*g**2*m**3*x - 1276*a**2*d**2*
e**5*g**2*m**2*x - 1680*a**2*d**2*e**5*g**2*m*x + a**2*d*e**6*f**2*m**6 +
27*a**2*d*e**6*f**2*m**5 + 295*a**2*d*e**6*f**2*m**4 + 1665*a**2*d*e**6*f*
*2*m**3 + 5104*a**2*d*e**6*f**2*m**2 + 8028*a**2*d*e**6*f**2*m + 5040*a**2
*d*e**6*f**2 + 2*a**2*d*e**6*f*g*m**6*x + 50*a**2*d*e**6*f*g*m**5*x + 490*
a**2*d*e**6*f*g*m**4*x + 2350*a**2*d*e**6*f*g*m**3*x + 5508*a**2*d*e**6*f*
g*m**2*x + 5040*a**2*d*e**6*f*g*m*x + a**2*d*e**6*g**2*m**6*x**2 + 23*a**2
*d*e**6*g**2*m**5*x**2 + 201*a**2*d*e**6*g**2*m**4*x**2 + 817*a**2*d*e**6*
g**2*m**3*x**2 + 1478*a**2*d*e**6*g**2*m**2*x**2 + 840*a**2*d*e**6*g**2*m*
x**2 + a**2*e**7*f**2*m**6*x + 27*a**2*e**7*f**2*m**5*x + 295*a**2*e**7*f*
*2*m**4*x + 1665*a**2*e**7*f**2*m**3*x + 5104*a**2*e**7*f**2*m**2*x + 8028
*a**2*e**7*f**2*m*x + 5040*a**2*e**7*f**2*x + 2*a**2*e**7*f*g*m**6*x**2 +
52*a**2*e**7*f*g*m**5*x**2 + 540*a**2*e**7*f*g*m**4*x**2 + 2840*a**2*e**7*
f*g*m**3*x**2 + 7858*a**2*e**7*f*g*m**2*x**2 + 10548*a**2*e**7*f*g*m*x**2
+ 5040*a**2*e**7*f*g*x**2 + a**2*e**7*g**2*m**6*x**3 + 25*a**2*e**7*g**...
```

3.157 $\int (d+ex)^m(f+gx)\left(a+cx^2\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 228

$$\begin{aligned}\int (d+ex)^m(f+gx)\left(a+cx^2\right)^2\,dx &= \frac{(cd^2+ae^2)^2(ef-dg)(d+ex)^{1+m}}{e^6(1+m)} \\ &\quad+\frac{(cd^2+ae^2)(ae^2g-cd(4ef-5dg))(d+ex)^{2+m}}{e^6(2+m)} \\ &\quad+\frac{2c(cd^2(3ef-5dg)+ae^2(ef-3dg))(d+ex)^{3+m}}{e^6(3+m)} \\ &\quad+\frac{2c(ae^2g-cd(2ef-5dg))(d+ex)^{4+m}}{e^6(4+m)} \\ &\quad+\frac{c^2(ef-5dg)(d+ex)^{5+m}}{e^6(5+m)}+\frac{c^2g(d+ex)^{6+m}}{e^6(6+m)}\end{aligned}$$

output
```
(a*e^2+c*d^2)^2*(-d*g+e*f)*(e*x+d)^(1+m)/e^6/(1+m)+(a*e^2+c*d^2)*(a*e^2*g-
c*d*(-5*d*g+4*e*f))*(e*x+d)^(2+m)/e^6/(2+m)+2*c*(c*d^2*(-5*d*g+3*e*f)+a*e^
2*(-3*d*g+e*f))*(e*x+d)^(3+m)/e^6/(3+m)+2*c*(a*e^2*g-c*d*(-5*d*g+2*e*f))*(
e*x+d)^(4+m)/e^6/(4+m)+c^2*(-5*d*g+e*f)*(e*x+d)^(5+m)/e^6/(5+m)+c^2*g*(e*x
+d)^(6+m)/e^6/(6+m)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.55

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)^2\,dx$$
$$=\frac{(d+ex)^{1+m}\left((ef-dg)(6+m)\left(e^4(1+m)(2+m)(3+m)(4+m)\left(a+cx^2\right)^2+4(cd^2+ae^2)(4+m)\right)\right.}{}$$

input
```
Integrate[(d + e*x)^m*(f + g*x)*(a + c*x^2)^2,x]
```

output
```
((d + e*x)^(1 + m)*((e*f - d*g)*(6 + m)*(e^4*(1 + m)*(2 + m)*(3 + m)*(4 +
m)*(a + c*x^2)^2 + 4*(c*d^2 + a*e^2)*(4 + m)*(a*e^2*(6 + 5*m + m^2) + c*(2
*d^2 - 2*d*e*(1 + m)*x + e^2*(2 + 3*m + m^2)*x^2)) - 4*c*d*(1 + m)*(d + e*
x)*(a*e^2*(12 + 7*m + m^2) + c*(2*d^2 - 2*d*e*(2 + m)*x + e^2*(6 + 5*m + m
^2)*x^2))) + g*(1 + m)*(d + e*x)*(e^4*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(a +
 c*x^2)^2 + 4*(c*d^2 + a*e^2)*(5 + m)*(a*e^2*(12 + 7*m + m^2) + c*(2*d^2 -
 2*d*e*(2 + m)*x + e^2*(6 + 5*m + m^2)*x^2)) - 4*c*d*(2 + m)*(d + e*x)*(a*
e^2*(20 + 9*m + m^2) + c*(2*d^2 - 2*d*e*(3 + m)*x + e^2*(12 + 7*m + m^2)*x
^2)))))/(e^6*(1 + m)*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2(f+gx)(d+ex)^m\,dx$$
$$\Big\downarrow \text{652}$$
$$\int \left(\frac{\left(ae^2+cd^2\right)^2(ef-dg)(d+ex)^m}{e^5}+\frac{\left(ae^2+cd^2\right)(d+ex)^{m+1}\left(ae^2g-cd(4ef-5dg)\right)}{e^5}+\frac{2c(d+ex)^{m+2}\left(ae^2\right.}{}\right.$$

$$\downarrow 2009$$

$$\frac{(ae^2+cd^2)^2(ef-dg)(d+ex)^{m+1}}{e^6(m+1)}+\frac{(ae^2+cd^2)(d+ex)^{m+2}(ae^2g-cd(4ef-5dg))}{e^6(m+2)}+\frac{2c(d+ex)^{m+3}(ae^2(ef-3dg)+cd^2(3ef-5dg))}{e^6(m+3)}+\frac{2c(d+ex)^{m+4}(ae^2g-cd(2ef-5dg))}{e^6(m+4)}+\frac{c^2(ef-5dg)(d+ex)^{m+5}}{e^6(m+5)}+\frac{c^2g(d+ex)^{m+6}}{e^6(m+6)}$$

input
```
Int[(d + e*x)^m*(f + g*x)*(a + c*x^2)^2,x]
```

output
```
((c*d^2 + a*e^2)^2*(e*f - d*g)*(d + e*x)^(1 + m))/(e^6*(1 + m)) + ((c*d^2
+ a*e^2)*(a*e^2*g - c*d*(4*e*f - 5*d*g))*(d + e*x)^(2 + m))/(e^6*(2 + m))
+ (2*c*(c*d^2*(3*e*f - 5*d*g) + a*e^2*(e*f - 3*d*g))*(d + e*x)^(3 + m))/(e
^6*(3 + m)) + (2*c*(a*e^2*g - c*d*(2*e*f - 5*d*g))*(d + e*x)^(4 + m))/(e^6
*(4 + m)) + (c^2*(e*f - 5*d*g)*(d + e*x)^(5 + m))/(e^6*(5 + m)) + (c^2*g*(
d + e*x)^(6 + m))/(e^6*(6 + m))
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1084 vs. $2(228) = 456$.

Time = 0.70 (sec) , antiderivative size = 1085, normalized size of antiderivative = 4.76

method	result	size
norman	Expression too large to display	1085
gosper	Expression too large to display	1275
orering	Expression too large to display	1278
risch	Expression too large to display	1647
parallelrisch	Expression too large to display	2517

input
```
int((e*x+d)^m*(g*x+f)*(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
g*c^2/(6+m)*x^6*exp(m*ln(e*x+d))+(a^2*e^4*g*m^4+2*a*c*d*e^3*f*m^4+18*a^2*e
^4*g*m^3-6*a*c*d^2*e^2*g*m^3+30*a*c*d*e^3*f*m^3+119*a^2*e^4*g*m^2-66*a*c*d
^2*e^2*g*m^2+148*a*c*d*e^3*f*m^2+12*c^2*d^3*e*f*m^2+342*a^2*e^4*g*m-180*a*
c*d^2*e^2*g*m+240*a*c*d*e^3*f*m-60*c^2*d^4*g*m+72*c^2*d^3*e*f*m+360*a^2*e^
4*g)/e^4/(m^5+20*m^4+155*m^3+580*m^2+1044*m+720)*x^2*exp(m*ln(e*x+d))+(a^2
*d*e^4*g*m^5+a^2*e^5*f*m^5+18*a^2*d*e^4*g*m^4+20*a^2*e^5*f*m^4-4*a*c*d^2*e
^3*f*m^4+119*a^2*d*e^4*g*m^3+155*a^2*e^5*f*m^3+12*a*c*d^3*e^2*g*m^3-60*a*c
*d^2*e^3*f*m^3+342*a^2*d*e^4*g*m^2+580*a^2*e^5*f*m^2+132*a*c*d^3*e^2*g*m^2
-296*a*c*d^2*e^3*f*m^2-24*c^2*d^4*e*f*m^2+360*a^2*d*e^4*g*m+1044*a^2*e^5*f
*m+360*a*c*d^3*e^2*g*m-480*a*c*d^2*e^3*f*m+120*c^2*d^5*g*m-144*c^2*d^4*e*f
*m+720*a^2*e^5*f)/e^5/(m^6+21*m^5+175*m^4+735*m^3+1624*m^2+1764*m+720)*x*e
xp(m*ln(e*x+d))+c*(2*a*e^2*g*m^2+c*d*e*f*m^2+22*a*e^2*g*m-5*c*d^2*g*m+6*c*
d*e*f*m+60*a*e^2*g)/e^2/(m^3+15*m^2+74*m+120)*x^4*exp(m*ln(e*x+d))+(d*g*m+
e*f*m+6*e*f)*c^2/e/(m^2+11*m+30)*x^5*exp(m*ln(e*x+d))-d*(-a^2*e^5*f*m^5+a^
2*d*e^4*g*m^4-20*a^2*e^5*f*m^4+18*a^2*d*e^4*g*m^3-155*a^2*e^5*f*m^3-4*a*c*
d^2*e^3*f*m^3+119*a^2*d*e^4*g*m^2-580*a^2*e^5*f*m^2+12*a*c*d^3*e^2*g*m^2-6
0*a*c*d^2*e^3*f*m^2+342*a^2*d*e^4*g*m-1044*a^2*e^5*f*m+132*a*c*d^3*e^2*g*m
-296*a*c*d^2*e^3*f*m-24*c^2*d^4*e*f*m+360*a^2*d*e^4*g-720*a^2*e^5*f+360*a*
c*d^3*e^2*g-480*a*c*d^2*e^3*f+120*c^2*d^5*g-144*c^2*d^4*e*f)/e^6/(m^6+21*m
^5+175*m^4+735*m^3+1624*m^2+1764*m+720)*exp(m*ln(e*x+d))+2*(a*d*e^2*g*m...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1419 vs. $2(228) = 456$.

Time = 0.10 (sec) , antiderivative size = 1419, normalized size of antiderivative = 6.22

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
(a^2*d*e^5*f*m^5 + (c^2*e^6*g*m^5 + 15*c^2*e^6*g*m^4 + 85*c^2*e^6*g*m^3 +
225*c^2*e^6*g*m^2 + 274*c^2*e^6*g*m + 120*c^2*e^6*g)*x^6 + (144*c^2*e^6*f
+ (c^2*e^6*f + c^2*d*e^5*g)*m^5 + 2*(8*c^2*e^6*f + 5*c^2*d*e^5*g)*m^4 + 5*
(19*c^2*e^6*f + 7*c^2*d*e^5*g)*m^3 + 10*(26*c^2*e^6*f + 5*c^2*d*e^5*g)*m^2
 + 12*(27*c^2*e^6*f + 2*c^2*d*e^5*g)*m)*x^5 + (20*a^2*d*e^5*f - a^2*d^2*e^
4*g)*m^4 + (360*a*c*e^6*g + (c^2*d*e^5*f + 2*a*c*e^6*g)*m^5 + (12*c^2*d*e^
5*f - (5*c^2*d^2*e^4 - 34*a*c*e^6)*g)*m^4 + (47*c^2*d*e^5*f - 2*(15*c^2*d^
2*e^4 - 107*a*c*e^6)*g)*m^3 + (72*c^2*d*e^5*f - (55*c^2*d^2*e^4 - 614*a*c*
e^6)*g)*m^2 + 6*(6*c^2*d*e^5*f - (5*c^2*d^2*e^4 - 132*a*c*e^6)*g)*m)*x^4 -
 (18*a^2*d^2*e^4*g - (4*a*c*d^3*e^3 + 155*a^2*d*e^5)*f)*m^3 + 2*(240*a*c*e
^6*f + (a*c*e^6*f + a*c*d*e^5*g)*m^5 + 2*(7*a*c*d*e^5*g - (c^2*d^2*e^4 - 9
*a*c*e^6)*f)*m^4 - ((18*c^2*d^2*e^4 - 121*a*c*e^6)*f - 5*(2*c^2*d^3*e^3 +
13*a*c*d*e^5)*g)*m^3 - 2*(2*(10*c^2*d^2*e^4 - 93*a*c*e^6)*f - (15*c^2*d^3*
e^3 + 56*a*c*d*e^5)*g)*m^2 - 4*((6*c^2*d^2*e^4 - 127*a*c*e^6)*f - 5*(c^2*d
^3*e^3 + 3*a*c*d*e^5)*g)*m)*x^3 + (20*(3*a*c*d^3*e^3 + 29*a^2*d*e^5)*f - (
12*a*c*d^4*e^2 + 119*a^2*d^2*e^4)*g)*m^2 + (360*a^2*e^6*g + (2*a*c*d*e^5*f
 + a^2*e^6*g)*m^5 + (32*a*c*d*e^5*f - (6*a*c*d^2*e^4 - 19*a^2*e^6)*g)*m^4
+ (2*(6*c^2*d^3*e^3 + 89*a*c*d*e^5)*f - (72*a*c*d^2*e^4 - 137*a^2*e^6)*g)*
m^3 + (4*(21*c^2*d^3*e^3 + 97*a*c*d*e^5)*f - (60*c^2*d^4*e^2 + 246*a*c*d^2
*e^4 - 461*a^2*e^6)*g)*m^2 + 6*(4*(3*c^2*d^3*e^3 + 10*a*c*d*e^5)*f - (1...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16458 vs. $2(212) = 424$.

Time = 3.91 (sec) , antiderivative size = 16458, normalized size of antiderivative = 72.18

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(g*x+f)*(c*x**2+a)**2,x)
```

output
```
Piecewise((d**m*(a**2*f*x + a**2*g*x**2/2 + 2*a*c*f*x**3/3 + a*c*g*x**4/2
+ c**2*f*x**5/5 + c**2*g*x**6/6), Eq(e, 0)), (-3*a**2*d*e**4*g/(60*d**5*e*
*6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**
10*x**4 + 60*e**11*x**5) - 12*a**2*e**5*f/(60*d**5*e**6 + 300*d**4*e**7*x
+ 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x*
*5) - 15*a**2*e**5*g*x/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**
2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 6*a*c*d**3*e*
*2*g/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*
x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 4*a*c*d**2*e**3*f/(60*d**5*e**6
 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10
*x**4 + 60*e**11*x**5) - 30*a*c*d**2*e**3*g*x/(60*d**5*e**6 + 300*d**4*e**
7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**1
1*x**5) - 20*a*c*d*e**4*f*x/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**
8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 60*a*c*d
*e**4*g*x**2/(60*d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d*
*2*e**9*x**3 + 300*d*e**10*x**4 + 60*e**11*x**5) - 40*a*c*e**5*f*x**2/(60*
d**5*e**6 + 300*d**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 30
0*d*e**10*x**4 + 60*e**11*x**5) - 60*a*c*e**5*g*x**3/(60*d**5*e**6 + 300*d
**4*e**7*x + 600*d**3*e**8*x**2 + 600*d**2*e**9*x**3 + 300*d*e**10*x**4 +
60*e**11*x**5) + 60*c**2*d**5*g*log(d/e + x)/(60*d**5*e**6 + 300*d**4*e...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(228) = 456$.

Time = 0.06 (sec) , antiderivative size = 575, normalized size of antiderivative = 2.52

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)^2\,dx$$

$$= \frac{(e^2(m+1)x^2 + demx - d^2)(ex+d)^m a^2 g}{(m^2+3\,m+2)e^2} + \frac{(ex+d)^{m+1}a^2 f}{e(m+1)}$$

$$+ \frac{2\,((m^2+3\,m+2)e^3x^3 + (m^2+m)de^2x^2 - 2\,d^2emx + 2\,d^3)(ex+d)^m acf}{(m^3+6\,m^2+11\,m+6)e^3}$$

$$+ \frac{2\,((m^3+6\,m^2+11\,m+6)e^4x^4 + (m^3+3\,m^2+2\,m)de^3x^3 - 3\,(m^2+m)d^2e^2x^2 + 6\,d^3emx - 6\,d^4)(ex}{(m^4+10\,m^3+35\,m^2+50\,m+24)e^4}$$

$$+ \frac{((m^4+10\,m^3+35\,m^2+50\,m+24)e^5x^5 + (m^4+6\,m^3+11\,m^2+6\,m)de^4x^4 - 4\,(m^3+3\,m^2+2\,m)d}{(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+1}$$

$$+ \frac{((m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120)e^6x^6 + (m^5+10\,m^4+35\,m^3+50\,m^2+24\,m)de^5x^5 -}{(m^6+21\,m^5+175}$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*a^2*g/((m^2 + 3*m + 2)*e^2)
+ (e*x + d)^(m + 1)*a^2*f/(e*(m + 1)) + 2*((m^2 + 3*m + 2)*e^3*x^3 + (m^2
+ m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m*a*c*f/((m^3 + 6*m^2 + 11
*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d
*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*a*c*
g/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) + ((m^4 + 10*m^3 + 35*m^2 + 50
*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2
 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e
*x + d)^m*c^2*f/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + ((
m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 3
5*m^3 + 50*m^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*
x^4 + 20*(m^3 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*
d^5*e*m*x - 120*d^6)*(e*x + d)^m*c^2*g/((m^6 + 21*m^5 + 175*m^4 + 735*m^3
+ 1624*m^2 + 1764*m + 720)*e^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2435 vs. 2(228) = 456.

Time = 0.13 (sec) , antiderivative size = 2435, normalized size of antiderivative = 10.68

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+a)^2,x, algorithm="giac")
```

output
```
((e*x + d)^m*c^2*e^6*g*m^5*x^6 + (e*x + d)^m*c^2*e^6*f*m^5*x^5 + (e*x + d)
^m*c^2*d*e^5*g*m^5*x^5 + 15*(e*x + d)^m*c^2*e^6*g*m^4*x^6 + (e*x + d)^m*c^
2*d*e^5*f*m^5*x^4 + 2*(e*x + d)^m*a*c*e^6*g*m^5*x^4 + 16*(e*x + d)^m*c^2*e
^6*f*m^4*x^5 + 10*(e*x + d)^m*c^2*d*e^5*g*m^4*x^5 + 85*(e*x + d)^m*c^2*e^6
*g*m^3*x^6 + 2*(e*x + d)^m*a*c*e^6*f*m^5*x^3 + 2*(e*x + d)^m*a*c*d*e^5*g*m
^5*x^3 + 12*(e*x + d)^m*c^2*d*e^5*f*m^4*x^4 - 5*(e*x + d)^m*c^2*d^2*e^4*g*
m^4*x^4 + 34*(e*x + d)^m*a*c*e^6*g*m^4*x^4 + 95*(e*x + d)^m*c^2*e^6*f*m^3*
x^5 + 35*(e*x + d)^m*c^2*d*e^5*g*m^3*x^5 + 225*(e*x + d)^m*c^2*e^6*g*m^2*x
^6 + 2*(e*x + d)^m*a*c*d*e^5*f*m^5*x^2 + (e*x + d)^m*a^2*e^6*g*m^5*x^2 - 4
*(e*x + d)^m*c^2*d^2*e^4*f*m^4*x^3 + 36*(e*x + d)^m*a*c*e^6*f*m^4*x^3 + 28
*(e*x + d)^m*a*c*d*e^5*g*m^4*x^3 + 47*(e*x + d)^m*c^2*d*e^5*f*m^3*x^4 - 30
*(e*x + d)^m*c^2*d^2*e^4*g*m^3*x^4 + 214*(e*x + d)^m*a*c*e^6*g*m^3*x^4 + 2
60*(e*x + d)^m*c^2*e^6*f*m^2*x^5 + 50*(e*x + d)^m*c^2*d*e^5*g*m^2*x^5 + 27
4*(e*x + d)^m*c^2*e^6*g*m*x^6 + (e*x + d)^m*a^2*e^6*f*m^5*x + (e*x + d)^m*
a^2*d*e^5*g*m^5*x + 32*(e*x + d)^m*a*c*d*e^5*f*m^4*x^2 - 6*(e*x + d)^m*a*c
*d^2*e^4*g*m^4*x^2 + 19*(e*x + d)^m*a^2*e^6*g*m^4*x^2 - 36*(e*x + d)^m*c^2
*d^2*e^4*f*m^3*x^3 + 242*(e*x + d)^m*a*c*e^6*f*m^3*x^3 + 20*(e*x + d)^m*c^
2*d^3*e^3*g*m^3*x^3 + 130*(e*x + d)^m*a*c*d*e^5*g*m^3*x^3 + 72*(e*x + d)^m
*c^2*d*e^5*f*m^2*x^4 - 55*(e*x + d)^m*c^2*d^2*e^4*g*m^2*x^4 + 614*(e*x + d
)^m*a*c*e^6*g*m^2*x^4 + 324*(e*x + d)^m*c^2*e^6*f*m*x^5 + 24*(e*x + d)^...
```

Mupad [B] (verification not implemented)

Time = 6.66 (sec) , antiderivative size = 1229, normalized size of antiderivative = 5.39

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
int((f + g*x)*(a + c*x^2)^2*(d + e*x)^m,x)
```

output
```
((d + e*x)^m*(720*a^2*d*e^5*f - 360*a^2*d^2*e^4*g - 120*c^2*d^6*g + 144*c^
2*d^5*e*f + 480*a*c*d^3*e^3*f - 360*a*c*d^4*e^2*g + 1044*a^2*d*e^5*f*m + 2
4*c^2*d^5*e*f*m + 580*a^2*d*e^5*f*m^2 + 155*a^2*d*e^5*f*m^3 + 20*a^2*d*e^5
*f*m^4 + a^2*d*e^5*f*m^5 - 342*a^2*d^2*e^4*g*m - 119*a^2*d^2*e^4*g*m^2 - 1
8*a^2*d^2*e^4*g*m^3 - a^2*d^2*e^4*g*m^4 + 296*a*c*d^3*e^3*f*m - 132*a*c*d^
4*e^2*g*m + 60*a*c*d^3*e^3*f*m^2 + 4*a*c*d^3*e^3*f*m^3 - 12*a*c*d^4*e^2*g*
m^2))/(e^6*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720)) +
 (x*(d + e*x)^m*(720*a^2*e^6*f + 580*a^2*e^6*f*m^2 + 155*a^2*e^6*f*m^3 + 2
0*a^2*e^6*f*m^4 + a^2*e^6*f*m^5 + 1044*a^2*e^6*f*m + 360*a^2*d*e^5*g*m + 1
20*c^2*d^5*e*g*m + 342*a^2*d*e^5*g*m^2 + 119*a^2*d*e^5*g*m^3 + 18*a^2*d*e^
5*g*m^4 + a^2*d*e^5*g*m^5 - 144*c^2*d^4*e^2*f*m - 24*c^2*d^4*e^2*f*m^2 - 4
80*a*c*d^2*e^4*f*m + 360*a*c*d^3*e^3*g*m - 296*a*c*d^2*e^4*f*m^2 - 60*a*c*
d^2*e^4*f*m^3 - 4*a*c*d^2*e^4*f*m^4 + 132*a*c*d^3*e^3*g*m^2 + 12*a*c*d^3*e
^3*g*m^3))/(e^6*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 72
0)) + (x^2*(m + 1)*(d + e*x)^m*(360*a^2*e^4*g + 119*a^2*e^4*g*m^2 + 18*a^2
*e^4*g*m^3 + a^2*e^4*g*m^4 + 342*a^2*e^4*g*m - 60*c^2*d^4*g*m + 72*c^2*d^3
*e*f*m + 12*c^2*d^3*e*f*m^2 + 240*a*c*d*e^3*f*m + 148*a*c*d*e^3*f*m^2 + 30
*a*c*d*e^3*f*m^3 + 2*a*c*d*e^3*f*m^4 - 180*a*c*d^2*e^2*g*m - 66*a*c*d^2*e^
2*g*m^2 - 6*a*c*d^2*e^2*g*m^3))/(e^4*(1764*m + 1624*m^2 + 735*m^3 + 175*m^
4 + 21*m^5 + m^6 + 720)) + (c^2*g*x^6*(d + e*x)^m*(274*m + 225*m^2 + 85...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1638, normalized size of antiderivative = 7.18

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)^2\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^m*(g*x+f)*(c*x^2+a)^2,x)
```

output

