Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/1.2.1.3/96-1.2.1.3-d1

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**151**]. This is test number [96].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (151)	0.00 (0)
Mathematica	100.00 (151)	0.00 (0)
Maple	96.03 (145)	3.97 (6)
Fricas	96.03 (145)	3.97 (6)
Reduce	83.44 (126)	16.56 (25)
Giac	48.34 (73)	51.66 (78)
Mupad	19.21 (29)	80.79 (122)
Sympy	7.95 (12)	92.05 (139)
Maxima	0.66 (1)	99.34 (150)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

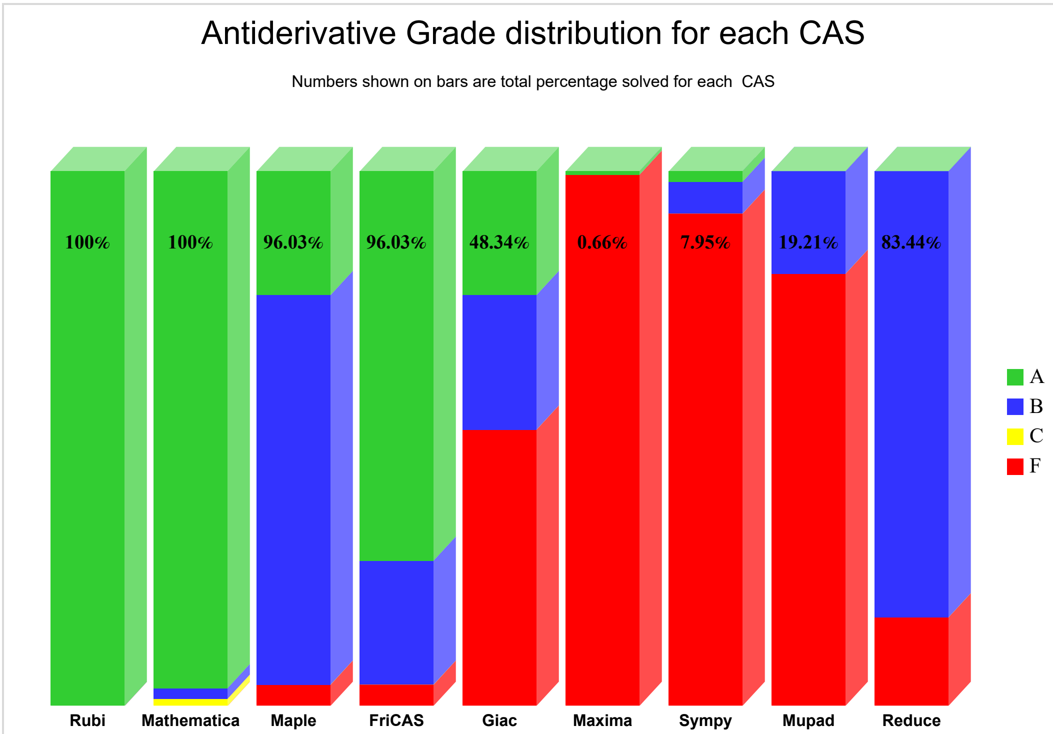
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

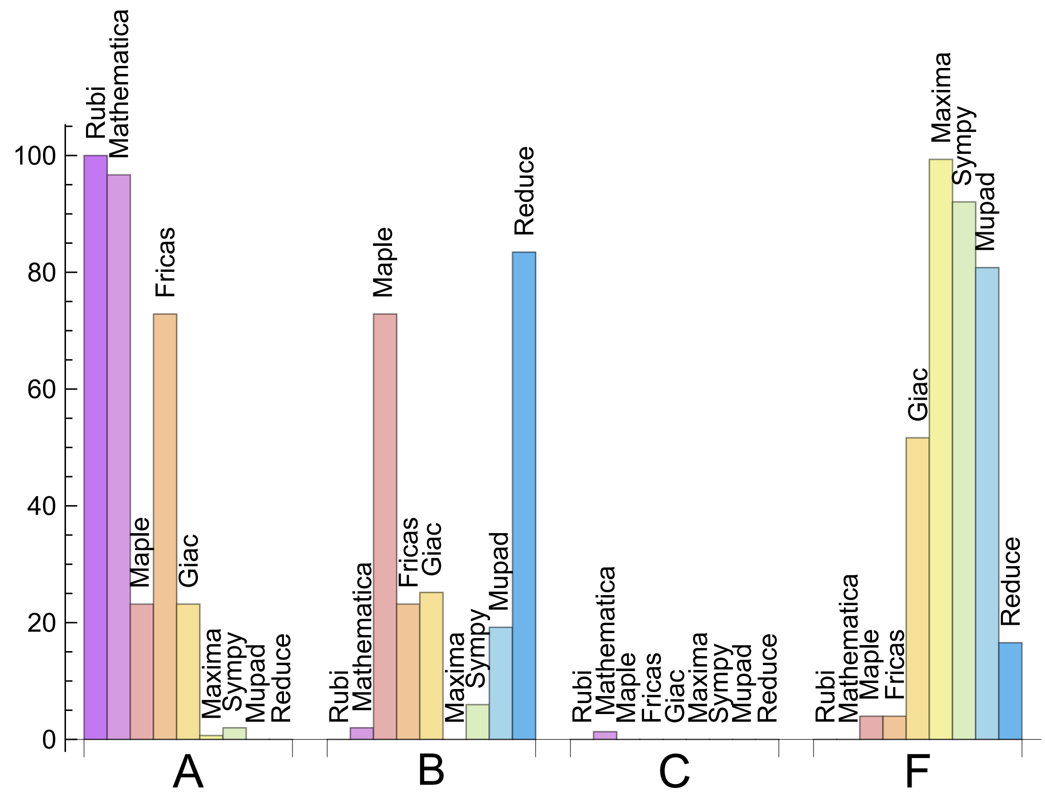
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	96.689	1.987	1.325	0.000
Fricas	72.848	23.179	0.000	3.974
Maple	23.179	72.848	0.000	3.974
Giac	23.179	25.166	0.000	51.656
Sympy	1.987	5.960	0.000	92.053
Maxima	0.662	0.000	0.000	99.338
Mupad	0.000	19.205	0.000	80.795
Reduce	0.000	83.444	0.000	16.556

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	6	100.00	0.00	0.00
Maple	6	100.00	0.00	0.00
Reduce	25	100.00	0.00	0.00
Giac	78	41.03	0.00	58.97
Mupad	122	0.00	100.00	0.00
Sympy	139	69.06	30.94	0.00
Maxima	150	34.00	0.00	66.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.03
Giac	0.23
Rubi	1.12
Mathematica	2.36
Maple	3.00
Reduce	6.01
Fricas	6.73
Mupad	7.14
Sympy	13.05

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	28.00	0.78	28.00	0.78
Mathematica	251.16	0.98	235.00	0.88
Rubi	309.42	1.11	281.00	1.12
Fricas	942.10	3.27	756.00	2.71
Sympy	1109.17	3.81	896.50	3.84
Giac	1175.78	3.86	508.00	1.79
Reduce	1202.12	4.03	822.00	3.25
Mupad	1986.90	7.37	544.00	3.56
Maple	3131.56	8.22	920.00	3.06

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

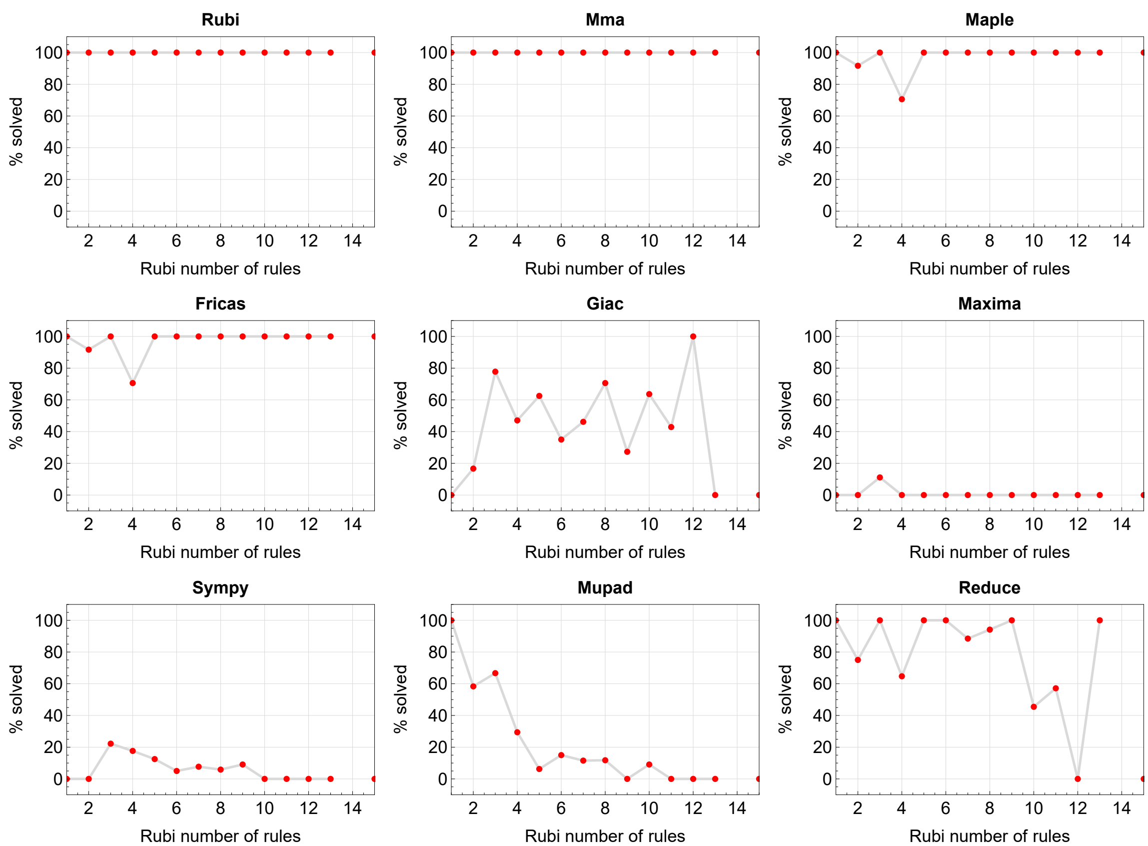


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

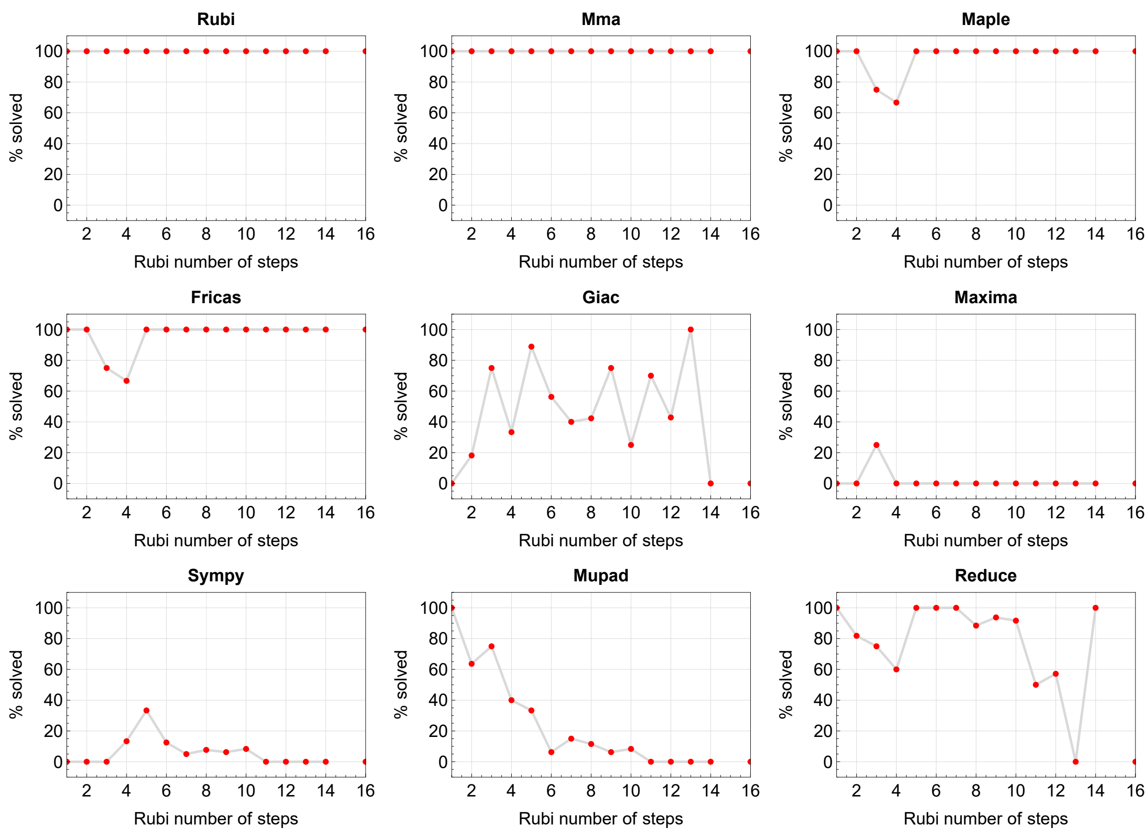


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

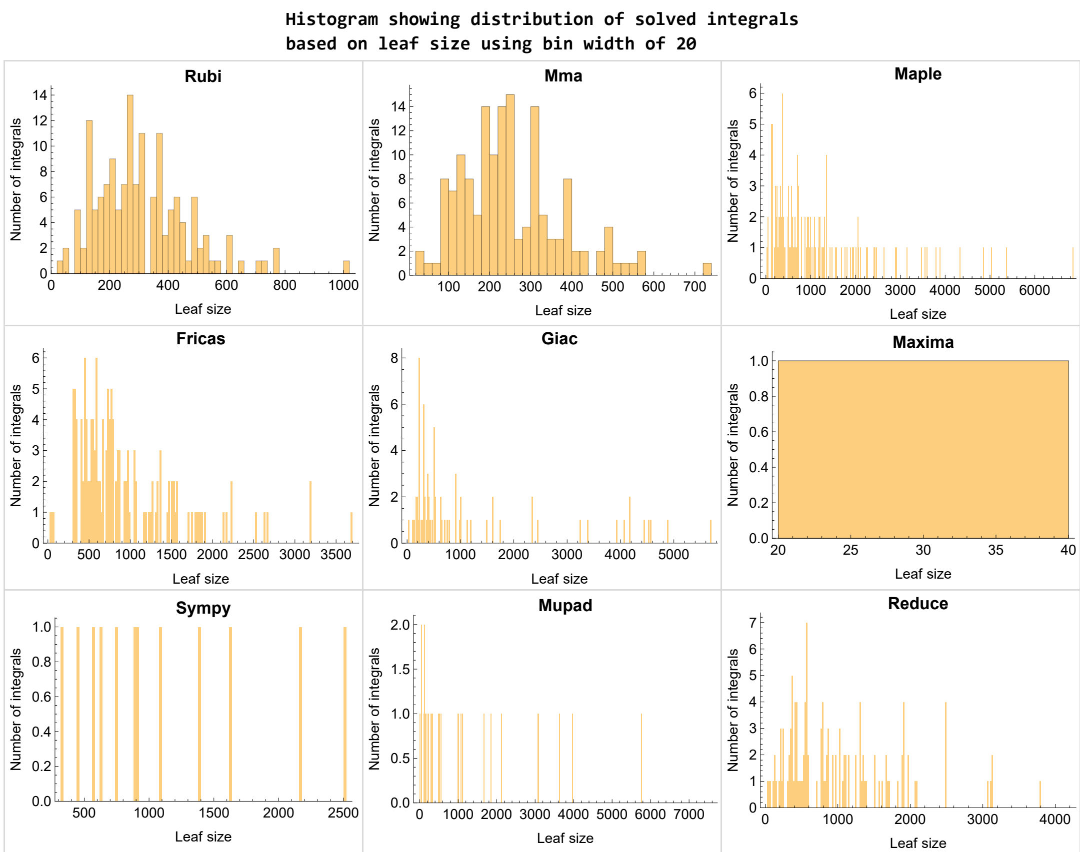


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

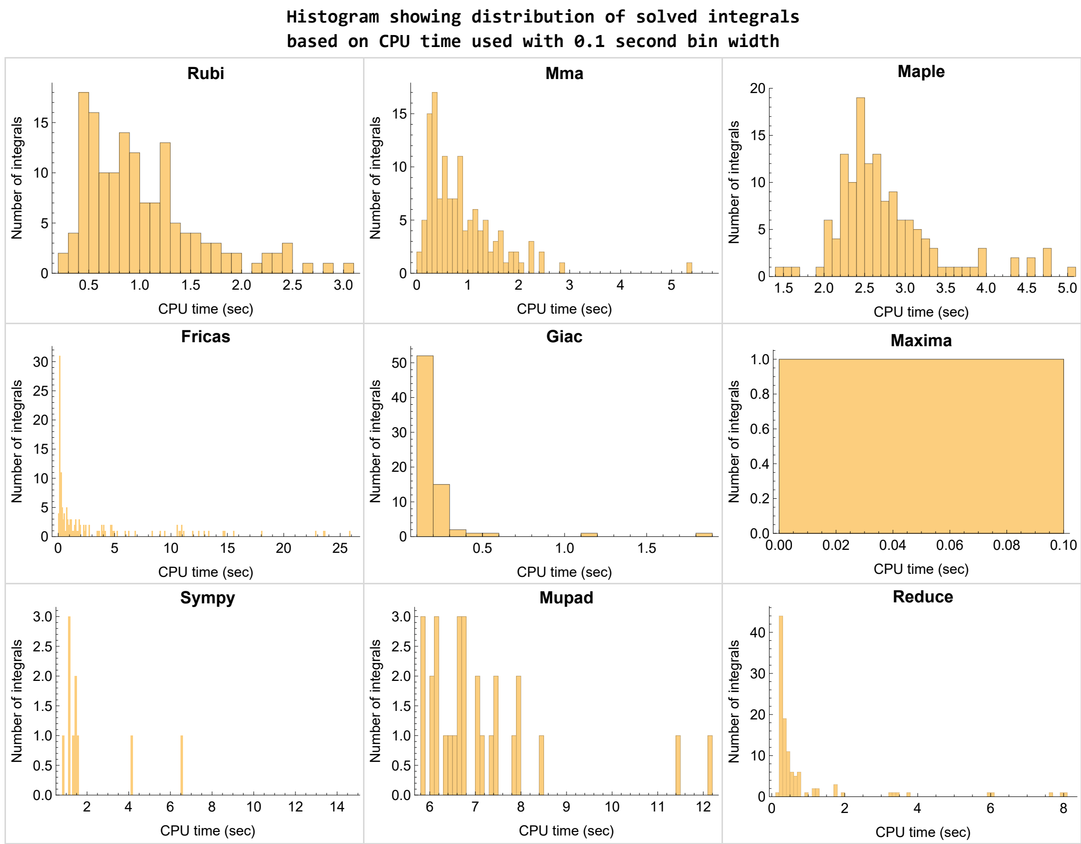


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

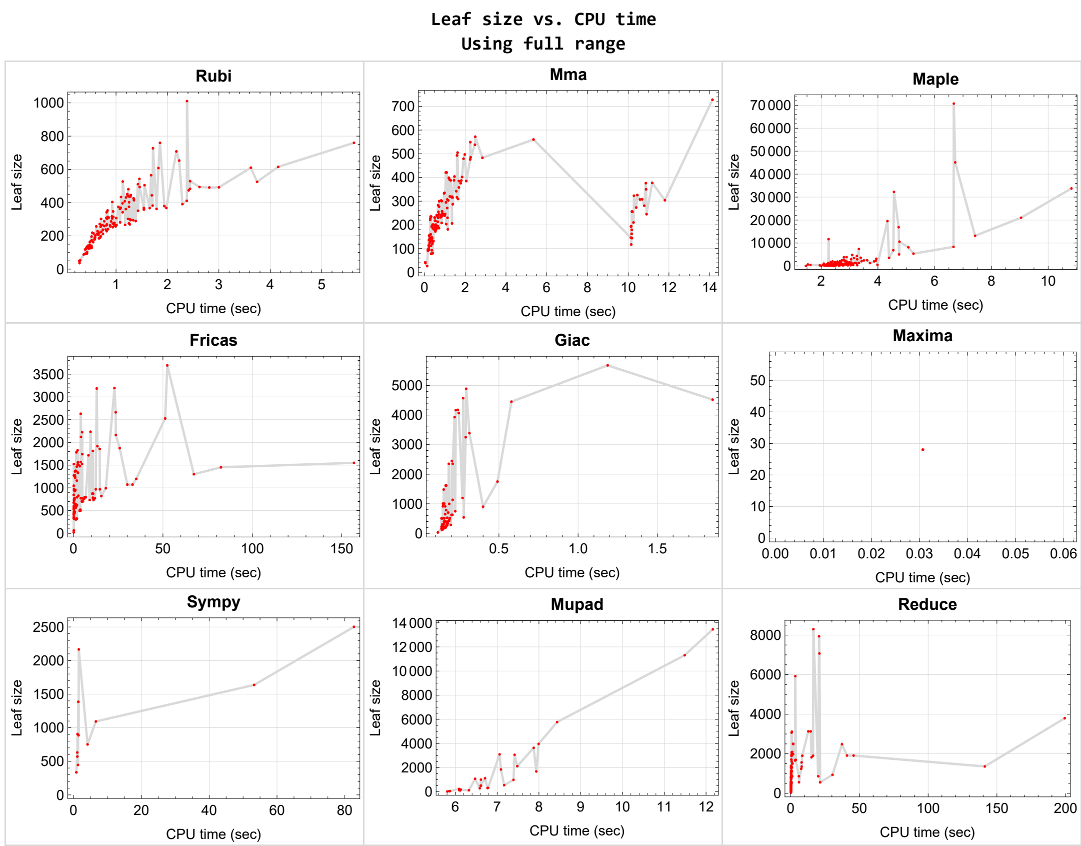


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{146, 147\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

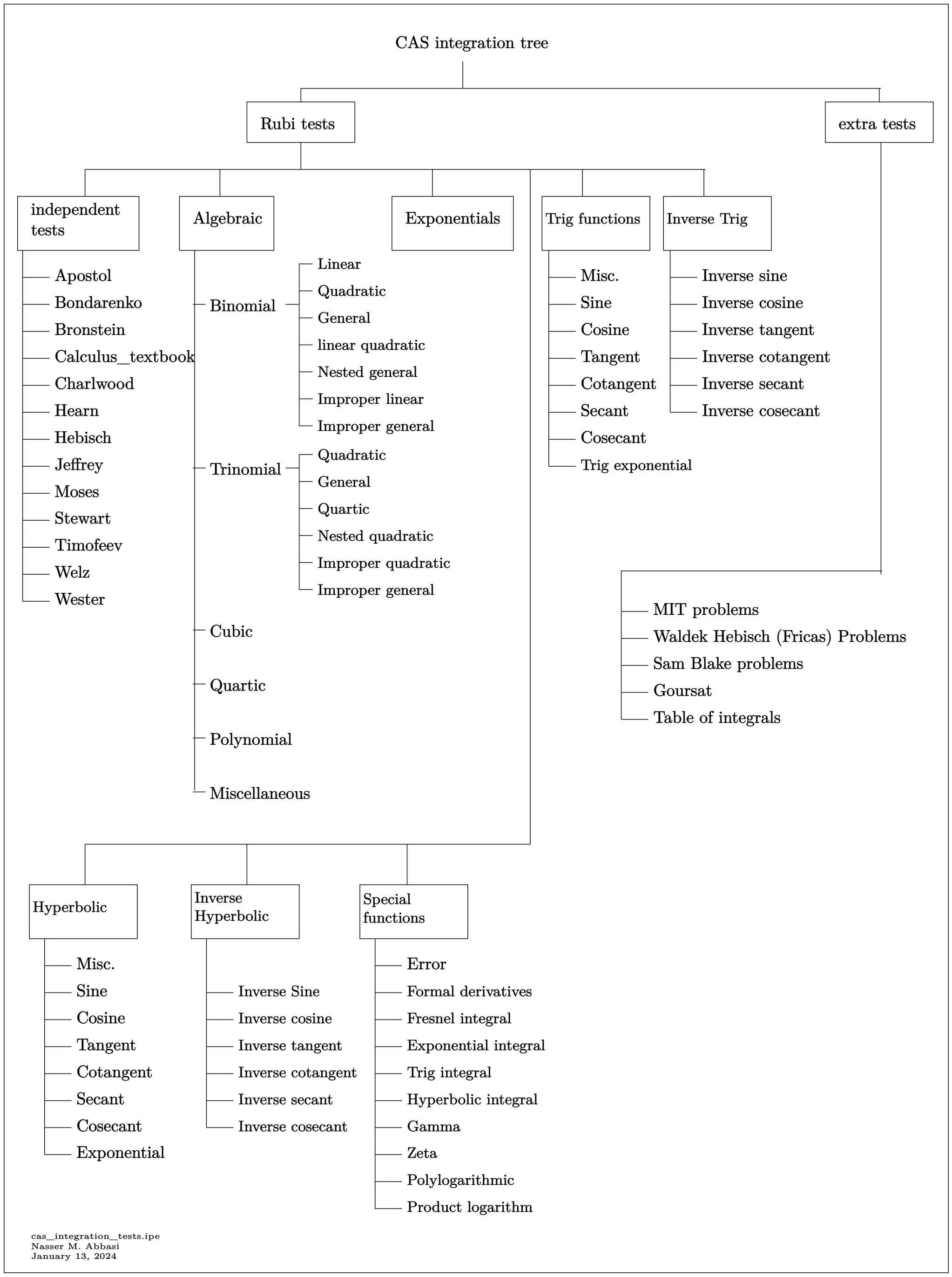
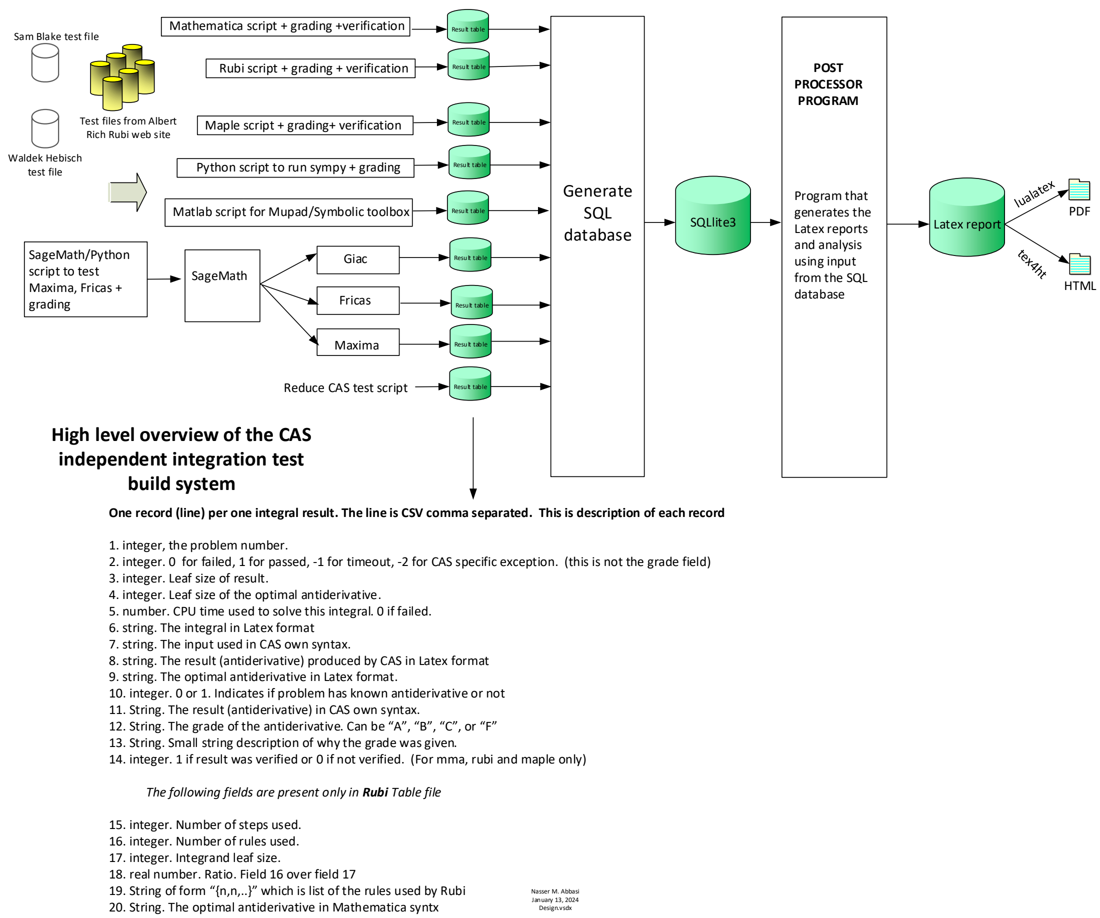


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151 }

B grade { 15, 76, 77 }

C grade { 43, 107 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 5, 14, 31, 51, 75, 76, 77, 81, 82, 83, 84, 85, 87, 91, 115, 116, 117, 118, 119, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 140, 141, 142, 145 }

B grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 78, 79, 80, 86, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 121, 122, 123, 130, 137, 138, 139, 143, 144 }

C grade { }

F normal fail { 146, 147, 148, 149, 150, 151 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 115 }

B grade { 90, 92, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

C grade { }

F normal fail { 146, 147, 148, 149, 150, 151 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 87 }

B grade { }

C grade { }

F normal fail { 16, 17, 18, 19, 24, 25, 26, 27, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 53, 54, 55, 56, 57, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 84, 85, 86, 118, 119, 120, 121, 128, 129, 136, 137, 146, 147, 148, 149, 150, 151 }

F(-1) timedout fail { }

F(-2) exception fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 28, 29, 30, 31, 32, 39, 40, 41, 42, 48, 49, 50, 51, 52, 61, 62, 63, 64, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 122, 123, 124, 125, 126, 127, 130, 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 143, 144, 145 }

Giac

A grade { 1, 2, 3, 4, 6, 11, 12, 13, 14, 23, 28, 29, 30, 31, 33, 39, 40, 41, 42, 48, 49, 50, 51, 53, 61, 62, 63, 64, 72, 73, 74, 75, 80, 81, 87 }

B grade { 7, 8, 9, 10, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 34, 35, 36, 37, 38, 54, 55, 56, 57, 58, 59, 60, 77, 78, 79, 122, 123, 124, 125, 126, 127, 128, 129 }

C grade { }

F normal fail { 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151 }

F(-1) timedout fail { }

F(-2) exception fail { 5, 15, 32, 43, 44, 45, 46, 47, 52, 65, 66, 67, 68, 69, 70, 71, 76, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 75, 76, 77, 83, 87, 90, 91, 115, 116, 117, 124, 125, 126, 127, 131, 132, 133, 134, 135, 140, 141, 142, 145 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 78, 79, 80, 81, 82, 84, 85, 86, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 118, 119, 120, 121, 122, 123, 128, 129, 130, 136, 137, 138, 139, 143, 144, 146, 147, 148, 149, 150, 151 }

F(-2) exception fail { }

Sympy

A grade { 29, 30, 31 }

B grade { 1, 2, 3, 4, 28, 72, 73, 74, 75 }

C grade { }

F normal fail { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 32, 33, 34, 42, 43, 52, 65, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144 }

F(-1) timedout fail { 19, 26, 27, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 145, 146, 147, 148, 149, 150, 151 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 120, 121, 123, 124, 125, 126, 127, 129, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 145 }

C grade { }

F normal fail { 38, 39, 53, 54, 58, 59, 60, 61, 62, 71, 103, 112, 119, 122, 128, 130, 131, 137, 144, 146, 147, 148, 149, 150, 151 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	451	385	1949	0	1044	2166	507	1033	1678
N.S.	1	1.02	0.87	4.40	0.00	2.36	4.89	1.14	2.33	3.79
time (sec)	N/A	1.220	1.688	2.401	0.000	0.143	1.549	0.155	0.355	7.937

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	353	307	1160	0	846	1386	398	790	985
N.S.	1	1.04	0.90	3.41	0.00	2.49	4.08	1.17	2.32	2.90
time (sec)	N/A	0.835	1.115	2.492	0.000	0.132	1.413	0.153	0.330	7.390

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	257	238	655	0	676	887	304	579	544
N.S.	1	1.06	0.98	2.70	0.00	2.78	3.65	1.25	2.38	2.24
time (sec)	N/A	0.616	0.818	2.188	0.000	0.105	1.472	0.143	0.253	7.167

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	219	179	373	0	532	571	222	400	307
N.S.	1	1.12	0.92	1.91	0.00	2.73	2.93	1.14	2.05	1.57
time (sec)	N/A	0.539	0.513	2.087	0.000	0.101	1.125	0.145	0.241	6.768

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	264	233	330	0	1337	0	0	428	291
N.S.	1	1.18	1.04	1.48	0.00	6.00	0.00	0.00	1.92	1.30
time (sec)	N/A	0.815	0.781	2.088	0.000	2.402	0.000	0.000	0.292	6.583

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	237	230	571	0	1241	0	327	527	320
N.S.	1	1.18	1.15	2.86	0.00	6.20	0.00	1.64	2.64	1.60
time (sec)	N/A	0.745	0.667	2.204	0.000	0.878	0.000	0.172	0.609	6.783

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	269	243	1035	0	1375	0	626	1089	0
N.S.	1	1.19	1.07	4.56	0.00	6.06	0.00	2.76	4.80	0.00
time (sec)	N/A	0.822	0.900	2.246	0.000	1.120	0.000	0.200	0.414	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	228	199	1340	0	560	0	1007	1116	0
N.S.	1	1.12	0.98	6.60	0.00	2.76	0.00	4.96	5.50	0.00
time (sec)	N/A	0.641	0.557	2.516	0.000	0.723	0.000	0.147	0.734	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	310	248	2109	0	702	0	1618	1664	0
N.S.	1	1.08	0.86	7.35	0.00	2.45	0.00	5.64	5.80	0.00
time (sec)	N/A	0.860	0.880	2.662	0.000	4.649	0.000	0.163	3.347	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	408	320	3583	0	874	0	2352	1910	0
N.S.	1	1.05	0.83	9.26	0.00	2.26	0.00	6.08	4.94	0.00
time (sec)	N/A	1.170	1.246	3.394	0.000	10.540	0.000	0.183	40.972	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	369	304	955	0	678	0	305	579	0
N.S.	1	1.12	0.92	2.89	0.00	2.05	0.00	0.92	1.75	0.00
time (sec)	N/A	1.091	11.803	2.470	0.000	0.179	0.000	0.148	0.314	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	259	245	512	0	536	0	224	400	0
N.S.	1	1.10	1.04	2.17	0.00	2.27	0.00	0.95	1.69	0.00
time (sec)	N/A	0.773	10.902	2.502	0.000	0.114	0.000	0.145	0.273	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	173	181	291	0	418	0	164	252	0
N.S.	1	1.11	1.16	1.87	0.00	2.68	0.00	1.05	1.62	0.00
time (sec)	N/A	0.530	1.145	2.254	0.000	0.098	0.000	0.137	0.246	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	131	112	131	0	337	0	114	135	0
N.S.	1	1.15	0.98	1.15	0.00	2.96	0.00	1.00	1.18	0.00
time (sec)	N/A	0.428	0.321	2.269	0.000	0.100	0.000	0.140	0.243	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	162	476	308	0	947	0	0	202	0
N.S.	1	1.18	3.47	2.25	0.00	6.91	0.00	0.00	1.47	0.00
time (sec)	N/A	0.556	2.244	2.342	0.000	0.244	0.000	0.000	0.266	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	137	117	708	0	355	0	220	372	0
N.S.	1	1.16	0.99	6.00	0.00	3.01	0.00	1.86	3.15	0.00
time (sec)	N/A	0.524	10.151	2.470	0.000	0.219	0.000	0.145	0.352	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	211	162	1353	0	442	0	505	966	0
N.S.	1	1.14	0.88	7.31	0.00	2.39	0.00	2.73	5.22	0.00
time (sec)	N/A	0.745	10.169	2.598	0.000	0.366	0.000	0.143	0.389	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	307	210	2059	0	558	0	912	1316	0
N.S.	1	1.14	0.78	7.65	0.00	2.07	0.00	3.39	4.89	0.00
time (sec)	N/A	1.003	10.261	3.425	0.000	1.051	0.000	0.161	0.782	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	421	273	3471	0	702	0	1483	1694	0
N.S.	1	1.13	0.73	9.33	0.00	1.89	0.00	3.99	4.55	0.00
time (sec)	N/A	1.349	10.393	3.157	0.000	5.255	0.000	0.151	3.743	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	351	241	728	0	736	0	901	862	0
N.S.	1	1.04	0.71	2.15	0.00	2.18	0.00	2.67	2.55	0.00
time (sec)	N/A	1.428	0.998	2.771	0.000	0.276	0.000	0.400	20.056	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	252	181	505	0	564	0	539	548	0
N.S.	1	0.98	0.71	1.97	0.00	2.20	0.00	2.11	2.14	0.00
time (sec)	N/A	0.938	0.575	2.455	0.000	0.173	0.000	0.277	0.245	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	178	129	345	0	414	0	285	332	0
N.S.	1	1.12	0.81	2.17	0.00	2.60	0.00	1.79	2.09	0.00
time (sec)	N/A	0.582	0.394	2.421	0.000	0.132	0.000	0.196	0.241	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	124	110	212	0	326	0	185	149	0
N.S.	1	1.13	1.00	1.93	0.00	2.96	0.00	1.68	1.35	0.00
time (sec)	N/A	0.420	0.241	2.513	0.000	0.122	0.000	0.146	0.244	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	127	110	523	0	333	0	200	380	0
N.S.	1	1.13	0.98	4.67	0.00	2.97	0.00	1.79	3.39	0.00
time (sec)	N/A	0.486	0.237	2.646	0.000	0.167	0.000	0.164	0.283	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	131	920	0	436	0	395	932	0
N.S.	1	1.00	0.71	4.97	0.00	2.36	0.00	2.14	5.04	0.00
time (sec)	N/A	0.669	0.442	2.755	0.000	0.364	0.000	0.158	0.424	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	266	185	1567	0	594	0	703	1897	0
N.S.	1	1.02	0.71	6.00	0.00	2.28	0.00	2.69	7.27	0.00
time (sec)	N/A	1.177	0.598	2.999	0.000	0.964	0.000	0.179	0.603	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	362	246	2275	0	772	0	1133	2498	0
N.S.	1	0.98	0.66	6.15	0.00	2.09	0.00	3.06	6.75	0.00
time (sec)	N/A	1.788	1.012	3.921	0.000	4.054	0.000	0.208	1.730	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	446	385	1426	0	1044	2502	513	1033	0
N.S.	1	1.03	0.89	3.29	0.00	2.41	5.76	1.18	2.38	0.00
time (sec)	N/A	1.287	2.053	2.579	0.000	0.140	82.710	0.152	0.352	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	347	303	797	0	846	1637	405	790	0
N.S.	1	1.03	0.90	2.36	0.00	2.51	4.86	1.20	2.34	0.00
time (sec)	N/A	0.943	1.172	2.556	0.000	0.112	53.248	0.161	0.323	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	255	236	483	0	676	1093	310	579	0
N.S.	1	1.05	0.97	1.98	0.00	2.77	4.48	1.27	2.37	0.00
time (sec)	N/A	0.701	0.847	2.453	0.000	0.105	6.582	0.152	0.286	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	215	180	230	0	532	751	229	400	0
N.S.	1	1.16	0.97	1.24	0.00	2.86	4.04	1.23	2.15	0.00
time (sec)	N/A	0.561	0.566	2.443	0.000	0.097	4.133	0.146	0.270	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	257	232	607	0	1327	0	0	425	0
N.S.	1	1.17	1.06	2.77	0.00	6.06	0.00	0.00	1.94	0.00
time (sec)	N/A	0.890	0.796	2.519	0.000	2.393	0.000	0.000	0.352	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	240	213	1300	0	1221	0	339	507	0
N.S.	1	1.18	1.05	6.40	0.00	6.01	0.00	1.67	2.50	0.00
time (sec)	N/A	0.845	0.652	2.685	0.000	0.902	0.000	0.178	0.591	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	263	243	2438	0	1375	0	626	1090	0
N.S.	1	1.17	1.08	10.84	0.00	6.11	0.00	2.78	4.84	0.00
time (sec)	N/A	0.883	0.828	2.728	0.000	1.319	0.000	0.206	0.502	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	225	201	4330	0	558	0	1005	1116	0
N.S.	1	1.16	1.04	22.32	0.00	2.88	0.00	5.18	5.75	0.00
time (sec)	N/A	0.715	0.562	3.258	0.000	0.752	0.000	0.155	0.706	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	304	247	7421	0	704	0	1618	1662	0
N.S.	1	1.09	0.89	26.69	0.00	2.53	0.00	5.82	5.98	0.00
time (sec)	N/A	0.926	0.895	3.327	0.000	4.620	0.000	0.167	3.251	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	402	317	10575	0	872	0	2352	1910	0
N.S.	1	1.06	0.84	27.98	0.00	2.31	0.00	6.22	5.05	0.00
time (sec)	N/A	1.255	1.321	4.759	0.000	10.972	0.000	0.208	45.784	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	481	505	402	16883	0	1072	0	3251	40	0
N.S.	1	1.05	0.84	35.10	0.00	2.23	0.00	6.76	0.08	0.00
time (sec)	N/A	1.554	1.845	4.730	0.000	30.082	0.000	0.290	200.041	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	474	350	1192	0	928	0	399	40	0
N.S.	1	1.15	0.85	2.90	0.00	2.26	0.00	0.97	0.10	0.00
time (sec)	N/A	2.412	10.880	2.994	0.000	0.621	0.000	0.174	200.133	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	368	281	937	0	730	0	319	862	0
N.S.	1	1.14	0.87	2.91	0.00	2.27	0.00	0.99	2.68	0.00
time (sec)	N/A	1.540	10.789	2.954	0.000	0.308	0.000	0.172	5.976	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	261	208	705	0	550	0	271	580	0
N.S.	1	1.12	0.89	3.01	0.00	2.35	0.00	1.16	2.48	0.00
time (sec)	N/A	0.977	0.841	2.848	0.000	0.176	0.000	0.174	0.291	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	186	148	392	0	414	0	224	347	0
N.S.	1	1.18	0.94	2.48	0.00	2.62	0.00	1.42	2.20	0.00
time (sec)	N/A	0.591	0.353	3.129	0.000	0.132	0.000	0.159	0.278	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	228	538	1316	0	1277	0	0	558	0
N.S.	1	1.18	2.79	6.82	0.00	6.62	0.00	0.00	2.89	0.00
time (sec)	N/A	0.818	2.477	3.333	0.000	0.750	0.000	0.000	0.322	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	190	144	2006	0	446	0	0	1374	0
N.S.	1	1.08	0.82	11.40	0.00	2.53	0.00	0.00	7.81	0.00
time (sec)	N/A	0.690	10.186	3.869	0.000	0.436	0.000	0.000	0.492	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	275	179	3143	0	588	0	0	1897	0
N.S.	1	0.99	0.64	11.27	0.00	2.11	0.00	0.00	6.80	0.00
time (sec)	N/A	1.249	10.177	3.944	0.000	1.242	0.000	0.000	0.675	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	379	256	5038	0	768	0	0	2498	0
N.S.	1	1.05	0.71	13.92	0.00	2.12	0.00	0.00	6.90	0.00
time (sec)	N/A	1.939	10.183	4.745	0.000	4.925	0.000	0.000	1.919	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	491	326	8132	0	968	0	0	3131	0
N.S.	1	1.02	0.67	16.84	0.00	2.00	0.00	0.00	6.48	0.00
time (sec)	N/A	2.814	10.450	5.073	0.000	12.426	0.000	0.000	12.810	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	559	543	549	1895	0	1524	0	784	1615	0
N.S.	1	0.97	0.98	3.39	0.00	2.73	0.00	1.40	2.89	0.00
time (sec)	N/A	1.455	2.250	2.571	0.000	0.178	0.000	0.166	0.739	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	434	479	1080	0	1272	0	648	1308	0
N.S.	1	0.99	1.10	2.47	0.00	2.91	0.00	1.48	2.99	0.00
time (sec)	N/A	1.097	1.915	2.588	0.000	0.155	0.000	0.160	0.477	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	342	388	673	0	1046	0	526	1033	0
N.S.	1	1.04	1.18	2.04	0.00	3.17	0.00	1.59	3.13	0.00
time (sec)	N/A	0.817	1.448	2.514	0.000	0.129	0.000	0.173	0.378	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	302	295	327	0	844	0	415	790	0
N.S.	1	1.17	1.14	1.26	0.00	3.26	0.00	1.60	3.05	0.00
time (sec)	N/A	0.709	1.053	2.414	0.000	0.136	0.000	0.158	0.359	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	414	342	997	0	1873	0	0	770	0
N.S.	1	1.14	0.94	2.75	0.00	5.17	0.00	0.00	2.13	0.00
time (sec)	N/A	1.349	1.558	2.499	0.000	25.834	0.000	0.000	0.492	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	361	309	2076	0	1717	0	473	40	0
N.S.	1	1.14	0.97	6.53	0.00	5.40	0.00	1.49	0.13	0.00
time (sec)	N/A	1.190	1.649	2.829	0.000	8.349	0.000	0.187	200.185	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	342	287	3893	0	1569	0	746	40	0
N.S.	1	1.11	0.93	12.68	0.00	5.11	0.00	2.43	0.13	0.00
time (sec)	N/A	1.129	1.388	2.895	0.000	3.968	0.000	0.224	200.040	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	375	316	6850	0	1741	0	1195	1564	0
N.S.	1	1.12	0.94	20.39	0.00	5.18	0.00	3.56	4.65	0.00
time (sec)	N/A	1.252	1.592	4.543	0.000	4.900	0.000	0.270	7.947	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	426	358	11685	0	1917	0	1750	1822	0
N.S.	1	1.14	0.96	31.33	0.00	5.14	0.00	4.69	4.88	0.00
time (sec)	N/A	1.341	1.687	2.260	0.000	13.337	0.000	0.491	15.175	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	317	271	19539	0	872	0	2449	1904	0
N.S.	1	1.17	1.00	71.83	0.00	3.21	0.00	9.00	7.00	0.00
time (sec)	N/A	0.919	1.115	4.335	0.000	10.518	0.000	0.203	16.310	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	396	404	32291	0	1072	0	3388	40	0
N.S.	1	1.07	1.09	87.51	0.00	2.91	0.00	9.18	0.11	0.00
time (sec)	N/A	1.126	1.471	4.565	0.000	33.004	0.000	0.314	200.141	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	494	497	45106	0	1300	0	4452	40	0
N.S.	1	1.02	1.03	93.39	0.00	2.69	0.00	9.22	0.08	0.00
time (sec)	N/A	1.459	1.978	6.724	0.000	67.307	0.000	0.579	200.057	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	611	608	572	70736	0	1550	0	5681	40	0
N.S.	1	1.00	0.94	115.77	0.00	2.54	0.00	9.30	0.07	0.00
time (sec)	N/A	1.825	2.492	6.675	0.000	156.881	0.000	1.187	200.043	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	506	610	376	1802	0	1160	0	518	40	0
N.S.	1	1.21	0.74	3.56	0.00	2.29	0.00	1.02	0.08	0.00
time (sec)	N/A	3.622	1.775	2.925	0.000	1.829	0.000	0.190	200.048	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	483	319	1471	0	936	0	424	40	0
N.S.	1	1.20	0.79	3.65	0.00	2.32	0.00	1.05	0.10	0.00
time (sec)	N/A	2.439	1.352	3.052	0.000	0.872	0.000	0.186	200.051	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	358	230	1061	0	724	0	367	862	0
N.S.	1	1.21	0.78	3.60	0.00	2.45	0.00	1.24	2.92	0.00
time (sec)	N/A	1.542	1.129	3.122	0.000	0.401	0.000	0.173	1.116	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	257	186	570	0	554	0	302	597	0
N.S.	1	1.40	1.02	3.11	0.00	3.03	0.00	1.65	3.26	0.00
time (sec)	N/A	0.994	0.712	3.115	0.000	0.218	0.000	0.172	0.316	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	295	235	2473	0	1509	0	0	791	0
N.S.	1	1.17	0.93	9.77	0.00	5.96	0.00	0.00	3.13	0.00
time (sec)	N/A	1.287	0.641	3.622	0.000	2.760	0.000	0.000	0.407	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	299	241	3549	0	1562	0	0	1711	0
N.S.	1	1.11	0.90	13.19	0.00	5.81	0.00	0.00	6.36	0.00
time (sec)	N/A	1.270	0.601	4.392	0.000	2.408	0.000	0.000	0.539	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	269	194	5365	0	592	0	0	1969	0
N.S.	1	1.10	0.80	21.99	0.00	2.43	0.00	0.00	8.07	0.00
time (sec)	N/A	1.268	0.655	5.252	0.000	1.567	0.000	0.000	0.684	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	368	238	8322	0	758	0	0	2498	0
N.S.	1	1.03	0.66	23.18	0.00	2.11	0.00	0.00	6.96	0.00
time (sec)	N/A	1.979	0.985	6.664	0.000	5.900	0.000	0.000	1.782	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	492	315	13161	0	968	0	0	3131	0
N.S.	1	1.04	0.67	27.94	0.00	2.06	0.00	0.00	6.65	0.00
time (sec)	N/A	3.001	1.376	7.424	0.000	14.737	0.000	0.000	14.609	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	607	615	387	21017	0	1198	0	0	3796	0
N.S.	1	1.01	0.64	34.62	0.00	1.97	0.00	0.00	6.25	0.00
time (sec)	N/A	4.153	1.807	9.046	0.000	35.080	0.000	0.000	199.307	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	755	760	483	33770	0	1452	0	0	40	0
N.S.	1	1.01	0.64	44.73	0.00	1.92	0.00	0.00	0.05	0.00
time (sec)	N/A	5.628	2.843	10.821	0.000	82.460	0.000	0.000	200.050	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	374	377	1200	0	676	906	306	579	0
N.S.	1	1.10	1.11	3.54	0.00	1.99	2.67	0.90	1.71	0.00
time (sec)	N/A	1.046	11.180	2.634	0.000	0.242	1.170	0.147	0.280	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	264	307	692	0	530	631	224	400	0
N.S.	1	1.11	1.29	2.91	0.00	2.23	2.65	0.94	1.68	0.00
time (sec)	N/A	0.714	10.579	2.470	0.000	0.146	1.105	0.148	0.256	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	170	203	374	0	416	445	164	252	0
N.S.	1	1.10	1.31	2.41	0.00	2.68	2.87	1.06	1.63	0.00
time (sec)	N/A	0.510	1.066	2.223	0.000	0.120	1.311	0.151	0.237	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	135	136	186	0	338	335	120	135	144
N.S.	1	1.16	1.17	1.60	0.00	2.91	2.89	1.03	1.16	1.24
time (sec)	N/A	0.459	0.303	2.323	0.000	0.102	0.837	0.142	0.211	6.116

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	162	485	132	0	975	0	0	212	118
N.S.	1	1.18	3.54	0.96	0.00	7.12	0.00	0.00	1.55	0.86
time (sec)	N/A	0.489	2.267	2.157	0.000	0.262	0.000	0.000	0.232	6.117

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	143	728	201	0	354	0	227	372	180
N.S.	1	1.19	6.07	1.68	0.00	2.95	0.00	1.89	3.10	1.50
time (sec)	N/A	0.483	14.138	2.278	0.000	0.179	0.000	0.137	0.272	6.126

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	219	193	399	0	448	0	508	966	0
N.S.	1	1.12	0.99	2.05	0.00	2.30	0.00	2.61	4.95	0.00
time (sec)	N/A	0.675	10.137	2.334	0.000	0.556	0.000	0.137	0.357	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	314	256	733	0	562	0	914	1316	0
N.S.	1	1.13	0.92	2.63	0.00	2.01	0.00	3.28	4.72	0.00
time (sec)	N/A	0.948	10.187	2.805	0.000	1.186	0.000	0.146	0.701	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	274	277	514	0	758	0	228	816	0
N.S.	1	1.07	1.08	2.01	0.00	2.96	0.00	0.89	3.19	0.00
time (sec)	N/A	0.938	0.714	2.778	0.000	0.383	0.000	0.165	0.228	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	202	201	259	0	586	0	182	573	0
N.S.	1	1.13	1.13	1.46	0.00	3.29	0.00	1.02	3.22	0.00
time (sec)	N/A	0.596	0.444	2.763	0.000	0.171	0.000	0.157	0.219	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	139	130	131	0	443	0	0	300	0
N.S.	1	1.11	1.04	1.05	0.00	3.54	0.00	0.00	2.40	0.00
time (sec)	N/A	0.450	0.283	2.424	0.000	0.159	0.000	0.000	0.198	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	42	50	0	59	0	0	71	50
N.S.	1	1.00	0.81	0.96	0.00	1.13	0.00	0.00	1.37	0.96
time (sec)	N/A	0.303	0.040	2.461	0.000	0.128	0.000	0.000	0.204	5.871

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	142	131	136	0	454	0	0	763	0
N.S.	1	1.14	1.05	1.09	0.00	3.63	0.00	0.00	6.10	0.00
time (sec)	N/A	0.502	0.265	2.397	0.000	0.299	0.000	0.000	0.309	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	206	201	270	0	610	0	0	1302	0
N.S.	1	1.03	1.00	1.35	0.00	3.05	0.00	0.00	6.51	0.00
time (sec)	N/A	0.655	0.394	2.944	0.000	0.748	0.000	0.000	0.476	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	281	283	545	0	792	0	0	2075	0
N.S.	1	0.95	0.96	1.84	0.00	2.68	0.00	0.00	7.01	0.00
time (sec)	N/A	1.080	0.667	2.906	0.000	1.941	0.000	0.000	0.716	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	26	21	28	22	0	30	36	19
N.S.	1	1.00	0.72	0.58	0.78	0.61	0.00	0.83	1.00	0.53
time (sec)	N/A	0.290	0.130	1.461	0.031	0.083	0.000	0.115	0.220	5.806

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	278	244	1275	0	778	0	0	525	0
N.S.	1	1.08	0.95	4.96	0.00	3.03	0.00	0.00	2.04	0.00
time (sec)	N/A	0.969	0.707	2.557	0.000	0.564	0.000	0.000	0.253	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	203	185	738	0	608	0	0	370	0
N.S.	1	1.12	1.02	4.05	0.00	3.34	0.00	0.00	2.03	0.00
time (sec)	N/A	0.600	0.411	2.441	0.000	0.221	0.000	0.000	0.217	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	141	144	404	0	470	0	0	195	234
N.S.	1	1.11	1.13	3.18	0.00	3.70	0.00	0.00	1.54	1.84
time (sec)	N/A	0.452	0.260	2.088	0.000	0.180	0.000	0.000	0.211	6.086

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	39	53	0	65	0	0	59	53
N.S.	1	1.00	0.78	1.06	0.00	1.30	0.00	0.00	1.18	1.06
time (sec)	N/A	0.285	0.046	2.003	0.000	0.150	0.000	0.000	0.204	5.854

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	144	133	287	0	479	0	0	455	0
N.S.	1	1.13	1.05	2.26	0.00	3.77	0.00	0.00	3.58	0.00
time (sec)	N/A	0.494	0.358	1.951	0.000	0.380	0.000	0.000	0.271	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	207	190	564	0	644	0	0	772	0
N.S.	1	0.99	0.91	2.70	0.00	3.08	0.00	0.00	3.69	0.00
time (sec)	N/A	0.658	0.479	2.070	0.000	1.045	0.000	0.000	0.392	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	302	255	984	0	828	0	0	1256	0
N.S.	1	0.94	0.79	3.06	0.00	2.57	0.00	0.00	3.90	0.00
time (sec)	N/A	1.233	0.684	2.096	0.000	2.714	0.000	0.000	0.521	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	367	243	2622	0	756	0	0	558	0
N.S.	1	1.14	0.75	8.12	0.00	2.34	0.00	0.00	1.73	0.00
time (sec)	N/A	1.654	0.865	2.799	0.000	0.501	0.000	0.000	21.451	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	265	180	1548	0	580	0	0	352	0
N.S.	1	1.11	0.75	6.48	0.00	2.43	0.00	0.00	1.47	0.00
time (sec)	N/A	1.010	0.486	2.691	0.000	0.267	0.000	0.000	0.252	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	179	137	879	0	438	0	0	219	0
N.S.	1	1.12	0.86	5.49	0.00	2.74	0.00	0.00	1.37	0.00
time (sec)	N/A	0.581	0.310	2.280	0.000	0.147	0.000	0.000	0.222	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	125	113	485	0	349	0	0	104	0
N.S.	1	1.13	1.02	4.37	0.00	3.14	0.00	0.00	0.94	0.00
time (sec)	N/A	0.421	0.209	3.984	0.000	0.130	0.000	0.000	0.246	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	128	113	427	0	358	0	0	242	0
N.S.	1	1.15	1.02	3.85	0.00	3.23	0.00	0.00	2.18	0.00
time (sec)	N/A	0.527	0.207	1.622	0.000	0.215	0.000	0.000	0.288	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	187	143	645	0	468	0	0	562	0
N.S.	1	1.13	0.87	3.91	0.00	2.84	0.00	0.00	3.41	0.00
time (sec)	N/A	0.680	0.331	1.531	0.000	0.592	0.000	0.000	0.436	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	289	193	1200	0	630	0	0	1140	0
N.S.	1	1.17	0.78	4.88	0.00	2.56	0.00	0.00	4.63	0.00
time (sec)	N/A	1.382	0.524	2.248	0.000	1.680	0.000	0.000	0.550	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	391	257	2040	0	798	0	0	1509	0
N.S.	1	1.18	0.78	6.16	0.00	2.41	0.00	0.00	4.56	0.00
time (sec)	N/A	2.292	0.864	2.698	0.000	6.227	0.000	0.000	1.235	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	494	376	4841	0	952	0	0	40	0
N.S.	1	1.14	0.87	11.18	0.00	2.20	0.00	0.00	0.09	0.00
time (sec)	N/A	2.623	10.867	3.093	0.000	1.057	0.000	0.000	200.062	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	382	308	2895	0	754	0	0	558	0
N.S.	1	1.14	0.92	8.67	0.00	2.26	0.00	0.00	1.67	0.00
time (sec)	N/A	1.711	10.685	3.270	0.000	0.495	0.000	0.000	6.067	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	267	206	1689	0	582	0	0	377	0
N.S.	1	1.09	0.84	6.92	0.00	2.39	0.00	0.00	1.55	0.00
time (sec)	N/A	0.966	1.352	2.487	0.000	0.277	0.000	0.000	0.321	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	190	143	960	0	447	0	0	230	0
N.S.	1	1.17	0.88	5.89	0.00	2.74	0.00	0.00	1.41	0.00
time (sec)	N/A	0.591	0.379	2.428	0.000	0.164	0.000	0.000	0.237	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	238	560	700	0	1360	0	0	365	0
N.S.	1	1.21	2.84	3.55	0.00	6.90	0.00	0.00	1.85	0.00
time (sec)	N/A	0.801	5.353	2.263	0.000	0.821	0.000	0.000	0.300	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	197	146	785	0	477	0	0	841	0
N.S.	1	1.12	0.83	4.46	0.00	2.71	0.00	0.00	4.78	0.00
time (sec)	N/A	0.705	10.134	2.526	0.000	0.717	0.000	0.000	0.441	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	294	194	1281	0	622	0	0	1140	0
N.S.	1	1.18	0.78	5.12	0.00	2.49	0.00	0.00	4.56	0.00
time (sec)	N/A	1.333	10.147	2.342	0.000	1.945	0.000	0.000	0.592	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	411	255	2256	0	792	0	0	1509	0
N.S.	1	1.21	0.75	6.64	0.00	2.33	0.00	0.00	4.44	0.00
time (sec)	N/A	2.376	10.209	2.852	0.000	6.834	0.000	0.000	1.296	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	525	323	3794	0	992	0	0	1901	0
N.S.	1	1.20	0.74	8.64	0.00	2.26	0.00	0.00	4.33	0.00
time (sec)	N/A	3.744	10.284	2.801	0.000	18.055	0.000	0.000	8.445	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	511	388	1923	0	2120	0	0	40	0
N.S.	1	1.17	0.89	4.42	0.00	4.87	0.00	0.00	0.09	0.00
time (sec)	N/A	1.426	1.286	2.879	0.000	4.130	0.000	0.000	200.036	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	431	300	1112	0	1782	0	0	1978	0
N.S.	1	1.21	0.84	3.12	0.00	5.01	0.00	0.00	5.56	0.00
time (sec)	N/A	1.194	0.853	2.875	0.000	1.586	0.000	0.000	1.796	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	309	218	639	0	1466	0	0	1329	0
N.S.	1	1.02	0.72	2.10	0.00	4.82	0.00	0.00	4.37	0.00
time (sec)	N/A	0.827	0.513	2.437	0.000	1.788	0.000	0.000	0.253	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	126	95	145	0	308	0	0	438	1071
N.S.	1	0.68	0.51	0.78	0.00	1.66	0.00	0.00	2.35	5.76
time (sec)	N/A	0.509	0.217	2.442	0.000	1.466	0.000	0.000	0.205	6.471

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	100	149	0	314	0	0	420	499
N.S.	1	1.00	0.72	1.08	0.00	2.28	0.00	0.00	3.04	3.62
time (sec)	N/A	0.478	0.197	2.221	0.000	1.427	0.000	0.000	0.209	6.605

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	95	138	0	306	0	0	323	120
N.S.	1	1.00	0.79	1.14	0.00	2.53	0.00	0.00	2.67	0.99
time (sec)	N/A	0.408	0.164	2.303	0.000	1.179	0.000	0.000	0.214	6.093

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	303	217	360	0	1476	0	0	3071	0
N.S.	1	1.06	0.76	1.26	0.00	5.16	0.00	0.00	10.74	0.00
time (sec)	N/A	0.827	0.538	2.345	0.000	3.531	0.000	0.000	0.665	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	437	303	716	0	1812	0	0	40	0
N.S.	1	1.10	0.76	1.80	0.00	4.56	0.00	0.00	0.10	0.00
time (sec)	N/A	1.278	0.821	2.604	0.000	10.844	0.000	0.000	200.030	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	565	390	1354	0	2162	0	0	5923	0
N.S.	1	1.10	0.76	2.64	0.00	4.22	0.00	0.00	11.57	0.00
time (sec)	N/A	1.682	1.240	2.685	0.000	23.672	0.000	0.000	3.421	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	657	708	493	2409	0	2526	0	0	7071	0
N.S.	1	1.08	0.75	3.67	0.00	3.84	0.00	0.00	10.76	0.00
time (sec)	N/A	2.173	1.598	3.505	0.000	51.267	0.000	0.000	20.926	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	653	396	1344	0	2630	0	4520	40	0
N.S.	1	1.41	0.86	2.91	0.00	5.69	0.00	9.78	0.09	0.00
time (sec)	N/A	2.228	1.204	3.197	0.000	4.001	0.000	1.850	200.034	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	404	300	868	0	2224	0	4572	2483	0
N.S.	1	0.99	0.73	2.12	0.00	5.42	0.00	11.15	6.06	0.00
time (sec)	N/A	0.928	0.724	2.835	0.000	4.798	0.000	0.274	37.319	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	362	123	227	0	495	0	4068	935	3963
N.S.	1	1.51	0.51	0.95	0.00	2.06	0.00	16.95	3.90	16.51
time (sec)	N/A	1.544	0.252	2.612	0.000	3.847	0.000	0.246	30.432	7.993

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	287	157	249	0	518	0	4176	828	3062
N.S.	1	1.28	0.70	1.11	0.00	2.30	0.00	18.56	3.68	13.61
time (sec)	N/A	0.905	0.247	2.379	0.000	3.880	0.000	0.240	0.442	7.417

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	213	154	244	0	517	0	4167	713	1844
N.S.	1	1.02	0.74	1.17	0.00	2.47	0.00	19.94	3.41	8.82
time (sec)	N/A	0.634	0.228	2.374	0.000	3.604	0.000	0.227	0.268	7.091

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	196	136	216	0	494	0	3930	561	1005
N.S.	1	1.08	0.75	1.19	0.00	2.73	0.00	21.71	3.10	5.55
time (sec)	N/A	0.523	0.217	2.637	0.000	3.409	0.000	0.220	0.285	6.611

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	481	300	590	0	2234	0	998	40	0
N.S.	1	1.24	0.77	1.52	0.00	5.74	0.00	2.57	0.10	0.00
time (sec)	N/A	1.240	0.727	2.696	0.000	9.494	0.000	0.186	200.035	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	760	399	943	0	2662	0	4890	7938	0
N.S.	1	1.48	0.77	1.83	0.00	5.17	0.00	9.50	15.41	0.00
time (sec)	N/A	1.855	1.196	3.019	0.000	23.564	0.000	0.293	20.655	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	527	421	1180	0	3186	0	0	40	0
N.S.	1	0.99	0.79	2.23	0.00	6.01	0.00	0.00	0.08	0.00
time (sec)	N/A	1.130	1.088	3.244	0.000	12.951	0.000	0.000	200.030	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	529	160	323	0	735	0	0	40	13455
N.S.	1	1.73	0.52	1.06	0.00	2.40	0.00	0.00	0.13	43.97
time (sec)	N/A	2.441	0.306	3.270	0.000	11.183	0.000	0.000	200.068	12.163

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	444	227	361	0	770	0	0	1359	11309
N.S.	1	1.25	0.64	1.02	0.00	2.17	0.00	0.00	3.83	31.86
time (sec)	N/A	1.696	0.284	2.651	0.000	11.929	0.000	0.000	141.203	11.489

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	362	235	373	0	785	0	0	1240	5771
N.S.	1	1.17	0.76	1.20	0.00	2.53	0.00	0.00	4.00	18.62
time (sec)	N/A	1.077	0.301	2.750	0.000	10.795	0.000	0.000	7.698	8.435

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	288	221	351	0	770	0	0	1067	3635
N.S.	1	1.01	0.78	1.24	0.00	2.71	0.00	0.00	3.76	12.80
time (sec)	N/A	0.771	0.281	2.793	0.000	10.906	0.000	0.000	0.415	7.873

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	271	193	307	0	733	0	0	852	2121
N.S.	1	1.12	0.80	1.27	0.00	3.04	0.00	0.00	3.54	8.80
time (sec)	N/A	0.683	0.350	3.039	0.000	9.001	0.000	0.000	0.319	7.482

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	727	421	901	0	3196	0	0	8301	0
N.S.	1	1.44	0.84	1.79	0.00	6.34	0.00	0.00	16.47	0.00
time (sec)	N/A	1.719	1.044	3.081	0.000	22.885	0.000	0.000	16.548	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	647	1011	505	1254	0	3694	0	0	40	0
N.S.	1	1.56	0.78	1.94	0.00	5.71	0.00	0.00	0.06	0.00
time (sec)	N/A	2.380	1.624	3.720	0.000	52.430	0.000	0.000	200.029	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	436	333	2901	0	1826	0	0	2091	0
N.S.	1	1.17	0.89	7.78	0.00	4.90	0.00	0.00	5.61	0.00
time (sec)	N/A	1.257	0.962	3.054	0.000	2.256	0.000	0.000	1.143	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	314	231	1777	0	1514	0	0	1395	0
N.S.	1	1.11	0.81	6.26	0.00	5.33	0.00	0.00	4.91	0.00
time (sec)	N/A	0.835	0.504	2.337	0.000	2.283	0.000	0.000	0.332	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	95	152	0	318	0	0	498	1114
N.S.	1	1.00	0.77	1.24	0.00	2.59	0.00	0.00	4.05	9.06
time (sec)	N/A	0.470	0.213	2.428	0.000	1.865	0.000	0.000	0.255	6.709

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	94	156	0	324	0	0	464	509
N.S.	1	1.00	0.66	1.09	0.00	2.27	0.00	0.00	3.24	3.56
time (sec)	N/A	0.481	0.195	2.131	0.000	1.891	0.000	0.000	0.247	6.612

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	90	146	0	316	0	0	346	120
N.S.	1	1.00	0.76	1.24	0.00	2.68	0.00	0.00	2.93	1.02
time (sec)	N/A	0.399	0.169	2.202	0.000	1.568	0.000	0.000	0.273	6.324

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	307	230	589	0	1524	0	0	3117	0
N.S.	1	1.20	0.90	2.31	0.00	5.98	0.00	0.00	12.22	0.00
time (sec)	N/A	0.812	0.496	2.196	0.000	4.726	0.000	0.000	0.926	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	442	335	1090	0	1856	0	0	38	0
N.S.	1	1.20	0.91	2.95	0.00	5.03	0.00	0.00	0.10	0.00
time (sec)	N/A	1.228	0.985	2.244	0.000	14.627	0.000	0.000	200.062	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	265	235	366	0	820	0	0	1349	3099
N.S.	1	0.67	0.60	0.93	0.00	2.09	0.00	0.00	3.43	7.89
time (sec)	N/A	0.737	0.331	2.682	0.000	15.568	0.000	0.000	8.039	7.058

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	172	0	0	0	0	0	0	0
N.S.	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	0.534	0.000	0.000	0.000	0.000	0.000	2.753	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	120	0	0	0	0	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.397	0.387	0.000	0.000	0.000	0.000	0.000	1.446	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	78	0	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.371	0.324	0.000	0.000	0.000	0.000	0.000	0.415	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.412	0.393	0.000	0.000	0.000	0.000	0.000	0.664	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	81	0	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.415	0.402	0.000	0.000	0.000	0.000	0.000	0.646	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	0	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.437	0.360	0.000	0.000	0.000	0.000	0.000	0.859	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [71] had the largest ratio of [.3750000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	8	1.02	38	0.211
2	A	7	6	1.04	38	0.158
3	A	5	4	1.06	36	0.111
4	A	5	4	1.12	35	0.114
5	A	8	7	1.18	38	0.184
6	A	8	7	1.18	38	0.184
7	A	8	7	1.19	38	0.184
8	A	5	4	1.12	38	0.105
9	A	7	6	1.08	38	0.158
10	A	9	8	1.05	38	0.211
11	A	9	8	1.12	40	0.200
12	A	7	6	1.10	40	0.150
13	A	5	4	1.11	38	0.105
14	A	4	3	1.15	37	0.081
15	A	8	7	1.18	40	0.175
16	A	5	4	1.16	40	0.100
17	A	7	6	1.14	40	0.150
18	A	9	8	1.14	40	0.200
19	A	11	10	1.13	40	0.250
20	A	9	8	1.04	40	0.200
21	A	8	7	0.98	40	0.175

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	6	5	1.12	38	0.132
23	A	6	5	1.13	37	0.135
24	A	6	5	1.13	40	0.125
25	A	7	6	1.00	40	0.150
26	A	8	7	1.02	40	0.175
27	A	10	9	0.98	40	0.225
28	A	10	9	1.03	40	0.225
29	A	8	7	1.03	40	0.175
30	A	6	5	1.05	38	0.132
31	A	5	4	1.16	37	0.108
32	A	9	8	1.17	40	0.200
33	A	9	8	1.18	40	0.200
34	A	9	8	1.17	40	0.200
35	A	6	5	1.16	40	0.125
36	A	8	7	1.09	40	0.175
37	A	10	9	1.06	40	0.225
38	A	12	11	1.05	40	0.275
39	A	12	11	1.15	40	0.275
40	A	9	8	1.14	40	0.200
41	A	8	7	1.12	38	0.184
42	A	6	5	1.18	37	0.135
43	A	9	8	1.18	40	0.200
44	A	7	6	1.08	40	0.150
45	A	8	7	0.99	40	0.175
46	A	10	9	1.05	40	0.225
47	A	12	11	1.02	40	0.275
48	A	11	10	0.97	40	0.250
49	A	9	8	0.99	40	0.200
50	A	7	6	1.04	38	0.158
51	A	6	5	1.17	37	0.135
52	A	11	10	1.14	40	0.250
53	A	11	10	1.14	40	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	11	10	1.11	40	0.250
55	A	11	10	1.12	40	0.250
56	A	11	10	1.14	40	0.250
57	A	7	6	1.17	40	0.150
58	A	9	8	1.07	40	0.200
59	A	11	10	1.02	40	0.250
60	A	13	12	1.00	40	0.300
61	A	13	12	1.21	40	0.300
62	A	12	11	1.20	40	0.275
63	A	9	8	1.21	38	0.211
64	A	8	7	1.40	37	0.189
65	A	10	9	1.17	40	0.225
66	A	10	9	1.11	40	0.225
67	A	8	7	1.10	40	0.175
68	A	10	9	1.03	40	0.225
69	A	12	11	1.04	40	0.275
70	A	14	13	1.01	40	0.325
71	A	16	15	1.01	40	0.375
72	A	8	7	1.10	38	0.184
73	A	6	5	1.11	38	0.132
74	A	4	3	1.10	36	0.083
75	A	4	3	1.16	35	0.086
76	A	7	6	1.18	38	0.158
77	A	4	3	1.19	38	0.079
78	A	6	5	1.12	38	0.132
79	A	8	7	1.13	38	0.184
80	A	8	7	1.07	40	0.175
81	A	5	4	1.13	40	0.100
82	A	5	4	1.11	38	0.105
83	A	1	1	1.00	37	0.027
84	A	6	5	1.14	40	0.125
85	A	7	6	1.03	40	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	8	7	0.95	40	0.175
87	A	3	3	1.00	21	0.143
88	A	7	6	1.08	38	0.158
89	A	7	6	1.12	38	0.158
90	A	4	3	1.11	36	0.083
91	A	2	2	1.00	35	0.057
92	A	6	5	1.13	38	0.132
93	A	7	6	0.99	38	0.158
94	A	8	7	0.94	38	0.184
95	A	9	8	1.14	40	0.200
96	A	8	7	1.11	40	0.175
97	A	6	5	1.12	38	0.132
98	A	4	3	1.13	37	0.081
99	A	6	5	1.15	40	0.125
100	A	7	6	1.13	40	0.150
101	A	8	7	1.17	40	0.175
102	A	10	9	1.18	40	0.225
103	A	11	10	1.14	40	0.250
104	A	10	9	1.14	40	0.225
105	A	7	6	1.09	38	0.158
106	A	6	5	1.17	37	0.135
107	A	9	8	1.21	40	0.200
108	A	7	6	1.12	40	0.150
109	A	8	7	1.18	40	0.175
110	A	10	9	1.21	40	0.225
111	A	12	11	1.20	40	0.275
112	A	8	7	1.17	40	0.175
113	A	8	7	1.21	40	0.175
114	A	6	5	1.02	40	0.125
115	A	2	2	0.68	40	0.050
116	A	2	2	1.00	38	0.053
117	A	2	2	1.00	37	0.054

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	7	6	1.06	40	0.150
119	A	8	7	1.10	40	0.175
120	A	10	9	1.10	40	0.225
121	A	12	11	1.08	40	0.275
122	A	4	4	1.41	40	0.100
123	A	9	8	0.99	40	0.200
124	A	7	7	1.51	40	0.175
125	A	5	5	1.28	40	0.125
126	A	3	3	1.02	38	0.079
127	A	3	3	1.08	37	0.081
128	A	2	2	1.24	40	0.050
129	A	2	2	1.48	40	0.050
130	A	11	10	0.99	40	0.250
131	A	10	10	1.73	40	0.250
132	A	8	8	1.25	40	0.200
133	A	6	6	1.17	40	0.150
134	A	4	4	1.01	38	0.105
135	A	4	4	1.12	37	0.108
136	A	2	2	1.44	40	0.050
137	A	2	2	1.56	40	0.050
138	A	8	7	1.17	38	0.184
139	A	7	6	1.11	38	0.158
140	A	2	2	1.00	38	0.053
141	A	2	2	1.00	36	0.056
142	A	2	2	1.00	35	0.057
143	A	7	6	1.20	38	0.158
144	A	8	7	1.20	38	0.184
145	A	4	4	0.67	40	0.100
146	A	4	4	1.00	34	0.118
147	A	4	4	1.00	32	0.125
148	A	3	2	1.00	27	0.074
149	A	4	4	1.00	34	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	4	4	1.00	34	0.118
151	A	4	4	1.00	34	0.118

CHAPTER 3

Listing of integrals

3.1 $\int x^3(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 443

$$\int x^3(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}\,dx =$$

$$-\frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^5d^5e^4}$$

$$+\frac{\left(cd^2-3ae^2\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{20c^2d^2e}$$

$$+\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6cd}$$

$$+\frac{\left(35c^3d^6+33ac^2d^4e^2+21a^2cd^2e^4-105a^3e^6-6cde\left(7c^2d^4+6acd^2e^2-21a^2e^4\right)x\right)\left(ade+\left(cd^2+ae^2\right)x\right.}{960c^4d^4e^3}$$

$$+\frac{\left(cd^2-ae^2\right)^3\left(7c^3d^6+15ac^2d^4e^2+21a^2cd^2e^4+21a^3e^6\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{512c^{11/2}d^{11/2}e^{9/2}}$$

output
```
-1/512*(-21*a^4*e^8+6*a^2*c^2*d^4*e^4+8*a*c^3*d^6*e^2+7*c^4*d^8)*(2*c*d*e*
x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/e^4+1/20*(-
3*a*e^2+c*d^2)*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/e+1/6*x
^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d+1/960*(35*c^3*d^6+33*a*c^2*
d^4*e^2+21*a^2*c*d^2*e^4-105*a^3*e^6-6*c*d*e*(-21*a^2*e^4+6*a*c*d^2*e^2+7*
c^2*d^4)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^4/d^4/e^3+1/512*(-a*
e^2+c*d^2)^3*(21*a^3*e^6+21*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+7*c^3*d^6)*arct
anh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/c^(11/2)/d^(11/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.87

$$\int x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$= \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(315a^5e^{10}-105a^4cde^8(5d+2ex)+6a^3c^2d^2e^6(13d^2+56dex+28e^2x^2)\right.}{}$$

input
```
Integrate[x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(315*a^5*e^10 - 10
5*a^4*c*d*e^8*(5*d + 2*e*x) + 6*a^3*c^2*d^2*e^6*(13*d^2 + 56*d*e*x + 28*e^
2*x^2) + 6*a^2*c^3*d^3*e^4*(9*d^3 - 6*d^2*e*x - 44*d*e^2*x^2 - 24*e^3*x^3)
 + a*c^4*d^4*e^2*(55*d^4 - 32*d^3*e*x + 24*d^2*e^2*x^2 + 224*d*e^3*x^3 + 1
28*e^4*x^4) + c^5*d^5*(-105*d^5 + 70*d^4*e*x - 56*d^3*e^2*x^2 + 48*d^2*e^3
*x^3 + 1664*d*e^4*x^4 + 1280*e^5*x^5)) + (15*(c*d^2 - a*e^2)^3*(7*c^3*d^6
+ 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*ArcTanh[(Sqrt[e]*Sqrt[
a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]])/(Sqrt[a*e + c*d*x]*Sqrt[d
 + e*x])))/(7680*c^(11/2)*d^(11/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1236, 27, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,dx$$

$$\Big\downarrow \ 1236$$

$$\frac{\int -\frac{3}{2}ex^2\left(2ade-\left(cd^2-3ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{6cde}+\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6cd}$$

$$\Big\downarrow \ 27$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6cd}-\frac{\int x^2\left(2ade-\left(cd^2-3ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{4cd}$$

$$\Big\downarrow \ 1236$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6cd}-\frac{\frac{\int \frac{1}{2}x\left(4ade\left(cd^2-3ae^2\right)+\left(7c^2d^4+6ace^2d^2-21a^2e^4\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{5cde}-\frac{1}{5}x^2\left(\frac{d}{e}-\frac{3ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)}{4cd}$$

$$\Big\downarrow \ 27$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6cd}-\frac{\frac{\int x\left(4ade\left(cd^2-3ae^2\right)+\left(7c^2d^4+6ace^2d^2-21a^2e^4\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{10cde}-\frac{1}{5}x^2\left(\frac{d}{e}-\frac{3ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^3}{4cd}$$

$$\Big\downarrow \ 1225$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6cd}-\frac{\frac{\frac{5\left(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8\right)\int\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16c^2d^2e^2}-\frac{\left(-105a^3e^6-6cdex\left(-21a^2e^4+6acd^2e^2+7c^2d^4\right)+21a^2cd^2e^4+33ac^2d^4e^2+35c^3d\right.}{24c^2d^2e^2}}{10cde}}{4cd}$$

$\Big\downarrow$ 1087

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6cd}-\frac{\frac{5\left(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}\right)}{16c^2d^2e^2}-\frac{\left(-105a^3e^6-\cdots\right.}{}}{10cde}$$

$\Big\downarrow$ 1092

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6cd}-\frac{\frac{5\left(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2ce\cdots}{\sqrt{cdex^2+\left(cd^2\cdots\right.}}}{4cde}\cdots\right.}{16c^2d^2e^2}\cdots}{10cde}$$

$\Big\downarrow$ 219

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6cd}-\frac{\frac{5\left(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\cdots\right.}{8c^{3/2}d^{3/2}e^{3/2}}\cdots\right.}{16c^2d^2e^2}\cdots}{10cde}$$

input

```
Int[x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(6*c*d) - (-1/5*((d/e
- (3*a*e)/(c*d))*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) + (-1/
24*((35*c^3*d^6 + 33*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 - 105*a^3*e^6 - 6*c*
d*e*(7*c^2*d^4 + 6*a*c*d^2*e^2 - 21*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2)^(3/2))/(c^2*d^2*e^2) + (5*(7*c^4*d^8 + 8*a*c^3*d^6*e^2 + 6*a
^2*c^2*d^4*e^4 - 21*a^4*e^8)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^
2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c^2*d^2*e^2))/(10
*c*d*e))/(4*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. $2(411) = 822$.

Time = 2.40 (sec) , antiderivative size = 1949, normalized size of antiderivative = 4.40

method	result	size
default	Expression too large to display	1949

input
```
int(x^3*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output

```
d*(1/5*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-7/10*(a*e^2+c*d^2
)/d/e/c*(1/4*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-5/8*(a*e^2+c*
d^2)/d/e/c*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-1/2*(a*e^2+c
*d^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)
^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c
*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*
c)^(1/2)))-1/4*a/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^
2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
)/(d*e*c)^(1/2)))-2/5*a/c*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e
/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((
1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2))/(d*e*c)^(1/2))))+e*(1/6*x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3
/2)/d/e/c-3/4*(a*e^2+c*d^2)/d/e/c*(1/5*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(3/2)/d/e/c-7/10*(a*e^2+c*d^2)/d/e/c*(1/4*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*
x^2*e)^(3/2)/d/e/c-5/8*(a*e^2+c*d^2)/d/e/c*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(3/2)/d/e/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(
a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d
^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1044, normalized size of antiderivative = 2.36

$$\int x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Too large to display}$$

input

```
integrate(x^3*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
[-1/30720*(15*(7*c^6*d^12 - 6*a*c^5*d^10*e^2 - 3*a^2*c^4*d^8*e^4 - 4*a^3*c
^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 42*a^5*c*d^2*e^10 - 21*a^6*e^12)*sqrt(c*
d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*
e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^11
*e + 55*a*c^5*d^9*e^3 + 54*a^2*c^4*d^7*e^5 + 78*a^3*c^3*d^5*e^7 - 525*a^4*
c^2*d^3*e^9 + 315*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + a*c^5*d^5*e^7)*x^4
+ 16*(3*c^6*d^8*e^4 + 14*a*c^5*d^6*e^6 - 9*a^2*c^4*d^4*e^8)*x^3 - 8*(7*c^6
*d^9*e^3 - 3*a*c^5*d^7*e^5 + 33*a^2*c^4*d^5*e^7 - 21*a^3*c^3*d^3*e^9)*x^2
+ 2*(35*c^6*d^10*e^2 - 16*a*c^5*d^8*e^4 - 18*a^2*c^4*d^6*e^6 + 168*a^3*c^3
*d^4*e^8 - 105*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x))/(c^6*d^6*e^5), -1/15360*(15*(7*c^6*d^12 - 6*a*c^5*d^10*e^2 - 3*a^2*
c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 42*a^5*c*d^2*e^10 -
 21*a^6*e^12)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^
2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^
11*e + 55*a*c^5*d^9*e^3 + 54*a^2*c^4*d^7*e^5 + 78*a^3*c^3*d^5*e^7 - 525*a^
4*c^2*d^3*e^9 + 315*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + a*c^5*d^5*e^7)*x^
4 + 16*(3*c^6*d^8*e^4 + 14*a*c^5*d^6*e^6 - 9*a^2*c^4*d^4*e^8)*x^3 - 8*(7*c
^6*d^9*e^3 - 3*a*c^5*d^7*e^5 + 33*a^2*c^4*d^5*e^7 - 21*a^3*c^3*d^3*e^9)...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2166 vs. $2(452) = 904$.

Time = 1.55 (sec) , antiderivative size = 2166, normalized size of antiderivative = 4.89

$$\int x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Piecewise(((-a*(-3*a*(7*a*d*e**2/6 + c*d**3 - (9*a*e**2/2 + 9*c*d**2/2)*(a
*e**3 + 2*c*d**2*e - e*(11*a*e**2/2 + 11*c*d**2/2)/6)/(5*c*d*e))/(4*c) - (
5*a*e**2/2 + 5*c*d**2/2)*(a*d**2*e - 4*a*(a*e**3 + 2*c*d**2*e - e*(11*a*e*
*2/2 + 11*c*d**2/2)/6)/(5*c) - (7*a*e**2/2 + 7*c*d**2/2)*(7*a*d*e**2/6 + c
*d**3 - (9*a*e**2/2 + 9*c*d**2/2)*(a*e**3 + 2*c*d**2*e - e*(11*a*e**2/2 +
11*c*d**2/2)/6)/(5*c*d*e))/(4*c*d*e))/(3*c*d*e))/(2*c) - (a*e**2 + c*d**2)
*(-2*a*(a*d**2*e - 4*a*(a*e**3 + 2*c*d**2*e - e*(11*a*e**2/2 + 11*c*d**2/2
)/6)/(5*c) - (7*a*e**2/2 + 7*c*d**2/2)*(7*a*d*e**2/6 + c*d**3 - (9*a*e**2/
2 + 9*c*d**2/2)*(a*e**3 + 2*c*d**2*e - e*(11*a*e**2/2 + 11*c*d**2/2)/6)/(5
*c*d*e))/(4*c*d*e))/(3*c) - (3*a*e**2/2 + 3*c*d**2/2)*(-3*a*(7*a*d*e**2/6
+ c*d**3 - (9*a*e**2/2 + 9*c*d**2/2)*(a*e**3 + 2*c*d**2*e - e*(11*a*e**2/2
 + 11*c*d**2/2)/6)/(5*c*d*e))/(4*c) - (5*a*e**2/2 + 5*c*d**2/2)*(a*d**2*e
- 4*a*(a*e**3 + 2*c*d**2*e - e*(11*a*e**2/2 + 11*c*d**2/2)/6)/(5*c) - (7*a
*e**2/2 + 7*c*d**2/2)*(7*a*d*e**2/6 + c*d**3 - (9*a*e**2/2 + 9*c*d**2/2)*(
a*e**3 + 2*c*d**2*e - e*(11*a*e**2/2 + 11*c*d**2/2)/6)/(5*c*d*e))/(4*c*d*e
))/(3*c*d*e))/(2*c*d*e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d
*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(
c*d*e), Ne(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c
*d**2)/(2*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-
a*e**2 - c*d**2)/(2*c*d*e))**2), True)) + sqrt(a*d*e + c*d*e*x**2 + x*(...
```

Maxima [F(-2)]

Exception generated.

$$\int x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.14

$$\int x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\frac{1}{7680}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(2\left(8\left(10\,ex+\frac{13\,c^5d^6e^5+ac^4d^4e^7}{c^5d^5e^5}\right)x+\frac{3\,c^5d^7e^4+14\,ac^4d^5e}{c^5d^5e^5}\right.\right.\right.\right.$$
$$-\frac{(7\,c^6d^{12}-6\,ac^5d^{10}e^2-3\,a^2c^4d^8e^4-4\,a^3c^3d^6e^6-15\,a^4c^2d^4e^8+42\,a^5cd^2e^{10}-21\,a^6e^{12})\log\left(\left|-cd^2-a\right.\right.}{1024\,\sqrt{cde}c^5d^5e^4}$$

input
```
integrate(x^3*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output
```
1/7680*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(10*e*x + (
13*c^5*d^6*e^5 + a*c^4*d^4*e^7)/(c^5*d^5*e^5))*x + (3*c^5*d^7*e^4 + 14*a*c
^4*d^5*e^6 - 9*a^2*c^3*d^3*e^8)/(c^5*d^5*e^5))*x - (7*c^5*d^8*e^3 - 3*a*c^
4*d^6*e^5 + 33*a^2*c^3*d^4*e^7 - 21*a^3*c^2*d^2*e^9)/(c^5*d^5*e^5))*x + (3
5*c^5*d^9*e^2 - 16*a*c^4*d^7*e^4 - 18*a^2*c^3*d^5*e^6 + 168*a^3*c^2*d^3*e^
8 - 105*a^4*c*d*e^10)/(c^5*d^5*e^5))*x - (105*c^5*d^10*e - 55*a*c^4*d^8*e^
3 - 54*a^2*c^3*d^6*e^5 - 78*a^3*c^2*d^4*e^7 + 525*a^4*c*d^2*e^9 - 315*a^5*
e^11)/(c^5*d^5*e^5)) - 1/1024*(7*c^6*d^12 - 6*a*c^5*d^10*e^2 - 3*a^2*c^4*d
^8*e^4 - 4*a^3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 42*a^5*c*d^2*e^10 - 21*a
^6*e^12)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^5*d^5*e^4)
```

Mupad [B] (verification not implemented)

Time = 7.94 (sec) , antiderivative size = 1678, normalized size of antiderivative = 3.79

$$\int x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\text{Too large to display}$$

input
```
int(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
((7*a*e^2 + 7*c*d^2)*((a*((x/2 + (a*e^2 + c*d^2)/(4*c*d*e))*(x*(a*e^2 + c*
d^2) + a*d*e + c*d*e*x^2)^(1/2) - (log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(
c*d*e)^(1/2) + a*e^2 + c*d^2 + 2*c*d*e*x)*((a*e^2 + c*d^2)^2/4 - a*c*d^2*e
^2))/(2*(c*d*e)^(3/2))))/(4*c) - (x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2))/(4*c*d*e) + (((log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2
) + a*e^2 + c*d^2 + 2*c*d*e*x)*((a*e^2 + c*d^2)^3 - 4*a*c*d^2*e^2*(a*e^2 +
 c*d^2)))/(16*(c*d*e)^(5/2)) + ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)*(8*c*d*e*(a*d*e + c*d*e*x^2) - 3*(a*e^2 + c*d^2)^2 + 2*c*d*e*x*(a*e^2
+ c*d^2)))/(24*c^2*d^2*e^2))*(5*a*e^2 + 5*c*d^2))/(8*c*d*e)))/(10*c*e) - (
2*a*d*((log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^
2 + 2*c*d*e*x)*((a*e^2 + c*d^2)^3 - 4*a*c*d^2*e^2*(a*e^2 + c*d^2)))/(16*(c
*d*e)^(5/2)) + ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(8*c*d*e*(a*
d*e + c*d*e*x^2) - 3*(a*e^2 + c*d^2)^2 + 2*c*d*e*x*(a*e^2 + c*d^2)))/(24*c
^2*d^2*e^2)))/(5*c) + (x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/
(6*c*d) + (x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(5*c*e) + (a
*e*((a*((x/2 + (a*e^2 + c*d^2)/(4*c*d*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d
*e*x^2)^(1/2) - (log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e
^2 + c*d^2 + 2*c*d*e*x)*((a*e^2 + c*d^2)^2/4 - a*c*d^2*e^2))/(2*(c*d*e)^(3
/2))))/(4*c) - (x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(4*c*d*e)
 + (((log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d...
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 1033, normalized size of antiderivative = 2.33

$$\int x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Too large to display}$$

input
```
int(x^3*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(315*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d*e**11 - 525*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**4*c**2*d**3*e**9 - 210*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a**4*c**2*d**2*e**10*x + 78*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**5
*e**7 + 336*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**4*e**8*x + 168*sq
rt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**3*e**9*x**2 + 54*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**2*c**4*d**7*e**5 - 36*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a**2*c**4*d**6*e**6*x - 264*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**
5*e**7*x**2 - 144*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**4*e**8*x**3
 + 55*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**9*e**3 - 32*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a*c**5*d**8*e**4*x + 24*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a*c**5*d**7*e**5*x**2 + 224*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**6*e*
*6*x**3 + 128*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**5*e**7*x**4 - 105*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**11*e + 70*sqrt(d + e*x)*sqrt(a*e +
 c*d*x)*c**6*d**10*e**2*x - 56*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**9*e
**3*x**2 + 48*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**8*e**4*x**3 + 1664*s
qrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**7*e**5*x**4 + 1280*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*c**6*d**6*e**6*x**5 - 315*sqrt(e)*sqrt(d)*sqrt(c)*log((sqr
t(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d*
*2))*a**6*e**12 + 630*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**5*c*d**2*...
```

3.2 $\int x^2(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 340

$$\int x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\frac{(cd^2-ae^2)\,(3c^2d^4+6acd^2e^2+7a^2e^4)\,(cd^2+ae^2+2cdex)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{128c^4d^4e^3}$$
$$+\frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{5cd}$$
$$-\frac{(15c^2d^4+12acd^2e^2-35a^2e^4-6cde(3cd^2-7ae^2)\,x)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{240c^3d^3e^2}$$
$$-\frac{(cd^2-ae^2)^3\,(3c^2d^4+6acd^2e^2+7a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128c^{9/2}d^{9/2}e^{7/2}}$$

output
```
1/128*(-a*e^2+c*d^2)*(7*a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*(2*c*d*e*x+a*e^2+
c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^3+1/5*x^2*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d-1/240*(15*c^2*d^4+12*a*c*d^2*e^2-35*a
^2*e^4-6*c*d*e*(-7*a*e^2+3*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/
2)/c^3/d^3/e^2-1/128*(-a*e^2+c*d^2)^3*(7*a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*
arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/c^(9/2)/d^(9/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.90

$$\int x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\frac{\sqrt{(ae+cdx)(d+ex)}\Big(\sqrt{c}\sqrt{d}\sqrt{e}(-105a^4e^8+10a^3cde^6(19d+7ex)-2a^2c^2d^2e^4(18d^2+61dex+28e^2x^2}{}$$

input
```
Integrate[x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-105*a^4*e^8 + 10
*a^3*c*d*e^6*(19*d + 7*e*x) - 2*a^2*c^2*d^2*e^4*(18*d^2 + 61*d*e*x + 28*e^
2*x^2) + 6*a*c^3*d^3*e^2*(-5*d^3 + 3*d^2*e*x + 16*d*e^2*x^2 + 8*e^3*x^3) +
 3*c^4*d^4*(15*d^4 - 10*d^3*e*x + 8*d^2*e^2*x^2 + 176*d*e^3*x^3 + 128*e^4*
x^4)) - (15*(c*d^2 - a*e^2)^3*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*ArcT
anh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(Sqrt[a*
e + c*d*x]*Sqrt[d + e*x])))/(1920*c^(9/2)*d^(9/2)*e^(7/2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\,dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{1}{2}ex\left(4ade-\left(3cd^2-7ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{5cde}+\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd}-\frac{\int x\left(4ade-\left(3cd^2-7ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{10cd}$$

$\downarrow$ 1225

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd}-\frac{\frac{\left(-35a^2e^4-6cdex\left(3cd^2-7ae^2\right)+12acd^2e^2+15c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24c^2d^2e^2}-\frac{5\left(cd^2-ae^2\right)\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\int\sqrt{cdex^2+\left(cd^2+a\right.}}{16c^2d^2e^2}}{10cd}$$

$\downarrow$ 1087

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd}-\frac{\frac{\left(-35a^2e^4-6cdex\left(3cd^2-7ae^2\right)+12acd^2e^2+15c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24c^2d^2e^2}-\frac{5\left(cd^2-ae^2\right)\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)}{}\right.}{}}{10cd}$$

$\downarrow$ 1092

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd}-\frac{\frac{\left(-35a^2e^4-6cdex\left(3cd^2-7ae^2\right)+12acd^2e^2+15c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24c^2d^2e^2}-\frac{5\left(cd^2-ae^2\right)\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)}{}\right.}{}}{10cd}$$

$\downarrow$ 219

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5cd}-\frac{\frac{\left(-35a^2e^4-6cdex\left(3cd^2-7ae^2\right)+12acd^2e^2+15c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24c^2d^2e^2}-\frac{5\left(cd^2-ae^2\right)\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)}{}\right.}{}}{10cd}$$

input
```
Int[x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*c*d) - (((15*c^2*d^
4 + 12*a*c*d^2*e^2 - 35*a^2*e^4 - 6*c*d*e*(3*c*d^2 - 7*a*e^2)*x)*(a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(24*c^2*d^2*e^2) - (5*(c*d^2 - a*e^2
)*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*
ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c^2*
d^2*e^2))/(10*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1159 vs. $2(312) = 624$.

Time $= 2.49$ (sec) , antiderivative size $= 1160$, normalized size of antiderivative $= 3.41$

method	result	size
default	Expression too large to display	1160

input
```
int(x^2*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
d*(1/4*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-5/8*(a*e^2+c*d^2)/d
/e/c*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-1/2*(a*e^2+c*d^2)/
d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c
*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/
2)))-1/4*a/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*
c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e
*c)^(1/2)))+e*(1/5*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-7/10*
(a*e^2+c*d^2)/d/e/c*(1/4*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-5
/8*(a*e^2+c*d^2)/d/e/c*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-
1/2*(a*e^2+c*d^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2
*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^
(1/2))/(d*e*c)^(1/2)))-1/4*a/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+
c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*
ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*
x^2*e)^(1/2))/(d*e*c)^(1/2)))-2/5*a/c*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(3/2)/d/e/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 846, normalized size of antiderivative = 2.49

$$\int x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
[-1/7680*(15*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*a^2*c^3*d^6*e^4 - 6*a^3*c^2
*d^4*e^6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^
2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d
*e^3)*x) - 4*(384*c^5*d^5*e^5*x^4 + 45*c^5*d^9*e - 30*a*c^4*d^7*e^3 - 36*a
^2*c^3*d^5*e^5 + 190*a^3*c^2*d^3*e^7 - 105*a^4*c*d*e^9 + 48*(11*c^5*d^6*e^
4 + a*c^4*d^4*e^6)*x^3 + 8*(3*c^5*d^7*e^3 + 12*a*c^4*d^5*e^5 - 7*a^2*c^3*d
^3*e^7)*x^2 - 2*(15*c^5*d^8*e^2 - 9*a*c^4*d^6*e^4 + 61*a^2*c^3*d^4*e^6 - 3
5*a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^
5*e^4), 1/3840*(15*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*a^2*c^3*d^6*e^4 - 6*a
^3*c^2*d^4*e^6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt
(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*
(384*c^5*d^5*e^5*x^4 + 45*c^5*d^9*e - 30*a*c^4*d^7*e^3 - 36*a^2*c^3*d^5*e^
5 + 190*a^3*c^2*d^3*e^7 - 105*a^4*c*d*e^9 + 48*(11*c^5*d^6*e^4 + a*c^4*d^4
*e^6)*x^3 + 8*(3*c^5*d^7*e^3 + 12*a*c^4*d^5*e^5 - 7*a^2*c^3*d^3*e^7)*x^2 -
 2*(15*c^5*d^8*e^2 - 9*a*c^4*d^6*e^4 + 61*a^2*c^3*d^4*e^6 - 35*a^3*c^2*d^2
*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(337) = 674$.

Time = 1.41 (sec) , antiderivative size = 1386, normalized size of antiderivative = 4.08

$$\int x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output

```
Piecewise(((-a*(a*d**2*e - 3*a*(a*e**3 + 2*c*d**2*e - e*(9*a*e**2/2 + 9*c*
d**2/2)/5)/(4*c) - (5*a*e**2/2 + 5*c*d**2/2)*(6*a*d*e**2/5 + c*d**3 - (7*a
*e**2/2 + 7*c*d**2/2)*(a*e**3 + 2*c*d**2*e - e*(9*a*e**2/2 + 9*c*d**2/2)/5
)/(4*c*d*e))/(3*c*d*e))/(2*c) - (a*e**2 + c*d**2)*(-2*a*(6*a*d*e**2/5 + c*
d**3 - (7*a*e**2/2 + 7*c*d**2/2)*(a*e**3 + 2*c*d**2*e - e*(9*a*e**2/2 + 9*
c*d**2/2)/5)/(4*c*d*e))/(3*c) - (3*a*e**2/2 + 3*c*d**2/2)*(a*d**2*e - 3*a*
(a*e**3 + 2*c*d**2*e - e*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c) - (5*a*e**2/2
+ 5*c*d**2/2)*(6*a*d*e**2/5 + c*d**3 - (7*a*e**2/2 + 7*c*d**2/2)*(a*e**3 +
 2*c*d**2*e - e*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c*d*e))/(3*c*d*e))/(2*c*d*
e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*
sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a
*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log
(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c
*d*e))**2), True)) + sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(e*x**
4/5 + x**3*(a*e**3 + 2*c*d**2*e - e*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c*d*e)
 + x**2*(6*a*d*e**2/5 + c*d**3 - (7*a*e**2/2 + 7*c*d**2/2)*(a*e**3 + 2*c*d
**2*e - e*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c*d*e))/(3*c*d*e) + x*(a*d**2*e
- 3*a*(a*e**3 + 2*c*d**2*e - e*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c) - (5*a*e
**2/2 + 5*c*d**2/2)*(6*a*d*e**2/5 + c*d**3 - (7*a*e**2/2 + 7*c*d**2/2)*(a*
e**3 + 2*c*d**2*e - e*(9*a*e**2/2 + 9*c*d**2/2)/5)/(4*c*d*e))/(3*c*d*e)...
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate(x^2*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.17

$$\int x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\frac{1}{1920}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(6\left(8\,ex+\frac{11\,c^4d^5e^4+ac^3d^3e^6}{c^4d^4e^4}\right)x+\frac{3\,c^4d^6e^3+12\,ac^3d^4e^5-7}{c^4d^4e^4}\right.\right.\right.$$
$$+\frac{(3\,c^5d^{10}-3\,ac^4d^8e^2-2\,a^2c^3d^6e^4-6\,a^3c^2d^4e^6+15\,a^4cd^2e^8-7\,a^5e^{10})\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{c\right.\right.\right.}{256\,\sqrt{cde}c^4d^4e^3}$$

input
```
integrate(x^2*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output
```
1/1920*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*(8*e*x + (11*c
^4*d^5*e^4 + a*c^3*d^3*e^6)/(c^4*d^4*e^4))*x + (3*c^4*d^6*e^3 + 12*a*c^3*d
^4*e^5 - 7*a^2*c^2*d^2*e^7)/(c^4*d^4*e^4))*x - (15*c^4*d^7*e^2 - 9*a*c^3*d
^5*e^4 + 61*a^2*c^2*d^3*e^6 - 35*a^3*c*d*e^8)/(c^4*d^4*e^4))*x + (45*c^4*d
^8*e - 30*a*c^3*d^6*e^3 - 36*a^2*c^2*d^4*e^5 + 190*a^3*c*d^2*e^7 - 105*a^4
*e^9)/(c^4*d^4*e^4)) + 1/256*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*a^2*c^3*d^6
*e^4 - 6*a^3*c^2*d^4*e^6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*log(abs(-c*d^2 -
 a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))))/(sqrt(c*d*e)*c^4*d^4*e^3)
```

Mupad [B] (verification not implemented)

Time = 7.39 (sec) , antiderivative size = 985, normalized size of antiderivative = 2.90

$$\int x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\text{Too large to display}$$

input
```
int(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
((7*a*e^2 + 7*c*d^2)*((a*((x/2 + (a*e^2 + c*d^2)/(4*c*d*e))*(x*(a*e^2 + c*
d^2) + a*d*e + c*d*e*x^2)^(1/2) - (log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(
c*d*e)^(1/2) + a*e^2 + c*d^2 + 2*c*d*e*x)*((a*e^2 + c*d^2)^2/4 - a*c*d^2*e
^2))/(2*(c*d*e)^(3/2))))/(4*c) - (x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(3/2))/(4*c*d*e) + (((log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2
) + a*e^2 + c*d^2 + 2*c*d*e*x)*((a*e^2 + c*d^2)^3 - 4*a*c*d^2*e^2*(a*e^2 +
 c*d^2)))/(16*(c*d*e)^(5/2)) + ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2)*(8*c*d*e*(a*d*e + c*d*e*x^2) - 3*(a*e^2 + c*d^2)^2 + 2*c*d*e*x*(a*e^2
+ c*d^2)))/(24*c^2*d^2*e^2))*(5*a*e^2 + 5*c*d^2))/(8*c*d*e)))/(10*c*d) - (
2*a*e*((log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^
2 + 2*c*d*e*x)*((a*e^2 + c*d^2)^3 - 4*a*c*d^2*e^2*(a*e^2 + c*d^2)))/(16*(c
*d*e)^(5/2)) + ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(8*c*d*e*(a*
d*e + c*d*e*x^2) - 3*(a*e^2 + c*d^2)^2 + 2*c*d*e*x*(a*e^2 + c*d^2)))/(24*c
^2*d^2*e^2)))/(5*c) + (x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(4
*c*e) - (a*d*((x/2 + (a*e^2 + c*d^2)/(4*c*d*e))*(x*(a*e^2 + c*d^2) + a*d*e
 + c*d*e*x^2)^(1/2) - (log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2)
 + a*e^2 + c*d^2 + 2*c*d*e*x)*((a*e^2 + c*d^2)^2/4 - a*c*d^2*e^2))/(2*(c*d
*e)^(3/2))))/(4*c) - (((log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2
) + a*e^2 + c*d^2 + 2*c*d*e*x)*((a*e^2 + c*d^2)^3 - 4*a*c*d^2*e^2*(a*e^2 +
 c*d^2)))/(16*(c*d*e)^(5/2)) + ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)...
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 790, normalized size of antiderivative = 2.32

$$\int x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Too large to display}$$

input
```
int(x^2*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
( - 105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d*e**9 + 190*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**3*c**2*d**3*e**7 + 70*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a**3*c**2*d**2*e**8*x - 36*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**5
*e**5 - 122*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**4*e**6*x - 56*sqr
t(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**7*x**2 - 30*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a*c**4*d**7*e**3 + 18*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c
**4*d**6*e**4*x + 96*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**5*e**5*x**2
 + 48*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e**6*x**3 + 45*sqrt(d +
e*x)*sqrt(a*e + c*d*x)*c**5*d**9*e - 30*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c*
*5*d**8*e**2*x + 24*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**7*e**3*x**2 +
528*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**6*e**4*x**3 + 384*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*c**5*d**5*e**5*x**4 + 105*sqrt(e)*sqrt(d)*sqrt(c)*log(
(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 -
c*d**2))*a**5*e**10 - 225*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e +
c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*c*d**2
*e**8 + 90*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d
)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c**2*d**4*e**6 + 30*s
qrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sq
rt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**3*d**6*e**4 + 45*sqrt(e)*sqrt(
d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*...
```

3.3 $\int x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 243

$$\begin{aligned}\int x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx &= -\frac{(cd^2-ae^2)\,(3cd^2+5ae^2)\,(cd^2+ae^2+2cdex)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{64c^3d^3e^2}\\ &\quad+\frac{(3cd^2-5ae^2+6cdex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{24c^2d^2e}\\ &\quad+\frac{(cd^2-ae^2)^3\,(3cd^2+5ae^2)\,\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{64c^{7/2}d^{7/2}e^{5/2}}\end{aligned}$$

output
```
-1/64*(-a*e^2+c*d^2)*(5*a*e^2+3*c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^2+1/24*(6*c*d*e*x-5*a*e^2+3*c*d^2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^2/d^2/e+1/64*(-a*e^2+c*d^2)^3*(5
*a*e^2+3*c*d^2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.98

$$\int x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(15a^3e^6-a^2cde^4(31d+10ex)+ac^2d^2e^2(9d^2+20dex+8e^2x^2)+c^3(-\right.}{192c^{7/2}d^{7/2}e^{5/2}}$$

input
```
Integrate[x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(15*a^3*e^6 - a^2*
c*d*e^4*(31*d + 10*e*x) + a*c^2*d^2*e^2*(9*d^2 + 20*d*e*x + 8*e^2*x^2) + c
^3*(-9*d^6 + 6*d^5*e*x + 72*d^4*e^2*x^2 + 48*d^3*e^3*x^3)) + (3*(c*d^2 - a
*e^2)^3*(3*c*d^2 + 5*a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*S
qrt[d]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(192*c^(7/2)*d
^(7/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,dx$$

$$\Big\downarrow \text{ 1225}$$

$$\frac{\left(-5ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24c^2d^2e}-\frac{\left(-\frac{5a^2e^4}{c^2d^2}+\frac{2ae^2}{c}+3d^2\right)\int\sqrt{cdex^2+(cd^2+ae^2)\,x+ade}dx}{16e}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{\left(-5ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24c^2d^2e}-\frac{\left(-\frac{5a^2e^4}{c^2d^2}+\frac{2ae^2}{c}+3d^2\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}\right)}{16e}$$

$$\Big\downarrow \; 1092$$

$$\frac{\left(-5ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24c^2d^2e}-\frac{\left(-\frac{5a^2e^4}{c^2d^2}+\frac{2ae^2}{c}+3d^2\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd}{\sqrt{cdex^2+\left(cd^2+c\ldots\right.}}}{4cde}\ldots\right.}{16e}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(-5ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24c^2d^2e}-\frac{\left(-\frac{5a^2e^4}{c^2d^2}+\frac{2ae^2}{c}+3d^2\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\ldots\right.}{16e}$$

input

```
Int[x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output

```
((3*c*d^2 - 5*a*e^2 + 6*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(
3/2))/(24*c^2*d^2*e) - ((3*d^2 + (2*a*e^2)/c - (5*a^2*e^4)/(c^2*d^2))*(((c
*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*
c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]
*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)
*d^(3/2)*e^(3/2))))/(16*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(219) = 438$.

Time = 2.19 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.70

method	result
default	$d\left(\frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{3dec}-\frac{\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{4cde}+\frac{\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\ln\left(\frac{1}{2}\right.}{}\right.}{2dec}\right.$

input

```
int(x*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERB
OSE)
```

output

```
d*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-1/2*(a*e^2+c*d^2)/d/e
/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/
d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*
x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))
)+e*(1/4*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-5/8*(a*e^2+c*d^2)
/d/e/c*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-1/2*(a*e^2+c*d^2
)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/
2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2
+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(
1/2)))-1/4*a/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/
2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d
*e*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 676, normalized size of antiderivative = 2.78

$$\int x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$$
$$= \left[-\frac{3\,(3\,c^4d^8-4\,ac^3d^6e^2-6\,a^2c^2d^4e^4+12\,a^3cd^2e^6-5\,a^4e^8)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2\right.}{}\right.$$
$$-\frac{3\,(3\,c^4d^8-4\,ac^3d^6e^2-6\,a^2c^2d^4e^4+12\,a^3cd^2e^6-5\,a^4e^8)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,cdex+cd^2}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+ac}\right.}{}$$

input
```
integrate(x*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
fricas")
```

output
```
[-1/768*(3*(3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2
*e^6 - 5*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +
 c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(48*c^4*d^4
*e^4*x^3 - 9*c^4*d^7*e + 9*a*c^3*d^5*e^3 - 31*a^2*c^2*d^3*e^5 + 15*a^3*c*d
*e^7 + 8*(9*c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^2 + 2*(3*c^4*d^6*e^2 + 10*a*c^3
*d^4*e^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x))/(c^4*d^4*e^3), -1/384*(3*(3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*
e^4 + 12*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^
2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(48*c^4*d^4*
e^4*x^3 - 9*c^4*d^7*e + 9*a*c^3*d^5*e^3 - 31*a^2*c^2*d^3*e^5 + 15*a^3*c*d*
e^7 + 8*(9*c^4*d^5*e^3 + a*c^3*d^3*e^5)*x^2 + 2*(3*c^4*d^6*e^2 + 10*a*c^3*
d^4*e^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
))/(c^4*d^4*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 887 vs. $2(238) = 476$.

Time = 1.47 (sec) , antiderivative size = 887, normalized size of antiderivative = 3.65

$$\int x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Piecewise(((-a*(5*a*d*e**2/4 + c*d**3 - (5*a*e**2/2 + 5*c*d**2/2)*(a*e**3
+ 2*c*d**2*e - e*(7*a*e**2/2 + 7*c*d**2/2)/4)/(3*c*d*e))/(2*c) - (a*e**2 +
 c*d**2)*(a*d**2*e - 2*a*(a*e**3 + 2*c*d**2*e - e*(7*a*e**2/2 + 7*c*d**2/2
)/4)/(3*c) - (3*a*e**2/2 + 3*c*d**2/2)*(5*a*d*e**2/4 + c*d**3 - (5*a*e**2/
2 + 5*c*d**2/2)*(a*e**3 + 2*c*d**2*e - e*(7*a*e**2/2 + 7*c*d**2/2)/4)/(3*c
*d*e))/(2*c*d*e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x +
2*sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e),
 Ne(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/
(2*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2
- c*d**2)/(2*c*d*e))**2), True)) + sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c
*d**2))*(e*x**3/4 + x**2*(a*e**3 + 2*c*d**2*e - e*(7*a*e**2/2 + 7*c*d**2/2
)/4)/(3*c*d*e) + x*(5*a*d*e**2/4 + c*d**3 - (5*a*e**2/2 + 5*c*d**2/2)*(a*e
**3 + 2*c*d**2*e - e*(7*a*e**2/2 + 7*c*d**2/2)/4)/(3*c*d*e))/(2*c*d*e) + (
a*d**2*e - 2*a*(a*e**3 + 2*c*d**2*e - e*(7*a*e**2/2 + 7*c*d**2/2)/4)/(3*c)
 - (3*a*e**2/2 + 3*c*d**2/2)*(5*a*d*e**2/4 + c*d**3 - (5*a*e**2/2 + 5*c*d*
*2/2)*(a*e**3 + 2*c*d**2*e - e*(7*a*e**2/2 + 7*c*d**2/2)/4)/(3*c*d*e))/(2*
c*d*e))/(c*d*e)), Ne(c*d*e, 0)), (2*(-a*c*d**4*e*(a*d*e + x*(a*e**2 + c*d*
*2))**(3/2)/(3*(a*e**2 + c*d**2)) + e*(a*d*e + x*(a*e**2 + c*d**2))**(7/2)
/(7*(a*e**2 + c*d**2)) + (a*d*e + x*(a*e**2 + c*d**2))**(5/2)*(-a*d*e**2 +
 c*d**3)/(5*(a*e**2 + c*d**2)))/(a*e**2 + c*d**2)**2, Ne(a*e**2 + c*d**...
```

Maxima [F(-2)]

Exception generated.

$$\int x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.25

$$\int x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$= \frac{1}{192}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(6\,ex+\frac{9\,c^3d^4e^3+ac^2d^2e^5}{c^3d^3e^3}\right)x+\frac{3\,c^3d^5e^2+10\,ac^2d^3e^4-5\,a^2cd}{c^3d^3e^3}\right.\right.$$
$$-\frac{(3\,c^4d^8-4\,ac^3d^6e^2-6\,a^2c^2d^4e^4+12\,a^3cd^2e^6-5\,a^4e^8)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2}\right.\right.\right.}{128\sqrt{cde}c^3d^3e^2}$$

input
```
integrate(x*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
giac")
```

output
```
1/192*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*e*x + (9*c^3*d^
4*e^3 + a*c^2*d^2*e^5)/(c^3*d^3*e^3))*x + (3*c^3*d^5*e^2 + 10*a*c^2*d^3*e^
4 - 5*a^2*c*d*e^6)/(c^3*d^3*e^3))*x - (9*c^3*d^6*e - 9*a*c^2*d^4*e^3 + 31*
a^2*c*d^2*e^5 - 15*a^3*e^7)/(c^3*d^3*e^3)) - 1/128*(3*c^4*d^8 - 4*a*c^3*d^
6*e^2 - 6*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 - 5*a^4*e^8)*log(abs(-c*d^2 -
 a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))))/(sqrt(c*d*e)*c^3*d^3*e^2)
```

Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 544, normalized size of antiderivative = 2.24

$$\int x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\frac{x\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e)^{3/2}}{4\,c\,d}$$

$$-\frac{a\,e\left(\left(\frac{x}{2}+\frac{c\,d^2+a\,e^2}{4\,c\,d\,e}\right)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e}-\frac{\ln\left(2\,\sqrt{(a\,e+c\,d\,x)\,(d+e\,x)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)\left(\frac{(c\,d}{}\right.}{2\,(c\,d\,e)^{3/2}}\right.}{4\,c}$$

$$-\frac{\left(\frac{\ln\left(2\,\sqrt{(a\,e+c\,d\,x)\,(d+e\,x)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)\left((c\,d^2+a\,e^2)^3-4\,a\,c\,d^2\,e^2\,(c\,d^2+a\,e^2)\right)}{16\,(c\,d\,e)^{5/2}}+\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,\left(8\,c\,d\,e\right.}{}\right.}{8\,c\,d}$$

$$+\frac{d\,\ln\left(2\,\sqrt{(a\,e+c\,d\,x)\;(d+e\,x)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)\,\left((c\,d^2+a\,e^2)^3-4\,a\,c\,d^2\,e^2\,(c\,d^2+a\,e^2\right.}{16\,(c\,d\,e)^{5/2}}$$

$$+\frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e}\,\left(8\,c\,d\,e\,(c\,d\,e\,x^2+a\,d\,e)-3\,(c\,d^2+a\,e^2)^2+2\,c\,d\,e\,x\,(c\,d^2+a\,e^2)\right)}{24\,c^2\,d\,e^2}$$

input
```
int(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
(x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(4*c*d) - (a*e*((x/2 + (
a*e^2 + c*d^2)/(4*c*d*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) -
(log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^2 + 2*c
*d*e*x)*((a*e^2 + c*d^2)^2/4 - a*c*d^2*e^2))/(2*(c*d*e)^(3/2))))/(4*c) - (
((log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^2 + 2*
c*d*e*x)*((a*e^2 + c*d^2)^3 - 4*a*c*d^2*e^2*(a*e^2 + c*d^2)))/(16*(c*d*e)^
(5/2)) + ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(8*c*d*e*(a*d*e +
c*d*e*x^2) - 3*(a*e^2 + c*d^2)^2 + 2*c*d*e*x*(a*e^2 + c*d^2)))/(24*c^2*d^2
*e^2))*(5*a*e^2 + 5*c*d^2))/(8*c*d) + (d*log(2*((a*e + c*d*x)*(d + e*x))^(
1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^2 + 2*c*d*e*x)*((a*e^2 + c*d^2)^3 - 4*a*c
*d^2*e^2*(a*e^2 + c*d^2)))/(16*(c*d*e)^(5/2)) + ((x*(a*e^2 + c*d^2) + a*d*
e + c*d*e*x^2)^(1/2)*(8*c*d*e*(a*d*e + c*d*e*x^2) - 3*(a*e^2 + c*d^2)^2 +
2*c*d*e*x*(a*e^2 + c*d^2)))/(24*c^2*d*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.38

$$\int x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$$

$$= \frac{15\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^3cd\,e^7-31\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c^2d^3e^5-10\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c^2d^2e^6x+9\sqrt{}}{}$$

input
```
int(x*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d*e**7 - 31*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a**2*c**2*d**3*e**5 - 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2
*c**2*d**2*e**6*x + 9*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**5*e**3 + 2
0*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**4*e**4*x + 8*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a*c**3*d**3*e**5*x**2 - 9*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c
**4*d**7*e + 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**6*e**2*x + 72*sqrt(
d + e*x)*sqrt(a*e + c*d*x)*c**4*d**5*e**3*x**2 + 48*sqrt(d + e*x)*sqrt(a*e
 + c*d*x)*c**4*d**4*e**4*x**3 - 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sq
rt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*
*4*e**8 + 36*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c*d**2*e**6 - 18*sq
rt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqr
t(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**2*d**4*e**4 - 12*sqrt(e)*sqrt(d
)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/
sqrt(a*e**2 - c*d**2))*a*c**3*d**6*e**2 + 9*sqrt(e)*sqrt(d)*sqrt(c)*log((s
qrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*
d**2))*c**4*d**8)/(192*c**4*d**4*e**3)
```

3.4 $\int (d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 195

$$\begin{aligned}\int (d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx &= \frac{(cd^2-ae^2)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^2d^2e} \\ &\quad+\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3cd}-\frac{(cd^2-ae^2)^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{3/2}}\end{aligned}$$

output
```
1/8*(-a*e^2+c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)/c^2/d^2/e+1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d-1/8*(-a
*e^2+c*d^2)^3*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.92

$$\int (d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(-3a^2e^4+2acde^2(4d+ex)+c^2d^2(3d^2+14dex+8e^2x^2))-\frac{3(cd^2-ae^2)^3\cdots}{\sqrt{c}\cdots}\right.}{24c^{5/2}d^{5/2}e^{3/2}}$$

input
```
Integrate[(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-3*a^2*e^4 + 2*a*
c*d*e^2*(4*d + e*x) + c^2*d^2*(3*d^2 + 14*d*e*x + 8*e^2*x^2)) - (3*(c*d^2
- a*e^2)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d
*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(24*c^(5/2)*d^(5/2)*e^(3/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,dx$$
$$\Big\downarrow \text{1160}$$
$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \sqrt{cdex^2+(cd^2+ae^2)\,x+ade}dx}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd}$$
$$\Big\downarrow \text{1087}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}\right)}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4cde}\right)}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\right)}{2d}+\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3cd}$$

input
```
Int[(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*c*d) + ((d^2 - (a*e^2)/c)
*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sq
rt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^
(3/2)*d^(3/2)*e^(3/2))))/(2*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(171) = 342$.

Time = 2.09 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.91

method	result
default	$d\left(\frac{(2cdxe+a\,e^2+c\,d^2)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}{4cde}+\frac{\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\ln\left(\frac{\frac{1}{2}a\,e^2+\frac{1}{2}c\,d^2+cdxe}{\sqrt{dec}}+\sqrt{ade+(a\,e^2+c\,d^2)x+c\right.}{8dec\sqrt{dec}}\right.$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
d*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d
/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x
*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+
e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-1/2*(a*e^2+c*d^2)/d/e
/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/
d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*
x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.73

$$\int (d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$

$$= \left[-\frac{3\,(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\,\sqrt{cdex^2+}\right.}{}\right.$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="fr
icas")
```

output
```
[-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d
*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e
) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 + 3*c^3*d^5*e + 8*
a*c^2*d^3*e^3 - 3*a^2*c*d*e^5 + 2*(7*c^3*d^4*e^2 + a*c^2*d^2*e^4)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^2), 1/48*(3*(c^3*d^6 -
3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c
*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(8*
c^3*d^3*e^3*x^2 + 3*c^3*d^5*e + 8*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5 + 2*(7*c^3
*d^4*e^2 + a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/
(c^3*d^3*e^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(182) = 364$.

Time = 1.13 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.93

$$\int (d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$$
$$= \begin{cases} \left(\frac{ex^2}{3} + \frac{x\left(ae^3+2cd^2e-\frac{e\left(\frac{5ae^2}{2}+\frac{5cd^2}{2}\right)}{3}\right)}{2cde} + \frac{\frac{4ade^2}{3}+cd^3-\frac{\left(\frac{3ae^2}{2}+\frac{3cd^2}{2}\right)\left(ae^3+2cd^2e-\frac{e\left(\frac{5ae^2}{2}+\frac{5cd^2}{2}\right)}{3}\right)}{2cde}}{cde}\right)\sqrt{ade+cdex^2+x(} \\ \frac{2\left(\frac{cd^3\left(ade+x\left(ae^2+cd^2\right)\right)^{\frac{3}{2}}}{3\left(ae^2+cd^2\right)}+\frac{e\left(ade+x\left(ae^2+cd^2\right)\right)^{\frac{5}{2}}}{5\left(ae^2+cd^2\right)}\right)}{ae^2+cd^2} \\ \sqrt{ade}\left(dx+\frac{ex^2}{2}\right) \end{cases}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Piecewise(((e*x**2/3 + x*(a*e**3 + 2*c*d**2*e - e*(5*a*e**2/2 + 5*c*d**2/2
)/3)/(2*c*d*e) + (4*a*d*e**2/3 + c*d**3 - (3*a*e**2/2 + 3*c*d**2/2)*(a*e**
3 + 2*c*d**2*e - e*(5*a*e**2/2 + 5*c*d**2/2)/3)/(2*c*d*e))/(c*d*e))*sqrt(a
*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)) + (a*d**2*e - a*(a*e**3 + 2*c*d**
2*e - e*(5*a*e**2/2 + 5*c*d**2/2)/3)/(2*c) - (a*e**2 + c*d**2)*(4*a*d*e**2
/3 + c*d**3 - (3*a*e**2/2 + 3*c*d**2/2)*(a*e**3 + 2*c*d**2*e - e*(5*a*e**2
/2 + 5*c*d**2/2)/3)/(2*c*d*e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 +
 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))
/sqrt(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e*
*2 - c*d**2)/(2*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(
x - (-a*e**2 - c*d**2)/(2*c*d*e))**2), True)), Ne(c*d*e, 0)), (2*(c*d**3*(
a*d*e + x*(a*e**2 + c*d**2))**(3/2)/(3*(a*e**2 + c*d**2)) + e*(a*d*e + x*(
a*e**2 + c*d**2))**(5/2)/(5*(a*e**2 + c*d**2)))/(a*e**2 + c*d**2), Ne(a*e*
*2 + c*d**2, 0)), (sqrt(a*d*e)*(d*x + e*x**2/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int (d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.14

$$\int (d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}\,dx$$
$$=\frac{1}{24}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\,ex+\frac{7\,c^2d^3e^2+acde^4}{c^2d^2e^2}\right)x+\frac{3\,c^2d^4e+8\,acd^2e^3-3\,a^2e^5}{c^2d^2e^2}\right)$$
$$+\frac{(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+\right.\right.\right.}{16\,\sqrt{cde}c^2d^2e}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="gi
ac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*e*x + (7*c^2*d^3*e^
2 + a*c*d*e^4)/(c^2*d^2*e^2))*x + (3*c^2*d^4*e + 8*a*c*d^2*e^3 - 3*a^2*e^5
)/(c^2*d^2*e^2)) + 1/16*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3
*e^6)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x
^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e)
```

Mupad [B] (verification not implemented)

Time = 6.77 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.57

$$\int (d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$$
$$= d\left(\frac{x}{2}+\frac{c\,d^2+a\,e^2}{4\,c\,d\,e}\right)\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}$$
$$-\frac{d\,\ln\left(2\,\sqrt{\left(a\,e+c\,d\,x\right)\,\left(d+e\,x\right)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)\,\left(\frac{{\left(c\,d^2+a\,e^2\right)}^2}{4}-a\,c\,d^2\,e^2\right)}{2\,{\left(c\,d\,e\right)}^{3/2}}$$
$$+\frac{e\,\ln\left(2\,\sqrt{\left(a\,e+c\,d\,x\right)\,\left(d+e\,x\right)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)\,\left({\left(c\,d^2+a\,e^2\right)}^3-4\,a\,c\,d^2\,e^2\,\left(c\,d^2+a\,e^2\right.\right.}{16\,{\left(c\,d\,e\right)}^{5/2}}$$
$$+\frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(8\,c\,d\,e\,\left(c\,d\,e\,x^2+a\,d\,e\right)-3\,{\left(c\,d^2+a\,e^2\right)}^2+2\,c\,d\,e\,x\,\left(c\,d^2+a\,e^2\right)\right)}{24\,c^2\,d^2\,e}$$

input
```
int((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
d*(x/2 + (a*e^2 + c*d^2)/(4*c*d*e))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2) - (d*log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 +
 c*d^2 + 2*c*d*e*x)*((a*e^2 + c*d^2)^2/4 - a*c*d^2*e^2))/(2*(c*d*e)^(3/2))
 + (e*log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^2
+ 2*c*d*e*x)*((a*e^2 + c*d^2)^3 - 4*a*c*d^2*e^2*(a*e^2 + c*d^2)))/(16*(c*d
*e)^(5/2)) + ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(8*c*d*e*(a*d*
e + c*d*e*x^2) - 3*(a*e^2 + c*d^2)^2 + 2*c*d*e*x*(a*e^2 + c*d^2)))/(24*c^2
*d^2*e)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.05

$$\int (d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}\,dx$$
$$=\frac{-3\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2cd\,e^5+8\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^3e^3+2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^2e^4x+3\sqrt{e}}{}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
( - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d*e**5 + 8*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a*c**2*d**3*e**3 + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*
d**2*e**4*x + 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e + 14*sqrt(d +
e*x)*sqrt(a*e + c*d*x)*c**3*d**4*e**2*x + 8*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*c**3*d**3*e**3*x**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e +
c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**6 -
 9*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c
)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**4 + 9*sqrt(e)*sqrt(
d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))
/sqrt(a*e**2 - c*d**2))*a*c**2*d**4*e**2 - 3*sqrt(e)*sqrt(d)*sqrt(c)*log((
sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c
*d**2))*c**3*d**6)/(24*c**3*d**3*e**2)
```

3.5 $\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 223

$$\begin{aligned}\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x}\,dx &= \frac{(5cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4cd}\\ &\quad+\frac{(3c^2d^4+6acd^2e^2-a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^{3/2}d^{3/2}\sqrt{e}}\\ &\quad-2\sqrt{a}d^{3/2}\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)\end{aligned}$$

output

```
1/4*(2*c*d*e*x+a*e^2+5*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d+
1/4*(-a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(
1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(1/2)-2*a^
(1/2)*d^(3/2)*e^(1/2)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x}\,dx$$
$$= \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(ae^2+cd(5d+2ex))-8\sqrt{a}c^{3/2}d^3e\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{ae+cd}}{\sqrt{a}\sqrt{e}\sqrt{d+e}}\right.\right.}{4c^{3/2}d^{3/2}\sqrt{e}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x,x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*(a*e^2 + c*d*(5*d + 2*e*x)) - 8*Sqrt[a]*c^(3/2)*d^3*e*ArcT
anh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] + (3*c^2*
d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c
]*Sqrt[d]*Sqrt[d + e*x])]))/(4*c^(3/2)*d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*
(d + e*x)])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{x}\,dx$$

$$\Big\downarrow \text{1231}$$

$$\frac{(ae^2+5cd^2+2cdex)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4cd}-\frac{\int -\frac{e(8aced^3+(3c^2d^4+6ace^2d^2-a^2e^4)x)}{2x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4cde}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{8aced^3+(3c^2d^4+6ace^2d^2-a^2e^4)x}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8cd}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(ae^2+5cd^2+2cdex\right)}{4cd}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+8acd^3e\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8cd}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(ae^2+5cd^2+2cdex\right)}{4cd}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}+8acd^3e\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{8cd}$$
$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(ae^2+5cd^2+2cdex\right)}{4cd}$$

$$\Big\downarrow \text{219}$$

$$\frac{8acd^3e\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{(-a^2e^4+6acd^2e^2+3c^2d^4)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}}{8cd}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(ae^2+5cd^2+2cdex\right)}{4cd}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{(-a^2e^4+6acd^2e^2+3c^2d^4)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}-16acd^3e\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+(cd^2+ae}{\sqrt{cdex^2+(cd^2+ae}}}{8cd}$$
$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(ae^2+5cd^2+2cdex\right)}{4cd}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(-a^2e^4+6acd^2e^2+3c^2d^4)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}-8\sqrt{a}cd^{5/2}\sqrt{e}\text{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex}}\right.}{8cd}$$
$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(ae^2+5cd^2+2cdex\right)}{4cd}$$

input
```
Int[((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x,x]
```

output
```
((5*c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(4*c*d) + (((3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(c*d^2 + a*e^2
 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])])/(Sqrt[c]*Sqrt[d]*Sqrt[e]) - 8*Sqrt[a]*c*d^(5/2)*Sqrt[e]*ArcT
anh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.48

method	result
default	$e\left(\frac{(2cdxe+a\,e^2+c\,d^2)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}{4cde}+\frac{\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\ln\left(\frac{\frac{1}{2}a\,e^2+\frac{1}{2}c\,d^2+cdxe}{\sqrt{dec}}+\sqrt{ade+(a\,e^2+c\,d^2)x+c\right.}{8dec\sqrt{dec}}\right.$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/x,x,method=_RETURNVERB
OSE)
```

output
```
e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d
/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x
*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+
d*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2
+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))
/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(
1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))
```

Fricas [A] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 1337, normalized size of antiderivative = 6.00

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x,x, algorithm="
fricas")
```

output
```
[1/16*(8*sqrt(a*d*e)*c^2*d^3*e*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e
^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
 + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - (3
*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*
x) + 4*(2*c^2*d^2*e^2*x + 5*c^2*d^3*e + a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x))/(c^2*d^2*e), 1/8*(4*sqrt(a*d*e)*c^2*d^3*e*log((8*a^2
*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*
c*d^3*e + a^2*d*e^3)*x)/x^2) - (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(
-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x
+ c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e
+ a*c*d*e^3)*x)) + 2*(2*c^2*d^2*e^2*x + 5*c^2*d^3*e + a*c*d*e^3)*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e), 1/16*(16*sqrt(-a*d*e)*c^2
*d^3*e*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e
 + a^2*d*e^3)*x)) - (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(c*d*e)*log(
8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8...
```

Sympy [F]

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}(d+ex)}{x}\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x,x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.58 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x}\,dx = d\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}$$
$$-d\ln\left(\frac{a\,e^2}{2}+\frac{c\,d^2}{2}+\frac{\sqrt{a\,d\,e}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{x}+\frac{a\,d\,e}{x}\right)\sqrt{a\,d\,e}$$
$$+e\left(\frac{x}{2}+\frac{c\,d^2+a\,e^2}{4\,c\,d\,e}\right)\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}$$
$$+\frac{d\ln\left(2\sqrt{(a\,e+c\,d\,x)\,(d+e\,x)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)(c\,d^2+a\,e^2)}{2\sqrt{c\,d\,e}}$$
$$-\frac{e\ln\left(2\sqrt{(a\,e+c\,d\,x)\,(d+e\,x)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)\left(\frac{(c\,d^2+a\,e^2)^2}{4}-a\,c\,d^2\,e^2\right)}{2\,(c\,d\,e)^{3/2}}$$

input
```
int(((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/x,x)
```

output
```
d*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) - d*log((a*e^2)/2 + (c*d^2
)/2 + ((a*d*e)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/x + (a
*d*e)/x)*(a*d*e)^(1/2) + e*(x/2 + (a*e^2 + c*d^2)/(4*c*d*e))*(x*(a*e^2 + c
*d^2) + a*d*e + c*d*e*x^2)^(1/2) + (d*log(2*((a*e + c*d*x)*(d + e*x))^(1/2
)*(c*d*e)^(1/2) + a*e^2 + c*d^2 + 2*c*d*e*x)*(a*e^2 + c*d^2))/(2*(c*d*e)^(
1/2)) - (e*log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c
*d^2 + 2*c*d*e*x)*((a*e^2 + c*d^2)^2/4 - a*c*d^2*e^2))/(2*(c*d*e)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x}\,dx$$
$$= \frac{\sqrt{ex+d}\,\sqrt{cdx+ae}\,acd\,e^3+5\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^3e+2\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^2e^2x+4\sqrt{e}\,\sqrt{d}\,\sqrt{a}\,\mathrm{lo}}{}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x,x)
```

output
```
(sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d*e**3 + 5*sqrt(d + e*x)*sqrt(a*e + c
*d*x)*c**2*d**3*e + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**2*e**2*x + 4
*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sq
rt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**2*d**3*e
+ 4*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)
*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**2*d**3
*e - 4*sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)
*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*c**2*d**3*e - sqrt
(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(
d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**4 + 6*sqrt(e)*sqrt(d)*sqrt(c)*log
((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 -
 c*d**2))*a*c*d**2*e**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e
+ c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**4
)/(4*c**2*d**2*e)
```

3.6 $\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 200

$$\begin{aligned}\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2}\,dx &= -\frac{(d-ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x} \\ &\quad+\frac{\sqrt{e}(3cd^2+ae^2)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{c}\sqrt{d}} \\ &\quad-\frac{\sqrt{d}(cd^2+3ae^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{a}\sqrt{e}}\end{aligned}$$

output
```
-(-e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x+e^(1/2)*(a*e^2+3*c*d^2
)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/c^(1/2)/d^(1/2)-d^(1/2)*(3*a*e^2+c*d^2)*arctanh(a^(1/2)*e^(1/2)*(
e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2}\,dx$$
$$= \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{a}\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}(-d+ex)\sqrt{d+ex}-\sqrt{c}d(cd^2+3ae^2)\,x\text{arctanh}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right.\right.}{\sqrt{a}\sqrt{c}\sqrt{d}\sqrt{e}x\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^2,x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[a]*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e
 + c*d*x]*(-d + e*x)*Sqrt[d + e*x] - Sqrt[c]*d*(c*d^2 + 3*a*e^2)*x*ArcTanh
[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] + Sqrt[a]*e*
(3*c*d^2 + a*e^2)*x*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*S
qrt[d + e*x])]))/(Sqrt[a]*Sqrt[c]*Sqrt[d]*Sqrt[e]*x*Sqrt[(a*e + c*d*x)*(d
+ e*x)])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x^2}\,dx$$
$$\Big\downarrow \text{1230}$$
$$-\frac{1}{2}\int -\frac{d(cd^2+3ae^2)+e(3cd^2+ae^2)x}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{(d-ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{2}\int \frac{d(cd^2+3ae^2)+e(3cd^2+ae^2)x}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{(d-ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}$$

$\downarrow$ 1269

$$\frac{1}{2}\left(e\left(ae^2+3cd^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+d\left(3ae^2+cd^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx\right)-\frac{(d-ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

$\downarrow$ 1092

$$\frac{1}{2}\left(d\left(3ae^2+cd^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+2e\left(ae^2+3cd^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{\ldots}{\sqrt{cde\ldots}}\right.$$
$$\frac{(d-ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

$\downarrow$ 219

$$\frac{1}{2}\left(d\left(3ae^2+cd^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{\sqrt{e}\left(ae^2+3cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cde\ldots}}\right.}{\sqrt{c}\sqrt{d}}\right.$$
$$\frac{(d-ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

$\downarrow$ 1154

$$\frac{1}{2}\left(\frac{\sqrt{e}\left(ae^2+3cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}}-2d\left(3ae^2+cd^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\ldots\right.$$
$$\frac{(d-ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{\sqrt{e}\left(ae^2+3cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}}-\frac{\sqrt{d}\left(3ae^2+cd^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+a\ldots}}\right.}{\sqrt{a}\sqrt{e}}\right.$$
$$\frac{(d-ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

input
```
Int[((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^2,x]
```

output
```
-(((d - e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x) + ((Sqrt[e]*(
3*c*d^2 + a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sq
rt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(Sqrt[c]*Sqrt[d]) - (
Sqrt[d]*(c*d^2 + 3*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]
*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(Sqrt[a]*S
qrt[e]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(170) = 340$.

Time = 2.20 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.86

method	result
default	$d\left(-\frac{(ade+(a e^2+c d^2)x+cd x^2e)^{\frac{3}{2}}}{adex}+\frac{(a e^2+c d^2)\left(\sqrt{ade+(a e^2+c d^2)x+cd x^2e}+\frac{(a e^2+c d^2)\ln\left(\frac{\frac{1}{2}a e^2+\frac{1}{2}c d^2+cdxe}{\sqrt{dec}}+\sqrt{ade+(a \cdots\right.}{2\sqrt{dec}} \cdots\right.}{2ad \cdots} \cdots\right.$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/x^2,x,method=_RETURNVE
RBOSE)
```

output
```
d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)/a/
d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e
^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)
^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+
a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*
e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+e*((a*d*e+(a*e^2+
c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*
e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*
d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))
```

Fricas [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 1241, normalized size of antiderivative = 6.20

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2,x, algorithm
="fricas")
```

output
```
[1/4*((3*c*d^2 + a*e^2)*x*sqrt(e/(c*d))*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 +
6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x + 4*(2*c^2*d^2*e*x +
 c^2*d^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e/(
c*d))) + (c*d^2 + 3*a*e^2)*x*sqrt(d/(a*e))*log((8*a^2*d^2*e^2 + (c^2*d^4 +
 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 8*(a*c*d^3*e + a^2*d*e^3)*x - 4*(2*a^2*d*e
^2 + (a*c*d^2*e + a^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(d/(a*e)))/x^2) + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x -
 d))/x, -1/4*(2*(3*c*d^2 + a*e^2)*x*sqrt(-e/(c*d))*arctan(1/2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-e/(c*d))
/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) - (c*d^2 + 3*a*e^2)*x*sqrt
(d/(a*e))*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 8
*(a*c*d^3*e + a^2*d*e^3)*x - 4*(2*a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(d/(a*e)))/x^2) - 4*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x - d))/x, 1/4*(2*(c*d^2 + 3*a*e^2)*
x*sqrt(-d/(a*e))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2
*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-d/(a*e))/(c*d^2*e*x^2 + a*d^2*e + (c*d^3
 + a*d*e^2)*x)) + (3*c*d^2 + a*e^2)*x*sqrt(e/(c*d))*log(8*c^2*d^2*e^2*x^2
+ c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x + 4*(2*c
^2*d^2*e*x + c^2*d^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*sqrt(e/(c*d))) + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x...
```

Sympy [F]

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}(d+ex)}{x^2}\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2}\,dx$$

$$= \sqrt{cdex^2+cd^2x+ae^2x+ade}e + \frac{(cd^3+3\,ade^2)\arctan\left(-\frac{\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}}{\sqrt{-ade}}\right)}{\sqrt{-ade}}$$
$$- \frac{(3\,cd^2e+ae^3)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{2\sqrt{cde}}$$
$$- \frac{\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)cd^3+\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)ade^2+2\sqrt{c}}{ade-\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)^2}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2,x, algorithm
="giac")
```

output
```
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*e + (c*d^3 + 3*a*d*e^2)*arctan
(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*
e))/sqrt(-a*d*e) - 1/2*(3*c*d^2*e + a*e^3)*log(abs(-c*d^2 - a*e^2 - 2*sqrt
(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/sq
rt(c*d*e) - ((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
*c*d^3 + (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a*d
*e^2 + 2*sqrt(c*d*e)*a*d^2*e)/(a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c
*d^2*x + a*e^2*x + a*d*e))^2)
```

Mupad [B] (verification not implemented)

Time = 6.78 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2}\,dx$$
$$= e\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e} - \frac{d\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e}}{x}$$
$$+ d\,\ln\left(2\sqrt{(a\,e+c\,d\,x)\;(d+e\,x)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)\sqrt{c\,d\,e}$$
$$- e\,\ln\left(\frac{a\,e^2}{2}+\frac{c\,d^2}{2}+\frac{\sqrt{a\,d\,e}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e}}{x}+\frac{a\,d\,e}{x}\right)\sqrt{a\,d\,e}$$
$$+ \frac{e\,\ln\left(2\sqrt{(a\,e+c\,d\,x)\;(d+e\,x)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)\;(c\,d^2+a\,e^2)}{2\sqrt{c\,d\,e}}$$
$$- \frac{d\,\ln\left(\frac{a\,e^2}{2}+\frac{c\,d^2}{2}+\frac{\sqrt{a\,d\,e}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{x}+\frac{a\,d\,e}{x}\right)\;(c\,d^2+a\,e^2)}{2\sqrt{a\,d\,e}}$$

input
```
int(((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/x^2,x)
```

output
```
e*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) - (d*(x*(a*e^2 + c*d^2) +
a*d*e + c*d*e*x^2)^(1/2))/x + d*log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d
*e)^(1/2) + a*e^2 + c*d^2 + 2*c*d*e*x)*(c*d*e)^(1/2) - e*log((a*e^2)/2 + (
c*d^2)/2 + ((a*d*e)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/x
 + (a*d*e)/x)*(a*d*e)^(1/2) + (e*log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*
d*e)^(1/2) + a*e^2 + c*d^2 + 2*c*d*e*x)*(a*e^2 + c*d^2))/(2*(c*d*e)^(1/2))
 - (d*log((a*e^2)/2 + (c*d^2)/2 + ((a*d*e)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*
e + c*d*e*x^2)^(1/2))/x + (a*d*e)/x)*(a*e^2 + c*d^2))/(2*(a*d*e)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.64

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2}\,dx$$
$$= \frac{-2\sqrt{ex+d}\,\sqrt{cdx+ae}\,ac\,d^2e+2\sqrt{ex+d}\,\sqrt{cdx+ae}\,acd\,e^2x+3\sqrt{e}\,\sqrt{d}\,\sqrt{a}\log\left(\sqrt{e}\,\sqrt{cdx+ae}-\sqrt{2\sqrt{c}}\right.}{}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d**2*e + 2*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a*c*d*e**2*x + 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e +
 c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*
sqrt(d + e*x))*a*c*d*e**2*x + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e
 + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c
)*sqrt(d + e*x))*c**2*d**3*x + 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(
a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqr
t(c)*sqrt(d + e*x))*a*c*d*e**2*x + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqr
t(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*s
qrt(c)*sqrt(d + e*x))*c**2*d**3*x - 3*sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e
)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e
+ 2*c*d*e*x)*a*c*d*e**2*x - sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d)*
sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*
x)*c**2*d**3*x + 2*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**3*x + 6*sq
rt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqr
t(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d**2*e*x)/(2*a*c*d*e*x)
```

3.7 $\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 227

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3}\,dx = -\frac{(2ade+(cd^2+5ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4aex^2} + 2\sqrt{c}\sqrt{d}e^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right) + \frac{(c^2d^4-6acd^2e^2-3a^2e^4)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4a^{3/2}\sqrt{d}e^{3/2}}$$

output
```
-1/4*(2*a*d*e+(5*a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a
/e/x^2+2*c^(1/2)*d^(1/2)*e^(3/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/4*(-3*a^2*e^4-6*a*c*d^2*e^2+c^2*
d^4)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))/a^(3/2)/d^(1/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3}\,dx$$
$$=\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(-\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(cd^2x+ae(2d+5ex))+(c^2d^4-6acd^2e^2-3a^2e^4)\,x\right.}{4a^{3/2}\sqrt{d}e^{3/2}x^2\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^3,x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d
*x]*Sqrt[d + e*x]*(c*d^2*x + a*e*(2*d + 5*e*x))) + (c^2*d^4 - 6*a*c*d^2*e^
2 - 3*a^2*e^4)*x^2*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sq
rt[d + e*x])] + 8*a^(3/2)*Sqrt[c]*d*e^3*x^2*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*
d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(4*a^(3/2)*Sqrt[d]*e^(3/2)*x^2*Sq
rt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{x^3}\,dx$$
$$\Big\downarrow \ 1229$$
$$-\frac{\int \frac{d(c^2d^4-6ace^2d^2-8ace^3xd-3a^2e^4)}{2x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4ade}-\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}\left(x\left(5ae^2+cd^2\right)+2ade\right)}{4aex^2}$$
$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{c^2d^4-6ace^2d^2-8ace^3xd-3a^2e^4}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8ae}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(x\left(5ae^2+cd^2\right)+2ade\right)}{4aex^2}$$

$$\Big\downarrow \ 1269$$

$$-\frac{\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-8acde^3\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8ae}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(x\left(5ae^2+cd^2\right)+2ade\right)}{4aex^2}$$

$$\Big\downarrow \ 1092$$

$$-\frac{\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-16acde^3\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x}}}{8ae}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(x\left(5ae^2+cd^2\right)+2ade\right)}{4aex^2}$$

$$\Big\downarrow \ 219$$

$$-\frac{\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-8a\sqrt{c}\sqrt{d}e^{5/2}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8ae}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(x\left(5ae^2+cd^2\right)+2ade\right)}{4aex^2}$$

$$\Big\downarrow \ 1154$$

$$-\frac{-2\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-8a\sqrt{c}\sqrt{d}e^{5/2}\text{arctanh}\left(\frac{}{2\sqrt{c}\sqrt{d}}\right)}{8ae}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(x\left(5ae^2+cd^2\right)+2ade\right)}{4aex^2}$$

$$\Big\downarrow \ 219$$

$$-\frac{-\frac{\left(-3a^2e^4-6acd^2e^2+c^2d^4\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{a}\sqrt{d}\sqrt{e}}-8a\sqrt{c}\sqrt{d}e^{5/2}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+}}\right)}{8ae}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(x\left(5ae^2+cd^2\right)+2ade\right)}{4aex^2}$$

input
```
Int[((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^3,x]
```

output
```
-1/4*((2*a*d*e + (c*d^2 + 5*a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(a*e*x^2) - (-8*a*Sqrt[c]*Sqrt[d]*e^(5/2)*ArcTanh[(c*d^2 + a*e^2
+ 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])] - ((c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTanh[(2*a*d*e + (c
*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])])/(Sqrt[a]*Sqrt[d]*Sqrt[e]))/(8*a*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. $2(193) = 386$.

Time = 2.25 (sec) , antiderivative size = 1035, normalized size of antiderivative = 4.56

method	result	size
default	Expression too large to display	1035

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/x^3,x,method=_RETURNVE
RBOSE)
```

output
```
d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(a*e^2+c*d^2
)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d
^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1
/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(
a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*c/a*(1/4*(2*c*
d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*
c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^
(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+1/2*c/a*((a
*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*
c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e
*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))+e*(-1/a/d/e/x*(a*d*e+(a*e^2+
c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c
)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*
e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*x^2*e)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*l
n((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*...
```

Fricas [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 1375, normalized size of antiderivative = 6.06

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3,x, algorithm
="fricas")
```

output
```
[1/16*(8*sqrt(c*d*e)*a^2*d*e^3*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c
*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - (c^2*d^4
 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2*d^2*e^2 + (c^2*d^
4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^
3)*x)/x^2) - 4*(2*a^2*d^2*e^2 + (a*c*d^3*e + 5*a^2*d*e^3)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d*e^2*x^2), -1/16*(16*sqrt(-c*d*e)*a^
2*d*e^3*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^
3*e + a*c*d*e^3)*x)) + (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(a*d*e)*x
^2*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a
*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*a^2*d^2*e^2 + (a*c*d^3*e
+ 5*a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d*e^2*
x^2), 1/8*(4*sqrt(c*d*e)*a^2*d*e^3*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6
*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*
c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - (c^2
*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d...
```

Sympy [F]

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}(d+ex)}{x^3}\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**3,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(193) = 386.

Time = 0.20 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.76

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3}\,dx$$
$$= -\frac{cde^2\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{\sqrt{cde}} - \frac{(c^2d^4-6\,acd^2e^2-3\,a^2e^4)\arctan\left(-\frac{\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}}{\sqrt{-ade}}\right)}{4\sqrt{-ade}ae} + \frac{\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)ac^2d^5e+2\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)a^2cd^3e^3}{}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3,x, algorithm
="giac")
```

output
```
-c*d*e^2*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/sqrt(c*d*e) - 1/4*(c^2*d^4 - 6*a*c*d
^2*e^2 - 3*a^2*e^4)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*
e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a*e) + 1/4*((sqrt(c*d*e)*x - s
qrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a*c^2*d^5*e + 2*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^2*c*d^3*e^3 - 3*(sqrt(c*
d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*d*e^5 - 8*sqrt(c
*d*e)*a^3*d^2*e^4 + (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))^3*c^2*d^4 + 10*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))^3*a*c*d^2*e^2 + 5*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a
*e^2*x + a*d*e))^3*a^2*e^4 + 8*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))^2*a*c*d^3*e + 16*sqrt(c*d*e)*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^2*d*e^3)/((a*d*e - (sq
rt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^2*a*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3}\,dx = \int \frac{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{x^3}\,dx$$

input
```
int(((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/x^3,x)
```

output
```
int(((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 1089, normalized size of antiderivative = 4.80

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3,x)
```

output
```
( - 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*d**2*e**4 - 10*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**3*d*e**5*x - 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d
**4*e**2 - 12*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d**3*e**3*x - 2*sqrt(
d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**5*e*x + 3*sqrt(e)*sqrt(d)*sqrt(a)*log
(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2)
 + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*e**6*x**2 + 9*sqrt(e)*sqrt(d)*sqrt(
a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c
*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**2*e**4*x**2 + 5*sqrt(e)*
sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e
 + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**4*e**2*x**2
 - sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*
sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**3*d**6*
x**2 + 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sq
rt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3
*e**6*x**2 + 9*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqr
t(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x)
)*a**2*c*d**2*e**4*x**2 + 5*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e +
 c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*
sqrt(d + e*x))*a*c**2*d**4*e**2*x**2 - sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)
*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sq...
```

3.8 $\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 203

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4}\,dx = \frac{(cd^2-ae^2)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8a^2de^2x^2} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3aex^3} - \frac{(cd^2-ae^2)^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8a^{5/2}d^{3/2}e^{5/2}}$$

output
```
1/8*(-a*e^2+c*d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/a^2/d/e^2/x^2-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a/e/x
^3-1/8*(-a*e^2+c*d^2)^3*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(3/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4}\,dx$$
$$=\frac{(-cd^2+ae^2)^3\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-3c^2d^4x^2+2acd^2ex(d+4ex)+a^2e^2(8d^2+14dex+3e^2x^2))}{(cd^2-ae^2)^3x^3}+\frac{3\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\cdots}}{\sqrt{a}\sqrt{\cdots}}\right.}{\sqrt{ae+cdx}\sqrt{d\cdots}}\right.}{24a^{5/2}d^{3/2}e^{5/2}}$$

input
```
Integrate[((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^4,x]
```

output
```
((-(c*d^2) + a*e^2)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt
[e]*(-3*c^2*d^4*x^2 + 2*a*c*d^2*e*x*(d + 4*e*x) + a^2*e^2*(8*d^2 + 14*d*e*
x + 3*e^2*x^2)))/((c*d^2 - a*e^2)^3*x^3) + (3*ArcTanh[(Sqrt[d]*Sqrt[a*e +
c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]
)))/(24*a^(5/2)*d^(3/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{x^4}\,dx$$
$$\Big\downarrow \; 1228$$
$$-\frac{1}{2}\left(\frac{cd^2}{ae}-e\right)\int \frac{\sqrt{cdex^2+(cd^2+ae^2)\,x+ade}}{x^3}dx-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3aex^3}$$
$$\Big\downarrow \; 1152$$

$$-\frac{1}{2}\left(\frac{cd^2}{ae}-e\right)\left(-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{4adex^2}\right.$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3aex^3}$$

$\downarrow$ 1154

$$-\frac{1}{2}\left(\frac{cd^2}{ae}-e\right)\left(\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+(cd^2+ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(a\right.}}{4adex}\right.$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3aex^3}$$

$\downarrow$ 219

$$-\frac{1}{2}\left(\frac{cd^2}{ae}-e\right)\left(\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right.}}{4adex^2}\right.$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3aex^3}$$

input
```
Int[((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^4,x]
```

output
```
-1/3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(a*e*x^3) - (((c*d^2)/(
a*e) - e)*(-1/4*((2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])/(a*d*e*x^2) + ((c*d^2 - a*e^2)^2*ArcTanh[(2*a*d*e + (c*d
^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])])/(8*a^(3/2)*d^(3/2)*e^(3/2))))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. $2(179) = 358$.

Time = 2.52 (sec) , antiderivative size = 1340, normalized size of antiderivative = 6.60

method	result	size
default	Expression too large to display	1340

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/x^4,x,method=_RETURNVE
RBOSE)
```

output
```
d*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c*d^2
)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(a*e^2
+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e
^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)
*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)
*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*c/a*(1/4
*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8
*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d
*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+1/2*c
/a*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^
2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
)/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^
(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))))+e*(-1/2/a/d/e/x^2*(a*
d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(
a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*
d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2
)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)...
```

Fricas [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.76

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4}\,dx$$
$$= \left[-\frac{3\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\sqrt{ade}x^3\log\left(\frac{8\,a^2d^2e^2+\left(c^2d^4+6\,acd^2e^2+a^2e^4\right)x^2+4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{x^2}\right.}{}\right.$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4,x, algorithm
="fricas")
```

output
```
[-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(a*d
*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^3*e^3 - (3*a*c
^2*d^5*e - 8*a^2*c*d^3*e^3 - 3*a^3*d*e^5)*x^2 + 2*(a^2*c*d^4*e^2 + 7*a^3*d
^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^2*e^3*x^3),
 1/48*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*d
*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*
e + a^2*d*e^3)*x)) - 2*(8*a^3*d^3*e^3 - (3*a*c^2*d^5*e - 8*a^2*c*d^3*e^3 -
 3*a^3*d*e^5)*x^2 + 2*(a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^2*e^3*x^3)]
```

Sympy [F]

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}(d+ex)}{x^4}\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**4,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1007 vs. $2(179) = 358$.

Time = 0.15 (sec) , antiderivative size = 1007, normalized size of antiderivative = 4.96

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4,x, algorithm
="giac")
```

output
```
1/8*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*arctan(-(sqrt(
c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqr
t(-a*d*e)*a^2*d*e^2) - 1/24*(3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))*a^2*c^3*d^8*e^2 + 39*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e))*a^3*c^2*d^6*e^4 + 9*(sqrt(c*d*e)*x - sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c*d^4*e^6 - 3*(sqrt(c*d*e)*x - sqr
t(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*d^2*e^8 + 16*sqrt(c*d*e)*a^4
*c*d^5*e^5 + 8*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e
))^3*a*c^3*d^7*e + 72*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e))^3*a^2*c^2*d^5*e^3 + 72*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
 + a*e^2*x + a*d*e))^3*a^3*c*d^3*e^5 + 8*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
 c*d^2*x + a*e^2*x + a*d*e))^3*a^4*d*e^7 + 48*sqrt(c*d*e)*(sqrt(c*d*e)*x -
 sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^2*c^2*d^6*e^2 + 48*sqrt(
c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^3
*c*d^4*e^4 - 3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e
))^5*c^3*d^6 + 9*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))^5*a*c^2*d^4*e^2 + 39*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^
2*x + a*d*e))^5*a^2*c*d^2*e^4 + 3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))^5*a^3*e^6 + 96*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d
*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^4*a^2*c*d^3*e^3 + 48*sqrt(c*d*e)*(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4}\,dx = \int \frac{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{x^4}\,dx$$

input
```
int(((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/x^4,x)
```

output
```
int(((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 1116, normalized size of antiderivative = 5.50

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4,x)
```

output

```
( - 16*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*d**3*e**5 - 28*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**4*d**2*e**6*x - 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4
*d*e**7*x**2 - 16*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**5*e**3 - 32*sq
rt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**4*e**4*x - 22*sqrt(d + e*x)*sqrt(a
*e + c*d*x)*a**3*c*d**3*e**5*x**2 - 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2
*c**2*d**6*e**2*x - 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**5*e**3
*x**2 + 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**7*e*x**2 - 3*sqrt(e)*s
qrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e
+ a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**4*e**8*x**3 + 6*sqr
t(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a
)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**2*e**6
*x**3 - 6*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*s
qrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c
**3*d**6*e**2*x**3 + 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*
x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(
d + e*x))*c**4*d**8*x**3 - 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e
+ c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)
*sqrt(d + e*x))*a**4*e**8*x**3 + 6*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqr
t(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*s
qrt(c)*sqrt(d + e*x))*a**3*c*d**2*e**6*x**3 - 6*sqrt(e)*sqrt(d)*sqrt(a)...
```

3.9 $\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 287

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5}\,dx = -\frac{(cd^2-ae^2)(5cd^2+3ae^2)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64a^3d^2e^3x^2} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{4aex^4} + \frac{(5cd^2-3ae^2)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{24a^2de^2x^3} + \frac{(cd^2-ae^2)^3(5cd^2+3ae^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{64a^{7/2}d^{5/2}e^{7/2}}$$

output
```
-1/64*(-a*e^2+c*d^2)*(3*a*e^2+5*c*d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d^2/e^3/x^2-1/4*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(3/2)/a/e/x^4+1/24*(-3*a*e^2+5*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(3/2)/a^2/d/e^2/x^3+1/64*(-a*e^2+c*d^2)^3*(3*a*e^2+5*c*d^2)*arcta
nh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/a^(7/2)/d^(5/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^5}\,dx$$
$$=\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(15c^3d^6x^3-ac^2d^4ex^2(10d+31ex)+a^2cd^2e^2x(8d^2+20dex+9e^2x^2)+3a^3e^3(16d^3+24d^2ex+2de^2x^2-}{x^4}\right.}{192a^{7/2}d^{5/2}e^{7/2}}$$

input
```
Integrate[((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^5,x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(15*c^3*d^6*x^3
 - a*c^2*d^4*e*x^2*(10*d + 31*e*x) + a^2*c*d^2*e^2*x*(8*d^2 + 20*d*e*x + 9
*e^2*x^2) + 3*a^3*e^3*(16*d^3 + 24*d^2*e*x + 2*d*e^2*x^2 - 3*e^3*x^3)))/x^
4) + (3*(c*d^2 - a*e^2)^3*(5*c*d^2 + 3*a*e^2)*ArcTanh[(Sqrt[d]*Sqrt[a*e +
c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]
)))/(192*a^(7/2)*d^(5/2)*e^(7/2))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{x^5}\,dx$$
$$\Big\downarrow \; 1237$$
$$-\frac{\int \frac{d(5cd^2+2cexd-3ae^2)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{2x^4}dx}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4aex^4}$$
$$\Big\downarrow \; 27$$
$$-\frac{\int \frac{(5cd^2+2cexd-3ae^2)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{x^4}dx}{8ae}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4aex^4}$$

$$\Big\downarrow \; 1228$$

$$-\frac{-\frac{\left(\frac{5c^2d^4}{a}-3ae^4-2cd^2e^2\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{2de}-\frac{\left(\frac{5cd}{ae}-\frac{3e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x^3}}{8ae}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4aex^4}$$

$$\Big\downarrow \; 1152$$

$$-\frac{-\frac{\left(\frac{5c^2d^4}{a}-3ae^4-2cd^2e^2\right)\left(-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{2de}-\frac{\left(\frac{5cd}{ae}-\frac{3e}{d}\right)\left(x\left(\cdots\right.\right.}{\cdots}}{8ae}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4aex^4}$$

$$\Big\downarrow \; 1154$$

$$-\frac{-\frac{\left(\frac{5c^2d^4}{a}-3ae^4-2cd^2e^2\right)\left(\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+c\cdots}}{4adex^2}\cdots\right.}{2de}\cdots}{8ae}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4aex^4}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{\left(\frac{5c^2d^4}{a}-3ae^4-2cd^2e^2\right)\left(\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{2de}-\cdots}{8ae}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4aex^4}$$

input
```
Int[((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^5,x]
```

output
```
-1/4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(a*e*x^4) - (-1/3*(((5*
c*d)/(a*e) - (3*e)/d)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/x^3 -
 (((5*c^2*d^4)/a - 2*c*d^2*e^2 - 3*a*e^4)*(-1/4*((2*a*d*e + (c*d^2 + a*e^2
)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^2) + ((c*d^2 -
a*e^2)^2*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*a^(3/2)*d^(3/2)*e^(3/2))
))/(2*d*e))/(8*a*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2108 vs. $2(259) = 518$.

Time = 2.66 (sec) , antiderivative size = 2109, normalized size of antiderivative = 7.35

method	result	size
default	Expression too large to display	2109

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/x^5,x,method=_RETURNVE
RBOSE)
```

output
```
d*(-1/4/a/d/e/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-5/8*(a*e^2+c*d^2
)/a/d/e*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2
+c*d^2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*
(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/
2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+
c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)
*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+
c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*c/
a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d
/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x
*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))
+1/2*c/a*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/
2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)
^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a
*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))))-1/4*c/a*(-1/2/a
/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(a*e^2+c*d^2)/a/d/e*(
-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e
*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+
1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/
(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)...
```

Fricas [A] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 702, normalized size of antiderivative = 2.45

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^5}\,dx$$

$$= \left[-\frac{3\,(5\,c^4d^8-12\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6-3\,a^4e^8)\sqrt{ade}x^4\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2-4\,\sqrt{}}{}\right.}{}\right.$$

$$-\frac{3\,(5\,c^4d^8-12\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6-3\,a^4e^8)\sqrt{-ade}x^4\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,ade+(}{2\,(acd^2e^2x^2+a^2d^2e^2+(acd^3}\right.}{}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5,x, algorithm
="fricas")
```

output
```
[-1/768*(3*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2
*e^6 - 3*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) +
 4*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e - 31*a^2*c^2*d^5*e^3 + 9*a^3*c*d^3*e^
5 - 9*a^4*d*e^7)*x^3 - 2*(5*a^2*c^2*d^6*e^2 - 10*a^3*c*d^4*e^4 - 3*a^4*d^2
*e^6)*x^2 + 8*(a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(a^4*d^3*e^4*x^4), -1/384*(3*(5*c^4*d^8 - 12*a*c^3*d^6
*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(-a*d*e)*x^4*a
rctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d
*e^3)*x)) + 2*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e - 31*a^2*c^2*d^5*e^3 + 9*a
^3*c*d^3*e^5 - 9*a^4*d*e^7)*x^3 - 2*(5*a^2*c^2*d^6*e^2 - 10*a^3*c*d^4*e^4
- 3*a^4*d^2*e^6)*x^2 + 8*(a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x))/(a^4*d^3*e^4*x^4)]
```

Sympy [F]

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^5}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}(d+ex)}{x^5}\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**5,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)/x**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. $2(259) = 518$.

Time = 0.16 (sec) , antiderivative size = 1618, normalized size of antiderivative = 5.64

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5,x, algorithm
="giac")
```

output
```
-1/64*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6
- 3*a^4*e^8)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
 a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^3*d^2*e^3) + 1/192*(15*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*c^4*d^11*e^3 + 348*(
sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c^3*d^9*e
^5 + 402*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5
*c^2*d^7*e^7 + 12*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*
d*e))*a^6*c*d^5*e^9 - 9*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))*a^7*d^3*e^11 + 128*sqrt(c*d*e)*a^5*c^2*d^8*e^6 + 73*(sqrt(c*d*
e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c^4*d^10*e^2 + 9
00*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c^3
*d^8*e^4 + 1854*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))^3*a^4*c^2*d^6*e^6 + 724*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*
e^2*x + a*d*e))^3*a^5*c*d^4*e^8 + 33*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d
^2*x + a*e^2*x + a*d*e))^3*a^6*d^2*e^10 + 384*sqrt(c*d*e)*(sqrt(c*d*e)*x -
 sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^3*c^3*d^9*e^3 + 1024*sqr
t(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a
^4*c^2*d^7*e^5 + 768*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
 + a*e^2*x + a*d*e))^2*a^5*c*d^5*e^7 - 55*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))^5*a*c^4*d^9*e + 132*(sqrt(c*d*e)*x - sqrt...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5}\,dx = \int \frac{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{x^5}\,dx$$

input
```
int(((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/x^5,x)
```

output
```
int(((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 3.35 (sec) , antiderivative size = 1664, normalized size of antiderivative = 5.80

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5,x)
```

output
```
( - 192*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*d**4*e**6 - 288*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**5*d**3*e**7*x - 24*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
**5*d**2*e**8*x**2 + 36*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*d*e**9*x**3 -
 192*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d**6*e**4 - 320*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**4*c*d**5*e**5*x - 104*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a**4*c*d**4*e**6*x**2 - 32*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**7
*e**3*x - 40*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**6*e**4*x**2 + 88
*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**5*e**5*x**3 + 40*sqrt(d + e*
x)*sqrt(a*e + c*d*x)*a**2*c**3*d**8*e**2*x**2 + 64*sqrt(d + e*x)*sqrt(a*e
+ c*d*x)*a**2*c**3*d**7*e**3*x**3 - 60*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c
**4*d**9*e*x**3 + 18*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x)
 - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))*a**5*e**10*x**4 - 6*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e +
 c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*
sqrt(d + e*x))*a**4*c*d**2*e**8*x**4 - 60*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt
(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c**2*d**4*e**6*x**4 + 36*sqrt(e)*sqrt(d)
*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e*
*2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c**3*d**6*e**4*x**4 + 4
2*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c...
```

3.10 $\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^6}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 387

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^6}\,dx = \frac{(cd^2-ae^2)(7c^2d^4+6acd^2e^2+3a^2e^4)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128a^4d^3e^4x^2} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5aex^5} + \frac{(7cd^2-3ae^2)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{40a^2de^2x^4} - \frac{(35c^2d^4-12acd^2e^2-15a^2e^4)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{240a^3d^2e^3x^3} - \frac{(cd^2-ae^2)^3(7c^2d^4+6acd^2e^2+3a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128a^{9/2}d^{7/2}e^{9/2}}$$

output
```
1/128*(-a*e^2+c*d^2)*(3*a^2*e^4+6*a*c*d^2*e^2+7*c^2*d^4)*(2*a*d*e+(a*e^2+c
*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^4/d^3/e^4/x^2-1/5*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a/e/x^5+1/40*(-3*a*e^2+7*c*d^2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^2/d/e^2/x^4-1/240*(-15*a^2*e^4-12*a*c*d
^2*e^2+35*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^3/d^2/e^3/x^3
-1/128*(-a*e^2+c*d^2)^3*(3*a^2*e^4+6*a*c*d^2*e^2+7*c^2*d^4)*arctanh(a^(1/2
)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(9/2)
/d^(7/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^6}\,dx$$
$$=\frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(105c^4d^8x^4-10ac^3d^6ex^3(7d+19ex)+2a^2c^2d^4e^2x^2(28d^2+61dex+18e^2x^2)-6a^3cd^2e^3x(8d^3+16d^2ex-}{x^5}\right.}{1920a^9}$$

input
```
Integrate[((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^6,x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(105*c^4*d^8*x^4
- 10*a*c^3*d^6*e*x^3*(7*d + 19*e*x) + 2*a^2*c^2*d^4*e^2*x^2*(28*d^2 + 61*d
*e*x + 18*e^2*x^2) - 6*a^3*c*d^2*e^3*x*(8*d^3 + 16*d^2*e*x + 3*d*e^2*x^2 -
 5*e^3*x^3) - 3*a^4*e^4*(128*d^4 + 176*d^3*e*x + 8*d^2*e^2*x^2 - 10*d*e^3*
x^3 + 15*e^4*x^4)))/x^5 - (15*(c*d^2 - a*e^2)^3*(7*c^2*d^4 + 6*a*c*d^2*e^2
 + 3*a^2*e^4)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d
+ e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(1920*a^(9/2)*d^(7/2)*e^(9/2
))
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1237, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x^6}\,dx$$

$$\Big\downarrow \text{1237}$$

$$-\frac{\int \frac{d(7cd^2+4cexd-3ae^2)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{2x^5}dx}{5ade}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5aex^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(7cd^2+4cexd-3ae^2)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{x^5}dx}{10ae}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5aex^5}$$

$$\Big\downarrow \; 1237$$

$$-\frac{-\frac{\int \frac{\left(35c^2d^4-12ace^2d^2+2ce\left(7cd^2-3ae^2\right)xd-15a^2e^4\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^4}dx}{4ade}-\frac{\left(\frac{7cd}{ae}-\frac{3e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4x^4}}{10ae}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5aex^5}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{\left(35c^2d^4-12ace^2d^2+2ce\left(7cd^2-3ae^2\right)xd-15a^2e^4\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^4}dx}{8ade}-\frac{\left(\frac{7cd}{ae}-\frac{3e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4x^4}}{10ae}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5aex^5}$$

$$\Big\downarrow \; 1228$$

$$-\frac{-\frac{-\frac{5\left(cd^2-ae^2\right)\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{2ade}-\frac{\left(\frac{35c^2d^4}{a}-15ae^4-12cd^2e^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3dex^3}}{8ade}-\frac{\left(\frac{7cd}{ae}-\frac{3e}{d}\right)\cdots}{\cdots}}{10ae}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5aex^5}$$

$$\Big\downarrow \; 1152$$

$$-\frac{-\frac{-\frac{5\left(cd^2-ae^2\right)\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{2ade}-\frac{\left(\frac{35c^2\cdots}{\cdots}\cdots\right.}{\cdots}}{8ade}\cdots}{10ae}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5aex^5}$$

$$\Big\downarrow \; 1154$$

$$-\frac{-\frac{-\frac{5\left(cd^2-ae^2\right)\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+\ldots\right.}}{4adex^2}\right.}{2ade}}{8ade}}{10ae}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5aex^5}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{-\frac{5\left(cd^2-ae^2\right)\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade\ldots}}{4adex^2}\right.}{2ade}}{8ade}}{10ae}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5aex^5}$$

input

```
Int[((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^6,x]
```

output

```
-1/5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(a*e*x^5) - (-1/4*(((7*
c*d)/(a*e) - (3*e)/d)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/x^4 -
 (-1/3*(((35*c^2*d^4)/a - 12*c*d^2*e^2 - 15*a*e^4)*(a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2)^(3/2))/(d*e*x^3) - (5*(c*d^2 - a*e^2)*(7*c^2*d^4 + 6*a*c*
d^2*e^2 + 3*a^2*e^4)*(-1/4*((2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^2) + ((c*d^2 - a*e^2)^2*ArcTanh[(2*a
*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2])])/(8*a^(3/2)*d^(3/2)*e^(3/2))))/(2*a*d*e))/(8*a*d*
e))/(10*a*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3582 vs. $2(355) = 710$.

Time = 3.39 (sec) , antiderivative size = 3583, normalized size of antiderivative = 9.26

method	result	size
default	Expression too large to display	3583

input

```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/x^6,x,method=_RETURNVE
RBOSE)
```

output

```
d*(-1/5/a/d/e/x^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-7/10*(a*e^2+c*d^
2)/a/d/e*(-1/4/a/d/e/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-5/8*(a*e^
2+c*d^2)/a/d/e*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2
*(a*e^2+c*d^2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/
2)-1/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(
3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*
(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2
+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+
(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x
))+2*c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^
2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^
(1/2)))+1/2*c/a*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)
*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)
*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))-1/4*c/a*
(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(a*e^2+c*d^2)/
a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2
)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2
*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*...
```

Fricas [A] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 874, normalized size of antiderivative = 2.26

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^6,x, algorithm
="fricas")
```

output
```
[-1/7680*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2*a^3*c^
2*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(a*d*e)*x^5*log((8*a^2*d^2*e
^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*
e + a^2*d*e^3)*x)/x^2) + 4*(384*a^5*d^5*e^5 - (105*a*c^4*d^9*e - 190*a^2*c
^3*d^7*e^3 + 36*a^3*c^2*d^5*e^5 + 30*a^4*c*d^3*e^7 - 45*a^5*d*e^9)*x^4 + 2
*(35*a^2*c^3*d^8*e^2 - 61*a^3*c^2*d^6*e^4 + 9*a^4*c*d^4*e^6 - 15*a^5*d^2*e
^8)*x^3 - 8*(7*a^3*c^2*d^7*e^3 - 12*a^4*c*d^5*e^5 - 3*a^5*d^3*e^7)*x^2 + 4
8*(a^4*c*d^6*e^4 + 11*a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x))/(a^5*d^4*e^5*x^5), 1/3840*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*
a^2*c^3*d^6*e^4 + 2*a^3*c^2*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(-
a*d*e)*x^5*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
 + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d
^3*e + a^2*d*e^3)*x)) - 2*(384*a^5*d^5*e^5 - (105*a*c^4*d^9*e - 190*a^2*c^
3*d^7*e^3 + 36*a^3*c^2*d^5*e^5 + 30*a^4*c*d^3*e^7 - 45*a^5*d*e^9)*x^4 + 2*
(35*a^2*c^3*d^8*e^2 - 61*a^3*c^2*d^6*e^4 + 9*a^4*c*d^4*e^6 - 15*a^5*d^2*e^
8)*x^3 - 8*(7*a^3*c^2*d^7*e^3 - 12*a^4*c*d^5*e^5 - 3*a^5*d^3*e^7)*x^2 + 48
*(a^4*c*d^6*e^4 + 11*a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x))/(a^5*d^4*e^5*x^5)]
```

Sympy [F]

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^6}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}(d+ex)}{x^6}\,dx$$

input
```
integrate((e*x+d)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**6,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)/x**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^6,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2352 vs. $2(355) = 710$.

Time = 0.18 (sec) , antiderivative size = 2352, normalized size of antiderivative = 6.08

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^6,x, algorithm
="giac")
```

output
```
1/128*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2*a^3*c^2*d^4*e
^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^4*d^3*e^4) -
1/1920*(105*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*
a^4*c^5*d^14*e^4 + 3615*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))*a^5*c^4*d^12*e^6 + 7770*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^
2*x + a*e^2*x + a*d*e))*a^6*c^3*d^10*e^8 + 3870*(sqrt(c*d*e)*x - sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^7*c^2*d^8*e^10 + 45*(sqrt(c*d*e)*x -
 sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^8*c*d^6*e^12 - 45*(sqrt(c*
d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^9*d^4*e^14 + 1280*
sqrt(c*d*e)*a^6*c^3*d^11*e^7 + 768*sqrt(c*d*e)*a^7*c^2*d^9*e^9 + 790*(sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c^5*d^13*e^
3 + 12570*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*
a^4*c^4*d^11*e^5 + 41820*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2
*x + a*d*e))^3*a^5*c^3*d^9*e^7 + 34420*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c
*d^2*x + a*e^2*x + a*d*e))^3*a^6*c^2*d^7*e^9 + 7470*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c*d^5*e^11 + 210*(sqrt(c*d*e
)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*d^3*e^13 + 3840*s
qrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2
*a^4*c^4*d^12*e^4 + 16640*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^6}\,dx = \int \frac{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{x^6}\,dx$$

input
```
int(((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/x^6,x)
```

output
```
int(((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 40.97 (sec) , antiderivative size = 1910, normalized size of antiderivative = 4.94

$$\int \frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^6}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^6,x)
```

output
```
( - 768*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*d**5*e**7 - 1056*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*a**6*d**4*e**8*x - 48*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a**6*d**3*e**9*x**2 + 60*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*d**2*e**10*x
**3 - 90*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*d*e**11*x**4 - 768*sqrt(d +
e*x)*sqrt(a*e + c*d*x)*a**5*c*d**7*e**5 - 1152*sqrt(d + e*x)*sqrt(a*e + c*
d*x)*a**5*c*d**6*e**6*x - 240*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d**5*
e**7*x**2 + 24*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d**4*e**8*x**3 - 30*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d**3*e**9*x**4 - 96*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**4*c**2*d**8*e**4*x - 80*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*a**4*c**2*d**7*e**5*x**2 + 208*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**2
*d**6*e**6*x**3 + 132*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**2*d**5*e**7*
x**4 + 112*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**9*e**3*x**2 + 104*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**8*e**4*x**3 - 308*sqrt(d + e*
x)*sqrt(a*e + c*d*x)*a**3*c**3*d**7*e**5*x**4 - 140*sqrt(d + e*x)*sqrt(a*e
 + c*d*x)*a**2*c**4*d**10*e**2*x**3 - 170*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a**2*c**4*d**9*e**3*x**4 + 210*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**1
1*e*x**4 - 45*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt
(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))
*a**6*e**12*x**5 + 75*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x
) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqr...
```

3.11 $\int \frac{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 330

$$\begin{aligned}
&\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx \\
&= \frac{1}{24}\left(\frac{a}{cd}-\frac{7d}{e^2}\right)x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}+\frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4e} \\
&\quad-\frac{(105c^3d^6-25ac^2d^4e^2-17a^2cd^2e^4-15a^3e^6-2cde(35c^2d^4-6acd^2e^2-5a^2e^4)\,x)\sqrt{ade+(cd^2+ae^2)}}{192c^3d^3e^4} \\
&\quad+\frac{(cd^2-ae^2)\,(35c^3d^6+15ac^2d^4e^2+9a^2cd^2e^4+5a^3e^6)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{64c^{7/2}d^{7/2}e^{9/2}}
\end{aligned}$$

output

```
1/24*(a/c/d-7*d/e^2)*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/4*x^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e-1/192*(105*c^3*d^6-25*a*c^2*d^4*e
^2-17*a^2*c*d^2*e^4-15*a^3*e^6-2*c*d*e*(-5*a^2*e^4-6*a*c*d^2*e^2+35*c^2*d^
4)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4+1/64*(-a*e^2+c*d
^2)*(5*a^3*e^6+9*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+35*c^3*d^6)*arctanh(c^(1/2
)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)
/d^(7/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 11.80 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.92

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$=\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\sqrt{c}\sqrt{d}\sqrt{e}(-15a^3e^6+a^2cde^4(-17d+10ex)+ac^2d^2e^2(-25d^2+12dex-8e^2x^2\right.}{19}$$

input
```
Integrate[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-15*a^3*e^6 + a
^2*c*d*e^4*(-17*d + 10*e*x) + a*c^2*d^2*e^2*(-25*d^2 + 12*d*e*x - 8*e^2*x^
2) + c^3*d^3*(105*d^3 - 70*d^2*e*x + 56*d*e^2*x^2 - 48*e^3*x^3))) + (3*Sqr
t[c*d]*Sqrt[c*d^2 - a*e^2]*(35*c^3*d^6 + 15*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^
4 + 5*a^3*e^6)*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c
*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2
- a*e^2)])))/(192*c^(7/2)*d^(7/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1215, 1236, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{d+ex}\,dx$$
$$\Big\downarrow \text{1215}$$
$$\int \frac{x^3(ae+cdx)}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$
$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{cdx^2(6ade+(7cd^2-ae^2)x)}{2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4cde}+\frac{x^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4e}$$

$\downarrow$ 27

$$\frac{x^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4e}-\frac{\int \frac{x^2(6ade+(7cd^2-ae^2)x)}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8e}$$

$\downarrow$ 1236

$$\frac{x^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4e}-\frac{\frac{\int -\frac{x\left(4ade\left(7cd^2-ae^2\right)+\left(35c^2d^4-6ace^2d^2-5a^2e^4\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3cde}+\frac{1}{3}x^2\left(\frac{7d}{e}-\frac{ae}{cd}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{8e}$$

$\downarrow$ 27

$$\frac{x^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4e}-\frac{\frac{1}{3}x^2\left(\frac{7d}{e}-\frac{ae}{cd}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}-\frac{\int \frac{x\left(4ade\left(7cd^2-ae^2\right)+\left(35c^2d^4-6ace^2d^2-5a^2e^4\right)x\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6cde}}{8e}$$

$\downarrow$ 1225

$$\frac{x^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4e}-\frac{\frac{1}{3}x^2\left(\frac{7d}{e}-\frac{ae}{cd}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}-\frac{\frac{3\left(cd^2-ae^2\right)\left(5a^3e^6+9a^2cd^2e^4+15ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8c^2d^2e^2}-\frac{(-\ldots}{\ldots}}{\ldots}}{8e}$$

$\downarrow$ 1092

$$\frac{x^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4e}-\frac{\frac{1}{3}x^2\left(\frac{7d}{e}-\frac{ae}{cd}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}-\frac{\frac{3\left(cd^2-ae^2\right)\left(5a^3e^6+9a^2cd^2e^4+15ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\ldots}{4c^2d^2e^2}\ldots}{\ldots}}{8e}$$

$\downarrow$ 219

$$\frac{x^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e} - \frac{\frac{1}{3}x^2\left(\frac{7d}{e}-\frac{ae}{cd}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2} - \frac{\frac{3\left(cd^2-ae^2\right)\left(5a^3e^6+9a^2cd^2e^4+15ac^2d^4e^2+35c^3d^6\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cd}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)}}\right.}{8c^{5/2}d^{5/2}e^{5/2}}}{}}{8e}$$

input
```
Int[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

output
```
(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e) - ((((7*d)/e - (a*
e)/(c*d))*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/3 - (-1/4*((105
*c^3*d^6 - 25*a*c^2*d^4*e^2 - 17*a^2*c*d^2*e^4 - 15*a^3*e^6 - 2*c*d*e*(35*
c^2*d^4 - 6*a*c*d^2*e^2 - 5*a^2*e^4)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])/(c^2*d^2*e^2) + (3*(c*d^2 - a*e^2)*(35*c^3*d^6 + 15*a*c^2*d^4*e
^2 + 9*a^2*c*d^2*e^4 + 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*S
qrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c
^(5/2)*d^(5/2)*e^(5/2)))/(6*c*d*e))/(8*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 954 vs. $2(302) = 604$.

Time = 2.47 (sec) , antiderivative size = 955, normalized size of antiderivative = 2.89

method	result
default	$\frac{d^2\left(\frac{(2cdxe+a\,e^2+c\,d^2)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}{4cde}+\frac{\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\ln\left(\frac{\frac{1}{2}a\,e^2+\frac{1}{2}c\,d^2+cdxe}{\sqrt{dec}}+\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}\right)}{8dec\sqrt{dec}}\right)}{e^3}$

input
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
d^2/e^3*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/
2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2
+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(
1/2))+1/e*(1/4*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-5/8*(a*e^2+
c*d^2)/d/e/c*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-1/2*(a*e^2
+c*d^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2
*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*
e*c)^(1/2)))-1/4*a/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*
e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/
2))/(d*e*c)^(1/2)))-d/e^2*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e
/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((
1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2))/(d*e*c)^(1/2)))-d^3/e^4*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1
/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.05

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$

$$= \left[-\frac{3\,(35\,c^4d^8-20\,ac^3d^6e^2-6\,a^2c^2d^4e^4-4\,a^3cd^2e^6-5\,a^4e^8)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a\right.}{}\right.$$

$$-\frac{3\,(35\,c^4d^8-20\,ac^3d^6e^2-6\,a^2c^2d^4e^4-4\,a^3cd^2e^6-5\,a^4e^8)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+c}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+}\right.}{}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/768*(3*(35*c^4*d^8 - 20*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^
2*e^6 - 5*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2
*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x
+ c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(48*c^4*d^
4*e^4*x^3 - 105*c^4*d^7*e + 25*a*c^3*d^5*e^3 + 17*a^2*c^2*d^3*e^5 + 15*a^3
*c*d*e^7 - 8*(7*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^2 + 2*(35*c^4*d^6*e^2 - 6*a
*c^3*d^4*e^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x))/(c^4*d^4*e^5), -1/384*(3*(35*c^4*d^8 - 20*a*c^3*d^6*e^2 - 6*a^2*c^
2*d^4*e^4 - 4*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e
)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(48*c^4
*d^4*e^4*x^3 - 105*c^4*d^7*e + 25*a*c^3*d^5*e^3 + 17*a^2*c^2*d^3*e^5 + 15*
a^3*c*d*e^7 - 8*(7*c^4*d^5*e^3 - a*c^3*d^3*e^5)*x^2 + 2*(35*c^4*d^6*e^2 -
6*a*c^3*d^4*e^4 - 5*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(c^4*d^4*e^5)]
```

Sympy [F]

$$\int \frac{x^3\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{d + ex}\,dx = \int \frac{x^3\sqrt{(d + ex)\,(ae + cdx)}}{d + ex}\,dx$$

input
```
integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(x**3*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.92

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$=\frac{1}{192}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\,x\left(\frac{6\,x}{e}-\frac{7\,c^3d^4e^2-ac^2d^2e^4}{c^3d^3e^4}\right)+\frac{35\,c^3d^5e-6\,ac^2d^3e^3-5\,a^2cde^5}{c^3d^3e^4}\right.\right.$$
$$-\frac{(35\,c^4d^8-20\,ac^3d^6e^2-6\,a^2c^2d^4e^4-4\,a^3cd^2e^6-5\,a^4e^8)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2}\right.\right.\right.}{128\sqrt{cde}c^3d^3e^4}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm
="giac")
```

output
```
1/192*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*x*(6*x/e - (7*c^3*
d^4*e^2 - a*c^2*d^2*e^4)/(c^3*d^3*e^4)) + (35*c^3*d^5*e - 6*a*c^2*d^3*e^3
- 5*a^2*c*d*e^5)/(c^3*d^3*e^4))*x - (105*c^3*d^6 - 25*a*c^2*d^4*e^2 - 17*a
^2*c*d^2*e^4 - 15*a^3*e^6)/(c^3*d^3*e^4)) - 1/128*(35*c^4*d^8 - 20*a*c^3*d
^6*e^2 - 6*a^2*c^2*d^4*e^4 - 4*a^3*c*d^2*e^6 - 5*a^4*e^8)*log(abs(-c*d^2 -
 a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))))/(sqrt(c*d*e)*c^3*d^3*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{x^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

input
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.75

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$= \frac{15\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^3cd\,e^7+17\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c^2d^3e^5-10\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c^2d^2e^6x+25}{}$$

input
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

output
```
(15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d*e**7 + 17*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a**2*c**2*d**3*e**5 - 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2
*c**2*d**2*e**6*x + 25*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**5*e**3 -
12*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**4*e**4*x + 8*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a*c**3*d**3*e**5*x**2 - 105*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*c**4*d**7*e + 70*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**6*e**2*x - 56*s
qrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**5*e**3*x**2 + 48*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*c**4*d**4*e**4*x**3 - 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e
)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2)
)*a**4*e**8 - 12*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) +
sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c*d**2*e**6 - 1
8*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)
*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**2*d**4*e**4 - 60*sqrt(e)*sq
rt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*
x))/sqrt(a*e**2 - c*d**2))*a*c**3*d**6*e**2 + 105*sqrt(e)*sqrt(d)*sqrt(c)*
log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**
2 - c*d**2))*c**4*d**8)/(192*c**4*d**4*e**5)
```

3.12 $\int \frac{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 236

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3e} + \frac{((5cd^2-3ae^2)\,(3cd^2+ae^2)-2cde(5cd^2-ae^2)\,x)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24c^2d^2e^3} - \frac{(cd^2-ae^2)\,(5c^2d^4+2acd^2e^2+a^2e^4)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{7/2}}$$

output
```
1/3*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e+1/24*((-3*a*e^2+5*c*d^2)
*(a*e^2+3*c*d^2)-2*c*d*e*(-a*e^2+5*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/c^2/d^2/e^3-1/8*(-a*e^2+c*d^2)*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4
)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 10.90 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.04

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$= \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(-3a^2e^4+2acde^2(-2d+ex)+c^2d^2(15d^2-10dex+8e^2x^2))-\frac{3\sqrt{cd}\sqrt{c}}{}\right.}{24c^{5/2}d^{5/2}e^{7/2}}$$

input
```
Integrate[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-3*a^2*e^4 + 2*a*
c*d*e^2*(-2*d + e*x) + c^2*d^2*(15*d^2 - 10*d*e*x + 8*e^2*x^2)) - (3*Sqrt[
c*d]*Sqrt[c*d^2 - a*e^2]*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcSinh[(Sq
rt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])
/(Sqrt[a*e + c*d*x]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)])))/(24*c^(5/2)*d
^(5/2)*e^(7/2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1215, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{d+ex}\,dx$$
$$\Big\downarrow \ 1215$$
$$\int \frac{x^2(ae+cdx)}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$
$$\Big\downarrow \ 1236$$

$$\frac{\int -\frac{cdx(4ade+(5cd^2-ae^2)x)}{2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3cde}+\frac{x^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3e}$$

$\Big\downarrow$ 27

$$\frac{x^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3e}-\frac{\int \frac{x(4ade+(5cd^2-ae^2)x)}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6e}$$

$\Big\downarrow$ 1225

$$\frac{x^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3e}-\frac{\frac{3(cd^2-ae^2)(a^2e^4+2acd^2e^2+5c^2d^4)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8c^2d^2e^2}-\frac{((5cd^2-3ae^2)(ae^2+3cd^2)-2cdex(5cd^2-ae^2))\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4c^2d^2e^2}}{6e}$$

$\Big\downarrow$ 1092

$$\frac{x^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3e}-\frac{\frac{3(cd^2-ae^2)(a^2e^4+2acd^2e^2+5c^2d^4)\int \frac{1}{4cde-\frac{(cd^2+2cexd+ae^2)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{4c^2d^2e^2}-\frac{((5cd^2-3ae^2)(ae^2+3cd^2)-2cdex(5cd^2-ae^2))\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4c^2d^2e^2}}{6e}$$

$\Big\downarrow$ 219

$$\frac{x^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3e}-\frac{\frac{3(cd^2-ae^2)(a^2e^4+2acd^2e^2+5c^2d^4)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{5/2}}-\frac{((5cd^2-3ae^2)(ae^2+3cd^2)-2cdex(5cd^2-ae^2))\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4c^2d^2e^2}}{6e}$$

input

```
Int[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

output

```
(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*e) - (-1/4*(((5*c*d^2
 - 3*a*e^2)*(3*c*d^2 + a*e^2) - 2*c*d*e*(5*c*d^2 - a*e^2)*x)*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*e^2) + (3*(c*d^2 - a*e^2)*(5*c^2*
d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt
[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(5
/2)*d^(5/2)*e^(5/2)))/(6*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(212) = 424$.