```
((d + e*x)**m*( - a**2*d**2*e**4*g*m**4 - 18*a**2*d**2*e**4*g*m**3 - 119*a
**2*d**2*e**4*g*m**2 - 342*a**2*d**2*e**4*g*m - 360*a**2*d**2*e**4*g + a**
2*d*e**5*f*m**5 + 20*a**2*d*e**5*f*m**4 + 155*a**2*d*e**5*f*m**3 + 580*a**
2*d*e**5*f*m**2 + 1044*a**2*d*e**5*f*m + 720*a**2*d*e**5*f + a**2*d*e**5*g
*m**5*x + 18*a**2*d*e**5*g*m**4*x + 119*a**2*d*e**5*g*m**3*x + 342*a**2*d*
e**5*g*m**2*x + 360*a**2*d*e**5*g*m*x + a**2*e**6*f*m**5*x + 20*a**2*e**6*
f*m**4*x + 155*a**2*e**6*f*m**3*x + 580*a**2*e**6*f*m**2*x + 1044*a**2*e**
6*f*m*x + 720*a**2*e**6*f*x + a**2*e**6*g*m**5*x**2 + 19*a**2*e**6*g*m**4*
x**2 + 137*a**2*e**6*g*m**3*x**2 + 461*a**2*e**6*g*m**2*x**2 + 702*a**2*e*
*6*g*m*x**2 + 360*a**2*e**6*g*x**2 - 12*a*c*d**4*e**2*g*m**2 - 132*a*c*d**
4*e**2*g*m - 360*a*c*d**4*e**2*g + 4*a*c*d**3*e**3*f*m**3 + 60*a*c*d**3*e*
*3*f*m**2 + 296*a*c*d**3*e**3*f*m + 480*a*c*d**3*e**3*f + 12*a*c*d**3*e**3
*g*m**3*x + 132*a*c*d**3*e**3*g*m**2*x + 360*a*c*d**3*e**3*g*m*x - 4*a*c*d
**2*e**4*f*m**4*x - 60*a*c*d**2*e**4*f*m**3*x - 296*a*c*d**2*e**4*f*m**2*x
 - 480*a*c*d**2*e**4*f*m*x - 6*a*c*d**2*e**4*g*m**4*x**2 - 72*a*c*d**2*e**
4*g*m**3*x**2 - 246*a*c*d**2*e**4*g*m**2*x**2 - 180*a*c*d**2*e**4*g*m*x**2
 + 2*a*c*d*e**5*f*m**5*x**2 + 32*a*c*d*e**5*f*m**4*x**2 + 178*a*c*d*e**5*f
*m**3*x**2 + 388*a*c*d*e**5*f*m**2*x**2 + 240*a*c*d*e**5*f*m*x**2 + 2*a*c*
d*e**5*g*m**5*x**3 + 28*a*c*d*e**5*g*m**4*x**3 + 130*a*c*d*e**5*g*m**3*x**
3 + 224*a*c*d*e**5*g*m**2*x**3 + 120*a*c*d*e**5*g*m*x**3 + 2*a*c*e**6*f...
```

3.158 $\int \frac{(d+ex)^m(a+cx^2)^2}{f+gx}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 237

$$\begin{aligned}\int \frac{(d+ex)^m(a+cx^2)^2}{f+gx}\,dx &= -\frac{c(ef+dg)(ce^2f^2+cd^2g^2+2ae^2g^2)(d+ex)^{1+m}}{e^4g^4(1+m)} \\ &\quad+\frac{c(2ae^2g^2+c(e^2f^2+2defg+3d^2g^2))(d+ex)^{2+m}}{e^4g^3(2+m)} \\ &\quad-\frac{c^2(ef+3dg)(d+ex)^{3+m}}{e^4g^2(3+m)}+\frac{c^2(d+ex)^{4+m}}{e^4g(4+m)} \\ &\quad+\frac{(cf^2+ag^2)^2(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{g(d+ex)}{ef-dg}\right)}{g^4(ef-dg)(1+m)}\end{aligned}$$

output
```
-c*(d*g+e*f)*(2*a*e^2*g^2+c*d^2*g^2+c*e^2*f^2)*(e*x+d)^(1+m)/e^4/g^4/(1+m)
+c*(2*a*e^2*g^2+c*(3*d^2*g^2+2*d*e*f*g+e^2*f^2))*(e*x+d)^(2+m)/e^4/g^3/(2+
m)-c^2*(3*d*g+e*f)*(e*x+d)^(3+m)/e^4/g^2/(3+m)+c^2*(e*x+d)^(4+m)/e^4/g/(4+
m)+(a*g^2+c*f^2)^2*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],-g*(e*x+d)/(-d*g
+e*f))/g^4/(-d*g+e*f)/(1+m)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^m (a+cx^2)^2}{f+gx}\,dx$$
$$= \frac{(d+ex)^{1+m}\left(-\frac{c(ef+dg)(ce^2f^2+cd^2g^2+2ae^2g^2)}{e^4(1+m)} + \frac{cg(2ae^2g^2+c(e^2f^2+2defg+3d^2g^2))(d+ex)}{e^4(2+m)} - \frac{c^2g^2(ef+3dg)(d+ex)^2}{e^4(3+m)} + \frac{c^2g^3(}{e^4(}\right.}{g^4}$$

input
```
Integrate[((d + e*x)^m*(a + c*x^2)^2)/(f + g*x),x]
```

output
```
((d + e*x)^(1 + m)*(-((c*(e*f + d*g)*(c*e^2*f^2 + c*d^2*g^2 + 2*a*e^2*g^2)
)/(e^4*(1 + m))) + (c*g*(2*a*e^2*g^2 + c*(e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2)
)*(d + e*x))/(e^4*(2 + m)) - (c^2*g^2*(e*f + 3*d*g)*(d + e*x)^2)/(e^4*(3 +
 m)) + (c^2*g^3*(d + e*x)^3)/(e^4*(4 + m)) + ((c*f^2 + a*g^2)^2*Hypergeome
tric2F1[1, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)*(1 +
m))))/g^4
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {652, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2 (d+ex)^m}{f+gx}\,dx$$
$$\Big\downarrow \text{652}$$
$$\int \left(-\frac{c(dg+ef)(d+ex)^m (2ae^2g^2+cd^2g^2+ce^2f^2)}{e^3g^4} + \frac{c(d+ex)^{m+1}(2ae^2g^2+c(3d^2g^2+2defg+e^2f^2))}{e^3g^3} + \right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{c(dg+ef)(d+ex)^{m+1}\left(2ae^2g^2+cd^2g^2+ce^2f^2\right)}{e^4g^4(m+1)}+\frac{c(d+ex)^{m+2}\left(2ae^2g^2+c\left(3d^2g^2+2defg+e^2f^2\right)\right)}{e^4g^3(m+2)}+\frac{\left(ag^2+cf^2\right)^2(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{g(d+ex)}{ef-dg}\right)}{g^4(m+1)(ef-dg)}-\frac{c^2(3dg+ef)(d+ex)^{m+3}}{e^4g^2(m+3)}+\frac{c^2(d+ex)^{m+4}}{e^4g(m+4)}$$

input
```
Int[((d + e*x)^m*(a + c*x^2)^2)/(f + g*x),x]
```

output
```
-((c*(e*f + d*g)*(c*e^2*f^2 + c*d^2*g^2 + 2*a*e^2*g^2)*(d + e*x)^(1 + m))/
(e^4*g^4*(1 + m))) + (c*(2*a*e^2*g^2 + c*(e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2)
)*(d + e*x)^(2 + m))/(e^4*g^3*(2 + m)) - (c^2*(e*f + 3*d*g)*(d + e*x)^(3 +
 m))/(e^4*g^2*(3 + m)) + (c^2*(d + e*x)^(4 + m))/(e^4*g*(4 + m)) + ((c*f^2
 + a*g^2)^2*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d +
 e*x))/(e*f - d*g))])/(g^4*(e*f - d*g)*(1 + m))
```

Defintions of rubi rules used

rule 652
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c
*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex+d)^m\,(c\,x^2+a)^2}{gx+f}dx$$

input
```
int((e*x+d)^m*(c*x^2+a)^2/(g*x+f),x)
```

output
```
int((e*x+d)^m*(c*x^2+a)^2/(g*x+f),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m (a+cx^2)^2}{f+gx}\,dx = \int \frac{(cx^2+a)^2(ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^2/(g*x+f),x, algorithm="fricas")
```

output
```
integral((c^2*x^4 + 2*a*c*x^2 + a^2)*(e*x + d)^m/(g*x + f), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m (a+cx^2)^2}{f+gx}\,dx = \int \frac{(a+cx^2)^2 (d+ex)^m}{f+gx}\,dx$$

input
```
integrate((e*x+d)**m*(c*x**2+a)**2/(g*x+f),x)
```

output
```
Integral((a + c*x**2)**2*(d + e*x)**m/(f + g*x), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m (a+cx^2)^2}{f+gx}\,dx = \int \frac{(cx^2+a)^2(ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^2/(g*x+f),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^2*(e*x + d)^m/(g*x + f), x)
```

Giac [F]

$$\int \frac{(d+ex)^m (a+cx^2)^2}{f+gx} \, dx = \int \frac{(cx^2+a)^2 (ex+d)^m}{gx+f} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^2/(g*x+f),x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^2*(e*x + d)^m/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+cx^2)^2}{f+gx} \, dx = \int \frac{(c\,x^2+a)^2 \,(d+e\,x)^m}{f+g\,x} \, dx$$

input
```
int(((a + c*x^2)^2*(d + e*x)^m)/(f + g*x),x)
```

output
```
int(((a + c*x^2)^2*(d + e*x)^m)/(f + g*x), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m (a+cx^2)^2}{f+gx} \, dx = \text{too large to display}$$

input
```
int((e*x+d)^m*(c*x^2+a)^2/(g*x+f),x)
```

output
```
((d + e*x)**m*a**2*d*e**3*g**4*m**4 + 10*(d + e*x)**m*a**2*d*e**3*g**4*m**
3 + 35*(d + e*x)**m*a**2*d*e**3*g**4*m**2 + 50*(d + e*x)**m*a**2*d*e**3*g*
*4*m + 24*(d + e*x)**m*a**2*d*e**3*g**4 - 2*(d + e*x)**m*a*c*d**2*e**2*f*g
**3*m**3 - 14*(d + e*x)**m*a*c*d**2*e**2*f*g**3*m**2 - 24*(d + e*x)**m*a*c
*d**2*e**2*f*g**3*m + 2*(d + e*x)**m*a*c*d*e**3*f**2*g**2*m**3 + 18*(d + e
*x)**m*a*c*d*e**3*f**2*g**2*m**2 + 52*(d + e*x)**m*a*c*d*e**3*f**2*g**2*m
+ 48*(d + e*x)**m*a*c*d*e**3*f**2*g**2 + 2*(d + e*x)**m*a*c*d*e**3*f*g**3*
m**4*x + 14*(d + e*x)**m*a*c*d*e**3*f*g**3*m**3*x + 24*(d + e*x)**m*a*c*d*
e**3*f*g**3*m**2*x - 2*(d + e*x)**m*a*c*e**4*f**2*g**2*m**4*x - 18*(d + e*
x)**m*a*c*e**4*f**2*g**2*m**3*x - 52*(d + e*x)**m*a*c*e**4*f**2*g**2*m**2*
x - 48*(d + e*x)**m*a*c*e**4*f**2*g**2*m*x + 2*(d + e*x)**m*a*c*e**4*f*g**
3*m**4*x**2 + 16*(d + e*x)**m*a*c*e**4*f*g**3*m**3*x**2 + 38*(d + e*x)**m*
a*c*e**4*f*g**3*m**2*x**2 + 24*(d + e*x)**m*a*c*e**4*f*g**3*m*x**2 - 6*(d
+ e*x)**m*c**2*d**4*f*g**3*m - 2*(d + e*x)**m*c**2*d**3*e*f**2*g**2*m**2 -
 8*(d + e*x)**m*c**2*d**3*e*f**2*g**2*m + 6*(d + e*x)**m*c**2*d**3*e*f*g**
3*m**2*x - (d + e*x)**m*c**2*d**2*e**2*f**3*g*m**3 - 7*(d + e*x)**m*c**2*d
**2*e**2*f**3*g*m**2 - 12*(d + e*x)**m*c**2*d**2*e**2*f**3*g*m + 2*(d + e*
x)**m*c**2*d**2*e**2*f**2*g**2*m**3*x + 8*(d + e*x)**m*c**2*d**2*e**2*f**2
*g**2*m**2*x - 3*(d + e*x)**m*c**2*d**2*e**2*f*g**3*m**3*x**2 - 3*(d + e*x
)**m*c**2*d**2*e**2*f*g**3*m**2*x**2 + (d + e*x)**m*c**2*d*e**3*f**4*m*...
```

3.159 $\int \frac{(d+ex)^m (a+cx^2)^2}{(f+gx)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 245

$$\int \frac{(d+ex)^m (a+cx^2)^2}{(f+gx)^2} dx = \frac{c(2ae^2g^2+c(3e^2f^2+2defg+d^2g^2))(d+ex)^{1+m}}{e^3g^4(1+m)} - \frac{2c^2(ef+dg)(d+ex)^{2+m}}{e^3g^3(2+m)} + \frac{c^2(d+ex)^{3+m}}{e^3g^2(3+m)} - \frac{4cf(cf^2+ag^2)(d+ex)^{1+m}}{eg^4m(f+gx)} + \frac{(cf^2+ag^2)(aeg^2m-cf(4dg-ef(4+m)))(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{g(d+ex)}{ef-dg}\right)}{g^4(ef-dg)^2m(1+m)}$$

output
```
c*(2*a*e^2*g^2+c*(d^2*g^2+2*d*e*f*g+3*e^2*f^2))*(e*x+d)^(1+m)/e^3/g^4/(1+m
)-2*c^2*(d*g+e*f)*(e*x+d)^(2+m)/e^3/g^3/(2+m)+c^2*(e*x+d)^(3+m)/e^3/g^2/(3
+m)-4*c*f*(a*g^2+c*f^2)*(e*x+d)^(1+m)/e/g^4/m/(g*x+f)+(a*g^2+c*f^2)*(a*e*g
^2*m-c*f*(4*d*g-e*f*(4+m)))*(e*x+d)^(1+m)*hypergeom([2, 1+m],[2+m],-g*(e*x
+d)/(-d*g+e*f))/g^4/(-d*g+e*f)^2/m/(1+m)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^m \left(a+cx^2\right)^2}{(f+gx)^2} \, dx = \frac{(d+ex)^{1+m} \left(\frac{c(2ae^2g^2+c(3e^2f^2+2defg+d^2g^2))}{e^3(1+m)} - \frac{2c^2g(ef+dg)(d+ex)}{e^3(2+m)} + \frac{c^2g^2(d+ex)^2}{e^3(3+m)} - \frac{4cf\left(cf^2+ag^2\right) \text{Hypergeometric2F1}\left(1,1\right.}{(ef-dg)(1+m)}\right.}{g^4}$$

input
```
Integrate[((d + e*x)^m*(a + c*x^2)^2)/(f + g*x)^2,x]
```

output
```
((d + e*x)^(1 + m)*((c*(2*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g + d^2*g^2))
)/(e^3*(1 + m)) - (2*c^2*g*(e*f + d*g)*(d + e*x))/(e^3*(2 + m)) + (c^2*g^2
*(d + e*x)^2)/(e^3*(3 + m)) - (4*c*f*(c*f^2 + a*g^2)*Hypergeometric2F1[1,
1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)*(1 + m)) + (e*(c
*f^2 + a*g^2)^2*Hypergeometric2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) +
 d*g)])/((e*f - d*g)^2*(1 + m))))/g^4
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {650, 25, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^2 (d+ex)^m}{(f+gx)^2} \, dx$$

$$\Big\downarrow \; 650$$

$$\frac{\int -\frac{(d+ex)^m\left(-\frac{c^2(dg-ef(m+1))f^3}{g^4}+\frac{c^2(ef-dg)x^2f}{g^2}-\frac{2ac(dg-ef(m+1))f}{g^2}+c^2\left(d-\frac{ef}{g}\right)x^3+a^2em-\frac{c(ef-dg)\left(cf^2+2ag^2\right)x}{g^3}\right)}{f+gx}dx}{ef-dg}+\frac{\left(ag^2+cf^2\right)^2(d+ex)^{m+1}}{g^4(f+gx)(ef-dg)}$$

$$\Big\downarrow \; 25$$

$$\frac{\left(ag^2+cf^2\right)^2(d+ex)^{m+1}}{g^4(f+gx)(ef-dg)}-\frac{\int \frac{(d+ex)^m\left(-\frac{c^2(dg-ef(m+1))f^3}{g^4}+\frac{c^2(ef-dg)x^2f}{g^2}-\frac{2ac(dg-ef(m+1))f}{g^2}+c^2\left(d-\frac{ef}{g}\right)x^3+a^2em-\frac{c(ef-dg)\left(cf^2+2ag^2\right)x}{g^3}\right)}{f+gx}dx}{ef-dg}$$

$$\Big\downarrow \; 2123$$

$$\frac{\left(ag^2+cf^2\right)^2(d+ex)^{m+1}}{g^4(f+gx)(ef-dg)}-\frac{\int \left(\frac{c(ef-dg)\left(-2ae^2g^2-c\left(3e^2f^2+2degf+d^2g^2\right)\right)(d+ex)^m}{e^2g^4}+\frac{\left(cf^2+ag^2\right)\left(aeg^2m-cf(4dg-ef(m+4))\right)(d+ex)^m}{g^4(f+gx)}+\frac{2c^2\left(e^2f^2-d^2g^2\right)(d+ex)^m}{e^2g^3}\right.}{ef-dg}$$

$$\Big\downarrow \; 2009$$

$$\frac{\left(ag^2+cf^2\right)^2(d+ex)^{m+1}}{g^4(f+gx)(ef-dg)}-\frac{-\frac{c(ef-dg)(d+ex)^{m+1}\left(2ae^2g^2+c\left(d^2g^2+2defg+3e^2f^2\right)\right)}{e^3g^4(m+1)}+\frac{\left(ag^2+cf^2\right)(d+ex)^{m+1}\left(aeg^2m-cf(4dg-ef(m+4))\right)\text{Hypergeometric2F1}\left(1,m+1\right.}{g^4(m+1)(ef-dg)}}{ef-dg}$$

input

```
Int[((d + e*x)^m*(a + c*x^2)^2)/(f + g*x)^2,x]
```

output

```
((c*f^2 + a*g^2)^2*(d + e*x)^(1 + m))/(g^4*(e*f - d*g)*(f + g*x)) - (-((c*
(e*f - d*g)*(2*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g + d^2*g^2))*(d + e*x)^
(1 + m))/(e^3*g^4*(1 + m))) + (2*c^2*(e*f - d*g)*(e*f + d*g)*(d + e*x)^(2
+ m))/(e^3*g^3*(2 + m)) - (c^2*(e*f - d*g)*(d + e*x)^(3 + m))/(e^3*g^2*(3
+ m)) + ((c*f^2 + a*g^2)*(a*e*g^2*m - c*f*(4*d*g - e*f*(4 + m)))*(d + e*x)
^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])
/(g^4*(e*f - d*g)*(1 + m)))/(e*f - d*g)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 650
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^2)^p, d + e*x
, x], R = PolynomialRemainder[(a + c*x^2)^p, d + e*x, x]}, Simp[R*(d + e*x)
^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))), x] + Simp[1/((m + 1)*(e
*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*ExpandToSum[(m + 1)*(e*f - d
*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g, n}, x] &&
 IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [F]

$$\int \frac{(ex+d)^m (c\,x^2+a)^2}{(gx+f)^2}dx$$

input
```
int((e*x+d)^m*(c*x^2+a)^2/(g*x+f)^2,x)
```

output
```
int((e*x+d)^m*(c*x^2+a)^2/(g*x+f)^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m (a+cx^2)^2}{(f+gx)^2}\,dx = \int \frac{(cx^2+a)^2(ex+d)^m}{(gx+f)^2}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^2/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral((c^2*x^4 + 2*a*c*x^2 + a^2)*(e*x + d)^m/(g^2*x^2 + 2*f*g*x + f^2)
, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m (a+cx^2)^2}{(f+gx)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*(c*x**2+a)**2/(g*x+f)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(d+ex)^m (a+cx^2)^2}{(f+gx)^2}\,dx = \int \frac{(cx^2+a)^2(ex+d)^m}{(gx+f)^2}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^2/(g*x+f)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^2*(e*x + d)^m/(g*x + f)^2, x)
```

Giac [F]

$$\int \frac{(d+ex)^m (a+cx^2)^2}{(f+gx)^2} \, dx = \int \frac{(cx^2+a)^2 (ex+d)^m}{(gx+f)^2} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^2/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^2*(e*x + d)^m/(g*x + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+cx^2)^2}{(f+gx)^2} \, dx = \int \frac{(c\,x^2+a)^2 \,(d+e\,x)^m}{(f+g\,x)^2} \, dx$$

input
```
int(((a + c*x^2)^2*(d + e*x)^m)/(f + g*x)^2,x)
```

output
```
int(((a + c*x^2)^2*(d + e*x)^m)/(f + g*x)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex)^m (a+cx^2)^2}{(f+gx)^2} \, dx = \text{too large to display}$$

input
```
int((e*x+d)^m*(c*x^2+a)^2/(g*x+f)^2,x)
```

output
```
( - (d + e*x)**m*a**2*d*e**3*g**4*m**4 - 6*(d + e*x)**m*a**2*d*e**3*g**4*m
**3 - 11*(d + e*x)**m*a**2*d*e**3*g**4*m**2 - 6*(d + e*x)**m*a**2*d*e**3*g
**4*m + 2*(d + e*x)**m*a*c*d**2*e**2*f*g**3*m**3 + 10*(d + e*x)**m*a*c*d**
2*e**2*f*g**3*m**2 + 12*(d + e*x)**m*a*c*d**2*e**2*f*g**3*m + 2*(d + e*x)*
*m*a*c*d**2*e**2*g**4*m**3*x + 10*(d + e*x)**m*a*c*d**2*e**2*g**4*m**2*x +
 12*(d + e*x)**m*a*c*d**2*e**2*g**4*m*x - 2*(d + e*x)**m*a*c*d*e**3*f**2*g
**2*m**3 - 14*(d + e*x)**m*a*c*d*e**3*f**2*g**2*m**2 - 32*(d + e*x)**m*a*c
*d*e**3*f**2*g**2*m - 24*(d + e*x)**m*a*c*d*e**3*f**2*g**2 - 2*(d + e*x)**
m*a*c*d*e**3*f*g**3*m**4*x - 12*(d + e*x)**m*a*c*d*e**3*f*g**3*m**3*x - 26
*(d + e*x)**m*a*c*d*e**3*f*g**3*m**2*x - 32*(d + e*x)**m*a*c*d*e**3*f*g**3
*m*x - 24*(d + e*x)**m*a*c*d*e**3*f*g**3*x + 2*(d + e*x)**m*a*c*d*e**3*g**
4*m**3*x**2 + 10*(d + e*x)**m*a*c*d*e**3*g**4*m**2*x**2 + 12*(d + e*x)**m*
a*c*d*e**3*g**4*m*x**2 + 2*(d + e*x)**m*a*c*e**4*f**2*g**2*m**4*x + 14*(d
+ e*x)**m*a*c*e**4*f**2*g**2*m**3*x + 32*(d + e*x)**m*a*c*e**4*f**2*g**2*m
**2*x + 24*(d + e*x)**m*a*c*e**4*f**2*g**2*m*x - 2*(d + e*x)**m*a*c*e**4*f
*g**3*m**4*x**2 - 10*(d + e*x)**m*a*c*e**4*f*g**3*m**3*x**2 - 12*(d + e*x)
**m*a*c*e**4*f*g**3*m**2*x**2 + 2*(d + e*x)**m*c**2*d**4*f*g**3*m + 2*(d +
 e*x)**m*c**2*d**4*g**4*m*x + 6*(d + e*x)**m*c**2*d**3*e*f**2*g**2*m - 2*(
d + e*x)**m*c**2*d**3*e*f*g**3*m**2*x + 6*(d + e*x)**m*c**2*d**3*e*f*g**3*
m*x - 2*(d + e*x)**m*c**2*d**3*e*g**4*m**2*x**2 + (d + e*x)**m*c**2*d**...
```

3.160 $\int \frac{(d+ex)^m \left(a+cx^2\right)^2}{(f+gx)^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 302

$$\int \frac{(d+ex)^m \left(a+cx^2\right)^2}{(f+gx)^3} \, dx = -\frac{c^2(3ef+dg)(d+ex)^{1+m}}{e^2g^4(1+m)} + \frac{c^2(d+ex)^{2+m}}{e^2g^3(2+m)} + \frac{\left(cf^2+ag^2\right)^2(d+ex)^{1+m}}{2g^4(ef-dg)(f+gx)^2} + \frac{2c\left(3cf^2+ag^2\right)(d+ex)^{1+m}}{eg^4m(f+gx)} + \frac{\left(a^2e^2g^4(1-m)m - 2acg^2\left(2d^2g^2-4defg(1+m)+e^2f^2\left(2+3m+m^2\right)\right) - c^2f^2\left(12d^2g^2-8defg(3+ \dots\right.\right.}{2g^4(ef-dg)^3m(\dots}$$

output

```
-c^2*(d*g+3*e*f)*(e*x+d)^(1+m)/e^2/g^4/(1+m)+c^2*(e*x+d)^(2+m)/e^2/g^3/(2+
m)+1/2*(a*g^2+c*f^2)^2*(e*x+d)^(1+m)/g^4/(-d*g+e*f)/(g*x+f)^2+2*c*(a*g^2+3
*c*f^2)*(e*x+d)^(1+m)/e/g^4/m/(g*x+f)+1/2*(a^2*e^2*g^4*(1-m)*m-2*a*c*g^2*(
2*d^2*g^2-4*d*e*f*g*(1+m)+e^2*f^2*(m^2+3*m+2))-c^2*f^2*(12*d^2*g^2-8*d*e*f
*g*(3+m)+e^2*f^2*(m^2+7*m+12)))*(e*x+d)^(1+m)*hypergeom([2, 1+m],[2+m],-g*
(e*x+d)/(-d*g+e*f))/g^4/(-d*g+e*f)^3/m/(1+m)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^m (a+cx^2)^2}{(f+gx)^3}\,dx$$

$$= \frac{(d+ex)^{1+m}\left(-\frac{c^2(3ef+dg)}{e^2(1+m)}+\frac{c^2g(d+ex)}{e^2(2+m)}+\frac{2c(3cf^2+ag^2)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{g(d+ex)}{-ef+dg}\right)}{(ef-dg)(1+m)}-\frac{4cef(cf^2+ag^2)\,\text{Hyperge}}{(e}\right.}{g^4}$$

input
```
Integrate[((d + e*x)^m*(a + c*x^2)^2)/(f + g*x)^3,x]
```

output
```
((d + e*x)^(1 + m)*(-((c^2*(3*e*f + d*g))/(e^2*(1 + m))) + (c^2*g*(d + e*x
))/(e^2*(2 + m)) + (2*c*(3*c*f^2 + a*g^2)*Hypergeometric2F1[1, 1 + m, 2 +
m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)*(1 + m)) - (4*c*e*f*(c*f^2
+ a*g^2)*Hypergeometric2F1[2, 1 + m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])
/((e*f - d*g)^2*(1 + m)) + (e^2*(c*f^2 + a*g^2)^2*Hypergeometric2F1[3, 1 +
 m, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)])/((e*f - d*g)^3*(1 + m))))/g^4
```

Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {650, 2124, 25, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2 (d+ex)^m}{(f+gx)^3}\,dx$$

$$\Big\downarrow \ 650$$

$$\frac{\int \frac{(d+ex)^m\left(\frac{c^2(2dg-ef(m+1))f^3}{g^4}-\frac{2c^2(ef-dg)x^2f}{g^2}+\frac{2ac(2dg-ef(m+1))f}{g^2}-2c^2\left(d-\frac{ef}{g}\right)x^3+a^2(e-em)+\frac{2c(ef-dg)\left(cf^2+2ag^2\right)x}{g^3}\right)}{(f+gx)^2}dx}{2(ef-dg)}+$$

$$\frac{\left(ag^2+cf^2\right)^2(d+ex)^{m+1}}{2g^4(f+gx)^2(ef-dg)}$$

$$\Big\downarrow \; 2124$$

$$\frac{\int -\frac{\frac{(d+ex)^m\left(a^2(1-m)me^2-\frac{2c^2(ef-dg)^2x^2}{g^2}-\frac{2ac\left(e^2\left(m^2+3m+2\right)f^2-4deg(m+1)f+2d^2g^2\right)}{g^2}-\frac{c^2f^2\left(e^2\left(m^2+7m+6\right)f^2-4deg(2m+3)f+6d^2g^2\right)}{g^4}+\frac{4c^2f(ef-\cdots}{g^3}\cdots\right.}{f+gx}}{ef-dg}\cdots}{2(ef-dg)}\cdots$$

$$\frac{\left(ag^2+cf^2\right)^2(d+ex)^{m+1}}{2g^4(f+gx)^2(ef-dg)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\left(ag^2+cf^2\right)(d+ex)^{m+1}\left(aeg^2(1-m)+cf(8dg-ef(m+7))\right)}{g^4(f+gx)(ef-dg)}-\int\frac{\frac{(d+ex)^m\left(a^2(1-m)me^2-\frac{2c^2(ef-dg)^2x^2}{g^2}-\frac{2ac\left(e^2\left(m^2+3m+2\right)f^2-4deg(m+1)f+2d^2g\cdots\right.}{g^2}\cdots\right.}{f+gx}}{ef-dg}\cdots}{2(ef-dg)}\cdots$$

$$\frac{\left(ag^2+cf^2\right)^2(d+ex)^{m+1}}{2g^4(f+gx)^2(ef-dg)}$$

$$\Big\downarrow \; 1195$$

$$\frac{\frac{\left(ag^2+cf^2\right)(d+ex)^{m+1}\left(aeg^2(1-m)+cf(8dg-ef(m+7))\right)}{g^4(f+gx)(ef-dg)}-\frac{\int\left(\frac{2c^2(ef-dg)^2(3ef+dg)(d+ex)^m}{eg^4}+\frac{\left(a^2e^2(1-m)mg^4-2ac\left(e^2\left(m^2+3m+2\right)f^2-4deg(m-\cdots\right.\right.}{\cdots}\cdots\right.}{\cdots}}{2(ef-dg)}\cdots$$

$$\frac{\left(ag^2+cf^2\right)^2(d+ex)^{m+1}}{2g^4(f+gx)^2(ef-dg)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\left(ag^2+cf^2\right)(d+ex)^{m+1}\left(aeg^2(1-m)+cf(8dg-ef(m+7))\right)}{g^4(f+gx)(ef-dg)}-\frac{\frac{(d+ex)^{m+1}\left(a^2e^2g^4(1-m)m-2acg^2\left(2d^2g^2-4defg(m+1)+e^2f^2\left(m^2+3m+2\right)\right)-c^2f^2\left(12\cdots\right.\right.}{g^4(m+1)\cdots}}{\cdots}}{2(ef-\cdots}$$

$$\frac{\left(ag^2+cf^2\right)^2(d+ex)^{m+1}}{2g^4(f+gx)^2(ef-dg)}$$

input
```
Int[((d + e*x)^m*(a + c*x^2)^2)/(f + g*x)^3,x]
```

output
```
((c*f^2 + a*g^2)^2*(d + e*x)^(1 + m))/(2*g^4*(e*f - d*g)*(f + g*x)^2) + ((
(c*f^2 + a*g^2)*(a*e*g^2*(1 - m) + c*f*(8*d*g - e*f*(7 + m)))*(d + e*x)^(1
 + m))/(g^4*(e*f - d*g)*(f + g*x)) - ((2*c^2*(e*f - d*g)^2*(3*e*f + d*g)*(
d + e*x)^(1 + m))/(e^2*g^4*(1 + m)) - (2*c^2*(e*f - d*g)^2*(d + e*x)^(2 +
m))/(e^2*g^3*(2 + m)) + ((a^2*e^2*g^4*(1 - m)*m - 2*a*c*g^2*(2*d^2*g^2 - 4
*d*e*f*g*(1 + m) + e^2*f^2*(2 + 3*m + m^2)) - c^2*f^2*(12*d^2*g^2 - 8*d*e*
f*g*(3 + m) + e^2*f^2*(12 + 7*m + m^2)))*(d + e*x)^(1 + m)*Hypergeometric2
F1[1, 1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))])/(g^4*(e*f - d*g)*(1 + m
)))/(e*f - d*g))/(2*(e*f - d*g))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 650
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^2)^p, d + e*x
, x], R = PolynomialRemainder[(a + c*x^2)^p, d + e*x, x]}, Simp[R*(d + e*x)
^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))), x] + Simp[1/((m + 1)*(e
*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*ExpandToSum[(m + 1)*(e*f - d
*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g, n}, x] &&
 IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [F]

$$\int \frac{(ex+d)^m (c\,x^2+a)^2}{(gx+f)^3}dx$$

input
```
int((e*x+d)^m*(c*x^2+a)^2/(g*x+f)^3,x)
```

output
```
int((e*x+d)^m*(c*x^2+a)^2/(g*x+f)^3,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m (a+cx^2)^2}{(f+gx)^3}\,dx = \int \frac{(cx^2+a)^2(ex+d)^m}{(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^2/(g*x+f)^3,x, algorithm="fricas")
```

output
```
integral((c^2*x^4 + 2*a*c*x^2 + a^2)*(e*x + d)^m/(g^3*x^3 + 3*f*g^2*x^2 +
3*f^2*g*x + f^3), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m (a+cx^2)^2}{(f+gx)^3}\,dx = \int \frac{(a+cx^2)^2 (d+ex)^m}{(f+gx)^3}\,dx$$

input
```
integrate((e*x+d)**m*(c*x**2+a)**2/(g*x+f)**3,x)
```

output
```
Integral((a + c*x**2)**2*(d + e*x)**m/(f + g*x)**3, x)
```

Maxima [F]

$$\int \frac{(d+ex)^m (a+cx^2)^2}{(f+gx)^3}\,dx = \int \frac{(cx^2+a)^2(ex+d)^m}{(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^2/(g*x+f)^3,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^2*(e*x + d)^m/(g*x + f)^3, x)
```

Giac [F]

$$\int \frac{(d+ex)^m (a+cx^2)^2}{(f+gx)^3}\,dx = \int \frac{(cx^2+a)^2(ex+d)^m}{(gx+f)^3}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^2/(g*x+f)^3,x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^2*(e*x + d)^m/(g*x + f)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+cx^2)^2}{(f+gx)^3} \, dx = \int \frac{\left(c\,x^2+a\right)^2 \,\left(d+e\,x\right)^m}{\left(f+g\,x\right)^3} \,dx$$

input
```
int(((a + c*x^2)^2*(d + e*x)^m)/(f + g*x)^3,x)
```

output
```
int(((a + c*x^2)^2*(d + e*x)^m)/(f + g*x)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex)^m (a+cx^2)^2}{(f+gx)^3} \, dx = \text{too large to display}$$

input
```
int((e*x+d)^m*(c*x^2+a)^2/(g*x+f)^3,x)
```

output

```
( - (d + e*x)**m*a**2*d*e**2*g**4*m**4 - 2*(d + e*x)**m*a**2*d*e**2*g**4*m
**3 + (d + e*x)**m*a**2*d*e**2*g**4*m**2 + 2*(d + e*x)**m*a**2*d*e**2*g**4
*m + 2*(d + e*x)**m*a*c*d**2*e*f*g**3*m**3 + 6*(d + e*x)**m*a*c*d**2*e*f*g
**3*m**2 + 4*(d + e*x)**m*a*c*d**2*e*f*g**3*m + 4*(d + e*x)**m*a*c*d**2*e*
g**4*m**3*x + 12*(d + e*x)**m*a*c*d**2*e*g**4*m**2*x + 8*(d + e*x)**m*a*c*
d**2*e*g**4*m*x - 2*(d + e*x)**m*a*c*d*e**2*f**2*g**2*m**3 - 10*(d + e*x)*
*m*a*c*d*e**2*f**2*g**2*m**2 - 16*(d + e*x)**m*a*c*d*e**2*f**2*g**2*m - 8*
(d + e*x)**m*a*c*d*e**2*f**2*g**2 - 2*(d + e*x)**m*a*c*d*e**2*f*g**3*m**4*
x - 10*(d + e*x)**m*a*c*d*e**2*f*g**3*m**3*x - 24*(d + e*x)**m*a*c*d*e**2*
f*g**3*m**2*x - 32*(d + e*x)**m*a*c*d*e**2*f*g**3*m*x - 16*(d + e*x)**m*a*
c*d*e**2*f*g**3*x + 4*(d + e*x)**m*a*c*d*e**2*g**4*m**3*x**2 + 8*(d + e*x)
**m*a*c*d*e**2*g**4*m**2*x**2 - 4*(d + e*x)**m*a*c*d*e**2*g**4*m*x**2 - 8*
(d + e*x)**m*a*c*d*e**2*g**4*x**2 + 2*(d + e*x)**m*a*c*e**3*f**2*g**2*m**4
*x + 10*(d + e*x)**m*a*c*e**3*f**2*g**2*m**3*x + 16*(d + e*x)**m*a*c*e**3*
f**2*g**2*m**2*x + 8*(d + e*x)**m*a*c*e**3*f**2*g**2*m*x - 2*(d + e*x)**m*
a*c*e**3*f*g**3*m**4*x**2 - 4*(d + e*x)**m*a*c*e**3*f*g**3*m**3*x**2 + 2*(
d + e*x)**m*a*c*e**3*f*g**3*m**2*x**2 + 4*(d + e*x)**m*a*c*e**3*f*g**3*m*x
**2 - 2*(d + e*x)**m*c**2*d**3*f**2*g**2*m**2 + 2*(d + e*x)**m*c**2*d**3*f
**2*g**2*m - 4*(d + e*x)**m*c**2*d**3*f*g**3*m**2*x + 4*(d + e*x)**m*c**2*
d**3*f*g**3*m*x - 2*(d + e*x)**m*c**2*d**3*g**4*m**2*x**2 + 2*(d + e*x)...
```

3.161 $\int \frac{(d+ex)^m(a+cx^2)}{(e+fx)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 202

$$\int \frac{(d+ex)^m(a+cx^2)}{(e+fx)^{3/2}}\,dx = \frac{2(ce^2+af^2)(d+ex)^{1+m}}{f^2(e^2-df)\sqrt{e+fx}} + \frac{2c(d+ex)^{1+m}\sqrt{e+fx}}{ef^2(3+2m)} + \frac{2(ae^2f^2(3+8m+4m^2)-c(d^2f^2+4de^2f(1+m)-4e^4(2+3m+m^2)))(d+ex)^{1+m}\sqrt{e+fx}\,\text{Hypergeo}}{ef^2(e^2-df)^2(3+2m)}$$

output
```
2*(a*f^2+c*e^2)*(e*x+d)^(1+m)/f^2/(-d*f+e^2)/(f*x+e)^(1/2)+2*c*(e*x+d)^(1+
m)*(f*x+e)^(1/2)/e/f^2/(3+2*m)+2*(a*e^2*f^2*(4*m^2+8*m+3)-c*(d^2*f^2+4*d*e
^2*f*(1+m)-4*e^4*(m^2+3*m+2)))*(e*x+d)^(1+m)*(f*x+e)^(1/2)*hypergeom([1, 3
/2+m],[3/2],e*(f*x+e)/(-d*f+e^2))/e/f^2/(-d*f+e^2)^2/(3+2*m)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^m(a+cx^2)}{(e+fx)^{3/2}}\,dx = \frac{2(d+ex)^m\left(\frac{f(d+ex)}{-e^2+df}\right)^{-m}\left(-3(ce^2+af^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-m,\frac{1}{2},\frac{e(e}{e^2}\right.\right.}{}$$

input
```
Integrate[((d + e*x)^m*(a + c*x^2))/(e + f*x)^(3/2),x]
```

output
```
(2*(d + e*x)^m*(-3*(c*e^2 + a*f^2)*Hypergeometric2F1[-1/2, -m, 1/2, (e*(e
+ f*x))/(e^2 - d*f)] - c*(e + f*x)*(6*e*Hypergeometric2F1[1/2, -m, 3/2, (e
*(e + f*x))/(e^2 - d*f)] - (e + f*x)*Hypergeometric2F1[3/2, -m, 5/2, (e*(e
 + f*x))/(e^2 - d*f)])))/(3*f^3*((f*(d + e*x))/(-e^2 + d*f))^m*Sqrt[e + f*
x])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {650, 27, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(d+ex)^m}{(e+fx)^{3/2}}\,dx$$

$$\Big\downarrow \text{650}$$

$$\frac{2\int \frac{(d+ex)^m\left(e\left(-2c(m+1)e^2+cdf-af^2(2m+1)\right)-c\left(d-\frac{e^2}{f}\right)f^2x\right)}{2f^2\sqrt{e+fx}}dx}{e^2-df}+\frac{2\left(a+\frac{ce^2}{f^2}\right)(d+ex)^{m+1}}{(e^2-df)\sqrt{e+fx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d+ex)^m\left(e\left(-2c(m+1)e^2+cdf-af^2(2m+1)\right)+cf\left(e^2-df\right)x\right)}{\sqrt{e+fx}}dx}{f^2(e^2-df)}+\frac{2\left(a+\frac{ce^2}{f^2}\right)(d+ex)^{m+1}}{(e^2-df)\sqrt{e+fx}}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{2c(e^2-df)\sqrt{e+fx}(d+ex)^{m+1}}{e(2m+3)}-\frac{\left(ae^2f^2(4m^2+8m+3)-c\left(d^2f^2+4de^2f(m+1)-4e^4(m^2+3m+2)\right)\right)\int \frac{(d+ex)^m}{\sqrt{e+fx}}dx}{e(2m+3)}}{f^2(e^2-df)}+\frac{2\left(a+\frac{ce^2}{f^2}\right)(d+ex)^{m+1}}{(e^2-df)\sqrt{e+fx}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\frac{2c(e^2-df)\sqrt{e+fx}(d+ex)^{m+1}}{e(2m+3)}-\frac{(d+ex)^m\left(-\frac{f(d+ex)}{e^2-df}\right)^{-m}\left(ae^2f^2\left(4m^2+8m+3\right)-c\left(d^2f^2+4de^2f(m+1)-4e^4\left(m^2+3m+2\right)\right)\right)\int\frac{\left(-\frac{exf}{e^2-df}-\frac{df}{e^2-}\right.}{\sqrt{e+fx}}}{e(2m+3)}}{f^2\left(e^2-df\right)}$$

$$\frac{2\left(a+\frac{ce^2}{f^2}\right)(d+ex)^{m+1}}{\left(e^2-df\right)\sqrt{e+fx}}$$

$$\Big\downarrow \text{79}$$

$$\frac{\frac{2c(e^2-df)\sqrt{e+fx}(d+ex)^{m+1}}{e(2m+3)}-\frac{2\sqrt{e+fx}(d+ex)^m\left(-\frac{f(d+ex)}{e^2-df}\right)^{-m}\left(ae^2f^2\left(4m^2+8m+3\right)-c\left(d^2f^2+4de^2f(m+1)-4e^4\left(m^2+3m+2\right)\right)\right)\text{Hyperge}}{ef(2m+3)}}{f^2\left(e^2-df\right)}$$

$$\frac{2\left(a+\frac{ce^2}{f^2}\right)(d+ex)^{m+1}}{\left(e^2-df\right)\sqrt{e+fx}}$$

input
```
Int[((d + e*x)^m*(a + c*x^2))/(e + f*x)^(3/2),x]
```

output
```
(2*(a + (c*e^2)/f^2)*(d + e*x)^(1 + m))/((e^2 - d*f)*Sqrt[e + f*x]) + ((2*
c*(e^2 - d*f)*(d + e*x)^(1 + m)*Sqrt[e + f*x])/(e*(3 + 2*m)) - (2*(a*e^2*f
^2*(3 + 8*m + 4*m^2) - c*(d^2*f^2 + 4*d*e^2*f*(1 + m) - 4*e^4*(2 + 3*m + m
^2)))*(d + e*x)^m*Sqrt[e + f*x]*Hypergeometric2F1[1/2, -m, 3/2, (e*(e + f*
x))/(e^2 - d*f)])/(e*f*(3 + 2*m)*(-((f*(d + e*x))/(e^2 - d*f)))^m))/(f^2*(
e^2 - d*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 650
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^2)^p, d + e*x
, x], R = PolynomialRemainder[(a + c*x^2)^p, d + e*x, x]}, Simp[R*(d + e*x)
^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))), x] + Simp[1/((m + 1)*(e
*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*ExpandToSum[(m + 1)*(e*f - d
*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g, n}, x] &&
 IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{(ex+d)^m\,(c\,x^2+a)}{(fx+e)^{\frac{3}{2}}}dx$$

input
```
int((e*x+d)^m*(c*x^2+a)/(f*x+e)^(3/2),x)
```

output
```
int((e*x+d)^m*(c*x^2+a)/(f*x+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m\,(a+cx^2)}{(e+fx)^{3/2}}\,dx=\int \frac{(cx^2+a)(ex+d)^m}{(fx+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)/(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)*sqrt(f*x + e)*(e*x + d)^m/(f^2*x^2 + 2*e*f*x + e^2),
x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^m\,(a+cx^2)}{(e+fx)^{3/2}}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*(c*x**2+a)/(f*x+e)**(3/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(d+ex)^m\,(a+cx^2)}{(e+fx)^{3/2}}\,dx=\int \frac{(cx^2+a)(ex+d)^m}{(fx+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)/(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)*(e*x + d)^m/(f*x + e)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^m\,(a+cx^2)}{(e+fx)^{3/2}}\,dx = \int \frac{(cx^2+a)(ex+d)^m}{(fx+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)/(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)*(e*x + d)^m/(f*x + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m\,(a+cx^2)}{(e+fx)^{3/2}}\,dx = \int \frac{(c\,x^2+a)\,\left(d+e\,x\right)^m}{\left(e+f\,x\right)^{3/2}}\,dx$$

input
```
int(((a + c*x^2)*(d + e*x)^m)/(e + f*x)^(3/2),x)
```

output
```
int(((a + c*x^2)*(d + e*x)^m)/(e + f*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m\,(a+cx^2)}{(e+fx)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((e*x+d)^m*(c*x^2+a)/(f*x+e)^(3/2),x)
```

output
```
(2*( - 4*sqrt(e + f*x)*(d + e*x)**m*a*d*e*f**2*m**2 - 8*sqrt(e + f*x)*(d +
 e*x)**m*a*d*e*f**2*m - 3*sqrt(e + f*x)*(d + e*x)**m*a*d*e*f**2 + 4*sqrt(e
 + f*x)*(d + e*x)**m*c*d**2*e*f*m + 2*sqrt(e + f*x)*(d + e*x)**m*c*d**2*f*
*2*m*x - 4*sqrt(e + f*x)*(d + e*x)**m*c*d*e**3*m - 8*sqrt(e + f*x)*(d + e*
x)**m*c*d*e**3 - 4*sqrt(e + f*x)*(d + e*x)**m*c*d*e**2*f*m**2*x - 2*sqrt(e
 + f*x)*(d + e*x)**m*c*d*e**2*f*m*x - 4*sqrt(e + f*x)*(d + e*x)**m*c*d*e**
2*f*x + 2*sqrt(e + f*x)*(d + e*x)**m*c*d*e*f**2*m*x**2 + sqrt(e + f*x)*(d
+ e*x)**m*c*d*e*f**2*x**2 + 4*sqrt(e + f*x)*(d + e*x)**m*c*e**4*m**2*x + 8
*sqrt(e + f*x)*(d + e*x)**m*c*e**4*m*x - 4*sqrt(e + f*x)*(d + e*x)**m*c*e*
*3*f*m**2*x**2 - 2*sqrt(e + f*x)*(d + e*x)**m*c*e**3*f*m*x**2 + 16*int((sq
rt(e + f*x)*(d + e*x)**m*x)/(4*d**2*e**2*f*m**2 + 8*d**2*e**2*f*m + 3*d**2
*e**2*f + 8*d**2*e*f**2*m**2*x + 16*d**2*e*f**2*m*x + 6*d**2*e*f**2*x + 4*
d**2*f**3*m**2*x**2 + 8*d**2*f**3*m*x**2 + 3*d**2*f**3*x**2 - 8*d*e**4*m**
3 - 16*d*e**4*m**2 - 6*d*e**4*m - 16*d*e**3*f*m**3*x - 28*d*e**3*f*m**2*x
- 4*d*e**3*f*m*x + 3*d*e**3*f*x - 8*d*e**2*f**2*m**3*x**2 - 8*d*e**2*f**2*
m**2*x**2 + 10*d*e**2*f**2*m*x**2 + 6*d*e**2*f**2*x**2 + 4*d*e*f**3*m**2*x
**3 + 8*d*e*f**3*m*x**3 + 3*d*e*f**3*x**3 - 8*e**5*m**3*x - 16*e**5*m**2*x
 - 6*e**5*m*x - 16*e**4*f*m**3*x**2 - 32*e**4*f*m**2*x**2 - 12*e**4*f*m*x*
*2 - 8*e**3*f**2*m**3*x**3 - 16*e**3*f**2*m**2*x**3 - 6*e**3*f**2*m*x**3),
x)*a*d**2*e**3*f**4*m**5 + 64*int((sqrt(e + f*x)*(d + e*x)**m*x)/(4*d**...
```

3.162 $\int (d+ex)^m(f+gx)^2\sqrt{a+cx^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 411

$$\int (d+ex)^m(f+gx)^2\sqrt{a+cx^2}\,dx = \frac{g^2(d+ex)^{1+m}\left(a+cx^2\right)^{3/2}}{ce(4+m)} + \frac{(cdg(3dg-2ef(4+m))-e^2(ag^2(1+m)-cf^2(4+m)))\,(d+ex)^{1+m}\sqrt{a+cx^2}\,\text{AppellF1}\left(1+m,-\right.}{ce^3(1+m)(4+m)\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}} - \frac{g(3dg-2ef(4+m))(d+ex)^{2+m}\sqrt{a+cx^2}\,\text{AppellF1}\left(2+m,-\frac{1}{2},-\frac{1}{2},3+m,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e^3(2+m)(4+m)\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}}$$

output
```
g^2*(e*x+d)^(1+m)*(c*x^2+a)^(3/2)/c/e/(4+m)+(c*d*g*(3*d*g-2*e*f*(4+m))-e^2
*(a*g^2*(1+m)-c*f^2*(4+m)))*(e*x+d)^(1+m)*(c*x^2+a)^(1/2)*AppellF1(1+m,-1/
2,-1/2,2+m,(e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2
)))/c/e^3/(1+m)/(4+m)/(1-(e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)))^(1/2)/(1-(e*x+d
)/(d+(-a)^(1/2)*e/c^(1/2)))^(1/2)-g*(3*d*g-2*e*f*(4+m))*(e*x+d)^(2+m)*(c*x
^2+a)^(1/2)*AppellF1(2+m,-1/2,-1/2,3+m,(e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)),(e
*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e^3/(2+m)/(4+m)/(1-(e*x+d)/(d-(-a)^(1/2)*e
/c^(1/2)))^(1/2)/(1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))^(1/2)
```

Mathematica [F]

$$\int (d+ex)^m (f+gx)^2 \sqrt{a+cx^2}\,dx = \int (d+ex)^m (f+gx)^2 \sqrt{a+cx^2}\,dx$$

input
```
Integrate[(d + e*x)^m*(f + g*x)^2*Sqrt[a + c*x^2],x]
```

output
```
Integrate[(d + e*x)^m*(f + g*x)^2*Sqrt[a + c*x^2], x]
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {743, 25, 27, 719, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}(f+gx)^2(d+ex)^m\,dx$$

$$\Big\downarrow \text{743}$$

$$\frac{\int -e(d+ex)^m\left(e\left(ag^2(m+1)-cf^2(m+4)\right)+cg(3dg-2ef(m+4))x\right)\sqrt{cx^2+a}dx}{ce^2(m+4)}+\frac{g^2\left(a+cx^2\right)^{3/2}(d+ex)^{m+1}}{ce(m+4)}$$

$$\Big\downarrow \text{25}$$

$$\frac{g^2\left(a+cx^2\right)^{3/2}(d+ex)^{m+1}}{ce(m+4)}-\frac{\int e(d+ex)^m\left(e\left(ag^2(m+1)-cf^2(m+4)\right)+cg(3dg-2ef(m+4))x\right)\sqrt{cx^2+a}dx}{ce^2(m+4)}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^2\left(a+cx^2\right)^{3/2}(d+ex)^{m+1}}{ce(m+4)}-\frac{\int (d+ex)^m\left(e\left(ag^2(m+1)-cf^2(m+4)\right)+cg(3dg-2ef(m+4))x\right)\sqrt{cx^2+a}dx}{ce(m+4)}$$

$\big\downarrow$ 719

$$\frac{g^2\left(a+cx^2\right)^{3/2}(d+ex)^{m+1}}{ce(m+4)}-\frac{\frac{cg(3dg-2ef(m+4))\int (d+ex)^{m+1}\sqrt{cx^2+a}dx}{e}-\frac{\left(cdg(3dg-2ef(m+4))-e^2\left(ag^2(m+1)-cf^2(m+4)\right)\right)\int (d+ex)^m\sqrt{cx^2+a}dx}{e}}{ce(m+4)}$$

$\big\downarrow$ 514

$$\frac{g^2\left(a+cx^2\right)^{3/2}(d+ex)^{m+1}}{ce(m+4)}-\frac{\frac{cg\sqrt{a+cx^2}(3dg-2ef(m+4))\int (d+ex)^{m+1}\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}}d(d+ex)}{e^2\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-ae}}{\sqrt{c}}+d}}}-\frac{\sqrt{a+cx^2}\left(cdg(3dg-2ef(m+4))-e^2\left(ag^2(m+1)-cf^2(m+4)\right)\right.}{e^2\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}}\sqrt{1}}}{ce(m+4)}$$

$\big\downarrow$ 150

$$\frac{g^2\left(a+cx^2\right)^{3/2}(d+ex)^{m+1}}{ce(m+4)}-\frac{\frac{cg\sqrt{a+cx^2}(d+ex)^{m+2}(3dg-2ef(m+4))\operatorname{AppellF1}\left(m+2,-\frac{1}{2},-\frac{1}{2},m+3,\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}\right)}{e^2(m+2)\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-ae}}{\sqrt{c}}+d}}}-\frac{\sqrt{a+cx^2}(d+ex)^{m+1}\operatorname{AppellF1}\left(m+1,-\frac{1}{2},-\frac{1}{2}\right.}{e^2}}{ce(m+4)}$$

input
```
Int[(d + e*x)^m*(f + g*x)^2*Sqrt[a + c*x^2],x]
```

output
```
(g^2*(d + e*x)^(1 + m)*(a + c*x^2)^(3/2))/(c*e*(4 + m)) - (-(((c*d*g*(3*d*
g - 2*e*f*(4 + m)) - e^2*(a*g^2*(1 + m) - c*f^2*(4 + m)))*(d + e*x)^(1 + m
)*Sqrt[a + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (d + e*x)/(d - (Sqrt[
-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e^2*(1 + m)*Sqrt[
1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c])]*Sqrt[1 - (d + e*x)/(d + (Sqrt[-a
]*e)/Sqrt[c])])) + (c*g*(3*d*g - 2*e*f*(4 + m))*(d + e*x)^(2 + m)*Sqrt[a +
 c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (d + e*x)/(d - (Sqrt[-a]*e)/Sqr
t[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e^2*(2 + m)*Sqrt[1 - (d + e
*x)/(d - (Sqrt[-a]*e)/Sqrt[c])]*Sqrt[1 - (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[
c])]))/(c*e*(4 + m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 743
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + c*x^2)^(p + 1)/
(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m + n + 2*p + 1))   I
nt[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c*e^n*(m + n + 2*p + 1)*(f + g*x)^
n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d + e*x)^(n - 2)*(a*e^2*(m +
 n - 1) - c*d^2*(m + n + 2*p + 1) - 2*c*d*e*(m + n + p)*x), x], x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && IGtQ[n, 1] && NeQ[m + n + 2*p + 1, 0]
```

Maple [F]

$$\int (ex+d)^m (gx+f)^2 \sqrt{c\,x^2+a}dx$$

input
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^(1/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int (d+ex)^m (f+gx)^2 \sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(gx+f)^2(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)*sqrt(c*x^2 + a)*(e*x + d)^m, x)
```