Time = 2.50 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.17

method	result
default	$\frac{\frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{3dec}-\frac{\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{4cde}+\frac{\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\ln\left(\frac{\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}}{\sqrt{dec}}\right.}{8dec\sqrt{d}}\right.}{2dec}}{e}$

input
```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-1/2*(a*e^2+c*d^2)/d
/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/
c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*
d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2
)))+d^2/e^3*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^
2)*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(
a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))-d/e^2*(1/4*(2*c*d*e*x+a*e^2+c*
d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e
^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.27

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$= \left[-\frac{3\,(5\,c^3d^6-3\,ac^2d^4e^2-a^2cd^2e^4-a^3e^6)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\,\sqrt{cdex^2+}\right.}{}\right.$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/96*(3*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d
*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e
) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 + 15*c^3*d^5*e - 4
*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5 - 2*(5*c^3*d^4*e^2 - a*c^2*d^2*e^4)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4), 1/48*(3*(5*c^3*d^6
 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-
c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(8
*c^3*d^3*e^3*x^2 + 15*c^3*d^5*e - 4*a*c^2*d^3*e^3 - 3*a^2*c*d*e^5 - 2*(5*c
^3*d^4*e^2 - a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
)/(c^3*d^3*e^4)]
```

Sympy [F]

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{x^2\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\,dx$$

input
```
integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(x**2*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.95

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$=\frac{1}{24}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\,x\left(\frac{4\,x}{e}-\frac{5\,c^2d^3e-acde^3}{c^2d^2e^3}\right)+\frac{15\,c^2d^4-4\,acd^2e^2-3\,a^2e^4}{c^2d^2e^3}\right)$$
$$+\frac{(5\,c^3d^6-3\,ac^2d^4e^2-a^2cd^2e^4-a^3e^6)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+\right.\right.\right.}{16\,\sqrt{cde}c^2d^2e^3}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm
="giac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*x*(4*x/e - (5*c^2*d^3*
e - a*c*d*e^3)/(c^2*d^2*e^3)) + (15*c^2*d^4 - 4*a*c*d^2*e^2 - 3*a^2*e^4)/(
c^2*d^2*e^3)) + 1/16*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^
6)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{x^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

input
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.69

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$= \frac{-3\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2cd\,e^5-4\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^3e^3+2\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^2e^4x+15\sqrt{e}}{}$$

input
```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

output
```
( - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d*e**5 - 4*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a*c**2*d**3*e**3 + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*
d**2*e**4*x + 15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e - 10*sqrt(d +
 e*x)*sqrt(a*e + c*d*x)*c**3*d**4*e**2*x + 8*sqrt(d + e*x)*sqrt(a*e + c*d*
x)*c**3*d**3*e**3*x**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e +
 c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**6
+ 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(
c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**4 + 9*sqrt(e)*sqrt
(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x)
)/sqrt(a*e**2 - c*d**2))*a*c**2*d**4*e**2 - 15*sqrt(e)*sqrt(d)*sqrt(c)*log
((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 -
 c*d**2))*c**3*d**6)/(24*c**3*d**3*e**4)
```

3.13 $\int \frac{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 156

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx = -\frac{(3cd^2-ae^2-2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4cde^2} + \frac{(cd^2-ae^2)(3cd^2+ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^{3/2}d^{3/2}e^{5/2}}$$

output
```
-1/4*(-2*c*d*e*x-a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/
d/e^2+1/4*(-a*e^2+c*d^2)*(a*e^2+3*c*d^2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e
^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.16

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$= \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(ae^2+cd(-3d+2ex))+\frac{(-6c^2d^4+4acd^2e^2+2a^2e^4)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\left(\sqrt{d-\frac{ae^2}{cd}}-\sqrt{d+\cdots}\right.}\right.}{\sqrt{ae+cdx}\sqrt{d+ex}}\right.}{4c^{3/2}d^{3/2}e^{5/2}}$$

input
```
Integrate[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e^2 + c*d*(-3*d
 + 2*e*x)) + ((-6*c^2*d^4 + 4*a*c*d^2*e^2 + 2*a^2*e^4)*ArcTanh[(Sqrt[e]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*(Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d + e*x]
))])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(4*c^(3/2)*d^(3/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1215, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{d+ex}\,dx$$
$$\Big\downarrow \; 1215$$
$$\int \frac{x(ae+cdx)}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$
$$\Big\downarrow \; 1225$$

$$-\frac{\left(\frac{a^2e^2}{c}+2ad^2-\frac{3cd^4}{e^2}\right)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-ae^2+3cd^2-2cdex\right)}{4cde^2}$$

$$\Big\downarrow \text{1092}$$

$$-\frac{\left(\frac{a^2e^2}{c}+2ad^2-\frac{3cd^4}{e^2}\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{4d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-ae^2+3cd^2-2cdex\right)}{4cde^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(\frac{a^2e^2}{c}+2ad^2-\frac{3cd^4}{e^2}\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8\sqrt{c}d^{3/2}\sqrt{e}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-ae^2+3cd^2-2cdex\right)}{4cde^2}$$

input
```
Int[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]
```

output
```
-1/4*((3*c*d^2 - a*e^2 - 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(c*d*e^2) - ((2*a*d^2 - (3*c*d^4)/e^2 + (a^2*e^2)/c)*ArcTanh[(c*d^2
 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])])/(8*Sqrt[c]*d^(3/2)*Sqrt[e])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(136) = 272$.

Time = 2.25 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.87

method	result
default	$\frac{\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{4cde}+\frac{\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\ln\left(\frac{\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}+cdxe}{\sqrt{dec}}+\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}\right)}{8dec\sqrt{dec}}}{e}-\frac{d\left(\right.}{}$

input
```
int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d),x,method=_RETURNVERB
OSE)
```

output
```
1/e*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c
/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d
*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)
)-d/e^2*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*l
n((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^
2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.68

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$= \Bigg[-\frac{(3\,c^2d^4-2\,acd^2e^2-a^2e^4)\sqrt{cde}\log\Big(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\,\sqrt{cdex^2+ade+(cd^2+}}{}$$
$$-\frac{(3\,c^2d^4-2\,acd^2e^2-a^2e^4)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-cde}}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+acde^3)x)}\right)-2\,(2\,c^2d^2e^2x-3}{8\,c^2d^2e^3}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="
fricas")
```

output
```
[-1/16*((3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^
2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a
*c*d*e^3)*x) - 4*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e + a*c*d*e^3)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3), -1/8*((3*c^2*d^4 - 2*a*c*d^
2*e^2 - a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c
*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(2*c^2*d^2*e^2*x - 3*c^2*d^3*e
+ a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3)]
```

Sympy [F]

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{x\sqrt{(d+ex)\,(ae+cdx)}}{d+ex}\,dx$$

input
```
integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(x*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.05

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$= \frac{1}{4}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(\frac{2\,x}{e}-\frac{3\,cd^2-ae^2}{cde^2}\right)$$
$$-\frac{(3\,c^2d^4-2\,acd^2e^2-a^2e^4)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{8\sqrt{cde}cde^2}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="
giac")
```

output
```
1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*x/e - (3*c*d^2 - a*e^2)
/(c*d*e^2)) - 1/8*(3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*log(abs(-c*d^2 - a
*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
 a*d*e))))/(sqrt(c*d*e)*c*d*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx=\int \frac{x\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{d+e\,x}\,dx$$

input
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x),x)
```

output
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.62

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx$$
$$=\frac{\sqrt{ex+d}\,\sqrt{cdx+ae}\,acd\,e^3-3\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^3e+2\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^2e^2x-\sqrt{e}\,\sqrt{d}\,\sqrt{c}\log}{}$$

input
```
int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

output
```
(sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d*e**3 - 3*sqrt(d + e*x)*sqrt(a*e + c
*d*x)*c**2*d**3*e + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**2*e**2*x - s
qrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sq
rt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**4 - 2*sqrt(e)*sqrt(d)*sqrt(c)*
log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**
2 - c*d**2))*a*c*d**2*e**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a
*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d
**4)/(4*c**2*d**2*e**3)
```

3.14 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 114

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e} - \frac{(cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}}$$

output
```
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e-(-a*e^2+c*d^2)*arctanh(c^(1/2)*d
^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(1/2)/d^
(1/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{e}+\frac{(-cd^2+ae^2)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{e^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[e] + ((-(c*d^2) + a*e^2)*ArcTanh[(Sqr
t[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[c]*Sqrt[d]
*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/e^(3/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1131, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}dx$$

$$\Big\downarrow \ 1131$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2e}$$

$$\Big\downarrow \ 1092$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{e}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x),x]
```

output
```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/e - ((c*d^2 - a*e^2)*ArcTanh[(
c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\ln\left(\frac{\frac{a\,e^{2}}{2}-\frac{c\,d^{2}}{2}+dec\left(x+\frac{d}{e}\right)}{\sqrt{dec}}+\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{dec}}}{e}$	131

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d),x,method=_RETURNVERBOS
E)
```

output
```
1/e*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1
/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*
d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.96

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{d + ex}\,dx$$
$$= \left[\frac{4\,\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}cde - (cd^2 - ae^2)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 + a^2e^4 + 4\,\sqrt{\right.}{4\,cde^2} \right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="fr
icas")
```

output
```
[1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e - (c*d^2 - a*e^2
)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 +
4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*
sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c*d*e^2), 1/2*(2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e + (c*d^2 - a*e^2)*sqrt(-c*d*e)*arcta
n(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e
^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*
x)))/(c*d*e^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{d + ex}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{d + ex}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex} dx = \frac{(cd^2 - ae^2)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{2\sqrt{cde}e} + \frac{\sqrt{cdex^2 + cd^2x + ae^2x + ade}}{e}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="gi
ac")
```

output
```
1/2*(c*d^2 - a*e^2)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*e) + sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{d + ex}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{d + e\,x}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{d + ex}\,dx$$
$$= \frac{\sqrt{ex+d}\,\sqrt{cdx+ae}\,cde + \sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a\,e^2 - \sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{}}{\sqrt{a\,e^2-c\,d^2}}\right)}{cd\,e^2}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)
```

output
```
(sqrt(d + e*x)*sqrt(a*e + c*d*x)*c*d*e + sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt
(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**
2))*a*e**2 - sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**2)/(c*d*e**2)
```

3.15 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 137

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x(d+ex)}\,dx = \frac{2\sqrt{c}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{e}} - \frac{2\sqrt{a}\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{d}}$$

output
```
2*c^(1/2)*d^(1/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2))/e^(1/2)-2*a^(1/2)*e^(1/2)*arctanh(a^(1/2)*e^(1/2)
*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/d^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 476 vs. $2(137) = 274$.

Time = 2.24 (sec) , antiderivative size = 476, normalized size of antiderivative = 3.47

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x(d + ex)}\,dx =$$
$$-\frac{2\sqrt{ae + cdx}\sqrt{d + ex}\left(-\left(\left(-\sqrt{c}d + \sqrt{cd^2 - ae^2}\right)\sqrt{-2cd^2 + ae^2 - 2\sqrt{c}d\sqrt{cd^2 - ae^2}}\arctan\left(\frac{\sqrt{-2cd^2+}}{\sqrt{a}\sqrt{c}}\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)),x]
```

output
```
(-2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-((-(Sqrt[c]*d) + Sqrt[c*d^2 - a*e^2]
)*Sqrt[-2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]]*ArcTan[(Sqrt[-2
*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]]*Sqrt[a*e + c*d*x])/(Sqrt
[a]*Sqrt[c]*Sqrt[d]*Sqrt[e]*(Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d + e*x]))]) +
 (Sqrt[c]*d + Sqrt[c*d^2 - a*e^2])*Sqrt[-2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqr
t[c*d^2 - a*e^2]]*ArcTan[(Sqrt[-2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 -
 a*e^2]]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[c]*Sqrt[d]*Sqrt[e]*(Sqrt[d - (a*
e^2)/(c*d)] - Sqrt[d + e*x]))] + 2*Sqrt[a]*Sqrt[c]*d*e*ArcTanh[(Sqrt[e]*Sq
rt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*(Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d + e*x]
))]))/(Sqrt[a]*Sqrt[d]*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1215, 1268, 140, 27, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{x(d + ex)}\,dx$$
$$\Big\downarrow \text{1215}$$
$$\int \frac{ae + cdx}{x\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}dx$$

$$\Big\downarrow \text{1268}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{\sqrt{ae+cdx}}{x\sqrt{d+ex}}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{140}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(cd\int \frac{1}{\sqrt{ae+cdx}\sqrt{d+ex}}dx+\int \frac{ae}{x\sqrt{ae+cdx}\sqrt{d+ex}}dx\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(cd\int \frac{1}{\sqrt{ae+cdx}\sqrt{d+ex}}dx+ae\int \frac{1}{x\sqrt{ae+cdx}\sqrt{d+ex}}dx\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{66}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(ae\int \frac{1}{x\sqrt{ae+cdx}\sqrt{d+ex}}dx+2cd\int \frac{1}{cd-\frac{e(ae+cdx)}{d+ex}}d\frac{\sqrt{ae+cdx}}{\sqrt{d+ex}}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(2cd\int \frac{1}{cd-\frac{e(ae+cdx)}{d+ex}}d\frac{\sqrt{ae+cdx}}{\sqrt{d+ex}}+2ae\int \frac{1}{\frac{ae(d+ex)}{ae+cdx}-d}d\frac{\sqrt{d+ex}}{\sqrt{ae+cdx}}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{e}}-\frac{2\sqrt{a}\sqrt{e}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sqrt{d+ex}}{\sqrt{d}\sqrt{ae+cdx}}\right)}{\sqrt{d}}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*((2*Sqrt[c]*Sqrt[d]*ArcTanh[(Sqrt[e]*Sqrt
[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/Sqrt[e] - (2*Sqrt[a]*Sqrt
[e]*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/
Sqrt[d]))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(109) = 218.

Time = 2.34 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.25

method	result
default	$\frac{\sqrt{ade+(a e^2+c d^2)x+cd x^2 e}+\frac{\left(a e^2+c d^2\right)\ln\left(\frac{\frac{1}{2}a e^2+\frac{1}{2}c d^2+cdxe}{\sqrt{dec}}+\sqrt{ade+\left(a e^2+c d^2\right)x+cd x^2 e}\right)}{2\sqrt{dec}}-\frac{ade\ln\left(\frac{2ade+\left(a e^2+c d^2\right)x+2\sqrt{ade}\sqrt{}}{x}\right)}{\sqrt{ade}}}{d}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/x/(e*x+d),x,method=_RETURNVERB
OSE)
```

output
```
1/d*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e
^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)
^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-1/d*((d*e*c*(x+d/e)^2+
(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*a*e^2-1/2*c*d^2+d*e
*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(
d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 947, normalized size of antiderivative = 6.91

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x(d+ex)}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="
fricas")
```

output
```
[1/2*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4
 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 1/2*sqrt(a*e/d)*log((8
*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d)
 + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2), -sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/
(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + 1/2*sqrt(a*e/d)*l
og((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a
*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2), sqrt(-a*e/d)*arctan(1/2*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a
*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + 1/2*sqrt(c*
d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*
e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x), -sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)
/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + sqrt(-a*e/d)*arc
tan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(-a*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)...
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x(d + ex)}\,dx = \int \frac{\sqrt{(d + ex)(ae + cdx)}}{x(d + ex)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x/(e*x+d),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x(d + ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x(d + ex)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x(d+ex)}\,dx=\int \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{x\,\left(d+e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x(d+ex)}\,dx$$
$$=\frac{\sqrt{e}\,\sqrt{d}\left(\sqrt{a}\log\left(\sqrt{e}\,\sqrt{cdx+ae}-\sqrt{2\sqrt{c}\,\sqrt{a}\,de+a\,e^2+c\,d^2}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}\right)e+\sqrt{a}\log\left(\sqrt{e}\,\sqrt{cdx+\right.\right.}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d),x)
```

output
```
(sqrt(e)*sqrt(d)*(sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*s
qrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*e + sqrt(a)
*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d
**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*e - sqrt(a)*log(2*sqrt(e)*sqrt(d)*sq
rt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)
*e + 2*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e
*x))/sqrt(a*e**2 - c*d**2))*d))/(d*e)
```

3.16 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 118

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2(d+ex)}\,dx=-\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{dx}-\frac{(cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{a}d^{3/2}\sqrt{e}}$$

output
```
-(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x-(-a*e^2+c*d^2)*arctanh(a^(1/2
)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(1/2)
/d^(3/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 10.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2(d+ex)}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{d}}{x}+\frac{(-cd^2+ae^2)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{a}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{d^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[d]/x) + ((-(c*d^2) + a*e^2)*ArcTanh
[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a]*Sq
rt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/d^(3/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1215, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x^2(d+ex)}dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{ae+cdx}{x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1228}$$

$$\frac{\left(cd^2-ae^2\right)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{\left(cd^2-ae^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{a}d^{3/2}\sqrt{e}}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)),x]
```

output
```
-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x)) - ((c*d^2 - a*e^2)*Ar
cTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[a]*d^(3/2)*Sqrt[e])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(102) = 204$.

Time = 2.47 (sec) , antiderivative size = 708, normalized size of antiderivative = 6.00

method	result
default	$-\frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{adex}+\frac{\left(a\,e^{2}+c\,d^{2}\right)\left(\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}+\frac{\left(a\,e^{2}+c\,d^{2}\right)\ln\left(\frac{\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}+cdxe}{\sqrt{dec}}+\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd}\right.}{2\sqrt{dec}}\right.}{2ade}$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/x^2/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output

```
1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)/
a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a
*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*
e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*
x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^
2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2
)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+e/d^2*((d*e*c*(
x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*a*e^2-1/2*
c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(1/2))/(d*e*c)^(1/2))-e/d^2*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*
(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2
+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+
(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x
))
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.01

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2(d+ex)}\,dx$$
$$=\left[-\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}ade+(cd^2-ae^2)\sqrt{ade}x\log\left(\frac{8a^2d^2e^2+(c^2d^4+6acd^2e^2+a^2e^4)x^2+4\sqrt{cdex^2+ade}}{}\right.}{4ad^2ex}\right.$$
$$-\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}ade-(cd^2-ae^2)\sqrt{-ade}x\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2ade+(cd^2+ae^2)}{2(acd^2e^2x^2+a^2d^2e^2+(acd^3e+a^2de^3)}\right.}{2ad^2ex}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e + (c*d^2 - a*e^
2)*sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*
x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2))/(a*d^2*e*x), -1/2
*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e - (c*d^2 - a*e^2)*sq
rt(-a*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c
*d^3*e + a^2*d*e^3)*x)))/(a*d^2*e*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2(d+ex)}\,dx=\int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{x^2\,(d+ex)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**2/(e*x+d),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**2*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^2(d + ex)}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)x^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(102) = 204$.

Time = 0.15 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^2(d + ex)}\,dx = \frac{(cd^2 - ae^2)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{\sqrt{-ade}d} - \frac{\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)cd^2 + \left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)ae^2 + 2\sqrt{cd}}{\left(ade - \left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)^2\right)d}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm
="giac")
```

output
```
(c*d^2 - a*e^2)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*d) - ((sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))*c*d^2 + (sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))*a*e^2 + 2*sqrt(c*d*e)*a*d*e)/((a*d*e - (sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^2(d + ex)}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{x^2\,(d + e\,x)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^2*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^2*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.15

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^2(d + ex)}\,dx$$
$$= \frac{-2\sqrt{ex + d}\,\sqrt{cdx + ae}\,ade - \sqrt{e}\,\sqrt{d}\,\sqrt{a}\log\left(\sqrt{e}\,\sqrt{cdx + ae} - \sqrt{2\sqrt{c}\,\sqrt{a}\,de + a\,e^2 + c\,d^2} + \sqrt{d}\,\sqrt{c}\,\sqrt{ex}\right.}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x)
```

output
```
( - 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*d*e - sqrt(e)*sqrt(d)*sqrt(a)*log(
sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*e**2*x + sqrt(e)*sqrt(d)*sqrt(a)*log(sq
rt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) +
sqrt(d)*sqrt(c)*sqrt(d + e*x))*c*d**2*x - sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt
(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))*a*e**2*x + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e
)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))*c*d**2*x + sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e
)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e
+ 2*c*d*e*x)*a*e**2*x - sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d)*sqrt
(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*c
*d**2*x)/(2*a*d**2*e*x)
```

3.17 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 185

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3(d+ex)}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2dx^2} - \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4x} + \frac{(cd^2-ae^2)\,(cd^2+3ae^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4a^{3/2}d^{5/2}e^{3/2}}$$

output

```
-1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x^2-1/4*(c/a/e-3*e/d^2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x+1/4*(-a*e^2+c*d^2)*(3*a*e^2+c*d^2)*
arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/a^(3/2)/d^(5/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 10.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3(d+ex)}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-cd^2x+ae(-2d+3ex))}{x^2}+\frac{(c^2d^4+2acd^2e^2-3a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{4a^{3/2}d^{5/2}e^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-(c*d^2*x) + a*e
*(-2*d + 3*e*x)))/x^2 + ((c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTanh[(Sq
rt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d
*x]*Sqrt[d + e*x])))/(4*a^(3/2)*d^(5/2)*e^(3/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1215, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \; 1215$$

$$\int \frac{ae+cdx}{x^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \; 1237$$

$$-\frac{\int -\frac{ae(cd^2-2cexd-3ae^2)}{2x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2ade}-\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2dx^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{cd^2-2cexd-3ae^2}{x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4d}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2dx^2}$$

$\Big\downarrow$ 1228

$$\frac{-\frac{\left(\frac{c^2d^4}{a}-3ae^4+2cd^2e^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2de}-\frac{\left(\frac{cd}{ae}-\frac{3e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{4d}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2dx^2}$$

$\Big\downarrow$ 1154

$$\frac{\frac{\left(\frac{c^2d^4}{a}-3ae^4+2cd^2e^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{de}-\frac{\left(\frac{cd}{ae}-\frac{3e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{4d}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2dx^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(\frac{c^2d^4}{a}-3ae^4+2cd^2e^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}d^{3/2}e^{3/2}}-\frac{\left(\frac{cd}{ae}-\frac{3e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{4d}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2dx^2}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)),x]
```

output
```
-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x^2) + (-((((c*d)/(a*e
) - (3*e)/d)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x) + (((c^2*d^4)
/a + 2*c*d^2*e^2 - 3*a*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[
a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[
a]*d^(3/2)*e^(3/2)))/(4*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1352 vs. $2(161) = 322$.

Time = 2.60 (sec) , antiderivative size = 1353, normalized size of antiderivative = 7.31

method	result	size
default	Expression too large to display	1353

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/x^3/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(a*e^2+c*d
^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c
*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln(
(1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*c/a*(1/4*(2*
c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*
a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c
)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+1/2*c/a*(
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/
2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d
*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))+e^2/d^3*((a*d*e+(a*e^2+c*d
^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/
(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e
/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2))/x))-e/d^2*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(
1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d
*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*...
```

Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.39

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^3(d + ex)}\,dx$$

$$= \left[-\frac{(c^2d^4 + 2\,acd^2e^2 - 3\,a^2e^4)\sqrt{ade}x^2 \log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2-4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,ade+(cd^2+ae}{x^2}\right.}{16\,a^2d^3e^2x^2}\right.$$

$$-\frac{(c^2d^4 + 2\,acd^2e^2 - 3\,a^2e^4)\sqrt{-ade}x^2 \arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,ade+(cd^2+ae^2)x)\sqrt{-ade}}{2\,(acd^2e^2x^2+a^2d^2e^2+(acd^3e+a^2de^3)x)}\right) + 2\,(2\,a^2d^2e^2}{8\,a^2d^3e^2x^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/16*((c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2*d
^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*
d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*a^2*d^2*e^2 + (a*c*d^3*e - 3*a^2*d*e^3)*
x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^2), -1/8*((
c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d
*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(2*a^
2*d^2*e^2 + (a*c*d^3*e - 3*a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x))/(a^2*d^3*e^2*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^3(d + ex)}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{x^3\,(d + ex)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**3/(e*x+d),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**3*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^3(d + ex)}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)x^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(161) = 322.

Time = 0.14 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^3(d + ex)}\,dx$$
$$= -\frac{(c^2d^4 + 2\,acd^2e^2 - 3\,a^2e^4)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{4\,\sqrt{-ade}ad^2e}$$
$$+ \frac{\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)ac^2d^5e + 10\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^2cd^3e}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm
="giac")
```

output
```
-1/4*(c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*arctan(-(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a*d^2*e
) + 1/4*((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a*c
^2*d^5*e + 10*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)
)*a^2*c*d^3*e^3 + 5*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))*a^3*d*e^5 + 8*sqrt(c*d*e)*a^3*d^2*e^4 + (sqrt(c*d*e)*x - sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*c^2*d^4 + 2*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a*c*d^2*e^2 - 3*(sqrt(c*d*e)*x - s
qrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*e^4 + 8*sqrt(c*d*e)*(sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a*c*d^3*e)/((a
*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^2*
a*d^2*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^3(d + ex)}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{x^3\,(d + e\,x)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^3*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^3*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 966, normalized size of antiderivative = 5.22

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^3(d + ex)}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x)
```

output

```
( - 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*d**2*e**4 + 6*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**3*d*e**5*x - 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d*
*4*e**2 + 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d**3*e**3*x - 2*sqrt(d
+ e*x)*sqrt(a*e + c*d*x)*a*c**2*d**5*e*x + 3*sqrt(e)*sqrt(d)*sqrt(a)*log(s
qrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*e**6*x**2 + sqrt(e)*sqrt(d)*sqrt(a)*l
og(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**
2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**2*e**4*x**2 - 3*sqrt(e)*sqrt
(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a
*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**4*e**2*x**2 - s
qrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt
(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**3*d**6*x**2
 + 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c
)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*e**
6*x**2 + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sq
rt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2
*c*d**2*e**4*x**2 - 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x
) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))*a*c**2*d**4*e**2*x**2 - sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(
a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*...
```

3.18 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 269

$$\begin{aligned}
\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4(d+ex)}\,dx
&= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3dx^3} - \frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12x^2} \\
&\quad + \frac{(3cd^2-5ae^2)\,(cd^2+3ae^2)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24a^2d^3e^2x} \\
&\quad - \frac{(cd^2-ae^2)\,(c^2d^4+2acd^2e^2+5a^2e^4)\,\mathrm{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8a^{5/2}d^{7/2}e^{5/2}}
\end{aligned}$$

output

```
-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x^3-1/12*(c/a/e-5*e/d^2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2+1/24*(-5*a*e^2+3*c*d^2)*(3*a*e^2
+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/x-1/8*(-a*e^2+
c*d^2)*(5*a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d
^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 10.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4(d+ex)}\,dx$$
$$=\frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(3c^2d^4x^2-2acd^2ex(d-2ex)+a^2e^2(-8d^2+10dex-15e^2x^2))}{x^3}-\frac{3(c^3d^6+ac^2d^4e^2+3a^2cd^2e^4-5a^3e^6)}{\sqrt{ae+cdx}\sqrt{d+}}\right.}{24a^{5/2}d^{7/2}e^{5/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^2*d^4*x^2 -
2*a*c*d^2*e*x*(d - 2*e*x) + a^2*e^2*(-8*d^2 + 10*d*e*x - 15*e^2*x^2)))/x^3
 - (3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTanh[(Sqr
t[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*
x]*Sqrt[d + e*x])))/(24*a^(5/2)*d^(7/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1215, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{x^4(d+ex)}\,dx$$
$$\Big\downarrow \text{1215}$$
$$\int \frac{ae+cdx}{x^4\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$
$$\Big\downarrow \text{1237}$$
$$-\frac{\int -\frac{ae(cd^2-4cexd-5ae^2)}{2x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3ade}-\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3dx^3}$$

$\downarrow$ 27

$$\frac{\int \frac{cd^2-4cexd-5ae^2}{x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3dx^3}$$

$\downarrow$ 1237

$$\frac{-\frac{\int \frac{\left(3cd^2-5ae^2\right)\left(cd^2+3ae^2\right)+2cde\left(cd^2-5ae^2\right)x}{2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(\frac{cd}{ae}-\frac{5e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2x^2}}{6d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3dx^3}$$

$\downarrow$ 27

$$\frac{-\frac{\int \frac{\left(3cd^2-5ae^2\right)\left(cd^2+3ae^2\right)+2cde\left(cd^2-5ae^2\right)x}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4ade}-\frac{\left(\frac{cd}{ae}-\frac{5e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2x^2}}{6d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3dx^3}$$

$\downarrow$ 1228

$$\frac{-\frac{-\frac{3\left(cd^2-ae^2\right)\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{adex}}{4ade}-\frac{\left(\frac{cd}{ae}-\frac{5e}{d}\right)\sqrt{x(ae^2-}}{2x}}{6d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3dx^3}$$

$\downarrow$ 1154

$$\frac{-\frac{\frac{3\left(cd^2-ae^2\right)\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}-\frac{\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{adex}}{4ade}}{6d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3dx^3}$$

$\downarrow$ 219

$$\frac{-\frac{-\frac{3\left(cd^2-ae^2\right)\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\operatorname{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{adex}}{4ade}-\left(\frac{ca}{ae}\cdots\right.}{6d}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3dx^3}$$

input

```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)),x]
```

output

```
-1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x^3) + (-1/2*(((c*d)/(
a*e) - (5*e)/d)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^2 - (-(((3*
c*d^2 - 5*a*e^2)*(c*d^2 + 3*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(a*d*e*x)) + (3*(c*d^2 - a*e^2)*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4
)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(3/2)*d^(3/2)*e^(3/2)))/(4*a*
d*e))/(6*d)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2058 vs. $2(241) = 482$.

Time = 3.42 (sec) , antiderivative size = 2059, normalized size of antiderivative = 7.65

method	result	size
default	Expression too large to display	2059

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/x^4/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c*d
^2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(a*e
^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a
*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^
2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*c/a*(1
/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1
/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/
(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+1/2
*c/a*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*
e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/
2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e
)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))))+e^2/d^3*(-1/a/d/e/x
*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+
c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1
/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2...
```

Fricas [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^4(d + ex)}\,dx$$
$$= \left[-\frac{3\left(c^3d^6 + ac^2d^4e^2 + 3\,a^2cd^2e^4 - 5\,a^3e^6\right)\sqrt{ade}x^3 \log\left(\frac{8\,a^2d^2e^2 + (c^2d^4 + 6\,acd^2e^2 + a^2e^4)x^2 + 4\,\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{x^2}\right.}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/96*(3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(a*d
*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^3*e^3 - (3*a*c
^2*d^5*e + 4*a^2*c*d^3*e^3 - 15*a^3*d*e^5)*x^2 + 2*(a^2*c*d^4*e^2 - 5*a^3*
d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^3)
, 1/48*(3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(-a*
d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3
*e + a^2*d*e^3)*x)) - 2*(8*a^3*d^3*e^3 - (3*a*c^2*d^5*e + 4*a^2*c*d^3*e^3
- 15*a^3*d*e^5)*x^2 + 2*(a^2*c*d^4*e^2 - 5*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^4(d + ex)}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{x^4\,(d + ex)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**4/(e*x+d),x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**4*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^4(d + ex)}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)x^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. 2(241) = 482.

Time = 0.16 (sec) , antiderivative size = 912, normalized size of antiderivative = 3.39

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm
="giac")
```

output
```
1/8*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*arctan(-(sqrt(
c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqr
t(-a*d*e)*a^2*d^3*e^2) - 1/24*(3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
 + a*e^2*x + a*d*e))*a^2*c^3*d^8*e^2 + 51*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))*a^3*c^2*d^6*e^4 + 105*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c*d^4*e^6 + 33*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*d^2*e^8 + 16*sqrt(c*d*e
)*a^4*c*d^5*e^5 + 48*sqrt(c*d*e)*a^5*d^3*e^7 + 8*(sqrt(c*d*e)*x - sqrt(c*d
*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a*c^3*d^7*e + 72*(sqrt(c*d*e)*x - s
qrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c^2*d^5*e^3 + 24*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c*d^3*e^5 - 4
0*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*d*e^
7 + 48*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))^2*a^2*c^2*d^6*e^2 + 144*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))^2*a^3*c*d^4*e^4 - 3*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*c^3*d^6 - 3*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a*c^2*d^4*e^2 - 9*(sqrt(c*d*e)*
x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^2*c*d^2*e^4 + 15*(sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^3*e^6)/((a*d
*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^3...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^4(d + ex)} dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{x^4\,(d + e\,x)} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^4*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^4*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 1316, normalized size of antiderivative = 4.89

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^4(d + ex)} dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x)
```

output
```
( - 16*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*d**3*e**5 + 20*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**4*d**2*e**6*x - 30*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**
4*d*e**7*x**2 - 16*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**5*e**3 + 16*s
qrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**4*e**4*x - 22*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a**3*c*d**3*e**5*x**2 - 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**
2*c**2*d**6*e**2*x + 14*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**5*e**
3*x**2 + 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**7*e*x**2 - 15*sqrt(e)
*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*
e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**4*e**8*x**3 - 6*s
qrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt
(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**2*e*
*6*x**3 + 12*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(
2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*
a**2*c**2*d**4*e**4*x**3 + 6*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e
+ c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)
*sqrt(d + e*x))*a*c**3*d**6*e**2*x**3 + 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt
(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))*c**4*d**8*x**3 - 15*sqrt(e)*sqrt(d)*sqrt(a)*l
og(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**
2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**4*e**8*x**3 - 6*sqrt(e)*sqrt(d)*...
```

3.19 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 372

$$\begin{aligned}\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^5(d+ex)}\,dx &= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4dx^4}-\frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24x^3}\\ &\quad+\frac{(5c^2d^4+6acd^2e^2-35a^2e^4)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{96a^2d^3e^2x^2}\\ &\quad-\frac{(15c^3d^6+17ac^2d^4e^2+25a^2cd^2e^4-105a^3e^6)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{192a^3d^4e^3x}\\ &\quad+\frac{(cd^2-ae^2)\,(5c^3d^6+9ac^2d^4e^2+15a^2cd^2e^4+35a^3e^6)\,\mathrm{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{64a^{7/2}d^{9/2}e^{7/2}}\end{aligned}$$

output
```
-1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x^4-1/24*(c/a/e-7*e/d^2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3+1/96*(-35*a^2*e^4+6*a*c*d^2*e^2+
5*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/x^2-1/192*(
-105*a^3*e^6+25*a^2*c*d^2*e^4+17*a*c^2*d^4*e^2+15*c^3*d^6)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d^4/e^3/x+1/64*(-a*e^2+c*d^2)*(35*a^3*e^6+15*
a^2*c*d^2*e^4+9*a*c^2*d^4*e^2+5*c^3*d^6)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d
^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(7/2)/d^(9/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 10.39 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{x^5(d+ex)}\,dx$$
$$=\frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-15c^3d^6x^3+ac^2d^4ex^2(10d-17ex)+a^2cd^2e^2x(-8d^2+12dex-25e^2x^2)+a^3e^3(-48d^3+56d^2ex-70de^{\cdots}}{x^4}\right.}{192a^{7/2}d^{9/2}e^{7/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^5*(d + e*x)),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-15*c^3*d^6*x^3
+ a*c^2*d^4*e*x^2*(10*d - 17*e*x) + a^2*c*d^2*e^2*x*(-8*d^2 + 12*d*e*x - 2
5*e^2*x^2) + a^3*e^3*(-48*d^3 + 56*d^2*e*x - 70*d*e^2*x^2 + 105*e^3*x^3)))
/x^4 + (3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*
e^6 - 35*a^4*e^8)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqr
t[d + e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(192*a^(7/2)*d^(9/2)*e^(
7/2))
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1215, 1237, 27, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x^5(d+ex)}\,dx$$
$$\Big\downarrow \; 1215$$
$$\int \frac{ae+cdx}{x^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$
$$\Big\downarrow \; 1237$$

$$-\frac{\int -\frac{ae\left(cd^2-6cexd-7ae^2\right)}{2x^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4ade}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{cd^2-6cexd-7ae^2}{x^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

$\Big\downarrow$ 1237

$$\frac{-\frac{\int \frac{5c^2d^4+6ace^2d^2+4ce\left(cd^2-7ae^2\right)xd-35a^2e^4}{2x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3ade}-\frac{\left(\frac{cd}{ae}-\frac{7e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3x^3}}{8d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int \frac{5c^2d^4+6ace^2d^2+4ce\left(cd^2-7ae^2\right)xd-35a^2e^4}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6ade}-\frac{\left(\frac{cd}{ae}-\frac{7e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3x^3}}{8d}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

$\Big\downarrow$ 1237

$$-\frac{-\frac{-\frac{\int \frac{15c^3d^6+17ac^2e^2d^4+25a^2ce^4d^2+2ce\left(5c^2d^4+6ace^2d^2-35a^2e^4\right)xd-105a^3e^6}{2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(\frac{5c^2d^4}{a}-35ae^4+6cd^2e^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2dex^2}}{6ade}-\left(\frac{cd}{ae}-\frac{7e}{d}\right)\cdots}{8d}$$
$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{-\frac{\int \frac{15c^3d^6+17ac^2e^2d^4+25a^2ce^4d^2+2ce\left(5c^2d^4+6ace^2d^2-35a^2e^4\right)xd-105a^3e^6}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4ade}-\frac{\left(\frac{5c^2d^4}{a}-35ae^4+6cd^2e^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2dex^2}}{6ade}-\left(\frac{cd}{ae}-\frac{7e}{d}\right)\cdots}{8d}$$
$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

$\Big\downarrow$ 1228

$$-\frac{-\frac{-\frac{-\frac{3\left(cd^2-ae^2\right)\left(35a^3e^6+15a^2cd^2e^4+9ac^2d^4e^2+5c^3d^6\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(-105a^3e^6+25a^2cd^2e^4+17ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+}}{adex}}{4ade}}{6ade}}{8d}$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

$\Big\downarrow$ 1154

$$-\frac{-\frac{-\frac{3\left(cd^2-ae^2\right)\left(35a^3e^6+15a^2cd^2e^4+9ac^2d^4e^2+5c^3d^6\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}-\frac{\left(-105a^3e^6+25a^2cd^2e^4+17ac^2d^4\right.}{}}{4ade}}{6ade}$$

$$8d$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

$\Big\downarrow$ 219

$$-\frac{-\frac{-\frac{3\left(cd^2-ae^2\right)\left(35a^3e^6+15a^2cd^2e^4+9ac^2d^4e^2+5c^3d^6\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(-105a^3e^6+25a^2cd^2e^4+17ac^2d^4e^2+15c^3d^6\right.}{adex}}{4ade}}{6ade}$$

$$8d$$

$$\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^4}$$

input

```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^5*(d + e*x)),x]
```

output

```
-1/4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x^4) + (-1/3*(((c*d)/(
a*e) - (7*e)/d)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^3 - (-1/2*(
((5*c^2*d^4)/a + 6*c*d^2*e^2 - 35*a*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(d*e*x^2) - (-(((15*c^3*d^6 + 17*a*c^2*d^4*e^2 + 25*a^2*c*d^2*
e^4 - 105*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x))
 + (3*(c*d^2 - a*e^2)*(5*c^3*d^6 + 9*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 35
*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(3/2)*d^(3/2)*e^(3/2)
))/(4*a*d*e))/(6*a*d*e))/(8*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3470 vs. 2(340) = 680.

Time = 3.16 (sec) , antiderivative size = 3471, normalized size of antiderivative = 9.33

method	result	size
default	Expression too large to display	3471

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/x^5/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/4/a/d/e/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-5/8*(a*e^2+c*d
^2)/a/d/e*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e
^2+c*d^2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/
4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+
1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^
2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^
2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*
c/a*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c
/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d
*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)
))+1/2*c/a*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((
1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*
(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))))-1/4*c/a*(-1/2
/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(a*e^2+c*d^2)/a/d/e
*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)/a/d
/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^
2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
)/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*...
```

Fricas [A] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 702, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^5(d + ex)}\,dx$$
$$= \left[-\frac{3\left(5c^4d^8 + 4ac^3d^6e^2 + 6a^2c^2d^4e^4 + 20a^3cd^2e^6 - 35a^4e^8\right)\sqrt{ade}\,x^4\log\left(\frac{8a^2d^2e^2 + (c^2d^4 + 6acd^2e^2 + a^2e^4)x^2 - 4\sqrt{}\right.}{}\right.$$
$$-\frac{3\left(5c^4d^8 + 4ac^3d^6e^2 + 6a^2c^2d^4e^4 + 20a^3cd^2e^6 - 35a^4e^8\right)\sqrt{-ade}\,x^4\arctan\left(\frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\,(2ade +}{2\left(acd^2e^2x^2 + a^2d^2e^2 + (acd\right.}\right.}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/768*(3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2
*e^6 - 35*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d
^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2)
+ 4*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 17*a^2*c^2*d^5*e^3 + 25*a^3*c*d^3*
e^5 - 105*a^4*d*e^7)*x^3 - 2*(5*a^2*c^2*d^6*e^2 + 6*a^3*c*d^4*e^4 - 35*a^4
*d^2*e^6)*x^2 + 8*(a^3*c*d^5*e^3 - 7*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^4), -1/384*(3*(5*c^4*d^8 + 4*a*c^3*
d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 35*a^4*e^8)*sqrt(-a*d*e)*
x^4*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e +
a^2*d*e^3)*x)) + 2*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 17*a^2*c^2*d^5*e^3
+ 25*a^3*c*d^3*e^5 - 105*a^4*d*e^7)*x^3 - 2*(5*a^2*c^2*d^6*e^2 + 6*a^3*c*d
^4*e^4 - 35*a^4*d^2*e^6)*x^2 + 8*(a^3*c*d^5*e^3 - 7*a^4*d^3*e^5)*x)*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^5(d + ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**5/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^5(d + ex)}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)x^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1483 vs. $2(340) = 680$.

Time $= 0.15$ (sec) , antiderivative size $= 1483$, normalized size of antiderivative $= 3.99$

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^5(d + ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm
="giac")
```

output
```
-1/64*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6
- 35*a^4*e^8)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^3*d^4*e^3) + 1/192*(15*(sqrt(c*d*e
)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*c^4*d^11*e^3 + 396*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c^3*d^9*
e^5 + 1170*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a
^5*c^2*d^7*e^7 + 1212*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e))*a^6*c*d^5*e^9 + 279*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
a*e^2*x + a*d*e))*a^7*d^3*e^11 + 128*sqrt(c*d*e)*a^5*c^2*d^8*e^6 + 256*sqr
t(c*d*e)*a^6*c*d^6*e^8 + 384*sqrt(c*d*e)*a^7*d^4*e^10 + 73*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c^4*d^10*e^2 + 980*(s
qrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c^3*d^8*
e^4 + 2238*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3
*a^4*c^2*d^6*e^6 + 292*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))^3*a^5*c*d^4*e^8 - 511*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
 + a*e^2*x + a*d*e))^3*a^6*d^2*e^10 + 384*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqr
t(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^3*c^3*d^9*e^3 + 1792*sqrt(c*
d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^4*c
^2*d^7*e^5 + 2432*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
a*e^2*x + a*d*e))^2*a^5*c*d^5*e^7 - 55*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^5(d + ex)}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{x^5\,\left(d + e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^5*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^5*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 1694, normalized size of antiderivative = 4.55

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^5(d+ex)}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x)
```

output
```
( - 192*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*d**4*e**6 + 224*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**5*d**3*e**7*x - 280*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a**5*d**2*e**8*x**2 + 420*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*d*e**9*x**3
 - 192*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d**6*e**4 + 192*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*a**4*c*d**5*e**5*x - 232*sqrt(d + e*x)*sqrt(a*e + c*d*
x)*a**4*c*d**4*e**6*x**2 + 320*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d**3
*e**7*x**3 - 32*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**7*e**3*x + 88
*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**6*e**4*x**2 - 168*sqrt(d + e
*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**5*e**5*x**3 + 40*sqrt(d + e*x)*sqrt(a*e
 + c*d*x)*a**2*c**3*d**8*e**2*x**2 - 128*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
**2*c**3*d**7*e**3*x**3 - 60*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**9*e
*x**3 + 210*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2
*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a
**5*e**10*x**4 + 90*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x)
- sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d +
 e*x))*a**4*c*d**2*e**8*x**4 - 156*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqr
t(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*s
qrt(c)*sqrt(d + e*x))*a**3*c**2*d**4*e**6*x**4 - 60*sqrt(e)*sqrt(d)*sqrt(a
)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*
d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c**3*d**6*e**4*x**4 - 54*sq...
```

3.20 $\int \frac{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 338

$$\begin{aligned}\int \frac{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^2}\,dx &= \frac{(35c^3d^6-25ac^2d^4e^2+5a^2cd^2e^4+a^3e^6-2cde(cd^2-ae^2)(5cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8c^2d^2e^4(cd^2-ae^2)}\\ &\quad+\frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{3cde^3}-\frac{2d^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{e^3(cd^2-ae^2)(d+ex)^2}\\ &\quad-\frac{(35c^3d^6-15ac^2d^4e^2-3a^2cd^2e^4-a^3e^6)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{9/2}}\end{aligned}$$

output
```
1/8*(35*c^3*d^6-25*a*c^2*d^4*e^2+5*a^2*c*d^2*e^4+a^3*e^6-2*c*d*e*(-a*e^2+c
*d^2)*(a*e^2+5*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e
^4/(-a*e^2+c*d^2)+1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e^3-2*d^
3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e^3/(-a*e^2+c*d^2)/(e*x+d)^2-1/8
*(-a^3*e^6-3*a^2*c*d^2*e^4-15*a*c^2*d^4*e^2+35*c^3*d^6)*arctanh(c^(1/2)*d^
(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(
5/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.71

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx$$

$$= \frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(-3a^2e^4(d+ex)-2acde^2(5d^2+4dex-e^2x^2)+c^2d^2(105d^3+35d^2ex-14de^2x^2+8e^3x^3))}{d+ex} - \frac{3(35c^3d^6-}{}\right.}{24c^{5/2}d^{5/2}e^{9/2}}$$

input
```
Integrate[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x)^2,x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(-3*a^2*e^4*(d +
e*x) - 2*a*c*d*e^2*(5*d^2 + 4*d*e*x - e^2*x^2) + c^2*d^2*(105*d^3 + 35*d^2
*e*x - 14*d*e^2*x^2 + 8*e^3*x^3)))/(d + e*x) - (3*(35*c^3*d^6 - 15*a*c^2*d
^4*e^2 - 3*a^2*c*d^2*e^4 - a^3*e^6)*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(S
qrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(24*c^
(5/2)*d^(5/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1213, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{1213}$$

$$\frac{2d^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{e^4(d+ex)} - \frac{\int \frac{-cdx^3e^4+(cd^2-ae^2)x^2e^3-d(cd^2-ae^2)xe^2+d^2(cd^2-ae^2)e}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^5}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-\frac{\frac{\int \frac{cd\left(11cd^2-ae^2\right)x^2e^4-2cd^2\left(3cd^2-5ae^2\right)xe^3+6cd^3\left(cd^2-ae^2\right)e^2}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3cde}-\frac{1}{3}e^3x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5}}{}$$

$$\Big\downarrow \; 27$$

$$\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-\frac{\frac{\int \frac{cd\left(11cd^2-ae^2\right)x^2e^4-2cd^2\left(3cd^2-5ae^2\right)xe^3+6cd^3\left(cd^2-ae^2\right)e^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6cde}-\frac{1}{3}e^3x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5}$$

$$\Big\downarrow \; 2192$$

$$\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-\frac{\frac{\frac{\int \frac{cde^3\left(2d\left(12c^2d^4-23ace^2d^2+a^2e^4\right)-e\left(3cd^2-ae^2\right)\left(19cd^2+3ae^2\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}e^3x\left(11cd^2-ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6cde}-\frac{1}{3}e^3x^2\sqrt{x\left(ae^2+cd^2\right)\ldots}}{e^5}$$

$$\Big\downarrow \; 27$$

$$\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-\frac{\frac{\frac{1}{4}e^2\int \frac{2d\left(12c^2d^4-23ace^2d^2+a^2e^4\right)-e\left(3cd^2-ae^2\right)\left(19cd^2+3ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{1}{2}e^3x\left(11cd^2-ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6cde}-\frac{1}{3}e^3x^2\sqrt{x\left(ae^2+cd^2\right)\ldots}}{e^5}$$

$$\Big\downarrow \; 1160$$

$$\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-\frac{\frac{\frac{1}{4}e^2\left(\frac{3\left(-a^3e^6-3a^2cd^2e^4-15ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}-\frac{\left(3cd^2-ae^2\right)\left(3ae^2+19cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cd}\right)+\frac{1}{2}e^3x\left(11cd^2-\ldots\right.}{6cde}}{e^5}$$

$$\Big\downarrow \; 1092$$

$$\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-$$

$$\frac{\frac{1}{4}e^2\left(\frac{3\left(-a^3e^6-3a^2cd^2e^4-15ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{cd}-\frac{\left(3cd^2-ae^2\right)\left(3ae^2+19cd^2\right)\sqrt{x\left(ae^2+cd^2\right)}}{cd}\right.}{6cde}}{e^5}$$

$\big\downarrow$ 219

$$\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-$$

$$\frac{\frac{1}{4}e^2\left(\frac{3\left(-a^3e^6-3a^2cd^2e^4-15ac^2d^4e^2+35c^3d^6\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}\sqrt{e}}-\frac{\left(3cd^2-ae^2\right)\left(3ae^2+19cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{cd}\right.}{6cde}}{e^5}$$

input
```
Int[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x)^2,x]
```

output
```
(2*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^4*(d + e*x)) - (-1/
3*(e^3*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + ((e^3*(11*c*d^2
- a*e^2)*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/2 + (e^2*(-(((3*c*
d^2 - a*e^2)*(19*c*d^2 + 3*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/(c*d)) + (3*(35*c^3*d^6 - 15*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4 - a^3*e^
6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(3/2)*d^(3/2)*Sqrt[e])))/4)/(6
*c*d*e))/e^5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. 2(312) = 624.

Time = 2.77 (sec) , antiderivative size = 728, normalized size of antiderivative = 2.15

method	result
default	$\frac{\frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{3dec}-\frac{\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{4cde}+\frac{\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\ln\left(\frac{\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}}{\sqrt{dec}}\right.}{8dec\sqrt{d}}\right.}{2dec}}{e^{2}}$

input
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^2,x,method=_RETURN
VERBOSE)
```

output
```
1/e^2*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/d/e/c-1/2*(a*e^2+c*d^2)
/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+
c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1
/2)))-2*d/e^3*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2
*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*
e*c)^(1/2))+3/e^4*d^2*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(
a*e^2-c*d^2)*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(
x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))-d^3/e^5*(-2/(a*e^2-c
*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+2*d*e*c/(a*e
^2-c*d^2)*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)
*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*
e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 736, normalized size of antiderivative = 2.18

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx$$
$$=\left[-\frac{3\,(35\,c^3d^7-15\,ac^2d^5e^2-3\,a^2cd^3e^4-a^3de^6+(35\,c^3d^6e-15\,ac^2d^4e^3-3\,a^2cd^2e^5-a^3e^7)x)\sqrt{cde}\log\left(\right.}{}\right.$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorit
hm="fricas")
```

output
```
[-1/96*(3*(35*c^3*d^7 - 15*a*c^2*d^5*e^2 - 3*a^2*c*d^3*e^4 - a^3*d*e^6 + (
35*c^3*d^6*e - 15*a*c^2*d^4*e^3 - 3*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c*d*e
)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e)
+ 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^4*x^3 + 105*c^3*d^6*e - 10
*a*c^2*d^4*e^3 - 3*a^2*c*d^2*e^5 - 2*(7*c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 +
 (35*c^3*d^5*e^2 - 8*a*c^2*d^3*e^4 - 3*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^6*x + c^3*d^4*e^5), 1/48*(3*(35*c^3*d
^7 - 15*a*c^2*d^5*e^2 - 3*a^2*c*d^3*e^4 - a^3*d*e^6 + (35*c^3*d^6*e - 15*a
*c^2*d^4*e^3 - 3*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c
*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(8*
c^3*d^3*e^4*x^3 + 105*c^3*d^6*e - 10*a*c^2*d^4*e^3 - 3*a^2*c*d^2*e^5 - 2*(
7*c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + (35*c^3*d^5*e^2 - 8*a*c^2*d^3*e^4 - 3
*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^6
*x + c^3*d^4*e^5)]
```

Sympy [F]

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx=\int \frac{x^3\sqrt{(d+ex)\,(ae+cdx)}}{(d+ex)^2}\,dx$$

input
```
integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**2,x)
```

output
```
Integral(x**3*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(312) = 624$.

Time = 0.40 (sec) , antiderivative size = 901, normalized size of antiderivative = 2.67

$$\int \frac{x^3\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorit
hm="giac")
```

output
```
1/24*(48*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*d^3*sgn(1/(e*x
+ d))*sgn(e)/(e^5*abs(e)) + 3*(35*c^3*d^6*sgn(1/(e*x + d))*sgn(e) - 15*a*c
^2*d^4*e^2*sgn(1/(e*x + d))*sgn(e) - 3*a^2*c*d^2*e^4*sgn(1/(e*x + d))*sgn(
e) - a^3*e^6*sgn(1/(e*x + d))*sgn(e))*arctan(sqrt(c*d*e - c*d^2*e/(e*x + d
) + a*e^3/(e*x + d))/sqrt(-c*d*e))/(sqrt(-c*d*e)*c^2*d^2*e^4*abs(e)) + (57
*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c^5*d^8*e^2*sgn(1/(e*x
+ d))*sgn(e) - 45*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a*c^4*
d^6*e^4*sgn(1/(e*x + d))*sgn(e) - 9*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3
/(e*x + d))*a^2*c^3*d^4*e^6*sgn(1/(e*x + d))*sgn(e) - 3*sqrt(c*d*e - c*d^2
*e/(e*x + d) + a*e^3/(e*x + d))*a^3*c^2*d^2*e^8*sgn(1/(e*x + d))*sgn(e) -
136*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*c^4*d^7*e*sgn(1/(e
*x + d))*sgn(e) + 120*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*
a*c^3*d^5*e^3*sgn(1/(e*x + d))*sgn(e) + 24*(c*d*e - c*d^2*e/(e*x + d) + a*
e^3/(e*x + d))^(3/2)*a^2*c^2*d^3*e^5*sgn(1/(e*x + d))*sgn(e) - 8*(c*d*e -
c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*a^3*c*d*e^7*sgn(1/(e*x + d))*sg
n(e) + 87*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(5/2)*c^3*d^6*sgn(
1/(e*x + d))*sgn(e) - 99*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(5/
2)*a*c^2*d^4*e^2*sgn(1/(e*x + d))*sgn(e) + 9*(c*d*e - c*d^2*e/(e*x + d) +
a*e^3/(e*x + d))^(5/2)*a^2*c*d^2*e^4*sgn(1/(e*x + d))*sgn(e) + 3*(c*d*e -
c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(5/2)*a^3*e^6*sgn(1/(e*x + d))*sgn...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx=\int \frac{x^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(d+e\,x\right)}^2}\,dx$$

input
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2,x)
```

output
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 20.06 (sec) , antiderivative size = 862, normalized size of antiderivative = 2.55

$$\int \frac{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x)
```

output
```
( - 24*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d**2*e**5 - 24*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**2*c*d*e**6*x - 80*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
c**2*d**4*e**3 - 64*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**3*e**4*x + 1
6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**2*e**5*x**2 + 840*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*c**3*d**6*e + 280*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3
*d**5*e**2*x - 112*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**4*e**3*x**2 + 6
4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**3*e**4*x**3 + 24*sqrt(e)*sqrt(d)
*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/s
qrt(a*e**2 - c*d**2))*a**3*d*e**6 + 24*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e
)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2)
)*a**3*e**7*x + 72*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**3*e**4 +
 72*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(
c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**5*x + 360*sqrt(e)*
sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d +
e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**5*e**2 + 360*sqrt(e)*sqrt(d)*sqrt(c
)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e
**2 - c*d**2))*a*c**2*d**4*e**3*x - 840*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(
e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2
))*c**3*d**7 - 840*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*...
```

3.21 $\int \frac{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 256

$$\begin{aligned}\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx &= -\frac{(15c^2d^4-8acd^2e^2+a^2e^4-2cde(cd^2-ae^2)\,x)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4cde^3\,(cd^2-ae^2)}\\ &\quad+\frac{2d^2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{e^2\,(cd^2-ae^2)\,(d+ex)^2}\\ &\quad+\frac{(15c^2d^4-6acd^2e^2-a^2e^4)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^{3/2}d^{3/2}e^{7/2}}\end{aligned}$$

output
```
-1/4*(15*c^2*d^4-8*a*c*d^2*e^2+a^2*e^4-2*c*d*e*(-a*e^2+c*d^2)*x)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^3/(-a*e^2+c*d^2)+2*d^2*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(3/2)/e^2/(-a*e^2+c*d^2)/(e*x+d)^2+1/4*(-a^2*e^4-6*a*c
*d^2*e^2+15*c^2*d^4)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.71

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae^2(d+ex)+cd(-15d^2-5dex+2e^2x^2))}{d+ex}+\frac{(15c^2d^4-6acd^2e^2-a^2e^4)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{4c^{3/2}d^{3/2}e^{7/2}}$$

input

```
Integrate[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x)^2,x]
```

output

```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e^2*(d + e*x)
+ c*d*(-15*d^2 - 5*d*e*x + 2*e^2*x^2)))/(d + e*x) + ((15*c^2*d^4 - 6*a*c*d
^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqr
t[d + e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(4*c^(3/2)*d^(3/2)*e^(7/
2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1213, 25, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{1213}$$

$$-\frac{\int -\frac{cdx^2e^3-(cd^2-ae^2)xe^2+d(cd^2-ae^2)e}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^4}-\frac{2d^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{cdx^2e^3-(cd^2-ae^2)xe^2+d(cd^2-ae^2)e}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^4}-\frac{2d^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 2192

$$\frac{\frac{\frac{\int \frac{cde^2\left(2d\left(2cd^2-3ae^2\right)-e\left(7cd^2-ae^2\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}e^2x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4}-\frac{2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{1}{4}e\int \frac{2d(2cd^2-3ae^2)-e(7cd^2-ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{1}{2}e^2x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4}-\frac{2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 1160

$$\frac{\frac{1}{4}e\left(\frac{\left(-a^2e^4-6acd^2e^2+15c^2d^4\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}-\left(7d-\frac{ae^2}{cd}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)+\frac{1}{2}e^2x\sqrt{x\,(ae}{e^4}$$
$$\frac{2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 1092

$$\frac{\frac{1}{4}e\left(\frac{\left(-a^2e^4-6acd^2e^2+15c^2d^4\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{cd}-\left(7d-\frac{ae^2}{cd}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+c}\right.}{e^4}$$
$$\frac{2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{4}e\left(\frac{\left(-a^2e^4-6acd^2e^2+15c^2d^4\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}\sqrt{e}}-\left(7d-\frac{ae^2}{cd}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{e^4}$$
$$\frac{2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)}$$

input
```
Int[(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x)^2,x]
```

output
```
(-2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^3*(d + e*x)) + ((e
^2*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/2 + (e*(-((7*d - (a*e^2)
/(c*d))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + ((15*c^2*d^4 - 6*a*
c*d^2*e^2 - a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d
]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(3/2)*d^(3/2
)*Sqrt[e])))/4)/e^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(234) = 468$.

Time = 2.46 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.97

method	result
default	$\frac{\frac{\left(2cdxe+a e^2+c d^2\right)\sqrt{ade+\left(a e^2+c d^2\right)x+cd x^2 e}}{4cde}+\frac{\left(4ac d^2 e^2-\left(a e^2+c d^2\right)^2\right)\ln\left(\frac{\frac{1}{2}a e^2+\frac{1}{2}c d^2+cdxe}{\sqrt{dec}}+\sqrt{ade+\left(a e^2+c d^2\right)x+cd x^2 e}\right)}{8dec\sqrt{dec}}}{e^2}+$ … d^2

input
```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^2,x,method=_RETURN
VERBOSE)
```

output
```
1/e^2*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c
*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/
2))+d^2/e^4*(-2/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+
d/e))^(3/2)+2*d*e*c/(a*e^2-c*d^2)*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1
/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))-2*d/e^3
*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*
a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2
)*(x+d/e))^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.20

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^2}\,dx$$
$$= \Bigg[-\frac{(15\,c^2d^5 - 6\,acd^3e^2 - a^2de^4 + (15\,c^2d^4e - 6\,acd^2e^3 - a^2e^5)x)\sqrt{cde}\log\Big(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 \cdots}{\cdots}$$
$$-\frac{(15\,c^2d^5 - 6\,acd^3e^2 - a^2de^4 + (15\,c^2d^4e - 6\,acd^2e^3 - a^2e^5)x)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cde \cdots}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d \cdots}\right)}{8\,(c^2d^2e \cdots}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorit
hm="fricas")
```

output
```
[-1/16*((15*c^2*d^5 - 6*a*c*d^3*e^2 - a^2*d*e^4 + (15*c^2*d^4*e - 6*a*c*d^
2*e^3 - a^2*e^5)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x
 + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(2*c^2*d^
2*e^3*x^2 - 15*c^2*d^4*e + a*c*d^2*e^3 - (5*c^2*d^3*e^2 - a*c*d*e^4)*x)*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^5*x + c^2*d^3*e^4),
-1/8*((15*c^2*d^5 - 6*a*c*d^3*e^2 - a^2*d*e^4 + (15*c^2*d^4*e - 6*a*c*d^2*
e^3 - a^2*e^5)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*
c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(2*c^2*d^2*e^3*x^2 - 15*c^2*d^
4*e + a*c*d^2*e^3 - (5*c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x))/(c^2*d^2*e^5*x + c^2*d^3*e^4)]
```

Sympy [F]

$$\int \frac{x^2\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^2}\,dx = \int \frac{x^2\sqrt{(d + ex)\,(ae + cdx)}}{(d + ex)^2}\,dx$$

input
```
integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**2,x)
```

output
```
Integral(x**2*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(234) = 468.

Time = 0.28 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.11

$$\int \frac{x^2\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d+ex)^2}\,dx =$$

$$-\frac{1}{4}\left(\frac{8\sqrt{cde - \frac{cd^2e}{ex+d} + \frac{ae^3}{ex+d}}d^2\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}{e^4|e|} + \frac{\left(15\,c^2d^4\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e) - 6\,acd^2e^2\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)\right.}{\sqrt{-cde}}\right.$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorit
hm="giac")
```

output
```
-1/4*(8*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*d^2*sgn(1/(e*x +
 d))*sgn(e)/(e^4*abs(e)) + (15*c^2*d^4*sgn(1/(e*x + d))*sgn(e) - 6*a*c*d^2
*e^2*sgn(1/(e*x + d))*sgn(e) - a^2*e^4*sgn(1/(e*x + d))*sgn(e))*arctan(sqr
t(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))/sqrt(-c*d*e))/(sqrt(-c*d*e)
*c*d*e^3*abs(e)) + (7*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c^
3*d^5*e*sgn(1/(e*x + d))*sgn(e) - 6*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3
/(e*x + d))*a*c^2*d^3*e^3*sgn(1/(e*x + d))*sgn(e) - sqrt(c*d*e - c*d^2*e/(
e*x + d) + a*e^3/(e*x + d))*a^2*c*d*e^5*sgn(1/(e*x + d))*sgn(e) - 9*(c*d*e
 - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*c^2*d^4*sgn(1/(e*x + d))*sgn
(e) + 10*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*a*c*d^2*e^2*s
gn(1/(e*x + d))*sgn(e) - (c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/
2)*a^2*e^4*sgn(1/(e*x + d))*sgn(e))/((c*d^2*e/(e*x + d) - a*e^3/(e*x + d))
^2*c*d*e^3*abs(e)))*abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx=\int \frac{x^2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(d+e\,x\right)}^2}\,dx$$

input
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2,x)
```

output
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.14

$$\int \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx$$
$$=\frac{\sqrt{ex+d}\,\sqrt{cdx+ae}\,ac\,d^2e^3+\sqrt{ex+d}\,\sqrt{cdx+ae}\,acd\,e^4x-15\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^4e-5\sqrt{ex+d}\,\sqrt{\cdots}}{\cdots}$$

input
```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x)
```

output
```
(sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d**2*e**3 + sqrt(d + e*x)*sqrt(a*e +
c*d*x)*a*c*d*e**4*x - 15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**4*e - 5*s
qrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**3*e**2*x + 2*sqrt(d + e*x)*sqrt(a*e
 + c*d*x)*c**2*d**2*e**3*x**2 - sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(
a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*
d*e**4 - sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*
sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**5*x - 6*sqrt(e)*sqrt
(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x)
)/sqrt(a*e**2 - c*d**2))*a*c*d**3*e**2 - 6*sqrt(e)*sqrt(d)*sqrt(c)*log((sq
rt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d
**2))*a*c*d**2*e**3*x + 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e +
 c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**5
+ 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt
(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**4*e*x + 2*sqrt(e)*sqrt(d
)*sqrt(c)*a*c*d**3*e**2 + 2*sqrt(e)*sqrt(d)*sqrt(c)*a*c*d**2*e**3*x - 10*s
qrt(e)*sqrt(d)*sqrt(c)*c**2*d**5 - 10*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**4*e*
x)/(4*c**2*d**2*e**4*(d + e*x))
```

3.22 $\int \frac{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 159

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e^2}+\frac{2d\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e^2(d+ex)} - \frac{(3cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{\sqrt{c}\sqrt{d}e^{5/2}}$$

output
```
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^2+2*d*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/e^2/(e*x+d)-(-a*e^2+3*c*d^2)*arctanh(e^(1/2)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/c^(1/2)/d^(1/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.81

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{e}(3d+ex)}{d+ex}-\frac{(3cd^2-ae^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{c}\sqrt{d}\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{e^{5/2}}$$

input
```
Integrate[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x)^2,x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[e]*(3*d + e*x))/(d + e*x) - ((3*c*d^
2 - a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e
*x])])/(Sqrt[c]*Sqrt[d]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/e^(5/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1213, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{(d+ex)^2}\,dx$$

$$\Big\downarrow \; 1213$$

$$\frac{2d\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{e^2(d+ex)}-\frac{\int \frac{e(cd^2-cexd-ae^2)}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^3}$$

$$\Big\downarrow \; 27$$

$$\frac{2d\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{e^2(d+ex)}-\frac{\int \frac{cd^2-cexd-ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^2}$$

$$\Big\downarrow \; 1160$$

$$\frac{2d\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2(d+ex)}-\frac{\frac{1}{2}\left(3cd^2-ae^2\right)\int\frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{2d\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2(d+ex)}-\frac{\left(3cd^2-ae^2\right)\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2d\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2(d+ex)}-\frac{\frac{\left(3cd^2-ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2}$$

input
```
Int[(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x)^2,x]
```

output
```
(2*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^2*(d + e*x)) - (-Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2] + ((3*c*d^2 - a*e^2)*ArcTanh[(c*d^
2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]))/e^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(141) = 282$.

Time = 2.42 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.17

method	result
default	$\frac{\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}+\frac{\left(a e^2-c d^2\right)\ln\left(\frac{\frac{a e^2}{2}-\frac{c d^2}{2}+dec\left(x+\frac{d}{e}\right)}{\sqrt{dec}}+\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}\right)}{2\sqrt{dec}}}{e^2}-\frac{d\left(-\frac{2\left(dec\left(x+\ldots\right.\right.}{\ldots}\right.}{\ldots}$

input
```
int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^2,x,method=_RETURNVE
RBOSE)
```

output
```
1/e^2*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln(
(1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-
c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))-d/e^3*(-2/(a*e^2-c*d^2)/(x+d/e)^2*(d
*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+2*d*e*c/(a*e^2-c*d^2)*((d*e*c*
(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*a*e^2-1/2
*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e)
)^(1/2))/(d*e*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.60

$$\int \frac{x\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^2}\,dx$$
$$= \Bigg[-\frac{(3\,cd^3 - ade^2 + (3\,cd^2e - ae^3)x)\sqrt{cde}\log\Big(8\,c^2d^2e^2x^2 + c^2d^4 + 6\,acd^2e^2 + a^2e^4 + 4\,\sqrt{cdex^2 + ade + \cdots}}{4\,(\cdots}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorithm
="fricas")
```

output
```
[-1/4*((3*c*d^3 - a*d*e^2 + (3*c*d^2*e - a*e^3)*x)*sqrt(c*d*e)*log(8*c^2*d
^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*
e + a*c*d*e^3)*x) - 4*(c*d*e^2*x + 3*c*d^2*e)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x))/(c*d*e^4*x + c*d^2*e^3), 1/2*((3*c*d^3 - a*d*e^2 + (3*c*d
^2*e - a*e^3)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c
*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(c*d*e^2*x + 3*c*d^2*e)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^4*x + c*d^2*e^3)]
```

Sympy [F]

$$\int \frac{x\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^2}\,dx = \int \frac{x\sqrt{(d + ex)\,(ae + cdx)}}{(d + ex)^2}\,dx$$

input
```
integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**2,x)
```

output
```
Integral(x*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(141) = 282$.

Time = 0.20 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.79

$$\int \frac{x\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d + ex)^2}\,dx$$
$$= \frac{\left(\frac{2\sqrt{cde - \frac{cd^2e}{ex+d} + \frac{ae^3}{ex+d}}\,d\,\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}{e^2|e|} + \frac{\left(3\,cd^2\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e) - ae^2\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)\right)\arctan\left(\frac{\sqrt{cde - \frac{cd^2e}{ex+d} + \frac{ae^3}{ex+d}}}{\sqrt{-cde}}\right)}{\sqrt{-cde}e|e|} + \sqrt{cde - \cdots}\right.}{e}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorithm
="giac")
```

output
```
(2*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*d*sgn(1/(e*x + d))*sg
n(e)/(e^2*abs(e)) + (3*c*d^2*sgn(1/(e*x + d))*sgn(e) - a*e^2*sgn(1/(e*x +
d))*sgn(e))*arctan(sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))/sqrt(
-c*d*e))/(sqrt(-c*d*e)*e*abs(e)) + (sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3
/(e*x + d))*c*d^2*sgn(1/(e*x + d))*sgn(e) - sqrt(c*d*e - c*d^2*e/(e*x + d)
 + a*e^3/(e*x + d))*a*e^2*sgn(1/(e*x + d))*sgn(e))/((c*d^2*e/(e*x + d) - a
*e^3/(e*x + d))*e*abs(e)))*abs(e)/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx=\int \frac{x\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{{\left(d+e\,x\right)}^2}\,dx$$

input
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2,x)
```

output
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.09

$$\int \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx$$
$$=\frac{12\sqrt{ex+d}\,\sqrt{cdx+ae}\,c\,d^2e+4\sqrt{ex+d}\,\sqrt{cdx+ae}\,cd\,e^2x+4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)ad}{}$$

input
```
int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x)
```

output
```
(12*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c*d**2*e + 4*sqrt(d + e*x)*sqrt(a*e +
c*d*x)*c*d*e**2*x + 4*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*d*e**2 + 4*sq
rt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqr
t(d + e*x))/sqrt(a*e**2 - c*d**2))*a*e**3*x - 12*sqrt(e)*sqrt(d)*sqrt(c)*l
og((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2
 - c*d**2))*c*d**3 - 12*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**2*e*x -
sqrt(e)*sqrt(d)*sqrt(c)*a*d*e**2 - sqrt(e)*sqrt(d)*sqrt(c)*a*e**3*x + 9*sq
rt(e)*sqrt(d)*sqrt(c)*c*d**3 + 9*sqrt(e)*sqrt(d)*sqrt(c)*c*d**2*e*x)/(4*c*
d*e**3*(d + e*x))
```

3.23 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 110

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^2}\,dx = -\frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e(d+ex)} + \frac{2\sqrt{c}\sqrt{d}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{e^{3/2}}$$

output
```
-2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e/(e*x+d)+2*c^(1/2)*d^(1/2)*arc
tanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{(d+ex)^2}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{e}}{d+ex}+\frac{\sqrt{c}\sqrt{d}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{e^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^2,x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[e]/(d + e*x)) + (Sqrt[c]*Sqrt[d]*
ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqr
t[a*e + c*d*x]*Sqrt[d + e*x])))/e^(3/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1125, 25, 27, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{(d+ex)^2}dx$$

$$\Big\downarrow \text{ 1125}$$

$$-\frac{\int -\frac{cde}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^2}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{cde}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^2}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{cd\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{2cd\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{e}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{e^{3/2}}-\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e(d+ex)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d + e*x)^2,x]
```

output
```
(-2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(d + e*x)) + (Sqrt[c]*
Sqrt[d]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/e^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(94) = 188.

Time = 2.51 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.93

method	result
default	$\frac{-\frac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{2}}+\frac{2dec\left(\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\ln\left(\frac{\frac{a\,e^{2}}{2}-\frac{c\,d^{2}}{2}+dec\left(x+\frac{d}{e}\right)}{\sqrt{dec}}+\sqrt{dec\left(x+\frac{d}{e}\right)\cdots}\right)}{2\sqrt{dec}}\cdots}{a\,e^{2}-c\,d^{2}}}{e^{2}}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/(e*x+d)^2,x,method=_RETURNVERB
OSE)
```

output
```
1/e^2*(-2/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^
(3/2)+2*d*e*c/(a*e^2-c*d^2)*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
+1/2*(a*e^2-c*d^2)*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d
*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.96

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx$$

$$= \left[\frac{(ex+d)\sqrt{\frac{cd}{e}}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\,(2\,cde^2x+cd^2e+ae^3)\sqrt{cdex^2+ade+(cd^2\cdots}\right.}{2\,(e^2x+de)}\right.$$

$$\left.-\frac{(ex+d)\sqrt{-\frac{cd}{e}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-\frac{cd}{e}}}{2\,(c^2d^2ex^2+acd^2e+(c^2d^3+acde^2)x)}\right)+2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{e^2x+de}\right]$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorithm="
fricas")
```

output
```
[1/2*((e*x + d)*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^2*x + d*e), -((e*x + d)*sqrt(-c*d/e
)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^
2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)
*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^2*x + d*e)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{(d+ex)^2}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**2,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^2} dx =$$

$$-2\left(\frac{cd\arctan\left(\frac{\sqrt{cde-\frac{cd^2e}{ex+d}+\frac{ae^3}{ex+d}}}{\sqrt{-cde}}\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{\sqrt{-cde}e|e|} + \frac{\sqrt{cde - \frac{cd^2e}{ex+d} + \frac{ae^3}{ex+d}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{e^2|e|} - \frac{\left(cde \operatorname{arc}\right.}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorithm="
giac")
```

output
```
-2*(c*d*arctan(sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))/sqrt(-c*d
*e))*sgn(1/(e*x + d))*sgn(e)/(sqrt(-c*d*e)*e*abs(e)) + sqrt(c*d*e - c*d^2*
e/(e*x + d) + a*e^3/(e*x + d))*sgn(1/(e*x + d))*sgn(e)/(e^2*abs(e)) - (c*d
*e*arctan(sqrt(c*d*e)/sqrt(-c*d*e)) + sqrt(c*d*e)*sqrt(-c*d*e))*sgn(1/(e*x
 + d))*sgn(e)/(sqrt(-c*d*e)*e^2*abs(e)))*abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^2} dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{(d + e\,x)^2} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^2,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(d+ex)^2}\,dx$$
$$= \frac{-2\sqrt{ex+d}\,\sqrt{cdx+ae}\,e+2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)d+2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}}{\sqrt{a\,e^2-c\,d^2}}\right.}{e^2\,(ex+d)}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x)
```

output
```
(2*( - sqrt(d + e*x)*sqrt(a*e + c*d*x)*e + sqrt(e)*sqrt(d)*sqrt(c)*log((sq
rt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d
**2))*d + sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)
*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*e*x - sqrt(e)*sqrt(d)*sqrt(
c)*d - sqrt(e)*sqrt(d)*sqrt(c)*e*x))/(e**2*(d + e*x))
```

3.24 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x(d+ex)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 112

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x(d+ex)^2}\,dx = \frac{2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d(d+ex)} - \frac{2\sqrt{a}\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{a}\sqrt{e}(d+ex)}\right)}{d^{3/2}}$$

output
```
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/(e*x+d)-2*a^(1/2)*e^(1/2)*arct
anh(d^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^(1/2)/e^(1/2)/(e*x+d
))/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x(d+ex)^2}\,dx = \frac{2\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{d}}{d+ex} - \frac{\sqrt{a}\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{d^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)^2),x]
```

output
```
(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[d]/(d + e*x) - (Sqrt[a]*Sqrt[e]*Arc
Tanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a
*e + c*d*x]*Sqrt[d + e*x])))/d^(3/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1214, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x(d+ex)^2}dx$$

$$\Big\downarrow \ 1214$$

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d(d+ex)}-\frac{\int -\frac{ae^3}{dx\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{ae^3}{dx\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^2}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d(d+ex)}$$

$$\Big\downarrow \ 27$$

$$\frac{ae\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{d}+\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d(d+ex)}$$

$$\Big\downarrow \ 1154$$

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d(d+ex)}-\frac{2ae\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{d}$$

$$\Big\downarrow \ 219$$

$$\frac{2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d(d+ex)}-\frac{\sqrt{a}\sqrt{e}\text{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{d^{3/2}}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x*(d + e*x)^2),x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d*(d + e*x)) - (Sqrt[a]*S
qrt[e]*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/d^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(96) = 192$.

Time = 2.65 (sec) , antiderivative size = 523, normalized size of antiderivative = 4.67

method	result
default	$\frac{\sqrt{ade+(a e^2+c d^2)x+cd x^2 e}+\frac{(a e^2+c d^2)\ln\left(\frac{\frac{1}{2}a e^2+\frac{1}{2}c d^2+cdxe}{\sqrt{dec}}+\sqrt{ade+(a e^2+c d^2)x+cd x^2 e}\right)}{2\sqrt{dec}}-\frac{ade\ln\left(\frac{2ade+(a e^2+c d^2)x+2\sqrt{ade}\sqrt{\cdots}}{x}\right.}{\sqrt{ade}}}{d^2}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/x/(e*x+d)^2,x,method=_RETURNVE
RBOSE)
```

output
```
1/d^2*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a
*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*
e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-1/d^2*((d*e*c*(x+d/e
)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*a*e^2-1/2*c*d^2
+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2
))/(d*e*c)^(1/2))-1/e/d*(-2/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^
2-c*d^2)*(x+d/e))^(3/2)+2*d*e*c/(a*e^2-c*d^2)*((d*e*c*(x+d/e)^2+(a*e^2-c*d
^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e)
)/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.97

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x(d+ex)^2}\,dx$$

$$= \left[\frac{(ex+d)\sqrt{\frac{ae}{d}}\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2-4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,ad^2e+(cd^3+ade^2)\,x)\,\sqrt{\frac{ae}{d}}+8\,(acd^3e+a^2de^3)\cdots}{x^2}\right.}{2\,(dex+d^2)}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d)^2,x, algorithm
="fricas")
```

output
```
[1/2*((e*x + d)*sqrt(a*e/d)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2
+ a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e
+ (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(d*e*x + d^2), ((e*x + d)*sq
rt(-a*e/d)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
 + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e
 + a^2*e^3)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(d*e*x +
d^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x(d + ex)^2}\,dx = \int \frac{\sqrt{(d + ex)\,(ae + cdx)}}{x\,(d + ex)^2}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x/(e*x+d)**2,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x(d + ex)^2}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)^2 x}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d)^2,x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)^2*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(96) = 192$.

Time = 0.16 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.79

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x(d + ex)^2} dx$$
$$= 2\left(\frac{a \arctan\left(\frac{\sqrt{cde - \frac{cd^2e}{ex+d} + \frac{ae^3}{ex+d}}d}{\sqrt{-ade}e}\right) \operatorname{sgn}\left(\frac{1}{ex+d}\right) \operatorname{sgn}(e)}{\sqrt{-ade}d|e|} - \frac{\left(ae^2 \arctan\left(\frac{\sqrt{cde}d}{\sqrt{-ade}e}\right) + \sqrt{-ade}\sqrt{cde}\right) \operatorname{sgn}\left(\frac{1}{ex+d}\right) \operatorname{sg}}{\sqrt{-ade}de^2|e|}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d)^2,x, algorithm
="giac")
```

output
```
2*(a*arctan(sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*d/(sqrt(-a*d
*e)*e))*sgn(1/(e*x + d))*sgn(e)/(sqrt(-a*d*e)*d*abs(e)) - (a*e^2*arctan(sq
rt(c*d*e)*d/(sqrt(-a*d*e)*e)) + sqrt(-a*d*e)*sqrt(c*d*e))*sgn(1/(e*x + d))
*sgn(e)/(sqrt(-a*d*e)*d*e^2*abs(e)) + sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e
^3/(e*x + d))*sgn(1/(e*x + d))*sgn(e)/(d*e^2*abs(e)))*e*abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x(d + ex)^2} dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{x\,(d + e\,x)^2} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x*(d + e*x)^2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x*(d + e*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 380, normalized size of antiderivative = 3.39

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x(d+ex)^2}\,dx$$
$$= \frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,de + \sqrt{e}\,\sqrt{d}\,\sqrt{a}\log\Bigl(\sqrt{e}\,\sqrt{cdx+ae} - \sqrt{2\sqrt{c}\,\sqrt{a}\,de + a\,e^2 + c\,d^2} + \sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d)^2,x)
```

output
```
(2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*d*e + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(
e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqr
t(d)*sqrt(c)*sqrt(d + e*x))*d*e + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt
(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sq
rt(c)*sqrt(d + e*x))*e**2*x + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e
 + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c
)*sqrt(d + e*x))*d*e + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(
d + e*x))*e**2*x - sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*s
qrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*d*e -
sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a
*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*e**2*x + 2*sqrt(e)*sqrt(d
)*sqrt(c)*d**2 + 2*sqrt(e)*sqrt(d)*sqrt(c)*d*e*x)/(d**2*e*(d + e*x))
```

3.25 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2(d+ex)^2} dx$

Optimal result

Integrand size = 40, antiderivative size = 185

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2(d+ex)^2} dx = \frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{adex(d+ex)^2} - \frac{(cd^2-3ae^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{a}\sqrt{e}(d+ex)}\right)}{\sqrt{a}d^{5/2}\sqrt{e}}$$

output
```
(c/a/e-3*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)-(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a/d/e/x/(e*x+d)^2-(-3*a*e^2+c*d^2)*arctanh(d
^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^(1/2)/e^(1/2)/(e*x+d))/a^
(1/2)/d^(5/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^2(d + ex)^2} dx$$
$$= \frac{\sqrt{(ae + cdx)(d + ex)}\left(-\frac{\sqrt{d}(d+3ex)}{x(d+ex)} + \frac{(-cd^2+3ae^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{a}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{d^{5/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)^2),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[d]*(d + 3*e*x))/(x*(d + e*x))) + (
(-(c*d^2) + 3*a*e^2)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*
Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/d^(5/
2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1214, 25, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{x^2(d + ex)^2} dx$$

$$\Big\downarrow \text{1214}$$

$$-\frac{\int -\frac{e^2\left(ae+d\left(c-\frac{ae^2}{d^2}\right)x\right)}{dx^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^2} - \frac{2e\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{d^2(d + ex)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{e^2\left(ae+d\left(c-\frac{ae^2}{d^2}\right)x\right)}{dx^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^2} - \frac{2e\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{d^2(d + ex)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{ae+d\left(c-\frac{ae^2}{d^2}\right)x}{x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{d}-\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2(d+ex)}$$

$$\Big\downarrow \text{1228}$$

$$\frac{\frac{1}{2}\left(cd-\frac{3ae^2}{d}\right)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{dx}}{d}-\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2(d+ex)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{-\left(\left(cd-\frac{3ae^2}{d}\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+(cd^2+ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}\right)-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{dx}}{d}-\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2(d+ex)}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{\left(cd-\frac{3ae^2}{d}\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}\sqrt{d}\sqrt{e}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{dx}}{d}-\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2(d+ex)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)^2),x]
```

output
```
(-2*e*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2*(d + e*x)) + (-(Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x)) - ((c*d - (3*a*e^2)/d)*Ar
cTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 919 vs. 2(167) = 334.

Time = 2.76 (sec) , antiderivative size = 920, normalized size of antiderivative = 4.97

method	result
default	$-\frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{adex}+\frac{\left(a\,e^{2}+c\,d^{2}\right)\left(\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}+\frac{\left(a\,e^{2}+c\,d^{2}\right)\ln\left(\frac{\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}+cdxe}{\sqrt{dec}}+\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd}\right.}{2\sqrt{dec}}\right.}{2ade}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/x^2/(e*x+d)^2,x,method=_RETURN
VERBOSE)
```

output
```
1/d^2*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2
)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2
*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^
(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*
d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*c/a*(1/4*(2*c*d*
e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*
d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1
/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+1/d^2*(-2/(a*
e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+2*d*e*c
/(a*e^2-c*d^2)*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c
*d^2)*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^
2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))-2/d^3*e*((a*d*e+(a*e^2+c*d
^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/
(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e
/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*e/d^3*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d
/e))^(1/2)+1/2*(a*e^2-c*d^2)*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c
)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.36

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2(d+ex)^2}\,dx$$

$$= \left[-\frac{\sqrt{ade}((cd^2e-3\,ae^3)x^2+(cd^3-3\,ade^2)x)\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2+4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,ad}{x^2}\right.}{4\,(ad^3e^2x^2+ad^4ex}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d)^2,x, algorit
hm="fricas")
```

output
```
[-1/4*(sqrt(a*d*e)*((c*d^2*e - 3*a*e^3)*x^2 + (c*d^3 - 3*a*d*e^2)*x)*log((
8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) +
8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(3*a*d*e^2*x + a*d^2*e)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^3*e^2*x^2 + a*d^4*e*x), 1/2*(sqrt(-a
*d*e)*((c*d^2*e - 3*a*e^3)*x^2 + (c*d^3 - 3*a*d*e^2)*x)*arctan(1/2*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a
*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(3*
a*d*e^2*x + a*d^2*e)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^3*e
^2*x^2 + a*d^4*e*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^2(d+ex)^2}\,dx = \int \frac{\sqrt{(d+ex)\,(ae+cdx)}}{x^2\,(d+ex)^2}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**2/(e*x+d)**2,x)
```

output
```
Integral(sqrt((d + e*x)*(a*e + c*d*x))/(x**2*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^2(d + ex)^2}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)^2 x^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d)^2,x, algorit
hm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)^2*x^2), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(167) = 334.

Time = 0.16 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.14

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^2(d + ex)^2}\,dx =$$

$$-\left(\frac{\left(cd^2\arctan\left(\frac{\sqrt{cded}}{\sqrt{-adee}}\right) - 3\,ae^2\arctan\left(\frac{\sqrt{cded}}{\sqrt{-adee}}\right) - 3\sqrt{-ade}\sqrt{cde}\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{\sqrt{-ade}d^2|e|} + \frac{2\sqrt{cde - \frac{cd^2e}{ex+\cdots}}}{\cdots}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d)^2,x, algorit
hm="giac")
```

output
```
-((c*d^2*arctan(sqrt(c*d*e)*d/(sqrt(-a*d*e)*e)) - 3*a*e^2*arctan(sqrt(c*d*
e)*d/(sqrt(-a*d*e)*e)) - 3*sqrt(-a*d*e)*sqrt(c*d*e))*sgn(1/(e*x + d))*sgn(
e)/(sqrt(-a*d*e)*d^2*abs(e)) + 2*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e
*x + d))*sgn(1/(e*x + d))*sgn(e)/(d^2*abs(e)) - (c*d^2*e*sgn(1/(e*x + d))*
sgn(e) - 3*a*e^3*sgn(1/(e*x + d))*sgn(e))*arctan(sqrt(c*d*e - c*d^2*e/(e*x
 + d) + a*e^3/(e*x + d))*d/(sqrt(-a*d*e)*e))/(sqrt(-a*d*e)*d^2*e*abs(e)) -
 (sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c*d^2*e*sgn(1/(e*x + d
))*sgn(e) - sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a*e^3*sgn(1/
(e*x + d))*sgn(e))/((a*e^3 - (c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))
*d)*d^2*abs(e)))*abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^2(d+ex)^2}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e}}{x^2\,{\left(d+e\,x\right)}^2}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^2*(d + e*x)^2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^2*(d + e*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 932, normalized size of antiderivative = 5.04

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^2(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d)^2,x)
```

output
```
( - 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*d**2*e**3 - 18*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**2*d*e**4*x - 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d**4
*e - 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d**3*e**2*x - 9*sqrt(e)*sqrt(d)
*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e*
*2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*d*e**4*x - 9*sqrt(e)*sq
rt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e +
 a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*e**5*x**2 + sqrt(e
)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d
*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**2*d**5*x + sqrt(
e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*
d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**2*d**4*e*x**2 -
 9*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*
sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*d*e**
4*x - 9*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqr
t(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*
e**5*x**2 + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2
*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c
**2*d**5*x + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(
2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*
c**2*d**4*e*x**2 + 9*sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d)*sqrt...
```

3.26 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3(d+ex)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 261

$$\begin{aligned}\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3(d+ex)^2}\,dx &= -\frac{(cd^2-15ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4ad^3(d+ex)} \\ &\quad -\frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2dx^2(d+ex)}-\frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4x(d+ex)} \\ &\quad +\frac{(c^2d^4+6acd^2e^2-15a^2e^4)\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{a}\sqrt{e}(d+ex)}\right)}{4a^{3/2}d^{7/2}e^{3/2}}\end{aligned}$$

output
```
-1/4*(-15*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^3/(e*x+
d)-1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x^2/(e*x+d)-1/4*(c/a/e-5*
e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x/(e*x+d)+1/4*(-15*a^2*e^4+
6*a*c*d^2*e^2+c^2*d^4)*arctanh(d^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/a^(1/2)/e^(1/2)/(e*x+d))/a^(3/2)/d^(7/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3(d+ex)^2}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-cd^2x(d+ex)+ae(-2d^2+5dex+15e^2x^2))}{x^2(d+ex)}+\frac{(c^2d^4+6acd^2e^2-15a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{ae+cdx}\sqrt{d+ex}}\right)}{4a^{3/2}d^{7/2}e^{3/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)^2),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-(c*d^2*x*(d + e
*x)) + a*e*(-2*d^2 + 5*d*e*x + 15*e^2*x^2)))/(x^2*(d + e*x)) + ((c^2*d^4 +
 6*a*c*d^2*e^2 - 15*a^2*e^4)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*
Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(4*a^(3/2)*d^
(7/2)*e^(3/2))
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1214, 25, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x^3(d+ex)^2}\,dx$$

$$\Big\downarrow \text{1214}$$

$$\frac{2e^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^3(d+ex)}-\frac{\int -\frac{-\frac{(cd^2-ae^2)x^2e^3}{d^3}+\frac{ae^3}{d}+\left(c-\frac{ae^2}{d^2}\right)xe^2}{x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{-\frac{(cd^2-ae^2)x^2e^3}{d^3}+\frac{ae^3}{d}+\left(c-\frac{ae^2}{d^2}\right)xe^2}{x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^2}+\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$\downarrow$ 2181

$$\frac{-\frac{\int -\frac{ae^3\left(cd^2-2e\left(3c-\frac{2ae^2}{d^2}\right)xd-7ae^2\right)}{2dx^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{e^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2d^2x^2}}{e^2}+\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$\downarrow$ 27

$$\frac{\frac{e^2\int \frac{cd^2-2e\left(3c-\frac{2ae^2}{d^2}\right)xd-7ae^2}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4d^2}-\frac{e^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2d^2x^2}}{e^2}+\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$\downarrow$ 1228

$$\frac{\frac{e^2\left(-\frac{\left(\frac{c^2d^4}{a}-15ae^4+6cd^2e^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2de}-\frac{\left(\frac{cd}{ae}-\frac{7e}{d}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}\right)}{4d^2}-\frac{e^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2d^2x^2}}{e^2}+\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$\downarrow$ 1154

$$\frac{\frac{e^2\left(\frac{\left(\frac{c^2d^4}{a}-15ae^4+6cd^2e^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{de}-\frac{\left(\frac{cd}{ae}-\frac{7e}{d}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}\right)}{4d^2}-\frac{e^2\sqrt{x(ae^2+cd}{2d}}{e^2}$$

$$\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$\downarrow$ 219

$$\frac{\frac{e^2\left(\frac{\left(\frac{c^2d^4}{a}-15ae^4+6cd^2e^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}d^{3/2}e^{3/2}}-\frac{\left(\frac{cd}{ae}-\frac{7e}{d}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}\right)}{4d^2}-\frac{e^2\sqrt{x(ae^2+cd^2)+ade+c}}{2d^2x^2}}{e^2}$$

$$\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^3*(d + e*x)^2),x]
```

output
```
(2*e^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^3*(d + e*x)) + (-1/
2*(e^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2*x^2) + (e^2*(-(((
(c*d)/(a*e) - (7*e)/d)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x) + (
((c^2*d^4)/a + 6*c*d^2*e^2 - 15*a*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*
x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
])/(2*Sqrt[a]*d^(3/2)*e^(3/2))))/(4*d^2))/e^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1566 vs. $2(233) = 466$.

Time = 3.00 (sec) , antiderivative size = 1567, normalized size of antiderivative = 6.00

method	result	size
default	Expression too large to display	1567

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/x^3/(e*x+d)^2,x,method=_RETURN
VERBOSE)
```

output
```
1/d^2*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(a*e^2+c
*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2
+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*l
n((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x
^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x
+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*c/a*(1/4*(
2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(
4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e
*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+1/2*c/a
*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+
1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/
(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1
/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))+3/d^4*e^2*((a*d*e+(a*e^2
+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x
*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a
*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-2/d^3*e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)
*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^
2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2
)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^...
```

Fricas [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 594, normalized size of antiderivative = 2.28

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^3(d+ex)^2}\,dx$$
$$= \left[-\frac{\left((c^2d^4e+6\,acd^2e^3-15\,a^2e^5)x^3+(c^2d^5+6\,acd^3e^2-15\,a^2de^4)x^2\right)\sqrt{ade}\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4\cdots}{\cdots}\right.}{\cdots}\right.$$
$$-\frac{\left((c^2d^4e+6\,acd^2e^3-15\,a^2e^5)x^3+(c^2d^5+6\,acd^3e^2-15\,a^2de^4)x^2\right)\sqrt{-ade}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)\cdots}}{2\,(acd^2e^2x^2+a^2d^2\cdots}\right.}{8\,(a^2\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d)^2,x, algorit
hm="fricas")
```

output
```
[-1/16*(((c^2*d^4*e + 6*a*c*d^2*e^3 - 15*a^2*e^5)*x^3 + (c^2*d^5 + 6*a*c*d
^3*e^2 - 15*a^2*d*e^4)*x^2)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*
a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/
x^2) + 4*(2*a^2*d^3*e^2 + (a*c*d^3*e^2 - 15*a^2*d*e^4)*x^2 + (a*c*d^4*e -
5*a^2*d^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^4*e^
3*x^3 + a^2*d^5*e^2*x^2), -1/8*(((c^2*d^4*e + 6*a*c*d^2*e^3 - 15*a^2*e^5)*
x^3 + (c^2*d^5 + 6*a*c*d^3*e^2 - 15*a^2*d*e^4)*x^2)*sqrt(-a*d*e)*arctan(1/
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)
) + 2*(2*a^2*d^3*e^2 + (a*c*d^3*e^2 - 15*a^2*d*e^4)*x^2 + (a*c*d^4*e - 5*a
^2*d^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^4*e^3*x
^3 + a^2*d^5*e^2*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^3(d + ex)^2}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**3/(e*x+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^3(d + ex)^2}\,dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)^2 x^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d)^2,x, algorit
hm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)^2*x^3), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(233) = 466$.

Time = 0.18 (sec) , antiderivative size = 703, normalized size of antiderivative = 2.69

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^3(d + ex)^2}\,dx = \text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d)^2,x, algorit
hm="giac")
```

output

```
1/4*(8*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*e*sgn(1/(e*x + d)
)*sgn(e)/(d^3*abs(e)) + (c^2*d^4*arctan(sqrt(c*d*e)*d/(sqrt(-a*d*e)*e)) +
6*a*c*d^2*e^2*arctan(sqrt(c*d*e)*d/(sqrt(-a*d*e)*e)) - 15*a^2*e^4*arctan(s
qrt(c*d*e)*d/(sqrt(-a*d*e)*e)) + sqrt(-a*d*e)*sqrt(c*d*e)*c*d^2 - 15*sqrt(
-a*d*e)*sqrt(c*d*e)*a*e^2)*sgn(1/(e*x + d))*sgn(e)/(sqrt(-a*d*e)*a*d^3*e*a
bs(e)) - (c^2*d^4*sgn(1/(e*x + d))*sgn(e) + 6*a*c*d^2*e^2*sgn(1/(e*x + d))
*sgn(e) - 15*a^2*e^4*sgn(1/(e*x + d))*sgn(e))*arctan(sqrt(c*d*e - c*d^2*e/
(e*x + d) + a*e^3/(e*x + d))*d/(sqrt(-a*d*e)*e))/(sqrt(-a*d*e)*a*d^3*e*abs
(e)) - (sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a*c^2*d^4*e^3*sg
n(1/(e*x + d))*sgn(e) + 6*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d)
)*a^2*c*d^2*e^5*sgn(1/(e*x + d))*sgn(e) - 7*sqrt(c*d*e - c*d^2*e/(e*x + d)
 + a*e^3/(e*x + d))*a^3*e^7*sgn(1/(e*x + d))*sgn(e) + (c*d*e - c*d^2*e/(e*
x + d) + a*e^3/(e*x + d))^(3/2)*c^2*d^5*sgn(1/(e*x + d))*sgn(e) - 10*(c*d*
e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*a*c*d^3*e^2*sgn(1/(e*x + d)
)*sgn(e) + 9*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*a^2*d*e^4
*sgn(1/(e*x + d))*sgn(e))/((a*e^3 - (c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*
x + d))*d)^2*a*d^3*abs(e)))*abs(e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^3(d + ex)^2}\,dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e}}{x^3\,{(d + e\,x)}^2}\,dx$$

input

```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^3*(d + e*x)^2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^3*(d + e*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 1897, normalized size of antiderivative = 7.27

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\,x + cdex^2}}{x^3(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d)^2,x)
```

output
```
( - 20*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*d**3*e**4 + 50*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**3*d**2*e**5*x + 150*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
*3*d*e**6*x**2 - 12*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d**5*e**2 + 20*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d**4*e**3*x + 80*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a**2*c*d**3*e**4*x**2 - 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
c**2*d**6*e*x - 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**5*e**2*x**2 +
75*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*
sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*d*e**
6*x**2 + 75*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2
*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a
**3*e**7*x**3 + 15*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) -
 sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d +
e*x))*a**2*c*d**3*e**4*x**2 + 15*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(
a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqr
t(c)*sqrt(d + e*x))*a**2*c*d**2*e**5*x**3 - 23*sqrt(e)*sqrt(d)*sqrt(a)*log
(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2)
 + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**5*e**2*x**2 - 23*sqrt(e)*sqrt(
d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*
e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**4*e**3*x**3 - 3*
sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*...
```

3.27 $\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4(d+ex)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 370

$$\begin{aligned}\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4(d+ex)^2}\,dx &= \frac{(3c^2d^4+10acd^2e^2-105a^2e^4)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24a^2d^4e(d+ex)} \\ &\quad - \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3dx^3(d+ex)} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12x^2(d+ex)} \\ &\quad + \frac{(3cd^2-7ae^2)\,(cd^2+5ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24a^2d^3e^2x(d+ex)} \\ &\quad - \frac{(c^3d^6+3ac^2d^4e^2+15a^2cd^2e^4-35a^3e^6)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{a}\sqrt{e}(d+ex)}\right)}{8a^{5/2}d^{9/2}e^{5/2}}\end{aligned}$$

output
```
1/24*(-105*a^2*e^4+10*a*c*d^2*e^2+3*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/a^2/d^4/e/(e*x+d)-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d
/x^3/(e*x+d)-1/12*(c/a/e-7*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/
x^2/(e*x+d)+1/24*(-7*a*e^2+3*c*d^2)*(5*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/x/(e*x+d)-1/8*(-35*a^3*e^6+15*a^2*c*d^2*e^4+
3*a*c^2*d^4*e^2+c^3*d^6)*arctanh(d^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)/a^(1/2)/e^(1/2)/(e*x+d))/a^(5/2)/d^(9/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4(d+ex)^2}\,dx$$
$$=\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-3c^2d^4x^2(d+ex)+2acd^2ex(d^2-4dex-5e^2x^2)+a^2e^2(8d^3-14d^2ex+35de^2x^2+105e^3x^3))}{x^3(d+ex)}-\frac{3(c^3}{}\right.}{24a^{5/2}d^{9/2}e^{5/2}}$$

input
```
Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)^2),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-3*c^2*d^4*x^2
*(d + e*x) + 2*a*c*d^2*e*x*(d^2 - 4*d*e*x - 5*e^2*x^2) + a^2*e^2*(8*d^3 -
14*d^2*e*x + 35*d*e^2*x^2 + 105*e^3*x^3)))/(x^3*(d + e*x))) - (3*(c^3*d^6
+ 3*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - 35*a^3*e^6)*ArcTanh[(Sqrt[d]*Sqrt[a
*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d +
 e*x])))/(24*a^(5/2)*d^(9/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {1214, 25, 2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x^4(d+ex)^2}\,dx$$

$$\Big\downarrow \ 1214$$

$$-\frac{\int -\frac{\frac{(cd^2-ae^2)x^3e^4}{d^4}-\frac{(cd^2-ae^2)x^2e^3}{d^3}+\frac{ae^3}{d}+\left(c-\frac{ae^2}{d^2}\right)xe^2}{x^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^2}-\frac{2e^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^4(d+ex)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\frac{(cd^2-ae^2)x^3e^4}{d^4}-\frac{(cd^2-ae^2)x^2e^3}{d^3}+\frac{ae^3}{d}+\left(c-\frac{ae^2}{d^2}\right)xe^2}{x^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^2}-\frac{2e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\Big\downarrow$ 2181

$$\frac{-\frac{\int -\frac{\frac{6a(cd^2-ae^2)x^2e^5}{d^3}-2a\left(5c-\frac{3ae^2}{d^2}\right)xe^4+\frac{a(cd^2-11ae^2)e^3}{d}}{2x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3ade}-\frac{e^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3d^2x^3}}{e^2}-\frac{2e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\frac{6a(cd^2-ae^2)x^2e^5}{d^3}-2a\left(5c-\frac{3ae^2}{d^2}\right)xe^4+a\left(cd-\frac{11ae^2}{d}\right)e^3}{x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6ade}-\frac{e^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3d^2x^3}}{e^2}-\frac{2e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\Big\downarrow$ 2181

$$\frac{\frac{-\frac{\int \frac{ae^3\left((cd^2-3ae^2)(3cd^2+19ae^2)+2de\left(\frac{12a^2e^4}{d^2}-23ace^2+c^2d^2\right)x\right)}{2dx^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2ade}-\frac{e^2\left(c-\frac{11ae^2}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2x^2}}{6ade}-\frac{e^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3d^2x^3}}{e^2}-\frac{2e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\Big\downarrow$ 27

$$\frac{\frac{-\frac{e^2\int \frac{(cd^2-3ae^2)(3cd^2+19ae^2)+2de\left(\frac{12a^2e^4}{d^2}-23ace^2+c^2d^2\right)x}{x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4d^2}-\frac{e^2\left(c-\frac{11ae^2}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2x^2}}{6ade}-\frac{e^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3d^2x^3}}{e^2}-\frac{2e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\Big\downarrow$ 1228

$$-\frac{\frac{e^2\left(-\frac{3\left(-35a^3e^6+15a^2cd^2e^4+3ac^2d^4e^2+c^3d^6\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(cd^2-3ae^2\right)\left(19ae^2+3cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{adex}\right)}{4d^2}-\frac{e^2\left(c-\frac{11ae\cdots}{d^2}\cdots\right.}{\cdots}}{6ade}\cdots}{e^2}$$

$$\frac{2e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\big\downarrow$ 1154

$$-\frac{\frac{e^2\left(\frac{3\left(-35a^3e^6+15a^2cd^2e^4+3ac^2d^4e^2+c^3d^6\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}-\frac{\left(cd^2-3ae^2\right)\left(19ae^2+3cd^2\right)\sqrt{x\left(ae^2+cd^2\right)\cdots}}{adex}\cdots\right.}{4d^2}\cdots}{6ade}\cdots}{e^2}$$

$$\frac{2e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\big\downarrow$ 219

$$-\frac{\frac{e^2\left(\frac{3\left(-35a^3e^6+15a^2cd^2e^4+3ac^2d^4e^2+c^3d^6\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(cd^2-3ae^2\right)\left(19ae^2+3cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{adex}\cdots\right.}{4d^2}\cdots}{6ade}\cdots}{e^2}$$

$$\frac{2e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

input
```
Int[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)^2),x]
```

output
```
(-2*e^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^4*(d + e*x)) + (-1
/3*(e^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2*x^3) + (-1/2*(e^
2*(c - (11*a*e^2)/d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^2 -
(e^2*(-(((c*d^2 - 3*a*e^2)*(3*c*d^2 + 19*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(a*d*e*x)) + (3*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 15*a^2*c*d
^2*e^4 - 35*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt
[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(3/2)*d^(3
/2)*e^(3/2))))/(4*d^2))/(6*a*d*e))/e^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2274 vs. $2(338) = 676$.

Time = 3.92 (sec) , antiderivative size = 2275, normalized size of antiderivative = 6.15

method	result	size
default	Expression too large to display	2275

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/x^4/(e*x+d)^2,x,method=_RETURN
VERBOSE)
```

output
```
1/d^2*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c
*d^2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(a
*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*
(a*e^2+c*d^2)/a/d/e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*
d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*
d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*c/a*
(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e
+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e
)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+1
/2*c/a*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*
a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(
1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d
*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))))+e^2/d^4*(-2/(a*e^
2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+2*d*e*c/(
a*e^2-c*d^2)*((d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+1/2*(a*e^2-c*d
^2)*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+
(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))-4/d^5*e^3*((a*d*e+(a*e^2+c*d
^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/
(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*...
```

Fricas [A] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 772, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{x^4(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d)^2,x, algorit
hm="fricas")
```

output
```
[-1/96*(3*((c^3*d^6*e + 3*a*c^2*d^4*e^3 + 15*a^2*c*d^2*e^5 - 35*a^3*e^7)*x
^4 + (c^3*d^7 + 3*a*c^2*d^5*e^2 + 15*a^2*c*d^3*e^4 - 35*a^3*d*e^6)*x^3)*sq
rt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^4*e^3 - (3*a
*c^2*d^5*e^2 + 10*a^2*c*d^3*e^4 - 105*a^3*d*e^6)*x^3 - (3*a*c^2*d^6*e + 8*
a^2*c*d^4*e^3 - 35*a^3*d^2*e^5)*x^2 + 2*(a^2*c*d^5*e^2 - 7*a^3*d^3*e^4)*x)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^5*e^4*x^4 + a^3*d^6*e
^3*x^3), 1/48*(3*((c^3*d^6*e + 3*a*c^2*d^4*e^3 + 15*a^2*c*d^2*e^5 - 35*a^3
*e^7)*x^4 + (c^3*d^7 + 3*a*c^2*d^5*e^2 + 15*a^2*c*d^3*e^4 - 35*a^3*d*e^6)*
x^3)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 +
 (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(8*a^3*d^4*e^3 - (3*a*c^2*d^5*e^2 + 10*a^
2*c*d^3*e^4 - 105*a^3*d*e^6)*x^3 - (3*a*c^2*d^6*e + 8*a^2*c*d^4*e^3 - 35*a
^3*d^2*e^5)*x^2 + 2*(a^2*c*d^5*e^2 - 7*a^3*d^3*e^4)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x))/(a^3*d^5*e^4*x^4 + a^3*d^6*e^3*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}{x^4(d + ex)^2}\, dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**4/(e*x+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)\, x + cdex^2}}{x^4(d + ex)^2}\, dx = \int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)^2 x^4}\, dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d)^2,x, algorit
hm="maxima")
```

output
```
integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)^2*x^4), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1133 vs. $2(338) = 676$.

Time = 0.21 (sec) , antiderivative size = 1133, normalized size of antiderivative = 3.06

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{x^4(d + ex)^2} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d)^2,x, algorit
hm="giac")
```

output
```
-1/24*(48*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*e^2*sgn(1/(e*x
 + d))*sgn(e)/(d^4*abs(e)) + (3*c^3*d^6*arctan(sqrt(c*d*e)*d/(sqrt(-a*d*e)
*e)) + 9*a*c^2*d^4*e^2*arctan(sqrt(c*d*e)*d/(sqrt(-a*d*e)*e)) + 45*a^2*c*d
^2*e^4*arctan(sqrt(c*d*e)*d/(sqrt(-a*d*e)*e)) - 105*a^3*e^6*arctan(sqrt(c*
d*e)*d/(sqrt(-a*d*e)*e)) + 3*sqrt(-a*d*e)*sqrt(c*d*e)*c^2*d^4 + 10*sqrt(-a
*d*e)*sqrt(c*d*e)*a*c*d^2*e^2 - 105*sqrt(-a*d*e)*sqrt(c*d*e)*a^2*e^4)*sgn(
1/(e*x + d))*sgn(e)/(sqrt(-a*d*e)*a^2*d^4*e^2*abs(e)) - 3*(c^3*d^6*sgn(1/(
e*x + d))*sgn(e) + 3*a*c^2*d^4*e^2*sgn(1/(e*x + d))*sgn(e) + 15*a^2*c*d^2*
e^4*sgn(1/(e*x + d))*sgn(e) - 35*a^3*e^6*sgn(1/(e*x + d))*sgn(e))*arctan(s
qrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*d/(sqrt(-a*d*e)*e))/(sqrt
(-a*d*e)*a^2*d^4*e^2*abs(e)) - (3*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(
e*x + d))*a^2*c^3*d^6*e^6*sgn(1/(e*x + d))*sgn(e) + 9*sqrt(c*d*e - c*d^2*e
/(e*x + d) + a*e^3/(e*x + d))*a^3*c^2*d^4*e^8*sgn(1/(e*x + d))*sgn(e) + 45
*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^4*c*d^2*e^10*sgn(1/(e
*x + d))*sgn(e) - 57*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^5
*e^12*sgn(1/(e*x + d))*sgn(e) + 8*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x
+ d))^(3/2)*a*c^3*d^7*e^3*sgn(1/(e*x + d))*sgn(e) - 24*(c*d*e - c*d^2*e/(e
*x + d) + a*e^3/(e*x + d))^(3/2)*a^2*c^2*d^5*e^5*sgn(1/(e*x + d))*sgn(e) -
 120*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*a^3*c*d^3*e^7*sgn
(1/(e*x + d))*sgn(e) + 136*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ade + (cd^2 + ae^2) x + cdex^2}}{x^4(d + ex)^2} dx = \int \frac{\sqrt{c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\;x + a\,d\,e}}{x^4\,(d + e\,x)^2} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^4*(d + e*x)^2),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(x^4*(d + e*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 2498, normalized size of antiderivative = 6.75

$$\int \frac{\sqrt{ade + (cd^2 + ae^2) x + cdex^2}}{x^4(d + ex)^2} dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d)^2,x)
```

output
```
( - 112*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*d**4*e**5 + 196*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**4*d**3*e**6*x - 490*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a**4*d**2*e**7*x**2 - 1470*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*d*e**8*x**
3 - 80*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**6*e**3 + 112*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*a**3*c*d**5*e**4*x - 238*sqrt(d + e*x)*sqrt(a*e + c*d*
x)*a**3*c*d**4*e**5*x**2 - 910*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**3
*e**6*x**3 - 20*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**7*e**2*x + 12
2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**6*e**3*x**2 + 142*sqrt(d +
e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**5*e**4*x**3 + 30*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a*c**3*d**8*e*x**2 + 30*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*
d**7*e**2*x**3 - 735*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x)
 - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))*a**4*d*e**8*x**3 - 735*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*
e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(
c)*sqrt(d + e*x))*a**4*e**9*x**4 - 210*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)
*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(
d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**3*e**6*x**3 - 210*sqrt(e)*sqrt(d)*sqrt
(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 +
c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**2*e**7*x**4 + 288*sqrt(
e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(...
```

3.28 $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 434

$$\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{\left(21c^4d^8-6a^2c^2d^4e^4-8a^3cd^2e^6-7a^4e^8\right)\left(cd^2+ae^2+2cdex\right)}{512c^4d^4e^5}$$

$$+\frac{1}{20}\left(\frac{a}{cd}-\frac{3d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}+\frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6e}$$

$$-\frac{\left(105c^3d^6-21ac^2d^4e^2-33a^2cd^2e^4-35a^3e^6-6cde\left(21c^2d^4-6acd^2e^2-7a^2e^4\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+\right.}{960c^3d^3e^4}$$

$$-\frac{\left(cd^2-ae^2\right)^3\left(21c^3d^6+21ac^2d^4e^2+15a^2cd^2e^4+7a^3e^6\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{512c^{9/2}d^{9/2}e^{11/2}}$$

output
```
1/512*(-7*a^4*e^8-8*a^3*c*d^2*e^6-6*a^2*c^2*d^4*e^4+21*c^4*d^8)*(2*c*d*e*x
+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^5+1/20*(a/
c/d-3*d/e^2)*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/6*x^3*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e-1/960*(105*c^3*d^6-21*a*c^2*d^4*e^2-33*a^
2*c*d^2*e^4-35*a^3*e^6-6*c*d*e*(-7*a^2*e^4-6*a*c*d^2*e^2+21*c^2*d^4)*x)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e^4-1/512*(-a*e^2+c*d^2)^3*(
7*a^3*e^6+15*a^2*c*d^2*e^4+21*a*c^2*d^4*e^2+21*c^3*d^6)*arctanh(c^(1/2)*d^
(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(9/2)/d^(
9/2)/e^(11/2)
```

Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.89

$$\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(-105a^5e^{10}+5a^4cde^8(11d+\right.}{}$$

input
```
Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-105*a^5*e^10 + 5
*a^4*c*d*e^8*(11*d + 14*e*x) + 2*a^3*c^2*d^2*e^6*(27*d^2 - 16*d*e*x - 28*e
^2*x^2) + 6*a^2*c^3*d^3*e^4*(13*d^3 - 6*d^2*e*x + 4*d*e^2*x^2 + 8*e^3*x^3)
 + a*c^4*d^4*e^2*(-525*d^4 + 336*d^3*e*x - 264*d^2*e^2*x^2 + 224*d*e^3*x^3
 + 1664*e^4*x^4) + c^5*d^5*(315*d^5 - 210*d^4*e*x + 168*d^3*e^2*x^2 - 144*
d^2*e^3*x^3 + 128*d*e^4*x^4 + 1280*e^5*x^5)) - (15*(c*d^2 - a*e^2)^3*(21*c
^3*d^6 + 21*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6)*ArcTanh[(Sqrt[c]
*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*S
qrt[d + e*x])))/(7680*c^(9/2)*d^(9/2)*e^(11/2))
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {1215, 1236, 27, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{d+ex}dx$$

$$\Big\downarrow \text{1215}$$

$$\int x^3(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{3}{2}cdx^2\left(2ade+\left(3cd^2-ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{6cde}+\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}$$

$$\Big\downarrow \; 27$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-\frac{\int x^2\left(2ade+\left(3cd^2-ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{4e}$$

$$\Big\downarrow \; 1236$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-\frac{\frac{\int -\frac{1}{2}x\left(4ade\left(3cd^2-ae^2\right)+\left(21c^2d^4-6ace^2d^2-7a^2e^4\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{5cde}+\frac{1}{5}x^2\left(\frac{3d}{e}-\frac{ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right.}{4e}$$

$$\Big\downarrow \; 27$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-\frac{\frac{1}{5}x^2\left(\frac{3d}{e}-\frac{ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}-\frac{\int x\left(4ade\left(3cd^2-ae^2\right)+\left(21c^2d^4-6ace^2d^2-7a^2e^4\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{10cde}}{4e}$$

$$\Big\downarrow \; 1225$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-\frac{\frac{1}{5}x^2\left(\frac{3d}{e}-\frac{ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}-\frac{\frac{5\left(-7a^4e^8-8a^3cd^2e^6-6a^2c^2d^4e^4+21c^4d^8\right)\int\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16c^2d^2e^2}-\frac{\left(-35a^3e\right.}{}}{}}{4e}$$

$$\Big\downarrow \; 1087$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-\frac{\frac{1}{5}x^2\left(\frac{3d}{e}-\frac{ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}-\frac{5\left(-7a^4e^8-8a^3cd^2e^6-6a^2c^2d^4e^4+21c^4d^8\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{4cde}\right.}{16c^2d^2e^2}}{}$$

$$\Big\downarrow \; 1092$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-$$

$$\frac{\frac{1}{5}x^2\left(\frac{3d}{e}-\frac{ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}-\frac{5\left(-7a^4e^8-8a^3cd^2e^6-6a^2c^2d^4e^4+21c^4d^8\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{4cde}\right.}{16c^2 \ldots}}{\ldots}$$

$$\Big\downarrow \; 219$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6e}-$$

$$\frac{\frac{1}{5}x^2\left(\frac{3d}{e}-\frac{ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}-\frac{5\left(-7a^4e^8-8a^3cd^2e^6-6a^2c^2d^4e^4+21c^4d^8\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+c}}{4cde}\right.}{16c^2d^2e^2 \ldots}}{\ldots}$$

input
```
Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

output
```
(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(6*e) - (((((3*d)/e - (
a*e)/(c*d))*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/5 - (-1/24*
((105*c^3*d^6 - 21*a*c^2*d^4*e^2 - 33*a^2*c*d^2*e^4 - 35*a^3*e^6 - 6*c*d*e
*(21*c^2*d^4 - 6*a*c*d^2*e^2 - 7*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2))/(c^2*d^2*e^2) + (5*(21*c^4*d^8 - 6*a^2*c^2*d^4*e^4 - 8*a
^3*c*d^2*e^6 - 7*a^4*e^8)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 +
 a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c^2*d^2*e^2))/(10*c*
d*e))/(4*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. $2(402) = 804$.

Time = 2.58 (sec) , antiderivative size = 1426, normalized size of antiderivative = 3.29

method	result	size
default	Expression too large to display	1426

input
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
d^2/e^3*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/
2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+
c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a
*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+1/e*(1/6*x*(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(5/2)/d/e/c-7/12*(a*e^2+c*d^2)/d/e/c*(1/5*(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/d/e/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/8*(2*c*d*e*
x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d
^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*l
n((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x
^2*e)^(1/2))/(d*e*c)^(1/2))))-1/6*a/c*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/
2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2
+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(
1/2))))-d/e^2*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/d/e/c-1/2*(a*e^
2+c*d^2)/d/e/c*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x
+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1044, normalized size of antiderivative = 2.41

$$\int \frac{x^3(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{d + ex}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/30720*(15*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 4*a^
3*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(c*
d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*
e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(1280*c^6*d^6*e^6*x^5 + 315*c^6*d^11
*e - 525*a*c^5*d^9*e^3 + 78*a^2*c^4*d^7*e^5 + 54*a^3*c^3*d^5*e^7 + 55*a^4*
c^2*d^3*e^9 - 105*a^5*c*d*e^11 + 128*(c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*x^4
- 16*(9*c^6*d^8*e^4 - 14*a*c^5*d^6*e^6 - 3*a^2*c^4*d^4*e^8)*x^3 + 8*(21*c^
6*d^9*e^3 - 33*a*c^5*d^7*e^5 + 3*a^2*c^4*d^5*e^7 - 7*a^3*c^3*d^3*e^9)*x^2
- 2*(105*c^6*d^10*e^2 - 168*a*c^5*d^8*e^4 + 18*a^2*c^4*d^6*e^6 + 16*a^3*c^
3*d^4*e^8 - 35*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x))/(c^5*d^5*e^6), 1/15360*(15*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 15*a^
2*c^4*d^8*e^4 + 4*a^3*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^10 -
 7*a^6*e^12)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2
*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(1280*c^6*d^6*e^6*x^5 + 315*c^6*d^1
1*e - 525*a*c^5*d^9*e^3 + 78*a^2*c^4*d^7*e^5 + 54*a^3*c^3*d^5*e^7 + 55*a^4
*c^2*d^3*e^9 - 105*a^5*c*d*e^11 + 128*(c^6*d^7*e^5 + 13*a*c^5*d^5*e^7)*x^4
 - 16*(9*c^6*d^8*e^4 - 14*a*c^5*d^6*e^6 - 3*a^2*c^4*d^4*e^8)*x^3 + 8*(21*c
^6*d^9*e^3 - 33*a*c^5*d^7*e^5 + 3*a^2*c^4*d^5*e^7 - 7*a^3*c^3*d^3*e^9)*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 984 vs. $2(442) = 884$.

Time = 82.71 (sec) , antiderivative size = 2502, normalized size of antiderivative = 5.76

$$\int \frac{x^3(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{d + ex}\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

output
```
a*e*Piecewise(((-a*(-3*a*(a*e**2/10 + c*d**2/10)/(4*c) - (5*a*e**2/2 + 5*c
*d**2/2)*(a*d*e/5 - (a*e**2/10 + c*d**2/10)*(7*a*e**2/2 + 7*c*d**2/2)/(4*c
*d*e))/(3*c*d*e))/(2*c) - (a*e**2 + c*d**2)*(-2*a*(a*d*e/5 - (a*e**2/10 +
c*d**2/10)*(7*a*e**2/2 + 7*c*d**2/2)/(4*c*d*e))/(3*c) - (3*a*e**2/2 + 3*c*
d**2/2)*(-3*a*(a*e**2/10 + c*d**2/10)/(4*c) - (5*a*e**2/2 + 5*c*d**2/2)*(a
*d*e/5 - (a*e**2/10 + c*d**2/10)*(7*a*e**2/2 + 7*c*d**2/2)/(4*c*d*e))/(3*c
*d*e))/(2*c*d*e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x +
2*sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e),
 Ne(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/
(2*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2
- c*d**2)/(2*c*d*e))**2), True)) + sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c
*d**2))*(x**4/5 + x**3*(a*e**2/10 + c*d**2/10)/(4*c*d*e) + x**2*(a*d*e/5 -
 (a*e**2/10 + c*d**2/10)*(7*a*e**2/2 + 7*c*d**2/2)/(4*c*d*e))/(3*c*d*e) +
x*(-3*a*(a*e**2/10 + c*d**2/10)/(4*c) - (5*a*e**2/2 + 5*c*d**2/2)*(a*d*e/5
 - (a*e**2/10 + c*d**2/10)*(7*a*e**2/2 + 7*c*d**2/2)/(4*c*d*e))/(3*c*d*e))
/(2*c*d*e) + (-2*a*(a*d*e/5 - (a*e**2/10 + c*d**2/10)*(7*a*e**2/2 + 7*c*d*
*2/2)/(4*c*d*e))/(3*c) - (3*a*e**2/2 + 3*c*d**2/2)*(-3*a*(a*e**2/10 + c*d*
*2/10)/(4*c) - (5*a*e**2/2 + 5*c*d**2/2)*(a*d*e/5 - (a*e**2/10 + c*d**2/10
)*(7*a*e**2/2 + 7*c*d**2/2)/(4*c*d*e))/(3*c*d*e))/(2*c*d*e))/(c*d*e)), Ne(
c*d*e, 0)), (2*(-a**3*d**3*e**3*(a*d*e + x*(a*e**2 + c*d**2))**(3/2)/3 ...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.18

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{1}{7680}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(2\left(8\left(10\,cdx+\frac{c^6d\right.\right.\right.\right.\right.$$
$$+\frac{(21\,c^6d^{12}-42\,ac^5d^{10}e^2+15\,a^2c^4d^8e^4+4\,a^3c^3d^6e^6+3\,a^4c^2d^4e^8+6\,a^5cd^2e^{10}-7\,a^6e^{12})\log\left(\left|-cd^2-ae^2\right.\right.}{1024\sqrt{cde}c^4d^4e^5}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm
="giac")
```

output
```
1/7680*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(10*c*d*x +
 (c^6*d^7*e^4 + 13*a*c^5*d^5*e^6)/(c^5*d^5*e^5))*x - (9*c^6*d^8*e^3 - 14*a
*c^5*d^6*e^5 - 3*a^2*c^4*d^4*e^7)/(c^5*d^5*e^5))*x + (21*c^6*d^9*e^2 - 33*
a*c^5*d^7*e^4 + 3*a^2*c^4*d^5*e^6 - 7*a^3*c^3*d^3*e^8)/(c^5*d^5*e^5))*x -
(105*c^6*d^10*e - 168*a*c^5*d^8*e^3 + 18*a^2*c^4*d^6*e^5 + 16*a^3*c^3*d^4*
e^7 - 35*a^4*c^2*d^2*e^9)/(c^5*d^5*e^5))*x + (315*c^6*d^11 - 525*a*c^5*d^9
*e^2 + 78*a^2*c^4*d^7*e^4 + 54*a^3*c^3*d^5*e^6 + 55*a^4*c^2*d^3*e^8 - 105*
a^5*c*d*e^10)/(c^5*d^5*e^5)) + 1/1024*(21*c^6*d^12 - 42*a*c^5*d^10*e^2 + 1
5*a^2*c^4*d^8*e^4 + 4*a^3*c^3*d^6*e^6 + 3*a^4*c^2*d^4*e^8 + 6*a^5*c*d^2*e^
10 - 7*a^6*e^12)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - s
qrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^4*d^4*e^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \int \frac{x^3\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{d+e\,x}\,dx$$

input
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
```

output
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1033, normalized size of antiderivative = 2.38

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \text{Too large to display}$$

input
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

output
```
( - 105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d*e**11 + 55*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**4*c**2*d**3*e**9 + 70*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a**4*c**2*d**2*e**10*x + 54*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**
5*e**7 - 32*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**4*e**8*x - 56*sqr
t(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**3*e**9*x**2 + 78*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**2*c**4*d**7*e**5 - 36*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a**2*c**4*d**6*e**6*x + 24*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**5*
e**7*x**2 + 48*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**4*e**8*x**3 -
525*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**9*e**3 + 336*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a*c**5*d**8*e**4*x - 264*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a*c**5*d**7*e**5*x**2 + 224*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**6*e*
*6*x**3 + 1664*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**5*e**7*x**4 + 315
*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**11*e - 210*sqrt(d + e*x)*sqrt(a*e
 + c*d*x)*c**6*d**10*e**2*x + 168*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**
9*e**3*x**2 - 144*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**8*e**4*x**3 + 12
8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**7*e**5*x**4 + 1280*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*c**6*d**6*e**6*x**5 + 105*sqrt(e)*sqrt(d)*sqrt(c)*log((
sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c
*d**2))*a**6*e**12 - 90*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**5*c*d**...
```

3.29 $\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 337

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = -\frac{(cd^2-ae^2)(7c^2d^4+6acd^2e^2+3a^2e^4)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128c^3d^3e^4} + \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5e} + \frac{(35c^2d^4-12acd^2e^2-15a^2e^4-6cde(7cd^2-3ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{240c^2d^2e^3} + \frac{(cd^2-ae^2)^3(7c^2d^4+6acd^2e^2+3a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128c^{7/2}d^{7/2}e^{9/2}}$$

output
```
-1/128*(-a*e^2+c*d^2)*(3*a^2*e^4+6*a*c*d^2*e^2+7*c^2*d^4)*(2*c*d*e*x+a*e^2
+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4+1/5*x^2*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e+1/240*(35*c^2*d^4-12*a*c*d^2*e^2-15*a^
2*e^4-6*c*d*e*(-3*a*e^2+7*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)/c^2/d^2/e^3+1/128*(-a*e^2+c*d^2)^3*(3*a^2*e^4+6*a*c*d^2*e^2+7*c^2*d^4)*a
rctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/c^(7/2)/d^(7/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.90

$$\int \frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(45a^4e^8-30a^3cde^6(d+ex)-\right.}{}$$

input
```
Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(45*a^4*e^8 - 30*a
^3*c*d*e^6*(d + e*x) - 6*a^2*c^2*d^2*e^4*(6*d^2 - 3*d*e*x - 4*e^2*x^2) + 2
*a*c^3*d^3*e^2*(95*d^3 - 61*d^2*e*x + 48*d*e^2*x^2 + 264*e^3*x^3) + c^4*d^
4*(-105*d^4 + 70*d^3*e*x - 56*d^2*e^2*x^2 + 48*d*e^3*x^3 + 384*e^4*x^4)) +
 (15*(c*d^2 - a*e^2)^3*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(Sq
rt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d
*x]*Sqrt[d + e*x])))/(1920*c^(7/2)*d^(7/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1215, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{d+ex}dx$$

$$\Big\downarrow \; 1215$$

$$\int x^2(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \; 1236$$

$$\frac{\int -\frac{1}{2}cdx\left(4ade+\left(7cd^2-3ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{5cde}+\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}$$

$\downarrow$ 27

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-\frac{\int x\left(4ade+\left(7cd^2-3ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{10e}$$

$\downarrow$ 1225

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-\frac{\frac{5\left(cd^2-ae^2\right)\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\int \sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16c^2d^2e^2}-\frac{\left(-15a^2e^4-6cdex\left(7cd^2-3ae^2\right)-12acd^2e^2+35c^2d^4\right)\left(x\left(ae^2+cd^2\right)+ade\right.}{24c^2d^2e^2}}{10e}$$

$\downarrow$ 1087

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-\frac{\frac{5\left(cd^2-ae^2\right)\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}\right)}{16c^2d^2e^2}-\frac{\left(-15a^2e^4-\right.}{}}{10e}$$

$\downarrow$ 1092

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-\frac{\frac{5\left(cd^2-ae^2\right)\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd}{\sqrt{cdex^2+\left(cd^2+\right.}}}{4cde}\right.}{16c^2d^2e^2}}{10e}$$

$\downarrow$ 219

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5e}-\frac{\frac{5\left(cd^2-ae^2\right)\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\right.}{16c^2d^2e^2}}{10e}$$

input
```
Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

output
```
(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(5*e) - (-1/24*((35*c^
2*d^4 - 12*a*c*d^2*e^2 - 15*a^2*e^4 - 6*c*d*e*(7*c*d^2 - 3*a*e^2)*x)*(a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(c^2*d^2*e^2) + (5*(c*d^2 - a*e^
2)*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*(((c*d^2 + a*e^2 + 2*c*d*e*x)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2
*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c^2
*d^2*e^2))/(10*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(309) = 618$.

Time = 2.56 (sec) , antiderivative size = 797, normalized size of antiderivative = 2.36

method	result
default	$\frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{2}}}{5dec}-\frac{\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\left(\frac{\left(2cdxe+a\,e\right.}{}\right.}{}\right.}{e}$

input
```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/d/e/c-1/2*(a*e^2+c*d^2)/d
/e/c*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/
c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d
^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^
2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a
*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))))+d^2/e^3*(1/3*(d*e*c*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*d*e*c*(x+d/e)+
a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^
2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e
*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))-d/e^2*(1/8*(2*c
*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*
a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/
e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 846, normalized size of antiderivative = 2.51

$$\int \frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/7680*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2*a^3*c^
2*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^
2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d
*e^3)*x) - 4*(384*c^5*d^5*e^5*x^4 - 105*c^5*d^9*e + 190*a*c^4*d^7*e^3 - 36
*a^2*c^3*d^5*e^5 - 30*a^3*c^2*d^3*e^7 + 45*a^4*c*d*e^9 + 48*(c^5*d^6*e^4 +
 11*a*c^4*d^4*e^6)*x^3 - 8*(7*c^5*d^7*e^3 - 12*a*c^4*d^5*e^5 - 3*a^2*c^3*d
^3*e^7)*x^2 + 2*(35*c^5*d^8*e^2 - 61*a*c^4*d^6*e^4 + 9*a^2*c^3*d^4*e^6 - 1
5*a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^
4*e^5), -1/3840*(15*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^2*c^3*d^6*e^4 + 2
*a^3*c^2*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqr
t(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2
*(384*c^5*d^5*e^5*x^4 - 105*c^5*d^9*e + 190*a*c^4*d^7*e^3 - 36*a^2*c^3*d^5
*e^5 - 30*a^3*c^2*d^3*e^7 + 45*a^4*c*d*e^9 + 48*(c^5*d^6*e^4 + 11*a*c^4*d^
4*e^6)*x^3 - 8*(7*c^5*d^7*e^3 - 12*a*c^4*d^5*e^5 - 3*a^2*c^3*d^3*e^7)*x^2
+ 2*(35*c^5*d^8*e^2 - 61*a*c^4*d^6*e^4 + 9*a^2*c^3*d^4*e^6 - 15*a^3*c^2*d^
2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^5)]
```

Sympy [A] (verification not implemented)

Time = 53.25 (sec) , antiderivative size = 1637, normalized size of antiderivative = 4.86

$$\int \frac{x^2(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{d + ex}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

output
```
a*e*Piecewise(((-a*(a*d*e/4 - (a*e**2/8 + c*d**2/8)*(5*a*e**2/2 + 5*c*d**2
/2)/(3*c*d*e))/(2*c) - (a*e**2 + c*d**2)*(-2*a*(a*e**2/8 + c*d**2/8)/(3*c)
 - (3*a*e**2/2 + 3*c*d**2/2)*(a*d*e/4 - (a*e**2/8 + c*d**2/8)*(5*a*e**2/2
+ 5*c*d**2/2)/(3*c*d*e))/(2*c*d*e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d
**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d*
*2)))/sqrt(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (
-a*e**2 - c*d**2)/(2*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*
d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*e))**2), True)) + sqrt(a*d*e + c*d*e*x*
*2 + x*(a*e**2 + c*d**2))*(x**3/4 + x**2*(a*e**2/8 + c*d**2/8)/(3*c*d*e) +
 x*(a*d*e/4 - (a*e**2/8 + c*d**2/8)*(5*a*e**2/2 + 5*c*d**2/2)/(3*c*d*e))/(
2*c*d*e) + (-2*a*(a*e**2/8 + c*d**2/8)/(3*c) - (3*a*e**2/2 + 3*c*d**2/2)*(
a*d*e/4 - (a*e**2/8 + c*d**2/8)*(5*a*e**2/2 + 5*c*d**2/2)/(3*c*d*e))/(2*c*
d*e))/(c*d*e)), Ne(c*d*e, 0)), (2*(a**2*d**2*e**2*(a*d*e + x*(a*e**2 + c*d
**2))**(3/2)/3 - 2*a*d*e*(a*d*e + x*(a*e**2 + c*d**2))**(5/2)/5 + (a*d*e +
 x*(a*e**2 + c*d**2))**(7/2)/7)/(a*e**2 + c*d**2)**3, Ne(a*e**2 + c*d**2,
0)), (x**3*sqrt(a*d*e)/3, True)) + c*d*Piecewise(((-a*(-3*a*(a*e**2/10 + c
*d**2/10)/(4*c) - (5*a*e**2/2 + 5*c*d**2/2)*(a*d*e/5 - (a*e**2/10 + c*d**2
/10)*(7*a*e**2/2 + 7*c*d**2/2)/(4*c*d*e))/(3*c*d*e))/(2*c) - (a*e**2 + c*d
**2)*(-2*a*(a*d*e/5 - (a*e**2/10 + c*d**2/10)*(7*a*e**2/2 + 7*c*d**2/2)/(4
*c*d*e))/(3*c) - (3*a*e**2/2 + 3*c*d**2/2)*(-3*a*(a*e**2/10 + c*d**2/10...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.20

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{1}{1920}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\left(6\left(8\,cdx+\frac{c^5d^6e^3-}{c}\right.\right.\right.\right.$$
$$-\frac{(7\,c^5d^{10}-15\,ac^4d^8e^2+6\,a^2c^3d^6e^4+2\,a^3c^2d^4e^6+3\,a^4cd^2e^8-3\,a^5e^{10})\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cdex}\right.\right.\right.}{256\sqrt{cde}c^3d^3e^4}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm
="giac")
```

output
```
1/1920*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*(8*c*d*x + (c^
5*d^6*e^3 + 11*a*c^4*d^4*e^5)/(c^4*d^4*e^4))*x - (7*c^5*d^7*e^2 - 12*a*c^4
*d^5*e^4 - 3*a^2*c^3*d^3*e^6)/(c^4*d^4*e^4))*x + (35*c^5*d^8*e - 61*a*c^4*
d^6*e^3 + 9*a^2*c^3*d^4*e^5 - 15*a^3*c^2*d^2*e^7)/(c^4*d^4*e^4))*x - (105*
c^5*d^9 - 190*a*c^4*d^7*e^2 + 36*a^2*c^3*d^5*e^4 + 30*a^3*c^2*d^3*e^6 - 45
*a^4*c*d*e^8)/(c^4*d^4*e^4)) - 1/256*(7*c^5*d^10 - 15*a*c^4*d^8*e^2 + 6*a^
2*c^3*d^6*e^4 + 2*a^3*c^2*d^4*e^6 + 3*a^4*c*d^2*e^8 - 3*a^5*e^10)*log(abs(
-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^3*d^3*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \int \frac{x^2\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{d+e\,x}\,dx$$

input
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
```

output
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 790, normalized size of antiderivative = 2.34

$$\int \frac{x^2 (ade + (cd^2 + ae^2) x + cdex^2)^{3/2}}{d + ex} \, dx = \text{Too large to display}$$

input

```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

output

```
(45*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d*e**9 - 30*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a**3*c**2*d**3*e**7 - 30*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3
*c**2*d**2*e**8*x - 36*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**5*e**5
 + 18*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**4*e**6*x + 24*sqrt(d +
e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**7*x**2 + 190*sqrt(d + e*x)*sqrt(a
*e + c*d*x)*a*c**4*d**7*e**3 - 122*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*
d**6*e**4*x + 96*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**5*e**5*x**2 + 5
28*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e**6*x**3 - 105*sqrt(d + e*
x)*sqrt(a*e + c*d*x)*c**5*d**9*e + 70*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5
*d**8*e**2*x - 56*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**7*e**3*x**2 + 48
*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**6*e**4*x**3 + 384*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*c**5*d**5*e**5*x**4 - 45*sqrt(e)*sqrt(d)*sqrt(c)*log((sqr
t(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d*
*2))*a**5*e**10 + 45*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x
) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*c*d**2*e**8
 + 30*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqr
t(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c**2*d**4*e**6 + 90*sqrt(e
)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**3*d**6*e**4 - 225*sqrt(e)*sqrt(d)*s
qrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/...
```

3.30 $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 244

$$\int \frac{x(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx = -\frac{\left(\frac{2ad^2}{c}-\frac{5d^4}{e^2}+\frac{3a^2e^2}{c^2}\right)(cd^2+ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{64d^2e} - \frac{(5cd^2-3ae^2-6cdex)\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{24cde^2} - \frac{(cd^2-ae^2)^3\,(5cd^2+3ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{64c^{5/2}d^{5/2}e^{7/2}}$$

output
```
-1/64*(2*a*d^2/c-5*d^4/e^2+3*a^2*e^2/c^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d^2/e-1/24*(-6*c*d*e*x-3*a*e^2+5*c*d^2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d/e^2-1/64*(-a*e^2+c*d^2)^3*(3*a*e
^2+5*c*d^2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.97

$$\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(-9a^3e^6+3a^2cde^4(3d+2ex)\right.}{}$$

input
```
Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-9*a^3*e^6 + 3*a^
2*c*d*e^4*(3*d + 2*e*x) + a*c^2*d^2*e^2*(-31*d^2 + 20*d*e*x + 72*e^2*x^2)
+ c^3*d^3*(15*d^3 - 10*d^2*e*x + 8*d*e^2*x^2 + 48*e^3*x^3)) - (3*(c*d^2 -
a*e^2)^3*(5*c*d^2 + 3*a*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt
[e]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(192*c^(5/2)*
d^(5/2)*e^(7/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1215, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{d+ex}dx$$

$$\Big\downarrow \text{1215}$$

$$\int x(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}dx$$

$$\Big\downarrow \text{1225}$$

$$-\frac{\left(\frac{3a^2e^2}{c}+2ad^2-\frac{5cd^4}{e^2}\right)\int \sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16d}-\frac{\left(-3ae^2+5cd^2-6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24cde^2}$$

$$\Big\downarrow \text{1087}$$

$$-\frac{\left(\frac{3a^2e^2}{c}+2ad^2-\frac{5cd^4}{e^2}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\int\frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8cde}\right)}{16d}-\frac{(-3ae^2+5cd^2-6cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{24cde^2}$$

$$\Big\downarrow \text{1092}$$

$$-\frac{\left(\frac{3a^2e^2}{c}+2ad^2-\frac{5cd^4}{e^2}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\int\frac{1}{4cde-\frac{(cd^2+2cexd+ae^2)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{cd^2+2cex\cdots}{\sqrt{cdex^2+(cd^2\cdots}}}{4cde}\cdots\right)}{16d}\cdots$$
$$\frac{(-3ae^2+5cd^2-6cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{24cde^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(\frac{3a^2e^2}{c}+2ad^2-\frac{5cd^4}{e^2}\right)\left(\frac{(ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cde}-\frac{(cd^2-ae^2)^2\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\cdots\right)}{16d}\cdots$$
$$\frac{(-3ae^2+5cd^2-6cdex)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}{24cde^2}$$

input
```
Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x),x]
```

output
```
-1/24*((5*c*d^2 - 3*a*e^2 - 6*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2))/(c*d*e^2) - ((2*a*d^2 - (5*c*d^4)/e^2 + (3*a^2*e^2)/c)*(((c*d^
2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d
*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sq
rt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^
(3/2)*e^(3/2))))/(16*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(220) = 440$.

Time = 2.45 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.98

method	result
default	$\frac{\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\left(\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{4cde}+\frac{\left(4ac\,d^{2}e \cdots\right.}{\cdots}\right.}{16dec}}{e}$

input
```
int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d),x,method=_RETURNVERB
OSE)
```

output
```
1/e*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c
/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^
2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))-d/e^2*(1/3*(d*e*c*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*d*e*c*(x+d/e)+a*e^
2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*
d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(
x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 676, normalized size of antiderivative = 2.77

$$\int \frac{x(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx = \left[-\frac{3\,(5\,c^4d^8-12\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6-3\,a^4e^8)\sqrt{cd \cdots}}{\cdots}\right.$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="
fricas")
```

output
```
[-1/768*(3*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2
*e^6 - 3*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +
 c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(48*c^4*d^4
*e^4*x^3 + 15*c^4*d^7*e - 31*a*c^3*d^5*e^3 + 9*a^2*c^2*d^3*e^5 - 9*a^3*c*d
*e^7 + 8*(c^4*d^5*e^3 + 9*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 10*a*c^3
*d^4*e^4 - 3*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x))/(c^3*d^3*e^4), 1/384*(3*(5*c^4*d^8 - 12*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*
e^4 + 4*a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2
*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c^4*d^4*e
^4*x^3 + 15*c^4*d^7*e - 31*a*c^3*d^5*e^3 + 9*a^2*c^2*d^3*e^5 - 9*a^3*c*d*e
^7 + 8*(c^4*d^5*e^3 + 9*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 - 10*a*c^3*d
^4*e^4 - 3*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
)/(c^3*d^3*e^4)]
```

Sympy [A] (verification not implemented)

Time = 6.58 (sec) , antiderivative size = 1093, normalized size of antiderivative = 4.48

$$\int \frac{x(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{d + ex}\,dx = \text{Too large to display}$$

input
```
integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

output
```
a*e*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e**2 + c*d**2)*(a*d*e/
3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(2*c*d*e))*
Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*
d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2)
**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x - (-a*e**2 -
 c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*e))**2), Tru
e)) + (x**2/3 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e) + (a*d*e/3 - (a*e**2/6 +
 c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d*e))*sqrt(a*d*e + c*d*
e*x**2 + x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)), (2*(-a*d*e*(a*d*e + x*(a*e**
2 + c*d**2))**(3/2)/3 + (a*d*e + x*(a*e**2 + c*d**2))**(5/2)/5)/(a*e**2 +
c*d**2)**2, Ne(a*e**2 + c*d**2, 0)), (x**2*sqrt(a*d*e)/2, True)) + c*d*Pie
cewise(((-a*(a*d*e/4 - (a*e**2/8 + c*d**2/8)*(5*a*e**2/2 + 5*c*d**2/2)/(3*
c*d*e))/(2*c) - (a*e**2 + c*d**2)*(-2*a*(a*e**2/8 + c*d**2/8)/(3*c) - (3*a
*e**2/2 + 3*c*d**2/2)*(a*d*e/4 - (a*e**2/8 + c*d**2/8)*(5*a*e**2/2 + 5*c*d
**2/2)/(3*c*d*e))/(2*c*d*e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2
*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/s
qrt(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2
 - c*d**2)/(2*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x
- (-a*e**2 - c*d**2)/(2*c*d*e))**2), True)) + sqrt(a*d*e + c*d*e*x**2 + x*
(a*e**2 + c*d**2))*(x**3/4 + x**2*(a*e**2/8 + c*d**2/8)/(3*c*d*e) + x*(...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.27

$$\int \frac{x(ade + (cd^2 + ae^2) x + cdex^2)^{3/2}}{d + ex} dx = \frac{1}{192} \sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2 \left(4 \left(6\, cdx + \frac{c^4d^5e^2 + 9\, ac}{c^3d^3e^3}\right.\right.\right.$$
$$+ \frac{(5\, c^4d^8 - 12\, ac^3d^6e^2 + 6\, a^2c^2d^4e^4 + 4\, a^3cd^2e^6 - 3\, a^4e^8) \log \left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + c}\right.\right.\right.}{128\, \sqrt{cde}c^2d^2e^3}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="
giac")
```

output
```
1/192*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*c*d*x + (c^4*d^
5*e^2 + 9*a*c^3*d^3*e^4)/(c^3*d^3*e^3))*x - (5*c^4*d^6*e - 10*a*c^3*d^4*e^
3 - 3*a^2*c^2*d^2*e^5)/(c^3*d^3*e^3))*x + (15*c^4*d^7 - 31*a*c^3*d^5*e^2 +
 9*a^2*c^2*d^3*e^4 - 9*a^3*c*d*e^6)/(c^3*d^3*e^3)) + 1/128*(5*c^4*d^8 - 12
*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 - 3*a^4*e^8)*log(abs(
-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(ade + (cd^2 + ae^2) x + cdex^2)^{3/2}}{d + ex} dx = \int \frac{x\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{d+e\,x} \,d x$$

input
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x),x)
```

output
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.37

$$\int \frac{x(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{-9\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^3cd\,e^7+9\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c^2d^3e^5}{}$$

input
```
int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

output
```
( - 9*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d*e**7 + 9*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a**2*c**2*d**3*e**5 + 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2
*c**2*d**2*e**6*x - 31*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**5*e**3 +
20*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**4*e**4*x + 72*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a*c**3*d**3*e**5*x**2 + 15*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*c**4*d**7*e - 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**6*e**2*x + 8*sq
rt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**5*e**3*x**2 + 48*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*c**4*d**4*e**4*x**3 + 9*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*
sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*
a**4*e**8 - 12*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c*d**2*e**6 - 18*
sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*s
qrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**2*d**4*e**4 + 36*sqrt(e)*sqrt
(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x)
)/sqrt(a*e**2 - c*d**2))*a*c**3*d**6*e**2 - 15*sqrt(e)*sqrt(d)*sqrt(c)*log
((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 -
 c*d**2))*c**4*d**8)/(192*c**3*d**3*e**4)
```

3.31 $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{d+ex}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 186

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{d+ex}\,dx = \frac{1}{8}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}+\frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}{3e}+\frac{\left(cd^2-ae^2\right)^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{5/2}}$$

output
```
1/8*(a/c/d-d/e^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)+1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/e+1/8*(-a*e^2+c*d^2)^3
*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{3/2}}{d + ex}\, dx = \frac{\sqrt{(ae + cdx)(d + ex)}\Big(\sqrt{c}\sqrt{d}\sqrt{e}(3a^2e^4 + 2acde^2(4d + 7ex) + c^2}{24c^{3/2}d^{3/}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(3*a^2*e^4 + 2*a*c
*d*e^2*(4*d + 7*e*x) + c^2*d^2*(-3*d^2 + 2*d*e*x + 8*e^2*x^2)) + (3*(c*d^2
 - a*e^2)^3*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*
d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(24*c^(3/2)*d^(3/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {1131, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{3/2}}{d + ex}\, dx$$

$$\Big\downarrow \text{1131}$$

$$\frac{\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{3/2}}{3e} - \frac{\left(cd^2 - ae^2\right)\int \sqrt{cdex^2 + \left(cd^2 + ae^2\right)x + ade}\,dx}{2e}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{3/2}}{3e} - \frac{\left(cd^2 - ae^2\right)\left(\frac{\left(ae^2 + cd^2 + 2cdex\right)\sqrt{x\left(ae^2 + cd^2\right) + ade + cdex^2}}{4cde} - \frac{\left(cd^2 - ae^2\right)^2 \int \frac{1}{\sqrt{cdex^2 + \left(cd^2 + ae^2\right)x + ade}}dx}{8cde}\right)}{2e}$$

$$\big\downarrow \text{1092}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4cde}\right)}{2e}$$

$$\big\downarrow \text{219}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{3/2}}\right)}{2e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x),x]
```

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(3*e) - ((c*d^2 - a*e^2)*(((
c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4
*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c
]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2
)*d^(3/2)*e^(3/2))))/(2*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.24

method	result
default	$\frac{\frac{\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\left(\frac{\left(2dec\left(x+\frac{d}{e}\right)+a\,e^{2}-c\,d^{2}\right)\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}{4dec}-\frac{\left(a\,e^{2}-c\,d^{2}\right)^{2}\ln\left(\frac{\frac{a\,e^{2}}{2}-}{}\right.}{}\right.}{2}}{e}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d),x,method=_RETURNVERBOS
E)
```

output
```
1/e*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(
1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+
d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e
))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1
/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.86

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{d+ex}\,dx=\left[-\frac{3\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+\right.}{}\right.$$

$$-\frac{3\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,cdex+cd^2+ae^2\right)\sqrt{-cde}}{2\left(c^2d^2e^2x^2+acd^2e^2+\left(c^2d^3e+acde^3\right)x\right)}\right)-2\left(8\,c^3\right.}{48\,c^2d^2e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="fr
icas")
```

output
```
[-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(c*d
*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e
) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 - 3*c^3*d^5*e + 8*
a*c^2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*d^4*e^2 + 7*a*c^2*d^2*e^4)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^3), -1/48*(3*(c^3*d^6 -
 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-
c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(8
*c^3*d^3*e^3*x^2 - 3*c^3*d^5*e + 8*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5 + 2*(c^3*
d^4*e^2 + 7*a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))
/(c^2*d^2*e^3)]
```

Sympy [A] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 751, normalized size of antiderivative = 4.04

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)
```

output
```
a*e*Piecewise(((x/2 + (a*e**2/4 + c*d**2/4)/(c*d*e))*sqrt(a*d*e + c*d*e*x*
*2 + x*(a*e**2 + c*d**2)) + (a*d*e/2 - (a*e**2/4 + c*d**2/4)*(a*e**2 + c*d
**2)/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)
*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (
a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*lo
g(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*
c*d*e))**2), True)), Ne(c*d*e, 0)), (2*(a*d*e + x*(a*e**2 + c*d**2))**(3/2
)/(3*(a*e**2 + c*d**2)), Ne(a*e**2 + c*d**2, 0)), (x*sqrt(a*d*e), True)) +
 c*d*Piecewise(((-a*(a*e**2/6 + c*d**2/6)/(2*c) - (a*e**2 + c*d**2)*(a*d*e
/3 - (a*e**2/6 + c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(2*c*d*e))
*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c
*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2
)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e))*log(x - (-a*e**2
- c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)/(2*c*d*e))**2), Tr
ue)) + (x**2/3 + x*(a*e**2/6 + c*d**2/6)/(2*c*d*e) + (a*d*e/3 - (a*e**2/6
+ c*d**2/6)*(3*a*e**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d*e))*sqrt(a*d*e + c*d
*e*x**2 + x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)), (2*(-a*d*e*(a*d*e + x*(a*e*
*2 + c*d**2))**(3/2)/3 + (a*d*e + x*(a*e**2 + c*d**2))**(5/2)/5)/(a*e**2 +
 c*d**2)**2, Ne(a*e**2 + c*d**2, 0)), (x**2*sqrt(a*d*e)/2, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.23

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \frac{1}{24}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\,cdx + \frac{c^3d^4e + 7\,ac^2d^2e^3}{c^2d^2e^2}\right)\right.$$
$$- \frac{(c^3d^6 - 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 - a^3e^6)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ad}\right.\right.\right.}{16\sqrt{cde}cde^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="gi
ac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*c*d*x + (c^3*d^4*e
+ 7*a*c^2*d^2*e^3)/(c^2*d^2*e^2))*x - (3*c^3*d^5 - 8*a*c^2*d^3*e^2 - 3*a^2
*c*d*e^4)/(c^2*d^2*e^2)) - 1/16*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e
^4 - a^3*e^6)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c*d*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{3/2}}{d + e\,x} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.15

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex}\,dx = \frac{3\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2cd\,e^5+8\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^3e^3+1}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)
```

output
```
(3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d*e**5 + 8*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a*c**2*d**3*e**3 + 14*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d*
*2*e**4*x - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e + 2*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*c**3*d**4*e**2*x + 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c
**3*d**3*e**3*x**2 - 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d
*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**6 + 9*
sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*s
qrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**4 - 9*sqrt(e)*sqrt(d)*
sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sq
rt(a*e**2 - c*d**2))*a*c**2*d**4*e**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqr
t(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d*
*2))*c**3*d**6)/(24*c**2*d**2*e**3)
```

3.32 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x(d+ex)} dx$

Optimal result

Integrand size = 40, antiderivative size = 219

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x(d+ex)} dx = \frac{(cd^2+5ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4e} - \frac{(c^2d^4-6acd^2e^2-3a^2e^4)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4\sqrt{c}\sqrt{d}e^{3/2}} - 2a^{3/2}\sqrt{d}e^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)$$

output
```
1/4*(2*c*d*e*x+5*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e-1/
4*(-3*a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/
2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(1/2)/d^(1/2)/e^(3/2)-2*a^(3
/2)*d^(1/2)*e^(3/2)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.06

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x(d + ex)}\,dx = \frac{\sqrt{ae + cdx}\sqrt{d + ex}\Big(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae + cdx}\sqrt{d + ex}(5ae^2 + cd(d +}{4}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*(5*a*e^2 + c*d*(d + 2*e*x)) + (-(c^2*d^4) + 6*a*c*d^2*e^2
+ 3*a^2*e^4)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c
*d*x])] - 8*a^(3/2)*Sqrt[c]*d*e^3*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/
(Sqrt[d]*Sqrt[a*e + c*d*x])]))/(4*Sqrt[c]*Sqrt[d]*e^(3/2)*Sqrt[(a*e + c*d*
x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1215, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{3/2}}{x(d + ex)}\,dx$$

$$\Big\downarrow \; 1215$$

$$\int \frac{(ae + cdx)\sqrt{x\left(ae^2 + cd^2\right) + ade + cdex^2}}{x}\,dx$$

$$\Big\downarrow \; 1231$$

$$\frac{\left(5ae^2 + cd^2 + 2cdex\right)\sqrt{x\left(ae^2 + cd^2\right) + ade + cdex^2}}{4e} - \frac{\int -\frac{cd\left(8a^2de^3 - \left(c^2d^4 - 6ace^2d^2 - 3a^2e^4\right)x\right)}{2x\sqrt{cdex^2 + (cd^2 + ae^2)x + ade}}\,dx}{4cde}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{8a^2de^3-(c^2d^4-6ace^2d^2-3a^2e^4)x}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8e}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(5ae^2+cd^2+2cdex)}{4e}$$

$\downarrow$ 1269

$$\frac{8a^2de^3\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-(-3a^2e^4-6acd^2e^2+c^2d^4)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8e}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(5ae^2+cd^2+2cdex)}{4e}$$

$\downarrow$ 1092

$$\frac{8a^2de^3\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-2(-3a^2e^4-6acd^2e^2+c^2d^4)\int \frac{1}{4cde-\frac{(cd^2+2cexd+ae^2)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+}}}{8e}$$
$$\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(5ae^2+cd^2+2cdex)}{4e}$$

$\downarrow$ 219

$$\frac{8a^2de^3\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{(-3a^2e^4-6acd^2e^2+c^2d^4)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}}{8e}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(5ae^2+cd^2+2cdex)}{4e}$$

$\downarrow$ 1154

$$\frac{-16a^2de^3\int \frac{1}{4ade-\frac{(2ade+(cd^2+ae^2)x)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{2ade+(cd^2+ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\frac{(-3a^2e^4-6acd^2e^2+c^2d^4)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ac}}\right.}{\sqrt{c}\sqrt{d}\sqrt{e}}}{8e}$$
$$\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(5ae^2+cd^2+2cdex)}{4e}$$

$\downarrow$ 219

$$\frac{-8a^{3/2}\sqrt{d}e^{5/2}\text{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)-\frac{(-3a^2e^4-6acd^2e^2+c^2d^4)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cd}}\right.}{\sqrt{c}\sqrt{d}\sqrt{e}}}{8e}$$
$$\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(5ae^2+cd^2+2cdex)}{4e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x]
```

output
```
((c*d^2 + 5*a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(4*e) + (-(((c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2
 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])])/(Sqrt[c]*Sqrt[d]*Sqrt[e])) - 8*a^(3/2)*Sqrt[d]*e^(5/2)*ArcTa
nh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(185) = 370$.

Time = 2.52 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.77

method	result
default	$\frac{\left(ade+\left(a e^2+c d^2\right)x+cd x^2 e\right)^{\frac{3}{2}}}{3}+\frac{\left(a e^2+c d^2\right)\left(\frac{\left(2cdxe+a e^2+c d^2\right)\sqrt{ade+\left(a e^2+c d^2\right)x+cd x^2 e}}{4cde}+\frac{\left(4ac d^2 e^2-\left(a e^2+c d^2\right)^2\right)\ln\left(\frac{\frac{1}{2}a e^2+\frac{1}{2}c d^2}{\sqrt{dec}}\cdots\right)}{8dec\sqrt{d}\cdots}\right)}{2}\cdots$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/x/(e*x+d),x,method=_RETURNVERB
OSE)
```

output
```
1/d*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2
*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4
*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*
c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+a*d*e*((a
*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*
c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e
*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))-1/d*(1/3*(d*e*c*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c
*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2
)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d
/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 1327, normalized size of antiderivative = 6.06

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="
fricas")
```

output
```
[1/16*(8*sqrt(a*d*e)*a*c*d*e^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e
^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
 + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - (c
^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*
x) + 4*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x))/(c*d*e^2), 1/8*(4*sqrt(a*d*e)*a*c*d*e^3*log((8*a^2*d
^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*
d^3*e + a^2*d*e^3)*x)/x^2) + (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(-c
*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +
c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e +
a*c*d*e^3)*x)) + 2*(2*c^2*d^2*e^2*x + c^2*d^3*e + 5*a*c*d*e^3)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^2), 1/16*(16*sqrt(-a*d*e)*a*c*d*e
^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a
^2*d*e^3)*x)) - (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(c*d*e)*log(8*c^
2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^...
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x(d + ex)}\,dx = \int \frac{((d + ex)\,(ae + cdx))^{\frac{3}{2}}}{x\,(d + ex)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x/(e*x+d),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x(d+ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x(d+ex)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x(d + ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{x\,\left(d + e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.94

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x(d + ex)}\,dx = \frac{5\sqrt{ex + d}\,\sqrt{cdx + ae}\,acd\,e^3 + \sqrt{ex + d}\,\sqrt{cdx + ae}\,c^2d^3e + 2\sqrt{ex}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d),x)
```

output
```
(5*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d*e**3 + sqrt(d + e*x)*sqrt(a*e + c
*d*x)*c**2*d**3*e + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**2*e**2*x + 4
*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sq
rt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c*d*e**3 +
 4*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*
sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c*d*e**3
 - 4*sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*s
qrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*a*c*d*e**3 + 3*sqrt(
e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**4 + 6*sqrt(e)*sqrt(d)*sqrt(c)*log(
(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 -
c*d**2))*a*c*d**2*e**2 - sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**4)/(
4*c*d*e**2)
```

3.33 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 203

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^2(d+ex)}\,dx = -\frac{(ae-cdx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x} + \frac{\sqrt{c}\sqrt{d}(cd^2+3ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{e}} - \frac{\sqrt{a}\sqrt{e}(3cd^2+ae^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{d}}$$

output
```
-(-c*d*x+a*e)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x+c^(1/2)*d^(1/2)*(3
*a*e^2+c*d^2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2))/e^(1/2)-a^(1/2)*e^(1/2)*(a*e^2+3*c*d^2)*arctanh(a^(1/
2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/d^(1/2
)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2(d+ex)}\,dx =$$
$$-\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{d}\sqrt{e}(ae-cdx)\sqrt{ae+cdx}\sqrt{d+ex}-\sqrt{c}d\left(cd^2+3ae^2\right)x\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)+\sqrt{}\right.}{\sqrt{d}\sqrt{e}x\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

output
```
-((Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[d]*Sqrt[e]*(a*e - c*d*x)*Sqrt[a*e
 + c*d*x]*Sqrt[d + e*x] - Sqrt[c]*d*(c*d^2 + 3*a*e^2)*x*ArcTanh[(Sqrt[c]*S
qrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])] + Sqrt[a]*e*(3*c*d^2 +
a*e^2)*x*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x
])]))/(Sqrt[d]*Sqrt[e]*x*Sqrt[(a*e + c*d*x)*(d + e*x)]))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1215, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^2(d+ex)}dx$$

$$\Big\downarrow \ 1215$$

$$\int \frac{(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x^2}dx$$

$$\Big\downarrow \ 1230$$

$$-\frac{1}{2}\int -\frac{ae\left(3cd^2+ae^2\right)+cd\left(cd^2+3ae^2\right)x}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}(ae-cdx)}{x}$$

$\downarrow$ 25

$$\frac{1}{2}\int \frac{ae\left(3cd^2+ae^2\right)+cd\left(cd^2+3ae^2\right)x}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{(ae-cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

$\downarrow$ 1269

$$\frac{1}{2}\left(cd\left(3ae^2+cd^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+ae\left(ae^2+3cd^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx\right)-\frac{(ae-cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

$\downarrow$ 1092

$$\frac{1}{2}\left(ae\left(ae^2+3cd^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+2cd\left(3ae^2+cd^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{\ldots}{\sqrt{c\ldots}}\right)-\frac{(ae-cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

$\downarrow$ 219

$$\frac{1}{2}\left(ae\left(ae^2+3cd^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{\sqrt{c}\sqrt{d}\left(3ae^2+cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade\ldots}}\right)}{\sqrt{e}}\right)-\frac{(ae-cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

$\downarrow$ 1154

$$\frac{1}{2}\left(\frac{\sqrt{c}\sqrt{d}\left(3ae^2+cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{e}}-2ae\left(ae^2+3cd^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+a\ldots}}\ldots\right)-\frac{(ae-cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{\sqrt{c}\sqrt{d}\left(3ae^2+cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{e}}-\frac{\sqrt{a}\sqrt{e}\left(ae^2+3cd^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\ldots\right)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+\ldots\right)}}\right)}{\sqrt{d}}\right)-\frac{(ae-cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x]
```

output
```
-(((a*e - c*d*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x) + ((Sqrt[
c]*Sqrt[d]*(c*d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c
]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[e] -
 (Sqrt[a]*Sqrt[e]*(3*c*d^2 + a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/
(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/
Sqrt[d])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. $2(173) = 346$.

Time = 2.68 (sec) , antiderivative size = 1300, normalized size of antiderivative = 6.40

method	result	size
default	Expression too large to display	1300

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/x^2/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)+3/2*(a*e^2+c*d^2)/
a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*
(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*
(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*
e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+a*d*e*(
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/
2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d
*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*x+a*e^
2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2
-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2
*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^
(1/2))/(d*e*c)^(1/2))))+e/d^2*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(
x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*
e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*
(x+d/e))^(1/2))/(d*e*c)^(1/2)))-e/d^2*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*l...
```

Fricas [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 1221, normalized size of antiderivative = 6.01

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^2(d + ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm
="fricas")
```

output

```
[1/4*((c*d^2 + 3*a*e^2)*sqrt(c*d/e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*
a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + (
3*c*d^2 + a*e^2)*sqrt(a*e/d)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e
^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2
*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2)
+ 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x, -1/4*(2*
(c*d^2 + 3*a*e^2)*sqrt(-c*d/e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*
c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) - (3*c*d^2 + a*e^2)*sqrt(a*e/d)*x*log(
(8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/
d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
 + a*e^2)*x)*(c*d*x - a*e))/x, 1/4*(2*(3*c*d^2 + a*e^2)*sqrt(-a*e/d)*x*arc
tan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(-a*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)
) + (c*d^2 + 3*a*e^2)*sqrt(c*d/e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*
c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x - a*e))/x, -1/2*((c*d...
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^2(d + ex)}\,dx = \int \frac{((d + ex)(ae + cdx))^{\frac{3}{2}}}{x^2(d + ex)}\,dx$$

input

```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**2/(e*x+d),x)
```

output

```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x**2*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2(d+ex)}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)x^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^2), x
)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.67

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^2(d+ex)}\,dx=\sqrt{cdex^2+cd^2x+ae^2x+ade}cd$$
$$+\frac{\left(3\,acd^2e+a^2e^3\right)\arctan\left(-\frac{\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}}{\sqrt{-ade}}\right)}{\sqrt{-ade}}$$
$$-\frac{\left(c^2d^3+3\,acde^2\right)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{2\sqrt{cde}}$$
$$-\frac{\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)acd^2e+\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)a^2e^3+2\sqrt{c}}{ade-\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm
="giac")
```

output
```
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*c*d + (3*a*c*d^2*e + a^2*e^3)*
arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt
(-a*d*e))/sqrt(-a*d*e) - 1/2*(c^2*d^3 + 3*a*c*d*e^2)*log(abs(-c*d^2 - a*e^
2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*
d*e))))/sqrt(c*d*e) - ((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))*a*c*d^2*e + (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))*a^2*e^3 + 2*sqrt(c*d*e)*a^2*d*e^2)/(a*d*e - (sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^2(d+ex)}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{x^2\,\left(d+e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.50

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^2(d+ex)}\,dx=\frac{-2\sqrt{ex+d}\,\sqrt{cdx+ae}\,ad\,e^2+2\sqrt{ex+d}\,\sqrt{cdx+ae}\,c\,d^2ex+\sqrt{e}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x)
```

output
```
( - 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*d*e**2 + 2*sqrt(d + e*x)*sqrt(a*e
+ c*d*x)*c*d**2*e*x + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x
) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))*a*e**3*x + 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x
) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))*c*d**2*e*x + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x
) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))*a*e**3*x + 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x
) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))*c*d**2*e*x - sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d)*sqrt(c
)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*a*e
**3*x - 3*sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e
*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*c*d**2*e*x + 6*
sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*s
qrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*d*e**2*x + 2*sqrt(e)*sqrt(d)*sqrt(c
)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e
**2 - c*d**2))*c*d**3*x)/(2*d*e*x)
```

3.34 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 225

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx = -\frac{\left(2ade+\left(5cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^2} + 2c^{3/2}d^{3/2}\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right) - \frac{\left(3c^2d^4+6acd^2e^2-a^2e^4\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\cdots}}{\sqrt{d}\sqrt{ade+(c\cdots}}\right.}{4\sqrt{a}d^{3/2}\sqrt{e}}$$

output
```
-1/4*(2*a*d*e+(a*e^2+5*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d
/x^2+2*c^(3/2)*d^(3/2)*e^(1/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))-1/4*(-a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^
4)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/a^(1/2)/d^(3/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.08

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^3(d+ex)}\,dx =$$
$$-\frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(5cd^2x+ae(2d+ex))-8\sqrt{a}c^{3/2}d^3ex^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{\cdots}}{\sqrt{e}\sqrt{ae\cdots}}\right.\right.}{4\sqrt{a}d^{3/2}\sqrt{e}x^2\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

output
```
-1/4*(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e +
c*d*x]*Sqrt[d + e*x]*(5*c*d^2*x + a*e*(2*d + e*x)) - 8*Sqrt[a]*c^(3/2)*d^3
*e*x^2*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])
] + (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*x^2*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqr
t[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])]))/(Sqrt[a]*d^(3/2)*Sqrt[e]*x^2*Sq
rt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1215, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^3(d+ex)}dx$$
$$\Big\downarrow \text{ 1215}$$
$$\int \frac{(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x^3}dx$$
$$\Big\downarrow \text{ 1229}$$

$$-\frac{\int -\frac{ae(3c^2d^4+8c^2exd^3+6ace^2d^2-a^2e^4)}{2x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4ade}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(x\left(ae^2+5cd^2\right)+2ade\right)}{4dx^2}$$

$\downarrow$ 27

$$\frac{\int \frac{3c^2d^4+8c^2exd^3+6ace^2d^2-a^2e^4}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8d}-\frac{\left(x\left(ae^2+5cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^2}$$

$\downarrow$ 1269

$$\frac{\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+8c^2d^3e\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8d}-\frac{\left(x\left(ae^2+5cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^2}$$

$\downarrow$ 1092

$$\frac{\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+16c^2d^3e\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{8d}-\frac{\left(x\left(ae^2+5cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^2}$$

$\downarrow$ 219

$$\frac{\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+8c^{3/2}d^{5/2}\sqrt{e}\,\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8d}-\frac{\left(x\left(ae^2+5cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^2}$$

$\downarrow$ 1154

$$\frac{8c^{3/2}d^{5/2}\sqrt{e}\,\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)-2\left(-a^2e^4+6acd^2e^2+3c^2d^4\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\ldots}{8d}-\frac{\left(x\left(ae^2+5cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^2}$$

$\downarrow$ 219

$$\frac{8c^{3/2}d^{5/2}\sqrt{e}\,\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)-\frac{(-a^2e^4+6acd^2e^2+3c^2d^4)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{a}\sqrt{d}\sqrt{e}}}{8d}$$

$$\frac{\left(x\left(ae^2+5cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4dx^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x]
```

output
```
-1/4*((2*a*d*e + (5*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(d*x^2) + (8*c^(3/2)*d^(5/2)*Sqrt[e]*ArcTanh[(c*d^2 + a*e^2 + 2*c
*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])] - ((3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2
+ a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])])/(Sqrt[a]*Sqrt[d]*Sqrt[e]))/(8*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2437 vs. $2(191) = 382$.

Time = 2.73 (sec) , antiderivative size = 2438, normalized size of antiderivative = 10.84

method	result	size
default	Expression too large to display	2438

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/x^3/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)+1/4*(a*e^2+c*d
^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)+3/2*(a*e^2+c
*d^2)/a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)
*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/
e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*
e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+a
*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*
e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/
2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e
)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*
x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d
^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*l
n((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x
^2*e)^(1/2))/(d*e*c)^(1/2)))+3/2*c/a*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((
1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2))/(d*e*c)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/
2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(...
```

Fricas [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 1375, normalized size of antiderivative = 6.11

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{3/2}}{x^3(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm
="fricas")
```

output
```
[1/16*(8*sqrt(c*d*e)*a*c*d^3*e*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c
*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - (3*c^2*d
^4 + 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2*d^2*e^2 + (c^2*d^
4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^
3)*x)/x^2) - 4*(2*a^2*d^2*e^2 + (5*a*c*d^3*e + a^2*d*e^3)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2), -1/16*(16*sqrt(-c*d*e)*a*c*
d^3*e*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*
x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*
e + a*c*d*e^3)*x)) + (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(a*d*e)*x^2
*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d
*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*a^2*d^2*e^2 + (5*a*c*d^3*e
+ a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^2)
, 1/8*(4*sqrt(c*d*e)*a*c*d^3*e*x^2*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c
*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*
e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + (3*c^2*d
^4 + 6*a*c*d^2*e^2 - a^2*e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/...
```

Sympy [F]

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{x^3(d+ex)}\,dx=\int \frac{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}{x^3\,(d+ex)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**3/(e*x+d),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x**3*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^3(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)x^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^3), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(191) = 382.

Time = 0.21 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.78

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^3(d + ex)}\,dx =$$
$$-\frac{c^2d^2e\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{\sqrt{cde}}$$
$$+\frac{(3c^2d^4 + 6acd^2e^2 - a^2e^4)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{4\sqrt{-ade}d}$$
$$-\frac{3\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)ac^2d^5e - 2\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^2cd^3e^3}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm
="giac")
```

output
```
-c^2*d^2*e*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/sqrt(c*d*e) + 1/4*(3*c^2*d^4 + 6*a
*c*d^2*e^2 - a^2*e^4)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*d) - 1/4*(3*(sqrt(c*d*e)*x -
 sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a*c^2*d^5*e - 2*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^2*c*d^3*e^3 - (sqrt(c*
d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*d*e^5 + 8*sqrt(c
*d*e)*a^2*c*d^4*e^2 - 5*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))^3*c^2*d^4 - 10*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e
^2*x + a*d*e))^3*a*c*d^2*e^2 - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))^3*a^2*e^4 - 16*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a*c*d^3*e - 8*sqrt(c*d*e)*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^2*d*e^3)/((a*d*e - (
sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^3(d + ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{x^3\,\left(d + e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 1090, normalized size of antiderivative = 4.84

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^3(d + ex)}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x)
```

output
```
( - 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*d**2*e**4 - 2*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**3*d*e**5*x - 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d*
*4*e**2 - 12*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d**3*e**3*x - 10*sqrt(
d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**5*e*x - sqrt(e)*sqrt(d)*sqrt(a)*log(s
qrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*e**6*x**2 + 5*sqrt(e)*sqrt(d)*sqrt(a)
*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d
**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**2*e**4*x**2 + 9*sqrt(e)*sq
rt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e +
 a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**4*e**2*x**2 +
 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*
sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**3*d**6*
x**2 - sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt
(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*e
**6*x**2 + 5*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(
2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*
a**2*c*d**2*e**4*x**2 + 9*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sq
rt(d + e*x))*a*c**2*d**4*e**2*x**2 + 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)
*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sq...
```

3.35 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 194

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx = -\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^2} - \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{3dx^3} + \frac{\left(cd^2-ae^2\right)^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8a^{3/2}d^{5/2}e^{3/2}}$$

output
```
-1/8*(c/a/e-e/d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/x^2-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^3+1/8*(-a*e
^2+c*d^2)^3*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.04

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx=\frac{\left(-cd^2+ae^2\right)^3\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(3c^2d^4x^2+2acd^2ex(7d+4e\cdots\right.}{\left(cd^2-ae^2\right)\cdots}\right.}{24a^{3/2}d^{5/2}e^{3/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

output
```
((-(c*d^2) + a*e^2)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[a]*Sqrt[d]*Sqrt
[e]*(3*c^2*d^4*x^2 + 2*a*c*d^2*e*x*(7*d + 4*e*x) + a^2*e^2*(8*d^2 + 2*d*e*
x - 3*e^2*x^2)))/((c*d^2 - a*e^2)^3*x^3) - (3*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqr
t[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]
)))/(24*a^(3/2)*d^(5/2)*e^(3/2))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1215, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^4(d+ex)}dx$$

$$\Big\downarrow \text{ 1215}$$

$$\int \frac{(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x^4}dx$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{\left(cd^2-ae^2\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{2d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3dx^3}$$

$$\Big\downarrow \text{ 1152}$$

$$\frac{\left(cd^2-ae^2\right)\left(-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4adex^2}\right)}{2d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3dx^3}$$

$$\Big\downarrow \textbf{1154}$$

$$\frac{\left(cd^2-ae^2\right)\left(\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4adex^2}\right)}{2d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3dx^3}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\left(cd^2-ae^2\right)\left(\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4adex^2}\right)}{2d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3dx^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x]
```

output
```
-1/3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d*x^3) + ((c*d^2 - a*e
^2)*(-1/4*((2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(a*d*e*x^2) + ((c*d^2 - a*e^2)^2*ArcTanh[(2*a*d*e + (c*d^2 + a
*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])])/(8*a^(3/2)*d^(3/2)*e^(3/2))))/(2*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4329 vs. $2(170) = 340$.

Time = 3.26 (sec) , antiderivative size = 4330, normalized size of antiderivative = 22.32

method	result	size
default	Expression too large to display	4330

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/x^4/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)-1/6*(a*e^2+c*d
^2)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)+1/4*(a*e
^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)+3/2*(a
*e^2+c*d^2)/a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+
c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+
c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1
/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((
1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*
(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))+4*c/a*(1/8*(2*
c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4
*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d
/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))))+3/2*c/a*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/
c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2))/(d*e*c)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)...
```

Fricas [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.88

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx = \left[-\frac{3\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\sqrt{ade}x^3\log\left(\frac{8\,a^2d^2e^2+(c\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{3\left(c^3d^6-3\,ac^2d^4e^2+3\,a^2cd^2e^4-a^3e^6\right)\sqrt{-ade}x^3\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,ade+(cd^2+ae^2)x)\sqrt{-ade}}{2\,(acd^2e^2x^2+a^2d^2e^2+(acd^3e+a^2de^3)x)}\right)+2\cdots}{48\,a^2d\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/96*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(a*d
*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^3*e^3 + (3*a*c
^2*d^5*e + 8*a^2*c*d^3*e^3 - 3*a^3*d*e^5)*x^2 + 2*(7*a^2*c*d^4*e^2 + a^3*d
^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^3),
 -1/48*(3*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*
d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3
*e + a^2*d*e^3)*x)) + 2*(8*a^3*d^3*e^3 + (3*a*c^2*d^5*e + 8*a^2*c*d^3*e^3
- 3*a^3*d*e^5)*x^2 + 2*(7*a^2*c*d^4*e^2 + a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**4/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^4(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)x^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^4), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1005 vs. $2(170) = 340$.

Time = 0.15 (sec) , antiderivative size = 1005, normalized size of antiderivative = 5.18

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^4(d + ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm
="giac")
```

output
```
-1/8*(c^3*d^6 - 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - a^3*e^6)*arctan(-(sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sq
rt(-a*d*e)*a*d^2*e) + 1/24*(3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
a*e^2*x + a*d*e))*a^2*c^3*d^8*e^2 - 9*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))*a^3*c^2*d^6*e^4 - 39*(sqrt(c*d*e)*x - sqrt(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c*d^4*e^6 - 3*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*d^2*e^8 - 16*sqrt(c*d*e)*a^4*
c*d^5*e^5 - 8*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)
)^3*a*c^3*d^7*e - 72*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
 a*d*e))^3*a^2*c^2*d^5*e^3 - 72*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
+ a*e^2*x + a*d*e))^3*a^3*c*d^3*e^5 - 8*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e))^3*a^4*d*e^7 - 48*sqrt(c*d*e)*(sqrt(c*d*e)*x -
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^3*c*d^4*e^4 - 48*sqrt(c*d
*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^4*d^
2*e^6 - 3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*
c^3*d^6 - 39*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
^5*a*c^2*d^4*e^2 - 9*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
 a*d*e))^5*a^2*c*d^2*e^4 + 3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a
*e^2*x + a*d*e))^5*a^3*e^6 - 48*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^
2 + c*d^2*x + a*e^2*x + a*d*e))^4*a*c^2*d^5*e - 96*sqrt(c*d*e)*(sqrt(c*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^4(d + ex)} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{3/2}}{x^4\,(d + e\,x)} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 1116, normalized size of antiderivative = 5.75

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^4(d + ex)} dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x)
```

output
```
( - 16*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*d**3*e**5 - 4*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**4*d**2*e**6*x + 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*
d*e**7*x**2 - 16*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**5*e**3 - 32*sqr
t(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**4*e**4*x - 10*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a**3*c*d**3*e**5*x**2 - 28*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2
*c**2*d**6*e**2*x - 22*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**5*e**3
*x**2 - 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**7*e*x**2 + 3*sqrt(e)*s
qrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e
+ a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**4*e**8*x**3 - 6*sqr
t(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a
)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**2*e**6
*x**3 + 6*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*s
qrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c
**3*d**6*e**2*x**3 - 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*
x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(
d + e*x))*c**4*d**8*x**3 + 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e
+ c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)
*sqrt(d + e*x))*a**4*e**8*x**3 - 6*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqr
t(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*s
qrt(c)*sqrt(d + e*x))*a**3*c*d**2*e**6*x**3 + 6*sqrt(e)*sqrt(d)*sqrt(a)...
```

3.36 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 278

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx = \frac{\left(cd^2-ae^2\right)\left(3cd^2+5ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+\right.}}{64a^2d^3e^2x^2}$$
$$-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4dx^4}-\frac{\left(\frac{3c}{ae}-\frac{5e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24x^3}$$
$$-\frac{\left(cd^2-ae^2\right)^3\left(3cd^2+5ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{64a^{5/2}d^{7/2}e^{5/2}}$$

output

```
1/64*(-a*e^2+c*d^2)*(5*a*e^2+3*c*d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^3/e^2/x^2-1/4*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(3/2)/d/x^4-1/24*(3*c/a/e-5*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)/x^3-1/64*(-a*e^2+c*d^2)^3*(5*a*e^2+3*c*d^2)*arctanh(a^(1/2)*e^(
1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7
/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.89

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(-9c^3d^6x^3+3ac^2d^4ex^2(2d+3ex)+a^2cd^2e^2x\right.}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-9*c^3*d^6*x^3
 + 3*a*c^2*d^4*e*x^2*(2*d + 3*e*x) + a^2*c*d^2*e^2*x*(72*d^2 + 20*d*e*x -
31*e^2*x^2) + a^3*e^3*(48*d^3 + 8*d^2*e*x - 10*d*e^2*x^2 + 15*e^3*x^3)))/x
^4) - (3*(c*d^2 - a*e^2)^3*(3*c*d^2 + 5*a*e^2)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sq
rt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x
])))/(192*a^(5/2)*d^(7/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1215, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^5(d+ex)}dx$$

$$\Big\downarrow \ 1215$$

$$\int \frac{(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x^5}dx$$

$$\Big\downarrow \ 1237$$

$$-\frac{\int -\frac{ae\left(3cd^2-2cexd-5ae^2\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^4}dx}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4dx^4}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(3cd^2-2cexd-5ae^2)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{x^4}dx}{8d} - \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4dx^4}$$

$\Big\downarrow$ 1228

$$\frac{-\frac{\left(\frac{3c^2d^4}{a}-5ae^4+2cd^2e^2\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{2de} - \frac{\left(\frac{3cd}{ae}-\frac{5e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x^3}}{8d} - \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4dx^4}$$

$\Big\downarrow$ 1152

$$\frac{-\frac{\left(\frac{3c^2d^4}{a}-5ae^4+2cd^2e^2\right)\left(-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{2de} - \frac{\left(\frac{3cd}{ae}-\frac{5e}{d}\right)\left(x\left(ae^2\right.\right.}{}}{8d} - \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4dx^4}$$

$\Big\downarrow$ 1154

$$\frac{-\frac{\left(\frac{3c^2d^4}{a}-5ae^4+2cd^2e^2\right)\left(\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right.}{2de}}{8d} - \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4dx^4}$$

$\Big\downarrow$ 219

$$\frac{-\frac{\left(\frac{3c^2d^4}{a}-5ae^4+2cd^2e^2\right)\left(\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{2de} - \frac{\left(\right.}{}}{8d} - \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4dx^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x]
```

output
```
-1/4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d*x^4) + (-1/3*(((3*c*
d)/(a*e) - (5*e)/d)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/x^3 - (
((3*c^2*d^4)/a + 2*c*d^2*e^2 - 5*a*e^4)*(-1/4*((2*a*d*e + (c*d^2 + a*e^2)*
x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^2) + ((c*d^2 - a*
e^2)^2*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*a^(3/2)*d^(3/2)*e^(3/2))))
/(2*d*e))/(8*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7420 vs. $2(250) = 500$.

Time = 3.33 (sec) , antiderivative size = 7421, normalized size of antiderivative = 26.69

method	result	size
default	Expression too large to display	7421

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/x^5/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 704, normalized size of antiderivative = 2.53

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^5(d + ex)} dx = \left[-\frac{3\,(3\,c^4d^8 - 4\,ac^3d^6e^2 - 6\,a^2c^2d^4e^4 + 12\,a^3cd^2e^6 - 5\,a^4e^8)\sqrt{ade}}{} \right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/768*(3*(3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2
*e^6 - 5*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) +
 4*(48*a^4*d^4*e^4 - (9*a*c^3*d^7*e - 9*a^2*c^2*d^5*e^3 + 31*a^3*c*d^3*e^5
 - 15*a^4*d*e^7)*x^3 + 2*(3*a^2*c^2*d^6*e^2 + 10*a^3*c*d^4*e^4 - 5*a^4*d^2
*e^6)*x^2 + 8*(9*a^3*c*d^5*e^3 + a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^4), 1/384*(3*(3*c^4*d^8 - 4*a*c^3*d^6*e
^2 - 6*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-a*d*e)*x^4*ar
ctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*
e^3)*x)) - 2*(48*a^4*d^4*e^4 - (9*a*c^3*d^7*e - 9*a^2*c^2*d^5*e^3 + 31*a^3
*c*d^3*e^5 - 15*a^4*d*e^7)*x^3 + 2*(3*a^2*c^2*d^6*e^2 + 10*a^3*c*d^4*e^4 -
 5*a^4*d^2*e^6)*x^2 + 8*(9*a^3*c*d^5*e^3 + a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^4*e^3*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^5(d + ex)} dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**5/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^5(d + ex)} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)x^5} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^5), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. 2(250) = 500.

Time = 0.17 (sec) , antiderivative size = 1618, normalized size of antiderivative = 5.82

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^5(d + ex)} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm
="giac")
```

output
```
1/64*(3*c^4*d^8 - 4*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 -
 5*a^4*e^8)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^2*d^3*e^2) - 1/192*(9*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*c^4*d^11*e^3 - 12*(sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c^3*d^9*e^5
- 402*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*c^
2*d^7*e^7 - 348*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))*a^6*c*d^5*e^9 - 15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))*a^7*d^3*e^11 - 128*sqrt(c*d*e)*a^6*c*d^6*e^8 - 33*(sqrt(c*d*e)*
x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c^4*d^10*e^2 - 724*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c^3*d^
8*e^4 - 1854*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
^3*a^4*c^2*d^6*e^6 - 900*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2
*x + a*d*e))^3*a^5*c*d^4*e^8 - 73*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))^3*a^6*d^2*e^10 - 768*sqrt(c*d*e)*(sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^4*c^2*d^7*e^5 - 1024*sqrt(c
*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^5*
c*d^5*e^7 - 384*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*
e^2*x + a*d*e))^2*a^6*d^3*e^9 - 33*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))^5*a*c^4*d^9*e - 596*(sqrt(c*d*e)*x - sqrt(c*d*e*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^5(d + ex)} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{3/2}}{x^5\,(d + e\,x)} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 1662, normalized size of antiderivative = 5.98

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^5(d+ex)}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x)
```

output
```
( - 192*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*d**4*e**6 - 32*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**5*d**3*e**7*x + 40*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
*5*d**2*e**8*x**2 - 60*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*d*e**9*x**3 -
192*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d**6*e**4 - 320*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**4*c*d**5*e**5*x - 40*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
**4*c*d**4*e**6*x**2 + 64*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d**3*e**7
*x**3 - 288*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**7*e**3*x - 104*sq
rt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**6*e**4*x**2 + 88*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**3*c**2*d**5*e**5*x**3 - 24*sqrt(d + e*x)*sqrt(a*e + c
*d*x)*a**2*c**3*d**8*e**2*x**2 + 36*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4
*d**9*e*x**3 - 30*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) -
sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e
*x))*a**5*e**10*x**4 + 42*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c
*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sq
rt(d + e*x))*a**4*c*d**2*e**8*x**4 + 36*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e
)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))*a**3*c**2*d**4*e**6*x**4 - 60*sqrt(e)*sqrt(d)*s
qrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2
 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c**3*d**6*e**4*x**4 - 6*s
qrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*s...
```

3.37 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^6(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 378

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^6(d+ex)}\,dx = -\frac{(cd^2-ae^2)(3c^2d^4+6acd^2e^2+7a^2e^4)(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128a^3d^4e^3x^2} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{5dx^5} - \frac{\left(\frac{3c}{ae}-\frac{7e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{40x^4} + \frac{(15c^2d^4+12acd^2e^2-35a^2e^4)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{240a^2d^3e^2x^3} + \frac{(cd^2-ae^2)^3(3c^2d^4+6acd^2e^2+7a^2e^4)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128a^{7/2}d^{9/2}e^{7/2}}$$

output
```
-1/128*(-a*e^2+c*d^2)*(7*a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*(2*a*d*e+(a*e^2+
c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d^4/e^3/x^2-1/5*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^5-1/40*(3*c/a/e-7*e/d^2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4+1/240*(-35*a^2*e^4+12*a*c*d^2*e^2+15*c^
2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^2/d^3/e^2/x^3+1/128*(-a*e
^2+c*d^2)^3*(7*a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*arctanh(a^(1/2)*e^(1/2)*(e
*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(7/2)/d^(9/2)/e^(
7/2)
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.84

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(45c^4d^8x^4-30ac^3d^6ex^3(d+ex)+6a^2c^2d^4e^2x\right.}{}\right.}{}$$

input

```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]
```

output

```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(45*c^4*d^8*x^4
 - 30*a*c^3*d^6*e*x^3*(d + e*x) + 6*a^2*c^2*d^4*e^2*x^2*(4*d^2 + 3*d*e*x -
 6*e^2*x^2) + 2*a^3*c*d^2*e^3*x*(264*d^3 + 48*d^2*e*x - 61*d*e^2*x^2 + 95*
e^3*x^3) + a^4*e^4*(384*d^4 + 48*d^3*e*x - 56*d^2*e^2*x^2 + 70*d*e^3*x^3 -
 105*e^4*x^4)))/x^5) + (15*(c*d^2 - a*e^2)^3*(3*c^2*d^4 + 6*a*c*d^2*e^2 +
7*a^2*e^4)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d
*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(1920*a^(7/2)*d^(9/2)*e^(7/2))
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {1215, 1237, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^6(d+ex)}dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x^6}dx$$

$$\Big\downarrow \text{1237}$$

$$-\frac{\int -\frac{ae\left(3cd^2-4cexd-7ae^2\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^5}dx}{5ade}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(3cd^2-4cexd-7ae^2)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{x^5}dx}{10d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

$$\Big\downarrow \text{1237}$$

$$-\frac{-\frac{\int \frac{\left(15c^2d^4+12ace^2d^2+2ce\left(3cd^2-7ae^2\right)xd-35a^2e^4\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^4}dx}{4ade}-\frac{\left(\frac{3cd}{ae}-\frac{7e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4x^4}}{10d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{\left(15c^2d^4+12ace^2d^2+2ce\left(3cd^2-7ae^2\right)xd-35a^2e^4\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^4}dx}{8ade}-\frac{\left(\frac{3cd}{ae}-\frac{7e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{4x^4}}{10d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

$$\Big\downarrow \text{1228}$$

$$-\frac{-\frac{-\frac{5\left(cd^2-ae^2\right)\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{2ade}-\frac{\left(\frac{15c^2d^4}{a}-35ae^4+12cd^2e^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3dex^3}}{8ade}-\frac{\left(\frac{3cd}{ae}-\frac{7e}{d}\right)(x}{}}{10d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

$$\Big\downarrow \text{1152}$$

$$-\frac{-\frac{5\left(cd^2-ae^2\right)\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{2ade}-\frac{\left(\frac{15c^2d^4}{a}\right.}{}}{8ade}}{10d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{-\frac{-\frac{5\left(cd^2-ae^2\right)\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+c\cdots\right.}}{4adex^2}\right.}{2ade}}{8ade}}{10d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{-\frac{5\left(cd^2-ae^2\right)\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+c\cdots}}{4adex^2}\right.}{2ade}}{8ade}}{10d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5dx^5}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]
```

output
```
-1/5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d*x^5) + (-1/4*(((3*c*
d)/(a*e) - (7*e)/d)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/x^4 - (
-1/3*(((15*c^2*d^4)/a + 12*c*d^2*e^2 - 35*a*e^4)*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(3/2))/(d*e*x^3) - (5*(c*d^2 - a*e^2)*(3*c^2*d^4 + 6*a*c*d^
2*e^2 + 7*a^2*e^4)*(-1/4*((2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^2) + ((c*d^2 - a*e^2)^2*ArcTanh[(2*a*d
*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])])/(8*a^(3/2)*d^(3/2)*e^(3/2))))/(2*a*d*e))/(8*a*d*e)
)/(10*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10574 vs. $2(346) = 692$.

Time = 4.76 (sec) , antiderivative size = 10575, normalized size of antiderivative = 27.98

method	result	size
default	Expression too large to display	10575

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/x^6/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 10.97 (sec) , antiderivative size = 872, normalized size of antiderivative = 2.31

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^6(d + ex)} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/7680*(15*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*a^2*c^3*d^6*e^4 - 6*a^3*c^2
*d^4*e^6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*sqrt(a*d*e)*x^5*log((8*a^2*d^2*e
^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*
e + a^2*d*e^3)*x)/x^2) + 4*(384*a^5*d^5*e^5 + (45*a*c^4*d^9*e - 30*a^2*c^3
*d^7*e^3 - 36*a^3*c^2*d^5*e^5 + 190*a^4*c*d^3*e^7 - 105*a^5*d*e^9)*x^4 - 2
*(15*a^2*c^3*d^8*e^2 - 9*a^3*c^2*d^6*e^4 + 61*a^4*c*d^4*e^6 - 35*a^5*d^2*e
^8)*x^3 + 8*(3*a^3*c^2*d^7*e^3 + 12*a^4*c*d^5*e^5 - 7*a^5*d^3*e^7)*x^2 + 4
8*(11*a^4*c*d^6*e^4 + a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x))/(a^4*d^5*e^4*x^5), -1/3840*(15*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*
a^2*c^3*d^6*e^4 - 6*a^3*c^2*d^4*e^6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*sqrt(
-a*d*e)*x^5*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*
d^3*e + a^2*d*e^3)*x)) + 2*(384*a^5*d^5*e^5 + (45*a*c^4*d^9*e - 30*a^2*c^3
*d^7*e^3 - 36*a^3*c^2*d^5*e^5 + 190*a^4*c*d^3*e^7 - 105*a^5*d*e^9)*x^4 - 2
*(15*a^2*c^3*d^8*e^2 - 9*a^3*c^2*d^6*e^4 + 61*a^4*c*d^4*e^6 - 35*a^5*d^2*e
^8)*x^3 + 8*(3*a^3*c^2*d^7*e^3 + 12*a^4*c*d^5*e^5 - 7*a^5*d^3*e^7)*x^2 + 4
8*(11*a^4*c*d^6*e^4 + a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x))/(a^4*d^5*e^4*x^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^6(d + ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**6/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{(ex+d)x^6}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^6), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2352 vs. 2(346) = 692.

Time = 0.21 (sec) , antiderivative size = 2352, normalized size of antiderivative = 6.22

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^6(d+ex)}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm
="giac")
```

output
```
-1/128*(3*c^5*d^10 - 3*a*c^4*d^8*e^2 - 2*a^2*c^3*d^6*e^4 - 6*a^3*c^2*d^4*e
^6 + 15*a^4*c*d^2*e^8 - 7*a^5*e^10)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^
2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^3*d^4*e^3) +
 1/1920*(45*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*
a^4*c^5*d^14*e^4 - 45*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
+ a*d*e))*a^5*c^4*d^12*e^6 - 3870*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))*a^6*c^3*d^10*e^8 - 7770*(sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^7*c^2*d^8*e^10 - 3615*(sqrt(c*d*e)*x -
 sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^8*c*d^6*e^12 - 105*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^9*d^4*e^14 - 768*
sqrt(c*d*e)*a^7*c^2*d^9*e^9 - 1280*sqrt(c*d*e)*a^8*c*d^7*e^11 - 210*(sqrt(
c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c^5*d^13*e^3
 - 7470*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^
4*c^4*d^11*e^5 - 34420*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x
 + a*d*e))^3*a^5*c^3*d^9*e^7 - 41820*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d
^2*x + a*e^2*x + a*d*e))^3*a^6*c^2*d^7*e^9 - 12570*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c*d^5*e^11 - 790*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*d^3*e^13 - 7680*sq
rt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*
a^5*c^3*d^10*e^6 - 23040*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^6(d + ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{x^6\,\left(d + e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 45.78 (sec) , antiderivative size = 1910, normalized size of antiderivative = 5.05

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^6(d+ex)}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x)
```

output
```
( - 768*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*d**5*e**7 - 96*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**6*d**4*e**8*x + 112*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
**6*d**3*e**9*x**2 - 140*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*d**2*e**10*x
**3 + 210*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*d*e**11*x**4 - 768*sqrt(d +
 e*x)*sqrt(a*e + c*d*x)*a**5*c*d**7*e**5 - 1152*sqrt(d + e*x)*sqrt(a*e + c
*d*x)*a**5*c*d**6*e**6*x - 80*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d**5*
e**7*x**2 + 104*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d**4*e**8*x**3 - 17
0*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d**3*e**9*x**4 - 1056*sqrt(d + e*
x)*sqrt(a*e + c*d*x)*a**4*c**2*d**8*e**4*x - 240*sqrt(d + e*x)*sqrt(a*e +
c*d*x)*a**4*c**2*d**7*e**5*x**2 + 208*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4
*c**2*d**6*e**6*x**3 - 308*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**2*d**5*
e**7*x**4 - 48*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**9*e**3*x**2 +
24*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**8*e**4*x**3 + 132*sqrt(d +
 e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**7*e**5*x**4 + 60*sqrt(d + e*x)*sqrt(a
*e + c*d*x)*a**2*c**4*d**10*e**2*x**3 - 30*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a**2*c**4*d**9*e**3*x**4 - 90*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**1
1*e*x**4 + 105*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqr
t(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x)
)*a**6*e**12*x**5 - 120*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d
*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*s...
```

3.38 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 481

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx = \frac{\left(7c^4d^8+8ac^3d^6e^2+6a^2c^2d^4e^4-21a^4e^8\right)\left(2ade+\left(cd^2+ae^2\right)x\right)}{512a^4d^5e^4x^2}$$

$$-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{6dx^6}-\frac{\left(\frac{c}{ae}-\frac{3e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{20x^5}$$

$$+\frac{\left(7c^2d^4+6acd^2e^2-21a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{160a^2d^3e^2x^4}$$

$$-\frac{\left(35c^3d^6+33ac^2d^4e^2+21a^2cd^2e^4-105a^3e^6\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{960a^3d^4e^3x^3}$$

$$-\frac{\left(cd^2-ae^2\right)^3\left(7c^3d^6+15ac^2d^4e^2+21a^2cd^2e^4+21a^3e^6\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{512a^{9/2}d^{11/2}e^{9/2}}$$

output
```
1/512*(-21*a^4*e^8+6*a^2*c^2*d^4*e^4+8*a*c^3*d^6*e^2+7*c^4*d^8)*(2*a*d*e+(
a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^4/d^5/e^4/x^2-1/
6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^6-1/20*(c/a/e-3*e/d^2)*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5+1/160*(-21*a^2*e^4+6*a*c*d^2*e^2+7*
c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^2/d^3/e^2/x^4-1/960*(-1
05*a^3*e^6+21*a^2*c*d^2*e^4+33*a*c^2*d^4*e^2+35*c^3*d^6)*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(3/2)/a^3/d^4/e^3/x^3-1/512*(-a*e^2+c*d^2)^3*(21*a^3*e^6+
21*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+7*c^3*d^6)*arctanh(a^(1/2)*e^(1/2)*(e*x+
d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(9/2)/d^(11/2)/e^(9/
2)
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.84

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(-105c^5d^{10}x^5+5ac^4d^8ex^4(14d+11ex)-2a^2\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-105*c^5*d^10*
x^5 + 5*a*c^4*d^8*e*x^4*(14*d + 11*e*x) - 2*a^2*c^3*d^6*e^2*x^3*(28*d^2 +
16*d*e*x - 27*e^2*x^2) + 6*a^3*c^2*d^4*e^3*x^2*(8*d^3 + 4*d^2*e*x - 6*d*e^
2*x^2 + 13*e^3*x^3) + a^4*c*d^2*e^4*x*(1664*d^4 + 224*d^3*e*x - 264*d^2*e^
2*x^2 + 336*d*e^3*x^3 - 525*e^4*x^4) + a^5*e^5*(1280*d^5 + 128*d^4*e*x - 1
44*d^3*e^2*x^2 + 168*d^2*e^3*x^3 - 210*d*e^4*x^4 + 315*e^5*x^5)))/x^6) - (
15*(c*d^2 - a*e^2)^3*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21
*a^3*e^6)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*
x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(7680*a^(9/2)*d^(11/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.275, Rules used = {1215, 1237, 27, 1237, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^7(d+ex)}dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae+cdx)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x^7}dx$$

$$\Big\downarrow \text{1237}$$

$$-\frac{\int -\frac{3ae\left(cd^2-2cexd-3ae^2\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^6}dx}{6ade}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(cd^2-2cexd-3ae^2\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^6}dx}{4d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$$\Big\downarrow \text{1237}$$

$$\frac{-\frac{\int \frac{\left(7c^2d^4+6ace^2d^2+4ce\left(cd^2-3ae^2\right)xd-21a^2e^4\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^5}dx}{5ade}-\frac{\left(\frac{cd}{ae}-\frac{3e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5x^5}}{4d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\left(7c^2d^4+6ace^2d^2+4ce\left(cd^2-3ae^2\right)xd-21a^2e^4\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^5}dx}{10ade}-\frac{\left(\frac{cd}{ae}-\frac{3e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{5x^5}}{4d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$$\Big\downarrow \text{1237}$$

$$-\frac{-\frac{-\frac{\int \frac{\left(35c^3d^6+33ac^2e^2d^4+21a^2ce^4d^2+2ce\left(7c^2d^4+6ace^2d^2-21a^2e^4\right)xd-105a^3e^6\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^4}dx}{4ade}-\frac{\left(\frac{7c^2d^4}{a}-21ae^4+6cd^2e^2\right)\left(x\left(ae^2+cd^2\right)\cdots}{4dex^4}}{10ade}\cdots}{4d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{\int \frac{\left(35c^3d^6+33ac^2e^2d^4+21a^2ce^4d^2+2ce\left(7c^2d^4+6ace^2d^2-21a^2e^4\right)xd-105a^3e^6\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^4}dx}{8ade}-\frac{\left(\frac{7c^2d^4}{a}-21ae^4+6cd^2e^2\right)\left(x\left(ae^2+cd^2\right)\cdots}{4dex^4}}{10ade}\cdots}{4d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$$\Big\downarrow \; 1228$$

$$-\frac{-\frac{-\frac{-\frac{5\left(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8\right)\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{2ade}-\frac{\left(-105a^3e^6+21a^2cd^2e^4+33ac^2d^4e^2+35c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\cdots}{3adex^3}}{8ade}\cdots}{10ade}\cdots}{4d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$$\Big\downarrow \; 1152$$

$$-\frac{-\frac{-\frac{5\left(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8\right)\left(-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{2ade}-\cdots}{8ade}\cdots}{10ade}\cdots$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$$\Big\downarrow \; 1154$$

$$-\frac{-\frac{-\frac{5\left(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8\right)\left(\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x(\cdots}}{4ade\cdots}\cdots\right)}{2ade}}{8ade}\cdots}{\cdots}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

$\downarrow$ 219

$$-\frac{-\frac{-\frac{\left(-105a^3e^6+21a^2cd^2e^4+33ac^2d^4e^2+35c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3adex^3}-\frac{5\left(-21a^4e^8+6a^2c^2d^4e^4+8ac^3d^6e^2+7c^4d^8\right)\left(\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\cdots\right)}{8\cdots}\cdots\right)}{8ade}}{10a\cdots}\cdots}{\cdots}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{6dx^6}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x]
```

output
```
-1/6*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d*x^6) + (-1/5*(((c*d)
/(a*e) - (3*e)/d)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/x^5 - (-1
/4*(((7*c^2*d^4)/a + 6*c*d^2*e^2 - 21*a*e^4)*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2))/(d*e*x^4) - (-1/3*((35*c^3*d^6 + 33*a*c^2*d^4*e^2 + 21*a
^2*c*d^2*e^4 - 105*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))
/(a*d*e*x^3) - (5*(7*c^4*d^8 + 8*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 21*a^
4*e^8)*(-1/4*((2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])/(a*d*e*x^2) + ((c*d^2 - a*e^2)^2*ArcTanh[(2*a*d*e + (c*d^2
+ a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])])/(8*a^(3/2)*d^(3/2)*e^(3/2))))/(2*a*d*e))/(8*a*d*e))/(10*a*d*e)
)/(4*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 16882 vs. $2(445) = 890$.

Time = 4.73 (sec) , antiderivative size = 16883, normalized size of antiderivative = 35.10

method	result	size
default	Expression too large to display	16883

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/x^7/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 30.08 (sec) , antiderivative size = 1072, normalized size of antiderivative = 2.23

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^7(d + ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/30720*(15*(7*c^6*d^12 - 6*a*c^5*d^10*e^2 - 3*a^2*c^4*d^8*e^4 - 4*a^3*c
^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 42*a^5*c*d^2*e^10 - 21*a^6*e^12)*sqrt(a*
d*e)*x^6*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(1280*a^6*d^6*e^6 - (1
05*a*c^5*d^11*e - 55*a^2*c^4*d^9*e^3 - 54*a^3*c^3*d^7*e^5 - 78*a^4*c^2*d^5
*e^7 + 525*a^5*c*d^3*e^9 - 315*a^6*d*e^11)*x^5 + 2*(35*a^2*c^4*d^10*e^2 -
16*a^3*c^3*d^8*e^4 - 18*a^4*c^2*d^6*e^6 + 168*a^5*c*d^4*e^8 - 105*a^6*d^2*
e^10)*x^4 - 8*(7*a^3*c^3*d^9*e^3 - 3*a^4*c^2*d^7*e^5 + 33*a^5*c*d^5*e^7 -
21*a^6*d^3*e^9)*x^3 + 16*(3*a^4*c^2*d^8*e^4 + 14*a^5*c*d^6*e^6 - 9*a^6*d^4
*e^8)*x^2 + 128*(13*a^5*c*d^7*e^5 + a^6*d^5*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x))/(a^5*d^6*e^5*x^6), 1/15360*(15*(7*c^6*d^12 - 6*a*c^
5*d^10*e^2 - 3*a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 +
42*a^5*c*d^2*e^10 - 21*a^6*e^12)*sqrt(-a*d*e)*x^6*arctan(1/2*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/
(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(1280*a^6
*d^6*e^6 - (105*a*c^5*d^11*e - 55*a^2*c^4*d^9*e^3 - 54*a^3*c^3*d^7*e^5 - 7
8*a^4*c^2*d^5*e^7 + 525*a^5*c*d^3*e^9 - 315*a^6*d*e^11)*x^5 + 2*(35*a^2*c^
4*d^10*e^2 - 16*a^3*c^3*d^8*e^4 - 18*a^4*c^2*d^6*e^6 + 168*a^5*c*d^4*e^8 -
 105*a^6*d^2*e^10)*x^4 - 8*(7*a^3*c^3*d^9*e^3 - 3*a^4*c^2*d^7*e^5 + 33*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^7(d + ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**7/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx=\int \frac{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{3}{2}}}{(ex+d)x^7}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^7), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3251 vs. $2(445) = 890$.

Time = 0.29 (sec) , antiderivative size = 3251, normalized size of antiderivative = 6.76

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^7(d+ex)}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm
="giac")
```

output
```
1/512*(7*c^6*d^12 - 6*a*c^5*d^10*e^2 - 3*a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e
^6 - 15*a^4*c^2*d^4*e^8 + 42*a^5*c*d^2*e^10 - 21*a^6*e^12)*arctan(-(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt
(-a*d*e)*a^4*d^5*e^4) - 1/7680*(105*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^
2*x + a*e^2*x + a*d*e))*a^5*c^6*d^17*e^5 - 90*(sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^6*c^5*d^15*e^7 - 15405*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^7*c^4*d^13*e^9 - 46140*(s
qrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^8*c^3*d^11*e
^11 - 46305*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*
a^9*c^2*d^9*e^13 - 14730*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2
*x + a*d*e))*a^10*c*d^7*e^15 - 315*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))*a^11*d^5*e^17 - 3072*sqrt(c*d*e)*a^8*c^3*d^12*e^10
- 6144*sqrt(c*d*e)*a^9*c^2*d^10*e^12 - 5120*sqrt(c*d*e)*a^10*c*d^8*e^14 -
595*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*c^
6*d^16*e^4 - 30210*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))^3*a^5*c^5*d^14*e^6 - 199425*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))^3*a^6*c^4*d^12*e^8 - 419500*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c^3*d^10*e^10 - 305925*(sqrt(
c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*c^2*d^8*e^12
 - 65010*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^7(d + ex)}\,dx = \int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{x^7\,\left(d+e\,x\right)} \,d x$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^7(d + ex)}\,dx = \int \frac{(ade + (a\,e^2 + c\,d^2)\,x + cde\,x^2)^{\frac{3}{2}}}{x^7\,(ex + d)}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x)
```

3.39 $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 411

$$\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^3}\,dx = -\frac{\left(7cd^2+ae^2\right)(ae+cdx)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^2d^2e^3} - \frac{\left(105c^3d^6-35ac^2d^4e^2-5a^2cd^2e^4-a^3e^6\right)\left(3cd^2-5ae^2-2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^2d^2e^5\left(cd^2-ae^2\right)} - \frac{2d^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{e^3\left(cd^2-ae^2\right)(d+ex)^3} + \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4cde^3(d+ex)} + \frac{3\left(cd^2-ae^2\right)\left(105c^3d^6-35ac^2d^4e^2-5a^2cd^2e^4-a^3e^6\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{64c^{5/2}d^{5/2}e^{11/2}}$$

output
```
-1/8*(a*e^2+7*c*d^2)*(c*d*x+a*e)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/c^2/d^2/e^3-1/64*(-a^3*e^6-5*a^2*c*d^2*e^4-35*a*c^2*d^4*e^2+105*c^3*d^6)*
(-2*c*d*e*x-5*a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d
^2/e^5/(-a*e^2+c*d^2)-2*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e^3/(-
a*e^2+c*d^2)/(e*x+d)^3+1/4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c/d/e^3
/(e*x+d)+3/64*(-a*e^2+c*d^2)*(-a^3*e^6-5*a^2*c*d^2*e^4-35*a*c^2*d^4*e^2+10
5*c^3*d^6)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(11/2)
```

Mathematica [A] (verified)

Time = 10.88 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.85

$$\int \frac{x^3(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^3}\,dx = \frac{\sqrt{(ae + cdx)(d + ex)}\Big(-\sqrt{c}\sqrt{d}\sqrt{e}(3a^3e^6(d + ex) + a^2cde^4(13}{}$$

input
```
Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^3,
x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[c]*Sqrt[d]*Sqrt[e]*(3*a^3*e^6*(d +
e*x) + a^2*c*d*e^4*(13*d^2 + 11*d*e*x - 2*e^2*x^2) - a*c^2*d^2*e^2*(315*d^
3 + 119*d^2*e*x - 44*d*e^2*x^2 + 24*e^3*x^3) + c^3*d^3*(315*d^4 + 105*d^3*
e*x - 42*d^2*e^2*x^2 + 24*d*e^3*x^3 - 16*e^4*x^4))) + (3*(c*d^2 - a*e^2)^(
3/2)*(105*c^3*d^6 - 35*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4 - a^3*e^6)*Sqrt[(c*
d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e +
c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])])/(Sqrt[c*d]*Sqrt[a*e + c*d*x])))/
(64*c^(5/2)*d^(5/2)*e^(11/2)*(d + e*x))
```

Rubi [A] (verified)

Time = 2.41 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.275, Rules used = {1213, 25, 2192, 27, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{1213}$$

$$-\frac{\int -\frac{c^2d^2x^4e^6-cd\left(cd^2-2ae^2\right)x^3e^5+\left(cd^2-ae^2\right)^2x^2e^4-d\left(cd^2-ae^2\right)^2xe^3+d^2\left(cd^2-ae^2\right)^2e^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^7}-\frac{2d^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{c^2d^2x^4e^6-cd(cd^2-2ae^2)x^3e^5+(cd^2-ae^2)^2x^2e^4-d(cd^2-ae^2)^2xe^3+d^2(cd^2-ae^2)^2e^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^7}-}{\frac{2d^3(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^5(d+ex)}}$$

$\Big\downarrow$ 2192

$$\frac{\frac{\int \frac{-3c^2d^2(5cd^2-3ae^2)x^3e^6+2cd(4c^2d^4-11ace^2d^2+4a^2e^4)x^2e^5-8cd^2(cd^2-ae^2)^2xe^4+8cd^3(cd^2-ae^2)^2e^3}{2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4cde}+\frac{1}{4}cde^5x^3\sqrt{x(ae^2+cd^2)+ad}}{e^7}}{\frac{2d^3(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^5(d+ex)}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{-3c^2d^2(5cd^2-3ae^2)x^3e^6+2cd(4c^2d^4-11ace^2d^2+4a^2e^4)x^2e^5-8cd^2(cd^2-ae^2)^2xe^4+8cd^3(cd^2-ae^2)^2e^3}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8cde}+\frac{1}{4}cde^5x^3\sqrt{x(ae^2+cd^2)+ad}}{e^7}}{\frac{2d^3(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^5(d+ex)}}$$

$\Big\downarrow$ 2192

$$\frac{\frac{\frac{\int \frac{3\left(c^2d^2(41c^2d^4-34ace^2d^2+a^2e^4)x^2e^6-4c^2d^3(4c^2d^4-13ace^2d^2+7a^2e^4)xe^5+16c^2d^4(cd^2-ae^2)^2e^4\right)}{2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3cde}-cde^5x^2(5cd^2-3ae^2)\sqrt{x(ae^2+cd^2)+ade+c}}{8cde}}{e^7}}{\frac{2d^3(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^5(d+ex)}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{c^2d^2(41c^2d^4-34ace^2d^2+a^2e^4)x^2e^6-4c^2d^3(4c^2d^4-13ace^2d^2+7a^2e^4)xe^5+16c^2d^4(cd^2-ae^2)^2e^4}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2cde}-cde^5x^2(5cd^2-3ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{8cde}}{e^7}}{\frac{2d^3(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^5(d+ex)}}$$

$\Big\downarrow$ 2192

$$\int \frac{c^2d^2e^5\left(2d\left(32c^3d^6-105ac^2e^2d^4+66a^2ce^4d^2-a^3e^6\right)-e\left(187c^3d^6-187ac^2e^2d^4+13a^2ce^4d^2+3a^3e^6\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx \quad \frac{}{2cde} + \frac{1}{2}cde^5x\left(a^2e^4-34acd^2e^2+41c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right.}$$

$$2cde \qquad 8cde \qquad e^7$$

$$\frac{2d^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

$\downarrow$ 27

$$\frac{1}{4}cde^4\int \frac{2d\left(32c^3d^6-105ac^2e^2d^4+66a^2ce^4d^2-a^3e^6\right)-e\left(187c^3d^6-187ac^2e^2d^4+13a^2ce^4d^2+3a^3e^6\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{1}{2}cde^5x\left(a^2e^4-34acd^2e^2+41c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+a}$$

$$2cde \qquad 8cde \qquad e^7$$

$$\frac{2d^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

$\downarrow$ 1160

$$\frac{1}{4}cde^4\left(\frac{3\left(cd^2-ae^2\right)\left(-a^3e^6-5a^2cd^2e^4-35ac^2d^4e^2+105c^3d^6\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}-\frac{\left(3a^3e^6+13a^2cd^2e^4-187ac^2d^4e^2+187c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)-}}{cd}\right.$$

$$2cde \qquad 8cde$$

$$\frac{2d^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

$\downarrow$ 1092

$$\frac{1}{4}cde^4\left(\frac{3\left(cd^2-ae^2\right)\left(-a^3e^6-5a^2cd^2e^4-35ac^2d^4e^2+105c^3d^6\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{cd}-\frac{\left(3a^3e^6+13a^2cd^2e^4-187ac^2\right.}{}\right.$$

$$2cde$$

$$\frac{2d^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

$\downarrow$ 219

$$\frac{\frac{\frac{1}{2}cde^5x\left(a^2e^4-34acd^2e^2+41c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}+\frac{1}{4}cde^4\left(\frac{3\left(cd^2-ae^2\right)\left(-a^3e^6-5a^2cd^2e^4-35ac^2d^4e^2+105c^3d^6\right)\text{arctanh}\left(\frac{ae^2+c\cdots}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2\cdots\right.}}\right.}{2c^{3/2}d^{3/2}\sqrt{e}}\cdots\right.}{2cde}\cdots}{8cde}$$

$$\frac{2d^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

input
```
Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^3,x]
```

output
```
(-2*d^3*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^5*
(d + e*x)) + ((c*d*e^5*x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/4
+ (-(c*d*e^5*(5*c*d^2 - 3*a*e^2)*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) + ((c*d*e^5*(41*c^2*d^4 - 34*a*c*d^2*e^2 + a^2*e^4)*x*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/2 + (c*d*e^4*(-(((187*c^3*d^6 - 187*a*c^2
*d^4*e^2 + 13*a^2*c*d^2*e^4 + 3*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])/(c*d)) + (3*(c*d^2 - a*e^2)*(105*c^3*d^6 - 35*a*c^2*d^4*e^2 -
5*a^2*c*d^2*e^4 - a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*
Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(3/2)*
d^(3/2)*Sqrt[e])))/4)/(2*c*d*e))/(8*c*d*e))/e^7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1191 vs. $2(381) = 762$.

Time = 2.99 (sec) , antiderivative size = 1192, normalized size of antiderivative = 2.90

method	result	size
default	Expression too large to display	1192

input
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^3,x,method=_RETURN
VERBOSE)
```

output

```
1/e^3*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)
/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*
d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e
^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))-3*d/e^4*(1/3*(d*e*c*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*d*e*c*(x+d/e)+
a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^
2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e
*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))+3/e^5*d^2*(2/(a
*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-6*d*e*
c/(a*e^2-c*d^2)*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*
e^2-c*d^2)*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^
2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+
d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
)/(d*e*c)^(1/2))))-d^3/e^6*(-2/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(5/2)+4*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)/(x+d/e)^
2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-6*d*e*c/(a*e^2-c*d^2)*(1/3
*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*d
*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1
/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d...
```

Fricas [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 928, normalized size of antiderivative = 2.26

$$\int \frac{x^3 (ade + (cd^2 + ae^2)\, x + cdex^2)^{3/2}}{(d + ex)^3}\, dx = \text{Too large to display}$$

input

```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorit
hm="fricas")
```

output
```
[1/256*(3*(105*c^4*d^9 - 140*a*c^3*d^7*e^2 + 30*a^2*c^2*d^5*e^4 + 4*a^3*c*
d^3*e^6 + a^4*d*e^8 + (105*c^4*d^8*e - 140*a*c^3*d^6*e^3 + 30*a^2*c^2*d^4*
e^5 + 4*a^3*c*d^2*e^7 + a^4*e^9)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*
x) + 4*(16*c^4*d^4*e^5*x^4 - 315*c^4*d^8*e + 315*a*c^3*d^6*e^3 - 13*a^2*c^
2*d^4*e^5 - 3*a^3*c*d^2*e^7 - 24*(c^4*d^5*e^4 - a*c^3*d^3*e^6)*x^3 + 2*(21
*c^4*d^6*e^3 - 22*a*c^3*d^4*e^5 + a^2*c^2*d^2*e^7)*x^2 - (105*c^4*d^7*e^2
- 119*a*c^3*d^5*e^4 + 11*a^2*c^2*d^3*e^6 + 3*a^3*c*d*e^8)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^7*x + c^3*d^4*e^6), -1/128*(3*(
105*c^4*d^9 - 140*a*c^3*d^7*e^2 + 30*a^2*c^2*d^5*e^4 + 4*a^3*c*d^3*e^6 + a
^4*d*e^8 + (105*c^4*d^8*e - 140*a*c^3*d^6*e^3 + 30*a^2*c^2*d^4*e^5 + 4*a^3
*c*d^2*e^7 + a^4*e^9)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x
^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(16*c^4*d^4*e^5*x^4 - 3
15*c^4*d^8*e + 315*a*c^3*d^6*e^3 - 13*a^2*c^2*d^4*e^5 - 3*a^3*c*d^2*e^7 -
24*(c^4*d^5*e^4 - a*c^3*d^3*e^6)*x^3 + 2*(21*c^4*d^6*e^3 - 22*a*c^3*d^4*e^
5 + a^2*c^2*d^2*e^7)*x^2 - (105*c^4*d^7*e^2 - 119*a*c^3*d^5*e^4 + 11*a^2*c
^2*d^3*e^6 + 3*a^3*c*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
)/(c^3*d^3*e^7*x + c^3*d^4*e^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 (ade + (cd^2 + ae^2) x + cdex^2)^{3/2}}{(d + ex)^3} \, dx = \text{Timed out}$$

input
```
integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^3} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.97

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^3} dx = \frac{1}{64}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\left(\frac{2\,cdx}{e^2} - \frac{5\,c^4d^5e^{16} - 3}{c^3d^3e}\right.\right.\right.$$
$$- \frac{2\,(c^2d^7 - 2\,acd^5e^2 + a^2d^3e^4)}{\left(\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)e + \sqrt{cde}d\right)e^5}$$
$$- \frac{3\,(105\,c^4d^8 - 140\,ac^3d^6e^2 + 30\,a^2c^2d^4e^4 + 4\,a^3cd^2e^6 + a^4e^8)\log\left(\left|cd^2 + ae^2 + 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2}\right.\right.\right.}{128\sqrt{cde}c^2d^2e^5}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorit
hm="giac")
```

output
```
1/64*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*c*d*x/e^2 - (5*c
^4*d^5*e^16 - 3*a*c^3*d^3*e^18)/(c^3*d^3*e^19))*x + (41*c^4*d^6*e^15 - 34*
a*c^3*d^4*e^17 + a^2*c^2*d^2*e^19)/(c^3*d^3*e^19))*x - (187*c^4*d^7*e^14 -
 187*a*c^3*d^5*e^16 + 13*a^2*c^2*d^3*e^18 + 3*a^3*c*d*e^20)/(c^3*d^3*e^19)
) - 2*(c^2*d^7 - 2*a*c*d^5*e^2 + a^2*d^3*e^4)/(((sqrt(c*d*e)*x - sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*e + sqrt(c*d*e)*d)*e^5) - 3/128*(105*c
^4*d^8 - 140*a*c^3*d^6*e^2 + 30*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^
8)*log(abs(c*d^2 + a*e^2 + 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
 c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^3} dx = \int \frac{x^3\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{\left(d + e\,x\right)^3} dx$$

input
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^3,x)
```

output
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^3, x)
```

Reduce [F]

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^3} dx = \int \frac{x^3(ade + (a\,e^2 + c\,d^2)\,x + cde\,x^2)^{\frac{3}{2}}}{(ex + d)^3} dx$$

input
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x)
```

output
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x)
```

3.40 $\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 322

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^3}\,dx = \frac{(ae+cdx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3cde^2} + \frac{(35c^2d^4-10acd^2e^2-a^2e^4)(3cd^2-5ae^2-2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24cde^4(cd^2-ae^2)} + \frac{2d^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{e^2(cd^2-ae^2)(d+ex)^3} - \frac{(cd^2-ae^2)(35c^2d^4-10acd^2e^2-a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{9/2}}$$

output
```
1/3*(c*d*x+a*e)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^2+1/24*(-a
^2*e^4-10*a*c*d^2*e^2+35*c^2*d^4)*(-2*c*d*e*x-5*a*e^2+3*c*d^2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^4/(-a*e^2+c*d^2)+2*d^2*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(5/2)/e^2/(-a*e^2+c*d^2)/(e*x+d)^3-1/8*(-a*e^2+c*d^2)*(-
a^2*e^4-10*a*c*d^2*e^2+35*c^2*d^4)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)
/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 10.79 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.87

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^3}dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(3a^2e^4(d+ex)-2acde^2(50d^2\right.}{}$$

input
```
Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^3,
x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(3*a^2*e^4*(d + e*
x) - 2*a*c*d*e^2*(50*d^2 + 19*d*e*x - 7*e^2*x^2) + c^2*d^2*(105*d^3 + 35*d
^2*e*x - 14*d*e^2*x^2 + 8*e^3*x^3)) - (3*(c*d^2 - a*e^2)^(3/2)*(35*c^2*d^4
 - 10*a*c*d^2*e^2 - a^2*e^4)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*ArcSinh
[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2
])])/(Sqrt[c*d]*Sqrt[a*e + c*d*x])))/(24*c^(3/2)*d^(3/2)*e^(9/2)*(d + e*x)
)
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1213, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{1213}$$

$$\frac{2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-\frac{\int \frac{-c^2d^2x^3e^5+cd\left(cd^2-2ae^2\right)x^2e^4-\left(cd^2-ae^2\right)^2xe^3+d\left(cd^2-ae^2\right)^2e^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^6}$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{2d^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^4(d+ex)}-\frac{\int \frac{c^2d^2(11cd^2-7ae^2)x^2e^5-2cd(3c^2d^4-8ace^2d^2+3a^2e^4)xe^4+6cd^2(cd^2-ae^2)^2e^3}{2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3cde}-\frac{1}{3}cde^4x^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^6}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2d^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^4(d+ex)}-\frac{\int \frac{c^2d^2(11cd^2-7ae^2)x^2e^5-2cd(3c^2d^4-8ace^2d^2+3a^2e^4)xe^4+6cd^2(cd^2-ae^2)^2e^3}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6cde}-\frac{1}{3}cde^4x^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^6}$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{2d^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^4(d+ex)}-\frac{\frac{\int \frac{c^2d^2e^4\left(2d(12c^2d^4-35ace^2d^2+19a^2e^4)-e(57c^2d^4-52ace^2d^2+3a^2e^4)x\right)}{2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2cde}+\frac{1}{2}cde^4x(11cd^2-7ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6cde}-\frac{1}{3}cde^4x^2\sqrt{x(\ldots}{e^6}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2d^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^4(d+ex)}-\frac{\frac{1}{4}cde^3\int \frac{2d(12c^2d^4-35ace^2d^2+19a^2e^4)-e(57c^2d^4-52ace^2d^2+3a^2e^4)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{1}{2}cde^4x(11cd^2-7ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6cde}-\frac{1}{3}cde^4x^2\sqrt{x(a\ldots}{e^6}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{2d^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^4(d+ex)}-\frac{\frac{1}{4}cde^3\left(\frac{3(cd^2-ae^2)(-a^2e^4-10acd^2e^2+35c^2d^4)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2cd}-\frac{(3a^2e^4-52acd^2e^2+57c^2d^4)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd}\right)+\frac{1}{2}cde^4x(11\ldots}{6cde}}{e^6}$$

$$\Big\downarrow \; 1092$$

$$\frac{2d^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^4(d+ex)}-$$
$$\frac{\frac{1}{4}cde^3\left(\frac{3(cd^2-ae^2)(-a^2e^4-10acd^2e^2+35c^2d^4)\int\frac{1}{4cde-\frac{(cd^2+2cexd+ae^2)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{cd}-\frac{(3a^2e^4-52acd^2e^2+57c^2d^4)\sqrt{x(ae^2+cd}}{cd}\right.}{6cde}}{e^6}$$

$\downarrow$ 219

$$\frac{2d^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^4(d+ex)}-$$
$$\frac{\frac{1}{4}cde^3\left(\frac{3(cd^2-ae^2)(-a^2e^4-10acd^2e^2+35c^2d^4)\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}\sqrt{e}}-\frac{(3a^2e^4-52acd^2e^2+57c^2d^4)\sqrt{x(ae^2+cd^2)+ade+cd}}{cd}\right.}{6cde}}{e^6}$$

input
```
Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^3,x]
```

output
```
(2*d^2*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^4*(
d + e*x)) - (-1/3*(c*d*e^4*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
) + ((c*d*e^4*(11*c*d^2 - 7*a*e^2)*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/2 + (c*d*e^3*(-(((57*c^2*d^4 - 52*a*c*d^2*e^2 + 3*a^2*e^4)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d)) + (3*(c*d^2 - a*e^2)*(35*c^2*
d^4 - 10*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqr
t[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(
3/2)*d^(3/2)*Sqrt[e])))/4)/(6*c*d*e))/e^6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(296) = 592$.

Time = 2.95 (sec) , antiderivative size = 937, normalized size of antiderivative = 2.91

method	result
default	$\frac{\frac{\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\left(\frac{\left(2dec\left(x+\frac{d}{e}\right)+a\,e^{2}-c\,d^{2}\right)\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}{4dec}-\frac{\left(a\,e^{2}-c\,d^{2}\right)^{2}\ln\left(\frac{\frac{a\,e^{2}}{2}-\cdots}{\cdots}\right)}{\cdots}\right)}{2}}{e^{3}}\cdots$

input
```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^3,x,method=_RETURN
VERBOSE)
```

output
```
1/e^3*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)
*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(
x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d
/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^
(1/2)))+d^2/e^5*(-2/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)
*(x+d/e))^(5/2)+4*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x
+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-6*d*e*c/(a*e^2-c*d^2)*(1/3*(d*e*c*(x+
d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*d*e*c*(x+d/e
)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*
e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d
*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))))-2*d/e^4*(2/
(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-6*d*
e*c/(a*e^2-c*d^2)*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(
a*e^2-c*d^2)*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*
e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^
2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/
2))/(d*e*c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 730, normalized size of antiderivative = 2.27

$$\int \frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^3}\,dx = \left[\frac{3\,(35\,c^3d^7-45\,ac^2d^5e^2+9\,a^2cd^3e^4+a^3de^6+(35\,c^3d^6e-45}{}\right.$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorit
hm="fricas")
```

output
```
[1/96*(3*(35*c^3*d^7 - 45*a*c^2*d^5*e^2 + 9*a^2*c*d^3*e^4 + a^3*d*e^6 + (3
5*c^3*d^6*e - 45*a*c^2*d^4*e^3 + 9*a^2*c*d^2*e^5 + a^3*e^7)*x)*sqrt(c*d*e)
*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(8*c^3*d^3*e^4*x^3 + 105*c^3*d^6*e - 100
*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 14*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 +
(35*c^3*d^5*e^2 - 38*a*c^2*d^3*e^4 + 3*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^6*x + c^2*d^3*e^5), 1/48*(3*(35*c^3*d
^7 - 45*a*c^2*d^5*e^2 + 9*a^2*c*d^3*e^4 + a^3*d*e^6 + (35*c^3*d^6*e - 45*a
*c^2*d^4*e^3 + 9*a^2*c*d^2*e^5 + a^3*e^7)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c
*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(8*
c^3*d^3*e^4*x^3 + 105*c^3*d^6*e - 100*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 14
*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + (35*c^3*d^5*e^2 - 38*a*c^2*d^3*e^4 +
3*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^
6*x + c^2*d^3*e^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.99

$$\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^3}\,dx = \frac{1}{24}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(\frac{4\,cdx}{e^2}-\frac{11\,c^3d^4e^{10}-7\,ac\right.\right.}{c^2d^2e^{13}}$$
$$+\frac{2\,(c^2d^6-2\,acd^4e^2+a^2d^2e^4)}{\left(\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)e+\sqrt{cde}d\right)e^4}$$
$$+\frac{(35\,c^3d^6-45\,ac^2d^4e^2+9\,a^2cd^2e^4+a^3e^6)\log\left(\left|cd^2+ae^2+2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+a\right.\right.\right.}{16\,\sqrt{cde}cde^4}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorit
hm="giac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*c*d*x/e^2 - (11*c^3
*d^4*e^10 - 7*a*c^2*d^2*e^12)/(c^2*d^2*e^13))*x + (57*c^3*d^5*e^9 - 52*a*c
^2*d^3*e^11 + 3*a^2*c*d*e^13)/(c^2*d^2*e^13)) + 2*(c^2*d^6 - 2*a*c*d^4*e^2
 + a^2*d^2*e^4)/(((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*
d*e))*e + sqrt(c*d*e)*d)*e^4) + 1/16*(35*c^3*d^6 - 45*a*c^2*d^4*e^2 + 9*a^
2*c*d^2*e^4 + a^3*e^6)*log(abs(c*d^2 + a*e^2 + 2*sqrt(c*d*e)*(sqrt(c*d*e)*
x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c*d*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(ade + (cd^2 + ae^2) x + cdex^2)^{3/2}}{(d + ex)^3} \, dx = \int \frac{x^2 \left(c\, d\, e\, x^2 + \left(c\, d^2 + a\, e^2\right)\, x + a\, d\, e\right)^{3/2}}{\left(d + e\, x\right)^3} \, dx$$

input
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^3,x)
```

output
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 862, normalized size of antiderivative = 2.68

$$\int \frac{x^2(ade + (cd^2 + ae^2) x + cdex^2)^{3/2}}{(d + ex)^3} \, dx = \text{Too large to display}$$

input
```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x)
```

output

```
(24*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d**2*e**5 + 24*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**2*c*d*e**6*x - 800*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*
*2*d**4*e**3 - 304*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**3*e**4*x + 11
2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**2*e**5*x**2 + 840*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*c**3*d**6*e + 280*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3
*d**5*e**2*x - 112*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**4*e**3*x**2 + 6
4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**3*e**4*x**3 - 24*sqrt(e)*sqrt(d)
*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/s
qrt(a*e**2 - c*d**2))*a**3*d*e**6 - 24*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e
)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2)
)*a**3*e**7*x - 216*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x)
 + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**3*e**4
- 216*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqr
t(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**5*x + 1080*sqrt(
e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**5*e**2 + 1080*sqrt(e)*sqrt(d)*sq
rt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt
(a*e**2 - c*d**2))*a*c**2*d**4*e**3*x - 840*sqrt(e)*sqrt(d)*sqrt(c)*log((s
qrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*
d**2))*c**3*d**7 - 840*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + ...
```

3.41 $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 234

$$\begin{aligned}\int \frac{x(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^3}\,dx = & -\frac{(5cd^2-ae^2)\,(3cd^2-5ae^2-2cdex)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4e^3\,(cd^2-ae^2)} \\ & -\frac{2d(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{e\,(cd^2-ae^2)\,(d+ex)^3} \\ & +\frac{3(cd^2-ae^2)\,(5cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4\sqrt{c}\sqrt{d}e^{7/2}}\end{aligned}$$

output
```
-1/4*(-a*e^2+5*c*d^2)*(-2*c*d*e*x-5*a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)/e^3/(-a*e^2+c*d^2)-2*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(5/2)/e/(-a*e^2+c*d^2)/(e*x+d)^3+3/4*(-a*e^2+c*d^2)*(-a*e^2+5*c*d^2)*arcta
nh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
)/c^(1/2)/d^(1/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.89

$$\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^3}\,dx = \frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{e}\sqrt{ae+cdx}(ae^2(13d+5ex)+cd(-15d^2-5dex+2e}{(d+ex)^2}\right.}{4e^{7/2}(ae+}$$

input
```
Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^3,x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[e]*Sqrt[a*e + c*d*x]*(a*e^2*(13*d
+ 5*e*x) + c*d*(-15*d^2 - 5*d*e*x + 2*e^2*x^2)))/(d + e*x)^2 - (6*(5*c^2*d
^4 - 6*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqr
t[e]*(Sqrt[-((c*d^2)/e) + a*e] - Sqrt[a*e + c*d*x]))])/(Sqrt[c]*Sqrt[d]*(d
 + e*x)^(3/2))))/(4*e^(7/2)*(a*e + c*d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1213, 25, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{1213}$$

$$-\frac{\int -\frac{c^2d^2x^2e^4-cd\left(cd^2-2ae^2\right)xe^3+\left(cd^2-ae^2\right)^2e^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^5}-\frac{2d\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{c^2d^2x^2e^4-cd(cd^2-2ae^2)xe^3+(cd^2-ae^2)^2e^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^5}-\frac{2d(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 2192

$$\frac{\frac{\int \frac{cde^3\left(2\left(cd^2-2ae^2\right)\left(2cd^2-ae^2\right)-cde\left(7cd^2-5ae^2\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}cde^3x\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^5}-\frac{2d(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{4}e^2\int \frac{2(cd^2-2ae^2)(2cd^2-ae^2)-cde(7cd^2-5ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{1}{2}cde^3x\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^5}-\frac{2d(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 1160

$$\frac{\frac{1}{4}e^2\left(\frac{3}{2}(cd^2-ae^2)(5cd^2-ae^2)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-(7cd^2-5ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}c}{e^5}-\frac{2d(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 1092

$$\frac{\frac{1}{4}e^2\left(3(cd^2-ae^2)(5cd^2-ae^2)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-(7cd^2-5ae^2)\sqrt{x(ae^2+cd^2}\right.}{e^5}-\frac{2d(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{4}e^2\left(\frac{3(cd^2-ae^2)(5cd^2-ae^2)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}-(7cd^2-5ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)}{e^5}-\frac{2d(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

input
```
Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^3,x]
```

output
```
(-2*d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^3*(d
 + e*x)) + ((c*d*e^3*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/2 + (e
^2*(-((7*c*d^2 - 5*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (
3*(c*d^2 - a*e^2)*(5*c*d^2 - a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2
*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2
*Sqrt[c]*Sqrt[d]*Sqrt[e])))/4)/e^5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 704 vs. $2(212) = 424$.

Time = 2.85 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.01

method	result
default	$\dfrac{\dfrac{2\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)^{2}}-\dfrac{6dec\left(\dfrac{\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+\dfrac{\left(a\,e^{2}-c\,d^{2}\right)\left(\dfrac{\left(2dec\left(x+\frac{d}{e}\right)+a\,e^{2}-c\,d^{2}\right)\sqrt{dec\left(x+\frac{d}{e}\right)^{2}\cdots}}{4dec}\cdots\right.}{\cdots}\right.}{\cdots}}{e^{3}\cdots}$

input
```
int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^3,x,method=_RETURNVE
RBOSE)
```

output
```
1/e^3*(2/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(
5/2)-6*d*e*c/(a*e^2-c*d^2)*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3
/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d
/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2
-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+
d/e))^(1/2))/(d*e*c)^(1/2))))-d/e^4*(-2/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+
d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+4*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)
/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-6*d*e*c/(a*e^2-c*
d^2)*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*
(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x
+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/
e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(
1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.35

$$\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^3}\,dx = \left[\frac{3\,(5\,c^2d^5-6\,acd^3e^2+a^2de^4+(5\,c^2d^4e-6\,acd^2e^3+a^2e^5)x)\sqrt{\cdots}}{\cdots}\right.$$

$$-\frac{3\,(5\,c^2d^5-6\,acd^3e^2+a^2de^4+(5\,c^2d^4e-6\,acd^2e^3+a^2e^5)x)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd\cdots}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+a\cdots}\right.}{8\,(cde^5x\cdots}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorithm
="fricas")
```

output
```
[1/16*(3*(5*c^2*d^5 - 6*a*c*d^3*e^2 + a^2*d*e^4 + (5*c^2*d^4*e - 6*a*c*d^2
*e^3 + a^2*e^5)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2
*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x
+ c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(2*c^2*d^2
*e^3*x^2 - 15*c^2*d^4*e + 13*a*c*d^2*e^3 - 5*(c^2*d^3*e^2 - a*c*d*e^4)*x)*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^5*x + c*d^2*e^4), -1/8
*(3*(5*c^2*d^5 - 6*a*c*d^3*e^2 + a^2*d*e^4 + (5*c^2*d^4*e - 6*a*c*d^2*e^3
+ a^2*e^5)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^
2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(2*c^2*d^2*e^3*x^2 - 15*c^2*d^4*e
+ 13*a*c*d^2*e^3 - 5*(c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x))/(c*d*e^5*x + c*d^2*e^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{(d + ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.16

$$\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^3}\,dx = \frac{1}{4}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(\frac{2\,cdx}{e^2}-\frac{7\,c^2d^3e^5-5\,acde^7}{cde^8}\right) - \frac{3\,(5\,c^2d^4-6\,acd^2e^2+a^2e^4)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{8\,\sqrt{cde}e^3} - \frac{2\left(\sqrt{cde}c^2d^5-2\,\sqrt{cde}acd^3e^2+\sqrt{cde}a^2de^4\right)}{\sqrt{cde}\left(\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)e+\sqrt{cde}d\right)e^3}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorithm
="giac")
```

output
```
1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*c*d*x/e^2 - (7*c^2*d^3*
e^5 - 5*a*c*d*e^7)/(c*d*e^8)) - 3/8*(5*c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4)*
log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c
*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*e^3) - 2*(sqrt(c*d*e)*c^2*d^5 -
2*sqrt(c*d*e)*a*c*d^3*e^2 + sqrt(c*d*e)*a^2*d*e^4)/(sqrt(c*d*e)*((sqrt(c*d
*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*e + sqrt(c*d*e)*d)*e^
3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^3} dx = \int \frac{x\,(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{3/2}}{(d + e\,x)^3} dx$$

input
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^3,x)
```

output
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2))/(d + e*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.48

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^3} dx = \frac{13\sqrt{ex + d}\,\sqrt{cdx + ae}\,ac\,d^2e^3 + 5\sqrt{ex + d}\,\sqrt{cdx + ae}\,acd\,e^4x -}{}$$

input
```
int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x)
```

output

```
(13*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d**2*e**3 + 5*sqrt(d + e*x)*sqrt(a
*e + c*d*x)*a*c*d*e**4*x - 15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**4*e
- 5*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**3*e**2*x + 2*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*c**2*d**2*e**3*x**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e
)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2)
)*a**2*d*e**4 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**5*x - 18*sq
rt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqr
t(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d**3*e**2 - 18*sqrt(e)*sqrt(d)*sqrt
(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a
*e**2 - c*d**2))*a*c*d**2*e**3*x + 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)
*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))
*c**2*d**5 + 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + s
qrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**4*e*x - sqrt(
e)*sqrt(d)*sqrt(c)*a**2*d*e**4 - sqrt(e)*sqrt(d)*sqrt(c)*a**2*e**5*x + 11*
sqrt(e)*sqrt(d)*sqrt(c)*a*c*d**3*e**2 + 11*sqrt(e)*sqrt(d)*sqrt(c)*a*c*d**
2*e**3*x - 10*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**5 - 10*sqrt(e)*sqrt(d)*sqrt(
c)*c**2*d**4*e*x)/(4*c*d*e**4*(d + e*x))
```

3.42 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 158

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^3}\,dx=\frac{3cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{e^2}-\frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{e(d+ex)^2}-\frac{3\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{e^{5/2}}$$

output

```
3*c*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^2-2*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(3/2)/e/(e*x+d)^2-3*c^(1/2)*d^(1/2)*(-a*e^2+c*d^2)*arctanh(c^(
1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/e^(5
/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.94

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{(d+ex)^3}\,dx=\frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{e}\left(-2ae^2+cd(3d+ex)\right)}{(ae+cdx)(d+ex)^2}-\frac{3\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)\text{arct}}{(ae+cdx)^{3/2}(}\right.}{e^{5/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^3,x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[e]*(-2*a*e^2 + c*d*(3*d + e*x)))/(
(a*e + c*d*x)*(d + e*x)^2) - (3*Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)*ArcTanh[(S
qrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)
^(3/2)*(d + e*x)^(3/2))))/e^(5/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {1125, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{1125}$$

$$-\frac{\int \frac{cde^2\left(cd^2-cexd-2ae^2\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^4}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \text{27}$$

$$-\frac{cd\int \frac{cd^2-cexd-2ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \text{1160}$$

$$-\frac{cd\left(\frac{3}{2}\left(cd^2-ae^2\right)\int\frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$\Big\downarrow$ 1092

$$-\frac{cd\left(3\left(cd^2-ae^2\right)\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$\Big\downarrow$ 219

$$-\frac{cd\left(\frac{3\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}-\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{e^2}-\frac{2\left(a-\frac{cd^2}{e^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(d + e*x)^3,x]
```

output
```
(-2*(a - (c*d^2)/e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*
x) - (c*d*(-Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2] + (3*(c*d^2 - a*e^
2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*Sqrt[e])))/e^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(140) = 280$.

Time = 3.13 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.48

method	result
default	$-\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^3}+ 4dec\left(\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^2}- 6dec\left(\frac{\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3}+\ldots\right.\right.$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/(e*x+d)^3,x,method=_RETURNVERB
OSE)
```

output
```
1/e^3*(-2/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^
(5/2)+4*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(5/2)-6*d*e*c/(a*e^2-c*d^2)*(1/3*(d*e*c*(x+d/e)^2+(a*
e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*
d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)
^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.62

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^3} dx = \left[-\frac{3(cd^3 - ade^2 + (cd^2e - ae^3)x)\sqrt{\frac{cd}{e}}\log\left(8c^2d^2e^2x^2 + c^2d^4 + 6\right.}{} \right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorithm="
fricas")
```

output
```
[-1/4*(3*(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(c*d/e)*log(8*c^2*d^2
*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e +
a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^
3*e + a*c*d*e^3)*x) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e
*x + 3*c*d^2 - 2*a*e^2))/(e^3*x + d*e^2), 1/2*(3*(c*d^3 - a*d*e^2 + (c*d^2
*e - a*e^3)*x)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2
*e + (c^2*d^3 + a*c*d*e^2)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x)*(c*d*e*x + 3*c*d^2 - 2*a*e^2))/(e^3*x + d*e^2)]
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^3} dx = \int \frac{((d + ex)(ae + cdx))^{\frac{3}{2}}}{(d + ex)^3} dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**3,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^3} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.42

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^3} dx = \frac{\sqrt{cdex^2 + cd^2x + ae^2x + ade}cd}{e^2} + \frac{3(c^2d^3 - acde^2)\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{2\sqrt{cde}e^2} + \frac{2\left(\sqrt{cde}c^2d^4 - 2\sqrt{cde}acd^2e^2 + \sqrt{cde}a^2e^4\right)}{\sqrt{cde}\left(\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)e + \sqrt{cde}d\right)e^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorithm="
giac")
```

output
```
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*c*d/e^2 + 3/2*(c^2*d^3 - a*c*d
*e^2)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x
^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*e^2) + 2*(sqrt(c*d*e)*c^2*
d^4 - 2*sqrt(c*d*e)*a*c*d^2*e^2 + sqrt(c*d*e)*a^2*e^4)/(sqrt(c*d*e)*((sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*e + sqrt(c*d*e)*d
)*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^3}\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{\left(d+e\,x\right)^3}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^3,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(d + e*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.20

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^3}\,dx = \frac{-8\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,e^3+12\sqrt{ex+d}\,\sqrt{cdx+ae}\,c\,d^2e+4\sqrt{e}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x)
```

output
```
( - 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*e**3 + 12*sqrt(d + e*x)*sqrt(a*e +
 c*d*x)*c*d**2*e + 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c*d*e**2*x + 12*sqrt(
e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))/sqrt(a*e**2 - c*d**2))*a*d*e**2 + 12*sqrt(e)*sqrt(d)*sqrt(c)*log(
(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 -
c*d**2))*a*e**3*x - 12*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d
*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**3 - 12*sq
rt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqr
t(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**2*e*x - 9*sqrt(e)*sqrt(d)*sqrt(c)*
a*d*e**2 - 9*sqrt(e)*sqrt(d)*sqrt(c)*a*e**3*x + 9*sqrt(e)*sqrt(d)*sqrt(c)*
c*d**3 + 9*sqrt(e)*sqrt(d)*sqrt(c)*c*d**2*e*x)/(4*e**3*(d + e*x))
```

3.43 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 193

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x(d+ex)^3}\,dx = -\frac{2\left(\frac{cd}{e}-\frac{ae}{d}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d+ex} - \frac{2a^{3/2}e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{a}\sqrt{e}(d+ex)}\right)}{d^{3/2}} + \frac{2c^{3/2}d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{e^{3/2}}$$

output
```
-2*(c*d/e-a*e/d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)-2*a^(3/2)
*e^(3/2)*arctanh(d^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^(1/2)/e
^(1/2)/(e*x+d))/d^(3/2)+2*c^(3/2)*d^(3/2)*arctanh(e^(1/2)*(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/e^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 538, normalized size of antiderivative = 2.79

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x(d+ex)^3}\,dx =$$

$$-\frac{2((ae+cdx)(d+ex))^{3/2}\left(cd^{5/2}\sqrt{e}(cd^2-ae^2)\sqrt{ae+cdx}+a^{3/2}e^3\left(\sqrt{a}e-i\sqrt{cd^2-ae^2}\right)\sqrt{cd^2-2ae^2-}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)^3),x]
```

output
```
(-2*((a*e + c*d*x)*(d + e*x))^(3/2)*(c*d^(5/2)*Sqrt[e]*(c*d^2 - a*e^2)*Sqr
t[a*e + c*d*x] + a^(3/2)*e^3*(Sqrt[a]*e - I*Sqrt[c*d^2 - a*e^2])*Sqrt[c*d^
2 - 2*a*e^2 - (2*I)*Sqrt[a]*e*Sqrt[c*d^2 - a*e^2]]*Sqrt[d + e*x]*ArcTan[(S
qrt[c*d^2 - 2*a*e^2 - (2*I)*Sqrt[a]*e*Sqrt[c*d^2 - a*e^2]]*Sqrt[d + e*x])/
(Sqrt[d]*Sqrt[e]*(Sqrt[-((c*d^2)/e) + a*e] - Sqrt[a*e + c*d*x]))] + a^(3/2
)*e^3*(Sqrt[a]*e + I*Sqrt[c*d^2 - a*e^2])*Sqrt[c*d^2 - 2*a*e^2 + (2*I)*Sqr
t[a]*e*Sqrt[c*d^2 - a*e^2]]*Sqrt[d + e*x]*ArcTan[(Sqrt[c*d^2 - 2*a*e^2 + (
2*I)*Sqrt[a]*e*Sqrt[c*d^2 - a*e^2]]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[e]*(Sqrt[
-((c*d^2)/e) + a*e] - Sqrt[a*e + c*d*x]))] + 2*c^(5/2)*d^5*Sqrt[d + e*x]*A
rcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*(Sqrt[-((c*d^2)/e) + a*e]
- Sqrt[a*e + c*d*x]))]))/(c*d^(7/2)*e^(3/2)*(a*e + c*d*x)^(3/2)*(d + e*x)^
2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1214, 25, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x(d+ex)^3}dx$$

$$\Big\downarrow \ 1214$$

$$-\frac{\int -\frac{e^3\left(c^2xd^3+a^2e^3\right)}{dx\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^4}-\frac{2\left(\frac{cd}{e}-\frac{ae}{d}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{e^3\left(c^2xd^3+a^2e^3\right)}{dx\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^4}-\frac{2\left(\frac{cd}{e}-\frac{ae}{d}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{c^2xd^3+a^2e^3}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{de}-\frac{2\left(\frac{cd}{e}-\frac{ae}{d}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \ 1269$$

$$\frac{a^2e^3\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+c^2d^3\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{de}-\frac{2\left(\frac{cd}{e}-\frac{ae}{d}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \ 1092$$

$$\frac{a^2e^3\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+2c^2d^3\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{de}-\frac{2\left(\frac{cd}{e}-\frac{ae}{d}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \ 219$$

$$\frac{a^2e^3\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{c^{3/2}d^{5/2}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{e}}}{de}-\frac{2\left(\frac{cd}{e}-\frac{ae}{d}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \ 1154$$

$$\frac{\frac{c^{3/2}d^{5/2}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{e}}-2a^2e^3\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+(cd^2+ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{de}-\frac{2\left(\frac{cd}{e}-\frac{ae}{d}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{c^{3/2}d^{5/2}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{e}}-\frac{a^{3/2}e^{5/2}\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{d}}}{de}-\frac{2\left(\frac{cd}{e}-\frac{ae}{d}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x*(d + e*x)^3),x]
```

output
```
(-2*((c*d)/e - (a*e)/d)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d +
e*x) + ((c^(3/2)*d^(5/2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sq
rt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[e] - (a^
(3/2)*e^(5/2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqr
t[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[d])/(d*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1315 vs. $2(163) = 326$.

Time = 3.33 (sec) , antiderivative size = 1316, normalized size of antiderivative = 6.82

method	result	size
default	Expression too large to display	1316

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/x/(e*x+d)^3,x,method=_RETURNVE
RBOSE)
```

output

```
1/d^3*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*
(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*
(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*
e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+a*d*e*(
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/
2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d
*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))-1/d^3*(1/3*(d*e*c*(x+d/e)^
2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1/4*(2*d*e*c*(x+d/e)+a*e
^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c
*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*
(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))-1/e/d^2*(2/(a*e^2-
c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-6*d*e*c/(a*
e^2-c*d^2)*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c
*d^2)*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d
^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c
*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*
e*c)^(1/2))))-1/e^2/d*(-2/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^2+(a*e^2-
c*d^2)*(x+d/e))^(5/2)+4*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)/(x+d/e)^2*(d*
e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-6*d*e*c/(a*e^2-c*d^2)*(1/3*(...
```

Fricas [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 1277, normalized size of antiderivative = 6.62

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x(d + ex)^3}\,dx = \text{Too large to display}$$

input

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d)^3,x, algorithm
="fricas")
```

output
```
[1/2*((c*d^2*e*x + c*d^3)*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*
a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + (
a*e^3*x + a*d*e^2)*sqrt(a*e/d)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e
^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2
*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2)
 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2))/(d*e^2*x
+ d^2*e), -1/2*(2*(c*d^2*e*x + c*d^3)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(
c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) - (a*e^3*x + a*d*e^2
)*sqrt(a*e/d)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2
 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d
*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d^2 - a*e^2))/(d*e^2*x + d^2*e), 1/2*(2*
(a*e^3*x + a*d*e^2)*sqrt(-a*e/d)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*e/d)/(a*c*d*e^2*x^2 +
a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + (c*d^2*e*x + c*d^3)*sqrt(c*d/e)*lo
g(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x +
 c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e)
+ 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*...
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x(d + ex)^3}\,dx = \int \frac{((d + ex)\,(ae + cdx))^{\frac{3}{2}}}{x\,(d + ex)^3}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x/(e*x+d)**3,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(x*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x(d+ex)^3}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex+d)^3 x}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d)^3,x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^3*x), x
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x(d+ex)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d)^3,x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x(d+ex)^3}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{3/2}}{x\,(d+e\,x)^3}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x*(d + e*x)^3),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x*(d + e*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.89

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x(d + ex)^3} dx = \frac{2\sqrt{ex + d}\,\sqrt{cdx + ae}\,ad\,e^3 - 2\sqrt{ex + d}\,\sqrt{cdx + ae}\,c\,d^3e + \sqrt{e}\,\sqrt{c}}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d)^3,x)
```

output
```
(2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*d*e**3 - 2*sqrt(d + e*x)*sqrt(a*e + c
*d*x)*c*d**3*e + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - s
qrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*
x))*a*d*e**3 + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqr
t(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x)
)*a*e**4*x + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(
2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*
a*d*e**3 + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*
sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*
e**4*x - sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*
x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*a*d*e**3 - sqrt(
e)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e +
c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*a*e**4*x + 2*sqrt(e)*sqrt(d)*s
qrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqr
t(a*e**2 - c*d**2))*c*d**4 + 2*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a
*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**3
*e*x + 2*sqrt(e)*sqrt(d)*sqrt(c)*a*d**2*e**2 + 2*sqrt(e)*sqrt(d)*sqrt(c)*a
*d*e**3*x - 2*sqrt(e)*sqrt(d)*sqrt(c)*c*d**4 - 2*sqrt(e)*sqrt(d)*sqrt(c)*c
*d**3*e*x)/(d**2*e**2*(d + e*x))
```

3.44 $\int \frac{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}{x^2(d+ex)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 176

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{x^2(d+ex)^3}\,dx = \frac{3\left(c-\frac{ae^2}{d^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d+ex} - \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{dx(d+ex)^2} - \frac{3\sqrt{a}\sqrt{e}(cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{a}\sqrt{e}(d+ex)}\right)}{d^{5/2}}$$

output
```
3*(c-a*e^2/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)-(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x/(e*x+d)^2-3*a^(1/2)*e^(1/2)*(-a*e^2+c*d^
2)*arctanh(d^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^(1/2)/e^(1/2)
/(e*x+d))/d^(5/2)
```

Mathematica [A] (verified)

Time = 10.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.82

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{x^2(d+ex)^3}\,dx = \frac{-\sqrt{d}(ae+cdx)\left(-2cd^2x+ae(d+3ex)\right)+3\sqrt{a}\sqrt{e}(-cd^2+ae^2)}{d^{5/2}x\sqrt{(ae+cdx)(d+e}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)^3),
x]
```

output
```
(-(Sqrt[d]*(a*e + c*d*x)*(-2*c*d^2*x + a*e*(d + 3*e*x))) + 3*Sqrt[a]*Sqrt[
e]*(-(c*d^2) + a*e^2)*x*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*S
qrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(d^(5/2)*x*Sqrt[(a*e +
 c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1214, 25, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^2(d+ex)^3}dx$$

$$\Big\downarrow \text{1214}$$

$$\frac{2\left(c-\frac{ae^2}{d^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}-\frac{\int -\frac{ae^5\left(ade+\left(2cd^2-ae^2\right)x\right)}{d^2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ae^5\left(ade+\left(2cd^2-ae^2\right)x\right)}{d^2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^4}+\frac{2\left(c-\frac{ae^2}{d^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \text{27}$$

$$\frac{ae\int\frac{ade+(2cd^2-ae^2)x}{x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{d^2}+\frac{2\left(c-\frac{ae^2}{d^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \textbf{1228}$$

$$\frac{ae\left(\frac{3}{2}\left(cd^2-ae^2\right)\int\frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}\right)}{d^2}+\frac{2\left(c-\frac{ae^2}{d^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \textbf{1154}$$

$$\frac{ae\left(-3\left(cd^2-ae^2\right)\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+(cd^2+ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}\right)}{d^2}+\frac{2\left(c-\frac{ae^2}{d^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{ae\left(-\frac{3(cd^2-ae^2)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}\sqrt{d}\sqrt{e}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}\right)}{d^2}+\frac{2\left(c-\frac{ae^2}{d^2}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d+ex}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)^3),x]
```

output

```
(2*(c - (a*e^2)/d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x
) + (a*e*(-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/x) - (3*(c*d^2 - a
*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[a]*Sqrt[d]*Sqrt[e])))/
d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2005 vs. $2(158) = 316$.

Time = 3.87 (sec) , antiderivative size = 2006, normalized size of antiderivative = 11.40

method	result	size
default	Expression too large to display	2006

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/x^2/(e*x+d)^3,x,method=_RETURN
VERBOSE)
```

output
```
1/d^3*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)+3/2*(a*e^2+c*d^2
)/a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)*(1/
4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/
8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(
d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+a*d*e
*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+
1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/
(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1
/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*e*x+a*
e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e
^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1
/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2))/(d*e*c)^(1/2))))+1/e/d^2*(-2/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+4*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)/(
x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-6*d*e*c/(a*e^2-c*d^
2)*(1/3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+1/2*(a*e^2-c*d^2)*(1
/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d
/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e)
)/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^...
```

Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.53

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^2(d + ex)^3}\,dx = \left[-\frac{3\left((cd^2e - ae^3)x^2 + (cd^3 - ade^2)x\right)\sqrt{\frac{ae}{d}}\log\left(\frac{8a^2d^2e^2 + (c^2d^4 + 6ac\right.}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d)^3,x, algorit
hm="fricas")
```

output
```
[-1/4*(3*((c*d^2*e - a*e^3)*x^2 + (c*d^3 - a*d*e^2)*x)*sqrt(a*e/d)*log((8*
a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d)
+ 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*(a*d*e - (2*c*d^2 - 3*a*e^2)*x))/(d^2*e*x^2 + d^3*x), 1/2*(3*((c
*d^2*e - a*e^3)*x^2 + (c*d^3 - a*d*e^2)*x)*sqrt(-a*e/d)*arctan(1/2*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a
*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) - 2*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e - (2*c*d^2 - 3*a*e^2)*x))/(d^2*e
*x^2 + d^3*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^2(d + ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**2/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade + (cd^2 + ae^2)\, x + cdex^2\right)^{3/2}}{x^2(d + ex)^3}\, dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{3}{2}}}{\left(ex + d\right)^3 x^2}\, dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d)^3,x, algorit
hm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^3*x^2),
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(ade + (cd^2 + ae^2)\, x + cdex^2\right)^{3/2}}{x^2(d + ex)^3}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d)^3,x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,1]%%%},[6,0]%%%}+%%%{%%{[%%%{-2,[0,1,0]%%%},0]:
[1,0,%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{x^2(d+ex)^3}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{x^2\,\left(d + e\,x\right)^3}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)^3),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 1374, normalized size of antiderivative = 7.81

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{3/2}}{x^2(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d)^3,x)
```

output
```
( - 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*d**2*e**4 - 18*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**2*d*e**5*x - 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d**4
*e**2 + 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d**3*e**3*x + 4*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*c**2*d**5*e*x - 9*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*
sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d
)*sqrt(c)*sqrt(d + e*x))*a**2*d*e**5*x - 9*sqrt(e)*sqrt(d)*sqrt(a)*log(sqr
t(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + s
qrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*e**6*x**2 + 6*sqrt(e)*sqrt(d)*sqrt(a)*l
og(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**
2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c*d**3*e**3*x + 6*sqrt(e)*sqrt(d)*sq
rt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2
+ c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c*d**2*e**4*x**2 + 3*sqrt(e)*
sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e
 + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**2*d**5*e*x + 3*sqr
t(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a
)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**2*d**4*e**2*x
**2 - 9*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqr
t(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*
d*e**5*x - 9*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(
2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x...
```

3.45 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^3(d+ex)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 279

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^3(d+ex)^3}\,dx=\frac{3(cd^2-5ae^2)\left(c-\frac{ae^2}{d^2}\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4ade(d+ex)}-\frac{\left(\frac{c}{ae}-\frac{5e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{4x(d+ex)^2}-\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{2adex^2(d+ex)^3}-\frac{3(cd^2-5ae^2)(cd^2-ae^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{a}\sqrt{e}(d+ex)}\right)}{4\sqrt{a}d^{7/2}\sqrt{e}}$$

output

```
3/4*(-5*a*e^2+c*d^2)*(c-a*e^2/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/a/d/e/(e*x+d)-1/4*(c/a/e-5*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
/x/(e*x+d)^2-1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/a/d/e/x^2/(e*x+d)
^3-3/4*(-5*a*e^2+c*d^2)*(-a*e^2+c*d^2)*arctanh(d^(1/2)*(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2)/a^(1/2)/e^(1/2)/(e*x+d))/a^(1/2)/d^(7/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 10.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.64

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)^3}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{d}\left(cd^2x(5d+13ex)+ae\left(2d^2-5dex-15e^2x^2\right)\right)}{x^2(d+ex)}-\cdots\right)}{4d^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)^3),
x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[d]*(c*d^2*x*(5*d + 13*e*x) + a*e*(
2*d^2 - 5*d*e*x - 15*e^2*x^2)))/(x^2*(d + e*x))) - (3*(c^2*d^4 - 6*a*c*d^2
*e^2 + 5*a^2*e^4)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqr
t[d + e*x])])/(Sqrt[a]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(4*d^(7/
2))
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1214, 25, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^3(d+ex)^3}dx$$

$$\Big\downarrow \; 1214$$

$$-\frac{\int -\frac{\frac{a^2e^6}{d}+\frac{a\left(2cd^2-ae^2\right)xe^5}{d^2}+\frac{\left(cd^2-ae^2\right)^2x^2e^4}{d^3}}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^4}-\frac{2e\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{a^2e^6}{d}+\frac{a\left(2cd^2-ae^2\right)xe^5}{d^2}+\frac{\left(cd^2-ae^2\right)^2x^2e^4}{d^3}}{x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^4}-\frac{2e\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$$\Big\downarrow \; 2181$$

$$\frac{-\frac{\int -\frac{ae^5\left(ae\left(5cd^2-7ae^2\right)+2d\left(\frac{2a^2e^4}{d^2}-5ace^2+2c^2d^2\right)x\right)}{2dx^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{ae^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2d^2x^2}}{e^4}-\frac{2e\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{e^4\int \frac{ae\left(5cd^2-7ae^2\right)+2d\left(\frac{2a^2e^4}{d^2}-5ace^2+2c^2d^2\right)x}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4d^2}-\frac{ae^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2d^2x^2}}{e^4}-\frac{2e\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$$\Big\downarrow \; 1228$$

$$\frac{\frac{e^4\left(\frac{3\left(cd^2-5ae^2\right)\left(cd^2-ae^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d}-\frac{\left(5cd^2-7ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}\right)}{4d^2}-\frac{ae^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2d^2x^2}}{e^4}-\frac{2e\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{e^4\left(-\frac{3\left(cd^2-5ae^2\right)\left(cd^2-ae^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{d}-\frac{\left(5cd^2-7ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}\right)}{4d^2}-ae^5\sqrt{x(a\cdots}}{e^4}-\frac{2e\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{e^4\left(-\frac{3\left(cd^2-5ae^2\right)\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}d^{3/2}\sqrt{e}}-\frac{\left(5cd^2-7ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}\right)}{4d^2}-\frac{ae^5\sqrt{x(ae^2+cd^2)+c}}{2d^2x^2}}{e^4}$$

$$\frac{2e\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)^3),x]
```

output
```
(-2*e*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^3*(d
 + e*x)) + (-1/2*(a*e^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2*
x^2) + (e^4*(-(((5*c*d^2 - 7*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(d*x)) - (3*(c*d^2 - 5*a*e^2)*(c*d^2 - a*e^2)*ArcTanh[(2*a*d*e + (c
*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])])/(2*Sqrt[a]*d^(3/2)*Sqrt[e])))/(4*d^2))/e^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3142 vs. $2(251) = 502$.

Time = 3.94 (sec) , antiderivative size = 3143, normalized size of antiderivative = 11.27

method	result	size
default	Expression too large to display	3143

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/x^3/(e*x+d)^3,x,method=_RETURN
VERBOSE)
```

output
```
1/d^3*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)+1/4*(a*e^2+c
*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)+3/2*(a*e^2
+c*d^2)/a/d/e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^
2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/
d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*
x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))
+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*
a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(
1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d
*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))+4*c/a*(1/8*(2*c*d*
e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c
*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2
+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c
*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2))/(d*e*c)^(1/2))))+3/2*c/a*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^
2*e)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*
d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln
((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^
2*e)^(1/2))/(d*e*c)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+
1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e...
```

Fricas [A] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.11

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^3(d+ex)^3}\,dx = \left[\frac{3\,((c^2d^4e - 6\,acd^2e^3 + 5\,a^2e^5)x^3 + (c^2d^5 - 6\,acd^3e^2 + 5\,a^2de^4)x}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d)^3,x, algorit
hm="fricas")
```

output
```
[1/16*(3*((c^2*d^4*e - 6*a*c*d^2*e^3 + 5*a^2*e^5)*x^3 + (c^2*d^5 - 6*a*c*d
^3*e^2 + 5*a^2*d*e^4)*x^2)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a
*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x
^2) - 4*(2*a^2*d^3*e^2 + (13*a*c*d^3*e^2 - 15*a^2*d*e^4)*x^2 + 5*(a*c*d^4*
e - a^2*d^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^4*e^
2*x^3 + a*d^5*e*x^2), 1/8*(3*((c^2*d^4*e - 6*a*c*d^2*e^3 + 5*a^2*e^5)*x^3
+ (c^2*d^5 - 6*a*c*d^3*e^2 + 5*a^2*d*e^4)*x^2)*sqrt(-a*d*e)*arctan(1/2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2
*(2*a^2*d^3*e^2 + (13*a*c*d^3*e^2 - 15*a^2*d*e^4)*x^2 + 5*(a*c*d^4*e - a^2
*d^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^4*e^2*x^3 +
 a*d^5*e*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^3(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**3/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^3(d+ex)^3}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex+d)^3x^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d)^3,x, algorit
hm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^3*x^3),
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d)^3,x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,3,9]%%%},[2,4]%%%}+%%%{%%%{-4,[1,5,7]%%%},[2,3]%%
%}+%%%{%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)^3}\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}{x^3\,\left(d+e\,x\right)^3}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)^3),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 1897, normalized size of antiderivative = 6.80

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^3(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d)^3,x)
```

output
```
( - 20*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*d**3*e**4 + 50*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**3*d**2*e**5*x + 150*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
*3*d*e**6*x**2 - 12*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d**5*e**2 - 20*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d**4*e**3*x - 40*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a**2*c*d**3*e**4*x**2 - 30*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
*c**2*d**6*e*x - 78*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**5*e**2*x**2
+ 75*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c
)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*d*e
**6*x**2 + 75*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt
(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))
*a**3*e**7*x**3 - 45*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x)
 - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))*a**2*c*d**3*e**4*x**2 - 45*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqr
t(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*s
qrt(c)*sqrt(d + e*x))*a**2*c*d**2*e**5*x**3 - 39*sqrt(e)*sqrt(d)*sqrt(a)*l
og(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**
2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**5*e**2*x**2 - 39*sqrt(e)*sqr
t(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e +
a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**4*e**3*x**3 +
9*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c...
```

3.46 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^4(d+ex)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 362

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^4(d+ex)^3}\,dx = -\frac{(3c^2d^4-100acd^2e^2+105a^2e^4)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24ad^4(d+ex)} - \frac{ae\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3dx^3(d+ex)} - \frac{7\left(c-\frac{ae^2}{d^2}\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12x^2(d+ex)} - \frac{\left(\frac{3c^2d^2}{a}-38ce^2+\frac{35ae^4}{d^2}\right)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24dex(d+ex)} + \frac{(cd^2-ae^2)(c^2d^4+10acd^2e^2-35a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{a}\sqrt{e}(d+ex)}\right)}{8a^{3/2}d^{9/2}e^{3/2}}$$

output

```
-1/24*(105*a^2*e^4-100*a*c*d^2*e^2+3*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2)/a/d^4/(e*x+d)-1/3*a*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/
d/x^3/(e*x+d)-7/12*(c-a*e^2/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x
^2/(e*x+d)-1/24*(3*c^2*d^2/a-38*c*e^2+35*a*e^4/d^2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/d/e/x/(e*x+d)+1/8*(-a*e^2+c*d^2)*(-35*a^2*e^4+10*a*c*d^2
*e^2+c^2*d^4)*arctanh(d^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^(1
/2)/e^(1/2)/(e*x+d))/a^(3/2)/d^(9/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 10.18 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.71

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{x^4(d+ex)^3}\,dx = \frac{\sqrt{ae+cdx}\Big(-\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}(3c^2d^4x^2(d+ex)+2acd^2ex(7$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)^3),
x]
```

output
```
(Sqrt[a*e + c*d*x]*(-(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*(3*c^2*d^4
*x^2*(d + e*x) + 2*a*c*d^2*e*x*(7*d^2 - 19*d*e*x - 50*e^2*x^2) + a^2*e^2*(
8*d^3 - 14*d^2*e*x + 35*d*e^2*x^2 + 105*e^3*x^3))) + 3*(c^3*d^6 + 9*a*c^2*
d^4*e^2 - 45*a^2*c*d^2*e^4 + 35*a^3*e^6)*x^3*Sqrt[d + e*x]*ArcTanh[(Sqrt[d
]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])]))/(24*a^(3/2)*d^(9/2
)*e^(3/2)*x^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.94 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {1214, 25, 2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{x^4(d+ex)^3}\,dx$$

$$\Big\downarrow \text{1214}$$

$$\frac{2e^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^4(d+ex)} - \frac{\int -\frac{\frac{a^2e^6}{d}-\frac{(cd^2-ae^2)^2x^3e^5}{d^4}+\frac{a(2cd^2-ae^2)xe^5}{d^2}+\frac{(cd^2-ae^2)^2x^2e^4}{d^3}}{x^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{a^2e^6}{d}-\frac{\left(cd^2-ae^2\right)^2x^3e^5}{d^4}+\frac{a\left(2cd^2-ae^2\right)xe^5}{d^2}+\frac{\left(cd^2-ae^2\right)^2x^2e^4}{d^3}}{x^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^4}+\frac{2e^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\big\downarrow$ 2181

$$\frac{-\frac{\int -\frac{-\frac{6a\left(cd^2-ae^2\right)^2x^2e^6}{d^3}+\frac{a^2\left(7cd^2-11ae^2\right)e^6}{d}+2a\left(\frac{3a^2e^4}{d^2}-8ace^2+3c^2d^2\right)xe^5}{2x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3ade}-\frac{ae^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3d^2x^3}}{e^4}+\frac{2e^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{-\frac{6a\left(cd^2-ae^2\right)^2x^2e^6}{d^3}+a^2\left(7cd-\frac{11ae^2}{d}\right)e^6+2a\left(\frac{3a^2e^4}{d^2}-8ace^2+3c^2d^2\right)xe^5}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6ade}-\frac{ae^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3d^2x^3}}{e^4}+\frac{2e^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\big\downarrow$ 2181

$$\frac{\frac{-\frac{\int -\frac{a^2e^6\left(3c^2d^4-52ace^2d^2-2e\left(\frac{12a^2e^4}{d^2}-35ace^2+19c^2d^2\right)xd+57a^2e^4\right)}{2dx^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{ae^5\left(7cd^2-11ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2d^2x^2}}{6ade}-\frac{ae^5\sqrt{x(ae^2+cd^2)+ade+}}{3d^2x^3}}{e^4}$$

$$\frac{2e^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{ae^5\int \frac{3c^2d^4-52ace^2d^2-2e\left(\frac{12a^2e^4}{d^2}-35ace^2+19c^2d^2\right)xd+57a^2e^4}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4d^2}-\frac{ae^5\left(7cd^2-11ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2d^2x^2}}{6ade}-\frac{ae^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3d^2x^3}}{e^4}$$

$$\frac{2e^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\big\downarrow$ 1228

$$\frac{\frac{\frac{ae^5\left(-\frac{3\left(cd^2-ae^2\right)\left(-35a^2e^4+10acd^2e^2+c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(\frac{3c^2d^4}{a}+57ae^4-52cd^2e^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dex}\right)}{4d^2}-\frac{ae^5\left(7cd^2-11\cdots\right.}{\cdots}}{6ade}}{e^4}$$

$$\frac{2e^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{ae^5\left(\frac{3\left(cd^2-ae^2\right)\left(-35a^2e^4+10acd^2e^2+c^2d^4\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}-\frac{\left(\frac{3c^2d^4}{a}+57ae^4-52cd^2e^2\right)\sqrt{x\left(ae^2+cd^2\right)+\cdots}}{dex}\cdots\right.}{4d^2}\cdots}{6ade}\cdots}{e^4}$$

$$\frac{2e^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{ae^5\left(\frac{3\left(cd^2-ae^2\right)\left(-35a^2e^4+10acd^2e^2+c^2d^4\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(\frac{3c^2d^4}{a}+57ae^4-52cd^2e^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dex}\cdots\right.}{4d^2}\cdots}{6ade}\cdots}{e^4}$$

$$\frac{2e^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)^3),x]
```

output
```
(2*e^2*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^4*(
d + e*x)) + (-1/3*(a*e^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2
*x^3) + (-1/2*(a*e^5*(7*c*d^2 - 11*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])/(d^2*x^2) + (a*e^5*(-((((3*c^2*d^4)/a - 52*c*d^2*e^2 + 57*a*e
^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d*e*x)) + (3*(c*d^2 - a*
e^2)*(c^2*d^4 + 10*a*c*d^2*e^2 - 35*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a
*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])])/(2*a^(3/2)*d^(3/2)*e^(3/2))))/(4*d^2))/(6*a*d*e))/e^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5037 vs. $2(330) = 660$.

Time = 4.74 (sec) , antiderivative size = 5038, normalized size of antiderivative = 13.92

method	result	size
default	Expression too large to display	5038

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/x^4/(e*x+d)^3,x,method=_RETURN
VERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 4.92 (sec) , antiderivative size = 768, normalized size of antiderivative = 2.12

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^4(d + ex)^3} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d)^3,x, algorit
hm="fricas")
```

output
```
[1/96*(3*((c^3*d^6*e + 9*a*c^2*d^4*e^3 - 45*a^2*c*d^2*e^5 + 35*a^3*e^7)*x^
4 + (c^3*d^7 + 9*a*c^2*d^5*e^2 - 45*a^2*c*d^3*e^4 + 35*a^3*d*e^6)*x^3)*sqr
t(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(8*a^3*d^4*e^3 + (3*a*
c^2*d^5*e^2 - 100*a^2*c*d^3*e^4 + 105*a^3*d*e^6)*x^3 + (3*a*c^2*d^6*e - 38
*a^2*c*d^4*e^3 + 35*a^3*d^2*e^5)*x^2 + 14*(a^2*c*d^5*e^2 - a^3*d^3*e^4)*x)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^5*e^3*x^4 + a^2*d^6*e
^2*x^3), -1/48*(3*((c^3*d^6*e + 9*a*c^2*d^4*e^3 - 45*a^2*c*d^2*e^5 + 35*a^
3*e^7)*x^4 + (c^3*d^7 + 9*a*c^2*d^5*e^2 - 45*a^2*c*d^3*e^4 + 35*a^3*d*e^6)
*x^3)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2
+ (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(8*a^3*d^4*e^3 + (3*a*c^2*d^5*e^2 - 100*
a^2*c*d^3*e^4 + 105*a^3*d*e^6)*x^3 + (3*a*c^2*d^6*e - 38*a^2*c*d^4*e^3 + 3
5*a^3*d^2*e^5)*x^2 + 14*(a^2*c*d^5*e^2 - a^3*d^3*e^4)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^5*e^3*x^4 + a^2*d^6*e^2*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^4(d + ex)^3} dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**4/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)^3}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}}{\left(ex+d\right)^3x^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d)^3,x, algorit
hm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^3*x^4),
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)^3}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d)^3,x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,4,11]%%%},[2,5]%%%}+%%%{%%%{-5,[1,6,9]%%%},[2,4]%
%%}+%%%{%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)^3}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}{x^4\,{\left(d+e\,x\right)}^3}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)^3),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^4*(d + e*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 2498, normalized size of antiderivative = 6.90

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^4(d+ex)^3}\,dx=\text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d)^3,x)
```

output

```
( - 112*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*d**4*e**5 + 196*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**4*d**3*e**6*x - 490*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a**4*d**2*e**7*x**2 - 1470*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*d*e**8*x**
3 - 80*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**6*e**3 - 56*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**3*c*d**5*e**4*x + 182*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*a**3*c*d**4*e**5*x**2 + 350*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**3*
e**6*x**3 - 140*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**7*e**2*x + 33
8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**6*e**3*x**2 + 958*sqrt(d +
e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**5*e**4*x**3 - 30*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a*c**3*d**8*e*x**2 - 30*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*
d**7*e**2*x**3 - 735*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x)
 - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))*a**4*d*e**8*x**3 - 735*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*
e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(
c)*sqrt(d + e*x))*a**4*e**9*x**4 + 420*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)
*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(
d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**3*e**6*x**3 + 420*sqrt(e)*sqrt(d)*sqrt
(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 +
c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**2*e**7*x**4 + 486*sqrt(
e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(...
```

3.47 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 483

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)^3}\,dx=\frac{\left(3c^3d^6+13ac^2d^4e^2-315a^2cd^2e^4+315a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right.}}{64a^2d^5e(d+ex)}$$

$$-\frac{ae\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4dx^4(d+ex)}-\frac{3\left(c-\frac{ae^2}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8x^3(d+ex)}$$

$$-\frac{\left(\frac{c^2d^2}{a}-22ce^2+\frac{21ae^4}{d^2}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32dex^2(d+ex)}$$

$$+\frac{\left(cd^2-ae^2\right)\left(3c^2d^4+14acd^2e^2-105a^2e^4\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64a^2d^4e^2x(d+ex)}$$

$$-\frac{3\left(cd^2-ae^2\right)\left(c^3d^6+5ac^2d^4e^2+35a^2cd^2e^4-105a^3e^6\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{a}\sqrt{e}(d+ex)}\right)}{64a^{5/2}d^{11/2}e^{5/2}}$$

output
```
1/64*(315*a^3*e^6-315*a^2*c*d^2*e^4+13*a*c^2*d^4*e^2+3*c^3*d^6)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^5/e/(e*x+d)-1/4*a*e*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)/d/x^4/(e*x+d)-3/8*(c-a*e^2/d^2)*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d)-1/32*(c^2*d^2/a-22*c*e^2+21*a*e^4/d^2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/e/x^2/(e*x+d)+1/64*(-a*e^2+c*d^2)*(
-105*a^2*e^4+14*a*c*d^2*e^2+3*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/a^2/d^4/e^2/x/(e*x+d)-3/64*(-a*e^2+c*d^2)*(-105*a^3*e^6+35*a^2*c*d^2*
e^4+5*a*c^2*d^4*e^2+c^3*d^6)*arctanh(d^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/a^(1/2)/e^(1/2)/(e*x+d))/a^(5/2)/d^(11/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 10.45 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.67

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{x^5(d+ex)^3}\,dx = \frac{\sqrt{ae+cdx}\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\left(3c^3d^6x^3(d+ex)+ac^2d^4ex^2(-\right.\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)^3),
x]
```

output
```
(Sqrt[a*e + c*d*x]*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*(3*c^3*d^6*x
^3*(d + e*x) + a*c^2*d^4*e*x^2*(-2*d^2 + 11*d*e*x + 13*e^2*x^2) - a^2*c*d^
2*e^2*x*(24*d^3 - 44*d^2*e*x + 119*d*e^2*x^2 + 315*e^3*x^3) + a^3*e^3*(-16
*d^4 + 24*d^3*e*x - 42*d^2*e^2*x^2 + 105*d*e^3*x^3 + 315*e^4*x^4)) - 3*(c^
4*d^8 + 4*a*c^3*d^6*e^2 + 30*a^2*c^2*d^4*e^4 - 140*a^3*c*d^2*e^6 + 105*a^4
*e^8)*x^4*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[
e]*Sqrt[d + e*x])]))/(64*a^(5/2)*d^(11/2)*e^(5/2)*x^4*Sqrt[(a*e + c*d*x)*(
d + e*x)])
```

Rubi [A] (verified)

Time = 2.81 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.275, Rules used = {1214, 25, 2181, 27, 2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^5(d+ex)^3}dx$$

$$\Big\downarrow \ 1214$$

$$-\frac{\int -\frac{\frac{\left(cd^2-ae^2\right)^2x^4e^6}{d^5}+\frac{a^2e^6}{d}-\frac{\left(cd^2-ae^2\right)^2x^3e^5}{d^4}+\frac{a\left(2cd^2-ae^2\right)xe^5}{d^2}+\frac{\left(cd^2-ae^2\right)^2x^2e^4}{d^3}}{x^5\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^4}-\frac{2e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\frac{\left(cd^2-ae^2\right)^2x^4e^6}{d^5}+\frac{a^2e^6}{d}-\frac{\left(cd^2-ae^2\right)^2x^3e^5}{d^4}+\frac{a\left(2cd^2-ae^2\right)xe^5}{d^2}+\frac{\left(cd^2-ae^2\right)^2x^2e^4}{d^3}}{x^5\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^4}-\frac{2e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$$\Big\downarrow \ 2181$$

$$\frac{-\frac{\int -\frac{\frac{8a\left(cd^2-ae^2\right)^2x^3e^7}{d^4}-\frac{8a\left(cd^2-ae^2\right)^2x^2e^6}{d^3}+\frac{3a^2\left(3cd^2-5ae^2\right)e^6}{d}+2a\left(\frac{4a^2e^4}{d^2}-11ace^2+4c^2d^2\right)xe^5}{2x^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4ade}-\frac{ae^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4d^2x^4}}{e^4}-\frac{2e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\frac{8a\left(cd^2-ae^2\right)^2x^3e^7}{d^4}-\frac{8a\left(cd^2-ae^2\right)^2x^2e^6}{d^3}+3a^2\left(3cd-\frac{5ae^2}{d}\right)e^6+2a\left(\frac{4a^2e^4}{d^2}-11ace^2+4c^2d^2\right)xe^5}{x^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{ae^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4d^2x^4}}{e^4}-\frac{2e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$\Big\downarrow$ 2181

$$\frac{\frac{-\frac{\int -\frac{3\left(\frac{16a^2\left(cd^2-ae^2\right)^2x^2e^8}{d^3}-4a^2\left(\frac{4a^2e^4}{d^2}-13ace^2+7c^2d^2\right)xe^7+\frac{a^2\left(c^2d^4-34ace^2d^2+41a^2e^4\right)e^6}{d}\right)}{2x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3ade}-\frac{ae^5\left(3cd^2-5ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2x^3}}{8ade}-\cdots}{e^4}-\frac{2e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\frac{16a^2\left(cd^2-ae^2\right)^2x^2e^8}{d^3}-4a^2\left(\frac{4a^2e^4}{d^2}-13ace^2+7c^2d^2\right)xe^7+\frac{a^2\left(c^2d^4-34ace^2d^2+41a^2e^4\right)e^6}{d}}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{ae^5\left(3cd^2-5ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2x^3}}{8ade}-ae^5\sqrt{x\cdots}}{e^4}-\frac{2e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$\Big\downarrow$ 2181

$$\frac{\frac{\frac{-\frac{\int \frac{a^2e^6\left(3c^3d^6+13ac^2e^2d^4-187a^2ce^4d^2+2e\left(-\frac{32a^3e^6}{d^2}+105a^2ce^4-66ac^2d^2e^2+c^3d^4\right)xd+187a^3e^6\right)}{2dx^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{ae^5\left(41a^2e^4-34acd^2e^2+c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade\cdots}}{2d^2x^2}}{2ade}\cdots}{8ade}\cdots}{e^4}-\frac{2e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{-\frac{ae^5\int \frac{3c^3d^6+13ac^2e^2d^4-187a^2ce^4d^2+2e\left(-\frac{32a^3e^6}{d^2}+105a^2ce^4-66ac^2d^2e^2+c^3d^4\right)xd+187a^3e^6}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4d^2}-\frac{ae^5\left(41a^2e^4-34acd^2e^2+c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cd\ldots}}{2d^2x^2}}{2ade}}{8ade}}{e^4}$$

$$\frac{2e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$$\Big\downarrow \; 1228$$

$$-\frac{-\frac{-\frac{ae^5\left(-\frac{3\left(cd^2-ae^2\right)\left(-105a^3e^6+35a^2cd^2e^4+5ac^2d^4e^2+c^3d^6\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(187a^3e^6-187a^2cd^2e^4+13ac^2d^4e^2+3c^3d^6\right)\sqrt{x\left(ae^2+cd^2\ldots\right.}}{adex}\ldots\right.}{4d^2}\ldots}{2ade}}{8ade}$$

$$\frac{2e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$$\Big\downarrow \; 1154$$

$$-\frac{-\frac{-\frac{ae^5\left(\frac{3\left(cd^2-ae^2\right)\left(-105a^3e^6+35a^2cd^2e^4+5ac^2d^4e^2+c^3d^6\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}-\frac{\left(187a^3e^6-187a^2cd^2e^4+13a\ldots\right.}{\ldots}\right.}{4d^2}\ldots}{2ade}}{8ade\ldots}$$

$$\frac{2e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{ae^5\left(41a^2e^4-34acd^2e^2+c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2d^2x^2}-\frac{ae^5\left(\frac{3\left(cd^2-ae^2\right)\left(-105a^3e^6+35a^2cd^2e^4+5ac^2d^4e^2+c^3d^6\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)\ldots}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+c\ldots\right.}}\right.}{2a^{3/2}d^{3/2}e^{3/2}}\ldots\right.}{4d\ldots}}{2ade}\ldots}{8ade}$$

$$\frac{2e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)^3),x]
```

output
```
(-2*e^3*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^5*
(d + e*x)) + (-1/4*(a*e^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^
2*x^4) + (-((a*e^5*(3*c*d^2 - 5*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])/(d^2*x^3)) + (-1/2*(a*e^5*(c^2*d^4 - 34*a*c*d^2*e^2 + 41*a^2*e^4
)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2*x^2) - (a*e^5*(-(((3*c
^3*d^6 + 13*a*c^2*d^4*e^2 - 187*a^2*c*d^2*e^4 + 187*a^3*e^6)*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x)) + (3*(c*d^2 - a*e^2)*(c^3*d^6 +
 5*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 - 105*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d
^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])])/(2*a^(3/2)*d^(3/2)*e^(3/2))))/(4*d^2))/(2*a*d*e))/(8*a*d*e)
)/e^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8131 vs. $2(447) = 894$.

Time = 5.07 (sec) , antiderivative size = 8132, normalized size of antiderivative = 16.84

method	result	size
default	Expression too large to display	8132

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/x^5/(e*x+d)^3,x,method=_RETURN
VERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 12.43 (sec) , antiderivative size = 968, normalized size of antiderivative = 2.00

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^5(d + ex)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d)^3,x, algorit
hm="fricas")
```

output
```
[1/256*(3*((c^4*d^8*e + 4*a*c^3*d^6*e^3 + 30*a^2*c^2*d^4*e^5 - 140*a^3*c*d
^2*e^7 + 105*a^4*e^9)*x^5 + (c^4*d^9 + 4*a*c^3*d^7*e^2 + 30*a^2*c^2*d^5*e^
4 - 140*a^3*c*d^3*e^6 + 105*a^4*d*e^8)*x^4)*sqrt(a*d*e)*log((8*a^2*d^2*e^2
 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e
+ a^2*d*e^3)*x)/x^2) - 4*(16*a^4*d^5*e^4 - (3*a*c^3*d^7*e^2 + 13*a^2*c^2*d
^5*e^4 - 315*a^3*c*d^3*e^6 + 315*a^4*d*e^8)*x^4 - (3*a*c^3*d^8*e + 11*a^2*
c^2*d^6*e^3 - 119*a^3*c*d^4*e^5 + 105*a^4*d^2*e^7)*x^3 + 2*(a^2*c^2*d^7*e^
2 - 22*a^3*c*d^5*e^4 + 21*a^4*d^3*e^6)*x^2 + 24*(a^3*c*d^6*e^3 - a^4*d^4*e
^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^6*e^4*x^5 + a^3
*d^7*e^3*x^4), 1/128*(3*((c^4*d^8*e + 4*a*c^3*d^6*e^3 + 30*a^2*c^2*d^4*e^5
 - 140*a^3*c*d^2*e^7 + 105*a^4*e^9)*x^5 + (c^4*d^9 + 4*a*c^3*d^7*e^2 + 30*
a^2*c^2*d^5*e^4 - 140*a^3*c*d^3*e^6 + 105*a^4*d*e^8)*x^4)*sqrt(-a*d*e)*arc
tan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e
^3)*x)) - 2*(16*a^4*d^5*e^4 - (3*a*c^3*d^7*e^2 + 13*a^2*c^2*d^5*e^4 - 315*
a^3*c*d^3*e^6 + 315*a^4*d*e^8)*x^4 - (3*a*c^3*d^8*e + 11*a^2*c^2*d^6*e^3 -
 119*a^3*c*d^4*e^5 + 105*a^4*d^2*e^7)*x^3 + 2*(a^2*c^2*d^7*e^2 - 22*a^3*c*
d^5*e^4 + 21*a^4*d^3*e^6)*x^2 + 24*(a^3*c*d^6*e^3 - a^4*d^4*e^5)*x)*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^6*e^4*x^5 + a^3*d^7*e^3*x...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^5(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**5/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^5(d+ex)^3}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex+d)^3x^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d)^3,x, algorit
hm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^3*x^5),
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{3/2}}{x^5(d+ex)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d)^3,x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,5,13]%%%},[2,6]%%%}+%%%{%%%{-6,[1,7,11]%%%},[2,5]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^5(d+ex)^3}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{3/2}}{x^5\,\left(d+e\,x\right)^3}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)^3),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 12.81 (sec) , antiderivative size = 3131, normalized size of antiderivative = 6.48

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^5(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d)^3,x)
```

output
```
( - 288*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*d**5*e**6 + 432*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**5*d**4*e**7*x - 756*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a**5*d**3*e**8*x**2 + 1890*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*d**2*e**9*
x**3 + 5670*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*d*e**10*x**4 - 224*sqrt(d
 + e*x)*sqrt(a*e + c*d*x)*a**4*c*d**7*e**4 - 96*sqrt(d + e*x)*sqrt(a*e + c
*d*x)*a**4*c*d**6*e**5*x + 204*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d**5
*e**6*x**2 - 672*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d**4*e**7*x**3 - 1
260*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d**3*e**8*x**4 - 336*sqrt(d + e
*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**8*e**3*x + 580*sqrt(d + e*x)*sqrt(a*e +
 c*d*x)*a**3*c**2*d**7*e**4*x**2 - 1468*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
*3*c**2*d**6*e**5*x**3 - 4176*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d*
*5*e**6*x**4 - 28*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**9*e**2*x**2
 + 208*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**8*e**3*x**3 + 236*sqrt
(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**7*e**4*x**4 + 42*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a*c**4*d**10*e*x**3 + 42*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
*c**4*d**9*e**2*x**4 + 2835*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e +
 c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*
sqrt(d + e*x))*a**5*d*e**10*x**4 + 2835*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e
)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))*a**5*e**11*x**5 - 1575*sqrt(e)*sqrt(d)*sqrt(...
```

3.48 $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 559

$$\int \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{d+ex}\,dx =$$

$$-\frac{3(cd^2-ae^2)^3\,(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6)\,(cd^2+ae^2+2cdex)\,\sqrt{ade+(cd^2+ae^2)\,x+c}}{16384c^5d^5e^6}$$

$$+\frac{(cd^2-ae^2)\,(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6)\,(cd^2+ae^2+2cdex)\,(ade+(cd^2+ae^2)\,x+cdex^2}{2048c^4d^4e^5}$$

$$+\frac{1}{112}\left(\frac{5a}{cd}-\frac{11d}{e^2}\right)x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}$$

$$+\frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{8e}$$

$$-\frac{(231c^3d^6-15ac^2d^4e^2-95a^2cd^2e^4-105a^3e^6-10cde(33c^2d^4-10acd^2e^2-15a^2e^4)\,x)\,(ade+(cd^2+ae^2}{4480c^3d^3e^4}$$

$$+\frac{3(cd^2-ae^2)^5\,(33c^3d^6+45ac^2d^4e^2+35a^2cd^2e^4+15a^3e^6)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{16384c^{11/2}d^{11/2}e^{13/2}}$$

output
```
-3/16384*(-a*e^2+c*d^2)^3*(15*a^3*e^6+35*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+33
*c^3*d^6)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/
c^5/d^5/e^6+1/2048*(-a*e^2+c*d^2)*(15*a^3*e^6+35*a^2*c*d^2*e^4+45*a*c^2*d^
4*e^2+33*c^3*d^6)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(3/2)/c^4/d^4/e^5+1/112*(5*a/c/d-11*d/e^2)*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(5/2)+1/8*x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e-1/4480*(2
31*c^3*d^6-15*a*c^2*d^4*e^2-95*a^2*c*d^2*e^4-105*a^3*e^6-10*c*d*e*(-15*a^2
*e^4-10*a*c*d^2*e^2+33*c^2*d^4)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)
/c^3/d^3/e^4+3/16384*(-a*e^2+c*d^2)^5*(15*a^3*e^6+35*a^2*c*d^2*e^4+45*a*c^
2*d^4*e^2+33*c^3*d^6)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(11/2)/d^(11/2)/e^(13/2)
```

Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 549, normalized size of antiderivative = 0.98

$$\int \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{d+ex}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\sqrt{c}\sqrt{d}\sqrt{e}(1575a^7e^{14}-525a^6cde^{12}(7d+\right.}{}$$

input
```
Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*(1575*a^7*e^14 - 5
25*a^6*c*d*e^12*(7*d + 2*e*x) + 35*a^5*c^2*d^2*e^10*(29*d^2 + 68*d*e*x + 2
4*e^2*x^2) + 5*a^4*c^3*d^3*e^8*(185*d^3 - 110*d^2*e*x - 376*d*e^2*x^2 - 14
4*e^3*x^3) + 5*a^3*c^4*d^4*e^6*(265*d^4 - 120*d^3*e*x + 80*d^2*e^2*x^2 + 3
20*d*e^3*x^3 + 128*e^4*x^4) + a^2*c^5*d^5*e^4*(-11193*d^5 + 7034*d^4*e*x -
 5488*d^3*e^2*x^2 + 4640*d^2*e^3*x^3 + 137600*d*e^4*x^4 + 103680*e^5*x^5)
+ a*c^6*d^6*e^2*(11445*d^6 - 7476*d^5*e*x + 5928*d^4*e^2*x^2 - 5056*d^3*e^
3*x^3 + 4480*d^2*e^4*x^4 + 212480*d*e^5*x^5 + 168960*e^6*x^6) + c^7*d^7*(-
3465*d^7 + 2310*d^6*e*x - 1848*d^5*e^2*x^2 + 1584*d^4*e^3*x^3 - 1408*d^3*e
^4*x^4 + 1280*d^2*e^5*x^5 + 87040*d*e^6*x^6 + 71680*e^7*x^7)) + (105*(c*d^
2 - a*e^2)^5*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^
6)*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(
Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(573440*c^(11/2)*d^(11/2)*e^(13/2))
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 543, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1215, 1236, 27, 1236, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{d+ex}dx$$

$$\Big\downarrow \text{1215}$$

$$\int x^3(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{1}{2}cdx^2\left(6ade+\left(11cd^2-5ae^2\right)x\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{8cde}+\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e}-\frac{\int x^2\left(6ade+\left(11cd^2-5ae^2\right)x\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{16e}$$

$$\Big\downarrow \text{1236}$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e}-\frac{\frac{\int -\frac{1}{2}x\left(4ade\left(11cd^2-5ae^2\right)+3\left(33c^2d^4-10ace^2d^2-15a^2e^4\right)x\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{7cde}+\frac{1}{7}x^2\left(\frac{11d}{e}-\frac{5ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ad\right.}{16e}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e}-\frac{\frac{1}{7}x^2\left(\frac{11d}{e}-\frac{5ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}-\frac{\int x\left(4ade\left(11cd^2-5ae^2\right)+3\left(33c^2d^4-10ace^2d^2-15a^2e^4\right)x\right)\left(cdex^2+\left(cd^2+ae^2\right)\right.}{14cde}}{16e}$$

$$\Big\downarrow \text{1225}$$

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e}-$$

$$\frac{\frac{1}{7}x^2\left(\frac{11d}{e}-\frac{5ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}-\frac{\frac{7\left(cd^2-ae^2\right)\left(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6\right)\int\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)}{8c^2d^2e^2}}{}}{16e}$$

$\downarrow$ 1087

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e}-$$

$$\frac{1}{7}x^2\left(\frac{11d}{e}-\frac{5ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}-\frac{\frac{7\left(cd^2-ae^2\right)\left(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+\right.\right.}{8cde}\right.}{8c^2d^2e^2}}{}$$

$\downarrow$ 1087

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e}-$$

$$\frac{1}{7}x^2\left(\frac{11d}{e}-\frac{5ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}-\frac{\frac{7\left(cd^2-ae^2\right)\left(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+\right.\right.}{8cde}\right.}{}}{}$$

$\downarrow$ 1092

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e}-$$

$$\frac{1}{7}x^2\left(\frac{11d}{e}-\frac{5ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}-\frac{\frac{7\left(cd^2-ae^2\right)\left(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+\right.\right.}{8cde}\right.}{}}{}$$

$\downarrow$ 219

$$\frac{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8e} -$$

$$\frac{1}{7}x^2\left(\frac{11d}{e}-\frac{5ae}{cd}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2} - \frac{7\left(cd^2-ae^2\right)\left(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+\right.\right.}{8cde}\right.}{}$$

input
```
Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

output
```
(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(8*e) - ((((11*d)/e -
(5*a*e)/(c*d))*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/7 - (-1/
20*((231*c^3*d^6 - 15*a*c^2*d^4*e^2 - 95*a^2*c*d^2*e^4 - 105*a^3*e^6 - 10*
c*d*e*(33*c^2*d^4 - 10*a*c*d^2*e^2 - 15*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2)^(5/2))/(c^2*d^2*e^2) + (7*(c*d^2 - a*e^2)*(33*c^3*d^6 +
45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*(((c*d^2 + a*e^2 + 2*c*d
*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*c*d*e) - (3*(c*d^2
 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d
*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e)))/(8*c^2*d^2*e^2))/(14*c*d*
e))/(16*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1894 vs. $2(523) = 1046$.

Time = 2.57 (sec) , antiderivative size = 1895, normalized size of antiderivative = 3.39

method	result	size
default	Expression too large to display	1895

input
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
d^2/e^3*(1/12*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5
/2)/c/d/e+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/8*(2*c*d*e*x+a*e^2
+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-
(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*
a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(
1/2))/(d*e*c)^(1/2)))+1/e*(1/8*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/
d/e/c-9/16*(a*e^2+c*d^2)/d/e/c*(1/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2
)/d/e/c-1/2*(a*e^2+c*d^2)/d/e/c*(1/12*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(5/2)/c/d/e+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e
/c*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/
d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+
c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e
^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))))-1/8*a/c*(1/12*(2*c*d*e*x+a
*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/c/d/e+5/24*(4*a*c*d^2*
e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/
4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/
8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e...
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1524, normalized size of antiderivative = 2.73

$$\int \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/2293760*(105*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*c^6*d^12*e^4
- 40*a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^10 - 20*a^6*c
^2*d^4*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*sqrt(c*d*e)*log(8*c^2*d^2*e
^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e +
a*c*d*e^3)*x) - 4*(71680*c^8*d^8*e^8*x^7 - 3465*c^8*d^15*e + 11445*a*c^7*d
^13*e^3 - 11193*a^2*c^6*d^11*e^5 + 1325*a^3*c^5*d^9*e^7 + 925*a^4*c^4*d^7*
e^9 + 1015*a^5*c^3*d^5*e^11 - 3675*a^6*c^2*d^3*e^13 + 1575*a^7*c*d*e^15 +
5120*(17*c^8*d^9*e^7 + 33*a*c^7*d^7*e^9)*x^6 + 1280*(c^8*d^10*e^6 + 166*a*
c^7*d^8*e^8 + 81*a^2*c^6*d^6*e^10)*x^5 - 128*(11*c^8*d^11*e^5 - 35*a*c^7*d
^9*e^7 - 1075*a^2*c^6*d^7*e^9 - 5*a^3*c^5*d^5*e^11)*x^4 + 16*(99*c^8*d^12*
e^4 - 316*a*c^7*d^10*e^6 + 290*a^2*c^6*d^8*e^8 + 100*a^3*c^5*d^6*e^10 - 45
*a^4*c^4*d^4*e^12)*x^3 - 8*(231*c^8*d^13*e^3 - 741*a*c^7*d^11*e^5 + 686*a^
2*c^6*d^9*e^7 - 50*a^3*c^5*d^7*e^9 + 235*a^4*c^4*d^5*e^11 - 105*a^5*c^3*d^
3*e^13)*x^2 + 2*(1155*c^8*d^14*e^2 - 3738*a*c^7*d^12*e^4 + 3517*a^2*c^6*d^
10*e^6 - 300*a^3*c^5*d^8*e^8 - 275*a^4*c^4*d^6*e^10 + 1190*a^5*c^3*d^4*e^1
2 - 525*a^6*c^2*d^2*e^14)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/
(c^6*d^6*e^7), -1/1146880*(105*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2
*c^6*d^12*e^4 - 40*a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e
^10 - 20*a^6*c^2*d^4*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*sqrt(-c*d*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.40

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm
="giac")
```

output
```
1/573440*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(10*(4*(1
4*c^2*d^2*e*x + (17*c^9*d^10*e^7 + 33*a*c^8*d^8*e^9)/(c^7*d^7*e^7))*x + (c
^9*d^11*e^6 + 166*a*c^8*d^9*e^8 + 81*a^2*c^7*d^7*e^10)/(c^7*d^7*e^7))*x -
(11*c^9*d^12*e^5 - 35*a*c^8*d^10*e^7 - 1075*a^2*c^7*d^8*e^9 - 5*a^3*c^6*d^
6*e^11)/(c^7*d^7*e^7))*x + (99*c^9*d^13*e^4 - 316*a*c^8*d^11*e^6 + 290*a^2
*c^7*d^9*e^8 + 100*a^3*c^6*d^7*e^10 - 45*a^4*c^5*d^5*e^12)/(c^7*d^7*e^7))*
x - (231*c^9*d^14*e^3 - 741*a*c^8*d^12*e^5 + 686*a^2*c^7*d^10*e^7 - 50*a^3
*c^6*d^8*e^9 + 235*a^4*c^5*d^6*e^11 - 105*a^5*c^4*d^4*e^13)/(c^7*d^7*e^7))
*x + (1155*c^9*d^15*e^2 - 3738*a*c^8*d^13*e^4 + 3517*a^2*c^7*d^11*e^6 - 30
0*a^3*c^6*d^9*e^8 - 275*a^4*c^5*d^7*e^10 + 1190*a^5*c^4*d^5*e^12 - 525*a^6
*c^3*d^3*e^14)/(c^7*d^7*e^7))*x - (3465*c^9*d^16*e - 11445*a*c^8*d^14*e^3
+ 11193*a^2*c^7*d^12*e^5 - 1325*a^3*c^6*d^10*e^7 - 925*a^4*c^5*d^8*e^9 - 1
015*a^5*c^4*d^6*e^11 + 3675*a^6*c^3*d^4*e^13 - 1575*a^7*c^2*d^2*e^15)/(c^7
*d^7*e^7)) - 3/32768*(33*c^8*d^16 - 120*a*c^7*d^14*e^2 + 140*a^2*c^6*d^12*
e^4 - 40*a^3*c^5*d^10*e^6 - 10*a^4*c^4*d^8*e^8 - 8*a^5*c^3*d^6*e^10 - 20*a
^6*c^2*d^4*e^12 + 40*a^7*c*d^2*e^14 - 15*a^8*e^16)*log(abs(-c*d^2 - a*e^2
- 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))))/(sqrt(c*d*e)*c^5*d^5*e^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{d+ex}\,dx=\int \frac{x^3\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{d+e\,x}\,dx$$

input
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
```

output
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 1615, normalized size of antiderivative = 2.89

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex} dx = \text{Too large to display}$$

input
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x)
```

output
```
(1575*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**7*c*d*e**15 - 3675*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**6*c**2*d**3*e**13 - 1050*sqrt(d + e*x)*sqrt(a*e + c*d
*x)*a**6*c**2*d**2*e**14*x + 1015*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c**
3*d**5*e**11 + 2380*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c**3*d**4*e**12*x
 + 840*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c**3*d**3*e**13*x**2 + 925*sqr
t(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**4*d**7*e**9 - 550*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a**4*c**4*d**6*e**10*x - 1880*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a**4*c**4*d**5*e**11*x**2 - 720*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**4
*d**4*e**12*x**3 + 1325*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**5*d**9*e**
7 - 600*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**5*d**8*e**8*x + 400*sqrt(d
 + e*x)*sqrt(a*e + c*d*x)*a**3*c**5*d**7*e**9*x**2 + 1600*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**3*c**5*d**6*e**10*x**3 + 640*sqrt(d + e*x)*sqrt(a*e + c
*d*x)*a**3*c**5*d**5*e**11*x**4 - 11193*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
*2*c**6*d**11*e**5 + 7034*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**6*d**10*
e**6*x - 5488*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**6*d**9*e**7*x**2 + 4
640*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**6*d**8*e**8*x**3 + 137600*sqrt
(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**6*d**7*e**9*x**4 + 103680*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*a**2*c**6*d**6*e**10*x**5 + 11445*sqrt(d + e*x)*sqrt(a
*e + c*d*x)*a*c**7*d**13*e**3 - 7476*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**
7*d**12*e**4*x + 5928*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**7*d**11*e**5...
```

3.49 $\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 437

$$\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx = \frac{\left(cd^2-ae^2\right)^3\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\sqrt{}}{1024c^4d^4e^5}$$
$$-\frac{\left(cd^2-ae^2\right)\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{384c^3d^3e^4}$$
$$+\frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7e}$$
$$+\frac{\left(63c^2d^4-20acd^2e^2-35a^2e^4-10cde\left(9cd^2-5ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{840c^2d^2e^3}$$
$$-\frac{\left(cd^2-ae^2\right)^5\left(9c^2d^4+10acd^2e^2+5a^2e^4\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{1024c^{9/2}d^{9/2}e^{11/2}}$$

output
```
1/1024*(-a*e^2+c*d^2)^3*(5*a^2*e^4+10*a*c*d^2*e^2+9*c^2*d^4)*(2*c*d*e*x+a*
e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^5-1/384*(-a*e
^2+c*d^2)*(5*a^2*e^4+10*a*c*d^2*e^2+9*c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c^3/d^3/e^4+1/7*x^2*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(5/2)/e+1/840*(63*c^2*d^4-20*a*c*d^2*e^2-35*a^2*e^4-10*c*
d*e*(-5*a*e^2+9*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/c^2/d^2/
e^3-1/1024*(-a*e^2+c*d^2)^5*(5*a^2*e^4+10*a*c*d^2*e^2+9*c^2*d^4)*arctanh(c
^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^
(9/2)/d^(9/2)/e^(11/2)
```

Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.10

$$\int \frac{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx = \frac{\left(cd^2-ae^2\right)^5\left((ae+cdx)(d+ex)\right)^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}\left(-525a^6e^{12}+350a^5cde\right.}{}\right.}{}$$

input
```
Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

output
```
((c*d^2 - a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[
e]*(-525*a^6*e^12 + 350*a^5*c*d*e^10*(4*d + e*x) - 35*a^4*c^2*d^2*e^8*(15*
d^2 + 26*d*e*x + 8*e^2*x^2) - 60*a^3*c^3*d^3*e^6*(10*d^3 - 5*d^2*e*x - 12*
d*e^2*x^2 - 4*e^3*x^3) + a^2*c^4*d^4*e^4*(3689*d^4 - 2332*d^3*e*x + 1824*d
^2*e^2*x^2 + 33520*d*e^3*x^3 + 23680*e^4*x^4) + 2*a*c^5*d^5*e^2*(-1680*d^5
 + 1099*d^4*e*x - 872*d^3*e^2*x^2 + 744*d^2*e^3*x^3 + 24320*d*e^4*x^4 + 18
560*e^5*x^5) + 3*c^6*d^6*(315*d^6 - 210*d^5*e*x + 168*d^4*e^2*x^2 - 144*d^
3*e^3*x^3 + 128*d^2*e^4*x^4 + 6400*d*e^5*x^5 + 5120*e^6*x^6)))/((c*d^2 - a
*e^2)^5*(a*e + c*d*x)*(d + e*x)) - (105*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^
2*e^4)*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])
])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(107520*c^(9/2)*d^(9/2)*e^(11/2
))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1215, 1236, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{d+ex}dx$$

$$\Big\downarrow \text{1215}$$

$$\int x^2(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}dx$$

$$\downarrow \text{1236}$$

$$\frac{\int -\frac{1}{2}cdx\left(4ade+\left(9cd^2-5ae^2\right)x\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{7cde}+\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7e}$$

$$\downarrow \text{27}$$

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7e}-\frac{\int x\left(4ade+\left(9cd^2-5ae^2\right)x\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{14e}$$

$$\downarrow \text{1225}$$

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7e}-\frac{\frac{7\left(cd^2-ae^2\right)\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{24c^2d^2e^2}-\frac{\left(-35a^2e^4-10cdex\left(9cd^2-5ae^2\right)-20acd^2e^2+63c^2d^4\right)\left(x\left(ae^2+cd^2\right)\right.}{60c^2d^2e^2}}{14e}$$

$$\downarrow \text{1087}$$

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7e}-\frac{\frac{7\left(cd^2-ae^2\right)\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\int \sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16cde}\right)}{24c^2d^2e^2}-\frac{(-35a}{}}{14e}$$

$$\downarrow \text{1087}$$

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7e}-\frac{\frac{7\left(cd^2-ae^2\right)\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{4cde}\right.}{16cde}\right.}{24c^2d^2e^2}}{14e}$$

$$\downarrow \text{1092}$$

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7e}-\frac{7\left(cd^2-ae^2\right)\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{4cde}\right.}{}\right.}{24c^2d^2e^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7e}-\frac{7\left(cd^2-ae^2\right)\left(5a^2e^4+10acd^2e^2+9c^2d^4\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{4cde}\right.}{}\right.}{24c^2d^2e^2}$$

input
```
Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

output
```
(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(7*e) - (-1/60*((63*c^
2*d^4 - 20*a*c*d^2*e^2 - 35*a^2*e^4 - 10*c*d*e*(9*c*d^2 - 5*a*e^2)*x)*(a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(c^2*d^2*e^2) + (7*(c*d^2 - a*e
^2)*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(((c*d^2 + a*e^2 + 2*c*d*e*x)
*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*c*d*e) - (3*(c*d^2 - a*
e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)
/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])
/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e)))/(24*c^2*d^2*e^2))/(14*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(405) = 810$.

Time = 2.59 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.47

method	result	size
default	Expression too large to display	1080

input
```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(1/7*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)/d/e/c-1/2*(a*e^2+c*d^2)/d
/e/c*(1/12*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)
/c/d/e+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/8*(2*c*d*e*x+a*e^2+c*
d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*
e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e
^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
))/(d*e*c)^(1/2)))))+d^2/e^3*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^
(5/2)+1/2*(a*e^2-c*d^2)*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x
+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*
e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/
2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c
)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))))-d/
e^2*(1/12*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/
c/d/e+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/8*(2*c*d*e*x+a*e^2+c*d
^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e
^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^
2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
)/(d*e*c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1272, normalized size of antiderivative = 2.91

$$\int \frac{x^2(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/430080*(105*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*c^5*d^10*e^4 - 15
*a^3*c^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10 + 15*a^6*c*d^2*e
^12 - 5*a^7*e^14)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +
 c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(15360*c^7*
d^7*e^7*x^6 + 945*c^7*d^13*e - 3360*a*c^6*d^11*e^3 + 3689*a^2*c^5*d^9*e^5
- 600*a^3*c^4*d^7*e^7 - 525*a^4*c^3*d^5*e^9 + 1400*a^5*c^2*d^3*e^11 - 525*
a^6*c*d*e^13 + 1280*(15*c^7*d^8*e^6 + 29*a*c^6*d^6*e^8)*x^5 + 128*(3*c^7*d
^9*e^5 + 380*a*c^6*d^7*e^7 + 185*a^2*c^5*d^5*e^9)*x^4 - 16*(27*c^7*d^10*e^
4 - 93*a*c^6*d^8*e^6 - 2095*a^2*c^5*d^6*e^8 - 15*a^3*c^4*d^4*e^10)*x^3 + 8
*(63*c^7*d^11*e^3 - 218*a*c^6*d^9*e^5 + 228*a^2*c^5*d^7*e^7 + 90*a^3*c^4*d
^5*e^9 - 35*a^4*c^3*d^3*e^11)*x^2 - 2*(315*c^7*d^12*e^2 - 1099*a*c^6*d^10*
e^4 + 1166*a^2*c^5*d^8*e^6 - 150*a^3*c^4*d^6*e^8 + 455*a^4*c^3*d^4*e^10 -
175*a^5*c^2*d^2*e^12)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5
*d^5*e^6), 1/215040*(105*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*c^5*d^10
*e^4 - 15*a^3*c^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10 + 15*a^
6*c*d^2*e^12 - 5*a^7*e^14)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2
*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15360*c^7*d^7*e^7*x^
6 + 945*c^7*d^13*e - 3360*a*c^6*d^11*e^3 + 3689*a^2*c^5*d^9*e^5 - 600*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.48

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \frac{1}{107520}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\left(2\left(8\left(10\left(12\,c^2a\right.\right.\right.\right.\right.\right.$$
$$+ \frac{(9\,c^7d^{14} - 35\,ac^6d^{12}e^2 + 45\,a^2c^5d^{10}e^4 - 15\,a^3c^4d^8e^6 - 5\,a^4c^3d^6e^8 - 9\,a^5c^2d^4e^{10} + 15\,a^6cd^2e^{12} - 5\,a^7e^{14})\log}{2048\sqrt{cde}c^4d^4e^5}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm
="giac")
```

output
```
1/107520*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(10*(12*c
^2*d^2*e*x + (15*c^8*d^9*e^6 + 29*a*c^7*d^7*e^8)/(c^6*d^6*e^6))*x + (3*c^8
*d^10*e^5 + 380*a*c^7*d^8*e^7 + 185*a^2*c^6*d^6*e^9)/(c^6*d^6*e^6))*x - (2
7*c^8*d^11*e^4 - 93*a*c^7*d^9*e^6 - 2095*a^2*c^6*d^7*e^8 - 15*a^3*c^5*d^5*
e^10)/(c^6*d^6*e^6))*x + (63*c^8*d^12*e^3 - 218*a*c^7*d^10*e^5 + 228*a^2*c
^6*d^8*e^7 + 90*a^3*c^5*d^6*e^9 - 35*a^4*c^4*d^4*e^11)/(c^6*d^6*e^6))*x -
(315*c^8*d^13*e^2 - 1099*a*c^7*d^11*e^4 + 1166*a^2*c^6*d^9*e^6 - 150*a^3*c
^5*d^7*e^8 + 455*a^4*c^4*d^5*e^10 - 175*a^5*c^3*d^3*e^12)/(c^6*d^6*e^6))*x
 + (945*c^8*d^14*e - 3360*a*c^7*d^12*e^3 + 3689*a^2*c^6*d^10*e^5 - 600*a^3
*c^5*d^8*e^7 - 525*a^4*c^4*d^6*e^9 + 1400*a^5*c^3*d^4*e^11 - 525*a^6*c^2*d
^2*e^13)/(c^6*d^6*e^6)) + 1/2048*(9*c^7*d^14 - 35*a*c^6*d^12*e^2 + 45*a^2*
c^5*d^10*e^4 - 15*a^3*c^4*d^8*e^6 - 5*a^4*c^3*d^6*e^8 - 9*a^5*c^2*d^4*e^10
 + 15*a^6*c*d^2*e^12 - 5*a^7*e^14)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*
e)*c^4*d^4*e^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \int \frac{x^2\,(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{d + e\,x}\,dx$$

input
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
```

output
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1308, normalized size of antiderivative = 2.99

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Too large to display}$$

input
```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x)
```

output
```
( - 525*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*c*d*e**13 + 1400*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*a**5*c**2*d**3*e**11 + 350*sqrt(d + e*x)*sqrt(a*e + c*
d*x)*a**5*c**2*d**2*e**12*x - 525*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**
3*d**5*e**9 - 910*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**3*d**4*e**10*x -
 280*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**3*d**3*e**11*x**2 - 600*sqrt(
d + e*x)*sqrt(a*e + c*d*x)*a**3*c**4*d**7*e**7 + 300*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a**3*c**4*d**6*e**8*x + 720*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**
3*c**4*d**5*e**9*x**2 + 240*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**4*d**4
*e**10*x**3 + 3689*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**5*d**9*e**5 - 2
332*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**5*d**8*e**6*x + 1824*sqrt(d +
e*x)*sqrt(a*e + c*d*x)*a**2*c**5*d**7*e**7*x**2 + 33520*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a**2*c**5*d**6*e**8*x**3 + 23680*sqrt(d + e*x)*sqrt(a*e + c*
d*x)*a**2*c**5*d**5*e**9*x**4 - 3360*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**
6*d**11*e**3 + 2198*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**6*d**10*e**4*x -
1744*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**6*d**9*e**5*x**2 + 1488*sqrt(d +
 e*x)*sqrt(a*e + c*d*x)*a*c**6*d**8*e**6*x**3 + 48640*sqrt(d + e*x)*sqrt(a
*e + c*d*x)*a*c**6*d**7*e**7*x**4 + 37120*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a*c**6*d**6*e**8*x**5 + 945*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**7*d**13*e -
 630*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**7*d**12*e**2*x + 504*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*c**7*d**11*e**3*x**2 - 432*sqrt(d + e*x)*sqrt(a*e + ...
```

3.50 $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 330

$$\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx = -\frac{\left(cd^2-ae^2\right)^3\left(7cd^2+5ae^2\right)\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512c^3d^3e^4} - \frac{\left(\frac{2ad^2}{c}-\frac{7d^4}{e^2}+\frac{5a^2e^2}{c^2}\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{192d^2e} - \frac{\left(7cd^2-5ae^2-10cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{60cde^2} + \frac{\left(cd^2-ae^2\right)^5\left(7cd^2+5ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{512c^{7/2}d^{7/2}e^{9/2}}$$

output
```
-1/512*(-a*e^2+c*d^2)^3*(5*a*e^2+7*c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4-1/192*(2*a*d^2/c-7*d^4/e^2+5*a
^2*e^2/c^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2
)/d^2/e-1/60*(-10*c*d*e*x-5*a*e^2+7*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(5/2)/c/d/e^2+1/512*(-a*e^2+c*d^2)^5*(5*a*e^2+7*c*d^2)*arctanh(c^(1/2)*
d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d
^(7/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.18

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex} dx = \frac{(cd^2 - ae^2)^5 ((ae + cdx)(d + ex))^{3/2} \left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(75a^5e^{10} - 5a^4cde^8(49d +}{}\right.}{}$$

input
```
Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

output
```
((c*d^2 - a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[
e]*(75*a^5*e^10 - 5*a^4*c*d*e^8*(49*d + 10*e*x) + 10*a^3*c^2*d^2*e^6*(15*d
^2 + 16*d*e*x + 4*e^2*x^2) - 6*a^2*c^3*d^3*e^4*(91*d^3 - 58*d^2*e*x - 564*
d*e^2*x^2 - 360*e^3*x^3) + a*c^4*d^4*e^2*(415*d^4 - 272*d^3*e*x + 216*d^2*
e^2*x^2 + 4448*d*e^3*x^3 + 3200*e^4*x^4) + c^5*d^5*(-105*d^5 + 70*d^4*e*x
- 56*d^3*e^2*x^2 + 48*d^2*e^3*x^3 + 1664*d*e^4*x^4 + 1280*e^5*x^5)))/((c*d
^2 - a*e^2)^5*(a*e + c*d*x)*(d + e*x)) + (15*(7*c*d^2 + 5*a*e^2)*ArcTanh[(
Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/((a*e + c*d*x
)^(3/2)*(d + e*x)^(3/2))))/(7680*c^(7/2)*d^(7/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1215, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{5/2}}{d + ex} dx$$

$$\Big\downarrow \text{1215}$$

$$\int x(ae + cdx)\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{3/2} dx$$

$$\Big\downarrow \text{1225}$$

$$-\frac{\left(\frac{5a^2e^2}{c}+2ad^2-\frac{7cd^4}{e^2}\right)\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{24d}-\frac{\left(-5ae^2+7cd^2-10cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{60cde^2}$$

$\Big\downarrow$ 1087

$$-\frac{\left(\frac{5a^2e^2}{c}+2ad^2-\frac{7cd^4}{e^2}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\int \sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16cde}\right)}{24d}-\frac{\left(-5ae^2+7cd^2-10cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{60cde^2}$$

$\Big\downarrow$ 1087

$$-\frac{\left(\frac{5a^2e^2}{c}+2ad^2-\frac{7cd^4}{e^2}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{4cde}\right.}{16cde}\right.}{24d}$$

$$\frac{\left(-5ae^2+7cd^2-10cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{60cde^2}$$

$\Big\downarrow$ 1092

$$-\frac{\left(\frac{5a^2e^2}{c}+2ad^2-\frac{7cd^4}{e^2}\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{4cde}\right.}{}\right.}{24d}$$

$$\frac{\left(-5ae^2+7cd^2-10cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{60cde^2}$$

$\Big\downarrow$ 219

$$-\frac{\left(\frac{5a^2e^2}{c}+2ad^2-\frac{7cd^4}{e^2}\right)\left(\frac{(ae^2+cd^2+2cdex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}{4cde}\right.}{}\right.}{24d}$$

$$\frac{\left(-5ae^2+7cd^2-10cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{60cde^2}$$

input
```
Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]
```

output
```
-1/60*((7*c*d^2 - 5*a*e^2 - 10*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(5/2))/(c*d*e^2) - ((2*a*d^2 - (7*c*d^4)/e^2 + (5*a^2*e^2)/c)*(((c*d
^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(8*
c*d*e) - (3*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(c*d
^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e)))/(24*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(302) = 604$.

Time = 2.51 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.04

method	result
default	$\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{2}}}{12cde}+\frac{5\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\left(\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac \cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d),x,method=_RETURNVERB
OSE)
```

output
```
1/e*(1/12*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/
c/d/e+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/8*(2*c*d*e*x+a*e^2+c*d
^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a*c*d^2*e^2-(a*e
^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*ln((1/2*a*e^
2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
)/(d*e*c)^(1/2))))-d/e^2*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2
)+1/2*(a*e^2-c*d^2)*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e
)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*
(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1
/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1
/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 1046, normalized size of antiderivative = 3.17

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Too large to display}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="
fricas")
```

output
```
[-1/30720*(15*(7*c^6*d^12 - 30*a*c^5*d^10*e^2 + 45*a^2*c^4*d^8*e^4 - 20*a^
3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 - 5*a^6*e^12)*sqrt(
c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*
d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(1280*c^6*d^6*e^6*x^5 - 105*c^6*d^
11*e + 415*a*c^5*d^9*e^3 - 546*a^2*c^4*d^7*e^5 + 150*a^3*c^3*d^5*e^7 - 245
*a^4*c^2*d^3*e^9 + 75*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + 25*a*c^5*d^5*e^
7)*x^4 + 16*(3*c^6*d^8*e^4 + 278*a*c^5*d^6*e^6 + 135*a^2*c^4*d^4*e^8)*x^3
- 8*(7*c^6*d^9*e^3 - 27*a*c^5*d^7*e^5 - 423*a^2*c^4*d^5*e^7 - 5*a^3*c^3*d^
3*e^9)*x^2 + 2*(35*c^6*d^10*e^2 - 136*a*c^5*d^8*e^4 + 174*a^2*c^4*d^6*e^6
+ 80*a^3*c^3*d^4*e^8 - 25*a^4*c^2*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x))/(c^4*d^4*e^5), -1/15360*(15*(7*c^6*d^12 - 30*a*c^5*d^10*
e^2 + 45*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 - 15*a^4*c^2*d^4*e^8 + 18*a^
5*c*d^2*e^10 - 5*a^6*e^12)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2
*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(1280*c^6*d^6*e^6*x^5
 - 105*c^6*d^11*e + 415*a*c^5*d^9*e^3 - 546*a^2*c^4*d^7*e^5 + 150*a^3*c^3*
d^5*e^7 - 245*a^4*c^2*d^3*e^9 + 75*a^5*c*d*e^11 + 128*(13*c^6*d^7*e^5 + 25
*a*c^5*d^5*e^7)*x^4 + 16*(3*c^6*d^8*e^4 + 278*a*c^5*d^6*e^6 + 135*a^2*c^4*
d^4*e^8)*x^3 - 8*(7*c^6*d^9*e^3 - 27*a*c^5*d^7*e^5 - 423*a^2*c^4*d^5*e^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex} \, dx = \text{Timed out}$$

input
```
integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.59

$$\int \frac{x(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{d + ex}\,dx = \frac{1}{7680}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\left(2\left(8\left(10\,c^2d^2ex + \right.\right.\right.\right.\right.$$
$$-\frac{(7\,c^6d^{12} - 30\,ac^5d^{10}e^2 + 45\,a^2c^4d^8e^4 - 20\,a^3c^3d^6e^6 - 15\,a^4c^2d^4e^8 + 18\,a^5cd^2e^{10} - 5\,a^6e^{12})\log\left(\left|-cd^2 - a\right.\right.}{1024\sqrt{cde}c^3d^3e^4}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="
giac")
```

output
```
1/7680*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*(10*c^2*d^2
*e*x + (13*c^7*d^8*e^5 + 25*a*c^6*d^6*e^7)/(c^5*d^5*e^5))*x + (3*c^7*d^9*e
^4 + 278*a*c^6*d^7*e^6 + 135*a^2*c^5*d^5*e^8)/(c^5*d^5*e^5))*x - (7*c^7*d^
10*e^3 - 27*a*c^6*d^8*e^5 - 423*a^2*c^5*d^6*e^7 - 5*a^3*c^4*d^4*e^9)/(c^5*
d^5*e^5))*x + (35*c^7*d^11*e^2 - 136*a*c^6*d^9*e^4 + 174*a^2*c^5*d^7*e^6 +
 80*a^3*c^4*d^5*e^8 - 25*a^4*c^3*d^3*e^10)/(c^5*d^5*e^5))*x - (105*c^7*d^1
2*e - 415*a*c^6*d^10*e^3 + 546*a^2*c^5*d^8*e^5 - 150*a^3*c^4*d^6*e^7 + 245
*a^4*c^3*d^4*e^9 - 75*a^5*c^2*d^2*e^11)/(c^5*d^5*e^5)) - 1/1024*(7*c^6*d^1
2 - 30*a*c^5*d^10*e^2 + 45*a^2*c^4*d^8*e^4 - 20*a^3*c^3*d^6*e^6 - 15*a^4*c
^2*d^4*e^8 + 18*a^5*c*d^2*e^10 - 5*a^6*e^12)*log(abs(-c*d^2 - a*e^2 - 2*sq
rt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/
(sqrt(c*d*e)*c^3*d^3*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex} dx = \int \frac{x\,(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{d + e\,x} dx$$

input
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x),x)
```

output
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 1033, normalized size of antiderivative = 3.13

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex} dx = \text{Too large to display}$$

input
```
int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x)
```

output
```
(75*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d*e**11 - 245*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**4*c**2*d**3*e**9 - 50*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
*4*c**2*d**2*e**10*x + 150*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**5*
e**7 + 160*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**4*e**8*x + 40*sqrt
(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**3*e**9*x**2 - 546*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**2*c**4*d**7*e**5 + 348*sqrt(d + e*x)*sqrt(a*e + c*d*x)
*a**2*c**4*d**6*e**6*x + 3384*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d*
*5*e**7*x**2 + 2160*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**4*e**8*x*
*3 + 415*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**9*e**3 - 272*sqrt(d + e
*x)*sqrt(a*e + c*d*x)*a*c**5*d**8*e**4*x + 216*sqrt(d + e*x)*sqrt(a*e + c*
d*x)*a*c**5*d**7*e**5*x**2 + 4448*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d
**6*e**6*x**3 + 3200*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d**5*e**7*x**4
 - 105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**11*e + 70*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*c**6*d**10*e**2*x - 56*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6
*d**9*e**3*x**2 + 48*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**8*e**4*x**3 +
 1664*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**6*d**7*e**5*x**4 + 1280*sqrt(d +
e*x)*sqrt(a*e + c*d*x)*c**6*d**6*e**6*x**5 - 75*sqrt(e)*sqrt(d)*sqrt(c)*lo
g((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2
- c*d**2))*a**6*e**12 + 270*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e
+ c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**5*c...
```

3.51 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 259

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx = \frac{3\left(cd^2-ae^2\right)^3\left(cd^2+ae^2+2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2e^3}$$
$$+\frac{1}{16}\left(\frac{a}{cd}-\frac{d}{e^2}\right)\left(cd^2+ae^2+2cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}$$
$$+\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{5e}-\frac{3\left(cd^2-ae^2\right)^5\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128c^{5/2}d^{5/2}e^{7/2}}$$

output
```
3/128*(-a*e^2+c*d^2)^3*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)/c^2/d^2/e^3+1/16*(a/c/d-d/e^2)*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5
/2)/e-3/128*(-a*e^2+c*d^2)^5*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.14

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{d+ex}\,dx = \frac{((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(-15a^4e^8+10a^3cde^6(7d+ex)+2a^2c^2d^2e^4(6}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(-15*a^4*e^8 +
10*a^3*c*d*e^6*(7*d + e*x) + 2*a^2*c^2*d^2*e^4*(64*d^2 + 233*d*e*x + 124*e
^2*x^2) + 2*a*c^3*d^3*e^2*(-35*d^3 + 23*d^2*e*x + 256*d*e^2*x^2 + 168*e^3*
x^3) + c^4*d^4*(15*d^4 - 10*d^3*e*x + 8*d^2*e^2*x^2 + 176*d*e^3*x^3 + 128*
e^4*x^4)))/((a*e + c*d*x)*(d + e*x)) - (15*(c*d^2 - a*e^2)^5*ArcTanh[(Sqrt
[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(3
/2)*(d + e*x)^(3/2))))/(640*c^(5/2)*d^(5/2)*e^(7/2))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1131, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{d+ex}\,dx$$

$$\Big\downarrow \ 1131$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-\frac{\left(cd^2-ae^2\right)\int \left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}dx}{2e}$$

$$\Big\downarrow \ 1087$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\int\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}dx}{16cde}\right)}{2e}$$

$\downarrow$ 1087

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\cdots}{\cdots}\cdots\right.}{16cde}\cdots\right.}{2e}$$

$\downarrow$ 1092

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\cdots}{\cdots}\cdots\right.}{16cde}\cdots\right.}{2e}$$

$\downarrow$ 219

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5e}-\frac{\left(cd^2-ae^2\right)\left(\frac{\left(ae^2+cd^2+2cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8cde}-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(ae^2+cd^2+2cdex\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cde}-\frac{\left(cd^2-ae^2\right)^2\cdots}{\cdots}\cdots\right.}{16cde}\cdots\right.}{2e}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]
```

output
```
(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(5*e) - ((c*d^2 - a*e^2)*(((
c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/
(8*c*d*e) - (3*(c*d^2 - a*e^2)^2*(((c*d^2 + a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d*e) - ((c*d^2 - a*e^2)^2*ArcTanh[(
c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2
+ a*e^2)*x + c*d*e*x^2])])/(8*c^(3/2)*d^(3/2)*e^(3/2))))/(16*c*d*e)))/(2*e
)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.26

method	result
default	$\frac{\frac{\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{5}+\frac{\left(a\,e^{2}-c\,d^{2}\right)\left(\frac{\left(2dec\left(x+\frac{d}{e}\right)+a\,e^{2}-c\,d^{2}\right)\left(dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{8dec}-\frac{3\left(a\,e^{2}-c\,d^{2}\right)^{2}\left(\frac{\left(2de\right.}{}\right.}{}\right.}{}}{}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d),x,method=_RETURNVERBOS
E)
```

output
```
1/e*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(
1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+
d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/
d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/
e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+
(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 844, normalized size of antiderivative = 3.26

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{d+ex}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="fr
icas")
```

output
```
[-1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2
*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 +
 c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^
3)*x) - 4*(128*c^5*d^5*e^5*x^4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2
*c^3*d^5*e^5 + 70*a^3*c^2*d^3*e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 +
21*a*c^4*d^4*e^6)*x^3 + 8*(c^5*d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3
*e^7)*x^2 - 2*(5*c^5*d^8*e^2 - 23*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*
a^3*c^2*d^2*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*
e^4), 1/1280*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3
*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d
*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(128*
c^5*d^5*e^5*x^4 + 15*c^5*d^9*e - 70*a*c^4*d^7*e^3 + 128*a^2*c^3*d^5*e^5 +
70*a^3*c^2*d^3*e^7 - 15*a^4*c*d*e^9 + 16*(11*c^5*d^6*e^4 + 21*a*c^4*d^4*e^
6)*x^3 + 8*(c^5*d^7*e^3 + 64*a*c^4*d^5*e^5 + 31*a^2*c^3*d^3*e^7)*x^2 - 2*(
5*c^5*d^8*e^2 - 23*a*c^4*d^6*e^4 - 233*a^2*c^3*d^4*e^6 - 5*a^3*c^2*d^2*e^8
)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.60

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \frac{1}{640}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\left(2\left(8c^2d^2ex + \frac{11c^6d^7e\right.\right.\right.\right.$$
$$+ \frac{3\,(c^5d^{10} - 5\,ac^4d^8e^2 + 10\,a^2c^3d^6e^4 - 10\,a^3c^2d^4e^6 + 5\,a^4cd^2e^8 - a^5e^{10})\log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x\right.\right.\right.}{256\sqrt{cde}c^2d^2e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="gi
ac")
```

output
```
1/640*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*c^2*d^2*e*x
+ (11*c^6*d^7*e^4 + 21*a*c^5*d^5*e^6)/(c^4*d^4*e^4))*x + (c^6*d^8*e^3 + 64
*a*c^5*d^6*e^5 + 31*a^2*c^4*d^4*e^7)/(c^4*d^4*e^4))*x - (5*c^6*d^9*e^2 - 2
3*a*c^5*d^7*e^4 - 233*a^2*c^4*d^5*e^6 - 5*a^3*c^3*d^3*e^8)/(c^4*d^4*e^4))*
x + (15*c^6*d^10*e - 70*a*c^5*d^8*e^3 + 128*a^2*c^4*d^6*e^5 + 70*a^3*c^3*d
^4*e^7 - 15*a^4*c^2*d^2*e^9)/(c^4*d^4*e^4)) + 3/256*(c^5*d^10 - 5*a*c^4*d^
8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^
10)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{d + e\,x}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 790, normalized size of antiderivative = 3.05

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x)
```

output
```
( - 15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d*e**9 + 70*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**3*c**2*d**3*e**7 + 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
**3*c**2*d**2*e**8*x + 128*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**5*
e**5 + 466*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**4*e**6*x + 248*sqr
t(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**3*e**7*x**2 - 70*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a*c**4*d**7*e**3 + 46*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c
**4*d**6*e**4*x + 512*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**5*e**5*x**
2 + 336*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**4*e**6*x**3 + 15*sqrt(d
+ e*x)*sqrt(a*e + c*d*x)*c**5*d**9*e - 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
c**5*d**8*e**2*x + 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**7*e**3*x**2 +
 176*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**5*d**6*e**4*x**3 + 128*sqrt(d + e*
x)*sqrt(a*e + c*d*x)*c**5*d**5*e**5*x**4 + 15*sqrt(e)*sqrt(d)*sqrt(c)*log(
(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 -
c*d**2))*a**5*e**10 - 75*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*c*d**2*
e**8 + 150*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d
)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c**2*d**4*e**6 - 150*
sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*s
qrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**3*d**6*e**4 + 75*sqrt(e)*sqrt
(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e...
```

3.52 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 362

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x(d+ex)}\,dx =$$

$$-\frac{\left(3c^3d^6-11ac^2d^4e^2-83a^2cd^2e^4-5a^3e^6+2cde\left(cd^2-5ae^2\right)\left(3cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cde}}{64cde^2}$$

$$+\frac{\left(3cd^2+11ae^2+6cdex\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{24e}$$

$$+\frac{\left(3c^4d^8-20ac^3d^6e^2+90a^2c^2d^4e^4+60a^3cd^2e^6-5a^4e^8\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{64c^{3/2}d^{3/2}e^{5/2}}$$

$$-2a^{5/2}d^{3/2}e^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)$$

output
```
-1/64*(3*c^3*d^6-11*a*c^2*d^4*e^2-83*a^2*c*d^2*e^4-5*a^3*e^6+2*c*d*e*(-5*a
*e^2+c*d^2)*(a*e^2+3*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d
/e^2+1/24*(6*c*d*e*x+11*a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)/e+1/64*(-5*a^4*e^8+60*a^3*c*d^2*e^6+90*a^2*c^2*d^4*e^4-20*a*c^3*d^6*e
^2+3*c^4*d^8)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)-2*a^(5/2)*d^(3/2)*e^(5/2)*arc
tanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2))
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.94

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x(d + ex)}\,dx = \frac{\sqrt{ae + cdx}\sqrt{d + ex}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae + cdx}\sqrt{d + ex}(15a^3e^6 + a^2cd\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*(15*a^3*e^6 + a^2*c*d*e^4*(337*d + 118*e*x) + a*c^2*d^2*e^
2*(57*d^2 + 244*d*e*x + 136*e^2*x^2) + c^3*(-9*d^6 + 6*d^5*e*x + 72*d^4*e^
2*x^2 + 48*d^3*e^3*x^3)) - 384*a^(5/2)*c^(3/2)*d^3*e^5*ArcTanh[(Sqrt[d]*Sq
rt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] + 3*(3*c^4*d^8 - 20*a*c^
3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*ArcTanh[(Sq
rt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(192*c^(3/2)*d
^(3/2)*e^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1215, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{5/2}}{x(d + ex)}\,dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae + cdx)\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{1231}$$

$$\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}-\frac{\int -\frac{cd\left(16a^2de^3-\left(cd^2-5ae^2\right)\left(3cd^2+ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x}dx}{8cde}$$

$\downarrow$ 27

$$\frac{\int \frac{\left(16a^2de^3-\left(cd^2-5ae^2\right)\left(3cd^2+ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x}dx}{16e}+\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

$\downarrow$ 1231

$$\frac{-\frac{\int -\frac{128a^3cd^3e^5+\left(3c^4d^8-20ac^3e^2d^6+90a^2c^2e^4d^4+60a^3ce^6d^2-5a^4e^8\right)x}{2x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cde}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(-5a^3e^6-83a^2cd^2e^4-11ac^2d^4e^2+2cdex\right.}{4cde}}{16e}$$

$$\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{128a^3cd^3e^5+\left(3c^4d^8-20ac^3e^2d^6+90a^2c^2e^4d^4+60a^3ce^6d^2-5a^4e^8\right)x}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}-\frac{\left(-5a^3e^6-83a^2cd^2e^4-11ac^2d^4e^2+2cdex\left(cd^2-5ae^2\right)\left(ae^2+3cd^2\right)+3c^3d^6\right.}{4cde}}{16e}$$

$$\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

$\downarrow$ 1269

$$\frac{\frac{128a^3cd^3e^5\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\left(-5a^4e^8+60a^3cd^2e^6+90a^2c^2d^4e^4-20ac^3d^6e^2+3c^4d^8\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}-\frac{\left(-5a^3e^6\right.}{}}{16e}$$

$$\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

$\downarrow$ 1092

$$\frac{\frac{128a^3cd^3e^5\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+2\left(-5a^4e^8+60a^3cd^2e^6+90a^2c^2d^4e^4-20ac^3d^6e^2+3c^4d^8\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2}{\sqrt{cdex^2+}}}{8cde}}{16e}$$

$$\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

$\Big\downarrow$ 219

$$\frac{\frac{128a^3cd^3e^5\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{\left(-5a^4e^8+60a^3cd^2e^6+90a^2c^2d^4e^4-20ac^3d^6e^2+3c^4d^8\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}}{8cde}}{16e}$$

$$\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

$\Big\downarrow$ 1154

$$\frac{\frac{\frac{\left(-5a^4e^8+60a^3cd^2e^6+90a^2c^2d^4e^4-20ac^3d^6e^2+3c^4d^8\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}-256a^3cd^3e^5\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ad}}}{8cde}}{16e}$$

$$\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{\left(-5a^4e^8+60a^3cd^2e^6+90a^2c^2d^4e^4-20ac^3d^6e^2+3c^4d^8\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}-128a^{5/2}cd^{5/2}e^{9/2}\text{arctanh}\left(\frac{x\left(a\right.}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x}}\right.}{8cde}}{16e}$$

$$\frac{\left(11ae^2+3cd^2+6cdex\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24e}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x]
```

output
```
((3*c*d^2 + 11*a*e^2 + 6*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2))/(24*e) + (-1/4*((3*c^3*d^6 - 11*a*c^2*d^4*e^2 - 83*a^2*c*d^2*e^4 -
5*a^3*e^6 + 2*c*d*e*(c*d^2 - 5*a*e^2)*(3*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d*e) + (((3*c^4*d^8 - 20*a*c^3*d^6*e^2 +
90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*ArcTanh[(c*d^2 + a*e^2
+ 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2])])/(Sqrt[c]*Sqrt[d]*Sqrt[e]) - 128*a^(5/2)*c*d^(5/2)*e^(9/2)*Arc
Tanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c*d*e))/(16*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(324) = 648$.

Time = 2.50 (sec) , antiderivative size = 997, normalized size of antiderivative = 2.75

method	result
default	$\frac{\frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{2}}}{5}+\frac{\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{\left(2cdxe+a\,e^{2}+c\,d^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}{8cde}+\frac{3\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\left(\frac{\left(2cdxe+a\,e^{2}\right.\cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}}{\cdots}$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x/(e*x+d),x,method=_RETURNVERB
OSE)
```

output
```
1/d*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)+1/2*(a*e^2+c*d^2)*(1/8*(2
*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(
4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/
d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*
x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*
x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+
c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*
ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*
x^2*e)^(1/2))/(d*e*c)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+
(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((2*
a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2))/x))))-1/d*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e
^2-c*d^2)*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2
-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e)+a*
e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-
c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c
*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 25.83 (sec) , antiderivative size = 1873, normalized size of antiderivative = 5.17

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{x(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="
fricas")
```

output
```
[1/768*(384*sqrt(a*d*e)*a^2*c^2*d^3*e^5*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*
a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/
x^2) - 3*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2
*e^6 - 5*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*
e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +
 c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(48*c^4*d^4
*e^4*x^3 - 9*c^4*d^7*e + 57*a*c^3*d^5*e^3 + 337*a^2*c^2*d^3*e^5 + 15*a^3*c
*d*e^7 + 8*(9*c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(3*c^4*d^6*e^2 + 122
*a*c^3*d^4*e^4 + 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(c^2*d^2*e^3), 1/384*(192*sqrt(a*d*e)*a^2*c^2*d^3*e^5*log((8*a^
2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a
*c*d^3*e + a^2*d*e^3)*x)/x^2) - 3*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c
^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d
*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c
^4*d^4*e^4*x^3 - 9*c^4*d^7*e + 57*a*c^3*d^5*e^3 + 337*a^2*c^2*d^3*e^5 + 15
*a^3*c*d*e^7 + 8*(9*c^4*d^5*e^3 + 17*a*c^3*d^3*e^5)*x^2 + 2*(3*c^4*d^6*e^2
 + 122*a*c^3*d^4*e^4 + 59*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + ...
```

Sympy [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x(d+ex)}\,dx=\int \frac{\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{5}{2}}}{x\left(d+ex\right)}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x/(e*x+d),x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(x*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x(d + ex)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x(d + ex)} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x(d + ex)} dx = \int \frac{(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{x\,(d + e\,x)} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.13

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x(d + ex)} dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d),x)
```

output
```
(15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d*e**7 + 337*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a**2*c**2*d**3*e**5 + 118*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
*2*c**2*d**2*e**6*x + 57*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**5*e**3
+ 244*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**4*e**4*x + 136*sqrt(d + e*
x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e**5*x**2 - 9*sqrt(d + e*x)*sqrt(a*e + c*
d*x)*c**4*d**7*e + 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**6*e**2*x + 72
*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**5*e**3*x**2 + 48*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*c**4*d**4*e**4*x**3 + 192*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt
(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c**2*d**3*e**5 + 192*sqrt(e)*sqrt(d)*sqr
t(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 +
 c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c**2*d**3*e**5 - 192*sqrt(e
)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c
*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*a**2*c**2*d**3*e**5 - 15*sqrt(e
)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))/sqrt(a*e**2 - c*d**2))*a**4*e**8 + 180*sqrt(e)*sqrt(d)*sqrt(c)*log
((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 -
 c*d**2))*a**3*c*d**2*e**6 + 270*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt
(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2
*c**2*d**4*e**4 - 60*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*...
```

3.53 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 318

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^2(d + ex)}\,dx = \frac{(c^2d^4 + 28acd^2e^2 + 19a^2e^4 + 2cde(cd^2 + 7ae^2)x)\sqrt{ade + (cd^2 + \cdots}}{8e}$$

$$- \frac{(3ae - cdx)(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{3x}$$

$$- \frac{(c^3d^6 - 15ac^2d^4e^2 - 45a^2cd^2e^4 - 5a^3e^6)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8\sqrt{c}\sqrt{d}e^{3/2}}$$

$$- a^{3/2}\sqrt{d}e^{3/2}(5cd^2 + 3ae^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d + ex)}{\sqrt{d}\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}\right)$$

output

```
1/8*(c^2*d^4+28*a*c*d^2*e^2+19*a^2*e^4+2*c*d*e*(7*a*e^2+c*d^2)*x)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e-1/3*(-c*d*x+3*a*e)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(3/2)/x-1/8*(-5*a^3*e^6-45*a^2*c*d^2*e^4-15*a*c^2*d^4*e^2+c^3
*d^6)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))/c^(1/2)/d^(1/2)/e^(3/2)-a^(3/2)*d^(1/2)*e^(3/2)*(3*a*e^2+5*c*
d^2)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.97

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^2(d+ex)}\,dx = \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(3a^2e^3(-8d+\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*(3*a^2*e^3*(-8*d + 11*e*x) + 2*a*c*d*e^2*x*(34*d + 13*e*x)
 + c^2*d^2*x*(3*d^2 + 14*d*e*x + 8*e^2*x^2)) - 24*a^(3/2)*Sqrt[c]*d*e^3*(5
*c*d^2 + 3*a*e^2)*x*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*S
qrt[d + e*x])] - 3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*
e^6)*x*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])
]))/(24*Sqrt[c]*Sqrt[d]*e^(3/2)*x*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1215, 1230, 25, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^2(d+ex)}\,dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1230}$$

$$-\frac{1}{2}\int -\frac{\left(ae\left(5cd^2+3ae^2\right)+cd\left(cd^2+7ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x}dx-\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$\big\downarrow$ 25

$$\frac{1}{2}\int \frac{\left(ae\left(5cd^2+3ae^2\right)+cd\left(cd^2+7ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x}dx-\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$\big\downarrow$ 1231

$$\frac{1}{2}\left(\frac{\left(19a^2e^4+2cdex\left(7ae^2+cd^2\right)+28acd^2e^2+c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4e}-\frac{\int -\frac{cd\left(8a^2de^3\left(5cd^2+3ae^2\right)-\left(\ldots\right.\right.}{2x\sqrt{cde\ldots}}\ldots}{\ldots}\right.$$

$$\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$\big\downarrow$ 27

$$\frac{1}{2}\left(\frac{\int \frac{8a^2de^3\left(5cd^2+3ae^2\right)-\left(c^3d^6-15ac^2e^2d^4-45a^2ce^4d^2-5a^3e^6\right)x}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8e}+\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(19a^2e^4+2cdex\left(7ae\ldots\right.\right.}{4e}\right.$$

$$\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$\big\downarrow$ 1269

$$\frac{1}{2}\left(\frac{8a^2de^3\left(3ae^2+5cd^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\left(-5a^3e^6-45a^2cd^2e^4-15ac^2d^4e^2+c^3d^6\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2\ldots\right.}}\ldots}{8e}\right.$$

$$\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$\big\downarrow$ 1092

$$\frac{1}{2}\left(\frac{8a^2de^3\left(3ae^2+5cd^2\right)\int\frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-2\left(-5a^3e^6-45a^2cd^2e^4-15ac^2d^4e^2+c^3d^6\right)\int\frac{}{4cde-\frac{\left(cd^2\right.}{cdex^2+}}}{8e}\right.$$

$$\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{8a^2de^3\left(3ae^2+5cd^2\right)\int\frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{(-5a^3e^6-45a^2cd^2e^4-15ac^2d^4e^2+c^3d^6)\text{arctanh}\left(\frac{ae^2+cd^2+2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd\right.}}\right.}{\sqrt{c}\sqrt{d}\sqrt{e}}}{8e}\right.$$

$$\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$\downarrow$ 1154

$$\frac{1}{2}\left(\frac{-16a^2de^3\left(3ae^2+5cd^2\right)\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+(cd^2+ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\frac{(-5a^3e^6-45a^2cd^2e^4-15ac^2d^4e^2+c^3d^6)\text{a}}{\sqrt{}}}{8e}\right.$$

$$\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(19a^2e^4+2cdex\left(7ae^2+cd^2\right)+28acd^2e^2+c^2d^4\right)}{4e}+\frac{-8a^{3/2}\sqrt{d}e^{5/2}\left(3ae^2+5c\right.}{}\right.$$

$$\frac{(3ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{3x}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x]
```

output
```
-1/3*((3*a*e - c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/x + (
((c^2*d^4 + 28*a*c*d^2*e^2 + 19*a^2*e^4 + 2*c*d*e*(c*d^2 + 7*a*e^2)*x)*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e) + (-(((c^3*d^6 - 15*a*c^2*
d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x
)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]
)/(Sqrt[c]*Sqrt[d]*Sqrt[e])) - 8*a^(3/2)*Sqrt[d]*e^(5/2)*(5*c*d^2 + 3*a*e^
2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*e))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2075 vs. $2(280) = 560$.

Time = 2.83 (sec) , antiderivative size = 2076, normalized size of antiderivative = 6.53

method	result	size
default	Expression too large to display	2076

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x^2/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)+5/2*(a*e^2+c*d^2)/
a/d/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)+1/2*(a*e^2+c*d^2)*(1/8*
(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16
*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/
c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2))/(d*e*c)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((
2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^
(1/2))/x))))+6*c/a*(1/12*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(5/2)/c/d/e+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/8*(2*c*
d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4*a
*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e
/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*...
```

Fricas [A] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 1717, normalized size of antiderivative = 5.40

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{x^2(d + ex)} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/96*(3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt
(c*d*e)*x*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt
(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 24*(5*a*c^2*d^3*e^3 + 3*a^2*c*d*e
^5)*sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)
*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(8*c^3*d^3*e^
3*x^3 - 24*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4)*x^2 + (3*c
^3*d^5*e + 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x))/(c*d*e^2*x), 1/48*(3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*
a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(-c*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e
^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 12*(5*a*c^2*d^3*e^3 +
 3*a^2*c*d*e^5)*sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 2*(
8*c^3*d^3*e^3*x^3 - 24*a^2*c*d^2*e^4 + 2*(7*c^3*d^4*e^2 + 13*a*c^2*d^2*e^4
)*x^2 + (3*c^3*d^5*e + 68*a*c^2*d^3*e^3 + 33*a^2*c*d*e^5)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^2*x), 1/96*(48*(5*a*c^2*d^3*e^3 + 3
*a^2*c*d*e^5)*sqrt(-a*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{x^2(d + ex)} dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**2/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{x^2(d + ex)} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)x^2} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^2), x
)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.49

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{x^2(d + ex)} dx = \frac{1}{24} \sqrt{cdex^2 + cd^2x + ae^2x + ade} \Bigg(2 \Bigg(4 c^2d^2ex + \frac{7 c^4d^5e^2 + 13 ac}{c^2d^2e^2}$$
$$+ \frac{(5 a^2cd^3e^2 + 3 a^3de^4) \arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{\sqrt{-ade}}$$
$$+ \frac{(c^3d^6 - 15 ac^2d^4e^2 - 45 a^2cd^2e^4 - 5 a^3e^6) \log\left(\left|-cd^2 - ae^2 - 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x +\right.\right.\right.}{16\sqrt{cde}e}$$
$$- \frac{\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^2cd^3e^2 + \left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^3de^4 + 2}{ade - \left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)^2}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm
="giac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*c^2*d^2*e*x + (7*c^
4*d^5*e^2 + 13*a*c^3*d^3*e^4)/(c^2*d^2*e^2))*x + (3*c^4*d^6*e + 68*a*c^3*d
^4*e^3 + 33*a^2*c^2*d^2*e^5)/(c^2*d^2*e^2)) + (5*a^2*c*d^3*e^2 + 3*a^3*d*e
^4)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/
sqrt(-a*d*e))/sqrt(-a*d*e) + 1/16*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d
^2*e^4 - 5*a^3*e^6)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*e) - ((sqrt(
c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^2*c*d^3*e^2 + (s
qrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*d*e^4 + 2*
sqrt(c*d*e)*a^3*d^2*e^3)/(a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^2(d+ex)}\,dx=\int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{x^2\,\left(d+e\,x\right)} \,d x$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^2(d+ex)}\,dx=\int \frac{\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cde\,x^{2}\right)^{\frac{5}{2}}}{x^{2}\left(ex+d\right)}d x$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x)
```

3.54 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 307

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^3(d+ex)}\,dx = -\frac{3(ae(3cd^2+ae^2)-cd(cd^2+3ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4x} - \frac{(ae-cdx)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{2x^2} + \frac{3\sqrt{c}\sqrt{d}(c^2d^4+10acd^2e^2+5a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4\sqrt{e}} - \frac{3\sqrt{a}\sqrt{e}(5c^2d^4+10acd^2e^2+a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4\sqrt{d}}$$

output
```
-3/4*(a*e*(a*e^2+3*c*d^2)-c*d*(3*a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/x-1/2*(-c*d*x+a*e)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/
x^2+3/4*c^(1/2)*d^(1/2)*(5*a^2*e^4+10*a*c*d^2*e^2+c^2*d^4)*arctanh(c^(1/2)
*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/e^(1/2)-
3/4*a^(1/2)*e^(1/2)*(a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*arctanh(a^(1/2)*e^(
1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/d^(1/2)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.93

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^3(d + ex)}\,dx = \frac{\sqrt{ae + cdx}\sqrt{d + ex}\left(\sqrt{d}\sqrt{e}\sqrt{ae + cdx}\sqrt{d + ex}(-9acdex(d - ex)\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d
 + e*x]*(-9*a*c*d*e*x*(d - e*x) + c^2*d^2*x^2*(5*d + 2*e*x) - a^2*e^2*(2*d
 + 5*e*x)) - 3*Sqrt[a]*e*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*x^2*ArcTan
h[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] + 3*Sqrt[c]
*d*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*x^2*ArcTanh[(Sqrt[e]*Sqrt[a*e +
c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(4*Sqrt[d]*Sqrt[e]*x^2*Sqrt[(a*
e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1215, 1230, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{5/2}}{x^3(d + ex)}\,dx$$

$$\Big\downarrow \ 1215$$

$$\int \frac{(ae + cdx)\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \ 1230$$

$$-\frac{3}{8}\int -\frac{2\left(ae\left(3cd^2+ae^2\right)+cd\left(cd^2+3ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^2}dx-$$
$$\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{4}\int \frac{\left(ae\left(3cd^2+ae^2\right)+cd\left(cd^2+3ae^2\right)x\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^2}dx-$$
$$\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{1230}$$

$$\frac{3}{4}\left(-\frac{1}{2}\int -\frac{ae\left(5c^2d^4+10ace^2d^2+a^2e^4\right)+cd\left(c^2d^4+10ace^2d^2+5a^2e^4\right)x}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\,(\right.$$
$$\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{ae\left(5c^2d^4+10ace^2d^2+a^2e^4\right)+cd\left(c^2d^4+10ace^2d^2+5a^2e^4\right)x}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{\left(ae\left(ae^2+3cd^2\right)-cdx\left(3ae^2+cd^2\right)\right.}{x}\right.$$
$$\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(cd\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+ae\left(a^2e^4+10acd^2e^2+5c^2d^4\right)\int \frac{}{x\sqrt{}}\right.\right.$$
$$\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(ae\left(a^2e^4+10acd^2e^2+5c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+2cd\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\int \frac{}{4}\right.\right.$$
$$\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}\left(\frac{1}{2}\left(ae\left(a^2e^4+10acd^2e^2+5c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{\sqrt{c}\sqrt{d}\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\text{a}\right.\right.$$

$$\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$\Big\downarrow$ 1154

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\sqrt{c}\sqrt{d}\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{e}}-2ae\left(a^2e^4+10acd^2e^2+5c^2d\right.\right.\right.$$

$$\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

$\Big\downarrow$ 219

$$\frac{3}{4}\left(\frac{1}{2}\left(\frac{\sqrt{c}\sqrt{d}\left(5a^2e^4+10acd^2e^2+c^2d^4\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{e}}-\frac{\sqrt{a}\sqrt{e}\left(a^2e^4+10acd^2e^2+5\right.}{}\right.\right.$$

$$\frac{(ae-cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{2x^2}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x]
```

output
```
-1/2*((a*e - c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/x^2 + (
3*(-(((a*e*(3*c*d^2 + a*e^2) - c*d*(c*d^2 + 3*a*e^2)*x)*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])/x) + ((Sqrt[c]*Sqrt[d]*(c^2*d^4 + 10*a*c*d^2*e^
2 + 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt
[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[e] - (Sqrt[a]*Sqrt
[e]*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e
^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])])/Sqrt[d])/2))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3892 vs. $2(267) = 534$.

Time = 2.90 (sec) , antiderivative size = 3893, normalized size of antiderivative = 12.68

method	result	size
default	Expression too large to display	3893

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x^3/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)+3/4*(a*e^2+c*d
^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)+5/2*(a*e^2+c
*d^2)/a/d/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)+1/2*(a*e^2+c*d^2)
*(1/8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/
e+3/16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2
+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)
+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2
)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x
^2*e)^(1/2))/x))))+6*c/a*(1/12*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(5/2)/c/d/e+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/8
*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/1
6*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^...
```

Fricas [A] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 1569, normalized size of antiderivative = 5.11

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm
="fricas")
```

output
```
[1/16*(3*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*sqrt(c*d/e)*x^2*log(8*c^2*
d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e
 + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2
*d^3*e + a*c*d*e^3)*x) + 3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*e
/d)*x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*
x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*c^2*d^2*e*x^3 -
2*a^2*d*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c*d^2*e + 5*a^2*e^3)*x)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, -1/16*(6*(c^2*d^4 + 10*
a*c*d^2*e^2 + 5*a^2*e^4)*sqrt(-c*d/e)*x^2*arctan(1/2*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e
*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) - 3*(5*c^2*d^4 + 10*a*c*d^2*e
^2 + a^2*e^4)*sqrt(a*e/d)*x^2*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*
e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) -
 4*(2*c^2*d^2*e*x^3 - 2*a^2*d*e^2 + (5*c^2*d^3 + 9*a*c*d*e^2)*x^2 - (9*a*c
*d^2*e + 5*a^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/x^2, 1
/16*(6*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*sqrt(-a*e/d)*x^2*arctan(1/2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(-a*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + 3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**3/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)x^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^3), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(267)=534$.

Time = 0.22 (sec) , antiderivative size = 746, normalized size of antiderivative = 2.43

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^3(d + ex)} \, dx = \frac{1}{4}\left(2\,c^2d^2ex + \frac{5\,c^3d^4e + 9\,ac^2d^2e^3}{cde}\right)\sqrt{cdex^2 + cd^2x + ae^2x + ade}$$
$$+ \frac{3\,(5\,ac^2d^4e + 10\,a^2cd^2e^3 + a^3e^5)\arctan\left(-\frac{\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}}{\sqrt{-ade}}\right)}{4\,\sqrt{-ade}}$$
$$- \frac{3\,(c^3d^5 + 10\,ac^2d^3e^2 + 5\,a^2cde^4)\log\left(\left|-cd^2 - ae^2 - 2\,\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)\right|\right)}{8\,\sqrt{cde}}$$
$$- \frac{7\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^2c^2d^5e^2 + 6\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)a^3cd^3e}{}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm
="giac")
```

output
```
1/4*(2*c^2*d^2*e*x + (5*c^3*d^4*e + 9*a*c^2*d^2*e^3)/(c*d*e))*sqrt(c*d*e*x
^2 + c*d^2*x + a*e^2*x + a*d*e) + 3/4*(5*a*c^2*d^4*e + 10*a^2*c*d^2*e^3 +
a^3*e^5)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))/sqrt(-a*d*e))/sqrt(-a*d*e) - 3/8*(c^3*d^5 + 10*a*c^2*d^3*e^2 + 5*a^2*
c*d*e^4)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*
e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/sqrt(c*d*e) - 1/4*(7*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^2*c^2*d^5*e^2 + 6*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*c*d^3*e^4 + 3*(
sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*d*e^6 + 1
6*sqrt(c*d*e)*a^3*c*d^4*e^3 + 8*sqrt(c*d*e)*a^4*d^2*e^5 - 9*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a*c^2*d^4*e - 18*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c*d^2*e^3 - 5
*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*e^5 -
 24*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))^2*a^2*c*d^3*e^2 - 16*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d
^2*x + a*e^2*x + a*d*e))^2*a^3*d*e^4)/(a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^3(d+ex)}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x^3\,\left(d+e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^3(d+ex)}\,dx=\int \frac{\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{5}{2}}}{x^3\left(ex+d\right)}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x)
```

3.55 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 336

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx = -\frac{\left(5c^2d^4+12acd^2e^2-a^2e^4-2cde\left(7cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8dx} - \frac{\left(4ade+3\left(3cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{12dx^3} + c^{3/2}d^{3/2}\sqrt{e}\left(3cd^2+5ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right) - \frac{\left(5c^3d^6+45ac^2d^4e^2+15a^2cd^2e^4\right.}{8}$$

output
```
-1/8*(5*c^2*d^4+12*a*c*d^2*e^2-a^2*e^4-2*c*d*e*(a*e^2+7*c*d^2)*x)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/x-1/12*(4*a*d*e+3*(a*e^2+3*c*d^2)*x)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^3+c^(3/2)*d^(3/2)*e^(1/2)*(5*a*e^
2+3*c*d^2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))-1/8*(-a^3*e^6+15*a^2*c*d^2*e^4+45*a*c^2*d^4*e^2+5*c^3*d^
6)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/a^(1/2)/d^(3/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.94

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^4(d + ex)} dx =$$
$$-\frac{\sqrt{ae + cdx}\sqrt{d + ex}\left(\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ae + cdx}\sqrt{d + ex}(3c^2d^3x^2(11d - 8ex) + 2acd^2ex(13d + 34ex) + a^2e^2(8\ldots\right.}{\ldots}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

output
```
-1/24*(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e +
 c*d*x]*Sqrt[d + e*x]*(3*c^2*d^3*x^2*(11*d - 8*e*x) + 2*a*c*d^2*e*x*(13*d
+ 34*e*x) + a^2*e^2*(8*d^2 + 14*d*e*x + 3*e^2*x^2)) + 3*(5*c^3*d^6 + 45*a*
c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*x^3*ArcTanh[(Sqrt[d]*Sqrt[a*e +
c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] - 24*Sqrt[a]*c^(3/2)*d^3*e*(3*c*d
^2 + 5*a*e^2)*x^3*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqr
t[d + e*x])]))/(Sqrt[a]*d^(3/2)*Sqrt[e]*x^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1215, 1229, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{x^4(d + ex)} dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae + cdx)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{x^4} dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{\int -\frac{ae\left(5c^2d^4+12ace^2d^2+2ce\left(7cd^2+ae^2\right)xd-a^2e^4\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^2}dx}{4ade}-\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(5c^2d^4+12ace^2d^2+2ce\left(7cd^2+ae^2\right)xd-a^2e^4\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^2}dx}{8d}-\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

$$\Big\downarrow \text{1230}$$

$$\frac{-\frac{1}{2}\int -\frac{5c^3d^6+45ac^2e^2d^4+8c^2e\left(3cd^2+5ae^2\right)xd^3+15a^2ce^4d^2-a^3e^6}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\left(-a^2e^4-2cdex\left(ae^2+7cd^2\right)+12acd^2e^2+\right.}{x}}{8d}$$
$$\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\int \frac{5c^3d^6+45ac^2e^2d^4+8c^2e\left(3cd^2+5ae^2\right)xd^3+15a^2ce^4d^2-a^3e^6}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{\left(-a^2e^4-2cdex\left(ae^2+7cd^2\right)+12acd^2e^2+5c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{x}}{8d}$$
$$\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{1}{2}\left(\left(-a^3e^6+15a^2cd^2e^4+45ac^2d^4e^2+5c^3d^6\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+8c^2d^3e\left(5ae^2+3cd^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+\right.}}\right.}{8d}$$
$$\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{1}{2}\left(\left(-a^3e^6+15a^2cd^2e^4+45ac^2d^4e^2+5c^3d^6\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+16c^2d^3e\left(5ae^2+3cd^2\right)\int \frac{}{4cde-\frac{\left(cd^2+\right.}{cdex^2+\left(\right.}}\right.}{8d}$$
$$\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{1}{2}\left(\left(-a^3e^6+15a^2cd^2e^4+45ac^2d^4e^2+5c^3d^6\right)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+8c^{3/2}d^{5/2}\sqrt{e}\left(5ae^2+3cd^2\right)\text{arctanh}\left(\right.\right.}{8d}$$

$$\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

$$\Big\downarrow \ 1154$$

$$\frac{\frac{1}{2}\left(8c^{3/2}d^{5/2}\sqrt{e}\left(5ae^2+3cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)-2\left(-a^3e^6+15a^2cd^2e^4+45ac^2d^4e^2+5c\right.\right.}{}$$

$$\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{1}{2}\left(8c^{3/2}d^{5/2}\sqrt{e}\left(5ae^2+3cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)-\frac{\left(-a^3e^6+15a^2cd^2e^4+45ac^2d^4e^2+5c^3d^6\right)\text{arctan}}{\sqrt{a}\sqrt{d}\sqrt{e}}\right.}{8d}$$

$$\frac{\left(3x\left(ae^2+3cd^2\right)+4ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{12dx^3}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x]
```

output
```
-1/12*((4*a*d*e + 3*(3*c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2))/(d*x^3) + (-(((5*c^2*d^4 + 12*a*c*d^2*e^2 - a^2*e^4 - 2*c*d*
e*(7*c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x) + (
8*c^(3/2)*d^(5/2)*Sqrt[e]*(3*c*d^2 + 5*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c
*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])] - ((5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*ArcT
anh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])])/(Sqrt[a]*Sqrt[d]*Sqrt[e]))/2)/(8*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6849 vs. $2(298) = 596$.

Time = 4.54 (sec) , antiderivative size = 6850, normalized size of antiderivative = 20.39

method	result	size
default	Expression too large to display	6850

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x^4/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 1741, normalized size of antiderivative = 5.18

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^4(d + ex)} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm
="fricas")
```

output
```
[1/96*(24*(3*a*c^2*d^5*e + 5*a^2*c*d^3*e^3)*sqrt(c*d*e)*x^3*log(8*c^2*d^2*
e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e +
 a*c*d*e^3)*x) - 3*(5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*
e^6)*sqrt(a*d*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e
^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2
+ a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(24*a*c^2*
d^4*e^2*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*c*d^3*e^3 + 3*a^3*d
*e^5)*x^2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^3), -1/96*(48*(3*a*c^2*d^5*e + 5*a^2*c*d
^3*e^3)*sqrt(-c*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*
e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 3*(5*c^3*d^6 + 45*a*c^2*d^4*e^2 + 15*a
^2*c*d^2*e^4 - a^3*e^6)*sqrt(a*d*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*
a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/
x^2) - 4*(24*a*c^2*d^4*e^2*x^3 - 8*a^3*d^3*e^3 - (33*a*c^2*d^5*e + 68*a^2*
c*d^3*e^3 + 3*a^3*d*e^5)*x^2 - 2*(13*a^2*c*d^4*e^2 + 7*a^3*d^2*e^4)*x)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^2*e*x^3), 1/48*(3*(5*c^3*d^
6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a*d*e)*x^3*arc...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**4/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)x^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^4), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1195 vs. $2(298)=596$.

Time = 0.27 (sec) , antiderivative size = 1195, normalized size of antiderivative = 3.56

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm
="giac")
```

output
```
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*c^2*d^2*e - 1/2*(3*c^3*d^4*e +
 5*a*c^2*d^2*e^3)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x -
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/sqrt(c*d*e) + 1/8*(5*c^3*d^
6 + 45*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - a^3*e^6)*arctan(-(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)
*d) - 1/24*(15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e
))*a^2*c^3*d^8*e^2 + 39*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))*a^3*c^2*d^6*e^4 + 45*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x
 + a*e^2*x + a*d*e))*a^4*c*d^4*e^6 - 3*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c
*d^2*x + a*e^2*x + a*d*e))*a^5*d^2*e^8 + 48*sqrt(c*d*e)*a^3*c^2*d^7*e^3 +
112*sqrt(c*d*e)*a^4*c*d^5*e^5 - 40*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))^3*a*c^3*d^7*e - 72*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c^2*d^5*e^3 - 24*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c*d^3*e^5 + 8*(sqrt(c*d*e)*
x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*d*e^7 - 144*sqrt(c*
d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^2*c
^2*d^6*e^2 - 240*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a
*e^2*x + a*d*e))^2*a^3*c*d^4*e^4 + 33*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))^5*c^3*d^6 + 153*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
+ c*d^2*x + a*e^2*x + a*d*e))^5*a*c^2*d^4*e^2 + 99*(sqrt(c*d*e)*x - sqr...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^4(d + ex)}\,dx = \int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{x^4\,\left(d+e\,x\right)} \,d x$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 7.95 (sec) , antiderivative size = 1564, normalized size of antiderivative = 4.65

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^4(d + ex)}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x)
```

output
```
( - 16*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*d**3*e**5 - 28*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**4*d**2*e**6*x - 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4
*d*e**7*x**2 - 16*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**5*e**3 - 80*sq
rt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**4*e**4*x - 142*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a**3*c*d**3*e**5*x**2 - 52*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
*2*c**2*d**6*e**2*x - 202*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**5*e
**3*x**2 + 48*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**4*e**4*x**3 - 6
6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**7*e*x**2 + 48*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a*c**3*d**6*e**2*x**3 - 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt
(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))*a**4*e**8*x**3 + 42*sqrt(e)*sqrt(d)*sqrt(a)*l
og(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**
2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**2*e**6*x**3 + 180*sqrt(e)*sq
rt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e +
 a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c**2*d**4*e**4*x**
3 + 150*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqr
t(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**
3*d**6*e**2*x**3 + 15*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x
) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))*c**4*d**8*x**3 - 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*...
```

3.56 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^5(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 373

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^5(d+ex)}\,dx =$$
$$-\frac{(2ade(5cd^2-ae^2)(cd^2+3ae^2)+(5c^3d^6+83ac^2d^4e^2+11a^2cd^2e^4-3a^3e^6)x)\sqrt{ade+(cd^2+ae^2)x+c}}{64ad^2ex^2}$$
$$-\frac{(6ade+(11cd^2+3ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{24dx^4}$$
$$+2c^{5/2}d^{5/2}e^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)+\frac{(5c^4d^8-60ac^3d^6e^2-90a^2c^2d^4e^4+20a^3cd^2e}{64a^{3/2}d}$$

output
```
-1/64*(2*a*d*e*(-a*e^2+5*c*d^2)*(3*a*e^2+c*d^2)+(-3*a^3*e^6+11*a^2*c*d^2*e
^4+83*a*c^2*d^4*e^2+5*c^3*d^6)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/
a/d^2/e/x^2-1/24*(6*a*d*e+(3*a*e^2+11*c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(3/2)/d/x^4+2*c^(5/2)*d^(5/2)*e^(3/2)*arctanh(c^(1/2)*d^(1/2)*(e*x
+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))+1/64*(-3*a^4*e^8+20*a
^3*c*d^2*e^6-90*a^2*c^2*d^4*e^4-60*a*c^3*d^6*e^2+5*c^4*d^8)*arctanh(a^(1/2
)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)
/d^(5/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.96

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^5(d + ex)} dx = \frac{\sqrt{ae + cdx}\sqrt{d + ex}\left(-\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ae + cdx}\sqrt{d + ex}(15c^3d^6x^3 + \right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-(Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d
*x]*Sqrt[d + e*x]*(15*c^3*d^6*x^3 + a*c^2*d^4*e*x^2*(118*d + 337*e*x) + a^
2*c*d^2*e^2*x*(136*d^2 + 244*d*e*x + 57*e^2*x^2) + 3*a^3*e^3*(16*d^3 + 24*
d^2*e*x + 2*d*e^2*x^2 - 3*e^3*x^3))) + 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 9
0*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*x^4*ArcTanh[(Sqrt[d]*Sqr
t[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])] + 384*a^(3/2)*c^(5/2)*d^5
*e^3*x^4*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x
])]))/(192*a^(3/2)*d^(5/2)*e^(3/2)*x^4*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1215, 1229, 27, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{5/2}}{x^5(d + ex)} dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae + cdx)\left(x\left(ae^2 + cd^2\right) + ade + cdex^2\right)^{3/2}}{x^5} dx$$

$$\Big\downarrow \text{1229}$$

$$-\frac{\int -\frac{ae\left(16c^2exd^3+\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{2x^3}dx}{8ade}-\frac{\left(x\left(3ae^2+11cd^2\right)+6ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$\downarrow$ 27

$$\frac{\int \frac{\left(16c^2exd^3+\left(5cd^2-ae^2\right)\left(cd^2+3ae^2\right)\right)\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{16d}-\frac{\left(x\left(3ae^2+11cd^2\right)+6ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$\downarrow$ 1229

$$\frac{-\frac{\int \frac{5c^4d^8-60ac^3e^2d^6-128ac^3e^3xd^5-90a^2c^2e^4d^4+20a^3ce^6d^2-3a^4e^8}{2x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(x\left(-3a^3e^6+11a^2cd^2e^4+83ac^2d^4e^2+5c^3d^6\right)\right.}{4adex^2}}{16d}$$
$$-\frac{\left(x\left(3ae^2+11cd^2\right)+6ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$\downarrow$ 27

$$\frac{-\frac{\int \frac{5c^4d^8-60ac^3e^2d^6-128ac^3e^3xd^5-90a^2c^2e^4d^4+20a^3ce^6d^2-3a^4e^8}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(x\left(-3a^3e^6+11a^2cd^2e^4+83ac^2d^4e^2+5c^3d^6\right)\right.}{4adex^2}}{16d}$$
$$-\frac{\left(x\left(3ae^2+11cd^2\right)+6ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$\downarrow$ 1269

$$\frac{-\frac{\left(-3a^4e^8+20a^3cd^2e^6-90a^2c^2d^4e^4-60ac^3d^6e^2+5c^4d^8\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-128ac^3d^5e^3\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\sqrt{x(ae}{}}{16d}$$
$$-\frac{\left(x\left(3ae^2+11cd^2\right)+6ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$\downarrow$ 1092

$$\frac{-\frac{\left(-3a^4e^8+20a^3cd^2e^6-90a^2c^2d^4e^4-60ac^3d^6e^2+5c^4d^8\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-256ac^3d^5e^3\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2}{\sqrt{cdex^2}}}{8ade}}{16d}$$
$$-\frac{\left(x\left(3ae^2+11cd^2\right)+6ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{\left(-3a^4e^8+20a^3cd^2e^6-90a^2c^2d^4e^4-60ac^3d^6e^2+5c^4d^8\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-128ac^{5/2}d^{9/2}e^{5/2}\text{arctanh}\left(\frac{ae^2+cd^2+2cde}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)\ldots}}\right.}{8ade}}{16d}$$

$$\frac{\left(x\left(3ae^2+11cd^2\right)+6ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$$\Big\downarrow \; 1154$$

$$-\frac{-2\left(-3a^4e^8+20a^3cd^2e^6-90a^2c^2d^4e^4-60ac^3d^6e^2+5c^4d^8\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}-128ac^{5/2}d^{9/2}e^{5/2}\text{arct}\ldots}{8ade}$$

$$\frac{\left(x\left(3ae^2+11cd^2\right)+6ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(x\left(-3a^3e^6+11a^2cd^2e^4+83ac^2d^4e^2+5c^3d^6\right)+2ade\left(5cd^2-ae^2\right)\left(3ae^2+cd^2\right)\right)}{4adex^2}-\frac{-\left(-3a^4e^8+20a^3cd^2e^6-90a^2c^2d^4e^4\ldots\right.}{\ldots}}{16d}$$

$$\frac{\left(x\left(3ae^2+11cd^2\right)+6ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{24dx^4}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x]
```

output
```
-1/24*((6*a*d*e + (11*c*d^2 + 3*a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2))/(d*x^4) + (-1/4*((2*a*d*e*(5*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^
2) + (5*c^3*d^6 + 83*a*c^2*d^4*e^2 + 11*a^2*c*d^2*e^4 - 3*a^3*e^6)*x)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^2) - (-128*a*c^(5/2)*d^(
9/2)*e^(5/2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])] - ((5*c^4*d^8 - 60*a*c^3*d
^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*ArcTanh[(2*a*d
*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])])/(Sqrt[a]*Sqrt[d]*Sqrt[e]))/(8*a*d*e))/(16*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11684 vs. $2(335) = 670$.

Time = 2.26 (sec) , antiderivative size = 11685, normalized size of antiderivative = 31.33

method	result	size
default	Expression too large to display	11685

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x^5/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 1917, normalized size of antiderivative = 5.14

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^5(d + ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm
="fricas")
```

output
```
[1/768*(384*sqrt(c*d*e)*a^2*c^2*d^5*e^3*x^4*log(8*c^2*d^2*e^2*x^2 + c^2*d^
4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) -
 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 -
 3*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2
+ a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(48
*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3 + 57*a^3*c*d^3*e^5 -
9*a^4*d*e^7)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*d^4*e^4 + 3*a^4*d^2*e
^6)*x^2 + 8*(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x))/(a^2*d^3*e^2*x^4), -1/768*(768*sqrt(-c*d*e)*a^2*c^2*d
^5*e^3*x^4*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e
*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3
*e + a*c*d*e^3)*x)) + 3*(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4
 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*sqrt(a*d*e)*x^4*log((8*a^2*d^2*e^2 + (c^2
*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d
*e^3)*x)/x^2) + 4*(48*a^4*d^4*e^4 + (15*a*c^3*d^7*e + 337*a^2*c^2*d^5*e^3
+ 57*a^3*c*d^3*e^5 - 9*a^4*d*e^7)*x^3 + 2*(59*a^2*c^2*d^6*e^2 + 122*a^3*c*
d^4*e^4 + 3*a^4*d^2*e^6)*x^2 + 8*(17*a^3*c*d^5*e^3 + 9*a^4*d^3*e^5)*x)*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^5(d + ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**5/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{(ex+d)x^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^5), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1750 vs. $2(335) = 670$.

Time = 0.49 (sec) , antiderivative size = 1750, normalized size of antiderivative = 4.69

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm
="giac")
```

output
```
-c^3*d^3*e^2*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/sqrt(c*d*e) - 1/64*(5*c^4*d^8 -
60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8)*arct
an(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*
d*e))/(sqrt(-a*d*e)*a*d^2*e) + 1/192*(15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
 c*d^2*x + a*e^2*x + a*d*e))*a^3*c^4*d^11*e^3 - 180*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c^3*d^9*e^5 + 114*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*c^2*d^7*e^7 + 60*(sq
rt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^6*c*d^5*e^9 -
 9*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^7*d^3*e
^11 - 512*sqrt(c*d*e)*a^5*c^2*d^8*e^6 - 55*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c^4*d^10*e^2 + 660*(sqrt(c*d*e)*x - s
qrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c^3*d^8*e^4 + 1374*(sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*c^2*d^6*e^6
 + 548*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^5
*c*d^4*e^8 + 33*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*
e))^3*a^6*d^2*e^10 + 2048*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))^2*a^4*c^2*d^7*e^5 + 768*sqrt(c*d*e)*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^5*c*d^5*e^7 + 73*(sq
rt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a*c^4*d^9*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{x^5(d+ex)}\,dx=\int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x^5\,\left(d+e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 15.18 (sec) , antiderivative size = 1822, normalized size of antiderivative = 4.88

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{x^5(d+ex)}\,dx=\text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x)
```

output
```
( - 192*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*d**4*e**6 - 288*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**5*d**3*e**7*x - 24*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
**5*d**2*e**8*x**2 + 36*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*d*e**9*x**3 -
 192*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d**6*e**4 - 832*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**4*c*d**5*e**5*x - 1000*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*a**4*c*d**4*e**6*x**2 - 192*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d**3*
e**7*x**3 - 544*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**7*e**3*x - 14
48*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**6*e**4*x**2 - 1576*sqrt(d
+ e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**5*e**5*x**3 - 472*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a**2*c**3*d**8*e**2*x**2 - 1408*sqrt(d + e*x)*sqrt(a*e + c*d
*x)*a**2*c**3*d**7*e**3*x**3 - 60*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d
**9*e*x**3 + 18*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sq
rt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x
))*a**5*e**10*x**4 - 102*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*
d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqr
t(d + e*x))*a**4*c*d**2*e**8*x**4 + 420*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e
)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))*a**3*c**2*d**4*e**6*x**4 + 900*sqrt(e)*sqrt(d)*
sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**
2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c**3*d**6*e**4*x**4 +...
```

3.57 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^6(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 272

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^6(d+ex)}\,dx = \frac{3(cd^2-ae^2)^3(2ade+(cd^2+ae^2)x)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{128a^2d^3e^2x^2}$$
$$-\frac{\left(\frac{c}{ae}-\frac{e}{d^2}\right)(2ade+(cd^2+ae^2)x)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{16x^4}$$
$$-\frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{5dx^5}-\frac{3(cd^2-ae^2)^5\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128a^{5/2}d^{7/2}e^{5/2}}$$

output
```
3/128*(-a*e^2+c*d^2)^3*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2)/a^2/d^3/e^2/x^2-1/16*(c/a/e-e/d^2)*(2*a*d*e+(a*e^2+c*d^2)*x
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4-1/5*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(5/2)/d/x^5-3/128*(-a*e^2+c*d^2)^5*arctanh(a^(1/2)*e^(1/2)*(e*x+
d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7/2)/e^(5/2
)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)}\,dx=\frac{\left(-cd^2+ae^2\right)^5\left((ae+cdx)(d+ex)\right)^{3/2}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(d+ex)^3\left(15a^4e^4-\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

output
```
((-(c*d^2) + a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*(-((Sqrt[a]*Sqrt[d]*
Sqrt[e]*(d + e*x)^3*(15*a^4*e^4 - (15*d^4*(a*e + c*d*x)^4)/(d + e*x)^4 + (
70*a*d^3*e*(a*e + c*d*x)^3)/(d + e*x)^3 + (128*a^2*d^2*e^2*(a*e + c*d*x)^2
)/(d + e*x)^2 - (70*a^3*d*e^3*(a*e + c*d*x))/(d + e*x)))/((-(c*d^2) + a*e^
2)^5*x^5*(a*e + c*d*x))) + (15*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a
]*Sqrt[e]*Sqrt[d + e*x])])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(640*a^
(5/2)*d^(7/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1215, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^6(d+ex)}dx$$

$$\Big\downarrow \; 1215$$

$$\int \frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \; 1228$$

$$\frac{\left(cd^2-ae^2\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^5}dx}{2d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{1152}$$

$$\frac{(cd^2-ae^2)\left(-\frac{3(cd^2-ae^2)^2\int \frac{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{x^3}dx}{16ade}-\frac{(x(ae^2+cd^2)+2ade)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{8adex^4}\right)}{2d}-\frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{1152}$$

$$\frac{(cd^2-ae^2)\left(-\frac{3(cd^2-ae^2)^2\left(-\frac{(cd^2-ae^2)^2\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8ade}-\frac{(x(ae^2+cd^2)+2ade)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4adex^2}\right)}{16ade}-\frac{(x(ae^2+c}{}\right)}{2d}-\frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{1154}$$

$$\frac{(cd^2-ae^2)\left(-\frac{3(cd^2-ae^2)^2\left(\frac{(cd^2-ae^2)^2\int \frac{1}{4ade-\frac{(2ade+(cd^2+ae^2)x)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{2ade+(cd^2+ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{4ade}-\frac{(x(ae^2+cd^2)+2ade)\sqrt{x(ae^2+cd^2)+a}}{4adex^2}\right.}{16ade}\right.}{2d}-\frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5dx^5}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(cd^2-ae^2\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{16ade}\right.}{2d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5dx^5}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x]
```

output
```
-1/5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d*x^5) + ((c*d^2 - a*e
^2)*(-1/8*((2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2))/(a*d*e*x^4) - (3*(c*d^2 - a*e^2)^2*(-1/4*((2*a*d*e + (c*d^2
+ a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^2) + ((c
*d^2 - a*e^2)^2*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*a^(3/2)*d^(3/2)*e
^(3/2))))/(16*a*d*e)))/(2*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 19538 vs. $2(244) = 488$.