Sympy [F]

$$\int (d+ex)^m(f+gx)^2\sqrt{a+cx^2}\,dx = \int \sqrt{a+cx^2}(d+ex)^m\,(f+gx)^2\,dx$$

input
```
integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)**m*(f + g*x)**2, x)
```

Maxima [F]

$$\int (d+ex)^m(f+gx)^2\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(gx+f)^2(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(g*x + f)^2*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m(f+gx)^2\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(gx+f)^2(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(g*x + f)^2*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^2\sqrt{a+cx^2}\,dx = \int (f+g\,x)^2\,\sqrt{c\,x^2+a}\,(d+e\,x)^m\,dx$$

input
```
int((f + g*x)^2*(a + c*x^2)^(1/2)*(d + e*x)^m,x)
```

output
```
int((f + g*x)^2*(a + c*x^2)^(1/2)*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m(f+gx)^2\sqrt{a+cx^2}\,dx = \int (ex+d)^m\,(gx+f)^2\,\sqrt{c\,x^2+a}dx$$

input
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^(1/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^(1/2),x)
```

3.163 $\int (d+ex)^m(f+gx)\sqrt{a+cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 318

$$\int (d+ex)^m(f+gx)\sqrt{a+cx^2}\,dx = \frac{(ef-dg)(d+ex)^{1+m}\sqrt{a+cx^2}\,\text{AppellF1}\left(1+m,-\frac{1}{2},-\frac{1}{2},2+m,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e^2(1+m)\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}} + \frac{g(d+ex)^{2+m}\sqrt{a+cx^2}\,\text{AppellF1}\left(2+m,-\frac{1}{2},-\frac{1}{2},3+m,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e^2(2+m)\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}}$$

output
```
(-d*g+e*f)*(e*x+d)^(1+m)*(c*x^2+a)^(1/2)*AppellF1(1+m,-1/2,-1/2,2+m,(e*x+d
)/(d-(-a)^(1/2)*e/c^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e^2/(1+m)/(1-
(e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)))^(1/2)/(1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)
))^(1/2)+g*(e*x+d)^(2+m)*(c*x^2+a)^(1/2)*AppellF1(2+m,-1/2,-1/2,3+m,(e*x+d
)/(d-(-a)^(1/2)*e/c^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e^2/(2+m)/(1-
(e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)))^(1/2)/(1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)
))^(1/2)
```

Mathematica [F]

$$\int (d+ex)^m(f+gx)\sqrt{a+cx^2}\,dx = \int (d+ex)^m(f+gx)\sqrt{a+cx^2}\,dx$$

input
```
Integrate[(d + e*x)^m*(f + g*x)*Sqrt[a + c*x^2],x]
```

output
```
Integrate[(d + e*x)^m*(f + g*x)*Sqrt[a + c*x^2], x]
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {719, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}(f+gx)(d+ex)^m\,dx$$

$$\Big\downarrow \text{719}$$

$$\frac{(ef-dg)\int (d+ex)^m\sqrt{cx^2+a}dx}{e}+\frac{g\int (d+ex)^{m+1}\sqrt{cx^2+a}dx}{e}$$

$$\Big\downarrow \text{514}$$

$$\frac{\sqrt{a+cx^2}(ef-dg)\int (d+ex)^m\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}}d(d+ex)}{e^2\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-ae}}{\sqrt{c}}+d}}}+\frac{g\sqrt{a+cx^2}\int (d+ex)^{m+1}\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}}d(d+ex)}{e^2\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-ae}}{\sqrt{c}}+d}}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{a+cx^2}(ef-dg)(d+ex)^{m+1}\operatorname{AppellF1}\left(m+1,-\frac{1}{2},-\frac{1}{2},m+2,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e^2(m+1)\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}}}+\frac{g\sqrt{a+cx^2}(d+ex)^{m+2}\operatorname{AppellF1}\left(m+2,-\frac{1}{2},-\frac{1}{2},m+3,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e^2(m+2)\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}}}$$

input
```
Int[(d + e*x)^m*(f + g*x)*Sqrt[a + c*x^2],x]
```

output
```
((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[a + c*x^2]*AppellF1[1 + m, -1/2, -1/2,
 2 + m, (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/
Sqrt[c])])/(e^2*(1 + m)*Sqrt[1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c])]*Sqr
t[1 - (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])]) + (g*(d + e*x)^(2 + m)*Sqrt[a
 + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (d + e*x)/(d - (Sqrt[-a]*e)/S
qrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e^2*(2 + m)*Sqrt[1 - (d +
 e*x)/(d - (Sqrt[-a]*e)/Sqrt[c])]*Sqrt[1 - (d + e*x)/(d + (Sqrt[-a]*e)/Sqr
t[c])])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int (ex+d)^m (gx+f) \sqrt{c\,x^2+a}dx$$

input
```
int((e*x+d)^m*(g*x+f)*(c*x^2+a)^(1/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)*(c*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int (d+ex)^m(f+gx)\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(gx+f)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + a)*(g*x + f)*(e*x + d)^m, x)
```

Sympy [F]

$$\int (d+ex)^m(f+gx)\sqrt{a+cx^2}\,dx = \int \sqrt{a+cx^2}(d+ex)^m\,(f+gx)\,dx$$

input
```
integrate((e*x+d)**m*(g*x+f)*(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)**m*(f + g*x), x)
```

Maxima [F]

$$\int (d+ex)^m(f+gx)\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(gx+f)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(g*x + f)*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m(f+gx)\sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(gx+f)(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(g*x + f)*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)\sqrt{a+cx^2}\,dx = \int (f+g\,x)\,\sqrt{c\,x^2+a}\,(d+e\,x)^m\,dx$$

input
```
int((f + g*x)*(a + c*x^2)^(1/2)*(d + e*x)^m,x)
```

output
```
int((f + g*x)*(a + c*x^2)^(1/2)*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m (f+gx)\sqrt{a+cx^2}\,dx = \left(\int (ex+d)^m \sqrt{c\,x^2+a}\,xdx\right)g + \left(\int (ex+d)^m \sqrt{c\,x^2+a}dx\right)f$$

input
```
int((e*x+d)^m*(g*x+f)*(c*x^2+a)^(1/2),x)
```

output
```
int((d + e*x)**m*sqrt(a + c*x**2)*x,x)*g + int((d + e*x)**m*sqrt(a + c*x**
2),x)*f
```

3.164 $\int (d+ex)^m \sqrt{a+cx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 154

$$\int (d+ex)^m \sqrt{a+cx^2}\,dx = \frac{(d+ex)^{1+m}\sqrt{a+cx^2}\,\text{AppellF1}\left(1+m,-\frac{1}{2},-\frac{1}{2},2+m,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(1+m)\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}}$$

output
```
(e*x+d)^(1+m)*(c*x^2+a)^(1/2)*AppellF1(1+m,-1/2,-1/2,2+m,(e*x+d)/(d-(-a)^(
1/2)*e/c^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e/(1+m)/(1-(e*x+d)/(d-(-
a)^(1/2)*e/c^(1/2)))^(1/2)/(1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03

$$\int (d+ex)^m \sqrt{a+cx^2}\,dx = \frac{(d+ex)^{1+m}\sqrt{a+cx^2}\,\text{AppellF1}\left(1+m,-\frac{1}{2},-\frac{1}{2},2+m,\frac{d+ex}{d-\sqrt{-\frac{a}{c}}e},\frac{d+ex}{d+\sqrt{-\frac{a}{c}}e}\right)}{e(1+m)\sqrt{\frac{e\left(\sqrt{-\frac{a}{c}}-x\right)}{d+\sqrt{-\frac{a}{c}}e}}\sqrt{\frac{e\left(\sqrt{-\frac{a}{c}}+x\right)}{-d+\sqrt{-\frac{a}{c}}e}}}$$

input
```
Integrate[(d + e*x)^m*Sqrt[a + c*x^2],x]
```

output
```
((d + e*x)^(1 + m)*Sqrt[a + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (d +
 e*x)/(d - Sqrt[-(a/c)]*e), (d + e*x)/(d + Sqrt[-(a/c)]*e)])/(e*(1 + m)*Sq
rt[(e*(Sqrt[-(a/c)] - x))/(d + Sqrt[-(a/c)]*e)]*Sqrt[(e*(Sqrt[-(a/c)] + x)
)/(-d + Sqrt[-(a/c)]*e)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}(d+ex)^m\,dx$$

$$\Big\downarrow \text{514}$$

$$\frac{\sqrt{a+cx^2}\int (d+ex)^m\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}d(d+ex)}{e\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{a+cx^2}(d+ex)^{m+1}\text{AppellF1}\left(m+1,-\frac{1}{2},-\frac{1}{2},m+2,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(m+1)\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}}}$$

input
```
Int[(d + e*x)^m*Sqrt[a + c*x^2],x]
```

output
```
((d + e*x)^(1 + m)*Sqrt[a + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (d +
 e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(
e*(1 + m)*Sqrt[1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c])]*Sqrt[1 - (d + e*x
)/(d + (Sqrt[-a]*e)/Sqrt[c])])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int (ex + d)^m \sqrt{c\,x^2 + a}dx$$

input
```
int((e*x+d)^m*(c*x^2+a)^(1/2),x)
```

output
```
int((e*x+d)^m*(c*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int (d+ex)^m\sqrt{a+cx^2}\,dx=\int \sqrt{cx^2+a}(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + a)*(e*x + d)^m, x)
```

Sympy [F]

$$\int (d+ex)^m\sqrt{a+cx^2}\,dx=\int \sqrt{a+cx^2}(d+ex)^m\,dx$$

input
```
integrate((e*x+d)**m*(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)**m, x)
```

Maxima [F]

$$\int (d+ex)^m\sqrt{a+cx^2}\,dx=\int \sqrt{cx^2+a}(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m \sqrt{a+cx^2}\,dx = \int \sqrt{cx^2+a}(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \sqrt{a+cx^2}\,dx = \int \sqrt{c\,x^2+a}\,(d+e\,x)^m\,dx$$

input
```
int((a + c*x^2)^(1/2)*(d + e*x)^m,x)
```

output
```
int((a + c*x^2)^(1/2)*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m \sqrt{a+cx^2}\,dx = \int (ex+d)^m\,\sqrt{c\,x^2+a}dx$$

input
```
int((e*x+d)^m*(c*x^2+a)^(1/2),x)
```

output
```
int((d + e*x)**m*sqrt(a + c*x**2),x)
```

3.165 $\int \frac{(d+ex)^m\sqrt{a+cx^2}}{f+gx}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(d+ex)^m\sqrt{a+cx^2}}{f+gx}\,dx = \text{Int}\left(\frac{(d+ex)^m\sqrt{a+cx^2}}{f+gx},x\right)$$

output
```
Defer(Int)((e*x+d)^m*(c*x^2+a)^(1/2)/(g*x+f),x)
```

Mathematica [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^m\sqrt{a+cx^2}}{f+gx}\,dx = \int \frac{(d+ex)^m\sqrt{a+cx^2}}{f+gx}\,dx$$

input
```
Integrate[((d + e*x)^m*Sqrt[a + c*x^2])/(f + g*x),x]
```

output
```
Integrate[((d + e*x)^m*Sqrt[a + c*x^2])/(f + g*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {744}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}(d+ex)^m}{f+gx} \, dx$$

$$\Big\downarrow \ 744$$

$$\int \frac{\sqrt{a+cx^2}(d+ex)^m}{f+gx}dx$$

input
```
Int[((d + e*x)^m*Sqrt[a + c*x^2])/(f + g*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(ex+d)^m \sqrt{c\,x^2+a}}{gx+f}dx$$

input
```
int((e*x+d)^m*(c*x^2+a)^(1/2)/(g*x+f),x)
```

output
```
int((e*x+d)^m*(c*x^2+a)^(1/2)/(g*x+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m\sqrt{a+cx^2}}{f+gx}\,dx = \int \frac{\sqrt{cx^2+a}(ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^(1/2)/(g*x+f),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + a)*(e*x + d)^m/(g*x + f), x)
```

Sympy [N/A]

Not integrable

Time = 7.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^m\sqrt{a+cx^2}}{f+gx}\,dx = \int \frac{\sqrt{a+cx^2}(d+ex)^m}{f+gx}\,dx$$

input
```
integrate((e*x+d)**m*(c*x**2+a)**(1/2)/(g*x+f),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x)**m/(f + g*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m\sqrt{a+cx^2}}{f+gx}\,dx=\int \frac{\sqrt{cx^2+a}(ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^(1/2)/(g*x+f),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^m/(g*x + f), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m\sqrt{a+cx^2}}{f+gx}\,dx=\int \frac{\sqrt{cx^2+a}(ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^(1/2)/(g*x+f),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(e*x + d)^m/(g*x + f), x)
```

Mupad [N/A]

Not integrable

Time = 6.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m\sqrt{a+cx^2}}{f+gx}\,dx=\int \frac{\sqrt{c\,x^2+a}\,{\left(d+e\,x\right)}^m}{f+g\,x}\,dx$$

input
```
int(((a + c*x^2)^(1/2)*(d + e*x)^m)/(f + g*x),x)
```

output
```
int(((a + c*x^2)^(1/2)*(d + e*x)^m)/(f + g*x), x)
```

Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m\sqrt{a+cx^2}}{f+gx}\,dx = \int \frac{(ex+d)^m\sqrt{c\,x^2+a}}{gx+f}dx$$

input
```
int((e*x+d)^m*(c*x^2+a)^(1/2)/(g*x+f),x)
```

output
```
int((e*x+d)^m*(c*x^2+a)^(1/2)/(g*x+f),x)
```

3.166 $\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 404

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+cx^2}}\,dx = \frac{g^2(d+ex)^{1+m}\sqrt{a+cx^2}}{ce(2+m)}$$

$$+\frac{(cdg(dg-2ef(2+m))-e^2(ag^2(1+m)-cf^2(2+m)))\,(d+ex)^{1+m}\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}\,\text{App}}{ce^3(1+m)(2+m)\sqrt{a+cx^2}}$$

$$-\frac{g(dg-2ef(2+m))(d+ex)^{2+m}\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}\,\text{AppellF1}\left(2+m,\frac{1}{2},\frac{1}{2},3+m,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d}{d+}\right.}{e^3(2+m)^2\sqrt{a+cx^2}}$$

output
```
g^2*(e*x+d)^(1+m)*(c*x^2+a)^(1/2)/c/e/(2+m)+(c*d*g*(d*g-2*e*f*(2+m))-e^2*(
a*g^2*(1+m)-c*f^2*(2+m)))*(e*x+d)^(1+m)*(1-(e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)
))^(1/2)*(1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))^(1/2)*AppellF1(1+m,1/2,1/2,2
+m,(e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/c/e^
3/(1+m)/(2+m)/(c*x^2+a)^(1/2)-g*(d*g-2*e*f*(2+m))*(e*x+d)^(2+m)*(1-(e*x+d)
/(d-(-a)^(1/2)*e/c^(1/2)))^(1/2)*(1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))^(1/2
)*AppellF1(2+m,1/2,1/2,3+m,(e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)),(e*x+d)/(d+(-a
)^(1/2)*e/c^(1/2)))/e^3/(2+m)^2/(c*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+cx^2}}\,dx=\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+cx^2}}\,dx$$

input
```
Integrate[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + c*x^2],x]
```

output
```
Integrate[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + c*x^2], x]
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {743, 25, 27, 719, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2(d+ex)^m}{\sqrt{a+cx^2}}\,dx$$

$$\Big\downarrow \; 743$$

$$\frac{\int -\frac{e(d+ex)^m\left(e\left(ag^2(m+1)-cf^2(m+2)\right)+cg(dg-2ef(m+2))x\right)}{\sqrt{cx^2+a}}dx}{ce^2(m+2)}+\frac{g^2\sqrt{a+cx^2}(d+ex)^{m+1}}{ce(m+2)}$$

$$\Big\downarrow \; 25$$

$$\frac{g^2\sqrt{a+cx^2}(d+ex)^{m+1}}{ce(m+2)}-\frac{\int \frac{e(d+ex)^m\left(e\left(ag^2(m+1)-cf^2(m+2)\right)+cg(dg-2ef(m+2))x\right)}{\sqrt{cx^2+a}}dx}{ce^2(m+2)}$$

$$\Big\downarrow \; 27$$

$$\frac{g^2\sqrt{a+cx^2}(d+ex)^{m+1}}{ce(m+2)}-\frac{\int \frac{(d+ex)^m\left(e\left(ag^2(m+1)-cf^2(m+2)\right)+cg(dg-2ef(m+2))x\right)}{\sqrt{cx^2+a}}dx}{ce(m+2)}$$

$$\Big\downarrow \; 719$$

$$\frac{g^2\sqrt{a+cx^2}(d+ex)^{m+1}}{ce(m+2)}-\frac{\frac{cg(dg-2ef(m+2))\int\frac{(d+ex)^{m+1}}{\sqrt{cx^2+a}}dx}{e}-\frac{(cdg(dg-2ef(m+2))-e^2(ag^2(m+1)-cf^2(m+2)))\int\frac{(d+ex)^m}{\sqrt{cx^2+a}}dx}{e}}{ce(m+2)}$$

$\downarrow$ 514

$$\frac{g^2\sqrt{a+cx^2}(d+ex)^{m+1}}{ce(m+2)}-\frac{\frac{cg\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}}(dg-2ef(m+2))\int\frac{(d+ex)^{m+1}}{\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}}d(d+ex)}{e^2\sqrt{a+cx^2}}-\frac{\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}}(cdg(dg-2ef(m+2))-}{}}{ce(m+2)}$$

$\downarrow$ 150

$$\frac{g^2\sqrt{a+cx^2}(d+ex)^{m+1}}{ce(m+2)}-\frac{\frac{cg(d+ex)^{m+2}\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}}(dg-2ef(m+2))\text{AppellF1}\left(m+2,\frac{1}{2},\frac{1}{2},m+3,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e^2(m+2)\sqrt{a+cx^2}}-\frac{(d+ex)^{m+1}\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1}}{}}{ce(m+2)}$$

input
```
Int[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + c*x^2],x]
```

output
```
(g^2*(d + e*x)^(1 + m)*Sqrt[a + c*x^2])/(c*e*(2 + m)) - (-(((c*d*g*(d*g -
2*e*f*(2 + m)) - e^2*(a*g^2*(1 + m) - c*f^2*(2 + m)))*(d + e*x)^(1 + m)*Sq
rt[1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c])]*Sqrt[1 - (d + e*x)/(d + (Sqrt
[-a]*e)/Sqrt[c])]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (d + e*x)/(d - (Sqrt[-a
]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e^2*(1 + m)*Sqrt[a
+ c*x^2])) + (c*g*(d*g - 2*e*f*(2 + m))*(d + e*x)^(2 + m)*Sqrt[1 - (d + e*
x)/(d - (Sqrt[-a]*e)/Sqrt[c])]*Sqrt[1 - (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c
])]*AppellF1[2 + m, 1/2, 1/2, 3 + m, (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]),
 (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e^2*(2 + m)*Sqrt[a + c*x^2]))/(c*
e*(2 + m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 743
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + c*x^2)^(p + 1)/
(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m + n + 2*p + 1))   I
nt[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c*e^n*(m + n + 2*p + 1)*(f + g*x)^
n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d + e*x)^(n - 2)*(a*e^2*(m +
 n - 1) - c*d^2*(m + n + 2*p + 1) - 2*c*d*e*(m + n + p)*x), x], x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && IGtQ[n, 1] && NeQ[m + n + 2*p + 1, 0]
```

Maple [F]

$$\int \frac{(ex+d)^m (gx+f)^2}{\sqrt{c\,x^2+a}}dx$$

input
```
int((e*x+d)^m*(g*x+f)^2/(c*x^2+a)^(1/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)^2/(c*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)^2(ex+d)^m}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)*(e*x + d)^m/sqrt(c*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^m (f+gx)^2}{\sqrt{a+cx^2}}\,dx$$

input
```
integrate((e*x+d)**m*(g*x+f)**2/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**m*(f + g*x)**2/sqrt(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+cx^2}}\,dx=\int \frac{(gx+f)^2(ex+d)^m}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^2*(e*x + d)^m/sqrt(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+cx^2}}\,dx=\int \frac{(gx+f)^2(ex+d)^m}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2*(e*x + d)^m/sqrt(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+cx^2}}\,dx=\int \frac{(f+g\,x)^2\,(d+e\,x)^m}{\sqrt{c\,x^2+a}}\,dx$$

input
```
int(((f + g*x)^2*(d + e*x)^m)/(a + c*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^2*(d + e*x)^m)/(a + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+cx^2}}\,dx = \left(\int \frac{(ex+d)^m}{\sqrt{c\,x^2+a}}dx\right) f^2 + \left(\int \frac{(ex+d)^m\,x^2}{\sqrt{c\,x^2+a}}dx\right) g^2 + 2\left(\int \frac{(ex+d)^m\,x}{\sqrt{c\,x^2+a}}dx\right) fg$$

input
```
int((e*x+d)^m*(g*x+f)^2/(c*x^2+a)^(1/2),x)
```

output
```
int((d + e*x)**m/sqrt(a + c*x**2),x)*f**2 + int(((d + e*x)**m*x**2)/sqrt(a
 + c*x**2),x)*g**2 + 2*int(((d + e*x)**m*x)/sqrt(a + c*x**2),x)*f*g
```

3.167 $\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 318

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+cx^2}}\,dx = \frac{(ef-dg)(d+ex)^{1+m}\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}\operatorname{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e^2(1+m)\sqrt{a+cx^2}} + \frac{g(d+ex)^{2+m}\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}\operatorname{AppellF1}\left(2+m,\frac{1}{2},\frac{1}{2},3+m,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e^2(2+m)\sqrt{a+cx^2}}$$

output
```
(-d*g+e*f)*(e*x+d)^(1+m)*(1-(e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)))^(1/2)*(1-(e*
x+d)/(d+(-a)^(1/2)*e/c^(1/2)))^(1/2)*AppellF1(1+m,1/2,1/2,2+m,(e*x+d)/(d-(
-a)^(1/2)*e/c^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e^2/(1+m)/(c*x^2+a)
^(1/2)+g*(e*x+d)^(2+m)*(1-(e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)))^(1/2)*(1-(e*x+
d)/(d+(-a)^(1/2)*e/c^(1/2)))^(1/2)*AppellF1(2+m,1/2,1/2,3+m,(e*x+d)/(d-(-a
)^(1/2)*e/c^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e^2/(2+m)/(c*x^2+a)^(
1/2)
```