Time = 4.34 (sec) , antiderivative size = 19539, normalized size of antiderivative = 71.83

method	result	size
default	Expression too large to display	19539

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x^6/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 10.52 (sec) , antiderivative size = 872, normalized size of antiderivative = 3.21

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^6(d + ex)} dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/2560*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2
*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(a*d*e)*x^5*log((8*a^2*d^2*e^2
+ (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e +
 a^2*d*e^3)*x)/x^2) + 4*(128*a^5*d^5*e^5 - (15*a*c^4*d^9*e - 70*a^2*c^3*d^
7*e^3 - 128*a^3*c^2*d^5*e^5 + 70*a^4*c*d^3*e^7 - 15*a^5*d*e^9)*x^4 + 2*(5*
a^2*c^3*d^8*e^2 + 233*a^3*c^2*d^6*e^4 + 23*a^4*c*d^4*e^6 - 5*a^5*d^2*e^8)*
x^3 + 8*(31*a^3*c^2*d^7*e^3 + 64*a^4*c*d^5*e^5 + a^5*d^3*e^7)*x^2 + 16*(21
*a^4*c*d^6*e^4 + 11*a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^
2)*x))/(a^3*d^4*e^3*x^5), 1/1280*(15*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*
c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*sqrt(-a*d*e
)*x^5*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e
+ a^2*d*e^3)*x)) - 2*(128*a^5*d^5*e^5 - (15*a*c^4*d^9*e - 70*a^2*c^3*d^7*e
^3 - 128*a^3*c^2*d^5*e^5 + 70*a^4*c*d^3*e^7 - 15*a^5*d*e^9)*x^4 + 2*(5*a^2
*c^3*d^8*e^2 + 233*a^3*c^2*d^6*e^4 + 23*a^4*c*d^4*e^6 - 5*a^5*d^2*e^8)*x^3
 + 8*(31*a^3*c^2*d^7*e^3 + 64*a^4*c*d^5*e^5 + a^5*d^3*e^7)*x^2 + 16*(21*a^
4*c*d^6*e^4 + 11*a^5*d^4*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x))/(a^3*d^4*e^3*x^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^6(d + ex)} dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**6/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^6(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)x^6}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^6), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2449 vs. $2(244) = 488$.

Time = 0.20 (sec) , antiderivative size = 2449, normalized size of antiderivative = 9.00

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^6(d + ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm
="giac")
```

output
```
3/128*(c^5*d^10 - 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 - 10*a^3*c^2*d^4*e^
6 + 5*a^4*c*d^2*e^8 - a^5*e^10)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^2*d^3*e^2) - 1/6
40*(15*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c
^5*d^14*e^4 - 75*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))*a^5*c^4*d^12*e^6 + 150*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*
e^2*x + a*d*e))*a^6*c^3*d^10*e^8 + 1130*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e))*a^7*c^2*d^8*e^10 + 75*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^8*c*d^6*e^12 - 15*(sqrt(c*d*e)*x -
 sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^9*d^4*e^14 + 256*sqrt(c*d*
e)*a^7*c^2*d^9*e^9 - 70*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))^3*a^3*c^5*d^13*e^3 + 350*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d
^2*x + a*e^2*x + a*d*e))^3*a^4*c^4*d^11*e^5 + 5700*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^5*c^3*d^9*e^7 + 7100*(sqrt(c*d*
e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^6*c^2*d^7*e^9 + 22
10*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c*d
^5*e^11 + 70*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
^3*a^8*d^3*e^13 + 2560*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))^2*a^6*c^2*d^8*e^8 + 2560*sqrt(c*d*e)*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^7*c*d^6*e^10 + 128*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^6(d + ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^6\,\left(d + e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 16.31 (sec) , antiderivative size = 1904, normalized size of antiderivative = 7.00

$$\int \frac{(ade + (cd^2 + ae^2)\, x + cdex^2)^{5/2}}{x^6(d + ex)}\, dx = \text{Too large to display}$$

input

```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x)
```

output

```
( - 256*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*d**5*e**7 - 352*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**6*d**4*e**8*x - 16*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
**6*d**3*e**9*x**2 + 20*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*d**2*e**10*x*
*3 - 30*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*d*e**11*x**4 - 256*sqrt(d + e
*x)*sqrt(a*e + c*d*x)*a**5*c*d**7*e**5 - 1024*sqrt(d + e*x)*sqrt(a*e + c*d
*x)*a**5*c*d**6*e**6*x - 1040*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d**5*
e**7*x**2 - 72*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d**4*e**8*x**3 + 110
*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d**3*e**9*x**4 - 672*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**4*c**2*d**8*e**4*x - 1520*sqrt(d + e*x)*sqrt(a*e + c
*d*x)*a**4*c**2*d**7*e**5*x**2 - 1024*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4
*c**2*d**6*e**6*x**3 - 116*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**2*d**5*
e**7*x**4 - 496*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**9*e**3*x**2 -
 952*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**8*e**4*x**3 - 396*sqrt(d
 + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**7*e**5*x**4 - 20*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a**2*c**4*d**10*e**2*x**3 - 110*sqrt(d + e*x)*sqrt(a*e + c*d
*x)*a**2*c**4*d**9*e**3*x**4 + 30*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**5*d
**11*e*x**4 - 15*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - s
qrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*
x))*a**6*e**12*x**5 + 60*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*
d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*...
```

3.58 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^7(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 369

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^7(d+ex)}\,dx =& -\frac{\left(cd^2-ae^2\right)^3\left(5cd^2+7ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{512a^3d^4e^3x^2}\\&+\frac{\left(cd^2-ae^2\right)\left(5cd^2+7ae^2\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{192a^2d^3e^2x^4}\\&-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{6dx^6}-\frac{\left(\frac{5c}{ae}-\frac{7e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{60x^5}\\&+\frac{\left(cd^2-ae^2\right)^5\left(5cd^2+7ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{512a^{7/2}d^{9/2}e^{7/2}}\end{aligned}$$

output

```
-1/512*(-a*e^2+c*d^2)^3*(7*a*e^2+5*c*d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d^4/e^3/x^2+1/192*(-a*e^2+c*d^2)*(7*
a*e^2+5*c*d^2)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(3/2)/a^2/d^3/e^2/x^4-1/6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/x^6-1
/60*(5*c/a/e-7*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5+1/512*(-
a*e^2+c*d^2)^5*(7*a*e^2+5*c*d^2)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(7/2)/d^(9/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.09

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^7(d+ex)}\,dx = \frac{(-cd^2+ae^2)^5\,((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(75c^5d^{10}x^5-5ac^4d^8ex^4}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

output
```
((-(c*d^2) + a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[a]*Sqrt[d]*Sq
rt[e]*(75*c^5*d^10*x^5 - 5*a*c^4*d^8*e*x^4*(10*d + 49*e*x) + 10*a^2*c^3*d^
6*e^2*x^3*(4*d^2 + 16*d*e*x + 15*e^2*x^2) + 6*a^3*c^2*d^4*e^3*x^2*(360*d^3
 + 564*d^2*e*x + 58*d*e^2*x^2 - 91*e^3*x^3) + a^4*c*d^2*e^4*x*(3200*d^4 +
4448*d^3*e*x + 216*d^2*e^2*x^2 - 272*d*e^3*x^3 + 415*e^4*x^4) + a^5*e^5*(1
280*d^5 + 1664*d^4*e*x + 48*d^3*e^2*x^2 - 56*d^2*e^3*x^3 + 70*d*e^4*x^4 -
105*e^5*x^5)))/((c*d^2 - a*e^2)^5*x^6*(a*e + c*d*x)*(d + e*x)) - (15*(5*c*
d^2 + 7*a*e^2)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d
 + e*x])])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(7680*a^(7/2)*d^(9/2)*e
^(7/2))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1215, 1237, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^7(d+ex)}dx$$

$$\Big\downarrow \text{ 1215}$$

$$\int \frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{ 1237}$$

$$-\frac{\int -\frac{ae(5cd^2-2cexd-7ae^2)\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/2}}{2x^6}dx}{6ade}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6dx^6}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(5cd^2-2cexd-7ae^2)\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/2}}{x^6}dx}{12d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6dx^6}$$

$\Big\downarrow$ 1228

$$\frac{-\frac{\left(\frac{5c^2d^4}{a}-7ae^4+2cd^2e^2\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^5}dx}{2de}-\frac{\left(\frac{5cd}{ae}-\frac{7e}{d}\right)\left(x(ae^2+cd^2)+ade+cdex^2\right)^{5/2}}{5x^5}}{12d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6dx^6}$$

$\Big\downarrow$ 1152

$$\frac{-\frac{\left(\frac{5c^2d^4}{a}-7ae^4+2cd^2e^2\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{16ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8adex^4}\right)}{2de}-\frac{\left(\frac{5cd}{ae}-\frac{7e}{d}\right)(x\ldots}{\ldots}}{12d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6dx^6}$$

$\Big\downarrow$ 1152

$$\frac{-\frac{\left(\frac{5c^2d^4}{a}-7ae^4+2cd^2e^2\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(-\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4adex^2}\right)}{16ade}-\frac{\left(x\left(\ldots\right.\right.}{\ldots}\right)}{2de}\ldots}{12d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6dx^6}$$

$\Big\downarrow$ 1154

$$-\frac{\left(\frac{5c^2d^4}{a}-7ae^4+2cd^2e^2\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(cd^2-ae^2\right)^2\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2\cdots\right.}}{4adex^2}\cdots\right)}{16ade}\cdots\right)}{2de}\cdots}{12d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6dx^6}$$

$$\Big\downarrow \; 219$$

$$-\frac{\left(\frac{5c^2d^4}{a}-7ae^4+2cd^2e^2\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade\cdots}}{4adex^2}\cdots\right)}{16ade}\cdots\right)}{2de}\cdots}{12d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6dx^6}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x]
```

output

```
-1/6*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d*x^6) + (-1/5*(((5*c*
d)/(a*e) - (7*e)/d)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/x^5 - (
((5*c^2*d^4)/a + 2*c*d^2*e^2 - 7*a*e^4)*(-1/8*((2*a*d*e + (c*d^2 + a*e^2)*
x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(a*d*e*x^4) - (3*(c*d^2
- a*e^2)^2*(-1/4*((2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(a*d*e*x^2) + ((c*d^2 - a*e^2)^2*ArcTanh[(2*a*d*e + (c*
d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])])/(8*a^(3/2)*d^(3/2)*e^(3/2))))/(16*a*d*e)))/(2*d*e))/(12*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32290 vs. $2(337) = 674$.

Time = 4.56 (sec) , antiderivative size = 32291, normalized size of antiderivative = 87.51

method	result	size
default	Expression too large to display	32291

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x^7/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 33.00 (sec) , antiderivative size = 1072, normalized size of antiderivative = 2.91

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d + ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/30720*(15*(5*c^6*d^12 - 18*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 20*a^
3*c^3*d^6*e^6 - 45*a^4*c^2*d^4*e^8 + 30*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(
a*d*e)*x^6*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 -
4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(1280*a^6*d^6*e^6 +
(75*a*c^5*d^11*e - 245*a^2*c^4*d^9*e^3 + 150*a^3*c^3*d^7*e^5 - 546*a^4*c^2
*d^5*e^7 + 415*a^5*c*d^3*e^9 - 105*a^6*d*e^11)*x^5 - 2*(25*a^2*c^4*d^10*e^
2 - 80*a^3*c^3*d^8*e^4 - 174*a^4*c^2*d^6*e^6 + 136*a^5*c*d^4*e^8 - 35*a^6*
d^2*e^10)*x^4 + 8*(5*a^3*c^3*d^9*e^3 + 423*a^4*c^2*d^7*e^5 + 27*a^5*c*d^5*
e^7 - 7*a^6*d^3*e^9)*x^3 + 16*(135*a^4*c^2*d^8*e^4 + 278*a^5*c*d^6*e^6 + 3
*a^6*d^4*e^8)*x^2 + 128*(25*a^5*c*d^7*e^5 + 13*a^6*d^5*e^7)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^4*d^5*e^4*x^6), -1/15360*(15*(5*c^6*d
^12 - 18*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 20*a^3*c^3*d^6*e^6 - 45*a^4
*c^2*d^4*e^8 + 30*a^5*c*d^2*e^10 - 7*a^6*e^12)*sqrt(-a*d*e)*x^6*arctan(1/2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x))
 + 2*(1280*a^6*d^6*e^6 + (75*a*c^5*d^11*e - 245*a^2*c^4*d^9*e^3 + 150*a^3*
c^3*d^7*e^5 - 546*a^4*c^2*d^5*e^7 + 415*a^5*c*d^3*e^9 - 105*a^6*d*e^11)*x^
5 - 2*(25*a^2*c^4*d^10*e^2 - 80*a^3*c^3*d^8*e^4 - 174*a^4*c^2*d^6*e^6 + 13
6*a^5*c*d^4*e^8 - 35*a^6*d^2*e^10)*x^4 + 8*(5*a^3*c^3*d^9*e^3 + 423*a^4...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^7(d + ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**7/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d+ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex+d)x^7}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^7), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3388 vs. 2(337) = 674.

Time = 0.31 (sec) , antiderivative size = 3388, normalized size of antiderivative = 9.18

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm
="giac")
```

output
```
-1/512*(5*c^6*d^12 - 18*a*c^5*d^10*e^2 + 15*a^2*c^4*d^8*e^4 + 20*a^3*c^3*d
^6*e^6 - 45*a^4*c^2*d^4*e^8 + 30*a^5*c*d^2*e^10 - 7*a^6*e^12)*arctan(-(sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(s
qrt(-a*d*e)*a^3*d^4*e^3) + 1/7680*(75*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*
d^2*x + a*e^2*x + a*d*e))*a^5*c^6*d^17*e^5 - 270*(sqrt(c*d*e)*x - sqrt(c*d
*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^6*c^5*d^15*e^7 + 225*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^7*c^4*d^13*e^9 + 15660*(
sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^8*c^3*d^11*
e^11 + 14685*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
*a^9*c^2*d^9*e^13 + 450*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*
x + a*d*e))*a^10*c*d^7*e^15 - 105*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))*a^11*d^5*e^17 + 3072*sqrt(c*d*e)*a^9*c^2*d^10*e^12 -
 425*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4*c
^6*d^16*e^4 + 1530*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))^3*a^5*c^5*d^14*e^6 + 75525*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))^3*a^6*c^4*d^12*e^8 + 203100*(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c^3*d^10*e^10 + 142065*(sqrt(c
*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*c^2*d^8*e^12
+ 28170*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^
9*c*d^6*e^14 + 595*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d + ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^7\,\left(d + e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d + ex)}\,dx = \int \frac{(ade + (a\,e^2 + c\,d^2)\,x + cde\,x^2)^{\frac{5}{2}}}{x^7\,(ex + d)}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x)
```

3.59 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^8(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 483

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^8(d+ex)}\,dx = \frac{\left(cd^2-ae^2\right)^3\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{}}{1024a^4d^5e^4x^2}$$
$$-\frac{\left(cd^2-ae^2\right)\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{384a^3d^4e^3x^4}$$
$$-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{7dx^7}-\frac{\left(\frac{5c}{ae}-\frac{9e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{84x^6}$$
$$+\frac{\left(35c^2d^4+20acd^2e^2-63a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{840a^2d^3e^2x^5}$$
$$-\frac{\left(cd^2-ae^2\right)^5\left(5c^2d^4+10acd^2e^2+9a^2e^4\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{1024a^{9/2}d^{11/2}e^{9/2}}$$

output
```
1/1024*(-a*e^2+c*d^2)^3*(9*a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*(2*a*d*e+(a*e
^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^4/d^5/e^4/x^2-1/384
*(-a*e^2+c*d^2)*(9*a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*(2*a*d*e+(a*e^2+c*d^2
)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/a^3/d^4/e^3/x^4-1/7*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/x^7-1/84*(5*c/a/e-9*e/d^2)*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(5/2)/x^6+1/840*(-63*a^2*e^4+20*a*c*d^2*e^2+35*c^2*d^4
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/a^2/d^3/e^2/x^5-1/1024*(-a*e^2+c
*d^2)^5*(9*a^2*e^4+10*a*c*d^2*e^2+5*c^2*d^4)*arctanh(a^(1/2)*e^(1/2)*(e*x+
d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(9/2)/d^(11/2)/e^(9/
2)
```

Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.03

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^8(d+ex)}\,dx = \frac{(-cd^2+ae^2)^5\,((ae+cdx)(d+ex))^{3/2}\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-525c^6d^{12}x^6+350ac^5\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

output
```
((-(c*d^2) + a*e^2)^5*((a*e + c*d*x)*(d + e*x))^(3/2)*((Sqrt[a]*Sqrt[d]*Sq
rt[e]*(-525*c^6*d^12*x^6 + 350*a*c^5*d^10*e*x^5*(d + 4*e*x) - 35*a^2*c^4*d
^8*e^2*x^4*(8*d^2 + 26*d*e*x + 15*e^2*x^2) + 60*a^3*c^3*d^6*e^3*x^3*(4*d^3
 + 12*d^2*e*x + 5*d*e^2*x^2 - 10*e^3*x^3) + a^4*c^2*d^4*e^4*x^2*(23680*d^4
 + 33520*d^3*e*x + 1824*d^2*e^2*x^2 - 2332*d*e^3*x^3 + 3689*e^4*x^4) + 2*a
^5*c*d^2*e^5*x*(18560*d^5 + 24320*d^4*e*x + 744*d^3*e^2*x^2 - 872*d^2*e^3*
x^3 + 1099*d*e^4*x^4 - 1680*e^5*x^5) + 3*a^6*e^6*(5120*d^6 + 6400*d^5*e*x
+ 128*d^4*e^2*x^2 - 144*d^3*e^3*x^3 + 168*d^2*e^4*x^4 - 210*d*e^5*x^5 + 31
5*e^6*x^6)))/((c*d^2 - a*e^2)^5*x^7*(a*e + c*d*x)*(d + e*x)) + (105*(5*c^2
*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sq
rt[a]*Sqrt[e]*Sqrt[d + e*x])])/((a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))))/(10
7520*a^(9/2)*d^(11/2)*e^(9/2))
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1215, 1237, 27, 1237, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^8(d+ex)}dx$$

$$\Big\downarrow \text{1215}$$

$$\int \frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{1237}$$

$$-\frac{\int -\frac{ae\left(5cd^2-4cexd-9ae^2\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{2x^7}dx}{7ade}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(5cd^2-4cexd-9ae^2\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^7}dx}{14d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{1237}$$

$$\frac{-\frac{\int \frac{\left(35c^2d^4+20ace^2d^2+2ce\left(5cd^2-9ae^2\right)xd-63a^2e^4\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{2x^6}dx}{6ade}-\frac{\left(\frac{5cd}{ae}-\frac{9e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6x^6}}{14d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\left(35c^2d^4+20ace^2d^2+2ce\left(5cd^2-9ae^2\right)xd-63a^2e^4\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^6}dx}{12ade}-\frac{\left(\frac{5cd}{ae}-\frac{9e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{6x^6}}{14d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7dx^7}$$

$$\Big\downarrow \text{1228}$$

$$-\frac{-\frac{7\left(cd^2-ae^2\right)\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\int\frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^5}dx}{2ade}-\frac{\left(\frac{35c^2d^4}{a}-63ae^4+20cd^2e^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5dex^5}}{12ade}-\frac{\left(\frac{5cd}{ae}-\frac{9e}{d}\cdots\right.}{\cdots}\cdots}{14d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7dx^7}$$

$\downarrow$ 1152

$$-\frac{-\frac{7\left(cd^2-ae^2\right)\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\int\frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{16ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{8adex^4}\right)}{2ade}-\frac{\left(3\cdots\right.}{\cdots}\cdots}{12ade}\cdots}{14d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7dx^7}$$

$\downarrow$ 1152

$$-\frac{-\frac{7\left(cd^2-ae^2\right)\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(ae^2+cd^2\right)+ade+\cdots}}{4adex^2}\cdots\right.}{16ade}\cdots\right.}{2ade}\cdots}{12ade}\cdots$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7dx^7}$$

$\downarrow$ 1154

$$-\frac{-\frac{7\left(cd^2-ae^2\right)\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4ade}-\frac{\left(x\left(ae^2+cd^2\right)+\cdots\right.}{\cdots}\right)}{16ade}\cdots\right.}{2ade}\cdots}{12\cdots}\cdots$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7dx^7}$$

$\downarrow$ 219

$$-\frac{-\frac{7\left(cd^2-ae^2\right)\left(9a^2e^4+10acd^2e^2+5c^2d^4\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(cd^2-ae^2\right)^2\operatorname{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x\left(\cdots\right.}}{4ade\cdots}\right)}{16ade}\cdots\right.}{2ade}\cdots}{12ade}\cdots$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7dx^7}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

output
```
-1/7*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d*x^7) + (-1/6*(((5*c*
d)/(a*e) - (9*e)/d)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/x^6 - (
-1/5*(((35*c^2*d^4)/a + 20*c*d^2*e^2 - 63*a*e^4)*(a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2)^(5/2))/(d*e*x^5) - (7*(c*d^2 - a*e^2)*(5*c^2*d^4 + 10*a*c*d
^2*e^2 + 9*a^2*e^4)*(-1/8*((2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2)^(3/2))/(a*d*e*x^4) - (3*(c*d^2 - a*e^2)^2*(-1/4*((2
*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
a*d*e*x^2) + ((c*d^2 - a*e^2)^2*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*S
qrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*a
^(3/2)*d^(3/2)*e^(3/2))))/(16*a*d*e)))/(2*a*d*e))/(12*a*d*e))/(14*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45105 vs. $2(447) = 894$.

Time = 6.72 (sec) , antiderivative size = 45106, normalized size of antiderivative = 93.39

method	result	size
default	Expression too large to display	45106

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x^8/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 67.31 (sec) , antiderivative size = 1300, normalized size of antiderivative = 2.69

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{x^8(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/430080*(105*(5*c^7*d^14 - 15*a*c^6*d^12*e^2 + 9*a^2*c^5*d^10*e^4 + 5*a
^3*c^4*d^8*e^6 + 15*a^4*c^3*d^6*e^8 - 45*a^5*c^2*d^4*e^10 + 35*a^6*c*d^2*e
^12 - 9*a^7*e^14)*sqrt(a*d*e)*x^7*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) +
 4*(15360*a^7*d^7*e^7 - (525*a*c^6*d^13*e - 1400*a^2*c^5*d^11*e^3 + 525*a^
3*c^4*d^9*e^5 + 600*a^4*c^3*d^7*e^7 - 3689*a^5*c^2*d^5*e^9 + 3360*a^6*c*d^
3*e^11 - 945*a^7*d*e^13)*x^6 + 2*(175*a^2*c^5*d^12*e^2 - 455*a^3*c^4*d^10*
e^4 + 150*a^4*c^3*d^8*e^6 - 1166*a^5*c^2*d^6*e^8 + 1099*a^6*c*d^4*e^10 - 3
15*a^7*d^2*e^12)*x^5 - 8*(35*a^3*c^4*d^11*e^3 - 90*a^4*c^3*d^9*e^5 - 228*a
^5*c^2*d^7*e^7 + 218*a^6*c*d^5*e^9 - 63*a^7*d^3*e^11)*x^4 + 16*(15*a^4*c^3
*d^10*e^4 + 2095*a^5*c^2*d^8*e^6 + 93*a^6*c*d^6*e^8 - 27*a^7*d^4*e^10)*x^3
 + 128*(185*a^5*c^2*d^9*e^5 + 380*a^6*c*d^7*e^7 + 3*a^7*d^5*e^9)*x^2 + 128
0*(29*a^6*c*d^8*e^6 + 15*a^7*d^6*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x))/(a^5*d^6*e^5*x^7), 1/215040*(105*(5*c^7*d^14 - 15*a*c^6*d^12*e
^2 + 9*a^2*c^5*d^10*e^4 + 5*a^3*c^4*d^8*e^6 + 15*a^4*c^3*d^6*e^8 - 45*a^5*
c^2*d^4*e^10 + 35*a^6*c*d^2*e^12 - 9*a^7*e^14)*sqrt(-a*d*e)*x^7*arctan(1/2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x))
 - 2*(15360*a^7*d^7*e^7 - (525*a*c^6*d^13*e - 1400*a^2*c^5*d^11*e^3 + 5...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^8(d + ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**8/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^8(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)x^8}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^8), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4452 vs. $2(447) = 894$.

Time = 0.58 (sec) , antiderivative size = 4452, normalized size of antiderivative = 9.22

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^8(d + ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm
="giac")
```

output
```
1/1024*(5*c^7*d^14 - 15*a*c^6*d^12*e^2 + 9*a^2*c^5*d^10*e^4 + 5*a^3*c^4*d^
8*e^6 + 15*a^4*c^3*d^6*e^8 - 45*a^5*c^2*d^4*e^10 + 35*a^6*c*d^2*e^12 - 9*a
^7*e^14)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^4*d^5*e^4) - 1/107520*(525*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^6*c^7*d^20*e^6 - 1575*
(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^7*c^6*d^18
*e^8 + 945*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a
^8*c^5*d^16*e^10 + 215565*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^
2*x + a*d*e))*a^9*c^4*d^14*e^12 + 431655*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
 c*d^2*x + a*e^2*x + a*d*e))*a^10*c^3*d^12*e^14 + 210315*(sqrt(c*d*e)*x -
sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^11*c^2*d^10*e^16 + 3675*(sq
rt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^12*c*d^8*e^18
 - 945*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^13*
d^6*e^20 + 30720*sqrt(c*d*e)*a^10*c^3*d^13*e^13 + 43008*sqrt(c*d*e)*a^11*c
^2*d^11*e^15 - 3500*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
a*d*e))^3*a^5*c^7*d^19*e^5 + 10500*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))^3*a^6*c^6*d^17*e^7 + 1068900*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c^5*d^15*e^9 + 4655700*(sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*c^4*d^13*e^
11 + 6225660*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^8(d + ex)}\,dx = \int \frac{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{x^8\,\left(d+e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{x^8(d+ex)}\,dx=\int \frac{\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{5}{2}}}{x^8\left(ex+d\right)}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x)
```

3.60 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 611

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^9(d+ex)}\,dx =$$

$$-\frac{3\left(cd^2-ae^2\right)^3\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\sqrt{ade+\left(cd^2+ae^2\right)x+}}{16384a^5d^6e^5x^2}$$

$$+\frac{\left(cd^2-ae^2\right)\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\left(2ade+\left(cd^2+ae^2\right)x\right)\left(ade+\left(cd^2+ae^2\right)x+cde\right.}{2048a^4d^5e^4x^4}$$

$$-\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{8dx^8}-\frac{\left(\frac{5c}{ae}-\frac{11e}{d^2}\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{112x^7}$$

$$+\frac{\left(15c^2d^4+10acd^2e^2-33a^2e^4\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{448a^2d^3e^2x^6}$$

$$-\frac{\left(105c^3d^6+95ac^2d^4e^2+15a^2cd^2e^4-231a^3e^6\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{4480a^3d^4e^3x^5}$$

$$+\frac{3\left(cd^2-ae^2\right)^5\left(15c^3d^6+35ac^2d^4e^2+45a^2cd^2e^4+33a^3e^6\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{16384a^{11/2}d^{13/2}e^{11/2}}$$

output
```
-3/16384*(-a*e^2+c*d^2)^3*(33*a^3*e^6+45*a^2*c*d^2*e^4+35*a*c^2*d^4*e^2+15
*c^3*d^6)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/a^5/d^6/e^5/x^2+1/2048*(-a*e^2+c*d^2)*(33*a^3*e^6+45*a^2*c*d^2*e^4+35*a*
c^2*d^4*e^2+15*c^3*d^6)*(2*a*d*e+(a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(3/2)/a^4/d^5/e^4/x^4-1/8*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2
)/d/x^8-1/112*(5*c/a/e-11*e/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x
^7+1/448*(-33*a^2*e^4+10*a*c*d^2*e^2+15*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(5/2)/a^2/d^3/e^2/x^6-1/4480*(-231*a^3*e^6+15*a^2*c*d^2*e^4+95*a*
c^2*d^4*e^2+105*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/a^3/d^4/e
^3/x^5+3/16384*(-a*e^2+c*d^2)^5*(33*a^3*e^6+45*a^2*c*d^2*e^4+35*a*c^2*d^4*
e^2+15*c^3*d^6)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/a^(11/2)/d^(13/2)/e^(11/2)
```

Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 572, normalized size of antiderivative = 0.94

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^9(d+ex)}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(1575c^7d^{14}x^7-525ac^6d^{12}ex^6(2d+7ex)+35a}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(1575*c^7*d^14*
x^7 - 525*a*c^6*d^12*e*x^6*(2*d + 7*e*x) + 35*a^2*c^5*d^10*e^2*x^5*(24*d^2
 + 68*d*e*x + 29*e^2*x^2) - 5*a^3*c^4*d^8*e^3*x^4*(144*d^3 + 376*d^2*e*x +
 110*d*e^2*x^2 - 185*e^3*x^3) + 5*a^4*c^3*d^6*e^4*x^3*(128*d^4 + 320*d^3*e
*x + 80*d^2*e^2*x^2 - 120*d*e^3*x^3 + 265*e^4*x^4) + a^5*c^2*d^4*e^5*x^2*(
103680*d^5 + 137600*d^4*e*x + 4640*d^3*e^2*x^2 - 5488*d^2*e^3*x^3 + 7034*d
*e^4*x^4 - 11193*e^5*x^5) + a^6*c*d^2*e^6*x*(168960*d^6 + 212480*d^5*e*x +
 4480*d^4*e^2*x^2 - 5056*d^3*e^3*x^3 + 5928*d^2*e^4*x^4 - 7476*d*e^5*x^5 +
 11445*e^6*x^6) + a^7*e^7*(71680*d^7 + 87040*d^6*e*x + 1280*d^5*e^2*x^2 -
1408*d^4*e^3*x^3 + 1584*d^3*e^4*x^4 - 1848*d^2*e^5*x^5 + 2310*d*e^6*x^6 -
3465*e^7*x^7)))/x^8) + (105*(c*d^2 - a*e^2)^5*(15*c^3*d^6 + 35*a*c^2*d^4*e
^2 + 45*a^2*c*d^2*e^4 + 33*a^3*e^6)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(S
qrt[a]*Sqrt[e]*Sqrt[d + e*x])]])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(57344
0*a^(11/2)*d^(13/2)*e^(11/2))
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1215, 1237, 27, 1237, 27, 1237, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^9(d+ex)}dx$$

$$\Big\downarrow \ 1215$$

$$\int \frac{(ae+cdx)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}{x^9}dx$$

$$\Big\downarrow \ 1237$$

$$-\frac{\int -\frac{ae\left(5cd^2-6cexd-11ae^2\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{2x^8}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(5cd^2-6cexd-11ae^2\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^8}dx}{16d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \ 1237$$

$$\frac{-\frac{\int \frac{\left(3\left(15c^2d^4+10ace^2d^2-33a^2e^4\right)+4cde\left(5cd^2-11ae^2\right)x\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{2x^7}dx}{7ade}-\frac{\left(\frac{5cd}{ae}-\frac{11e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7x^7}}{16d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\int \frac{\left(3\left(15c^2d^4+10ace^2d^2-33a^2e^4\right)+4cde\left(5cd^2-11ae^2\right)x\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^7}dx}{14ade}-\frac{\left(\frac{5cd}{ae}-\frac{11e}{d}\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{7x^7}}{16d}-\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$$\Big\downarrow \ 1237$$

$$-\frac{-\frac{-\frac{\int \frac{3\left(105c^3d^6+95ac^2e^2d^4+15a^2ce^4d^2+2ce\left(15c^2d^4+10ace^2d^2-33a^2e^4\right)xd-231a^3e^6\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{2x^6}dx}{6ade}-\frac{\left(\frac{15c^2d^4}{a}-33ae^4+10cd^2e^2\right)\left(x\ldots\right.}{2dex\ldots}}{14ade}\ldots}{16d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$\downarrow$ 27

$$-\frac{-\frac{-\frac{\int \frac{\left(105c^3d^6+95ac^2e^2d^4+15a^2ce^4d^2+2ce\left(15c^2d^4+10ace^2d^2-33a^2e^4\right)xd-231a^3e^6\right)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^6}dx}{4ade}-\frac{\left(\frac{15c^2d^4}{a}-33ae^4+10cd^2e^2\right)\left(x\left(\ldots\right.\right.}{2dex^6\ldots}}{14ade}\ldots}{16d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$\downarrow$ 1228

$$-\frac{-\frac{-\frac{-\frac{7\left(cd^2-ae^2\right)\left(33a^3e^6+45a^2cd^2e^4+35ac^2d^4e^2+15c^3d^6\right)\int \frac{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}{x^5}dx}{2ade}-\frac{\left(-231a^3e^6+15a^2cd^2e^4+95ac^2d^4e^2+105c^3d^6\right)\left(x\left(ae^2+c\ldots\right.\right.}{5adex^5}}{4ade}\ldots}{14ade}\ldots}{16d}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$\downarrow$ 1152

$$-\frac{-\frac{-\frac{7\left(cd^2-ae^2\right)\left(33a^3e^6+45a^2cd^2e^4+35ac^2d^4e^2+15c^3d^6\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\int \frac{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}{x^3}dx}{16ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\left(x\left(ae^2+cd^2\right)+ade+c\ldots\right.}{8adex^4}\ldots\right)}{2ade}\ldots}{4ade}\ldots}{14a\ldots}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$\downarrow$ 1152

$$-\frac{-\frac{-\frac{7\left(cd^2-ae^2\right)\left(33a^3e^6+45a^2cd^2e^4+35ac^2d^4e^2+15c^3d^6\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(-\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{\left(x\left(ae^2+cd^2\right)+2ade\right)\sqrt{x}}{4ade}\right.}{16ade}\right.}{2ade}}{}}{}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$\downarrow$ 1154

$$-\frac{-\frac{-\frac{7\left(cd^2-ae^2\right)\left(33a^3e^6+45a^2cd^2e^4+35ac^2d^4e^2+15c^3d^6\right)\left(-\frac{3\left(cd^2-ae^2\right)^2\left(\frac{\left(cd^2-ae^2\right)^2\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ad}}}{4ade}\right.}{16ade}\right.}{2ade}}{}}{}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

$\downarrow$ 219

$$3\left(cd^2-ae^2\right.$$

$$7\left(cd^2-ae^2\right)\left(33a^3e^6+45a^2cd^2e^4+35ac^2d^4e^2+15c^3d^6\right)\left(-\right.$$

$$-\frac{-\frac{-\frac{\left(-231a^3e^6+15a^2cd^2e^4+95ac^2d^4e^2+105c^3d^6\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{5adex^5}-}{}}{}$$

$$\frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{8dx^8}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x]
```

output
```
-1/8*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d*x^8) + (-1/7*(((5*c*
d)/(a*e) - (11*e)/d)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/x^7 -
(-1/2*(((15*c^2*d^4)/a + 10*c*d^2*e^2 - 33*a*e^4)*(a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2)^(5/2))/(d*e*x^6) - (-1/5*((105*c^3*d^6 + 95*a*c^2*d^4*e^2
+ 15*a^2*c*d^2*e^4 - 231*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(5/2))/(a*d*e*x^5) - (7*(c*d^2 - a*e^2)*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 4
5*a^2*c*d^2*e^4 + 33*a^3*e^6)*(-1/8*((2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(a*d*e*x^4) - (3*(c*d^2 - a*e^2)^2
*(-1/4*((2*a*d*e + (c*d^2 + a*e^2)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(a*d*e*x^2) + ((c*d^2 - a*e^2)^2*ArcTanh[(2*a*d*e + (c*d^2 + a*e^
2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])])/(8*a^(3/2)*d^(3/2)*e^(3/2))))/(16*a*d*e)))/(2*a*d*e))/(4*a*d*e))/(14
*a*d*e))/(16*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1215
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + b*x +
 c*x^2)^(p - 1), x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 -
 b*d*e + a*e^2, 0] && GtQ[p, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 70735 vs. 2(571) = 1142.

Time = 6.68 (sec) , antiderivative size = 70736, normalized size of antiderivative = 115.77

method	result	size
default	Expression too large to display	70736

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x^9/(e*x+d),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 156.88 (sec) , antiderivative size = 1550, normalized size of antiderivative = 2.54

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^9(d + ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm
="fricas")
```

output
```
[-1/2293760*(105*(15*c^8*d^16 - 40*a*c^7*d^14*e^2 + 20*a^2*c^6*d^12*e^4 +
8*a^3*c^5*d^10*e^6 + 10*a^4*c^4*d^8*e^8 + 40*a^5*c^3*d^6*e^10 - 140*a^6*c^
2*d^4*e^12 + 120*a^7*c*d^2*e^14 - 33*a^8*e^16)*sqrt(a*d*e)*x^8*log((8*a^2*
d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c
*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(71680*a^8*d^8*e^8 + (1575*a*c^7*d^15*e -
3675*a^2*c^6*d^13*e^3 + 1015*a^3*c^5*d^11*e^5 + 925*a^4*c^4*d^9*e^7 + 1325
*a^5*c^3*d^7*e^9 - 11193*a^6*c^2*d^5*e^11 + 11445*a^7*c*d^3*e^13 - 3465*a^
8*d*e^15)*x^7 - 2*(525*a^2*c^6*d^14*e^2 - 1190*a^3*c^5*d^12*e^4 + 275*a^4*
c^4*d^10*e^6 + 300*a^5*c^3*d^8*e^8 - 3517*a^6*c^2*d^6*e^10 + 3738*a^7*c*d^
4*e^12 - 1155*a^8*d^2*e^14)*x^6 + 8*(105*a^3*c^5*d^13*e^3 - 235*a^4*c^4*d^
11*e^5 + 50*a^5*c^3*d^9*e^7 - 686*a^6*c^2*d^7*e^9 + 741*a^7*c*d^5*e^11 - 2
31*a^8*d^3*e^13)*x^5 - 16*(45*a^4*c^4*d^12*e^4 - 100*a^5*c^3*d^10*e^6 - 29
0*a^6*c^2*d^8*e^8 + 316*a^7*c*d^6*e^10 - 99*a^8*d^4*e^12)*x^4 + 128*(5*a^5
*c^3*d^11*e^5 + 1075*a^6*c^2*d^9*e^7 + 35*a^7*c*d^7*e^9 - 11*a^8*d^5*e^11)
*x^3 + 1280*(81*a^6*c^2*d^10*e^6 + 166*a^7*c*d^8*e^8 + a^8*d^6*e^10)*x^2 +
 5120*(33*a^7*c*d^9*e^7 + 17*a^8*d^7*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x))/(a^6*d^7*e^6*x^8), -1/1146880*(105*(15*c^8*d^16 - 40*a*c^7
*d^14*e^2 + 20*a^2*c^6*d^12*e^4 + 8*a^3*c^5*d^10*e^6 + 10*a^4*c^4*d^8*e^8
+ 40*a^5*c^3*d^6*e^10 - 140*a^6*c^2*d^4*e^12 + 120*a^7*c*d^2*e^14 - 33*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^9(d + ex)}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**9/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^9(d + ex)}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)x^9}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^9), x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5681 vs. 2(571) = 1142.

Time = 1.19 (sec) , antiderivative size = 5681, normalized size of antiderivative = 9.30

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^9(d + ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm
="giac")
```

output
```
-3/16384*(15*c^8*d^16 - 40*a*c^7*d^14*e^2 + 20*a^2*c^6*d^12*e^4 + 8*a^3*c^
5*d^10*e^6 + 10*a^4*c^4*d^8*e^8 + 40*a^5*c^3*d^6*e^10 - 140*a^6*c^2*d^4*e^
12 + 120*a^7*c*d^2*e^14 - 33*a^8*e^16)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e
*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^5*d^6*e^5
) + 1/573440*(1575*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))*a^7*c^8*d^23*e^7 - 4200*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x +
 a*e^2*x + a*d*e))*a^8*c^7*d^21*e^9 + 2100*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2
 + c*d^2*x + a*e^2*x + a*d*e))*a^9*c^6*d^19*e^11 + 1147720*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^10*c^5*d^17*e^13 + 344169
0*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^11*c^4*d
^15*e^15 + 3444840*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a
*d*e))*a^12*c^3*d^13*e^17 + 1132180*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^
2*x + a*e^2*x + a*d*e))*a^13*c^2*d^11*e^19 + 12600*(sqrt(c*d*e)*x - sqrt(c
*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^14*c*d^9*e^21 - 3465*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^15*d^7*e^23 + 163840*s
qrt(c*d*e)*a^11*c^4*d^16*e^14 + 327680*sqrt(c*d*e)*a^12*c^3*d^14*e^16 + 22
9376*sqrt(c*d*e)*a^13*c^2*d^12*e^18 - 12075*(sqrt(c*d*e)*x - sqrt(c*d*e*x^
2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^6*c^8*d^22*e^6 + 32200*(sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^7*c^7*d^20*e^8 + 571830
0*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^8*c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^9(d + ex)}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^9\,\left(d + e\,x\right)}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^9(d + ex)} dx = \int \frac{(ade + (a e^2 + c d^2) x + cde x^2)^{\frac{5}{2}}}{x^9 (ex + d)} dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x)
```

3.61 $\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 506

$$\int \frac{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^4}\,dx = \frac{\left(231c^3d^6-63ac^2d^4e^2-7a^2cd^2e^4-a^3e^6\right)(ae+cdx)^2\sqrt{ade+}}{80c^2d^2e^4\left(cd^2-ae^2\right)}$$
$$-\frac{3\left(9cd^2+ae^2\right)(ae+cdx)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40c^2d^2e^3}$$
$$+\frac{\left(231c^3d^6-63ac^2d^4e^2-7a^2cd^2e^4-a^3e^6\right)\left(3cd^2-5ae^2-2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{128c^2d^2e^6}$$
$$-\frac{2d^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{e^3\left(cd^2-ae^2\right)(d+ex)^4}+\frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{7/2}}{5cde^3(d+ex)^2}$$
$$-\frac{3\left(cd^2-ae^2\right)^2\left(231c^3d^6-63ac^2d^4e^2-7a^2cd^2e^4-a^3e^6\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{128c^{5/2}d^{5/2}e^{13/2}}$$

output
```
1/80*(-a^3*e^6-7*a^2*c*d^2*e^4-63*a*c^2*d^4*e^2+231*c^3*d^6)*(c*d*x+a*e)^2
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^4/(-a*e^2+c*d^2)-3/40*(
a*e^2+9*c*d^2)*(c*d*x+a*e)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d
^2/e^3+1/128*(-a^3*e^6-7*a^2*c*d^2*e^4-63*a*c^2*d^4*e^2+231*c^3*d^6)*(-2*c
*d*e*x-5*a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^
6-2*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/e^3/(-a*e^2+c*d^2)/(e*x+d)
^4+1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/c/d/e^3/(e*x+d)^2-3/128*(-a
*e^2+c*d^2)^2*(-a^3*e^6-7*a^2*c*d^2*e^4-63*a*c^2*d^4*e^2+231*c^3*d^6)*arct
anh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/c^(5/2)/d^(5/2)/e^(13/2)
```

Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.74

$$\int \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^4}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(-15a^4e^8(d+ex)-10a^3cde^6(8d^2+7dex-e^2\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^4,
x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*((Sqrt[c]*Sqrt[d]*Sqrt[e]*(-15*a^4*e^8*(d +
 e*x) - 10*a^3*c*d*e^6*(8*d^2 + 7*d*e*x - e^2*x^2) + 2*a^2*c^2*d^2*e^4*(16
59*d^3 + 662*d^2*e*x - 233*d*e^2*x^2 + 124*e^3*x^3) - 2*a*c^3*d^3*e^2*(336
0*d^4 + 1197*d^3*e*x - 459*d^2*e^2*x^2 + 256*d*e^3*x^3 - 168*e^4*x^4) + c^
4*d^4*(3465*d^5 + 1155*d^4*e*x - 462*d^3*e^2*x^2 + 264*d^2*e^3*x^3 - 176*d
*e^4*x^4 + 128*e^5*x^5)))/(d + e*x) - (15*(c*d^2 - a*e^2)^2*(231*c^3*d^6 -
 63*a*c^2*d^4*e^2 - 7*a^2*c*d^2*e^4 - a^3*e^6)*ArcTanh[(Sqrt[e]*Sqrt[a*e +
 c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x
])))/(640*c^(5/2)*d^(5/2)*e^(13/2))
```

Rubi [A] (verified)

Time = 3.62 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.21, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1213, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(x \left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^4} \, dx$$

$$\Big\downarrow \text{1213}$$

$$\frac{2d^3 \left(cd^2-ae^2\right)^2 \sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}{e^6(d+ex)}-\frac{\int \frac{-c^3d^3x^5e^8+c^2d^2\left(cd^2-3ae^2\right)x^4e^7-cd\left(c^2d^4-3ace^2d^2+3a^2e^4\right)x^3e^6+\left(cd^2-ae^2\right)^3x^2e^5-d\left(cd^2-ae^2\right)^3xe^4+d^2\left(cd^2-ae^2\right)^3e^3}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^9}$$

$$\Big\downarrow \text{2192}$$

$$\frac{2d^3 \left(cd^2-ae^2\right)^2 \sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}{e^6(d+ex)}-\frac{\frac{\int \frac{c^3d^3\left(19cd^2-21ae^2\right)x^4e^8-2c^2d^2\left(5c^2d^4-19ace^2d^2+15a^2e^4\right)x^3e^7+10cd\left(cd^2-ae^2\right)^3x^2e^6-10cd^2\left(cd^2-ae^2\right)^3xe^5+10cd^3\left(cd^2-ae^2\right)^3e^4}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5cde}-\frac{1}{5}c^2d^2e^7x \cdots}{e^9}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d^3 \left(cd^2-ae^2\right)^2 \sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}{e^6(d+ex)}-\frac{\frac{\int \frac{c^3d^3\left(19cd^2-21ae^2\right)x^4e^8-2c^2d^2\left(5c^2d^4-19ace^2d^2+15a^2e^4\right)x^3e^7+10cd\left(cd^2-ae^2\right)^3x^2e^6-10cd^2\left(cd^2-ae^2\right)^3xe^5+10cd^3\left(cd^2-ae^2\right)^3e^4}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{10cde}-\frac{1}{5}c^2d^2e^7x \cdots}{e^9}$$

$$\Big\downarrow \text{2192}$$

$$\frac{2d^3 \left(cd^2-ae^2\right)^2 \sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}{e^6(d+ex)}-\frac{\frac{\frac{\int \frac{-3c^3d^3\left(71c^2d^4-106ace^2d^2+31a^2e^4\right)x^3e^8+2c^2d^2\left(40c^3d^6-177ac^2e^2d^4+183a^2ce^4d^2-40a^3e^6\right)x^2e^7-80c^2d^3\left(cd^2-ae^2\right)^3xe^6+80c^2d^4\left(cd^2-ae^2\right)^3e^5}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cde}+\frac{1}{4} \cdots}{10cde} \cdots}{e^9}$$

$\downarrow$ 27

$$\frac{2d^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^6(d+ex)}-\frac{\frac{\frac{\int \frac{-3c^3d^3\left(71c^2d^4-106ace^2d^2+31a^2e^4\right)x^3e^8+2c^2d^2\left(40c^3d^6-177ac^2e^2d^4+183a^2ce^4d^2-40a^3e^6\right)x^2e^7-80c^2d^3\left(cd^2-ae^2\right)^3xe^6+80c^2d^4\left(cd^2-ae^2\right)^3e^5}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}+\frac{1}{4}\cdots}{10cde}}{e^9}$$

$\downarrow$ 2192

$$\frac{2d^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^6(d+ex)}-\frac{\frac{\frac{\int \frac{3\left(c^3d^3\left(515c^3d^6-883ac^2e^2d^4+357a^2ce^4d^2-5a^3e^6\right)x^2e^8-4c^3d^4\left(40c^3d^6-191ac^2e^2d^4+226a^2ce^4d^2-71a^3e^6\right)xe^7+160c^3d^5\left(cd^2-ae^2\right)^3e^6\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3cde}-c^2d^2e^7x^2\cdots}{8cde}}{10cde}\cdots$$

$\downarrow$ 27

$$\frac{2d^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^6(d+ex)}-\frac{\frac{\frac{\int \frac{c^3d^3\left(515c^3d^6-883ac^2e^2d^4+357a^2ce^4d^2-5a^3e^6\right)x^2e^8-4c^3d^4\left(40c^3d^6-191ac^2e^2d^4+226a^2ce^4d^2-71a^3e^6\right)xe^7+160c^3d^5\left(cd^2-ae^2\right)^3e^6}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}-c^2d^2e^7x^2\left(31a\cdots\right.}{8cde}}{10cde}\cdots$$

$\downarrow$ 2192

$$\frac{2d^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^6(d+ex)}-\frac{\frac{\frac{\int \frac{c^3d^3e^7\left(2d\left(320c^4d^8-1475ac^3e^2d^6+1843a^2c^2e^4d^4-677a^3ce^6d^2+5a^4e^8\right)-e\left(2185c^4d^8-4160ac^3e^2d^6+2038a^2c^2e^4d^4-80a^3ce^6d^2-15a^4e^8\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}c^2d^2e^7\cdots}{2cde}}{8cde}\cdots$$

$\downarrow$ 27

$$\frac{2d^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^6(d+ex)}-\frac{\frac{\frac{1}{4}c^2d^2e^6\int \frac{2d\left(320c^4d^8-1475ac^3e^2d^6+1843a^2c^2e^4d^4-677a^3ce^6d^2+5a^4e^8\right)-e\left(2185c^4d^8-4160ac^3e^2d^6+2038a^2c^2e^4d^4-80a^3ce^6d^2-15a^4e^8\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{1}{2}c^2d^2e^7x\cdots}{2cde}}{8cde}\cdots$$

$\Big\downarrow$ 1160

$$\frac{2d^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^6(d+ex)}-$$

$$\frac{\frac{1}{4}c^2d^2e^6\left(\frac{15\left(cd^2-ae^2\right)^2\left(-a^3e^6-7a^2cd^2e^4-63ac^2d^4e^2+231c^3d^6\right)\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}-\frac{\left(-15a^4e^8-80a^3cd^2e^6+2038a^2c^2d^4e^4-4160ac^3d^6e^2+\right.}{cd}\right.}{2cde}$$

$\Big\downarrow$ 1092

$$\frac{2d^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^6(d+ex)}-$$

$$\frac{\frac{1}{4}c^2d^2e^6\left(\frac{15\left(cd^2-ae^2\right)^2\left(-a^3e^6-7a^2cd^2e^4-63ac^2d^4e^2+231c^3d^6\right)\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{cd}-\frac{\left(-15a^4e^8-80a^3cd^2e^6\right.}{}\right.}{2c}$$

$\Big\downarrow$ 219

$$\frac{2d^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^6(d+ex)}-$$

$$\frac{\frac{1}{2}c^2d^2e^7x\left(-5a^3e^6+357a^2cd^2e^4-883ac^2d^4e^2+515c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}+\frac{1}{4}c^2d^2e^6\left(\frac{15\left(cd^2-ae^2\right)^2\left(-a^3e^6-7a^2cd^2e^4-63ac^2d^4e^2+231c^3d^6\right)\text{ar}}{2c^{3/2}d^{3/2}\sqrt{}}\right.}{2cde}$$

input
```
Int[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^4,x]
```

output
```
(2*d^3*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^6
*(d + e*x)) - (-1/5*(c^2*d^2*e^7*x^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) + ((c^2*d^2*e^7*(19*c*d^2 - 21*a*e^2)*x^3*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/4 + (-(c^2*d^2*e^7*(71*c^2*d^4 - 106*a*c*d^2*e^2 + 31
*a^2*e^4)*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + ((c^2*d^2*e^7
*(515*c^3*d^6 - 883*a*c^2*d^4*e^2 + 357*a^2*c*d^2*e^4 - 5*a^3*e^6)*x*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/2 + (c^2*d^2*e^6*(-(((2185*c^4*d^8
 - 4160*a*c^3*d^6*e^2 + 2038*a^2*c^2*d^4*e^4 - 80*a^3*c*d^2*e^6 - 15*a^4*e
^8)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d)) + (15*(c*d^2 - a*e
^2)^2*(231*c^3*d^6 - 63*a*c^2*d^4*e^2 - 7*a^2*c*d^2*e^4 - a^3*e^6)*ArcTanh
[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(3/2)*d^(3/2)*Sqrt[e])))/4)/(2*c*d*e))/(
8*c*d*e))/(10*c*d*e))/e^9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1801 vs. $2(472) = 944$.

Time = 2.92 (sec) , antiderivative size = 1802, normalized size of antiderivative = 3.56

method	result	size
default	Expression too large to display	1802

input
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^4,x,method=_RETURN
VERBOSE)
```

output
```
1/e^4*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)
*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(
x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2
)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/
d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^
2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))))-3*d/e^5*(2/3/(a*e^2-c*d^2
)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)-10/3*d*e*c/(a*e^
2-c*d^2)*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d
^2)*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2
)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*
d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)
^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))))+3*d^2/e^6*(2/(a*e^2-c
*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)-8*d*e*c/(a*e
^2-c*d^2)*(2/3/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d
/e))^(7/2)-10/3*d*e*c/(a*e^2-c*d^2)*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x
+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d
*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/
4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/
e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/...
```

Fricas [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 1160, normalized size of antiderivative = 2.29

$$\int \frac{x^3(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorit
hm="fricas")
```

output
```
[-1/2560*(15*(231*c^5*d^11 - 525*a*c^4*d^9*e^2 + 350*a^2*c^3*d^7*e^4 - 50*
a^3*c^2*d^5*e^6 - 5*a^4*c*d^3*e^8 - a^5*d*e^10 + (231*c^5*d^10*e - 525*a*c
^4*d^8*e^3 + 350*a^2*c^3*d^6*e^5 - 50*a^3*c^2*d^4*e^7 - 5*a^4*c*d^2*e^9 -
a^5*e^11)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 +
 a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^
2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(128*c^5*d^5*e^6
*x^5 + 3465*c^5*d^10*e - 6720*a*c^4*d^8*e^3 + 3318*a^2*c^3*d^6*e^5 - 80*a^
3*c^2*d^4*e^7 - 15*a^4*c*d^2*e^9 - 16*(11*c^5*d^6*e^5 - 21*a*c^4*d^4*e^7)*
x^4 + 8*(33*c^5*d^7*e^4 - 64*a*c^4*d^5*e^6 + 31*a^2*c^3*d^3*e^8)*x^3 - 2*(
231*c^5*d^8*e^3 - 459*a*c^4*d^6*e^5 + 233*a^2*c^3*d^4*e^7 - 5*a^3*c^2*d^2*
e^9)*x^2 + (1155*c^5*d^9*e^2 - 2394*a*c^4*d^7*e^4 + 1324*a^2*c^3*d^5*e^6 -
 70*a^3*c^2*d^3*e^8 - 15*a^4*c*d*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x))/(c^3*d^3*e^8*x + c^3*d^4*e^7), 1/1280*(15*(231*c^5*d^11 - 525
*a*c^4*d^9*e^2 + 350*a^2*c^3*d^7*e^4 - 50*a^3*c^2*d^5*e^6 - 5*a^4*c*d^3*e^
8 - a^5*d*e^10 + (231*c^5*d^10*e - 525*a*c^4*d^8*e^3 + 350*a^2*c^3*d^6*e^5
 - 50*a^3*c^2*d^4*e^7 - 5*a^4*c*d^2*e^9 - a^5*e^11)*x)*sqrt(-c*d*e)*arctan
(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^
2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x
)) + 2*(128*c^5*d^5*e^6*x^5 + 3465*c^5*d^10*e - 6720*a*c^4*d^8*e^3 + 3318*
a^2*c^3*d^6*e^5 - 80*a^3*c^2*d^4*e^7 - 15*a^4*c*d^2*e^9 - 16*(11*c^5*d^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^4}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.02

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^4}\,dx = \frac{1}{640}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\left(2\left(\frac{8c^2d^2x}{e^2} - \frac{19c^6d}{}\right.\right.\right.\right.$$
$$+ \frac{2(c^3d^9 - 3ac^2d^7e^2 + 3a^2cd^5e^4 - a^3d^3e^6)}{\left(\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)e + \sqrt{cde}d\right)e^6}$$
$$+ \frac{3(231c^5d^{10} - 525ac^4d^8e^2 + 350a^2c^3d^6e^4 - 50a^3c^2d^4e^6 - 5a^4cd^2e^8 - a^5e^{10})\log\left(\left|cd^2 + ae^2 + 2\sqrt{cde}\left(\sqrt{}\right.\right.\right.}{256\sqrt{cde}c^2d^2e^6}$$

input
```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorit
hm="giac")
```

output
```
1/640*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(2*(8*c^2*d^2*x/e^
2 - (19*c^6*d^7*e^23 - 21*a*c^5*d^5*e^25)/(c^4*d^4*e^26))*x + (71*c^6*d^8*
e^22 - 106*a*c^5*d^6*e^24 + 31*a^2*c^4*d^4*e^26)/(c^4*d^4*e^26))*x - (515*
c^6*d^9*e^21 - 883*a*c^5*d^7*e^23 + 357*a^2*c^4*d^5*e^25 - 5*a^3*c^3*d^3*e
^27)/(c^4*d^4*e^26))*x + (2185*c^6*d^10*e^20 - 4160*a*c^5*d^8*e^22 + 2038*
a^2*c^4*d^6*e^24 - 80*a^3*c^3*d^4*e^26 - 15*a^4*c^2*d^2*e^28)/(c^4*d^4*e^2
6)) + 2*(c^3*d^9 - 3*a*c^2*d^7*e^2 + 3*a^2*c*d^5*e^4 - a^3*d^3*e^6)/(((sqr
t(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*e + sqrt(c*d*e)*
d)*e^6) + 3/256*(231*c^5*d^10 - 525*a*c^4*d^8*e^2 + 350*a^2*c^3*d^6*e^4 -
50*a^3*c^2*d^4*e^6 - 5*a^4*c*d^2*e^8 - a^5*e^10)*log(abs(c*d^2 + a*e^2 + 2
*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
))/(sqrt(c*d*e)*c^2*d^2*e^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^4}\,dx=\int \frac{x^3\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}{{\left(d+e\,x\right)}^4}\,dx$$

input
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^4,x)
```

output
```
int((x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^4, x)
```

Reduce [F]

$$\int \frac{x^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^4}\,dx=\int \frac{x^3(ade+(a\,e^2+c\,d^2)\,x+cde\,x^2)^{\frac{5}{2}}}{(ex+d)^4}dx$$

input
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x)
```

output
```
int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x)
```

3.62 $\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 403

$$\begin{aligned}
\int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^4}\,dx &= \\
&-\frac{(63c^2d^4-14acd^2e^2-a^2e^4)(ae+cdx)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24cde^3(cd^2-ae^2)} \\
&+\frac{(ae+cdx)^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4cde^2} \\
&-\frac{5(63c^2d^4-14acd^2e^2-a^2e^4)(3cd^2-5ae^2-2cdex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{192cde^5} \\
&+\frac{2d^2(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{e^2(cd^2-ae^2)(d+ex)^4} \\
&+\frac{5(cd^2-ae^2)^2(63c^2d^4-14acd^2e^2-a^2e^4)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{64c^{3/2}d^{3/2}e^{11/2}}
\end{aligned}$$

output
```
-1/24*(-a^2*e^4-14*a*c*d^2*e^2+63*c^2*d^4)*(c*d*x+a*e)^2*(a*d*e+(a*e^2+c*d
^2)*x+c*d*e*x^2)^(1/2)/c/d/e^3/(-a*e^2+c*d^2)+1/4*(c*d*x+a*e)^3*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^2-5/192*(-a^2*e^4-14*a*c*d^2*e^2+63*c^
2*d^4)*(-2*c*d*e*x-5*a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/c/d/e^5+2*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)/e^2/(-a*e^2+c*d^2)
/(e*x+d)^4+5/64*(-a*e^2+c*d^2)^2*(-a^2*e^4-14*a*c*d^2*e^2+63*c^2*d^4)*arct
anh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
))/c^(3/2)/d^(3/2)/e^(11/2)
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.79

$$\int \frac{x^2(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^4}\,dx = \frac{(cd^2-ae^2)^2\left((ae+cdx)(d+ex)\right)^{5/2}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{e}(15a^3e^6(d+ex)+a^2cde^4\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^4,
x]
```

output
```
((c*d^2 - a*e^2)^2*((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[c]*Sqrt[d]*Sqrt[
e]*(15*a^3*e^6*(d + e*x) + a^2*c*d*e^4*(-839*d^2 - 337*d*e*x + 118*e^2*x^2
) + a*c^2*d^2*e^2*(1785*d^3 + 637*d^2*e*x - 244*d*e^2*x^2 + 136*e^3*x^3) -
 3*c^3*d^3*(315*d^4 + 105*d^3*e*x - 42*d^2*e^2*x^2 + 24*d*e^3*x^3 - 16*e^4
*x^4)))/((c*d^2 - a*e^2)^2*(a*e + c*d*x)^2*(d + e*x)^3) + (15*(63*c^2*d^4
- 14*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*S
qrt[d]*Sqrt[d + e*x])])/((a*e + c*d*x)^(5/2)*(d + e*x)^(5/2))))/(192*c^(3/
2)*d^(3/2)*e^(11/2))
```

Rubi [A] (verified)

Time = 2.44 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.275, Rules used = {1213, 25, 2192, 27, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{1213}$$

$$-\frac{\int -\frac{c^3d^3x^4e^7-c^2d^2\left(cd^2-3ae^2\right)x^3e^6+cd\left(c^2d^4-3ace^2d^2+3a^2e^4\right)x^2e^5-\left(cd^2-ae^2\right)^3xe^4+d\left(cd^2-ae^2\right)^3e^3}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^8}-\frac{2d^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{c^3d^3x^4e^7-c^2d^2\left(cd^2-3ae^2\right)x^3e^6+cd\left(c^2d^4-3ace^2d^2+3a^2e^4\right)x^2e^5-\left(cd^2-ae^2\right)^3xe^4+d\left(cd^2-ae^2\right)^3e^3}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^8}-\frac{2d^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\int \frac{-c^3d^3\left(15cd^2-17ae^2\right)x^3e^7+2c^2d^2\left(4c^2d^4-15ace^2d^2+12a^2e^4\right)x^2e^6-8cd\left(cd^2-ae^2\right)^3xe^5+8cd^2\left(cd^2-ae^2\right)^3e^4}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cde}+\frac{1}{4}c^2d^2e^6x^3\sqrt{x\left(ae^2+cd^2\right)\ldots}}{e^8}$$
$$\frac{2d^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{-c^3d^3\left(15cd^2-17ae^2\right)x^3e^7+2c^2d^2\left(4c^2d^4-15ace^2d^2+12a^2e^4\right)x^2e^6-8cd\left(cd^2-ae^2\right)^3xe^5+8cd^2\left(cd^2-ae^2\right)^3e^4}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}+\frac{1}{4}c^2d^2e^6x^3\sqrt{x\left(ae^2+cd^2\right)\ldots}}{e^8}$$
$$\frac{2d^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{\frac{\int \frac{c^3d^3\left(123c^2d^4-190ace^2d^2+59a^2e^4\right)x^2e^7-4c^2d^2\left(12c^3d^6-51ac^2e^2d^4+53a^2ce^4d^2-12a^3e^6\right)xe^6+48c^2d^3\left(cd^2-ae^2\right)^3e^5}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3cde}-\frac{1}{3}c^2d^2e^6x^2\left(15cd^2-17ae^2\right)\sqrt{}}{8cde}}{e^8}$$

$$\frac{2d^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{c^3d^3\left(123c^2d^4-190ace^2d^2+59a^2e^4\right)x^2e^7-4c^2d^2\left(12c^3d^6-51ac^2e^2d^4+53a^2ce^4d^2-12a^3e^6\right)xe^6+48c^2d^3\left(cd^2-ae^2\right)^3e^5}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6cde}-\frac{1}{3}c^2d^2e^6x^2\left(15cd^2-17ae^2\right)\sqrt{}}{8cde}}{e^8}$$

$$\frac{2d^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{\frac{\frac{\int \frac{c^3d^3e^6\left(2d\left(96c^3d^6-411ac^2e^2d^4+478a^2ce^4d^2-155a^3e^6\right)-e\left(561c^3d^6-1017ac^2e^2d^4+455a^2ce^4d^2-15a^3e^6\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}c^2d^2e^6x\left(59a^2e^4-190acd^2e^2+123c^2d^4\right.}{6cde}}{8cde}}{e^8}$$

$$\frac{2d^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{1}{4}c^2d^2e^5\int \frac{2d\left(96c^3d^6-411ac^2e^2d^4+478a^2ce^4d^2-155a^3e^6\right)-e\left(561c^3d^6-1017ac^2e^2d^4+455a^2ce^4d^2-15a^3e^6\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{1}{2}c^2d^2e^6x\left(59a^2e^4-190acd^2e^2+123c^2d^4\right)}{6cde}}{8cde}}{e^8}$$

$$\frac{2d^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{\frac{1}{4}c^2d^2e^5\left(\frac{15\left(cd^2-ae^2\right)^2\left(-a^2e^4-14acd^2e^2+63c^2d^4\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}-\frac{\left(-15a^3e^6+455a^2cd^2e^4-1017ac^2d^4e^2+561c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ad}}{cd}\right.}{6cde}}{8cde}$$

$$\frac{2d^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

$\downarrow$ 1092

$$\frac{\frac{1}{4}c^2d^2e^5\left(\frac{15\left(cd^2-ae^2\right)^2\left(-a^2e^4-14acd^2e^2+63c^2d^4\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{cd}-\frac{\left(-15a^3e^6+455a^2cd^2e^4-1017ac^2d^4e\right.}{}\right.}{6cde}$$

$$\frac{2d^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

$\downarrow$ 219

$$\frac{\frac{1}{2}c^2d^2e^6x\left(59a^2e^4-190acd^2e^2+123c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}+\frac{1}{4}c^2d^2e^5\left(\frac{15\left(cd^2-ae^2\right)^2\left(-a^2e^4-14acd^2e^2+63c^2d^4\right)\text{arctanh}\left(\frac{ae^2+cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2-\right.}}\right.}{2c^{3/2}d^{3/2}\sqrt{e}}\right.}{6cde}$$

$$\frac{2d^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^5(d+ex)}$$

input
```
Int[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^4,x]
```

output
```
(-2*d^2*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^
5*(d + e*x)) + ((c^2*d^2*e^6*x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/4 + (-1/3*(c^2*d^2*e^6*(15*c*d^2 - 17*a*e^2)*x^2*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2]) + ((c^2*d^2*e^6*(123*c^2*d^4 - 190*a*c*d^2*e^2 + 5
9*a^2*e^4)*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/2 + (c^2*d^2*e^5
*(-(((561*c^3*d^6 - 1017*a*c^2*d^4*e^2 + 455*a^2*c*d^2*e^4 - 15*a^3*e^6)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c*d)) + (15*(c*d^2 - a*e^2)^2
*(63*c^2*d^4 - 14*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*
x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
])/(2*c^(3/2)*d^(3/2)*Sqrt[e])))/4)/(6*c*d*e))/(8*c*d*e))/e^8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1470 vs. $2(373) = 746$.

Time = 3.05 (sec), antiderivative size = 1471, normalized size of antiderivative = 3.65

method	result	size
default	Expression too large to display	1471

input
```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^4,x,method=_RETURN
VERBOSE)
```

output
```
1/e^4*(2/3/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(7/2)-10/3*d*e*c/(a*e^2-c*d^2)*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e
))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c
*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2
*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^
(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*
e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))))
)+d^2/e^6*(-2/(a*e^2-c*d^2)/(x+d/e)^4*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/
e))^(7/2)+6*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^
2+(a*e^2-c*d^2)*(x+d/e))^(7/2)-8*d*e*c/(a*e^2-c*d^2)*(2/3/(a*e^2-c*d^2)/(x
+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)-10/3*d*e*c/(a*e^2-c*
d^2)*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*
(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x
+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)
/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d
/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))))))-2*d/e^5*(2/(a*e^2-c*d^2
)/(x+d/e)^3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)-8*d*e*c/(a*e^2-c
*d^2)*(2/3/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(7/2)-10/3*d*e*c/(a*e^2-c*d^2)*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+...
```

Fricas [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 936, normalized size of antiderivative = 2.32

$$\int \frac{x^2(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorit
hm="fricas")
```

output
```
[-1/768*(15*(63*c^4*d^9 - 140*a*c^3*d^7*e^2 + 90*a^2*c^2*d^5*e^4 - 12*a^3*
c*d^3*e^6 - a^4*d*e^8 + (63*c^4*d^8*e - 140*a*c^3*d^6*e^3 + 90*a^2*c^2*d^4
*e^5 - 12*a^3*c*d^2*e^7 - a^4*e^9)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 +
c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3
)*x) - 4*(48*c^4*d^4*e^5*x^4 - 945*c^4*d^8*e + 1785*a*c^3*d^6*e^3 - 839*a^
2*c^2*d^4*e^5 + 15*a^3*c*d^2*e^7 - 8*(9*c^4*d^5*e^4 - 17*a*c^3*d^3*e^6)*x^
3 + 2*(63*c^4*d^6*e^3 - 122*a*c^3*d^4*e^5 + 59*a^2*c^2*d^2*e^7)*x^2 - (315
*c^4*d^7*e^2 - 637*a*c^3*d^5*e^4 + 337*a^2*c^2*d^3*e^6 - 15*a^3*c*d*e^8)*x
)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^7*x + c^2*d^3*e^
6), -1/384*(15*(63*c^4*d^9 - 140*a*c^3*d^7*e^2 + 90*a^2*c^2*d^5*e^4 - 12*a
^3*c*d^3*e^6 - a^4*d*e^8 + (63*c^4*d^8*e - 140*a*c^3*d^6*e^3 + 90*a^2*c^2*
d^4*e^5 - 12*a^3*c*d^2*e^7 - a^4*e^9)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e
)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(48*c^4
*d^4*e^5*x^4 - 945*c^4*d^8*e + 1785*a*c^3*d^6*e^3 - 839*a^2*c^2*d^4*e^5 +
15*a^3*c*d^2*e^7 - 8*(9*c^4*d^5*e^4 - 17*a*c^3*d^3*e^6)*x^3 + 2*(63*c^4*d^
6*e^3 - 122*a*c^3*d^4*e^5 + 59*a^2*c^2*d^2*e^7)*x^2 - (315*c^4*d^7*e^2 - 6
37*a*c^3*d^5*e^4 + 337*a^2*c^2*d^3*e^6 - 15*a^3*c*d*e^8)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*e^7*x + c^2*d^3*e^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^4}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.05

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^4}\,dx = \frac{1}{192}\sqrt{cdex^2 + cd^2x + ae^2x + ade}\left(2\left(4\left(\frac{6c^2d^2x}{e^2} - \frac{15c^5d^6e^{16}}{c^{\cdots}}\right.\right.\right.$$
$$- \frac{2(c^3d^8 - 3ac^2d^6e^2 + 3a^2cd^4e^4 - a^3d^2e^6)}{\left(\left(\sqrt{cde}x - \sqrt{cdex^2 + cd^2x + ae^2x + ade}\right)e + \sqrt{cde}d\right)e^5}$$
$$- \frac{5(63c^4d^8 - 140ac^3d^6e^2 + 90a^2c^2d^4e^4 - 12a^3cd^2e^6 - a^4e^8)\log\left(\left|cd^2 + ae^2 + 2\sqrt{cde}\left(\sqrt{cde}x - \sqrt{cdex^2}\cdots\right.\right.\right.}{128\sqrt{cde}cde^5}$$

input
```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorit
hm="giac")
```

output
```
1/192*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*(6*c^2*d^2*x/e^2 -
 (15*c^5*d^6*e^16 - 17*a*c^4*d^4*e^18)/(c^3*d^3*e^19))*x + (123*c^5*d^7*e^
15 - 190*a*c^4*d^5*e^17 + 59*a^2*c^3*d^3*e^19)/(c^3*d^3*e^19))*x - (561*c^
5*d^8*e^14 - 1017*a*c^4*d^6*e^16 + 455*a^2*c^3*d^4*e^18 - 15*a^3*c^2*d^2*e
^20)/(c^3*d^3*e^19)) - 2*(c^3*d^8 - 3*a*c^2*d^6*e^2 + 3*a^2*c*d^4*e^4 - a^
3*d^2*e^6)/(((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
*e + sqrt(c*d*e)*d)*e^5) - 5/128*(63*c^4*d^8 - 140*a*c^3*d^6*e^2 + 90*a^2*
c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 - a^4*e^8)*log(abs(c*d^2 + a*e^2 + 2*sqrt(c
*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqr
t(c*d*e)*c*d*e^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^4} dx = \int \frac{x^2 (c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,x + a\,d\,e)^{5/2}}{(d + e\,x)^4} dx$$

input
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^4,x)
```

output
```
int((x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^4, x)
```

Reduce [F]

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^4} dx = \int \frac{x^2(ade + (a\,e^2 + c\,d^2)\,x + cde\,x^2)^{\frac{5}{2}}}{(ex + d)^4} dx$$

input
```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x)
```

output
```
int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x)
```

3.63 $\int \frac{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 295

$$\int \frac{x(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{(d+ex)^4}\,dx = \frac{(7cd^2-ae^2)\,(ae+cdx)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3e^2\,(cd^2-ae^2)}$$
$$+\frac{5(7cd^2-ae^2)\,(3cd^2-5ae^2-2cdex)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24e^4}$$
$$-\frac{2d(ade+(cd^2+ae^2)\,x+cdex^2)^{7/2}}{e\,(cd^2-ae^2)\,(d+ex)^4}$$
$$-\frac{5(cd^2-ae^2)^2\,(7cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8\sqrt{c}\sqrt{d}e^{9/2}}$$

output

```
1/3*(-a*e^2+7*c*d^2)*(c*d*x+a*e)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/e^2/(-a*e^2+c*d^2)+5/24*(-a*e^2+7*c*d^2)*(-2*c*d*e*x-5*a*e^2+3*c*d^2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^4-2*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(7/2)/e/(-a*e^2+c*d^2)/(e*x+d)^4-5/8*(-a*e^2+c*d^2)^2*(-a*e^2+7*c*d^2
)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2))/c^(1/2)/d^(1/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.78

$$\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^4}\,dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{e}(3a^2e^4(27d+11ex)-2acde^2(95d^2+34dex-13e^2 \ldots}{(ae+cdx)^2(\ldots}\right.}{\ldots}$$

input
```
Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^4,x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[e]*(3*a^2*e^4*(27*d + 11*e*x) - 2*
a*c*d*e^2*(95*d^2 + 34*d*e*x - 13*e^2*x^2) + c^2*d^2*(105*d^3 + 35*d^2*e*x
 - 14*d*e^2*x^2 + 8*e^3*x^3)))/((a*e + c*d*x)^2*(d + e*x)^3) - (15*(c*d^2
- a*e^2)^2*(7*c*d^2 - a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*
Sqrt[d]*Sqrt[d + e*x])])/(Sqrt[c]*Sqrt[d]*(a*e + c*d*x)^(5/2)*(d + e*x)^(5
/2))))/(24*e^(9/2))
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1213, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{(d+ex)^4}dx$$

$$\Big\downarrow \ \color{blue}{1213}$$

$$\frac{2d\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-\frac{\int \frac{-c^3d^3x^3e^6+c^2d^2\left(cd^2-3ae^2\right)x^2e^5-cd\left(c^2d^4-3ace^2d^2+3a^2e^4\right)xe^4+\left(cd^2-ae^2\right)^3e^3}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^7}$$

$$\Big\downarrow \ \color{blue}{2192}$$

$$\frac{\frac{2d\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-\frac{\frac{\int\frac{c^3d^3\left(11cd^2-13ae^2\right)x^2e^6-2c^2d^2\left(3c^2d^4-11ace^2d^2+9a^2e^4\right)xe^5+6cd\left(cd^2-ae^2\right)^3e^4}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3cde}-\frac{1}{3}c^2d^2e^5x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^7}}{}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-\frac{\frac{\int\frac{c^3d^3\left(11cd^2-13ae^2\right)x^2e^6-2c^2d^2\left(3c^2d^4-11ace^2d^2+9a^2e^4\right)xe^5+6cd\left(cd^2-ae^2\right)^3e^4}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6cde}-\frac{1}{3}c^2d^2e^5x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^7}$$

$$\Big\downarrow \text{2192}$$

$$\frac{2d\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-\frac{\frac{\frac{\int\frac{c^2d^2e^5\left(2\left(12c^3d^6-47ac^2e^2d^4+49a^2ce^4d^2-12a^3e^6\right)-cde\left(57c^2d^4-94ace^2d^2+33a^2e^4\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}c^2d^2e^5x\left(11cd^2-13ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6cde}-}{e^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-\frac{\frac{\frac{1}{4}cde^4\int\frac{2\left(12c^3d^6-47ac^2e^2d^4+49a^2ce^4d^2-12a^3e^6\right)-cde\left(57c^2d^4-94ace^2d^2+33a^2e^4\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{1}{2}c^2d^2e^5x\left(11cd^2-13ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6cde}-\frac{1}{3}}{e^7}$$

$$\Big\downarrow \text{1160}$$

$$\frac{2d\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^4(d+ex)}-\frac{\frac{\frac{1}{4}cde^4\left(\frac{15}{2}\left(cd^2-ae^2\right)^2\left(7cd^2-ae^2\right)\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\left(33a^2e^4-94acd^2e^2+57c^2d^4\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}c^2d^2e^5x\left(11cd^2\right.}{6cde}}{e^7}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2d(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^4(d+ex)}-\frac{\frac{\frac{1}{4}cde^4\left(15(cd^2-ae^2)^2(7cd^2-ae^2)\int\frac{1}{4cde-\frac{(cd^2+2cexd+ae^2)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-(33a^2e^4-94acd^2e^2+57c^2d^4)\sqrt{x(ae^2+cd^2)+ade}\right.}{6cde}}{e^7}$$

$$\Big\downarrow \; 219$$

$$\frac{2d(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^4(d+ex)}-\frac{\frac{\frac{1}{4}cde^4\left(\frac{15(cd^2-ae^2)^2(7cd^2-ae^2)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}-(33a^2e^4-94acd^2e^2+57c^2d^4)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+}{6cde}}{e^7}$$

input
```
Int[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^4,x]
```

output
```
(2*d*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^4*(
d + e*x)) - (-1/3*(c^2*d^2*e^5*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2]) + ((c^2*d^2*e^5*(11*c*d^2 - 13*a*e^2)*x*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])/2 + (c*d*e^4*(-((57*c^2*d^4 - 94*a*c*d^2*e^2 + 33*a^2*e^4
)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (15*(c*d^2 - a*e^2)^2*(7*
c*d^2 - a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt
[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*Sqrt
[e])))/4)/(6*c*d*e))/e^7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1060 vs. $2(269) = 538$.

Time = 3.12 (sec) , antiderivative size = 1061, normalized size of antiderivative = 3.60

method	result	size
default	Expression too large to display	1061

input
```
int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^4,x,method=_RETURNVE
RBOSE)
```

output
```
1/e^4*(2/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(
7/2)-8*d*e*c/(a*e^2-c*d^2)*(2/3/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(
a*e^2-c*d^2)*(x+d/e))^(7/2)-10/3*d*e*c/(a*e^2-c*d^2)*(1/5*(d*e*c*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8*(2*d*e*c*(x+d/e)+a*e^
2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c
*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d
^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1
/2))/(d*e*c)^(1/2))))))-d/e^5*(-2/(a*e^2-c*d^2)/(x+d/e)^4*(d*e*c*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(7/2)+6*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)/(x+d/
e)^3*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)-8*d*e*c/(a*e^2-c*d^2)*(
2/3/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)-
10/3*d*e*c/(a*e^2-c*d^2)*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2
)+1/2*(a*e^2-c*d^2)*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e
)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*
(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1
/8*(a*e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1
/2)+(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))))))
```

Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 724, normalized size of antiderivative = 2.45

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^4}\,dx = \left[-\frac{15\,(7\,c^3d^7 - 15\,ac^2d^5e^2 + 9\,a^2cd^3e^4 - a^3de^6 + (7\,c^3d^6e - 15\,c\right.$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorithm
="fricas")
```

output
```
[-1/96*(15*(7*c^3*d^7 - 15*a*c^2*d^5*e^2 + 9*a^2*c*d^3*e^4 - a^3*d*e^6 + (
7*c^3*d^6*e - 15*a*c^2*d^4*e^3 + 9*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c*d*e)
*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^4*x^3 + 105*c^3*d^6*e - 190
*a*c^2*d^4*e^3 + 81*a^2*c*d^2*e^5 - 2*(7*c^3*d^4*e^3 - 13*a*c^2*d^2*e^5)*x
^2 + (35*c^3*d^5*e^2 - 68*a*c^2*d^3*e^4 + 33*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e^6*x + c*d^2*e^5), 1/48*(15*(7*c^3*d
^7 - 15*a*c^2*d^5*e^2 + 9*a^2*c*d^3*e^4 - a^3*d*e^6 + (7*c^3*d^6*e - 15*a*
c^2*d^4*e^3 + 9*a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*
d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(8*c
^3*d^3*e^4*x^3 + 105*c^3*d^6*e - 190*a*c^2*d^4*e^3 + 81*a^2*c*d^2*e^5 - 2*
(7*c^3*d^4*e^3 - 13*a*c^2*d^2*e^5)*x^2 + (35*c^3*d^5*e^2 - 68*a*c^2*d^3*e^
4 + 33*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c*d*e
^6*x + c*d^2*e^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^4} dx = \text{Timed out}$$

input
```
integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^4} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.24

$$\int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^4}\,dx = \frac{1}{24}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(\frac{4c^2d^2x}{e^2}-\frac{11c^4d^5e^{10}-13\right.\right.$$
$$+\frac{5(7c^3d^6-15ac^2d^4e^2+9a^2cd^2e^4-a^3e^6)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+}\right.\right.\right.}{16\sqrt{cde}e^4}$$
$$+\frac{2\left(\sqrt{cde}c^3d^7-3\sqrt{cde}ac^2d^5e^2+3\sqrt{cde}a^2cd^3e^4-\sqrt{cde}a^3de^6\right)}{\sqrt{cde}\left(\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)e+\sqrt{cde}d\right)e^4}$$

input
```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorithm
="giac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*c^2*d^2*x/e^2 - (11
*c^4*d^5*e^10 - 13*a*c^3*d^3*e^12)/(c^2*d^2*e^13))*x + (57*c^4*d^6*e^9 - 9
4*a*c^3*d^4*e^11 + 33*a^2*c^2*d^2*e^13)/(c^2*d^2*e^13)) + 5/16*(7*c^3*d^6
- 15*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - a^3*e^6)*log(abs(-c*d^2 - a*e^2 - 2
*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))
))/(sqrt(c*d*e)*e^4) + 2*(sqrt(c*d*e)*c^3*d^7 - 3*sqrt(c*d*e)*a*c^2*d^5*e^
2 + 3*sqrt(c*d*e)*a^2*c*d^3*e^4 - sqrt(c*d*e)*a^3*d*e^6)/(sqrt(c*d*e)*((sq
rt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*e + sqrt(c*d*e)
*d)*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^4}\,dx = \int \frac{x\,(c\,d\,e\,x^2 + (c\,d^2 + a\,e^2)\,\,x + a\,d\,e)^{5/2}}{(d + e\,x)^4}\,dx$$

input
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^4,x)
```

output
```
int((x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2))/(d + e*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 862, normalized size of antiderivative = 2.92

$$\int \frac{x(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^4}\,dx = \text{Too large to display}$$

input
```
int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x)
```

output

```
(648*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d**2*e**5 + 264*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**2*c*d*e**6*x - 1520*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
*c**2*d**4*e**3 - 544*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**3*e**4*x +
 208*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**2*e**5*x**2 + 840*sqrt(d +
e*x)*sqrt(a*e + c*d*x)*c**3*d**6*e + 280*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c
**3*d**5*e**2*x - 112*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**4*e**3*x**2
+ 64*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**3*e**4*x**3 + 120*sqrt(e)*sqr
t(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x
))/sqrt(a*e**2 - c*d**2))*a**3*d*e**6 + 120*sqrt(e)*sqrt(d)*sqrt(c)*log((s
qrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*
d**2))*a**3*e**7*x - 1080*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e +
c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**3
*e**4 - 1080*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**5*x + 180
0*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)
*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**5*e**2 + 1800*sqrt(e)*sqr
t(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x
))/sqrt(a*e**2 - c*d**2))*a*c**2*d**4*e**3*x - 840*sqrt(e)*sqrt(d)*sqrt(c)
*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e*
*2 - c*d**2))*c**3*d**7 - 840*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt...
```

3.64 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 183

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{(d+ex)^4}\,dx = -\frac{5cd\left(3cd^2-5ae^2-2cdex\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4e^3} - \frac{2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{e(d+ex)^3} + \frac{15\sqrt{c}\sqrt{d}\left(cd^2-ae^2\right)^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4e^{7/2}}$$

output
```
-5/4*c*d*(-2*c*d*e*x-5*a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/e^3-2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/e/(e*x+d)^3+15/4*c^(1/2)
*d^(1/2)*(-a*e^2+c*d^2)^2*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^4}\,dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{e}(-8a^2e^4+acde^2(25d+9ex)+c^2d^2(-15d^2-5dex+}{(ae+cdx)^2(d+ex)^3}\right.}{4e^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^4,x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[e]*(-8*a^2*e^4 + a*c*d*e^2*(25*d +
 9*e*x) + c^2*d^2*(-15*d^2 - 5*d*e*x + 2*e^2*x^2)))/((a*e + c*d*x)^2*(d +
e*x)^3) + (15*Sqrt[c]*Sqrt[d]*(c*d^2 - a*e^2)^2*ArcTanh[(Sqrt[c]*Sqrt[d]*S
qrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/((a*e + c*d*x)^(5/2)*(d + e*x)
^(5/2))))/(4*e^(7/2))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.40, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1125, 25, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{(d+ex)^4}dx$$

$$\Big\downarrow \; 1125$$

$$-\frac{\int -\frac{c^3d^3x^2e^5-c^2d^2(cd^2-3ae^2)xe^4+cd(c^2d^4-3ace^2d^2+3a^2e^4)e^3}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^6} - \frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{c^3d^3x^2e^5-c^2d^2(cd^2-3ae^2)xe^4+cd(c^2d^4-3ace^2d^2+3a^2e^4)e^3}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^6}-\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}}{}$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{\int \frac{c^2d^2e^4\left(2\left(2cd^2-3ae^2\right)\left(cd^2-2ae^2\right)-cde\left(7cd^2-9ae^2\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}c^2d^2e^4x\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^6}-\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{4}cde^3\int \frac{2(2cd^2-3ae^2)(cd^2-2ae^2)-cde(7cd^2-9ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{1}{2}c^2d^2e^4x\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^6}-\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{1}{4}cde^3\left(\frac{15}{2}(cd^2-ae^2)^2\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-(7cd^2-9ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}c^2d^2e^4x\sqrt{}}{e^6}-\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{1}{4}cde^3\left(15(cd^2-ae^2)^2\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-(7cd^2-9ae^2)\sqrt{x(ae^2+cd^2)+ade}\right.}{e^6}-\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{4}cde^3\left(\frac{15(cd^2-ae^2)^2\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}-(7cd^2-9ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}\right)+\frac{1}{2}c}{e^6}$$

$$\frac{2(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^3(d+ex)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x)^4,x]
```

output
```
(-2*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^3*(d
 + e*x)) + ((c^2*d^2*e^4*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/2
+ (c*d*e^3*(-((7*c*d^2 - 9*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]) + (15*(c*d^2 - a*e^2)^2*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c
]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c
]*Sqrt[d]*Sqrt[e])))/4)/e^6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(161) = 322$.

Time = 3.12 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.11

method	result
	$6dec\left(\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^3}-\frac{8dec\left(\frac{2\left(dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{3\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^2}-\right.}{}\right.$ 1

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/(e*x+d)^4,x,method=_RETURNVERB
OSE)
```

output
```
1/e^4*(-2/(a*e^2-c*d^2)/(x+d/e)^4*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^
(7/2)+6*d*e*c/(a*e^2-c*d^2)*(2/(a*e^2-c*d^2)/(x+d/e)^3*(d*e*c*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(7/2)-8*d*e*c/(a*e^2-c*d^2)*(2/3/(a*e^2-c*d^2)/(x+d/e
)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)-10/3*d*e*c/(a*e^2-c*d^2)
*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*(a*e^2-c*d^2)*(1/8
*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e
))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e
/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*e^2-c*d^2)^2/d/e/c
*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d*e*c*(x+d/e)^2+(a*
e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2)))))))
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.03

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^4}\,dx = \left[\frac{15\,(c^2d^5-2\,acd^3e^2+a^2de^4+(c^2d^4e-2\,acd^2e^3+a^2e^5)x)\sqrt{\frac{cd}{e}}\ \cdots}{\cdots}\right.$$

$$-\frac{15\,(c^2d^5-2\,acd^3e^2+a^2de^4+(c^2d^4e-2\,acd^2e^3+a^2e^5)x)\sqrt{-\frac{cd}{e}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+\cdots}{2\,(c^2d^2ex^2+acd^2e+(c^2d^3+acde^2)\cdots}\right.}{8\,(e\cdots}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorithm="
fricas")
```

output
```
[1/16*(15*(c^2*d^5 - 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e - 2*a*c*d^2*e^
3 + a^2*e^5)*x)*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(2*c^2*d^
2*e^2*x^2 - 15*c^2*d^4 + 25*a*c*d^2*e^2 - 8*a^2*e^4 - (5*c^2*d^3*e - 9*a*c
*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(e^4*x + d*e^3), -
1/8*(15*(c^2*d^5 - 2*a*c*d^3*e^2 + a^2*d*e^4 + (c^2*d^4*e - 2*a*c*d^2*e^3
+ a^2*e^5)*x)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*
e + (c^2*d^3 + a*c*d*e^2)*x)) - 2*(2*c^2*d^2*e^2*x^2 - 15*c^2*d^4 + 25*a*c
*d^2*e^2 - 8*a^2*e^4 - (5*c^2*d^3*e - 9*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x))/(e^4*x + d*e^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^4}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{(d + ex)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.65

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^4}\,dx = \frac{1}{4}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(\frac{2\,c^2d^2x}{e^2}-\frac{7\,c^3d^4e^5-9\,ac^2d^2e^7}{cde^8}\right)$$
$$-\frac{15\,(c^3d^5-2\,ac^2d^3e^2+a^2cde^4)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{8\,\sqrt{cde}e^3}$$
$$-\frac{2\left(\sqrt{cde}c^3d^6-3\,\sqrt{cde}ac^2d^4e^2+3\,\sqrt{cde}a^2cd^2e^4-\sqrt{cde}a^3e^6\right)}{\sqrt{cde}\left(\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)e+\sqrt{cde}d\right)e^3}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorithm="
giac")
```

output
```
1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*c^2*d^2*x/e^2 - (7*c^3*
d^4*e^5 - 9*a*c^2*d^2*e^7)/(c*d*e^8)) - 15/8*(c^3*d^5 - 2*a*c^2*d^3*e^2 +
a^2*c*d*e^4)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(
c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*e^3) - 2*(sqrt(c*d*
e)*c^3*d^6 - 3*sqrt(c*d*e)*a*c^2*d^4*e^2 + 3*sqrt(c*d*e)*a^2*c*d^2*e^4 - s
qrt(c*d*e)*a^3*e^6)/(sqrt(c*d*e)*((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*
x + a*e^2*x + a*d*e))*e + sqrt(c*d*e)*d)*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{{\left(d+e\,x\right)}^4}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^4,x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(d + e*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 597, normalized size of antiderivative = 3.26

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^4}\,dx = \frac{-8\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2e^5+25\sqrt{ex+d}\,\sqrt{cdx+ae}\,ac\,d^2e^3+9\cdots}{}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x)
```

output
```
( - 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*e**5 + 25*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a*c*d**2*e**3 + 9*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d*e**4*x
- 15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**4*e - 5*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*c**2*d**3*e**2*x + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**2*
e**3*x**2 + 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*d*e**4 + 15*sqrt(
e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**5*x - 30*sqrt(e)*sqrt(d)*sqrt(c)*l
og((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2
 - c*d**2))*a*c*d**3*e**2 - 30*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a
*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d*
*2*e**3*x + 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**5 + 15*sqrt(e)
*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d +
 e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**4*e*x - 10*sqrt(e)*sqrt(d)*sqrt(c)*a
**2*d*e**4 - 10*sqrt(e)*sqrt(d)*sqrt(c)*a**2*e**5*x + 20*sqrt(e)*sqrt(d)*s
qrt(c)*a*c*d**3*e**2 + 20*sqrt(e)*sqrt(d)*sqrt(c)*a*c*d**2*e**3*x - 10*sqr
t(e)*sqrt(d)*sqrt(c)*c**2*d**5 - 10*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**4*e*x)
/(4*e**4*(d + e*x))
```

3.65 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x(d+ex)^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 253

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{x(d+ex)^4}\,dx = \frac{c(3cd^2-2ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e^2} - \frac{2\left(\frac{cd}{e}-\frac{ae}{d}\right)(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{(d+ex)^2} - \frac{2a^{5/2}e^{5/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{a}\sqrt{e}(d+ex)}\right)}{d^{3/2}} - \frac{c^{3/2}d^{3/2}(3cd^2-5ae^2)\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{e^{5/2}}$$

output
```
c*(-2*a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^2-2*(c*d/e-
a*e/d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^2-2*a^(5/2)*e^(5/2)
*arctanh(d^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^(1/2)/e^(1/2)/(
e*x+d))/d^(3/2)-c^(3/2)*d^(3/2)*(-5*a*e^2+3*c*d^2)*arctanh(e^(1/2)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.93

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x(d+ex)^4}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\Big(\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(-4acd^2e^2+2a$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)^4),x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d +
 e*x]*(-4*a*c*d^2*e^2 + 2*a^2*e^4 + c^2*d^3*(3*d + e*x)) - 2*a^(5/2)*e^5*(
d + e*x)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x
])] - c^(3/2)*d^3*(3*c*d^2 - 5*a*e^2)*(d + e*x)*ArcTanh[(Sqrt[e]*Sqrt[a*e
+ c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(d^(3/2)*e^(5/2)*Sqrt[a*e + c
*d*x]*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {1214, 25, 2184, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x(d+ex)^4}dx$$

$$\Big\downarrow \text{ 1214}$$

$$\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{de^2(d+ex)}-\frac{\int -\frac{\frac{a^3e^9}{d}+c^3d^3x^2e^5-c^2d^2\left(cd^2-3ae^2\right)xe^4}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^6}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\frac{a^3e^9}{d}+c^3d^3x^2e^5-c^2d^2\left(cd^2-3ae^2\right)xe^4}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^6}+\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{de^2(d+ex)}$$

$$\Big\downarrow \text{2184}$$

$$\frac{\frac{\frac{\int \frac{ce^5\left(2a^3e^5-c^2d^3\left(3cd^2-5ae^2\right)x\right)}{2x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cde}+c^2d^2e^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^6}+\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{de^2(d+ex)}}{}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{e^4\int \frac{2a^3e^5-c^2d^3\left(3cd^2-5ae^2\right)x}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d}+c^2d^2e^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^6}+\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{de^2(d+ex)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{e^4\left(2a^3e^5\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-c^2d^3\left(3cd^2-5ae^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx\right)}{2d}+c^2d^2e^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^6}+\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{de^2(d+ex)}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{e^4\left(2a^3e^5\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-2c^2d^3\left(3cd^2-5ae^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\right)}{2d}+c^2d^2e^4\sqrt{x\left(a\right.}}{e^6}+\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{de^2(d+ex)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{e^4\left(2a^3e^5\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{c^{3/2}d^{5/2}\left(3cd^2-5ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{e}}\right)}{2d}+c^2d^2e^4\sqrt{x\left(ae^2+c\ldots\right.}}{e^6}$$

$$\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{de^2(d+ex)}$$

$\downarrow$ 1154

$$\frac{\frac{e^4\left(-4a^3e^5\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}-\frac{c^{3/2}d^{5/2}\left(3cd^2-5ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{e}}\right)}{2d}}{e^6}$$

$$\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{de^2(d+ex)}$$

$\downarrow$ 219

$$\frac{\frac{e^4\left(-\frac{2a^{5/2}e^{9/2}\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{d}}-\frac{c^{3/2}d^{5/2}\left(3cd^2-5ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{e}}\right)}{2d}+c^2\ldots}{e^6}$$

$$\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{de^2(d+ex)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x*(d + e*x)^4),x]
```

output
```
(2*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d*e^2*(
d + e*x)) + (c^2*d^2*e^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2] + (e^
4*(-((c^(3/2)*d^(5/2)*(3*c*d^2 - 5*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e
*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)])/Sqrt[e]) - (2*a^(5/2)*e^(9/2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2
*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sq
rt[d]))/(2*d))/e^6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2472 vs. $2(221) = 442$.

Time = 3.62 (sec) , antiderivative size = 2473, normalized size of antiderivative = 9.77

method	result	size
default	Expression too large to display	2473

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x/(e*x+d)^4,x,method=_RETURNVE
RBOSE)
```

output
```
1/d^4*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)+1/2*(a*e^2+c*d^2)*(1/8*
(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16
*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/
c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2))/(d*e*c)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln((
2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^
(1/2))/x))))-1/d^4*(1/5*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)+1/2*
(a*e^2-c*d^2)*(1/8*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(3/2)-3/16*(a*e^2-c*d^2)^2/d/e/c*(1/4*(2*d*e*c*(x+d/e
)+a*e^2-c*d^2)/d/e/c*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/8*(a*
e^2-c*d^2)^2/d/e/c*ln((1/2*a*e^2-1/2*c*d^2+d*e*c*(x+d/e))/(d*e*c)^(1/2)+(d
*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(d*e*c)^(1/2))))-1/e/d^3*(2/3
/(a*e^2-c*d^2)/(x+d/e)^2*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(7/2)-...
```

Fricas [A] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 1509, normalized size of antiderivative = 5.96

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x(d + ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d)^4,x, algorithm
="fricas")
```

output
```
[-1/4*((3*c^2*d^5 - 5*a*c*d^3*e^2 + (3*c^2*d^4*e - 5*a*c*d^2*e^3)*x)*sqrt(
c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*
d*e^2*x + c*d^2*e + a*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(c*d/e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 2*(a^2*e^5*x + a^2*d*e^4)*sqrt(a
*e/d)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)
*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(c^2*d^3*e*x + 3*c^2*
d^4 - 4*a*c*d^2*e^2 + 2*a^2*e^4)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x))/(d*e^3*x + d^2*e^2), 1/2*((3*c^2*d^5 - 5*a*c*d^3*e^2 + (3*c^2*d^4*e -
5*a*c*d^2*e^3)*x)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*
d^2*e + (c^2*d^3 + a*c*d*e^2)*x)) + (a^2*e^5*x + a^2*d*e^4)*sqrt(a*e/d)*lo
g((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*
e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 2*(c^2*d^3*e*x + 3*c^2*d^4 - 4*
a*c*d^2*e^2 + 2*a^2*e^4)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(d*e
^3*x + d^2*e^2), 1/4*(4*(a^2*e^5*x + a^2*d*e^4)*sqrt(-a*e/d)*arctan(1/2*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sq
rt(-a*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) - (3*c^2
*d^5 - 5*a*c*d^3*e^2 + (3*c^2*d^4*e - 5*a*c*d^2*e^3)*x)*sqrt(c*d/e)*log...
```

Sympy [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x(d+ex)^4}\,dx = \int \frac{((d+ex)\,(ae+cdx))^{\frac{5}{2}}}{x\,(d+ex)^4}\,dx$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x/(e*x+d)**4,x)
```

output
```
Integral(((d + e*x)*(a*e + c*d*x))**(5/2)/(x*(d + e*x)**4), x)
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x(d + ex)^4}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^4 x}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d)^4,x, algorithm
="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^4*x), x
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x(d + ex)^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d)^4,x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x(d + ex)^4}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x\,{\left(d + e\,x\right)}^4}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x*(d + e*x)^4),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x*(d + e*x)^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 791, normalized size of antiderivative = 3.13

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x(d + ex)^4}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x/(e*x+d)^4,x)
```

output
```
(8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*d*e**5 - 16*sqrt(d + e*x)*sqrt(a*e
 + c*d*x)*a*c*d**3*e**3 + 12*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**5*e +
 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**4*e**2*x + 4*sqrt(e)*sqrt(d)*sq
rt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2
+ c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*d*e**5 + 4*sqrt(e)*sqrt(d)
*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e*
*2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*e**6*x + 4*sqrt(e)*sqrt
(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a
*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*d*e**5 + 4*sqrt(e)*s
qrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e
+ a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*e**6*x - 4*sqrt(e
)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c
*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*a**2*d*e**5 - 4*sqrt(e)*sqrt(d)
*sqrt(a)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2
*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*a**2*e**6*x + 20*sqrt(e)*sqrt(d)*sqrt(c)
*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e*
*2 - c*d**2))*a*c*d**4*e**2 + 20*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt
(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*
d**3*e**3*x - 12*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) +
sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**6 - 12*sq...
```

3.66 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 269

$$\begin{aligned}\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)^4}\,dx =& -\frac{\left(2cd^2-3ae^2\right)\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{d^2e(d+ex)} \\ & -\frac{ae\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{dx(d+ex)^2} \\ & -\frac{a^{3/2}e^{3/2}\left(5cd^2-3ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{a}\sqrt{e}(d+ex)}\right)}{d^{5/2}} \\ & +\frac{2c^{5/2}d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{c}\sqrt{d}(d+ex)}\right)}{e^{3/2}}\end{aligned}$$

output
```
-(-3*a*e^2+2*c*d^2)*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/d^2/e/(e*x+d)-a*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x/(e*x+d)^2-a
^(3/2)*e^(3/2)*(-3*a*e^2+5*c*d^2)*arctanh(d^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)/a^(1/2)/e^(1/2)/(e*x+d))/d^(5/2)+2*c^(5/2)*d^(5/2)*arctanh
(e^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^(1/2)/d^(1/2)/(e*x+d))/
e^(3/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.90

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)^4}\,dx=\frac{\sqrt{(ae+cdx)(d+ex)}\left(-\sqrt{d}\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}(2c^2d^4x-4ac\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)^4),
x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-(Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d
 + e*x]*(2*c^2*d^4*x - 4*a*c*d^2*e^2*x + a^2*e^3*(d + 3*e*x))) + a^(3/2)*e
^3*(-5*c*d^2 + 3*a*e^2)*x*(d + e*x)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(S
qrt[a]*Sqrt[e]*Sqrt[d + e*x])] + 2*c^(5/2)*d^5*x*(d + e*x)*ArcTanh[(Sqrt[e
]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(d^(5/2)*e^(3/2)*x
*Sqrt[a*e + c*d*x]*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {1214, 25, 2181, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^2(d+ex)^4}\,dx$$

$$\Big\downarrow \ 1214$$

$$-\frac{\int -\frac{\frac{a^3e^9}{d}+\frac{a^2\left(3cd^2-ae^2\right)xe^8}{d^2}+c^3d^3x^2e^5}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^6}-\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2e(d+ex)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\frac{a^3e^9}{d}+\frac{a^2\left(3cd^2-ae^2\right)xe^8}{d^2}+c^3d^3x^2e^5}{x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^6}-\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2e(d+ex)}$$

$\downarrow$ 2181

$$\frac{-\frac{\int -\frac{ae^6\left(2c^3xd^5+a^2e^3\left(5cd^2-3ae^2\right)\right)}{2dx\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ade}-\frac{a^2e^8\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^2x}}{e^6}-\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2e(d+ex)}$$

$\downarrow$ 27

$$\frac{\frac{e^5\int \frac{2c^3xd^5+a^2e^3\left(5cd^2-3ae^2\right)}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d^2}-\frac{a^2e^8\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^2x}}{e^6}-\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2e(d+ex)}$$

$\downarrow$ 1269

$$\frac{\frac{e^5\left(a^2e^3\left(5cd^2-3ae^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+2c^3d^5\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx\right)}{2d^2}-\frac{a^2e^8\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^2x}}{e^6}-\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2e(d+ex)}$$

$\downarrow$ 1092

$$\frac{\frac{e^5\left(a^2e^3\left(5cd^2-3ae^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+4c^3d^5\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\right)}{2d^2}-\frac{a^2e^8\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^2x}}{e^6}-\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2e(d+ex)}$$

$\downarrow$ 219

$$\frac{\frac{e^5\left(a^2e^3\left(5cd^2-3ae^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2c^{5/2}d^{9/2}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{e}}\right)}{2d^2}-\frac{a^2e^8\sqrt{x(ae^2+cd^2)+ade}}{d^2x}}{e^6}$$

$$\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2e(d+ex)}$$

$\downarrow$ 1154

$$\frac{\frac{e^5\left(\frac{2c^{5/2}d^{9/2}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{e}}-2a^2e^3\left(5cd^2-3ae^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\right)}{2d^2}}{e^6}$$

$$\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2e(d+ex)}$$

$\downarrow$ 219

$$\frac{\frac{e^5\left(\frac{2c^{5/2}d^{9/2}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{e}}-\frac{a^{3/2}e^{5/2}\left(5cd^2-3ae^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{d}}\right)}{2d^2}-\frac{a^2e}{}}{e^6}$$

$$\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2e(d+ex)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)^4),x]
```

output
```
(-2*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2*e*
(d + e*x)) + (-((a^2*e^8*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2
*x)) + (e^5*((2*c^(5/2)*d^(9/2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqr
t[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/Sqrt[e
] - (a^(3/2)*e^(5/2)*(5*c*d^2 - 3*a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2
)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])])/Sqrt[d]))/(2*d^2))/e^6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3548 vs. $2(237) = 474$.

Time = 4.39 (sec) , antiderivative size = 3549, normalized size of antiderivative = 13.19

method	result	size
default	Expression too large to display	3549

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x^2/(e*x+d)^4,x,method=_RETURN
VERBOSE)
```

output
```
1/d^4*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(7/2)+5/2*(a*e^2+c*d^2
)/a/d/e*(1/5*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)+1/2*(a*e^2+c*d^2)*(1/
8*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/
16*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)
^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d
^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+a*d*e*(1/3*(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/
e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+a*d*e*((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^
(1/2)+1/2*(a*e^2+c*d^2)*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*
d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)-a*d*e/(a*d*e)^(1/2)*ln
((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2))/x))))+6*c/a*(1/12*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(5/2)/c/d/e+5/24*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/8*(2*
c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)/c/d/e+3/16*(4
*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d/e/c*(1/4*(2*c*d*e*x+a*e^2+c*d^2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)/c/d/e+1/8*(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/d
/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2...
```

Fricas [A] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 1562, normalized size of antiderivative = 5.81

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)^4}\,dx=\text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d)^4,x, algorit
hm="fricas")
```

output
```
[1/4*(2*(c^2*d^4*e*x^2 + c^2*d^5*x)*sqrt(c*d/e)*log(8*c^2*d^2*e^2*x^2 + c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*(2*c*d*e^2*x + c*d^2*e + a*e^3)*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d/e) + 8*(c^2*d^3*e + a*c*d*e
^3)*x) - ((5*a*c*d^2*e^3 - 3*a^2*e^5)*x^2 + (5*a*c*d^3*e^2 - 3*a^2*d*e^4)*
x)*sqrt(a*e/d)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^
2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e + (c*d^3 + a*
d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(a^2*d*e^3 +
 (2*c^2*d^4 - 4*a*c*d^2*e^2 + 3*a^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x))/(d^2*e^2*x^2 + d^3*e*x), -1/4*(4*(c^2*d^4*e*x^2 + c^2*d^5*x
)*sqrt(-c*d/e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c
*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d/e)/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3
 + a*c*d*e^2)*x)) + ((5*a*c*d^2*e^3 - 3*a^2*e^5)*x^2 + (5*a*c*d^3*e^2 - 3*
a^2*d*e^4)*x)*sqrt(a*e/d)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 +
a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d^2*e +
(c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(
a^2*d*e^3 + (2*c^2*d^4 - 4*a*c*d^2*e^2 + 3*a^2*e^4)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x))/(d^2*e^2*x^2 + d^3*e*x), 1/2*(((5*a*c*d^2*e^3 -
3*a^2*e^5)*x^2 + (5*a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*sqrt(-a*e/d)*arctan(1/2*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(-a*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) + (...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^2(d+ex)^4}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**2/(e*x+d)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^2(d + ex)^4}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^4 x^2}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d)^4,x, algorit
hm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^4*x^2),
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^2(d + ex)^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d)^4,x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[0,0,13]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[6,6
]%%%}+%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^2(d + ex)^4}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^2\,\left(d + e\,x\right)^4}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)^4),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 1711, normalized size of antiderivative = 6.36

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^2(d + ex)^4}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d)^4,x)
```

output

```
( - 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*d**2*e**6 - 18*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**3*d*e**7*x - 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d
**4*e**4 + 18*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d**3*e**5*x - 4*sqrt(
d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**5*e**3*x - 4*sqrt(d + e*x)*sqrt(a*e +
 c*d*x)*c**3*d**7*e*x - 9*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c
*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sq
rt(d + e*x))*a**3*d*e**7*x - 9*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*
e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(
c)*sqrt(d + e*x))*a**3*e**8*x**2 + 12*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*
sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d
)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**3*e**5*x + 12*sqrt(e)*sqrt(d)*sqrt(a)*l
og(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**
2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**2*e**6*x**2 + 5*sqrt(e)*sqrt
(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a
*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**5*e**3*x + 5*sq
rt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(
a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**4*e**
4*x**2 - 9*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*
sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*
*3*d*e**7*x - 9*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) +...
```

3.67 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^3(d+ex)^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 244

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{x^3(d+ex)^4}\,dx = \frac{15(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4d^3(d+ex)} - \frac{5\left(c-\frac{ae^2}{d^2}\right)(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}{4x(d+ex)^2} - \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{2dx^2(d+ex)^3} - \frac{15\sqrt{a}\sqrt{e}(cd^2-ae^2)^2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{a}\sqrt{e}(d+ex)}\right)}{4d^{7/2}}$$

output

```
15/4*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d^3/(e*x+d)-
5/4*(c-a*e^2/d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d)^2-1/2*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/d/x^2/(e*x+d)^3-15/4*a^(1/2)*e^(1/
2)*(-a*e^2+c*d^2)^2*arctanh(d^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/a^(1/2)/e^(1/2)/(e*x+d))/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.80

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^3(d+ex)^4}\,dx=\frac{((ae+cdx)(d+ex))^{5/2}\left(\frac{\sqrt{d}\left(8c^2d^4x^2-acd^2ex(9d+25ex)+a^2e^2\left(-2d^2+5dex+\right.\right.}{x^2(ae+cdx)^2(d+ex)^3}\right.}{4d^{7/2}}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)^4),
x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*((Sqrt[d]*(8*c^2*d^4*x^2 - a*c*d^2*e*x*(9
*d + 25*e*x) + a^2*e^2*(-2*d^2 + 5*d*e*x + 15*e^2*x^2)))/(x^2*(a*e + c*d*x
)^2*(d + e*x)^3) - (15*Sqrt[a]*Sqrt[e]*(c*d^2 - a*e^2)^2*ArcTanh[(Sqrt[d]*
Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/((a*e + c*d*x)^(5/2)*
(d + e*x)^(5/2))))/(4*d^(7/2))
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1214, 25, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^3(d+ex)^4}\,dx$$

$$\Big\downarrow \text{1214}$$

$$\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}-\frac{\int -\frac{\frac{a^3e^9}{d}+\frac{a^2\left(3cd^2-ae^2\right)xe^8}{d^2}+\frac{a\left(3c^2d^4-3ace^2d^2+a^2e^4\right)x^2e^7}{d^3}}{x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^6}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{a^3e^9}{d}+\frac{a^2\left(3cd^2-ae^2\right)xe^8}{d^2}+\frac{a\left(3c^2d^4-3ace^2d^2+a^2e^4\right)x^2e^7}{d^3}}{x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^6}+\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$\Big\downarrow$ 2181

$$\frac{-\frac{\int -\frac{a^2e^8\left(ae\left(9cd^2-7ae^2\right)+2d\left(\frac{2a^2e^4}{d^2}-7ace^2+6c^2d^2\right)x\right)}{2dx^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{a^2e^8\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2d^2x^2}}{e^6}+\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$\Big\downarrow$ 27

$$\frac{\frac{ae^7\int \frac{ae\left(9cd^2-7ae^2\right)+2d\left(\frac{2a^2e^4}{d^2}-7ace^2+6c^2d^2\right)x}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4d^2}-\frac{a^2e^8\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2d^2x^2}}{e^6}+\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$\Big\downarrow$ 1228

$$\frac{\frac{ae^7\left(\frac{15\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d}-\frac{\left(9cd^2-7ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}\right)}{4d^2}-\frac{a^2e^8\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2d^2x^2}}{e^6}+\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$\Big\downarrow$ 1154

$$\frac{\frac{ae^7\left(-\frac{15\left(cd^2-ae^2\right)^2\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{d}-\frac{\left(9cd^2-7ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}\right)}{4d^2}-\frac{a^2e^8\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2d^2x^2}}{e^6}+\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

$\Big\downarrow$ 219

$$\frac{\frac{ae^7\left(-\frac{15\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}d^{3/2}\sqrt{e}}-\frac{\left(9cd^2-7ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}\right)}{4d^2}-\frac{a^2e^8\sqrt{x(ae^2+cd^2)+ade+cd}}{2d^2x^2}}{e^6}$$

$$\frac{2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)^4),x]
```

output
```
(2*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^3*(d
+ e*x)) + (-1/2*(a^2*e^8*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2
*x^2) + (a*e^7*(-(((9*c*d^2 - 7*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])/(d*x)) - (15*(c*d^2 - a*e^2)^2*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2
)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])])/(2*Sqrt[a]*d^(3/2)*Sqrt[e])))/(4*d^2))/e^6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5364 vs. $2(216) = 432$.

Time = 5.25 (sec) , antiderivative size = 5365, normalized size of antiderivative = 21.99

method	result	size
default	Expression too large to display	5365

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x^3/(e*x+d)^4,x,method=_RETURN
VERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.43

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^3(d + ex)^4}\,dx = \left[\frac{15\left((c^2d^4e - 2\,acd^2e^3 + a^2e^5)x^3 + (c^2d^5 - 2\,acd^3e^2 + a^2de^4)x^2\right)}{}\right.$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d)^4,x, algorit
hm="fricas")
```

output
```
[1/16*(15*((c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)*x^3 + (c^2*d^5 - 2*a*c*d^
3*e^2 + a^2*d*e^4)*x^2)*sqrt(a*e/d)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*
d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*
a*d^2*e + (c*d^3 + a*d*e^2)*x)*sqrt(a*e/d) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/
x^2) - 4*(2*a^2*d^2*e^2 - (8*c^2*d^4 - 25*a*c*d^2*e^2 + 15*a^2*e^4)*x^2 +
(9*a*c*d^3*e - 5*a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
)/(d^3*e*x^3 + d^4*x^2), 1/8*(15*((c^2*d^4*e - 2*a*c*d^2*e^3 + a^2*e^5)*x^
3 + (c^2*d^5 - 2*a*c*d^3*e^2 + a^2*d*e^4)*x^2)*sqrt(-a*e/d)*arctan(1/2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqr
t(-a*e/d)/(a*c*d*e^2*x^2 + a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)) - 2*(2*a^
2*d^2*e^2 - (8*c^2*d^4 - 25*a*c*d^2*e^2 + 15*a^2*e^4)*x^2 + (9*a*c*d^3*e -
 5*a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(d^3*e*x^3 +
 d^4*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^3(d + ex)^4}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**3/(e*x+d)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^3(d+ex)^4}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex+d)^4x^3}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d)^4,x, algorit
hm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^4*x^3),
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^3(d+ex)^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d)^4,x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,3,9]%%%},[2,4]%%%}+%%%{%%%{-4,[1,5,7]%%%},[2,3]%%
%}+%%%{%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^3(d + ex)^4} dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^3\,\left(d + e\,x\right)^4} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)^4),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 1969, normalized size of antiderivative = 8.07

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^3(d + ex)^4} dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d)^4,x)
```

output
```
( - 20*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*d**3*e**5 + 50*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**3*d**2*e**6*x + 150*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
*3*d*e**7*x**2 - 12*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d**5*e**3 - 60*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d**4*e**4*x - 160*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**2*c*d**3*e**5*x**2 - 54*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a*c**2*d**6*e**2*x - 70*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**5*e**3*x
**2 + 48*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**7*e*x**2 + 75*sqrt(e)*sqr
t(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e +
a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*d*e**7*x**2 + 75*sq
rt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(
a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*e**8*x**3
- 105*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(
c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*
d**3*e**5*x**2 - 105*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x)
 - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))*a**2*c*d**2*e**6*x**3 - 15*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqr
t(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*s
qrt(c)*sqrt(d + e*x))*a*c**2*d**5*e**3*x**2 - 15*sqrt(e)*sqrt(d)*sqrt(a)*l
og(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**
2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**4*e**4*x**3 + 45*sqrt(e)*...
```

3.68 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^4(d+ex)^4} dx$

Optimal result

Integrand size = 40, antiderivative size = 359

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^4(d+ex)^4} dx = \frac{5(cd^2-7ae^2)(cd^2-ae^2)^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8ad^4e(d+ex)} - \frac{5(cd^2-7ae^2)\left(c-\frac{ae^2}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{24adex(d+ex)^2} - \frac{\left(\frac{c}{ae}-\frac{7e}{d^2}\right)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{12x^2(d+ex)^3} - \frac{(ade+(cd^2+ae^2)x+cdex^2)^{7/2}}{3adex^3(d+ex)^4} - \frac{5(cd^2-7ae^2)(cd^2-ae^2)^2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{a}\sqrt{e}(d+ex)}\right)}{8\sqrt{a}d^{9/2}\sqrt{e}}$$

output
```
5/8*(-7*a*e^2+c*d^2)*(-a*e^2+c*d^2)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/a/d^4/e/(e*x+d)-5/24*(-7*a*e^2+c*d^2)*(c-a*e^2/d^2)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(3/2)/a/d/e/x/(e*x+d)^2-1/12*(c/a/e-7*e/d^2)*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d)^3-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(7/2)/a/d/e/x^3/(e*x+d)^4-5/8*(-7*a*e^2+c*d^2)*(-a*e^2+c*d^2)^2*arcta
nh(d^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^(1/2)/e^(1/2)/(e*x+d)
)/a^(1/2)/d^(9/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.66

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^4(d+ex)^4}\,dx=\frac{((ae+cdx)(d+ex))^{5/2}\left(-\frac{\sqrt{d}\left(3c^2d^4x^2(11d+27ex)+2acd^2ex\left(13d^2-34dex-9\right.\right.}{x^3(ae+cdx}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)^4),
x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*(-((Sqrt[d]*(3*c^2*d^4*x^2*(11*d + 27*e*x
) + 2*a*c*d^2*e*x*(13*d^2 - 34*d*e*x - 95*e^2*x^2) + a^2*e^2*(8*d^3 - 14*d
^2*e*x + 35*d*e^2*x^2 + 105*e^3*x^3)))/(x^3*(a*e + c*d*x)^2*(d + e*x)^3))
+ (15*(c*d^2 - a*e^2)^2*(-(c*d^2) + 7*a*e^2)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c
*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a]*Sqrt[e]*(a*e + c*d*x)^(5
/2)*(d + e*x)^(5/2))))/(24*d^(9/2))
```

Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {1214, 25, 2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^4(d+ex)^4}\,dx$$

$$\Big\downarrow \ 1214$$

$$-\frac{\int -\frac{\frac{a^3e^9}{d}+\frac{a^2\left(3cd^2-ae^2\right)xe^8}{d^2}+\frac{a\left(3c^2d^4-3ace^2d^2+a^2e^4\right)x^2e^7}{d^3}+\frac{\left(cd^2-ae^2\right)^3x^3e^6}{d^4}}{x^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^6}-\frac{2e\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\frac{a^3e^9}{d}+\frac{a^2\left(3cd^2-ae^2\right)xe^8}{d^2}+\frac{a\left(3c^2d^4-3ace^2d^2+a^2e^4\right)x^2e^7}{d^3}+\frac{\left(cd^2-ae^2\right)^3x^3e^6}{d^4}}{x^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^6}-\frac{2e\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\Big\downarrow$ 2181

$$\frac{-\frac{\int -\frac{\frac{a^3\left(13cd^2-11ae^2\right)e^9}{d}+2a^2\left(\frac{3a^2e^4}{d^2}-11ace^2+9c^2d^2\right)xe^8+\frac{6a\left(cd^2-ae^2\right)^3x^2e^7}{d^3}}{2x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3ade}-\frac{a^2e^8\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3d^2x^3}}{e^6}-\frac{2e\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\frac{a^3\left(13cd^2-11ae^2\right)e^9}{d}+2a^2\left(\frac{3a^2e^4}{d^2}-11ace^2+9c^2d^2\right)xe^8+\frac{6a\left(cd^2-ae^2\right)^3x^2e^7}{d^3}}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6ade}-\frac{a^2e^8\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3d^2x^3}}{e^6}-\frac{2e\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\Big\downarrow$ 2181

$$\frac{\frac{-\frac{\int -\frac{a^2e^8\left(ae\left(33c^2d^4-94ace^2d^2+57a^2e^4\right)+2d\left(-\frac{12a^3e^6}{d^2}+47a^2ce^4-49ac^2d^2e^2+12c^3d^4\right)x\right)}{2dx^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{a^2e^8\left(13cd^2-11ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2d^2x^2}}{6ade}-\frac{a^2e^8}{}\cdots}{e^6}-\frac{2e\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{ae^7\int \frac{ae\left(33c^2d^4-94ace^2d^2+57a^2e^4\right)+2d\left(-\frac{12a^3e^6}{d^2}+47a^2ce^4-49ac^2d^2e^2+12c^3d^4\right)x}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4d^2}-\frac{a^2e^8\left(13cd^2-11ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2d^2x^2}}{6ade}-\frac{a^2e^8\sqrt{x(ae}{}\cdots}{e^6}-\frac{2e\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$\Big\downarrow$ 1228

$$\frac{\frac{ae^7\left(\frac{15\left(cd^2-7ae^2\right)\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d}-\frac{\left(57a^2e^4-94acd^2e^2+33c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}\right)}{4d^2}-\frac{a^2e^8\left(13cd^2-11ae^2\right)\sqrt{x\left(a\cdots\right.}}{2d^2x^2\cdots}}{\frac{6ade}{e^6}}$$

$$\frac{2e\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{ae^7\left(-\frac{15\left(cd^2-7ae^2\right)\left(cd^2-ae^2\right)^2\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{d}-\frac{\left(57a^2e^4-94acd^2e^2+33c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}\cdots\right.}{4d^2}}{\frac{6ade}{e^6}}$$

$$\frac{2e\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{ae^7\left(-\frac{\left(57a^2e^4-94acd^2e^2+33c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}-\frac{15\left(cd^2-7ae^2\right)\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}d^{3/2}\sqrt{e}}\right)}{4d^2}-\frac{a^2e^8\left(\cdots\right.}{\cdots}}{\frac{6ade}{e^6}}$$

$$\frac{2e\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^4(d+ex)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)^4),x]
```

output
```
(-2*e*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^4*
(d + e*x)) + (-1/3*(a^2*e^8*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(
d^2*x^3) + (-1/2*(a^2*e^8*(13*c*d^2 - 11*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(d^2*x^2) + (a*e^7*(-(((33*c^2*d^4 - 94*a*c*d^2*e^2 + 5
7*a^2*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d*x)) - (15*(c*d^
2 - 7*a*e^2)*(c*d^2 - a*e^2)^2*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sq
rt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sq
rt[a]*d^(3/2)*Sqrt[e])))/(4*d^2))/(6*a*d*e))/e^6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8321 vs. $2(327) = 654$.

Time = 6.66 (sec) , antiderivative size = 8322, normalized size of antiderivative = 23.18

method	result	size
default	Expression too large to display	8322

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x^4/(e*x+d)^4,x,method=_RETURN
VERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 5.90 (sec) , antiderivative size = 758, normalized size of antiderivative = 2.11

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^4(d + ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d)^4,x, algorit
hm="fricas")
```

output
```
[-1/96*(15*((c^3*d^6*e - 9*a*c^2*d^4*e^3 + 15*a^2*c*d^2*e^5 - 7*a^3*e^7)*x
^4 + (c^3*d^7 - 9*a*c^2*d^5*e^2 + 15*a^2*c*d^3*e^4 - 7*a^3*d*e^6)*x^3)*sqr
t(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^4*e^3 + (81*a
*c^2*d^5*e^2 - 190*a^2*c*d^3*e^4 + 105*a^3*d*e^6)*x^3 + (33*a*c^2*d^6*e -
68*a^2*c*d^4*e^3 + 35*a^3*d^2*e^5)*x^2 + 2*(13*a^2*c*d^5*e^2 - 7*a^3*d^3*e
^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^5*e^2*x^4 + a*d^6
*e*x^3), 1/48*(15*((c^3*d^6*e - 9*a*c^2*d^4*e^3 + 15*a^2*c*d^2*e^5 - 7*a^3
*e^7)*x^4 + (c^3*d^7 - 9*a*c^2*d^5*e^2 + 15*a^2*c*d^3*e^4 - 7*a^3*d*e^6)*x
^3)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2
*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 +
(a*c*d^3*e + a^2*d*e^3)*x)) - 2*(8*a^3*d^4*e^3 + (81*a*c^2*d^5*e^2 - 190*a
^2*c*d^3*e^4 + 105*a^3*d*e^6)*x^3 + (33*a*c^2*d^6*e - 68*a^2*c*d^4*e^3 + 3
5*a^3*d^2*e^5)*x^2 + 2*(13*a^2*c*d^5*e^2 - 7*a^3*d^3*e^4)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*d^5*e^2*x^4 + a*d^6*e*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^4(d + ex)^4}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**4/(e*x+d)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^4(d + ex)^4}\,dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^4 x^4}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d)^4,x, algorit
hm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^4*x^4),
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^4(d + ex)^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d)^4,x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,4,11]%%%},[2,5]%%%}+%%%{%%%{-5,[1,6,9]%%%},[2,4]%
%%}+%%%{%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{x^4(d+ex)^4}\,dx = \int \frac{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{5/2}}{x^4\,\left(d+e\,x\right)^4}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)^4),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^4*(d + e*x)^4), x)
```

Reduce [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 2498, normalized size of antiderivative = 6.96

$$\int \frac{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}{x^4(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d)^4,x)
```

output
```
( - 112*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*d**4*e**5 + 196*sqrt(d + e*x)
*sqrt(a*e + c*d*x)*a**4*d**3*e**6*x - 490*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a**4*d**2*e**7*x**2 - 1470*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*d*e**8*x**
3 - 80*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**6*e**3 - 224*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*a**3*c*d**5*e**4*x + 602*sqrt(d + e*x)*sqrt(a*e + c*d*
x)*a**3*c*d**4*e**5*x**2 + 1610*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**
3*e**6*x**3 - 260*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**7*e**2*x +
218*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**6*e**3*x**2 + 766*sqrt(d
+ e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**5*e**4*x**3 - 330*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*a*c**3*d**8*e*x**2 - 810*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c
**3*d**7*e**2*x**3 - 735*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*
d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqr
t(d + e*x))*a**4*d*e**8*x**3 - 735*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqr
t(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*s
qrt(c)*sqrt(d + e*x))*a**4*e**9*x**4 + 1050*sqrt(e)*sqrt(d)*sqrt(a)*log(sq
rt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) +
sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**3*e**6*x**3 + 1050*sqrt(e)*sqrt(d
)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e
**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**2*e**7*x**4 + 180
*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)...
```

3.69 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 471

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)^4}\,dx = -\frac{\left(15c^3d^6-839ac^2d^4e^2+1785a^2cd^2e^4-945a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{192ad^5(d+ex)} - \frac{ae\left(11cd^2-9ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24d^2x^3(d+ex)} - \frac{\left(59cd^2-63ae^2\right)\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{96d^3x^2(d+ex)} - \frac{\left(cd^2-ae^2\right)\left(15c^2d^4-322acd^2e^2+315a^2e^4\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{192ad^4ex(d+ex)} - \frac{ae\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{4dx^4(d+ex)^2} + \frac{5\left(cd^2-ae^2\right)^2\left(c^2d^4+14acd^2e^2-63a^2e^4\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{a}\sqrt{e}(d+ex)}\right)}{64a^{3/2}d^{11/2}e^{3/2}}$$

output
```
-1/192*(-945*a^3*e^6+1785*a^2*c*d^2*e^4-839*a*c^2*d^4*e^2+15*c^3*d^6)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^5/(e*x+d)-1/24*a*e*(-9*a*e^2+11*c*
d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d^2/x^3/(e*x+d)-1/96*(-63*a*e
^2+59*c*d^2)*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d^3/x^
2/(e*x+d)-1/192*(-a*e^2+c*d^2)*(315*a^2*e^4-322*a*c*d^2*e^2+15*c^2*d^4)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^4/e/x/(e*x+d)-1/4*a*e*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^4/(e*x+d)^2+5/64*(-a*e^2+c*d^2)^2*(-63*a
^2*e^4+14*a*c*d^2*e^2+c^2*d^4)*arctanh(d^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)/a^(1/2)/e^(1/2)/(e*x+d))/a^(3/2)/d^(11/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.67

$$\int \frac{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}{x^5(d+ex)^4}\,dx = \frac{((ae+cdx)(d+ex))^{5/2}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(15c^3d^6x^3(d+ex)+ac^2d^4ex^2(118d^2-337}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)^4),
x]
```

output
```
(((a*e + c*d*x)*(d + e*x))^(5/2)*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(15*c^3*d^6*x
^3*(d + e*x) + a*c^2*d^4*e*x^2*(118*d^2 - 337*d*e*x - 839*e^2*x^2) + a^2*c
*d^2*e^2*x*(136*d^3 - 244*d^2*e*x + 637*d*e^2*x^2 + 1785*e^3*x^3) - 3*a^3*
e^3*(-16*d^4 + 24*d^3*e*x - 42*d^2*e^2*x^2 + 105*d*e^3*x^3 + 315*e^4*x^4))
)/(x^4*(a*e + c*d*x)^2*(d + e*x)^3)) + (15*(c*d^2 - a*e^2)^2*(c^2*d^4 + 14
*a*c*d^2*e^2 - 63*a^2*e^4)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sq
rt[e]*Sqrt[d + e*x])])/((a*e + c*d*x)^(5/2)*(d + e*x)^(5/2))))/(192*a^(3/2
)*d^(11/2)*e^(3/2))
```

Rubi [A] (verified)

Time = 3.00 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.275, Rules used = {1214, 25, 2181, 27, 2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^5(d+ex)^4}dx$$

$$\Big\downarrow \text{1214}$$

$$\frac{2e^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}-\frac{\int -\frac{\frac{a^3e^9}{d}+\frac{a^2\left(3cd^2-ae^2\right)xe^8}{d^2}-\frac{\left(cd^2-ae^2\right)^3x^4e^7}{d^5}+\frac{a\left(3c^2d^4-3ace^2d^2+a^2e^4\right)x^2e^7}{d^3}+\frac{\left(cd^2-ae^2\right)^3x^3e^6}{d^4}}{x^5\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^6}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{a^3e^9}{d}+\frac{a^2\left(3cd^2-ae^2\right)xe^8}{d^2}-\frac{\left(cd^2-ae^2\right)^3x^4e^7}{d^5}+\frac{a\left(3c^2d^4-3ace^2d^2+a^2e^4\right)x^2e^7}{d^3}+\frac{\left(cd^2-ae^2\right)^3x^3e^6}{d^4}}{x^5\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{e^6}+\frac{2e^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$$\Big\downarrow \text{2181}$$

$$\frac{-\frac{\int -\frac{\frac{a^3\left(17cd^2-15ae^2\right)e^9}{d}-\frac{8a\left(cd^2-ae^2\right)^3x^3e^8}{d^4}+2a^2\left(\frac{4a^2e^4}{d^2}-15ace^2+12c^2d^2\right)xe^8+\frac{8a\left(cd^2-ae^2\right)^3x^2e^7}{d^3}}{2x^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4ade}-\frac{a^2e^8\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4d^2x^4}}{e^6}+\frac{2e^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\frac{a^3\left(17cd^2-15ae^2\right)e^9}{d}-\frac{8a\left(cd^2-ae^2\right)^3x^3e^8}{d^4}+2a^2\left(\frac{4a^2e^4}{d^2}-15ace^2+12c^2d^2\right)xe^8+\frac{8a\left(cd^2-ae^2\right)^3x^2e^7}{d^3}}{x^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{a^2e^8\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4d^2x^4}}{e^6}+\frac{2e^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$\Big\downarrow$ 2181

$$\frac{\frac{-\frac{\int -\frac{-\frac{48a^2\left(cd^2-ae^2\right)^3x^2e^9}{d^3}+\frac{a^3\left(59c^2d^4-190ace^2d^2+123a^2e^4\right)e^9}{d}+4a^2\left(-\frac{12a^3e^6}{d^2}+51a^2ce^4-53ac^2d^2e^2+12c^3d^4\right)xe^8}{2x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3ade}-\frac{a^2e^8\left(17cd^2-15ae^2\right)\sqrt{x\left(ae^2+\cdots\right.}}{3d^2x^3}}{8ade}\cdots}{e^6}+\frac{2e^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int -\frac{-\frac{48a^2\left(cd^2-ae^2\right)^3x^2e^9}{d^3}+\frac{a^3\left(59c^2d^4-190ace^2d^2+123a^2e^4\right)e^9}{d}+4a^2\left(-\frac{12a^3e^6}{d^2}+51a^2ce^4-53ac^2d^2e^2+12c^3d^4\right)xe^8}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6ade}-\frac{a^2e^8\left(17cd^2-15ae^2\right)\sqrt{x\left(ae^2+cd^2\cdots\right.}}{3d^2x^3}}{8ade}\cdots}{e^6}+\frac{2e^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$\Big\downarrow$ 2181

$$\frac{\frac{\frac{-\frac{\int -\frac{a^3e^9\left(15c^3d^6-455ac^2e^2d^4+1017a^2ce^4d^2-2e\left(-\frac{96a^3e^6}{d^2}+411a^2ce^4-478ac^2d^2e^2+155c^3d^4\right)xd-561a^3e^6\right)}{2dx^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{a^2e^8\left(123a^2e^4-190acd^2e^2+59c^2d^4\right)\sqrt{x\left(\cdots\right.}}{2d^2x^2}}{6ade}\cdots}{8ade}\cdots}{e^6}+\frac{2e^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\frac{a^2e^8\int \frac{15c^3d^6-455ac^2e^2d^4+1017a^2ce^4d^2-2e\left(-\frac{96a^3e^6}{d^2}+411a^2ce^4-478ac^2d^2e^2+155c^3d^4\right)xd-561a^3e^6}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4d^2}-\frac{a^2e^8\left(123a^2e^4-190acd^2e^2+59c^2d^4\right)\sqrt{x\left(ae^2+\ldots\right.}}{2d^2x^2}}{6ade}}{8ade}}{e^6}$$

$$\frac{2e^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$\Big\downarrow$ 1228

$$\frac{\frac{\frac{a^2e^8\left(-\frac{15\left(-63a^2e^4+14acd^2e^2+c^2d^4\right)\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(-561a^3e^6+1017a^2cd^2e^4-455ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade\ldots}}{adex}\right.}{4d^2}}{6ade}}{8ade}$$

$$\frac{2e^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$\Big\downarrow$ 1154

$$\frac{\frac{\frac{a^2e^8\left(\frac{15\left(cd^2-ae^2\right)^2\left(-63a^2e^4+14acd^2e^2+c^2d^4\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}-\frac{\left(-561a^3e^6+1017a^2cd^2e^4-455ac^2d^4e^2\ldots\right.}{ade\ldots}\right.}{4d^2}}{6ade}}{8ad\ldots}$$

$$\frac{2e^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{a^2e^8\left(\frac{15\left(cd^2-ae^2\right)^2\left(-63a^2e^4+14acd^2e^2+c^2d^4\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(-561a^3e^6+1017a^2cd^2e^4-455ac^2d^4e^2+15c^3d^6\right)\sqrt{\ldots}}{adex}\right.}{4d^2}}{6ade}}{8ade}$$

$$\frac{2e^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^5(d+ex)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)^4),x]
```

output
```
(2*e^2*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^5
*(d + e*x)) + (-1/4*(a^2*e^8*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(d^2*x^4) + (-1/3*(a^2*e^8*(17*c*d^2 - 15*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(d^2*x^3) + (-1/2*(a^2*e^8*(59*c^2*d^4 - 190*a*c*d^2*e
^2 + 123*a^2*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2*x^2) +
 (a^2*e^8*(-(((15*c^3*d^6 - 455*a*c^2*d^4*e^2 + 1017*a^2*c*d^2*e^4 - 561*a
^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x)) + (15*(c*d
^2 - a*e^2)^2*(c^2*d^4 + 14*a*c*d^2*e^2 - 63*a^2*e^4)*ArcTanh[(2*a*d*e + (
c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2])])/(2*a^(3/2)*d^(3/2)*e^(3/2))))/(4*d^2))/(6*a*d*e))/(8*a*d
*e))/e^6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 13160 vs. $2(435) = 870$.

Time = 7.42 (sec) , antiderivative size = 13161, normalized size of antiderivative = 27.94

method	result	size
default	Expression too large to display	13161

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x^5/(e*x+d)^4,x,method=_RETURN
VERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 14.74 (sec) , antiderivative size = 968, normalized size of antiderivative = 2.06

$$\int \frac{(ade + (cd^2 + ae^2)\,x + cdex^2)^{5/2}}{x^5(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d)^4,x, algorit
hm="fricas")
```

output
```
[-1/768*(15*((c^4*d^8*e + 12*a*c^3*d^6*e^3 - 90*a^2*c^2*d^4*e^5 + 140*a^3*
c*d^2*e^7 - 63*a^4*e^9)*x^5 + (c^4*d^9 + 12*a*c^3*d^7*e^2 - 90*a^2*c^2*d^5
*e^4 + 140*a^3*c*d^3*e^6 - 63*a^4*d*e^8)*x^4)*sqrt(a*d*e)*log((8*a^2*d^2*e
^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*
e + a^2*d*e^3)*x)/x^2) + 4*(48*a^4*d^5*e^4 + (15*a*c^3*d^7*e^2 - 839*a^2*c
^2*d^5*e^4 + 1785*a^3*c*d^3*e^6 - 945*a^4*d*e^8)*x^4 + (15*a*c^3*d^8*e - 3
37*a^2*c^2*d^6*e^3 + 637*a^3*c*d^4*e^5 - 315*a^4*d^2*e^7)*x^3 + 2*(59*a^2*
c^2*d^7*e^2 - 122*a^3*c*d^5*e^4 + 63*a^4*d^3*e^6)*x^2 + 8*(17*a^3*c*d^6*e^
3 - 9*a^4*d^4*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*d^
6*e^3*x^5 + a^2*d^7*e^2*x^4), -1/384*(15*((c^4*d^8*e + 12*a*c^3*d^6*e^3 -
90*a^2*c^2*d^4*e^5 + 140*a^3*c*d^2*e^7 - 63*a^4*e^9)*x^5 + (c^4*d^9 + 12*a
*c^3*d^7*e^2 - 90*a^2*c^2*d^5*e^4 + 140*a^3*c*d^3*e^6 - 63*a^4*d*e^8)*x^4)
*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*
c*d^3*e + a^2*d*e^3)*x)) + 2*(48*a^4*d^5*e^4 + (15*a*c^3*d^7*e^2 - 839*a^2
*c^2*d^5*e^4 + 1785*a^3*c*d^3*e^6 - 945*a^4*d*e^8)*x^4 + (15*a*c^3*d^8*e -
 337*a^2*c^2*d^6*e^3 + 637*a^3*c*d^4*e^5 - 315*a^4*d^2*e^7)*x^3 + 2*(59*a^
2*c^2*d^7*e^2 - 122*a^3*c*d^5*e^4 + 63*a^4*d^3*e^6)*x^2 + 8*(17*a^3*c*d^6*
e^3 - 9*a^4*d^4*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)^4}\,dx=\text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**5/(e*x+d)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)^4}\,dx=\int \frac{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}}{\left(ex+d\right)^4x^5}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d)^4,x, algorit
hm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^4*x^5),
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^5(d+ex)^4}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d)^4,x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,5,13]%%%},[2,6]%%%}+%%%{%%%{-6,[1,7,11]%%%},[2,5]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{x^5(d + ex)^4} dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^5\,\left(d + e\,x\right)^4} dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)^4),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^5*(d + e*x)^4), x)
```

Reduce [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 3131, normalized size of antiderivative = 6.65

$$\int \frac{(ade + (cd^2 + ae^2) x + cdex^2)^{5/2}}{x^5(d + ex)^4} dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d)^4,x)
```

output
```
( - 864*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*d**5*e**6 + 1296*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*a**5*d**4*e**7*x - 2268*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*a**5*d**3*e**8*x**2 + 5670*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*d**2*e**
9*x**3 + 17010*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*d*e**10*x**4 - 672*sqr
t(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d**7*e**4 - 1440*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a**4*c*d**6*e**5*x + 2628*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*
c*d**5*e**6*x**2 - 7056*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d**4*e**7*x
**3 - 18900*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c*d**3*e**8*x**4 - 1904*s
qrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**2*d**8*e**3*x + 1292*sqrt(d + e*x)*
sqrt(a*e + c*d*x)*a**3*c**2*d**7*e**4*x**2 - 2852*sqrt(d + e*x)*sqrt(a*e +
 c*d*x)*a**3*c**2*d**6*e**5*x**3 - 9888*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
*3*c**2*d**5*e**6*x**4 - 1652*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d*
*9*e**2*x**2 + 4448*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**8*e**3*x*
*3 + 11476*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**3*d**7*e**4*x**4 - 210*
sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**4*d**10*e*x**3 - 210*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a*c**4*d**9*e**2*x**4 + 8505*sqrt(e)*sqrt(d)*sqrt(a)*log(s
qrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**5*d*e**10*x**4 + 8505*sqrt(e)*sqrt(d)*s
qrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2
 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**5*e**11*x**5 - 12285*sqr...
```

3.70 $\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 607

$$
\begin{aligned}
&\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)^4}\,dx = \frac{\left(15c^4d^8+80ac^3d^6e^2-3318a^2c^2d^4e^4+6720a^3cd^2e^6-3465a^4e^8\right)}{640a^2d^6e(d+ex)} \\
&-\frac{ae\left(13cd^2-11ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{40d^2x^4(d+ex)} \\
&-\frac{\left(31cd^2-33ae^2\right)\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{80d^3x^3(d+ex)} \\
&-\frac{\left(cd^2-ae^2\right)\left(5c^2d^4-228acd^2e^2+231a^2e^4\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{320ad^4ex^2(d+ex)} \\
&+\frac{\left(cd^2-ae^2\right)\left(15c^3d^6+85ac^2d^4e^2-1239a^2cd^2e^4+1155a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{640a^2d^5e^2x(d+ex)} \\
&-\frac{ae\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}{5dx^5(d+ex)^2} \\
&-\frac{3\left(cd^2-ae^2\right)^2\left(c^3d^6+7ac^2d^4e^2+63a^2cd^2e^4-231a^3e^6\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{\sqrt{a}\sqrt{e}(d+ex)}\right)}{128a^{5/2}d^{13/2}e^{5/2}}
\end{aligned}
$$

output
```
1/640*(-3465*a^4*e^8+6720*a^3*c*d^2*e^6-3318*a^2*c^2*d^4*e^4+80*a*c^3*d^6*
e^2+15*c^4*d^8)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^6/e/(e*x+d)-
1/40*a*e*(-11*a*e^2+13*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d^2/
x^4/(e*x+d)-1/80*(-33*a*e^2+31*c*d^2)*(-a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/d^3/x^3/(e*x+d)-1/320*(-a*e^2+c*d^2)*(231*a^2*e^4-228*a
*c*d^2*e^2+5*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^4/e/x^2/
(e*x+d)+1/640*(-a*e^2+c*d^2)*(1155*a^3*e^6-1239*a^2*c*d^2*e^4+85*a*c^2*d^4
*e^2+15*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^5/e^2/x/(e*
x+d)-1/5*a*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^5/(e*x+d)^2-3/128
*(-a*e^2+c*d^2)^2*(-231*a^3*e^6+63*a^2*c*d^2*e^4+7*a*c^2*d^4*e^2+c^3*d^6)*
arctanh(d^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^(1/2)/e^(1/2)/(e
*x+d))/a^(5/2)/d^(13/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.64

$$\int \frac{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}{x^6(d+ex)^4}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(-15c^4d^8x^4(d+ex)+10ac^3d^6ex^3\left(d^2-7dex-\right.\right.}{}\right.}{}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)^4),
x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-15*c^4*d^8*x^
4*(d + e*x) + 10*a*c^3*d^6*e*x^3*(d^2 - 7*d*e*x - 8*e^2*x^2) + 2*a^2*c^2*d
^4*e^2*x^2*(124*d^3 - 233*d^2*e*x + 662*d*e^2*x^2 + 1659*e^3*x^3) + 2*a^3*
c*d^2*e^3*x*(168*d^4 - 256*d^3*e*x + 459*d^2*e^2*x^2 - 1197*d*e^3*x^3 - 33
60*e^4*x^4) + a^4*e^4*(128*d^5 - 176*d^4*e*x + 264*d^3*e^2*x^2 - 462*d^2*e
^3*x^3 + 1155*d*e^4*x^4 + 3465*e^5*x^5)))/(x^5*(d + e*x))) - (15*(c*d^2 -
a*e^2)^2*(c^3*d^6 + 7*a*c^2*d^4*e^2 + 63*a^2*c*d^2*e^4 - 231*a^3*e^6)*ArcT
anh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(Sqrt[a*
e + c*d*x]*Sqrt[d + e*x])))/(640*a^(5/2)*d^(13/2)*e^(5/2))
```

Rubi [A] (verified)

Time = 4.15 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {1214, 25, 2181, 27, 2181, 27, 2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^6(d+ex)^4}dx$$

$$\Big\downarrow \; 1214$$

$$-\frac{\int -\frac{\frac{a^3e^9}{d}+\frac{\left(cd^2-ae^2\right)^3x^5e^8}{d^6}+\frac{a^2\left(3cd^2-ae^2\right)xe^8}{d^2}-\frac{\left(cd^2-ae^2\right)^3x^4e^7}{d^5}+\frac{a\left(3c^2d^4-3ace^2d^2+a^2e^4\right)x^2e^7}{d^3}+\frac{\left(cd^2-ae^2\right)^3x^3e^6}{d^4}}{x^6\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^6}-\frac{2e^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^6(d+ex)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{a^3e^9}{d}+\frac{\left(cd^2-ae^2\right)^3x^5e^8}{d^6}+\frac{a^2\left(3cd^2-ae^2\right)xe^8}{d^2}-\frac{\left(cd^2-ae^2\right)^3x^4e^7}{d^5}+\frac{a\left(3c^2d^4-3ace^2d^2+a^2e^4\right)x^2e^7}{d^3}+\frac{\left(cd^2-ae^2\right)^3x^3e^6}{d^4}}{x^6\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^6}-\frac{2e^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^6(d+ex)}$$

$$\Big\downarrow \; 2181$$

$$\frac{-\frac{\int -\frac{\frac{10a\left(cd^2-ae^2\right)^3x^4e^9}{d^5}+\frac{a^3\left(21cd^2-19ae^2\right)e^9}{d}-\frac{10a\left(cd^2-ae^2\right)^3x^3e^8}{d^4}+2a^2\left(\frac{5a^2e^4}{d^2}-19ace^2+15c^2d^2\right)xe^8+\frac{10a\left(cd^2-ae^2\right)^3x^2e^7}{d^3}}{2x^5\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{5ade}-\frac{a^2e^8\sqrt{x(ae^2+}}{5}}{e^6}-\frac{2e^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^6(d+ex)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\frac{10a\left(cd^2-ae^2\right)^3x^4e^9}{d^5}+\frac{a^3\left(21cd^2-19ae^2\right)e^9}{d}-\frac{10a\left(cd^2-ae^2\right)^3x^3e^8}{d^4}+2a^2\left(\frac{5a^2e^4}{d^2}-19ace^2+15c^2d^2\right)xe^8+\frac{10a\left(cd^2-ae^2\right)^3x^2e^7}{d^3}}{x^5\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{10ade}-\frac{a^2e^8\sqrt{x(ae^2+cd^2}}{5d^2x^5}}{e^6}$$

$$\frac{2e^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^6(d+ex)}$$

$\downarrow$ 2181

$$\frac{\frac{-\frac{\int -\frac{\frac{80a^2\left(cd^2-ae^2\right)^3x^3e^{10}}{d^4}-\frac{80a^2\left(cd^2-ae^2\right)^3x^2e^9}{d^3}+\frac{3a^3\left(31c^2d^4-106ace^2d^2+71a^2e^4\right)e^9}{d}+2a^2\left(-\frac{40a^3e^6}{d^2}+177a^2ce^4-183ac^2d^2e^2+40c^3d^4\right)xe^8}{2x^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4ade}-\frac{a^2e^8}{}}{10ade}}{e^6}$$

$$\frac{2e^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^6(d+ex)}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\frac{80a^2\left(cd^2-ae^2\right)^3x^3e^{10}}{d^4}-\frac{80a^2\left(cd^2-ae^2\right)^3x^2e^9}{d^3}+\frac{3a^3\left(31c^2d^4-106ace^2d^2+71a^2e^4\right)e^9}{d}+2a^2\left(-\frac{40a^3e^6}{d^2}+177a^2ce^4-183ac^2d^2e^2+40c^3d^4\right)xe^8}{x^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8ade}-\frac{a^2e^8(21}{}}{10ade}}{e^6}$$

$$\frac{2e^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^6(d+ex)}$$

$\downarrow$ 2181

$$\frac{\frac{\frac{-\frac{\int -\frac{3\left(\frac{160a^3\left(cd^2-ae^2\right)^3x^2e^{11}}{d^3}-4a^3\left(-\frac{40a^3e^6}{d^2}+191a^2ce^4-226ac^2d^2e^2+71c^3d^4\right)xe^{10}+\frac{a^3\left(5c^3d^6-357ac^2e^2d^4+883a^2ce^4d^2-515a^3e^6\right)e^9}{d}\right)}{2x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3ade}-\frac{a^2e^8(71}{}}{8ade}}{10ade}}{e^6}$$

$$\frac{2e^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^6(d+ex)}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\frac{\int \frac{\frac{160a^3\left(cd^2-ae^2\right)^3x^2e^{11}}{d^3}-4a^3\left(-\frac{40a^3e^6}{d^2}+191a^2ce^4-226ac^2d^2e^2+71c^3d^4\right)xe^{10}+\frac{a^3\left(5c^3d^6-357ac^2e^2d^4+883a^2ce^4d^2-515a^3e^6\right)e^9}{d}}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{a^2e^8\left(71a^2e^4-10\cdots\right.}{\cdots}}{8ade}\cdots}{10ade}\cdots}{e^6}$$

$$\frac{2e^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^6(d+ex)}$$

$\downarrow$ 2181

$$\frac{\frac{\frac{-\frac{\int \frac{a^3e^9\left(d\left(15c^4d^8+80ac^3e^2d^6-2038a^2c^2e^4d^4+4160a^3ce^6d^2-2185a^4e^8\right)+2e\left(5c^4d^8-677ac^3e^2d^6+1843a^2c^2e^4d^4-1475a^3ce^6d^2+320a^4e^8\right)x\right)}{2d^2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{a^2e^8\left(-5\cdots\right.}{\cdots}}{2ade}\cdots}{8ade}\cdots}{\cdots}$$

$$\frac{2e^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^6(d+ex)}$$

$\downarrow$ 27

$$\frac{\frac{\frac{-\frac{a^2e^8\int \frac{15c^4d^9+80ac^3e^2d^7-2038a^2c^2e^4d^5+4160a^3ce^6d^3-2185a^4e^8d+2e\left(5c^4d^8-677ac^3e^2d^6+1843a^2c^2e^4d^4-1475a^3ce^6d^2+320a^4e^8\right)x}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4d^3}-\frac{a^2e^8\left(-515a^3\cdots\right.}{\cdots}}{2ade}\cdots}{8ade}\cdots}{\cdots}$$

$$\frac{2e^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^6(d+ex)}$$

$\downarrow$ 1228

$$\frac{\frac{-\frac{a^2e^8\left(-\frac{15\left(-231a^3e^6+63a^2cd^2e^4+7ac^2d^4e^2+c^3d^6\right)\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ae}-\frac{\left(-2185a^4e^8+4160a^3cd^2e^6-2038a^2c^2d^4e^4+80ac^3d^6\cdots\right.}{aex}\cdots\right.}{4d^3}\cdots}{2ade}\cdots}{\cdots}$$

$$\frac{2e^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^6(d+ex)}$$

$\downarrow$ 1154

$$-\frac{a^2e^8\left(\frac{15\left(cd^2-ae^2\right)^2\left(-231a^3e^6+63a^2cd^2e^4+7ac^2d^4e^2+c^3d^6\right)\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ae}-\frac{\left(-2185a^4e^8+4160a^3cd^2\cdots\right.}{}\right)}{\frac{4d^3}{2ade}}$$

$$\frac{2e^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^6(d+ex)}$$

$\downarrow$ 219

$$-\frac{a^2e^8\left(-515a^3e^6+883a^2cd^2e^4-357ac^2d^4e^2+5c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2d^2x^2}-\frac{a^2e^8\left(\frac{15\left(cd^2-ae^2\right)^2\left(-231a^3e^6+63a^2cd^2e^4+7ac^2d^4e^2+c^3d^6\right)\operatorname{arctan}\cdots}{2a^{3/2}\sqrt{d}e^{3/2}}\cdots\right)}{2ade}$$

$$\frac{2e^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^6(d+ex)}$$

input

```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)^4),x]
```

output

```
(-2*e^3*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^
6*(d + e*x)) + (-1/5*(a^2*e^8*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
/(d^2*x^5) + (-1/4*(a^2*e^8*(21*c*d^2 - 19*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(d^2*x^4) + (-((a^2*e^8*(31*c^2*d^4 - 106*a*c*d^2*e^2
 + 71*a^2*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2*x^3)) + (
-1/2*(a^2*e^8*(5*c^3*d^6 - 357*a*c^2*d^4*e^2 + 883*a^2*c*d^2*e^4 - 515*a^3
*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2*x^2) - (a^2*e^8*(-
(((15*c^4*d^8 + 80*a*c^3*d^6*e^2 - 2038*a^2*c^2*d^4*e^4 + 4160*a^3*c*d^2*e
^6 - 2185*a^4*e^8)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*e*x)) +
 (15*(c*d^2 - a*e^2)^2*(c^3*d^6 + 7*a*c^2*d^4*e^2 + 63*a^2*c*d^2*e^4 - 231
*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(3/2)*Sqrt[d]*e^(3/2)
)))/(4*d^3))/(2*a*d*e))/(8*a*d*e))/(10*a*d*e))/e^6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 21016 vs. $2(567) = 1134$.

Time = 9.05 (sec) , antiderivative size = 21017, normalized size of antiderivative = 34.62

method	result	size
default	Expression too large to display	21017

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x^6/(e*x+d)^4,x,method=_RETURN
VERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 35.08 (sec) , antiderivative size = 1198, normalized size of antiderivative = 1.97

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^6(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d)^4,x, algorit
hm="fricas")
```

output
```
[-1/2560*(15*((c^5*d^10*e + 5*a*c^4*d^8*e^3 + 50*a^2*c^3*d^6*e^5 - 350*a^3
*c^2*d^4*e^7 + 525*a^4*c*d^2*e^9 - 231*a^5*e^11)*x^6 + (c^5*d^11 + 5*a*c^4
*d^9*e^2 + 50*a^2*c^3*d^7*e^4 - 350*a^3*c^2*d^5*e^6 + 525*a^4*c*d^3*e^8 -
231*a^5*d*e^10)*x^5)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2
*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d
*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) +
4*(128*a^5*d^6*e^5 - (15*a*c^4*d^9*e^2 + 80*a^2*c^3*d^7*e^4 - 3318*a^3*c^2
*d^5*e^6 + 6720*a^4*c*d^3*e^8 - 3465*a^5*d*e^10)*x^5 - (15*a*c^4*d^10*e +
70*a^2*c^3*d^8*e^3 - 1324*a^3*c^2*d^6*e^5 + 2394*a^4*c*d^4*e^7 - 1155*a^5*
d^2*e^9)*x^4 + 2*(5*a^2*c^3*d^9*e^2 - 233*a^3*c^2*d^7*e^4 + 459*a^4*c*d^5*
e^6 - 231*a^5*d^3*e^8)*x^3 + 8*(31*a^3*c^2*d^8*e^3 - 64*a^4*c*d^6*e^5 + 33
*a^5*d^4*e^7)*x^2 + 16*(21*a^4*c*d^7*e^4 - 11*a^5*d^5*e^6)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^7*e^4*x^6 + a^3*d^8*e^3*x^5), 1/12
80*(15*((c^5*d^10*e + 5*a*c^4*d^8*e^3 + 50*a^2*c^3*d^6*e^5 - 350*a^3*c^2*d
^4*e^7 + 525*a^4*c*d^2*e^9 - 231*a^5*e^11)*x^6 + (c^5*d^11 + 5*a*c^4*d^9*e
^2 + 50*a^2*c^3*d^7*e^4 - 350*a^3*c^2*d^5*e^6 + 525*a^4*c*d^3*e^8 - 231*a^
5*d*e^10)*x^5)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2
*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(128*a^5*d^6*e^5 - (15*a*c^4*d^
9*e^2 + 80*a^2*c^3*d^7*e^4 - 3318*a^3*c^2*d^5*e^6 + 6720*a^4*c*d^3*e^8 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(ade + \left(cd^2 + ae^2\right)x + cdex^2\right)^{5/2}}{x^6(d+ex)^4}\,dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**6/(e*x+d)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{x^6(d+ex)^4}\,dx = \int \frac{\left(cdex^2 + ade + (cd^2 + ae^2)x\right)^{\frac{5}{2}}}{\left(ex+d\right)^4 x^6}\,dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d)^4,x, algorit
hm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^4*x^6),
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{x^6(d+ex)^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d)^4,x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,6,15]%%%},[2,7]%%%}+%%%{%%%{-7,[1,8,13]%%%},[2,6]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^6(d+ex)^4}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^6\,\left(d+e\,x\right)^4}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)^4),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)^4), x)
```

Reduce [B] (verification not implemented)

Time = 199.31 (sec) , antiderivative size = 3796, normalized size of antiderivative = 6.25

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^6(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d)^4,x)
```

output

```
( - 2816*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*d**6*e**7 + 3872*sqrt(d + e*
x)*sqrt(a*e + c*d*x)*a**6*d**5*e**8*x - 5808*sqrt(d + e*x)*sqrt(a*e + c*d*
x)*a**6*d**4*e**9*x**2 + 10164*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*d**3*e
**10*x**3 - 25410*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*d**2*e**11*x**4 - 7
6230*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**6*d*e**12*x**5 - 2304*sqrt(d + e*x
)*sqrt(a*e + c*d*x)*a**5*c*d**8*e**5 - 4224*sqrt(d + e*x)*sqrt(a*e + c*d*x
)*a**5*c*d**7*e**6*x + 6512*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d**6*e*
*7*x**2 - 11880*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d**5*e**8*x**3 + 31
878*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**5*c*d**4*e**9*x**4 + 85470*sqrt(d +
 e*x)*sqrt(a*e + c*d*x)*a**5*c*d**3*e**10*x**5 - 6048*sqrt(d + e*x)*sqrt(a
*e + c*d*x)*a**4*c**2*d**9*e**4*x + 3760*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a
**4*c**2*d**8*e**5*x**2 - 6272*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**2*d
**7*e**6*x**3 + 13964*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**2*d**6*e**7*
x**4 + 47964*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*c**2*d**5*e**8*x**5 - 44
64*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**10*e**3*x**2 + 8168*sqrt(d
 + e*x)*sqrt(a*e + c*d*x)*a**3*c**3*d**9*e**4*x**3 - 22292*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**3*c**3*d**8*e**5*x**4 - 57964*sqrt(d + e*x)*sqrt(a*e +
 c*d*x)*a**3*c**3*d**7*e**6*x**5 - 180*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**
2*c**4*d**11*e**2*x**3 + 1590*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d*
*10*e**3*x**4 + 1770*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**4*d**9*e**...
```

3.71 $\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^7(d+ex)^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 755

$$\begin{aligned}
&\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^7(d+ex)^4}\,dx = \\
&-\frac{(75c^5d^{10}+295ac^4d^8e^2+1230a^2c^3d^6e^4-45234a^3c^2d^4e^6+88935a^4cd^2e^8-45045a^5e^{10})\sqrt{ade+(cd^2+a}}{7680a^3d^7e^2(d+ex)} \\
&-\frac{ae(15cd^2-13ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{60d^2x^5(d+ex)} \\
&-\frac{(135cd^2-143ae^2)(cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{480d^3x^4(d+ex)} \\
&-\frac{(cd^2-ae^2)(5c^2d^4-418acd^2e^2+429a^2e^4)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{960ad^4ex^3(d+ex)} \\
&+\frac{(cd^2-ae^2)(25c^3d^6+105ac^2d^4e^2-3069a^2cd^2e^4+3003a^3e^6)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3840a^2d^5e^2x^2(d+ex)} \\
&-\frac{(cd^2-ae^2)(75c^4d^8+320ac^3d^6e^2+1350a^2c^2d^4e^4-16632a^3cd^2e^6+15015a^4e^8)\sqrt{ade+(cd^2+ae^2)x+}}{7680a^3d^6e^3x(d+ex)} \\
&-\frac{ae(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{6dx^6(d+ex)^2} \\
&+\frac{(cd^2-ae^2)^2(5c^4d^8+28ac^3d^6e^2+126a^2c^2d^4e^4+924a^3cd^2e^6-3003a^4e^8)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex}}{\sqrt{a}\sqrt{e}(d+ex)}\right.}{512a^{7/2}d^{15/2}e^{7/2}}
\end{aligned}$$

output
```
-1/7680*(-45045*a^5*e^10+88935*a^4*c*d^2*e^8-45234*a^3*c^2*d^4*e^6+1230*a^
2*c^3*d^6*e^4+295*a*c^4*d^8*e^2+75*c^5*d^10)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)/a^3/d^7/e^2/(e*x+d)-1/60*a*e*(-13*a*e^2+15*c*d^2)*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/d^2/x^5/(e*x+d)-1/480*(-143*a*e^2+135*c*d^2)*(-
a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d^3/x^4/(e*x+d)-1/960
*(-a*e^2+c*d^2)*(429*a^2*e^4-418*a*c*d^2*e^2+5*c^2*d^4)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)/a/d^4/e/x^3/(e*x+d)+1/3840*(-a*e^2+c*d^2)*(3003*a^3*
e^6-3069*a^2*c*d^2*e^4+105*a*c^2*d^4*e^2+25*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/a^2/d^5/e^2/x^2/(e*x+d)-1/7680*(-a*e^2+c*d^2)*(15015*a^
4*e^8-16632*a^3*c*d^2*e^6+1350*a^2*c^2*d^4*e^4+320*a*c^3*d^6*e^2+75*c^4*d^
8)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d^6/e^3/x/(e*x+d)-1/6*a*e*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/d/x^6/(e*x+d)^2+1/512*(-a*e^2+c*d^2
)^2*(-3003*a^4*e^8+924*a^3*c*d^2*e^6+126*a^2*c^2*d^4*e^4+28*a*c^3*d^6*e^2+
5*c^4*d^8)*arctanh(d^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^(1/2)
/e^(1/2)/(e*x+d))/a^(7/2)/d^(15/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 2.84 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.64

$$\int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{x^7(d+ex)^4}\,dx = \frac{\sqrt{(ae+cdx)(d+ex)}\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(75c^5d^{10}x^5(d+ex)+5ac^4d^8ex^4(-10d^2+49d\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)^4),
x]
```

output
```
(Sqrt[(a*e + c*d*x)*(d + e*x)]*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(75*c^5*d^10*x^
5*(d + e*x) + 5*a*c^4*d^8*e*x^4*(-10*d^2 + 49*d*e*x + 59*e^2*x^2) + 10*a^2
*c^3*d^6*e^2*x^3*(4*d^3 - 16*d^2*e*x + 103*d*e^2*x^2 + 123*e^3*x^3) + 6*a^
3*c^2*d^4*e^3*x^2*(360*d^4 - 564*d^3*e*x + 1058*d^2*e^2*x^2 - 2997*d*e^3*x
^3 - 7539*e^4*x^4) + a^4*c*d^2*e^4*x*(3200*d^5 - 4448*d^4*e*x + 6776*d^3*e
^2*x^2 - 12144*d^2*e^3*x^3 + 31647*d*e^4*x^4 + 88935*e^5*x^5) + a^5*e^5*(1
280*d^6 - 1664*d^5*e*x + 2288*d^4*e^2*x^2 - 3432*d^3*e^3*x^3 + 6006*d^2*e^
4*x^4 - 15015*d*e^5*x^5 - 45045*e^6*x^6)))/(x^6*(d + e*x))) + (15*(c*d^2 -
 a*e^2)^2*(5*c^4*d^8 + 28*a*c^3*d^6*e^2 + 126*a^2*c^2*d^4*e^4 + 924*a^3*c*
d^2*e^6 - 3003*a^4*e^8)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[
e]*Sqrt[d + e*x])])/(Sqrt[a*e + c*d*x]*Sqrt[d + e*x])))/(7680*a^(7/2)*d^(1
5/2)*e^(7/2))
```

Rubi [A] (verified)

Time = 5.63 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1214, 25, 2181, 27, 2181, 27, 2181, 27, 2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}{x^7(d+ex)^4}dx$$

$$\Big\downarrow \text{1214}$$

$$\frac{2e^4\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^7(d+ex)}-\frac{\int -\frac{-\frac{\left(cd^2-ae^2\right)^3x^6e^9}{d^7}+\frac{a^3e^9}{d}+\frac{\left(cd^2-ae^2\right)^3x^5e^8}{d^6}+\frac{a^2\left(3cd^2-ae^2\right)xe^8}{d^2}-\frac{\left(cd^2-ae^2\right)^3x^4e^7}{d^5}+\frac{a\left(3c^2d^4-3ace^2d^2+a^2e^4\right)x^2e^7}{d^3}+\frac{\left(cd^2-ae^2\right)^3x^3e^6}{d^4}}{x^7\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^6}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\frac{\left(cd^2-ae^2\right)^3x^6e^9}{d^7}+\frac{a^3e^9}{d}+\frac{\left(cd^2-ae^2\right)^3x^5e^8}{d^6}+\frac{a^2\left(3cd^2-ae^2\right)xe^8}{d^2}-\frac{\left(cd^2-ae^2\right)^3x^4e^7}{d^5}+\frac{a\left(3c^2d^4-3ace^2d^2+a^2e^4\right)x^2e^7}{d^3}+\frac{\left(cd^2-ae^2\right)^3x^3e^6}{d^4}}{x^7\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^6}+\frac{2e^4\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^7(d+ex)}$$

$\Big\downarrow$ 2181

$$-\frac{\int -\frac{-\frac{12a(cd^2-ae^2)^3x^5e^{10}}{d^6}+\frac{12a(cd^2-ae^2)^3x^4e^9}{d^5}+\frac{a^3(25cd^2-23ae^2)e^9}{d}-\frac{12a(cd^2-ae^2)^3x^3e^8}{d^4}+2a^2\left(\frac{6a^2e^4}{d^2}-23ace^2+18c^2d^2\right)xe^8+\frac{12a(cd^2-ae^2)^3x^2}{d^3}}{2x^6\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{6ade}$$

$$e^6$$

$$\frac{2e^4(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^7(d+ex)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{-\frac{12a(cd^2-ae^2)^3x^5e^{10}}{d^6}+\frac{12a(cd^2-ae^2)^3x^4e^9}{d^5}+\frac{a^3(25cd^2-23ae^2)e^9}{d}-\frac{12a(cd^2-ae^2)^3x^3e^8}{d^4}+2a^2\left(\frac{6a^2e^4}{d^2}-23ace^2+18c^2d^2\right)xe^8+\frac{12a(cd^2-ae^2)^3x^2e^7}{d^3}}{x^6\sqrt{cdex^2+(cd^2+ae^2)x+ade}}d}{12ade}$$

$$e^6$$

$$\frac{2e^4(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^7(d+ex)}$$

$\Big\downarrow$ 2181

$$\frac{-\frac{\int -\frac{-\frac{120a^2(cd^2-ae^2)^3x^4e^{11}}{d^5}+\frac{120a^2(cd^2-ae^2)^3x^3e^{10}}{d^4}-\frac{120a^2(cd^2-ae^2)^3x^2e^9}{d^3}+\frac{a^3(135c^2d^4-478ace^2d^2+327a^2e^4)e^9}{d}+8a^2\left(-\frac{15a^3e^6}{d^2}+68a^2ce^4-70ac\right.}{2x^5\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{5ade}}{12ade}$$

$$e^6$$

$$\frac{2e^4(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^7(d+ex)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{-\frac{120a^2(cd^2-ae^2)^3x^4e^{11}}{d^5}+\frac{120a^2(cd^2-ae^2)^3x^3e^{10}}{d^4}-\frac{120a^2(cd^2-ae^2)^3x^2e^9}{d^3}+\frac{a^3(135c^2d^4-478ace^2d^2+327a^2e^4)e^9}{d}+8a^2\left(-\frac{15a^3e^6}{d^2}+68a^2ce^4-70ac^2d^2\right.}{x^5\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{10ade}}{12ade}$$

$$e^6$$

$$\frac{2e^4(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^7(d+ex)}$$

$\Big\downarrow$ 2181

$$-\frac{\int -\frac{3\left(-\frac{320a^3\left(cd^2-ae^2\right)^3x^3e^{12}}{d^4}+\frac{320a^3\left(cd^2-ae^2\right)^3x^2e^{11}}{d^3}-2a^3\left(-\frac{160a^3e^6}{d^2}+807a^2ce^4-958ac^2d^2e^2+295c^3d^4\right)xe^{10}+\frac{a^3\left(5c^3d^6-693ac^2e^2d^4+1803a^2ce^4\cdots\right)}{d}\cdots\right.}{2x^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\cdots}{4ade}\cdots$$

$$\cdots 10ade \quad 12ade$$

$$\frac{2e^4\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^7(d+ex)}$$

$$\Big\downarrow \; 27$$

$$\frac{3\int -\frac{-\frac{320a^3\left(cd^2-ae^2\right)^3x^3e^{12}}{d^4}+\frac{320a^3\left(cd^2-ae^2\right)^3x^2e^{11}}{d^3}-2a^3\left(-\frac{160a^3e^6}{d^2}+807a^2ce^4-958ac^2d^2e^2+295c^3d^4\right)xe^{10}+\frac{a^3\left(5c^3d^6-693ac^2e^2d^4+1803a^2ce^4d^2-\cdots\right)}{d}\cdots}{x^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\cdots}{8ade}\cdots$$

$$\cdots 10ade \quad 12ade$$

$$\frac{2e^4\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^7(d+ex)}$$

$$\Big\downarrow \; 2181$$

$$\frac{3\left(-\frac{\int \frac{\frac{1920a^4\left(cd^2-ae^2\right)^3x^2e^{13}}{d^3}+\frac{4a^3\left(5c^4d^8-1173ac^3e^2d^6+3243a^2c^2e^4d^4-2523a^3ce^6d^2+480a^4e^8\right)xe^{10}}{d^2}+\frac{a^3\left(25c^4d^8+100ac^3e^2d^6-5946a^2c^2e^4d^4+13284\cdots\right)}{d}\cdots}{2x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\cdots}{3ade}\cdots\right.}{8ade}\cdots$$

$$\frac{2e^4\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^7(d+ex)}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(-\frac{\int \frac{\frac{1920a^4\left(cd^2-ae^2\right)^3x^2e^{13}}{d^3}+\frac{4a^3\left(5c^4d^8-1173ac^3e^2d^6+3243a^2c^2e^4d^4-2523a^3ce^6d^2+480a^4e^8\right)xe^{10}}{d^2}+\frac{a^3\left(25c^4d^8+100ac^3e^2d^6-5946a^2c^2e^4d^4+13284\cdots\right)}{d}\cdots}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\cdots}{6ade}\cdots\right.}{8ade}\cdots$$

$$\frac{2e^4\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^7(d+ex)}$$

$\downarrow$ 2181

$$3\left(-\frac{-\frac{\int \frac{a^3e^9\left(d\left(75c^5d^{10}+295ac^4e^2d^8+1230a^2c^3e^4d^6-29874a^3c^2e^6d^4+58215a^4ce^8d^2-29685a^5e^{10}\right)+2e\left(25c^5d^{10}+100ac^4e^2d^8-9786a^2c^3e^4d^6+24804a^3c^2e\right.\right.}{2d^2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{2ade}}{6ade}\right.$$

$$\frac{2e^4\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^7(d+ex)}$$

$\downarrow$ 27

$$3\left(-\frac{-\frac{a^2e^8\int \frac{75c^5d^{11}+295ac^4e^2d^9+1230a^2c^3e^4d^7-29874a^3c^2e^6d^5+58215a^4ce^8d^3-29685a^5e^{10}d+2e\left(25c^5d^{10}+100ac^4e^2d^8-9786a^2c^3e^4d^6+24804a^3c^2e^6d^4\right.}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4d^3}}{6ade}\right.$$

$$\frac{2e^4\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^7(d+ex)}$$

$\downarrow$ 1228

$$3\left(-\frac{-\frac{a^2e^8\left(-\frac{15\left(-3003a^4e^8+924a^3cd^2e^6+126a^2c^2d^4e^4+28ac^3d^6e^2+5c^4d^8\right)\left(cd^2-ae^2\right)^2\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ae}-\frac{\left(-29685a^5e^{10}+58215a^4cd^2e\right.}{}\right.}{4d^3}}{}\right.$$

$$\frac{2e^4\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^7(d+ex)}$$

$\downarrow$ 1154

$$3\left(-\frac{a^2e^8\left(\frac{15\left(cd^2-ae^2\right)^2\left(-3003a^4e^8+924a^3cd^2e^6+126a^2c^2d^4e^4+28ac^3d^6e^2+5c^4d^8\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ae}-\right.}{4d^3}\right.$$

$$\frac{2e^4\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^7(d+ex)}$$

$\downarrow$ 219

$$3\left(-\frac{a^2e^8\left(-1083a^3e^6+1803a^2cd^2e^4-693ac^2d^4e^2+5c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3d^2x^3}-\frac{a^2e^8\left(-7335a^4e^8+13284a^3cd^2e^6-5946a^2c^2d^4e^4+100ac^3d^6e^2+25c\right.}{2d^2x^2}\right.$$

$$\frac{2e^4\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^7(d+ex)}$$

input
```
Int[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)^4),x]
```

output
```
(2*e^4*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^7
*(d + e*x)) + (-1/6*(a^2*e^8*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/
(d^2*x^6) + (-1/5*(a^2*e^8*(25*c*d^2 - 23*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])/(d^2*x^5) + (-1/4*(a^2*e^8*(135*c^2*d^4 - 478*a*c*d^2*
e^2 + 327*a^2*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2*x^4)
+ (3*(-1/3*(a^2*e^8*(5*c^3*d^6 - 693*a*c^2*d^4*e^2 + 1803*a^2*c*d^2*e^4 -
1083*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2*x^3) - (-1
/2*(a^2*e^8*(25*c^4*d^8 + 100*a*c^3*d^6*e^2 - 5946*a^2*c^2*d^4*e^4 + 13284
*a^3*c*d^2*e^6 - 7335*a^4*e^8)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(d^2*x^2) - (a^2*e^8*(-(((75*c^5*d^10 + 295*a*c^4*d^8*e^2 + 1230*a^2*c^3
*d^6*e^4 - 29874*a^3*c^2*d^4*e^6 + 58215*a^4*c*d^2*e^8 - 29685*a^5*e^10)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*e*x)) + (15*(c*d^2 - a*e^2)
^2*(5*c^4*d^8 + 28*a*c^3*d^6*e^2 + 126*a^2*c^2*d^4*e^4 + 924*a^3*c*d^2*e^6
 - 3003*a^4*e^8)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*
Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(3/2)*Sqrt[d]*
e^(3/2))))/(4*d^3))/(6*a*d*e)))/(8*a*d*e))/(10*a*d*e))/(12*a*d*e))/e^6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 33769 vs. $2(711) = 1422$.

Time = 10.82 (sec) , antiderivative size = 33770, normalized size of antiderivative = 44.73

method	result	size
default	Expression too large to display	33770

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2)/x^7/(e*x+d)^4,x,method=_RETURN
VERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 82.46 (sec) , antiderivative size = 1452, normalized size of antiderivative = 1.92

$$\int \frac{\left(ade + (cd^2 + ae^2)\,x + cdex^2\right)^{5/2}}{x^7(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d)^4,x, algorit
hm="fricas")
```

output
```
[-1/30720*(15*((5*c^6*d^12*e + 18*a*c^5*d^10*e^3 + 75*a^2*c^4*d^8*e^5 + 70
0*a^3*c^3*d^6*e^7 - 4725*a^4*c^2*d^4*e^9 + 6930*a^5*c*d^2*e^11 - 3003*a^6*
e^13)*x^7 + (5*c^6*d^13 + 18*a*c^5*d^11*e^2 + 75*a^2*c^4*d^9*e^4 + 700*a^3
*c^3*d^7*e^6 - 4725*a^4*c^2*d^5*e^8 + 6930*a^5*c*d^3*e^10 - 3003*a^6*d*e^1
2)*x^6)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^
4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 +
 a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(1280*a^6*d
^7*e^6 + (75*a*c^5*d^11*e^2 + 295*a^2*c^4*d^9*e^4 + 1230*a^3*c^3*d^7*e^6 -
 45234*a^4*c^2*d^5*e^8 + 88935*a^5*c*d^3*e^10 - 45045*a^6*d*e^12)*x^6 + (7
5*a*c^5*d^12*e + 245*a^2*c^4*d^10*e^3 + 1030*a^3*c^3*d^8*e^5 - 17982*a^4*c
^2*d^6*e^7 + 31647*a^5*c*d^4*e^9 - 15015*a^6*d^2*e^11)*x^5 - 2*(25*a^2*c^4
*d^11*e^2 + 80*a^3*c^3*d^9*e^4 - 3174*a^4*c^2*d^7*e^6 + 6072*a^5*c*d^5*e^8
 - 3003*a^6*d^3*e^10)*x^4 + 8*(5*a^3*c^3*d^10*e^3 - 423*a^4*c^2*d^8*e^5 +
847*a^5*c*d^6*e^7 - 429*a^6*d^4*e^9)*x^3 + 16*(135*a^4*c^2*d^9*e^4 - 278*a
^5*c*d^7*e^6 + 143*a^6*d^5*e^8)*x^2 + 128*(25*a^5*c*d^8*e^5 - 13*a^6*d^6*e
^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^4*d^8*e^5*x^7 + a^4
*d^9*e^4*x^6), -1/15360*(15*((5*c^6*d^12*e + 18*a*c^5*d^10*e^3 + 75*a^2*c^
4*d^8*e^5 + 700*a^3*c^3*d^6*e^7 - 4725*a^4*c^2*d^4*e^9 + 6930*a^5*c*d^2*e^
11 - 3003*a^6*e^13)*x^7 + (5*c^6*d^13 + 18*a*c^5*d^11*e^2 + 75*a^2*c^4*d^9
*e^4 + 700*a^3*c^3*d^7*e^6 - 4725*a^4*c^2*d^5*e^8 + 6930*a^5*c*d^3*e^10...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d + ex)^4} dx = \text{Timed out}$$

input
```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**7/(e*x+d)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d + ex)^4} dx = \int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^4 x^7} dx$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d)^4,x, algorit
hm="maxima")
```

output
```
integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^4*x^7),
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d + ex)^4} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d)^4,x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,7,17]%%%},[2,8]%%%}+%%%{%%%{-8,[1,9,15]%%%},[2,7]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d+ex)^4}\,dx = \int \frac{\left(c\,d\,e\,x^2 + \left(c\,d^2 + a\,e^2\right)\,x + a\,d\,e\right)^{5/2}}{x^7\,\left(d + e\,x\right)^4}\,dx$$

input
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)^4),x)
```

output
```
int((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)^4), x)
```

Reduce [F]

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d+ex)^4}\,dx = \int \frac{\left(ade + \left(a\,e^2 + c\,d^2\right)x + cde\,x^2\right)^{\frac{5}{2}}}{x^7\left(ex + d\right)^4}dx$$

input
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d)^4,x)
```

output
```
int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d)^4,x)
```

3.72 $\int \frac{x^3(d+ex)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 339

$$\int \frac{x^3(d+ex)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{(cd^2-7ae^2)x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{24c^2d^2e} + \frac{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4cd} + \frac{(15c^3d^6+17ac^2d^4e^2+25a^2cd^2e^4-105a^3e^6-2cde(5c^2d^4+6acd^2e^2-35a^2e^4)x)\sqrt{ade+(cd^2+ae^2)}}{192c^4d^4e^3} - \frac{(cd^2-ae^2)(5c^3d^6+9ac^2d^4e^2+15a^2cd^2e^4+35a^3e^6)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{64c^{9/2}d^{9/2}e^{7/2}}$$

output

```
1/24*(-7*a*e^2+c*d^2)*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/
e+1/4*x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d+1/192*(15*c^3*d^6+17
*a*c^2*d^4*e^2+25*a^2*c*d^2*e^4-105*a^3*e^6-2*c*d*e*(-35*a^2*e^4+6*a*c*d^2
*e^2+5*c^2*d^4)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^3-1/6
4*(-a*e^2+c*d^2)*(35*a^3*e^6+15*a^2*c*d^2*e^4+9*a*c^2*d^4*e^2+5*c^3*d^6)*a
rctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/c^(9/2)/d^(9/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 11.18 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.11

$$\int \frac{x^3(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{(d+ex)\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}(-105a^4e^7+5a^3cde^5(5d-7ex)+a^2c^2d^2e^3(17d^2+13dex+14e^2x^2)+ac^3\right.}{}$$

input
```
Integrate[(x^3*(d + e*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
((d + e*x)*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*
(-105*a^4*e^7 + 5*a^3*c*d*e^5*(5*d - 7*e*x) + a^2*c^2*d^2*e^3*(17*d^2 + 13
*d*e*x + 14*e^2*x^2) + a*c^3*d^3*e*(15*d^3 + 7*d^2*e*x - 4*d*e^2*x^2 - 8*e
^3*x^3) + c^4*d^4*x*(15*d^3 - 10*d^2*e*x + 8*d*e^2*x^2 + 48*e^3*x^3)) - 3*
Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*(5*c^3*d^6 + 9*a*c^2*d^4*e^2 + 15*a^2*c*d^2*
e^4 + 35*a^3*e^6)*Sqrt[a*e + c*d*x]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[
a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(192*c^(9/2)*d^(9/2)*e^(7
/2)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1236, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \; 1236$$

$$\frac{\int -\frac{ex^2\left(6ade-\left(cd^2-7ae^2\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cde}+\frac{x^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4cd}$$

$$\Big\downarrow \; 27$$

$$\frac{x^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cd}-\frac{\int \frac{x^2\left(6ade-\left(cd^2-7ae^2\right)x\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cd}$$

$$\Big\downarrow \; 1236$$

$$\frac{x^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cd}-\frac{\frac{\int \frac{x\left(4ade\left(cd^2-7ae^2\right)+\left(5c^2d^4+6ace^2d^2-35a^2e^4\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3cde}-\frac{1}{3}x^2\left(\frac{d}{e}-\frac{7ae}{cd}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8cd}$$

$$\Big\downarrow \; 27$$

$$\frac{x^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cd}-\frac{\frac{\int \frac{x\left(4ade\left(cd^2-7ae^2\right)+\left(5c^2d^4+6ace^2d^2-35a^2e^4\right)x\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6cde}-\frac{1}{3}x^2\left(\frac{d}{e}-\frac{7ae}{cd}\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{8cd}$$

$$\Big\downarrow \; 1225$$

$$\frac{x^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cd}-\frac{\frac{\frac{3\left(cd^2-ae^2\right)\left(35a^3e^6+15a^2cd^2e^4+9ac^2d^4e^2+5c^3d^6\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8c^2d^2e^2}-\frac{\left(-105a^3e^6-2cdex\left(-35a^2e^4+6acd^2e^2+5c^2d^4\right)+25a^2cd^2e^4+17ac^2d^4\right.}{4c^2d^2e^2}}{6cde}}{8cd}$$

$$\Big\downarrow \; 1092$$

$$\frac{x^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cd}-\frac{\frac{\frac{3\left(cd^2-ae^2\right)\left(35a^3e^6+15a^2cd^2e^4+9ac^2d^4e^2+5c^3d^6\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4c^2d^2e^2}-\frac{\left(-105a^3e^6-2cdex\left(-35a^2e^4+6acd^2e^{\cdots}\right.\right.}{}}{6cde}}{8cd}$$

$$\Big\downarrow \; 219$$

$$\frac{x^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4cd}-\frac{\frac{\frac{3\left(cd^2-ae^2\right)\left(35a^3e^6+15a^2cd^2e^4+9ac^2d^4e^2+5c^3d^6\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{5/2}}-\frac{\left(-105a^3e^6-2cdex\left(-35a^2e^4+6acd^2e^2+5c^2d^4\right)+\cdots\right.}{}}{6cde}}{8cd}$$

input
```
Int[(x^3*(d + e*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c*d) - (-1/3*((d/e -
(7*a*e)/(c*d))*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (-1/4*((
15*c^3*d^6 + 17*a*c^2*d^4*e^2 + 25*a^2*c*d^2*e^4 - 105*a^3*e^6 - 2*c*d*e*(
5*c^2*d^4 + 6*a*c*d^2*e^2 - 35*a^2*e^4)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])/(c^2*d^2*e^2) + (3*(c*d^2 - a*e^2)*(5*c^3*d^6 + 9*a*c^2*d^4*
e^2 + 15*a^2*c*d^2*e^4 + 35*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(
2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(
8*c^(5/2)*d^(5/2)*e^(5/2)))/(6*c*d*e))/(8*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1199 vs. $2(311) = 622$.

Time = 2.63 (sec) , antiderivative size = 1200, normalized size of antiderivative = 3.54

method	result	size
default	Expression too large to display	1200

input
```
int(x^3*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
d*(1/3*x^2/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-5/6*(a*e^2+c*d^2)
/d/e/c*(1/2*x/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/4*(a*e^2+c*d
^2)/d/e/c*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^
2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-1/2*a/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*
x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))
-2/3*a/c*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2
)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+e*(1/4*x^3/d/e/c*(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2)-7/8*(a*e^2+c*d^2)/d/e/c*(1/3*x^2/d/e/c*(a*d*e+(a*e^2+
c*d^2)*x+c*d*x^2*e)^(1/2)-5/6*(a*e^2+c*d^2)/d/e/c*(1/2*x/d/e/c*(a*d*e+(a*e
^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/4*(a*e^2+c*d^2)/d/e/c*(1/d/e/c*(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^
2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^
(1/2))-1/2*a/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2/3*a/c*(1/d/e/c*(a*d*e+(a*e^2
+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2
+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(
1/2)))-3/4*a/c*(1/2*x/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/4*(a
*e^2+c*d^2)/d/e/c*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*...
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.99

$$\int \frac{x^3(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[-\frac{3\left(5\,c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+20\,a^3cd^2e^6-35\,a^4e^8\right)\sqrt{cde}\,\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a\right.}{}\right.$$

input
```
integrate(x^3*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
[-1/768*(3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2
*e^6 - 35*a^4*e^8)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2
*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x
+ c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(48*c^4*d^
4*e^4*x^3 + 15*c^4*d^7*e + 17*a*c^3*d^5*e^3 + 25*a^2*c^2*d^3*e^5 - 105*a^3
*c*d*e^7 + 8*(c^4*d^5*e^3 - 7*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 + 6*a*
c^3*d^4*e^4 - 35*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x))/(c^5*d^5*e^4), 1/384*(3*(5*c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d
^4*e^4 + 20*a^3*c*d^2*e^6 - 35*a^4*e^8)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)
/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(48*c^4*
d^4*e^4*x^3 + 15*c^4*d^7*e + 17*a*c^3*d^5*e^3 + 25*a^2*c^2*d^3*e^5 - 105*a
^3*c*d*e^7 + 8*(c^4*d^5*e^3 - 7*a*c^3*d^3*e^5)*x^2 - 2*(5*c^4*d^6*e^2 + 6*
a*c^3*d^4*e^4 - 35*a^2*c^2*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x))/(c^5*d^5*e^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(343) = 686$.

Time = 1.17 (sec) , antiderivative size = 906, normalized size of antiderivative = 2.67

$$\int \frac{x^3(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Piecewise(((-a*(-3*a*e/(4*c) - (d - (7*a*e**2/2 + 7*c*d**2/2)/(4*c*d))*(5*
a*e**2/2 + 5*c*d**2/2)/(3*c*d*e))/(2*c) - (a*e**2 + c*d**2)*(-2*a*(d - (7*
a*e**2/2 + 7*c*d**2/2)/(4*c*d))/(3*c) - (3*a*e**2/2 + 3*c*d**2/2)*(-3*a*e/
(4*c) - (d - (7*a*e**2/2 + 7*c*d**2/2)/(4*c*d))*(5*a*e**2/2 + 5*c*d**2/2)/
(3*c*d*e))/(2*c*d*e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*
x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d
*e), Ne(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d*
*2)/(2*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e
**2 - c*d**2)/(2*c*d*e))**2), True)) + sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2
 + c*d**2))*(x**3/(4*c*d) + x**2*(d - (7*a*e**2/2 + 7*c*d**2/2)/(4*c*d))/(
3*c*d*e) + x*(-3*a*e/(4*c) - (d - (7*a*e**2/2 + 7*c*d**2/2)/(4*c*d))*(5*a*
e**2/2 + 5*c*d**2/2)/(3*c*d*e))/(2*c*d*e) + (-2*a*(d - (7*a*e**2/2 + 7*c*d
**2/2)/(4*c*d))/(3*c) - (3*a*e**2/2 + 3*c*d**2/2)*(-3*a*e/(4*c) - (d - (7*
a*e**2/2 + 7*c*d**2/2)/(4*c*d))*(5*a*e**2/2 + 5*c*d**2/2)/(3*c*d*e))/(2*c*
d*e))/(c*d*e)), Ne(c*d*e, 0)), (2*(-a**3*c*d**6*e**3*sqrt(a*d*e + x*(a*e**
2 + c*d**2))/(a*e**2 + c*d**2) + e*(a*d*e + x*(a*e**2 + c*d**2))**(9/2)/(9
*(a*e**2 + c*d**2)) + (a*d*e + x*(a*e**2 + c*d**2))**(7/2)*(-3*a*d*e**2 +
c*d**3)/(7*(a*e**2 + c*d**2)) + (a*d*e + x*(a*e**2 + c*d**2))**(5/2)*(3*a*
*2*d**2*e**3 - 3*a*c*d**4*e)/(5*(a*e**2 + c*d**2)) + (a*d*e + x*(a*e**2 +
c*d**2))**(3/2)*(-a**3*d**3*e**4 + 3*a**2*c*d**5*e**2)/(3*(a*e**2 + c*d...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.90

$$\int \frac{x^3(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{1}{192}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\left(4\,x\left(\frac{6\,x}{cd}+\frac{c^3d^4e^2-7\,ac^2d^2e^4}{c^4d^4e^3}\right)-\frac{5\,c^3d^5e+6\,ac^2d^3e^3-35\,a^2cde^5}{c^4d^4e^3}\right.\right.$$
$$+\frac{(5\,c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+20\,a^3cd^2e^6-35\,a^4e^8)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2}\right.\right.\right.}{128\,\sqrt{cde}c^4d^4e^3}$$

input
```
integrate(x^3*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output
```
1/192*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*(4*x*(6*x/(c*d) + (c^
3*d^4*e^2 - 7*a*c^2*d^2*e^4)/(c^4*d^4*e^3)) - (5*c^3*d^5*e + 6*a*c^2*d^3*e
^3 - 35*a^2*c*d*e^5)/(c^4*d^4*e^3))*x + (15*c^3*d^6 + 17*a*c^2*d^4*e^2 + 2
5*a^2*c*d^2*e^4 - 105*a^3*e^6)/(c^4*d^4*e^3)) + 1/128*(5*c^4*d^8 + 4*a*c^3
*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 35*a^4*e^8)*log(abs(-c*d
^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e
^2*x + a*d*e))))/(sqrt(c*d*e)*c^4*d^4*e^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{x^3\,(d+e\,x)}{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int((x^3*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
int((x^3*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.71

$$\int \frac{x^3(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{-105\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^3cd\,e^7+25\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c^2d^3e^5+70\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2c^2d^2e^6x+}{}$$

input
```
int(x^3*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
( - 105*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d*e**7 + 25*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**2*c**2*d**3*e**5 + 70*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
a**2*c**2*d**2*e**6*x + 17*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**5*e**
3 - 12*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**4*e**4*x - 56*sqrt(d + e*
x)*sqrt(a*e + c*d*x)*a*c**3*d**3*e**5*x**2 + 15*sqrt(d + e*x)*sqrt(a*e + c
*d*x)*c**4*d**7*e - 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**6*e**2*x +
8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**4*d**5*e**3*x**2 + 48*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*c**4*d**4*e**4*x**3 + 105*sqrt(e)*sqrt(d)*sqrt(c)*log((sq
rt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d
**2))*a**4*e**8 - 60*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x
) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c*d**2*e**6
 - 18*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqr
t(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**2*d**4*e**4 - 12*sqrt(e
)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))/sqrt(a*e**2 - c*d**2))*a*c**3*d**6*e**2 - 15*sqrt(e)*sqrt(d)*sqrt(
c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*
e**2 - c*d**2))*c**4*d**8)/(192*c**5*d**5*e**4)
```

3.73 $\int \frac{x^2(d+ex)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 238

$$\int \frac{x^2(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3cd} - \frac{\left((3cd^2-5ae^2)\,(cd^2+3ae^2)-2cde(cd^2-5ae^2)\,x\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24c^3d^3e^2} + \frac{(cd^2-ae^2)\,(c^2d^4+2acd^2e^2+5a^2e^4)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{7/2}d^{7/2}e^{5/2}}$$

output

```
1/3*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d-1/24*((-5*a*e^2+3*c*d^
2)*(3*a*e^2+c*d^2)-2*c*d*e*(-5*a*e^2+c*d^2)*x)*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)/c^3/d^3/e^2+1/8*(-a*e^2+c*d^2)*(5*a^2*e^4+2*a*c*d^2*e^2+c^2*d
^4)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2))/c^(7/2)/d^(7/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 10.58 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.29

$$\int \frac{x^2(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx =$$
$$-\frac{(d+ex)\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}(-15a^3e^5+a^2cde^3(4d-5ex)+c^3d^3x(3d^2-2dex-8e^2x^2)+ac^2d^2e(3d\right.}{24c^{7/2}d^{7/2}e^{5/2}\sqrt{\frac{cd(d+ex}{cd^2-ae}}}$$

input
```
Integrate[(x^2*(d + e*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
-1/24*((d + e*x)*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*
e^2)]*(-15*a^3*e^5 + a^2*c*d*e^3*(4*d - 5*e*x) + c^3*d^3*x*(3*d^2 - 2*d*e*
x - 8*e^2*x^2) + a*c^2*d^2*e*(3*d^2 + 2*d*e*x + 2*e^2*x^2)) - 3*Sqrt[c*d]*
Sqrt[c*d^2 - a*e^2]*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*Sqrt[a*e + c*d*x
]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^
2 - a*e^2])]))/(c^(7/2)*d^(7/2)*e^(5/2)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^
2)]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{\int -\frac{ex\,(4ade-(cd^2-5ae^2)x)}{2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3cde}+\frac{x^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{3cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}-\frac{\int \frac{x\left(4ade-\left(cd^2-5ae^2\right)x\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6cd}$$

$\Big\downarrow$ 1225

$$\frac{x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}-\frac{\frac{\left(\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)-2cdex\left(cd^2-5ae^2\right)\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4c^2d^2e^2}-\frac{3\left(cd^2-ae^2\right)\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}d}{8c^2d^2e^2}}{6cd}$$

$\Big\downarrow$ 1092

$$\frac{x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}-\frac{\frac{\left(\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)-2cdex\left(cd^2-5ae^2\right)\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4c^2d^2e^2}-\frac{3\left(cd^2-ae^2\right)\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+}}}{4c^2d^2e^2}}{6cd}$$

$\Big\downarrow$ 219

$$\frac{x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3cd}-\frac{\frac{\left(\left(3cd^2-5ae^2\right)\left(3ae^2+cd^2\right)-2cdex\left(cd^2-5ae^2\right)\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4c^2d^2e^2}-\frac{3\left(cd^2-ae^2\right)\left(5a^2e^4+2acd^2e^2+c^2d^4\right)\text{arctanh}\left(\frac{ae^2+cd}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2\right.}}\right.}{8c^{5/2}d^{5/2}e^{5/2}}}{6cd}$$

input
```
Int[(x^2*(d + e*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d) - ((((3*c*d^2 -
5*a*e^2)*(c*d^2 + 3*a*e^2) - 2*c*d*e*(c*d^2 - 5*a*e^2)*x)*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2])/(4*c^2*d^2*e^2) - (3*(c*d^2 - a*e^2)*(c^2*d^4
 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[
c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(8*c^(5/
2)*d^(5/2)*e^(5/2)))/(6*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 691 vs. $2(214) = 428$.

Time = 2.47 (sec) , antiderivative size = 692, normalized size of antiderivative = 2.91

method	result
default	$d\left(\frac{x\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}{2dec}-\frac{3(a\,e^{2}+c\,d^{2})\left(\frac{\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{dec}-\frac{\left(a\,e^{2}+c\,d^{2}\right)\ln\left(\frac{\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}+cdxe}{\sqrt{dec}}+\sqrt{ade+\left(a\,e^{2}+\right.}\right.}{2dec\sqrt{dec}}\right.}{4dec}\right.$

input
```
int(x^2*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
d*(1/2*x/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/4*(a*e^2+c*d^2)/d
/e/c*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/
e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-1/2*a/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/
(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+e*(1
/3*x^2/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-5/6*(a*e^2+c*d^2)/d/e
/c*(1/2*x/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/4*(a*e^2+c*d^2)/
d/e/c*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d
/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-1/2*a/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)
/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2/3
*a/c*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/
e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.23

$$\int \frac{x^2(d+ex)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$
$$= \left[-\frac{3\,(c^3d^6+ac^2d^4e^2+3\,a^2cd^2e^4-5\,a^3e^6)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\,\sqrt{cdex^2+\cdots}\right.}{\cdots}\right.$$
$$-\frac{3\,(c^3d^6+ac^2d^4e^2+3\,a^2cd^2e^4-5\,a^3e^6)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-cde}}{2\,(c^2d^2e^2x^2+acd^2e^2+(c^2d^3e+acde^3)x)}\right)-2\,(\cdots}{48\,c^4\cdots}$$

input
```
integrate(x^2*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
[-1/96*(3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(c*d
*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e
) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^3*d^3*e^3*x^2 - 3*c^3*d^5*e - 4*
a*c^2*d^3*e^3 + 15*a^2*c*d*e^5 + 2*(c^3*d^4*e^2 - 5*a*c^2*d^2*e^4)*x)*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e^3), -1/48*(3*(c^3*d^6
+ a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*sqrt(-c*d*e)*arctan(1/2*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(
-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(
8*c^3*d^3*e^3*x^2 - 3*c^3*d^5*e - 4*a*c^2*d^3*e^3 + 15*a^2*c*d*e^5 + 2*(c^
3*d^4*e^2 - 5*a*c^2*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
))/(c^4*d^4*e^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(230) = 460$.

Time = 1.11 (sec) , antiderivative size = 631, normalized size of antiderivative = 2.65

$$\int \frac{x^2(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= \begin{cases} \left(-\frac{a\left(d-\frac{\frac{5ae^2}{2}+\frac{5cd^2}{2}}{3cd}\right)}{2c}-\frac{(ae^2+cd^2)\left(-\frac{2ae}{3c}-\frac{\left(d-\frac{\frac{5ae^2}{2}+\frac{5cd^2}{2}}{3cd}\right)\left(\frac{3ae^2}{2}+\frac{3cd^2}{2}\right)}{2cde}\right)}{2cde}\right)\left(\left(\begin{cases}\frac{\log\left(ae^2+cd^2+2cdex+2\sqrt{cde}\sqrt{ade+cdex^2+}\right.}{\sqrt{cde}} \\ \frac{\left(x-\frac{-ae^2-cd^2}{2cde}\right)\log\left(x-\frac{-ae^2-cd^2}{2cde}\right)}{\sqrt{cde\left(x-\frac{-ae^2-cd^2}{2cde}\right)^2}}\end{cases}\right.\right. \\ \frac{2\left(\frac{a^2cd^5e^2\sqrt{ade+x(ae^2+cd^2)}}{ae^2+cd^2}+\frac{e(ade+x(ae^2+cd^2))^{\frac{7}{2}}}{7(ae^2+cd^2)}+\frac{(ade+x(ae^2+cd^2))^{\frac{5}{2}}(-2ade^2+cd^3)}{5(ae^2+cd^2)}+\frac{(ade+x(ae^2+cd^2))^{\frac{3}{2}}(a^2d^2e^3-2acd^4e)}{3(ae^2+cd^2)}\right)}{(ae^2+cd^2)^3} \\ \frac{\frac{dx^3}{3}+\frac{ex^4}{4}}{\sqrt{ade}} \end{cases}$$

input
```
integrate(x**2*(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Piecewise(((-a*(d - (5*a*e**2/2 + 5*c*d**2/2)/(3*c*d))/(2*c) - (a*e**2 + c
*d**2)*(-2*a*e/(3*c) - (d - (5*a*e**2/2 + 5*c*d**2/2)/(3*c*d))*(3*a*e**2/2
 + 3*c*d**2/2)/(2*c*d*e))/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*
d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt
(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 -
c*d**2)/(2*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (
-a*e**2 - c*d**2)/(2*c*d*e))**2), True)) + sqrt(a*d*e + c*d*e*x**2 + x*(a*
e**2 + c*d**2))*(x**2/(3*c*d) + x*(d - (5*a*e**2/2 + 5*c*d**2/2)/(3*c*d))/
(2*c*d*e) + (-2*a*e/(3*c) - (d - (5*a*e**2/2 + 5*c*d**2/2)/(3*c*d))*(3*a*e
**2/2 + 3*c*d**2/2)/(2*c*d*e))/(c*d*e)), Ne(c*d*e, 0)), (2*(a**2*c*d**5*e*
*2*sqrt(a*d*e + x*(a*e**2 + c*d**2))/(a*e**2 + c*d**2) + e*(a*d*e + x*(a*e
**2 + c*d**2))**(7/2)/(7*(a*e**2 + c*d**2)) + (a*d*e + x*(a*e**2 + c*d**2)
)**(5/2)*(-2*a*d*e**2 + c*d**3)/(5*(a*e**2 + c*d**2)) + (a*d*e + x*(a*e**2
 + c*d**2))**(3/2)*(a**2*d**2*e**3 - 2*a*c*d**4*e)/(3*(a*e**2 + c*d**2)))/
(a*e**2 + c*d**2)**3, Ne(a*e**2 + c*d**2, 0)), ((d*x**3/3 + e*x**4/4)/sqrt
(a*d*e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.94

$$\int \frac{x^2(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{1}{24}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(2\,x\left(\frac{4\,x}{cd}+\frac{c^2d^3e-5\,acde^3}{c^3d^3e^2}\right)-\frac{3\,c^2d^4+4\,acd^2e^2-15\,a^2e^4}{c^3d^3e^2}\right)$$
$$-\frac{(c^3d^6+ac^2d^4e^2+3\,a^2cd^2e^4-5\,a^3e^6)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+\ldots}\right)\right|\right)}{16\sqrt{cde}c^3d^3e^2}$$

input
```
integrate(x^2*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output
```
1/24*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*x*(4*x/(c*d) + (c^2*d^
3*e - 5*a*c*d*e^3)/(c^3*d^3*e^2)) - (3*c^2*d^4 + 4*a*c*d^2*e^2 - 15*a^2*e^
4)/(c^3*d^3*e^2)) - 1/16*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^
3*e^6)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^3*d^3*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{x^2\,(d+e\,x)}{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int((x^2*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
int((x^2*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.68

$$\int \frac{x^2(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{15\sqrt{ex+d}\,\sqrt{cdx+ae}\,a^2cd\,e^5-4\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^3e^3-10\sqrt{ex+d}\,\sqrt{cdx+ae}\,a\,c^2d^2e^4x-3\sqrt{e}}{}$$

input
```
int(x^2*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(15*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d*e**5 - 4*sqrt(d + e*x)*sqrt(a
*e + c*d*x)*a*c**2*d**3*e**3 - 10*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d
**2*e**4*x - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e + 2*sqrt(d + e*
x)*sqrt(a*e + c*d*x)*c**3*d**4*e**2*x + 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*
c**3*d**3*e**3*x**2 - 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**6 +
9*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)
*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**4 + 3*sqrt(e)*sqrt(d
)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/
sqrt(a*e**2 - c*d**2))*a*c**2*d**4*e**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((s
qrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*
d**2))*c**3*d**6)/(24*c**4*d**4*e**3)
```

3.74 $\int \frac{x(d+ex)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 155

$$\int \frac{x(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{(cd^2-3ae^2+2cdex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2e} - \frac{(cd^2-ae^2)\,(cd^2+3ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^{5/2}d^{5/2}e^{3/2}}$$

output
```
1/4*(2*c*d*e*x-3*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/
d^2/e-1/4*(-a*e^2+c*d^2)*(3*a*e^2+c*d^2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e
^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.31

$$\int \frac{x(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= \frac{\sqrt{c}\sqrt{d}\sqrt{e}(d+ex)\left(-3a^2e^3+acde(d-ex)+c^2d^2x(d+2ex)\right)+2(c^2d^4+2acd^2e^2-3a^2e^4)\sqrt{ae+cdx}\sqrt{}}{4c^{5/2}d^{5/2}e^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(x*(d + e*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[d]*Sqrt[e]*(d + e*x)*(-3*a^2*e^3 + a*c*d*e*(d - e*x) + c^2*d
^2*x*(d + 2*e*x)) + 2*(c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*Sqrt[a*e + c*d
*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*(Sq
rt[d - (a*e^2)/(c*d)] - Sqrt[d + e*x]))])/(4*c^(5/2)*d^(5/2)*e^(3/2)*Sqrt[
(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \; 1225$$

$$\frac{\left(-3ae^2+cd^2+2cdex\right)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{4c^2d^2e}-\frac{\left(cd^2-ae^2\right)\left(3ae^2+cd^2\right)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8c^2d^2e}$$

$$\Big\downarrow \; 1092$$

$$\frac{(-3ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4c^2d^2e}-\frac{(cd^2-ae^2)(3ae^2+cd^2)\int\frac{1}{4cde-\frac{(cd^2+2cexd+ae^2)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{4c^2d^2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{(-3ae^2+cd^2+2cdex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4c^2d^2e}-\frac{(cd^2-ae^2)(3ae^2+cd^2)\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{3/2}}$$

input
```
Int[(x*(d + e*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
((c*d^2 - 3*a*e^2 + 2*c*d*e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
)/(4*c^2*d^2*e) - ((c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^
2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])])/(8*c^(5/2)*d^(5/2)*e^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(135) = 270.

Time = 2.22 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.41

method	result
default	$d\left(\frac{\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}}{dec}-\frac{(a\,e^{2}+c\,d^{2})\ln\left(\frac{\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}+cdxe}{\sqrt{dec}}+\sqrt{ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e}\right)}{2dec\sqrt{dec}}\right)+e\left(\frac{x\sqrt{ade+(a\,e^{2}}{2\ldots}\right.$

input
```
int(x*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
d*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c
*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2))/(d*e*c)^(1/2))+e*(1/2*x/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^
2*e)^(1/2)-3/4*(a*e^2+c*d^2)/d/e/c*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)
^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-1/2*a/c*ln(
(1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.68

$$\int \frac{x(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\left[-\frac{(c^2d^4+2\,acd^2e^2-3\,a^2e^4)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+4\,\sqrt{cdex^2+ade+(cd^2+\ldots}\right.}{\ldots}\right.$$

input
```
integrate(x*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
fricas")
```

output
```
[-1/16*((c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*sqrt(c*d*e)*log(8*c^2*d^2*e^
2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a
*c*d*e^3)*x) - 4*(2*c^2*d^2*e^2*x + c^2*d^3*e - 3*a*c*d*e^3)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^2), 1/8*((c^2*d^4 + 2*a*c*d^2*e
^2 - 3*a^2*e^4)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*
d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(2*c^2*d^2*e^2*x + c^2*d^3*e - 3
*a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^3*d^3*e^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(151) = 302$.

Time = 1.31 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.87

$$\int \frac{x(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \begin{cases} \left(-\frac{ae}{2c}-\frac{\left(d-\frac{\frac{3ae^2}{2}+\frac{3cd^2}{2}}{2cd}\right)(ae^2+cd^2)}{2cde}\right)\left(\begin{cases}\frac{\log\left(ae^2+cd^2+2cdex+2\sqrt{cde}\sqrt{ade+cdex^2+x(ae^2+cd^2)}\right)}{\sqrt{cde}} & \text{for } ade-\frac{(ae^2+cd^2)^2}{4cde} \\ \frac{\left(x-\frac{-ae^2-cd^2}{2cde}\right)\log\left(x-\frac{-ae^2-cd^2}{2cde}\right)}{\sqrt{cde\left(x-\frac{-ae^2-cd^2}{2cde}\right)^2}} & \text{otherwise}\end{cases}\right. \\ \frac{2\left(-\frac{acd^4e\sqrt{ade+x\left(ae^2+cd^2\right)}}{ae^2+cd^2}+\frac{e\left(ade+x\left(ae^2+cd^2\right)\right)^{\frac{5}{2}}}{5\left(ae^2+cd^2\right)}+\frac{\left(ade+x\left(ae^2+cd^2\right)\right)^{\frac{3}{2}}\left(-ade^2+cd^3\right)}{3\left(ae^2+cd^2\right)}\right)}{(ae^2+cd^2)^2} \\ \frac{\frac{dx^2}{2}+\frac{ex^3}{3}}{\sqrt{ade}} \end{cases}$$

input
```
integrate(x*(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Piecewise(((-a*e/(2*c) - (d - (3*a*e**2/2 + 3*c*d**2/2)/(2*c*d))*(a*e**2 +
 c*d**2)/(2*c*d*e))*Piecewise((log(a*e**2 + c*d**2 + 2*c*d*e*x + 2*sqrt(c*
d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/sqrt(c*d*e), Ne(a*d*e
 - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e**2 - c*d**2)/(2*c*d*e)
)*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*(x - (-a*e**2 - c*d**2)
/(2*c*d*e))**2), True)) + (x/(2*c*d) + (d - (3*a*e**2/2 + 3*c*d**2/2)/(2*c
*d))/(c*d*e))*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)), Ne(c*d*e, 0)
), (2*(-a*c*d**4*e*sqrt(a*d*e + x*(a*e**2 + c*d**2))/(a*e**2 + c*d**2) + e
*(a*d*e + x*(a*e**2 + c*d**2))**(5/2)/(5*(a*e**2 + c*d**2)) + (a*d*e + x*(
a*e**2 + c*d**2))**(3/2)*(-a*d*e**2 + c*d**3)/(3*(a*e**2 + c*d**2)))/(a*e*
*2 + c*d**2)**2, Ne(a*e**2 + c*d**2, 0)), ((d*x**2/2 + e*x**3/3)/sqrt(a*d*
e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.06

$$\int \frac{x(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{1}{4}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(\frac{2\,x}{cd}+\frac{cd^2-3\,ae^2}{c^2d^2e}\right)$$
$$+\frac{(c^2d^4+2\,acd^2e^2-3\,a^2e^4)\log\left(\left|-cd^2-ae^2-2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{8\,\sqrt{cde}c^2d^2e}$$

input
```
integrate(x*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
giac")
```

output
```
1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*x/(c*d) + (c*d^2 - 3*a*
e^2)/(c^2*d^2*e)) + 1/8*(c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*log(abs(-c*d
^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e
^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex)}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{x\,(d+e\,x)}{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int((x*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
int((x*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.63

$$\int \frac{x(d+ex)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$
$$= \frac{-3\sqrt{ex+d}\,\sqrt{cdx+ae}\,acd\,e^3+\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^3e+2\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^2e^2x+3\sqrt{e}\,\sqrt{d}\,\sqrt{c}}{}$$

input
```
int(x*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
( - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d*e**3 + sqrt(d + e*x)*sqrt(a*e
+ c*d*x)*c**2*d**3*e + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**2*e**2*x
+ 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(
c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**4 - 2*sqrt(e)*sqrt(d)*sqr
t(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(
a*e**2 - c*d**2))*a*c*d**2*e**2 - sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqr
t(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**
2*d**4)/(4*c**3*d**3*e**2)
```

3.75 $\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 116

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cd} + \frac{(cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{3/2}d^{3/2}\sqrt{e}}$$

output
```
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d+(-a*e^2+c*d^2)*arctanh(c^(1/2)
*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/
d^(3/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.17

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)(d+ex)+(cd^2-ae^2)\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{c^{3/2}d^{3/2}\sqrt{e}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(d + e*x) + (c*d^2 - a*e^2)*Sqrt[a*
e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*
Sqrt[a*e + c*d*x])])/(c^(3/2)*d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e*x)
])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2d}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{d}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(d^2-\frac{ae^2}{c}\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{c}d^{3/2}\sqrt{e}}+\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd}$$

input
```
Int[(d + e*x)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2],x]
```

output
```
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(c*d) + ((d^2 - (a*e^2)/c)*Arc
Tanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*d^(3/2)*Sqrt[e])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.60

method	result
default	$\frac{d\ln\left(\frac{\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}+cdxe}{\sqrt{dec}}+\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}\right)}{\sqrt{dec}}+e\left(\frac{\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{dec}-\frac{\left(a\,e^{2}+c\,d^{2}\right)\ln\left(\frac{\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}+cd}{\sqrt{dec}}\right.}{2de}\right.$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
d*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2))/(d*e*c)^(1/2)+e*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)
^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1
/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.91

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[\frac{4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}cde-(cd^2-ae^2)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4-4\,\sqrt{\right.}{4\,c^2d^2e}\right.$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="fr
icas")
```

output
```
[1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e - (c*d^2 - a*e^2
)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 -
4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*
sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c^2*d^2*e), 1/2*(2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e - (c*d^2 - a*e^2)*sqrt(-c*d*e)*arc
tan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a
*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3
)*x)))/(c^2*d^2*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(107) = 214$.

Time = 0.84 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.89

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \begin{cases} \left(d-\frac{ae^2+cd^2}{2cd}\right)\left(\begin{cases} \frac{\log\left(ae^2+cd^2+2cdex+2\sqrt{cde}\sqrt{ade+cdex^2+x(ae^2+cd^2)}\right)}{\sqrt{cde}} & \text{for } ade-\frac{(ae^2+cd^2)^2}{4cde}\neq 0 \\ \frac{\left(x-\frac{-ae^2-cd^2}{2cde}\right)\log\left(x-\frac{-ae^2-cd^2}{2cde}\right)}{\sqrt{cde\left(x-\frac{-ae^2-cd^2}{2cde}\right)^2}} & \text{otherwise} \end{cases}\right)+\frac{\sqrt{ade+cdex^2 \ldots}}{\ldots} \\ \frac{2d\sqrt{ade+x(ae^2+cd^2)}+\frac{2e\left(-ade\sqrt{ade+x(ae^2+cd^2)}+\frac{(ade+x(ae^2+cd^2))^{\frac{3}{2}}}{3}\right)}{ae^2+cd^2}}{ae^2+cd^2} \\ \frac{dx+\frac{ex^2}{2}}{\sqrt{ade}} \end{cases}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Piecewise(((d - (a*e**2 + c*d**2)/(2*c*d))*Piecewise((log(a*e**2 + c*d**2
+ 2*c*d*e*x + 2*sqrt(c*d*e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))
)/sqrt(c*d*e), Ne(a*d*e - (a*e**2 + c*d**2)**2/(4*c*d*e), 0)), ((x - (-a*e
**2 - c*d**2)/(2*c*d*e))*log(x - (-a*e**2 - c*d**2)/(2*c*d*e))/sqrt(c*d*e*
(x - (-a*e**2 - c*d**2)/(2*c*d*e))**2), True)) + sqrt(a*d*e + c*d*e*x**2 +
 x*(a*e**2 + c*d**2))/(c*d), Ne(c*d*e, 0)), ((2*d*sqrt(a*d*e + x*(a*e**2 +
 c*d**2)) + 2*e*(-a*d*e*sqrt(a*d*e + x*(a*e**2 + c*d**2)) + (a*d*e + x*(a*
e**2 + c*d**2))**(3/2)/3)/(a*e**2 + c*d**2))/(a*e**2 + c*d**2), Ne(a*e**2
+ c*d**2, 0)), ((d*x + e*x**2/2)/sqrt(a*d*e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = -\frac{(cd^2-ae^2)\log\left(\left|-cd^2-ae^2-2\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{2\sqrt{cde}cd} + \frac{\sqrt{cdex^2+cd^2x+ae^2x+ade}}{cd}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="gi
ac")
```

output
```
-1/2*(c*d^2 - a*e^2)*log(abs(-c*d^2 - a*e^2 - 2*sqrt(c*d*e)*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c*d) + sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)/(c*d)
```

Mupad [B] (verification not implemented)

Time = 6.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.24

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{c\,d} - \frac{a\,e^3\,\ln\left(2\,\sqrt{\left(a\,e+c\,d\,x\right)\,\left(d+e\,x\right)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)}{2\,{\left(c\,d\,e\right)}^{3/2}} + \frac{c\,d^2\,e\,\ln\left(2\,\sqrt{\left(a\,e+c\,d\,x\right)\,\left(d+e\,x\right)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)}{2\,{\left(c\,d\,e\right)}^{3/2}}$$

input
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2),x)
```

output
```
(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(c*d) - (a*e^3*log(2*((a*e +
 c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^2 + 2*c*d*e*x))/(2*(c
*d*e)^(3/2)) + (c*d^2*e*log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2
) + a*e^2 + c*d^2 + 2*c*d*e*x))/(2*(c*d*e)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16

$$\int \frac{d+ex}{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{\sqrt{ex+d}\,\sqrt{cdx+ae}\,cde-\sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a\,e^2+\sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{}}{\sqrt{a\,e^2-c\,d^2}}\right.}{c^2d^2e}$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(sqrt(d + e*x)*sqrt(a*e + c*d*x)*c*d*e - sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt
(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**
2))*a*e**2 + sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**2)/(c**2*d**2*e)
```

3.76 $\int \frac{d+ex}{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 137

$$\int \frac{d+ex}{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{c}\sqrt{d}} - \frac{2\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{a}\sqrt{e}}$$

output
```
2*e^(1/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/c^(1/2)/d^(1/2)-2*d^(1/2)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)
/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(1/2)/e^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 485 vs. $2(137) = 274$.

Time = 2.27 (sec) , antiderivative size = 485, normalized size of antiderivative = 3.54

$$\int \frac{d+ex}{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx =$$

$$-\frac{2\sqrt{ae+cdx}\sqrt{d+ex}\left(\sqrt{c}d\left(-\left(\left(-\sqrt{c}d+\sqrt{cd^2-ae^2}\right)\sqrt{-2cd^2+ae^2-2\sqrt{c}d\sqrt{cd^2-ae^2}}\arctan\left(\sqrt{\right.\right.\right.\right.}{}$$

input
```
Integrate[(d + e*x)/(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(-2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(Sqrt[c]*d*(-((-(Sqrt[c]*d) + Sqrt[c*d
^2 - a*e^2])*Sqrt[-2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]]*ArcT
an[(Sqrt[-2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]]*Sqrt[a*e + c*
d*x])/(Sqrt[a]*Sqrt[c]*Sqrt[d]*Sqrt[e]*(Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d +
 e*x]))]) + (Sqrt[c]*d + Sqrt[c*d^2 - a*e^2])*Sqrt[-2*c*d^2 + a*e^2 + 2*Sq
rt[c]*d*Sqrt[c*d^2 - a*e^2]]*ArcTan[(Sqrt[-2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*S
qrt[c*d^2 - a*e^2]]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[c]*Sqrt[d]*Sqrt[e]*(S
qrt[d - (a*e^2)/(c*d)] - Sqrt[d + e*x]))]) + 2*a^(3/2)*e^3*ArcTanh[(Sqrt[e
]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*(Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d +
e*x]))]))/(a^(3/2)*Sqrt[c]*Sqrt[d]*e^(5/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1268, 140, 27, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \ 1268$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{\sqrt{d+ex}}{x\sqrt{ae+cdx}}dx}{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \textbf{140}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(e\int \frac{1}{\sqrt{ae+cdx}\sqrt{d+ex}}dx+\int \frac{d}{x\sqrt{ae+cdx}\sqrt{d+ex}}dx\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(e\int \frac{1}{\sqrt{ae+cdx}\sqrt{d+ex}}dx+d\int \frac{1}{x\sqrt{ae+cdx}\sqrt{d+ex}}dx\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \textbf{66}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(d\int \frac{1}{x\sqrt{ae+cdx}\sqrt{d+ex}}dx+2e\int \frac{1}{cd-\frac{e(ae+cdx)}{d+ex}}d\frac{\sqrt{ae+cdx}}{\sqrt{d+ex}}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \textbf{104}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(2e\int \frac{1}{cd-\frac{e(ae+cdx)}{d+ex}}d\frac{\sqrt{ae+cdx}}{\sqrt{d+ex}}+2d\int \frac{1}{\frac{ae(d+ex)}{ae+cdx}-d}d\frac{\sqrt{d+ex}}{\sqrt{ae+cdx}}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{c}\sqrt{d}}-\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sqrt{d+ex}}{\sqrt{d}\sqrt{ae+cdx}}\right)}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(d + e*x)/(x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*((2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[a*e + c
*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(Sqrt[c]*Sqrt[d]) - (2*Sqrt[d]*Ar
cTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(Sqrt[
a]*Sqrt[e])))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{e \ln\left(\frac{\frac{1}{2} a\,e^{2}+\frac{1}{2} c\,d^{2}+c d x e}{\sqrt{d e c}}+\sqrt{a d e+\left(a\,e^{2}+c\,d^{2}\right) x+c d\,x^{2} e}\right)}{\sqrt{d e c}}-\frac{d \ln\left(\frac{2 a d e+\left(a\,e^{2}+c\,d^{2}\right) x+2 \sqrt{a d e}\,\sqrt{a d e+\left(a\,e^{2}+c\,d^{2}\right) x+c d\,x^{2} e}}{x}\right)}{\sqrt{a d e}}$	132

input

```
int((e*x+d)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERB
OSE)
```

output

```
e*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2))/(d*e*c)^(1/2)-d/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+
2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 975, normalized size of antiderivative = 7.12

$$\int \frac{d+ex}{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
fricas")
```

output
```
[1/2*sqrt(e/(c*d))*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e
^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x + 4*(2*c^2*d^2*e*x + c^2*d^3 + a*c*d*e^2)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e/(c*d))) + 1/2*sqrt(d/(
a*e))*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 8*(a*
c*d^3*e + a^2*d*e^3)*x - 4*(2*a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)*sqrt(c*
d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(d/(a*e)))/x^2), -sqrt(-e/(c*d))*
arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2
+ a*e^2)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) + 1
/2*sqrt(d/(a*e))*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*
x^2 + 8*(a*c*d^3*e + a^2*d*e^3)*x - 4*(2*a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)
*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(d/(a*e)))/x^2), sqrt(
-d/(a*e))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(-d/(a*e))/(c*d^2*e*x^2 + a*d^2*e + (c*d^3 + a*d*
e^2)*x)) + 1/2*sqrt(e/(c*d))*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e
^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x + 4*(2*c^2*d^2*e*x + c^2*d^3 +
a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e/(c*d))), sqr
t(-d/(a*e))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(-d/(a*e))/(c*d^2*e*x^2 + a*d^2*e + (c*d^3 + a*
d*e^2)*x)) - sqrt(-e/(c*d))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*...
```

Sympy [F]

$$\int \frac{d+ex}{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{d+ex}{x\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)/x/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral((d + e*x)/(x*sqrt((d + e*x)*(a*e + c*d*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.86

$$\int \frac{d+ex}{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{e\ln\left(2\sqrt{(a\,e+c\,d\,x)\,(d+e\,x)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)}{\sqrt{c\,d\,e}}$$
$$-\frac{d\ln\left(\frac{a\,e^2}{2}+\frac{c\,d^2}{2}+\frac{\sqrt{a\,d\,e}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{x}+\frac{a\,d\,e}{x}\right)}{\sqrt{a\,d\,e}}$$

input
```
int((d + e*x)/(x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
(e*log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^2 + 2
*c*d*e*x))/(c*d*e)^(1/2) - (d*log((a*e^2)/2 + (c*d^2)/2 + ((a*d*e)^(1/2)*(
x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/x + (a*d*e)/x))/(a*d*e)^(1/2
)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.55

$$\int \frac{d+ex}{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{\sqrt{e}\,\sqrt{d}\left(\sqrt{a}\log\left(\sqrt{e}\,\sqrt{cdx+ae}-\sqrt{2\sqrt{c}\,\sqrt{a}\,de+a\,e^2+c\,d^2}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}\right)cd+\sqrt{a}\log\left(\sqrt{e}\,\sqrt{cdx}\right.\right.}{}$$

input
```
int((e*x+d)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(sqrt(e)*sqrt(d)*(sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*s
qrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c*d + sqrt(
a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c
*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c*d - sqrt(a)*log(2*sqrt(e)*sqrt(d
)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*
e*x)*c*d + 2*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt
(d + e*x))/sqrt(a*e**2 - c*d**2))*a*e))/(a*c*d*e)
```

3.77 $\int \frac{d+ex}{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 120

$$\int \frac{d+ex}{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{aex} + \frac{(cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{3/2}\sqrt{d}e^{3/2}}$$

output
```
-(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/e/x+(-a*e^2+c*d^2)*arctanh(a^(1
/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/
2)/d^(1/2)/e^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 728 vs. 2(120) = 240.

Time = 14.14 (sec) , antiderivative size = 728, normalized size of antiderivative = 6.07

$$\int \frac{d+ex}{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= \frac{\sqrt{ae+cdx}\sqrt{d+ex}\left(\frac{\sqrt{a}\sqrt{e}\sqrt{ae+cdx}\left(ae^2\left(-3d-3ex+\sqrt{d-\frac{ae^2}{cd}}\sqrt{d+ex}\right)+c\left(4d^3+d^2\left(5ex-4\sqrt{d-\frac{ae^2}{cd}}\sqrt{d+ex}\right)+d\left(e^2x^2-3e\sqrt{d-\cdots}\right.\right.\right.}{x\left(cd^2\left(4\sqrt{d-\frac{ae^2}{cd}}-4\sqrt{d+ex}\right)+cdex\left(3\sqrt{d-\frac{ae^2}{cd}}-\sqrt{d+ex}\right)+ae^2\left(-\sqrt{d-\frac{ae^2}{cd}}+3\sqrt{d+ex}\right)\right)}\cdots\right.}{\cdots}$$

input
```
Integrate[(d + e*x)/(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*((Sqrt[a]*Sqrt[e]*Sqrt[a*e + c*d*x]*(a*e^
2*(-3*d - 3*e*x + Sqrt[d - (a*e^2)/(c*d)]*Sqrt[d + e*x]) + c*(4*d^3 + d^2*
(5*e*x - 4*Sqrt[d - (a*e^2)/(c*d)]*Sqrt[d + e*x]) + d*(e^2*x^2 - 3*e*Sqrt[
d - (a*e^2)/(c*d)]*x*Sqrt[d + e*x]))))/(x*(c*d^2*(4*Sqrt[d - (a*e^2)/(c*d)
] - 4*Sqrt[d + e*x]) + c*d*e*x*(3*Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d + e*x])
 + a*e^2*(-Sqrt[d - (a*e^2)/(c*d)] + 3*Sqrt[d + e*x]))) - (Sqrt[c*d^2 - a*
e^2]*(c*d^2 - a*e^2 + Sqrt[c]*d*Sqrt[c*d^2 - a*e^2])*ArcTan[(Sqrt[-2*c*d^2
 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sq
rt[c]*Sqrt[d]*Sqrt[e]*(Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d + e*x]))])/(Sqrt[d
]*Sqrt[-2*c*d^2 + a*e^2 - 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]]) - (Sqrt[c*d^2
- a*e^2]*(-(c*d^2) + a*e^2 + Sqrt[c]*d*Sqrt[c*d^2 - a*e^2])*ArcTan[(Sqrt[-
2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]]*Sqrt[a*e + c*d*x])/(Sqr
t[a]*Sqrt[c]*Sqrt[d]*Sqrt[e]*(Sqrt[d - (a*e^2)/(c*d)] - Sqrt[d + e*x]))])/
(Sqrt[d]*Sqrt[-2*c*d^2 + a*e^2 + 2*Sqrt[c]*d*Sqrt[c*d^2 - a*e^2]])))/(a^(3
/2)*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \; 1228$$

$$-\frac{1}{2}\left(\frac{cd^2}{ae}-e\right)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{aex}$$

$$\Big\downarrow \; 1154$$

$$\left(\frac{cd^2}{ae}-e\right)\int \frac{1}{4ade-\frac{(2ade+(cd^2+ae^2)x)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{2ade+(cd^2+ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{aex}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(\frac{cd^2}{ae}-e\right)\text{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{a}\sqrt{d}\sqrt{e}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{aex}$$

input
```
Int[(d + e*x)/(x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(a*e*x)) + (((c*d^2)/(a*e) -
 e)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[a]*Sqrt[d]*Sqrt[e])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.68

method	result
default	$d\left(-\frac{\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}{adex}+\frac{(a\,e^2+c\,d^2)\ln\left(\frac{2ade+\left(a\,e^2+c\,d^2\right)x+2\sqrt{ade}\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}{x}\right)}{2ade\sqrt{ade}}\right)-\frac{e\ln\left(\frac{2ade+}{}\right.}{}$

input
```
int((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)/a/
d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)
*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.95

$$\int \frac{d+ex}{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\left[-\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}ade+(cd^2-ae^2)\sqrt{ade}x\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2-4\sqrt{cdex^2+ade}}{}\right.}{4\,a^2de^2x}\right.$$
$$-\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}ade+(cd^2-ae^2)\sqrt{-ade}x\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,ade+(cd^2+ae^2)}{2\,(acd^2e^2x^2+a^2d^2e^2+(acd^3e+a^2de^3)x}\right.}{2\,a^2de^2x}$$

input
```
integrate((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
[-1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e + (c*d^2 - a*e^
2)*sqrt(a*d*e)*x*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*
x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2))/(a^2*d*e^2*x), -1
/2*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e + (c*d^2 - a*e^2)*
sqrt(-a*d*e)*x*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a
*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a
*c*d^3*e + a^2*d*e^3)*x)))/(a^2*d*e^2*x)]
```

Sympy [F]

$$\int \frac{d+ex}{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{d+ex}{x^2\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)/x**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral((d + e*x)/(x**2*sqrt((d + e*x)*(a*e + c*d*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(104) = 208$.

Time = 0.14 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.89

$$\int \frac{d+ex}{x^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= -\frac{(cd^2-ae^2)\arctan\left(-\frac{\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}}{\sqrt{-ade}}\right)}{\sqrt{-ade}ae}$$
$$-\frac{\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)cd^2+\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)ae^2+2\sqrt{cd}}{\left(ade-\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)^2\right)ae}$$

input
```
integrate((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output
```
-(c*d^2 - a*e^2)*arctan(-(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2
*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a*e) - ((sqrt(c*d*e)*x - sqrt(c*d
*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*c*d^2 + (sqrt(c*d*e)*x - sqrt(c*d*e*x
^2 + c*d^2*x + a*e^2*x + a*d*e))*a*e^2 + 2*sqrt(c*d*e)*a*d*e)/((a*d*e - (s
qrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)*a*e)
```

Mupad [B] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.50

$$\int \frac{d+ex}{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{d\,\mathrm{atanh}\left(\frac{\frac{x\,(c\,d^2+a\,e^2)}{2}+a\,d\,e}{\sqrt{a\,d\,e}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\right)\,(c\,d^2+a\,e^2)}{2\,(a\,d\,e)^{3/2}} - \frac{\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{a\,e\,x} - \frac{e\,\ln\left(\frac{a\,e^2}{2}+\frac{c\,d^2}{2}+\frac{\sqrt{a\,d\,e}\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}{x}+\frac{a\,d\,e}{x}\right)}{\sqrt{a\,d\,e}}$$

input
```
int((d + e*x)/(x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
(d*atanh(((x*(a*e^2 + c*d^2))/2 + a*d*e)/((a*d*e)^(1/2)*(x*(a*e^2 + c*d^2)
 + a*d*e + c*d*e*x^2)^(1/2)))*(a*e^2 + c*d^2))/(2*(a*d*e)^(3/2)) - (x*(a*e
^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)/(a*e*x) - (e*log((a*e^2)/2 + (c*d^2
)/2 + ((a*d*e)^(1/2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/x + (a
*d*e)/x))/(a*d*e)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.10

$$\int \frac{d+ex}{x^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \frac{-2\sqrt{ex+d}\,\sqrt{cdx+ae}\,ade+\sqrt{e}\,\sqrt{d}\,\sqrt{a}\,\log\left(\sqrt{e}\,\sqrt{cdx+ae}-\sqrt{2\sqrt{c}\,\sqrt{a}\,de+a\,e^2+c\,d^2}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex}\right.}{}$$

input
```
int((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
( - 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*d*e + sqrt(e)*sqrt(d)*sqrt(a)*log(
sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*e**2*x - sqrt(e)*sqrt(d)*sqrt(a)*log(sq
rt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) +
sqrt(d)*sqrt(c)*sqrt(d + e*x))*c*d**2*x + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt
(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))*a*e**2*x - sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e
)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))*c*d**2*x - sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e
)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e
+ 2*c*d*e*x)*a*e**2*x + sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d)*sqrt
(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*c
*d**2*x)/(2*a**2*d*e**2*x)
```

3.78 $\int \frac{d+ex}{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 195

$$\int \frac{d+ex}{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2aex^2}+\frac{(3cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4a^2de^2x} - \frac{(cd^2-ae^2)\,(3cd^2+ae^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4a^{5/2}d^{3/2}e^{5/2}}$$

output
```
-1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/e/x^2+1/4*(-a*e^2+3*c*d^2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d/e^2/x-1/4*(-a*e^2+c*d^2)*(a*
e^2+3*c*d^2)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(3/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 10.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.99

$$\int \frac{d+ex}{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{-\sqrt{a}\sqrt{d}\sqrt{e}(d+ex)\left(-3c^2d^3x^2+acdex(-d+ex)+a^2e^2(2d+ex)\right)-\left(3c^2d^4-2acd^2e^2-a^2e^4\right)x^2\sqrt{ae}}{4a^{5/2}d^{3/2}e^{5/2}x^2\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)/(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(-(Sqrt[a]*Sqrt[d]*Sqrt[e]*(d + e*x)*(-3*c^2*d^3*x^2 + a*c*d*e*x*(-d + e*x
) + a^2*e^2*(2*d + e*x))) - (3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*x^2*Sqrt
[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*S
qrt[e]*Sqrt[d + e*x])])/(4*a^(5/2)*d^(3/2)*e^(5/2)*x^2*Sqrt[(a*e + c*d*x)*
(d + e*x)])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$
$$\Big\downarrow \; 1237$$
$$-\frac{\int \frac{d(3cd^2+2cexd-ae^2)}{2x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2ade}-\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2aex^2}$$
$$\Big\downarrow \; 27$$
$$-\frac{\int \frac{3cd^2+2cexd-ae^2}{x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4ae}-\frac{\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{2aex^2}$$

$$\Big\downarrow \text{1228}$$

$$-\frac{-\frac{\left(\frac{3c^2d^4}{a}-ae^4-2cd^2e^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2de}-\frac{\left(\frac{3cd}{ae}-\frac{e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{4ae}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2aex^2}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{\frac{\left(\frac{3c^2d^4}{a}-ae^4-2cd^2e^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{de}-\frac{\left(\frac{3cd}{ae}-\frac{e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{4ae}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2aex^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\left(\frac{3c^2d^4}{a}-ae^4-2cd^2e^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}d^{3/2}e^{3/2}}-\frac{\left(\frac{3cd}{ae}-\frac{e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{4ae}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2aex^2}$$

input

```
Int[(d + e*x)/(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output

```
-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(a*e*x^2) - (-((((3*c*d)/
(a*e) - e/d)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x) + (((3*c^2*d^
4)/a - 2*c*d^2*e^2 - a*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[
a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[
a]*d^(3/2)*e^(3/2)))/(4*a*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(171) = 342$.

Time = 2.33 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.05

method	result
default	$d\left(-\frac{\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}{2ade\,x^2}-\frac{3(a\,e^2+c\,d^2)\left(-\frac{\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}{adex}+\frac{\left(a\,e^2+c\,d^2\right)\ln\left(\frac{2ade+\left(a\,e^2+c\,d^2\right)x+2\sqrt{ade}\,\sqrt{a}}{x}\right.}{2ade\sqrt{ade}}\right.}{4ade}\right.$

input
```
int((e*x+d)/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/4*(a*e^2+c*d^2
)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d
^2)/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+1/2*c/a/(a*d*e)^(1/2)*ln((2*a*d*e+(
a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)
)+e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)/
a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))
```

Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.30

$$\int \frac{d+ex}{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$=\left[-\frac{(3\,c^2d^4-2\,acd^2e^2-a^2e^4)\sqrt{ade}x^2\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2+4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,ade+(cd^2+ae}{x^2}\right.}{16\,a^3d^2e^3x^2}\right.$$

input
```
integrate((e*x+d)/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
[-1/16*((3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*sqrt(a*d*e)*x^2*log((8*a^2*d
^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*
d^3*e + a^2*d*e^3)*x)/x^2) + 4*(2*a^2*d^2*e^2 - (3*a*c*d^3*e - a^2*d*e^3)*
x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*d^2*e^3*x^2), 1/8*((3
*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*sqrt(-a*d*e)*x^2*arctan(1/2*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*
e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(2*a^2
*d^2*e^2 - (3*a*c*d^3*e - a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x))/(a^3*d^2*e^3*x^2)]
```

Sympy [F]

$$\int \frac{d+ex}{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{d+ex}{x^3\sqrt{(d+ex)\,(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)/x**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral((d + e*x)/(x**3*sqrt((d + e*x)*(a*e + c*d*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(171) = 342$.

Time = 0.14 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.61

$$\int \frac{d+ex}{x^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$
$$= \frac{(3\,c^2d^4-2\,acd^2e^2-a^2e^4)\arctan\left(-\frac{\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}}{\sqrt{-ade}}\right)}{4\,\sqrt{-ade}a^2de^2}$$
$$+\frac{5\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)ac^2d^5e+10\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)a^2cd}{}$$

input
```
integrate((e*x+d)/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output
```
1/4*(3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*arctan(-(sqrt(c*d*e)*x - sqrt(c*
d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sqrt(-a*d*e)*a^2*d*e^
2) + 1/4*(5*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*
a*c^2*d^5*e + 10*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d
*e))*a^2*c*d^3*e^3 + (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x +
 a*d*e))*a^3*d*e^5 + 8*sqrt(c*d*e)*a^2*c*d^4*e^2 - 3*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*c^2*d^4 + 2*(sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a*c*d^2*e^2 + (sqrt(c*d*e)*x
- sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*e^4 + 8*sqrt(c*d*e)*(
sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^2*d*e^3)/
((a*d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)
^2*a^2*d*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{d+e\,x}{x^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)/(x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int((d + e*x)/(x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 966, normalized size of antiderivative = 4.95

$$\int \frac{d+ex}{x^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
( - 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*d**2*e**4 - 2*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**3*d*e**5*x - 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d*
*4*e**2 + 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d**3*e**3*x + 6*sqrt(d
+ e*x)*sqrt(a*e + c*d*x)*a*c**2*d**5*e*x - sqrt(e)*sqrt(d)*sqrt(a)*log(sqr
t(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + s
qrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*e**6*x**2 - 3*sqrt(e)*sqrt(d)*sqrt(a)*l
og(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**
2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**2*e**4*x**2 + sqrt(e)*sqrt(d
)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e
**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**4*e**2*x**2 + 3*s
qrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt
(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**3*d**6*x**2
 - sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*
sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*e**6*
x**2 - 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sq
rt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2
*c*d**2*e**4*x**2 + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x)
+ sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d +
 e*x))*a*c**2*d**4*e**2*x**2 + 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(
a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*...
```

3.79 $\int \frac{d+ex}{x^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 279

$$\begin{aligned}\int \frac{d+ex}{x^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx &= -\frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3aex^3}+\frac{(5cd^2-ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{12a^2de^2x^2}\\ &\quad-\frac{(5cd^2-3ae^2)\,(3cd^2+ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{24a^3d^2e^3x}\\ &\quad+\frac{(cd^2-ae^2)\,(5c^2d^4+2acd^2e^2+a^2e^4)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8a^{7/2}d^{5/2}e^{7/2}}\end{aligned}$$

output
```
-1/3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/e/x^3+1/12*(-a*e^2+5*c*d^2)
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d/e^2/x^2-1/24*(-3*a*e^2+5*c*
d^2)*(a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d^2/e^3/x
+1/8*(-a*e^2+c*d^2)*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctanh(a^(1/2)*e^(1
/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(7/2)/d^(5/
2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 10.19 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.92

$$\int \frac{d+ex}{x^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{-\sqrt{a}\sqrt{d}\sqrt{e}(d+ex)\left(15c^3d^5x^3+ac^2d^3ex^2(5d-4ex)+a^3e^3(8d^2+2dex-3e^2x^2)-a^2cde^2x(2d^2+2dex\right.}{24a^{7/2}d^{5/2}e^{7/2}x^3\sqrt{(ae+}}$$

input
```
Integrate[(d + e*x)/(x^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(-(Sqrt[a]*Sqrt[d]*Sqrt[e]*(d + e*x)*(15*c^3*d^5*x^3 + a*c^2*d^3*e*x^2*(5*
d - 4*e*x) + a^3*e^3*(8*d^2 + 2*d*e*x - 3*e^2*x^2) - a^2*c*d*e^2*x*(2*d^2
+ 2*d*e*x + 3*e^2*x^2))) + 3*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4
- a^3*e^6)*x^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e +
 c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(24*a^(7/2)*d^(5/2)*e^(7/2)*x^3
*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \ 1237$$

$$-\frac{\int \frac{d(5cd^2+4cexd-ae^2)}{2x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3ade}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3aex^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{5cd^2+4cexd-ae^2}{x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6ae}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3aex^3}$$

$\Big\downarrow$ 1237

$$-\frac{-\frac{\int \frac{\left(5cd^2-3ae^2\right)\left(3cd^2+ae^2\right)+2cde\left(5cd^2-ae^2\right)x}{2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(\frac{5cd}{ae}-\frac{e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2x^2}}{6ae}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3aex^3}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{\left(5cd^2-3ae^2\right)\left(3cd^2+ae^2\right)+2cde\left(5cd^2-ae^2\right)x}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4ade}-\frac{\left(\frac{5cd}{ae}-\frac{e}{d}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2x^2}}{6ae}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3aex^3}$$

$\Big\downarrow$ 1228

$$-\frac{-\frac{-\frac{3\left(cd^2-ae^2\right)\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(5cd^2-3ae^2\right)\left(ae^2+3cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{adex}}{4ade}-\frac{\left(\frac{5cd}{ae}-\frac{e}{d}\right)\sqrt{x(ae\cdots}}{\cdots}}{6ae}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3aex^3}$$

$\Big\downarrow$ 1154

$$-\frac{-\frac{\frac{3\left(cd^2-ae^2\right)\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}-\frac{\left(5cd^2-3ae^2\right)\left(ae^2+3cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+c\cdots}}{adex}}{4ade}\cdots}{6ae}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3aex^3}$$

$\Big\downarrow$ 219

$$-\frac{-\frac{\frac{3\left(cd^2-ae^2\right)\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{3/2}d^{3/2}e^{3/2}}-\frac{\left(5cd^2-3ae^2\right)\left(ae^2+3cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{adex}}{4ade}-\cdots}{6ae}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3aex^3}$$

input
```
Int[(d + e*x)/(x^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
-1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(a*e*x^3) - (-1/2*(((5*c*
d)/(a*e) - e/d)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/x^2 - (-(((5*
c*d^2 - 3*a*e^2)*(3*c*d^2 + a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2])/(a*d*e*x)) + (3*(c*d^2 - a*e^2)*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4
)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(3/2)*d^(3/2)*e^(3/2)))/(4*a*
d*e))/(6*a*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 732 vs. $2(251) = 502$.

Time = 2.80 (sec) , antiderivative size = 733, normalized size of antiderivative = 2.63

method	result
default	$d\left(-\frac{\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}{3ade\,x^3}-\frac{5(a\,e^2+c\,d^2)\left(-\frac{\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}{2ade\,x^2}-\frac{3\left(a\,e^2+c\,d^2\right)\left(-\frac{\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}{adex}+\dots\right)}{\dots}\right)}{\dots}\right.$

input
```
int((e*x+d)/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
d*(-1/3/a/d/e/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-5/6*(a*e^2+c*d^2
)/a/d/e*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/4*(a*e^2
+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e
^2+c*d^2)/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+1/2*c/a/(a*d*e)^(1/2)*ln((2*a
*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/
2))/x))-2/3*c/a*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a
*e^2+c*d^2)/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2
)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))+e*(-1/2/a/d/e/x^2*(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x*(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)/a/d/e/(a*d*e)^(1/2)*ln
((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2))/x))+1/2*c/a/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(
1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))
```

Fricas [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 562, normalized size of antiderivative = 2.01

$$\int \frac{d+ex}{x^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= \left[-\frac{3\,(5\,c^3d^6-3\,ac^2d^4e^2-a^2cd^2e^4-a^3e^6)\sqrt{ade}x^3\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2-4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{x^2}\right.}{}\right.$$

$$-\frac{3\,(5\,c^3d^6-3\,ac^2d^4e^2-a^2cd^2e^4-a^3e^6)\sqrt{-ade}x^3\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,ade+(cd^2+ae^2)x)\sqrt{-ade}}{2\,(acd^2e^2x^2+a^2d^2e^2+(acd^3e+a^2de^3)x)}\right)}{48}$$

input
```
integrate((e*x+d)/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
[-1/96*(3*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(a*d
*e)*x^3*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^3*d^3*e^3 + (15*a*
c^2*d^5*e - 4*a^2*c*d^3*e^3 - 3*a^3*d*e^5)*x^2 - 2*(5*a^2*c*d^4*e^2 - a^3*
d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^4*d^3*e^4*x^3)
, -1/48*(3*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*sqrt(-a
*d*e)*x^3*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
+ (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^
3*e + a^2*d*e^3)*x)) + 2*(8*a^3*d^3*e^3 + (15*a*c^2*d^5*e - 4*a^2*c*d^3*e^
3 - 3*a^3*d*e^5)*x^2 - 2*(5*a^2*c*d^4*e^2 - a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x))/(a^4*d^3*e^4*x^3)]
```

Sympy [F]

$$\int \frac{d+ex}{x^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\int \frac{d+ex}{x^4\sqrt{(d+ex)(ae+cdx)}}\,dx$$

input
```
integrate((e*x+d)/x**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral((d + e*x)/(x**4*sqrt((d + e*x)*(a*e + c*d*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x^4\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(251) = 502$.

Time = 0.15 (sec) , antiderivative size = 914, normalized size of antiderivative = 3.28

$$\int \frac{d+ex}{x^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output

```
-1/8*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*arctan(-(sqrt
(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))/sqrt(-a*d*e))/(sq
rt(-a*d*e)*a^3*d^2*e^3) - 1/24*(33*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2
*x + a*e^2*x + a*d*e))*a^2*c^3*d^8*e^2 + 105*(sqrt(c*d*e)*x - sqrt(c*d*e*x
^2 + c*d^2*x + a*e^2*x + a*d*e))*a^3*c^2*d^6*e^4 + 51*(sqrt(c*d*e)*x - sqr
t(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^4*c*d^4*e^6 + 3*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*a^5*d^2*e^8 + 48*sqrt(c*d*
e)*a^3*c^2*d^7*e^3 + 16*sqrt(c*d*e)*a^4*c*d^5*e^5 - 40*(sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a*c^3*d^7*e + 24*(sqrt(c*d*e)
*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^2*c^2*d^5*e^3 + 72*(
sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^3*c*d^3*e
^5 + 8*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^3*a^4
*d*e^7 + 144*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2
*x + a*d*e))^2*a^3*c*d^4*e^4 + 48*sqrt(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*
x^2 + c*d^2*x + a*e^2*x + a*d*e))^2*a^4*d^2*e^6 + 15*(sqrt(c*d*e)*x - sqrt
(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*c^3*d^6 - 9*(sqrt(c*d*e)*x - sq
rt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a*c^2*d^4*e^2 - 3*(sqrt(c*d*e
)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^2*c*d^2*e^4 - 3*(sq
rt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^5*a^3*e^6)/((a*
d*e - (sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))^2)^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{x^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{d+e\,x}{x^4\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int((d + e*x)/(x^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int((d + e*x)/(x^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 1316, normalized size of antiderivative = 4.72

$$\int \frac{d+ex}{x^4\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
( - 16*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*d**3*e**5 - 4*sqrt(d + e*x)*sq
rt(a*e + c*d*x)*a**4*d**2*e**6*x + 6*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*
d*e**7*x**2 - 16*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**5*e**3 + 16*sqr
t(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**4*e**4*x + 14*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a**3*c*d**3*e**5*x**2 + 20*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2
*c**2*d**6*e**2*x - 22*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**5*e**3
*x**2 - 30*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**7*e*x**2 + 3*sqrt(e)*
sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e
 + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**4*e**8*x**3 + 6*sq
rt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(
a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**2*e**
6*x**3 + 12*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2
*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a
**2*c**2*d**4*e**4*x**3 - 6*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e +
 c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*
sqrt(d + e*x))*a*c**3*d**6*e**2*x**3 - 15*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt
(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))*c**4*d**8*x**3 + 3*sqrt(e)*sqrt(d)*sqrt(a)*lo
g(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2
) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**4*e**8*x**3 + 6*sqrt(e)*sqrt(d)*s...
```

3.80 $\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 256

$$\begin{aligned}\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx &= -\frac{3(3cd^2+ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^2d^2e^3}\\ &\quad-\frac{2d^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e^3\,(cd^2-ae^2)\,(d+ex)}+\frac{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2cde^3}\\ &\quad+\frac{3(5c^2d^4+2acd^2e^2+a^2e^4)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^{5/2}d^{5/2}e^{7/2}}\end{aligned}$$

output
```
-3/4*(a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3-2
*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e^3/(-a*e^2+c*d^2)/(e*x+d)+1/
2*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^3+3/4*(a^2*e^4+2*a
*c*d^2*e^2+5*c^2*d^4)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{\sqrt{c}\sqrt{d}\sqrt{e}(3a^3e^5(d+ex)+a^2cde^3(4d^2+5dex+e^2x^2)+c^3d^4x(-15d^2-5dex+2e^2x^2)-ac^2d^2e(15d^3+}{4c^{5/2}d^{5/2}e^{7/2}\left(cd^2-a\right.}$$

input
```
Integrate[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(Sqrt[c]*Sqrt[d]*Sqrt[e]*(3*a^3*e^5*(d + e*x) + a^2*c*d*e^3*(4*d^2 + 5*d*e
*x + e^2*x^2) + c^3*d^4*x*(-15*d^2 - 5*d*e*x + 2*e^2*x^2) - a*c^2*d^2*e*(1
5*d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3)) + 3*(5*c^3*d^6 - 3*a*c^2*d^4*e
^2 - a^2*c*d^2*e^4 - a^3*e^6)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqr
t[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(4*c^(5/2)*d^(5/
2)*e^(7/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1213, 25, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{1213}$$

$$-\frac{\int -\frac{d^2-exd+e^2x^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^3}-\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d^2-exd+e^2x^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^3}-\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{\int \frac{e\left(2d\left(2cd^2-ae^2\right)-e\left(7cd^2+3ae^2\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{ex\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd}}{e^3}-\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{2d\left(2cd^2-ae^2\right)-e\left(7cd^2+3ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cd}+\frac{ex\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd}}{e^3}-\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{\frac{3\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cd}-\left(\frac{3ae^2}{cd}+7d\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cd}+\frac{ex\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd}}{e^3}-\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{\frac{3\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{cd}-\left(\frac{3ae^2}{cd}+7d\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cd}+\frac{ex\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd}}{e^3}-\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{3\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}\sqrt{e}}-\left(\frac{3ae^2}{cd}+7d\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4cd}+\frac{ex\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2cd}}{e^3}-\frac{2d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^3(d+ex)\left(cd^2-ae^2\right)}$$

input
```
Int[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(-2*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^3*(c*d^2 - a*e^2)*
(d + e*x)) + ((e*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c*d) +
(-((7*d + (3*a*e^2)/(c*d))*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) +
(3*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*
x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
])/(2*c^(3/2)*d^(3/2)*Sqrt[e]))/(4*c*d))/e^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(230) = 460$.

Time = 2.78 (sec) , antiderivative size = 514, normalized size of antiderivative = 2.01

method	result
default	$\frac{d^2 \ln\left(\frac{\frac{1}{2}a e^2+\frac{1}{2}c d^2+cdxe}{\sqrt{dec}}+\sqrt{ade+(a e^2+c d^2)x+cd x^2 e}\right)}{e^3\sqrt{dec}}+\frac{x\sqrt{ade+\left(a e^2+c d^2\right)x+cd x^2 e}}{2dec}-\frac{3\left(a e^2+c d^2\right)\left(\frac{\sqrt{ade+\left(a e^2+c d^2\right)x+cd x^2}}{dec}\right.}{}$

input
```
int(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
d^2/e^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)+1/e*(1/2*x/d/e/c*(a*d*e+(a*e^2+c*d^2)*
x+c*d*x^2*e)^(1/2)-3/4*(a*e^2+c*d^2)/d/e/c*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)
/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-1/2
*a/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-d/e^2*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d
*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+2*d^3/
e^4/(a*e^2-c*d^2)/(x+d/e)*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 758, normalized size of antiderivative = 2.96

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \left[\frac{3\,(5\,c^3d^7-3\,ac^2d^5e^2-a^2cd^3e^4-a^3de^6+(5\,c^3d^6e-3\,ac^2d^4e^3-a^2cd^2e^5-a^3e^7)x)\sqrt{cde}\log\left(8\,c^2d^2e^2x\right.}{}\right.$$
$$-\frac{3\,(5\,c^3d^7-3\,ac^2d^5e^2-a^2cd^3e^4-a^3de^6+(5\,c^3d^6e-3\,ac^2d^4e^3-a^2cd^2e^5-a^3e^7)x)\sqrt{-cde}\arctan\left(\frac{\sqrt{c}\right.}{}$$

input
```
integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
[1/16*(3*(5*c^3*d^7 - 3*a*c^2*d^5*e^2 - a^2*c*d^3*e^4 - a^3*d*e^6 + (5*c^3
*d^6*e - 3*a*c^2*d^4*e^3 - a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(c*d*e)*log(8*c
^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*
d^3*e + a*c*d*e^3)*x) - 4*(15*c^3*d^6*e - 4*a*c^2*d^4*e^3 - 3*a^2*c*d^2*e^
5 - 2*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x^2 + (5*c^3*d^5*e^2 - 2*a*c^2*d^3*e^4
 - 3*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^6
*e^4 - a*c^3*d^4*e^6 + (c^4*d^5*e^5 - a*c^3*d^3*e^7)*x), -1/8*(3*(5*c^3*d^
7 - 3*a*c^2*d^5*e^2 - a^2*c*d^3*e^4 - a^3*d*e^6 + (5*c^3*d^6*e - 3*a*c^2*d
^4*e^3 - a^2*c*d^2*e^5 - a^3*e^7)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c
^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(15*c^3*d^6
*e - 4*a*c^2*d^4*e^3 - 3*a^2*c*d^2*e^5 - 2*(c^3*d^4*e^3 - a*c^2*d^2*e^5)*x
^2 + (5*c^3*d^5*e^2 - 2*a*c^2*d^3*e^4 - 3*a^2*c*d*e^6)*x)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^6*e^4 - a*c^3*d^4*e^6 + (c^4*d^5*e^5 -
 a*c^3*d^3*e^7)*x)]
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{x^3}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(x**3/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{1}{4}\sqrt{cdex^2+cd^2x+ae^2x+ade}\left(\frac{2\,x}{cde^2}-\frac{7\,cd^2e^5+3\,ae^7}{c^2d^2e^8}\right)$$
$$-\frac{2\,d^3}{\left(\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)e+\sqrt{cde}d\right)e^3}$$
$$-\frac{3\,(5\,c^2d^4+2\,acd^2e^2+a^2e^4)\log\left(\left|cd^2+ae^2+2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{8\,\sqrt{cde}c^2d^2e^3}$$

input
```
integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output
```
1/4*sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)*(2*x/(c*d*e^2) - (7*c*d^2*
e^5 + 3*a*e^7)/(c^2*d^2*e^8)) - 2*d^3/(((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 +
c*d^2*x + a*e^2*x + a*d*e))*e + sqrt(c*d*e)*d)*e^3) - 3/8*(5*c^2*d^4 + 2*a
*c*d^2*e^2 + a^2*e^4)*log(abs(c*d^2 + a*e^2 + 2*sqrt(c*d*e)*(sqrt(c*d*e)*x
 - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(sqrt(c*d*e)*c^2*d^2*e^3
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\int \frac{x^3}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 816, normalized size of antiderivative = 3.19

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Too large to display}$$

input
```
int(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
( - 3*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c*d**2*e**5 - 3*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**2*c*d*e**6*x - 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**
2*d**4*e**3 - 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**3*e**4*x + 2*sqr
t(d + e*x)*sqrt(a*e + c*d*x)*a*c**2*d**2*e**5*x**2 + 15*sqrt(d + e*x)*sqrt
(a*e + c*d*x)*c**3*d**6*e + 5*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**5*e*
*2*x - 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**3*d**4*e**3*x**2 + 3*sqrt(e)*s
qrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e
*x))/sqrt(a*e**2 - c*d**2))*a**3*d*e**6 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((s
qrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*
d**2))*a**3*e**7*x + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d
*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**3*e*
*4 + 3*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sq
rt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**5*x + 9*sqrt(e)
*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d +
 e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**5*e**2 + 9*sqrt(e)*sqrt(d)*sqrt(c)
*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e*
*2 - c*d**2))*a*c**2*d**4*e**3*x - 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)
*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))
*c**3*d**7 - 15*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + s
qrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**3*d**6*e*x + sq...
```

3.81 $\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 178

$$\begin{aligned}\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx &= \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{cde^2}+\frac{2d^2\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{e^2(cd^2-ae^2)(d+ex)}\\ &\quad-\frac{(3cd^2+ae^2)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{3/2}d^{3/2}e^{5/2}}\end{aligned}$$

output
```
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c/d/e^2+2*d^2*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)/e^2/(-a*e^2+c*d^2)/(e*x+d)-(a*e^2+3*c*d^2)*arctanh(c^(
1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3
/2)/d^(3/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.13

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{\sqrt{c}\sqrt{d}\sqrt{e}(-a^2e^3(d+ex)+c^2d^3x(3d+ex)+acde(3d^2-e^2x^2))-(3c^2d^4-2acd^2e^2-a^2e^4)\sqrt{ae+cdx}}{c^{3/2}d^{3/2}e^{5/2}\,(cd^2-ae^2)\,\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[x^2/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(Sqrt[c]*Sqrt[d]*Sqrt[e]*(-(a^2*e^3*(d + e*x)) + c^2*d^3*x*(3*d + e*x) + a
*c*d*e*(3*d^2 - e^2*x^2)) - (3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*Sqrt[a*e
 + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[
d]*Sqrt[d + e*x])])/(c^(3/2)*d^(3/2)*e^(5/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c
*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1213, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$
$$\Big\downarrow \text{1213}$$
$$\frac{2d^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{e^2(d+ex)\,(cd^2-ae^2)}-\frac{\int \frac{d-ex}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e^2}$$
$$\Big\downarrow \text{1160}$$

$$\frac{\frac{2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2(d+ex)\left(cd^2-ae^2\right)}-}{\frac{\frac{1}{2}\left(\frac{ae^2}{cd}+3d\right)\int\frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd}}{e^2}}$$

$$\Big\downarrow \textbf{1092}$$

$$\frac{\frac{2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2(d+ex)\left(cd^2-ae^2\right)}-}{\frac{\left(\frac{ae^2}{cd}+3d\right)\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd}}{e^2}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\frac{2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e^2(d+ex)\left(cd^2-ae^2\right)}-}{\frac{\frac{\left(\frac{ae^2}{cd}+3d\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd}}{e^2}}$$

input
```
Int[x^2/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e^2*(c*d^2 - a*e^2)*(
d + e*x)) - (-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(c*d)) + ((3*d
+ (a*e^2)/(c*d))*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sq
rt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*Sq
rt[e]))/e^2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.46

method	result
default	$\frac{\frac{\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{dec}-\frac{\left(a\,e^{2}+c\,d^{2}\right)\ln\left(\frac{\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}+cdxe}{\sqrt{dec}}+\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}\right)}{2dec\sqrt{dec}}}{e}-\frac{2d^{2}\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x\right.}}{e^{3}\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}$

input
```
int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(1/d/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e
/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c
*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2*d^2/e^3/(a*e^2-c*d^2)/(x+d/e)*(d*e*c*(x+
d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-d/e^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)
/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 586, normalized size of antiderivative = 3.29

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$

$$= \left[\frac{(3\,c^2d^5-2\,acd^3e^2-a^2de^4+(3\,c^2d^4e-2\,acd^2e^3-a^2e^5)x)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2\right.}{}\right.$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
[1/4*((3*c^2*d^5 - 2*a*c*d^3*e^2 - a^2*d*e^4 + (3*c^2*d^4*e - 2*a*c*d^2*e^
3 - a^2*e^5)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c
*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(3*c^2*d^4*e
- a*c*d^2*e^3 + (c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x))/(c^3*d^5*e^3 - a*c^2*d^3*e^5 + (c^3*d^4*e^4 - a*c^2*d^2*e^
6)*x), 1/2*((3*c^2*d^5 - 2*a*c*d^3*e^2 - a^2*d*e^4 + (3*c^2*d^4*e - 2*a*c*
d^2*e^3 - a^2*e^5)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2
+ a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(3*c^2*d^4*e - a*c*d^2*e^3
 + (c^2*d^3*e^2 - a*c*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
))/(c^3*d^5*e^3 - a*c^2*d^3*e^5 + (c^3*d^4*e^4 - a*c^2*d^2*e^6)*x)]
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{x^2}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(x**2/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$
$$= \frac{2\,d^2}{\left(\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)e+\sqrt{cde}d\right)e^2} + \frac{(3\,cd^2+ae^2)\log\left(\left|cd^2+ae^2+2\,\sqrt{cde}\left(\sqrt{cde}x-\sqrt{cdex^2+cd^2x+ae^2x+ade}\right)\right|\right)}{2\,\sqrt{cde}cde^2} + \frac{\sqrt{cdex^2+cd^2x+ae^2x+ade}}{cde^2}$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output
```
2*d^2/(((sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))*e +
sqrt(c*d*e)*d)*e^2) + 1/2*(3*c*d^2 + a*e^2)*log(abs(c*d^2 + a*e^2 + 2*sqrt
(c*d*e)*(sqrt(c*d*e)*x - sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e))))/(s
qrt(c*d*e)*c*d*e^2) + sqrt(c*d*e*x^2 + c*d^2*x + a*e^2*x + a*d*e)/(c*d*e^2
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx = \int \frac{x^2}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.22

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{4\sqrt{ex+d}\,\sqrt{cdx+ae}\,ac\,d^2e^3+4\sqrt{ex+d}\,\sqrt{cdx+ae}\,acd\,e^4x-12\sqrt{ex+d}\,\sqrt{cdx+ae}\,c^2d^4e-4\sqrt{ex+d}}{}$$

input
```
int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d**2*e**3 + 4*sqrt(d + e*x)*sqrt(a*
e + c*d*x)*a*c*d*e**4*x - 12*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**4*e -
 4*sqrt(d + e*x)*sqrt(a*e + c*d*x)*c**2*d**3*e**2*x - 4*sqrt(e)*sqrt(d)*sq
rt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt
(a*e**2 - c*d**2))*a**2*d*e**4 - 4*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sq
rt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*
*2*e**5*x - 8*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqr
t(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d**3*e**2 - 8*sqrt(
e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d**2*e**3*x + 12*sqrt(e)*sqrt(d)*sqrt(
c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*
e**2 - c*d**2))*c**2*d**5 + 12*sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a
*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d
**4*e*x - sqrt(e)*sqrt(d)*sqrt(c)*a**2*d*e**4 - sqrt(e)*sqrt(d)*sqrt(c)*a*
*2*e**5*x + 2*sqrt(e)*sqrt(d)*sqrt(c)*a*c*d**3*e**2 + 2*sqrt(e)*sqrt(d)*sq
rt(c)*a*c*d**2*e**3*x - 9*sqrt(e)*sqrt(d)*sqrt(c)*c**2*d**5 - 9*sqrt(e)*sq
rt(d)*sqrt(c)*c**2*d**4*e*x)/(4*c**2*d**2*e**3*(a*d*e**2 + a*e**3*x - c*d*
*3 - c*d**2*e*x))
```

3.82 $\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 125

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{2d\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{e\,(cd^2-ae^2)\,(d+ex)}+\frac{2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}}$$

output
```
-2*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e/(-a*e^2+c*d^2)/(e*x+d)+2*ar
ctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2))/c^(1/2)/d^(1/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.04

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\left(-\frac{d^{3/2}\sqrt{e}(ae+cdx)}{cd^2-ae^2}+\frac{\sqrt{ae+cdx}\sqrt{d+ex}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{c}}\right)}{\sqrt{d}e^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[x/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*(-((d^(3/2)*Sqrt[e]*(a*e + c*d*x))/(c*d^2 - a*e^2)) + (Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[
d + e*x])])/Sqrt[c]))/(Sqrt[d]*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1213, 25, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \text{1213}$$

$$-\frac{\int -\frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e}-\frac{2d\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)(cd^2-ae^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{e}-\frac{2d\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)(cd^2-ae^2)}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2\int \frac{1}{4cde-\frac{(cd^2+2cexd+ae^2)^2}{cdex^2+(cd^2+ae^2)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{e}-\frac{2d\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)(cd^2-ae^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}}-\frac{2d\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)(cd^2-ae^2)}$$

input
```
Int[x/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(-2*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(c*d^2 - a*e^2)*(d +
 e*x)) + ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[c]*Sqrt[d]*e^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2}a\,e^{2}+\frac{1}{2}c\,d^{2}+cdxe}{\sqrt{dec}}+\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}\right)}{e\sqrt{dec}}+\frac{2d\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}{e^{2}\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}$	131

input
```
int(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)+2*d/e^2/(a*e^2-c*d^2)/(x+d/e)*(d*e*c*(x+d/
e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.54

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\left[-\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}cd^2e-(cd^3-ade^2+(cd^2e-ae^3)x)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\right.}{2\,(c^2d^4e^2-acd^2e^4+(}\right.$$
$$-\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}cd^2e+(cd^3-ade^2+(cd^2e-ae^3)x)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+a}}{2\,(c^2d^2e^2x^2+ac}\right.}{c^2d^4e^2-acd^2e^4+(c^2d^3e^3-acde^5)x}$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
fricas")
```

output
```
[-1/2*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d^2*e - (c*d^3 - a*
d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 +
 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(
2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x))/(c^
2*d^4*e^2 - a*c*d^2*e^4 + (c^2*d^3*e^3 - a*c*d*e^5)*x), -(2*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*c*d^2*e + (c*d^3 - a*d*e^2 + (c*d^2*e - a*e^
3)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*
(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 +
(c^2*d^3*e + a*c*d*e^3)*x)))/(c^2*d^4*e^2 - a*c*d^2*e^4 + (c^2*d^3*e^3 - a
*c*d*e^5)*x)]
```

Sympy [F]

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{x}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(x/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[0,0,5]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,2]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{x}{(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.40

$$\int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$$
$$= \frac{2\sqrt{ex+d}\,\sqrt{cdx+ae}\,c\,d^2e+2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)ad\,e^2+2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\cdots}{\sqrt{a\,e^2\cdots}}\right.}{}$$

input
```
int(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*(sqrt(d + e*x)*sqrt(a*e + c*d*x)*c*d**2*e + sqrt(e)*sqrt(d)*sqrt(c)*log
((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 -
 c*d**2))*a*d*e**2 + sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x
) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*e**3*x - sqrt(
e)*sqrt(d)*sqrt(c)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))/sqrt(a*e**2 - c*d**2))*c*d**3 - sqrt(e)*sqrt(d)*sqrt(c)*log((sqrt
(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**
2))*c*d**2*e*x + sqrt(e)*sqrt(d)*sqrt(c)*c*d**3 + sqrt(e)*sqrt(d)*sqrt(c)*
c*d**2*e*x))/(c*d*e**2*(a*d*e**2 + a*e**3*x - c*d**3 - c*d**2*e*x))
```

3.83 $\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 52

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{(cd^2-ae^2)\,(d+ex)}$$

output
```
2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\frac{2(ae+cdx)}{(cd^2-ae^2)\,\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*(a*e + c*d*x))/((c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}dx$$

$$\Big\downarrow \textbf{1123}$$

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{(d+ex)(cd^2-ae^2)}$$

input
```
Int[1/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d^2 - a*e^2)*(d + e*x)
)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result	size
trager	$-\frac{2\sqrt{c d \,x^{2} e+a \,e^{2} x+c \,d^{2} x+a d e}}{\left(a \,e^{2}-c \,d^{2}\right) \left(e x+d\right)}$	50
gosper	$-\frac{2 \left(c d x+a e\right)}{\left(a \,e^{2}-c \,d^{2}\right) \sqrt{c d \,x^{2} e+a \,e^{2} x+c \,d^{2} x+a d e}}$	51
orering	$-\frac{2 \left(c d x+a e\right)}{\left(a \,e^{2}-c \,d^{2}\right) \sqrt{a d e+\left(a \,e^{2}+c \,d^{2}\right) x+c d \,x^{2} e}}$	52
default	$-\frac{2 \sqrt{d e c \left(x+\frac{d}{e}\right)^{2}+\left(a \,e^{2}-c \,d^{2}\right) \left(x+\frac{d}{e}\right)}}{e \left(a \,e^{2}-c \,d^{2}\right) \left(x+\frac{d}{e}\right)}$	65

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-2/(a*e^2-c*d^2)/(e*x+d)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{cd^3-ade^2+(cd^2e-ae^3)x}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
fricas")
```

output
```
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c*d^3 - a*d*e^2 + (c*d^2*e
- a*e^3)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\int \frac{1}{\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,1]%%%},[2]%%%}+%%%{%%{[%%%{-2,[0,1,0]%%%},0]:[1
,0,%%%{-1
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(a\,e^2-c\,d^2\right)\,\left(d+e\,x\right)}$$

input
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
-(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((a*e^2 - c*d^2)*(d + e
*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{-2\sqrt{ex+d}\,\sqrt{cdx+ae}\,e-2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,d-2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,ex}{e\left(a\,e^3x-c\,d^2ex+ad\,e^2-c\,d^3\right)}$$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
( - 2*(sqrt(d + e*x)*sqrt(a*e + c*d*x)*e + sqrt(e)*sqrt(d)*sqrt(c)*d + sqr
t(e)*sqrt(d)*sqrt(c)*e*x))/(e*(a*d*e**2 + a*e**3*x - c*d**3 - c*d**2*e*x))
```

3.84 $\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 125

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{2e\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{d\,(cd^2-ae^2)\,(d+ex)}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{\sqrt{a}d^{3/2}\sqrt{e}}$$

output
```
-2*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/d/(-a*e^2+c*d^2)/(e*x+d)-2*ar
ctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2))/a^(1/2)/d^(3/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \frac{2\left(-\frac{\sqrt{d}e^{3/2}(ae+cdx)}{cd^2-ae^2}-\frac{\sqrt{ae+cdx}\sqrt{d+ex}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{ae+cdx}}{\sqrt{a}\sqrt{e}\sqrt{d+ex}}\right)}{\sqrt{a}}\right)}{d^{3/2}\sqrt{e}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/(x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*(-((Sqrt[d]*e^(3/2)*(a*e + c*d*x))/(c*d^2 - a*e^2)) - (Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[
d + e*x])])/Sqrt[a]))/(d^(3/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1214, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}dx$$

$$\Big\downarrow \textbf{1214}$$

$$-\int -\frac{1}{dx\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \textbf{25}$$

$$\int \frac{1}{dx\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{d}-\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \textbf{1154}$$

$$-\frac{2\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{d}-\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{\text{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{a}d^{3/2}\sqrt{e}}-\frac{2e\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d(d+ex)\left(cd^2-ae^2\right)}$$

input
```
Int[1/(x*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(-2*e*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d*(c*d^2 - a*e^2)*(d +
 e*x)) - ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(Sqrt[a]*d^(3/2)*Sqrt[e])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09

method	result	size
default	$-\frac{\ln\left(\frac{2ade+\left(a\,e^{2}+c\,d^{2}\right)x+2\sqrt{ade}\,\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{x}\right)}{d\sqrt{ade}}+\frac{2\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}{d\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}$	136

input
```
int(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
-1/d/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e
^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)+2/d/(a*e^2-c*d^2)/(x+d/e)*(d*e*c*(x+d/e)^
2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.63

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\left[-\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}ade^2-(cd^3-ade^2+(cd^2e-ae^3)x)\sqrt{ade}\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+c\cdots}{\cdots}\right.}{2\,(acd^5e-a^2d^3e^3+(acd^4e^2-a^2d^2e^4)\cdots}\right.$$
$$-\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}ade^2-(cd^3-ade^2+(cd^2e-ae^3)x)\sqrt{-ade}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+c\cdots}}{2\,(acd^2e^2x^2+c\cdots}\right.}{acd^5e-a^2d^3e^3+(acd^4e^2-a^2d^2e^4)x}$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="fricas")
```

output
```
[-1/2*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e^2 - (c*d^3 - a*
d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6
*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)
/x^2))/(a*c*d^5*e - a^2*d^3*e^3 + (a*c*d^4*e^2 - a^2*d^2*e^4)*x), -(2*sqrt
(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e^2 - (c*d^3 - a*d*e^2 + (c*d^
2*e - a*e^3)*x)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^
2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)))/(a*c*d^5*e - a^2*d^3*e^3 + (a*c*d
^4*e^2 - a^2*d^2*e^4)*x)]
```

Sympy [F]

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,1,5]%%%},[2,2]%%%}+%%%{%%%{-2,[1,3,3]%%%},[2,1]%%
%}+%%%{%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x\,(d+e\,x)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}\,dx$$

input
```
int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 763, normalized size of antiderivative = 6.10

$$\int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
(2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*d*e**2 + sqrt(e)*sqrt(d)*sqrt(a)*log(
sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*d*e**2 + sqrt(e)*sqrt(d)*sqrt(a)*log(sq
rt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) +
sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*e**3*x - sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt
(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))*c*d**3 - sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*
sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d
)*sqrt(c)*sqrt(d + e*x))*c*d**2*e*x + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*
sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d
)*sqrt(c)*sqrt(d + e*x))*a*d*e**2 + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sq
rt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*
sqrt(c)*sqrt(d + e*x))*a*e**3*x - sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt
(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sq
rt(c)*sqrt(d + e*x))*c*d**3 - sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e
 + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c
)*sqrt(d + e*x))*c*d**2*e*x - sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d
)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*
e*x)*a*d*e**2 - sqrt(e)*sqrt(d)*sqrt(a)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt
(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*a*e**3...
```

3.85 $\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 200

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = -\frac{(cd^2-3ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{ad^2\,(cd^2-ae^2)\,(d+ex)} - \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{adex(d+ex)} + \frac{(cd^2+3ae^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{3/2}d^{5/2}e^{3/2}}$$

output
```
-(-3*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^2/(-a*e^2+c*
d^2)/(e*x+d)-(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d/e/x/(e*x+d)+(3*a*
e^2+c*d^2)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\frac{\sqrt{a}\sqrt{d}\sqrt{e}(-c^2d^3x(d+ex)+a^2e^3(d+3ex)-acde(d^2-3e^2x^2))+(c^2d^4+2acd^2e^2-3a^2e^4)\,x\sqrt{ae+cd}}{a^{3/2}d^{5/2}e^{3/2}\,(cd^2-ae^2)\,x\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/(x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(Sqrt[a]*Sqrt[d]*Sqrt[e]*(-(c^2*d^3*x*(d + e*x)) + a^2*e^3*(d + 3*e*x) - a
*c*d*e*(d^2 - 3*e^2*x^2)) + (c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*x*Sqrt[a
*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqr
t[e]*Sqrt[d + e*x])])/(a^(3/2)*d^(5/2)*e^(3/2)*(c*d^2 - a*e^2)*x*Sqrt[(a*e
 + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1214, 25, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \text{1214}$$

$$\frac{2e^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{d^2(d+ex)\,(cd^2-ae^2)}-\int -\frac{d-ex}{d^2x^2\sqrt{cdex^2+(cd^2+ae^2)\,x+ade}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{d-ex}{d^2x^2\sqrt{cdex^2+(cd^2+ae^2)\,x+ade}}dx+\frac{2e^2\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}{d^2(d+ex)\,(cd^2-ae^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{d-ex}{x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{d^2}+\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1228$$

$$\frac{-\frac{1}{2}\left(\frac{cd^2}{ae}+3e\right)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{aex}}{d^2}+\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\left(\frac{cd^2}{ae}+3e\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+(cd^2+ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{aex}}{d^2}+\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(\frac{cd^2}{ae}+3e\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}\sqrt{d}\sqrt{e}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{aex}}{d^2}+\frac{2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2(d+ex)\left(cd^2-ae^2\right)}$$

input
```
Int[1/(x^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(2*e^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2*(c*d^2 - a*e^2)*(
d + e*x)) + (-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(a*e*x)) + (((c
*d^2)/(a*e) + 3*e)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d
]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[a]*Sqrt[d
]*Sqrt[e]))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.35

method	result
default	$\frac{-\frac{\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{adex}+\frac{\left(a\,e^{2}+c\,d^{2}\right)\ln\left(\frac{2ade+\left(a\,e^{2}+c\,d^{2}\right)x+2\sqrt{ade}\,\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}{x}\right)}{2ade\sqrt{ade}}}{d}-\frac{2e\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+(a\,e^{2}-c}}{d^{2}(a\,e^{2}-c\,d^{2})\left(x+\right.}$

input
```
int(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
1/d*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)/
a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-2*e/d^2/(a*e^2-c*d^2)/(x+d/e)*(d*e*c*(x
+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+e/d^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^
2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 610, normalized size of antiderivative = 3.05

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$=\left[\frac{\sqrt{ade}((c^2d^4e+2\,acd^2e^3-3\,a^2e^5)x^2+(c^2d^5+2\,acd^3e^2-3\,a^2de^4)x)\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2+4}{}\right.}{4\,((a^2cd^5e}\right.$$
$$-\frac{\sqrt{-ade}((c^2d^4e+2\,acd^2e^3-3\,a^2e^5)x^2+(c^2d^5+2\,acd^3e^2-3\,a^2de^4)x)\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2\,a}{2\,(acd^2e^2x^2+a^2d^2e^2+}\right.}{2\,((a^2cd^5e^3-a^3d^3e^5)x^2+(}$$

input
```
integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
[1/4*(sqrt(a*d*e)*((c^2*d^4*e + 2*a*c*d^2*e^3 - 3*a^2*e^5)*x^2 + (c^2*d^5
+ 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) -
 4*(a*c*d^4*e - a^2*d^2*e^3 + (a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^2*c*d^5*e^3 - a^3*d^3*e^5)*x^2 + (a^2*
c*d^6*e^2 - a^3*d^4*e^4)*x), -1/2*(sqrt(-a*d*e)*((c^2*d^4*e + 2*a*c*d^2*e^
3 - 3*a^2*e^5)*x^2 + (c^2*d^5 + 2*a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*arctan(1/2
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x))
 + 2*(a*c*d^4*e - a^2*d^2*e^3 + (a*c*d^3*e^2 - 3*a^2*d*e^4)*x)*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^2*c*d^5*e^3 - a^3*d^3*e^5)*x^2 + (a^
2*c*d^6*e^2 - a^3*d^4*e^4)*x)]
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x^2\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x^2), x
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,0,1]%%%},[6,0]%%%}+%%%{%%{[%%%{-2,[0,1,0]%%%},0]:
[1,0,%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x^2\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1302, normalized size of antiderivative = 6.51

$$\int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output
```
( - 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*d**2*e**3 - 6*sqrt(d + e*x)*sqr
t(a*e + c*d*x)*a**2*d*e**4*x + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d**4*
e + 2*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*c*d**3*e**2*x - 3*sqrt(e)*sqrt(d)*
sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**
2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*d*e**4*x - 3*sqrt(e)*sqr
t(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e +
a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*e**5*x**2 + 2*sqrt(
e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*
d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c*d**3*e**2*x +
2*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*s
qrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c*d**2*e*
*3*x**2 + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*s
qrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**
2*d**5*x + sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*
sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c*
*2*d**4*e*x**2 - 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) +
 sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d +
e*x))*a**2*d*e**4*x - 3*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d
*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt
(d + e*x))*a**2*e**5*x**2 + 2*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(...
```

3.86 $\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 296

$$\begin{aligned}\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx &= \frac{(3cd^2-5ae^2)\,(cd^2+3ae^2)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4a^2d^3e\,(cd^2-ae^2)\,(d+ex)} \\ &\quad - \frac{\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2adex^2(d+ex)} + \frac{\left(\frac{5a}{d^2}+\frac{3c}{e^2}\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4a^2x(d+ex)} \\ &\quad - \frac{3(c^2d^4+2acd^2e^2+5a^2e^4)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4a^{5/2}d^{7/2}e^{5/2}}\end{aligned}$$

output
```
1/4*(-5*a*e^2+3*c*d^2)*(3*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)/a^2/d^3/e/(-a*e^2+c*d^2)/(e*x+d)-1/2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)/a/d/e/x^2/(e*x+d)+1/4*(5*a/d^2+3*c/e^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)/a^2/x/(e*x+d)-3/4*(5*a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)*arctanh(
a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a
^(5/2)/d^(7/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx$$
$$= \frac{\sqrt{a}\sqrt{d}\sqrt{e}(3c^3d^5x^2(d+ex)+a^3e^4(2d^2-5dex-15e^2x^2)+ac^2d^3ex(d^2+5dex+4e^2x^2)-a^2cde^2(2d^3-}{4a^{5/2}d^{7/2}e^{5/2}\left(cd^2-a\right.}$$

input
```
Integrate[1/(x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
(Sqrt[a]*Sqrt[d]*Sqrt[e]*(3*c^3*d^5*x^2*(d + e*x) + a^3*e^4*(2*d^2 - 5*d*e
*x - 15*e^2*x^2) + a*c^2*d^3*e*x*(d^2 + 5*d*e*x + 4*e^2*x^2) - a^2*c*d*e^2
*(2*d^3 - 4*d^2*e*x + d*e^2*x^2 + 15*e^3*x^3)) - 3*(c^3*d^6 + a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*x^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTa
nh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(4*a^(5/2
)*d^(7/2)*e^(5/2)*(c*d^2 - a*e^2)*x^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1214, 25, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\,dx$$

$$\Big\downarrow \; 1214$$

$$-\int -\frac{\frac{e^2x^2}{d^3}-\frac{ex}{d^2}+\frac{1}{d}}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{2e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^3(d+ex)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 25$$

$$\int \frac{\frac{e^2x^2}{d^3}-\frac{ex}{d^2}+\frac{1}{d}}{x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{2e^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^3(d+ex)(cd^2-ae^2)}$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int \frac{\frac{7ae^2}{d}+2\left(c-\frac{2ae^2}{d^2}\right)xe+3cd}{2x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2ad^2ex^2}-\frac{2e^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^3(d+ex)(cd^2-ae^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\frac{7ae^2}{d}+2\left(c-\frac{2ae^2}{d^2}\right)xe+3cd}{x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2ad^2ex^2}-\frac{2e^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^3(d+ex)(cd^2-ae^2)}$$

$$\Big\downarrow \text{1228}$$

$$-\frac{-\frac{3\left(\frac{c^2d^2}{a}+\frac{5ae^4}{d^2}+2ce^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2e}-\frac{\left(\frac{3c}{ae}+\frac{7e}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{4ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2ad^2ex^2}-\frac{2e^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^3(d+ex)(cd^2-ae^2)}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{\frac{3\left(\frac{c^2d^2}{a}+\frac{5ae^4}{d^2}+2ce^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{e}-\frac{\left(\frac{3c}{ae}+\frac{7e}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{4ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2ad^2ex^2}-\frac{2e^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^3(d+ex)(cd^2-ae^2)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{3\left(\frac{c^2d^2}{a}+\frac{5ae^4}{d^2}+2ce^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}\sqrt{d}e^{3/2}}-\frac{\left(\frac{3c}{ae}+\frac{7e}{d^2}\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{4ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2ad^2ex^2}-\frac{2e^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{d^3(d+ex)(cd^2-ae^2)}$$

input
```
Int[1/(x^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]
```

output
```
-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(a*d^2*e*x^2) - (2*e^3*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^3*(c*d^2 - a*e^2)*(d + e*x))
 - (-((((3*c)/(a*e) + (7*e)/d^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2])/x) + (3*((c^2*d^2)/a + 2*c*e^2 + (5*a*e^4)/d^2)*ArcTanh[(2*a*d*e + (c*
d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])])/(2*Sqrt[a]*Sqrt[d]*e^(3/2)))/(4*a*d*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1214
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[ExpandToS
um[(((-d)^n*(-2*c*d + b*e)^(-m - 1))/(e^n*x^n) - ((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/(Sqrt[a + b*x + c*x^2]/x^n), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && ILtQ[n, 0] &&
EqQ[m + p, -3/2]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(268) = 536$.

Time = 2.91 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.84

method	result
default	$\frac{-\frac{\sqrt{ade+\left(a e^{2}+c d^{2}\right) x+c d x^{2} e}}{2 a d e\, x^{2}}-\frac{3\left(a e^{2}+c d^{2}\right)\left(-\frac{\sqrt{ade+\left(a e^{2}+c d^{2}\right) x+c d x^{2} e}}{a d e x}+\frac{\left(a e^{2}+c d^{2}\right) \ln \left(\frac{2 a d e+\left(a e^{2}+c d^{2}\right) x+2 \sqrt{a d e} \sqrt{a d e+\left(a e^{2}+c \cdots\right.}}{x}\right.}{2 a d e \sqrt{a d e}}\cdots\right.}{4 a d e}\cdots}{d}$

input
```
int(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
1/d*(-1/2/a/d/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/4*(a*e^2+c*d
^2)/a/d/e*(-1/a/d/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c
*d^2)/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+1/2*c/a/(a*d*e)^(1/2)*ln((2*a*d*e
+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/
x))-e^2/d^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)-e/d^2*(-1/a/d/e/x*(a*d*e+(a*e^2+c*
d^2)*x+c*d*x^2*e)^(1/2)+1/2*(a*e^2+c*d^2)/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+
(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x
))+2*e^2/d^3/(a*e^2-c*d^2)/(x+d/e)*(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 792, normalized size of antiderivative = 2.68

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="fricas")
```

output
```
[1/16*(3*((c^3*d^6*e + a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 5*a^3*e^7)*x^3 +
(c^3*d^7 + a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 - 5*a^3*d*e^6)*x^2)*sqrt(a*d*e)
*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d
*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(2*a^2*c*d^5*e^2 - 2*a^3*d^3*e
^4 - (3*a*c^2*d^5*e^2 + 4*a^2*c*d^3*e^4 - 15*a^3*d*e^6)*x^2 - (3*a*c^2*d^6
*e + 2*a^2*c*d^4*e^3 - 5*a^3*d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x))/((a^3*c*d^6*e^4 - a^4*d^4*e^6)*x^3 + (a^3*c*d^7*e^3 - a^4*d^5*
e^5)*x^2), 1/8*(3*((c^3*d^6*e + a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 - 5*a^3*e^
7)*x^3 + (c^3*d^7 + a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 - 5*a^3*d*e^6)*x^2)*sq
rt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e
 + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d
^3*e + a^2*d*e^3)*x)) - 2*(2*a^2*c*d^5*e^2 - 2*a^3*d^3*e^4 - (3*a*c^2*d^5*
e^2 + 4*a^2*c*d^3*e^4 - 15*a^3*d*e^6)*x^2 - (3*a*c^2*d^6*e + 2*a^2*c*d^4*e
^3 - 5*a^3*d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*
c*d^6*e^4 - a^4*d^4*e^6)*x^3 + (a^3*c*d^7*e^3 - a^4*d^5*e^5)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x^3\sqrt{(d+ex)\,(ae+cdx)}\,(d+ex)}\,dx$$

input
```
integrate(1/x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x^3), x
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,3,9]%%%},[2,4]%%%}+%%%{%%%{-4,[1,5,7]%%%},[2,3]%%
%}+%%%{%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \int \frac{1}{x^3\,\left(d+e\,x\right)\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}\,dx$$

input
```
int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)),x)
```

output
```
int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 2075, normalized size of antiderivative = 7.01

$$\int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)
```

output

```
( - 20*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**4*d**3*e**6 + 50*sqrt(d + e*x)*s
qrt(a*e + c*d*x)*a**4*d**2*e**7*x + 150*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
*4*d*e**8*x**2 + 8*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**5*e**4 + 10*s
qrt(d + e*x)*sqrt(a*e + c*d*x)*a**3*c*d**4*e**5*x + 50*sqrt(d + e*x)*sqrt(
a*e + c*d*x)*a**3*c*d**3*e**6*x**2 + 12*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a*
*2*c**2*d**7*e**2 - 42*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**6*e**3
*x - 54*sqrt(d + e*x)*sqrt(a*e + c*d*x)*a**2*c**2*d**5*e**4*x**2 - 18*sqrt
(d + e*x)*sqrt(a*e + c*d*x)*a*c**3*d**8*e*x - 18*sqrt(d + e*x)*sqrt(a*e +
c*d*x)*a*c**3*d**7*e**2*x**2 + 75*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt
(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sq
rt(c)*sqrt(d + e*x))*a**4*d*e**8*x**2 + 75*sqrt(e)*sqrt(d)*sqrt(a)*log(sqr
t(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + s
qrt(d)*sqrt(c)*sqrt(d + e*x))*a**4*e**9*x**3 - 42*sqrt(e)*sqrt(d)*sqrt(a)*
log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d*
*2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c**2*d**5*e**4*x**2 - 42*sqrt(e)
*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*
e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c**2*d**4*e**5*
x**3 - 24*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*s
qrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c
**3*d**7*e**2*x**2 - 24*sqrt(e)*sqrt(d)*sqrt(a)*log(sqrt(e)*sqrt(a*e + ...
```

3.87 $\int \frac{x^2}{(-1+x)\sqrt{-3+2x+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{x^2}{(-1+x)\sqrt{-3+2x+x^2}}\,dx = \sqrt{-3+2x+x^2}+\frac{\sqrt{-3+2x+x^2}}{2(1-x)}$$

output
```
(x^2+2*x-3)^(1/2)+(x^2+2*x-3)^(1/2)/(2-2*x)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{(-1+x)\sqrt{-3+2x+x^2}}\,dx = \frac{(-3+2x)\sqrt{-3+2x+x^2}}{2(-1+x)}$$

input
```
Integrate[x^2/((-1 + x)*Sqrt[-3 + 2*x + x^2]),x]
```

output
```
((-3 + 2*x)*Sqrt[-3 + 2*x + x^2])/(2*(-1 + x))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1213, 25, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x-1)\sqrt{x^2+2x-3}}dx$$

$$\Big\downarrow \text{1213}$$

$$\frac{\sqrt{x^2+2x-3}}{2(1-x)}-\int -\frac{x+1}{\sqrt{x^2+2x-3}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{x+1}{\sqrt{x^2+2x-3}}dx+\frac{\sqrt{x^2+2x-3}}{2(1-x)}$$

$$\Big\downarrow \text{1104}$$

$$\frac{\sqrt{x^2+2x-3}}{2(1-x)}+\sqrt{x^2+2x-3}$$

input
```
Int[x^2/((-1 + x)*Sqrt[-3 + 2*x + x^2]),x]
```

output
```
Sqrt[-3 + 2*x + x^2] + Sqrt[-3 + 2*x + x^2]/(2*(1 - x))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1213
```
Int[(x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^
2)^(p_), x_Symbol] :> Simp[-2*(-d)^n*e^(2*m - n + 3)*(Sqrt[a + b*x + c*x^2]
/((-2*c*d + b*e)^(m + 2)*(d + e*x))), x] - Simp[e^(2*m - n + 2)   Int[Expan
dToSum[((-d)^n*(-2*c*d + b*e)^(-m - 1) - e^n*x^n*((-c)*d + b*e + c*e*x)^(-m
 - 1))/(d + e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && IGtQ[n, 0] && EqQ[m
+ p, -3/2]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{(3+x)(2x-3)}{2\sqrt{x^2+2x-3}}$	21
orering	$\frac{(3+x)(2x-3)}{2\sqrt{x^2+2x-3}}$	21
trager	$\frac{(2x-3)\sqrt{x^2+2x-3}}{2x-2}$	23
risch	$\frac{2x^2+3x-9}{2\sqrt{x^2+2x-3}}$	23
default	$\sqrt{x^2+2x-3}-\frac{\sqrt{(x-1)^2+4x-4}}{2(x-1)}$	31

input
```
int(x^2/(x-1)/(x^2+2*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(3+x)*(2*x-3)/(x^2+2*x-3)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{(-1+x)\sqrt{-3+2x+x^2}}\,dx = \frac{\sqrt{x^2+2\,x-3}(2\,x-3)}{2\,(x-1)}$$

input
```
integrate(x^2/(x-1)/(x^2+2*x-3)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(x^2 + 2*x - 3)*(2*x - 3)/(x - 1)
```

Sympy [F]

$$\int \frac{x^2}{(-1+x)\sqrt{-3+2x+x^2}}\,dx = \int \frac{x^2}{\sqrt{(x-1)\,(x+3)}\,(x-1)}\,dx$$

input
```
integrate(x**2/(x-1)/(x**2+2*x-3)**(1/2),x)
```

output
```
Integral(x**2/(sqrt((x - 1)*(x + 3))*(x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{(-1+x)\sqrt{-3+2x+x^2}}\,dx = \sqrt{x^2+2\,x-3} - \frac{\sqrt{x^2+2\,x-3}}{2\,(x-1)}$$

input
```
integrate(x^2/(x-1)/(x^2+2*x-3)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 + 2*x - 3) - 1/2*sqrt(x^2 + 2*x - 3)/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{(-1+x)\sqrt{-3+2x+x^2}}\,dx = \sqrt{x^2+2\,x-3} + \frac{2}{x-\sqrt{x^2+2\,x-3}-1}$$

input
```
integrate(x^2/(x-1)/(x^2+2*x-3)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x^2 + 2*x - 3) + 2/(x - sqrt(x^2 + 2*x - 3) - 1)
```

Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{x^2}{(-1+x)\sqrt{-3+2x+x^2}}\,dx = \frac{\left(x-\frac{3}{2}\right)\,\sqrt{x^2+2\,x-3}}{x-1}$$

input
```
int(x^2/((x - 1)*(2*x + x^2 - 3)^(1/2)),x)
```

output
```
((x - 3/2)*(2*x + x^2 - 3)^(1/2))/(x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(-1+x)\sqrt{-3+2x+x^2}}\,dx = \frac{2\sqrt{x^2+2x-3}\,x-3\sqrt{x^2+2x-3}-3x+3}{2x-2}$$

input
```
int(x^2/(x-1)/(x^2+2*x-3)^(1/2),x)
```

output
```
(2*sqrt(x**2 + 2*x - 3)*x - 3*sqrt(x**2 + 2*x - 3) - 3*x + 3)/(2*(x - 1))
```

3.88 $\int \frac{x^3(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 257

$$\int \frac{x^3(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2a^3e^3(d+ex)}{c^3d^3\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(3cd^2+7ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^3d^3e^2} + \frac{x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2c^2d^2e} + \frac{3\left(c^2d^4+2acd^2e^2+5a^2e^4\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^{7/2}d^{7/2}e^{5/2}}$$

output
```
2*a^3*e^3*(e*x+d)/c^3/d^3/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)-1/4*(7*a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d
^3/e^2+1/2*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e+3/4*(5*a^2*
e^4+2*a*c*d^2*e^2+c^2*d^4)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.95

$$\int \frac{x^3(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{-\sqrt{c}\sqrt{d}\sqrt{e}(d+ex)\left(-15a^3e^5+a^2cde^3(4d-5ex)+c^3d^4x(3d-\right.}{}$$

input
```
Integrate[(x^3*(d + e*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-(Sqrt[c]*Sqrt[d]*Sqrt[e]*(d + e*x)*(-15*a^3*e^5 + a^2*c*d*e^3*(4*d - 5*e
*x) + c^3*d^4*x*(3*d - 2*e*x) + a*c^2*d^2*e*(3*d^2 + 2*d*e*x + 2*e^2*x^2))
) + 3*(c^3*d^6 + a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 5*a^3*e^6)*Sqrt[a*e + c
*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[
a*e + c*d*x])])/(4*c^(7/2)*d^(7/2)*e^(5/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d
*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1211, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)}{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int \frac{a^2e^6-acdxe^5+c^2d^2x^2e^4}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{c^3d^3e^4}+\frac{2a^3e^3(d+ex)}{c^3d^3\,(cd^2-ae^2)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\int -\frac{cde^4\left(2ae\left(cd^2-2ae^2\right)+cd\left(3cd^2+7ae^2\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}cde^3x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3e^4}+\frac{2a^3e^3(d+ex)}{c^3d^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 27

$$\frac{\frac{1}{2}cde^3x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-\frac{1}{4}e^3\int\frac{2ae(cd^2-2ae^2)+cd(3cd^2+7ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{c^3d^3e^4}+\frac{2a^3e^3(d+ex)}{c^3d^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 1160

$$\frac{\frac{1}{2}cde^3x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-\frac{1}{4}e^3\left(\frac{(7ae^2+3cd^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}-\frac{3(5a^2e^4+2acd^2e^2+c^2d^4)\int\frac{}{\sqrt{cdex^2+(cd}}}{2e}\right.}{c^3d^3e^4}$$

$$\frac{2a^3e^3(d+ex)}{c^3d^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 1092

$$\frac{\frac{1}{2}cde^3x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-\frac{1}{4}e^3\left(\frac{(7ae^2+3cd^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}-\frac{3(5a^2e^4+2acd^2e^2+c^2d^4)\int\frac{}{4cde-\frac{(cd}{cdex^2}}}{}\right.}{c^3d^3e^4}$$

$$\frac{2a^3e^3(d+ex)}{c^3d^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 219

$$\frac{2a^3e^3(d+ex)}{c^3d^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+$$

$$\frac{\frac{1}{2}cde^3x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-\frac{1}{4}e^3\left(\frac{(7ae^2+3cd^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}-\frac{3(5a^2e^4+2acd^2e^2+c^2d^4)\text{arctanh}\left(\frac{}{2\sqrt{}}\right.}{2\sqrt{c}\sqrt{de}}\right.}{c^3d^3e^4}$$

input
```
Int[(x^3*(d + e*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(2*a^3*e^3*(d + e*x))/(c^3*d^3*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) + ((c*d*e^3*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/2 - (e^3*(((3*c*d^2 + 7*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2])/e - (3*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 +
 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))))/4)/(c^3*d^3*e^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1274 vs. $2(231) = 462$.

Time = 2.56 (sec) , antiderivative size = 1275, normalized size of antiderivative = 4.96

method	result	size
default	Expression too large to display	1275

input
```
int(x^3*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
d*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e
/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e
/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(
2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/
2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2*a/c*(-1/d/e/c
/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*
e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)))+e*(1/2*x^3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-5/4*(a
*e^2+c*d^2)/d/e/c*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(
a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(
a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^
2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*
d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*
x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))
-2*a/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e
/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)))-3/2*a/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^
2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^
2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(...
```

Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 778, normalized size of antiderivative = 3.03

$$\int \frac{x^3(d+ex)}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \left[\frac{3\left(ac^3d^6e+a^2c^2d^4e^3+3a^3cd^2e^5-5a^4e^7+(c^4d^7+ac^3d^5e^2+3\right.}{}\right.$$

$$-\frac{3\left(ac^3d^6e+a^2c^2d^4e^3+3a^3cd^2e^5-5a^4e^7+(c^4d^7+ac^3d^5e^2+3a^2c^2d^3e^4-5a^3cde^6)x\right)\sqrt{-cde}\arctan\left(\frac{\sqrt{}}{}\right.}{}$$

input
```
integrate(x^3*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[1/16*(3*(a*c^3*d^6*e + a^2*c^2*d^4*e^3 + 3*a^3*c*d^2*e^5 - 5*a^4*e^7 + (c
^4*d^7 + a*c^3*d^5*e^2 + 3*a^2*c^2*d^3*e^4 - 5*a^3*c*d*e^6)*x)*sqrt(c*d*e)
*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*
x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) +
 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(3*a*c^3*d^5*e^2 + 4*a^2*c^2*d^3*e^4 - 1
5*a^3*c*d*e^6 - 2*(c^4*d^5*e^2 - a*c^3*d^3*e^4)*x^2 + (3*c^4*d^6*e + 2*a*c
^3*d^4*e^3 - 5*a^2*c^2*d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x))/(a*c^5*d^6*e^4 - a^2*c^4*d^4*e^6 + (c^6*d^7*e^3 - a*c^5*d^5*e^5)*x),
 -1/8*(3*(a*c^3*d^6*e + a^2*c^2*d^4*e^3 + 3*a^3*c*d^2*e^5 - 5*a^4*e^7 + (c
^4*d^7 + a*c^3*d^5*e^2 + 3*a^2*c^2*d^3*e^4 - 5*a^3*c*d*e^6)*x)*sqrt(-c*d*e
)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^
2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*
d*e^3)*x)) + 2*(3*a*c^3*d^5*e^2 + 4*a^2*c^2*d^3*e^4 - 15*a^3*c*d*e^6 - 2*(
c^4*d^5*e^2 - a*c^3*d^3*e^4)*x^2 + (3*c^4*d^6*e + 2*a*c^3*d^4*e^3 - 5*a^2*
c^2*d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c^5*d^6*e^
4 - a^2*c^4*d^4*e^6 + (c^6*d^7*e^3 - a*c^5*d^5*e^5)*x)]
```

Sympy [F]

$$\int \frac{x^3(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{x^3(d+ex)}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**3*(d + e*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex)}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[4,4,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,0]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{x^3\,(d+e\,x)}{{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)}^{3/2}}\,dx$$

input
```
int((x^3*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((x^3*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.04

$$\int \frac{x^3(d+ex)}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{15\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a^3e^6-9\sqrt{e}}{}$$

input
```
int(x^3*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(15*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**6 - 9*s
qrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) +
sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e**4 - 3
*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**4*e**2 -
 3*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x
) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**3*d**6 - 10*s
qrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**3*e**6 + 3*sqrt(e)*sqrt(d)*sqr
t(c)*sqrt(a*e + c*d*x)*a**2*c*d**2*e**4 - sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e
 + c*d*x)*c**3*d**6 - 15*sqrt(d + e*x)*a**3*c*d*e**6 + 4*sqrt(d + e*x)*a**
2*c**2*d**3*e**4 - 5*sqrt(d + e*x)*a**2*c**2*d**2*e**5*x + 3*sqrt(d + e*x)
*a*c**3*d**5*e**2 + 2*sqrt(d + e*x)*a*c**3*d**4*e**3*x + 2*sqrt(d + e*x)*a
*c**3*d**3*e**4*x**2 + 3*sqrt(d + e*x)*c**4*d**6*e*x - 2*sqrt(d + e*x)*c**
4*d**5*e**2*x**2)/(4*sqrt(a*e + c*d*x)*c**4*d**4*e**3*(a*e**2 - c*d**2))
```

3.89 $\int \frac{x^2(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 182

$$\int \frac{x^2(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2a^2e^2(d+ex)}{c^2d^2\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2e} - \frac{\left(cd^2+3ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{5/2}d^{5/2}e^{3/2}}$$

output
```
-2*a^2*e^2*(e*x+d)/c^2/d^2/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e-(3*a*e^2+c*d^2)*
arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/c^(5/2)/d^(5/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.02

$$\int \frac{x^2(d+ex)}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{e}(d+ex)\left(-3a^2e^3+c^2d^3x+acde(d-ex)\right)-\left(c^2d^4+2a\cdots\right.}{c^{5/2}d^{5/2}e^{3/2}\left(cd^2-ae^2\right)\sqrt{(\cdots}}$$

input
```
Integrate[(x^2*(d + e*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(Sqrt[c]*Sqrt[d]*Sqrt[e]*(d + e*x)*(-3*a^2*e^3 + c^2*d^3*x + a*c*d*e*(d -
e*x)) - (c^2*d^4 + 2*a*c*d^2*e^2 - 3*a^2*e^4)*Sqrt[a*e + c*d*x]*Sqrt[d + e
*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/
(c^(5/2)*d^(5/2)*e^(3/2)*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1211, 25, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 1211$$

$$\frac{\int -\frac{e^2(ae-cdx)}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{c^2d^2e^2}-\frac{2a^2e^2(d+ex)}{c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{e^2(ae-cdx)}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{c^2d^2e^2}-\frac{2a^2e^2(d+ex)}{c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{ae-cdx}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{c^2d^2}-\frac{2a^2e^2(d+ex)}{c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1160

$$-\frac{\frac{\left(3ae^2+cd^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2e}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}}{c^2d^2}-\frac{2a^2e^2(d+ex)}{c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1092

$$-\frac{\frac{\left(3ae^2+cd^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{e}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}}{c^2d^2}-\frac{2a^2e^2(d+ex)}{c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 219

$$-\frac{2a^2e^2(d+ex)}{c^2d^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\frac{(3ae^2+cd^2)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}}{c^2d^2}$$

input
```
Int[(x^2*(d + e*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*a^2*e^2*(d + e*x))/(c^2*d^2*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2]) - (-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/e) + (
(c*d^2 + 3*a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e
^(3/2)))/(c^2*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 737 vs. 2(164) = 328.

Time = 2.44 (sec) , antiderivative size = 738, normalized size of antiderivative = 4.05

method	result
default	$d\left(-\frac{x}{dec\sqrt{ade+(a e^2+c d^2)x+cd x^2 e}}-\frac{(a e^2+c d^2)\left(-\frac{1}{dec\sqrt{ade+\left(a e^2+c d^2\right)x+cd x^2 e}}-\frac{\left(a e^2+c d^2\right)\left(2cdxe+a e^2+c d^{\cdots}\right.}{dec\left(4ac d^2 e^2-\left(a e^2+c d^2\right)^2\right)\sqrt{ade+\left(a e^{2\cdots}\right.}}\right.}{2dec}\right.$

input
```
int(x^2*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
d*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/
c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2
*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)
*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2
)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+e*(x^2/d/e/c/(a*
d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*
d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*
d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+
c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(
1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2
+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2*a/c*(-1/d/e/c/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*
c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 608, normalized size of antiderivative = 3.34

$$\int \frac{x^2(d+ex)}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \left[\frac{(ac^2d^4e+2\,a^2cd^2e^3-3\,a^3e^5+(c^3d^5+2\,ac^2d^3e^2-3\,a^2cde^4)x)\cdot}{}\right.$$

input
```
integrate(x^2*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[1/4*((a*c^2*d^4*e + 2*a^2*c*d^2*e^3 - 3*a^3*e^5 + (c^3*d^5 + 2*a*c^2*d^3*
e^2 - 3*a^2*c*d*e^4)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*
c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d
*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(a*c^
2*d^3*e^2 - 3*a^2*c*d*e^4 + (c^3*d^4*e - a*c^2*d^2*e^3)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x))/(a*c^4*d^5*e^3 - a^2*c^3*d^3*e^5 + (c^5*d^6*
e^2 - a*c^4*d^4*e^4)*x), 1/2*((a*c^2*d^4*e + 2*a^2*c*d^2*e^3 - 3*a^3*e^5 +
 (c^3*d^5 + 2*a*c^2*d^3*e^2 - 3*a^2*c*d*e^4)*x)*sqrt(-c*d*e)*arctan(1/2*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt
(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*
(a*c^2*d^3*e^2 - 3*a^2*c*d*e^4 + (c^3*d^4*e - a*c^2*d^2*e^3)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c^4*d^5*e^3 - a^2*c^3*d^3*e^5 + (c^5
*d^6*e^2 - a*c^4*d^4*e^4)*x)]
```

Sympy [F]

$$\int \frac{x^2(d+ex)}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^2(d+ex)}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**2*(d + e*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[3,3,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,0]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx=\int \frac{x^2\,(d+e\,x)}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int((x^2*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((x^2*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.03

$$\int \frac{x^2(d+ex)}{\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx=\frac{-12\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a^2e^4+8\sqrt{}$$

input
```
int(x^2*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 12*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**4 +
8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x)
 + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d**2*e**2 + 4
*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**4 + 9*sqrt
(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*e**4 - 2*sqrt(e)*sqrt(d)*sqrt(c
)*sqrt(a*e + c*d*x)*a*c*d**2*e**2 + sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d
*x)*c**2*d**4 + 12*sqrt(d + e*x)*a**2*c*d*e**4 - 4*sqrt(d + e*x)*a*c**2*d*
*3*e**2 + 4*sqrt(d + e*x)*a*c**2*d**2*e**3*x - 4*sqrt(d + e*x)*c**3*d**4*e
*x)/(4*sqrt(a*e + c*d*x)*c**3*d**3*e**2*(a*e**2 - c*d**2))
```

3.90 $\int \frac{x(d+ex)}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 127

$$\int \frac{x(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\frac{2ae(d+ex)}{cd\,(cd^2-ae^2)\,\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{3/2}d^{3/2}\sqrt{e}}$$

output
```
2*a*e*(e*x+d)/c/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2
*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/c^(3/2)/d^(3/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.13

$$\int \frac{x(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\frac{2\left(a\sqrt{c}\sqrt{d}e^{3/2}(d+ex)+(cd^2-ae^2)\,\sqrt{ae+cdx}\sqrt{d+ex}\operatorname{arctanh}\right.}{c^{3/2}d^{3/2}\sqrt{e}\,(cd^2-ae^2)\,\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(x*(d + e*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(2*(a*Sqrt[c]*Sqrt[d]*e^(3/2)*(d + e*x) + (c*d^2 - a*e^2)*Sqrt[a*e + c*d*x
]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e
+ c*d*x])]))/(c^(3/2)*d^(3/2)*Sqrt[e]*(c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(
d + e*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1211, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd}+\frac{2ae(d+ex)}{cd\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{cd}+\frac{2ae(d+ex)}{cd\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{c^{3/2}d^{3/2}\sqrt{e}}+\frac{2ae(d+ex)}{cd\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(x*(d + e*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(2*a*e*(d + e*x))/(c*d*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) + ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(c^(3/2)*d^(3/2)*Sqrt[e])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(111) = 222$.

Time = 2.09 (sec) , antiderivative size = 404, normalized size of antiderivative = 3.18

method	result
default	$d\left(-\frac{1}{dec\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}-\frac{(a\,e^2+c\,d^2)(2cdxe+a\,e^2+c\,d^2)}{dec\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}\right)+e\left(-\frac{}{dec\sqrt{ade+(a\,e^2}}\right.$

input
```
int(x*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
d*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2
*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)
*x+c*d*x^2*e)^(1/2))+e*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1
/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(
a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)
/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+
c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1
/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(111) = 222$.

Time = 0.18 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.70

$$\int \frac{x(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \left[\frac{4\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}acde^2+\left(acd^2e-a^2e^3+\left(c^2d^3\right.\right.}{}\right.$$

input
```
integrate(x*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
fricas")
```

output
```
[1/2*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*c*d*e^2 + (a*c*d^2*e
 - a^2*e^3 + (c^2*d^3 - a*c*d*e^2)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 +
c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3
)*x))/(a*c^3*d^4*e^2 - a^2*c^2*d^2*e^4 + (c^4*d^5*e - a*c^3*d^3*e^3)*x), (
2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*c*d*e^2 - (a*c*d^2*e - a^2
*e^3 + (c^2*d^3 - a*c*d*e^2)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^
2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)))/(a*c^3*d^4*e^2 - a^
2*c^2*d^2*e^4 + (c^4*d^5*e - a*c^3*d^3*e^3)*x)]
```

Sympy [F]

$$\int \frac{x(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{x(d+ex)}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x*(d + e*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[1,1,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,2]
%%%}+%%%{
```

Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.84

$$\int \frac{x(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\frac{e\ln\left(2\sqrt{(a\,e+c\,d\,x)\,(d+e\,x)}\,\sqrt{c\,d\,e}+a\,e^2+c\,d^2+2\,c\,d\,e\,x\right)}{(c\,d\,e)^{3/2}}+\frac{d\,(2\,x\,(c\,d^2+a\,e^2)+4\,a\,d\,e)}{\left((c\,d^2+a\,e^2)^2-4\,a\,c\,d^2\,e^2\right)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}-\frac{x\left(\frac{(c\,d^2+a\,e^2)^2}{2}-a\,c\,d^2\,e^2\right)+\frac{a\,d\,e\,(c\,d^2+a\,e^2)}{2}}{c\,d\,\left(\frac{(c\,d^2+a\,e^2)^2}{4}-a\,c\,d^2\,e^2\right)\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}}$$

input
```
int((x*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
(e*log(2*((a*e + c*d*x)*(d + e*x))^(1/2)*(c*d*e)^(1/2) + a*e^2 + c*d^2 + 2
*c*d*e*x))/(c*d*e)^(3/2) + (d*(2*x*(a*e^2 + c*d^2) + 4*a*d*e))/(((a*e^2 +
c*d^2)^2 - 4*a*c*d^2*e^2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)) -
 (x*((a*e^2 + c*d^2)^2/2 - a*c*d^2*e^2) + (a*d*e*(a*e^2 + c*d^2))/2)/(c*d*
((a*e^2 + c*d^2)^2/4 - a*c*d^2*e^2)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.54

$$\int \frac{x(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\frac{2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a\,e^2-2\sqrt{e}\,\sqrt{}$$

input
```
int(x*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
(2*(sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*e**2 - sqrt(e
)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(
d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**2 - sqrt(e)*sqrt(d)*
sqrt(c)*sqrt(a*e + c*d*x)*a*e**2 - sqrt(d + e*x)*a*c*d*e**2))/(sqrt(a*e +
c*d*x)*c**2*d**2*e*(a*e**2 - c*d**2))
```

3.91 $\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 50

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2(d+ex)}{\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
(-2*e*x-2*d)/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2(d+ex)}{\left(cd^2-ae^2\right)\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*(d + e*x))/((c*d^2 - a*e^2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1124, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1124}$$

$$e^2\int 0dx-\frac{2(d+ex)}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{24}$$

$$-\frac{2(d+ex)}{\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*(d + e*x))/((c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

method	result
trager	$\frac{2\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}{(a\,e^2-c\,d^2)(cdx+ae)}$
gosper	$\frac{2(ex+d)^2(cdx+ae)}{(a\,e^2-c\,d^2)(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$
orering	$\frac{2(ex+d)^2(cdx+ae)}{(a\,e^2-c\,d^2)(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{3}{2}}}$
default	$\frac{2d(2cdxe+a\,e^2+c\,d^2)}{\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}+e\left(-\frac{1}{dec\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}-\frac{(a\,e^2+c\,d^2)(2c}{dec\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right.}\right.$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
2/(a*e^2-c*d^2)/(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=-\frac{2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{acd^2e-a^2e^3+(c^2d^3-acde^2)x}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="fr
icas")
```

output
```
-2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c*d^2*e - a^2*e^3 + (c^2
*d^3 - a*c*d*e^2)*x)
```

Sympy [F]

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{d+ex}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[1,1,0]%%%},[2,0]%%%}+%%%{%%{[%%%{-2,[0,0,1]%%%},0]:
[1,0,%%%{
```

Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}}{\left(a\,e+c\,d\,x\right)\,\left(a\,e^2-c\,d^2\right)}$$

input
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((a*e + c*d*x)*(a*e^2 -
c*d^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}+2\sqrt{ex+d}\,cd}{\sqrt{cdx+ae}\,cd\left(a\,e^2-c\,d^2\right)}$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*(sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x) + sqrt(d + e*x)*c*d))/(sqrt(
a*e + c*d*x)*c*d*(a*e**2 - c*d**2))
```

3.92 $\int \frac{d+ex}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 127

$$\int \frac{d+ex}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2cd(d+ex)}{ae\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{3/2}\sqrt{d}e^{3/2}}$$

output
```
2*c*d*(e*x+d)/a/e/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-2
*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2))/a^(3/2)/d^(1/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int \frac{d+ex}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2\left(\frac{\sqrt{a}cd^{3/2}\sqrt{e}(d+ex)}{cd^2-ae^2}-\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sqrt{d+ex}}{\sqrt{d}\sqrt{ae+cdx}}\right)\right)}{a^{3/2}\sqrt{d}e^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)/(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*((Sqrt[a]*c*d^(3/2)*Sqrt[e]*(d + e*x))/(c*d^2 - a*e^2) - Sqrt[a*e + c*d
*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*
e + c*d*x])]))/(a^(3/2)*Sqrt[d]*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1212, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1212}$$

$$\frac{2cd(d+ex)}{ae\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-cde^3\int -\frac{1}{acde^4x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx$$

$$\Big\downarrow \text{25}$$

$$cde^3\int \frac{1}{acde^4x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2cd(d+ex)}{ae\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ae}+\frac{2cd(d+ex)}{ae\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{2cd(d+ex)}{ae\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ae}$$

$$\Big\downarrow \text{219}$$

$$\frac{2cd(d+ex)}{ae\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{a^{3/2}\sqrt{d}e^{3/2}}$$

input
```
Int[(d + e*x)/(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*c*d*(d + e*x))/(a*e*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) - ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[
e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(a^(3/2)*Sqrt[d]*e^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1212
```
Int[((x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)
^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*d - b*e)^n*((d + e
*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*x^2])), x] - Simp[e^2/c^(m +
n - 1)   Int[ExpandToSum[(c^(m + n - 1)*(d + e*x)^(m - 1) - ((c*d - b*e)^n*
(2*c*d - b*e)^(m - 1))/(e^n*x^n))/(c*d - b*e - c*e*x), x]/(Sqrt[a + b*x + c
*x^2]/x^n), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^
2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(111) = 222$.

Time = 1.95 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.26

method	result
default	$\frac{2e(2cdxe+a e^2+c d^2)}{\left(4ac d^2e^2-(a e^2+c d^2)^2\right)\sqrt{ade+(a e^2+c d^2)x+cd x^2e}}+d\left(\frac{1}{ade\sqrt{ade+(a e^2+c d^2)x+cd x^2e}}-\frac{(a e^2+c d^2)(2cd}{ade\left(4ac d^2e^2-(a e^2+c d^2)^2\right)}\right.$

input
```
int((e*x+d)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
2*e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*x^2*e)^(1/2)+d*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/
2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2
)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a
*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/
2))/x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(111) = 222$.

Time = 0.38 (sec) , antiderivative size = 479, normalized size of antiderivative = 3.77

$$\int \frac{d+ex}{x\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \left[\frac{4\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}acd^2e+(acd^2e-a^2e^3+(c^2d^3\right.$$

input
```
integrate((e*x+d)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
fricas")
```

output
```
[1/2*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*c*d^2*e + (a*c*d^2*e
 - a^2*e^3 + (c^2*d^3 - a*c*d*e^2)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*
d*e^3)*x)/x^2))/(a^3*c*d^3*e^3 - a^4*d*e^5 + (a^2*c^2*d^4*e^2 - a^3*c*d^2*
e^4)*x), (2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*c*d^2*e + (a*c*d
^2*e - a^2*e^3 + (c^2*d^3 - a*c*d*e^2)*x)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*
d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)))/(a^3*c*
d^3*e^3 - a^4*d*e^5 + (a^2*c^2*d^4*e^2 - a^3*c*d^2*e^4)*x)]
```

Sympy [F]

$$\int \frac{d+ex}{x\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{d+ex}{x\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)/x/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)/(x*((d + e*x)*(a*e + c*d*x))**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[1,1,5]%%%},[2,3]%%%}+%%%{%%%{-2,[2,3,3]%%%},[2,2]%%
%}+%%%{%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{d+e\,x}{x\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int((d + e*x)/(x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int((d + e*x)/(x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 455, normalized size of antiderivative = 3.58

$$\int \frac{d+ex}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{\sqrt{e}\,\sqrt{d}\,\sqrt{a}\,\sqrt{cdx+ae}\,\log\left(\sqrt{e}\,\sqrt{cdx+ae}-\sqrt{2\sqrt{c}\,\sqrt{a}\,de+a\,e^2}\right.}{}$$

input
```
int((e*x+d)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) -
 sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d +
e*x))*a*e**2 - sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(
a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqr
t(c)*sqrt(d + e*x))*c*d**2 + sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log
(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2)
 + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*e**2 - sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*
e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*
e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c*d**2 - sqrt(e)*sqrt(d)*s
qrt(a)*sqrt(a*e + c*d*x)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(
a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*a*e**2 + sqrt(e)*sqrt(d)
*sqrt(a)*sqrt(a*e + c*d*x)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqr
t(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*c*d**2 - 2*sqrt(e)*sqr
t(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*d*e - 2*sqrt(d + e*x)*a*c*d**2*e)/(sqrt(a
*e + c*d*x)*a**2*d*e**2*(a*e**2 - c*d**2))
```

3.93 $\int \frac{d+ex}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 209

$$\int \frac{d+ex}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{1}{aex\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{3c^2d^4-a^2e^4+cde\left(3cd^2-ae^2\right)x}{a^2de^2\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\left(3cd^2+ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{5/2}d^{3/2}e^{5/2}}$$

output
```
-1/a/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(3*c^2*d^4-a^2*e^4+c*d*e*
(-a*e^2+3*c*d^2)*x)/a^2/d/e^2/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)+(a*e^2+3*c*d^2)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(3/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.91

$$\int \frac{d+ex}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{\sqrt{a}\sqrt{d}\sqrt{e}(d+ex)\left(a^2e^3-3c^2d^3x+acde(-d+ex)\right)+\left(3c^2d^4\right.}{a^{5/2}d^{3/2}e^{5/2}\left(cd^2-ae^2\right)}$$

input
```
Integrate[(d + e*x)/(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(Sqrt[a]*Sqrt[d]*Sqrt[e]*(d + e*x)*(a^2*e^3 - 3*c^2*d^3*x + a*c*d*e*(-d +
e*x)) + (3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*x*Sqrt[a*e + c*d*x]*Sqrt[d +
 e*x]*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])]
)/(a^(5/2)*d^(3/2)*e^(5/2)*(c*d^2 - a*e^2)*x*Sqrt[(a*e + c*d*x)*(d + e*x)]
)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1212, 25, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1212}$$

$$-c^2d^2e^4\int -\frac{ae-cdx}{a^2c^2d^2e^6x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{2c^2d^2(d+ex)}{a^2e^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{25}$$

$$c^2d^2e^4\int \frac{ae-cdx}{a^2c^2d^2e^6x^2\sqrt{cdex^2+(cd^2+ae^2)\,x+ade}}dx-\frac{2c^2d^2(d+ex)}{a^2e^2\,(cd^2-ae^2)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{ae-cdx}{x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{a^2e^2}-\frac{2c^2d^2(d+ex)}{a^2e^2\,(cd^2-ae^2)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \; 1228$$

$$\frac{-\frac{(ae^2+3cd^2)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{dx}}{a^2e^2}-\frac{2c^2d^2(d+ex)}{a^2e^2\,(cd^2-ae^2)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{(ae^2+3cd^2)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{d}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{dx}}{a^2e^2}-\frac{2c^2d^2(d+ex)}{a^2e^2\,(cd^2-ae^2)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(ae^2+3cd^2)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}d^{3/2}\sqrt{e}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{dx}}{a^2e^2}-\frac{2c^2d^2(d+ex)}{a^2e^2\,(cd^2-ae^2)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

input
```
Int[(d + e*x)/(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*c^2*d^2*(d + e*x))/(a^2*e^2*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2]) + (-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x))
 + ((3*c*d^2 + a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqr
t[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[a]*d^(
3/2)*Sqrt[e]))/(a^2*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1212
```
Int[((x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)
^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*d - b*e)^n*((d + e
*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*x^2])), x] - Simp[e^2/c^(m +
n - 1)   Int[ExpandToSum[(c^(m + n - 1)*(d + e*x)^(m - 1) - ((c*d - b*e)^n*
(2*c*d - b*e)^(m - 1))/(e^n*x^n))/(c*d - b*e - c*e*x), x]/(Sqrt[a + b*x + c
*x^2]/x^n), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^
2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(191) = 382$.

Time = 2.07 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.70

method	result
default	$d\left(-\frac{1}{adex\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}-\frac{3(a\,e^2+c\,d^2)\left(\frac{1}{ade\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}-\frac{\left(a\,e^2+c\,d^2\right)\left(2cdxe+a\,e^2+c\right.}{ade\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)\sqrt{ade+\left(a\,e\right.}}\right.}{2ade}\right.$

input
```
int((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
d*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/a/
d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(
2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(
a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-4*c/a*(2*c*d*e*x
+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x
^2*e)^(1/2))+e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d
^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c
*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))
```

Fricas [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 644, normalized size of antiderivative = 3.08

$$\int \frac{d+ex}{x^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \left[\frac{\sqrt{ade}((3\,c^3d^5-2\,ac^2d^3e^2-a^2cde^4)x^2+(3\,ac^2d^4e-2\,a^2cd^2e\right.$$

$$-\frac{\sqrt{-ade}((3\,c^3d^5-2\,ac^2d^3e^2-a^2cde^4)x^2+(3\,ac^2d^4e-2\,a^2cd^2e^3-a^3e^5)x)\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2}{2\,(acd^2e^2x^2+a^2d^2e^2}\right.}{2\,((a^3c^2d^5e^3-a^4cd^3e^5)x^2+}$$

input
```
integrate((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[1/4*(sqrt(a*d*e)*((3*c^3*d^5 - 2*a*c^2*d^3*e^2 - a^2*c*d*e^4)*x^2 + (3*a*
c^2*d^4*e - 2*a^2*c*d^2*e^3 - a^3*e^5)*x)*log((8*a^2*d^2*e^2 + (c^2*d^4 +
6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x
)/x^2) - 4*(a^2*c*d^3*e^2 - a^3*d*e^4 + (3*a*c^2*d^4*e - a^2*c*d^2*e^3)*x)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c^2*d^5*e^3 - a^4*c*d^
3*e^5)*x^2 + (a^4*c*d^4*e^4 - a^5*d^2*e^6)*x), -1/2*(sqrt(-a*d*e)*((3*c^3*
d^5 - 2*a*c^2*d^3*e^2 - a^2*c*d*e^4)*x^2 + (3*a*c^2*d^4*e - 2*a^2*c*d^2*e^
3 - a^3*e^5)*x)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (
a*c*d^3*e + a^2*d*e^3)*x)) + 2*(a^2*c*d^3*e^2 - a^3*d*e^4 + (3*a*c^2*d^4*e
 - a^2*c*d^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c^
2*d^5*e^3 - a^4*c*d^3*e^5)*x^2 + (a^4*c*d^4*e^4 - a^5*d^2*e^6)*x)]
```

Sympy [F]

$$\int \frac{d+ex}{x^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{d+ex}{x^2\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)/x**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)/(x**2*((d + e*x)*(a*e + c*d*x))**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[1,1,0]%%%},[6,0]%%%}+%%%{%%{[%%%{-2,[0,0,1]%%%},0]:
[1,0,%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{x^2 (ade+(cd^2+ae^2) x+cdex^2)^{3/2}}\,dx = \int \frac{d+e\,x}{x^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int((d + e*x)/(x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int((d + e*x)/(x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 772, normalized size of antiderivative = 3.69

$$\int \frac{d+ex}{x^2 (ade+(cd^2+ae^2) x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x
) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))*a**2*e**4*x - 2*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqr
t(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + s
qrt(d)*sqrt(c)*sqrt(d + e*x))*a*c*d**2*e**2*x + 3*sqrt(e)*sqrt(d)*sqrt(a)*
sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d
*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**2*d**4*x - sqrt(
e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(
2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*
a**2*e**4*x - 2*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt
(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sq
rt(c)*sqrt(d + e*x))*a*c*d**2*e**2*x + 3*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e
+ c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e*
*2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**2*d**4*x + sqrt(e)*sqrt(d
)*sqrt(a)*sqrt(a*e + c*d*x)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sq
rt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*a**2*e**4*x + 2*sqrt(
e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d
+ e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*a*c*d**2*e**
2*x - 3*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(2*sqrt(e)*sqrt(d)*sq
rt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e...
```

3.94 $\int \frac{d+ex}{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 322

$$\int \frac{d+ex}{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{1}{2aex^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{5cd^2+ae^2}{4a^2de^2x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{15c^3d^6+ac^2d^4e^2-5a^2cd^2e^4-3a^3e^6+cde(5cd^2-3ae^2)\left(3cd^2+ae^2\right)x}{4a^3d^2e^3\left(cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{3(5c^2d^4+2acd^2e^2+a^2e^4)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4a^{7/2}d^{5/2}e^{7/2}}$$

output
```
-1/2/a/e/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/4*(a*e^2+5*c*d^2)/a
^2/d/e^2/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/4*(15*c^3*d^6+a*c^2*d
^4*e^2-5*a^2*c*d^2*e^4-3*a^3*e^6+c*d*e*(-3*a*e^2+5*c*d^2)*(a*e^2+3*c*d^2)*
x)/a^3/d^2/e^3/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4*
(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(7/2)/d^(5/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.79

$$\int \frac{d+ex}{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{\sqrt{a}\sqrt{d}\sqrt{e}(d+ex)\left(15c^3d^5x^2+ac^2d^3ex(5d-4ex)+a^3e^4(2d-\right.}{}$$

input
```
Integrate[(d + e*x)/(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(Sqrt[a]*Sqrt[d]*Sqrt[e]*(d + e*x)*(15*c^3*d^5*x^2 + a*c^2*d^3*e*x*(5*d -
4*e*x) + a^3*e^4*(2*d - 3*e*x) - a^2*c*d*e^2*(2*d^2 + 2*d*e*x + 3*e^2*x^2)
) - 3*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*x^2*Sqrt[a*e
 + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*S
qrt[a*e + c*d*x])])/(4*a^(7/2)*d^(5/2)*e^(7/2)*(c*d^2 - a*e^2)*x^2*Sqrt[(a
*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1212, 25, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \ 1212$$

$$\frac{2c^3d^3(d+ex)}{a^3e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-c^3d^3e^5\int -\frac{\frac{x^2}{a^3cde^8}-\frac{x}{a^2c^2d^2e^7}+\frac{1}{ac^3d^3e^6}}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx$$

$$\Big\downarrow \ 25$$

$$c^3d^3e^5\int \frac{\frac{x^2}{a^3cde^8}-\frac{x}{a^2c^2d^2e^7}+\frac{1}{ac^3d^3e^6}}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2c^3d^3(d+ex)}{a^3e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 2181$$

$$c^3d^3e^5\left(-\frac{\int \frac{ae\left(7cd^2+3ae^2\right)-2cd\left(2cd^2-ae^2\right)x}{2a^2c^3d^3e^7x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2a^2c^3d^4e^7x^2}\right)+\frac{2c^3d^3(d+ex)}{a^3e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 27$$

$$c^3d^3e^5\left(-\frac{\int \frac{ae\left(7cd^2+3ae^2\right)-2cd\left(2cd^2-ae^2\right)x}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4a^3c^3d^4e^8}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2a^2c^3d^4e^7x^2}\right)+\frac{2c^3d^3(d+ex)}{a^3e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1228$$

$$c^3d^3e^5\left(-\frac{-\frac{3\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d}-\frac{\left(3ae^2+7cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}}{4a^3c^3d^4e^8}-\frac{\sqrt{x\left(ae^2+cd^2\right)\cdots}}{2a^2c^3d^4\cdots}\right.$$

$$\frac{2c^3d^3(d+ex)}{a^3e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1154$$

$$c^3d^3e^5\left(-\frac{\frac{3\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{d}-\frac{\left(3ae^2+7cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{dx}}{4a^3c^3d^4e^8}\cdots\right.$$

$$\frac{2c^3d^3(d+ex)}{a^3e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 219$$

$$\frac{2c^3d^3(d+ex)}{a^3e^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+$$
$$c^3d^3e^5\left(-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2a^2c^3d^4e^7x^2}-\frac{\frac{3\left(a^2e^4+2acd^2e^2+5c^2d^4\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}d^{3/2}\sqrt{e}}-\frac{(3ae^2+7}{}}{4a^3c^3d^4e^8}\right.$$

input
```
Int[(d + e*x)/(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*c^3*d^3*(d + e*x))/(a^3*e^3*(c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) + c^3*d^3*e^5*(-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]/(a^2*c^3*d^4*e^7*x^2) - (-(((7*c*d^2 + 3*a*e^2)*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])/(d*x)) + (3*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^
4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a
*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(2*Sqrt[a]*d^(3/2)*Sqrt[e]))/(4*a
^3*c^3*d^4*e^8))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1212
```
Int[((x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)
^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*d - b*e)^n*((d + e
*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*x^2])), x] - Simp[e^2/c^(m +
n - 1)   Int[ExpandToSum[(c^(m + n - 1)*(d + e*x)^(m - 1) - ((c*d - b*e)^n*
(2*c*d - b*e)^(m - 1))/(e^n*x^n))/(c*d - b*e - c*e*x), x]/(Sqrt[a + b*x + c
*x^2]/x^n), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^
2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(294) = 588$.

Time = 2.10 (sec) , antiderivative size = 984, normalized size of antiderivative = 3.06

method	result
default	$d\left(-\frac{1}{2ade\,x^2\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}-\frac{5(a\,e^2+c\,d^2)\left(-\frac{1}{adex\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}-\frac{3\left(a\,e^2+c\,d^2\right)\left(\frac{1}{ade\sqrt{ade+\left(a\,e^2+c\,d\right.}}\right.}{}\right.}{}\right.$

input

```
int((e*x+d)/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
d*(-1/2/a/d/e/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-5/4*(a*e^2+c*d^2
)/a/d/e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d
^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a
/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2
+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)
*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-4*c/a*(2*c
*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2))-3/2*c/a*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)
^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*
d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
))/x)))+e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c
*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)
/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-4*c/a*(2
*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)
*x+c*d*x^2*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 828, normalized size of antiderivative = 2.57

$$\int \frac{d+ex}{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[1/16*(3*((5*c^4*d^7 - 3*a*c^3*d^5*e^2 - a^2*c^2*d^3*e^4 - a^3*c*d*e^6)*x^
3 + (5*a*c^3*d^6*e - 3*a^2*c^2*d^4*e^3 - a^3*c*d^2*e^5 - a^4*e^7)*x^2)*sqr
t(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*
sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(2*a^3*c*d^4*e^3 - 2*a
^4*d^2*e^5 - (15*a*c^3*d^6*e - 4*a^2*c^2*d^4*e^3 - 3*a^3*c*d^2*e^5)*x^2 -
(5*a^2*c^2*d^5*e^2 - 2*a^3*c*d^3*e^4 - 3*a^4*d*e^6)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x))/((a^4*c^2*d^6*e^4 - a^5*c*d^4*e^6)*x^3 + (a^5*c*
d^5*e^5 - a^6*d^3*e^7)*x^2), 1/8*(3*((5*c^4*d^7 - 3*a*c^3*d^5*e^2 - a^2*c^
2*d^3*e^4 - a^3*c*d*e^6)*x^3 + (5*a*c^3*d^6*e - 3*a^2*c^2*d^4*e^3 - a^3*c*
d^2*e^5 - a^4*e^7)*x^2)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*
x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(2*a^3*c*d^4*e^3 - 2*a
^4*d^2*e^5 - (15*a*c^3*d^6*e - 4*a^2*c^2*d^4*e^3 - 3*a^3*c*d^2*e^5)*x^2 -
(5*a^2*c^2*d^5*e^2 - 2*a^3*c*d^3*e^4 - 3*a^4*d*e^6)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x))/((a^4*c^2*d^6*e^4 - a^5*c*d^4*e^6)*x^3 + (a^5*c*
d^5*e^5 - a^6*d^3*e^7)*x^2)]
```

Sympy [F]

$$\int \frac{d+ex}{x^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx=\int \frac{d+ex}{x^3\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)/x**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)/(x**3*((d + e*x)*(a*e + c*d*x))**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[1,1,11]%%%},[2,7]%%%}+%%%{%%%{-4,[2,3,9]%%%},[2,6]%
%%}+%%%{%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{x^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{d+e\,x}{x^3\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

input
```
int((d + e*x)/(x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int((d + e*x)/(x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 1256, normalized size of antiderivative = 3.90

$$\int \frac{d+ex}{x^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(9*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x)
 - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))*a**4*e**8*x**2 + 24*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(
sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**2*e**6*x**2 + 42*sqrt(e)*sqrt(d
)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)
*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c**2
*d**4*e**4*x**2 - 75*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)
*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(
d)*sqrt(c)*sqrt(d + e*x))*c**4*d**8*x**2 + 9*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(
a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e +
a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**4*e**8*x**2 + 24*sqrt
(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt
(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))
*a**3*c*d**2*e**6*x**2 + 42*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(
sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c**2*d**4*e**4*x**2 - 75*sqrt(e)*sqr
t(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt
(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**4*d
**8*x**2 - 9*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(2*sqrt(e)*sq...
```

3.95 $\int \frac{x^3(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 323

$$\begin{aligned}\int \frac{x^3(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx &= \frac{2a^3e^3(d+ex)}{c^4d^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ &\quad - \frac{\left(cd^2-3ae^2\right)\left(3cd^2+19ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24c^4d^4e^2} \\ &\quad + \frac{\left(cd^2-11ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12c^3d^3e} \\ &\quad + \frac{x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^2d^2} \\ &\quad + \frac{\left(c^3d^6+3ac^2d^4e^2+15a^2cd^2e^4-35a^3e^6\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8c^{9/2}d^{9/2}e^{5/2}}\end{aligned}$$

output

```
2*a^3*e^3*(e*x+d)/c^4/d^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/24*(-3
*a*e^2+c*d^2)*(19*a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c
^4/d^4/e^2+1/12*(-11*a*e^2+c*d^2)*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2
)/c^3/d^3/e+1/3*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2+1/8*(-
35*a^3*e^6+15*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2+c^3*d^6)*arctanh(c^(1/2)*d^(1/
2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(9/2)/d^(9/2
)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.75

$$\int \frac{x^3(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{-\sqrt{c}\sqrt{d}\sqrt{e}(d+ex)\left(-105a^3e^5+5a^2cde^3(2d-7ex)+c^3d^3x(3d^2\right.}{}$$

input
```
Integrate[(x^3*(d + e*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),
x]
```

output
```
(-(Sqrt[c]*Sqrt[d]*Sqrt[e]*(d + e*x)*(-105*a^3*e^5 + 5*a^2*c*d*e^3*(2*d -
7*e*x) + c^3*d^3*x*(3*d^2 - 2*d*e*x - 8*e^2*x^2) + a*c^2*d^2*e*(3*d^2 + 8*
d*e*x + 14*e^2*x^2))) + 3*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 -
35*a^3*e^6)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[
d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(24*c^(9/2)*d^(9/2)*e^(5/2)*Sqrt[(
a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1211, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)^2}{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int \frac{a^2(cd^2-ae^2)e^7+c^3d^3x^3e^6-acd(cd^2-ae^2)xe^6+c^2d^2(cd^2-ae^2)x^2e^5}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{c^4d^4e^5}+\frac{2a^3e^3(d+ex)}{c^4d^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\int \frac{6a^2cd\left(cd^2-ae^2\right)e^8-2ac^2d^2\left(5cd^2-3ae^2\right)xe^7+c^3d^3\left(cd^2-11ae^2\right)x^2e^6}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3cde}+\frac{1}{3}c^2d^2e^5x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^4d^4e^5}+\frac{2a^3e^3(d+ex)}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{6a^2cd\left(cd^2-ae^2\right)e^8-2ac^2d^2\left(5cd^2-3ae^2\right)xe^7+c^3d^3\left(cd^2-11ae^2\right)x^2e^6}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6cde}+\frac{1}{3}c^2d^2e^5x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^4d^4e^5}+\frac{2a^3e^3(d+ex)}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 2192

$$\frac{\frac{\frac{\int -\frac{c^2d^2e^6\left(2ae\left(c^2d^4-23ace^2d^2+12a^2e^4\right)+cd\left(cd^2-3ae^2\right)\left(3cd^2+19ae^2\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}c^2d^2e^5x\left(cd^2-11ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6cde}+\frac{1}{3}c^2d^2e^5x^2\sqrt{}}{c^4d^4e^5}$$

$$\frac{2a^3e^3(d+ex)}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{1}{2}c^2d^2e^5x\left(cd^2-11ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}-\frac{1}{4}cde^5\int \frac{2ae\left(c^2d^4-23ace^2d^2+12a^2e^4\right)+cd\left(cd^2-3ae^2\right)\left(3cd^2+19ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6cde}+\frac{1}{3}c^2d^2e^5x^2\sqrt{x(}}{c^4d^4e^5}$$

$$\frac{2a^3e^3(d+ex)}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1160

$$\frac{\frac{\frac{1}{2}c^2d^2e^5x\left(cd^2-11ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}-\frac{1}{4}cde^5\left(\frac{\left(cd^2-3ae^2\right)\left(19ae^2+3cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{3\left(-35a^3e^6+15a^2cd^2e^4+3ac^2d^4e^2\right.}{}\right.}{6cde}}{c^4d^4e^5}$$

$$\frac{2a^3e^3(d+ex)}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1092

$$\frac{\frac{\frac{1}{2}c^2d^2e^5x(cd^2-11ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}-\frac{1}{4}cde^5\left(\frac{(cd^2-3ae^2)(19ae^2+3cd^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}-\frac{3(-35a^3e^6+15a^2cd^2e^4+3ac^2d^4e^2\cdots}{\cdots}\right.}{6cde}}{c^4d^4e^5}$$

$$\frac{2a^3e^3(d+ex)}{c^4d^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2a^3e^3(d+ex)}{c^4d^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+$$

$$\frac{\frac{\frac{1}{2}c^2d^2e^5x(cd^2-11ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}-\frac{1}{4}cde^5\left(\frac{(cd^2-3ae^2)(19ae^2+3cd^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}-\frac{3(-35a^3e^6+15a^2cd^2e^4+3ac^2d^4e^2\cdots}{\cdots}\right.}{6cde}}{c^4d^4e^5}$$

input
```
Int[(x^3*(d + e*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(2*a^3*e^3*(d + e*x))/(c^4*d^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
) + ((c^2*d^2*e^5*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/3 + ((c
^2*d^2*e^5*(c*d^2 - 11*a*e^2)*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
])/2 - (c*d*e^5*(((c*d^2 - 3*a*e^2)*(3*c*d^2 + 19*a*e^2)*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2])/e - (3*(c^3*d^6 + 3*a*c^2*d^4*e^2 + 15*a^2*c*d
^2*e^4 - 35*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d
]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d
]*e^(3/2))))/4)/(6*c*d*e))/(c^4*d^4*e^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2621 vs. 2(293) = 586.

Time = 2.80 (sec) , antiderivative size = 2622, normalized size of antiderivative = 8.12

method	result	size
default	Expression too large to display	2622

input
```
int(x^3*(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
d^2*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d
/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d
/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c
*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(
1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2*a/c*(-1/d/e
/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+
a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^
2*e)^(1/2)))+e^2*(1/3*x^4/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-7/
6*(a*e^2+c*d^2)/d/e/c*(1/2*x^3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/
2)-5/4*(a*e^2+c*d^2)/d/e/c*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^
2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c
*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*
c)^(1/2))-2*a/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c
*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))-3/2*a/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)
*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d...
```

Fricas [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 756, normalized size of antiderivative = 2.34

$$\int \frac{x^3(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \left[-\frac{3\,(ac^3d^6e+3\,a^2c^2d^4e^3+15\,a^3cd^2e^5-35\,a^4e^7+(c^4d^7+3\,ac^3\right.$$

$$-\frac{3\,(ac^3d^6e+3\,a^2c^2d^4e^3+15\,a^3cd^2e^5-35\,a^4e^7+(c^4d^7+3\,ac^3d^5e^2+15\,a^2c^2d^3e^4-35\,a^3cde^6)x)\sqrt{-cde}\,\mathrm{ar}}{}$$

input
```
integrate(x^3*(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[-1/96*(3*(a*c^3*d^6*e + 3*a^2*c^2*d^4*e^3 + 15*a^3*c*d^2*e^5 - 35*a^4*e^7
 + (c^4*d^7 + 3*a*c^3*d^5*e^2 + 15*a^2*c^2*d^3*e^4 - 35*a^3*c*d*e^6)*x)*sq
rt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sq
rt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt
(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(8*c^4*d^4*e^3*x^3 - 3*a*c^3*d^
5*e^2 - 10*a^2*c^2*d^3*e^4 + 105*a^3*c*d*e^6 + 2*(c^4*d^5*e^2 - 7*a*c^3*d^
3*e^4)*x^2 - (3*c^4*d^6*e + 8*a*c^3*d^4*e^3 - 35*a^2*c^2*d^2*e^5)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^6*d^6*e^3*x + a*c^5*d^5*e^4), -
1/48*(3*(a*c^3*d^6*e + 3*a^2*c^2*d^4*e^3 + 15*a^3*c*d^2*e^5 - 35*a^4*e^7 +
 (c^4*d^7 + 3*a*c^3*d^5*e^2 + 15*a^2*c^2*d^3*e^4 - 35*a^3*c*d*e^6)*x)*sqrt
(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x
 + c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e
 + a*c*d*e^3)*x)) - 2*(8*c^4*d^4*e^3*x^3 - 3*a*c^3*d^5*e^2 - 10*a^2*c^2*d^
3*e^4 + 105*a^3*c*d*e^6 + 2*(c^4*d^5*e^2 - 7*a*c^3*d^3*e^4)*x^2 - (3*c^4*d
^6*e + 8*a*c^3*d^4*e^3 - 35*a^2*c^2*d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x))/(c^6*d^6*e^3*x + a*c^5*d^5*e^4)]
```

Sympy [F]

$$\int \frac{x^3(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{x^3(d+ex)^2}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**3*(d + e*x)**2/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[5,5,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,0]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{x^3\,{(d+e\,x)}^2}{{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)}^{3/2}}\,dx$$

input
```
int((x^3*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((x^3*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 21.45 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.73

$$\int \frac{x^3(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{-840\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a^3e^6+36$$

input
```
int(x^3*(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 840*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e +
c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**6 +
 360*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d
*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e*
*4 + 72*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e +
c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d**4
*e**2 + 24*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e
 + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**3*d**
6 + 525*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**3*e**6 - 135*sqrt(e)*
sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d**2*e**4 - 9*sqrt(e)*sqrt(d)*sqr
t(c)*sqrt(a*e + c*d*x)*a*c**2*d**4*e**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a
*e + c*d*x)*c**3*d**6 + 840*sqrt(d + e*x)*a**3*c*d*e**6 - 80*sqrt(d + e*x)
*a**2*c**2*d**3*e**4 + 280*sqrt(d + e*x)*a**2*c**2*d**2*e**5*x - 24*sqrt(d
 + e*x)*a*c**3*d**5*e**2 - 64*sqrt(d + e*x)*a*c**3*d**4*e**3*x - 112*sqrt(
d + e*x)*a*c**3*d**3*e**4*x**2 - 24*sqrt(d + e*x)*c**4*d**6*e*x + 16*sqrt(
d + e*x)*c**4*d**5*e**2*x**2 + 64*sqrt(d + e*x)*c**4*d**4*e**3*x**3)/(192*
sqrt(a*e + c*d*x)*c**5*d**5*e**3)
```

3.96 $\int \frac{x^2(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 239

$$\int \frac{x^2(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = -\frac{2a^2e^2(d+ex)}{c^3d^3\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} + \frac{(cd^2-7ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{4c^3d^3e} + \frac{x\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{2c^2d^2} - \frac{(c^2d^4+6acd^2e^2-15a^2e^4)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^{7/2}d^{7/2}e^{3/2}}$$

output
```
-2*a^2*e^2*(e*x+d)/c^3/d^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/4*(-7
*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e+1/2*x*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2-1/4*(-15*a^2*e^4+6*a*c*d^2*e^2
+c^2*d^4)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.75

$$\int \frac{x^2(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{e}(d+ex)\left(-15a^2e^3+acde(d-5ex)+c^2d^2x(d+2ex)\right)}{4c^{7/2}d^{7/2}e^{3/2}\sqrt{}}$$

input

```
Integrate[(x^2*(d + e*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),
x]
```

output

```
(Sqrt[c]*Sqrt[d]*Sqrt[e]*(d + e*x)*(-15*a^2*e^3 + a*c*d*e*(d - 5*e*x) + c^
2*d^2*x*(d + 2*e*x)) - (c^2*d^4 + 6*a*c*d^2*e^2 - 15*a^2*e^4)*Sqrt[a*e + c
*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[
a*e + c*d*x])])/(4*c^(7/2)*d^(7/2)*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1211, 25, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)^2}{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}dx$$

$$\Big\downarrow \ 1211$$

$$\frac{\int -\frac{-c^2d^2x^2e^4+a(cd^2-ae^2)e^4-cd(cd^2-ae^2)xe^3}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{c^3d^3e^3}-\frac{2a^2e^2(d+ex)}{c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{-c^2d^2x^2e^4+a(cd^2-ae^2)e^4-cd(cd^2-ae^2)xe^3}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{c^3d^3e^3}-\frac{2a^2e^2(d+ex)}{c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \ 2192$$

$$-\frac{\frac{\int \frac{cde^4\left(2ae\left(3cd^2-2ae^2\right)-cd\left(cd^2-7ae^2\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}-\frac{1}{2}cde^3x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3e^3}-\frac{2a^2e^2(d+ex)}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{1}{4}e^3\int \frac{2ae(3cd^2-2ae^2)-cd(cd^2-7ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{1}{2}cde^3x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3e^3}-\frac{2a^2e^2(d+ex)}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1160$$

$$-\frac{\frac{1}{4}e^3\left(\frac{\left(-15a^2e^4+6acd^2e^2+c^2d^4\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2e}-\frac{\left(cd^2-7ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}\right)-\frac{1}{2}cde^3x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3e^3}-\frac{2a^2e^2(d+ex)}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1092$$

$$-\frac{\frac{1}{4}e^3\left(\frac{\left(-15a^2e^4+6acd^2e^2+c^2d^4\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{e}-\frac{\left(cd^2-7ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}\right)-\frac{1}{2}cde^3x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3e^3}-\frac{2a^2e^2(d+ex)}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{1}{4}e^3\left(\frac{\left(-15a^2e^4+6acd^2e^2+c^2d^4\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}-\frac{\left(cd^2-7ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e}\right)-\frac{1}{2}cde^3x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3e^3}-\frac{2a^2e^2(d+ex)}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(x^2*(d + e*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*a^2*e^2*(d + e*x))/(c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]) - (-1/2*(c*d*e^3*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (e^3*
(-(((c*d^2 - 7*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/e) + ((
c^2*d^4 + 6*a*c*d^2*e^2 - 15*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/
(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/
(2*Sqrt[c]*Sqrt[d]*e^(3/2))))/4)/(c^3*d^3*e^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1547 vs. $2(213) = 426$.

Time = 2.69 (sec) , antiderivative size = 1548, normalized size of antiderivative = 6.48

method	result	size
default	Expression too large to display	1548

input
```
int(x^2*(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
d^2*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/
e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*
(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1
/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+e^2*(1/2*x^3/d
/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-5/4*(a*e^2+c*d^2)/d/e/c*(x^2/
d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/
d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/
d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e
*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d
*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2*a/c*(-1/d/e/c/(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d
^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
)))-3/2*a/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c
*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2
)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d
*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+2*d*e
*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d...
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.43

$$\int \frac{x^2(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \left[-\frac{(ac^2d^4e+6\,a^2cd^2e^3-15\,a^3e^5+(c^3d^5+6\,ac^2d^3e^2-15\,a^2cde^4\right.$$

input
```
integrate(x^2*(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[-1/16*((a*c^2*d^4*e + 6*a^2*c*d^2*e^3 - 15*a^3*e^5 + (c^3*d^5 + 6*a*c^2*d
^3*e^2 - 15*a^2*c*d*e^4)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 +
6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2
*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(
2*c^3*d^3*e^2*x^2 + a*c^2*d^3*e^2 - 15*a^2*c*d*e^4 + (c^3*d^4*e - 5*a*c^2*
d^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^2*x +
a*c^4*d^4*e^3), 1/8*((a*c^2*d^4*e + 6*a^2*c*d^2*e^3 - 15*a^3*e^5 + (c^3*d^
5 + 6*a*c^2*d^3*e^2 - 15*a^2*c*d*e^4)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e
)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) + 2*(2*c^3*
d^3*e^2*x^2 + a*c^2*d^3*e^2 - 15*a^2*c*d*e^4 + (c^3*d^4*e - 5*a*c^2*d^2*e^
3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e^2*x + a*c^4*
d^4*e^3)]
```

Sympy [F]

$$\int \frac{x^2(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{x^2(d+ex)^2}{((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**2*(d + e*x)**2/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[4,4,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,0]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{x^2\,(d+e\,x)^2}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}\,dx$$

input
```
int((x^2*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((x^2*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.47

$$\int \frac{x^2(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{15\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a^2e^4-6\sqrt{e}$$

input
```
int(x^2*(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(15*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**4 - 6*s
qrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) +
sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d**2*e**2 - sqrt
(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqr
t(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**4 - 10*sqrt(e)*
sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*e**4 + 2*sqrt(e)*sqrt(d)*sqrt(c)*sq
rt(a*e + c*d*x)*a*c*d**2*e**2 - 15*sqrt(d + e*x)*a**2*c*d*e**4 + sqrt(d +
e*x)*a*c**2*d**3*e**2 - 5*sqrt(d + e*x)*a*c**2*d**2*e**3*x + sqrt(d + e*x)
*c**3*d**4*e*x + 2*sqrt(d + e*x)*c**3*d**3*e**2*x**2)/(4*sqrt(a*e + c*d*x)
*c**4*d**4*e**2)
```

3.97 $\int \frac{x(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 160

$$\int \frac{x(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2ae(d+ex)}{c^2d^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2}+\frac{\left(cd^2-3ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{5/2}d^{5/2}\sqrt{e}}$$

output
```
2*a*e*(e*x+d)/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2+(-3*a*e^2+c*d^2)*arctanh(c^(1/2)*d^(1/
2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2
)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.86

$$\int \frac{x(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{\sqrt{c}\sqrt{d}\sqrt{e}(3ae+cdx)(d+ex)+\left(cd^2-3ae^2\right)\sqrt{ae+cdx}\sqrt{d+ex}}{c^{5/2}d^{5/2}\sqrt{e}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(x*(d + e*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(Sqrt[c]*Sqrt[d]*Sqrt[e]*(3*a*e + c*d*x)*(d + e*x) + (c*d^2 - 3*a*e^2)*Sqr
t[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt
[e]*Sqrt[a*e + c*d*x])])/(c^(5/2)*d^(5/2)*Sqrt[e]*Sqrt[(a*e + c*d*x)*(d +
e*x)])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1211, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)^2}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int \frac{e\left(cd^2+cexd-ae^2\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^2d^2e}+\frac{2ae(d+ex)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{cd^2+cexd-ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^2d^2}+\frac{2ae(d+ex)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{1}{2}\left(cd^2-3ae^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^2d^2}+\frac{2ae(d+ex)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\left(cd^2-3ae^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}+\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^2d^2}+\frac{2ae(d+ex)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(cd^2-3ae^2)\operatorname{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}+\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{c^2d^2}+\frac{2ae(d+ex)}{c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

input
```
Int[(x*(d + e*x)^2)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(2*a*e*(d + e*x))/(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) +
(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2] + ((c*d^2 - 3*a*e^2)*ArcTanh[
(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]))/(c^2*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 878 vs. $2(142) = 284$.

Time = 2.28 (sec) , antiderivative size = 879, normalized size of antiderivative = 5.49

method	result
default	$d^2\left(-\frac{1}{dec\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}-\frac{(a\,e^2+c\,d^2)(2cdxe+a\,e^2+c\,d^2)}{dec\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}\right)+e^2\left(\frac{}{dec\sqrt{ade+(a\,e^2}}\right.$

input
```
int(x*(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
d^2*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*
(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2))+e^2*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^
2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c
*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*
c)^(1/2))-2*a/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c
*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))+2*d*e*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e
^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1
/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2))/(d*e*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.74

$$\int \frac{x(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \left[-\frac{\left(acd^2e-3\,a^2e^3+\left(c^2d^3-3\,acde^2\right)x\right)\sqrt{cde}\log\left(8\,c^2d^2e^2x^2+c\right.}{}\right.$$

$$-\frac{\left(acd^2e-3\,a^2e^3+\left(c^2d^3-3\,acde^2\right)x\right)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,cdex+cd^2+ae^2\right)\sqrt{-cde}}{2\left(c^2d^2e^2x^2+acd^2e^2+\left(c^2d^3e+acde^3\right)x\right)}\right)-2\left(c^2d^2e\right.}{2\left(c^4d^4ex+ac^3d^3e^2\right)}$$

input
```
integrate(x*(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[-1/4*((a*c*d^2*e - 3*a^2*e^3 + (c^2*d^3 - 3*a*c*d*e^2)*x)*sqrt(c*d*e)*log
(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(
c^2*d^3*e + a*c*d*e^3)*x) - 4*(c^2*d^2*e*x + 3*a*c*d*e^2)*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x))/(c^4*d^4*e*x + a*c^3*d^3*e^2), -1/2*((a*c*d^2
*e - 3*a^2*e^3 + (c^2*d^3 - 3*a*c*d*e^2)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*
d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(c^2
*d^2*e*x + 3*a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^4*
d^4*e*x + a*c^3*d^3*e^2)]
```

Sympy [F]

$$\int \frac{x(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{x(d+ex)^2}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x*(d + e*x)**2/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{2,[3,3,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,2]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{x\,{(d+e\,x)}^2}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

input
```
int((x*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((x*(d + e*x)^2)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.37

$$\int \frac{x(d+ex)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{-12\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a\,e^2+4\sqrt{e}$$

input
```
int(x*(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 12*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*e**2 + 4*s
qrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) +
sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**2 + 9*sqrt(e)*s
qrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*e**2 - sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e
 + c*d*x)*c*d**2 + 12*sqrt(d + e*x)*a*c*d*e**2 + 4*sqrt(d + e*x)*c**2*d**2
*e*x)/(4*sqrt(a*e + c*d*x)*c**3*d**3*e)
```

3.98 $\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 111

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=-\frac{2(d+ex)}{cd\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{c^{3/2}d^{3/2}}$$

output
```
(-2*e*x-2*d)/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2*e^(1/2)*arctanh
(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/
c^(3/2)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^2}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{-2\sqrt{c}\sqrt{d}(d+ex)+2\sqrt{e}\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{e}\sqrt{ae+cdx}}\right)}{c^{3/2}d^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[c]*Sqrt[d]*(d + e*x) + 2*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*
ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(c^(
3/2)*d^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1124, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1124}$$

$$\frac{e\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd}-\frac{2(d+ex)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2e\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{cd}-\frac{2(d+ex)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{e}\,\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{c^{3/2}d^{3/2}}-\frac{2(d+ex)}{cd\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^2/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*(d + e*x))/(c*d*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (Sqrt[e
]*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*
e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(c^(3/2)*d^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(97) = 194$.

Time = 3.98 (sec) , antiderivative size = 485, normalized size of antiderivative = 4.37

method	result
default	$\frac{2d^2(2cdxe+a\,e^2+c\,d^2)}{\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}+e^2\left(-\frac{x}{dec\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}-\frac{(a\,e^2+c\,d^2)\left(-\frac{}{dec\sqrt{ade+(}}\right.}{}\right.$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
2*d^2*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2)+e^2*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+
c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1
/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(
d*e*c)^(1/2))+2*d*e*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e
^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a
*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.14

$$\int \frac{(d+ex)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \left[\frac{(cdx+ae)\sqrt{\frac{e}{cd}}\log\left(8\,c^2d^2e^2x^2+c^2d^4+6\,acd^2e^2+a^2e^4+8\,(c^2\right.}{} \right.$$

$$\left. -\frac{(cdx+ae)\sqrt{-\frac{e}{cd}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-\frac{e}{cd}}}{2\,(cde^2x^2+ade^2+(cd^2e+ae^3)x)}\right)+2\,\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{c^2d^2x+acde}\right]$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
fricas")
```

output
```
[1/2*((c*d*x + a*e)*sqrt(e/(c*d))*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*
d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x + 4*(2*c^2*d^2*e*x + c^2*d
^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e/(c*d)))
 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^2*d^2*x + a*c*d*e), -
((c*d*x + a*e)*sqrt(-e/(c*d))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e
^2 + (c*d^2*e + a*e^3)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
)/(c^2*d^2*x + a*c*d*e)]
```

Sympy [F]

$$\int \frac{(d+ex)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{(d+ex)^2}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**2/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[1,1,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,2]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(d+e\,x)^2}{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((d + e*x)^2/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{2\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)-2\sqrt{e}\,\sqrt{d}\,\sqrt{c}}{\sqrt{cdx+ae}\,c^2d^2}$$

input
```
int((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*(sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2)) - sqrt(e)*sqrt(
d)*sqrt(c)*sqrt(a*e + c*d*x) - sqrt(d + e*x)*c*d))/(sqrt(a*e + c*d*x)*c**2
*d**2)
```

3.99 $\int \frac{(d+ex)^2}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 111

$$\int \frac{(d+ex)^2}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2(d+ex)}{ae\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{2\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{3/2}e^{3/2}}$$

output
```
2*(e*x+d)/a/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-2*d^(1/2)*arctanh(a^
(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(
3/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^2}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2\sqrt{a}\sqrt{e}(d+ex)-2\sqrt{d}\sqrt{ae+cdx}\sqrt{d+ex}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}\sqrt{d+ex}}{\sqrt{d}\sqrt{ae+cdx}}\right)}{a^{3/2}e^{3/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^2/(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*Sqrt[a]*Sqrt[e]*(d + e*x) - 2*Sqrt[d]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*A
rcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(a^(3
/2)*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1212, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{x\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1212}$$

$$\frac{2(d+ex)}{ae\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-e^2\int -\frac{d}{ae^3x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx$$

$$\Big\downarrow \text{25}$$

$$e^2\int \frac{d}{ae^3x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2(d+ex)}{ae\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ae}+\frac{2(d+ex)}{ae\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{2(d+ex)}{ae\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2d\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ae}$$

$$\Big\downarrow \text{219}$$

$$\frac{2(d+ex)}{ae\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\sqrt{d}\,\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{a^{3/2}e^{3/2}}$$

input
```
Int[(d + e*x)^2/(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*(d + e*x))/(a*e*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (Sqrt[d]
*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(a^(3/2)*e^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1212
```
Int[((x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)
^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*d - b*e)^n*((d + e
*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*x^2])), x] - Simp[e^2/c^(m +
n - 1)   Int[ExpandToSum[(c^(m + n - 1)*(d + e*x)^(m - 1) - ((c*d - b*e)^n*
(2*c*d - b*e)^(m - 1))/(e^n*x^n))/(c*d - b*e - c*e*x), x]/(Sqrt[a + b*x + c
*x^2]/x^n), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^
2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(95) = 190$.

Time = 1.62 (sec) , antiderivative size = 427, normalized size of antiderivative = 3.85

method	result
default	$d^2\left(\frac{1}{ade\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}-\frac{(a\,e^2+c\,d^2)(2cdxe+a\,e^2+c\,d^2)}{ade\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}-\frac{\ln\left(\frac{2ade+\left(a\,e^2+c\,d^2\right)x+2}{\right.}}{}$

input
```
int((e*x+d)^2/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
d^2*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(
2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(
a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+e^2*(-1/d/e/c/(a
*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2
+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^
(1/2))+4*d*e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.23

$$\int \frac{(d+ex)^2}{x\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \left[\frac{(cdx+ae)\sqrt{\frac{d}{ae}}\log\left(\frac{8\,a^2d^2e^2+(c^2d^4+6\,acd^2e^2+a^2e^4)x^2+8\,(acd^3e+a^2de^3)}{}\right.}{}\right.$$

input
```
integrate((e*x+d)^2/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[1/2*((c*d*x + a*e)*sqrt(d/(a*e))*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^
2*e^2 + a^2*e^4)*x^2 + 8*(a*c*d^3*e + a^2*d*e^3)*x - 4*(2*a^2*d*e^2 + (a*c
*d^2*e + a^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(d/(a
*e)))/x^2) + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c*d*e*x + a
^2*e^2), ((c*d*x + a*e)*sqrt(-d/(a*e))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e +
 (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-d/(a*e))/(c*d^2*e*
x^2 + a*d^2*e + (c*d^3 + a*d*e^2)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x))/(a*c*d*e*x + a^2*e^2)]
```

Sympy [F]

$$\int \frac{(d+ex)^2}{x\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^2}{x\,((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/x/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**2/(x*((d + e*x)*(a*e + c*d*x))**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{x\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[1,1,5]%%%},[2,3]%%%}+%%%{%%%{-2,[2,3,3]%%%},[2,2]%%
%}+%%%{%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(d+e\,x)^2}{x\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int((d + e*x)^2/(x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int((d + e*x)^2/(x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex)^2}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,\sqrt{a}\,\sqrt{cdx+ae}\,\log\left(\sqrt{e}\,\sqrt{cdx+ae}-\sqrt{2\sqrt{c}\,\sqrt{a}\,de+a\,e^{2}}\right.}{}$$

input
```
int((e*x+d)^2/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) -
 sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d +
e*x))*c*d + sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e
 + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c
)*sqrt(d + e*x))*c*d - sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(2*sqr
t(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d
*e + 2*c*d*e*x)*c*d + 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*e + 2*
sqrt(d + e*x)*a*c*d*e)/(sqrt(a*e + c*d*x)*a**2*c*d*e**2)
```

3.100 $\int \frac{(d+ex)^2}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 165

$$\int \frac{(d+ex)^2}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2cd(d+ex)}{a^2e^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{a^2e^2x} + \frac{\left(3cd^2-ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{5/2}\sqrt{d}e^{5/2}}$$

output
```
-2*c*d*(e*x+d)/a^2/e^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/e^2/x+(-a*e^2+3*c*d^2)*arctanh(a^(1/2)*e^
(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(
1/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^2}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{-\sqrt{a}\sqrt{d}\sqrt{e}(ae+3cdx)(d+ex)+\left(3cd^2-ae^2\right)x\sqrt{ae+cdx}\sqrt{}}{a^{5/2}\sqrt{d}e^{5/2}x\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^2/(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(-(Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + 3*c*d*x)*(d + e*x)) + (3*c*d^2 - a*e^2)*
x*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/
(Sqrt[d]*Sqrt[a*e + c*d*x])])/(a^(5/2)*Sqrt[d]*e^(5/2)*x*Sqrt[(a*e + c*d*x
)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1212, 25, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1212}$$

$$-cde^3\int -\frac{ade-\left(cd^2-ae^2\right)x}{a^2cde^5x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{2cd(d+ex)}{a^2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{25}$$

$$cde^3\int \frac{ade-\left(cd^2-ae^2\right)x}{a^2cde^5x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{2cd(d+ex)}{a^2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{ade-\left(cd^2-ae^2\right)x}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{a^2e^2}-\frac{2cd(d+ex)}{a^2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1228}$$

$$\frac{-\frac{1}{2}\left(3cd^2-ae^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{x}}{a^2e^2}-\frac{2cd(d+ex)}{a^2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{\left(3cd^2-ae^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{a^2e^2}-\frac{2cd(d+ex)}{a^2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\left(3cd^2-ae^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}\sqrt{d}\sqrt{e}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}}{a^2e^2}-\frac{2cd(d+ex)}{a^2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^2/(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*c*d*(d + e*x))/(a^2*e^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) +
 (-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/x) + ((3*c*d^2 - a*e^2)*Ar
cTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]))/(a^2*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1212
```
Int[((x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)
^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*d - b*e)^n*((d + e
*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*x^2])), x] - Simp[e^2/c^(m +
n - 1)   Int[ExpandToSum[(c^(m + n - 1)*(d + e*x)^(m - 1) - ((c*d - b*e)^n*
(2*c*d - b*e)^(m - 1))/(e^n*x^n))/(c*d - b*e - c*e*x), x]/(Sqrt[a + b*x + c
*x^2]/x^n), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^
2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(147) = 294$.