Mathematica [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+cx^2}}\,dx=\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+cx^2}}\,dx$$

input
```
Integrate[((d + e*x)^m*(f + g*x))/Sqrt[a + c*x^2],x]
```

output
```
Integrate[((d + e*x)^m*(f + g*x))/Sqrt[a + c*x^2], x]
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {719, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)(d+ex)^m}{\sqrt{a+cx^2}}\,dx$$

$$\Big\downarrow \text{719}$$

$$\frac{(ef-dg)\int \frac{(d+ex)^m}{\sqrt{cx^2+a}}dx}{e}+\frac{g\int \frac{(d+ex)^{m+1}}{\sqrt{cx^2+a}}dx}{e}$$

$$\Big\downarrow \text{514}$$

$$\frac{(ef-dg)\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-ae}}{\sqrt{c}}+d}}\int \frac{(d+ex)^m}{\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}}}d(d+ex)}{e^2\sqrt{a+cx^2}}+\frac{g\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-ae}}{\sqrt{c}}+d}}\int \frac{(d+ex)^{m+1}}{\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}}}d(d+ex)}{e^2\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{150}$$

$$\frac{(ef-dg)(d+ex)^{m+1}\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}}\operatorname{AppellF1}\left(m+1,\frac{1}{2},\frac{1}{2},m+2,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e^2(m+1)\sqrt{a+cx^2}}+\frac{g(d+ex)^{m+2}\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}}\operatorname{AppellF1}\left(m+2,\frac{1}{2},\frac{1}{2},m+3,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e^2(m+2)\sqrt{a+cx^2}}$$

input
```
Int[((d + e*x)^m*(f + g*x))/Sqrt[a + c*x^2],x]
```

output
```
((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c
])]*Sqrt[1 - (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])]*AppellF1[1 + m, 1/2, 1/
2, 2 + m, (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e
)/Sqrt[c])])/(e^2*(1 + m)*Sqrt[a + c*x^2]) + (g*(d + e*x)^(2 + m)*Sqrt[1 -
 (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c])]*Sqrt[1 - (d + e*x)/(d + (Sqrt[-a]*e
)/Sqrt[c])]*AppellF1[2 + m, 1/2, 1/2, 3 + m, (d + e*x)/(d - (Sqrt[-a]*e)/S
qrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e^2*(2 + m)*Sqrt[a + c*x^
2])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int \frac{(ex+d)^m (gx+f)}{\sqrt{c\,x^2+a}}dx$$

input
```
int((e*x+d)^m*(g*x+f)/(c*x^2+a)^(1/2),x)
```

output
```
int((e*x+d)^m*(g*x+f)/(c*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)(ex+d)^m}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((g*x + f)*(e*x + d)^m/sqrt(c*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^m\,(f+gx)}{\sqrt{a+cx^2}}\,dx$$

input
```
integrate((e*x+d)**m*(g*x+f)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**m*(f + g*x)/sqrt(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)(ex+d)^m}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x + f)*(e*x + d)^m/sqrt(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+cx^2}}\,dx = \int \frac{(gx+f)(ex+d)^m}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)*(e*x + d)^m/sqrt(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+cx^2}}\,dx = \int \frac{(f+g\,x)\,(d+e\,x)^m}{\sqrt{c\,x^2+a}}\,dx$$

input
```
int(((f + g*x)*(d + e*x)^m)/(a + c*x^2)^(1/2),x)
```

output
```
int(((f + g*x)*(d + e*x)^m)/(a + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+cx^2}}\,dx = \left(\int \frac{(ex+d)^m}{\sqrt{c\,x^2+a}}dx\right) f + \left(\int \frac{(ex+d)^m\,x}{\sqrt{c\,x^2+a}}dx\right) g$$

input
```
int((e*x+d)^m*(g*x+f)/(c*x^2+a)^(1/2),x)
```

output
```
int((d + e*x)**m/sqrt(a + c*x**2),x)*f + int(((d + e*x)**m*x)/sqrt(a + c*x
**2),x)*g
```

3.168 $\int \frac{(d+ex)^m}{\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 154

$$\int \frac{(d+ex)^m}{\sqrt{a+cx^2}}\,dx = \frac{(d+ex)^{1+m}\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}\operatorname{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(1+m)\sqrt{a+cx^2}}$$

output
```
(e*x+d)^(1+m)*(1-(e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)))^(1/2)*(1-(e*x+d)/(d+(-a
)^(1/2)*e/c^(1/2)))^(1/2)*AppellF1(1+m,1/2,1/2,2+m,(e*x+d)/(d-(-a)^(1/2)*e
/c^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e/(1+m)/(c*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^m}{\sqrt{a+cx^2}}\,dx = \frac{\sqrt{\frac{e\left(\sqrt{-\frac{a}{c}}-x\right)}{d+\sqrt{-\frac{a}{c}}e}}\sqrt{\frac{e\left(\sqrt{-\frac{a}{c}}+x\right)}{-d+\sqrt{-\frac{a}{c}}e}}(d+ex)^{1+m}\operatorname{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{d+ex}{d-\sqrt{-\frac{a}{c}}e},\frac{d+ex}{d+\sqrt{-\frac{a}{c}}e}\right)}{e(1+m)\sqrt{a+cx^2}}$$

input
```
Integrate[(d + e*x)^m/Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[(e*(Sqrt[-(a/c)] - x))/(d + Sqrt[-(a/c)]*e)]*Sqrt[(e*(Sqrt[-(a/c)] +
 x))/(-d + Sqrt[-(a/c)]*e)]*(d + e*x)^(1 + m)*AppellF1[1 + m, 1/2, 1/2, 2
+ m, (d + e*x)/(d - Sqrt[-(a/c)]*e), (d + e*x)/(d + Sqrt[-(a/c)]*e)])/(e*(
1 + m)*Sqrt[a + c*x^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{\sqrt{a+cx^2}} \, dx$$

$$\Big\downarrow \textbf{514}$$

$$\frac{\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}}\int \frac{(d+ex)^m}{\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}}}d(d+ex)}{e\sqrt{a+cx^2}}$$

$$\Big\downarrow \textbf{150}$$

$$\frac{(d+ex)^{m+1}\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}}\sqrt{1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}}\text{AppellF1}\left(m+1,\frac{1}{2},\frac{1}{2},m+2,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(m+1)\sqrt{a+cx^2}}$$

input
```
Int[(d + e*x)^m/Sqrt[a + c*x^2],x]
```

output
```
((d + e*x)^(1 + m)*Sqrt[1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c])]*Sqrt[1 -
 (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])]*AppellF1[1 + m, 1/2, 1/2, 2 + m, (d
 + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])
/(e*(1 + m)*Sqrt[a + c*x^2])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int \frac{(ex+d)^m}{\sqrt{c\,x^2+a}}dx$$

input
```
int((e*x+d)^m/(c*x^2+a)^(1/2),x)
```

output
```
int((e*x+d)^m/(c*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m/sqrt(c*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m}{\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^m}{\sqrt{a+cx^2}}\,dx$$

input
```
integrate((e*x+d)**m/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**m/sqrt(a + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/sqrt(c*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+a}}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/sqrt(c*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{\sqrt{a+cx^2}}\,dx = \int \frac{(d+e\,x)^m}{\sqrt{c\,x^2+a}}\,dx$$

input
```
int((d + e*x)^m/(a + c*x^2)^(1/2),x)
```

output
```
int((d + e*x)^m/(a + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{c\,x^2+a}}dx$$

input
```
int((e*x+d)^m/(c*x^2+a)^(1/2),x)
```

output
```
int((d + e*x)**m/sqrt(a + c*x**2),x)
```

3.169 $\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+cx^2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+cx^2}} \, dx = \operatorname{Int}\left(\frac{(d+ex)^m}{(f+gx)\sqrt{a+cx^2}}, x\right)$$

output
```
Defer(Int)((e*x+d)^m/(g*x+f)/(c*x^2+a)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 2.75 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+cx^2}} \, dx = \int \frac{(d+ex)^m}{(f+gx)\sqrt{a+cx^2}} \, dx$$

input
```
Integrate[(d + e*x)^m/((f + g*x)*Sqrt[a + c*x^2]),x]
```

output
```
Integrate[(d + e*x)^m/((f + g*x)*Sqrt[a + c*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {744}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{\sqrt{a+cx^2}(f+gx)} \, dx$$
$$\Big\downarrow \; 744$$
$$\int \frac{(d+ex)^m}{\sqrt{a+cx^2}(f+gx)}dx$$

input
```
Int[(d + e*x)^m/((f + g*x)*Sqrt[a + c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(ex+d)^m}{(gx+f)\sqrt{c\,x^2+a}}dx$$

input
```
int((e*x+d)^m/(g*x+f)/(c*x^2+a)^(1/2),x)
```

output
```
int((e*x+d)^m/(g*x+f)/(c*x^2+a)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+a}(gx+f)}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + a)*(e*x + d)^m/(c*g*x^3 + c*f*x^2 + a*g*x + a*f), x)
```

Sympy [N/A]

Not integrable

Time = 6.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+cx^2}}\,dx = \int \frac{(d+ex)^m}{\sqrt{a+cx^2}\,(f+gx)}\,dx$$

input
```
integrate((e*x+d)**m/(g*x+f)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**m/(sqrt(a + c*x**2)*(f + g*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+a}(gx+f)}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/(sqrt(c*x^2 + a)*(g*x + f)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+a}(gx+f)}\,dx$$

input
```
integrate((e*x+d)^m/(g*x+f)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/(sqrt(c*x^2 + a)*(g*x + f)), x)
```

Mupad [N/A]

Not integrable

Time = 10.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+cx^2}}\,dx = \int \frac{(d+e\,x)^m}{(f+g\,x)\,\sqrt{c\,x^2+a}}\,dx$$

input
```
int((d + e*x)^m/((f + g*x)*(a + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x)^m/((f + g*x)*(a + c*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 200.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+cx^2}}\,dx = \int \frac{(ex+d)^m}{(gx+f)\sqrt{c\,x^2+a}}dx$$

input
```
int((e*x+d)^m/(g*x+f)/(c*x^2+a)^(1/2),x)
```

output
```
int((e*x+d)^m/(g*x+f)/(c*x^2+a)^(1/2),x)
```

3.170 $\int \frac{(d+cdx)^{-q}(f-cfx)^{3/2}}{\sqrt{1-c^2x^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 107

$$\int \frac{(d+cdx)^{-q}(f-cfx)^{3/2}}{\sqrt{1-c^2x^2}}\,dx = \frac{4f(d+cdx)^{1-q}\sqrt{f-cfx}}{cd(1-2q)\sqrt{1-c^2x^2}} - \frac{2f(d+cdx)^{2-q}\sqrt{f-cfx}}{cd^2(3-2q)\sqrt{1-c^2x^2}}$$

output
```
4*f*(c*d*x+d)^(1-q)*(-c*f*x+f)^(1/2)/c/d/(1-2*q)/(-c^2*x^2+1)^(1/2)-2*f*(c
*d*x+d)^(2-q)*(-c*f*x+f)^(1/2)/c/d^2/(3-2*q)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 4.74 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.69

$$\int \frac{(d+cdx)^{-q}(f-cfx)^{3/2}}{\sqrt{1-c^2x^2}}\,dx = \frac{2f(1+cx)(d+cdx)^{-q}\sqrt{f-cfx}(5-cx+2q(-1+cx))}{c(-3+2q)(-1+2q)\sqrt{1-c^2x^2}}$$

input
```
Integrate[(f - c*f*x)^(3/2)/((d + c*d*x)^q*Sqrt[1 - c^2*x^2]),x]
```

output
```
(2*f*(1 + c*x)*Sqrt[f - c*f*x]*(5 - c*x + 2*q*(-1 + c*x)))/(c*(-3 + 2*q)*(
-1 + 2*q)*(d + c*d*x)^q*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {707, 696}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-cfx)^{3/2}(cdx+d)^{-q}}{\sqrt{1-c^2x^2}}dx$$

$$\Big\downarrow \text{ 707}$$

$$2f\int \frac{(cxd+d)^{-q}\sqrt{f-cfx}}{\sqrt{1-c^2x^2}}dx-\frac{2f^2\sqrt{1-c^2x^2}(cdx+d)^{1-q}}{cd(3-2q)\sqrt{f-cfx}}$$

$$\Big\downarrow \text{ 696}$$

$$\frac{4f^2\sqrt{1-c^2x^2}(cdx+d)^{-q}}{c(1-2q)\sqrt{f-cfx}}-\frac{2f^2\sqrt{1-c^2x^2}(cdx+d)^{1-q}}{cd(3-2q)\sqrt{f-cfx}}$$

input
```
Int[(f - c*f*x)^(3/2)/((d + c*d*x)^q*Sqrt[1 - c^2*x^2]),x]
```

output
```
(-2*f^2*(d + c*d*x)^(1 - q)*Sqrt[1 - c^2*x^2])/(c*d*(3 - 2*q)*Sqrt[f - c*f
*x]) + (4*f^2*Sqrt[1 - c^2*x^2])/(c*(1 - 2*q)*(d + c*d*x)^q*Sqrt[f - c*f*x
])
```

Defintions of rubi rules used

rule 696
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*((a + c*x^2)^(
p + 1)/(c*(m - n - 1))), x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && EqQ
[c*d^2 + a*e^2, 0] && EqQ[m + p, 0] && EqQ[e*f + d*g, 0] && NeQ[m - n - 1,
0]
```

rule 707
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*((a + c*x
^2)^(p + 1)/(c*g*(n + p + 2))), x] - Simp[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2))   Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m
+ p - 1, 0] &&  !LtQ[n, -1] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.70

method	result	size
gosper	$-\frac{2(-cfx+f)^{\frac{3}{2}}(2cqx-cx-2q+5)(cx+1)(cdx+d)^{-q}}{\sqrt{-c^{2}x^{2}+1}\,(cx-1)c(4q^{2}-8q+3)}$	75
orering	$-\frac{2(-cfx+f)^{\frac{3}{2}}(2cqx-cx-2q+5)(cx+1)(cdx+d)^{-q}}{\sqrt{-c^{2}x^{2}+1}\,(cx-1)c(4q^{2}-8q+3)}$	75

input
```
int((-c*f*x+f)^(3/2)/((c*d*x+d)^q)/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-2*(-c*f*x+f)^(3/2)*(2*c*q*x-c*x-2*q+5)*(c*x+1)/(-c^2*x^2+1)^(1/2)/((c*d*x
+d)^q)/(c*x-1)/c/(4*q^2-8*q+3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.87

$$\int \frac{(d+cdx)^{-q}(f-cfx)^{3/2}}{\sqrt{1-c^2x^2}}\,dx = -\frac{2\sqrt{-c^2x^2+1}\sqrt{-cfx+f}(2\,fq-(2\,cfq-cf)x-5\,f)}{(4\,cq^2-8\,cq-(4\,c^2q^2-8\,c^2q+3\,c^2)x+3\,c)(cdx+d)^q}$$

input
```
integrate((-c*f*x+f)^(3/2)/((c*d*x+d)^q)/(-c^2*x^2+1)^(1/2),x, algorithm="
fricas")
```

output
```
-2*sqrt(-c^2*x^2 + 1)*sqrt(-c*f*x + f)*(2*f*q - (2*c*f*q - c*f)*x - 5*f)/(
(4*c*q^2 - 8*c*q - (4*c^2*q^2 - 8*c^2*q + 3*c^2)*x + 3*c)*(c*d*x + d)^q)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{-q}(f-cfx)^{3/2}}{\sqrt{1-c^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate((-c*f*x+f)**(3/2)/((c*d*x+d)**q)/(-c**2*x**2+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.66

$$\int \frac{(d+cdx)^{-q}(f-cfx)^{3/2}}{\sqrt{1-c^2x^2}}\,dx = \frac{2\left(c^2f^{\frac{3}{2}}(2q-1)x^2+4cf^{\frac{3}{2}}x-f^{\frac{3}{2}}(2q-5)\right)}{(4q^2-8q+3)\sqrt{cx+1}(cx+1)^q cd^q}$$

input
```
integrate((-c*f*x+f)^(3/2)/((c*d*x+d)^q)/(-c^2*x^2+1)^(1/2),x, algorithm="
maxima")
```

output
```
2*(c^2*f^(3/2)*(2*q - 1)*x^2 + 4*c*f^(3/2)*x - f^(3/2)*(2*q - 5))/((4*q^2
- 8*q + 3)*sqrt(c*x + 1)*(c*x + 1)^q*c*d^q)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.69

$$\int \frac{(d+cdx)^{-q}(f-cfx)^{3/2}}{\sqrt{1-c^2x^2}}\,dx = -\frac{2f^2\left(\frac{4\sqrt{2}\sqrt{d^2f}}{(4q^2|d|-8q|d|+3|d|)(2d)^q}+\frac{\frac{2\sqrt{((cfx-f)d+2df)d}d}{(2q-1)\left(\frac{(cfx-f)d+2df}{f}\right)^q}-\frac{((cfx-f)d+2df)\sqrt{((cfx-f)d+2df)d}}{f(2q-3)\left(\frac{(cfx-f)d+2df}{f}\right)^q}}{d|d|}\right)}{c|f|}$$

input
```
integrate((-c*f*x+f)^(3/2)/((c*d*x+d)^q)/(-c^2*x^2+1)^(1/2),x, algorithm="
giac")
```

output
```
-2*f^2*(4*sqrt(2)*sqrt(d^2*f)/((4*q^2*abs(d) - 8*q*abs(d) + 3*abs(d))*(2*d
)^q) + (2*sqrt(((c*f*x - f)*d + 2*d*f)*d)*d/((2*q - 1)*(((c*f*x - f)*d + 2
*d*f)/f)^q) - ((c*f*x - f)*d + 2*d*f)*sqrt(((c*f*x - f)*d + 2*d*f)*d)/(f*(
2*q - 3)*(((c*f*x - f)*d + 2*d*f)/f)^q))/(d*abs(d)))/(c*abs(f))
```

Mupad [B] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07

$$\int \frac{(d+cdx)^{-q}(f-cfx)^{3/2}}{\sqrt{1-c^2x^2}}\,dx = \frac{\frac{8\,f\,x\,\sqrt{f-c\,f\,x}}{4\,q^2-8\,q+3}-\frac{2\,f\,(2\,q-5)\,\sqrt{f-c\,f\,x}}{c\,(4\,q^2-8\,q+3)}+\frac{2\,c\,f\,x^2\,(2\,q-1)\,\sqrt{f-c\,f\,x}}{4\,q^2-8\,q+3}}{\sqrt{1-c^2\,x^2}\,(d+c\,d\,x)^q}$$

input
```
int((f - c*f*x)^(3/2)/((1 - c^2*x^2)^(1/2)*(d + c*d*x)^q),x)
```

output
```
((8*f*x*(f - c*f*x)^(1/2))/(4*q^2 - 8*q + 3) - (2*f*(2*q - 5)*(f - c*f*x)^
(1/2))/(c*(4*q^2 - 8*q + 3)) + (2*c*f*x^2*(2*q - 1)*(f - c*f*x)^(1/2))/(4*
q^2 - 8*q + 3))/((1 - c^2*x^2)^(1/2)*(d + c*d*x)^q)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int \frac{(d+cdx)^{-q}(f-cfx)^{3/2}}{\sqrt{1-c^2x^2}}\,dx = \frac{2\sqrt{f}\,\sqrt{cx+1}\,f(2cqx-cx-2q+5)}{d^q\,(cx+1)^q\,c\,(4q^2-8q+3)}$$

input
```
int((-c*f*x+f)^(3/2)/((c*d*x+d)^q)/(-c^2*x^2+1)^(1/2),x)
```

output
```
(2*sqrt(f)*sqrt(c*x + 1)*f*(2*c*q*x - c*x - 2*q + 5))/(d**q*(c*x + 1)**q*c
*(4*q**2 - 8*q + 3))
```

3.171 $\int \frac{(d+ex)^{3/2}}{(d-ex)^{4/3}\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 84

$$\int \frac{(d+ex)^{3/2}}{(d-ex)^{4/3}\sqrt{d^2-e^2x^2}}\,dx=\frac{12d\sqrt{d+ex}}{5e\sqrt[3]{d-ex}\sqrt{d^2-e^2x^2}}+\frac{6(d-ex)^{2/3}\sqrt{d+ex}}{e\sqrt{d^2-e^2x^2}}$$

output
```
12/5*d*(e*x+d)^(1/2)/e/(-e*x+d)^(1/3)/(-e^2*x^2+d^2)^(1/2)+6*(-e*x+d)^(2/3
)*(e*x+d)^(1/2)/e/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex)^{3/2}}{(d-ex)^{4/3}\sqrt{d^2-e^2x^2}}\,dx=\frac{6(7d-5ex)\sqrt{d+ex}}{5e\sqrt[3]{d-ex}\sqrt{d^2-e^2x^2}}$$

input
```
Integrate[(d + e*x)^(3/2)/((d - e*x)^(4/3)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(6*(7*d - 5*e*x)*Sqrt[d + e*x])/(5*e*(d - e*x)^(1/3)*Sqrt[d^2 - e^2*x^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {706}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(d-ex)^{4/3}\sqrt{d^2-e^2x^2}}\,dx$$

$$\Big\downarrow \text{706}$$

$$\text{Indeterminate}$$

input
```
Int[(d + e*x)^(3/2)/((d - e*x)^(4/3)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
Indeterminate
```

Defintions of rubi rules used

rule 706
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Simp[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g)))   Int[(d + e*x)^(m - 1)*(f
 + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
 && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + p - 1, 0] && LtQ[n, -1] && IntegerQ[2*
p]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.51

method	result
gosper	$\frac{6(-5ex+7d)\sqrt{ex+d}}{5(-ex+d)^{\frac{1}{3}}e\sqrt{-e^{2}x^{2}+d^{2}}}$
orering	$\frac{6(-5ex+7d)\sqrt{ex+d}}{5(-ex+d)^{\frac{1}{3}}e\sqrt{-e^{2}x^{2}+d^{2}}}$
risch	$\frac{6(-ex+d)^{\frac{2}{3}}\left((-ex+d)^{2}\right)^{\frac{1}{6}}\sqrt{ex+d}\left((-e^{2}x^{2}+d^{2})^{3}\right)^{\frac{1}{6}}\left(-\frac{(ex-d)^{2}\left(e^{2}x^{2}-d^{2}\right)^{3}}{(ex+d)^{3}}\right)^{\frac{1}{6}}}{e\left(-(ex-d)^{5}\right)^{\frac{1}{6}}\sqrt{-e^{2}x^{2}+d^{2}}\left((ex-d)^{2}\right)^{\frac{1}{6}}\left(-(e^{2}x^{2}-d^{2})^{3}\right)^{\frac{1}{6}}}+\frac{12d\left((-ex+d)^{2}\right)^{\frac{1}{6}}\sqrt{ex+d}\left((-e^{2}x^{2}+d^{2})^{3}\right)^{\frac{1}{6}}\cdots}{5e\left((-ex+d)^{5}\right)^{\frac{1}{6}}(-ex+d)^{\frac{1}{3}}\sqrt{-e^{2}x^{2}+d^{2}}\left((e\cdots\right.}$

input
```
int((e*x+d)^(3/2)/(-e*x+d)^(4/3)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
6/5/(-e*x+d)^(1/3)*(-5*e*x+7*d)*(e*x+d)^(1/2)/e/(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^{3/2}}{(d-ex)^{4/3}\sqrt{d^2-e^2x^2}}\,dx = -\frac{6\sqrt{-e^2x^2+d^2}(5\,ex-7\,d)\sqrt{ex+d}(-ex+d)^{\frac{2}{3}}}{5\left(e^4x^3-de^3x^2-d^2e^2x+d^3e\right)}$$

input
```
integrate((e*x+d)^(3/2)/(-e*x+d)^(4/3)/(-e^2*x^2+d^2)^(1/2),x, algorithm="
fricas")
```

output
```
-6/5*sqrt(-e^2*x^2 + d^2)*(5*e*x - 7*d)*sqrt(e*x + d)*(-e*x + d)^(2/3)/(e^
4*x^3 - d*e^3*x^2 - d^2*e^2*x + d^3*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(d-ex)^{4/3}\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{-(-d+ex)\,(d+ex)}\,(d-ex)^{\frac{4}{3}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(-e*x+d)**(4/3)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)/(sqrt(-(-d + e*x)*(d + e*x))*(d - e*x)**(4/3)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.25

$$\int \frac{(d+ex)^{3/2}}{(d-ex)^{4/3}\sqrt{d^2-e^2x^2}}\,dx = -\frac{6\,(5\,ex-7\,d)}{5\,(-ex+d)^{\frac{5}{6}}e}$$

input
```
integrate((e*x+d)^(3/2)/(-e*x+d)^(4/3)/(-e^2*x^2+d^2)^(1/2),x, algorithm="
maxima")
```

output
```
-6/5*(5*e*x - 7*d)/((-e*x + d)^(5/6)*e)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.32

$$\int \frac{(d+ex)^{3/2}}{(d-ex)^{4/3}\sqrt{d^2-e^2x^2}}\,dx = \frac{6\left(5\,(-ex+d)^{\frac{1}{6}}+\frac{2\,d}{(-ex+d)^{\frac{5}{6}}}\right)}{5\,e}$$

input
```
integrate((e*x+d)^(3/2)/(-e*x+d)^(4/3)/(-e^2*x^2+d^2)^(1/2),x, algorithm="
giac")
```

output
```
6/5*(5*(-e*x + d)^(1/6) + 2*d/(-e*x + d)^(5/6))/e
```

Mupad [B] (verification not implemented)

Time = 6.73 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^{3/2}}{(d-ex)^{4/3}\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{42\,d\,\sqrt{d+e\,x}}{5\,e^3}-\frac{6\,x\,\sqrt{d+e\,x}}{e^2}\right)}{x^2\,{(d-e\,x)}^{1/3}-\frac{d^2\,{(d-e\,x)}^{1/3}}{e^2}}$$

input
```
int((d + e*x)^(3/2)/((d^2 - e^2*x^2)^(1/2)*(d - e*x)^(4/3)),x)
```

output
```
-((d^2 - e^2*x^2)^(1/2)*((42*d*(d + e*x)^(1/2))/(5*e^3) - (6*x*(d + e*x)^(
1/2))/e^2))/(x^2*(d - e*x)^(1/3) - (d^2*(d - e*x)^(1/3))/e^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.25

$$\int \frac{(d+ex)^{3/2}}{(d-ex)^{4/3}\sqrt{d^2-e^2x^2}}\,dx = \frac{-6ex+\frac{42d}{5}}{(-ex+d)^{\frac{5}{6}}\,e}$$

input
```
int((e*x+d)^(3/2)/(-e*x+d)^(4/3)/(-e^2*x^2+d^2)^(1/2),x)
```

output
```
(6*(7*d - 5*e*x))/(5*(d - e*x)**(5/6)*e)
```

3.172 $\int (d+ex)^m (f+gx)^n \left(a+cx^2\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 233

$$\begin{aligned}
&\int (d+ex)^m (f+gx)^n \left(a+cx^2\right) \, dx \\
&= -\frac{c(ef(2+m)+dg(4+m+2n))(d+ex)^{1+m}(f+gx)^{1+n}}{e^2g^2(2+m+n)(3+m+n)} \\
&\quad + \frac{c(d+ex)^{2+m}(f+gx)^{1+n}}{e^2g(3+m+n)} \\
&\quad + \frac{(c(ef(1+m)+dg(1+n))(ef(2+m)+dg(4+m+2n))+g(2+m+n)\left(ae^2g(3+m+n)-cd(ef\right.}{e^2g^2(ef-dg)(1+m)(}
\end{aligned}$$

output