Time = 1.53 (sec) , antiderivative size = 645, normalized size of antiderivative = 3.91

method	result
default	$\frac{2e^2(2cdxe+a e^2+c d^2)}{\left(4ac d^2e^2-(a e^2+c d^2)^2\right)\sqrt{ade+(a e^2+c d^2)x+cd x^2e}}+d^2\left(-\frac{1}{adex\sqrt{ade+(a e^2+c d^2)x+cd x^2e}}-\frac{3(a e^2+c d^2)\left(\frac{\cdots}{ade\sqrt{ade+\cdots}}\cdots\right.}{\cdots}\cdots\right.$

input
```
int((e*x+d)^2/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
2*e^2*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2)+d^2*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*l
n((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2))/x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+2*d*e*(1/a/d/e/(a*d*e+(a*e^2+c*
d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c
*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/
(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*
d^2)*x+c*d*x^2*e)^(1/2))/x))
```

Fricas [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.84

$$\int \frac{(d+ex)^2}{x^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \left[-\frac{\sqrt{ade}\left((3\,c^2d^3-acde^2)x^2+(3\,acd^2e-a^2e^3)x\right)\log\left(\frac{8\,a^2d^2e^2 \cdots}{\cdots}\right)\cdots}{\cdots}\right.$$

$$-\frac{\sqrt{-ade}\left((3\,c^2d^3-acde^2)x^2+(3\,acd^2e-a^2e^3)x\right)\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,ade+(cd^2+ae^2)x)\sqrt{-ade}}{2\,(acd^2e^2x^2+a^2d^2e^2+(acd^3e+a^2de^3)x)}\right)+2\cdots}{2\,(a^3cd^2e^3x^2+a^4de^4x)}$$

input
```
integrate((e*x+d)^2/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[-1/4*(sqrt(a*d*e)*((3*c^2*d^3 - a*c*d*e^2)*x^2 + (3*a*c*d^2*e - a^2*e^3)*
x)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a
*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(3*a*c*d^2*e*x + a^2*d*e^2)*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^3*c*d^2*e^3*x^2 + a^4*d*e^
4*x), -1/2*(sqrt(-a*d*e)*((3*c^2*d^3 - a*c*d*e^2)*x^2 + (3*a*c*d^2*e - a^2
*e^3)*x)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
 (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3
*e + a^2*d*e^3)*x)) + 2*(3*a*c*d^2*e*x + a^2*d*e^2)*sqrt(c*d*e*x^2 + a*d*e
 + (c*d^2 + a*e^2)*x))/(a^3*c*d^2*e^3*x^2 + a^4*d*e^4*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^2}{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^2}{x^2((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/x**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**2/(x**2*((d + e*x)*(a*e + c*d*x))**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{x^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[1,1,0]%%%},[6,0]%%%}+%%%{%%{[%%%{-2,[0,0,1]%%%},0]:
[1,0,%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(d+e\,x)^2}{x^2\,\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int((d + e*x)^2/(x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int((d + e*x)^2/(x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.41

$$\int \frac{(d+ex)^2}{x^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,\sqrt{a}\,\sqrt{cdx+ae}\,\log\left(\sqrt{e}\,\sqrt{cdx+ae}-\sqrt{2\sqrt{c}\,\sqrt{a}\,de+a\,e\right.}{}$$

input
```
int((e*x+d)^2/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) -
 sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d +
e*x))*a**2*e**4*x - 9*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e
)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt
(d)*sqrt(c)*sqrt(d + e*x))*c**2*d**4*x + sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e
+ c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e*
*2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*e**4*x - 9*sqrt(e)*sqrt
(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(
c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**2*d*
*4*x - sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(2*sqrt(e)*sqrt(d)*sqr
t(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*
a**2*e**4*x + 9*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(2*sqrt(e)*sq
rt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*
c*d*e*x)*c**2*d**4*x + 10*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*d
*e**3*x + 6*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d**3*e*x - 2*sqr
t(d + e*x)*a**3*d*e**4 - 6*sqrt(d + e*x)*a**2*c*d**3*e**2 - 6*sqrt(d + e*x
)*a**2*c*d**2*e**3*x - 18*sqrt(d + e*x)*a*c**2*d**4*e*x)/(2*sqrt(a*e + c*d
*x)*a**3*d*e**3*x*(a*e**2 + 3*c*d**2))
```

3.101 $\int \frac{(d+ex)^2}{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 246

$$\int \frac{(d+ex)^2}{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2c^2d^2(d+ex)}{a^3e^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2a^2e^2x^2} + \frac{\left(7cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4a^3de^3x} - \frac{\left(15c^2d^4-6acd^2e^2-a^2e^4\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4a^{7/2}d^{3/2}e^{7/2}}$$

output
```
2*c^2*d^2*(e*x+d)/a^3/e^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/e^2/x^2+1/4*(-a*e^2+7*c*d^2)*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d/e^3/x-1/4*(-a^2*e^4-6*a*c*d^2*e^2
+15*c^2*d^4)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/a^(7/2)/d^(3/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^2}{x^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{-\sqrt{a}\sqrt{d}\sqrt{e}(d+ex)\left(-15c^2d^3x^2+acdex(-5d+ex)+a^2e^2(2d\right.}{4a^{7/2}d^{3/2}}$$

input
```
Integrate[(d + e*x)^2/(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(-(Sqrt[a]*Sqrt[d]*Sqrt[e]*(d + e*x)*(-15*c^2*d^3*x^2 + a*c*d*e*x*(-5*d +
e*x) + a^2*e^2*(2*d + e*x))) - (15*c^2*d^4 - 6*a*c*d^2*e^2 - a^2*e^4)*x^2*
Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(S
qrt[d]*Sqrt[a*e + c*d*x])])/(4*a^(7/2)*d^(3/2)*e^(7/2)*x^2*Sqrt[(a*e + c*d
*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1212, 25, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{x^3\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \ 1212$$

$$\frac{2c^2d^2(d+ex)}{a^3e^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - c^2d^2e^4\int -\frac{\frac{(cd^2-ae^2)x^2}{a^3cde^7}-\frac{(cd^2-ae^2)x}{a^2c^2d^2e^6}+\frac{1}{ac^2de^5}}{x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx$$

$$\Big\downarrow \ 25$$

$$c^2d^2e^4\int \frac{\frac{(cd^2-ae^2)x^2}{a^3cde^7}-\frac{(cd^2-ae^2)x}{a^2c^2d^2e^6}+\frac{1}{ac^2de^5}}{x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx + \frac{2c^2d^2(d+ex)}{a^3e^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \ 2181$$

$$c^2d^2e^4\left(-\frac{\int \frac{ae(7cd^2-ae^2)-2cd(2cd^2-3ae^2)x}{2a^2c^2de^6x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2ade}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2a^2c^2d^2e^6x^2}\right)+\frac{2c^2d^2(d+ex)}{a^3e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 27$$

$$c^2d^2e^4\left(-\frac{\int \frac{ae(7cd^2-ae^2)-2cd(2cd^2-3ae^2)x}{x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4a^3c^2d^2e^7}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2a^2c^2d^2e^6x^2}\right)+\frac{2c^2d^2(d+ex)}{a^3e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1228$$

$$c^2d^2e^4\left(-\frac{-\frac{\left(-a^2e^4-6acd^2e^2+15c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d}-\frac{(7cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{dx}}{4a^3c^2d^2e^7}-\frac{\sqrt{x\left(ae^2+cd^2\right)+\cdots}}{2a^2c^2d^{2}\cdots}\right.$$

$$\frac{2c^2d^2(d+ex)}{a^3e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1154$$

$$c^2d^2e^4\left(-\frac{\frac{\left(-a^2e^4-6acd^2e^2+15c^2d^4\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{d}-\frac{(7cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{dx}}{4a^3c^2d^2e^7}\right.$$

$$\frac{2c^2d^2(d+ex)}{a^3e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 219$$

$$\frac{2c^2d^2(d+ex)}{a^3e^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+$$
$$c^2d^2e^4\left(-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2a^2c^2d^2e^6x^2}-\frac{\frac{(-a^2e^4-6acd^2e^2+15c^2d^4)\text{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{a}d^{3/2}\sqrt{e}}-\frac{(7cd^2-\cdots}{\cdots}}{4a^3c^2d^2e^7}\right.$$

input
```
Int[(d + e*x)^2/(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*c^2*d^2*(d + e*x))/(a^3*e^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
) + c^2*d^2*e^4*(-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(a^2*c^2
*d^2*e^6*x^2) - (-(((7*c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/(d*x)) + ((15*c^2*d^4 - 6*a*c*d^2*e^2 - a^2*e^4)*ArcTanh[(2*a*d*e
 + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])])/(2*Sqrt[a]*d^(3/2)*Sqrt[e]))/(4*a^3*c^2*d^2*e^7))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1212
```
Int[((x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)
^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*d - b*e)^n*((d + e
*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*x^2])), x] - Simp[e^2/c^(m +
n - 1)   Int[ExpandToSum[(c^(m + n - 1)*(d + e*x)^(m - 1) - ((c*d - b*e)^n*
(2*c*d - b*e)^(m - 1))/(e^n*x^n))/(c*d - b*e - c*e*x), x]/(Sqrt[a + b*x + c
*x^2]/x^n), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^
2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1199 vs. $2(220) = 440$.

Time = 2.25 (sec) , antiderivative size = 1200, normalized size of antiderivative = 4.88

method	result	size
default	Expression too large to display	1200

input
```
int((e*x+d)^2/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
d^2*(-1/2/a/d/e/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-5/4*(a*e^2+c*d
^2)/a/d/e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c
*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)
/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-4*c/a*(2
*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)
*x+c*d*x^2*e)^(1/2))-3/2*c/a*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^
2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*
a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2))/x)))+e^2*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^
2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a
*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*
d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*d*e*
(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/
e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*
c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*
d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-4*c/a*(2*c*d*e*...
```

Fricas [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 630, normalized size of antiderivative = 2.56

$$\int \frac{(d+ex)^2}{x^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \Bigg[-\frac{((15\,c^3d^5-6\,ac^2d^3e^2-a^2cde^4)x^3+(15\,ac^2d^4e-6\,a^2cd^2e^3}{}$$

input
```
integrate((e*x+d)^2/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[-1/16*(((15*c^3*d^5 - 6*a*c^2*d^3*e^2 - a^2*c*d*e^4)*x^3 + (15*a*c^2*d^4*
e - 6*a^2*c*d^2*e^3 - a^3*e^5)*x^2)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*
d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*
e^3)*x)/x^2) + 4*(2*a^3*d^2*e^3 - (15*a*c^2*d^4*e - a^2*c*d^2*e^3)*x^2 - (
5*a^2*c*d^3*e^2 - a^3*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
))/(a^4*c*d^3*e^4*x^3 + a^5*d^2*e^5*x^2), 1/8*(((15*c^3*d^5 - 6*a*c^2*d^3*
e^2 - a^2*c*d*e^4)*x^3 + (15*a*c^2*d^4*e - 6*a^2*c*d^2*e^3 - a^3*e^5)*x^2)
*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*
c*d^3*e + a^2*d*e^3)*x)) - 2*(2*a^3*d^2*e^3 - (15*a*c^2*d^4*e - a^2*c*d^2*
e^3)*x^2 - (5*a^2*c*d^3*e^2 - a^3*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x))/(a^4*c*d^3*e^4*x^3 + a^5*d^2*e^5*x^2)]
```

Sympy [F]

$$\int \frac{(d+ex)^2}{x^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{(d+ex)^2}{x^3\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/x**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**2/(x**3*((d + e*x)*(a*e + c*d*x))**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{x^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{x^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[1,1,11]%%%},[2,7]%%%}+%%%{%%%{-4,[2,3,9]%%%},[2,6]%
%%}+%%%{%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{(d+e\,x)^2}{x^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int((d + e*x)^2/(x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int((d + e*x)^2/(x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 1140, normalized size of antiderivative = 4.63

$$\int \frac{(d+ex)^2}{x^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^2/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 3*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d
*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt
(d + e*x))*a**3*e**6*x**2 - 23*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*l
og(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**
2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**2*e**4*x**2 + 15*sqrt(e)*sqr
t(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt
(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2
*d**4*e**2*x**2 + 75*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)
*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(
d)*sqrt(c)*sqrt(d + e*x))*c**3*d**6*x**2 - 3*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(
a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e +
a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*e**6*x**2 - 23*sqrt
(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt
(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))
*a**2*c*d**2*e**4*x**2 + 15*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(
sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**4*e**2*x**2 + 75*sqrt(e)*sqrt(d
)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)
*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**3*d**6
*x**2 + 3*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(2*sqrt(e)*sqrt(...
```

3.102 $\int \frac{(d+ex)^2}{x^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 331

$$\int \frac{(d+ex)^2}{x^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2c^3d^3(d+ex)}{a^4e^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3a^2e^2x^3}+\frac{\left(11cd^2-ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12a^3de^3x^2}-\frac{\left(3cd^2-ae^2\right)\left(19cd^2+3ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24a^4d^2e^4x}+\frac{\left(35c^3d^6-15ac^2d^4e^2-3a^2cd^2e^4-a^3e^6\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8a^{9/2}d^{5/2}e^{9/2}}$$

output
```
-2*c^3*d^3*(e*x+d)/a^4/e^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/3*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/e^2/x^3+1/12*(-a*e^2+11*c*d^2)*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d/e^3/x^2-1/24*(-a*e^2+3*c*d^2)*
(3*a*e^2+19*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^4/d^2/e^4/x+1
/8*(-a^3*e^6-3*a^2*c*d^2*e^4-15*a*c^2*d^4*e^2+35*c^3*d^6)*arctanh(a^(1/2)*
e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(9/2)/d
^(5/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^2}{x^4\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{-\sqrt{a}\sqrt{d}\sqrt{e}(d+ex)\left(105c^3d^5x^3+5ac^2d^3ex^2(7d-2ex)+a^3e^3\right.}{}$$

input
```
Integrate[(d + e*x)^2/(x^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(-(Sqrt[a]*Sqrt[d]*Sqrt[e]*(d + e*x)*(105*c^3*d^5*x^3 + 5*a*c^2*d^3*e*x^2*
(7*d - 2*e*x) + a^3*e^3*(8*d^2 + 2*d*e*x - 3*e^2*x^2) - a^2*c*d*e^2*x*(14*
d^2 + 8*d*e*x + 3*e^2*x^2))) + 3*(35*c^3*d^6 - 15*a*c^2*d^4*e^2 - 3*a^2*c*
d^2*e^4 - a^3*e^6)*x^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[a]*Sq
rt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(24*a^(9/2)*d^(5/2)*e^(
9/2)*x^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 2.29 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {1212, 25, 2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{x^4\left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1212}$$

$$-c^3d^3e^5\int -\frac{-\frac{(cd^2-ae^2)x^3}{a^4cde^9}+\frac{(cd^2-ae^2)x^2}{a^3c^2d^2e^8}-\frac{(cd^2-ae^2)x}{a^2c^3d^3e^7}+\frac{1}{ac^3d^2e^6}}{x^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{2c^3d^3(d+ex)}{a^4e^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 25}$$

$$c^3d^3e^5\int \frac{-\frac{(cd^2-ae^2)x^3}{a^4cde^9}+\frac{(cd^2-ae^2)x^2}{a^3c^2d^2e^8}-\frac{(cd^2-ae^2)x}{a^2c^3d^3e^7}+\frac{1}{ac^3d^2e^6}}{x^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{2c^3d^3(d+ex)}{a^4e^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 2181

$$c^3d^3e^5\left(-\frac{\int \frac{\frac{6\left(cd^2-ae^2\right)x^2}{a^3ce^8}-\frac{2\left(3cd^2-5ae^2\right)x}{a^2c^2de^7}+\frac{\frac{11c}{a}-\frac{e^2}{d^2}}{c^3e^6}}{2x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3a^2c^3d^3e^7x^3}\right)-\frac{2c^3d^3(d+ex)}{a^4e^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 27

$$c^3d^3e^5\left(-\frac{\int \frac{\frac{6\left(cd^2-ae^2\right)x^2}{a^3ce^8}-\frac{2\left(3cd^2-5ae^2\right)x}{a^2c^2de^7}+\frac{\frac{11c}{a}-\frac{e^2}{d^2}}{c^3e^6}}{x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3a^2c^3d^3e^7x^3}\right)-\frac{2c^3d^3(d+ex)}{a^4e^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 2181

$$c^3d^3e^5\left(-\frac{-\frac{\int \frac{a^2de\left(-\frac{3ae^4}{d^2}-10ce^2+\frac{57c^2d^2}{a}\right)-2c\left(12c^2d^4-23ace^2d^2+a^2e^4\right)x}{2a^2c^3de^7x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(11cd^2-ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2a^2c^3d^3e^7x^2}}{6ade}-\frac{\sqrt{x\,(ae^2+}}{3a}\right.$$

$$-\frac{2c^3d^3(d+ex)}{a^4e^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 27

$$c^3d^3e^5\left(-\frac{-\frac{\int \frac{ae\left(-\frac{3a^2e^4}{d}-10acde^2+57c^2d^3\right)-2c\left(12c^2d^4-23ace^2d^2+a^2e^4\right)x}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4a^3c^3d^2e^8}-\frac{\left(11cd^2-ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2a^2c^3d^3e^7x^2}}{6ade}-\frac{\sqrt{x\left(ae^2+\cdots\right.}}{3a\cdots}\right.$$

$$\frac{2c^3d^3(d+ex)}{a^4e^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\big\downarrow$ 1228

$$c^3d^3e^5\left(-\frac{-\frac{-\frac{3\left(-a^3e^6-3a^2cd^2e^4-15ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2d^2}-\frac{\left(3cd^2-ae^2\right)\left(3ae^2+19cd^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{d^2x}}{4a^3c^3d^2e^8}-\cdots}{6ade}\right.$$

$$\frac{2c^3d^3(d+ex)}{a^4e^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\big\downarrow$ 1154

$$c^3d^3e^5\left(-\frac{-\frac{\frac{3\left(-a^3e^6-3a^2cd^2e^4-15ac^2d^4e^2+35c^3d^6\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{d^2}-\frac{\left(3cd^2-ae^2\right)\left(3ae^2+19cd^2\right)\sqrt{x\cdots}}{d^2x}}{4a^3c^3d^2e^8}\cdots}{6ade}\right.$$

$$\frac{2c^3d^3(d+ex)}{a^4e^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\big\downarrow$ 219

$$c^3d^3e^5\left(-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3a^2c^3d^3e^7x^3}-\frac{-\frac{\left(11cd^2-ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2a^2c^3d^3e^7x^2}-\frac{\frac{3\left(-a^3e^6-3a^2cd^2e^4-15ac^2d^4e^2+35c^3d^6\right)\cdots}{2\cdots}}{\cdots}}{\cdots}\right.$$

$$\frac{2c^3d^3(d+ex)}{a^4e^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^2/(x^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*c^3*d^3*(d + e*x))/(a^4*e^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]) + c^3*d^3*e^5*(-1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(a^2*c^
3*d^3*e^7*x^3) - (-1/2*((11*c*d^2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])/(a^2*c^3*d^3*e^7*x^2) - (-(((3*c*d^2 - a*e^2)*(19*c*d^2 + 3*
a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d^2*x)) + (3*(35*c^3*
d^6 - 15*a*c^2*d^4*e^2 - 3*a^2*c*d^2*e^4 - a^3*e^6)*ArcTanh[(2*a*d*e + (c*
d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2])])/(2*Sqrt[a]*d^(5/2)*Sqrt[e]))/(4*a^3*c^3*d^2*e^8))/(6*a*d*e
))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1212
```
Int[((x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)
^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*d - b*e)^n*((d + e
*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*x^2])), x] - Simp[e^2/c^(m +
n - 1)   Int[ExpandToSum[(c^(m + n - 1)*(d + e*x)^(m - 1) - ((c*d - b*e)^n*
(2*c*d - b*e)^(m - 1))/(e^n*x^n))/(c*d - b*e - c*e*x), x]/(Sqrt[a + b*x + c
*x^2]/x^n), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^
2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2039 vs. $2(301) = 602$.

Time = 2.70 (sec) , antiderivative size = 2040, normalized size of antiderivative = 6.16

method	result	size
default	Expression too large to display	2040

input
```
int((e*x+d)^2/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
d^2*(-1/3/a/d/e/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-7/6*(a*e^2+c*d
^2)/a/d/e*(-1/2/a/d/e/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-5/4*(a*e
^2+c*d^2)/a/d/e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a
*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+
c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^
2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-4*
c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*x^2*e)^(1/2))-3/2*c/a*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^
2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*
ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2))/x)))-4/3*c/a*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^
2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d
*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
)/x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))+e^2*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)...
```

Fricas [A] (verification not implemented)

Time = 6.23 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.41

$$\int \frac{(d+ex)^2}{x^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[-1/96*(3*((35*c^4*d^7 - 15*a*c^3*d^5*e^2 - 3*a^2*c^2*d^3*e^4 - a^3*c*d*e^
6)*x^4 + (35*a*c^3*d^6*e - 15*a^2*c^2*d^4*e^3 - 3*a^3*c*d^2*e^5 - a^4*e^7)
*x^3)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)
*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(8*a^4*d^3*e^
4 + (105*a*c^3*d^6*e - 10*a^2*c^2*d^4*e^3 - 3*a^3*c*d^2*e^5)*x^3 + (35*a^2
*c^2*d^5*e^2 - 8*a^3*c*d^3*e^4 - 3*a^4*d*e^6)*x^2 - 2*(7*a^3*c*d^4*e^3 - a
^4*d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^5*c*d^4*e^5
*x^4 + a^6*d^3*e^6*x^3), -1/48*(3*((35*c^4*d^7 - 15*a*c^3*d^5*e^2 - 3*a^2*
c^2*d^3*e^4 - a^3*c*d*e^6)*x^4 + (35*a*c^3*d^6*e - 15*a^2*c^2*d^4*e^3 - 3*
a^3*c*d^2*e^5 - a^4*e^7)*x^3)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^
2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(8*a^4*d^3*e^4 +
 (105*a*c^3*d^6*e - 10*a^2*c^2*d^4*e^3 - 3*a^3*c*d^2*e^5)*x^3 + (35*a^2*c^
2*d^5*e^2 - 8*a^3*c*d^3*e^4 - 3*a^4*d*e^6)*x^2 - 2*(7*a^3*c*d^4*e^3 - a^4*
d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^5*c*d^4*e^5*x^
4 + a^6*d^3*e^6*x^3)]
```

Sympy [F]

$$\int \frac{(d+ex)^2}{x^4 (ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^2}{x^4 \left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/x**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**2/(x**4*((d + e*x)*(a*e + c*d*x))**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{x^4 (ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2}{x^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^2/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[1,1,14]%%%},[2,9]%%%}+%%%{%%%{-5,[2,3,12]%%%},[2,8]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{x^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{\left(d+e\,x\right)^2}{x^4\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

input
```
int((d + e*x)^2/(x^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int((d + e*x)^2/(x^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 1509, normalized size of antiderivative = 4.56

$$\int \frac{(d+ex)^2}{x^4 \left(ade+(cd^2+ae^2)\, x+cdex^2\right)^{3/2}}\, dx = \text{Too large to display}$$

input
```
int((e*x+d)^2/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(15*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x
) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))*a**4*e**8*x**3 + 66*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log
(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2)
 + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**2*e**6*x**3 + 288*sqrt(e)*sqrt
(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(
c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*
*2*d**4*e**4*x**3 - 210*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt
(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))*a*c**3*d**6*e**2*x**3 - 735*sqrt(e)*sqrt(d)*s
qrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sq
rt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**4*d**8*x*
*3 + 15*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sq
rt(d + e*x))*a**4*e**8*x**3 + 66*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)
*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d
**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**2*e**6*x**3 + 288*sqrt(e)*
sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*s
qrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**
2*c**2*d**4*e**4*x**3 - 210*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*l...
```

3.103 $\int \frac{x^3(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 433

$$\int \frac{x^3(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2a^3e^3\left(cd^2-ae^2\right)(d+ex)}{c^5d^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{\left(3c^3d^6+13ac^2d^4e^2-187a^2cd^2e^4+187a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{64c^5d^5e^2} + \frac{\left(c^2d^4-34acd^2e^2+41a^2e^4\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{32c^4d^4e} + \frac{\left(3cd^2-5ae^2\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{8c^3d^3} + \frac{ex^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^2d^2} + \frac{3\left(cd^2-ae^2\right)\left(c^3d^6+5ac^2d^4e^2+35a^2cd^2e^4-105a^3e^6\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{64c^{11/2}d^{11/2}e^{5/2}}$$

output
```
2*a^3*e^3*(-a*e^2+c*d^2)*(e*x+d)/c^5/d^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)-1/64*(187*a^3*e^6-187*a^2*c*d^2*e^4+13*a*c^2*d^4*e^2+3*c^3*d^6)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/e^2+1/32*(41*a^2*e^4-34*a*c*d
^2*e^2+c^2*d^4)*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e+1/8*(-
5*a*e^2+3*c*d^2)*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3+1/4*e
*x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2+3/64*(-a*e^2+c*d^2)*(
-105*a^3*e^6+35*a^2*c*d^2*e^4+5*a*c^2*d^4*e^2+c^3*d^6)*arctanh(c^(1/2)*d^(
1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(11/2)/d^(
11/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 10.87 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.87

$$\int \frac{x^3(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx =$$
$$-\frac{(d+ex)\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}(315a^4e^7-105a^3cde^5(3d-ex)+a^2c^2d^2e^3(13d^2-119dex-42e^2x^2)+c^4c\right.}{}$$

input
```
Integrate[(x^3*(d + e*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),
x]
```

output
```
-1/64*((d + e*x)*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*
e^2)]*(315*a^4*e^7 - 105*a^3*c*d*e^5*(3*d - e*x) + a^2*c^2*d^2*e^3*(13*d^2
 - 119*d*e*x - 42*e^2*x^2) + c^4*d^4*x*(3*d^3 - 2*d^2*e*x - 24*d*e^2*x^2 -
 16*e^3*x^3) + a*c^3*d^3*e*(3*d^3 + 11*d^2*e*x + 44*d*e^2*x^2 + 24*e^3*x^3
)) - 3*Sqrt[c*d]*Sqrt[c*d^2 - a*e^2]*(c^3*d^6 + 5*a*c^2*d^4*e^2 + 35*a^2*c
*d^2*e^4 - 105*a^3*e^6)*Sqrt[a*e + c*d*x]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]
*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*d^2 - a*e^2])]))/(c^(11/2)*d^(11/2)*
e^(5/2)*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*Sqrt[(a*e + c*d*x)*(d + e*x)
])
```

Rubi [A] (verified)

Time = 2.62 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1211, 2192, 27, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)^3}{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1211$$

$$\frac{\int \frac{c^4d^4x^4e^8+a^2\left(cd^2-ae^2\right)^2e^8+c^3d^3\left(2cd^2-ae^2\right)x^3e^7-acd\left(cd^2-ae^2\right)^2xe^7+c^2d^2\left(cd^2-ae^2\right)^2x^2e^6}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^5d^5e^6}+$$

$$\frac{2a^3e^3(d+ex)\left(cd^2-ae^2\right)}{c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 2192

$$\frac{\frac{\int \frac{8a^2cd\left(cd^2-ae^2\right)^2e^9+3c^4d^4\left(3cd^2-5ae^2\right)x^3e^8-8ac^2d^2\left(cd^2-ae^2\right)^2xe^8+2c^3d^3\left(4c^2d^4-11ace^2d^2+4a^2e^4\right)x^2e^7}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4cde}+\frac{1}{4}c^3d^3e^7x^3\sqrt{x\left(ae^2+cd^2\right)}}{c^5d^5e^6}$$

$$\frac{2a^3e^3(d+ex)\left(cd^2-ae^2\right)}{c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{8a^2cd\left(cd^2-ae^2\right)^2e^9+3c^4d^4\left(3cd^2-5ae^2\right)x^3e^8-8ac^2d^2\left(cd^2-ae^2\right)^2xe^8+2c^3d^3\left(4c^2d^4-11ace^2d^2+4a^2e^4\right)x^2e^7}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8cde}+\frac{1}{4}c^3d^3e^7x^3\sqrt{x\left(ae^2+cd^2\right)}}{c^5d^5e^6}$$

$$\frac{2a^3e^3(d+ex)\left(cd^2-ae^2\right)}{c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 2192

$$\frac{\frac{\frac{\int \frac{3\left(16a^2c^2d^2\left(cd^2-ae^2\right)^2e^{10}-4ac^3d^3\left(7c^2d^4-13ace^2d^2+4a^2e^4\right)xe^9+c^4d^4\left(c^2d^4-34ace^2d^2+41a^2e^4\right)x^2e^8\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3cde}+c^3d^3e^7x^2\left(3cd^2-5ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+}}{8cde}}{c^5d^5e^6}$$

$$\frac{2a^3e^3(d+ex)\left(cd^2-ae^2\right)}{c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{16a^2c^2d^2\left(cd^2-ae^2\right)^2e^{10}-4ac^3d^3\left(7c^2d^4-13ace^2d^2+4a^2e^4\right)xe^9+c^4d^4\left(c^2d^4-34ace^2d^2+41a^2e^4\right)x^2e^8}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+c^3d^3e^7x^2\left(3cd^2-5ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade}}{8cde}}{c^5d^5e^6}$$

$$\frac{2a^3e^3(d+ex)\left(cd^2-ae^2\right)}{c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 2192

$$\frac{\frac{\frac{\frac{\int -\frac{c^3d^3e^8\left(2ae\left(c^3d^6-66ac^2e^2d^4+105a^2ce^4d^2-32a^3e^6\right)+cd\left(3c^3d^6+13ac^2e^2d^4-187a^2ce^4d^2+187a^3e^6\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}c^3d^3e^7x\left(41a^2e^4-34acd^2e^2+c^2d^4\right)\sqrt{x\left(ae^2\cdots\right.}}{2cde}}{8cde}}{c^5d^5e^6}$$

$$\frac{2a^3e^3(d+ex)\left(cd^2-ae^2\right)}{c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\frac{1}{2}c^3d^3e^7x\left(41a^2e^4-34acd^2e^2+c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-\frac{1}{4}c^2d^2e^7\int\frac{2ae\left(c^3d^6-66ac^2e^2d^4+105a^2ce^4d^2-32a^3e^6\right)+cd\left(3c^3d^6+13ac^2e^2d^4-187a^2ce^4\cdots\right.}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\cdots}{2cde}}{8cde}}{c^5d^5e^6}$$

$$\frac{2a^3e^3(d+ex)\left(cd^2-ae^2\right)}{c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 1160

$$\frac{\frac{\frac{1}{2}c^3d^3e^7x\left(41a^2e^4-34acd^2e^2+c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-\frac{1}{4}c^2d^2e^7\left(\frac{\left(187a^3e^6-187a^2cd^2e^4+13ac^2d^4e^2+3c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{3\left(cd^2\cdots\right.}{\cdots}\right.}{2cde}}{8cde}$$

$$\frac{2a^3e^3(d+ex)\left(cd^2-ae^2\right)}{c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 1092

$$\frac{\frac{1}{2}c^3d^3e^7x\left(41a^2e^4-34acd^2e^2+c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-\frac{1}{4}c^2d^2e^7\left(\frac{\left(187a^3e^6-187a^2cd^2e^4+13ac^2d^4e^2+3c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{3\left(cd^2\cdots\right.}{\cdots}\right.}{2cde}$$

$$\frac{2a^3e^3(d+ex)\left(cd^2-ae^2\right)}{c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 219

$$\frac{2a^3e^3(d+ex)\left(cd^2-ae^2\right)}{c^5d^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+$$

$$\frac{\frac{\frac{1}{2}c^3d^3e^7x\left(41a^2e^4-34acd^2e^2+c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}-\frac{1}{4}c^2d^2e^7\left(\frac{\left(187a^3e^6-187a^2cd^2e^4+13ac^2d^4e^2+3c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}-\frac{3\left(cd^2-\right.}{}\right.}{2cde}}{8}$$

input
```
Int[(x^3*(d + e*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(2*a^3*e^3*(c*d^2 - a*e^2)*(d + e*x))/(c^5*d^5*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) + ((c^3*d^3*e^7*x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])/4 + (c^3*d^3*e^7*(3*c*d^2 - 5*a*e^2)*x^2*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2] + ((c^3*d^3*e^7*(c^2*d^4 - 34*a*c*d^2*e^2 + 41*a^2*e^4
)*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/2 - (c^2*d^2*e^7*(((3*c^3
*d^6 + 13*a*c^2*d^4*e^2 - 187*a^2*c*d^2*e^4 + 187*a^3*e^6)*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/e - (3*(c*d^2 - a*e^2)*(c^3*d^6 + 5*a*c^2*d^
4*e^2 + 35*a^2*c*d^2*e^4 - 105*a^3*e^6)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x
)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]
)/(2*Sqrt[c]*Sqrt[d]*e^(3/2))))/4)/(2*c*d*e))/(8*c*d*e))/(c^5*d^5*e^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4840 vs. $2(399) = 798$.

Time = 3.09 (sec) , antiderivative size = 4841, normalized size of antiderivative = 11.18

method	result	size
default	Expression too large to display	4841

input
```
int(x^3*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
d^3*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d
/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d
/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c
*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(
1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2*a/c*(-1/d/e
/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+
a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^
2*e)^(1/2)))+e^3*(1/4*x^5/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-9/
8*(a*e^2+c*d^2)/d/e/c*(1/3*x^4/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/
2)-7/6*(a*e^2+c*d^2)/d/e/c*(1/2*x^3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2)-5/4*(a*e^2+c*d^2)/d/e/c*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+
1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/
(d*e*c)^(1/2))-2*a/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*
e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(
a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))-3/2*a/c*(-x/d/e/c/(a*d*e+(a*e^...
```

Fricas [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 952, normalized size of antiderivative = 2.20

$$\int \frac{x^3(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[1/256*(3*(a*c^4*d^8*e + 4*a^2*c^3*d^6*e^3 + 30*a^3*c^2*d^4*e^5 - 140*a^4*
c*d^2*e^7 + 105*a^5*e^9 + (c^5*d^9 + 4*a*c^4*d^7*e^2 + 30*a^2*c^3*d^5*e^4
- 140*a^3*c^2*d^3*e^6 + 105*a^4*c*d*e^8)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*
x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c
*d*e^3)*x) + 4*(16*c^5*d^5*e^4*x^4 - 3*a*c^4*d^7*e^2 - 13*a^2*c^3*d^5*e^4
+ 315*a^3*c^2*d^3*e^6 - 315*a^4*c*d*e^8 + 24*(c^5*d^6*e^3 - a*c^4*d^4*e^5)
*x^3 + 2*(c^5*d^7*e^2 - 22*a*c^4*d^5*e^4 + 21*a^2*c^3*d^3*e^6)*x^2 - (3*c^
5*d^8*e + 11*a*c^4*d^6*e^3 - 119*a^2*c^3*d^4*e^5 + 105*a^3*c^2*d^2*e^7)*x)
*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^7*d^7*e^3*x + a*c^6*d^6*e
^4), -1/128*(3*(a*c^4*d^8*e + 4*a^2*c^3*d^6*e^3 + 30*a^3*c^2*d^4*e^5 - 140
*a^4*c*d^2*e^7 + 105*a^5*e^9 + (c^5*d^9 + 4*a*c^4*d^7*e^2 + 30*a^2*c^3*d^5
*e^4 - 140*a^3*c^2*d^3*e^6 + 105*a^4*c*d*e^8)*x)*sqrt(-c*d*e)*arctan(1/2*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqr
t(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2
*(16*c^5*d^5*e^4*x^4 - 3*a*c^4*d^7*e^2 - 13*a^2*c^3*d^5*e^4 + 315*a^3*c^2*
d^3*e^6 - 315*a^4*c*d*e^8 + 24*(c^5*d^6*e^3 - a*c^4*d^4*e^5)*x^3 + 2*(c^5*
d^7*e^2 - 22*a*c^4*d^5*e^4 + 21*a^2*c^3*d^3*e^6)*x^2 - (3*c^5*d^8*e + 11*a
*c^4*d^6*e^3 - 119*a^2*c^3*d^4*e^5 + 105*a^3*c^2*d^2*e^7)*x)*sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^7*d^7*e^3*x + a*c^6*d^6*e^4)]
```

Sympy [F]

$$\int \frac{x^3(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{x^3(d+ex)^3}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**3*(d + e*x)**3/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[6,6,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,0]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{x^3\,(d+e\,x)^3}{\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\;x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int((x^3*(d + e*x)^3)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((x^3*(d + e*x)^3)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{x^3(ex+d)^3}{(ade+(a\,e^2+c\,d^2)\,x+cde\,x^2)^{\frac{3}{2}}}dx$$

input
```
int(x^3*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
int(x^3*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

3.104 $\int \frac{x^2(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 334

$$\begin{aligned}\int \frac{x^2(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx &= -\frac{2a^2e^2\left(cd^2-ae^2\right)(d+ex)}{c^4d^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ &+ \frac{\left(3c^2d^4-52acd^2e^2+57a^2e^4\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24c^4d^4e} \\ &+ \frac{\left(7cd^2-11ae^2\right)x\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12c^3d^3} \\ &+ \frac{ex^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^2d^2} \\ &- \frac{\left(cd^2-ae^2\right)\left(c^2d^4+10acd^2e^2-35a^2e^4\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{8c^{9/2}d^{9/2}e^{3/2}}\end{aligned}$$

output
```
-2*a^2*e^2*(-a*e^2+c*d^2)*(e*x+d)/c^4/d^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)+1/24*(57*a^2*e^4-52*a*c*d^2*e^2+3*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)/c^4/d^4/e+1/12*(-11*a*e^2+7*c*d^2)*x*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)/c^3/d^3+1/3*e*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/c^2/d^2-1/8*(-a*e^2+c*d^2)*(-35*a^2*e^4+10*a*c*d^2*e^2+c^2*d^4)*arctan
h(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))
/c^(9/2)/d^(9/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 10.68 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.92

$$\int \frac{x^2(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{(d+ex)\left(\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{\frac{cd(d+ex)}{cd^2-ae^2}}(105a^3e^5-5a^2cde^3(20d-7ex)+c\right.}{}$$

input
```
Integrate[(x^2*(d + e*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),
x]
```

output
```
((d + e*x)*(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[(c*d*(d + e*x))/(c*d^2 - a*e^2)]*
(105*a^3*e^5 - 5*a^2*c*d*e^3*(20*d - 7*e*x) + a*c^2*d^2*e*(3*d^2 - 38*d*e*
x - 14*e^2*x^2) + c^3*d^3*x*(3*d^2 + 14*d*e*x + 8*e^2*x^2)) - 3*Sqrt[c*d]*
Sqrt[c*d^2 - a*e^2]*(c^2*d^4 + 10*a*c*d^2*e^2 - 35*a^2*e^4)*Sqrt[a*e + c*d
*x]*ArcSinh[(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c*d]*Sqrt[c*
d^2 - a*e^2])]))/(24*c^(9/2)*d^(9/2)*e^(3/2)*Sqrt[(c*d*(d + e*x))/(c*d^2 -
 a*e^2)]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {1211, 25, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)^3}{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int -\frac{-c^3d^3x^3e^6+a(cd^2-ae^2)^2e^5-c^2d^2(2cd^2-ae^2)x^2e^5-cd(cd^2-ae^2)^2xe^4}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{c^4d^4e^4}-\frac{2a^2e^2(d+ex)(cd^2-ae^2)}{c^4d^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{-c^3d^3x^3e^6+a\left(cd^2-ae^2\right)^2e^5-c^2d^2\left(2cd^2-ae^2\right)x^2e^5-cd\left(cd^2-ae^2\right)^2xe^4}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^4d^4e^4}-\frac{2a^2e^2(d+ex)\left(cd^2-ae^2\right)}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 2192$$

$$-\frac{\frac{\int \frac{6acd\left(cd^2-ae^2\right)^2e^6-c^3d^3\left(7cd^2-11ae^2\right)x^2e^6-2c^2d^2\left(3c^2d^4-8ace^2d^2+3a^2e^4\right)xe^5}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3cde}-\frac{1}{3}c^2d^2e^5x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^4d^4e^4}-\frac{2a^2e^2(d+ex)\left(cd^2-ae^2\right)}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{6acd\left(cd^2-ae^2\right)^2e^6-c^3d^3\left(7cd^2-11ae^2\right)x^2e^6-2c^2d^2\left(3c^2d^4-8ace^2d^2+3a^2e^4\right)xe^5}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6cde}-\frac{1}{3}c^2d^2e^5x^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^4d^4e^4}-\frac{2a^2e^2(d+ex)\left(cd^2-ae^2\right)}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 2192$$

$$-\frac{\frac{\frac{\int \frac{c^2d^2e^6\left(2ae\left(19c^2d^4-35ace^2d^2+12a^2e^4\right)-cd\left(3c^2d^4-52ace^2d^2+57a^2e^4\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}-\frac{1}{2}c^2d^2e^5x\left(7cd^2-11ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6cde}-\frac{1}{3}c^2d^2e^5}{c^4d^4e^4}-\frac{2a^2e^2(d+ex)\left(cd^2-ae^2\right)}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{1}{4}cde^5\int \frac{2ae\left(19c^2d^4-35ace^2d^2+12a^2e^4\right)-cd\left(3c^2d^4-52ace^2d^2+57a^2e^4\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{1}{2}c^2d^2e^5x\left(7cd^2-11ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6cde}-\frac{1}{3}c^2d^2e^5x^2}{c^4d^4e^4}-\frac{2a^2e^2(d+ex)\left(cd^2-ae^2\right)}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1160$$

$$-\frac{\frac{\frac{1}{4}cde^5\left(\frac{3\left(cd^2-ae^2\right)\left(-35a^2e^4+10acd^2e^2+c^2d^4\right)\int\frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2e}-\frac{\left(57a^2e^4-52acd^2e^2+3c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{e}\right)-\frac{1}{2}c^2d^2e^5}{6cde}}{c^4d^4e^4}$$

$$\frac{2a^2e^2(d+ex)\left(cd^2-ae^2\right)}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1092}$$

$$-\frac{\frac{\frac{1}{4}cde^5\left(\frac{3\left(cd^2-ae^2\right)\left(-35a^2e^4+10acd^2e^2+c^2d^4\right)\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{e}-\frac{\left(57a^2e^4-52acd^2e^2+3c^2d^4\right)\sqrt{x\left(ae^2+\right.}}{e}}{6cde}}{c^4d^4e^4}$$

$$\frac{2a^2e^2(d+ex)\left(cd^2-ae^2\right)}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\frac{1}{4}cde^5\left(\frac{3\left(cd^2-ae^2\right)\left(-35a^2e^4+10acd^2e^2+c^2d^4\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}-\frac{\left(57a^2e^4-52acd^2e^2+3c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+}}{e}}{6cde}}{c^4d^4e^4}$$

$$\frac{2a^2e^2(d+ex)\left(cd^2-ae^2\right)}{c^4d^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(x^2*(d + e*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*a^2*e^2*(c*d^2 - a*e^2)*(d + e*x))/(c^4*d^4*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2]) - (-1/3*(c^2*d^2*e^5*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2]) + (-1/2*(c^2*d^2*e^5*(7*c*d^2 - 11*a*e^2)*x*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2]) + (c*d*e^5*(-(((3*c^2*d^4 - 52*a*c*d^2*e^2 +
 57*a^2*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/e) + (3*(c*d^2 -
 a*e^2)*(c^2*d^4 + 10*a*c*d^2*e^2 - 35*a^2*e^4)*ArcTanh[(c*d^2 + a*e^2 + 2
*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])])/(2*Sqrt[c]*Sqrt[d]*e^(3/2))))/4)/(6*c*d*e))/(c^4*d^4*e^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2894 vs. $2(304) = 608$.

Time = 3.27 (sec) , antiderivative size = 2895, normalized size of antiderivative = 8.67

method	result	size
default	Expression too large to display	2895

input
```
int(x^2*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
d^3*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/
e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*
(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1
/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+e^3*(1/3*x^4/d
/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-7/6*(a*e^2+c*d^2)/d/e/c*(1/2*
x^3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-5/4*(a*e^2+c*d^2)/d/e/c*
(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c
*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c
*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*
c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)
+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2*a/c*(-1/d/e/c/(
a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^
2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)
^(1/2)))-3/2*a/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*
e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+
c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*
e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)...
```

Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 754, normalized size of antiderivative = 2.26

$$\int \frac{x^2(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \left[\frac{3\left(ac^3d^6e+9\,a^2c^2d^4e^3-45\,a^3cd^2e^5+35\,a^4e^7+\left(c^4d^7+9\,ac^3d^5\right.\right.}{}\right.$$

input
```
integrate(x^2*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[1/96*(3*(a*c^3*d^6*e + 9*a^2*c^2*d^4*e^3 - 45*a^3*c*d^2*e^5 + 35*a^4*e^7
+ (c^4*d^7 + 9*a*c^3*d^5*e^2 - 45*a^2*c^2*d^3*e^4 + 35*a^3*c*d*e^6)*x)*sqr
t(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(
c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(8*c^4*d^4*e^3*x^3 + 3*a*c^3*d^5
*e^2 - 100*a^2*c^2*d^3*e^4 + 105*a^3*c*d*e^6 + 14*(c^4*d^5*e^2 - a*c^3*d^3
*e^4)*x^2 + (3*c^4*d^6*e - 38*a*c^3*d^4*e^3 + 35*a^2*c^2*d^2*e^5)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^6*d^6*e^2*x + a*c^5*d^5*e^3), 1
/48*(3*(a*c^3*d^6*e + 9*a^2*c^2*d^4*e^3 - 45*a^3*c*d^2*e^5 + 35*a^4*e^7 +
(c^4*d^7 + 9*a*c^3*d^5*e^2 - 45*a^2*c^2*d^3*e^4 + 35*a^3*c*d*e^6)*x)*sqrt(
-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x
+ c*d^2 + a*e^2)*sqrt(-c*d*e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e
+ a*c*d*e^3)*x)) + 2*(8*c^4*d^4*e^3*x^3 + 3*a*c^3*d^5*e^2 - 100*a^2*c^2*d^
3*e^4 + 105*a^3*c*d*e^6 + 14*(c^4*d^5*e^2 - a*c^3*d^3*e^4)*x^2 + (3*c^4*d^
6*e - 38*a*c^3*d^4*e^3 + 35*a^2*c^2*d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (
c*d^2 + a*e^2)*x))/(c^6*d^6*e^2*x + a*c^5*d^5*e^3)]
```

Sympy [F]

$$\int \frac{x^2(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{x^2(d+ex)^3}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**2*(d + e*x)**3/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{1,[5,5,0]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,0]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{x^2\,(d+e\,x)^3}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}\,dx$$

input
```
int((x^2*(d + e*x)^3)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((x^2*(d + e*x)^3)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.67

$$\int \frac{x^2(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{-840\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a^3e^6+10}{}$$

input
```
int(x^2*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 840*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e +
c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**6 +
 1080*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**2*e
**4 - 216*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e
+ c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c**2*d*
*4*e**2 - 24*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a
*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**3*d
**6 + 525*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**3*e**6 - 615*sqrt(e
)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d**2*e**4 + 87*sqrt(e)*sqrt(d)*
sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**4*e**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*sqr
t(a*e + c*d*x)*c**3*d**6 + 840*sqrt(d + e*x)*a**3*c*d*e**6 - 800*sqrt(d +
e*x)*a**2*c**2*d**3*e**4 + 280*sqrt(d + e*x)*a**2*c**2*d**2*e**5*x + 24*sq
rt(d + e*x)*a*c**3*d**5*e**2 - 304*sqrt(d + e*x)*a*c**3*d**4*e**3*x - 112*
sqrt(d + e*x)*a*c**3*d**3*e**4*x**2 + 24*sqrt(d + e*x)*c**4*d**6*e*x + 112
*sqrt(d + e*x)*c**4*d**5*e**2*x**2 + 64*sqrt(d + e*x)*c**4*d**4*e**3*x**3)
/(192*sqrt(a*e + c*d*x)*c**5*d**5*e**2)
```

3.105 $\int \frac{x(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 244

$$\int \frac{x(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2ae(cd^2-ae^2)(d+ex)}{c^3d^3\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{(5cd^2-7ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4c^3d^3} + \frac{ex\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{2c^2d^2} + \frac{3(cd^2-5ae^2)(cd^2-ae^2)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^{7/2}d^{7/2}\sqrt{e}}$$

output
```
2*a*e*(-a*e^2+c*d^2)*(e*x+d)/c^3/d^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)+1/4*(-7*a*e^2+5*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3+
1/2*e*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2+3/4*(-5*a*e^2+c*d^
2)*(-a*e^2+c*d^2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.84

$$\int \frac{x(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{e}(d+ex)\left(-15a^2e^3+acde(13d-5ex)+c^2d^2x(5d+2e\right.}{4c^{7/2}d}$$

input
```
Integrate[(x*(d + e*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(Sqrt[c]*Sqrt[d]*Sqrt[e]*(d + e*x)*(-15*a^2*e^3 + a*c*d*e*(13*d - 5*e*x) +
 c^2*d^2*x*(5*d + 2*e*x)) - 6*(c^2*d^4 - 6*a*c*d^2*e^2 + 5*a^2*e^4)*Sqrt[a
*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]
*(Sqrt[-((c*d^2)/e) + a*e] - Sqrt[a*e + c*d*x]))])/(4*c^(7/2)*d^(7/2)*Sqrt
[e]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1211, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)^3}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int \frac{c^2d^2x^2e^4+cd\left(2cd^2-ae^2\right)xe^3+\left(cd^2-ae^2\right)^2e^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^3d^3e^2}+\frac{2ae(d+ex)\left(cd^2-ae^2\right)}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\int \frac{cde^3\left(2\left(cd^2-2ae^2\right)\left(2cd^2-ae^2\right)+cde\left(5cd^2-7ae^2\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}+\frac{1}{2}cde^3x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3e^2}+\frac{2ae(d+ex)\left(cd^2-ae^2\right)}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{4}e^2\int \frac{2\left(cd^2-2ae^2\right)\left(2cd^2-ae^2\right)+cde\left(5cd^2-7ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{1}{2}cde^3x\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{c^3d^3e^2}+\frac{2ae(d+ex)\left(cd^2-ae^2\right)}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1160$$

$$\frac{\frac{1}{4}e^2\left(\frac{3}{2}\left(cd^2-5ae^2\right)\left(cd^2-ae^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\left(5cd^2-7ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)+\frac{1}{2}c}{c^3d^3e^2}$$
$$\frac{2ae(d+ex)\left(cd^2-ae^2\right)}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1092$$

$$\frac{\frac{1}{4}e^2\left(3\left(cd^2-5ae^2\right)\left(cd^2-ae^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}+\left(5cd^2-7ae^2\right)\sqrt{x\left(ae^2+cd^2\right.}\right.}{c^3d^3e^2}$$
$$\frac{2ae(d+ex)\left(cd^2-ae^2\right)}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{1}{4}e^2\left(\frac{3\left(cd^2-5ae^2\right)\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}+\left(5cd^2-7ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{c^3d^3e^2}$$
$$\frac{2ae(d+ex)\left(cd^2-ae^2\right)}{c^3d^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(x*(d + e*x)^3)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(2*a*e*(c*d^2 - a*e^2)*(d + e*x))/(c^3*d^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]) + ((c*d*e^3*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/2
 + (e^2*((5*c*d^2 - 7*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2] +
 (3*(c*d^2 - 5*a*e^2)*(c*d^2 - a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/
(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/
(2*Sqrt[c]*Sqrt[d]*Sqrt[e])))/4)/(c^3*d^3*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1688 vs. $2(218) = 436$.

Time = 2.49 (sec) , antiderivative size = 1689, normalized size of antiderivative = 6.92

method	result	size
default	Expression too large to display	1689

input
```
int(x*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
d^3*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*
(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2))+e^3*(1/2*x^3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2)-5/4*(a*e^2+c*d^2)/d/e/c*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+
1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/
(d*e*c)^(1/2))-2*a/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*
e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(
a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))-3/2*a/c*(-x/d/e/c/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*
c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/
c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+3*d*e^2*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2...
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.39

$$\int \frac{x(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \left[\frac{3\left(ac^2d^4e-6\,a^2cd^2e^3+5\,a^3e^5+\left(c^3d^5-6\,ac^2d^3e^2+5\,a^2cde^4\right)x\right.}{}\right.$$

$$-\frac{3\left(ac^2d^4e-6\,a^2cd^2e^3+5\,a^3e^5+\left(c^3d^5-6\,ac^2d^3e^2+5\,a^2cde^4\right)x\right)\sqrt{-cde}\arctan\left(\frac{\sqrt{cdex^2+ade+\left(cd^2+ae^2\right)x}\left(2\,cde\right.}{2\left(c^2d^2e^2x^2+acd^2e^2+\left(c^2d\right.\right.}\right.}{8\left(c^5d^5\right.}$$

input
```
integrate(x*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[1/16*(3*(a*c^2*d^4*e - 6*a^2*c*d^2*e^3 + 5*a^3*e^5 + (c^3*d^5 - 6*a*c^2*d
^3*e^2 + 5*a^2*c*d*e^4)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6
*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*
c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) + 4*(2
*c^3*d^3*e^2*x^2 + 13*a*c^2*d^3*e^2 - 15*a^2*c*d*e^4 + 5*(c^3*d^4*e - a*c^
2*d^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e*x +
a*c^4*d^4*e^2), -1/8*(3*(a*c^2*d^4*e - 6*a^2*c*d^2*e^3 + 5*a^3*e^5 + (c^3*
d^5 - 6*a*c^2*d^3*e^2 + 5*a^2*c*d*e^4)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d
*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*
e)/(c^2*d^2*e^2*x^2 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3)*x)) - 2*(2*c^3
*d^3*e^2*x^2 + 13*a*c^2*d^3*e^2 - 15*a^2*c*d*e^4 + 5*(c^3*d^4*e - a*c^2*d^
2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(c^5*d^5*e*x + a*c^
4*d^4*e^2)]
```

Sympy [F]

$$\int \frac{x(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{x(d+ex)^3}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x*(d + e*x)**3/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{8,[4,4,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,2]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex)^3}{(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{x\,(d+e\,x)^3}{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}\,dx$$

input
```
int((x*(d + e*x)^3)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((x*(d + e*x)^3)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.55

$$\int \frac{x(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{15\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a^2e^4-18\sqrt{e}$$

input
```
int(x*(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(15*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*e**4 - 18*
sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*c*d**2*e**2 + 3*s
qrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) +
sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c**2*d**4 - 10*sqrt(
e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*e**4 + 11*sqrt(e)*sqrt(d)*sqrt(c
)*sqrt(a*e + c*d*x)*a*c*d**2*e**2 - sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d
*x)*c**2*d**4 - 15*sqrt(d + e*x)*a**2*c*d*e**4 + 13*sqrt(d + e*x)*a*c**2*d
**3*e**2 - 5*sqrt(d + e*x)*a*c**2*d**2*e**3*x + 5*sqrt(d + e*x)*c**3*d**4*
e*x + 2*sqrt(d + e*x)*c**3*d**3*e**2*x**2)/(4*sqrt(a*e + c*d*x)*c**4*d**4*
e)
```

3.106 $\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 163

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = -\frac{2(d+ex)^2}{cd\sqrt{ade+(cd^2+ae^2)x+cdex^2}}+\frac{3e\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{c^2d^2}+\frac{3\sqrt{e}(cd^2-ae^2)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{5/2}d^{5/2}}$$

output
```
-2*(e*x+d)^2/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+3*e*(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2+3*e^(1/2)*(-a*e^2+c*d^2)*arctanh(c^(1/2
)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)
/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^3}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{-\sqrt{c}\sqrt{d}(d+ex)\left(-3ae^2+cd(2d-ex)\right)+3\sqrt{e}\left(cd^2-ae^2\right)\sqrt{ae -}}{c^{5/2}d^{5/2}\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-(Sqrt[c]*Sqrt[d]*(d + e*x)*(-3*a*e^2 + c*d*(2*d - e*x))) + 3*Sqrt[e]*(c*
d^2 - a*e^2)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt
[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(c^(5/2)*d^(5/2)*Sqrt[(a*e + c*d*
x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {1124, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1124}$$

$$\frac{\int \frac{e\left(2cd^2+cexd-ae^2\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^2d^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\int \frac{2cd^2+cexd-ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{c^2d^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{e\left(\frac{3}{2}\left(cd^2-ae^2\right)\int\frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{c^2d^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{e\left(3\left(cd^2-ae^2\right)\int\frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}+\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{c^2d^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{e\left(\frac{3\left(cd^2-ae^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{e}}+\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}\right)}{c^2d^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{c^2d^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2),x]
```

output
```
(-2*(c*d^2 - a*e^2)*(d + e*x))/(c^2*d^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) + (e*(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2] + (3*(c*d^2 -
 a*e^2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[c]*Sqrt[d]*Sqrt[e])))/
(c^2*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(145) = 290$.

Time = 2.43 (sec) , antiderivative size = 960, normalized size of antiderivative = 5.89

method	result
default	$\frac{2d^3(2cdxe+a e^2+c d^2)}{\left(4ac d^2e^2-(a e^2+c d^2)^2\right)\sqrt{ade+(a e^2+c d^2)x+cd x^2e}} + e^3\left(\frac{x^2}{dec\sqrt{ade+(a e^2+c d^2)x+cd x^2e}} - \frac{3(a e^2+c d^2)\left(-\frac{}{dec\sqrt{ade+(a}}\right.}{}\right.$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
2*d^3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2)+e^3*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2
+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+
1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/
(d*e*c)^(1/2))-2*a/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*
e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(
a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))+3*d*e^2*(-x/d/e/c/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*
c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/
c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2))/(d*e*c)^(1/2))+3*d^2*e*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-
(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.74

$$\int \frac{(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \left[-\frac{3\left(acd^2e-a^2e^3+(c^2d^3-acde^2)x\right)\sqrt{\frac{e}{cd}}\log\left(8\,c^2d^2e^2x^2+c^2d\right.}{}\right.$$

$$-\frac{3\left(acd^2e-a^2e^3+(c^2d^3-acde^2)x\right)\sqrt{-\frac{e}{cd}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2\,cdex+cd^2+ae^2)\sqrt{-\frac{e}{cd}}}{2\,(cde^2x^2+ade^2+(cd^2e+ae^3)x)}\right)-2\sqrt{cdex^2}}{2\left(c^3d^3x+ac^2d^2e\right)}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
fricas")
```

output
```
[-1/4*(3*(a*c*d^2*e - a^2*e^3 + (c^2*d^3 - a*c*d*e^2)*x)*sqrt(e/(c*d))*log
(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*
c*d*e^3)*x - 4*(2*c^2*d^2*e*x + c^2*d^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*sqrt(e/(c*d))) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(c*d*e*x - 2*c*d^2 + 3*a*e^2))/(c^3*d^3*x + a*c^2*d^2*e), -1/2
*(3*(a*c*d^2*e - a^2*e^3 + (c^2*d^3 - a*c*d*e^2)*x)*sqrt(-e/(c*d))*arctan(
1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2
)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e^2 + (c*d^2*e + a*e^3)*x)) - 2*sqrt(c
*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*x - 2*c*d^2 + 3*a*e^2))/(c^3*
d^3*x + a*c^2*d^2*e)]
```

Sympy [F]

$$\int \frac{(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{(d+ex)^3}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**3/((d + e*x)*(a*e + c*d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{2,[3,3,4]%%%},0]:[1,0,%%%{-1,[1,1,1]%%%}]%%},[2,2]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{{\left(d+e\,x\right)}^3}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

input
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2),x)
```

output
```
int((d + e*x)^3/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex)^3}{\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\frac{-12\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\log\left(\frac{\sqrt{e}\,\sqrt{cdx+ae}+\sqrt{d}\,\sqrt{c}\,\sqrt{ex+d}}{\sqrt{a\,e^2-c\,d^2}}\right)a\,e^2+12\sqrt{}}{}$$

input
```
int((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 12*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*e**2 + 12*
sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*c*d**2 + 9*sqrt(e)*
sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*e**2 - 9*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(
a*e + c*d*x)*c*d**2 + 12*sqrt(d + e*x)*a*c*d*e**2 - 8*sqrt(d + e*x)*c**2*d
**3 + 4*sqrt(d + e*x)*c**2*d**2*e*x)/(4*sqrt(a*e + c*d*x)*c**3*d**3)
```

3.107 $\int \frac{(d+ex)^3}{x(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 197

$$\int \frac{(d+ex)^3}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2\left(cd^2-ae^2\right)(d+ex)}{acde\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2e^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{3/2}d^{3/2}} - \frac{2d^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{3/2}e^{3/2}}$$

output
```
2*(-a*e^2+c*d^2)*(e*x+d)/a/c/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2
*e^(3/2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)-2*d^(3/2)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/
d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/e^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.35 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.84

$$\int \frac{(d+ex)^3}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{2\left(\sqrt{c}\sqrt{d}\left(\sqrt{a}\sqrt{e}(-cd^2+ae^2)\,(d+ex)+\sqrt{d}\left(\sqrt{a}e-i\sqrt{cd^2-ae^2}\right)\sqrt{cd^2-2ae^2-2i\sqrt{a}e\sqrt{cd^2-ae^2}}\sqrt{\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[(d + e*x)^3/(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*(Sqrt[c]*Sqrt[d]*(Sqrt[a]*Sqrt[e]*(-(c*d^2) + a*e^2)*(d + e*x) + Sqrt[
d]*(Sqrt[a]*e - I*Sqrt[c*d^2 - a*e^2])*Sqrt[c*d^2 - 2*a*e^2 - (2*I)*Sqrt[a
]*e*Sqrt[c*d^2 - a*e^2]]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTan[(Sqrt[c*d^
2 - 2*a*e^2 - (2*I)*Sqrt[a]*e*Sqrt[c*d^2 - a*e^2]]*Sqrt[d + e*x])/(Sqrt[d]
*Sqrt[e]*(Sqrt[-((c*d^2)/e) + a*e] - Sqrt[a*e + c*d*x]))] + Sqrt[d]*(Sqrt[
a]*e + I*Sqrt[c*d^2 - a*e^2])*Sqrt[c*d^2 - 2*a*e^2 + (2*I)*Sqrt[a]*e*Sqrt[
c*d^2 - a*e^2]]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTan[(Sqrt[c*d^2 - 2*a*e
^2 + (2*I)*Sqrt[a]*e*Sqrt[c*d^2 - a*e^2]]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[e]*
(Sqrt[-((c*d^2)/e) + a*e] - Sqrt[a*e + c*d*x]))]) + 2*a^(3/2)*e^3*Sqrt[a*e
 + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*(
Sqrt[-((c*d^2)/e) + a*e] - Sqrt[a*e + c*d*x]))]))/(a^(3/2)*c^(3/2)*d^(3/2)
*e^(3/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1212, 25, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{x\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1212}$$

$$\frac{2(d+ex)\left(\frac{d}{ae}-\frac{e}{cd}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{e\int -\frac{cd^3+ae^3x}{ae^2x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd}$$

$$\Big\downarrow \; 25$$

$$\frac{e\int \frac{cd^3+ae^3x}{ae^2x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{cd}+\frac{2(d+ex)\left(\frac{d}{ae}-\frac{e}{cd}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{cd^3+ae^3x}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{acde}+\frac{2(d+ex)\left(\frac{d}{ae}-\frac{e}{cd}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1269$$

$$\frac{ae^3\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+cd^3\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{acde}+\frac{2(d+ex)\left(\frac{d}{ae}-\frac{e}{cd}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1092$$

$$\frac{2ae^3\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}+cd^3\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{acde}+\frac{2(d+ex)\left(\frac{d}{ae}-\frac{e}{cd}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{cd^3\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{ae^{5/2}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}}}{acde}+\frac{2(d+ex)\left(\frac{d}{ae}-\frac{e}{cd}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{ae^{5/2}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}}-2cd^3\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+(cd^2+ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}}{acde}+\frac{2(d+ex)\left(\frac{d}{ae}-\frac{e}{cd}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{ae^{5/2}\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}}-\frac{cd^{5/2}\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{a}\sqrt{e}}}{acde}+\frac{2(d+ex)\left(\frac{d}{ae}-\frac{e}{cd}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^3/(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*(d/(a*e) - e/(c*d))*(d + e*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2] + ((a*e^(5/2)*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*S
qrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(Sqrt[c]*Sqrt[d]) -
(c*d^(5/2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e
]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(Sqrt[a]*Sqrt[e]))/(a*c*d
*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1212
```
Int[((x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)
^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*d - b*e)^n*((d + e
*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*x^2])), x] - Simp[e^2/c^(m +
n - 1)   Int[ExpandToSum[(c^(m + n - 1)*(d + e*x)^(m - 1) - ((c*d - b*e)^n*
(2*c*d - b*e)^(m - 1))/(e^n*x^n))/(c*d - b*e - c*e*x), x]/(Sqrt[a + b*x + c
*x^2]/x^n), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^
2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(167) = 334$.

Time = 2.26 (sec) , antiderivative size = 700, normalized size of antiderivative = 3.55

method	result
default	$d^3\left(\frac{1}{ade\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}-\frac{(a\,e^2+c\,d^2)(2cdxe+a\,e^2+c\,d^2)}{ade\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}-\frac{\ln\left(\frac{2ade+\left(a\,e^2+c\,d^2\right)x+2}{}\right.}{}\right.$

input
```
int((e*x+d)^3/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
d^3*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(
2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(
a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+e^3*(-x/d/e/c/(a
*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a
*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2
+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^
(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+6*d^2*e*(2*c*d*e*x+a*e^2+c*d^2
)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+
3*d*e^2*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/
e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*x^2*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 1360, normalized size of antiderivative = 6.90

$$\int \frac{(d+ex)^3}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[1/2*((a*c*d*e^2*x + a^2*e^3)*sqrt(e/(c*d))*log(8*c^2*d^2*e^2*x^2 + c^2*d^
4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x + 4*(2*c^2*d^2*e
*x + c^2*d^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt
(e/(c*d))) + (c^2*d^3*x + a*c*d^2*e)*sqrt(d/(a*e))*log((8*a^2*d^2*e^2 + (c
^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 8*(a*c*d^3*e + a^2*d*e^3)*x - 4*(2
*a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(d/(a*e)))/x^2) + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(c*d^2 - a*e^2))/(a*c^2*d^2*e*x + a^2*c*d*e^2), -1/2*(2*(a*c*d*e^2*x + a
^2*e^3)*sqrt(-e/(c*d))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-e/(c*d))/(c*d*e^2*x^2 + a*d*e^2 + (c
*d^2*e + a*e^3)*x)) - (c^2*d^3*x + a*c*d^2*e)*sqrt(d/(a*e))*log((8*a^2*d^2
*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 8*(a*c*d^3*e + a^2*d*e^3)
*x - 4*(2*a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x)*sqrt(d/(a*e)))/x^2) - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(c*d^2 - a*e^2))/(a*c^2*d^2*e*x + a^2*c*d*e^2), 1/2*(2*(c^2*d^3
*x + a*c*d^2*e)*sqrt(-d/(a*e))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2
+ a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-d/(a*e))/(c*d^2*e*x^2 + a*
d^2*e + (c*d^3 + a*d*e^2)*x)) + (a*c*d*e^2*x + a^2*e^3)*sqrt(e/(c*d))*log(
8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c
*d*e^3)*x + 4*(2*c^2*d^2*e*x + c^2*d^3 + a*c*d*e^2)*sqrt(c*d*e*x^2 + a*...
```

Sympy [F]

$$\int \frac{(d+ex)^3}{x\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^3}{x\,((d+ex)\,(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/x/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**3/(x*((d + e*x)*(a*e + c*d*x))**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{x\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{x\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^3/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{x\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{{\left(d+e\,x\right)}^3}{x\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

input
```
int((d + e*x)^3/(x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int((d + e*x)^3/(x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)^3}{x\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\frac{\sqrt{e}\,\sqrt{d}\,\sqrt{a}\,\sqrt{cdx+ae}\,\log\left(\sqrt{e}\,\sqrt{cdx+ae}-\sqrt{2\sqrt{c}\,\sqrt{a}\,de+a\,e^2}\right.}{}$$

input
```
int((e*x+d)^3/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) -
 sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d +
e*x))*c**2*d**3 + sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sq
rt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*
sqrt(c)*sqrt(d + e*x))*c**2*d**3 - sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*
x)*log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(
c)*sqrt(a)*d*e + 2*c*d*e*x)*c**2*d**3 + 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e
 + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/
sqrt(a*e**2 - c*d**2))*a**2*e**3 - 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*
d*x)*a**2*e**3 + 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d**2*e -
2*sqrt(d + e*x)*a**2*c*d*e**3 + 2*sqrt(d + e*x)*a*c**2*d**3*e)/(sqrt(a*e +
 c*d*x)*a**2*c**2*d**2*e**2)
```

3.108 $\int \frac{(d+ex)^3}{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 176

$$\int \frac{(d+ex)^3}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2(cd^2-ae^2)\left(d+ex\right)}{a^2e^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{d\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{a^2e^2x}+\frac{3\sqrt{d}(cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{5/2}e^{5/2}}$$

output
```
-2*(-a*e^2+c*d^2)*(e*x+d)/a^2/e^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-
d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/e^2/x+3*d^(1/2)*(-a*e^2+c*d^
2)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2))/a^(5/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 10.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^3}{x^2 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{-\sqrt{a}\sqrt{e}(d+ex)\left(3cd^2x+ae(d-2ex)\right)+3\sqrt{d}(cd^2-ae^2)\,x\sqrt{}}{a^{5/2}e^{5/2}x\sqrt{(ae+cdx)(d+}}$$

input
```
Integrate[(d + e*x)^3/(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(-(Sqrt[a]*Sqrt[e]*(d + e*x)*(3*c*d^2*x + a*e*(d - 2*e*x))) + 3*Sqrt[d]*(c
*d^2 - a*e^2)*x*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e
+ c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(a^(5/2)*e^(5/2)*x*Sqrt[(a*e +
 c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1212, 25, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{x^2 \left(x(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 1212$$

$$e^2\left(-\int -\frac{d\left(ade-\left(cd^2-2ae^2\right)x\right)}{a^2e^4x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx\right)-\frac{2(d+ex)\left(cd^2-ae^2\right)}{a^2e^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \; 25$$

$$e^2\int \frac{d\left(ade-\left(cd^2-2ae^2\right)x\right)}{a^2e^4x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{2(d+ex)\left(cd^2-ae^2\right)}{a^2e^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{d\int \frac{ade-(cd^2-2ae^2)x}{x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{a^2e^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{a^2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1228}$$

$$\frac{d\left(-\frac{3}{2}\left(cd^2-ae^2\right)\int \frac{1}{x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}\right)}{a^2e^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{a^2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{d\left(3\left(cd^2-ae^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+(cd^2+ae^2)x}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}\right)}{a^2e^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{a^2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{3(cd^2-ae^2)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2\sqrt{a}\sqrt{d}\sqrt{e}}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{x}\right)}{a^2e^2}-\frac{2(d+ex)\left(cd^2-ae^2\right)}{a^2e^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^3/(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*(c*d^2 - a*e^2)*(d + e*x))/(a^2*e^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) + (d*(-(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/x) + (3*(c*
d^2 - a*e^2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt
[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*Sqrt[a]*Sqrt[d]*Sqrt
[e])))/(a^2*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1212
```
Int[((x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)
^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*d - b*e)^n*((d + e
*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*x^2])), x] - Simp[e^2/c^(m +
n - 1)   Int[ExpandToSum[(c^(m + n - 1)*(d + e*x)^(m - 1) - ((c*d - b*e)^n*
(2*c*d - b*e)^(m - 1))/(e^n*x^n))/(c*d - b*e - c*e*x), x]/(Sqrt[a + b*x + c
*x^2]/x^n), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^
2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(158) = 316$.

Time = 2.53 (sec) , antiderivative size = 785, normalized size of antiderivative = 4.46

method	result
default	$d^3\left(-\frac{1}{adex\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}-\frac{3(a\,e^2+c\,d^2)\left(\frac{1}{ade\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}-\frac{\left(a\,e^2+c\,d^2\right)\left(2cdxe+a\,e^2+\right.}{ade\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)\sqrt{ade+\left(a\right.}}\right.}{2ade}\right.$

input
```
int((e*x+d)^3/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
d^3*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/
a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e
*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-4*c/a*(2*c*d*e
*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2))+e^3*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^
2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*
d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+6*d*e^2*(2*c*d*e*x+a*e^2+c*d^2)/(4*a
*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+3*d^2*
e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*
c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*
d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))
```

Fricas [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.71

$$\int \frac{(d+ex)^3}{x^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \left[-\frac{3\left((c^2d^3-acde^2)x^2+(acd^2e-a^2e^3)x\right)\sqrt{\frac{d}{ae}}\log\left(\frac{8a^2d^2e^2+(c\ldots}{\ldots}\right)\ldots}{\ldots}, -\frac{3\left((c^2d^3-acde^2)x^2+(acd^2e-a^2e^3)x\right)\sqrt{-\frac{d}{ae}}\arctan\left(\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\,(2ade+(cd^2+ae^2)x)\sqrt{-\frac{d}{ae}}}{2\,(cd^2ex^2+ad^2e+(cd^3+ade^2)x)}\right)+2\sqrt{\ldots}}{2\,(a^2cde^2x^2+a^3e^3x)}\right]$$

input
```
integrate((e*x+d)^3/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[-1/4*(3*((c^2*d^3 - a*c*d*e^2)*x^2 + (a*c*d^2*e - a^2*e^3)*x)*sqrt(d/(a*e
))*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 8*(a*c*d
^3*e + a^2*d*e^3)*x - 4*(2*a^2*d*e^2 + (a*c*d^2*e + a^2*e^3)*x)*sqrt(c*d*e
*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(d/(a*e)))/x^2) + 4*sqrt(c*d*e*x^2 +
 a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (3*c*d^2 - 2*a*e^2)*x))/(a^2*c*d*e^2*
x^2 + a^3*e^3*x), -1/2*(3*((c^2*d^3 - a*c*d*e^2)*x^2 + (a*c*d^2*e - a^2*e^
3)*x)*sqrt(-d/(a*e))*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-d/(a*e))/(c*d^2*e*x^2 + a*d^2*e + (c
*d^3 + a*d*e^2)*x)) + 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e
 + (3*c*d^2 - 2*a*e^2)*x))/(a^2*c*d*e^2*x^2 + a^3*e^3*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^3}{x^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(d+ex)^3}{x^2\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/x**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**3/(x**2*((d + e*x)*(a*e + c*d*x))**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^3/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[1,1,0]%%%},[6,0]%%%}+%%%{%%{[%%%{-2,[0,0,1]%%%},0]:
[1,0,%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{x^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{(d+e\,x)^3}{x^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int((d + e*x)^3/(x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int((d + e*x)^3/(x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 841, normalized size of antiderivative = 4.78

$$\int \frac{(d+ex)^3}{x^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^3/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(3*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x)
 - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))*a**2*c*d*e**4*x + 6*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(
sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**3*e**2*x - 9*sqrt(e)*sqrt(d)*sq
rt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqr
t(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**3*d**5*x +
 3*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x)
 + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))*a**2*c*d*e**4*x + 6*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(
sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**3*e**2*x - 9*sqrt(e)*sqrt(d)*sq
rt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqr
t(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**3*d**5*x -
 3*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(2*sqrt(e)*sqrt(d)*sqrt(c)
*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*c*d*e*x)*a**2
*c*d*e**4*x - 6*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(2*sqrt(e)*sq
rt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(c)*sqrt(a)*d*e + 2*
c*d*e*x)*a*c**2*d**3*e**2*x + 9*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*
log(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(d + e*x)*sqrt(a*e + c*d*x) + 2*sqrt(...
```

3.109 $\int \frac{(d+ex)^3}{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 250

$$\int \frac{(d+ex)^3}{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2cd(cd^2-ae^2)\,(d+ex)}{a^3e^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{d\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2a^2e^2x^2}+\frac{\left(7cd^2-5ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4a^3e^3x}-\frac{3(cd^2-ae^2)\,(5cd^2-ae^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4a^{7/2}\sqrt{d}e^{7/2}}$$

output
```
2*c*d*(-a*e^2+c*d^2)*(e*x+d)/a^3/e^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)-1/2*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/e^2/x^2+1/4*(-5*a*e^2
+7*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/e^3/x-3/4*(-a*e^2+c*
d^2)*(-a*e^2+5*c*d^2)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(7/2)/d^(1/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 10.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^3}{x^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{-\sqrt{a}\sqrt{d}\sqrt{e}(d+ex)\left(-15c^2d^3x^2+a^2e^2(2d+5ex)+acdex(-5\right.}{4a^{7/2}\sqrt{}}$$

input
```
Integrate[(d + e*x)^3/(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(-(Sqrt[a]*Sqrt[d]*Sqrt[e]*(d + e*x)*(-15*c^2*d^3*x^2 + a^2*e^2*(2*d + 5*e
*x) + a*c*d*e*x*(-5*d + 13*e*x))) - 3*(5*c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4
)*x^2*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/
(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(4*a^(7/2)*Sqrt[d]*e^(7/2)*x^2*Sqrt[(a*e
 + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1212, 25, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{x^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1212}$$

$$\frac{2cd(d+ex)\left(cd^2-ae^2\right)}{a^3e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-cde^3\int -\frac{\frac{\left(cd^2-ae^2\right)^2x^2}{a^3cde^6}-\frac{\left(cd^2-2ae^2\right)x}{a^2ce^5}+\frac{d}{ace^4}}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx$$

$$\Big\downarrow \text{25}$$

$$cde^3\int \frac{\frac{\left(cd^2-ae^2\right)^2x^2}{a^3cde^6}-\frac{\left(cd^2-2ae^2\right)x}{a^2ce^5}+\frac{d}{ace^4}}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx+\frac{2cd(d+ex)\left(cd^2-ae^2\right)}{a^3e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 2181

$$cde^3\left(-\frac{\int \frac{d\left(\frac{7d^2}{a}-\frac{5e^2}{c}\right)-2\left(\frac{2cd^4}{a^2e^4}-\frac{5d^2}{ae^2}+\frac{2}{c}\right)e^3x}{2e^4x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2ade}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2a^2ce^5x^2}\right)+\frac{2cd(d+ex)\left(cd^2-ae^2\right)}{a^3e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 27

$$cde^3\left(-\frac{\int \frac{d\left(\frac{7d^2}{a}-\frac{5e^2}{c}\right)-2\left(\frac{2cd^4}{a^2e^4}-\frac{5d^2}{ae^2}+\frac{2}{c}\right)e^3x}{x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4ade^5}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2a^2ce^5x^2}\right)+\frac{2cd(d+ex)\left(cd^2-ae^2\right)}{a^3e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 1228

$$cde^3\left(-\frac{-\frac{3\left(cd^2-ae^2\right)\left(5cd^2-ae^2\right)\int\frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2a^2ce}-\frac{\left(7cd^2-5ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{a^2cex}}{4ade^5}-\frac{\sqrt{x\left(ae^2+cd^2\right)+a}}{2a^2ce^5x^2}\right.$$

$$\frac{2cd(d+ex)\left(cd^2-ae^2\right)}{a^3e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 1154

$$cde^3\left(-\frac{\frac{3\left(cd^2-ae^2\right)\left(5cd^2-ae^2\right)\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{a^2ce}-\frac{\left(7cd^2-5ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{a^2cex}}{4ade^5}-\right.$$

$$\frac{2cd(d+ex)\left(cd^2-ae^2\right)}{a^3e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 219

$$\frac{2cd(d+ex)\left(cd^2-ae^2\right)}{a^3e^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+ cde^3\left(-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2a^2ce^5x^2}-\frac{\frac{3\left(cd^2-ae^2\right)\left(5cd^2-ae^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{5/2}c\sqrt{d}e^{3/2}}-\frac{(7cd^2-5ae^2}{}}{4ade^5}\right.$$

input
```
Int[(d + e*x)^3/(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*c*d*(c*d^2 - a*e^2)*(d + e*x))/(a^3*e^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]) + c*d*e^3*(-1/2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/
(a^2*c*e^5*x^2) - (-(((7*c*d^2 - 5*a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2])/(a^2*c*e*x)) + (3*(c*d^2 - a*e^2)*(5*c*d^2 - a*e^2)*ArcTanh[(
2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(5/2)*c*Sqrt[d]*e^(3/2)))/(4*a*d*e^5))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1212
```
Int[((x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)
^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*d - b*e)^n*((d + e
*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*x^2])), x] - Simp[e^2/c^(m +
n - 1)   Int[ExpandToSum[(c^(m + n - 1)*(d + e*x)^(m - 1) - ((c*d - b*e)^n*
(2*c*d - b*e)^(m - 1))/(e^n*x^n))/(c*d - b*e - c*e*x), x]/(Sqrt[a + b*x + c
*x^2]/x^n), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^
2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1280 vs. $2(224) = 448$.

Time = 2.34 (sec) , antiderivative size = 1281, normalized size of antiderivative = 5.12

method	result	size
default	Expression too large to display	1281

input
```
int((e*x+d)^3/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
2*e^3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2)+d^3*(-1/2/a/d/e/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2)-5/4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c
*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-
(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(
1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2))/x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))-3/2*c/a*(1/a/d/e/(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2
)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-
1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))+3*d*e^2*(1/a/d/e/(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^
2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*
d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2))/x))+3*d^2*e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x
^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^
2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e
^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1...
```

Fricas [A] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 622, normalized size of antiderivative = 2.49

$$\int \frac{(d+ex)^3}{x^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \left[\frac{3\left(\left(5\,c^3d^5-6\,ac^2d^3e^2+a^2cde^4\right)x^3+\left(5\,ac^2d^4e-6\,a^2cd^2e^3+\right.\right.}{}\right.$$

input
```
integrate((e*x+d)^3/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[1/16*(3*((5*c^3*d^5 - 6*a*c^2*d^3*e^2 + a^2*c*d*e^4)*x^3 + (5*a*c^2*d^4*e
 - 6*a^2*c*d^2*e^3 + a^3*e^5)*x^2)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d
^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e
^3)*x)/x^2) - 4*(2*a^3*d^2*e^3 - (15*a*c^2*d^4*e - 13*a^2*c*d^2*e^3)*x^2 -
 5*(a^2*c*d^3*e^2 - a^3*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x))/(a^4*c*d^2*e^4*x^3 + a^5*d*e^5*x^2), 1/8*(3*((5*c^3*d^5 - 6*a*c^2*d^3
*e^2 + a^2*c*d*e^4)*x^3 + (5*a*c^2*d^4*e - 6*a^2*c*d^2*e^3 + a^3*e^5)*x^2)
*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*
d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^2 + (a*
c*d^3*e + a^2*d*e^3)*x)) - 2*(2*a^3*d^2*e^3 - (15*a*c^2*d^4*e - 13*a^2*c*d
^2*e^3)*x^2 - 5*(a^2*c*d^3*e^2 - a^3*d*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x))/(a^4*c*d^2*e^4*x^3 + a^5*d*e^5*x^2)]
```

Sympy [F]

$$\int \frac{(d+ex)^3}{x^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{(d+ex)^3}{x^3\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/x**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**3/(x**3*((d + e*x)*(a*e + c*d*x))**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{x^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{x^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^3/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[1,1,11]%%%},[2,7]%%%}+%%%{%%%{-4,[2,3,9]%%%},[2,6]%
%%}+%%%{%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{x^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{(d+e\,x)^3}{x^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int((d + e*x)^3/(x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int((d + e*x)^3/(x^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 1140, normalized size of antiderivative = 4.56

$$\int \frac{(d+ex)^3}{x^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^3/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(9*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x)
 - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))*a**3*e**6*x**2 - 39*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(
sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**2*e**4*x**2 - 45*sqrt(e)*sqrt(d
)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)
*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d*
*4*e**2*x**2 + 75*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sq
rt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*
sqrt(c)*sqrt(d + e*x))*c**3*d**6*x**2 + 9*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e
 + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e
**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*e**6*x**2 - 39*sqrt(e)
*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*
sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*
*2*c*d**2*e**4*x**2 - 45*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqr
t(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + s
qrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**4*e**2*x**2 + 75*sqrt(e)*sqrt(d)*s
qrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sq
rt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**3*d**6*x*
*2 - 9*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(2*sqrt(e)*sqrt(d)*...
```

3.110 $\int \frac{(d+ex)^3}{x^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 340

$$\int \frac{(d+ex)^3}{x^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2c^2d^2(cd^2-ae^2)\,(d+ex)}{a^4e^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{d\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3a^2e^2x^3}+\frac{(11cd^2-7ae^2)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{12a^3e^3x^2}-\frac{(57c^2d^4-52acd^2e^2+3a^2e^4)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24a^4de^4x}+\frac{(cd^2-ae^2)\,(35c^2d^4-10acd^2e^2-a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8a^{9/2}d^{3/2}e^{9/2}}$$

output
```
-2*c^2*d^2*(-a*e^2+c*d^2)*(e*x+d)/a^4/e^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)-1/3*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/e^2/x^3+1/12*(-7
*a*e^2+11*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/e^3/x^2-1/24*
(3*a^2*e^4-52*a*c*d^2*e^2+57*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2)/a^4/d/e^4/x+1/8*(-a*e^2+c*d^2)*(-a^2*e^4-10*a*c*d^2*e^2+35*c^2*d^4)*ar
ctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1
/2))/a^(9/2)/d^(3/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 10.21 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^3}{x^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{-\sqrt{a}\sqrt{d}\sqrt{e}(d+ex)\left(105c^3d^5x^3+5ac^2d^3ex^2(7d-20ex)+a^2c\right.}{}$$

input
```
Integrate[(d + e*x)^3/(x^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(-(Sqrt[a]*Sqrt[d]*Sqrt[e]*(d + e*x)*(105*c^3*d^5*x^3 + 5*a*c^2*d^3*e*x^2*
(7*d - 20*e*x) + a^2*c*d*e^2*x*(-14*d^2 - 38*d*e*x + 3*e^2*x^2) + a^3*e^3*
(8*d^2 + 14*d*e*x + 3*e^2*x^2))) + 3*(35*c^3*d^6 - 45*a*c^2*d^4*e^2 + 9*a^
2*c*d^2*e^4 + a^3*e^6)*x^3*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d
]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(24*a^(9/2)*d^(3/2)
*e^(9/2)*x^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {1212, 25, 2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{x^4\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1212}$$

$$-c^2d^2e^4\int -\frac{-\frac{\left(cd^2-ae^2\right)^2x^3}{a^4cde^8}+\frac{\left(cd^2-ae^2\right)^2x^2}{a^3c^2d^2e^7}-\frac{\left(cd^2-2ae^2\right)x}{a^2c^2de^6}+\frac{1}{ac^2e^5}}{x^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx-\frac{2c^2d^2(d+ex)\left(cd^2-ae^2\right)}{a^4e^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 25}$$

$$c^2d^2e^4\int \frac{-\frac{(cd^2-ae^2)^2x^3}{a^4cde^8}+\frac{(cd^2-ae^2)^2x^2}{a^3c^2d^2e^7}-\frac{(cd^2-2ae^2)x}{a^2c^2de^6}+\frac{1}{ac^2e^5}}{x^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{2c^2d^2(d+ex)(cd^2-ae^2)}{a^4e^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 2181

$$c^2d^2e^4\left(-\frac{\int \frac{\frac{6(cd^2-ae^2)^2x^2}{a^3ce^7}-\frac{2(3c^2d^4-8ace^2d^2+3a^2e^4)x}{a^2c^2de^6}+\frac{11cd^2-7ae^2}{ac^2e^5}}{2x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3a^2c^2de^6x^3}\right)-\frac{2c^2d^2(d+ex)(cd^2-ae^2)}{a^4e^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 27

$$c^2d^2e^4\left(-\frac{\int \frac{\frac{6(cd^2-ae^2)^2x^2}{a^3ce^7}-\frac{2(3c^2d^4-8ace^2d^2+3a^2e^4)x}{a^2c^2de^6}+\frac{11cd^2-7ae^2}{ac^2e^5}}{x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{6ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3a^2c^2de^6x^3}\right)-\frac{2c^2d^2(d+ex)(cd^2-ae^2)}{a^4e^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 2181

$$c^2d^2e^4\left(-\frac{-\frac{\int \frac{\frac{57d^4}{a}-\frac{52e^2d^2}{c}-2\left(\frac{12cd^4}{a^2e^4}-\frac{35d^2}{ae^2}+\frac{19}{c}\right)e^3xd+\frac{3ae^4}{c^2}}{2e^5x^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{2ade}-\frac{(11cd^2-7ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2a^2c^2de^6x^2}}{6ade}-\frac{\sqrt{x(ae^2+cd^2)+ad}}{3a^2c^2de^6x^3}\right.$$

$$-\frac{2c^2d^2(d+ex)(cd^2-ae^2)}{a^4e^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 27

$$c^2d^2e^4\left(-\frac{-\frac{\int \frac{\frac{57d^4}{a}-\frac{52e^2d^2}{c}-2\left(\frac{12cd^4}{a^2e^4}-\frac{35d^2}{ae^2}+\frac{19}{c}\right)e^3xd+\frac{3ae^4}{c^2}}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4ade^6}-\frac{\left(11cd^2-7ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2a^2c^2de^6x^2}}{6ade}-\frac{\sqrt{x\left(ae^2+cd^2\right)+ac}}{3a^2c^2de^6x^3}\right.$$

$$\frac{2c^2d^2(d+ex)\left(cd^2-ae^2\right)}{a^4e^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1228$$

$$c^2d^2e^4\left(-\frac{-\frac{-\frac{3\left(cd^2-ae^2\right)\left(-a^2e^4-10acd^2e^2+35c^2d^4\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2a^2c^2de}-\frac{\left(3a^2e^4-52acd^2e^2+57c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{a^2c^2dex}}{4ade^6}-}{6ade}\right.$$

$$\frac{2c^2d^2(d+ex)\left(cd^2-ae^2\right)}{a^4e^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1154$$

$$c^2d^2e^4\left(-\frac{-\frac{\frac{3\left(cd^2-ae^2\right)\left(-a^2e^4-10acd^2e^2+35c^2d^4\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{a^2c^2de}-\frac{\left(3a^2e^4-52acd^2e^2+57c^2d^4\right)\sqrt{x}}{a^2c^2dex}}{4ade^6}}{6ade}\right.$$

$$\frac{2c^2d^2(d+ex)\left(cd^2-ae^2\right)}{a^4e^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 219$$

$$c^2d^2e^4\left(-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{3a^2c^2de^6x^3}-\frac{-\frac{\left(11cd^2-7ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2a^2c^2de^6x^2}-\frac{3\left(cd^2-ae^2\right)\left(-a^2e^4-10acd^2e^2+35c^2d^4\right)}{2a^5}}{}\right.$$

$$\frac{2c^2d^2(d+ex)\left(cd^2-ae^2\right)}{a^4e^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[(d + e*x)^3/(x^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*c^2*d^2*(c*d^2 - a*e^2)*(d + e*x))/(a^4*e^4*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2]) + c^2*d^2*e^4*(-1/3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]/(a^2*c^2*d*e^6*x^3) - (-1/2*((11*c*d^2 - 7*a*e^2)*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])/(a^2*c^2*d*e^6*x^2) - (-(((57*c^2*d^4 - 52*a
*c*d^2*e^2 + 3*a^2*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a^2*
c^2*d*e*x)) + (3*(c*d^2 - a*e^2)*(35*c^2*d^4 - 10*a*c*d^2*e^2 - a^2*e^4)*A
rcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(5/2)*c^2*d^(3/2)*e^(3/2)))/(4*a
*d*e^6))/(6*a*d*e))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1212
```
Int[((x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)
^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*d - b*e)^n*((d + e
*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*x^2])), x] - Simp[e^2/c^(m +
n - 1)   Int[ExpandToSum[(c^(m + n - 1)*(d + e*x)^(m - 1) - ((c*d - b*e)^n*
(2*c*d - b*e)^(m - 1))/(e^n*x^n))/(c*d - b*e - c*e*x), x]/(Sqrt[a + b*x + c
*x^2]/x^n), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^
2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2255 vs. $2(310) = 620$.

Time = 2.85 (sec) , antiderivative size = 2256, normalized size of antiderivative = 6.64

method	result	size
default	Expression too large to display	2256

input
```
int((e*x+d)^3/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
d^3*(-1/3/a/d/e/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-7/6*(a*e^2+c*d
^2)/a/d/e*(-1/2/a/d/e/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-5/4*(a*e
^2+c*d^2)/a/d/e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a
*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+
c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^
2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-4*
c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*x^2*e)^(1/2))-3/2*c/a*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^
2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*
ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2))/x)))-4/3*c/a*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^
2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d
*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
)/x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))+e^3*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*
d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e...
```

Fricas [A] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 792, normalized size of antiderivative = 2.33

$$\int \frac{(d+ex)^3}{x^4 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[1/96*(3*((35*c^4*d^7 - 45*a*c^3*d^5*e^2 + 9*a^2*c^2*d^3*e^4 + a^3*c*d*e^6
)*x^4 + (35*a*c^3*d^6*e - 45*a^2*c^2*d^4*e^3 + 9*a^3*c*d^2*e^5 + a^4*e^7)*
x^3)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*
x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*
e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(8*a^4*d^3*e^4
 + (105*a*c^3*d^6*e - 100*a^2*c^2*d^4*e^3 + 3*a^3*c*d^2*e^5)*x^3 + (35*a^2
*c^2*d^5*e^2 - 38*a^3*c*d^3*e^4 + 3*a^4*d*e^6)*x^2 - 14*(a^3*c*d^4*e^3 - a
^4*d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^5*c*d^3*e^5
*x^4 + a^6*d^2*e^6*x^3), -1/48*(3*((35*c^4*d^7 - 45*a*c^3*d^5*e^2 + 9*a^2*
c^2*d^3*e^4 + a^3*c*d*e^6)*x^4 + (35*a*c^3*d^6*e - 45*a^2*c^2*d^4*e^3 + 9*
a^3*c*d^2*e^5 + a^4*e^7)*x^3)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^
2*e^2*x^2 + a^2*d^2*e^2 + (a*c*d^3*e + a^2*d*e^3)*x)) + 2*(8*a^4*d^3*e^4 +
 (105*a*c^3*d^6*e - 100*a^2*c^2*d^4*e^3 + 3*a^3*c*d^2*e^5)*x^3 + (35*a^2*c
^2*d^5*e^2 - 38*a^3*c*d^3*e^4 + 3*a^4*d*e^6)*x^2 - 14*(a^3*c*d^4*e^3 - a^4
*d^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^5*c*d^3*e^5*x
^4 + a^6*d^2*e^6*x^3)]
```

Sympy [F]

$$\int \frac{(d+ex)^3}{x^4\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{(d+ex)^3}{x^4\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/x**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**3/(x**4*((d + e*x)*(a*e + c*d*x))**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{x^4\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{x^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^3/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[1,1,14]%%%},[2,9]%%%}+%%%{%%%{-5,[2,3,12]%%%},[2,8]
%%%}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{x^4\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{\left(d+e\,x\right)^3}{x^4\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

input
```
int((d + e*x)^3/(x^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int((d + e*x)^3/(x^4*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 1509, normalized size of antiderivative = 4.44

$$\int \frac{(d+ex)^3}{x^4\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^3/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 15*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*
d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqr
t(d + e*x))*a**4*e**8*x**3 - 156*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)
*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d
**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**2*e**6*x**3 + 486*sqrt(e)*
sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*s
qrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**
2*c**2*d**4*e**4*x**3 + 420*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(
sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2)
+ sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**3*d**6*e**2*x**3 - 735*sqrt(e)*sqrt(
d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c
)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*c**4*d**
8*x**3 - 15*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e
 + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c
)*sqrt(d + e*x))*a**4*e**8*x**3 - 156*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c
*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2
+ c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**2*e**6*x**3 + 486*sqr
t(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqr
t(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x)
)*a**2*c**2*d**4*e**4*x**3 + 420*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d...
```

3.111 $\int \frac{(d+ex)^3}{x^5\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 439

$$\int \frac{(d+ex)^3}{x^5\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{2c^3d^3(cd^2-ae^2)(d+ex)}{a^5e^5\sqrt{ade+(cd^2+ae^2)x+cdex^2}} - \frac{d\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{4a^2e^2x^4} + \frac{(5cd^2-3ae^2)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{8a^3e^3x^3} - \frac{(41c^2d^4-34acd^2e^2+a^2e^4)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{32a^4de^4x^2} + \frac{(187c^3d^6-187ac^2d^4e^2+13a^2cd^2e^4+3a^3e^6)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{64a^5d^2e^5x} - \frac{3(cd^2-ae^2)(105c^3d^6-35ac^2d^4e^2-5a^2cd^2e^4-a^3e^6)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{64a^{11/2}d^{5/2}e^{11/2}}$$

output
```
2*c^3*d^3*(-a*e^2+c*d^2)*(e*x+d)/a^5/e^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)-1/4*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/e^2/x^4+1/8*(-3*a
*e^2+5*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/e^3/x^3-1/32*(a^
2*e^4-34*a*c*d^2*e^2+41*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a
^4/d/e^4/x^2+1/64*(3*a^3*e^6+13*a^2*c*d^2*e^4-187*a*c^2*d^4*e^2+187*c^3*d^
6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^5/d^2/e^5/x-3/64*(-a*e^2+c*d^
2)*(-a^3*e^6-5*a^2*c*d^2*e^4-35*a*c^2*d^4*e^2+105*c^3*d^6)*arctanh(a^(1/2)
*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(11/2)
/d^(5/2)/e^(11/2)
```

Mathematica [A] (verified)

Time = 10.28 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)^3}{x^5 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{d}\sqrt{e}(d+ex)\left(315c^4d^7x^4+105ac^3d^5ex^3(d-3ex)+a^2c^2\right.}{}$$

input
```
Integrate[(d + e*x)^3/(x^5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(Sqrt[a]*Sqrt[d]*Sqrt[e]*(d + e*x)*(315*c^4*d^7*x^4 + 105*a*c^3*d^5*e*x^3*
(d - 3*e*x) + a^2*c^2*d^3*e^2*x^2*(-42*d^2 - 119*d*e*x + 13*e^2*x^2) + a^4
*e^4*(-16*d^3 - 24*d^2*e*x - 2*d*e^2*x^2 + 3*e^3*x^3) + a^3*c*d*e^3*x*(24*
d^3 + 44*d^2*e*x + 11*d*e^2*x^2 + 3*e^3*x^3)) - 3*(105*c^4*d^8 - 140*a*c^3
*d^6*e^2 + 30*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)*x^4*Sqrt[a*e +
c*d*x]*Sqrt[d + e*x]*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(Sqrt[a]*Sqrt[e]*
Sqrt[d + e*x])])/(64*a^(11/2)*d^(5/2)*e^(11/2)*x^4*Sqrt[(a*e + c*d*x)*(d +
 e*x)])
```

Rubi [A] (verified)

Time = 3.74 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {1212, 25, 2181, 27, 2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{x^5 (x(ae^2+cd^2)+ade+cdex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1212}$$

$$\frac{2c^3d^3(d+ex)(cd^2-ae^2)}{a^5e^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - c^3d^3e^5\int -\frac{\frac{(cd^2-ae^2)^2x^4}{a^5cde^{10}} - \frac{(cd^2-ae^2)^2x^3}{a^4c^2d^2e^9} + \frac{(cd^2-ae^2)^2x^2}{a^3c^3d^3e^8} - \frac{(cd^2-2ae^2)x}{a^2c^3d^2e^7} + \frac{1}{ac^3de^6}}{x^5\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx$$

$\big\downarrow$ 25

$$c^3d^3e^5\int \frac{\frac{(cd^2-ae^2)^2x^4}{a^5cde^{10}}-\frac{(cd^2-ae^2)^2x^3}{a^4c^2d^2e^9}+\frac{(cd^2-ae^2)^2x^2}{a^3c^3d^3e^8}-\frac{(cd^2-2ae^2)x}{a^2c^3d^2e^7}+\frac{1}{ac^3de^6}}{x^5\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx+\frac{2c^3d^3(d+ex)(cd^2-ae^2)}{a^5e^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 2181

$$c^3d^3e^5\left(-\frac{\int \frac{-\frac{8(cd^2-ae^2)^2x^3}{a^4ce^9}+\frac{8(cd^2-ae^2)^2x^2}{a^3c^2de^8}-\frac{2(4c^2d^4-11acе^2d^2+4a^2e^4)x}{a^2c^3d^2e^7}+\frac{3(5cd^2-3ae^2)}{ac^3de^6}}{2x^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{4ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4a^2c^3d^2e^7x^4}\right)+\frac{2c^3d^3(d+ex)(cd^2-ae^2)}{a^5e^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 27

$$c^3d^3e^5\left(-\frac{\int \frac{-\frac{8(cd^2-ae^2)^2x^3}{a^4ce^9}+\frac{8(cd^2-ae^2)^2x^2}{a^3c^2de^8}-\frac{2(4c^2d^4-11ace^2d^2+4a^2e^4)x}{a^2c^3d^2e^7}+\frac{3(5cd^2-3ae^2)}{ac^3de^6}}{x^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{8ade}-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4a^2c^3d^2e^7x^4}\right)+\frac{2c^3d^3(d+ex)(cd^2-ae^2)}{a^5e^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 2181

$$c^3d^3e^5\left(-\frac{-\frac{\int \frac{3\left(\frac{16d(cd^2-ae^2)^2x^2}{a^3ce^8}-\frac{4(4c^2d^4-13ace^2d^2+7a^2e^4)x}{a^2c^2e^7}+\frac{41c^2d^4-34ace^2d^2+a^2e^4}{ac^3de^6}\right)}{2x^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3ade}-\frac{(5cd^2-3ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{a^2c^3d^2e^7x^3}}{8ade}-\cdots\right)+\frac{2c^3d^3(d+ex)(cd^2-ae^2)}{a^5e^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\big\downarrow$ 27

$$c^3d^3e^5\left(-\frac{-\frac{\int \frac{\frac{16d\left(cd^2-ae^2\right)^2x^2}{a^3ce^8}-\frac{4\left(4c^2d^4-13ace^2d^2+7a^2e^4\right)x}{a^2c^2e^7}+\frac{41c^2d^4-34ace^2d^2+a^2e^4}{ac^3de^6}}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(5cd^2-3ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{a^2c^3d^2e^7x^3}}{8ade}-\sqrt{\cdots}\right.$$

$$\frac{2c^3d^3(d+ex)\left(cd^2-ae^2\right)}{a^5e^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 2181$$

$$c^3d^3e^5\left(-\frac{-\frac{-\frac{\int \frac{\left(\frac{187d^6}{ae^6}-\frac{187d^4}{ce^4}+\frac{13ad^2}{c^2e^2}+\frac{3a^2}{c^3}\right)e+2d\left(-\frac{32cd^6}{a^2e^6}+\frac{105d^4}{ae^4}-\frac{66d^2}{ce^2}+\frac{a}{c^2}\right)x}{2dex^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(a^2e^4-34acd^2e^2+41c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2a^2c^3d^2e^7x^2}}{2ade}-\frac{(\cdots}{\cdots}}{8ade}\right.$$

$$\frac{2c^3d^3(d+ex)\left(cd^2-ae^2\right)}{a^5e^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 27$$

$$c^3d^3e^5\left(-\frac{-\frac{-\frac{\int \frac{\frac{187d^6}{ae^5}-\frac{187d^4}{ce^3}+\frac{13ad^2}{c^2e}+2\left(-\frac{32cd^6}{a^2e^6}+\frac{105d^4}{ae^4}-\frac{66d^2}{ce^2}+\frac{a}{c^2}\right)xd+\frac{3a^2e}{c^3}}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4ad^2e^2}-\frac{\left(a^2e^4-34acd^2e^2+41c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{2a^2c^3d^2e^7x^2}}{2ade}-\frac{(5c\cdots}{\cdots}}{8ade}\right.$$

$$\frac{2c^3d^3(d+ex)\left(cd^2-ae^2\right)}{a^5e^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1228$$

$$c^3d^3e^5\left(-\frac{-\frac{-\frac{-\frac{3\left(cd^2-ae^2\right)\left(-a^3e^6-5a^2cd^2e^4-35ac^2d^4e^2+105c^3d^6\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2a^2c^3de^6}-\frac{\left(3a^3e^6+13a^2cd^2e^4-187ac^2d^4e^2+187c^3d^6\right)\sqrt{\cdots}}{a^2c^3de^6x}}{4ad^2e^2}\cdots}{2ade}\cdots}{8ade}\right.$$

$$\frac{2c^3d^3(d+ex)\left(cd^2-ae^2\right)}{a^5e^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1154}$$

$$c^3d^3e^5\left(-\frac{-\frac{-\frac{3\left(cd^2-ae^2\right)\left(-a^3e^6-5a^2cd^2e^4-35ac^2d^4e^2+105c^3d^6\right)\int\frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{a^2c^3de^6}-\frac{\left(3a^3e^6+13a^2cd^2\cdots\right.}{\cdots}}{4ad^2e^2}\cdots}{2ade}\cdots\right.$$

$$\frac{2c^3d^3(d+ex)\left(cd^2-ae^2\right)}{a^5e^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2c^3d^3(d+ex)\left(cd^2-ae^2\right)}{a^5e^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+$$

$$c^3d^3e^5\left(-\frac{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{4a^2c^3d^2e^7x^4}-\frac{-\frac{\left(5cd^2-3ae^2\right)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{a^2c^3d^2e^7x^3}-\frac{-\frac{\left(a^2e^4-34acd^2e^2+41c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)\cdots}}{2a^2c^3d^2e^7x^2}\cdots}{\cdots}}{\cdots}\right.$$

input

```
Int[(d + e*x)^3/(x^5*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output

```
(2*c^3*d^3*(c*d^2 - a*e^2)*(d + e*x))/(a^5*e^5*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) + c^3*d^3*e^5*(-1/4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]/(a^2*c^3*d^2*e^7*x^4) - (-(((5*c*d^2 - 3*a*e^2)*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])/(a^2*c^3*d^2*e^7*x^3)) - (-1/2*((41*c^2*d^4 - 3
4*a*c*d^2*e^2 + a^2*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a^2
*c^3*d^2*e^7*x^2) - (-(((187*c^3*d^6 - 187*a*c^2*d^4*e^2 + 13*a^2*c*d^2*e^
4 + 3*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a^2*c^3*d*e^6
*x)) + (3*(c*d^2 - a*e^2)*(105*c^3*d^6 - 35*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^
4 - a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt
[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(5/2)*c^3*d^(3/2)*
e^(13/2)))/(4*a*d^2*e^2))/(2*a*d*e))/(8*a*d*e))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1212
```
Int[((x_)^(n_.)*((d_.) + (e_.)*(x_))^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)
^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*d - b*e)^n*((d + e
*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*x^2])), x] - Simp[e^2/c^(m +
n - 1)   Int[ExpandToSum[(c^(m + n - 1)*(d + e*x)^(m - 1) - ((c*d - b*e)^n*
(2*c*d - b*e)^(m - 1))/(e^n*x^n))/(c*d - b*e - c*e*x), x]/(Sqrt[a + b*x + c
*x^2]/x^n), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^
2, 0] && IGtQ[m, 0] && ILtQ[n, 0]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3793 vs. $2(405) = 810$.

Time = 2.80 (sec) , antiderivative size = 3794, normalized size of antiderivative = 8.64

method	result	size
default	Expression too large to display	3794

input
```
int((e*x+d)^3/x^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
d^3*(-1/4/a/d/e/x^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-9/8*(a*e^2+c*d
^2)/a/d/e*(-1/3/a/d/e/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-7/6*(a*e
^2+c*d^2)/a/d/e*(-1/2/a/d/e/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-5/
4*(a*e^2+c*d^2)/a/d/e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-
3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(
a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)
/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e
+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/
x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))-3/2*c/a*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c
*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2
-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^
(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*x^2*e)^(1/2))/x)))-4/3*c/a*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)
^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c
*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln(
(2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)
^(1/2))/x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))-5/4*c/a*(-1/2/a/d/e/x^2/(a*d*...
```

Fricas [A] (verification not implemented)

Time = 18.06 (sec) , antiderivative size = 992, normalized size of antiderivative = 2.26

$$\int \frac{(d+ex)^3}{x^5 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3/x^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[1/256*(3*((105*c^5*d^9 - 140*a*c^4*d^7*e^2 + 30*a^2*c^3*d^5*e^4 + 4*a^3*c
^2*d^3*e^6 + a^4*c*d*e^8)*x^5 + (105*a*c^4*d^8*e - 140*a^2*c^3*d^6*e^3 + 3
0*a^3*c^2*d^4*e^5 + 4*a^4*c*d^2*e^7 + a^5*e^9)*x^4)*sqrt(a*d*e)*log((8*a^2
*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*
c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(16*a^5*d^4*e^5 - (315*a*c^4*d^8*e - 315*
a^2*c^3*d^6*e^3 + 13*a^3*c^2*d^4*e^5 + 3*a^4*c*d^2*e^7)*x^4 - (105*a^2*c^3
*d^7*e^2 - 119*a^3*c^2*d^5*e^4 + 11*a^4*c*d^3*e^6 + 3*a^5*d*e^8)*x^3 + 2*(
21*a^3*c^2*d^6*e^3 - 22*a^4*c*d^4*e^5 + a^5*d^2*e^7)*x^2 - 24*(a^4*c*d^5*e
^4 - a^5*d^3*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^6*c*d
^4*e^6*x^5 + a^7*d^3*e^7*x^4), 1/128*(3*((105*c^5*d^9 - 140*a*c^4*d^7*e^2
+ 30*a^2*c^3*d^5*e^4 + 4*a^3*c^2*d^3*e^6 + a^4*c*d*e^8)*x^5 + (105*a*c^4*d
^8*e - 140*a^2*c^3*d^6*e^3 + 30*a^3*c^2*d^4*e^5 + 4*a^4*c*d^2*e^7 + a^5*e^
9)*x^4)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(a*c*d^2*e^2*x^2 + a^2*d^2*e^
2 + (a*c*d^3*e + a^2*d*e^3)*x)) - 2*(16*a^5*d^4*e^5 - (315*a*c^4*d^8*e - 3
15*a^2*c^3*d^6*e^3 + 13*a^3*c^2*d^4*e^5 + 3*a^4*c*d^2*e^7)*x^4 - (105*a^2*
c^3*d^7*e^2 - 119*a^3*c^2*d^5*e^4 + 11*a^4*c*d^3*e^6 + 3*a^5*d*e^8)*x^3 +
2*(21*a^3*c^2*d^6*e^3 - 22*a^4*c*d^4*e^5 + a^5*d^2*e^7)*x^2 - 24*(a^4*c*d^
5*e^4 - a^5*d^3*e^6)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a...
```

Sympy [F]

$$\int \frac{(d+ex)^3}{x^5 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{(d+ex)^3}{x^5 ((d+ex)(ae+cdx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/x**5/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral((d + e*x)**3/(x**5*((d + e*x)*(a*e + c*d*x))**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{x^5\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3/x^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3}{x^5\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+d)^3/x^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[1,1,17]%%%},[2,11]%%%}+%%%{%%%{-6,[2,3,15]%%%},[2,1
0]%%%}+%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{x^5\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{(d+e\,x)^3}{x^5\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int((d + e*x)^3/(x^5*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int((d + e*x)^3/(x^5*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 8.45 (sec) , antiderivative size = 1901, normalized size of antiderivative = 4.33

$$\int \frac{(d+ex)^3}{x^5\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^3/x^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(21*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x
) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
 + e*x))*a**5*e**10*x**4 + 111*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*l
og(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**
2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**4*c*d**2*e**8*x**4 + 738*sqrt(e)*sq
rt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqr
t(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*
c**2*d**4*e**6*x**4 - 2130*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(s
qrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c**3*d**6*e**4*x**4 - 1575*sqrt(e)*sq
rt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqr
t(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**
4*d**8*e**2*x**4 + 2835*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt
(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))*c**5*d**10*x**4 + 21*sqrt(e)*sqrt(d)*sqrt(a)*
sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(2*sqrt(c)*sqrt(a)*d
*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**5*e**10*x**4 + 1
11*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x)
 + sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))*a**4*c*d**2*e**8*x**4 + 738*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + ...
```

3.112 $\int \frac{x^5}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 435

$$\int \frac{x^5}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2a^5e^5}{c^5d^5\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$-\frac{\left(13cd^2+7ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{4c^3d^3e^4}$$
$$+\frac{2(c^5d^{10}+3a^5e^{10})\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^5d^5e^4\left(cd^2-ae^2\right)^2(d+ex)^2}$$
$$-\frac{2(10c^5d^{10}-15ac^4d^8e^2-3a^5e^{10})\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^4d^4e^4\left(cd^2-ae^2\right)^3(d+ex)}$$
$$+\frac{(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{2c^2d^2e^4}$$
$$+\frac{5(7c^2d^4+6acd^2e^2+3a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4c^{7/2}d^{7/2}e^{9/2}}$$

output
```
2*a^5*e^5/c^5/d^5/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)-1/4*(7*a*e^2+13*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/
d^3/e^4+2/3*(3*a^5*e^10+c^5*d^10)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/
c^5/d^5/e^4/(-a*e^2+c*d^2)^2/(e*x+d)^2-2/3*(-3*a^5*e^10-15*a*c^4*d^8*e^2+1
0*c^5*d^10)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^4/(-a*e^2+c*
d^2)^3/(e*x+d)+1/2*(e*x+d)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2
/e^4+5/4*(3*a^2*e^4+6*a*c*d^2*e^2+7*c^2*d^4)*arctanh(c^(1/2)*d^(1/2)*(e*x+
d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(9/2
)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{-\frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)\left(-45a^5e^9(d+ex)^2+15a^4cde^7(2d-ex)(d+ex)^2+6a^3c^2d\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[x^5/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-45*a^5*e^9*(d + e*x)^2 + 15*a^
4*c*d*e^7*(2*d - e*x)*(d + e*x)^2 + 6*a^3*c^2*d^2*e^5*(d + e*x)^2*(6*d^2 +
 2*d*e*x + e^2*x^2) + c^5*d^8*x*(105*d^3 + 140*d^2*e*x + 21*d*e^2*x^2 - 6*
e^3*x^3) - 2*a^2*c^3*d^4*e^3*(95*d^4 + 111*d^3*e*x - 6*d^2*e^2*x^2 - 9*d*e
^3*x^3 + 9*e^4*x^4) + a*c^4*d^6*e*(105*d^4 - 50*d^3*e*x - 237*d^2*e^2*x^2
- 48*d*e^3*x^3 + 18*e^4*x^4)))/(c*d^2 - a*e^2)^3) + 15*(7*c^2*d^4 + 6*a*c*
d^2*e^2 + 3*a^2*e^4)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTanh[(Sqrt[c]*
Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(12*c^(7/2)*d^(7/2)*e
^(9/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1244, 27, 1233, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 1244$$

$$-\frac{2\int -\frac{e^2x^3\left(8ade+\left(7cd^2-3ae^2\right)x\right)}{2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3e^3\left(cd^2-ae^2\right)}-\frac{2dx^4}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{x^3\left(8ade+\left(7cd^2-3ae^2\right)x\right)}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3e\left(cd^2-ae^2\right)}-\frac{2dx^4}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1233$$

$$\frac{\frac{2\int \frac{x\left(4ade\left(7c^2d^4-12ace^2d^2-3a^2e^4\right)+\left(35c^3d^6-61ac^2e^2d^4+9a^2ce^4d^2-15a^3e^6\right)x\right)}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cde\left(cd^2-ae^2\right)^2}-\frac{2x^2\left(ade\left(-3a^2e^4-12acd^2e^2+7c^2d^4\right)+x\left(-3a^3e^6-a^2cd^2e^4-\right)\right)}{cde\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex}}}{3e\left(cd^2-ae^2\right)}$$
$$\frac{2dx^4}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{x\left(4ade\left(7c^2d^4-12ace^2d^2-3a^2e^4\right)+\left(35c^3d^6-61ac^2e^2d^4+9a^2ce^4d^2-15a^3e^6\right)x\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cde\left(cd^2-ae^2\right)^2}-\frac{2x^2\left(ade\left(-3a^2e^4-12acd^2e^2+7c^2d^4\right)+x\left(-3a^3e^6-a^2cd^2e^4-1\right)\right)}{cde\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}}{3e\left(cd^2-ae^2\right)}$$
$$\frac{2dx^4}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1225$$

$$\frac{\frac{\frac{15\left(cd^2-ae^2\right)^3\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{8c^2d^2e^2}-\frac{\left(-45a^4e^8+30a^3cd^2e^6+36a^2c^2d^4e^4-2cdex\left(-15a^3e^6+9a^2cd^2e^4-61ac^2d^4e^2+35c\right.\right.}{4c^2d^2e^2}}{cde(cd^2-ae^2)^2}}{3e\left(cd^2-\right.}$$

$$\frac{2dx^4}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1092

$$\frac{\frac{15\left(cd^2-ae^2\right)^3\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{4c^2d^2e^2}-\frac{\left(-45a^4e^8+30a^3cd^2e^6+36a^2c^2d^4e^4-2cdex\left(-15a\right.\right.}{}}{cde(cd^2-ae^2)^2}$$

$$\frac{2dx^4}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{15\left(cd^2-ae^2\right)^3\left(3a^2e^4+6acd^2e^2+7c^2d^4\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{5/2}}-\frac{\left(-45a^4e^8+30a^3cd^2e^6+36a^2c^2d^4e^4-2cdex\left(-15a^3e^6+9a^2cd^2\right.\right.}{}}{cde(cd^2-ae^2)^2}}{3e}$$

$$\frac{2dx^4}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[x^5/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*d*x^4)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ((-2*x^2*(a*d*e*(7*c^2*d^4 - 12*a*c*d^2*e^2 - 3*a^2*e^4) +
(7*c^3*d^6 - 11*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(c*d*e*(c*d
^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (-1/4*((105*c
^4*d^8 - 190*a*c^3*d^6*e^2 + 36*a^2*c^2*d^4*e^4 + 30*a^3*c*d^2*e^6 - 45*a^
4*e^8 - 2*c*d*e*(35*c^3*d^6 - 61*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 15*a^3*
e^6)*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*e^2) + (15*(
c*d^2 - a*e^2)^3*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTanh[(c*d^2 +
a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)
*x + c*d*e*x^2])])/(8*c^(5/2)*d^(5/2)*e^(5/2)))/(c*d*e*(c*d^2 - a*e^2)^2))
/(3*e*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1244
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d
_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(e*f - d*g))*(f + g*x)^(n - 1)*((a +
b*x + c*x^2)^(p + 1)/(p*(2*c*d - b*e)*(d + e*x))), x] + Simp[1/(p*e^2*(2*c*
d - b*e))   Int[(f + g*x)^(n - 2)*(a + b*x + c*x^2)^p*Simp[b*e*g*((-e)*f +
d*g + e*f*n - d*g*n - e*f*p) + c*(d^2*g^2*(n - 1) - d*e*f*g*n + e^2*f^2*(2*
p + 1)) - e*g*(b*e*g*p - c*(e*f*n - d*g*n + 2*e*f*p))*x, x], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1] && LtQ[p, -1] && EqQ[c*d^2 - b*d*
e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1922 vs. $2(401) = 802$.

Time = 2.88 (sec) , antiderivative size = 1923, normalized size of antiderivative = 4.42

method	result	size
default	Expression too large to display	1923

input
```
int(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
2*d^4/e^5*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/e*(1/2*x^3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2)-5/4*(a*e^2+c*d^2)/d/e/c*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x
+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e
^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1
/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2))/(d*e*c)^(1/2))-2*a/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^
(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))-3/2*a/c*(-x/d/e/c/(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d
^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*
d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+d^2/e^3*(-x/d/e/c/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*
c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/
c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(401) = 802$.

Time = 4.13 (sec) , antiderivative size = 2120, normalized size of antiderivative = 4.87

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[1/48*(15*(7*a*c^5*d^12*e - 15*a^2*c^4*d^10*e^3 + 6*a^3*c^3*d^8*e^5 + 2*a^
4*c^2*d^6*e^7 + 3*a^5*c*d^4*e^9 - 3*a^6*d^2*e^11 + (7*c^6*d^11*e^2 - 15*a*
c^5*d^9*e^4 + 6*a^2*c^4*d^7*e^6 + 2*a^3*c^3*d^5*e^8 + 3*a^4*c^2*d^3*e^10 -
 3*a^5*c*d*e^12)*x^3 + (14*c^6*d^12*e - 23*a*c^5*d^10*e^3 - 3*a^2*c^4*d^8*
e^5 + 10*a^3*c^3*d^6*e^7 + 8*a^4*c^2*d^4*e^9 - 3*a^5*c*d^2*e^11 - 3*a^6*e^
13)*x^2 + (7*c^6*d^13 - a*c^5*d^11*e^2 - 24*a^2*c^4*d^9*e^4 + 14*a^3*c^3*d
^7*e^6 + 7*a^4*c^2*d^5*e^8 + 3*a^5*c*d^3*e^10 - 6*a^6*d*e^12)*x)*sqrt(c*d*
e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*
e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e)
 + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(105*a*c^5*d^11*e^2 - 190*a^2*c^4*d^9*
e^4 + 36*a^3*c^3*d^7*e^6 + 30*a^4*c^2*d^5*e^8 - 45*a^5*c*d^3*e^10 - 6*(c^6
*d^9*e^4 - 3*a*c^5*d^7*e^6 + 3*a^2*c^4*d^5*e^8 - a^3*c^3*d^3*e^10)*x^4 + 3
*(7*c^6*d^10*e^3 - 16*a*c^5*d^8*e^5 + 6*a^2*c^4*d^6*e^7 + 8*a^3*c^3*d^4*e^
9 - 5*a^4*c^2*d^2*e^11)*x^3 + (140*c^6*d^11*e^2 - 237*a*c^5*d^9*e^4 + 12*a
^2*c^4*d^7*e^6 + 66*a^3*c^3*d^5*e^8 - 45*a^5*c*d*e^12)*x^2 + (105*c^6*d^12
*e - 50*a*c^5*d^10*e^3 - 222*a^2*c^4*d^8*e^5 + 84*a^3*c^3*d^6*e^7 + 45*a^4
*c^2*d^4*e^9 - 90*a^5*c*d^2*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x))/(a*c^7*d^12*e^6 - 3*a^2*c^6*d^10*e^8 + 3*a^3*c^5*d^8*e^10 - a^4*c^
4*d^6*e^12 + (c^8*d^11*e^7 - 3*a*c^7*d^9*e^9 + 3*a^2*c^6*d^7*e^11 - a^3*c^
5*d^5*e^13)*x^3 + (2*c^8*d^12*e^6 - 5*a*c^7*d^10*e^8 + 3*a^2*c^6*d^8*e^...
```

Sympy [F]

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^5}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(x**5/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**5/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^5}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
integrate(x^5/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^5}{(d+e\,x)\,\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(x^5/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(x^5/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^5}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{x^5}{(ex+d)\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}dx$$

input
```
int(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
int(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

3.113 $\int \frac{x^4}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 356

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = -\frac{2a^4e^4}{c^4d^4(cd^2-ae^2)(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{c^2d^2e^3} - \frac{2(c^4d^8+3a^4e^8)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3c^4d^4e^3(cd^2-ae^2)^2(d+ex)^2} + \frac{2(7c^4d^8-12ac^3d^6e^2-3a^4e^8)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3c^3d^3e^3(cd^2-ae^2)^3(d+ex)} - \frac{(5cd^2+3ae^2)\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{5/2}d^{5/2}e^{7/2}}$$

output
```
-2*a^4*e^4/c^4/d^4/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3-2/3*(3*a^4*e^8
+c^4*d^8)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^3/(-a*e^2+c*d^
2)^2/(e*x+d)^2+2/3*(-3*a^4*e^8-12*a*c^3*d^6*e^2+7*c^4*d^8)*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^3/(-a*e^2+c*d^2)^3/(e*x+d)-(3*a*e^2+5*c
*d^2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e
*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)\left(-9a^4e^7(d+ex)^2+3a^3cde^5(3d-ex)(d+ex)^2+c^4d^7x(15d^2\right.}{}$$

input
```
Integrate[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-9*a^4*e^7*(d + e*x)^2 + 3*a^3*c*
d*e^5*(3*d - e*x)*(d + e*x)^2 + c^4*d^7*x*(15*d^2 + 20*d*e*x + 3*e^2*x^2)
+ a*c^3*d^5*e*(15*d^3 - 11*d^2*e*x - 39*d*e^2*x^2 - 9*e^3*x^3) + a^2*c^2*d
^3*e^3*(-31*d^3 - 33*d^2*e*x + 9*d*e^2*x^2 + 9*e^3*x^3)))/(c*d^2 - a*e^2)^
3 - 3*(5*c*d^2 + 3*a*e^2)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTanh[(Sqr
t[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])])/(3*c^(5/2)*d^(5/
2)*e^(7/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1244, 27, 1233, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex)\left(x\,(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 1244$$

$$-\frac{2\int -\frac{e^2x^2\left(6ade+(5cd^2-3ae^2)x\right)}{2\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/2}}dx}{3e^3\left(cd^2-ae^2\right)}-\frac{2dx^3}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{x^2\left(6ade+(5cd^2-3ae^2)x\right)}{\left(cdex^2+(cd^2+ae^2)x+ade\right)^{3/2}}dx}{3e\left(cd^2-ae^2\right)}-\frac{2dx^3}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{1233}$$

$$\frac{\frac{2\int \frac{2ade\left(5c^2d^4-10ace^2d^2-3a^2e^4\right)+\left(15c^3d^6-31ac^2e^2d^4+9a^2ce^4d^2-9a^3e^6\right)x}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cde(cd^2-ae^2)^2}-\frac{2x\left(ade\left(-3a^2e^4-10acd^2e^2+5c^2d^4\right)+x\left(-3a^3e^6-a^2cd^2e^4-9ac^2\right.\right.}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3e\left(cd^2-ae^2\right)}$$

$$\frac{2dx^3}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{2ade\left(5c^2d^4-10ace^2d^2-3a^2e^4\right)+\left(15c^3d^6-31ac^2e^2d^4+9a^2ce^4d^2-9a^3e^6\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cde(cd^2-ae^2)^2}-\frac{2x\left(ade\left(-3a^2e^4-10acd^2e^2+5c^2d^4\right)+x\left(-3a^3e^6-a^2cd^2e^4-9ac^2d^4\right.\right.}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3e\left(cd^2-ae^2\right)}$$

$$\frac{2dx^3}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{\frac{\left(-9a^3e^6+9a^2cd^2e^4-31ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cde}-\frac{3\left(cd^2-ae^2\right)^3\left(3ae^2+5cd^2\right)\int \frac{1}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2cde}}{cde(cd^2-ae^2)^2}-\frac{2x\left(ade\left(-3a^2e^4-1\right.\right.}{c}}{3e\left(cd^2-ae^2\right)}$$

$$\frac{2dx^3}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{\frac{\left(-9a^3e^6+9a^2cd^2e^4-31ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cde}-\frac{3\left(cd^2-ae^2\right)^3\left(3ae^2+5cd^2\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x}}}{cde}}{cde(cd^2-ae^2)^2}}{3e\left(cd^2-ae^2\right)}$$

$$\frac{2dx^3}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\left(-9a^3e^6+9a^2cd^2e^4-31ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{cde}-\frac{3\left(cd^2-ae^2\right)^3\left(3ae^2+5cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}e^{3/2}}}{cde(cd^2-ae^2)^2} - \cdots}{3e\left(cd^2-ae^2\right)}$$
$$-\frac{2dx^3}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*d*x^3)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ((-2*x*(a*d*e*(5*c^2*d^4 - 10*a*c*d^2*e^2 - 3*a^2*e^4) + (5
*c^3*d^6 - 9*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(c*d*e*(c*d^2
- a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (((15*c^3*d^6 -
31*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 9*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2])/(c*d*e) - (3*(c*d^2 - a*e^2)^3*(5*c*d^2 + 3*a*e^2)*ArcT
anh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(3/2)*d^(3/2)*e^(3/2)))/(c*d*e*(c*d^2
 - a*e^2)^2))/(3*e*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1244
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d
_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(e*f - d*g))*(f + g*x)^(n - 1)*((a +
b*x + c*x^2)^(p + 1)/(p*(2*c*d - b*e)*(d + e*x))), x] + Simp[1/(p*e^2*(2*c*
d - b*e))   Int[(f + g*x)^(n - 2)*(a + b*x + c*x^2)^p*Simp[b*e*g*((-e)*f +
d*g + e*f*n - d*g*n - e*f*p) + c*(d^2*g^2*(n - 1) - d*e*f*g*n + e^2*f^2*(2*
p + 1)) - e*g*(b*e*g*p - c*(e*f*n - d*g*n + 2*e*f*p))*x, x], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1] && LtQ[p, -1] && EqQ[c*d^2 - b*d*
e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1111 vs. $2(330) = 660$.

Time = 2.88 (sec) , antiderivative size = 1112, normalized size of antiderivative = 3.12

method	result	size
default	Expression too large to display	1112

input
```
int(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/d
/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d
/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c
*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(
1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2*a/c*(-1/d/e
/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+
a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^
2*e)^(1/2)))+d^2/e^3*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*
e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(
a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+d^4/e^5*(-2/3/(a*e^2-c*d^2)/(x+d/e
)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d*e*c/(a*e^2-c*d^2)^3*
(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2
))-2*d^3/e^4*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-d/e^2*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^
2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/
2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)
^(1/2))/(d*e*c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(330) = 660$.

Time = 1.59 (sec) , antiderivative size = 1782, normalized size of antiderivative = 5.01

$$\int \frac{x^4}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[1/12*(3*(5*a*c^4*d^10*e - 12*a^2*c^3*d^8*e^3 + 6*a^3*c^2*d^6*e^5 + 4*a^4*
c*d^4*e^7 - 3*a^5*d^2*e^9 + (5*c^5*d^9*e^2 - 12*a*c^4*d^7*e^4 + 6*a^2*c^3*
d^5*e^6 + 4*a^3*c^2*d^3*e^8 - 3*a^4*c*d*e^10)*x^3 + (10*c^5*d^10*e - 19*a*
c^4*d^8*e^3 + 14*a^3*c^2*d^4*e^7 - 2*a^4*c*d^2*e^9 - 3*a^5*e^11)*x^2 + (5*
c^5*d^11 - 2*a*c^4*d^9*e^2 - 18*a^2*c^3*d^7*e^4 + 16*a^3*c^2*d^5*e^6 + 5*a
^4*c*d^3*e^8 - 6*a^5*d*e^10)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^
4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) +
 4*(15*a*c^4*d^9*e^2 - 31*a^2*c^3*d^7*e^4 + 9*a^3*c^2*d^5*e^6 - 9*a^4*c*d^
3*e^8 + 3*(c^5*d^8*e^3 - 3*a*c^4*d^6*e^5 + 3*a^2*c^3*d^4*e^7 - a^3*c^2*d^2
*e^9)*x^3 + (20*c^5*d^9*e^2 - 39*a*c^4*d^7*e^4 + 9*a^2*c^3*d^5*e^6 + 3*a^3
*c^2*d^3*e^8 - 9*a^4*c*d*e^10)*x^2 + (15*c^5*d^10*e - 11*a*c^4*d^8*e^3 - 3
3*a^2*c^3*d^6*e^5 + 15*a^3*c^2*d^4*e^7 - 18*a^4*c*d^2*e^9)*x)*sqrt(c*d*e*x
^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c^6*d^11*e^5 - 3*a^2*c^5*d^9*e^7 + 3*a
^3*c^4*d^7*e^9 - a^4*c^3*d^5*e^11 + (c^7*d^10*e^6 - 3*a*c^6*d^8*e^8 + 3*a^
2*c^5*d^6*e^10 - a^3*c^4*d^4*e^12)*x^3 + (2*c^7*d^11*e^5 - 5*a*c^6*d^9*e^7
 + 3*a^2*c^5*d^7*e^9 + a^3*c^4*d^5*e^11 - a^4*c^3*d^3*e^13)*x^2 + (c^7*d^1
2*e^4 - a*c^6*d^10*e^6 - 3*a^2*c^5*d^8*e^8 + 5*a^3*c^4*d^6*e^10 - 2*a^4*c^
3*d^4*e^12)*x), 1/6*(3*(5*a*c^4*d^10*e - 12*a^2*c^3*d^8*e^3 + 6*a^3*c^2*d^
6*e^5 + 4*a^4*c*d^4*e^7 - 3*a^5*d^2*e^9 + (5*c^5*d^9*e^2 - 12*a*c^4*d^7...
```

Sympy [F]

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{x^4}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}\,(d+ex)}\,dx$$

input
```
integrate(x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**4/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^4}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
integrate(x^4/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^4}{(d+e\,x)\,\left(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(x^4/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(x^4/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 1978, normalized size of antiderivative = 5.56

$$\int \frac{x^4}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 72*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*d**2*e*
*8 - 144*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e +
 c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*d*e**
9*x - 72*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e +
 c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*e**10
*x**2 + 96*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e
 + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c*d
**4*e**6 + 192*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt
(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3
*c*d**3*e**7*x + 96*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)
*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))
*a**3*c*d**2*e**8*x**2 + 144*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log
((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 -
 c*d**2))*a**2*c**2*d**6*e**4 + 288*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d
*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a
*e**2 - c*d**2))*a**2*c**2*d**5*e**5*x + 144*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(
a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x
))/sqrt(a*e**2 - c*d**2))*a**2*c**2*d**4*e**6*x**2 - 288*sqrt(e)*sqrt(d)*s
qrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(...
```

3.114 $\int \frac{x^3}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 304

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{2a^3e^3}{c^3d^3(cd^2-ae^2)(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} + \frac{2(c^3d^6+3a^3e^6)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3c^3d^3e^2(cd^2-ae^2)^2(d+ex)^2} - \frac{2(4c^3d^6-9ac^2d^4e^2-3a^3e^6)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{3c^2d^2e^2(cd^2-ae^2)^3(d+ex)} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{3/2}d^{3/2}e^{5/2}}$$

output
```
2*a^3*e^3/c^3/d^3/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)+2/3*(3*a^3*e^6+c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3
/d^3/e^2/(-a*e^2+c*d^2)^2/(e*x+d)^2-2/3*(-3*a^3*e^6-9*a*c^2*d^4*e^2+4*c^3*
d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^2/(-a*e^2+c*d^2)^3/
(e*x+d)+2*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{2\left(-\frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)\left(-3a^3e^5(d+ex)^2+c^3d^6x(3d+4ex)-a^2cd^3e^3(8d+9e\right.}{(cd^2-ae^2)^3}\right.}{3c^{3/2}d^{3/2}}$$

input
```
Integrate[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*(-((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-3*a^3*e^5*(d + e*x)^2 + c^3
*d^6*x*(3*d + 4*e*x) - a^2*c*d^3*e^3*(8*d + 9*e*x) + a*c^2*d^4*e*(3*d^2 -
4*d*e*x - 9*e^2*x^2)))/(c*d^2 - a*e^2)^3) + 3*(a*e + c*d*x)^(3/2)*(d + e*x
)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x]
)]))/(3*c^(3/2)*d^(3/2)*e^(5/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1244, 27, 1224, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1244}$$

$$-\frac{2\int -\frac{e^2x\left(4ade+3\left(cd^2-ae^2\right)x\right)}{2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3e^3\left(cd^2-ae^2\right)}-\frac{2dx^2}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{x\left(4ade+3\left(cd^2-ae^2\right)x\right)}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3e\left(cd^2-ae^2\right)}-\frac{2dx^2}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 1224}$$

$$\frac{3\left(\frac{d}{e}-\frac{ae}{cd}\right)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx-\frac{2\left(x\left(-3a^3e^6-a^2cd^2e^4-7ac^2d^4e^2+3c^3d^6\right)+ade\left(cd^2-3ae^2\right)\left(ae^2+3cd^2\right)\right)}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3e\left(cd^2-ae^2\right)}-\frac{2dx^2}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1092$$

$$\frac{6\left(\frac{d}{e}-\frac{ae}{cd}\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+ae^2}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\frac{2\left(x\left(-3a^3e^6-a^2cd^2e^4-7ac^2d^4e^2+3c^3d^6\right)+ade\left(cd^2-3ae^2\right)\left(ae^2\right.\right.}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3e\left(cd^2-ae^2\right)}$$
$$\frac{2dx^2}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{3\left(\frac{d}{e}-\frac{ae}{cd}\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}-\frac{2\left(x\left(-3a^3e^6-a^2cd^2e^4-7ac^2d^4e^2+3c^3d^6\right)+ade\left(cd^2-3ae^2\right)\left(ae^2+3cd^2\right)\right)}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3e\left(cd^2-ae^2\right)}-\frac{2dx^2}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*d*x^2)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ((-2*(a*d*e*(c*d^2 - 3*a*e^2)*(3*c*d^2 + a*e^2) + (3*c^3*d^
6 - 7*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(c*d*e*(c*d^2 - a*e^2
)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (3*(d/e - (a*e)/(c*d))*
ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(Sqrt[c]*Sqrt[d]*Sqrt[e]))/(3*e*(c*d^2
 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1224
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c))   Int[(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -
1] &&  !(IntegerQ[p] && NeQ[a, 0] && NiceSqrtQ[b^2 - 4*a*c])
```

rule 1244
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d
_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(e*f - d*g))*(f + g*x)^(n - 1)*((a +
b*x + c*x^2)^(p + 1)/(p*(2*c*d - b*e)*(d + e*x))), x] + Simp[1/(p*e^2*(2*c*
d - b*e))   Int[(f + g*x)^(n - 2)*(a + b*x + c*x^2)^p*Simp[b*e*g*((-e)*f +
d*g + e*f*n - d*g*n - e*f*p) + c*(d^2*g^2*(n - 1) - d*e*f*g*n + e^2*f^2*(2*
p + 1)) - e*g*(b*e*g*p - c*(e*f*n - d*g*n + 2*e*f*p))*x, x], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1] && LtQ[p, -1] && EqQ[c*d^2 - b*d*
e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(280) = 560$.

Time = 2.44 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.10

method	result
default	$\frac{2d^2(2cdxe+a\,e^2+c\,d^2)}{e^3\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}+\frac{-\frac{x}{dec\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}-\frac{\left(a\,e^2+c\,d^2\right)\left(-\frac{1}{dec\sqrt{ade+\left(a\,e^2+c\,d^2\right)\cdots}}\right.}{\cdots}}{\cdots}$

input
```
int(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
2*d^2/e^3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)+1/e*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x
^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x
^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*
e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e
^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
))/(d*e*c)^(1/2))-d/e^2*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-
(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))-d^3/e^4*(-2/3/(a*e^2-c*d^2)/(x+
d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d*e*c/(a*e^2-c*d^2)
^3*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(
1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(280) = 560$.

Time = 1.79 (sec) , antiderivative size = 1466, normalized size of antiderivative = 4.82

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[1/6*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 +
 (c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (
2*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^
9)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 -
2*a^4*d*e^8)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^
2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c
*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(3*a*c^3*d^7*
e^2 - 8*a^2*c^2*d^5*e^4 - 3*a^3*c*d^3*e^6 + (4*c^4*d^7*e^2 - 9*a*c^3*d^5*e
^4 - 3*a^3*c*d*e^8)*x^2 + (3*c^4*d^8*e - 4*a*c^3*d^6*e^3 - 9*a^2*c^2*d^4*e
^5 - 6*a^3*c*d^2*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a*c
^5*d^10*e^4 - 3*a^2*c^4*d^8*e^6 + 3*a^3*c^3*d^6*e^8 - a^4*c^2*d^4*e^10 + (
c^6*d^9*e^5 - 3*a*c^5*d^7*e^7 + 3*a^2*c^4*d^5*e^9 - a^3*c^3*d^3*e^11)*x^3
+ (2*c^6*d^10*e^4 - 5*a*c^5*d^8*e^6 + 3*a^2*c^4*d^6*e^8 + a^3*c^3*d^4*e^10
 - a^4*c^2*d^2*e^12)*x^2 + (c^6*d^11*e^3 - a*c^5*d^9*e^5 - 3*a^2*c^4*d^7*e
^7 + 5*a^3*c^3*d^5*e^9 - 2*a^4*c^2*d^3*e^11)*x), -1/3*(3*(a*c^3*d^8*e - 3*
a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*c^3*d
^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3*d^6
*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9 - a*c^3
*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)*sqrt(-c*d
*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x +...
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^3}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**3/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^3}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
integrate(x^3/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^3}{(d+e\,x)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(x^3/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1329, normalized size of antiderivative = 4.37

$$\int \frac{x^3}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*(3*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*d**2*e**
6 + 6*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*d*e**7*x
 + 3*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d
*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*e**8*x**2
 - 9*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d
*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**4*e*
*4 - 18*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e +
c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*d**3
*e**5*x - 9*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*
e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c*
d**2*e**6*x**2 + 9*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*
sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*
a*c**2*d**6*e**2 + 18*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(
e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2
))*a*c**2*d**5*e**3*x + 9*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((s
qrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*
d**2))*a*c**2*d**4*e**4*x**2 - 3*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)
*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a...
```

3.115 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 186

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = -\frac{2x^2}{(cd^2-ae^2)\,(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} - \frac{8d^2\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3e\,(cd^2-ae^2)^2\,(d+ex)^2} + \frac{8d(cd^2-3ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3e\,(cd^2-ae^2)^3\,(d+ex)}$$

output
```
-2*x^2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-8/3*
d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e/(-a*e^2+c*d^2)^2/(e*x+d)^2+8
/3*d*(-3*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/e/(-a*e^2+c*
d^2)^3/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.51

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{2(ae+cdx)^3\left(d^2-\frac{6ade(d+ex)}{ae+cdx}-\frac{3a^2e^2(d+ex)^2}{(ae+cdx)^2}\right)}{3\left(cd^2-ae^2\right)^3\left((ae+cdx)(d+ex)\right)^{3/2}}$$

input
```
Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*(a*e + c*d*x)^3*(d^2 - (6*a*d*e*(d + e*x))/(a*e + c*d*x) - (3*a^2*e^2*(
d + e*x)^2)/(a*e + c*d*x)^2))/(3*(c*d^2 - a*e^2)^3*((a*e + c*d*x)*(d + e*x
))^(3/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.68, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1243, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1243}$$

$$\frac{2x^2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{4ae\int \frac{x}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{ 1158}$$

$$\frac{2x^2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8ae\left(x\left(ae^2+cd^2\right)+2ade\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*x^2)/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2]) - (8*a*e*(2*a*d*e + (c*d^2 + a*e^2)*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[
a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1243
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/(
(d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(2*c*d - b*e))*(f + g*x)^n*((a + b*
x + c*x^2)^(p + 1)/(e*p*(b^2 - 4*a*c)*(d + e*x))), x] + Simp[n*((a*g*(2*c*d
 - b*e) - c*f*(b*d - 2*a*e))/(d*e*p*(b^2 - 4*a*c)))   Int[(f + g*x)^(n - 1)
*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[n, 1] && LtQ[p, -1] && EqQ[n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.78

method	result
gosper	$\frac{2(cdx+ae)(3a^2e^4x^2+6acd^2e^2x^2-c^2d^4x^2+12a^2de^3x+4acd^3ex+8d^2e^2a^2)}{3(e^6a^3-3d^2e^4a^2c+3d^4e^2ac^2-d^6c^3)(cdx^2e+ae^2x+cd^2x+ade)^{\frac{3}{2}}}$
orering	$\frac{2(3a^2e^4x^2+6acd^2e^2x^2-c^2d^4x^2+12a^2de^3x+4acd^3ex+8d^2e^2a^2)(cdx+ae)}{3(e^6a^3-3d^2e^4a^2c+3d^4e^2ac^2-d^6c^3)(ade+(ae^2+cd^2)x+cdx^2e)^{\frac{3}{2}}}$
trager	$\frac{2(3a^2e^4x^2+6acd^2e^2x^2-c^2d^4x^2+12a^2de^3x+4acd^3ex+8d^2e^2a^2)\sqrt{cdx^2e+ae^2x+cd^2x+ade}}{3(a^2e^4-2acd^2e^2+c^2d^4)(ex+d)^2(ae^2-cd^2)(cdx+ae)}$
default	$\frac{-\frac{1}{dec\sqrt{ade+\left(ae^2+cd^2\right)x+cdx^2e}}-\frac{\left(ae^2+cd^2\right)\left(2cdxe+ae^2+cd^2\right)}{dec\left(4acd^2e^2-\left(ae^2+cd^2\right)^2\right)\sqrt{ade+\left(ae^2+cd^2\right)x+cdx^2e}}}{e}+\frac{d^2\left(-\frac{2}{3\left(ae^2-cd^2\right)\left(x+\frac{d}{e}\right)\sqrt{dec\left(x+\frac{d}{e}\right)^2}}\right.}{}$

input
```
int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
2/3*(c*d*x+a*e)*(3*a^2*e^4*x^2+6*a*c*d^2*e^2*x^2-c^2*d^4*x^2+12*a^2*d*e^3*
x+4*a*c*d^3*e*x+8*a^2*d^2*e^2)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^
3*d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.66

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{2\left(8\,a^2d^2e^2-(c^2d^4-6\,acd^2e^2-3\,a^2e^4)x^2+4\,(aca\right.}{3\left(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+(c^4d^7e^2-3\,ac^3d^5e^4+3\,a^2c^2d^3e^6-a^3cde^8)x^3+(2\,c^4d^8e-\right.}$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
-2/3*(8*a^2*d^2*e^2 - (c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*x^2 + 4*(a*c*d
^3*e + 3*a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^3*
d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 -
 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5
*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d
^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^2}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}\,(d+ex)}\,dx$$

input
```
integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**2/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F]

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
integrate(x^2/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x
)
```

Mupad [B] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 1071, normalized size of antiderivative = 5.76

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{4\,c\,d^3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a}}{3\,\left(a^3\,d\,e^7+x\,a^3\,e^8-3\,a^2\,c\,d^3\,e^5-3\,x\,a^2\,c\,d^2\,e^6+3\,a\,c^2\,c\right.}$$
$$-\frac{2\,d^2\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}}{3\,a^2\,d^2\,e^5+6\,a^2\,d\,e^6\,x+3\,a^2\,e^7\,x^2-6\,a\,c\,d^4\,e^3-12\,a\,c\,d^3\,e^4\,x-6\,a\,c\,d^2\,e^5\,x^2+3\,c^2\,d^6\,e+6\,c^2\,d^5\,e^2\,x+3}$$
$$-\frac{4\,a\,d\,e^2\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}}{3\,\left(a^3\,d\,e^7+x\,a^3\,e^8-3\,a^2\,c\,d^3\,e^5-3\,x\,a^2\,c\,d^2\,e^6+3\,a\,c^2\,d^5\,e^3+3\,x\,a\,c^2\,d^4\,e^4-c^3\,d^7\,e-x\,c^3\,d^6\,e^2\right)}$$
$$+\frac{2\,c^4\,d^7\,x}{\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$
$$+\frac{22\,a^3\,c\,d^2\,e^5}{3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$
$$-\frac{28\,a^2\,c^2\,d^4\,e^3}{3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$
$$+\frac{2\,a\,c^3\,d^6\,e}{\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$
$$+\frac{10\,a^2\,c^2\,d^3\,e^4\,x}{3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$
$$+\frac{2\,a^3\,c\,d\,e^6\,x}{\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$
$$-\frac{22\,a\,c^3\,d^5\,e^2\,x}{3\,\sqrt{c\,d^2\,x+c\,d\,e\,x^2+a\,d\,e+a\,e^2\,x}\,\left(a^4\,c\,d\,e^9-4\,a^3\,c^2\,d^3\,e^7+6\,a^2\,c^3\,d^5\,e^5-4\,a\,c^4\,d^7\,e^3+c^5\,d^9\,e\right)}$$

input
```
int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(4*c*d^3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*(a^3*d*e^7 - c^
3*d^7*e + a^3*e^8*x + 3*a*c^2*d^5*e^3 - 3*a^2*c*d^3*e^5 - c^3*d^6*e^2*x +
3*a*c^2*d^4*e^4*x - 3*a^2*c*d^2*e^6*x)) - (2*d^2*(a*d*e + a*e^2*x + c*d^2*
x + c*d*e*x^2)^(1/2))/(3*c^2*d^6*e + 3*a^2*d^2*e^5 + 3*a^2*e^7*x^2 + 6*c^2
*d^5*e^2*x + 3*c^2*d^4*e^3*x^2 - 6*a*c*d^4*e^3 + 6*a^2*d*e^6*x - 12*a*c*d^
3*e^4*x - 6*a*c*d^2*e^5*x^2) - (4*a*d*e^2*(a*d*e + a*e^2*x + c*d^2*x + c*d
*e*x^2)^(1/2))/(3*(a^3*d*e^7 - c^3*d^7*e + a^3*e^8*x + 3*a*c^2*d^5*e^3 - 3
*a^2*c*d^3*e^5 - c^3*d^6*e^2*x + 3*a*c^2*d^4*e^4*x - 3*a^2*c*d^2*e^6*x)) +
 (2*c^4*d^7*x)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*e -
 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9)) +
 (22*a^3*c*d^2*e^5)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*
d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*
e^9)) - (28*a^2*c^2*d^4*e^3)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1
/2)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 +
 a^4*c*d*e^9)) + (2*a*c^3*d^6*e)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^
(1/2)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7
 + a^4*c*d*e^9)) + (10*a^2*c^2*d^3*e^4*x)/(3*(a*d*e + a*e^2*x + c*d^2*x +
c*d*e*x^2)^(1/2)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*
c^2*d^3*e^7 + a^4*c*d*e^9)) + (2*a^3*c*d*e^6*x)/((a*d*e + a*e^2*x + c*d^2*
x + c*d*e*x^2)^(1/2)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 -...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.35

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{-4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,a^2d^2e^4-8\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+a}}{}$$

input
```
int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
(2*( - 6*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*d**2*e**4 - 12*sqr
t(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*d*e**5*x - 6*sqrt(e)*sqrt(d)*s
qrt(c)*sqrt(a*e + c*d*x)*a**2*e**6*x**2 - 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a
*e + c*d*x)*c**2*d**6 - 4*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d
**5*e*x - 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d**4*e**2*x**2
+ 8*sqrt(d + e*x)*a**2*c*d**3*e**4 + 12*sqrt(d + e*x)*a**2*c*d**2*e**5*x +
 3*sqrt(d + e*x)*a**2*c*d*e**6*x**2 + 4*sqrt(d + e*x)*a*c**2*d**4*e**3*x +
 6*sqrt(d + e*x)*a*c**2*d**3*e**4*x**2 - sqrt(d + e*x)*c**3*d**5*e**2*x**2
))/(3*sqrt(a*e + c*d*x)*c*d*e**2*(a**3*d**2*e**6 + 2*a**3*d*e**7*x + a**3*
e**8*x**2 - 3*a**2*c*d**4*e**4 - 6*a**2*c*d**3*e**5*x - 3*a**2*c*d**2*e**6
*x**2 + 3*a*c**2*d**6*e**2 + 6*a*c**2*d**5*e**3*x + 3*a*c**2*d**4*e**4*x**
2 - c**3*d**8 - 2*c**3*d**7*e*x - c**3*d**6*e**2*x**2))
```

3.116 $\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 138

$$\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2d}{3e\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2\left(cd^2+3ae^2\right)\left(cd^2+ae^2+2cdex\right)}{3e\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-2/3*d/e/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2/
3*(3*a*e^2+c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2
+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.72

$$\int \frac{x}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2(c^2d^3x(3d+2ex)+a^2e^3(2d+3ex)+2acde(3d^2+5de}{3\left(cd^2-ae^2\right)^3(d+ex)\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[x/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*(c^2*d^3*x*(3*d + 2*e*x) + a^2*e^3*(2*d + 3*e*x) + 2*a*c*d*e*(3*d^2 + 5
*d*e*x + 3*e^2*x^2)))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d
 + e*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1220, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{\left(3ae^2+cd^2\right)\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3e\left(cd^2-ae^2\right)}-\frac{2d}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{2\left(3ae^2+cd^2\right)\left(ae^2+cd^2+2cdex\right)}{3e\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2d}{3e(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[x/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*d)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]) + (2*(c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*e*(c*d^2 -
 a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.08

method	result
gosper	$-\frac{2(cdx+ae)(6x^2acd\,e^3+2x^2c^2d^3e+3a^2e^4x+10ac\,d^2e^2x+3c^2d^4x+2a^2d\,e^3+6ac\,d^3e)}{3(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3)(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$
orering	$-\frac{2(6x^2acd\,e^3+2x^2c^2d^3e+3a^2e^4x+10ac\,d^2e^2x+3c^2d^4x+2a^2d\,e^3+6ac\,d^3e)(cdx+ae)}{3(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3)(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{3}{2}}}$
trager	$-\frac{2(6x^2acd\,e^3+2x^2c^2d^3e+3a^2e^4x+10ac\,d^2e^2x+3c^2d^4x+2a^2d\,e^3+6ac\,d^3e)\sqrt{cd\,x^2e+a\,e^2x+c\,d^2x+ade}}{3(a^2e^4-2ac\,d^2e^2+c^2d^4)(ex+d)^2(a\,e^2-c\,d^2)(cdx+ae)}$
default	$\frac{4cdxe+2a\,e^2+2c\,d^2}{e\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}-\frac{d\left(-\frac{2}{3\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}+\frac{8d\cdots}{3\left(a\,e^2-c\,d^2\right)\cdots}\right.}{e^2}$

input
```
int(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
-2/3*(c*d*x+a*e)*(6*a*c*d*e^3*x^2+2*c^2*d^3*e*x^2+3*a^2*e^4*x+10*a*c*d^2*e
^2*x+3*c^2*d^4*x+2*a^2*d*e^3+6*a*c*d^3*e)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2
*d^4*e^2-c^3*d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(130) = 260.

Time = 1.43 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.28

$$\int \frac{x}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{2\,(6\,acd}{3\,(ac^3d^8e-3\,a^2c^2d^6e^3+3\,a^3cd^4e^5-a^4d^2e^7+(c^4d^7e^2-}$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
fricas")
```

output
```
2/3*(6*a*c*d^3*e + 2*a^2*d*e^3 + 2*(c^2*d^3*e + 3*a*c*d*e^3)*x^2 + (3*c^2*
d^4 + 10*a*c*d^2*e^2 + 3*a^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e
^2)*x)/(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 +
(c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2
*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9
)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2
*a^4*d*e^8)*x)
```

Sympy [F]

$$\int \frac{x}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x}{\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F]

$$\int \frac{x}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
giac")
```

output
```
integrate(x/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 499, normalized size of antiderivative = 3.62

$$\int \frac{x}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{4\,a^2\,d\,e^3\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}+6\,a^2\,e^4\,x\,\sqrt{\dots}}{-3\,a^4\,d^2\,e^7-6\,a^4\,d\,e^8\,\dots}$$

input
```
int(x/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(4*a^2*d*e^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 6*a^2*e^4*x*(
x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 6*c^2*d^4*x*(x*(a*e^2 + c*d
^2) + a*d*e + c*d*e*x^2)^(1/2) + 4*c^2*d^3*e*x^2*(x*(a*e^2 + c*d^2) + a*d*
e + c*d*e*x^2)^(1/2) + 12*a*c*d^3*e*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2) + 20*a*c*d^2*e^2*x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) +
 12*a*c*d*e^3*x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(3*c^4*d^
9*x - 3*a^4*d^2*e^7 - 3*a^4*e^9*x^2 + 9*a^3*c*d^4*e^5 + 6*c^4*d^8*e*x^2 -
9*a^2*c^2*d^6*e^3 + 3*c^4*d^7*e^2*x^3 + 3*a*c^3*d^8*e - 6*a^4*d*e^8*x + 9*
a^2*c^2*d^4*e^5*x^2 + 9*a^2*c^2*d^3*e^6*x^3 - 3*a*c^3*d^7*e^2*x + 15*a^3*c
*d^3*e^6*x - 3*a^3*c*d*e^8*x^3 - 9*a^2*c^2*d^5*e^4*x - 15*a*c^3*d^6*e^3*x^
2 + 3*a^3*c*d^2*e^7*x^2 - 9*a*c^3*d^5*e^4*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 420, normalized size of antiderivative = 3.04

$$\int \frac{x}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,a\,d^2e^2+8\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,a}{}$$

input
```
int(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*(6*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*d**2*e**2 + 12*sqrt(e)*s
qrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*d*e**3*x + 6*sqrt(e)*sqrt(d)*sqrt(c)*sq
rt(a*e + c*d*x)*a*e**4*x**2 + 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*
c*d**4 + 4*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d**3*e*x + 2*sqrt(e
)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d**2*e**2*x**2 - 2*sqrt(d + e*x)*a**
2*d*e**4 - 3*sqrt(d + e*x)*a**2*e**5*x - 6*sqrt(d + e*x)*a*c*d**3*e**2 - 1
0*sqrt(d + e*x)*a*c*d**2*e**3*x - 6*sqrt(d + e*x)*a*c*d*e**4*x**2 - 3*sqrt
(d + e*x)*c**2*d**4*e*x - 2*sqrt(d + e*x)*c**2*d**3*e**2*x**2))/(3*sqrt(a*
e + c*d*x)*e*(a**3*d**2*e**6 + 2*a**3*d*e**7*x + a**3*e**8*x**2 - 3*a**2*c
*d**4*e**4 - 6*a**2*c*d**3*e**5*x - 3*a**2*c*d**2*e**6*x**2 + 3*a*c**2*d**
6*e**2 + 6*a*c**2*d**5*e**3*x + 3*a*c**2*d**4*e**4*x**2 - c**3*d**8 - 2*c*
*3*d**7*e*x - c**3*d**6*e**2*x**2))
```

3.117 $\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 121

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2}{3\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{8cd(cd^2+ae^2+2cdex)}{3\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
2/3/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-8/3*c*d
*(2*c*d*e*x+a*e^2+c*d^2)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \frac{1}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2a^2e^4-4acde^2(3d+2ex)-2c^2d^2(3d^2+12dex+8e^2x^2}{3\left(cd^2-ae^2\right)^3(d+ex)\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*a^2*e^4 - 4*a*c*d*e^2*(3*d + 2*e*x) - 2*c^2*d^2*(3*d^2 + 12*d*e*x + 8*e
^2*x^2))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{4cd\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)}+\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[1/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
) - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14

method	result	size
gosper	$-\frac{2(cdx+ae)\left(-8x^{2}c^{2}d^{2}e^{2}-4xacd\,e^{3}-12x\,c^{2}d^{3}e+a^{2}e^{4}-6ac\,d^{2}e^{2}-3c^{2}d^{4}\right)}{3(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3})(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{3}{2}}}$	138
orering	$-\frac{2\left(-8x^{2}c^{2}d^{2}e^{2}-4xacd\,e^{3}-12x\,c^{2}d^{3}e+a^{2}e^{4}-6ac\,d^{2}e^{2}-3c^{2}d^{4}\right)(cdx+ae)}{3(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3})(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{3}{2}}}$	139
default	$\frac{-\frac{2}{3\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}+\frac{8dec\left(2dec\left(x+\frac{d}{e}\right)+a\,e^{2}-c\,d^{2}\right)}{3\left(a\,e^{2}-c\,d^{2}\right)^{3}\sqrt{dec\left(x+\frac{d}{e}\right)^{2}+\left(a\,e^{2}-c\,d^{2}\right)\left(x+\frac{d}{e}\right)}}}{e}$	146
trager	$-\frac{2\left(-8x^{2}c^{2}d^{2}e^{2}-4xacd\,e^{3}-12x\,c^{2}d^{3}e+a^{2}e^{4}-6ac\,d^{2}e^{2}-3c^{2}d^{4}\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{3(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4})(ex+d)^{2}(a\,e^{2}-c\,d^{2})(cdx+ae)}$	146

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
-2/3*(c*d*x+a*e)*(-8*c^2*d^2*e^2*x^2-4*a*c*d*e^3*x-12*c^2*d^3*e*x+a^2*e^4-
6*a*c*d^2*e^2-3*c^2*d^4)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-c^3*d^6)
/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(113) = 226$.

Time = 1.18 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.53

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{2\left(8c^2d^2e^2x^2+3c^2d^4+6acd^2e^2-a^2e^4+4\left(3c^2d\right.\right.}{3\left(ac^3d^8e-3a^2c^2d^6e^3+3a^3cd^4e^5-a^4d^2e^7+(c^4d^7e^2-3ac^3d^5e^4+3a^2c^2d^3e^6-a^3cde^8)x^3+(2c^4d^8e-\right.}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
fricas")
```

output
```
-2/3*(8*c^2*d^2*e^2*x^2 + 3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4 + 4*(3*c^2*d
^3*e + a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^3*d^
8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3
*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a
*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^9
 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F]

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="
giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{2\,\sqrt{c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e}\,\left(-a^2\,e^4+6\,a\,c\,d^2\,e^2\right.}{3\,\left(a\,e+c\,d\,x\right)\,\left(a\,e^2-\right.}$$

input
```
int(1/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(3*c^2*d^4 - a^2*e^4 + 8*
c^2*d^2*e^2*x^2 + 6*a*c*d^2*e^2 + 12*c^2*d^3*e*x + 4*a*c*d*e^3*x))/(3*(a*e
 + c*d*x)*(a*e^2 - c*d^2)^3*(d + e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.67

$$\int \frac{1}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{-\frac{16\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,c\,d^3}{3} - \frac{32\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,c\,d^2ex}{3} - \frac{16\sqrt{e}\,\sqrt{d}}{}}{\sqrt{cdx+ae}\,\left(a^3e^8x^2-3a^2c\,d^2e^6x^2+3a\,c^2d^4\right.}$$

input
```
int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*( - 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d**3 - 16*sqrt(e)*sqr
t(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d**2*e*x - 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt
(a*e + c*d*x)*c*d*e**2*x**2 - sqrt(d + e*x)*a**2*e**4 + 6*sqrt(d + e*x)*a*
c*d**2*e**2 + 4*sqrt(d + e*x)*a*c*d*e**3*x + 3*sqrt(d + e*x)*c**2*d**4 + 1
2*sqrt(d + e*x)*c**2*d**3*e*x + 8*sqrt(d + e*x)*c**2*d**2*e**2*x**2))/(3*s
qrt(a*e + c*d*x)*(a**3*d**2*e**6 + 2*a**3*d*e**7*x + a**3*e**8*x**2 - 3*a*
*2*c*d**4*e**4 - 6*a**2*c*d**3*e**5*x - 3*a**2*c*d**2*e**6*x**2 + 3*a*c**2
*d**6*e**2 + 6*a*c**2*d**5*e**3*x + 3*a*c**2*d**4*e**4*x**2 - c**3*d**8 -
2*c**3*d**7*e*x - c**3*d**6*e**2*x**2))
```

3.118 $\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 286

$$\begin{aligned}\int \frac{1}{x(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx &= \frac{2cd}{ae\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}} \\ &+ \frac{2(3cd^2+ae^2)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3ad\left(cd^2-ae^2\right)^2(d+ex)^2} \\ &+ \frac{2(3cd^2-ae^2)\left(cd^2+3ae^2\right)\sqrt{ade+(cd^2+ae^2)\,x+cdex^2}}{3ad^2\left(cd^2-ae^2\right)^3(d+ex)} \\ &- \frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{3/2}d^{5/2}e^{3/2}}\end{aligned}$$

output
```
2*c*d/a/e/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2
/3*(a*e^2+3*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d/(-a*e^2+c*d
^2)^2/(e*x+d)^2+2/3*(-a*e^2+3*c*d^2)*(3*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/a/d^2/(-a*e^2+c*d^2)^3/(e*x+d)-2*arctanh(a^(1/2)*e^(1/2
)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(5/2)
/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.76

$$\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)\left(-3c^3d^5(d+ex)^2+a^3e^6(4d+3ex)-ac^2d^3e^3x(9d+8e\right.}{\left(-cd^2+ae^2\right)^3}\right.}{3a^{3/2}d^5}$$

input
```
Integrate[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-3*c^3*d^5*(d + e*x)^2 + a^3*e
^6*(4*d + 3*e*x) - a*c^2*d^3*e^3*x*(9*d + 8*e*x) + a^2*c*d*e^4*(-9*d^2 - 4
*d*e*x + 3*e^2*x^2)))/(-(c*d^2) + a*e^2)^3 - 3*(a*e + c*d*x)^(3/2)*(d + e*
x)^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x
])]))/(3*a^(3/2)*d^(5/2)*e^(3/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1246, 27, 1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1246}$$

$$\frac{2\int \frac{e\left(3\left(cd^2-ae^2\right)-4cdex\right)}{2x\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3de\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3\left(cd^2-ae^2\right)-4cdex}{x\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1235}$$

$$\frac{\frac{2(-3a^3e^6+7a^2cd^2e^4+ac^2d^4e^2+cdex(3cd^2-ae^2)(3ae^2+cd^2)+3c^3d^6)}{ade(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\int -\frac{3\left(cd^2-ae^2\right)^3}{2x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ade(cd^2-ae^2)^2}}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\left(cd^2-ae^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ade}+\frac{2(-3a^3e^6+7a^2cd^2e^4+ac^2d^4e^2+cdex(3cd^2-ae^2)(3ae^2+cd^2)+3c^3d^6)}{ade(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{2(-3a^3e^6+7a^2cd^2e^4+ac^2d^4e^2+cdex(3cd^2-ae^2)(3ae^2+cd^2)+3c^3d^6)}{ade(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{6\left(cd^2-ae^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{2(-3a^3e^6+7a^2cd^2e^4+ac^2d^4e^2+cdex(3cd^2-ae^2)(3ae^2+cd^2)+3c^3d^6)}{ade(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{3(cd^2-ae^2)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{a^{3/2}d^{3/2}e^{3/2}}}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3d\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input

```
Int[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2)^(3/2)) + ((2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 7*a^2*c*d^2*e^4 - 3*a^3*
e^6 + c*d*e*(3*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2)*x))/(a*d*e*(c*d^2 - a*e^2)
^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (3*(c*d^2 - a*e^2)*ArcTa
nh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])])/(a^(3/2)*d^(3/2)*e^(3/2)))/(3*d*(c*d^2 -
a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1246
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> Simp[(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p*
((c*d - b*e - c*e*x)/(p*(2*c*d - b*e)*(e*f - d*g))), x] + Simp[1/(p*(2*c*d
- b*e)*(e*f - d*g))   Int[(f + g*x)^n*(a + b*x + c*x^2)^p*(b*e*g*(n + p + 1
) + c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x), x], x]
/; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ
[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.26

method	result
default	$\frac{\frac{1}{ade\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}-\frac{\left(a\,e^{2}+c\,d^{2}\right)\left(2cdxe+a\,e^{2}+c\,d^{2}\right)}{ade\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}-\frac{\ln\left(\frac{2ade+\left(a\,e^{2}+c\,d^{2}\right)x+2\sqrt{ade}\,\sqrt{ade+\left(a\,e^{2}+\cdots\right.}}{x}\right.}{ade\sqrt{ade}}}{d}$

input
```
int(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/d*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(
2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(
a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-1/d*(-2/3/(a*e^2
-c*d^2)/(x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d*e*c/(a
*e^2-c*d^2)^3*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)
*(x+d/e))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(262) = 524$.

Time = 3.53 (sec) , antiderivative size = 1476, normalized size of antiderivative = 5.16

$$\int \frac{1}{x(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
[1/6*(3*(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 +
 (c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (
2*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^
9)*x^2 + (c^4*d^9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 -
2*a^4*d*e^8)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2
+ a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(3*
a*c^3*d^8*e + 9*a^3*c*d^4*e^5 - 4*a^4*d^2*e^7 + (3*a*c^3*d^6*e^3 + 8*a^2*c
^2*d^4*e^5 - 3*a^3*c*d^2*e^7)*x^2 + (6*a*c^3*d^7*e^2 + 9*a^2*c^2*d^5*e^4 +
 4*a^3*c*d^3*e^6 - 3*a^4*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2
)*x))/(a^3*c^3*d^11*e^3 - 3*a^4*c^2*d^9*e^5 + 3*a^5*c*d^7*e^7 - a^6*d^5*e^
9 + (a^2*c^4*d^10*e^4 - 3*a^3*c^3*d^8*e^6 + 3*a^4*c^2*d^6*e^8 - a^5*c*d^4*
e^10)*x^3 + (2*a^2*c^4*d^11*e^3 - 5*a^3*c^3*d^9*e^5 + 3*a^4*c^2*d^7*e^7 +
a^5*c*d^5*e^9 - a^6*d^3*e^11)*x^2 + (a^2*c^4*d^12*e^2 - a^3*c^3*d^10*e^4 -
 3*a^4*c^2*d^8*e^6 + 5*a^5*c*d^6*e^8 - 2*a^6*d^4*e^10)*x), 1/3*(3*(a*c^3*d
^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 -
3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*
a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5 + a^3*c*d^2*e^7 - a^4*e^9)*x^2 + (c^4*d^
9 - a*c^3*d^7*e^2 - 3*a^2*c^2*d^5*e^4 + 5*a^3*c*d^3*e^6 - 2*a^4*d*e^8)*x)*
sqrt(-a*d*e)*arctan(1/2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*...
```

Sympy [F]

$$\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(x*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{3}{2}}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x), x
)
```

Giac [F]

$$\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{3}{2}}(ex+d)x}\,dx$$

input
```
integrate(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x\,\left(d+e\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

input
```
int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(1/(x*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 3071, normalized size of antiderivative = 10.74

$$\int \frac{1}{x(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(3*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x)
 - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))*a**3*d**2*e**6 + 6*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(s
qrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*d*e**7*x + 3*sqrt(e)*sqrt(d)*sqrt(a)*
sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d
*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*e**8*x**2 - 9*
sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) -
sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e
*x))*a**2*c*d**4*e**4 - 18*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(s
qrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**3*e**5*x - 9*sqrt(e)*sqrt(d)*sqr
t(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt
(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c*d**2*e*
*6*x**2 + 9*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e
 + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c
)*sqrt(d + e*x))*a*c**2*d**6*e**2 + 18*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e +
c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2
 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**2*d**5*e**3*x + 9*sqrt(e)
*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt...
```

3.119 $\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 397

$$\begin{aligned}
&\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \\
&-\frac{c(3cd^2-ae^2)}{a^2e^2\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&-\frac{1}{adex(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&-\frac{\left(9c^2d^4-6acd^2e^2+5a^2e^4\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3a^2d^2e\left(cd^2-ae^2\right)^2(d+ex)^2} \\
&-\frac{\left(9c^3d^6-9ac^2d^4e^2+31a^2cd^2e^4-15a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3a^2d^3e\left(cd^2-ae^2\right)^3(d+ex)} \\
&+\frac{\left(3cd^2+5ae^2\right)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{5/2}d^{7/2}e^{5/2}}
\end{aligned}$$

output
```
-c*(-a*e^2+3*c*d^2)/a^2/e^2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)-1/a/d/e/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
-1/3*(5*a^2*e^4-6*a*c*d^2*e^2+9*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)/a^2/d^2/e/(-a*e^2+c*d^2)^2/(e*x+d)^2-1/3*(-15*a^3*e^6+31*a^2*c*d^2*
e^4-9*a*c^2*d^4*e^2+9*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2
/d^3/e/(-a*e^2+c*d^2)^3/(e*x+d)+(5*a*e^2+3*c*d^2)*arctanh(a^(1/2)*e^(1/2)*
(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(7/2)/e
^(5/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)\left(-9c^4d^7x(d+ex)^2-3ac^3d^5e(d-3ex)(d+ex)^2+a^4e^7\right.}{}}{}$$

input
```
Integrate[1/(x^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-9*c^4*d^7*x*(d + e*x)^2 - 3*a*
c^3*d^5*e*(d - 3*e*x)*(d + e*x)^2 + a^4*e^7*(3*d^2 + 20*d*e*x + 15*e^2*x^2
) + a^2*c^2*d^3*e^3*(9*d^3 + 9*d^2*e*x - 33*d*e^2*x^2 - 31*e^3*x^3) - a^3*
c*d*e^5*(9*d^3 + 39*d^2*e*x + 11*d*e^2*x^2 - 15*e^3*x^3)))/((-(c*d^2) + a*
e^2)^3*x)) + 3*(3*c*d^2 + 5*a*e^2)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*Arc
Tanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(3*a^(5
/2)*d^(7/2)*e^(5/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1246, 27, 1235, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 1246$$

$$\frac{2\int \frac{e\left(3cd^2-6cexd-5ae^2\right)}{2x^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3de\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3cd^2-6cexd-5ae^2}{x^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \ 1235$$

$$\frac{\frac{2\left(-5a^3e^6+cdex\left(-5a^2e^4+10acd^2e^2+3c^2d^4\right)+9a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)}{adex\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\int -\frac{9c^3d^6-9ac^2e^2d^4+31a^2ce^4d^2+2ce\left(3c^2d^4+10ace^2d^2-5a^2e^4\right)xd-\cdots}{2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\cdots}{ade\left(cd^2-ae^2\right)^2}}{3d\left(cd^2-ae^2\right)}\cdots$$

$$\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{9c^3d^6-9ac^2e^2d^4+31a^2ce^4d^2+2ce\left(3c^2d^4+10ace^2d^2-5a^2e^4\right)xd-15a^3e^6}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ade\left(cd^2-ae^2\right)^2}+\frac{2\left(-5a^3e^6+cdex\left(-5a^2e^4+10acd^2e^2+3c^2d^4\right)+9a^2cd^2e^4+ac^2d^4e^2+3\cdots\right.}{adex\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}}{3d\left(cd^2-ae^2\right)}\cdots$$

$$\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \ 1228$$

$$\frac{\frac{-\frac{3\left(5ae^2+3cd^2\right)\left(cd^2-ae^2\right)^3\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(-15a^3e^6+31a^2cd^2e^4-9ac^2d^4e^2+9c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{adex}}{ade\left(cd^2-ae^2\right)^2}+\frac{2\left(-5a^3e^6+cde\cdots\right.}{\cdots}}{3d\left(cd^2-ae^2\right)}\cdots$$

$$\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \ 1154$$

$$\frac{\frac{\frac{3\left(cd^2-ae^2\right)^3\left(5ae^2+3cd^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}-\frac{\left(-15a^3e^6+31a^2cd^2e^4-9ac^2d^4e^2+9c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+}}{adex}}{ade(cd^2-ae^2)^2}}{3d\left(cd^2-ae^2\right)}$$

$$\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{2\left(-5a^3e^6+cdex\left(-5a^2e^4+10acd^2e^2+3c^2d^4\right)+9a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)}{adex(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\frac{3\left(cd^2-ae^2\right)^3\left(5ae^2+3cd^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade-}}\right.}{2a^{3/2}d^{3/2}e^{3/2}}}{ade}}{3d\left(cd^2-ae^2\right)}$$

$$\frac{2e(ae+cdx)}{3dx\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input

```
Int[1/(x^2*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output

```
(-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x*(a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2)^(3/2)) + ((2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 5*a^
3*e^6 + c*d*e*(3*c^2*d^4 + 10*a*c*d^2*e^2 - 5*a^2*e^4)*x))/(a*d*e*(c*d^2 -
 a*e^2)^2*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (-(((9*c^3*d^6
- 9*a*c^2*d^4*e^2 + 31*a^2*c*d^2*e^4 - 15*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2])/(a*d*e*x)) + (3*(c*d^2 - a*e^2)^3*(3*c*d^2 + 5*a*e^2
)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])])/(2*a^(3/2)*d^(3/2)*e^(3/2)))/(a*d*
e*(c*d^2 - a*e^2)^2))/(3*d*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1246
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> Simp[(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p*
((c*d - b*e - c*e*x)/(p*(2*c*d - b*e)*(e*f - d*g))), x] + Simp[1/(p*(2*c*d
- b*e)*(e*f - d*g))   Int[(f + g*x)^n*(a + b*x + c*x^2)^p*(b*e*g*(n + p + 1
) + c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x), x], x]
/; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ
[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n, 0]
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.80

method	result
default	$\frac{-\frac{1}{a d e x \sqrt{a d e+\left(a e^{2}+c d^{2}\right) x+c d x^{2} e}}-\frac{3\left(a e^{2}+c d^{2}\right)\left(\frac{1}{a d e \sqrt{a d e+\left(a e^{2}+c d^{2}\right) x+c d x^{2} e}}-\frac{\left(a e^{2}+c d^{2}\right)\left(2 c d x e+a e^{2}+c d^{2}\right)}{a d e\left(4 a c d^{2} e^{2}-\left(a e^{2}+c d^{2}\right)^{2}\right) \sqrt{a d e+\left(a e^{2}+c d^{2}\right) x+c d}}\right.}{2 a d e}}{d}$

input
```
int(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
1/d*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/
a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e
*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-4*c/a*(2*c*d*e
*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*x^2*e)^(1/2))+e/d^2*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c
*d^2)*(x+d/e))^(1/2)+8/3*d*e*c/(a*e^2-c*d^2)^3*(2*d*e*c*(x+d/e)+a*e^2-c*d^
2)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))-e/d^2*(1/a/d/e/(a*d*e+(a
*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)
/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1
/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a
*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. 2(371) = 742.

Time = 10.84 (sec) , antiderivative size = 1812, normalized size of antiderivative = 4.56

$$\int \frac{1}{x^2(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[1/12*(3*((3*c^5*d^9*e^2 - 4*a*c^4*d^7*e^4 - 6*a^2*c^3*d^5*e^6 + 12*a^3*c^
2*d^3*e^8 - 5*a^4*c*d*e^10)*x^4 + (6*c^5*d^10*e - 5*a*c^4*d^8*e^3 - 16*a^2
*c^3*d^6*e^5 + 18*a^3*c^2*d^4*e^7 + 2*a^4*c*d^2*e^9 - 5*a^5*e^11)*x^3 + (3
*c^5*d^11 + 2*a*c^4*d^9*e^2 - 14*a^2*c^3*d^7*e^4 + 19*a^4*c*d^3*e^8 - 10*a
^5*d*e^10)*x^2 + (3*a*c^4*d^10*e - 4*a^2*c^3*d^8*e^3 - 6*a^3*c^2*d^6*e^5 +
 12*a^4*c*d^4*e^7 - 5*a^5*d^2*e^9)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^
2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 +
 a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*
d*e^3)*x)/x^2) - 4*(3*a^2*c^3*d^9*e^2 - 9*a^3*c^2*d^7*e^4 + 9*a^4*c*d^5*e^
6 - 3*a^5*d^3*e^8 + (9*a*c^4*d^8*e^3 - 9*a^2*c^3*d^6*e^5 + 31*a^3*c^2*d^4*
e^7 - 15*a^4*c*d^2*e^9)*x^3 + (18*a*c^4*d^9*e^2 - 15*a^2*c^3*d^7*e^4 + 33*
a^3*c^2*d^5*e^6 + 11*a^4*c*d^3*e^8 - 15*a^5*d*e^10)*x^2 + (9*a*c^4*d^10*e
- 3*a^2*c^3*d^8*e^3 - 9*a^3*c^2*d^6*e^5 + 39*a^4*c*d^4*e^7 - 20*a^5*d^2*e^
9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a^3*c^4*d^11*e^5 - 3*
a^4*c^3*d^9*e^7 + 3*a^5*c^2*d^7*e^9 - a^6*c*d^5*e^11)*x^4 + (2*a^3*c^4*d^1
2*e^4 - 5*a^4*c^3*d^10*e^6 + 3*a^5*c^2*d^8*e^8 + a^6*c*d^6*e^10 - a^7*d^4*
e^12)*x^3 + (a^3*c^4*d^13*e^3 - a^4*c^3*d^11*e^5 - 3*a^5*c^2*d^9*e^7 + 5*a
^6*c*d^7*e^9 - 2*a^7*d^5*e^11)*x^2 + (a^4*c^3*d^12*e^4 - 3*a^5*c^2*d^10*e^
6 + 3*a^6*c*d^8*e^8 - a^7*d^6*e^10)*x), -1/6*(3*((3*c^5*d^9*e^2 - 4*a*c^4*
d^7*e^4 - 6*a^2*c^3*d^5*e^6 + 12*a^3*c^2*d^3*e^8 - 5*a^4*c*d*e^10)*x^4 ...
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x^2\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(x**2*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^2),
 x)
```

Giac [F]

$$\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}(ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^2),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x^2\,\left(d+e\,x\right)\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

input
```
int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\,(ex+d)\,\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}\,dx$$

input
```
int(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
int(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

3.120 $\int \frac{1}{x^3(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 512

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{c(15c^2d^4-7a^2e^4)}{4a^3de^3(cd^2-ae^2)(d+ex)\sqrt{ade+(cd^2+ae^2)x+c}}$$
$$-\frac{1}{2adex^2(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$
$$+\frac{\frac{7a}{d^2}+\frac{5c}{e^2}}{4a^2x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}$$
$$+\frac{(45c^3d^6-15ac^2d^4e^2-33a^2cd^2e^4+35a^3e^6)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12a^3d^3e^2(cd^2-ae^2)^2(d+ex)^2}$$
$$+\frac{(45c^4d^8-30ac^3d^6e^2-36a^2c^2d^4e^4+190a^3cd^2e^6-105a^4e^8)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{12a^3d^4e^2(cd^2-ae^2)^3(d+ex)}$$
$$-\frac{5(3c^2d^4+6acd^2e^2+7a^2e^4)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{4a^{7/2}d^{9/2}e^{7/2}}$$

output
```
1/4*c*(-7*a^2*e^4+15*c^2*d^4)/a^3/d/e^3/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/2/a/d/e/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)+1/4*(7*a/d^2+5*c/e^2)/a^2/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)+1/12*(35*a^3*e^6-33*a^2*c*d^2*e^4-15*a*c^2*d^4*e^2+45*c
^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d^3/e^2/(-a*e^2+c*d^2)
^2/(e*x+d)^2+1/12*(-105*a^4*e^8+190*a^3*c*d^2*e^6-36*a^2*c^2*d^4*e^4-30*a*
c^3*d^6*e^2+45*c^4*d^8)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^3/d^4/e^
2/(-a*e^2+c*d^2)^3/(e*x+d)-5/4*(7*a^2*e^4+6*a*c*d^2*e^2+3*c^2*d^4)*arctanh
(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/
a^(7/2)/d^(9/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)(-45c^5d^9x^2(d+ex)^2-15ac^4d^7ex(d-2ex)(d+ex)^2+6a}{}$$

input
```
Integrate[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-45*c^5*d^9*x^2*(d + e*x)^2 - 15*
a*c^4*d^7*e*x*(d - 2*e*x)*(d + e*x)^2 + 6*a^2*c^3*d^5*e^2*(d + e*x)^2*(d^2
 + 2*d*e*x + 6*e^2*x^2) + a^5*e^8*(-6*d^3 + 21*d^2*e*x + 140*d*e^2*x^2 + 1
05*e^3*x^3) - 2*a^3*c^2*d^3*e^4*(9*d^4 - 9*d^3*e*x - 6*d^2*e^2*x^2 + 111*d
*e^3*x^3 + 95*e^4*x^4) + a^4*c*d*e^6*(18*d^4 - 48*d^3*e*x - 237*d^2*e^2*x^
2 - 50*d*e^3*x^3 + 105*e^4*x^4)))/((-(c*d^2) + a*e^2)^3*x^2) - 15*(3*c^2*d
^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTan
h[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(12*a^(7/2
)*d^(9/2)*e^(7/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {1246, 27, 1235, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1246}$$

$$\frac{2\int \frac{e\left(3cd^2-8cexd-7ae^2\right)}{2x^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3de\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3cd^2-8cexd-7ae^2}{x^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1235}$$

$$\frac{\frac{2\left(-7a^3e^6+cdex\left(-7a^2e^4+12acd^2e^2+3c^2d^4\right)+11a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)}{adex^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\int -\frac{15c^3d^6-9ac^2e^2d^4+61a^2ce^4d^2+4ce\left(3c^2d^4+12ace^2d^2-7a^2e^4\right)x\cdots}{2x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\cdots}{ade\left(cd^2-ae^2\right)^2}}{3d\left(cd^2-ae^2\right)}$$

$$\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{15c^3d^6-9ac^2e^2d^4+61a^2ce^4d^2+4ce\left(3c^2d^4+12ace^2d^2-7a^2e^4\right)xd-35a^3e^6}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ade\left(cd^2-ae^2\right)^2}+\frac{2\left(-7a^3e^6+cdex\left(-7a^2e^4+12acd^2e^2+3c^2d^4\right)+11a^2cd^2e^4+ac^2d^4e^2\cdots\right)}{adex^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}}{3d\left(cd^2-ae^2\right)}$$

$$\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1237}$$

$$-\frac{-\frac{\int \frac{45c^4d^8-30ac^3e^2d^6-36a^2c^2e^4d^4+190a^3ce^6d^2+2ce\left(15c^3d^6-9ac^2e^2d^4+61a^2ce^4d^2-35a^3e^6\right)xd-105a^4e^8}{2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(-35a^3e^6+61a^2cd^2e^4-9ac^2d^4e^2+15c^3d^6\right)}{2adex^2}}{ade(cd^2-ae^2)^2}}{3d\left(cd^2-ae^2\right)}$$

$$\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{45c^4d^8-30ac^3e^2d^6-36a^2c^2e^4d^4+190a^3ce^6d^2+2ce\left(15c^3d^6-9ac^2e^2d^4+61a^2ce^4d^2-35a^3e^6\right)xd-105a^4e^8}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4ade}-\frac{\left(-35a^3e^6+61a^2cd^2e^4-9ac^2d^4e^2+15c^3d^6\right)}{2adex^2}}{ade(cd^2-ae^2)^2}}{3d\left(cd^2-ae^2\right)}$$

$$\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 1228

$$-\frac{-\frac{-\frac{15\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\left(cd^2-ae^2\right)^3\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(-105a^4e^8+190a^3cd^2e^6-36a^2c^2d^4e^4-30ac^3d^6e^2+45c^4d^8\right)\sqrt{x\left(ae^2+cd^2\right)+}}{adex}}{4ade}}{ade(cd^2-ae^2)^2}}{3d}$$

$$\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 1154

$$-\frac{\frac{\frac{15\left(cd^2-ae^2\right)^3\left(7a^2e^4+6acd^2e^2+3c^2d^4\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}-\frac{\left(-105a^4e^8+190a^3cd^2e^6-36a^2c^2d^4e^4-30ac^3d^6\right.}{adex}}{4ade}}{ade(cd^2-ae^2)^2}}{}$$

$$\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{2\left(-7a^3e^6+cdex\left(-7a^2e^4+12acd^2e^2+3c^2d^4\right)+11a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)}{adex^2\left(cd^2-ae^2\right)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{-\frac{\left(-35a^3e^6+61a^2cd^2e^4-9ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{2adex^2}}{}$$

$$\frac{2e(ae+cdx)}{3dx^2\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2)) + ((2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 11*a^2*c*d^2*e^4 - 7
*a^3*e^6 + c*d*e*(3*c^2*d^4 + 12*a*c*d^2*e^2 - 7*a^2*e^4)*x))/(a*d*e*(c*d^
2 - a*e^2)^2*x^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (-1/2*((15
*c^3*d^6 - 9*a*c^2*d^4*e^2 + 61*a^2*c*d^2*e^4 - 35*a^3*e^6)*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^2) - (-(((45*c^4*d^8 - 30*a*c^3*d^
6*e^2 - 36*a^2*c^2*d^4*e^4 + 190*a^3*c*d^2*e^6 - 105*a^4*e^8)*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x)) + (15*(c*d^2 - a*e^2)^3*(3*c^2
*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2
*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(2
*a^(3/2)*d^(3/2)*e^(3/2)))/(4*a*d*e))/(a*d*e*(c*d^2 - a*e^2)^2))/(3*d*(c*d
^2 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1246
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> Simp[(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p*
((c*d - b*e - c*e*x)/(p*(2*c*d - b*e)*(e*f - d*g))), x] + Simp[1/(p*(2*c*d
- b*e)*(e*f - d*g))   Int[(f + g*x)^n*(a + b*x + c*x^2)^p*(b*e*g*(n + p + 1
) + c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x), x], x]
/; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ
[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1353 vs. $2(476) = 952$.

Time = 2.68 (sec) , antiderivative size = 1354, normalized size of antiderivative = 2.64

method	result	size
default	Expression too large to display	1354

input
```
int(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
1/d*(-1/2/a/d/e/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-5/4*(a*e^2+c*d
^2)/a/d/e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c
*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)
/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^
2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-4*c/a*(2
*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)
*x+c*d*x^2*e)^(1/2))-3/2*c/a*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^
2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*
a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2))/x)))+e^2/d^3*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+
c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^
2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-e/
d^2*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/
a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e
*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-4*c/a*(2*c*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(476) = 952$.

Time = 23.67 (sec) , antiderivative size = 2162, normalized size of antiderivative = 4.22

$$\int \frac{1}{x^3(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[1/48*(15*((3*c^6*d^11*e^2 - 3*a*c^5*d^9*e^4 - 2*a^2*c^4*d^7*e^6 - 6*a^3*c
^3*d^5*e^8 + 15*a^4*c^2*d^3*e^10 - 7*a^5*c*d*e^12)*x^5 + (6*c^6*d^12*e - 3
*a*c^5*d^10*e^3 - 7*a^2*c^4*d^8*e^5 - 14*a^3*c^3*d^6*e^7 + 24*a^4*c^2*d^4*
e^9 + a^5*c*d^2*e^11 - 7*a^6*e^13)*x^4 + (3*c^6*d^13 + 3*a*c^5*d^11*e^2 -
8*a^2*c^4*d^9*e^4 - 10*a^3*c^3*d^7*e^6 + 3*a^4*c^2*d^5*e^8 + 23*a^5*c*d^3*
e^10 - 14*a^6*d*e^12)*x^3 + (3*a*c^5*d^12*e - 3*a^2*c^4*d^10*e^3 - 2*a^3*c
^3*d^8*e^5 - 6*a^4*c^2*d^6*e^7 + 15*a^5*c*d^4*e^9 - 7*a^6*d^2*e^11)*x^2)*s
qrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 -
4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x
)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(6*a^3*c^3*d^10*e^3
- 18*a^4*c^2*d^8*e^5 + 18*a^5*c*d^6*e^7 - 6*a^6*d^4*e^9 - (45*a*c^5*d^10*e
^3 - 30*a^2*c^4*d^8*e^5 - 36*a^3*c^3*d^6*e^7 + 190*a^4*c^2*d^4*e^9 - 105*a
^5*c*d^2*e^11)*x^4 - (90*a*c^5*d^11*e^2 - 45*a^2*c^4*d^9*e^4 - 84*a^3*c^3*
d^7*e^6 + 222*a^4*c^2*d^5*e^8 + 50*a^5*c*d^3*e^10 - 105*a^6*d*e^12)*x^3 -
(45*a*c^5*d^12*e - 66*a^3*c^3*d^8*e^5 - 12*a^4*c^2*d^6*e^7 + 237*a^5*c*d^4
*e^9 - 140*a^6*d^2*e^11)*x^2 - 3*(5*a^2*c^4*d^11*e^2 - 8*a^3*c^3*d^9*e^4 -
 6*a^4*c^2*d^7*e^6 + 16*a^5*c*d^5*e^8 - 7*a^6*d^3*e^10)*x)*sqrt(c*d*e*x^2
+ a*d*e + (c*d^2 + a*e^2)*x))/((a^4*c^4*d^12*e^6 - 3*a^5*c^3*d^10*e^8 + 3*
a^6*c^2*d^8*e^10 - a^7*c*d^6*e^12)*x^5 + (2*a^4*c^4*d^13*e^5 - 5*a^5*c^3*d
^11*e^7 + 3*a^6*c^2*d^9*e^9 + a^7*c*d^7*e^11 - a^8*d^5*e^13)*x^4 + (a^4...
```

Sympy [F]

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x^3\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(1/x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(x**3*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^3),
 x)
```

Giac [F]

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^3),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{x^3\,\left(d+e\,x\right)\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(1/(x^3*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 5923, normalized size of antiderivative = 11.57

$$\int \frac{1}{x^3(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(315*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*
x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(
d + e*x))*a**6*d**2*e**12*x**2 + 630*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*
d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 +
 c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**6*d*e**13*x**3 + 315*sqrt(e)*
sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*s
qrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**
6*e**14*x**4 - 570*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*s
qrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)
*sqrt(c)*sqrt(d + e*x))*a**5*c*d**4*e**10*x**2 - 1140*sqrt(e)*sqrt(d)*sqrt
(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(
a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**5*c*d**3*e**
11*x**3 - 570*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a
*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt
(c)*sqrt(d + e*x))*a**5*c*d**2*e**12*x**4 + 45*sqrt(e)*sqrt(d)*sqrt(a)*sqr
t(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e
+ a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**4*c**2*d**6*e**8*x*
*2 + 90*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c
*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sq
rt(d + e*x))*a**4*c**2*d**5*e**9*x**3 + 45*sqrt(e)*sqrt(d)*sqrt(a)*sqrt...
```

3.121 $\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 657

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx =$$
$$-\frac{c(35c^3d^6+5ac^2d^4e^2-3a^2cd^2e^4-21a^3e^6)}{8a^4d^2e^4\left(cd^2-ae^2\right)(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$-\frac{1}{3adex^3(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$+\frac{\frac{9a}{d^2}+\frac{7c}{e^2}}{12a^2x^2(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$-\frac{35c^2d^4+54acd^2e^2+63a^2e^4}{24a^3d^3e^3x(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$-\frac{\left(105c^4d^8-20ac^3d^6e^2-42a^2c^2d^4e^4-84a^3cd^2e^6+105a^4e^8\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{24a^4d^4e^3\left(cd^2-ae^2\right)^2(d+ex)^2}$$
$$-\frac{\left(105c^5d^{10}-55ac^4d^8e^2-54a^2c^3d^6e^4-78a^3c^2d^4e^6+525a^4cd^2e^8-315a^5e^{10}\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cd}}{24a^4d^5e^3\left(cd^2-ae^2\right)^3(d+ex)}$$
$$+\frac{5(7c^3d^6+15ac^2d^4e^2+21a^2cd^2e^4+21a^3e^6)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{8a^{9/2}d^{11/2}e^{9/2}}$$

output
```
-1/8*c*(-21*a^3*e^6-3*a^2*c*d^2*e^4+5*a*c^2*d^4*e^2+35*c^3*d^6)/a^4/d^2/e^
4/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/3/a/d/e
/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/12*(9*a/d^2+7*c/e^2
)/a^2/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-1/24*(63*a^2*e^4
+54*a*c*d^2*e^2+35*c^2*d^4)/a^3/d^3/e^3/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)-1/24*(105*a^4*e^8-84*a^3*c*d^2*e^6-42*a^2*c^2*d^4*e^4-20*a
*c^3*d^6*e^2+105*c^4*d^8)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^4/d^4/
e^3/(-a*e^2+c*d^2)^2/(e*x+d)^2-1/24*(-315*a^5*e^10+525*a^4*c*d^2*e^8-78*a^
3*c^2*d^4*e^6-54*a^2*c^3*d^6*e^4-55*a*c^4*d^8*e^2+105*c^5*d^10)*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^4/d^5/e^3/(-a*e^2+c*d^2)^3/(e*x+d)+5/8*(21
*a^3*e^6+21*a^2*c*d^2*e^4+15*a*c^2*d^4*e^2+7*c^3*d^6)*arctanh(a^(1/2)*e^(1
/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(9/2)/d^(11
/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 493, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^4(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{d}\sqrt{e}\left(-105c^6d^{11}x^3(d+ex)^2-5ac^5d^9ex^2(7d-11ex)(d+ex)^2+a^2c^4d^7\cdots\right.}{\cdots}$$

input
```
Integrate[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
((Sqrt[a]*Sqrt[d]*Sqrt[e]*(-105*c^6*d^11*x^3*(d + e*x)^2 - 5*a*c^5*d^9*e*x
^2*(7*d - 11*e*x)*(d + e*x)^2 + a^2*c^4*d^7*e^2*x*(d + e*x)^2*(14*d^2 + 23
*d*e*x + 54*e^2*x^2) - 2*a^3*c^3*d^5*e^3*(d + e*x)^2*(4*d^3 + 4*d^2*e*x -
9*d*e^2*x^2 - 39*e^3*x^3) + a^6*e^9*(8*d^4 - 18*d^3*e*x + 63*d^2*e^2*x^2 +
 420*d*e^3*x^3 + 315*e^4*x^4) + a^4*c^2*d^3*e^5*(24*d^5 - 12*d^4*e*x + 62*
d^3*e^2*x^2 + 3*d^2*e^3*x^3 - 636*d*e^4*x^4 - 525*e^5*x^5) - a^5*c*d*e^7*(
24*d^5 - 40*d^4*e*x + 135*d^3*e^2*x^2 + 651*d^2*e^3*x^3 + 105*d*e^4*x^4 -
315*e^5*x^5)))/((c*d^2 - a*e^2)^3*x^3*(d + e*x)) + 15*(7*c^3*d^6 + 15*a*c^
2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]
*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(24
*a^(9/2)*d^(11/2)*e^(9/2)*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.275, Rules used = {1246, 27, 1235, 27, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1246}$$

$$\frac{2\int \frac{e\left(3\left(cd^2-3ae^2\right)-10cdex\right)}{2x^4\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3de\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3\left(cd^2-3ae^2\right)-10cdex}{x^4\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3d\left(cd^2-ae^2\right)}-\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1235}$$

$$\frac{\frac{2\left(-9a^3e^6+cdex\left(-9a^2e^4+14acd^2e^2+3c^2d^4\right)+13a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)}{adex^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2\int -\frac{3\left(7c^3d^6-3ac^2e^2d^4+33a^2ce^4d^2+2ce\left(3c^2d^4+14ace^2d^2-9a^2e^4\right)\right.}{2x^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade\left(cd^2-ae^2\right)^2}}{3d\left(cd^2-ae^2\right)}$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{7c^3d^6-3ac^2e^2d^4+33a^2ce^4d^2+2ce\left(3c^2d^4+14ace^2d^2-9a^2e^4\right)xd-21a^3e^6}{x^4\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ade\left(cd^2-ae^2\right)^2}+\frac{2\left(-9a^3e^6+cdex\left(-9a^2e^4+14acd^2e^2+3c^2d^4\right)+13a^2cd^2e^4+ac^2d^4e^2\right.}{adex^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}}{3d\left(cd^2-ae^2\right)}$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \text{1237}$$

$$\frac{3\left(-\frac{\int \frac{35c^4d^8-16ac^3e^2d^6-18a^2c^2e^4d^4+168a^3ce^6d^2+4ce\left(7c^3d^6-3ac^2e^2d^4+33a^2ce^4d^2-21a^3e^6\right)xd-105a^4e^8}{2x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{3ade}-\frac{\left(-21a^3e^6+33a^2cd^2e^4-3ac^2d^4e^2+7c^3d^6\right.}{3adex^3}\right.}{ade\left(cd^2-ae^2\right)^2}$$

$$3d\left(cd^2-ae^2\right)$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(-\frac{\int \frac{35c^4d^8-16ac^3e^2d^6-18a^2c^2e^4d^4+168a^3ce^6d^2+4ce\left(7c^3d^6-3ac^2e^2d^4+33a^2ce^4d^2-21a^3e^6\right)xd-105a^4e^8}{x^3\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{6ade}-\frac{\left(-21a^3e^6+33a^2cd^2e^4-3ac^2d^4e^2+7c^3d^6\right.}{3adex^3}\right.}{ade\left(cd^2-ae^2\right)^2}$$

$$3d\left(cd^2-ae^2\right)$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1237$$

$$\frac{3\left(-\frac{-\frac{\int \frac{105c^5d^{10}-55ac^4e^2d^8-54a^2c^3e^4d^6-78a^3c^2e^6d^4+525a^4ce^8d^2+2ce\left(35c^4d^8-16ac^3e^2d^6-18a^2c^2e^4d^4+168a^3ce^6d^2-105a^4e^8\right)xd-315a^5e^{10}}{2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{(-\cdots}{\cdots}}{6ade}\right.}{ade(cd\cdots}$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(-\frac{-\frac{\int \frac{105c^5d^{10}-55ac^4e^2d^8-54a^2c^3e^4d^6-78a^3c^2e^6d^4+525a^4ce^8d^2+2ce\left(35c^4d^8-16ac^3e^2d^6-18a^2c^2e^4d^4+168a^3ce^6d^2-105a^4e^8\right)xd-315a^5e^{10}}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{4ade}-\frac{(-\cdots}{\cdots}}{6ade}\right.}{ade(cd\cdots}$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1228$$

$$3\left(-\frac{-\frac{-\frac{15\left(21a^3e^6+21a^2cd^2e^4+15ac^2d^4e^2+7c^3d^6\right)\left(cd^2-ae^2\right)^3\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(-315a^5e^{10}+525a^4cd^2e^8-78a^3c^2d^4e^6-54a^2c^3d^6e^4-\cdots\right.}{adex\cdots}}{4ade}\cdots}{6ade}\cdots\right.$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1154$$

$$3\left(-\frac{-\frac{15\left(cd^2-ae^2\right)^3\left(21a^3e^6+21a^2cd^2e^4+15ac^2d^4e^2+7c^3d^6\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}-\frac{\left(-315a^5e^{10}+525a^4cd^2e^8\cdots\right.}{\cdots}}{4ade}\cdots\right.$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{2\left(-9a^3e^6+cdex\left(-9a^2e^4+14acd^2e^2+3c^2d^4\right)+13a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)}{adex^3\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{3\left(-\frac{\left(-21a^3e^6+33a^2cd^2e^4-3ac^2d^4e^2+7c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+\cdots}}{3adex^3}\cdots\right.}{\cdots}$$

$$\frac{2e(ae+cdx)}{3dx^3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^3*(a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2)^(3/2)) + ((2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 13*a^2*c*d^2*e^4 - 9
*a^3*e^6 + c*d*e*(3*c^2*d^4 + 14*a*c*d^2*e^2 - 9*a^2*e^4)*x))/(a*d*e*(c*d^
2 - a*e^2)^2*x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (3*(-1/3*(
(7*c^3*d^6 - 3*a*c^2*d^4*e^2 + 33*a^2*c*d^2*e^4 - 21*a^3*e^6)*Sqrt[a*d*e +
 (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^3) - (-1/2*((35*c^4*d^8 - 16*a*c
^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 + 168*a^3*c*d^2*e^6 - 105*a^4*e^8)*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x^2) - (-(((105*c^5*d^10 - 55
*a*c^4*d^8*e^2 - 54*a^2*c^3*d^6*e^4 - 78*a^3*c^2*d^4*e^6 + 525*a^4*c*d^2*e
^8 - 315*a^5*e^10)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d*e*x))
 + (15*(c*d^2 - a*e^2)^3*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4
+ 21*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqr
t[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(3/2)*d^(3/2)*e^(
3/2)))/(4*a*d*e))/(6*a*d*e)))/(a*d*e*(c*d^2 - a*e^2)^2))/(3*d*(c*d^2 - a*e
^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1246
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((
d_) + (e_.)*(x_)), x_Symbol] :> Simp[(f + g*x)^(n + 1)*(a + b*x + c*x^2)^p*
((c*d - b*e - c*e*x)/(p*(2*c*d - b*e)*(e*f - d*g))), x] + Simp[1/(p*(2*c*d
- b*e)*(e*f - d*g))   Int[(f + g*x)^n*(a + b*x + c*x^2)^p*(b*e*g*(n + p + 1
) + c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x), x], x]
/; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ
[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2408 vs. $2(617) = 1234$.

Time = 3.50 (sec) , antiderivative size = 2409, normalized size of antiderivative = 3.67

method	result	size
default	Expression too large to display	2409

input
```
int(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
1/d*(-1/3/a/d/e/x^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-7/6*(a*e^2+c*d
^2)/a/d/e*(-1/2/a/d/e/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-5/4*(a*e
^2+c*d^2)/a/d/e*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a
*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+
c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^
2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-4*
c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+
c*d^2)*x+c*d*x^2*e)^(1/2))-3/2*c/a*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^
2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*
ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2))/x)))-4/3*c/a*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^
2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d
*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)
)/x))-4*c/a*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))+e^2/d^3*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^
2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. $2(617) = 1234$.

Time = 51.27 (sec) , antiderivative size = 2526, normalized size of antiderivative = 3.84

$$\int \frac{1}{x^4(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output

```
[1/96*(15*((7*c^7*d^13*e^2 - 6*a*c^6*d^11*e^4 - 3*a^2*c^5*d^9*e^6 - 4*a^3*
c^4*d^7*e^8 - 15*a^4*c^3*d^5*e^10 + 42*a^5*c^2*d^3*e^12 - 21*a^6*c*d*e^14)
*x^6 + (14*c^7*d^14*e - 5*a*c^6*d^12*e^3 - 12*a^2*c^5*d^10*e^5 - 11*a^3*c^
4*d^8*e^7 - 34*a^4*c^3*d^6*e^9 + 69*a^5*c^2*d^4*e^11 - 21*a^7*e^15)*x^5 +
(7*c^7*d^15 + 8*a*c^6*d^13*e^2 - 15*a^2*c^5*d^11*e^4 - 10*a^3*c^4*d^9*e^6
- 23*a^4*c^3*d^7*e^8 + 12*a^5*c^2*d^5*e^10 + 63*a^6*c*d^3*e^12 - 42*a^7*d*
e^14)*x^4 + (7*a*c^6*d^14*e - 6*a^2*c^5*d^12*e^3 - 3*a^3*c^4*d^10*e^5 - 4*
a^4*c^3*d^8*e^7 - 15*a^5*c^2*d^6*e^9 + 42*a^6*c*d^4*e^11 - 21*a^7*d^2*e^13
)*x^3)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4
)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 +
a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(8*a^4*c^3*d
^11*e^4 - 24*a^5*c^2*d^9*e^6 + 24*a^6*c*d^7*e^8 - 8*a^7*d^5*e^10 + (105*a*
c^6*d^12*e^3 - 55*a^2*c^5*d^10*e^5 - 54*a^3*c^4*d^8*e^7 - 78*a^4*c^3*d^6*e
^9 + 525*a^5*c^2*d^4*e^11 - 315*a^6*c*d^2*e^13)*x^5 + (210*a*c^6*d^13*e^2
- 75*a^2*c^5*d^11*e^4 - 131*a^3*c^4*d^9*e^6 - 174*a^4*c^3*d^7*e^8 + 636*a^
5*c^2*d^5*e^10 + 105*a^6*c*d^3*e^12 - 315*a^7*d*e^14)*x^4 + (105*a*c^6*d^1
4*e + 15*a^2*c^5*d^12*e^3 - 114*a^3*c^4*d^10*e^5 - 106*a^4*c^3*d^8*e^7 - 3
*a^5*c^2*d^6*e^9 + 651*a^6*c*d^4*e^11 - 420*a^7*d^2*e^13)*x^3 + (35*a^2*c^
5*d^13*e^2 - 51*a^3*c^4*d^11*e^4 + 6*a^4*c^3*d^9*e^6 - 62*a^5*c^2*d^7*e^8
+ 135*a^6*c*d^5*e^10 - 63*a^7*d^3*e^12)*x^2 - 2*(7*a^3*c^4*d^12*e^3 - 1...
```

Sympy [F]

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x^4\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(1/x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(x**4*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{3}{2}}(ex+d)x^4}\,dx$$

input
```
integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^4),
 x)
```

Giac [F]

$$\int \frac{1}{x^4(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{\left(cdex^2+ade+(cd^2+ae^2)x\right)^{\frac{3}{2}}(ex+d)x^4}\,dx$$

input
```
integrate(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^4),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x^4\ (d+e\,x)\ \left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(1/(x^4*(d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 20.93 (sec) , antiderivative size = 7071, normalized size of antiderivative = 10.76

$$\int \frac{1}{x^4(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
( - 630*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c
*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sq
rt(d + e*x))*a**7*d**2*e**14*x**3 - 1260*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e
+ c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e*
*2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**7*d*e**15*x**4 - 630*sqrt
(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt
(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))
*a**7*e**16*x**5 + 945*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(
e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqr
t(d)*sqrt(c)*sqrt(d + e*x))*a**6*c*d**4*e**12*x**3 + 1890*sqrt(e)*sqrt(d)*
sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*s
qrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**6*c*d**3
*e**13*x**4 + 945*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sq
rt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*
sqrt(c)*sqrt(d + e*x))*a**6*c*d**2*e**14*x**5 + 180*sqrt(e)*sqrt(d)*sqrt(a
)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)
*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**5*c**2*d**6*e*
*10*x**3 + 360*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(
a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqr
t(c)*sqrt(d + e*x))*a**5*c**2*d**5*e**11*x**4 + 180*sqrt(e)*sqrt(d)*sqr...
```

3.122 $\int \frac{x^5}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 462

$$\int \frac{x^5}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2a^5e^5}{c^5d^5\left(cd^2-ae^2\right)(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}$$
$$+\frac{\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{c^2d^2e^4}+\frac{2(c^5d^{10}+5a^5e^{10})\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5c^5d^5e^4\left(cd^2-ae^2\right)^2(d+ex)^3}$$
$$-\frac{2(16c^5d^{10}-25ac^4d^8e^2-15a^5e^{10})\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^4d^4e^4\left(cd^2-ae^2\right)^3(d+ex)^2}$$
$$+\frac{2(58c^5d^{10}-175ac^4d^8e^2+150a^2c^3d^6e^4+15a^5e^{10})\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^3d^3e^4\left(cd^2-ae^2\right)^4(d+ex)}$$
$$-\frac{\left(7cd^2+3ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{5/2}d^{5/2}e^{9/2}}$$

output
```
2*a^5*e^5/c^5/d^5/(-a*e^2+c*d^2)/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^4+2/5*(5*a^5*e^
10+c^5*d^10)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^5/d^5/e^4/(-a*e^2+c
*d^2)^2/(e*x+d)^3-2/15*(-15*a^5*e^10-25*a*c^4*d^8*e^2+16*c^5*d^10)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^4/(-a*e^2+c*d^2)^3/(e*x+d)^2+2/
15*(15*a^5*e^10+150*a^2*c^3*d^6*e^4-175*a*c^4*d^8*e^2+58*c^5*d^10)*(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4/(-a*e^2+c*d^2)^4/(e*x+d)-(3*a
*e^2+7*c*d^2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(5/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(d+ex)^2 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)\left(45a^5e^9(d+ex)^3-15a^4cde^7(4d-ex)(d+ex)^3+c^5d^9x(105\right.}{}$$

input
```
Integrate[x^5/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(45*a^5*e^9*(d + e*x)^3 - 15*a^4*c
*d*e^7*(4*d - e*x)*(d + e*x)^3 + c^5*d^9*x*(105*d^3 + 245*d^2*e*x + 161*d*
e^2*x^2 + 15*e^3*x^3) + a*c^4*d^7*e*(105*d^4 - 95*d^3*e*x - 637*d^2*e^2*x^
2 - 515*d*e^3*x^3 - 60*e^4*x^4) + 2*a^3*c^2*d^3*e^5*(173*d^4 + 380*d^3*e*x
 + 195*d^2*e^2*x^2 - 45*d*e^3*x^3 - 30*e^4*x^4) + 2*a^2*c^3*d^5*e^3*(-170*
d^4 - 226*d^3*e*x + 145*d^2*e^2*x^2 + 255*d*e^3*x^3 + 45*e^4*x^4)))/((c*d^
2 - a*e^2)^4*(d + e*x)) - 15*(7*c*d^2 + 3*a*e^2)*(a*e + c*d*x)^(3/2)*(d +
e*x)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d
*x])])/(15*c^(5/2)*d^(5/2)*e^(9/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.41, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1267, 27, 2167, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(d+ex)^2 \left(x \left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \ 1267$$

$$\frac{\int \frac{-\left(\left(7cd^2+3ae^2\right)x^4e^5\right)-4d\left(2cd^2+3ae^2\right)x^3e^4-2d^2\left(cd^2+9ae^2\right)x^2e^3+2d^3\left(cd^2-6ae^2\right)xe^2+d^4\left(cd^2-3ae^2\right)e}{2(d+ex)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{cde^6}+\frac{(d+ex)^2}{cde^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{-\left(\left(7cd^2+3ae^2\right)x^4e^5\right)-4d\left(2cd^2+3ae^2\right)x^3e^4-2d^2\left(cd^2+9ae^2\right)x^2e^3+2d^3\left(cd^2-6ae^2\right)xe^2+d^4\left(cd^2-3ae^2\right)e}{(d+ex)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{2cde^6}+\frac{(d+ex)^2}{cde^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 2167$$

$$\frac{\int \left(-\frac{2ced^6}{(d+ex)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}+\frac{10ced^5}{(d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}-\frac{20ced^4}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}+\frac{20ce(d+ex}{(cdex^2+(cd^2+ae^2}\right.}{2cde^6}$$

$$\frac{(d+ex)^2}{cde^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{e^{3/2}\left(3ae^2+7cd^2\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{c^{3/2}d^{3/2}}+\frac{32c^3d^8e\left(ae^2+cd^2+2cdex\right)}{5\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8c^2d^7}{5(d+ex)\left(cd^2-ae^2\right)^2\sqrt{x(a}}}{}$$

$$\frac{(d+ex)^2}{cde^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[x^5/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(d + e*x)^2/(c*d*e^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + ((-4*c
*d^6*e)/(5*(c*d^2 - a*e^2)*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2]) - (8*c^2*d^7*e)/(5*(c*d^2 - a*e^2)^2*(d + e*x)*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2]) + (20*c*d^5*e)/(3*(c*d^2 - a*e^2)*(d + e*x)*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (40*c*d^3*e*(d + e*x))/((c*d^
2 - a*e^2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*e*(7*d + (3*a
*e^2)/(c*d))*(d + e*x))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2] + (32*
c^3*d^8*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(5*(c*d^2 - a*e^2)^4*Sqrt[a*d*e + (
c*d^2 + a*e^2)*x + c*d*e*x^2]) - (80*c^2*d^6*e*(c*d^2 + a*e^2 + 2*c*d*e*x)
)/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (40*
c*d^4*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/((c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2]) - (e^(3/2)*(7*c*d^2 + 3*a*e^2)*ArcTanh[(c*d^2 +
 a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2])])/(c^(3/2)*d^(3/2)))/(2*c*d*e^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2167
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*x + c*x^2)^p, (d + e*x)^m*Pq, x]
, x] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&
 EqQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[m + Expon[Pq, x] + 2*p + 1, 0] && ILt
Q[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1343 vs. $2(432) = 864$.

Time = 3.20 (sec) , antiderivative size = 1344, normalized size of antiderivative = 2.91

method	result	size
default	Expression too large to display	1344

input
```
int(x^5/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
1/e^2*(x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2)
/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)
/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e
/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)
^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))-2*a/c*(-1/d
/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*
x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*
x^2*e)^(1/2)))-8*d^3/e^5*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d
^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-2*d/e^3*(-x/d/e/c/(a*d*e+(a
*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a
*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)
/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+
1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+3*d^2/e^4*(-1/d/e/c/(a*d*e+(a*e^2+c*d
^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*
d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+5/e^6*d^
4*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2
)+8/3*d*e*c/(a*e^2-c*d^2)^3*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(1/2))-d^5/e^7*(-2/5/(a*e^2-c*d^2)/(x+d/e)^2/(d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1308 vs. $2(432) = 864$.

Time = 4.00 (sec) , antiderivative size = 2630, normalized size of antiderivative = 5.69

$$\int \frac{x^5}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[1/60*(15*(7*a*c^5*d^13*e - 25*a^2*c^4*d^11*e^3 + 30*a^3*c^3*d^9*e^5 - 10*
a^4*c^2*d^7*e^7 - 5*a^5*c*d^5*e^9 + 3*a^6*d^3*e^11 + (7*c^6*d^11*e^3 - 25*
a*c^5*d^9*e^5 + 30*a^2*c^4*d^7*e^7 - 10*a^3*c^3*d^5*e^9 - 5*a^4*c^2*d^3*e^
11 + 3*a^5*c*d*e^13)*x^4 + (21*c^6*d^12*e^2 - 68*a*c^5*d^10*e^4 + 65*a^2*c
^4*d^8*e^6 - 25*a^4*c^2*d^4*e^10 + 4*a^5*c*d^2*e^12 + 3*a^6*e^14)*x^3 + 3*
(7*c^6*d^13*e - 18*a*c^5*d^11*e^3 + 5*a^2*c^4*d^9*e^5 + 20*a^3*c^3*d^7*e^7
 - 15*a^4*c^2*d^5*e^9 - 2*a^5*c*d^3*e^11 + 3*a^6*d*e^13)*x^2 + (7*c^6*d^14
 - 4*a*c^5*d^12*e^2 - 45*a^2*c^4*d^10*e^4 + 80*a^3*c^3*d^8*e^6 - 35*a^4*c^
2*d^6*e^8 - 12*a^5*c*d^4*e^10 + 9*a^6*d^2*e^12)*x)*sqrt(c*d*e)*log(8*c^2*d
^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e
+ (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*
e + a*c*d*e^3)*x) + 4*(105*a*c^5*d^12*e^2 - 340*a^2*c^4*d^10*e^4 + 346*a^3
*c^3*d^8*e^6 - 60*a^4*c^2*d^6*e^8 + 45*a^5*c*d^4*e^10 + 15*(c^6*d^10*e^4 -
 4*a*c^5*d^8*e^6 + 6*a^2*c^4*d^6*e^8 - 4*a^3*c^3*d^4*e^10 + a^4*c^2*d^2*e^
12)*x^4 + (161*c^6*d^11*e^3 - 515*a*c^5*d^9*e^5 + 510*a^2*c^4*d^7*e^7 - 90
*a^3*c^3*d^5*e^9 - 15*a^4*c^2*d^3*e^11 + 45*a^5*c*d*e^13)*x^3 + (245*c^6*d
^12*e^2 - 637*a*c^5*d^10*e^4 + 290*a^2*c^4*d^8*e^6 + 390*a^3*c^3*d^6*e^8 -
 135*a^4*c^2*d^4*e^10 + 135*a^5*c*d^2*e^12)*x^2 + (105*c^6*d^13*e - 95*a*c
^5*d^11*e^3 - 452*a^2*c^4*d^9*e^5 + 760*a^3*c^3*d^7*e^7 - 165*a^4*c^2*d^5*
e^9 + 135*a^5*c*d^3*e^11)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x...
```

Sympy [F]

$$\int \frac{x^5}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{x^5}{((d+ex)(ae+cdx))^{\frac{3}{2}}(d+ex)^2}\,dx$$

input
```
integrate(x**5/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**5/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4520 vs. $2(432) = 864$.

Time = 1.85 (sec) , antiderivative size = 4520, normalized size of antiderivative = 9.78

$$\int \frac{x^5}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
1/15*(2*(45*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c^18*d^39*e^
30*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 - 870*sqrt(c*d*e - c*d^2*e/(e*x + d)
 + a*e^3/(e*x + d))*a*c^17*d^37*e^32*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 +
7950*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^2*c^16*d^35*e^34*
abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 - 45600*sqrt(c*d*e - c*d^2*e/(e*x + d)
+ a*e^3/(e*x + d))*a^3*c^15*d^33*e^36*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 +
 183900*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^4*c^14*d^31*e^
38*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 - 553560*sqrt(c*d*e - c*d^2*e/(e*x +
 d) + a*e^3/(e*x + d))*a^5*c^13*d^29*e^40*abs(e)*sgn(1/(e*x + d))^4*sgn(e)
^4 + 1288560*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^6*c^12*d^
27*e^42*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 - 2371200*sqrt(c*d*e - c*d^2*e/
(e*x + d) + a*e^3/(e*x + d))*a^7*c^11*d^25*e^44*abs(e)*sgn(1/(e*x + d))^4*
sgn(e)^4 + 3496350*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^8*c
^10*d^23*e^46*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 - 4161300*sqrt(c*d*e - c*
d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^9*c^9*d^21*e^48*abs(e)*sgn(1/(e*x + d
))^4*sgn(e)^4 + 4006860*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*
a^10*c^8*d^19*e^50*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 - 3113760*sqrt(c*d*e
 - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^11*c^7*d^17*e^52*abs(e)*sgn(1/(e
*x + d))^4*sgn(e)^4 + 1938300*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x
+ d))*a^12*c^6*d^15*e^54*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 - 953400*sq...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{x^5}{(d+e\,x)^2\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}\,dx$$

input
```
int(x^5/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(x^5/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^5}{(d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{x^5}{\left(ex+d\right)^2 \left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}dx$$

input
```
int(x^5/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
int(x^5/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

3.123 $\int \frac{x^4}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 410

$$\int \frac{x^4}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2a^4e^4}{c^4d^4\left(cd^2-ae^2\right)(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2(c^4d^8+5a^4e^8)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5c^4d^4e^3\left(cd^2-ae^2\right)^2(d+ex)^3} + \frac{2(11c^4d^8-20ac^3d^6e^2-15a^4e^8)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^3d^3e^3\left(cd^2-ae^2\right)^3(d+ex)^2} - \frac{2(23c^4d^8-80ac^3d^6e^2+90a^2c^2d^4e^4+15a^4e^8)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15c^2d^2e^3\left(cd^2-ae^2\right)^4(d+ex)} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{3/2}d^{3/2}e^{7/2}}$$

output
```
-2*a^4*e^4/c^4/d^4/(-a*e^2+c*d^2)/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)-2/5*(5*a^4*e^8+c^4*d^8)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/
c^4/d^4/e^3/(-a*e^2+c*d^2)^2/(e*x+d)^3+2/15*(-15*a^4*e^8-20*a*c^3*d^6*e^2+
11*c^4*d^8)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^3/(-a*e^2+c*
d^2)^3/(e*x+d)^2-2/15*(15*a^4*e^8+90*a^2*c^2*d^4*e^4-80*a*c^3*d^6*e^2+23*c
^4*d^8)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/e^3/(-a*e^2+c*d^2)
^4/(e*x+d)+2*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.73

$$\int \frac{x^4}{(d+ex)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{2\left(-\frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)\left(15a^4e^7(d+ex)^3+c^4d^8x\left(15d^2+35dex+23e^2x^2\right)+\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[x^4/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(2*(-((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(15*a^4*e^7*(d + e*x)^3 + c^4
*d^8*x*(15*d^2 + 35*d*e*x + 23*e^2*x^2) + a^3*c*d^3*e^5*(73*d^2 + 160*d*e*
x + 90*e^2*x^2) + a*c^3*d^6*e*(15*d^3 - 20*d^2*e*x - 106*d*e^2*x^2 - 80*e^
3*x^3) + a^2*c^2*d^4*e^3*(-55*d^3 - 56*d^2*e*x + 80*d*e^2*x^2 + 90*e^3*x^3
)))/((c*d^2 - a*e^2)^4*(d + e*x))) + 15*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2
)*ArcTanh[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])]))/(
15*c^(3/2)*d^(3/2)*e^(7/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1268, 109, 27, 167, 27, 162, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex)^2 \left(x \left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \ \textbf{1268}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{x^4}{(ae+cdx)^{3/2}(d+ex)^{7/2}}dx}{\sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ \textbf{109}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^3}{cd(d+ex)^{5/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{2\int \frac{x^2\left(6ade-\left(cd^2-ae^2\right)x\right)}{2\sqrt{ae+cdx}(d+ex)^{7/2}}dx}{cd(cd^2-ae^2)}\right)}{\sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^3}{cd(d+ex)^{5/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{\int \frac{x^2\left(6ade-\left(cd^2-ae^2\right)x\right)}{\sqrt{ae+cdx}(d+ex)^{7/2}}dx}{cd(cd^2-ae^2)}\right)}{\sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ \textbf{167}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^3}{cd(d+ex)^{5/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{\frac{2dx^2\left(5ae^2+cd^2\right)\sqrt{ae+cdx}}{5e(d+ex)^{5/2}\left(cd^2-ae^2\right)}-\frac{2\int \frac{x\left(5x\left(cd^2-ae^2\right)^2+4ade\left(cd^2+5ae^2\right)\right)}{2\sqrt{ae+cdx}(d+ex)^{5/2}}dx}{5e\left(cd^2-ae^2\right)}}{cd(cd^2-ae^2)}\right)}{\sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^3}{cd(d+ex)^{5/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{\frac{2dx^2\left(5ae^2+cd^2\right)\sqrt{ae+cdx}}{5e(d+ex)^{5/2}\left(cd^2-ae^2\right)}-\frac{\int \frac{x\left(5x\left(cd^2-ae^2\right)^2+4ade\left(cd^2+5ae^2\right)\right)}{\sqrt{ae+cdx}(d+ex)^{5/2}}dx}{5e\left(cd^2-ae^2\right)}}{cd(cd^2-ae^2)}\right)}{\sqrt{x \left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ \textbf{162}$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^3}{cd(d+ex)^{5/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{\frac{2dx^2\left(5ae^2+cd^2\right)\sqrt{ae+cdx}}{5e(d+ex)^{5/2}\left(cd^2-ae^2\right)}-\frac{\frac{5\left(cd^2-ae^2\right)^2\int\frac{1}{\sqrt{ae+cdx}\sqrt{d+ex}}dx}{e^2}-\frac{2d\sqrt{ae+cdx}\left(2ex\left(1\cdots\right.\right.}{\cdots}}{cd(cd^2\cdots}\right.}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 66

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^3}{cd(d+ex)^{5/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{\frac{2dx^2\left(5ae^2+cd^2\right)\sqrt{ae+cdx}}{5e(d+ex)^{5/2}\left(cd^2-ae^2\right)}-\frac{\frac{10\left(cd^2-ae^2\right)^2\int\frac{1}{cd-\frac{e(ae+cdx)}{d+ex}}d\frac{\sqrt{ae+cdx}}{\sqrt{d+ex}}}{e^2}-\frac{2d\sqrt{ae+cd\cdots}}{\cdots}}{cd\cdots}\right.}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\downarrow$ 221

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^3}{cd(d+ex)^{5/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{\frac{2dx^2\left(5ae^2+cd^2\right)\sqrt{ae+cdx}}{5e(d+ex)^{5/2}\left(cd^2-ae^2\right)}-\frac{\frac{10\left(cd^2-ae^2\right)^2\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{c}\sqrt{d}e^{5/2}}-\frac{2d\sqrt{ae+cdx}\cdots}{\cdots}}{cd\cdots}\right.}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[x^4/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*((2*a*e*x^3)/(c*d*(c*d^2 - a*e^2)*Sqrt[a*
e + c*d*x]*(d + e*x)^(5/2)) - ((2*d*(c*d^2 + 5*a*e^2)*x^2*Sqrt[a*e + c*d*x
])/(5*e*(c*d^2 - a*e^2)*(d + e*x)^(5/2)) - ((-2*d*Sqrt[a*e + c*d*x]*(d*(15
*c^3*d^6 - 55*a*c^2*d^4*e^2 + 73*a^2*c*d^2*e^4 + 15*a^3*e^6) + 2*e*(10*c^3
*d^6 - 37*a*c^2*d^4*e^2 + 36*a^2*c*d^2*e^4 + 15*a^3*e^6)*x))/(3*e^2*(c*d^2
 - a*e^2)^2*(d + e*x)^(3/2)) + (10*(c*d^2 - a*e^2)^2*ArcTanh[(Sqrt[e]*Sqrt
[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(Sqrt[c]*Sqrt[d]*e^(5/2))
)/(5*e*(c*d^2 - a*e^2)))/(c*d*(c*d^2 - a*e^2))))/Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(382) = 764$.

Time = 2.84 (sec) , antiderivative size = 868, normalized size of antiderivative = 2.12

method	result
default	$\frac{-\frac{x}{dec\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}-\frac{\left(a\,e^{2}+c\,d^{2}\right)\left(-\frac{1}{dec\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}-\frac{\left(a\,e^{2}+c\,d^{2}\right)\left(2cdxe+a\,e^{2}+c\,d^{2}\right)}{dec\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x}}\right.}{2dec}}{e^{2}}$

input
```
int(x^4/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
1/e^2*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/
d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/
c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*
d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^
(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+d^4/e^6*(-2/
5/(a*e^2-c*d^2)/(x+d/e)^2/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-6/
5*d*e*c/(a*e^2-c*d^2)*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-
c*d^2)*(x+d/e))^(1/2)+8/3*d*e*c/(a*e^2-c*d^2)^3*(2*d*e*c*(x+d/e)+a*e^2-c*d
^2)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)))+6*d^2/e^4*(2*c*d*e*x+a
*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2
*e)^(1/2)-2*d/e^3*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2
+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d
*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))-4/e^5*d^3*(-2/3/(a*e^2-c*d^2)/(x+d/e)
/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d*e*c/(a*e^2-c*d^2)^3*(
2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. 2(382) = 764.

Time = 4.80 (sec) , antiderivative size = 2224, normalized size of antiderivative = 5.42

$$\int \frac{x^4}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[1/30*(15*(a*c^4*d^11*e - 4*a^2*c^3*d^9*e^3 + 6*a^3*c^2*d^7*e^5 - 4*a^4*c*
d^5*e^7 + a^5*d^3*e^9 + (c^5*d^9*e^3 - 4*a*c^4*d^7*e^5 + 6*a^2*c^3*d^5*e^7
 - 4*a^3*c^2*d^3*e^9 + a^4*c*d*e^11)*x^4 + (3*c^5*d^10*e^2 - 11*a*c^4*d^8*
e^4 + 14*a^2*c^3*d^6*e^6 - 6*a^3*c^2*d^4*e^8 - a^4*c*d^2*e^10 + a^5*e^12)*
x^3 + 3*(c^5*d^11*e - 3*a*c^4*d^9*e^3 + 2*a^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*
e^7 - 3*a^4*c*d^3*e^9 + a^5*d*e^11)*x^2 + (c^5*d^12 - a*c^4*d^10*e^2 - 6*a
^2*c^3*d^8*e^4 + 14*a^3*c^2*d^6*e^6 - 11*a^4*c*d^4*e^8 + 3*a^5*d^2*e^10)*x
)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 +
4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*
sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(15*a*c^4*d^10*e^2 - 55*a^2
*c^3*d^8*e^4 + 73*a^3*c^2*d^6*e^6 + 15*a^4*c*d^4*e^8 + (23*c^5*d^9*e^3 - 8
0*a*c^4*d^7*e^5 + 90*a^2*c^3*d^5*e^7 + 15*a^4*c*d*e^11)*x^3 + (35*c^5*d^10
*e^2 - 106*a*c^4*d^8*e^4 + 80*a^2*c^3*d^6*e^6 + 90*a^3*c^2*d^4*e^8 + 45*a^
4*c*d^2*e^10)*x^2 + (15*c^5*d^11*e - 20*a*c^4*d^9*e^3 - 56*a^2*c^3*d^7*e^5
 + 160*a^3*c^2*d^5*e^7 + 45*a^4*c*d^3*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*
d^2 + a*e^2)*x))/(a*c^6*d^13*e^5 - 4*a^2*c^5*d^11*e^7 + 6*a^3*c^4*d^9*e^9
- 4*a^4*c^3*d^7*e^11 + a^5*c^2*d^5*e^13 + (c^7*d^11*e^7 - 4*a*c^6*d^9*e^9
+ 6*a^2*c^5*d^7*e^11 - 4*a^3*c^4*d^5*e^13 + a^4*c^3*d^3*e^15)*x^4 + (3*c^7
*d^12*e^6 - 11*a*c^6*d^10*e^8 + 14*a^2*c^5*d^8*e^10 - 6*a^3*c^4*d^6*e^12 -
 a^4*c^3*d^4*e^14 + a^5*c^2*d^2*e^16)*x^3 + 3*(c^7*d^13*e^5 - 3*a*c^6*d...
```

Sympy [F]

$$\int \frac{x^4}{(d+ex)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{x^4}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}\,(d+ex)^2}\,dx$$

input
```
integrate(x**4/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**4/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)^2 (ade + (cd^2+ae^2) x + cdex^2)^{3/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4572 vs. 2(382) = 764.

Time = 0.27 (sec) , antiderivative size = 4572, normalized size of antiderivative = 11.15

$$\int \frac{x^4}{(d+ex)^2 (ade + (cd^2+ae^2) x + cdex^2)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate(x^4/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
-2/15*(15*a^4*e^5*abs(e)/((c^5*d^9*sgn(1/(e*x + d))*sgn(e) - 4*a*c^4*d^7*e
^2*sgn(1/(e*x + d))*sgn(e) + 6*a^2*c^3*d^5*e^4*sgn(1/(e*x + d))*sgn(e) - 4
*a^3*c^2*d^3*e^6*sgn(1/(e*x + d))*sgn(e) + a^4*c*d*e^8*sgn(1/(e*x + d))*sg
n(e))*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))) - (15*sqrt(c*d*e)
*c^4*d^8*abs(e)*arctan(sqrt(c*d*e)/sqrt(-c*d*e)) - 60*sqrt(c*d*e)*a*c^3*d^
6*e^2*abs(e)*arctan(sqrt(c*d*e)/sqrt(-c*d*e)) + 90*sqrt(c*d*e)*a^2*c^2*d^4
*e^4*abs(e)*arctan(sqrt(c*d*e)/sqrt(-c*d*e)) - 60*sqrt(c*d*e)*a^3*c*d^2*e^
6*abs(e)*arctan(sqrt(c*d*e)/sqrt(-c*d*e)) + 15*sqrt(c*d*e)*a^4*e^8*abs(e)*
arctan(sqrt(c*d*e)/sqrt(-c*d*e)) + 23*sqrt(-c*d*e)*c^4*d^8*abs(e) - 80*sqr
t(-c*d*e)*a*c^3*d^6*e^2*abs(e) + 90*sqrt(-c*d*e)*a^2*c^2*d^4*e^4*abs(e) +
15*sqrt(-c*d*e)*a^4*e^8*abs(e))*sgn(1/(e*x + d))*sgn(e)/(sqrt(c*d*e)*sqrt(
-c*d*e)*c^5*d^9*e^3 - 4*sqrt(c*d*e)*sqrt(-c*d*e)*a*c^4*d^7*e^5 + 6*sqrt(c*
d*e)*sqrt(-c*d*e)*a^2*c^3*d^5*e^7 - 4*sqrt(c*d*e)*sqrt(-c*d*e)*a^3*c^2*d^3
*e^9 + sqrt(c*d*e)*sqrt(-c*d*e)*a^4*c*d*e^11) + (15*sqrt(c*d*e - c*d^2*e/(
e*x + d) + a*e^3/(e*x + d))*c^18*d^38*e^26*abs(e)*sgn(1/(e*x + d))^4*sgn(e
)^4 - 300*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a*c^17*d^36*e^
28*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 + 2850*sqrt(c*d*e - c*d^2*e/(e*x + d
) + a*e^3/(e*x + d))*a^2*c^16*d^34*e^30*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4
 - 17040*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^3*c^15*d^32*e
^32*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 + 71700*sqrt(c*d*e - c*d^2*e/(e*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{x^4}{{\left(d+e\,x\right)}^2\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}} \,d x$$

input
```
int(x^4/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(x^4/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 37.32 (sec) , antiderivative size = 2483, normalized size of antiderivative = 6.06

$$\int \frac{x^4}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^4/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(30*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*
x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*d**3*e**8
+ 90*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d
*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*d**2*e**9
*x + 90*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e +
c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*d*e**1
0*x**2 + 30*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*
e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*e*
*11*x**3 - 120*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt
(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3
*c*d**5*e**6 - 360*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*
sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*
a**3*c*d**4*e**7*x - 360*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sq
rt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d
**2))*a**3*c*d**3*e**8*x**2 - 120*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x
)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e
**2 - c*d**2))*a**3*c*d**2*e**9*x**3 + 180*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*
e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))
/sqrt(a*e**2 - c*d**2))*a**2*c**2*d**7*e**4 + 540*sqrt(e)*sqrt(d)*sqrt(c)*
sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt...
```

3.124 $\int \frac{x^3}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 240

$$\int \frac{x^3}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2x^3}{\left(cd^2-ae^2\right)(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{12dx^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5\left(cd^2-ae^2\right)^2(d+ex)^3} + \frac{16ad^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5\left(cd^2-ae^2\right)^3(d+ex)^2} - \frac{16ad(cd^2-3ae^2)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5\left(cd^2-ae^2\right)^4(d+ex)}$$

output

```
-2*x^3/(-a*e^2+c*d^2)/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+12
/5*d*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^2/(e*x+d)^
3+16/5*a*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^3/(e*x
+d)^2-16/5*a*d*(-3*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-
a*e^2+c*d^2)^4/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.51

$$\int \frac{x^3}{(d+ex)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{2(ae+cdx)^5\left(d^3-\frac{5ad^2e(d+ex)}{ae+cdx}+\frac{15a^2de^2(d+ex)^2}{(ae+cdx)^2}+\frac{5a^3e^3(d\cdots}{(ae+c\cdots}\right.}{5\left(cd^2-ae^2\right)^4\left((ae+cdx)(d+ex)\right)^{5/2}}$$

input
```
Integrate[x^3/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(2*(a*e + c*d*x)^5*(d^3 - (5*a*d^2*e*(d + e*x))/(a*e + c*d*x) + (15*a^2*d*
e^2*(d + e*x)^2)/(a*e + c*d*x)^2 + (5*a^3*e^3*(d + e*x)^3)/(a*e + c*d*x)^3
))/(5*(c*d^2 - a*e^2)^4*((a*e + c*d*x)*(d + e*x))^(5/2))
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.51, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1267, 27, 2169, 27, 1220, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 1267$$

$$-\frac{\int \frac{\left(5cd^2+ae^2\right)x^2e^3+2d\left(2cd^2+ae^2\right)xe^2+d^2\left(cd^2+ae^2\right)e}{2(d+ex)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{cde^4}-\frac{1}{cde^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\left(5cd^2+ae^2\right)x^2e^3+2d\left(2cd^2+ae^2\right)xe^2+d^2\left(cd^2+ae^2\right)e}{(d+ex)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{2cde^4}-\frac{1}{cde^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 2169$$

$$-\frac{-\frac{\int \frac{e^4\left(d\left(c^2d^4+12ace^2d^2+3a^2e^4\right)+3e\left(cd^2+ae^2\right)\left(3cd^2+ae^2\right)x\right)}{2(d+ex)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{2cde^3}-\frac{e\left(\frac{ae^2}{cd}+5d\right)}{2(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{2cde^4}-\frac{1}{cde^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{e\int \frac{d\left(c^2d^4+12ace^2d^2+3a^2e^4\right)+3e\left(cd^2+ae^2\right)\left(3cd^2+ae^2\right)x}{(d+ex)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{4cd}-\frac{e\left(\frac{ae^2}{cd}+5d\right)}{2(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{2cde^4}-\frac{1}{cde^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1220

$$-\frac{-\frac{e\left(-\frac{3\left(5a^3e^6+15a^2cd^2e^4-5ac^2d^4e^2+c^3d^6\right)\int \frac{1}{(d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{5\left(cd^2-ae^2\right)}-\frac{16c^2d^5}{5(d+ex)^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{4cd}-\frac{}{2(d+}}{2cde^4}$$

$$\frac{1}{cde^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1129

$$-\frac{-\frac{e\left(-\frac{3\left(5a^3e^6+15a^2cd^2e^4-5ac^2d^4e^2+c^3d^6\right)\left(\frac{4cd\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)}+\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{5\left(cd^2-ae^2\right)}-\frac{}{5(d+ex)^2\left(c\right.}\right.}{4cd}}{2cde^4}$$

$$\frac{1}{cde^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1088

$$-\frac{-\frac{e\left(-\frac{3\left(5a^3e^6+15a^2cd^2e^4-5ac^2d^4e^2+c^3d^6\right)\left(\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{5\left(cd^2-ae^2\right)}-\frac{\cdots}{5(d+ex)^2\left(\cdots\right.}\cdots\right.}{4cd}\cdots}{2cde^4}$$

$$\frac{1}{cde^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[x^3/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
-(1/(c*d*e^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])) - (-1/2*(e*(5*d
 + (a*e^2)/(c*d)))/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
 - (e*((-16*c^2*d^5)/(5*(c*d^2 - a*e^2)*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]) - (3*(c^3*d^6 - 5*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4
+ 5*a^3*e^6)*(2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(5*(c*d^2 - a*e^2))))/(4*c
*d))/(2*c*d*e^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.95

method	result
gosper	$\frac{2(cdx+ae)(5a^3e^6x^3+15a^2c\,d^2e^4x^3-5a\,c^2d^4e^2x^3+c^3d^6x^3+30a^3d\,e^5x^2+20a^2c\,d^3e^3x^2-2a\,c^2d^5e\,x^2+40a^3d^2e^4x+8a^2c\,d^4e^2x+16a}{5(ex+d)(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8)(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$
orering	$\frac{2(5a^3e^6x^3+15a^2c\,d^2e^4x^3-5a\,c^2d^4e^2x^3+c^3d^6x^3+30a^3d\,e^5x^2+20a^2c\,d^3e^3x^2-2a\,c^2d^5e\,x^2+40a^3d^2e^4x+8a^2c\,d^4e^2x+16a^3d^3e^3)(c}{5(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8)(ex+d)(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{3}{2}}}$
trager	$\frac{2(5a^3e^6x^3+15a^2c\,d^2e^4x^3-5a\,c^2d^4e^2x^3+c^3d^6x^3+30a^3d\,e^5x^2+20a^2c\,d^3e^3x^2-2a\,c^2d^5e\,x^2+40a^3d^2e^4x+8a^2c\,d^4e^2x+16a^3d^3e^3)\sqrt{c}}{5(cdx+ae)(a\,e^2-c\,d^2)(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3)(ex+d)^3}$
default	$\frac{-\frac{1}{dec\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}-\frac{\left(a\,e^2+c\,d^2\right)\left(2cdxe+a\,e^2+c\,d^2\right)}{dec\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}}{e^2}-\frac{4d(2cdxe+a\,e^2+c\,d^2}{e^3\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\sqrt{ade+(a}}$

input
```
int(x^3/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
2/5*(c*d*x+a*e)*(5*a^3*e^6*x^3+15*a^2*c*d^2*e^4*x^3-5*a*c^2*d^4*e^2*x^3+c^
3*d^6*x^3+30*a^3*d*e^5*x^2+20*a^2*c*d^3*e^3*x^2-2*a*c^2*d^5*e*x^2+40*a^3*d
^2*e^4*x+8*a^2*c*d^4*e^2*x+16*a^3*d^3*e^3)/(e*x+d)/(a^4*e^8-4*a^3*c*d^2*e^
6+6*a^2*c^2*d^4*e^4-4*a*c^3*d^6*e^2+c^4*d^8)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*
d*e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(226) = 452.

Time = 3.85 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.06

$$\int \frac{x^3}{(d+ex)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{}{5\left(ac^4d^{11}e-4\,a^2c^3d^9e^3+6\,a^3c^2d^7e^5-4\,a^4cd^5e^7+a^5d^3\right.}$$

input
```
integrate(x^3/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
2/5*(16*a^3*d^3*e^3 + (c^3*d^6 - 5*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 5*a^
3*e^6)*x^3 - 2*(a*c^2*d^5*e - 10*a^2*c*d^3*e^3 - 15*a^3*d*e^5)*x^2 + 8*(a^
2*c*d^4*e^2 + 5*a^3*d^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x
)/(a*c^4*d^11*e - 4*a^2*c^3*d^9*e^3 + 6*a^3*c^2*d^7*e^5 - 4*a^4*c*d^5*e^7
+ a^5*d^3*e^9 + (c^5*d^9*e^3 - 4*a*c^4*d^7*e^5 + 6*a^2*c^3*d^5*e^7 - 4*a^3
*c^2*d^3*e^9 + a^4*c*d*e^11)*x^4 + (3*c^5*d^10*e^2 - 11*a*c^4*d^8*e^4 + 14
*a^2*c^3*d^6*e^6 - 6*a^3*c^2*d^4*e^8 - a^4*c*d^2*e^10 + a^5*e^12)*x^3 + 3*
(c^5*d^11*e - 3*a*c^4*d^9*e^3 + 2*a^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7 - 3*
a^4*c*d^3*e^9 + a^5*d*e^11)*x^2 + (c^5*d^12 - a*c^4*d^10*e^2 - 6*a^2*c^3*d
^8*e^4 + 14*a^3*c^2*d^6*e^6 - 11*a^4*c*d^4*e^8 + 3*a^5*d^2*e^10)*x)
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{x^3}{((d+ex)(ae+cdx))^{\frac{3}{2}}(d+ex)^2}\,dx$$

input
```
integrate(x**3/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**3/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4068 vs. $2(226) = 452$.

Time = 0.25 (sec) , antiderivative size = 4068, normalized size of antiderivative = 16.95

$$\int \frac{x^3}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate(x^3/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output

```
2/5*(5*a^3*e^4*abs(e)/((c^4*d^8*sgn(1/(e*x + d))*sgn(e) - 4*a*c^3*d^6*e^2*
sgn(1/(e*x + d))*sgn(e) + 6*a^2*c^2*d^4*e^4*sgn(1/(e*x + d))*sgn(e) - 4*a^
3*c*d^2*e^6*sgn(1/(e*x + d))*sgn(e) + a^4*e^8*sgn(1/(e*x + d))*sgn(e))*sqr
t(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))) - (c^3*d^6*abs(e) - 5*a*c^
2*d^4*e^2*abs(e) + 15*a^2*c*d^2*e^4*abs(e) + 5*a^3*e^6*abs(e))*sgn(1/(e*x
+ d))*sgn(e)/(sqrt(c*d*e)*c^4*d^8*e^2 - 4*sqrt(c*d*e)*a*c^3*d^6*e^4 + 6*sq
rt(c*d*e)*a^2*c^2*d^4*e^6 - 4*sqrt(c*d*e)*a^3*c*d^2*e^8 + sqrt(c*d*e)*a^4*
e^10) + (15*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^2*c^16*d^3
3*e^26*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 - 240*sqrt(c*d*e - c*d^2*e/(e*x
+ d) + a*e^3/(e*x + d))*a^3*c^15*d^31*e^28*abs(e)*sgn(1/(e*x + d))^4*sgn(e
)^4 + 1800*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^4*c^14*d^29
*e^30*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 - 8400*sqrt(c*d*e - c*d^2*e/(e*x
+ d) + a*e^3/(e*x + d))*a^5*c^13*d^27*e^32*abs(e)*sgn(1/(e*x + d))^4*sgn(e
)^4 + 27300*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^6*c^12*d^2
5*e^34*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 - 65520*sqrt(c*d*e - c*d^2*e/(e*
x + d) + a*e^3/(e*x + d))*a^7*c^11*d^23*e^36*abs(e)*sgn(1/(e*x + d))^4*sgn
(e)^4 + 120120*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^8*c^10*
d^21*e^38*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 - 171600*sqrt(c*d*e - c*d^2*e
/(e*x + d) + a*e^3/(e*x + d))*a^9*c^9*d^19*e^40*abs(e)*sgn(1/(e*x + d))^4*
sgn(e)^4 + 193050*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^1...
```

Mupad [B] (verification not implemented)

Time = 7.99 (sec) , antiderivative size = 3963, normalized size of antiderivative = 16.51

$$\int \frac{x^3}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx = \text{Too large to display}$$

input
```
int(x^3/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(2*d^3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(5*a^2*d^3*e^6 + 5*c
^2*d^7*e^2 + 5*a^2*e^9*x^3 + 15*a^2*d^2*e^7*x + 15*a^2*d*e^8*x^2 + 15*c^2*
d^6*e^3*x + 15*c^2*d^5*e^4*x^2 + 5*c^2*d^4*e^5*x^3 - 10*a*c*d^5*e^4 - 30*a
*c*d^4*e^5*x - 30*a*c*d^3*e^6*x^2 - 10*a*c*d^2*e^7*x^3) - (12*d^2*(a*d*e +
 a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(5*(3*a^2*d^2*e^6 + 3*c^2*d^6*e^2 +
 3*a^2*e^8*x^2 + 6*c^2*d^5*e^3*x + 3*c^2*d^4*e^4*x^2 - 6*a*c*d^4*e^4 + 6*a
^2*d*e^7*x - 12*a*c*d^3*e^5*x - 6*a*c*d^2*e^6*x^2)) - (6*c^2*d^6*(a*d*e +
a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(5*(3*a^4*d^2*e^10 + 3*c^4*d^10*e^2
+ 3*a^4*e^12*x^2 - 12*a*c^3*d^8*e^4 - 12*a^3*c*d^4*e^8 + 6*c^4*d^9*e^3*x +
 18*a^2*c^2*d^6*e^6 + 3*c^4*d^8*e^4*x^2 + 6*a^4*d*e^11*x + 18*a^2*c^2*d^4*
e^8*x^2 - 24*a*c^3*d^7*e^5*x - 24*a^3*c*d^3*e^9*x + 36*a^2*c^2*d^5*e^7*x -
 12*a*c^3*d^6*e^6*x^2 - 12*a^3*c*d^2*e^10*x^2)) - (6*c*d^4*(a*d*e + a*e^2*
x + c*d^2*x + c*d*e*x^2)^(1/2))/(5*(3*a^3*d^2*e^8 - 3*c^3*d^8*e^2 + 3*a^3*
e^10*x^2 + 9*a*c^2*d^6*e^4 - 9*a^2*c*d^4*e^6 - 6*c^3*d^7*e^3*x - 3*c^3*d^6
*e^4*x^2 + 6*a^3*d*e^9*x + 18*a*c^2*d^5*e^5*x - 18*a^2*c*d^3*e^7*x + 9*a*c
^2*d^4*e^6*x^2 - 9*a^2*c*d^2*e^8*x^2)) + (16*c^3*d^7*(a*d*e + a*e^2*x + c*
d^2*x + c*d*e*x^2)^(1/2))/(15*(a^5*d*e^12 + a^5*e^13*x - c^5*d^11*e^2 + 5*
a*c^4*d^9*e^4 - 5*a^4*c*d^3*e^10 - c^5*d^10*e^3*x - 10*a^2*c^3*d^7*e^6 + 1
0*a^3*c^2*d^5*e^8 + 5*a*c^4*d^8*e^5*x - 5*a^4*c*d^2*e^11*x - 10*a^2*c^3*d^
6*e^7*x + 10*a^3*c^2*d^4*e^9*x)) + (24*c*d^3*(a*d*e + a*e^2*x + c*d^2*x...
```

Reduce [B] (verification not implemented)

Time = 30.43 (sec) , antiderivative size = 935, normalized size of antiderivative = 3.90

$$\int \frac{x^3}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx = \text{Too large to display}$$

input
```
int(x^3/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 15*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**3*d**3*e**6 - 45*sqrt(
e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**3*d**2*e**7*x - 45*sqrt(e)*sqrt(d)
*sqrt(c)*sqrt(a*e + c*d*x)*a**3*d*e**8*x**2 - 15*sqrt(e)*sqrt(d)*sqrt(c)*s
qrt(a*e + c*d*x)*a**3*e**9*x**3 - 15*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*
d*x)*a**2*c*d**5*e**4 - 45*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*
c*d**4*e**5*x - 45*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d**3*e
**6*x**2 - 15*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d**2*e**7*x
**3 - 5*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**7*e**2 - 15*sq
rt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**6*e**3*x - 15*sqrt(e)*sq
rt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**5*e**4*x**2 - 5*sqrt(e)*sqrt(d)*
sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**4*e**5*x**3 + 3*sqrt(e)*sqrt(d)*sqrt(c
)*sqrt(a*e + c*d*x)*c**3*d**9 + 9*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x
)*c**3*d**8*e*x + 9*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**7*e*
*2*x**2 + 3*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**6*e**3*x**3
+ 32*sqrt(d + e*x)*a**3*c*d**4*e**6 + 80*sqrt(d + e*x)*a**3*c*d**3*e**7*x
+ 60*sqrt(d + e*x)*a**3*c*d**2*e**8*x**2 + 10*sqrt(d + e*x)*a**3*c*d*e**9*
x**3 + 16*sqrt(d + e*x)*a**2*c**2*d**5*e**5*x + 40*sqrt(d + e*x)*a**2*c**2
*d**4*e**6*x**2 + 30*sqrt(d + e*x)*a**2*c**2*d**3*e**7*x**3 - 4*sqrt(d + e
*x)*a*c**3*d**6*e**4*x**2 - 10*sqrt(d + e*x)*a*c**3*d**5*e**5*x**3 + 2*sqr
t(d + e*x)*c**4*d**7*e**3*x**3)/(5*sqrt(a*e + c*d*x)*c*d*e**3*(a**4*d**...
```

3.125 $\int \frac{x^2}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 225

$$\int \frac{x^2}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2d^2}{5e^2\left(cd^2-ae^2\right)(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$-\frac{4d(2cd^2-5ae^2)}{15e^2\left(cd^2-ae^2\right)^2(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$+\frac{2(c^2d^4-10acd^2e^2-15a^2e^4)\left(cd^2+ae^2+2cdex\right)}{15e^2\left(cd^2-ae^2\right)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
2/5*d^2/e^2/(-a*e^2+c*d^2)/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)-4/15*d*(-5*a*e^2+2*c*d^2)/e^2/(-a*e^2+c*d^2)^2/(e*x+d)/(a*d*e+(a*e^2+c*
d^2)*x+c*d*e*x^2)^(1/2)+2/15*(-15*a^2*e^4-10*a*c*d^2*e^2+c^2*d^4)*(2*c*d*e
*x+a*e^2+c*d^2)/e^2/(-a*e^2+c*d^2)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{(d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2\left(-c^3d^5x^2(5d+2ex)+a^3e^4\left(8d^2+20dex+15e^2x^2\right)+ac^2d^3ex\left(20d^2+49dex+20e^2x^2\right)+a^2cde^2\left(40d^3\right.\right.}{15\left(cd^2-ae^2\right)^4(d+ex)^2\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[x^2/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(-2*(-(c^3*d^5*x^2*(5*d + 2*e*x)) + a^3*e^4*(8*d^2 + 20*d*e*x + 15*e^2*x^2
) + a*c^2*d^3*e*x*(20*d^2 + 49*d*e*x + 20*e^2*x^2) + a^2*c*d*e^2*(40*d^3 +
 104*d^2*e*x + 85*d*e^2*x^2 + 30*e^3*x^3)))/(15*(c*d^2 - a*e^2)^4*(d + e*x
)^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.28, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1267, 27, 1220, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)^2 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1267}$$

$$-\frac{\int \frac{e\left(d\left(cd^2+3ae^2\right)+e\left(5cd^2+3ae^2\right)x\right)}{2(d+ex)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{2cde^3}-\frac{1}{2cde^2(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{d\left(cd^2+3ae^2\right)+e\left(5cd^2+3ae^2\right)x}{(d+ex)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{4cde^2}-\frac{1}{2cde^2(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1220}$$

$$-\frac{\frac{(-15a^2e^4-10acd^2e^2+c^2d^4)\int \frac{1}{(d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{5(cd^2-ae^2)}-\frac{8cd^3}{5(d+ex)^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{4cde^2}-\frac{1}{2cde^2(d+ex)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{\frac{(-15a^2e^4-10acd^2e^2+c^2d^4)\left(\frac{4cd\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)}+\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{5(cd^2-ae^2)}-\frac{8}{5(d+ex)^2(cd^2-ae^2)\sqrt{}}}{4cde^2}-\frac{1}{2cde^2(d+ex)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{\frac{(-15a^2e^4-10acd^2e^2+c^2d^4)\left(\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{5(cd^2-ae^2)}-\frac{}{5(d+ex)^2(cd^2-ae^2)\sqrt{}}}{4cde^2}-\frac{1}{2cde^2(d+ex)\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

input
```
Int[x^2/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
-1/2*1/(c*d*e^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (
(-8*c*d^3)/(5*(c*d^2 - a*e^2)*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
 c*d*e*x^2]) + ((c^2*d^4 - 10*a*c*d^2*e^2 - 15*a^2*e^4)*(2/(3*(c*d^2 - a*e
^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*c*d*(c*d^2
 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
 + c*d*e*x^2])))/(5*(c*d^2 - a*e^2)))/(4*c*d*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.11

method	result
gosper	$-\frac{2(cdx+ae)(30a^2cd\,e^5x^3+20a\,c^2d^3e^3x^3-2c^3d^5e\,x^3+15a^3e^6x^2+85a^2c\,d^2e^4x^2+49a\,c^2d^4e^2x^2-5c^3d^6x^2+20a^3d\,e^5x+104a^2c\,d^3e^3}{15(ex+d)(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8)(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$
orering	$-\frac{2(30a^2cd\,e^5x^3+20a\,c^2d^3e^3x^3-2c^3d^5e\,x^3+15a^3e^6x^2+85a^2c\,d^2e^4x^2+49a\,c^2d^4e^2x^2-5c^3d^6x^2+20a^3d\,e^5x+104a^2c\,d^3e^3x+20a\,c^2}{15(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8)(ex+d)(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{3}{2}}}$
trager	$-\frac{2(30a^2cd\,e^5x^3+20a\,c^2d^3e^3x^3-2c^3d^5e\,x^3+15a^3e^6x^2+85a^2c\,d^2e^4x^2+49a\,c^2d^4e^2x^2-5c^3d^6x^2+20a^3d\,e^5x+104a^2c\,d^3e^3x+20a\,c^2}{15(cdx+ae)(a\,e^2-c\,d^2)(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3)(ex+d)^3}$
default	$\frac{4cdxe+2a\,e^2+2c\,d^2}{e^2\left(4ac\,d^2e^2-(a\,e^2+c\,d^2)^2\right)\sqrt{ade+(a\,e^2+c\,d^2)x+cd\,x^2e}}+\frac{d^2\left(-\frac{2}{5\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^2\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}-\frac{6dec\left(-\right.}{}\right.}{}$

input
```
int(x^2/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/15*(c*d*x+a*e)*(30*a^2*c*d*e^5*x^3+20*a*c^2*d^3*e^3*x^3-2*c^3*d^5*e*x^3
+15*a^3*e^6*x^2+85*a^2*c*d^2*e^4*x^2+49*a*c^2*d^4*e^2*x^2-5*c^3*d^6*x^2+20
*a^3*d*e^5*x+104*a^2*c*d^3*e^3*x+20*a*c^2*d^5*e*x+8*a^3*d^2*e^4+40*a^2*c*d
^4*e^2)/(e*x+d)/(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a*c^3*d^6*e^2
+c^4*d^8)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(213) = 426.

Time = 3.88 (sec) , antiderivative size = 518, normalized size of antiderivative = 2.30

$$\int \frac{x^2}{(d+ex)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx =$$

$$-\frac{2\,(40\,a^2cd^4e^2+8\,a^3d^2e^4}{15\,(ac^4d^{11}e-4\,a^2c^3d^9e^3+6\,a^3c^2d^7e^5-4\,a^4cd^5e^7+a^5d^3e^9+(c^5d^9e^3-4\,ac^4d^7e^5+6\,a^2c^3d^5e^7-4\,a^3c^2d^3}$$

input
```
integrate(x^2/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
-2/15*(40*a^2*c*d^4*e^2 + 8*a^3*d^2*e^4 - 2*(c^3*d^5*e - 10*a*c^2*d^3*e^3
- 15*a^2*c*d*e^5)*x^3 - (5*c^3*d^6 - 49*a*c^2*d^4*e^2 - 85*a^2*c*d^2*e^4 -
 15*a^3*e^6)*x^2 + 4*(5*a*c^2*d^5*e + 26*a^2*c*d^3*e^3 + 5*a^3*d*e^5)*x)*s
qrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^4*d^11*e - 4*a^2*c^3*d^9*e
^3 + 6*a^3*c^2*d^7*e^5 - 4*a^4*c*d^5*e^7 + a^5*d^3*e^9 + (c^5*d^9*e^3 - 4*
a*c^4*d^7*e^5 + 6*a^2*c^3*d^5*e^7 - 4*a^3*c^2*d^3*e^9 + a^4*c*d*e^11)*x^4
+ (3*c^5*d^10*e^2 - 11*a*c^4*d^8*e^4 + 14*a^2*c^3*d^6*e^6 - 6*a^3*c^2*d^4*
e^8 - a^4*c*d^2*e^10 + a^5*e^12)*x^3 + 3*(c^5*d^11*e - 3*a*c^4*d^9*e^3 + 2
*a^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7 - 3*a^4*c*d^3*e^9 + a^5*d*e^11)*x^2 +
 (c^5*d^12 - a*c^4*d^10*e^2 - 6*a^2*c^3*d^8*e^4 + 14*a^3*c^2*d^6*e^6 - 11*
a^4*c*d^4*e^8 + 3*a^5*d^2*e^10)*x)
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{x^2}{((d+ex)(ae+cdx))^{\frac{3}{2}}(d+ex)^2}\,dx$$

input
```
integrate(x**2/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**2/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4176 vs. $2(213) = 426$.

Time = 0.24 (sec) , antiderivative size = 4176, normalized size of antiderivative = 18.56

$$\int \frac{x^2}{(d+ex)^2 (ade + (cd^2+ae^2)x + cdex^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(x^2/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
-2/15*(15*a^2*c*d*e^3*abs(e)/((c^4*d^8*sgn(1/(e*x + d))*sgn(e) - 4*a*c^3*d
^6*e^2*sgn(1/(e*x + d))*sgn(e) + 6*a^2*c^2*d^4*e^4*sgn(1/(e*x + d))*sgn(e)
 - 4*a^3*c*d^2*e^6*sgn(1/(e*x + d))*sgn(e) + a^4*e^8*sgn(1/(e*x + d))*sgn(
e))*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))) + 2*(c^3*d^5*abs(e)
 - 10*a*c^2*d^3*e^2*abs(e) - 15*a^2*c*d*e^4*abs(e))*sgn(1/(e*x + d))*sgn(e
)/(sqrt(c*d*e)*c^4*d^8*e - 4*sqrt(c*d*e)*a*c^3*d^6*e^3 + 6*sqrt(c*d*e)*a^2
*c^2*d^4*e^5 - 4*sqrt(c*d*e)*a^3*c*d^2*e^7 + sqrt(c*d*e)*a^4*e^9) + (30*sq
rt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a*c^17*d^34*e^20*abs(e)*sg
n(1/(e*x + d))^4*sgn(e)^4 - 465*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*
x + d))*a^2*c^16*d^32*e^22*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 + 3360*sqrt(
c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^3*c^15*d^30*e^24*abs(e)*sgn
(1/(e*x + d))^4*sgn(e)^4 - 15000*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e
*x + d))*a^4*c^14*d^28*e^26*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 + 46200*sqr
t(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^5*c^13*d^26*e^28*abs(e)*s
gn(1/(e*x + d))^4*sgn(e)^4 - 103740*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3
/(e*x + d))*a^6*c^12*d^24*e^30*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 + 174720
*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^7*c^11*d^22*e^32*abs(
e)*sgn(1/(e*x + d))^4*sgn(e)^4 - 223080*sqrt(c*d*e - c*d^2*e/(e*x + d) + a
*e^3/(e*x + d))*a^8*c^10*d^20*e^34*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 + 21
4500*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^9*c^9*d^18*e^3...
```

Mupad [B] (verification not implemented)

Time = 7.42 (sec) , antiderivative size = 3062, normalized size of antiderivative = 13.61

$$\int \frac{x^2}{(d+ex)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^2/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(((6*a*e^2 - 10*c*d^2)/(15*e*(a*e^2 - c*d^2)^3) - (4*c*d^2)/(5*e*(a*e^2 -
c*d^2)^3))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - (((d
*((d*((8*c^3*d^4*e)/(15*(a*e^2 - c*d^2)^5) - (8*c^2*d^2*e*(2*a*e^2 - c*d^2
))/(15*(a*e^2 - c*d^2)^5)))/e + (2*c*d*(5*a^2*e^4 + c^2*d^4 - 10*a*c*d^2*e
^2))/(5*(a*e^2 - c*d^2)^5)))/e - (12*a^3*e^6 + 6*c^3*d^6 - 24*a*c^2*d^4*e^
2 - 18*a^2*c*d^2*e^4)/(15*e*(a*e^2 - c*d^2)^5))*(x*(a*e^2 + c*d^2) + a*d*e
 + c*d*e*x^2)^(1/2))/(d + e*x) - (((2*c*d^3 + 2*a*d*e^2)/(5*(a*e^2 - c*d^2
)^2*(3*a*e^3 - 3*c*d^2*e)) + (d*((2*a*e^3 - 2*c*d^2*e)/(5*(a*e^2 - c*d^2)^
2*(3*a*e^3 - 3*c*d^2*e)) - (4*c*d^2*e)/(5*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c
*d^2*e))))/e)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 +
 (((d*((d*((e^2*(2*c^2*d^3 - 6*a*c*d*e^2))/(5*(a*e^2 - c*d^2)^2*(3*a^2*e^5
 + 3*c^2*d^4*e - 6*a*c*d^2*e^3)) + (4*c^2*d^3*e^2)/(5*(a*e^2 - c*d^2)^2*(3
*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e^3))))/e + (e^2*(12*a^2*e^3 - 24*a*c*d
^2*e))/(5*(a*e^2 - c*d^2)^2*(3*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e^3))))/e
 + (12*a^2*d*e^4)/(5*(a*e^2 - c*d^2)^2*(3*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^
2*e^3)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + ((x*
(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x*((a*(((a*e^2 + c*d^2)*((8*c^
5*d^5*e^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*
e^3 + a^2*c*d*e^5)) - (16*c^5*d^5*e^3*(3*a*e^2 - c*d^2))/(15*(a*e^2 - c*d^
2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (16*a*c^5...
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 828, normalized size of antiderivative = 3.68

$$\int \frac{x^2}{(d+ex)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^2/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*(30*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*d**3*e**4 + 90*sqrt(
e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*d**2*e**5*x + 90*sqrt(e)*sqrt(d)
*sqrt(c)*sqrt(a*e + c*d*x)*a**2*d*e**6*x**2 + 30*sqrt(e)*sqrt(d)*sqrt(c)*s
qrt(a*e + c*d*x)*a**2*e**7*x**3 + 20*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*
d*x)*a*c*d**5*e**2 + 60*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d**4
*e**3*x + 60*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d**3*e**4*x**2
+ 20*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d**2*e**5*x**3 - 2*sqrt
(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d**7 - 6*sqrt(e)*sqrt(d)*sqrt(c
)*sqrt(a*e + c*d*x)*c**2*d**6*e*x - 6*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c
*d*x)*c**2*d**5*e**2*x**2 - 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*
*2*d**4*e**3*x**3 - 8*sqrt(d + e*x)*a**3*d**2*e**6 - 20*sqrt(d + e*x)*a**3
*d*e**7*x - 15*sqrt(d + e*x)*a**3*e**8*x**2 - 40*sqrt(d + e*x)*a**2*c*d**4
*e**4 - 104*sqrt(d + e*x)*a**2*c*d**3*e**5*x - 85*sqrt(d + e*x)*a**2*c*d**
2*e**6*x**2 - 30*sqrt(d + e*x)*a**2*c*d*e**7*x**3 - 20*sqrt(d + e*x)*a*c**
2*d**5*e**3*x - 49*sqrt(d + e*x)*a*c**2*d**4*e**4*x**2 - 20*sqrt(d + e*x)*
a*c**2*d**3*e**5*x**3 + 5*sqrt(d + e*x)*c**3*d**6*e**2*x**2 + 2*sqrt(d + e
*x)*c**3*d**5*e**3*x**3))/(15*sqrt(a*e + c*d*x)*e**2*(a**4*d**3*e**8 + 3*a
**4*d**2*e**9*x + 3*a**4*d*e**10*x**2 + a**4*e**11*x**3 - 4*a**3*c*d**5*e*
*6 - 12*a**3*c*d**4*e**7*x - 12*a**3*c*d**3*e**8*x**2 - 4*a**3*c*d**2*e**9
*x**3 + 6*a**2*c**2*d**7*e**4 + 18*a**2*c**2*d**6*e**5*x + 18*a**2*c**2...
```

3.126 $\int \frac{x}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 209

$$\int \frac{x}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = -\frac{2d}{5e\left(cd^2-ae^2\right)(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2(cd^2+5ae^2)}{15e\left(cd^2-ae^2\right)^2(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{8cd(cd^2+5ae^2)\left(cd^2+ae^2+2cdex\right)}{15e\left(cd^2-ae^2\right)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-2/5*d/e/(-a*e^2+c*d^2)/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-
2/15*(5*a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)+8/15*c*d*(5*a*e^2+c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)/e/(-a*e^2+c
*d^2)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.74

$$\int \frac{x}{(d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2(-a^3e^5(2d+5ex)+c^3d^4x(15d^2+20dex+8e^2x^2)+c}{15\left(cd^2-a\right.}$$

input
```
Integrate[x/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*(-(a^3*e^5*(2*d + 5*e*x)) + c^3*d^4*x*(15*d^2 + 20*d*e*x + 8*e^2*x^2) +
 a^2*c*d*e^3*(20*d^2 + 49*d*e*x + 20*e^2*x^2) + a*c^2*d^2*e*(30*d^3 + 85*d
^2*e*x + 104*d*e^2*x^2 + 40*e^3*x^3)))/(15*(c*d^2 - a*e^2)^4*(d + e*x)^2*S
qrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {1220, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)^2 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{\left(5ae^2+cd^2\right)\int \frac{1}{(d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{5e\left(cd^2-ae^2\right)}-\frac{2d}{5e(d+ex)^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{\left(5ae^2+cd^2\right)\left(\frac{4cd\int\frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3(cd^2-ae^2)}+\frac{2}{3(d+ex)(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{5e\left(cd^2-ae^2\right)}-\frac{2d}{5e(d+ex)^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1088

$$-\frac{2d}{5e(d+ex)^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\left(5ae^2+cd^2\right)\left(\frac{2}{3(d+ex)(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{8cd(ae^2+cd^2+2cdex)}{3(cd^2-ae^2)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{5e\left(cd^2-ae^2\right)}$$

input
```
Int[x/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*d)/(5*e*(c*d^2 - a*e^2)*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) - ((c*d^2 + 5*a*e^2)*(2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(
3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(5*e*(c
*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.17

method	result
gosper	$-\frac{2(cdx+ae)(-40x^3ac^2d^2e^4-8x^3c^3d^4e^2-20x^2a^2cde^5-104x^2ac^2d^3e^3-20x^2c^3d^5e+5e^6a^3x-49d^2e^4a^2cx-85d^4e^2ac^2x-15d^6c^3}{15(ex+d)(a^4e^8-4a^3cd^2e^6+6a^2c^2d^4e^4-4ac^3d^6e^2+c^4d^8)(cdx^2e+ae^2x+cd^2x+ade)^{\frac{3}{2}}}$
orering	$-\frac{2(-40x^3ac^2d^2e^4-8x^3c^3d^4e^2-20x^2a^2cde^5-104x^2ac^2d^3e^3-20x^2c^3d^5e+5e^6a^3x-49d^2e^4a^2cx-85d^4e^2ac^2x-15d^6c^3x+2a^3de^5}{15(a^4e^8-4a^3cd^2e^6+6a^2c^2d^4e^4-4ac^3d^6e^2+c^4d^8)(ex+d)(ade+(ae^2+cd^2)x+cdx^2e)^{\frac{3}{2}}}$
trager	$-\frac{2(-40x^3ac^2d^2e^4-8x^3c^3d^4e^2-20x^2a^2cde^5-104x^2ac^2d^3e^3-20x^2c^3d^5e+5e^6a^3x-49d^2e^4a^2cx-85d^4e^2ac^2x-15d^6c^3x+2a^3de^5}{15(cdx+ae)(ae^2-cd^2)(e^6a^3-3d^2e^4a^2c+3d^4e^2ac^2-d^6c^3)(ex+d)^3}$
default	$\frac{-\frac{2}{3\left(ae^2-cd^2\right)\left(x+\frac{d}{e}\right)\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(ae^2-cd^2\right)\left(x+\frac{d}{e}\right)}}+\frac{8dec\left(2dec\left(x+\frac{d}{e}\right)+ae^2-cd^2\right)}{3\left(ae^2-cd^2\right)^3\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(ae^2-cd^2\right)\left(x+\frac{d}{e}\right)}}}{e^2} - \frac{d\left(-\frac{}{5\left(ae^2-cd^2\right)\left(x+\frac{d}{e}\right)}\right.}{}$

input

```
int(x/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output

```
-2/15*(c*d*x+a*e)*(-40*a*c^2*d^2*e^4*x^3-8*c^3*d^4*e^2*x^3-20*a^2*c*d*e^5*
x^2-104*a*c^2*d^3*e^3*x^2-20*c^3*d^5*e*x^2+5*a^3*e^6*x-49*a^2*c*d^2*e^4*x-
85*a*c^2*d^4*e^2*x-15*c^3*d^6*x+2*a^3*d*e^5-20*a^2*c*d^3*e^3-30*a*c^2*d^5*
e)/(e*x+d)/(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d^4*e^4-4*a*c^3*d^6*e^2+c^4*
d^8)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(197) = 394$.

Time = 3.60 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.47

$$\int \frac{x}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{}{15\left(ac^4d^{11}e-4\,a^2c^3d^9e^3+6\,a^3c^2d^7e^5-4\,a^4cd^5e^7+a^5d\right.}$$

input
```
integrate(x/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
2/15*(30*a*c^2*d^5*e + 20*a^2*c*d^3*e^3 - 2*a^3*d*e^5 + 8*(c^3*d^4*e^2 + 5
*a*c^2*d^2*e^4)*x^3 + 4*(5*c^3*d^5*e + 26*a*c^2*d^3*e^3 + 5*a^2*c*d*e^5)*x
^2 + (15*c^3*d^6 + 85*a*c^2*d^4*e^2 + 49*a^2*c*d^2*e^4 - 5*a^3*e^6)*x)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^4*d^11*e - 4*a^2*c^3*d^9*e^3
 + 6*a^3*c^2*d^7*e^5 - 4*a^4*c*d^5*e^7 + a^5*d^3*e^9 + (c^5*d^9*e^3 - 4*a*
c^4*d^7*e^5 + 6*a^2*c^3*d^5*e^7 - 4*a^3*c^2*d^3*e^9 + a^4*c*d*e^11)*x^4 +
(3*c^5*d^10*e^2 - 11*a*c^4*d^8*e^4 + 14*a^2*c^3*d^6*e^6 - 6*a^3*c^2*d^4*e^
8 - a^4*c*d^2*e^10 + a^5*e^12)*x^3 + 3*(c^5*d^11*e - 3*a*c^4*d^9*e^3 + 2*a
^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7 - 3*a^4*c*d^3*e^9 + a^5*d*e^11)*x^2 + (
c^5*d^12 - a*c^4*d^10*e^2 - 6*a^2*c^3*d^8*e^4 + 14*a^3*c^2*d^6*e^6 - 11*a^
4*c*d^4*e^8 + 3*a^5*d^2*e^10)*x)
```

Sympy [F]

$$\int \frac{x}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{x}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}\,(d+ex)^2}\,dx$$

input
```
integrate(x/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4167 vs. 2(197) = 394.

Time = 0.23 (sec) , antiderivative size = 4167, normalized size of antiderivative = 19.94

$$\int \frac{x}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
2/15*(15*a*c^2*d^2*e^3*abs(e)/((c^4*d^8*sgn(1/(e*x + d))*sgn(e) - 4*a*c^3*
d^6*e^2*sgn(1/(e*x + d))*sgn(e) + 6*a^2*c^2*d^4*e^4*sgn(1/(e*x + d))*sgn(e
) - 4*a^3*c*d^2*e^6*sgn(1/(e*x + d))*sgn(e) + a^4*e^8*sgn(1/(e*x + d))*sgn
(e))*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))) - 8*(c^3*d^4*e*abs
(e) + 5*a*c^2*d^2*e^3*abs(e))*sgn(1/(e*x + d))*sgn(e)/(sqrt(c*d*e)*c^4*d^8
 - 4*sqrt(c*d*e)*a*c^3*d^6*e^2 + 6*sqrt(c*d*e)*a^2*c^2*d^4*e^4 - 4*sqrt(c*
d*e)*a^3*c*d^2*e^6 + sqrt(c*d*e)*a^4*e^8) + (15*sqrt(c*d*e - c*d^2*e/(e*x
+ d) + a*e^3/(e*x + d))*c^18*d^35*e^10*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4
- 210*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a*c^17*d^33*e^12*a
bs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 + 1320*sqrt(c*d*e - c*d^2*e/(e*x + d) +
a*e^3/(e*x + d))*a^2*c^16*d^31*e^14*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 - 4
800*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^3*c^15*d^29*e^16*a
bs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 + 10500*sqrt(c*d*e - c*d^2*e/(e*x + d) +
 a*e^3/(e*x + d))*a^4*c^14*d^27*e^18*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 -
10920*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^5*c^13*d^25*e^20
*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 - 10920*sqrt(c*d*e - c*d^2*e/(e*x + d)
 + a*e^3/(e*x + d))*a^6*c^12*d^23*e^22*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4
+ 68640*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^7*c^11*d^21*e^
24*abs(e)*sgn(1/(e*x + d))^4*sgn(e)^4 - 150150*sqrt(c*d*e - c*d^2*e/(e*x +
 d) + a*e^3/(e*x + d))*a^8*c^10*d^19*e^26*abs(e)*sgn(1/(e*x + d))^4*sgn...
```

Mupad [B] (verification not implemented)

Time = 7.09 (sec) , antiderivative size = 1844, normalized size of antiderivative = 8.82

$$\int \frac{x}{(d+ex)^2 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(((e*(2*a*e^2 + 2*c*d^2))/(5*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)) - (4
*c*d^2*e)/(5*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2)
+ a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + (((d*((8*c^3*d^4*e)/(15*(a*e^2 -
 c*d^2)^5) - (16*c^2*d^2*e*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^5)))/e + (
2*c*d*(a*e^2 + c*d^2)^2)/(5*(a*e^2 - c*d^2)^5))*(x*(a*e^2 + c*d^2) + a*d*e
 + c*d*e*x^2)^(1/2))/(d + e*x) + (((d*((e^2*(10*c^2*d^3 + 6*a*c*d*e^2))/(5
*(a*e^2 - c*d^2)^2*(3*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e^3)) - (4*c^2*d^3
*e^2)/(5*(a*e^2 - c*d^2)^2*(3*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e^3))))/e
- (12*a^2*e^5)/(5*(a*e^2 - c*d^2)^2*(3*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e
^3)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + ((x*(a*
e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*((a*(((a*e^2 + c*d^2)*((8*c^5*d^5*
e^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
a^2*c*d*e^5)) - (16*c^5*d^5*e^3*(3*a*e^2 + 2*c*d^2))/(15*(a*e^2 - c*d^2)^4
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) - (16*a*c^5*d^6*e^
4)/(15*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (2
*c^2*d^2*e^2*(30*c^4*d^6 + 16*a*c^3*d^4*e^2 + 10*a^2*c^2*d^2*e^4))/(15*(a*
e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^4*d^4*e
^2*(a*e^2 + c*d^2)*(3*a*e^2 + 2*c*d^2))/(15*(a*e^2 - c*d^2)^4*(c^3*d^5*e -
 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - x*((a*((8*c^5*d^5*e^3*(a*e^2 + c*d^
2))/(15*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) ...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 713, normalized size of antiderivative = 3.41

$$\int \frac{x}{(d+ex)^2\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{-\frac{16\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,ac\,d^4e^2}{3} - 16\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,ac\,c$$

input
```
int(x/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*( - 40*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d**4*e**2 - 120*sq
rt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d**3*e**3*x - 120*sqrt(e)*sqrt
(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d**2*e**4*x**2 - 40*sqrt(e)*sqrt(d)*sqrt
(c)*sqrt(a*e + c*d*x)*a*c*d*e**5*x**3 - 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e
 + c*d*x)*c**2*d**6 - 24*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d*
*5*e*x - 24*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d**4*e**2*x**2
- 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d**3*e**3*x**3 - 2*sqrt
(d + e*x)*a**3*d*e**6 - 5*sqrt(d + e*x)*a**3*e**7*x + 20*sqrt(d + e*x)*a**
2*c*d**3*e**4 + 49*sqrt(d + e*x)*a**2*c*d**2*e**5*x + 20*sqrt(d + e*x)*a**
2*c*d*e**6*x**2 + 30*sqrt(d + e*x)*a*c**2*d**5*e**2 + 85*sqrt(d + e*x)*a*c
**2*d**4*e**3*x + 104*sqrt(d + e*x)*a*c**2*d**3*e**4*x**2 + 40*sqrt(d + e*
x)*a*c**2*d**2*e**5*x**3 + 15*sqrt(d + e*x)*c**3*d**6*e*x + 20*sqrt(d + e*
x)*c**3*d**5*e**2*x**2 + 8*sqrt(d + e*x)*c**3*d**4*e**3*x**3))/(15*sqrt(a*
e + c*d*x)*e*(a**4*d**3*e**8 + 3*a**4*d**2*e**9*x + 3*a**4*d*e**10*x**2 +
a**4*e**11*x**3 - 4*a**3*c*d**5*e**6 - 12*a**3*c*d**4*e**7*x - 12*a**3*c*d
**3*e**8*x**2 - 4*a**3*c*d**2*e**9*x**3 + 6*a**2*c**2*d**7*e**4 + 18*a**2*
c**2*d**6*e**5*x + 18*a**2*c**2*d**5*e**6*x**2 + 6*a**2*c**2*d**4*e**7*x**
3 - 4*a*c**3*d**9*e**2 - 12*a*c**3*d**8*e**3*x - 12*a*c**3*d**7*e**4*x**2
- 4*a*c**3*d**6*e**5*x**3 + c**4*d**11 + 3*c**4*d**10*e*x + 3*c**4*d**9*e*
*2*x**2 + c**4*d**8*e**3*x**3))
```

3.127 $\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 181

$$\int \frac{1}{(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2}{5\left(cd^2-ae^2\right)(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}+\frac{4cd}{5\left(cd^2-ae^2\right)^2(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{16c^2d^2(cd^2+ae^2+2cdex)}{5\left(cd^2-ae^2\right)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output

```
2/5/(-a*e^2+c*d^2)/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+4/5*c
*d/(-a*e^2+c*d^2)^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-16/5*c
^2*d^2*(2*c*d*e*x+a*e^2+c*d^2)/(-a*e^2+c*d^2)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d
*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.75

$$\int \frac{1}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = -\frac{2(a^3e^6-a^2cde^4(5d+2ex)+ac^2d^2e^2(15d^2+20dex+8e^2x^2)+c^3d^3(5d^3+30d^2ex+40de^2x^2+16e^3x^3))}{5\,(cd^2-ae^2)^4\,(d+ex)^2\sqrt{(ae+cdx)(d+ex)}}$$

input
```
Integrate[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*(a^3*e^6 - a^2*c*d*e^4*(5*d + 2*e*x) + a*c^2*d^2*e^2*(15*d^2 + 20*d*e*
x + 8*e^2*x^2) + c^3*d^3*(5*d^3 + 30*d^2*e*x + 40*d*e^2*x^2 + 16*e^3*x^3))
)/(5*(c*d^2 - a*e^2)^4*(d + e*x)^2*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 \left(x\,(ae^2+cd^2)+ade+cdex^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1129$$

$$\frac{6cd\int \frac{1}{(d+ex)(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{5\,(cd^2-ae^2)}+\frac{2}{5(d+ex)^2\,(cd^2-ae^2)\,\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \; 1129$$

$$\frac{6cd\left(\frac{4cd\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3(cd^2-ae^2)}+\frac{2}{3(d+ex)(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{5\left(cd^2-ae^2\right)}+\frac{2}{5(d+ex)^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{6cd\left(\frac{2}{3(d+ex)(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{8cd(ae^2+cd^2+2cdex)}{3(cd^2-ae^2)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{5\left(cd^2-ae^2\right)}+\frac{2}{5(d+ex)^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[1/((d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
2/(5*(c*d^2 - a*e^2)*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]) + (6*c*d*(2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3
*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(5*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.19

method	result
gosper	$-\frac{2(cdx+ae)(16c^3d^3e^3x^3+8x^2a\,c^2d^2e^4+40c^3d^4e^2x^2-2x\,a^2cd\,e^5+20xa\,c^2d^3e^3+30c^3d^5ex+e^6a^3-5d^2e^4a^2c+15d^4e^2a\,c^2+5d^6c^3)}{5(ex+d)(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8)(cd\,x^2e+a\,e^2x+c\,d^2x+ade)^{\frac{3}{2}}}$
orering	$-\frac{2(16c^3d^3e^3x^3+8x^2a\,c^2d^2e^4+40c^3d^4e^2x^2-2x\,a^2cd\,e^5+20xa\,c^2d^3e^3+30c^3d^5ex+e^6a^3-5d^2e^4a^2c+15d^4e^2a\,c^2+5d^6c^3)(cdx+ae)}{5(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e^4-4a\,c^3d^6e^2+c^4d^8)(ex+d)(ade+(a\,e^2+c\,d^2)x+cd\,x^2e)^{\frac{3}{2}}}$
trager	$-\frac{2(16c^3d^3e^3x^3+8x^2a\,c^2d^2e^4+40c^3d^4e^2x^2-2x\,a^2cd\,e^5+20xa\,c^2d^3e^3+30c^3d^5ex+e^6a^3-5d^2e^4a^2c+15d^4e^2a\,c^2+5d^6c^3)\sqrt{cd\,x^2e+}}{5(cdx+ae)(a\,e^2-c\,d^2)(e^6a^3-3d^2e^4a^2c+3d^4e^2a\,c^2-d^6c^3)(ex+d)^3}$
default	$\frac{-\frac{2}{5\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^2\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}-\frac{6dec\left(-\frac{2}{3\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}+\frac{8dec}{3\left(a\,e^2-c\,d^2\right)^3}\right.}{5\left(a\,e^2-c\,d^2\right)}}{e^2}$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
-2/5*(c*d*x+a*e)*(16*c^3*d^3*e^3*x^3+8*a*c^2*d^2*e^4*x^2+40*c^3*d^4*e^2*x^
2-2*a^2*c*d*e^5*x+20*a*c^2*d^3*e^3*x+30*c^3*d^5*e*x+a^3*e^6-5*a^2*c*d^2*e^
4+15*a*c^2*d^4*e^2+5*c^3*d^6)/(e*x+d)/(a^4*e^8-4*a^3*c*d^2*e^6+6*a^2*c^2*d
^4*e^4-4*a*c^3*d^6*e^2+c^4*d^8)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. 2(169) = 338.

Time = 3.41 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.73

$$\int \frac{1}{(d+ex)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{2\left(16\,c^3 d\right.}{5\left(ac^4d^{11}e-4\,a^2c^3d^9e^3+6\,a^3c^2d^7e^5-4\,a^4cd^5e^7+a^5d^3e^9+\left(c^5d^9e^3-4\,ac^4d^7e^5+6\,a^2c^3d^5e^7-4\,a^3c^2d^3e\right.\right.}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
-2/5*(16*c^3*d^3*e^3*x^3 + 5*c^3*d^6 + 15*a*c^2*d^4*e^2 - 5*a^2*c*d^2*e^4
+ a^3*e^6 + 8*(5*c^3*d^4*e^2 + a*c^2*d^2*e^4)*x^2 + 2*(15*c^3*d^5*e + 10*a
*c^2*d^3*e^3 - a^2*c*d*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
/(a*c^4*d^11*e - 4*a^2*c^3*d^9*e^3 + 6*a^3*c^2*d^7*e^5 - 4*a^4*c*d^5*e^7 +
 a^5*d^3*e^9 + (c^5*d^9*e^3 - 4*a*c^4*d^7*e^5 + 6*a^2*c^3*d^5*e^7 - 4*a^3*
c^2*d^3*e^9 + a^4*c*d*e^11)*x^4 + (3*c^5*d^10*e^2 - 11*a*c^4*d^8*e^4 + 14*
a^2*c^3*d^6*e^6 - 6*a^3*c^2*d^4*e^8 - a^4*c*d^2*e^10 + a^5*e^12)*x^3 + 3*(
c^5*d^11*e - 3*a*c^4*d^9*e^3 + 2*a^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7 - 3*a
^4*c*d^3*e^9 + a^5*d*e^11)*x^2 + (c^5*d^12 - a*c^4*d^10*e^2 - 6*a^2*c^3*d^
8*e^4 + 14*a^3*c^2*d^6*e^6 - 11*a^4*c*d^4*e^8 + 3*a^5*d^2*e^10)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}\,(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3930 vs. $2(169) = 338$.

Time = 0.22 (sec) , antiderivative size = 3930, normalized size of antiderivative = 21.71

$$\int \frac{1}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
2/5*(16*c^3*d^3*e*abs(e)*sgn(1/(e*x + d))*sgn(e)/(sqrt(c*d*e)*c^4*d^8 - 4*
sqrt(c*d*e)*a*c^3*d^6*e^2 + 6*sqrt(c*d*e)*a^2*c^2*d^4*e^4 - 4*sqrt(c*d*e)*
a^3*c*d^2*e^6 + sqrt(c*d*e)*a^4*e^8) - (5*c^3*d^3/((c^4*d^8*e^2*sgn(1/(e*x
 + d))*sgn(e) - 4*a*c^3*d^6*e^4*sgn(1/(e*x + d))*sgn(e) + 6*a^2*c^2*d^4*e^
6*sgn(1/(e*x + d))*sgn(e) - 4*a^3*c*d^2*e^8*sgn(1/(e*x + d))*sgn(e) + a^4*
e^10*sgn(1/(e*x + d))*sgn(e))*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x
+ d))) + (15*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c^18*d^34*e
^22*sgn(1/(e*x + d))^4*sgn(e)^4 - 240*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e
^3/(e*x + d))*a*c^17*d^32*e^24*sgn(1/(e*x + d))^4*sgn(e)^4 + 1800*sqrt(c*d
*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^2*c^16*d^30*e^26*sgn(1/(e*x +
d))^4*sgn(e)^4 - 8400*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^
3*c^15*d^28*e^28*sgn(1/(e*x + d))^4*sgn(e)^4 + 27300*sqrt(c*d*e - c*d^2*e/
(e*x + d) + a*e^3/(e*x + d))*a^4*c^14*d^26*e^30*sgn(1/(e*x + d))^4*sgn(e)^
4 - 65520*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^5*c^13*d^24*
e^32*sgn(1/(e*x + d))^4*sgn(e)^4 + 120120*sqrt(c*d*e - c*d^2*e/(e*x + d) +
 a*e^3/(e*x + d))*a^6*c^12*d^22*e^34*sgn(1/(e*x + d))^4*sgn(e)^4 - 171600*
sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^7*c^11*d^20*e^36*sgn(1
/(e*x + d))^4*sgn(e)^4 + 193050*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*
x + d))*a^8*c^10*d^18*e^38*sgn(1/(e*x + d))^4*sgn(e)^4 - 171600*sqrt(c*d*e
 - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^9*c^9*d^16*e^40*sgn(1/(e*x + ...
```

Mupad [B] (verification not implemented)

Time = 6.61 (sec) , antiderivative size = 1005, normalized size of antiderivative = 5.55

$$\int \frac{1}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(((16*c^3*d^4*e)/(15*(a*e^2 - c*d^2)^5) - (8*c^2*d^2*e*(a*e^2 + c*d^2))/(1
5*(a*e^2 - c*d^2)^5))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d +
e*x) - (((e^2*(10*c^2*d^3 - 18*a*c*d*e^2))/(5*(a*e^2 - c*d^2)^2*(3*a^2*e^5
 + 3*c^2*d^4*e - 6*a*c*d^2*e^3)) + (8*c^2*d^3*e^2)/(5*(a*e^2 - c*d^2)^2*(3
*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e^3)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d
*e*x^2)^(1/2))/(d + e*x)^2 - (2*e^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2
)^(1/2))/((d + e*x)^3*(5*a^2*e^5 + 5*c^2*d^4*e - 10*a*c*d^2*e^3)) - ((x*((
32*a*c^5*d^6*e^4)/(15*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2
*c*d*e^5)) - ((a*e^2 + c*d^2)*((16*c^5*d^5*e^3*(a*e^2 + c*d^2))/(15*(a*e^2
 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*c^5*d^5*e^3*
(3*a*e^2 - 11*c*d^2))/(15*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 +
 a^2*c*d*e^5))))/(c*d*e) + (2*c^2*d^2*e^2*(30*c^4*d^6 - 104*a*c^3*d^4*e^2
+ 58*a^2*c^2*d^2*e^4))/(15*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3
+ a^2*c*d*e^5)) + (4*c^4*d^4*e^2*(a*e^2 + c*d^2)*(3*a*e^2 - 11*c*d^2))/(15
*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))) - (a*((16
*c^5*d^5*e^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d
^3*e^3 + a^2*c*d*e^5)) + (8*c^5*d^5*e^3*(3*a*e^2 - 11*c*d^2))/(15*(a*e^2 -
 c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c + (c*d*e*(a*e^2
 + c*d^2)*(30*c^4*d^6 - 104*a*c^3*d^4*e^2 + 58*a^2*c^2*d^2*e^4))/(15*(a*e^
2 - c*d^2)^4*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))*(x*(a*e^2 + ...
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 561, normalized size of antiderivative = 3.10

$$\int \frac{1}{(d+ex)^2 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{\frac{32\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,c^2d^5}{5} + \frac{96\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,c^2d^4ex}{5} + \frac{96\sqrt{e}\,\sqrt{d}}{}}{\sqrt{cdx+ae}\,\left(a^4e^{11}x^3 - 4a^3c\,d^2e^9x^3 + 6a^2c^2d^4\right.}$$

input
```
int(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output

```
(2*(16*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d**5 + 48*sqrt(e)*sq
rt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d**4*e*x + 48*sqrt(e)*sqrt(d)*sqrt(c)
*sqrt(a*e + c*d*x)*c**2*d**3*e**2*x**2 + 16*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a
*e + c*d*x)*c**2*d**2*e**3*x**3 - sqrt(d + e*x)*a**3*e**6 + 5*sqrt(d + e*x
)*a**2*c*d**2*e**4 + 2*sqrt(d + e*x)*a**2*c*d*e**5*x - 15*sqrt(d + e*x)*a*
c**2*d**4*e**2 - 20*sqrt(d + e*x)*a*c**2*d**3*e**3*x - 8*sqrt(d + e*x)*a*c
**2*d**2*e**4*x**2 - 5*sqrt(d + e*x)*c**3*d**6 - 30*sqrt(d + e*x)*c**3*d**
5*e*x - 40*sqrt(d + e*x)*c**3*d**4*e**2*x**2 - 16*sqrt(d + e*x)*c**3*d**3*
e**3*x**3))/(5*sqrt(a*e + c*d*x)*(a**4*d**3*e**8 + 3*a**4*d**2*e**9*x + 3*
a**4*d*e**10*x**2 + a**4*e**11*x**3 - 4*a**3*c*d**5*e**6 - 12*a**3*c*d**4*
e**7*x - 12*a**3*c*d**3*e**8*x**2 - 4*a**3*c*d**2*e**9*x**3 + 6*a**2*c**2*
d**7*e**4 + 18*a**2*c**2*d**6*e**5*x + 18*a**2*c**2*d**5*e**6*x**2 + 6*a**
2*c**2*d**4*e**7*x**3 - 4*a*c**3*d**9*e**2 - 12*a*c**3*d**8*e**3*x - 12*a*
c**3*d**7*e**4*x**2 - 4*a*c**3*d**6*e**5*x**3 + c**4*d**11 + 3*c**4*d**10*
e*x + 3*c**4*d**9*e**2*x**2 + c**4*d**8*e**3*x**3))
```

3.128 $\int \frac{1}{x(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 389

$$\int \frac{1}{x(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2cd}{ae\left(cd^2-ae^2\right)(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$+\frac{2(5cd^2+ae^2)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5ad\left(cd^2-ae^2\right)^2(d+ex)^3}$$
$$+\frac{2(15c^2d^4+14acd^2e^2-5a^2e^4)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15ad^2\left(cd^2-ae^2\right)^3(d+ex)^2}$$
$$+\frac{2(15c^3d^6+73ac^2d^4e^2-55a^2cd^2e^4+15a^3e^6)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15ad^3\left(cd^2-ae^2\right)^4(d+ex)}$$
$$-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{3/2}d^{7/2}e^{3/2}}$$

output
```
2*c*d/a/e/(-a*e^2+c*d^2)/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
+2/5*(a*e^2+5*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d/(-a*e^2+c
*d^2)^2/(e*x+d)^3+2/15*(-5*a^2*e^4+14*a*c*d^2*e^2+15*c^2*d^4)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^2/(-a*e^2+c*d^2)^3/(e*x+d)^2+2/15*(15*a^3*
e^6-55*a^2*c*d^2*e^4+73*a*c^2*d^4*e^2+15*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)/a/d^3/(-a*e^2+c*d^2)^4/(e*x+d)-2*arctanh(a^(1/2)*e^(1/2)*(
e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(3/2)/d^(7/2)/e^
(3/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.77

$$\int \frac{1}{x(d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)\left(15c^4d^7(d+ex)^3+a^4e^8\left(23d^2+35dex+15e^2x^2\right)+ac\right.}{}\right.}{}$$

input
```
Integrate[1/(x*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(2*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(15*c^4*d^7*(d + e*x)^3 + a^4*e
^8*(23*d^2 + 35*d*e*x + 15*e^2*x^2) + a*c^3*d^5*e^3*x*(90*d^2 + 160*d*e*x
+ 73*e^2*x^2) + a^2*c^2*d^3*e^4*(90*d^3 + 80*d^2*e*x - 56*d*e^2*x^2 - 55*e
^3*x^3) - a^3*c*d*e^6*(80*d^3 + 106*d^2*e*x + 20*d*e^2*x^2 - 15*e^3*x^3)))
/((c*d^2 - a*e^2)^4*(d + e*x)) - 15*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*Ar
cTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])]))/(15*a
^(3/2)*d^(7/2)*e^(3/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)^2 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1259}$$

$$\int \left(-\frac{e}{d^2(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{e}{d(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}+\frac{}{d^2x\left(x\left(ae^2\right.\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\operatorname{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{a^{3/2}d^{7/2}e^{3/2}}+\frac{2\left(a^2e^4+cdex\left(ae^2+cd^2\right)+c^2d^4\right)}{ad^3e\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{16c^2de\left(ae^2+cd^2+2cdex\right)}{5\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{8ce\left(ae^2+cd^2+2cdex\right)}{3d\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{4ce}{5(d+ex)\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2e}{3d^2(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2e}{5d(d+ex)^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[1/(x*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*e)/(5*d*(c*d^2 - a*e^2)*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) - (4*c*e)/(5*(c*d^2 - a*e^2)^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2]) - (2*e)/(3*d^2*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (16*c^2*d*e*(c*d^2 + a*e^2 + 2*c*d*e*
x))/(5*(c*d^2 - a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (8
*c*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*d*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d
^2 + a*e^2)*x + c*d*e*x^2]) + (2*(c^2*d^4 + a^2*e^4 + c*d*e*(c*d^2 + a*e^2
)*x))/(a*d^3*e*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]) - ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(a^(3/2)*d^(7/2)*e^(3/2))
```

Defintions of rubi rules used

rule 1259
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x
)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &&
 EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && (ILtQ[n, 0] || (IGtQ[n, 0]
&& ILtQ[p + 1/2, 0])) &&  !IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.52

method	result
default	$\frac{\frac{1}{ade\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}-\frac{\left(a\,e^2+c\,d^2\right)\left(2cdxe+a\,e^2+c\,d^2\right)}{ade\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}-\frac{\ln\left(\frac{2ade+\left(a\,e^2+c\,d^2\right)x+2\sqrt{ade}\,\sqrt{ade+\left(a\,e^2+\right.}}{x}\right.}{ade\sqrt{ade}}}{d^2}$

input
```
int(1/x/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
1/d^2*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e
*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-1/d^2*(-2/3/(a
*e^2-c*d^2)/(x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d*e*
c/(a*e^2-c*d^2)^3*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e^2-c*
d^2)*(x+d/e))^(1/2))-1/e/d*(-2/5/(a*e^2-c*d^2)/(x+d/e)^2/(d*e*c*(x+d/e)^2+
(a*e^2-c*d^2)*(x+d/e))^(1/2)-6/5*d*e*c/(a*e^2-c*d^2)*(-2/3/(a*e^2-c*d^2)/(
x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d*e*c/(a*e^2-c*d^
2)^3*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))
^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. 2(361) = 722.

Time = 9.49 (sec) , antiderivative size = 2234, normalized size of antiderivative = 5.74

$$\int \frac{1}{x(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
[1/30*(15*(a*c^4*d^11*e - 4*a^2*c^3*d^9*e^3 + 6*a^3*c^2*d^7*e^5 - 4*a^4*c*
d^5*e^7 + a^5*d^3*e^9 + (c^5*d^9*e^3 - 4*a*c^4*d^7*e^5 + 6*a^2*c^3*d^5*e^7
 - 4*a^3*c^2*d^3*e^9 + a^4*c*d*e^11)*x^4 + (3*c^5*d^10*e^2 - 11*a*c^4*d^8*
e^4 + 14*a^2*c^3*d^6*e^6 - 6*a^3*c^2*d^4*e^8 - a^4*c*d^2*e^10 + a^5*e^12)*
x^3 + 3*(c^5*d^11*e - 3*a*c^4*d^9*e^3 + 2*a^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*
e^7 - 3*a^4*c*d^3*e^9 + a^5*d*e^11)*x^2 + (c^5*d^12 - a*c^4*d^10*e^2 - 6*a
^2*c^3*d^8*e^4 + 14*a^3*c^2*d^6*e^6 - 11*a^4*c*d^4*e^8 + 3*a^5*d^2*e^10)*x
)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2
 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2
)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) + 4*(15*a*c^4*d^11*e
+ 90*a^3*c^2*d^7*e^5 - 80*a^4*c*d^5*e^7 + 23*a^5*d^3*e^9 + (15*a*c^4*d^8*e
^4 + 73*a^2*c^3*d^6*e^6 - 55*a^3*c^2*d^4*e^8 + 15*a^4*c*d^2*e^10)*x^3 + (4
5*a*c^4*d^9*e^3 + 160*a^2*c^3*d^7*e^5 - 56*a^3*c^2*d^5*e^7 - 20*a^4*c*d^3*
e^9 + 15*a^5*d*e^11)*x^2 + (45*a*c^4*d^10*e^2 + 90*a^2*c^3*d^8*e^4 + 80*a^
3*c^2*d^6*e^6 - 106*a^4*c*d^4*e^8 + 35*a^5*d^2*e^10)*x)*sqrt(c*d*e*x^2 + a
*d*e + (c*d^2 + a*e^2)*x))/(a^3*c^4*d^15*e^3 - 4*a^4*c^3*d^13*e^5 + 6*a^5*
c^2*d^11*e^7 - 4*a^6*c*d^9*e^9 + a^7*d^7*e^11 + (a^2*c^5*d^13*e^5 - 4*a^3*
c^4*d^11*e^7 + 6*a^4*c^3*d^9*e^9 - 4*a^5*c^2*d^7*e^11 + a^6*c*d^5*e^13)*x^
4 + (3*a^2*c^5*d^14*e^4 - 11*a^3*c^4*d^12*e^6 + 14*a^4*c^3*d^10*e^8 - 6*a^
5*c^2*d^8*e^10 - a^6*c*d^6*e^12 + a^7*d^4*e^14)*x^3 + 3*(a^2*c^5*d^15*e...
```

Sympy [F]

$$\int \frac{1}{x(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

input
```
integrate(1/x/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(x*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}(ex+d)^2x}\,dx$$

input
```
integrate(1/x/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^2*x),
 x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(361) = 722$.

Time = 0.19 (sec) , antiderivative size = 998, normalized size of antiderivative = 2.57

$$\int \frac{1}{x(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
2/15*((15*c^4*d^4*e^3/(sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a
*sgn(1/(e*x + d))*sgn(e)) + 15*(c^4*d^8*e^3 - 4*a*c^3*d^6*e^5 + 6*a^2*c^2*
d^4*e^7 - 4*a^3*c*d^2*e^9 + a^4*e^11)*arctan(sqrt(c*d*e - c*d^2*e/(e*x + d
) + a*e^3/(e*x + d))*d/(sqrt(-a*d*e)*e))/(sqrt(-a*d*e)*a*d^3*e*sgn(1/(e*x
+ d))*sgn(e)) + (90*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c^2*
d^6*e^4*sgn(1/(e*x + d))^4*sgn(e)^4 - 60*sqrt(c*d*e - c*d^2*e/(e*x + d) +
a*e^3/(e*x + d))*a*c*d^4*e^6*sgn(1/(e*x + d))^4*sgn(e)^4 + 15*sqrt(c*d*e -
 c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^2*d^2*e^8*sgn(1/(e*x + d))^4*sgn(e
)^4 - 20*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))^(3/2)*c*d^5*e^3*sgn
(1/(e*x + d))^4*sgn(e)^4 + 5*(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))
^(3/2)*a*d^3*e^5*sgn(1/(e*x + d))^4*sgn(e)^4 + 3*(c*d*e - c*d^2*e/(e*x + d
) + a*e^3/(e*x + d))^(5/2)*d^4*e^2*sgn(1/(e*x + d))^4*sgn(e)^4)/(d^5*sgn(1
/(e*x + d))^5*sgn(e)^5))*e^4*abs(e)/(c*d^2*e^2 - a*e^4)^4 - (15*sqrt(c*d*e
)*c^4*d^8*abs(e)*arctan(sqrt(c*d*e)*d/(sqrt(-a*d*e)*e)) - 60*sqrt(c*d*e)*a
*c^3*d^6*e^2*abs(e)*arctan(sqrt(c*d*e)*d/(sqrt(-a*d*e)*e)) + 90*sqrt(c*d*e
)*a^2*c^2*d^4*e^4*abs(e)*arctan(sqrt(c*d*e)*d/(sqrt(-a*d*e)*e)) - 60*sqrt(
c*d*e)*a^3*c*d^2*e^6*abs(e)*arctan(sqrt(c*d*e)*d/(sqrt(-a*d*e)*e)) + 15*sq
rt(c*d*e)*a^4*e^8*abs(e)*arctan(sqrt(c*d*e)*d/(sqrt(-a*d*e)*e)) + 15*sqrt(
-a*d*e)*c^4*d^7*e*abs(e) + 73*sqrt(-a*d*e)*a*c^3*d^5*e^3*abs(e) - 55*sqrt(
-a*d*e)*a^2*c^2*d^3*e^5*abs(e) + 15*sqrt(-a*d*e)*a^3*c*d*e^7*abs(e))*sg...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)^2 \left(ade + (cd^2+ae^2)x + cdex^2\right)^{3/2}}\,dx = \int \frac{1}{x\,(d+e\,x)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(1/(x*(d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(1/(x*(d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x(d+ex)^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{x\left(ex+d\right)^2\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
int(1/x/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

3.129 $\int \frac{1}{x^2(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 515

$$\begin{aligned}
&\int \frac{1}{x^2(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \\
&-\frac{c(3cd^2-ae^2)}{a^2e^2\left(cd^2-ae^2\right)(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&-\frac{1}{adex(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&-\frac{\left(15c^2d^4-10acd^2e^2+7a^2e^4\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5a^2d^2e\left(cd^2-ae^2\right)^2(d+ex)^3} \\
&-\frac{\left(45c^3d^6-45ac^2d^4e^2+83a^2cd^2e^4-35a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15a^2d^3e\left(cd^2-ae^2\right)^3(d+ex)^2} \\
&-\frac{\left(45c^4d^8-60ac^3d^6e^2+346a^2c^2d^4e^4-340a^3cd^2e^6+105a^4e^8\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15a^2d^4e\left(cd^2-ae^2\right)^4(d+ex)} \\
&+\frac{\left(3cd^2+7ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{5/2}d^{9/2}e^{5/2}}
\end{aligned}$$

output
```
-c*(-a*e^2+3*c*d^2)/a^2/e^2/(-a*e^2+c*d^2)/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)-1/a/d/e/x/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)-1/5*(7*a^2*e^4-10*a*c*d^2*e^2+15*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)/a^2/d^2/e/(-a*e^2+c*d^2)^2/(e*x+d)^3-1/15*(-35*a^3*e^6+83*a^2
*c*d^2*e^4-45*a*c^2*d^4*e^2+45*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)/a^2/d^3/e/(-a*e^2+c*d^2)^3/(e*x+d)^2-1/15*(105*a^4*e^8-340*a^3*c*d^2
*e^6+346*a^2*c^2*d^4*e^4-60*a*c^3*d^6*e^2+45*c^4*d^8)*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)/a^2/d^4/e/(-a*e^2+c*d^2)^4/(e*x+d)+(7*a*e^2+3*c*d^2)*a
rctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/a^(5/2)/d^(9/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)\left(45c^5d^9x(d+ex)^3+15ac^4d^7e(d-4ex)(d+ex)^3+a^5e \cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[1/(x^2*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
),x]
```

output
```
(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(45*c^5*d^9*x*(d + e*x)^3 + 15*a
*c^4*d^7*e*(d - 4*e*x)*(d + e*x)^3 + a^5*e^9*(15*d^3 + 161*d^2*e*x + 245*d
*e^2*x^2 + 105*e^3*x^3) + 2*a^3*c^2*d^3*e^5*(45*d^4 + 255*d^3*e*x + 145*d^
2*e^2*x^2 - 226*d*e^3*x^3 - 170*e^4*x^4) - a^4*c*d*e^7*(60*d^4 + 515*d^3*e
*x + 637*d^2*e^2*x^2 + 95*d*e^3*x^3 - 105*e^4*x^4) + 2*a^2*c^3*d^5*e^3*(-3
0*d^4 - 45*d^3*e*x + 195*d^2*e^2*x^2 + 380*d*e^3*x^3 + 173*e^4*x^4)))/((c*
d^2 - a*e^2)^4*x*(d + e*x))) + 15*(3*c*d^2 + 7*a*e^2)*(a*e + c*d*x)^(3/2)*
(d + e*x)^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e
+ c*d*x])])/(15*a^(5/2)*d^(9/2)*e^(5/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.48, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1259}$$

$$\int \left(\frac{e^2}{d^2(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}+\frac{1}{d^2x^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}+\frac{}{d^3(d+ex)\left(x\left(ae^2\right.\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{a^{3/2}d^{9/2}\sqrt{e}}+\frac{3\left(ae^2+cd^2\right)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2a^{5/2}d^{9/2}e^{5/2}}-\frac{4\left(a^2e^4+cdex\left(ae^2+cd^2\right)+c^2d^4\right)}{ad^4\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\left(3a^2e^4-2acd^2e^2+3c^2d^4\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{a^2d^4e^2x\left(cd^2-ae^2\right)^2}+\frac{2\left(a^2e^4+cdex\left(ae^2+cd^2\right)+c^2d^4\right)}{ad^3ex\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{16c^2e^2\left(ae^2+cd^2+2cdex\right)}{5\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{16ce^2\left(ae^2+cd^2+2cdex\right)}{3d^2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{4ce^2}{5d(d+ex)\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{2e^2}{5d^2(d+ex)^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{4e^2}{3d^3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[1/(x^2*(d + e*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*e^2)/(5*d^2*(c*d^2 - a*e^2)*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]) + (4*c*e^2)/(5*d*(c*d^2 - a*e^2)^2*(d + e*x)*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2]) + (4*e^2)/(3*d^3*(c*d^2 - a*e^2)*(d + e*x)*S
qrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (16*c^2*e^2*(c*d^2 + a*e^2 +
 2*c*d*e*x))/(5*(c*d^2 - a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]) - (16*c*e^2*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*d^2*(c*d^2 - a*e^2)^3*Sqr
t[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (4*(c^2*d^4 + a^2*e^4 + c*d*e*
(c*d^2 + a*e^2)*x))/(a*d^4*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) + (2*(c^2*d^4 + a^2*e^4 + c*d*e*(c*d^2 + a*e^2)*x))/(a*d^3
*e*(c*d^2 - a*e^2)^2*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((3*
c^2*d^4 - 2*a*c*d^2*e^2 + 3*a^2*e^4)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])/(a^2*d^4*e^2*(c*d^2 - a*e^2)^2*x) + (2*ArcTanh[(2*a*d*e + (c*d^2 +
 a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2])])/(a^(3/2)*d^(9/2)*Sqrt[e]) + (3*(c*d^2 + a*e^2)*ArcTanh[(2*a*d*e
 + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])])/(2*a^(5/2)*d^(9/2)*e^(5/2))
```

Defintions of rubi rules used

rule 1259
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x
)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &&
 EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && (ILtQ[n, 0] || (IGtQ[n, 0]
&& ILtQ[p + 1/2, 0])) &&  !IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 943, normalized size of antiderivative = 1.83

method	result
default	$\frac{-\frac{1}{adex\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}-\frac{3\left(a\,e^{2}+c\,d^{2}\right)\left(\frac{1}{ade\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}-\frac{\left(a\,e^{2}+c\,d^{2}\right)\left(2cdxe+a\,e^{2}+c\,d^{2}\right)}{ade\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd}}\right.}{2ade}}{d^{2}}$

input
```
int(1/x^2/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETU
RNVERBOSE)
```

output
```
1/d^2*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2
)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d
/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x
+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-4*c/a*(2*c*d
*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*x^2*e)^(1/2))+1/d^2*(-2/5/(a*e^2-c*d^2)/(x+d/e)^2/(d*e*c*(x+d/e)^2+(a*e
^2-c*d^2)*(x+d/e))^(1/2)-6/5*d*e*c/(a*e^2-c*d^2)*(-2/3/(a*e^2-c*d^2)/(x+d/
e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d*e*c/(a*e^2-c*d^2)^3
*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/
2)))-2/d^3*e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2
)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*
e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d
^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))+2*e/d^3
*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
+8/3*d*e*c/(a*e^2-c*d^2)^3*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+
(a*e^2-c*d^2)*(x+d/e))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1321 vs. $2(485) = 970$.

Time = 23.56 (sec) , antiderivative size = 2662, normalized size of antiderivative = 5.17

$$\int \frac{1}{x^2(d+ex)^2\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algor
ithm="fricas")
```

output
```
[1/60*(15*((3*c^6*d^11*e^3 - 5*a*c^5*d^9*e^5 - 10*a^2*c^4*d^7*e^7 + 30*a^3
*c^3*d^5*e^9 - 25*a^4*c^2*d^3*e^11 + 7*a^5*c*d*e^13)*x^5 + (9*c^6*d^12*e^2
 - 12*a*c^5*d^10*e^4 - 35*a^2*c^4*d^8*e^6 + 80*a^3*c^3*d^6*e^8 - 45*a^4*c^
2*d^4*e^10 - 4*a^5*c*d^2*e^12 + 7*a^6*e^14)*x^4 + 3*(3*c^6*d^13*e - 2*a*c^
5*d^11*e^3 - 15*a^2*c^4*d^9*e^5 + 20*a^3*c^3*d^7*e^7 + 5*a^4*c^2*d^5*e^9 -
 18*a^5*c*d^3*e^11 + 7*a^6*d*e^13)*x^3 + (3*c^6*d^14 + 4*a*c^5*d^12*e^2 -
25*a^2*c^4*d^10*e^4 + 65*a^4*c^2*d^6*e^8 - 68*a^5*c*d^4*e^10 + 21*a^6*d^2*
e^12)*x^2 + (3*a*c^5*d^13*e - 5*a^2*c^4*d^11*e^3 - 10*a^3*c^3*d^9*e^5 + 30
*a^4*c^2*d^7*e^7 - 25*a^5*c*d^5*e^9 + 7*a^6*d^3*e^11)*x)*sqrt(a*d*e)*log((
8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2
 + a*d*e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) +
8*(a*c*d^3*e + a^2*d*e^3)*x)/x^2) - 4*(15*a^2*c^4*d^12*e^2 - 60*a^3*c^3*d^
10*e^4 + 90*a^4*c^2*d^8*e^6 - 60*a^5*c*d^6*e^8 + 15*a^6*d^4*e^10 + (45*a*c
^5*d^10*e^4 - 60*a^2*c^4*d^8*e^6 + 346*a^3*c^3*d^6*e^8 - 340*a^4*c^2*d^4*e
^10 + 105*a^5*c*d^2*e^12)*x^4 + (135*a*c^5*d^11*e^3 - 165*a^2*c^4*d^9*e^5
+ 760*a^3*c^3*d^7*e^7 - 452*a^4*c^2*d^5*e^9 - 95*a^5*c*d^3*e^11 + 105*a^6*
d*e^13)*x^3 + (135*a*c^5*d^12*e^2 - 135*a^2*c^4*d^10*e^4 + 390*a^3*c^3*d^8
*e^6 + 290*a^4*c^2*d^6*e^8 - 637*a^5*c*d^4*e^10 + 245*a^6*d^2*e^12)*x^2 +
(45*a*c^5*d^13*e - 15*a^2*c^4*d^11*e^3 - 90*a^3*c^3*d^9*e^5 + 510*a^4*c^2*
d^7*e^7 - 515*a^5*c*d^5*e^9 + 161*a^6*d^3*e^11)*x)*sqrt(c*d*e*x^2 + a*d...
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x^2\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)^2}\,dx$$

input
```
integrate(1/x**2/(e*x+d)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(x**2*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(ex+d\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algor
ithm="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^2*x^2
), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4890 vs. 2(485) = 970.

Time = 0.29 (sec) , antiderivative size = 4890, normalized size of antiderivative = 9.50

$$\int \frac{1}{x^2(d+ex)^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^2/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algor
ithm="giac")
```

output
```
-1/15*(e^7*(2*(150*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*c^18*
d^42*e^30*sgn(1/(e*x + d))^4*sgn(e)^4 - 2550*sqrt(c*d*e - c*d^2*e/(e*x + d
) + a*e^3/(e*x + d))*a*c^17*d^40*e^32*sgn(1/(e*x + d))^4*sgn(e)^4 + 20445*
sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^2*c^16*d^38*e^34*sgn(1
/(e*x + d))^4*sgn(e)^4 - 102720*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*
x + d))*a^3*c^15*d^36*e^36*sgn(1/(e*x + d))^4*sgn(e)^4 + 362400*sqrt(c*d*e
 - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^4*c^14*d^34*e^38*sgn(1/(e*x + d)
)^4*sgn(e)^4 - 953400*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^
5*c^13*d^32*e^40*sgn(1/(e*x + d))^4*sgn(e)^4 + 1938300*sqrt(c*d*e - c*d^2*
e/(e*x + d) + a*e^3/(e*x + d))*a^6*c^12*d^30*e^42*sgn(1/(e*x + d))^4*sgn(e
)^4 - 3113760*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^7*c^11*d
^28*e^44*sgn(1/(e*x + d))^4*sgn(e)^4 + 4006860*sqrt(c*d*e - c*d^2*e/(e*x +
 d) + a*e^3/(e*x + d))*a^8*c^10*d^26*e^46*sgn(1/(e*x + d))^4*sgn(e)^4 - 41
61300*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^9*c^9*d^24*e^48*
sgn(1/(e*x + d))^4*sgn(e)^4 + 3496350*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e
^3/(e*x + d))*a^10*c^8*d^22*e^50*sgn(1/(e*x + d))^4*sgn(e)^4 - 2371200*sqr
t(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^11*c^7*d^20*e^52*sgn(1/(e
*x + d))^4*sgn(e)^4 + 1288560*sqrt(c*d*e - c*d^2*e/(e*x + d) + a*e^3/(e*x
+ d))*a^12*c^6*d^18*e^54*sgn(1/(e*x + d))^4*sgn(e)^4 - 553560*sqrt(c*d*e -
 c*d^2*e/(e*x + d) + a*e^3/(e*x + d))*a^13*c^5*d^16*e^56*sgn(1/(e*x + d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x^2\,(d+e\,x)^2\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,d$$

input
```
int(1/(x^2*(d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*(d + e*x)^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 20.66 (sec) , antiderivative size = 7938, normalized size of antiderivative = 15.41

$$\int \frac{1}{x^2(d+ex)^2\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
( - 315*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c
*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sq
rt(d + e*x))*a**6*d**3*e**12*x - 945*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*
d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 +
 c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**6*d**2*e**13*x**2 - 945*sqrt(
e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(
2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*
a**6*d*e**14*x**3 - 315*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt
(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))*a**6*e**15*x**4 + 1230*sqrt(e)*sqrt(d)*sqrt(a
)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)
*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**5*c*d**5*e**10
*x + 3690*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e +
 c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*
sqrt(d + e*x))*a**5*c*d**4*e**11*x**2 + 3690*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(
a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e +
a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**5*c*d**3*e**12*x**3 +
 1230*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d
*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt
(d + e*x))*a**5*c*d**2*e**13*x**4 - 1725*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a...
```

3.130 $\int \frac{x^5}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 530

$$\int \frac{x^5}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2a^5e^5}{c^5d^5\left(cd^2-ae^2\right)(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}$$
$$+\frac{2(c^5d^{10}+7a^5e^{10})\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7c^5d^5e^4\left(cd^2-ae^2\right)^2(d+ex)^4}$$
$$-\frac{2(22c^5d^{10}-35ac^4d^8e^2-35a^5e^{10})\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35c^4d^4e^4\left(cd^2-ae^2\right)^3(d+ex)^3}$$
$$+\frac{2(122c^5d^{10}-385ac^4d^8e^2+350a^2c^3d^6e^4+105a^5e^{10})\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{105c^3d^3e^4\left(cd^2-ae^2\right)^4(d+ex)^2}$$
$$-\frac{2(176c^5d^{10}-805ac^4d^8e^2+1400a^2c^3d^6e^4-1050a^3c^2d^4e^6-105a^5e^{10})\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{105c^2d^2e^4\left(cd^2-ae^2\right)^5(d+ex)}$$
$$+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{3/2}d^{3/2}e^{9/2}}$$

output
```
2*a^5*e^5/c^5/d^5/(-a*e^2+c*d^2)/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)+2/7*(7*a^5*e^10+c^5*d^10)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
/c^5/d^5/e^4/(-a*e^2+c*d^2)^2/(e*x+d)^4-2/35*(-35*a^5*e^10-35*a*c^4*d^8*e^
2+22*c^5*d^10)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^4/d^4/e^4/(-a*e^2
+c*d^2)^3/(e*x+d)^3+2/105*(105*a^5*e^10+350*a^2*c^3*d^6*e^4-385*a*c^4*d^8*
e^2+122*c^5*d^10)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^4/(-a*
e^2+c*d^2)^4/(e*x+d)^2-2/105*(-105*a^5*e^10-1050*a^3*c^2*d^4*e^6+1400*a^2*
c^3*d^6*e^4-805*a*c^4*d^8*e^2+176*c^5*d^10)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(1/2)/c^2/d^2/e^4/(-a*e^2+c*d^2)^5/(e*x+d)+2*arctanh(c^(1/2)*d^(1/2)*(
e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(3/2)/d^(3/2)/e^
(9/2)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{(d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2\left(-\frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)\left(15cd^6e^3(ae+cdx)^4+21c^2d^7e^2(ae+cdx)^3(d+ex)-\right.}{}\right.}{}$$

input
```
Integrate[x^5/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(2*(-((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(15*c*d^6*e^3*(a*e + c*d*x)^4
 + 21*c^2*d^7*e^2*(a*e + c*d*x)^3*(d + e*x) - 105*a*c*d^5*e^4*(a*e + c*d*x
)^3*(d + e*x) + 35*c^3*d^8*e*(a*e + c*d*x)^2*(d + e*x)^2 - 175*a*c^2*d^6*e
^3*(a*e + c*d*x)^2*(d + e*x)^2 + 350*a^2*c*d^4*e^5*(a*e + c*d*x)^2*(d + e*
x)^2 + 105*c^4*d^9*(a*e + c*d*x)*(d + e*x)^3 - 525*a*c^3*d^7*e^2*(a*e + c*
d*x)*(d + e*x)^3 + 1050*a^2*c^2*d^5*e^4*(a*e + c*d*x)*(d + e*x)^3 - 1050*a
^3*c*d^3*e^6*(a*e + c*d*x)*(d + e*x)^3 - 105*a^5*e^9*(d + e*x)^4))/((c*d^2
 - a*e^2)^5*(d + e*x)^2)) + 105*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTan
h[(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])/(Sqrt[e]*Sqrt[a*e + c*d*x])]))/(105*c^(3
/2)*d^(3/2)*e^(9/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 527, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1268, 109, 27, 167, 27, 167, 27, 162, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(d+ex)^3 \left(x \left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \ 1268$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\int \frac{x^5}{(ae+cdx)^{3/2}(d+ex)^{9/2}}dx}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 109$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^4}{cd(d+ex)^{7/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{2\int \frac{x^3\left(8ade-\left(cd^2-ae^2\right)x\right)}{2\sqrt{ae+cdx}(d+ex)^{9/2}}dx}{cd(cd^2-ae^2)}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^4}{cd(d+ex)^{7/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{\int \frac{x^3\left(8ade-\left(cd^2-ae^2\right)x\right)}{\sqrt{ae+cdx}(d+ex)^{9/2}}dx}{cd(cd^2-ae^2)}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 167$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^4}{cd(d+ex)^{7/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{\frac{2dx^3\left(7ae^2+cd^2\right)\sqrt{ae+cdx}}{7e(d+ex)^{7/2}\left(cd^2-ae^2\right)}-\frac{2\int \frac{x^2\left(7x\left(cd^2-ae^2\right)^2+6ade\left(cd^2+7ae^2\right)\right)}{2\sqrt{ae+cdx}(d+ex)^{7/2}}dx}{7e\left(cd^2-ae^2\right)}}{cd(cd^2-ae^2)}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^4}{cd(d+ex)^{7/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{\frac{2dx^3\left(7ae^2+cd^2\right)\sqrt{ae+cdx}}{7e(d+ex)^{7/2}\left(cd^2-ae^2\right)}-\frac{\int \frac{x^2\left(7x\left(cd^2-ae^2\right)^2+6ade\left(cd^2+7ae^2\right)\right)}{\sqrt{ae+cdx}(d+ex)^{7/2}}dx}{7e\left(cd^2-ae^2\right)}}{cd(cd^2-ae^2)}\right)}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 167

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^4}{cd(d+ex)^{7/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{\frac{2dx^3\left(7ae^2+cd^2\right)\sqrt{ae+cdx}}{7e(d+ex)^{7/2}\left(cd^2-ae^2\right)}-\frac{-\frac{2\int -\frac{x\left(35x\left(cd^2-ae^2\right)^3+4ade\left(7c^2d^4-20ace^2d^2-35a^2 \cdots\right.\right.}{2\sqrt{ae+cdx}(d+ex)^{5/2}}}{5e\left(cd^2-ae^2\right)} \cdots}{7e\left(cd^2- \cdots\right.}}{cd(cd^2-ae^2)} \cdots\right.}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2} \cdots}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^4}{cd(d+ex)^{7/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{\frac{2dx^3\left(7ae^2+cd^2\right)\sqrt{ae+cdx}}{7e(d+ex)^{7/2}\left(cd^2-ae^2\right)}-\frac{\frac{\int \frac{x\left(35x\left(cd^2-ae^2\right)^3+4ade\left(7c^2d^4-20ace^2d^2-35a^2e^4\right)\right)}{\sqrt{ae+cdx}(d+ex)^{5/2}} \cdots}{5e\left(cd^2-ae^2\right)} \cdots}{7e\left(cd^2-a \cdots\right.}}{cd(cd^2-ae^2)} \cdots\right.}{\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2} \cdots}$$

$\Big\downarrow$ 162

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^4}{cd(d+ex)^{7/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{\frac{2dx^3\left(7ae^2+cd^2\right)\sqrt{ae+cdx}}{7e(d+ex)^{7/2}\left(cd^2-ae^2\right)}-\frac{\frac{35\left(cd^2-ae^2\right)^3\int \frac{1}{\sqrt{ae+cdx}\sqrt{d+ex}}dx}{e^2}-\frac{2\sqrt{ae+cdx}\left(-105c \cdots\right.}{\cdots}}{\cdots}}{\cdots} \cdots\right.}{\sqrt{x\left(ae \cdots\right.}}$$

$\Big\downarrow$ 66

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^4}{cd(d+ex)^{7/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{\frac{2dx^3\left(7ae^2+cd^2\right)\sqrt{ae+cdx}}{7e(d+ex)^{7/2}\left(cd^2-ae^2\right)}-\frac{\frac{70\left(cd^2-ae^2\right)^3\int\frac{1}{cd-\frac{e(ae+cdx)}{d+ex}}d\frac{\sqrt{ae+cdx}}{\sqrt{d+ex}}}{e^2}-\frac{2\sqrt{ae+cdx}}{\cdots}}{\cdots}}{\cdots}\right)}{\sqrt{x(\cdots}}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(\frac{2aex^4}{cd(d+ex)^{7/2}(cd^2-ae^2)\sqrt{ae+cdx}}-\frac{\frac{2dx^3\left(7ae^2+cd^2\right)\sqrt{ae+cdx}}{7e(d+ex)^{7/2}\left(cd^2-ae^2\right)}-\frac{\frac{70\left(cd^2-ae^2\right)^3\text{arctanh}\left(\frac{\sqrt{e}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{c}\sqrt{d}e^{5/2}}-\frac{2\sqrt{ae+cdx}(\cdots}{\cdots}}{\cdots}}{\cdots}\right)}{\sqrt{x(\cdots}}$$

input
```
Int[x^5/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*((2*a*e*x^4)/(c*d*(c*d^2 - a*e^2)*Sqrt[a*
e + c*d*x]*(d + e*x)^(7/2)) - ((2*d*(c*d^2 + 7*a*e^2)*x^3*Sqrt[a*e + c*d*x
])/(7*e*(c*d^2 - a*e^2)*(d + e*x)^(7/2)) - ((-2*d*(7*c^2*d^4 - 20*a*c*d^2*
e^2 - 35*a^2*e^4)*x^2*Sqrt[a*e + c*d*x])/(5*e*(c*d^2 - a*e^2)*(d + e*x)^(5
/2)) + ((-2*Sqrt[a*e + c*d*x]*(105*c^4*d^10 - 490*a*c^3*d^8*e^2 + 896*a^2*
c^2*d^6*e^4 - 790*a^3*c*d^4*e^6 - 105*a^4*d^2*e^8 + 2*e*(70*c^4*d^9 - 329*
a*c^3*d^7*e^2 + 607*a^2*c^2*d^5*e^4 - 435*a^3*c*d^3*e^6 - 105*a^4*d*e^8)*x
))/(3*e^2*(c*d^2 - a*e^2)^2*(d + e*x)^(3/2)) + (70*(c*d^2 - a*e^2)^3*ArcTa
nh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(Sqrt[c]*
Sqrt[d]*e^(5/2)))/(5*e*(c*d^2 - a*e^2)))/(7*e*(c*d^2 - a*e^2)))/(c*d*(c*d^
2 - a*e^2))))/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1179 vs. 2(498) = 996.

Time = 3.24 (sec) , antiderivative size = 1180, normalized size of antiderivative = 2.23

method	result	size
default	Expression too large to display	1180

input
```
int(x^5/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
1/e^3*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*d^2)/
d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/d/e/
c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*
d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*e*c)^
(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2))+12*d^2/e^5*(
2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(1/2)-3*d/e^4*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(
1/2)-(a*e^2+c*d^2)/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d
^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))-10/e^6*d^3*(-2/3/(a*e^2-c*
d^2)/(x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d*e*c/(a*e^
2-c*d^2)^3*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x
+d/e))^(1/2))+5/e^7*d^4*(-2/5/(a*e^2-c*d^2)/(x+d/e)^2/(d*e*c*(x+d/e)^2+(a*
e^2-c*d^2)*(x+d/e))^(1/2)-6/5*d*e*c/(a*e^2-c*d^2)*(-2/3/(a*e^2-c*d^2)/(x+d
/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d*e*c/(a*e^2-c*d^2)^
3*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1
/2)))-d^5/e^8*(-2/7/(a*e^2-c*d^2)/(x+d/e)^3/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)
*(x+d/e))^(1/2)-8/7*d*e*c/(a*e^2-c*d^2)*(-2/5/(a*e^2-c*d^2)/(x+d/e)^2/(d*e
*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-6/5*d*e*c/(a*e^2-c*d^2)*(-2/3/(a
*e^2-c*d^2)/(x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d*e*
c/(a*e^2-c*d^2)^3*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1586 vs. $2(498) = 996$.

Time = 12.95 (sec) , antiderivative size = 3186, normalized size of antiderivative = 6.01

$$\int \frac{x^5}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{x^5}{(d+ex)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{x^5}{((d+ex)\,(ae+cdx))^{\frac{3}{2}}\,(d+ex)^3}\,dx$$

input
```
integrate(x**5/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**5/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(d+ex)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^5}{(d+ex)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{x^5}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)^3}\,dx$$

input
```
integrate(x^5/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(x^5/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^3),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{x^5}{(d+e\,x)^3\,\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)^{3/2}}\,dx$$

input
```
int(x^5/((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(x^5/((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^5}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{x^5}{(ex+d)^3\,(ade+(a\,e^2+c\,d^2)\,x+cde\,x^2)^{\frac{3}{2}}}dx$$

input
```
int(x^5/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
int(x^5/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

3.131 $\int \frac{x^4}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 306

$$\begin{aligned}\int \frac{x^4}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=&-\frac{2x^4}{\left(cd^2-ae^2\right)(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\\&+\frac{16dx^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7\left(cd^2-ae^2\right)^2(d+ex)^4}-\frac{96adex^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35\left(cd^2-ae^2\right)^3(d+ex)^3}\\&-\frac{128a^2d^2e\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35\left(cd^2-ae^2\right)^4(d+ex)^2}\\&+\frac{128a^2de(cd^2-3ae^2)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35\left(cd^2-ae^2\right)^5(d+ex)}\end{aligned}$$

output
```
-2*x^4/(-a*e^2+c*d^2)/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+16
/7*d*x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^2/(e*x+d)^
4-96/35*a*d*e*x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^3
/(e*x+d)^3-128/35*a^2*d^2*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^
2+c*d^2)^4/(e*x+d)^2+128/35*a^2*d*e*(-3*a*e^2+c*d^2)*(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^5/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.52

$$\int \frac{x^4}{(d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2(ae+cdx)^5 \left(5d^4-\frac{28ad^3e(d+ex)}{ae+cdx}+\frac{70a^2d^2e^2(d+ex)^2}{(ae+cdx)^2}-\frac{140a\ldots}{(\ldots}\right.}{35\left(cd^2-ae^2\right)^5 (d+ex)^2((ae+cdx)(d\ldots}$$

input
```
Integrate[x^4/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(2*(a*e + c*d*x)^5*(5*d^4 - (28*a*d^3*e*(d + e*x))/(a*e + c*d*x) + (70*a^2
*d^2*e^2*(d + e*x)^2)/(a*e + c*d*x)^2 - (140*a^3*d*e^3*(d + e*x)^3)/(a*e +
 c*d*x)^3 - (35*a^4*e^4*(d + e*x)^4)/(a*e + c*d*x)^4))/(35*(c*d^2 - a*e^2)
^5*(d + e*x)^2*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 2.44 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.73, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1267, 27, 2169, 27, 2169, 27, 1220, 1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d+ex)^3 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1267}$$

$$-\frac{\int \frac{\left(7cd^2+ae^2\right)x^3e^4+3d\left(3cd^2+ae^2\right)x^2e^3+d^2\left(5cd^2+3ae^2\right)xe^2+d^3\left(cd^2+ae^2\right)e}{2(d+ex)^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{cde^5}-\frac{1}{cde^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{(7cd^2+ae^2)x^3e^4+3d(3cd^2+ae^2)x^2e^3+d^2(5cd^2+3ae^2)xe^2+d^3(cd^2+ae^2)e}{(d+ex)^3(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{2cde^5}-\frac{1}{cde^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 2169

$$-\frac{-\frac{\int \frac{3(3cd^2+ae^2)^2x^2e^7+2d(11c^2d^4+18ace^2d^2+3a^2e^4)xe^6+3d^2(c^2d^4+6ace^2d^2+a^2e^4)e^5}{2(d+ex)^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{2cde^4}-\frac{e\left(\frac{ae^2}{cd}+7d\right)}{2(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{2cde^5}-\frac{1}{cde^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{3(3cd^2+ae^2)^2x^2e^7+2d(11c^2d^4+18ace^2d^2+3a^2e^4)xe^6+3d^2(c^2d^4+6ace^2d^2+a^2e^4)e^5}{(d+ex)^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{4cde^4}-\frac{e\left(\frac{ae^2}{cd}+7d\right)}{2(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{2cde^5}-\frac{1}{cde^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 2169

$$-\frac{-\frac{-\frac{\int \frac{3e^8\left(d\left(3c^3d^6+15ac^2e^2d^4+25a^2ce^4d^2+5a^3e^6\right)+e\left(19c^3d^6+15ac^2e^2d^4+25a^2ce^4d^2+5a^3e^6\right)x\right)}{2(d+ex)^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3cde^3}-\frac{e^5\left(ae^2+3cd^2\right)^2}{cd(d+ex)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}}{4cde^4}-\frac{\cdots}{2(d+e\cdots}}{2cde^5}-\frac{1}{cde^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{-\frac{e^5\int \frac{d\left(3c^3d^6+15ac^2e^2d^4+25a^2ce^4d^2+5a^3e^6\right)+e\left(19c^3d^6+15ac^2e^2d^4+25a^2ce^4d^2+5a^3e^6\right)x}{(d+ex)^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{2cd}-\frac{e^5\left(ae^2+3cd^2\right)^2}{cd(d+ex)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}}{4cde^4}-\frac{\cdots}{2(d+ex)\cdots}}{2cde^5}-\frac{1}{cde^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 1220

$$-\frac{-\frac{-\frac{e^5\left(\frac{\left(-35a^4e^8-140a^3cd^2e^6+70a^2c^2d^4e^4-28ac^3d^6e^2+5c^4d^8\right)\int\frac{1}{(d+ex)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{7\left(cd^2-ae^2\right)}-\frac{32c^3d^7}{7(d+ex)^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade\cdots}}\cdots\right)}{2cd}\cdots}{4cde^4}\cdots}{2cde^5\cdots}$$

$$\frac{1}{cde^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1129$$

$$-\frac{-\frac{-\frac{e^5\left(\frac{\left(-35a^4e^8-140a^3cd^2e^6+70a^2c^2d^4e^4-28ac^3d^6e^2+5c^4d^8\right)\left(\frac{6cd\int\frac{1}{(d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{5\left(cd^2-ae^2\right)}+\frac{2}{5(d+ex)^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+\cdots}}\cdots\right)}{7\left(cd^2-ae^2\right)}\cdots\right)}{2cd}\cdots}{4cde^4}\cdots}{2cde\cdots}$$

$$\frac{1}{cde^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1129$$

$$-\frac{-\frac{-\frac{e^5\left(\frac{\left(-35a^4e^8-140a^3cd^2e^6+70a^2c^2d^4e^4-28ac^3d^6e^2+5c^4d^8\right)\left(\frac{6cd\left(\frac{4cd\int\frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)}+\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+a\cdots}}\cdots\right)}{5\left(cd^2-ae^2\right)}\cdots\right)}{7\left(cd^2-ae^2\right)}\cdots\right)}{2cd}\cdots}{4\cdots}\cdots}{\cdots}$$

$$\frac{1}{cde^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \ 1088$$

$$-\frac{-\frac{-\frac{e^5\left(\frac{\left(-35a^4e^8-140a^3cd^2e^6+70a^2c^2d^4e^4-28ac^3d^6e^2+5c^4d^8\right)\left(\frac{6cd\left(\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+a}}\right)}{5\left(cd^2-ae^2\right)}\right.}{7\left(cd^2-ae^2\right)}\right.}{2cd}}{}}{}$$

$$\frac{1}{cde^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[x^4/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
-(1/(c*d*e^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])) - (-1/2*(e*(7*d
 + (a*e^2)/(c*d)))/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
 - (-((e^5*(3*c*d^2 + a*e^2)^2)/(c*d*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e
^2)*x + c*d*e*x^2])) - (e^5*((-32*c^3*d^7)/(7*(c*d^2 - a*e^2)*(d + e*x)^3*
Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + ((5*c^4*d^8 - 28*a*c^3*d^6*
e^2 + 70*a^2*c^2*d^4*e^4 - 140*a^3*c*d^2*e^6 - 35*a^4*e^8)*(2/(5*(c*d^2 -
a*e^2)*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (6*c*d*(
2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
) - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])))/(5*(c*d^2 - a*e^2))))/(7*(c*d^2 - a*e^2)
)))/(2*c*d))/(4*c*d*e^4))/(2*c*d*e^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 3.27 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.06

method	result
gosper	$\frac{2(cdx+ae)(35a^4e^8x^4+140a^3c\,d^2e^6x^4-70a^2c^2d^4e^4x^4+28a\,c^3d^6e^2x^4-5c^4d^8x^4+280a^4d\,e^7x^3+280a^3c\,d^3e^5x^3-56a^2c^2d^5e^3x^3+8a\,c\cdots}{35(ex+d)^2(a^5e^{10}-5a^4c\,d^2e^8+10a^3c^2d^4e^6-10a^2c^3d^6e^4+5a\,c^4d^8e^2-\cdots}$
trager	$\frac{2(35a^4e^8x^4+140a^3c\,d^2e^6x^4-70a^2c^2d^4e^4x^4+28a\,c^3d^6e^2x^4-5c^4d^8x^4+280a^4d\,e^7x^3+280a^3c\,d^3e^5x^3-56a^2c^2d^5e^3x^3+8a\,c^3d^7e\,x^3+\cdots}{35(cdx+ae)(a\,e^2-c\,d^2)(a^4e^8-4a^3c\,d^2e^6+6a^2c^2\cdots}$
orering	$\frac{2(35a^4e^8x^4+140a^3c\,d^2e^6x^4-70a^2c^2d^4e^4x^4+28a\,c^3d^6e^2x^4-5c^4d^8x^4+280a^4d\,e^7x^3+280a^3c\,d^3e^5x^3-56a^2c^2d^5e^3x^3+8a\,c^3d^7e\,x^3+\cdots}{35(a^5e^{10}-5a^4c\,d^2e^8+10a^3c^2d^4e^6-10a^2c^3d^6e^4+5a\,c^4d^8e^2-c^5d^{10})(e\cdots}$
default	$\frac{-\frac{1}{dec\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}-\frac{\left(a\,e^2+c\,d^2\right)\left(2cdxe+a\,e^2+c\,d^2\right)}{dec\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}}{e^3}+\frac{d^4\left(-\frac{2}{7\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^3\sqrt{dec\left(x+\frac{d}{e}\right)\cdots}}\cdots\right)}{\cdots}$

input
```
int(x^4/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
2/35*(c*d*x+a*e)*(35*a^4*e^8*x^4+140*a^3*c*d^2*e^6*x^4-70*a^2*c^2*d^4*e^4*
x^4+28*a*c^3*d^6*e^2*x^4-5*c^4*d^8*x^4+280*a^4*d*e^7*x^3+280*a^3*c*d^3*e^5
*x^3-56*a^2*c^2*d^5*e^3*x^3+8*a*c^3*d^7*e*x^3+560*a^4*d^2*e^6*x^2+224*a^3*
c*d^4*e^4*x^2-16*a^2*c^2*d^6*e^2*x^2+448*a^4*d^3*e^5*x+64*a^3*c*d^5*e^3*x+
128*a^4*d^4*e^4)/(e*x+d)^2/(a^5*e^10-5*a^4*c*d^2*e^8+10*a^3*c^2*d^4*e^6-10
*a^2*c^3*d^6*e^4+5*a*c^4*d^8*e^2-c^5*d^10)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*
e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(288) = 576$.

Time = 11.18 (sec) , antiderivative size = 735, normalized size of antiderivative = 2.40

$$\int \frac{x^4}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx =$$

$$-\frac{}{35\,(ac^5d^{14}e - 5\,a^2c^4d^{12}e^3 + 10\,a^3c^3d^{10}e^5 - 10\,a^4c^2d^8e^7 + 5\,a^5cd^6e^9 - a^6d^4e^{11} + (c^6d^{11}e^4 - 5\,ac^5d^9e^6 + 10}$$

input
```
integrate(x^4/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
-2/35*(128*a^4*d^4*e^4 - (5*c^4*d^8 - 28*a*c^3*d^6*e^2 + 70*a^2*c^2*d^4*e^
4 - 140*a^3*c*d^2*e^6 - 35*a^4*e^8)*x^4 + 8*(a*c^3*d^7*e - 7*a^2*c^2*d^5*e
^3 + 35*a^3*c*d^3*e^5 + 35*a^4*d*e^7)*x^3 - 16*(a^2*c^2*d^6*e^2 - 14*a^3*c
*d^4*e^4 - 35*a^4*d^2*e^6)*x^2 + 64*(a^3*c*d^5*e^3 + 7*a^4*d^3*e^5)*x)*sqr
t(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^5*d^14*e - 5*a^2*c^4*d^12*e^
3 + 10*a^3*c^3*d^10*e^5 - 10*a^4*c^2*d^8*e^7 + 5*a^5*c*d^6*e^9 - a^6*d^4*e
^11 + (c^6*d^11*e^4 - 5*a*c^5*d^9*e^6 + 10*a^2*c^4*d^7*e^8 - 10*a^3*c^3*d^
5*e^10 + 5*a^4*c^2*d^3*e^12 - a^5*c*d*e^14)*x^5 + (4*c^6*d^12*e^3 - 19*a*c
^5*d^10*e^5 + 35*a^2*c^4*d^8*e^7 - 30*a^3*c^3*d^6*e^9 + 10*a^4*c^2*d^4*e^1
1 + a^5*c*d^2*e^13 - a^6*e^15)*x^4 + 2*(3*c^6*d^13*e^2 - 13*a*c^5*d^11*e^4
 + 20*a^2*c^4*d^9*e^6 - 10*a^3*c^3*d^7*e^8 - 5*a^4*c^2*d^5*e^10 + 7*a^5*c*
d^3*e^12 - 2*a^6*d*e^14)*x^3 + 2*(2*c^6*d^14*e - 7*a*c^5*d^12*e^3 + 5*a^2*
c^4*d^10*e^5 + 10*a^3*c^3*d^8*e^7 - 20*a^4*c^2*d^6*e^9 + 13*a^5*c*d^4*e^11
 - 3*a^6*d^2*e^13)*x^2 + (c^6*d^15 - a*c^5*d^13*e^2 - 10*a^2*c^4*d^11*e^4
+ 30*a^3*c^3*d^9*e^6 - 35*a^4*c^2*d^7*e^8 + 19*a^5*c*d^5*e^10 - 4*a^6*d^3*
e^12)*x)
```

Sympy [F]

$$\int \frac{x^4}{(d+ex)^3 (ade + (cd^2+ae^2)x + cdex^2)^{3/2}}\,dx = \int \frac{x^4}{((d+ex)(ae+cdx))^{\frac{3}{2}} (d+ex)^3}\,dx$$

input
```
integrate(x**4/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**4/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(d+ex)^3 (ade + (cd^2+ae^2)x + cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F]

$$\int \frac{x^4}{(d+ex)^3 (ade + (cd^2+ae^2)x + cdex^2)^{3/2}}\,dx = \int \frac{x^4}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)^3}\,dx$$

input
```
integrate(x^4/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(x^4/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^3),
 x)
```

Mupad [B] (verification not implemented)

Time = 12.16 (sec) , antiderivative size = 13455, normalized size of antiderivative = 43.97

$$\int \frac{x^4}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx = \text{Too large to display}$$

input
```
int(x^4/((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(32*d^3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(7*(5*a^2*d^3*e^7 +
 5*c^2*d^7*e^3 + 5*a^2*e^10*x^3 + 15*a^2*d^2*e^8*x + 15*a^2*d*e^9*x^2 + 15
*c^2*d^6*e^4*x + 15*c^2*d^5*e^5*x^2 + 5*c^2*d^4*e^6*x^3 - 10*a*c*d^5*e^5 -
 30*a*c*d^4*e^6*x - 30*a*c*d^3*e^7*x^2 - 10*a*c*d^2*e^8*x^3)) - (2*d^4*(a*
d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(7*a^2*d^4*e^7 + 7*c^2*d^8*e^3
 + 7*a^2*e^11*x^4 + 28*a^2*d^3*e^8*x + 28*a^2*d*e^10*x^3 + 28*c^2*d^7*e^4*
x + 42*a^2*d^2*e^9*x^2 + 42*c^2*d^6*e^5*x^2 + 28*c^2*d^5*e^6*x^3 + 7*c^2*d
^4*e^7*x^4 - 14*a*c*d^6*e^5 - 56*a*c*d^5*e^6*x - 84*a*c*d^4*e^7*x^2 - 56*a
*c*d^3*e^8*x^3 - 14*a*c*d^2*e^9*x^4) + (626*c^5*d^11*(a*d*e + a*e^2*x + c*
d^2*x + c*d*e*x^2)^(1/2))/(105*(a^7*d*e^17 + a^7*e^18*x - c^7*d^15*e^3 + 7
*a*c^6*d^13*e^5 - 7*a^6*c*d^3*e^15 - c^7*d^14*e^4*x - 21*a^2*c^5*d^11*e^7
+ 35*a^3*c^4*d^9*e^9 - 35*a^4*c^3*d^7*e^11 + 21*a^5*c^2*d^5*e^13 + 7*a*c^6
*d^12*e^6*x - 7*a^6*c*d^2*e^16*x - 21*a^2*c^5*d^10*e^8*x + 35*a^3*c^4*d^8*
e^10*x - 35*a^4*c^3*d^6*e^12*x + 21*a^5*c^2*d^4*e^14*x)) - (494*c^2*d^5*(a
*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(105*(a^4*d*e^11 + a^4*e^12*x
 + c^4*d^9*e^3 - 4*a*c^3*d^7*e^5 - 4*a^3*c*d^3*e^9 + c^4*d^8*e^4*x + 6*a^2
*c^2*d^5*e^7 - 4*a*c^3*d^6*e^6*x - 4*a^3*c*d^2*e^10*x + 6*a^2*c^2*d^4*e^8*
x)) + (6*c^2*d^7*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(7*(5*a^4*
d^3*e^11 + 5*c^4*d^11*e^3 + 5*a^4*e^14*x^3 - 20*a*c^3*d^9*e^5 - 20*a^3*c*d
^5*e^9 + 15*a^4*d^2*e^12*x + 15*a^4*d*e^13*x^2 + 15*c^4*d^10*e^4*x + 30...
```

Reduce [F]

$$\int \frac{x^4}{(d+ex)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \int \frac{x^4}{(ex+d)^3\,(ade+(a\,e^2+c\,d^2)\,x+cde\,x^2)^{\frac{3}{2}}}dx$$

input
```
int(x^4/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
int(x^4/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

3.132 $\int \frac{x^3}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 355

$$\int \frac{x^3}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2cdx^4}{ae\left(cd^2-ae^2\right)(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} - \frac{2\left(7cd^2+ae^2\right)x^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7ae\left(cd^2-ae^2\right)^2(d+ex)^4} + \frac{12\left(7cd^2+ae^2\right)x^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35\left(cd^2-ae^2\right)^3(d+ex)^3} + \frac{16ad\left(7cd^2+ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35\left(cd^2-ae^2\right)^4(d+ex)^2} - \frac{16a\left(cd^2-3ae^2\right)\left(7cd^2+ae^2\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35\left(cd^2-ae^2\right)^5(d+ex)}$$

output

```
2*c*d*x^4/a/e/(-a*e^2+c*d^2)/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)-2/7*(a*e^2+7*c*d^2)*x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/e/(
-a*e^2+c*d^2)^2/(e*x+d)^4+12/35*(a*e^2+7*c*d^2)*x^2*(a*d*e+(a*e^2+c*d^2)*x
+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^3/(e*x+d)^3+16/35*a*d*(a*e^2+7*c*d^2)*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^4/(e*x+d)^2-16/35*a*(-
3*a*e^2+c*d^2)*(a*e^2+7*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a
*e^2+c*d^2)^5/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \frac{2(c^4d^7x^3(7d+2ex)-2ac^3d^5ex^2(7d^2+24dex+7e^2x^2)}{}$$

input
```
Integrate[x^3/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(2*(c^4*d^7*x^3*(7*d + 2*e*x) - 2*a*c^3*d^5*e*x^2*(7*d^2 + 24*d*e*x + 7*e^
2*x^2) + a^4*e^5*(16*d^3 + 56*d^2*e*x + 70*d*e^2*x^2 + 35*e^3*x^3) + 2*a^2
*c^2*d^3*e^2*x*(28*d^3 + 97*d^2*e*x + 119*d*e^2*x^2 + 35*e^3*x^3) + 2*a^3*
c*d*e^3*(56*d^4 + 200*d^3*e*x + 259*d^2*e^2*x^2 + 140*d*e^3*x^3 + 35*e^4*x
^4)))/(35*(c*d^2 - a*e^2)^5*(d + e*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1267, 27, 2169, 27, 1220, 1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d+ex)^3 (x(ae^2+cd^2)+ade+cdex^2)^{3/2}}dx$$

$$\Big\downarrow \text{1267}$$

$$-\frac{\int \frac{3(3cd^2+ae^2)x^2e^3+6d(cd^2+ae^2)xe^2+d^2(cd^2+3ae^2)e}{2(d+ex)^3(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{2cde^4}-\frac{1}{2cde^3(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{3(3cd^2+ae^2)x^2e^3+6d(cd^2+ae^2)xe^2+d^2(cd^2+3ae^2)e}{(d+ex)^3(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{4cde^4}-\frac{1}{2cde^3(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 2169

$$-\frac{-\frac{\int \frac{3e^4\left(d\left(c^2d^4+10ace^2d^2+5a^2e^4\right)+e\left(9c^2d^4+10ace^2d^2+5a^2e^4\right)x\right)}{2(d+ex)^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3cde^3}-\frac{e\left(\frac{ae^2}{cd}+3d\right)}{(d+ex)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{4cde^4}-\frac{1}{2cde^3(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{e\int \frac{d\left(c^2d^4+10ace^2d^2+5a^2e^4\right)+e\left(9c^2d^4+10ace^2d^2+5a^2e^4\right)x}{(d+ex)^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{2cd}-\frac{e\left(\frac{ae^2}{cd}+3d\right)}{(d+ex)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{4cde^4}-\frac{1}{2cde^3(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 1220

$$-\frac{-\frac{e\left(-\frac{\left(35a^3e^6+35a^2cd^2e^4-7ac^2d^4e^2+c^3d^6\right)\int \frac{1}{(d+ex)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{7\left(cd^2-ae^2\right)}-\frac{16c^2d^5}{7(d+ex)^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{2cd}-\frac{\ldots}{(d+\ldots}}{4cde^4}$$

$$\frac{1}{2cde^3(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 1129

$$-\frac{-\frac{e\left(-\frac{\left(35a^3e^6+35a^2cd^2e^4-7ac^2d^4e^2+c^3d^6\right)\left(\frac{6cd\int \frac{1}{(d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{5\left(cd^2-ae^2\right)}+\frac{2}{5(d+ex)^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{7\left(cd^2-ae^2\right)}-\frac{\ldots}{7(d\ldots}\ldots\right)}{2cd}\ldots}{4cde^4}$$

$$\frac{1}{2cde^3(d+ex)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$\Big\downarrow$ 1129

$$-\frac{-\frac{e\left(-\frac{\left(35a^3e^6+35a^2cd^2e^4-7ac^2d^4e^2+c^3d^6\right)\left(\frac{6cd\left(\frac{4cd\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)}+\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{5\left(cd^2-ae^2\right)}+\frac{\cdots}{5(d+e\cdots}\right.}{7\left(cd^2-ae^2\right)}\right.}{2cd}}{4cde^4}\cdots$$

$$\frac{1}{2cde^3(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\big\downarrow$ 1088

$$-\frac{-\frac{e\left(-\frac{\left(35a^3e^6+35a^2cd^2e^4-7ac^2d^4e^2+c^3d^6\right)\left(\frac{6cd\left(\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{5\left(cd^2-ae^2\right)}+\frac{\cdots}{5(d+\cdots}\right.}{7\left(cd^2-ae^2\right)}\right.}{2cd}}{4cde^4}\cdots$$

$$\frac{1}{2cde^3(d+ex)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[x^3/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
-1/2*1/(c*d*e^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (
-((e*(3*d + (a*e^2)/(c*d)))/((d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2])) - (e*((-16*c^2*d^5)/(7*(c*d^2 - a*e^2)*(d + e*x)^3*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((c^3*d^6 - 7*a*c^2*d^4*e^2 + 35*a^2*
c*d^2*e^4 + 35*a^3*e^6)*(2/(5*(c*d^2 - a*e^2)*(d + e*x)^2*Sqrt[a*d*e + (c*
d^2 + a*e^2)*x + c*d*e*x^2]) + (6*c*d*(2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e
*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(
5*(c*d^2 - a*e^2))))/(7*(c*d^2 - a*e^2))))/(2*c*d))/(4*c*d*e^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.02

method	result
gosper	$-\frac{2(cdx+ae)(70a^3cd e^7x^4+70a^2c^2d^3e^5x^4-14a c^3d^5e^3x^4+2c^4d^7 e x^4+35a^4e^8x^3+280a^3c d^2e^6x^3+238a^2c^2d^4e^4x^3-48a c^3d^6e^2x^3+\cdots}{35(ex+d)^2(a^5e^{10}-5a^4c d^2e^8+10a^3c^2d^4e^6-10\cdots}$
trager	$-\frac{2(70a^3cd e^7x^4+70a^2c^2d^3e^5x^4-14a c^3d^5e^3x^4+2c^4d^7 e x^4+35a^4e^8x^3+280a^3c d^2e^6x^3+238a^2c^2d^4e^4x^3-48a c^3d^6e^2x^3+7c^4d^8x^3-\cdots}{35(cdx+ae)(a e^2-c d^2)(a\cdots}$
orering	$-\frac{2(70a^3cd e^7x^4+70a^2c^2d^3e^5x^4-14a c^3d^5e^3x^4+2c^4d^7 e x^4+35a^4e^8x^3+280a^3c d^2e^6x^3+238a^2c^2d^4e^4x^3-48a c^3d^6e^2x^3+7c^4d^8x^3-\cdots}{35(a^5e^{10}-5a^4c d^2e^8+10a^3c^2d^4e^6-10a^2c^3d^6e^4\cdots}$
default	$\frac{4cdxe+2a e^2+2c d^2}{e^3\left(4ac d^2e^2-(a e^2+c d^2)^2\right)\sqrt{ade+(a e^2+c d^2)x+cd x^2e}}-\frac{3d\left(-\frac{2}{3\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a e^2-c d^2\right)\left(x+\frac{d}{e}\right)}}+\frac{\cdots}{3\left(a e^2-c\cdots\right.}\cdots\right.}{e^4}$

input
```
int(x^3/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/35*(c*d*x+a*e)*(70*a^3*c*d*e^7*x^4+70*a^2*c^2*d^3*e^5*x^4-14*a*c^3*d^5*
e^3*x^4+2*c^4*d^7*e*x^4+35*a^4*e^8*x^3+280*a^3*c*d^2*e^6*x^3+238*a^2*c^2*d
^4*e^4*x^3-48*a*c^3*d^6*e^2*x^3+7*c^4*d^8*x^3+70*a^4*d*e^7*x^2+518*a^3*c*d
^3*e^5*x^2+194*a^2*c^2*d^5*e^3*x^2-14*a*c^3*d^7*e*x^2+56*a^4*d^2*e^6*x+400
*a^3*c*d^4*e^4*x+56*a^2*c^2*d^6*e^2*x+16*a^4*d^3*e^5+112*a^3*c*d^5*e^3)/(e
*x+d)^2/(a^5*e^10-5*a^4*c*d^2*e^8+10*a^3*c^2*d^4*e^6-10*a^2*c^3*d^6*e^4+5*
a*c^4*d^8*e^2-c^5*d^10)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(337) = 674$.

Time = 11.93 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.17

$$\int \frac{x^3}{(d+ex)^3 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{}{35\left(ac^5d^{14}e-5\,a^2c^4d^{12}e^3+10\,a^3c^3d^{10}e^5-10\,a^4c^2d^8e^7\right.}$$

input
```
integrate(x^3/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
2/35*(112*a^3*c*d^5*e^3 + 16*a^4*d^3*e^5 + 2*(c^4*d^7*e - 7*a*c^3*d^5*e^3
+ 35*a^2*c^2*d^3*e^5 + 35*a^3*c*d*e^7)*x^4 + (7*c^4*d^8 - 48*a*c^3*d^6*e^2
 + 238*a^2*c^2*d^4*e^4 + 280*a^3*c*d^2*e^6 + 35*a^4*e^8)*x^3 - 2*(7*a*c^3*
d^7*e - 97*a^2*c^2*d^5*e^3 - 259*a^3*c*d^3*e^5 - 35*a^4*d*e^7)*x^2 + 8*(7*
a^2*c^2*d^6*e^2 + 50*a^3*c*d^4*e^4 + 7*a^4*d^2*e^6)*x)*sqrt(c*d*e*x^2 + a*
d*e + (c*d^2 + a*e^2)*x)/(a*c^5*d^14*e - 5*a^2*c^4*d^12*e^3 + 10*a^3*c^3*d
^10*e^5 - 10*a^4*c^2*d^8*e^7 + 5*a^5*c*d^6*e^9 - a^6*d^4*e^11 + (c^6*d^11*
e^4 - 5*a*c^5*d^9*e^6 + 10*a^2*c^4*d^7*e^8 - 10*a^3*c^3*d^5*e^10 + 5*a^4*c
^2*d^3*e^12 - a^5*c*d*e^14)*x^5 + (4*c^6*d^12*e^3 - 19*a*c^5*d^10*e^5 + 35
*a^2*c^4*d^8*e^7 - 30*a^3*c^3*d^6*e^9 + 10*a^4*c^2*d^4*e^11 + a^5*c*d^2*e^
13 - a^6*e^15)*x^4 + 2*(3*c^6*d^13*e^2 - 13*a*c^5*d^11*e^4 + 20*a^2*c^4*d^
9*e^6 - 10*a^3*c^3*d^7*e^8 - 5*a^4*c^2*d^5*e^10 + 7*a^5*c*d^3*e^12 - 2*a^6
*d*e^14)*x^3 + 2*(2*c^6*d^14*e - 7*a*c^5*d^12*e^3 + 5*a^2*c^4*d^10*e^5 + 1
0*a^3*c^3*d^8*e^7 - 20*a^4*c^2*d^6*e^9 + 13*a^5*c*d^4*e^11 - 3*a^6*d^2*e^1
3)*x^2 + (c^6*d^15 - a*c^5*d^13*e^2 - 10*a^2*c^4*d^11*e^4 + 30*a^3*c^3*d^9
*e^6 - 35*a^4*c^2*d^7*e^8 + 19*a^5*c*d^5*e^10 - 4*a^6*d^3*e^12)*x)
```

Sympy [F]

$$\int \frac{x^3}{(d+ex)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x^3}{\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)^3}\,dx$$

input
```
integrate(x**3/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**3/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(d+ex)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F]

$$\int \frac{x^3}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{x^3}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)^3}\,dx$$

input
```
integrate(x^3/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(x^3/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^3),
 x)
```

Mupad [B] (verification not implemented)

Time = 11.49 (sec) , antiderivative size = 11309, normalized size of antiderivative = 31.86

$$\int \frac{x^3}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^3/((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(2*d^3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(7*a^2*d^4*e^6 + 7*c
^2*d^8*e^2 + 7*a^2*e^10*x^4 + 28*a^2*d^3*e^7*x + 28*a^2*d*e^9*x^3 + 28*c^2
*d^7*e^3*x + 42*a^2*d^2*e^8*x^2 + 42*c^2*d^6*e^4*x^2 + 28*c^2*d^5*e^5*x^3
+ 7*c^2*d^4*e^6*x^4 - 14*a*c*d^6*e^4 - 56*a*c*d^5*e^5*x - 84*a*c*d^4*e^6*x
^2 - 56*a*c*d^3*e^7*x^3 - 14*a*c*d^2*e^8*x^4) - (16*d^2*(a*d*e + a*e^2*x +
 c*d^2*x + c*d*e*x^2)^(1/2))/(7*(5*a^2*d^3*e^6 + 5*c^2*d^7*e^2 + 5*a^2*e^9
*x^3 + 15*a^2*d^2*e^7*x + 15*a^2*d*e^8*x^2 + 15*c^2*d^6*e^3*x + 15*c^2*d^5
*e^4*x^2 + 5*c^2*d^4*e^5*x^3 - 10*a*c*d^5*e^4 - 30*a*c*d^4*e^5*x - 30*a*c*
d^3*e^6*x^2 - 10*a*c*d^2*e^7*x^3)) + (32*a^5*e^10*(a*d*e + a*e^2*x + c*d^2
*x + c*d*e*x^2)^(1/2))/(35*(a^7*d*e^16 + a^7*e^17*x - c^7*d^15*e^2 + 7*a*c
^6*d^13*e^4 - 7*a^6*c*d^3*e^14 - c^7*d^14*e^3*x - 21*a^2*c^5*d^11*e^6 + 35
*a^3*c^4*d^9*e^8 - 35*a^4*c^3*d^7*e^10 + 21*a^5*c^2*d^5*e^12 + 7*a*c^6*d^1
2*e^5*x - 7*a^6*c*d^2*e^15*x - 21*a^2*c^5*d^10*e^7*x + 35*a^3*c^4*d^8*e^9*
x - 35*a^4*c^3*d^6*e^11*x + 21*a^5*c^2*d^4*e^13*x)) - (10*c^5*d^10*(a*d*e
+ a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*(a^7*d*e^16 + a^7*e^17*x - c^7*
d^15*e^2 + 7*a*c^6*d^13*e^4 - 7*a^6*c*d^3*e^14 - c^7*d^14*e^3*x - 21*a^2*c
^5*d^11*e^6 + 35*a^3*c^4*d^9*e^8 - 35*a^4*c^3*d^7*e^10 + 21*a^5*c^2*d^5*e^
12 + 7*a*c^6*d^12*e^5*x - 7*a^6*c*d^2*e^15*x - 21*a^2*c^5*d^10*e^7*x + 35*
a^3*c^4*d^8*e^9*x - 35*a^4*c^3*d^6*e^11*x + 21*a^5*c^2*d^4*e^13*x)) + (2*a
^2*e^4*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(7*(a^4*d*e^10 + ...
```

Reduce [B] (verification not implemented)

Time = 141.20 (sec) , antiderivative size = 1359, normalized size of antiderivative = 3.83

$$\int \frac{x^3}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^3/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*(70*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**3*d**4*e**6 + 280*sqrt
(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**3*d**3*e**7*x + 420*sqrt(e)*sqrt(
d)*sqrt(c)*sqrt(a*e + c*d*x)*a**3*d**2*e**8*x**2 + 280*sqrt(e)*sqrt(d)*sqr
t(c)*sqrt(a*e + c*d*x)*a**3*d*e**9*x**3 + 70*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(
a*e + c*d*x)*a**3*e**10*x**4 + 70*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x
)*a**2*c*d**6*e**4 + 280*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*
d**5*e**5*x + 420*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d**4*e*
*6*x**2 + 280*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d**3*e**7*x
**3 + 70*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d**2*e**8*x**4 -
 14*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**8*e**2 - 56*sqrt(e
)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**7*e**3*x - 84*sqrt(e)*sqrt(d
)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**6*e**4*x**2 - 56*sqrt(e)*sqrt(d)*sqr
t(c)*sqrt(a*e + c*d*x)*a*c**2*d**5*e**5*x**3 - 14*sqrt(e)*sqrt(d)*sqrt(c)*
sqrt(a*e + c*d*x)*a*c**2*d**4*e**6*x**4 + 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a
*e + c*d*x)*c**3*d**10 + 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*
d**9*e*x + 12*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**8*e**2*x**
2 + 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**7*e**3*x**3 + 2*sq
rt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**6*e**4*x**4 - 16*sqrt(d +
e*x)*a**4*d**3*e**8 - 56*sqrt(d + e*x)*a**4*d**2*e**9*x - 70*sqrt(d + e*x)
*a**4*d*e**10*x**2 - 35*sqrt(d + e*x)*a**4*e**11*x**3 - 112*sqrt(d + e*...
```

3.133 $\int \frac{x^2}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 310

$$\int \frac{x^2}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\frac{2d^2}{7e^2\left(cd^2-ae^2\right)(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$-\frac{4d(3cd^2-7ae^2)}{35e^2\left(cd^2-ae^2\right)^2(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$-\frac{2(c^2d^4-14acd^2e^2-35a^2e^4)}{105e^2\left(cd^2-ae^2\right)^3(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$+\frac{8cd(c^2d^4-14acd^2e^2-35a^2e^4)\left(cd^2+ae^2+2cdex\right)}{105e^2\left(cd^2-ae^2\right)^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
2/7*d^2/e^2/(-a*e^2+c*d^2)/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)-4/35*d*(-7*a*e^2+3*c*d^2)/e^2/(-a*e^2+c*d^2)^2/(e*x+d)^2/(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)-2/105*(-35*a^2*e^4-14*a*c*d^2*e^2+c^2*d^4)/e^2/(
-a*e^2+c*d^2)^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+8/105*c*d*
(-35*a^2*e^4-14*a*c*d^2*e^2+c^2*d^4)*(2*c*d*e*x+a*e^2+c*d^2)/e^2/(-a*e^2+c
*d^2)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2(c^4d^6x^2(35d^2+28dex+8e^2x^2)+a^4e^6(8d^2+28dex+}{}$$

input
```
Integrate[x^2/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(2*(c^4*d^6*x^2*(35*d^2 + 28*d*e*x + 8*e^2*x^2) + a^4*e^6*(8*d^2 + 28*d*e*
x + 35*e^2*x^2) - 4*a*c^3*d^4*e*x*(35*d^3 + 119*d^2*e*x + 97*d*e^2*x^2 + 2
8*e^3*x^3) - 4*a^3*c*d*e^4*(28*d^3 + 97*d^2*e*x + 119*d*e^2*x^2 + 35*e^3*x
^3) - 2*a^2*c^2*d^2*e^2*(140*d^4 + 518*d^3*e*x + 711*d^2*e^2*x^2 + 518*d*e
^3*x^3 + 140*e^4*x^4)))/(105*(c*d^2 - a*e^2)^5*(d + e*x)^3*Sqrt[(a*e + c*d
*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1267, 27, 1220, 1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1267}$$

$$-\frac{\int \frac{e\left(d\left(cd^2+5ae^2\right)+e\left(7cd^2+5ae^2\right)x\right)}{2(d+ex)^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3cde^3}-\frac{1}{3cde^2(d+ex)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{d\left(cd^2+5ae^2\right)+e\left(7cd^2+5ae^2\right)x}{(d+ex)^3\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{6cde^2}-\frac{1}{3cde^2(d+ex)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1220

$$-\frac{\frac{\left(-35a^2e^4-14acd^2e^2+c^2d^4\right)\int \frac{1}{(d+ex)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{7(cd^2-ae^2)}-\frac{12cd^3}{7(d+ex)^3(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{6cde^2}-\frac{1}{3cde^2(d+ex)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1129

$$-\frac{\frac{\left(-35a^2e^4-14acd^2e^2+c^2d^4\right)\left(\frac{6cd\int \frac{1}{(d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{5\left(cd^2-ae^2\right)}+\frac{2}{5(d+ex)^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{7(cd^2-ae^2)}-\frac{}{7(d+ex)^3(cd^2-}}{6cde^2}$$
$$\frac{1}{3cde^2(d+ex)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1129

$$-\frac{\frac{\left(-35a^2e^4-14acd^2e^2+c^2d^4\right)\left(\frac{6cd\left(\frac{4cd\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)}+\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{5\left(cd^2-ae^2\right)}+\frac{}{5(d+ex)^2\left(cd^2-ae^2\right)\sqrt{}}}{7(cd^2-ae^2)}}{6cde^2}$$
$$\frac{1}{3cde^2(d+ex)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$\Big\downarrow$ 1088

$$-\frac{\frac{\left(-35a^2e^4-14acd^2e^2+c^2d^4\right)\left(\frac{6cd\left(\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{5\left(cd^2-ae^2\right)}+\frac{}{5(d+ex)^2\left(cd^2-ae^2\right)}}{7(cd^2-ae^2)}}{6cde^2}$$
$$\frac{1}{3cde^2(d+ex)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[x^2/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
-1/3*1/(c*d*e^2*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) -
 ((-12*c*d^3)/(7*(c*d^2 - a*e^2)*(d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*
x + c*d*e*x^2]) + ((c^2*d^4 - 14*a*c*d^2*e^2 - 35*a^2*e^4)*(2/(5*(c*d^2 -
a*e^2)*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (6*c*d*(
2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]
) - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e +
(c*d^2 + a*e^2)*x + c*d*e*x^2])))/(5*(c*d^2 - a*e^2))))/(7*(c*d^2 - a*e^2)
))/(6*c*d*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.20

method	result
gosper	$-\frac{2(cdx+ae)\left(-280a^2c^2d^2e^6x^4-112a\,c^3d^4e^4x^4+8c^4d^6e^2x^4-140a^3cd\,e^7x^3-1036a^2c^2d^3e^5x^3-388a\,c^3d^5e^3x^3+28c^4d^7e\,x^3+35a^4e\right.}{105(ex+d)^2\left(a^5e^{10}-5a^4c\,d^2e^8+10a\right.}$
trager	$-\frac{2\left(-280a^2c^2d^2e^6x^4-112a\,c^3d^4e^4x^4+8c^4d^6e^2x^4-140a^3cd\,e^7x^3-1036a^2c^2d^3e^5x^3-388a\,c^3d^5e^3x^3+28c^4d^7e\,x^3+35a^4e^8x^2-476c\right.}{105(cdx+ae)(}$
orering	$-\frac{2\left(-280a^2c^2d^2e^6x^4-112a\,c^3d^4e^4x^4+8c^4d^6e^2x^4-140a^3cd\,e^7x^3-1036a^2c^2d^3e^5x^3-388a\,c^3d^5e^3x^3+28c^4d^7e\,x^3+35a^4e^8x^2-476c\right.}{105\left(a^5e^{10}-5a^4c\,d^2e^8+10a^3c^2d^4e^6\right.}$
default	$\frac{-\frac{2}{3\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}+\frac{8dec\left(2dec\left(x+\frac{d}{e}\right)+a\,e^2-c\,d^2\right)}{3\left(a\,e^2-c\,d^2\right)^3\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}}{e^3}+\frac{d^2\left(-\frac{}{7\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right.}\right.}{}$

input
```
int(x^2/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/105*(c*d*x+a*e)*(-280*a^2*c^2*d^2*e^6*x^4-112*a*c^3*d^4*e^4*x^4+8*c^4*d
^6*e^2*x^4-140*a^3*c*d*e^7*x^3-1036*a^2*c^2*d^3*e^5*x^3-388*a*c^3*d^5*e^3*
x^3+28*c^4*d^7*e*x^3+35*a^4*e^8*x^2-476*a^3*c*d^2*e^6*x^2-1422*a^2*c^2*d^4
*e^4*x^2-476*a*c^3*d^6*e^2*x^2+35*c^4*d^8*x^2+28*a^4*d*e^7*x-388*a^3*c*d^3
*e^5*x-1036*a^2*c^2*d^5*e^3*x-140*a*c^3*d^7*e*x+8*a^4*d^2*e^6-112*a^3*c*d^
4*e^4-280*a^2*c^2*d^6*e^2)/(e*x+d)^2/(a^5*e^10-5*a^4*c*d^2*e^8+10*a^3*c^2*
d^4*e^6-10*a^2*c^3*d^6*e^4+5*a*c^4*d^8*e^2-c^5*d^10)/(c*d*e*x^2+a*e^2*x+c*
d^2*x+a*d*e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(294) = 588$.

Time = 10.79 (sec) , antiderivative size = 785, normalized size of antiderivative = 2.53

$$\int \frac{x^2}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx =$$

$$-\frac{}{105\,(ac^5d^{14}e-5\,a^2c^4d^{12}e^3+10\,a^3c^3d^{10}e^5-10\,a^4c^2d^8e^7+5\,a^5cd^6e^9-a^6d^4e^{11}+(c^6d^{11}e^4-5\,ac^5d^9e^6+1}$$

input
```
integrate(x^2/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
-2/105*(280*a^2*c^2*d^6*e^2 + 112*a^3*c*d^4*e^4 - 8*a^4*d^2*e^6 - 8*(c^4*d
^6*e^2 - 14*a*c^3*d^4*e^4 - 35*a^2*c^2*d^2*e^6)*x^4 - 4*(7*c^4*d^7*e - 97*
a*c^3*d^5*e^3 - 259*a^2*c^2*d^3*e^5 - 35*a^3*c*d*e^7)*x^3 - (35*c^4*d^8 -
476*a*c^3*d^6*e^2 - 1422*a^2*c^2*d^4*e^4 - 476*a^3*c*d^2*e^6 + 35*a^4*e^8)
*x^2 + 4*(35*a*c^3*d^7*e + 259*a^2*c^2*d^5*e^3 + 97*a^3*c*d^3*e^5 - 7*a^4*
d*e^7)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^5*d^14*e - 5*a^
2*c^4*d^12*e^3 + 10*a^3*c^3*d^10*e^5 - 10*a^4*c^2*d^8*e^7 + 5*a^5*c*d^6*e^
9 - a^6*d^4*e^11 + (c^6*d^11*e^4 - 5*a*c^5*d^9*e^6 + 10*a^2*c^4*d^7*e^8 -
10*a^3*c^3*d^5*e^10 + 5*a^4*c^2*d^3*e^12 - a^5*c*d*e^14)*x^5 + (4*c^6*d^12
*e^3 - 19*a*c^5*d^10*e^5 + 35*a^2*c^4*d^8*e^7 - 30*a^3*c^3*d^6*e^9 + 10*a^
4*c^2*d^4*e^11 + a^5*c*d^2*e^13 - a^6*e^15)*x^4 + 2*(3*c^6*d^13*e^2 - 13*a
*c^5*d^11*e^4 + 20*a^2*c^4*d^9*e^6 - 10*a^3*c^3*d^7*e^8 - 5*a^4*c^2*d^5*e^
10 + 7*a^5*c*d^3*e^12 - 2*a^6*d*e^14)*x^3 + 2*(2*c^6*d^14*e - 7*a*c^5*d^12
*e^3 + 5*a^2*c^4*d^10*e^5 + 10*a^3*c^3*d^8*e^7 - 20*a^4*c^2*d^6*e^9 + 13*a
^5*c*d^4*e^11 - 3*a^6*d^2*e^13)*x^2 + (c^6*d^15 - a*c^5*d^13*e^2 - 10*a^2*
c^4*d^11*e^4 + 30*a^3*c^3*d^9*e^6 - 35*a^4*c^2*d^7*e^8 + 19*a^5*c*d^5*e^10
 - 4*a^6*d^3*e^12)*x)
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{x^2}{((d+ex)(ae+cdx))^{\frac{3}{2}}(d+ex)^3}\,dx$$

input
```
integrate(x**2/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x**2/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F]

$$\int \frac{x^2}{(d+ex)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\int \frac{x^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)^3}\,dx$$

input
```
integrate(x^2/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(x^2/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^3),
 x)
```

Mupad [B] (verification not implemented)

Time = 8.44 (sec) , antiderivative size = 5771, normalized size of antiderivative = 18.62

$$\int \frac{x^2}{(d+ex)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(x^2/((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(((10*a*e^2 - 14*c*d^2)/(35*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)) - (4*
c*d^2)/(7*(a*e^2 - c*d^2)^2*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a
*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 - (((d*((16*c^2*d^2)/(35*(a*e^2 - c*d
^2)^4) - (8*c^3*d^4)/(35*(a*e^2 - c*d^2)^5)))/e - (34*c^3*d^5 - 188*a*c^2*
d^3*e^2 + 82*a^2*c*d*e^4)/(105*e*(a*e^2 - c*d^2)^5))*(x*(a*e^2 + c*d^2) +
a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - (((d*((d*((d*((8*c^4*d^5*e^3)/(35*(a
*e^2 - c*d^2)^4*(3*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e^3)) - (8*c^3*d^3*e^
3*(3*a*e^2 - c*d^2))/(35*(a*e^2 - c*d^2)^4*(3*a^2*e^5 + 3*c^2*d^4*e - 6*a*
c*d^2*e^3))))/e + (e^2*(26*c^4*d^6 - 124*a*c^3*d^4*e^2 + 74*a^2*c^2*d^2*e^
4))/(35*(a*e^2 - c*d^2)^4*(3*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e^3))))/e +
 (4*a*c*d*e^3*(22*c^2*d^4 - 21*a^2*e^4 + 15*a*c*d^2*e^2))/(35*(a*e^2 - c*d
^2)^4*(3*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e^3))))/e - (e^2*(84*a^3*c*d^2*
e^4 - 96*a^4*e^6 + 44*a^2*c^2*d^4*e^2))/(35*(a*e^2 - c*d^2)^4*(3*a^2*e^5 +
 3*c^2*d^4*e - 6*a*c*d^2*e^3)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1
/2))/(d + e*x)^2 - (((2*c*d^3 + 2*a*d*e^2)/(7*(a*e^2 - c*d^2)^2*(5*a*e^3 -
 5*c*d^2*e)) + (d*((2*a*e^3 - 2*c*d^2*e)/(7*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5
*c*d^2*e)) - (4*c*d^2*e)/(7*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e))))/e)*
(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 + (((16*a^3*e^6
 + 6*c^3*d^6 - 32*a*c^2*d^4*e^2 - 22*a^2*c*d^2*e^4)/(35*(a*e^2 - c*d^2)^4*
(3*a*e^3 - 3*c*d^2*e)) - (d*((d*((8*c^3*d^4*e^2)/(35*(a*e^2 - c*d^2)^4*...
```

Reduce [B] (verification not implemented)

Time = 7.70 (sec) , antiderivative size = 1240, normalized size of antiderivative = 4.00

$$\int \frac{x^2}{(d+ex)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^2/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*( - 280*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d**5*e**4 - 11
20*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d**4*e**5*x - 1680*sqr
t(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d**3*e**6*x**2 - 1120*sqrt(e
)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d**2*e**7*x**3 - 280*sqrt(e)*sq
rt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d*e**8*x**4 - 112*sqrt(e)*sqrt(d)*s
qrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**7*e**2 - 448*sqrt(e)*sqrt(d)*sqrt(c)*sq
rt(a*e + c*d*x)*a*c**2*d**6*e**3*x - 672*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e
+ c*d*x)*a*c**2*d**5*e**4*x**2 - 448*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*
d*x)*a*c**2*d**4*e**5*x**3 - 112*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)
*a*c**2*d**3*e**6*x**4 + 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*
d**9 + 32*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**8*e*x + 48*sqr
t(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**7*e**2*x**2 + 32*sqrt(e)*sq
rt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**6*e**3*x**3 + 8*sqrt(e)*sqrt(d)*sq
rt(c)*sqrt(a*e + c*d*x)*c**3*d**5*e**4*x**4 - 8*sqrt(d + e*x)*a**4*d**2*e*
*8 - 28*sqrt(d + e*x)*a**4*d*e**9*x - 35*sqrt(d + e*x)*a**4*e**10*x**2 + 1
12*sqrt(d + e*x)*a**3*c*d**4*e**6 + 388*sqrt(d + e*x)*a**3*c*d**3*e**7*x +
 476*sqrt(d + e*x)*a**3*c*d**2*e**8*x**2 + 140*sqrt(d + e*x)*a**3*c*d*e**9
*x**3 + 280*sqrt(d + e*x)*a**2*c**2*d**6*e**4 + 1036*sqrt(d + e*x)*a**2*c*
*2*d**5*e**5*x + 1422*sqrt(d + e*x)*a**2*c**2*d**4*e**6*x**2 + 1036*sqrt(d
 + e*x)*a**2*c**2*d**3*e**7*x**3 + 280*sqrt(d + e*x)*a**2*c**2*d**2*e**...
```

3.134 $\int \frac{x}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 284

$$\begin{aligned}
&\int \frac{x}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \\
&-\frac{2d}{7e\left(cd^2-ae^2\right)(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&-\frac{2(cd^2+7ae^2)}{35e\left(cd^2-ae^2\right)^2(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&-\frac{4cd(cd^2+7ae^2)}{35e\left(cd^2-ae^2\right)^3(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\
&+\frac{16c^2d^2(cd^2+7ae^2)\left(cd^2+ae^2+2cdex\right)}{35e\left(cd^2-ae^2\right)^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}
\end{aligned}$$

output

```
-2/7*d/e/(-a*e^2+c*d^2)/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-
2/35*(7*a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^2/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)-4/35*c*d*(7*a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^3/(e*x+d)/(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+16/35*c^2*d^2*(7*a*e^2+c*d^2)*(2*c*d*e*x
+a*e^2+c*d^2)/e/(-a*e^2+c*d^2)^5/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.78

$$\int \frac{x}{(d+ex)^3 \left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \frac{2(a^4e^7(2d+7ex)-2a^3cde^5(7d^2+24dex+7e^2x^2)+c^4\cdots}{}$$

input
```
Integrate[x/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*(a^4*e^7*(2*d + 7*e*x) - 2*a^3*c*d*e^5*(7*d^2 + 24*d*e*x + 7*e^2*x^2) +
 c^4*d^5*x*(35*d^3 + 70*d^2*e*x + 56*d*e^2*x^2 + 16*e^3*x^3) + 2*a^2*c^2*d
^2*e^3*(35*d^3 + 119*d^2*e*x + 97*d*e^2*x^2 + 28*e^3*x^3) + 2*a*c^3*d^3*e*
(35*d^4 + 140*d^3*e*x + 259*d^2*e^2*x^2 + 200*d*e^3*x^3 + 56*e^4*x^4)))/(3
5*(c*d^2 - a*e^2)^5*(d + e*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1220, 1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)^3 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$-\frac{\left(7ae^2+cd^2\right)\int \frac{1}{(d+ex)^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{7e\left(cd^2-ae^2\right)}-\frac{2d}{7e(d+ex)^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \text{1129}$$

$$-\frac{(7ae^2+cd^2)\left(\frac{6cd\int \frac{1}{(d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{5(cd^2-ae^2)}+\frac{2}{5(d+ex)^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{7e\left(cd^2-ae^2\right)}-\frac{2d}{7e(d+ex)^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \textbf{1129}$$

$$-\frac{(7ae^2+cd^2)\left(\frac{6cd\left(\frac{4cd\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)}+\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{5(cd^2-ae^2)}+\frac{2}{5(d+ex)^2(cd^2-ae^2)\sqrt{x(ae^2}}\right)}{7e\left(cd^2-ae^2\right)}-\frac{2d}{7e(d+ex)^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \textbf{1088}$$

$$-\frac{2d}{7e(d+ex)^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{(7ae^2+cd^2)\left(\frac{6cd\left(\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{5(cd^2-ae^2)}+\frac{2}{5(d+ex)^2(cd^2-ae^2)\sqrt{x(ae^2+}}\right)}{7e\left(cd^2-ae^2\right)}$$

input

```
Int[x/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output

```
(-2*d)/(7*e*(c*d^2 - a*e^2)*(d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) - ((c*d^2 + 7*a*e^2)*(2/(5*(c*d^2 - a*e^2)*(d + e*x)^2*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (6*c*d*(2/(3*(c*d^2 - a*e^2)*(d + e
*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*c*d*(c*d^2 + a*e^2 +
 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2]))/(5*(c*d^2 - a*e^2))))/(7*e*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.24

method	result
gosper	$-\frac{2(cdx+ae)(112a\,c^3d^3e^5x^4+16c^4d^5e^3x^4+56a^2c^2d^2e^6x^3+400a\,c^3d^4e^4x^3+56c^4d^6e^2x^3-14a^3cd\,e^7x^2+194a^2c^2d^3e^5x^2+518a\,c^3d^5}{35(ex+d)^2(a^5e^{10}-5a^4c\,d^2e^8+10a^3c^2d^4e^6-10a^2c^3}$
trager	$-\frac{2(112a\,c^3d^3e^5x^4+16c^4d^5e^3x^4+56a^2c^2d^2e^6x^3+400a\,c^3d^4e^4x^3+56c^4d^6e^2x^3-14a^3cd\,e^7x^2+194a^2c^2d^3e^5x^2+518a\,c^3d^5e^3x^2+70}{35(cdx+ae)(a\,e^2-c\,d^2)(a^4e^8-}$
orering	$-\frac{2(112a\,c^3d^3e^5x^4+16c^4d^5e^3x^4+56a^2c^2d^2e^6x^3+400a\,c^3d^4e^4x^3+56c^4d^6e^2x^3-14a^3cd\,e^7x^2+194a^2c^2d^3e^5x^2+518a\,c^3d^5e^3x^2+70}{35(a^5e^{10}-5a^4c\,d^2e^8+10a^3c^2d^4e^6-10a^2c^3d^6e^4+5a}$
default	$\frac{-\frac{2}{5\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^2\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}-\frac{6dec\left(-\frac{2}{3\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}+\frac{8dec}{3\left(a\,e^2-c\,d^2\right)^3}\right.}{5\left(a\,e^2-c\,d^2\right)}}{e^3}$

input
```
int(x/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
-2/35*(c*d*x+a*e)*(112*a*c^3*d^3*e^5*x^4+16*c^4*d^5*e^3*x^4+56*a^2*c^2*d^2
*e^6*x^3+400*a*c^3*d^4*e^4*x^3+56*c^4*d^6*e^2*x^3-14*a^3*c*d*e^7*x^2+194*a
^2*c^2*d^3*e^5*x^2+518*a*c^3*d^5*e^3*x^2+70*c^4*d^7*e*x^2+7*a^4*e^8*x-48*a
^3*c*d^2*e^6*x+238*a^2*c^2*d^4*e^4*x+280*a*c^3*d^6*e^2*x+35*c^4*d^8*x+2*a^
4*d*e^7-14*a^3*c*d^3*e^5+70*a^2*c^2*d^5*e^3+70*a*c^3*d^7*e)/(e*x+d)^2/(a^5
*e^10-5*a^4*c*d^2*e^8+10*a^3*c^2*d^4*e^6-10*a^2*c^3*d^6*e^4+5*a*c^4*d^8*e^
2-c^5*d^10)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(268) = 536$.