```
-c*(e*f*(2+m)+d*g*(4+m+2*n))*(e*x+d)^(1+m)*(g*x+f)^(1+n)/e^2/g^2/(2+m+n)/(
3+m+n)+c*(e*x+d)^(2+m)*(g*x+f)^(1+n)/e^2/g/(3+m+n)+(c*(e*f*(1+m)+d*g*(1+n)
)*(e*f*(2+m)+d*g*(4+m+2*n))+g*(2+m+n)*(a*e^2*g*(3+m+n)-c*d*(e*f*(2+m)+d*g*
(1+n))))*(e*x+d)^(1+m)*(g*x+f)^(1+n)*hypergeom([1, 2+m+n],[2+m],-g*(e*x+d)
/(-d*g+e*f))/e^2/g^2/(-d*g+e*f)/(1+m)/(2+m+n)/(3+m+n)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.75

$$\int (d+ex)^m(f+gx)^n\left(a+cx^2\right)\,dx$$
$$=\frac{(d+ex)^{1+m}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}\left(c(ef-dg)^2\,\text{Hypergeometric2F1}\left(1+m,-2-n,2+m,\frac{g(d+ex)}{-ef+dg}\right)+\right.}{}$$

input
```
Integrate[(d + e*x)^m*(f + g*x)^n*(a + c*x^2),x]
```

output
```
((d + e*x)^(1 + m)*(f + g*x)^n*(c*(e*f - d*g)^2*Hypergeometric2F1[1 + m, -
2 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e*(2*c*f*(-(e*f) + d*g)*Hype
rgeometric2F1[1 + m, -1 - n, 2 + m, (g*(d + e*x))/(-(e*f) + d*g)] + e*(c*f
^2 + a*g^2)*Hypergeometric2F1[1 + m, -n, 2 + m, (g*(d + e*x))/(-(e*f) + d*
g)])))/(e^3*g^2*(1 + m)*((e*(f + g*x))/(e*f - d*g))^n)
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {651, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(d+ex)^m(f+gx)^n\,dx$$

$$\Big\downarrow \text{651}$$

$$\frac{\int (d+ex)^m(f+gx)^n\left(ag(m+n+3)e^2-c(ef(m+2)+dg(m+2n+4))xe-cd(ef(m+2)+dg(n+1))\right)dx}{e^2g(m+n+3)}$$
$$\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{e^2g(m+n+3)}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{\left(ae^2g^2\left(m^2+m(2n+5)+n^2+5n+6\right)+c\left(d^2g^2\left(n^2+3n+2\right)+2defg(m+1)(n+1)+e^2f^2\left(m^2+3m+2\right)\right)\right)\int(d+ex)^m(f+gx)^ndx}{g(m+n+2)}-\frac{c(d+ex)^{m+1}(f+}{}}{e^2g(m+n+3)}$$

$$\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{e^2g(m+n+3)}$$

$\downarrow$ 80

$$\frac{\frac{(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}\left(ae^2g^2\left(m^2+m(2n+5)+n^2+5n+6\right)+c\left(d^2g^2\left(n^2+3n+2\right)+2defg(m+1)(n+1)+e^2f^2\left(m^2+3m+2\right)\right)\right)\int(d+ex)^m\left(\frac{ef}{ef-dg}+\right.}{g(m+n+2)}}{e^2g(m+n+3)}$$

$$\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{e^2g(m+n+3)}$$

$\downarrow$ 79

$$\frac{\frac{(d+ex)^{m+1}(f+gx)^n\left(\frac{e(f+gx)}{ef-dg}\right)^{-n}\left(ae^2g^2\left(m^2+m(2n+5)+n^2+5n+6\right)+c\left(d^2g^2\left(n^2+3n+2\right)+2defg(m+1)(n+1)+e^2f^2\left(m^2+3m+2\right)\right)\right)\text{Hypergeo}}{eg(m+1)(m+n+2)}}{e^2g(m+n+3)}$$

$$\frac{c(d+ex)^{m+2}(f+gx)^{n+1}}{e^2g(m+n+3)}$$

input
```
Int[(d + e*x)^m*(f + g*x)^n*(a + c*x^2),x]
```

output
```
(c*(d + e*x)^(2 + m)*(f + g*x)^(1 + n))/(e^2*g*(3 + m + n)) + (-((c*(e*f*(
2 + m) + d*g*(4 + m + 2*n))*(d + e*x)^(1 + m)*(f + g*x)^(1 + n))/(g*(2 + m
 + n))) + ((a*e^2*g^2*(6 + m^2 + 5*n + n^2 + m*(5 + 2*n)) + c*(e^2*f^2*(2
+ 3*m + m^2) + 2*d*e*f*g*(1 + m)*(1 + n) + d^2*g^2*(2 + 3*n + n^2)))*(d +
e*x)^(1 + m)*(f + g*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((g*(d + e*x
))/(e*f - d*g))])/(e*g*(1 + m)*(2 + m + n)*((e*(f + g*x))/(e*f - d*g))^n))
/(e^2*g*(3 + m + n))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 651
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2*p + 1))   Int
[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^
2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1)
, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && IGtQ[p, 0] &&  !IntegerQ[m]
 &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

Maple [F]

$$\int (ex + d)^m (gx + f)^n (c x^2 + a) \, dx$$

input
```
int((e*x+d)^m*(g*x+f)^n*(c*x^2+a),x)
```

output
```
int((e*x+d)^m*(g*x+f)^n*(c*x^2+a),x)
```

Fricas [F]

$$\int (d+ex)^m (f+gx)^n \left(a+cx^2\right)\,dx = \int (cx^2+a)(ex+d)^m (gx+f)^n\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+a),x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)*(e*x + d)^m*(g*x + f)^n, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m (f+gx)^n \left(a+cx^2\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*(g*x+f)**n*(c*x**2+a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (d+ex)^m (f+gx)^n \left(a+cx^2\right)\,dx = \int (cx^2+a)(ex+d)^m (gx+f)^n\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+a),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)*(e*x + d)^m*(g*x + f)^n, x)
```

Giac [F]

$$\int (d+ex)^m(f+gx)^n\left(a+cx^2\right)\,dx = \int (cx^2+a)(ex+d)^m(gx+f)^n\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^n*(c*x^2+a),x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)*(e*x + d)^m*(g*x + f)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^n\left(a+cx^2\right)\,dx = \int (f+g\,x)^n\,\left(c\,x^2+a\right)\,\left(d+e\,x\right)^m\,dx$$

input
```
int((f + g*x)^n*(a + c*x^2)*(d + e*x)^m,x)
```

output
```
int((f + g*x)^n*(a + c*x^2)*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m(f+gx)^n\left(a+cx^2\right)\,dx = \text{too large to display}$$

input
```
int((e*x+d)^m*(g*x+f)^n*(c*x^2+a),x)
```

output
```
((f + g*x)**n*(d + e*x)**m*a*d*e**2*f*g**2*m**3 + 3*(f + g*x)**n*(d + e*x)
**m*a*d*e**2*f*g**2*m**2*n + 5*(f + g*x)**n*(d + e*x)**m*a*d*e**2*f*g**2*m
**2 + 3*(f + g*x)**n*(d + e*x)**m*a*d*e**2*f*g**2*m*n**2 + 10*(f + g*x)**n
*(d + e*x)**m*a*d*e**2*f*g**2*m*n + 6*(f + g*x)**n*(d + e*x)**m*a*d*e**2*f
*g**2*m + (f + g*x)**n*(d + e*x)**m*a*d*e**2*f*g**2*n**3 + 5*(f + g*x)**n*
(d + e*x)**m*a*d*e**2*f*g**2*n**2 + 6*(f + g*x)**n*(d + e*x)**m*a*d*e**2*f
*g**2*n + (f + g*x)**n*(d + e*x)**m*a*d*e**2*g**3*m**2*n*x + 2*(f + g*x)**
n*(d + e*x)**m*a*d*e**2*g**3*m*n**2*x + 5*(f + g*x)**n*(d + e*x)**m*a*d*e*
*2*g**3*m*n*x + (f + g*x)**n*(d + e*x)**m*a*d*e**2*g**3*n**3*x + 5*(f + g*
x)**n*(d + e*x)**m*a*d*e**2*g**3*n**2*x + 6*(f + g*x)**n*(d + e*x)**m*a*d*
e**2*g**3*n*x + (f + g*x)**n*(d + e*x)**m*a*e**3*f*g**2*m**3*x + 2*(f + g*
x)**n*(d + e*x)**m*a*e**3*f*g**2*m**2*n*x + 5*(f + g*x)**n*(d + e*x)**m*a*
e**3*f*g**2*m**2*x + (f + g*x)**n*(d + e*x)**m*a*e**3*f*g**2*m*n**2*x + 5*
(f + g*x)**n*(d + e*x)**m*a*e**3*f*g**2*m*n*x + 6*(f + g*x)**n*(d + e*x)**
m*a*e**3*f*g**2*m*x + (f + g*x)**n*(d + e*x)**m*c*d**3*f*g**2*m*n + 2*(f +
 g*x)**n*(d + e*x)**m*c*d**3*f*g**2*m - (f + g*x)**n*(d + e*x)**m*c*d**3*g
**3*m*n**2*x - 2*(f + g*x)**n*(d + e*x)**m*c*d**3*g**3*m*n*x - 2*(f + g*x)
**n*(d + e*x)**m*c*d**2*e*f**2*g*m*n - (f + g*x)**n*(d + e*x)**m*c*d**2*e*
f*g**2*m**2*n*x - 2*(f + g*x)**n*(d + e*x)**m*c*d**2*e*f*g**2*m**2*x + 2*(
f + g*x)**n*(d + e*x)**m*c*d**2*e*f*g**2*m*n**2*x + (f + g*x)**n*(d + e...
```

3.173 $\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 430

$$\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)^p\,dx=\frac{g^2(d+ex)^{1+m}\left(a+cx^2\right)^{1+p}}{ce(3+m+2p)}$$

$$+\frac{(2cdg(dg(1+p)-ef(3+m+2p))-e^2(ag^2(1+m)-cf^2(3+m+2p)))\,(d+ex)^{1+m}\left(a+cx^2\right)^p\Bigg(1}{ce^3(1+m)(3+m+}$$

$$-\frac{2g(dg(1+p)-ef(3+m+2p))(d+ex)^{2+m}\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\text{AppellF1}\Big(}{e^3(2+m)(3+m+2p)}$$

output

```
g^2*(e*x+d)^(1+m)*(c*x^2+a)^(p+1)/c/e/(3+m+2*p)+(2*c*d*g*(d*g*(p+1)-e*f*(3
+m+2*p))-e^2*(a*g^2*(1+m)-c*f^2*(3+m+2*p)))*(e*x+d)^(1+m)*(c*x^2+a)^p*Appe
llF1(1+m,-p,-p,2+m,(e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)),(e*x+d)/(d+(-a)^(1/2)*
e/c^(1/2)))/c/e^3/(1+m)/(3+m+2*p)/((1-(e*x+d)/(d-(-a)^(1/2)*e/c^(1/2)))^p)
/((1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))^p)-2*g*(d*g*(p+1)-e*f*(3+m+2*p))*(e
*x+d)^(2+m)*(c*x^2+a)^p*AppellF1(2+m,-p,-p,3+m,(e*x+d)/(d-(-a)^(1/2)*e/c^(
1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e^3/(2+m)/(3+m+2*p)/((1-(e*x+d)/(d
-(-a)^(1/2)*e/c^(1/2)))^p)/((1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))^p)
```

Mathematica [F]

$$\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)^p\,dx=\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)^p\,dx$$

input
```
Integrate[(d + e*x)^m*(f + g*x)^2*(a + c*x^2)^p,x]
```

output
```
Integrate[(d + e*x)^m*(f + g*x)^2*(a + c*x^2)^p, x]
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {743, 25, 27, 719, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2\left(a+cx^2\right)^p(d+ex)^m\,dx$$

$$\Big\downarrow \text{743}$$

$$\frac{\int -e(d+ex)^m\left(e\left(ag^2(m+1)-cf^2(m+2p+3)\right)+2cg(dg(p+1)-ef(m+2p+3))x\right)\left(cx^2+a\right)^p\,dx}{ce^2(m+2p+3)}+\frac{g^2\left(a+cx^2\right)^{p+1}(d+ex)^{m+1}}{ce(m+2p+3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{g^2\left(a+cx^2\right)^{p+1}(d+ex)^{m+1}}{ce(m+2p+3)}-\frac{\int e(d+ex)^m\left(e\left(ag^2(m+1)-cf^2(m+2p+3)\right)+2cg(dg(p+1)-ef(m+2p+3))x\right)\left(cx^2+a\right)^p\,dx}{ce^2(m+2p+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^2\left(a+cx^2\right)^{p+1}(d+ex)^{m+1}}{ce(m+2p+3)}-\frac{\int (d+ex)^m\left(e\left(ag^2(m+1)-cf^2(m+2p+3)\right)+2cg(dg(p+1)-ef(m+2p+3))x\right)\left(cx^2+a\right)^p dx}{ce(m+2p+3)}$$

$$\Big\downarrow \; 719$$

$$\frac{g^2\left(a+cx^2\right)^{p+1}(d+ex)^{m+1}}{ce(m+2p+3)}-\frac{\frac{2cg(dg(p+1)-ef(m+2p+3))\int (d+ex)^{m+1}\left(cx^2+a\right)^p dx}{e}-\frac{\left(2cdg(dg(p+1)-ef(m+2p+3))-e^2\left(ag^2(m+1)-cf^2(m+2p+3)\right)\right)\int (d+ex)^m\left(cx^2+a\right.}{e}}{ce(m+2p+3)}$$

$$\Big\downarrow \; 514$$

$$\frac{g^2\left(a+cx^2\right)^{p+1}(d+ex)^{m+1}}{ce(m+2p+3)}-\frac{\frac{2cg\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p}(dg(p+1)-ef(m+2p+3))\int (d+ex)^{m+1}\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^p\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^p d(d+ex)}{e^2}-\frac{(a+}{}}{}$$

$$\Big\downarrow \; 150$$

$$\frac{g^2\left(a+cx^2\right)^{p+1}(d+ex)^{m+1}}{ce(m+2p+3)}-\frac{\frac{2cg\left(a+cx^2\right)^p(d+ex)^{m+2}\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p}(dg(p+1)-ef(m+2p+3))\operatorname{AppellF1}\left(m+2,-p,-p,m+3,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e^2(m+2)}-}{}$$

input
```
Int[(d + e*x)^m*(f + g*x)^2*(a + c*x^2)^p,x]
```

output
```
(g^2*(d + e*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*e*(3 + m + 2*p)) - (-(((2*c
*d*g*(d*g*(1 + p) - e*f*(3 + m + 2*p)) - e^2*(a*g^2*(1 + m) - c*f^2*(3 + m
 + 2*p)))*(d + e*x)^(1 + m)*(a + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (
d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])]
)/(e^2*(1 + m)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1 - (d + e*x)
/(d + (Sqrt[-a]*e)/Sqrt[c]))^p)) + (2*c*g*(d*g*(1 + p) - e*f*(3 + m + 2*p)
)*(d + e*x)^(2 + m)*(a + c*x^2)^p*AppellF1[2 + m, -p, -p, 3 + m, (d + e*x)
/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e^2*(
2 + m)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1 - (d + e*x)/(d + (S
qrt[-a]*e)/Sqrt[c]))^p))/(c*e*(3 + m + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 743
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + c*x^2)^(p + 1)/
(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m + n + 2*p + 1))   I
nt[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c*e^n*(m + n + 2*p + 1)*(f + g*x)^
n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d + e*x)^(n - 2)*(a*e^2*(m +
 n - 1) - c*d^2*(m + n + 2*p + 1) - 2*c*d*e*(m + n + p)*x), x], x], x] /; F
reeQ[{a, c, d, e, f, g, m, p}, x] && IGtQ[n, 1] && NeQ[m + n + 2*p + 1, 0]
```

Maple [F]

$$\int (ex+d)^m (gx+f)^2 (c\,x^2+a)^p \,dx$$

input
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^p,x)
```

output
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^m (f+gx)^2 \left(a+cx^2\right)^p \,dx = \int (gx+f)^2 (cx^2+a)^p (ex+d)^m \,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)*(c*x^2 + a)^p*(e*x + d)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m (f+gx)^2 \left(a+cx^2\right)^p \,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)^p\,dx=\int (gx+f)^2\left(cx^2+a\right)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((g*x + f)^2*(c*x^2 + a)^p*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)^p\,dx=\int (gx+f)^2\left(cx^2+a\right)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((g*x + f)^2*(c*x^2 + a)^p*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)^2\left(a+cx^2\right)^p\,dx=\int (f+g\,x)^2\,\left(c\,x^2+a\right)^p\,(d+e\,x)^m\,dx$$

input
```
int((f + g*x)^2*(a + c*x^2)^p*(d + e*x)^m,x)
```

output
```
int((f + g*x)^2*(a + c*x^2)^p*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m (f+gx)^2 \left(a+cx^2\right)^p \,dx = \text{too large to display}$$

input
```
int((e*x+d)^m*(g*x+f)^2*(c*x^2+a)^p,x)
```

output
```
( - (d + e*x)**m*(a + c*x**2)**p*a**2*e**3*g**2*m**2 - 2*(d + e*x)**m*(a +
 c*x**2)**p*a**2*e**3*g**2*m*p - 3*(d + e*x)**m*(a + c*x**2)**p*a**2*e**3*
g**2*m - 2*(d + e*x)**m*(a + c*x**2)**p*a**2*e**3*g**2*p - 2*(d + e*x)**m*
(a + c*x**2)**p*a**2*e**3*g**2 + (d + e*x)**m*(a + c*x**2)**p*a*c*d**2*e*g
**2*m**2 - (d + e*x)**m*(a + c*x**2)**p*a*c*d**2*e*g**2*m + 4*(d + e*x)**m
*(a + c*x**2)**p*a*c*d*e**2*f*g*m**2 + 12*(d + e*x)**m*(a + c*x**2)**p*a*c
*d*e**2*f*g*m*p + 14*(d + e*x)**m*(a + c*x**2)**p*a*c*d*e**2*f*g*m + 8*(d
+ e*x)**m*(a + c*x**2)**p*a*c*d*e**2*f*g*p**2 + 16*(d + e*x)**m*(a + c*x**
2)**p*a*c*d*e**2*f*g*p + 6*(d + e*x)**m*(a + c*x**2)**p*a*c*d*e**2*f*g + 2
*(d + e*x)**m*(a + c*x**2)**p*a*c*d*e**2*g**2*m*p*x + 4*(d + e*x)**m*(a +
c*x**2)**p*a*c*d*e**2*g**2*p**2*x + 4*(d + e*x)**m*(a + c*x**2)**p*a*c*d*e
**2*g**2*p*x + (d + e*x)**m*(a + c*x**2)**p*a*c*e**3*f**2*m**2 + 4*(d + e*
x)**m*(a + c*x**2)**p*a*c*e**3*f**2*m*p + 5*(d + e*x)**m*(a + c*x**2)**p*a
*c*e**3*f**2*m + 4*(d + e*x)**m*(a + c*x**2)**p*a*c*e**3*f**2*p**2 + 10*(d
 + e*x)**m*(a + c*x**2)**p*a*c*e**3*f**2*p + 6*(d + e*x)**m*(a + c*x**2)**
p*a*c*e**3*f**2 - 2*(d + e*x)**m*(a + c*x**2)**p*c**2*d**3*g**2*m*p*x - 2*
(d + e*x)**m*(a + c*x**2)**p*c**2*d**3*g**2*m*x + 2*(d + e*x)**m*(a + c*x*
*2)**p*c**2*d**2*e*f*g*m**2*x + 4*(d + e*x)**m*(a + c*x**2)**p*c**2*d**2*e
*f*g*m*p*x + 6*(d + e*x)**m*(a + c*x**2)**p*c**2*d**2*e*f*g*m*x + (d + e*x
)**m*(a + c*x**2)**p*c**2*d**2*e*g**2*m**2*x**2 + 2*(d + e*x)**m*(a + c...
```

3.174 $\int (d+ex)^m(f+gx)\left(a+cx^2\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 314

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)^p\,dx = \frac{(ef-dg)(d+ex)^{1+m}\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\operatorname{AppellF1}\left(1+m,-p,-p,2+m,\frac{}{d}\right.}{e^2(1+m)} + \frac{g(d+ex)^{2+m}\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\operatorname{AppellF1}\left(2+m,-p,-p,3+m,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\right.}{e^2(2+m)}$$

output
```
(-d*g+e*f)*(e*x+d)^(1+m)*(c*x^2+a)^p*AppellF1(1+m,-p,-p,2+m,(e*x+d)/(d-(-a
)^(1/2)*e/c^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e^2/(1+m)/((1-(e*x+d)
/(d-(-a)^(1/2)*e/c^(1/2)))^p)/((1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))^p)+g*(
e*x+d)^(2+m)*(c*x^2+a)^p*AppellF1(2+m,-p,-p,3+m,(e*x+d)/(d-(-a)^(1/2)*e/c^
(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e^2/(2+m)/((1-(e*x+d)/(d-(-a)^(1/
2)*e/c^(1/2)))^p)/((1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))^p)
```

Mathematica [F]

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)^p\,dx=\int (d+ex)^m(f+gx)\left(a+cx^2\right)^p\,dx$$

input
```
Integrate[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,x]
```

output
```
Integrate[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {719, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(a+cx^2\right)^p(d+ex)^m\,dx$$

$$\Big\downarrow \text{ 719}$$

$$\frac{(ef-dg)\int (d+ex)^m\left(cx^2+a\right)^p dx}{e}+\frac{g\int (d+ex)^{m+1}\left(cx^2+a\right)^p dx}{e}$$

$$\Big\downarrow \text{ 514}$$

$$\frac{\left(a+cx^2\right)^p(ef-dg)\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-ae}}{\sqrt{c}}+d}\right)^{-p}\int (d+ex)^m\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^p\left(1-\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^p d(d+ex)}{e^2}$$
$$\frac{g\left(a+cx^2\right)^p\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{\frac{\sqrt{-ae}}{\sqrt{c}}+d}\right)^{-p}\int (d+ex)^{m+1}\left(1-\frac{d+ex}{d-\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^p\left(1-\frac{d+ex}{d+\frac{\sqrt{-ae}}{\sqrt{c}}}\right)^p d(d+ex)}{e^2}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{(a+cx^2)^p (ef-dg)(d+ex)^{m+1} \left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p} \left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p} \text{AppellF1}\left(m+1,-p,-p,m+2,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\right.}{e^2(m+1)}$$

$$\frac{g(a+cx^2)^p (d+ex)^{m+2} \left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p} \left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p} \text{AppellF1}\left(m+2,-p,-p,m+3,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e^2(m+2)}$$

input
```
Int[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,x]
```

output
```
((e*f - d*g)*(d + e*x)^(1 + m)*(a + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m
, (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c
])])/(e^2*(1 + m)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1 - (d + e
*x)/(d + (Sqrt[-a]*e)/Sqrt[c]))^p) + (g*(d + e*x)^(2 + m)*(a + c*x^2)^p*Ap
pellF1[2 + m, -p, -p, 3 + m, (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*
x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e^2*(2 + m)*(1 - (d + e*x)/(d - (Sqrt[-a]
*e)/Sqrt[c]))^p*(1 - (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int (ex+d)^m (gx+f) \left(c\,x^2+a\right)^p dx$$

input
```
int((e*x+d)^m*(g*x+f)*(c*x^2+a)^p,x)
```

output
```
int((e*x+d)^m*(g*x+f)*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)^p\,dx = \int (gx+f)\left(cx^2+a\right)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((g*x + f)*(c*x^2 + a)^p*(e*x + d)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)^p\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(g*x+f)*(c*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)^p\,dx=\int (gx+f)\left(cx^2+a\right)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((g*x + f)*(c*x^2 + a)^p*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)^p\,dx=\int (gx+f)\left(cx^2+a\right)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(g*x+f)*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((g*x + f)*(c*x^2 + a)^p*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)^p\,dx=\int (f+g\,x)\,\left(c\,x^2+a\right)^p\,(d+e\,x)^m\,dx$$

input
```
int((f + g*x)*(a + c*x^2)^p*(d + e*x)^m,x)
```

output
```
int((f + g*x)*(a + c*x^2)^p*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m(f+gx)\left(a+cx^2\right)^p\,dx = \text{too large to display}$$

input
```
int((e*x+d)^m*(g*x+f)*(c*x^2+a)^p,x)
```

output
```
(2*(d + e*x)**m*(a + c*x**2)**p*a*d*e*g*m + 2*(d + e*x)**m*(a + c*x**2)**p
*a*d*e*g*p + (d + e*x)**m*(a + c*x**2)**p*a*d*e*g + (d + e*x)**m*(a + c*x*
*2)**p*a*e**2*f*m + 2*(d + e*x)**m*(a + c*x**2)**p*a*e**2*f*p + 2*(d + e*x
)**m*(a + c*x**2)**p*a*e**2*f + (d + e*x)**m*(a + c*x**2)**p*c*d**2*g*m*x
+ (d + e*x)**m*(a + c*x**2)**p*c*d*e*f*m*x + 2*(d + e*x)**m*(a + c*x**2)**
p*c*d*e*f*p*x + 2*(d + e*x)**m*(a + c*x**2)**p*c*d*e*f*x + (d + e*x)**m*(a
 + c*x**2)**p*c*d*e*g*m*x**2 + 2*(d + e*x)**m*(a + c*x**2)**p*c*d*e*g*p*x*
*2 + (d + e*x)**m*(a + c*x**2)**p*c*d*e*g*x**2 - 2*int(((d + e*x)**m*(a +
c*x**2)**p*x**2)/(a*d*m**2 + 4*a*d*m*p + 3*a*d*m + 4*a*d*p**2 + 6*a*d*p +
2*a*d + a*e*m**2*x + 4*a*e*m*p*x + 3*a*e*m*x + 4*a*e*p**2*x + 6*a*e*p*x +
2*a*e*x + c*d*m**2*x**2 + 4*c*d*m*p*x**2 + 3*c*d*m*x**2 + 4*c*d*p**2*x**2
+ 6*c*d*p*x**2 + 2*c*d*x**2 + c*e*m**2*x**3 + 4*c*e*m*p*x**3 + 3*c*e*m*x**
3 + 4*c*e*p**2*x**3 + 6*c*e*p*x**3 + 2*c*e*x**3),x)*a*c*d*e**2*g*m**4 - 12
*int(((d + e*x)**m*(a + c*x**2)**p*x**2)/(a*d*m**2 + 4*a*d*m*p + 3*a*d*m +
 4*a*d*p**2 + 6*a*d*p + 2*a*d + a*e*m**2*x + 4*a*e*m*p*x + 3*a*e*m*x + 4*a
*e*p**2*x + 6*a*e*p*x + 2*a*e*x + c*d*m**2*x**2 + 4*c*d*m*p*x**2 + 3*c*d*m
*x**2 + 4*c*d*p**2*x**2 + 6*c*d*p*x**2 + 2*c*d*x**2 + c*e*m**2*x**3 + 4*c*
e*m*p*x**3 + 3*c*e*m*x**3 + 4*c*e*p**2*x**3 + 6*c*e*p*x**3 + 2*c*e*x**3),x
)*a*c*d*e**2*g*m**3*p - 7*int(((d + e*x)**m*(a + c*x**2)**p*x**2)/(a*d*m**
2 + 4*a*d*m*p + 3*a*d*m + 4*a*d*p**2 + 6*a*d*p + 2*a*d + a*e*m**2*x + 4...
```

3.175 $\int (d+ex)^m \left(a+cx^2\right)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 152

$$\int (d+ex)^m \left(a+cx^2\right)^p \, dx$$
$$= \frac{(d+ex)^{1+m}\left(a+cx^2\right)^p \left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p}\left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p} \operatorname{AppellF1}\left(1+m,-p,-p,2+m,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d\cdots}{d+\cdots}\right.}{e(1+m)}$$

output
```
(e*x+d)^(1+m)*(c*x^2+a)^p*AppellF1(1+m,-p,-p,2+m,(e*x+d)/(d-(-a)^(1/2)*e/c
^(1/2)),(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))/e/(1+m)/((1-(e*x+d)/(d-(-a)^(1/2
)*e/c^(1/2)))^p)/((1-(e*x+d)/(d+(-a)^(1/2)*e/c^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03