Time = 10.91 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.71

$$\int \frac{x}{(d+ex)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \frac{}{35\,(ac^5d^{14}e-5\,a^2c^4d^{12}e^3+10\,a^3c^3d^{10}e^5-10\,a^4c^2d^8e^7}$$

input
```
integrate(x/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
2/35*(70*a*c^3*d^7*e + 70*a^2*c^2*d^5*e^3 - 14*a^3*c*d^3*e^5 + 2*a^4*d*e^7
 + 16*(c^4*d^5*e^3 + 7*a*c^3*d^3*e^5)*x^4 + 8*(7*c^4*d^6*e^2 + 50*a*c^3*d^
4*e^4 + 7*a^2*c^2*d^2*e^6)*x^3 + 2*(35*c^4*d^7*e + 259*a*c^3*d^5*e^3 + 97*
a^2*c^2*d^3*e^5 - 7*a^3*c*d*e^7)*x^2 + (35*c^4*d^8 + 280*a*c^3*d^6*e^2 + 2
38*a^2*c^2*d^4*e^4 - 48*a^3*c*d^2*e^6 + 7*a^4*e^8)*x)*sqrt(c*d*e*x^2 + a*d
*e + (c*d^2 + a*e^2)*x)/(a*c^5*d^14*e - 5*a^2*c^4*d^12*e^3 + 10*a^3*c^3*d^
10*e^5 - 10*a^4*c^2*d^8*e^7 + 5*a^5*c*d^6*e^9 - a^6*d^4*e^11 + (c^6*d^11*e
^4 - 5*a*c^5*d^9*e^6 + 10*a^2*c^4*d^7*e^8 - 10*a^3*c^3*d^5*e^10 + 5*a^4*c^
2*d^3*e^12 - a^5*c*d*e^14)*x^5 + (4*c^6*d^12*e^3 - 19*a*c^5*d^10*e^5 + 35*
a^2*c^4*d^8*e^7 - 30*a^3*c^3*d^6*e^9 + 10*a^4*c^2*d^4*e^11 + a^5*c*d^2*e^1
3 - a^6*e^15)*x^4 + 2*(3*c^6*d^13*e^2 - 13*a*c^5*d^11*e^4 + 20*a^2*c^4*d^9
*e^6 - 10*a^3*c^3*d^7*e^8 - 5*a^4*c^2*d^5*e^10 + 7*a^5*c*d^3*e^12 - 2*a^6*
d*e^14)*x^3 + 2*(2*c^6*d^14*e - 7*a*c^5*d^12*e^3 + 5*a^2*c^4*d^10*e^5 + 10
*a^3*c^3*d^8*e^7 - 20*a^4*c^2*d^6*e^9 + 13*a^5*c*d^4*e^11 - 3*a^6*d^2*e^13
)*x^2 + (c^6*d^15 - a*c^5*d^13*e^2 - 10*a^2*c^4*d^11*e^4 + 30*a^3*c^3*d^9*
e^6 - 35*a^4*c^2*d^7*e^8 + 19*a^5*c*d^5*e^10 - 4*a^6*d^3*e^12)*x)
```

Sympy [F]

$$\int \frac{x}{(d+ex)^3 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \int \frac{x}{\left((d+ex)(ae+cdx)\right)^{\frac{3}{2}}(d+ex)^3}\,dx$$

input
```
integrate(x/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(x/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F]

$$\int \frac{x}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{x}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)^3}\,dx$$

input
```
integrate(x/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
integrate(x/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^3), x
)
```

Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 3635, normalized size of antiderivative = 12.80

$$\int \frac{x}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x/((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(((d*((d*((8*c^4*d^5*e^3)/(35*(a*e^2 - c*d^2)^4*(3*a^2*e^5 + 3*c^2*d^4*e -
 6*a*c*d^2*e^3)) - (24*c^3*d^3*e^3*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*
(3*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e^3))))/e + (e^2*(70*c^4*d^6 + 24*a*c
^3*d^4*e^2 - 22*a^2*c^2*d^2*e^4))/(35*(a*e^2 - c*d^2)^4*(3*a^2*e^5 + 3*c^2
*d^4*e - 6*a*c*d^2*e^3))))/e + (4*a^2*c*d*e^5*(27*a*e^2 - 35*c*d^2))/(35*(
a*e^2 - c*d^2)^4*(3*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e^3)))*(x*(a*e^2 + c
*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + (((e*(2*a*e^2 + 2*c*d^2))/
(7*(a*e^2 - c*d^2)^2*(5*a*e^3 - 5*c*d^2*e)) - (4*c*d^2*e)/(7*(a*e^2 - c*d^
2)^2*(5*a*e^3 - 5*c*d^2*e)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)
)/(d + e*x)^3 + (((12*c^3*d^4 + 12*a*c^2*d^2*e^2)/(35*(a*e^2 - c*d^2)^5) -
 (8*c^3*d^4)/(35*(a*e^2 - c*d^2)^5))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2))/(d + e*x) + (((d*((8*c^3*d^4*e^2)/(35*(a*e^2 - c*d^2)^4*(3*a*e^3
 - 3*c*d^2*e)) - (8*c^2*d^2*e^2*(2*a*e^2 + 3*c*d^2))/(35*(a*e^2 - c*d^2)^4
*(3*a*e^3 - 3*c*d^2*e))))/e + (e*(10*c^3*d^5 + 16*a*c^2*d^3*e^2 + 6*a^2*c*
d*e^4))/(35*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2) +
 a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + (((d*((e^2*(14*c^2*d^3 + 6*a*c*d*
e^2))/(7*(a*e^2 - c*d^2)^2*(5*a^2*e^5 + 5*c^2*d^4*e - 10*a*c*d^2*e^3)) - (
4*c^2*d^3*e^2)/(7*(a*e^2 - c*d^2)^2*(5*a^2*e^5 + 5*c^2*d^4*e - 10*a*c*d^2*
e^3))))/e - (16*a^2*e^5)/(7*(a*e^2 - c*d^2)^2*(5*a^2*e^5 + 5*c^2*d^4*e - 1
0*a*c*d^2*e^3)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*...
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 1067, normalized size of antiderivative = 3.76

$$\int \frac{x}{(d+ex)^3\,(ade+(cd^2+ae^2)\,x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*(112*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**6*e**2 + 448*s
qrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**5*e**3*x + 672*sqrt(e)*
sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**4*e**4*x**2 + 448*sqrt(e)*sqrt
(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**3*e**5*x**3 + 112*sqrt(e)*sqrt(d)*
sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**2*e**6*x**4 + 16*sqrt(e)*sqrt(d)*sqrt(
c)*sqrt(a*e + c*d*x)*c**3*d**8 + 64*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d
*x)*c**3*d**7*e*x + 96*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**6
*e**2*x**2 + 64*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**5*e**3*x
**3 + 16*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**4*e**4*x**4 - 2
*sqrt(d + e*x)*a**4*d*e**8 - 7*sqrt(d + e*x)*a**4*e**9*x + 14*sqrt(d + e*x
)*a**3*c*d**3*e**6 + 48*sqrt(d + e*x)*a**3*c*d**2*e**7*x + 14*sqrt(d + e*x
)*a**3*c*d*e**8*x**2 - 70*sqrt(d + e*x)*a**2*c**2*d**5*e**4 - 238*sqrt(d +
 e*x)*a**2*c**2*d**4*e**5*x - 194*sqrt(d + e*x)*a**2*c**2*d**3*e**6*x**2 -
 56*sqrt(d + e*x)*a**2*c**2*d**2*e**7*x**3 - 70*sqrt(d + e*x)*a*c**3*d**7*
e**2 - 280*sqrt(d + e*x)*a*c**3*d**6*e**3*x - 518*sqrt(d + e*x)*a*c**3*d**
5*e**4*x**2 - 400*sqrt(d + e*x)*a*c**3*d**4*e**5*x**3 - 112*sqrt(d + e*x)*
a*c**3*d**3*e**6*x**4 - 35*sqrt(d + e*x)*c**4*d**8*e*x - 70*sqrt(d + e*x)*
c**4*d**7*e**2*x**2 - 56*sqrt(d + e*x)*c**4*d**6*e**3*x**3 - 16*sqrt(d + e
*x)*c**4*d**5*e**4*x**4))/(35*sqrt(a*e + c*d*x)*e*(a**5*d**4*e**10 + 4*a**
5*d**3*e**11*x + 6*a**5*d**2*e**12*x**2 + 4*a**5*d*e**13*x**3 + a**5*e*...
```

3.135 $\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 241

$$\int \frac{1}{(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2}{7\left(cd^2-ae^2\right)(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$+\frac{16cd}{35\left(cd^2-ae^2\right)^2(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$+\frac{32c^2d^2}{35\left(cd^2-ae^2\right)^3(d+ex)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$
$$-\frac{128c^3d^3(cd^2+ae^2+2cdex)}{35\left(cd^2-ae^2\right)^5\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
2/7/(-a*e^2+c*d^2)/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+16/35
*c*d/(-a*e^2+c*d^2)^2/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+32
/35*c^2*d^2/(-a*e^2+c*d^2)^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/
2)-128/35*c^3*d^3*(2*c*d*e*x+a*e^2+c*d^2)/(-a*e^2+c*d^2)^5/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.80

$$\int \frac{1}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx = -\frac{2(-5a^4e^8+4a^3cde^6(7d+2ex)-2a^2c^2d^2e^4(35d^2+28dex+8e^2x^2)+4ac^3d^3e^2(35d^3+70d^2ex+56de^2x}{35(cd^2-ae^2)^5(d+ex)^3\sqrt{(ae+cdx)(}}$$

input
```
Integrate[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*(-5*a^4*e^8 + 4*a^3*c*d*e^6*(7*d + 2*e*x) - 2*a^2*c^2*d^2*e^4*(35*d^2
+ 28*d*e*x + 8*e^2*x^2) + 4*a*c^3*d^3*e^2*(35*d^3 + 70*d^2*e*x + 56*d*e^2*
x^2 + 16*e^3*x^3) + c^4*d^4*(35*d^4 + 280*d^3*e*x + 560*d^2*e^2*x^2 + 448*
d*e^3*x^3 + 128*e^4*x^4)))/(35*(c*d^2 - a*e^2)^5*(d + e*x)^3*Sqrt[(a*e + c
*d*x)*(d + e*x)])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {1129, 1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3 (x(ae^2+cd^2)+ade+cdex^2)^{3/2}} dx$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{8cd\int \frac{1}{(d+ex)^2(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{7(cd^2-ae^2)}+\frac{2}{7(d+ex)^3(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{8cd\left(\frac{6cd\int \frac{1}{(d+ex)\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{5(cd^2-ae^2)}+\frac{2}{5(d+ex)^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{7\left(cd^2-ae^2\right)}+\frac{2}{7(d+ex)^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1129$$

$$\frac{8cd\left(\frac{6cd\left(\frac{4cd\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)}+\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{5(cd^2-ae^2)}+\frac{2}{5(d+ex)^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{7\left(cd^2-ae^2\right)}+\frac{2}{7(d+ex)^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

$$\Big\downarrow \; 1088$$

$$\frac{8cd\left(\frac{6cd\left(\frac{2}{3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{8cd\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{5(cd^2-ae^2)}+\frac{2}{5(d+ex)^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{7\left(cd^2-ae^2\right)}+\frac{2}{7(d+ex)^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input

```
Int[1/((d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output

```
2/(7*(c*d^2 - a*e^2)*(d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^
2]) + (8*c*d*(2/(5*(c*d^2 - a*e^2)*(d + e*x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2
)*x + c*d*e*x^2]) + (6*c*d*(2/(3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*c*d*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*
d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])))/(5*(c*d^2 -
a*e^2))))/(7*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p + 
2], 0]
```

Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.27

method	result
gosper	$-\frac{2(cdx+ae)\left(-128c^4d^4e^4x^4-64a\,c^3d^3e^5x^3-448c^4d^5e^3x^3+16a^2c^2d^2e^6x^2-224a\,c^3d^4e^4x^2-560c^4d^6e^2x^2-8a^3cd\,e^7x+56a^2c^2d^3e^5\right.}{35(ex+d)^2\left(a^5e^{10}-5a^4c\,d^2e^8+10a^3c^2d^4e^6-10a^2c^3d^6e^4+5a\,c^4d^8e^2-c^5d\right.}$
default	$\frac{-\frac{2}{7\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^3\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}-\frac{8dec\left(-\frac{2}{5\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)^2\sqrt{dec\left(x+\frac{d}{e}\right)^2+\left(a\,e^2-c\,d^2\right)\left(x+\frac{d}{e}\right)}}-\frac{6dec\left(-\frac{}{3\left(a\right.}\right.}{}\right)}{}}{e^3}$
trager	$-\frac{2\left(-128c^4d^4e^4x^4-64a\,c^3d^3e^5x^3-448c^4d^5e^3x^3+16a^2c^2d^2e^6x^2-224a\,c^3d^4e^4x^2-560c^4d^6e^2x^2-8a^3cd\,e^7x+56a^2c^2d^3e^5x-280a\,c\right.}{35(cdx+ae)(a\,e^2-c\,d^2)\left(a^4e^8-4a^3c\,d^2e^6+6a^2c^2d^4e\right.}$
orering	$-\frac{2\left(-128c^4d^4e^4x^4-64a\,c^3d^3e^5x^3-448c^4d^5e^3x^3+16a^2c^2d^2e^6x^2-224a\,c^3d^4e^4x^2-560c^4d^6e^2x^2-8a^3cd\,e^7x+56a^2c^2d^3e^5x-280a\,c\right.}{35\left(a^5e^{10}-5a^4c\,d^2e^8+10a^3c^2d^4e^6-10a^2c^3d^6e^4+5a\,c^4d^8e^2-c^5d^{10}\right)(ex+d\right.}$

input
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
-2/35*(c*d*x+a*e)*(-128*c^4*d^4*e^4*x^4-64*a*c^3*d^3*e^5*x^3-448*c^4*d^5*e
^3*x^3+16*a^2*c^2*d^2*e^6*x^2-224*a*c^3*d^4*e^4*x^2-560*c^4*d^6*e^2*x^2-8*
a^3*c*d*e^7*x+56*a^2*c^2*d^3*e^5*x-280*a*c^3*d^5*e^3*x-280*c^4*d^7*e*x+5*a
^4*e^8-28*a^3*c*d^2*e^6+70*a^2*c^2*d^4*e^4-140*a*c^3*d^6*e^2-35*c^4*d^8)/(
e*x+d)^2/(a^5*e^10-5*a^4*c*d^2*e^8+10*a^3*c^2*d^4*e^6-10*a^2*c^3*d^6*e^4+5
*a*c^4*d^8*e^2-c^5*d^10)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(225) = 450$.

Time = 9.00 (sec) , antiderivative size = 733, normalized size of antiderivative = 3.04

$$\int \frac{1}{(d+ex)^3 \left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{}{35\left(ac^5d^{14}e-5a^2c^4d^{12}e^3+10a^3c^3d^{10}e^5-10a^4c^2d^8e^7+5a^5cd^6e^9-a^6d^4e^{11}+\left(c^6d^{11}e^4-5ac^5d^9e^6+10\right.\right.}$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="fricas")
```

output
```
-2/35*(128*c^4*d^4*e^4*x^4 + 35*c^4*d^8 + 140*a*c^3*d^6*e^2 - 70*a^2*c^2*d
^4*e^4 + 28*a^3*c*d^2*e^6 - 5*a^4*e^8 + 64*(7*c^4*d^5*e^3 + a*c^3*d^3*e^5)
*x^3 + 16*(35*c^4*d^6*e^2 + 14*a*c^3*d^4*e^4 - a^2*c^2*d^2*e^6)*x^2 + 8*(3
5*c^4*d^7*e + 35*a*c^3*d^5*e^3 - 7*a^2*c^2*d^3*e^5 + a^3*c*d*e^7)*x)*sqrt(
c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^5*d^14*e - 5*a^2*c^4*d^12*e^3
+ 10*a^3*c^3*d^10*e^5 - 10*a^4*c^2*d^8*e^7 + 5*a^5*c*d^6*e^9 - a^6*d^4*e^1
1 + (c^6*d^11*e^4 - 5*a*c^5*d^9*e^6 + 10*a^2*c^4*d^7*e^8 - 10*a^3*c^3*d^5*
e^10 + 5*a^4*c^2*d^3*e^12 - a^5*c*d*e^14)*x^5 + (4*c^6*d^12*e^3 - 19*a*c^5
*d^10*e^5 + 35*a^2*c^4*d^8*e^7 - 30*a^3*c^3*d^6*e^9 + 10*a^4*c^2*d^4*e^11
+ a^5*c*d^2*e^13 - a^6*e^15)*x^4 + 2*(3*c^6*d^13*e^2 - 13*a*c^5*d^11*e^4 +
 20*a^2*c^4*d^9*e^6 - 10*a^3*c^3*d^7*e^8 - 5*a^4*c^2*d^5*e^10 + 7*a^5*c*d^
3*e^12 - 2*a^6*d*e^14)*x^3 + 2*(2*c^6*d^14*e - 7*a*c^5*d^12*e^3 + 5*a^2*c^
4*d^10*e^5 + 10*a^3*c^3*d^8*e^7 - 20*a^4*c^2*d^6*e^9 + 13*a^5*c*d^4*e^11 -
 3*a^6*d^2*e^13)*x^2 + (c^6*d^15 - a*c^5*d^13*e^2 - 10*a^2*c^4*d^11*e^4 +
30*a^3*c^3*d^9*e^6 - 35*a^4*c^2*d^7*e^8 + 19*a^5*c*d^5*e^10 - 4*a^6*d^3*e^
12)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{\left((d+ex)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)^3}\,dx$$

input
```
integrate(1/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F]

$$\int \frac{1}{(d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(ex+d\right)^3}\,dx$$

input
```
integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm
="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^3), x
)
```

Mupad [B] (verification not implemented)

Time = 7.48 (sec) , antiderivative size = 2121, normalized size of antiderivative = 8.80

$$\int \frac{1}{(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
(((d*((24*c^4*d^5*e^3)/(35*(a*e^2 - c*d^2)^4*(3*a^2*e^5 + 3*c^2*d^4*e - 6*
a*c*d^2*e^3)) + (4*c^3*d^3*e^3*(17*a*e^2 - 29*c*d^2))/(35*(a*e^2 - c*d^2)^
4*(3*a^2*e^5 + 3*c^2*d^4*e - 6*a*c*d^2*e^3))))/e - (e^2*(70*c^4*d^6 - 256*
a*c^3*d^4*e^2 + 162*a^2*c^2*d^2*e^4))/(35*(a*e^2 - c*d^2)^4*(3*a^2*e^5 + 3
*c^2*d^4*e - 6*a*c*d^2*e^3)))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2
))/(d + e*x)^2 - (((e^2*(14*c^2*d^3 - 26*a*c*d*e^2))/(7*(a*e^2 - c*d^2)^2*
(5*a^2*e^5 + 5*c^2*d^4*e - 10*a*c*d^2*e^3)) + (12*c^2*d^3*e^2)/(7*(a*e^2 -
 c*d^2)^2*(5*a^2*e^5 + 5*c^2*d^4*e - 10*a*c*d^2*e^3)))*(x*(a*e^2 + c*d^2)
+ a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^3 - (((d*((16*c^5*d^6*e^2)/(35*(a*e^
2 - c*d^2)^7) + (16*c^4*d^4*e^2*(7*a*e^2 - 13*c*d^2))/(105*(a*e^2 - c*d^2)
^7)))/e + (4*c^3*d^3*e*(23*c^2*d^4 - 17*a^2*e^4 + 6*a*c*d^2*e^2))/(105*(a*
e^2 - c*d^2)^7))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)
+ (((24*c^3*d^4*e^2)/(35*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e)) - (12*c^
2*d^2*e^2*(a*e^2 + c*d^2))/(35*(a*e^2 - c*d^2)^4*(3*a*e^3 - 3*c*d^2*e)))*(
x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 - (2*e^2*(x*(a*e
^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/((d + e*x)^4*(7*a^2*e^5 + 7*c^2*d^
4*e - 14*a*c*d^2*e^3)) - (((a*(((a*e^2 + c*d^2)*((16*c^7*d^7*e^4*(a*e^2 +
c*d^2))/(35*(a*e^2 - c*d^2)^6*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))
 + (32*c^7*d^7*e^4*(4*a*e^2 - 13*c*d^2))/(105*(a*e^2 - c*d^2)^6*(c^3*d^5*e
 - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/(c*d*e) + (16*c^6*d^6*e^3*(27*a^2*...
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 852, normalized size of antiderivative = 3.54

$$\int \frac{1}{(d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(2*( - 128*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**7 - 512*sqrt(
e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**6*e*x - 768*sqrt(e)*sqrt(d)*s
qrt(c)*sqrt(a*e + c*d*x)*c**3*d**5*e**2*x**2 - 512*sqrt(e)*sqrt(d)*sqrt(c)
*sqrt(a*e + c*d*x)*c**3*d**4*e**3*x**3 - 128*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(
a*e + c*d*x)*c**3*d**3*e**4*x**4 - 5*sqrt(d + e*x)*a**4*e**8 + 28*sqrt(d +
 e*x)*a**3*c*d**2*e**6 + 8*sqrt(d + e*x)*a**3*c*d*e**7*x - 70*sqrt(d + e*x
)*a**2*c**2*d**4*e**4 - 56*sqrt(d + e*x)*a**2*c**2*d**3*e**5*x - 16*sqrt(d
 + e*x)*a**2*c**2*d**2*e**6*x**2 + 140*sqrt(d + e*x)*a*c**3*d**6*e**2 + 28
0*sqrt(d + e*x)*a*c**3*d**5*e**3*x + 224*sqrt(d + e*x)*a*c**3*d**4*e**4*x*
*2 + 64*sqrt(d + e*x)*a*c**3*d**3*e**5*x**3 + 35*sqrt(d + e*x)*c**4*d**8 +
 280*sqrt(d + e*x)*c**4*d**7*e*x + 560*sqrt(d + e*x)*c**4*d**6*e**2*x**2 +
 448*sqrt(d + e*x)*c**4*d**5*e**3*x**3 + 128*sqrt(d + e*x)*c**4*d**4*e**4*
x**4))/(35*sqrt(a*e + c*d*x)*(a**5*d**4*e**10 + 4*a**5*d**3*e**11*x + 6*a*
*5*d**2*e**12*x**2 + 4*a**5*d*e**13*x**3 + a**5*e**14*x**4 - 5*a**4*c*d**6
*e**8 - 20*a**4*c*d**5*e**9*x - 30*a**4*c*d**4*e**10*x**2 - 20*a**4*c*d**3
*e**11*x**3 - 5*a**4*c*d**2*e**12*x**4 + 10*a**3*c**2*d**8*e**6 + 40*a**3*
c**2*d**7*e**7*x + 60*a**3*c**2*d**6*e**8*x**2 + 40*a**3*c**2*d**5*e**9*x*
*3 + 10*a**3*c**2*d**4*e**10*x**4 - 10*a**2*c**3*d**10*e**4 - 40*a**2*c**3
*d**9*e**5*x - 60*a**2*c**3*d**8*e**6*x**2 - 40*a**2*c**3*d**7*e**7*x**3 -
 10*a**2*c**3*d**6*e**8*x**4 + 5*a*c**4*d**12*e**2 + 20*a*c**4*d**11*e*...
```

3.136 $\int \frac{1}{x(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 504

$$\int \frac{1}{x(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2cd}{ae\left(cd^2-ae^2\right)(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex}}$$

$$+\frac{2(7cd^2+ae^2)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7ad\left(cd^2-ae^2\right)^2(d+ex)^4}$$

$$+\frac{2(35c^2d^4+20acd^2e^2-7a^2e^4)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35ad^2\left(cd^2-ae^2\right)^3(d+ex)^3}$$

$$+\frac{2(105c^3d^6+185ac^2d^4e^2-133a^2cd^2e^4+35a^3e^6)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{105ad^3\left(cd^2-ae^2\right)^4(d+ex)^2}$$

$$+\frac{2(105c^4d^8+790ac^3d^6e^2-896a^2c^2d^4e^4+490a^3cd^2e^6-105a^4e^8)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{105ad^4\left(cd^2-ae^2\right)^5(d+ex)}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{3/2}d^{9/2}e^{3/2}}$$

output
```
2*c*d/a/e/(-a*e^2+c*d^2)/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
+2/7*(a*e^2+7*c*d^2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d/(-a*e^2+c
*d^2)^2/(e*x+d)^4+2/35*(-7*a^2*e^4+20*a*c*d^2*e^2+35*c^2*d^4)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^2/(-a*e^2+c*d^2)^3/(e*x+d)^3+2/105*(35*a^3
*e^6-133*a^2*c*d^2*e^4+185*a*c^2*d^4*e^2+105*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)
*x+c*d*e*x^2)^(1/2)/a/d^3/(-a*e^2+c*d^2)^4/(e*x+d)^2+2/105*(-105*a^4*e^8+4
90*a^3*c*d^2*e^6-896*a^2*c^2*d^4*e^4+790*a*c^3*d^6*e^2+105*c^4*d^8)*(a*d*e
+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a/d^4/(-a*e^2+c*d^2)^5/(e*x+d)-2*arctanh
(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/
a^(3/2)/d^(9/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.84

$$\int \frac{1}{x(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \frac{2\left(-\frac{\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)\left(15ad^3e^6(ae+cdx)^4-105acd^4e^5(ae+cdx)^3(d+e\right.}{}\right.}{}$$

input
```
Integrate[1/(x*(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),
x]
```

output
```
(2*(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(15*a*d^3*e^6*(a*e + c*d*x)^4
 - 105*a*c*d^4*e^5*(a*e + c*d*x)^3*(d + e*x) + 21*a^2*d^2*e^7*(a*e + c*d*x
)^3*(d + e*x) + 350*a*c^2*d^5*e^4*(a*e + c*d*x)^2*(d + e*x)^2 - 175*a^2*c*
d^3*e^6*(a*e + c*d*x)^2*(d + e*x)^2 + 35*a^3*d*e^8*(a*e + c*d*x)^2*(d + e*
x)^2 - 1050*a*c^3*d^6*e^3*(a*e + c*d*x)*(d + e*x)^3 + 1050*a^2*c^2*d^4*e^5
*(a*e + c*d*x)*(d + e*x)^3 - 525*a^3*c*d^2*e^7*(a*e + c*d*x)*(d + e*x)^3 +
 105*a^4*e^9*(a*e + c*d*x)*(d + e*x)^3 - 105*c^5*d^9*(d + e*x)^4))/((c*d^2
 - a*e^2)^5*(d + e*x)^2)) - 105*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*ArcTan
h[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])]))/(105*a^(3
/2)*d^(9/2)*e^(3/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.44, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d+ex)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1259}$$

$$\int \left(-\frac{e}{d^2(d+ex)^2\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{e}{d(d+ex)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{}{d^3(d+ex)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{a^{3/2}d^{9/2}e^{3/2}}+\frac{2\left(a^2e^4+cdex\left(ae^2+cd^2\right)+c^2d^4\right)}{ad^4e\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{128c^3d^2e\left(ae^2+cd^2+2cdex\right)}{35\left(cd^2-ae^2\right)^5\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{16c^2e\left(ae^2+cd^2+2cdex\right)}{5\left(cd^2-ae^2\right)^4\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{32c^2de}{35(d+ex)\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}+\frac{8ce\left(ae^2+cd^2+2cdex\right)}{3d^2\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{4ce}{5d(d+ex)\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{16ce}{35(d+ex)^2\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2e}{5d^2(d+ex)^2\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2e}{7d(d+ex)^3\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2e}{3d^3(d+ex)\left(cd^2-ae^2\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}$$

input
```
Int[1/(x*(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(-2*e)/(7*d*(c*d^2 - a*e^2)*(d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) - (16*c*e)/(35*(c*d^2 - a*e^2)^2*(d + e*x)^2*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2]) - (2*e)/(5*d^2*(c*d^2 - a*e^2)*(d + e*x)^2*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (32*c^2*d*e)/(35*(c*d^2 - a*e^2
)^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (4*c*e)/(5*d*
(c*d^2 - a*e^2)^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) -
 (2*e)/(3*d^3*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c
*d*e*x^2]) + (128*c^3*d^2*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(35*(c*d^2 - a*e^
2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (16*c^2*e*(c*d^2 + a*e
^2 + 2*c*d*e*x))/(5*(c*d^2 - a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d
*e*x^2]) + (8*c*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*d^2*(c*d^2 - a*e^2)^3*Sq
rt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*(c^2*d^4 + a^2*e^4 + c*d*e
*(c*d^2 + a*e^2)*x))/(a*d^4*e*(c*d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^
2)*x + c*d*e*x^2]) - ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt
[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(a^(3/2)*d^(9/2)
*e^(3/2))
```

Defintions of rubi rules used

rule 1259
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x
)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &&
 EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && (ILtQ[n, 0] || (IGtQ[n, 0]
&& ILtQ[p + 1/2, 0])) &&  !IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 901, normalized size of antiderivative = 1.79

method	result
default	$\frac{\frac{1}{ade\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}-\frac{\left(a\,e^2+c\,d^2\right)\left(2cdxe+a\,e^2+c\,d^2\right)}{ade\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)\sqrt{ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e}}-\frac{\ln\left(\frac{2ade+\left(a\,e^2+c\,d^2\right)x+2\sqrt{ade}\,\sqrt{ade+\left(a\,e^2+\right.}}{x}\right.}{ade\sqrt{ade}}}{d^3}$

input
```
int(1/x/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
1/d^3*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e
*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d
^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2
*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-1/d^3*(-2/3/(a
*e^2-c*d^2)/(x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d*e*
c/(a*e^2-c*d^2)^3*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e^2-c*
d^2)*(x+d/e))^(1/2))-1/e/d^2*(-2/5/(a*e^2-c*d^2)/(x+d/e)^2/(d*e*c*(x+d/e)^
2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-6/5*d*e*c/(a*e^2-c*d^2)*(-2/3/(a*e^2-c*d^2)
/(x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d*e*c/(a*e^2-c*
d^2)^3*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e
))^(1/2)))-1/e^2/d*(-2/7/(a*e^2-c*d^2)/(x+d/e)^3/(d*e*c*(x+d/e)^2+(a*e^2-c
*d^2)*(x+d/e))^(1/2)-8/7*d*e*c/(a*e^2-c*d^2)*(-2/5/(a*e^2-c*d^2)/(x+d/e)^2
/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-6/5*d*e*c/(a*e^2-c*d^2)*(-2
/3/(a*e^2-c*d^2)/(x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3
*d*e*c/(a*e^2-c*d^2)^3*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e
^2-c*d^2)*(x+d/e))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1588 vs. $2(472) = 944$.

Time = 22.89 (sec) , antiderivative size = 3196, normalized size of antiderivative = 6.34

$$\int \frac{1}{x(d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{x(d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{x\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{3}{2}}\left(d+ex\right)^3}\,dx$$

input
```
integrate(1/x/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(x*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{x(d+ex)^3 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{3}{2}}\left(ex+d\right)^3 x}\,dx$$

input
```
integrate(1/x/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^3*x),
 x)
```

Giac [F]

$$\int \frac{1}{x(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)^3 x}\,dx$$

input
```
integrate(1/x/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^3*x),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \int \frac{1}{x\,(d+e\,x)^3\,(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)^{3/2}}\,dx$$

input
```
int(1/(x*(d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(1/(x*(d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 16.55 (sec) , antiderivative size = 8301, normalized size of antiderivative = 16.47

$$\int \frac{1}{x(d+ex)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
(105*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*
x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(
d + e*x))*a**5*d**4*e**10 + 420*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*
log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d*
*2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**5*d**3*e**11*x + 630*sqrt(e)*sqrt(
d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c
)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**5*d**
2*e**12*x**2 + 420*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*s
qrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)
*sqrt(c)*sqrt(d + e*x))*a**5*d*e**13*x**3 + 105*sqrt(e)*sqrt(d)*sqrt(a)*sq
rt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e
 + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**5*e**14*x**4 - 525
*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) -
 sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d +
e*x))*a**4*c*d**6*e**8 - 2100*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*lo
g(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2
) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**4*c*d**5*e**9*x - 3150*sqrt(e)*sqrt(
d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c
)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**4*c*d
**4*e**10*x**2 - 2100*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqr...
```

3.137 $\int \frac{1}{x^2(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 647

$$\int \frac{1}{x^2(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx =$$

$$-\frac{c(3cd^2-ae^2)}{a^2e^2\left(cd^2-ae^2\right)(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$-\frac{1}{adex(d+ex)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

$$-\frac{\left(21c^2d^4-14acd^2e^2+9a^2e^4\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{7a^2d^2e\left(cd^2-ae^2\right)^2(d+ex)^4}$$

$$-\frac{3(35c^3d^6-35ac^2d^4e^2+53a^2cd^2e^4-21a^3e^6)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35a^2d^3e\left(cd^2-ae^2\right)^3(d+ex)^3}$$

$$-\frac{\left(105c^4d^8-140ac^3d^6e^2+422a^2c^2d^4e^4-364a^3cd^2e^6+105a^4e^8\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{35a^2d^4e\left(cd^2-ae^2\right)^4(d+ex)^2}$$

$$-\frac{\left(105c^5d^{10}-175ac^4d^8e^2+1474a^2c^3d^6e^4-2198a^3c^2d^4e^6+1365a^4cd^2e^8-315a^5e^{10}\right)\sqrt{ade+\left(cd^2+ae^2\right)}}{35a^2d^5e\left(cd^2-ae^2\right)^5(d+ex)}$$

$$+\frac{3(cd^2+3ae^2)\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{5/2}d^{11/2}e^{5/2}}$$

output
```
-c*(-a*e^2+3*c*d^2)/a^2/e^2/(-a*e^2+c*d^2)/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2)-1/a/d/e/x/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)-1/7*(9*a^2*e^4-14*a*c*d^2*e^2+21*c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)/a^2/d^2/e/(-a*e^2+c*d^2)^2/(e*x+d)^4-3/35*(-21*a^3*e^6+53*a^2
*c*d^2*e^4-35*a*c^2*d^4*e^2+35*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)/a^2/d^3/e/(-a*e^2+c*d^2)^3/(e*x+d)^3-1/35*(105*a^4*e^8-364*a^3*c*d^2
*e^6+422*a^2*c^2*d^4*e^4-140*a*c^3*d^6*e^2+105*c^4*d^8)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2)/a^2/d^4/e/(-a*e^2+c*d^2)^4/(e*x+d)^2-1/35*(-315*a^5*
e^10+1365*a^4*c*d^2*e^8-2198*a^3*c^2*d^4*e^6+1474*a^2*c^3*d^6*e^4-175*a*c^
4*d^8*e^2+105*c^5*d^10)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/a^2/d^5/e/
(-a*e^2+c*d^2)^5/(e*x+d)+3*(3*a*e^2+c*d^2)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)
/d^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(11/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 505, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2(d+ex)^3\left(ade+(cd^2+ae^2)x+cdex^2\right)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)\left(-105c^6d^{11}x(d+ex)^4-35ac^5d^9e(d-5ex)(d+ex)^4+a\right.}{}$$

input
```
Integrate[1/(x^2*(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)
),x]
```

output
```
((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(-105*c^6*d^11*x*(d + e*x)^4 - 35*
a*c^5*d^9*e*(d - 5*e*x)*(d + e*x)^4 + a^6*e^11*(35*d^4 + 528*d^3*e*x + 121
8*d^2*e^2*x^2 + 1050*d*e^3*x^3 + 315*e^4*x^4) + a^2*c^4*d^7*e^3*(175*d^5 +
 350*d^4*e*x - 1750*d^3*e^2*x^2 - 5250*d^2*e^3*x^3 - 4809*d*e^4*x^4 - 1474
*e^5*x^5) + a^4*c^2*d^3*e^7*(350*d^5 + 3675*d^4*e*x + 6328*d^3*e^2*x^2 + 2
062*d^2*e^3*x^3 - 2366*d*e^4*x^4 - 1365*e^5*x^5) - a^5*c*d*e^9*(175*d^5 +
2289*d^4*e*x + 4790*d^3*e^2*x^2 + 3346*d^2*e^3*x^3 + 315*d*e^4*x^4 - 315*e
^5*x^5) + 2*a^3*c^3*d^5*e^5*(-175*d^5 - 1225*d^4*e*x - 1050*d^3*e^2*x^2 +
1834*d^2*e^3*x^3 + 2953*d*e^4*x^4 + 1099*e^5*x^5)))/((c*d^2 - a*e^2)^5*x*(
d + e*x)^2) + 105*(c*d^2 + 3*a*e^2)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*Ar
cTanh[(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])/(Sqrt[d]*Sqrt[a*e + c*d*x])])/(35*a^
(5/2)*d^(11/2)*e^(5/2)*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 1011, normalized size of antiderivative = 1.56, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d+ex)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1259}$$

$$\int \left(\frac{e^2}{d^2(d+ex)^3\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}+\frac{3e^2}{d^4(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{}{d^4x\left(x\left(ae^2+\right.\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{128de^2(cd^2+2cexd+ae^2)c^3}{35(cd^2-ae^2)^5\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\frac{32e^2(cd^2+2cexd+ae^2)c^2}{5d(cd^2-ae^2)^4\sqrt{cdex^2+(cd^2+ae^2)x+ade}}+\frac{32e^2c^2}{35(cd^2-ae^2)^3(d+ex)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\frac{8e^2(cd^2+2cexd+ae^2)c}{d^3(cd^2-ae^2)^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}+\frac{8e^2c}{5d^2(cd^2-ae^2)^2(d+ex)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}+\frac{16e^2c}{35d(cd^2-ae^2)^2(d+ex)^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}+\frac{3(cd^2+ae^2)\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}\right)}{2a^{5/2}d^{11/2}e^{5/2}}+\frac{3\operatorname{arctanh}\left(\frac{2ade+(cd^2+ae^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{cdex^2+(cd^2+ae^2)x+ade}}\right)}{a^{3/2}d^{11/2}\sqrt{e}}-\frac{(3c^2d^4-2ace^2d^2+3a^2e^4)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}{a^2d^5e^2(cd^2-ae^2)^2x}+\frac{2(c^2d^4+ce(cd^2+ae^2)xd+a^2e^4)}{ad^4e(cd^2-ae^2)^2x\sqrt{cdex^2+(cd^2+ae^2)x+ade}}-\frac{6(c^2d^4+ce(cd^2+ae^2)xd+a^2e^4)}{ad^5(cd^2-ae^2)^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}+\frac{2e^2}{d^4(cd^2-ae^2)(d+ex)\sqrt{cdex^2+(cd^2+ae^2)x+ade}}+\frac{4e^2}{5d^3(cd^2-ae^2)(d+ex)^2\sqrt{cdex^2+(cd^2+ae^2)x+ade}}+\frac{2e^2}{7d^2(cd^2-ae^2)(d+ex)^3\sqrt{cdex^2+(cd^2+ae^2)x+ade}}$$

input
```
Int[1/(x^2*(d + e*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

output
```
(2*e^2)/(7*d^2*(c*d^2 - a*e^2)*(d + e*x)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x
+ c*d*e*x^2]) + (16*c*e^2)/(35*d*(c*d^2 - a*e^2)^2*(d + e*x)^2*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (4*e^2)/(5*d^3*(c*d^2 - a*e^2)*(d + e*
x)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (32*c^2*e^2)/(35*(c*d^
2 - a*e^2)^3*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (8*c
*e^2)/(5*d^2*(c*d^2 - a*e^2)^2*(d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]) + (2*e^2)/(d^4*(c*d^2 - a*e^2)*(d + e*x)*Sqrt[a*d*e + (c*d^2 +
 a*e^2)*x + c*d*e*x^2]) - (128*c^3*d*e^2*(c*d^2 + a*e^2 + 2*c*d*e*x))/(35*
(c*d^2 - a*e^2)^5*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (32*c^2*e
^2*(c*d^2 + a*e^2 + 2*c*d*e*x))/(5*d*(c*d^2 - a*e^2)^4*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2]) - (8*c*e^2*(c*d^2 + a*e^2 + 2*c*d*e*x))/(d^3*(c*
d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (6*(c^2*d^4
+ a^2*e^4 + c*d*e*(c*d^2 + a*e^2)*x))/(a*d^5*(c*d^2 - a*e^2)^2*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*(c^2*d^4 + a^2*e^4 + c*d*e*(c*d^2 +
 a*e^2)*x))/(a*d^4*e*(c*d^2 - a*e^2)^2*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2]) - ((3*c^2*d^4 - 2*a*c*d^2*e^2 + 3*a^2*e^4)*Sqrt[a*d*e + (c*d^2
 + a*e^2)*x + c*d*e*x^2])/(a^2*d^5*e^2*(c*d^2 - a*e^2)^2*x) + (3*ArcTanh[(
2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2])])/(a^(3/2)*d^(11/2)*Sqrt[e]) + (3*(c*d^2 + a*e^
2)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqr...
```

Defintions of rubi rules used

rule 1259
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x
)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &&
 EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[m, 0] && (ILtQ[n, 0] || (IGtQ[n, 0]
&& ILtQ[p + 1/2, 0])) &&  !IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. 2(613) = 1226.

Time = 3.72 (sec) , antiderivative size = 1254, normalized size of antiderivative = 1.94

method	result	size
default	Expression too large to display	1254

input
```
int(1/x^2/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2),x,method=_RETU
RNVERBOSE)
```

output
```
1/d^3*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-3/2*(a*e^2+c*d^2
)/a/d/e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d
/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x
+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x))-4*c/a*(2*c*d
*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*x^2*e)^(1/2))+1/e/d^2*(-2/7/(a*e^2-c*d^2)/(x+d/e)^3/(d*e*c*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(1/2)-8/7*d*e*c/(a*e^2-c*d^2)*(-2/5/(a*e^2-c*d^2)/(x+
d/e)^2/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-6/5*d*e*c/(a*e^2-c*d^
2)*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/
2)+8/3*d*e*c/(a*e^2-c*d^2)^3*(2*d*e*c*(x+d/e)+a*e^2-c*d^2)/(d*e*c*(x+d/e)^
2+(a*e^2-c*d^2)*(x+d/e))^(1/2))))-3/d^4*e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^
2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)
^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+
c*d*x^2*e)^(1/2))/x))+3*e/d^4*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(d*e*c*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d*e*c/(a*e^2-c*d^2)^3*(2*d*e*c*(x+d/e)+a
*e^2-c*d^2)/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))+2/d^3*(-2/5/(a*
e^2-c*d^2)/(x+d/e)^2/(d*e*c*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-6/5*d*e
*c/(a*e^2-c*d^2)*(-2/3/(a*e^2-c*d^2)/(x+d/e)/(d*e*c*(x+d/e)^2+(a*e^2-c*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1837 vs. 2(613) = 1226.

Time = 52.43 (sec) , antiderivative size = 3694, normalized size of antiderivative = 5.71

$$\int \frac{1}{x^2(d+ex)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{x^2(d+ex)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{x^2\left((d+ex)\,(ae+cdx)\right)^{\frac{3}{2}}(d+ex)^3}\,dx$$

input
```
integrate(1/x**2/(e*x+d)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

output
```
Integral(1/(x**2*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{x^2(d+ex)^3\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{3/2}}\,dx = \int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)^3x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algor
ithm="maxima")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^3*x^2
), x)
```

Giac [F]

$$\int \frac{1}{x^2(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{3}{2}}(ex+d)^3x^2}\,dx$$

input
```
integrate(1/x^2/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algor
ithm="giac")
```

output
```
integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)^3*x^2
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x^2\,{\left(d+e\,x\right)}^3\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{3/2}}\,dx$$

input
```
int(1/(x^2*(d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*(d + e*x)^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2(d+ex)^3\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}\,dx=\int \frac{1}{x^2\,(ex+d)^3\,(ade+(a\,e^2+c\,d^2)\,x+cde\,x^2)^{\frac{3}{2}}}dx$$

input
```
int(1/x^2/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

output
```
int(1/x^2/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

3.138 $\int \frac{x^4(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 373

$$\int \frac{x^4(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2aex^3(d+ex)}{3cd\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2x\left(ade\left(3c^2d^4+10acd^2e^2-5a^2e^4\right)+\left(3c^3d^6+ac^2d^4e^2+9a^2cd^2e^4-5a^3e^6\right)x\right)}{3c^2d^2e\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\left(9c^3d^6-9ac^2d^4e^2+31a^2cd^2e^4-15a^3e^6\right)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{3c^3d^3e^2\left(cd^2-ae^2\right)^3} - \frac{\left(3cd^2+5ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}\right)}{c^{7/2}d^{7/2}e^{5/2}}$$

output
```
2/3*a*e*x^3*(e*x+d)/c/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)-2/3*x*(a*d*e*(-5*a^2*e^4+10*a*c*d^2*e^2+3*c^2*d^4)+(-5*a^3*e^6+9*a^2*
c*d^2*e^4+a*c^2*d^4*e^2+3*c^3*d^6)*x)/c^2/d^2/e/(-a*e^2+c*d^2)^3/(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/3*(-15*a^3*e^6+31*a^2*c*d^2*e^4-9*a*c^2*d
^4*e^2+9*c^3*d^6)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^3/d^3/e^2/(-a*
e^2+c*d^2)^3-(5*a*e^2+3*c*d^2)*arctanh(c^(1/2)*d^(1/2)*(e*x+d)/e^(1/2)/(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(7/2)/d^(7/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.89

$$\int \frac{x^4(d+ex)}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)(d+ex)^2(-15a^5e^8(d+ex)+3c^5d^8x^2(3d+ex)+a^4cde^6(31d^2+11dex-2}{}$$

input
```
Integrate[(x^4*(d + e*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(d + e*x)^2*(-15*a^5*e^8*(d + e*x)
 + 3*c^5*d^8*x^2*(3*d + e*x) + a^4*c*d*e^6*(31*d^2 + 11*d*e*x - 20*e^2*x^2
) - 3*a*c^4*d^6*e*x*(-6*d^2 + d*e*x + 3*e^2*x^2) - 3*a^3*c^2*d^2*e^4*(3*d^
3 - 11*d^2*e*x - 13*d*e^2*x^2 + e^3*x^3) + 3*a^2*c^3*d^4*e^2*(3*d^3 - 5*d^
2*e*x - 3*d*e^2*x^2 + 3*e^3*x^3)))/(c*d^2 - a*e^2)^3 - 3*(3*c*d^2 + 5*a*e^
2)*(a*e + c*d*x)^(5/2)*(d + e*x)^(5/2)*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])
/(Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(3*c^(7/2)*d^(7/2)*e^(5/2)*((a*e + c*d*
x)*(d + e*x))^(5/2))
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1233, 27, 1233, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex)}{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1233}$$

$$\frac{2\int -\frac{e(cd^2-ae^2)x^2(6ade-(3cd^2-5ae^2)x)}{2(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3cde\,(cd^2-ae^2)^2}+\frac{2aex^3(d+ex)}{3cd\,(cd^2-ae^2)\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2aex^3(d+ex)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{\int \frac{x^2\left(6ade-\left(3cd^2-5ae^2\right)x\right)}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3cd\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1233$$

$$\frac{2aex^3(d+ex)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{\frac{2\int -\frac{2ade\left(3c^2d^4+10ace^2d^2-5a^2e^4\right)+\left(9c^3d^6-9ac^2e^2d^4+31a^2ce^4d^2-15a^3e^6\right)x}{2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{cde\left(cd^2-ae^2\right)^2}+\frac{2x\left(ade\left(-5a^2e^4+10acd^2e^2+3c^2d^4\right)+x\left(-5a^3e^6+9a^2cd^2e^4+ac \cdots\right)\right)}{cde\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}}{3cd\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{2aex^3(d+ex)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{\frac{2x\left(ade\left(-5a^2e^4+10acd^2e^2+3c^2d^4\right)+x\left(-5a^3e^6+9a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)\right)}{cde\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\int \frac{2ade\left(3c^2d^4+10ace^2d^2-5a^2e^4\right)+\left(9c^3d^6-9ac^2e^2d^4+31a^2ce^4d^2 \cdots\right)}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}} \cdots}{cde\left(cd^2-ae^2\right)^2}}{3cd\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1160$$

$$\frac{2aex^3(d+ex)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{\frac{2x\left(ade\left(-5a^2e^4+10acd^2e^2+3c^2d^4\right)+x\left(-5a^3e^6+9a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)\right)}{cde\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\frac{\left(-15a^3e^6+31a^2cd^2e^4-9ac^2d^4e^2+9c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cde \cdots}}{cde} \cdots}{cde\left(cd^2 \cdots\right)}}{3cd\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1092$$

$$\frac{2aex^3(d+ex)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{\frac{2x\left(ade\left(-5a^2e^4+10acd^2e^2+3c^2d^4\right)+x\left(-5a^3e^6+9a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)\right)}{cde\left(cd^2-ae^2\right)^2\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{\left(-15a^3e^6+31a^2cd^2e^4-9ac^2d^4e^2+9c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cde \cdots}}{cde} \cdots}{3cd\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{2aex^3(d+ex)}{3cd\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-$$

$$\frac{\frac{2x\left(ade\left(-5a^2e^4+10acd^2e^2+3c^2d^4\right)+x\left(-5a^3e^6+9a^2cd^2e^4+ac^2d^4e^2+3c^3d^6\right)\right)}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{\frac{\left(-15a^3e^6+31a^2cd^2e^4-9ac^2d^4e^2+9c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cde}}{cde}}{}}{3cd\left(cd^2-ae^2\right)}$$

input
```
Int[(x^4*(d + e*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(2*a*e*x^3*(d + e*x))/(3*c*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) - ((2*x*(a*d*e*(3*c^2*d^4 + 10*a*c*d^2*e^2 - 5*a^2*e^4)
+ (3*c^3*d^6 + a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 5*a^3*e^6)*x))/(c*d*e*(c*
d^2 - a*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (((9*c^3*d^6
 - 9*a*c^2*d^4*e^2 + 31*a^2*c*d^2*e^4 - 15*a^3*e^6)*Sqrt[a*d*e + (c*d^2 +
a*e^2)*x + c*d*e*x^2])/(c*d*e) - (3*(c*d^2 - a*e^2)^3*(3*c*d^2 + 5*a*e^2)*
ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e
+ (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*c^(3/2)*d^(3/2)*e^(3/2)))/(c*d*e*(c
*d^2 - a*e^2)^2))/(3*c*d*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2900 vs. $2(347) = 694$.

Time = 3.05 (sec) , antiderivative size = 2901, normalized size of antiderivative = 7.78

method	result	size
default	Expression too large to display	2901

input
```
int(x^4*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVE
RBOSE)
```

output
```
d*(-1/3*x^3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c*d^2
)/d/e/c*(-x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d
^2)/d/e/c*(-1/2*x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(a*e^2
+c*d^2)/d/e/c*(-1/3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e
^2+c*d^2)/d/e/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^
2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^
2+c*d^2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1
/2)))+1/2*a/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)
/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+
c*d^2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2
)))+2*a/c*(-1/3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c
*d^2)/d/e/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(
a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*
d^2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))
))+1/d/e/c*(-x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/2*(a*e^2+c*
d^2)/d/e/c*(-1/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)
/d/e/c*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/d/e/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*x*e)/(d*
e*c)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/(d*e*c)^(1/2)))+e*(x^4
/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-5/2*(a*e^2+c*d^2)/d/e/c*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(347) = 694$.

Time = 2.26 (sec) , antiderivative size = 1826, normalized size of antiderivative = 4.90

$$\int \frac{x^4(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="fricas")
```

output
```
[1/12*(3*(3*a^2*c^4*d^9*e^2 - 4*a^3*c^3*d^7*e^4 - 6*a^4*c^2*d^5*e^6 + 12*a
^5*c*d^3*e^8 - 5*a^6*d*e^10 + (3*c^6*d^10*e - 4*a*c^5*d^8*e^3 - 6*a^2*c^4*
d^6*e^5 + 12*a^3*c^3*d^4*e^7 - 5*a^4*c^2*d^2*e^9)*x^3 + (3*c^6*d^11 + 2*a*
c^5*d^9*e^2 - 14*a^2*c^4*d^7*e^4 + 19*a^4*c^2*d^3*e^8 - 10*a^5*c*d*e^10)*x
^2 + (6*a*c^5*d^10*e - 5*a^2*c^4*d^8*e^3 - 16*a^3*c^3*d^6*e^5 + 18*a^4*c^2
*d^4*e^7 + 2*a^5*c*d^2*e^9 - 5*a^6*e^11)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*
x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c
*d*e^3)*x) + 4*(9*a^2*c^4*d^8*e^3 - 9*a^3*c^3*d^6*e^5 + 31*a^4*c^2*d^4*e^7
 - 15*a^5*c*d^2*e^9 + 3*(c^6*d^9*e^2 - 3*a*c^5*d^7*e^4 + 3*a^2*c^4*d^5*e^6
 - a^3*c^3*d^3*e^8)*x^3 + (9*c^6*d^10*e - 3*a*c^5*d^8*e^3 - 9*a^2*c^4*d^6*
e^5 + 39*a^3*c^3*d^4*e^7 - 20*a^4*c^2*d^2*e^9)*x^2 + (18*a*c^5*d^9*e^2 - 1
5*a^2*c^4*d^7*e^4 + 33*a^3*c^3*d^5*e^6 + 11*a^4*c^2*d^3*e^8 - 15*a^5*c*d*e
^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/(a^2*c^7*d^11*e^5 - 3
*a^3*c^6*d^9*e^7 + 3*a^4*c^5*d^7*e^9 - a^5*c^4*d^5*e^11 + (c^9*d^12*e^4 -
3*a*c^8*d^10*e^6 + 3*a^2*c^7*d^8*e^8 - a^3*c^6*d^6*e^10)*x^3 + (c^9*d^13*e
^3 - a*c^8*d^11*e^5 - 3*a^2*c^7*d^9*e^7 + 5*a^3*c^6*d^7*e^9 - 2*a^4*c^5*d^
5*e^11)*x^2 + (2*a*c^8*d^12*e^4 - 5*a^2*c^7*d^10*e^6 + 3*a^3*c^6*d^8*e^8 +
 a^4*c^5*d^6*e^10 - a^5*c^4*d^4*e^12)*x), 1/6*(3*(3*a^2*c^4*d^9*e^2 - 4*a^
3*c^3*d^7*e^4 - 6*a^4*c^2*d^5*e^6 + 12*a^5*c*d^3*e^8 - 5*a^6*d*e^10 + (...
```

Sympy [F]

$$\int \frac{x^4(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\int \frac{x^4(d+ex)}{((d+ex)\,(ae+cdx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral(x**4*(d + e*x)/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x^4(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)x^4}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="giac")
```

output
```
integrate((e*x + d)*x^4/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{x^4\,(d+e\,x)}{{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}\,dx$$

input
```
int((x^4*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
int((x^4*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 2091, normalized size of antiderivative = 5.61

$$\int \frac{x^4(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(x^4*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
( - 120*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e +
c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**5*d*e**9
 - 120*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**5*e**10*x
 + 288*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c
*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*c*d**3*
e**7 + 168*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e
 + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*c*d
**2*e**8*x - 120*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sq
rt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a*
*4*c*d*e**9*x**2 - 144*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt
(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**
2))*a**3*c**2*d**5*e**5 + 144*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*lo
g((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2
- c*d**2))*a**3*c**2*d**4*e**6*x + 288*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e +
c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqr
t(a*e**2 - c*d**2))*a**3*c**2*d**3*e**7*x**2 - 96*sqrt(e)*sqrt(d)*sqrt(c)*
sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**3*d**7*e**3 - 240*sqrt(e)*sqrt(d)*s
qrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(...
```

3.139 $\int \frac{x^3(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 284

$$\int \frac{x^3(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2aex^2(d+ex)}{3cd\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2(ade(cd^2-ae^2)\left(3cd^2-ae^2\right)\left(cd^2+3ae^2\right)+\left(cd^2-ae^2\right)\left(3c^3d^6+ac^2d^4e^2+7a^2cd^2e^4-3a^3e^6\right)x)}{3c^2d^2e\left(cd^2-ae^2\right)^4\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}(d+ex)}{\sqrt{e}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{c^{5/2}d^{5/2}e^{3/2}}$$

output
```
2/3*a*e*x^2*(e*x+d)/c/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
3/2)-2/3*(a*d*e*(-a*e^2+c*d^2)*(-a*e^2+3*c*d^2)*(3*a*e^2+c*d^2)+(-a*e^2+c*
d^2)*(-3*a^3*e^6+7*a^2*c*d^2*e^4+a*c^2*d^4*e^2+3*c^3*d^6)*x)/c^2/d^2/e/(-a
*e^2+c*d^2)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2*arctanh(c^(1/2)*d^
(1/2)*(e*x+d)/e^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/c^(5/2)/d^(
5/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.81

$$\int \frac{x^3(d+ex)}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{2\left(-\frac{\sqrt{c}\sqrt{d}\sqrt{e}(ae+cdx)(d+ex)^4\left(-a^3cde^4+\frac{3c^2d^5(ae+cdx)^2}{(d+ex)^2}+\frac{9a^2cd^2e^3(ae+cdx)}{d+ex}-\frac{3a^3e^5}{\cdots}\right.}{(cd^2-ae^2)^3}\cdots\right.}{3c^{5/2}d^{5/2}e^{3/2}((ae+c\cdots}$$

input
```
Integrate[(x^3*(d + e*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(2*(-((Sqrt[c]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(d + e*x)^4*(-(a^3*c*d*e^4) +
 (3*c^2*d^5*(a*e + c*d*x)^2)/(d + e*x)^2 + (9*a^2*c*d^2*e^3*(a*e + c*d*x))
/(d + e*x) - (3*a^3*e^5*(a*e + c*d*x))/(d + e*x)))/(c*d^2 - a*e^2)^3) + 3*
(a*e + c*d*x)^(5/2)*(d + e*x)^(5/2)*ArcTanh[(Sqrt[e]*Sqrt[a*e + c*d*x])/(S
qrt[c]*Sqrt[d]*Sqrt[d + e*x])]))/(3*c^(5/2)*d^(5/2)*e^(3/2)*((a*e + c*d*x)
*(d + e*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1233, 27, 27, 1224, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)}{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}dx$$

$$\Big\downarrow \; 1233$$

$$\frac{2\int -\frac{ex\left(4ade(cd^2-ae^2)-3(cd^2-ae^2)^2x\right)}{2(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3cde\,(cd^2-ae^2)^2}+\frac{2aex^2(d+ex)}{3cd\,(cd^2-ae^2)\,(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2aex^2(d+ex)}{3cd\,(cd^2-ae^2)\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}-\frac{\int \frac{(cd^2-ae^2)x(4ade-3(cd^2-ae^2)x)}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3cd\,(cd^2-ae^2)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{2aex^2(d+ex)}{3cd\,(cd^2-ae^2)\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}-\frac{\int \frac{x(4ade-3(cd^2-ae^2)x)}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3cd\,(cd^2-ae^2)}$$

$$\Big\downarrow \; 1224$$

$$\frac{2aex^2(d+ex)}{3cd\,(cd^2-ae^2)\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}-\frac{\frac{2(x(-3a^3e^6+7a^2cd^2e^4+ac^2d^4e^2+3c^3d^6)+ade(3cd^2-ae^2)(3ae^2+cd^2))}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-3\left(\frac{d}{e}-\frac{ae}{cd}\right)\int \frac{1}{\sqrt{cdex^2+(cd^2+ae^2)x+ade}}dx}{3cd\,(cd^2-ae^2)}$$

$$\Big\downarrow \; 1092$$

$$\frac{2aex^2(d+ex)}{3cd\,(cd^2-ae^2)\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}-\frac{\frac{2(x(-3a^3e^6+7a^2cd^2e^4+ac^2d^4e^2+3c^3d^6)+ade(3cd^2-ae^2)(3ae^2+cd^2))}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-6\left(\frac{d}{e}-\frac{ae}{cd}\right)\int \frac{1}{4cde-\frac{\left(cd^2+2cexd+ae^2\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{cd^2+2cexd+c}{\sqrt{cdex^2+(cd^2+ae^2}}}{3cd\,(cd^2-ae^2)}$$

$$\Big\downarrow \; 219$$

$$\frac{2aex^2(d+ex)}{3cd\,(cd^2-ae^2)\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}-\frac{\frac{2(x(-3a^3e^6+7a^2cd^2e^4+ac^2d^4e^2+3c^3d^6)+ade(3cd^2-ae^2)(3ae^2+cd^2))}{cde(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{3\left(\frac{d}{e}-\frac{ae}{cd}\right)\text{arctanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{e}}}{3cd\,(cd^2-ae^2)}$$

input
```
Int[(x^3*(d + e*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(2*a*e*x^2*(d + e*x))/(3*c*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x +
c*d*e*x^2)^(3/2)) - ((2*(a*d*e*(3*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (3*c^
3*d^6 + a*c^2*d^4*e^2 + 7*a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(c*d*e*(c*d^2 - a
*e^2)^2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (3*(d/e - (a*e)/(c*
d))*ArcTanh[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*
d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(Sqrt[c]*Sqrt[d]*Sqrt[e]))/(3*c*d*
(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1224
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c))   Int[(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -
1] &&  !(IntegerQ[p] && NeQ[a, 0] && NiceSqrtQ[b^2 - 4*a*c])
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1776 vs. $2(262) = 524$.

Time = 2.34 (sec) , antiderivative size = 1777, normalized size of antiderivative = 6.26

method	result	size
default	Expression too large to display	1777

input
```
int(x^3*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVE
RBOSE)
```

output
```
d*(-x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)/d/
e/c*(-1/2*x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(a*e^2+c*d^2
)/d/e/c*(-1/3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c*d
^2)/d/e/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*
d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^
2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+
1/2*a/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*
e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)
^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))+2*
a/c*(-1/3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c*d^2)/
d/e/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2
)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))))+e*(
-1/3*x^3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c*d^2)/d
/e/c*(-x^2/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+1/2*(a*e^2+c*d^2)
/d/e/c*(-1/2*x/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/4*(a*e^2+c*
d^2)/d/e/c*(-1/3/d/e/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+
c*d^2)/d/e/c*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c
*d^2)^2)^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(262) = 524$.

Time = 2.28 (sec) , antiderivative size = 1514, normalized size of antiderivative = 5.33

$$\int \frac{x^3(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="fricas")
```

output
```
[1/6*(3*(a^2*c^3*d^7*e^2 - 3*a^3*c^2*d^5*e^4 + 3*a^4*c*d^3*e^6 - a^5*d*e^8
 + (c^5*d^8*e - 3*a*c^4*d^6*e^3 + 3*a^2*c^3*d^4*e^5 - a^3*c^2*d^2*e^7)*x^3
 + (c^5*d^9 - a*c^4*d^7*e^2 - 3*a^2*c^3*d^5*e^4 + 5*a^3*c^2*d^3*e^6 - 2*a^
4*c*d*e^8)*x^2 + (2*a*c^4*d^8*e - 5*a^2*c^3*d^6*e^3 + 3*a^3*c^2*d^4*e^5 +
a^4*c*d^2*e^7 - a^5*e^9)*x)*sqrt(c*d*e)*log(8*c^2*d^2*e^2*x^2 + c^2*d^4 +
6*a*c*d^2*e^2 + a^2*e^4 + 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2
*c*d*e*x + c*d^2 + a*e^2)*sqrt(c*d*e) + 8*(c^2*d^3*e + a*c*d*e^3)*x) - 4*(
3*a^2*c^3*d^6*e^3 + 8*a^3*c^2*d^4*e^5 - 3*a^4*c*d^2*e^7 + (3*c^5*d^8*e + 9
*a^2*c^3*d^4*e^5 - 4*a^3*c^2*d^2*e^7)*x^2 + (6*a*c^4*d^7*e^2 + 9*a^2*c^3*d
^5*e^4 + 4*a^3*c^2*d^3*e^6 - 3*a^4*c*d*e^8)*x)*sqrt(c*d*e*x^2 + a*d*e + (c
*d^2 + a*e^2)*x))/(a^2*c^6*d^10*e^4 - 3*a^3*c^5*d^8*e^6 + 3*a^4*c^4*d^6*e^
8 - a^5*c^3*d^4*e^10 + (c^8*d^11*e^3 - 3*a*c^7*d^9*e^5 + 3*a^2*c^6*d^7*e^7
 - a^3*c^5*d^5*e^9)*x^3 + (c^8*d^12*e^2 - a*c^7*d^10*e^4 - 3*a^2*c^6*d^8*e
^6 + 5*a^3*c^5*d^6*e^8 - 2*a^4*c^4*d^4*e^10)*x^2 + (2*a*c^7*d^11*e^3 - 5*a
^2*c^6*d^9*e^5 + 3*a^3*c^5*d^7*e^7 + a^4*c^4*d^5*e^9 - a^5*c^3*d^3*e^11)*x
), -1/3*(3*(a^2*c^3*d^7*e^2 - 3*a^3*c^2*d^5*e^4 + 3*a^4*c*d^3*e^6 - a^5*d*
e^8 + (c^5*d^8*e - 3*a*c^4*d^6*e^3 + 3*a^2*c^3*d^4*e^5 - a^3*c^2*d^2*e^7)*
x^3 + (c^5*d^9 - a*c^4*d^7*e^2 - 3*a^2*c^3*d^5*e^4 + 5*a^3*c^2*d^3*e^6 - 2
*a^4*c*d*e^8)*x^2 + (2*a*c^4*d^8*e - 5*a^2*c^3*d^6*e^3 + 3*a^3*c^2*d^4*e^5
 + a^4*c*d^2*e^7 - a^5*e^9)*x)*sqrt(-c*d*e)*arctan(1/2*sqrt(c*d*e*x^2 +...
```

Sympy [F]

$$\int \frac{x^3(d+ex)}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{x^3(d+ex)}{((d+ex)(ae+cdx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral(x**3*(d + e*x)/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex)}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x^3(d+ex)}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)x^3}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="giac")
```

output
```
integrate((e*x + d)*x^3/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex)}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{x^3\,(d+e\,x)}{{(c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e)}^{5/2}}\,dx$$

input
```
int((x^3*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
int((x^3*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1395, normalized size of antiderivative = 4.91

$$\int \frac{x^3(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(x^3*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*(3*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*d*e**7 +
 3*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x
) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**4*e**8*x - 9*
sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x) +
 sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c*d**3*e**5 -
6*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*d*x)
 + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c*d**2*e**6*
x + 3*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e + c*
d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**3*c*d*e**7
*x**2 + 9*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt(a*e
+ c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2*c**2
*d**5*e**3 - 9*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqrt(e)*sqrt
(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d**2))*a**2
*c**2*d**3*e**5*x**2 - 3*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sq
rt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d
**2))*a*c**3*d**7*e + 6*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log((sqr
t(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2 - c*d*
*2))*a*c**3*d**6*e**2*x + 9*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*log(
(sqrt(e)*sqrt(a*e + c*d*x) + sqrt(d)*sqrt(c)*sqrt(d + e*x))/sqrt(a*e**2...
```

3.140 $\int \frac{x^2(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 123

$$\int \frac{x^2(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2x^2(d+ex)}{3\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{8d(2ade+\left(cd^2+ae^2\right)x)}{3\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
-2/3*x^2*(e*x+d)/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+8/
3*d*(2*a*d*e+(a*e^2+c*d^2)*x)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{x^2(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx=\frac{2(d+ex)\left(3c^2d^4x^2+6acd^2ex(2d+ex)+a^2e^2(8d^2+4dex-e^2x\right.}{3\left(cd^2-ae^2\right)^3((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[(x^2*(d + e*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(2*(d + e*x)*(3*c^2*d^4*x^2 + 6*a*c*d^2*e*x*(2*d + e*x) + a^2*e^2*(8*d^2 +
 4*d*e*x - e^2*x^2)))/(3*(c*d^2 - a*e^2)^3*((a*e + c*d*x)*(d + e*x))^(3/2)
)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.053$, Rules used = {1227, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)}{(x\,(ae^2+cd^2)+ade+cdex^2)^{5/2}}dx$$

$$\Big\downarrow \ 1227$$

$$\frac{4d\int \frac{x}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3\,(cd^2-ae^2)}-\frac{2x^2(d+ex)}{3\,(cd^2-ae^2)\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \ 1158$$

$$\frac{8d(x\,(ae^2+cd^2)+2ade)}{3\,(cd^2-ae^2)^3\sqrt{x\,(ae^2+cd^2)+ade+cdex^2}}-\frac{2x^2(d+ex)}{3\,(cd^2-ae^2)\,(x\,(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

input
```
Int[(x^2*(d + e*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*x^2*(d + e*x))/(3*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x
^2)^(3/2)) + (8*d*(2*a*d*e + (c*d^2 + a*e^2)*x))/(3*(c*d^2 - a*e^2)^3*Sqrt
[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1227
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(b*f - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[m*((b*(
e*f + d*g) - 2*(c*d*f + a*e*g))/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m
 - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.24

method	result	size
gosper	$-\frac{2(ex+d)^{2}(cdx+ae)\left(-a^{2}e^{4}x^{2}+6ac\,d^{2}e^{2}x^{2}+3c^{2}d^{4}x^{2}+4a^{2}d\,e^{3}x+12ac\,d^{3}ex+8d^{2}e^{2}a^{2}\right)}{3(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3})(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{5}{2}}}$	152
trager	$-\frac{2\left(-a^{2}e^{4}x^{2}+6ac\,d^{2}e^{2}x^{2}+3c^{2}d^{4}x^{2}+4a^{2}d\,e^{3}x+12ac\,d^{3}ex+8d^{2}e^{2}a^{2}\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{3(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4})(cdx+ae)^{2}(a\,e^{2}-c\,d^{2})(ex+d)}$	153
orering	$-\frac{2\left(-a^{2}e^{4}x^{2}+6ac\,d^{2}e^{2}x^{2}+3c^{2}d^{4}x^{2}+4a^{2}d\,e^{3}x+12ac\,d^{3}ex+8d^{2}e^{2}a^{2}\right)(ex+d)^{2}(cdx+ae)}{3(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3})(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{5}{2}}}$	153
default	Expression too large to display	1155

input
```
int(x^2*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVE
RBOSE)
```

output
```
-2/3*(e*x+d)^2*(c*d*x+a*e)*(-a^2*e^4*x^2+6*a*c*d^2*e^2*x^2+3*c^2*d^4*x^2+4
*a^2*d*e^3*x+12*a*c*d^3*e*x+8*a^2*d^2*e^2)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^
2*d^4*e^2-c^3*d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(115) = 230$.

Time = 1.87 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.59

$$\int \frac{x^2(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{2\,(8\,a^2d}{3\,(a^2c^3d^7e^2-3\,a^3c^2d^5e^4+3\,a^4cd^3e^6-a^5de^8+(c^5d^8e-3\,ac^4d^6e}$$

input
```
integrate(x^2*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="fricas")
```

output
```
2/3*(8*a^2*d^2*e^2 + (3*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*x^2 + 4*(3*a*c*
d^3*e + a^2*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a^2*c^3
*d^7*e^2 - 3*a^3*c^2*d^5*e^4 + 3*a^4*c*d^3*e^6 - a^5*d*e^8 + (c^5*d^8*e -
3*a*c^4*d^6*e^3 + 3*a^2*c^3*d^4*e^5 - a^3*c^2*d^2*e^7)*x^3 + (c^5*d^9 - a*
c^4*d^7*e^2 - 3*a^2*c^3*d^5*e^4 + 5*a^3*c^2*d^3*e^6 - 2*a^4*c*d*e^8)*x^2 +
 (2*a*c^4*d^8*e - 5*a^2*c^3*d^6*e^3 + 3*a^3*c^2*d^4*e^5 + a^4*c*d^2*e^7 -
a^5*e^9)*x)
```

Sympy [F]

$$\int \frac{x^2(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{x^2(d+ex)}{((d+ex)\,(ae+cdx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral(x**2*(d + e*x)/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x^2(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \int \frac{(ex+d)x^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="giac")
```

output
```
integrate((e*x + d)*x^2/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.71 (sec) , antiderivative size = 1114, normalized size of antiderivative = 9.06

$$\int \frac{x^2(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
(4*a^2*e^3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*(c^5*d^8*x -
3*a^2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 + a*c^4*d^7*e - a^4*c*d*e^7 - 3*a*c^
4*d^6*e^2*x + 3*a^2*c^3*d^4*e^4*x - a^3*c^2*d^2*e^6*x)) - (2*a^2*e^2*(a*d*
e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2))/(3*c^5*d^7*x^2 + 3*a^2*c^3*d^5*e
^2 - 6*a^3*c^2*d^3*e^4 + 3*a^4*c*d*e^6 + 6*a*c^4*d^6*e*x + 3*a^2*c^3*d^3*e
^4*x^2 - 12*a^2*c^3*d^4*e^3*x + 6*a^3*c^2*d^2*e^5*x - 6*a*c^4*d^5*e^2*x^2)
 + (2*a^3*d*e^6)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*e
 - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9))
 + (2*a^3*e^7*x)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9*e
 - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9))
 + (22*a*c^2*d^5*e^2)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^
5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*
d*e^9)) - (28*a^2*c*d^3*e^4)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1
/2)*(c^5*d^9*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 +
 a^4*c*d*e^9)) - (4*a*c*d^2*e*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2
))/(3*(c^5*d^8*x - 3*a^2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 + a*c^4*d^7*e - a
^4*c*d*e^7 - 3*a*c^4*d^6*e^2*x + 3*a^2*c^3*d^4*e^4*x - a^3*c^2*d^2*e^6*x))
 + (2*c^3*d^6*e*x)/((a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1/2)*(c^5*d^9
*e - 4*a*c^4*d^7*e^3 + 6*a^2*c^3*d^5*e^5 - 4*a^3*c^2*d^3*e^7 + a^4*c*d*e^9
)) + (10*a*c^2*d^4*e^3*x)/(3*(a*d*e + a*e^2*x + c*d^2*x + c*d*e*x^2)^(1...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 498, normalized size of antiderivative = 4.05

$$\int \frac{x^2(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{\frac{4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,a^3d\,e^5}{3}+\frac{4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,a^3e^6x}{3}+\frac{4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,a^2}{3}$$

input
```
int(x^2*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*(2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**3*d*e**5 + 2*sqrt(e)*sq
rt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**3*e**6*x + 2*sqrt(e)*sqrt(d)*sqrt(c)*sq
rt(a*e + c*d*x)*a**2*c*d**2*e**4*x + 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e +
c*d*x)*a**2*c*d*e**5*x**2 + 6*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*
c**2*d**5*e + 6*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**4*e**2
*x + 6*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**6*x + 6*sqrt(e)*s
qrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**5*e*x**2 - 8*sqrt(d + e*x)*a**2*c
**2*d**4*e**3 - 4*sqrt(d + e*x)*a**2*c**2*d**3*e**4*x + sqrt(d + e*x)*a**2
*c**2*d**2*e**5*x**2 - 12*sqrt(d + e*x)*a*c**3*d**5*e**2*x - 6*sqrt(d + e*
x)*a*c**3*d**4*e**3*x**2 - 3*sqrt(d + e*x)*c**4*d**6*e*x**2))/(3*sqrt(a*e
+ c*d*x)*c**2*d**2*e*(a**4*d*e**7 + a**4*e**8*x - 3*a**3*c*d**3*e**5 - 2*a
**3*c*d**2*e**6*x + a**3*c*d*e**7*x**2 + 3*a**2*c**2*d**5*e**3 - 3*a**2*c*
*2*d**3*e**5*x**2 - a*c**3*d**7*e + 2*a*c**3*d**6*e**2*x + 3*a*c**3*d**5*e
**3*x**2 - c**4*d**8*x - c**4*d**7*e*x**2))
```

3.141 $\int \frac{x(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 143

$$\int \frac{x(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2ae(d+ex)}{3cd\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{2\left(3cd^2+ae^2\right)\left(cd^2+ae^2+2cdex\right)}{3cd\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
2/3*a*e*(e*x+d)/c/d/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
-2/3*(a*e^2+3*c*d^2)*(2*c*d*e*x+a*e^2+c*d^2)/c/d/(-a*e^2+c*d^2)^3/(a*d*e+(
a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.66

$$\int \frac{x(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)\left(3a^2e^3(2d+ex)+3c^2d^3x(d+2ex)+2acde\left(d^2+5dex+e^2x^2\right)\right)}{3\left(cd^2-ae^2\right)^3\left((ae+cdx)(d+ex)\right)^{3/2}}$$

input
```
Integrate[(x*(d + e*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)*(3*a^2*e^3*(2*d + e*x) + 3*c^2*d^3*x*(d + 2*e*x) + 2*a*c*d*e
*(d^2 + 5*d*e*x + e^2*x^2)))/(3*(c*d^2 - a*e^2)^3*((a*e + c*d*x)*(d + e*x)
)^(3/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1218, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)}{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{(ae^2+3cd^2)\int \frac{1}{(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3cd(cd^2-ae^2)}+\frac{2ae(d+ex)}{3cd(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{2ae(d+ex)}{3cd(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}-\frac{2(ae^2+3cd^2)(ae^2+cd^2+2cdex)}{3cd(cd^2-ae^2)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

input
```
Int[(x*(d + e*x))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(2*a*e*(d + e*x))/(3*c*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2)) - (2*(3*c*d^2 + a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*c*d*(
c*d^2 - a*e^2)^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.09

method	result
gosper	$\frac{2(ex+d)^{2}(cdx+ae)(2x^{2}acd\,e^{3}+6x^{2}c^{2}d^{3}e+3a^{2}e^{4}x+10ac\,d^{2}e^{2}x+3c^{2}d^{4}x+6a^{2}d\,e^{3}+2ac\,d^{3}e)}{3(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3})(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade)^{\frac{5}{2}}}$
trager	$\frac{2(2x^{2}acd\,e^{3}+6x^{2}c^{2}d^{3}e+3a^{2}e^{4}x+10ac\,d^{2}e^{2}x+3c^{2}d^{4}x+6a^{2}d\,e^{3}+2ac\,d^{3}e)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{3(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4})(cdx+ae)^{2}(a\,e^{2}-c\,d^{2})(ex+d)}$
orering	$\frac{2(2x^{2}acd\,e^{3}+6x^{2}c^{2}d^{3}e+3a^{2}e^{4}x+10ac\,d^{2}e^{2}x+3c^{2}d^{4}x+6a^{2}d\,e^{3}+2ac\,d^{3}e)(ex+d)^{2}(cdx+ae)}{3(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3})(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{5}{2}}}$
default	$d\left(-\frac{1}{3dec(ade+(a\,e^{2}+c\,d^{2})x+cd\,x^{2}e)^{\frac{3}{2}}}-\frac{(a\,e^{2}+c\,d^{2})\left(\frac{\frac{4}{3}cdxe+\frac{2}{3}a\,e^{2}+\frac{2}{3}c\,d^{2}}{\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}+\frac{\cdots}{3\left(4ac\,d^{2}e^{2}-\left(a\,\cdots\right.\right.}\cdots\right)}{2dec}\cdots\right.$

input
```
int(x*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
2/3*(e*x+d)^2*(c*d*x+a*e)*(2*a*c*d*e^3*x^2+6*c^2*d^3*e*x^2+3*a^2*e^4*x+10*
a*c*d^2*e^2*x+3*c^2*d^4*x+6*a^2*d*e^3+2*a*c*d^3*e)/(a^3*e^6-3*a^2*c*d^2*e^
4+3*a*c^2*d^4*e^2-c^3*d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(135) = 270$.

Time = 1.89 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.27

$$\int \frac{x(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx =$$

$$-\frac{2\,(2\,acd^3e+6\,a^2de^3+2\,(3\,c^2d^3e+acde^3)x^2+(3\,c^2d^4}{3\,(a^2c^3d^7e^2-3\,a^3c^2d^5e^4+3\,a^4cd^3e^6-a^5de^8+(c^5d^8e-3\,ac^4d^6e^3+3\,a^2c^3d^4e^5-a^3c^2d^2e^7)x^3+(c^5d^9-}$$

input
```
integrate(x*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
fricas")
```

output
```
-2/3*(2*a*c*d^3*e + 6*a^2*d*e^3 + 2*(3*c^2*d^3*e + a*c*d*e^3)*x^2 + (3*c^2
*d^4 + 10*a*c*d^2*e^2 + 3*a^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*
e^2)*x)/(a^2*c^3*d^7*e^2 - 3*a^3*c^2*d^5*e^4 + 3*a^4*c*d^3*e^6 - a^5*d*e^8
 + (c^5*d^8*e - 3*a*c^4*d^6*e^3 + 3*a^2*c^3*d^4*e^5 - a^3*c^2*d^2*e^7)*x^3
 + (c^5*d^9 - a*c^4*d^7*e^2 - 3*a^2*c^3*d^5*e^4 + 5*a^3*c^2*d^3*e^6 - 2*a^
4*c*d*e^8)*x^2 + (2*a*c^4*d^8*e - 5*a^2*c^3*d^6*e^3 + 3*a^3*c^2*d^4*e^5 +
a^4*c*d^2*e^7 - a^5*e^9)*x)
```

Sympy [F]

$$\int \frac{x(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{x(d+ex)}{((d+ex)\,(ae+cdx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral(x*(d + e*x)/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{(ex+d)x}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
giac")
```

output
```
integrate((e*x + d)*x/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.61 (sec) , antiderivative size = 509, normalized size of antiderivative = 3.56

$$\int \frac{x(d+ex)}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \frac{12\,a^2\,d\,e^3\,\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}+6\,a^2\,e^4\,x\,\sqrt{c\,d\,e\,x^2}}{3\,a^5\,d\,e^8+3\,a^5\,e^9\,x-9\,a^4\,c\,d^3\,e^6}$$

input
```
int((x*(d + e*x))/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
(12*a^2*d*e^3*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 6*a^2*e^4*x*
(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2) + 6*c^2*d^4*x*(x*(a*e^2 + c*
d^2) + a*d*e + c*d*e*x^2)^(1/2) + 12*c^2*d^3*e*x^2*(x*(a*e^2 + c*d^2) + a*
d*e + c*d*e*x^2)^(1/2) + 4*a*c*d^3*e*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^
2)^(1/2) + 20*a*c*d^2*e^2*x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)
+ 4*a*c*d*e^3*x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(3*a^5*d*
e^8 + 3*a^5*e^9*x - 3*c^5*d^9*x^2 - 9*a^4*c*d^3*e^6 - 3*c^5*d^8*e*x^3 - 3*
a^2*c^3*d^7*e^2 + 9*a^3*c^2*d^5*e^4 - 6*a*c^4*d^8*e*x + 9*a^2*c^3*d^5*e^4*
x^2 - 15*a^3*c^2*d^3*e^6*x^2 - 9*a^2*c^3*d^4*e^5*x^3 + 3*a^3*c^2*d^2*e^7*x
^3 - 3*a^4*c*d^2*e^7*x + 6*a^4*c*d*e^8*x^2 + 15*a^2*c^3*d^6*e^3*x - 9*a^3*
c^2*d^4*e^5*x + 3*a*c^4*d^7*e^2*x^2 + 9*a*c^4*d^6*e^3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.24

$$\int \frac{x(d+ex)}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{-\frac{4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,a^2 d\,e^3}{3}-\frac{4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,a^2 e^4 x}{3}-4\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cd}}{}$$

input
```
int(x*(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*( - 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*d*e**3 - 2*sqrt(e)
*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*e**4*x - 6*sqrt(e)*sqrt(d)*sqrt(c)
*sqrt(a*e + c*d*x)*a*c*d**3*e - 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x
)*a*c*d**2*e**2*x - 2*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c*d*e**3
*x**2 - 6*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d**4*x - 6*sqrt(e
)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**2*d**3*e*x**2 + 6*sqrt(d + e*x)*a**
2*c*d**2*e**3 + 3*sqrt(d + e*x)*a**2*c*d*e**4*x + 2*sqrt(d + e*x)*a*c**2*d
**4*e + 10*sqrt(d + e*x)*a*c**2*d**3*e**2*x + 2*sqrt(d + e*x)*a*c**2*d**2*
e**3*x**2 + 3*sqrt(d + e*x)*c**3*d**5*x + 6*sqrt(d + e*x)*c**3*d**4*e*x**2
))/(3*sqrt(a*e + c*d*x)*c*d*(a**4*d*e**7 + a**4*e**8*x - 3*a**3*c*d**3*e**
5 - 2*a**3*c*d**2*e**6*x + a**3*c*d*e**7*x**2 + 3*a**2*c**2*d**5*e**3 - 3*
a**2*c**2*d**3*e**5*x**2 - a*c**3*d**7*e + 2*a*c**3*d**6*e**2*x + 3*a*c**3
*d**5*e**3*x**2 - c**4*d**8*x - c**4*d**7*e*x**2))
```

3.142 $\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 118

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{2(d+ex)}{3\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} + \frac{8e(cd^2+ae^2+2cdex)}{3\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}$$

output
```
1/3*(-2*e*x-2*d)/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+8/
3*e*(2*c*d*e*x+a*e^2+c*d^2)/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.76

$$\int \frac{d+ex}{\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2(d+ex)\left(3a^2e^4+6acde^2(d+2ex)+c^2d^2\left(-d^2+4dex+8e^2x^2\right)\right.}{3\left(cd^2-ae^2\right)^3((ae+cdx)(d+ex))^{3/2}}$$

input
```
Integrate[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(2*(d + e*x)*(3*a^2*e^4 + 6*a*c*d*e^2*(d + 2*e*x) + c^2*d^2*(-d^2 + 4*d*e*
x + 8*e^2*x^2)))/(3*(c*d^2 - a*e^2)^3*((a*e + c*d*x)*(d + e*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1159, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 1159$$

$$-\frac{4e\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3\left(cd^2-ae^2\right)}-\frac{2(d+ex)}{3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \ 1088$$

$$\frac{8e\left(ae^2+cd^2+2cdex\right)}{3\left(cd^2-ae^2\right)^3\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}-\frac{2(d+ex)}{3\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[(d + e*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x))/(3*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^
(3/2)) + (8*e*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*(c*d^2 - a*e^2)^3*Sqrt[a*d*e
 + (c*d^2 + a*e^2)*x + c*d*e*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.24

method	result
gosper	$-\frac{2(ex+d)^{2}(cdx+ae)\left(8x^{2}c^{2}d^{2}e^{2}+12xacd\,e^{3}+4x\,c^{2}d^{3}e+3a^{2}e^{4}+6ac\,d^{2}e^{2}-c^{2}d^{4}\right)}{3\left(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3}\right)\left(cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade\right)^{\frac{5}{2}}}$
trager	$-\frac{2\left(8x^{2}c^{2}d^{2}e^{2}+12xacd\,e^{3}+4x\,c^{2}d^{3}e+3a^{2}e^{4}+6ac\,d^{2}e^{2}-c^{2}d^{4}\right)\sqrt{cd\,x^{2}e+a\,e^{2}x+c\,d^{2}x+ade}}{3\left(a^{2}e^{4}-2ac\,d^{2}e^{2}+c^{2}d^{4}\right)(cdx+ae)^{2}\left(a\,e^{2}-c\,d^{2}\right)(ex+d)}$
orering	$-\frac{2\left(8x^{2}c^{2}d^{2}e^{2}+12xacd\,e^{3}+4x\,c^{2}d^{3}e+3a^{2}e^{4}+6ac\,d^{2}e^{2}-c^{2}d^{4}\right)(ex+d)^{2}(cdx+ae)}{3\left(e^{6}a^{3}-3d^{2}e^{4}a^{2}c+3d^{4}e^{2}a\,c^{2}-d^{6}c^{3}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{5}{2}}}$
default	$d\left(\frac{\frac{4}{3}cdxe+\frac{2}{3}a\,e^{2}+\frac{2}{3}c\,d^{2}}{\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}+\frac{16dec\left(2cdxe+a\,e^{2}+c\,d^{2}\right)}{3\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)^{2}\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}\right)+e$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/3*(e*x+d)^2*(c*d*x+a*e)*(8*c^2*d^2*e^2*x^2+12*a*c*d*e^3*x+4*c^2*d^3*e*x
+3*a^2*e^4+6*a*c*d^2*e^2-c^2*d^4)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2
-c^3*d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(110) = 220$.

Time = 1.57 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.68

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{2\,(8\,c^2}{3\,(a^2c^3d^7e^2-3\,a^3c^2d^5e^4+3\,a^4cd^3e^6-a^5de^8+(c^5d^8e-3\,ac^4d^6e}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="fr
icas")
```

output
```
2/3*(8*c^2*d^2*e^2*x^2 - c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4 + 4*(c^2*d^3*
e + 3*a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a^2*c^3*d
^7*e^2 - 3*a^3*c^2*d^5*e^4 + 3*a^4*c*d^3*e^6 - a^5*d*e^8 + (c^5*d^8*e - 3*
a*c^4*d^6*e^3 + 3*a^2*c^3*d^4*e^5 - a^3*c^2*d^2*e^7)*x^3 + (c^5*d^9 - a*c^
4*d^7*e^2 - 3*a^2*c^3*d^5*e^4 + 5*a^3*c^2*d^3*e^6 - 2*a^4*c*d*e^8)*x^2 + (
2*a*c^4*d^8*e - 5*a^2*c^3*d^6*e^3 + 3*a^3*c^2*d^4*e^5 + a^4*c*d^2*e^7 - a^
5*e^9)*x)
```

Sympy [F]

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \int \frac{d+ex}{((d+ex)(ae+cdx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral((d + e*x)/((d + e*x)*(a*e + c*d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \int \frac{ex+d}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((e*x + d)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.02

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = -\frac{2\sqrt{c\,d\,e\,x^2+(c\,d^2+a\,e^2)\,x+a\,d\,e}\,(3\,a^2\,e^4+6\,a\,c\,d^2\,e^2+12\,a\,c\,d\,e^3\,x-c^2\,d^4+4\,c^2\,d^3\,e\,x+8\,c^2\,d^2\,e^2\,x^2)}{3\,(a\,e+c\,d\,x)^2\,(a\,e^2-c\,d^2)^3\,(d+e\,x)}$$

input
```
int((d + e*x)/(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2),x)
```

output
```
-(2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(3*a^2*e^4 - c^2*d^4 + 8
*c^2*d^2*e^2*x^2 + 6*a*c*d^2*e^2 + 4*c^2*d^3*e*x + 12*a*c*d*e^3*x))/(3*(a*
e + c*d*x)^2*(a*e^2 - c*d^2)^3*(d + e*x))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.93

$$\int \frac{d+ex}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}\,dx = \frac{\frac{16\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,ad\,e^2}{3}+\frac{16\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,a\,e^3x}{3}+\frac{16\sqrt{e}\,\sqrt{d}\,\sqrt{c}\,\sqrt{cdx+ae}\,c}{3}}{\sqrt{cdx+ae}\,(a^3cd\,e^7x^2-3a^2c^2d^3e^5x^2+3}$$

input
```
int((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*(8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*d*e**2 + 8*sqrt(e)*sqrt(
d)*sqrt(c)*sqrt(a*e + c*d*x)*a*e**3*x + 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e
 + c*d*x)*c*d**2*e*x + 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*d*e**
2*x**2 - 3*sqrt(d + e*x)*a**2*e**4 - 6*sqrt(d + e*x)*a*c*d**2*e**2 - 12*sq
rt(d + e*x)*a*c*d*e**3*x + sqrt(d + e*x)*c**2*d**4 - 4*sqrt(d + e*x)*c**2*
d**3*e*x - 8*sqrt(d + e*x)*c**2*d**2*e**2*x**2))/(3*sqrt(a*e + c*d*x)*(a**
4*d*e**7 + a**4*e**8*x - 3*a**3*c*d**3*e**5 - 2*a**3*c*d**2*e**6*x + a**3*
c*d*e**7*x**2 + 3*a**2*c**2*d**5*e**3 - 3*a**2*c**2*d**3*e**5*x**2 - a*c**
3*d**7*e + 2*a*c**3*d**6*e**2*x + 3*a*c**3*d**5*e**3*x**2 - c**4*d**8*x -
c**4*d**7*e*x**2))
```

3.143 $\int \frac{d+ex}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 255

$$\int \frac{d+ex}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\frac{2cd(d+ex)}{3ae\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}}+\frac{2\left(3c^3d^6-7ac^2d^4e^2-a^2cd^2e^4-3a^3e^6+cde\left(cd^2-3ae^2\right)\left(3cd^2+ae^2\right)x\right)}{3a^2de^2\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{5/2}d^{3/2}e^{5/2}}$$

output
```
2/3*c*d*(e*x+d)/a/e/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
+2/3*(3*c^3*d^6-7*a*c^2*d^4*e^2-a^2*c*d^2*e^4-3*a^3*e^6+c*d*e*(-3*a*e^2+c*
d^2)*(a*e^2+3*c*d^2)*x)/a^2/d/e^2/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+
c*d*e*x^2)^(1/2)-2*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(1/2))/a^(5/2)/d^(3/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.90

$$\int \frac{d+ex}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \frac{2\left(\frac{\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)(d+ex)^4\left(-ac^3d^4e+\frac{3a^2e^5(ae+cdx)^2}{(d+ex)^2}-\frac{3c^3d^5(ae+cdx)}{d+ex}+\frac{9ac^2d^3e^2}{d+ \cdots}\right.}{\left(-cd^2+ae^2\right)^3} \cdots\right.}{3a^{5/2}d^{3/2}e^{5/2}((ae+ \cdots}$$

input
```
Integrate[(d + e*x)/(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output
```
(2*((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(d + e*x)^4*(-(a*c^3*d^4*e) + (
3*a^2*e^5*(a*e + c*d*x)^2)/(d + e*x)^2 - (3*c^3*d^5*(a*e + c*d*x))/(d + e*
x) + (9*a*c^2*d^3*e^2*(a*e + c*d*x))/(d + e*x)))/(-(c*d^2) + a*e^2)^3 - 3*
(a*e + c*d*x)^(5/2)*(d + e*x)^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a*e + c*d*x])/(S
qrt[a]*Sqrt[e]*Sqrt[d + e*x])]))/(3*a^(5/2)*d^(3/2)*e^(5/2)*((a*e + c*d*x)
*(d + e*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1235, 27, 1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 1235$$

$$\frac{2cd(d+ex)}{3ae\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{2\int -\frac{d\left(cd^2-ae^2\right)\left(3\left(cd^2-ae^2\right)+4cdex\right)}{2x\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3ade\left(cd^2-ae^2\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3\left(cd^2-ae^2\right)+4cdex}{x\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3ae\left(cd^2-ae^2\right)}+\frac{2cd(d+ex)}{3ae\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow 1235$$

$$\frac{\frac{2\left(-3a^3e^6-a^2cd^2e^4-7ac^2d^4e^2+cdex(cd^2-3ae^2)(ae^2+3cd^2)+3c^3d^6\right)}{ade(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\int -\frac{3\left(cd^2-ae^2\right)^3}{2x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ade(cd^2-ae^2)^2}}{3ae\left(cd^2-ae^2\right)}+\frac{2cd(d+ex)}{3ae\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{3\left(cd^2-ae^2\right)\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ade}+\frac{2\left(-3a^3e^6-a^2cd^2e^4-7ac^2d^4e^2+cdex(cd^2-3ae^2)(ae^2+3cd^2)+3c^3d^6\right)}{ade(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3ae\left(cd^2-ae^2\right)}+\frac{2cd(d+ex)}{3ae\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow 1154$$

$$\frac{\frac{2\left(-3a^3e^6-a^2cd^2e^4-7ac^2d^4e^2+cdex(cd^2-3ae^2)(ae^2+3cd^2)+3c^3d^6\right)}{ade(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{6\left(cd^2-ae^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}}{3ae\left(cd^2-ae^2\right)}+\frac{2cd(d+ex)}{3ae\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow 219$$

$$\frac{\frac{2\left(-3a^3e^6-a^2cd^2e^4-7ac^2d^4e^2+cdex(cd^2-3ae^2)(ae^2+3cd^2)+3c^3d^6\right)}{ade(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{3(cd^2-ae^2)\text{arctanh}\left(\frac{x\left(ae^2+cd^2\right)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}\right)}{a^{3/2}d^{3/2}e^{3/2}}}{3ae\left(cd^2-ae^2\right)}+\frac{2cd(d+ex)}{3ae\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[(d + e*x)/(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output
```
(2*c*d*(d + e*x))/(3*a*e*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2)^(3/2)) + ((2*(3*c^3*d^6 - 7*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e
^6 + c*d*e*(c*d^2 - 3*a*e^2)*(3*c*d^2 + a*e^2)*x))/(a*d*e*(c*d^2 - a*e^2)^
2*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (3*(c*d^2 - a*e^2)*ArcTan
h[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c
*d^2 + a*e^2)*x + c*d*e*x^2])])/(a^(3/2)*d^(3/2)*e^(3/2)))/(3*a*e*(c*d^2 -
 a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(233) = 466$.

Time = 2.20 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.31

method	result
default	$e\left(\frac{\frac{4}{3}cdxe+\frac{2}{3}a\,e^{2}+\frac{2}{3}c\,d^{2}}{\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)\left(ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e\right)^{\frac{3}{2}}}+\frac{16dec\left(2cdxe+a\,e^{2}+c\,d^{2}\right)}{3\left(4ac\,d^{2}e^{2}-\left(a\,e^{2}+c\,d^{2}\right)^{2}\right)^{2}\sqrt{ade+\left(a\,e^{2}+c\,d^{2}\right)x+cd\,x^{2}e}}\right)+d$

input
```
int((e*x+d)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
e*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e
^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)^2*
(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+d*(1/3/a/
d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c*d^2)/a/d/e*(2/3*(
2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2
)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)^2*(2*c*d*e
*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/a/d/e*(1/a/d/e/
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e
^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e
)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 752 vs. $2(233) = 466$.

Time = 4.73 (sec) , antiderivative size = 1524, normalized size of antiderivative = 5.98

$$\int \frac{d+ex}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
fricas")
```

output
```
[1/6*(3*(a^2*c^3*d^7*e^2 - 3*a^3*c^2*d^5*e^4 + 3*a^4*c*d^3*e^6 - a^5*d*e^8
 + (c^5*d^8*e - 3*a*c^4*d^6*e^3 + 3*a^2*c^3*d^4*e^5 - a^3*c^2*d^2*e^7)*x^3
 + (c^5*d^9 - a*c^4*d^7*e^2 - 3*a^2*c^3*d^5*e^4 + 5*a^3*c^2*d^3*e^6 - 2*a^
4*c*d*e^8)*x^2 + (2*a*c^4*d^8*e - 5*a^2*c^3*d^6*e^3 + 3*a^3*c^2*d^4*e^5 +
a^4*c*d^2*e^7 - a^5*e^9)*x)*sqrt(a*d*e)*log((8*a^2*d^2*e^2 + (c^2*d^4 + 6*
a*c*d^2*e^2 + a^2*e^4)*x^2 - 4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*d^3*e + a^2*d*e^3)*x)/
x^2) + 4*(4*a^2*c^3*d^7*e^2 - 9*a^3*c^2*d^5*e^4 - 3*a^5*d*e^8 + (3*a*c^4*d
^7*e^2 - 8*a^2*c^3*d^5*e^4 - 3*a^3*c^2*d^3*e^6)*x^2 + (3*a*c^4*d^8*e - 4*a
^2*c^3*d^6*e^3 - 9*a^3*c^2*d^4*e^5 - 6*a^4*c*d^2*e^7)*x)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x))/(a^5*c^3*d^9*e^5 - 3*a^6*c^2*d^7*e^7 + 3*a^7*c
*d^5*e^9 - a^8*d^3*e^11 + (a^3*c^5*d^10*e^4 - 3*a^4*c^4*d^8*e^6 + 3*a^5*c^
3*d^6*e^8 - a^6*c^2*d^4*e^10)*x^3 + (a^3*c^5*d^11*e^3 - a^4*c^4*d^9*e^5 -
3*a^5*c^3*d^7*e^7 + 5*a^6*c^2*d^5*e^9 - 2*a^7*c*d^3*e^11)*x^2 + (2*a^4*c^4
*d^10*e^4 - 5*a^5*c^3*d^8*e^6 + 3*a^6*c^2*d^6*e^8 + a^7*c*d^4*e^10 - a^8*d
^2*e^12)*x), 1/3*(3*(a^2*c^3*d^7*e^2 - 3*a^3*c^2*d^5*e^4 + 3*a^4*c*d^3*e^6
 - a^5*d*e^8 + (c^5*d^8*e - 3*a*c^4*d^6*e^3 + 3*a^2*c^3*d^4*e^5 - a^3*c^2*
d^2*e^7)*x^3 + (c^5*d^9 - a*c^4*d^7*e^2 - 3*a^2*c^3*d^5*e^4 + 5*a^3*c^2*d^
3*e^6 - 2*a^4*c*d*e^8)*x^2 + (2*a*c^4*d^8*e - 5*a^2*c^3*d^6*e^3 + 3*a^3*c^
2*d^4*e^5 + a^4*c*d^2*e^7 - a^5*e^9)*x)*sqrt(-a*d*e)*arctan(1/2*sqrt(c*...
```

Sympy [F]

$$\int \frac{d+ex}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\int \frac{d+ex}{x\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)/x/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral((d + e*x)/(x*((d + e*x)*(a*e + c*d*x))**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*e^2-c*d^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{d+ex}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \int \frac{ex+d}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((e*x+d)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="
giac")
```

output
```
integrate((e*x + d)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{x\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \int \frac{d+e\,x}{x\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}}\,dx$$

input
```
int((d + e*x)/(x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)),x)
```

output
```
int((d + e*x)/(x*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 3117, normalized size of antiderivative = 12.22

$$\int \frac{d+ex}{x\,(ade+(cd^2+ae^2)\,x+cdex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(3*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x)
 - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d
+ e*x))*a**4*d*e**7 + 3*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt
(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sq
rt(d)*sqrt(c)*sqrt(d + e*x))*a**4*e**8*x - 9*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(
a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e +
a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d**3*e**5 - 6*sqr
t(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqr
t(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x)
)*a**3*c*d**2*e**6*x + 3*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqr
t(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + s
qrt(d)*sqrt(c)*sqrt(d + e*x))*a**3*c*d*e**7*x**2 + 9*sqrt(e)*sqrt(d)*sqrt(
a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a
)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a**2*c**2*d**5*e
**3 - 9*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c
*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e**2 + c*d**2) + sqrt(d)*sqrt(c)*sq
rt(d + e*x))*a**2*c**2*d**3*e**5*x**2 - 3*sqrt(e)*sqrt(d)*sqrt(a)*sqrt(a*e
 + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2*sqrt(c)*sqrt(a)*d*e + a*e
**2 + c*d**2) + sqrt(d)*sqrt(c)*sqrt(d + e*x))*a*c**3*d**7*e + 6*sqrt(e)*s
qrt(d)*sqrt(a)*sqrt(a*e + c*d*x)*log(sqrt(e)*sqrt(a*e + c*d*x) - sqrt(2...
```

3.144 $\int \frac{d+ex}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 369

$$\int \frac{d+ex}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{1}{aex\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{5c^2d^4-3a^2e^4+cde(5cd^2-3ae^2)x}{3a^2de^2\left(cd^2-ae^2\right)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} - \frac{15c^4d^8-26ac^3d^6e^2-2a^2c^2d^4e^4+6a^3cd^2e^6-9a^4e^8+cde(15c^3d^6-31ac^2d^4e^2+9a^2cd^2e^4-9a^3e^6)x}{3a^3d^2e^3\left(cd^2-ae^2\right)^3\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} + \frac{\left(5cd^2+3ae^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{e}(d+ex)}{\sqrt{d}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}\right)}{a^{7/2}d^{5/2}e^{7/2}}$$

output
```
-1/a/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-1/3*(5*c^2*d^4-3*a^2*e^4+
c*d*e*(-3*a*e^2+5*c*d^2)*x)/a^2/d/e^2/(-a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(3/2)-1/3*(15*c^4*d^8-26*a*c^3*d^6*e^2-2*a^2*c^2*d^4*e^4+6*a^
3*c*d^2*e^6-9*a^4*e^8+c*d*e*(-9*a^3*e^6+9*a^2*c*d^2*e^4-31*a*c^2*d^4*e^2+1
5*c^3*d^6)*x)/a^3/d^2/e^3/(-a*e^2+c*d^2)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(1/2)+(3*a*e^2+5*c*d^2)*arctanh(a^(1/2)*e^(1/2)*(e*x+d)/d^(1/2)/(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/a^(7/2)/d^(5/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.91

$$\int \frac{d+ex}{x^2 \left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = -\frac{\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)(d+ex)^2\left(-15c^5d^8x^2(d+ex)+3a^5e^8(d+3ex)-3a^4cde^6\left(3d^2+de\right.\right.}{}$$

input
```
Integrate[(d + e*x)/(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output
```
(-((Sqrt[a]*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)*(d + e*x)^2*(-15*c^5*d^8*x^2*(d
+ e*x) + 3*a^5*e^8*(d + 3*e*x) - 3*a^4*c*d*e^6*(3*d^2 + d*e*x - 6*e^2*x^2)
 + a*c^4*d^6*e*x*(-20*d^2 + 11*d*e*x + 31*e^2*x^2) - 3*a^2*c^3*d^4*e^2*(d^
3 - 13*d^2*e*x - 11*d*e^2*x^2 + 3*e^3*x^3) + 3*a^3*c^2*d^2*e^4*(3*d^3 - 3*
d^2*e*x - 5*d*e^2*x^2 + 3*e^3*x^3)))/((-(c*d^2) + a*e^2)^3*x)) + 3*(5*c*d^
2 + 3*a*e^2)*(a*e + c*d*x)^(5/2)*(d + e*x)^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a*e
 + c*d*x])/(Sqrt[a]*Sqrt[e]*Sqrt[d + e*x])])/(3*a^(7/2)*d^(5/2)*e^(7/2)*((
a*e + c*d*x)*(d + e*x))^(5/2))
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1235, 27, 1235, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^2 \left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{2cd(d+ex)}{3aex\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}-\frac{2\int -\frac{d\left(cd^2-ae^2\right)\left(5cd^2+6cexd-3ae^2\right)}{2x^2\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3ade\left(cd^2-ae^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5cd^2+6cexd-3ae^2}{x^2(cdex^2+(cd^2+ae^2)x+ade)^{3/2}}dx}{3ae\left(cd^2-ae^2\right)}+\frac{2cd(d+ex)}{3aex\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 1235

$$\frac{\frac{2(-3a^3e^6+cdex(-3a^2e^4-10acd^2e^2+5c^2d^4)-a^2cd^2e^4-9ac^2d^4e^2+5c^3d^6)}{adex(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\int -\frac{15c^3d^6-31ac^2e^2d^4+9a^2ce^4d^2+2ce\left(5c^2d^4-10ace^2d^2-3a^2e^4\right)xd\cdots}{2x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}\cdots}{ade(cd^2-ae^2)^2}}{3ae\left(cd^2-ae^2\right)}$$
$$\frac{2cd(d+ex)}{3aex\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{15c^3d^6-31ac^2e^2d^4+9a^2ce^4d^2+2ce\left(5c^2d^4-10ace^2d^2-3a^2e^4\right)xd-9a^3e^6}{x^2\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{ade(cd^2-ae^2)^2}+\frac{2(-3a^3e^6+cdex(-3a^2e^4-10acd^2e^2+5c^2d^4)-a^2cd^2e^4-9ac^2d^4e^2+5\cdots}{adex(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}}{3ae\left(cd^2-ae^2\right)}$$
$$\frac{2cd(d+ex)}{3aex\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 1228

$$\frac{\frac{-\frac{3\left(3ae^2+5cd^2\right)\left(cd^2-ae^2\right)^3\int \frac{1}{x\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}dx}{2ade}-\frac{\left(-9a^3e^6+9a^2cd^2e^4-31ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+cdex^2}}{adex}}{ade(cd^2-ae^2)^2}+\frac{2(-3a^3e^6+cde\cdots}{\cdots}}{3ae\left(cd^2-ae^2\right)}$$
$$\frac{2cd(d+ex)}{3aex\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 1154

$$\frac{\frac{\frac{3\left(cd^2-ae^2\right)^3\left(3ae^2+5cd^2\right)\int \frac{1}{4ade-\frac{\left(2ade+\left(cd^2+ae^2\right)x\right)^2}{cdex^2+\left(cd^2+ae^2\right)x+ade}}d\frac{2ade+\left(cd^2+ae^2\right)x}{\sqrt{cdex^2+\left(cd^2+ae^2\right)x+ade}}}{ade}-\frac{\left(-9a^3e^6+9a^2cd^2e^4-31ac^2d^4e^2+15c^3d^6\right)\sqrt{x\left(ae^2+cd^2\right)+ade+\cdots}}{adex}}{ade(cd^2-ae^2)^2}\cdots}{3ae\left(cd^2-ae^2\right)}$$
$$\frac{2cd(d+ex)}{3aex\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{2(-3a^3e^6+cdex(-3a^2e^4-10acd^2e^2+5c^2d^4)-a^2cd^2e^4-9ac^2d^4e^2+5c^3d^6)}{adex(cd^2-ae^2)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}+\frac{\frac{3(cd^2-ae^2)^3(3ae^2+5cd^2)\text{arctanh}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade\cdots}}\right.}{2a^{3/2}d^{3/2}e^{3/2}}\cdots}{ade\cdots}}{3ae\left(cd^2-ae^2\right)}$$

$$\frac{2cd(d+ex)}{3aex\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input
```
Int[(d + e*x)/(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output
```
(2*c*d*(d + e*x))/(3*a*e*(c*d^2 - a*e^2)*x*(a*d*e + (c*d^2 + a*e^2)*x + c*
d*e*x^2)^(3/2)) + ((2*(5*c^3*d^6 - 9*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3
*e^6 + c*d*e*(5*c^2*d^4 - 10*a*c*d^2*e^2 - 3*a^2*e^4)*x))/(a*d*e*(c*d^2 -
a*e^2)^2*x*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (-(((15*c^3*d^6
- 31*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 9*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*
e^2)*x + c*d*e*x^2])/(a*d*e*x)) + (3*(c*d^2 - a*e^2)^3*(5*c*d^2 + 3*a*e^2)
*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d
*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])])/(2*a^(3/2)*d^(3/2)*e^(3/2)))/(a*d*e
*(c*d^2 - a*e^2)^2))/(3*a*e*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1089 vs. $2(345) = 690$.

Time = 2.24 (sec) , antiderivative size = 1090, normalized size of antiderivative = 2.95

method	result	size
default	Expression too large to display	1090

input
```
int((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVE
RBOSE)
```

output
```
d*(-1/a/d/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-5/2*(a*e^2+c*d^2)/a/
d/e*(1/3/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c*d^2)/a
/d/e*(2/3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)
^2*(2*c*d*e*x+a*e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/a/d/
e*(1/a/d/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*
c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*
x+c*d*x^2*e)^(1/2)-1/a/d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*
d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))-4*c/a*(2/3*(2*c*d
*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c
*d*x^2*e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)^2*(2*c*d*e*x+a*
e^2+c*d^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)))+e*(1/3/a/d/e/(a*d*e+(
a*e^2+c*d^2)*x+c*d*x^2*e)^(3/2)-1/2*(a*e^2+c*d^2)/a/d/e*(2/3*(2*c*d*e*x+a*
e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*
e)^(3/2)+16/3*d*e*c/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)^2*(2*c*d*e*x+a*e^2+c*d
^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2))+1/a/d/e*(1/a/d/e/(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2)-(a*e^2+c*d^2)/a/d/e*(2*c*d*e*x+a*e^2+c*d^2)/(4
*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(1/2)-1/a/
d/e/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^
2+c*d^2)*x+c*d*x^2*e)^(1/2))/x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(345) = 690$.

Time = 14.63 (sec) , antiderivative size = 1856, normalized size of antiderivative = 5.03

$$\int \frac{d+ex}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="fricas")
```

output
```
[1/12*(3*((5*c^6*d^10*e - 12*a*c^5*d^8*e^3 + 6*a^2*c^4*d^6*e^5 + 4*a^3*c^3
*d^4*e^7 - 3*a^4*c^2*d^2*e^9)*x^4 + (5*c^6*d^11 - 2*a*c^5*d^9*e^2 - 18*a^2
*c^4*d^7*e^4 + 16*a^3*c^3*d^5*e^6 + 5*a^4*c^2*d^3*e^8 - 6*a^5*c*d*e^10)*x^
3 + (10*a*c^5*d^10*e - 19*a^2*c^4*d^8*e^3 + 14*a^4*c^2*d^4*e^7 - 2*a^5*c*d
^2*e^9 - 3*a^6*e^11)*x^2 + (5*a^2*c^4*d^9*e^2 - 12*a^3*c^3*d^7*e^4 + 6*a^4
*c^2*d^5*e^6 + 4*a^5*c*d^3*e^8 - 3*a^6*d*e^10)*x)*sqrt(a*d*e)*log((8*a^2*d
^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 4*sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(a*d*e) + 8*(a*c*
d^3*e + a^2*d*e^3)*x)/x^2) - 4*(3*a^3*c^3*d^8*e^3 - 9*a^4*c^2*d^6*e^5 + 9*
a^5*c*d^4*e^7 - 3*a^6*d^2*e^9 + (15*a*c^5*d^9*e^2 - 31*a^2*c^4*d^7*e^4 + 9
*a^3*c^3*d^5*e^6 - 9*a^4*c^2*d^3*e^8)*x^3 + (15*a*c^5*d^10*e - 11*a^2*c^4*
d^8*e^3 - 33*a^3*c^3*d^6*e^5 + 15*a^4*c^2*d^4*e^7 - 18*a^5*c*d^2*e^9)*x^2
+ (20*a^2*c^4*d^9*e^2 - 39*a^3*c^3*d^7*e^4 + 9*a^4*c^2*d^5*e^6 + 3*a^5*c*d
^3*e^8 - 9*a^6*d*e^10)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x))/((a
^4*c^5*d^11*e^5 - 3*a^5*c^4*d^9*e^7 + 3*a^6*c^3*d^7*e^9 - a^7*c^2*d^5*e^11
)*x^4 + (a^4*c^5*d^12*e^4 - a^5*c^4*d^10*e^6 - 3*a^6*c^3*d^8*e^8 + 5*a^7*c
^2*d^6*e^10 - 2*a^8*c*d^4*e^12)*x^3 + (2*a^5*c^4*d^11*e^5 - 5*a^6*c^3*d^9*
e^7 + 3*a^7*c^2*d^7*e^9 + a^8*c*d^5*e^11 - a^9*d^3*e^13)*x^2 + (a^6*c^3*d^
10*e^6 - 3*a^7*c^2*d^8*e^8 + 3*a^8*c*d^6*e^10 - a^9*d^4*e^12)*x), -1/6*(3*
((5*c^6*d^10*e - 12*a*c^5*d^8*e^3 + 6*a^2*c^4*d^6*e^5 + 4*a^3*c^3*d^4*e...
```

Sympy [F]

$$\int \frac{d+ex}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\int \frac{d+ex}{x^2\left(\left(d+ex\right)\left(ae+cdx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)/x**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Integral((d + e*x)/(x**2*((d + e*x)*(a*e + c*d*x))**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{d+ex}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \int \frac{ex+d}{\left(cdex^2+ade+\left(cd^2+ae^2\right)x\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="giac")
```

output
```
integrate((e*x + d)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*x^2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \int \frac{d+e\,x}{x^2\,{\left(c\,d\,e\,x^2+\left(c\,d^2+a\,e^2\right)\,x+a\,d\,e\right)}^{5/2}} \,d x$$

input
```
int((d + e*x)/(x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)),x)
```

output
```
int((d + e*x)/(x^2*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{d+ex}{x^2\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx=\int \frac{ex+d}{x^2\left(ade+\left(a\,e^2+c\,d^2\right)x+cde\,x^2\right)^{\frac{5}{2}}}dx$$

input
```
int((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
int((e*x+d)/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

3.145 $\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 393

$$\begin{aligned}\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = & -\frac{2a^2e^2}{3c^2d^2\left(cd^2-ae^2\right)(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{3/2}} \\ & +\frac{4ae(3cd^2+ae^2)}{3c^2d^2\left(cd^2-ae^2\right)^2(d+ex)^2\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}} \\ & +\frac{2(c^2d^4+10acd^2e^2+5a^2e^4)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{5c^2d^2\left(cd^2-ae^2\right)^3(d+ex)^3} \\ & +\frac{8(c^2d^4+10acd^2e^2+5a^2e^4)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15cd\left(cd^2-ae^2\right)^4(d+ex)^2} \\ & +\frac{16(c^2d^4+10acd^2e^2+5a^2e^4)\sqrt{ade+\left(cd^2+ae^2\right)x+cdex^2}}{15\left(cd^2-ae^2\right)^5(d+ex)}\end{aligned}$$

output
```
-2/3*a^2*e^2/c^2/d^2/(-a*e^2+c*d^2)/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x
^2)^(3/2)+4/3*a*e*(a*e^2+3*c*d^2)/c^2/d^2/(-a*e^2+c*d^2)^2/(e*x+d)^2/(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+2/5*(5*a^2*e^4+10*a*c*d^2*e^2+c^2*d^4)*
(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/c^2/d^2/(-a*e^2+c*d^2)^3/(e*x+d)^3
+8/15*(5*a^2*e^4+10*a*c*d^2*e^2+c^2*d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)
^(1/2)/c/d/(-a*e^2+c*d^2)^4/(e*x+d)^2+16/15*(5*a^2*e^4+10*a*c*d^2*e^2+c^2*
d^4)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(-a*e^2+c*d^2)^5/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.60

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{2(c^4d^6x^2(15d^2+20dex+8e^2x^2)+a^4e^6(8d^2+20dex+$$

input
```
Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output
```
(2*(c^4*d^6*x^2*(15*d^2 + 20*d*e*x + 8*e^2*x^2) + a^4*e^6*(8*d^2 + 20*d*e*
x + 15*e^2*x^2) + 4*a^3*c*d*e^4*(20*d^3 + 53*d^2*e*x + 45*d*e^2*x^2 + 15*e
^3*x^3) + 4*a*c^3*d^4*e*x*(15*d^3 + 45*d^2*e*x + 53*d*e^2*x^2 + 20*e^3*x^3
) + 2*a^2*c^2*d^2*e^2*(20*d^4 + 110*d^3*e*x + 189*d^2*e^2*x^2 + 110*d*e^3*
x^3 + 20*e^4*x^4)))/(15*(c*d^2 - a*e^2)^5*(d + e*x)*((a*e + c*d*x)*(d + e*
x))^(3/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1244, 27, 1159, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 1244$$

$$-\frac{2\int -\frac{e^2(2ade-(cd^2+5ae^2)x)}{2(cdex^2+(cd^2+ae^2)x+ade)^{5/2}}dx}{5e^3\left(cd^2-ae^2\right)}-\frac{2dx}{5e(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2ade-(cd^2+5ae^2)x}{(cdex^2+(cd^2+ae^2)x+ade)^{5/2}}dx}{5e\left(cd^2-ae^2\right)}-\frac{2dx}{5e(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1159$$

$$\frac{-\frac{4(5a^2e^4+10acd^2e^2+c^2d^4)\int \frac{1}{\left(cdex^2+\left(cd^2+ae^2\right)x+ade\right)^{3/2}}dx}{3(cd^2-ae^2)^2}-\frac{2\left(x\left(5a^2e^4+10acd^2e^2+c^2d^4\right)+4ade\left(3ae^2+cd^2\right)\right)}{3(cd^2-ae^2)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}}{5e\left(cd^2-ae^2\right)}-\frac{2dx}{5e(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

$$\Big\downarrow \; 1088$$

$$\frac{\frac{8(5a^2e^4+10acd^2e^2+c^2d^4)\left(ae^2+cd^2+2cdex\right)}{3(cd^2-ae^2)^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}-\frac{2\left(x\left(5a^2e^4+10acd^2e^2+c^2d^4\right)+4ade\left(3ae^2+cd^2\right)\right)}{3(cd^2-ae^2)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}}{5e\left(cd^2-ae^2\right)}-\frac{2dx}{5e(d+ex)\left(cd^2-ae^2\right)\left(x\left(ae^2+cd^2\right)+ade+cdex^2\right)^{3/2}}$$

input

```
Int[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)),x]
```

output

```
(-2*d*x)/(5*e*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2)^(3/2)) + ((-2*(4*a*d*e*(c*d^2 + 3*a*e^2) + (c^2*d^4 + 10*a*c*d^2*e^2
 + 5*a^2*e^4)*x))/(3*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*
x^2)^(3/2)) + (8*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^2 + 2
*c*d*e*x))/(3*(c*d^2 - a*e^2)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2
]))/(5*e*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1244
```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_))/((d
_) + (e_.)*(x_)), x_Symbol] :> Simp[(-(e*f - d*g))*(f + g*x)^(n - 1)*((a +
b*x + c*x^2)^(p + 1)/(p*(2*c*d - b*e)*(d + e*x))), x] + Simp[1/(p*e^2*(2*c*
d - b*e))   Int[(f + g*x)^(n - 2)*(a + b*x + c*x^2)^p*Simp[b*e*g*((-e)*f +
d*g + e*f*n - d*g*n - e*f*p) + c*(d^2*g^2*(n - 1) - d*e*f*g*n + e^2*f^2*(2*
p + 1)) - e*g*(b*e*g*p - c*(e*f*n - d*g*n + 2*e*f*p))*x, x], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && IGtQ[n, 1] && LtQ[p, -1] && EqQ[c*d^2 - b*d*
e + a*e^2, 0]
```

Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.93

method	result
gosper	$-\frac{2(cdx+ae)\left(40a^2c^2d^2e^6x^4+80a\,c^3d^4e^4x^4+8c^4d^6e^2x^4+60a^3cd\,e^7x^3+220a^2c^2d^3e^5x^3+212a\,c^3d^5e^3x^3+20c^4d^7e\,x^3+15a^4e^8x^2+1\right.}{15\left(a^5e^{10}-5a^4c\,d^2e^8+10a^3c^2d^4e^6-1\right.}$
orering	$-\frac{2\left(40a^2c^2d^2e^6x^4+80a\,c^3d^4e^4x^4+8c^4d^6e^2x^4+60a^3cd\,e^7x^3+220a^2c^2d^3e^5x^3+212a\,c^3d^5e^3x^3+20c^4d^7e\,x^3+15a^4e^8x^2+180a^3c\,d^2e\right.}{15\left(a^5e^{10}-5a^4c\,d^2e^8+10a^3c^2d^4e^6-1\right.}$
trager	$-\frac{2\left(40a^2c^2d^2e^6x^4+80a\,c^3d^4e^4x^4+8c^4d^6e^2x^4+60a^3cd\,e^7x^3+220a^2c^2d^3e^5x^3+212a\,c^3d^5e^3x^3+20c^4d^7e\,x^3+15a^4e^8x^2+180a^3c\,d^2e\right.}{15\left(a^4e^8-4a^3c\,d^2e^6\right.}$
default	$\frac{-\frac{1}{3dec\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{3}{2}}}-\frac{\left(a\,e^2+c\,d^2\right)\left(\frac{\frac{4}{3}cdxe+\frac{2}{3}a\,e^2+\frac{2}{3}c\,d^2}{\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)\left(ade+\left(a\,e^2+c\,d^2\right)x+cd\,x^2e\right)^{\frac{3}{2}}}+\frac{16dec\left(2cdx\right.}{3\left(4ac\,d^2e^2-\left(a\,e^2+c\,d^2\right)^2\right)}\right.}{2dec}}{e}$

input
```
int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*x^2*e)^(5/2),x,method=_RETURNVE
RBOSE)
```

output
```
-2/15*(c*d*x+a*e)*(40*a^2*c^2*d^2*e^6*x^4+80*a*c^3*d^4*e^4*x^4+8*c^4*d^6*e
^2*x^4+60*a^3*c*d*e^7*x^3+220*a^2*c^2*d^3*e^5*x^3+212*a*c^3*d^5*e^3*x^3+20
*c^4*d^7*e*x^3+15*a^4*e^8*x^2+180*a^3*c*d^2*e^6*x^2+378*a^2*c^2*d^4*e^4*x^
2+180*a*c^3*d^6*e^2*x^2+15*c^4*d^8*x^2+20*a^4*d*e^7*x+212*a^3*c*d^3*e^5*x+
220*a^2*c^2*d^5*e^3*x+60*a*c^3*d^7*e*x+8*a^4*d^2*e^6+80*a^3*c*d^4*e^4+40*a
^2*c^2*d^6*e^2)/(a^5*e^10-5*a^4*c*d^2*e^8+10*a^3*c^2*d^4*e^6-10*a^2*c^3*d^
6*e^4+5*a*c^4*d^8*e^2-c^5*d^10)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(373) = 746$.

Time = 15.57 (sec) , antiderivative size = 820, normalized size of antiderivative = 2.09

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \frac{}{15\left(a^2c^5d^{13}e^2-5\,a^3c^4d^{11}e^4+10\,a^4c^3d^9e^6-10\,a^5c^2d^7e^8\right.}$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="fricas")
```

output
```
2/15*(40*a^2*c^2*d^6*e^2 + 80*a^3*c*d^4*e^4 + 8*a^4*d^2*e^6 + 8*(c^4*d^6*e
^2 + 10*a*c^3*d^4*e^4 + 5*a^2*c^2*d^2*e^6)*x^4 + 4*(5*c^4*d^7*e + 53*a*c^3
*d^5*e^3 + 55*a^2*c^2*d^3*e^5 + 15*a^3*c*d*e^7)*x^3 + 3*(5*c^4*d^8 + 60*a*
c^3*d^6*e^2 + 126*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 + 5*a^4*e^8)*x^2 + 4*
(15*a*c^3*d^7*e + 55*a^2*c^2*d^5*e^3 + 53*a^3*c*d^3*e^5 + 5*a^4*d*e^7)*x)*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a^2*c^5*d^13*e^2 - 5*a^3*c^4*
d^11*e^4 + 10*a^4*c^3*d^9*e^6 - 10*a^5*c^2*d^7*e^8 + 5*a^6*c*d^5*e^10 - a^
7*d^3*e^12 + (c^7*d^12*e^3 - 5*a*c^6*d^10*e^5 + 10*a^2*c^5*d^8*e^7 - 10*a^
3*c^4*d^6*e^9 + 5*a^4*c^3*d^4*e^11 - a^5*c^2*d^2*e^13)*x^5 + (3*c^7*d^13*e
^2 - 13*a*c^6*d^11*e^4 + 20*a^2*c^5*d^9*e^6 - 10*a^3*c^4*d^7*e^8 - 5*a^4*c
^3*d^5*e^10 + 7*a^5*c^2*d^3*e^12 - 2*a^6*c*d*e^14)*x^4 + (3*c^7*d^14*e - 9
*a*c^6*d^12*e^3 + a^2*c^5*d^10*e^5 + 25*a^3*c^4*d^8*e^7 - 35*a^4*c^3*d^6*e
^9 + 17*a^5*c^2*d^4*e^11 - a^6*c*d^2*e^13 - a^7*e^15)*x^3 + (c^7*d^15 + a*
c^6*d^13*e^2 - 17*a^2*c^5*d^11*e^4 + 35*a^3*c^4*d^9*e^6 - 25*a^4*c^3*d^7*e
^8 - a^5*c^2*d^5*e^10 + 9*a^6*c*d^3*e^12 - 3*a^7*d*e^14)*x^2 + (2*a*c^6*d^
14*e - 7*a^2*c^5*d^12*e^3 + 5*a^3*c^4*d^10*e^5 + 10*a^4*c^3*d^8*e^7 - 20*a
^5*c^2*d^6*e^9 + 13*a^6*c*d^4*e^11 - 3*a^7*d^2*e^13)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)x+cdex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(a*e^2-c*d^2)>0)', see `assume
?` for mor
```

Giac [F]

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \int \frac{x^2}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}(ex+d)}\,dx$$

input
```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm
="giac")
```

output
```
integrate(x^2/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(e*x + d)), x
)
```

Mupad [B] (verification not implemented)

Time = 7.06 (sec) , antiderivative size = 3099, normalized size of antiderivative = 7.89

$$\int \frac{x^2}{(d+ex)\left(ade+(cd^2+ae^2)\,x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(x^2/((d + e*x)*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(5/2)),x)
```

output
```
(((6*a*e^2 - 10*c*d^2)/(15*(a*e^2 - c*d^2)^4) - (4*c*d^2)/(5*(a*e^2 - c*d^
2)^4))*(x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x) - (((d*((e
*(2*a*e^3 - 2*c*d^2*e))/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)) - (4*c
*d^2*e^2)/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e))))/e + (e*(2*c*d^3 +
2*a*d*e^2))/(5*(a*e^2 - c*d^2)^3*(3*a*e^3 - 3*c*d^2*e)))*(x*(a*e^2 + c*d^2
) + a*d*e + c*d*e*x^2)^(1/2))/(d + e*x)^2 + ((x*(a*e^2 + c*d^2) + a*d*e +
c*d*e*x^2)^(1/2)*(x*((((12*c^3*d^3*e^2)/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e -
2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2 + c*d^2))/(5*(a*e^
2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)))*(a*e^2 + c*d^2)
)/(c*d*e) - (6*c^2*d^2*e*(a*e^2 + c*d^2))/(5*(a*e^2 - c*d^2)^2*(c^3*d^5*e
- 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) + (8*a*c^3*d^4*e^3)/(5*(a*e^2 - c*d^2)^3
*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (2*c^2*d^2*e*(46*a^2*e^4 +
 4*c^2*d^4 + 66*a*c*d^2*e^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d
^3*e^3 + a^2*c*d*e^5))) + (a*((12*c^3*d^3*e^2)/(5*(a*e^2 - c*d^2)^2*(c^3*d
^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5)) - (4*c^3*d^3*e^2*(a*e^2 + c*d^2))/(
5*(a*e^2 - c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/c - (c*
d*(a*e^2 + c*d^2)*(46*a^2*e^4 + 4*c^2*d^4 + 66*a*c*d^2*e^2))/(15*(a*e^2 -
c*d^2)^3*(c^3*d^5*e - 2*a*c^2*d^3*e^3 + a^2*c*d*e^5))))/((a*e + c*d*x)*(d
+ e*x)) + ((x*(a*e^2 + c*d^2) + a*d*e + c*d*e*x^2)^(1/2)*(x*((a*(((a*e^2 +
 c*d^2)*((4*c^4*d^4*e^3*(a*e^2 + c*d^2))/(15*(a*e^2 - c*d^2)^3*(c^3*d^5...
```

Reduce [B] (verification not implemented)

Time = 8.04 (sec) , antiderivative size = 1349, normalized size of antiderivative = 3.43

$$\int \frac{x^2}{(d+ex)\left(ade+\left(cd^2+ae^2\right)x+cdex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)
```

output
```
(2*(40*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**3*d**3*e**5 + 120*sqrt
(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**3*d**2*e**6*x + 120*sqrt(e)*sqrt(
d)*sqrt(c)*sqrt(a*e + c*d*x)*a**3*d*e**7*x**2 + 40*sqrt(e)*sqrt(d)*sqrt(c)
*sqrt(a*e + c*d*x)*a**3*e**8*x**3 + 80*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e +
c*d*x)*a**2*c*d**5*e**3 + 280*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*
*2*c*d**4*e**4*x + 360*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d*
*3*e**5*x**2 + 200*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d**2*e
**6*x**3 + 40*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a**2*c*d*e**7*x**4
 + 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**7*e + 104*sqrt(e)
*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**6*e**2*x + 264*sqrt(e)*sqrt(d
)*sqrt(c)*sqrt(a*e + c*d*x)*a*c**2*d**5*e**3*x**2 + 248*sqrt(e)*sqrt(d)*sq
rt(c)*sqrt(a*e + c*d*x)*a*c**2*d**4*e**4*x**3 + 80*sqrt(e)*sqrt(d)*sqrt(c)
*sqrt(a*e + c*d*x)*a*c**2*d**3*e**5*x**4 + 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(
a*e + c*d*x)*c**3*d**8*x + 24*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c*
*3*d**7*e*x**2 + 24*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**6*e*
*2*x**3 + 8*sqrt(e)*sqrt(d)*sqrt(c)*sqrt(a*e + c*d*x)*c**3*d**5*e**3*x**4
- 8*sqrt(d + e*x)*a**4*d**2*e**7 - 20*sqrt(d + e*x)*a**4*d*e**8*x - 15*sqr
t(d + e*x)*a**4*e**9*x**2 - 80*sqrt(d + e*x)*a**3*c*d**4*e**5 - 212*sqrt(d
 + e*x)*a**3*c*d**3*e**6*x - 180*sqrt(d + e*x)*a**3*c*d**2*e**7*x**2 - 60*
sqrt(d + e*x)*a**3*c*d*e**8*x**3 - 40*sqrt(d + e*x)*a**2*c**2*d**6*e**3...
```

3.146 $\int (gx)^n(d+ex)^2 \left(ad+(bd+ae)x+bex^2\right)^p \, dx$

Optimal result

Integrand size = 34, antiderivative size = 93

$$\int (gx)^n(d+ex)^2 \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \frac{d^2(gx)^{1+n}\left(1+\frac{bx}{a}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}\left(ad+(bd+ae)x+bex^2\right)^p \text{AppellF1}\left(1+n,-p,-2-p,2+n,-\frac{bx}{a},-\frac{e}{}\right.}{g(1+n)}$$

output
```
d^2*(g*x)^(1+n)*(a*d+(a*e+b*d)*x+b*e*x^2)^p*AppellF1(1+n,-p,-2-p,2+n,-b*x/
a,-e*x/d)/g/(1+n)/((1+b*x/a)^p)/((1+e*x/d)^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.85

$$\int (gx)^n(d+ex)^2 \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \frac{x(gx)^n\left(1+\frac{bx}{a}\right)^{-p}((a+bx)(d+ex))^p\left(1+\frac{ex}{d}\right)^{-p}\left(d^2(6+5n+n^2)\text{AppellF1}\left(1+n,-p,-p,2+n,-\frac{bx}{a}\right.\right.}{}$$

input
```
Integrate[(g*x)^n*(d + e*x)^2*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output
```
(x*(g*x)^n*((a + b*x)*(d + e*x))^p*(d^2*(6 + 5*n + n^2)*AppellF1[1 + n, -p
, -p, 2 + n, -((b*x)/a), -((e*x)/d)] + e*(1 + n)*x*(2*d*(3 + n)*AppellF1[2
 + n, -p, -p, 3 + n, -((b*x)/a), -((e*x)/d)] + e*(2 + n)*x*AppellF1[3 + n,
 -p, -p, 4 + n, -((b*x)/a), -((e*x)/d)])))/((1 + n)*(2 + n)*(3 + n)*(1 + (
b*x)/a)^p*(1 + (e*x)/d)^p)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1268, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 (gx)^n \left(x(ae+bd)+ad+bex^2\right)^p dx$$

$$\Big\downarrow \text{1268}$$

$$(a+bx)^{-p}(d+ex)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p\int (gx)^n(a+bx)^p(d+ex)^{p+2}dx$$

$$\Big\downarrow \text{152}$$

$$\left(\frac{bx}{a}+1\right)^{-p}(d+ex)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p\int (gx)^n\left(\frac{bx}{a}+1\right)^p(d+ex)^{p+2}dx$$

$$\Big\downarrow \text{152}$$

$$d^2\left(\frac{bx}{a}+1\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p\int (gx)^n\left(\frac{bx}{a}+1\right)^p\left(\frac{ex}{d}+1\right)^{p+2}dx$$

$$\Big\downarrow \text{150}$$

$$\frac{d^2(gx)^{n+1}\left(\frac{bx}{a}+1\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \text{AppellF1}\left(n+1,-p,-p-2,n+2,-\frac{bx}{a},-\frac{ex}{d}\right)}{g(n+1)}$$

input
```
Int[(g*x)^n*(d + e*x)^2*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output
```
(d^2*(g*x)^(1 + n)*(a*d + (b*d + a*e)*x + b*e*x^2)^p*AppellF1[1 + n, -p, -
2 - p, 2 + n, -((b*x)/a), -((e*x)/d)])/(g*(1 + n)*(1 + (b*x)/a)^p*(1 + (e*
x)/d)^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int (gx)^n (ex+d)^2 \left(ad+(ae+bd)x+be\,x^2\right)^p dx$$

input
```
int((g*x)^n*(e*x+d)^2*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output
```
int((g*x)^n*(e*x+d)^2*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

Fricas [F]

$$\int (gx)^n(d+ex)^2\left(ad+(bd+ae)x+bex^2\right)^p\,dx$$
$$=\int (ex+d)^2\left(bex^2+ad+(bd+ae)x\right)^p(gx)^n\,dx$$

input
```
integrate((g*x)^n*(e*x+d)^2*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="fric
as")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)*(b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x)
^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (gx)^n(d+ex)^2\left(ad+(bd+ae)x+bex^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((g*x)**n*(e*x+d)**2*(a*d+(a*e+b*d)*x+b*e*x**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (gx)^n(d+ex)^2\left(ad+(bd+ae)x+bex^2\right)^p\,dx$$
$$=\int (ex+d)^2\left(bex^2+ad+(bd+ae)x\right)^p(gx)^n\,dx$$

input
```
integrate((g*x)^n*(e*x+d)^2*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="maxi
ma")
```

output
```
integrate((e*x + d)^2*(b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x)^n, x)
```

Giac [F]

$$\int (gx)^n(d+ex)^2 \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \int (ex+d)^2 (bex^2+ad+(bd+ae)x)^p (gx)^n \, dx$$

input
```
integrate((g*x)^n*(e*x+d)^2*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="giac
")
```

output
```
integrate((e*x + d)^2*(b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^n(d+ex)^2 \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \int (g\,x)^n\,(d+e\,x)^2\,\left(b\,e\,x^2+(a\,e+b\,d)\,x+a\,d\right)^p \,dx$$

input
```
int((g*x)^n*(d + e*x)^2*(a*d + x*(a*e + b*d) + b*e*x^2)^p,x)
```

output
```
int((g*x)^n*(d + e*x)^2*(a*d + x*(a*e + b*d) + b*e*x^2)^p, x)
```

Reduce [F]

$$\int (gx)^n(d+ex)^2\left(ad+(bd+ae)x+bex^2\right)^p\,dx=\text{too large to display}$$

input
```
int((g*x)^n*(e*x+d)^2*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output
```
(g**n*(x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a**3*d*e**2*n**2*p + x**n*
(a*d + a*e*x + b*d*x + b*e*x**2)**p*a**3*d*e**2*n*p**2 + 3*x**n*(a*d + a*e
*x + b*d*x + b*e*x**2)**p*a**3*d*e**2*n*p + x**n*(a*d + a*e*x + b*d*x + b*
e*x**2)**p*a**3*d*e**2*p**2 + 2*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a
**3*d*e**2*p - x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a**3*e**3*n**2*p*x
 - 2*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a**3*e**3*n*p**2*x - 2*x**n*
(a*d + a*e*x + b*d*x + b*e*x**2)**p*a**3*e**3*n*p*x - x**n*(a*d + a*e*x +
b*d*x + b*e*x**2)**p*a**3*e**3*p**3*x - 2*x**n*(a*d + a*e*x + b*d*x + b*e*
x**2)**p*a**3*e**3*p**2*x - 2*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a**
2*b*d**2*e*n**2*p - 6*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a**2*b*d**2
*e*n*p**2 - 8*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a**2*b*d**2*e*n*p -
 6*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a**2*b*d**2*e*p**2 - 6*x**n*(a
*d + a*e*x + b*d*x + b*e*x**2)**p*a**2*b*d**2*e*p + x**n*(a*d + a*e*x + b*
d*x + b*e*x**2)**p*a**2*b*d*e**2*n**2*p*x + 6*x**n*(a*d + a*e*x + b*d*x +
b*e*x**2)**p*a**2*b*d*e**2*n*p**2*x + 4*x**n*(a*d + a*e*x + b*d*x + b*e*x*
*2)**p*a**2*b*d*e**2*n*p*x + 5*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a*
*2*b*d*e**2*p**3*x + 4*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a**2*b*d*e
**2*p**2*x + x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a**2*b*e**3*n**2*p*x
**2 + 3*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a**2*b*e**3*n*p**2*x**2 +
 x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a**2*b*e**3*n*p*x**2 + 2*x**n...
```

3.147 $\int (gx)^n(d+ex) \left(ad+(bd+ae)x+bex^2\right)^p \, dx$

Optimal result

Integrand size = 32, antiderivative size = 91

$$\int (gx)^n(d+ex) \left(ad+(bd+ae)x+bex^2\right)^p \, dx$$
$$= \frac{d(gx)^{1+n} \left(1+\frac{bx}{a}\right)^{-p} \left(1+\frac{ex}{d}\right)^{-p} \left(ad+(bd+ae)x+bex^2\right)^p \operatorname{AppellF1}\left(1+n,-p,-1-p,2+n,-\frac{bx}{a},-\frac{ex}{d}\right)}{g(1+n)}$$

output
```
d*(g*x)^(1+n)*(a*d+(a*e+b*d)*x+b*e*x^2)^p*AppellF1(1+n,-p,-1-p,2+n,-b*x/a,
-e*x/d)/g/(1+n)/((1+b*x/a)^p)/((1+e*x/d)^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.32

$$\int (gx)^n(d+ex) \left(ad+(bd+ae)x+bex^2\right)^p \, dx$$
$$= \frac{x(gx)^n \left(1+\frac{bx}{a}\right)^{-p} ((a+bx)(d+ex))^p \left(1+\frac{ex}{d}\right)^{-p} \left(d(2+n) \operatorname{AppellF1}\left(1+n,-p,-p,2+n,-\frac{bx}{a},-\frac{ex}{d}\right) \cdots\right.}{(1+n)(2+n)}$$

input
```
Integrate[(g*x)^n*(d + e*x)*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output
```
(x*(g*x)^n*((a + b*x)*(d + e*x))^p*(d*(2 + n)*AppellF1[1 + n, -p, -p, 2 +
n, -((b*x)/a), -((e*x)/d)] + e*(1 + n)*x*AppellF1[2 + n, -p, -p, 3 + n, -(
(b*x)/a), -((e*x)/d)]))/((1 + n)*(2 + n)*(1 + (b*x)/a)^p*(1 + (e*x)/d)^p)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1268, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)(gx)^n \left(x(ae+bd)+ad+bex^2\right)^p dx$$

$$\Big\downarrow \text{ 1268}$$

$$(a+bx)^{-p}(d+ex)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (gx)^n(a+bx)^p(d+ex)^{p+1}dx$$

$$\Big\downarrow \text{ 152}$$

$$\left(\frac{bx}{a}+1\right)^{-p}(d+ex)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (gx)^n\left(\frac{bx}{a}+1\right)^p(d+ex)^{p+1}dx$$

$$\Big\downarrow \text{ 152}$$

$$d\left(\frac{bx}{a}+1\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (gx)^n\left(\frac{bx}{a}+1\right)^p\left(\frac{ex}{d}+1\right)^{p+1}dx$$

$$\Big\downarrow \text{ 150}$$

$$\frac{d(gx)^{n+1}\left(\frac{bx}{a}+1\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \text{AppellF1}\left(n+1,-p,-p-1,n+2,-\frac{bx}{a},-\frac{ex}{d}\right)}{g(n+1)}$$

input
```
Int[(g*x)^n*(d + e*x)*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output
```
(d*(g*x)^(1 + n)*(a*d + (b*d + a*e)*x + b*e*x^2)^p*AppellF1[1 + n, -p, -1
- p, 2 + n, -((b*x)/a), -((e*x)/d)])/(g*(1 + n)*(1 + (b*x)/a)^p*(1 + (e*x)
/d)^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int (gx)^n (ex+d) \left(ad+(ae+bd)x+be\,x^2\right)^p dx$$

input
```
int((g*x)^n*(e*x+d)*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output
```
int((g*x)^n*(e*x+d)*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

Fricas [F]

$$\int (gx)^n(d+ex)\left(ad+(bd+ae)x+bex^2\right)^p\,dx$$
$$=\int (ex+d)\left(bex^2+ad+(bd+ae)x\right)^p(gx)^n\,dx$$

input
```
integrate((g*x)^n*(e*x+d)*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="fricas
")
```

output
```
integral((e*x + d)*(b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (gx)^n(d+ex)\left(ad+(bd+ae)x+bex^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate((g*x)**n*(e*x+d)*(a*d+(a*e+b*d)*x+b*e*x**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (gx)^n(d+ex)\left(ad+(bd+ae)x+bex^2\right)^p\,dx$$
$$=\int (ex+d)\left(bex^2+ad+(bd+ae)x\right)^p(gx)^n\,dx$$

input
```
integrate((g*x)^n*(e*x+d)*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="maxima
")
```

output
```
integrate((e*x + d)*(b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x)^n, x)
```

Giac [F]

$$\int (gx)^n(d+ex)\left(ad+(bd+ae)x+bex^2\right)^p\,dx = \int (ex+d)\left(bex^2+ad+(bd+ae)x\right)^p(gx)^n\,dx$$

input
```
integrate((g*x)^n*(e*x+d)*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^n(d+ex)\left(ad+(bd+ae)x+bex^2\right)^p\,dx = \int (g\,x)^n\,(d+e\,x)\,\left(b\,e\,x^2+(a\,e+b\,d)\,x+a\,d\right)^p\,dx$$

input
```
int((g*x)^n*(d + e*x)*(a*d + x*(a*e + b*d) + b*e*x^2)^p,x)
```

output
```
int((g*x)^n*(d + e*x)*(a*d + x*(a*e + b*d) + b*e*x^2)^p, x)
```

Reduce [F]

$$\int (gx)^n(d+ex)\left(ad+(bd+ae)x+bex^2\right)^p\,dx = \text{too large to display}$$

input
```
int((g*x)^n*(e*x+d)*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output

```
(g**n*( - x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a**2*d*e*n*p - x**n*(a*
d + a*e*x + b*d*x + b*e*x**2)**p*a**2*d*e*p + x**n*(a*d + a*e*x + b*d*x +
b*e*x**2)**p*a**2*e**2*n*p*x + x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a*
*2*e**2*p**2*x + x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a*b*d**2*n*p + 4
*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a*b*d**2*p**2 + 3*x**n*(a*d + a*
e*x + b*d*x + b*e*x**2)**p*a*b*d**2*p + x**n*(a*d + a*e*x + b*d*x + b*e*x*
*2)**p*a*b*d*e*n**2*x + 5*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a*b*d*e
*n*p*x + 2*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a*b*d*e*n*x + 4*x**n*(
a*d + a*e*x + b*d*x + b*e*x**2)**p*a*b*d*e*p**2*x + 2*x**n*(a*d + a*e*x +
b*d*x + b*e*x**2)**p*a*b*d*e*p*x + x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**
p*a*b*e**2*n**2*x**2 + 3*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a*b*e**2
*n*p*x**2 + x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a*b*e**2*n*x**2 + 2*x
**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a*b*e**2*p**2*x**2 + x**n*(a*d + a
*e*x + b*d*x + b*e*x**2)**p*a*b*e**2*p*x**2 + x**n*(a*d + a*e*x + b*d*x +
b*e*x**2)**p*b**2*d**2*n**2*x + 4*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p
*b**2*d**2*n*p*x + 2*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*b**2*d**2*n*
x + 3*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*b**2*d**2*p**2*x + 2*x**n*(
a*d + a*e*x + b*d*x + b*e*x**2)**p*b**2*d**2*p*x + x**n*(a*d + a*e*x + b*d
*x + b*e*x**2)**p*b**2*d*e*n**2*x**2 + 3*x**n*(a*d + a*e*x + b*d*x + b*e*x
**2)**p*b**2*d*e*n*p*x**2 + x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*b*...
```

3.148 $\int (gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 88

$$\int (gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \frac{(gx)^{1+n} \left(1+\frac{bx}{a}\right)^{-p} \left(1+\frac{ex}{d}\right)^{-p} \left(ad+(bd+ae)x+bex^2\right)^p \operatorname{AppellF1}\left(1+n,-p,-p,2+n,-\frac{bx}{a},-\frac{ex}{d}\right)}{g(1+n)}$$

output
```
(g*x)^(1+n)*(a*d+(a*e+b*d)*x+b*e*x^2)^p*AppellF1(1+n,-p,-p,2+n,-b*x/a,-e*x
/d)/g/(1+n)/((1+b*x/a)^p)/((1+e*x/d)^p)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int (gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \frac{x(gx)^n \left(\frac{a+bx}{a}\right)^{-p} \left(\frac{d+ex}{d}\right)^{-p} ((a+bx)(d+ex))^p \operatorname{AppellF1}\left(1+n,-p,-p,2+n,-\frac{bx}{a},-\frac{ex}{d}\right)}{1+n}$$

input
```
Integrate[(g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output
```
(x*(g*x)^n*((a + b*x)*(d + e*x))^p*AppellF1[1 + n, -p, -p, 2 + n, -((b*x)/
a), -((e*x)/d)])/((1 + n)*((a + b*x)/a)^p*((d + e*x)/d)^p)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^n \left(x(ae+bd)+ad+bex^2\right)^p \, dx$$

$$\Big\downarrow \text{1179}$$

$$\frac{\left(\frac{bx}{a}+1\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (gx)^n \left(\frac{bx}{a}+1\right)^p \left(\frac{ex}{d}+1\right)^p d(gx)}{g}$$

$$\Big\downarrow \text{150}$$

$$\frac{(gx)^{n+1}\left(\frac{bx}{a}+1\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \text{AppellF1}\left(n+1,-p,-p,n+2,-\frac{bx}{a},-\frac{ex}{d}\right)}{g(n+1)}$$

input
```
Int[(g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output
```
((g*x)^(1 + n)*(a*d + (b*d + a*e)*x + b*e*x^2)^p*AppellF1[1 + n, -p, -p, 2
 + n, -((b*x)/a), -((e*x)/d)])/(g*(1 + n)*(1 + (b*x)/a)^p*(1 + (e*x)/d)^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int (gx)^n \left(ad+(ae+bd)\,x+be\,x^2\right)^p dx$$

input
```
int((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output
```
int((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

Fricas [F]

$$\int (gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \,dx = \int \left(bex^2+ad+(bd+ae)x\right)^p (gx)^n \,dx$$

input
```
integrate((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="fricas")
```

output
```
integral((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (gx)^n \left(ad + (bd + ae)x + bex^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((g*x)**n*(a*d+(a*e+b*d)*x+b*e*x**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (gx)^n \left(ad + (bd + ae)x + bex^2\right)^p \, dx = \int \left(bex^2 + ad + (bd + ae)x\right)^p (gx)^n \, dx$$

input
```
integrate((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x)^n, x)
```

Giac [F]

$$\int (gx)^n \left(ad + (bd + ae)x + bex^2\right)^p \, dx = \int \left(bex^2 + ad + (bd + ae)x\right)^p (gx)^n \, dx$$

input
```
integrate((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="giac")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \int (g\,x)^n \left(b\,e\,x^2+(a\,e+b\,d)\,x+a\,d\right)^p dx$$

input
```
int((g*x)^n*(a*d + x*(a*e + b*d) + b*e*x^2)^p,x)
```

output
```
int((g*x)^n*(a*d + x*(a*e + b*d) + b*e*x^2)^p, x)
```

Reduce [F]

$$\int (gx)^n \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \text{too large to display}$$

input
```
int((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output

```
(g**n*(2*x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a*d*p + x**n*(a*d + a*e*
x + b*d*x + b*e*x**2)**p*a*e*n*x + x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**
p*a*e*p*x + x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*b*d*n*x + x**n*(a*d +
 a*e*x + b*d*x + b*e*x**2)**p*b*d*p*x + int((x**n*(a*d + a*e*x + b*d*x + b
*e*x**2)**p*x)/(a**2*d*e*n**2 + 3*a**2*d*e*n*p + a**2*d*e*n + 2*a**2*d*e*p
**2 + a**2*d*e*p + a**2*e**2*n**2*x + 3*a**2*e**2*n*p*x + a**2*e**2*n*x +
2*a**2*e**2*p**2*x + a**2*e**2*p*x + a*b*d**2*n**2 + 3*a*b*d**2*n*p + a*b*
d**2*n + 2*a*b*d**2*p**2 + a*b*d**2*p + 2*a*b*d*e*n**2*x + 6*a*b*d*e*n*p*x
 + 2*a*b*d*e*n*x + 4*a*b*d*e*p**2*x + 2*a*b*d*e*p*x + a*b*e**2*n**2*x**2 +
 3*a*b*e**2*n*p*x**2 + a*b*e**2*n*x**2 + 2*a*b*e**2*p**2*x**2 + a*b*e**2*p
*x**2 + b**2*d**2*n**2*x + 3*b**2*d**2*n*p*x + b**2*d**2*n*x + 2*b**2*d**2
*p**2*x + b**2*d**2*p*x + b**2*d*e*n**2*x**2 + 3*b**2*d*e*n*p*x**2 + b**2*
d*e*n*x**2 + 2*b**2*d*e*p**2*x**2 + b**2*d*e*p*x**2),x)*a**3*e**3*n**3*p +
 4*int((x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*x)/(a**2*d*e*n**2 + 3*a**
2*d*e*n*p + a**2*d*e*n + 2*a**2*d*e*p**2 + a**2*d*e*p + a**2*e**2*n**2*x +
 3*a**2*e**2*n*p*x + a**2*e**2*n*x + 2*a**2*e**2*p**2*x + a**2*e**2*p*x +
a*b*d**2*n**2 + 3*a*b*d**2*n*p + a*b*d**2*n + 2*a*b*d**2*p**2 + a*b*d**2*p
 + 2*a*b*d*e*n**2*x + 6*a*b*d*e*n*p*x + 2*a*b*d*e*n*x + 4*a*b*d*e*p**2*x +
 2*a*b*d*e*p*x + a*b*e**2*n**2*x**2 + 3*a*b*e**2*n*p*x**2 + a*b*e**2*n*x**
2 + 2*a*b*e**2*p**2*x**2 + a*b*e**2*p*x**2 + b**2*d**2*n**2*x + 3*b**2*...
```

3.149 $\int \frac{(gx)^n(ad+(bd+ae)x+bex^2)^p}{d+ex}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 93

$$\int \frac{(gx)^n\left(ad+(bd+ae)x+bex^2\right)^p}{d+ex}\,dx = \frac{(gx)^{1+n}\left(1+\frac{bx}{a}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}\left(ad+(bd+ae)x+bex^2\right)^p\operatorname{AppellF1}\left(1+n,-p,1-p,2+n,-\frac{bx}{a},-\frac{ex}{d}\right)}{dg(1+n)}$$

output
```
(g*x)^(1+n)*(a*d+(a*e+b*d)*x+b*e*x^2)^p*AppellF1(1+n,-p,1-p,2+n,-b*x/a,-e*
x/d)/d/g/(1+n)/((1+b*x/a)^p)/((1+e*x/d)^p)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{(gx)^n\left(ad+(bd+ae)x+bex^2\right)^p}{d+ex}\,dx = \frac{x(gx)^n\left(1+\frac{bx}{a}\right)^{-p}((a+bx)(d+ex))^p\left(1+\frac{ex}{d}\right)^{-p}\operatorname{AppellF1}\left(1+n,-p,1-p,2+n,-\frac{bx}{a},-\frac{ex}{d}\right)}{d+dn}$$

input
```
Integrate[((g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p)/(d + e*x),x]
```

output
```
(x*(g*x)^n*((a + b*x)*(d + e*x))^p*AppellF1[1 + n, -p, 1 - p, 2 + n, -((b*
x)/a), -((e*x)/d)])/((d + d*n)*(1 + (b*x)/a)^p*(1 + (e*x)/d)^p)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1268, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^n \left(x(ae+bd)+ad+bex^2\right)^p}{d+ex} \, dx$$

$$\Big\downarrow \text{1268}$$

$$(a+bx)^{-p}(d+ex)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (gx)^n(a+bx)^p(d+ex)^{p-1}dx$$

$$\Big\downarrow \text{152}$$

$$\left(\frac{bx}{a}+1\right)^{-p}(d+ex)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (gx)^n\left(\frac{bx}{a}+1\right)^p(d+ex)^{p-1}dx$$

$$\Big\downarrow \text{152}$$

$$\frac{\left(\frac{bx}{a}+1\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (gx)^n\left(\frac{bx}{a}+1\right)^p\left(\frac{ex}{d}+1\right)^{p-1}dx}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{(gx)^{n+1}\left(\frac{bx}{a}+1\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \text{AppellF1}\left(n+1,-p,1-p,n+2,-\frac{bx}{a},-\frac{ex}{d}\right)}{dg(n+1)}$$

input
```
Int[((g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p)/(d + e*x),x]
```

output
```
((g*x)^(1 + n)*(a*d + (b*d + a*e)*x + b*e*x^2)^p*AppellF1[1 + n, -p, 1 - p
, 2 + n, -((b*x)/a), -((e*x)/d)])/(d*g*(1 + n)*(1 + (b*x)/a)^p*(1 + (e*x)/
d)^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(gx)^n (ad + (ae + bd) x + be x^2)^p}{ex + d} dx$$

input
```
int((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d),x)
```

output
```
int((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(gx)^n (ad + (bd + ae)x + bex^2)^p}{d + ex} dx = \int \frac{(bex^2 + ad + (bd + ae)x)^p (gx)^n}{ex + d} dx$$

input
```
integrate((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d),x, algorithm="fricas
")
```

output
```
integral((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x)^n/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(gx)^n\,(ad+(bd+ae)x+bex^2)^p}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate((g*x)**n*(a*d+(a*e+b*d)*x+b*e*x**2)**p/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(gx)^n\,(ad+(bd+ae)x+bex^2)^p}{d+ex}\,dx = \int \frac{(bex^2+ad+(bd+ae)x)^p(gx)^n}{ex+d}\,dx$$

input
```
integrate((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d),x, algorithm="maxima
")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x)^n/(e*x + d), x)
```

Giac [F]

$$\int \frac{(gx)^n\,(ad+(bd+ae)x+bex^2)^p}{d+ex}\,dx = \int \frac{(bex^2+ad+(bd+ae)x)^p(gx)^n}{ex+d}\,dx$$

input
```
integrate((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x)^n/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^n (ad+(bd+ae)x+bex^2)^p}{d+ex}\,dx=\int \frac{(g\,x)^n\,\left(b\,e\,x^2+\left(a\,e+b\,d\right)\,x+a\,d\right)^p}{d+e\,x}\,dx$$

input
```
int(((g*x)^n*(a*d + x*(a*e + b*d) + b*e*x^2)^p)/(d + e*x),x)
```

output
```
int(((g*x)^n*(a*d + x*(a*e + b*d) + b*e*x^2)^p)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{(gx)^n (ad+(bd+ae)x+bex^2)^p}{d+ex}\,dx=\text{too large to display}$$

input
```
int((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d),x)
```

output

```
(g**n*(x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a - int((x**n*(a*d + a*e*x
 + b*d*x + b*e*x**2)**p*x)/(a**2*d*e*n + a**2*d*e*p + a**2*e**2*n*x + a**2
*e**2*p*x + a*b*d**2*n + a*b*d**2*p + 2*a*b*d*e*n*x + 2*a*b*d*e*p*x + a*b*
e**2*n*x**2 + a*b*e**2*p*x**2 + b**2*d**2*n*x + b**2*d**2*p*x + b**2*d*e*n
*x**2 + b**2*d*e*p*x**2),x)*a**2*b*e**2*n*p - int((x**n*(a*d + a*e*x + b*d
*x + b*e*x**2)**p*x)/(a**2*d*e*n + a**2*d*e*p + a**2*e**2*n*x + a**2*e**2*
p*x + a*b*d**2*n + a*b*d**2*p + 2*a*b*d*e*n*x + 2*a*b*d*e*p*x + a*b*e**2*n
*x**2 + a*b*e**2*p*x**2 + b**2*d**2*n*x + b**2*d**2*p*x + b**2*d*e*n*x**2
+ b**2*d*e*p*x**2),x)*a**2*b*e**2*p**2 + int((x**n*(a*d + a*e*x + b*d*x +
b*e*x**2)**p*x)/(a**2*d*e*n + a**2*d*e*p + a**2*e**2*n*x + a**2*e**2*p*x +
 a*b*d**2*n + a*b*d**2*p + 2*a*b*d*e*n*x + 2*a*b*d*e*p*x + a*b*e**2*n*x**2
 + a*b*e**2*p*x**2 + b**2*d**2*n*x + b**2*d**2*p*x + b**2*d*e*n*x**2 + b**
2*d*e*p*x**2),x)*a*b**2*d*e*n**2 + int((x**n*(a*d + a*e*x + b*d*x + b*e*x*
*2)**p*x)/(a**2*d*e*n + a**2*d*e*p + a**2*e**2*n*x + a**2*e**2*p*x + a*b*d
**2*n + a*b*d**2*p + 2*a*b*d*e*n*x + 2*a*b*d*e*p*x + a*b*e**2*n*x**2 + a*b
*e**2*p*x**2 + b**2*d**2*n*x + b**2*d**2*p*x + b**2*d*e*n*x**2 + b**2*d*e*
p*x**2),x)*a*b**2*d*e*n*p + int((x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*
x)/(a**2*d*e*n + a**2*d*e*p + a**2*e**2*n*x + a**2*e**2*p*x + a*b*d**2*n +
 a*b*d**2*p + 2*a*b*d*e*n*x + 2*a*b*d*e*p*x + a*b*e**2*n*x**2 + a*b*e**2*p
*x**2 + b**2*d**2*n*x + b**2*d**2*p*x + b**2*d*e*n*x**2 + b**2*d*e*p*x*...
```

3.150 $\int \frac{(gx)^n(ad+(bd+ae)x+bex^2)^p}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 93

$$\int \frac{(gx)^n\left(ad+(bd+ae)x+bex^2\right)^p}{(d+ex)^2}\,dx = \frac{(gx)^{1+n}\left(1+\frac{bx}{a}\right)^{-p}\left(1+\frac{ex}{d}\right)^{-p}\left(ad+(bd+ae)x+bex^2\right)^p \operatorname{AppellF1}\left(1+n,-p,2-p,2+n,-\frac{bx}{a},-\frac{ex}{d}\right)}{d^2g(1+n)}$$

output
```
(g*x)^(1+n)*(a*d+(a*e+b*d)*x+b*e*x^2)^p*AppellF1(1+n,-p,2-p,2+n,-b*x/a,-e*
x/d)/d^2/g/(1+n)/((1+b*x/a)^p)/((1+e*x/d)^p)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

$$\int \frac{(gx)^n\left(ad+(bd+ae)x+bex^2\right)^p}{(d+ex)^2}\,dx = \frac{x(gx)^n\left(1+\frac{bx}{a}\right)^{-p}((a+bx)(d+ex))^p\left(1+\frac{ex}{d}\right)^{-p}\operatorname{AppellF1}\left(1+n,-p,2-p,2+n,-\frac{bx}{a},-\frac{ex}{d}\right)}{d^2(1+n)}$$

input
```
Integrate[((g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p)/(d + e*x)^2,x]
```

output
```
(x*(g*x)^n*((a + b*x)*(d + e*x))^p*AppellF1[1 + n, -p, 2 - p, 2 + n, -((b*
x)/a), -((e*x)/d)])/(d^2*(1 + n)*(1 + (b*x)/a)^p*(1 + (e*x)/d)^p)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1268, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^n \left(x(ae+bd)+ad+bex^2\right)^p}{(d+ex)^2} \, dx$$

$$\Big\downarrow \text{1268}$$

$$(a+bx)^{-p}(d+ex)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (gx)^n(a+bx)^p(d+ex)^{p-2}dx$$

$$\Big\downarrow \text{152}$$

$$\left(\frac{bx}{a}+1\right)^{-p}(d+ex)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (gx)^n\left(\frac{bx}{a}+1\right)^p(d+ex)^{p-2}dx$$

$$\Big\downarrow \text{152}$$

$$\frac{\left(\frac{bx}{a}+1\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (gx)^n\left(\frac{bx}{a}+1\right)^p\left(\frac{ex}{d}+1\right)^{p-2}dx}{d^2}$$

$$\Big\downarrow \text{150}$$

$$\frac{(gx)^{n+1}\left(\frac{bx}{a}+1\right)^{-p}\left(\frac{ex}{d}+1\right)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \text{AppellF1}\left(n+1,-p,2-p,n+2,-\frac{bx}{a},-\frac{ex}{d}\right)}{d^2g(n+1)}$$

input
```
Int[((g*x)^n*(a*d + (b*d + a*e)*x + b*e*x^2)^p)/(d + e*x)^2,x]
```

output
```
((g*x)^(1 + n)*(a*d + (b*d + a*e)*x + b*e*x^2)^p*AppellF1[1 + n, -p, 2 - p
, 2 + n, -((b*x)/a), -((e*x)/d)])/(d^2*g*(1 + n)*(1 + (b*x)/a)^p*(1 + (e*x
)/d)^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int \frac{(gx)^n (ad + (ae + bd) x + be\, x^2)^p}{(ex + d)^2} dx$$

input
```
int((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d)^2,x)
```

output
```
int((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{(gx)^n (ad + (bd + ae)x + bex^2)^p}{(d + ex)^2} dx = \int \frac{(bex^2 + ad + (bd + ae)x)^p (gx)^n}{(ex + d)^2} dx$$

input
```
integrate((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d)^2,x, algorithm="fric
as")
```

output
```
integral((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x)^n/(e^2*x^2 + 2*d*e*x + d^
2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(gx)^n \left(ad+(bd+ae)x+bex^2\right)^p}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x)**n*(a*d+(a*e+b*d)*x+b*e*x**2)**p/(e*x+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(gx)^n \left(ad+(bd+ae)x+bex^2\right)^p}{(d+ex)^2}\,dx = \int \frac{\left(bex^2+ad+(bd+ae)x\right)^p (gx)^n}{(ex+d)^2}\,dx$$

input
```
integrate((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d)^2,x, algorithm="maxi
ma")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x)^n/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{(gx)^n \left(ad+(bd+ae)x+bex^2\right)^p}{(d+ex)^2}\,dx = \int \frac{\left(bex^2+ad+(bd+ae)x\right)^p (gx)^n}{(ex+d)^2}\,dx$$

input
```
integrate((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d)^2,x, algorithm="giac
")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(g*x)^n/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^n \left(ad+(bd+ae)x+bex^2\right)^p}{(d+ex)^2}\,dx=\int \frac{(g\,x)^n\,\left(b\,e\,x^2+(a\,e+b\,d)\,x+a\,d\right)^p}{(d+e\,x)^2}\,dx$$

input
```
int(((g*x)^n*(a*d + x*(a*e + b*d) + b*e*x^2)^p)/(d + e*x)^2,x)
```

output
```
int(((g*x)^n*(a*d + x*(a*e + b*d) + b*e*x^2)^p)/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{(gx)^n \left(ad+(bd+ae)x+bex^2\right)^p}{(d+ex)^2}\,dx=\text{too large to display}$$

input
```
int((g*x)^n*(a*d+(a*e+b*d)*x+b*e*x^2)^p/(e*x+d)^2,x)
```

output

```
(g**n*(x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a - int((x**n*(a*d + a*e*x
 + b*d*x + b*e*x**2)**p*x)/(a**2*d**2*e*n + a**2*d**2*e*p - a**2*d**2*e +
2*a**2*d*e**2*n*x + 2*a**2*d*e**2*p*x - 2*a**2*d*e**2*x + a**2*e**3*n*x**2
 + a**2*e**3*p*x**2 - a**2*e**3*x**2 + a*b*d**3*n + a*b*d**3*p + 3*a*b*d**
2*e*n*x + 3*a*b*d**2*e*p*x - a*b*d**2*e*x + 3*a*b*d*e**2*n*x**2 + 3*a*b*d*
e**2*p*x**2 - 2*a*b*d*e**2*x**2 + a*b*e**3*n*x**3 + a*b*e**3*p*x**3 - a*b*
e**3*x**3 + b**2*d**3*n*x + b**2*d**3*p*x + 2*b**2*d**2*e*n*x**2 + 2*b**2*
d**2*e*p*x**2 + b**2*d*e**2*n*x**3 + b**2*d*e**2*p*x**3),x)*a**2*b*d*e**2*
n*p - int((x**n*(a*d + a*e*x + b*d*x + b*e*x**2)**p*x)/(a**2*d**2*e*n + a*
*2*d**2*e*p - a**2*d**2*e + 2*a**2*d*e**2*n*x + 2*a**2*d*e**2*p*x - 2*a**2
*d*e**2*x + a**2*e**3*n*x**2 + a**2*e**3*p*x**2 - a**2*e**3*x**2 + a*b*d**
3*n + a*b*d**3*p + 3*a*b*d**2*e*n*x + 3*a*b*d**2*e*p*x - a*b*d**2*e*x + 3*
a*b*d*e**2*n*x**2 + 3*a*b*d*e**2*p*x**2 - 2*a*b*d*e**2*x**2 + a*b*e**3*n*x
**3 + a*b*e**3*p*x**3 - a*b*e**3*x**3 + b**2*d**3*n*x + b**2*d**3*p*x + 2*
b**2*d**2*e*n*x**2 + 2*b**2*d**2*e*p*x**2 + b**2*d*e**2*n*x**3 + b**2*d*e*
*2*p*x**3),x)*a**2*b*d*e**2*p**2 + int((x**n*(a*d + a*e*x + b*d*x + b*e*x*
*2)**p*x)/(a**2*d**2*e*n + a**2*d**2*e*p - a**2*d**2*e + 2*a**2*d*e**2*n*x
 + 2*a**2*d*e**2*p*x - 2*a**2*d*e**2*x + a**2*e**3*n*x**2 + a**2*e**3*p*x*
*2 - a**2*e**3*x**2 + a*b*d**3*n + a*b*d**3*p + 3*a*b*d**2*e*n*x + 3*a*b*d
**2*e*p*x - a*b*d**2*e*x + 3*a*b*d*e**2*n*x**2 + 3*a*b*d*e**2*p*x**2 - ...
```

3.151 $\int (gx)^n(d+ex)^m \left(ad+(bd+ae)x+bex^2\right)^p \, dx$

Optimal result

Integrand size = 34, antiderivative size = 103

$$\int (gx)^n(d+ex)^m \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \frac{(gx)^{1+n} \left(1+\frac{bx}{a}\right)^{-p} (d+ex)^m \left(1+\frac{ex}{d}\right)^{-m-p} \left(ad+(bd+ae)x+bex^2\right)^p \text{AppellF1}\left(1+n,-p,-m-p,2+\right.}{g(1+n)}$$

output
```
(g*x)^(1+n)*(e*x+d)^m*(1+e*x/d)^(-m-p)*(a*d+(a*e+b*d)*x+b*e*x^2)^p*AppellF
1(1+n,-p,-m-p,2+n,-b*x/a,-e*x/d)/g/(1+n)/((1+b*x/a)^p)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int (gx)^n(d+ex)^m \left(ad+(bd+ae)x+bex^2\right)^p \, dx = \frac{x(gx)^n \left(\frac{a+bx}{a}\right)^{-p} (d+ex)^m \left(\frac{d+ex}{d}\right)^{-m-p} ((a+bx)(d+ex))^p \text{AppellF1}\left(1+n,-p,-m-p,2+n,-\frac{bx}{a},-\right.}{1+n}$$

input
```
Integrate[(g*x)^n*(d + e*x)^m*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output
```
(x*(g*x)^n*(d + e*x)^m*((d + e*x)/d)^(-m - p)*((a + b*x)*(d + e*x))^p*Appe
llF1[1 + n, -p, -m - p, 2 + n, -((b*x)/a), -((e*x)/d)])/((1 + n)*((a + b*x
)/a)^p)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1268, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^n (d+ex)^m \left(x(ae+bd)+ad+bex^2\right)^p \, dx$$

$$\Big\downarrow \text{1268}$$

$$(a+bx)^{-p}(d+ex)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (gx)^n (a+bx)^p (d+ex)^{m+p} dx$$

$$\Big\downarrow \text{152}$$

$$\left(\frac{bx}{a}+1\right)^{-p}(d+ex)^{-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (gx)^n \left(\frac{bx}{a}+1\right)^p (d+ex)^{m+p} dx$$

$$\Big\downarrow \text{152}$$

$$\left(\frac{bx}{a}+1\right)^{-p}(d+ex)^m \left(\frac{ex}{d}+1\right)^{-m-p}\left(x(ae+bd)+ad+bex^2\right)^p \int (gx)^n \left(\frac{bx}{a}+1\right)^p \left(\frac{ex}{d}+1\right)^{m+p} dx$$

$$\Big\downarrow \text{150}$$

$$\frac{(gx)^{n+1}\left(\frac{bx}{a}+1\right)^{-p}(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m-p}\left(x(ae+bd)+ad+bex^2\right)^p \text{AppellF1}\left(n+1,-p,-m-p,n+2,-\frac{bx}{a}\right.}{g(n+1)}$$

input
```
Int[(g*x)^n*(d + e*x)^m*(a*d + (b*d + a*e)*x + b*e*x^2)^p,x]
```

output
```
((g*x)^(1 + n)*(d + e*x)^m*(1 + (e*x)/d)^(-m - p)*(a*d + (b*d + a*e)*x + b
*e*x^2)^p*AppellF1[1 + n, -p, -m - p, 2 + n, -((b*x)/a), -((e*x)/d)])/(g*(
1 + n)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1268
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((d
 + e*x)^FracPart[p]*(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f
 + g*x)^n*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

Maple [F]

$$\int (gx)^n (ex+d)^m \left(ad+(ae+bd)x+be\,x^2\right)^p dx$$

input
```
int((g*x)^n*(e*x+d)^m*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output
```
int((g*x)^n*(e*x+d)^m*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

Fricas [F]

$$\int (gx)^n(d+ex)^m\left(ad+(bd+ae)x+bex^2\right)^p\,dx = \int \left(bex^2+ad+(bd+ae)x\right)^p(ex+d)^m(gx)^n\,dx$$

input
```
integrate((g*x)^n*(e*x+d)^m*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="fric
as")
```

output
```
integral((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(e*x + d)^m*(g*x)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (gx)^n(d+ex)^m\left(ad+(bd+ae)x+bex^2\right)^p\,dx = \text{Timed out}$$

input
```
integrate((g*x)**n*(e*x+d)**m*(a*d+(a*e+b*d)*x+b*e*x**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (gx)^n(d+ex)^m\left(ad+(bd+ae)x+bex^2\right)^p\,dx = \int \left(bex^2+ad+(bd+ae)x\right)^p(ex+d)^m(gx)^n\,dx$$

input
```
integrate((g*x)^n*(e*x+d)^m*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="maxi
ma")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(e*x + d)^m*(g*x)^n, x)
```

Giac [F]

$$\int (gx)^n(d+ex)^m\left(ad+(bd+ae)x+bex^2\right)^p\,dx = \int (bex^2+ad+(bd+ae)x)^p(ex+d)^m(gx)^n\,dx$$

input
```
integrate((g*x)^n*(e*x+d)^m*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x, algorithm="giac
")
```

output
```
integrate((b*e*x^2 + a*d + (b*d + a*e)*x)^p*(e*x + d)^m*(g*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^n(d+ex)^m\left(ad+(bd+ae)x+bex^2\right)^p\,dx = \int (g\,x)^n\,(d+e\,x)^m\,\left(b\,e\,x^2+(a\,e+b\,d)\,x+a\,d\right)^p\,dx$$

input
```
int((g*x)^n*(d + e*x)^m*(a*d + x*(a*e + b*d) + b*e*x^2)^p,x)
```

output
```
int((g*x)^n*(d + e*x)^m*(a*d + x*(a*e + b*d) + b*e*x^2)^p, x)
```

Reduce [F]

$$\int (gx)^n(d+ex)^m\left(ad+(bd+ae)x+bex^2\right)^p\,dx = \text{too large to display}$$

input
```
int((g*x)^n*(e*x+d)^m*(a*d+(a*e+b*d)*x+b*e*x^2)^p,x)
```

output

```
(g**n*(x**n*(d + e*x)**m*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a*d*m + 2*x**
n*(d + e*x)**m*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a*d*p + x**n*(d + e*x)*
*m*(a*d + a*e*x + b*d*x + b*e*x**2)**p*a*e*m*x + x**n*(d + e*x)**m*(a*d +
a*e*x + b*d*x + b*e*x**2)**p*a*e*n*x + x**n*(d + e*x)**m*(a*d + a*e*x + b*
d*x + b*e*x**2)**p*a*e*p*x + x**n*(d + e*x)**m*(a*d + a*e*x + b*d*x + b*e*
x**2)**p*b*d*n*x + x**n*(d + e*x)**m*(a*d + a*e*x + b*d*x + b*e*x**2)**p*b
*d*p*x + int((x**n*(d + e*x)**m*(a*d + a*e*x + b*d*x + b*e*x**2)**p*x)/(a*
*2*d*e*m**2 + 2*a**2*d*e*m*n + 3*a**2*d*e*m*p + a**2*d*e*m + a**2*d*e*n**2
 + 3*a**2*d*e*n*p + a**2*d*e*n + 2*a**2*d*e*p**2 + a**2*d*e*p + a**2*e**2*
m**2*x + 2*a**2*e**2*m*n*x + 3*a**2*e**2*m*p*x + a**2*e**2*m*x + a**2*e**2
*n**2*x + 3*a**2*e**2*n*p*x + a**2*e**2*n*x + 2*a**2*e**2*p**2*x + a**2*e*
*2*p*x + a*b*d**2*m*n + a*b*d**2*m*p + a*b*d**2*n**2 + 3*a*b*d**2*n*p + a*
b*d**2*n + 2*a*b*d**2*p**2 + a*b*d**2*p + a*b*d*e*m**2*x + 3*a*b*d*e*m*n*x
 + 4*a*b*d*e*m*p*x + a*b*d*e*m*x + 2*a*b*d*e*n**2*x + 6*a*b*d*e*n*p*x + 2*
a*b*d*e*n*x + 4*a*b*d*e*p**2*x + 2*a*b*d*e*p*x + a*b*e**2*m**2*x**2 + 2*a*
b*e**2*m*n*x**2 + 3*a*b*e**2*m*p*x**2 + a*b*e**2*m*x**2 + a*b*e**2*n**2*x*
*2 + 3*a*b*e**2*n*p*x**2 + a*b*e**2*n*x**2 + 2*a*b*e**2*p**2*x**2 + a*b*e*
*2*p*x**2 + b**2*d**2*m*n*x + b**2*d**2*m*p*x + b**2*d**2*n**2*x + 3*b**2*
d**2*n*p*x + b**2*d**2*n*x + 2*b**2*d**2*p**2*x + b**2*d**2*p*x + b**2*d*e
*m*n*x**2 + b**2*d*e*m*p*x**2 + b**2*d*e*n**2*x**2 + 3*b**2*d*e*n*p*x**...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                  print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0])  #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:])  #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file