$$\int (d+ex)^m \left(a+cx^2\right)^p \, dx$$
$$= \frac{\left(\frac{e\left(\sqrt{-\frac{a}{c}}-x\right)}{d+\sqrt{-\frac{a}{c}}e}\right)^{-p}\left(\frac{e\left(\sqrt{-\frac{a}{c}}+x\right)}{-d+\sqrt{-\frac{a}{c}}e}\right)^{-p}(d+ex)^{1+m}\left(a+cx^2\right)^p \operatorname{AppellF1}\left(1+m,-p,-p,2+m,\frac{d+ex}{d-\sqrt{-\frac{a}{c}}e},\frac{d+e\cdots}{d+\sqrt{\cdots}}\right.}{e(1+m)}$$

input
```
Integrate[(d + e*x)^m*(a + c*x^2)^p,x]
```

output
```
((d + e*x)^(1 + m)*(a + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (d + e*x)/
(d - Sqrt[-(a/c)]*e), (d + e*x)/(d + Sqrt[-(a/c)]*e)])/(e*(1 + m)*((e*(Sqr
t[-(a/c)] - x))/(d + Sqrt[-(a/c)]*e))^p*((e*(Sqrt[-(a/c)] + x))/(-d + Sqrt
[-(a/c)]*e))^p)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^p (d+ex)^m \, dx$$

$$\Big\downarrow \text{514}$$

$$\frac{(a+cx^2)^p \left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p} \left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p} \int (d+ex)^m \left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^p \left(1-\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^p d(d+ex)}{e}$$

$$\Big\downarrow \text{150}$$

$$\frac{(a+cx^2)^p (d+ex)^{m+1} \left(1-\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}}\right)^{-p} \left(1-\frac{d+ex}{\frac{\sqrt{-a}e}{\sqrt{c}}+d}\right)^{-p} \text{AppellF1}\left(m+1,-p,-p,m+2,\frac{d+ex}{d-\frac{\sqrt{-a}e}{\sqrt{c}}},\frac{d+ex}{d+\frac{\sqrt{-a}e}{\sqrt{c}}}\right)}{e(m+1)}$$

input
```
Int[(d + e*x)^m*(a + c*x^2)^p,x]
```

output
```
((d + e*x)^(1 + m)*(a + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (d + e*x)/
(d - (Sqrt[-a]*e)/Sqrt[c]), (d + e*x)/(d + (Sqrt[-a]*e)/Sqrt[c])])/(e*(1 +
 m)*(1 - (d + e*x)/(d - (Sqrt[-a]*e)/Sqrt[c]))^p*(1 - (d + e*x)/(d + (Sqrt
[-a]*e)/Sqrt[c]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int (ex+d)^m \left(c\,x^2+a\right)^p dx$$

input
```
int((e*x+d)^m*(c*x^2+a)^p,x)
```

output
```
int((e*x+d)^m*(c*x^2+a)^p,x)
```

Fricas [F]

$$\int (d+ex)^m \left(a+cx^2\right)^p \,dx = \int \left(cx^2+a\right)^p (ex+d)^m \,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)^p*(e*x + d)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \left(a+cx^2\right)^p \,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(c*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^m \left(a+cx^2\right)^p \,dx = \int \left(cx^2+a\right)^p (ex+d)^m \,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^p*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m \left(a+cx^2\right)^p \,dx = \int \left(cx^2+a\right)^p (ex+d)^m \,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^p*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \left(a+cx^2\right)^p \, dx = \int \left(c\,x^2+a\right)^p\,\left(d+e\,x\right)^m \,dx$$

input
```
int((a + c*x^2)^p*(d + e*x)^m,x)
```

output
```
int((a + c*x^2)^p*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m \left(a+cx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((e*x+d)^m*(c*x^2+a)^p,x)
```

output
```
((d + e*x)**m*(a + c*x**2)**p*a*e + (d + e*x)**m*(a + c*x**2)**p*c*d*x - i
nt(((d + e*x)**m*(a + c*x**2)**p*x**2)/(a*d*m + 2*a*d*p + a*d + a*e*m*x +
2*a*e*p*x + a*e*x + c*d*m*x**2 + 2*c*d*p*x**2 + c*d*x**2 + c*e*m*x**3 + 2*
c*e*p*x**3 + c*e*x**3),x)*a*c*e**2*m**2 - 4*int(((d + e*x)**m*(a + c*x**2)
**p*x**2)/(a*d*m + 2*a*d*p + a*d + a*e*m*x + 2*a*e*p*x + a*e*x + c*d*m*x**
2 + 2*c*d*p*x**2 + c*d*x**2 + c*e*m*x**3 + 2*c*e*p*x**3 + c*e*x**3),x)*a*c
*e**2*m*p - int(((d + e*x)**m*(a + c*x**2)**p*x**2)/(a*d*m + 2*a*d*p + a*d
 + a*e*m*x + 2*a*e*p*x + a*e*x + c*d*m*x**2 + 2*c*d*p*x**2 + c*d*x**2 + c*
e*m*x**3 + 2*c*e*p*x**3 + c*e*x**3),x)*a*c*e**2*m - 4*int(((d + e*x)**m*(a
 + c*x**2)**p*x**2)/(a*d*m + 2*a*d*p + a*d + a*e*m*x + 2*a*e*p*x + a*e*x +
 c*d*m*x**2 + 2*c*d*p*x**2 + c*d*x**2 + c*e*m*x**3 + 2*c*e*p*x**3 + c*e*x*
*3),x)*a*c*e**2*p**2 - 2*int(((d + e*x)**m*(a + c*x**2)**p*x**2)/(a*d*m +
2*a*d*p + a*d + a*e*m*x + 2*a*e*p*x + a*e*x + c*d*m*x**2 + 2*c*d*p*x**2 +
c*d*x**2 + c*e*m*x**3 + 2*c*e*p*x**3 + c*e*x**3),x)*a*c*e**2*p + int(((d +
 e*x)**m*(a + c*x**2)**p*x**2)/(a*d*m + 2*a*d*p + a*d + a*e*m*x + 2*a*e*p*
x + a*e*x + c*d*m*x**2 + 2*c*d*p*x**2 + c*d*x**2 + c*e*m*x**3 + 2*c*e*p*x*
*3 + c*e*x**3),x)*c**2*d**2*m**2 + 2*int(((d + e*x)**m*(a + c*x**2)**p*x**
2)/(a*d*m + 2*a*d*p + a*d + a*e*m*x + 2*a*e*p*x + a*e*x + c*d*m*x**2 + 2*c
*d*p*x**2 + c*d*x**2 + c*e*m*x**3 + 2*c*e*p*x**3 + c*e*x**3),x)*c**2*d**2*
m*p + int(((d + e*x)**m*(a + c*x**2)**p*x**2)/(a*d*m + 2*a*d*p + a*d + ...
```

3.176 $\int \frac{(d+ex)^m(a+cx^2)^p}{f+gx}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(d+ex)^m (a+cx^2)^p}{f+gx}\,dx = \text{Int}\left(\frac{(d+ex)^m (a+cx^2)^p}{f+gx},x\right)$$

output
```
Defer(Int)((e*x+d)^m*(c*x^2+a)^p/(g*x+f),x)
```

Mathematica [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^m (a+cx^2)^p}{f+gx}\,dx = \int \frac{(d+ex)^m (a+cx^2)^p}{f+gx}\,dx$$

input
```
Integrate[((d + e*x)^m*(a + c*x^2)^p)/(f + g*x),x]
```

output
```
Integrate[((d + e*x)^m*(a + c*x^2)^p)/(f + g*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {744}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^p (d+ex)^m}{f+gx} \, dx$$

$$\Big\downarrow \; 744$$

$$\int \frac{(a+cx^2)^p (d+ex)^m}{f+gx} dx$$

input
```
Int[((d + e*x)^m*(a + c*x^2)^p)/(f + g*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(ex+d)^m (c\,x^2+a)^p}{gx+f} dx$$

input
```
int((e*x+d)^m*(c*x^2+a)^p/(g*x+f),x)
```

output
```
int((e*x+d)^m*(c*x^2+a)^p/(g*x+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^m (a+cx^2)^p}{f+gx}\,dx = \int \frac{(cx^2+a)^p (ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^p/(g*x+f),x, algorithm="fricas")
```

output
```
integral((c*x^2 + a)^p*(e*x + d)^m/(g*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (a+cx^2)^p}{f+gx}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(c*x**2+a)**p/(g*x+f),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^m (a+cx^2)^p}{f+gx}\,dx = \int \frac{(cx^2+a)^p (ex+d)^m}{gx+f}\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^p/(g*x+f),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^p*(e*x + d)^m/(g*x + f), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^m (a+cx^2)^p}{f+gx} \, dx = \int \frac{(cx^2+a)^p (ex+d)^m}{gx+f} \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+a)^p/(g*x+f),x, algorithm="giac")
```

output
```
integrate((c*x^2 + a)^p*(e*x + d)^m/(g*x + f), x)
```

Mupad [N/A]

Not integrable

Time = 6.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^m (a+cx^2)^p}{f+gx} \, dx = \int \frac{(c\,x^2+a)^p\,(d+e\,x)^m}{f+g\,x} \, dx$$

input
```
int(((a + c*x^2)^p*(d + e*x)^m)/(f + g*x),x)
```

output
```
int(((a + c*x^2)^p*(d + e*x)^m)/(f + g*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 6068, normalized size of antiderivative = 252.83

$$\int \frac{(d+ex)^m (a+cx^2)^p}{f+gx} dx = \text{Too large to display}$$

input
```
int((e*x+d)^m*(c*x^2+a)^p/(g*x+f),x)
```

output
```
((d + e*x)**m*(a + c*x**2)**p*d - 2*int(((d + e*x)**m*(a + c*x**2)**p*x**3
)/(2*a*d**2*f*g*p + 2*a*d**2*g**2*p*x + a*d*e*f**2*m + 2*a*d*e*f**2*p + a*
d*e*f*g*m*x + 4*a*d*e*f*g*p*x + 2*a*d*e*g**2*p*x**2 + a*e**2*f**2*m*x + 2*
a*e**2*f**2*p*x + a*e**2*f*g*m*x**2 + 2*a*e**2*f*g*p*x**2 + 2*c*d**2*f*g*p
*x**2 + 2*c*d**2*g**2*p*x**3 + c*d*e*f**2*m*x**2 + 2*c*d*e*f**2*p*x**2 + c
*d*e*f*g*m*x**3 + 4*c*d*e*f*g*p*x**3 + 2*c*d*e*g**2*p*x**4 + c*e**2*f**2*m
*x**3 + 2*c*e**2*f**2*p*x**3 + c*e**2*f*g*m*x**4 + 2*c*e**2*f*g*p*x**4),x)
*c*d**2*e*g**2*m*p - int(((d + e*x)**m*(a + c*x**2)**p*x**3)/(2*a*d**2*f*g
*p + 2*a*d**2*g**2*p*x + a*d*e*f**2*m + 2*a*d*e*f**2*p + a*d*e*f*g*m*x + 4
*a*d*e*f*g*p*x + 2*a*d*e*g**2*p*x**2 + a*e**2*f**2*m*x + 2*a*e**2*f**2*p*x
 + a*e**2*f*g*m*x**2 + 2*a*e**2*f*g*p*x**2 + 2*c*d**2*f*g*p*x**2 + 2*c*d**
2*g**2*p*x**3 + c*d*e*f**2*m*x**2 + 2*c*d*e*f**2*p*x**2 + c*d*e*f*g*m*x**3
 + 4*c*d*e*f*g*p*x**3 + 2*c*d*e*g**2*p*x**4 + c*e**2*f**2*m*x**3 + 2*c*e**
2*f**2*p*x**3 + c*e**2*f*g*m*x**4 + 2*c*e**2*f*g*p*x**4),x)*c*d*e**2*f*g*m
**2 + 4*int(((d + e*x)**m*(a + c*x**2)**p*x**3)/(2*a*d**2*f*g*p + 2*a*d**2
*g**2*p*x + a*d*e*f**2*m + 2*a*d*e*f**2*p + a*d*e*f*g*m*x + 4*a*d*e*f*g*p*
x + 2*a*d*e*g**2*p*x**2 + a*e**2*f**2*m*x + 2*a*e**2*f**2*p*x + a*e**2*f*g
*m*x**2 + 2*a*e**2*f*g*p*x**2 + 2*c*d**2*f*g*p*x**2 + 2*c*d**2*g**2*p*x**3
 + c*d*e*f**2*m*x**2 + 2*c*d*e*f**2*p*x**2 + c*d*e*f*g*m*x**3 + 4*c*d*e*f*
g*p*x**3 + 2*c*d*e*g**2*p*x**4 + c*e**2*f**2*m*x**3 + 2*c*e**2*f**2*p*x...
```

3.177 $\int (c+dx)^{-4-2p}(e+fx)^3 \left(a+bx^2\right)^p \, dx$

Optimal result

Integrand size = 28, antiderivative size = 726

$$\int (c+dx)^{-4-2p}(e+fx)^3 \left(a+bx^2\right)^p \, dx = -\frac{3f^2(de-cf)(c+dx)^{-3-2p}\left(a+bx^2\right)^{1+p}}{bd^2}$$
$$+\frac{(de-cf)\left(3ad^2f^2(3+2p)-b(d^2e^2-2cdef-2c^2f^2(4+3p))\right)(c+dx)^{-3-2p}\left(a+bx^2\right)^{1+p}}{bd^2\left(bc^2+ad^2\right)(3+2p)}$$
$$-\frac{(de-cf)^2\left(3ad^2f(3+2p)+bc(2de(2+p)+cf(5+4p))\right)(c+dx)^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{2d^2\left(bc^2+ad^2\right)^2(1+p)(3+2p)}$$
$$-\frac{f^3(c+dx)^{-2p}\left(a+bx^2\right)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\right.}{2d^4p}$$
$$-\frac{(de-cf)\left(3a^2d^4f^2(3+2p)+b^2c^2(d^2e^2+cdef+c^2f^2)(3+2p)-abd^2(d^2e^2-2c^2f^2(4+3p)-cdef(\right.}{d^3}$$

output
```
-3*f^2*(-c*f+d*e)*(d*x+c)^(-3-2*p)*(b*x^2+a)^(p+1)/b/d^2+(-c*f+d*e)*(3*a*d
^2*f^2*(3+2*p)-b*(d^2*e^2-2*c*d*e*f-2*c^2*f^2*(4+3*p)))*(d*x+c)^(-3-2*p)*(
b*x^2+a)^(p+1)/b/d^2/(a*d^2+b*c^2)/(3+2*p)-1/2*(-c*f+d*e)^2*(3*a*d^2*f*(3+
2*p)+b*c*(2*d*e*(2+p)+c*f*(5+4*p)))*(b*x^2+a)^(p+1)/d^2/(a*d^2+b*c^2)^2/(p
+1)/(3+2*p)/((d*x+c)^(2*p+2))-1/2*f^3*(b*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*
p,(d*x+c)/(c-(-a)^(1/2)*d/b^(1/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d^4/p
/((d*x+c)^(2*p))/((1-(d*x+c)/(c-(-a)^(1/2)*d/b^(1/2)))^p)/((1-(d*x+c)/(c+(
-a)^(1/2)*d/b^(1/2)))^p)-(-c*f+d*e)*(3*a^2*d^4*f^2*(3+2*p)+b^2*c^2*(c^2*f^
2+c*d*e*f+d^2*e^2)*(3+2*p)-a*b*d^2*(d^2*e^2-2*c^2*f^2*(4+3*p)-c*d*e*f*(11+
6*p)))*((-a)^(1/2)-b^(1/2)*x)*(d*x+c)^(-1-2*p)*(b*x^2+a)^p*hypergeom([-p,
-1-2*p],[-2*p],2*(-a)^(1/2)*b^(1/2)*(d*x+c)/(b^(1/2)*c-(-a)^(1/2)*d)/((-a)
^(1/2)-b^(1/2)*x))/d^3/(b^(1/2)*c+(-a)^(1/2)*d)/(a*d^2+b*c^2)^2/(1+2*p)/(3
+2*p)/((-(b^(1/2)*c+(-a)^(1/2)*d)*((-a)^(1/2)+b^(1/2)*x)/(b^(1/2)*c-(-a)^(
1/2)*d)/((-a)^(1/2)-b^(1/2)*x))^p)
```

Mathematica [F]

$$\int (c+dx)^{-4-2p}(e+fx)^3\left(a+bx^2\right)^p\,dx=\int (c+dx)^{-4-2p}(e+fx)^3\left(a+bx^2\right)^p\,dx$$

input
```
Integrate[(c + d*x)^(-4 - 2*p)*(e + f*x)^3*(a + b*x^2)^p,x]
```

output
```
Integrate[(c + d*x)^(-4 - 2*p)*(e + f*x)^3*(a + b*x^2)^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3\left(a+bx^2\right)^p(c+dx)^{-2p-4}\,dx$$
$$\Big\downarrow \ 744$$
$$\int (e+fx)^3\left(a+bx^2\right)^p(c+dx)^{-2p-4}dx$$

input
```
Int[(c + d*x)^(-4 - 2*p)*(e + f*x)^3*(a + b*x^2)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

Maple [F]

$$\int (dx+c)^{-4-2p} (fx+e)^3 (b\,x^2+a)^p\,dx$$

input
```
int((d*x+c)^(-4-2*p)*(f*x+e)^3*(b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(-4-2*p)*(f*x+e)^3*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^{-4-2p}(e+fx)^3\left(a+bx^2\right)^p\,dx=\int (fx+e)^3(bx^2+a)^p(dx+c)^{-2\,p-4}\,dx$$

input
```
integrate((d*x+c)^(-4-2*p)*(f*x+e)^3*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3)*(b*x^2 + a)^p*(d*x + c)
^(-2*p - 4), x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{-4-2p}(e+fx)^3\left(a+bx^2\right)^p\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(-4-2*p)*(f*x+e)**3*(b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^{-4-2p}(e+fx)^3\left(a+bx^2\right)^p\,dx = \int (fx+e)^3\left(bx^2+a\right)^p(dx+c)^{-2\,p-4}\,dx$$

input
```
integrate((d*x+c)^(-4-2*p)*(f*x+e)^3*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((f*x + e)^3*(b*x^2 + a)^p*(d*x + c)^(-2*p - 4), x)
```

Giac [F]

$$\int (c+dx)^{-4-2p}(e+fx)^3\left(a+bx^2\right)^p\,dx = \int (fx+e)^3\left(bx^2+a\right)^p(dx+c)^{-2\,p-4}\,dx$$

input
```
integrate((d*x+c)^(-4-2*p)*(f*x+e)^3*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((f*x + e)^3*(b*x^2 + a)^p*(d*x + c)^(-2*p - 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{-4-2p}(e+fx)^3\left(a+bx^2\right)^p\,dx=\int \frac{(e+f\,x)^3\,(b\,x^2+a)^p}{(c+d\,x)^{2\,p+4}}\,dx$$

input
```
int(((e + f*x)^3*(a + b*x^2)^p)/(c + d*x)^(2*p + 4),x)
```

output
```
int(((e + f*x)^3*(a + b*x^2)^p)/(c + d*x)^(2*p + 4), x)
```

Reduce [F]

$$\begin{aligned}&\int (c+dx)^{-4-2p}(e+fx)^3\left(a+bx^2\right)^p\,dx\\&=\left(\int \frac{(b\,x^2+a)^p}{(dx+c)^{2p}\,c^4+4\,(dx+c)^{2p}\,c^3dx+6\,(dx+c)^{2p}\,c^2d^2x^2+4\,(dx+c)^{2p}\,c\,d^3x^3+(dx+c)^{2p}\,d^4x^4}dx\right)e^3\\&\quad+\left(\int \frac{(b\,x^2+a)^p\,x^3}{(dx+c)^{2p}\,c^4+4\,(dx+c)^{2p}\,c^3dx+6\,(dx+c)^{2p}\,c^2d^2x^2+4\,(dx+c)^{2p}\,c\,d^3x^3+(dx+c)^{2p}\,d^4x^4}dx\right)\\&\quad+3\left(\int \frac{(b\,x^2+a)^p\,x^2}{(dx+c)^{2p}\,c^4+4\,(dx+c)^{2p}\,c^3dx+6\,(dx+c)^{2p}\,c^2d^2x^2+4\,(dx+c)^{2p}\,c\,d^3x^3+(dx+c)^{2p}\,d^4x^4}dx\right.\\&\quad+3\left(\int \frac{(b\,x^2+a)^p\,x}{(dx+c)^{2p}\,c^4+4\,(dx+c)^{2p}\,c^3dx+6\,(dx+c)^{2p}\,c^2d^2x^2+4\,(dx+c)^{2p}\,c\,d^3x^3+(dx+c)^{2p}\,d^4x^4}dx\right.\end{aligned}$$

input
```
int((d*x+c)^(-4-2*p)*(f*x+e)^3*(b*x^2+a)^p,x)
```

output
```
int((a + b*x**2)**p/((c + d*x)**(2*p)*c**4 + 4*(c + d*x)**(2*p)*c**3*d*x +
 6*(c + d*x)**(2*p)*c**2*d**2*x**2 + 4*(c + d*x)**(2*p)*c*d**3*x**3 + (c +
 d*x)**(2*p)*d**4*x**4),x)*e**3 + int(((a + b*x**2)**p*x**3)/((c + d*x)**(
2*p)*c**4 + 4*(c + d*x)**(2*p)*c**3*d*x + 6*(c + d*x)**(2*p)*c**2*d**2*x**
2 + 4*(c + d*x)**(2*p)*c*d**3*x**3 + (c + d*x)**(2*p)*d**4*x**4),x)*f**3 +
 3*int(((a + b*x**2)**p*x**2)/((c + d*x)**(2*p)*c**4 + 4*(c + d*x)**(2*p)*
c**3*d*x + 6*(c + d*x)**(2*p)*c**2*d**2*x**2 + 4*(c + d*x)**(2*p)*c*d**3*x
**3 + (c + d*x)**(2*p)*d**4*x**4),x)*e*f**2 + 3*int(((a + b*x**2)**p*x)/((
c + d*x)**(2*p)*c**4 + 4*(c + d*x)**(2*p)*c**3*d*x + 6*(c + d*x)**(2*p)*c*
*2*d**2*x**2 + 4*(c + d*x)**(2*p)*c*d**3*x**3 + (c + d*x)**(2*p)*d**4*x**4
),x)*e**2*f
```

3.178 $\int (c+dx)^{-3-2p}(e+fx)^2\left(a+bx^2\right)^p\,dx$

Optimal result

Integrand size = 28, antiderivative size = 467

$$\int (c+dx)^{-3-2p}(e+fx)^2\left(a+bx^2\right)^p\,dx = -\frac{(de-cf)^2(c+dx)^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{2d\left(bc^2+ad^2\right)(1+p)}$$

$$-\frac{f^2(c+dx)^{-2p}\left(a+bx^2\right)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\right.}{2d^3p}$$

$$-\frac{(de-cf)\left(2ad^2f+bc(de+cf)\right)\left(\sqrt{-a}-\sqrt{b}x\right)\left(-\frac{\left(\sqrt{b}c+\sqrt{-a}d\right)\left(\sqrt{-a}+\sqrt{b}x\right)}{\left(\sqrt{b}c-\sqrt{-a}d\right)\left(\sqrt{-a}-\sqrt{b}x\right)}\right)^{-p}(c+dx)^{-1-2p}\left(a+bx^2\right)}{d^2\left(\sqrt{b}c+\sqrt{-a}d\right)\left(bc^2+ad^2\right)(1+2}$$

output
```
-1/2*(-c*f+d*e)^2*(b*x^2+a)^(p+1)/d/(a*d^2+b*c^2)/(p+1)/((d*x+c)^(2*p+2))-
1/2*f^2*(b*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(d*x+c)/(c-(-a)^(1/2)*d/b^(1
/2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d^3/p/((d*x+c)^(2*p))/((1-(d*x+c)/(
c-(-a)^(1/2)*d/b^(1/2)))^p)/((1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^p)-(-c*f
+d*e)*(2*a*d^2*f+b*c*(c*f+d*e))*((-a)^(1/2)-b^(1/2)*x)*(d*x+c)^(-1-2*p)*(b
*x^2+a)^p*hypergeom([-p, -1-2*p],[-2*p],2*(-a)^(1/2)*b^(1/2)*(d*x+c)/(b^(1
/2)*c-(-a)^(1/2)*d)/((-a)^(1/2)-b^(1/2)*x))/d^2/(b^(1/2)*c+(-a)^(1/2)*d)/(
a*d^2+b*c^2)/(1+2*p)/((-(b^(1/2)*c+(-a)^(1/2)*d)*((-a)^(1/2)+b^(1/2)*x)/(b
^(1/2)*c-(-a)^(1/2)*d)/((-a)^(1/2)-b^(1/2)*x))^p)
```

Mathematica [F]

$$\int (c+dx)^{-3-2p}(e+fx)^2\left(a+bx^2\right)^p\,dx=\int (c+dx)^{-3-2p}(e+fx)^2\left(a+bx^2\right)^p\,dx$$

input
```
Integrate[(c + d*x)^(-3 - 2*p)*(e + f*x)^2*(a + b*x^2)^p,x]
```

output
```
Integrate[(c + d*x)^(-3 - 2*p)*(e + f*x)^2*(a + b*x^2)^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2\left(a+bx^2\right)^p(c+dx)^{-2p-3}\,dx$$
$$\Big\downarrow \text{ 744}$$
$$\int (e+fx)^2\left(a+bx^2\right)^p(c+dx)^{-2p-3}dx$$

input
```
Int[(c + d*x)^(-3 - 2*p)*(e + f*x)^2*(a + b*x^2)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

Maple [F]

$$\int (dx+c)^{-3-2p} (fx+e)^2 \left(b\,x^2+a\right)^p dx$$

input
```
int((d*x+c)^(-3-2*p)*(f*x+e)^2*(b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(-3-2*p)*(f*x+e)^2*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^{-3-2p}(e+fx)^2 \left(a+bx^2\right)^p \, dx = \int (fx+e)^2 \left(bx^2+a\right)^p (dx+c)^{-2p-3} \, dx$$

input
```
integrate((d*x+c)^(-3-2*p)*(f*x+e)^2*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((f^2*x^2 + 2*e*f*x + e^2)*(b*x^2 + a)^p*(d*x + c)^(-2*p - 3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{-3-2p}(e+fx)^2 \left(a+bx^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(-3-2*p)*(f*x+e)**2*(b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^{-3-2p}(e+fx)^2\left(a+bx^2\right)^p\,dx=\int (fx+e)^2\left(bx^2+a\right)^p(dx+c)^{-2p-3}\,dx$$

input
```
integrate((d*x+c)^(-3-2*p)*(f*x+e)^2*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((f*x + e)^2*(b*x^2 + a)^p*(d*x + c)^(-2*p - 3), x)
```

Giac [F]

$$\int (c+dx)^{-3-2p}(e+fx)^2\left(a+bx^2\right)^p\,dx=\int (fx+e)^2\left(bx^2+a\right)^p(dx+c)^{-2p-3}\,dx$$

input
```
integrate((d*x+c)^(-3-2*p)*(f*x+e)^2*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((f*x + e)^2*(b*x^2 + a)^p*(d*x + c)^(-2*p - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{-3-2p}(e+fx)^2\left(a+bx^2\right)^p\,dx=\int \frac{(e+f\,x)^2\,(b\,x^2+a)^p}{(c+d\,x)^{2\,p+3}}\,dx$$

input
```
int(((e + f*x)^2*(a + b*x^2)^p)/(c + d*x)^(2*p + 3),x)
```

output
```
int(((e + f*x)^2*(a + b*x^2)^p)/(c + d*x)^(2*p + 3), x)
```

Reduce [F]

$$\int (c+dx)^{-3-2p}(e+fx)^2\left(a+bx^2\right)^p\,dx$$
$$= \left(\int \frac{\left(b\,x^2+a\right)^p}{\left(dx+c\right)^{2p}c^3+3\left(dx+c\right)^{2p}c^2dx+3\left(dx+c\right)^{2p}c\,d^2x^2+\left(dx+c\right)^{2p}d^3x^3}dx\right)e^2$$
$$+\left(\int \frac{\left(b\,x^2+a\right)^p x^2}{\left(dx+c\right)^{2p}c^3+3\left(dx+c\right)^{2p}c^2dx+3\left(dx+c\right)^{2p}c\,d^2x^2+\left(dx+c\right)^{2p}d^3x^3}dx\right)f^2$$
$$+2\left(\int \frac{\left(b\,x^2+a\right)^p x}{\left(dx+c\right)^{2p}c^3+3\left(dx+c\right)^{2p}c^2dx+3\left(dx+c\right)^{2p}c\,d^2x^2+\left(dx+c\right)^{2p}d^3x^3}dx\right)ef$$

input
```
int((d*x+c)^(-3-2*p)*(f*x+e)^2*(b*x^2+a)^p,x)
```

output
```
int((a + b*x**2)**p/((c + d*x)**(2*p)*c**3 + 3*(c + d*x)**(2*p)*c**2*d*x +
 3*(c + d*x)**(2*p)*c*d**2*x**2 + (c + d*x)**(2*p)*d**3*x**3),x)*e**2 + in
t(((a + b*x**2)**p*x**2)/((c + d*x)**(2*p)*c**3 + 3*(c + d*x)**(2*p)*c**2*
d*x + 3*(c + d*x)**(2*p)*c*d**2*x**2 + (c + d*x)**(2*p)*d**3*x**3),x)*f**2
 + 2*int(((a + b*x**2)**p*x)/((c + d*x)**(2*p)*c**3 + 3*(c + d*x)**(2*p)*c
**2*d*x + 3*(c + d*x)**(2*p)*c*d**2*x**2 + (c + d*x)**(2*p)*d**3*x**3),x)*
e*f
```

3.179 $\int (c+dx)^{-2-2p}(e+fx)\left(a+bx^2\right)^p\,dx$

Optimal result

Integrand size = 26, antiderivative size = 377

$$\int (c+dx)^{-2-2p}(e+fx)\left(a+bx^2\right)^p\,dx =$$

$$-\frac{f(c+dx)^{-2p}\left(a+bx^2\right)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{\ }{c-}\right.}{2d^2p}$$

$$-\frac{(de-cf)\left(\sqrt{-a}-\sqrt{b}x\right)\left(-\frac{\left(\sqrt{b}c+\sqrt{-a}d\right)\left(\sqrt{-a}+\sqrt{b}x\right)}{\left(\sqrt{b}c-\sqrt{-a}d\right)\left(\sqrt{-a}-\sqrt{b}x\right)}\right)^{-p}(c+dx)^{-1-2p}\left(a+bx^2\right)^p\operatorname{Hypergeometric2F1}\left(\right.}{d\left(\sqrt{b}c+\sqrt{-a}d\right)(1+2p)}$$

output
```
-1/2*f*(b*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(d*x+c)/(c-(-a)^(1/2)*d/b^(1/
2)),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d^2/p/((d*x+c)^(2*p))/((1-(d*x+c)/(c
-(-a)^(1/2)*d/b^(1/2)))^p)/((1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^p)-(-c*f+
d*e)*((-a)^(1/2)-b^(1/2)*x)*(d*x+c)^(-1-2*p)*(b*x^2+a)^p*hypergeom([-p, -1
-2*p],[-2*p],2*(-a)^(1/2)*b^(1/2)*(d*x+c)/(b^(1/2)*c-(-a)^(1/2)*d)/((-a)^(
1/2)-b^(1/2)*x))/d/(b^(1/2)*c+(-a)^(1/2)*d)/(1+2*p)/((-(b^(1/2)*c+(-a)^(1/
2)*d)*((-a)^(1/2)+b^(1/2)*x)/(b^(1/2)*c-(-a)^(1/2)*d)/((-a)^(1/2)-b^(1/2)*
x))^p)
```

Mathematica [F]

$$\int (c+dx)^{-2-2p}(e+fx)\left(a+bx^2\right)^p\,dx=\int (c+dx)^{-2-2p}(e+fx)\left(a+bx^2\right)^p\,dx$$

input
```
Integrate[(c + d*x)^(-2 - 2*p)*(e + f*x)*(a + b*x^2)^p,x]
```

output
```
Integrate[(c + d*x)^(-2 - 2*p)*(e + f*x)*(a + b*x^2)^p, x]
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {719, 489, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\left(a+bx^2\right)^p(c+dx)^{-2p-2}\,dx$$

$$\Big\downarrow \text{ 719}$$

$$\frac{(de-cf)\int (c+dx)^{-2(p+1)}\left(bx^2+a\right)^p dx}{d}+\frac{f\int (c+dx)^{-2p-1}\left(bx^2+a\right)^p dx}{d}$$

$$\Big\downarrow \text{ 489}$$

$$\frac{f\int (c+dx)^{-2p-1}\left(bx^2+a\right)^p dx}{d}-$$

$$\frac{\left(\sqrt{-a}-\sqrt{b}x\right)\left(a+bx^2\right)^p(de-cf)(c+dx)^{-2p-1}\left(-\frac{\left(\sqrt{-a}+\sqrt{b}x\right)\left(\sqrt{-a}d+\sqrt{b}c\right)}{\left(\sqrt{-a}-\sqrt{b}x\right)\left(\sqrt{b}c-\sqrt{-a}d\right)}\right)^{-p}\text{Hypergeometric2F1}\Big(-2p-1}{d(2p+1)\left(\sqrt{-a}d+\sqrt{b}c\right)}$$

$$\Big\downarrow \text{ 514}$$

$$\frac{f\left(a+bx^2\right)^p\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p}\int(c+dx)^{-2p-1}\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^p\left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^p d(c+dx)}{d^2}-\frac{\left(\sqrt{-a}-\sqrt{b}x\right)\left(a+bx^2\right)^p(de-cf)(c+dx)^{-2p-1}\left(-\frac{\left(\sqrt{-a}+\sqrt{b}x\right)\left(\sqrt{-a}d+\sqrt{b}c\right)}{\left(\sqrt{-a}-\sqrt{b}x\right)\left(\sqrt{b}c-\sqrt{-a}d\right)}\right)^{-p}\text{Hypergeometric2F1}\left(-2p-1\right.}{d(2p+1)\left(\sqrt{-a}d+\sqrt{b}c\right)}$$

$\downarrow$ 150

$$-\frac{f\left(a+bx^2\right)^p(c+dx)^{-2p}\left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p}\left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{2d^2p}$$
$$\frac{\left(\sqrt{-a}-\sqrt{b}x\right)\left(a+bx^2\right)^p(de-cf)(c+dx)^{-2p-1}\left(-\frac{\left(\sqrt{-a}+\sqrt{b}x\right)\left(\sqrt{-a}d+\sqrt{b}c\right)}{\left(\sqrt{-a}-\sqrt{b}x\right)\left(\sqrt{b}c-\sqrt{-a}d\right)}\right)^{-p}\text{Hypergeometric2F1}\left(-2p-1\right.}{d(2p+1)\left(\sqrt{-a}d+\sqrt{b}c\right)}$$

input
```
Int[(c + d*x)^(-2 - 2*p)*(e + f*x)*(a + b*x^2)^p,x]
```

output
```
-1/2*(f*(a + b*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (c + d*x)/(c - (Sqrt
[-a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])/(d^2*p*(c + d*x)^
(2*p)*(1 - (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sq
rt[-a]*d)/Sqrt[b]))^p) - ((d*e - c*f)*(Sqrt[-a] - Sqrt[b]*x)*(c + d*x)^(-1
 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sq
rt[b]*(c + d*x))/((Sqrt[b]*c - Sqrt[-a]*d)*(Sqrt[-a] - Sqrt[b]*x))])/(d*(S
qrt[b]*c + Sqrt[-a]*d)*(1 + 2*p)*(-(((Sqrt[b]*c + Sqrt[-a]*d)*(Sqrt[-a] +
Sqrt[b]*x))/((Sqrt[b]*c - Sqrt[-a]*d)*(Sqrt[-a] - Sqrt[b]*x))))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [F]

$$\int (dx+c)^{-2p-2} (fx+e) \left(b\,x^2+a\right)^p dx$$

input
```
int((d*x+c)^(-2*p-2)*(f*x+e)*(b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(-2*p-2)*(f*x+e)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^{-2-2p}(e+fx)\left(a+bx^2\right)^p\,dx = \int (fx+e)\left(bx^2+a\right)^p(dx+c)^{-2\,p-2}\,dx$$

input
```
integrate((d*x+c)^(-2-2*p)*(f*x+e)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((f*x + e)*(b*x^2 + a)^p*(d*x + c)^(-2*p - 2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{-2-2p}(e+fx)\left(a+bx^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)**(-2-2*p)*(f*x+e)*(b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^{-2-2p}(e+fx)\left(a+bx^2\right)^p\,dx=\int (fx+e)\left(bx^2+a\right)^p(dx+c)^{-2\,p-2}\,dx$$

input
```
integrate((d*x+c)^(-2-2*p)*(f*x+e)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((f*x + e)*(b*x^2 + a)^p*(d*x + c)^(-2*p - 2), x)
```

Giac [F]

$$\int (c+dx)^{-2-2p}(e+fx)\left(a+bx^2\right)^p\,dx=\int (fx+e)\left(bx^2+a\right)^p(dx+c)^{-2\,p-2}\,dx$$

input
```
integrate((d*x+c)^(-2-2*p)*(f*x+e)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((f*x + e)*(b*x^2 + a)^p*(d*x + c)^(-2*p - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{-2-2p}(e+fx)\left(a+bx^2\right)^p\,dx=\int \frac{(e+f\,x)\,\left(b\,x^2+a\right)^p}{(c+d\,x)^{2\,p+2}}\,dx$$

input
```
int(((e + f*x)*(a + b*x^2)^p)/(c + d*x)^(2*p + 2),x)
```

output
```
int(((e + f*x)*(a + b*x^2)^p)/(c + d*x)^(2*p + 2), x)
```

Reduce [F]

$$\begin{aligned}&\int (c+dx)^{-2-2p}(e+fx)\left(a+bx^2\right)^p\,dx\\&=\left(\int \frac{\left(b\,x^2+a\right)^p}{(dx+c)^{2p}\,c^2+2\,(dx+c)^{2p}\,cdx+(dx+c)^{2p}\,d^2x^2}dx\right)e\\&\quad+\left(\int \frac{\left(b\,x^2+a\right)^p x}{(dx+c)^{2p}\,c^2+2\,(dx+c)^{2p}\,cdx+(dx+c)^{2p}\,d^2x^2}dx\right)f\end{aligned}$$

input
```
int((d*x+c)^(-2-2*p)*(f*x+e)*(b*x^2+a)^p,x)
```

output
```
int((a + b*x**2)**p/((c + d*x)**(2*p)*c**2 + 2*(c + d*x)**(2*p)*c*d*x + (c
 + d*x)**(2*p)*d**2*x**2),x)*e + int(((a + b*x**2)**p*x)/((c + d*x)**(2*p)
*c**2 + 2*(c + d*x)**(2*p)*c*d*x + (c + d*x)**(2*p)*d**2*x**2),x)*f
```

3.180 $\int (c+dx)^{-1-2p} \left(a+bx^2\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int (c+dx)^{-1-2p} \left(a+bx^2\right)^p \, dx = -\frac{(c+dx)^{-2p} \left(a+bx^2\right)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\right.}{2dp}$$

output
```
-1/2*(b*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(d*x+c)/(c-(-a)^(1/2)*d/b^(1/2)
),(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))/d/p/((d*x+c)^(2*p))/((1-(d*x+c)/(c-(-a
)^(1/2)*d/b^(1/2)))^p)/((1-(d*x+c)/(c+(-a)^(1/2)*d/b^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03

$$\int (c+dx)^{-1-2p} \left(a+bx^2\right)^p \, dx = -\frac{\left(\frac{d\left(\sqrt{-\frac{a}{b}}-x\right)}{c+\sqrt{-\frac{a}{b}}d}\right)^{-p} \left(\frac{d\left(\sqrt{-\frac{a}{b}}+x\right)}{-c+\sqrt{-\frac{a}{b}}d}\right)^{-p} (c+dx)^{-2p} \left(a+bx^2\right)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c+dx}{c-\sqrt{-\frac{a}{b}}d},\right.}{2dp}$$

input
```
Integrate[(c + d*x)^(-1 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-1/2*((a + b*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (c + d*x)/(c - Sqrt[-(
a/b)]*d), (c + d*x)/(c + Sqrt[-(a/b)]*d)])/(d*p*((d*(Sqrt[-(a/b)] - x))/(c
 + Sqrt[-(a/b)]*d))^p*((d*(Sqrt[-(a/b)] + x))/(-c + Sqrt[-(a/b)]*d))^p*(c
+ d*x)^(2*p))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^p (c+dx)^{-2p-1}\,dx$$

$$\Big\downarrow \ 514$$

$$\frac{(a+bx^2)^p \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p} \int (c+dx)^{-2p-1} \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{p} \left(1-\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{p} d(c+dx)}{d}$$

$$\Big\downarrow \ 150$$

$$-\frac{(a+bx^2)^p (c+dx)^{-2p} \left(1-\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}}\right)^{-p} \left(1-\frac{c+dx}{\frac{\sqrt{-a}d}{\sqrt{b}}+c}\right)^{-p} \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,\frac{c+dx}{c-\frac{\sqrt{-a}d}{\sqrt{b}}},\frac{c+dx}{c+\frac{\sqrt{-a}d}{\sqrt{b}}}\right)}{2dp}$$

input
```
Int[(c + d*x)^(-1 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-1/2*((a + b*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (c + d*x)/(c - (Sqrt[-
a]*d)/Sqrt[b]), (c + d*x)/(c + (Sqrt[-a]*d)/Sqrt[b])])/(d*p*(c + d*x)^(2*p
)*(1 - (c + d*x)/(c - (Sqrt[-a]*d)/Sqrt[b]))^p*(1 - (c + d*x)/(c + (Sqrt[-
a]*d)/Sqrt[b]))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

Maple [F]

$$\int (dx+c)^{-1-2p} \left(b\,x^2+a\right)^p dx$$

input
```
int((d*x+c)^(-1-2*p)*(b*x^2+a)^p,x)
```

output
```
int((d*x+c)^(-1-2*p)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^{-1-2p} \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p (dx+c)^{-2\,p-1} \, dx$$

input
```
integrate((d*x+c)^(-1-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x + c)^(-2*p - 1), x)
```

Sympy [F]

$$\int (c+dx)^{-1-2p}\left(a+bx^2\right)^p\,dx=\int \left(a+bx^2\right)^p(c+dx)^{-2p-1}\,dx$$

input
```
integrate((d*x+c)**(-1-2*p)*(b*x**2+a)**p,x)
```

output
```
Integral((a + b*x**2)**p*(c + d*x)**(-2*p - 1), x)
```

Maxima [F]

$$\int (c+dx)^{-1-2p}\left(a+bx^2\right)^p\,dx=\int \left(bx^2+a\right)^p(dx+c)^{-2\,p-1}\,dx$$

input
```
integrate((d*x+c)^(-1-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x + c)^(-2*p - 1), x)
```

Giac [F]

$$\int (c+dx)^{-1-2p}\left(a+bx^2\right)^p\,dx=\int \left(bx^2+a\right)^p(dx+c)^{-2\,p-1}\,dx$$

input
```
integrate((d*x+c)^(-1-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x + c)^(-2*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{-1-2p}\left(a+bx^2\right)^p\,dx=\int \frac{{\left(b\,x^2+a\right)}^p}{{\left(c+d\,x\right)}^{2\,p+1}}\,dx$$

input
```
int((a + b*x^2)^p/(c + d*x)^(2*p + 1),x)
```

output
```
int((a + b*x^2)^p/(c + d*x)^(2*p + 1), x)
```

Reduce [F]

$$\int (c+dx)^{-1-2p}\left(a+bx^2\right)^p\,dx=\int \frac{(b\,x^2+a)^p}{(dx+c)^{2p}\,c+(dx+c)^{2p}\,dx}dx$$

input
```
int((d*x+c)^(-1-2*p)*(b*x^2+a)^p,x)
```

output
```
int((a + b*x**2)**p/((c + d*x)**(2*p)*c + (c + d*x)**(2*p)*d*x),x)
```

3.181 $\int \frac{(c+dx)^{-2p}(a+bx^2)^p}{e+fx} dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+dx)^{-2p}(a+bx^2)^p}{e+fx} dx = \text{Int}\left(\frac{(c+dx)^{-2p}(a+bx^2)^p}{e+fx}, x\right)$$

output
```
Defer(Int)((b*x^2+a)^p/((d*x+c)^(2*p))/(f*x+e),x)
```

Mathematica [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^{-2p}(a+bx^2)^p}{e+fx} dx = \int \frac{(c+dx)^{-2p}(a+bx^2)^p}{e+fx} dx$$

input
```
Integrate[(a + b*x^2)^p/((c + d*x)^(2*p)*(e + f*x)),x]
```

output
```
Integrate[(a + b*x^2)^p/((c + d*x)^(2*p)*(e + f*x)), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {744}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p (c+dx)^{-2p}}{e+fx} \, dx$$

$$\Big\downarrow \; 744$$

$$\int \frac{(a+bx^2)^p (c+dx)^{-2p}}{e+fx}dx$$

input
```
Int[(a + b*x^2)^p/((c + d*x)^(2*p)*(e + f*x)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(b\,x^2+a)^p\,(dx+c)^{-2p}}{fx+e}dx$$

input
```
int((b*x^2+a)^p/((d*x+c)^(2*p))/(f*x+e),x)
```

output
```
int((b*x^2+a)^p/((d*x+c)^(2*p))/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx)^{-2p}(a+bx^2)^p}{e+fx}\,dx = \int \frac{(bx^2+a)^p}{(fx+e)(dx+c)^{2p}}\,dx$$

input
```
integrate((b*x^2+a)^p/((d*x+c)^(2*p))/(f*x+e),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/((f*x + e)*(d*x + c)^(2*p)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{-2p}(a+bx^2)^p}{e+fx}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p/((d*x+c)**(2*p))/(f*x+e),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx)^{-2p}(a+bx^2)^p}{e+fx}\,dx = \int \frac{(bx^2+a)^p}{(fx+e)(dx+c)^{2p}}\,dx$$

input
```
integrate((b*x^2+a)^p/((d*x+c)^(2*p))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/((f*x + e)*(d*x + c)^(2*p)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx)^{-2p}(a+bx^2)^p}{e+fx}\,dx = \int \frac{(bx^2+a)^p}{(fx+e)(dx+c)^{2p}}\,dx$$

input
```
integrate((b*x^2+a)^p/((d*x+c)^(2*p))/(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/((f*x + e)*(d*x + c)^(2*p)), x)
```

Mupad [N/A]

Not integrable

Time = 7.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx)^{-2p}(a+bx^2)^p}{e+fx}\,dx = \int \frac{(b\,x^2+a)^p}{(e+f\,x)\,(c+d\,x)^{2\,p}}\,dx$$

input
```
int((a + b*x^2)^p/((e + f*x)*(c + d*x)^(2*p)),x)
```

output
```
int((a + b*x^2)^p/((e + f*x)*(c + d*x)^(2*p)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{(c+dx)^{-2p}\left(a+bx^2\right)^p}{e+fx}\,dx = \int \frac{\left(b\,x^2+a\right)^p}{\left(dx+c\right)^{2p}e+\left(dx+c\right)^{2p}fx}dx$$

input
```
int((b*x^2+a)^p/((d*x+c)^(2*p))/(f*x+e),x)
```

output
```
int((a + b*x**2)**p/((c + d*x)**(2*p)*e + (c + d*x)**(2*p)*f*x),x)
```

3.182 $\int \frac{(c+dx)^{1-2p}(a+bx^2)^p}{(e+fx)^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(c+dx)^{1-2p}\left(a+bx^2\right)^p}{(e+fx)^2}\,dx=\text{Int}\left(\frac{(c+dx)^{1-2p}\left(a+bx^2\right)^p}{(e+fx)^2},x\right)$$

output
```
Defer(Int)((d*x+c)^(1-2*p)*(b*x^2+a)^p/(f*x+e)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)^{1-2p}\left(a+bx^2\right)^p}{(e+fx)^2}\,dx=\int \frac{(c+dx)^{1-2p}\left(a+bx^2\right)^p}{(e+fx)^2}\,dx$$

input
```
Integrate[((c + d*x)^(1 - 2*p)*(a + b*x^2)^p)/(e + f*x)^2,x]
```

output
```
Integrate[((c + d*x)^(1 - 2*p)*(a + b*x^2)^p)/(e + f*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {744}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p (c+dx)^{1-2p}}{(e+fx)^2} dx$$

$$\Big\downarrow \text{744}$$

$$\int \frac{(a+bx^2)^p (c+dx)^{1-2p}}{(e+fx)^2} dx$$

input
```
Int[((c + d*x)^(1 - 2*p)*(a + b*x^2)^p)/(e + f*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^{1-2p} (b\,x^2+a)^p}{(fx+e)^2} dx$$

input
```
int((d*x+c)^(1-2*p)*(b*x^2+a)^p/(f*x+e)^2,x)
```

output
```
int((d*x+c)^(1-2*p)*(b*x^2+a)^p/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{(c+dx)^{1-2p}\left(a+bx^2\right)^p}{(e+fx)^2}\,dx=\int \frac{(bx^2+a)^p(dx+c)^{-2\,p+1}}{(fx+e)^2}\,dx$$

input
```
integrate((d*x+c)^(1-2*p)*(b*x^2+a)^p/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x + c)^(-2*p + 1)/(f^2*x^2 + 2*e*f*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{1-2p}\left(a+bx^2\right)^p}{(e+fx)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)**(1-2*p)*(b*x**2+a)**p/(f*x+e)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)^{1-2p}(a+bx^2)^p}{(e+fx)^2}\,dx = \int \frac{(bx^2+a)^p(dx+c)^{-2\,p+1}}{(fx+e)^2}\,dx$$

input
```
integrate((d*x+c)^(1-2*p)*(b*x^2+a)^p/(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x + c)^(-2*p + 1)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)^{1-2p}(a+bx^2)^p}{(e+fx)^2}\,dx = \int \frac{(bx^2+a)^p(dx+c)^{-2\,p+1}}{(fx+e)^2}\,dx$$

input
```
integrate((d*x+c)^(1-2*p)*(b*x^2+a)^p/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x + c)^(-2*p + 1)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 8.75 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)^{1-2p}(a+bx^2)^p}{(e+fx)^2}\,dx = \int \frac{(b\,x^2+a)^p\,(c+d\,x)^{1-2\,p}}{(e+f\,x)^2}\,dx$$

input
```
int(((a + b*x^2)^p*(c + d*x)^(1 - 2*p))/(e + f*x)^2,x)
```

output
```
int(((a + b*x^2)^p*(c + d*x)^(1 - 2*p))/(e + f*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 4.36

$$\int \frac{(c+dx)^{1-2p}(a+bx^2)^p}{(e+fx)^2}\,dx = \left(\int \frac{(b\,x^2+a)^p}{(dx+c)^{2p}e^2+2\,(dx+c)^{2p}efx+(dx+c)^{2p}f^2x^2}dx\right)c + \left(\int \frac{(b\,x^2+a)^p\,x}{(dx+c)^{2p}e^2+2\,(dx+c)^{2p}efx+(dx+c)^{2p}f^2x^2}dx\right)d$$

input
```
int((d*x+c)^(1-2*p)*(b*x^2+a)^p/(f*x+e)^2,x)
```

output
```
int((a + b*x**2)**p/((c + d*x)**(2*p)*e**2 + 2*(c + d*x)**(2*p)*e*f*x + (c
 + d*x)**(2*p)*f**2*x**2),x)*c + int(((a + b*x**2)**p*x)/((c + d*x)**(2*p)
*e**2 + 2*(c + d*x)**(2*p)*e*f*x + (c + d*x)**(2*p)*f**2*x**2),x)*d
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                 convert(leaf_count_result,string),"$ vs. $2(",
                                 convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file